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10

NELSON WA MATHS

for the **Australian Curriculum**

Revised 1st Edition

Stephen Corcoran | Ross Brodie | Stephen Swift | Sue Garner

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Stephen Corcoran
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Ross Brodie has worked as a classroom teacher and Mathematics Head of Department at a number of regional and metropolitan secondary schools. He has taught Mathematics at Years 8 to 12. Ross brings a wide range of experience from education and other sectors to the writing of mathematics student books.

Stephen Swift started teaching Mathematics, Science and Computing in 1973 and has taught at all levels from Years 7 to12 in several states, in urban and country schools until retiring in 2010 from the role of Mathematics Head of Department at Wellington Point State High School in Brisbane.

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The **Digital Resources Team** includes Sue Garner, Alan Preston, Allason McNamara, Dawn Fernandez, Francesca Moloney, Frank Moya, Greg Neal, Ian Bull, Roger Walter and Tobias Cooper.

Contents

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About this book

At the beginning of each chapter

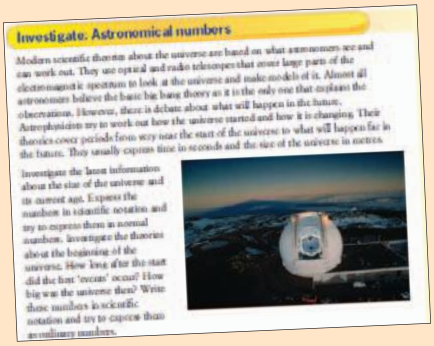
- Clear outline of chapter contents
- Strong links to the curriculum
- Prior learning provides a quick check of understanding for the individualisation of instructional sequence

Contents
1.1 Statistical measures
1.2 Organising data
1.3 Comparing samples and populations
Chapter summary
Chapter review

Prior learning Chapter 1
MAT08SPPL00001
Parent guide Chapter 1
MAT08SPPG00001
Curriculum guide Chapter 1
MAT08SPCU00001

Australian Curriculum statements

Data representation and interpretation
Explore the practicalities and implications of obtaining representative data using a variety of investigative processes.
Explore the variation of means and proportions in representative data.
Investigate the effect of individual data values, including outliers, on the mean and median.



Investigate: Astronomical numbers

Investigate the latest information about the size of the universe and its current age. Express the numbers in scientific notation and try to express them as normal numbers. Investigate the theories about the beginning of the universe. How long after the start did the first 'events' occur? How big was the universe then? Write these numbers in scientific notation and try to express them as ordinary numbers.

- Parent guide provides vital information for parents
- Curriculum guide provides a rationale in words that students will understand
- Clear sectioning that is consistent with the exercises
- Many sections begin with a concrete investigation, with teacher support materials provided
- Intriguing aids for students provided

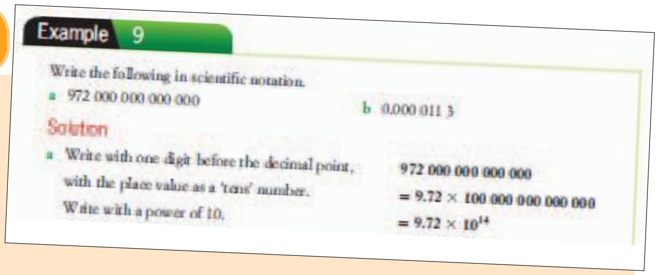
In each chapter

- Examples are clearly set out with explanations on the left and expected student answers in bold on the right
- Colour-coded examples make it easy for teachers to direct students

  Green – nearly all students will master

  Gold – most students will master

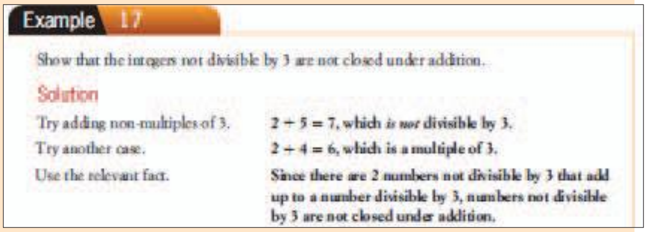
  Blue – some students should master
- Examples are sequenced in a natural learning progression that promotes chunking and effective use of working memory
- Clear examples enable parents to assist their student with confidence that they are 'doing it the way the teacher explained it'
- Important boxes highlight key terms and concepts



Example 9

Write the following in scientific notation.
a 972 000 000 000 000 b 0.000 011 3

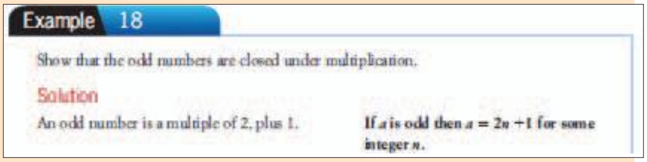
Solution
a Write with one digit before the decimal point, with the place value as a 'tens' number. Write with a power of 10.
972 000 000 000 000
= 9.72 × 100 000 000 000 000
= 9.72×10^{14}



Example 17

Show that the integers not divisible by 3 are not closed under addition.

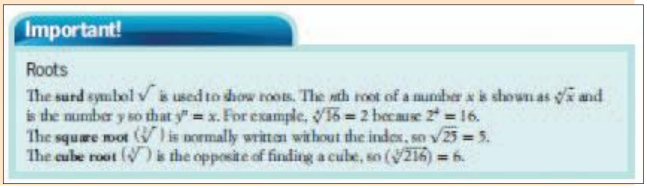
Solution
Try adding non-multiples of 3. **2 + 5 = 7, which is *not* divisible by 3.**
Try another case. **2 + 4 = 6, which is a multiple of 3.**
Use the relevant fact. **Since there are 2 numbers not divisible by 3 that add up to a number divisible by 3, numbers not divisible by 3 are not closed under addition.**



Example 18

Show that the odd numbers are closed under multiplication.

Solution
An odd number is a multiple of 2, plus 1. **If a is odd then $a = 2n + 1$ for some integer n.**



Important!

Roots
The **surd** symbol $\sqrt{\ }$ is used to show roots. The nth root of a number x is shown as $\sqrt[n]{x}$ and is the number y so that $y^n = x$. For example, $\sqrt[4]{16} = 2$ because $2^4 = 16$.
The **square root** ($\sqrt[2]{\ }$) is normally written without the index, so $\sqrt{25} = 5$.
The **cube root** ($\sqrt[3]{\ }$) is the opposite of finding a cube, so $(\sqrt[3]{216}) = 6$.

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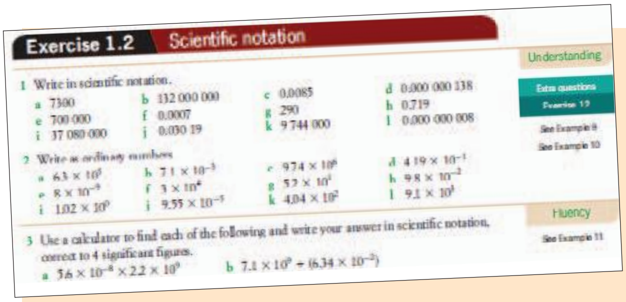
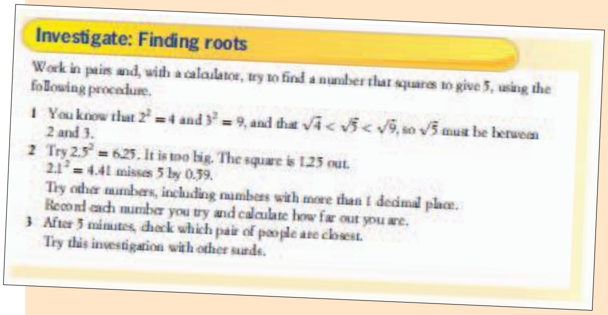


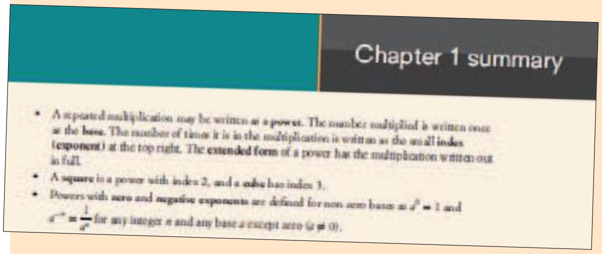
- Initial exercise questions focus on understanding or fluency and refer directly back to examples
- Questions are categorised according to the four proficiency strands – understanding, fluency, problem solving and reasoning
- Questions and parts are colour coded to be consistent with the examples

- Colour is used pedagogically to enhance learning



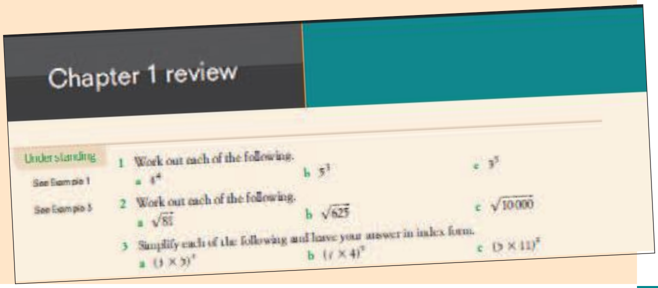
- There is an abundance of carefully designed investigations proven to aid deep understanding

At the end of each chapter



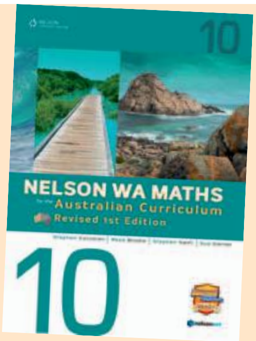
- A chapter summary that re-states all important concepts

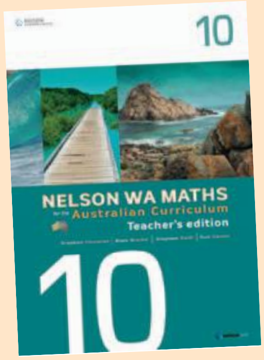
- A comprehensive review set out with example references and consistent with exercise layout
- Answers to all questions are given at the back of the book



Other books in the series

- There are five books in this Years 7 to 10 series, which has been written specifically for the Australian Mathematics Curriculum, including two books for Year 10 (see back cover).
- There is also a printed Teacher's Edition of each textbook with additional guidance and advice on implementing the Australian Mathematics Curriculum in the classroom.
- Accompanying each printed textbook is a digital textbook called the NelsonNetBook (see opposite page).





Icons in the textbook

- In the outside margins of your textbook you will find numerous icons for resources that can be accessed through the NelsonNet website or through the NelsonNetBook.

Alternative method Moving the decimal	Explains different ways of solving a maths problem	Puzzle sheet Find the number	Makes learning maths methods and skills fun
Animated example Stem-and-leaf plots	Shows how to tackle difficult problems	Quiz Expressions	Tests and marks your knowledge of a topic
CAS calculator exercise Quadratic equations	Develops your CAS calculator skills	Teacher notes Restaurant tables	Gives guidance for your teacher
Curriculum guide Chapter 6	Explains what you need to know and be able to do	Technology GeoGebra: Exterior angle of a triangle	Provides a spreadsheet or GeoGebra activity
Extra questions Exercise 6.1	Allows you to do additional practice and drill	TLF learning object Exploring order of operations (L6543)	Provides interactive ways of learning about maths*
Maths clip Statistics	Gives the 'big picture' view of your maths topic	Video tutorial Algebraic expressions	Fully explains maths methods and skills
Maths dictionary	Provides illustrated explanations of all terms	Weblink Algebra masterclass	Links to a website that makes maths relevant
Parent guide Chapter 6	Provides guidance for your parents to help you	Worked solutions Exercise 6.1	Gives solutions steps for selected problems
Prior learning Chapter 6	Finds out what you already know and can do	Worksheet Writing a rule	Supplies tasks to be done digitally or by using a pen

* Your teacher will explain how to access The Learning Federation learning objects.

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NelsonNet Website

- Go to www.nelsonnet.com.au to log in or find out more.
- The first screen you will see on the student website is called ‘Chapter resources’.
- Click on a chapter and a list of different resources types will appear.
- Click on a resource type and a list of specific resources will pop up.
- Clicking on one of those resources will open a pdf file, start a video, or some special maths software.
- Use the blue tabs on the left of the screen to access calculator resources and the maths dictionary.

NelsonNetBook

- This is a web-based ebook that you can customise to meet your own learning needs.
- The icons with the blue NelsonNet logo are ‘hotspots’. Click on an icon to open that resource.
- The tools on the vertical toolbar allow you to personalise pages in a variety of ways, including voice recordings, drawings and links to favourite websites. They also enable you to zoom in and out.
- The tools on the horizontal toolbar allow you to navigate around your ebook and change settings.
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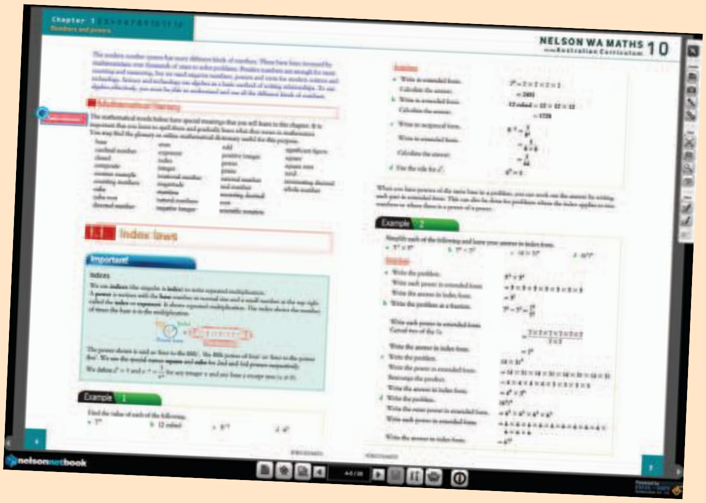

Please note that complimentary access to NelsonNet and the NelsonNetBook is only available to teachers who use the accompanying student textbook as a core educational resource in their classroom. Contact your sales representative for information about access codes and conditions.

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Text

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Metric system

Symbols

millimetre — mm
centimetre — cm
metre — m
kilometre — km
hectare — ha
millilitre — mL
litre — L
kilolitre — kL
megalitre — ML
milligram — mg
gram — g
kilogram — kg
tonne — t

Length

10 mm = 1 cm
100 cm = 1 m
1000 m = 1 km

Area

100 mm^2 = 1 cm^2
10 000 cm^2 = 1 m^2
10 000 m^2 = 1 ha
100 ha = 1 km^2

Capacity

1000 mL = 1 L
1000 L = 1 kL
1000 kL = 1 ML

Mass

1000 mg = 1 g
1000 g = 1 kg
1000 kg = 1 t

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Symbols and abbreviations

$\therefore$	therefore
$=$	is equal to
$\neq$	is not equal to
$\approx$, $\sim$, $\doteqdot$, $\simeq$	is approximately equal to
$>$	is greater than
$\not>$	is not greater than
$\geq$	is greater than or equal to
$<$	is less than
$\not<$	is not less than
$\leq$	is less than or equal to
$\propto$	is proportional to
$5.3\overline{7214}$ or $5.3\dot{7}21\dot{4}$	shows $5.3721472147214\ldots$
1 : 100	Scale of 1 to 100
BC	before Christ
BCE	before the common era
AD	Anno Domini
CE	common era
QED	*quod erat demonstrandum*
RTP	Required to prove
LHS	Left-hand side
RHS	Right-hand side
log	common logarithm
$\log_b x$	logarithm of x to the base b
$\sum$	the sum of
$\triangle ABC$	triangle ABC
$\angle ABC$	angle ABC
$\equiv$, $\cong$	is congruent to
$\vert\vert\vert$, $\sim$	is similar to
$A \leftrightarrow P$	A corresponds to P
	right angle
	supplementary angles
	angles at a point
	angles in a triangle
	angles in a quadrilateral
	angles in an isosceles triangle
	alternate angles
$\parallel$	is parallel to

$\perp$	is perpendicular to
%	percentage
$\frac{a}{b}$	fraction, division $a \div b$ or ratio $a : b$
$\pm$	plus or minus
$\sqrt{\ }$	square root
$\sqrt[3]{\ }$	cube root
$\sqrt[n]{\ }$	nth root
45°	45 degrees
$\bar{x}$	mean of sample, average
μ	mean of population
Q_1, $Q1$	first quartile
Q_3, $Q3$	third quartile
IQR	interquartile range
σ, σ_n	standard deviation of population
s, σ_{n-1}	standard deviation of sample
m	gradient
c	y-intercept
π	pi $\approx 3.141592653589793238\ldots$
$\{\}$, $\varnothing$	null set, empty set
$\subseteq$	subset of
$\subset$	proper subset of
$\in$	is a member of
$\notin$	is not a member of
$\cup$	union of sets
$\cap$	intersection of sets
$\Rightarrow$	it follows that
$\sin\theta$	sine ratio of angle θ
$\tan\theta$	tangent ratio of angle θ
$\cos\theta$	cosine ratio of angle θ
$P(A\vert B)$	conditional probability of A given B
	complementary angles
	vertically opposite angles
	exterior angle of a triangle
A	opposite angles in a parallelogram
	corresponding angles
	co-interior (allied) angles

Greek alphabet

A, α alpha
B, β beta
Γ, γ gamma
Δ, δ delta
E, ε epsilon
Z, ζ zeta
H, η eta
Θ, θ theta
I, ι iota
K, κ kappa
Λ, λ lambda
M, μ mu
N, ν nu
Ξ, ξ xi
O, o omicron
Π, π pi
P, ρ rho
Σ, σ sigma
T, τ tau
Y, υ upsilon
Φ, φ phi
X, χ chi
Ψ, ψ psi
Ω, ω omega

Number and algebra

1 Numbers and powers

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Curriculum guide
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Australian Curriculum statements

Real numbers
Define rational and irrational numbers and perform operations with surds and fractional indices. (10A) (ACMNA264)

Patterns and algebra
Simplify algebraic products and quotients using index laws. (ACMNA231)

Video tutorial
Numbers and powers
MAT10NAVT00001

Weblink
The number 42
MAT10NAWB00001

The modern number system has many different kinds of numbers. These have been invented by mathematicians over thousands of years to solve problems. Positive numbers are enough for most counting and measuring, but we need negative numbers, powers and roots for modern science and technology. Science and technology use algebra as a basic method for writing relationships. To use algebra effectively, you must be able to understand and use all the different kinds of numbers.

Mathematical literacy

Maths dictionary
MAT10ASDI00001

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

base	even	odd	significant figure
cardinal number	exponent	positive integer	square
closed	index	power	square root
composite	integer	prime	surd
counter example	irrational number	rational number	terminating decimal
counting numbers	magnitude	real number	whole number
cube	mantissa	recurring decimal	
cube root	natural numbers	root	
directed number	negative integer	scientific notation	

1.1 Index laws

Important!

Weblink
Math is fun: Exponents
MAT10NAWB00001

Indices

We use **indices** (the singular is **index**) to write repeated multiplication.
A **power** is written with the **base** number in normal size and a small number at the top right called the **index** or **exponent**. It shows repeated multiplication. The index shows the number of times the base is in the multiplication.

Base Index

$$4^5 = 4 \times 4 \times 4 \times 4 \times 4$$

Power form Extended form

The power shown is said as 'four to the fifth', 'the fifth power of four' or 'four to the power five'. We use the special names **square** and **cube** for 2nd and 3rd powers respectively.

We define $a^0 = 1$ and $a^{-n} = \dfrac{1}{a^n}$ for any integer n and any base a except zero ($a \neq 0$).

Example 1

Find the value of each of the following.

a 7^4 **b** 12 cubed **c** 8^{-2} **d** 6^0

9780170361941

Solution

a Write in extended form. $7^4 = 7 \times 7 \times 7 \times 7$

Calculate the answer. $= 2401$

b Write in extended form. $12 \text{ cubed} = 12 \times 12 \times 12$

Calculate the answer. $= 1728$

c Write in reciprocal form. $8^{-2} = \frac{1}{8^2}$

Write in extended form. $= \frac{1}{8 \times 8}$

Calculate the answer. $= \frac{1}{64}$

d Use the rule for a^0. $6^0 = 1$

When you have powers of the same base in a problem, you can work out the answer by writing each part in extended form. This can also be done for problems where the index applies to two numbers or where there is a power of a power.

Example 2

Simplify each of the following and leave your answer in index form.

a $5^3 \times 5^4$ **b** $7^6 \div 7^2$ **c** $(4 \times 3)^4$ **d** $(6^3)^4$

Solution

a Write the problem. $5^3 \times 5^4$

Write each power in extended form. $= 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

Write the answer in index form. $= 5^7$

b Write the problem as a fraction. $7^6 \div 7^2 = \frac{7^6}{7^2}$

Write each power in extended form.
Cancel two of the 7s. $= \frac{7 \times 7 \times 7 \times 7 \times \cancel{7} \times \cancel{7}}{\cancel{7} \times \cancel{7}}$

Write the answer in index form. $= 7^4$

c Write the problem. $(4 \times 3)^4$

Write the power in extended form. $= (4 \times 3) \times (4 \times 3) \times (4 \times 3) \times (4 \times 3)$

Rearrange the product. $= 4 \times 4 \times 4 \times 4 \times 3 \times 3 \times 3 \times 3$

Write the answer in index form. $= 4^4 \times 3^4$

d Write the problem. $(6^3)^4$

Write the outer power in extended form. $= 6^3 \times 6^3 \times 6^3 \times 6^3$

Write each power in extended form. $= 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times$
$6 \times 6 \times 6$

Write the answer in index form. $= 6^{12}$

Important!

Index laws

For any bases a and b and any indices m and n:

1 $a^m \times a^n = a^{m+n}$

2 $a^m \div a^n = a^{m-n}$ (for $m > n$)

3 $(a^m)^n = a^{mn}$

4 $(ab)^n = a^n b^n$

Note: m and n are integers and $a \neq 0$, $b \neq 0$.

The opposite of finding a power is finding a **root**.

Important!

Roots

The **surd** symbol $\sqrt{\ }$ is used to show roots. The nth root of a number x is shown as $\sqrt[n]{x}$ and is the number y so that $y^n = x$. For example, $\sqrt[4]{16} = 2$ because $2^4 = 16$.
The **square root** $(\sqrt[2]{\ })$ is normally written without the index, so $\sqrt{25} = 5$.
The **cube root** $(\sqrt[3]{\ })$ is the opposite of finding a cube, so $(\sqrt[3]{216}) = 6$.

It makes sense to write a root as a fractional power, because of the index law that $(a^m)^n = a^{mn}$. This works for $\sqrt[4]{16} = 2$ as $(16^{\frac{1}{4}})^4 = 16^{\frac{1}{4} \times 4} = 16^1 = 16$, and $2^4 = 16$ so $\sqrt[4]{16} = 16^{\frac{1}{4}}$.

Example 3

Puzzle sheet
Indices squaresaw
MAT10NAPS00002

Find each of the following.

a $\sqrt{121}$ **b** $\sqrt[5]{1024}$ **c** $81^{\frac{1}{2}}$ **d** $32^{\frac{1}{5}}$

Solution

a Think of the square that gives 121. $\mathbf{11^2 = 121}$

Write the answer. $\mathbf{\sqrt{121} = 11}$

b Try some possibilities. $\mathbf{3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243}$

$\mathbf{4^5 = 4 \times 4 \times 4 \times 4 \times 4 = 1024}$

Write the answer. $\mathbf{\sqrt[5]{1024} = 4}$

c Write in surd form. $\mathbf{81^{\frac{1}{2}} = \sqrt{81}}$

Think of the square that gives 81. $\mathbf{9^2 = 81}$

Write the answer. $\mathbf{81^{\frac{1}{2}} = 9}$

d Try a possibility. $\mathbf{2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32}$

Write the answer. $\mathbf{32^{\frac{1}{5}} = 2}$

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The power key on your calculator could be shown as $x^{\blacksquare}$, a^b or y^x. The nth root could be shown as $\sqrt[n]{\ }$, $\sqrt[x]{\ }$ or $x^{1/n}$. Many calculators also have square and square root keys shown as x^2 and $\sqrt{\ }$. In most cases, roots are obtained using the 2nd function key.
The order in which you need to enter powers may vary between calculators.

Example 4

Use a calculator to find

a 16^3 **b** $\sqrt{127}$ **c** 7^{-3}
d $\sqrt[4]{500}$ **e** $5^{1.7}$

Solution

a Enter as 16 y^x 3 = | 16^3 4096

Write the answer. | $\mathbf{16^3 = 4096}$

b Enter as $\sqrt{\ }$ 127 = | $\sqrt{127}$ 11.26942767

Round and write the answer. | $\sqrt{127} \approx 11.269$

c Enter as 7 y^x (−) 3 = | 7^{-3} 0.00292

Round and write the answer. | $7^{-3} \approx 0.002\ 92$

d Enter as 4 $\sqrt[x]{\ }$ 500 = | $^4\sqrt{500}$ 4.728708045

Round and write the answer. | $\sqrt[4]{500} \approx 4.729$

e Enter as 5 y^x 1.7 = | $5^{1.7}$ 15.42584657

Round and write the answer. | $5^{1.7} \approx 15.426$

Example 5

Use the index laws to simplify the following, leaving your answers with positive indices.

a $7^3 \times 7^8$ **b** $14^{15} \div 14^8$ **c** $5^3 \div 5^{10}$
d $(3^5)^3$ **e** $(5^2)^{-4}$ **f** $7^{11} \div 7^{11}$

Solution

a Write the problem. $7^3 \times 7^8$

Use index law 1. $= 7^{3+8}$

Work out the answer in index form. $= 7^{11}$

b Write the problem. $14^{15} \div 14^8$

Use index law 2. $= 14^{15-8}$

Work out the answer in index form. $= 14^7$

Worksheet
Numbers and indices
MAT10NAWK00001

Puzzle sheet
Powers and roots
MAT10NAPS00001

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c Write the problem. $5^3 \div 5^{10}$

Use index law 2. $= 5^{3-10}$

Work out the answer in index form. $= 5^{-7}$

Change to a positive index. $= \dfrac{1}{5^7}$

d Write the problem. $(3^5)^3$

Use index law 3. $= 3^{5\times 3}$

Work out the answer in index form. $= 3^{15}$

e Write the problem. $(5^2)^{-4}$

Use index law 3. $= 5^{2\times -4}$

Work out the answer in index form. $= 5^{-8}$

Change to a positive index. $= \dfrac{1}{5^8}$

f Write the problem. $7^{11} \div 7^{11}$

Use index law 2. $= 7^{11-11}$

Work out the answer in index form. $= 7^0$

Simplify using the definition $a^0 = 1$. $= 1$

Some expressions need to be simplified using more than one index law. The normal order of operations still applies, although you may have to insert brackets where they are only implied in the original expression.

Example 6

Simplify each of the following expressions and leave in index form.

a $(5^2 \times 3^{-3})^4$ **b** $\dfrac{5^3 \times 3^2}{(7^{-4} \times 5^2)^3}$ **c** $\left(\dfrac{7}{4}\right)^2 \div \left(\dfrac{7^{-2}}{5}\right)^3$

Solution

a Write the problem. $(5^2 \times 3^{-3})^4$

Use index law 4. $= (5^2)^4 \times (3^{-3})^4$

Use index law 3. $= 5^8 \times 3^{-12}$

Write the answer with positive powers. $= \dfrac{5^8}{3^{12}}$

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b Write the problem. $\dfrac{5^3 \times 3^2}{(7^{-4} \times 5^2)^3}$

Move the denominator to the top and change the sign of the power. $= 5^3 \times 3^2 \times (7^{-4} \times 5^2)^{-3}$

Use index law 4. $= 5^3 \times 3^2 \times (7^{-4})^{-3} \times (5^2)^{-3}$

Use index law 3 to remove the brackets. $= 5^3 \times 3^2 \times 7^{12} \times 5^{-6}$

Apply index law 1 to the common bases. $= 3^2 \times 5^{-3} \times 7^{12}$

Write the answer as a fraction with positive powers. $= \dfrac{3^2 \times 7^{12}}{5^3}$

c Write the problem. $\left(\dfrac{7}{4}\right)^2 \div \left(\dfrac{7^{-2}}{5}\right)^3$

Use index law 3. $= \dfrac{7^2}{4^2} \div \dfrac{7^{-6}}{5^3}$

Invert the 2nd fraction as a result of changing the division sign to a multiplication sign. $= \dfrac{7^2}{4^2} \times \dfrac{5^3}{7^{-6}}$

Use index law 2 on the common base of 7 and change base 4 to its lowest prime base. $= \dfrac{7^{2+6}}{(2^2)^2} \times \dfrac{5^3}{1}$

Tidy up and use index law 3. $= \dfrac{7^8 \times 5^3}{2^4}$

Example 7

Simplify each of the following and leave in index form.

a $30^4 \times 9 \times 15^{-2}$

b $\dfrac{27^3 \times 9^5}{(3^{-4} \times 9^3)^2}$

Solution

a Write the problem. $30^4 \times 9 \times 15^{-2}$

Rewrite the bases as products of prime factors. $= (2 \times 3 \times 5)^4 \times (3 \times 3) \times (3 \times 5)^{-2}$

Use index law 3 and tidy up where possible. $= 2^4 \times 3^4 \times 5^4 \times 3^2 \times 3^{-2} \times 5^{-2}$

Use index law 1 with the common bases. $= 2^4 \times 3^{4+2-2} \times 5^{4-2}$

Work out the answer in index form. $= 2^4 \times 3^4 \times 5^2$

b Write the problem. $\dfrac{27^3 \times 9^5}{(3^{-4} \times 9^3)^2}$

Move the denominator to the top and change the sign of the power. $= 27^3 \times 9^5 \times (3^{-4} \times 9^3)^{-2}$

Use index laws 3 and 4 to remove the brackets. $= 27^3 \times 9^5 \times 3^8 \times 9^{-6}$

Rewrite the bases as products of prime factors. $= (3 \times 3 \times 3)^3 \times (3 \times 3)^5 \times 3^8 \times (3 \times 3)^{-6}$

Tidy up where possible inside the brackets. $= (3^3)^3 \times (3^2)^5 \times 3^8 \times (3^2)^{-6}$

Use index law 3. $= 3^9 \times 3^{10} \times 3^8 \times 3^{-12}$

Use index law 1. $= 3^{9+10+8-12}$

Work out the answer in index form. $= 3^{15}$

The index laws can also be used with algebraic expressions.

Example 8

Animated example
Index laws
MAT10NAAE00001

CAS TI-Nspire exercise
Numbers and powers
MAT10NATI00001

CAS ClassPad exercise
Numbers and powers
MAT10NACP00001

Simplify each of the following expressions. Write your answers with positive powers.

a $(x^2y^3)^4$ **b** $a^5 \times a^{-6} \div a^7$ **c** $(4m^{-2}n^4) \div (64m^{-3}n^6)$

Solution

a Write the problem. $(x^2y^3)^4$

Use index laws 3 and 4. $= x^8y^{12}$

b Write the problem. $a^5 \times a^{-6} \div a^7$

Use index laws 1 and 2. $= a^{5+-6-7}$

$= a^{-8}$

Write as a fraction with a positive power. $= \dfrac{1}{a^8}$

c Write the problem. $(4m^{-2}n^4) \div (64m^{-3}n^6)$

Change the 64 to 4^3 so that the numbers have the same base. $= (4m^{-2}n^4) \div (4^3m^{-3}n^6)$

Use index laws 1 and 2. $= 4^{1-3}m^{-2+3}n^{4-6}$

Simplify. $= 4^{-2}mn^{-2}$

Write as a fraction with positive powers. $= \dfrac{m}{4^2n^2}$

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Exercise 1.1 Index laws

Understanding

Extra questions
Exercise 1.1
MAT10NAEQ00001

See Example 1

1 Work out each of the following.

a 4^3 **b** 7^5 **c** 4^5 **d** 5^3
e 6^4 **f** 2^{10} **g** 2^7 **h** 15^2
i 11^4 **j** 8^3 **k** 4^{-5} **l** 11^0
m 7^{-2} **n** 13^{-1} **o** 7^{-3} **p** 12^{-2}
q 115^0 **r** 5^{-4}

See Example 4

2 Use a calculator to find each of the following.

a 16^4 **b** 12^3 **c** 16^2
d 17^3 **e** 24^4 **f** 19^5
g 23^2 **h** 53^3 **i** 17^4

See Example 3

3 Work out each of the following.

a $\sqrt{64}$ **b** $\sqrt{25}$ **c** $\sqrt{100}$
d $\sqrt{400}$ **e** $\sqrt{169}$ **f** $\sqrt{144}$
g $\sqrt{225}$ **h** $\sqrt{10\,000}$ **i** $\sqrt{16\,000\,000}$

See Example 4

4 Use a calculator to find each of the following, correct to three decimal places.

a $\sqrt{50}$ **b** $\sqrt{250}$ **c** $\sqrt{148}$
d $\sqrt{85}$ **e** $\sqrt{7}$ **f** $\sqrt{48}$
g $\sqrt{412}$ **h** $\sqrt{3300}$ **i** $\sqrt{575\,000}$

See Example 2

5 Simplify each of the following and leave your answer in index form.

a $4^3 \times 4^7$ **b** $7^6 \times 7$ **c** $11^3 \times 11^3$
d $5^4 \times 5^5$ **e** $10^4 \times 10^{12}$ **f** $9^5 \times 9^3 \times 9$
g $14^3 \times 14^2 \times 14^7$ **h** $3^7 \times 3 \times 3^5$ **i** $15^3 \times 15^3 \times 15^3$

6 Simplify each of the following and leave your answer in index form.

a $8^5 \div 8^3$ **b** $5^6 \div 5^2$ **c** $7^5 \div 7$
d $11^8 \div 11^7$ **e** $6^{12} \div 6^{10}$ **f** $15^7 \div 15^7$
g $13^9 \div 13^4$ **h** $14^{15} \div 14^5$ **i** $19^3 \div 19^2$

7 Simplify each of the following and leave your answer in index form.

a $(4 \times 5)^3$ **b** $(6 \times 4)^7$ **c** $(7 \times 11)^4$
d $(6 \times 7)^8$ **e** $(11 \times 13)^0$ **f** $(2 \times 7)^{12}$

8 Simplify each of the following and leave your answer in index form.

a $(5^4)^3$ **b** $(7^2)^5$ **c** $(3^7)^4$
d $(11^{12})^4$ **e** $(1^4)^{16}$ **f** $(5^5)^5$

See Example 3

9 Find each of the following.

a $\sqrt[3]{125}$ **b** $\sqrt{169}$ **c** $\sqrt[4]{16}$
d $\sqrt[3]{216}$ **e** $\sqrt[6]{729}$ **f** $\sqrt[4]{256}$
g $\sqrt[7]{10\,000\,000}$ **h** $\sqrt{289}$ **i** $\sqrt[3]{27\,000}$

See Example 4

10 Use a calculator to find each of the following, correct to four decimal places.

a 11^{-3} **b** $\sqrt{58}$ **c** $\sqrt[4]{79}$
d 2.9^4 **e** $\sqrt{29.73}$ **f** $\sqrt[3]{257.8}$
g 6.7^{-1} **h** 7.89^{-3} **i** $\sqrt[5]{1.05}$

11 Calculate each expression correct to three decimal places where necessary.

a $5^{1.8}$ **b** $6^{1.7}$ **c** $81^{0.5}$
d $7^{1.6}$ **e** $1.4^{2.6}$ **f** $3.77^{4.01}$

See Example 5

12 Use the index laws to simplify the following, leaving your answers with positive indices.

a $4^3 \times 4^7$ **b** $7^6 \times 7^0$ **c** $15^3 \times 15^3$
d $6^4 \times 6^{15}$ **e** $10^{14} \times 10^{12}$ **f** $9^5 \times 9^3 \times 9^8$
g $14^3 \times 14^{12} \times 14^1$ **h** $3^6 \times 3^6 \times 3^5$ **i** $25^{13} \times 25^3 \times 25^4$

13 Use the index laws to simplify the following, leaving your answers with positive indices.

a $8^5 \div 8^3$ **b** $3^6 \div 3^2$ **c** $7^{15} \div 7^0$
d $11^{28} \div 11^{17}$ **e** $6^{12} \div 6^{12}$ **f** $15^7 \div 15^6$
g $14^{29} \div 14^{23}$ **h** $17^{15} \div 17^{13}$ **i** $12^{13} \div 12^9$

14 Use the index laws to simplify the following, leaving your answers with positive indices.

a $(7 \times 5)^4$ **b** $(6 \times 3)^6$ **c** $(9 \times 11)^4$
d $(6 \times 8)^7$ **e** $(12 \times 13)^8$ **f** $(2 \times 10)^9$

15 Use the index laws to simplify the following, leaving your answers with positive indices.

a $(4^7)^3$ **b** $(8^3)^5$ **c** $(5^7)^9$
d $(12^5)^{11}$ **e** $(9^{40})^6$ **f** $(3^3)^{33}$

Fluency

16 Use the index laws to simplify the following, leaving your answers with positive indices.

a $7^{10} \times 7^{25}$ **b** $6^{-7} \times 6^3$ **c** $7^8 \div 7^8$
d $(4^{-9})^5$ **e** $(8^{-7})^{-1}$ **f** $(6^{-3} \times 5^6)^3$
g $(7^{-1} \times 8^4)^{-5}$ **h** $(11^{-3} \times 7^4)^{-2} \times 7^5$ **i** $8^6 \times 8^{-4} \div 8^{-3}$

See Example 3

17 Find each of the following.

a $25^{\frac{1}{2}}$ **b** $125^{\frac{1}{3}}$ **c** $343^{\frac{1}{3}}$
d $243^{\frac{1}{5}}$ **e** $64^{\frac{1}{2}}$ **f** $64^{\frac{1}{6}}$
g $216^{\frac{1}{3}}$ **h** $81^{\frac{1}{4}}$ **i** $625^{\frac{1}{4}}$

18 Use a calculator to find each of the following, correct to 2 decimal places where necessary.

a $50^{\frac{1}{2}}$ **b** $236^{\frac{1}{3}}$ **c** $28^{\frac{1}{3}}$ **d** $560^{\frac{1}{5}}$
e $95^{\frac{1}{2}}$ **f** $2^{\frac{1}{6}}$ **g** $4^{1.8}$ **h** $3^{1.8}$
i $16^{0.5}$ **j** $5^{1.6}$ **k** $1.4^{2.6}$ **l** $3.75^{4.1}$

Problem solving

See Example 6

Worked solution

Exercise 1.1

MAT10NAWS00001

19 Simplify each of the following expressions and leave them in index form.

a $(4^2 \times 11^{-2})^3$ **b** $(3^{-2} \times 7^4)^2$ **c** $(5^2 \times 16^{-3})^{-3}$

d $\dfrac{8^3 \times 5^2}{(5^4 \times 2^2)^3}$ **e** $\dfrac{243^{-3} \times 5^2}{(5^{-3} \times 27^{-2})^3}$ **f** $\dfrac{5^{-2} \times 7^{-2}}{(5^{-4} \times 7^2)^{-5}}$

g $\left(\dfrac{7}{4^4}\right)^3 \times \left(\dfrac{4^2}{7}\right)^4$ **h** $\left(\dfrac{7}{8}\right)^2 \div \left(\dfrac{7^{-2}}{16}\right)^3$ **i** $\left(\dfrac{7}{5}\right)^6 \div \left(\dfrac{7^{-2}}{5^{-2}}\right)^{-3}$

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20 Simplify each of the following expressions and leave them in index form.

a $18^2 \times 12 \times 27^{-2}$ b $21^3 \times 7^{-3} \times 49^{-2}$ c $15^4 \times 125 \times 50^{-2} \times 15^{-3}$

d $\dfrac{5^3 \times 2^{20}}{(5^{-4} \times 16^3)^2}$ e $\dfrac{3^{-3} \times 5^5}{(5^{-4} \times 15^3)^{-3}}$ f $\dfrac{5^{-9} \times 10^8}{(20^{-4} \times 25^3)^{-2}}$

Worked solution
Exercise 1.1
MAT10NAWS00001
See Example 7

21 Simplify each of the following expressions. Write your answers with positive powers.

a $(x^4y^3)^3$ b $(xy^{-5})^4$ c $(x^{-12}y^{-1})^{-3}$

d $b^{14} \times b^3 \div b^{17}$ e $a^{-11} \times a^{-5} \div a^{-9}$ f $c^{14} \div c^{-6} \times c^{-12}$

g $(27m^{-3}n^4) \div (81m^{-2}n)$ h $(25p^4q^{-3}) \div (625p^4q^{-7})$ i $(1024r^{-2}s^4t^3) \div (64r^{-3}s^6t)$

See Example 8
Worked solution
Exercise 1.1
MAT10NAWS00001

1.2 Scientific notation

Very large and very small numbers are usually written in **scientific** (standard) **notation**.

Important!

Scientific notation

Numbers written in scientific notation are shown with a **mantissa** from 1 to less than 10, multiplied by a power of 10. The **order of magnitude** of the number is the power. For example:

$25\,000\,000 = 2.5 \times 10^7$ has mantissa 2.5 and order 7.

mantissa order

$0.006\,37 = 6.37 \times 10^{-3}$ has mantissa 6.37 and order ${}^{-}3$.

TLF Learning object
Exploring powers of 10 (L6548)
MAT10NAIN00001

Example 9

Write the following in scientific notation.

a 972 000 000 000 000 b 0.000 011 3

Solution

a Write with one digit before the decimal point, with the place value as a 'tens' number. Write with a power of 10.

972 000 000 000 000
$= \mathbf{9.72 \times 100\,000\,000\,000\,000}$
$= \mathbf{9.72 \times 10^{14}}$

b Write with one digit before the decimal point, with the place value as a 'tens' number. Write with a power of 10.

0.000 011 3
$= \mathbf{1.13 \times 0.000\,01}$
$= \mathbf{1.13 \times 10^{-5}}$

Some people change to scientific notation by moving the decimal point to make a number from 1 to less than 10 and counting how many places it has to move.

TLF Learning object
Exploring exponents and scientific notation (L6550)
MAT10NAIN00001

Example 10

Write the following as ordinary numbers

a 8.25×10^5 **b** 7.1×10^{-3}

Solution

a Write the problem. 8.25×10^5

Write the power as a 'tens' number. $= 8.25 \times 100\,000$

Write as an ordinary number. $= 825\,000$

b Write the problem. 7.1×10^{-3}

Write the power as a 'tens' number. $= 7.1 \times 0.001$

Write as an ordinary number. $= 0.0071$

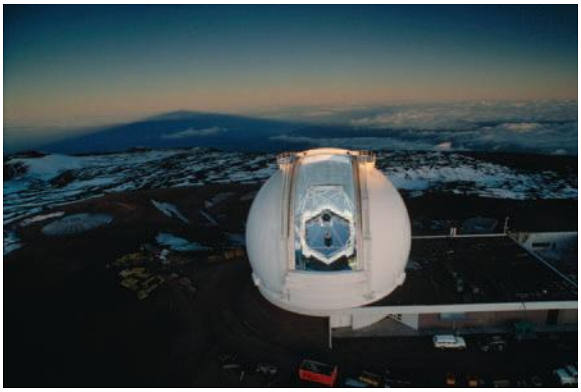
Most calculators have an EXP key to enter numbers in scientific notation. They automatically show large or small answers in scientific notation and can be set to show all answers in scientific notation. The display varies between calculators, but generally shows the order of magnitude as a superscript or after an E.

Weblink
Scale of the Universe
MAT10NAWB00001

Investigate: Astronomical numbers

Modern scientific theories about the universe are based on what astronomers see and can work out. They use optical and radio telescopes that cover large parts of the electromagnetic spectrum to look at the universe and make models of it. Almost all astronomers believe the basic big bang theory as it is the only one that explains the observations. However, there is debate about what will happen in the future. Astrophysicists try to work out how the universe started and how it is changing. Their theories cover periods from very near the start of the universe to what will happen far in the future. They usually express time in seconds and the size of the universe in metres.

Investigate the latest information about the size of the universe and its current age. Express the numbers in scientific notation and try to express them in normal numbers. Investigate the theories about the beginning of the universe. How long after the start did the first 'events' occur? How big was the universe then? Write these numbers in scientific notation and try to express them as ordinary numbers.



Example 11

Use a calculator to work out the following.

a $9.7 \times 10^{-6} \times 7.1 \times 10^{15}$

b $\dfrac{5.016 \times 10^{27} \times 3214}{3.52 \times 10^{-21}}$

Solution

a Enter 9.7 EXP (–) 6 × 7.1 EXP 15 =

$9.7\times10^{-6}\times7.1\times10^{15}$
6.887×10^{10}

Write the answer. $\mathbf{9.7 \times 10^{-6} \times 7.1 \times 10^{15} = 6.887 \times 10^{10}}$

b Enter 5.016 EXP 27 × 3214 ÷ 3.52 EXP (–) 21 =

$5.016\times10^{27}\times3214\div3.52\times10^{-21}$
4.57995×10^{51}

Round and write the answer. $\mathbf{\dfrac{5.016 \times 10^{27} \times 3214}{3.52 \times 10^{-21}} \approx 4.580 \times 10^{51}}$

CAS TI-Nspire exercise
Numbers and powers
MAT10NATI00001

CAS ClassPad exercise
Numbers and powers
MAT10NACP00001

Puzzle sheet
Scientific notation
MAT10NAPS00003

A measurement accurate to the nearest metre could be stated as 57 m, 0.057 km or 57 000 mm. Each of these has the same accuracy, and this is very clear if they are written in scientific notation as 5.7×10^1 m, 5.7×10^{-2} km or 5.7×10^4 mm.

Important!

Accuracy

The accuracy of a number can be stated as the **number of significant figures**, which is the number of digits in the mantissa of a number written in scientific notation.

Example 12

Which is the most accurately written number out of 0.0005, 27 800 and 35 000 000?

Solution

Think of each number in scientific notation.

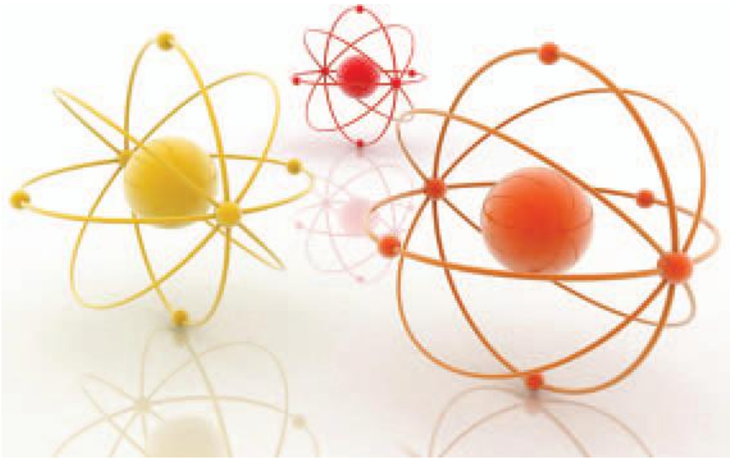
$\mathbf{0.0005 = 5.0 \times 10^{-4}}$

$\mathbf{27\,800 = 2.78 \times 10^{4}}$

$\mathbf{35\,000\,000 = 3.5 \times 10^{7}}$

Think of the number of significant figures. **5.0 has 1 digit, 2.78 has 3 digits, 3.5 has 2 digits.**

Write the answer. **27 800 is the most accurately written.**



Example 13

The kinetic energy of an object is given by the formula $E = \frac{1}{2}mv^2$. If m is in kg and v is in m/s, then E is in Joules. What is the kinetic energy of an electron (mass $= 9.01 \times 10^{-32}$ kg) travelling at a fifth of the speed of light (speed of light, $c = 3 \times 10^8$ m/s)?

Solution

Find the velocity (v) of the electron.

$$\frac{c}{5} = \frac{3 \times 10^8}{5}$$
$$= 6.0 \times 10^7 \text{ m/s}$$

Write the formula.

$$E = \frac{1}{2}mv^2$$

Substitute in the appropriate values.
(Check your units)

$$= \frac{9.01 \times 10^{-32} \times (6.0 \times 10^7)^2}{2}$$

Use index laws 3 and 4.

$$= \frac{9.01 \times 10^{-32} \times 6.0^2 \times 10^{14}}{2}$$

Use the index laws, tidy up and simplify.

$$= \frac{9.01 \times \cancel{36}^{18} \times 10^{-18}}{\cancel{2}^{1}}$$
$$= 9.01 \times 18 \times 10^{-18}$$

On your calculator enter 9.01 [×] 18 [×] 10 [y^x] [(–)] 18.

9.01×18×10^{-18}
1.6218×10^{-16}

Write your answer.

The kinetic energy of the electron is 1.6218×10^{-16} Joules.

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Exercise 1.2 Scientific notation

Understanding

Extra questions
Exercise 1.2
MAT10NAEQ00002

1 Write in scientific notation.

a 7300	b 132 000 000	c 0.0085	d 0.000 000 138
e 700 000	f 0.0007	g 290	h 0.719
i 37 080 000	j 0.030 19	k 9 744 000	l 0.000 000 008

See Example 9

See Example 10

2 Write as ordinary numbers.

a 6.3×10^5	b 7.1×10^{-3}	c 9.74×10^6	d 4.19×10^{-1}
e 8×10^{-9}	f 3×10^4	g 5.2×10^1	h 9.8×10^{-2}
i 1.02×10^0	j 9.55×10^{-5}	k 4.04×10^2	l 9.1×10^3

Fluency

See Example 11

3 Use a calculator to find each of the following and write your answer in scientific notation, correct to 4 significant figures.

a $5.6 \times 10^{-8} \times 2.2 \times 10^9$
b $7.1 \times 10^9 \div (6.34 \times 10^{-2})$
c $8.64 \times 10^7 \times 4.9 \times 10^{11}$
d $9.6 \times 10^{-3} \times 2.09 \times 10^{-12}$
e $8.11 \times 10^4 \div (3.61 \times 10^{-9})$
f $4.74 \times 10^{-3} \div (1.9 \times 10^{15})$

4 State the number of significant figures in each of the following.

a 34 700	b 0.000 65	c 900
d 0.000 006	e 40	f 8.65×10^{11}
g 9.047×10^{-9}	h 405 060 000	i 810 000

See Example 12

5 Choose the most accurately written figure in each set of numbers.

a 52, 4000, 8.6, 975
b 703, 8.8, 0.000 74, 38 000
c 6.047, 0.0007, 149 000, 9500
d 0.004, 97 000, 3 058 000, 58.3
e 29, 347.6, 8.95, 0.054
f 9 000 000, 27 000, 0.0006, 0.000 000 755
g 2, 200, 200.5, 0.0055
h 0.050 57, 2.54, 2500, 25 600

6 Use a calculator to find each of the following and write the answer in scientific notation, correct to 4 significant figures.

a $\dfrac{5861 \times 2 \times 10^8}{3.58 \times 10^{-5}}$
b $\dfrac{7.13 \times 10^{-8} \times 1.16 \times 10^6}{4925}$
c $\dfrac{3.1 \times 10^8 \times 8.2 \times 10^{14}}{6.8 \times 10^{13} \times 0.1067}$
d $\dfrac{7.85 \times 10^{11} \times 7.84 \times 10^{11}}{7.85 \times 10^{-11} - 7.84 \times 10^{-11}}$
e $\dfrac{6.101 \times 10^{-10} \times 7.232 \times 10^{-3}}{12.113 \times 10^{-5} + 8.001 \times 10^{-7}}$
f $\dfrac{4.71 \times 10^{26} \times 2.95 \times 10^{27}}{3.5 \times 10^{29} \times \pi^2}$

Problem solving

Worked solution
Exercise 1.2
MAT10NAWS00002

7 An electron's mass is about 9.109×10^{-31} kg and a proton's mass is about 1.673×10^{-27} kg. Which is heavier and by how many times?

Worked solution
Exercise 1.2
MAT10NAWS00002

8 The radius of an electron is less than 1 attometre and the radius of a proton is 862 attometres, where 1 attometre is equivalent to 10^{-18} metres.

a Determine the maximum volume of an electron.
b Calculate the volume of a proton.
c Compare the volumes of each.

See Example 13

9 The Large Hadron Collider accelerates protons around a 27 km long circular tunnel under the border of Switzerland and France. They reach speeds of 0.999 999 991 times the speed of light. At this speed, the mass of a proton is $\sqrt{\dfrac{1}{1-\left(\frac{v}{c}\right)^2}}$ times its rest mass. The speed of light is 3×10^8 m/s and the rest mass of a proton is 1.673×10^{-27} kg. What is its mass at:

a one quarter the speed of light $\left(\frac{v}{c} = 0.25\right)$?

b nine-tenths the speed of light $\left(\frac{v}{c} = 0.9\right)$?

c 0.999 999 991 times the speed of light?

Reasoning

Worked solution
Exercise 1.2
MAT10NAWS00002

10 The mass of the Earth is 5.9722×10^{24} kg and its diameter is 12 756 200 m. The mass of Jupiter is 1.8987×10^{27} kg and its diameter is 142 984 km. Considering them as spheres, find the:

a volume of the Earth

b volume of Jupiter

c density of the Earth

d density of Jupiter.

e Explain whether you would consider the Earth or Jupiter to be 'heavier'.

1.3 Real numbers

A **recurring decimal** is a decimal that has one or more digits that repeat forever. We show that digits are repeating by putting a bar (or dots) over the repeating digits. A **terminating decimal** is a decimal that is not recurring, so comes to an end.

Important!

Recurring decimals and fractions

To change a recurring decimal to a fraction:

- multiply the decimal by 10^n, where n is the number of recurring digits
- subtract the number
- divide by $10^n - 1$ and simplify if possible.

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Example 14

Change $0.\overline{114}$ to a fraction.

Solution

Note how many figures recur.	**It has 3 digits recurring.**
Write the number out to show the repetition.	**Number** $= 0.114\ 114\ 114\ 114\ \ldots$
Multiply by 1000 for the 3 digits.	$1000 \times$ **number** $= 114.114\ 114\ \ldots$
Subtract the number.	$999 \times$ **number** $= 114$
Divide by 999.	**Number** $= \dfrac{114}{999}$
Cancel if possible.	$= \dfrac{38}{333}$
Write the answer.	$0.\overline{114} = \dfrac{38}{333}$

Puzzle sheet
Rational numbers
MAT10NAPS00004

Sometimes the number does not recur from immediately after the decimal point. In this case, multiply by a power of 10 so that it does in order to make it easier.

Example 15

Change $0.07\overline{123}$ to a fraction.

Solution

Multiply by 100 to make it recur just after the decimal point.	$100 \times$ **number** $= 7.\overline{123}$
Note how many digits recur.	**It has 3 recurring digits.**
Write the number out.	$100 \times$ **number** $= 7.123\ 123\ \ldots$
Multiply by 1000 for the repeating digits.	$1000 \times 100 \times$ **number** $= 7123.123\ 123\ \ldots$
Subtract.	$999 \times 100 \times$ **number** $= 7116$
Divide by 999×100.	**Number** $= \dfrac{7116}{99\,900}$
Cancel if possible.	$\dfrac{7116}{99\,900} = \dfrac{593}{8325}$
Write the answer.	$0.07\overline{123} = \dfrac{593}{8325}$

All fractions can be converted to terminating or recurring decimals and vice versa.

Important!

Rational, irrational and real numbers

A **rational** number can be written as the ratio of two integers, such as $5.31 = \frac{531}{100}$, $^{-}7 = \frac{14}{^{-}2}$, or $0.3\overline{571428} = \frac{5}{14}$. Every rational number can also be written as a recurring or terminating decimal.

An **irrational number** is not rational. It cannot be written as a terminating or recurring decimal.

An irrational number such as $\sqrt{2}$ or π cannot be expressed as a fraction.

The rational and irrational numbers completely fill the number line and, together, make the **real numbers**.

Example 16

State whether each of the following numbers is probably rational or irrational.

a 0.3333 ... **b** 3.0128 **c** 1.256 386 42 ... **d** 4.256565 ...

Solution

a This looks like it is actually $0.\overline{3}$. **0.3333 ... is probably rational.**

b This is a terminating decimal. **3.0128 is rational.**

c This does not recur or terminate. **1.256 386 42 ... is probably irrational.**

d This looks like it is actually $4.2\overline{56}$. **4.256 565 ... is probably rational.**

Investigate: Finding roots

Work in pairs and, with a calculator, try to find a number that squares to give 5, using the following procedure.

1 You know that $2^2 = 4$ and $3^2 = 9$, and that $\sqrt{4} < \sqrt{5} < \sqrt{9}$, so $\sqrt{5}$ must be between 2 and 3.

2 Try $2.5^2 = 6.25$. It is too big. The square is 1.25 out.
$2.1^2 = 4.41$ misses 5 by 0.59.
Try other numbers, including numbers with more than 1 decimal place.
Record each number you try and calculate how far out you are.

3 After 5 minutes, check which pair of people are closest.
Try this investigation with other surds.

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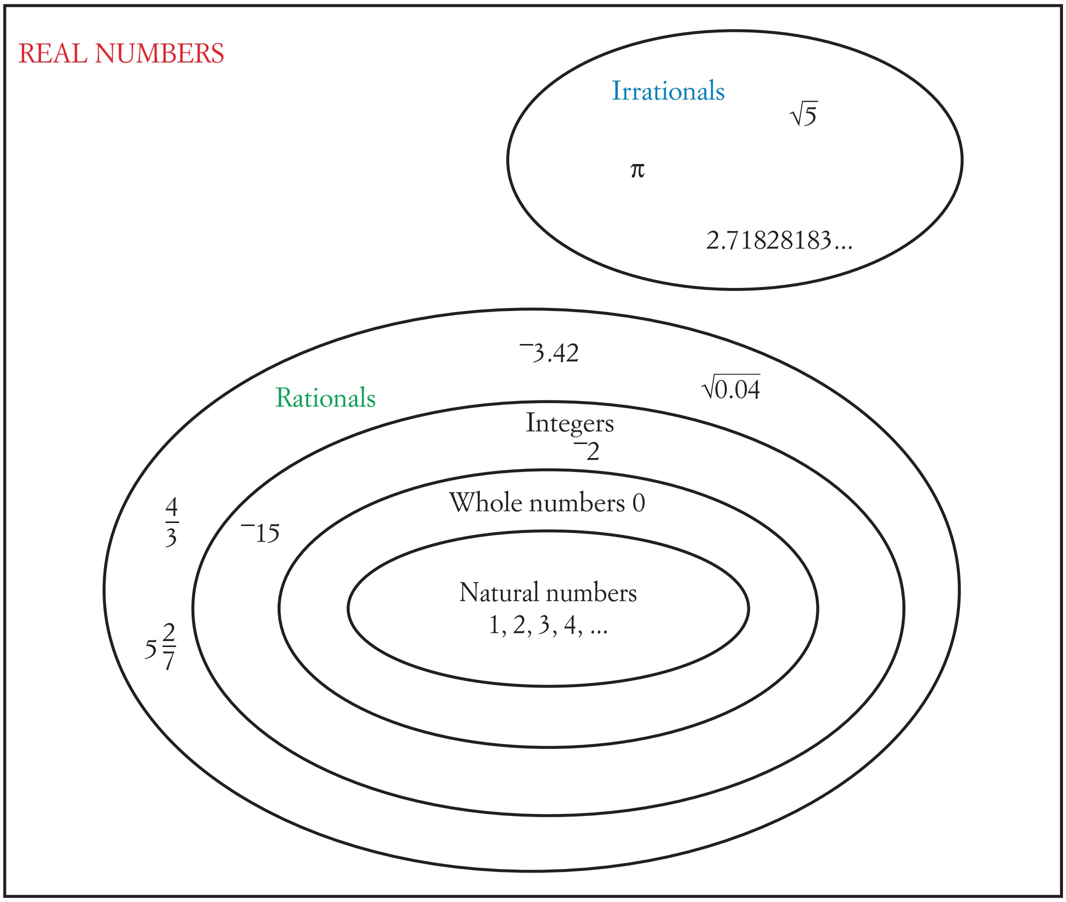
Important!

Types of numbers

You already know quite a few kinds of numbers. The **positive integers, counting** or **natural numbers** are 1, 2, 3, 4, 5, 6, 7, . . . and so on. You know that **even** numbers divide exactly by 2, and **odd** numbers don't. **Prime** numbers have exactly two different factors: themselves and 1. **Composite** numbers have more than two factors.
When we include zero and the **negative integers** we get the **integers**. The counting numbers are sometimes called **cardinal numbers** and the integers are often called **whole numbers**.



If we multiply two counting numbers, we get another one. Mathematicians say that the counting numbers are **closed** for multiplication. Not every subtraction of counting numbers gives a counting number. We say that the counting numbers are **not closed** for subtraction.
If you *always* get another number from the same set for an operation, the set of numbers is closed for that operation.
To show that something is *not true*, all you have to do is find one example to show it doesn't work. This is called a **counter-example**.
An even number can always be written in the form $2n$, where n is an integer. Similarly, any multiple of 3 can be written as $3n$ and any number that has a remainder of 4 when divided by 5 can be written as $5n + 4$.

Example 17

Show that the integers not divisible by 3 are not closed under addition.

Solution

Try adding non-multiples of 3.	**$2 + 5 = 7$, which *is not* divisible by 3.**
Try another case.	**$2 + 4 = 6$, which is a multiple of 3.**
Use the relevant fact.	**Since there are 2 numbers not divisible by 3 that add up to a number divisible by 3, numbers not divisible by 3 are not closed under addition.**

Investigate: Computer numbers

When the first electronic computers were being built, John von Neumann (1903–1957) (see the right-hand side of the photo) analysed the problems involved and developed astonishing insights into how they should be constructed. He also built his own machine to use on mathematical problems. He is generally credited with suggesting that electronic computers should use **binary numbers**. In binary arithmetic, the problem $5 + 3$ is written as $101 + 11$. This can be machine coded as on-off-on plus on-on. Investigate binary numbers and how they are used in computers. What are **octal** and **hexadecimal** numbers? How are they used?

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To show that something is true, it has to be proven for every possible case.

Example 18

Show that the odd numbers are closed under multiplication.

Solution

An odd number is a multiple of 2, plus 1.	**If a is odd then $a = 2n + 1$ for some integer n.**
To *prove* that it is closed, you have to show that the product of *any* two odd numbers is odd.	**Let a and b be any two odd numbers.**
Write the odd numbers in a useful form.	**Choose integers n and m so that $a = 2n + 1$ and $b = 2m + 1$.**
Write the product.	$a \times b = a(2m + 1)$
Expand using the distributive law.	$= 2am + a$
Write the last a as $2n + 1$.	$= 2am + 2n + 1$
Use the common factor of 2.	$= 2(am + n) + 1$
State the kind of number it is.	**$2(am + n) + 1$ is an odd number because it is in the form $2p + 1$, where p is an integer.**
State the result.	**Odd numbers are closed under multiplication.**

Exercise 1.3 Real numbers

Understanding

Extra questions

Exercise 1.3

MAT10NAEQ00003

See Example 14

1 Change each of the following to a fraction.

a $0.\overline{235}$ **b** $0.\overline{414}$ **c** $0.\overline{314}$

d $0.\overline{672}$ **e** $0.\overline{621}$ **f** $0.\overline{2244}$

g $0.\overline{1}$ **h** $0.\overline{13}$ **i** $0.0\overline{444}$

2 Change each of the following to a decimal.

a $\frac{5}{6}$ **b** $\frac{5}{12}$ **c** $\frac{31}{99}$

d $\frac{19}{16}$ **e** $\frac{18}{37}$ **f** $\frac{628}{303}$

g $\frac{3}{7}$ **h** $\frac{1003}{8325}$ **i** $\frac{902}{999}$

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Fluency

See Example 15

3 Change each of the following to a fraction.

a $0.0\overline{712}$ b $0.0\overline{123}$ c $0.0\overline{215}$
d $0.06\overline{714}$ e $0.06\overline{069}$ f $0.00\overline{2124}$

Worked solution
Exercise 1.3
MAT10NAWS00003

See Example 16

4 State whether each of the following is probably rational or irrational.

a 12.5 b 0.111 ... c 1.57183 ...
d 2.181 81 ... e 1.123 42 ... f 1.356
g 7.272 73 ... h 3.522 22 ... i 156.5
j 0.856 66 ... k 124.387 219 ... l 414.214 12 ...

Problem solving

See Example 17

5 Show that the integers not divisible by 3 are not closed under subtraction.

6 Show that the set {1, 2, 3} is not closed under addition.

7 Show that the set {$^-1$, 0, 1} is not closed under addition.

8 Show that the prime numbers are not closed under either addition or multiplication.

9 Show that the multiples of 5 are not closed under division.

10 Show that numbers that have a remainder of 2 when divided by 3 are not closed under addition.

11 Show that numbers that have a remainder of 2 when divided by 3 are not closed under subtraction.

Reasoning

See Example 18

Worked solution
Exercise 1.3
MAT10NAWS00003

Worked solution
Exercise 1.3
MAT10NAWS00003

12 Show that the odd numbers are not closed under addition.

13 Show that the set {$^-1$, 1} is closed under multiplication.

14 Show that the set {$^-1$, 0, 1} is closed under multiplication.

15 Show that the odd numbers are not closed under subtraction.

16 Show that the even numbers are closed under multiplication.

17 Show that the even numbers are not closed under division.

18 A new operation called malition (symbol #) is made up. Two numbers are malited by adding their product and sum. For example, $3 \# 4 = (3 \times 4) + (3 + 4) = 19$.

a Are the even numbers closed under malition?
b Are the odd numbers closed under malition?
c Are the integers closed under malition?

19 Make up an operation that is closed for prime numbers.

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Chapter 1 summary

Quiz
Numbers and powers
MAT10NAQZ00001

- A repeated multiplication may be written as a **power**. The number multiplied is written once as the **base**. The number of times it is in the multiplication is written as the small **index (exponent)** at the top right. The **extended form** of a power has the multiplication written out in full.
- A **square** is a power with index 2, and a **cube** has index 3.
- Powers with **zero** and **negative exponents** are defined for non-zero bases as $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$ for any integer n and any base a except zero ($a \neq 0$).
- The index laws are that for any numbers a and b and any indices m and n:

  1 $a^m \times a^n = a^{m+n}$
  2 $a^m \div a^n = a^{m-n}$ (for $m > n$)
  3 $(a^m)^n = a^{mn}$
  4 $(ab)^n = a^n b^n$

  Note: m and n are integers and $a \neq 0$, $b \neq 0$.
- The nth **root** of a number x is written using the **surd** symbol as $\sqrt[n]{x}$. It is a number y that must be raised to the nth power to give x, so $y^n = x$. The **square root**, $\sqrt{x}$, is written without the index number. The third root, $\sqrt[3]{\ }$, is called the **cube root.**
- Square roots, cube roots and so on can be written as powers using fractions such as $x^{\frac{1}{2}}$, $x^{\frac{1}{3}}$, and so on.
- Numbers written in **scientific notation (standard notation)** are shown with a **mantissa** from 1 to less than 10, multiplied by a power of 10. The **order of magnitude** of the number is the power.
- The accuracy of a number can be given as the **number of significant figures,** which is the number of digits in the mantissa of a number written in scientific notation.
- A **recurring decimal** is a decimal that has one or more digits that repeat forever. We show that the digits are repeating by putting a bar (or dots) over the repeating digits. A **terminating decimal** is a decimal that is not recurring, but comes to an end.
- A **rational** number can be written as a fraction. It is also a terminating or recurring decimal.
- An **irrational number** is not rational. It cannot be written as a terminating or recurring decimal. An irrational number cannot be expressed as a fraction.
- The rational and irrational numbers together make the **real** numbers and fill the number line.
- The **positive integers, counting** or **natural numbers** are 1, 2, 3, 4, 5, 6, 7, ... **Even** numbers divide exactly by 2, and **odd** numbers don't. **Prime** numbers have exactly two different factors, themselves and 1. **Composite** numbers have more than two factors.
- If you *always* get another number of the same kind for an operation, those numbers are **closed** for that operation.

Chapter 1 review

Understanding

See Example 1

1 Work out each of the following.

a 4^4 **b** 5^3 **c** 3^5

See Example 3

2 Work out each of the following.

a $\sqrt{81}$ **b** $\sqrt{625}$ **c** $\sqrt{10\,000}$

3 Simplify each of the following and leave your answer in index form.

a $(3 \times 5)^4$ **b** $(7 \times 4)^5$ **c** $(5 \times 11)^3$

See Example 3

4 Find each of the following.

a $\sqrt[3]{343}$ **b** $\sqrt{225}$ **c** $\sqrt[4]{81}$

5 Use the index laws to simplify the following, leaving your answers with positive indices.

a $7^5 \div 7^3$ **b** $23^{13} \times 23^3 \times 23^{-4}$ **c** $(11^5)^{12}$

See Example 9

6 Write in scientific notation.

a 9300 **b** 185 000 000 **c** 0.000 95

See Example 10

7 Write as ordinary numbers.

a 7.03×10^5 **b** 8.11×10^{-4} **c** 9.603×10^7

See Example 14

8 Change each of the following to a fraction.

a $0.\overline{173}$ **b** $0.\overline{513}$ **c** $0.\overline{216}$

Fluency

9 Use the index laws to simplify the following, leaving your answers with positive indices.

a $5^{10} \times 5^{27}$ **b** $6^{-11} \times 6^3$ **c** $5^7 \div 5^7$

See Example 11

10 Use a calculator to find each of the following and write your answer in scientific notation, correct to 4 significant figures.

a $4.6 \times 10^{-7} \times 5.2 \times 10^9$ **b** $7.01 \times 10^9 \div (8.34 \times 10^{-2})$

11 State the number of significant figures in each of the following.

a 4 700 **b** 0.010 63 **c** 700

12 Use a calculator to find each of the following, correct to 2 decimal places.

a $40^{\frac{1}{2}}$ **b** $736^{\frac{1}{3}}$ **c** $39^{\frac{1}{4}}$

See Example 12

13 Choose the most accurately written figure in each set of numbers.

a 72, 4000, 9.6, 875 **b** 7.3, 9906, 0.000 54, 48 200

14 Use a calculator to find each of the following and write the answer in scientific notation, correct to 4 significant figures.

a $\dfrac{4789 \times 2 \times 10^5}{4.58 \times 10^{-7}}$ **b** $\dfrac{8.14 \times 10^{-8} \times 1.06 \times 10^7}{575}$

See Example 15

15 Change each of the following to a fraction.

a $0.07\overline{114}$ **b** $0.03\overline{069}$ **c** $0.00\overline{7724}$

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16 State whether each of the following is probably rational or irrational. *See* Example 16

a 0.666 ... **b** 67.501 **c** 1.236 067 977 ...
d 4.192 192 ... **e** 6.283 185 307 ... **f** 1.7706

Problem solving

17 Simplify each of the following expressions and leave in index form. *See* Example 6

a $(7^2 \times 5^{-3})^3$ **b** $\dfrac{729^{-3} \times 5^2}{(5^{-3} \times 27^{-2})^3}$ **c** $\left(\dfrac{49}{11}\right)^2 \div \left(\dfrac{7^{-2}}{22}\right)^4$

18 Simplify each of the following expressions and leave in index form. *See* Example 7

a $16^3 \times 4^{-3} \times 48^{-2}$ **b** $\dfrac{5^{-3} \times 3^5}{(3^{-4} \times 15^3)^{-4}}$ **c** $\dfrac{5^{-8} \times 20^8}{(10^{-4} \times 25^3)^{-2}}$

19 Simplify each of the following expressions. Write your answers with positive powers. *See* Example 8

a $(xy^{-2})^{-4}$ **b** $a^{-12} \times a^5 \div a^{-2}$ **c** $(36p^4q^{-3}) \div (216p^4q^{-2})$

20 The force of gravity is given by $F = \dfrac{Gm_1m_2}{r^2}$ newtons, where $G = 6.6730 \times 10^{-11}$, m_1 and m_2 are the masses of the objects in kg and r is the distance between them in m. *See* Example 13
The masses of the Earth and Moon are 5.9722×10^{24} kg and 7.3477×10^{22} kg respectively.
The average distance from Earth to the Moon is 384 403 000 m.
Find the force of attraction between the Earth and the Moon.

21 Show that numbers that have a remainder of 2 when divided by 3 are not closed under multiplication. *See* Example 17

Reasoning

22 Show that the even numbers are closed under addition. *See* Example 18

Statistics and probability

2 Statistics

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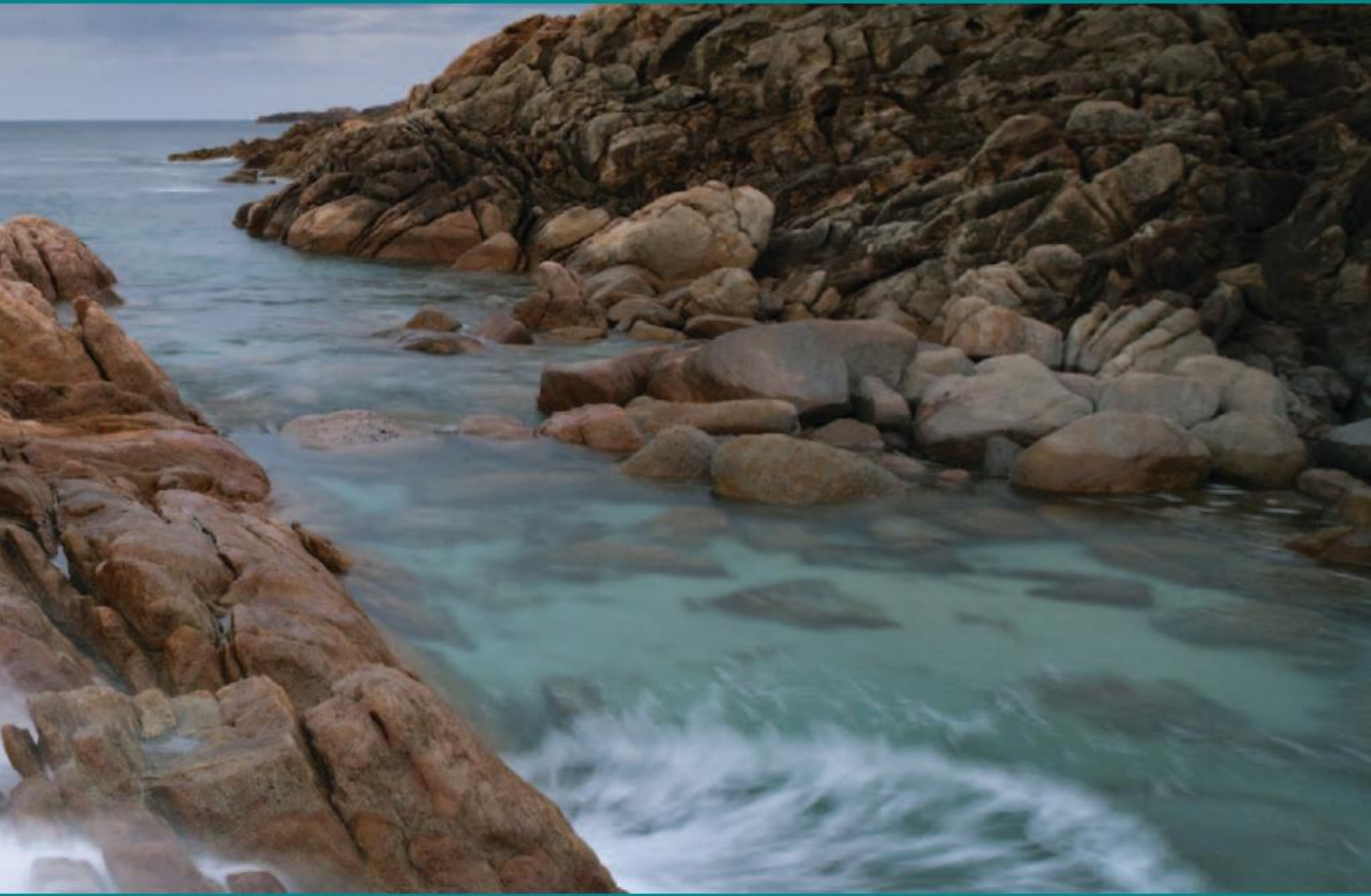

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MAT10SPPL00002

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Australian Curriculum statements

Data representation and interpretation

Determine quartiles and interquartile range. (ACMSP248)

Construct and interpret box plots and use them to compare data sets. (ACMSP249)

Compare shapes of box plots to corresponding histograms and dot plots. (ACMSP250)

Use scatter plots to investigate and comment on relationships between two numerical variables. (ACMSP251)

Investigate and describe bivariate numerical data where the independent variable is time. (ACMSP252)

Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data. (ACMSP253)

Video tutorial
Statistics
MAT10SPVT00002

Weblink
Kaggle: Data science competitions
MAT10SPWB00002

The modern world is an information world. Governments, credit bureaus, banks and marketeers collect and use information about how we live, where we live, what we buy and what we desire. You can tap into information at home and elsewhere at any time using the Internet. We use statistics to collect, display and analyse information.

Statistics is used to work out the needs of particular parts of the Australian population, as well as what is likely to happen in the future. In medicine, statistics is used to investigate diseases and their treatment. Governments and agencies use statistics to decide where to put new developments, and businesses use them to design ad campaigns and to target consumer groups for sales of particular products. Investment firms, trusts and fund managers use statistics to decide what stocks they should buy on the share market and how much they are willing to pay for particular shares.

Mathematical literacy

TLF Learning object
Australia's population dataset (M007102)
MAT10SPIN00002

Maths dictionary
MAT10ASDI00001

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

back-to-back	fair	nominal	scatter plot
bias	first quartile	non-compliant	side-by-side
bimodal	five number summary	numerical	skewed
bivariate	interpolation	ordinal	spread
box-and-whisker plot	interquartile range	outlier	strong relationship
boxplot	line of best fit	population	survey
categorical	lower quartile	positive relationship	symmetrical
central tendency	mean	positive skew	third quartile
continuous	median	quartile	two-way table
cumulative frequency	mode	range	univariate
discrete	negative relationship	relationship	upper quartile
extrapolation	negative skew	sample	weak relationship

2.1 Calculating and interpreting data measures

Important!

Types of data

Categorical data places the information in categories such as colour, sex and nationality. There are two kinds of categorical data.

1 **Nominal** data has no numeric meaning or order.
2 **Ordinal** data has a natural order, but does not represent a measurement.

Numerical data consists of numbers. There are two kinds of numerical data.

1 **Continuous** data can take any value between the smallest and largest values, so we may need to show how accurate it is.
2 **Discrete** data can only have particular values. In most cases, the values are whole numbers.

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There are many ways to measure numerical data, but few for categorical data.

Important!

Data measures

The **mode** is the score with the highest frequency. It is the most common score. When there are two scores with equal highest frequencies, we say that the distribution is **bimodal**. The mode can be used for any type of data.

The **median** is the *middle* score. To find the median, all scores are arranged in order from smallest to largest and the middle score is chosen. If there are an even number of scores, then the average of the middle two scores is used. When there are a large number of scores, the progressive total of frequencies (the **cumulative frequency**) is used to work out the median.

The median is the $\frac{n+1}{2}$th score. The median can be used for ordinal, discrete or continuous data.

The **mean** ($\bar{x}$) is the **average** score. It is calculated by adding all the scores and dividing by the number of scores. It can be used for discrete or continuous data, but may give unsuitable values for discrete data.

For numeric data, the **range** is the difference between the highest and lowest scores. For categorical data, it is the number of different scores.

The **quartiles** of a frequency distribution divide it into 4 equal sections.

The **first quartile** (**lower quartile**) is the score that has a quarter (25%) of the scores below it. Its symbol is $Q1$ or Q_1.

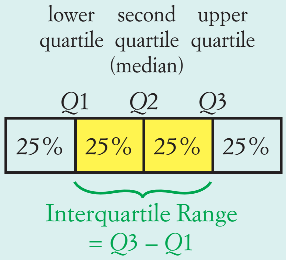
The **second quartile** is the score that has two quarters (50%) of the scores below it. It is another name for the median. It can be written as $Q2$ or Q_2.

The **third quartile** (**upper quartile**) is the score that has three quarters (75%) of the scores below it. Its symbol is $Q3$ or Q_3.

The **interquartile range** (IQR) is the difference between the third and first quartiles. It can be worked out for ordinal and numeric data.

$$\text{IQR} = Q3 - Q1$$

The mode, mean and median are all **measures of central tendency**, while the range and interquartile range are **measures of spread**.



The mathematical symbol Σ, which means 'the sum of' can be used to write the formula for the mean as $\bar{x} = \frac{\Sigma x}{n} = \frac{\Sigma x}{\Sigma f}$, where x stands for a score, n is the number of scores and f stands for a frequency.

Example 1

The hair colours of a class of Year 10 students in Ireland were as follows.
Dark Brown, Red, Blonde, Dark Brown, Blonde, Celtic Bronze, Black, Blonde, Dark Brown, Light Brown, Light Brown, Dark Brown, Black, Light Brown, Blonde, Black, Blonde, Light Brown, Blonde, Blonde, Red, Dark Brown, Dark Brown, Blonde, Light Brown, Blonde, Strawberry Blonde
Find the mode and range.




Solution

Make a frequency table of the colours.

Colour	Frequency
Black	3
Blonde	9
Celtic Bronze	1
Dark Brown	6
Light Brown	5
Red	2
Strawberry Blonde	1
Total	27

State the colour with the highest frequency. **The mode is Blonde.**

How many different colours are there? **The range is 7.**

Example 2

A class of Year 10 students were ask to rank takeaway chicken, chips, hamburgers, kebabs and pizza in order of their preference from 1 to 5, with 1 being the best. The results for pizza are as follows.
1, 2, 1, 2, 1, 5, 2, 2, 4, 4, 4, 1, 4, 1, 1, 3, 4, 3, 3, 3, 3, 4, 2, 2, 2, 3

a Find the median and mode for pizza.
b Which is the best measure of pizza's popularity?
c Is this sample sufficient to make good conclusions?

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Solution

a Arrange the results in order. **1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 5**

There are more 2s than anything else. **The mode is second preference.**

The middle 2 scores are 2 and 3. $\textbf{Median} = \frac{2+3}{2} = 2.5$

b The data is ordinal. **It is not sensible to use the mean, so the best measure is the median. Pizza is 2nd or 3rd in popularity.**

c Only the pizza results are given. **The results are incomplete, and the sample is quite small, so you cannot make good conclusions.**

The preferences in Example **2** are not really numbers. Preferences are nominal data, so it is not sensible to calculate a mean.
A table with frequencies can be used to find the mean and median. In that case, the total of all the scores is the total of the score × frequency column. This is written as Σxf.

Example 3

CAS TI-Nspire exercise
Statistics
MAT10SPTI00002

CAS ClassPad exercise
Statistics
MAT10SPCP00002

Technology worksheet
Excel worksheet: Finding the mean for grouped data
MAT10SPCT00001

The marks of some Year 10 students for an English essay were as follows.
10, 16, 12, 13, 10, 13, 10, 18, 11, 15, 9, 17, 7, 14, 11, 13, 10, 19, 11, 15, 8, 16, 12, 19, 10, 13, 9
Find the mean, median, mode and range for the essay marks.

Solution

Make a frequency table with score × frequency and cumulative frequency columns.

Score x	Frequency f	xf	Cumulative frequency
7	1	7	1
8	1	8	2
9	2	18	4
10	5	50	9
11	3	33	12
12	2	24	14
13	4	52	18
14	1	14	19
15	2	30	21
16	2	32	23
17	1	17	24
18	1	18	25
19	2	38	27
Totals	27	341	

10 has the highest frequency. **The mode is 10.**

There are 27 scores, so the median is the 14th score. There are 13 scores on either side of it. **The median is 12.**

Find the total.	$\Sigma xf = 341$
We have to find the mean of 27 scores.	**Mean,** $\bar{x} = \frac{\Sigma x}{\Sigma f} = \frac{341}{27} \approx 12.63$
The lowest mark is 7 and the highest is 19.	**Range** $= 19 - 7 = 12$

Strictly speaking, it is only possible to find quartiles exactly when the total frequency is a multiple of 4. As a result, there is some disagreement among statisticians about how quartiles should be calculated. Different methods may give slightly different values for the interquartile range.

Important!

Finding quartiles

To find the first quartile, find the median of the first half of the scores (*below* the median). For a frequency table, this is the $\frac{n+1}{4}$th score for n odd and the $\frac{n+2}{4}$th score for n even.

To find the third quartile, find the median of the second half of the scores (*above* the median). For a frequency table, this is the $\frac{3n+3}{4}$th score for n odd and the $\frac{3n+2}{4}$th score for n even.

Example 4

Video tutorial
Interquartile range
MAT10SPVT10003

Worksheet
Analysing data 1
MAT10SPWK00004

Find the median, quartiles and interquartile range for each of the following sets of scores.

a 13, 19, 14, 17, 14, 16, 12, 21, 15, 16, 15, 24, 14, 18, 14, 20, 14, 18, 13, 18, 14, 26, 15, 22

b 8, 17, 11, 17, 11, 15, 10, 12, 11, 13, 12, 12, 7, 14, 7, 17, 8, 20, 12

c

x	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
f	1	1	0	2	0	2	3	2	3	1	1	3	4	4	1	1	3	1	0	1

d

x	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
f	1	2	2	3	2	2	11	1	3	2	4	1	0	4	3

Solution

a Put the 24 scores in order and divide into two.	**12, 13, 13, 14, 14, 14, 14, 14, 14, 15, 15, 15 | 16, 16, 17, 18, 18, 18, 19, 20, 21, 22, 24, 26**
Find the median.	**Median,** $Q2 = \frac{15+16}{2} = 15.5$
Split the first half of the scores.	**12, 13, 13, 14, 14, 14 | 14, 14, 14, 15, 15, 15**
Find the first quartile.	**First quartile,** $Q1 = \frac{14+14}{2} = 14$
Split the second half of the scores.	**16, 16, 17, 18, 18, 18 | 19, 20, 21, 22, 24, 26**
Find the third quartile.	**Third quartile,** $Q3 = \frac{18+19}{2} = 18.5$
Find the interquartile range.	**IQR** $= Q3 - Q1 = 18.5 - 14 = 4.5$
Write the results.	**Median** $= 15.5$, $Q1 = 14$, $Q3 = 18.5$, **IQR** $= 4.5$

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b Put the 19 scores in order and divide in two.

7, 7, 8, 8, 10, 11, 11, 11, 12, 12, 12, 12, 13, 14, 15, 17, 17, 17, 20

Find the median.

Median, $Q2 = 12$

Divide the scores *before* the median in two.

7, 7, 8, 8, 10, 11, 11, 11, 12

Find the first quartile.

First quartile, $Q1 = 10$

Divide the scores *after* the median into two.

12, 12, 13, 14, 15, 17, 17, 17, 20

Find the third quartile.

Third quartile, $Q3 = 15$

Find the interquartile range.

IQR $= Q3 - Q1 = 15 - 10 = 5$

Write the results.

Median $= 12$, $Q1 = 10$, $Q3 = 15$, IQR $= 5$

c Rearrange the table so that the scores are arranged vertically and include a cumulative frequency column.

$\Sigma f = 34$, so there are 34 scores.

Score x	Frequency f	Cumulative frequency
11	1	1
12	1	2
13	0	2
14	2	4
15	0	4
16	2	6
17	3	9
18	2	11
19	3	14
20	1	15
21	1	16
22	3	19
23	4	23
24	4	27
25	1	28
26	1	29
27	3	32
28	1	33
29	0	33
30	1	34
Total	34	

The median is the $\frac{34+1}{2} = 17.5$th score.

The 17th and 18th scores are both 22, so median $= 22$.

34 is even, so use $\frac{n+2}{4}$ for $Q1$.

$Q1$ is the $\frac{34+2}{4} = 9$th score.

Find $Q1$.

The 9th score is the last 17, so $Q1 = 17$.

34 is even, so use $\frac{3n+2}{4}$ for $Q3$.

$Q3$ is the $\frac{3 \times 34 + 2}{4} = 26$th score.

Find $Q3$.

The 26th score is the third 24, so $Q3 = 24$.

Find the interquartile range. IQR $= Q3 - Q1 = 24 - 17 = 7$

Write the results. **Median $= 22$, $Q1 = 17$, $Q3 = 24$, IQR $= 7$**

d Rearrange the table so that the scores are arranged vertically and add a cumulative frequency column.

$\Sigma f = 41$, so there are 34 scores.

Score x	Frequency f	Cumulative frequency
9	1	1
10	2	3
11	2	5
12	3	8
13	2	10
14	2	12
15	11	23
16	1	24
17	3	27
18	2	29
19	4	33
20	1	34
21	0	34
22	4	38
23	3	41
Total	41	

The median is the $\frac{41+1}{2} = 21$st score. **The 21st score is the third last 15, so median $= 15$.**

41 is odd, so use $\frac{n+1}{4}$ for $Q1$. **$Q1$ is the $\frac{41+1}{4} = 10.5$th score.**

Find $Q1$. **The 10th score is the second 13 and the 11th score is the first 14, so**

$$Q1 = \frac{13+14}{2} = 13.5$$

41 is odd, so use $\frac{3n+3}{4}$ for $Q3$. **$Q3$ is the $\frac{3 \times 41 + 3}{4} = 31.5$th score.**

Find $Q3$. **The 31st and 32nd scores are the middle 19s, so $Q3 = \frac{19+19}{2} = 19$**

Find the interquartile range. **IQR $= Q3 - Q1 = 19 - 13.5 = 5.5$**

Write the results. **Median $= 15$, $Q1 = 13.5$, $Q3 = 19$, IQR $= 5.5$**

Examination of the minimum, first quartile, median, and maximum of a set of data can indicate important information about the distribution.

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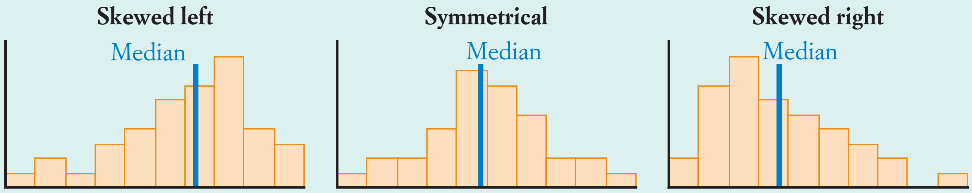
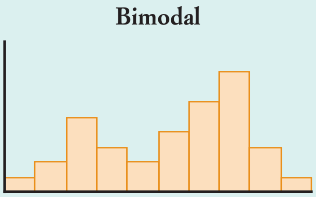
Important!

Five number summary and distribution shapes

The five numbers; the minimum, first quartile, median, third quartile and maximum ($Q0$, $Q1$, $Q2$, $Q3$, $Q4$) of a frequency distribution are collectively known as the **five number summary**.
A **symmetrical** distribution is roughly the same shape on either side of the middle.
A **skewed** distribution is spread out more on one side of the median than the other.
A distribution that is **skewed to the left** (a **negative skew**) is spread out more below the median than above the median.
If the scores are spread out more on the right, it is **skewed to the right** (a **positive skew**).



Some distributions have two high parts. These are called **bimodal** distributions, although they don't always have two actual modes.



You can use the five number summary to see if a distribution is likely to be skewed or symmetrical. A skewed distribution will have greater distances on one side of the median than the other.

Example 5

Technology worksheet
Excel worksheet: Five number summary
MAT10SPCT00002

Technology worksheet
Excel worksheet: Skewness
MAT10SPCT00005

a Find the five number summary for 30, 39, 30, 36, 34, 37, 25, 39, 21, 35, 33, 38, 33, 35, 34, 36, 30, 36, 35, 37, 32, 38, 33, 40, 35, 40, 22, 37, 33, 37.
b Comment on the five number summary for **a**.
c Find the five number summary for 7, 15, 10, 17, 7, 15, 5, 17, 10, 20, 8, 13, 7, 15, 6, 17, 10, 15, 8, 12, 6, 14, 12, 14, 12.
d Comment on the five number summary for **c**.

Solution

a Write the scores in order. — **21, 22, 25, 30, 30, 30, 32, 33, 33, 33, 33, 34, 34, 35, 35, 35, 35, 36, 36, 36, 37, 37, 37, 37, 38, 38, 39, 39, 40, 40**

Divide into 4 parts. — **21, 22, 25, 30, 30, 30, 32, 33, 33, 33, 33, 34, 34, 35, 35 | 35, 35, 36, 36, 36, 37, 37, 37, 37, 38, 38, 39, 39, 40, 40**

Write the five number summary. — **The five number summary is 21, 33, 35, 37, 40**

b Look at the position of the median. — **35 − 21 = 14 and 40 − 35 = 5, so the bottom half of the range extends farther than the top half.**

Comment on the distribution. — **The distribution is probably skewed to left.**

c Write the scores in order. — **5, 6, 6, 7, 7, 7, 8, 8, 10, 10, 10, 12, 12, 12, 13, 14, 14, 15, 15, 15, 15, 17, 17, 17, 20**

Divide into 4 parts. — **5, 6, 6, 7, 7, 7 | 8, 8, 10, 10, 10, 12, 12, 12, 13, 14, 14, 15, 15 | 15, 15, 17, 17, 17, 20**

Write the five number summary. — **The five number summary is 5, 7.5, 12, 15, 20**

d Look at the position of the median. — **20 − 12 = 8 and 12 − 5 = 7, so the distribution is fairly balanced either side of the median.**

Comment on the distribution. — **The distribution is probably symmetrical.**

Investigate: Use of technology

You can use digital technology to find the mean, median, mode, range, quartiles and interquartile range of a set of data. Scientific calculators can usually be used for at least the mean. Some will calculate all of these measures. Unfortunately, different technologies use quite different methods for data entry and retrieval of the information. They may also use different methods to calculate the quartiles. The following small sets of data can be used to work out which method your calculator uses to find the quartiles.
Use both data sets to find which method your calculator uses.

Set A: 4, 8, 12, 16, 20		Set B: 4, 8, 12, 16, 20, 24			*Q*1		*Q*3	
*Q*1	*Q*3	*Q*1	*Q*3	Method	*n* odd	*n* even	*n* odd	*n* even
6	18	7	21	1	$(n+1)/4$	$(n+1)/4$	$(3n+3)/4$	$(3n+3)/4$
8	16	8	20	2	$(n+3)/4$	$(n+2)/4$	$(3n+1)/4$	$(3n+2)/4$
6	18	8	20	3	$(n+1)/4$	$(n+2)/4$	$(3n+3)/4$	$(3n+2)/4$
4	16	8	20	4	$(n+1)/4$	$(n+1)/4$	$(3n+3)/4$	$(3n+3)/4$
8	16	9	19	5	$(n+3)/4$	$(n+3)/4$	$(3n+1)/4$	$(3n+1)/4$

In this book, method 3 is used to calculate quartiles.

For typical numerical data, the mean is used as the measure of the centre because it is calculated using all of the data. The range and interquartile range use only a few data points.
The mean and median do not make sense for nominal data, even if it is coded by numbers. While the mean uses all the scores, its value can be dramatically changed by one or two very large or small scores. The median is not changed much by large or small scores.
You should use the measure of central tendency that suits your purpose. The mode is the *most popular* score, the median has *half the values on either side* and the mean is based on the *total*. The median should be used for numeric data where a few very large or very small values may distort the value of the mean.

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Example 6

Which measure of central tendency should you use for incomes in each of the following cases?

a You want to know whether the people in a town can afford to build a public pool.

b You want to know whether many people in a town will be able to afford to buy paintings for their homes.

c You want to know what tax rate is paid by the most people in a town.

Solution

a In this case, it is the income of the whole town that is important.
The mean income should be used because it is about all the people.

b In this case, it is the income of a proportion of the people that is important.
The median income should be used to work out how many people will have spare money.

c The tax rate is determined by the income bracket.
The mode should be used to work out the most common income bracket.

It is important for you to know that people often think something with a number must be true. A media report that says a **survey** showed that 64% of Australians were opposed to the use of nuclear energy could be very significant. However, if the survey was badly done, or only a very small sample, it would not necessarily be true. The result by itself does not tell you whether you should take any notice of it. Larger samples are more reliable than smaller samples.

Example 7

A current affairs TV program interviewed robbery victims who were upset by the sentences the courts gave their attackers. They asked people to SMS two numbers to say Yes or No to mandatory sentences of 5 years for armed robbery. The next night they reported that 70% of the people who phoned in supported the proposal. Coincidentally, a phone survey of 2000 people spread across the state on the same subject by a polling company reported that only 38% supported mandatory sentencing, with an uncertainty of 4%. The TV host said their own survey had received 5320 SMSs, and they had made sure no one could ring twice. Explain the differences in the results and suggest which is most likely to be correct.

Solution

Only voters can directly influence laws.
The surveys should be directed to people of voting age, as their views determine elections and laws.

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Phone surveys are not as good as interviews or personally delivered forms.	**Neither method is particularly good as you do not really know who is answering a phone call, so you don't know if the person is an adult.**
Groups with a particular view may coordinate survey responses if they can.	**A ring-up TV survey is liable to be targeted by pressure groups making organised responses, so it is possible for the results to be very biased.**
Polling companies choose a representative sample.	**The group of people picked by the polling organisation is probably an unbiased group.**
The uncertainty is worked out from the size of the sample.	**The 4% uncertainty assumes that the group of people that answered are mostly those picked.**
	The program most likely biased viewer's opinions.
Write a conclusion.	**Only people who are very interested will ring up the TV station, so the polling survey is more likely to be correct.**

Exercise 2.1 Calculating and interpreting data measures

Understanding

Extra questions
Exercise 2.1
MAT10SPEQ00004

See Example 1

1 Find the mode and range for each of the following sets of data.

a Yellow, Yellow, Black, Green, White, Green, Blue, Black, White, Green, Yellow, Green, Yellow, White, Blue, Green, Blue, Orange, Blue, Red, Red, Yellow, Yellow, Red

b Germany, Germany, Greece, Greece, Germany, Greece, Holland, India, Australia, Italy, Australia, Vietnam, Vietnam, Australia, Australia, Germany, New Zealand, Australia, Holland

c Mountain, Desert, Riverine, Plains, Coastal, Forest, Coastal, Desert, Desert, Desert, Coastal, Desert, Desert, Forest, Riverine, Plains, Forest, Desert, Riverine, Mountain, Forest, Plains, Plains

d Oliver, Jack, Noah, Noah, Ethan, Joshua, Cooper, William, Ethan, William, William, Lachlan, Oliver, Cooper, Jack, Joshua, Lachlan, Thomas, Noah, Lachlan, Jack, Jack, Noah, Oliver, Jack, Cooper, Thomas, Lachlan

e Charlotte, Olivia, Isabella, Ruby, Olivia, Ruby, Chloe, Isabella, Olivia, Charlotte, Olivia, Ruby, Ruby, Chloe, Isabella, Olivia, Chloe, Charlotte

See Example 2

2 Find the median and mode for each of the following sets of ordinal data.

a 6, 5, 2, 6, 3, 4, 6, 4, 3, 4, 4, 3, 4, 2, 2, 3, 7, 2, 4, 2, 4, 7, 4

b B, C, C, C, E, C, C, E, E, E, A, C, D, B, A, C, C, C, D, D, C, C, D, C, E

c N, A, A, SA, D, N, A, D, N, N, A, D, A, A, SD, A, A, SD, N, A, A, A, A, A, A, A, A, N, N, A, SA, N, N, N, N, A, D, N, A, A

d **, **, ****, ****+, ****, ****, ****, *, ***, ***, ***+, ****, **, ***+, ***+, ***, ***+, *, *, ****+, **, ***, ***, **, ***+, ***, ***+, ****+, ***, **, **, ***, *, ***, **

e PG, PG, M, M, X, PG, MA15+, X, PG, M, M, MA15+, M, G, M, PG, X, PG, M, PG, G, MA15+, MA15+, X, PG, PG, PG, G, M, M

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3 Find the mean (to one decimal place), median, mode and range for each of the following sets of data. *See* Example 3

a 7, 3, 2, 1, 9, 10, 3, 10, 11

b 8, 11, 8, 16, 7, 19, 9, 13, 7, 14, 8, 10

c

Score	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Frequency	1	3	3	2	1	4	5	6	4	3	3	2	2	2	1	0	0	2

d

Score	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
Frequency	6	3	6	5	6	13	12	5	5	1	8	2	4	4	2	1	1	3	1	1

e

Score	12	13	14	15	16	17	18	19
Frequency	4	19	18	10	6	6	3	4

4 Find the median, quartiles and interquartile range for each of the following sets of data. *See* Example 4

a 26, 28, 22, 30, 23, 27, 23, 28, 22, 27, 19, 27, 26, 28, 21, 28, 21, 27, 25, 27, 24

b 4, 6, 2, 7, 5, 7, 6, 10, 2, 7, 3, 6, 4, 7, 4, 8, 5, 10, 5, 8, 6, 6, 2, 7, 3

c 30, 37, 33, 35, 30, 37, 32, 41, 31, 38, 33, 35, 34, 35, 34, 35, 33, 35, 32, 35

d 14, 15, 14, 16, 14, 18, 14, 17, 15, 15, 14, 18, 14, 15, 13, 16, 12, 18, 14, 16, 15, 15, 12, 17, 12, 16, 15, 16

e 41, 48, 41, 46, 43, 46, 44, 47, 30, 45, 40, 51, 43, 45, 37, 46, 39, 48, 41, 46, 43, 48, 41, 49, 41, 48

5 Find the median, quartiles and interquartile range for each of the following sets of data.

a

Score	2	3	4	5	6	7	8	9	10
Frequency	2	6	6	5	5	15	13	5	2

b

Score	15	16	17	18	19	20	21	22	23	24	25
Frequency	1	1	0	3	3	9	7	17	19	3	1

c

Score	3	4	5	6	7	8	9
Frequency	1	8	12	17	12	2	2

d

Score	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Frequency	2	3	3	3	1	7	6	3	7	8	2	6	7	6	5	2	2

e

Score	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Frequency	2	5	2	8	7	12	3	3	2	5	4	2	3	3	3	3	1	2	2	0	1

Fluency

6 Find the five number summary for each of the following distributions. *See* Example 5

a 30, 31, 30, 32, 28, 31, 29, 32, 27, 33, 27, 31, 30, 41, 29, 36, 26

b 12, 12, 9, 18, 6, 15, 4, 13, 6, 16, 6, 16, 2, 15, 5, 12, 3, 15, 6, 15, 11, 15, 11, 20, 4, 14, 9

c 15, 20, 14, 24, 13, 35, 11, 21, 14, 24, 11, 21, 15, 16, 14, 23, 7, 17, 14, 16, 11, 22

d 8, 14, 11, 14, 9, 16, 8, 14, 9, 14, 9, 13, 13, 16, 11, 14, 8, 13, 9, 15, 1

e 55, 60, 37, 60, 50, 61, 51, 73, 55, 62, 50, 60, 57, 58, 54, 68, 58, 65, 57, 59, 52, 61, 52, 59, 57

7 Find the five number summary for each of the following distributions.

a

Score	30	31	32	33	34	35	36	37	38	39
Frequency	3	1	2	7	11	7	4	2	3	1

b

Score	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Frequency	1	0	7	8	9	7	7	5	0	4	3	1	2	1

c

Score	8	9	10	11	12	13	14	15	16	17	18
Frequency	2	5	8	7	4	4	8	4	3	4	4
Score	19	20	21	22	23	24	25	26	27	28	29
Frequency	2	0	0	1	1	0	1	0	0	1	1

d

Score	101	102	103	104	105	106	107	108	109
Frequency	3	3	7	10	7	3	4	2	1

e

Score	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Frequency	1	1	1	3	9	6	6	11	5	5	4	5	3	1

See Example 6

8 Which data measure(s) should be used in each of the following cases?
- a You want to know what you are likely to earn after a few years working in a local furniture factory.
- b A caterer is going to set up a sandwich bar in a local industrial estate. In their first week, what size iced coffee should they stock in their drinks fridge?
- c A security company is doing weekly payrolls in cash on-site for an orchardist to pay his casual pickers, who earn a variety of amounts.
- d A band is ordering tour t-shirts to sell at their concerts. They only want to bother with one size.
- e How much can you expect families to spend at a theme park?

9 Five friends were playing Yahtzee. At the end of the game Peter came first with a score of 233, Simon came second with 186, Paul and Andrew came equal third with 165 and David came last with 108. Find the ranges of the names, places and scores.

Problem solving

10 The table shows the mass of fruit from each plant in a new variety of tomatoes.

Mass of fruit (kg)	3	4	5	6	7	8	9	10	11	12
Frequency	1	9	11	8	5	7	4	1	3	1

- a What is the typical weight of fruit?
- b How much fruit would you expect from 5000 plants?
- c Is the sample sufficient to make good conclusions?

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11 The rally lengths of tennis games in the state titles were as follows. Find the mean, median, mode and range of the rally lengths.

Worked solutions
Exercise 2.1
MAT10SPWS00004

Number of shots played	1	2	3	4	5	6	7	8	9
Frequency	80	63	95	42	47	39	24	13	4

a An average game has 6 rallies. How many rally shots would you expect in 5 games?

b An average set has 20 games. How many rally shots would there be in an typical set?

c Is this sample sufficient to make good conclusions?

12 A real estate company is doing a report for its shareholders, predicting the year ahead. What measure of house prices should they use to work out each of the following?

a Earnings from commissions on sales.

b Typical prices of houses they will offer for sale.

c Amounts earned by their salespeople, who work on commission.

d Amount earned from their ‘prestige properties’ division.

e Amount earned by their ‘family specialist’ division.

13 The mean, median and mode of the incomes of the 97 employees, including the management, at a large warehouse-type hardware store were \$1240, \$945 and \$830. The lowest wage was \$730.

a What would be the effect of a 10% increase in everyone’s wages on the four amounts above?

b What would be the effect of a \$100 increase?

c Estimate the five number summary for the original wages.

Reasoning

14 The information below shows the ratings of accommodation in the Brisbane CBD. Explain the typical rating.

15 Explain the positions of the median, mean and mode when a distribution is skewed to the right.

16 20 people have a median height of 170 cm and a mean height of 169.4 cm. They are joined by a basketball team of 6 players with an average height of 180 cm. Explain what happens to the mean and median.

Worked solutions
Exercise 2.1
MAT10SPWS00004

17 Comment on the five number summaries for each of the following distributions.

a 6, 16, 7, 16, 6, 13, 5, 17, 6, 13, 9, 13, 8, 10, 9, 10, 6, 20, 7, 12, 7, 15, 8, 17

b 164, 172, 163, 179, 163, 170, 163, 183, 164, 179, 163, 165, 163, 167, 164, 171, 164, 168, 162, 178

c 8, 13, 3, 13, 10, 10, 0, 16, 0, 11, 10, 14, 8, 13, 8, 11, 9, 11, 10, 11, 7, 13, 8, 19

d 39, 50, 36, 48, 41, 47, 37, 51, 41, 45, 45, 47, 38, 50, 43, 47, 43, 47, 40, 49, 44, 46, 35, 49, 44, 48, 35, 46, 44, 47, 45, 51

e 14, 15, 15, 16, 15, 18, 14, 20, 13, 18, 13, 17, 15, 19, 13, 19, 15, 17

See Example 7

18 The 'Top 40' hits from different radio stations can be different. One radio station compiles its 'Top 40' hits from a survey of the sales of single CDs in a music store in the city that regularly sells about 3000 singles each week. A chain of stores in suburban shopping centres sells about the same number of singles, but gets a different 'Top 40' that a suburban radio station uses. Explain how the stations can get different 'Top 40's in the same city.

Worked solutions
Exercise 2.1
MAT10SPWS00004

19 Polling companies try to predict the way that voters will react at an election. Different polls commonly get different results, and frequently the results they get are different to actual election results. There are 150 electorates in Australia of almost equal size. One polling company conducts a poll of 3000 people by phone. The people are chosen at random from the electoral rolls, spread evenly through Australia. Another does a poll of 100 electorates in urban areas, also using 3000 people. The polls are conducted three days before the election, and they both predict a victory (by 20 seats and 10 seats respectively) for the party that is actually defeated by 15 seats at the election. Explain how these properly conducted surveys by reputable organisations can get the result completely wrong.

20 The Australian Bureau of Statistics regularly checks the earnings of Australians. It has found that when it uses information from employers or employees it gets different results, even when it surveys employees of the employers that it surveys. Explain how this could be so.

2.2 Constructing and interpreting data displays

Statistical data displays are used for different reasons. Some are best suited to communicating the data, while others are more suitable for comparison or analysis.

Weblink
Visualising how a population reaches 7 billion
MAT10SPWB00002

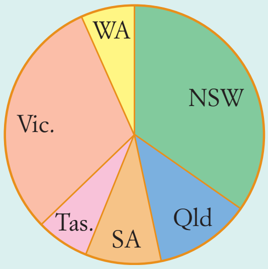
TLF Learning object
Graph investigator (L5903)
MAT10SPIN00002

Important!

Types of data displays

Circle graphs (pie graphs or **sector graphs), picture graphs (pictograms** or **pictographs)** and **divided bar charts** give you an excellent impression of overall **relationships** between different aspects of statistical information. In a circle graph, the angles of the sectors show the parts of a total amount. In a divided bar graph, the lengths of the sections show the parts of the total amount.

House of Representatives, 1901
(Total = 75 seats)



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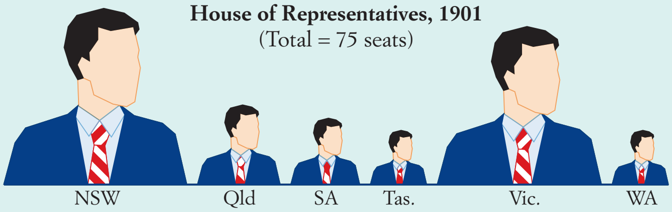
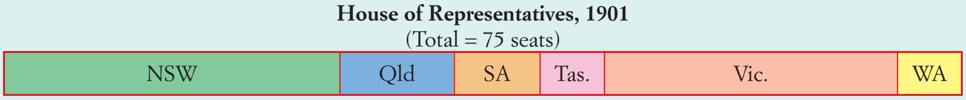
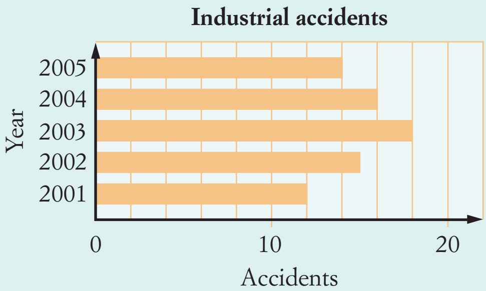
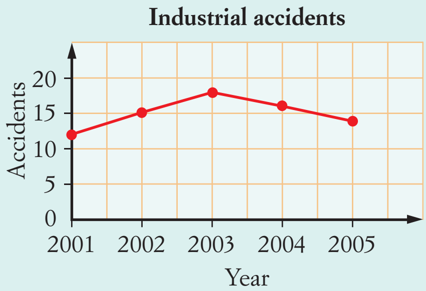


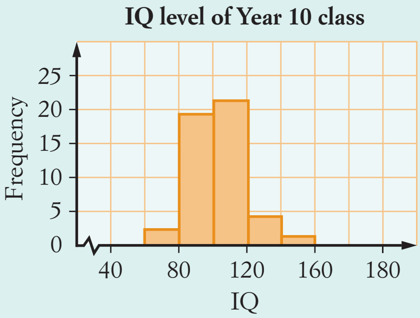


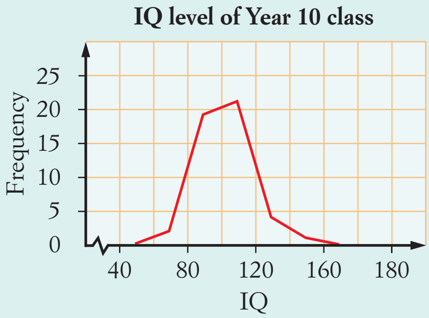
You can read **bar charts**, **column graph** and **line graphs** quite accurately, so they are useful for making predictions or finding precise relationships.





Histograms and **polygons** are used to show frequencies of grouped data. Strictly speaking, a histogram is a column graph of continuous data, so if the data is discrete it should be called a column graph. There is no gap between the columns because the data is continuous. The score groups (such 25–29, 30–34, etc.) are called **classes.** The actual upper and lower boundaries of a class are called the **upper class limit** and **lower class limit.** You put the points of **frequency polygons** in the centres of class groups. It is usual to show frequencies of zero before the first class and after the last class. Some people also call a line graph of discrete data frequencies a frequency polygon.





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TLF Learning object

Graph investigator: Homework hours (L5909)

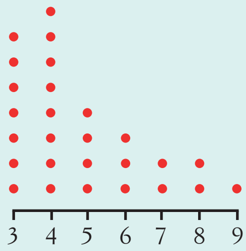
MAT10SPIN00003

TLF Learning object

Graph investigator: Reaction times (L10338)

MAT10SPIN00003

You use **dot plots** and **stem-and-leaf plots** to get a quick idea of the nature of a distribution. You can make them while you are still collecting data and add new data to them as you go.



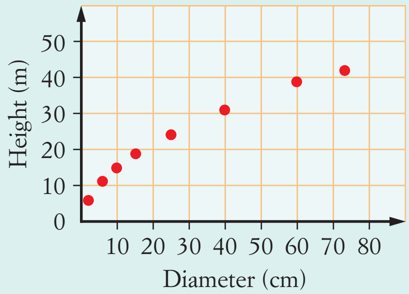
Golden-oldies concert audience ages

Key: 5 | 9 = 59 years old

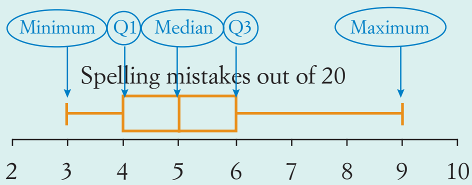
2	5
3	2 9
4	3 6
5	4 5 9 9
6	0 2 4 7 7 8 9
7	3 6 7
8	1

A **scatter plot (scatter graph)** is used to show bivariate data, particularly where the variables could be related.
For convenience, the **scales** of a graph or chart usually go up in 1s, 2s, 4s, 5s, 10s and so on.

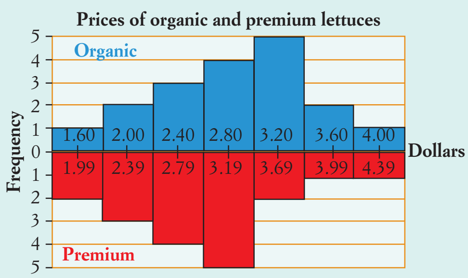
Tree trunk dimensions



A **box-and-whisker plot** (**boxplot**) is used to show the spread of data. You draw a rectangle from the lower quartile to the upper quartile, divided at the median. Then rule lines from the quartiles to the maximum and minimum. You can make these clearer with short vertical lines. Put the scale underneath.



Back-to-back or **side-by-side** (**parallel**) data displays are used to compare different distributions.



Reaction times (Key: 6|2 = 0.26 s = 2|6)

Left hand		Right hand
9 9 6 6	2	3 6 6 7 8 9 9 9 9
9 8 7 7 6 5 5 4 4 4 2 2 1 1	3	1 1 2 2 2 2 5 5 6 6 7 9
8 8 5 5 3 3 2 2	4	0 0 4
7	5	4 4
	6	6

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Example 8

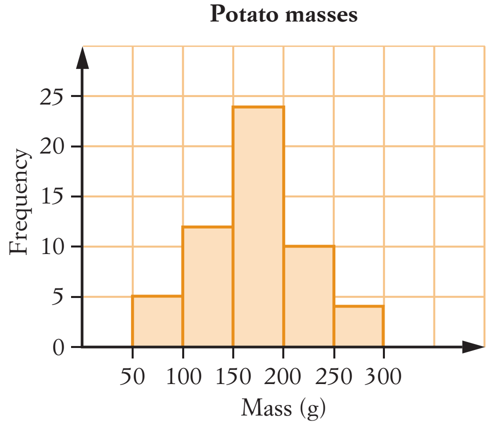
a Draw a histogram for the following table of the masses of potatoes.

Mass (g)	50–99	100–149	150–199	200–249	250–299
Frequency	5	12	24	10	4

b Comment on the shape of the distribution.

Solution

a When you draw a histogram, you should use the true class limits. In this case, since the masses are obviously rounded to the nearest gram, the true class limits are 49.5, 99.5, 149.5, 199.5, 249.5 and 299.5 grams. Draw the histogram, making sure it is clear that the class limits are just before 50, 100, 150, 200, 250 and 300.



b The distribution is spread evenly on either side of the centre.

The distribution is symmetrical.

Worksheet
Analysing data 2
MAT10SPWK00005

Puzzle sheet
Statistics: Love and truth
MAT10SPPS00005

Example 9

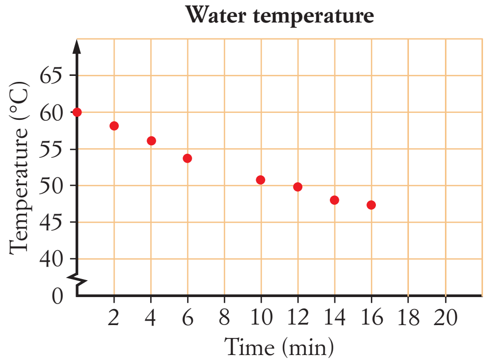
The table below shows the temperature of a beaker of hot water as measured at 2-minute intervals by students working in groups.

Time (min)	0	2	4	6	8	10	12	14	16
Temperature (°C)	60	58	56	54	45	51	50	48	47

a Draw a scatter graph of the results.
b Does the temperature fall evenly?

Solution

a The temperature at the eighth minute has clearly been misread. Discard it as non-compliant and draw the graph.



b Overall, it looks as if the temperature falls about $1\frac{1}{2}$°C every 2 minutes.

The temperature appears to fall evenly over the time it was measured.

CAS TI-Nspire exercise
Statistics
MAT10SPTI00002

CAS ClassPad exercise
Statistics
MAT10SPCP00002

Worksheet
Scatter plots
MAT10SPWK00002

Video tutorial
Box-and-whisker plots
MAT10SPVT10004

Technology worksheet
Excel worksheet: Five number summary
MAT10SPCT00002

Example 10

a Draw a box and whisker plot of the following data.
26, 27, 23, 27, 24, 27, 23, 28, 24, 27, 22, 27, 26, 27, 25, 28, 26, 26, 26, 26

b Comment on the distribution.

Solution

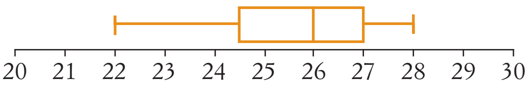
a Write the data in order and divide into 4 parts.

22, 23, 23, 24, 24 | 25, 26, 26, 26, 26 | 26, 26, 27, 27, 27 | 27, 27, 27, 28, 28

Write the 5 number summary.

The 5 number summary is 22, 24.5, 26, 27, 28

Draw the boxplot.



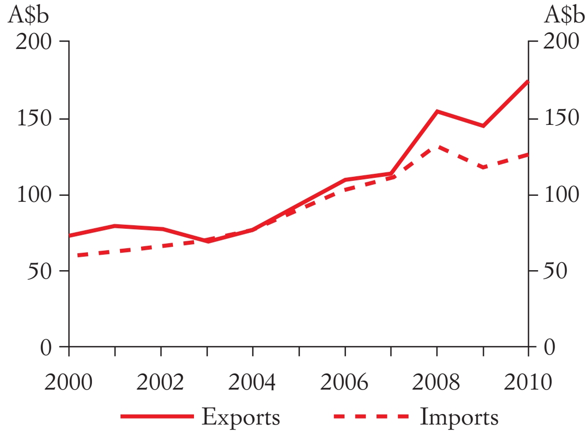
b $Q1$ is farther from the median than $Q3$, and the minimum is farther from the median than the maximum.

The distribution is negatively skewed.

Investigate: Trade with Asia

The following graph appeared in the report *Australia's trade with East Asia*, published by the Department of foreign Affairs and Trade.

Australia's trade in goods and services with East Asia



Based on ABS trade data on DFAT STARS database and ABS catalogue 5368.0.

In the report it says that:

- The value of total trade in goods and services increased 14.9 per cent to $302.9 billion in 2010 – since 2005, total trade values have increased by an average of 10.2 per cent per annum.
- In comparison, Australia's total trade with the world rose 9.6 per cent to $552.4 billion.
- East Asia accounted for 54.8 per cent of Australia's total trade in goods and services.

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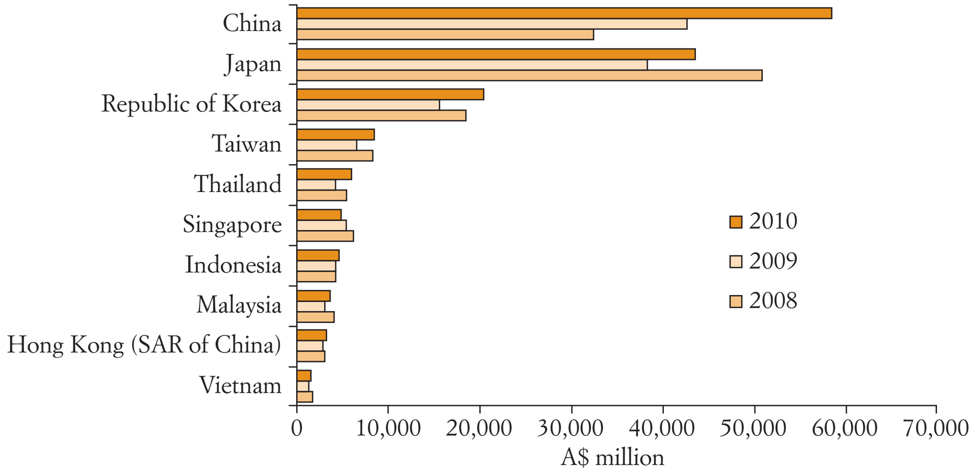
The whole report is available at www.dfat.gov.au. Use the internet to look up the report and investigate the basis of the statistics that are quoted in the report. You may also need to investigate the statistics that relate to the report compiled by the ABS.

The graph below of Merchandise Exports also appears in the report.



What does it show about changes of Australian exports to Asia?

In both the DFAT and ABS statistics, some data that has been collected is not available. It says that it is 'commercial in confidence' or just 'confidential in ABS trade statistics'. What does this mean, and why is some data withheld? Try to find the value of Australian wheat and gold exports during the last 20 years.

Example 11

Technology: GeoGebra: Boxplot and dot plot

MAT10SPTC00002

Compare a boxplot to the dot plot and histogram of this data.
5, 9, 6, 10, 5, 6, 3, 6, 5, 10, 3, 9, 5, 8, 5, 13, 5, 12, 6, 13, 6, 14, 5, 8, 5, 13, 5

Solution

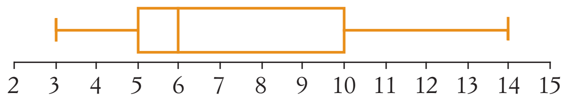
Arrange in order and divide into 4 groups.

3, 3, 5, 5, 5, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 8, 8, 9, 9, 10, 10, 12, 13, 13, 13, 14

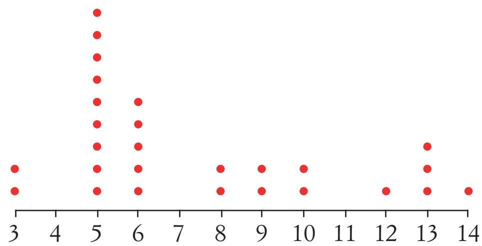
Write the 5 number summary.

The 5 number summary is 3, 5, 6, 10, 14

Draw a boxplot.



Draw a dot plot.



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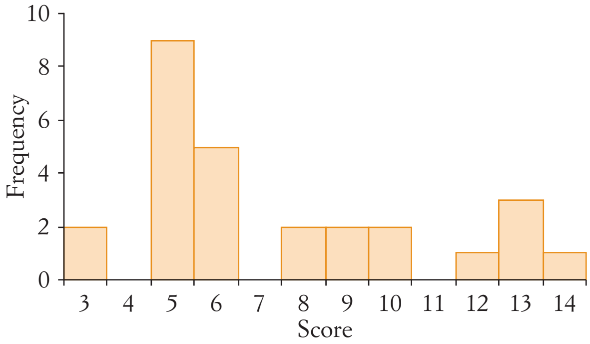
Draw a histogram.



Compare the shapes.

The dot plot and histogram spread out to the same extent as the boxplot and have their highest parts between the first quartile and the median, where the box-and-whisker plot has its shortest section.

Back-to-back or parallel (side-by-side) plots are used when

- a **population** is divided naturally into two parts, such as male and female
- there are different populations
- information is obtained at different times about the same population.

Example 12

a Make a back-to-back stem-and-leaf plot of the following information about the Maths and Science marks of a Year 10 class.
Maths: 50, 70, 46, 71, 37, 72, 40, 70, 53, 72, 45, 56, 55, 55, 35, 61, 34, 67, 39, 69, 22, 84, 52, 57
Science: 41, 63, 55, 59, 42, 79, 21, 74, 44, 77, 48, 92, 46, 81, 54, 56, 24, 66, 28, 86, 53, 71, 39, 83

b What can you say about the marks in Maths and Science?

c Comment on the means, medians and ranges of the distributions.

Solution

a The lowest mark is 21 and the highest is 92, so make the stems go from 2 to 9. The units will be the 'leaves'. Put the Maths marks on the left and the Science marks on the right.

Maths and Science marks

Maths Key: 6|5 = 56		Science Key: 5|6 = 56
2	2	1 4 8
9 7 5 4	3	9
6 5 0	4	1 2 3 6 8
7 6 5 5 3 2 0	5	3 4 5 6 9
9 7 1	6	3 6
2 2 1 0 0	7	1 4 7 9
4	8	1 3 6
	9	2

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b The centres, spreads and shapes of the plots are about the same. — **The class' performance in Maths was about the same as in Science, although there were more high scores in Science.**

c Find the total of the Maths results. — **Total = 1312**

Find the mean of the Maths results. — **Maths mean** $= \frac{1312}{24} \approx 54.7$

Use the plot to find the median. — **Maths median** $= \frac{55 + 55}{2} = 55$

Find the range. — **Maths range** $= 84 - 22 = 62$

Find the total of the Science results. — **Total = 1382**

Find the mean of the Science results. — **Science mean** $= \frac{1382}{24} \approx 57.6$

Use the plot to find the median. — **Science median** $= \frac{55 + 56}{2} = 55.5$

Find the range. — **Science range** $= 92 - 21 = 71$

Compare the results. — **The medians for Maths and Science were almost the same, but the Science average was a little higher and the Science range was greater. Looking more closely at the distributions, $Q3$ was higher for Science, which explains the higher mean.**

Exercise 2.2 Constructing and interpreting data displays

Understanding

Extra questions
Exercise 2.2
MAT10SPEQ00005

See Example 9

1 Make a stem-and-leaf plot of the following times for drivers to stop at a red light.

4.2 3.0 1.9 3.7 4.6 2.6 3.5 3.8 4.1 3.7 3.4 3.1 2.7 0.2 2.1
1.8 4.3 3.4 3.6 2.2 3.1 5.0 2.6 24 1.9 2.6 2.7 2.8 3.4 2.3

2 Draw a stemplot of the following marks that students said they received for a university entrance exam.

56 40 38 40 72 92 63 88 69 78 66 87 60 72 64
44 60 320 35 36 52 58 46 84 75 57 69 63 46 51
95 66 84 42 54 56 37 62 50 46 50 91 88

3 Draw scatter plots for each of the following tables.

a

x	7	22	15	12	25	8	18	5	10
y	10	36	18	20	42	12	28	2	12

b

x	18	27	10	12	15	22	30	5	8	20	24
y	3.2	3.8	17	12.8	8	6.8	5	17	15.2	5	2.6

c

x	25	7	15	25	22	18	5	9	12	13	6
y	62	13	27	57	53	46	7	19	18	21	5

See Example 10

4 Draw box-and-whisker plots for each of the sets of data.

a 26, 27, 25, 28, 25, 28, 24, 28, 24, 27, 24, 30, 25, 28, 26, 28, 26, 27, 23, 28, 24, 30, 27, 30

b 5, 7, 5, 8, 5, 9, 5, 9, 7, 8, 4, 8, 6, 8, 4, 9, 7, 10, 4, 8, 7, 7, 2, 9, 6, 9, 6

c 23, 77, 57, 86, 53, 78, 36, 67, 20, 79, 34, 61, 11, 80, 57, 67, 35, 74, 49, 97, 43, 65, 25, 62, 30

Fluency

See Example 8

5 The data below shows the energy values of foods eaten by a large sample of Australians.

Energy (kJ)	0–49	50–99	100–149	150–199	200–249	250–299	300–349
Frequency	7	15	20	23	14	8	3

a Draw a histogram of the results.

b Comment on the shape of the distribution.

6 The data below shows the prices of commonly available computers, including secondhand machines, notebooks and laptops.

Price ($)	0–499	500–999	1000–1499	1500–1999
Frequency	6	28	41	45
Price ($)	2000–2499	2500–2999	3000–3499	3500–3999
Frequency	48	40	20	7

a Draw a histogram.

b Comment on the shape of the distribution.

See Example 11

7 Compare box-and-whisker plots of each of the following sets of data to histograms and dot plots of the same data.

a 14, 19, 13, 18, 14, 15, 12, 16, 14, 15, 15, 21, 10, 19, 15, 19, 11, 17, 13, 16, 11, 18, 11, 17, 14, 15, 13, 21, 15, 15, 15, 17, 14, 16, 10, 17

b 100, 102, 98, 104, 99, 102, 101, 103, 99, 104, 101, 104, 100, 103, 98, 102, 101, 103, 99, 102, 102, 102, 100, 102, 96, 104, 102, 103, 100

c 21, 28, 22, 24, 21, 26, 21, 24, 22, 26, 22, 28, 23, 28, 21, 23, 23, 25, 22, 23, 20, 24, 22

Problem solving

8 The data below shows the numbers of customers and total sales in a music store over a period of 2 weeks.

Day	M	T	W	Th	F	S
Customers	41	52	53	59	70	63
Sales ($)	1705	2620	2525	3005	3150	2555
Day	M	T	W	Th	F	S
Customers	38	48	56	62	69	65
Sales ($)	1990	1880	2240	3210	2755	2435

a Draw a column graph of the sales over the two weeks.

b Can you make any conclusion from the graph?

c Draw a scatter graph of the customers and sales.

d What can you conclude from the scatter graph?

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9 The table below shows the minimum and maximum temperatures in a Queensland town over a period of 2 weeks.

Worked solutions
Exercise 2.2
MAT10SPWS00005

Min (°C)	16	18	20	22	18	19	20	17	15	18	22	23	19	17
Max (°C)	25	28	34	34	27	31	31	28	27	27	36	34	29	26

a Draw a scatter plot to show the results.

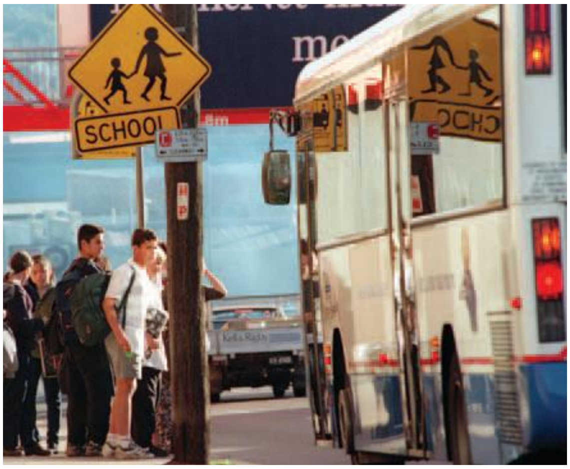
b Does there seem to be a relationship between the minimum and the maximum?

10 The information below shows the times taken to travel to school reported by some Year 10 students in Victoria and Western Australia.

See Example 12

a Use a back-to-back stem-and-leaf plot to compare the results.

b Compare the distributions using the means, medians and ranges.



Victorian students: 15, 18, 50, 40, 1, 15, 10, 22, 10, 5, 35, 10, 45, 20, 15, 5, 15, 25, 10, 10, 9, 25, 10, 37, 4, 10, 20, 20

Western Australian students: 30, 10, 15, 20, 20, 10, 3, 5, 10, 20, 2, 10, 10, 3, 36, 50, 20, 10, 5, 15

11 The information below shows the 2011 weekly running costs of a range of small and medium cars from the RAC WA, rounded to the nearest dollar.

a Use a back-to-back stemplot (with costs in \$5 groups) to compare them.

b Use the means, medians and ranges to compare the distributions.

Small cars \$cost/week: 42, 49, 48, 51, 47, 49, 46, 42, 56, 51, 39, 49, 61, 44, 32, 42, 41

Medium cars \$cost/week: 52, 51, 60, 54, 57, 60, 40, 60, 52, 45, 55, 56, 62

12 Use a box-and-whisker plot to comment on the following times taken by some Year 10 students to answer an online questionnaire. The times were automatically recorded by the website.

13, 31, 17, 14, 22, 22, 28, 20, 8, 31, 12, 11, 15, 21, 10, 20, 16, 19, 8, 18, 21, 16, 22, 16, 18, 26, 11, 26, 18, 10, 23, 17, 6, 12, 24, 19, 16, 19, 27

13 Use a box-and-whisker plot to comment on the following times that people said they spend watching TV over a week.

4, 14, 5, 13, 6, 22, 5, 8, 22, 3, 3, 3, 10, 1, 0, 8, 8, 10, 15, 15, 2, 1, 5, 5, 11, 1, 6, 15, 5, 10, 2, 1, 1, 2, 7, 3, 4, 4, 3, 18, 21, 1, 8, 2, 10, 3, 2, 2, 8, 8, 15, 23, 4, 2, 2, 20, 7, 5, 21

Worked solutions
Exercise 2.2
MAT10SPWS00005

Reasoning

14 Many websites automatically record data. The online sales recorded each day by such an Australian hobby-shop site was displayed as a box-and-whisker plot each week for the online manager. Explain whether you think it would provide good feedback.

Worked solutions
Exercise 2.2
MAT10SPWS00005

15 A customer service toll-free line had some automated and some operator parts. One of the automated parts allowed people to state whether they were happy with the service provided. Customers were asked to 'stay online' after their query was answered to complete this survey question. They could say they were Very happy, Happy, Satisfied, Not satisfied or Very unhappy. Explain whether a boxplot of the responses on a scale from 1 to 5 would be a good way to show the results.

2.3 Analysing data

The first step in analysing data is really to determine whether the data has been collected in a reasonable way. If the data is poor, then any conclusions or decisions made from it will also be of poor quality.
You should remember the terms below from your work in previous years.

Important!

Data quality

A **fair sample** is collected from a sample that is like the rest of the population. A **biased sample** is not typical of the population. **Biased methods** and **biased questions** are likely to influence results in a particular direction. **Fair** ones are not likely to influence results in a particular way. Fair questions with fair samples will not give reliable results if the sample is too small. A clear pattern in results of a good survey does suggest that a sample is sufficient. However, conclusions drawn from surveys where frequencies are less than 5 should be regarded with suspicion.
Non-compliant data is obviously wrong, like someone's age being 164.
Outliers are data items that are a long way from the rest of the data, but that could be correct. A simple criterion for an outlier is that it lies beyond 1.5 IQRs below the first quartile or above the third quartile.

If data is collected in a biased way, contains many non-compliant items or lots of outliers, or is collected using very small samples, then it is likely to be of poor quality.

Example 13

A survey of shoppers about their holiday plans is conducted as a competition. Survey forms are left in fast food outlets and prizes go to the first 3 names drawn. Respondents are asked questions about their income, family, desired holiday destinations and the kinds of holidays they prefer. They complete the forms by filling in their name, phone number, email and address and put them in boxes at the shops. Explain why this is likely to give poor quality data.

Solution

Many people will not bother doing it.	**There will be a non-response bias–only people interested in the prizes will do the survey.**
People are often reluctant to give their true income, or may not even know it.	**Responses are likely to be inaccurate.**

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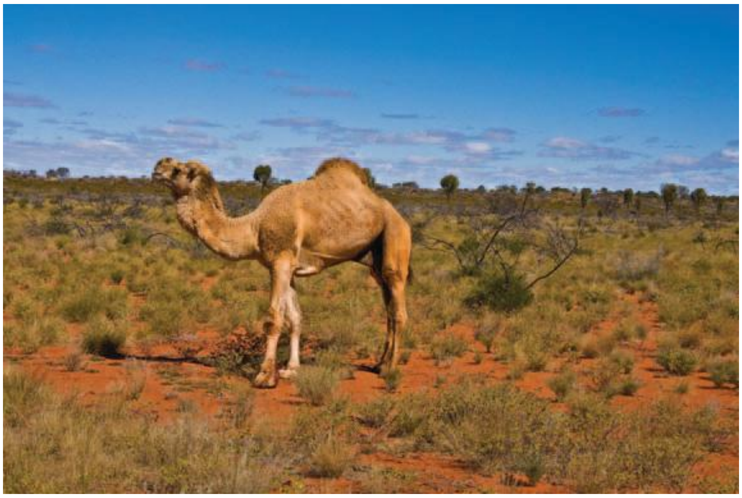
People are also reluctant to give their contact details.	**Responses are unlikely to be able to be checked.**
People who really want the prizes may complete multiple forms.	**Multiple entries will cause bias.**
Only people who buy fast food will be in the shops.	**The survey will be biased towards those who often buy fast food.**
Summarise the problems.	**The survey answers are likely to be inaccurate, and the sample is quite biased in a number of different ways.**

Investigate: Unintended consequences

Research using statistics can be used to predict what will happen in the future. This can lead to actions to change what is anticipated, or to avoid particular results. Unfortunately, the outcomes of actions are not always what we mean them to be. For example, camels were imported into Australia to carry materials and supplies for work on the railway line to Alice Springs. When they were no longer needed, they were released and have since become a problem in central Australia.

Work in groups to consider what the unintended effects of each of the following actions were or might be.



- Keeping cats as pets in Australia
- Importation of *Bufo marinus* (cane toad) from South America to Queensland for the control of cane beetles
- The one-child policy in China
- Planting of eucalyptus trees for shade in California and Italy
- Closure of phosphate mines in Nauru
- Use of fluorocarbons and chlorofluorocarbons in refrigerators
- Prohibition of the use of alcohol in the United States between 1919 and 1933

Example 14

Use the interquartile range to determine if any of the following sets of data have outliers.

a 25, 33, 30, 30, 29, 31, 26, 30, 24, 32, 22, 17, 28, 30, 30

b 31, 91, 55, 59, 52, 82, 43, 62, 30, 57, 24, 96, 29, 70, 34, 73, 8, 80, 45, 72, 53, 59, 39, 75, 29

c 11, 12, 9, 12, 11, 13, 12, 14, 11, 14, 7, 13, 9, 23, 9, 18, 9, 15, 12, 16

Solution

a Arrange in order.
17, 22, 24, 25, 26, 28, 29, 30, 30, 30, 30, 30, 31, 32, 33

Divide into 4 parts.
17, 22, 24, 25, 26, 28, 29, 30, 30, 30, 30, 30, 31, 32, 33

Find $Q1$, $Q3$ and the IQR.
$Q1 = 25$, $Q3 = 30$, IQR $= 5$

Find the outlier limits.
Lower limit for outliers $= 25 - 1.5 \times 5 = 18.5$
Upper limit for outliers $= 30 + 1.5 \times 5 = 37.5$

State the result.
17 is an outlier because it is more than 1.5 times the IQR below $Q1$.

b Arrange in order.
8, 24, 29, 29, 30, 31, 34, 39, 43, 45, 52, 53, 55, 57, 59, 59, 62, 70, 72, 73, 75, 80, 82, 91, 96

Divide into 4 parts.
8, 24, 29, 29, 30, 31 | 34, 39, 43, 45, 52, 53, 55, 57, 59, 59, 62, 70, 72 | 73, 75, 80, 82, 91, 96

Find $Q1$, $Q3$ and the IQR.
$Q1 = 32.5$, $Q3 = 72.5$, IQR $= 40$

Find the outlier limits.
Lower limit for outliers $= 32.5 - 1.5 \times 40 < 0$
Upper limit for outliers $= 32.5 + 1.5 \times 40 = 112.5$

State the result.
There are no outliers.

c Arrange in order
7, 9, 9, 9, 9, 11, 11, 11, 12, 12, 12, 12, 13, 13, 14, 14, 15, 16, 18, 23

Divide into 4 parts.
7, 9, 9, 9, 9 | 11, 11, 11, 12, 12 | 12, 12, 13, 13, 14 | 14, 15, 16, 18, 23

Find $Q1$, $Q3$ and the IQR.
$Q1 = 10$, $Q3 = 14$, IQR $= 4$

Find the outlier limits.
Lower limit for outliers $= 10 - 1.5 \times 4 = 4$
Upper limit for outliers $= 14 + 1.5 \times 4 = 20$

State the result.
23 is an outlier because it is more than 1.5 times the IQR above $Q3$.

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Bivariate data has two values for each item. For example, collecting the ages and heights of children together makes a bivariate data set. Each child has two values that are tied together. Bivariate data can have both values categorical, one categorical and one numeric, or both numeric data. When one or both values are categorical, a **two-way table** is often the best way to analyse the data.

Example 15

Worksheet
Analysing data 2
MAT10SPWK00005

Worksheet
Statistics
MAT10SPWK00003

Puzzle sheet
Statistical tables
MAT10SPPS00006

The table below shows the results of a survey of Year 10 students about smoking and colds. Some information is missing.

		Number of colds last year					
		0	**1**	**2**	**3**	**4**	**Total**
Smoking	**Smokers**	0	3		6	2	
	Non-smokers	4		8	4	1	27
	Total	4	13		10	3	43

a Complete the missing parts of the table.
b Did smokers or non-smokers have more colds?
c Calculate the percentages of smokers and non-smokers who had more than 2 colds.
d Does smoking appear to affect health?
e Are the conclusions of this survey likely to be reliable?

Solution

a Complete the 'Total' column on the right. **Total smokers = 43 − 27 = 16**

Complete the '1 cold' column. **Non-smokers with 1 cold = 13 − 3 = 10**

Check the non-smokers total. **4 + 10 + 8 + 4 + 1 = 27 ✓OK**

Complete the 'smokers' row. **Smokers with 2 colds = 16 − 11 = 5**

Complete the '2 colds' column. **5 + 8 = 13**

Check the 'total' row. **4 + 13 + 13 + 10 + 3 = 43 ✓OK**

Show the complete table.

		Number of colds last year					
		0	**1**	**2**	**3**	**4**	**Total**
Smoking	**Smokers**	0	3	5	6	2	**16**
	Non-smokers	4	10	8	4	1	**27**
	Total	**4**	**13**	**13**	**10**	**3**	**43**

b Work out the numbers who had more than 2 colds. **Smokers with colds = 16 − 0 = 16**
Non-smokers with colds = 27 − 4 = 23

Write the answer. **More non-smokers than smokers had 2 colds.**

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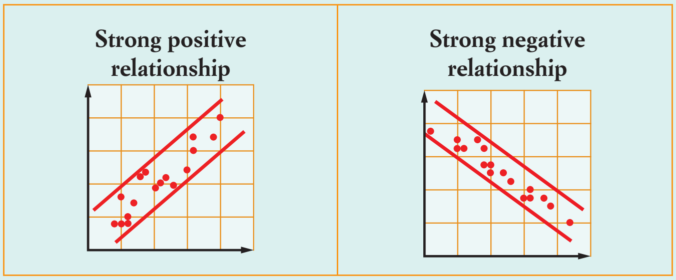
c	Find the percentage of smokers with more than 2 colds.	**Smokers with more than 2 colds** $= \frac{8}{16} \times 100\%$ $= 50\%$
	Find the percentage of non-smokers with more than 2 colds.	**Non-smokers with more than 2 colds** $= \frac{5}{27} \times 100\%$ $\approx 18.5\%$
	Write the answer.	**50% of smokers and 18.5% of non-smokers had more than 2 colds.**
d	Use the table and the percentages.	**All the smokers had colds but some non-smokers didn't, as well as a much greater percentage of smokers having multiple colds, so smoking does appear to affect health.**
e	Half the cells have frequencies less than 5.	**Conclusions about smoking and getting no colds have too small a sample.**
	Consider joining parts together.	**The lowest frequency for 'more than 2 colds' and '2 or less colds' is 5 (for non-smokers).**
	Assess the reliability.	**The conclusions are actually based on 'more than 2 colds' or not, so they do appear to be reasonable.**

Scatter plots are the best way to analyse numerical bivariate data. Variables that have a linear relationship will make a straight line when plotted. Most statistical variables do not make a true straight line, but may still be related. For example, you know that tall people are usually heavier than short people, but this is not always true. A person is not necessarily heavier than someone shorter than them, but they are more likely to be heavier than lighter.

Important!

Scatter plots

If the points on a scatter plot lie between sloping parallel lines, you can say that there is a relationship between the variables.
If the lines have a positive slope, you say it is a **positive relationship**. The variables generally change in the same way; when one goes up, so does the other.
If the lines have a negative slope, you say that it is a **negative relationship**. The variables generally change in the opposite way; when one goes up, the other goes down.
The parallel lines are close together for a **strong relationship**. If they are well separated, you say that there is only a **weak relationship**.



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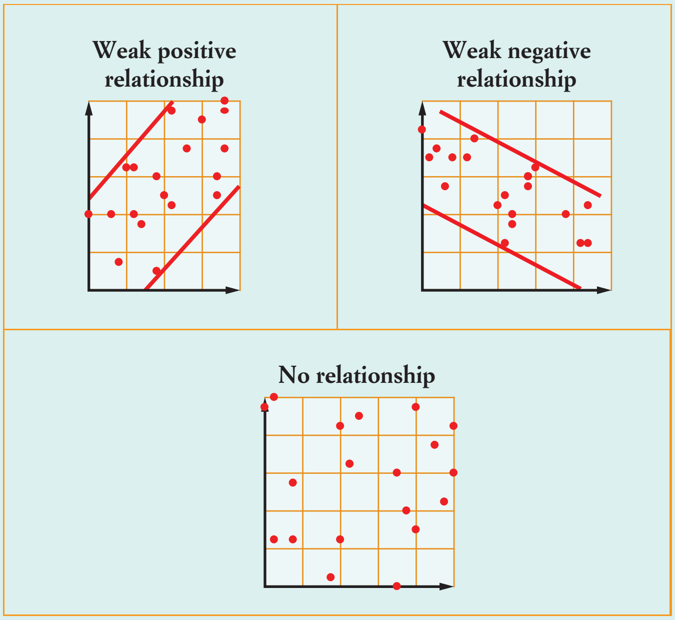
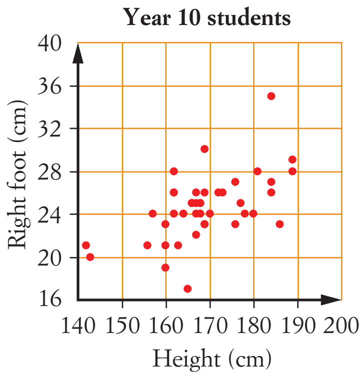


Example 16

The information below shows the heights and right foot lengths of some Year 10 students. Use a scatter plot to comment on the relationship. The data is in the form (height, foot length), in cm.
(157, 24), (167, 25), (156, 21), (172, 26), (163, 21), (181, 28), (180, 24), (167, 24), (169, 30), (177, 25), (162, 24), (165, 17), (167, 22), (189, 29), (168, 25), (169, 23), (184, 35), (162, 26), (170, 24), (142, 21), (169, 26), (167, 26), (186, 23), (178, 24), (160, 21), (166, 25), (169, 23), (168, 24), (173, 26), (164, 24), (143, 20), (162, 28), (189, 28), (166, 25), (176, 23), (184, 26), (168, 25), (184, 27), (160, 23), (162, 26), (160, 19), (176, 27), (166, 25)

Solution

Plot the points on graph paper, choosing the scales so that the points nearly fill the space.



Look at the pattern and make a conclusion.

There appears to be a weak relationship between height and the length of the right foot for Year 10 students.

Worksheet
Scatter plots
MAT10SPWK00002

CAS TI-Nspire exercise
Statistics
MAT10SPTI00002

CAS ClassPad exercise
Statistics
MAT10SPCP00002

Strong relationships can be used to predict one variable from the other.

Technology Worksheet
Excel worksheet: Line of best fit
MAT10SPCT00003

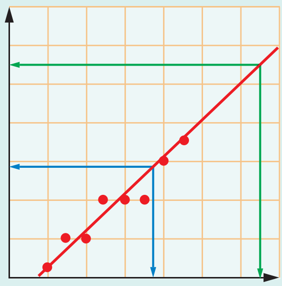
CAS TI-Nspire exercise
Statistics
MAT10SPTI00002

CAS ClassPad exercise
Statistics
MAT10SPCP00002

Important!

Prediction from a scatter plot

A **line of best fit** is a line that shows the general pattern of points on a graph.
Interpolation is the prediction of a value within the range of data.
Extrapolation is the prediction of a value outside the range of the data.



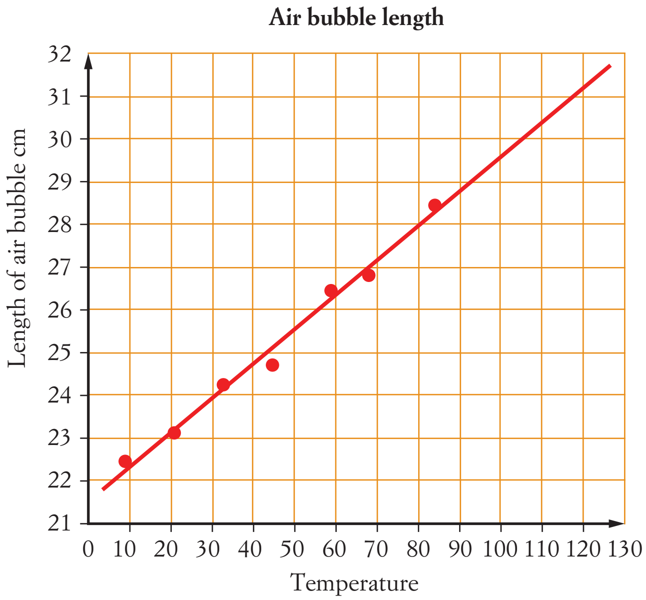
Example 17

Students heated a thin glass tube with one end sealed and the other bent down in an oil bath. They measured the length of a bubble of air trapped in the tube at different temperatures and obtained the following results. Plot the points and predict the length if the oil could be safely heated to 120°C.

Temperature °C	9	21	33	45	59	68	84
Length (cm)	22.4	23.1	24.2	24.7	26.4	26.8	28.4

Solution

Plot the points and draw a line of best fit past 120°.



Read the value from the line of best fit.

If it were possible, the air bubble would have a length of about 31.2 cm at 120°C.

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Investigate: Australian biodiversity

Australia is recognised as one of the most biologically diverse areas of the world. However, many species have become extinct in a short time in Australia. Many of these extinctions are the result of human activity. It is even possible that human disturbance resulted in the extinction of Australian mega fauna. However, the rate of extinction since European settlement has been very high. Australia has the *worst* mammal extinction record in the world – 27 mammals have become extinct in the last 200 years. No other country or continent has such a tragic record of mammal extinctions.

- Investigate extinctions of mammals in Australia
- Investigate extinctions of birds in Australia
- Investigate extinctions of plants in Australia
- How are the extinctions and the reduction of biodiversity spread over time?

A strong relationship is not necessarily a straight line.

Example 18

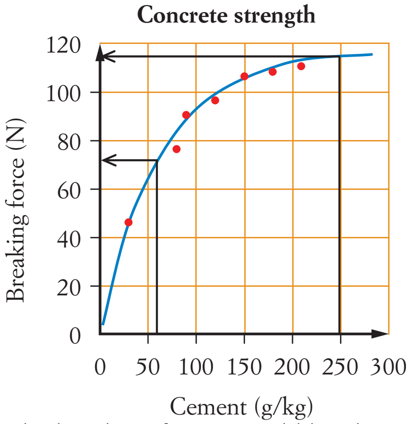
The table below shows the amount of cement powder used in different mixes for concrete blocks and the compressive force (in newtons) needed to break the block.

Cement (g/kg)	30	80	90	120	150	180	210
Breaking force (N)	46	76	90	96	106	108	110

a Draw a scatter plot of the information.
b Use the graph to predict the breaking forces needed for 60 g/kg and 250 g/kg cement.

Solution

a Plot the points with the cement axis extended enough to make the required predictions.
Put in a line of best fit.



b Use vertical and horizontal lines to read the required values from the line of best fit.

Write the answers to an appropriate accuracy.

The breaking forces would be about 70 N for 60 g/kg cement and about 115 N for 250 g/kg cement.

Parallel boxplots are a quick and straightforward way to compare sets of data. The boxplots are drawn on the same scale, but above each other. You need to label the boxplots so that they can easily be identified.

Example 19

Animated example
Analysing data
MAT10SPAE00002

CAS TI-Nspire exercise
Statistics
MAT10SPTI00002

CAS ClassPad exercise
Statistics
MAT10SPCP00002

a Draw parallel boxplots of the English results below of two Year 10 classes.
Class 1: 56, 70, 55, 80, 51, 66, 29, 69, 59, 80, 47, 84, 41, 70, 30, 78, 50, 60, 92, 42, 67, 64, 72, 60, 77
Class 2: 43, 53, 33, 65, 44, 53, 33, 64, 37, 67, 47, 52, 15, 67, 23, 57, 49

b Compare the results of the classes.

c Compare the distributions using the means, medians, quartiles and ranges.

Solution

a Arrange the results of class 1 in order and divide into 4 parts.

29, 30, 41, 42, 47, 50 | 51, 55, 56, 59, 60, 60, 64, 66, 67, 69, 70, 70, 72 | 77, 78, 80, 80, 84, 92

Work out the 5 number summary.

The 5 number summary is 29, 50.5, 64, 74.5, 92

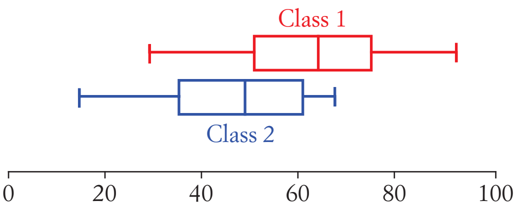
Arrange the results of class 2 in order and divide into 4 parts

15, 23, 33, 33 | 37, 43, 44, 47, 49, 52, 53, 53, 57 | 64, 65, 67, 67

Work out the 5 number summary.

The 5 number summary is 15, 35, 49, 60.5, 67

Use the 5 number summaries to draw the box-and-whisker plots on top of each other.



b The box is further up the scale for class 1 than class 2.

Class 1 generally did better than class 2.

c Find the mean for Class 1.

Class 1 mean $= \frac{1549}{29} \approx 62.0$

Find the mean for Class 2.

Class 2 mean $= \frac{802}{17} \approx 47.2$

Compare the results of the classes.

***Q*0, *Q*1, *Q*2, *Q*3, *Q*4 and the mean are all lower for class 2 than class 1, so class 2 really has lower results than class 1. Class 1 also has a higher range.**

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Investigate: Aboriginal and Torres Strait Islander age distribution

The table below shows the age distributions of Aboriginal and Torres Strait Islander people and non-Indigenous Australians.

Indigenous and non-Indigenous populations in Australia by age, 30 June 2006							
Age	**0–4**	**5–9**	**10–14**	**15–19**	**20–24**	**25–29**	**30–34**
ATSI	64 426	65 136	64 687	54 943	44 779	36 866	36 283
non-ATSI	1 245 656	1 275 073	1 335 467	1 360 262	1 427 079	1 367 064	1 453 521
Age	**35–39**	**40–44**	**45–49**	**50–54**	**55–59**	**60–64**	**65–69**
ATSI	34 760	30 251	25 073	19 812	14 423	9 689	6 477
non-ATSI	1 490 096	1 502 303	1 468 357	1 342 491	1 257 071	979 653	774 942
Age	**70–74**	**75–79**	**80–84**	**85 and over**	**Total**		
ATSI	4 291	2 634	1 394	1 119	517 043		
non-ATSI	626 539	549 915	404 354	320 994	20 180 837		

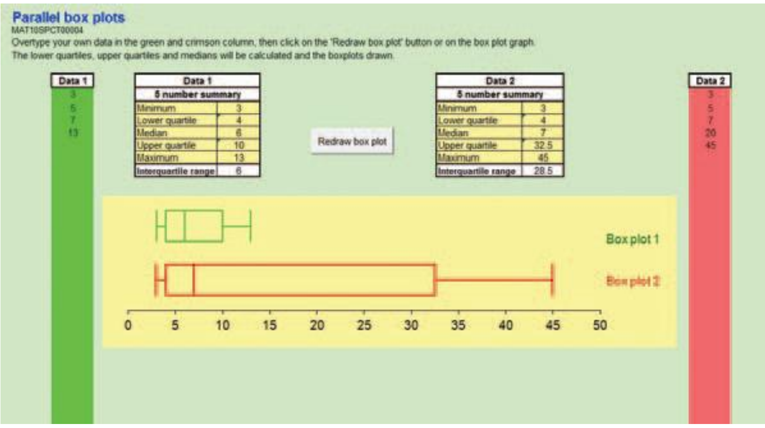
Source: ABS

- Use the table to find the 5 number summaries of the age distributions for each group. Your teacher might want you to estimate the positions of the quartiles within the age groups.
- Draw parallel boxplots of the age distributions.
- What can you conclude?
- What implications does this have for Government health policy?

Technology Parallel boxplots

Technology worksheet
Excel worksheet: Parallel boxplots
MAT10SPCT00004

You can use an interactive spreadsheet to draw parallel box-and-whisker plots like the one in Example **19**. You can download the spreadsheet from NelsonNet, or your teacher may have already downloaded it to the school computers for you to use.



Investigate: Examining media reports

Read the following media report from the ABC

Indigenous people 'living longer'

May 25, 2009

NEW figures released by the Australian Bureau of Statistics show Indigenous life expectancy is longer than previously thought.

The ABS announced today that the life expectancy gap between Indigenous and non-Indigenous people is about 10 years - down from previous estimates of almost 17 years.

But the new results are not necessarily a reflection of better health amongst Indigenous people.

ABS assistant director of demography, Matthew Montgomery, says a new method, which better accounts for Indigenous deaths, has been used to compile the latest figures.

"ABS certainly intends to use that direct method into the future because it provides us with a better estimate," he said.

"The previous estimates, really there was some uncertainty, [because] we had to make quite a bit of assumptions about the method.

"The current method allows us to stand back and let the data speak for itself."

The new data shows Indigenous life expectancy across Australia for men and women is 67.2 years and 72.9 years respectively - almost 10 years below non-Indigenous life expectancy.

The Northern Territory has the worst Indigenous life expectancy of any state or territory, at just 61.5 years for men and 69.2 years for women.

NSW recorded the highest life expectancy for Indigenous men and women, with 69.9 years and 75 years respectively.

- What is meant by 'life expectancy'?
- Does the report mean that Aboriginal health has improved?
- Find out the life expectancy of all Australians.
- How is life expectancy worked out?
- Does life expectancy have any commercial value or is it only of interest to governments?

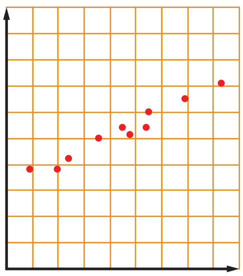
Exercise 2.3 Analysing data

Fluency

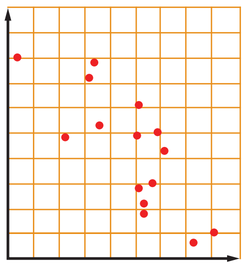
Extra questions
Exercise 2.3
MAT10SPEQ00006

1 State whether any relationship is indicated by each of the following scatter plots, and if so, what kind of relationship is shown.

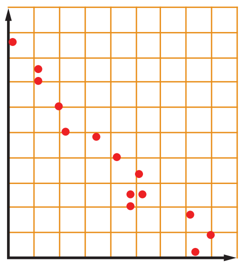
a



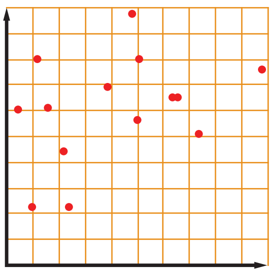
b



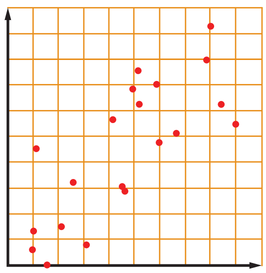
c



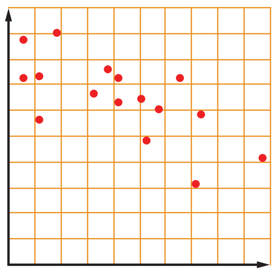
d



e



f



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Fluency

2 Use the interquartile range to determine if any of the following sets of positive data have outliers.
See Example 14

a 8, 12, 10, 12, 8, 10, 9, 20, 8, 12, 10, 12, 7, 14, 9, 10, 5, 12, 5, 10

b 21, 30, 24, 35, 23, 27, 26, 33, 26, 28, 24, 26, 23, 27, 24, 27, 26, 27, 16, 34, 21, 26, 22, 27, 20

c 21, 26, 20, 26, 23, 27, 25, 26, 24, 27, 21, 25, 25, 27, 20, 25, 19, 26, 23, 25, 23, 29, 21, 27

d 12, 15, 13, 15, 13, 19, 12, 19, 13, 15, 11, 14, 5, 16, 12, 17, 10, 16, 11

e 46, 62, 49, 51, 48, 51, 45, 52, 46, 56, 48, 51, 50, 51, 45, 54, 47, 54, 47, 52, 49, 54, 46, 52, 48, 51, 36, 51, 49, 54, 44, 50

3 Draw scatter plots for each of the following sets of points and state whether a relationship is shown. If a relationship is shown, state the nature of the relationship.
See Example 16

a (10, 53), (11, 49), (14, 49), (22, 39), (23, 46), (28, 33), (35, 40), (37, 34), (43, 24), (43, 27), (50, 26), (53, 21), (57, 16)

b (6, 11), (8, 14), (13, 43), (13, 41), (18, 49), (22, 1), (28, 12), (30, 53), (36, 27), (38, 55), (41, 22), (50, 3), (50, 22), (59, 17), (34, 9), (17, 35), (37, 21), (41, 9), (39, 52)

c (1, 10), (9, 22), (15, 23), (21, 20), (24, 38), (24, 26), (26, 51), (30, 28), (37, 45), (37, 55), (45, 36), (46, 47), (48, 53), (57, 49), (15, 22), (8, 32), (17, 17), (1, 12), (30, 30), (30, 36), (29, 27)

4 Draw parallel boxplots of each of the following sets of data.
See Example 19

a **Set 1:** 11, 16, 9, 14, 12, 13, 10, 12, 7, 16, 9, 14, 5, 15, 12, 14, 11, 12

Set 2: 8, 9, 7, 11, 6, 9, 9, 9, 7, 16, 7, 12, 8, 15, 8, 15, 5, 9

b **Set 1:** 23, 26, 23, 28, 25, 29, 24, 25, 25, 25, 25, 26, 22, 26, 23, 27, 23, 28, 24, 29, 23, 25

Set 2: 25, 26, 22, 31, 24, 28, 20, 28, 25, 32, 16, 25, 21, 27, 24, 27, 24, 29, 20, 28, 25, 29, 24, 26

c **Set 1:** 6, 10, 3, 6, 4, 9, 4, 7, 5, 10, 5, 8, 2, 8, 3, 9, 5, 6, 5, 9, 5, 7, 4, 6, 4

Set 2: 5, 9, 7, 8, 3, 8, 3, 9, 8, 9, 7, 9, 7, 8, 7, 9, 7, 8, 4, 9, 7, 8, 6, 10, 7, 8, 6

5 Make scatter plots of the following sets of data and draw lines of best fit.
See Examples 17, 18

a

x	30	35	40	45	50	55	60	65	70	75
y	5	3.5	4	6.5	7	7.5	6	6.5	7	9.5

b

x	30	35	40	45	50	55	60	65	70	75
y	7	4.5	4	5.5	5	2.5	2	1.5	3	2.5

c

x	2	4	6	8	10	12	14	16	18	20
y	2.1	3.2	3.6	3.8	3.4	3.3	3.6	3.2	2.1	1.4

Problem solving

See Example 15

6 The table below shows the numbers of accidents in which drivers of different age groups were involved. According to the report, it shows that 'older drivers are safer than younger drivers'.

		Accidents in last 2 years					
		1	**2**	**3**	**4**	**5**	**Total**
Age group	**17–21**		24		14	12	
	22–26	16	14		9	9	60
	27–31	9		7	4	2	29
	32–41	4	2	2	0	0	8
	Over 41	4	2	0	0	0	6
	Total	49		42	27	23	

a Complete the missing parts of the table.
b Are older or younger drivers involved in more accidents?
c Find the percentages of drivers with accidents in each age-group who have had 4 or more accidents in the last 2 years.
d Do older or younger drivers seem more prone to multiple accidents?
e Are the conclusions of this survey likely to be reliable?

7 The table below shows the 'reading ages' of boys and girls in Year 10.

		Reading age						
		12	**13**	**14**	**15**	**16**	**17**	**Total**
Sex	**Male**	14	17		31	24		122
	Female		10	17		29		
	Total	21		41	67			240

a Complete the missing parts of the table
b Are there more boys or girls who are good readers in Year 10?
c Do boys or girls appear to be better readers in Year 10?
d Are the conclusions of this survey likely to be reliable?

8 The data below shows the heights and arms spans of a sample of students in Year 10.
(157, 167), (167, 165), (156, 153), (172, 173), (163, 164), (30, 162), (181, 185), (180, 180), (167, 167), (169, 169), (190, 88), (177, 176), (162, 156), (165, 165), (167, 160), (189, 189), (168, 167), (169, 159), (184, 182), (175, 174), (162, 162), (170, 160), (142, 155), (169, 168), (167, 167), (186, 183), (133, 166), (178, 174), (160, 153), (166, 167), (169, 173), (168, 166), (173, 172), (164, 166), (143, 153), (162, 155)
a Make a scatter plot.
b Does there appear to be any relationship?

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9 The data below shows the ages and heights of some Australian school students on 1 January 2011.
(9, 142), (15, 175), (13, 167), (12, 150), (14, 172), (10, 134), (12, 145), (15, 163), (13, 162), (11, 134), (11, 138), (14, 157), (12, 144), (14, 171), (12, 155), (9, 144), (14, 165), (12, 168), (13, 156), (13, 150), (12, 164), (10, 147), (16, 157), (14, 163), (19, 173), (11, 136), (17, 174), (15, 160), (17, 156), (17, 165), (15, 176), (13, 152), (12, 158), (8, 135), (15, 172), (16, 167), (16, 168), (11, 149), (13, 158), (12, 150), (10, 146), (18, 175), (13, 159), (16, 150), (17, 186), (8, 250), (10, 146), (14, 169), (16, 174), (11, 148)

a Make a scatter plot.
b Does there appear to be any relationship?
c What would you expect to find if you collected ages and heights for university students?

Worked solutions
Exercise 2.3
MAT10SPWS00006

10 Two sprinters have the following training times over 100 m (in seconds).

Fred:	12.3	12.4	12.5	12.3	12.4	12.8	13.1	12.1	12.7	12.5
Sam:	12.1	12.0	12.0	16.8	12.1	12.2	12.4	12.1	12.3	12.4

Worked solutions
Exercise 2.3
MAT10SPWS00006

a Draw parallel box-and-whisker plots for both sprinters on the same scale.
b Compare the sprinters using the means, medians, quartiles and ranges.
c Which sprinter should be chosen for the school track-and-field team? Give reasons.

11 The weekly rental prices (in dollars) of houses in Brisbane, advertised one Saturday morning, were as follows.
Inner southern suburbs: 275, 340, 385, 330, 290, 235, 345, 300, 400, 265, 235, 250, 310, 300, 340, 330
Inner northern suburbs: 500, 450, 325, 390, 280, 230, 360, 275, 400, 285, 298, 480, 340, 390, 290, 470, 600, 290

a Draw parallel box-and-whisker plots for both areas on the same scale.
b Compare rental costs in inner northern and inner southern suburbs.
c Compare the areas using the means, medians, quartiles and ranges.

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12 The marks of some students for Physics and Chemistry are shown below.
Physics: 60, 81, 74, 45, 44, 44, 34, 59, 48, 74, 43, 48, 62, 46, 59, 64, 64, 55, 55, 45, 71, 59, 51, 55, 67
Chemistry: 80, 65, 56, 67, 70, 53, 76, 68, 43, 59, 43, 61, 43, 80, 74, 53, 52, 58, 49, 67, 62, 37, 65, 58, 46, 65, 65

a Draw parallel box plots for the data.
b Compare the results.
c Compare the classes using the means, medians, quartiles and ranges.

Worked solutions
Exercise 2.3
MAT10SPWS00006
See Example 17

13 The germination rate for parsnip seeds stored for different times is shown below.

Time (weeks)	6	8	10	12	14	16
Germination (%)	75	83	71	67	67	55

a Draw a scatter plot to show the results and put in a line of best fit.
b What happens to the germination rate as time goes on?
c What would the germination rate be if the seed was stored for only 1 week?
d What would the germination rate be if the seed was stored for 20 weeks?

See Example 19

14 The table below shows the amount of potassium iodide (KI) used up in a reaction and the time taken for the reaction (with other quantities kept constant).

KI amount (g)	5	10	15	20	25	30	35	40	45	50
Time (s)	19.5	8.5	5.2	4	5	2.3	1.4	1	2.7	1.5

a Draw a scatter plot of the information and add a line of best fit.
b Use the graph to predict the time of reaction if 70 g of KI is used.

15 The records for the women's high jump are shown below.

Year	1932	1943	1956	1958	1960	1961	1974	1977	1984
Record (m)	1.65	1.71	1.75	1.80	1.85	1.90	1.95	2.00	2.07

a Draw a scatter plot of the information and add a line of best fit.
b Use the graph to predict the record at the Sydney Olympic Games in 2000. Find out whether this prediction was correct.

Reasoning

16 Explain why the information in question **6** is not useful for determining whether older or younger drivers are more accident prone.

See Example 13

17 A marketing firm placed survey forms inside the plastic wrapping of adult board games asking about the kinds of games people liked. The makers of the board game wanted to find out how to expand their market and appeal to a wider range of people. The survey could be sent back using a prepaid envelope. Explain whether you think this was a good way to do the survey.

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18 To find the kinds of child care holiday programs that children want, students in Year 3 at a school near the childcare centre were asked to pick from a list of 10 different activities they would like to do in the holidays. The centre planned to offer local parents a special deal to try to replace business lost when people went away with their children for the holidays. Explain whether you think this method was a good way to get the information.

19 Explain whether or not the data in question **13** could be used to set a 'use-by' date for parsnip seed.

20 Explain why records like those in question **15** are not very good for prediction of future records.

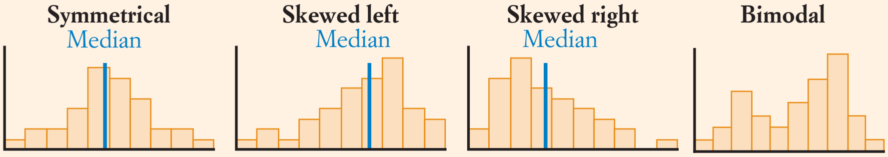
Chapter 2 summary

Quiz
Statistics
MAT10SPQZ00002

- **Categorical** data separates information into different (named) categories: **nominal** data has no numeric meaning or order, **ordinal** data has a natural order, but does not represent a measurement.
- **Numerical** data consists of numbers: **continuous** data can take any value between the smallest and largest values; **discrete** data can only have particular values. In most cases, the values are whole numbers.
- The **mean** $\bar{x}$ is the **average** score. It is the total divided by the number of scores.
- The **median** is the middle score. It is found by arranging all scores in order to choose the middle score. **Cumulative frequency** is the progressive total of frequencies and can be used to work out the median.
- The **mode** is the score with the highest frequency. A **bimodal** distribution has two scores with equal highest frequencies.
- The mean, median and mode are called **measures of central tendency**.
- The mathematical symbol Σ means 'the sum of' and can be used to write the formula for the mean as $\bar{x} = \dfrac{\Sigma x}{n} = \dfrac{\Sigma x}{\Sigma f}$, where x stands for a score, n is the number of scores and f stands for a frequency.
- The mean is most often used as the typical score. However, if there are a few high or low scores that would distort the mean, you should use the median. The mode should be used in cases where the most common score is needed.
- The **range** is the difference between the highest and lowest scores and measures the **spread** of the data.
- The **quartiles** of a frequency distribution divide it into quarters.
- The **first quartile** (**lower quartile**) is the score that has a quarter (25%) of the scores below it. Its symbol is $Q1$ or Q_1. For a finite distribution of n scores, it is the median of the scores below the median, or the $\dfrac{n+1}{4}$th score for n odd and the $\dfrac{n+2}{4}$th score for n even.
- The **second quartile** is the score that has two quarters (50%) of the scores below it. It is another name for the median. It can be written as $Q2$ or Q_2.
- The **third quartile** (**upper quartile**) is the score that has three quarters (75%) of the scores below it. Its symbol is $Q3$ or Q_3. For a finite distribution of n scores, it is the median of the scores above the median, or the $\dfrac{3n+3}{4}$th score for n odd and the $\dfrac{3n+2}{4}$th score for n even.
- The **interquartile range** (IQR) is the difference between the third and first quartiles.

$$\text{IQR} = Q3 - Q1$$

- The **five number summary** is the minimum, first quartile, median, third quartile and maximum ($Q0$, $Q1$, $Q2$, $Q3$, $Q4$) of a frequency distribution.
- A **symmetrical** distribution is roughly the same shape on either side of the middle.
- A distribution is **skewed to the left** (**negatively skewed**) if the scores are spread out more below the median than above the median. If the scores are spread out more on the right, it is **skewed to the right** (**positively skewed**).
- **Bimodal** distributions have two high parts, although they don't always have two actual modes.



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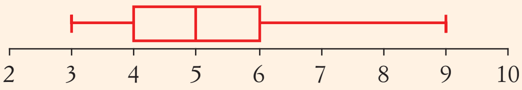
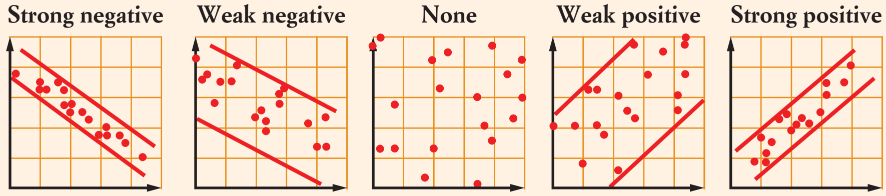
Chapter 2 summary

- A **box-and-whisker plot** (**boxplot**) uses the 5 number summary to show how data is distributed. You put a rectangle stretching from the lower quartile to the upper quartile, divided at the median and draw horizontal lines from the quartiles to the maximum and minimum. A scale is shown under the drawing.

Spelling mistakes out of 20



- **Back-to-back** or **side-by-side** data displays are used to compare different distributions.
- A **fair sample** is like the whole **population**. A **biased sample** is not typical of the population.
- **Biased methods** and **biased questions** are likely to influence results in a particular direction. **Fair** ones are not likely to influence results in a particular way.
- **Non-compliant data** is obviously wrong, like someone's age being 164.
- An **outlier** is a score that has a very different value to the rest of a set of data. It can be defined as being more than 1.5 times the interquartile range below the first quartile or above the third quartile.
- Data where each item has two measurements that can be represented as variables is called **bivariate data.**
- A **scatter plot** of bivariate data has values that are plotted as coordinates. One variable is on the horizontal axis and the other one is on the vertical axis. It is used to look for relationships between variables.
- If the points on a scatter plot lie between sloping parallel lines, you can say that there is a **relationship** between the variables. If the lines have a positive slope, you say it is a **positive** relationship, but for a **negative** relationship they slope down. The variables generally change in the same way in a positive relationship but in opposite ways in a negative relationship.
- The parallel lines are close together for a **strong** relationship but are well separated when the relationship is **weak**.



- A **line of best fit** is a line that shows the general pattern of points on a graph.
- **Interpolation** is the prediction of a value within the range of data.
- **Extrapolation** is the prediction of a value outside the range of the data.

Chapter 2 review

Understanding

See Example 1

1 Find the mode and range for the following set of data.
Macaroni, Spaghetti, Fettuccini, Lasagne, Lasagne, Macaroni, Spaghetti, Fusilli, Spaghetti, Spaghetti, Macaroni, Fettuccini, Lasagne, Spaghetti, Fettuccini, Macaroni, Lasagne, Macaroni, Lasagne, Gnocchi, Spaghetti, Fettuccini, Spaghetti, Gnocchi

See Example 2

2 Find the median and mode for the following set of data.
Very poor, Very poor, OK, Poor, Very poor, Very Good, Very poor, Poor, OK, OK, OK, Poor, Good, Very poor, Poor, Very poor, Very poor, Poor, Very poor, Good, Good, Very Good, Very Good, Very poor, Good, Good, Very Good

See Example 3

3 Find the mean, median, mode and range for each of the following sets of data.

a 5, 13, 10, 12, 10, 13, 11, 14, 10, 13, 7, 12, 7, 13, 10, 13, 8, 12, 9, 12, 5

b

Score	25	26	27	28	29	30	31	32	33	34	35	36	37
Frequency	5	1	4	1	3	9	16	13	17	9	5	5	2

4 Make a stem-and-leaf plot of the times below that Year 10 students say that they got up on Monday morning.
7:00, 5:30, 6:30, 7:30, 7:30, 8:00, 8:30, 8:00, 7:45, 7:00, 7:30, 7:00, 7:15, 6:15, 7:30, 7:00, 7:45, 7:30, 6:45, 7:30, 7:30, 7:00, 6:15, 6:30, 8:00, 7:00, 6:00, 8:00, 6:30, 6:30

See Example 9

5 Make scatter plots of the following sets of data.

a

x	15	20	25	30	35	40	45	50	55
y	45	39	38	36	34	37	31	29	32

b

x	3	7	11	15	19	23	27	31	35	39
y	26	38	68	67	80	78	66	52	31	8

See Example 4

6 Find the median, quartiles and interquartile range for each of the following sets of data.

a 3, 7, 4, 9, 3, 9, 6, 8, 7, 7, 2, 10, 7, 9, 7, 7, 5, 7, 4, 8

b

Score	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Frequency	2	2	1	0	5	1	7	7	3	5	4	9	4	7	4	4	3	1	3

See Example 10

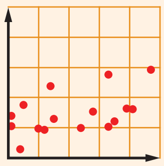
7 Draw box-and-whisker plots for each of these sets of data.

a 8, 9, 9, 12, 10, 13, 14, 8, 4, 5, 7, 10, 7, 9, 8, 7, 5, 7

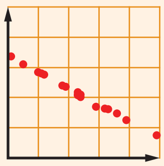
b 26, 25, 24, 20, 24, 15, 19, 26, 27, 31, 21, 26, 16, 29, 22, 22, 15, 29

8 State whether any relationship is indicated by each of the following scatter plots, and if so, the kind of relationship that is shown.

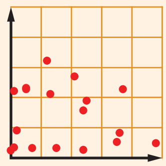
a



b



c



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Fluency

9 The following table shows the masses of a group of people. See Example 8

Mass (kg)	40–49	50–59	60–69	70–79	80–89	90–99	100–109	110–119
Frequency	4	7	16	14	6	2	1	4

a Draw a histogram of the results.
b Comment on the shape of the distribution.

10 Find the five number summary for each of the following distributions. See Example 5
a 45, 54, 49, 50, 44, 54, 46, 52, 41, 53, 42, 56, 43, 53, 39, 50, 47, 55, 47, 54, 42
b

Score	12	13	14	15	16	17	18	19	20
Frequency	1	2	7	15	26	12	13	4	1

11 Which data measure(s) should be used in each of the following cases? See Example 6
a A car yard wants to know how much it will get from selling 15 cars.
b An up-market credit card provider wants to know if plumbers earn enough to be worth targeting in a marketing campaign.

12 Compare a box-and-whisker plot of the following set of data to a histogram and dot plot of the same data. See Example 11
10, 10, 7, 11, 6, 11, 7, 12, 7, 14, 10, 10, 9, 13, 6, 12, 6, 10, 8, 7, 6, 14, 6, 12, 5, 11, 8, 14, 5, 11, 6, 10, 8, 12, 9

13 Use the interquartile range to determine if any of the following sets of positive data have outliers. See Example 14
a 8, 11, 9, 11, 9, 12, 9, 12, 4, 11, 7, 13, 9, 12, 9, 13, 10, 11, 10, 14
b 13, 27, 13, 23, 19, 23, 15, 25, 16, 21, 15, 27, 12, 22, 19, 27, 17, 23, 12, 26, 15, 25, 19, 20, 17

14 Draw scatter plots for each of the following sets of points and state whether a relationship is shown. If a relationship is shown, state the nature of the relationship. See Example 16
a (11, 57), (24, 65), (35, 75), (48, 83), (60, 98), (73, 110), (21, 64), (24, 63), (29, 71), (28, 74), (25, 63), (40, 74), (5, 46), (17, 61), (12, 50), (6, 50), (22, 59), (20, 54), (15, 58),
b (20, 75), (19, 71), (37, 9), (49, 5), (24, 52), (35, 49), (31, 14), (35, 46), (13, 96), (28, 12), (34, 29), (4, 91), (15, 68), (2, 112), (25, 32), (21, 62), (26, 43), (50, 3), (50, 22), (59, 17), (34, 9), (17, 35), (37, 21), (41, 9), (39, 52)

15 Draw parallel boxplots of the following sets of data. See Example 19
Set 1: 8, 8, 4, 9, 8, 9, 5, 13, 7, 9, 5, 18, 7, 11, 3, 12, 4, 18, 5, 11, 5, 19, 5, 14
Set 2: 7, 15, 12, 15, 8, 17, 5, 14, 11, 14, 12, 13, 12, 17, 2, 14, 9, 15, 8, 15, 13, 13, 10, 16, 7, 15

16 Make scatter plots of the following sets of data and draw lines of best fit. See Examples 17, 18
a

x	1.3	2.5	3.1	4.4	5.4	6.6
y	12.6	18.3	27.4	38.3	51.1	68.1

b

x	1.5	3.9	6	10.7	11.3	12.4	15.8	18.8
y	4.3	6.5	10.1	10.9	14.1	18.7	17.6	24.7

Problem solving

17 The numbers of damaged apples in cases of 40 apples were recorded as follows.

Number of damaged apples	0	1	2	3	4	5
Number of cases	12	10	7	5	4	2

a What is the typical number of damaged apples in a case?
b How many damaged apples would you expect to get in 74 cases?

See Example 12

18 Two greyhounds have the following race times (in seconds) over the same meetings.
Houndog: 53, 59, 61, 51, 59, 53, 58, 44, 47, 48, 73, 73, 70, 66, 60, 60, 50, 71, 37, 64
Chaser: 64, 53, 60, 61, 66, 69, 82, 68, 64, 68, 76, 59, 57, 58, 71, 83, 68, 77, 64, 76
a Make a back-to-back stemplot for the data.
b Compare the greyhounds.
c Compare the greyhounds using the means, medians and ranges and comment on your results.

See Example 15

19 This table shows the heights and errors in a spelling test of a group of 15-year-olds.

		Errors in spelling					
		1	**2**	**3**	**4**	**5**	**Total**
Height (cm)	**150–159**	0	1		5	1	10
	160–169	8	6		2	9	34
	170–179						
	180–189	2	3		2	2	11
	Over 189	2	0		1	1	4
	Total	23	12		17	22	94

a Complete the missing parts of the table.
b Do taller students appear to make more or fewer spelling errors than shorter students?
c Are the conclusions of this survey likely to be reliable?

See Example 17

20 The following table shows the numbers of new sports utilities sold in Australia.

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Number ('000s)	113	126	149	163	187	194	185	216	208	205	252

a Draw a scatter plot to show the results.
b What kind of relationship is shown, if any?
c Put in a line of best fit and predict the number of sales in 2014.

21 Two classes of students sat the same exam, one first thing in the morning and the other last thing in the afternoon. Their results are shown below:
Morning class: 3, 15, 10, 12, 14, 15, 15, 9, 10, 9, 13, 15, 11, 13, 12, 14, 13, 13, 14, 15, 17, 14, 5, 13, 8, 9, 17, 11, 11, 17
Afternoon class: 3, 11, 8, 6, 7, 9, 3, 11, 11, 8, 12, 12, 3, 12, 9, 5, 5, 5, 13, 5, 4, 12, 10, 13, 16, 12, 12, 17, 9, 6
a Make parallel box-and-whisker plots for the classes.
b Does either class appear to have had an advantage?
c Compare the morning and afternoon distributions using the means, medians, quartiles and ranges and comment.

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Reasoning

22 Explain the positions of the median and the mean in a negatively skewed distribution.

23 A class of 20 Year 10 students has an average mass of 72 kg. The median mass is 68 kg. Three more students join the class. These three have an average mass of 80 kg and a median mass of 78 kg. Explain what will happen to the mean and median for the class.

24 What are other explanations for the results in question **21**?

25 The 5 number summary of a distribution is 12, 17, 19, 20, 22. Comment on the distribution and explain how you would estimate the mean.

26 Two market research companies undertake surveys for different local chocolate makers to find the sizes of Easter eggs that they should make for the next Easter season. One maker rings 1000 people from different areas of the city and asks them to state the mass of chocolate they would want to have in Easter eggs. The other company takes sample eggs to 100 selected houses in the suburbs. The first company gets higher weights than the second. Explain which you think would be the most reliable. *See* Examples 7, 13

27 How reliable is the prediction for 2014 in question **20**?

Number and algebra

3 Algebra

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Contents

Prior learning
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MAT10NAPL00003

Parent guide
Chapter 3
MAT10NAPG00003

Curriculum guide
Chapter 3
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Australian Curriculum statements

Patterns and algebra

Factorise algebraic expressions by taking out a common algebraic factor. (ACMNA230)
Simplify algebraic products and quotients using index laws. (ACMNA231)
Apply the four operations to simple algebraic fractions with numerical denominators. (ACMNA232)
Substitute values into formulas to determine an unknown. (ACMNA234) AC

Video tutorial
Algebra
MAT10NAVT00003

Weblink
What is Algebra, really?
MAT10NAWB00003

Algebra has a long history. As far back as 2000 BCE, the ancient Babylonians developed a form of algebra. Evidence of equations recorded on clay tablets shows that the Babylonians used algebra to solve problems.

Like much of our mathematics, algebra came into modern use through the Muslim culture of more than a thousand years ago. Muhammad ibn Mūsā al-Khwārizmī (about 780–850) of the 'House of Wisdom' in Baghdad used the phrase *al-jabr* to describe some basic operations in the solution of equations. It has become the word algebra and his name is immortalised in the word 'algorithm' that describes the arithmetic methods he detailed in his book *Al-Kitāb al-mukhtaṣar fī ḥisāb al-jabr wa-l-muqābala*.

In algebra, unknown quantities are represented by variables. Variables are used in rules or equations that can then be used to model difficult problems.

Mathematical literacy

Maths dictionary
MAT10ASDI00001

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

- algebraic expression
- algebraic fraction
- arithmetic expression
- binomial brackets
- coefficient
- collecting like terms
- common factor
- constant
- difference of two squares
- distributive law
- evaluate
- expand
- expression
- factorise
- formula
- highest common factor
- like terms
- lowest common denominator
- perfect square
- simplification
- simplify
- square of a difference
- square of a sum
- subject
- substitution
- term
- transpose
- variable

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3.1 Simplifying expressions

You have already done quite a bit of algebra in previous years. You should already be familiar with most of the terms below.

Important!

Variables, expressions and formulas

A **variable** is a letter or symbol that stands for a number. A **constant** is a number.
An **expression** has variables and/or numbers connected by arithmetic operations like $+$, $\div$ and powers. An expression with numbers only is an **arithmetic expression**, while one with variables is an **algebraic expression**.
The numbers that are multiplied by the variables are called **coefficients**, the parts separated from the rest by $+$ or $-$ are called **terms** and a number on its own is called a **constant term**. In algebra, you usually leave out the multiply symbol $\times$ between a constant and a variable or between variables. So the expression $5x + 8$ means $5 \times x + 8$ and has 2 terms, namely $5x$ and 8. Here, the variable is x, the constant term is 8 and the coefficient in the first term is 5.
To **evaluate** an algebraic expression you use **substitution**. You put in values for the variables (**substitute** values) and work out the answer. When $x = {}^{-}3$ is substituted into the expression $5x + 8$, its value is $^{-}7$.
A **formula** is an equation with a variable on the left and an algebraic expression on the right. The variable on the left is the **subject** of the formula. The value of the subject variable is calculated by evaluating the expression on the right using given values for the variables in the expression. When the values $A = 12$ and $h = 8$ are substituted into the formula $V = \frac{1}{3}Ah$, you get an answer of 32 for the subject V. This formula can be **transposed** so that the subject is h. It then becomes $h = \frac{3V}{A}$.

You may need to change word sentences into algebraic expressions.

Example 1

Write an expression for each of the following.

a The sum of $3a$ and 5
b 10 less than the product of 7 and b
c The quotient of x^2 and $4c$

Solution

a 'Sum' means to add. $\quad \mathbf{3a + 5}$

b 'Product' means to multiply and 'less than' means to subtract. $\quad \mathbf{7b - 10}$

c 'Quotient' means to divide. $\quad \dfrac{x^2}{4c}$

Important!

Like terms

We call the terms in an algebraic expression with exactly the same variables **like terms**. Each variable has to be to the same power. Like terms can be added or subtracted to **simplify** the expression. This is called **collecting like terms**. You will find that it is easier to put the variables in alphabetical order in each term. When there are different powers of a variable in different terms you should put the terms in order of the power. If possible, put a positive term first in your answer.

Example 2

CAS TI-Nspire exercise
Algebra
MAT10NATI00003

CAS ClassPad exercise
Algebra
MAT10NACP00003

Simplify the following if possible.

a $2ac + 5ca$

b $3b^2 + 4ab - 5bb$

c $6pq - 6qt - 2ptqp + 4tq + 8$

d $m^2 + 2m^3 - 3m^2 - 7m^3$

Solution

a Write the expression. $\quad 2ac + 5ca$

Use the commutative law of multiplication to put the variables in alphabetical order. $\quad = 2ac + 5ac$

Collect like terms. $\quad = 7ac$

b Write the expression. $\quad 3b^2 + 4ab - 5bb$

Write $b \times b$ in index notation. $\quad = 3b^2 + 4ab - 5b^2$

Collect like terms. $\quad = 4ab - 2b^2$

c Write the expression. $\quad 6pq - 6qt - 2ptqp + 4tq + 8$

Use the commutative law and index notation. $\quad = 6pq - 6qt - 2p^2qt + 4qt + 8$

The only like terms are ^-6qt and $4qt$. $\quad = 6pq - 2qt - 2p^2qt + 8$

d Write the expression. $\quad m^2 + 2m^3 - 3m^2 - 7m^3$

m^2 means $1m^2$, and m^3 is different.
Collect like terms. $\quad = {}^-2m^2 - 5m^3$

You can use the index laws when simplifying expressions involving multiplication and division.

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Example 3

Simplify each of the following.

a $8eq^2 \times 5e^3q^3$ **b** $10n^2x^5 \div 15n^4x^4$ **c** $4b^4u^2r^{-3} \times 10b^3u^{-4}r^4$

d $18k^{-3}p^5u^3 \div 6k^4p^{-2}u$ **e** $3x \times (yz^2)^5 \times 5x^{-3} \times (2y^{-3})^{-3}$

Solution

a Write with index 1 for the first *e*.	$8e^1q^2 \times 5e^3q^3$
Add indices and multiply coefficients.	$= 8 \times 5 \times e^{1+3} \times q^{2+3}$
Simplify.	$= 40e^4q^5$
b Write in fraction format, with positive indices.	$\dfrac{10n^2x^5}{15n^4x^4}$
Cancel coefficients, subtract denominator indices.	$= \dfrac{{}^{2}\cancel{10}n^{2-4}x^{5-4}}{{}_{3}\cancel{15}}$
Simplify.	$= \dfrac{2n^{-2}x^1}{3}$
Move the negative index to the denominator and write x^1 as x.	$= \dfrac{2x}{3n^2}$
c Write the problem.	$4b^4u^2r^{-3} \times 10b^3u^{-4}r^4$
Add indices and multiply coefficients. Write the variables in alphabetical order.	$= 4 \times 10 \times b^{4+3} \times r^{-3+4} \times u^{2+{}^-4}$
Simplify.	$= 40b^7r^1u^{-2}$
Move the negative index to the denominator and write r^1 as just r.	$= \dfrac{40b^7r}{u^2}$
d Write in fraction format, with the power of 1 written explicitly.	$\dfrac{18k^{-3}p^5u^3}{6k^4p^{-2}u^1}$
Cancel coefficients, subtract denominator indices.	$= 3 \times k^{-3-4} \times p^{5-{}^-2} \times u^{3-1}$
Simplify.	$= 3k^{-7} \times p^7 \times u^2$
Move the negative index to the denominator to leave with positive indices.	$= \dfrac{3p^7u^2}{k^7}$
e Write the problem.	$3x \times (yz^2)^5 \times 5x^{-3} \times (2y^{-3})^{-3}$
Simplify the powers of powers.	$= 3x \times y^5z^{10} \times 5x^{-3} \times 2^{-3}y^9$
Add indices and multiply coefficients where possible. Write variables in alphabetical order.	$= 15 \times 2^{-3}x^{-2}y^{14}z^{10}$
Write in fraction format with positive indices and evaluate 2^3.	$= \dfrac{15y^{14}z^{10}}{8x^2}$

In some cases, you may need to collect like terms in the numerators or denominators when simplifying multiplications or divisions of expressions.

Example 4

Simplify each of the following.

a $\dfrac{12x - 4x}{5xz - xz} \times (3a + 2a)$

b $\dfrac{11pqr + 9pqr}{24mrw - 9mrw} \times \dfrac{18wm - 10wm}{4v + 3v}$

Solution

a Write the second part in fraction format and insert brackets. $\dfrac{(12x - 4x)}{(5xz - xz)} \times \dfrac{(3a + 2a)}{1}$

Collect like terms. $= \dfrac{8x}{4xz} \times \dfrac{5a}{1}$

Cancel the numbers and the x. $= \dfrac{2}{z} \times \dfrac{5a}{1}$

Simplify. $= \dfrac{10a}{z}$

b Write the problem. $\dfrac{11pqr + 9pqr}{24mrw - 9mrw} \times \dfrac{18wm - 10wm}{4v + 3v}$

Collect like terms. $= \dfrac{20pqr}{15mrw} \times \dfrac{8wm}{7v}$

Cancel the numbers, m, w and r. $= \dfrac{4pq}{3} \times \dfrac{8}{7v}$

Simplify. $= \dfrac{32pq}{21v}$

Investigate: Space stations

Astronauts in a space station need to be able to move around in comfort. They work best under the same gravitational conditions as on Earth. Mathematicians discovered that, if the space station rotated at the appropriate speed, the gravity of Earth could be simulated on board. This is the formula that they worked out to simulate Earth's gravity:

$$N = \frac{42}{\pi}\sqrt{\frac{5}{r}}$$

where N = rotations per minute of the space station and r = the radius of the space station in metres.

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Use the formula to answer these questions.

1 A space station with a radius of 11 m is launched. Calculate the number of rotations per minute that the space station must make to simulate Earth's gravity.

2 An American space station has the same gravity on board as Earth's. It has a radius of 35 m. How many rotations per minute does the space station make?

3 A Russian space lab has a radius of 6.2 m. In orbit it rotates 15 times every minute.
   a Would the people on board have trouble moving about? Explain.
   b If a problem exists, how could it be corrected?

Research the International Space Station. Does it use artificial gravity? Can you think of reasons why they wouldn't use artificial gravity on a space station?

Example 5

Substitute the values given in each of the formulas below to find the values of the subject.

a $s = ut + \frac{1}{2}at^2$, for $u = 10$, $t = 3$ and $a = 8$

b $T = \sqrt{2P + 4B}$, for $P = 1.5$ and $B = 5.5$

Solution

a Write the formula.	$\boldsymbol{s = ut + \frac{1}{2}at^2}$
Substitute $u = 10$, $t = 3$ and $a = 8$.	$= \mathbf{10 \times 3 + 0.5 \times 8 \times 3 \times 3}$
Evaluate.	$= \mathbf{66}$
b Write the formula.	$\boldsymbol{T = \sqrt{2P + 4B}}$
Substitute $P = 1.5$ and $B = 5.5$.	$= \mathbf{\sqrt{2 \times 1.5 + 4 \times 5.5}}$
Evaluate.	$= \mathbf{5}$

If you want to find the value of a variable that is needed to give a particular value for a formula, you can rearrange the formula first. If you wanted to find multiple values, this would save time anyway. To rearrange the formula, you use inverse operations. You may have used inverse operations to solve equations in previous years. Remember that they are done in reverse to the usual order of operations.

CAS TI-Nspire exercise
Algebra
MAT10NATI00003

CAS ClassPad exercise
Algebra
MAT10NACP00003

Example 6

Substitute the given values in each of the following formulas and solve the resulting equation for the remaining variable.

a $A = \frac{1}{2}bh$, $A = 26\text{ cm}^2$, $b = 4\text{ cm}$

b $F = ma$, $F = 30\text{ N}$, $m = 6\text{ kg}$

c $V = \dfrac{\pi r^2 h}{3}$, $V = 201\text{ cm}^3$, $r = 4\text{ cm}$

Solution

a Substitute the values in the formula. $\quad 26 = \dfrac{1}{2} \times b \times 4$

Simplify the RHS. $\quad 26 = 2b$

Divide by 2. $\quad b = 13\text{ cm}$

b Substitute the values in the formula. $\quad 30 = 6 \times a$

Divide by 6. $\quad a = 5\text{ m/s}^2$

c Substitute the values in the formula. $\quad 201 = \dfrac{\pi \times 4^2 \times h}{3}$

Multiply both sides by 3 $\quad 16\pi \times h = 603$

Use your calculator to divide by (16π). $\quad h = 11.996...$

Round and write the answer. $\quad h \approx 12\text{ cm}$

Video tutorial
Changing the subject of a formula
MAT10NAVT10005

Example 7

Make x the subject of each of the following formulas.

a $\sqrt{\dfrac{x}{b}} = r + t$ **b** $v^2 - x^2 = 2as$

Solution

a Write the formula. $\quad \sqrt{\dfrac{x}{b}} = r + t$

Square both sides to get rid of the root in the x term. $\quad \left(\sqrt{\dfrac{x}{b}}\right)^2 = (r + t)^2$

Simplify. $\quad \dfrac{x}{b} = (r + t)^2$

Multiply both sides by b to isolate x. $\quad x = b(r + t)^2$

b Write the formula. $\quad v^2 - x^2 = 2as$

Take v^2 from both sides to isolate the x^2 term. $\quad -x^2 = 2as - v^2$

Multiply both sides by $^-1$ to make the x^2 term positive. $\quad x^2 = v^2 - 2as$

Take square roots of both sides to get x on its own. $\quad x = \sqrt{v^2 - 2as}$

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Exercise 3.1 Simplifying expressions

Understanding

Extra questions
Exercise 3.1
MAT10NAEQ00007
See Example 1

Worksheet
Algebra 1
MAT10NAWK00006

Worksheet
Algebra 3
MAT10NAWK00008

1 Write expressions for the following.

a The sum of 12 and y
b Twice g plus 4
c 6 less than $5d$
d 3 more than $5b$
e The product of $4c$ and v
f The quotient of $12h$ and 5

2 Write expressions for the following.

a 5 more than the product of 7 and c
b The sum of x squared and 8
c 3 times the sum of $6d$ and 2
d Half the sum of $7b$ and g
e Twice the difference of $9m$ and 1
f The quotient of 4 less than $3a$ and 5

3 Write the following using mathematical symbols.

a A number is increased by 20
b Seven less than a number
c Half a number is diminished by 5
d A number is doubled and then increased by 17
e Three more than twice a number
f A number is increased by a third of itself
g A number is reduced by 7 and then doubled
h Two-thirds of a number is added to 15

4 Simplify if possible.

See Example 2

a $3a + 5a - 2a$
b $4n - 3n + n$
c $9k + 3k + 2k$
d $7f - 2f - 3f$
e $z + 2z - 3z$
f $11m - 8m + 3m$
g $2g + g + 4g$
h $8s - 4s - s$
i $7r - 5r + 2r$
j $11k + 4k + 8k$
k $7j - 3j - j$
l $n + n + n$
m $b - b + b$
n $e + 5e - 3e$
o $10m - 6m + 5m$

5 Simplify if possible.

a $4t + 3s - 2t + 5s$
b $7m + 5 - 2 + 4m$
c $5p - 2p + 4q + 6p$
d $4a + 2b - a + 3c$
e $5w - 6u + 5y - 6z$
f $3j + 5i + 3i - j$
g $8e - 3e + e + 9$
h $2y + 4m - y + z + 5m$
i $3r + 2s + 5r + 3t + 4s$
j $2m + 3m + 5u + 8u - m$
k $3f + 8 + 5f - 4$
l $1 + y + 8 + 9y - 5$
m $3x + 2y - x + 7y$
n $6a + 12b - 4a - 10b$
o $9m + 2m^2 - 4mn$
p $11pq - 7qp + 4pq$
q $4xy + 7c - 2xy - 4c$
r $3a + 5bd - bd + 2a$

6 Simplify if possible.

a $4x - 10x + 5x$
b $^{-}6r - 4r + 8r$
c $v - 5v + v$
d $2t - 3t - 4t$
e $8c - 5c - 9c$
f $^{-}4d - 9d + 2d$
g $-i + {}^{-}5i - {}^{-}3i$
h $6y - {}^{-}3y + y$
i $8d - 7e - 4d + 3e$
j $m + n - 3m + n$
k $3x - 4y + 8x - y$
l $5t - 3r - 4r + t$

m $^-4y + 2 - 3y - 1$ **n** $4g - 6h + g - 5h$ **o** $7k + 4k - 5m - 4m$
p $11w - 12w + 13w - 14w$ **q** $3h + {}^-8h - 3c - 5c$ **r** $^-3s - 8y - s + {}^-4y + 4s$
s $x - y - 5x + 7y - 6$ **t** $4m - 6k + 4m + 6k$ **u** $3q - 5r + 4z - 8$

See Example 5

7 Find the value of A in each of the following formulas if $x = 3$, $y = 2$ and $z = 5$.
a $A = 2x + 3y$ **b** $A = 3x - 4y$ **c** $A = 5(x + 2y)$
d $A = 3(2x - z)$ **e** $A = x^2 + y^2$ **f** $A = z^2 - y^2$
g $A = 3(x + y)^2$ **h** $A = x(y + z)^2$ **i** $A = 3xz - 2y^2$

8 If $y = mx + c$, find y when:
a $m = {}^-1, x = 4, c = 3$ **b** $m = 1, x = {}^-5, c = {}^-2$
c $m = 2, x = {}^-3, c = {}^-5$ **d** $m = {}^-2, x = {}^-3, c = 4$
e $m = 0, x = 9, c = {}^-3$ **f** $m = {}^-8, x = 0, c = 14$
g $m = {}^-4, x = {}^-1, c = 2$ **h** $m = {}^-1.5, x = {}^-4, c = 5$

Fluency

9 Write down expressions that show each of the following.
a 5 less than double the number
b the sum of 4 and the number all multiplied by 6
c the square of the number decreased by 10
d half of the number increased by 2
e the product of 3 and 4 more than the number
f the quotient of 8 less than the number and 7
g the product of 5 more than twice the number and 3
h the square of 4 less than 7 times the number
i half the difference between 4 times the number and 9
j the product of 3 more than the number and 3 less than the number

10 Simplify if possible.
a $7c^2 + 4r + 3c^2 - 9r$ **b** $12m^2 - 7g - 8m^2 + 5g$
c $4x^2 - 3x + 2 + 6x^2 - 5x$ **d** $3ac^2 + 4ce^2 - 7ac^2 - 4ce$
e $7b^2 - 3ab + 4 - 10b^2 + 5ab - 8$ **f** $3a^2 + 2ac - a^2 - 5ac$
g $4mn + 2m \times 3n^2 - m \times n$ **h** $3m \times 2m + 5m^3 - 4mn + mn$
i $g^2 + 5g^3 + g \times 3g - 7$ **j** $ab \times 5ab - 3ab^2 + 7ab$
k $3p \times 2q \times 4r + p^2 \times 8qr$ **l** $p \times q + q \times r + r \times p$
m $8pq^2 - 6pq + 4p^2q + q^2p$ **n** $2m \times 3f \times 4r^2 - 4m \times 5$
o $6y \times 5xyz - 3xyz^2 + xy^2z$ **p** $16a \div 4 \times 5a + a^2 - 6a^3$
q $^-4a \times 3b + {}^-6a \times {}^-4b - ab$ **r** $7t - 4t^3 + 6t \times 4t - 8t \times 6$

11 Find the values of the following formulas for the values given.
a $M = 5c + 2d$ when $c = 2$ and $d = 5$ **b** $p = 4e - 3q$ when $e = 8$ and $q = 3$
c $a = m(3m - 4)$ when $m = 5$ **d** $d = 2s(3y - s)$ when $s = 6$ and $y = 4$
e $h = \dfrac{7g - 4r}{3}$ when $g = 6$ and $r = 9$ **f** $v = \dfrac{2w + 5z}{4w - 3z}$ when $w = 3$ and $z = 2$
g $y = 4x^2 - 5x + 3$ when $x = 3$ **h** $s = 4ut - t^2 + 5u$ when $u = 6$ and $t = 7$

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12 Substitute the given values in each of the following formulas and solve the resulting equation for the remaining variable. See Example 6

a $A = bh$, $A = 45\text{ m}^2$, $h = 3\text{ m}$

b $s = vt$, $s = 100\text{ m}$, $v = 8\text{ m/s}$

c $V = bhw$, $V = 450\text{ cm}^3$, $h = 9\text{ m}$, $w = 10\text{ m}$

d $V = \pi r^2 h$, $V = 600\text{ cm}^3$, $r = 5\text{ cm}$

e $A = \dfrac{(a+b)h}{2}$, $A = 400\text{ m}^2$, $a = 15\text{ m}$, $b = 23\text{ m}$

f $s = ut + \frac{1}{2}at^2$, $a = 9.8\text{ m/s}^2$, $s = 50\text{ m}$, $t = 3\text{ s}$

g $E = \frac{1}{2}mc^2$, $E = 1.2 \times 10^9\text{ J}$, $c = 3 \times 10^8\text{ m/s}$ (answer in kg)

h $PV = nRT$, $P = 1$, $V = 22.4$, $T = 293$, $n = 1$ (answer in atm-L/mole/K)

13 Transpose the following formulas so that the letter shown in brackets is the subject. See Example 7

a $v = u(2 - 3at)$ (t)

b $G = k(I - ct)$ (c)

c $m = \dfrac{b-a}{c-a}$ (b)

d $a = \dfrac{u + at}{s}$ (u)

e $\dfrac{a}{b} = \dfrac{c}{d}$ (d)

f $v = \sqrt{\dfrac{a-b}{c}}$ (a)

g $a^2 = 2b + c^2$ (b)

h $d = \sqrt{bc} - 5$ (c)

14 Simplify each of the following. See Example 3

a $4a^2c^2k^3 \times 9a^3c^2k^3$

b $3hux^3 \times 2hux^3$

c $6k^3p^3r^3 \times 5k^3p^2r^3$

d $e^3ny \times 8e^3n^2$

e $5cn \times 4cn^2w$

f $4a^2g^3n^2 \times 3a^2g^3n^2$

g $2ah^2k^3 \times 7ah^3k^2$

h $5a^2h^3v^2 \times 2a^2hv$

i $9e^3p^2t^2 \times 7ept$

j $6h^2ru \times 7h^2ru^2$

k $8bhk^2 \times b^3h^3$

l $5bq^2t^3 \times 9b^3q^3t$

m $63k^2p^2t^2 \div 54k^3p$

n $2e^2u^2v^3 \div 5eu^3v$

o $24a^3g^2 \div 21a^3g^3u^3$

p $5b^2p^2 \div 45b^2p^3u^2$

q $15c^2q^2 \div 10c^2q^2v$

r $42ab^2h \div 35a^2b^2h^3$

s $24c^2g^2k^2 \div 4c^2g^2k$

t $12h^2u^3y^3 \div 14h^2uy^2$

u $12c^3u^3v^2 \div 32cu$

v $60a^3c^3h^2 \div 10a^2c^3h^3$

w $72g^2nu \div 64g^3nu$

x $45m^3qu^3 \div 54m^2q^3u$

y $54b^2c^2 \div 9bcn^3$

z $15q^2v^2 \div 25qv$

15 Simplify each of the following.

a $9h^7n^2u^6 \div 6h^2n^9u^8$

b $4r^3t^{10}w^2 \div 5r^3t^2w$

c $27e^3p^6t^4 \div 45e^{10}p^{10}t^7$

d $2a^{12}c^9 \times 1a^7c^9$

e $4b^5q^6 \times 5b^9q^5v^3$

f $25a^8pu^7 \div 10a^{12}p^9u^3$

g $4n^4p^{11}v^9 \times n^8p^7v^{10}$

h $4a^3t^{10}y^2 \times 3a^{12}t^2y^9$

i $16m^{12}q^7r^9 \div 8mq^2r^4$

j $40h^9m^{11}u^{10} \div 10h^6m^9u^8$

k $3b^8c^4h^9 \times 5b^{12}c$

l $2a^2b^8p^6 \times 1a^4b^6p^8$

m $12k^{12}u^8w^7 \div 3ku^5w^9$

n $b^{10}h^{11}r^7 \times 1b^7h^8r^6$

o $16k^9n^{10}p^7 \div 20k^8n^8p^{11}$

p $15a^8g^6m^9 \div 12a^5g^{12}m^{11}$

q $2a^2c^9 \times 1a^5c^4g^{11}$

r $3r^5t^4u^6 \div 15r^{11}t^{12}u^3$

s $24b^5e^3n^8 \div 8b^6e^5$

t $21e^7m^2u^{11} \div 35e^8m^8u^{12}$

u $28b^4m^4t^7 \div 35b^{11}m^5t^6$

v $16q^{11}t^8w^7 \div 24q^{12}t^9w^8$

w $4b^8h^8m^3 \times 3b^{12}h^8m^4$

x $5b^{11}g^{12} \times 4b^{11}g^6v^{12}$

y $e^6p^{12}q \times 5e^8p^4q^4$

z $4g^2n^{12}v^5 \times 3g^7n^6v^6$

16 Simplify each of the following.

a $a^{-4}b^5 \times {}^{-}5a^2b^2g^4$
b $15h^{-1}k^{-3}x^3 \div 20h^3k^4x^2$
c $56a^{-4}c^5m^{-5} \div 8a^3c^4m^4$
d $24ce^3h^{-4} \div {}^{-}54c^{-5}eh^{-2}$
e $^{-}3a^{-3}n^{-1} \times 4a^5np^3$
f $^{-}5k^{-3}p^{-3}r^{-5} \div {}^{-}5kp^5r^{-3}$
g $^{-}3b^{-4}r^{-3}t^4 \div 27b^{-5}t^3$
h $b^5 \times {}^{-}9b^2p^{-2}q^2$
i $40a^5c^{-5}m^3 \div {}^{-}56a^{-2}c^2m^2$
j $^{-}21b^{-1}n^4 \div {}^{-}7b^2n^{-1}r$
k $4ae^{-3}k \times a^{-4}e^{-1}k^{-4}$
l $8c^3q \div {}^{-}20c^3q^5t^{-4}$
m $^{-}3b^{-5}h^3k^{-5} \div 2b^{-2}h^5k^5$
n $8b^5k^{-1}p^{-3} \div {}^{-}40b^{-3}k^{-3}p^3$
o $8m^5pr^2 \times 9m^2p^4r^3$
p $2a^3c^{-4}v^4 \times a^3c^3v$
q $6c^{-1}g^{-3}p^3 \times {}^{-}7c^5g^3p^{-3}$
r $^{-}12m^2q^4t^{-5} \div 28m^{-4}q^{-1}$
s $7b^5e^2k^4 \div {}^{-}4b^{-5}e^{-1}k^3$
t $^{-}20b^5h^5v^4 \div 5b^{-1}h^{-3}v^2$
u $^{-}2a^{-2}e^{-1}m^{-5} \times 3a^{-1}e^5m^4$
v $7e^2n \times 8e^3m^{-1}n^2$
w $5g^{-3}k^{-2}t \times 7g^4k^{-3}t^{-4}$
x $42e^{-5}g^{-5}m^{-5} \div 7e^3g^5m^3$
y $20c^4g^4h^5 \div {}^{-}4c^{-4}g^{-3}$
z $5g^{-3}k^3c^4 \times -g^4k^{-3}c^{-5}$

"SOMETIMES IT DOES, SOMETIMES IT DOESN'T."

17 Simplify each of the following.

a $^{-}35a^{-1}(e^{-1}k)^3 \div {}^{-}15a^3e^{-2}$
b $3a(c^{-2}u^2)^{-3} \times {}^{-}4(a^{-2}c^{-3})^{-2}u^{-3}$
c $^{-}6(n^2r^3u^3)^{-2} \times 7(n^{-3}r^{-3}u^2)^{-1}$
d $28a^{-3}(k^{-2})^3 \div 12a^{-2}(k^{-2}u^{-1})^{-2}$
e $^{-}8(c^{-2}t^3)^2 \div {}^{-}24(c^{-2}t^{-2})^{-3}u^{-2}$
f $36a^{-3}(k^{-1}t^{-2})^{-1} \div 30(a^{-3}k^{-2})^{-3}t^{-1}$
g $7(c^2k^{-2})^4m \times {}^{-}6c^2(k^3m^{-2})^{-1}$
h $^{-}2h^{-1}(p^3)^3 \times {}^{-}7h^{-1}(p^{-2}q^2)^3$
i $15c^{-2}(gm)^{-3} \div {}^{-}27(c^2g^3)^{-2}m^{-2}$
j $3b^3(p^3r^{-1})^{-2} \times 8(bpr^3)^{-1}$
k $^{-}3(a^2g^3k^{-2})^4 \times 8(a^{-3}g^2)^{-3}k^2$
l $^{-}5a^{-1}(c^{-2}e)^3 \div 2(a^{-2}c^{-3})^{-4}$

18 Simplify each of the following.

a $^{-}12e^7k^{-8}u^{12}z^3 \times -e^{-2}k^{-8}u^{10}z^{-6}$
b $24m^{-4}n^9v \div 20m^7v^{-12}w^{-11}$
c $12a^{-2}m^{-8}u^2v^{-2} \times {}^{-}5a^{11}u^{-12}v^5$
d $70kn^7w^{-11}z^{-10} \div {}^{-}60k^{11}n^{10}w^{-12}z^4$
e $^{-}6a^{-10}e^{10}z \div {}^{-}24a^{11}e^{-5}z^7$
f $^{-}7g^{11}mr^{-11} \times 5g^{-3}m^3r^{-11}v^{-11}$
g $^{-}15a^{10}k^8z^6 \div {}^{-}25a^4k^{-7}y^6z^5$
h $3q^{12}u^{-4}v^9w^{-11} \times 10q^{-3}u^{-1}vw^7$
i $^{-}21g^3k^{10}w^{-11}x^{12} \div {}^{-}30g^{-2}k^{-3}w^{-12}x^{-5}$
j $4g^{10}m^{-4}n^{-1} \times 7g^{-12}m^{-9}n^{-10}u^{-9}$
k $^{-}8b^{-3}h^5k^{-9} \div {}^{-}20b^{11}eh^3k^{-11}$
l $^{-}12n^{12}p^2r^3x^4 \times {}^{-}11n^{11}r^{-11}x^{-6}$
m $^{-}12a^2eq^9 \times {}^{-}13a^8eq^{10}$
n $7b^{-3}m^{-5}w^{-10}z^8 \times 9b^{-9}m^4w^5z^{-10}$
o $14e^6k^2m^{-12}t^{-5} \div {}^{-}91e^{10}k^2m^8t^{-6}$
p $4c^4q^{-5}r^8 \times {}^{-}7c^9q^{-7}$
q $12a^2k^{-11}m^7u^{-3} \times a^{-12}m^4u^{-11}$
r $^{-}54h^{-11}q^{-1}v^{10}x^{-5} \div {}^{-}30h^6q^{-2}v^{-5}x^8$

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s $72n^{10}q^{-7}t^8u^{-9} \div {}^{-}66n^{12}t^3$ **t** $^{-}66a^{-9}n^7r^{-1}v^7 \div {}^{-}6a^5n^3v$

u $90c^{-10}m^{-6}p^{-6} \div 9c^7m^3p^{-8}q^{-6}$ **v** $^{-}13e^{12}q^{-9}u^{-8} \div -e^{-12}p^7q^{-2}u^{-10}$

w $32a^9y^{-8}z^{-3} \div {}^{-}104a^4n^9z^{-9}$ **x** $48g^3h^{-12}m^{-10}u^4 \div 42g^{-11}h^6m^{-5}$

y $^{-}7c^{-10}e^{11}n^{-7}q^{-7} \times 13c^6n^{10}q^{-12}$ **z** $70k^{-9}n^3ty^{-6} \div 130k^{11}n^{-8}t^{11}y^{10}$

19 Simplify each of the following. *See* Example 4

a $\dfrac{9u-7u}{9vu+6vu} \times v(qu+5qu)$ **b** $\dfrac{5x-3x}{10w-7w} \times \dfrac{3w+w}{3wx+2wx}$

c $\dfrac{10n-4n}{4k^2+3k^2} \times \dfrac{8tk-6tk}{2n^2+n^2}$ **d** $\dfrac{3xy+7xy}{2x^2+5x^2} \times \dfrac{9mx-7mx}{7ay-2ay}$

e $\dfrac{9ae-7ae}{9wz-6wz} \times \dfrac{7ez+8ez}{8gw+6gw}$ **f** $\dfrac{15kx^2-13kx^2}{15uw+11uw} \times \dfrac{13w^2+7w^2}{13x^2-11x^2}$

Problem solving

20 Write expressions for the perimeters of each of the following rectangles in terms of its width (w).

a The length is twice the width.

b The length is 4 cm more than the width.

c The width is 3 m less than the length.

d The width is one-third of the length.

e The length is 4 mm longer than twice the width.

f The length is 5 cm shorter than three times the width.

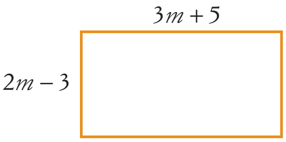
g The width is 7 m shorter than one-quarter of the length.

h The length decreased by 12 mm is equal to the width.

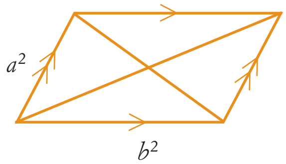
Worked solutions
Exercise 3.1
MAT10NAWS00007

21 Find expressions for the perimeters of the following shapes and simplify your expressions.

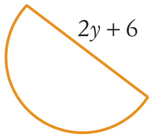
a



b



c



Worked solutions
Exercise 3.1
MAT10NAWS00007

22 Find the value of the nominated variable in each of the following formulas.

a u if $v = u + at$ and $v = 12$, $a = 2$ and $t = 3$

b m if $k = m(1 - ct)$ and $k = 28$, $c = 4$ and $t = 2$

c a if $s = ut + \frac{1}{2}at^2$ and $s = 26$, $u = 8$ and $t = 2$

d E if $I = \dfrac{E - V}{R}$ and $I = 4.2$, $V = 7$ and $R = 2.5$

e h if $d = 8\sqrt{\dfrac{h}{a}}$, $d = 24$ and $a = 5$

f g if $B = \sqrt{2gh}$ and $B = 20$ and $h = 20$

g t if $s = \dfrac{(u+v)t}{2}$ and $s = 15$, $u = 3$ and $v = 5$

h c if $a^2 = b^2 + c^2$ and $a = 24$ and $b = 10$

i w if $v = \dfrac{w-u}{gr}$ and $v = 14$, $u = 6.8$, $g = 9.8$ and $r = 2.5$

j r if $p = ghr + B$ and $p = 980$, $g = 9.8$, $h = 15$ and $B = 24.5$

Worked solutions
Exercise 3.1
MAT10NAWS00007

Reasoning

23 Explain the difference between a formula and an equation.

3.2 Expansion and factorisation

As you saw last year, you use the **distributive law** to **expand** brackets.

Important!

The distributive law

The product of the sum (or difference) of two numbers with another number is the same as the sum (or difference) of the products of that number with each separately.
This can be written in symbols as:

$$a \times (b + c) = a \times b + a \times c$$

$$\text{and} \quad a \times (b - c) = a \times b - a \times c.$$

The distributive law can be summarised as: $a(b \pm c) = ab \pm ac$.

When you are **expanding brackets**, you may want to write a subtraction as the addition of a negative. This helps you to avoid mistakes with signs.

Example 8

CAS TI-Nspire exercise
Algebra
MAT10NATI00003

CAS ClassPad exercise
Algebra
MAT10NACP00003

Simplify each of the following.

a $4(a + 2)$ **b** $2m(4 - 3m)$ **c** $^{-}3k(4k - 7)$

Solution

a Write the problem. $\quad \mathbf{4(a + 2)}$

Use the distributive law. $\quad = \mathbf{4 \times a + 4 \times 2}$

Simplify. $\quad = \mathbf{4a + 8}$

b Write the subtraction as adding a negative. $\quad \mathbf{2m(4 + {}^{-}3m)}$

Use the distributive law. $\quad = \mathbf{2m \times 4 + 2m \times {}^{-}3m}$

Simplify. $\quad = \mathbf{8m + {}^{-}6m^2}$

Write in the usual way. $\quad = \mathbf{8m - 6m^2}$

c Write the subtraction as adding a negative. $\quad \mathbf{{}^{-}3k(4k + {}^{-}7)}$

Use the distributive law. $\quad = \mathbf{{}^{-}3k \times 4k + {}^{-}3k \times {}^{-}7}$

Simplify. $\quad = \mathbf{{}^{-}12k^2 + 21k}$

Write with the positive term first. $\quad = \mathbf{21k - 12k^2}$

You may also need to use the index laws when expanding brackets.

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Example 9

Simplify each of the following.

a $6b^5k^4(6b^6 - 5v^3)$ **b** $^-5b^5z^{-6}(2b^{-2} - 9z^4)$

Solution

a Write the problem. $\mathbf{6b^5k^4(6b^6 - 5v^3)}$

Use the distributive law. $\mathbf{= 6b^5k^4 \times 6b^6 + 6b^5k^4 \times {}^-5v^3}$

Simplify and use the index law. $\mathbf{= 36b^{5+6}k^4 - 30b^5k^4v^3}$

$\mathbf{= 36b^{11}k^4 - 30b^5k^4v^3}$

b Write the subtraction as adding a negative. $\mathbf{{}^-5b^5z^{-6}(2b^{-2} + {}^-9z^4)}$

Use the distributive law. $\mathbf{= {}^-5b^5z^{-6} \times 2b^{-2} + {}^-5b^5z^{-6} \times {}^-9z^4}$

Simplify, using the index laws. $\mathbf{= {}^-10b^{5+{}^-2}z^{-6} + 45b^5z^{-6+4}}$

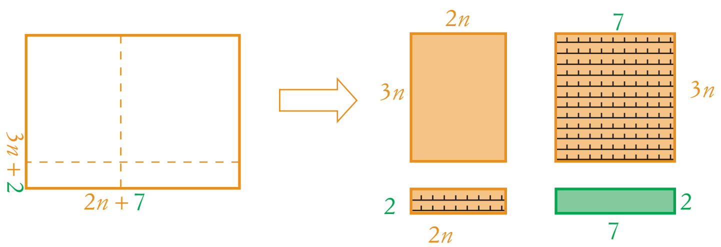
Simplify the indices and write the positive term first. $\mathbf{= 45b^5z^{-2} - 10b^3z^{-6}}$

Two brackets multiplied together are called **binomial brackets**. These can be expanded in several different ways.

Investigate: Binomial brackets

The Area method

You expand the binomial brackets $(3n + 2)(2n + 7)$ by considering the area of a rectangle with a height of $(3n + 2)$ and width $(2n + 7)$.
Then divide the height and width into their parts to make smaller rectangles as shown below.



Area $= (3n + 2)(2n + 7)$
Total area $= 3n \times 2n + 3n \times 7 + 2 \times 2n + 2 \times 7$
The areas are the same, so $(3n + 2)(2n + 7) = 6n^2 + 21n + 4n + 14$

$= 6n^2 + 25n + 14$

Technology: GeoGebra
Simplifying expressions
MAT10NATC00003

Weblink
Expanding brackets using the grid method
MAT10NAWB00003

The Table method

You use a table instead of a rectangle, but still multiply widths and heights. Then you put the answers in the table as shown on the right.

	$2n$	$+7$
$3n$	$6n^2$	$21n$
$+2$	$4n$	14

Then add the answers to give

$$\begin{aligned}(3n+2)(2n+7) &= 6n^2 + 21n + 4n + 14 \\ &= 6n^2 + 25n + 14\end{aligned}$$

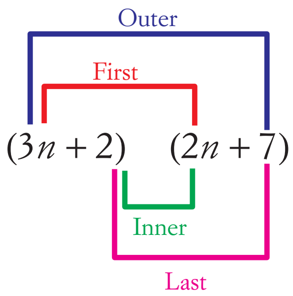
The Distributive Law method

You just apply the distributive law twice, so this is a purely algebraic method.

$$\begin{aligned}&(3n+2)(2n+7) \\ &= 3n(2n+7) + 2(2n+7) \\ &= 3n \times 2n + 3n \times 7 + 2 \times 2n + 2 \times 7 \\ &= 6n^2 + 21n + 4n + 14 \\ &= 6n^2 + 25n + 14\end{aligned}$$

The FOIL method

For this method, you just multiply the terms in each bracket in the order **F**irst terms, **O**uter terms, **I**nner terms and **L**ast terms.



For $(3n+2)(2n+7)$, you get:

First terms: $3n \times 2n = 6n^2$
Outer terms: $3n \times 7 = 21n$
Inner terms: $2 \times 2n = 4n$
Last terms: $2 \times 7 = 14$

Try each method for yourself.
Even though there is no such thing as a negative length, you can use negative lengths on the sides of the Area model for this investigation.
Your teacher might want you to use one method more than the others, but generally different people find that they prefer different models.

The FOIL method only works with binomial brackets. If there are more than 2 terms in one of the brackets, this method doesn't work. The distributive law method is the most general method, and applies to expansions of all kinds of brackets.

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Example 10

Simplify each of the following by expanding the brackets.

a $(x + 2)(x + 3)$ **b** $(a + 5)(a - 4)$ **c** $(k - 5)(k - 4)$

Solution

a Write the problem. $(x + 2)(x + 3)$

Expand using your chosen method. $= x^2 + 2x + 3x + 6$

Simplify. $= x^2 + 5x + 6$

b Write the problem. $(a + 5)(a - 4)$

Expand using your chosen method. $= a^2 - 4a + 5a - 20$

Simplify. $= a^2 + a - 20$

c Write the problem. $(k - 5)(k - 4)$

Expand using your chosen method, being very careful with signs. $= k^2 - 4k - 5k + 20$

Simplify. $= k^2 - 9k + 20$

The methods work just as well when there is a second variable. If the variables are not in the same order in both brackets, you should change the order before starting the expansion.

Example 11

Expand and simplify each of the following.

a $(3x + 2y)(4x - 3y)$ **b** $(4 - 5c)(2c - 3)$ **c** $(3p - 4m)(3m + 4p)$

Solution

a Write the problem. $(3x + 2y)(4x - 3y)$

Expand using your chosen method. $= 12x^2 - 9xy + 8xy - 6y^2$

Simplify. $= 12x^2 - xy - 6y^2$

b Write the problem. $(4 - 5c)(2c - 3)$

Change the order in the first bracket. $= (^-5c + 4)(2c - 3)$

Expand using your chosen method. $= {}^-10c^2 + 15c + 8c - 12$

Simplify, and write the positive term first. $= 23c - 10c^2 - 12$

c Write the problem. $(3p - 4m)(3m + 4p)$

Changing the order in the second bracket is easier than the first bracket. $= (3p - 4m)(4p + 3m)$

Expand using your chosen method. $= 12p^2 + 9mp - 16mp - 12m^2$

Simplify. $= 12p^2 - 7mp - 12m^2$

CAS TI-Nspire exercise
Algebra
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CAS ClassPad exercise
Algebra
MAT10NACP00003

When you are expanding several sets of brackets, you will probably have to collect like terms to finish the simplification.

Puzzle sheet
The distributive law
MAT10NAPS00008

Example 12

Expand and simplify each of the following.

a $3a(2a+5)-2(a-8)$

b $2e(a+3e-6h)-h(9a-4e+4h)-5a(7a-3e+8h)$

c $(8r-9m)(r-4m)-(9r-m)(6r+m)$

d $8m^{-5}p(9h^{-1}m^5p^{-6}-h^2m^{-3}+9m^{-3}p^4)-7h^{-1}m^{-3}(h^3m^{-5}p-4m^3p^{-5}-5hp^4)$

Solution

a Write the problem. $\quad 3a(2a+5)-2(a-8)$

Expand the brackets, being careful with signs. $\quad =6a^2+15a-2a+16$

Simplify. $\quad =6a^2+13a+16$

b Write the problem. $\quad 2e(a+3e-6h)-h(9a-4e+4h)-5a(7a-3e+8h)$

Expand the brackets, being careful with signs. $\quad =2ae+6e^2-12eh-9ah+4eh-4h^2-35a^2+15ae-40ah$

Simplify. $\quad =6e^2-35a^2-4h^2+17ae-49ah-8eh$

c Write the problem. $\quad (8r-9m)(r-4m)-(9r-m)(6r+m)$

Expand the brackets, being careful with signs. $\quad =8r^2-32mr-9mr+36m^2-54r^2-9mr+6mr+m^2$

Simplify. $\quad =37m^2-44mr-46r^2$

d Write the problem. $\quad 8m^{-5}p(9h^{-1}m^5p^{-6}-h^2m^{-3}+9m^{-3}p^4)-7h^{-1}m^{-3}(h^3m^{-5}p-4m^3p^{-5}-5hp^4)$

Expand the brackets, being careful with signs and powers. $\quad =72h^{-1}p^{-5}-8h^2m^{-8}p+72m^{-8}p^5-7h^2m^{-8}p+28h^{-1}p^{-5}+35m^{-3}p^4$

Simplify. $\quad =100h^{-1}p^{-5}-15h^2m^{-8}p+35m^{-3}p^4+72m^{-8}p^5$

Investigate: Special expansions

Expand each of the following expressions.

a $(x+3)(x-3)$ **b** $(x-4)(x+4)$ **c** $(4x+1)(4x-1)$
d $(5x-1)(5x+1)$ **e** $(3x+7)(3x-7)$

Write a rule for the expansions of $(x+y)(x-y)$ and $(x-y)(x+y)$.
Now expand the following expressions. Note that you can write $(x+1)^2=(x+1)(x+1)$.

a $(x+1)^2$ **b** $(x-3)^2$ **c** $(x+4)^2$ **d** $(5x-3)^2$ **e** $(7x+5)^2$

Write rules for the expansions of $(x+y)^2$ and $(x-y)^2$.

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You should learn the special expansions below.

Important!

Special expansions

The special expansion $(x + y)(x - y) = x^2 - y^2$ is called the **difference of two squares**.
The special expansion $(x + y)^2 = x^2 + 2xy + y^2$ is called the **square of a sum**.
$(x - y)^2 = x^2 - 2xy + y^2$ is called the **square of a difference**.
Sometimes they are both just called **perfect squares**.

Example 13

Expand each of the following expressions.

a $(m + 4)^2$ **b** $(p - 3)^2$ **c** $(r - 6)(r + 6)$

Solution

a Identify the product. $(m + 4)^2$ **is a square of a sum**

Apply the pattern. $(m + 4)^2 = m^2 + 2 \times m \times 4 + 4^2$

Simplify. $= m^2 + 8m + 16$

b Identify the product. $(p - 4)^2$ **is a square of a difference**

Apply the pattern. $(p - 4)^2 = p^2 - 2 \times p \times 4 + 4^2$

Simplify. $= p^2 - 8p + 16$

c Identify the product. $(r - 6)(r + 6)$ **is a difference of squares**

Apply the pattern. $(r - 6)(r + 6) = r^2 - 6^2$

Simplify. $= r^2 - 36$

Video tutorial
Special binomial products
MAT10NAVT10006

CAS TI-Nspire exercise
Algebra
MAT10NATI00003

CAS ClassPad exercise
Algebra
MAT10NACP00003

Example 14

Expand each of the following expressions.

a $(3a + 5c)(3a - 5c)$ **b** $(4q - 7r)^2$ **c** $(7y + 5)^2$

Solution

a Identify the product. $(3a + 5c)(3a - 5c)$ **is a difference of squares**

Apply the pattern. $(3a - 5c)(3a + 5c) = (3a)^2 - (5c)^2$

Simplify. $= 9a^2 - 25c^2$

b Identify the product. $(4q - 7r)^2$ **is a square of a difference**

Apply the pattern. $(4q - 7r)^2 = (4q)^2 - 2 \times 4q \times 7r + (7r)^2$

Simplify. $= 16q^2 - 56qr + 49r^2$

c Identify the product. $(7y + 5)^2$ **is a square of a sum**

Apply the pattern. $(7y + 5)^2 = (7y)^2 + 2 \times 7y \times 5 + 5^2$

Simplify. $= 49y^2 + 70y + 25$

Puzzle sheet
Expanding binomials
MAT10NAPS00007

The opposite of expansion of brackets is **factorisation**. In the simplest kind of factorisation, you find a common factor of the terms and use the distributive law in reverse to make brackets.

Important!

Factorisation

Factorisation is the opposite process to multiplying out brackets. In the **common factor** method, the **highest common factor** of the terms is put outside the brackets, leaving the other factors of each term inside the brackets.

Sometimes the terms containing variables in an expression are all negative. In this case, it is normal to use a negative common factor. In other cases you should make the common factor positive.

Example 15

Factorise each of the following.

a $12p + 18$ **b** $28u - 35v + 42$ **c** $24 - 30w - 36v$

Solution

a Write the problem. $\mathbf{12p + 18}$

Write each term as a product of the HCF, 6. $= \mathbf{6 \times 2p + 6 \times 3}$

Use the distributive law in reverse. $= \mathbf{6(2p + 3)}$

b Write the problem. $\mathbf{28u - 35v + 42}$

Write each term as a product of the HCF, 7. $= \mathbf{7 \times 4u - 7 \times 5v + 7 \times 6}$

Use the distributive law in reverse. $= \mathbf{7(4u - 5v + 6)}$

c Write the problem. $\mathbf{24 - 30w - 36v}$

Choose $^-6$ as the HCF, since all the variable terms are negative. $= \mathbf{{}^-6 \times {}^-4 + {}^-6 \times 5w + {}^-6 \times 6v}$

Use the distributive law in reverse. $= \mathbf{{}^-6({}^-4 + 5w + 6v)}$

Rearrange the terms in the brackets to start with a positive term. $= \mathbf{{}^-6(5w + 6v - 4)}$

The highest common factor might involve one or more variables, and it could also include powers.

Example 16

Factorise each of the following.

a $24mnp + 18mp - 2mpq$ **b** $56q^2h^3 - 8eq^5h^7 - 16e^3q^5h^4$

Solution

a Write the problem. $\mathbf{24mnp + 18mp - 2mpq}$

Think of the terms as products of the HCF, $2mp$. $= \mathbf{2mp \times 12n + 2mp \times 9 - 2mp \times q}$

Use the distributive law in reverse. $= \mathbf{2mp(12n + 9 - q)}$

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b The HCF includes 8, q^2 and h^3. | $56q^2h^3 - 8eq^5h^7 - 16e^3q^5h^4$

Think of the terms as products of the HCF, $4q^2h^3$. | $= 8q^2h^3 \times 7 - 8q^2h^3 \times eq^3h^4 - 8q^2h^3 \times 2e^3q^3h$

Use the distributive law in reverse. | $= 8q^2h^3(7 - eq^3h^4 - 2e^3q^3h)$

When you factorise expressions with terms that have negative powers, it is usual to make sure that all the powers inside the brackets are positive. This means that you will include the most negative powers of each variable in the highest common factor.

Example 17

Factorise $9x^9y^{-5}z^3 - 12x^4y^{-2}z^{-1}$.

Solution

Write the problem and think of powers in the HCF.
$$9x^9y^{-5}z^3 - 12x^4y^{-2}z^{-1}$$

Think of the terms as products of the HCF, $3x^4y^{-5}z^{-1}$.
$$= 3x^4y^{-5}z^{-1} \times 3x^5z^4 - 3x^4y^{-5}z^{-1} \times 4y^3$$

Use the distributive law in reverse.
$$= 3x^4y^{-5}z^{-1}(3x^5z^4 - 4y^3)$$

Write the negative powers as a denominator.
$$= \frac{3x^4(3x^5z^4 - 4y^3)}{y^5z}$$

Exercise 3.2 Expansion and factorisation

Understanding

Extra questions
Exercise 3.2
MAT10NAEQ00008

1 Expand and simplify each of the following.

See Example 8
See Example 10

a $a(a + 5)$ **b** $m(m - p)$ **c** $g(2g + 3f)$
d $z(4z - 3w)$ **e** $x(3x + 2y)$ **f** $c(8c - 7)$
g $-m(5m - 3n)$ **h** $-f(2f - 5h)$ **i** $-d(2d + 3e)$
j $^{-}4c(2c - 7)$ **k** $^{-}3n(2n + 6)$ **l** $^{-}5k(3c - 2k)$

2 Expand and simplify each of the following.

a $(x + 4)(x + 3)$ **b** $(1 + b)(2 + b)$ **c** $(a + 4)(a + 5)$
d $(6 + a)(2 + a)$ **e** $(y + 1)(y + 6)$ **f** $(a + 1)(a + 2)$
g $(m + 5)(m + 7)$ **h** $(a + 2)(a + 6)$ **i** $(n + 2)(n + 4)$

3 Factorise each of the following.

See Example 15

a $^{-}70q - 30$ **b** $75q - 40$ **c** $28y - 35$
d $44a - 56$ **e** $^{-}72v - 81$ **f** $130c - 143$
g $^{-}90c - 75$ **h** $91t - 117$ **i** $^{-}45u - 165$
j $^{-}18n - 26$ **k** $^{-}28w - 210$ **l** $70r + 126$
m $48b + 20$ **n** $90g + 63$ **o** $^{-}22x - 12$

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p $78k + 143$ **q** $^{-}60u - 156$ **r** $20p + 5$
s $^{-}66v - 77$ **t** $80h + 120$ **u** $^{-}143x - 55$
v $33e + 39$ **w** $77m - 35$ **x** $140t + 154$
y $^{-}7a - 35$ **z** $55k - 10$

See Example 11

4 Expand each of the following.

a $(a + 1)(2a + 3)$ **b** $(2a + 1)(a + 3)$ **c** $(3x + 2)(x + 4)$
d $(2y + 3)(4y + 5)$ **e** $(5g + 1)(7g - 2)$ **f** $(3a + 2)(6a - 2)$
g $(7m - 1)(7m + 1)$ **h** $(3x - 2)(5x - 3)$ **i** $(5y + 4)(5y + 4)$
j $(4y + 3)(3y + 4)$ **k** $(7n + 5)(7n - 5)$ **l** $(4 - 2c)(2 + 4c)$

See Example 13

5 Complete each of the following using special expansions.

a $(a + b)^2 = a^2 + 2ab + \ldots$ **b** $(x + y)^2 = x^2 + \ldots + y^2$
c $(w - r)^2 = w^2 - 2rw + \ldots$ **d** $(a + 2)^2 = a^2 + 4a + \ldots$
e $(x - 3)^2 = x^2 - \ldots + 9$ **f** $(m - 5)^2 = \ldots - 10m + 25$
g $(g + 7)^2 = g^2 + 14g + \ldots$ **h** $(6 + b)^2 = 36 + \ldots + b^2$
i $(3 - n)^2 = \ldots - 6n + n^2$ **j** $(k + 8)^2 = k^2 + 16k + \ldots$

Fluency

6 Expand each of the following.

a $3(a^2 - 3a - 6)$ **b** $7(x - 3y + 4z)$ **c** $4(x^2 + 5x + 7)$
d $6(2m - n + 3p)$ **e** $3m(m - 3n - 2p)$ **f** $2g(4 - 3g + a)$
g $^{-}4p(p - 3q + 2r)$ **h** $^{-}3x(x + 3y - 5z)$ **i** $^{-}9y(y - 2x + 5z)$
j $x(x^2 + 4x - 3)$ **k** $2y(y^2 - 7y + 6)$ **l** $4cd(2c^2 + 3d - 3)$

7 Simplify each of the following.

a $(x + 2)(x + 3)$ **b** $(x + 1)(x + 4)$ **c** $(a + 3)(a + 5)$
d $(x - 3)(x + 6)$ **e** $(a - 10)(a - 9)$ **f** $(m - 7)(m + 11)$
g $(x + 5)(x - 5)$ **h** $(y - 7)(y + 3)$ **i** $(x + 12)(x + 5)$
j $(k - 9)(k + 11)$ **k** $(x + 6)(x - 5)$ **l** $(2 + n)(9 - n)$

See Example 12

8 Simplify each of the following.

a $4(x + 5) + 9(x + 3)$ **b** $7(2a + 3) + 6(3a - 1)$ **c** $10(3d + 5) - 7$
d $5(3a + 4) + 3(a - 3)$ **e** $6(m - 7) + 4(m + 10)$ **f** $5(b - 2) - (3 - 4b)$
g $5(a + 6) - 3(2a + 1)$ **h** $2(3x + 7) - 4(7 + x)$ **i** $7(3x - 2) - 3(3 - 4x)$

9 Simplify each of the following.

a $x(x + 6) + x(x + 3)$ **b** $b(b + 2) + b(b + 5)$
c $m(m - 5) - m(m - 3)$ **d** $c(c + 9) - c(c - 8)$
e $y(4y - 3) - y(4 - y)$ **f** $h(2h - 3) - h(5 + 2h)$
g $a(5a - 3) + a(5 - 4a)$ **h** $c(5c - 3) + c(2 - 8c)$
i $2p(3p - 5) - 4p(7 - 6p)$ **j** $3x(5 - 2x) - 2x(x - 6)$

10 Simplify each of the following.

a $3(2x + 4y - z) + 2(x - 3y + 5z)$ **b** $2(3a - 4b + c) + 5(a + 2b - c)$
c $5(5a + 2b - 3c) - 2(a - 2b - c)$ **d** $3(2x - 3y - 4z) + 7(x - y + z)$
e $x(x^2 - 8x - 4) + 3x(x^2 - 6x - 5)$ **f** $a(a^2 - 5a + 1) + 3a(a^2 - 2a + 7)$
g $2x(3x^2 - 2x - 4) + 6x(x^2 - 2x + 3)$ **h** $9a(a^2 + 2a - 8) + 3a(a^2 - 4a - 2)$
i $3x(5x^2 - x + 2) - 2x(x^2 + 2x + 4)$ **j** $6a(4 - 2a + 3a^2) - 5a(2a^2 - 7 - 3a)$

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11 Factorise each of the following.

a $^{-}144b - 156e - 12$ **b** $156p - 36u - 36$ **c** $42x + 45k + 18$
d $144m - 156t + 108$ **e** $35z + 77m - 28$ **f** $^{-}90m - 63e - 72$
g $^{-}18b - 33g - 6$ **h** $24q + 26w + 30$ **i** $48x - 20e + 24$
j $^{-}84t - 35x - 91$ **k** $130p - 90n - 110$ **l** $^{-}60w - 44c - 52$
m $12a - 45h + 3$ **n** $5p - 55r + 35$ **o** $84q - 54e + 30$
p $^{-}104c - 8p - 16$ **q** $120x - 180m + 195$ **r** $40t + 88a - 104$
s $105h + 135y + 135$ **t** $110k - 120m + 20$ **u** $5y - 75r - 40$
v $26h - 4x - 24$ **w** $42v - 182n - 112$ **x** $40r - 60m - 75$
y $40c + 52m + 36$ **z** $16k - 120x - 48$

12 Simplify each of the following. *See* Example 9

a $6h^{-6}m(2h^{-4} - 9m^4)$ **b** $^{-}3n^{-5}x^3(6x^{-5} + 5z^3)$ **c** $6t^3w^{-3}(8x^4 - 5t^{-6})$
d $2a^{-1}c(6a^{-6} - 7w^{-6})$ **e** $3b^{-1}x^{-6}(5 - 7x^4)$ **f** $2b^6n^{-1}(4n^6 - 3b)$
g $5a^{-1}k^6(8a^{-3} - 5k^{-2})$ **h** $8a^2b^6(6b^{-1} + 7k^7)$ **i** $-k^6t^4(4x^{-1} + 3)$
j $4b^{-1}c^{-5}(2b^{-7} + 7z^{-1})$ **k** $3g^{-2}p^4(8r^{-1} + p^4)$ **l** $7m^{-2}q^{-4}(6 + 7m^5)$
m $5k^3m^{-5}(5k^3 + 3m^{-2})$ **n** $5h^{-6}k^{-2}(7k^4 + 3p^7)$ **o** $^{-}5k^{-7}r^4(9v + 5k^{-5})$
p $8e^6h^{-3}(3e^{-5} - 5p^5)$ **q** $7h^{-3}r^{-1}(4z^{-2} + 5r^{-3})$ **r** $^{-}4a^{-6}y^{-3}(9y^{-6} + 4a^{-7})$
s $9k^2(2e^2 + k^{-3})$ **t** $5u^{-3}(4u^{-6} + 1)$ **u** $6e^2n^{-3}(5v - 8e^{-3})$
v $5r^6t^6(7r^2 - 2y^{-5})$ **w** $^{-}8c^{-5}t(2z^6 + 9)$ **x** $-n^{-3}t^3(8t + 7n^{-2})$
y $8m^{-2}w^4(9m^3 + 4w^2)$ **z** $2g^5t^{-4}(2t^7 + 9u^2)$

13 Simplify each of the following.

a $(a - 3)(4 + 2a)$ **b** $(5 - 2x)(3 - x)$ **c** $(3 - 5a)(4 + a)$
d $(k - 4)(2k - 5)$ **e** $(^{-}2m + 1)(3 - m)$ **f** $(3x + 1)(7 - 2x)$
g $(2p + q)(^{-}2p - q)$ **h** $(2n - 5m)(3m + n)$ **i** $(2x + 3y)(3x + 2y)$
j $(4x - 3y)(2y - 7x)$ **k** $(8a - 5b)(7b - 2a)$ **l** $(3y + x)(-x - y)$
m $(8u + 12e)(8u - 12e)$ **n** $(8g + 5v)(9g + 4v)$ **o** $(8e + 11t)(10e + 11t)$
p $(11a - 9x)(7a + 5x)$ **q** $(11x - 2r)(15x - 1r)$ **r** $(14c + 3z)(9c + 1z)$
s $(14x - 13m)(14x - 3m)$ **t** $(8x + 9p)(7x + 15p)$ **u** $(2v + 1c)(3v - 4c)$
v $(12p - 5z)(5p + 4z)$ **w** $(7y - 13r)(15y + 1r)$ **x** $(2m - 15q)(11m - 12q)$
y $(2z + 13u)(7z - 15u)$ **z** $(14z - 1a)(5z + 3a)$

14 Simplify each of the following.

a $(a + b)^2$ **b** $(x - y)^2$ **c** $(t + 2)^2$
d $(g + 3)^2$ **e** $(9 - h)^2$ **f** $(a + 6)^2$
g $(m + 11)^2$ **h** $(10 - b)^2$ **i** $(f - g)^2$
j $(u + 9)^2$ **k** $(8 + v)^2$ **l** $(y - 13)^2$

15 Simplify each of the following.

a $(a + 2)(a - 2)$ **b** $(x - 3)(x + 3)$ **c** $(e + 5)(e - 5)$
d $(n + 4)(n - 4)$ **e** $(j - 9)(j + 9)$ **f** $(q + 4)(q - 4)$
g $(h + 2)(h - 2)$ **h** $(5 + w)(w - 5)$ **i** $(a + r)(a - r)$
j $(p + t)(t - p)$ **k** $(y - g)(g + y)$ **l** $(a - p)(a + p)$

16 Simplify each of the following.

a $^{-}7c(6c + 5q - 5x) - q(c - 4q - 6x) - 6x(9c + 7q - 2x)$

b $5n(7n + 4t + 7x) + x(6n - t + 4x) + 8t(7n + 8t - 6x)$

c $9e(5a - 7e + 6m) + 3a(5a + 1e + 5m) + m(4a - e - 6m)$

d $u(p + 1u - 7y) + 5y(3p + u - 5y) + p(3p - 5u + 4y)$
e $^{-}8y(k + 5x + 9y) + 6x(5k - 7x - 9y) + 2k(4k - 9x + 6y)$
f $^{-}2p(8a - 5e - 7p) - 2a(7a - 9e + 6p) + e(7a + 4e + 8p)$
g $^{-}6a(3a + 2m - 6t) - 4m(5a - 6m + t) - 5t(7a - 6m + 9t)$
h $8e(9e - 2u + 8y) - 4y(3e - u + 9y) + 3u(4e - 7u - 4y)$
i $-r(5q + 4r + u) - 9q(5q - 7r + u) + 9u(7q - 8r - 2u)$
j $^{-}8w(6k - 7w + 7z) - 5z(4k - 3w - 4z) - 7k(8k - 9w - 6z)$
k $^{-}9v(7p + 9r + 8v) - 9r(5p + 1r - 4v) + 5p(2p + 1r + 6v)$
l $^{-}6u(7b + 2m - 6u) + 3b(3b - 5m + 3u) + 5m(5b - 2m - 8u)$
m $3g(9g - 8h - 2x) + 3h(7g + 9h + 9x) - 5x(9g - 8h + 3x)$
n $9e(2e - 3g - 4v) - 9v(9e - 5g - 6v) + 6g(6e - 5g - 7v)$
o $5p(c + 9p + 9r) - c(9c - 1p + 4r) - 8r(4c + p + 7r)$
p $^{-}4u(4b - 5u + 5v) - v(7b + 8u - 6v) + 7b(b - 4u - 2v)$
q $2e + 7h - 2m - 6h(8e + 9h - 3m) - 9e(2e - 7h + m)$
r $48a + 5e + 8z + a(5a + 2e + z) - 6e(a + 4e + 9z)$
s $3e(2e - 5h - 6q) - 2h(3e + 7h - 3q) - 8q(7e - 4h - 8q)$
t $^{-}2b(9b + 4p + 6y) + 2y(2b + 7p + 4y) - 6p(b - 4p + 8y)$
u $3u(5q + 6u + 5w) + 9q(6q - 1u - 7w) + 7w(8q + 3u - 5w)$
v $7b(a + 5b + 8u) + 5u(4a + 7b - 6u) - a(a - 6b + 9u)$
w $28a + 5c + 5h + 5c(7a + 2c - 9h) - 4a(a - 2c - 7h)$
x $^{-}5p(7h + 8n - p) + 3h(9h - 1n + 7p) + 7n(6h - 1n - 7p)$
y $^{-}5a(3a + 7c - 6n) - c(6a - 7c + 7n) + 2n(a - 3c + 8n)$
z $q(4q - 5t + 7x) + 5x(9q - 5t - 6x) - 6t(7q - 4t + 5x)$

17 Simplify each of the following.

a $(6u - t)(6u - 7t) + (7u - 8t)(7u - 6t)$
b $(5y + 4g)(7y + 3g) - (3y + 4g)(8y + 9g)$
c $(5p - 8n)(6p - 5n) - (p - 6n)(3p + 2n)$
d $(3b - 2p)(6b + p) - (7b - 9p)(8b - p)$
e $(3c + 2g)(9c + 7g) - (2c - 3g)(8c + 9g)$
f $(6t - 7a)(9t - 7a) - (t - a)(5t - 8a)$
g $(8c + q)(2c - q) + (6c + 5q)(3c - 7q)$
h $(3p + 5v)(7p - 8v) + (9p + 5v)(6p + v)$
i $(8q + r)(2q - 7r) + (7q - 2r)(q - 7r)$
j $(5h + 7y)(7h - 4y) + (6h + 5y)(4h + 7y)$
k $(7z + 2h)(9z + h) - (3z + 5h)(5z - 9h)$
l $(4r - 7n)(7r + 4n) - (5r - 8n)(r - 2n)$
m $(9h + 7q)(5h + 6q) + (5h + 8q)(h + 9q)$
n $(7v + 4t)(4v + 3t) - (8v + 3t)(3v + 4t)$
o $(3m - 2y)(8m + 5y) - (7m + 3y)(5m - 2y)$
p $(5x + b)(9x - 2b) - (9x - b)(3x + 7b)$
q $(a - 6b)(7a - b) + (3a + 2b)(6a - b)$
r $(4c - p)(7c - 9p) - (8c - p)(c - 4p)$
s $(7a - 6h)(9a - 7h) - (9a + 2h)(7a - 4h)$
t $(3g - 7h)(5g + 9h) + (3g - 5h)(2g + 9h)$
u $(2n - 3c)(5n - 6c) + (n + 6c)(n - 5c)$
v $(4q - a)(9q - 8a) + (5q - 4a)(8q + 7a)$
w $(4h + 9p)(7h + 9p) - (3h - 5p)(7h - 9p)$
x $(9e + 4z)(7e + 4z) - (7e + 9z)(e - 5z)$
y $(8c + 3w)(8c + 7w) - (3c + 2w)(6c - w)$
z $(3u + 2h)(u - 3h) + (3u - 5h)(u - 4h)$

18 Simplify each of the following.

a $8q^{-7}r^{7}(9q^{-3}r^{-2}v^{2} - 8q^{2}v + 7r^{-3}v^{-2}) - 6r^{-2}v^{2}(q^{-7}r^{6}v^{-4} + 4q^{-10}r^{7} + 9r^{7}v)$
b $6k^{7}r^{4}(2k^{-2}r^{-4}u^{-1} + 2r^{-2}u^{-4} - 7k^{-1}u^{3}) - 9r^{-4}u^{-1}(3k^{7}r^{6}u^{-3} + 4k^{5}r^{4} - 5r^{4}u^{3})$
c $8k^{-3}q^{4}(9k^{3}q^{2}t - 7kt^{6} + 5q^{3}t^{2}) + q^{2}t(7k^{-3}q^{5}t + 9q^{4}t^{6} - 8k^{0}q^{4})$
d $8t^{6}u^{-6}(7g^{-6}tu^{12} - 6g^{-1}t^{2} + 5t^{2}u^{3}) - 7g^{-6}t^{2}(7g^{5}t^{6}u^{-6} + 6g^{-6}u^{3} + 9t^{5}u^{6})$
e $u^{-7}y^{3}(4ru^{2}y^{-10} - r^{-6}u^{2} - 2u^{2}y^{6}) + 7ru^{2}(4r^{-7}u^{-7}y^{3} - 6u^{-7}y^{-7} + 9r^{3}y^{6})$

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f $3h^3q(7b^{-5}h^3q^2 - h^3q^2 - 3b^{-2}h^3) - 6b^{-5}h^3(6b^3h^3q - bq^2 + h^3q^3)$

g $2a^{-4}n^7(6a^4n^5v^{-5} - a^{-5}v + 6n^4v^5) + 8n^5v^{-5}(9a^{-4}n^6v^{10} - 7a^0n^7 - 2n^7v)$

h ${}^{-}3m^{-6}t^5(4m^3t^6z^{-6} - 8t^3z^6 + 5m^{-6}z^5) + 8t^6z^{-6}(3m^{-6}t^2z^{12} - m^{-3}t^5 - t^5z^5)$

i $4g^2k^{-2}(4g^3k^{-3}u^2 + g^2u - k^3u^{-3}) + 5k^{-3}u^2(3g^2k^4u^{-5} - 3k^{-2}u - g^5k^{-2})$

j ${}^{-}8u^5w^{-1}(7b^2u^{-9}w^6 + b^5u^{-7} - 9u^{-7}w) - 9b^2u^{-7}(3b^3u^5w^{-1} - 2b^{-1}w - 4u^3w^5)$

k $7t^6w^{-4}(2q^4t^7w^{10} - q^{11}t^6 + 5t^6w^{-2}) - 7q^4t^6(2q^7t^6w^{-4} - 2t^7w^6 + q^{-4}w^{-2})$

l $q^{-5}t^{-5}(4g^{-3}q^{-2} + 6q^{-4}t^5 + 9g^{-6}q^{-4}) - 6g^{-3}q^{-4}(5g^{-3}q^{-5}t^{-5} - 3g^{-5}t^5 - 9q^{-3}t^{-5})$

m ${}^{-}6g^2n^2(g^2n^{-2}p^2 + g^2p^4 - 7n^2p^{-2}) - 7n^{-2}p^2(2g^2n^6p^{-4} - g^4n^2 - 3n^2p^4)$

n ${}^{-}8a^4w^2(9a^{-4}w^2y + w^{-4}y^2 + 8ay^4) + 9w^2y(3a^4w^{-4}y - 8a^0w^2 - 3w^2y^4)$

o $7e^5n^4(3e^{-4}n^6t^{-1} - 8e^{-1}t^{-6} + 9n^{-4}t^6) + 9n^6t^{-1}(6e^5n^{-6}t^7 + 5n^4t^{-6} - 7en^4)$

p ${}^{-}6n^{-3}t(6h^3n^4t^{-4} + 5h^8n^{-4} + 9n^{-4}t^3) + 5h^3n^{-4}(9h^5n^{-3}t + 2ht^3 + 8n^5t^{-3})$

q $5m^2p(8a^{-1}m^6p + 7a^4m^3 + 2m^3p^{-2}) + 6a^{-1}m^3(a^5m^2p + 9m^5p^2 - ap^{-2})$

r $9p^{-4}y^4(9g^{-4}py^{-8} - 7p^{-6}y^3 + 5g^{-2}p^{-6}) - 9g^{-4}p^{-6}(8g^2p^{-4}y^4 - 3g^4y^3 + 6p^2y^{-4})$

s $9h^6y^{-3}(8h^4y^3z - 3hz^6 - 2y^4z^3) - 3y^3z(h^6y^{-2}z^2 + 9h^{10}y^{-3} + y^{-3}z^6)$

t $k^{-7}n^7(8k^4n^7q^6 + 3n^4q^7 + 5k^6q^3) - 5n^7q^6(3k^{-7}n^4q - 8k^{-3}n^7 + 7n^7q^3)$

u $6k^5q^{-2}(9k^{-2}q^{-4}r - 4kr^6 - 5q^{-2}r^{-4}) + 9q^{-4}r(k^5r^{-5} + 6q^{-2}r^6 + 4k^3q^{-2})$

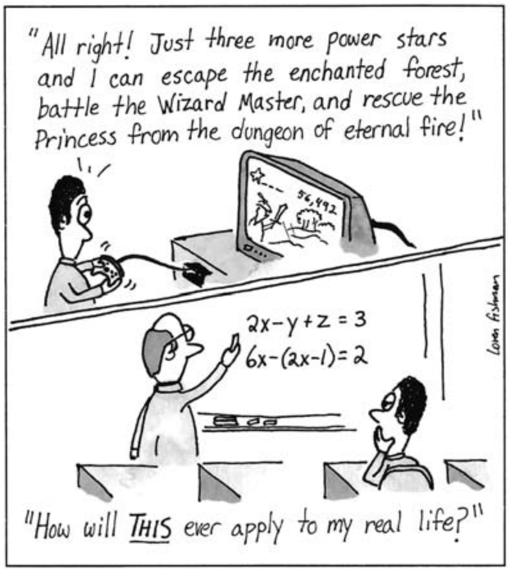
v $p^5x^2(6a^7p^4x^3 + a^9p^7) + 2a^7p^7(5a^2p^5x^2 - 2a^2x^7 + 5p^2x^5)$

w ${}^{-}6m^3r^2(4a^{-7}m^{-7}r + 3a^{-5}m^{-6} - 7m^{-6}r^{-6}) - 9a^{-7}m^{-6}(4a^2m^3r^2 + 6m^2r^3 + a^2r^{-6})$

x ${}^{-}7k^4w(5e^4k^{-6}w^3 - 4k^{-5}w^2 + e^7k^{-5}) - 4e^4k^{-5}(7e^3k^4w - 3ew^2 - 4k^3w^4)$

y $2e^5r^{-7}(3e^6r^{-2}v + 4ev^{-3} + 6r^6v^{-2}) - 7r^{-2}v(9e^5rv^{-3} - 4e^{11}r^{-7} - 3r^{-7}v^{-3})$

z $3a^2b^{-6}(3a^{-4}b^{-5}u^{-1} - 7b^{-4}u^{-5} - 7a^{-1}u^6) - 9b^{-5}u^{-1}(3a^2b^{-5}u^{-4} - 7a^{-2}b^{-6} - 7b^{-6}u^6)$



19 Factorise each of the following. *See* Example 16

a $20a^2 - 18ap + 18au$	**b** $126bk + 140by + 210b^2$	**c** $3ey + 11e^2 + 7eh$
d ${}^{-}8c^2 - 15cv - 10ct$	**e** $169by + 182bv + 195b^2$	**f** $70gh + 168g^2 + 140gz$
g $10a^2 - 10av + 150az$	**h** ${}^{-}99cn - 154cw - 110c^2$	**i** $16at - 120a^2 - 40aq$
j $84b^2 - 7bu - 49bh$	**k** $10cr - 12cp - 12c^2$	**l** ${}^{-}12ev - 24e^2 - 108ew$
m $5k^2 + 45kr - 65kv$	**n** $77xy - 49xz - 56x^2$	**o** $12bu + 2b^2 + 14bm$
p ${}^{-}75n^2 - 20nt - 55np$	**q** ${}^{-}99cz - 77cm - 121c^2$	**r** ${}^{-}36np - 39n^2 - 12nq$

s $48a^2 + 88ae + 48ay$ **t** $30ah - 22az - 28a^2$ **u** $50ev - 80e^2 - 110em$
v $132a^2 - 12ay - 156am$ **w** $^{-}13kw - 3kp - 6k^2$ **x** $4kp - 36k^2 - 60kx$
y $98u^2 - 168uv - 126ux$ **z** $15gq - 105gu - 120g^2$

20 Factorise each of the following.

a $^{-}168e^3k^6q^3 - 182e^2k^6q^5 - 196e^7k^2q^7$
b $44w^3y^3z^{10} - 143w^4yz^6 + 66w^6y^2z^5$
c $^{-}35m^6n^5u^7 - 63m^9n^6u^5 - 70m^4n^5u^9$
d $30g^6r^3y^4 - 28g^3r^6y^3 - 18g^6r^8y$
e $^{-}28c^5y^3z^6 - 49c^8y^5z^3 - 35c^6y^3z^6$
f $70h^2m^9u - 110h^4m^3u^3 + 110hm^7u^4$
g $30g^6p^4q^5 + 14gp^5q^7 - 24g^6p^4q^6$
h $4a^2r^9z^2 - a^7r^4z^3 + 3a^6r^7z$
i $20a^3m^4p^3 + 24a^7m^8p^2 - 28a^2m^8p^6$
j $110b^5m^2q^4 - 121bm^6q^5 + 110b^3m^7q^2$
k $120an^5x^4 + 8a^6n^{10}x - 32a^2n^5x^2$
l $66c^5p^{13}v^3 - 55c^5p^5v^7 - 77cp^{10}v^4$
m $77b^6e^2h^3 + 70be^4h^5 - 105b^6e^2h^7$
n $75e^5r^7z^4 + 120e^{10}r^4z^6 + 150e^8r^8z^2$
o $22h^7x^4y^8 - 55h^9x^6y^4 - 66h^4x^4y^5$
p $105a^4hu^7 + 7a^2h^3u^4 - 28a^6h^6u^2$
q $^{-}35e^2tz^{10} - 56e^5t^5z^5 - 28e^3tz^8$
r $26h^7q^{10}u^5 + 12h^3u^9 + 14h^2qu^9$
s $^{-}130e^5v^2x^3 - 140e^4v^5x^5 - 120e^9v^2x^5$
t $5a^5k^9t^7 + 12a^8k^5t^6 + 4a^8k^{10}t^3$
u $52c^6x^5y^5 - 39c^8x^6y - 130c^5x^7y^6$
v $105e^4u^2z^7 - 60eu^3z^5 + 105e^6u^5z^3$
w $7t^2xz^7 - 3t^4x^2z^5 + 9t^7xz^{10}$
x $^{-}88g^6t^9x^4 - 99g^2tx^5 - 121g^2tx^6$
y $^{-}16g^3xz^4 - 8g^3xz^7 - 32g^6xz^8$
z $^{-}91p^8r^5 - 104b^5p^3r^6 - 91b^4p^5r^3$

See Example 14

21 Simplify each of the following.

a $(2a + 3)^2$ **b** $(3x + 1)^2$ **c** $(5m - 2)^2$
d $(3h - 7)^2$ **e** $(8u - 3)^2$ **f** $(2 - 9k)^2$
g $(5 + 4d)^2$ **h** $(9k - 2)^2$ **i** $(2a + 5m)^2$
j $(4g + 3s)^2$ **k** $(7h + 2k)^2$ **l** $(8d - q)^2$

22 Simplify each of the following.

a $(2b + 1)(2b - 1)$ **b** $(3c - 2)(3c + 2)$ **c** $(5m - 3)(5m + 3)$
d $(1 + 2x)(1 - 2x)$ **e** $(4m + 1)(4m - 1)$ **f** $(8g - 3)(8g + 3)$
g $(2a - p)(2a + p)$ **h** $(c - 3d)(c + 3d)$ **i** $(8d + 2)(8d - 2)$
j $(8r - 5y)(8r + 5y)$ **k** $(7y + 4d)(7y - 4d)$ **l** $(6 - 3p)(6 + 3p)$

See Example 17

23 Factorise each of the following.

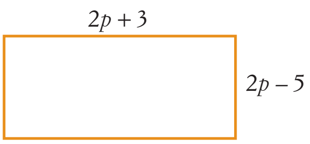
a $3a^4g^4t^6 + 13a^{-3}g^7t^{12} + 13a^{-2}g^2t^8$
b $165q^{-1}v^{-3}z^8 - 135v^{-2}z^5 + 210q^{-4}v^{-1}z^7$
c $^{-}98c^{12}u^9y^{-3} - 21c^{10}u^9y^{-4} - 28c^6u^9y$
d $18m^9q^{-6}u^2 - 8m^3q^{-1}u^5 + 16m^9q^{-2}u$
e $13m^{-1}t^{13}z^5 + 195m^{-2}t^4z^{11} + 52m^{-3}t^5z^6$
f $42h^{-1}p^5q^{-7} - 15hp^{-6}q^{-4} + 3h^{-6}p^{-2}q^{-3}$
g $^{-}24e^{-2}h^{-3}p^{-7} + 10e^{-6}h^{-2}p^{-2} - 22e^{-4}h^{-3}p^{-2}$
h $45h^{-2}q^7z + 126h^2q^7 - 36hq^{12}z^{-4}$
i $^{-}20a^6g^8x^{-3} - 130a^9g^{12}x^{-6} - 150a^5g^{11}x^{-3}$
j $^{-}225n^{-5}q^{-6}r^3 + 30n^{-7}r^8 + 105n^{-6}q^{-6}r^8$

Problem solving

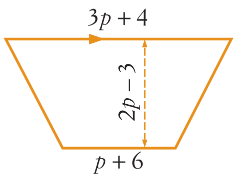
Worked solutions
Exercise 3.2
MAT10NAWS00008

24 Find expressions for the areas of the following shapes and simplify your expressions.

a



b



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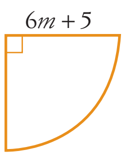

Worked solutions
Exercise 3.2
MAT10NAWS00008

25 **a** Expand $(n + \frac{1}{2})^2$.

b Use your result to find the value of the following, without using a calculator.

i $(3\frac{1}{2})^2$ **ii** $(19\frac{1}{2})^2$

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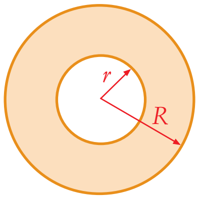
Reasoning

26 An *annulus* is the area enclosed by concentric circles, as shown by the shaded area on the right.
Show that the area is given by $A = \pi(R - r)(R + r)$



Worked solutions
Exercise 3.2
MAT10NAWS00008

3.3 Algebraic fractions

Important!

Algebraic fractions

These are fractions that involve algebraic expressions, rather than only numbers. The algebraic part may be in the denominator or numerator. **Algebraic fractions** are added, subtracted, multiplied and divided in the same way as ordinary fractions.

Remember that when you add and subtract fractions, you must use a common denominator. You should use the **lowest common denominator** (LCD).

Example 18

Video tutorial
Algebraic fractions
MAT10NAVT10007

Simplify each of the following.

a $\frac{a}{4} + \frac{b}{6}$ **b** $\frac{g}{2} - \frac{g}{5}$ **c** $\frac{m}{7} \times \frac{n}{3}$ **d** $\frac{p}{3} \div \frac{q}{5}$

Solution

a Write the problem and think of $\frac{1}{4} + \frac{1}{6}$. $\quad \frac{a}{4} + \frac{b}{6}$

Do like $\frac{1}{4} + \frac{1}{6} = \frac{3 \times 1}{3 \times 4} + \frac{2 \times 1}{2 \times 6}$ with LCD = 12. $\quad = \frac{3 \times a}{3 \times 4} + \frac{2 \times b}{2 \times 6}$

Write over the common denominator. $\quad = \frac{3a + 2b}{12}$

b Write the problem and think of $\frac{1}{2} - \frac{1}{5}$. $\quad \frac{g}{2} - \frac{g}{5}$

Do like $\frac{5 \times 1}{5 \times 2} - \frac{2 \times 1}{2 \times 5}$ with LCD = 10. $\quad = \frac{5 \times g}{5 \times 2} - \frac{2 \times g}{2 \times 5}$

Write over the common denominator. $\quad = \frac{5g - 2g}{10}$

Simplify. $\quad = \frac{3g}{10}$

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c Write the problem and think of $\frac{2}{7} \times \frac{5}{3}$. $\quad \frac{m}{7} \times \frac{n}{3}$

Multiply the tops and multiply the bottoms. $\quad = \frac{m \times n}{7 \times 3}$

Simplify. $\quad = \frac{mn}{21}$

d Write the problem and think of $\frac{2}{3} \div \frac{7}{5}$. $\quad \frac{p}{3} \div \frac{q}{5}$

Change to multiplication and turn the second fraction upside down. $\quad = \frac{p}{3} \times \frac{5}{q}$

Multiply the tops and multiply the bottoms. $\quad = \frac{p \times 5}{3 \times q}$

Simplify. $\quad = \frac{5p}{3q}$

Example 19

CAS TI-Nspire exercise
Algebra
MAT10NATI00003

CAS ClassPad exercise
Algebra
MAT10NACP00003

Simplify $\frac{6h+1}{8} + \frac{4h-7}{12}$.

Solution

Write the problem and think of $\frac{1}{8} + \frac{1}{12}$. $\quad \frac{6h+1}{8} + \frac{4h-7}{12}$

Do like $\frac{1}{8} + \frac{1}{12} = \frac{3 \times 1}{3 \times 8} + \frac{2 \times 1}{2 \times 12}$ with LCD $= 24$. $\quad = \frac{3 \times (6h+1)}{3 \times 8} + \frac{2 \times (4h-7)}{2 \times 12}$

Write over the common denominator. $\quad = \frac{3(6h+1) + 2(4h-7)}{24}$

Expand the brackets. $\quad = \frac{18h + 3 + 8h - 14}{24}$

Collect like terms. $\quad = \frac{26h - 11}{24}$

Example 20

Simplify $\frac{9m+2n}{6} - \frac{8m-5n}{5}$.

Solution

Write the problem and think of $\frac{1}{6} - \frac{1}{5}$. $\quad \frac{9m+2n}{6} - \frac{8m-5n}{5}$

Do like $\frac{1}{6} - \frac{1}{5} = \frac{5 \times 1}{5 \times 6} - \frac{6 \times 1}{6 \times 5}$ with LCD $= 30$. $\quad = \frac{5 \times (9m+2n)}{5 \times 6} - \frac{6 \times (8m-5n)}{6 \times 5}$

Write over the common denominator. $\quad = \frac{5(9m+2n) - 6(8m-5n)}{30}$

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Expand the brackets.	$= \dfrac{45m + 10n - 48m + 30n}{30}$
Collect like terms.	$= \dfrac{{}^{-}3m + 40n}{30}$
Write the positive term first.	$= \dfrac{40n - 3m}{30}$

Remember that when you multiply fractions, you multiply the numerators and multiply the denominators.

Example 21

Animated example
Algebraic fractions
MAT10NAAE00003

Simplify $\dfrac{5p + 3q}{6} \times \dfrac{3p - q}{5}$

Solution

Write the problem with brackets.	$\dfrac{(5p + 3q)}{6} \times \dfrac{(3p - q)}{5}$
Multiply the tops and multiply the bottoms.	$= \dfrac{(5p + 3q)(3p - q)}{6 \times 5}$
Expand the brackets.	$= \dfrac{15p^2 + 4pq - 3q^2}{30}$

Remember that to divide by a fraction, you multiply by its reciprocal.

Example 22

Simplify $\dfrac{2k + 3}{5} \div \dfrac{k - 4}{3}$.

Solution

Write the problem with brackets.	$\dfrac{(2k + 3)}{5} \div \dfrac{(k - 4)}{3}$
Change to multiplication and turn the second fraction upside down.	$= \dfrac{(2k + 3)}{5} \times \dfrac{3}{(k - 4)}$
Multiply the tops and multiply the bottoms.	$= \dfrac{3(2k + 3)}{5(k - 4)}$
Expand the brackets.	$= \dfrac{6k + 9}{5k - 20}$

You can use index laws when multiplying and dividing algebraic fractions.

Example 23

Simplify each of these:

a $\dfrac{3x^4y^3}{4z^5} \times \dfrac{6x^5z^8}{9y^7}$

b $\dfrac{a^3c}{4b^6} \div \dfrac{8a^4}{3bc^2}$

Solution

a Write the problem. $\dfrac{3x^4y^3}{4z^5} \times \dfrac{6x^5z^8}{9y^7}$

Separate the numeric and variable parts. $= \dfrac{3 \times 6 \times x^4y^3x^5z^8}{4 \times 9 \times y^7z^5}$

Simplify the numeric part and use index laws for the variables. $= \dfrac{x^{4+5}y^{3-7}z^{8-5}}{2}$

Simplify the indices. $= \dfrac{x^9y^{-4}z^3}{2}$

Change the negative index to positive on the bottom. $= \dfrac{x^9z^3}{2y^4}$

b Write the problem. $\dfrac{a^3c}{4b^6} \div \dfrac{8a^4}{3bc^2}$

Change to multiplication by the inverse. $\dfrac{a^3c}{4b^6} \times \dfrac{3bc^2}{8a^4}$

Separate the numeric and variable parts. $= \dfrac{3 \times a^3cbc^2}{4 \times 8 \times b^6a^4}$

Simplify the numeric part and use index laws for the variables (remember $c = c^1$). $= \dfrac{3 \times a^{3-4}c^{1+2}b^{1-6}}{32}$

Simplify the indices. $= \dfrac{3 \times a^{-1}c^3b^{-5}}{32}$

Change the negative index to positive on the bottom. $= \dfrac{3c^3}{32ab^5}$

Exercise 3.3 Algebraic fractions

Understanding

Extra questions
Exercise 3.3
MAT10NAEQ00009

Worksheet
Algebra 4
MAT10NAWK00009

See Example 19

1 Simplify each of the following.

a $\dfrac{3a}{4} + \dfrac{2y}{3}$ **b** $\dfrac{2g}{9} + \dfrac{m}{3}$ **c** $\dfrac{7m}{5} + \dfrac{n}{10}$

d $\dfrac{2c}{9} - \dfrac{3k}{2}$ **e** $\dfrac{9c}{8} - \dfrac{r}{10}$ **f** $\dfrac{3g}{4} - \dfrac{7y}{5}$

g $\dfrac{5b}{6} \times \dfrac{n}{8}$ **h** $\dfrac{c}{12} \times \dfrac{p}{24}$ **i** $\dfrac{e}{14} \times \dfrac{k}{4}$

j $\dfrac{4g}{9} \div \dfrac{7u}{4}$ **k** $\dfrac{9e}{2} \div \dfrac{6h}{7}$ **l** $\dfrac{7m}{8} \div \dfrac{9v}{7}$

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m $\frac{7p}{4}+\frac{p}{12}$
n $\frac{q}{10}+\frac{q}{20}$
o $\frac{2a}{7}+\frac{a}{6}$
p $\frac{9m}{8}-\frac{m}{12}$
q $\frac{5q}{6}-\frac{2q}{9}$
r $\frac{c}{12}-\frac{c}{3}$
s $\frac{9b}{4}\times\frac{7b}{6}$
t $\frac{5c}{3}\times\frac{c}{12}$
u $\frac{3a}{4}\times\frac{7a}{2}$
v $\frac{a}{4}\div\frac{6a}{5}$
w $\frac{5c}{4}\div\frac{5c}{3}$
x $\frac{m}{15}\div 4m$
y $\frac{13b}{15}-\frac{13b}{10}$
z $\frac{9b}{2}-\frac{14b}{9}$

Fluency

2 Simplify each of the following.

See Example 18

a $\frac{8h-9}{7}+\frac{9h-5}{6}$
b $\frac{g-4}{6}+\frac{4g-5}{10}$
c $\frac{9k+8}{5}+\frac{8k+5}{2}$
d $\frac{h+7}{7}+\frac{8h-9}{4}$
e $\frac{4g-7}{9}+\frac{8g-9}{2}$
f $\frac{6b+1}{6}+\frac{4b-5}{12}$
g $\frac{4g-1}{5}+\frac{8g+5}{3}$
h $\frac{7n+3}{3}+\frac{8n-7}{6}$
i $\frac{2a+1}{12}+\frac{6a-1}{6}$
j $\frac{2a-7}{8}+\frac{6a-5}{10}$
k $\frac{7b-6g}{8}+\frac{8b+5g}{10}$
l $\frac{a+e}{3}+\frac{3a+8e}{9}$
m $\frac{2h-9y}{2}+\frac{4h+y}{3}$
n $\frac{8r+5t}{3}+\frac{2r+7t}{4}$
o $\frac{3g-x}{10}+\frac{4g+9x}{8}$
p $\frac{4g-7m}{5}+\frac{7g-4m}{2}$
q $\frac{8q-9r}{10}+\frac{4q+r}{5}$
r $\frac{7h-5t}{3}+\frac{h+t}{12}$
s $\frac{9g-8x}{5}+\frac{5g+8x}{10}$
t $\frac{6c-e}{4}+\frac{6c-e}{6}$

3 Simplify each of the following.

See Example 20

a $\frac{2r+7}{12}-\frac{6r-7}{18}$
b $\frac{7r+3}{4}-\frac{2r-9}{5}$
c $\frac{6e+1}{4}-\frac{6e-5}{6}$
d $\frac{5k-6}{6}-\frac{8k-7}{3}$
e $\frac{9v-1}{8}-\frac{2v-7}{4}$
f $\frac{n+9}{3}-\frac{4n+9}{15}$
g $\frac{7c+4}{3}-\frac{6c+1}{8}$
h $\frac{7h-1}{6}-\frac{8h-7}{5}$
i $\frac{5h-9}{2}-\frac{4h+5}{5}$
j $\frac{9k-2}{12}-\frac{2k-1}{3}$
k $\frac{e+8v}{9}-\frac{8e+5v}{6}$
l $\frac{7c+9t}{4}-\frac{3c-5t}{6}$
m $\frac{2u+3v}{10}-\frac{u-3v}{6}$
n $\frac{k+7t}{6}-\frac{4k-5t}{3}$
o $\frac{9g+7r}{5}-\frac{8g+9r}{10}$
p $\frac{9c+2p}{8}-\frac{6c-7p}{10}$
q $\frac{2c+5p}{2}-\frac{4c-p}{10}$
r $\frac{2e+3t}{8}-\frac{9e-7t}{5}$
s $\frac{p-3q}{12}-\frac{2p-3q}{2}$
t $\frac{9g-2v}{9}-\frac{4g+9v}{6}$

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See Example 21

4 Simplify each of the following.

a $\frac{3q-7}{8}\times\frac{7q-4}{5}$ **b** $\frac{e+8}{7}\times\frac{6e+7}{5}$ **c** $\frac{8a-7}{9}\times\frac{4a+3}{5}$

d $\frac{2e-3}{4}\times\frac{3e+7}{12}$ **e** $\frac{7g-6}{3}\times\frac{5g-2}{2}$ **f** $\frac{7c+9}{9}\times\frac{5c+8}{6}$

g $\frac{b-9}{5}\times\frac{4b+9}{2}$ **h** $\frac{3a+1}{9}\times\frac{a-9}{12}$ **i** $\frac{6e+5}{9}\times\frac{9e-1}{6}$

j $\frac{3b-5}{8}\times\frac{6b-5}{5}$ **k** $\frac{3p+5q}{3}\times\frac{7p+5q}{6}$ **l** $\frac{8b-7c}{5}\times\frac{4b-5c}{4}$

m $\frac{3n-4q}{12}\times\frac{8n-q}{18}$ **n** $\frac{7c+6m}{3}\times\frac{c+5m}{4}$ **o** $\frac{5n-2p}{2}\times\frac{n-8p}{6}$

p $\frac{6a+7e}{7}\times\frac{a+5e}{4}$ **q** $\frac{5c+6g}{6}\times\frac{c+2g}{4}$ **r** $\frac{5g-3n}{4}\times\frac{9g+7n}{8}$

s $\frac{3p-2q}{12}\times\frac{6p-7q}{2}$ **t** $\frac{9g+2u}{6}\times\frac{8g+u}{8}$

See Example 22

5 Simplify each of the following.

a $\frac{5n-9}{6}\div\frac{2n-3}{7}$ **b** $\frac{9p-7}{2}\div\frac{6p+5}{7}$ **c** $\frac{9p+1}{9}\div\frac{8p-7}{8}$

d $\frac{2a+1}{8}\div\frac{9a-7}{7}$ **e** $\frac{4c+3}{8}\div\frac{7c+4}{5}$ **f** $\frac{4h+1}{3}\div\frac{7h-4}{7}$

g $\frac{8u-3}{4}\div\frac{4u-5}{3}$ **h** $\frac{7e+9}{4}\div\frac{8e-3}{5}$ **i** $\frac{9a-2}{7}\div\frac{7a-2}{5}$

j $\frac{2a-7}{6}\div\frac{3a+5}{5}$ **k** $\frac{5n-6p}{9}\div\frac{9n+5p}{7}$ **l** $\frac{8c-7h}{2}\div\frac{9c+4h}{9}$

m $\frac{9e+4p}{3}\div\frac{5e+3p}{7}$ **n** $\frac{e+9n}{8}\div\frac{6e+7n}{3}$ **o** $\frac{9a+7h}{7}\div\frac{2a-7h}{8}$

p $\frac{9a+8e}{5}\div\frac{6a+e}{3}$ **q** $\frac{4g+7v}{3}\div\frac{6g+v}{8}$ **r** $\frac{g-9h}{6}\div\frac{8g-7h}{7}$

s $\frac{3h+5p}{5}\div\frac{9h-8p}{7}$ **t** $\frac{b-3r}{3}\div\frac{8b+5r}{5}$

6 Simplify each of the following.

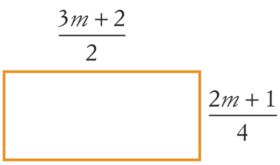
a $\frac{8b^5g}{12k^6y^2}\times\frac{2bk^6}{10g^6y}$ **b** $\frac{6n^5r^5}{3u^2}\div\frac{8n}{2r^3u^7}$ **c** $\frac{m^3u^6}{4v^7w^3}\div\frac{8u^4}{3mv^2w^5}$ **d** $\frac{5b^2c^7}{12q^5w}\times\frac{9b^5q^6w^2}{10c^7}$

Problem solving

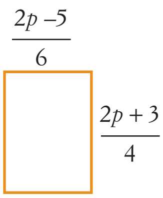
Worked solutions
Exercise 3.3
MAT10NAWS00009

7 Find the perimeters of the following shapes.

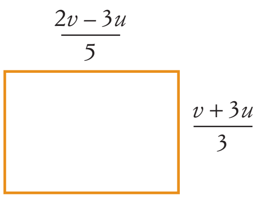
a



b



c



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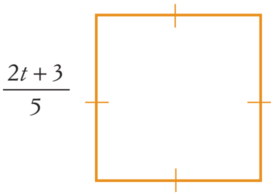
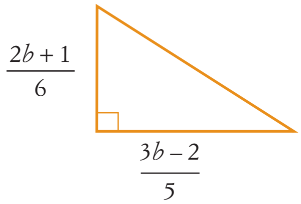
8 Find the areas of the following shapes.

Worked solutions
Exercise 3.3
MAT10NAWS00009

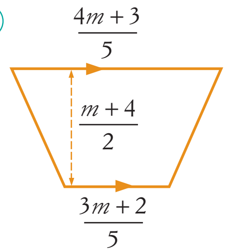
a



b



c



9 The simplifications below may have errors. For each one, if there is an error, identify the line in which the *first* error occurs and state the correct answer.

Reasoning

a
$$\frac{9q+2r}{6}+\frac{7q+5r}{8}$$
$$=\frac{8(9q+2r)}{24}+\frac{6(7q+5r)}{24} \quad \text{(A)}$$
$$=\frac{72q+16r+42q+5r}{24} \quad \text{(B)}$$
$$=\frac{114q+21r}{24} \quad \text{(C)}$$

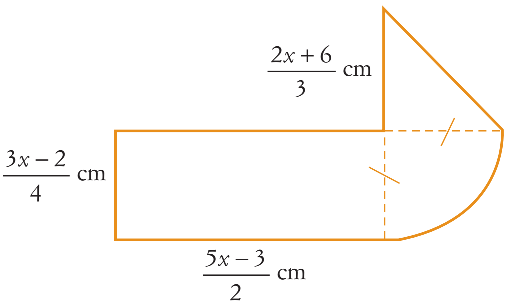
b
$$\frac{4b+5h}{4}-\frac{6b+h}{3}$$
$$=\frac{3(4b+5h)}{12}-\frac{4(6b+h)}{12} \quad \text{(A)}$$
$$=\frac{12b+15h-24b+4h}{12} \quad \text{(B)}$$
$$=\frac{15h-12b}{12} \quad \text{(C)}$$

c
$$\frac{9h-8r}{4}\times\frac{4h+5r}{5}$$
$$=\frac{(9h-8r)(4h+5r)}{20} \quad \text{(A)}$$
$$=\frac{36h^2-32hr+45hr-40r^2}{20} \quad \text{(B)}$$
$$=\frac{36h^2+13hr-40r^2}{20} \quad \text{(C)}$$

d
$$\frac{3k-7u}{5}\div\frac{5k+u}{9}$$
$$=\frac{3k-7u}{5}\times\frac{9}{5k+u} \quad \text{(A)}$$
$$=\frac{9(3k-7u)}{5(5k+u)} \quad \text{(B)}$$
$$=\frac{27k-7u}{25k+5u} \quad \text{(C)}$$

10 Explain the steps in finding the area of the shape below.

Worked solutions
Exercise 3.3
MAT10NAWS00009



Chapter 3 summary

Quiz
Algebra
MAT10NAQZ00003

- A **variable** is a letter or symbol that stands for a number. A **constant** is a number.
- An **expression** has variables and/or numbers connected by arithmetic operations like $+$, $\div$ and powers. An expression with numbers only is an **arithmetic expression**, while one with variables is an **algebraic expression**.
- The numbers in an expression that are multiplied by variables are called **coefficients**, the parts separated from the rest by $+$ or $-$ are called **terms** and a number on its own is called a **constant term**.
- To **evaluate** an algebraic expression you **substitute** values for the variables and work out (**evaluate**) the answer.
- A **formula** is an equation with a variable on the left and an algebraic expression on the right. The variable on the left is the **subject** of the formula. A formula is **transposed** using inverse operations to change the subject of the formula. A formula is evaluated by substituting values for the variables.
- Algebraic terms with exactly the same variables are called **like terms**. Like terms can be added or subtracted to **simplify** the expression. This is called **collecting like terms**.
- When you multiply or divide algebraic terms, the coefficients and each different variable are multiplied or divided separately.
- The **distributive law** states that the product of the sum (or difference) of two numbers with another number is the same as the sum (or difference) of the products of that number with each one separately. This can be written in symbols as:
$$a \times (b + c) = a \times b + a \times c$$
and
$$a \times (b - c) = a \times b - a \times c.$$
- **Expansion of brackets** means the removal of brackets using the distributive law. It is also called **simplification**.
- **Binomial products** are double brackets like $(x + 2)(x - 5)$. They can be expanded using the distributive law several times.
- The expansion $(x - y)(x + y) = x^2 - y^2$ is called the **difference of two squares**.
- The expansion $(x + y)^2 = x^2 + 2xy + y^2$ is called the **square of a sum**.
- The expansion $(x - y)^2 = x^2 - 2xy + y^2$ is called the **square of a difference**.
- The **highest common factor** of two algebraic terms is the largest expression that will divide exactly into the given terms.
- **Factorisation** is the opposite process to multiplying out brackets. In the **common factor** method, the highest common factor of the terms is put outside the brackets, leaving the other factors of each term inside the brackets.
- An **algebraic fraction** is a fraction that has at least one algebraic expression. They are simplified in the same way as normal fractions. The top line of an algebraic fraction sometimes factorises to make a factor that will cancel with the denominator.

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Understanding

1 Write expressions for each of the following using mathematical symbols.
a Eight less than x
b The product of a number and 8
c Four less than three times a number
d The product of four less than a number and three

See Example 1

Worksheet
Algebra 2
MAT10NAWK00007

2 Simplify if possible.
a $4p + 8p - 5p$
b $3h + t - 5h + 3t$
c $4xy + 3x^2 - 3y - xy - 5x^2$

See Example 2

3 Simplify each of the following.
a $m(m - 3)$
b $^{-}2k(4m - 3k)$
c $(x + 2)(x + 5)$
d $(4e - 5)(e + 3)$

See Examples 8, 10, 11

4 Factorise the following.
a $30b + 18$
b $^{-}9m - 21$

See Example 15

5 $a = 3$, $b = 5$ and $c = {}^{-}2$. Find the value of $D = ab - 5c$.

See Example 5

6 Simplify each of the following.
a $\frac{9q}{8} + \frac{8w}{3}$
b $\frac{4p}{9} - \frac{p}{6}$
c $\frac{14b}{9} \times \frac{13b}{12}$
d $\frac{14r}{9} \div \frac{4t}{13}$

See Example 18

Fluency

7 Simplify each of the following.
a $54n^2tu^2 \div 63ntu^3$
b $6g^3k^2r^3 \times 7gk$

See Example 3

Algebra is included as a muse in this Vatican artwork. Some of the other 'muses' included are Medicina (Medicine), Jurisprudentia (Justice), Historia (History), Poesis (Poetry) and Musica (Music).

See Examples 6, 7

8 Transpose the following formulas so that the letter shown in brackets is the subject.

a $A = 2\pi rs \quad (r)$

b $Z = \dfrac{x - u}{s} \quad (u)$

See Example 12

9 Simplify each of the following.

a $3(4t + 3) + 4(2t + 1)$

b $v(3v + 7) - 4v(5v - 6)$

c $3mp^2t(mt - 3mp + 2pt)$

d $3k(5 - 4k + 2k^2) - 3k(4k^2 - 5 - k)$

See Example 9

10 Simplify each of the following.

a $^-8b^2c^{-3}(5g^5 + 14b^{-5})$

b $a(7a + 4c - 3x) - 9x(8a - c - 4x) - c(3a - 7c + 5x)$

c $(7g + p)(3g + 5p) - (8g - 5p)(5g + 4p)$

d $4m^{10}u^{-3}(3g^{10}m^{-11}u^{13} + m^9 - 8m^9u^3) + 2g^{10}m^9(7g^{-10}m^{10}u^{-3} - 2g^{-3}u^3)$

See Example 13

11 Simplify each of the following using special expansions.

a $(4 - a)(4 + a)$

b $(h + 3)^2$

See Examples 16, 17

12 Factorise each of the following.

a $42gn + 196gw - 210g^2$

b $63ar^{10}w^6 - 135a^4r^5w^5 + 99a^2r^9w^4$

c $24n^3y^3 + 88n^{-5}y^{-2}z^{-3} + 40n^{-6}y^3z^{-2}$

See Examples 19–23

13 Simplify each of the following.

a $\dfrac{3q + 8r}{7} + \dfrac{8q - 3r}{6}$

b $\dfrac{5p + 9}{4} - \dfrac{3p + 1}{5}$

c $\dfrac{8b - 9v}{8} \times \dfrac{5b - 4v}{2}$

d $\dfrac{p + 1}{6} \div \dfrac{8p - 9}{5}$

14 Simplify each of the following.

a $^-35b^3t^{-1}u^{-3} \div {}^-45b^5t^{-4}u^{-2}$

b $5e^{-1}h^{-4}r^2 \times {}^-8e^4h^{-5}r^{-2}$

c $^-5(g^3p^{-1})^3x \times {}^-4g^{-5}(p^4x^{-3})^{-2}$

d $\dfrac{48g^4h^4}{25k^7} \div \dfrac{15kh^2}{8g^2}$

e $54e^{-11}t^{-5}u^4y^{-12} \div {}^-63e^{-4}t^6u^{-10}y^{-7}$

f $9b^7gt^{12}v^{-12} \times {}^-7b^6g^{-11}t^{-9}v^4$

See Example 4

15 Simplify $\dfrac{9ev^2 - 4ev^2}{10w^2v - w^2v} \times \dfrac{2e^2w^2v + 4e^2w^2v}{7wve - 5wve}$.

See Example 14

16 Simplify each of the following.

a $(7 - 3q)^2$

b $(6y - 5)(6y + 5)$

c $(5h + 2v)^2$

Problem solving

17 Find the value of the nominated variable in each of the following formulas.

a n if $PV = nRT$ and $P = 1100$, $V = 4.2$, $R = 70.2$ and $T = 324$

b a if $v = 5$, $u = 3$, $s = 6$ and $v = \sqrt{u^2 + 2as}$

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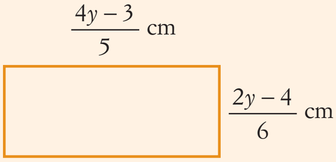
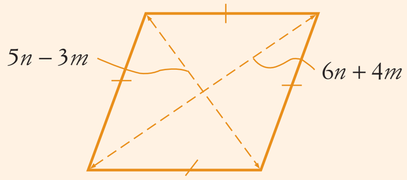
18 Find an expression for the perimeter of this shape.

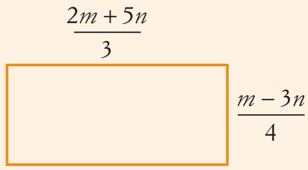


19 Find expressions for the areas of the shapes below and simplify your expressions.

a



b



Reasoning

20 The simplifications below may have errors. For each one, if there is an error, identify the line in which the *first* error occurs and state the correct answer.

a

$$\frac{5m+n}{6}-\frac{3m-4n}{4}$$

$$=\frac{2(5m+n)}{12}-\frac{3(3m-4n)}{12} \quad \text{(A)}$$

$$=\frac{10m+2n-9m-12n}{12} \quad \text{(B)}$$

$$=\frac{m-10n}{12} \quad \text{(C)}$$

b

$$\frac{3b-2c}{4}\times\frac{4b-3c}{3}$$

$$=\frac{3(3b-2c)}{12}\times\frac{4(4b-3c)}{12} \quad \text{(A)}$$

$$=\frac{(9b-6c)(16b-12c)}{144} \quad \text{(B)}$$

$$=\frac{144b^2-204bc-72c^2}{144} \quad \text{(C)}$$

$$=\frac{12(12b^2-17bc-6c^2)}{144} \quad \text{(D)}$$

$$=\frac{12b^2-17bc-6c^2}{12} \quad \text{(E)}$$

21 Expand $(10n+5)^2$ and use your expansion to find a short cut for the calculation of $15^2, 25^2, 35^2, \ldots$

Measurement and geometry

4 Area and volume

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Contents

Prior learning
Chapter 4
MAT10MGPL00004

Parent guide
Chapter 4
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Curriculum guide
Chapter 4
MAT10MGCU00004

Australian Curriculum statements

Using units of measurement
Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids. (ACMMG242)

Video tutorial
Area and Volume
MAT10MGVT00004

Weblink
Shape, space and measures
MAT10MGWB00004

Weblink
The Pentagon
MAT10MGWB00004

Measuring length and calculating area and volume are important skills for everyday living. Imagine that you need to buy new carpet for your bedroom. How much carpet will you need to buy? Another area problem that you may need to deal with is calculating how many cans of paint to buy to cover the walls of your room. Suppose you want to build a garden bed. How much soil will you need? How much fertilizer will you need to add to the soil? You could use volume to find out how much cement mix it will take to pour a walkway or how much sand is needed to fill a sandbox.

Before measuring devices such as rulers were invented, parts of the human body were used to measure distances. This meant that people had ready access to a measuring device that was accurate enough for most practical uses.

Most ancient civilisations used lengths like a foot (about 30 cm) to measure short distances. They used something like one marching pace or stride (about 90 cm) for longer distances. Carpenters and traders measured lengths of timber using the cubit (about 45 cm)—the distance from the elbow to the tip of the longest finger. Can you imagine how complicated it would be to order the timber for a building project in cubits?

Today, measuring lengths and calculating with them is much simpler because we use units from the metric system to express these quantities.

Mathematical literacy

Maths dictionary
MAT10ASDI00001

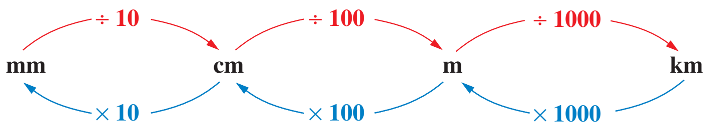
The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

apex	cylinder	millimetre	square millimetre
area	edges	net	square centimetre
base	face	parallelogram	square metre
capacity	hectare	prism	square kilometre
centimetre	hemisphere	pyramid	surface area
circle	hypotenuse	Pythagoras' theorem	three-dimensional
cone	kilometre	rectangle	triangle
cubic centimetre	kite	rhombus	trapezium
cubic metre	litre	slant height	two-dimensional
cubic millimetre	metre	sphere	volume
curved surface area	metric system		vertex

4.1 Solving area problems

In the **metric system**, the standard unit of length is the **metre** (**m**). Other common units of length are shown below.

You often need to change from one unit to another. When changing to a *larger* unit, *divide*. When changing to a *smaller* unit, *multiply*. The arrows in the diagram below show the directions in which you move the decimal point when changing from one unit to another.



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The **area** of a flat shape measures the size of the region enclosed by the boundary of the shape.

In the metric system, small areas are measured in:

- **square millimetres** (**mm^2**)—the area of a square 1 mm long and 1 mm wide, or
- **square centimetres** (**cm^2**)—the area of a square 1 cm long and 1 cm wide.

Larger areas are measured in:

- **square metres** (**m^2**)—the area of a square 1 m long and 1 m wide.

Very large areas are measured in:

- **hectares** (**ha**)—the area of a square 100 m long and 100 m wide, or
- **square kilometres** (**km^2**)—the area of a square 1 km long and 1 km wide.

You can convert between metric area units.

Important!

Metric area unit conversions

$1 \text{ cm}^2 = 10 \text{ mm} \times 10 \text{ mm} = 100 \text{ mm}^2$

$1 \text{ m}^2 = 100 \text{ cm} \times 100 \text{ cm} = 10\,000 \text{ cm}^2 = 1\,000\,000 \text{ mm}^2$

$1 \text{ ha} = 100 \text{ m} \times 100 \text{ m} = 10\,000 \text{ m}^2$

$1 \text{ km}^2 = 1000 \text{ m} \times 1000 \text{ m} = 1\,000\,000 \text{ m}^2 = 100 \text{ ha}$

If you are changing to a *larger* unit, *divide*. If changing to a *smaller* unit, *multiply*.

$$\begin{tikzcd}
\mathbf{mm^2} \arrow[r, bend left, "\div 100"] & \mathbf{cm^2} \arrow[l, bend left, "\times 100"] \arrow[r, bend left, "\div 10\,000"] & \mathbf{m^2} \arrow[l, bend left, "\times 10\,000"] \arrow[r, bend left, "\div 10\,000"] & \mathbf{ha} \arrow[l, bend left, "\times 10\,000"] \arrow[r, bend left, "\div 100"] & \mathbf{km^2} \arrow[l, bend left, "\times 100"]
\end{tikzcd}$$

Example 1

Convert the measurements to the units shown in brackets.

a $26\,500 \text{ m}^2$ (ha)

b 3.215 km^2 (ha)

Solution

a Write the measurement. $\quad \mathbf{26\,500 \text{ m}^2}$

Changing to a *larger* unit, so *divide*. $\quad \mathbf{= 26\,500 \div 10\,000 \text{ ha}}$

$\mathbf{= 2.65 \text{ ha}}$

b Write the measurement. $\quad \mathbf{3.215 \text{ km}^2}$

Changing to a *smaller* unit, so *multiply*. $\quad \mathbf{= 3.215 \times 100 \text{ ha}}$

$\mathbf{= 321.5 \text{ ha}}$

CAS TI-Nspire exercise
Area and volume
MAT10MGTI00004

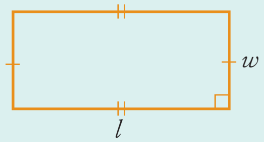
CAS ClassPad exercise
Area and volume
MAT10MGCP00004

For some plane shapes there are rules that can be used to calculate areas. The most common ones are shown here.

Important!

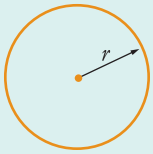
Area rules

Rectangle



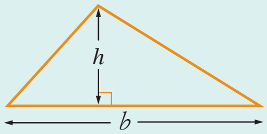
$A = l \times w$

Circle



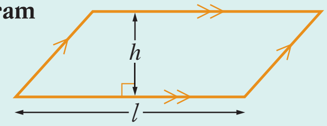
$A = \pi r^2$

Triangle



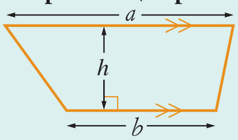
$A = \frac{1}{2}bh$

Parallelogram



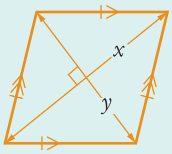
$A = l \times h$

Trapezium (trapezoid)

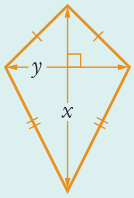


$A = \frac{1}{2} \times (a + b)h$

Rhombus



Kite

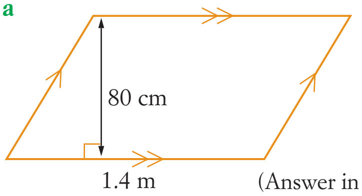


$A = \frac{1}{2}xy = \frac{1}{2}$(product of diagonals)

Example 2

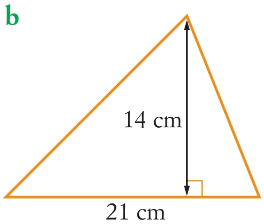
Find the areas of these shapes.

a



(Answer in m^2.)

b



Solution

a Write the area rule for a parallelogram.	$A = l \times h$
Substitute the given measurements.	$= 1.4 \text{ m} \times 80 \text{ cm}$
Change both units to metres.	$= 1.4 \text{ m} \times 0.8 \text{ m}$
Evaluate.	$= 1.12 \text{ m}^2$
State the result.	**The parallelogram has an area of 1.12 m^2.**

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b Write the area rule for a triangle. $A = \frac{1}{2}bh$

Substitute the given measurements and cancel. $= \frac{1}{\cancel{2}_1} \times 21 \times \cancel{14}^7$

Evaluate. $= 147\text{ cm}^2$

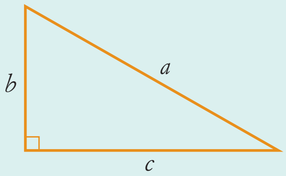
State the result. **The triangle has an area of 147 cm^2.**

In order to calculate the area of a triangle, we need its height. The height of a triangle is not always given. **Pythagoras' theorem** can be used to help calculate areas involving right-angled triangles.

Important!

Pythagoras' theorem

For a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.
For this triangle: $a^2 = b^2 + c^2$



Teacher notes
Proof of Pythagoras' theorem
MAT10MGTN00008

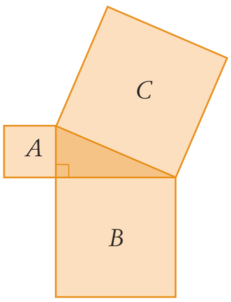
Pythagorean triples (or **triads**) such as (3, 4, 5), (5, 12, 13) and (7, 24, 25) can sometimes be used as a 'short cut' in calculations with right-angled triangles.

Investigate: Pythagoras' theorem

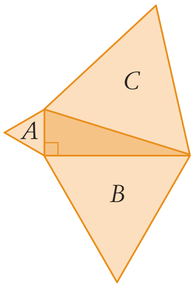
Pythagoras' theorem tells us that in diagram **a** below:

Area of A + area of B = area of C

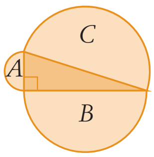
a



b



c



1 Diagram **b** shows a right-angled triangle with an equilateral triangle constructed on each side. Is it true to say that: area of A + area of B = area of C?
2 Diagram **c** shows a right-angled triangle with a semicircle constructed on each side. Is it true to say that: area of A + area of B = area of C?
3 Investigate other demonstrations of Pythagoras' theorem.

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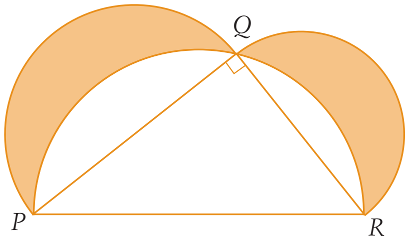
Lunes

Triangle PQR is drawn so that PR is the diameter of a semicircle and Q lies on the circumference, as shown here.
Any triangle drawn in this way will be right-angled.
Semicircles are constructed using PQ and QR as diameters, as shown.
The shaded areas formed are called **lunes**.

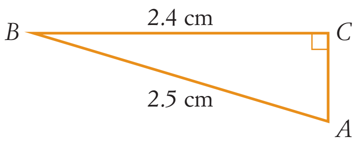


How does the sum of the areas of the lunes compare with the area of $\triangle PQR$?

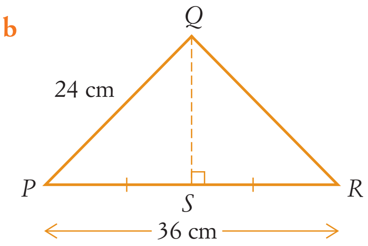
Example 3

Find the area of each of these triangles.

a



b



Solution

a Apply Pythagoras' theorem to the triangle. $\qquad BA^2 = CA^2 + BC^2$

Substitute for known sides. $\qquad 2.5^2 = CA^2 + 2.4^2$

Evaluate. $\qquad 6.25 = CA^2 + 5.76$

Reverse the equation. $\qquad CA^2 + 5.76 = 6.25$

Subtract 5.76 from both sides. $\qquad CA^2 = 6.25 - 5.76$

$\qquad = 0.49 \text{ cm}^2$

Take the square root. $\qquad CA = 0.7 \text{ cm}$

Write the rule for the area of a triangle. $\qquad A = \frac{1}{2}bh$

Substitute for known values and cancel. $\qquad = \frac{1}{{}_1\cancel{2}} \times \cancel{2.4}^{1.2} \times 0.7$

Evaluate. $\qquad = 0.84 \text{ cm}^2$

State the result. $\qquad$ **The area of $\triangle ACB$ is 0.84 cm^2.**

b The height of $\triangle PQR$ is QS. QS needs to be calculated.

Apply Pythagoras' theorem to $\triangle PQS$. $\qquad PQ^2 = QS^2 + PS^2$

Substitute for known sides. $\left(PS = \frac{1}{2}PR\right)$ $\qquad 24^2 = QS^2 + 18^2$

Evaluate. $\qquad 576 = QS^2 + 324$

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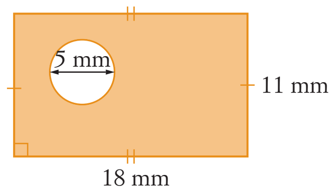
Reverse the equation.	$QS^2 + 324 = 576$
Subtract 324 from both sides.	$QS^2 = 576 - 324$
	$= 252$
Keep the square root on your calculator.	$QS = 15.8745\ldots$
Write the rule for the area of a triangle.	$A = \frac{1}{2}bh$
Substitute for known values and cancel.	$= \frac{1}{\cancel{2}_1} \times \cancel{36}^{18} \times 15.8745\ldots$
Use the value on your calculator.	$= 285.741\ldots\text{ cm}^2$
Round and state the result.	**The area of $\triangle PQR$ is about 286 cm^2.**

Some complex shapes are formed by combining simple plane shapes. When finding the area of these figures, mark on your diagram how the shape has been formed.

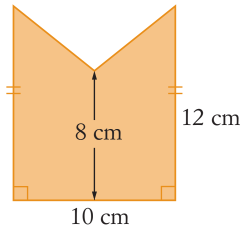
Example 4

Find the areas of the following shaded regions.

a



b



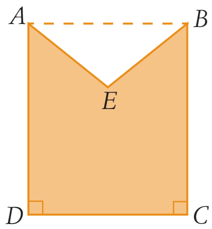
Solution

a The shaded region is a rectangle with a circular unshaded region.

Begin with the rectangle.	**Rectangle area** $= l \times w$
Substitute for l and w.	$= 18 \times 11$
Evaluate, including units.	$= 198\text{ mm}^2$
Write the area rule for a circle.	**Circle area** $= \pi r^2$
Use $r = \frac{D}{2}$ and substitute for r.	$= \pi \times 2.5^2$
Keep the value on your calculator.	$= 19.634\ldots\text{ mm}^2$
Find the shaded area.	**Shaded area = rectangle area − circle area**
Use the value on your calculator.	$= 198\text{ mm}^2 - 19.634\ldots\text{ mm}^2$
Evaluate.	$= 178.365\ldots\text{ mm}^2$
Round off.	$\approx 178\text{ mm}^2$

b There are *many* ways to work out this area. Here is *one*.

Name the vertices of the region.



Begin with rectangle $ABCD$. | **Area of $ABCD$ $= l \times w$**

Substitute. | $= 12 \times 10$

Evaluate. | $= 120\ \text{cm}^2$

Find the area of $\triangle ABE$. | $A = \frac{1}{2}bh$

Find the height, then substitute and cancel. | $= \frac{1}{\cancel{2}_1} \times \cancel{10}^5 \times 4$

Evaluate. | $= 20\ \text{cm}^2$

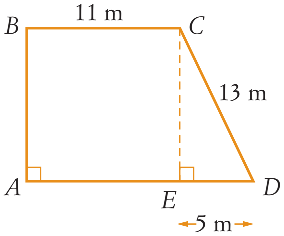
Calculate the shaded region. | **Shaded area = area of $ABCD$ − area of $\triangle ABE$**

Substitute. | $= 120\ \text{cm}^2 - 20\ \text{cm}^2$

Evaluate. | $= 100\ \text{cm}^2$

Example 5

Calculate the area of this figure.



Solution

First find CE using Pythagoras' theorem. | $13^2 = CE^2 + 5^2$

Evaluate. | $169 = CE^2 + 25$

Reverse the equation and subtract 25 from both sides. | $CE^2 = 169 - 25$

$= 144\ \text{m}^2$

Take the square root of both sides. | $CE = 12\ \text{m}$

Now find the area of rectangle $ABCE$. | **Area of $ABCE$ $= l \times w$**

Substitute. | $= 11 \times 12$

Evaluate. | $= 132\ \text{m}^2$

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Find the area of $\triangle CDE$.	$\boldsymbol{A = \frac{1}{2}bh}$
Substitute and cancel.	$\boldsymbol{= \frac{1}{{}_{1}\cancel{2}} \times 5 \times \cancel{12}^{6}}$
Evaluate.	$\boldsymbol{= 30\ \mathrm{m}^2}$
Calculate the total area.	**Total area = area of *ABCE* + area of** $\boldsymbol{\triangle CDE}$
Substitute.	$\boldsymbol{= 132\ \mathrm{m}^2 + 30\ \mathrm{m}^2}$
Evaluate.	$\boldsymbol{= 162\ \mathrm{m}^2}$

Exercise 4.1 Solving area problems

Extra questions
Exercise 4.1
MAT10MGEQ00010

A calculator should be used where appropriate in this exercise. For calculations involving π, use the π key on your calculator and answer correct to 1 decimal place.

Understanding

See Example 1

Puzzle sheet
Area
MAT10MGPS00010

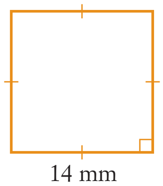
1 Convert the measurements below to the units shown in brackets.

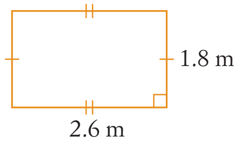
a 7.3 ha (m^2) **b** 48 000 m^2 (km^2) **c** 57 ha (km^2)
d 0.025 km^2 (ha) **e** 0.008 27 ha (m^2) **f** 0.003 42 km^2 (m^2)

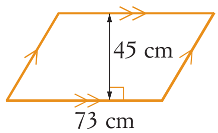
2 Find the area of each of the following.

a Square floor tile of side 12 cm
b Rectangular birthday card 18 cm by 11 cm
c Rectangular table top 1.5 m by 0.9 m
d Australian flag 2 m by 1.3 m
e Parallelogram of base 12.5 mm and height 5.6 mm
f Rhombus with diagonals of 1.7 m and 2.8 m
g Kite with diagonals of 26 cm and 44 cm

3 Find the areas of the following shapes.

a


b


c


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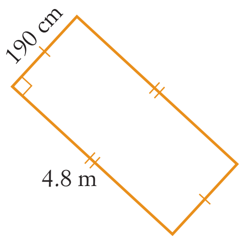
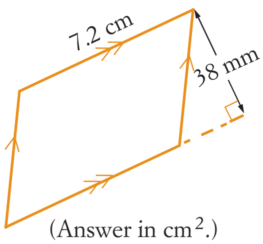
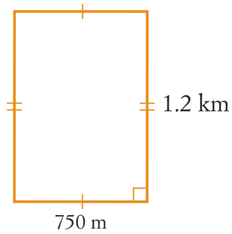


(Answer in m^2.)

e


(Answer in cm^2.)

f


(Answer in km^2.)

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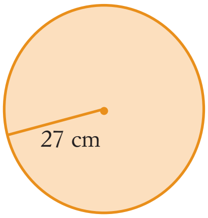
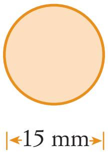
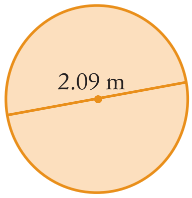
g



h



i



4 Find the area of each of the following.

a Triangular scarf with base 1.5 m and height 0.8 m

b Circular lid with a radius of 25 mm

c Triangular wall tile with base 25 cm and height 12 cm

d Trapezoidal table top with parallel sides 1.2 m and 1.8 m and height 80 cm

e Compact disc with a diameter of 12 cm

f Circular tablecloth with a radius of 1.5 m

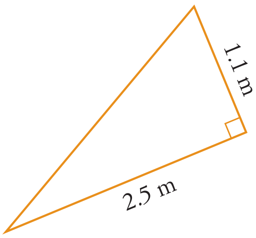
g Kite with diagonals of 95 cm and 46 cm

h Rhombus shaped rug with diagonals of 3.8 m and 2.1 m

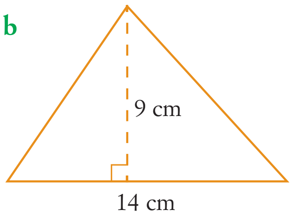
See Example 2

5 Find the areas of the following shapes.

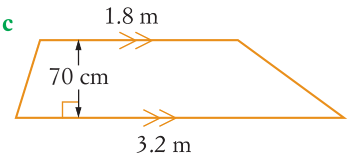
a



b

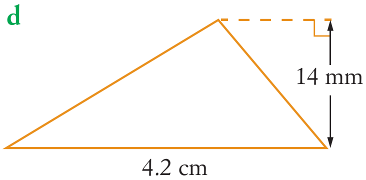


c



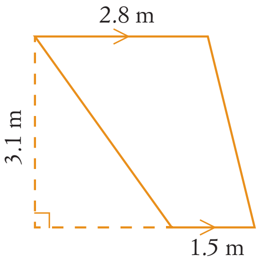
(Answer in m^2.)

d



(Answer in cm^2.)

e



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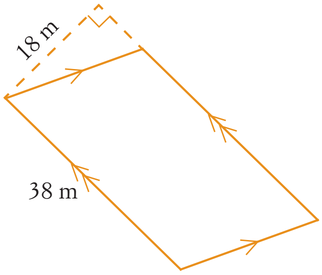
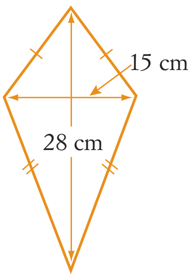


g



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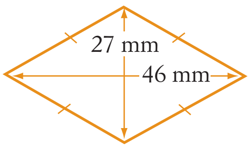
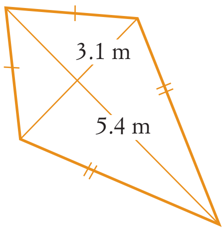


i



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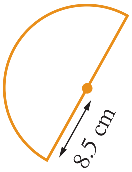
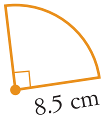
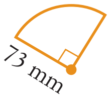
Fluency

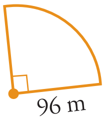
6 Find the areas of the following shapes.

a


b


c


d


e


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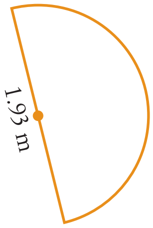
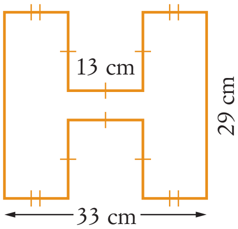
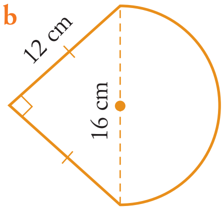
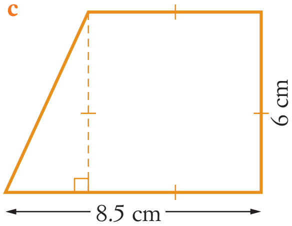


7 Work out the missing measurements, then find the areas of these shapes.

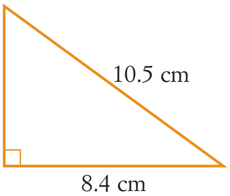
a


b


c


8 Find the areas of these triangles.

Worked solutions
Exercise 4.1
MAT10MGWS00010
See Example 3

a


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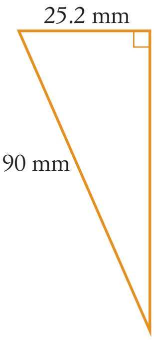
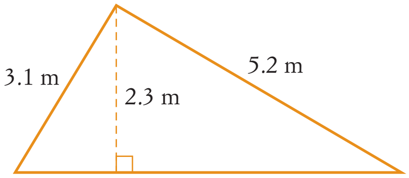
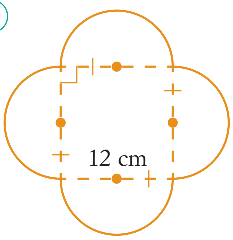
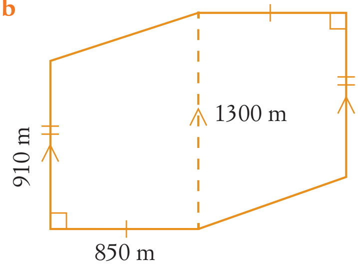


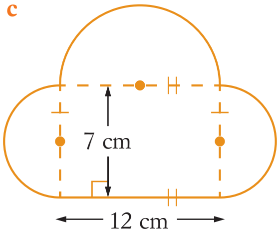
c


9 Find the areas of the following compound shapes.

Worked solutions
Exercise 4.1
MAT10MGWS00010
See Example 3

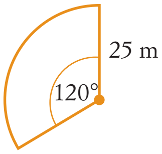
a


b


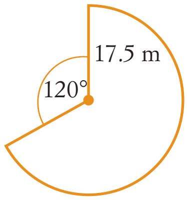
c


10 Find the areas of the following shapes.

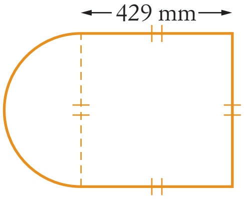
a



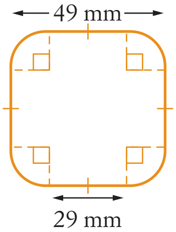
b



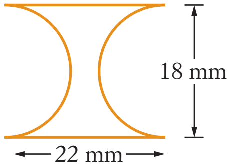
c



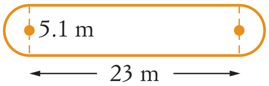
d



e



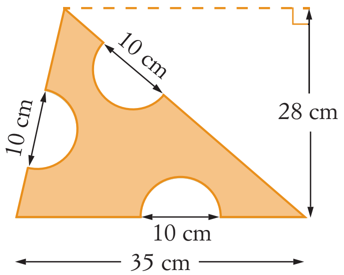
f



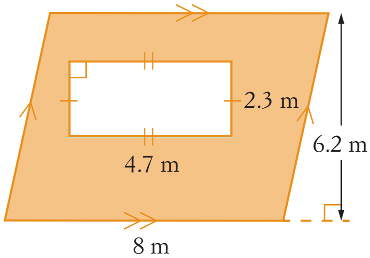
Worked solutions
Exercise 4.1
MAT10MGWS00010
See Example 3

11 Find the areas of the following shaded regions.

a

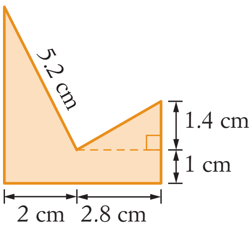


b

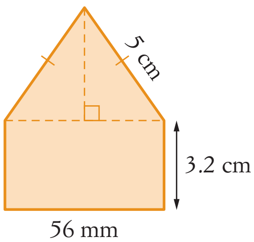


12 Find the areas of the following shapes.

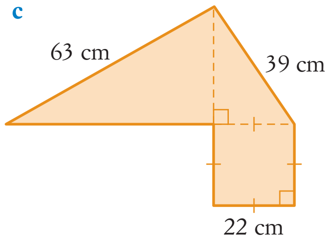
a



b



c



Problem solving

13 A circular plate has an area of 707 cm^2. What is its radius?

14 A square piece of metal has an area of 110.25 cm^2. What are its dimensions?

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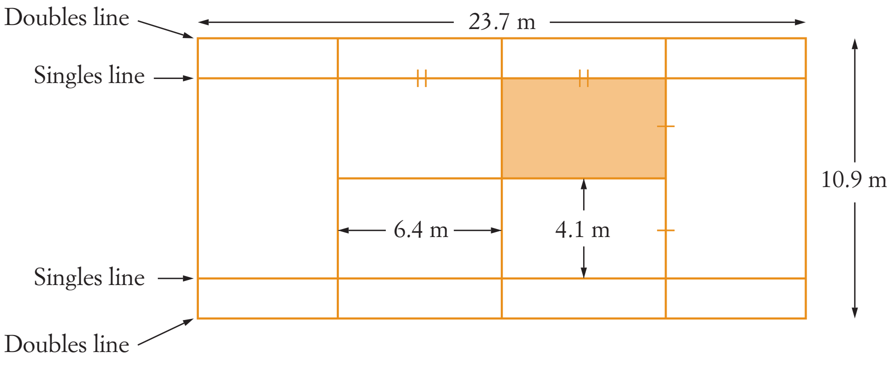
15 Here is a diagram of a full-sized tennis court.



a What area (shaded) does a player have to serve into?

b Find the total area for play in a singles match.

c Find the extra area that is available for a doubles match.

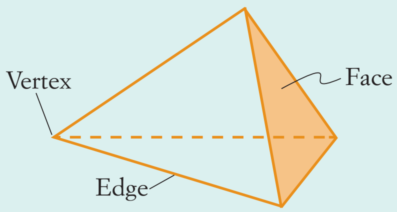
4.2 Surface area

In the previous section of this chapter we found the areas of a range of **two-dimensional (2D)** shapes. Two-dimensional shapes are flat and have length and width but no depth.
Three-dimensional (3D) shapes occupy space and have length, width and depth.
Solids are 3D figures. The **faces** of a 3D shape are its separate outside surfaces. The lines where the faces meet are called **edges** and the corners where the edges meet are called **vertices.** The cross-section and/or the faces are used to name 3D shapes.

Important!

3D shapes

The **faces** of a 3D shape separate the interior from the exterior.
The lines where the faces meet are called **edges**.
The corners where edges meet are called **vertices**
(the singular form is **vertex**).

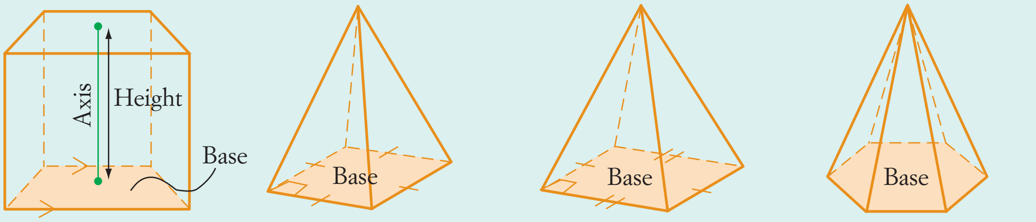


Weblink
K-6 Geometric Shapes
MAT10MGWB00004

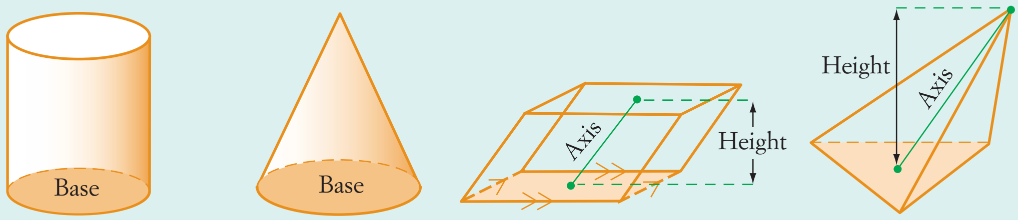
Important!

Classifying 3D Shapes

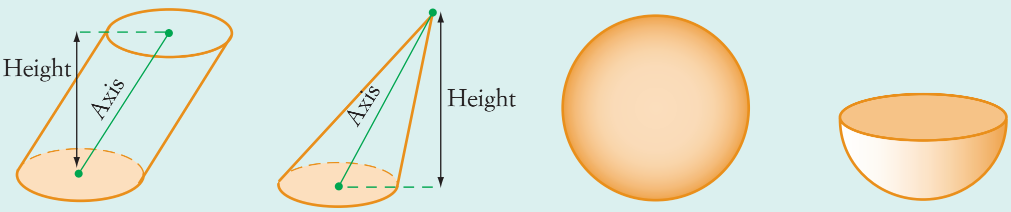
A **prism** has constant polygonal cross-sections in one direction. All the cross-sections parallel to one face (the **base)** are the same shape and size as the base.
A **pyramid** has a base that is a polygon. The other faces are triangles that all meet at a **vertex** called the **apex.** The cross-sections parallel to the base are different sizes, so it is not a prism.
The **axis** of a prism or pyramid is a line passing through the centres of all the cross-sections parallel to its base. The height of a prism or pyramid is measured at right angles (90°) to the base. **Slant** prisms and pyramids have the axis at an angle to the height. The axis *is* the height for **right** prisms and pyramids.
A **parallelepiped** is a slant parallelogram-based prism. Its faces are all parallelograms.
A **cylinder** is like a prism with a circular base. A **cone** is like a pyramid with a circular base.
A **sphere** is a ball shape. A **hemisphere** is half a sphere.



Trapezoidal prism **Square pyramid** **Rectangular pyramid** **Hexagonal pyramid**



Cylinder **Cone** **Parallelepiped** **Slant triangular pyramid**



Slant cylinder **Slant cone** **Sphere** **Hemisphere**

Other 3D shapes made from polygons are called **hedrons.** Hedrons must have flat faces.
They are named using Greek words for the number of faces: **tetra** = **4, hexa** = **6, octa** = **8, deca** = **10, dodeca** =**12** and **icosa** = 20.
There are just five **Platonic solids.** These are regular hedrons in which every face is the same and every vertex is the same.

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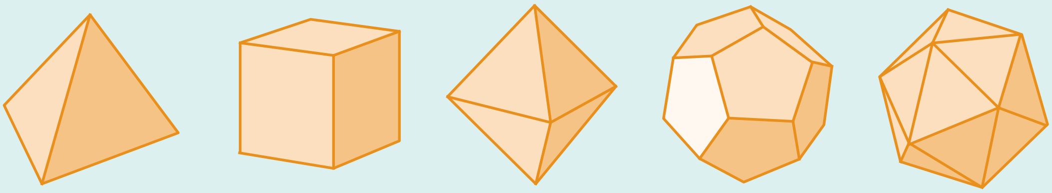
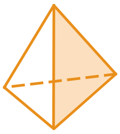
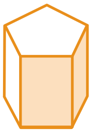
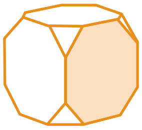


Regular tetrahedron | Cube or regular hexahedron | Regular octahedron | Regular dodecahedron | Regular icosahedron

Example 6

Name the following shapes.

a 
b 
c 
d 

Solution

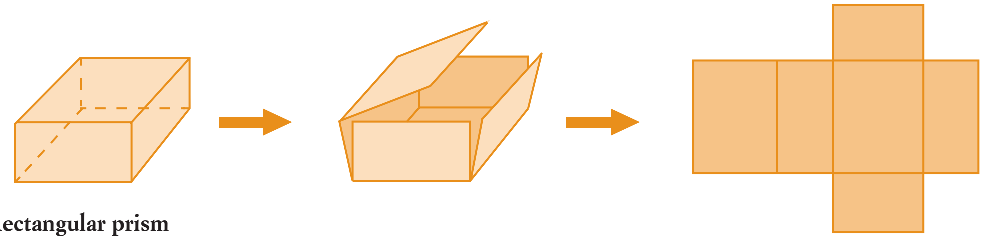
a This is like a prism but it has a circular base. **Cylinder.**

b This is a pyramid with a triangular base. It is also a hedron with 4 faces. **Triangular pyramid or tetrahedron.**

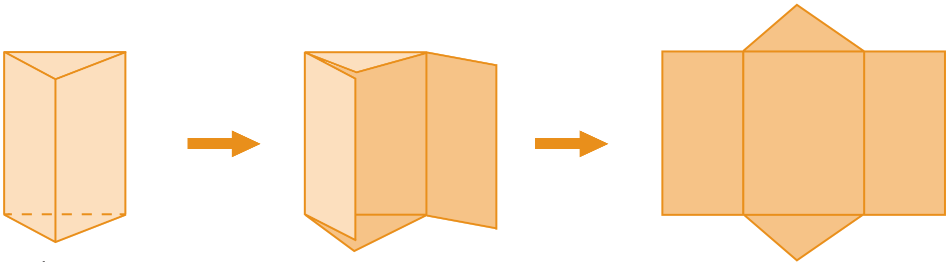
c This is a prism with a base that is a pentagon. **Pentagonal prism**

d This is a hedron with 14 faces. **14-sided hedron**

Some people would call Example **6d** a **truncated cube**. It is not regular because it has a mixture of differently shaped faces.
The **net** of a 3D shape is a 2D shape that can be unfolded from the 3D shape, or folded up to make the 3D shape.



Rectangular prism **Net**

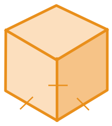


Triangular prism **Net**

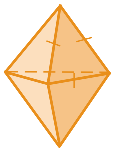
Example 7

Draw nets of the following shapes.

a



b

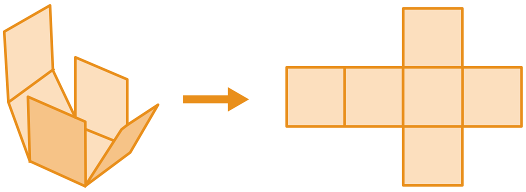


Solution

a This solid is a cube.

Imagine unfolding the cube.

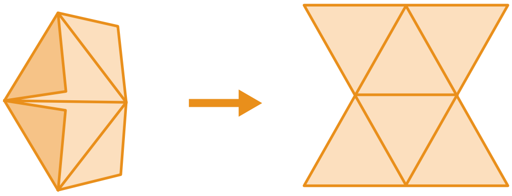
All the faces are squares.



b 3 triangles meet at the top and bottom vertices, but 4 meet at the others. It is an irregular hexahedron.

Imagine unfolding the hexahedron.

The faces are all equilateral triangles.



The area of a 3D shape is known as its **surface area**.

Important!

Surface area

The surface area of a 3D shape is the total area of all its faces.

The net of a 3D shape is useful for finding its surface area.

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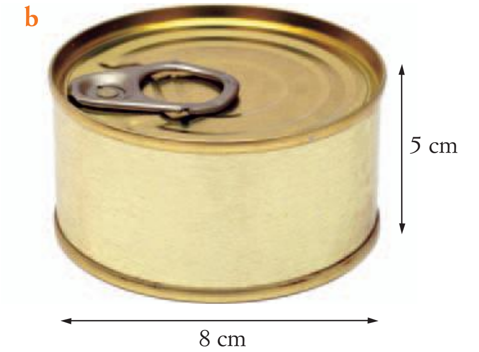
Example 8

Find the surface area of each of the following shapes.

a

b



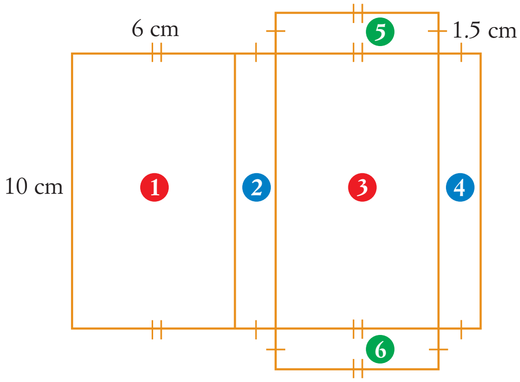
Worksheet
Area and volume 1
MAT10MGWK00010

Worksheet
Area and volume 3
MAT10MGWK00012

Puzzle sheet
Surface area
MAT10MGPS00009

Solution

a First draw a net of the shape.
Write in the dimensions you know.
Number the faces.
There are 6 rectangular faces altogether.
Rectangle 1 = rectangle 3
Rectangle 2 = rectangle 4
Rectangle 5 = rectangle 6



Find the area of face 1 (or 3). | **Face 1** $= l \times w$

Substitute for l and w. | $= 10\text{ cm} \times 6\text{ cm}$

$= 60\text{ cm}^2$

Find the area of face 2 (or 4). | **Face 2** $= l \times w$

Substitute for l and w. | $= 10\text{ cm} \times 1.5\text{ cm}$

$= 15\text{ cm}^2$

Find the area of face 5 (or 6). | **Face 5** $= l \times w$

Substitute for l and w. | $= 6\text{ cm} \times 1.5\text{ cm}$

$= 9\text{ cm}^2$

Write the rule for surface area (SA). | **SA = sum of the areas of all faces**

Use the fact that some rectangles are equal. | $= 2 \times \text{face } 1 + 2 \times \text{face } 2 + 2 \times \text{face } 5$

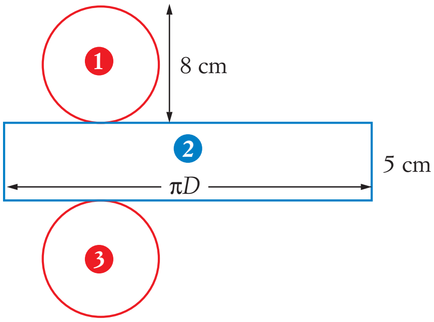
Substitute areas previously calculated. | $= (2 \times 60 + 2 \times 15 + 2 \times 9)\text{ cm}^2$

Evaluate. | $= 168\text{ cm}^2$

b First draw a net of the shape.

Write in the dimensions you know. Number the faces.

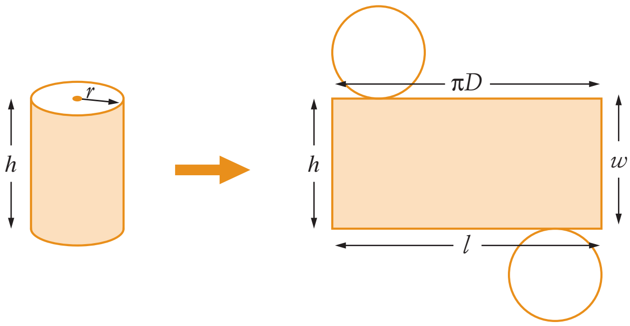
There are only 3 faces: a rectangle and 2 equal-sized circles.



Find the area of the circles.	**Face 1** $= \pi r^2$
Substitute for known values.	$= \pi \times (4\text{ cm})^2$
Store as π × 4 x^2 STO A	πx4²→A 50.26548246
Round, but keep the full value in A.	**Face 1** $\approx$ **50.3 cm²**
Find the area of the rectangle.	**Face 2** $= l \times w$
Substitute for l and w. Here, l is the circumference of the circle.	$= \pi D \times 5\text{ cm}$
	$= \pi \times 8\text{ cm} \times 5\text{ cm}$
Store as π × 8 × 5 STO B	πx8x5→B 125.6637061
Round, but keep the full value in B.	**Face 2** $\approx$ **125.7 cm²**
Write the rule for surface area (SA).	**SA = sum of the areas of all faces**
Use the fact that 2 faces are equal.	$= 2 \times$ **face 1** $+$ **face 2**
Substitute for areas previously calculated.	$\approx (2 \times 50.3 + 125.7)\text{ cm}^2$
Enter as 2 × A + B =	2xA+B 226.1946711
Round to the nearest cm².	$\approx$ **226 cm²**

It is usually more convenient to use a rule for working out the surface area of a cylinder. The curved area of a cylinder, without the top and bottom, is called the **curved surface area** (**CSA**). From the previous example, you can see that the curved surface area is calculated as follows:

$$\begin{aligned} \text{CSA} &= l \times w \\ &= \pi D \times h \\ &= \pi \times 2r \times h \\ &= 2\pi r h \end{aligned}$$



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Important!

Curved surface area of a cylinder

Curved surface area (CSA) of a cylinder $= 2\pi rh$

Example 9

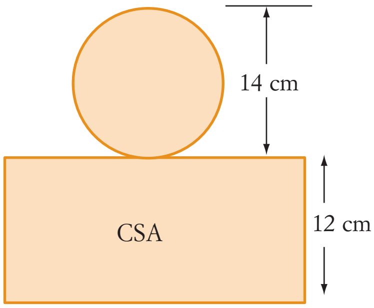
A cylinder is 12 cm high and has a radius of 7 cm. Find the surface area of the cylinder if it has a base but no lid.

Solution

First draw a net of the shape.

Write in the dimensions you know.

There are only 2 faces: the CSA of the cylinder and a circular end.



Write the rule for the curved surface area.	**CSA** $= 2\pi rh$
Substitute known information.	$= \mathbf{2 \times \pi \times 7\ cm \times 12\ cm}$
Enter as 2 [×] [π] [×] 7 [×] 12 [STO] [A]	2×π×7×12→A 527.7875658
Round, but keep the full value in A.	**CSA** $\approx$ **527.8 cm²**
Calculate the area of the base.	**Area of base** $= \pi r^2$
Substitute known information.	$= \pi \times (\mathbf{7\ cm})^2$
Enter as [π] [×] 7 [x^2] [STO] [B]	π×7²→B 153.93804
Round, but keep the full value in B.	**Area of base** $\approx$ **153.9 cm²**
Find the surface area (SA).	**SA** $=$ **CSA** $+$ **area of base**
Substitute the calculated values.	$\approx$ **527.8 cm²** $+$ **153.9 cm²**
Enter as A [+] B [=]	A+B 681.7256058
Round to the nearest cm².	$\approx$ **682 cm²**

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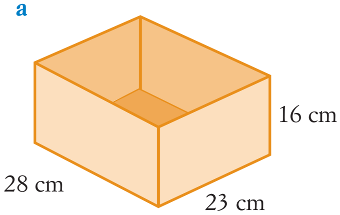
A net may be used to form an open solid or a closed solid. This tissue box is an example of a **closed solid**.

This can with the lid removed is an example of an **open solid**.

Example 10

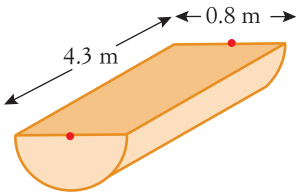
Calculate the surface area of each of the following solids.

a



Open box

b



Closed half-cylinder

Solution

a Draw the net for the open box.

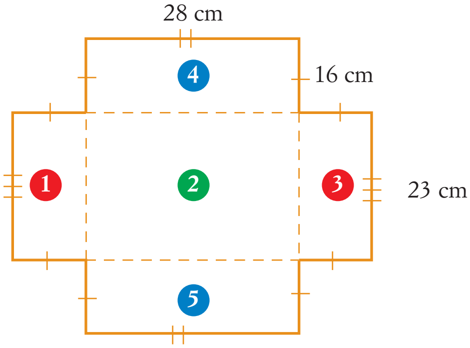
Write in the dimensions you know.

Number the faces.

There are 5 rectangular faces altogether.

Rectangle 1 = rectangle 3

Rectangle 4 = rectangle 5



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Find the area of face 1 (or 3).
Substitute for l and w.

$$\begin{aligned}\text{Face 1} &= l \times w\\ &= 23\text{ cm} \times 16\text{ cm}\\ &= 368\text{ cm}^2\end{aligned}$$

Find the area of face 4 (or 5).
Substitute for l and w.

$$\begin{aligned}\text{Face 4} &= l \times w\\ &= 28\text{ cm} \times 16\text{ cm}\\ &= 448\text{ cm}^2\end{aligned}$$

Find the area of face 2.
Substitute for l and w.

$$\begin{aligned}\text{Face 2} &= l \times w\\ &= 28\text{ cm} \times 23\text{ cm}\\ &= 644\text{ cm}^2\end{aligned}$$

Write the rule for surface area (SA).
Use the fact that some rectangles are equal.
Substitute areas previously calculated.
Evaluate.

$$\begin{aligned}\text{SA} &= \text{sum of the areas of all faces}\\ &= 2 \times \text{face 1} + \text{face 2} + 2 \times \text{face 4}\\ &= (2 \times 368 + 644 + 2 \times 448)\text{ cm}^2\\ &= 2276\text{ cm}^2\end{aligned}$$

b First draw a net of the shape.

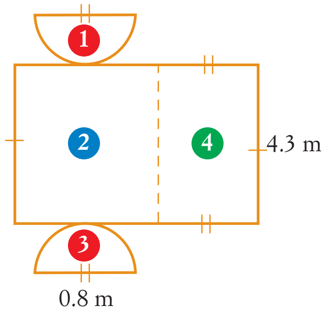
Write in the dimensions you know.
Number the faces.

There are 4 faces: 2 rectangles and 2 equal-sized semicircles.

Faces 1 and 3 are the semicircular ends of the half-cylinder.

Face 2 is the curved surface area (CSA) of the half-cylinder.

Face 4 is the top of the half-cylinder.



Find the area of the semicircles.

$$\text{Face 1} = \tfrac{1}{2}\pi r^2$$

Cancel and substitute for known values.
Here $r = \frac{1}{2} \times 0.8 = 0.4$

$$= \tfrac{1}{2} \times \pi \times 0.4^2$$

Store as
0.5 [×] [π] [×] 0.4 [x^2] [STO] [A].

```
0.5×π×0.4²→A
        0.2513274123
```

$\text{Face 1} = 0.251\,327\ldots\text{ m}^2$

Write the rule for the curved surface area of the half-cylinder.
Cancel.
Substitute known information.

$$\begin{aligned}\text{Face 2} &= \tfrac{1}{2} \times 2\pi rh\\ &= \pi rh\\ &= \pi \times 0.4\text{ m} \times 4.3\text{ m}\end{aligned}$$

Store as [π] [×] 0.4 [×] 4.3 [STO] [B]

```
π×0.4×4.3→B
         5.403539364
```

$\text{Face 2} = 5.403\,53\ldots\text{ m}^2$

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Find the area of the remaining rectangle.	Face 4 $= l \times w$
Substitute for l and w. Here, l is the diameter of the semicircle and w is the height of the half-cylinder.	$= 0.8\ \text{m} \times 4.3\ \text{m}$
	Face 4 $= 3.44\ \text{m}^2$
Write the rule for surface area (SA).	**SA = sum of the areas of all faces**
Use the fact that 2 faces are equal.	$= 2 \times$ face 1 + face 2 + face 4
Enter as 2 [×] A [+] B [+] 3.44 [=]	2×A+B+3.44 9.346194189
Evaluate.	$\approx 9.35\ \text{m}^2$

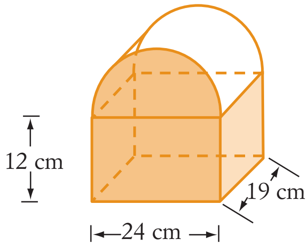
For part **b** of the previous example, it should be noted that the areas of faces 1, 2 and 3 were not rounded before the calculation of the surface area. This ensures that the final answer is more accurate than it would have been if these values had been rounded.

Many solids are formed by combining two or more different solids.

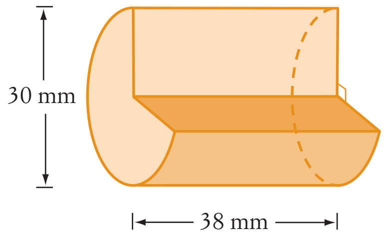
Example 11

Calculate the surface area of each of the following solids correct to one decimal place.

a



b



Solution

a This solid consists of a half-cylinder mounted on a rectangular prism.

The surface area of the shape will include the 3 faces of the half-cylinder and 5 faces of the rectangular prism.

Calculate the curved surface of the half-cylinder. CSA is the curved surface of a cylinder.	**Curved surface of half-cylinder** $= \frac{1}{2}\text{CSA}$
Substitute the rule for CSA.	$= \frac{1}{2} \times 2\pi r h$
Substitute for known values. The diameter is 24 cm.	$= \frac{1}{2} \times 2\pi \times 12 \times 19$
Cancel and evaluate.	$= 716.283\ldots\ \text{cm}^2$
The other two faces are semicircles.	**Area of semicircular faces** $= \pi r^2$

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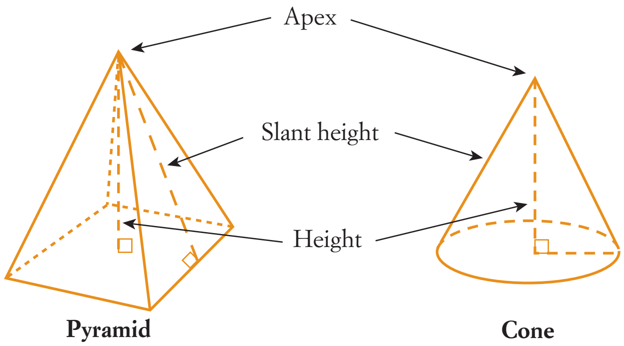
Substitute for known values.	$= \pi \times 12 \times 12$
Evaluate.	$= 452.389\ldots \text{ cm}^2$
Now calculate the area of the 5 rectangular faces of the prism.	**Area of rectangular faces** $= (24 \times 19) + 2 \times (12 \times 24) + 2 \times (19 \times 12)$
Evaluate.	$= 1488 \text{ cm}^2$
Calculate the surface area.	$\text{SA} = 716.283\ldots + 452.389\ldots + 1488$
Evaluate.	$= 2656.67\ldots \text{ cm}^2$
Round off and state the result.	**The surface area of the solid is approximately 2656.7 cm^2.**

b This solid consists of a closed cylinder with a portion removed.

Calculate the portion of the cylinder that has been removed.	**Portion removed** $= \dfrac{90^\circ}{360^\circ} = \dfrac{1}{4}$
The surface area of the solid consists of $\frac{3}{4}$ of the CSA for a full cylinder, two ends that are each $\frac{3}{4}$ of a full circle and two equal rectangular faces.	
Calculate the curved surface of the solid.	**Curved surface of solid** $= \frac{3}{4}\text{CSA}$
Substitute the rule for CSA.	$= \frac{3}{4} \times 2\pi rh$
Substitute for known values. The diameter is 30 mm.	$= \dfrac{3}{{}_1\cancel{2}\cancel{4}} \times \cancel{2}^1 \pi \times 15 \times \cancel{38}^{19}$
Cancel and evaluate.	$= 2686.06\ldots \text{ mm}^2$
The two end faces are three-quarter circles.	**Area of end faces** $= 2 \times \frac{3}{4} \times \pi r^2$
Substitute for known values and cancel.	$= \frac{3}{2} \times \pi \times 15 \times 15$
Evaluate.	$= 1060.28\ldots \text{ mm}^2$
Now calculate the area of the rectangular faces of the solid.	**Area of rectangular faces** $= 2 \times (15 \times 38)$
Evaluate.	$= 1140 \text{ mm}^2$
Calculate the surface area.	$\text{SA} = 2686.06\ldots + 1060.28\ldots + 1140$
Evaluate.	$= 4886.34\ldots \text{ mm}^2$
Round off and state the result.	**The surface area of the solid is approximately 4886.3 mm^2.**

A **pyramid** is a solid made up of a base and triangular faces that meet at an **apex**, which is also called the **vertex** of the pyramid. The **slant height** is the perpendicular distance from the vertex down a face, as shown in the diagram below. A **cone** is a solid with a circular base and an apex. The height, h, of the cone is measured perpendicular to the base and the slant height is the distance from the edge of the base to the apex along the face of the cone.

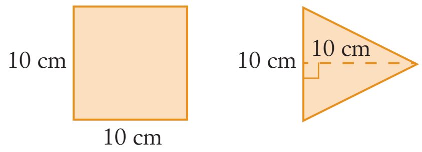


Investigate: Surface areas of pyramids and cones

You will need cardboard, scissors, tape, a protractor, a compass and a ruler.

Pyramids

1 First, make a square pyramid using the dimensions shown here.



2 Next, construct a pyramid using only equilateral triangles.

3 Now, use the pyramids you have made to answer the following.

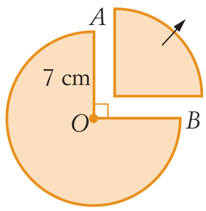
a What are the differences between a triangular pyramid and a square pyramid?

b How are these two types of pyramid similar?

c What would you call a pyramid with a six-sided base?

Cones

1 Draw a circle with a radius of 7 cm. Mark in two radii, OA and OB, so that $\angle AOB$ is 90°, as shown. Cut out the 90° sector and put it to one side. Join up the cut edges with some tape to form a cone.



2 Repeat this procedure, but this time take out a larger sector. How are the two cones different? How are they similar?

3 Make a list of household objects that have a conical shape. (They may only be part of a cone.)

4 Find out what the *Venturi effect* has to do with cones.

5 What are the advantages and disadvantages of making paper cups in a conical shape?

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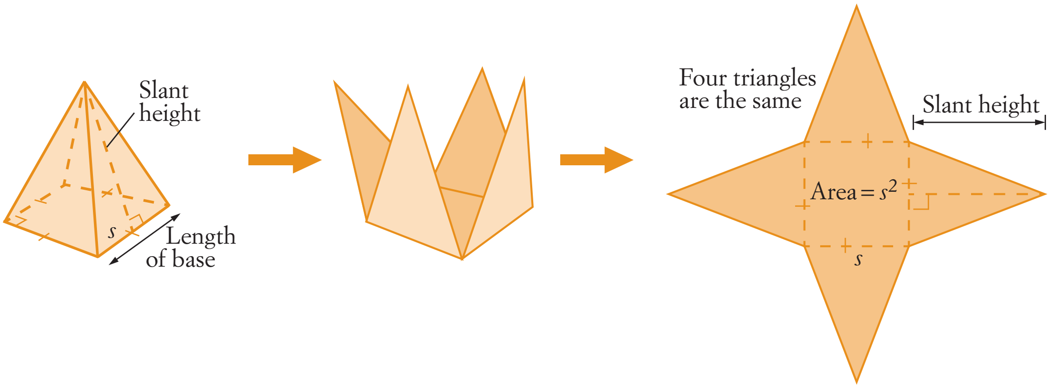
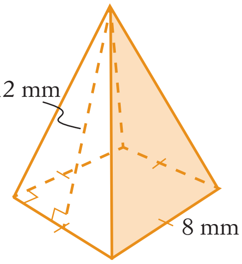
When you work out the surface area of a square pyramid, you need to know the length of the base and the slant height.



Example 12

Video tutorial
Surface area of a pyramid
MAT10MGVT10008

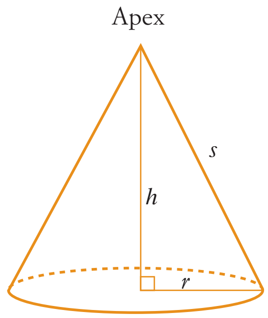
Find the surface area of this square pyramid.



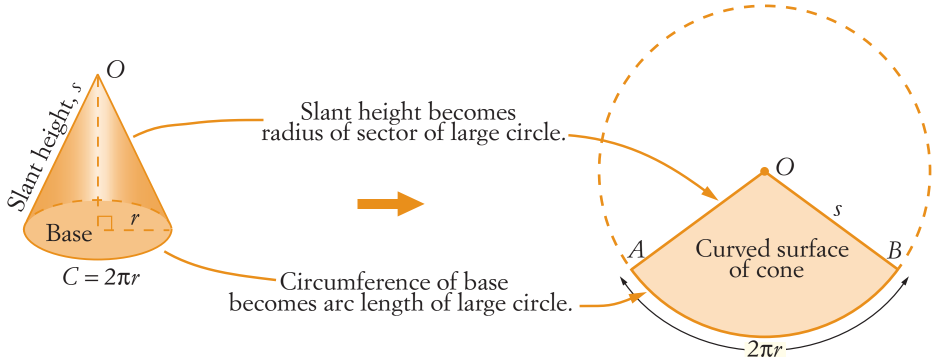
Solution

The base is a square.	**Area of base** $= l^2$
Substitute for l.	$= 8\text{ mm} \times 8\text{ mm}$
Evaluate.	$= 64\text{ mm}^2$
Each side is a triangle.	**Area of side** $= \frac{1}{2}bh$
Substitute for b and h.	$= \frac{1}{2} \times 8 \times 12\text{ mm}^2$
Evaluate.	$= 48\text{ mm}^2$
Write a rule for the surface area.	**Surface area** $=$ **area of base** $+ 4 \times$ **area of side**
Substitute.	$= 64\text{ mm}^2 + 4 \times 48\text{ mm}^2$
Evaluate.	$= 256\text{ mm}^2$

A **cone** is a solid with a circular base and an apex. The height, h, of the cone is measured perpendicular to the base and the slant height, s, is the distance from the edge of the base to the apex along the face of the cone.



The net of a cone consists of the curved surface and the circular base as shown here. The base is a circle with radius, r. The curved surface is the sector of a circle with radius, s, and arc length AB. The length of AB must be the same as the circumference of the base, i.e. $2\pi r$.



We can develop a formula for the area of a cone using this information.

$$\frac{\text{Area of sector } AOB}{\text{Area of large circle}} = \frac{\text{arc length } AB}{\text{circumference of large circle}}$$

$$\frac{\text{Area of sector } AOB}{\pi s^2} = \frac{2\pi r}{2\pi s} = \frac{r}{s}$$

$$\text{Area of sector } AOB = \frac{r}{s} \times \pi s^2$$

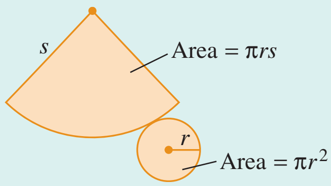
$$= \pi rs$$

The total surface area of the cone is the area of the base and the area of the sector ABC.

Important!

Surface area of a cone

Surface area of a cone $= \pi r^2 + \pi rs$
where $r =$ radius of the base of the cone and
$s =$ slant height of the cone



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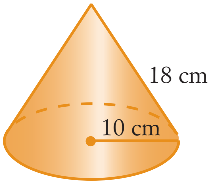
Example 13

Find the surface area of this cone.



Solution

Write the rule for surface area. $\mathbf{SA} = \pi rs + \pi r^2$

Substitute known values. $= \pi \times 10\ \text{cm} \times 18\ \text{cm} + \pi \times (10\ \text{cm})^2$

Evaluate. $= 565.486\ldots\ \text{cm}^2 + 314.159\ldots\ \text{cm}^2$

Round to the nearest cm^2. $\approx 880\ \text{cm}^2$

Investigate: Surface area of a sphere

Work in groups of four or five students. Each group will need an orange, some paper towels and a knife to cut the orange in half.
Finding a rule for the surface area of a sphere can be done using some sophisticated mathematics. It can also be approximated using some basic reasoning.

- Squeeze and mould your orange to make it shaped like a sphere as much as possible.
- Cut your orange in half. The circle formed by the edge of the orange is called the **great circle** of the orange. This is the circle with the same radius as the radius of the sphere.

- Trace a great circle of your orange on a paper towel.
- Discuss with the members of your group how many of these great circles you think you can cover with pieces of your orange's peel.
- Trace four more great circles on your paper towels.
- Now carefully tear off pieces of the orange peel, each about 4 or 5 cm^2.
- Use the pieces of peel you have torn off to cover as many circles as possible. Place the pieces, one at a time, in the great circle you drew. Be careful not to overlap pieces or leave gaps. You may need to tear off some smaller pieces to entirely cover each circle without gaps or overlaps.
- Record the number of great circles covered by the orange peel.

Work as a group to use your results to answer the following questions. Make sure you record your answers and can explain your conclusions because each group member needs to be prepared to explain your group's thinking to the whole class.

1. Write a mathematical expression for the area covered for each great circle. What does r in your expression stand for?
2. How many circles did you cover in all?
3. Based on your results, write a rule or equation that gives the surface area of your orange.
4. Do you think your equation will work for finding the surface area of any sphere? Explain your reasoning.

Now summarise your group's findings and conclusions.
Your teacher will ask all groups to share their findings and conclusions in turn.
Don't forget to clean up!

From the previous investigation, you should have seen that the surface area of a sphere is $4\pi r^2$. A hemisphere is half a sphere and may be either open or closed. An open hemisphere has no circular base while a closed hemisphere does. You can also use the formula for the surface area of a sphere to calculate the surface of a hemisphere – with or without a base.

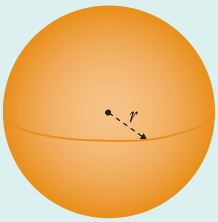
$$\begin{aligned}\text{SA of an open hemisphere} &= \frac{1}{2}\text{SA of a sphere}\\ &= \frac{1}{2} \times 4\pi r^2\\ &= 2\pi r^2\end{aligned}$$

$$\begin{aligned}\text{SA of a closed hemisphere} &= \text{SA of an open hemisphere} + \text{area of circular base}\\ &= 2\pi r^2 + \pi r^2\\ &= 3\pi r^2\end{aligned}$$

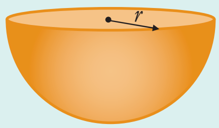
Important!

Surface area of a sphere

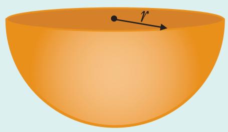
SA of a sphere $= 4\pi r^2$



SA of a closed hemisphere $= 3\pi r^2$



SA of an open hemisphere $= 2\pi r^2$



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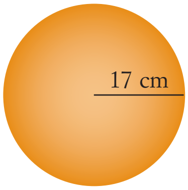
Example 14

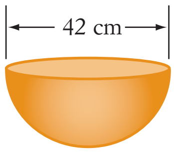
Video tutorial
Surface area of a cone and sphere
MAT10MGVT10009

Calculate the surface areas of each of the following.

a



b



Closed hemisphere

Solution

a Write the rule for the SA of a sphere. $\quad \mathbf{SA} = 4\pi r^2$

Substitute for r. $\quad = 4 \times \pi \times 17^2$

Evaluate. $\quad = 3631.68\ldots$

Round off. $\quad \approx 3631.7 \text{ cm}^2$

b Write the rule for the SA of a sphere. $\quad \mathbf{SA} = 3\pi r^2$

Substitute for r. Diameter = 42 cm. $\quad = 3 \times \pi \times 21^2$

Evaluate. $\quad = 4156.32\ldots$

Round off. $\quad \approx 4156.3 \text{ cm}^2$

Exercise 4.2 Surface area

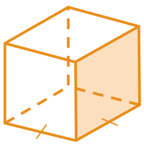
A calculator should be used where appropriate in this exercise. For calculations involving π, use the π key on your calculator and answer correct to 1 decimal place.

Understanding

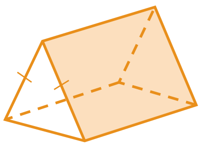
Extra questions
Exercise 4.2
MAT10MGEQ00011
See Example 4
See Example 6

1 Name the following shapes.

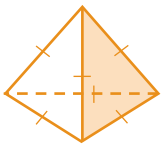
a



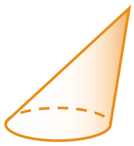
b



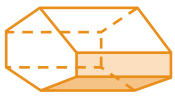
c



d



e



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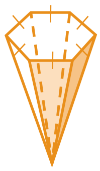
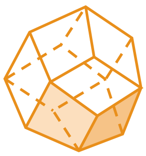


g



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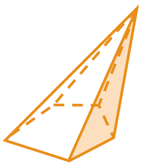

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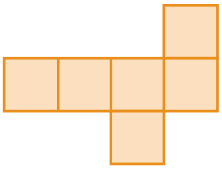
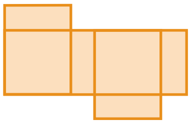
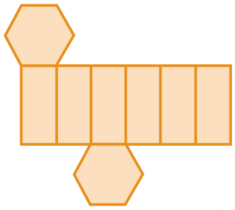
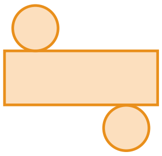
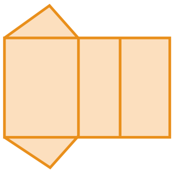
2 Identify the shapes whose nets are shown below.

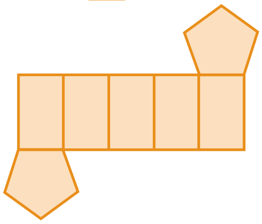
a


b


c


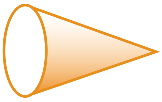
d


e


f


See Example 7

3 Draw nets for the following shapes.

a


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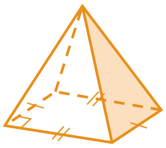
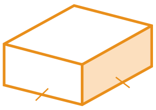


c


d


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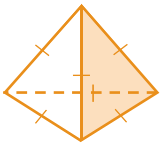

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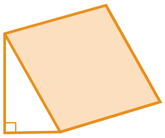
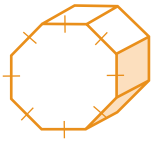
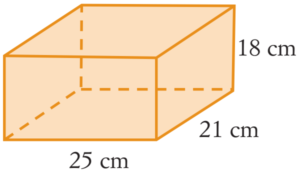


g


See Example 8

4 Draw nets of the following shapes, including all necessary measurements.

a


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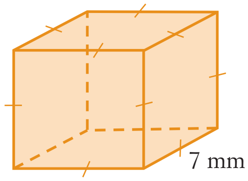

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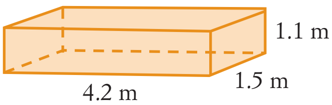

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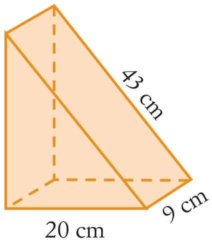
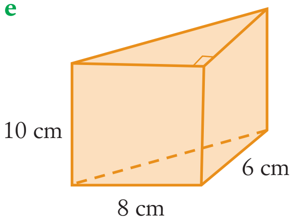


e


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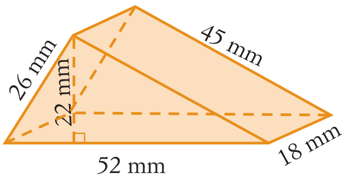

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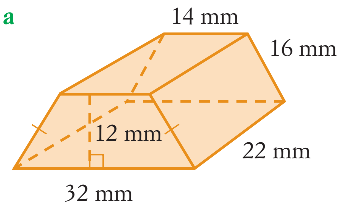
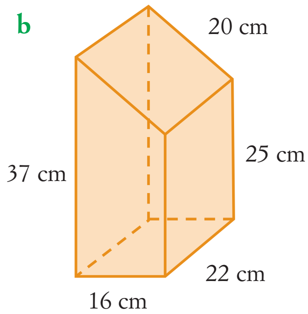
5 Draw nets of the following shapes, including all necessary measurements.

a

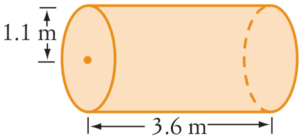


b



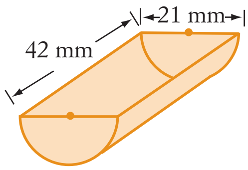
6 Draw nets of the following shapes, including all necessary measurements. See Example 8, 10

a



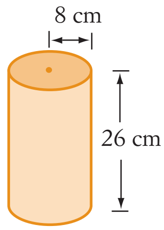
Closed cylinder

b



Open half-cylinder

c

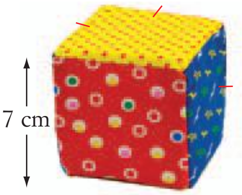


Cylinder (open both ends)

7 Draw nets of the following shapes, including all necessary measurements. See Example 8

a

b



c

8 Draw nets of the following shapes, including all necessary measurements.

a

b

c

See Example 8

Fluency

9 Use the nets you drew in question **4** to find the surface area of each shape. See Example 8

10 Use the nets you drew in question **5** to find the surface area of each shape. See Example 8

11 Use the nets you drew in question **6** to find the surface area of each shape. See Example 8

12 Use the nets you drew in question **7** to find the surface area of each shape.

See Example 8

13 Use the nets you drew in question **8** to find the surface area of each shape.

14 Calculate the surface area of rectangular prisms with the following dimensions.

a length 12 cm, width 8 cm, height 17 cm
b length 24 mm, width 19 mm, height 11 mm
c length 2.3 m, width 1.3 m, height 0.9 m
d length 35 cm, width 17 cm, height 42 cm
e length 1.7 m, width 46 cm, height 1.4 m
f length 22 cm, width 76 mm, height 19 cm

See Example 9

15 Calculate the curved surface area of the cylinders with the following dimensions. Give your answers correct to two decimal places.

a radius 7 cm, height 11 cm
b diameter 32 mm, height 42 mm
c radius 1.7 m, height 3.3 m
d radius 5.7 cm, height 12.4 cm
e diameter 63 cm, height 2.3 m
f radius 85 mm, height 22.7 cm

Worked solutions
Exercise 4.2
MAT10MGWS00011
See Example 9

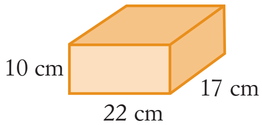
16 Calculate the surface area of the closed cylinders with the following dimensions. Give your answers correct to two decimal places.

a radius 15 cm, height 22 cm
b radius 2.8 m, height 3.1 m
c diameter 61 mm, height 89 mm
d radius 41 cm, height 86 cm
e radius 92 cm, height 1.7 m
f diameter 33 mm, height 7.9 cm

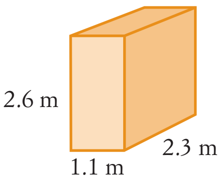
See Example 8

17 Calculate the surface area of each of the following solids.

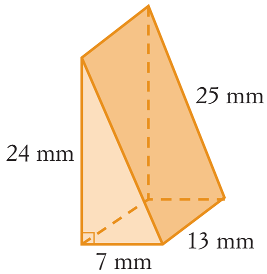
a



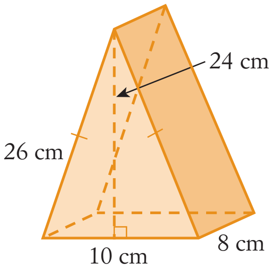
b



c



d



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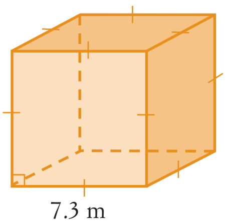
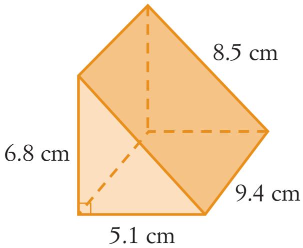


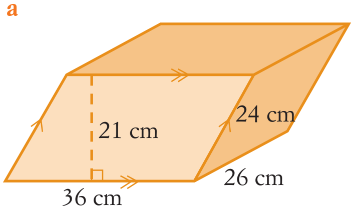
f



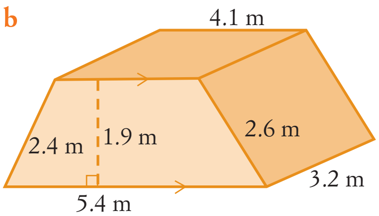
Worked solutions
Exercise 4.2
MAT10MGWS00011

18 Calculate the surface area of each the following solids.

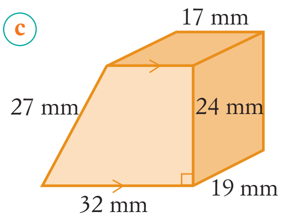
a



b



c



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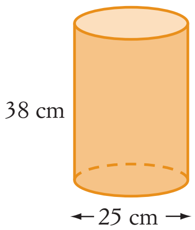
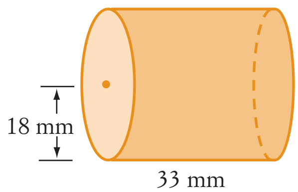
19 Calculate the surface area of each the following solids. *See* Examples 8, 10

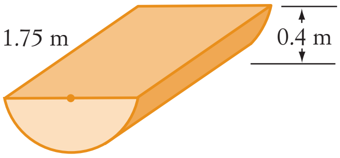
a **Closed cylinder**



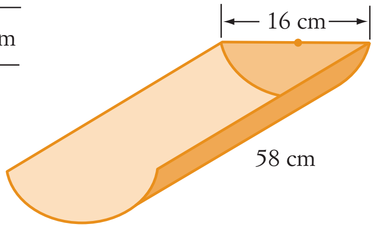
b **Cylinder, open both ends**



c **Closed half-cylinder**



d **Open half-cylinder, one end open**



20 Calculate the surface area of each of the following pyramids.

a square base measuring 20 cm by 20 cm and a slant height of 24 cm

b square base measuring 25 mm by 25 mm and a slant height of 38 mm

c rectangular base measuring 2.2 m by 4.1 m and a slant height of 3 m

21 Calculate the surface area of each of the following cones.

a radius 7 cm, slant height 24 cm

b diameter 9 mm, slant height 12 mm

c diameter 5 m, slant height 6 m

d radius 6.8 cm, slant height 17.2 cm

22 Calculate the surface area of each of the following.

a sphere with a radius of 5 cm

b sphere with a diameter of 28 mm

c open hemisphere with a radius of 3.2 m

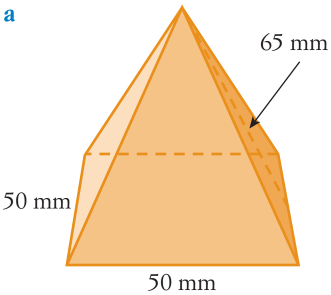
d closed hemisphere with a diameter of 43 cm

e open hemisphere with a diameter of 1.9 m

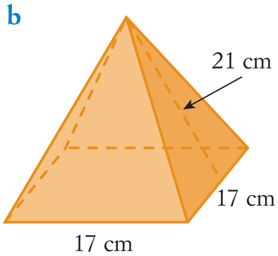
f closed hemisphere with a diameter of 63 mm

23 Calculate the surface area of each of the following pyramids. *See* Example 12

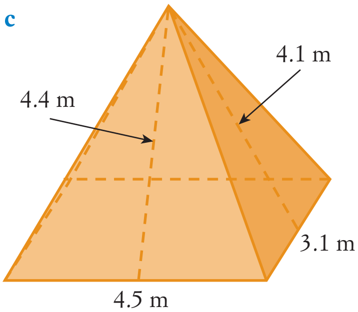
a



b



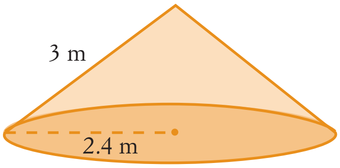
c



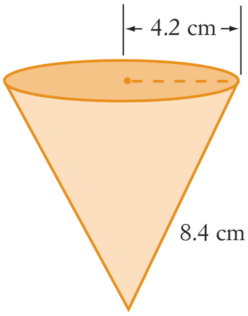
See Example 13

24 Calculate the surface area of each of the following cones.

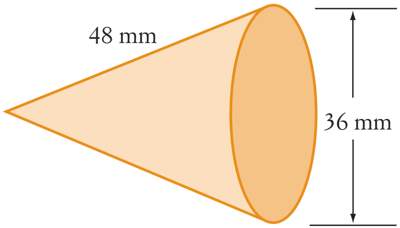
a



b



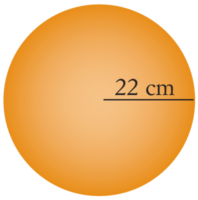
c



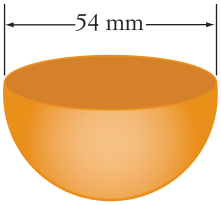
See Example 14

25 Calculate the surface area of each of the following.

a

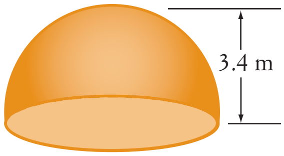


b



Open hemisphere

c



Closed hemisphere

Problem solving

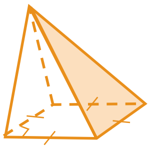
26 Sketch the following shapes.

a Cube
b Right trapezoidal prism
c Slant rectangular prism
d Slant cone
e Right square pyramid
f Slant rectangular pyramid

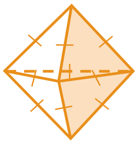
See Example 7

27 Draw nets for the following shapes.

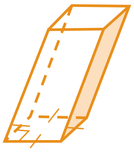
a



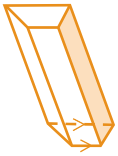
b



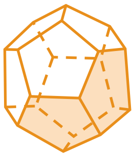
c



d



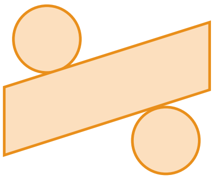
e



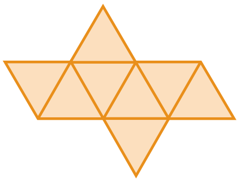
f

28 Identify the shapes whose nets are shown below.

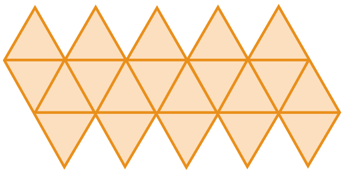
a



b



c



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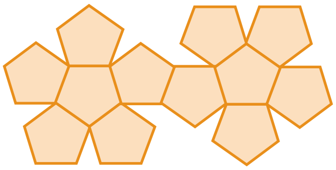

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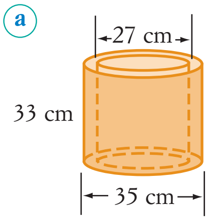
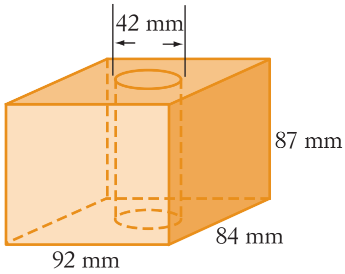
29 The surfaces of each of the following solids need to be coated with a special paint. In each case, calculate the total area that will be coated.

Worked solutions
Exercise 4.2
MAT10MGWS00011
See Example 11

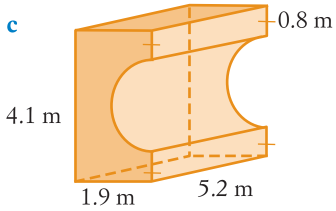
a



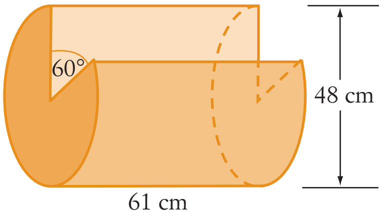
b



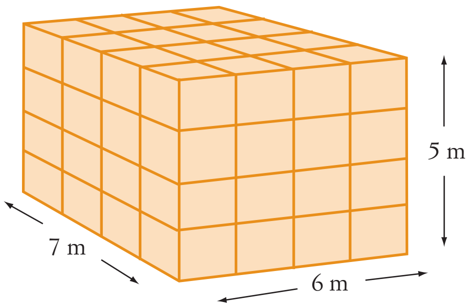
c



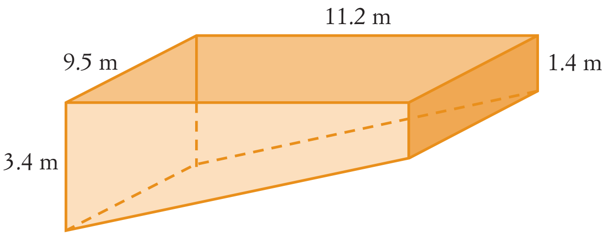
d



30 This pile of hay bales has to be covered to protect it from the weather. What area of tarpaulin is needed?



31 An in-ground swimming pool is in the shape of a trapezoidal prism as shown below.

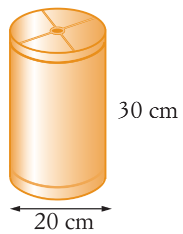


a Calculate correct to one decimal place the length of the sloping bottom edge of the pool.

b Calculate the surface area of the inside of the pool.

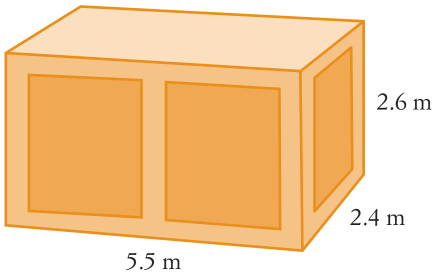
c Find the cost of coating the inside of the pool with paint that comes in 4 L tins if the tins cost \$125 and each litre of paint covers 8 m^2.

32 This lampshade needs to be re-covered. Find the area of material required if 5% wastage must be allowed for seams and overlaps.



Reasoning

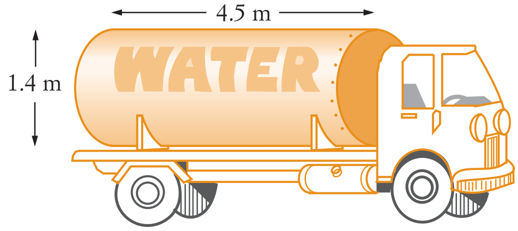
33 This container is used to transport goods by sea. It has to be painted all over.



a Find the area to be painted.

b Find the cost of painting the container if a contractor quotes $57/m^2 for materials and labour.

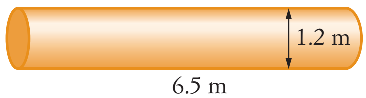
34 This water tanker is used to help compact new road base. The water tank on the truck needs repainting.



a Find the area to be painted (including the ends).

b Find the number of litres of paint needed if the coverage is 9 m^2/L.

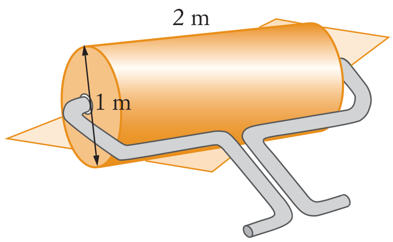
35 A rectangular sheet of steel has to be bent and welded to form this pipe.



a Find the dimensions of the smallest sheet of steel that could be used.

b Find the cost of constructing the pipe if 1.35 m^2 must be allowed for overlap and materials and labour cost $75/m^2.

36 This roller is used to roll a cricket pitch 20 m long and 2 m wide. How many revolutions will the roller make in rolling the pitch?



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4.3 Volumes of shapes

The **volume** of an object is the amount of space that it occupies. Volume is measured in **cubic** units. The units we usually use for volume are **cubic millimetres (mm^3), cubic centimetres (cm^3)** and **cubic metres (m^3). Capacity** is another word for volume but it usually refers to the amount of space inside a container. Capacity can be measured using volume units, but it is usually measured in **litres (L)**. Multiples of litres are shown below.

TLF Learning object
Turn up the volume (R11505)
MAT10MGIN00004

Important!

Volume and capacity units

1 millilitre (mL) = 1 cm^3
1 litre (L) = 1000 mL = 1000 cm^3
1 kilolitre (kL) = 1000 L = 1 m^3
1 megalitre (ML) = 1000 kL = 1000 m^3

TLF Learning object
Inside a cubic metre (L164)
MAT10MGIN00004

You can use the following when converting units. If you are changing to a *larger* unit, *divide*. If changing to a *smaller* unit, *multiply*.

$$\begin{tikzcd}
\textbf{mL (cm}^3\textbf{)} \arrow[r, bend left, "\div 1000"] & \textbf{L} \arrow[l, bend left, "\times 1000"] \arrow[r, bend left, "\div 1000"] & \textbf{kL (m}^3\textbf{)} \arrow[l, bend left, "\times 1000"] \arrow[r, bend left, "\div 1000"] & \textbf{ML} \arrow[l, bend left, "\times 1000"]
\end{tikzcd}$$

Example 15

Convert the following measurements to the units shown in brackets.

a 34 mL (cm^3)
b 1.405 m^3 (L)
c 750 m^3 (ML)
d 0.004 55 kL (cm^3)

Solution

a Write the measurement. **34 mL**
These are equal units. **= 34 cm^3**

b Write the measurement. **1.405 m^3**
Changing to a *smaller* unit, so *multiply*. **= 1.405 × 1000 L**
Evaluate. **= 1405 L**

c Write the measurement. **750 m^3**
Since 1 m^3 = 1 kL, change to kL first. **= 750 kL**
Changing to a *larger* unit, so *divide*. **= 750 ÷ 1000 ML**
Evaluate. **= 0.75 ML**

d Write the measurement. **0.004 55 kL**

Changing to a *smaller* unit, so *multiply*. = **0.004 55 × 1000 L**

Evaluate. = **4.55 L**

Changing to a *smaller* unit, so *multiply*. = **4.55 × 1000 mL**

Evaluate. = **4550 mL**

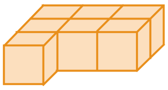
Use 1 cm^3 = 1 mL. = **4550 cm^3**

Investigate: Volume formulas

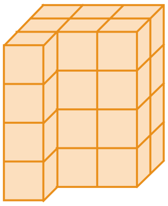
You will need centicubes, an open cylindrical container (such as a jam tin), light cardboard, scissors, a compass, some tape, some sand and a ruler. Work in groups of three to five.

Prisms

1 A centicube has an edge length of 1 cm, so the volume is 1 cm^3 and the area of each face is 1 cm^2. Use some centicubes to form a shape with a polygon base, similar to the one shown here.



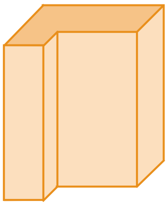
2 Use more centicubes to make copies of this shape and place them one on top of the other to form a prism, as shown here.



3 Find the volume of the prism by counting the centicubes and write down the result.

4 Work out the area of the polygon base and the height of the prism and record these.

5 Draw the prism you have formed without marking in the centicubes, as shown here. Mark in the area of the base and the height.

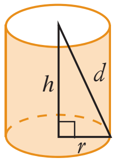


6 Complete the rule below for the volume of the prism.
Volume = … × …

Cones

1 Measure the radius, r, and height, h, of the cylinder.

2 Measure d as shown in the diagram on the right or work it out using Pythagoras' theorem. The measurement d is the radius of part of a circle that will form the curved surface of a cone.



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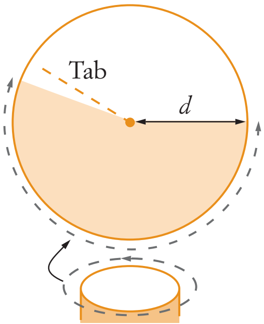
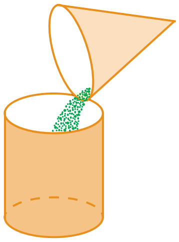
3 Draw a circle with radius d on cardboard.
4 Measure or calculate the circumference of the cylinder. This will be the length of the arc needed to form the cone.
5 Mark this distance around the edge of the circle you have drawn. Now cut out the sector, adding a small tab for taping.



6 Form the cone by taping it up. Check to see that the cone fits neatly into the cylinder.
7 Now fill the cone with sand and empty it into the cylinder. How many times do you need to do this to fill the cylinder?
8 The formula for the volume of a cylinder is $V = \pi r^2 h$.
9 Use this information to work out a formula for the volume of a cone. Compare your formula with others in the class.



Square-based pyramids

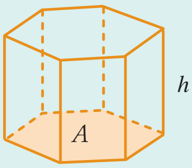
Try using the same procedure to find a formula for the volume of a square-based pyramid.

The volumes of common shapes can be worked out using formulas.

Important!

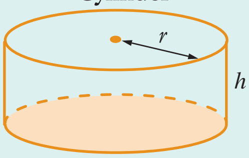
Volume formulas

Prism



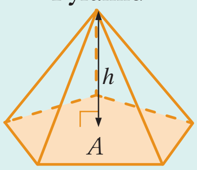
Volume = area of base × height
$V = Ah$

Cylinder



Volume = area of base × height
$V = \pi r^2 h$

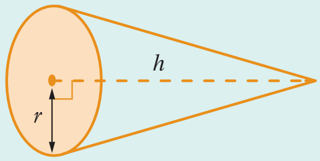
Pyramid



Volume $= \frac{1}{3}$ area of base $\times$ height

$V = \frac{1}{3}Ah$ or $V = \frac{Ah}{3}$

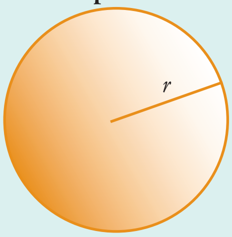
Cone



Volume $= \frac{1}{3}$ area of base $\times$ height

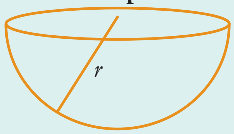
$V = \frac{1}{3}\pi r^2 h$ or $V = \frac{\pi r^2 h}{3}$

Sphere



Volume $= \frac{4}{3}\pi r^3$

Hemisphere



Volume $= \frac{2}{3}\pi r^3$

Technology worksheet

Excel worksheet: Volume calculator

MAT10MGCT00006

Worksheet

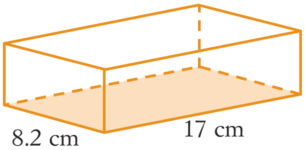
Area and volume 4

MAT10MGWK00013

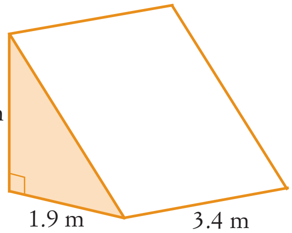
Example 16

Find the volume of each of these prisms.

a



b



Solution

a The base is a rectangle.	$A = l \times w$
Substitute the values.	$= 17 \times 8.2$
Evaluate.	$= 139.4\ \text{cm}^2$
Write the rule for the volume of a prism.	$V = Ah$
Substitute the values.	$= 139.4\ \text{cm}^2 \times 4.3\ \text{cm}$
Evaluate.	$= 599.42\ \text{cm}^3$
Round to 2 figures accuracy.	**The volume is about 600 cm^3.**

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b The base is a triangle. $A = \frac{1}{2}bh$

Substitute the values. $= \frac{1}{2} \times 1.9 \times 2.6$

Evaluate. $= 2.47\ \text{m}^2$

Write the rule for the volume of a prism. $V = Ah$

Substitute the values. $= 2.47\ \text{m}^2 \times 3.4\ \text{m}$

Evaluate. $= 8.398\ \text{m}^3$

Round to 2 figures accuracy. **The volume is about 8.4 m^3.**

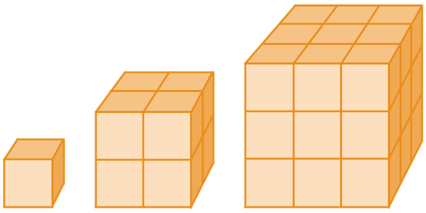
Investigate: Cubes

Task 1

You will need about 80 identical cubic blocks, such as centicubes, to complete this task.

We can consider each cube as having a length of 1 unit.

1 Use the blocks to make up larger cubes with edges of 2 units, 3 units and so on. Sketch each of the larger cubes.



2 For each cube, with edge x units, separate the cube into x sets of blocks. This needs to be done so that there is an odd number of blocks in each set, and so that the odd numbers are consecutive. Record the results in a table similar to the one shown below.

Edge length, x	Number of unit cubes	Consecutive odd sum
1	$1^3 = 1$	1
2	$2^3 = 8$	$3 + 5$
3	$3^3 = 27$	$7 + \ldots + \ldots$
4	…	…

3 Extend the table and describe any patterns that you see.

Task 2

Here is another activity you can undertake using identical blocks (centicubes). You will also need some paint that washes off. As an alternative, you could use a good sketch.

1 Build or sketch a cube made up of 8 identical blocks (edge length 2 blocks). Colour the outer faces of the cube a different colour to the remaining faces of the blocks.

2 How many of the blocks have:

a three faces coloured differently?

b two faces coloured differently?

c one face coloured differently?

d all faces unchanged (no faces coloured differently)?

3 Build and colour cubes with edge lengths of 3, 4, 5, etc. blocks, and repeat observations **a** to **d**. Record your results in a table similar to the one shown below.

		a	b	c	d
		Number of faces coloured differently			
Edge length	**Total blocks**	**3**	**2**	**1**	**0**
2	8	8	0	0	0
3	27	8	12	…	…
4	64	8	…	…	…
5	…	…	…	…	…

4 What number patterns can you see in columns **a**, **b**, **c** and **d**?
5 Divide each of the numbers in column **c** by 6. What do you see?

Task 3

1 Use the cubes formed in Task 1 to complete the following table.

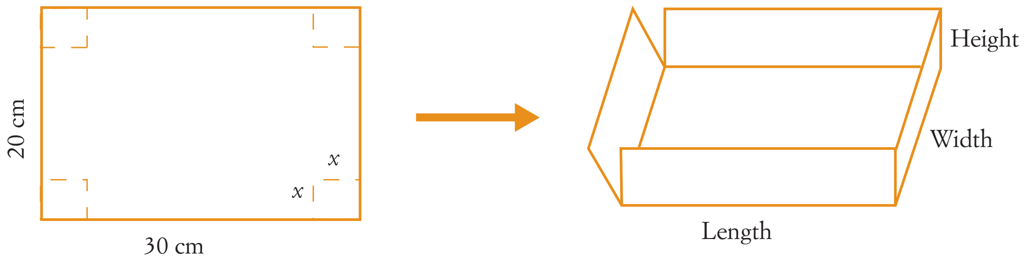
Edge length, x	1	2	3	4	…
Volume, V	1	8	27	…	…
Surface area, A	6	24	…	…	…
$V : A$	0.16	0.3	…	…	…

2 Draw a graph of volume (V) (horizontal axis) against $10 \times V: A$ (vertical axis).
3 What do you conclude?

Technology Maximum volume

Technology worksheet
Excel worksheet: Volume of a box
MAT10MGCT00007

A sheet of cardboard measures 30 cm by 20 cm.
If equal sized squares are cut from each corner, the cardboard can be folded and fastened to form an open box.



Let's see how the volume of the box changes as the size of the cut out corner varies.

1 Set up a spreadsheet as shown here.

	A	B	C	D	E	F	G
1	Volume of box						
2							
3	x	Length	Width	Height	Volume		
4							
5							
6							

2 The cells A4, A5, A6, … will contain the various lengths of the sides of the square corners that are cut out as shown in the diagram. Enter 0 in cell A4. What is the greatest value of x? Why?

3 Enter '=A4+1' in cell A5 and then copy the formula into cells A6 to A14.

4 Cells B4 to B14 will contain the various lengths of the box. Work out the formula that should be entered into cell B4. Copy the formula into cells B5 to B14.

5 Cells C4 to C14 will contain the various widths of the box. Work out the formula that should be entered into cell C4. Copy the formula into cells C5 to C14.

6 Cells D4 to D14 will contain the various heights of the box. What measurement is the same as the height of the box? Enter a formula into cell D4 and copy it into cells D5 to D14.

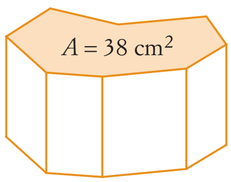
7 Cells E4 to E14 will contain the volumes of the box. Enter a formula into cell E4 and copy it into cells E5 to E14.

8 What are the dimensions of the box with the largest volume?

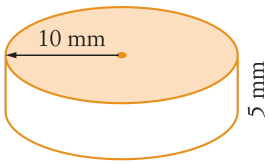
Example 17

Find the volume of each of these solids to the nearest whole number.

a



b

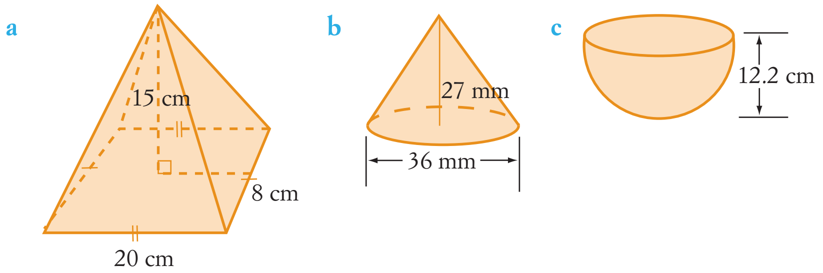


Solution

a Write the prism formula.	$V = Ah$
Substitute the values.	$= \mathbf{38\ cm^2 \times 6\ cm}$
Evaluate.	$= \mathbf{228\ cm^3}$
Write the answer.	**The volume is 228 cm³.**
b Write the cylinder formula.	$V = \pi r^2 h$
Substitute $r = 10$ mm and $h = 5$ mm.	$= \pi \times (10\ \text{mm})^2 \times 5\ \text{mm}$
Use your calculator for π.	$= 1570.79\ldots$
Round off.	$\approx 1571\ \text{mm}^3$
Write the answer.	**The volume is about 1571 mm³.**

Example 18

Find the volume of each of these solids.



Worksheet
Area and volume 2
MAT10MGWK00011

Solution

a Write the pyramid formula. $V = \frac{1}{3}Ah$

The base is a rectangle. $= \frac{1}{3} \times l \times w \times h$

Substitute $l = 20$ cm, $w = 8$ cm and $h = 15$ cm. $= \frac{1}{3} \times 20 \times 8 \times 15 \text{ cm}^3$

Evaluate. $= 800 \text{ cm}^3$

Write the answer. **The volume is 800 cm^3.**

b Write the cone formula. $V = \frac{1}{3}\pi r^2 h$

Substitute the values. $D = 38$ mm. $= \frac{1}{3} \times \pi \times 18^2 \times 27$

Evaluate. $= 9160.88 \ldots \text{ mm}^3$

Round off and write the answer. **The volume is about 9161 mm^3.**

c Write the open hemisphere formula. $V = \frac{2}{3}\pi r^3$

Substitute the values. $= \frac{2}{3} \times \pi \times 12.2^3$

Evaluate. $= 3803.10 \ldots \text{ cm}^3$

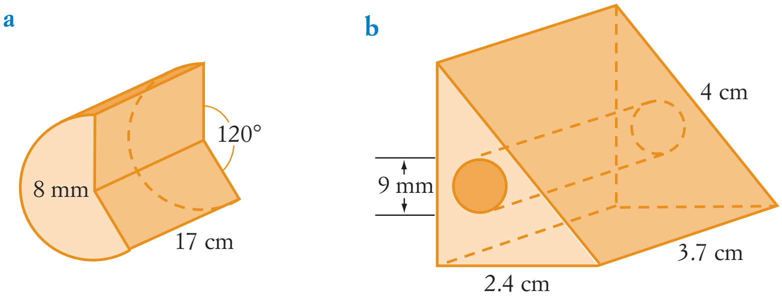
Round off and write the answer. **The volume is about 3803 cm^3.**

It is possible to calculate the volume of solids that are formed by combining two or more different solids.

Example 19

Animated example
Volumes of shapes
MAT10MGAE00004

Calculate the volume of each of the following solids correct to one decimal place.



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Solution

a This solid consists of a cylinder with a 120° sector removed. This means that the remaining part is a 240° sector.

Calculate the area of cross section. **Area of cross-section** $= \frac{240°}{360°} \times \pi r^2$

Cancel. $= \frac{\cancel{240°}^{2}}{\cancel{360°}_{3}} \times \pi r^2$

Substitute for r. $= \frac{2}{3} \times \pi \times 8^2$

Evaluate. $= 134.041\ldots \text{ cm}^2$

Use the rule for the volume of a prism. $V = Ah$

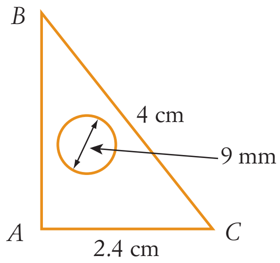
Substitute. $= 134.041\ldots \times 17 \text{ cm}^3$

Evaluate. $= 2278.70\ldots \text{ cm}^3$

Round off and state the result. **The volume of the solid is approximately 2278.7 cm³.**

b The cross-section is a triangle with a circle removed.

Draw a diagram to represent the cross-section.



Use Pythagoras' theorem to calculate the height of the triangle. $BC^2 = BA^2 + AC^2$

Substitute for known values. $4^2 = BA^2 + 2.4^2$

Evaluate. $16 = BA^2 + 5.76$

Reverse the equation. $BA^2 + 5.76 = 16$

Subtract 5.76 from both sides. $BA^2 = 10.24$

Take the square root of both sides. $BA = 3.2 \text{ cm}$

Calculate the area of the triangle. $A = \frac{1}{2}bh$

Substitute for known values. $= \frac{1}{2} \times 2.4 \times 3.2$

Evaluate. $= 3.84 \text{ cm}^2$

Calculate the area of the circle. $A = \pi r^2$

Substitute using $r = 0.45$ cm. $= \pi \times 0.45^2$

Evaluate. $= 0.636\,17\ldots \text{ cm}^2$

Calculate the area of the cross-section.	**Area of cross-section** **= area of triangle − area of circle**
Substitute for the areas.	$= 3.84 - 0.636\ 17\ldots\ \text{cm}^2$
Evaluate.	$= 3.203\ 82\ldots\ \text{cm}^2$
Calculate the volume of the solid.	$V = Ah$
Substitute.	$= 3.203\ 82\ldots \times 3.7\ \text{cm}^3$
Evaluate.	$= 11.854\ 16\ldots\ \text{mm}^2$
Round off and state the result.	**The volume of the solid is approximately 11.9 cm^3.**

Exercise 4.3 Volumes of shapes

Understanding

A calculator should be used where appropriate in this exercise. For calculations involving π, use the π key on your calculator and answer correct to 1 decimal place.

Extra questions
Exercise 4.3
MAT10MGEQ00012

See Example 15

1 Convert each of the following to the unit indicated.

a 2400 mL to L **b** 3.2 L to cm^3 **c** 7.5 L to mL
d 730 mL to L **e** 41 cm^3 to mL **f** 8.2 m^3 to L
g 432 mL to cm^3 **h** 3.24 L to m^3 **i** 2320 cm^3 to L

See Example 15

2 Convert each of the following to the unit indicated.

a 5400 mL to kL **b** 0.027 m^3 to cm^3 **c** 4300 cm^3 to kL
d 354 ML to m^3 **e** 0.025 m^3 to L **f** 0.84 kL to cm^3

3 The following table relates to the dimensions of a range of rectangular prisms. Complete the table.

	Length	Width	Height	Volume
a	15 cm	12 cm	8 cm	…
b	4 m	3 m	…	120 m^3
c	…	11 mm	22 mm	4114 mm^3
d	26.4 cm	17.5 cm	14.4 cm	…
e	6.7 m	…	1.5 m	32.16 m^3

Fluency

4 Find the volume of:

a a cube with a side of 3 cm

b a rectangular prism measuring 4 cm by 8 cm by 6 cm

c a rectangular prism measuring 1.2 m by 0.9 m by 0.6 m

d a rectangular prism measuring 33 cm by 12 cm by 50 cm

e a rectangular block of cheese measuring 13 cm by 8 cm by 3 cm.

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5 Find the volumes of the cylinders with the following dimensions.

a radius 17 cm, height 22 cm
b diameter 20 mm, height 15 mm
c diameter 2.4 m, height 1.9 m
d radius 8.3 cm, height 5.2 cm
e radius 0.75 m, height 3.6 m
f diameter 14 mm, height 37 mm

6 Find the volume of:

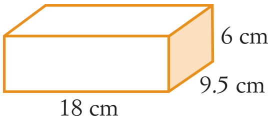
a a conical pile of sugar with a height of 3.4 m and a radius of 4.8 m
b a square-based pyramid with a base measuring 13 cm by 13 cm and a height of 11 cm
c a rectangular-based pyramid with a base measuring 2.1 m by 1.7 m and a height of 3.3 m
d a conical funnel with a height of 26 cm and a diameter of 18 cm
e a rectangular-based pyramid with a base measuring 42 mm by 21 mm and a height of 11 mm.

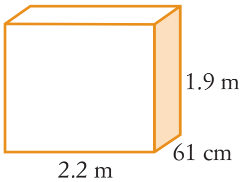
7 Find the volume of:

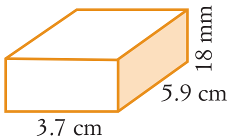
a a tennis ball with a radius of 3.5 cm
b a golf ball with a diameter of 43 mm
c a basketball with a radius of 12.7 cm
d a spherical meteorological balloon with a diameter of 2.7 m
e half an orange with a diameter of 9 cm.

8 Find the volumes of the following prisms.

See Example 16

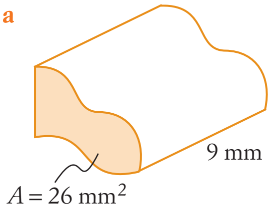
a


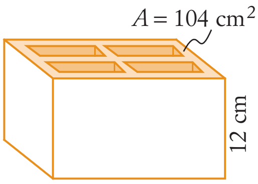
b


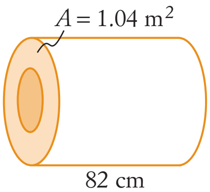
c


9 Find the volumes of the following solids.

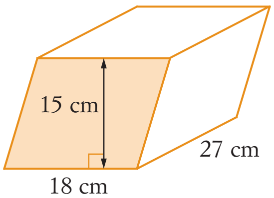
See Examples 16, 17

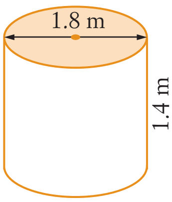
a


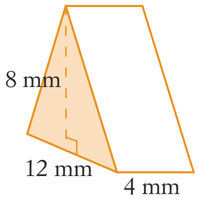
b


c


(Answer in m^3.)

d


e


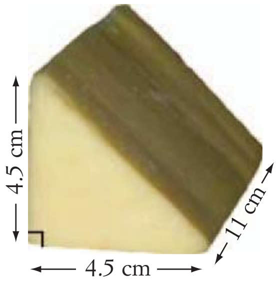
f


See Examples 16, 17

10 Find the volume of each of the following.

a

b

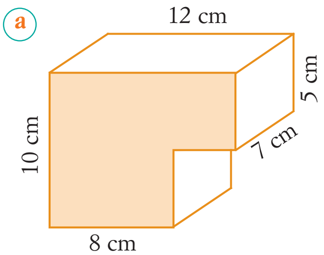


c

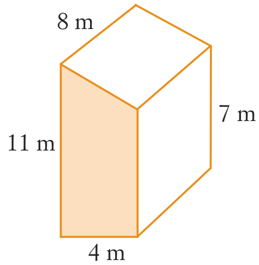
Worked solutions
Exercise 4.3
MAT10MGWS00012

11 Find the volumes of the following solids.

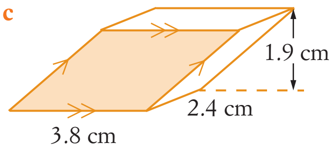
a



b



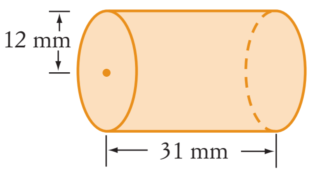
c



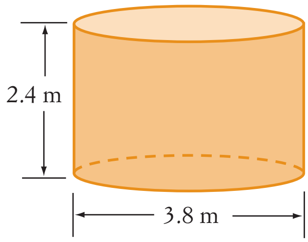
See Example 17

12 Find the volumes of the following solids.

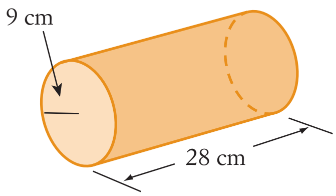
a



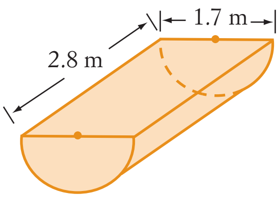
b



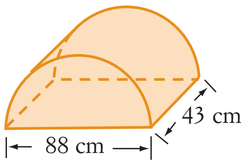
c



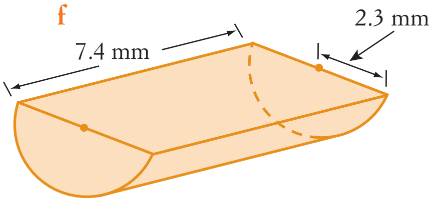
d



e



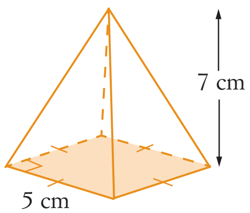
f



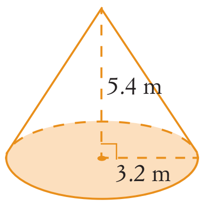
Worked solutions
Exercise 4.3
MAT10MGWS00012
See Example 18

13 Find the volumes of the following solids.

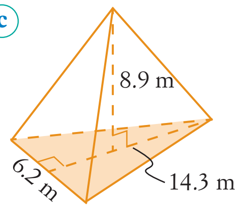
a



b



c



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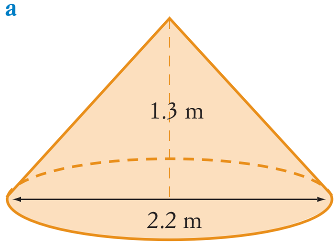
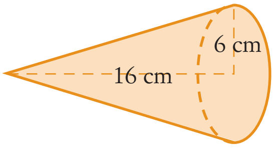
14 Find the volume of each of the following cones.

See Example 18

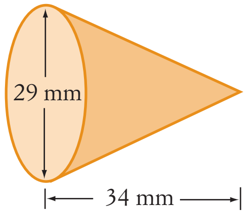
a



b



c



15 Find the volume of each of the following.

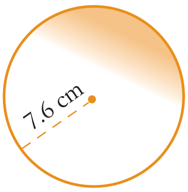
Worked solutions

Exercise 4.3

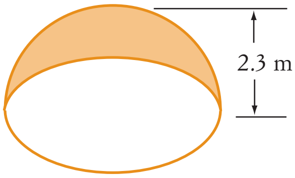
MAT10MGWS00012

See Example 18

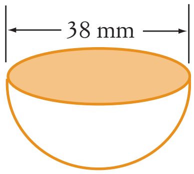
a



b



c

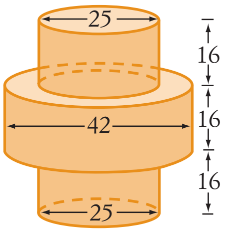


Problem solving

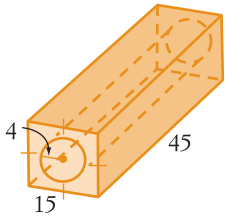
16 Each of the following solids is made of plastic and manufactured using an infusion moulding process. Calculate the volume of plastic in each. All dimensions are shown in centimetres.

See Example 19

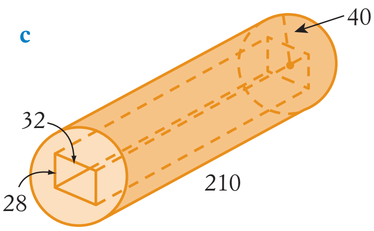
a



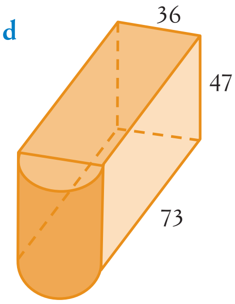
b



c



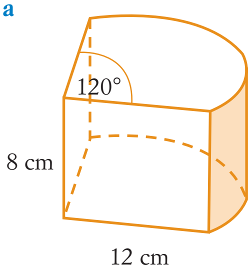
d



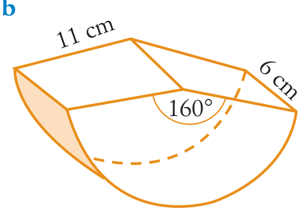
17 Find the volume of each of the following solids.

See Example 19

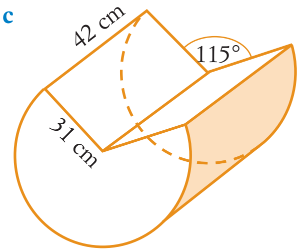
a



b

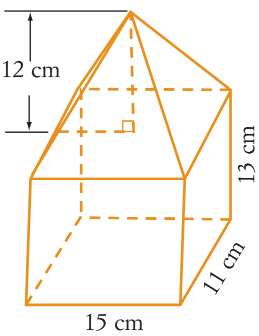


c

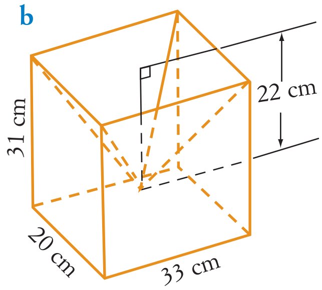


18 Find the volume of each of the following solids.

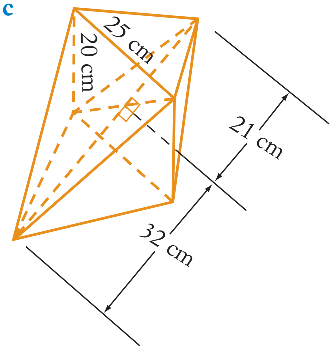
a



b

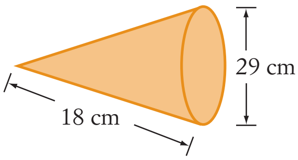


c

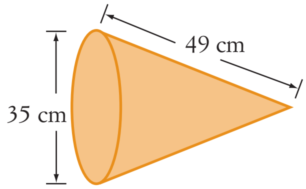


19 Calculate the volume of each of the following cones.

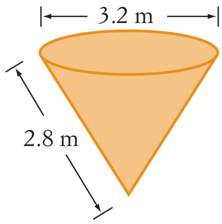
a



b

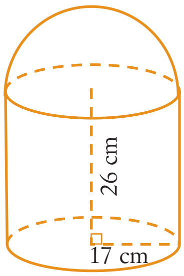


c

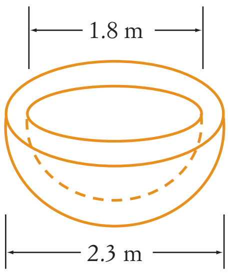


20 Calculate the volume of each of the following solids.

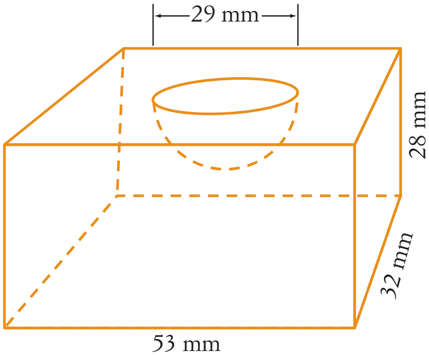
a



b

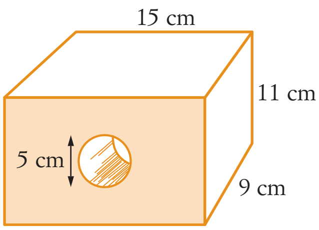


c



See Example 19

21 This metal block has had a circular hole drilled through it. Calculate the volume of metal remaining in the block.



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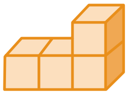
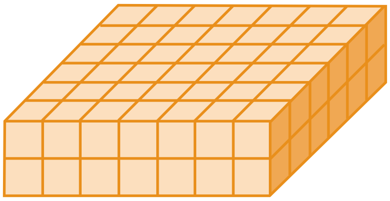
Reasoning

22 Each  represents 1 cm^3

What is the greatest number of these solids

 that can be fitted into the solid on the right?

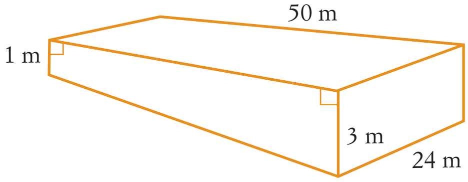
(The shapes must be wholly contained within the solid, but may form part of the solid's surface.)



23 **a** Find the volume of this swimming pool in cubic metres.

b What is the capacity of the pool in kilolitres?

c The pool is filled using a pipe with a cross-sectional area of 0.02 m^2. How long will this take if the water flows through the pipe at 2 m per second?



24 Tennis balls with a radius of 3.5 cm are packed into a rectangular carton for shipment to a sports store. The carton has a base measuring 35 cm by 70 cm and is 70 cm high. The balls are packed in layers in the carton.

a What is the maximum number of tennis balls that will fit in each layer?

b When the carton is full, how many tennis balls will it hold?

c What percentage of space in the carton is occupied by the tennis balls?

25 The planets in our solar system can be considered to be spherical. The Earth has a diameter of approximately 12 742 km.

a Jupiter has a diameter of approximately 142 981 km. How many times would the volume of the Earth fit into the volume of Jupiter?

b If the volume of the Earth is 17.823 times the volume of Mercury, what is Mercury's diameter?

Chapter 4 summary

Quiz

Area and volume

MAT10MGQZ00004

- The metric system has the **metre** (**m**) as its standard unit of length. Other units of length are the **kilometre** (**km**), the **centimetre** (**cm**) and the **millimetre** (**mm**). If you are changing to a larger unit, divide. If changing to a smaller unit, multiply.
- The **area** of a flat shape is the size of the region enclosed by its boundary. The units of area in the metric system are the **square millimetre (mm^2)**, the **square centimetre (cm^2)**, the **square metre (m^2)**, the **hectare (ha)** and the **square kilometre (km^2)**. If you are changing to a *larger* unit, *divide*. If changing to a *smaller* unit, *multiply*.
- There are rules that can be used to calculate areas of some plane shapes.

  Area of rectangle = length × width or $A = lw$

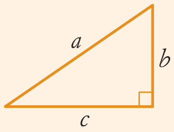
  Area of circle = π × radius^2 or $A = \pi r^2$

  Area of triangle = $\frac{1}{2}$ × base × height or $A = \frac{1}{2}bh$

  Area of parallelogram = base × height or $A = bh$

  Area of trapezium = average of parallel sides × height or $A = \frac{1}{2}a + bh$

  Area of rhombus or kite = $\frac{1}{2}$ × (product of diagonals)
- **Pythagoras' theorem** can be used to help calculate the lengths of sides in right-angled triangles. Pythagoras' theorem states that, for a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. For this triangle:



$$a^2 = b^2 + c^2$$

- **Two-dimensional (2D) shapes** are flat and have length and width but no depth. **Three-dimensional (3D) shapes** occupy space and have length, width and depth. The **faces** of a 3D shape separate the interior from the exterior. The lines where faces meet are called **edges** and the corners where edges meet are called **vertices** (singular **vertex**).
- The **net** of a 3D shape is the 2D shape that can be folded to form the 3D shape.
- The **surface area (SA)** of a 3D shape is the total area of all its faces.
- The **curved surface area (CSA)** of a cylinder = $2\pi rh$.
- A **pyramid** is a solid made up of a base and triangular faces that meet at an **apex**, which is also called the **vertex** of the pyramid. The height, h, of a pyramid is the perpendicular distance from the apex to the base. The **slant height**, s, of a pyramid is the height of the triangular face measured along the face.
- A **cone** is a solid with a circular base and an apex. The height, h, of the cone is measured perpendicular to the base and the slant height, s, is the distance from the edge of the base to the apex along the face of the cone. The surface area of a cone = $\pi r^2 + \pi rs$ where r = the radius of the cone.
- The surface area of a **sphere** = $4\pi r^2$. A **hemisphere** is half a sphere. The surface area of a closed hemisphere = $3\pi r^2$ and the surface area of an open hemisphere = $2\pi r^2$.

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- The **volume** of an object is the amount of space that it occupies. **Capacity** is another word for volume but it usually refers to the amount of space inside a container. The units we usually use for volume are the **cubic millimetre (mm^3)**, **cubic centimetre (cm^3)** and **cubic metre (m^3)**. Capacity can be measured using volume units, but it is usually measured in **litres (L)**. If you are changing to a *larger* unit, *divide*. If you are changing to a *smaller* unit, *multiply*.
- The volumes of common shapes can be worked out using formulas.

Volume of prism = area of base × height or $V = Ah$

Volume of cylinder = area of base × height or $V = \pi r^2$

Volume of pyramid = $\frac{1}{3}$ × area of base × height or $V = \frac{1}{3}Ah$

Volume of cone = $\frac{1}{3}$ × area of base × height or $V = \frac{1}{3}\pi r^2$

Volume of sphere = $\frac{4}{3}\pi r^3$

Volume of hemisphere = $\frac{2}{3}\pi r^3$

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Understanding

See Example 1

1 Convert the measurements below to the units shown in brackets.

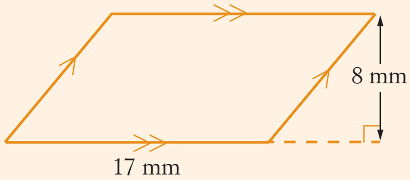
a 56 700 m^2 (km^2) **b** 3.87 ha (m^2)
c 57 ha (km^2) **d** 0.025 km^2 (ha)

2 Find the area of each of the following.

a a square of side 17 cm
b a rectangle measuring 24 mm by 15 mm
c a parallelogram with a base of 21.4 m and a height of 15.8 m
d a rhombus with diagonals of 28 cm and 16 cm
e a kite with diagonals of 51 mm and 13 mm

See Example 2

3 Find the area of this figure.

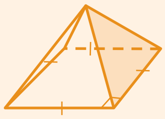


See Example 6

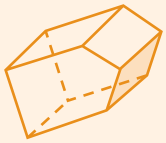
4 Identify the shapes shown below.

a

b



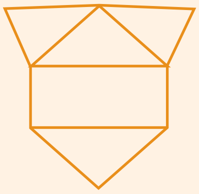
c



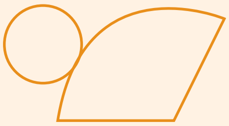
See Example 7

5 Identify the shapes whose nets are shown below.

a



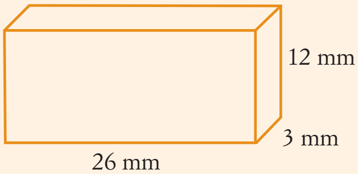
b



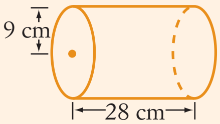
See Examples 8, 10

6 Draw a net for this shape, including any dimensions shown.

a

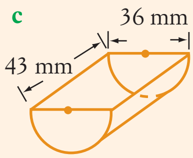


b



Closed cylinder

c



Open half-cylinder

See Example 15

7 Convert each of the following to the unit indicated.

a 2.75 L to mL **b** 5.43 m^3 to L **c** 271 mL to cm^3
d 3600 mL to kL **e** 0.049 m^3 to cm^3 **f** 2800 cm^3 to kL

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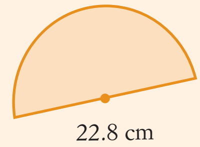
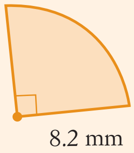
Fluency

8 Find the areas of the following shapes.

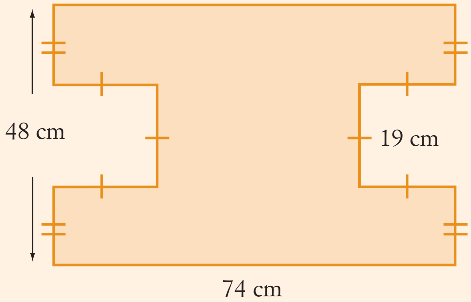
a



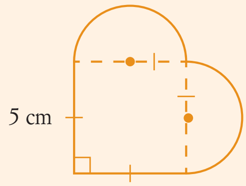
b



9 Work out the missing measurements, then find the areas of this shape.

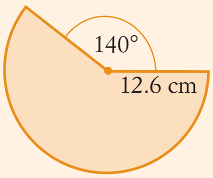


10 Find the area of this shape.

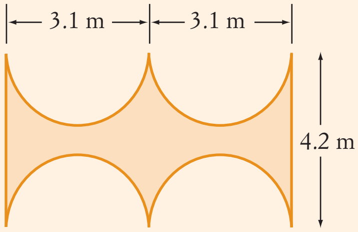


11 Find the areas of the following shapes.

a



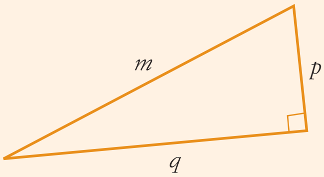
b



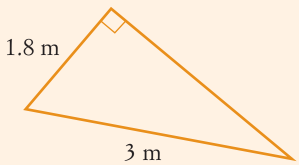
12 Use Pythagoras' theorem and the triangle on the right to:

a complete $m^2 = \ldots + \ldots$

b find p if $m = 5$ and $q = 4.8$.



13 Find the area of this shape. See Example 3



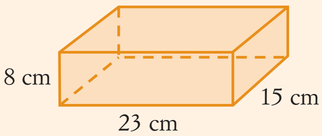
14 Calculate the surface area of the solid shown in question **6a**. See Example 8

15 Calculate the surface area of the solid shown in question **6b**. See Example 8

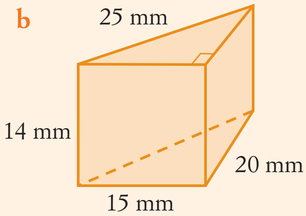
See Example 8 **16** Calculate the surface area of the solid shown in question **6c**.

See Example 8 **17** Find the surface area of these solids.

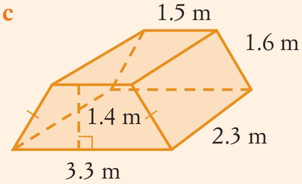
a



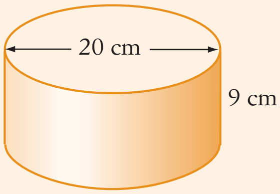
b



c



See Example 8 **18** **a** Draw a net for this shape, including any dimensions shown.
b Calculate the surface area.



See Example 9 **19** Calculate the curved surface area of the cylinders with the following dimensions. Give your answers correct to one decimal place.

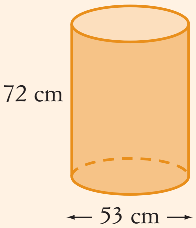
a radius 12 cm, height 9 cm
b diameter 18 mm, height 11 mm

See Example 9 **20** Calculate the surface area of the closed cylinders with the following dimensions. Give your answers correct to one decimal place.

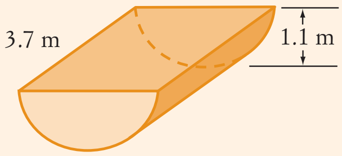
a diameter 9 cm, height 14 cm
b radius 1.6 m, height 2.5 m

See Examples 8, 10 **21** Calculate the surface area of each the following solids.

a **Closed cylinder**



b **Closed half-cylinder**



22 Calculate the surface area of each of the following.

a square-based pyramid measuring 12 cm by 12 cm and a slant height of 14 cm
b cone with diameter 18 mm, slant height 14 mm
c sphere with a radius of 1.7 m
d closed hemisphere with a diameter of 26 cm

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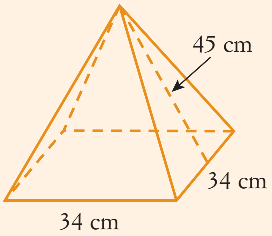
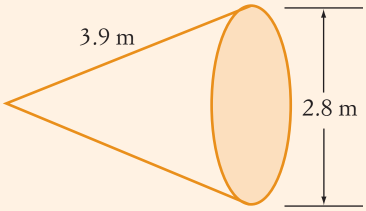
23 Calculate the surface area of each of the following.

See Examples 12, 13, 14

a

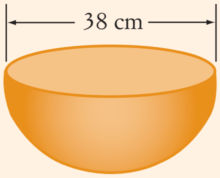


b



c

d

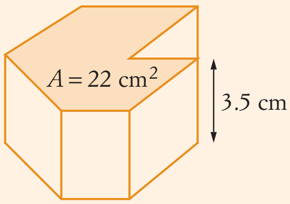


Closed hemisphere

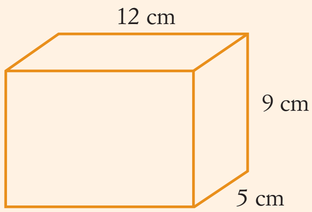
24 Find the volume of:

a a cube with a side of 3 cm
b a rectangular prism measuring 4 cm by 8 cm by 6 cm
c a cylinder with a radius of 17 cm and a height of 22 mm
d a cone with a height of 3.4 m and a radius of 4.8 m
e a rectangular-based pyramid with a base measuring 2.1 m by 1.7 m and a height of 3.3 m
f a sphere with a diameter of 3.5 cm

25 Find the volume of this prism.



26 Find the volume of this object.



27 Find the volume of a box measuring 1.3 m by 2.2 m by 0.8 m.

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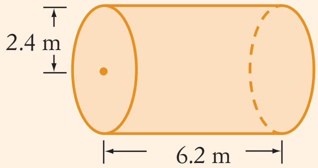
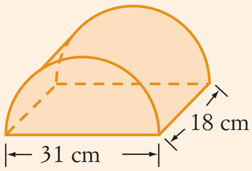
See Examples 17, 18

28 Find the volumes of the following solids.

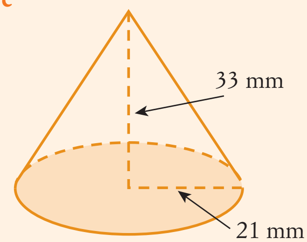
a



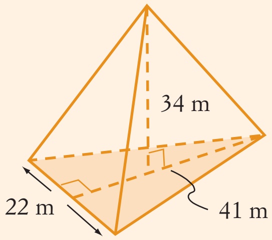
b



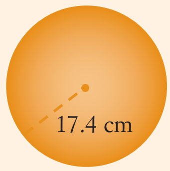
c



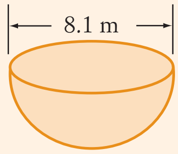
d



e



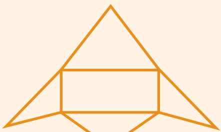
f



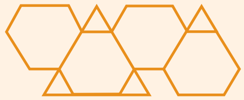
Problem solving

29 Identify the shapes whose nets are shown below.

a

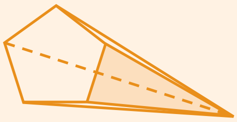


b

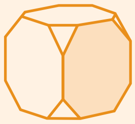


30 Draw nets for the following shapes.

a



b

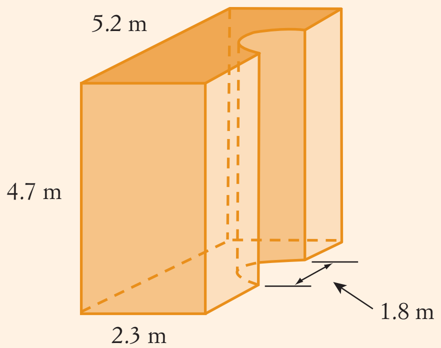


31 A circular disc has an area of 908 mm^2. What is its radius?

32 A circle has a diameter of 16 m. What is the perimeter of a 60° sector?

See Example 11

33 The surfaces of this solid need to be painted. Calculate the total area that will be painted.



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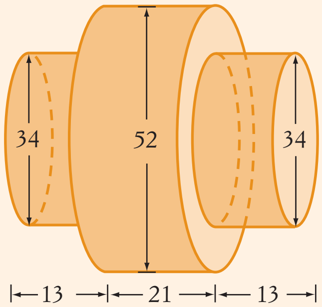
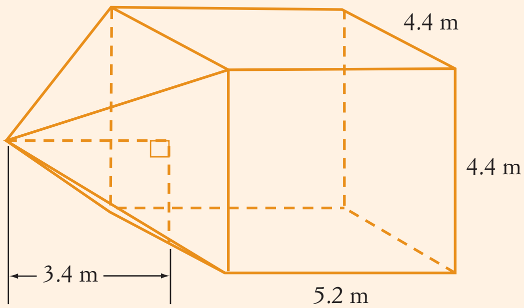
34 The following solid is made of plastic and manufactured using an infusion moulding process. Calculate the volume of plastic in the solid if all dimensions are shown in centimetres.

See Example 19

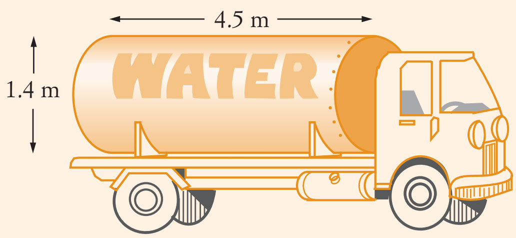


35 Find the volume of this solid.

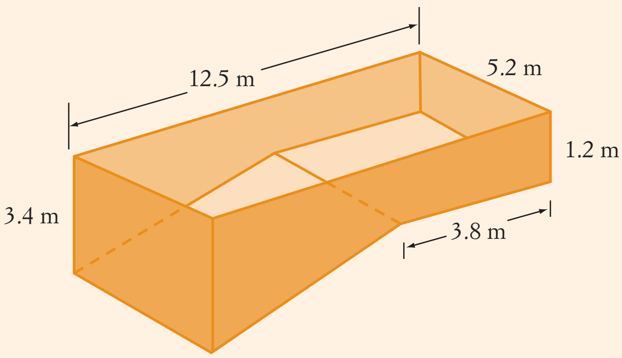


Reasoning

36 This water tanker is able to carry a maximum load of 5 tonnes up a steep hill. Will it be able to make it up the hill if the tanker is full of water? (1 cm^3 of water has a mass of 1 g.)



37 A newly constructed in-ground swimming pool is shown here.



a Calculate the volume of the swimming pool.

b A hose with a flow rate of 225 L per minute is used to fill the pool. How long will this take if the swimming pool is filled to a depth 15 cm from the top?

Number and algebra

5 Coordinate geometry

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Contents

Prior learning
Chapter 5
MAT10NAPL00005

Parent guide
Chapter 5
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Curriculum guide
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MAT10NACU00005

Australian Curriculum statements

Linear and non-linear relationships

Solve problems involving parallel and perpendicular lines. (ACMNA238)

Explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technology as appropriate. (ACMNA239)

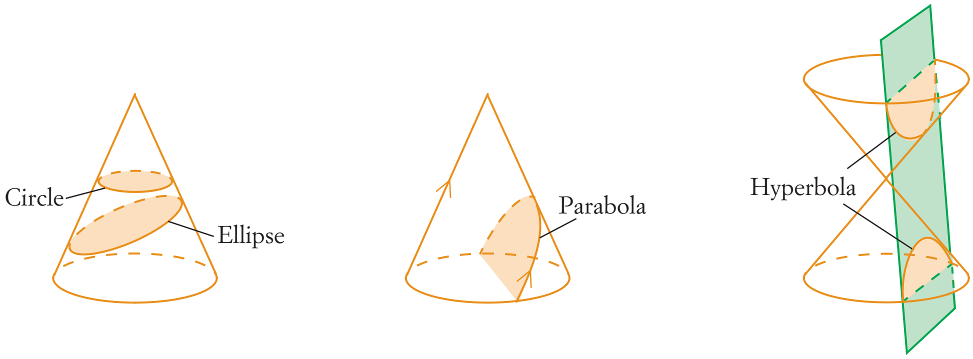
Video tutorial
Coordinate geometry
MAT10NAVT00005

We can use maps and street directories to describe and locate places we want to find. It is also important to be able to describe and locate positions in mathematics. Many cars are now fitted with global positioning systems to locate places. These systems rely on mathematical descriptions of positions and satellite technology.

In mathematics, positions are described using coordinates. You are already familiar with using Cartesian coordinates to plot and describe the position of points on the Cartesian plane. René Descartes (1596–1650) is credited with developing the Cartesian system, which still forms the foundation for graphing many centuries after his death.

Coordinates are also used in maps and they have important applications in science. The use of coordinates is essential in the study of vectors. They form the basis for much of the physics of movement. Without coordinates, it would be very difficult to study speed, acceleration, force and mechanics.

Pierre de Fermat, who knew Descartes, greatly extended this work and worked out the shapes and equations of many different curves, including the curves that are obtained by cutting a slice through a cone – the conic sections.



Mathematical literacy

Maths dictionary
MAT10ASDI00001

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

asymptote
Cartesian coordinates
Cartesian plane
circle
distance formula
exponential function
falling
gradient
gradient–intercept form
gradient–point form
intercepts
linear equation
linear function
linear model
line of best fit
maximum
midpoint
minimum
model
origin
parabola
parallel lines
perpendicular lines
quadrants
quadratic function
rectangular coordinates
rising
roots
slope
standard form
trendline
turning point
vertex
x-axis
x-coordinate
x-intercept
y-axis
y-intercept
y-coordinate
zeros

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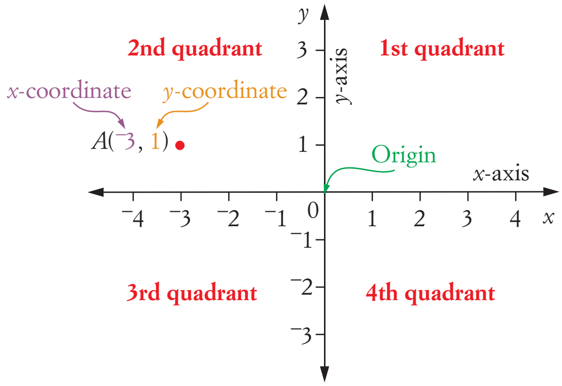
5.1 Distances and midpoints in the plane

You have already seen that points can be plotted on a number lattice known as the **Cartesian plane.**

This is done using numbers on the x-axis and y-axis called **coordinates.** The x-coordinate is always first. A series of points may be plotted to give a shape, or to show the graph of a function.

The diagram below shows the main features of the Cartesian coordinate system.

Cartesian coordinate system

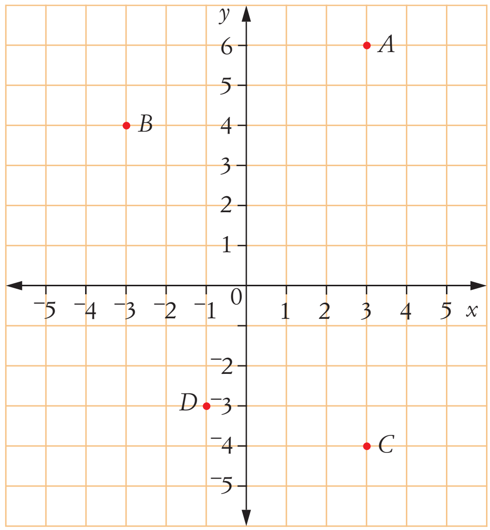


Example 1

Plot the points $A(3, 6)$, $B(^-3, 4)$, $C(3, ^-4)$ and $D(^-1, ^-3)$.

Solution

Draw up a set of axes.
Label the horizontal axis x and the vertical axis y.
The x-coordinate is always first.
It is the horizontal distance.
The y-coordinate is second.
It is the vertical distance.
Plot the points.



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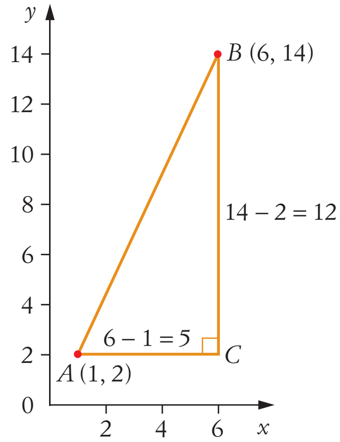
Cartesian coordinates are also called **rectangular coordinates.**
We can calculate the distance between two points on the Cartesian plane using Pythagoras' theorem.
Suppose there are two points $A(1, 2)$ and $B(6, 14)$.
Once these have been plotted, extra lines need to be drawn in to form the right-angled triangle ABC shown on the right.
In $\triangle ABC$:



$$AC = 6 - 1 = 5$$
$$BC = 14 - 2 = 12$$

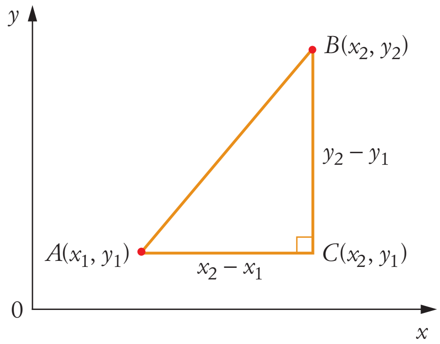
Using Pythagoras' theorem:

$$AB^2 = AC^2 + BC^2$$
$$AB^2 = 5^2 + 12^2$$

So

$$\begin{aligned} AB &= \sqrt{5^2 + 12^2} \\ &= \sqrt{169} \\ &= 13 \end{aligned}$$

Now suppose $A = (x_1, y_1)$ and $B = (x_2, y_2)$, as shown in the diagram on the right.



$$AC = x_2 - x_1$$
$$BC = y_2 - y_1$$
$$AB^2 = AC^2 + BC^2$$
$$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

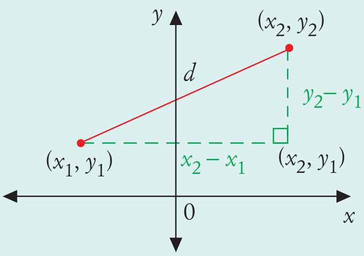
This is known as the **distance formula.**

Important!

Distance formula

The distance between two points (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



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Example 2

Find the distance between points (3, 7) and ($^-4$, 10).

Solution

Identify the two points. **Let $(x_1, y_1) = (3, 7)$ and $(x_2, y_2) = (^-4, 10)$.**

Write the distance formula. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute for x_1, y_1, x_2 and y_2. $= \sqrt{(^-4 - 3)^2 + (10 - 7)^2}$

Evaluate the brackets. $= \sqrt{(^-7)^2 + 3^2}$

$= \sqrt{58}$

Evaluate and round off. **≈ 7.62 units**

Video tutorial

Distance, midpoint and gradient formulas

MAT10NAVT00010

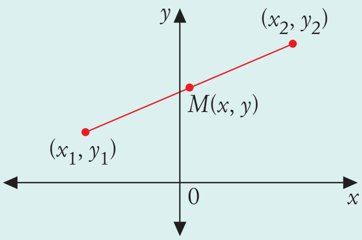
We can calculate the **midpoint** of two points using averages. The x-coordinate of the midpoint will be the average of the x-coordinates, and the y-coordinate will be the average of the y-coordinates.

Important!

Midpoint

The **midpoint,** M, of two points (x_1, y_1) and (x_2, y_2) is given by

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



Example 3

Find the midpoint of (3, 6) and ($^-5$, 8).

Solution

Identify the two points. **Let $(x_1, y_1) = (3, 6)$ and $(x_2, y_2) = (^-5, 8)$.**

Write the midpoint formula. $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Calculate the x-coordinate of M. $x_M = \frac{x_1 + x_2}{2} = \frac{3 + {^-5}}{2} = {^-1}$

Calculate the y-coordinate of M. $y_M = \frac{y_1 + y_2}{2} = \frac{6 + 8}{2} = 7$

State the result. **The midpoint of (3, 6) and ($^-5$, 8) is ($^-1$, 7).**

Technology worksheet

Excel worksheet: Midpoint and distance between two points

MAT10NACT00008

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Example 4

The midpoint of A and B is $(3, ^-4)$. If $A = (5, 1)$, find B.

Solution

Identify the two points.	Let $A = (x_1, y_1) = (5, 1)$ and $B = (x_2, y_2)$.
Write the midpoint formula.	$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Substitute for x_1, y_1, x_2 and y_2.	$= \left(\frac{5 + x_2}{2}, \frac{1 + y_2}{2}\right)$
Use the fact that $M = (3, ^-4)$.	$3 = \frac{5 + x_2}{2}$
Multiply both sides by 2.	$3 \times 2 = \frac{5 + x_2}{2} \times 2$
Cancel and reverse the equation.	$5 + x_2 = 6$
Solve for x_2.	$x_2 = 1$
Use the fact that $M = (3, ^-4)$.	$^-4 = \frac{1 + y_2}{2}$
Multiply both sides by 2.	$^-4 \times 2 = \frac{1 + y_2}{2} \times 2$
Cancel and reverse the equation.	$1 + y_2 = ^-8$
Solve for y_2.	$y_2 = ^-9$
State the result.	B is $(1, ^-9)$.

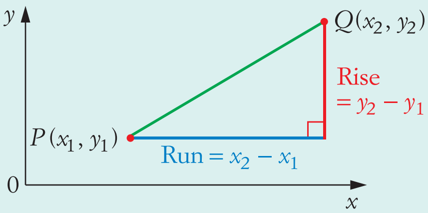
An important property of a segment or line is its steepness. This is called the **slope** or **gradient**. It measures how much the segment or line goes up for each step along the x-axis. We can work out the gradient using the **rise** (vertical distance) and **run** (horizontal distance) of a line between two points.

Important!

Gradient

If $P(x_1,y_1)$ and $Q(x_2,y_2)$ are points on a straight line:

- the **rise** from P to Q is the y-distance $y_2 - y_1$;
- the **run** from P to Q is the x-distance $x_2 - x_1$.



We use the symbol m to represent **gradient** (slope).
The gradient of the line PQ has the symbol m_{PQ} and is worked out using the formula

$$\text{Gradient} = \frac{\text{rise}}{\text{run}} \quad \text{or} \quad m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$$

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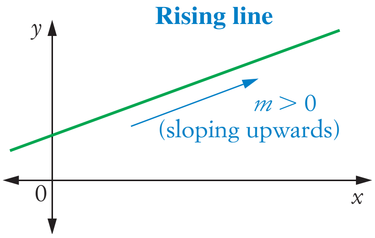
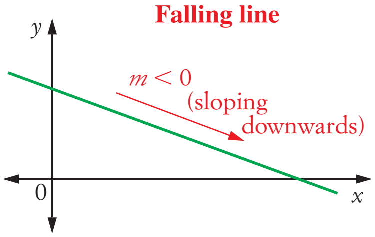
Lines that rise from left to right are described as **rising** and have a *positive gradient* ($m > 0$). **Falling** lines fall from left to right and have a *negative gradient* ($m < 0$).

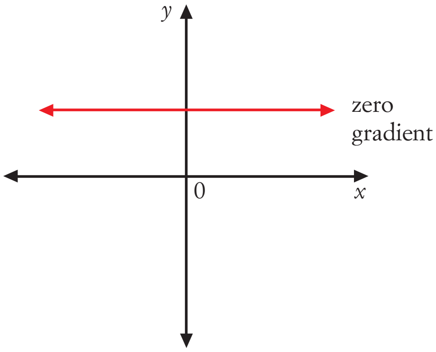




For a horizontal or flat line, the rise is 0.

So, gradient $= \dfrac{0}{\text{run}} = 0$.

This means a horizontal line has a **zero gradient**.

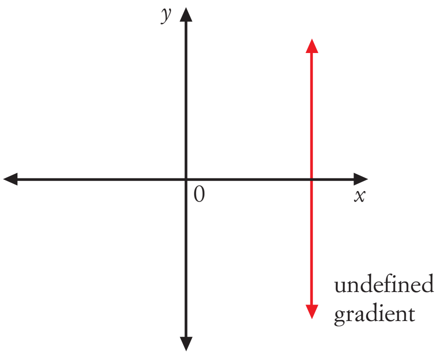


In a vertical line, the run is 0.

So, gradient $= \dfrac{\text{rise}}{0}$.

But division by zero is undefined.

A vertical line has an **undefined gradient**. Some people say that the gradient is **infinite**, but this is imprecise.



Example 5

Worksheet
Gradient, midpoint and distance
MAT10NAWK00014

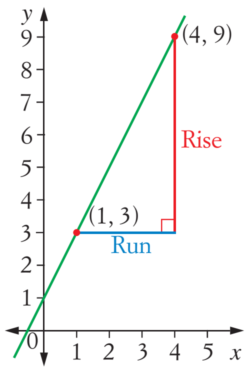
Work out the gradient of the line through (1, 3) and (4, 9).

Solution

Sketch a diagram.

$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$

So we need to work out the vertical and horizontal distances.



Calculate the rise.	**Rise** $= 9 - 3 = 6$
Calculate the run.	**Run** $= 4 - 1 = 3$
Write the rule for gradient, substitute in the known values and evaluate.	**Gradient** $= \dfrac{\text{rise}}{\text{run}} = \dfrac{6}{3} = 2$
State the result.	**The gradient of the line is 2.**

You may also wish to calculate the gradient without sketching a diagram.

Example 6

Work out the gradient of the segment joining $({}^-2, 5)$ and $(7, {}^-4)$.

Solution

Identify the points.	**Let** $({}^-2, 5) = (x_1, y_1)$ **and** $(7, {}^-4) = (x_2, y_2)$.
Write the rule for gradient.	$m = \dfrac{y_2 - y_1}{x_2 - x_1}$
Substitute in the known values.	$= \dfrac{{}^-4 - 5}{7 - {}^-2}$
Evaluate.	$= \dfrac{{}^-9}{9}$
	$= {}^-1$
State the result.	**The gradient of the segment is** ${}^-1$.

The rules for calculating lengths, midpoints and gradients can be useful in problem solving.

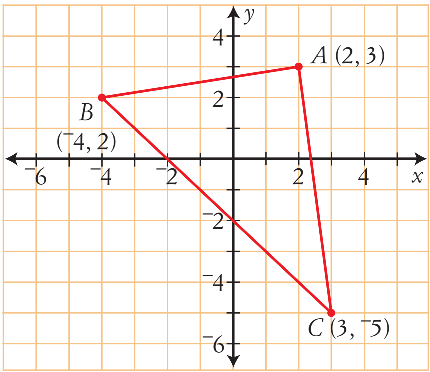
Example 7

Determine if the triangle whose vertices are $A(2, 3)$, $B({}^-4, 2)$ and $C(3, {}^-5)$ is scalene, isosceles or equilateral.

Solution

Sketch a diagram.

You need to compare the lengths of the sides of the triangle.



State the distance formula. $\quad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

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Calculate the length of AB using $A(2, 3) = (x_1, y_1)$ and $B(^-4, 2) = (x_2, y_2)$.	$AB = \sqrt{(^-4 - 2)^2 + (2 - 3)^2} = \sqrt{37}$
Calculate the length of BC using $B(^-4, 2) = (x_1, y_1)$ and $C(3, ^-5) = (x_2, y_2)$.	$BC = \sqrt{(3 - ^-4)^2 + (^-5 - 2)^2} = \sqrt{98}$
Calculate the length of AC using $A(2, 3) = (x_1, y_1)$ and $C(3, ^-5) = (x_2, y_2)$.	$AC = \sqrt{(3 - 2)^2 + (^-5 - 3)^2} = \sqrt{65}$
All sides of $\triangle ABC$ are of different lengths.	**$\triangle ABC$ is scalene.**

Exercise 5.1 Distances and midpoints in the plane

Understanding

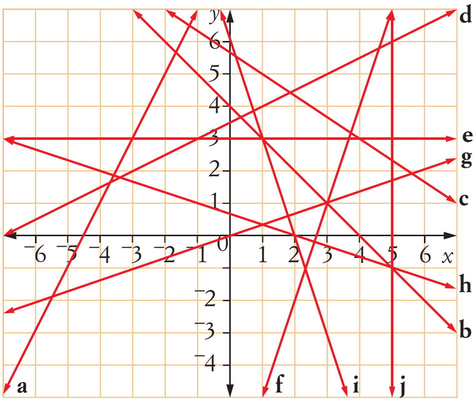
Extra questions
Exercise 5.1
MAT10NAEQ00013

See Example 1

1 Plot the following points on one set of coordinate axes.

$A(3, 5)$	$B(^-2, 4)$	$C(3, ^-2)$	$D(^-5, ^-1)$	$E(4, 0)$
$F(0, ^-4)$	$G(^-1, 3)$	$H(^-2, 0)$	$I(0, 6)$	$J(3, 3)$

2 Classify the lines shown below as either rising, falling or neither.

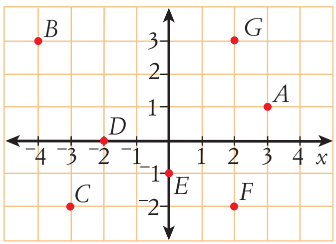


3 Without using a diagram, state in which quadrant of the Cartesian plane each of the following points lie.

a $(1, 7)$ **b** $(^-4, 2)$ **c** $(^-9, ^-14)$ **d** $(^-9, 15)$
e $(^-4, ^-9)$ **f** $(12, ^-1)$ **g** $(^-4, 11)$ **h** $(3, 13)$

4 Use this diagram only to find the coordinates of the midpoint of the line segment:

a GF **b** BG **c** CF **d** EF
e GE **f** BD **g** BC **h** BF



Fluency

See Example 2

5 Find the distance between these points.

- **a** $(3, 6)$ and $(5, 8)$
- **b** $({}^-2, 4)$ and $(4, 0)$
- **c** $(7, {}^-4)$ and $({}^-5, 1)$
- **d** $(8, 6)$ and $({}^-7, 13)$
- **e** $({}^-3, {}^-5)$ and $(6, 4)$
- **f** $(6, 2)$ and $({}^-3, {}^-7)$
- **g** $(4, 1)$ and $({}^-2, {}^-4)$
- **h** $({}^-3, 5)$ and $({}^-1, {}^-7)$
- **i** $(6, 1)$ and $(3, {}^-7)$
- **j** $(5, {}^-5)$ and $({}^-3, 8)$

See Example 3

6 Find the midpoint of these points.

- **a** $(3, {}^-2)$ and $(7, 4)$
- **b** $(1, 5)$ and $({}^-3, {}^-4)$
- **c** $(4, 5)$ and $({}^-4, 7)$
- **d** $(3, {}^-3)$ and $({}^-2, 4)$
- **e** $({}^-3, 6)$ and $({}^-7, 4)$
- **f** $({}^-3, 9)$ and $(4, {}^-6)$
- **g** $({}^-7, 0)$ and $(0, 5)$
- **h** $(0, 0)$ and $(9, {}^-4)$
- **i** $({}^-3, {}^-9)$ and $({}^-6, {}^-7)$
- **j** $(4, 2)$ and $({}^-3, {}^-8)$

See Examples 5, 6

7 Find the gradient of each line shown in question **2**.

See Examples 5, 6

8 Find the slope of the line through each of these pairs of points.

- **a** $(3, 5)$ and $(5, 11)$
- **b** $(1, 6)$ and $(4, 0)$
- **c** $({}^-1, {}^-3)$ and $(2, 3)$
- **d** $(3, {}^-6)$ and $(4, 1)$
- **e** $(4, 2)$ and $({}^-4, 1)$
- **f** $(0, 3)$ and $(5, {}^-7)$
- **g** $({}^-3, 4)$ and $(3, {}^-2)$
- **h** $(6, 6)$ and $(7, 7)$
- **i** $({}^-2, 3)$ and $({}^-6, 3)$
- **j** $({}^-4, 0)$ and $(0, {}^-3)$
- **k** $({}^-4, {}^-5)$ and $({}^-4, 6)$
- **l** $(2, 4)$ and $(4, 1)$

Worked solutions
Exercise 5.1
MAT10NAWS00013
See Examples 2, 3, 5

9 For the intervals joining each of these pairs of points find:

- **i** the length (leave in surd form if necessary)
- **ii** the midpoint
- **iii** the gradient

- **a** $(1, 3)$ and $(6, 7)$
- **b** $(4, 4)$ and $(7, 1)$
- **c** $({}^-3, 1)$ and $({}^-5, 6)$
- **d** $(8, 2)$ and $({}^-5, 3)$
- **e** $({}^-7, 5)$ and $(0, 2)$
- **f** $(1, 9)$ and $({}^-8, 2)$
- **g** $(9, {}^-6)$ and $({}^-5, 8)$
- **h** $(11, 2)$ and $(6, {}^-3)$
- **i** $({}^-5, {}^-1)$ and $({}^-4, {}^-7)$
- **j** $({}^-3, {}^-1)$ and $({}^-8, 2)$
- **k** $({}^-5, 0)$ and $(0, {}^-4)$
- **l** $({}^-6, 1)$ and $(6, {}^-1)$

Problem solving

10 The midpoint of AB is $({}^-5, 3)$. If A is $(2, 9)$, find B.

Worked solutions
Exercise 5.1
MAT10NAWS00013

11 The midpoint of PQ is $(8, 7)$. If P is $(5, 10)$, find Q.

12 The midpoint of RT is $({}^-4, {}^-5)$. If T is $({}^-10, 3)$, find R.

See Example 4

13 A pentagon $ABCDE$ has vertices $(3, 4)$, $(7, {}^-1)$, $(8, {}^-4)$, $(3, {}^-2)$ and $({}^-3, 5)$. Find the lengths of its sides.

See Example 4

See Example 4

14 A quadrilateral has vertices $(1, {}^-1)$, $(2, 2)$, $(5, 2)$ and $(2, {}^-3)$. Find the lengths of its sides and the lengths of its diagonals.

Reasoning

15 A quadrilateral has vertices $G({}^-1, 1)$, $H(2, 3)$, $I(3, {}^-1)$ and $J(0, {}^-3)$.

- **a** Find the midpoints of the diagonals.
- **b** What type of quadrilateral is $GHIJ$?

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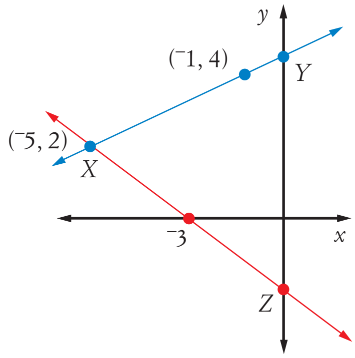
16 Calculate the area of $\triangle XYZ$ shown below.



17 The diagonals of a parallelogram bisect each other. $ABCD$ is a parallelogram where $A = (^-1, {}^-1)$, $B = (3, 0)$ and $C = (1, {}^-3)$ and AC is a diagonal. Use midpoints to find point D.

18 $A(3, 4)$, $B(1, {}^-3)$ and $C(^-2, 1)$ are the vertices of a triangle.

- **a** Find the midpoints D and E of AB and AC respectively.
- **b** Show that $DE \parallel BC$.
- **c** Show that $DE = \frac{1}{2}BC$.

19 A triangle has vertices $(^-2, 3)$, $(1, 1)$ and $(^-1, {}^-2)$.

- **a** Find the lengths of its sides.
- **b** What type of triangle is it?

20 A quadrilateral has vertices $(3, 1)$, $(1, {}^-3)$, $(^-3, {}^-5)$ and $(^-1, {}^-1)$.

- **a** Find the lengths of its sides.
- **b** What type of quadrilateral is it?

21 Points $(^-1, 0)$, $(0, {}^-2)$ and $(4, 1)$ are three vertices of a parallelogram. Use midpoints to find the three possible positions of the fourth vertex.

Worked solutions
Exercise 5.1
MAT10NAWS00013
See Example 7

5.2 Straight-line graphs

Linear functions have graphs that are straight lines. The equation of a linear function is called a **linear equation.** When you draw the graph of a linear equation, you really only need two points, but it is a good idea to plot a third one as a check.

Important!

Linear equations

The equation of a straight line (a **linear equation)** in the Cartesian system may have an x term, a y term and a number.
The following are all examples of linear equations:

$$2x - 3y + 7 = 0 \quad y = \frac{2x}{3} - 5 \quad 4y = 9 \quad 3x = 11 - 5y \quad x = \frac{3}{7}$$

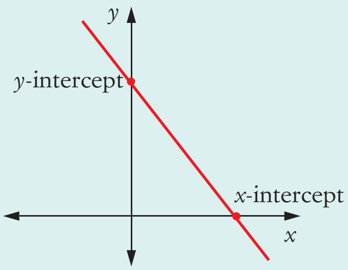
Equations involving higher powers of variables (e.g. x^2, y^3) or roots of variables (e.g. $\sqrt{x}$, $\sqrt[3]{y}$) are not linear equations, nor are equations with variables that are part of the denominator of a fraction (e.g. $\frac{5}{2x}$, $\frac{7}{y}$).

The graph of a linear function will always cut at least one of the Cartesian axes.

Important!

Intercepts

The **intercepts** of a straight line are the points where it crosses the axes of the Cartesian plane.
The x-**intercept** is where the line crosses the x-axis and the y-**intercept** is where it crosses the y-axis.



The term 'intercept' is also used to refer to the distance from the origin to the intersection of the line and the axis. The x-intercept can mean the value of x when $y = 0$. The y-intercept is the value of y when $x = 0$.

Example 8

CAS TI-Nspire exercise
Coordinate geometry
MAT10NATI00005

CAS ClassPad exercise
Coordinate geometry
MAT10NACP00005

TLF Learning object
Grapher (L3531)
MAT10NAIN00005

Draw $y = 3x - 4$ for $^{-}2 \leq x \leq 5$ and use the graph to find the intercepts.

Solution

Since $y = 3x - 4$ is a linear equation, we will find three points.

Construct a table of values for x and y using any three x values from $^{-}2$ to 5.

$\boldsymbol{y = 3x - 4}$

x	$^{-}2$	1	5
y			

Now calculate the corresponding y values.

x	$y = 3x - 4$	(x, y)
$^{-}2$	$3 \times {}^{-}2 - 4 = {}^{-}6 - 4 = {}^{-}10$	$({}^{-}2, {}^{-}10)$
1	$3 \times 1 - 4 = 3 - 4 = {}^{-}1$	$(1, {}^{-}1)$
5	$3 \times 5 - 4 = 15 - 4 = 11$	$(5, 11)$

x	$^{-}2$	1	5
y	$^{-}10$	$^{-}1$	11

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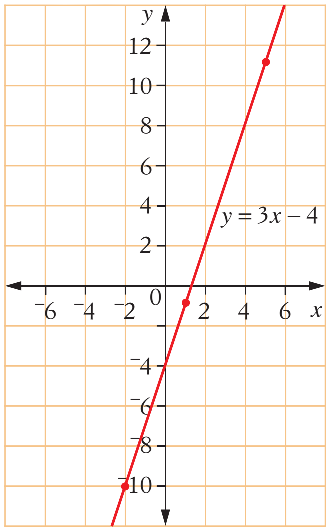
Plot the points and draw a straight line through them.

Find where the line cuts the axes.

You need to measure the x-intercept.



State the result.

The x-intercept is about (1.3, 0).

The y-intercept is (0, $^{-}4$).

In the previous example, the exact value of the x-intercept can be calculated as follows.

For the x-intercept, $y = 0$.	$0 = 3x - 4$
Reverse the equation.	$3x - 4 = 0$
Add 4 to both sides.	$3x = 4$
Divide both sides by 3.	$x = \frac{4}{3} = 1\frac{1}{3}$

So, the x-intercept is $\left(1\frac{1}{3}, 0\right)$.

It is usually easier to rearrange an equation into the form '$y = \ldots$' before you calculate the y values that correspond to given x values.

Example 9

CAS TI-Nspire exercise
Coordinate geometry
MAT10NATI00005

CAS ClassPad exercise
Coordinate geometry
MAT10NACP00005

Draw the graph of $2x + 3y - 8 = 0$ for $^{-}5 \le x \le 5$ and find the intercepts.

Solution

Write the equation.	$\mathbf{2x + 3y - 8 = 0}$
Undo the $-$ 8 by adding 8 to both sides.	$\mathbf{2x + 3y - 8 + 8 = 0 + 8}$
Simplify.	$\mathbf{2x + 3y = 8}$
Undo the $+\ 2x$ by subtracting $2x$ from both sides.	$\mathbf{2x + 3y - 2x = 8 - 2x}$
Simplify.	$\mathbf{3y = 8 - 2x}$
Undo the $\times$ 3 by dividing both sides by 3.	$\mathbf{\frac{3y}{3} = \frac{8}{3} - \frac{2x}{3}}$
Simplify and rearrange.	$\mathbf{y = \frac{^{-}2}{3}x + \frac{8}{3}}$

Draw up a table of values for x and y with $^{-}5 \leq x \leq 5$. You can make it easier by choosing x values that give whole-number y values.

x	$^{-}5$	1	4
y			

Now calculate the corresponding y values.

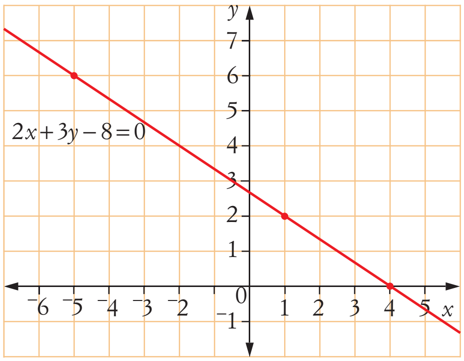
x	$y = \frac{^{-}2}{3}x + \frac{8}{3}$	(x, y)
$^{-}5$	$\frac{^{-}2}{3} \times {^{-}5} + \frac{8}{3} = \frac{10}{3} + \frac{8}{3} = 6$	$(^{-}5, 6)$
1	$\frac{^{-}2}{3} \times 1 + \frac{8}{3} = \frac{^{-}2}{3} + \frac{8}{3} = 2$	$(1, 2)$
4	$\frac{^{-}2}{3} \times 4 + \frac{8}{3} = \frac{^{-}8}{3} + \frac{8}{3} = 0$	$(4, 0)$

x	$^{-}5$	1	4
y	6	2	0

Plot the points and draw a straight line through them.

Read the intercepts from the graph.

For the y-intercept, you may need to measure the distance up the axis and use the scale for the axis.



State the result.

The y-intercept is about $(0, 2\frac{2}{3})$ and the x-intercept is $(4, 0)$.

To work out whether a point is on a line or not, we can graph the equation of the line and check. However, we won't always have time to draw a graph, so we need a faster way. We can substitute the x- and y-coordinates of the point into the equation. If they work (**satisfy** the equation), then the point is on the line. If they don't satisfy the equation, then the point is not on the line.

Example 10

Is $A(^{-}1, 2)$ on the line $3x - 2y + 7 = 0$?

Solution

Write the equation.

$$3x - 2y + 7 = 0$$

Substitute the coordinates of $A(^{-}1, 2)$ into the LHS side of the equation and evaluate.

$$\begin{aligned} 3 \times {^{-}1} - 2 \times 2 + 7 &= {^{-}3} - 4 + 7 \\ &= {^{-}7} + 7 \\ &= 0 \end{aligned}$$

This is the same as the RHS (0), so the coordinates satisfy the equation.

$A(^{-}1, 2)$ is on the line $3x - 2y + 7 = 0$.

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Example 11

a Choose two points on the line $2y + 3x = 6$ for $^-2 \leq x \leq 4$ and draw the graph.

b Calculate the gradient.

c Find the intercepts.

CAS TI-Nspire exercise
Coordinate geometry
MAT10NATI00005

CAS ClassPad exercise
Coordinate geometry
MAT10NACP00005

Solution

a Write the equation. This is a linear equation, so all we need is two points and we can draw the graph. $\quad \mathbf{2y + 3x = 6}$

Choose any x value from $^-2$ to 4. $\quad$ **Let $x = 4$.**

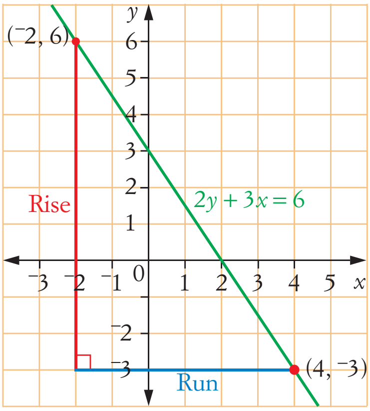
Substitute to find the corresponding y value. $\quad \mathbf{2y + 3 \times 4 = 6}$

Evaluate.
$$2y = {}^-6$$
$$y = {}^-3$$

State the coordinates of the point. $\quad$ **$(4, {}^-3)$ lies on $2y + 3x = 6$**

Follow the same procedure for (say) $x = {}^-2$. $\quad$ **$({}^-2, 6)$ lies on $2y + 3x = 6$**

Use these points to draw the graph.



b Use the two points to calculate the gradient. $\quad$ **Let $(x_1, y_1) = ({}^-2, 6)$ and $(x_2, y_2) = (4, {}^-3)$.**

Write the gradient formula.
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute.
$$= \frac{{}^-3 - 6}{4 - {}^-2}$$

Evaluate.
$$= \frac{{}^-9}{6} = \frac{{}^-3}{2}$$

State the result. $\quad$ **Gradient $= \frac{{}^-3}{2}$**

c Locate the intercepts on the graph. $\quad$ **The x-intercept is $(2, 0)$ and the y-intercept is $(0, 3)$.**

Investigate: Equations of lines

Technology worksheet
Excel worksheet: Drawing linear graphs
MAT10NACT00009

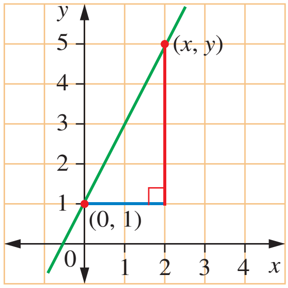
TLF Learning object
Grapher (L3531)
MAT10NAIN00005

1 Draw graphs of the lines represented by the following equations.

a $y = 2x + 5$ **b** $y = -x - 3$ **c** $y = {}^{-}3x + 4$
d $y = 3x - 1$ **e** $y = \frac{1}{2}x - 3$ **f** $y = {}^{-}\frac{2}{3}x + 1$

2 Work out the gradient and y-intercept for each.
3 Compare the values of the gradient and y-intercept with the equations for the lines.
4 What do you notice? Does this always work? Can you explain why?

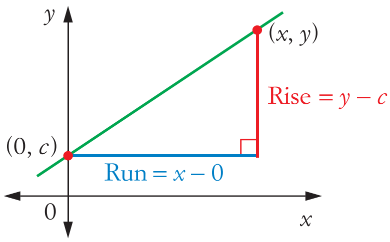
In the above investigation, you should have seen that the equation of a straight line can be written using its gradient and y-intercept. The gradient (slope) is the coefficient of the x term and the y-intercept is the constant term. We usually use m as the symbol for gradient and c as the symbol for the y-intercept. This can be shown to be true in all cases as follows.
Suppose that a straight line has a slope of 2 and y-intercept (0, 1), so $c = 1$. We can draw the graph of the line as shown below with the point (x, y) somewhere on the line.
Let $(x_1, y_1) = (0, 1)$ and $(x_2, y_2) = (x, y)$.



Here $$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$2 = \frac{y - 1}{x - 0}$$
So $$\frac{y - 1}{x} = 2$$
$$y - 1 = 2x$$
$$y = 2x + 1$$

For the *general case*, suppose that a straight line has slope m and y-intercept $(0, c)$. We can draw a graph of the general case using the point (x, y) and the other information as shown here.

Here $$m = \frac{\text{rise}}{\text{run}} = \frac{y - c}{x - 0}$$
So $$\frac{y - c}{x} = m$$
$$y - c = mx$$
$$y = mx + c$$



This is called the **gradient–intercept form** of the equation of a line.
Another way of writing the equation of a line is known as the **standard form.** It is written with no fractions and all the terms on one side. The standard form of the equation $y = 2x + 1$ is $2x - y + 1 = 0$.

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Important!

Forms of the equation of a line

The **gradient–intercept** (or **slope–intercept**) **form** of the equation of a straight line is $y = mx + c$ where m is the gradient and c is the y-intercept.
The **standard** (or **general**) **form** of the equation of a straight line is $ax + by + c = 0$ where a, b and c are constants (numbers).

Example 12

Write the standard form equation of the line with:

a gradient 7 and y-intercept $^-2$

b gradient $^-\frac{3}{4}$ and y-intercept (0, 5).

Puzzle sheet
Equations in gradient form
MAT10NAPS00011

Solution

a Write the equation in gradient–intercept form. $\boldsymbol{y = mx + c}$

We know that $m = 7$ and $c = {^-2}$. $\boldsymbol{y = 7x + {^-2}}$

Simplify. $\boldsymbol{y = 7x - 2}$

To write the equation in standard form, subtract $7x$ from both sides and then add 2. $\boldsymbol{y - 7x + 2 = 0}$

b Write the equation in gradient–intercept form. $\boldsymbol{y = mx + c}$

We know that $m = {^-\frac{3}{4}}$ and $c = 5$. $\boldsymbol{y = {^-\frac{3}{4}}x + 5}$

Multiply all terms by 4 (the denominator of the x term). $\boldsymbol{y \times 4 = {^-\frac{3}{4}}x \times 4 + 5 \times 4}$

Simplify. $\boldsymbol{4y = {^-3}x + 20}$

To write the equation in standard form, add $3x$ to both sides and then subtract 20. $\boldsymbol{4y + 3x - 20 = 0}$

Example 13

What are the gradient and the y-intercept of each of these lines?

a $y = {^-4}x + 9$

b $3x + 2y - 6 = 0$

Video tutorial
The gradient–intercept formula
MAT10NAVT10011

Solution

a Write the equation. $\boldsymbol{y = {^-4}x + 9}$

Compare with the gradient–intercept form. $\boldsymbol{y = mx + c}$

You can see that $m = {^-4}$ and $c = 9$. **Gradient = $^-4$ and y-intercept = (0, 9).**

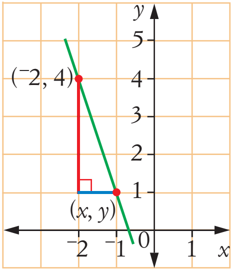
b We first need to write the equation in gradient–intercept form.

Write the equation. $\boldsymbol{3x + 2y - 6 = 0}$

Subtract $3x$ from both sides and then add 6.	$\mathbf{2y = 6 - 3x}$
Divide throughout by 2 (the coefficient of y).	$\mathbf{\frac{2y}{2} = \frac{6}{2} - \frac{3x}{2}}$
Simplify.	$\mathbf{y = 3 - \frac{3}{2}x}$
Write the equation with the x term first.	$\mathbf{y = {}^{-}\frac{3}{2}x + 3}$
Compare with the gradient–intercept form.	$\mathbf{y = mx + c}$
You can see that $m = {}^{-}\frac{3}{2}$ and $c = 3$.	**Gradient** $= {}^{-}\frac{3}{2}$ **and** $\mathbf{y}$**-intercept** $= \mathbf{(0, 3)}$**.**

We can also work out the equation of a line using the gradient and one point on the line. Suppose a straight line has slope $m = {}^{-}3$ and passes through the point $({}^{-}2, 4)$. The equation uses (x, y) for points on the line.

Let $(x_1, y_1) = ({}^{-}2, 4)$ and $(x_2, y_2) = (x, y)$.



Here $\quad m = \dfrac{y - y_1}{x - x_1}$

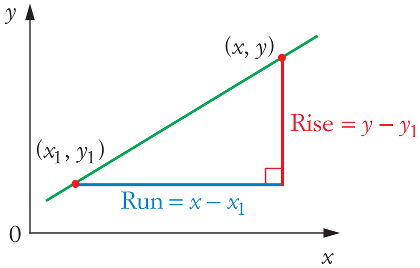
$${}^{-}3 = \frac{y - 4}{x - {}^{-}2}$$

So $\quad \dfrac{y - 4}{x - {}^{-}2} = {}^{-}3$

$$y - 4 = {}^{-}3(x - {}^{-}2)$$

In standard form, this equation is written as $3x + y + 2 = 0$.

For the *general case*, suppose that a straight line has slope m and passes through the point (x_1, y_1). The equation uses (x, y) for points on the line.



Here $\quad m = \dfrac{\text{rise}}{\text{run}} = \dfrac{y - y_1}{x - x_1}$

So $\quad \dfrac{y - y_1}{x - x_1} = m$

$$y - y_1 = m(x - x_1)$$

This is called the **gradient–point form** of the equation of a line.

Important!

Gradient–point form of the equation of a line

The **gradient–point form** of the equation of a line is given by

$$y - y_1 = m(x - x_1)$$

where m is the gradient and (x_1, y_1) is a point on the line.

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Example 14

Work out the standard form equation of a straight line with slope $\frac{2}{3}$ that passes through the point $(^{-}2, 1)$.

Video tutorial
The gradient–point formula
MAT10NAVT00012

Solution

Write the gradient–point equation of a line. $\quad y - y_1 = m(x - x_1)$

Substitute using $(x_1, y_1) = (^{-}2, 1)$ and $m = \frac{2}{3}$. $\quad y - 1 = \frac{2}{3}(x - {}^{-}2)$

Simplify. $\quad y - 1 = \frac{2}{3}(x + 2)$

Multiply throughout by 3. $\quad 3(y - 1) = 2(x + 2)$

Expand both sides. $\quad 3y - 3 = 2x + 4$

$$3y - 2x - 7 = 0$$

Rearrange into standard form. $\quad 2x - 3y + 7 = 0$

It is also possible to calculate the equation of a line if you know two points through which it passes.

Example 15

Find the equation of a line passing through (1, 3) and $(4, {}^{-}3)$.

CAS TI-Nspire exercise
Coordinate geometry
MAT10NATI00005

CAS ClassPad exercise
Coordinate geometry
MAT10NACP00005

Solution

Identify the coordinates. $\quad$ **Let** $(x_1, y_1) = (1, 3)$ **and** $(x_2, y_2) = (4, {}^{-}3)$.

First find the gradient.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{^{-}3 - 3}{4 - 1} = \frac{^{-}6}{3} = {}^{-}2$$

Now you know the gradient and a point through which the line passes.

Write the gradient–point formula. $\quad y - y_1 = m(x - x_1)$

Substitute using $(x_1, y_1) = (1, 3)$ and $m = {}^{-}2$. $\quad y - 3 = {}^{-}2(x - 1)$

Expand. $\quad y - 3 = {}^{-}2x + 2$

Rearrange into standard form. $\quad 2x + y - 5 = 0$

In Example **15**, you could have used point $(4, {}^{-}3)$ instead of (1, 3) to obtain the equation of the line.

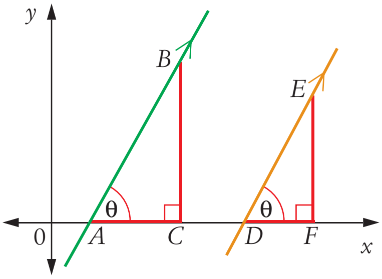
Parallel and perpendicular lines

Technology: GeoGebra
Perpendicular lines
MAT10NATC00005

The gradients of parallel and perpendicular lines are closely related.

Suppose two lines AB and DE are parallel. Then the angles they make with the x-axis are the same, as shown in this diagram.

If we draw in the extra lines BC and EF, we can work out the gradients.



$$\triangle ABC \,|||\, \triangle DEF$$

So $$\frac{BC}{AC} = \frac{EF}{DF}$$

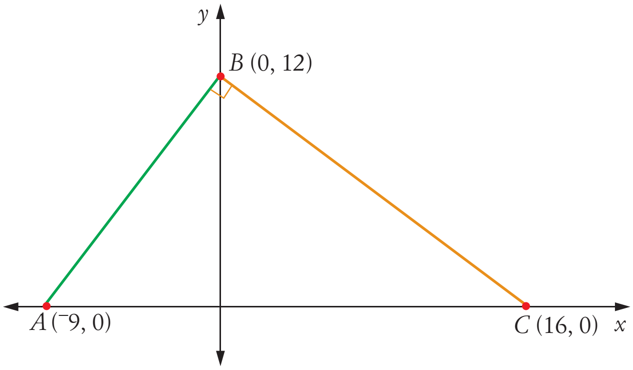
But $$m_{AB} = \frac{BC}{AC} \text{ and } m_{DE} = \frac{EF}{DF}$$

So $$m_{AB} = m_{DE}$$

Suppose two lines AB and BC are perpendicular. Then they make a right angle at the intersection B, as shown here.

First calculate the gradient of AB.

$$\begin{aligned} m_{AB} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{12 - 0}{0 - 9} \\ &= \tfrac{4}{3} \end{aligned}$$



Now calculate the gradient of BC.

$$\begin{aligned} m_{BC} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{0 - 12}{16 - 0} \\ &= \tfrac{^-3}{4} \end{aligned}$$

So $m_{AB} \times m_{BC} = \frac{4}{3} \times \frac{^-3}{4} = {}^-1$

It is possible to show that this result applies to all pairs of lines that are perpendicular.

Teacher notes
Perpendicular lines
MAT10NATN00002

Important!

Parallel and perpendicular lines

The gradients m_1 and m_2 of **parallel lines** are the same.

For parallel lines: $m_1 = m_2$

The gradients m_1 and m_2 of **perpendicular lines** have a product of $^-1$. They are negative reciprocals.

For perpendicular lines: $m_1 \times m_2 = {}^-1$

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Example 16

Worksheet
Parallel and perpendicular lines
MAT10NAWK00015

Show that $2x - y + 8 = 0$ and $4x = 2y + 5$ are parallel.

Solution

If we can show that the gradients are the same, the lines must be parallel. Therefore, we need to find the gradient of each line. This can be done by writing the equations of the lines in gradient–intercept form.

Write the equation of the first line.	$\mathbf{2x - y + 8 = 0}$
Rearrange into the form $y = mx + c$.	$\mathbf{y = 2x + 8}$
Compare with $y = mx + c$ to find the gradient.	**Gradient of $2x - y + 8 = 0$ is 2.**
Write the equation of the second line.	$\mathbf{4x = 2y + 5}$
Rearrange into the form $y = mx + c$.	$\mathbf{y = 2x - 2\frac{1}{2}}$
Compare with $y = mx + c$ to find the gradient.	**Gradient of $4x = 2y + 5$ is 2.**
The gradients of the two lines are equal.	**The lines are parallel.**

It is not possible to calculate the gradient of a line parallel to the y-axis. Lines parallel to the x-axis have a slope of 0. The equations of these lines can be written as long as you know any point on the lines.

Important!

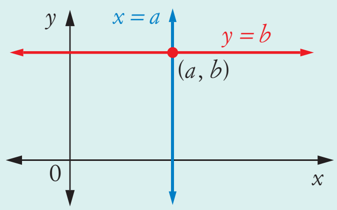
Lines parallel to the axes

The horizontal line parallel to the x-axis through (a, b) is

$$y = b.$$

The vertical line parallel to the y-axis through (a, b) is

$$x = a.$$



Example 17

Write the equation of:

a the vertical line through $(^-4, 7)$

b the horizontal line through $(^-2, 6)$.

Solution

a The y value can be anything but the x value must be $^-4$. Write the equation.	**The equation is $x = {}^-4$.**
b The x value can be anything but the y value must be 6. Write the equation.	**The equation is $y = 6$.**

Puzzle sheet
Gradients of parallel and perpendicular lines
MAT10NAPS00012

Puzzle sheet
Equations of parallel lines
MAT10NAPS00013

Example 18

Find the equation of the line through $(^-2, 5)$ that is:

a parallel to $2x + 3y - 4 = 0$ **b** perpendicular to $2x + 3y - 4 = 0$.

Solution

a Write the equation. $\quad \mathbf{2x + 3y - 4 = 0}$

Rearrange into gradient–intercept form. $\quad \mathbf{y = {}^-\tfrac{2}{3}x + 1\tfrac{1}{3}}$

Compare with $y = mx + c$. $\quad \mathbf{m = {}^-\tfrac{2}{3}}$

A parallel line will have the same gradient, m.

Write the gradient–point equation of the line. $\quad \mathbf{y - y_1 = m(x - x_1)}$

We know $m = {}^-\frac{2}{3}$ and $(x_1, y_1) = (^-2, 5)$. $\quad \mathbf{y - 5 = {}^-\tfrac{2}{3}(x - {}^-2)}$

Multiply throughout by 3. $\quad \mathbf{3(y - 5) = {}^-2(x + 2)}$

Expand both sides. $\quad \mathbf{3y - 15 = {}^-2x - 4}$

Rearrange into standard form. $\quad \mathbf{2x + 3y - 11 = 0}$

b Write the rule for perpendicular lines. $\quad \mathbf{m_1 \times m_2 = {}^-1}$

Substitute the gradient of $2x + 3y - 4 = 0$. $\quad \mathbf{{}^-\tfrac{2}{3} \times m_2 = {}^-1}$

Multiply both sides by ${}^-\frac{3}{2}$. $\quad \mathbf{m_2 = \tfrac{3}{2}}$

A line perpendicular to $2x + 3y - 4 = 0$ will have the gradient m_2.

Write the gradient–point equation of the line. $\quad \mathbf{y - y_1 = m(x - x_1)}$

We know $m = \frac{3}{2}$ and $(x_1, y_1) = (^-2, 5)$. $\quad \mathbf{y - 5 = \tfrac{3}{2}(x - {}^-2)}$

Multiply throughout by 2. $\quad \mathbf{2(y - 5) = 3(x + 2)}$

Expand both sides. $\quad \mathbf{2y - 10 = 3x + 6}$

Rearrange into standard form. $\quad \mathbf{3x - 2y + 16 = 0}$

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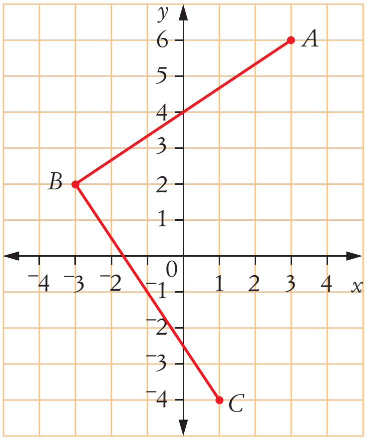
Example 19

Plot the points $A(3, 6)$, $B(^{-}3, 2)$ and $C(1, ^{-}4)$ and show that $\angle ABC = 90°$.

Solution

Plot the points.

Join AB and BC.



Write the rule for gradient. $\quad \boldsymbol{m = \dfrac{y_2 - y_1}{x_2 - x_1}}$

For AB, use $(x_1, y_1) = (3, 6)$ and $(x_2, y_2) = (^{-}3, 2)$. $\quad \boldsymbol{m_{AB} = \dfrac{2 - 6}{^{-}3 - 3} = \dfrac{^{-}4}{^{-}6} = \dfrac{2}{3}}$

For BC, use $(x_1, y_1) = (^{-}3, 2)$ and $(x_2, y_2) = (1, ^{-}4)$. $\quad \boldsymbol{m_{BC} = \dfrac{^{-}4 - 2}{1 - {}^{-}3} = \dfrac{^{-}6}{4} = \dfrac{^{-}3}{2}}$

Find the product of the gradients. $\quad \boldsymbol{m_{AB} \times m_{BC} = \dfrac{2}{3} \times \dfrac{^{-}3}{2} = {}^{-}1}$

So $m_1 \times m_2 = {}^{-}1$. $\quad$ **AB and BC are perpendicular, so $\angle ABC = 90°$.**

We can **model** real patterns and relationships using algebraic equations, tables of values or graphs. Often, models will provide only approximations of real-life situations. However, this will usually be close enough for most practical purposes. Relationships that can be represented by functions of the form $y = mx + c$ are called **linear models**.

Example 20

The height, h cm, of a seedling is given by:
$h = 2d + 10$ where d is the number of days since it was planted.

a Complete a table of values for the function for $0 \leq d \leq 8$.
b Use the table to draw a graph of the height function.
c What type of function is the height function?
d Compare the height function with the gradient–intercept form of a linear function ($y = mx + c$) and explain what the values 2 and 10 represent.

Solution

a Draw up the table of values for d and h using any three values of d from 0 to 8.

$\boldsymbol{h = 2d + 10}$

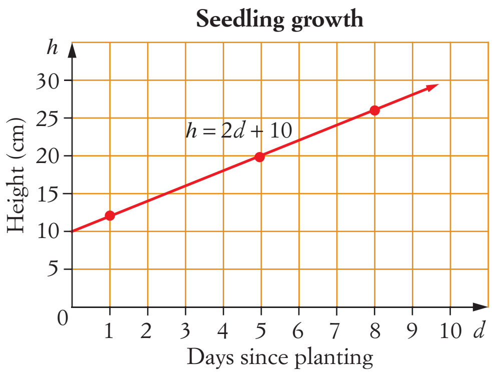
d	1	5	8
h			

Calculate the corresponding values of h.

d	$h = 2d + 10$	(d, h)
1	$2 \times 1 + 10 = 12$	(1, 12)
5	$2 \times 5 + 10 = 20$	(5, 20)
8	$2 \times 8 + 10 = 26$	(8, 26)

d	1	5	8
h	12	20	26

b Plot the points and join them. Label the axes and add a title for the graph.



c The graph is a straight line.

The function $h = 2d + 10$ is linear.

d Write the height function.

$\boldsymbol{h = 2d + 10}$

Write the gradient–intercept form.

$\boldsymbol{y = mx + c}$

Compare the variables.

$\boldsymbol{y \leftrightarrow h}$ **and** $\boldsymbol{x \leftrightarrow d}$

For the height function, the height (h) depends on the number of days after planting (d), so d is the independent variable and h is the dependent variable.

The horizontal axis shows the independent variable and the vertical axis shows the dependent variable.

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Compare the x-coefficients and the constant terms.

In the height function, 2 is the gradient of the graph and 10 is the intercept on the vertical (h) axis, i.e. the height when the seedling was planted ($d = 0$).

In the previous example, the gradient of the graph for the height function was positive. This means that the value of h (height) increases as d (the number of days) increases.

Example 21

A small business buys $20 000 worth of computer equipment. For tax purposes, the value, $\$V$, of the equipment decreases (depreciates) over time according to the equation:

$$V = 20\,000 - 4000t$$

where t is the number of years since purchase.

a Complete a table of values for the function for $0 \leq t \leq 6$.

b Use the table to draw a graph of the value function.

c When will the computer equipment be worthless for tax purposes?

Solution

a Draw up the table of values for t and V using any three values of t from 0 to 6.

$V = 20\,000 - 4000t$

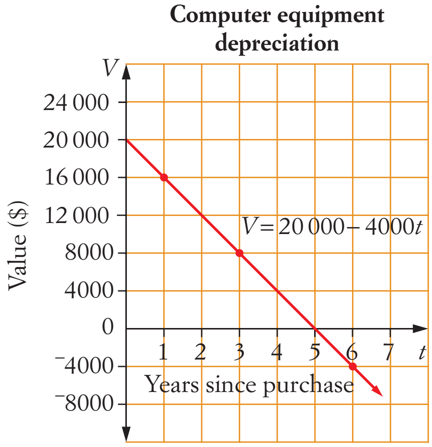
t	1	3	6
V			

Calculate the corresponding values of V.

t	$V = 20\,000 - 4000t$	(t, V)
1	$20\,000 - 4000 \times 1 = 16\,000$	(1, 16 000)
3	$20\,000 - 4000 \times 3 = 8000$	(3, 8000)
6	$20\,000 - 4000 \times 6 = {}^{-}4000$	(6, $^{-}4000$)

t	1	3	6
V	16 000	8000	$^{-}4000$

b Plot the points and join them. Label the axes and add a title for the graph.



c Find the value of t when $V = 0$.

$t = 5$ **years**

State the result.

The equipment will be worthless for tax purposes after 5 years.

In real-life situations, observed data usually does not exactly fit the linear model $y = mx + c$. When real data is graphed, we may need to draw a **line of best fit** for the plotted points in order to see the relationship.

Important!

Line of best fit

The line of best fit for data points:

- represents most of the points as closely as possible
- should have approximately the same number of points above the line as below
- is drawn so that the perpendicular distances between the points and the line are as small as possible.

The line of best fit may be used to make predictions of the values of variables. **Interpolation** is the prediction of values within the range of data. **Extrapolation** is the prediction of values outside the range of data.

Example 22

Samantha's height was measured over a period of 8 years. The table below shows the data.

Age, *a* (years)	1	3	4	6	7	8
Height, *h* (cm)	75	98	102	119	120	131

a Plot the data and draw in a line of best fit.
b Use the graph to estimate Samantha's height when she was born.
c Find the equation of the line of best fit.
d Use the equation of the line of best fit to estimate Samantha's height when she turns 12.

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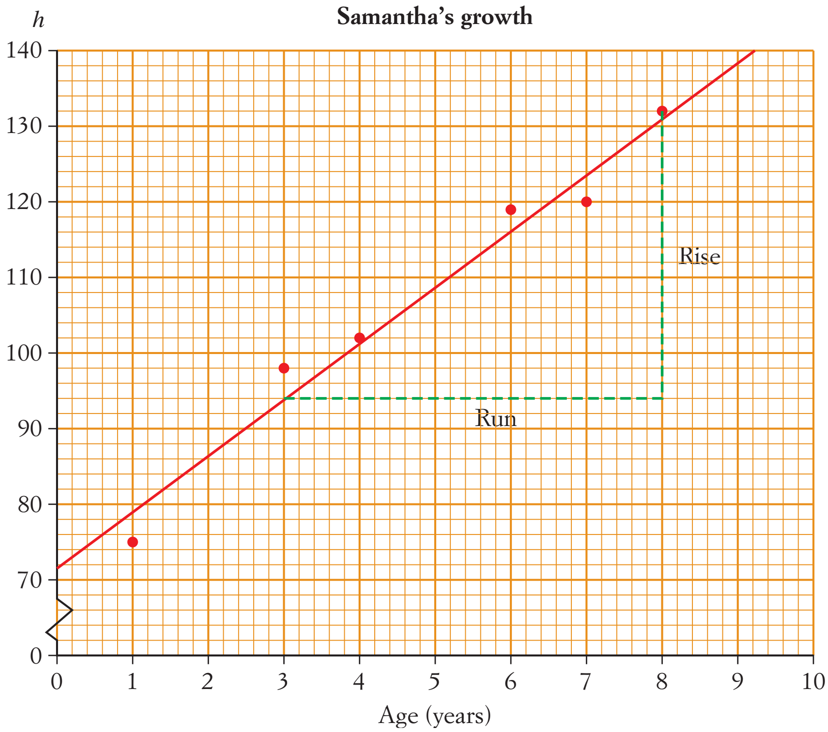
e Use the equation to estimate Samantha's height when she turns 30 and comment on the result.

Solution

a Draw up a set of axes and plot the points. Label the axes and add a title for the graph. Use a transparent ruler to draw in the line of best fit.



b Find the point where the line of best fit cuts the vertical axis. Use the scale to calculate the value of h when $a = 0$.

$h \approx 71$ **cm when** $a = 0$

Write the result.

Samantha's height was about 71 cm when she was born.

c This is a linear function, so it can be written in the form $y = mx + c$ where m is the gradient and c is the intercept on the vertical axis.

For this height function, the independent variable is age (a) and the dependent variable is height (h). — $y \leftrightarrow h$ **and** $x \leftrightarrow a$

We can read the intercept on the vertical (h) axis from the graph. — **Vertical axis intercept = (0, 71)**

We need to find two points on the line of best fit to calculate its gradient. Locate two points on the line. — **(3, 94) and (8,131) are on the line.**

Write the rule for the gradient. — $\text{Gradient} = \dfrac{\text{rise}}{\text{run}}$

Substitute using the chosen points. — $= \dfrac{131 - 94}{8 - 3}$

Evaluate. The graph is rising, so the gradient is positive. — $= \dfrac{37}{5}$

$= 7.4$

Write the equation based on $y = mx + c$. — $h = 7.4a + 71$

d Substitute $a = 12$. — $= 7.4 \times 12 + 71$

Evaluate. — $= 159.8$ **cm**

State the result. — **Samantha will be about 160 cm high when she turns 12.**

e Substitute $a = 30$. — $h = 7.4 \times 30 + 71$

Evaluate. — $= 293$ **cm**

State the result and comment on it. — **The graph predicts that Samantha will be about 293 cm high when she turns 30. This is not possible. The answer obtained from the formula shows that there are limitations to how useful models of this kind can be. We all stop growing after a certain age but the formula doesn't take this into account.**

Technology Lines of best fit

Technology worksheet
Excel worksheet: Line of best fit
MAT10SPCT00003

Let's see how Excel can be used to insert a line of best fit for plotted data.
The population of a small town is recorded as follows.

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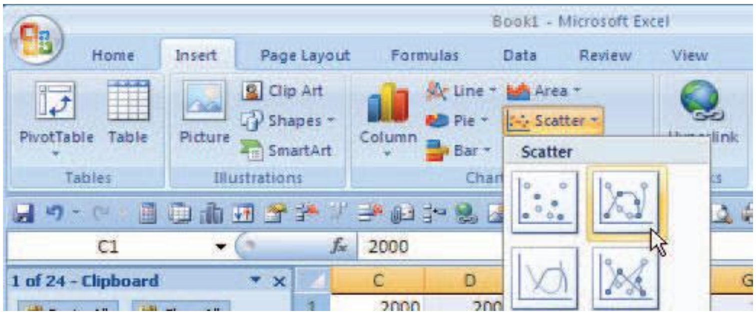
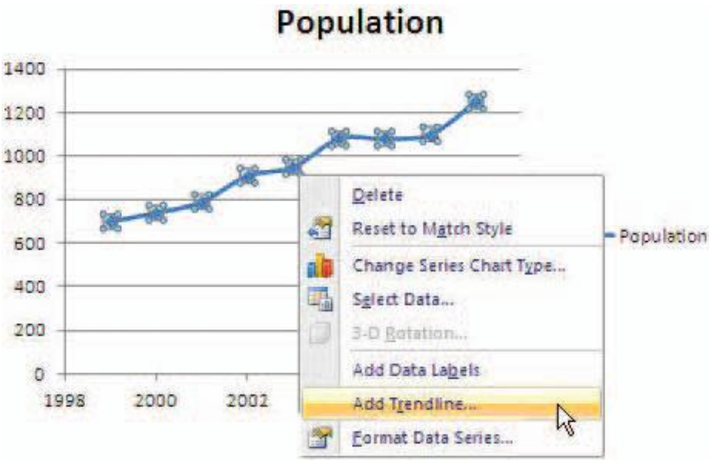
Year	1999	2000	2001	2002	2003	2004	2005	2006	2007
Population	700	740	790	910	950	1080	1080	1100	1250

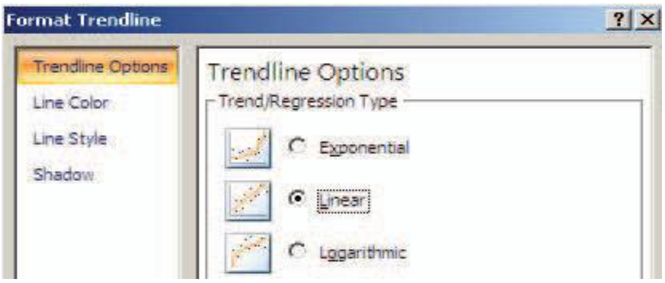
Enter the information from the table into a spreadsheet. Type 'Year' into cell A1 and 'Population' into cell A2. Then enter the years from the first row of the table into cells B1 to J1 and the population values into cells B2 to J2.
Select cells B1 to J2, then under the 'Insert' tab, click on 'Scatter' and select 'Scatter with smooth lines and markers'.



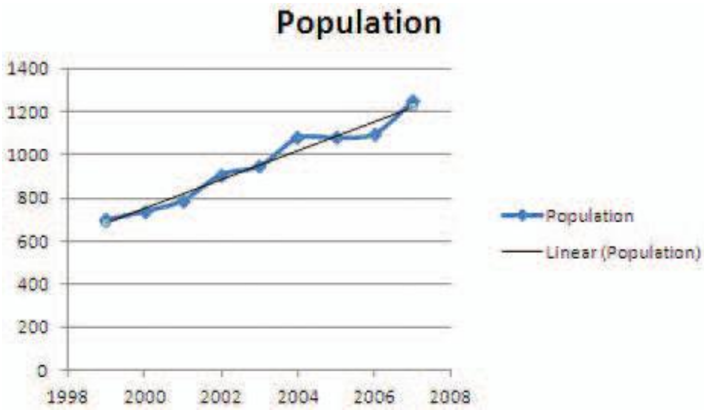
Now place the cursor on one of the data points and press the right-hand button ('right click') on the mouse. Select the Add Trendline option. A **trendline** is actually a line of best fit.



Select 'Linear' and then click on 'Close'.



The trendline will then be inserted as shown below.



If you right-click on the trendline and select the Format Trendline option, you will be able to experiment with the look of the trendline.

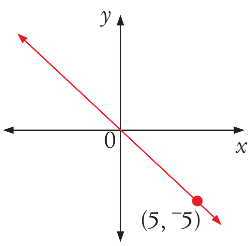
Exercise 5.2 Straight-line graphs

Understanding

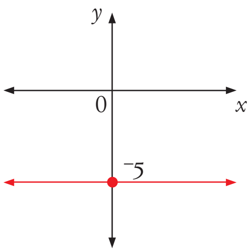
Extra questions
Exercise 5.2
MAT10NAEQ00014

1 The graphs below are not drawn to scale.

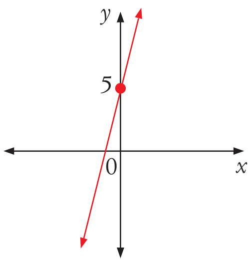
a



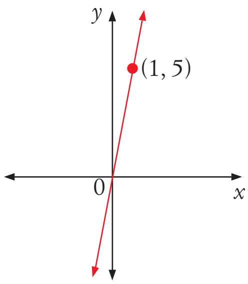
b



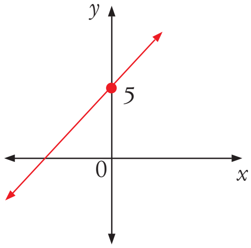
c



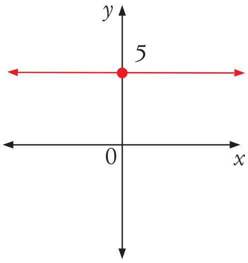
d



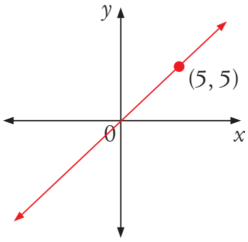
e



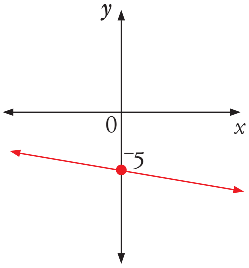
f



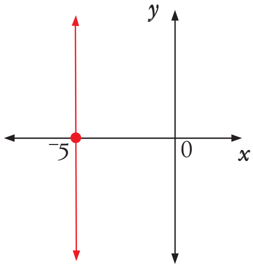
g



h



i



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Choose the most appropriate equation for each one from the following list.

i $y = x + 5$ **ii** $y = 5$ **iii** $x = 5$
iv $y = 5 - x$ **v** $y = 5x$ **vi** $y = x - 5$
vii $y = {}^{-}5$ **viii** $y = 5x + 5$ **ix** $y = x$
x $x = {}^{-}5$ **xi** $y = -x$ **xii** $y = {}^{-}\frac{1}{2}x - 5$

2 Draw each of the following and state the intercepts from the graph if possible. *See* Example 8

a $y = 2x - 3$ for ${}^{-}3 \le x \le 6$ **b** $y = {}^{-}3$ for ${}^{-}4 \le x \le 4$
c $x = 1$ **d** $y = 2x + 1$ for ${}^{-}5 \le x \le 5$
e $y = -x + 2$ for ${}^{-}6 \le x \le 6$ **f** $y = {}^{-}2x - 3$ for ${}^{-}6 \le x \le 3$
g $y = 3x - 1$ for ${}^{-}2 \le x \le 3$ **h** $x = {}^{-}3$
i $y = 3x + 4$ for ${}^{-}4 \le x \le 2$ **j** $y = -x - 3$ for ${}^{-}6 \le x \le 6$

3 Work out whether each of the points is on the given line. *See* Example 10

a $3x + 2y + 8 = 0$	$({}^{-}4, 2)$	$(3, 4)$	$({}^{-}2, 1)$	$(2, {}^{-}7)$	$(4, 0)$
b $y = x - 3$	$({}^{-}1, 2)$	$(1, {}^{-}2)$	$(3, 0)$	$({}^{-}2, 5)$	$({}^{-}3, {}^{-}6)$
c $y = 2x + 6$	$(0, {}^{-}6)$	$(2, 8)$	$(1, 14)$	$({}^{-}2, 2)$	$(3, 12)$
d $y = {}^{-}3x + 2$	$(1, 1)$	$({}^{-}1, 3)$	$(3, {}^{-}7)$	$(0, {}^{-}2)$	$({}^{-}4, 14)$
e $3x - y - 6 = 0$	$({}^{-}4, 6)$	$(0, {}^{-}3)$	$(2, 0)$	$(10, {}^{-}12)$	$({}^{-}4, {}^{-}18)$

4 Draw each of the following for ${}^{-}4 \le x \le 4$ and state the intercepts if possible. *See* Example 9

a $3x - 2y + 6 = 0$ **b** $4x - 3y = 12$ **c** $x + 3y - 4 = 0$
d $2x + 2y + 5 = 0$ **e** $6x - 2y + 3 = 0$ **f** $5x + 3y = 15$

5 Find the equation of the line parallel to the x-axis that passes through each point. *See* Example 17

a $({}^{-}2, 4)$ **b** $(3, 5)$ **c** $(2, {}^{-}1)$ **d** $(0, 5)$
e $(4, 0)$ **f** $({}^{-}1, {}^{-}4)$ **g** $(6, 1)$ **h** $({}^{-}4, 7)$
i $(7, {}^{-}3)$ **j** $({}^{-}8, {}^{-}8)$ **k** $(3, 3)$ **l** $(5, 1)$

6 Find the equation of the line parallel to the y-axis that passes through each point.

a $({}^{-}2, 4)$ **b** $(3, 5)$ **c** $(2, {}^{-}1)$ **d** $(0, 5)$
e $(4, 0)$ **f** $({}^{-}3, {}^{-}4)$ **g** $(6, 1)$ **h** $({}^{-}4, 7)$
i $(7, {}^{-}3)$ **j** $({}^{-}8, {}^{-}8)$ **k** $(3, 3)$ **l** $(5, 1)$

Fluency

7 For each of the following functions, draw the line, then: *See* Example 11

i choose two points and calculate the gradient
ii find the intercepts if possible.

a $y = 3x - 2$ **b** $y = x + 4$ **c** $y = {}^{-}2x + 3$
d $x = {}^{-}1$ **e** $y = {}^{-}\frac{1}{4}x + 1$ **f** $2y = 3x - 4$
g $2y = 4x + 1$ **h** $3y = {}^{-}2x - 3$ **i** $5y = {}^{-}7x + 8$
j $4x - y + 8 = 0$ **k** $x + 3y - 4 = 0$ **l** $4x - 3y = 0$

8 Write in gradient–intercept form the equation of the line with:

a gradient 2 and y-intercept $(0, 4)$ **b** slope ${}^{-}3$ and y-intercept $(0, 1)$
c y-intercept ${}^{-}6$ and slope 5 **d** gradient 3 and y-intercept ${}^{-}2$
e y-intercept $(0, {}^{-}3)$ and slope 0 **f** slope ${}^{-}4$ and y-intercept 0.

See Example 12

9 Write in standard form the equation of the line with:

- **a** gradient 4 and y-intercept $^{-}1$
- **b** gradient $\frac{2}{3}$ and y-intercept $(0, ^{-}2)$
- **c** y-intercept $^{-}1$ and slope $^{-}\frac{4}{3}$
- **d** y-intercept $(0, 2)$ and gradient $\frac{5}{2}$
- **e** gradient $3\frac{1}{3}$ and y-intercept $^{-}4$
- **f** y-intercept $\frac{5}{4}$ and slope $^{-}1\frac{3}{4}$.

See Example 13

10 Without drawing a graph, work out the gradient and y-intercept of each line.

- **a** $y = 2x + 5$
- **b** $y = 5 - 3x$
- **c** $y = -x + 1$
- **d** $y = 4 - 7x$
- **e** $y = x + 2$
- **f** $y = {}^{-}8x$
- **g** $x - 2y + 1 = 0$
- **h** $7x + y - 14 = 0$
- **i** $x - 3y + 6 = 0$
- **j** $3y - 8 = {}^{-}4x$
- **k** $7x = 3y$

See Example 14

11 Find the equation, in both gradient–intercept and standard form, of the line:

- **a** with gradient 2, passing through $(2, 5)$
- **b** through $(^{-}1, 4)$ with slope $^{-}1$
- **c** through $(^{-}3, ^{-}2)$ with slope 1
- **d** with gradient $^{-}4$, passing through $(^{-}1, ^{-}4)$
- **e** through $(4, 3)$ with gradient $^{-}\frac{3}{4}$
- **f** through $(^{-}3, ^{-}4)$ with slope $\frac{7}{3}$.

Worked solutions
Exercise 5.2
MAT10NAWS00014

12 Find the equation of the line passing through $(2, 3)$ and $(2, ^{-}1)$.

13 Find the equation of the line with slope $^{-}\frac{2}{3}$ that passes through $(0, 4)$, in both gradient–intercept and standard forms.

14 Work out the general and slope–intercept forms of the equation of the line with:

- **a** x-intercept 2 and y-intercept 1
- **b** x-intercept $^{-}3$ and y-intercept 4
- **c** x-intercept 4 and y-intercept $^{-}2$
- **d** x-intercept $^{-}1$ and y-intercept 4
- **e** x-intercept $(3, 0)$ and y-intercept $(0, ^{-}8)$
- **f** intercepts $(^{-}5, 0)$ and $(0, ^{-}3)$
- **g** intercepts $(0, ^{-}1)$ and $(5, 0)$
- **h** intercepts $(0, 6)$ and $(^{-}4, 0)$
- **i** both intercepts $^{-}4$
- **j** both intercepts 2.

15 A line passes through $(0, 4)$ and $(2, 8)$.

- **a** What is the slope of the line?
- **b** What is the y-intercept?
- **c** Write its equation in standard form.

See Example 15

16 Find the equation of the line through each of these pairs of points, in both general and slope–intercept forms.

- **a** $(4, 6)$ and $(5, 3)$
- **b** $(3, 1)$ and $(^{-}3, ^{-}5)$
- **c** $(0, 3)$ and $(2, 7)$
- **d** $(^{-}1, ^{-}3)$ and $(^{-}4, 6)$
- **e** $(3, 4)$ and $(6, ^{-}8)$
- **f** $(^{-}2, 5)$ and $(2, 1)$
- **g** $(2, 1)$ and $(^{-}3, ^{-}4)$
- **h** $(4, 2)$ and $(6, 10)$
- **i** $(3, ^{-}4)$ and $(^{-}5, 1)$
- **j** $(^{-}2, ^{-}6)$ and $(^{-}4, ^{-}2)$
- **k** $(3, 6)$ and $(^{-}2, ^{-}4)$
- **l** $(6, ^{-}1)$ and $(4, ^{-}5)$

17 Use the most appropriate method to find the general form of the equation of each line.

- **a** Gradient 2 and y-intercept $^{-}4$
- **b** Slope $^{-}\frac{2}{3}$ and passing through $(4, ^{-}3)$
- **c** Passing through $(5, 1)$ and $(^{-}4, ^{-}5)$
- **d** Passing through $(2, 1)$ and $(2, 8)$
- **e** Slope 4 and intercept $(0, 2)$
- **f** Slope $^{-}2$ and intercept $(5, 0)$

Worked solutions
Exercise 5.2
MAT10NAWS00014
See Example 16

18 Without drawing graphs, work out whether the following pairs of lines are parallel.

- **a** $2x + 3y + 8 = 0$ and $6y = 5 - 4x$
- **b** $2x - y - 4 = 0$ and $y = 2x + 6$
- **c** $5y - 2x = 3$ and $2y - 5x + 6 = 0$
- **d** $3y = 4x - 9$ and $y - \frac{4}{3}x + 4 = 10$
- **e** $y = 4 - 2\frac{1}{2}x$ and $2y + 5x - 2 = 0$
- **f** $3y + 5x = 9$ and $5y - 3x + 6 = 0$

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19 Find the equations of the lines through $(^-2, 5)$ parallel and perpendicular to $y = \frac{3}{5}x - 4$.

See Example 18

20 Find the equations of the lines through $(5, {}^-4)$ parallel and perpendicular to $y = 4x + 7$.

21 Find the equations of the lines through $(^-2, 5)$ parallel and perpendicular to $3x + 2y + 5 = 0$.

22 Find the equations of the lines through $(^-1, {}^-5)$ parallel and perpendicular to $x - 4y + 3 = 0$.

Worked solutions
Exercise 5.2
MAT10NAWS00014

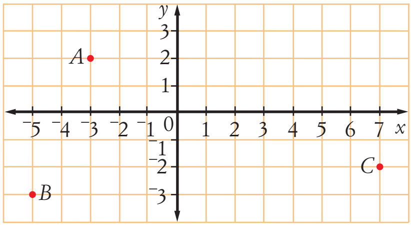
23 Show that $2x + 3y + 8 = 0$ and $6y = 5 - 4x$ are parallel.

24 Show that $4x + 5y - 6 = 0$ and $10x = 8y - 5$ are perpendicular.

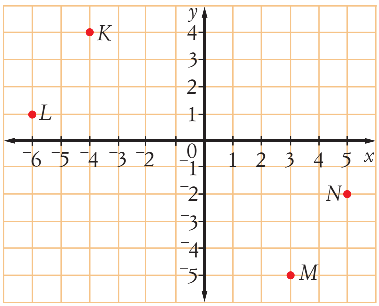
Problem solving

See Example 19

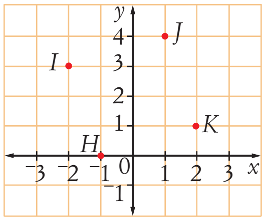
25 Show that $A(^-3, 2)$, $B(^-5, {}^-3)$ and $C(7, {}^-2)$ form a right-angled triangle and find its area.



26 Show that $K(^-4, 4)$, $L(^-6, 1)$, $M(3, {}^-5)$ and $N(5, {}^-2)$ are the vertices of a rectangle and find its area.



27 Show that $H(^-1, 0)$, $I(^-2, 3)$, $J(1, 4)$ and $K(2, 1)$ are the vertices of a square.



28 $J = (^-3, {}^-1)$, $K = (^-1, 0)$ and $L = (1, {}^-4)$. Show that $\angle JKL = 90°$.

29 The volume, V, in litres, of petrol remaining in a portable generator is given by:

$$V = 8 - \frac{3}{2}h$$

where h is the number of hours after the generator starts with a full tank.

a Construct a table of values for the function for $0 \le h \le 6$.

b Use the table to draw a graph of the function.

c What type of function is it?

d Use the graph to find when there is 3 L of petrol left in the generator.

e How many litres of petrol are left in the generator after 2 hours?

f What does the value of the function at $h = 0$ represent?

g What does the value of the function where the graph crosses the h-axis represent?

See Examples 20, 21

30 The cost, C, in dollars, of hiring a clown for a birthday party is given by:

$$C = 50 + 40h$$

where h is the number of hours for which the clown is hired.

a Draw a graph of the function for $0 \leq h \leq 8$.

b What type of function is it?

c Use the graph to find the cost of hiring the clown for 3 hours.

d For how long could the clown be hired with \$200 if the time may be charged in fractions of an hour?

e What does the value 50 in the function $C = 50 + 40h$ represent?

f What does the value 40 in the function $C = 50 + 40h$ represent?

See Examples 20, 21

31 A minibus can carry 20 passengers and a coach can carry 70 passengers. A total of 250 pupils are to be taken on an excursion. Write the number of minibuses used as x and the number of coaches as y.

a How many pupils can be carried on x minibuses and y coaches?

b Write an equation for the number of minibuses and the number of coaches that could be used to carry 250 pupils.

c Draw the graph for this equation.

d What are the exact values of x and y that can be used for 250 pupils?

e Are any other possibilities suggested by the graph?

f On the same graph, draw lines to show the equations for 150 pupils and 400 pupils.

g What do you notice about the graphs?

See Examples 20, 21

32 A coffee wholesaler spends a maximum of \$20 000 a week on coffee. He can buy coffee from Africa for \$15/kg and coffee from Papua New Guinea for \$9/kg.

a Write an equation for the number of kilograms of coffee from Africa (a) and the number of kilograms of coffee from Papua New Guinea (p) he can buy for \$20 000.

b Construct a table of values for a and p and use it to draw a graph of the equation.

c If the supply of Papua New Guinea coffee one week is restricted to 800 kg, use the graph to work out how much African coffee the wholesaler can buy.

See Examples 20, 21

33 A movie theatre is filled to capacity with 600 people. When the movie finishes, people start leaving at the rate of 80 people per minute.

a Express P, the number of people in the theatre, as a function of t, the number of minutes after the movie ends.

b Draw a graph of the function.

c Use the graph to find how long after the movie ends the theatre will still be half-full.

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Reasoning

34 Find the equation of the line that is the perpendicular bisector of the segment joining the points $K(6, 10)$ and $L(^{-}18, 4)$.

35 The heights and weights of a group of people who are considered to be close to the 'ideal' weight for their height are measured and recorded as follows.

See Example 22

Height, *h* (cm)	140	147	153	159	163	173	179	182	187	193
Weight, *w* (kg)	47	48	56	55	63	66	68	75	74	80

a Plot the data and draw in the line of best fit—the 'ideal' height-for-weight line.
b Use the graph to interpolate the 'ideal' weight for a person who is 170 cm tall.
c Use the graph to extrapolate the 'ideal' height of a person who weighs 85 kg.
d Find the equation of the line of best fit.
e Use the equation of the line of best fit to check the answers you found in parts **b** and **c**.

36 The air temperature, T, in °C, at various heights above sea level is recorded as follows.

See Example 22

Height, *h* (m)	0	1400	2500	4400	6000	6900	9400
Air temperature, *T* (°C)	15	9	$^{-}7$	$^{-}14$	$^{-}20$	$^{-}32$	$^{-}43$

a Plot the data and draw in the line of best fit.
b Use the graph to find the likely air temperature at 5000 m above sea level.
c Use the graph to find the likely height above sea level if the air temperature is $^{-}55$°C.
d Find the equation of the line of best fit.
e Use the equation of the line of best fit to find the likely air temperature at 8000 m above sea level.
f Use the equation of the line of best fit to find the likely height above sea level if the air temperature is $^{-}60$°C.

5.3 Modelling curves

TLF Learning object
Grapher (L3531)
MAT10NAIN00005

So far, the work of this chapter has focused on linear functions and their equations. You will now look at some non-linear functions. You have previously looked at a class of functions called **quadratics**. **Quadratic functions** have the form $y = ax^2 + bx + c$ and the graph of a quadratic function is called a **parabola**.

When you draw the graph of a linear function, a minimum of two points is required. When drawing the graph of a non-linear function, more points are required in order to represent it on the Cartesian plane.

Investigate: Graphing quadratics

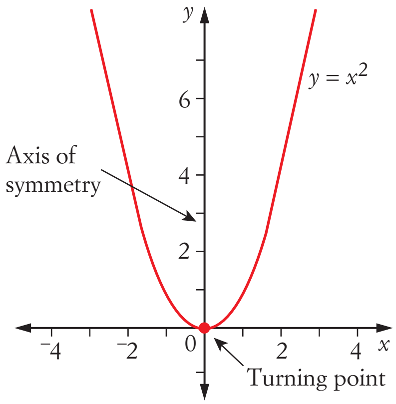
1 Copy and complete the table of values for $y = x^2$.

x	$^-3$	$^-2$	$^-1$	$^-0.5$	$^-0.25$	0	0.25	0.5	1	2	3
y											

2 Draw x- and y-axes on a sheet of grid paper. Use the scale 1 cm : 1 unit for both axes and label the x-axis from $^-4$ to 4 and the y-axis from $^-1$ to 10.
3 Plot each of the points from the table of values onto the grid paper.
4 Join the points with a smooth curve.
5 Describe the shape of the graph.

In the previous investigation you drew a basic parabola – the graph of the function $y = x^2$. The main features of this parabola are described below.

- It is symmetrical and the y-axis is the axis of symmetry. The left and right sides are mirror images of each other.
- The point (0, 0) is called the **turning point** or **vertex** of this parabola.
- The 'U' shape of this parabola is described as being **concave upwards**.



Technology worksheet
Excel worksheet: Investigating parabolas 1
MAT10NACT00010

Important!

The graph of the parabola $y = ax^2$

For the graph of $y = ax^2$, the value of a (the coefficient of x^2) determines whether the parabola is:

- 'wide' or 'narrow'
- concave upwards or downwards.

As the size of a increases, the graph of the parabola becomes 'narrower' and as the size of a decreases, the graph 'widens'. When $a > 0$, the parabola is concave upwards and when $a < 0$, the parabola is concave downwards (it is reflected in the x-axis). The parabolas below illustrate this.

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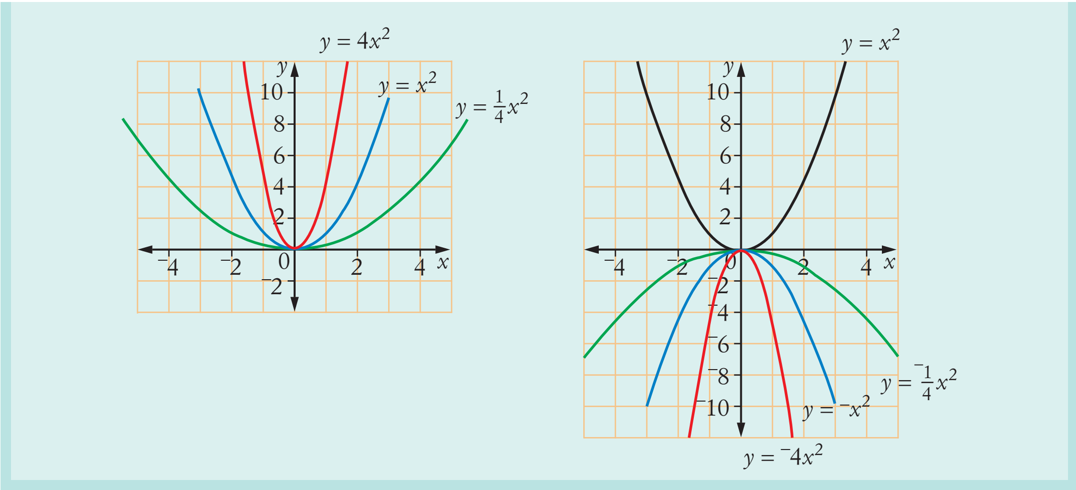
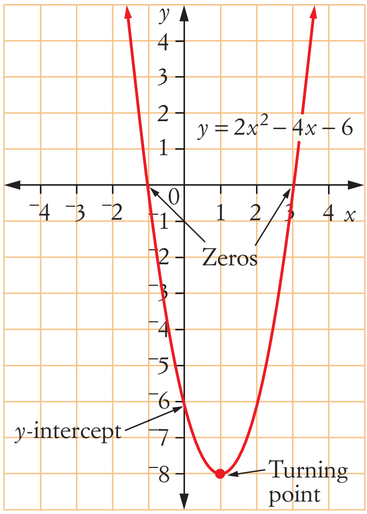


The general form of a quadratic function is $y = ax^2 + bx + c$. The graph of the quadratic $y = 2x^2 - 4x + 6$ shown here can be used to illustrate the main features of the graphs of quadratic functions.



- Parabolas are symmetrical.
- The **zeros** ${^-}1$ and 3 are where the graph crosses the x-axis. The zeros are also known as the **roots** of the graph. Not all parabolas have zeros. The root is the value of x that makes $y = 0$.
- The **y-intercept** $(0, {^-}6)$ is where the graph crosses the y-axis. All parabolas have a y-intercept.
- The **turning point** or **vertex** of this parabola is a **minimum** at $(1, {^-}8)$. The parabola is translated 1 to the right and 8 down from $y = x^2$.
- The turning point of a parabola must be a **minimum** or a **maximum**. A minimum is the point where the gradient of the parabola changes from negative to positive. A maximum is the point where the gradient of the parabola changes from positive to negative.

TLF Learning object
Eaglecat: Parabola (L10095)
MAT10NAIN00005

Weblink
Parabola
MAT10NAWB00005

TLF Learning object
Graphing functions (R11253)
MAT10NAIN00005

Example 23

Draw the graph of $y = x^2 + 2x + 6$ for ${^-}3 \le x \le 2$ and identify its main features.

Solution

Draw up a table of values for ${^-}3 \le x \le 2$ and calculate the corresponding y values.

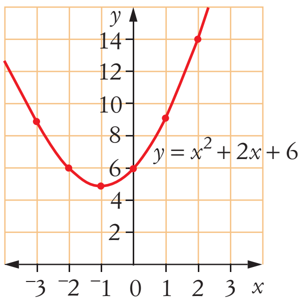
$$y = x^2 + 2x + 6$$

x	${^-}3$	${^-}2$	${^-}1$	0	1	2
x^2	9	4	1	0	1	4
$+ 2x$	${^-}6$	${^-}4$	${^-}2$	0	2	4
$+ 6$	6	6	6	6	6	6
y	9	6	5	6	9	14

CAS TI-Nspire exercise
Coordinate geometry
MAT10NATI00005

CAS ClassPad exercise
Coordinate geometry
MAT10NACP00005

Plot the points and join them with a smooth curve. Continue the curve so that it is symmetrical.



Look at the graph to identify the main features.

There is a turning point at ($^{-}1$, 5).
The turning point is a minimum.
The y-intercept is (0, 6).

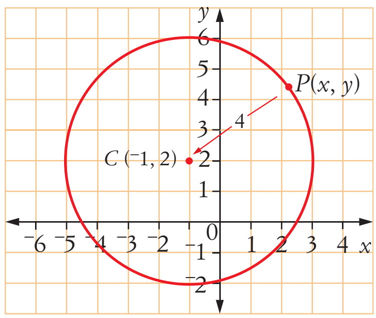
State the result.

The minimum is at ($^{-}1$, 5) and the y-intercept is at (0, 6).

You can also use spreadsheets to investigate non-linear functions such as quadratics.

Circles

A **circle** is a curve in which the distance from the centre to the circumference (the radius) is always the same. Suppose a circle has a radius of 4 and a centre at $C(^{-}1, 2)$, as shown here. If we use $P(x, y)$ for a point on the circumference, we can use the distance formula to find the equation of a circle.



$$PC = \sqrt{(x - {}^{-}1)^2 + (y - 2)^2}$$

But $\quad PC = 4$

So $\sqrt{(x - {}^{-}1)^2 + (y - 2)^2} = 4$

Squaring, we get: $(x + 1)^2 + (y - 2)^2 = 16$

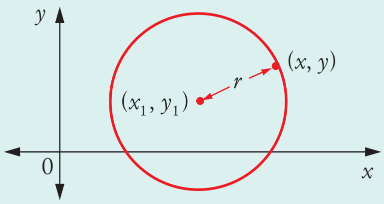
This is the equation of the circle with a radius of 4 and a centre at ($^{-}1$, 2).
We can generalise this to state the equation of any circle.

Important!

Equation of a circle

The **equation of a circle** with centre (x_1, y_1) and radius r is given by:

$$(x - x_1)^2 + (y - y_1)^2 = r^2$$



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Example 24

A circle has centre $(3, {}^{-}2)$ and radius 5.

a Find the equation of the circle.
b Sketch the circle.
c Determine whether point (6, 2) is on the circle.
d Determine whether point $({}^{-}2, 1)$ is on the circle.

Solution

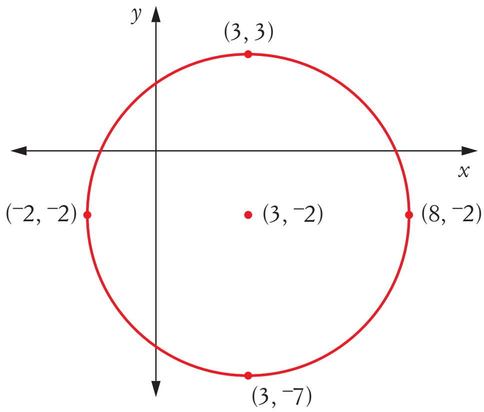
a Write the general equation of a circle. $\quad$ $\mathbf{(x - x_1)^2 + (y - y_1)^2 = r^2}$

Substitute using $(x_1, y_1) = (3, {}^{-}2)$ and $r = 5$. $\quad$ $\mathbf{(x - 3)^2 + (y - {}^{-}2)^2 = 5^2}$

Simplify. $\quad$ $\mathbf{(x - 3)^2 + (y + 2)^2 = 25}$

b Work out the coordinates needed. $\quad$ With radius 5 the circle will reach down to $(3, {}^{-}7)$, up to $(3, 3)$, back to $({}^{-}2, {}^{-}2)$ and across to $(8, 2)$

Draw the axes, mark the centre and extreme points and sketch the circle.



c If (6, 2) lies on the circle, $x = 6$ and $y = 2$ will satisfy the equation of the circle.

Write the equation of the circle. $\quad$ $\mathbf{(x - 3)^2 + (y + 2)^2 = 25}$

Substitute using $(x, y) = (6, 2)$ in the LHS. $\quad$ $\mathbf{LHS = (6 - 3)^2 + (2 + 2)^2}$

Evaluate. $\quad$ $\mathbf{= 9 + 16}$
$\mathbf{= 25}$

LHS = RHS, so state the result. $\quad$ **(6, 2) lies on** $\mathbf{(x - 3)^2 + (y + 2)^2 = 25}$**.**

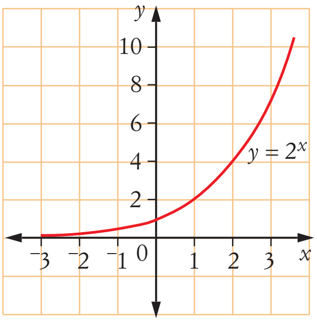
d Write the equation of the circle. $\quad$ $\mathbf{(x - 3)^2 + (y + 2)^2 = 25}$

Substitute using $(x, y) = ({}^{-}2, 1)$ in the LHS. $\quad$ $\mathbf{LHS = ({}^{-}2 - 3)^2 + (1 + 2)^2}$

Evaluate. $\quad$ $\mathbf{= 25 + 9}$
$\mathbf{= 34}$

LHS $\neq$ RHS, so state the result. $\quad$ **(−2, 1) does not lie on the circle.**

Functions that have the general form $y = a^x$ are known as **exponential functions**.
The graph of $y = 2^x$ is shown here. We can use it to list the features of the graph of an exponential function as follows.



- The slope of the graph is always positive.
- The graph intersects the y-axis at $x = 0$.
- There is no axis of symmetry.
- As x gets smaller, y gets closer to the x-axis and the graph becomes flat. The x-axis is an **asymptote**.
- As x gets larger, y gets much larger and the graph becomes very steep.

Example 25

Weblink
The most important video
MAT10NAWB00005

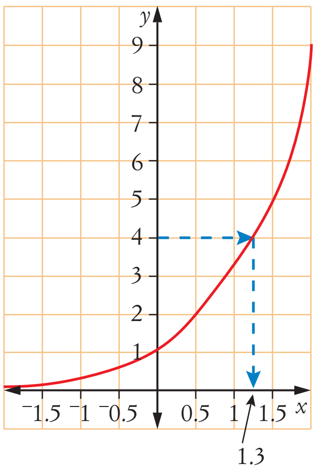
a Draw a graph of the exponential function $y = 3^x$ for $^{-}2 \le x \le 2$.
b Use the graph to estimate, correct to one decimal place, the value of x for which $3^x = 4$.

Solution

a Complete a table of values using your calculator for x between $^{-}2$ and 2.

x	$^{-}2$	$^{-}1$	0	1	2
$y = 3^x$	0.111 . . .	0.333 . . .	1	3	9

Plot the points and join them with a smooth curve.



b Locate the point on the graph where $y = 4$. Find the corresponding x value.

When $y = 4$, $x \approx 1.3$.

State the result.

$3^x = 4$ when $x \approx 1.3$.

It is obvious that the graph of $y = 3^x + 2$ will be the same general shape as the graph of $y = 3^x$ but it will be moved *up* by 2 units, so the asymptote will be $y = 2$ instead of $y = 0$. An exponential function can be sketched by finding a few points and using the general shape.

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Example 26

Animated example
Modelling curves
MAT10NAAE00005

Sketch the graph of $y = 4^x + 1$

Solution

Find the values for $^-1$, 0 and 1 as a guide.

x	$^-1$	0	1
$y = 4^x + 1$	1.25	2	5

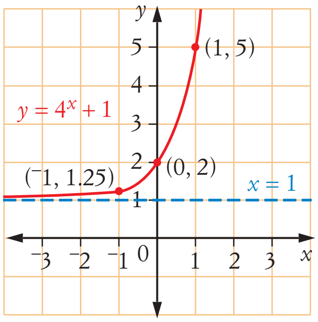
State the asymptote.

The asymptote will be $y = 1$

State the general shape.

The shape will be like, say, $y = 2^x$, but steeper.

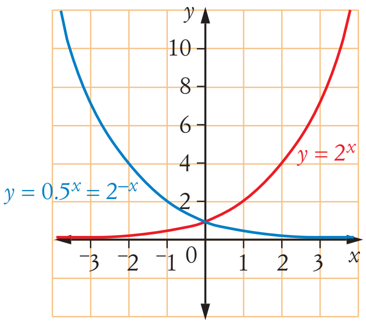
Show the important points, the asymptote and sketch the graph.



Consider the function $y = 0.5^x$. This exponential function is closely related to $y = 2^x$. $0.5 = \frac{1}{2} = 2^{-1}$, so $0.5^x = 2^{-x}$. The values of $y = 0.5^x = 2^{-x}$ are related to those of $y = 2^x$ as shown in the table below. You can check the table on your calculator.

x	$^-3$	$^-2$	$^-1$	0	1	2	3
$y = 2^x$	$\frac{1}{8} = 0.125$	$\frac{1}{4} = 0.25$	$\frac{1}{2} = 0.5$	1	2	4	8
$y = 2^{-x}$	8	4	2	1	$\frac{1}{2} = 0.5$	$\frac{1}{4} = 0.25$	$\frac{1}{8} = 0.125$

Both graphs are shown on the same set of axes below.



The graph of $y = 2^{-x}$ is the reflection of $y = 2^x$ in the y-axis. Its features are shown below.

- The slope of the graph is always negative.
- The graph intersects the y-axis at $x = 0$.
- There is no axis of symmetry.

- As x gets larger, y gets close to the x-axis and the graph becomes flat. The x-axis is an asymptote.
- As x gets smaller, y gets much larger and the graph becomes very steep.

An exponential function with a base less than 1 (a negative exponent) can also be sketched using the general shape and a few points.

Example 27

Sketch the graph of $y = 0.25^x - 2$.

Solution

Find the values for $^{-}1$, 0 and 1 as a guide.

x	$^{-}1$	0	1
$y = 0.25^x - 2$	2	$^{-}1$	$^{-}1.75$

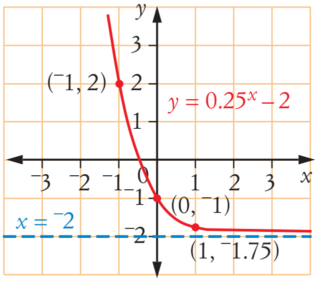
State the asymptote.

The asymptote will be $y = {}^{-}2$

State the general shape.

The shape will be like, say, $y = 2^{-x}$, but steeper.

Show the important points, the asymptote and sketch the graph.



Exercise 5.3 Modelling curves

Your teacher may ask you to use a graphics calculator for some of these questions.

Understanding

Extra questions

Exercise 5.3

MAT10NAEQ00015

1 For each of the following equations, state whether the graph is a straight line, a parabola, a circle, an exponential function or none of these.

a $3x + 2y = 5$ **b** $y = {}^{-}3$ **c** $x^2 + y^2 = 49$

d $x = 17$ **e** $y = 3x^2$ **f** $y = 5 - x^2$

g $y = 5^x$ **h** $x^2 + y^2 = 36$ **i** $y = 4x - 2$

j $y = 2x^2 + 5$ **k** $3x - 2y - 15 = 0$ **l** $y = x^3 + 2x + 12$

m $y = x$ **n** $y = 3^x$ **o** $y = 22 - 4x^2$

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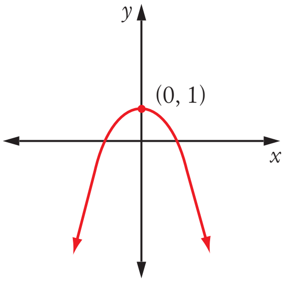
Worksheet
Matching parabolas with their equations
MAT10NAWK00016

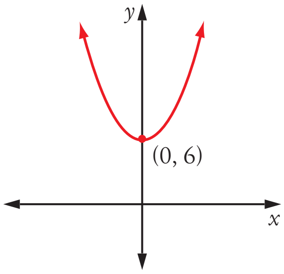
2 Match each of these equations with a parabola from those shown below.

a $y = 1 + x^2$	**b** $y = 10 - x^2$	**c** $y = x^2 - 6$
d $y = {}^{-}10 - x^2$	**e** $y = x^2$	**f** $y = 6 - x^2$
g $y = x^2 + 6$	**h** $y = -x^2 + 1$	**i** $y = x^2 - 10$
j $y = -x^2$	**k** $y = -x^2 - 6$	**l** $y = x^2 + 10$

A



B



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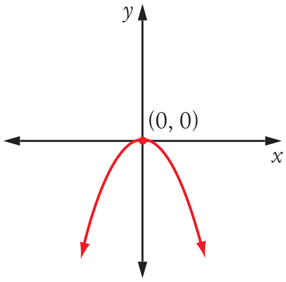
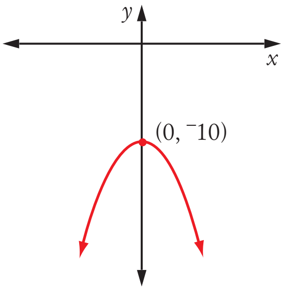
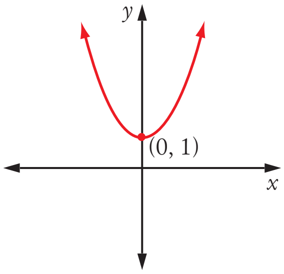


D



E



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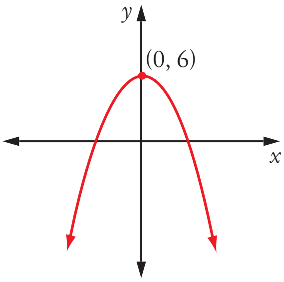

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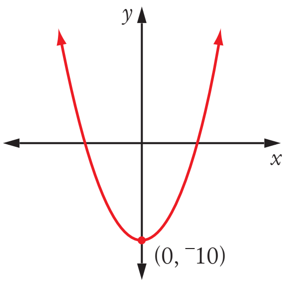

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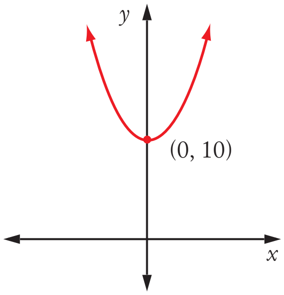

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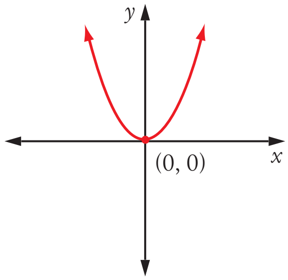

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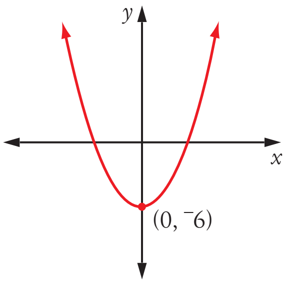
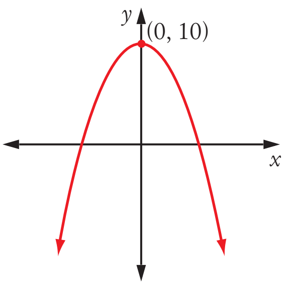
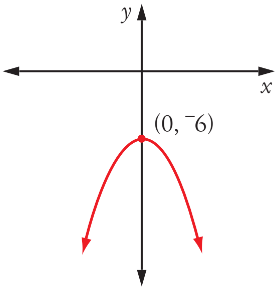


K

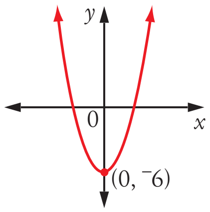


L



3 Match each of these equations with a graph from those shown below.

a $y = 2^x$ **b** $x^2 + y^2 = 9$ **c** $y = 2 - x^2$
d $y = -x$ **e** $y = x^2 + 4$ **f** $x^2 + y^2 = 16$
g $y = 3x^2$ **h** $y = 4^x$ **i** $y = x^2 - 6$

A


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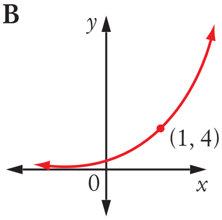

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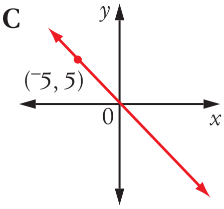

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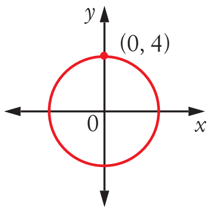

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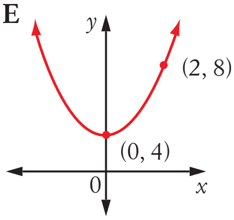

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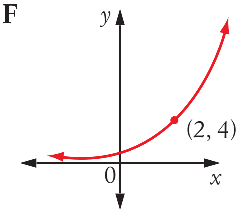

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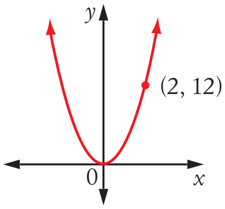

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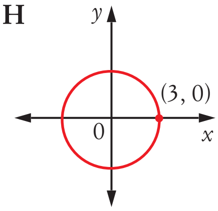
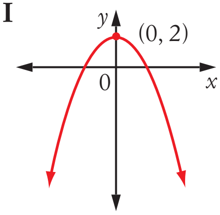


I


4 For each pair of parabolas described below, which one will be 'wider' when compared to the other?

a $y = x^2$ or $y = 4x^2$ **b** $y = -x^2$ or $y = {}^-2x^2$
c $y = x^2$ or $y = \frac{1}{2}x^2$ **d** $y = 2x^2$ or $y = \frac{1}{5}x^2$
e $y = {}^-3x^2$ or $y = \frac{1}{3}x^2$ **f** $y = 0.25x^2$ or $y = 4x^2$

Worked solutions
Exercise 5.3
MAT10NAWS00015

5 **a** Sketch the graphs of the following parabolas on the same set of axes.

i $y = x^2$ **ii** $y = (x + 2)^2$ **iii** $y = (x - 2)^2$

b Describe how the value of a affects the graph of the parabola whose equation is $y = (x + a)^2$.

Fluency

6 **a** Draw graphs of the following quadratics for ${}^-4 \leq x \leq 4$ on the same set of axes.

i $y = x^2$ **ii** $y = x^2 + 3$ **iii** $y = x^2 + {}^-2$

b For each of the parabolas in part **a:**

i what is the equation of the axis of symmetry?
ii what are the coordinates of the vertex?
iii what is the minimum value?

c Describe how the constant a changes the graph of the parabola with the equation $y = x^2 + a$.

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7 Write down the equation of the circle with:

See Example 24

Worked solutions
Exercise 5.3
MAT10NAWS00015

a centre (4, 5) and radius 3
b centre ($^-2$, 4) and radius 8
c centre (3, $^-6$) and radius 5
d centre (0, 0) and radius 9
e centre ($^-3$, $^-5$) and radius 2

8 For circles with the following equations, find:

Worked solutions
Exercise 5.3
MAT10NAWS00015

i the coordinates of the centre
ii the length of the radius

a $x^2 + y^2 = 16$
b $x^2 + y^2 = 1$
c $(x + 2)^2 + (y - 7)^2 = 25$
d $(x + 5)^2 + (y + 3)^2 = 9$
e $(x - 6)^2 + (y + 9)^2 = 121$
f $(x - 4)^2 + (y - 8)^2 = 64$
g $(x + 3)^2 + (y + 2)^2 = 100$
h $(x + 10)^2 + (y - 4)^2 = 225$

9 Sketch each of the following circles on suitable axes.

a $x^2 + (y + 4)^2 = 16$
b $(x - 3)^2 + y^2 = 36$
c $(x + 2)^2 + (y - 1)^2 = 81$
d $(x - 3)^2 + (y - 3)^2 = 49$
e $(x + 1)^2 + (y + 5)^2 = 25$
f $(x - 8)^2 + (y + 8)^2 = 64$

10 Consider the exponential function $y = 2^x$.

See Example 25

a Draw the graph of $y = 2^x$ for $^-3 \leq x \leq 3$.
b Use the graph to estimate, correct to one decimal place, the value of:
   i 2^0
   ii 2^1
   iii $2^{0.5}$
   iv $2^{1.6}$
c Use the graph to estimate, correct to one decimal place, the value of x if:
   i $2^x = 1$
   ii $2^x = 4$
   iii $2^x = 7$
   iv $2^x = 0.8$

11 Consider the exponential function $y = 3^x$.

a Draw the graph of $y = 3^x$ for $^-2 \leq x \leq 2$.
b Use the graph to estimate, correct to one decimal place, the value of:
   i 3^0
   ii $3^{1.5}$
   iii $3^{0.6}$
   iv $3^{1.8}$
c Use the graph to estimate, correct to one decimal place, the value of x if:
   i $3^x = 3$
   ii $3^x = 5$
   iii $3^x = 0.4$
   iv $3^x = 2.7$

12 Sketch each of the following on suitable axes.

See Example 26

a $y = 3^x - 2$
b $y = 2^x + 3$
c $y = 4^x - 1$
d $y = 2^x - 3$
e $y = 3^x + 4$
f $y = 5^x + 1$

13 Draw a graph of each of the following quadratic functions. For each graph:

i show the zeros (if possible), the y-intercept and the turning point
ii identify the turning point as a maximum or minimum.

a $y = x^2 - 4x - 5$
b $y = -x^2 - x + 12$
c $y = x^2 - 6x + 5$
d $y = 3x^2 + 24x + 36$
e $y = 2x^2 - 14x + 20$
f $y = {}^-2x^2 + 4x + 6$
g $y = -x^2 + 2x - 1$
h $y = x^2 - 6x + 9$
i $y = 2x^2 - 4x + 2$
j $y = 3x^2 + 6x + 21$

14 Each of the parabolas described below has an equation in the form $y = x^2 + c$ or $y = -x^2 + c$. Use the description to write the equation of the parabola in each case.

a it has a vertex at (0, 0) and is concave upwards

b it has a maximum at (0, 0) and is concave downwards

c the axis of symmetry is $x = 0$ and there is a minimum at $y = {}^{-}2$

d it is concave downwards and has a vertex at (0, 3)

e it has a vertex at $(0, \frac{1}{2})$ and is concave upwards

f the y-axis is the axis of symmetry and it has a maximum at $y = \frac{^{-}2}{3}$

g it has a turning point at $(0, {}^{-}6)$ and is concave downwards

See Example 24

15 Which, if any, of the following points lie on each of the circles in question **7**?

$A(4, 8)$ $B(3, {}^{-}2)$ $C(6, 4)$ $D(1, 4)$ $E({}^{-}5, {}^{-}5)$

See Example 26

16 Sketch each of the following on suitable axes.

a $y = 0.2^x - 2$ **b** $y = 0.5^x + 3$ **c** $y = 0.4^x - 1$

d $y = 2^{-x} - 1$ **e** $y = 5^{-x} + 2$ **f** $y = 3^{-x} + 1$

Problem solving

17 An object is dropped from the top of a tall building. The height (h) in metres of the object at any time (t) seconds is given by:

$$h = 105 - 4.8t^2$$

a Draw a graph of the equation for $0 \le t \le 5$.

b How tall is the building?

c How far has the object fallen after 2 seconds?

d How long does it take for the object to hit the ground?

18 The distance (d) a car travels after the brakes are applied at various speeds (s) is given by:

$$d = \frac{3s^2}{500}$$

where d is in metres and s is in km/h. Draw a graph of the equation for $0 \le s \le 100$ and use it to answer the following questions.

a How far does the car travel after the brakes are applied at a speed of 38 km/h?

b How far does the car travel after the brakes are applied at a speed of 75 km/h?

c What was the speed of the car if it took 30 m to stop after the brakes were applied?

19 The equation $(x - 15)^2 + (y + 8)^2 = 289$ is a circle.

a Is the origin on, inside or outside the circle?

b Is the point (4, 1) on, inside or outside the circle?

c Is the point (1, 4) on, inside or outside the circle?

d How can you easily tell whether a point is inside or outside a given circle?

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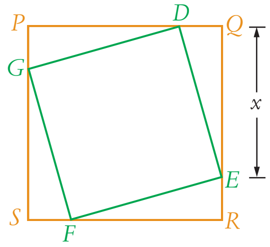
Reasoning

20 A square $DEFG$ is drawn inside a larger square $PQRS$ that has a side length of 1 unit, as shown here.



a If $PD = QE = RF = SG = x$ units, show that $DQ = (1 - x)$ units.

b Show that the area of the smaller square is given by $A = 2x^2 - 2x + 1$

c Draw a graph of this equation.

d Use your graph to find the value of x at which the area of the smaller square is a minimum. What is the minimum area of the smaller square?

21 A group of environmentalists set up a project to track the population size of the possums in a native forest. The size of the population is given by:

$$P = 50 \times 1.06^n$$

where n is the number of months after the project was established.

a What was the population of possums when the project started?

b How many possums were present after 12 months?

c How many possums were present after 30 months?

d Draw a graph of P against n for $0 \leq n \leq 50$.

e How long did it take for the population to reach 500?

22 A population of cockroaches is discovered in a warehouse. The size of the population is given by:

$$C = 200 \times 1.12^n$$

where n is the number of weeks after the cockroaches were discovered.

a How many cockroaches were in the warehouse when they were discovered?

b How many cockroaches were present after 10 weeks?

c How many cockroaches were present after 20 weeks?

d Draw a graph of C against n for $0 \leq n \leq 30$.

e How long did it take for the population to reach 3000?

23 A hobby farmer wants to build a rectangular enclosure for chickens. He has enough material to build an enclosure with a perimeter of 60 m.

a If the length of the enclosure is l and the width is w, complete the expression: $l = \ldots - \ldots$

b Write an equation for A in terms of w.

c Draw a graph of A against w for $0 \leq w \leq 30$.

d What are the dimensions of the enclosure that will contain a maximum area?

Chapter 5 summary

Quiz
Coordinate geometry
MAT10NAQZ00005

- On the **Cartesian plane**, points are identified by their *x*-**coordinate** and *y*-**coordinate.** The *x*-**axis** and *y*-**axis** cross at right angles at the **origin** to form four **quadrants. Cartesian (rectangular) coordinates** are shown in round brackets with the *x*-coordinate first.
- The graph of a function, $f(x)$, is shown by substituting x values into the function and plotting $(x, f(x))$ on the Cartesian plane. A table of values is often used.
- A **linear function** has a graph that is a straight line. The rule for a linear function is called a **linear equation**. In the Cartesian system a linear equation may have an x term, a y term and a number (constant).
- The distance between two points (x_1, y_1) and (x_2, y_2) is given by the **distance formula:**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- The **midpoint**, M, of two points (x_1, y_1) and (x_2, y_2) is given by

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

- The points where the graph of a function crosses the axes are called the **intercepts.** (The term 'intercept' can also refer to the distance from the origin to the point where the graph cuts the axis.)
- The **gradient (slope)** of a straight line has the symbol m and is given by the formula:

  $\text{Gradient} = m = \dfrac{\text{rise}}{\text{run}} = \dfrac{y_2 - y_1}{x_2 - x_1}$ where (x_1, y_1) and (x_2, y_2) are points on the line.
- Upward-sloping (rising) lines have a positive gradient and downward-sloping (falling) lines have a negative gradient. **Parallel lines** have the same gradient.
- The **standard form** of the equation for a straight line is $ax + by + c = 0$. It has no fractions and all the terms are on one side of the equals sign.
- The **gradient–intercept** form is $y = mx + c$, where m is the gradient and c is the y-intercept.
- The equation of a horizontal line through (a, b), parallel to the x-axis, is $y = b$. The equation of a vertical line through (a, b), parallel to the y-axis, is $x = a$.
- The **gradient–point form** of the equation of a line is $y - y_1 = m(x - x_1)$, where m is the gradient and (x_1, y_1) is a point on the line.
- Lines with gradients m_1 and m_2 are **perpendicular** if $m_1 \times m_2 = {}^{-}1$ and are **parallel** if $m_1 = m_2$.
- A **model** is a representation of real-life patterns and relationships. A relationship that can be represented by a function of the form $y = mx + c$ is called a **linear model.**
- For real-life data, we may need to draw a **line of best fit (trendline)** for the plotted points in order to see the relationship.
- A **quadratic function** has the form $y = ax^2 + bx + c$ and forms a graph that has a shape called a **parabola.** A parabola is symmetrical about the **turning point** or **vertex,** which may be a **maximum** or **minimum.** The **zeros** or **roots** of a quadratic are where the graph cuts the x-axis.
- The **equation of a circle** with centre (x_1, y_1) and radius r is

$$(x - x_1)^2 + (y - y_1)^2 = r^2.$$

- Functions that have the general form $y = a^x$ are called **exponential functions**. Exponential functions have a horizontal line that is an **asymptote**. For $a < 1$ (or $a > 1$ and a negative exponent), the function approaches the asymptote as x becomes large and positive. For $a > 1$, the function approaches the asymptote when x is large and negative. The function does not touch the asymptote.

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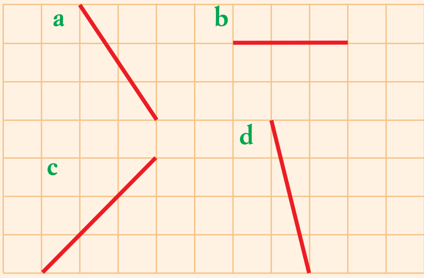
Understanding

1 Plot the points $A(3, {}^-4)$, $B({}^-4, 2)$ and $C({}^-2, {}^-4)$.

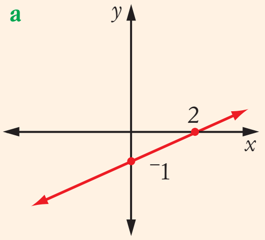
2 Use the graphs on the right to calculate the gradient of each of the segments.

See Example 1

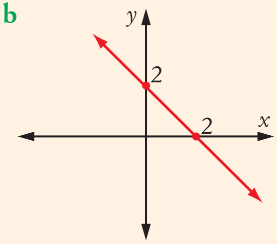


3 The graphs below are not drawn to scale.

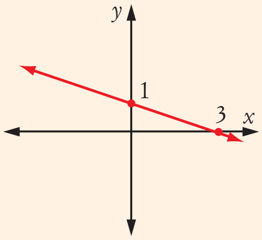
a



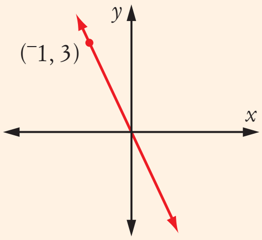
b



c



d



Choose the most appropriate equation for each one from the following list.

i $y = {}^-\frac{1}{3}x + 1$ **ii** $y = {}^-3x$ **iii** $y = \frac{1}{2}x - 1$ **iv** $y = -x + 2$

4 Draw $y = {}^-1$ for ${}^-3 \le x \le 3$.

5 Draw $y = 3x - 4$ for ${}^-2 \le x \le 2$.

6 State whether each of the following is a linear equation.

a $3y = 7 - 5x$ **b** $\frac{x}{5} + 2y = 8$ **c** $y = \frac{3}{x} + 4$ **d** $y = x^2 - 4$

7 Draw the line $y = 2x - 3$ for ${}^-3 \le x \le 3$ and find the intercepts. *See* Example 8

8 Determine whether $(5, 6)$ is on the line $y = 2x - 3$. *See* Example 9

9 Find the equation of the line parallel to the x-axis passing through $({}^-4, 7)$. *See* Example 16

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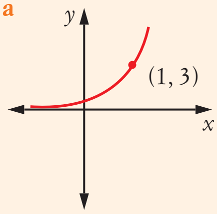
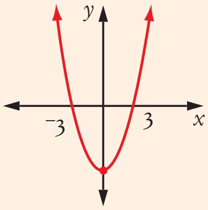
10 The graphs below are not drawn to scale.
Match the equation with the graph for quadratic, circular and exponential functions.

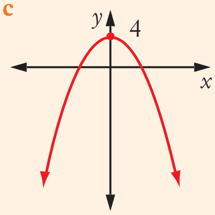
a



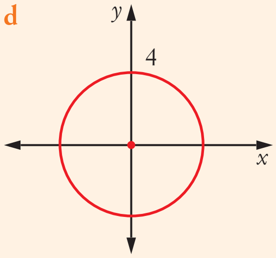
b



c



d



Choose the most appropriate equation for each one from the following list.

i $y = x^2 - 9$ **ii** $y = 3^x$

iii $x^2 + y^2 = 16$ **iv** $y = -x^2 + 4$

Fluency

11 Find the distance between points $(^-2, 4)$ and $(3, ^-8)$.

See Example 2

12 Find the midpoint of the line joining $(^-2, 4)$ and $(3, ^-8)$.

See Example 3

13 Find the slope of the line passing through $(^-2, 4)$ and $(1, ^-2)$.

See Examples 5, 6

14 Find the equation of the line with a gradient of 3 and y-intercept $(0, ^-1)$.

15 Without drawing a graph, find the gradient and y-intercept of the line $x - 2y + 4 = 0$.

See Example 12

16 A line with gradient 3 passes through the point $(^-2, 5)$. Find the equation of the line in gradient–intercept and standard forms.

See Example 13

17 Find the equation of the line passing through $(^-2, 3)$ with gradient $\frac{^-2}{3}$, in standard form.

18 A line passes through the points $(2, ^-1)$ and $(^-4, 5)$. Find the equation of the line in gradient–intercept and standard forms.

See Example 14

19 Without drawing a graph, determine whether $3x - 4y + 2 = 0$ and $8x + 6y + 1 = 0$ are parallel, perpendicular or neither.

See Example 15

20 Find the equation of the line parallel to $y = 3x + 4$, passing through $(^-1, ^-5)$.

See Example 17

21 Find the equations of the lines parallel and perpendicular to $y = 3x + 4$, passing through $(^-1, ^-5)$, in standard form.

22 Find the equation of a circle with radius 4 and centre $(^-1, 3)$.

See Example 24

23 Sketch $(x - 3)^2 + (y + 2)^2 = 9$ on suitable axes.

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24 **a** Sketch the graph of the exponential function $y = 4^x$ for $^{-}2 \le x \le 2$. *See* Example 26

b Use the graph to estimate, correct to one decimal place, the value of $4^{0.8}$.

c Use the graph to estimate, correct to one decimal place, the value of x if $4^x = 6$.

25 Sketch $y = 4^x - 3$ on suitable axes. *See* Example 26

26 **a** Draw the graph of $y = x^2 - x - 6$ and use it to identify the main features of the parabola. *See* Example 23

b Identify the turning point as a maximum or minimum.

27 Sketch $y = 4^{-x} + 2$ on suitable axes. *See* Example 27

Problem solving

28 The midpoint of AB is $(4, {}^{-}1)$. If $A = (6, {}^{-}3)$, find point B. *See* Example 4

29 The speed of a car after passing a checkpoint is given by the function: *See* Examples 20, 21

$$S = 10 + 3T$$

where S is the speed in m/s and T is the time in seconds after passing the checkpoint.

a Draw a graph of the function for $0 \le T \le 10$.

b Use the graph to find when the speed of the car will reach 30 m/s.

c Use the graph to find the speed of the car after 4 seconds.

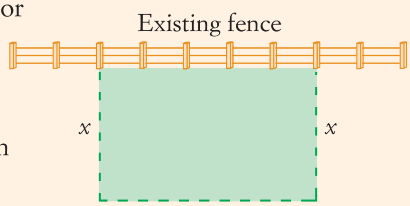
30 The owner of a fast-food outlet has a total of \$200 to spend on hamburger patties and frankfurters for hotdogs each day. Each hamburger patty costs \$1.60, while each frankfurter costs 60c. *See* Example 20, 21

a Write an equation for the number of hamburger patties (h) and frankfurters (f) that the owner could buy.

b Complete a table of values for h and f and use it to draw a graph of the equation.

c If the owner knows that she usually sells 70 hamburgers a day, how many frankfurters should she buy?

31 A hobby farmer wants to fence off a rectangular pen for chickens using an existing fence as one boundary for the pen, as shown. She has only 16 m of fencing material and wants to use it all to form the pen.



a If the side of the chicken pen is x metres long, form an equation for the area, A, of the pen.

b Draw a graph of the area.

c What is the maximum area of the pen?

d Find the dimensions of the pen that give the maximum area.

Reasoning

32 The hearing ability, h, measured in hearing units, of people of various ages, a, was measured as follows. *See* Example 22

Age, *a* (years)	8	24	43	51	62	77
Hearing ability, *h*	96	69	64	49	32	28

a Plot the points and draw a line of best fit for the data.

b Find the equation of the line of best fit.

c Use the equation of the line of best fit to calculate the likely hearing ability of a 30-year-old.

d At what age would you expect a person's hearing ability to reach 20 hearing units?

Statistics and probability

6 Probability

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Contents

Prior learning
Chapter 6
MAT10SPPL00006

Parent guide
Chapter 6
MAT10SPPG00006

Curriculum guide
Chapter 6
MAT10SPCU00006

Australian Curriculum statements

Chance

Describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence. (ACMSP246)

Use the language of 'if … then, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language. (ACMSP247)

Video tutorial
Probability
MAT10SPVT00006

Weblink
Jonny Heeley's masterclass on probability
MAT10SPWB00006

Radioactive materials are an important part of our modern world. They are used in medicine to treat cancer and to trace blood on its journey around the body to treat cardiovascular diseases. The theory of radioactivity uses mathematical probability theory. Indeed, much of modern physics relies on probability to explain the atomic and subatomic theories about matter. While it is certain that uranium will decay to give other elements, it is chance that determines which atoms of the uranium decay. The probabilities are worked out using energy states within the uranium nucleus. Calculating the statistics from the probabilities produces the theory of atomic decay. Probability and statistics are both important to understanding the certainties of physics, as well as their uses in our society as a whole.

Mathematical literacy

Maths dictionary
MAT10ASDI00001

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

and	experimental probability	null set	theoretical probability
assuming	favourable outcome	or	tree diagram
certain	frequency	outcome	trial
complement	given	probability	two-way table
conditional probability	if … then	relative frequency	union
dependent	impossible	sample point	universal set
element	independent	sample space	Venn diagram
event	intersection	simplified tree diagram	
expected frequency	mutually exclusive		

6.1 Probabilities of single events

Technology worksheet
Excel worksheet: Long run proportion
MAT10SPCT00023

You already know how to calculate probabilities from frequencies. You should also know that although you can use probability to predict the proportion of times something will happen *in the long run*, it will not tell you what will definitely happen next. The probability of getting a tail the next time you toss a normal coin is always 0.5. It doesn't matter if you have tossed 20 heads in a row, the probability is still 0.5. While there is natural variation, the greater the number of trials, the closer the theoretical and experimental probabilities will be.

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Investigate: Lotto simulation

Australia, like many countries, has various Lotto games where you choose 4 or more numbers up to 36, 40, 50, 70 or 80. They are marketed as Powerball, Gold Lotto, Lotto Strike, Keno, Oz Lotto, Australian Soccer Pools, Intralot, Super 66, Cash 3 and so on. If some or all of your numbers 'come up', you win a prize. Most people who play these games do so in the hope of winning the major prize or jackpot. The probability of winning the major prize varies with the format but is usually less than 0.000 001.
The game below, *Class Lotto*, only uses ten numbers.

Teacher notes
Lotto simulation
MAT10SPTN00003

CAS TI-Nspire exercise
Probability
MAT10SPTI00006

CAS ClassPad exercise
Probability
MAT10SPCP00006

Rules for *Class Lotto*

1 Each person in the class chooses 4 numbers from 0 to 9 to put on their entry form.
2 You need to put the game number on your form before it is collected.
3 The entry forms are collected and the draw is done.
4 Your teacher will use a ten-sided die or a computer to pick the winning numbers.
5 The winner (if there is one) is the person whose numbers come up.

Class Lotto entry form	Game number: _____
Name: ______________________________	
Circle four numbers:	
	0 1 2 3 4 5 6 7 8 9

- Play the game until someone in the class wins.
- Then keep going until someone wins again.
- Would you need to go this long again until you got another winner?
- Try it out to see how long it takes.
- What is the experimental probability of winning *Class Lotto*?
- Compare the experimental probability with the theoretical probability (your teacher will tell you what it is).

The probability of winning *Class Lotto* is about 5000 to 20 000 times as good as winning any of the commercial games. This means that, in the long run, you would win *Class Lotto* at least 5000 times before you won a commercial lotto game.

Important!

Probability

Experimental probability uses real data to work out chances or likelihoods. Each item of data is called a **trial**. **Theoretical probability** is based on a list that contains all the possible outcomes. The list is called the **sample space**. Every outcome in the sample space is listed exactly as many times as it is possible for it to occur. A single outcome is called an **element** or **sample point**.

An **event** is a collection of outcomes, whether they come from trials or a sample space. This set could have any number of elements, from none up to the total number.

The outcomes in an event for which you are calculating the probability are called **favourable outcomes**. The number of elements in an event is written as n**(event)** or is the **frequency** of the event.

The probability of an event is a number from 0 to 1, written as P**(event)** and worked out using the formulas below, provided that all elements of the sample space have an equal chance of occurring.

$$P(\text{event}) = \frac{\text{Frequency of event}}{\text{Total Frequency}} = \frac{\text{Number of favourable outcomes}}{\text{Number of trials}}$$

$$\text{or } P(\text{event}) = \frac{n(\text{event})}{n(\text{sample space})}$$

Experimental probabilities are also called **relative frequencies**. Probabilities can be written as fractions, decimals or percentages. An event that has a probability of 0 can never happen and is said to be **impossible**. An event with a probability of 1 always happens and is called **certain**.

Probability measures the expected proportion of the time an event will occur. **Expected frequency** is the product of the probability and the number of attempts made in the future.

An event E is composed of elements of the sample space. The **complement** is the event that consists of the rest of the sample space, and is written as E′ or $\overline{\text{E}}$. The sum of the probability of an event and its complement is 1, so

$$\begin{aligned} P(\text{E}) + P(\text{E}') &= 1 \\ P(\text{E}') &= 1 - P(\text{E}) \\ \text{and} \quad P(\text{E}) &= 1 - P(\text{E}') \end{aligned}$$

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Example 1

The masses of 100 g chocolate bars were measured as they came off the production line. The results (in grams) were as follows:

100	99	99	103	100	100	101
102	102	100	101	100	101	101
102	99	99	98	100		

a What is the probability of a bar being underweight?
b How many underweight bars would you expect to get in a carton of 2000 bars?
c How many bars would you need to buy to end up with 30 overweight bars?

Solution

a Check the numbers. — **There are 19 bars, 5 underweight and 8 overweight.**

Write the formula. — **Probablity** $= \dfrac{\text{favourable outcomes}}{\text{trials}}$

Substitute values. — $= \dfrac{5}{19} \approx 0.263$

b Multiply by the number of trials. — **Expected frequency** $= \dfrac{5}{19} \times 2000 \approx 526$

Write the answer. — **There would be about 526 underweight bars from 2000.**

c Write an equation for the number of trials needed. — $30 = \dfrac{8}{19} \times t$

Solve to find t. — $t = \dfrac{30 \times 19}{8} \approx 71$

Write the answer in a sentence. — **You would need about 71 bars to find 30 overweight ones.**

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Example 2

Worksheet
Single event probabilities
MAT10SPWK00018

Peter, Louisa, Jessica, David and Hannah play 'eenie, meenie, minie, mo' to decide which two will go in the front on a theme park ride.

a List the sample space for the front seats.
b What is the probability that two girls will be at the front?
c What is the probability that there will be at least one boy at the front?

Solution

a Use the first letters of their names to list the sample space. It doesn't matter who is first.

Sample space = {PL, PJ, PD, PH, LJ, LD, LH, JD, JH, DH}

b There are 3 combinations with 2 girls: LJ, LH and JH.

$P(\text{two girls}) = \frac{3}{10} = 0.3 = 30\%$

Write the answer.

The probability of two girls in front is 30%.

c 'At least one boy' is the complement of 'two girls'.

$P(\text{at least one boy in front}) = 1 - 0.3 = 0.7$

Write the answer.

The probability of at least one boy being at the front is 70%.

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Investigate: Archery targets

An archery target is divided into 5 colours with a thin line in the middle of each colour. Points are given for arrows that land in each zone as shown in the diagram on the right. The inner-ring in the centre of the yellow circle is only used for tie-breaks and still scores 10. The whole target is 122 cm or 80 cm in diameter.

If you hit the target, what is the probability of scoring 6?

- Assuming the target is 80 cm in diameter, what is the area of the whole target?
- What are the diameter and radius of the 6 point circle?
- What is the area of the 6 point circle?
- What is the probability of scoring 6?
- What are the probabilities of the other scores, assuming that you hit the target?

In the investigation above, there are an infinite number of places where an arrow can land on a target. It is still possible to work out probabilities in situations like this where there are an infinite number of possible events, so you cannot list a sample space. Mathematically defining probability for infinite sets is quite difficult. However, you can still calculate *probability* as a *ratio*. You need to be able to find the fraction of the favourable events to all the events using area or some other method.

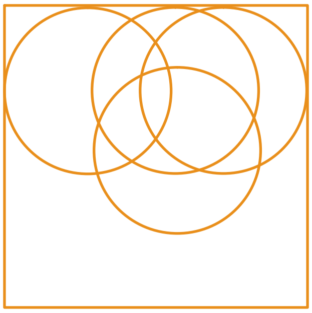
Example 3

Ten-cent coins (diameter 23.6 mm) are thrown onto a small table that has been divided into squares of side length 4 cm. If the coin lands completely inside a square you get 50c back. What is the probability you will win?

Solution

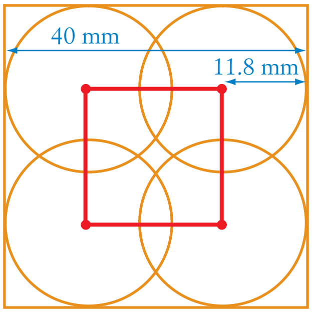
Draw a square and show some places where the coin could land and be completely inside.

It is immediately clear that the centre of the circle is restricted to a much smaller square.



Draw a diagram showing the square where the coin's centre could land.

For the coin to be completely inside the square, the smaller square for the centres must be $40 - 2 \times 11.8$ mm wide.



Calculate the area that the centre of the coin can land in to be completely inside.

$$\textbf{Area inside} = 16.4 \times 16.4 \text{ mm}^2 = 269 \text{ mm}^2$$

Calculate the area of the whole square.

$$\textbf{Area of square} = 40 \times 40 \text{ mm}^2 = 1600 \text{ mm}^2$$

Find the probability of landing completely inside as the ratio of the areas.

$$P(\text{inside}) = \frac{269}{1600} \simeq 0.168$$

Write the answer.

The probability of winning is about 17%.

If coins landed too close to the edge of the table in Example **3**, they would fall off. However, in sideshow alley games like this, it wouldn't make any difference to the probability.

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Exercise 6.1 Probabilities of single events

Fluency

Extra questions
Exercise 6.1
MAT10SPEQ00016
See Example 1

1 The masses in grams of standard-sized letters were measured by the post office and the following data was obtained.

16, 14, 12, 10, 10, 10, 20, 6, 30, 22, 18, 4, 12, 8, 10, 6, 20, 30, 6, 22, 6, 16, 8, 14, 8, 20, 8, 8, 4, 10, 18, 6, 8, 6, 28, 14, 10, 10, 8, 30

a Standard letters over 20 g are too heavy. What is the probability of a letter being overweight?

b From 20 000 standard letters, how many would you expect to be overweight?

2 In June, Brisbane has an average of 8.7, Sydney 12.5, Melbourne 15.4, Adelaide 14.9, Perth 15.1, Darwin 0.6 and Hobart 14.5 rainy days.

a What is the probability of a rainy day in June in each capital?

b If June is typical of the winter, how many rainy days would you expect in Melbourne over winter?

c How many days in Brisbane would you expect to be rain-free in winter?

3 In a particular oyster bed off the Western Australian coast, a diver found 7 pearls in 58 oysters.

a What was the probability of finding a pearl in an oyster?

b What was the probability of not getting a pearl?

c How many pearls would you expect to find in 400 oysters from this bed?

d How many oysters would you have to open to get 400 pearls?

4 A box of 10 dice contains 3 green, 5 red and 2 blue dice.

a What is the probability of choosing a blue die?

b What is the probability of choosing a red die?

5 A sale table has 4 red, 3 green and 8 blue pairs of thongs on it. A customer buys one pair of thongs at random.

a What is the probability that the thongs bought are green?

b What is the probability that they are blue?

6 A bag contains 3 hockey balls, 5 cricket balls and 2 vigoro balls. One ball is taken out at random.

a Which is the most likely ball to be taken out?

b What is the probability of taking out a hockey ball?

c If this was repeated 200 times, how many times would expect to get a vigoro ball?

7 There are 52 cards in a normal pack. The deck of cards is shuffled and then 'Cut' by selecting one card, which is then replaced.

a What is the probability of cutting a heart?

b What is n(jack)?

c What is P(jack)?

d What is the probability of cutting a picture card?

e What is the probability of cutting an ace?

f What is the probability of cutting a number card?

g If the cards were cut 60 times, how many court cards (K, Q, J) would you expect to get?

8 Classify each of the following events according to whether its probability is 0, nearly 0, 1, nearly 1, or in between.

a Getting a royal routine (A-K-Q-J-10 of the same suit) in poker

b You scoring at least 60% in Maths

c An Australian winning the world sailing championship

d Australia beating the West Indies in cricket

e You living until at least your 20th birthday

f You living until at least your 120th birthday

See Example 2

9 Esmeralda has 3 sun-caps (red, yellow and blue) and 2 colours of zinc cream (pink and yellow) for her face.

a Show the sample space of combinations of caps and cream.

b What is the probability that Esmeralda has a matching sun-cap and zinc cream, if she puts on both at random?

See Example 3

10 A game has a table divided into 5 cm squares for throwing Australian 20 cent coins. You win if you land the coin completely inside a square. Australian 20 cent coins are 28.5 mm in diameter. What is the probability of winning?

Problem solving

11 An archer kept records of his performance. He scored 10 for a bulls-eye and smaller scores for the other circles. The results of his target practice were:

7, 8, 6, 7, 4, 8, 4, 7, 10, 7, 5, 1, 6, 7, 10, 5, 10, 8, 9, 9
6, 8, 7, 8, 8, 3, 6, 7, 0, 7, 8, 10, 6, 10, 7, 7, 8, 7, 4, 5
4, 7, 9, 7, 7, 7, 6, 7, 5, 6, 10, 10, 7, 7, 8, 7, 5, 7, 10, 3
7, 6, 4, 6, 1, 10, 10, 8, 1, 8, 9, 6, 2, 8, 8, 6, 10, 5, 7, 4
8, 5, 5, 7, 8, 8, 10, 9, 7, 10, 6, 8, 2, 7, 9, 10, 6, 8, 8, 7

a What was the probability of getting a bulls-eye?

b How many bulls-eyes would he expect from 45 arrows?

c How many arrows would he have to shoot to get 25 bulls-eyes?

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Worked solutions
Exercise 6.1
MAT10SPWS00016

12 Insurance companies use records of deaths to work out suitable rates for life insurance. The table on the right shows the numbers of deaths from 50 000 men and 50 000 women at certain ages in 1 year.

Age at death	Number of deaths	
	Males	Females
20	32	13
30	41	19
40	67	39
50	161	94
60	348	205

Source: From ABS mortality tables

a What is the probability that a 30-year-old man will die in the next year?

b From 53 000 40-year-old men, how many would you expect to die in the next year?

c Work out a recommended premium for \$10 000 life insurance coverage for a 50-year-old man for 1 year, assuming that 40% of the premium is taken by costs.

d Work out the premium for a 60-year-old woman on the same basis.

e A leading insurance company quotes a premium of \$23.44 per month on the internet for \$200 000 cover on a 40-year-old female non-smoker. Use the probability of death from the table to decide whether this is a reasonable premium.

f Another insurer quotes \$79.05 per fortnight on an internet calculator for \$200 000 cover for a 60-year-old male non-smoking electrician. Use the probability of death from the table to decide whether this is a reasonable premium.

13 A drawer contains 3 green socks and 2 yellow socks. Two socks are taken out at random.

a List the sample space using the abbreviations Gl, G2, G3, Yl and Y2 for the socks.

b Which event is more likely: getting matching-colour socks or unmatched socks?

c What is the probability of getting 2 green socks?

d What is the probability of getting 2 yellow socks?

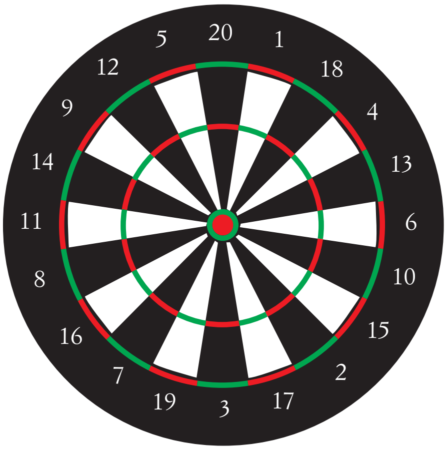
Worked solutions
Exercise 6.1
MAT10SPWS00016

14 A dartboard is 451 mm in diameter, and the scoring area has a diameter of 340 mm. The inner bull has a diameter of 12.7 mm and the outer bull a diameter of 31.8 mm. The outside of the treble ring has a diameter of 214 mm and the double and treble rings are both of width 9 mm. Beginners generally struggle to consistently hit the dartboard. Assuming that you hit the board calculate as a percentage correct to two decimal places the probability of:

a scoring

b hitting the treble ring

c scoring double 3



Reasoning

15 In question **10**, explain why it doesn't matter how big the table is. What about if you could have another go if your coin fell off? Explain your answer.

Worked solutions
Exercise 6.1
MAT10SPWS00016

6.2 Probabilities of combined events

Many probability questions are concerned with combined events. The simplest ways of considering combined events are by using **two-way tables**, **Venn diagrams** and **tree diagrams**.

Important!

Use of probability diagrams

You will find two-way tables most useful for situations having two dimensions. Situations with two variables are of this kind.
Tree diagrams are most useful for situations in stages. If one part occurs before another, or you can consider it like that, then a tree diagram is a useful way to visualise the problem.
You would obviously use Venn diagrams when a situation can be thought of in terms of combinations of sets. They are particularly useful for situations involving 'and/or' combinations.

Example 4

TLF Learning object
Performance of diagnostic tests (R12187)
MAT10SPIN00006

The information in the table below measured reaction times and the time (in hours) students said they spent watching TV on school nights.

	Reaction time (seconds)		
TV hours	0.2–0.29	0.3–0.39	0.4+
0–5	37	102	65
6–15	29	80	57
> 15	11	29	24

a What is the probability that a student has a reaction time under 0.3 seconds?
b What is the probability that a student who watches more than 15 hours of TV on school nights has a reaction time under 0.3 seconds?
c What is the probability that a student who watches less than 6 hours of TV on school nights has a reaction time under 0.3 seconds?
d Compare the probabilities of reaction times under 0.3 seconds of students who watch less than 6 hours of TV on school nights with those who watch more than 15 hours of TV on school nights.

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Solution

a Work out the total number. — **Number of trials** $= 434$

Find the number less than 0.3 s. — $n(< 0.3 \text{ s}) = 77$

Find the probability. — $P(< 0.3\text{s}) = \frac{77}{434} = \frac{11}{62} \approx 0.1774$

b Write the number of trials. — $n(\text{watch more than 15 hrs of TV}) = 64$

Find the probability. — $P(< 0.3\text{s with more than 15 hrs}) = \frac{11}{64} \approx 0.1719$

c Write the number of trials. — $n(\text{watch less than 6 hrs of TV}) = 204$

Find the probability. — $P(< 0.3\text{s with less than 6 hrs}) = \frac{37}{204} \approx 0.1814$

d 17% and 18% are nearly the same. — **There is little difference between the reaction times under 0.3 seconds of students who watch less than 6 hours and those who watch more than 15 hours of TV on school nights.**

Two-way tables are also useful in theoretical probability.

Example 5

Puzzle sheet
Dice probability
MAT10SPPS00017

Tetrahedral dice have four faces numbered 1, 2, 3 and 4. Two such dice are rolled and the numbers they land on are added together.

a What is the probability of getting a total of 6?

b How many totals of 6 would you expect from 130 rolls?

Solution

a Make a two-way table, and work out the totals.

There are $n = 16$ totals in the sample space.

	1	**2**	**3**	**4**
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

Calculate the probability. — $P(6) = \frac{n(6)}{n} = \frac{3}{16} = 0.1875$

b Multiply by the number of trials. — **Expected number** $= 0.1875 \times 130$

Work out the answer. — $= 24.275$

Write the answer. — **There would be about 24 totals of 6 from 130 rolls.**

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For events involving removal of objects from a group, you need to decide whether or not the situation involves **replacement**. Tree diagrams where there is no replacement have fewer branches at each stage. Dealing cards is a simple example that can be considered as one with no replacement. Those involving replacement have the same number of branches at each node.

Example 6

Puzzle sheet

Combined events: Tree diagrams

MAT10SPPS00015

A bag contains 2 red and 3 blue marbles. One marble is taken out at random, and then another, without replacement.

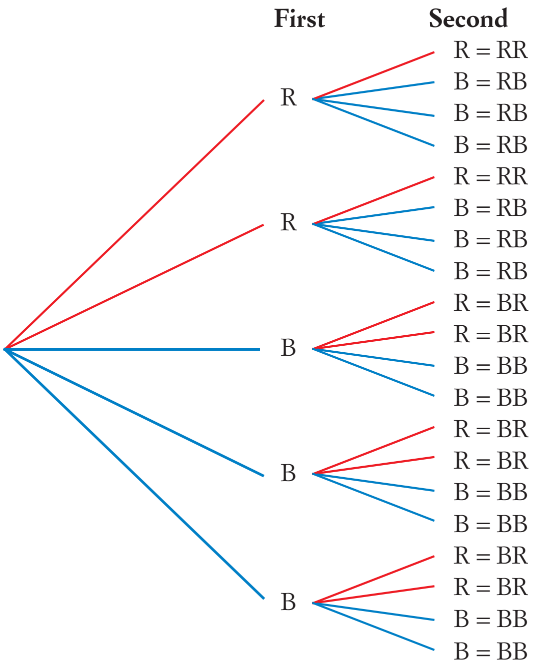
a Use a tree diagram to show the sample space.

b Find the probability that the 2 marbles taken out are different colours.

Solution

a Use the branches to show the results for the marbles. Since the first marble is not replaced, there are only 4 branches for the second marble. In each case, the colour of the first marble makes a difference to the possibilities for the second.

There are 20 elements in the sample space.



b Write the formula.
$$P(\text{different}) = \frac{n(\text{different})}{n(\text{sample space})}$$

Substitute values, including RB *and* BR.
$$= \frac{12}{20} = \frac{3}{5} = 0.6$$

Write the answer.
The probability of getting different coloured marbles is 0.6 $\left(\frac{3}{5}\text{ or } 60\%\right)$.

Even if the marbles were drawn simultaneously, Example **6** would be done in exactly the same way because they could be considered to be drawn in order.

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Tree diagrams can be extended to cope with three-stage events.

Example 7

Video tutorial
Tree diagrams
MAT10SPVT10013

Three coins are tossed together.

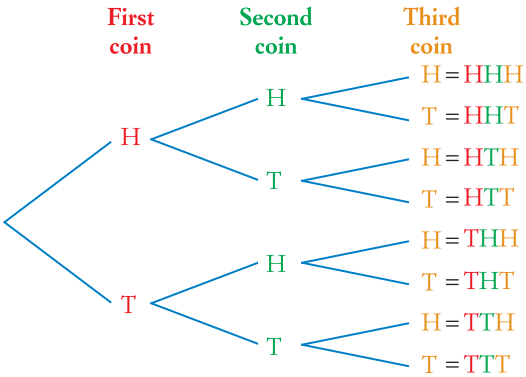
a Use a tree diagram to show the sample space.

b What is the probability of getting at least 2 heads?

Solution

a It makes no difference whether the coins are tossed at the same time or one after the other. You can draw the tree diagram as if they are tossed in succession.

There are 8 elements in the sample space.



b Write the formula.

$$P(\text{at least 2 heads}) = \frac{n(\text{at least 2 heads})}{n(\text{sample space})}$$

Include 2 heads and 3 heads.

$$= \frac{4}{8} = \frac{1}{2} = 0.5$$

When people say '2 heads' in situations like Example **7**, they usually mean 'exactly 2 heads'. This is different from 'at least 2 heads', and different again from '2 heads in a row'. You can see from the tree diagram that $P(\text{exactly 2 heads}) = \frac{3}{8}$ and $P(\text{2 heads in a row}) = \frac{1}{4}$. It is not really necessary to show the elements of the sample space on the end of the tree diagram, so the HHH, HHT, HTH, etc. can be omitted on the end of the tree diagram.

When there are more than a few possibilities, you will find the size of a tree diagram becomes quite large. You can combine branches, provided you keep track of the number of branches at each node.

Important!

Simplified tree diagrams

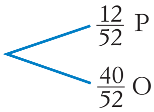
A **simplified tree diagram** shows repeated branches on a normal tree diagram as single branches. You write a fraction in the combined branch. The numerator shows the number of repeated branches that have been combined. The denominator shows the original number of branches before combination.

Example 8

Three cards are dealt from a normal 52-card pack after it is shuffled. Use a simplified tree diagram to find the probability that they are all picture cards (jacks, queens or kings).

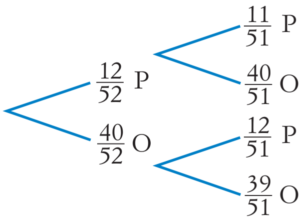
Solution

For the first card, combine the 12 picture card branches and the 40 other card branches from the 52 cards at the node.



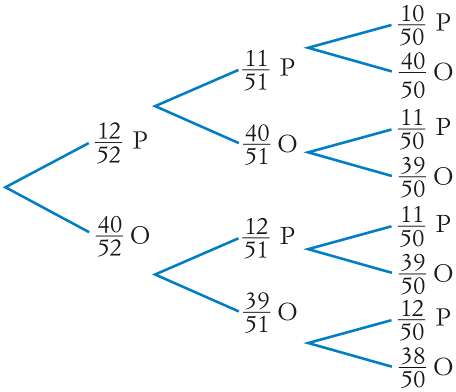
This is a non-replacement question, so the second card is chosen from 51 cards.

- On the picture branch, 11 are picture cards and there are still 40 non-pictures.
- On the other branch there are still 12 picture cards but only 39 others.



There are 50 cards left to pick from.

- On the first node there are 10 picture cards and 40 others left.
- On the second node there are 11 picture cards and 39 others left.
- On the third node there are also 11 picture cards and 39 others left.
- On the last node there are 12 picture cards and 38 others left.



You want the P-P-P combined branches. These really have 12 × 11 × 10 branches from the total of 52 × 51 × 50 branches altogether in an unsimplified diagram.

$$P(\text{3 pictures}) = \frac{12 \times 11 \times 10}{52 \times 51 \times 50}$$

Work out the answer.

$$= \frac{11}{1105} \approx .009955$$

Round and write the answer.

The probability of getting 3 picture cards is just under 1%.

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It makes a large difference whether the cards are replaced or not. In Example **8**, the probabilities in the second stage are affected by the result of the first stage, but in Example **7** the first stage does not affect the second.

Important!

Dependent and independent events

Two events are **independent** if the probability of one is *not* affected by the occurrence of the other.
Dependent events are not **independent**.

Investigate: Babies' names

In NSW, the probability that a newborn baby will be named Michael is obviously independent of the probability that a newborn in Queensland whose parents are not known to the parents of the NSW couple will be named Michael. Suppose that on a particular day 25 babies are born at a NSW hospital and 23 are born in a Queensland hospital. Three of the NSW babies are named Michael and 2 of the Queensland babies are named Michael. What is the probability that if one baby from each of NSW and Queensland is chosen at random they will both be named Michael?

- In how many ways can you choose a baby named Michael from Queensland?
- For each of the ways that a baby named Michael is chosen from Queensland, how many ways can a baby named Michael be chosen from NSW?
- How many ways altogether does this mean that babies named Michael can be chosen from both Queensland and NSW?
- For each of the ways that a baby named Michael is chosen from Queensland, how many ways can a baby *not* named Michael be chosen from NSW?
- How many ways altogether does this mean that a baby named Michael can be chosen from Queensland and one not named Michael can be chosen from NSW?
- In how many ways can you choose a baby *not* named Michael from Queensland?
- For each of the ways that a baby *not* named Michael is chosen from Queensland, how many ways can a baby named Michael be chosen from NSW?
- How many ways altogether does this mean that a baby *not* named Michael can be chosen from Queensland and one named Michael can be chosen from NSW?
- For each of the ways that a baby *not* named Michael is chosen from Queensland, how many ways can a baby *not* named Michael be chosen from NSW?
- How many ways altogether does this mean that a baby *not* named Michael can be chosen from Queensland and one *not* named Michael can be chosen from NSW?

Teacher notes
Qld v. NSW
MAT10SPTN00004

Puzzle sheet
Combined events: Two-way tables
MAT10SPPS00014

Puzzle sheet
Conditional probability: Two-way tables
MAT10SPPS00018

- Complete the two-way table below.

		New South Wales		
		Michael	Not Michael	Total
Queensland	Michael			
	Not Michael			
	Totals			575

Work out the following.

- The probability of a baby named Michael being chosen from Queensland.
- The probability of a baby named Michael being chosen from NSW.
- The probability of a baby named Michael being chosen from Queensland and a baby named Michael being chosen from NSW.
- The product of the probability of a baby named Michael being chosen from Queensland and the probability of a baby named Michael being chosen from NSW.

What do you find?

From the investigation above, it is clear that the probability of a combination of independent events is easily calculated from the individual probabilities.

Important!

Probability of a combination of independent events

The probability of two independent events A and B both occurring is the product of their individual probabilities.
For *independent* events A and B, $P(\text{A and B}) = P(\text{A}) \times P(\text{B})$.

Example 9

Two dice are rolled together. What is the probability that they both land on 1?

Solution

The results for the dice are independent.

$$\boldsymbol{P}(\textbf{1st is 1 and 2nd is 1}) = \boldsymbol{P}(\textbf{1st is 1}) \times \boldsymbol{P}(\textbf{2nd is 1})$$

Substitute the values.

$$= \frac{1}{6} \times \frac{1}{6}$$

Calculate the answer.

$$= \frac{1}{36}$$

The commonly used English words '**and**' and '**or**' are often used as if they mean the same thing. In mathematics the difference between them is very important.

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Important!

The meanings of 'and' and 'or'

In mathematics, '**… and …**' means *both*, but '**… or …** ' means *at least one*.
'and' corresponds to the **intersection** of sets, while 'or' corresponds to the **union** of sets.
The intersection of two sets A and B, $A \cap B$, is the set that contains the elements in both A and B.
The union of two sets A and B, $A \cup B$, is the set that contains the elements in at least one of A or B.

Example 10

Animated example
Analysing data
MAT10SPAE00006

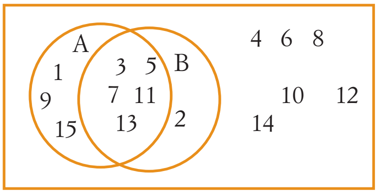
Video tutorial
Venn diagrams
MAT10SPVT10014

a Draw a Venn diagram to show the numbers from 1 to 15 inclusive. Show set A = {odd numbers} and B = {prime numbers}. Use your diagram to find the following probabilities for a number from 1 to 15.

b It is odd and prime. **c** It is odd or prime. **d** It is even or prime.

Solution

a Show all the numbers in the universal set (the numbers from 1 to 15) in the rectangle. Put the numbers in the appropriate parts of the Venn diagram. Remember that 2 is the first prime number.



b Write the formula using set notation.

$$P(\textbf{odd and prime}) = \frac{n(A \cap B)}{n(U)}$$

Substitute values (there are 5 in the intersection).

$$= \frac{5}{15} = \frac{1}{3}$$

c Write the formula.

$$P(\textbf{odd or prime}) = \frac{n(A \cup B)}{n(U)}$$

Substitute values.

$$= \frac{9}{15} = \frac{3}{5}$$

d Write the formula.

$$P(\textbf{even or prime}) = \frac{n(\overline{A} \cup B)}{n(U)}$$

Substitute values.

$$= \frac{12}{15} = \frac{4}{5}$$

In the example above, the universal set contained all the numbers from 1 to 15. A *universal set* is the set of all elements under consideration, denoted by U.
The link between and, or, intersection and union allows you to work out a formula for combinations of probabilities.

Important!

Probability of a combination involving and/or

The probability of the combined event (A or B) is given by the formula

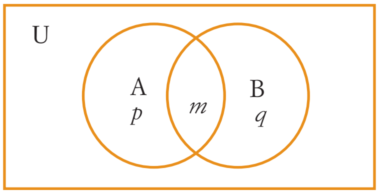
$$P(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A and B}).$$

Technology: Geogebra:
Probability and Venn diagrams
MAT10SPTC00006

Puzzle sheet
And/or problems
MAT10SPPS00016

This is easily shown with a Venn diagram of events A and B with a sample space (universal set U) having u members.

Suppose there are m elements in $A \cap B$, p elements in A that are not in B and q elements that are in B but not in A. Show all the numbers in the diagram.



Obviously $n(A \cup B) = p + m + q$

Then

$$P(\text{A or B}) = \frac{n(A \cup B)}{n(U)} = \frac{m + p + q}{u}$$

Now add 0 to the top line in the form $m - m$.

$$= \frac{m + p + q + m - m}{u}$$

$$= \frac{(m + p) + (q + m) - m}{u}$$

$$= \frac{n(A) + n(B) - n(A \cap B)}{u}$$

$$= \frac{n(A)}{u} + \frac{n(B)}{u} - \frac{n(A \cap B)}{u}$$

But $\frac{n(A)}{u} = P(A)$, $\frac{n(B)}{u} = P(B)$ and $\frac{n(A \cap B)}{u} = P(\text{A and B})$, so

$P(\text{A or B}) = P(A) + P(B) - P(\text{A and B})$ **QED**

Example 11

When they were children, 20% of adults had measles, 30% had chickenpox and 12% had both. What is the probability that an adult chosen at random had childhood measles or chickenpox?

Solution

Write the formula.	$\boldsymbol{P(\text{A or B}) = P(A) + P(B) - P(\text{A and B})}$
Substitute values.	$\boldsymbol{= 0.2 + 0.3 - 0.12 = 0.38}$
Write the answer.	**The probability is 38%.**

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Exercise 6.2 Probabilities of combined events

Understanding

Extra questions
Exercise 6.2
MAT10SPEQ00017

1 Two normal coins are tossed together. What is the probability that they are both heads?

2 Two weighted coins are tossed together. They have a 60% chance of landing with heads up. What is the probability that they are both heads?

3 What is the probability that 3 normal coins tossed together will all be heads?

See Example 9

4 What is the probability that 4 normal coins tossed together will all be heads?

5 If 3 normal dice are tossed, what is the probability of getting a total of 18?

6 Three dice have the dot for the '1' drilled out and lead inserted so that the chance of getting a six increases to 25%.

a What is the probability that when 2 are tossed you get a double six?

b What is the probability that when all three are tossed the total is 18?

7 A number is chosen at random from the numbers 1 to 50. What is the probability that it is a multiple of 3 or 4? See Example 11

8 A card is chosen at random from a normal pack. What is the probability that it is a number card (not A, K, Q, J) or a heart?

Fluency

9 A normal die and a tetrahedral die numbered 1–4 are rolled together and the numbers they land on are added together.
Use a two-way table to find each of the following probabilities. See Example 5

a The total is 9.

b At least one of the dice lands on 3.

c The total is 9 and one of the dice lands on 3.

d The total is 9 or one of the dice lands on 3.

e The total is 6.

f One of the dice lands on a number less than 3.

g The total is 6 and one of the dice lands on a number less than 3.

h The total is 6 or one of the dice lands on a number less than 3.

10 A bag contains 2 yellow and 3 black marbles. Two marbles are drawn at random, with replacement. See Example 6

a Use a tree diagram to show the sample space.

Find the probability that the marbles are:

b both yellow c both black d different colours.

11 Repeat question **10** without replacement.

12 Two red, two blue and two yellow marbles are placed in a jar. Two marbles are then chosen at random.

a If the first is replaced before the second is chosen, what is the probability that:

i they will be the same colour?

ii they will be different colours?

iii at least one marble will be yellow?

b If there is no replacement, what is the probability that:
  i they will be the same colour?
  ii they will be different colours?
  iii at least one marble will be blue?

See Example 10

13 Draw a Venn diagram to show the numbers from 1 to 30. Let set A = {multiples of 3} and set B = {even numbers}.
Use your diagram to find the following probabilities for a number from 1 to 30.
a It is even and divisible by 3.
b It is a multiple of 3 or even.
c It is a multiple of 3 or even but not both.

14 a Draw a Venn diagram to show the names of the days of the week and months of the year. Let A = {Names containing the letter ‘t’}, B = {Names starting with ‘m’} and C = {Names with at least two vowels}.
Use your diagram to find the following probabilities for the name of a day of the week or month of the year.
b It has at least two vowels and a ‘t’.
c It has at least two vowels or a ‘t’.
d It starts with ‘m’ and has a ‘t’
e It starts with ‘m’ or has a ‘t’.
f It starts with ‘m’ and has at least two vowels.
g It starts with ‘m’ or has at least two vowels.
h It does not have ‘t’ or start with ‘m’
i It does not have at least two vowels or start with ‘m’.
j It does not have at least two vowels or a ‘t’.

See Example 7

15 A family is selected at random from families having 3 children.
a Use a tree diagram to show the sample space for the genders of the children.
b What is the probability that 2 of the children are girls?
c What is the probability that at least 2 of the children are girls?
d What is the probability that all the children are boys?

See Example 8

16 A caterer prepares 4 salad, 3 ham and 5 vegemite sandwiches for mixed plates. A guest takes 2 sandwiches at random from a plate no-one else has touched yet.
a Use a simplified tree diagram to show the sample space.
b What is the probability that they are both ham sandwiches?
c What is the probability that they are both salad sandwiches?
d What is the probability that they are both vegemite sandwiches?

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e What is the probability that one is ham and the other is a salad sandwich?
f What is the probability that one is ham and the other is a vegemite sandwich?
g What is the probability that one is vegemite and the other is a salad sandwich?
h What is the probability that they are both the same kind of sandwich?
i What is the probability that they are different kinds of sandwich?

17 A card is cut from a normal 52-card pack and the suit is noted. The card is replaced and the process is twice repeated.
a Draw a simplified tree diagram to show the sample space for spades.
b Find the probability that all 3 cards are spades.
c Find the probability of getting at most 1 spade.

18 A multiple-choice test has 4 questions, each having 5 responses. Only one of the responses is correct in each question.
a Draw a simplified tree diagram to show correct and incorrect responses.
b Find the probability of getting all 4 questions correct by chance.
c Find the probability of getting at least 2 questions correct by chance.
d If 50 people did the test, how many would you expect to score over 50% by chance?

19 A university English literature library has 15 000 books: 2000 of them are about Shakespeare's works, 5000 of them are about plays and 1300 are about Shakespeare's plays. A book is selected at random by a silverfish.
a What is the probability it is about Shakespeare's plays?
b What is the probability it is about Shakespeare or plays?
c What is the probability that it is about neither Shakespeare nor plays?

20 A collection of 'silver' coins consists of three 20-cent, ten 10-cent, five 5-cent and twelve 50-cent coins. Two coins are chosen at random.
a Use a simplified tree diagram to show the sample space.
b What is the probability of getting two coins the same?
c What is the probability of getting different coins?
d What is the probability of getting a 50-cent coin?
e What is the probability of getting a 20-cent coin?
f What is the probability of getting a 50-cent and a 20-cent coin?
g What is the probability of getting a 20-cent coin or a 50 cent coin?
h What is the probability of getting a 20-cent coin or a 50 cent coin, but not both?
i What is the probability of ending up with more than 25 cents?
j What is the probability of ending up with less than 25 cents?

Problem solving

21 A card is cut from a normal pack and the suit is noted. The card is replaced and the procedure is repeated.
a What is the probability that both cards will be the same suit?
b What is the probability that both cards will be clubs?
c What is the probability that the cards will be different suits?
d What is the probability that at least one card will be a spade?
e From 400 such trials, how many times would you expect to get the same suit?

See Example 4

22 The table below shows the results of a survey of Year 10 students about smoking and colds.

		Number of colds last year					
		0	**1**	**2**	**3**	**4**	**Total**
Smoking	**Smokers**	0	3	5	6	2	16
	Non-smokers	4	10	8	4	1	27
	Total	4	13	13	10	3	43

a What is the probability that a smoker had more than 2 colds?

b What is the probability that a non-smoker had more than 2 colds?

c Compare the probabilities that smokers and non-smokers have many colds.

23 The table below shows the numbers of accidents in which drivers of different age groups were involved.

		Accidents in last 2 years					
		1	**2**	**3**	**4**	**5**	**Total**
Age group	**17–21**	16	24	21	14	12	87
	22–26	16	14	12	9	9	60
	27–31	9	7	7	4	2	29
	32–41	4	2	2	0	0	8
	Over 41	4	2	0	0	0	6
	Total	49	49	42	27	23	190

a What is the probability that a driver under 27 had exactly one accident in the last 2 years?

b What is the probability that a driver aged 27 or more had exactly one accident in the last 2 years?

c Compare the probabilities of younger and older drivers having accidents.

Worked solutions
Exercise 6.2
MAT10SPWS00017

24 The table below shows the age-specific deaths of the Indigenous (ATSI) and non-indigenous (non-ATSI) populations of Australia in 2008.

%s	**0**	**1–14**	**15–24**	**25–34**	**35–44**	**45–54**	**55–64**	**65+ years**	**Total**
ATSI	105	47	124	170	266	366	405	848	2331
Non-ATSI	792	310	782	1183	2112	4598	8925	80 287	98 989

Source: ABS

In the following, assume that the 2008 figures apply to all years.

a What is the percentage probability of an indigenous person living to 65?

b What is the percentage probability of a non-indigenous person living to 65?

c Compare the likelihood of indigenous and non-indigenous people living to age 65 and comment on the result.

25 Two multiple-choice questions have answers labelled **A**, **B**, **C** and **D**. Only one answer is correct in each question.

a Work out the probabilities of getting 0, 1 or 2 questions right by just guessing (say **A** is the answer to both questions).

b From 100 people doing the test, how many would you expect to guess each of 0, 1 or 2 questions correctly?

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26 What is the probability that a number made up of the digits 4, 7, 8 and 3, each used once only, is more than 4500?

Worked solutions
Exercise 6.2
MAT10SPWS00017

27 Two cards are dealt from a well-shuffled pack.

a What is the probability that both cards will be clubs?

b What is the probability that both cards will be the same suit?

c What is the probability that the cards will be different suits?

d What is the probability that at least one card will be a spade?

e From 400 such trials, how many times would you expect to get the same suit?

28 David and Peter regularly play tennis. Over a long period, David has won 40% of the games they play. What is his probability of winning at least 2 of the next three games they play?

29 In the game of poker, a flush is all 5 cards of the same suit.

a What is the chance of being dealt a flush in hearts?

b What is the probability of being dealt a flush?

30 Twenty people are trapped in a lift: 8 have blue eyes and 9 have brown eyes; 5 have blond hair and 7 have black hair; 1 person has red hair and the rest are brunettes. The red-haired person is the only person with green eyes, there are 2 blue-eyed blonds and 2 hazel-eyed brunettes. Three black-haired people have blue eyes. One person is chosen at random to climb up the shaft to get help. Find each of the following probabilities.

Worked solutions
Exercise 6.2
MAT10SPWS00017

a It is someone with blue eyes and blond hair.

b It is someone with blue eyes or blond hair.

c It is someone with black hair and brown eyes.

d It is someone with black hair or brown eyes.

e It is someone with brunette hair or brown eyes.

Reasoning

31 A stack of timber at a yard has planks of lengths 3.6, 3.9, 4.2, 4.5, 4.8 and 5.1 m, with ends facing outwards. There are 10 planks of each length. Find the probability of selecting 3 lengths greater than 4.5 m long by chance and explain your reasoning.

32 There are 20 bags of sugar on a shelf and 4 of them have breaks in the bags, so the sugar leaks. Explain how to find the probability of getting 3 good bags by chance.

33 In a production line, the rotor, bearings and case are put together to make an electric motor. The rotor incorporates the commutator and the case has the field coils. The tolerance of manufacture of the parts means that the probability that the rotor will not fit the case properly is 3%, the probability that the bearings will not fit the case properly is 2% and that the rotor will not fit the bearings properly is 1%. The motor will still work if any of these things happen, but it will be a 'lemon' and shake when it runs and will wear out very quickly. Explain why the probability of getting a lemon is not 6%.

6.3 Conditional probability

Weblink
Conditional probability and independent events
MAT10SPWB00006

Investigate: Probability of two events both occurring

You can use simplified tree diagrams to find the probability of two particular items selected from a group of items, whether the items are replaced or not. Consider the case of two marbles being chosen from a collection of 3 red and 2 blue marbles. This can be done with or without replacement.

- Use a tree diagram to calculate the probability of the first marble being red and the second marble being blue if the first is replaced before the second is drawn.
- Use a tree diagram to calculate the probability of the first marble being red and the second marble being blue if the first is *not* replaced before the second is drawn.
- What is the probability of the first marble being red?
- With replacement, what is the probability of the second marble being blue?
- Compare this with the probability of the first marble being blue.
- Without replacement, what is the probability of the second marble being blue?
- How is this different from the probability of the first marble being blue?

In many practical situations, you will know that events are *not* independent. It is important to be able to deal with situations where we have dependent events. **Conditional probability** allows you to calculate probabilities for dependent events.

Important!

Conditional probability

The conditional probability of event A given event B, written as $P(A|B)$, is the probability that A occurs, assuming that B occurs. In $P(A|B)$, the sample space is restricted to event B. The words **given**, **assuming**, **knowing**, **if … then**, **provided**, are often used in conditional probability.

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Example 12

In draw poker, you are dealt 5 cards and get to throw some away and get new ones before you see who has the highest hand. The discards are usually shuffled back into the pack before the new cards are dealt out to the players who discarded. If you are dealt 4 hearts and a spade and discard the spade, what is the probability of getting another heart to make a flush?

Solution

How many cards are left in the restricted sample space?

There are 48 unknown cards that could be in the reshuffled pack.

How many of the remaining cards are favourable?

There are 9 hearts left in the unknown cards.

Find the conditional probability.

$P(\text{5th heart}|\text{4 hearts}) = \frac{9}{48} = \frac{3}{16} = 0.1875$

Write the answer.

There is an 18.75% chance of getting a flush.

Example 13

The table below shows the results of a survey of the times taken by Year 8 students to get to school and the transport they used. What is the probability that a student who walks takes less than 15 minutes to get to school?

	0–4 min	5–9 min	10–14 min	15–19 min	20–29 min	30–39 min	40–59 min	60+ min
Bike	0	0	2	1	0	0	0	0
Bus	2	1	7	4	16	22	14	5
Car	11	25	13	10	11	6	4	1
Other	0	1	0	1	1	1	0	2
Train/tram	0	0	1	2	1	2	6	0
Walk	11	5	3	2	5	1	0	0

Solution

What is the conditional probability?

We want $P(\text{less than 15}|\text{walk})$.

Focus on the important line of the table.

	0–4 min	5–9 min	10–14 min	15–19 min	20–29 min	30–39 min	40–59 min	60+ min
Bike	0	0	2	1	0	0	0	0
Bus	2	1	7	4	16	22	14	5
Car	11	25	13	10	11	6	4	1
Other	0	1	0	1	1	1	0	2
Train/tram	0	0	1	2	1	2	6	0
Walk	11	5	3	2	5	1	0	0

What is the restricted sample space?	**Walking = 11 + 5 + 3 + 2 + 5 + 1 = 27**
How many *of these* take less than 15 minutes?	**n(under 15) = 11 + 5 + 3 = 19**
Find the probability.	**P(under 15 min|walking) $= \frac{19}{27} \approx 0.7037$**
Write the answer.	**A Year 10 student who walks to school has a probability of about 70% of getting there in under 15 minutes.**

Common errors when calculating conditional probabilities include: doing the problem backwards, not restricting the sample space correctly and using all of event A instead of just the part that is inside event B for the calculation of $P(A|B)$.

Example 14

A student incorrectly used the table in Example **13** and found that the probability that someone who travelled by bus took at least an hour to get to school was 0.625. What is the correct answer and what did the student do incorrectly?

Solution

What is the conditional probability?	**We want P(60 + |bus).**
What is the restricted sample space?	**Bus = 2 + 1 + 7 + 4 + 16 + 22 + 14 + 5 = 71**
How many *of these* take 60+ minutes?	**n(60+) = 5**
Find the probability.	**P(60+|bus) $= \frac{5}{71} \approx 0.07042$**
Write 0.625 as a fraction.	**$0.625 = \frac{5}{8}$**
What did the student do?	**The student used 60+ as the sample space.**
Write the answer.	**The correct probability is about 0.07; the student actually found the probability that someone who took at least an hour travelled by bus.**

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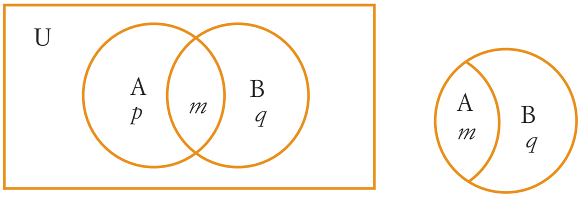

Full sample space Restricted sample space

As shown in Example **14**, it is very important when calculating conditional probabilities to be clear about the restricted sample space and what part of this is favourable.

Conditional probability is easily related to the probabilities of A, B and A ∩ B using a Venn diagram. Using the same setup we used previously, assume that a sample space has u elements, there are m elements in A ∩ B, p elements in A that are not in B and q elements that are in B but not in A. The Venn diagram of this situation is shown above.

For this situation, $P(\text{A}|\text{B})$ can be viewed as a probability with the sample space restricted to the set B, since we are assuming that B occurs. This is shown on the right of the main Venn diagram above. The number of elements in the restricted set A is now m, so $P(\text{A}|\text{B}) = \dfrac{m}{m+q}$.

We can multiply this by 1 in the form $1 = \dfrac{u}{u}$ to give the following.

$$\begin{aligned}
P(\text{A}|\text{B}) &= \frac{m}{m+q} \\
&= \frac{m}{m+q} \times \frac{u}{u} \\
&= \frac{mu}{u(m+q)} \\
&= \frac{m}{u} \times \frac{u}{m+q} \\
&= \frac{m}{u} \div \frac{m+q}{u}
\end{aligned}$$

But $P(\text{A and B}) = \dfrac{m}{u}$ and $P(\text{B}) = \dfrac{m+q}{u}$.

This means that we have $P(\text{A}|\text{B}) = P(\text{A} \cap \text{B}) \div P(\text{B})$ or $P(\text{A}|\text{B}) = \dfrac{P(\text{A and B})}{P(\text{B})}$.

It doesn't make any difference whether you consider $P(\text{A}|\text{B})$ or $P(\text{B}|\text{A})$, the analysis is still the same, so $P(\text{B}|\text{A}) = \dfrac{P(\text{A and B})}{P(\text{A})}$. This also leads to a mathematical definition of independence, since you can rewrite the formula as

$$P(\text{A and B}) = P(\text{A}|\text{B}) \times P(\text{B}) \text{ or } P(\text{A and B}) = P(\text{B}|\text{A}) \times P(\text{A})$$
.

Important!

Conditional probability and independence

The events A and B are independent if and only if $P(\text{A}|\text{B}) = P(\text{A})$. It follows that $P(\text{B}|\text{A}) = P(\text{B})$.
For dependent events A and B, the conditional probability of A given B is given by $P(\text{A}|\text{B}) = \dfrac{P(\text{A and B})}{P(\text{B})}$. This is also expressed as $P(\text{A and B}) = P(\text{A}|\text{B}) \times P(\text{B})$ for practical purposes.

Conditional probability can be used instead of tree diagrams to deal with many problems. When you want the probability of two events both occurring, you can use the conditional probability formula for $P(\text{A and B})$.

Example 15

Video tutorial
Conditional probability
MAT10SPVT10015

What is the probability of being dealt two aces in a row from a shuffled pack of cards?

Solution

Write the desired probability as an 'and' statement.

$P(\text{2 aces}) = P(\text{2nd card is an ace and 1st card is an ace})$

Use the conditional probability formula.

$= P(\text{2nd card is an ace}|\text{1st card an an ace}) \times P(\text{1st is an ace})$

There are 4 aces in a pack of 52 cards.

$P(\text{1st card is an ace}) = \frac{4}{52} = \frac{1}{13}$

After an ace is taken out there are 3 aces left out of 51 cards.

$P(\text{2nd card is an ace}|\text{1st card is an ace}) = \frac{3}{51} = \frac{1}{17}$

Use the formula.

$P(\text{2 aces}) = \frac{1}{13} \times \frac{1}{17} = \frac{1}{221} \approx 0.004\,525$

Write the answer.

The probability of being dealt 2 aces in a row from a normal pack of cards is about 0.45%.

Exercise 6.3 Conditional probability

Understanding

Extra questions
Exercise 6.3
MAT10SPEQ00018
See Example 12

1 Two normal dice are thrown and the total is calculated.
  a Given that the total is 7, what is the probability that one of the dice shows 2?
  b Given that one of the dice shows 2, what is the probability that the total is 7?
  c Given that the total is 8, what is the probability of a double?
  d Given a double, what is the probability of getting a total of 8?
  e Given a double, what is the probability that the total is 10?
  f Given a total of 10, what is the probability of a double?
  g Given that the total is 5, what is the probability that one of the dice shows 3?
  h Given that one of the dice shows 3, what is the probability that the total is 5?

2 A box of coloured crayons has 4 red, 2 black, 4 yellow, 2 blue, 3 green and 5 purple crayons left in it. Students are allowed to use 2 crayons at a time, so a student takes 2 crayons at random.
  a Given that one crayon is black, what is the probability that the other crayon is black?
  b If one crayon is yellow, what is the probability that the other one is purple?
  c Having a red crayon, what is the probability that the other one is also red?
  d Assuming that a crayon is green, what is the probability that the other one is black?

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e What is the probability that the second crayon is yellow if you know that the first one is red?

f Providing that the first crayon is purple, what is the probability that the other one is yellow?

g If one crayon is green, what is the probability that the other one is red?

h If one is red, what is the probability that the other one is green?

i If one is black, what is the probability that the other one is blue?

j If one is purple, what is the probability that the other one is also purple?

3 Instead of taking 2 crayons, the student in question **2** took 3 crayons.

a If 2 of the crayons are green, what is the probability that the other one is yellow?

b If 2 of the crayons are yellow and green, what is the probability that the other one is green?

c If 2 of the crayons are yellow, what is the probability that the other one is also yellow?

d If 2 of the crayons are red, what is the probability that the other one is also red?

e If 2 of the crayons are red and yellow, what is the probability that the other one is yellow?

f If 2 of the crayons are red and yellow, what is the probability that the other one is red?

g If 2 of the crayons are red, what is the probability that the other one is yellow?

h If 2 of the crayons are yellow, what is the probability that the other one is red?

i If one crayon is red, what is the probability that the other two are blue?

j If one crayon is blue, what is the probability that the other two are red?

Fluency

4 In draw poker, find the probability of drawing one card to an inside straight. That is, if you have something like 7, 8, 10, J and discard a 4, what is the probability of drawing a 9 to get a straight?

5 In draw poker, what is the probability of drawing to an outside straight? That is, if you keep 8, 9, 10, J, what is the probability of drawing either a 7 or a queen to get a straight?

6 Using the table from Example **13**, find each of the following probabilities. *See* Example 13

a The probability that a student who went to school by car took less than 5 minutes to get there.

b The probability that a student who took less than 5 minutes to get to school went by car.

c The probability that a student who went by train or tram took at least 40 minutes to get to school.

d The probability that a student who took at least 40 minutes to get to school went by train or tram.

e The probability that a student who took the bus took less than 10 minutes to get to school.

f The probability that a student who took less than 10 minutes to get to school went by bus.

g The probability that a student who walked to school took less than 15 minutes to get there.

h The probability that a student who took less than 15 minutes to get to school walked there.

i The probability that a student who caught the bus took at least half an hour to get to school.

j The probability that a student who took at least half an hour to get to school went by bus.

Problem solving

Worked solutions
Exercise 6.3
MAT10SPWS00018

7 You are dealt a draw poker hand of A A K 7 3. You could try for 2 pairs by keeping the aces and king to get A A K K x or A A K x x. You could also try for 3 aces (A A A x y) by throwing the king as well. Which is more likely?

Worked solutions
Exercise 6.3
MAT10SPWS00018

8 The table below shows the results of a large survey of students' eye and hair colours.

		Hair colour			
		Black	Brown	Red	Blond
Eye colour	Brown	68	119	26	7
	Blue	20	74	17	94
	Hazel	15	54	14	10
	Green	5	29	14	16

a Find the probability that someone with black hair has brown eyes.
b Find the probability that someone with blond hair has blue eyes.
c Find the probability that someone with red hair has green eyes.
d Use your results to comment on the stereotypes that people with black hair have brown eyes, blond-haired people have blue eyes and redheads have green eyes.
e People with brown eyes are said to have black hair. Use probability to comment on this stereotype.

Reasoning

Worked solutions
Exercise 6.3
MAT10SPWS00018
See Example 14

9 A student incorrectly worked out that the probability of getting a total of 4 from rolling two dice when one of the dice showed a 3 was $\frac{3}{11}$. Find the correct answer and explain what the student did wrong.

10 Using the table in question **8**, a student worked out that the probability of someone with brown hair having blue eyes was $\frac{74}{205}$. Another student said it was $\frac{37}{291}$ and a third said it was $\frac{74}{119}$. The teacher said they were all wrong. Find the correct answer and explain their errors.

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Chapter 6 summary

Quiz
Probability
MAT10SPQZ00006

- **Experimental probability** uses real data to work out probability. Each item of data is called a **trial**.
- **Theoretical probability** is based on a list containing all the possible outcomes called the **sample space**. Every outcome in the sample space is listed exactly as many times as it occurs. A single outcome is called an **element** or **sample point**.
- An **event** is a collection of outcomes, whether they come from trials or a sample space. The outcomes in an event for which you are calculating the probability are called **favourable outcomes**. The number of elements in an event is written as n**(event)** and is the **frequency** of the outcome.
- The probability of an event is a number from 0 to 1, written as P**(event)** and worked out using the formulas below.

$$P(\text{event}) = \frac{\text{Frequency of event}}{\text{Total frequency}} = \frac{\text{Number of favourable outcomes}}{\text{Number of trials}}$$

$$\text{or } P(\text{event}) = \frac{n(\text{event})}{n(\text{sample space})}$$

- Experimental probabilities are also called **relative frequencies**.
- Probabilities can be written as fractions, decimals or percentages. An event that has a probability of 0 can never happen and is said to be **impossible**. An event with a probability of 1 always happens and is called **certain**.
- **Expected frequency** is the product of the probability and the number of future attempts.
- For any event E, the **complement** of the event is shown as E′ or $\overline{\text{E}}$. The sum of the probabilities of an event and its complement is 1.

$$\begin{aligned} P(\text{E}) + P(\text{E}') &= 1 \\ \text{so } P(\text{E}') &= 1 - P(\text{E}) \\ \text{and } P(\text{E}) &= 1 - P(\text{E}') \end{aligned}$$

- **Two-way tables** are most useful for situations with two variables. **Tree diagrams** are most useful for events in stages. **Venn diagrams** are most useful for combinations of sets, particularly and/or situations.
- A **simplified tree diagram** shows repeated branches on a normal tree diagram as single branches, with a fraction showing the number of branches that have been combined, out of the total number at that join.
- Two events are **independent** if the probability of one is *not* affected by the occurrence of the other. For independent events A and B, $P(\text{A and B}) = P(\text{A}) \times P(\text{B})$.
- Dependent events are not **independent**.
- The **intersection** of two sets A and B, $A \cap B$, is the set that contains the elements in both A and B.
- The **union** of two sets A and B, $A \cup B$, is the set that contains the elements in at least one of A or B.

Chapter 6 summary

- In mathematics, '**... and ...**' means *both*, but '**... or ...** ' means *at least one*. 'and' corresponds to the intersection of sets, while 'or' corresponds to the union.
- The probability of (A or B) and (B or A) are related by

$$P(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A and B}).$$

- The **conditional probability** of event A given event B, written as $P(\text{A}|\text{B})$, is the probability that A occurs, assuming that B occurs. In $P(\text{A}|\text{B})$, the sample space is restricted to event B.
- The words **given**, **assuming**, **knowing**, **if ... then**, **provided**, are often used in conditional probability.
- Events A and B are independent if and only if $P(\text{A}|\text{B}) = P(\text{A})$. It follows that $P(\text{B}|\text{A}) = P(\text{B})$.
- For dependent events A and B, the conditional probability of A given B is given by

$$P(\text{A}|\text{B}) = \frac{P(\text{A and B})}{P(\text{B})}.$$

This is also expressed as

$$P(\text{A and B}) = P(\text{A}|\text{B}) \times P(\text{B})$$

for practical purposes.

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Chapter 6 review

Understanding

See Example 9

1 When a cricket bat is spun up into the air, it has equal chances of landing with the flat side up or down. What is the probability that if a cricket bat is spun up three times, it will land flat side down all three times?

2 40% of the milk sold in a supermarket is ordinary white homogenised milk.
  a Find the probability that two customers buying milk have bought ordinary white milk.
  b What is the probability that both have bought other kinds of milk?
  c What is the probability that one has ordinary milk and the other not?

See Example 12

3 In the card game Pontoon, an ace counts as 1 or 11, picture cards count as 10 and number cards count as their number value. You get your cards 1 at a time, and after the first 2, you can decide whether to keep going or not. You try to get as close to a total of 21 as possible without busting. Busting means going over 21, and if you bust you lose. Pontoon means getting a total of 21 with only 2 cards and you then win automatically, otherwise your total has to be bigger than the dealer's total to win.
  a If your first card is a king, what is the probability of getting pontoon?
  b If your first card is an ace, what is the probability of getting pontoon?
  c If the first two cards you get are a 7 and a 10, what is the probability of getting 20 or 21 on your third card?
  d If the first two cards you are dealt are a 6 and a king, what is the probability of busting on your third card?
  e If the first two cards are both 9s, what is the probability of busting on your third card?
  f What is the largest sum that you can have on the first two cards before the chance of busting on your third card is 50%?

See Example 11

4 A number is chosen from 1 to 25. What is the probability that it is divisible by 2 or 3?

See Example 15

5 There are 3 raspberry, 4 lime and 5 pineapple iceblocks left in the fridge. 2 iceblocks are taken out at random. Find each of the following probabilities for the iceblocks.
  a They are both raspberry.
  b They are both lime.
  c They are both pineapple.
  d They are different flavours.

Fluency

See Example 1

6 Two classes of Year 10 students were asked about the money they received last week. 14 said that they didn't get any, 10 said that they got pocket money, 18 said that they got paid for work and the other 12 said that they got money from something else.
  a What was the probability of a Year 10 student doing paid work?
  b What was the probability of a Year 10 student not working?
  c If you asked another 120 students, how many would you expect to get pocket money?

7 The two classes in question **6** had 24 boys and 30 girls. If you choose a student at random, what is the probability that the student is female?

8 Estimate the probability of each of the following events.
  a Rain in the Simpson desert next week.
  b Rain at the South pole next week
  c Rain somewhere in the world next week.
  d Rain on the West coast of Tasmania next week.

See Example 5

9 Two spinners with equal sectors numbered 1–5 are spun and the numbers they land on are added together. Use a two-way table to find each of the following probabilities.

a The total is 6.
b One of the spinners lands on 4.
c The total is 6 and one of the spinners lands on 4.
d The total is 6 or one of the spinners lands on 4.
e One of the spinners lands on a number less than 3.
f The total is 4.
g One of the spinners lands on a number less than 3 and the total is 4.
h One of the spinners lands on a number less than 3 or the total is 4.

See Example 6

10 A box of fluoro lamps has two 9 W and two 15 W lamps in it. Two lamps are taken out in the dark because both lamps in the room have blown.

a Use a tree diagram to show the sample space.

Find the probabilities of each of the following.

b The lamps are both 9W.
c The lamps are different.

11 Repeat question **10** if the first lamp is replaced before the second is taken out.

See Example 10

12 **a** Draw a Venn diagram to show the years from 2005 to 2025. Let A = {leap years} and B = {Years that include the digit 1}. Use your diagram to find the following probabilities for an arbitrary year.
b It is a leap year.
c It includes the digit 1.
d It is a leap year that includes the digit 1.
e It is a leap year or it includes the digit 1.
f It does not include the digit 1 but is a leap year.

See Example 2

13 Tingting has 2 blue, 2 red and 1 black biros in her pencil case. She takes out 2 biros without looking.

a Show the sample space for biros taken out, using B1 and B2 for blue and just B for black.
b What is the probability that she gets a red and blue biro?

See Example 7

14 If a couple have blood types A and O, then their children can also have blood types A or O. Such a couple have 4 children.

a Use a tree diagram to show the sample space for the blood types of the children.
b What is the probability that 3 of the children have blood type A?
c What is the probability that at least 2 of the children have blood type A?
d What is the probability they are all of type O?

See Example 8

15 A bag contains 3 black, 5 red and 6 green counters. Two counters are shaken from the bag.

a Draw a simplified tree diagram to show the sample space.
b Find the probability that both counters are red.
c Find the probability of getting a red counter and then a black counter.
d Find the probability that the first is red and the second is green.
e Find the probability that one is red and the other is green.
f What is the probability that the counters are different?

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16 The table below shows where Year 10 students said they received money from last week. *See* Example 13

Year 10 money source

	Work	None	Pocket money	Chores	Gift	Other
Female	9	6	6	2	1	20
Male	8	5	4	1	1	3

a What is the probability that a boy received money from work?
b What is the probability that a student who received pocket money was female?
c What is the probability that a student received no money?
d What is the probability that a girl received no money?

17 A draughts board has squares that are 3 cm square and pieces that are 2 cm in diameter. In a game, you throw a draughts piece onto the board but if it goes completely off the board, you do it again. What is the probability that it crosses the line between 2 or more squares? *See* Example 3

18 A shop specialising in sports shoes has 21 kinds of cross-trainers, 15 kinds of sprinting runners, 12 kinds of long distance runners, 16 kinds of soccer boots, 12 kinds of rugby boots, 18 kinds of Aussie Rules boots, 6 kinds of bowling shoes and 60 other kinds of sports shoes. A pair of shoes is chosen at random.
a What is the probability that they are football boots?
b What is the probability that they are suitable for running or bowling?

19 A Year 10 class has 10 Australian-born students, 7 from Asia, 4 from Europe and 5 from Africa. Two students are chosen at random to represent the class.
a Use a simplified tree diagram to show the sample space.
b What is the probability that the students are both Asians?
c What is the probability that they are from different continents?
d What is the probability that they are European and African?
e What is the probability that they are European or African?

Problem solving

20 Students in a typical Year 10 class said that they woke up at these times yesterday morning.
7:30, 7:00, 6:30, 5:30, 7:30, 7:00, 7:30, 8:00, 6:30, 7:30, 7:00, 7:30, 7:00, 6:30, 8:00, 8:30, 7:30, 7:30, 7:00, 7:30, 6:00, 6:30, 6:30, 7:00, 7:00, 7:30, 6:30, 6:30, 8:00, 8:00
One student is chosen at random.
a What is the probability that the student woke up at 7:00 a.m.?
b From 200 Year 10 students, how many would you expect to get up at 8:00 a.m.?
c How many students would you need to ask to find 40 who got up before 7:00 a.m.?

21 The table below shows the heights and errors in spelling of a group of 16-year-olds.

		Errors in spelling				
		1	2	3	4	5
Height (cm)	150–159	0	1	3	5	1
	160–169	8	6	9	2	9
	170–179	11	2	6	7	9
	180–189	2	3	2	2	2
	Over 189	2	0	0	1	1

a What is the probability that a student shorter than 170 cm made more than 2 errors?
b What is the probability that a student at least 170 cm tall made more than 2 errors?
c Compare the probability of spelling errors between shorter and taller students.

22 A box of car fuses has four 10-amp and two 5-amp fuses left in it. Two fuses are shaken out of the box.
a List the sample space for the two fuses taken out.
b What is the probability that they will both be the same current rating?

23 A test has 3 true-false questions and 2 multiple-choice questions with options **A**, **B**, **C**, **D** and **E**. Only one answer is correct for each question.
a What is the probability of guessing all 5 questions correctly?
b What is the probability of guessing 2 of the true-false questions and 1 of the multiple-choice questions correctly?
c What is the probability of guessing 1 of the true-false and both multiple-choice questions correctly?
d What is the probability of getting them all wrong?

24 What is the probability that a 3-digit number using the digits 4, 3 and 2 is greater than 320?

25 The table below shows the results of a large survey of Year 10 students who received money in the fifth week of Term 2.

Year 10 money and source

	Work	Pocket money	Chores	Gift	Other
\$1–\$9	1	2	2	0	4
\$10–\$19	1	11	3	0	1
\$20–\$49	10	13	6	3	8
\$50–\$99	24	10	2	2	4
\$100–\$199	20	3	0	3	1
\$200+	11	1	0	0	3

Note: 46 of the students surveyed did not receive money from any source.
a What is the probability that a student who worked got less than \$100?
b What is the probability that a student who got less than \$100 worked for it?
c What is the probability that a student who received money from a gift got less than \$100?
d Compare the probability of getting more than \$99 from gifts to working.

26 A beginner at pool can generally hit the ball he is aiming for so that its centre goes within 10 cm of where he wants it to, as long the white ball is less than 30 cm from the coloured ball and the coloured ball is less than 40 cm from the point it is being aimed for. On a particular table the balls are 60 mm in diameter and the pockets have an effective width of 90 mm. Provided that the ball is not hit too hard, and overlaps the edge of the pocket by less than 10 mm, it will go in. What is the probability that the beginner will sink a ball he is aiming for if the white and the coloured ball are within 30 cm and 40 cm as outlined?

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27 A class of Year 10 students are asked about their intentions to study Science in Year 11. Of the 24 students asked, 14 say they are going to do Biology, 10 are going to do Chemistry, and 8 are going to do Physics. There are 2 students who are going to do all three subjects. There are 2 students doing only Physics and another 2 doing only Chemistry. There is one student not doing Chemistry who is doing both Physics and Biology. Find each of the following probabilities if you select a student at random.

a The student is doing none of the three subjects.
b The student is doing only Biology.
c The student is doing Chemistry and Physics.
d The student is doing Biology or Physics.

Reasoning

28 In a school library, DVDs recorded from educational broadcasts have call numbers on the edges facing out from the shelves, so you normally look up the call number on the computer to access a recording. On a rainy day one of the sports teachers is looking for a recording about basketball or netball and knows which shelf these are on, so doesn't bother looking up the call number. There are actually 8 basketball recordings, 6 netball recordings, 12 football recordings, 9 cricket recordings and 20 recordings of other sports on that shelf. If they have been jumbled up, explain the probability that the first 3 recordings picked out by the teacher are basketball or netball.

29 A student said that you could get 4 heads, 3 heads and a tail, 2 heads and 2 tails, 1 tail and 3 heads or 4 tails from tossing 4 coins, so the probability of getting 4 heads is 20%. Explain what is wrong with this reasoning and find the correct answer.

30 Another student said that when two dice are tossed, and one of the dice shows a 4, the probability of getting a total of 7 was $\frac{6}{12} = 50\%$. This is because there are six ways of getting a total of 7, six ways the first dice could show 4, and six ways the second die could show 4. Explain the correct answer and what is wrong with this reasoning.

31 Using the table in question **25** someone said that a student who got pocket money had a probability of 40% of getting at least \$10 but less than \$20. What is the correct answer and what is wrong with this reasoning?

32 In question **17**, explain the difference it would make to the probability if the piece has to be rethrown when it does not land fully on the board. Remember that a draughts board has 8 squares each way.

33 In question **27**, a class member said there was an 80% chance that a student intending to study Chemistry also intended to take Physics. Explain if this was correct, and if not, what is the right answer?

Number and algebra

7 Advanced algebra and functions

9780170361941

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Australian Curriculum statements

Patterns and algebra
Expand binomial products and factorise monic quadratic expressions using a variety of strategies. (ACMNA233)

Linear and non-linear relationships
Explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technology as appropriate. (ACMNA239)

Video tutorial
Advanced algebra and functions
MAT10NAVT00007

Changing the format of algebraic expressions allows you to see connections that are otherwise hidden. This is one of the key skills of algebra. Expansion and factorisation are the main methods used, apart from simplification of terms. These are essential in solving nonlinear equations and in many other aspects of mathematics in your senior years. Expansion and factorisation do not have many direct applications, but they are essential enabling skills that allow you to do lots of other things.

Mathematical literacy

Maths dictionary
MAT10ASDI00001

Weblink
Algebra help
MAT10NAWB00007

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

coefficient	dummy variable	integer	square of a difference
common factor	expansion	monic quadrilateral	square of a sum
cross method	expression	non-monic quadrilateral	term
decomposition method	factor	perfect square	trinomial
difference of two squares	factorise	quadratic	turning point
	grouping factorisation	sketch	zeros

7.1 Algebraic patterns and groups

You have already learnt some special expansions that allow you to quickly simplify expressions. They can be used for factorisation as well as for simplification.

Important!

Special expansions

The special **expansion** $(x + y)(x - y) = x^2 - y^2$ is called the **difference of two squares**.
The special expansion $(x + y)^2 = x^2 + 2xy + y^2$ is called the **square of a sum**.
The special expansion $(x - y)^2 = x^2 - 2xy + y^2$ is called the **square of a difference**.
Sometimes they are just called **perfect squares**.

When you use the difference of two squares it doesn't matter whether you put the subtraction bracket or the addition bracket first.

Example 1

Puzzle sheet
Difference of two perfect squares
MAT10NAPS00019

Factorise each of the following expressions.

a $p^2 - 16$ **b** $4x^2 - 9y^2$ **c** $25 - (m + n)^2$

Solution

a Write the problem. $\quad \boldsymbol{p^2 - 16}$

Write in the difference of two squares pattern. $\quad \boldsymbol{= p^2 - 4^2}$

Use the difference of two squares. $\quad \boldsymbol{= (p + 4)(p - 4)}$

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b Write the problem. $4x^2 - 9y^2$

Write in the difference of two squares pattern. $= (2x)^2 - (3y)^2$

Use the difference of two squares. $= (2x - 3y)(2x + 3y)$

c Write the problem. $25 - (m + n)^2$

Write in the difference of two squares pattern. $= 5^2 - (m + n)^2$

Use the difference of two squares. $= [5 - (m + n)][5 + (m + n)]$

Simplify the internal brackets. $= (5 - m - n)(5 + m + n)$

For perfect squares, remember that in the centre you must have double the product of the terms that are squared. Remember also that the sign of this term determines whether it is a sum or difference.

Example 2

Factorise each of the following expressions.

a $m^2 + 2mn + n^2$

b $p^2 - 2p + 1$

c $9c^2 - 6cg + g^2$

d $4x^2 + 20x + 25$

Solution

a Write the problem. $m^2 + 2mn + n^2$

Write as $(\text{first})^2 \pm 2 \times \text{first} \times \text{second} + (\text{second})^2$ $= m^2 + 2 \times m \times n + n^2$

Use $x^2 + 2xy + y^2 = (x + y)^2$. $= (m + n)^2$

b Write the problem. $p^2 - 2p + 1$

Write as $(\text{first})^2 \pm 2 \times \text{first} \times \text{second} + (\text{second})^2$ $= p^2 - 2 \times p \times 1 + 1^2$

Use $x^2 - 2xy + y^2 = (x - y)^2$. $= (p - 1)^2$

c Write the problem. $9c^2 - 6cg + g^2$

Write as $(\text{first})^2 \pm 2 \times \text{first} \times \text{second} + (\text{second})^2$ $= (3c)^2 - 2 \times 3c \times g + g^2$

Use $x^2 - 2xy + y^2 = (x - y)^2$. $= (3c - g)^2$

d Write the problem. $4x^2 + 20x + 25$

Write as $(\text{first})^2 \pm 2 \times \text{first} \times \text{second} + (\text{second})^2$ $= (2x)^2 + 2 \times 2x \times 5 + 5^2$

Use $x^2 + 2xy + y^2 = (x + y)^2$. $= (2x + 5)^2$

CAS TI-Nspire exercise
Advanced algebra and functions
MAT10NATI00007

CAS ClassPad exercise
Advanced algebra and functions
MAT10NACP00007

Puzzle sheet
Perfect squares
MAT10NAPS00020

Investigate: Graphs of perfect squares

You have already seen that the graphs of $y = x^2$ and $y = -x^2$ are parabolas. What about the graphs of perfect squares?
For the function $y = x^2 - 6x + 9$, do the following.

- Factorise $x^2 - 6x + 9$.
- Complete the table of values below.

x	$^-2$	$^-1$	0	1	2	3	4	5	6	7	8
y											

- Plot the points and join with a smooth curve.
- What shape is formed?
- Compare the graph to that of $y = x^2$.
- Write a conclusion.

Repeat the steps above for the functions $y = x^2 + 4x + 4$, $y = 2x - x^2 - 1$ and $y = -x^2 - 4x - 4$.
What can you conclude about the graphs of perfect squares?

Important!

Graphs of perfect squares

The graph of the perfect square $y = (x - a)^2$ is a parabola with its **turning point** at $(a, 0)$. It is shifted a units to the right compared to $y = x^2$.
The graph of the perfect square $y = (x + a)^2$ is a parabola with its turning point at $(-a, 0)$. It is shifted a units to the left compared to $y = x^2$.
$y = -x^2$, $y = -(x - a)^2$ and $y = -(x + a)^2$ are upside down (reflected in the x-axis) compared to $y = x^2$.

Example 3

Use factorisation to help sketch the graph of $y = {}^-8x - x^2 - 16$.

Solution

Write the function in the perfect square order.

$$\mathbf{y = {}^-8x - x^2 - 16}$$
$$\mathbf{= -(x^2 + 8x + 16)}$$

Factorise the function.

$$\mathbf{= -(x + 4)^2}$$

State the relationship to $y = x^2$.

$y = -(x + 4)^2$ is shifted 4 units to the left and upside down compared to $y = x^2$. Its turning point is $(^-4, 0)$.

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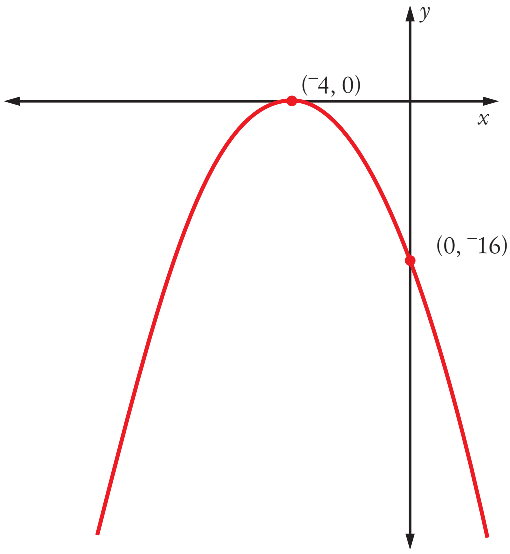
Sketch the graph, including the y-intercept at $(0, {}^{-}16)$.



In some cases you can group terms together to make a **common factor** that is a group. This is called **grouping factorisation** and is common when there are 4 terms.

Example 4

Factorise each of the following expressions.

a $mx - nx + 3m - 3n$

b $5a^2 - 4ab + 4b - 5a$

Solution

a Write the problem. $\quad \boldsymbol{mx - nx + 3m - 3n}$

Factorise the first and last pairs of terms. $\quad = \boldsymbol{x(m - n) + 3(m - n)}$

Use $(m - n)$ as a common factor. $\quad = \boldsymbol{(x + 3)(m - n)}$

b Write the problem. $\quad \boldsymbol{5a^2 - 4ab + 4b - 5a}$

Make a common factor of $(5a - 4b)$ with the pairs. $\quad = \boldsymbol{a(5a - 4b) - 1(5a - 4b)}$

Use $(5a - 4b)$ as a common factor. $\quad = \boldsymbol{(a - 1)(5a - 4b)}$

CAS TI-Nspire exercise
Advanced algebra and functions
MAT10NATI00007

CAS ClassPad exercise
Advanced algebra and functions
MAT10NACP00007

Puzzle sheet
Grouping
MAT10NAPS00021

You should check that both parts in Example **4** can also be done by rearranging the terms. You can rearrange part **a** as $mx + 3m - nx - 3n$ to get the common factor $(x + 3)$. You can rearrange part **b** as $5a^2 - 5a - 4ab + 4b$ to get the common factor $(a - 1)$. It doesn't matter which way around you do it, so you can pick whichever way seems easier to you.

Exercise 7.1 Algebraic patterns and groups

Understanding

Extra questions
Exercise 7.1
MAT10NAEQ00019

See Example 1

1 Copy and complete each of the following factorisations.

a $x^2 - y^2 = (x + y)(\ldots\ldots)$
b $16 - a^2 = (4 + a)(\ldots\ldots)$
c $64 - b^2 = (\ldots\ldots)(8 - b)$
d $w^2 - 49 = (w + 7)(\ldots\ldots)$
e $p^2 - q^2 = (\ldots\ldots)(p - q)$
f $a^2 - 25 = (\ldots\ldots)(a - 5)$
g $c^2 - 81 = (\ldots\ldots)(c - 9)$
h $x^2 - 121 = (\ldots\ldots)(x - 11)$
i $36 - d^2 = (6 + d)(\ldots\ldots)$
j $100 - m^2 = (10 + m)(\ldots\ldots)$

See Example 2

2 Copy and complete each of the following factorisations.

a $m^2 + 2mn + n^2 = (\ldots + n)^2$
b $x^2 - 2xy + y^2 = (x - \ldots)^2$
c $a^2 + 4a + 4 = (\ldots + 2)^2$
d $w^2 + 6w + 9 = (w + \ldots)^2$
e $h^2 - 2hk + k^2 = (\ldots - k)^2$
f $b^2 - 10b + 25 = (b - \ldots)^2$
g $n^2 + 12n + 36 = (n + \ldots)^2$
h $m^2 - 12m + 36 = (m - \ldots)^2$
i $x^2 - 14x + 49 = (x - \ldots)^2$
j $k^2 + 18k + 81 = (k + \ldots)^2$

3 Factorise each of the following.

a $a^2 - m^2$
b $x^2 - 1$
c $c^2 - 9$
d $y^2 - 16$
e $25 - w^2$
f $49 - a^2$
g $b^2 - 100$
h $g^2 - 81$
i $169 - n^2$
j $x^2 - 64$

4 Factorise each of the following.

a $a^2 + 2ab + b^2$
b $m^2 - 2mn + n^2$
c $h^2 - 6h + 9$
d $x^2 + 20x + 100$
e $p^2 + 10p + 25$
f $g^2 - 12g + 36$
g $x^2 - 14x + 49$
h $h^2 + 6h + 9$
i $25 - 10n + n^2$
j $p^2 + 8p + 16$
k $9 - 6x + x^2$
l $y^2 + 18y + 81$

See Example 4

5 Factorise each of the following.

a $6b + ab + 6m + am$
b $^-7g - dg + 7y + dy$
c $^-5a - ac + 5b + bc$
d $m^2 + mn + mq + nq$
e $ab - 3b + a^3 - 3a^2$
f $m^2 + 2m^3 - 6m - 3$
g $ax + ay + bx + by$
h $xy - xz + ay - az$
i $ad + bd - ap - bp$

Fluency

6 Factorise each of the following.

a $4a^2 - 25b^2$
b $16x^2 - 9y^2$
c $49m^2 - 16n^2$
d $9a^2 - 64b^2$
e $4x^2 - 121y^2$
f $36x^2 - 25y^2$
g $144m^2 - 25n^2$
h $64p^2 - 49q^2$
i $81a^2 - 16b^2$

7 Factorise each of the following.

a $4x^2 - 4x + 1$
b $9x^2 + 6x + 1$
c $16g^2 - 8g + 1$
d $4a^2 - 4ab + b^2$
e $9p^2 + 6pq + q^2$
f $25x^2 - 10xy + y^2$
g $x^2 + 16bx + 64b^2$
h $b^2 + 22bc + 121c^2$
i $a^2 - 18ab + 81b^2$
j $a^2 - 16ab + 64b^2$
k $9h^2 + 6hk + k^2$
l $16g^2 + 8g + 1$
m $4m^2 - 4mn + n^2$
n $16p^2 + 8pq + q^2$
o $25x^2 - 10x + 1$
p $49y^2 - 56y + 16$

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8 Factorise each of the following.

Worked solutions
Exercise 7.1
MAT10NAWS00019

a $(2a)^2 + 12a + 3^2$
b $(3x)^2 - 30x + 5^2$
c $(4m)^2 - 24m + 3^2$
d $9n^2 + 12n + 4$
e $25k^2 - 40k + 16$
f $4a^2 - 28a + 49$
g $4m^2 - 24m + 36$
h $81x^2 + 36x + 4$
i $16a^2 - 40a + 25$
j $64m^2 + 112m + 49$

9 Factorise each of the following.

Worked solutions
Exercise 7.1
MAT10NAWS00019

a $2xy^2 - y^2 - 2x + 1$
b $m^2n^2 + 1 - n - m^2n$
c $hx + gx + jx + hz + gz + jz$
d $ad - ac + cg + cf - df - dg$
e $a^2 - b^2 + 3a - 3b$
f $4x^2 + x - 16x - 4$
g $ac + 4d + ad + 4c$
h $x^2y + 3xy - 4x - 12$
i $a^2b - 1 - a + ab$
j $6a^3 - 12a^2 - 4a + 8$

10 Factorise each of the following.

Worked solutions
Exercise 7.1
MAT10NAWS00019

a $(a - 3)^2 - 1$
b $(a + b)^2 - 9$
c $(x - 7)^2 - 16$
d $4 - (2m - 3)^2$
e $4 - (4a - 7)^2$
f $(x - 3)^2 - 25$
g $36(x - 1)^2 - 25(x + 2)^2$
h $64(a + 1)^2 - 9(a - 3)^2$
i $4(3x - 5)^2 - 49(2x - 3)^2$
j $(x - y)^2 - (x + 2y)^2$
k $(a + b)^2 - (a - 2b)^2$

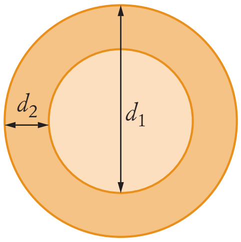
Problem solving

11 Use factorisation to help sketch the graphs of each of the following.

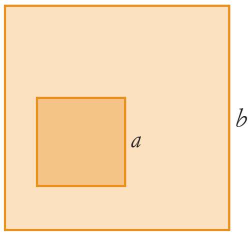
See Example 3

a $y = x^2 - 2x + 1$
b $y = -x^2 - 6x - 9$
c $y = x^2 + 2x + 1$
d $y = x^2 + 6x + 9$
e $y = x^2 - 8x + 16$
f $y = x^2 + 8x + 16$
g $y = 10x - x^2 - 25$
h $y = 2x - x^2 - 1$

12 An **annulus** is the shape left when a circle is cut out of the centre of another circle, as shown in the diagram. Find a formula for the area of the annulus in terms of d_1 and d_2 shown on the diagram. (*Hint:* Start with the areas of the circles in terms of the radii.)



13 This diagram shows two squares whose sides are a metres and b metres.



a What is the area of the larger square?
b What is the area of the smaller square?
c Write an expression for the area of the shaded region.
d Factorise the expression you wrote in part **c**.
e If $a = 37$ m and $b = 137$ m, use both of the expressions you wrote to find the area of the shaded region. Which calculation was easier?

Reasoning

14 Show that the volume of concrete used to make a concrete pipe of outer diameter D, inner diameter d and length L is given by $V = \frac{1}{4}\pi L(D + d)(D - d)$.

7.2 Quadratic expressions

Quadratic functions have the format $y = ax^2 + bx + c$ and their graphs are parabolas.

Important!

Quadratic expression

A **quadratic expression** in the variable x has the format $ax^2 + bx + c$, where a, b and c are real numbers. Quadratic expressions are often just called quadratics. They are also called **trinomials** since they have three terms. A quadratic where $a = 1$, $x^2 + bx + c$ is called a **monic quadratic**.

You have already seen that quadratics are produced by the expansion of binomial brackets, such as

$$(a + 5)(a - 4) = a^2 + a - 20 \quad \text{or} \quad (5c - 4)(2c - 3) = 10c^2 - 23c + 12$$

It is not immediately clear how you can do the reverse, that is, factorise quadratics.

Example 5

Video tutorial
Factorising quadratic expressions 1
MAT10NAVT10016

Complete each of the following factorisations and check by expansion.

a $p^2 + 8p + 15 = (p + 3)(p + \ldots)$ **b** $m^2 + m - 6 = (m + 3)(m - \ldots)$

c $x^2 - 7x + 6 = (x - 1)(x - \ldots)$ **d** $a^2 - 6a - 16 = (a - 8)(a + \ldots)$

Solution

a Write the partial answer. $\quad (p + 3)(p + \ldots)$

15 = 3 × 5, so try 5. $\quad = (p + 3)(p + 5)$

Expand the brackets. $\quad = p(p + 5) + 3(p + 5)$

Keep going. $\quad = p^2 + 5p + 3p + 15$

Simplify. $\quad = p^2 + 8p + 15$ ✓OK

Write the whole answer. $\quad p^2 + 8p + 15 = (p + 3)(p + 5)$

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b Write the partial answer. $(m + 3)(m - \ldots)$

$^{-}6 = 3 \times {}^{-}2$, so try $^{-}2$. $= (m + 3)(m - 2)$

Expand the brackets. $= m(m - 2) + 3(m - 2)$

Keep going. $= m^2 - 2m + 3m - 6$

Simplify. $= m^2 + m - 6$ ✓OK

Write the whole answer. $m^2 + m - 6 = (m + 3)(m - 2)$

c Write the partial answer. $(x - 1)(x - \ldots)$

$6 = {}^{-}1 \times {}^{-}6$, so try $^{-}6$. $= (x - 1)(x - 6)$

Expand the brackets. $= x(x - 6) - 1(x - 6)$

Keep going. $= x^2 - 6x - x + 6$

Simplify. $= x^2 - 7x + 6$ ✓OK

Write the whole answer. $x^2 - 7x + 6 = (x - 1)(x - 6)$

d Write the partial answer. $(a - 8)(a + \ldots)$

$^{-}16 = {}^{-}8 \times 2$, so try 2. $= (a - 8)(a + 2)$

Expand the brackets. $= a(a + 2) - 8(a + 2)$

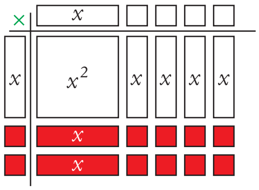
Keep going. $= a^2 + 2a - 8a - 16$

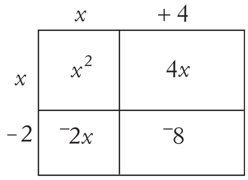
Simplify. $= a^2 - 6a - 16$ ✓OK

Write the whole answer. $a^2 - 6a - 16 = (a - 8)(a + 2)$

Investigate: Factorisation of quadratics I

You may have seen previously that you can model expansion of binomial brackets with squares to represent x^2 and oblongs of the same length to represent x. By colouring them on the back, you can also show $-x^2$ and $-x$. You can represent numbers using small squares of the same width as the oblongs to represent 1. This method is shown below for $(x - 2)(x + 4)$. The *area* method is shown beside it.



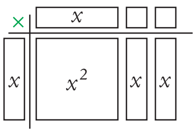


The expansion of $(x - 2)(x + 4)$ is $x^2 + 2x - 8$, as can be seen from either model.

How can you start with the expansion and end up with the model, using the squares and oblongs?

Looking at the first 2 terms of $x^2 + 2x - 8$, $x^2 + 2x = x(x + 2)$.

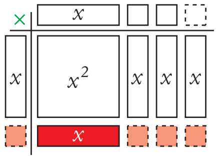
Make a model of $x(x + 2) = x^2 + 2x$.



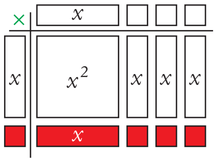
Teacher notes
Factorisation of quadratics I
MAT10NATN00005

Weblink
Factorising with concrete materials
MAT10NAWB00007

This gives you the x^2 term and *some* of the xs. It doesn't give any units.
To improve the model you add zero in the form $x - x$, with the positive x on the top and the negative x down the side.



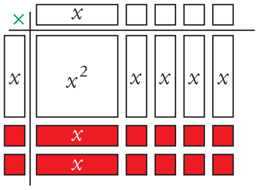
Inside the lines you now have $x^2 - x + 3x$.
To make the model correct you need to put a unit on the top, a negative unit down the side and some units inside the lines.
Now count the units at the side, on the top and inside the lines.
Does the number of units inside the lines match the number of units in the problem ($^{-}8$)?



Units table:

Side	Top	Inside	OK?
$^{-}1$	3	$^{-}3$	✗

Add another zero in the form $x - x$, putting the negative x down the side and the positive x on the top.
You now have $x^2 - 2x + 4x$ inside the lines.
Put in extra units to make the expansion work.

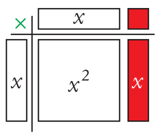


Units table:

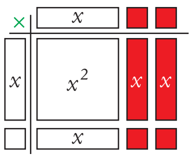
Side	Top	Inside	OK?
$^{-}1$	3	$^{-}3$	✗
$^{-}2$	4	$^{-}8$	✓

Count the units down the side, on the top and inside the lines.
Does it match $^{-}8$?
Yes!
The factorisation is, unsurprisingly, $x^2 + 2x - 8 = x^2 - 2x + 4x - 8 = (x - 2)(x + 4)$
Obviously, any quadratic with signs like the one above can be done in the same way.
The signs in a quadratic can be distributed in any of 4 ways: $ax^2 + bx - c$, $ax^2 - bx + c$, $ax^2 + bx + c$ or $ax^2 - bx - c$.

Let's look at factorising $x^2 - x - 12$.
Start with a model of $x^2 - x = x(x - 1)$.
In this case, you need to add zero again, but will have to put the negative on the top and the positive down the side.



This will make $x^2 - 2x + x$ inside the lines. You will also need to put in extra units to make the model work. Draw up a table of the units.
Do the units match the units ($^{-}12$) in the quadratic?



Units table:

Side	Top	Inside	OK?
1	$^{-}2$	$^{-}2$	✗

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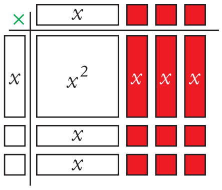
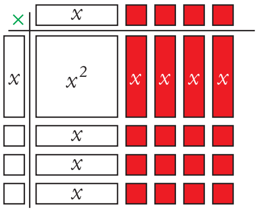
No, so keep going by adding another zero.



Units table:

Side	Top	Inside	OK?
1	$^{-}2$	$^{-}2$	✗
2	$^{-}3$	$^{-}6$	✗

You now have $x^2 - 3x + 2x$ inside the lines. What about the units? Do they match the quadratic?
No, you need $^{-}12$.
Go again.

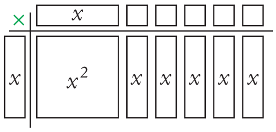


Units table:

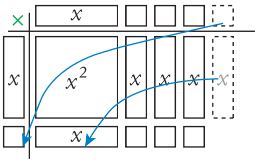
Side	Top	Inside	OK?
1	$^{-}2$	$^{-}2$	✗
2	$^{-}3$	$^{-}6$	✗
3	$^{-}4$	$^{-}12$	✓

You now have $x^2 - 4x + 3x$ inside the lines. Do the units match the quadratic?
Yes! There are $^{-}12$.
The factorisation is $x^2 - x - 12 = x^2 - 4x + 3x - 12 = (x - 4)(x + 3)$

Now let's look at $x^2 + 5x + 6$.
Make a model of $x^2 + 5x = x(x + 5)$
This gives the x^2 term and *all* of the xs.
You need some units. Adding $x - x$ will make the units inside the lines all negative.



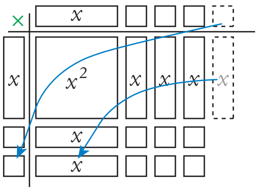
Move one of the xs from the top to the side instead of adding zero.
Put in the units to make the model work.



Units table:

Side	Top	Inside	OK?
1	4	4	✗

Do the units match the 6 in the quadratic?
No, so move another one.



Units table:

Side	Top	Inside	OK?
1	4	4	✗
2	3	6	✓

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Do the units match the 6 in the quadratic? Yes!
The factorisation is $x^2 + 5x + 6 = x^2 + 2x + 3x + 6 = (x + 2)(x + 3)$.
Try working through the model for $x^2 - 8x + 12$. You need to remember that when the units are negative *both* on the top and at the side, they will be *positive* inside the lines.
Starting with $x^2 - 8x = x(x - 8)$ you will have to move $-x$ from the top to the side.
You should end up with the last line of the units table being | $^-2$ | $^-6$ | 12 | ✓ |.
What is the factorisation?
Your teacher will probably want you to do some more models before you go on.

In the investigation above, you will have noticed that the last column and row numbers in the units table always *add up* to the **coefficient** of x and *multiply* to give the constant. The side number is the number in the first bracket and the top number is the number in the second bracket.
If there is a factorisation, this always works when x^2 has a coefficient of 1.
The numbers u and v that give $x^2 + bx + c = (x + u)(x + v)$ are such that $uv = c$ and $u + v = b$.

Example 6

Factorise each of the following.

a $a^2 + 7a + 12$ **b** $x^2 + 9x + 8$

Solution

a Write the problem. $\quad \boldsymbol{a^2 + 7a + 12}$

You want two numbers that multiply to 12 and sum to 7. $\quad \mathbf{2 \times 6 = 12 \text{ but } 2 + 6 \neq 7}$

Try some other numbers. $\quad \mathbf{3 \times 4 = 12 \text{ and } 3 + 4 = 7}$ ✓**OK**

Rewrite the expression using $3a$ and $4a$. $\quad \boldsymbol{a^2 + 7a + 12 = a^2 + 3a + 4a + 12}$

Factorise each pair. $\quad \boldsymbol{= a(a + 3) + 4(a + 3)}$

Use $(a + 3)$ as the common factor. $\quad \boldsymbol{= (a + 3)(a + 4)}$

b Write the problem. $\quad \boldsymbol{x^2 + 9x + 8}$

You want two numbers that multiply to 8 and sum to 9. $\quad \mathbf{2 \times 4 = 8 \text{ but } 2 + 4 \neq 9}$

Try some other numbers. $\quad \mathbf{1 \times 8 = 8 \text{ and } 1 + 8 = 9}$ ✓**OK**

Rewrite the expression using $1x$ and $8x$. $\quad \boldsymbol{x^2 + 9x + 8 = x^2 + x + 8x + 8}$

Factorise each pair. $\quad \boldsymbol{= x(x + 1) + 8(x + 1)}$

Use $(x + 1)$ as the common factor. $\quad \boldsymbol{= (x + 1)(x + 8)}$

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When you rewrite the middle term in the quadratic, it is always easier to *write the smallest term first*. If one term is negative and the other is positive, the smallest is the negative term.

Example 7

Factorise each of the following.

a $x^2 + x - 6$ **b** $a^2 - 2a - 15$ **c** $y^2 - 6y + 8$

Solution

a Write the problem. $x^2 + x - 6$

You want two numbers that multiply to $^-6$ and sum to 1. $2 \times {}^-3 = {}^-6$ but $2 + {}^-3 = {}^-1$

Swap signs to change the sign of $^-1$. $^-2 \times 3 = {}^-6$ and $^-2 + 3 = 1$ ✓OK

Rewrite the expression with the negative first. $x^2 + x - 6 = x^2 - 2x + 3x - 6$

Factorise each pair. $= x(x - 2) + 3(x - 2)$

Use $(x - 2)$ as the common factor. $= (x - 2)(x + 3)$

b Write the problem. $a^2 - 2a - 15$

You want two numbers that multiply to $^-15$ and sum to $^-2$. $^-5 \times 3 = {}^-6$ and $^-5 + 3 = {}^-2$ ✓OK

Rewrite the expression with the negative first. $a^2 - 2a - 15 = a^2 - 5a + 3a - 15$

Factorise each pair. $= a(a - 5) + 3(a - 5)$

Use $(a - 5)$ as the common factor. $= (a - 5)(a + 3)$

c Write the problem. $y^2 - 6y + 8$

You want two numbers with product 8 and sum $^-6$. $^-2 \times {}^-4 = {}^-8$ and $^-2 + {}^-4 = {}^-6$ ✓OK

Rewrite the expression with the smallest first. $y^2 - 6y + 8 = y^2 - 4y - 2y + 8$

Factorise each pair, being careful with signs. $= y(y - 4) - 2(y - 4)$

Use $(y - 4)$ as the common factor. $= (y - 4)(y - 2)$

CAS TI-Nspire exercise
Advanced algebra and functions
MAT10NATI00007

CAS ClassPad exercise
Advanced algebra and functions
MAT10NACP00007

Technology worksheet
Excel worksheet: Factorising trinomials
MAT10NACT00011

You will have noticed that once you have found the right two numbers, then with quadratics like those in Examples **6** and **7** you can go straight to the answer.

Technology worksheet
Excel worksheet: Investigating parabolas 2
MAT10NACT00012

Technology: GeoGebra
Parabolas
MAT10NATC00007

TLF Learning object
Grapher (L3531)
MAT10NAIN00007

TLF Learning object
Graphing functions (R11253)
MAT10NAIN00007

Investigate: Graphs of quadratics

In the last section of the text, you looked at sketching perfect squares. How can you use factorisation to help sketch quadratics?
For the function $y = x^2 - 5x - 6$, do the following.

- Factorise $x^2 - 5x - 6$.
- Complete the table of values below.

x	$^-2$	$^-1$	0	1	2	3	4	5	6	7	8
y											

- Plot the points and join with a smooth curve.
- What shape is formed?
- Compare the graph to that of $y = x^2$.
- Where is the turning point?
- What are the zeros (x-intercepts)?
- Write a conclusion.

Repeat the steps above for the functions $y = x^2 + x + 6$, $y = 2x - x^2 - 8$ and $y = -x^2 - 5x - 4$. What can you conclude about the graphs of quadratics?

Important!

Graphs of quadratics

The graph of the quadratic $f(x) = (x + a)(x + b)$ is a parabola with its **turning point** at $\left(\frac{-a-b}{2}, f\left(\frac{-a-b}{2}\right)\right)$. It is just the graph of $y = x^2$ shifted $\frac{a+b}{2}$ units to the left and $f\left(\frac{-a-b}{2}\right)$ units up or down, depending on its sign.

If $\frac{a+b}{2}$ is negative, then the graph is shifted to the right. If the whole function is negative (i.e., the coefficient of x^2 is negative), then the graph is turned upside down. The **zeros** of the function, where it passes through the x-axis, are at $(-a, 0)$ and $(-b, 0)$.

Example 8

Worksheet
Parabolas
MAT10NAWK00019

Use factorisation to help sketch the graph of $y = {}^-6x - x^2 - 5$.

Solution

Rearrange the function to write as a quadratic.

$$\begin{aligned} y &= {}^-6x - x^2 - 5 \\ &= -(x^2 + 6x + 5) \end{aligned}$$

Use $1 \times 5 = 5$ and $1 + 5 = 6$ to factorise the quadratic.

$$= -(x + 1)(x + 5)$$

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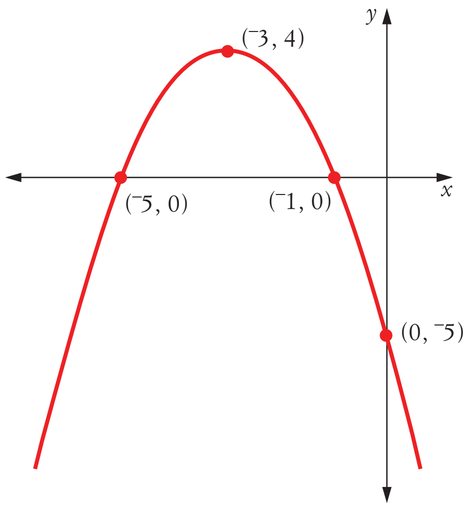
Use the characteristics of the factorised function to state important points.

The graph is shifted $\frac{1+5}{2} = 3$ units to the left. It has its turning point at $(^{-}3, 4)$ and zeros at $(^{-}5, 0)$ and $(^{-}1, 0)$. The y-intercept is at $(0, {}^{-}5)$.

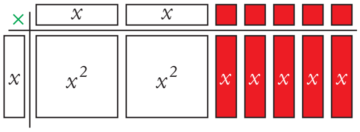
Sketch the graph.



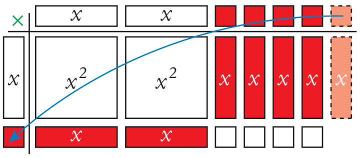
What happens with quadratics where the coefficient of x^2 is not 1 (**non-monic quadrilaterals**)?

Investigate: Factorisation of quadratics II

You can use the previous model to factorise quadratics where the coefficient of x is not 1, but it is a little more tedious. Let's try factorising $2x^2 - 5x - 3$.
Looking at the first 2 terms of $2x^2 - 5x - 3$, $2x^2 - 5x = x(2x - 5)$.
Make a model of $x(2x - 5) = 2x^2 - 5x$.
This gives you the x^2 terms and all the xs.
It doesn't give any units.



Move one of the units from the top to the side.
You have to put some units inside the lines to make the model work.
But these will be positive as $^{-}1 \times {}^{-}1 = 1$.



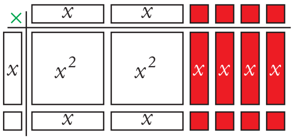
Units table:

Side	Top	Inside	OK?
$^{-}1$	$^{-}4$	4	✗

There are now too many $-x$s. To make the units inside the lines negative, change the sign of the unit at the side.

Teacher notes
Factorisation of quadratics 2
MAT10NATN00006

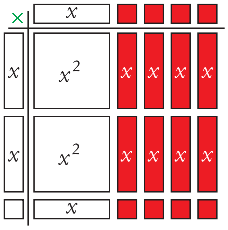
Now there aren't enough negative xs and there are too many negative units. If you move more to the side, it gets worse.



Units table:

Side	Top	Inside	OK?
$^{-}1$	$^{-}4$	4	✗
1	$^{-}4$	$^{-}4$	✗

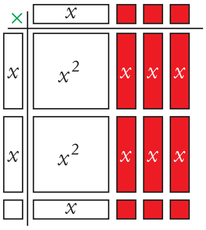
Try swapping x from the top to the side. You'll also have to move an x^2.



Units table:

Side	Top	Inside	OK?
$^{-}1$	$^{-}4$	4	✗
1	$^{-}4$	$^{-}4$	✗
1	$^{-}4$	$^{-}4$	✗

Now there are too many $-x$s *and* too many $^{-}1$s.
Try taking one of the units off the top.



Units table:

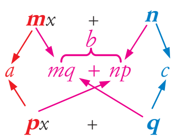
Column	Row	Inside	OK?
$^{-}1$	$^{-}4$	4	✗
1	$^{-}4$	$^{-}4$	✗
1	$^{-}4$	$^{-}4$	✗
1	$^{-}3$	$^{-}3$	✓

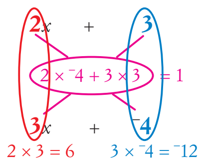
That works!
But the units table by itself would not have shown you the steps to get to this.
To get the correct model, you do need to use a bit of trial and error.
The factorisation is $2x^2 - 6x + x - 3 = (2x + 1)(x - 3)$.
Your teacher may want you to try some more models.

While $(2x + 3)(3x - 4) = 6x^2 + x - 12$, factorising $6x^2 + x - 12$ is difficult.
Assume that $ax^2 + bx + c$ factorises to give $(mx + n)(px + q)$.
$(mx + n)(px + q) = mpx^2 + mqx + npx + nq = mpx^2 + (mq + np)x + nq$
so $ax^2 + bx + c = mpx^2 + (mq + np)x + nq$
This means that $a = mp$, $c = nq$ and $b = mq + np$.

Cross method

To use the **cross method** you arrange numbers that might be m, n, p and q in a square as shown on the right.
The correct arrangement for $6x^2 + x - 12$ is shown beside it on the far right.
The end products give a and c.
The cross products give b.





Decomposition method

We can multiply a and c to get $ac = mp \times nq = mpnq = mq \times np$.
Write $s = mq$ and $t = np$.

Then $st = ac$ and $s + t = b$, so $ax^2 + bx + c$ $= ax^2 + sx + tx + c$ will be the expansion of $(mx + n)\ (px + q)$.

For $6x^2 + x - 12$, $ac = {}^{-}72$ and $b = 1$.
$^{-}8 \times 9 = {}^{-}72$ and $^{-}8 + 9 = 1$, so $s = {}^{-}8$ and $t = 9$, so use $x = {}^{-}8x + 9x$

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For the **decomposition method**, you need to find two numbers s and t that multiply to give ac and add to give b.

Then you decompose bx into $sx + tx$ to factorise the quadratic using grouping.

$$6x^2 + x - 12 = 6x^2 - 8x + 9x - 12$$
$$= 2x(3x - 4) + 3(3x - 4)$$
$$= (2x + 3)(3x - 4)$$

Example 9

Factorise $6p^2 + 17p + 10$.

Solution

Cross method

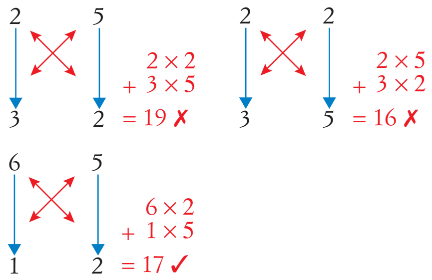
Make a cross with numbers vertically arranged that multiply to give 6 and 10 and try the cross products.

Try 2×3 and 5×2 first.

Try them with the 5 and 2 switched over.

Try 6×1 and 5×2.

It gives the right answer for b.



Write the answer.

$$\mathbf{6p^2 + 17p + 10 = (6p + 5)(p + 2)}$$

Decomposition method

What numbers do you need?

$ac = 6 \times 10 = 60$ and $b = 17$, so you need to find u and v so that $uv = 60$ and $u + v = 17$.

Try some out until you find the right ones.

$6 \times 10 = 60$ but $6 + 10 \neq 17$ ✗

$5 \times 12 = 60$ and $5 + 12 = 17$ ✓

Write the quadratic using 5 and 12.

$$\mathbf{6p^2 + 17p + 10 = 6p^2 + 5p + 12p + 10}$$

Factorise each pair.

$$\mathbf{= p(6p + 5) + 2(6p + 5)}$$

Use $(6p + 5)$ as the common factor.

$$\mathbf{= (6p + 5)(p + 2)}$$

The decomposition method is the exact reverse of the expansion of brackets, while the cross method uses more obvious trial and error to find the correct factorisation. The quadratic $4x^2 + 7x + 2$ does not factorise. It is harder to see that a quadratic will not factorise from the cross method than the decomposition method.

Your teacher may have a preference as to which method you use, or may leave it up to you. If you use the decomposition method, it is easiest to put the *smallest* of u and v *first*, just as with the simpler quadratics.

CAS TI-Nspire exercise
Advanced algebra and functions
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CAS ClassPad exercise
Advanced algebra and functions
MAT10NACP00007

Video tutorial
Factorising quadratic expressions 2
MAT10NAVT10017

Puzzle sheet
Trinominoes
MAT10NAPS00022

Example 10

Factorise each of the following.

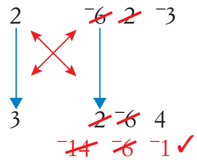
a $6x^2 - x - 12$ **b** $3x^2 - 11x + 10$ **c** $4x^2 - 3x - 7$

Solution

Cross method

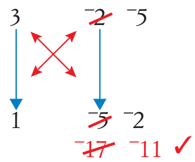
a Make a cross with numbers that multiply to give 6 and $^{-}12$.

Work out the cross products and write the answers underneath the cross, moving across as you try new ones.



Write the answer. **$6x^2 - x - 12 = (2x - 3)(3x + 4)$**

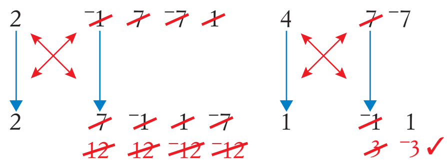
b Make a cross with numbers that multiply to give 3 and 10. The cross product is negative so the numbers that give 10 both have to be negative.



Write the answer. **$3x^2 - 11x + 10 = (3x - 5)(x - 2)$**

c Make a cross with numbers that multiply to give 4 and $^{-}7$. Try different numbers and do the cross products as before.

2×2 doesn't work. Try 4×1 on a new cross.



Write the answer. **$4x^2 - 3x - 7 = (4x - 7)(x + 1)$**

Decomposition method

a What numbers do you need? **$ac = 6 \times {}^{-}12 = {}^{-}72$ and $b = {}^{-}1$, so you need to find u and v so that $uv = {}^{-}72$ and $u + v = {}^{-}1$.**

Try some out. **$8 \times {}^{-}9 = {}^{-}72$ and $8 + {}^{-}9 = {}^{-}1$ ✓**

Write the quadratic using $^{-}9$ and 8. **$6x^2 - x - 12 = 6x^2 - 9x + 8x - 12$**

Factorise each pair. **$= 3x(2x - 3) + 4(2x - 3)$**

Use $(2x - 3)$ as the common factor. **$= (2x - 3)(3x + 4)$**

b What numbers do you need? **$ac = 3 \times 10 = 30$ and $b = {}^{-}11$, so you need to find u and v so that $uv = 30$ and $u + v = {}^{-}11$.**

They must both be negative. **$^{-}3 \times {}^{-}10 = 30$ but $^{-}3 + {}^{-}10 \neq {}^{-}11$ ✗**

$^{-}6 \times {}^{-}5 = 30$ and $^{-}6 + {}^{-}5 = {}^{-}11$ ✓

Write the quadratic using $^{-}6$ and $^{-}5$. **$3x^2 - 11x + 10 = 3x^2 - 6x - 5x + 10$**

Factorise each pair (watch signs). **$= 3x(x - 2) - 5(x - 2)$**

Use $(x - 2)$ as the common factor. **$= (x - 2)(3x - 5)$**

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c What numbers do you need? — $ac = 4 \times {}^-7 = {}^-28$ **and** $b = {}^-3$**, so you need to find** u **and** v **so that** $uv = {}^-28$ **and** $u + v = {}^-3$**.**

Try them out. The first one doesn't give the answer we need, swap signs. — $7 \times {}^-4 = {}^-28$ **but** $7 + {}^-4 \neq {}^-3$ ✗

$^-7 \times 4 = {}^-28$ **and** $^-7 + 4 = {}^-3$ ✓

Write the quadratic using $^-7$ and 4. — $4x^2 - 3x - 7 = 4x^2 - 7x + 4x - 7$

Factorise each pair, using 1 in front of the second bracket. — $= x(4x - 7) + 1(4x - 7)$

Use $(4x - 7)$ as the common factor. — $= (4x - 7)(x + 1)$

You can factorise quadratic expressions with two variables in exactly the same way as quadratic functions with one variable.

Example 11

Animated example
Quadratic expressions
MAT10NAAE00007

Factorise each of the following.

a $v^2 - 3vw - 10w^2$ **b** $4p^2 + 8pq - 21q^2$

Solution

a Write the problem. — $v^2 - 3vw - 10w^2$

You want numbers with product $^-10$ and sum $^-3$. — $^-2 \times 5 = {}^-10$ **but** $^-2 + 5 = 3$

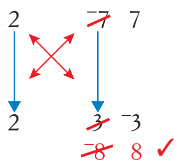
$^-5 \times 2 = {}^-10$ **and** $^-5 + 2 = {}^-3$ ✓**OK**

Write the answer. — $v^2 - 3vw - 10w^2 = (v - 5w)(v + 2w)$

b Cross method

Make a cross with numbers that multiply to give 4 and $^-21$.

Work out the cross products and write the answers underneath the cross, moving across as you try new ones.



Write the answer. — $4p^2 + 8pq - 21q^2 = (2p + 7q)(2p - 3q)$

Decomposition method

What numbers do you need? — $ac = 4 \times {}^-21 = {}^-84$ **and** $b = 8$**, so you need to find** u **and** v **so that** $uv = {}^-84$ **and** $u + v = 8$**.**

Try some out. — $^-7 \times 12 = {}^-84$ **but** $^-7 + 12 \neq 8$ ✗

$^-6 \times 14 = {}^-84$ **and** $^-6 + 14 = 8$ ✓

Write the quadratic using $^-6$ and 14. — $4p^2 + 8pq - 21q^2 = 4p^2 - 6pq + 14pq - 21q^2$

Factorise each pair. — $= 2p(2p - 3q) + 7q(2p - 3q)$

Use $(2x - 3)$ as the common factor. — $= (2p - 3q)(2p + 7q)$

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Exercise 7.2 Quadratic expressions

Understanding

Extra questions
Exercise 7.2
MAT10NAEQ00020

See Example 5

1 Complete each of the following factorisations and check by expansion.

a $a^2 + 5a + 4 = (a + 4)(a + \ldots)$ **b** $x^2 + 6x + 5 = (x + \ldots)(x + 1)$
c $y^2 + 7y + 10 = (y + 2)(y + \ldots)$ **d** $m^2 + 10m + 9 = (m + 1)(m + \ldots)$
e $p^2 - 5p + 4 = (p - 4)(p - \ldots)$ **f** $b^2 - 7b + 12 = (b - \ldots)(b - 4)$
g $a^2 - 3a - 4 = (a - \ldots)(a + 1)$ **h** $x^2 - 2x - 8 = (x + 2)(x - \ldots)$
i $y^2 + y - 12 = (y - 3)(y + \ldots)$ **j** $a^2 + 8a - 20 = (a - \ldots)(a + 10)$

See Example 6

2 Factorise each of the following.

a $g^2 + 8g + 12$ **b** $h^2 + 5h + 6$ **c** $k^2 + 13k + 42$ **d** $x^2 + 5x + 6$
e $t^2 + 7t + 6$ **f** $z^2 + 11z + 30$ **g** $t^2 + 15t + 54$ **h** $y^2 + 7y + 12$
i $a^2 + 11a + 28$ **j** $w^2 + 12w + 35$

If you are going to use the 'cross' method, answer question **3**.
If you are going to use the decomposition method, answer questions **4** and **5**.

3 a Which 'cross' diagram will give the factors of $2x^2 + 17x + 8$?

A $\begin{array}{cc} 2x & 4 \\ x & 2 \end{array}$ **B** $\begin{array}{cc} 2x & 8 \\ x & 1 \end{array}$ **C** $\begin{array}{cc} 2x & 1 \\ x & 8 \end{array}$ **D** $\begin{array}{cc} 2x & 2 \\ x & 4 \end{array}$

b Which 'cross' diagram will give the factors of $6x^2 + 7x - 5$?

A $\begin{array}{cc} 3x & 5 \\ 2x & ^{-}1 \end{array}$ **B** $\begin{array}{cc} 3x & ^{-}5 \\ 2x & 1 \end{array}$ **C** $\begin{array}{cc} 6x & 1 \\ x & ^{-}5 \end{array}$ **D** $\begin{array}{cc} 3x & ^{-}1 \\ 2x & 5 \end{array}$

c Which 'cross' diagram will give the factors of $4x^2 - 16x + 7$?

A $\begin{array}{cc} 4x & ^{-}7 \\ x & ^{-}1 \end{array}$ **B** $\begin{array}{cc} 2x & 7 \\ 2x & 1 \end{array}$ **C** $\begin{array}{cc} 4x & ^{-}1 \\ x & ^{-}7 \end{array}$ **D** $\begin{array}{cc} 2x & ^{-}1 \\ 2x & ^{-}7 \end{array}$

d Which 'cross' diagram will give the factors of $12x^2 - 17x + 6$?

A $\begin{array}{cc} 2x & ^{-}3 \\ 6x & ^{-}2 \end{array}$ **B** $\begin{array}{cc} 3x & ^{-}6 \\ 4x & ^{-}1 \end{array}$ **C** $\begin{array}{cc} 6x & ^{-}6 \\ x & ^{-}1 \end{array}$ **D** $\begin{array}{cc} 4x & ^{-}3 \\ 3x & ^{-}2 \end{array}$

e Which 'cross' diagram will give the factors of $8x^2 - 2x - 15$?

A $\begin{array}{cc} 4x & ^{-}3 \\ 2x & 5 \end{array}$ **B** $\begin{array}{cc} 8x & ^{-}3 \\ x & 5 \end{array}$ **C** $\begin{array}{cc} 2x & ^{-}3 \\ 4x & 5 \end{array}$ **D** $\begin{array}{cc} x & 1 \\ 8x & ^{-}15 \end{array}$

4 Find two numbers that:

a multiply to give 15 and add to give 8 **b** multiply to give $^{-}27$ and add to give $^{-}6$
c multiply to give $^{-}40$ and add to give $^{-}3$ **d** add to give 12 and multiply to give 32
e add to give $^{-}5$ and multiply to give $^{-}36$ **f** multiply to give 64 and add to give $^{-}16$
g add to give 2 and multiply to give $^{-}15$ **h** add to give $^{-}3$ and multiply to give $^{-}54$.

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5 Each quadratic below has been 'decomposed'. Complete the factorisations.

a $2x(x-1)+7(x-1)$ **b** $3a(2a+3)-2(2a+3)$
c $4x(3x-5)+3(3x-5)$ **d** $5y(y-4)+6(y-4)$
e $7x(2x+1)+3(2x+1)$ **f** $3x(3x-1)-1(3x-1)$
g $5a(4a+3)-2(4a+3)$ **h** $5x(2x-7)+(2x-7)$
i $6a(a-3)-(a-3)$ **j** $4x(2x-9)+3(2x-9)$

Fluency

6 Factorise each of the following.

See Examples 6, 7

a a^2+4a+3 **b** x^2+9x+8 **c** y^2+7y+6
d m^2+5m+6 **e** x^2+6x+8 **f** $p^2+9p+20$
g $p^2+10p+16$ **h** $g^2+8g+15$ **i** $x^2+10x+24$
j $x^2+13x+30$ **k** $a^2+14a+40$ **l** $m^2+17m+42$
m x^2-3x+2 **n** a^2-4a+3 **o** y^2-5y+6
p $x^2-10x+9$ **q** $y^2-7y+10$ **r** $a^2-10a+16$
s $x^2-10x+21$ **t** $w^2-11w+18$ **u** $a^2-11a+24$
v $a^2-18a+32$ **w** $y^2-16y+15$ **x** $b^2-12b+35$

7 Factorise each of the following.

a b^2-b-6 **b** a^2-a-20 **c** x^2-x-12
d y^2-y-42 **e** c^2-2c-3 **f** a^2-2a-8
g $n^2-2n-15$ **h** m^2-7m-8 **i** $a^2-3a-10$
j $x^2+3x-40$ **k** $y^2-4y-12$ **l** $b^2-6b-27$
m p^2+5p-6 **n** k^2+3k-4 **o** a^2+a-6
p c^2+c-20 **q** $x^2+3x-40$ **r** y^2+5y-6
s $a^2+4a-12$ **t** b^2+6b-7 **u** $k^2+6k-16$
v $p^2+4p-32$ **w** $a^2+2a-35$ **x** $x^2+5x-36$

8 Factorise each of the following.

See Example 11

a $e^2-14en+45n^2$ **b** $w^2+8wt-105t^2$ **c** $a^2-8ag+15g^2$
d $b^2+3bv-108v^2$ **e** $p^2+3pe-4e^2$ **f** $c^2-8cm-33m^2$
g $z^2-10zm-11m^2$ **h** $y^2+6yn-55n^2$ **i** $g^2+7ge-78e^2$
j $g^2+2gp-99p^2$ **k** $h^2+21hw+98w^2$ **l** $z^2+29zq+210q^2$
m $z^2-11zu-60u^2$ **n** $x^2+19xz+84z^2$ **o** $q^2+3qn-28n^2$
p $p^2-2pr-3r^2$ **q** $x^2-14xv+48v^2$ **r** $k^2+4kp+3p^2$
s $h^2-3hz-108z^2$ **t** $c^2-23cp+130p^2$ **u** $z^2-6zr-40r^2$
v $t^2-6tp-27p^2$ **w** $g^2+10gq-24q^2$ **x** $k^2-14km+24m^2$

9 Factorise each of the following.

See Example 9

a $2x^2+9x+9$ **b** $2a^2+9a+10$ **c** $2x^2+13x+15$
d $3m^2+14m+8$ **e** $3x^2+20x+12$ **f** $3b^2+13b+12$
g $5x^2+22x+8$ **h** $5c^2+14c+9$ **i** $4x^2+12x+9$
j $4b^2+13b+3$ **k** $5p^2+7p+2$ **l** $5h^2+11h+2$
m $6a^2+7a+2$ **n** $6x^2+13x+2$ **o** $3y^2+8y+4$
p $6x^2+23x+20$ **q** $12n^2+29n+14$ **r** $8x^2+18x+9$
s $12y^2+35y+25$ **t** $16w^2+24w+9$ **u** $15x^2+28x+12$
v $32r^2+52r+15$ **w** $12a^2+22a+8$ **x** $42x^2+73x+28$

See Example 10

10 Factorise each of the following.

a $2x^2 - 9x - 5$	**b** $5a^2 - a - 4$	**c** $2x^2 - 7x - 15$
d $2m^2 - m - 1$	**e** $3x^2 - 2x - 1$	**f** $6b^2 - 5b - 1$
g $2x^2 - 5x - 3$	**h** $2h^2 - 3h - 2$	**i** $6y^2 - 11y - 2$
j $5x^2 - 9x - 2$	**k** $9n^2 - 17n - 2$	**l** $10x^2 - 19x - 2$
m $2a^2 - 7a - 4$	**n** $7x^2 + 13x - 2$	**o** $10y^2 + 19y - 2$
p $4x^2 + 15x - 4$	**q** $5x^2 + 14x - 3$	**r** $2b^2 + 5b - 3$
s $3x^2 + 8x - 3$	**t** $4w^2 + 11w - 3$	**u** $5x^2 + 19x - 4$
v $6r^2 + 23r - 4$	**w** $2x^2 + 7x - 4$	**x** $2a^2 + 9a - 5$

11 Factorise each of the following.

a $5x^2 + 39x - 8$	**b** $3a^2 - 10a - 8$	**c** $7b^2 - 16b - 15$
d $5x^2 + 7x - 6$	**e** $3y^2 + 19y + 6$	**f** $5x^2 - 24x + 27$
g $7f^2 - 54f + 35$	**h** $11k^2 - 89k + 8$	**i** $12a^2 + 32a - 35$
j $20y^2 - 13y - 84$	**k** $24x^2 + 110x + 121$	**l** $6d^2 - d - 77$
m $2x^2 + x - 10$	**n** $30m^2 - 31m - 44$	**o** $6x^2 - 13x - 5$
p $6c^2 + 7c - 20$	**q** $9x^2 + 6x + 1$	**r** $18a^2 + 21a - 4$
s $12p^2 + 20p - 25$	**t** $15x^2 + 7x - 2$	**u** $8a^2 - 10a - 3$
v $7x^2 - 27x - 4$	**w** $9n^2 + 35n - 4$	**x** $6x^2 + x - 77$

12 Factorise each of the following.

a $27z^2 + 12zn - 32n^2$	**b** $27b^2 + 60bg + 28g^2$	**c** $84p^2 - 16pe - 5e^2$
d $5v^2 - 34vk - 99k^2$	**e** $40b^2 + 94bn + 55n^2$	**f** $15m^2 + 2mp - 8p^2$
g $98x^2 + 189xu + 55u^2$	**h** $72p^2 - 198pn + 121n^2$	**i** $48x^2 - 46xg - 77g^2$
j $30v^2 - 229vn + 30n^2$	**k** $24m^2 - 110mh + 75h^2$	**l** $45z^2 - 57zm - 4m^2$
m $112b^2 + 150bh + 27h^2$	**n** $63y^2 - 34ya - 80a^2$	**o** $24m^2 - 52mr + 24r^2$
p $104n^2 - 67na - 10a^2$	**q** $143u^2 + 87ug - 90g^2$	**r** $110e^2 - 41eu - 126u^2$
s $180x^2 - 153xp - 14p^2$	**t** $195t^2 - 92tc - 7c^2$	**u** $36r^2 - 72rk + 11k^2$
v $3x^2 + 41xb - 14b^2$	**w** $99z^2 - 170zn + 39n^2$	**x** $8t^2 + 27tp + 9p^2$

Problem solving

Worked solutions
Exercise 7.2
MAT10NAWS00020

13 How many integer values of b are there for which $x^2 + bx + 6$ can be factorised using integer values?

14 How many integer values of b are there for which $x^2 + bx - 12$ can be factorised using integer values?

See Example 8

15 Use factorisation to help sketch the graph of each of the following.

a $y = x^2 - 2x - 3$	**b** $y = -x^2 - 4x - 3$	**c** $y = x^2 + 6x + 8$
d $y = x^2 - 6x + 8$	**e** $y = x^2 + 4x + 3$	**f** $y = x^2 - 5x + 4$
g $y = 5x - x^2 - 6$	**h** $y = x^2 - x - 6$	

Reasoning

Worked solutions
Exercise 7.2
MAT10NAWS00020

16 How many whole number values of c between 1 and 100 can you find so that $x^2 - x - c$ can be factorised using whole numbers?

17 How many whole number values of c between 1 and 100 can you find so that $x^2 + 2x - c$ can be factorised using whole numbers?

Worked solutions
Exercise 7.2
MAT10NAWS00020

18 How many integer values of a can you find, strictly between 100 and 200, so that $ax^2 + x - 6$ can be factorised using integers?

19 For what values of c are there an odd number of values of b for which $x^2 + bx + c$ can be factorised using integer values?

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7.3 Other expressions and functions

All the expressions and functions you have looked at so far in this chapter have had just two factors. In most cases, each factor has been a group so you have ended up with binomial brackets. Combinations of the types you have already looked at may give more than two factors or enable you to factorise expressions with higher powers.
Common factors should always be removed before any factorisation is done.

CAS TI-Nspire exercise
Advanced algebra and functions
MAT10NATI00007

CAS ClassPad exercise
Advanced algebra and functions
MAT10NACP00007

Example 12

Factorise each of the following.

a $3t^2 - 48$ **b** $2m^3n - 8mn^3$ **c** $5x^3 - 30x^2 + 45x$

Solution

a Write the expression. $\mathbf{3t^2 - 48}$

Take out the common factor. $\mathbf{= 3(t^2 - 16)}$

Use the difference of two squares. $\mathbf{= 3(t - 4)(t + 4)}$

b Write the expression. $\mathbf{2m^3n - 8mn^3}$

Take out the common factor. $\mathbf{= 2mn(m^2 - 4n^2)}$

Use the difference of two squares. $\mathbf{= 2mn(m + 2n)(m - 2n)}$

c Write the expression. $\mathbf{5x^3 - 30x^2 + 45x}$

Take out the common factor. $\mathbf{= 5x(x^2 - 6x + 9)}$

Use perfect squares. $\mathbf{= 5x(x - 3)^2}$

Quadratics will not always be given in the usual order, so check any three-term expressions to see if they are quadratics.

Example 13

Express each of the following as products of the simplest possible factors.

a $6x^2 - 30x + 24$ **b** $5 + 6a^2 - 13a$ **c** $8ab^2 - 20ab - 12a$

Solution

a Write the expression. $\mathbf{6x^2 - 30x + 24}$

Take out the common factor. $\mathbf{= 6(x^2 - 5x + 4)}$

Use $^-4 \times {}^-1 = 4$ and $^-4 + {}^-1 = {}^-5$ to factorise the quadratic. $\mathbf{= 6(x - 4)(x - 1)}$

b Write the expression. $\mathbf{5 + 6a^2 - 13a}$

Put in the normal quadratic order. $= \mathbf{6a^2 - 13a + 5}$

Factorise as a quadratic. $= \mathbf{(2a - 1)(3a - 5)}$

c Write the expression. $\mathbf{8ab^2 - 20ab - 12a}$

Take out the common factor. $= \mathbf{4a(2b^2 - 5b - 3)}$

Factorise as a quadratic. $= \mathbf{4a(b - 3)(2b + 1)}$

Occasionally, you can rewrite powers as squares. You may be able to substitute a **dummy variable** for an expression to simplify a factorisation. In this case, remember to substitute the original expression back in and simplify.

Example 14

Puzzle sheet
Mixed factorisations
MAT10NAPS00023

Factorise each of the following.

a $(e - 3)^2 - 25$ **b** $a^4 - 16$ **c** $x^4 - 5x^2 - 36$ **d** $2(b - 1)^2 + 5(b - 1) + 2$

Solution

a Write the expression. $\mathbf{(e - 3)^2 - 25}$

Use the dummy variable $p = (e - 3)$. $= \mathbf{p^2 - 25}$

Factorise as a difference of squares. $= \mathbf{(p - 5)(p + 5)}$

Substitute the dummy variable $p = (e - 3)$. $= \mathbf{[(e - 3) - 5][(e - 3) + 5]}$

Simplify the internal brackets. $= \mathbf{(e - 3 - 5)(e - 3 + 5)}$

Complete the simplification. $= \mathbf{(e - 8)(e + 2)}$

b Write the expression. $\mathbf{a^4 - 16}$

Express as a difference of squares. $= \mathbf{(a^2)^2 - 4^2}$

Factorise as a difference of squares. $= \mathbf{(a^2 + 4)(a^2 - 4)}$

Factorise $(a^2 - 4)$ as a difference of squares. $= \mathbf{(a^2 + 4)(a + 2)(a - 2)}$

c Write the expression. $\mathbf{x^4 - 5x^2 - 36}$

Express as a quadratic in x^2. $= \mathbf{(x^2)^2 - 5x^2 - 36}$

Use $^-9 \times 4 = 36$ and $^-9 + 4 = {}^-5$
to factorise the quadratic. $= \mathbf{(x^2 - 9)(x^2 + 4)}$

Factorise $(x^2 - 9)$ as a difference of squares. $= \mathbf{(x - 3)(x + 3)(x^2 + 4)}$

d Write the expression. $\mathbf{2(b - 1)^2 + 5(b - 1) + 2}$

Use the dummy variable $p = (b - 1)$. $= \mathbf{2p^2 + 5p + 2}$

Factorise as a quadratic. $= \mathbf{(2p + 1)(p + 2)}$

Substitute the dummy variable $p = (b - 1)$. $= \mathbf{[2(b - 1) + 1][(b - 1) + 2]}$

Simplify the internal brackets. $= \mathbf{(2b - 2 + 1)(b - 1 + 2)}$

Complete the simplification. $= \mathbf{(2b - 1)(b + 1)}$

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You may be able to use a special pattern or reorganisation to do a grouping factorisation.

Example 15

Factorise each of the following.

a $a^3 + 4 - 4a - a^2$

b $a^2 - 2ab + b^2 - c^2$

Solution

a Write the expression. $\quad \boldsymbol{a^3 + 4 - 4a - a^2}$

Rearrange to make groups. $\quad \boldsymbol{= a^3 - a^2 - 4a + 4}$

Factorise each pair. $\quad \boldsymbol{= a^2(a - 1) - 4(a - 1)}$

Use $(a - 1)$ as a common factor. $\quad \boldsymbol{= (a - 1)(a^2 - 4)}$

Factorise $(a^2 - 4)$ as a difference of squares. $\quad \boldsymbol{= (a - 1)(a - 2)(a + 2)}$

b Write the expression. $\quad \boldsymbol{a^2 - 2ab + b^2 - c^2}$

Use perfect squares for the first 3 terms. $\quad \boldsymbol{= (a - b)^2 - c^2}$

Use the dummy variable $p = (a - b)$. $\quad \boldsymbol{= p^2 - c^2}$

Use the difference of squares. $\quad \boldsymbol{= (p - c)(p + c)}$

Substitute the dummy variable $p = (a - b)$. $\quad \boldsymbol{= [(a - b) - c][(a - b) + c]}$

Simplify the internal brackets. $\quad \boldsymbol{= (a - b - c)(a - b + c)}$

Exercise 7.3 Other expressions and functions

Fluency

Extra questions

Exercise 7.3

MAT10NAEQ00021

See Example 12

1 Factorise each of the following.

a $3x^2 - 12y^2$ b $5a^2 - 45b^2$ c $8m^2 - 32n^2$ d $3a^3b - 27ab^3$

e $6x^3y - 24xy^3$ f $7p^3q - 63pq^3$ g $28n^3 - 7n$ h $27x^3 - 3x$

i $16h^3 - 4h$ j $18x^2 - 2$

See Example 13

2 Factorise each of the following expressions.

a $2n^2 - 12nk + 16k^2$ b $7n^2 - 91ny + 252y^2$

c $5q^2 - 65qt + 210t^2$ d $4u^2 + 4ut - 48t^2$

e $3w^2 + 42wm + 147m^2$ f $8n^2 + 24nw - 224w^2$

g $7z^2 - 42zw + 56w^2$ h $6a^2 - 78ac + 240c^2$

i $3t^2 + 6th - 72h^2$ j $8a^2 - 16am - 120m^2$

3 Factorise each of the following.

a $48w^2 - 56wc - 392c^2$ b $128m^2 - 624mx + 216x^2$

c $12q^2 - 45qm + 27m^2$ d $42c^2 + 74cm - 56m^2$

e $16p^2 - 100pr + 126r^2$ f $49a^2 - 203am - 210m^2$

g $36n^2 + 108nq + 81q^2$ h $56c^2 + 42cv - 245v^2$

i $16t^2 - 142tw - 18w^2$ j $216h^2 + 12hq - 140q^2$

Problem solving

See Example 14

4 Factorise each of the following expressions.

a $(a-3)^2-1$
b $(a+b)^2-9$
c $(x-7)^2-16$
d $32-8(2m-3)^2$
e $36-9(4a-7)^2$
f $5(x-3)^2-125$
g $36(x-1)^2-25(x+2)^2$
h $64(a+1)^2-9(a-3)^2$
i $4(3x-5)^2-49(2x-3)^2$
j $3(x-y)^2-3(x+2y)^2$
k $2(a+b)^2-2(a-2b)^2$
l a^4-b^4
m x^4-5x^2-36
n a^4-2a^2-63
o $80a^4-5n^4$

See Example 15

5 Factorise the following.

a $5a^3+4a+5a^2+4$
b $b^3-5b^2+2b-10$
c $5m^3-m^2+15m-3$
d $a^2-9+ab-3b$
e $3x^2-12y^2+x-2y$
f $6+a^3+6a+a^2$
g $x^2+2xy+y^2-a^2$
h y^2-a^2-6a-9
i $9x^2-p^2+2p-1$
j $36k^2+12k+1-25h^2$

Worked solutions
Exercise 7.3
MAT10NAWS00021

6 Factorise each of the following.

a $4x^2y^2-x^2+2xy-y^2$
b $9a^2b^2-x^2-2px-p^2$
c $x^2+2xy+y^2-z^2$
d $m^2-2mn+n^2-p^2$
e $z^2-x^2-2xy-y^2$
f $a^2-2ab+b^2-4$
g $p^2+2pq+q^2-25$
h $a^2-4-2ab+b^2$
i $m^2+9-6m-16k^2$
j $m^2+25-10m-16c^2$
k $r^2-s^2-16-8s$
l $a^2+12cm-4c^2-9m^2$
m $16x^2+y^2-z^2-8xy$
n $a^4-b^2+25-10a^2$

Worked solutions
Exercise 7.3
MAT10NAWS00021

7 Factorise each of the following.

a $4a^2-28a+48$
b $10x^2+50x+60$
c $20m^2-125$
d $12a^4-75a^2$
e $8m^2-32n^2p^2$
f $(a-b)^2+5(a-b)+4$
g $4(a+b)^2-(a-b)^2$
h x^4+x^2-20
i $6ay^2+5ay+a$
j $64-64y-48y^2$
k $12x^2y^4-14xy^3-6y^2$
l $3(2-h)^2+11(2-h)+6$
m $9x^4-2x^2-32$

Reasoning

8 Use factorisation to find short cuts for the following calculations.

a 21^2 b 19^2 c 31^2 d 29^2 e 101^2

Worked solutions
Exercise 7.3
MAT10NAWS00021

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Quiz

Advanced algebra and functions

MAT10NAQZ00007

- The **difference of two squares** is the special expansion $(x + y)(x - y) = x^2 - y^2$.
- The **square of a sum** is the expansion $(x + y)^2 = x^2 + 2xy + y^2$ and the **square of a difference** is the expansion $(x - y)^2 = x^2 - 2xy + y^2$. These are often called **perfect squares**.
- The difference of two squares and perfect squares are used in reverse to factorise expressions.
- A **grouping factorisation** is common when there are 4 terms and works by rearranging terms if necessary to obtain a grouped term as a common factor.
- A **quadratic expression** in the variable x has the format $ax^2 + bx + c$, where a, b and c are real numbers. Quadratic expressions are often just called quadratics or **trinomials** because they have three terms.
- If a quadratic with a unit coefficient of x^2 can be factorised you use the numbers that add to give b and multiply to give c. The numbers u and v that give $x^2 + bx + c = (x + u)(x + v)$ are such that $uv = c$ and $u + v = b$.
- You can use factorisation to help sketch quadratics. The perfect square $y = (x + a)^2$ is a parabola with its turning point at $(-a, 0)$. It is just a graph of $y = x^2$ shifted a units to the left. If a is negative, it is shifted to the right.
- The graph of a quadratic that can be factorised to $f(x) = (x + a)(x + b)$ is a parabola with its turning point at $\left(\frac{-a-b}{2}, f\left(\frac{-a-b}{2}\right)\right)$. It is just a graph of $y = x^2$ shifted $\frac{a+b}{2}$ units to the left and $f\left(\frac{-a-b}{2}\right)$ units up or down, depending on its sign. If $\frac{a+b}{2}$ is negative, then the graph is shifted to the right. If the whole function is negative (i.e., the coefficient of x^2 is negative), the graph is turned upside down. The zeros of the function, where it passes through the x-axis, are at $(-a, 0)$ and $(-b, 0)$.
- For coefficients of x^2 other than 1, there are two methods to find the factors of a quadratic. Factors always *add up* to give the coefficient of x and *multiply* to give the constant.
- In the **cross method** of factorisation of a quadratic $ax^2 + bx + c$, to find the factors $(mx + n)(px + q)$, you use a square to find the factors m, p, n and q as shown on the right.

$$\begin{matrix} m & & n \\ & \times & \\ p & & q \end{matrix} \qquad \begin{aligned} mp &= a, \\ nq &= c, \\ mq + pn &= b \end{aligned}$$

- In the **decomposition method** of factorising quadratics, you find numbers u and v such that the product $uv = ac$ and the sum $u + v = b$. The term bx is then decomposed as $bx = ux + vx$ to make a grouping factorisation.
- To factorise general expressions, first remove common factors. If there are three terms, look for a quadratic or perfect square. If there are four terms, look for a grouping factorisation. You may also be able to write higher powers as squares or use a **dummy variable** to make it easier to factorise an expression.

Chapter 7 review

Understanding

See Examples 1, 2

1 Copy and complete each of the following factorisations.

a $m^2 - p^2 = (m + \ldots)(m - \ldots)$ b $x^2 - 6x + 9 = (x - \ldots)^2$ c $g^2 + 10g + 25 = (g + \ldots)^2$

2 Factorise each of the following.

a $r^2 - 16$ b $h^2 + 12h + 36$ c $k^2 - 14k + 49$

See Example 4

3 Factorise each of the following.

a $x(3x - y) + 2y(3x - y)$ b $ab + ac + bd + cd$ c $x^2 - 3xy - 6xz + 18yz$

See Example 5

4 Complete each of the following factorisations and check by expansion.

a $m^2 + 7m + 12 = (m + 4)(m + \ldots)$ b $x^2 - 6x - 16 = (x - \ldots)(x + 2)$

See Example 6

5 Factorise each of the following.

a $k^2 + 10k + 16$ b $g^2 + 11g + 28$

Fluency

6 Factorise each of the following.

a $25g^2 - 36t^2$ b $16u^2 - 56u + 49$ c $9m^2 + 6mn + n^2$

7 Factorise each of the following.

a $7m^3 - 7m^2 + 4am - 4a$ b $36 - (m - n)^2$

See Examples 6, 7

8 Factorise each of the following.

a $x^2 + x - 42$ b $p^2 - 2p - 15$ c $k^2 - 10k + 16$ d $a^2 + 13a + 36$

See Example 9

9 Factorise each of the following.

a $y^2 + 11yk + 18k^2$ b $m^2 + 8mz + 7z^2$ c $n^2 + nz - 132z^2$

See Examples 12, 13

10 Factorise each of the following expressions.

a $5x^3 - 80xy^2$ b $2m^2 + 6ma - 56a^2$

c $3q^2 - 3qt - 216t^2$ d $7g^2 - 14gm - 168m^2$

See Example 11

11 Factorise each of the following.

a $g^2 - 13gp + 30p^2$ b $48u^2 + 14ug - 49g^2$ c $15t^2 + 62tb + 63b^2$

See Examples 9, 10

12 Factorise each of the following.

a $10u^2 + 41u + 40$ b $28b^2 + 9b - 9$

c $56a^2 - 51a - 27$ d $4a^2 - 35a + 49$

Problem solving

See Examples 3, 8

13 Use factorisation to help sketch the graphs of each of the following.

a $y = x^2 + 10x + 25$ b $y = x^2 - 3x - 4$

14 How many integer values of b are there for which $x^2 + bx + 12$ can be factorised using integer values?

15 How many integer values of b are there for which $x^2 + bx - 20$ can be factorised using integer values?

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16 Factorise each of the following expressions. *See* Examples 14, 15

a $63 - 7(3a + 5)^2$

b $3x^2 - 12 + xy - 2y$

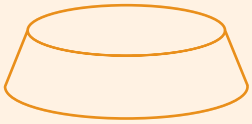
c $3x^4y^2 - 12x^2y^3 + 15x^3y^4$

d $16p^4 - 81q^2$

e $6(p - 4)^2 - 11(p - 4) - 10$

Reasoning

17 A *frustum* is formed by cutting the top off a cone parallel to the base. Show that for a frustum with height h, base radius R and top radius r, the angle θ between the base and the sloping side is given by $\tan\theta = \dfrac{h}{R - r}$. Hence find expressions for the height of the full cone, the height of the part cut off and show that the volume of the frustum is $V = \pi h(R^2 + Rr + r^2)$.



18 How many whole number values of c between 1 and 100 can you find so that $x^2 - 3x - c$ can be factorised using whole numbers?

19 How many integer values of a can you find, strictly between 100 and 200, so that $ax^2 + 2x - 10$ can be factorised using integers?

Measurement and geometry

8 Geometry

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Contents

Prior learning
Chapter 8
MAT10MGPL00008

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Australian Curriculum statements

Geometric reasoning

Formulate proofs involving congruent triangles and angle properties. (ACMMG243)

Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes. (ACMMG244)

Video tutorial
Geometry
MAT10MGVT00008

Geometry is about shapes, lines, space and angles and the relationships between them. The geometry of flat surfaces was first studied in ancient Greece by the mathematician Euclid and this field of study is now known as Euclidean geometry in recognition of his work. Euclid was the founder of *deductive geometry* – the process of arriving at new geometric facts from previously known facts by using logical reasoning. In geometry, a written logical argument is called a *proof*. Geometry has many uses. It is used whenever we are dealing with the size, shape, volume, or position of an object. As a school subject, it helps students to develop logical reasoning skills. Geometry forms the basis of computer-aided design (CAD) which is now used to draw almost all building plans. Geometry also underpins the field of 2D and 3D computer animation which is widely used in the gaming and film industries.

Mathematical literacy

Maths dictionary
MAT10ASDI00001

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

acute-angled triangle
allied angles
alternate angles
angle sum of a quadrilateral
angles at a point
anticlockwise
centre of magnification
centre of rotation
centre of symmetry
clockwise
cointerior angles
complementary angles
congruent
congruent figures
corresponding angles
deductive geometry
diagonally opposite angles of a parallelogram
dilation
enlargement
equilateral triangle
exterior angle of a triangle
irregular polygon
isosceles triangle
kite
line of reflection
magnification
obtuse-angled triangle
parallelogram
plane shape
polygon
quadrilateral
reduction
rectangle
reflection
regular polygon
rhombus
right-angled triangle
rotation
scalene triangle
similar figures
square
sum of angles in a triangle
supplementary angles
transformation
translation
transversal
trapezium
two-dimensional
vertex
vertically opposite angles

8.1 Properties of shapes

Plane shapes (two-dimensional or 2D figures) can be drawn on a flat surface.
A **transformation** changes a shape (an **object**) or its position to form an **image.**

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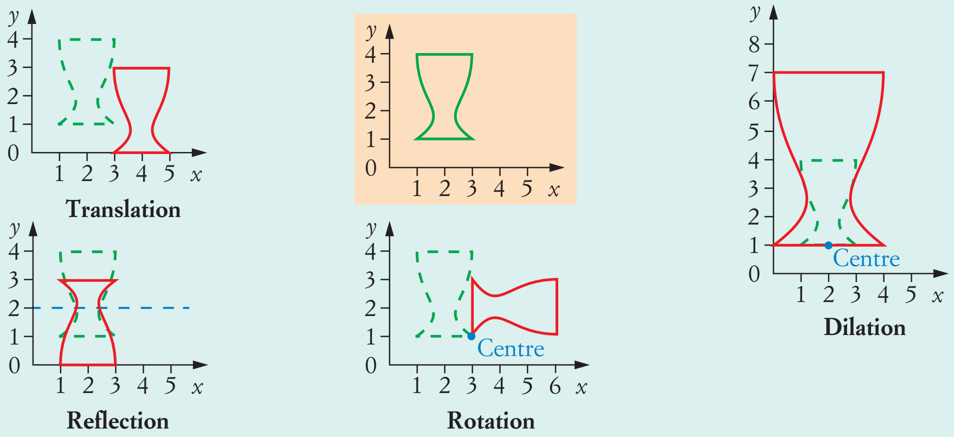
Important!

Transformations

A **translation** is a sliding movement. The vertices of the object are all changed by the same amount to form the image.
A **rotation** is a turn **(clockwise** or **anticlockwise)** through an angle. Each point stays the same distance from the **centre of rotation.**
A dilation is a change of size. The **enlargement** or **reduction** is measured by the scale factor or **magnification.** The **centre of magnification** keeps the same coordinates.
When a shape is **reflected,** all points swap to the other side of the **line of reflection.**

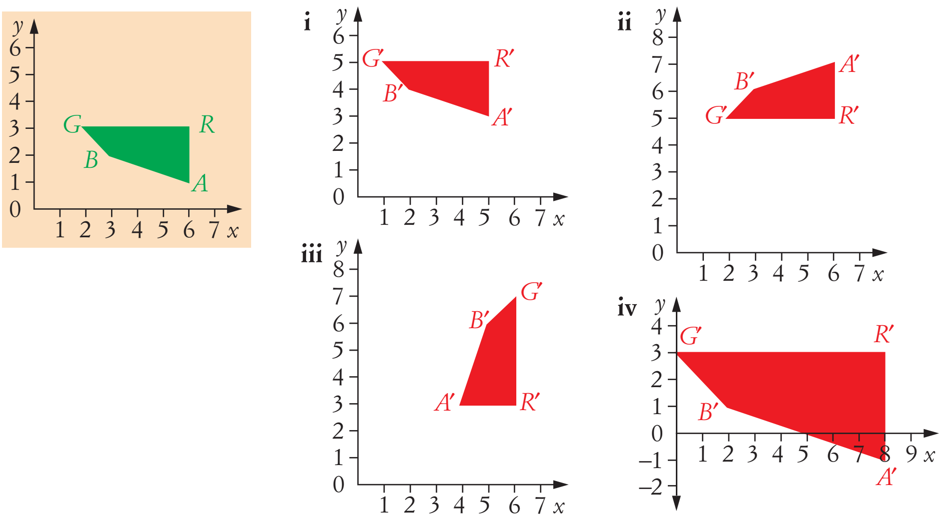


Example 1

Puzzle sheet
Transformations
MAT10MGPS00024

The diagram below shows transformations of the figure *GRAB*.

a Use coordinates to describe each transformation.
b Fully describe each transformation in words.



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Solution

a Write the coordinates of the original figure and the coordinates of the new figure.

i $G(2, 3) \to G'(1, 5)$, $R(6, 3) \to R'(5, 5)$
$A(6,1) \to A'(5, 3)$, $B(3, 2) \to B'(2, 4)$

ii $G(2, 3) \to G'(2, 5)$, $R(6, 3) \to R'(6, 5)$
$A(6,1) \to A'(6, 7)$, $B(3, 2) \to B'(3, 6)$

iii $G(2, 3) \to G'(6, 7)$, $R(6, 3) \to R'(6, 3)$
$A(6,1) \to A'(4, 3)$, $B(3, 2) \to B'(5, 6)$

iv $G(2, 3) \to G'(0, 3)$, $R(6, 3) \to R'(8, 3)$
$A(6, 1) \to A'(8, ^-1)$, $B(3, 2) \to B'(2, 1)$

b **i** The figure has slid up 2 and left 1. **A translation up 2 and left 1.**

ii The figure is reflected upwards. **A reflection in the line $y = 4$.**

iii The figure has turned, but R is the same. **A rotation 90° clockwise about R.**

iv The figure is twice as big, and all points are twice as far from (4, 3) as they were. **A dilation with factor 2 and centre (4, 3).**

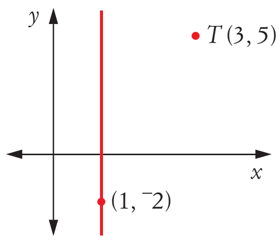
Example 2

What will be the coordinates of $T(3, 5)$ after a reflection in the vertical line passing through $(1, ^-2)$?

Solution

Make a sketch.

T is 2 units to the right of the vertical line passing through $(1, ^-2)$.



T will swap from 2 units to the right of (1, 5) to 2 units to the left of (1, 5), as this is the point opposite T across the line of reflection. **$T(3, 5) \to T'(^-1, 5)$**

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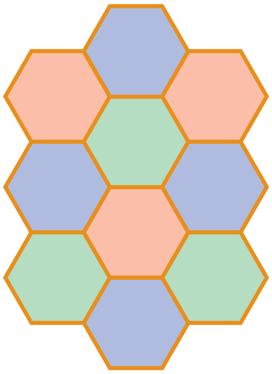
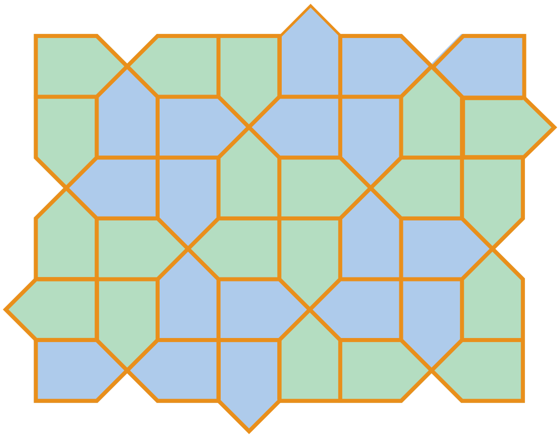
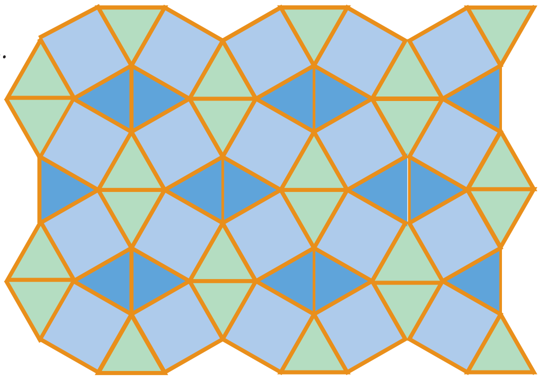
Investigate: Tessellations

A **tessellation** is a pattern that covers an area with a repeating pattern. The pattern formed is symmetrical, but the 'tiles' that make the pattern are not necessarily symmetrical.
A **regular tessellation** is made using regular polygons of the same size.





A **semi-regular tessellation** is made using combinations of (different) regular polygons.



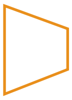
An **irregular tessellation** is made using congruent irregular polygons.
You can see examples of tessellations in floor and wall tiling and in artwork.

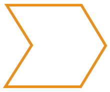
1 Locate examples of tessellations in magazines or on the Internet and classify them as regular, irregular or semi-regular.

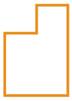
2 Make a chart to display the various examples of tessellations you have found.

3 Create separate tessellations on an A4 sheet of paper using each of the following shapes (use colours).

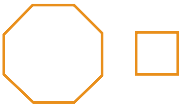
a 

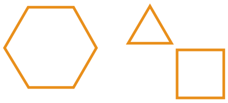
b 

c 

d 

4 Create a semi-regular tessellation on an A4 sheet of paper with each group of shapes, using colours as well.

a 

b 

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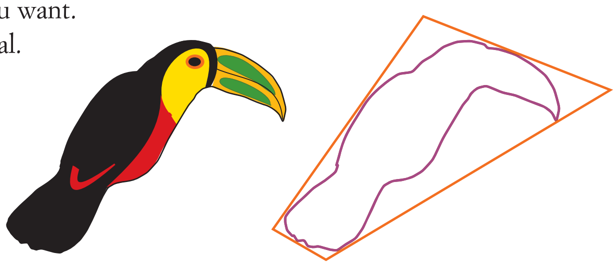
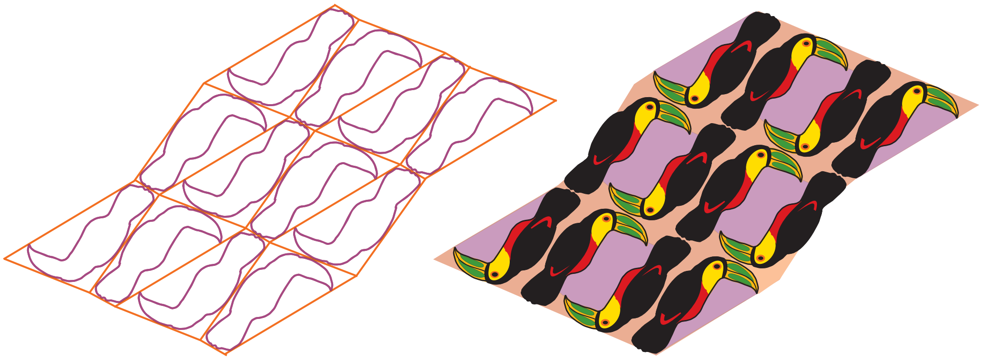
Teacher notes
Quasi-tessellations
MAT10MGTN00007

You can create drawings that are like tessellations, but have recognisable shapes included, by changing the desired shapes into approximate polygons to begin with. It is simplest to use a quadrilateral.

5 Make an outline of the shape you want.
6 Fit the outline into a quadrilateral.



7 On an A3 sheet of paper make a tessellation of copies of the quadrilateral (with an outline).
8 Fill in the spaces with some neutral colours.
9 Colour your shapes.



Your teacher may wish to display some of your finished products in the classroom.

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Most of the important plane shapes have straight sides.

Technology: GeoGebra
Naming polygons
MAT10MGTC00008

Important!

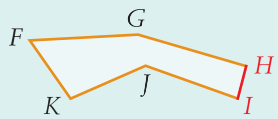
Polygons

Polygons are closed figures with straight sides. A polygon is classified by its number of sides and the nature of its angles and sides.

Number of sides	Name of polygon
3	**Tri**angle
4	**Quad**rilateral
5	**Pent**agon
6	**Hex**agon
7	**Hept**agon

Number of sides	Name of polygon
8	**Oct**agon
9	**Non**agon
10	**Dec**agon
12	**Dodec**agon
20	**Icos**agon

Regular polygons have equal sides and equal angles.
In an **irregular polygon,** at least one of the angles or sides is different from the others.
All the vertices of a **convex** polygon point outwards.



Concave polygons have at least one inward-pointing vertex, so at least one internal angle is greater than 180°.
Polygons are named by writing the **vertices** (corners) in order. These are normally shown by capital letters. The sides of a polygon are named using the two vertices at the ends.
FGHIJK is a concave hexagon. It is irregular. The **red** side is named ***HI***.

Triangles and quadrilaterals are shapes that are commonly used in geometry. It is important for you to know their properties.

Triangles and quadrilaterals

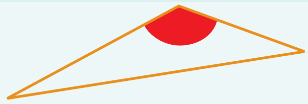
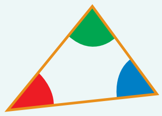
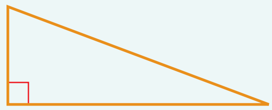
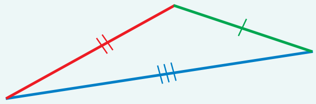
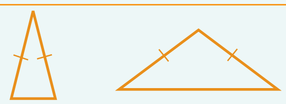
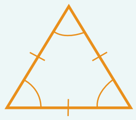
Triangles and some quadrilaterals have special names.

Weblink
Names of angles
MAT10MGWB00008

Important!

Triangles

Triangles can be classified by their angles or by their sides. The sides of triangles are often named using the lower case letter of the opposite vertex.

Name	Property	Example
Obtuse-angled triangle	• An obtuse angle (between 90° and 180°)	
Acute-angled triangle	• All acute angles (less than 90°)	
Right-angled triangle	• A right angle (90°)	
Scalene triangle	• All sides different	
Isosceles triangle	• Two sides equal	
Equilateral triangle	• All sides equal and all angles equal (60°) (regular)	

Quadrilaterals

It is possible to define the special quadrilaterals as follows.
A **trapezium** is a quadrilateral with at least one pair of opposite sides parallel.
A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.
A **rectangle** is a parallelogram with one angle a right angle.
A **square** is a rectangle with two adjacent sides equal in length.
A **rhombus** is a parallelogram with two adjacent sides equal in length.
A **kite** is a convex quadrilateral with two pairs of equal adjacent sides.

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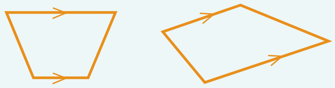
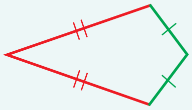
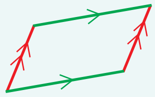
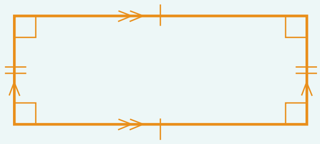
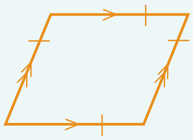
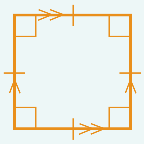
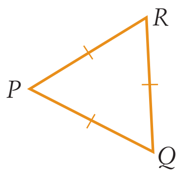
In addition to these definitions, special quadrilaterals have a number of other properties.

Name	Property	Example
Trapezium (trapezoid)	• One pair of parallel sides	
Kite	• Two pairs of equal adjacent sides • One pair of opposite angles equal • One axis of symmetry • Diagonals intersect at right angles	
Parallelogram	• Opposite sides parallel • Opposite sides equal • Opposite angles equal • Diagonals bisect each other • Has rotational symmetry	
Rectangle	• Opposite sides parallel • Opposite sides equal • All angles are right angles • Diagonals are equal in length • Diagonals bisect each other • Two axes of symmetry • Has rotational symmetry	
Rhombus	• Opposite sides parallel • All sides equal • Opposite angles equal • Diagonals bisect each other at right angles • Diagonals bisect the angles of the rhombus • Two axes of symmetry • Has rotational symmetry	
Square	• Opposite sides parallel • All sides equal • All angles are right angles • Diagonals equal in length • Diagonals bisect each other at right angles • Diagonals bisect the angles of the square • Four axes of symmetry • Has rotational symmetry	

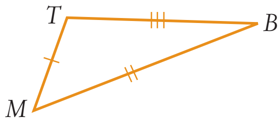
Example 3

Name and classify the following shapes.

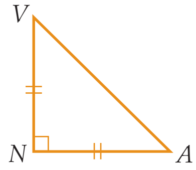
a



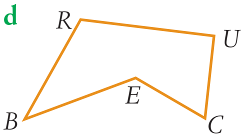
b



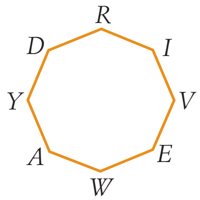
c



d



e



Solution

a This shape has 3 equal sides. ***PRQ* is an acute-angled equilateral triangle.**

b This shape has 3 different sides and an angle between 90° and 180°. ***MTB* is an obtuse-angled scalene triangle.**

c This shape has 3 sides, 2 of which are equal, and one angle is a right angle (90°). ***VAN* is a right-angled isosceles triangle.**

d This shape has 5 sides and an inward-pointing vertex at *E*. ***BRUCE* is an (irregular) concave pentagon.**

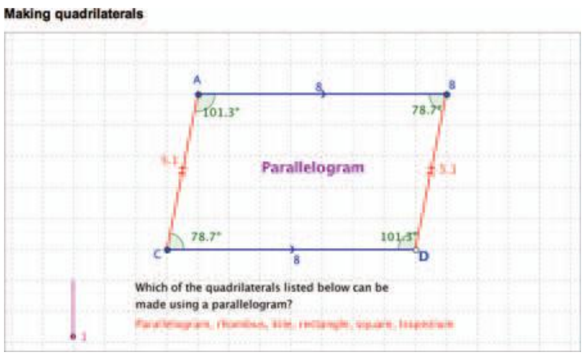
e This shape has 8 sides and it looks as if all the sides and all the angles are equal. ***DRIVEWAY* is a regular (convex) octagon.**

Technology activity

Technology: GeoGebra
Making quadrilaterals
MAT10MGTC00012

Weblink
Sum of interior angles of a quadrilateral
MAT10MGWB00008

The GeoGebra activity 'Making quadrilaterals' is available on the NelsonNet website. It will help you to understand the properties of quadrilaterals and their relationship to each other.



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Example 4

Worksheet

Quadrilaterals: True or false

MAT10MGWK00020

a Is a rectangle a square?
b Is a square a parallelogram?
c A parallelogram has equal diagonals. What kind of quadrilateral is it?
d Give three possible properties, any one of which will ensure that a parallelogram is a rhombus.

Solution

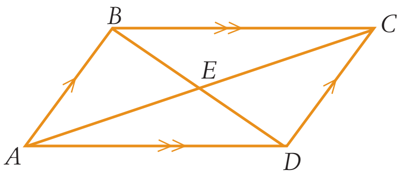
a A rectangle has opposite sides equal but adjacent sides are not necessarily equal.

A square has all sides equal. **A rectangle is not a square.**

b A square has both pairs of opposite sides equal and parallel.

A parallelogram has both pairs of opposite sides parallel. **A square is a parallelogram.**

c Draw the situation.



The diagonals are equal and bisect each other. **From the information, $EA = EB = EC = ED$**

What does this mean about the triangles? **$\triangle ABE$, $\triangle BCE$, $\triangle CDE$ and $\triangle DAE$ are all isosceles so $\angle EBC = \angle ECB$ and $\angle ECD = \angle EDC$.**

Use the parallel lines. **$\angle ABD = \angle EDC$ (alternate angles), so $\angle ABD = \angle ECD$**

Now consider $\angle ABC$ and $\angle BCD$.

$$\begin{aligned}\angle ABC &= \angle EDC + \angle EBC = \angle ECD + \angle ECB \\ &= \angle BCD\end{aligned}$$

Use the fact they are cointerior. **But $\angle ABC + \angle BCD = 180°$**

State the conclusion. **Thus $\angle ABC = \angle BCD = 90°$ and the parallelogram must be a rectangle.**

d A parallelogram has adjacent sides equal. **If two adjacent sides are equal they must all be equal so it is a rhombus.**

The diagonals bisect each other. **If the diagonals are perpendicular then all the triangles formed are congruent (SAS), so it must be a rhombus.**

The angles between a diagonal and side are alternate angles. **If a diagonal bisects the corner angle, then the triangle it forms is isosceles, so its sides are equal and the shape must be a rhombus.**

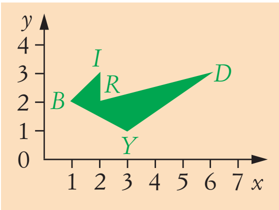
Exercise 8.1 Properties of shapes

Understanding

Extra questions
Exercise 8.1
MAT10MGEQ00022

See Example 1

1 Use coordinates to describe each of the transformations shown below.



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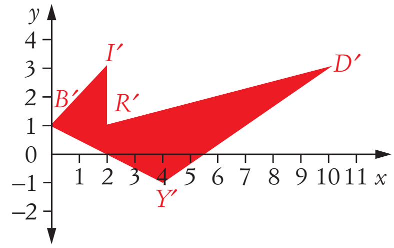

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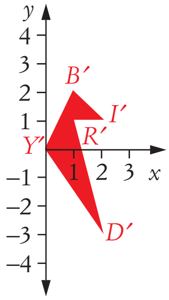

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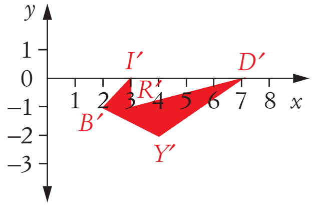

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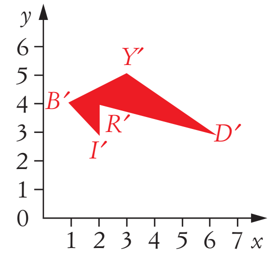

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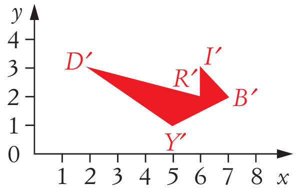

See Example 3

2 Name and classify the following shapes.

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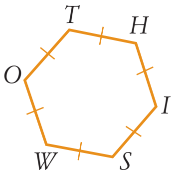

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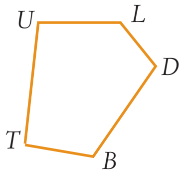

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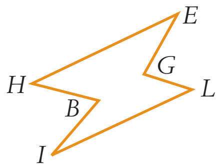

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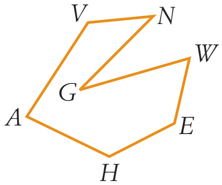

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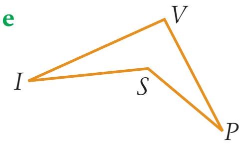

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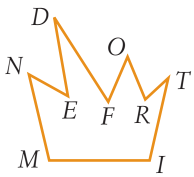

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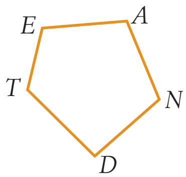

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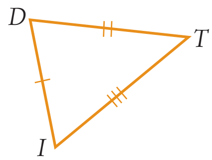

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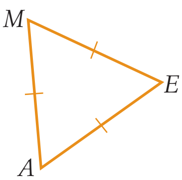

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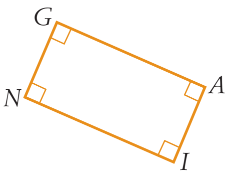

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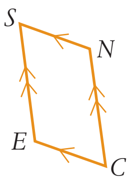

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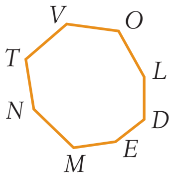

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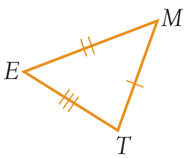

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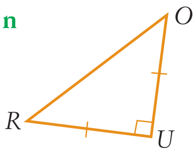

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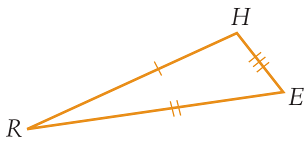

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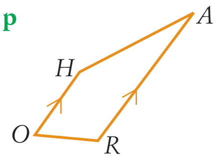

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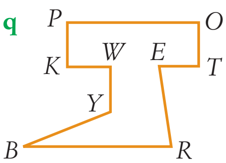

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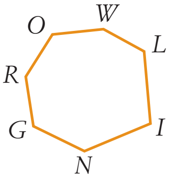

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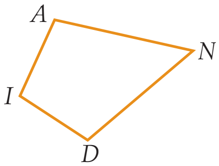

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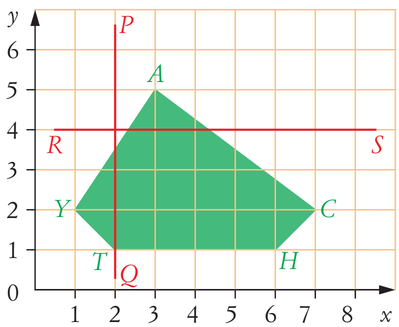
Fluency

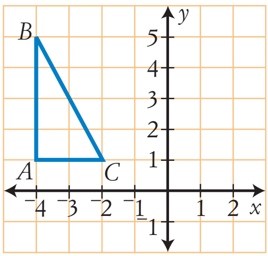
3 Copy the shape shown onto grid paper and show a:

- **a** translation 3 up and 4 left
- **b** translation 2 down and 3 right
- **c** rotation 90° clockwise with centre (2, 1)
- **d** rotation 90° anticlockwise with centre (7, 2)
- **e** reflection in the line PQ
- **f** reflection in the line RS
- **g** magnification with factor 3 and centre (2, 3)
- **h** reduction with factor 0.5 and centre (1, 2).

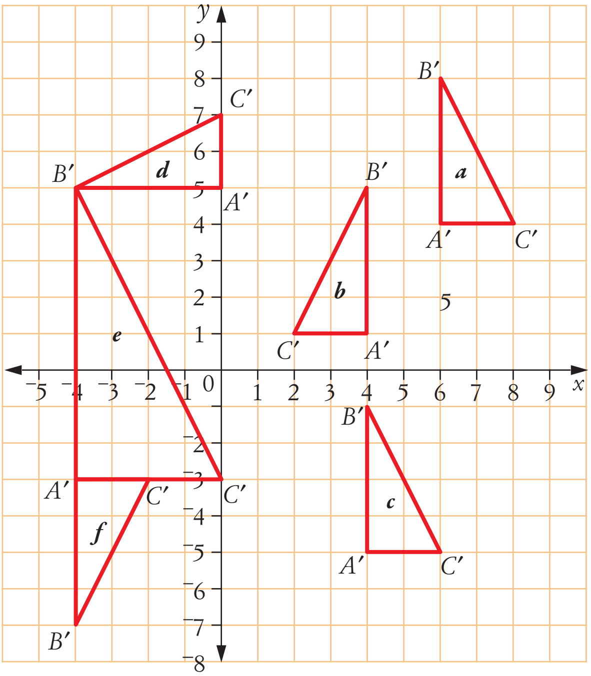


4 The shape ABC shown here has been transformed by a single transformation to produce images ***a***, ***b***, ***c***, ***d***, ***e*** and ***f*** shown below.

See Example 1



Describe each transformation in words.



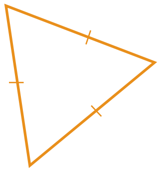
5 Fully describe each of the transformations in question **1** in words.

Worked solutions
Exercise 8.1
MAT10MGWS00022
See Example 2

6 Find the coordinates of $P(1, 3)$, $Q(4, 6)$, $R(5, 3)$ and $S(2, 2)$ after a:

a translation 1 left and 2 down
b translation 3 up and 2 right
c reflection in the line $x = 3$
d reflection in the line $y = 1$
e rotation 180° clockwise with centre (3, 2)
f rotation 90° anticlockwise with centre (1, 3)
g magnification with factor 2 and centre (3, 5)
h reduction with factor $\frac{1}{2}$ and centre (2, 4).

7 Use the properties of triangles and quadrilaterals to name each of the following shapes.

a

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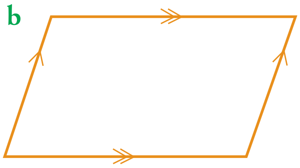
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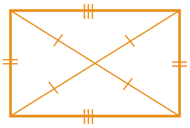
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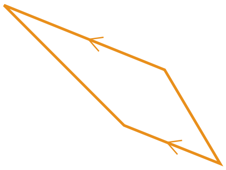
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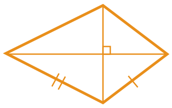
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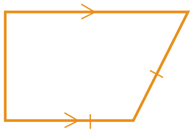
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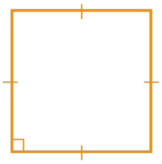
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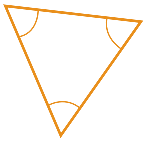
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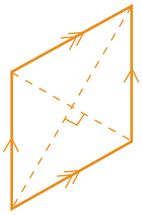
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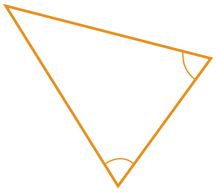
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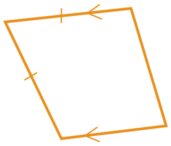
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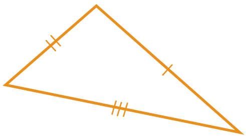

See Example 4

8 State whether each of the following is true (T) or false (F).

a The diagonals of a square, a rhombus and a rectangle bisect each other.
b The angles of a rectangle are equal to 90°.
c A kite and an isosceles triangle both have only one axis of symmetry.
d An isosceles triangle is an equilateral triangle.
e A rhombus has two axes of symmetry.
f Opposite sides of a kite are equal.
g The sides opposite the equal angles of an isosceles triangle are equal.
h Adjacent sides of a parallelogram are equal.
i A square is a rhombus.
j A scalene triangle has rotational symmetry.
k The diagonals of a square are equal.
l The diagonals of a rhombus bisect each other at right angles.
m The diagonals of a kite are equal.
n An equilateral triangle has only two axes of symmetry.

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Problem solving

Worked solutions
Exercise 8.1
MAT10MGWS00022

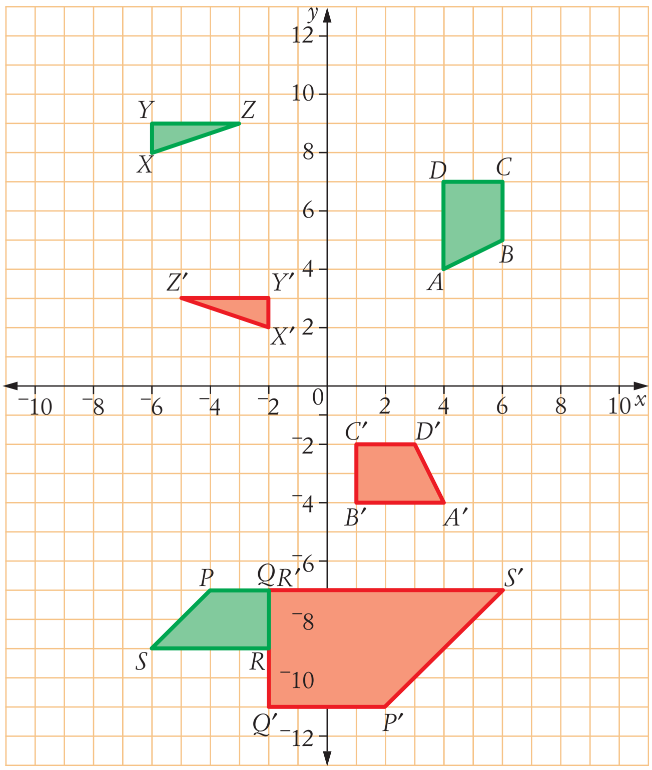
9 An object with vertices $A(1, ^-3)$, $B(^-4, 1)$, $C(^-3, 7)$, $D(3, 7)$ and $E(4, 3)$ is reflected in the line $y = x$. Find the coordinates of the image $A'B'C'D'E'$.

10 An object with vertices $X(^-3, 1)$, $Y(^-3, 4)$ and $Z(^-1, 4)$ is subjected to a number of separate transformations. The coordinates of the image of XYZ after each transformation are given below. In each case, fully describe the transformation made to XYZ to produce $X'Y'Z'$.

- **a** $X(^-3, 1) \rightarrow X'(^-2, 8)$, $Y(^-3, 4) \rightarrow Y'(1, 8)$, $Z(^-1, 4) \rightarrow Z'(1, 6)$
- **b** $X(^-3, 1) \rightarrow X'(5, 6)$, $Y(^-3, 4) \rightarrow Y'(5, 9)$, $Z(^-1, 4) \rightarrow Z'(7, 9)$
- **c** $X(^-3, 1) \rightarrow X'(^-7, ^-7)$, $Y(^-3, 4) \rightarrow Y'(^-7, ^-4)$, $Z(^-1, 4) \rightarrow Z'(^-5, ^-4)$
- **d** $X(^-3, 1) \rightarrow X'(3, 1)$, $Y(^-3, 4) \rightarrow Y'(3, 4)$, $Z(^-1, 4) \rightarrow Z'(1, 4)$
- **e** $X(^-3, 1) \rightarrow X'(^-3, ^-1)$, $Y(^-3, 4) \rightarrow Y'(^-3, ^-4)$, $Z(^-1, 4) \rightarrow Z'(^-1, ^-4)$
- **f** $X(^-3, 1) \rightarrow X'(^-7, 1)$, $Y(^-3, 4) \rightarrow Y'(^-7, 4)$, $Z(^-1, 4) \rightarrow Z'(^-9, 4)$
- **g** $X(^-3, 1) \rightarrow X'(^-3, 11)$, $Y(^-3, 4) \rightarrow Y'(^-3, 8)$, $Z(^-1, 4) \rightarrow Z'(^-1, 8)$
- **h** $X(^-3, 1) \rightarrow X'(^-3, ^-5)$, $Y(^-3, 4) \rightarrow Y'(^-3, 4)$, $Z(^-1, 4) \rightarrow Z'(3, 4)$

Reasoning

11 Objects $ABCD$, XYZ and $PQRS$ are shown in the diagram below. In each case, the object has undergone one transformation followed by another to form images $A'B'C'D'$, $X'Y'Z'$ and $P'Q'R'S'$ respectively.



Fully describe the two transformations that have been used in each case.

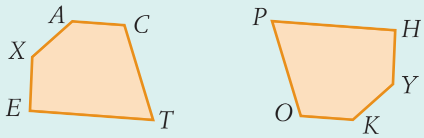
8.2 Similarity and congruence

You looked at a number of transformations in the previous section of this book. When an object is subjected to a translation, reflection or rotation, its image is exactly the same shape and size – the object and image are identical. However, when an object is subjected to a dilation, the image is the same shape but a different size. It is either enlarged or reduced by a scale factor. In geometry there are special terms to describe these situations.

Important!

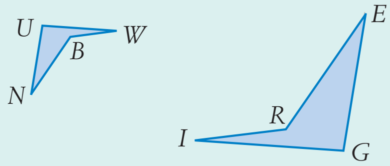
Similarity and congruence

Congruent shapes are exact copies of each other. The figures are exactly the same shape and size. The symbol for congruency is ≡.



ACTEX ≡ KOPHY

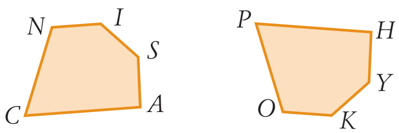
Similar figures are exactly the same shape but not necessarily the same size. The symbol for similarity is |||.



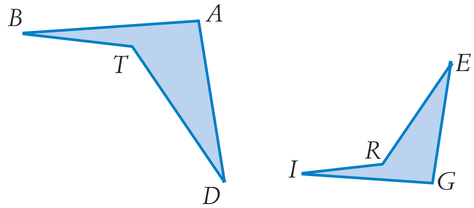
UWBN ||| GIRE

When naming similar or congruent figures, corresponding vertices are named in the same order (as above). For congruent and similar figures, the **corresponding angles** in each figure are the same.
The sides opposite corresponding angles are called **corresponding** sides.
In similar shapes, corresponding angles are equal *and* the ratio of corresponding sides is equal. This ratio is called the **scale factor**. The lengths of one shape are multiplied by the scale factor to work out the corresponding lengths in the other shape. In congruent figures they are the same, so the scale factor is 1.

Even when a shape must be flipped over to form a mirror image, it is considered congruent or similar to another. This is shown in the following diagrams.



INCAS ≡ KOPHY



ABTD ||| GIRE

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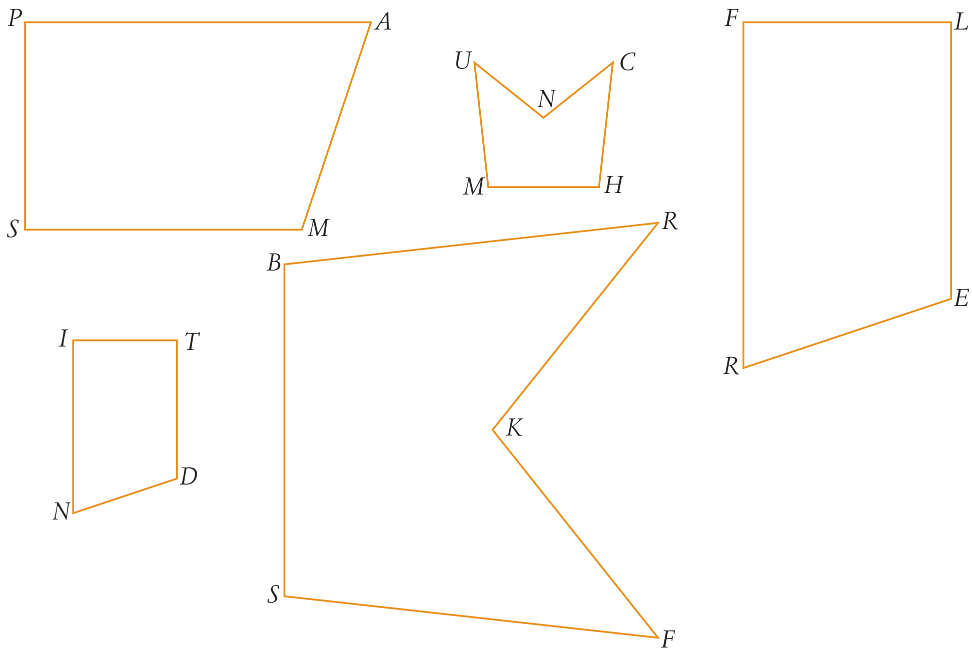
Example 5

TLF Learning object
Measures: Similar shapes (L2309)
MAT10MGIN00008

Measure the sides and angles of the following shapes. Name any shapes that are either congruent or similar. (Don't include congruent shapes as similar shapes.) Work out the scale factors of similar shapes.



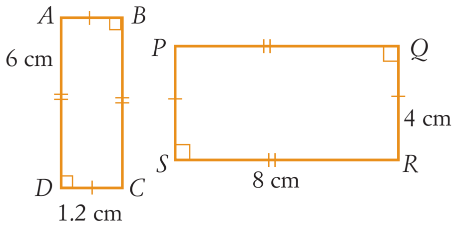
Solution

Compare shapes by measuring corresponding angles and lengths of corresponding sides.	
SPAM and *LFRE* are mirror images that are exactly the same shape and size.	***SPAM* ≡ *LFRE***
MUNCH and *BRKFS* are the same shape, but different sizes.	***MUNCH* ||| *BRKFS***
BR = 54 mm = 3 × 18 mm = 3 × *MU*.	**The scale factor is 3.**
SPAM and *TIND* are mirror images that are the same shape but different sizes.	***SPAM* ||| *TIND***
SP = 30 mm and *TI* = 15 mm.	**The scale factor is $\frac{1}{2}$ (0.5).**
SPAM ≡ *LFRE*, so *LFRE* and *TIND* are also similar. Each side in *TIND* is half the length of the corresponding side in *LFRE*.	***LFRE* ||| *TIND* (scale factor = 0.5)**

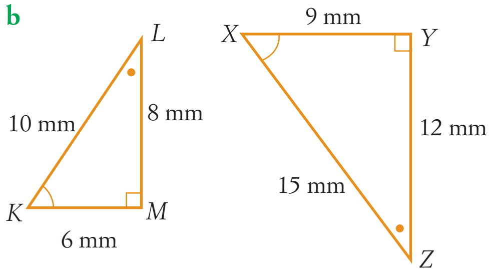
Example 6

Without direct measurement determine if the following pairs of figures are similar.

a



b



Solution

a The figures are both rectangles.

The angles of *ABCD* and *PQRS* are the same.

Find the ratio of corresponding sides.

$$\frac{AB}{QR} = \frac{1.2\text{ cm}}{4\text{ cm}} = \frac{12}{40} = \frac{3}{10}$$

$$\frac{AD}{PQ} = \frac{6\text{ cm}}{8\text{ cm}} = \frac{3}{4}$$

The ratio of corresponding sides is not equal.

***ABCD* and *PQRS* are not similar as the ratio of corresponding sides is not equal.**

b Measure the angles of $\triangle KLM$ and $\triangle XYZ$.

The corresponding angles of $\triangle KLM$ and $\triangle XYZ$ are equal, so the triangles have the same shape.

Find the ratio of corresponding sides.

$$\frac{KM}{XY} = \frac{6\text{ mm}}{9\text{ mm}} = \frac{2}{3}$$

$$\frac{ML}{YZ} = \frac{8\text{ mm}}{12\text{ mm}} = \frac{2}{3}$$

$$\frac{KL}{XZ} = \frac{10\text{ mm}}{15\text{ mm}} = \frac{2}{3}$$

$\triangle KLM$ and $\triangle XYZ$ are similar as they are the same shape and the ratio of corresponding sides is equal.

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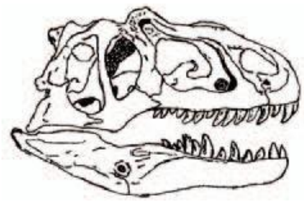
A scale drawing can be used as a two-dimensional representation of an object. The scale drawing and the view of the object are similar figures.

Example 7

Here is a scale drawing of the skull of a dinosaur called Allosaurus fragilis – a large meat-eating predator. The scale used is 1 : 20. What is the actual width of the dinosaur's skull?



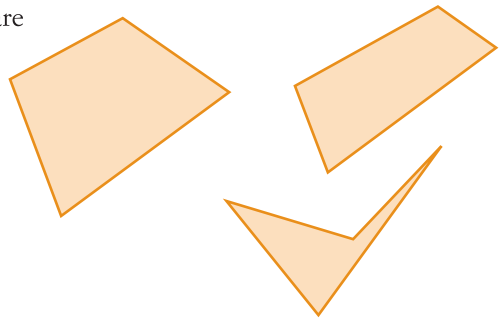
Solution

Measure the width of the skull on the drawing.	**Drawing skull width = 4 cm**
Write the scale.	**Scale = 1 : 20 =** $\frac{1}{20}$
1 : 20 means that 1 cm on the drawing represents 20 cm on the object.	$\frac{1}{20} = \frac{\text{drawing distance}}{\text{object distance}}$
Substitute for drawing skull width.	$\frac{1}{20} = \frac{4\text{ cm}}{\text{object distance}}$
Cross multiply.	**Object distance × 1 = 4 cm × 20**
Evaluate.	**= 80 cm**
State the result.	**The dinosaur's skull is 80 cm wide.**

Investigate: Angles and sides

Of these three quadrilaterals, two have the same sides and two have the same angles, but none are congruent or similar.

Draw each of the following pairs so that they are neither congruent nor similar, if you can.



1 Two convex quadrilaterals with the same sides in the same order
2 Two pentagons with the same sides in the same order
3 Two pentagons with the same angles in the same order
4 Two triangles with the same sides
5 Two triangles with the same angles

Discuss your findings as a class. Are there any general rules?

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You can use similar figures to find unknown sides.

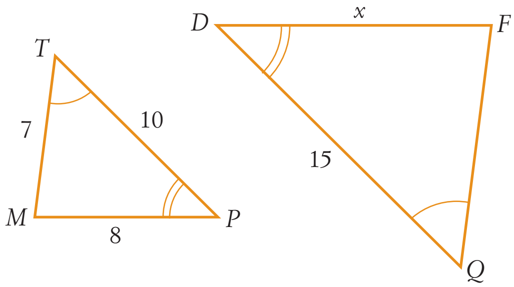
CAS TI-Nspire exercise
Geometry
MAT10MGTI00008

CAS ClassPad exercise
Geometry
MAT10MGCP00008

Puzzle sheet
Similar triangles
MAT10MGPS00025

Example 8

The triangles shown are similar. Find the value of x.



Solution

Name the vertices in corresponding order. $\triangle MTP \mathrel{|||} \triangle FQD$

Find the scale factor using corresponding sides.

$$\text{Scale factor} = \frac{DQ}{PT} = \frac{15}{10} = 1.5$$

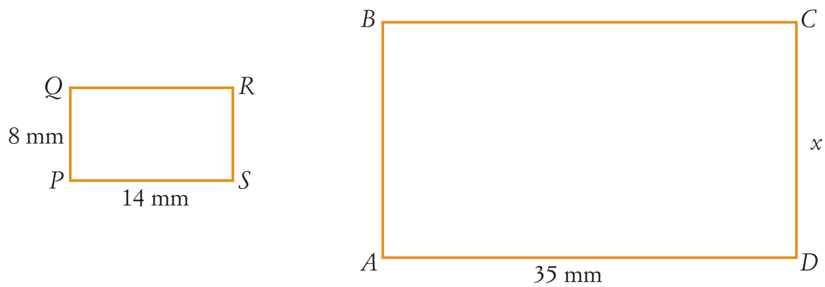
Use the scale factor to find x from the corresponding side.

$$\begin{aligned} x &= 1.5 \times MP \\ &= 1.5 \times 8 \\ &= 12 \end{aligned}$$

In the previous example, the scale factor was used to calculate the length of the unknown side of the similar triangle. You can also use the fact that the ratio of corresponding sides of similar figures is equal to calculate unknown sides.

Example 9

If $PQRS \mathrel{|||} ABCD$, calculate the value of x.



Solution

Identify corresponding sides. $PQ \leftrightarrow AB$, $QR \leftrightarrow BC$, $RS \leftrightarrow CD$ and $SP \leftrightarrow DA$

Write a statement using the ratio of corresponding sides. $\dfrac{PQ}{AB} = \dfrac{SP}{DA}$

Substitute for known values. $\dfrac{8}{x} = \dfrac{14}{35}$

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Cross multiply.	$8 \times 35 = 14 \times x$
Reverse the equation.	$14 \times x = 8 \times 35$
Divide both sides by 14 and cancel.	$\dfrac{\cancel{14}^{1} \times x}{\cancel{14}_{1}} = \dfrac{\cancel{8}^{4} \times \cancel{35}^{5}}{\cancel{14}_{\cancel{7}_{1}}}$
Evaluate.	$= 20$
State the result including the unit.	$x = 20$ **mm**

Similar triangles can be used to solve real world problems.

Example 10

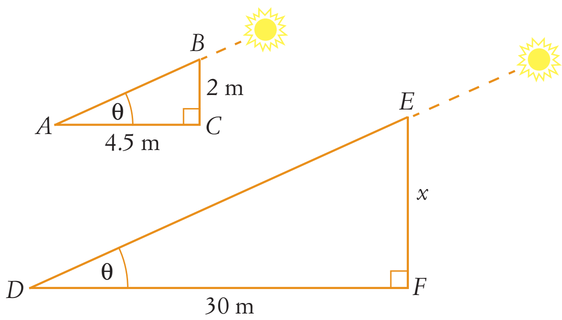
Alternative methods
Similar triangles
MAT10MGAM00001

An electricity pole casts a shadow of 30 m. At the same time, a 2 m stick held vertically casts a 4.5 m shadow. Find the height of the pole.

Solution

Draw a diagram, putting in all the information.

The angle of the sun will be the same in both cases, because it is at the same time.



Two angles are the same, so the third is also the same.	$\triangle ABC \mathrel{\vert\vert\vert} \triangle DEF$ (AAA)
Find the scale factor, using corresponding sides.	Scale factor $= \dfrac{DF}{AC} = \dfrac{30}{4.5}$ $= 6\dfrac{2}{3}$
Use the scale factor to find x from the corresponding side.	$x = 6\frac{2}{3} \times BC$ $= 6\frac{2}{3} \times 2$ ≈ 13.3 **m**
Write the answer.	**The pole is about 13.3 m high.**

All polygons can be divided into triangles, so congruent and similar triangles are used extensively in geometric proofs.

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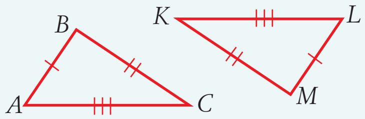
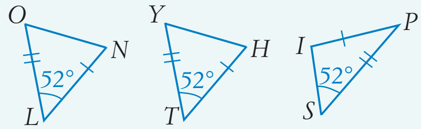
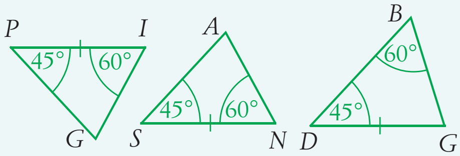
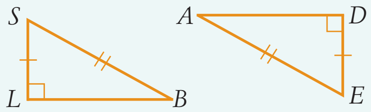
TLF Learning object
Congruent triangles (L3517)
MAT10MGIN00008

There are four tests (proofs) of triangle congruency.

It is important to understand the difference between a **demonstration** and a **proof**. Lying one triangle exactly on top of the other is a very powerful demonstration of congruency, but does not prove they are congruent. A proof uses a logical sequence of steps, as shown in the next two examples. These use the congruency and similarity tests.

Important!

Triangle congruency tests

Relationship	Example
Side/side/side (SSS) The three corresponding sides are equal.	 $\triangle ABC \equiv \triangle LMK$ (SSS)
Side/angle/side (SAS) Two sides and the *enclosed* angle are equal. (The angle must be between the sides.)	 $\triangle OLN \equiv \triangle YTH$ (SAS)
Angle/side/angle (ASA) or angle/angle/side (AAS) Two angles and the *corresponding* side are equal. (The side must be in the same position in relation to the angles in each triangle.)	 $\triangle PIG \equiv \triangle SNA$ (ASA)
Right angle/hypotenuse/side (RHS) The hypotenuse and a corresponding side are equal in two right-angled triangles.	 $\triangle SBL \equiv \triangle EAD$ (RHS)

Note that, in the second example above, $\triangle IPS$ may not be congruent with $\triangle OLN$ and $\triangle YTH$ because the angle is not enclosed by the sides. In the third example, $\triangle BGD$ has the known side in a different position relative to the angles, compared with $\triangle PIG$ and $\triangle SNA$.

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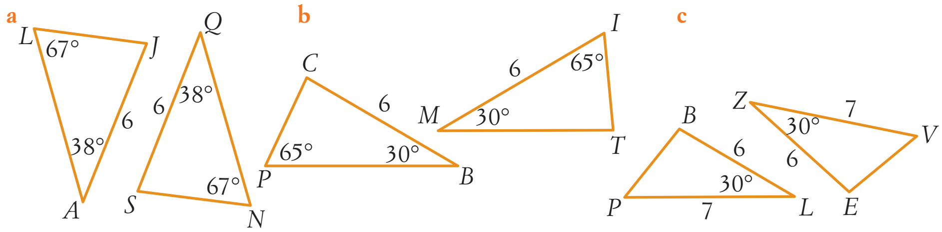
Example 11

Worksheet
Congruent triangle tests
MAT10MGWK00022

Prove congruence for any of the following pairs of triangles that are congruent.



Solution

a It looks like ASA. Do an angle. $\angle JLA = 67° = \angle SNQ$

Do the side. $AJ = 6 = QS$

Do the other angle. $\angle LAJ = 38° = \angle NQS$

State the result and the test used. $\triangle JLA \equiv \triangle SNQ$ (ASA)

b In $\triangle PCB$ 6 is opposite 65°, but in $\triangle JMT$ it is opposite 85°. **Two angles are equal but the corresponding side is not. The triangles are not necessarily congruent.**

c It looks like SAS. Do a side. $BL = 6 = EZ$

Do the angle. $\angle BLP = 30° = \angle EZV$

Do the other side. $LP = 7 = ZV$

State the result and the test used. $\triangle BLP \equiv \triangle EZV$ (SAS)

In Example **11**, the triangles in part **b** *could* be congruent but we cannot be sure they are because we cannot prove it. It would be equally wrong to say they are not congruent as to say they are congruent because we cannot prove it either way.

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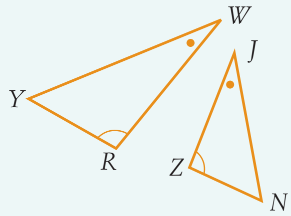
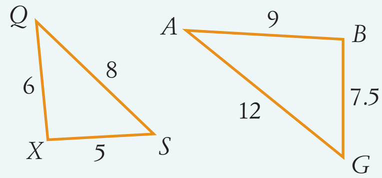
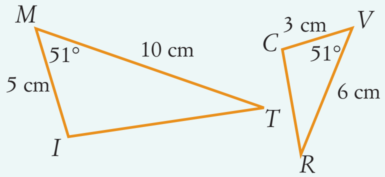
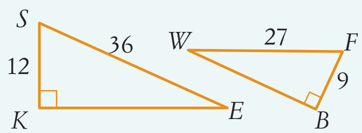
Important!

Triangle similarity tests

Relationship	Example
Angle/angle/angle (AAA) or **Angle/angle (AA)** The angles of each of the triangles are the same. Two angles the same is good enough because the third must then be the same anyway, so some people just use AA as the test. In the example, $\triangle YWR \mid\mid\mid \triangle NJZ$ (AAA).	
Side/side/side (SSS) The three corresponding sides of the triangles are in the same ratios. In the example, $\triangle QSX \mid\mid\mid \triangle AGB$ (SSS).	
Side/angle/side (SAS) Two corresponding sides of the triangles are in the same ratio and the enclosed angles are equal. In the example, $\triangle IMT \mid\mid\mid \triangle CVR$ (SAS).	
Right angle/hypotenuse/side (RHS) The hypotenuse and a corresponding side are in the same ratio in two right-angled triangles. In the example, $\triangle SKE \mid\mid\mid \triangle FBW$ (RHS).	

In proving that the ratios of corresponding sides are equal it doesn't matter whether you use the ratios between corresponding sides within the triangles or corresponding sides between the triangles (the scale factor).

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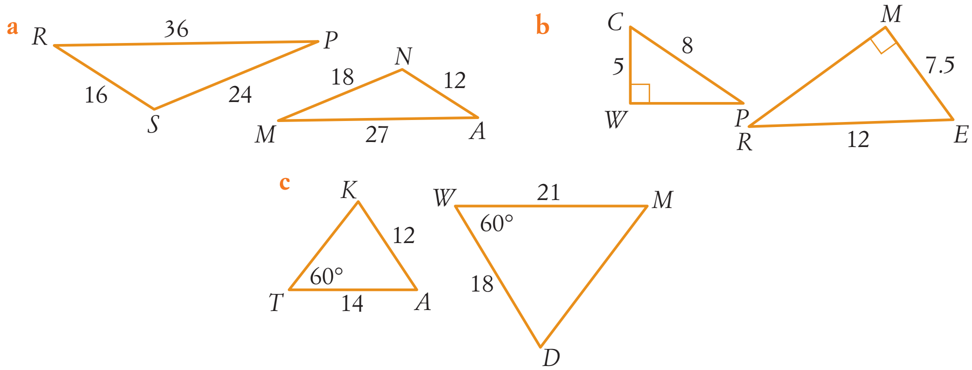
Example 12

Prove similarity for any of the following pairs of triangles which are similar.



Solution

a It might be SSS. The scale factor is the easiest for this one. $\quad RP : PS : SR = 36 : 24 : 16$

Multiply by the scale factor $\frac{3}{4}$. $\quad = 27 : 18 : 12$

Replace with side names. $\quad = AM : MN : NA$

State the result and the test used. $\quad \triangle RPS \text{ ||| } \triangle AMN$ **(SSS)**

b It might be RHS. Do the angle first. $\quad \angle PWC = \angle MER = 90°$

Do the ratio for the first triangle. $\quad WC : CP = 5 : 8$

Simplify the ratio for the other triangle. $\quad ME : ER = 7.5 : 12 = 15 : 24 = 5 : 8$

State the ratio equality. $\quad WC : CP = 5 : 8 = ME : ER$

State the result and the test used. $\quad \triangle PWC \text{ ||| } \triangle MER$ **(RHS)**

c It might be SAS. **In $\triangle KAT$, the 60° angle is opposite a known side, but in $\triangle WMD$ it is between the known sides.**

The positions are not corresponding. **The triangles are not necessarily similar.**

In part **a** of Example **12** you could compare the ratios of the sides within the triangles. For $\triangle RPS$ the ratios are $\frac{RP}{PS} = \frac{36}{24} = 3 : 2$, $\frac{RS}{SP} = \frac{16}{24} = 2 : 3$ and $\frac{RS}{RP} = \frac{16}{36} = 4 : 9$, while for $\triangle AMN$ the ratios are $\frac{AM}{NM} = \frac{27}{18} = 3 : 2$, $\frac{AN}{MN} = \frac{12}{18} = 2 : 3$ and $\frac{AN}{NM} = \frac{12}{27} = 4 : 9$, so the ratios of corresponding sides are equal. In fact, two comparisons for each triangle are enough, because then the third must be the same anyway.

In part **b**, you could also use the scale factor for the sides, which works out to be $1\frac{1}{2}$.

Exercise 8.2 Similarity and congruence

Understanding

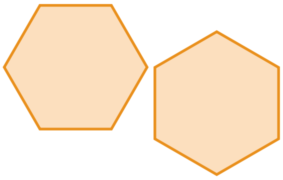
Extra questions

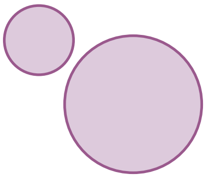
Exercise 8.2

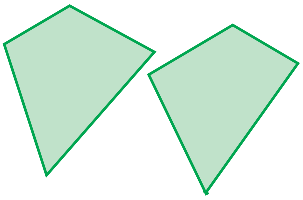
MAT10MGEQ00023

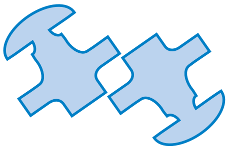
See Example 5

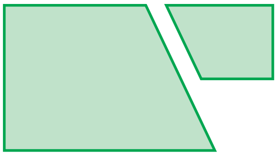
1 State whether each pair of shapes is congruent, similar or neither. Work out the scale factor for similar shapes.

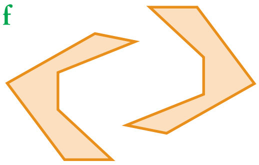
a 

b 

c 

d 

e 

f 

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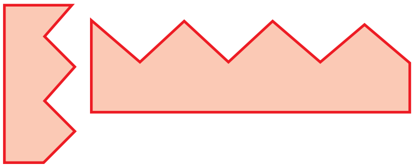
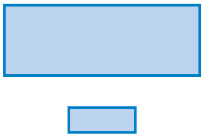
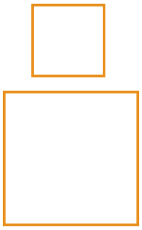
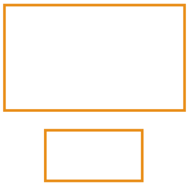
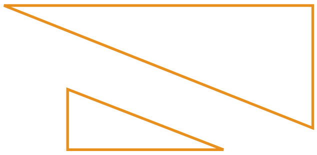
h 

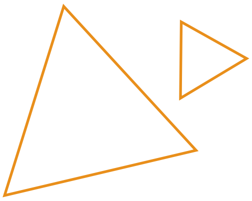
i 

2 Using measurement, find out which of the following pairs of figures are similar.

a 

b 

c 

d 

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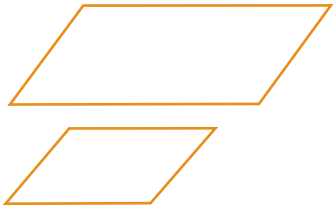
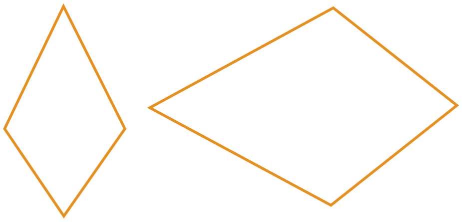
f 

g 

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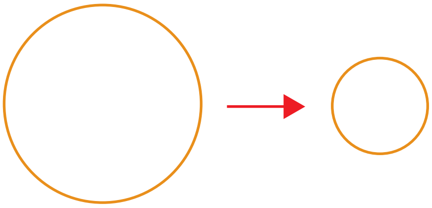
3 Each of the following pairs of shapes is similar. By measuring with a ruler, find the scale factor of the enlargement or reduction for each of these pairs of diagrams.

a



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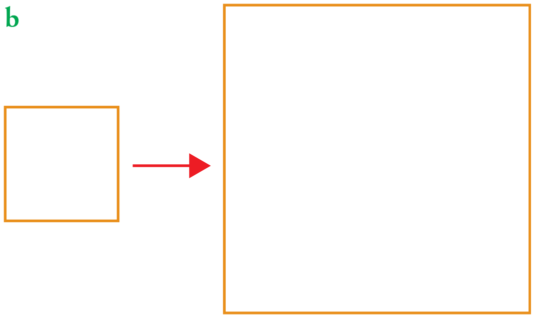

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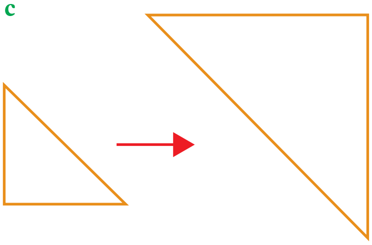

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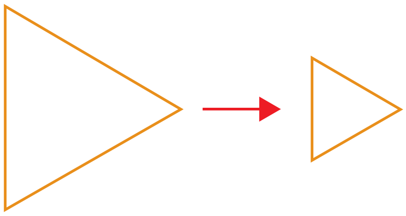
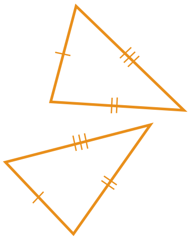


4 Each of the following pairs are congruent. Which congruency test (SSS, SAS, AAS or RHS) does each pair of triangles satisfy?

a



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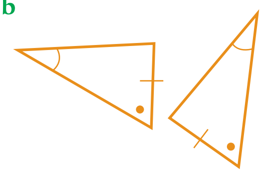

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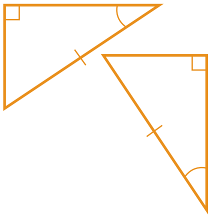

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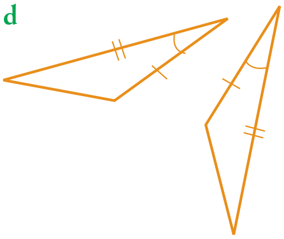

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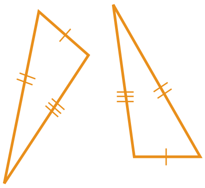

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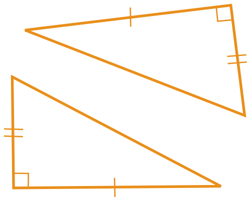

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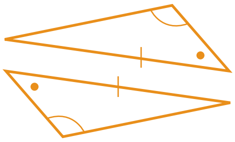

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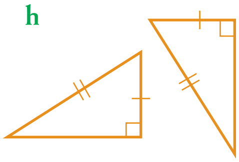

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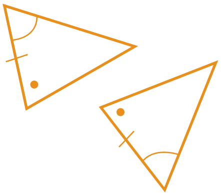
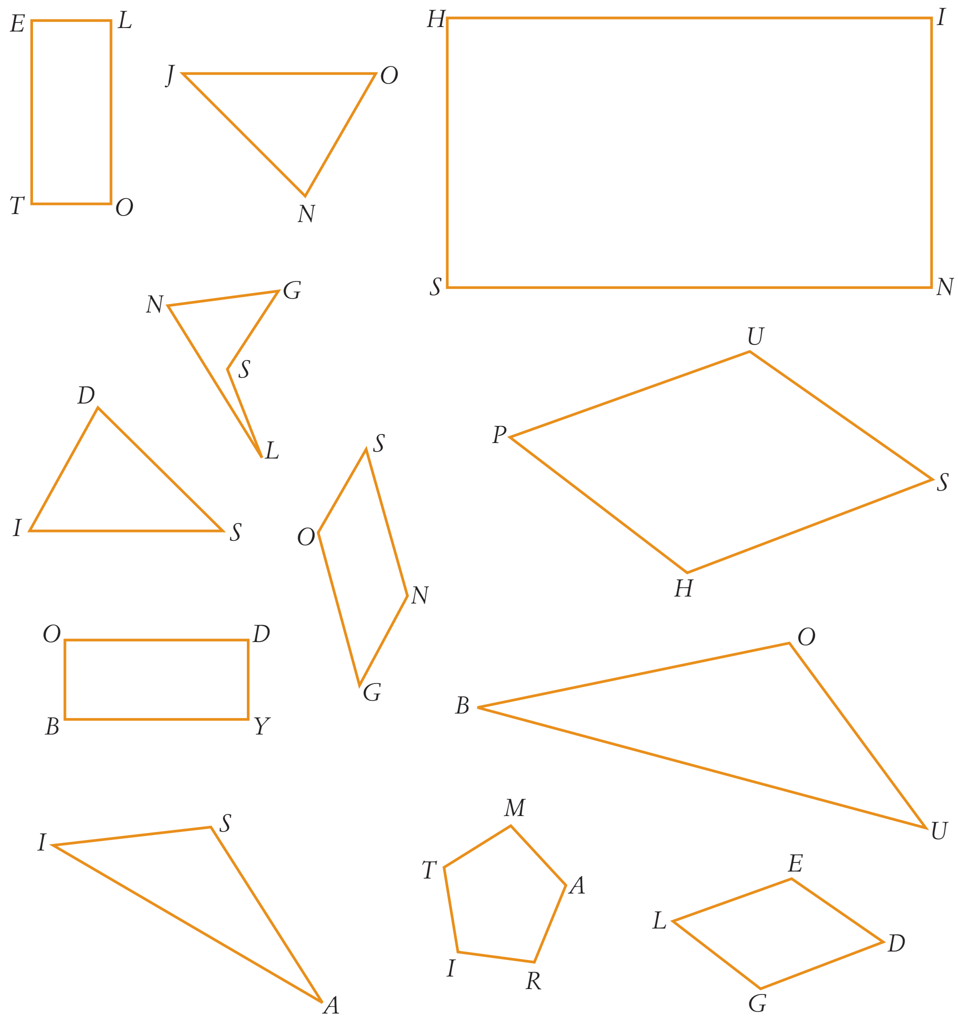


Fluency

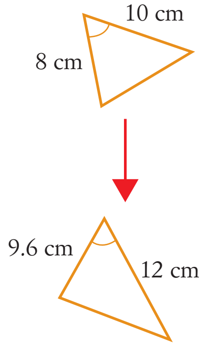
5 Name similar and congruent shapes below. Work out the scale factor for similar shapes.



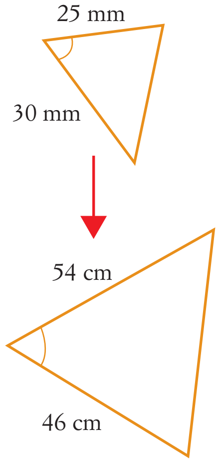
See Example 6

6 The following pairs of figures are not drawn to scale. In each case, determine if the shapes are similar. For similar shapes, calculate the scale factor.

a



b



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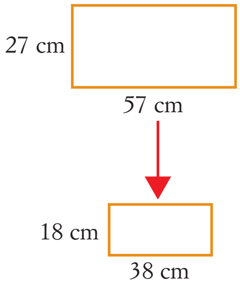
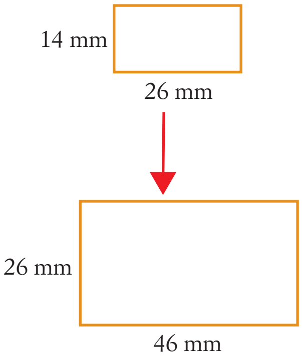
c

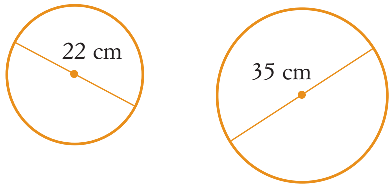


d

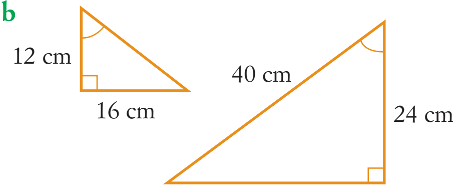


7 The following pairs of figures are not drawn to scale. Work out which ones are similar.

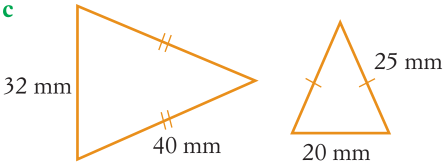
a



b



c



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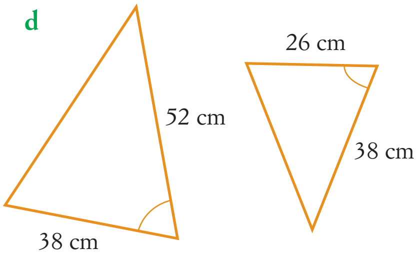

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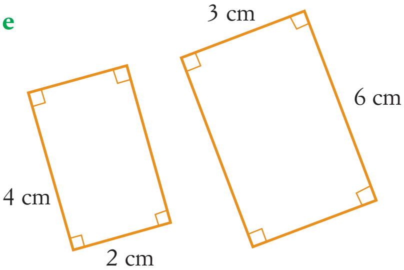

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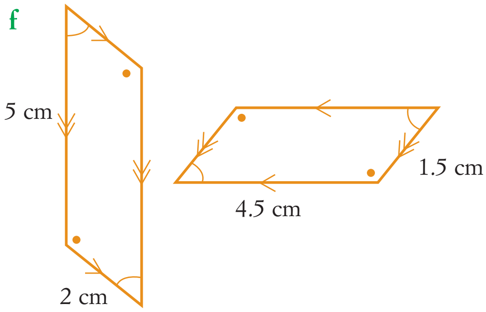
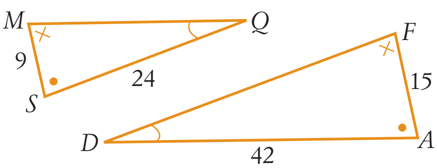


8 Name any pairs of triangles below that are similar.

a



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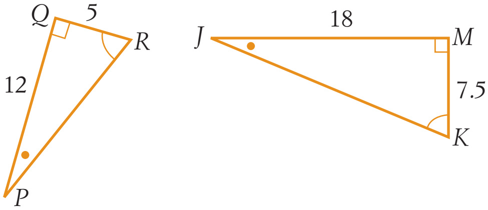
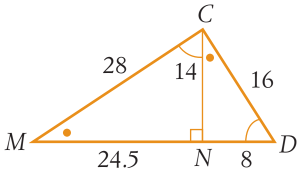


c



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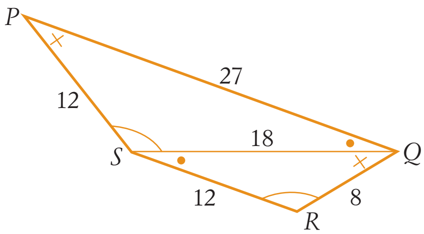


9 **a** Draw $\triangle ABC$ with $AB = 60$ mm, $BC = 80$ mm and $CA = 40$ mm.

b Draw $\triangle HQP$ with $HQ = 90$ mm, $QP = 120$ mm and $HP = 60$ mm.

c Are the matching sides of the triangles you drew in parts **a** and **b** in the same ratio?

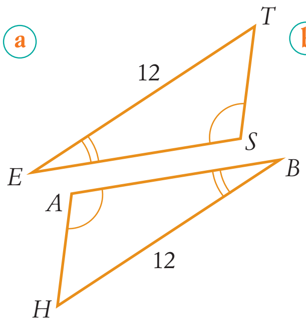
d Are the matching angles of the triangles equal?

e Are $\triangle ABC$ and $\triangle HQP$ similar? Why?

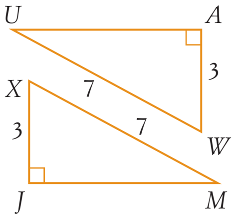
Worked solutions
Exercise 8.2
MAT10MGWS00023
See Example 11

10 Prove congruence for any of the following pairs of triangles that are congruent.

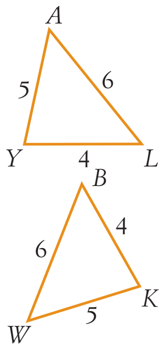
a



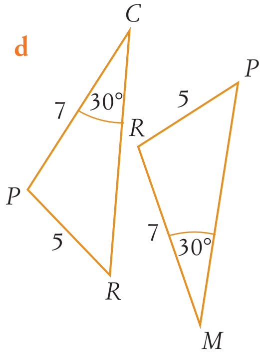
b



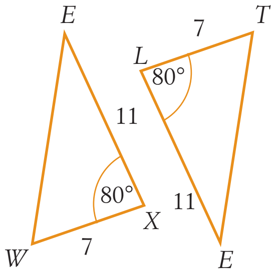
c



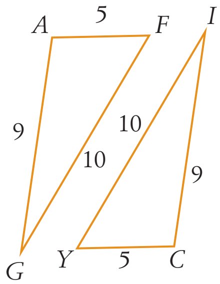
d



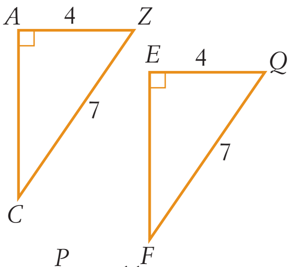
e



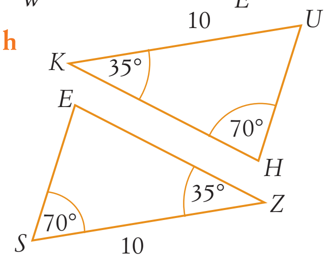
f



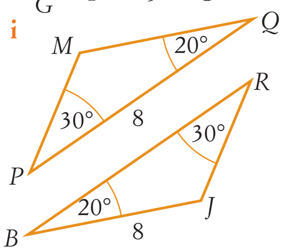
g



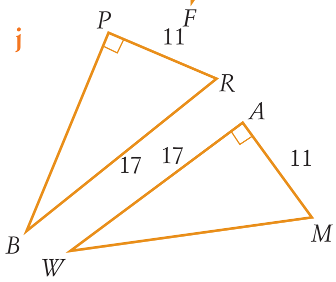
h



i



j



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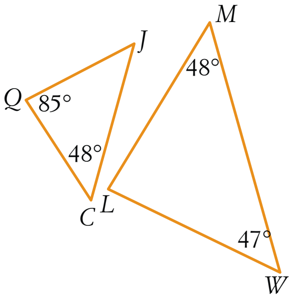
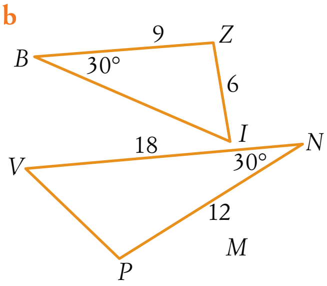
11 Prove similarity for any of the following pairs of triangles that are similar. *See* Example 12

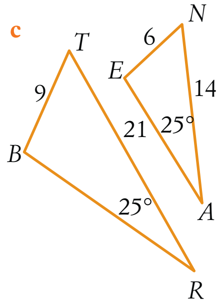
a



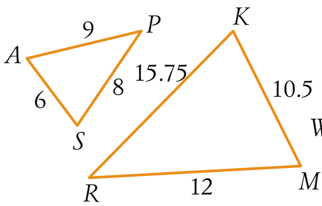
b



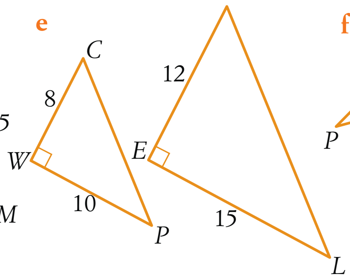
c



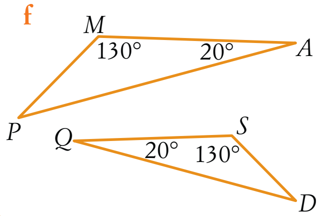
d



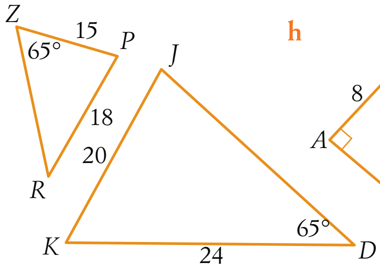
e



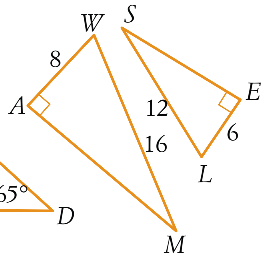
f



g



h



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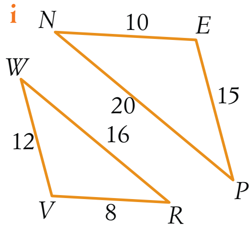
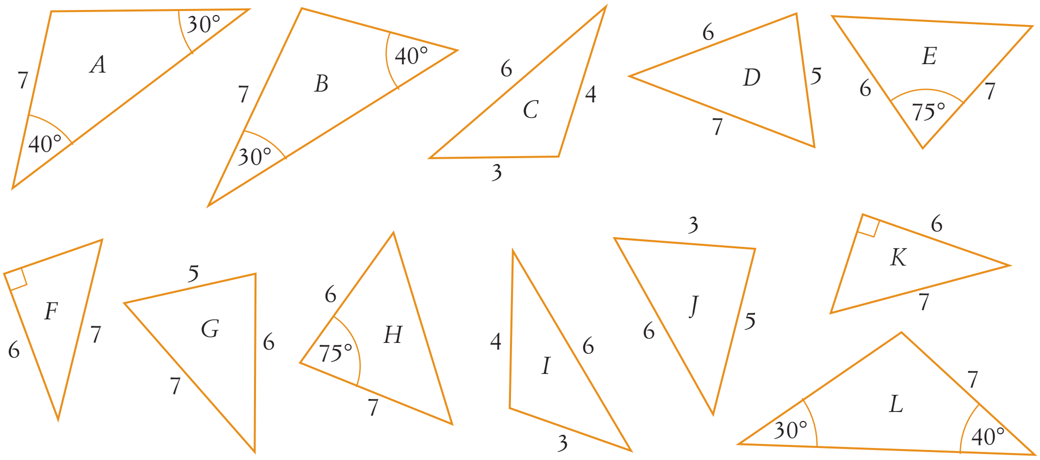
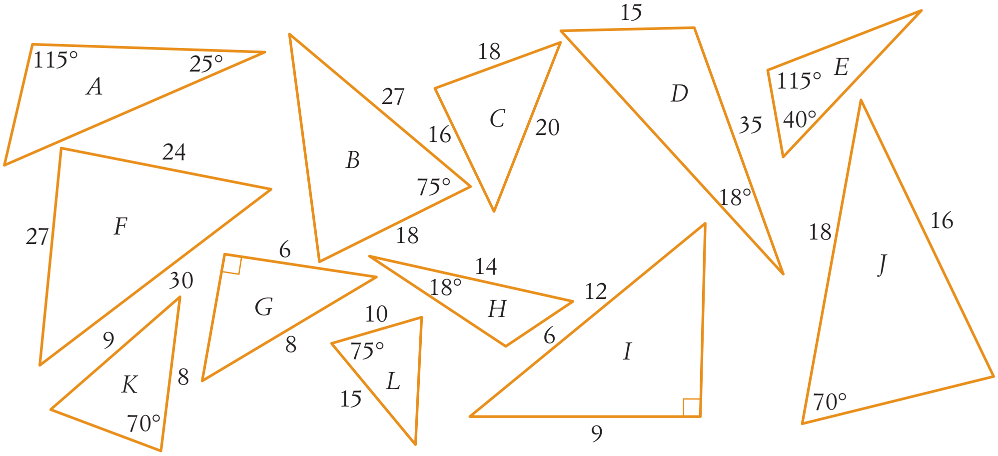


12 Choose the pairs of congruent triangles below and state the test used in each case.



13 Choose the pairs of similar triangles below and state the test used in each case.



Problem solving

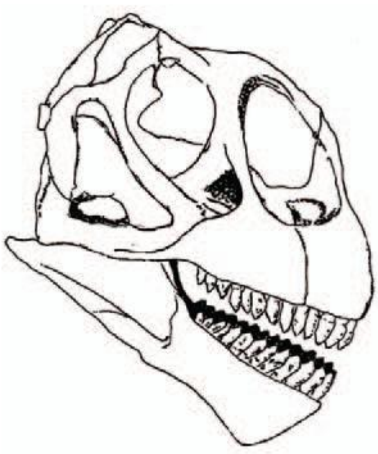
14 Answer true (T) or false (F) to each of the following statements.

a All squares are similar.
b The sides of similar figures are in the same ratio.
c All rhombuses are similar.
d All circles are similar.
e The angles of similar figures are equal.
f All parallelograms are similar.
g Any two isosceles triangles are always similar.
h All equilateral triangles are similar.
i All hexagons are similar.
j Since the angles of two rectangles are equal, they must be similar.
k All right-angled triangles are similar.
l All regular octagons are similar.
m If two triangles have two pairs of matching angles equal, they are similar.

See Example 7

15 This is a scale drawing of the skull of a dinosaur called *Camarasaurus lentus* – a large herbivore that reached a length of about 15 m. The scale used is 1 : 25. Use the scale drawing to answer the following questions.

a What is the actual width of the dinosaur's skull?
b What is the actual height of the dinosaur's skull?
c What is the length of the longest tooth that is visible in the jaw of the dinosaur?



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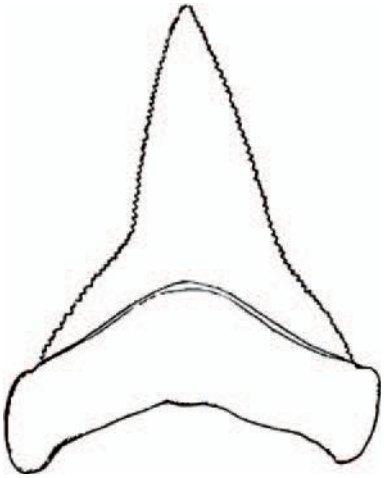
16 This is a scale drawing of the tooth of a shark. The scale used is 4 : 1. Use the scale drawing to answer the following questions.

a What is the actual width of the shark's tooth?

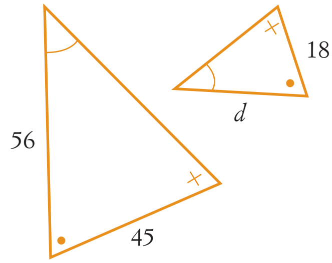
b What is the actual height of the shark's tooth?



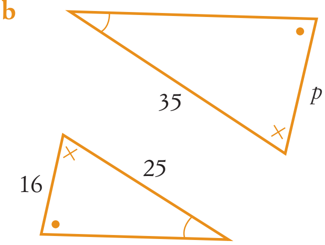
17 A photograph measures 105 mm wide by 60 mm high. An enlargement has a height of 20 cm. What is its width?

18 Find the value of the pronumeral in each of the following pairs of similar triangles. *See* Example 8

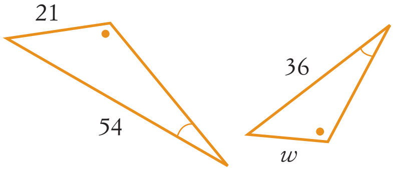
a



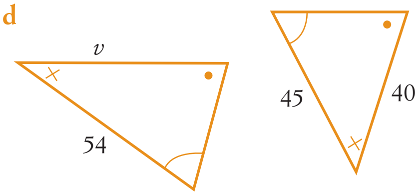
b



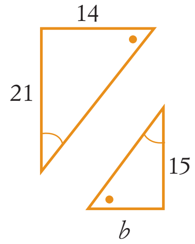
c



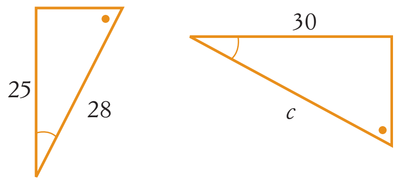
d



e



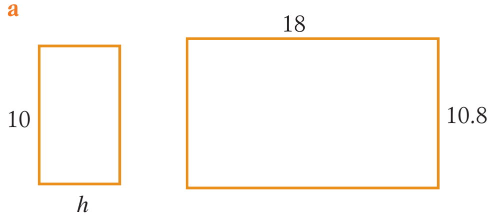
f



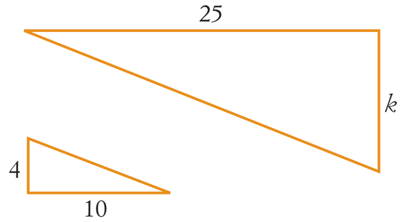
See Example 9

19 Find the value of the pronumerals in the following pairs of similar figures.

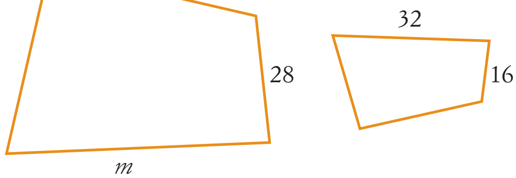
a



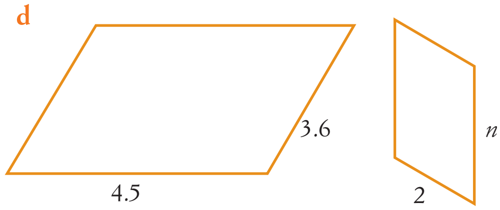
b



c

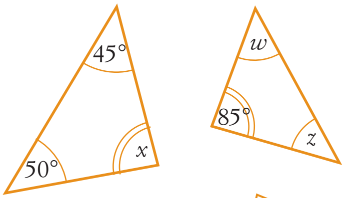


d

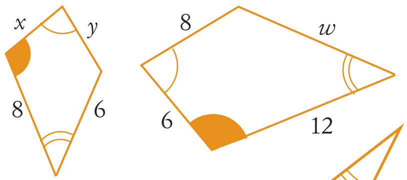


20 The pairs of figures below are similar. Calculate the values of the pronumerals.

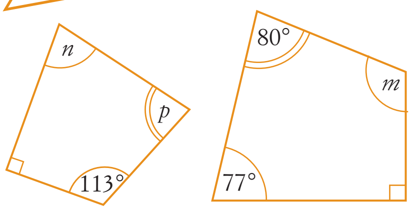
a



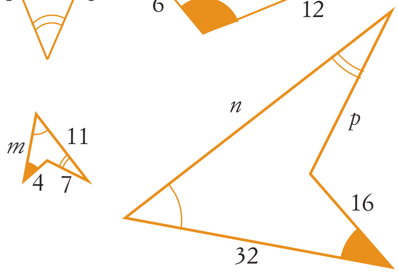
b



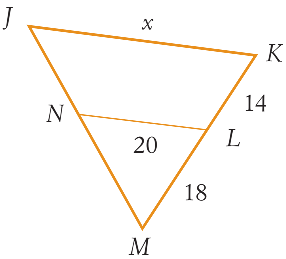
c



d



21 $\triangle MLN$ is similar to $\triangle MKJ$. Find x, correct to one decimal place.



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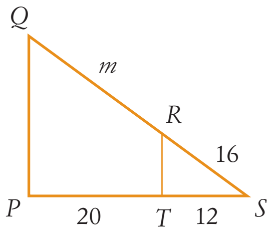
22 $\triangle QSP$ is similar to $\triangle RST$. Find m, correct to one decimal place.

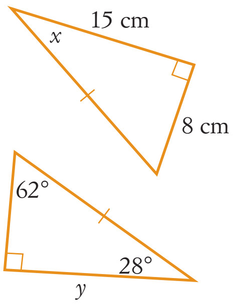


Worked solutions
Exercise 8.2
MAT10MGWS00023

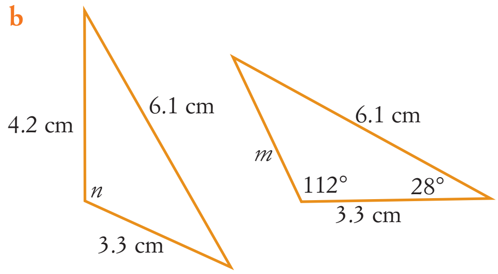
23 For these pairs of congruent triangles, find the value of each pronumeral.

Worked solutions
Exercise 8.2
MAT10MGWS00023

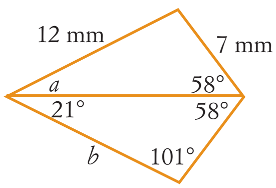
a



b



c



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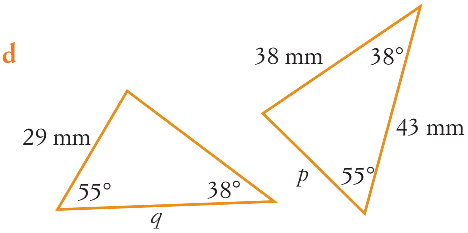

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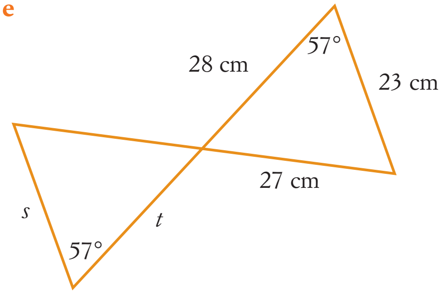

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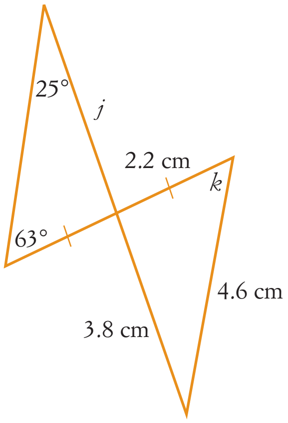
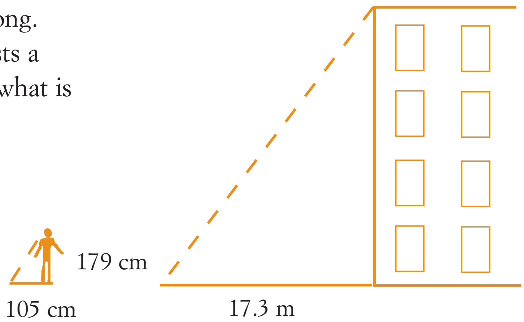


24 At 1:30 p.m., Jose casts a shadow 105 cm long. At the same time a building next to him casts a shadow 17.3 m long. If Jose is 179 cm tall, what is the height of the building?

See Example 10



25 A right-angled triangle has sides that measure 14 cm, 48 cm and 50 cm. A similar triangle has a hypotenuse that measures 20 cm. Find the length of its shortest side.

Reasoning

26 Donna held a ruler vertically 60 cm away from her face and found that, when she was 3.4 m from her friend, she could line up the top of the 30 cm ruler with the top of the friend's head and the bottom of the ruler with the friend's feet. How tall (to the nearest centimetre) is the friend?

27 A tree casts a shadow 18 m long. At the same time, a 3 m stick held vertically casts a shadow of 4 m. How high is the tree?

8.3 Geometric problems and proofs

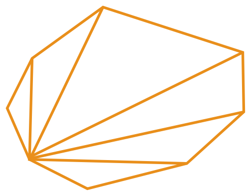
Since the angles of a triangle add up to 180°, we can easily find the sums of angles of other polygons by dividing the polygon into triangles.

Weblink
Interior angles of polygons
MAT10MGWB00008

Weblink
Polygons - quadrilaterals
MAT10MGWB00008

Example 13

Find the sum of the internal angles of an octagon.



Solution

Draw an octagon and divide the inside into triangles that have the same vertices as the octagon.

Count the triangles. **Octagon angles = 6 triangles**

Work out the total. $= 6 \times 180°$

$= 1080°$

Write the answer. **The angles of an octagon add up to 1080°.**

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In problems like the following examples, there is often more than one way to find the solution. In some cases, you will need to use more than one rule in steps to find a value. Sometimes you may have to use algebra.

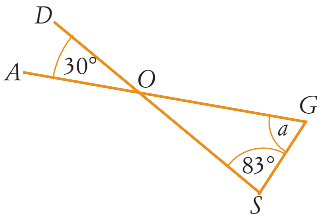
Example 14

Worksheet
Angle fact cards
MAT10MGWK00021

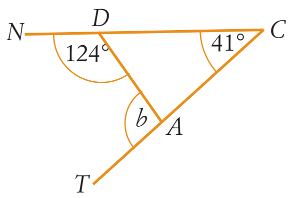
Puzzle sheet
Finding angles
MAT10MGPS00026

Work out the angles shown by letters below. Give reasons for each step in words and include the code.

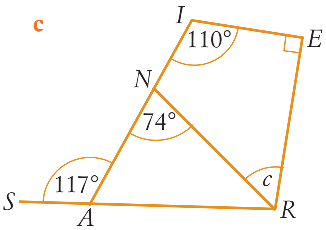
a



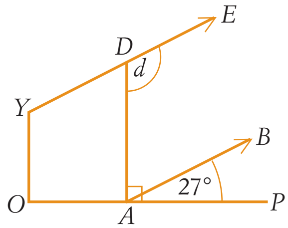
b



c



d



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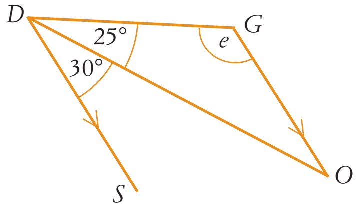


Solution

a $\angle GOS$ is vertically opposite 30°. $\quad \angle GOS = 30°$

The angle sum of a triangle is 180°.

$$a = 180° - (83° + 30°)$$
$$= 67°$$

b The exterior angle is the sum of the interior opposite angles.

$$124° = 41° + \angle DAC$$
$$\angle DAC = 83°$$

b is supplementary with $\angle DAC = 83°$.

$$b + 83° = 180°$$
$$b = 97°$$

c $\angle INR$ is supplementary with 74°.

$$\angle INR + 74° = 180°$$
$$\angle INR = 106°$$

The angle sum of quadrilateral $NIER$ is 360°.

$$c = 360° - (106° + 110° + 90°)$$
$$= 54°$$

d $\angle BAD$ is complementary with 27°.

$$\angle BAD + 27° = 90°$$
$$\angle BAD = 63°$$

d is cointerior with $\angle BAD = 63°$.

$$d + 63° = 180°$$
$$d = 117°$$

e $\angle DOG$ is alternate to 30°. $\quad \angle DOG = 30°$

The angle sum of a triangle is 180°.

$$e = 180° - (25° + 30°)$$
$$= 125°$$

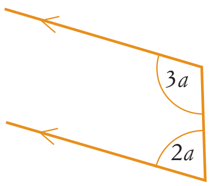
CAS TI-Nspire exercise
Geometry
MAT10MGTI00008

CAS ClassPad exercise
Geometry
MAT10MGCP00008

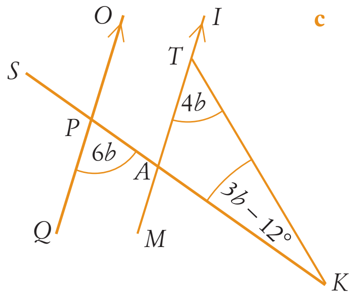
Example 15

Use algebra to find the values of the variables below. Give reasons and show your working.

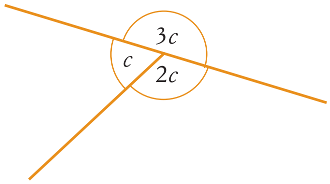
a



b



c



Solution

a $3a$ and $2a$ are cointerior. $\quad$ $\mathbf{3a + 2a = 180°}$

Simplify. $\quad$ $\mathbf{5a = 180°}$

Divide by 5. $\quad$ $\mathbf{a = 36°}$

b $\angle PAT$ is alternate to $6b$. $\quad$ $\mathbf{\angle PAT = 6b}$

The exterior angle $\angle PAT = 6b$ is the sum of the interior opposite angles. $\quad$ $\mathbf{6b = 4b + (3b - 12°)}$

Simplify. $\quad$ $\mathbf{6b = 7b - 12°}$

Solve for b. $\quad$ $\mathbf{b = 12°}$

c c, $2c$ and $3c$ are angles at a point. $\quad$ $\mathbf{c + 2c + 3c = 360°}$

Simplify. $\quad$ $\mathbf{6c = 360°}$

Divide by 6. $\quad$ $\mathbf{c = 60°}$

The work on angles that you have just completed is part of **deductive geometry**. In deductive geometry, we use logical steps to find and prove properties of shapes. You must be able to give a reason for every step and set it out as shown below.

Important!

Geometric proof

A geometric proof is set out under three headings as follows:

To prove: A statement of what must be proved.

Construction: A diagram with any extra lines drawn in.

Proof: The logical steps, with the reasons for each step.

At the end of the proof the initials **QED** are added, to show that the proof is complete. These are an abbreviation for *quod erat demonstrandum*, which is Latin for ‘which was to be proved’.

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In the following examples, triangle congruency is used with the angle properties you have already studied. Construction lines are added to the diagram wherever appropriate.

Example 16

Prove that, if the diagonal of a quadrilateral bisects both angles, then the figure is a kite.

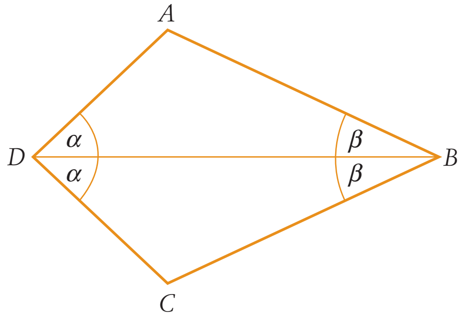
Solution

Draw a labelled sketch that can be used to represent the situation.
State the way that it is to be drawn.
The sketch will need to show a quadrilateral with a diagonal drawn in. The diagonal needs to bisect the angles of the quadrilateral.

Construction

Draw quadrilateral *ABCD* with the diagonal *BD*.

Show the angles as being bisected.



Re-state what has to be proven using the labelled diagram.

A kite has two pairs of equal adjacent sides. This means you have to prove that $AD = CD$ and $AB = CB$. This will be true if $\triangle DAB \equiv \triangle DCB$.

To prove

If the diagonal *BD* of a quadrilateral *ABCD* bisects angles *B* and *D*, then *ABCD* is a kite.

Proof

$\angle D$ is bisected.	$\angle ADB = \angle CDB$ **(given)**
BD is a common side.	$BD = BD$ **(common side)**
$\angle B$ is bisected.	$\angle ABD = \angle CBD$ **(given)**
Use ASA to prove that the triangles are congruent.	**So** $\triangle DAB \equiv \triangle DCB$ **(ASA)**
Corresponding sides are equal.	$AD = DC$ **and** $AB = CB$
ABCD has two pairs of adjacent sides equal.	***ABCD* is a kite.** QED

Video tutorial
Congruent triangles proofs
MAT10MGVT10019

Animated example
Geometric problems and proofs
MAT10MGAE00008

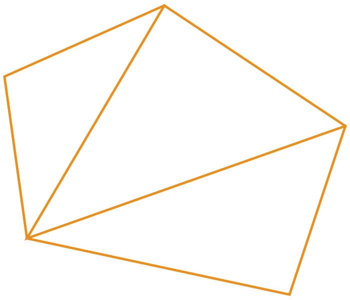
Exercise 8.3 Geometric problems and proofs

Understanding

Extra questions
Exercise 8.3
MAT10MGEQ00024

See Example 13

1 **a** Draw a pentagon and draw all the possible diagonals from one vertex.
b How many diagonals did you draw?
c How many triangles did you form?
d Calculate the sum of the interior angles of a pentagon by finding the total angle sum of the triangles.



See Example 13

2 Find the sum of the internal angles of a polygon with:

a 7 sides	**b** 12 sides	**c** 18 sides
d 21 sides	**e** 25 sides	**f** 30 sides

3 Find the number of sides for a polygon that has an angle sum of:

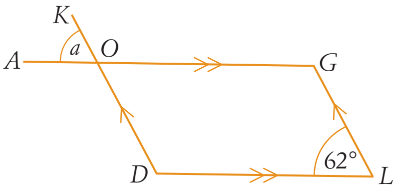
a 1620°	**b** 1980°	**c** 1260°
d 2520°	**e** 3600°	**f** 4140°

Fluency

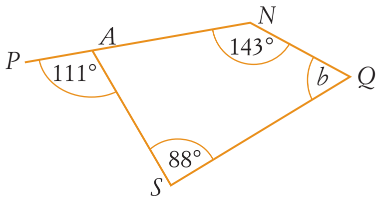
See Example 14

4 Work out the angles shown by letters below. Give reasons for each step in words and include the code.

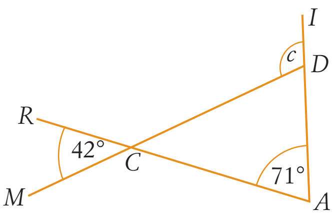
a



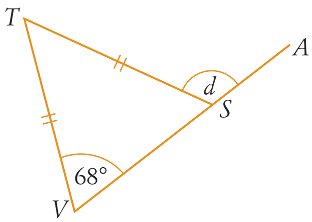
b



c



d



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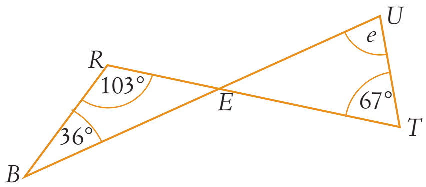
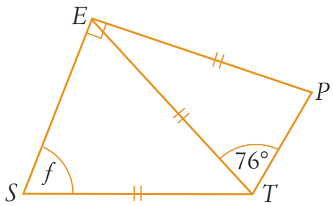
e



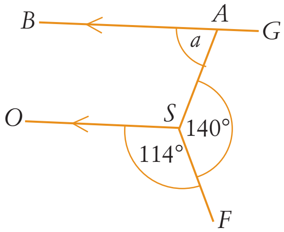
f



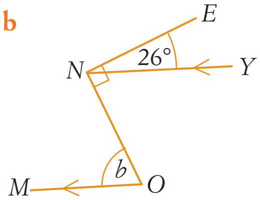
5 Work out the angles shown by letters below. Give reasons for each step in words and include the code.

Worked solutions
Exercise 8.3
MAT10MGWS00024

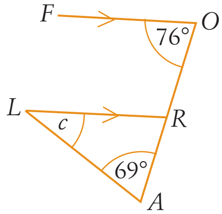
a



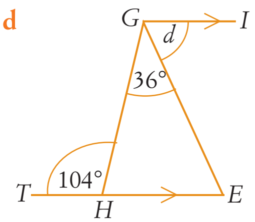
b



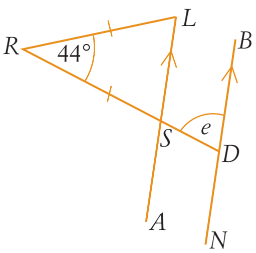
c



d



e



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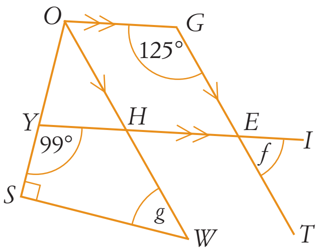
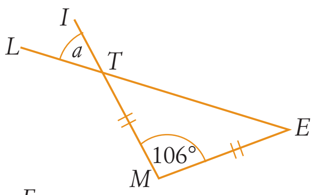
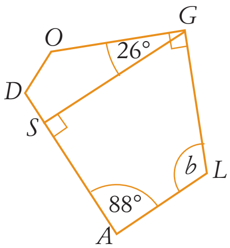


6 Work out the angles shown by letters below. Give reasons for each step in words and include the code.

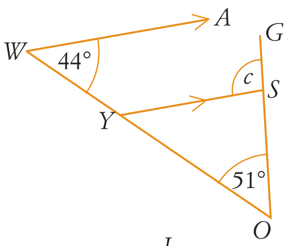
a



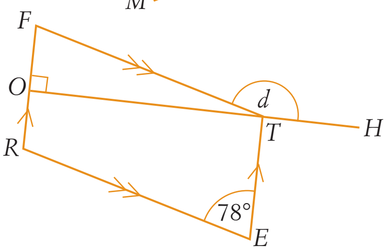
b



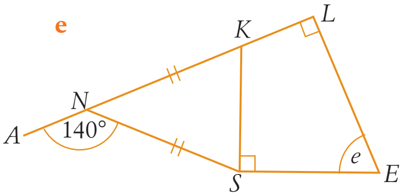
c



d



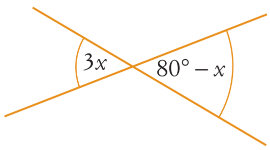
e

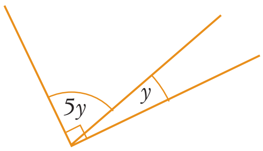


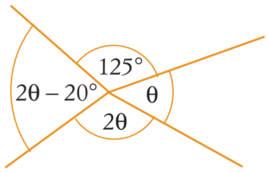
7 Use algebra to find the values of the variables below. Give reasons and show your working.

Problem solving

See Example 15

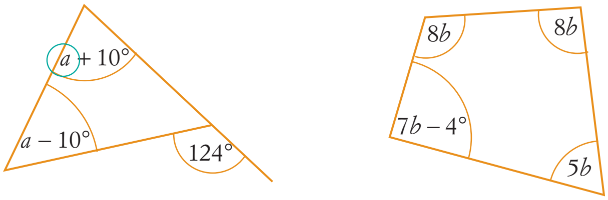




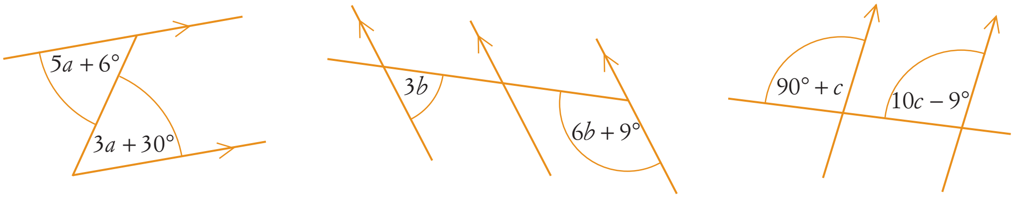


Worked solutions
Exercise 8.3
MAT10MGWS00024

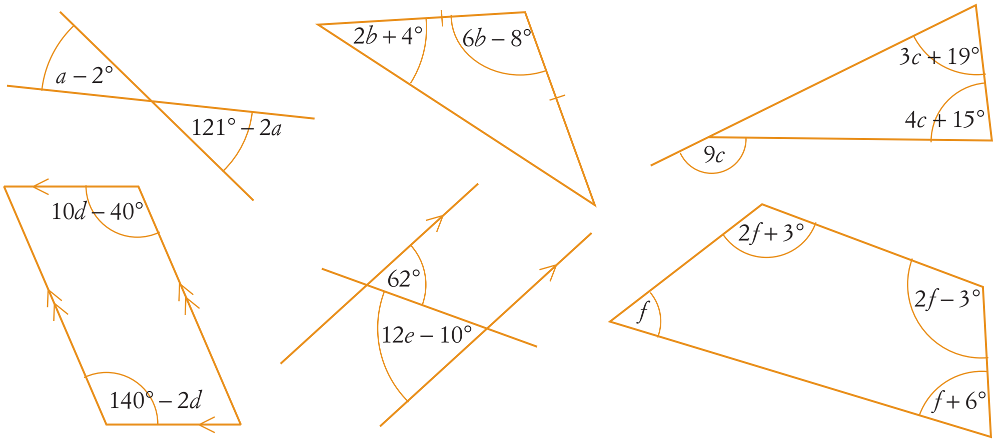
8 Use algebra to find the values of the variables on the right. Give reasons and show your working.



9 Use algebra to find the values of the variables below. Give reasons and show your working.



10 Use algebra to find the values of the variables below. Give reasons and show your working.



11 Find the size of the interior angles of a regular polygon that has:

a 28 sides **b** 8 sides **c** 12 sides **d** 45 sides

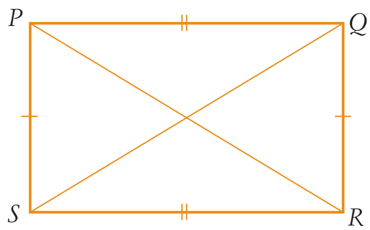
12 Find the number of sides in a regular polygon that has each of its interior angles equal to:

a 140° **b** 150° **c** 175° **d** 165°

Reasoning

Worked solutions
Exercise 8.3
MAT10MGWS00024
See Example 16

13 $PQRS$ is a rectangle. Prove that $\triangle PRS \equiv \triangle QSR$ and hence show that the diagonals of a rectangle are congruent.



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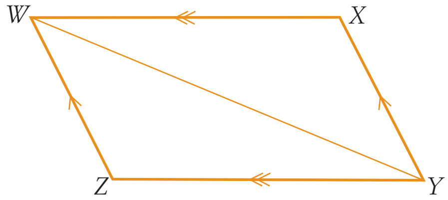
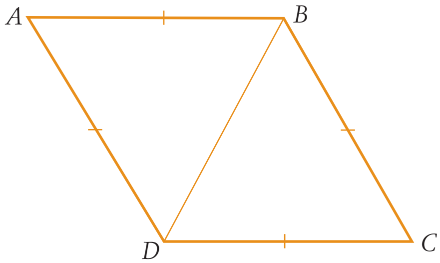
14 *WXYZ* is a parallelogram. Prove that $\triangle WXY \equiv \triangle YZW$ and hence show that the opposite sides of a parallelogram are equal.

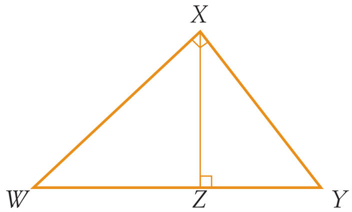


15 Use the results of the previous question to prove that the diagonals of a parallelogram bisect each other.

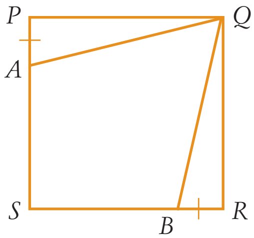
16 *ABCD* is a rhombus. Prove that $\triangle ABD \equiv \triangle CBD$ and hence show that the opposite angles of a rhombus are bisected by the diagonal.



17 Prove that in the triangle on the right, *WXZ*, *WYX* and *XYZ* are similar. Hint: Show that all three triangles have the same angles.



18 *PQRS* is a square and $PA = RB$. Prove that $QA = QB$.



19 Prove that a quadrilateral with equal opposite sides is a parallelogram. (*Hint*: Show that alternate angles are equal to prove that the sides are parallel.)

20 Prove that a quadrilateral with one pair of sides equal and parallel must be a parallelogram.

Chapter 8 summary

Quiz
Geometry
MAT10MGQZ00008

- **Plane shapes** or **two-dimensional (2D) figures** can be drawn on a flat surface.
- A **transformation** changes the position or appearance of a shape. Coordinates can be used to show a transformation. A **translation** is a sliding movement. A **rotation** is a turn, **clockwise** or **anticlockwise**, around the **centre of rotation**. A **reflection** swaps all the points across the **line of reflection**, like a mirror. A **dilation** is an enlargement or reduction. The **magnification** is the enlargement factor, while the **centre of magnification** stays in the same position.
- Most important plane shapes are **polygons,** which have straight sides. The name of a polygon is determined by the number of sides that it has. The corners are called **vertices. Regular** polygons have equal sides and equal angles. **Irregular** polygons have at least one angle or side that is different. **Convex** figures point outwards at the vertices but **concave** figures have at least one inward bend.
- **Equilateral** triangles have three equal sides, **isosceles** triangles have two, and **scalene** triangles have none. The angle names for triangles are **acute-, right-** and **obtuse-angled.**
- **Quadrilaterals** have four sides and are categorised by their side and angle properties. The **square** and **rectangle** have 90° angles. A **trapezium (trapezoid)** has one pair of parallel sides. A **parallelogram** has two pairs of parallel sides. A kite has two pairs of adjacent sides equal. A **rhombus** and a square have all sides the same.
- **Congruent** (≡) figures are the same shape and size. There are four tests for **congruent triangles**: **SSS**, **SAS**, **ASA** (or **AAS**) and **RHS**.
- **Similar** (|||) figures are the same shape but not necessarily the same size. Similar triangles have equal corresponding angles and the ratios of corresponding sides are equal. The scale factor measures the relative sizes of similar shapes.
- There are four tests for **similar triangles**: **AAA**, **SSS**, **SAS** and **RHS**.
- **Deductive geometry** uses logical steps, which are formally set out, to prove geometric properties. The letters **QED** are put at the end of a proof to show that it is finished.

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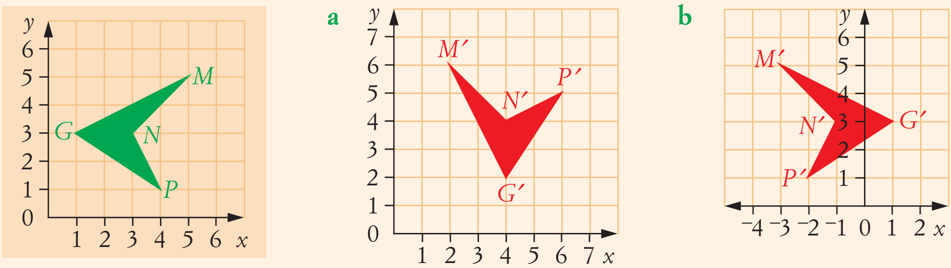
Chapter 8 review

Understanding

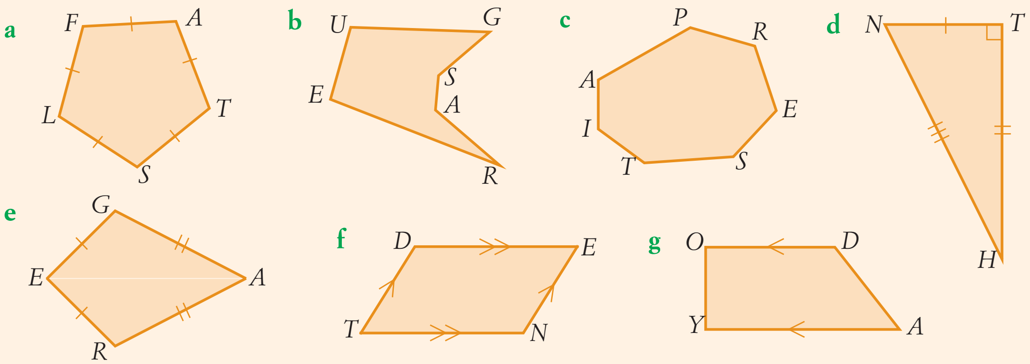
1 Use coordinates to describe each of the transformations shown below.

See Example 1



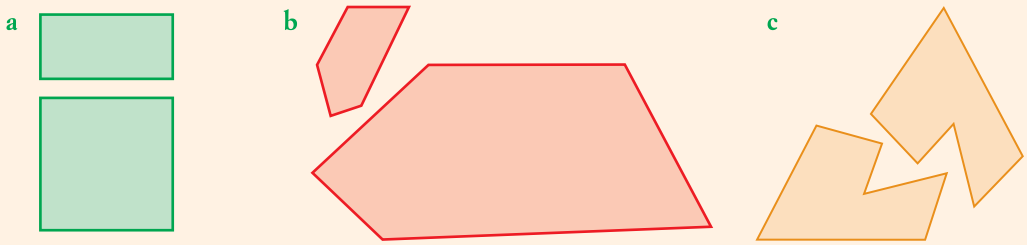
2 Name and classify the following shapes.

See Example 3



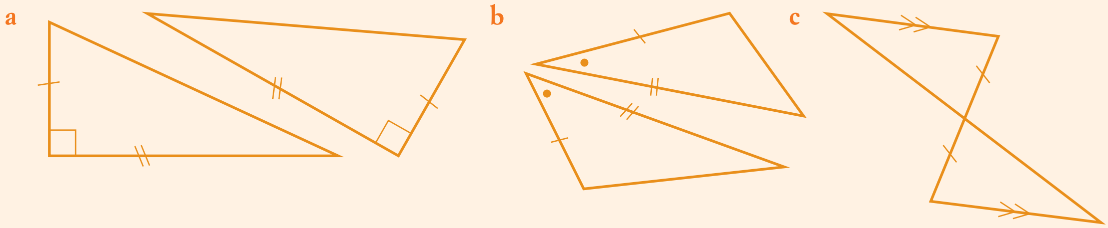
3 State whether each pair of shapes is congruent, similar or neither.

See Example 5



4 State which congruency test can be used to prove that each of the following pairs of triangles is congruent.

See Example 11



Fluency

5 Fully describe each of the transformations in question **1** in words.

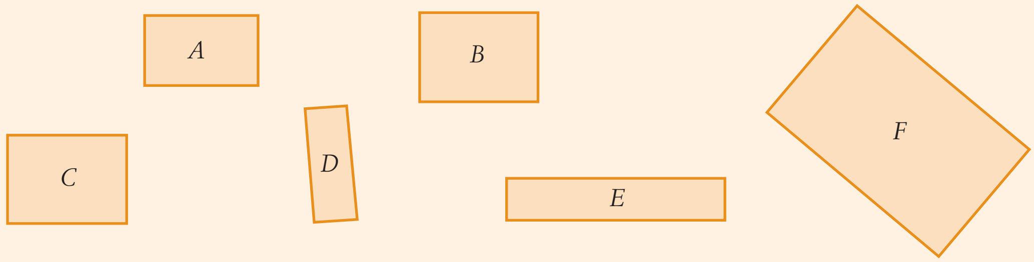
6 Find the coordinates of $M(3, 5)$ after a:

See Example 2

a translation 3 up and 4 left

b rotation 90° clockwise around (2, 3).

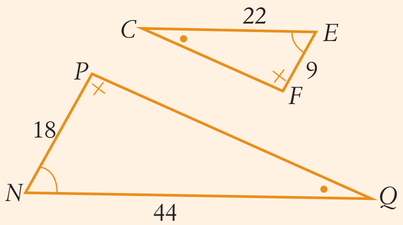
7 Name any similar and congruent shapes below. Work out the scale factors of similar shapes.



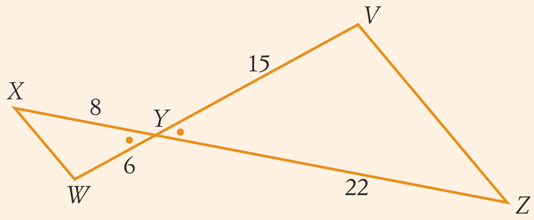
See Example 6

8 Without direct measurement determine if the following pairs of figures are similar. State the scale factor for any similar figures.

a



b



9 Work out the scale factor for any similar shapes found in question **3**.

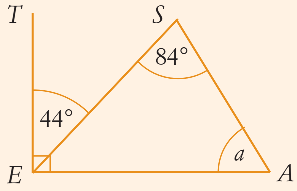
See Example 13

10 Find the sum of the internal angles of an octagon.

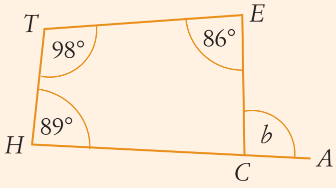
See Example 14

11 Find the angles shown by letters. Give reasons for each step in words and include the code.

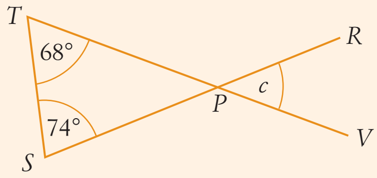
a



b



c



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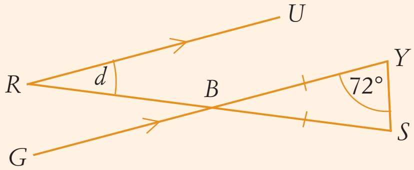

See Example 4

12 State if each of the following is true or false.

a A rhombus is a square

b All circles are similar

c The diagonals of squares intersect at 90°

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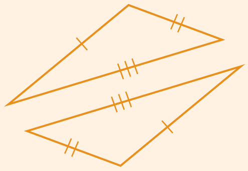
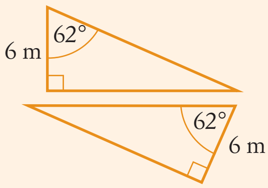
13 State whether each pair of triangles is congruent and state the test used.

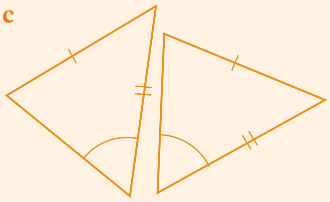
a



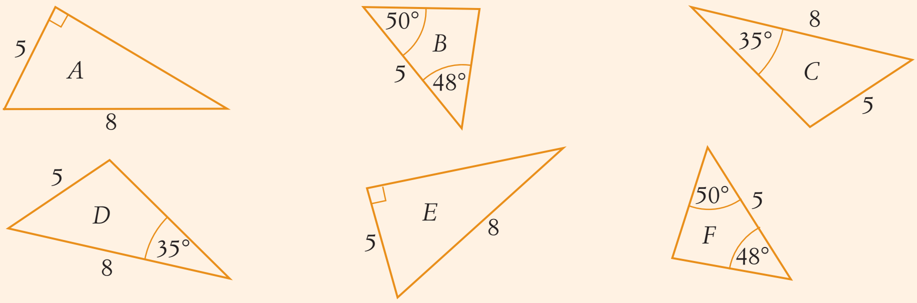
b



c

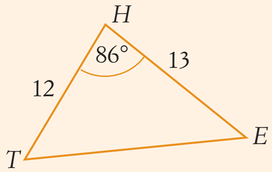


14 Prove congruence for any of the following pairs of triangles that are congruent.

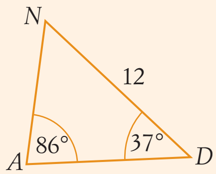


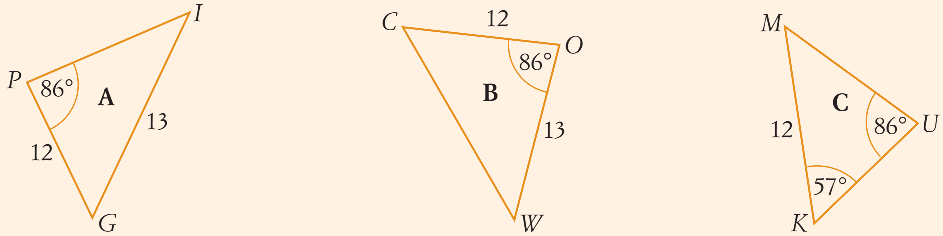
15 Match triangles **a** and **b** with a congruent triangle from **A**, **B** and **C** below and state the test used in each case.

a



b





Problem solving

See Example 7

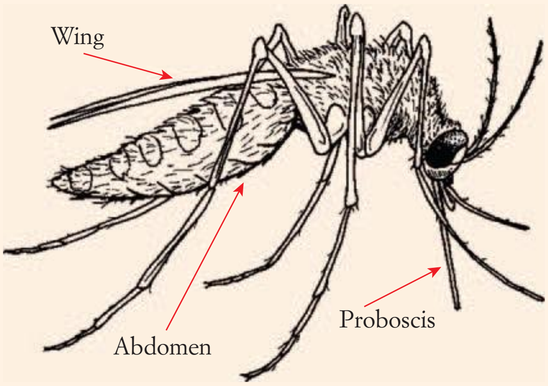
16 This scooter is 201 cm long.

a Calculate the scale for this photo.

b What is the actual diameter of the front wheel?

c What is the actual length of the seat?

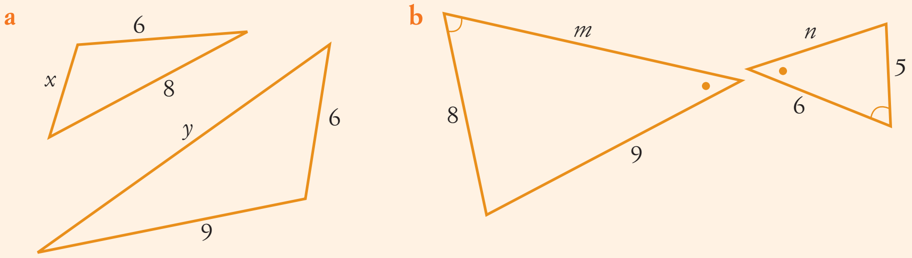
17 This is a scale drawing of a mosquito. The scale used is 6 : 1. Use the scale drawing to answer the following questions.



a What is the greatest actual width of the mosquito's abdomen?
b How long is the mosquito's proboscis?
c How long is the mosquito's wing?

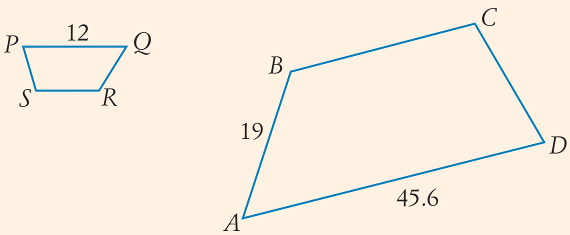
See Example 8

18 Find the values of the unknowns in each of the following pairs of similar triangles.

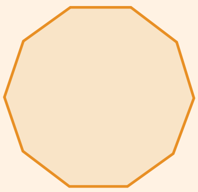


See Example 9

19 The shapes below are similar. Find the value of QR.



20 Find the internal angle of a regular decagon.



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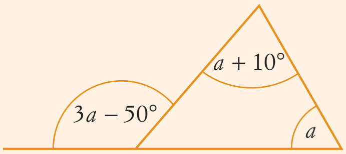
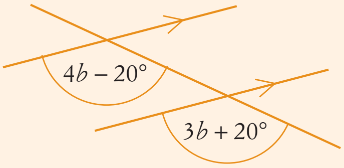
21 Use algebra to find the values of these variables. Give reasons and show your working.

See Example 15

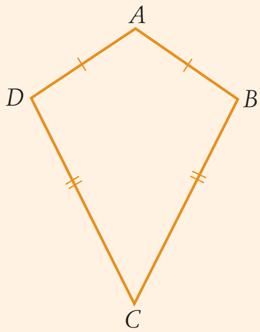




Reasoning

22 Construct the diagonal AC in the kite $ABCD$ on the right. Prove that $\triangle ABC \equiv \triangle ADC$ and hence show that the angles between unequal sides of a kite are equal.

See Example 16



23 A tree casts a shadow 28 m long at the same time that a 4½ m stick casts a 6 m shadow. How tall is the tree?

See Example 10

24 Prove that, if the diagonally opposite angles of a quadrilateral are equal, then the figure must be a parallelogram. (*Hint*: Use the angle sum of a quadrilateral.)

Number and algebra

9 Money and finance

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MAT10NAPL00009

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Australian Curriculum statements

Money and financial mathematics
Connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies. (ACMNA229)

Video tutorial
Money and finance
MAT10NAVT00009

Everyone borrows money at some time and nearly everybody invests money. If you have an account with a financial institution such as a bank, building society or credit union that pays interest you are actually investing money with the institution. In Australia, workers and employers invest money in their superannuation. In fact, superannuation funds are huge investors on behalf of their members, so if you are working for someone in Australia, you are automatically an investor. The amount you pay for the use of someone else's money when you have a loan is the interest. When you borrow money you would prefer the interest rate to be low, but when you invest money you would like it to be high. In this chapter, you will learn more about how interest works.

Mathematical literacy

Maths dictionary
MAT10ASDI00001

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

average daily balance	defaulting	mortgage	repayment
cash-out	flat-rate interest	nominal interest rate	rest
comparison rate	growth factor	on terms	simple interest
compound interest	instalment	pay-day lender	vendor finance
daily rate	interest	principal	
	interest rate	reducing interest	

9.1 Interest calculation

The extra amount you pay back for a loan, or the extra amount you get when you invest money is called **interest**. Interest is calculated in two different ways. While interest is calculated for loans and investments, it is easiest to consider them both as loans. when you invest money, you are effectively lending it to someone else.

Important!

Interest

Interest is the charge made on a loan. The ratio of the interest for one year to the amount borrowed is called the **interest rate** and is normally written as a decimal or percentage. It is also called the **nominal interest rate**.

The amount lent or borrowed is called the **principal**.

The amount that has to be paid back each week or month for a loan on **terms** is called the **instalment** or **repayment**.

Simple interest is calculated using the whole loan for the whole time it is borrowed. It is also called **flat-rate interest**.

Compound interest is calculated at regular intervals called **rests**, and added to the loan, with the result that the principal increases at each rest. If you make regular payments on a loan at compound interest, the interest is usually calculated before the payment is subtracted from the principal.

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Pay-day lenders lend small amounts of cash for short periods of time. The amounts lent are less than banks and other financial institutions would lend.

Example 1

Annie borrowed \$1500 from a 'pay-day' lender to tide her over and had to pay back \$1550 a fortnight later.

a What was the interest paid?
b What would it be for a year?
c What was the interest rate as a percentage?

Solution

a The interest is the extra amount paid. **Interest = \$1550 − \$1500 = \$50**

b There are 26 fortnights in a year. **Interest for 1 year = 26 × \$50 = \$1300**

c Calculate as a percentage of the loan. **Interest rate** $= \frac{1300}{1500} \times 100\% \approx 86.7\%$

Write the answer. **The interest rate is about 87%.**

When things are advertised on terms, the interest may not be given, is hidden in the fine print or is disguised as 'rental'.

Example 2

A 55″ 3D TV+Blu-ray home theatre pack is advertised at \$1999 after cashback or for weekly rental of \$29.25. The rental terms are given on another page as based on a 36-month contract.

a What is the total amount paid for 'rental'?
b How much is the interest for 1 year?
c Calculate the interest rate, assuming you own the home theatre after the 3 years.

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Solution

a There are 52 weeks in a year.

Total paid $= \$29.25 \times 52 \times 3 = \$4563$

b Calculate the total interest.

Total interest $= \$4563 - \$1999 = \$2564$

There are 3 years in the 36 months.

Interest for 1 year $= \$2564 \div 3 \approx \$854.67$

c Calculate the interest as a percentage of the value of the goods (the amount lent).

Interest rate $= \dfrac{854.\overline{6}}{1999} \times 100\% \approx 42.75\%$

State the nominal interest rate.

The nominal interest rate is about 43%.

In some cases, the interest paid is stated as a low percentage, but there are 'other charges' that inflate the amount paid.

Example 3

CAS TI-Nspire exercise
Money and finance
MAT10NATI00009

CAS ClassPad exercise
Money and finance
MAT10NACP00009

Worksheet
Loan repayments
MAT10NAWK00025

A car dealer can arrange finance for Shona to buy a car for \$6700 over 2 years with monthly payments.
The stated interest rate is 12% but there is also an establishment cost of \$300, \$200 a year loan insurance and the car dealer gets a commission of \$250.

a What is the actual cost of the car?

b What is the real interest rate, with all charges taken into account?

Solution

a Calculate the interest.

Work out the percentage.

$$\textbf{Interest for 1 year} = 12\% \text{ of } \$6700 = 0.12 \times \$6700 = \$804$$

Calculate the total interest.

$$\textbf{Total interest} = 2 \times \$804 = \$1608$$

Calculate the other charges.

$$\textbf{Other charges} = \$300 + 2 \times \$200 + \$250 = \$950$$

Find the total cost.

$$\textbf{Total amount} = \$6700 + \$1608 + \$950 = \$9258$$

Write the answer.

The actual cost of the car is \$9258

b Find the real interest.

$$\textbf{Real interest} = \$9258 - \$6700 = \$2558$$

Find the real interest for 1 year.

$$\textbf{Real interest per year} = \$2558 \div 2 = \$1279$$

Express as a percentage.

$$\textbf{Real interest rate} = \frac{1279}{6700} \times 100\% \approx 19.1\%$$

Write the answer.

The real interest rate is about 19%.

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When you work out simple interest, you multiply the interest rate by the amount and then by the period of the loan. This can be written as a formula.

Important!

Simple interest formula

The simple interest I on the principal P at interest rate i (or $r\%$) for n years is given by

$$I = Pin = \frac{Prn}{100}.$$

Example 4

Bobby borrowed \$5400 at 9% simple interest over 3 years. How much does it cost him in interest?

Solution

Write the formula.	$\boldsymbol{I = \frac{Prn}{100}}$
Write the values of the variables.	$\boldsymbol{P = \$5400, r = 9, n = 3}$
Substitute the values.	**Interest** $\boldsymbol{= \frac{\$4500 \times 9 \times 3}{100}}$
Calculate the answer.	$\boldsymbol{= \$1215}$
Write the answer.	**He has to pay \$1215 interest.**

CAS TI-Nspire exercise
Money and finance
MAT10NATI00009

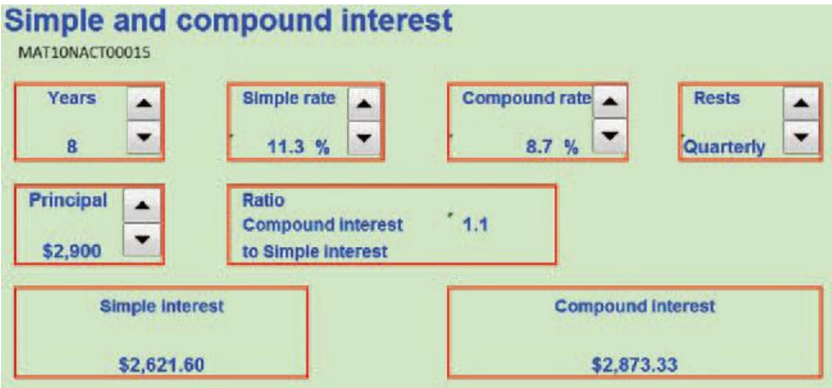
CAS ClassPad exercise
Money and finance
MAT10NACP00009

Worksheet
Simple interest table
MAT10NAWK00023

Puzzle sheet
Simple interest
MAT10NAPS00027

Because compound interest is added to the principal each time it is calculated, you need to work out the amount for each interest period separately. If no other information is given, then the rest is assumed to be a year.

Investigate: Compound and simple interest



You can use a spreadsheet to compare compound and simple interest.

Technology worksheet
Excel worksheet: Simple and compound interest calculator
MAT10NACT00015

Technology: GeoGebra
Compound and simple interest investments
MAT10NATC00009

Weblink
Accounting study guide
MAT10NAWB00009

The spreadsheet 'Simple and compound interest' can be found on the NelsonNet website. Download and open the spreadsheet and use it to find the simple interest rate that gives the same amount of interest as each of the following.

- 10% compounded annually for 3 years
- 10% compounded monthly for 3 years
- 10% compounded monthly for 6 years
- 15% compounded weekly for 4 years

Is there a simple way to work out a simple interest rate that gives the same interest as a compound interest rate?

CAS TI-Nspire exercise
Money and finance
MAT10NATI00009

CAS ClassPad exercise
Money and finance
MAT10NACP00009

Example 5

Work out the interest on $2000 at 7% compound interest for 4 years.

Solution

Find the interest for the first year.	**Interest for 1 year = 0.07 × \$2000** **= \$140**
Add to the amount.	**Total after 1 year = \$2140**
Find the interest for the 2nd year.	**Interest for 2nd year = 0.07 × \$2140** **= \$149.80**
Add to the total.	**Total after 2 years = \$2289.80**
Find the interest for the 3rd year.	**Interest for 3rd year = 0.07 × \$2289.80** **≈ \$160.29**
Add to the total.	**Total after 3 years ≈ \$2450.09**
Find the interest for the 4th year.	**Interest for 4th year ≈ 0.07 × \$2450.09** **≈ \$171.51**
Add to the total.	**Total after 4 years ≈ \$2621.59**
Find the total interest.	**Interest ≈ \$2621.59 − \$2000** **= \$621.59**

In Example **5**, you should keep full values in your calculator and round the final amount from the exact total ($2621.59202). You should not use rounded values in calculations. In Australia, financial institutions are not allowed to round amounts in interest calculations.

You can save time in calculations by finding the amount at the end of each year as a percentage of the amount at the beginning of the year. You can think of this as a **growth factor**. You can also use the constant multiplier function on your calculator.

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Example 6

Work out the compound interest on \$6000 at 6.5% for 4 years.

Solution

Find the growth factor.

$$\text{Growth factor} = 100\% + 6.5\% = 106.5\% = 1.065$$

Find the amount after 1 year.

$$\text{Amount after 1 year} = 1.065 \times \$6000 = \$6390$$

Find the amount after 2 years.

$$\text{Amount after 2 years} = 1.065 \times \$6390 = \$6805.35$$

Find the amount after 3 years.

$$\text{Amount after 3 years} = 1.065 \times \$6805.35 \approx \$7247.70$$

Find the amount after 4 years.

$$\text{Amount after 4 years} \approx 1.065 \times \$7247.70 \approx \$7718.80$$

Find the interest.

$$\text{Interest} = \$7718.80 - \$6000 = \$1718.80$$

Puzzle sheet
Compound interest with annual rests
MAT10NAPS00028

In Examples **5** and **6** you should compare the compound interest to the simple interest that you would get with the same time and percentage.
The formula for compound interest is more complicated than that for simple interest. In Example **6**, you should be able to see that the final amount could be worked out as \$6000 × $(1.065)^4$. The formula works the same way.

Important!

Compound interest formula

The formula for the amount accumulated by compound interest is

$$A = P(1+i)^n = P\left(1+\frac{r}{100}\right)^n,$$

where A is the amount after n years, P is the principal and i (or r%) is the interest rate.

Example 7

Use the formula to find the compound interest on \$2450 at 6.3% for 5 years.

Solution

Write the formula.	$A = P(1+i)^n$
Write the values of the variables.	$P = \$2450,\ i = 0.063,\ n = 5$
Substitute the values.	$\text{Amount} = 2450(1+0.063)^5$
Simplify the brackets to the growth factor.	$= 2450 \times 1.063^5$
Find the amount.	≈ 3325.31
Find the interest.	$\text{Interest} \approx \$3325.31 - \$2450$
	$= \$875.31$
Write the answer.	**The compound interest on \$2450 at 6.3% for 5 years is \$875.31.**

In cases where the rest period is less than a year, the compound interest rate has to be adjusted. It is simplest to adjust the rate and the period using the number of rests in a year.

Video tutorial
Compound interest
MAT10NAVT10021

CAS TI-Nspire exercise
Money and finance
MAT10NATI00009

CAS ClassPad exercise
Money and finance
MAT10NACP00009

Puzzle sheet
Compound interest with non-annual rests
MAT10NAPS00029

Example 8

Find the amount after 6 months when \$894 is invested at 5.7% compounding each month.

Solution

Change the interest rate to monthly.	$\text{Monthly rate} = \dfrac{0.057}{12} = 0.00475$
Write as the growth factor.	$\text{Monthly growth factor} = 1.00475$
Find the amount after 1 month.	$\text{Amount after 1 month} = \$894 \times 1.00475$
	$\approx \$898.25$
Do two months.	$\text{Amount after 2 months} \approx \$898.25 \times 1.00475$
	$\approx \$902.51$
Do three months.	$\text{Amount after 3 months} \approx \$902.51 \times 1.00475$
	$\approx \$906.80$
Do four months.	$\text{Amount after 4 months} \approx \$906.80 \times 1.00475$
	$\approx \$911.11$
Do five months.	$\text{Amount after 5 months} \approx \$911.11 \times 1.00475$
	$\approx \$915.44$
Do six months.	$\text{Amount after 6 months} \approx \$915.44 \times 1.00475$
	$\approx \$919.78$
Enter on your calculator.	894 [=] [×] 1.00475 [=] [=] [=] [=] [=] [=]

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The display for the last two steps should look like this if you have used the constant multiplier correctly.

```
ANS×1.00475
        915.4351691
```

```
ANS×1.00475
        919.7834862
```

Write the answer. **\$894 amounts to \$919.78 after 6 months at 5.7% compounding monthly.**

When you use the constant multiplier on your calculator, you need to be careful about counting the number of times you press the equals sign.

Important!

The formula for the amount accumulated by compound interest for non-annual rests is

$$A = P\left(1 + \frac{i}{k}\right)^{kt} = P\left(1 + \frac{r}{100k}\right)^{kt},$$

where A is the amount after kt rests, P is the principal, i (or r%) is the interest rate, k is the number of rests in a year and t is the number of years.

Example 9

Use the formula to find the interest on \$2450 invested at 6.3% compounding monthly for 5 years.

Solution

Write the formula. $\boldsymbol{A = P\left(1 + \frac{i}{k}\right)^{kt}}$

Write the values of the variables. $\boldsymbol{P = 2450,\ i = 0.063,\ k = 12,\ t = 5}$

Substitute the values. $\textbf{Amount} = \mathbf{2450\left(1 + \frac{0.063}{12}\right)^{12\times 5}}$

Simplify. $\mathbf{= 2450 \times 1.00525^{60}}$

Calculate the answer. $\mathbf{\approx 3354.37}$

Enter as 2450 (1 + 0.063 ÷ 12) ^ (12 × 5) =

```
2450(1+0.063÷12)^(12×5)
            3354.370198
```

Find the interest. **Interest = \$3354.37 − \$2450**

= \$904.37

Write the answer. **The interest after 5 years is \$904.37.**

Notice that the answer to Example **9** is more than Example **7**, so the interest from monthly rests is more than from annual rests at the same nominal interest rate. Compounding daily would increase the amount even more, but only by another \$2.67.

Technology worksheet

Excel worksheet: Comparing interest rates

MAT10NACT00014

Exercise 9.1 Interest calculation

Understanding

Extra questions
Exercise 9.1
MAT10NAEQ00025

See Example 4
See Example 6

1 Use the simple interest formula to calculate the interest for each of the following.
- a $590 at 12% for 3 years
- b $2940 at 8% for 5 years
- c $6800 at 15% for 6 years
- d $458 at 9.6% for 4 years
- e $7320 at 24.7% for 2 years

2 Find the growth factor for each of the following amounts of compound interest.
- a 8% compound interest
- b 6.4% compound interest
- c 3.8% compound interest
- d 12.4% compound interest
- e 9.6% compound interest

See Example 8

3 Find the growth factor for each of the following cases.
- a 8.1% compound interest with monthly rests.
- b 6.5% compound interest with weekly rests.
- c 5.9% compound interest with quarterly rests.
- d 12.6% compound interest with monthly rests.
- e 9.1% compound interest with fortnightly rests.

Fluency

See Example 1

4 Find the interest rate for each of the following payday loans.
- a $300 for 5 days with charges of $75 and $1.95.
- b $400 for 9 days with a charge of $110.
- c $300 for 5 days with charges of $45 and $60.
- d $600 for 3 weeks with charges of $45 and an extra $30 per week.
- e $500 for 4 days with a charge of $27.50 per $100.

See Example 3

5 Find the real simple interest rate for each of the following vendor finance agreements.
- a $26 000 for a car with weekly payments of $342 a week over 3 years.
- b $7800 for a used car with stated interest of only 12% over 3 years, an establishment fee of $800 and commission of $590.

See Examples 5, 6

6 Calculate the amount and interest for each of the following.
- a $3000 at 12% compound interest for 3 years
- b $4900 at 8% compound interest for 4 years
- c $6200 at 6.4% compound interest for 4 years
- d $8400 at 9.6% compound interest for 3 years
- e $12 300 at 10% compound interest for 4 years

See Example 8

7 Calculate the amount and interest for each of the following.
- a $4000 at 9.6% compound interest for 6 months with monthly rests.
- b $3800 at 8.7% compound interest for 4 months with monthly rests.
- c $4600 at 10.8% compound interest for 7 months with monthly rests.
- d $8400 at 7.2% compound interest for 12 months with monthly rests.
- e $7200 at 5.7% compound interest for 9 months with monthly rests.

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8 Use the compound interest formula to calculate the amount and interest for each of the following. *See* Example 7

a $5000 at 7.5% compound interest for 3 years

b $7000 at 8.6% compound interest for 4 years

c $12 300 at 9.2% compound interest for 5 years

d $6200 at 5.8% compound interest for 3 years

e $4920 at 4.9% compound interest for 3 years

9 Use the compound interest formula to calculate the amount and interest for each of the following. *See* Example 9

a $8000 at 8.4% compound interest for 5 years with monthly rests.

b $4000 at 9.1% compound interest for 3 years with weekly rests.

c $7000 at 9.3% compound interest for 4 years with monthly rests.

d $9200 at 6.8% compound interest for 4 years with quarterly rests.

e $5740 at 7.2% compound interest for 3 years with monthly rests.

Problem solving

10 You 'rent' a notebook computer worth $399 for $5.10 a week on a 36-month contract. At the end of the time you buy it for $50.

Worked solutions Exercise 9.1 MAT10NAWS00025

See Example 2

a What is the total cost?

b How much is the interest for 1 year?

c What is the interest rate?

11 You rented a 60 inch flat screen TV sold for $1388 at a 'rent' of $17.65 a week on a 36-month contract. At the end of the time you purchased it for $400. What was the total cost and the real interest rate?

Worked solutions Exercise 9.1 MAT10NAWS00025

12 Yoda bought a bedroom suite for $2890 with 'nothing to pay' for 2 years. At the end of the 2 years, the interest is calculated back to the start of the time at 19%. There is also an 'accounting' fee of $15 per month. She actually paid it off after 3 years. What was the real amount she paid and the real interest rate?

Worked solutions Exercise 9.1 MAT10NAWS00025

Reasoning

13 Find the time taken for an amount to double using compound interest at rates of 7%, 10%, 14% and 20% using $1000 as the initial amount. Can you give a rough rule for the time taken to double an amount under compound interest?

14 Calculate the return on $1000 for 1 year at 5%, 10%, 15% and 20% compound interest, compounding annually, quarterly, monthly, weekly and daily and comment on the difference that compounding at shorter intervals makes to the interest gained.

9.2 Investing money

There are many different ways to invest money. Most investments are calculated at compound interest. Only very large investors on the short-term money market typically invest at simple interest. This kind of investment might be something like \$600 000 for 5 days. The market is used by banks and other financial institutions, as well as by wealthy individuals who have money that they are not due to use for a short time.

Example 10

\$700 000 is invested at 4.24% for 8 days. How much interest is earned?

Solution

The term is a fraction of a year.	**Time** $= \frac{8}{365} \approx$ **0.02192 years.**
Write the formula.	$I = Pin$
Write the values of the variables.	$P =$ **\$700 000,** $i = 0.0424$, $n \approx 0.02192$
Substitute the values.	**Interest** $\approx 700\,000 \times 0.0424 \times 0.02192$
Work out the answer.	≈ 650.52
Write the answer.	**The interest is \$650.52**

Intermediate calculations such as the term in Example **10** should be kept in your calculator and used for later calculations. Do not use the rounded amounts.
Most savings accounts calculate interest on minimum monthly balance. The interest may be paid monthly, 6-monthly or annually. This is effectively a sum of simple interest calculations.

Example 11

The interest for Sandy's savings account is paid at the end of June on minimum monthly balances at a rate of 3%. Use the minimum balances below to find Sandy's interest for the financial year.

Month	Jul	Aug	Sep	Oct	Nov	Dec
Min. balance	\$42.50	\$127.38	\$149.80	\$265.20	\$474.90	\$28.30
Month	Jan	Feb	Mar	Apr	May	Jun
Min. balance	\$248.90	\$521.40	\$672.90	\$783.60	\$985.68	\$32.40

Solution

Calculate the interest rate for a month.	**Monthly rate** $= \frac{0.03}{12} = 0.0025$
Find the interest for July.	**July interest** $= 0.0025 \times \$42.50 \approx \$0.11$
Find the interest for August.	**August interest** $= 0.0025 \times \$127.38 \approx \$0.32$
Find the interest for September.	**September interest** $= 0.0025 \times \$149.80 \approx \$0.37$
Find the interest for October.	**October interest** $= 0.0025 \times \$265.20 \approx \$0.66$

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Find the interest for November.	**November interest** $= 0.0025 \times \$474.90 \approx \$1.19$
Find the interest for December.	**December interest** $= 0.0025 \times \$28.30 \approx \$0.07$
Find the interest for January.	**January interest** $= 0.0025 \times \$248.90 \approx \$0.62$
Find the interest for February.	**February interest** $= 0.0025 \times \$521.40 \approx \$1.30$
Find the interest for March.	**March interest** $= 0.0025 \times \$672.90 \approx \$1.68$
Find the interest for April.	**April interest** $= 0.0025 \times \$783.60 \approx \$1.96$
Find the interest for May.	**May interest** $= 0.0025 \times \$985.68 \approx \$2.46$
Find the interest for June.	**June interest** $= 0.0025 \times \$32.40 \approx \$0.08$
Add the (exact) amounts to find the total.	**Total** $\approx \$0.11 + \$0.32 + \ldots = \$10.83$
Write the answer.	**Sandy received $10.83 interest for the year.**

Term deposits attract much greater interest than savings accounts, but usually you cannot get your money back until the end of the term. If you take your money out early because of hardship you will receive no interest. Term deposits are normally offered for terms from about 3 months up to about 5 years. You normally have to put a minimum amount of about $5000 into a term deposit.

Example 12

Kim put $7500 into a term deposit at 5.4% for 3 months. How much is it worth at the end of the term?

Solution

Calculate the growth factor.	**Growth factor** $= 1 + \dfrac{0.054}{4} = 1.0135$
Multiply the principal by the growth factor.	**Amount after 3 months** $= \$7500 \times 1.0135$
Calculate the amount.	$= \$7601.25$
Write the answer.	**It is worth $7601.25 at the end of the term.**

When you set up a term deposit for 2 or more years, there are different ways the interest is paid. Some term deposits pay it into another account at fixed intervals such as a month or quarter. Others will simply treat it as compounding interest, reinvesting it at the end of each year.

Example 13

Tom put an inheritance of \$15 300 into a term deposit for 4 years at 5.8%. He could have the interest paid into another account at the end of each year, or have it reinvested in the term deposit.

a How much would he get each year it he chooses the first option?

b How much would the investment be worth after 4 years if he had it reinvested?

Solution

a Calculate the interest.	**Interest $= 0.058 \times$ \$15 300**
Work out the answer.	**$=$ \$887.40**
Write the answer.	**He would get \$887.40 each year.**
b Work out the growth factor.	**Growth factor $= 1 + 0.058 = 1.058$**
Enter as 15300 [=] [×] 1.058 [=] [=] [=] [=].	ANS×1.058 19170.5292
Write the answer.	**It would be worth \$19 170.53 after 4 years.**

It is easy to lose track of the number of times you press the equals sign when calculating compound interest, so use the compound interest formula for large numbers of rests.

Example 14

Inar invested \$16 900 with a mortgage broker at 6.9% calculated monthly for 2 years. How much did she get back?

Solution

Write the formula.	$A = P\left(1 + \frac{i}{k}\right)^{kt}$
Write the values of the variables.	$P = 16\ 900,\ i = 0.069,\ k = 12,\ t = 2$
Substitute the values.	**Amount** $= 16\ 900\left(1 + \frac{0.069}{12}\right)^{12 \times 2}$
Simplify.	$= 16\ 900 \times 1.00575^{24}$
Calculate the answer.	≈ 3354.37
Enter as 16900 [×] 1.00575 [∧] 24 [=].	16900×1.00575^24 19393.12043
Write the answer.	**Inar got back \$19 393.12.**

Instead of investing money in a bank account or some other kind of investment, you can buy something that you think will increase in value. This is more risky than term deposits, but no investment is without risk. Generally, a higher return means a higher risk. If you invest in something with high returns, you have a greater risk of losing your money.

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Sometimes you will be interested in how long it takes for an investment to reach a particular value.

Example 15

How long would it take an investment of \$3000 to reach a value of \$5000 at 15% simple interest?

Solution

Work out the amount of interest needed.	**Interest = \$5000 − \$3000 = \$2000**
Write the simple interest formula.	$I = Pin$
Write the values of the variables.	$I = 2000,\ P = 3000,\ i = 0.15$
Substitute values in the formula.	$2000 = 3000 \times 0.15 \times n$
Simplify the equation.	$450n = 2000$
Divide both sides by 450.	$n = \frac{2000}{450} = 4.4\overline{4} \approx 4$ **years and 5.3 months**
Write the answer, allowing for different circumstances.	**It would take about 4 years and 5 months, or 6 years if it has to be a whole number of years.**

Example 16

How long would it take an investment of \$5000 to reach a value of \$8000 at 9% compound interest?

Solution

Work out the growth factor.	**Growth factor = 1.09**
Multiply by the growth factor.	**Amount after 1 year = \$5000 × 1.09 = \$5450**
Multiply by the growth factor again.	**Amount after 2 years = \$5450 × 1.09 = \$5940.50**
Multiply by the growth factor again.	**Amount after 3 years = \$5940.50 × 1.09 ≈ \$6475.15**
Multiply by the growth factor again.	**Amount after 4 years = \$6475.15 × 1.09 ≈ \$7057.91**
Multiply by the growth factor again.	**Amount after 5 years = \$7057.91 × 1.09 ≈ \$7693.12**
Multiply by the growth factor again.	**Amount after 6 years = \$7693.12 × 1.09 ≈ \$8385.50**
Write the answer, allowing for different circumstances.	**It would take about $5\frac{1}{2}$ years, or 6 years if it has to be a whole number of years.**

In Example **16** you would actually use the constant factor method on your calculator and count the number of times you had to press the = button.

Exercise 9.2 Investing money

Fluency

Extra questions
Exercise 9.2
MAT10NAEQ00026
See Example 11

1 Find the interest earned in a financial year on a savings account that pays interest of 2.4% on minimum monthly balances if the balances are as shown in the table below.

Month	Jul	Aug	Sep	Oct	Nov	Dec
Balance	$267.59	$543.00	$555.43	$520.83	$551.17	$503.95
Month	Jan	Feb	Mar	Apr	May	Jun
Balance	$547.00	$569.18	$558.88	$507.61	$585.11	$298.01

2 Find the interest earned in a financial year on a savings account that pays interest of 3.3% on minimum monthly balances if the balances are as shown in the table below.

Month	Jul	Aug	Sep	Oct	Nov	Dec
Balance	$1701.44	$1868.21	$2303.47	$1802.33	$2261.56	$2472.81
Month	Jan	Feb	Mar	Apr	May	Jun
Balance	$ 202.61	$ 398.29	$1812.48	$ 491.75	$ 485.88	$ 307.84

3 Find the interest earned in a calendar year on a savings account that pays interest of 2.7% on minimum monthly balances if the balances are as shown in the table below.

Month	Jan	Feb	Mar	Apr	May	Jun
Balance	$2203.76	$ 381.81	$2256.72	$2380.21	$366.63	$677.40
Month	Jul	Aug	Sep	Oct	Nov	Dec
Balance	$ 478.36	$2378.85	$2223.04	$ 334.64	$583.62	$596.11

See Example 12

4 How much is a term deposit of $8300 at 5.6% for 6 months worth at the end of the term?

5 How much is a term deposit of $12 400 at 4.8% for 3 months worth at the end of the term?

6 How much is a term deposit of $10 500 at 6.1% for 9 months worth at the end of the term?

See Example 10

7 $850 000 is invested at 3.2% for 7 days. How much interest is earned?

8 $1 500 000 is invested at 3.7% for 6 days. How much interest is earned?

9 $590 000 is invested at 2.8% for 9 days. How much interest is earned?

See Example 13

10 Antoinette put $18 700 into a term deposit for 4 years at 6.3%. The interest was calculated and paid every six months. How much does she get paid into her savings account every 6 months from the term deposit?

11 Andrew has the income from a trust fund paid into his savings account every year. The trust fund is for $24 000 and is earning interest at the rate of 5.2%. How much does he get each year from the fund?

12 Carmen put $7200 into a term deposit at 6.25% for 3 years. How much does she get back when the term deposit matures (i.e. at the end of the term)?

13 Colin was given $6000 in settlement for an injury claim. He decided to put it into a term deposit at 5.75% for 5 years. What does he get back at the end of the time?

See Example 14

14 What is an investment of $6400 at 4.8% compounding monthly worth after 4 years?

15 What is an investment of $13 900 at 6.8% compounding quarterly worth after 3 years?

16 What is an investment of $8900 at 5.9% compounding weekly worth after 5 years?

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Problem solving

See Example 15

17 How long would it take an investment of \$2500 to reach a value of \$4000 at 12% simple interest?

18 How long would it take an investment of \$4000 to reach a value of \$8000 at 15% simple interest?

19 How long would it take an investment of \$2800 to reach a value of \$6000 at 14.8% simple interest?

See Example 16

20 How long would it take an investment of \$2000 to reach a value of \$3500 at 12% compound interest?

21 How long would it take an investment of \$4500 to reach a value of \$9000 at 15% compound interest?

22 How long would it take an investment of \$1500 to reach a value of \$4300 at 8.9% compound interest?

Worked solutions Exercise 9.2 MAT10NAWS00026

Reasoning

See Example 17

23 Use a principal of \$1000 to find the simple interest rate that gives the same interest as compound interest of 12% over 3 years, logically explaining your steps.

24 Find the simple interest rate that gives the same interest as 10% compounding semi-annually over 5 years, logically explaining your steps.

Worked solutions Exercise 9.2 MAT10NAWS00026

25 Find the interest rate that, compounding quarterly, gives the same interest as 15% simple interest over 3 years, logically explaining your steps.

Worked solutions Exercise 9.2 MAT10NAWS00026

26 Find the interest rate that, compounding monthly, gives the same interest as 18% simple interest over 5 years, logically explaining your steps.

9.3 Borrowing money

There are many different ways to borrow money. Generally speaking, the easier it is to borrow, the higher the interest rate and other charges will be. Very few people are able to avoid borrowing money, at least for major purchases such as a car or house. Ultimately, you will be better off financially if you can avoid borrowing money wherever possible. If you have to borrow money, you should take the time to become familiar with the rates and conditions.

Payday lenders offer very short term loans (up to a few weeks) of small amounts from \$100 to about \$1500. They often disguise interest as fees and charges, so you need to compare all costs when obtaining credit. Most require you to allow them access to a savings account for repayments. It can be difficult for you to revoke this access.

If you don't pay a loan according to the conditions, this is called **defaulting**, and usually attracts extra charges.

Example 17

Animated example
Borrowing money
MAT10NAAE00009

A cash loans business offers loans of up to \$500 at no interest for up to 3 weeks, but charge a fee of \$45 per loan and \$30 per week for the period of the loan. If you default on the loan, they apply a default cost of \$50, then roll over the loan and charges to a new loan for 1 week.

On Thursday, Mandy agreed to repay a loan of \$300 on her next payday, which was on Friday week. Unfortunately she had forgotten about the electricity bill, so couldn't pay the loan until her next payday, a fortnight later.

a Work out the total cost if she paid the loan on time.
b Work out the cost by the time she had actually paid it.
c Work out the interest rate.

Solution

a State the length of the loan.	**The loan is more than a week so counts as 2 weeks.**
Work out the total cost.	$\text{Cost} = \$45 + 2 \times \$30 = \$105$
b Work out the default cost.	$\text{Second loan cost} = \$50 + \$45 + \$30 = \$125$
Work out the 2nd default cost.	$\text{Third loan cost} = \$50 + \$45 + \$30 = \$125$
Find the total cost.	$\text{Total cost} = \$105 + \$125 + \$125 = \$355$
c Work out the actual loan time.	$\text{Time of loan} = 8 \text{ days} + 14 \text{ days} = 22 \text{ days}$
Work out the interest for 1 day.	$\text{Interest for 1 day} = \$355 \div 22 = \$16.13636\ldots$
Work out the interest for a year.	$\text{Interest for 1 year} = \$16.13636\ldots \times 365 = 5889.7727\ldots$
Work out the interest rate.	$\text{Interest rate} = \dfrac{5889.7727\ldots}{300} \times 100\% \approx 1963\%$

In Example **17**, you will notice that in the end, Mandy had to pay back more than double the amount she borrowed. If you cannot avoid defaulting on a payment, you should talk to the lender, preferably before it is due. Many lenders will try to help you by rescheduling payments. If you don't tell them about your problems they will assume the worst and apply the maximum penalties. Credit card companies usually allow you to get a cash advance when you buy goods (a **cash-out**), but charge you a high interest rate for this money. You may also have to pay an extra charge for the shop to allow you to use a credit card. Credit card interest is calculated on the **average daily balance** of your account, using the **daily rate** for the number of days in the billing period. This means that you actually pay interest on purchases or cash advances for the number of days left in the billing period for each purchase that you make.

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Example 18

Dan bought some groceries worth \$35 from his corner store on 20 June and got a cash-out of \$200. The shop charged 2.5% extra for using the credit card. His credit card's billing period is until the end of each month and is due 5 days later, on the 5th of the next month. The card costs \$75/year and has a credit charge of 11% for purchases and 24% for cash advances.

a How much did the credit cost him?
b What was overall interest rate for the groceries?
c What was overall interest rate for the cash advance?

Solution

a Work out the extra for groceries.

$$\text{Groceries extra} = 0.025 \times \$35 \approx \$0.88$$

Work out the total for groceries.

$$\text{Groceries total} \approx \$35 + \$0.88 = \$35.88$$

Work out the daily rate for purchases.

$$\text{Purchases daily rate} = \frac{0.11}{365} = 0.000301\ldots$$

There are 30 days in June.

$$\text{Days left in billing period} = 30 - 20 = 10$$

Work out the interest for groceries.

$$\text{Groceries interest} \approx \$35.88 \times 10 \times 0.000301\ldots \approx \$0.11$$

Work out the groceries credit cost.

$$\text{Total groceries credit cost} \approx 0.98$$

Work out the extra for the cash.

$$\text{Cash extra} = 0.025 \times \$200 = \$5$$

Work out the total for the cash.

$$\text{Cash total} = \$200 + \$5 = \$205$$

Work out the daily rate for cash.

$$\text{Cash daily rate} = \frac{0.24}{365} = 0.0006575\ldots$$

Work out the interest for cash.

$$\text{Cash interest} = \$205 \times 10 \times 0.000657\ldots \approx \$1.35$$

Work out the cash credit cost.

$$\text{Total cash credit cost} \approx \$5 + \$1.35 = \$6.35$$

Work out the total credit cost.

$$\text{Total credit cost} \approx \$0.98 + \$6.35 = \$7.33$$

Write the answer.

The total cost of the credit was \$7.33.

b Work out the cost for 1 day.

$$\text{Cost for 1 day} \approx \$0.98 \div 10 = \$0.098$$

Work out the cost for a year.

$$\text{Cost for a year} \approx \$0.098 \times 365 \approx \$35.88$$

Work out the interest rate.

$$\text{Interest rate} \approx \frac{35.88}{35} \times 100\% \approx 102.5\%$$

State the answer.

The interest rate on the groceries was about 102.5%.

c Work out the cost for 1 day. **Cost for 1 day** $\approx \$6.35 \div 10$
$= \$0.635$

Work out the cost for a year. **Cost for a year** $\approx \$0.635 \times 365$
$\approx \$231.70$

Work out the interest rate. **Interest rate** $\approx \dfrac{231.70}{200} \times 100\%$
$\approx 115.9\%$

State the answer. **The interest rate on the cash advance was about 115.9%.**

The actual interest rate in Example **18** is very high because of the shop's charge for using the credit card. Credit card providers also charge very high fees for defaulting on payments. In addition, they normally keep charging interest at a higher rate on all purchases until the debt is paid in full.
If you want to avoid paying high charges for Credit cards, then:

- don't use them if you will be charged for credit
- don't use them for cash advances
- use them at the end of the billing period if you have to use them.

Because so many merchants now charge for the use of credit cards, it is better to use a debit card when making purchases. When you do this, you are using your own money so you don't pay any interest.
If you make a major purchase such as furniture or a car and can't pay cash, you should avoid using **vendor finance**. Vendor finance means that the shop or company you are buying from arranges the loan. As you have already seen in the first section of this chapter, the charges for vendor finance are usually quite high. A personal loan from a financial institution will almost always be cheaper. You need to make sure that you can afford the repayments. The costs for defaults on loans are always high. Legally, if you default on a debt, the lender can get a court to sell your possessions to get their money back. In this case, the court bailiffs can enter your home and take your things. However, reputable companies will usually try to help you avoid this.

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Example 19

Ying has a personal loan of $12 000 to buy a car over 3 years at 11% flat-rate interest. Work out the following.

a The total interest **b** The total she has to pay **c** Her monthly repayments

Solution

a Write the formula.	$I = \frac{Prn}{100}$
Write the values of the variables.	$P = \$12\ 000,\ r = 11,\ n = 3$
Substitute the values.	$\text{Interest} = \frac{\$12\ 000 \times 11 \times 3}{100}$
Calculate the answer.	$= \$3960$
b Add the interest and the loan.	$\text{Total to pay} = \$12\ 000 + \$3960$ $= \$15\ 960$
c Find the number of payments.	$\text{Number of repayments} = 12 \times 3 = 36$
Divide to find the amount.	$\text{Repayment} = \frac{\$15\ 960}{36}$ $\approx \$443.33$
Write the answer.	**Ying will have to pay $443.33 a month for 3 years.**

TLF Learning object
Maths and the car: Loan calculator (L1449)
MAT10NAIN00009

Personal loans can be secured or unsecured. Example **20** would probably involve a secured loan. This means that if you default on the loan, the lender could repossess and sell the car to get their money back. Secured loans have lower interest rates than unsecured loans. You are usually required to have comprehensive insurance on a car purchased using a secured loan.

Example 20

David has an unsecured personal loan of $7500 so he can pay for a landscape gardening course. The loan is at 16% simple interest over 2 years. What are his fortnightly repayments?

Solution

Write the formula.	$I = Pin$
Write the values of the variables.	$P = \$7500,\ i = 0.16,\ n = 2$
Substitute the values and work out the interest.	$\text{Interest} = \$7500 \times 0.16 \times 2 = \$2400$
Work out the total to repay.	$\text{Total to pay back} = \$7500 + \$2400$ $= \$9900$
There are 26 fortnights in a year.	$\text{Number of payments} = 26 \times 2 = 52$
Divide to find the amount.	$\text{Repayment} = \frac{\$9900}{52} \approx \$190.38$
Write the answer.	**David will have to pay $190.38 a fortnight.**

A house is the biggest purchase that most people ever make. Banks and other institutions advertise their interest rates very aggressively. You need to look carefully at all the fees involved, which can include establishment fees of up to \$2000, accounting fees of up to \$20 a month, loan insurance, etc. The **comparison rate** is supposed to take all these costs into account to allow you to compare loans.

A loan for purchasing a house is called a **mortgage** if it involves using the property as security. Mortgages are calculated using compound interest, but because the payments you make exceed the interest payable, both, the interest and the amount owed gradually decrease. This explains why it is often called **reducing interest**. Mortgage loans are for very long periods and are secured by the property itself. However, like any loan, if you default and the property is sold for less than what you owe, you will still have to pay the outstanding amount.

Worksheet
Amount owing on a mortgage
MAT10NAWK00026

Weblink
Home loan repayment calculator
MAT10NAWB00009

Example 21

Ezra and Kate have just borrowed \$300 000 at 7.5% reducing interest to buy their first home. They have to pay \$2100 a month for the mortgage. How much have they paid off after 6 months?

Solution

Find the monthly rate.	**Monthly interest rate** $= 7.5\% \div 12 = 0.625\%$
Find the 1st month's interest.	**Interest** $= 0.00625 \times \$300\ 000 = \$1875$
Find the amount owed.	**Balance** $= \$300\ 000 + \$1875 - \$2100$ $= \$299\ 775$
Find the 2nd month's interest.	**Interest** $= 0.00625 \times \$299\ 775 \approx \$1873.59$
Find the amount owed.	**Balance** $\approx \$299\ 775 + \$1873.59 - \$2100$ $\approx \$299\ 548.59$
Find the 3rd month's interest.	**Interest** $\approx 0.00625 \times \$299\ 548.59 \approx \$1872.18$
Find the amount owed.	**Balance** $\approx \$299\ 548.59 + \$1872.18 - \$2100$ $\approx \$299\ 320.77$
Find the 4th month's interest.	**Interest** $\approx 0.00625 \times \$299\ 320.77 \approx \$1870.75$
Find the amount owed.	**Balance** $\approx \$299\ 320.77 + \$1870.75 - \$2100$ $\approx \$299\ 091.53$
Find the 5th month's interest.	**Interest** $\approx 0.00625 \times \$299\ 091.53$ $\approx \$1869.32$
Find the amount owed.	**Balance** $\approx \$299\ 091.53 + \$1869.32 - \$2100$ $\approx \$298\ 860.85$
Find the 6th month's interest.	**Interest** $\approx 0.00625 \times \$298\ 860.85$ $\approx \$1867.88$
Find the amount owed.	**Balance** $\approx \$298\ 860.85 + \$1867.88 - \$2100$ $\approx \$298\ 628.73$
Find the amount paid off.	**Amount paid off** $= \$300\ 000 - \$298\ 628.73$ $= \$1371.27$

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In Example **21**, although the couple have paid 6 × \$2100 = \$12 600 in the first six months, they have only reduced their mortgage by \$1371.27. It will take them 30 years to pay the loan back and by that time they will have paid \$756 000 altogether. Banks, building societies and other lenders do some checking to make sure they will get their money back, but they don't have to check if you can afford to pay the mortgage. In recent years, some lenders have badly miscalculated and lost a lot of money by lending money to people who cannot pay. Many overseas banks have actually gone bust as a result of such poor lending practices such as lending money to people on the dole.

Calculation of mortgage payments is quite complicated, but the table below gives the costs for some typical loans. There are many mortgage calculators available on the internet.

Monthly payments for mortgages on \$1000

Years	Interest rate							
	5%	5.5%	6%	6.5%	7%	7.5%	8%	8.5%
10	10.6066	10.8526	11.1021	11.3548	11.611	11.8702	12.1328	12.3986
15	7.9079	8.1708	8.4386	8.7111	8.9883	9.2701	9.5565	9.8474
20	6.5996	6.8789	7.1643	7.4557	7.753	8.0559	8.3644	8.6782
25	5.8459	6.1409	6.443	6.7521	7.0678	7.3899	7.7182	8.0523
30	5.3682	5.6779	5.9955	6.3207	6.653	6.9921	7.3376	7.6891

Example 22

Worksheet
Mortgage exercise
MAT10NAWK00024

a What would a mortgage of \$280 000 over 20 years at 6.5% cost each month?
b How much interest would be paid over the life of the mortgage?

Solution

a Use the 6.5% payment for 20 years in the table above. — **Payment for \$1000 = \$7.4557/month**

Multiply by 280 for \$280 000. — **Payment for loan = \$7.4557 × 280/month**

Find the answer. — **≈ 2087.60**

Write the answer. — **It would cost \$2087.60 each month.**

b Multiply the payment by the number of them. — **Total paid = \$2087.60 × 12 × 20**

Calculate the answer. — **= \$501 024**

Subtract the principal to get the interest. — **Interest = \$501 024 − \$280 000**
= \$221 024

Write the answer. — **The total interest paid would be \$221 024.**

How do you know whether you can afford to make payments on a loan without doing a detailed budget? A good rule of thumb is the 25% rule: you can probably afford to spend 25% of your income (after tax) on repayments. Some financial institutions lift this to 30% for high income earners.

Example 23

Erica and Martha have a combined income of \$2120 a week after tax. They have commitments of \$360 in existing payments. How much could they afford to borrow using each of the following loans?

a 20-year loan at 6.5% **b** 25-year loan at 6.5% **c** 30-year loan at 6.5%

Solution

a Change to yearly after-tax income.	**Yearly income** $= \$2120 \times 52$ $= \$110\ 240$
Change to monthly after-tax income.	**Monthly income** $= \$110\ 240 \div 12$ $\approx \$9186.67$
Divide by 4 to get 25%.	**Total payments** $= 9186.67 \div 4$ $\approx \$2296.67$
Subtract commitments.	**Affordable mortgage** $= \$2296.67 - \$360$ $= \$1936.67$
Use the 6.5% column for 20 years.	**Payment for \$1000** $= \$7.4557$
Divide to find the maximum loan.	**Maximum amount** $= \$1936.67 \div \$7.4557$
Round *down* to the nearest whole.	≈ 259
Write the answer.	**They could afford a 20-year loan of \$259 000.**
b Use the 6.5% column for 25 years.	**Payment for \$1000** $= \$6.7521$
Divide to find the maximum loan.	**Maximum amount** $= \$1936.67 \div \$6.7521$
Round *down* to the nearest whole.	≈ 286
Write the answer.	**They could afford a 25-year loan of \$286 000.**
c Use the 6.5% column for 30 years.	**Payment for \$1000** $= \$6.3207$
Divide to find the maximum loan.	**Maximum amount** $= \$1936.67 \div \$6.3207$
Round *down* to the nearest whole.	≈ 306
Write the answer.	**They could afford a 30-year loan of \$306 000.**

Conservative lenders like banks will allow you to borrow up to about 90% of the value of a property. Mortgage lenders who allow you to borrow a greater proportion charge higher interest rates. You also need to allow enough to cover legal and transfer costs.

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Exercise 9.3 Borrowing money

Fluency

Extra questions

Exercise 9.3

MAT10NAEQ00027

See Example 19

1 Constance has a personal loan for \$13 000 to buy a car over 3 years at 11.5%. Work out the following.

a The total interest **b** The total to repay **c** The monthly repayments

2 Mohammed borrowed \$17 500 over 4 years at 10.8% flat rate interest. Work out the following.

a The total interest **b** The total to repay **c** The fortnightly repayments

3 Zahra has a loan of \$10 900 at 11.7% over 2 years. Work out the following.

a The total interest **b** The total to repay **c** The monthly repayments

See Example 20

4 Find the interest payable and repayments on each of the following unsecured personal loans.

a \$12 000 at 17% interest over 2 years with monthly repayments

b \$16 600 at 16.8% interest over 3 years with fortnightly repayments

c \$11 400 at 19.3% interest over 2 years with weekly repayments

d \$22 000 at 15.8% interest over 4 years with monthly repayments

e \$15 800 at 17.5% interest over 3 years with fortnightly repayments

See Example 22

5 For each of the following mortgage loans, use the table on page 369 to find the amount that must be paid each month to borrow \$1000.

a A loan over 25 years at 7% **b** A loan over 30 years at 6.5%

c A loan over 20 years at 5.5% **d** A loan over 25 years at 7.5%

e A loan over 30 years at 8%

6 Use the table on page 369 to find the monthly payment for each of the following mortgages.

a \$320 000 over 30 years at 8% interest

b \$480 000 over 25 years at 7.5% interest

c \$290 000 over 20 years at 6.5% interest

d \$520 000 over 30 years at 7% interest

e \$860 000 over 30 years at 6.5% interest

7 For each loan in question **6**, work out the amount of interest that is paid altogether.

Problem solving

See Example 17

8 Callum borrowed \$400 from a payday lender for 4 days. The charge was \$25 per \$100 borrowed plus \$2 a day.

a Work out the amount he has to pay back.

b Work out the interest rate.

9 Dorothea borrowed \$500 from a cash loans provider for 6 days at a charge of \$45 to set up the loan and \$20 per hundred dollars or part thereof. She was told that if she paid it back late she would be charged for another loan, for the amount due and a penalty of \$60, and given 5 days to pay the new loan.

a Work out the amount she has to pay back.

b Work out the interest rate if it is paid on time.

c Work out the amount she has to pay if she is late.

d Work out the interest rate if the loan is paid 5 days after it was due.

Worked solutions
Exercise 9.3
MAT10NAWS00027

10 Andrew borrowed $450 from an internet lender for 2 weeks at a rate of $25 per hundred dollars or part thereof and $10 per week of the loan. Defaulting would incur a penalty of $80 and a new loan would be set up for a week for the total amount.

a Work out the amount he has to pay back.

b Work out the interest rate if it is paid on time.

c Work out the amount he has to pay if he is late.

d Work out the interest rate if it is paid a week (7 days) after it was due.

See Example 18

11 Phoong's credit card costs $90 a year and has an interest rate of 12.6% for purchases and 25.7% for cash advances. His billing period is from the 16th of one month to the 15th of the next and his account is due on the 22nd of each month. He used his credit card at an ATM to get $400 cash on 20 October. Since the ATM was from another bank he was charged a flat fee of $5 on the transaction. On the same day, he used his credit card to buy a takeaway that cost him $23.65 and had to pay a fee of $2 for using his credit card. Work out each of the following.

a The total cost of the cash advance (note that the $5 fee is not subject to interest).

b The interest rate for the cash advance.

c The total cost of the takeaway.

d The interest rate on the takeaway.

12 Anja's credit card costs $150 a year and has an interest rate of 10.8% on purchases and 20.5% on cash advances. Her billing period ends on the 12th of each month and the account is due on the 26th of the month. She has an 'interest-free' period of up to 45 days, which means that if she pays her account on or before the due date she pays no interest on purchases. If she doesn't pay on time she pays interest for purchases and a penalty of 5% of the outstanding amount, then outstanding amounts are charged at the cash rate until paid. Interest is always charged on cash advances. Interest is charged on the '45 days'. For the account due in November she had a total of $690 in purchases including $12 in merchant credit charges and $500 in cash advances, having paid 2.5% on the $300 cash advance and $4 for the other one. She paid this account on time. However, she didn't pay the account due in December until the 3rd of January. This account had $2424 (including $15 in merchant credit charges) in purchases and $900 in cash advances, plus merchant fees of $18 for the cash advances.

a Explain how you would calculate the average daily balance in this case.

b How much did she pay on 28 November?

c What was the interest rate on her cash?

d How much was due on 26 December?

e How much did she have to pay on 3 January?

f What was the overall interest rate?

13 Simon bought $84.60 worth of groceries on the 15th of the month and was going to get a cash-out of $100 when he paid. However, when the checkout operator said "Credit card charge is 3%, OK?" he changed his mind and paid cash. His card doesn't have an annual fee but the interest charge for both purchases and cash advances is 26%. His billing period finishes on the 25th of the month.

a How much did he save by changing his mind?

b What would the interest rate have been if he had done as he planned?

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14 Dick and Jane just bought a new house with a mortgage of \$440 000 at a variable rate of 7.5% over 25 years. Use the table on page 369 to find out the monthly payment on their mortgage and work out how much they have paid off after the first 4 months.

See Example 21

15 Peta and Konrad borrowed \$360 000 to buy their first house at a mortgage rate of 6.5% over 25 years. They chose to make higher payments of \$3136 a month because they were told this would reduce the term to only 15 years if they could keep it up.

Worked solutions
Exercise 9.3
MAT10NAWS00027

a How much was left to pay after the first 6 months?

b How much extra were they paying each month?

c How much would they save over the term of the loan?

16 Carmelita and James have a combined income of \$2920 a week after tax. They have commitments of \$474 a month in existing payments. How much could they afford to borrow using each of the following loans?

See Example 23

a 20-year loan at 6% **b** 25-year loan at 6% **c** 30-year loan at 6%

17 Rajendra and Julia have a combined income of \$2570 a week after tax. They have commitments of only \$136 a month in existing payments. How much could they afford to borrow using each of the following loans?

a 15-year loan at 7.5% **b** 20-year loan at 7.5% **c** 30-year loan at 7.5%

18 Instead of a standard mortgage in question **14**, Dick and Jane could have chosen an 'easy-start' mortgage that had reduced payments of only \$2000 a month for the first 6 months. If they had chosen this mortgage, what would they owe after the first 6 months? Approximately what would the payments be after the low start, assuming that the 25 years started after that?

Reasoning

19 Alistair has a choice of credit cards. He normally uses a debit card and only intends to use his credit card in an emergency. If you were Alistair, explain which of the following you would choose.

Worked solutions
Exercise 9.3
MAT10NAWS00027

A No annual fee, interest rate of 25% on purchases and cash advances.

B Annual fee of \$40, interest rate of 12% on purchases and 24% on cash advances.

C Annual fee of \$150, interest rate of 10% on purchases and 21% on cash advances.

D Annual fee of \$250, interest rate of 10% on purchases and 21% on cash advances, with an interest-free period of 'up to 45 days'.

20 Tonya has the choice of credit cards given in question **19**, but she normally buys about \$1500 worth of things a month and pays on time for her purchases about 75% of the time. She never gets cash-outs. When she is late, she always pays off her card the next month. Explain which card you think she should choose.

Chapter 9 summary

Quiz
Money and finance
MAT10NAQZ00009

- **Interest** is the charge made on a loan. The ratio of the interest for one year to the amount borrowed is called the **interest rate** and is normally written as a decimal or percentage. It is also called the **nominal interest rate**.
- The amount lent or borrowed is called the **principal.**
- The amount that has to be paid back each week or month for a loan on **terms** is called the **instalment** or **repayment.**
- **Simple interest** is calculated using the whole loan for the whole time it is borrowed. It is also called **flat-rate interest.**
- **Compound interest** is calculated at regular intervals called **rests,** and added to the loan, with the result that the principal increases at each rest. If you make regular payments on a loan at compound interest, the interest is usually calculated before the payment is subtracted from the principal.
- Compound interest on a loan is often called **reducing interest**.
- A **pay-day lender** lends small amounts of cash for short periods of time, usually until the borrower's next pay-day.
- The **simple interest formula** for the interest I on the principal P at interest rate i (or $r\%$) for n rests is $I = Pin = \frac{Prn}{100}$.
- The **growth factor** is the quantity $1 + \frac{i}{k}$ you multiply the amount accumulated at compound interest by at each rest to get the new amount, where i is the interest rate and k is the number of rests in a year.
- You can use the constant multiplier function on your calculator to help calculate compound interest.
- The **compound interest formula** is $A = P(1+i)^n = P\left(1 + \frac{r}{100}\right)^n$, where A is the amount after n years, P is the principal and i (or $r\%$) is the interest rate. For non-annual rests the formula is $A = P\left(1 + \frac{i}{k}\right)^{kt} = P\left(1 + \frac{r}{100k}\right)^{kt}$, where A is the amount after kt rests, k is the number of rests in a year and t is the number of years.
- Failure to pay something according to the contract, such as a loan, is called **defaulting**.
- A **cash-out** is cash obtained with a credit or debit card when paying for purchases.
- The **average daily balance** of a credit account is the average of the amounts at the end of each day over the billing period.
- The **daily rate** of interest is the nominal interest rate divided by 365 (the days in a year).
- **Vendor finance** is a loan that is arranged by the seller of goods.
- The **comparison rate** for a loan takes account of fees and charges, in addition to the stated interest rate, and is supposed to make it easier to compare the rates of different providers.
- A **mortgage** is a secured loan for the purpose of buying property.

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Chapter 9 review

Understanding

1 Use the simple interest formula to calculate the interest for each of the following. *See Example 4*

a $760 at 13% for 4 years b $2624 at 6.8% for 3 years

2 Find the growth factor for each of the following cases of compound interest. *See Example 6*

a 7.5% compound interest b 9.7 compound interest
c 3.8% compound interest

3 Find the growth factor for each of the following cases of compound interest. *See Example 8*
a 5.1% compound interest with monthly rests.
b 7.8% compound interest with weekly rests.

Fluency

4 Find the interest rate for each of the following payday loans. *See Example 1*
a $520 for 5 days with a charge of $25 per $100 or part thereof.
b $380 for 9 days with a charge of $50, plus $30 a week or part thereof.

5 Find the real simple interest rate for a car available for $19 990 or weekly payments of $199.90 a week over 4 years. *See Example 3*

6 Find the interest earned in a financial year on a savings account that pays interest of 2.1% on minimum monthly balances if the balances are as shown in the table below. *See Example 11*

Month	Jul	Aug	Sep	Oct	Nov	Dec
Balance	$2317.56	$2256.27	$470.29	$412.39	$2093.56	$3064.87
Month	Jan	Feb	Mar	Apr	May	Jun
Balance	$2185.10	$ 265.82	$310.88	$400.70	$ 222.39	$ 418.17

7 How much is a term deposit of $14 700 at 6.3% for 9 months worth at the end of the term? *See Example 12*

8 Afua has a personal loan for $17 000 to buy a car over 4 years at 11.5%. Work out the following. *See Example 19*

a The total interest b The total to repay c The monthly repayments

9 Find the interest payable and repayments on each of the following unsecured personal loans. *See Example 20*
a $7000 at 16.5% interest over 18 months with monthly repayments
b $19 500 over 4 years at 18.6% interest with fortnightly repayments

10 Calculate the amount and interest for each of the following. *See Examples 5, 6*
a $8000 at 8% compound interest for 3 years
b $7900 at 6.7% compound interest for 4 years

11 Calculate the amount and total interest for $12 300 at 7.8% compound interest for 5 months with monthly rests. *See Example 8*

12 $940 000 is invested at 2.9% for 6 days on the short-term money market. How much interest is earned? *See Example 10*

13 A term deposit of $26 400 for 4 years pays 7.2%. The interest is calculated and paid into a savings account every quarter. How much is paid each quarter? *See Example 13*

14 A term deposit of $21 000 over 3 years pays 5.75% interest. How much is it worth at the end of the term?

See Example 22

15 Use the table on page 369 to find the amount that must be paid each month on a mortgage to borrow \$1000 at 8% interest over 25 years.

16 Use the table on page 369 to find the monthly payment for a mortgage of \$355 000 over 30 years at 6.5%. How much interest is paid altogether on the mortgage?

See Example 7

17 Use the formula to calculate the amount and interest for \$6240 at 7.3% compound interest for 5 years.

See Example 9

18 Use the compound interest formula to calculate the amount and interest for \$24 000 at 6.5% compounding fortnightly for 8 years.

See Example 14

19 What is an investment of \$12 300 at 6.9% compounding monthly worth after 5 years?

Problem solving

See Example 2

20 Derek 'rented' a lounge suite costing \$2499 for \$31.99 a week on a 48-month contract. Halfway through the contract he paid \$300 to upgrade to a newer suite costing \$2199, with a new 3-year contract for \$25 a week. At the end of the contract he took up the offer to buy the suite for \$150.

a What was the interest rate for the first suite, assuming that the \$300 would have been enough to buy it outright after the 2 years?

b What was the interest rate for the second contract?

See Example 15

21 How many years would it take an investment of \$7200 to reach a value of at least \$12 000 at 14% simple interest?

See Example 17

22 Corey borrowed \$360 from a payday lender for 5 days. The charge was \$25 per \$100 or part thereof borrowed plus an establishment fee of \$20. Unfortunately he could not pay on time and had to pay a penalty of \$75, and the loan and costs were rolled over into a new loan that was paid on his next payday, 7 days later.

a Work out the amount he had to pay back in the first instance.

b Work out the interest rate on the first loan.

c How much did he eventually have to pay back?

d What was the overall interest rate?

See Example 16

23 How many years would it take an investment of \$8500 to reach a value of at least \$12 000 at 4.6% compound interest?

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24 Wilhemina's credit card costs \$120 a year and has an interest rate of 11.4% for purchases and 23.2% for cash advances. Her billing period ends on the 20th of each month. One month she bought a new outfit for \$420 on the 2nd and got a cash advance of \$200 on the 13th. She had to pay merchant charges of \$5 and 3.5% respectively to use her credit card for these transactions. *See* Example 18

Work out each of the following.

a The total cost of the new clothes.
b The interest rate for the new clothes.
c The total cost of the cash advance.
d The interest rate for the cash advance.

25 Carol bought a new house with a mortgage of \$325 000 at a variable rate of 6% over 25 years. Use the table on page 369 to find the monthly payment on her mortgage and work out how much she has paid off after the first 5 months. *See* Example 21

26 Stevie has an income of \$2420 a week after tax. He has commitments of \$220 a month in existing payments. How much could he afford to borrow using each of the following loans? *See* Example 23

a 20-year loan at 6.5%
b 25-year loan at 6.5%
c 30-year loan at 6.5%

Reasoning

27 Find the number of years it takes for an amount to at least triple with a compound interest rate of 10%. Explain your method.

28 Find the simple interest rate that will give the same return as an investment at 12% interest compounded over 4 years, clearly explaining your steps.

29 Find the interest rate that, compounding monthly, gives the same interest as 24% simple interest over 2 years, logically explaining your steps.

30 Colleen has a choice of the following credit cards. She uses her credit card for an average of \$2400 in purchases and cash advances of about \$1000 a month. Explain which of the following you would recommend for her.

A No annual fee, interest rate of 24% on purchases and cash advances.
B Annual fee of \$30, interest rate of 14% on purchases and 24% on cash advances.
C Annual fee of \$120, interest rate of 12% on purchases and 21% on cash advances.
D Annual fee of \$200, interest rate of 12% on purchases and 21% on cash advances, with an interest-free period of 'up to 45 days'.

Measurement and geometry

10 Trigonometry

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Contents

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MAT10MGPL00010

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Curriculum guide
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Australian Curriculum statements

Pythagoras and trigonometry

Solve right-angled triangle problems including those involving direction and angles of elevation and depression. (ACMMG245)

Use the unit circle to define trigonometry functions, and graph them with and without the use of digital technologies. (10A) (ACMMG274)

Apply Pythagoras' theorem and trigonometry to solving three-dimensional problems in right-angled triangles. (10A) (ACMMG276)

Video tutorial
Trigonometry
MAT10MGVT00010

Sumerian astronomers were the first to use angle measure, by dividing circles into 360 degrees. They also studied the ratios of sides of similar triangles. They were followed by the Babylonians who extended the study of the ratios of the sides of similar triangles to discover some of the major properties of these ratios. It was, however, the ancient Greeks who transformed trigonometry into a formal field of mathematics. In recognition of the work of the Greeks, the word trigonometry is derived from the Greek roots *trigon*—triangle and *metron*—measure.

Trigonometry is a branch of mathematics that uses the relationships between the sides and angles of triangles. The earliest applications of trigonometry were in the fields of astronomy, navigation and surveying. By using trigonometry, it became possible to calculate distances that could not be measured directly, such as the distance between the Earth and the Moon, or the height of a very tall structure. The pyramids of Egypt and the megaliths of Stonehenge in England were built using a detailed knowledge of angles.

Applications of trigonometry are found in physics and virtually all branches of engineering, particularly in the study of periodic phenomena, such as sound vibrations and the flow of alternating current.

Mathematical literacy

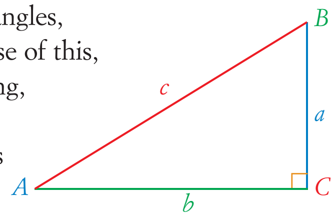
Maths dictionary
MAT10ASDI00001

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

angle of depression	composite figures	origin	sine (sin)
angle of elevation	cosine (cos)	polygon	trigonometric (trig) ratio
bearing	direction	Pythagoras' theorem	tangent (tan)
Cartesian plane	hypotenuse	Pythagorean triad	three dimensions
compass	line of sight	Pythagorean triple	two dimensions
compass rose	opposite side	right-angled	unit circle

10.1 Pythagoras' theorem

Any straight-sided closed figure **(polygon)** can be divided into triangles, and these can always be divided into right-angled triangles. Because of this, right-angled triangles have always been used as the basis of building, surveying and navigation.



There is an accepted naming convention for triangles. The vertices are named using capital letters, and the opposite sides have the same lower-case letter.

In $\triangle ABC$:

a is opposite $\angle A$, b is opposite $\angle B$ and c is opposite $\angle C$

and $AB = c$, $BC = a$ and $AC = b$.

It is also customary to indicate the right angle with a small square in the corner. In a right-angled triangle, the longest side is the **hypotenuse**. The hypotenuse is opposite the right angle.

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Important!

Pythagoras' theorem

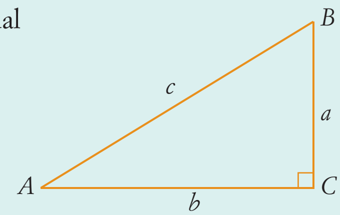
In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

So for $\triangle ABC$:

$$AB^2 = BC^2 + AC^2$$

or

$$c^2 = a^2 + b^2$$



TLF Learning object
Exploring the Pythagorean theorem (L6559)
MAT10MGIN00010

In Chapter 4, you saw how Pythagoras' theorem could be used to find the lengths of unknown sides when calculating the areas of triangles and other shapes.

Example 1

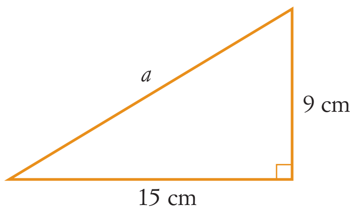
Teacher notes
Proof of Pythagoras' theorem
MAT10MGTN00008

CAS TI-Nspire exercise
Trigonometry
MAT10MGTI00010

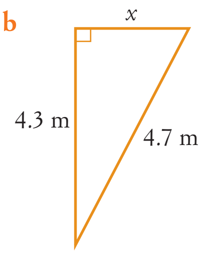
CAS ClassPad exercise
Trigonometry
MAT10MGCP00010

Find the unknown side in each of the following triangles.

a



b



Solution

a Apply Pythagoras' theorem to the triangle. $\quad a^2 = 15^2 + 9^2$

Evaluate. $\quad = 225 + 81$

$= 306$

Take the square root of both sides. $\quad a = 17.4928 \ldots$ cm

Round off and state the result. **The unknown side is about 17.5 cm.**

b Apply Pythagoras' theorem to the triangle. $\quad 4.7^2 = 4.3^2 + x^2$

Evaluate. $\quad 22.09 = 18.49 + x^2$

Reverse the equation. $\quad x^2 + 18.49 = 22.09$

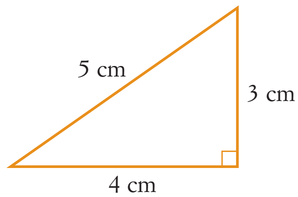
Subtract 18.49 from both sides. $\quad x^2 = 22.09 - 18.49$

$= 3.6$

Take the square root of both sides. $\quad x = 1.8973 \ldots$ m

Round off and state the result. **The unknown side is about 1.9 m.**

There are some right-angled triangles in which the lengths of the sides are whole numbers. Consider the triangle shown here.
You can see that:



$$5^2 = 4^2 + 3^2$$
$$25 = 16 + 9$$

Any three whole numbers that satisfy Pythagoras' theorem (e.g. 3, 4, 5) are known as a **Pythagorean triple** (or **triad**). It can easily be shown that any multiple of a Pythagorean triple is also a Pythagorean triple. For example,

$10^2 = 8^2 + 6^2$ because $100 = 64 + 36$, so 6, 8, 10 is a Pythagorean triple
$15^2 = 12^2 + 9^2$ because $225 = 144 + 81$, so 9, 12, 15 is a Pythagorean triple

and so on.

Puzzle sheet
Pythagorean triads
MAT10MGPS00030

Teacher notes
Pythagorean triples
MAT10MGTN00009

Important!

Pythagorean triples (or triads)

Common Pythagorean triples include:

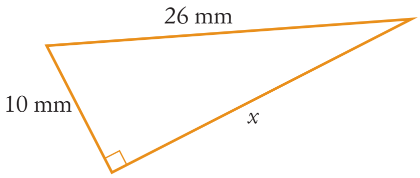
3, 4, 5   5, 12, 13   8, 15, 17   7, 24, 25

Any multiple of these Pythagorean triples will also be a Pythagorean triple.

Pythagorean triples can sometimes be used as a 'short cut' when solving triangles.

Example 2

Find the unknown side in this triangle.



Solution

Look at the factors of the side lengths.

The shortest side is 5×2 and the hypotenuse is 13×2. **There is a 5, 12, 13 triple.**

The sides are 2 times the triple 5, 12, 13. $\boldsymbol{x = 12 \times 2 = 24}$ **mm**

State the result. **The unknown side is 24 mm.**

You can use Pythagoras' theorem to check whether a triangle is right-angled. Builders often use Pythagoras' theorem to check whether corners are square.

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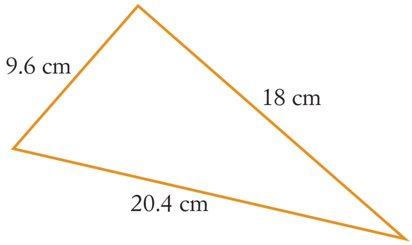
Example 3

Is the triangle shown here right-angled?



Solution

If the triangle is right-angled, the longest side will be the hypotenuse and Pythagoras' theorem will apply, so $20.4^2 = 9.6^2 + 18^2$.

Find the sum of the squares of the shorter sides. $\mathbf{9.6^2 + 18^2 = 92.16 + 324}$

$\mathbf{= 416.16}$

Find the square of the longest side. $\mathbf{20.4^2 = 416.16}$

Compare the results. $\mathbf{20.4^2 = 9.6^2 + 18^2}$

This satisfies Pythagoras' theorem. **The triangle is right-angled.**

Pythagoras' theorem has many practical applications.

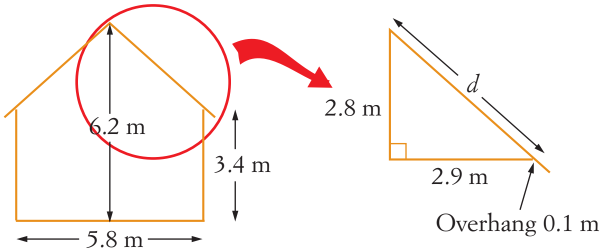
Example 4

The front of this shed is 5.8 m wide. The sides are 3.4 m high and the highest point of the roof is 6.2 m above its base. The roof of the shed is made from sheets of iron that overhang by 10 cm on both sides.
How long are the sheets of iron?

Solution

Draw a diagram and add dimensions.

Change the overhang to the same units (m).



Apply Pythagoras' theorem to the triangle.	$d^2 = 2.9^2 + 2.8^2$
Evaluate.	$= 8.41 + 7.84$
	$= 16.25$
Take the square root of both sides.	$d = 4.0311... \approx 4.03$ m
The overhang needs to be added.	**Sheet length** $\approx 4.03 + 0.1$ m
Round off and state the result.	**The roof sheets are about 4.13 m (4130 mm) long.**

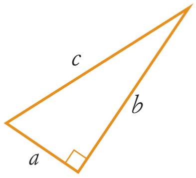
Exercise 10.1 Pythagoras' theorem

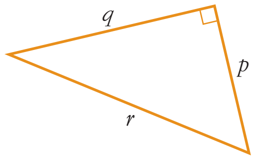
In this exercise, answers should be rounded to 1 decimal place where necessary.

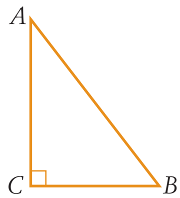
Understanding

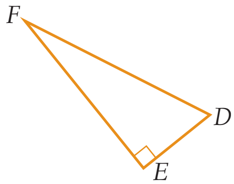
Extra questions
Exercise 10.1
MAT10MGEQ00028

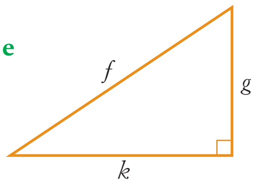
1 Name the hypotenuse for each of the following triangles.

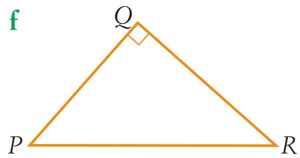
a


b


c


d


e


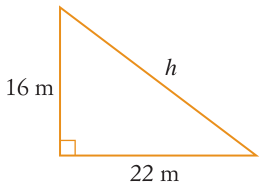
f


2 State Pythagoras' theorem for each of the triangles shown in question **1**.

Fluency

See Example 1

3 Calculate the hypotenuse in each of the following triangles.

a


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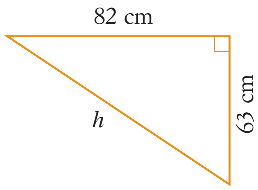

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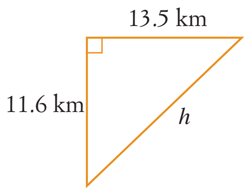

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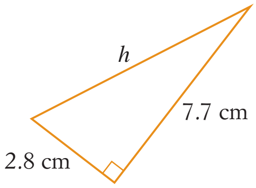

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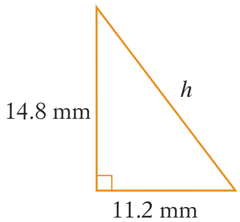

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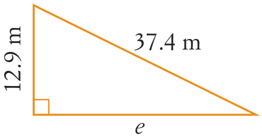
4 Calculate the unknown side in each of the following triangles.

a



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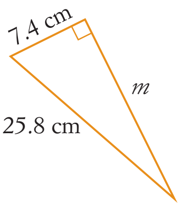

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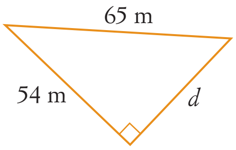

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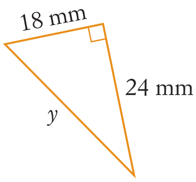

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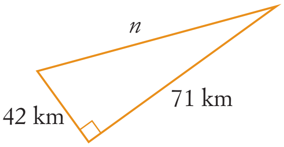

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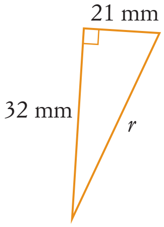

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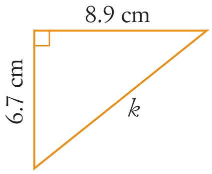

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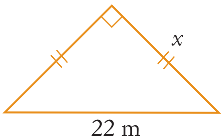
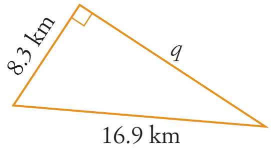


i



5 Use Pythagorean triples to find the unknown side in each of the following triangles.

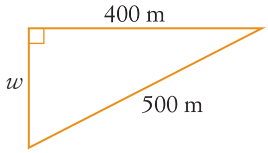
Worked solutions

Exercise 10.1

MAT10MGWS00028

See Example 2

a



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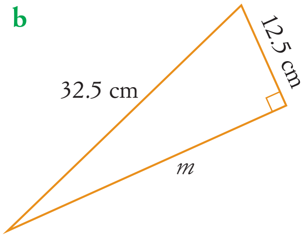

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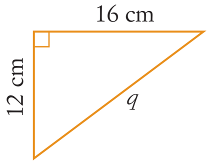

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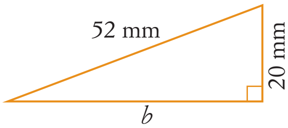

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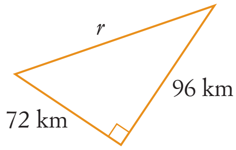
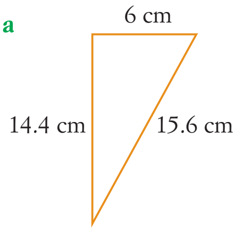


6 Which of the following are right-angled triangles?

See Example 3

a



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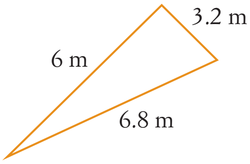

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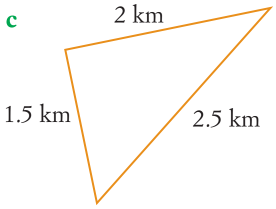

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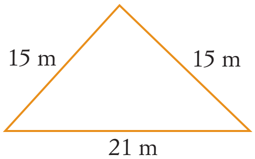

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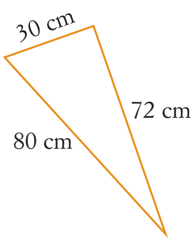

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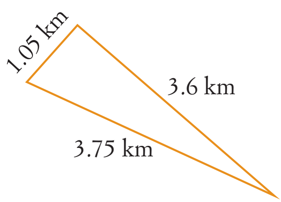
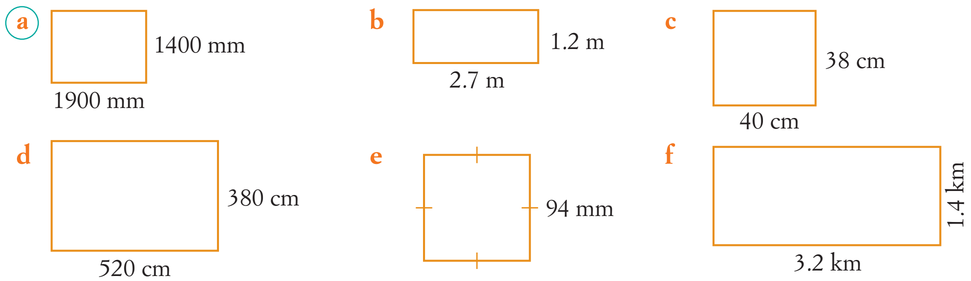


Problem solving

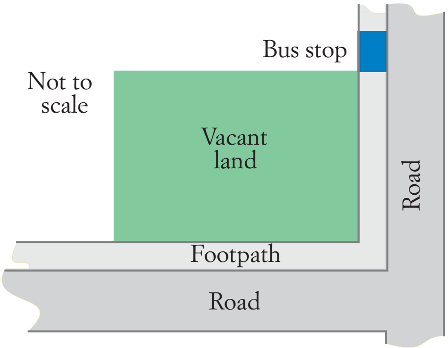
Worked solutions
Exercise 10.1
MAT10MGWS00028

7 Find the length of the diagonals in each of the following rectangles and squares.



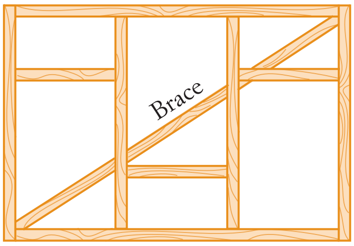
Worked solutions
Exercise 10.1
MAT10MGWS00028

8 A vacant corner block of land measures 32 m by 22 m. Instead of going around the footpath, many people walk diagonally across the unfenced block to reach a bus stop.
How much further will they need to walk when the block is fenced?



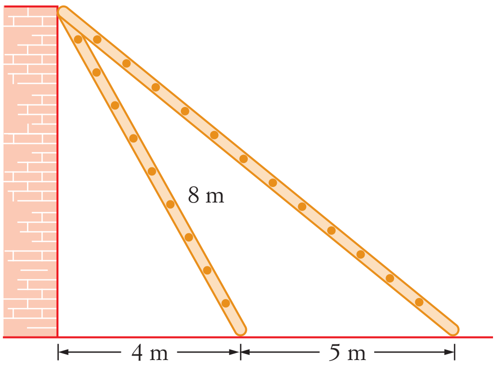
See Example 4

9 A frame is being constructed for a wall 3.5 m long and 2.4 m high. The wall requires a brace as shown here. How long should the brace be?

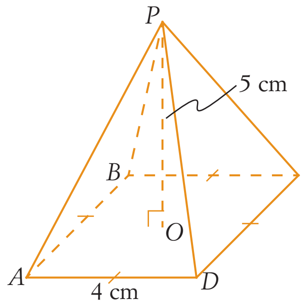


Puzzle sheet
Pythagorean two-step problems
MAT10MGPS00031

10 Two ladders are resting at the top of a wall as shown below. The shorter ladder is 8 m long and its bottom end is 4 m from the base of the wall. The end of the longer ladder is 5 m from the end of the shorter ladder on the ground. How long is the longer ladder?



11 *ABCDP* is a pyramid with a square base of side 4 cm. The height of the pyramid (*OP*) is 5 cm. Calculate the length of the edge *PA*.



Reasoning

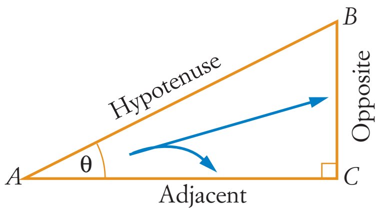
12 A cyclist travels 35 km north of her starting point and then travels 46 km due west. How far is she from her starting point 'as the crow flies'?

13 A rectangular room measuring 4 m by 8 m is to be used for a home theatre. The walls of the room are 3 m high. The projector is to be mounted on the ceiling halfway between the longer side walls and facing one of the 4 m wide walls where the screen will be mounted. The screen measures 1.6 m by 0.9 m and is set up with its centre in the centre of the 4 m wide wall. If the optimum distance for the projector is 7 m from the centre of the screen, how far along the ceiling from the wall on which the screen is mounted should the projector be installed?

10.2 Trigonometric ratios

TLF Learning object
Exploring trigonometry (L6561)
MAT10MGIN00010

You have previously done some work with trigonometric ratios in Year 9. We often shorten the word 'trigonometric' to just 'trig'. It's easier to say.
The diagram below shows a right-angled triangle, △*ABC*. An angle has been marked with the Greek letter θ (theta). Greek letters are often used to represent angles in trigonometry because of the enormous contribution that Greek mathematicians made to this field of study.



In △*ABC*:

- *AB* is called the **hypotenuse**. The hypotenuse is always the longest side. It is also the side opposite the right angle.
- *AC* is called the side **adjacent** to θ. (Adjacent means 'next to'.)
- *BC* is called the side **opposite** θ.

There are three trig ratios—**sine, cosine** and **tangent**.

Important!

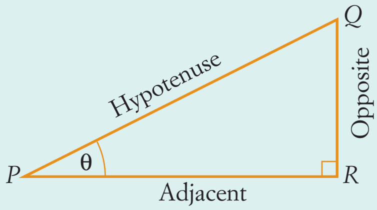
Trig ratios

In any right-angled triangle PQR,

$$\text{sine } \theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{QR}{PQ}$$

$$\text{cosine } \theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{PR}{PQ}$$

$$\text{tangent } \theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{QR}{PR}$$



The names of the trig ratios are usually shortened as follows:
sine $\Longleftrightarrow$ **sin** cosine $\Longleftrightarrow$ **cos** tangent $\Longleftrightarrow$ **tan**
The acronym 'SOH CAH TOA' may help you to remember these ratios.

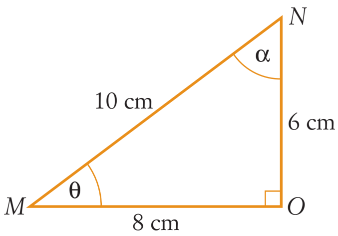
Important!

sin = opp/hyp	cos = adj/hyp	tan = opp/adj
S O H	C A H	T O A
Some Old Hams	Can't Always Hide	Their Old Age

Example 5

For $\triangle MNO$:

a name the hypotenuse
b name the side opposite α
c name the side adjacent to θ
d Complete: $\sin\,\theta = \frac{\text{opposite}}{\ldots\ldots\ldots\ldots} = \frac{NO}{\ldots\ldots}$
e Express $\cos\,\alpha$ as a ratio in simplest form.
f Express $\tan\,\theta$ as a ratio in simplest form.



Solution

a MN is the longest side (it is also opposite the right angle). ***MN* is the hypotenuse.**

b Imagine drawing a line from α across the triangle. ***MO* is opposite α.**

c Both MN and MO are next to θ but MN is the hypotenuse. ***MO* is adjacent to θ.**

d Refer to the rule for the sin ratio.

$$\sin\,\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

Substitute the side names.

$$= \frac{NO}{MN}$$

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e Refer to the rule for the cos ratio. $\cos\alpha = \frac{\text{adjacent}}{\text{hypotenuse}}$

Substitute information from the diagram. $= \frac{6\text{ cm}}{10\text{ cm}}$

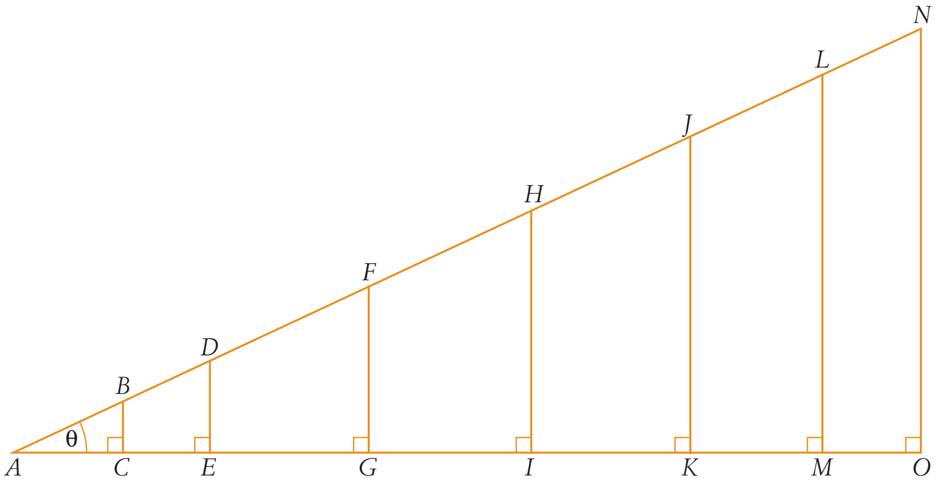
Simplify. $= \frac{3}{5}$

f *NO* is the opposite side for θ. $\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{6}{4} = \frac{3}{2}$

Investigate: Trig ratios

Worksheet
Investigating trigonometric ratios
MAT10MGWK00027

1 Look at the following diagram. How many right-angled triangles can you see?



2 Copy the table below and then complete it by measurement and calculation. Measure all lengths to the nearest mm and express any calculations correct to 3 decimal places.

$\triangle ABC$	$\triangle ADE$	$\triangle AFG$	$\triangle AHI$	$\triangle AJK$	$\triangle ALM$	$\triangle ANO$
$AB = \ldots$	$AD = \ldots$	$AF = \ldots$	$AH = \ldots$	$AJ = \ldots$	$AL = \ldots$	$AN = \ldots$
$AC = \ldots$	$AE = \ldots$	$AG = \ldots$	$AI = \ldots$	$AK = \ldots$	$AM = \ldots$	$AO = \ldots$
$BC = \ldots$	$DE = \ldots$	$FG = \ldots$	$HI = \ldots$	$JK = \ldots$	$LM = \ldots$	$NO = \ldots$
$\frac{BC}{AB} = \ldots$	$\frac{DE}{AD} = \ldots$	$\frac{FG}{AF} = \ldots$	$\frac{HI}{AH} = \ldots$	$\frac{JK}{AJ} = \ldots$	$\frac{LM}{AL} = \ldots$	$\frac{NO}{AN} = \ldots$
$\frac{AC}{AB} = \ldots$	$\frac{AE}{AD} = \ldots$	$\frac{AG}{AF} = \ldots$	$\frac{AI}{AH} = \ldots$	$\frac{AK}{AJ} = \ldots$	$\frac{AM}{AL} = \ldots$	$\frac{AO}{AN} = \ldots$
$\frac{BC}{AC} = \ldots$	$\frac{DE}{AE} = \ldots$	$\frac{FG}{AG} = \ldots$	$\frac{HI}{AI} = \ldots$	$\frac{JK}{AK} = \ldots$	$\frac{LM}{AM} = \ldots$	$\frac{NO}{AO} = \ldots$

3 Measure the size of θ using your protractor.

4 Locate the trig functions on your calculator. Your teacher will help you if you need help to use these functions. Use your calculator to find the values of sin θ, cos θ and tan θ.

5 Check back with the values in the table you have just completed. What do you notice?

6 What do the values in each row represent?

You saw in the previous investigation that the values of sin θ, cos θ and tan θ remain constant for any value of θ, no matter how long the sides of the triangle in which θ occurs. Because of this fact, it is possible to program calculators and computers with the values for sin θ, cos θ and tan θ. A table of trig ratios is provided below for you to use, but it is more accurate to use the trigonometric functions on a scientific calculator. Different calculators use slightly different logic, so you will need to make sure that you can use the trig functions on *your own calculator*. Make sure that your calculator is in the *degree mode*.

Trigonometric ratios

Deg.	Sin	Cos	Tan	Deg.	Sin	Cos	Tan	Deg.	Sin	Cos	Tan
0	0.000	1.000	0.000	**30**	0.500	0.866	0.577	**60**	0.866	0.500	1.732
1	0.017	1.000	0.017	**31**	0.515	0.857	0.601	**61**	0.875	0.485	1.804
2	0.035	0.999	0.035	**32**	0.530	0.848	0.625	**62**	0.883	0.469	1.881
3	0.052	0.999	0.052	**33**	0.545	0.839	0.649	**63**	0.891	0.454	1.963
4	0.070	0.998	0.070	**34**	0.559	0.829	0.675	**64**	0.899	0.438	2.050
5	0.087	0.996	0.087	**35**	0.574	0.819	0.700	**65**	0.906	0.423	2.145
6	0.105	0.995	0.105	**36**	0.588	0.809	0.727	**66**	0.914	0.407	2.246
7	0.122	0.993	0.123	**37**	0.602	0.799	0.754	**67**	0.921	0.391	2.356
8	0.139	0.990	0.141	**38**	0.616	0.788	0.781	**68**	0.927	0.375	2.475
9	0.156	0.988	0.158	**39**	0.629	0.777	0.810	**69**	0.934	0.358	2.605
10	0.174	0.985	0.176	**40**	0.643	0.766	0.839	**70**	0.940	0.342	2.747
11	0.191	0.982	0.194	**41**	0.656	0.755	0.869	**71**	0.946	0.326	2.904
12	0.208	0.978	0.213	**42**	0.669	0.743	0.900	**72**	0.951	0.309	3.078
13	0.225	0.974	0.231	**43**	0.682	0.731	0.933	**73**	0.956	0.292	3.271
14	0.242	0.970	0.249	**44**	0.695	0.719	0.966	**74**	0.961	0.276	3.487
15	0.259	0.966	0.268	**45**	0.707	0.707	1.000	**75**	0.966	0.259	3.732
16	0.276	0.961	0.287	**46**	0.719	0.695	1.036	**76**	0.970	0.242	4.011
17	0.292	0.956	0.306	**47**	0.731	0.682	1.072	**77**	0.974	0.225	4.331
18	0.309	0.951	0.325	**48**	0.743	0.669	1.111	**78**	0.978	0.208	4.705
19	0.326	0.946	0.344	**49**	0.755	0.656	1.150	**79**	0.982	0.191	5.145
20	0.342	0.940	0.364	**50**	0.766	0.643	1.192	**80**	0.985	0.174	5.671
21	0.358	0.934	0.384	**51**	0.777	0.629	1.235	**81**	0.988	0.156	6.314
22	0.375	0.927	0.404	**52**	0.788	0.616	1.280	**82**	0.990	0.139	7.115
23	0.391	0.921	0.424	**53**	0.799	0.602	1.327	**83**	0.993	0.122	9.144
24	0.407	0.914	0.445	**54**	0.809	0.588	1.376	**84**	0.995	0.105	9.514
25	0.423	0.906	0.466	**55**	0.819	0.574	1.428	**85**	0.996	0.087	11.430
26	0.438	0.899	0.488	**56**	0.829	0.559	1.483	**86**	0.998	0.070	14.301
27	0.454	0.891	0.510	**57**	0.839	0.545	1.540	**87**	0.999	0.052	19.081
28	0.469	0.883	0.532	**58**	0.848	0.530	1.600	**88**	0.999	0.035	28.636
29	0.485	0.875	0.554	**59**	0.857	0.515	1.664	**89**	1.000	0.017	57.290
								90	1.000	0.000	n.d.

Note that all values in the trig table (except for the trig ratios for 0° and 90°, and the values for sin 30°, tan 45° and cos 60°) are approximate as each has been rounded off to 3 decimal places.

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Example 6

Find the value of tan 58°, correct to 3 decimal places.

Solution

Using a calculator:

Enter as tan 58 =

tan(58) 1.600334529

Round off. **tan 58° ≈ 1.600**

Using the trig table:

Locate 58 in the degree column and read the tan value. **tan 58° ≈ 1.600**

If you know the value of a trig ratio for an angle, you can work out the angle.

Example 7

If tan β = 0.67, find the value of angle β.

Solution

Using a calculator:

Enter as SHIFT tan 0.67 =

$\tan^{-1}(0.67)$ 33.82208522

Round off. **β ≈ 33.8°**

Using the trig table:

Locate values in the tan column close to 0.67 and read the degree values.

tan 33° = 0.649
tan 34° = 0.675

Select the degree value whose tan is closest to 0.67. **β ≈ 34°**

You can use the lengths of the sides of right-angled triangles and trig ratios to find the values of angles in the triangles.

CAS TI-Nspire exercise
Trigonometry
MAT10MGTI00010

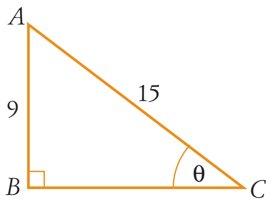
CAS ClassPad exercise
Trigonometry
MAT10MGCP00010

Puzzle sheet
Finding an unknown angle
MAT10MGPS00033

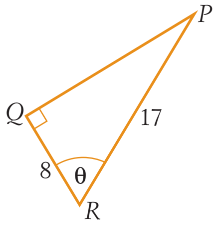
Example 8

Find the value of θ in each of the triangles shown here.

a



b



Solution

a *AB* and *AC* are the known sides of △*ABC*. *AB* is opposite θ and *AC* is the hypotenuse, so use sin.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin\theta = \frac{AB}{AC}$$

Substitute the side lengths. $= \frac{9}{15}$

Simplify. $= 0.6$

Use your calculator to find θ. $\theta \approx 36.9°$

b *QR* and *PR* are the known sides of △*PQR*. *QR* is adjacent to θ and *PR* is the hypotenuse, so use cos.

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{QR}{PR}$$

Substitute the side lengths. $= \frac{8}{17}$

Simplify. $= 0.4705...$

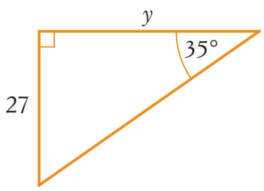
Use your calculator to find θ. $\theta \approx 61.9°$

Trig ratios can also be used to calculate the lengths of sides in right-angled triangles. You need to work out which sides (opposite, adjacent or hypotenuse) the known and required sides are.

Puzzle sheet
Trigonometry equations
MAT10MGPS00032

Example 9

Find the value of *y* in this triangle.



Solution

The known side, 27, is opposite 35° and *y* is adjacent to 35°, so use tan.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 35° = \frac{27}{y}$$

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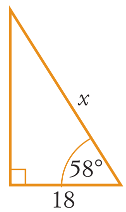
Multiply both sides by y.	$y \times \tan 35° = \dfrac{27}{y} \times y$
Simplify.	$y \tan 35° = 27$
Divide both sides by tan 35°.	$\dfrac{y \times \tan 35°}{\tan 35°} = \dfrac{27}{\tan 35°}$
Simplify.	$y = \dfrac{27}{\tan 35°}$
Evaluate. Enter as 27 ÷ tan 35 =	27÷tan(35) 38.55999618
Round off and state the result.	$y \approx 38.6$

Example 10

Find the value of x in this triangle.



Puzzle sheet
Solving triangles
MAT10MGPS00034

Solution

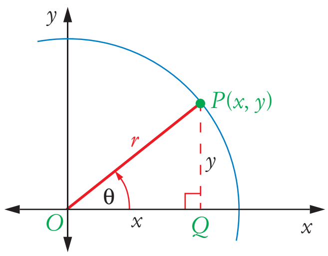
The known side, 18, is adjacent to 58° and x is the hypotenuse, so use cos.	$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
	$\cos 58° = \dfrac{18}{x}$
Multiply both sides by x.	$x \cos 58° = 18$
Divide both sides by cos 58°.	$x = \dfrac{18}{\cos 58°}$
Evaluate. Enter as: 18 ÷ cos 58 =	18÷cos(58) 33.96743847
Round off and state the result.	$x \approx 34$

Now we will take a more detailed look at the values of trig functions. The diagram below shows a point $P(x, y)$ in the first quadrant of the Cartesian plane. By constructing the right-angled $\triangle OPQ$ as shown, we can write:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \cos\theta = \frac{x}{r}$$

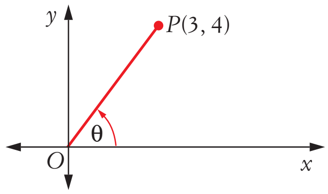
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \qquad \tan\theta = \frac{y}{x}$$



Example 11

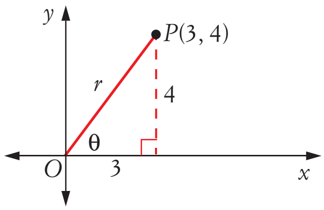
P is the point (3, 4). OP makes an angle of θ with the x-axis as shown here.

a $\sin\theta$ **b** $\cos\theta$ **c** $\tan\theta$



Solution

a Complete the right-angled triangle and label OP as r.



Because P is (3, 4), you can mark in the lengths of the other sides of the triangle as shown.

We need to know the length of the hypotenuse to calculate $\sin\theta$.
Use Pythagoras' theorem.

$$\begin{aligned} r^2 &= 3^2 + 4^2 \\ &= 9 + 16 \\ &= 25 \end{aligned}$$

Take the square root of each side. $r = 5$

Use the rule for $\sin\theta$ where θ is drawn in the Cartesian plane. $\sin\theta = \dfrac{y}{r}$

Substitute known values. $\sin\theta = \dfrac{4}{5}$

b Use the rule for $\cos\theta$ where θ is drawn in the Cartesian plane. $\cos\theta = \dfrac{x}{r}$

Substitute known values. $\cos\theta = \dfrac{3}{5}$

c Use the rule for $\tan\theta$ where θ is drawn in the Cartesian plane. $\tan\theta = \dfrac{y}{x}$

Substitute known values. $\tan\theta = \dfrac{4}{3}$

The definitions for $\sin\theta$, $\cos\theta$ and $\tan\theta$ used in the previous example are true for points in all four quadrants of the Cartesian plane (i.e. they work for angles greater than 90°).

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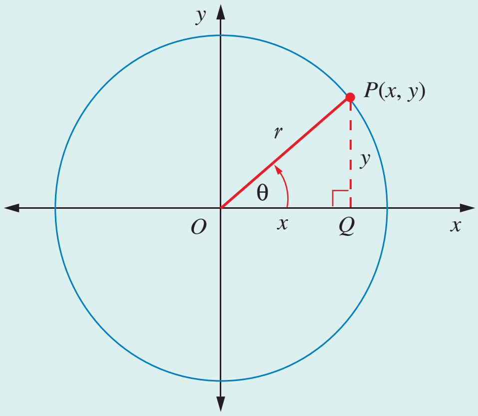
Important!

Trigonometric ratios and the Cartesian plane

For any point $P(x, y)$ on a circle of radius r with centre at the origin:

$\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$ and $\tan\theta = \frac{y}{x}$



Weblink

The interactive Mathematics classroom: Trigonometry

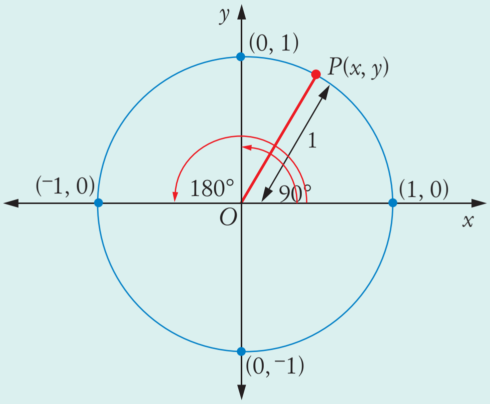
MAT10MGWB00010

We can use the **unit circle** to help us understand more about trig ratios.

Important!

Unit circle

The unit circle has its centre at the origin and a radius of 1.



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Example 12

Use the unit circle to find the value of:

a $\sin 0°$ **b** $\cos 180°$ **c** $\tan 180°$ **d** $\sin 270°$ **e** $\cos 90°$

Solution

a Use the definition for $\sin \theta$. $\sin \theta = \dfrac{y}{r}$

Locate 0° on the unit circle and substitute values. $\sin 0° = \dfrac{0}{1}$

Evaluate. $\sin 0° = 0$

b Use the definition for $\cos \theta$. $\cos \theta = \dfrac{x}{r}$

Locate 180° on the unit circle and substitute values. $\cos 180° = \dfrac{^{-}1}{1}$

Evaluate. $\cos 180° = {}^{-}1$

c Use the definition for $\tan \theta$. $\tan \theta = \dfrac{y}{x}$

Locate 180° on the unit circle and substitute values. $\tan 180° = \dfrac{0}{^{-}1}$

Evaluate. $\tan 180° = 0$

d Locate 270° on the unit circle and use the definition for $\sin \theta$. $\sin 270° = \dfrac{^{-}1}{1}$

Evaluate. $\sin 270° = {}^{-}1$

e Locate 90° on the unit circle and use the definition for $\cos \theta$. $\cos 90° = \dfrac{0}{1}$

Evaluate. $\cos 90° = 0$

Use your calculator to check the values of the ratios in Example **12**.

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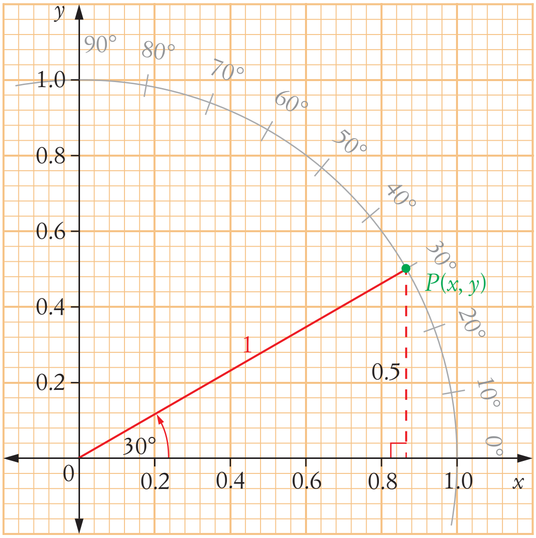
Let's look at the unit circle in a little more detail. The following diagram shows the unit circle in the first quadrant.



In this case, the radius of the unit circle makes an angle of 30° with the x-axis.
Using the diagram, $P(x, y) = (0.86, 0.50)$.
Now,

$$\sin 30^\circ = \frac{y}{r} = \frac{0.5}{1} = 0.5$$

$$\cos 30^\circ = \frac{x}{r} = \frac{0.86}{1} = 0.86$$

The fact that the unit circle has a radius of 1 leads to a general result, as shown below.

Important!

Sin and cos with the unit circle

For any unit circle:

$\sin\theta = y$-coordinate $\cos\theta = x$-coordinate

Teacher notes
Special trig ratios
MAT10MGTN00010

Technology: GeoGebra
Angles of any magnitude
MAT10MGTC00010

Weblink
Trigonometric graphs
MAT10MGWB00010

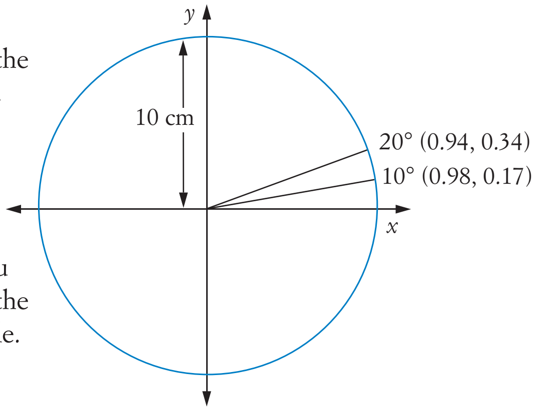
Technology worksheet
Excel worksheet: Trigonometric graphs
MAT10MGCT00016

Investigate: The graphs of trigonometric functions

In this investigation you are going to draw a large unit circle and use it to help draw the graphs of the trig ratios from 0° to 360°. You will need some sheets of graph paper (2 mm), a pair of compasses, a ruler and a protractor.

Step 1:

- Draw a circle with a radius of 10 cm on one sheet of graph paper. This is your unit circle. This means that for your circle 1 cm = 0.1 of a unit.
- Draw in the x-axis and y-axis, making sure that the circle passes through the points (1, 0), (0, 1), ($^-1$, 0) and (0, $^-1$).
- Now use your protractor to mark the angles 10°, 20°, 30°, ... , 360° on the circumference of the unit circle. (Remember: 360° = 0°.)
- Write the coordinates of each point you have marked on the circumference of the unit circle to 2 decimal places if possible.



Step 2:

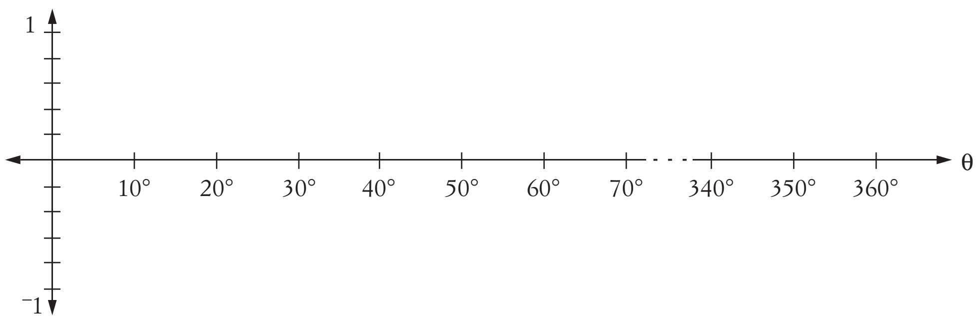
- Prepare a table like the one below:

θ	10°	20°	30°	40°	...	330°	340°	350°	360°
sin θ					...				
cos θ					...				

- Use the coordinates you marked on the unit circle to complete the table. (Remember: for the unit circle, sin θ = x-coordinate of P and cos θ = y-coordinate of P.)

Step 3:

- Now we can draw the graph of y = sin θ for values of θ between 0° and 360°. Draw the horizontal axis (degrees) and vertical axis (units) like this:



- Use the table you completed in Step 2 to plot the points and then join the points with a smooth curve to draw the graph of y = sin θ.
- Now repeat this process for y = cos θ. Use the same set of axes you used to draw the graph of y = sin θ. But use a different colour.
- Describe the similarities and differences between the graphs of y = sin θ and y = cos θ.

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Step 4:

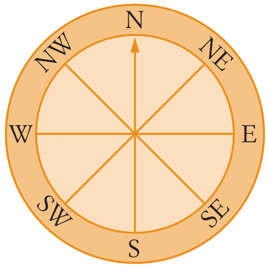
- You know that the value of tan 90° is not defined. This makes drawing the graph of $y = \tan\theta$ a bit more interesting.
- Prepare a table like the one below:

θ	0°	10°	20°	30°	...	70°	80°	85°	89°
tan θ					...				

- Use your calculator to complete the table.
- Now draw up a set of axes similar to the ones you used to plot the graphs of $y = \sin\theta$ and $y = \cos\theta$ but you will need to extend the vertical axis so that it can be used to plot the values of $\tan\theta$ for $\theta > 45°$.
- Use the table you have just completed to plot the points and then join the points with a smooth curve to draw the graph of $y = \tan\theta$ for $0° \leq \theta < 90°$.
- Just as the value of tan 90° is not defined, so too is the value of tan 270° not defined. Use your calculator to explore values of $\tan\theta$ just greater than 90°, just less than 270° and just greater than 270°.
- Use the above results to draw the graph of $y = \tan\theta$ for $0° \leq \theta \leq 360°$.

Bearings and direction

A **compass** is used to find directions. Compasses are used by many different types of people. People who hike, sail or fly often rely on a compass for their safety.
A simplified compass face (or **compass rose**) is shown here. The major directions, N (north), S (south), E (east) and W (west), are further divided by the directions NE (north-east), NW (north-west), SE (south-east) and SW (south-west). Because there are 360° in a circle, there are 90° between N and W, N and E, etc. Similarly, there are 45° between N and NE, E and SE and so on. Captain Cook used a compass that was divided into many more directions, such as north-north-east and north-east by north. The divisions were about the same accuracy as a sailing ship could be steered.

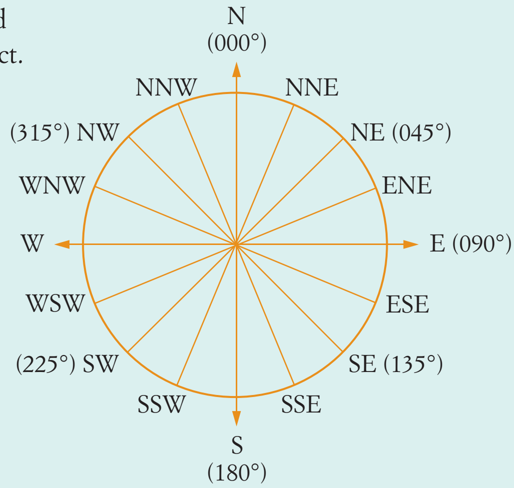


Directions are given in terms of the basic compass points N, S, E and W.
There are two commonly accepted methods of stating directions. For instance, the direction NE can be stated as:
N 45° E OR 45° north of east
(Face N and turn 45° E.) (Face E and turn 45° N.)
We will use *the first method* in the remainder of this book.
You may have heard people say that they need to 'get their bearings' when they are not quite sure in which direction they should go. In navigation, the term **bearing** has a special meaning related to direction.

Important!

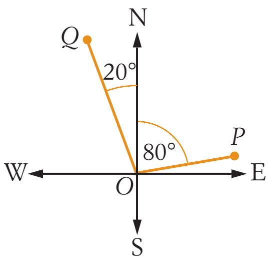
Bearings

The **bearing** of an object is the angle measured in a *clockwise* direction from *north* to the object. Bearings are written as three-digit numbers followed by the degree symbol (°). The diagram on the right shows the bearings of the major compass points.



Example 13

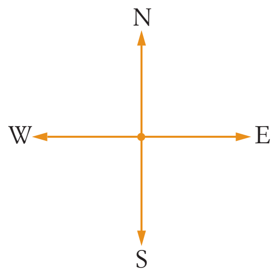
State the direction and bearing of P and Q from O as shown in the diagram below.



Solution

P is 80° east of north.	***P* is in the direction N 80° E.**
P is 80° clockwise from north.	**The bearing of *P* is 080°.**
Q is 20° west of north.	***Q* is in the direction N 20° W.**
Q is 340° clockwise from north.	**The bearing of *Q* is 340°.**

A simplified compass rose like the one shown here is usually drawn when working with bearings.



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Example 14

Sketch the following bearings.

a 306°

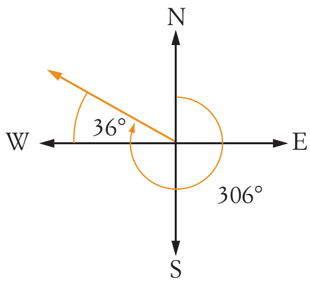
b 221°

Solution

a Sketch a simplified compass rose.

$306° = 270° + 36°$

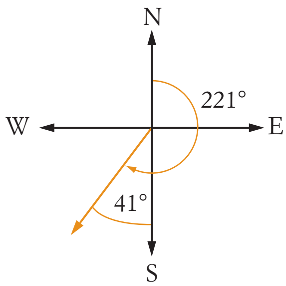
Draw in an arrow from the origin at an angle of 36° clockwise from west.



b Sketch a compass rose.

$221° = 180° + 41°$

Draw in an arrow from the origin at an angle of 41° clockwise from south.



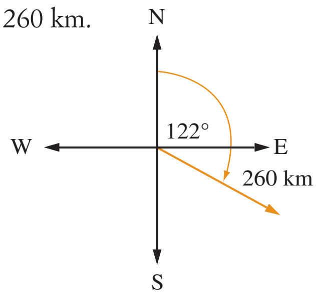
Example 15

Video tutorial
Bearings
MAT10MGVT10022

A plane leaves a runway and remains on a bearing of 122° for 260 km.

a How far south of the runway is the plane?

b What is the bearing of the runway from the plane?



Solution

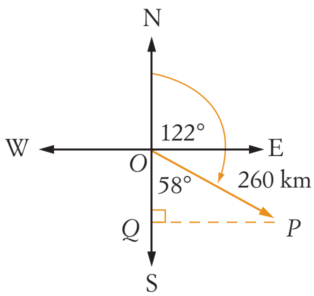
a Mark in the origin (O) and the position of the plane (P) on the sketch.

Draw in PQ to form the right-angled triangle OPQ.

We need to know the distance represented by OQ.

We know $OP = 260$ km.

$\angle POQ$ and 122° are supplementary, so $\angle POQ = 58°$.



For $\triangle POQ$, OP is the hypotenuse and OQ is adjacent to $\angle POQ$, so use the cos ratio.

$$\cos 58° = \frac{OQ}{260}$$

Multiply both sides by 260.

$$260 \times \cos 58° = OQ$$

Reverse the equation and evaluate by entering:

260 × cos 58 =

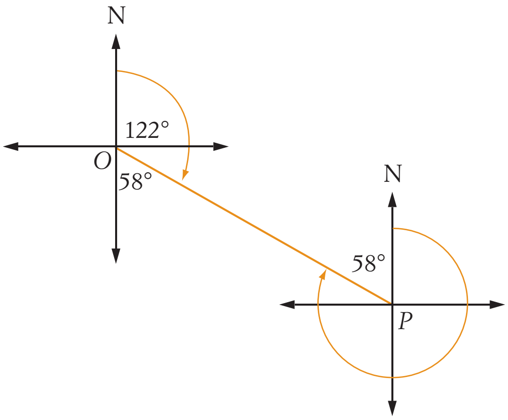
```
260xcos(58)
        137.7790087
```

Round off and state the result.

The plane is about 137.8 km south of the runway.

b Draw a compass rose with the centre at P.

The angle between OP and due north at P must be 58° as it is an alternate angle with the angle between OP and due south at O.



Calculate the bearing.

Bearing of the runway from the plane

$$= 360° - 58°$$
$$= 302°$$

Exercise 10.2 Trigonometric ratios

Understanding

Extra questions

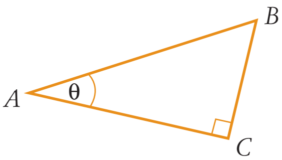
Exercise 10.2

MAT10MGEQ00029

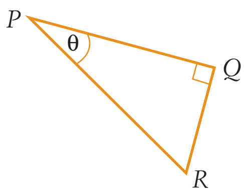
1 For each of the triangles shown here, name:

i the side opposite θ **ii** the side adjacent to θ **iii** the hypotenuse

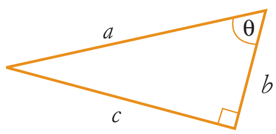
a



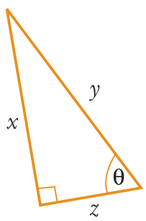
b



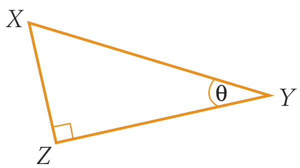
c



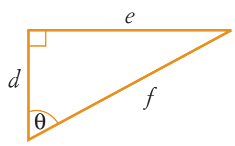
d



e



f



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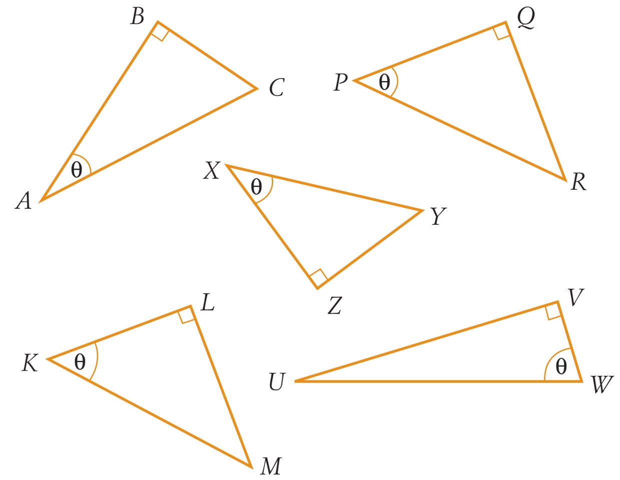
2 For each of the triangles shown in question **1**, state:

a $\sin\theta$ **b** $\cos\theta$ **c** $\tan\theta$

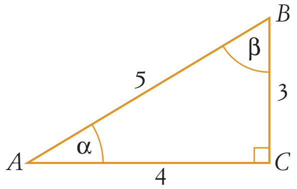
3 Copy and complete this table using the triangles on the right.

Triangle	$\sin\theta$	$\cos\theta$	$\tan\theta$
ABC	$\frac{\dots}{AC}$	$\frac{AB}{\dots}$	$\frac{\dots}{\dots}$
PQR	$\frac{\dots}{\dots}$	$\frac{\dots}{PR}$	$\frac{QR}{\dots}$
XYZ	$\frac{YZ}{\dots}$	$\frac{\dots}{\dots}$	$\frac{\dots}{XZ}$
KLM	$\frac{\dots}{\dots}$	$\frac{\dots}{\dots}$	$\frac{\dots}{\dots}$
UVW	$\frac{\dots}{\dots}$	$\frac{\dots}{\dots}$	$\frac{\dots}{\dots}$



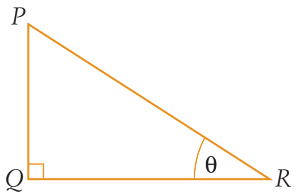
4 Use $\triangle ABC$ to write the following as fractions. See Example 5

a $\sin\alpha$ **b** $\cos\beta$ **c** $\tan\beta$
d $\sin\beta$ **e** $\tan\alpha$ **f** $\cos\alpha$



5 For $\triangle PQR$, which trig ratio of θ is equal to:

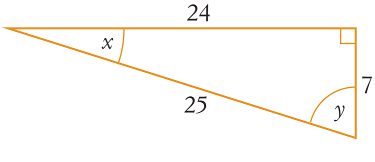
a $\frac{PQ}{PR}$? **b** $\frac{QR}{PR}$? **c** $\frac{PQ}{QR}$?



6 Use this diagram to complete the following.

a $\sin x = \frac{7}{\dots}$ **b** $\tan y = \frac{24}{\dots}$ **c** $\cos y = \frac{\dots}{25}$

d $\sin y = \frac{24}{\dots}$ **e** $\cos x = \frac{\dots}{25}$ **f** $\tan x = \frac{\dots}{24}$



7 Find the value of each of the following, correct to 3 decimal places where necessary. See Example 6

a sin 20° **b** tan 45° **c** cos 80° **d** cos 21°
e sin 90° **f** tan 9° **g** tan 17° **h** sin 33°
i cos 77° **j** cos 89° **k** tan 30° **l** sin 60°

8 Use the table of trigonometric ratios on page 390 to find the value of each of the following, correct to 3 decimal places.

a sin 20° **b** cos 42° **c** sin 17° **d** cos 45°
e sin 90° **f** cos 33° **g** sin 80° **h** cos 90°
i sin 45° **j** cos 55°

See Example 7

9 Find the value of θ, to the nearest degree.

a $\sin\theta = 0.970$	**b** $\tan\theta = 0.500$	**c** $\cos\theta = 0.150$
d $\tan\theta = 0.98$	**e** $\sin\theta = 0.15$	**f** $\cos\theta = 0.90$
g $\sin\theta = 0.123$	**h** $\cos\theta = 0.68$	**i** $\tan\theta = 0.034$
j $\tan\theta = 0.6878$	**k** $\sin\theta = 0.487$	**l** $\cos\theta = 0.66$

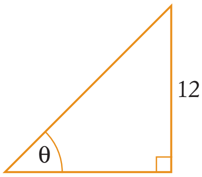
10 Use the table of trigonometric ratios on page 390 to find the value of θ (to the nearest degree).

a $\sin\theta = 0.970$	**b** $\sin\theta = 0.58$	**c** $\cos\theta = 0.123$
d $\cos\theta = 0.5$	**e** $\sin\theta = 0.15$	**f** $\cos\theta = 0.68$
g $\sin\theta = 0.252$	**h** $\cos\theta = 0.91$	**i** $\sin\theta = 0.034$

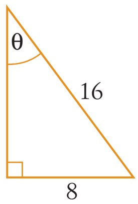
Fluency

11 Find the value of the trig ratio named in each of the following, correct to 3 decimal places.

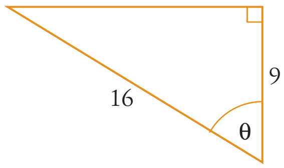
a $\tan\theta$



b $\sin\theta$



c $\cos\theta$



See Example 8

12 Use the values of the trig ratios in question **11** to find the value of θ in each case. (Give answers to the nearest degree.)

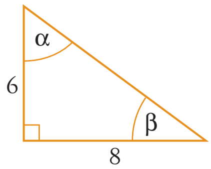
13 For each triangle below:

i state the values of $\sin\alpha$, $\cos\alpha$ and $\tan\alpha$ as ratios in simplest form

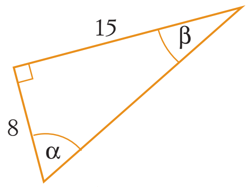
ii state the values of $\sin\beta$, $\cos\beta$ and $\tan\beta$ as ratios in simplest form

iii find the values of α and β to the nearest degree.

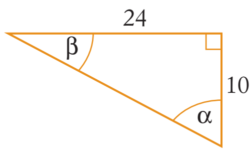
a



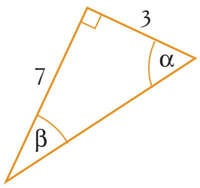
b



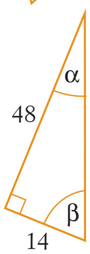
c



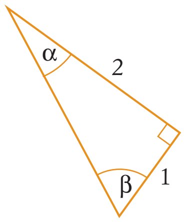
d



e



f



14 Calculate the following, correct to 3 decimal places where necessary.

a $20 \times \sin 30°$	**b** $15 \times \cos 70°$	**c** $\tan 45° \times 18$
d $55 \times \sin 80°$	**e** $37 \times \tan 27°$	**f** $\cos 69° \times 108$

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15 Find the values of the following, correct to 1 decimal place.

a $\dfrac{10}{\sin 30°}$ **b** $\dfrac{25}{\cos 30°}$ **c** $\dfrac{32}{\cos 45°}$

d $\dfrac{19}{\sin 67°}$ **e** $\dfrac{47}{\sin 58°}$ **f** $\dfrac{83}{\cos 12°}$

16 Find the value of the unknown, correct to 1 decimal place.

a $\tan 30° = \dfrac{a}{40}$ **b** $\sin 60° = \dfrac{b}{100}$ **c** $\sin 45° = \dfrac{d}{18}$

d $\cos 28° = \dfrac{y}{142}$ **e** $\sin 87° = \dfrac{x}{15}$ **f** $\tan 12° = \dfrac{n}{21.4}$

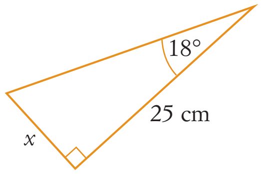
17 Find the value of the unknown in each of the following, correct to 1 decimal place.

a $\sin 35° = \dfrac{18}{a}$ **b** $\cos 22° = \dfrac{72}{d}$ **c** $\cos 47° = \dfrac{48}{m}$

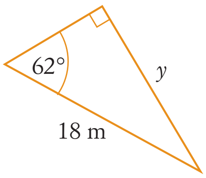
d $\sin 78° = \dfrac{36}{n}$ **e** $\sin 18° = \dfrac{21}{y}$ **f** $\cos 67° = \dfrac{88}{x}$

18 Find the value of the unknown side in each of these triangles, correct to 1 decimal place. See Example 9

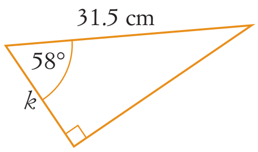
a



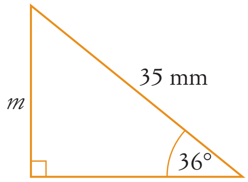
b



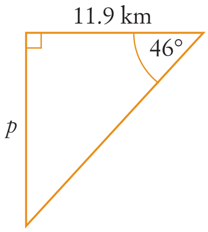
c



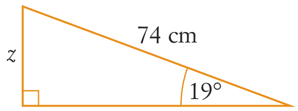
d



e

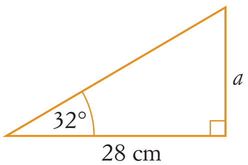


f

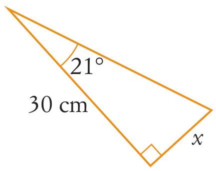


19 Find the value of the unknown side in each triangle, correct to 1 decimal place.

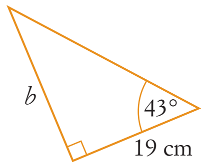
a



b



c



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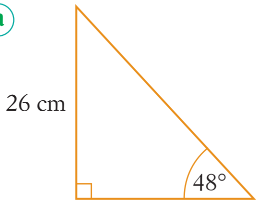
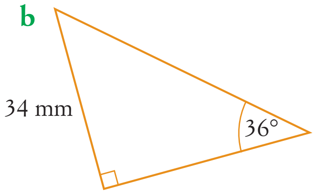
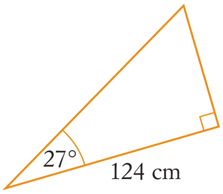
Worked solutions
Exercise 10.2
MAT10MGWS00029
See Example 10

20 Find the value of the hypotenuse in each of these triangles, correct to 2 decimal places.

a


b


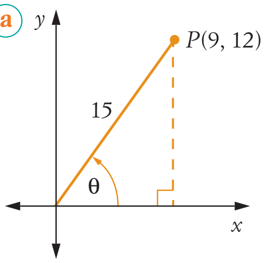
c


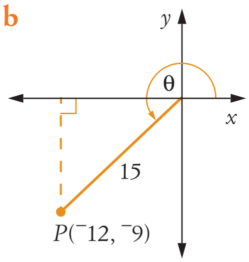
See Example 11

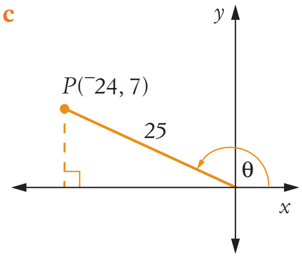
21 For each of the following:

i state the values of x, y and r if (x, y) are the coordinates of P and r is its distance from the origin

ii use the values of x, y and r to write the values of $\sin\theta$, $\cos\theta$ and $\tan\theta$.

a


b


c


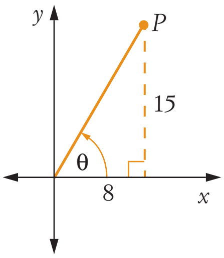
Worked solutions
Exercise 10.2
MAT10MGWS00029

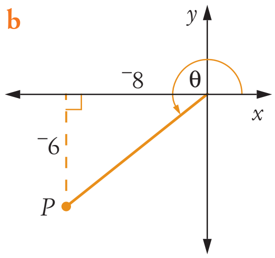
22 For each of the following:

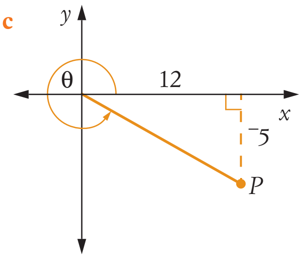
i state the x-coordinate and y-coordinate of P

ii calculate r using Pythagoras' theorem

iii write the values of $\sin\theta$, $\cos\theta$ and $\tan\theta$.

a


b


c


See Example 12

23 Use the unit circle shown on page 395 to calculate the values of the following trig ratios, if possible.

a sin 180°	**b** cos 90°	**c** tan 180°	**d** cos 0°
e cos 270°	**f** sin 360°	**g** tan 270°	**h** cos 180°
i sin 0°	**j** cos 360°		

24 Use the diagram of the first quadrant of the unit circle on page 397 to find the values of the following. (Give approximate values where necessary.)

a sin 60°	**b** cos 60°	**c** cos 30°	**d** sin 80°
e tan 45°	**f** sin 10°	**g** cos 80°	**h** sin 70°
i tan 30°	**j** tan 80°		

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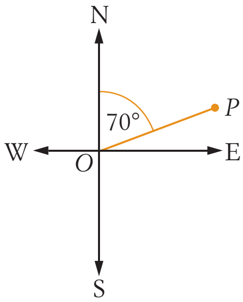
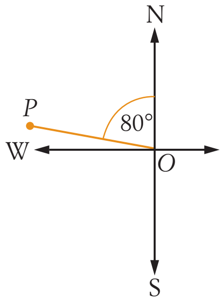
25 For each of the following, state the bearing of P from O. *See* Example 13

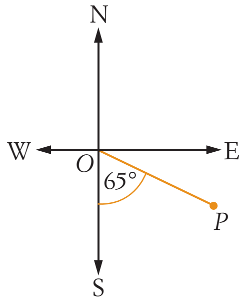
a



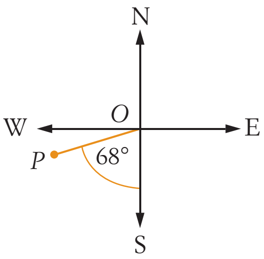
b



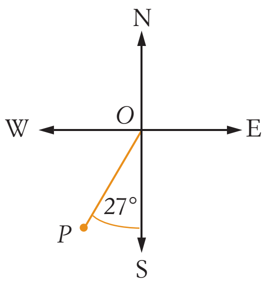
c



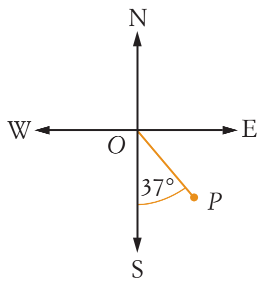
d



e



f



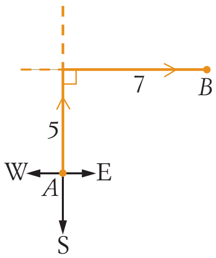
26 For each of the situations shown in the previous question, state the bearing of O from P.

27 Sketch the following bearings. *See* Example 14

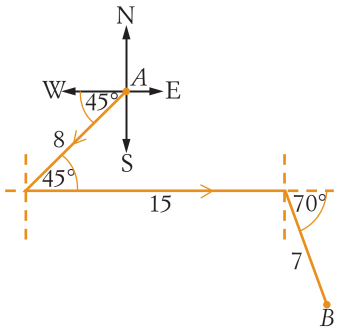
a 245° **b** 337° **c** 043° **d** 121°
e 208° **f** 114° **g** 316° **h** 261°

28 A number of paths are shown below. Each has A as its starting point. Describe each of the paths shown in these diagrams by giving the direction and distance of each leg.

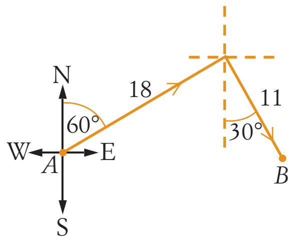
a



b



c

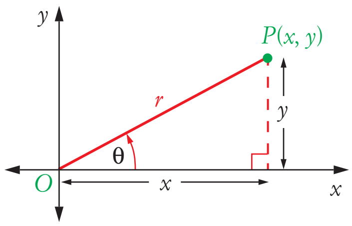


Problem solving

29 In each of the following cases, P is a point on the plane. O is the origin and θ is the angle between OP and the x-axis. Draw a diagram to represent each case, then:

i find $\sin\theta$

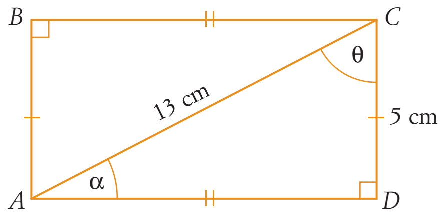
ii use this value to calculate the value of θ, to the nearest degree.



a $P = (6, 8)$ **b** $P = (5, 12)$ **c** $P = (^{-}7, 24)$
d $P = (4, ^{-}3)$ **e** $P = (24, 10)$ **f** $P = (^{-}9, ^{-}12)$
g $P = (1, \sqrt{3})$ **h** $P = (2, \sqrt{5})$

Worked solutions
Exercise 10.2
MAT10MGWS00029

30 In rectangle *ABCD*, *AC* is 13 cm and the short sides are 5 cm long. Find the values of α and θ, to the nearest degree.



31 Use the scale 1 cm : 10 km to make an accurate drawing of each path, from *P* to *Q*.

a 20 km due S, then 40 km SW

b 30 km N 60° W, then 45 km N 30° E

c 80 km N 10° E, then 30 km due S

d 50 km due W, then 20 km due S, then 60 km N 60° E

e 40 km due E, then 30 km N 30° E, then 25 km S 60° E

32 Use your drawings from question **31** to find in each case:

i the bearing of the end point from the starting point

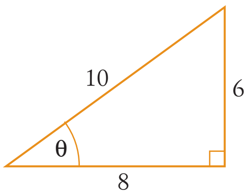
ii the distance between the end point and the starting point.

33 For each of the following triangles, find the value of:

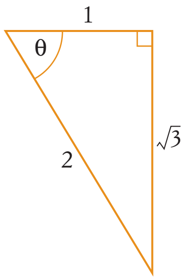
i $\sin\theta$ **ii** $\cos\theta$ **iii** $\sin^2\theta + \cos^2\theta$.

Hint: In part **iii**, $\sin^2\theta = (\sin\theta)^2$.

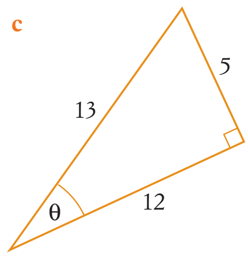
a



b



c

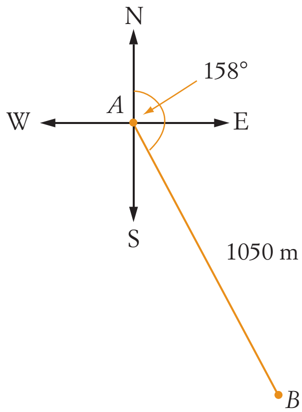


See Example 15

34 A hiker leaves a base camp (*A*) and remains on a bearing of 158° for 1050 m.

a How far east of the base camp is the hiker?

b What is the bearing of the base camp from the hiker?



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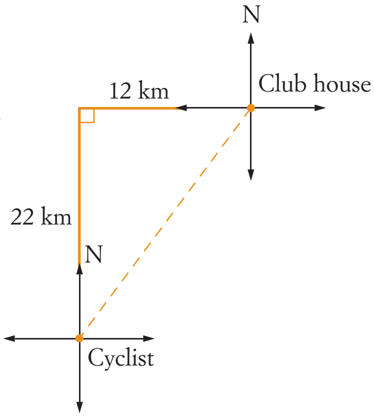
35 A cyclist begins from a club house and travels due west for 12 km, then due south for 22 km.

a How far is the cyclist from the club house?

b What is the bearing of the cyclist from the club house?



Reasoning

36 Two ships leave the same port. The first travels for 220 km on a bearing of 168° while the second ship travels for 186 km on a bearing of 256°.

a How far apart are the two ships?

b Calculate the bearing of the second ship from the first.

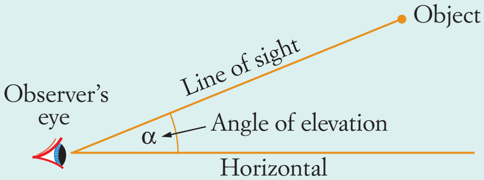
10.3 2D and 3D applications of trigonometry

Many practical problems in navigation, surveying and engineering can be solved using right-angled triangles. Trigonometry can be used to measure distances in these triangles that would be unmeasurable any other way.
Angles of elevation and depression are commonly encountered in trig problems.

Important!

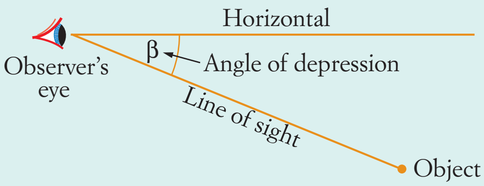
Angle of elevation

The **angle of elevation** of an object from an observer is the angle between the horizontal and the **line of sight** *up* to the object.



Angle of depression

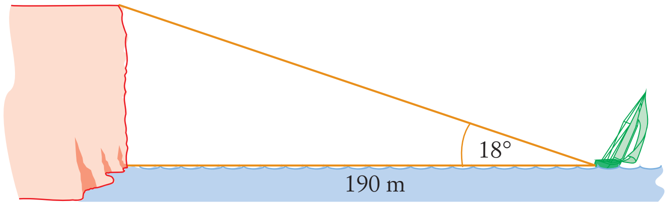
The **angle of depression** of an object from an observer is the angle between the horizontal and the **line of sight** *down* to the object.



Example 16

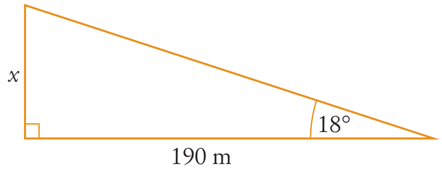
Video tutorial
Angles of elevation and depression
MAT10MGVT00023

The angle of elevation from a yacht to the top of a cliff is 18°. If the yacht is 190 m from the base of the cliff, find how high the cliff is.



Solution

Draw a simplified sketch of the information.
Let the height of the cliff be x m.



The known and required sides are adjacent to and opposite 18°. Use tan 18°.

$$\tan 18^\circ = \frac{\text{opposite}}{\text{adjacent}} = \frac{x}{190}$$

Multiply both sides by 190 and reverse the equation.

$$x = 190 \times \tan 18^\circ$$

Enter 190 × tan 18 =

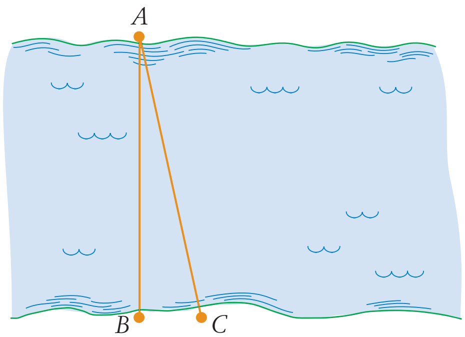
```
190xtan(18)
                61.73474228
```

State the result.

The cliff is about 61.7 m high.

Example 17

A survey locates points A, B and C on the banks of a river as shown here. BC is measured to be 17.3 m and $\angle ACB$ is found to be 78°. Calculate the width of the river (AB) where AB is a line straight across the river, and is perpendicular to line BC.



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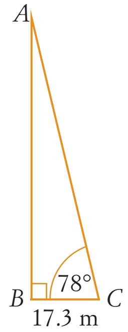
Solution

First draw a sketch and include all the supplied information.

We know the side adjacent to $\angle ACB$ and we want to find the opposite side.



Use tan 78°. $\quad \tan 78^\circ = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{AB}{17.3}$

Multiply both sides by 17.3 and reverse the equation. $\quad AB = 17.3 \times \tan 78^\circ$

Enter 17.3 [×] [tan] 78 [=]

```
17.3xtan(78)
             81.39010089
```

State the result. **The river is about 81.4 m wide.**

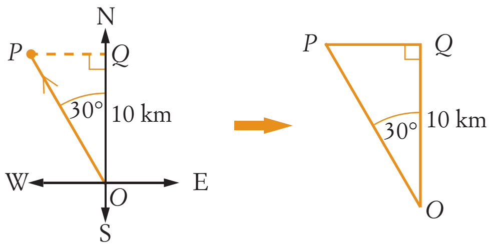
Example 18

A cyclist travels in the direction N 30° W until she is 10 km north of her starting point. Calculate how far the cyclist has travelled.

Solution

First draw a sketch and highlight the information required.

We know the adjacent side (OQ) and want to find the hypotenuse (OP).



Use cos 30°. $\quad \cos 30^\circ = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{OQ}{OP}$

Substitute known information. $\quad \cos 30^\circ = \dfrac{10}{OP}$

Multiply both sides by OP. $\quad OP \times \cos 30^\circ = 10$

Divide both sides by cos 30°. $\quad OP = \dfrac{10}{\cos 30^\circ}$

Enter 10 [÷] [cos] 30 [=]

```
10÷cos(30)
             11.54700538
```

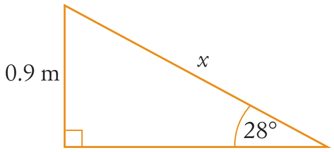
State the result. **The cyclist has travelled about 11.5 km.**

Example 19

An overhead sign is positioned 0.9 m from the top of a steel framework and is supported by a wire. If the angle between the wire and the sign is 28°, how much wire is needed?

Solution

Draw a simplified diagram using this information. Let the length of the wire be x m.



We know the opposite side and want to find the hypotenuse.

Use sin 28°.

$$\sin 28° = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{0.9}{x}$$

Multiply both sides by x.

$$x \sin 28° = 0.9$$

Divide both sides by sin 28°.

$$x = \frac{0.9}{\sin 28°}$$

Enter 0.9 ÷ sin 28 =

```
0.9÷sin(28)
        1.917049021
```

State the result.

About 1.9 m of wire is needed.

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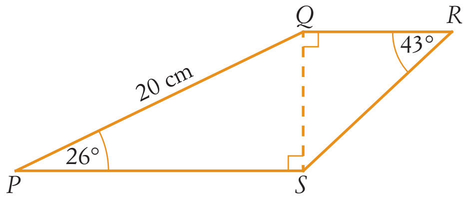
Some figures are composed of two or more right-angled triangles joined by a common side. These are sometimes called **composite figures.** When dealing with composite figures, you will usually have to find a length or angle from one triangle first and then transfer that information to the other triangle.

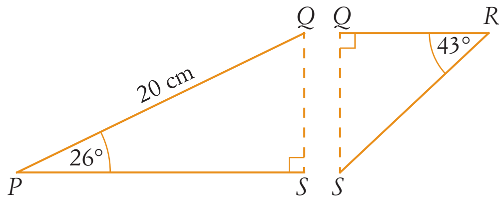
Example 20

Find the length of SR in this diagram.



Solution

Draw the diagram as two separate triangles.



We need to find QS in $\triangle PQS$ before we can find SR in $\triangle QRS$.

Look at $\triangle PQS$. We know the hypotenuse and we want to find the opposite side, QS.

Use sin 26°.

$$\sin 26° = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{QS}{20}$$

Multiply both sides by 20 and reverse the equation.

$$QS = 20 \sin 26°$$

Enter 20 [×] [sin] 26 [=]

Keep the value in your calculator.

```
20×sin(26)
        8.767422936
```

Look at $\triangle QRS$. We know the side opposite 43° and we want to find the hypotenuse, SR.

Use sin 43°.

$$\sin 43° = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{8.7674...}{SR}$$

Multiply both sides by SR.

$$SR \times \sin 43° = 8.7674...$$

Divide both sides by sin 43°.

$$SR = \frac{8.7674...}{\sin 43°}$$

Enter 8.7674 ... [÷] [sin] 43 [=]

```
8.7674÷sin(43)
        12.85545613
```

State the result.

SR is about 12.9 cm long.

The trigonometric problems solved so far in this chapter have involved using triangles only in two dimensions (2D). Problems involving triangles in three dimensions (3D) are a little more complex and need to be simplified before they can be solved using trigonometry.

Important!

Solving 3D trigonometric problems

To solve three-dimensional problems:

- Use a line box drawing to help visualise the problem.
- Represent the 3D information by drawing it in the line box including all relevant information.
- Label points on the diagram.
- 'Unfold' the diagram and draw the triangles flat.
- Label the unknown sides and angles.
- Use trigonometric ratios to find the unknowns.
- Write the solution to the problem in an appropriate form.

Example 21

From a certain point A, a mountain peak due north has an angle of elevation of 20°. From another point B, 2 km east of A and on the same level as A, the bearing of the peak is N 40° W. Find the height of the peak above the level of A and B.

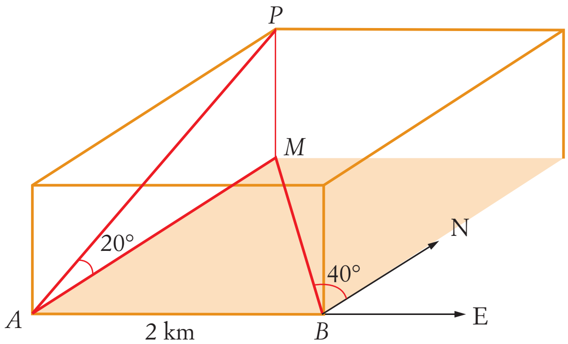
Solution

Draw a line box.

Make a sketch inside the box to make a 3D drawing.

Call the foot of the mountain M and the peak P.

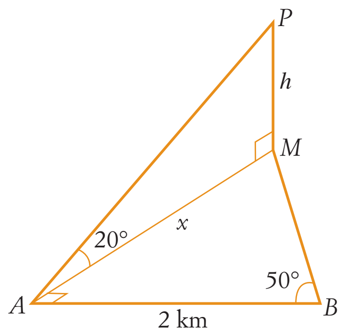
Show north and east at B. You don't have to show all of the major directions because this could make your diagram too confusing.



$\angle MBN = 40°$ and $\angle ABN = 90°$, so $\angle ABM = 50°$

Now redraw the diagram without the parts of the box you don't need.

Let the height of the mountain be h and the let x be the distance from A to the foot of the mountain directly below P.



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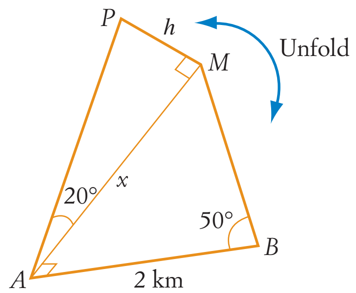
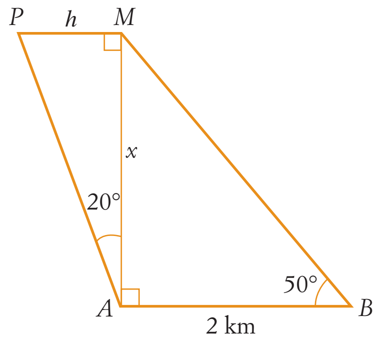
Now 'unfold' the diagram.

First lie $\triangle APM$ flat.



Then lie $\triangle MAB$ flat.



Use tan in $\triangle MAB$ to find x. $\quad \tan 50° = \dfrac{x}{2}$

Rearrange but don't calculate. $\quad x = 2 \tan 50°$

Now use tan in $\triangle PAM$ to find h. $\quad \tan 20° = \dfrac{h}{x}$

Rearrange to isolate h. $\quad h = x \tan 20°$

Substitute for x. $\quad = 2 \tan 50° \times \tan 20°$

Evaluate. $\quad = 0.86752...$

Round off. $\quad \approx 0.868$ km

Write the answer. **The mountain peak is about 868 m high.**

Example 22

Hassan observes that the top of a transmission tower in the direction N 38° E is at an angle of elevation of 12°. Fatima is 375 m due east of Hassan, and she says the bearing of the tower is 308°. Find the height of the tower.

Animated example
2D and 3D applications of trigonometry
MAT10MGAE00010

Solution

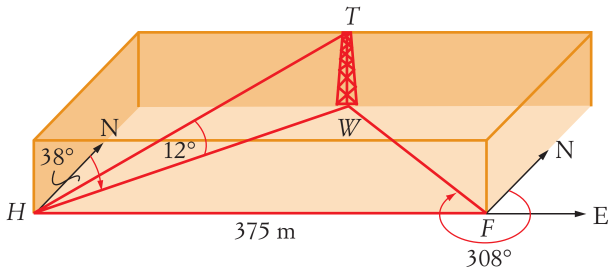
Draw a line box.

Make a sketch inside the box.

Call Hassan's position H.

Call Fatima's position F.

Show north and east.



Call the bottom of the tower W.

Call the top of the tower T.

$\angle WHN = 38°$ and $\angle FHN = 90°$, so $\angle FHW = 52°$.

The other bearing shows that $\angle WFH = 38°$.

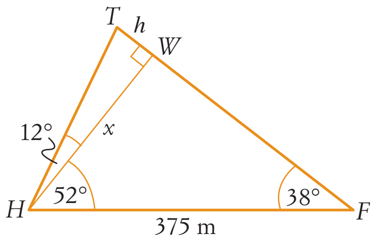
$\angle TWH = 90°$.
Unfold and draw the triangles flat.

Call the height h.

Call the common side x.

The angles in $\triangle WHF$ add up to 180°.

Thus $\angle HWF = 90°$.



Use sin in $\triangle WHF$ to find x. $\quad \mathbf{\sin 38° = \dfrac{x}{375}}$

Rearrange but don't calculate. $\quad \mathbf{x = 375 \sin 38°}$

Now use tan in $\triangle THW$ to find h. $\quad \mathbf{\tan 12° = \dfrac{h}{x}}$

Rearrange and isolate h. $\quad \mathbf{h = x \tan 12°}$

Substitute for x. $\quad \mathbf{= 375 \sin 38° \times \tan 12°}$

Enter as
375 [×] [sin] [(] 38 [)] [×] [tan] [(] 12 [)] [=]

```
375×sin(38)×tan(12)
                49.07358238
```

Round off. $\quad \mathbf{\approx 49.1\ m}$

Write the answer. **The tower is about 49.1 m high.**

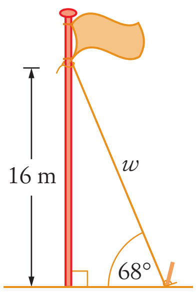
Exercise 10.3 2D and 3D applications of trigonometry

Problem solving

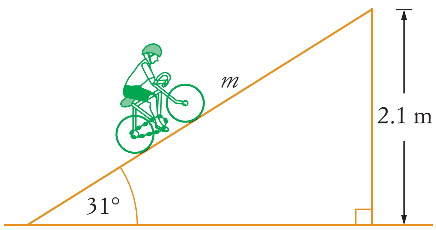
Extra questions
Exercise 10.3
MAT10MGEQ00030

1 Find the unknown length in each of the following triangles, correct to 2 decimal places.

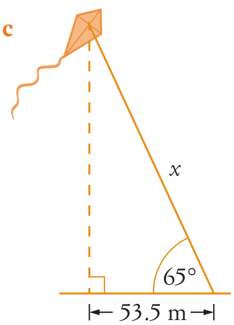
a



b



c



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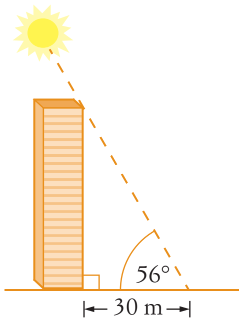
2 The shadow of a building is 30 m long when the angle of elevation of the sun is 56°.
  a Calculate the height of the building.
  b How long will the shadow be when the angle of elevation of the sun becomes 32°?

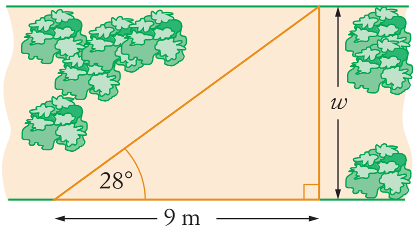


Worked solutions
Exercise 10.3
MAT10MGWS00030
See Example 16

3 An observer in a lighthouse, 19 m high, sights a boat at sea at an angle of depression of 18°. The base of the lighthouse is at sea-level. How far is the boat from the base of the lighthouse?

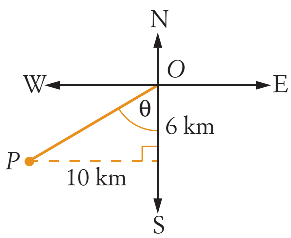
4 A helicopter flying at a height of 1800 m is directly overhead. After 15 minutes, it is still 1800 m above the ground, but its angle of elevation is 8°. Calculate:
  a the distance travelled in that time, to the nearest 100 m
  b the speed of the helicopter in km/h.

5 A nature strip was surveyed and measurements were recorded as shown here. What is the width of the nature strip?



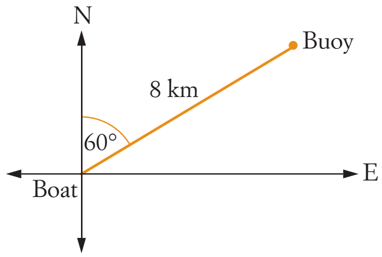
See Example 17

6 Point P is 10 km west and 6 km south of O.
  a Find θ.
  b Find the bearing from O to P

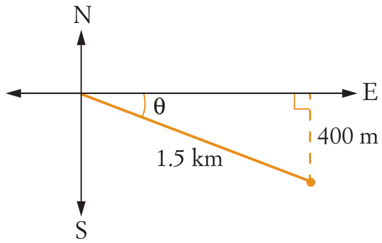


Worked solutions
Exercise 10.3
MAT10MGWS00030
See Example 18

7 The bearing of a buoy from a boat is 060°. If the boat and the buoy are 8 km apart, how far east of the boat is the buoy?



8 A cyclist travels for 1.5 km in a straight line so that she is 400 m south of her starting point. What is the bearing of the cyclist from her starting point, to the nearest degree?

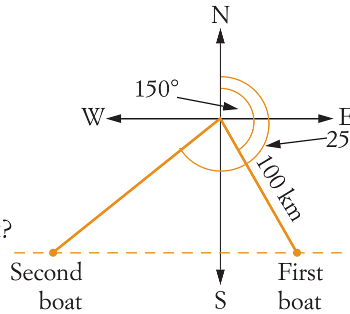


9 Two boats leave port at the same time. The first boat travels on a bearing of 150°, while the second boat travels on a bearing of 250°. After the first boat has travelled 100 km, it is due east of the second boat.

a How far east of its starting point is the first boat?

b How far south of its starting point is the second boat?

c What is the distance between the boats?

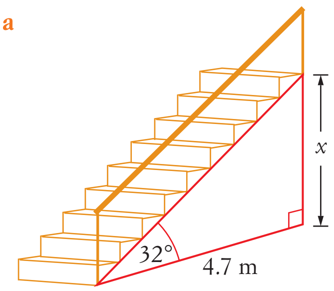


10 A plane flies for 500 km on a bearing of 205°. How far south of its starting point is it now?

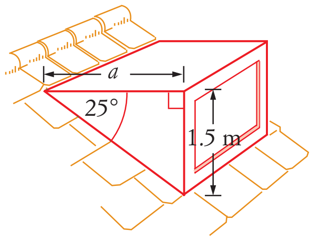
See Example 19

11 Find the unknown length in each of the following triangles, correct to 2 decimal places.

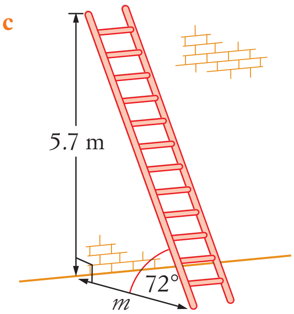
a



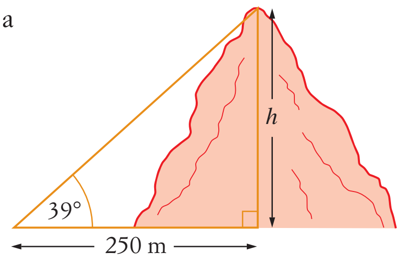
b



c



12 A surveyor measures the angle to the top of a peak from a point 250 m away to be 39°. How high is the peak?



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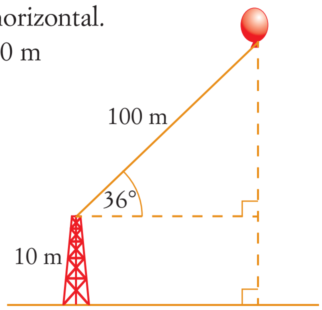
13 A balloon is tied to a string that makes an angle of 36° with the horizontal. The string is 100 m long, and the bottom end is tied to a tower 10 m above the ground. How high above the ground is the balloon?



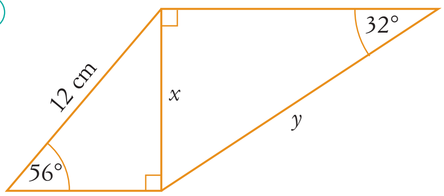
14 A park has two flagpoles 20 m apart. One pole is 12 m high, the other is 21 m. Calculate the angle of depression from the top of the taller pole to the top of the shorter one.

Reasoning

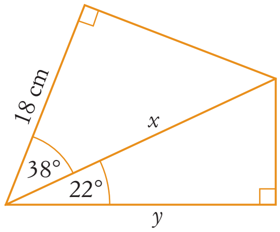
Worked solutions
Exercise 10.3
MAT10MGWS00030
See Example 20

15 In each of the following, first find x and then find y, correct to 1 decimal place.

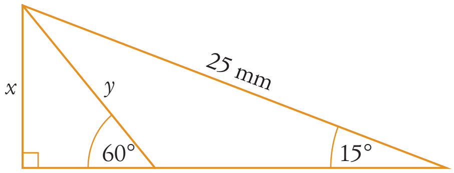
a



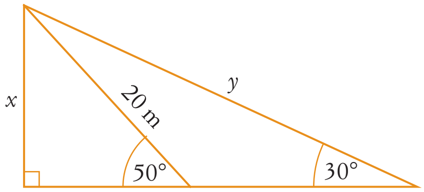
b



c



d



16 A hiker is walking up a 35° slope. After walking 30 m up the slope from the base of a tree, she notices that the top of the tree is level with her eyes. If she is 1.7 m tall, how high is the tree?

17 A flagpole 12 m high is temporarily erected for an Australia Day ceremony. It is held in place by two wires: one 15 m long fastened due west of the pole and the other 17 m long fastened due south of the pole. Both supporting wires are attached to the top of the flagpole. Calculate:

- **a** the distance that each wire is from the pole at the point where it meets the ground
- **b** the angle of inclination that each wire makes with the horizontal
- **c** the distance between the points where the wires meet the ground.

18 From a point on a level straight road running due east, an observer finds that the summit of a mountain bears due north and that the angle of elevation is 11°30′. From a second point on the road 3.2 km due east of the first point, the observer finds that the bearing of the summit is N 25° W. Find the height of the summit of the mountain above the level of the road.

See Examples 21, 22

19 From a lighthouse 150 m above sea level, the lighthouse keeper observes a boat due east at an angle of depression of 25° and another boat due south at an angle of depression of 32°.

- **a** Find the distance that separates the boats.
- **b** Find the bearing of the second boat from the first.

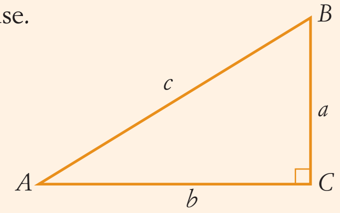
20 A wall 2 m high and 60 m long stands in a vertical plane running east and west. Calculate the width and area of the shadow cast by the wall on level ground when the sun is due north at an elevation of 40°47′.

Chapter 10 summary

Quiz
Trigonometry
MAT10MGQZ00010

- In a right-angled triangle, the longest side is called the hypotenuse. It is opposite the right angle. Here, AB or c is the hypotenuse. **Pythagoras' theorem** applies to all right-angled triangles:

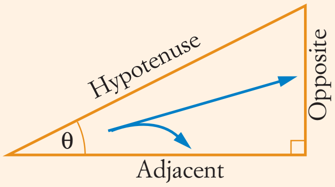
  $AB^2 = BC^2 + AC^2$ or $c^2 = a^2 + b^2$



- Any three whole numbers that satisfy Pythagoras' theorem are called a **Pythagorean triple** (or **triad**), e.g. 3, 4, 5 and 5, 12, 13.
- **Trigonometric (trig)** ratios can be used to find the values of angles and the lengths of unknown sides in right-angled triangles.
- The **trig ratios** are defined as:

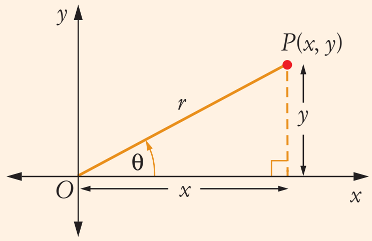
$$\textbf{sin}\ \theta = \frac{\text{opposite side}}{\text{hypotenuse}} \qquad \textbf{cos}\ \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\textbf{tan}\ \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$



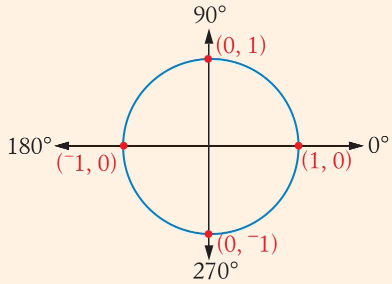
- The trig ratios can be defined using the Cartesian plane as shown on the right.

$$\sin\ \theta = \frac{y}{r} \qquad \cos\ \theta = \frac{x}{r} \qquad \tan\ \theta = \frac{y}{x}$$

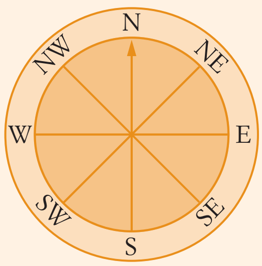


- The **unit circle** has a radius of 1 unit and its centre at the origin.

- We can use the unit circle to find the trig ratios of 90°, 180°, 270° and 360° (0°).



- A **compass** is used to find **directions** and **bearings.** The major directions are N (north), S (south), E (east), W (west), NE (northeast), NW (north-west), SE (south-east) and SW (south-west).



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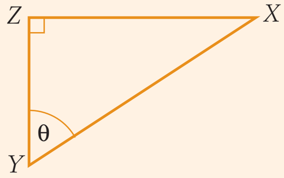
Chapter 10 summary

- The **bearing** of an object is the angle measured in a clockwise direction from north to the object. A bearing is written as a three-digit number of degrees (°).
- The **angle of elevation** of an object from an observer is the angle between the horizontal and the line of sight *up* to the object. The **angle of depression** of an object from an observer is the angle between the horizontal and the line of sight *down* to the object.
- To solve **three-dimensional** problems:
  - A line box drawing may help to visualise the problem.
  - Make a 3D drawing including triangles and all relevant information.
  - Label points on the drawing.
  - 'Unfold' the diagram and draw the triangles flat.
  - Label the unknowns and intermediate sides and angles.
  - Use the methods of right-angled triangles to find the unknowns.
  - Write the solution to the problem in an appropriate form.

Chapter 10 review

Understanding

1 **a** Name the hypotenuse in $\triangle XYZ$.
b State Pythagoras' theorem for $\triangle XYZ$.



2 Refer to $\triangle XYZ$ in question **1** to name:
a the side opposite θ **b** the side adjacent to θ.

3 Refer to $\triangle XYZ$ in question **1** to state:
a $\sin\theta$ **b** $\cos\theta$ **c** $\tan\theta$.

See Examples 5, 6, 7

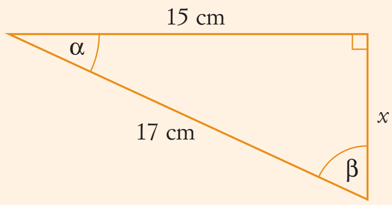
4 **a** Find the value of $\tan 59°$, correct to 3 decimal places.
b Find θ, to the nearest degree, if $\tan\theta = 0.685$.

5 **a** Find the value of $\sin 31°$, correct to 3 decimal places.
b Find θ, to the nearest degree, if $\sin\theta = 0.438$.
c Use $\triangle PQR$ in question **7** to find the value of $\cos\alpha$.

Fluency

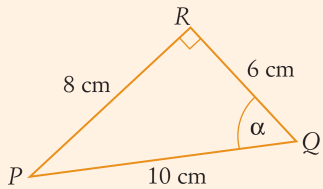
See Example 1

6 Use Pythagoras' theorem to find the unknown side in this triangle.



See Example 8

7 Use $\triangle PQR$ to:
a write the value of $\tan\alpha$ in simplest form
b find the value of α to the nearest degree.



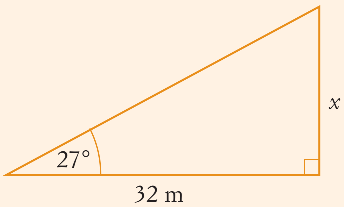
8 Calculate the values of α and β in the triangle shown in question **6**, to the nearest degree.

9 Find the values of the unknowns, correct to 1 decimal place.
a $a = 17\tan 52°$ **b** $\tan 38° = \dfrac{b}{12}$

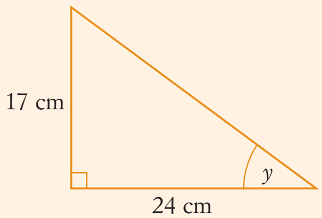
See Example 9

10 Use the tan ratio to find the values of the unknowns in the following triangles, correct to 1 decimal place.

a



b



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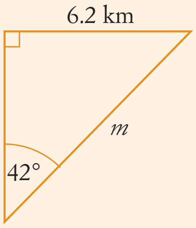
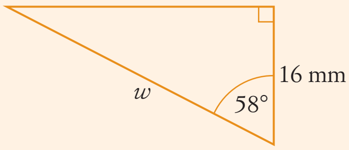
11 Find the values of the unknowns in the following triangles, correct to 1 decimal place.

a



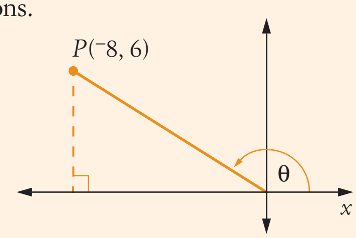
b



12 Use the diagram on the right to answer the following questions.

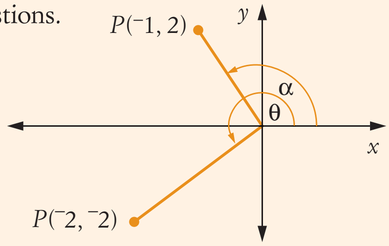
See Example 11

- **a** What is the value of sin θ?
- **b** What is the value of cos θ?
- **c** What is the value of θ, to the nearest degree?



13 Use the diagram on the right to answer the following questions.

- **a** What is the value of sin θ?
- **b** What is the value of tan θ?
- **c** What is the value of α, to the nearest degree?
- **d** What is the value of θ?

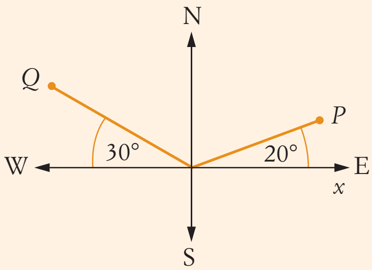


14 Use the drawing of a unit circle on page 395 to state the value of:

See Example 12

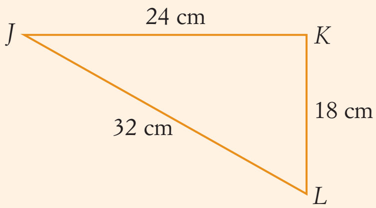
- **a** sin 90°
- **b** cos 270°
- **c** tan 270°
- **d** sin 0°

15 Write the bearings of P and Q.



Problem solving

16 Is △JKL acute-, obtuse- or right-angled?

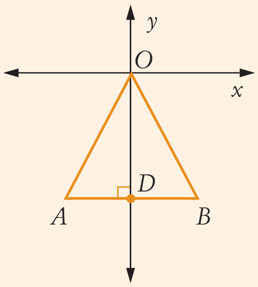


17 Use the scale 1 cm : 10 km to draw the path of a car that leaves point A and travels 20 km due north, then 40 km on a bearing of 315°, then 35 km on a bearing of 210°, finishing at point B.

18 Use the drawing you made in question **17** to find:

a the bearing from A to B **b** the distance from A to B.

19 In the diagram on the right, $\triangle OAB$ is equilateral and symmetrical about the y-axis.
$OA = OB = AB = 2$.



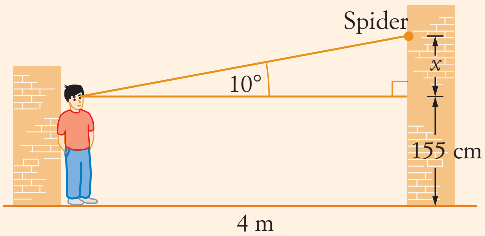
Find (leave as a square root)

a AD **b** OD **c** $\angle AOD$

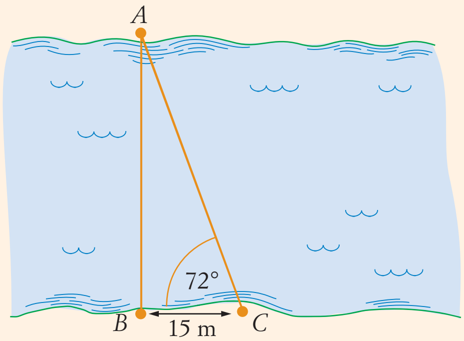
Use this diagram to find the value of

d sin 240° **e** cos 240° **f** tan 240°
g sin 300° **h** cos ($^-$60°) **i** tan ($^-$60°)

20 Sam's eye level is 155 cm above the floor. When he stands against a wall, he can see a spider on an opposite wall at an angle of elevation of 10°. If the walls are 4 m apart, how high on the wall is the spider?



21 Three pegs, A, B and C, have been placed on the banks of a river as shown. B and C are 15 m apart and $\angle ACB$ is 72°. How wide is the river?



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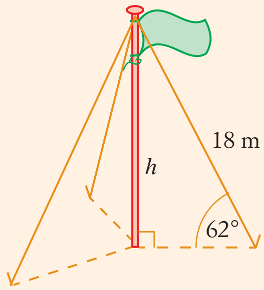
22 A flagpole is held vertically by three wires, each 18 m long. Each wire makes an angle of 62° with the horizontal. How high is the pole (correct to 1 decimal place)?

See Example 19



23 A surveyor on the beach finds that the angle of elevation of the top of a sand dune is 22°. After she walks along the beach towards the dune, the angle increases to 25°. On a detailed map of the area, the height of the dune is marked as 120 m. How far has the surveyor walked along the beach?

Reasoning

24 A spider and a fly are in the same room. The spider is hungry and the fly is asleep. The room is 15 m long, 6 m wide and 6 m high. The spider is on an end wall, 0.5 m from the ceiling and halfway between the side walls. The fly is on the other end wall, 0.5 m from the floor and halfway between the side walls. The spider wants to reach the fly but must crawl on the walls, ceiling and floor to get to its prey. What is the shortest route the spider might take? (*Hint*: The answer is *not* 21 m.)

25 Cheryl has borrowed a clinometer from school and when she was on the beach at Surfer's Paradise she measured the angle of elevation of the top of a very tall building due west as 15°. Because the tide was out, her eye was 2 m above sea level. She then walked 150 m due north along the beach. The top of the building was then in the direction S 72° W. How high is the building?

Number and algebra

11 Equations

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Australian Curriculum statements

Linear and non-linear relationships

Solve problems involving linear equations, including those derived from formulas. (ACMNA235)

Solve linear inequalities and graph their solutions on a number line. (ACMNA236)

Solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology. (ACMNA237)

Solve linear equations involving simple algebraic fractions. (ACMNA238)

Solve simple quadratic equations using a range of strategies. (ACMNA241)

Video tutorial
Equations
MAT10NAVT00011

Solving equations is a key part in finding answers to many mathematical problems. All kinds of mathematical equations rely on the skills developed in solving linear equations. This also applies to systems of mathematical equations and the solution of inequalities. In this chapter, you will extend your understanding of linear equations and look at solving simple inequalities and quadratic equations. You will also look at the simplest kind of equation system: simultaneous linear equations in 2 unknowns.

Mathematical literacy

Maths dictionary
MAT10ASDI00001

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

coefficient	intersection	quadratic formula	solve
completing the square	inverse operation	RHS	substitution method
constant	LHS	satisfy	variable
elimination method	linear equation	simultaneous equations	zero product rule
equation	linear inequality	simultaneous solution	
factorisation	order of operations	solution	
inequation	quadratic equation		

11.1 Linear equations and inequations

In previous years, you solved equations using methods such as inspection, backtracking and modelling. Equations are formally solved by using **inverse operations**. This allows you to solve more complex types than other methods.

Important!

Equations and inverse operations

An **equation** is a mathematical sentence. It has an equals sign with expressions on both sides. The simplest algebraic equations are **linear** equations with *one variable*. When plotted for different values of the variable, a linear equation produces a straight line.
A **solution** of an algebraic equation is a value of the variable that makes the equation true. The process of finding the solution to an equation is called **solving** an equation. We say that a solution **satisfies** the equation.
Inverse operations are shown below.

Operation	Symbol	Inverse operation	Symbol
Addition	$+$	Subtract	$-$
Subtraction	$-$	Add	$+$
Multiplication	$\times$	Divide	$\div$
Division	$\div$	Multiply	$\times$

To solve an equation:

- Aim to get the variable on its own on the left-hand side of the equals sign
- Do inverse operations in the reverse order to the normal order of operations
- Whatever you do to one side, do the same to the other side of the equation
- Show your steps with the equals signs neatly under each other

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You can solve most equations with only one operation by inspection, so we will start with equations with two operations.

Example 1

Solve the following equations.

a $4a + 17 = 5$ **b** $\frac{m}{4} - 6 = 8\frac{1}{2}$ **c** $10 - 5n = {}^{-}27$

Solution

a Write the equation. $\mathbf{4a + 17 = 5}$

The side with the variable has a multiplication and an addition. Undo the addition first, to reverse the order of operations. $\mathbf{4a + 17 - 17 = 5 - 17}$

Simplify both sides. $\mathbf{4a = {}^{-}12}$

Now undo the multiplication by dividing. $\mathbf{\frac{4a}{4} = \frac{{}^{-}12}{4}}$

Simplify to get the answer. $\mathbf{a = {}^{-}3}$

b Write the equation. $\mathbf{\frac{m}{4} - 6 = 8\frac{1}{2}}$

Get rid of the $-$ 6 by adding 6 before undoing division. $\mathbf{\frac{m}{4} - 6 + 6 = 8\frac{1}{2} + 6}$

Simplify both sides. $\mathbf{\frac{m}{4} = 14\frac{1}{2}}$

Now undo the division by multiplying by 4. $\mathbf{\frac{m}{4} \times 4 = 14\frac{1}{2} \times 4}$

Simplify to get the answer. $\mathbf{m = 58}$

c Write the equation. $\mathbf{10 - 5n = {}^{-}27}$

You have to get rid of the 10 first to work in reverse order of operations. $\mathbf{10 - 5n - 10 = {}^{-}27 - 10}$

Simplify both sides. $\mathbf{{}^{-}5n = {}^{-}37}$

Now undo the multiplication by dividing by $^{-}5$. $\mathbf{\frac{{}^{-}5n}{{}^{-}5} = \frac{{}^{-}37}{{}^{-}5}}$

Simplify to get the answer. $\mathbf{n = 7\frac{2}{5}}$

Weblink
The balance method
MAT10NAWB00011

CAS TI-Nspire exercise
Equations
MAT10NATI00011

CAS ClassPad exercise
Equations
MAT10NACP00011

In Example **1** part **c** some people would prefer to multiply the whole problem by $^{-}1$ first to change the $^{-}5n$ to $5n$, making the equation $5n - 10 = 27$. The final answer would still be $7\frac{2}{5}$.

The next type of equation has variables on both sides. Move all the variable terms to one side first. You can choose the side so that the coefficient of the variable ends up positive. Then you can solve the equation as you would normally.

Example 2

Solve the following equations.

a $5y + 6 = 9y - 21$ **b** $4 - 3y = 15 - 8y$

Solution

a Write the equation. $5y + 6 = 9y - 21$

Move the variables to the RHS. $5y + 6 - 5y = 9y - 21 - 5y$

Simplify. $6 = 4y - 21$

Reverse the equation and add 21. $4y - 21 + 21 = 6 + 21$

Simplify. $4y = 27$

Divide by 4. $\frac{4y}{4} = \frac{27}{4}$

Simplify to get the answer. $y = 6\frac{3}{4}$

b Write the equation. $4 - 3y = 15 - 8y$

Move the variables left and the constants right. $4 - 3y - 4 + 8y = 15 - 8y - 4 + 8y$

Simplify. $5y = 11$

Divide by 5. $\frac{5y}{5} = \frac{11}{5}$

Simplify to get the answer. $y = 2\frac{1}{5}$

When you have an equation with brackets, work in reverse order by multiplying out the brackets first.

Example 3

Puzzle sheet
Solving linear equations 1
MAT10NAPS00035

What are the solutions to the following equations?

a $5 = 21 - 7(1 - x)$ **b** $3(2a - 5) = 2(a - 7)$

Solution

a Write the problem. $5 = 21 - 7(1 - x)$

Multiply out the brackets, being careful with signs. $5 = 21 - 7 + 7x$

Simplify and reverse the equation. $7x + 14 = 5$

Move the constant to the right. $7x + 14 - 14 = 5 - 14$

Simplify. $7x = {}^-9$

Divide by 7. $\frac{7x}{7} = \frac{{}^-9}{7}$

Simplify to get the answer. $x = {}^-1\frac{2}{7}$

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b Write the equation. $3(2a - 5) = 2(a - 7)$

Multiply out the brackets, being careful with signs. $6a - 15 = 2a - 14$

Move the variables left and the constants right. $6a - 15 + 15 - 2a = 2a - 14 + 15 - 2a$

Simplify $4a = 1$

Divide by 4. $\frac{4a}{4} = \frac{1}{4}$

Simplify to get the answer. $a = \frac{1}{4}$

You may be tempted to miss out steps to 'save time' by doing them in your head. It actually takes you longer if you do that, and you are more likely to make mistakes.
When you have an equation with fractions, get rid of the fractions first. Do this by multiplying *every term* by the lowest common denominator. If you have grouped terms in a numerator, leave it grouped with brackets. Multiply out the brackets *after* you've got rid of the fractions.

Example 4

Video tutorial
Equations with algebraic fractions
MAT10NAVT10026

Puzzle sheet
Solving linear equations 2
MAT10NAPS00036

Solve the following.

a $\frac{2m}{3} - \frac{m}{2} = 2$ **b** $\frac{2a+4}{5} = \frac{2}{3}$ **c** $\frac{2n+1}{3} - \frac{3n-2}{2} = {}^{-}5$

Solution

a Write the problem. $\frac{2m}{3} - \frac{m}{2} = 2$

Multiply each term by the lowest common denominator, 6. $6 \times \frac{2m}{3} - 6 \times \frac{m}{2} = 6 \times 2$

Simplify by cancelling the denominators. $2 \times 2m - 3 \times m = 12$

Simplify each term. $4m - 3m = 12$

Simplify the variables. $m = 12$

b Write the equation with brackets around the grouped denominator. $\frac{(2a+4)}{5} = \frac{2}{3}$

Multiply each term by the lowest common denominator, 15. $15 \times \frac{(2a+4)}{5} = 15 \times \frac{2}{3}$

Simplify by cancelling the denominators. $3(2a + 4) = 5 \times 2$

Multiply out the brackets. $6a + 12 = 10$

Undo the + 12 by subtraction. $6a + 12 - 12 = 10 - 12$

Simplify $6a = {}^{-}2$

Divide by 6. $\frac{6a}{6} = \frac{{}^{-}2}{6}$

Simplify to get the answer. $a = {}^{-}\frac{1}{3}$

c Write the equation with brackets around the grouped numerators. $\frac{(2n+1)}{3} - \frac{(3n-2)}{2} = {}^{-}5$

Multiply each term by the lowest common denominator, 6. $6 \times \frac{(2n+1)}{3} - 6 \times \frac{(3n-2)}{2} = 6 \times {}^{-}5$

Simplify by cancelling the denominators. $2(2n+1) - 3(3n-2) = {}^{-}30$

Multiply out the brackets. $4n + 2 - 9n + 6 = {}^{-}30$

Simplify the LHS. ${}^{-}5n + 8 = {}^{-}30$

Subtract 8 from both sides. ${}^{-}5n + 8 - 8 = {}^{-}30 - 8$

Simplify. ${}^{-}5n = {}^{-}38$

Divide by ${}^{-}5$. $\frac{{}^{-}5n}{{}^{-}5} = \frac{{}^{-}38}{{}^{-}5}$

Simplify to get the answer. $n = 7\frac{3}{5}$

In some cases, equations with variables as denominators are actually linear equations. For these equations, you can still multiply by the lowest common denominator.

Example 5

Solve $\frac{5}{3a} - \frac{3}{5a} = \frac{4}{5}$.

Solution

Write the equation. $\frac{5}{3a} - \frac{3}{5a} = \frac{4}{5}$

Multiply each term by the lowest common denominator, $15a$. $15a \times \frac{5}{3a} - 15a \times \frac{3}{5a} = 15a \times \frac{4}{5}$

Simplify by cancelling the denominators. $5 \times 5 - 3 \times 3 = 3a \times 4$

Simplify both sides and reverse the equation. $12a = 25 - 9 = 16$

Divide by 12. $\frac{12a}{12} = \frac{16}{12}$

Simplify to get the answer. $a = 1\frac{1}{3}$

Many linear equations arise from substitution into formulas to find the values of one of the variables.

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Example 6

A can manufacturer has to make cans of diameter 9 cm with a capacity of 415 mL. What height do they need to be?

Solution

Change the volume to cubic units.	**415 mL = 415 cm^3**
Write the formula for the volume of a cylinder.	$\boldsymbol{V = \pi r^2 h}$
Substitute the values $v = 415$, $r = 4.5$.	$\mathbf{415 = \pi \times 4.5^2 \times} \boldsymbol{h}$
Simplify and reverse the equation.	$\mathbf{20.25\pi} \boldsymbol{h} \mathbf{= 415}$
Divide by 20.25π.	$\dfrac{\mathbf{20.25\pi} \boldsymbol{h}}{\mathbf{20.25\pi}} = \dfrac{\mathbf{415}}{\mathbf{20.25\pi}}$
Enter as 415 ÷ (20.25 × π) =	415÷(20.25×π) 6.523387791
Write the answer	**The can needs to be about 6.5 cm high.**

Linear equations can be used to solve a wide range of problems. You should always start by choosing a name for the unknown. x is traditional, but you could use something else to avoid confusion with $\times$. When solving problems, you should check that your answer satisfies the conditions, not just the equation.

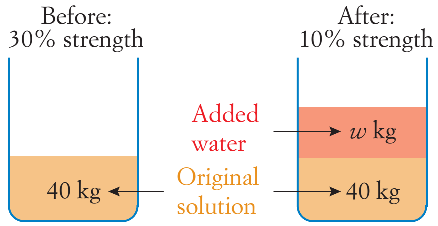
Example 7

A soft-drink plant's basic sugar solution is at a concentration of 30% by mass. How much water must be added to 40 kg of the solution to make the strength of the solution 10% before adding the flavour?

Solution

Choose w for the mass of water that needs to be added.

Picture in your mind the solution and the added water before and after it is diluted.



Write your choice.	**Let the mass of water added =** $\boldsymbol{w}$ **kg.**
Write down an expression for the amount of sugar.	**Mass of sugar = 30% of 40 kg**
Do the calculation.	**= 0.3 × 40 kg** **= 12 kg**
Work out the final mass.	**Total mass = 40 +** $\boldsymbol{w}$ **kg**
Write down an expression for the amount of sugar at the end.	**Mass of sugar = 10% of (40 +** $\boldsymbol{w}$**) kg**

Write as an equation.	$\mathbf{12 = 0.1 \times (40 + w)}$
Multiply out the brackets.	$\mathbf{12 = 4 + 0.1w}$
Reverse the equation and subtract 4.	$\mathbf{4 + 0.1w - 4 = 12 - 4}$
Simplify.	$\mathbf{0.1w = 8}$
Multiply by 10 to get w.	$\mathbf{10 \times 0.1w = 10 \times 8}$
Simplify to get the answer.	$\mathbf{w = 80}$
Check the result.	**Amount of sugar = 12 kg**
Find the total amount of solution.	**Total mass = 40 kg + 80 kg = 120 kg**
Check the percentage.	**Concentration of sugar** $= \frac{12}{120} \times 100\% = 10\%$ **OK✓**
Write the answer.	**80 kg of water must be added to make it 10%.**

An equation has an equals sign that shows both sides are the same size. When you compare two expressions, one may be smaller, equal or bigger than the other. When they are equal you get an equation. When they are not necessarily equal you get an **inequality**.

Important!

Linear inequalities

A **linear inequality** is like a linear equation, but instead of having an equals sign, it has one of the signs $>$, $<$, $\geq$ or $\leq$. It is also called an **inequation**.

Inequations can be simplified in the same way as you solve equations. As long as you do the same thing to both sides, you can do anything you like to an equation. You can add and subtract anything you like from both sides of an inequality. However, when you multiply or divide by a negative number you have to change the inequality.

Important!

Properties of inequalities

For any inequality $x > y$,

1 $x + a > y + a$ for any number a.

2 $x - a > y - a$ for any number a.

3 $ax > ay$ for any *positive* number a.

4 $ax < ay$ for any *negative* number a.

5 $\frac{x}{a} > \frac{y}{a}$ for any *positive* number a.

6 $\frac{x}{a} < \frac{y}{a}$ for any *negative* number a.

The same laws hold for $\geq$, $<$ and $\leq$.

It is normal to show the solutions to an inequality as a graph on the number line.

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Example 8

For each of the following, solve the inequality and show your solution on a number line.

a $m + 3 \le 8$ **b** $4x > {}^{-}7$ **c** ${}^{-}5n \ge 12$

d $v - 5 < 7$ **e** $\frac{p}{{}^{-}3} < {}^{-}11$

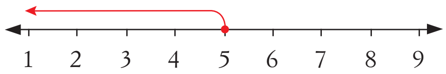
Solution

a Write the inequality. $\boldsymbol{m + 3 \le 8}$

This is like $m + 3 = 8$, so take 3 from both sides. $\boldsymbol{m + 3 - 3 \le 8 - 3}$

Simplify both sides. $\boldsymbol{m \le 5}$

Show the answer on a number line. Remember to show $\le$ by a closed circle. Show the set going down from 5.

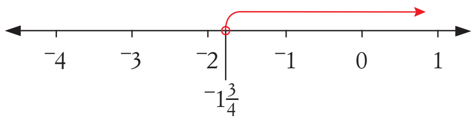


b Write the inequality. $\boldsymbol{4x > {}^{-}7}$

This is like $4x = {}^{-}7$, so divide both sides by 4. $\boldsymbol{\frac{4x}{4} > \frac{{}^{-}7}{4}}$

Simplify both sides. $\boldsymbol{x > {}^{-}1\frac{3}{4}}$

Show the answer on a number line. Remember to show > by an open circle. Show the set going up from ${}^{-}1\frac{3}{4}$.

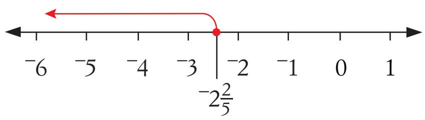


c Write the inequality. $\boldsymbol{{}^{-}5n \ge 12}$

This is like ${}^{-}5n = 12$, so divide both sides by ${}^{-}5$. $\boldsymbol{\frac{{}^{-}5n}{{}^{-}5} \le \frac{12}{{}^{-}5}}$

Reverse the sign for division by a negative.
Simplify both sides. $\boldsymbol{n \le {}^{-}2\frac{2}{5}}$

Show the answer on a number line. Remember to show $\le$ by a closed circle. Show the set going down from ${}^{-}2\frac{2}{5}$.

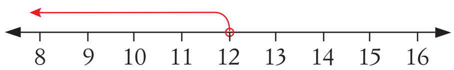


d Write the inequality. $\boldsymbol{v - 5 < 7}$

This is like $v - 5 = 7$, so add 5 to both sides. $\boldsymbol{v - 5 + 5 < 7 + 5}$

Simplify both sides. $\boldsymbol{v < 12}$

Show the answer on a number line. Remember to show < by an open circle. Show the set going down from 12.

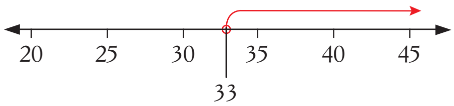


e Write the inequality. $\frac{p}{^{-}3} < {}^{-}11$

This is like $\frac{p}{^{-}3} = {}^{-}11$, so multiply by $^{-}3$. *Reverse the sign* for multiplication by a negative. $^{-}3 \times \frac{p}{^{-}3} > {}^{-}3 \times {}^{-}11$

Simplify both sides. $p > 33$

Show the answer on a number line. Remember to show > with an open circle. Show the set going up from 33.



Inequalities with two operations are done in the same way as linear equations.

Example 9

CAS TI-Nspire exercise
Equations
MAT10NATI00011
CAS ClassPad exercise
Equations
MAT10NACP00011

For each of the following, solve the inequality and show your solution on a number line.

a $7 - 3g \leq 15$ **b** $4w - 9 < 2$

Solution

a Write the inequality. $7 - 3g \leq 15$

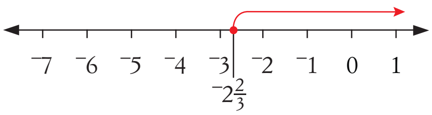
Subtract 7 from both sides. $7 - 3g - 7 \leq 15 - 7$

Simplify both sides. $^{-}3g \leq 8$

Divide both sides by $^{-}3$ and *reverse the sign* for division by a negative. $\frac{^{-}3g}{^{-}3} \geq \frac{8}{^{-}3}$

Simplify to get the answer. $g \geq {}^{-}2\frac{2}{3}$

Show the answer on a number line.



b Write the problem. $4w - 9 < 2$

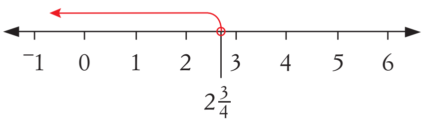
Add 9 to both sides. $4w - 9 + 9 < 2 + 9$

Simplify both sides. $4w < 11$

Divide both sides by 4. $\frac{4w}{4} < \frac{11}{4}$

Simplify to get the answer. $w < 2\frac{3}{4}$

Show the answer on a number line.



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Linear inequalities with variables on both sides of the inequality, brackets or fractions can also be solved like linear equations.

Example 10

For each of the following, solve the inequality and show your solution on a number line.

a $4p + 5 > 7p - 8$ **b** $6(3r - 7) - 8 \leq 7(3r - 4) - 46$

c $\dfrac{m-7}{12} - \dfrac{m-1}{4} \geq 1$

Solution

a Write the inequality. $\mathbf{4p + 5 > 7p - 8}$

Move the variables right and the constants left. $\mathbf{4p + 5 - 4p + 8 > 7p - 8 - 4p + 8}$

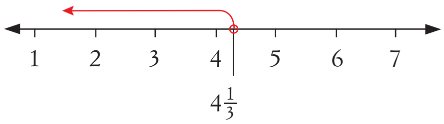
Simplify both sides. $\mathbf{13 > 3p}$

Reverse the inequality, including the $>$ sign. $\mathbf{3p < 13}$

Divide by 3. $\mathbf{\dfrac{3p}{3} < \dfrac{13}{3}}$

Simplify to get the answer. $\mathbf{p < 4\tfrac{1}{3}}$

Show the answer on a number line.



b Write the inequality. $\mathbf{6(3r - 7) - 8 \leq 7(3r - 4) - 46}$

Expand the brackets. $\mathbf{18r - 42 - 8 \leq 21r - 28 - 46}$

Simplify both sides. $\mathbf{18r - 50 \leq 21r - 74}$

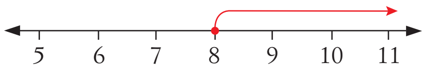
Move the variables right and the constants left. $\mathbf{18r - 50 - 18r + 74 \leq 21r - 74 - 18r + 74}$

Simplify both sides. $\mathbf{24 \leq 3r}$

Reverse the inequality. $\mathbf{3r \geq 24}$

Divide by 3. $\mathbf{r \geq 8}$

Show the answer on a number line.



c Write the inequality with brackets around the grouped numerators. $\frac{(m-7)}{12} - \frac{(m-1)}{4} \geq 1$

Multiply each term by the lowest common denominator, 12. $12 \times \frac{(m-7)}{12} - 12 \times \frac{(m-1)}{4} \geq 12 \times 1$

Simplify by cancelling the denominators. $1(m-7) - 3(m-1) \geq 12$

Multiply out the brackets. $m - 7 - 3m + 3 \geq 12$

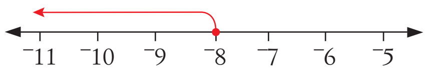
Simplify the LHS. $^-2m - 4 \geq 12$

Add 4 to both sides. $^-2m \geq 16$

Divide by $^-2$ and reverse the $\geq$ sign. $\frac{^-2m}{^-2} \leq \frac{16}{^-2}$

Simplify to get the answer. $m \leq {}^-8$

Show the answer on a number line.



You can use simple inequalities to solve some types of problems.

Example 11

Anton wants to go in-line skating while on holiday. He has \$43 available and it costs \$7 an hour to hire the skates, pads and helmet. Hire is available in multiples of a quarter of an hour. He also has to leave a \$20 deposit. Write and solve an inequality to find the maximum time he can spend skating.

Solution

Choose a variable for the time. **Let the time skating = t hours**

Write an expression for the money. **Money needed = $7t + 20$**

Write an inequality for the money. $7t + 20 \leq 43$

Solve the inequality. $7t + 20 - 20 \leq 43 - 20$

Simplify both sides. $7t \leq 23$

Divide by 7. $\frac{7t}{7} \leq \frac{23}{7}$

Simplify to find the answer. $t \leq 3\frac{2}{7}$

Change $\frac{2}{7}$ hours to minutes. $\frac{2}{7}\text{ h} = \frac{2}{7} \times 60 \approx 17$ **minutes**

State the maximum. t **= 3 hours 15 minutes**

Check the answer. **Money needed = \$7 × $3\frac{1}{4}$ + \$20**
= \$42.75 ✓**OK**

Write the answer. **Anton can hire skates for up to 3 h 15 minutes.**

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You can prove the properties of inequalities by using the rules for addition, subtraction, multiplication and division of positive and negative numbers.

Example 12

If $x \le y$ and $a < 0$, prove that $\frac{x}{a} \ge \frac{y}{a}$.

Solution

Write the requirement.	**RTP:** $x \le y, a < 0 \Rightarrow \frac{x}{a} \le \frac{y}{a}$
Put the Proof heading.	**Proof**
Write the inequality.	$x \le y$
Write in terms of a number property.	$\Rightarrow x - y$ **is negative or zero.**
Use the fact that the quotient of two negative numbers is positive.	$\Rightarrow \frac{x-y}{a}$ **is positive or zero.**
Use fraction properties.	$\Rightarrow \frac{x}{a} - \frac{y}{a} \ge 0$
Write as an inequality.	$\Rightarrow \frac{x}{a} \ge \frac{y}{a}$ **QED**

Exercise 11.1 Linear equations and inequations

Understanding

Extra questions
Exercise 11.1
MAT10NAEQ00031

1 Solve each of the following equations.

a $a + 7.6 = {}^{-}12.5$ b $b - 4.3 = {}^{-}5.8$ c $5m = {}^{-}100$
d $7 + h = {}^{-}13$ e $\frac{k}{8} = {}^{-}7$ f $3.7 + y = {}^{-}9.8$
g $\frac{x}{10} = {}^{-}12$ h $h - 17 = {}^{-}12$ i $0.8n = {}^{-}4$
j $x + 7 = 4$ k $m - 5 = {}^{-}7$ l $12n = {}^{-}30$
m $\frac{n}{14} = {}^{-}2.5$ n $14d = {}^{-}35$ o $18.4 + a = 6.9$
p $6a - 6 = {}^{-}12$ q $\frac{e}{5} + 15 = 5$ r $2.2 - 3r = 5.8$
s $2m + 8.4 = 4.6$ t $1.4 - \frac{4.2d}{5} = 9.8$

2 For each of the following, solve the inequality and show your solution on a number line.

See Example 8

a $r + 9 \ge 6$ b $r + 3 > 0$ c $b + 1 \le {}^{-}8$ d $v + 1 > 4$
e $g + 8 \ge {}^{-}5$ f $h - 6 \ge 1$ g $w - 2 \le {}^{-}9$ h $m - 8 < {}^{-}4$
i $x - 7 \le 0$ j $e - 1 < 0$

3 For each of the following, solve the inequality and show your solution on a number line.

a ${}^{-}6a \le {}^{-}24$ b $7m \ge 28$ c $6w < 36$ d ${}^{-}5m \ge {}^{-}35$
e $3q > 24$ f $\frac{e}{-3} > {}^{-}3$ g $\frac{r}{9} \ge 5$ h $\frac{h}{8} > 1$
i $\frac{v}{9} \le {}^{-}7$ j $\frac{e}{-2} < 1$

4 For each of the following, solve the inequality and show your solution on a number line.

a $7b \le {}^{-}8$ b ${}^{-}5x > 2$ c ${}^{-}8e \le {}^{-}2$ d $8q > 0$ e ${}^{-}7z > {}^{-}6$
f $\frac{h}{-3} \ge {}^{-}1$ g $\frac{e}{-5} < {}^{-}4$ h $\frac{g}{-5} \ge {}^{-}5$ i $\frac{z}{-9} > 5$ j $\frac{t}{-5} > {}^{-}7$

Fluency

See Example 1

5 Solve each of the following equations.

a $2a + 9 = 3$ b $4b + 16 = 4$ c $7m + 26 = {}^-9$

d $5 + 4n = {}^-23$ e $9k + 5 = {}^-40$ f $\frac{m}{3} + 5 = 2$

g $\frac{r}{2} + 11 = 5$ h $3 = 2h + 17$ i $\frac{w}{5} + 14 = 6$

j $\frac{3c}{2} + 22 = 10$ k $3b + 24 = 15$ l $\frac{4d}{3} + 13 = 5$

See Example 2

6 Find the roots of each of the following equations.

a $15a = 18a + 27$ b $12m = 5m - 35$

c $2a + 3 = 5a + 9$ d $5 - m = 19 + 6m$

e $4x + 9 = 2x - 21$ f $2g + 5 = 13 + 6g$

g $3k + 2 = 26 + 7k$ h $3w + 3 = 9w + 27$

i $10 - 3x = -x + 9$ j $4a - 3 = 2a - 9$

k $8m - 7 = 2m - 17$ l $15x - 9 = 12x - 15$

m $7r + 12 = {}^-18 - 3r$ n $11g - 2 = 10g - 12$

o $8 + 4w = 12 + 6w$ p $8m + 9 = 4m - 11$

q $5w - 3 = \frac{1}{2}w - 15$ r $4x - 6 = 1\frac{1}{2}x - 16$

s $6d - 25 = 8d + 7$ t ${}^-9 - 2b = 3b + 11$

u $14 + 2n = 2 - n$

See Example 3

7 Solve each of the following equations.

a $2(a + 4) = {}^-18$ b $3(3 - b) = 21$ c $6(n + 3) = {}^-24$

d $9(-m + 2) = 27$ e $8(y + 2) = {}^-32$ f ${}^-9(x + 4) = 54$

g $2({}^-3d + 1) = {}^-28$ h $3(3 - 4a) = 33$ i $5(3 - 2w) = 45$

j $8(6 - 3k) = 56$ k $12(3 - 2x) = 60$ l $5(3w - 6) = 8w - 2$

m $3(4x - 5) = 2x - 25$ n $5(2a + 1) = 3a - 23$ o $3(4k + 5) = 4(7 + 2k)$

p $3(b - 2) = 2(b + 3)$ q $2(5y + 2) = 4(2y + 5)$ r $5(2m + 7) = 3(2m + 3)$

s $2(n + 4) = 3(n - 2)$ t $6(2x + 1) = 5(2x + 3)$ u $7(a - 3) = 3(2a - 7)$

8 Solve each of the following equations.

a $33 = 7(1 + 2a) - a$ b $21 = 28 + 7(2 - b)$

c $5m + 2(10 - 2m) = 15$ d $9k + 6(5 - 2k) = 0$

e ${}^-16 = y - 3(y + 2)$ f ${}^-5 = 11x - 5(x + 4)$

g ${}^-6 = 3d + 5(d + 2)$ h $0 = 15 + 2w - 2(3 - w)$

i $5(n - 3) - 3(n + 4) = {}^-1$ j $4(1 - 2x) - 3(2 - 3x) = 7 - 2x$

See Example 4

9 Solve each of the following equations.

a $\frac{x}{2} + \frac{x}{4} = 12$ b $\frac{a}{3} + \frac{a}{6} = 7$ c $\frac{b}{4} + \frac{b}{3} = 7$

d $\frac{2d}{5} + \frac{d}{2} = 4\frac{1}{2}$ e $\frac{a}{2} - 4 = \frac{a}{6}$ f $\frac{x}{4} + 2 = \frac{x}{5} + 1$

g $b + \frac{b}{7} = \frac{b}{2} + 9$ h $\frac{m}{3} - 4 = \frac{3m}{5}$ i $\frac{3n}{5} - \frac{2n}{3} = n + 16$

j $\frac{k}{5} + 3 = k - 1$ k $\frac{2d}{5} + 3 = \frac{d}{2}$ l $\frac{2x}{3} + \frac{3}{4} = \frac{x}{3}$

10 Find the roots of these equations.

a $\frac{a+6}{6}=2$ b $\frac{b-7}{4}=3$ c $\frac{2h-3}{5}={}^{-}1$

d $\frac{k+3}{8}-\frac{k}{6}=0$ e $\frac{2n+3}{5}-\frac{n+2}{4}={}^{-}1$ f $\frac{b+3}{3}-\frac{b}{4}=1\frac{3}{4}$

11 For each of the following, solve the inequality and show your solution on a number line. See Example 9

a $5n-3>9$ b $2p-6>{}^{-}1$ c $3c+7>0$ d ${}^{-}6e-5\le 0$

e $8k-9<4$ f $9y-9<{}^{-}7$ g ${}^{-}9c-6<9$ h $3k-1\le{}^{-}4$

i ${}^{-}6z+2>5$ j ${}^{-}3u+8>0$

12 For each of the following, solve the inequality and show your solution on a number line. See Example 10

a ${}^{-}4q+1\le 13-6q$ b $12b+8<9b+29$

c ${}^{-}4u-1>{}^{-}33-8u$ d $2e+8\ge 1-5e$

e $8(2p-6)-6>6(3p-10)-4$ f $6(2k-10)-9<4(2k-7)-29$

g $7(2b-4)-7\le 9(3b-3)+57$ h $4(3w-9)-7\ge 5(w-4)-72$

i $\frac{8c+7}{3}-\frac{2c-9}{2}>{}^{-}1\frac{5}{6}$ j $\frac{a+4}{12}+\frac{2a-5}{18}\le\frac{1}{4}$

k $\frac{g-3}{6}-\frac{3g+4}{7}\ge{}^{-}5$ l $\frac{g+8}{9}-\frac{6g+7}{5}>{}^{-}10$

13 Find the roots of these equations. See Example 5

a $\frac{1}{a}+\frac{3}{a}=4$ b $\frac{2}{3b}-\frac{2}{5b}=\frac{1}{10}$ c $\frac{6}{p-5}={}^{-}2$

d $\frac{12}{b+1}=3$ e $\frac{7}{2m-4}=1$ f $\frac{3}{5-2c}={}^{-}1$

g $\frac{8}{3g-5}={}^{-}4$ h $\frac{8}{5n-2}={}^{-}9$ i $\frac{-x}{3}-\frac{x+4}{6}={}^{-}10$

Problem solving

14 The length of a room is 4 m more than 3 times its width. If the perimeter is 28 m, how wide is the room? See Example 6

15 Trent jogged for 1 hour and walked for 2 hours. If he jogged three times as fast as he walked, and he travelled 20 km, how fast did he walk? See Example 7

16 Gail has $8 worth of 5-cent and 10-cent coins. If the number of 10-cent coins is 20 less than twice the number of 5-cent coins, how many coins are there altogether?

Worked solutions
Exercise 11.1
MAT10NAWS00031

17 A square garden has a 1 m wide walkway put around it. As a result, the area of the plot is decreased by 96 m^2. What were the dimensions of the original garden plot?

18 When three consecutive odd numbers are added and this total is doubled, the result is 186. What are the numbers?

Worked solutions
Exercise 11.1
MAT10NAWS00031

19 The temperature in the USA is always given in degrees Fahrenheit, while we always use Celsius. The formula for conversion of Fahrenheit to Celsius is $c = \frac{5}{9}(f - 32)$.

a Use the formula to write an equation and find the temperature in Fahrenheit that would have been reported in the US when Melbourne had a record temperature in 2009 of 46.4°.

b Find the temperature x that is the same on both scales. Find whether or not this temperature is possible in each of the United States and Australia.

20 1 kg of a 40% by mass solution of sodium hydroxide (NaOH) is to be diluted to 1 molar NaOH solution. This has 40 g of NaOH per litre of water. How much water must be added, and how much solution will be made?

See Example 11

21 Waldo is buying Kiwi fruit vines for his back fence. He has to plant at least one male plant for every 4 female plants to make sure he gets good fertilisation. The male plants only produce about a third of the fruit of the female plants, but they are harder to grow, so they cost \$12 each while the female plants only cost \$8 each. Write an inequality and find the maximum number of plants Waldo can get for \$200, if he gets the minimum number of male plants.

22 Janet has \$29.50. Write an inequality and solve it to find the largest quantity of walnuts she can buy at a price of \$8.40/kg.

23 A special applicator for in-situ wall cavity insulation production costs \$32, and the tubes of "compressed" insulation foam cost \$7 each. You only need one applicator for a job, but it has to be disposed of afterwards. Each tube of "compressed" insulation foam will do 3 square metres of wall cavity. What area of wall can be done for \$300?

24 A big tile/paver cutter costs \$120 a day to hire, and the blades for the cutter cost \$45 each. A landscaper hires the cutter and estimates that he needs to cut 450 pavers for a big job at a garden centre. Each blade will do a maximum of about 70 pavers. The landscaper reckons that if they leave all the cutting until the rest of the pavers are done, they will be able to cut the ones that need doing in one day. Write an inequality and find the minimum cost of the cutter and blades.

Reasoning

25 If $x \geq y$ and $a < 0$, prove that $\frac{x}{a} \leq \frac{y}{a}$.

See Example 12

26 If $x < y$ and $a > 0$, prove that $\frac{x}{a} < \frac{y}{a}$.

Worked solutions
Exercise 11.1
MAT10NAWS00031

27 If $x > 0$, prove that $x + \frac{a^2}{x} \geq 2a$ for any number a. Hint: Consider $(x - a)^2$.

28 If $x < 0$, prove that $x + \frac{a^2}{x} \leq 2a$ for any number a.

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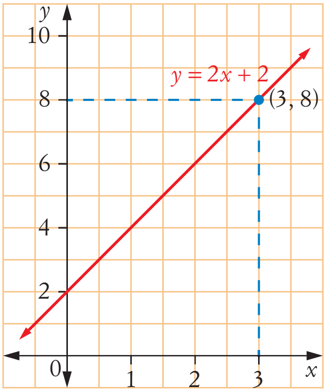
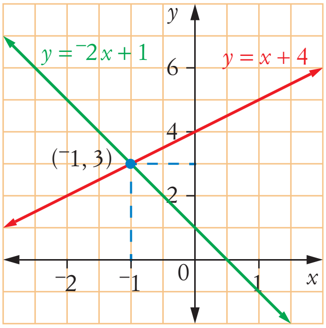
11.2 Simultaneous equations



Look at the graph of the function $y = 2x + 2$.
Any ordered pair that lies on the graph satisfies the function.
There are an infinite number of solutions to this equation.
Here, the point (3, 8) lies on the graph.
So (3, 8) must satisfy the equation $y = 2x + 2$.
Let's check by substituting.
When $x = 3$:
$y = 2 \times 3 + 2 = 6 + 2 = 8$
So (3, 8) satisfies the equation $y = 2x + 2$.



Now look at the two linear functions that are graphed below.
The graphs cross at the point ($^-1$, 3). This means that ($^-1$, 3) should satisfy both functions because it lies on the graph of each.
Let's check.
($^-1$, 3) satisfies $y = x + 4$ because, when $x = {}^-1$,

$$y = {}^-1 + 4 = 3$$

($^-1$, 3) also satisfies $y = {}^-2x + 1$ because, when $x = {}^-1$,

$$y = {}^-2 \times {}^-1 + 1 = 2 + 1 = 3$$

The solution $x = {}^-1$ and $y = 3$ to the equations $y = x + 4$ and $y = {}^-2x + 1$ satisfies both equations *at the same time*.
We can regard $y = 2x + 2$ as a function where y is the independent variable in order to sketch its graph. At the same time, we can think of it as an equation with two variables.
Similarly, $y = x + 4$ is both a function and an equation with two variables.
The point ($^-1$, 3) is both the intersection of the lines and a solution of both equations, where $x = {}^-1$ and $y = 3$.

Important!

Simultaneous equations

Equations considered *at the same time* make an equation system called **simultaneous equations**.
A solution of these equations satisfies all the equations at the same time and is referred to as a **simultaneous solution**. The simultaneous solution of two equations in two unknowns is the intersection of their lines on a graph.

Technology: GeoGebra
Simultaneous equations
MAT10NATC00011

CAS TI-Nspire exercise
Equations
MAT10NATI00011

CAS ClassPad exercise
Equations
MAT10NACP00011

Worksheet
Solving simultaneous equations by graphing
MAT10NAWK00028

Example 13

Find the solution of the following simultaneous equations by graphing.

$3y - 2x = 3$

$x + 2y = 16$

Solution

Write the equations and number them for convenience.

$\mathbf{3y - 2x = 3}$ [1]
$\mathbf{x + 2y = 16}$ [2]

Choose some convenient points to sketch the line for [1].

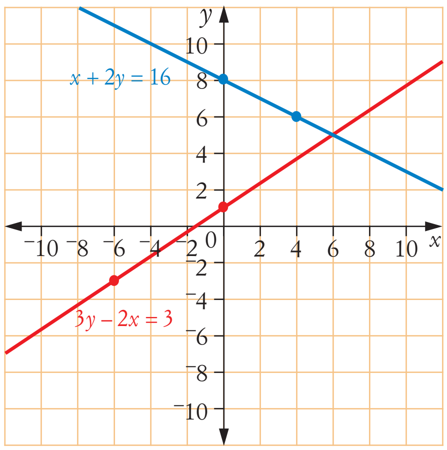
For $x = 0$, $y = 1$, so $(0, 1)$ is on the line.
For $x = {}^{-}6$, $y = {}^{-}3$, so $({}^{-}6, {}^{-}3)$ is on the line.

Choose some convenient points to sketch the line for [2].

For $x = 0$, $y = 0$, so $(0, 8)$ is on the line.
For $x = 4$, $y = 6$, so $(4, 6)$ is on the line.

Use the points to show both lines on graph paper.

The lines intersect at the point $(6, 5)$.



Write the answer.

The simultaneous solution is $x = 6$ and $y = 5$.

Using graphs to solve simultaneous equations can take a long time. You may also find it difficult to find the exact point of intersection, so it is not always particularly accurate. It is often quicker and more accurate to use an algebraic method.

Technology worksheet
Excel worksheet: Solving simultaneous equations
MAT10NACT00017

Important!

Substitution method

When you use the **substitution method** you use one equation to express one variable in terms of the other. Then you substitute this into the other equation to make a linear equation in one variable.
Solve this and use the expression to find the value of the substituted variable.
Check your solution in both *original* equations.

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Example 14

Solve each of these pairs of simultaneous equations by substitution.

a $2b - a = 3$
$a + 3b = 7$

b $4x + 3y = 8$
$3x + 2y = 7$

Solution

a Write the equations and number them for convenience.	$2b - a = 3 \quad [1]$ $a + 3b = 7 \quad [2]$
It is easy to rearrange [1] to make a the subject.	$a = 2b - 3 \quad [1]$
Use brackets and substitute [3] into [2].	$(2b - 3) + 3b = 7$
Get rid of the brackets.	$2b - 3 + 3b = 7$
Simplify and move the 3 to the RHS.	$5b - 3 + 3 = 7 + 3$
Simplify.	$5b = 10$
Divide by 5 to get b.	$b = 2$
Substitute b into [3] find a.	$a = 2b - 3 = 2 \times 2 - 3 = 1$
Check the values in [1].	$2b - a = 2 \times 2 - 1 = 3$ ✓OK
Substitute in [2].	$a + 3b = 1 + 3 \times 2 = 7$ ✓OK
Write the answer.	$a = 1$ and $b = 2$
b Write the equations and number them for convenience.	$4x + 3y = 8 \quad [1]$ $3x + 2y = 7 \quad [2]$
Rearrange [2] to isolate y, the variable with the smallest coefficient.	$y = \dfrac{7 - 3x}{2} \quad [3]$
Use brackets and substitute [3] into [1].	$4x + 3 \times \dfrac{(7 - 3x)}{2} = 8$
Get rid of the denominator by multiplying by 2.	$2 \times 4x + 2 \times 3 \times \dfrac{(7 - 3x)}{2} = 2 \times 8$
Simplify both sides.	$8x + 3(7 - 3x) = 16$
Multiply out the brackets.	$8x + 21 - 9x = 16$
Simplify again.	$21 - x = 16$
Move the x to the right and the 16 to the left.	$21 - x + x - 16 = 16 + x - 16$
Simplify and reverse the equation.	$x = 5$
Substitute in [3].	$y = \dfrac{7 - 3 \times 5}{2} = {}^{-}4$
Check the values in [1].	$4x + 3y = 4 \times 5 + 3 \times {}^{-}4 = 8$ ✓OK
Check the values in [2].	$3x + 2y = 3 \times 5 + 2 \times {}^{-}4 = 7$ ✓OK
Write the answer.	$x = 5$ and $y = {}^{-}4$

Puzzle sheet
Simultaneous equations by substitution
MAT10NAPS00039

Obviously, a pair of equations where one of the variables has a coefficient of 1 or $^{-}1$ is much easier to solve by substitution than a pair where no variable has a coefficient of 1. If you find the answer to the first variable is a fraction, it is very easy for you to make a mistake.

Important!

Elimination method

In the **elimination method** you make the coefficients of one variable opposites by multiplying the equations by suitable constants. The coefficient in one equation is the same size but the opposite sign to the other equation. Then add the equations to make an equation with only one variable. Solve this and find the other value.
Check your solution in both *original* equations.

Example 15

Video tutorial
Simultaneous equations
MAT10NAVT10027

Weblink
Simultaneous equations quizzes
MAT10NAWB00011

Solve these simultaneous equations by elimination.

a $2a - b = 3$
$a + b = 3$

b $3x + y = 3$
$x + y = {}^{-}1$

c $3x + 2y = {}^{-}1$
$5x + 3y = {}^{-}3$

d $9p + 4q = 3$
$15p + 28q = {}^{-}11$

Solution

a Write the equations and number them for convenience.

$$\mathbf{2a - b = 3} \quad [1]$$
$$\mathbf{a + b = 3} \quad [2]$$

Since the coefficients of b are already opposites, just add the equations.

$$\mathbf{2a - b = 3} \quad [1]$$
$$\mathbf{a + b = 3} \quad [2]$$
$$\mathbf{3a + 0 = 6} \quad [3]$$

Now solve equation [3].

$$\frac{\mathbf{3a}}{\mathbf{3}} = \frac{\mathbf{6}}{\mathbf{3}}$$

Simplify.

$$\mathbf{a = 2}$$

Substitute $a = 2$ into [2].

$$\mathbf{2 + b = 3}$$

Solve for b.

$$\mathbf{b = 1}$$

Check the values in [1].

$$\mathbf{2a - b = 2 \times 2 - 1 = 3} \ \checkmark\text{OK}$$

Check the values in [2].

$$\mathbf{a + b = 2 + 1 = 3} \ \checkmark\text{OK}$$

Write the answer.

$$\mathbf{a = 2 \text{ and } b = 1}$$

b Write the equations and number them for convenience.

$$\mathbf{3x + y = 3} \quad [1]$$
$$\mathbf{x + y = {}^{-}1} \quad [2]$$

Multiply equation [2] by $^{-}1$ to make the coefficients of y opposites and add the equations.

$$\mathbf{3x + y = 3} \quad [1]$$
$$\mathbf{-x - y = 1} \quad [2] \times {}^{-}\mathbf{1}$$
$$\mathbf{2x \quad = 4} \quad [3]$$

Now solve equation [3].

$$\mathbf{x = 2}$$

Substitute $x = 2$ into [2].

$$\mathbf{2 + y = {}^{-}1}$$

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Solve for y.	$y = {}^{-}3$
Check the values in [1].	$3x + y = 3 \times 2 + {}^{-}3 = 3$ ✓OK
Check the values in [2].	$x + y = 2 + {}^{-}3 = {}^{-}1$ ✓OK
Write the answer.	$x = 2$ and $y = {}^{-}3$
c Write the equations and number them for convenience.	$3x + 2y = {}^{-}1 \quad [1]$ $5x + 3y = {}^{-}3 \quad [2]$
Multiply equation [1] by 3 and equation [2] by ${}^{-}2$ to make the coefficients of y opposites and add the equations.	$9x + 6y = {}^{-}3 \quad [1] \times 3$ ${}^{-}10x - 6y = 6 \quad [2] \times {}^{-}2$ $-x = 3 \quad [3]$
Now solve equation [3].	$x = {}^{-}3$
Substitute $x = {}^{-}3$ into [1].	$3 \times {}^{-}3 + 2y = {}^{-}1$
Simplify.	${}^{-}9 + 2y = {}^{-}1$
Add 9 to both sides.	${}^{-}9 + 2y + 9 = {}^{-}1 + 9$
Simplify	$2y = 8$
Solve for y.	$y = 4$
Check the values in [1].	$3x + 2y = 3 \times {}^{-}3 + 2 \times 4$ $= {}^{-}9 + 8$ $= {}^{-}1$ ✓OK
Check the values in [2].	$5x + 3y = 5 \times {}^{-}3 + 3 \times 4$ $= {}^{-}15 + 12$ $= {}^{-}3$ ✓OK
Write the answer.	$x = {}^{-}3$ and $y = 4$
d Write the equations and number them for convenience.	$9p + 4q = 3 \quad [1]$ $15p + 28q = {}^{-}11 \quad [2]$
Multiply equation [1] by 5 and equation [2] by ${}^{-}3$ to make the coefficients of p opposites and add the equations.	$45p + 20q = 15 \quad [1] \times 5$ ${}^{-}45p - 84q = 33 \quad [2] \times {}^{-}3$ ${}^{-}64q = 48 \quad [3]$
Now solve equation [3].	$q = \frac{48}{{}^{-}64} = {}^{-}\frac{3}{4}$
Substitute $q = {}^{-}\frac{3}{4}$ into [1].	$9p + 4 \times {}^{-}\frac{3}{4} = 3$
Simplify.	$9p - 3 = 3$
Add 3 to both sides.	$9p = 6$
Solve for p.	$p = \frac{6}{9} = \frac{2}{3}$
Check the values in [1].	$9p + 4q$ $= 9 \times \frac{2}{3} + 4 \times {}^{-}\frac{3}{4}$ $= 6 - 3 = 3$ ✓OK

Check the values in [2].	$15p + 28q$
	$= 15 \times \frac{2}{3} + 28 \times {}^{-}\frac{3}{4}$
	$= 10 - 21 = {}^{-}11$ ✓OK
Write the answer.	$p = \frac{2}{3}$ and $q = {}^{-}\frac{3}{4}$

As with ordinary linear equations, you should get rid of fractions before solving simultaneous equations.

Example 16

Puzzle sheet
Simultaneous equations
MAT10NAPS00041

Solve the simultaneous equations $\frac{a-1}{2} - \frac{b}{3} = 1$ and $a - \frac{b+2}{5} = 4$.

Solution

Write the equations and number them for convenience.	$\frac{a-1}{2} - \frac{b}{3} = 1$	[1]
	$a - \frac{b+2}{5} = 4$	[2]
Multiply [1] by the LCD, 6.	$6 \times \frac{(a-1)}{2} - 6 \times \frac{b}{3} = 6 \times 1$	
Cancel the denominators.	$3(a-1) - 2 \times b = 6$	
Multiply out the brackets.	$3a - 3 - 2b = 6$	
Simplify.	$3a - 2b = 9$	[3]
Multiply [2] by 5.	$5 \times a - 5 \times \frac{(b+2)}{5} = 5 \times 4$	
Cancel the denominator.	$5a - 1(b+2) = 20$	
Multiply out the brackets.	$5a - b - 2 = 20$	
Simplify.	$5a - b = 22$	[4]
Multiply [4] by ${}^{-}2$ to make the coefficients of b opposites and add the equations.	${}^{-}10a + 2b = {}^{-}44$	[4] $\times$ ${}^{-}2$
	$3a - 2b = 9$	[3]
	${}^{-}7a = {}^{-}35$	[5]
Solve equation [5].	$a = 5$	
Substitute $a = 5$ into [4].	$5 \times 5 - b = 22$	
Simplify.	$25 - b = 22$	
Solve for b.	$b = 3$	
Check the values in the *original* [1].	$\frac{a-1}{2} - \frac{b}{3} = \frac{5-1}{2} - \frac{3}{3} = 2 + 1 = 3$ ✓OK	
Check the values in the *original* [2].	$a - \frac{b+2}{5} = 5 - \frac{3+2}{5} = 5 - 1 = 4$ ✓OK	
Write the answer.	**The solution is $a = 5$ and $b = 3$.**	

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When you are solving problems involving simultaneous equations you follow the same basic steps as for all problem solving: creating a mathematical model, solving the maths and checking the solution.

Important!

Solving problems with simultaneous equations

1. Read the information carefully
2. Choose variable names for the unknown quantities
3. Write equations
4. Solve the equations using the most convenient method
5. Check the solution using the original information
6. State the solution in the same terms as the problem

Example 17

Animated example
Simultaneous equations
MAT10NAAE00011

A school play was attended by a total of 400 people. Adults were charged \$5 and children \$2. Ticket receipts were \$1100. How many adults attended?

Solution

Choose sensible variable names.	**Let the number of adults be a and the number of children be c.**
Write an equation for the attendance.	$a + c = 400$ [1]
Write an equation for the receipts.	$5a + 2c = 1100$ [2]
Rearrange [1] to make c the subject.	$c = 400 - a$ [3]
Use brackets to substitute [3] into [2].	$5a + 2(400 - a) = 1100$
Multiply out the brackets.	$5a + 800 - 2a = 1100$
Simplify and subtract 800.	$3a + 800 - 800 = 1100 - 800$
Simplify.	$3a = 300$
Solve for a.	$a = 100$
Substitute $a = 100$ into [1].	$100 + c = 400$
Solve for c.	$c = 300$
Check with the *original* information.	**Attendance** $= 100 + 300 = 400$ ✓**OK**
Check the receipts.	**Receipts** $= \$5 \times 100 + \$2 \times 300$ $= \$500 + \$600$ $= \$1100$
State the answer in the terms of the question.	**100 adults attended the play.**

Investigate: Cramer's rule

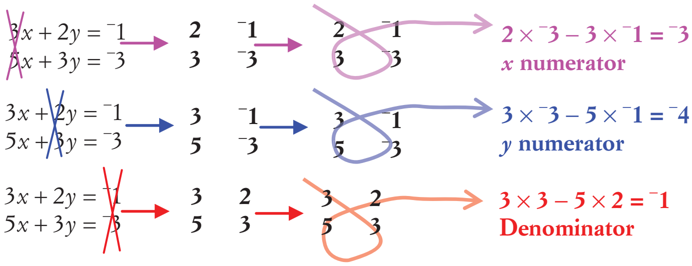
You can have more than two simultaneous equations with more than 2 unknowns. For example, the following system has 5 equations and 4 variables.

$$\begin{aligned} 8h + 14i + 9j + 9k &= 25 \\ 6h + 11i + 3j + 10k &= 13 \\ 4h + 6i - 12j + 11k &= {}^{-}11 \\ h + 2i + 3j &= 5 \\ 3h + 3i - 2j + 3k &= 5 \end{aligned}$$

You can check that the solution to this system is $h = 5$, $i = {}^{-}3$, $j = 2$ and $k = 1$.
Cramer's rule is an advanced formula that gives the solution to n equations in n variables. It is rarely used for large systems because the number of calculations is so large. Approximate methods are used instead. However, for 2 equations in 2 unknowns the rule can be applied in a simple way.
You need to put both equations in the form $ax + by = c$.
Then you use cross multiplication and subtraction as shown below.
Consider the equations

$$\begin{aligned} 3x + 2y &= {}^{-}1 \\ 5x + 3y &= {}^{-}3 \end{aligned}$$

Progressively cross out a column and cross multiply.



Change the sign of the x numerator to find the values of x and y.

$$x = \frac{-({}^{-}3)}{{}^{-}1}, \qquad y = \frac{{}^{-}4}{{}^{-}1}$$

This gives $x = {}^{-}3$ and $y = 4$, as in Example **15** part **c**.
Check the result of Cramer's rule for Example **15** part **d**. You should get the same answer after cancelling down the fractions to their simplest form.

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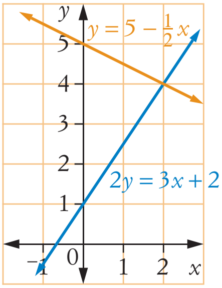
Exercise 11.2 Simultaneous equations

Understanding

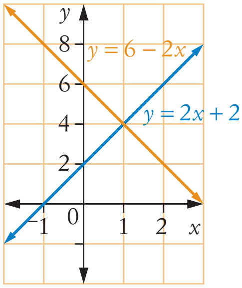
Extra questions
Exercise 11.2
MAT10NAEQ00032

1 Use the following graphs to find simultaneous solutions to:

a $y = 5 - \frac{1}{2}x$ and $2y = 3x + 2$



b $y = 6 - 2x$ and $y = 2x + 2$



2 Find the solution of the following pairs of simultaneous equations by graphing.

a $y = x$ and $y = 2x + 1$
b $y = x + 3$ and $y = 2x + 4$
c $y = -x + 2$ and $y = 2x - 1$
d $y = 2x - 7$ and $y = -x - 1$
e $y = {}^{-}2x + 2$ and $y = 3x + 7$

Fluency

3 Find the solution of the following pairs of simultaneous equations by graphing.

See Example 13

a $2x + y = 4$ and $x - y = 2$
b $3x - y = 7$ and $2x + y = 3$
c $10x - 5y = 50$ and $x + y = 2$
d $3x + 4y = {}^{-}2$ and $x - 4y = {}^{-}6$
e $4x + y = 23$ and $x + y = 8$

4 Find the solution of the following pairs of simultaneous equations by substitution.

See Example 14

a $x = y + 3$, $x + 2y = 9$
b $y = 3x$, $2x - y = {}^{-}6$
c $b = 2a - 3$, $a + 3b = 5$
d $3a + b = 8$, $b = a - 12$
e $x + 2y = 6$, $x = 3y - 9$
f $c = 3d - 1$, $5c - 7d = 19$
g $c = d + 1$, $2c + d = 14$
h $c = 7 - 2d$, $3c - 2d = 9$
i $x = 1 - y$, $3 = y + 2x$

5 Find the solution of the following pairs of simultaneous equations by elimination.

See Example 15

a $x + y = 5$, $x - y = 1$
b $2x + y = 12$, $3x - y = 13$
c $5m - 2n = 13$, $m + n = 4$
d $4a - b = 10$, $a + 3b = 9$
e $4m - n = 6$, $3m + 2n = {}^{-}1$
f $5x + y = 12$, $3x + 2y = 3$
g $3x + 2y = 6$, $2x - 3y = 17$
h $5a + 2b = {}^{-}8$, $a - 3b = {}^{-}5$
i $2m + 3n = {}^{-}1$, $m + 4n = 2$

6 Solve the following pairs of simultaneous equations.

See Example 16

a $\frac{x}{2} + y = 0$, $x - y = 6$
b $\frac{a}{3} + b = 4$, $a - b = 0$
c $m + \frac{n}{3} = 1$, $m - n = 5$

d $\dfrac{3x}{2} - \dfrac{y}{4} = 2$
$2x + y = 8$

e $\dfrac{3m}{2} - \dfrac{2n}{3} = 5$
$m + n = {}^{-}1$

f $\dfrac{c}{2} - \dfrac{d}{3} = \dfrac{4}{3}$
$\dfrac{c}{3} + \dfrac{d}{3} = 2$

g $x + \dfrac{y+1}{2} = 5$
$\dfrac{x-2}{2} - y = 0$

h $\dfrac{a-1}{2} + b = 1$
$\dfrac{b-1}{2} + a = 4$

i $\dfrac{y-x}{3} - \dfrac{x-y}{2} = {}^{-}5$
$\dfrac{x}{6} + \dfrac{2y}{3} = \dfrac{{}^{-}2}{3}$

Problem solving

7 The total cost of tickets for 2 adults and 3 children to see a show was \$20. It cost \$31 for the tickets for a family group with 3 adults and 5 children. Find the cost of each type of ticket.

Worked solutions
Exercise 11.2
MAT10NAWS00032

8 The heights of water (h cm) in two containers after a certain time (t seconds) are given by the following equations:
Container A: $h = 2t$ Container B: $h = 3t - 20$
Find, graphically, the time when the two containers have the same height of water.

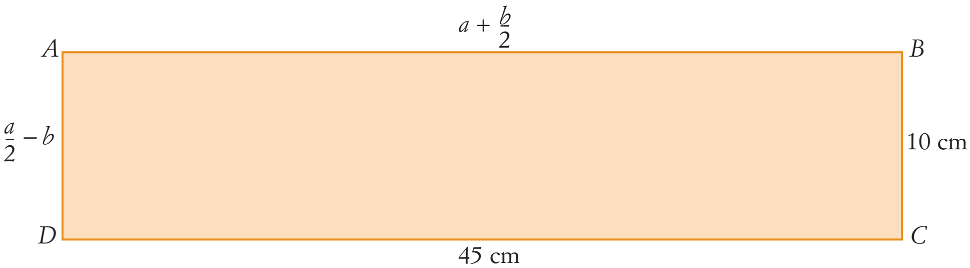
9 The length of a rectangular room is 2 m longer than its width. If the perimeter of the room is 68 m, find its length.

10 Two workers laid 250 bricks. If Sam had laid twice as many as he did and Peta had laid half as many as she did, there would have been 50 bricks left over. How many bricks did each lay?

Worked solutions
Exercise 11.2
MAT10NAWS00032

11 I bought three trays of strawberries and four trays of cherries for \$18 at the local fruit shop. My friend bought four trays of strawberries and three trays of cherries for \$17. How much did we pay for each tray of strawberries and cherries?

12 $ABCD$ is a rectangle. Use this diagram to find the values of a and b.



13 Ross, Trish and Helen are comparing the number of DVDs that they own. Ross and Trish own the same number of DVDs but Helen has fewer than both of them. Ross has five less than six times the number of Helen's DVDs, while Trish has one more than three times Helen's number of DVDs. How many DVDs does Helen have?

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Worked solutions
Exercise 11.2
MAT10NAWS00032

14 Jackie and Patrick are shearing at the same shed. One day, Jackie worked for 8 hours and Patrick worked for 9 hours. Between them they sheared 320 sheep. The next day they sheared 272 sheep, but Patrick knocked off after only 6 hours, while Jackie worked for 8 hours again. Assuming that each man sheared at a consistent rate, find the number of sheep they could each shear in an hour.

11.3 Quadratic equations

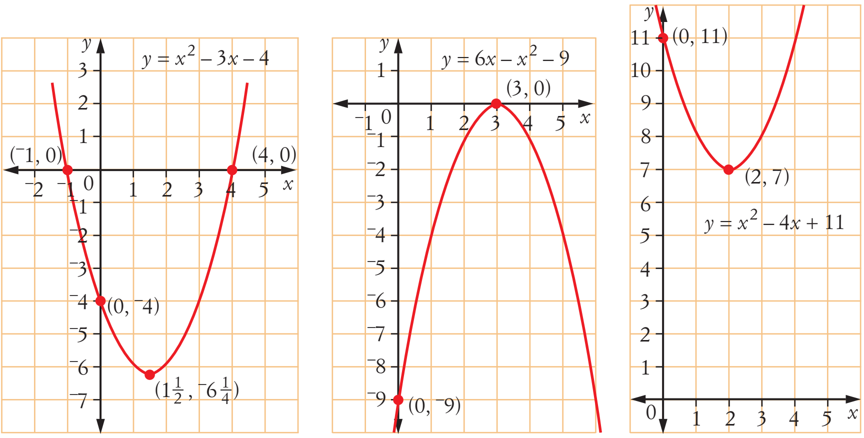
You have already studied quadratic expressions. **Quadratic equations** are the result of putting a quadratic expression equal to zero.

Important!

Quadratic equations

A quadratic equation is one that can be expressed in the form $ax^2 + bx + c = 0$, where x is the variable and a, b and c are constants.

You have already seen that quadratic functions produce parabolas and practiced drawing the graphs of quadratic functions. Consider the following quadratic equations: $x^2 - 3x - 4 = 0$, $6x - x^2 - 9 = 0$ and $x^2 - 4x + 11 = 0$. The corresponding quadratic functions are: $y = x^2 - 3x - 4$, $y = 6x - x^2 - 9$, and $y = x^2 - 4x + 11$. The first two expressions factorise to give $(x - 4)(x + 1)$ and $-(x - 3)^2$, but the third expression doesn't factorise. It is actually equal to $(x - 2)^2 + 7$.
You can use the methods from Chapters 5 and 7 to sketch the graphs as shown below.



The first graph has two zeros, the second only one and the third none. This means that the corresponding quadratic equations have two solutions, one solution and no solutions.

While you can find the solutions to quadratic equations by graphing, as above, it is quicker and more accurate to find them using algebra.
The first method just uses the square root to find solutions.

Example 18

Solve each of the following equations.

a $m^2 = 16$ **b** $3x^2 = 75$

Solution

a Write the equation. $\quad \mathbf{m^2 = 16}$

Write the meaning of the equation. **The square of m is 16.**

Write the answers. $\quad \mathbf{m = {}^-4 \text{ or } m = 4}$

b Write the equation. $\quad \mathbf{3x^2 = 75}$

Divide by 3. $\quad \mathbf{\dfrac{3x^2}{3} = \dfrac{75}{3}}$

Simplify. $\quad \mathbf{x^2 = 25}$

Write in the square root form. $\quad \mathbf{x = \pm\sqrt{25} = \pm 5}$

Separate the $\pm$ into $+$ and $-$. $\quad \mathbf{x = {}^-5 \text{ or } x = 5}$

Instead of using the phrase 'the square of m is 16', you could write $m = \pm\sqrt{16}$. It is necessary to write the $\pm$ because, by itself, $\sqrt{\ }$ is taken to mean a *positive* number. You may need to isolate the square before you can do this.

Example 19

Video tutorial
Simple quadratic equations
MAT10NAVT10028

Solve each of the following equations.

a $(x - 8)^2 = 9$ **b** $(x + 3)^2 = 1$

Solution

a Write the equation. $\quad \mathbf{(x - 8)^2 = 9}$

Write in the square root form. $\quad \mathbf{x - 8 = \pm\sqrt{9} = \pm 3}$

Separate the $\pm$ into $+$ and $-$. $\quad \mathbf{x - 8 = {}^-3 \text{ or } x - 8 = 3}$

Solve each part. $\quad \mathbf{x - 8 + 8 = {}^-3 + 8 \text{ or } x - 8 + 8 = 3 + 8}$

Write the answers. $\quad \mathbf{x = 5 \text{ or } x = 11}$

b Write the equation. $\quad \mathbf{(x + 3)^2 = 1}$

Write in the square root form. $\quad \mathbf{x + 3 = \pm\sqrt{1} = \pm 1}$

Separate the $\pm$ into $+$ and $-$. $\quad \mathbf{x + 3 = {}^-1 \text{ or } x + 3 = 1}$

Solve each part. $\quad \mathbf{x + 3 - 3 = {}^-1 - 3 \text{ or } x + 3 - 3 = 1 - 3}$

Write the answers. $\quad \mathbf{x = {}^-4 \text{ or } x = {}^-2}$

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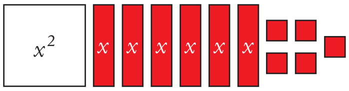
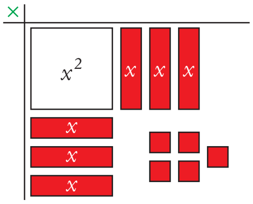
In principle, any quadratic equation can be solved by changing it to the form $(x \pm p)^2 = q$. You should remember that the expansion of a perfect square is given by $(x + y)^2 = x^2 + 2xy + y^2$ or for a difference by $(x - y)^2 = x^2 - 2xy + y^2$.
Thus $(x \pm p)^2 = x^2 \pm 2xp + p^2$ so $(x \pm p)^2 = q$ is the same as $x^2 \pm 2xp + p^2 = q$. This is used in reverse to change quadratic equations to the form $(x \pm p)^2 = q$.

Investigate: Completion of the square

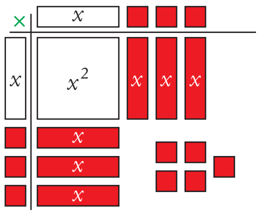
You can use a physical model to express any quadratic in the form $(x \pm p)^2 \pm q$. You will need some small squares to represent units, strips to represent x and a larger square to represent x^2. They need to be white on one side and coloured on the reverse to represent negatives. You may have used some in Chapter 7 to model factorisation of quadratics.
Consider the quadratic expression $x^2 - 6x - 5$.
You can model this as shown here.



Make a cross to show multiplication and arrange the x^2 and xs so that there are an equal number of xs to the right and below the x^2. Leave the units to one side for the moment.



To get the x^2 and xs you will obviously need an x and 3 negative units at both the top and side.



But the negative units at the top and side multiply to give *positive* units inside. You will need to put in 9 positive units and another 9 negative units so the total is still $x^2 - 6x - 5$.
This gives the model shown below.
The model now shows

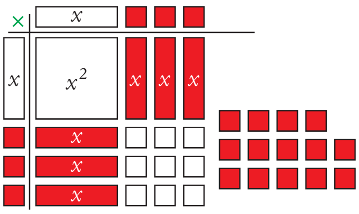
$x^2 - 6x + 9 - 14$.

The arrangement represents

$(x - 3)(x - 3) - 14$

which is $(x - 3)^2 - 14$.
This shows that

$x^2 - 6x - 5 = (x - 3)^2 - 14$.



Your teacher will probably give you some other problems to try modelling in the way shown above.

Important!

Completing the square

You can **complete the square** in a quadratic equation by:

1. dividing (if necessary) to make the coefficient of x^2 equal to 1
2. moving the constant to the RHS
3. adding half the square of the coefficient of x to both sides
4. factorising the LHS as a perfect square

Example 20

CAS TI-Nspire exercise
Equations
MAT10NATI00011

CAS ClassPad exercise
Equations
MAT10NACP00011

Solve the following quadratic equations by completing the square.

a $e^2 + 10e = 0$ **b** $x^2 - 6x + 5 = 0$ **c** $3k^2 + 6k - 24 = 0$

Solution

a

Write the equation.	$e^2 + 10e = 0$
Start at step 3 by adding 5^2 to both sides.	$e^2 + 10e + 5^2 = 0 + 5^2$
Factorise the LHS and simplify the RHS.	$(e + 5)^2 = 25$
Write in the square root form.	$e + 5 = \pm\sqrt{25} = -5$ **or** 5
Separate the $\pm$ into $+$ and $-$.	$e + 5 = {}^{-}5$ **or** $e + 5 = 5$
Solve each part.	$e + 5 - 5 = {}^{-}5 - 5$ **or** $e + 5 - 5 = 5 - 5$
Write the answers.	$e = {}^{-}10$ **or** $e = 0$

b

Write the equation.	$x^2 - 6x + 5 = 0$
Start at step 2 by taking 5 from both sides.	$x^2 - 6x + 5 - 5 = 0 - 5$
Simplify.	$x^2 - 6x = {}^{-}5$
Add the square of half of 6 to both sides.	$x^2 - 6x + 3^2 = {}^{-}5 + 3^2$
Factorise the LHS and simplify the RHS.	$(x - 3)^2 = {}^{-}5 + 9 = 4$
Write in the square root form.	$x - 3 = \pm\sqrt{4} = -2$ **or** 2
Separate the $\pm$ into $+$ and $-$.	$x - 3 = {}^{-}2$ **or** $x - 3 = 2$
Solve each part.	$x - 3 + 3 = {}^{-}2 + 3$ **or** $x - 3 + 3 = 2 + 3$
Write the answers.	$x = 1$ **or** $x = 5$

c

Write the equation.	$3k^2 + 6k - 24 = 0$
Make the coefficient of k^2 equal to 1 by dividing through by 3.	$\frac{3k^2}{3} + \frac{6k}{3} - \frac{24}{3} = \frac{0}{3}$
Simplify.	$k^2 + 2k - 8 = 0$
Add the constant 8 to both sides.	$k^2 + 2k - 8 + 8 = 0 + 8$
Simplify.	$k^2 + 2k = 8$

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Add the square of half of 2 to both sides.	$k^2 + 2k + 1^2 = 8 + 1^2$
Factorise the LHS and simplify the RHS.	$(k + 1)^2 = 9$
Write in the square root form.	$k + 1 = \pm\sqrt{9} = {}^-3$ or 3
Separate the $\pm$ into $+$ and $-$.	$k + 1 = {}^-3$ or $k + 1 = 3$
Solve each part.	$k + 1 - 1 = {}^-3 - 1$ or $k + 1 - 1 = 3 - 1$
Write the answers.	$k = {}^-4$ or $k = 2$

In some cases you will have a fraction involved in your answer.

Example 21

Solve $2m^2 + 5m - 12 = 0$ by completing the square.

Solution

Write the equation.	$2m^2 + 5m - 12 = 0$
Make the coefficient of m^2 equal to 1 by dividing through by 2.	$\frac{2m^2}{2} + \frac{5m}{2} - \frac{12}{2} = 0$
Simplify.	$m^2 + 2\frac{1}{2}m - 6 = 0$
Add the constant 6 to both sides.	$m^2 + 2\frac{1}{2}m - 6 + 6 = 0 + 6$
Simplify.	$m^2 + 2\frac{1}{2}m = 6$
Add the square of half of $\frac{5}{2}$ to both sides.	$m^2 + \frac{5}{2}m + \left(\frac{5}{4}\right)^2 = 6 + \left(\frac{5}{4}\right)^2$
Factorise the LHS and simplify the RHS.	$\left(m + \frac{5}{4}\right)^2 = 6 + \frac{25}{16} = \frac{96 + 25}{16} = \frac{121}{16}$
Write in the square root form.	$m + \frac{5}{4} = \pm\sqrt{\frac{121}{16}} = \pm\frac{11}{4}$
Separate the $\pm$ into $+$ and $-$.	$m + \frac{5}{4} = {}^-\frac{11}{4}$ or $m + \frac{5}{4} = \frac{11}{4}$
Solve each part.	$m + \frac{5}{4} - \frac{5}{4} = {}^-\frac{11}{4} - \frac{5}{4}$ or $m + \frac{5}{4} - \frac{5}{4} = \frac{11}{4} - \frac{5}{4}$
Complete the answers.	$m = \frac{{}^-16}{4}$ or $m = \frac{6}{4}$
Simplify to get the final answers.	$m = {}^-4$ or $m = 1\frac{1}{2}$

In many cases, it is easier to solve a quadratic equation by factorisation than by completing the square. When solving by factorisation it is necessary to use the following rule.

Important!

Zero product rule

If $ab = 0$ then at least one of a and b must be zero. That is, $a = 0$ or $b = 0$.

Example 22

Solve each of the following.

a $m(m - 7) = 0$ **b** $(a + 7)(a - 3) = 0$

Solution

a Write the problem.	$\boldsymbol{m(m-7)=0}$
Use the zero product rule.	$\boldsymbol{m=0 \text{ or } m-7=0}$
Solve for m.	$\boldsymbol{m=0 \text{ or } m=7}$
b Write the equation.	$\boldsymbol{(a+7)(a-3)=0}$
Use the zero product rule.	$\boldsymbol{a+7=0 \text{ or } a-3=0}$
Solve for a.	$\boldsymbol{a={}^{-}7 \text{ or } a=3}$

Example 23

Video tutorial
Quadratic equations by factorising
MAT10NAVT10029

Puzzle sheet
Factorising quadratic equations
MAT10NAPS00037

Solve each of the following by factorisation.

a $3m^2 = 6m$ **b** $x^2 + 5x + 6 = 0$ **c** $p^2 = 5p + 24$
d $n(n + 2) = 8$ **e** $2x^2 - x - 15 = 0$ **f** $(x - 1)^2 + 5(x - 1) - 6 = 0$

Solution

a Write the equation.	$\boldsymbol{3m^2=6m}$
Put all terms on the LHS.	$\boldsymbol{3m^2-6m=0}$
Simplify by dividing all terms by 3.	$\boldsymbol{m^2-2m=0}$
Factorise.	$\boldsymbol{m(m-2)=0}$
Use the zero product rule.	$\boldsymbol{m=0 \text{ or } m-2=0}$
Solve for m.	$\boldsymbol{m=0 \text{ or } m=2}$

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b	Write the equation.	$x^2 + 5x + 6 = 0$
	Factorise.	$(x + 3)(x + 2) = 0$
	Use the zero product rule.	$x + 3 = 0$ or $x + 2 = 0$
	Solve for x.	$x = {}^-3$ or $x = {}^-2$
c	Write the equation.	$p^2 = 5p + 24$
	Put all terms on the LHS.	$p^2 - 5p - 24 = 0$
	Factorise.	$(p + 3)(p - 8) = 0$
	Use the zero product rule.	$p + 3 = 0$ or $p - 8 = 0$
	Solve for p.	$p = {}^-3$ or $p = 8$
d	Write the equation.	$n(n + 2) = 8$
	Multiply out the brackets	$n^2 + 2n = 8$
	Put all terms on the LHS.	$n^2 + 2n - 8 = 0$
	Factorise.	$(n + 4)(n - 2) = 0$
	Use the zero product rule.	$n + 4$ or $n - 2 = 0$
	Solve for n.	$n = {}^-4$ or $n = 2$
e	Write the equation.	$2x^2 - x - 15 = 0$
	Find the product and sum numbers (see pages 278–279).	$ac = 2 \times {}^-15 = {}^-30$ and $b = {}^-1$
	Use $5 \times {}^-6 = {}^-30$ and $5 + {}^-6 = {}^-1$ to factorise.	$2x^2 + 5x - 6x - 15 = 0$
	Group and use the common factors.	$x(2x + 5) - 3(2x + 5) = 0$
	Complete the factorisation.	$(2x + 5)(x - 3) = 0$
	Use the zero product rule.	$2x + 5 = 0$ or $x - 3 = 0$
	Solve for x.	$x = {}^-2\frac{1}{2}$ or $x = 3$
f	Write the equation.	$(x - 1)^2 + 5(x - 1) - 6 = 0$
	Substitute $y = (x - 1)$.	$y^2 + 5y - 6 = 0$
	Factorise.	$(y + 6)(y - 1) = 0$
	Substitute $(x - 1)$ back for y.	$[(x - 1) + 6][(x - 1) - 1] = 0$
	Remove the inner brackets.	$(x - 1 + 6)(x - 1 - 1) = 0$
	Simplify.	$(x + 5)(x - 2) = 0$
	Use the zero product rule.	$x + 5 = 0$ or $x - 2 = 0$
	Solve for x.	$x = {}^-5$ or $x = 2$

Not all quadratics will factorise or the factorisation may not be obvious. In this case you could always use completion of the square, but you may find that method tedious.

The following rule is actually worked out by completing the square of the general equation $ax^2 + bx + c = 0$.

Important!

The quadratic formula

The solutions to the quadratic equation $ax^2 + bx + c = 0$ where a, b and c are constants and $a \neq 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 24

Technology worksheet

Excel worksheet: The quadratic formula

MAT10NACT00018

Video tutorial

The quadratic formula

MAT10NAVT10030

Puzzle sheet

Quadratic formula

MAT10NAPS00038

Solve each of the following equations using the quadratic formula.

a $x^2 - 3x + 2 = 0$ **b** $6x^2 + x - 2 = 0$ **c** $x^2 = 1 - 6x$

Solution

a Write the problem. $\quad x^2 - 3x + 2 = 0$

Compare with $ax^2 + bx + c = 0$. $\quad a = 1, b = {}^-3, c = 2$

Write the formula. $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Substitute values. $\quad = \dfrac{-({}^-3) \pm \sqrt{({}^-3)^2 - 4 \times 1 \times 2}}{2 \times 1}$

Simplify the terms. $\quad = \dfrac{3 \pm \sqrt{9 - 8}}{2}$

Separate the $\pm$ into $+$ and $-$. $\quad = \dfrac{3 - 1}{2}$ or $\dfrac{3 + 1}{2}$

Write the final answers. $\quad x = 1$ or $x = 2$

b Write the equation. $\quad 6x^2 + x - 2 = 0$

Compare with $ax^2 + bx + c = 0$. $\quad a = 6, b = 1, c = {}^-2$

Write the formula. $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Substitute values. $\quad = \dfrac{-1 \pm \sqrt{1^2 - 4 \times 6 \times {}^-2}}{2 \times 6}$

Simplify the terms. $\quad = \dfrac{{}^-1 \pm \sqrt{1 + 48}}{12}$

Separate the $\pm$ into $+$ and $-$. $\quad = \dfrac{{}^-1 - 7}{12}$ or $\dfrac{{}^-1 + 7}{12}$

Write the final answers. $\quad x = {}^-\frac{2}{3}$ or $x = \frac{1}{2}$

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c Write the problem. $x^2 = 1 - 6x$

Put all the terms on the LHS. $x^2 + 6x - 1 = 0$

Compare with $ax^2 + bx + c = 0$. $a = 1, b = 6, c = {}^-1$

Write the formula. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Substitute values. $= \dfrac{{}^-6 \pm \sqrt{6^2 - 4 \times 1 \times {}^-1}}{2 \times 1}$

Simplify the terms. $= \dfrac{{}^-6 \pm \sqrt{36 + 4}}{2}$

Simplify the square root term. $= \dfrac{{}^-6 \pm \sqrt{40}}{2}$

Simplify the surd. $= \dfrac{{}^-6 \pm 2\sqrt{10}}{2}$

Factorise the numerator. $= \dfrac{{}^-2(3 \mp \sqrt{10})}{2}$

Cancel and separate the $\pm$ into $+$ and $-$. $= {}^-3 - \sqrt{10}$ or ${}^-3 + \sqrt{10}$

Find approximations. $\approx {}^-6.162$ or 0.162

Write the final answers. $x = -\sqrt{10} - 3$ or $x = \sqrt{10} - 3$

Using quadratic equations to model and solve problems requires the same approach as any other kind of problem solving. For example, you should always draw a diagram to show physical situations. You should always choose sensible variable names. The only thing that makes it different is the fact that the mathematical model is a quadratic equation. Sometimes a quadratic model may produce a false solution that has to be discarded when you check the solutions with the original information.

Example 25

Worksheet
Problems involving quadratic equations
MAT10NAWK00029

A ball is propelled upwards so that its height (h) in metres after t seconds is given by the formula $h = 30t - 5t^2$. Find the time taken for the ball to reach a height of 25 m.

Solution

Write the equation. $h = 30t - 5t^2$

State the required value of h. $h = 25$

Substitute in the equation. $25 = 30t - 5t^2$

Rearrange the equation as a standard quadratic. $5t^2 - 30t + 25 = 0$

Divide by 5 to simplify the equation. $t^2 - 6t + 5 = 0$

Factorise the quadratic. $(t - 1)(t - 5) = 0$

Use the zero product rule. $t - 1 = 0$ or $t - 5 = 0$

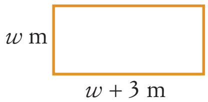
Solve for t. $t = 1$ or $t = 5$

Write the relevant original information.	$h = 30t - 5t^2$
Check the first answer.	**At $t = 1$ s,**
Calculate the height at $t = 1$ s.	$h = 30 \times 1 - 5 \times 1^2$ $= 30 - 5$ m $= 25$ m ✓OK
Check the second answer.	**At $t = 5$ s,**
Calculate the height at $t = 5$ s.	$h = 30 \times 5 - 5 \times 5^2$ $= 150 - 125$ m $= 25$ m ✓OK
State your answer.	**The ball takes 1 second to reach a height of 25 m on the way up and it is again at 25 m another 4 seconds later on the way down.**

Example 26

A rectangular room has a length that is 3 m longer than its width. If the area of the room is 54 m^2, what is the length?

Solution

Draw a diagram. Label the width as w. The length is then $w + 3$. Label the length as $w + 3$.	
Write an expression for the area of the room.	$A = lw = w(w + 3)$
Substitute the value of A.	$w(w + 3) = 54$
Multiply out the brackets.	$w^2 + 3w = 54$
Rearrange the equation as a standard quadratic.	$w^2 + 3w - 54 = 0$
Factorise the quadratic.	$(w + 9)(w - 6) = 0$
Use the zero product rule.	$w + 9 = 0$ **or** $w - 6 = 0$
Solve for w.	$w = {}^{-}9$ **or** $w = 6$
Write the relevant original information.	**w is a width.**
Check the first answer.	**A negative width makes no sense.**
Check the second answer.	**If the width = 6 m, the length is 9 m, so the area is 54 m^2** ✓OK
Write the answer to the problem.	**The length of the room is 9 m.**

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Exercise 11.3 Quadratic equations

Understanding

Extra questions
Exercise 11.3
MAT10NAEQ00033

See Example 18

1 Solve each of the following equations.

a $x^2 = 4$ b $a^2 = 49$ c $b^2 = 121$
d $25 = h^2$ e $x^2 = 81$ f $2a^2 = 8$
g $3x^2 = 27$ h $8b^2 = 200$ i $12x^2 = 300$
j $k^2 = \frac{9}{49}$ k $16n^2 = 81$ l $32m^2 = 18$

See Example 19

2 Solve each of the following equations.

a $(z + 2)^2 = 9$ b $(h - 10)^2 = 25$ c $(e - 5)^2 = 81$ d $(k - 7)^2 = 100$
e $(c - 8)^2 = 100$ f $(q + 3)^2 = 16$ g $(q + 7)^2 = 9$ h $(b - 9)^2 = 144$
i $(y - 11)^2 = 144$ j $(m - 9)^2 = 81$

See Example 22

3 Solve each of the following equations.

a $x(x + 1) = 0$ b $(x + 2)x = 0$ c $(x - 3)(x + 1) = 0$
d $(f - 4)(f + 9) = 0$ e $(x - 10)(x + 3) = 0$ f $(n - 4)(n + 3) = 0$
g $(b + 8)b = 0$ h $(x - 3)^2 = 0$ i $(2m + 3)(m - 4) = 0$
j $(2k + 3)(k - 3) = 0$ k $(4w - 5)(5w + 7) = 0$ l $(10x + 1)(x - 2) = 0$

Fluency

See Examples 20, 21

4 Solve each of the following equations by completing the square.

a $v^2 - 2v - 3 = 0$ b $a^2 - 17a + 72 = 0$ c $r^2 - r - 30 = 0$
d $n^2 + 12n + 35 = 0$ e $w^2 + 2w - 15 = 0$ f $y^2 + 7y - 30 = 0$
g $k^2 - 23k + 130 = 0$ h $g^2 - 9g + 20 = 0$ i $c^2 - 4c - 165 = 0$
j $30n^2 + 61n + 30 = 0$ k $15b^2 - 13b - 6 = 0$ l $32m^2 - 108m + 81 = 0$
m $27x^2 - 60x - 32 = 0$

See Example 23

5 Solve each of the following equations by factorisation.

a $x^2 + 9x + 20 = 0$ b $a^2 + 8a + 12 = 0$ c $x^2 - 8x + 7 = 0$
d $m^2 - 3m - 28 = 0$ e $n^2 - 3n - 40 = 0$ f $a^2 + 2a - 63 = 0$
g $m^2 + 5m - 24 = 0$ h $d^2 - 18d + 81 = 0$ i $a^2 + 11a + 28 = 0$
j $x^2 + x = 0$ k $x^2 + 3x = 0$ l $a^2 - 2a = 0$
m $3d^2 + 6d = 0$ n $y^2 = 4y$ o $x^2 = 6x$
p $2x^2 + 10x + 12 = 0$ q $a^2 + a = 20$ r $x^2 - 4x = 45$
s $b^2 - 8b = {}^-7$ t $h^2 + 2h = 35$ u $5a + 6 = a^2$
v $u^2 + 6 = 5u$ w $2b = 8b^2$ x $d - 3d^2 = 0$
y $2n - 5n^2 = 0$

See Example 24

6 Solve each of the following equations using the quadratic formula.

a $2x^2 - 3x + 1 = 0$ b $3x^2 + 2x - 1 = 0$ c $4x^2 + 7x - 2 = 0$
d $5x^2 - 17x + 6 = 0$ e $3x^2 - 2x - 8 = 0$ f $2x^2 - 13x + 15 = 0$
g $2x^2 + 13x + 20 = 0$ h $2x^2 - 5x - 12 = 0$ i $3x^2 - 8x - 35 = 0$
j $5x^2 - 3x - 2 = 0$ k $4x^2 + 11x + 7 = 0$ l $2x^2 + 3x - 20 = 0$

7 Solve each of the following equations by factorisation.

a $2x^2 + x - 1 = 0$ **b** $3x^2 + 5x + 2 = 0$ **c** $3x^2 - 14x + 15 = 0$
d $3x^2 - 11x + 6 = 0$ **e** $2c^2 - 5c - 33 = 0$ **f** $3a^2 + 2a - 21 = 0$
g $7a^2 + 25a + 12 = 0$ **h** $9h^2 + 36h + 11 = 0$ **i** $5x^2 - 7x - 6 = 0$
j $6a^2 - 7a + 1 = 0$ **k** $8m^2 - 26m + 15 = 0$ **l** $6m^2 + 7m - 3 = 0$
m $12h^2 - 8h - 15 = 0$ **n** $14m^2 - 55m + 21 = 0$ **o** $9x^2 - 12x + 4 = 0$
p $6m^2 - 13m + 5 = 0$ **q** $4c^2 - 19c - 30 = 0$ **r** $4b^2 + 21b - 18 = 0$
s $4a^2 - 4a = 3$ **t** $7b^2 + 48b = 7$ **u** $2m^2 - 10 = m$
v $2g^2 - 21 = 11g$ **w** $2c^2 - 11c = {}^-12$

8 Solve each of the following equations using the quadratic formula and leave your answers in surd form.

a $x^2 + 4x + 1 = 0$ **b** $x^2 - 7x + 2 = 0$ **c** $x^2 + 6x + 1 = 0$
d $x^2 - 6x + 3 = 0$ **e** $2x^2 - 4x - 5 = 0$ **f** $2x^2 + 6x + 1 = 0$
g $3x^2 + 6x + 2 = 0$ **h** $4x^2 - x - 1 = 0$ **i** $5x^2 - 10x + 3 = 0$
j $4x^2 - 3x - 2 = 0$ **k** $2x^2 + 4x - 1 = 0$ **l** $5x^2 + 2x - 1 = 0$

9 Solve each of the following equations using the quadratic formula and write your answers correct to 2 decimal places.

a $x^2 + 3x + 1 = 0$ **b** $a^2 - 7a + 1 = 0$ **c** $b^2 + 5b + 3 = 0$
d $3w^2 - 9w + 2 = 0$ **e** $2n^2 + 4n - 1 = 0$ **f** $2x^2 - 7x + 4 = 0$
g $x + 4 = x^2$ **h** $2x^2 + x = 2$ **i** $6 = 3n^2 + 2n$
j $3w^2 + 9w = {}^-5$ **k** $5c + 2 = 3c^2$

Problem solving

10 Solve each of the following equations.

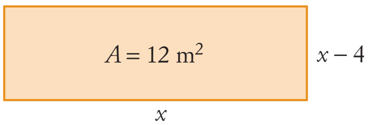
a $(x - 1)^2 + 3(x - 1) + 2 = 0$ **b** $(x + 2)^2 + 4(x + 2) + 3 = 0$
c $(y + 1)^2 - 5(y + 1) - 6 = 0$ **d** $(x - 5)^2 - 7(x - 5) - 60 = 0$
e $(b + 3)^2 + 4(b + 3) - 45 = 0$ **f** $3(x - 2)^2 + 7(x - 2) + 2 = 0$
g $4(w + 4)^2 - 19(w + 4) - 5 = 0$ **h** $2(x - 7)^2 + 13(x - 7) - 24 = 0$
i $4(m - 6)^2 - 12(m - 6) - 7 = 0$ **j** $8(3 + x)^2 - 18(3 + x) + 9 = 0$

See Example 25

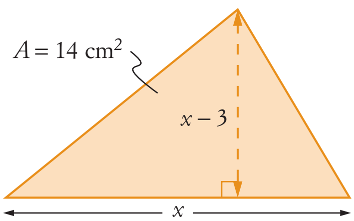
11 **a** The sum of a positive whole number and its square is 56. What is the number?
b The sum of a positive integer and its square is 110. What is the number?
c When a number is subtracted from its square, the result is 90. What is the number?

See Example 26

12 Find the dimensions of this rectangle if its area is 12 m^2.



13 The area of this triangle is 14 cm^2.
What is the length of its base?



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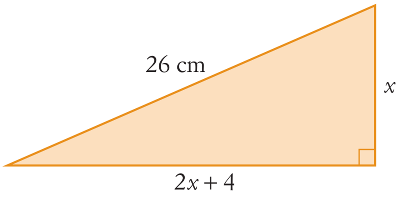
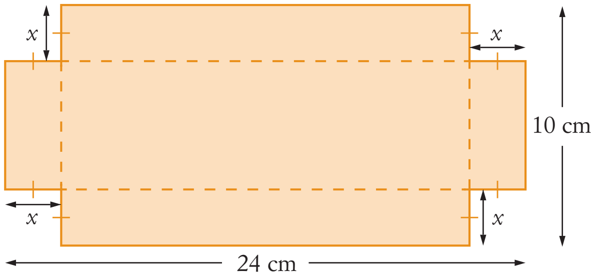
14 Use Pythagoras' theorem to find the dimensions of this right-angled triangle.

Worked solutions
Exercise 11.3
MAT10NAWS00033



15 This shape is the net for a box that is formed when the net is folded along the dotted lines. What are the dimensions of the box if the area of its base is 120 cm^2?



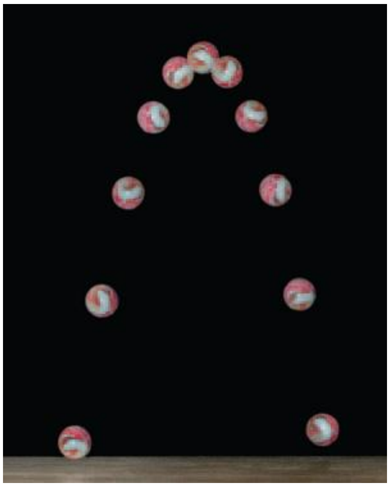
16 The sum of the first n integers is given by the formula $S = \frac{1}{2}n(n + 1)$.

Worked solutions
Exercise 11.3
MAT10NAWS00033

Find how many integers must be added to obtain a sum of:

a 45   b 78   c 120

17 A ball thrown upwards reached a height, h metres, after t seconds according to the formula $h = 38t - 5t^2$. At what time did the ball first reach a height of 48 m?



18 Peta decides to make a box to hold some newspaper clippings for an assignment. She starts with a rectangular piece of cardboard that is 3 times as long as it is wide. Peta then cuts out squares of side 10 cm from each corner and folds up the ends to form a box. She calculates that the box has a volume of 1200 cm^3. What are the dimensions of the box?

Reasoning

19 a Complete the square for the quadratic equation $x^2 + 6x + 10 = 0$.

b Hence show that there are no real answers on the number line.

c Use part **a** to find the turning point of $y = x^2 + 6x + 10$.

20 Complete the square for the quadratic equation $x^2 + bx + c = 0$ and show that the result is consistent with the quadratic formula.

Worked solutions
Exercise 11.3
MAT10NAWS00033

21 Complete the square for the quadratic equation $ax^2 + bx + c = 0$.

22 Show that any quadratic equation $ax^2 + bx + c = 0$, where $b^2 < 4ac$, can have no real answers on the number line.

Chapter 11 summary

Quiz
Equations
MAT10NAQZ00011

- An **equation** is a mathematical sentence with an equals sign and expressions on each side.
- A **linear** equation produces a straight line when the expression is calculated and plotted for different values of the variable.
- A **solution** of an algebraic equation is a value of the variable that makes the equation true. A solution **satisfies** the equation.
- **Inverse operations** are opposite operations such as addition/subtraction and multiplication/division.
- To solve a linear equation isolate the variable by doing inverse operations in the reverse order to the normal order of operations. Do the same thing to each side of the equation.
- A **linear inequality** is like a linear equation, but instead of having an equals sign, it has one of the signs $>$, $<$, $\geq$ or $\leq$. It is also called an **inequation**.
- For any inequality $x > y$:
  - $x + a > y + a$ for any number a.
  - $x - a > y - a$ for any number a.
  - $ax > ay$ for any *positive* number a
  - $ax < ay$ for any *negative* number a.
  - $\frac{x}{a} > \frac{y}{a}$ for any *positive* number a.
  - $\frac{x}{a} < \frac{y}{a}$ for any *negative* number a.

  The same laws hold for $\geq$, $<$ and $\leq$.
- Equations considered *at the same time* make an equation system called **simultaneous equations**.
- A **simultaneous solution** is a solution that satisfies all the equations at the same time.
- The simultaneous solution of two equations in two unknowns is the intersection of their lines on a graph.
- To solve simultaneous equations by the **substitution method** you use one equation to express one variable in terms of the other. Then you substitute this into the other equation to make a linear equation in one variable. Solve this and then find the other variable.
- To solve simultaneous equations by the **elimination method** you make the coefficients of one variable opposites by multiplying the equations by suitable constants. Then add the equations to make an equation with only one variable. Solve this and then find the other variable.
- A **quadratic equation** is one that can be expressed in the form $ax^2 + bx + c = 0$, where x is the variable and a, b and c are constants.
- You can use **completion of the square** to solve a quadratic equation by:
  1 dividing (if necessary) to make the coefficient of x^2 equal to 1
  2 moving the constant to the RHS
  3 adding half the square of the coefficient of x to both sides
  4 factorising the LHS as a perfect square
- You can solve quadratic equations by factorisation using the **zero product rule:** if $ab = 0$ then $a = 0$ or $b = 0$.
- The solutions to a quadratic equation $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, provided $a \neq 0$.

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Understanding

1 Solve each of the following equations.
a $m + 12 = 7$ b $4g = {}^-24$ c $7 - p = 18$ d $\frac{36}{d} = {}^-9$

2 Solve the equations $y = 2x - 3$ and $y = 9 - x$ by graphing.

3 For each of the following, solve the inequality and show your solution on a number line. See Example 8
a ${}^-7y \geq 9$ b $-q + 7 \leq 0$ c $\frac{q}{4} < 9$ d $n - 3 > {}^-7$

4 Solve each of the following equations. See Example 18
a $y^2 = 9$ b $5t^2 = 80$ c $25g^2 = 16$

5 Solve the equation $(v + 7)^2 = 144$ See Example 19

6 Solve each of the following equations. See Example 22
a $x(x + 5) = 0$ b $(a - 4)(a + 7) = 0$ c $(3m + 2)(2m - 5) = 0$

Fluency

7 Solve each of the following equations. See Example 1
a $\frac{z}{7} + 5 = 9$ b $8t - 6 = {}^-54$

8 Find the solution of the equations $y + 2x = 10$ and $2y - 3x + 8 = 0$ by graphing. See Example 13

9 Find the roots of each of the following equations. See Examples 2, 3
a $5b - 3 = 2b - 12$ b $\frac{2c - 3}{5} = 3$ c $24 = 18 + 6(5 - b)$

10 Solve each of the following equations. See Example 4
a $\frac{k}{6} + \frac{k}{4} = \frac{1}{2}$ b $\frac{5c}{2} + \frac{c}{9} = \frac{47}{3}$ c $\frac{2g + 3}{4} - \frac{g - 5}{3} = 2\frac{5}{6}$

11 For each of the following, solve the inequality and show your solution on a number line. See Example 9
a ${}^-5x + 2 < {}^-5$ b $6h + 4 \leq 3$

12 For each of the following, solve the inequality and show your solution on a number line. See Example 10
a $7x - 7 > {}^-13 - 2x$ b $3(5z - 2) - 6 \leq 4(4z - 3) + 4$
c $\frac{5x - 3}{5} - \frac{6x - 7}{4} < \frac{{}^-1}{10}$

13 Find the solution of the simultaneous equations $a + 7b = 19$ and $2a - 3b = {}^-13$ by substitution. See Example 14

14 Find the solution of the simultaneous equations $7u + 4w = 0$ and $9u + 5w = {}^-1$ by elimination. See Example 15

15 Solve each of the following equations by completing the square. See Examples 20, 21
a $y^2 + 10y + 21 = 0$ b $a^2 + 8a - 20 = 0$ c $15b^2 - 13b - 6 = 0$

16 Solve each of the following equations by factorisation. See Example 23
a $x^2 - 5x = 0$ b $w^2 - 13w + 42 = 0$ c $y^2 - 7y - 18 = 0$
d $c^2 + 6c = 16$ e $28h^2 - 15h = 25$ f $28g^2 = 9g - 4$

17 Solve the equation $4m^2 + 7m - 2 = 0$ using the quadratic formula. See Example 24

18 Solve $2x^2 + 7x + 1 = 0$ using the quadratic formula and express your answers both in surd and decimal form.

19 Solve $\frac{3}{2a} + \frac{1}{a} = 5$

See Example 16

20 Solve the following pairs of simultaneous equations.

a $\frac{a}{6} - \frac{b}{2} = 1$
$\frac{6a}{5} + \frac{3b}{10} = 3$

b $\frac{x + 2y}{3} + \frac{x - 3y}{2} = 5$
$\frac{2x}{5} - \frac{x - 3y}{4} = 1$

Problem solving

See Example 6

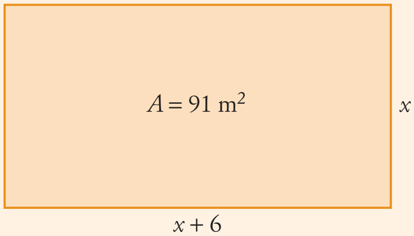
21 A rectangular garden bed is 0.5 m longer than it is wide. Find its dimensions if its perimeter is 9 m.

See Example 7

22 Liam is twice as old as his son. Together their ages total 78. Write an equation and find their ages.

See Example 26

23 A rectangle has a length that is 6 m longer than its width. Find its dimensions if its area is 91 m^2.



24 A woman is three times as old as her daughter, but in 10 years she will only be twice as old as her daughter. Write simultaneous equations to find their present ages.

25 Solve the following simultaneous equations:
$\frac{1}{d} + \frac{3}{g} = 10$ and $\frac{2}{d} + \frac{4}{g} = 16$

26 Solve $3(x - 5)^2 - 4(x - 5) - 7 = 0$.

Reasoning

See Example 25

27 A lead sinker weight thrown into water sinks so that its depth is given by $d = 3t^2 - 6t$. Find the time that it takes to sink to a depth of 24 metres and explain your reasoning.

28 Complete the square for the equation $x^2 + 4x + 9 = 0$ and hence show that there are no real answers on the number line.

See Example 12

29 If $x > y$ and $a < 0$, prove that $\frac{x}{a} < \frac{y}{a}$.

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Chapter 1

Exercise 1.1

1 **a** 64 **b** 16 807 **c** 1024
d 125 **e** 1296 **f** 1024
g 128 **h** 225 **i** 14 641
j 512 **k** $\frac{1}{1024}$ **l** 1
m $\frac{1}{49}$ **n** $\frac{1}{13}$ **o** $\frac{1}{343}$
p $\frac{1}{144}$ **q** 1 **r** $\frac{1}{625}$

2 **a** 65 536 **b** 1728 **c** 256
d 4913 **e** 331 776 **f** 2 476 099
g 529 **h** 148 877 **i** 83 521

3 **a** 8 **b** 5 **c** 10
d 20 **e** 13 **f** 12
g 15 **h** 100 **i** 4000

4 **a** 7.071 **b** 15.811 **c** 12.166
d 9.220 **e** 2.646 **f** 6.928
g 20.298 **h** 57.446 **i** 758.288

5 **a** 4^{10} **b** 7^7 **c** 11^6
d 5^9 **e** 10^{16} **f** 9^9
g 14^{12} **h** 3^{13} **i** 15^9

6 **a** 8^2 **b** 5^4 **c** 7^4
d 11 **e** 6^2 **f** 1
g 13^5 **h** 14^{10} **i** 19

7 **a** $4^3 \times 5^3$ **b** $6^7 \times 4^7$ **c** $7^4 \times 11^4$
d $6^8 \times 7^8$ **e** 1 **f** $2^{12} \times 7^{12}$

8 **a** 5^{12} **b** 7^{10} **c** 3^{28}
d 11^{48} **e** 1 **f** 5^{25}

9 **a** 5 **b** 13 **c** 2
d 6 **e** 3 **f** 4
g 10 **h** 17 **i** 30

10 **a** 0.0008 **b** 7.6158 **c** 2.9813
d 70.7281 **e** 5.4525 **f** 6.3645
g 0.1493 **h** 0.0020 **i** 1.0098

11 **a** 18.119 **b** 21.031 **c** 9
d 22.499 **e** 2.398 **f** 204.705

12 **a** 4^{10} **b** 7^6 **c** 15^6
d 6^{19} **e** 10^{26} **f** 9^{16}
g 14^{16} **h** 3^{17} **i** 25^{20}

13 **a** 8^2 **b** 3^4 **c** 7^{15}
d 11^{11} **e** 1 **f** 15
g 14^6 **h** 17^2 **i** 12^4

14 **a** $7^4 \times 5^4$ **b** $6^6 \times 3^6$ **c** $9^4 \times 11^4$
d $6^7 \times 8^7$ **e** $12^8 \times 13^8$ **f** $2^9 \times 10^9$

15 **a** 4^{21} **b** 8^{15} **c** 5^{63}
d 12^{55} **e** 9^{240} **f** 3^{99}

16 **a** 7^{35} **b** $\frac{1}{6^4}$ **c** 1
d $\frac{1}{4^{45}}$ **e** 8^7 **f** $\frac{5^{18}}{6^9}$
g $\frac{7^5}{8^{20}}$ **h** $\frac{11^6}{7^3}$ **i** 8^5

17 **a** 5 **b** 5 **c** 7
d 3 **e** 8 **f** 2
g 6 **h** 3 **i** 5

18 **a** 7.07 **b** 6.18 **c** 3.04
d 3.55 **e** 9.75 **f** 1.12
g 12.13 **h** 7.22 **i** 4
j 13.13 **k** 2.40 **l** 225.70

19 **a** $\frac{4^6}{11^6}$ **b** $\frac{7^8}{3^4}$ **c** $\frac{2^{36}}{5^6}$
d $\frac{2^3}{5^{10}}$ **e** $3^3 \times 5^{11}$ **f** $\frac{7^8}{5^{22}}$
g $\frac{1}{2^8 \times 7}$ **h** $7^8 \times 2^6$ **i** 1

20 **a** $\frac{2^4}{3}$ **b** $\frac{3^3}{7^4}$ **c** $\frac{3}{2^2}$
d $\frac{5^{11}}{2^4}$ **e** $3^6 \times 5^2$ **f** $\frac{5^3}{2^8}$

21 **a** $x^{12}y^9$ **b** $\frac{x^4}{y^{20}}$ **c** $x^{36}y^3$
d 1 **e** $\frac{1}{a^7}$ **f** c^8
g $\frac{n^3}{3m}$ **h** $\frac{q^4}{5^2}$ **i** $\frac{2^4rt^2}{s^2}$

Exercise 1.2

1 **a** 7.3×10^3 **b** 1.32×10^8 **c** 8.5×10^{-3}
d 1.38×10^{-7} **e** 7×10^5 **f** 7×10^{-4}
g 2.9×10^2 **h** 7.19×10^{-1} **i** 3.708×10^7
j 3.019×10^{-2} **k** 9.744×10^6 **l** 8×10^{-9}

2 **a** 630 000 **b** 0.0071 **c** 9 740 000
d 0.419 **e** 0.000 000 008 **f** 30 000
g 52 **h** 0.098 **i** 1.02
j 0.000 0955 **k** 404 **l** 9100

Answers

3 a 1.232×10^2 b 1.120×10^{11}
c 4.234×10^{19} d 2.006×10^{-14}
e 2.247×10^{13} f 2.495×10^{-18}

4 a 3 b 2 c 1
d 1 e 1 f 3
g 4 h 5 i 2

5 a 975 b 703 c 6.047
d 3 058 000 e 347.6 f 0.000 000 755
g 200.5 h 0.050 57

6 a 3.274×10^{16} b 1.679×10^{-5}
c 3.504×10^{10} d 6.154×10^{36}
e 3.619×10^{-8} f 4.022×10^{23}

7 A proton is approximately 1837 times more massive or heavier than an electron.

8 a 4.19×10^{-54} m^3 b 2.68×10^{-45} m^3
c A proton's volume is at least 640 million times that of an electron.

9 a 1.728×10^{-27} kg b 3.838×10^{-27} kg
c 1.247×10^{-23} kg

10 a 1.0868×10^{21} m^3 b 1.5306×10^{24} m^3
c 5495 kg/m^3 d 1240 kg/m^3
e Jupiter is about 318 times heavier than the Earth.

Exercise 1.3

1 a $\frac{235}{999}$ b $\frac{46}{111}$ c $\frac{314}{999}$
d $\frac{224}{333}$ e $\frac{23}{37}$ f $\frac{68}{303}$
g $\frac{1}{9}$ h $\frac{13}{99}$ i $\frac{2}{45}$

2 a $0.8\overline{3}$ b $0.41\overline{66}$ c $0.\overline{31}$
d 1.1875 e $0.\overline{486}$ f $2.\overline{0726}$
g $0.\overline{428571}$ h $0.12\overline{048}$ i $0.\overline{902}$

3 a $\frac{356}{4995}$ b $\frac{41}{3330}$ c $\frac{43}{1998}$
d $\frac{559}{8325}$ e $\frac{2021}{33\,300}$ f $\frac{59}{27\,775}$

4 a Rational b Rational c Irrational
d Rational e Irrational f Rational
g Irrational h Rational i Rational
j Rational k Irrational l Irrational

5–19 Check your answer with your teacher.

Chapter review

1 a 256 b 125 c 243

2 a 9 b 25 c 100

3 a $3^4 \times 5^4$ b $4^5 \times 7^5$ c $5^3 \times 11^3$

4 a 7 b 15 c 3

5 a 7^2 b 23^{12} c 11^{60}

6 a 9.3×10^3 b 1.85×10^8 c 9.5×10^{-4}

7 a 703 000 b 0.000811 c 96 030 000

8 a $\frac{173}{999}$ b $\frac{19}{37}$ c $\frac{8}{37}$

9 a 5^{37} b $\frac{1}{6^8}$ c 1

10 a 2.392×10^3 b 8.405×10^{10}

11 a 2 b 4 c 1

12 a 6.32 b 9.03 c 2.50

13 a 875 b 9906

14 a 2.091×10^{15} b 1.501×10^{-3}

15 a $\frac{7107}{99\,900}$ b $\frac{511}{16\,650}$ c $\frac{1931}{249\,975}$

16 a Rational b Rational c Irrational
d Rational e Irrational f Rational

17 a $\frac{7^6}{5^9}$ b 5^{11} c $7^{12} \times 11^2 \times 2^4$

18 a $\frac{1}{2^2 \times 3^2}$ b 3×5^9 c $2^8 \times 5^4$

19 a $\frac{y^8}{x^4}$ b $\frac{1}{a^5}$ c $\frac{1}{6q}$

20 1.9817×10^{20} N

21 Check your answer with your teacher.

22 Check your answer with your teacher.

Chapter 2

Exercise 2.1

1 a Yellow, 6 b Australia, 5 c Desert, 7
d Jack, 5 e Olivia, 5

2 a Median = 4, mode = 4
b Median = C, mode = C
c Median = A, mode = A
d Median = ***, mode = ***
e Median = M, mode = PG

3 a Mean = 6.2, median = 7, modes = 3, 10, range = 10
b Mean ≈ 10.8, median = 9.5, mode = 8, range = 12
c Mean ≈ 10.3, median = 10, mode = 10, range = 17
d Mean ≈ 37.0, median = 36, mode = 35, range = 19
e Mean ≈ 14.6, median = 14, mode = 13, range = 7

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4 **a** Median = 26, $Q1$ = 22.5, $Q3$ = 27.5, IQR = 5
b Median = 6, $Q1$ = 4, $Q3$ = 7, IQR = 3
c Median = 34.5, $Q1$ = 32.5, $Q3$ = 35, IQR = 2.5
d Median = 15, $Q1$ = 14, $Q3$ = 16, IQR = 2
e Median = 44.5, $Q1$ = 41, $Q3$ = 47, IQR = 6

5 **a** Median = 7, $Q1$ = 5, $Q3$ = 8, IQR = 3
b Median = 22, $Q1$ = 20, $Q3$ = 23, IQR = 3
c Median = 6, $Q1$ = 5, $Q3$ = 7, IQR = 2
d Median = 12, $Q1$ = 8, $Q3$ = 15, IQR = 7
e Median = 46, $Q1$ = 44, $Q3$ = 51.5, IQR = 7.5

6 **a** 26, 28.5, 30, 32, 41 **b** 2, 6, 12, 15, 20
c 7, 14, 15.5, 21, 35 **d** 1, 9, 13, 14, 16
e 37, 53, 58, 60.5, 73

7 **a** 30, 33, 34, 35.5, 39 **b** 3, 6, 8, 10, 16
c 8, 10.5, 13.5, 16.5, 29 **d** 101, 103, 104, 105.5, 109
e 23, 27.5, 30, 32, 36

8 **a** Mode **b** Mode **c** Mean
d Mode **e** Median

9 5, 4 and 125 resp.

10 **a** 6.48 kg **b** 32 400 kg
c Only 50 plants were checked and there is no information about the other factors that affect fruit production. However, the pattern is clear so it is probably sufficient.

11 **a** 106 **b** 423
c 407 rallies seems a lot and there is a clear pattern, so it is probably sufficient.

12 **a** Mean **b** Median **c** Mean
d An amount higher than the median, say $Q3$.
e An amount lower than the median, say $Q1$.

13 **a** The amounts increase by 10% to \$1364, \$1039.50, \$913 and \$803.
b The amounts increase by \$100 to \$1340, \$1045, \$930 and \$830.
c About 730, 850, 945, 1500, 3000. The 850, 1500 and 3000 could vary.

14 ★★★★

15 The median will be lower than the mean, and the mode is probably lower than the median.

16 The mean increases to about 171.8 cm and the median probably increases to about 173 cm.

17 **a** Summary: 5, 7, 9.5, 14.5, 20; probably skewed right.
b Summary: 162, 163, 164.5, 171.5, 183; probably skewed right.
c Summary: 0, 8, 10, 13, 19; probably symmetrical.
d Summary: 35, 41, 45, 47.5, 51, probably symmetrical.
e Summary: 13, 14, 15, 18, 20; probably skewed right.

18 Suburban stores could have younger customers who like different music from those that shop in the city.

19 The vote might change in 3 days, only 20 or 30 people are surveyed in each electorate, a small % error could make a big difference in seats.

20 Employers would definitely supply the gross wages, but employees might give their wages after tax or other deductions, or not want to give their actual earnings.

Exercise 2.2

1 0.2 and 24 are non-compliant, so should not be used.

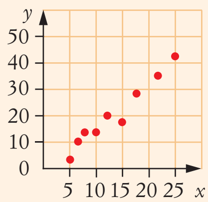
Stopping times (s)

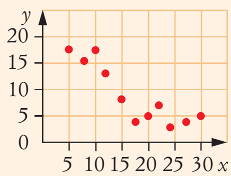
Stem	Leaf
1	8 9 9
2	1 2 3 6 6 6 7 7 8
3	0 1 1 4 4 4 5 6 7 7 8
4	1 2 3 6
5	0

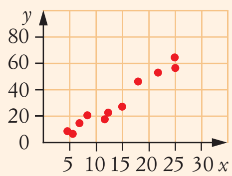
2 320 is non-compliant, so should not be used.

University entrance marks

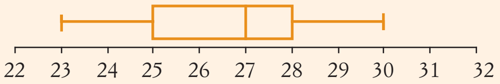
Stem	Leaf
3	5 6 7 8
4	0 0 2 4 6 6 6
5	0 0 1 2 4 6 6 7 8
6	0 0 2 3 3 4 6 6 9 9
7	2 2 5 8
8	4 4 7 8 8
9	1 2 5

3 **a**


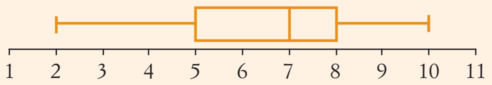
b


c


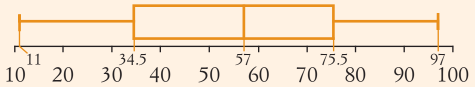
4 a



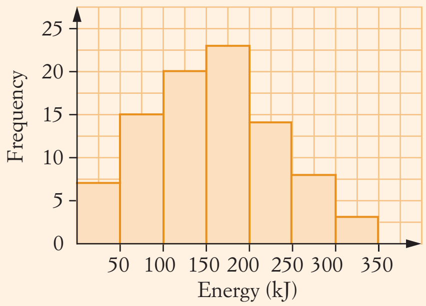
b



c

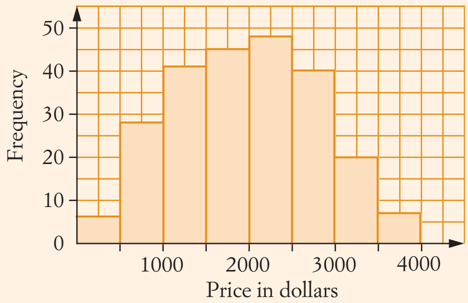


5 a **Food energy consumption**



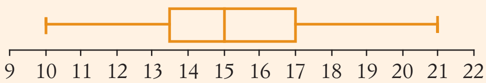
b The distribution is skewed a little to the right.

6 a **Computer prices**

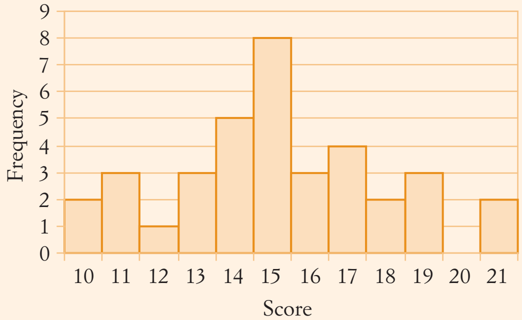


b The shape is fairly symmetrical.

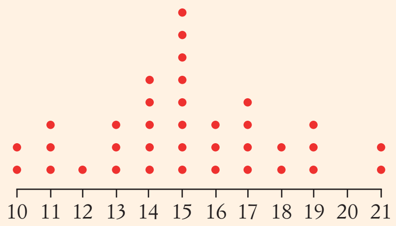
7 a **Box-and-whisker**



Histogram

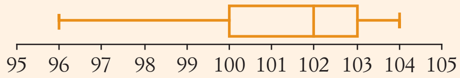


Dot plot

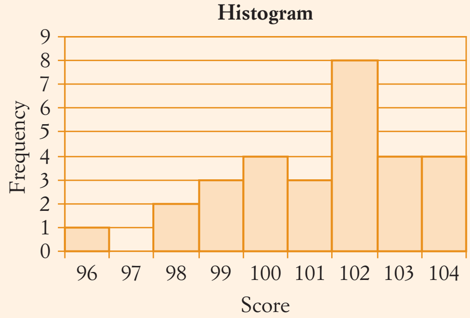


The box-and-whisker plot, histogram and dot plot all show a fairly symmetrical distribution.

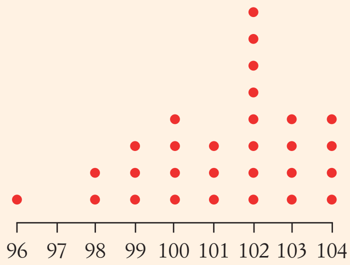
b **Box-and-whisker**



Histogram

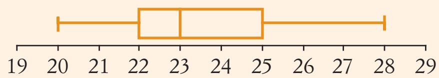


Dot plot

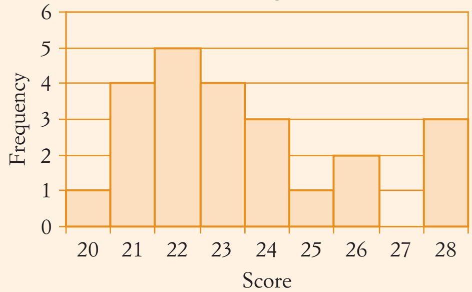


The box-and-whisker plot, histogram and dot plot all show a distribution skewed to the left.

c **Box-and-whisker**



Histogram



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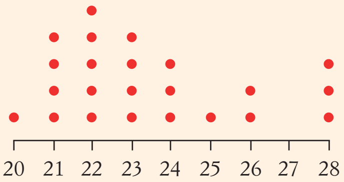
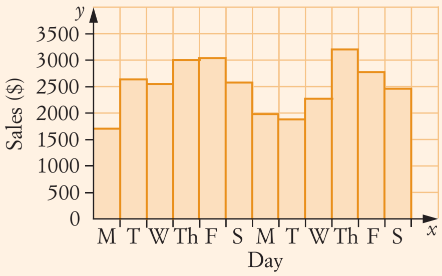
Dot plot



The box-and-whisker plot, histogram and dot plot all show a distribution skewed to the right.

8 a

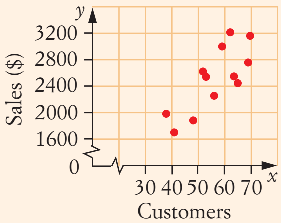
Music store sales



b Their biggest sales occur on Thursdays and Fridays, and their smallest occur on Mondays.

c

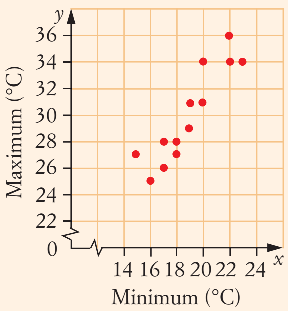
Music store sales



d Greater numbers of customers and higher sales go together.

9 a

Temperatures in Qld town



b Yes, they are higher or lower together.

10 a There does not seem to be any real difference between Victoria and Western Australia.

Time taken by Year 10 students to travel to school (minutes)

Key: 5|1 = 15 minutes = 1|5

Vic		WA
9 5 5 4 1	0	2 3 3 5 5
8 5 5 5 5 0 0 0 0 0 0 0	1	0 0 0 0 0 0 5 5
5 5 2 0 0 0	2	0 0 0 0
7 5	3	0 6
5 0	4	
0	5	0

b Victoria: mean ≈ 18.2, median = 15, range = 49, WA: mean = 15.2, median = 10, range = 48. The means and medians are higher for Victoria, so there is actually a difference, although the ranges are almost the same.

11 a Medium size cars generally cost more to run than small cars.

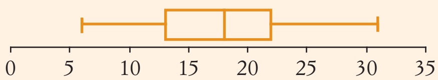
Car running costs ($/week)

Key: 1|5 = $51 = 5|1

Small		Medium
2	3	
9	3	
4 2 2 2 1	4	0
9 9 9 8 7 6	4	5
1 1	5	1 2 2 4
6	5	5 6 7
1	6	0 0 0 2

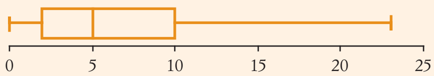
b Small: mean ≈ $46.41, median = $47, range = 29; Medium: mean ≈ $54.15, median = $55, range = 22. There is a real difference in running costs, although the medium cars have a lower range of costs.

12 The times, ranging from 6 to 31 minutes, are fairly symmetrical about the median.



13 The times watching TV are skewed to the right.

Time watching TV



14 It would give a good summary of sales for each month, but the manager would probably want more detail of the kinds of things sold.

15 A scale like this does not provide much discrimination, so a boxplot is unlikely to give much information. It is only 5 points anyway, and these are very likely to be the same as the 5 number summary, so it wouldn't

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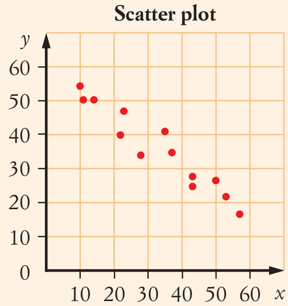
provide any real feedback. Very unhappy customers are mostly not going to 'stay on the line' anyway.

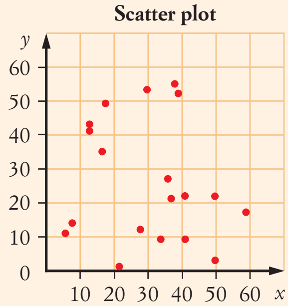
Exercise 2.3

1 a Strong positive b Weak negative
c Strong negative d No relationship
e Weak positive f Weak negative

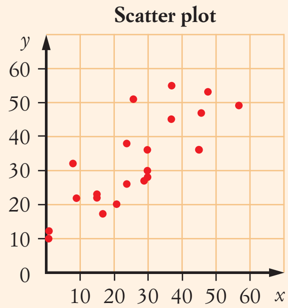
2 a 20 b 16, 34, 35 c None
d 5 e 36, 62

3 a


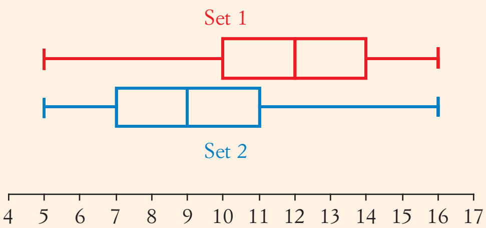
Strong negative

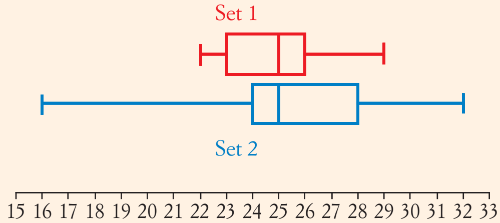
b


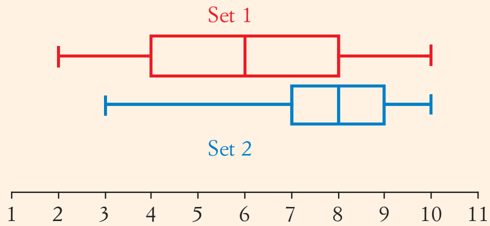
No relationship

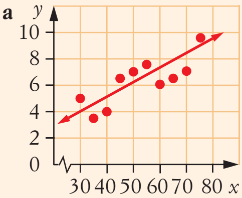
c


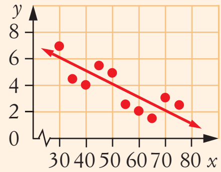
Weak positive

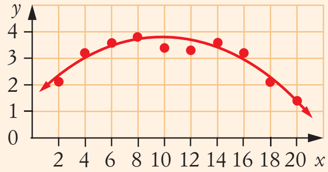
4 a


b


c


5 a


b


c


6 a

		Accidents in last 2 years					
		1	**2**	**3**	**4**	**5**	**Total**
Agegroup	**17–21**	16	24	21	14	12	87
	22–26	16	14	12	9	9	60
	27–31	9	7	7	4	2	29
	32–41	4	2	2	0	0	8
	Over 41	4	2	0	0	0	6
	Total	**49**	**49**	**42**	**27**	**23**	**190**

b Younger drivers
c 17–21, 30%; 22–26, 30%; 27–31, 21%; 32–41, 0%, Over 41, 0%.
d Younger drivers seem more prone to multiple accidents.
e The numbers for 5-year age groups over 26 are a bit low. However, if they are classified as older (over 26) and younger (26 or less) then the sample is sufficient to get good conclusions.

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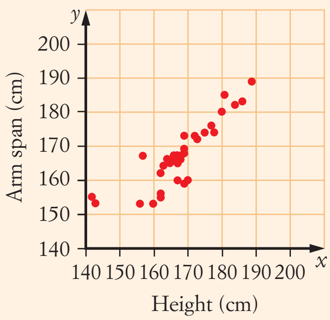
7 a

		Reading age						
		12	**13**	**14**	**15**	**16**	**17**	**Total**
Sex	**Male**	14	17	24	31	24	12	122
	Female	7	10	17	36	29	19	118
	Total	**21**	**27**	**41**	**67**	**53**	**31**	**240**

b More girls are good readers (reading age over 15).
c Girls appear better, 41% to 30%.
d All of the frequencies are reasonable, so conclusions are likely to be reliable.

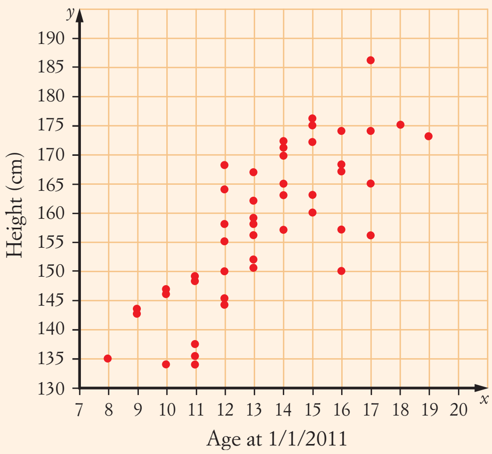
8 a (30, 162) discarded as non-compliant.

Arm span and height



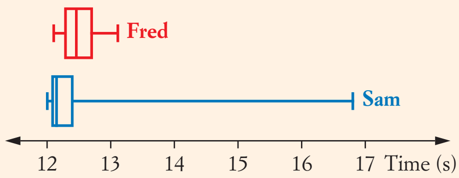
b There is a strong positive relationship; taller students have larger arm spans.

9 a **Age and height**



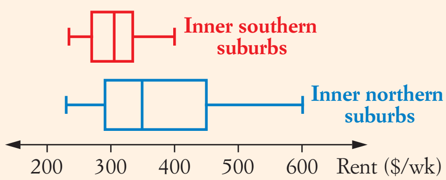
b There is a weak positive relationship. Older students are often taller.
c You would not expect to find any relationship as people have mostly finished growing when they reach university.

10 a **Sprinters' times over 100 m**



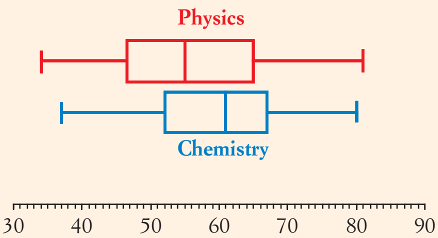
b Sam's mean (12.64) is almost the same as Fred's (12.51), despite one really bad time that makes his range bigger. His median is much lower, at 12.15 to 12.45.
c Sam, because he is faster, except for one time when he may have been sick

11 a **Weekly rents, Brisbane**



b Rental costs in the inner northern suburbs are generally higher, but they are also spread over a much greater range than rental costs in the inner southern suburbs.
c The mean ($370 to $308), the quartiles and the median are all higher for the northern suburbs so it is quite a bit more expensive.

12 a



b Students generally performed better in Chemistry than in Physics.
c The mean for chemistry (59.8) is higher than that for Physics (56.3), so this reinforces the conclusion from the median and quartiles of the boxplot.

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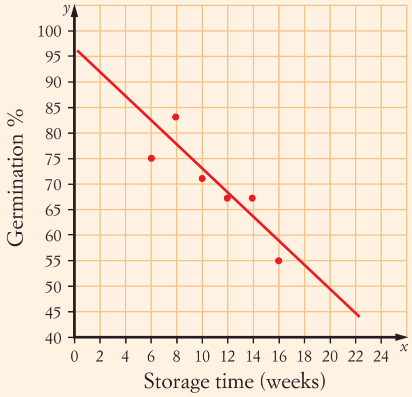
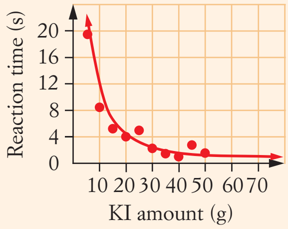
13 a **Parsnip seed germination**



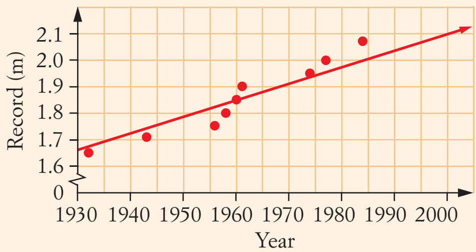
b As the storage time increases, the germination rate decreases.
c Probably about 94%.
d Probably about 49%.

14 a **Potassium iodide reaction**



b Less than 1 s.

15 a **Women's high-jump records**



b About 2.1 m (2000 result was 2.01 m).

16 There is no information about the number of drivers in each group or how far they generally drive.

17 The people who bought the board games are already board game buyers, so the marketing firm will not find out how to appeal to a wider group. It is a good approach if they want to find out about existing buyers.

18 It may be a reasonable approach, but they probably should ask students from lower grades as well.

19 It could be used to set a 'use-by' date, but you would need to decide on an acceptable germination rate.

20 Extrapolation is always difficult, and in this case changes of equipment or methods could make substantial differences.

Chapter review

1 Spaghetti, 7

2 Median = Poor, Mode = Very poor

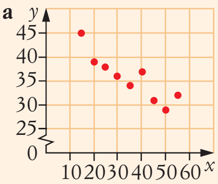
3 a Mean ≈ 10.43, Median = 11, Mode = 13, Range = 9
b Mean ≈ 31.7, Median = 32, Mode = 33, Range = 12

4

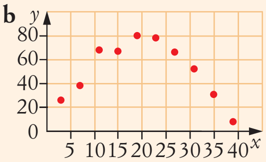
Rising times of Year 10 students on Monday
Key: 6|30 = 6:30 a.m.

5	30
6	0 15 15 30 30 30 30 45
7	0 0 0 0 0 0 15 30 30 30 30 30 30 30 45 45
8	0 0 0 0 30

5 a

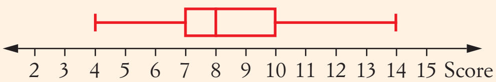


b

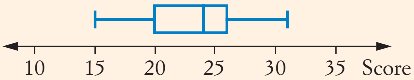


6 a Median = 7, $Q1 = 4.5$, $Q3 = 8$, IQR = 3.5
b Median = 12, $Q1 = 8.5$, $Q3 = 15$, IQR = 6.5

7 a

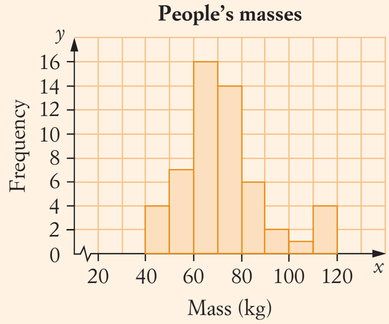


b



8 a Weak positive **b** Strong negative **c** None

9 a **People's masses**



b Most people have masses between 50 and 90 kg.

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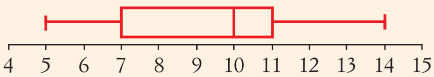
10 a 39, 43.5, 49, 53.5, 56 **b** 12, 15, 16, 17, 20

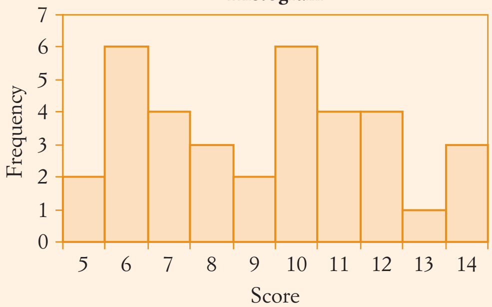
11 a Mean price **b** Median income

12

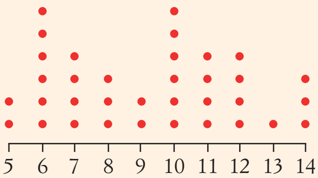
Box-and-whisker



Histogram



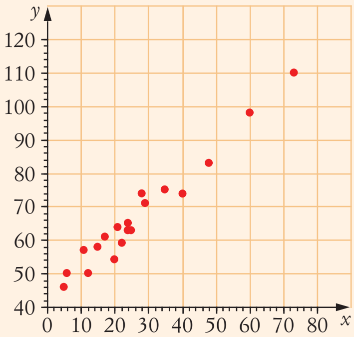
Dot plot



The histogram and dot plot show a bimodal distribution consistent with the box-and-whisker plot between 5 and 7 and between 10 and 11.

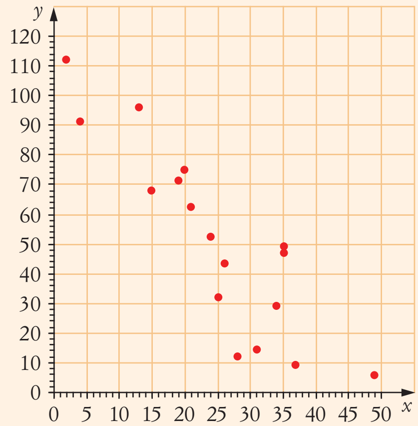
13 a 4 **b** No outliers.

14 a



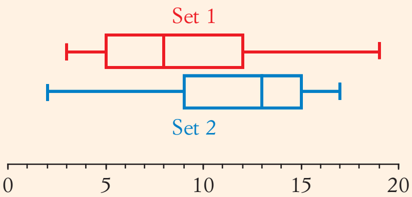
Strong positive

b

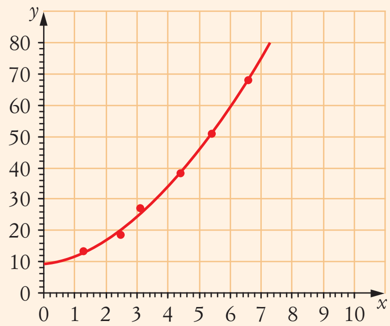


Strong negative

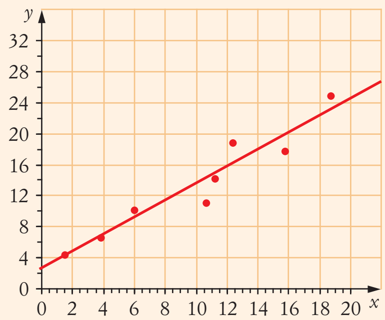
15



16 a



b



17 a 1 apple **b** About 120 apples (using the mean).

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18 a

Greyhound times (s) Key 3|5 = 5 3 = 5 |3 seconds

Houndog		Chaser
7	3	
8 7 4	4	
9 9 8 3 3 1 0	5	3 7 8 9
6 4 1 0 0	6	0 1 4 4 4 6 8 8 8 9
3 3 1 0	7	1 6 6 7
	8	2 3

b Houndog is generally faster.

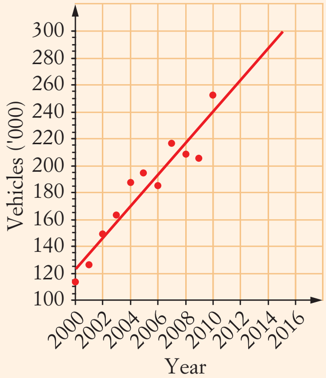
c The means (57.85 to 67.2 s), and medians (59 to 67 s) reinforce the conclusion from the stem-and-leaf plot. The ranges (36 to 30 s) are similar.

19 a

		Errors in spelling					
		1	**2**	**3**	**4**	**5**	**Total**
Height (cm)	150–159	0	1	3	5	1	**10**
	160–169	8	6	9	2	9	**34**
	170–179	11	2	6	7	9	**35**
	180–189	2	3	2	2	2	**11**
	Over 189	2	0	0	1	1	**4**
	Total	**23**	**12**	**20**	**17**	**22**	**94**

b There is no pattern. The small number of students over 180 cm and under 160 cm suggest that the conclusions may not be reliable. However, if they are classified as over 170 and under 170 cm, then there is sufficient data to suggest there is no pattern.

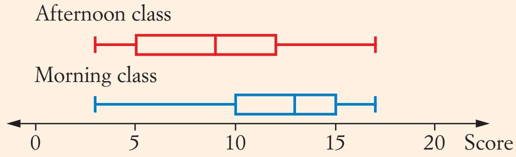
20 a **New sports utilities sales in Australia**



b Yes, strong positive relationship.

c About 290 000.

21 a **Exam results**



b The morning class appears to have had an advantage.

c The means (12.2 to 9.0) reinforce the conclusion from the medians and quartiles in the boxplots.

22 The mean would be lower than the median because the low values would affect it more than the median.

23 The new mean would be $(72 \times 20 + 3 \times 80) \div 23 \approx 73$ kg. The median would probably not change because one of the new students must have a lowish weight.

24 The morning class might just be a better class.

25 The distribution is skewed to the left so the mean would be lower than the median. A quarter of the results are in each of the ranges 12–17, 17–19, 19–20, 20–22, so this gives an overall mean of about 18.5.

26 The first company has a much larger sample than the second, but their method is poor. People would have a much better idea of sizes from the second survey, so it is more likely to be accurate. A large sample will not compensate for poor methods.

27 If there were changes in costs, the economic situation or fashion, the prediction could be quite wrong.

Chapter 3

Exercise 3.1

1 a $12 + y$ **b** $2g + 4$ **c** $5d - 6$
d $3 + 5b$ **e** $4cv$ **f** $\frac{12h}{5}$

2 a $5 + 7c$ **b** $x^2 + 8$ **c** $3(6d + 2)$
d $\frac{1}{2}(7b + g)$ **e** $2(9m - 1)$ **f** $\frac{3a - 4}{5}$

3 a $n + 20$ **b** $n - 7$ **c** $\frac{1}{2}n - 5$
d $2n + 17$ **e** $2n + 3$ **f** $n + \frac{1}{3}n$
g $2(n - 7)$ **h** $\frac{2}{3}n + 15$

4 a $6a$ **b** $2n$ **c** $14k$ **d** $2f$
e 0 **f** $6m$ **g** $7g$ **h** $3s$
i $4r$ **j** $23k$ **k** $3j$ **l** $3n$
m b **n** $3e$ **o** $9m$

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5 **a** $2t + 8s$ **b** $11m + 3$
c $9p + 4q$ **d** $3a + 2b + 3c$
e $5w - 6u + 5y - 6z$ **f** $2j + 8i$
g $6e + 9$ **h** $y + 9m + z$
i $8r + 6s + 3t$ **j** $4m + 13u$
k $8f + 4$ **l** $4 + 10y$
m $2x + 9y$ **n** $2a + 2b$
o $9m + 2m^2 - 4mn$ **p** $8pq$
q $2xy + 3c$ **r** $5a + 4bd$

6 **a** $-x$ **b** $^{-}2r$ **c** $^{-}3v$
d $^{-}5t$ **e** $^{-}6c$ **f** $^{-}11d$
g $^{-}3i$ **h** $10y$ **i** $4d - 4e$
j $2n - 2m$ **k** $11x - 5y$ **l** $6t - 7r$
m $^{-}7y + 1$ **n** $5g - 11h$ **o** $11k - 9m$
p $^{-}2w$ **q** $^{-}5h - 8c$ **r** $^{-}12y$
s $6y - 4x - 6$ **t** $8m$ **u** $3q - 5r + 4z - 8$

7 **a** $A = 12$ **b** $A = 1$ **c** $A = 35$
d $A = 3$ **e** $A = 13$ **f** $A = 21$
g $A = 75$ **h** $A = 147$ **i** $A = 37$

8 **a** $y = {}^{-}1$ **b** $y = {}^{-}7$ **c** $y = {}^{-}11$ **d** $y = 10$
e $y = {}^{-}3$ **f** $y = 14$ **g** $y = 6$ **h** $y = 11$

9 **a** $2n - 5$ **b** $6(n + 4)$ **c** $n^2 - 10$
d $\frac{1}{2}n + 2$ **e** $3(n + 4)$ **f** $\dfrac{n - 8}{7}$
g $3(2n + 5)$ **h** $(7n - 4)^2$ **i** $\frac{1}{2}(4n - 9)$
j $(n + 3)(n - 3)$

10 **a** $10c^2 - 5r$ **b** $4m^2 - 2g$
c $10x^2 - 8x + 2$ **d** $4ce^2 - 4ac^2 - 4ce$
e $2ab - 3b^2 - 4$ **f** $2a^2 - 3ac$
g $3mn + 6mn^2$ **h** $6m^2 + 5m^3 - 3mn$
i $4g^2 + 5g^3 - 7$ **j** $5a^2b^2 - 3ab^2 + 7ab$
k $24pqr + 8p^2qr$ **l** $pq + qr + pr$
m $9pq^2 - 6pq + 4p^2q$ **n** $24fmr^2 - 20m$
o $31xy^2z - 3xyz^2$ **p** $21a^2 - 6a^3$
q $11ab$ **r** $24t^2 - 4t^3 - 41t$

11 **a** $m = 20$ **b** $p = 23$ **c** $a = 55$ **d** $d = 72$
e $h = 2$ **f** $v = 2\frac{2}{3}$ **g** $v = 24$ **h** $s = 149$

12 **a** $b = 15$ m **b** $t = 12.5$ s
c $b = 5$ m **d** $h \approx 7.6$ cm
e $h \approx 21.05$ m **f** $u \approx 1.97$ m/s
g $m \approx 2.67 \times 10^{-8}$ kg **h** $R \approx 0.0765$ atm-L/mole/K

13 **a** $t = \dfrac{2u - v}{3au}$ **b** $c = \dfrac{Ik - G}{kt}$
c $b = m(c - a) + a$ **d** $u = sa - at$ or $a(s - t)$
e $d = \dfrac{bc}{a}$ **f** $a = cv^2 + b$
g $b = \dfrac{a^2 - c^2}{2}$ **h** $c = \dfrac{(d + 5)^2}{b}$

14 **a** $36a^5c^4k^6$ **b** $6h^2u^2x^6$ **c** $30k^6p^5r^6$
d $8e^6n^3y$ **e** $20c^2n^3w$ **f** $12a^4g^6n^4$
g $14a^2h^5k^5$ **h** $10a^4h^4v^3$ **i** $63e^4p^3t^3$
j $42h^4r^2u^3$ **k** $8b^4h^4k^2$ **l** $45b^4q^5t^4$
m $\dfrac{7pt^2}{6k}$ **n** $\dfrac{2ev^2}{5u}$ **o** $\dfrac{8}{7gu^3}$
p $\dfrac{1}{9pu^2}$ **q** $\dfrac{3}{2v}$ **r** $\dfrac{6}{5ah^2}$
s $6k$ **t** $\dfrac{6u^2y}{7}$ **u** $\dfrac{3c^2u^2v^2}{8}$
v $\dfrac{6a}{h}$ **w** $\dfrac{9}{8g}$ **x** $\dfrac{5mu^2}{6q^2}$
y $\dfrac{6bc}{n^3}$ **z** $\dfrac{3qv}{5}$

15 **a** $\dfrac{3h^5}{2n^7u^2}$ **b** $\dfrac{4t^8w}{5}$ **c** $\dfrac{3}{5e^7p^4t^3}$
d $2a^{19}c^{18}$ **e** $20b^{14}q^{11}v^3$ **f** $\dfrac{5u^4}{2a^4p^8}$
g $4n^{12}p^{18}v^{19}$ **h** $12a^{15}t^{12}y^{11}$ **i** $2m^{11}q^5r^5$
j $4h^3m^2u^2$ **k** $15b^{20}c^5h^9$ **l** $2a^6b^{14}p^{14}$
m $\dfrac{4k^{11}u^3}{w^2}$ **n** $b^{17}h^{19}r^{13}$ **o** $\dfrac{4kn^2}{5p^4}$
p $\dfrac{5a^3}{4g^6m^2}$ **q** $2a^7c^{13}g^{11}$ **r** $\dfrac{u^3}{5r^6t^8}$
s $\dfrac{3n^8}{be^2}$ **t** $\dfrac{3}{5em^6u}$ **u** $\dfrac{4t}{5b^7m}$
v $\dfrac{2}{3qtw}$ **w** $12b^{20}h^{16}m^7$ **x** $20b^{22}g^{18}v^{12}$
y $5e^{14}p^{16}q^5$ **z** $12g^9n^{18}v^{11}$

16 **a** $\dfrac{^{-}5b^7g^4}{a^2}$ **b** $\dfrac{3x}{4h^4k^7}$ **c** $\dfrac{7c}{a^7m^9}$
d $\dfrac{^{-}4c^6e^2}{9h^2}$ **e** $^{-}12a^2p^3$ **f** $\dfrac{1}{k^4p^8r^2}$
g $\dfrac{-bt}{9r^3}$ **h** $\dfrac{^{-}9b^7q^2}{p^2}$ **i** $\dfrac{^{-}5a^7m}{7c^7}$
j $\dfrac{3n^5}{b^3r}$ **k** $\dfrac{4}{a^3e^4k^3}$ **l** $\dfrac{^{-}2t^4}{5q^4}$
m $\dfrac{^{-}3}{2b^3h^2k^{10}}$ **n** $\dfrac{^{-}b^8k^2}{5p^6}$ **o** $72m^7p^5r^5$
p $\dfrac{2a^6v^5}{c}$ **q** $^{-}42c^4$ **r** $\dfrac{^{-}3m^6q^5}{7t^5}$
s $\dfrac{^{-}7b^{10}e^3k}{4}$ **t** $^{-}4b^6h^8v^2$ **u** $\dfrac{^{-}6e^4}{a^3m}$
v $\dfrac{56e^5n^3}{m}$ **w** $\dfrac{35g}{k^5t^3}$ **x** $\dfrac{6}{e^8g^{10}m^8}$
y $^{-}5c^8g^7h^5$ **z** $\dfrac{^{-}5g}{c}$

17 a $\frac{7k^3}{3a^4e}$ **b** $\frac{^{-}12a^5c^{12}}{u^9}$ **c** $\frac{^{-}42}{nr^3u^8}$

d $\frac{7}{3ak^{10}u^2}$ **e** $\frac{u^2}{3c^{10}}$ **f** $\frac{6t^3}{5a^{12}k^5}$

g $\frac{^{-}42c^{10}m^3}{k^{11}}$ **h** $\frac{14p^3q^6}{h^2}$ **i** $\frac{^{-}5c^2g^3}{9m}$

j $\frac{24b^2}{p^7r}$ **k** $\frac{^{-}24a^{17}g^6}{k^6}$ **l** $\frac{^{-}5e^3}{2a^9c^{18}}$

18 a $\frac{12e^5u^{22}}{k^{16}z^3}$ **b** $\frac{6n^9v^{13}w^{11}}{5m^{11}}$ **c** $\frac{^{-}60a^9v^3}{m^8u^{10}}$

d $\frac{^{-}7w}{6k^{10}n^3z^{14}}$ **e** $\frac{e^{15}}{4a^{21}z^6}$ **f** $\frac{^{-}35g^8m^4}{r^{22}v^{11}}$

g $\frac{3a^6k^{15}z}{5y^6}$ **h** $\frac{30q^9v^{10}}{u^5w^4}$ **i** $\frac{7g^5k^{13}wx^{17}}{10}$

j $\frac{28}{g^2m^{13}n^{11}u^9}$ **k** $\frac{2h^2k^2}{5b^{14}e}$ **l** $\frac{132n^{23}p^2}{r^8x^2}$

m $156a^{10}e^2q^{19}$ **n** $\frac{63}{b^{12}mw^5z^2}$ **o** $\frac{^{-}2t}{13e^4m^{20}}$

p $\frac{^{-}28c^{13}r^8}{q^{12}}$ **q** $\frac{12m^{11}}{a^{10}k^{11}u^{14}}$ **r** $\frac{9qv^{15}}{5h^{17}x^{13}}$

s $\frac{^{-}12t^5}{11n^2q^7u^9}$ **t** $\frac{11n^4v^6}{a^{14}r}$ **u** $\frac{10p^2q^6}{c^{17}m^9}$

v $\frac{13e^{24}u^2}{p^7q^7}$ **w** $\frac{^{-}4a^5z^6}{13n^9y^8}$ **x** $\frac{8g^{14}u^4}{7h^{18}m^5}$

y $\frac{^{-}91e^{11}n^3}{c^4q^{19}}$ **z** $\frac{7n^{11}}{13k^{20}t^{10}y^{16}}$

19 a $\frac{4qu}{5}$ **b** $\frac{8}{15w}$ **c** $\frac{4t}{7kn}$

d $\frac{4m}{7a}$ **e** $\frac{5ae^2}{7gw^2}$ **f** $\frac{10kw}{13u}$

20 a $P = 6w$ **b** $P = (4w + 8)$ cm
c $P = (4w + 6)$ m **d** $P = 8w$
e $P = (6w + 8)$ mm **f** $P = (8w - 10)$ cm
g $P = (10w + 56)$ m **h** $P = (4w + 24)$ mm

21 a $10m + 4$ **b** $2a^2 + 2b^2$ **c** $\pi y + 2y + 3\pi + 6$

22 a $u = 6$ **b** $m = {}^{-}4$ **c** $a = 5$
d $E = 17.5$ **e** $h = 45$ **f** $g = 10$
g $t = 3.75$ **h** $c \approx 21.8$ (or $2\sqrt{119}$)
i $w = 349.8$ **j** $r = 6.5$

23 A formula has more than one variable and is an equation that usually has one of the variables by itself on the left-hand side. An equation is just a statement of equality and could have any number of variables, including none. However, most simple equations have one variable.

Exercise 3.2

1 a $a^2 + 5a$ **b** $m^2 - mp$ **c** $2g^2 + 3fg$
d $4z^2 - 3wz$ **e** $3x^2 + 2xy$ **f** $8c^2 - 7c$
g $^{-}5m^2 + 3mn$ **h** $^{-}2f^2 + 5fh$ **i** $^{-}2d^2 - 3de$
j $^{-}8c^2 + 28c$ **k** $^{-}6n^2 - 18n$ **l** $^{-}15ck + 10k^2$

2 a $x^2 + 7x + 12$ **b** $2 + 3b + b^2$
c $a^2 + 9a + 20$ **d** $12 + 8a + a^2$
e $y^2 + 7y + 6$ **f** $a^2 + 3a + 2$
g $m^2 + 12m + 35$ **h** $a^2 + 8a + 12$
i $n^2 + 6n + 8$

3 a $^{-}10(7q + 3)$ **b** $5(15q - 8)$ **c** $7(4y - 5)$
d $4(11a - 14)$ **e** $^{-}9(8v + 9)$ **f** $13(10c - 11)$
g $^{-}15(6c + 5)$ **h** $13(7t - 9)$ **i** $^{-}15(3u + 11)$
j $^{-}2(9n + 13)$ **k** $^{-}14(2w + 15)$ **l** $14(5r + 9)$
m $4(12b + 5)$ **n** $9(10g + 7)$ **o** $^{-}2(11x + 6)$
p $13(6k + 11)$ **q** $^{-}12(5u + 13)$ **r** $5(4p + 1)$
s $^{-}11(6v + 7)$ **t** $40(2h + 3)$ **u** $^{-}11(13x + 5)$
v $3(11e + 13)$ **w** $7(11m - 5)$ **z** $14(10t + 11)$
y $^{-}7(a + 5)$ **z** $5(11k - 2)$

4 a $2a^2 + 5a + 3$ **b** $2a^2 + 7a + 3$
c $3x^2 + 14x + 8$ **d** $8y^2 + 22y + 15$
e $35g^2 - 3g - 2$ **f** $18a^2 + 6a - 4$
g $49m^2 - 1$ **h** $15x^2 - 19x + 6$
i $25y^2 + 40y + 16$ **j** $12y^2 + 25y + 12$
k $49n^2 - 25$ **l** $8 + 12c - 8c^2$

5 a $(a + b)^2 = a^2 + 2ab + b^2$
b $(x + y)^2 = x^2 + 2xy + y^2$
c $(w - r)^2 = w^2 - 2rw + r^2$
d $(a + 2)^2 = a^2 + 4a + 4$
e $(x - 3)^2 = x^2 - 6x + 9$
f $(m - 5)^2 = m^2 - 10m + 25$
g $(g + 7)^2 = g^2 + 14g + 49$
h $(6 + b)^2 = 36 + 12b + b^2$
i $(3 - n)^2 = 9 - 6n + n^2$
j $(k + 8)^2 = k^2 + 16k + 64$

6 a $3a^2 - 9a - 18$ **b** $7x - 21y + 28z$
c $4x^2 + 20x + 28$ **d** $12m - 6n + 18p$
e $3m^2 - 9mn - 6mp$ **f** $8g - 6g^2 + 2ag$
g $^{-}4p^2 + 12pq - 8pr$ **h** $^{-}3x^2 - 9xy + 15xz$
i $^{-}9y^2 + 18xy - 45yz$ **j** $x^3 + 4x^2 - 3x$
k $2y^3 - 14y^2 + 12y$ **l** $8c^3d + 12cd^2 - 12cd$

7 a $x^2 + 5x + 6$ **b** $x^2 + 5x + 4$
c $a^2 + 8a + 15$ **d** $x^2 + 3x - 18$
e $a^2 - 19a + 90$ **f** $m^2 + 4m - 77$
g $x^2 - 25$ **h** $y^2 - 4y - 21$
i $x^2 + 17x + 60$ **j** $k^2 + 2k - 99$
k $x^2 + x - 30$ **l** $18 + 7n - n^2$

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8 a $13x + 47$ **b** $32a + 15$ **c** $30d + 43$
d $18a + 11$ **e** $10m - 2$ **f** $9b - 13$
g $27 - a$ **h** $2x - 14$ **i** $33x - 23$

9 a $2x^2 + 9x$ **b** $2b^2 + 7b$ **c** ${}^-2m$
d $17c$ **e** $5y^2 - 7y$ **f** ${}^-8h$
g $a^2 + 2a$ **h** ${}^-3c^2 - c$ **i** $30p^2 - 38p$
j ${}^-8x^2 + 27x$

10 a $8x + 6y + 7z$ **b** $11a + 2b - 3c$
c $23a + 14b - 13c$ **d** $13x - 16y - 5z$
e $4x^3 - 26x^2 - 19x$ **f** $4a^3 - 11a^2 + 22a$
g $12x^3 - 16x^2 + 10x$ **h** $12a^3 + 6a^2 - 78a$
i $13x^3 - 7x^2 - 2x$ **j** $8a^3 + 3a^2 + 59a$

11 a ${}^-12(12b + 13e + 1)$ **b** $12(13p - 3u - 3)$
c $3(14x + 15k + 6)$ **d** $12(12m - 13t + 9)$
e $7(5z + 11m - 4)$ **f** ${}^-9(10m + 7e + 8)$
g ${}^-3(6b + 11g + 2)$ **h** $2(12q + 13w + 15)$
i $4(12x - 5e + 6)$ **j** ${}^-7(12t + 5x + 13)$
k $10(13p - 9n - 11)$ **l** ${}^-4(15w + 11c + 13)$
m $3(4a - 15h + 1)$ **n** $5(p - 11r + 7)$
o $6(14q - 9e + 5)$ **p** ${}^-8(13c + p + 2)$
q $15(8x - 12m + 13)$ **r** $8(5t + 11a - 13)$
s $15(7h + 9y + 9)$ **t** $10(11k - 12m + 2)$
u $5(y - 15r - 8)$ **v** $2(13h - 2x - 12)$
w $14(3v - 13n - 8)$ **x** $5(8r - 12m - 15)$
y $4(10c + 13m + 9)$ **z** $8(2k - 15x - 6)$

12 a $12h^{-10}m - 54h^{-6}m^5$ **b** ${}^-18n^{-5}x^{-2} - 15n^{-5}x^3z^3$
c $48t^3w^{-3}x^4 - 30t^{-3}w^{-3}$ **d** $12a^{-7}c - 14a^{-1}cw^{-6}$
e $15b^{-1}x^{-6} - 21b^{-1}x^{-2}$ **f** $8b^6n^5 - 6b^7n^{-1}$
g $40a^{-4}k^6 - 25a^{-1}k^4$ **h** $48a^2b^5 + 56a^2b^6k^7$
i ${}^-4k^6t^4x^{-1} - 3k^6t^4$ **j** $8b^{-8}c^{-5} + 28b^{-1}c^{-5}z^{-1}$
k $24g^{-2}p^4r^{-1} + 3g^{-2}p^8$ **l** $42m^{-2}q^{-4} + 49m^3q^{-4}$
m $25k^6m^{-5} + 15k^3m^{-7}$ **n** $35h^{-6}k^2 + 15h^{-6}k^{-2}p^7$
o ${}^-45k^{-7}r^4v - 25k^{-12}r^4$ **p** $24eh^{-3} - 40e^6h^{-3}p^5$
q $28h^{-3}r^{-1}z^{-2} + 35h^{-3}r^{-4}$ **r** ${}^-36a^{-6}y^{-9} - 16a^{-13}y^{-3}$
s $18e^2k^2 + 9k^{-1}$ **t** $20u^{-9} + 5u^{-3}$
u $30e^2n^{-3}v - 48e^{-1}n^{-3}$ **v** $35r^8t^6 - 10r^6t^6y^{-5}$
w ${}^-16c^{-5}tz^6 - 72c^{-5}t$ **x** ${}^-8n^{-3}t^4 - 7n^{-5}t^3$
y $72mw^4 + 32m^{-2}w^6$ **z** $4g^5t^3 + 18g^5t^{-4}u^2$

13 a $2a^2 - 12 - 2a$ **b** $2x^2 - 11x + 15$
c ${}^-5a^2 - 17a + 12$ **d** $2k^2 - 13k + 20$
e $2m^2 - 7m + 3$ **f** ${}^-6x^2 + 19x + 7$
g ${}^-4p^2 - 4pq - q^2$ **h** $2n^2 + mn - 15m^2$
i $6x^2 + 13xy + 6y^2$ **j** ${}^-28x^2 + 29xy - 6y^2$
k ${}^-16a^2 + 66ab - 35b^2$ **l** ${}^-4xy - 3y^2 - x^2$
m $64u^2 - 144e^2$ **n** $72g^2 + 77gv + 20v^2$
o $80e^2 + 198et + 121t^2$ **p** $77a^2 - 8ax - 45x^2$
q $165x^2 - 41xr + 2r^2$ **r** $126c^2 + 41cz + 3z^2$
s $196x^2 - 224xm + 39m^2$ **t** $56x^2 + 183xp + 135p^2$
u $6v^2 - 5vc - 4c^2$ **v** $60p^2 + 23pz - 20z^2$
w $105y^2 - 188yr - 13r^2$ **x** $22m^2 - 189mq + 180q^2$
y $14z^2 + 61zu - 195u^2$ **z** $70z^2 + 37za - 3a^2$

14 a $a^2 + 2ab + b^2$ **b** $x^2 - 2xy + y^2$
c $t^2 + 4t + 4$ **d** $g^2 + 6g + 9$
e $81 - 18h + h^2$ **f** $a^2 + 12a + 36$
g $m^2 + 22m + 121$ **h** $100 - 20b + b^2$
i $f^2 - 2fg + g^2$ **j** $u^2 + 18u + 81$
k $64 + 16v + v^2$ **l** $y^2 - 26y + 169$

15 a $a^2 - 4$ **b** $x^2 - 9$ **c** $e^2 - 25$
d $n^2 - 16$ **e** $j^2 - 81$ **f** $q^2 - 16$
g $h^2 - 4$ **h** $w^2 - 25$ **i** $a^2 - r^2$
j $t^2 - p^2$ **k** $y^2 - g^2$ **l** $a^2 - p^2$

16 a ${}^-42c^2 + 4q^2 + 12x^2 - 36cq - 19cx - 36qx$
b $35n^2 + 64t^2 + 4x^2 + 76nt + 41nx - 49tx$
c $15a^2 - 63e^2 - 6m^2 + 48ae + 19am + 53em$
d $3p^2 + u^2 - 25y^2 - 4pu + 19py - 2uy$
e $8k^2 - 42x^2 - 72y^2 + 12kx + 4ky - 94xy$
f ${}^-14a^2 + 4e^2 + 14p^2 + 25ae - 28ap + 18ep$
g ${}^-18a^2 + 24m^2 - 45t^2 - 32am + at + 26mt$
h $72e^2 - 21u^2 - 36y^2 - 4eu + 52ey - 8uy$
i ${}^-45q^2 - 4r^2 - 18u^2 + 58qr + 54qu - 73ru$
j ${}^-56k^2 + 56w^2 + 20z^2 + 15kw + 22kz - 41wz$
k $10p^2 - 9r^2 - 72v^2 - 40pr - 33pv - 45rv$
l $9b^2 - 10m^2 + 36u^2 + 10bm - 33bu - 52mu$
m $27g^2 + 27h^2 - 15x^2 - 3gh - 51gx + 67hx$
n $18e^2 - 30g^2 + 54v^2 + 9eg - 117ev + 3gv$
o ${}^-9c^2 + 45p^2 - 56r^2 + 6cp - 36cr + 37pr$
p $7b^2 + 20u^2 + 6v^2 - 44bu - 21bv - 28uv$
q ${}^-18e^2 - 54h^2 + 2e + 7h - 2m + 15eh - 9em + 18hm$
r $5a^2 - 24e^2 + 48a + 5e + 8z - 4ae + az - 54ez$
s $6e^2 - 14h^2 + 64q^2 - 21eh - 74eq + 38hq$
t ${}^-18b^2 + 24p^2 + 8y^2 - 14bp - 8by - 34py$
u $54q^2 + 18u^2 - 35w^2 + 6qu - 7qw + 36uw$
v ${}^-a^2 + 35b^2 - 30u^2 + 13ab + 11au + 91bu$
w ${}^-4a^2 + 10c^2 + 28a + 5c + 5h + 43ac + 28ah - 45ch$
x $27h^2 - 7n^2 + 5p^2 + 39hn - 14hp - 89np$
y ${}^-15a^2 + 7c^2 + 16n^2 - 41ac + 32an - 13cn$
z $4q^2 + 24t^2 - 30x^2 - 47qt + 52qx - 55tx$

17 a $85u^2 - 146ut + 55t^2$ **b** $11y^2 - 16yg - 24g^2$
c $27p^2 - 57pn + 52n^2$ **d** ${}^-38b^2 + 70bp - 11p^2$
e $11c^2 + 45cg + 41g^2$ **f** $49t^2 - 92ta + 41a^2$

g $34c^2 - 33cq - 36q^2$ **h** $75p^2 + 50pv - 35v^2$
i $23q^2 - 105qr + 7r^2$ **j** $59h^2 + 91hy + 7y^2$
k $48z^2 + 27zh + 47h^2$ **l** $23r^2 - 15rn - 44n^2$
m $50h^2 + 142hq + 114q^2$ **n** $4v^2 - 4vt$
o ${}^{-}11m^2 - 2my - 4y^2$ **p** $18x^2 - 61xb + 5b^2$
q $25a^2 - 34ab + 4b^2$ **r** $20c^2 - 10cp + 5p^2$
s $50h^2 - 81ah$ **t** $21g^2 + 9gh - 108h^2$
u $11n^2 - 26nc - 12c^2$ **v** $76q^2 - 38qa - 20a^2$
w $7h^2 + 161hp + 36p^2$ **x** $56e^2 + 90ez + 61z^2$
y $46c^2 + 71cw + 23w^2$ **z** $6u^2 - 24uh + 14h^2$

18 a $48q^{-10}r^5v^2 + 50q^{-7}r^4v^{-2} - 64q^{-5}r^7v - 54r^5v^3$
b ${}^{-}24k^5u^{-1} - 15k^7r^2u^{-4} - 42k^6r^4u^3 + 45u^2$
c $64q^6t + 47k^{-3}q^7t^2 - 56k^{-2}q^4t^6 + 9q^6t^7$
d ${}^{-}97g^{-1}t^8u^{-6} - 7g^{-6}t^7u^6 - 42g^{-12}t^2u^3 + 40t^8u^{-3}$
e $27r^{-6}u^{-5}y^3 - 38ru^{-5}y^{-7} + 63r^4u^2y^6 - 2u^{-5}y^9$
f ${}^{-}45b^{-2}h^6q + 15b^{-5}h^6q^3 + 6b^{-4}h^3q^2 - 3h^6q^3$
g ${}^{-}44n^{12}v^{-5} + 84a^{-4}n^{11}v^5 - 2a^{-9}n^7v - 16n^{12}v^{-4}$
h ${}^{-}20m^{-3}t^{11}z^{-6} + 48m^{-6}t^8z^6 - 15m^{-12}t^5z^5 - 8t^{11}z^{-1}$
i $11g^5k^{-5}u^2 + 11g^2ku^{-3} + 4g^4k^{-2}u - 15k^{-5}u^3$
j ${}^{-}35b^5u^{-2}w^{-1} - 20b^2u^{-4}w^5 + 18bu^{-7}w + 72u^{-2}$
k ${}^{-}21q^{11}t^{12}w^{-4} + 28q^4t^{13}w^6 - 7t^6w^{-2} + 35t^{12}w^{-6}$
l ${}^{-}21g^{-6}q^{-9}t^{-5} + 58g^{-3}q^{-7}t^{-5} + 18g^{-8}q^{-4}t^5 + 6q^{-9}$
m $g^4p^2 + 28g^2n^4p^{-2} - 6g^4n^2p^4 + 21p^6$
n ${}^{-}144w^4y + 19a^4w^{-2}y^2 - 64a^5w^2y^4 - 27w^4y^5$
o ${}^{-}42e^1n^{10}t^{-1} + 117e^5t^6 - 56e^4n^4t^{-6} + 45n^{10}t^{-7}$
p $15h^8n^{-7}t + 4h^3nt^{-3} + 10h^4n^{-4}t^3 - 54n^{-7}t^4$
q $41a^4m^5p + 94a^{-1}m^8p^2 - 6m^3p^{-2} + 10m^5p^{-1}$
r ${}^{-}27g^{-2}p^{-10}y^4 - 54g^{-4}p^{-4}y^{-4} + 27gp^{-6}y^3$
$- 63p^{-10}y^7 + 81g^{-4}p^{-3}y^{-4}$
s $45h^{10}z - 21h^6y^1z^3 - 27h^7y^{-3}z^6 - 3z^7$
t $48k^{-3}n^{14}q^6 - 12k^{-7}n^{11}q^7 + 5k^{-1}n^7q^3 - 35n^{14}q^9$
u $90k^3q^{-6}r - 21k^5q^{-4}r^{-4} - 24k^6q^{-2}r^6 + 54q^{-6}r^7$
v $11a^9p^{12}x^2 + 16a^7p^9x^5 - 4a^9p^7x^7$
w ${}^{-}54a^{-5}m^{-3}r^2 - 78a^{-7}m^{-4}r^3 - 9a^{-5}m^{-6}r^{-6}$
$+ 42m^{-3}r^{-4}$
x ${}^{-}35e^7k^{-1}w - 19e^4k^{-2}w^4 + 12e^5k^{-5}w^2 + 28k^{-1}w^3$
y $34e^{11}r^{-9}v - 51e^5r^{-1}v^{-2} + 8e^6r^{-7}v^{-3} + 21r^{-9}v^{-2}$
z $72a^{-2}b^{-11}u^{-1} - 48a^2b^{-10}u^{-5} - 21a^1b^{-6}u^6$
$+ 63b^{-11}u^5$

19 a $2a(10a - 9p + 9u)$ **b** $14b(9k + 10y + 15b)$
c $e(3y + 11e + 7h)$ **d** $-c(8c + 15v + 10t)$
e $13b(13y + 14v + 15b)$ **f** $14g(5h + 12g + 10z)$
g $10a(a - v + 15z)$ **h** ${}^{-}11c(9n + 14w + 10c)$
i $8a(2t - 15a - 5q)$ **j** $7b(12b - u - 7h)$
k $2c(5r - 6p - 6c)$ **l** ${}^{-}12e(v + 2e + 9w)$
m $5k(k + 9r - 13v)$ **n** $7x(11y - 7z - 8x)$
o $2b(6u + b + 7m)$ **p** ${}^{-}5n(15n + 4t + 11p)$
q ${}^{-}11c(9z + 7m + 11c)$ **r** ${}^{-}3n(12p + 13n + 4q)$
s $8a(6a + 11e + 6y)$ **t** $2a(15h - 11z - 14a)$
u $10e(5v - 8e - 11m)$ **v** $12a(11a - y - 13m)$
w $-k(13w + 3p + 6k)$ **x** $4k(p - 9k - 15x)$
y $14u(7u - 12v - 9x)$ **z** $15g(q - 7u - 8g)$

20 a ${}^{-}14e^2k^2q^3(12ek^4 + 13k^4q^2 + 14e^5q^4)$
b $11w^3yz^5(4y^2z^5 - 13zw + 6yw^3)$
c ${}^{-}7m^4n^5u^5(5u^2m^2 + 9m^5n + 10u^4)$
d $2g^3r^3y(15g^3y^3 - 14y^2r^3 - 9g^3r^5)$
e ${}^{-}7c^5y^3z^3(4z^3 + 7y^2c^3 + 5z^3c)$
f $10hm^3u(7m^6h - 11h^3u^2 + 11m^4u^3)$
g $2gp^4q^5(15g^5 + 7pq^2 - 12g^5q)$
h $a^2r^4z(4r^5z - z^2a^5 + 3r^3a^4)$
i $4a^2m^4p^2(5pa + 6a^5m^4 - 7p^4m^4)$
j $11bm^2q^2(10b^4q^2 - 11q^3m^4 + 10b^2m^5)$
k $8an^5x(15x^3 + n^5a^5 - 4xa)$
l $11cp^5v^3(6p^8c^4 - 5c^4v^4 - 7p^5v)$
m $7be^2h^3(11b^5 + 10e^2h^2 - 15b^5h^4)$
n $15e^5r^4z^2(5r^3z^2 + 8z^4e^5 + 10r^4e^3)$
o $11h^4x^4y^4(2y^4h^3 - 5h^5x^2 - 6y)$
p $7a^2hu^2(15a^2u^5 + u^2h^2 - 4a^4h^5)$
q ${}^{-}7e^2tz^5(5z^5 + 8t^4e^3 + 4z^3e)$
r $2h^2u^5(13q^{10}h^5 + 6hu^4 + 7qu^4)$
s ${}^{-}10e^4v^2x^3(13e + 14v^3x^2 + 12e^5x^2)$
t $a^5k^5t^3(5k^4t^4 + 12t^3a^3 + 4k^5a^3)$
u $13c^5x^5y(4y^4c - 3c^3x - 10y^5x^2)$
v $15eu^2z^3(7e^3z^4 - 4z^2u + 7e^5u^3)$
w $t^2xz^5(7z^2 - 3xt^2 + 9z^5t^5)$
x ${}^{-}11g^2tx^4(8t^8g^4 + 9x + 11x^2)$
y ${}^{-}8g^3xz^4(2 + z^3 + 4g^3z^4)$
z ${}^{-}13p^3r^3(7p^5r^2 + 8r^3b^5 + 7p^2b^4)$

21 a $4a^2 + 12a + 9$ **b** $9x^2 + 6x + 1$
c $25m^2 - 20m + 4$ **d** $9h^2 - 42h + 49$
e $64u^2 - 48u + 9$ **f** $81k^2 - 36k + 4$
g $16d^2 + 40d + 25$ **h** $81k^2 - 36k + 4$
i $4a^2 + 20am + 25m^2$ **j** $16g^2 + 24gs + 9s^2$
k $49h^2 + 28hk + 4k^2$ **l** $64d^2 - 16dq + q^2$

22 a $4b^2 - 1$ **b** $9c^2 - 4$ **c** $25m^2 - 9$
d $1 - 4x^2$ **e** $16m^2 - 1$ **f** $64g^2 - 9$
g $4a^2 - p^2$ **h** $c^2 - 9d^2$ **i** $64d^2 - 4$
j $64r^2 - 25y^2$ **k** $49y^2 - 16d^2$ **l** $36 - 9p^2$

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23 a $\dfrac{g^2t^6(3a^7g^2 + 13g^5t^6 + 13at^2)}{a^3}$

b $\dfrac{15z^5(11z^3q^3 - 9q^4v + 14z^2v^2)}{q^4v^3}$

c $\dfrac{{}^-7c^6u^9(14yc^6 + 3c^4 + 4y^5)}{y^4}$

d $\dfrac{2m^3u(9m^6u - 4u^4q^5 + 8m^6q^4)}{q^6}$

e $\dfrac{13t^4z^5(t^9m^2 + 15mz^6 + 4tz)}{m^3}$

f $\dfrac{3(14p^{11}h^5 - 5h^7q^3 + p^4q^4)}{h^6p^6q^7}$

g $\dfrac{2(5hp^5 - 12e^4 - 11e^2p^5)}{e^6h^3p^7}$

h $\dfrac{9q^7(5z^5 + 14z^4h^4 - 4q^5h^3)}{h^2z^4}$

i $\dfrac{{}^-10a^5g^8(2x^3a + 13a^4g^4 + 15x^3g^3)}{x^6}$

j $\dfrac{15r^3(2q^6r^5 - 15n^2 + 7nr^5)}{n^7q^6}$

24 a $4p^2 - 4p - 15$ **b** $4p^2 + 4p - 15$

c $\dfrac{\pi(36m^2 + 60m + 25)}{4}$

25 a $(n + \frac{1}{2})^2 = n^2 + n + \frac{1}{4} = n(n+1) + \frac{1}{4}$

b i $(3\frac{1}{2})^2 = 3 \times 4 + \frac{1}{4} = 12\frac{1}{4}$

ii $(19\frac{1}{2})^2 = 19 \times 20 + \frac{1}{4} = 380\frac{1}{4}$

26 Area of outside circle $= \pi R^2$ and the area of the inside circle $= \pi r^2$.
Area of annulus $= \pi R^2 - \pi r^2$.
Now $\pi(R - r)(R - r) = \pi[R(R + r) - r(R + r)]$
$= \pi(R^2 + Rr - Rr - r^2) = \pi R^2 - \pi r^2$ QED.

Exercise 3.3

1 a $\dfrac{9a + 8y}{12}$ **b** $\dfrac{2g + 3m}{9}$ **c** $\dfrac{14m + n}{10}$

d $\dfrac{4c - 27k}{18}$ **e** $\dfrac{45c - 4r}{40}$ **f** $\dfrac{15g - 28y}{20}$

g $\dfrac{5bn}{48}$ **h** $\dfrac{cp}{288}$ **i** $\dfrac{ek}{56}$

j $\dfrac{16g}{63u}$ **k** $\dfrac{21e}{4h}$ **l** $\dfrac{49m}{72v}$

m $\dfrac{11p}{6}$ **n** $\dfrac{3q}{20}$ **o** $\dfrac{19a}{42}$

p $\dfrac{25m}{24}$ **q** $\dfrac{11q}{18}$ **r** $\dfrac{{}^-c}{4}$

s $\dfrac{21b^2}{8}$ **t** $\dfrac{5c^2}{36}$ **u** $\dfrac{21a^2}{8}$

v $\dfrac{5}{24}$ **w** $\dfrac{3}{4}$ **x** $\dfrac{1}{60}$

y $\dfrac{{}^-13b}{30}$ **z** $\dfrac{53b}{18}$

2 a $\dfrac{111h - 89}{42}$ **b** $\dfrac{17g - 35}{30}$ **c** $\dfrac{58k + 41}{10}$

d $\dfrac{60h - 35}{28}$ **e** $\dfrac{80g - 95}{18}$ **f** $\dfrac{16b - 3}{12}$

g $\dfrac{52g + 22}{15}$ **h** $\dfrac{22n - 1}{6}$ **i** $\dfrac{14a - 1}{12}$

j $\dfrac{34a - 55}{40}$ **k** $\dfrac{67b - 10g}{40}$ **l** $\dfrac{6a + 11e}{9}$

m $\dfrac{14h - 25y}{6}$ **n** $\dfrac{38r + 41t}{12}$ **o** $\dfrac{32g + 41x}{40}$

p $\dfrac{43g - 34m}{10}$ **q** $\dfrac{16q - 7r}{10}$ **r** $\dfrac{29h - 19t}{12}$

s $\dfrac{23g - 8x}{10}$ **t** $\dfrac{30c - 5e}{12}$

3 a $\dfrac{35 - 6r}{36}$ **b** $\dfrac{27r + 51}{20}$ **c** $\dfrac{6e + 13}{12}$

d $\dfrac{8 - 11k}{6}$ **e** $\dfrac{5v + 13}{8}$ **f** $\dfrac{n + 36}{15}$

g $\dfrac{38c + 29}{24}$ **h** $\dfrac{37 - 13h}{30}$ **i** $\dfrac{17h - 55}{10}$

j $\dfrac{k + 2}{12}$ **k** $\dfrac{v - 22e}{18}$ **l** $\dfrac{15c + 37t}{12}$

m $\dfrac{u + 24v}{30}$ **n** $\dfrac{17t - 7k}{6}$ **o** $\dfrac{2g + r}{2}$

p $\dfrac{21c + 38p}{40}$ **q** $\dfrac{3c + 13p}{5}$ **r** $\dfrac{71t - 62e}{40}$

s $\dfrac{15q - 11p}{12}$ **t** $\dfrac{6g - 31v}{18}$

4 a $\dfrac{21q^2 - 61q + 28}{40}$ **b** $\dfrac{6e^2 + 55e + 56}{35}$

c $\dfrac{32a^2 - 4a - 21}{45}$ **d** $\dfrac{6e^2 + 5e - 21}{48}$

e $\dfrac{35g^2 - 44g + 12}{6}$ **f** $\dfrac{35c^2 + 101c + 72}{54}$

g $\dfrac{4b^2 - 27b - 81}{10}$ **h** $\dfrac{3a^2 - 26a - 9}{108}$

i $\dfrac{54e^2 + 39e - 5}{54}$ **j** $\dfrac{18b^2 - 45b + 25}{40}$

k $\dfrac{21p^2 + 50pq + 25q^2}{18}$ **l** $\dfrac{32b^2 - 68bc + 35c^2}{20}$

m $\dfrac{24n^2 - 35nq + 4q^2}{216}$ **n** $\dfrac{7c^2 + 41cm + 30m^2}{12}$

o $\dfrac{5n^2-42np+16p^2}{12}$ **p** $\dfrac{6a^2+37ae+35e^2}{28}$

q $\dfrac{5c^2+16cg+12g^2}{24}$ **r** $\dfrac{45g^2+8gn-21n^2}{32}$

s $\dfrac{18p^2-33pq+14q^2}{24}$ **t** $\dfrac{72g^2+25gu+2u^2}{48}$

5 a $\dfrac{35n-63}{12n-18}$ **b** $\dfrac{63p-49}{12p+10}$ **c** $\dfrac{72p+8}{72p-63}$

d $\dfrac{14a+7}{72a-56}$ **e** $\dfrac{20c+15}{56c+32}$ **f** $\dfrac{28h+7}{21h-12}$

g $\dfrac{24u-9}{16u-20}$ **h** $\dfrac{35e+45}{32e-12}$ **i** $\dfrac{45a-10}{49a-14}$

j $\dfrac{10a-35}{18a+30}$ **k** $\dfrac{35n-42p}{81n+45p}$ **l** $\dfrac{72c-63h}{18c+8h}$

m $\dfrac{63e+28p}{15e+9p}$ **n** $\dfrac{3e+27n}{48e+56n}$ **o** $\dfrac{72a+56h}{14a-49h}$

p $\dfrac{27a+24e}{30a+5e}$ **q** $\dfrac{32g+56v}{18g+3v}$ **r** $\dfrac{7g-63h}{48g-42h}$

s $\dfrac{21h+35p}{45h-40p}$ **t** $\dfrac{5b-15r}{24b+15r}$

6 a $\dfrac{2b^6}{15g^5y^3}$ **b** $\dfrac{n^4r^8u^5}{2}$

c $\dfrac{3m^4u^2w^2}{32v^5}$ **d** $\dfrac{3b^7qw}{8}$

7 a $\dfrac{8m+5}{2}$ **b** $\dfrac{10p-1}{6}$ **c** $\dfrac{22v+12u}{15}$

8 a $\dfrac{4t^2+12t+9}{25}$ **b** $\dfrac{6b^2-b-2}{60}$ **c** $\dfrac{7m^2+33m+20}{20}$

9 a A, $\dfrac{57q+23r}{24}$ **b** B, $\dfrac{11h-12b}{12}$

c No error. **d** C, $\dfrac{27k-63u}{25k+5u}$

10 Find the areas of each of the parts (Rectangle, triangle and quarter circle) and add to get $\dfrac{(408+27\pi)x^2-(344+36\pi)x+48+12\pi}{192}$ cm^2.

Chapter review

1 a $x-8$ **b** $8n$ **c** $3x-4$ **d** $3(x-4)$

2 a $7p$ **b** $4t-2h$ **c** $3xy-2x^2-3y$

3 a m^2-3m **b** $6k^2-8km$
c $x^2+7x+10$ **d** $4e^2+7e-15$

4 a $6(5b+3)$ **b** $^-3(3m+7)$

5 $D=25$

6 a $\dfrac{27q+64w}{24}$ **b** $\dfrac{5p}{18}$ **c** $\dfrac{91b^2}{54}$ **d** $\dfrac{91r}{18t}$

7 a $\dfrac{6n}{7u}$ **b** $42g^4k^3r^3$

8 a $r=\dfrac{A}{2\pi s}$ **b** $u=x-Zs$

9 a $20t+13$ **b** $31v-17v^2$
c $3m^2p^2t^2-9m^2p^3t+6mp^3t^2$ **d** $30k-9k^2-6k^3$

10 a $^-40b^2c^{-3}g^5-112b^{-3}c^{-3}$
b $7a^2+7c^2+36x^2+ac-75ax+4cx$
c $^-19g^2+31gp+25p^2$
d $18m^{19}u^{-3}+12g^{10}m^{-1}u^{10}-4g^7m^9u^3-32m^{19}$

11 a $16-a^2$ **b** h^2+6h+9

12 a $14g(3n+14w-15g)$
b $9ar^5w^4(7r^5w^2-15wa^3+11r^4a)$
c $\dfrac{8(3n^9y^5z^3+11n+5y^5z)}{n^6y^2z^3}$

13 a $\dfrac{74q+27r}{42}$ **b** $\dfrac{13p+41}{20}$

c $\dfrac{40b^2-77bv+36v^2}{16}$ **d** $\dfrac{5p+5}{48p-54}$

14 a $\dfrac{7t^3}{9b^2u}$ **b** $\dfrac{^-40e^3}{h^9}$ **c** $\dfrac{20g^4x^7}{p^{11}}$

d $\dfrac{128g^6h^2}{125k^8}$ **e** $\dfrac{^-6u^{14}}{7e^7t^{11}y^5}$ **f** $\dfrac{^-63b^{13}t^3}{g^{10}v^8}$

15 $\dfrac{5e^2v}{3w}$

16 a $49-42q+9q^2$ **b** $36y^2-25$
c $25h^2+20hv+4v^2$

17 a $n\approx 0.203$ **b** $a=\frac{4}{3}$

18 $\dfrac{34y-38}{15}$ cm

19 a $15n^2+mn-6m^2$ **b** $\dfrac{2m^2-mn-15n^2}{12}$

20 a B, $\dfrac{m+14n}{12}$
b A is unnecessary, also error in C, $\dfrac{12b^2-17bc+6c^2}{12}$

21 $(10n+5)^2=100n^2+100n+25=100n(n+1)+25$. To find the square of a number ending in 5, multiply the tens digit by one more than the tens digit and put 25 on the end of it. For 65^2, $6\times 7=42$, so $65^2=4225$.

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Chapter 4

Exercise 4.1

1 a 73 000 m^2 **b** 0.048 km^2 **c** 0.57 km^2
d 2.5 ha **e** 82.7 m^2 **f** 3420 m^2

2 a 144 cm^2 **b** 198 cm^2 **c** 1.35 m^2
d 2.6 m^2 **e** 70 mm^2 **f** 2.38 m^2
g 572 cm^2

3 a 196 mm^2 **b** 4.68 m^2 **c** 3285 cm^2
d 9.12 m^2 **e** 27.36 cm^2 **f** 0.9 km^2
g 2290.2 cm^2 **h** 176.7 mm^2 **i** 3.4 m^2

4 a 0.6 m^2 **b** 1963.5 mm^2 **c** 150 cm^2
d 1.2 m^2 **e** 113.1cm^2 **f** 7.1 m^2
g 2185 cm^2 **h** 3.99 m^2

5 a 1.375 m^2 **b** 63 cm^2 **c** 1.75 m^2
d 2.94 cm^2 **e** 6.665 m^2 **f** 684 m^2
g 210 cm^2 **h** 621 mm^2 **i** 8.37 m^2

6 a 1045.6 mm^2 **b** 113.5 cm^2 **c** 56.7 cm^2
d 4185.4 mm^2 **e** 7238.2 m^2 **f** 1.5 m^2

7 a 619 cm^2 **b** 172.5 cm^2 **c** 43.5 m^2

8 a 26.46 cm^2 **b** 1088.64 mm^2 **c** 7.75 m^2

9 a 370.2 cm^2 **b** 1 878 500 m^2 (1.8785 km^2)
c 179.0 cm^2

10 a 654.5 m^2 **b** 641.4 m^2
c 256 313.7 mm^2 **d** 2315.2 cm^2
e 141.5 mm^2 **f** 137.7 m^2

11 a 372.2 cm^2 **b** 38.79 m^2

12 a 11.56 cm^2 **b** 29.4 cm^2 **c** 1710.1 cm^2

13 15.0 cm

14 10.5 cm by 10.5 cm

15 a 26.24 m^2 **b** 194.34 m^2 **c** 63.99 m^2

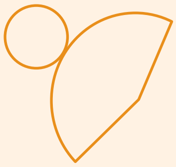
Exercise 4.2

1 a Cube or regular hexahedron
b Triangular prism
c Tetrahedron **d** Slant cone
e Hexagonal prism **f** Hexagonal pyramid
g Dodecahedron **h** Slant pentagonal pyramid

2 a Cube or regular hexahedron
b Rectangular prism
c Hexagonal prism **d** Cylinder
e Triangular prism **f** Pentagonal prism

3 Other arrangements of nets are possible.

a



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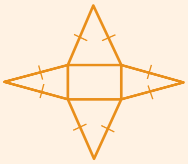

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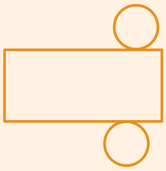

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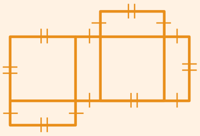

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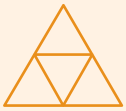

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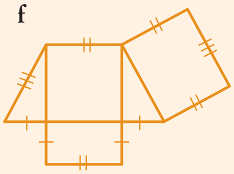

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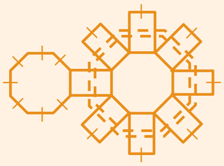
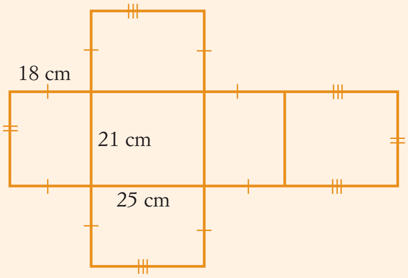


4 a



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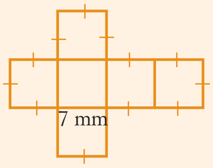

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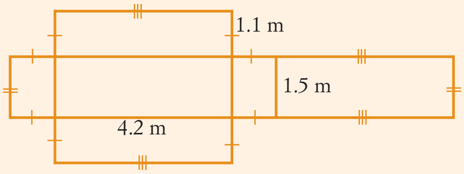

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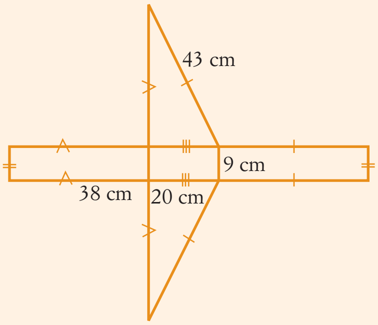

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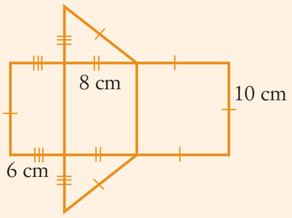

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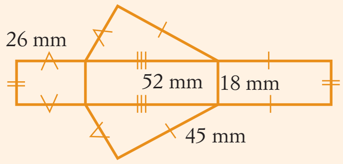

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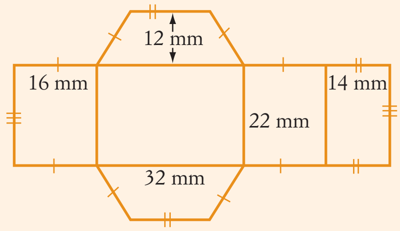

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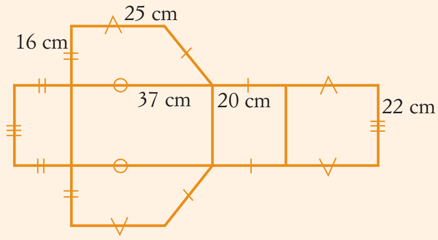

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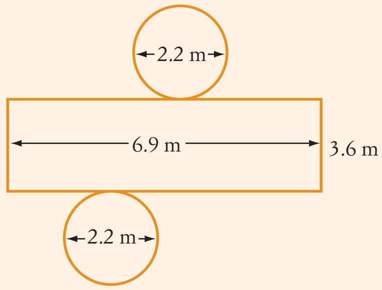

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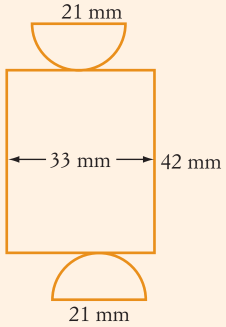

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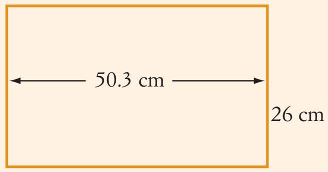
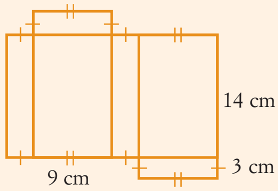


7 a



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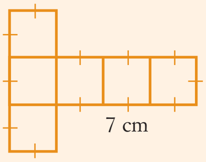

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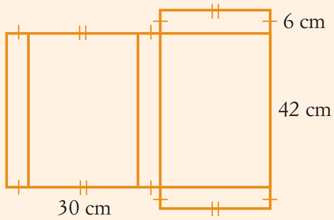
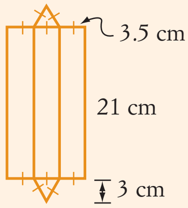


8 a



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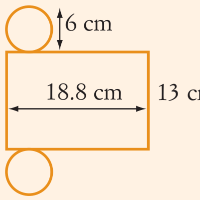

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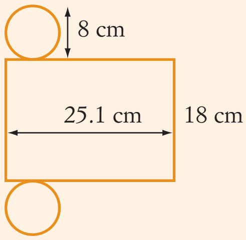

9 a 2706 cm^2 **b** 294 mm^2 **c** 25.14 m^2
d 1670.9 cm^2 **e** 288 cm^2 **f** 3358 mm^2

10 a 2268 mm^2 **b** 3148 cm^2

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11 a 32.5 m^2 **b** 1731.8 mm^2 **c** 1306.9 cm^2

12 a 390 cm^2 **b** 294 cm^2 **c** 3384 cm^2

13 a 231 cm^2 **b** 301.6 cm^2 **c** 552.9 cm^2

14 a 872 cm^2 **b** 1858 mm^2 **c** 12.46 m^2
d 5558 cm^2 **e** 7.612 m^2 **f** 1459.2 cm^2

15 a 483.8 cm^2 **b** 4222.3 mm^2 **c** 35.2 m^2
d 444.1 cm^2 **e** 45 521.7 cm^2 **f** 1212.3 cm^2

16 a 3487.2 cm^2 **b** 103.8 m^2
c 22 900.6 mm^2 **d** 32 716.5 cm^2
e 151 449.9 cm^2 **f** 9900.7 mm^2

17 a 1528 cm^2 **b** 22.74 m^2 **c** 896 mm^2
d 736 cm^2 **e** 319.74 m^2 **f** 226.44 cm^2

18 a 4632 cm^2 **b** 64.45 m^2 **c** 3076 mm^2

19 a 3966.3 cm^2 **b** 3732.2 mm^2
c 4.1 m^2 **d** 1558.2 cm^2

20 a 1360 cm^2 **b** 2525 mm^2 **c** 27.92 m^2

21 a 681.7 cm^2 **b** 233.3 mm^2
c 66.8 m^2 **d** 512.7 cm^2

22 a 314.2 cm^2 **b** 2463 mm^2
c 64.3 m^2 **d** 4356.6 cm^2
e 5.7 m^2 **f** 9351.7 mm^2

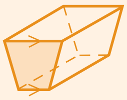
23 a 9000 mm^2 **b** 1003 cm^2 **c** 46.46 m^2

24 a 40.7 m^2 **b** 166.3 cm^2 **c** 3732.2 mm^2

25 a 6082.1 cm^2 **b** 4580.4 mm^2 **c** 109.0 m^2

26 Other shapes are possible.

a

b

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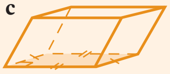
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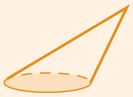
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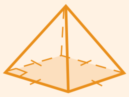
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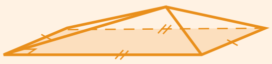
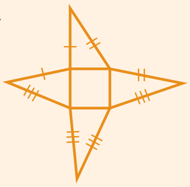


27 Other arrangements of nets are possible.

a

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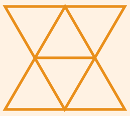
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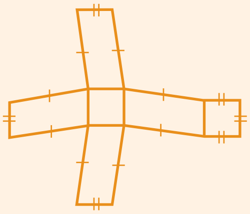
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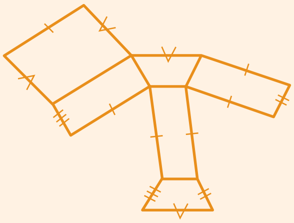
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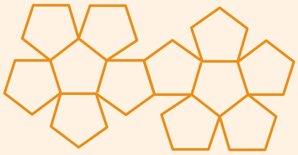
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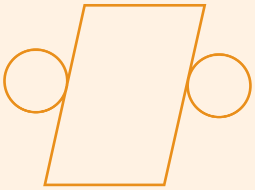

28 a Slant cylinder **b** Regular octahedron
c Regular icosahedron **d** Regular dodecahedron

29 a 7206.8 cm^2 **b** 54 788.5 mm^2
c 80.5 m^2 **d** 13 609.4 cm^2

30 172 m^2

31 a 11.4 m **b** 207.66 m^2
c 25.9 L is required so 7 tins are needed at a cost of $875.

32 1979.2 cm^2

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33 a 67.48 m^2 **b** \$3846.36

34 a 22.9 m^2 **b** 2.5 L

35 a 3.8 m × 6.5 m **b** \$1953.75

36 7 full revolutions (6.4)

Exercise 4.3

1 a 2.4 L **b** 3200 cm^3 **c** 7500 mL
d 0.73 L **e** 41 mL **f** 8200 L
g 432 cm^3 **h** 0.003 24 m^3 **i** 2.32 L

2 a 0.0054 kL **b** 27 000 cm^3 **c** 0.0043 kL
d 354 000 m^3 **e** 25 L **f** 840 000 cm^3

3

	Length	Width	Height	Volume
a	15 cm	12 cm	8 cm	1440 cm^3
b	4 m	3 m	10 m	120 m^3
c	17 mm	11 mm	22 mm	4114 mm^3
d	26.4 cm	17.5 cm	14.4 cm	6652.8 cm^3
e	6.7 m	3.2 m	1.5 m	32.16 m^3

4 a 27 cm^3 **b** 192 cm^3 **c** 0.648 m^3
d 19 800 cm^3 **e** 312 cm^3

5 a 19 974.2 cm^3 **b** 4712.4 mm^3 **c** 8.6 m^3
d 1125.4 cm^3 **e** 6.4 m^3 **f** 5695.7 mm^3

6 a 82.0 m^3 **b** 619.7 cm^3 **c** 3.927 m^3
d 2205.4 cm^3 **e** 3234 mm^3

7 a 179.6 cm^3 **b** 41 629.8 mm^3 **c** 8580.2 cm^3
d 10.3 m^3 **e** 190.9 cm^3

8 a 1026 cm^3 **b** 2.5498 m^3
c 39.294 cm^3

9 a 234 cm^3 **b** 1248 cm^3 **c** 0.8528 m^3
d 7290 cm^3 **e** 3.6 m^3 **f** 192 mm^3

10 a 110.25 cm^3 **b** 111.4 cm^3 **c** 500.3 cm^3

11 a 700 cm^3 **b** 288 m^3 **c** 17.3 cm^3

12 a 14 024.1 mm^3 **b** 27.2 m^3 **c** 7125.1 cm^3
d 3.2 m^3 **e** 130 765.7 cm^3 **f** 104.1 mm^3

13 a 58.3 cm^3 **b** 57.9 m^3 **c** 131.5 m^3

14 a 1.6 m^3 **b** 603.2 cm^3 **c** 7485.9 mm^3

15 a 1838.8 cm^3 **b** 25.5 m^3 **c** 14 365.5 mm^3

16 a 37 875.0 cm^3 **b** 7863.1 cm^3
c 867 415.1 cm^3 **d** 147 436.1 cm^3

17 a 1206.4 cm^3 **b** 1013.7 cm^3 **c** 86 295.1 cm^3

18 a 2805 cm^3 **b** 15 620 cm^3 **c** 8833.3 cm^3

19 a 2348.2 cm^3 **b** 14 678.1 cm^3 **c** 6.16 m^3

20 a 33 895.7 cm^3 **b** 1.7 m^3 **c** 410 103.0 mm^3

21 1308.3 cm^3

22 20 solids

23 a 2400 m^3 **b** 2400 kL **c** $16\frac{2}{3}$ hours

24 a 50 **b** 500 **c** About 52%

25 a About 1413 times. **b** About 4878 km.

Chapter review

1 a 0.0567 km^2 **b** 38 700 m^2
c 0.57 km^2 **d** 2.5 ha

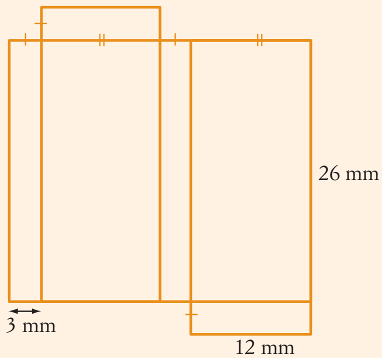
2 a 289 cm^2 **b** 360 cm^2 **c** 338.12 m^2
d 224 cm^2 **e** 331.5 mm^2

3 136 mm^2

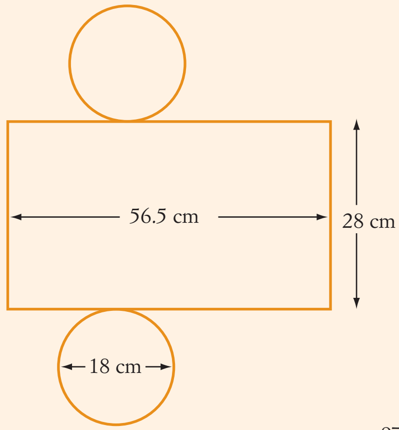
4 a Cylinder **b** Square pyramid **c** Pentagonal prism

5 a Rectangular pyramid **b** Cone

6 a



b



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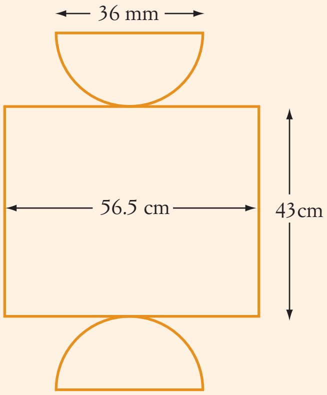
c



7 a 2750 mL **b** 5430 L
c 271 cm^3 **d** 0.003 600 kL
e 49 000 cm^3 **f** 0.002 800 kL

8 a 204.1 cm^2 **b** 52.8 mm^2

9 2830 cm^2

10 44.6 cm^2

11 a 304.8 cm^2 **b** 10.9 m^2

12 a $m^2 = p^2 + q^2$ **b** $p = 1.4$

13 2.16 m^2

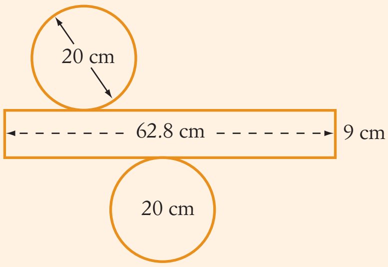
14 852 mm^2

15 2092.3 cm^2

16 3449.5 mm^2

17 a 1298 cm^2 **b** 1140 mm^2 **c** 25.12 m^2

18 a



b 1193.8 cm^2

19 a 678.6 cm^2 **b** 622.0 mm^2

20 a 523.1 cm^2 **b** 41.2 m^2

21 a 16 400.7 cm^2 **b** 24.7 m^2

22 a 480 cm^2 **b** 650.3 mm^2
c 36.3 m^2 **d** 1592.8 cm^2

23 a 4216 cm^2 **b** 23.3 m^2
c 3631.7 mm^2 **d** 3402.3 cm^2

24 a 27 cm^3 **b** 192 cm^3
c 19 974.2 mm^3 **d** 82 m^3
e 3.927 m^3 **f** 22.4 cm^3

25 77 cm^3

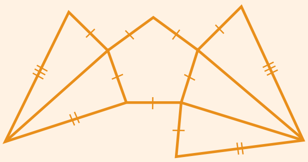
26 540 cm^3

27 2.288 m^3

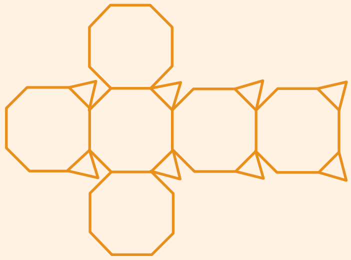
28 a 112.2 m^3 **b** 6792.9 cm^3
c 15 239.9 mm^3 **d** 5111.3 m^3
e 22 066.6 cm^3 **f** 139.1 m^3

29 a Slant rectangular pyramid
b Octahedron (truncated tetrahedron)

30 a



b



31 17 mm

32 24.4 m

33 96.7 m^2

34 68 204 cm^3

35 122.6 cm^3

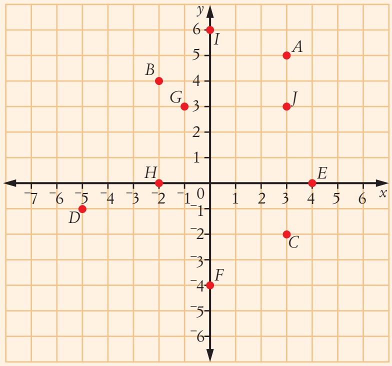
36 No, the water will weigh about 6.9 tonnes.

37 a 127.8 m^3 **b** 8 h 45 min.

Chapter 5

Exercise 5.1

1



2 **a**, **d**, **f**, **g** are rising; **b**, **c**, **h**, **i**, are falling; **e** and **j** are neither.

3 **a** and **h** are in the first quadrant; **b, d** and **g** are in the second quadrant; **c** and **e** are in the third quadrant; **f** is in the fourth quadrant.

4 **a** $(2, \frac{1}{2})$ **b** $(^-1, 3)$ **c** $(^-\frac{1}{2}, {}^-2)$
d $(1, {}^-1\frac{1}{2})$ **e** $(1, 1)$ **f** $(^-3, 1\frac{1}{2})$
g $(^-3\frac{1}{2}, \frac{1}{2})$ **h** $(^-1, \frac{1}{2})$

5 **a** $\sqrt{8} \approx 2.83$ units **b** $\sqrt{52} \approx 7.21$ units
c 13 units **d** $\sqrt{274} \approx 16.55$ units
e $\sqrt{162} \approx 12.73$ units **f** $\sqrt{162} \approx 12.73$ units
g $\sqrt{61} \approx 7.81$ units **h** $\sqrt{148} \approx 12.17$ units
i $\sqrt{73} \approx 8.54$ units **j** $\sqrt{233} \approx 15.26$ units

6 **a** $(5, 1)$ **b** $\left(^-1, \frac{1}{2}\right)$ **c** $(0, 6)$ **d** $\left(\frac{1}{2}, \frac{1}{2}\right)$
e $(^-5, 5)$ **f** $\left(\frac{1}{2}, 1\frac{1}{2}\right)$ **g** $\left(^-3\frac{1}{2}, 2\frac{1}{2}\right)$ **h** $\left(4\frac{1}{2}, {}^-2\right)$
i $\left(^-4\frac{1}{2}, {}^-8\right)$ **j** $\left(\frac{1}{2}, {}^-3\right)$

7 **a** 2 **b** $^-1$ **c** $^-\frac{2}{3}$
d $\frac{1}{2}$ **e** 0 **f** 3
g $\frac{1}{3}$ **h** $^-\frac{1}{3}$ **i** $^-3$
j Not defined (division by 0)

8 **a** 3 **b** $^-2$ **c** 2
d 7 **e** $\frac{1}{8}$ **f** $^-2$
g $^-1$ **h** 1 **i** 0
j $^-\frac{3}{4}$ **k** Not defined (division by 0)
l $^-\frac{3}{2}$

9 **a** **i** $\sqrt{41}$ **ii** $(3.5, 5)$ **iii** $\frac{4}{5}$
b **i** $\sqrt{18}$ **ii** $(5.5, 2.5)$ **iii** $^-1$
c **i** $\sqrt{29}$ **ii** $(^-4, 3.5)$ **iii** $^-2\frac{1}{2}$
d **i** $\sqrt{170}$ **ii** $(1.5, 2.5)$ **iii** $^-\frac{1}{13}$
e **i** $\sqrt{58}$ **ii** $(^-3.5, 3.5)$ **iii** $^-\frac{3}{7}$
f **i** $\sqrt{130}$ **ii** $(^-3.5, 5.5)$ **iii** $\frac{7}{9}$
g **i** $\sqrt{392}$ **ii** $(2, 1)$ **iii** $^-1$
h **i** $\sqrt{50}$ **ii** $(8.5, {}^-0.5)$ **iii** 1
i **i** $\sqrt{37}$ **ii** $(^-4.5, {}^-4)$ **iii** $^-6$
j **i** $\sqrt{34}$ **ii** $(^-5.5, 0.5)$ **iii** $^-\frac{3}{5}$
k **i** $\sqrt{41}$ **ii** $(^-2.5, {}^-2)$ **iii** $^-\frac{4}{5}$
l **i** $\sqrt{148}$ **ii** $(0, 0)$ **iii** $^-\frac{1}{6}$

10 B is $(^-12, {}^-3)$.

11 Q is $(11, 4)$.

12 R is $(2, {}^-13)$.

13 $\sqrt{41}$, $\sqrt{10}$, $\sqrt{29}$, $\sqrt{85}$ and $\sqrt{37}$ units.

14 Sides: $\sqrt{10}$, 3, $\sqrt{34}$ and $\sqrt{5}$ units. Diagonals both 5 units.

15 **a** Both $(1, 0)$. **b** Parallelogram

16 18.75 units^2

17 $D = (^-3, {}^-4)$

18 **a** $D\left(2, \frac{1}{2}\right)$, $E\left(\frac{1}{2}, 2\frac{1}{2}\right)$
b $m_{DE} = m_{BC} = {}^-\frac{4}{3}$
c $DE = \frac{5}{2}$ units, $BC = 5$ units.

19 **a** $\sqrt{13}$, $\sqrt{13}$ and $\sqrt{26}$ units.
b Isosceles triangle

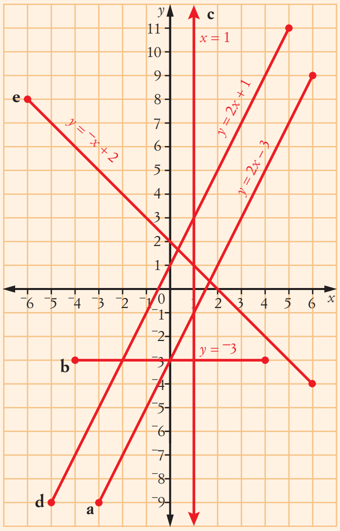
20 **a** $\sqrt{20}$, $\sqrt{20}$, $\sqrt{20}$ and $\sqrt{20}$ units.
b Rhombus

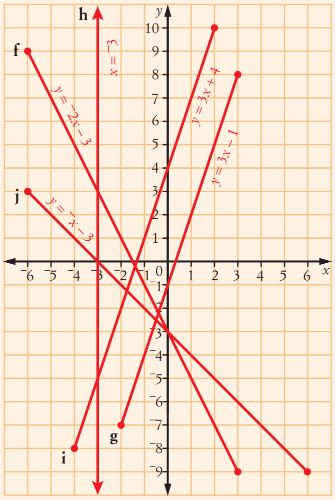
21 $(^-5, {}^-3)$, $(5, {}^-1)$ or $(3, 3)$.

Exercise 5.2

1 **a** xi **b** vii **c** viii
d v **e** i **f** ii
g ix **h** xii **i** x

2





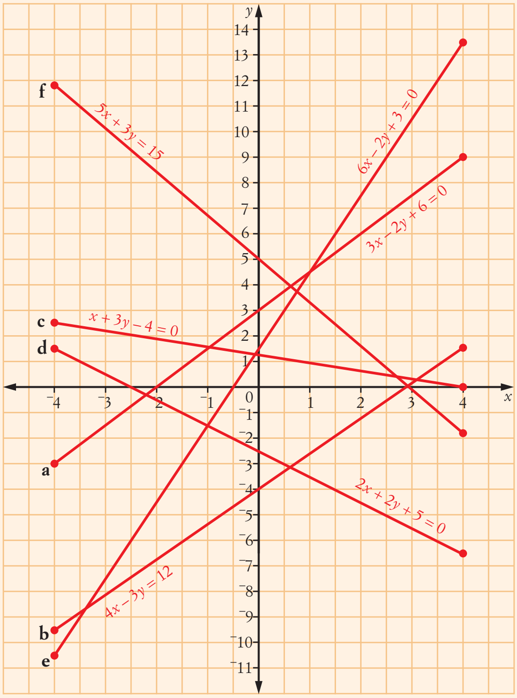
Answers are in the order: x-intercept, y-intercept.

a $\left(1\frac{1}{2}, 0\right)$ and $(0, {}^{-}3)$.
b No x-intercept, $(0, {}^{-}3)$.
c $(1, 0)$, no y-intercept.
d $\left({}^{-}\frac{1}{2}, 0\right)$ and $(0, 1)$.
e $(2, 0)$ and $(0, 2)$.
f $\left({}^{-}1\frac{1}{2}, 0\right)$ and $(0, {}^{-}3)$.
g $\left(\frac{1}{3}, 0\right)$ and $(0, {}^{-}1)$.
h $({}^{-}3, 0)$, no y-intercept.
i $\left({}^{-}1\frac{1}{3}, 0\right)$ and $(0, 4)$.
j $({}^{-}3, 0)$ and $(0, {}^{-}3)$.

3

a	Yes	No	No	Yes	No
b	No	Yes	Yes	No	Yes
c	No	No	No	Yes	Yes
d	No	No	Yes	No	Yes
e	No	No	Yes	No	Yes

4



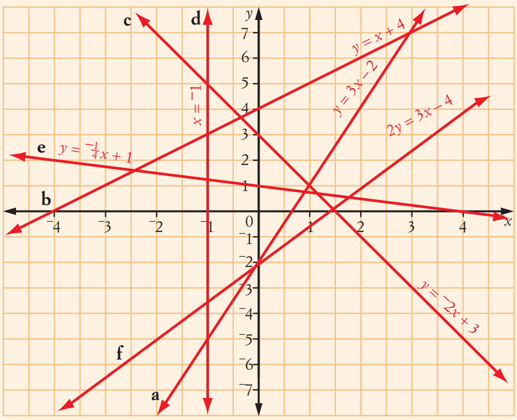
Answers are in the order: x-intercept, y-intercept.

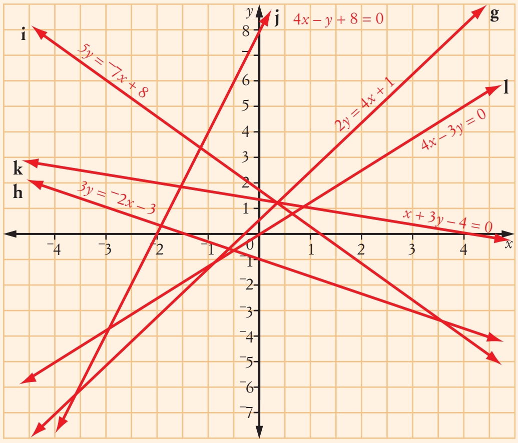
a $({}^{-}2, 0)$ and $(0, 3)$
b $(3, 0)$ and $(0, {}^{-}4)$
c $(4, 0)$ and $\left(0, 1\frac{1}{3}\right)$
d $\left({}^{-}2\frac{1}{2}, 0\right)$ and $\left(0, {}^{-}2\frac{1}{2}\right)$
e $\left(\frac{{}^{-}1}{2}, 0\right)$ and $\left(0, 1\frac{1}{2}\right)$
f $(3, 0)$ and $(0, 5)$

5
a $y = 4$ **b** $y = 5$ **c** $y = {}^{-}1$
d $y = 5$ **e** $y = 0$ **f** $y = {}^{-}4$
g $y = 1$ **h** $y = 7$ **i** $y = {}^{-}3$
j $y = {}^{-}8$ **k** $y = 3$ **l** $y = 1$

6
a $x = {}^{-}2$ **b** $x = 3$ **c** $x = 2$
d $x = 0$ **e** $x = 4$ **f** $x = {}^{-}3$
g $x = 6$ **h** $x = {}^{-}4$ **i** $x = 7$
j $x = {}^{-}8$ **k** $x = 3$ **l** $x = 5$

7





Intercepts are in the order: x-intercept, y-intercept.

a	**i** 3	**ii** $\frac{2}{3}, {}^{-}2$	
b	**i** 1	**ii** ${}^{-}4, 4$	
c	**i** ${}^{-}2$	**ii** $1\frac{1}{2}, 3$	
d	**i** Not defined	**ii** ${}^{-}1$, no y-intercept	
e	**i** ${}^{-}\frac{1}{4}$	**ii** $4, 1$	
f	**i** $1\frac{1}{2}$	**ii** $1\frac{1}{3}, {}^{-}2$	
g	**i** 2	**ii** ${}^{-}\frac{1}{4}, \frac{1}{2}$	
h	**i** ${}^{-}\frac{2}{3}$	**ii** ${}^{-}1\frac{1}{2}, {}^{-}1$	
i	**i** ${}^{-}1\frac{2}{5}$	**ii** $1\frac{1}{7}, 1\frac{3}{5}$	
j	**i** 4	**ii** ${}^{-}2, 8$	
k	**i** ${}^{-}\frac{1}{3}$	**ii** $4, 1\frac{1}{3}$	
l	**i** $1\frac{1}{3}$	**ii** $0, 0$	

8 a $y = 2x + 4$ **b** $y = {}^{-}3x + 1$
c $y = 5x - 6$ **d** $y = 3x - 2$
e $y = {}^{-}3$ **f** $y = {}^{-}4x$

9 a $4x - y - 1 = 0$ **b** $2x - 3y - 6 = 0$
c $4x + 3y + 3 = 0$ **d** $5x - 2y + 4 = 0$
e $10x - 3y - 12 = 0$ **f** $7x + 4y - 5 = 0$

10 Answers are in the order: gradient, y-intercept.
a $2, 5$ **b** ${}^{-}3, 5$ **c** ${}^{-}1, 1$
d ${}^{-}7, 4$ **e** $1, 2$ **f** ${}^{-}8, 0$
g $\frac{1}{2}, \frac{1}{2}$ **h** ${}^{-}7, 14$ **i** $\frac{1}{3}, 2$
j ${}^{-}\frac{4}{3}, 2\frac{2}{3}$ **k** $\frac{7}{3}, 0$

11 a $y = 2x + 1, 2x - y + 1 = 0$
b $y = -x + 3, x + y - 3 = 0$
c $y = x + 1, x - y + 1 = 0$
d $y = {}^{-}4x - 8, 4x + y + 8 = 0$
e $y = {}^{-}\frac{3}{4}x + 6, 3x + 4y - 24 = 0$
f $y = \frac{7}{3}x + 3, 7x - 3y + 9 = 0$

12 $x = 2$

13 $y = {}^{-}\frac{2}{3}x + 4, 2x + 3y - 12 = 0$

14 a $x + 2y - 2 = 0, y = {}^{-}\frac{1}{2}x + 1$
b $4x - 3y + 12 = 0, y = \frac{4}{3}x + 4$
c $x - 2y - 4 = 0, y = \frac{1}{2}x - 2$
d $4x - y + 4 = 0, y = 4x + 4$
e $8x - 3y - 24 = 0, y = \frac{8}{3}x - 8$
f $3x + 5y + 15 = 0, y = {}^{-}\frac{3}{5}x - 3$
g $x - 5y - 5 = 0, y = \frac{1}{5}x - 1$
h $3x - 2y + 12 = 0, y = \frac{3}{2}x + 6$
i $x + y + 4 = 0, y = {}^{-}x - 4$
j $x + y - 2 = 0, y = {}^{-}x + 2$

15 a 2 **b** $(0, 4)$ **c** $2x - y + 4 = 0$

16 a $3x + y - 18 = 0, y = {}^{-}3x + 18$
b $x - y - 2 = 0, y = x - 2$
c $2x - y + 3 = 0, y = 2x + 3$
d $3x + y + 6 = 0, y = {}^{-}3x - 6$
e $4x + y - 16 = 0, y = {}^{-}4x + 16$
f $x + y - 3 = 0, y = -x + 3$
g $x - y - 1 = 0, y = x - 1$
h $4x - y - 14 = 0, y = 4x - 14$
i $5x + 8y + 17 = 0, y = {}^{-}\frac{5}{8}x - 2\frac{1}{8}$
j $2x + y + 10 = 0, y = {}^{-}2x - 10$
k $2x - y = 0, y = 2x$
l $2x - y - 13 = 0, y = 2x - 13$

17 a $2x - y - 4 = 0$ **b** $2x + 3y + 1 = 0$
c $2x - 3y - 7 = 0$ **d** $x = 2$
e $4x - y + 2 = 0$ **f** $2x + y - 10 = 0$

18 a Parallel **b** Parallel **c** Not parallel
d Parallel **e** Parallel **f** Not parallel

For questions 19–22, the equation of the parallel line is given first.

19 $3x - 5y + 31 = 0, 5x + 3y - 5 = 0$

20 $4x - y - 24 = 0, x + 4y + 11 = 0$

21 $3x + 2y - 4 = 0, 2x - 3y + 19 = 0$

22 $x - 4y - 19 = 0, 4x + y + 9 = 0$

23 $m_1 = m_2 = {}^{-}\frac{2}{3}$

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24 $m_1 \times m_2 = {}^{-}\frac{4}{5} \times \frac{5}{4} = {}^{-}1$

25 $m_{AB} \times m_{AC} = \frac{5}{2} \times {}^{-}\frac{2}{5} = {}^{-}1$; Area $= 29$ units^2

26 $m_{LK} = m_{MN} = \frac{3}{2}$; $m_{KN} = m_{LM} = {}^{-}\frac{2}{3}$;
$m_{LK} \times m_{KN} = {}^{-}1$; $LK = MN = \sqrt{13}$ units;
$KN = LM = 3\sqrt{13}$ units. Area $= 39$ units^2

27 $m_{HI} = m_{KJ} = {}^{-}3$, $m_{IJ} = m_{HK} = \frac{1}{3}$;
$m_{HI} \times m_{IJ} = {}^{-}3 \times \frac{1}{3} = {}^{-}1$;
and $HI = IJ = JK = KH = \sqrt{10}$ units.

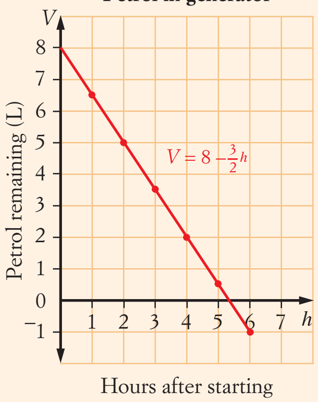
28 $m_{JK} \times m_{KL} = \frac{1}{2} \times {}^{-}2 = {}^{-}1$

29 a

h	0	1	2	3	4	5	6
V	8	6.5	5	3.5	2	0.5	$^{-}1$

b

Petrol in generator



c Linear function

d After $3\frac{1}{3}$ hours.

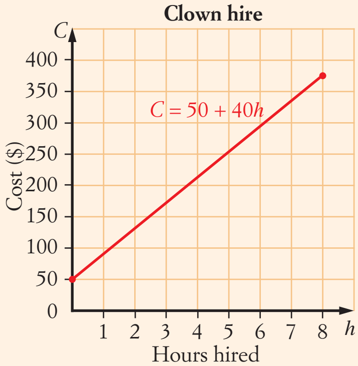
e 5 L

f The volume of petrol in the generator before it is started.

g The number of hours when the generator runs out of petrol.

30 a

Clown hire



b Linear function.

c \$170

d 3.75 hours

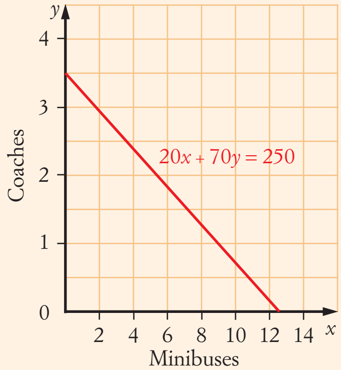
e The cost of hiring the clown regardless of how many hours he or she works. It is sometimes called a service fee or call-out fee.

f The hourly rate of pay for the clown.

31 a $20x + 70y$ pupils

b $20x + 70y = 250$

c Buses to carry 250 pupils

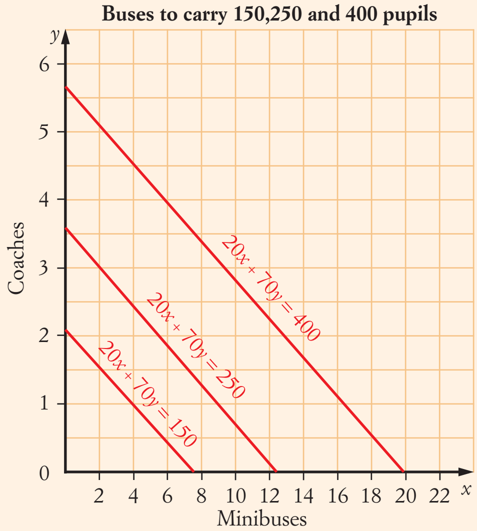


d 9 minibuses and 1 coach, or 2 minibuses and 3 coaches.

e There are other possibilities but these would carry approximately 250 pupils.

f

Buses to carry 150,250 and 400 pupils

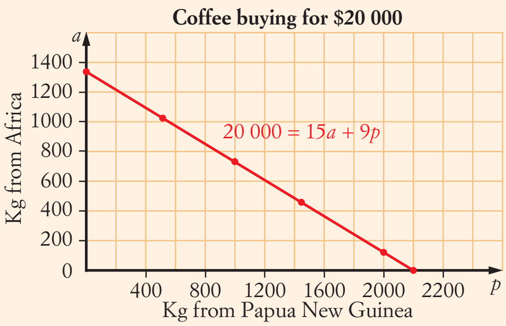


g The graphs are parallel.

32 a $20\,000 = 15a + 9p$

b

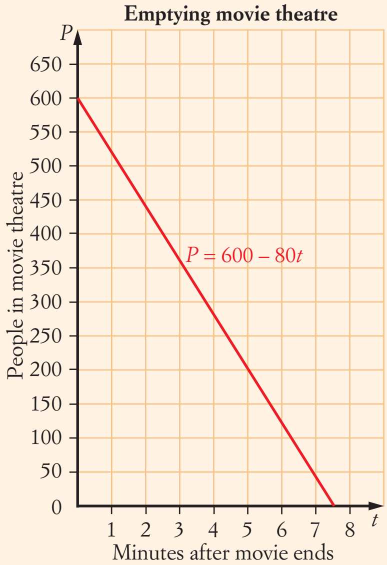
p	0	500	1000	1500	2000
a	1333	1033	733	433	133



c About 850 kg of African coffee.

33 a $P = 600 - 80t$

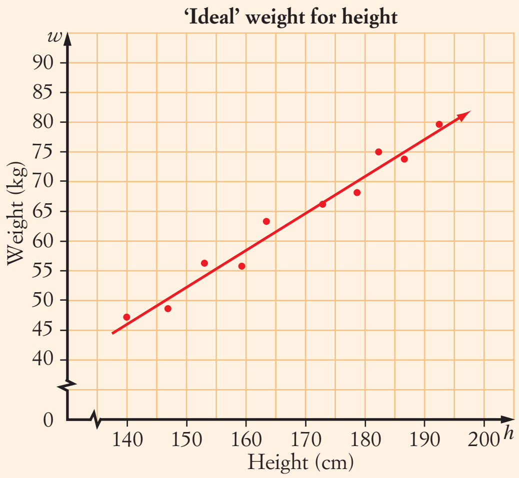
b



c About 3.8 minutes.

34 $4x - y + 17 = 0$

35 a



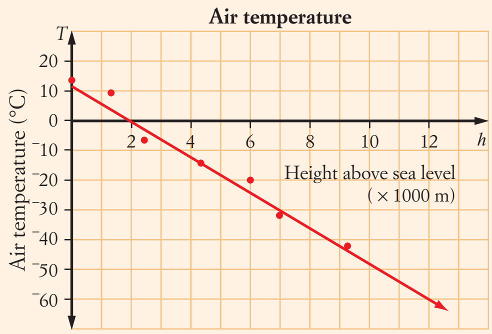
b About 65 kg.

c About 202 cm.

d $w = 0.63h - 43$ (Answers may vary slightly).

e 64.1 kg (using equation) and 65 kg (from graph); 203 cm (using equation) and 202 cm (from graph).

36 a



b About 17°C.

c About 11 100 m.

d $T = {}^{-}0.0063h + 15$ (Answers may vary slightly).

e $^{-}35.4$°C

f 11 900 m

Exercise 5.3

1 Straight lines: **a**, **b**, **d**, **m**, **i**, **k**
Parabolas: **e**, **f**, **j**, **o**
Circles: **c**, **h**
Exponentials: **g**, **n**
None: **l**

2

a E	**b** K	**c** J
d D	**e** I	**f** F
g B	**h** A	**i** G
j C	**k** L	**l** H

3

a F	**b** H	**c** I
d C	**e** E	**f** D
g G	**h** B	**i** A

4 a $y = x^2$ **b** $y = -x^2$

c $y = \frac{1}{2}x^2$ **d** $y = \frac{1}{5}x^2$

e $y = \frac{1}{3}x^2$ **f** $y = 0.25x^2$

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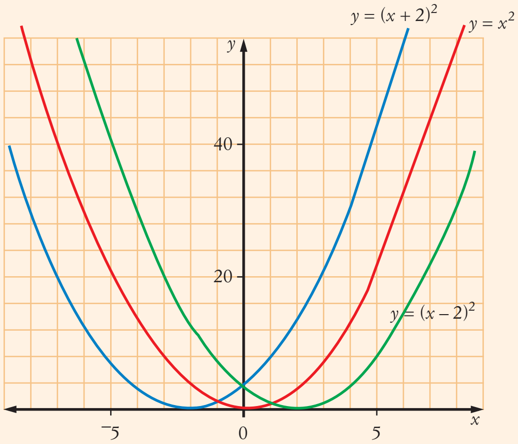
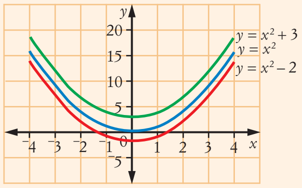
5 a



b If $a > 0$, the graph shifts to the left. If $a < 0$, the graph shifts to the right.

6 a



b **i** For all: $x = 0$
For parts **ii** and **iii**, answers are stated in the order: $y = x^2$, $y = x^2 + 3$, $y = x^2 + {}^-2$

ii $(0, 0)$, $(0, 3)$, $(0, {}^-2)$

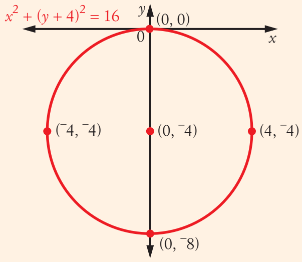
iii $0, 3, {}^-2$

c When $a > 0$, the graph is shifted up compared with the graph of $y = x^2$.
When $a < 0$, the graph is shifted down compared with the graph of $y = x^2$.

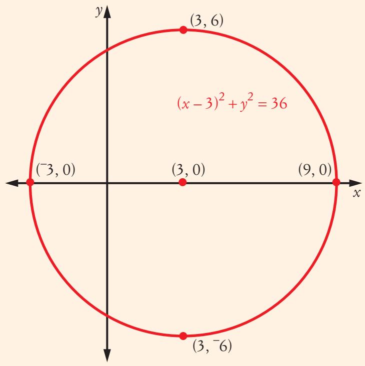
7 a $(x - 4)^2 + (y - 5)^2 = 9$
b $(x + 2)^2 + (y - 4)^2 = 64$
c $(x - 3)^2 + (y + 6)^2 = 25$
d $x^2 + y^2 = 81$
e $(x + 3)^2 + (y + 5)^2 = 4$

8 a **i** $(0, 0)$ **ii** 4
b **i** $(0, 0)$ **ii** 1
c **i** $({}^-2, 7)$ **ii** 5
d **i** $({}^-5, {}^-3)$ **ii** 3
e **i** $(6, {}^-9)$ **ii** 11
f **i** $(4, 8)$ **ii** 8
g **i** $({}^-3, {}^-2)$ **ii** 10
h **i** $({}^-10, 4)$ **ii** 15

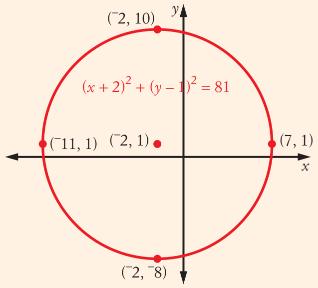
9 a



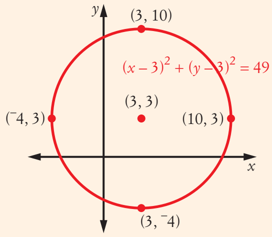
b



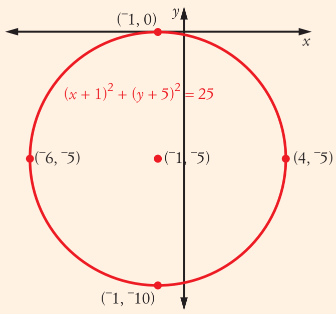
c



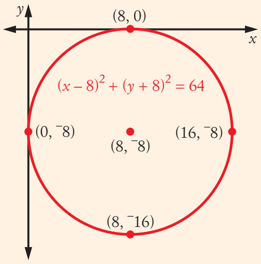
d



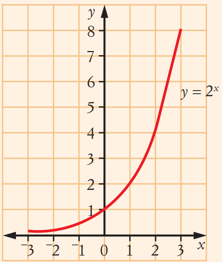
e



f



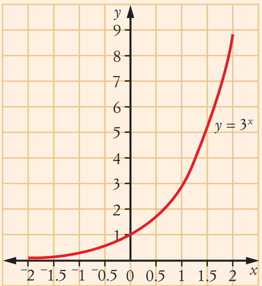
10 a



b **i** 1 **ii** 2 **iii** 1.4 **iv** 3

c **i** 0 **ii** 2 **iii** 2.8 **iv** $^{-}0.3$

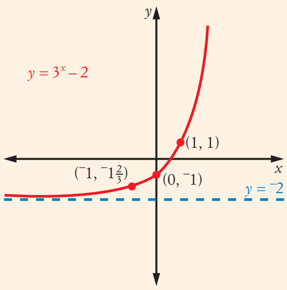
11 a



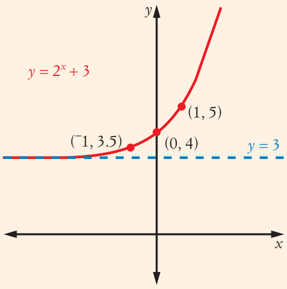
b **i** 1 **ii** 5.2 **iii** 1.9 **iv** 7.2

c **i** 1 **ii** 1.5 **iii** $^{-}0.8$ **iv** 0.9

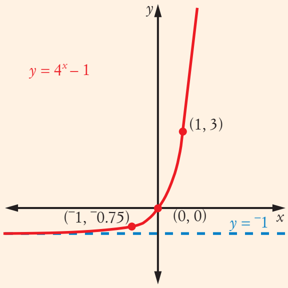
12 a



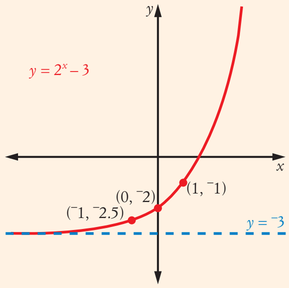
b



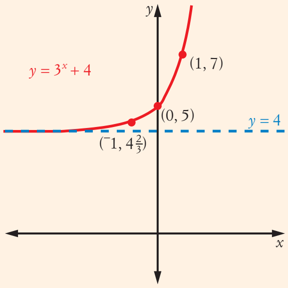
c



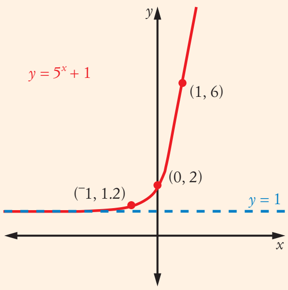
d



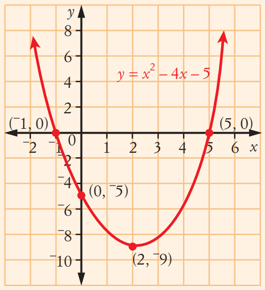
e



f



13 a **i**



ii Minimum

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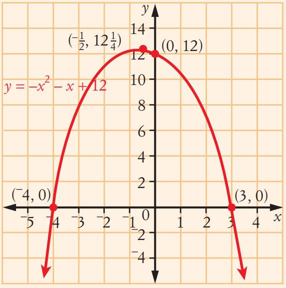
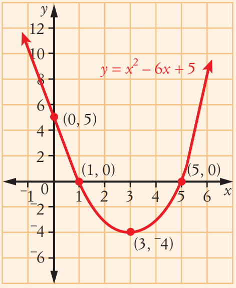
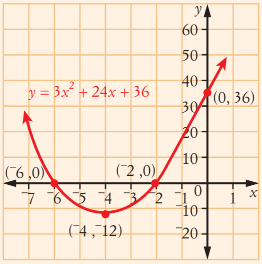
b **i** 

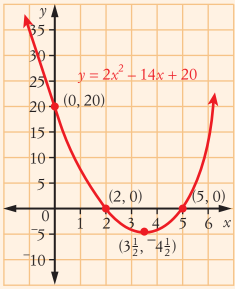
ii Maximum

c **i** 

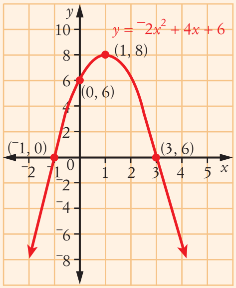
ii Minimum

d **i** 

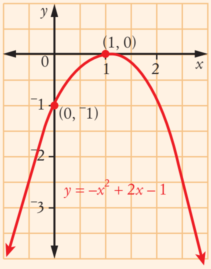
ii Minimum

e **i** 

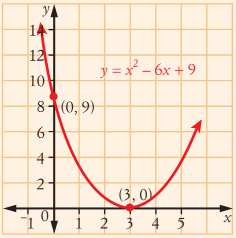
ii Minimum

f **i** 

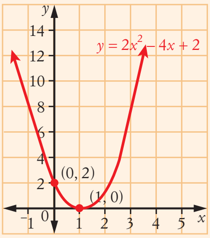
ii Maximum

g **i** 

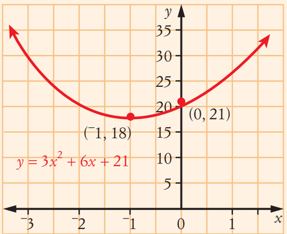
ii Maximum

h **i** 

ii Minimum

i **i** 

ii Minimum

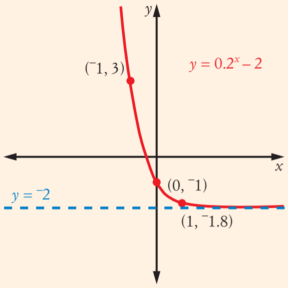
j **i** 

ii Minimum

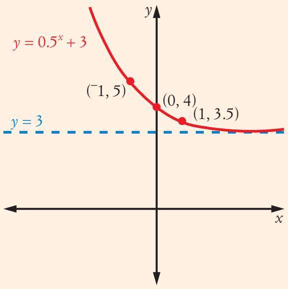
14 a $y = x^2$ **b** $y = -x^2$ **c** $y = x^2 - 2$
d $y = -x^2 + 3$ **e** $y = x^2 + \frac{1}{2}$ **f** $y = x^2 - \frac{2}{3}$
g $y = -x^2 - 6$

15 a A **b** C **c** None
d None **e** E

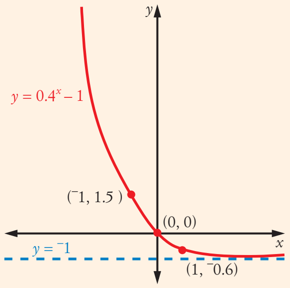
16 a



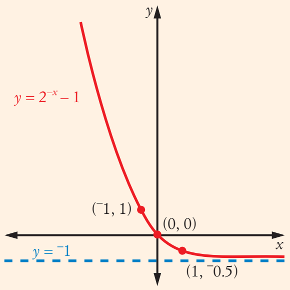
b



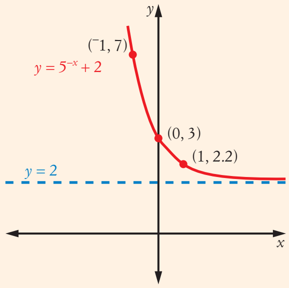
c



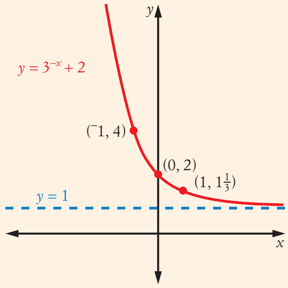
d



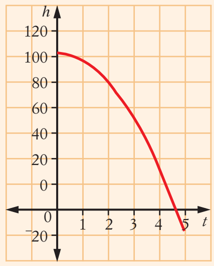
e



f



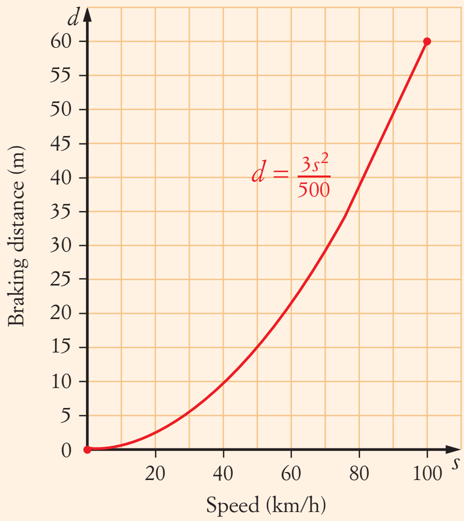
17 a



b 105 m **c** 19.2 m **d** 4.7 s

18

Car braking



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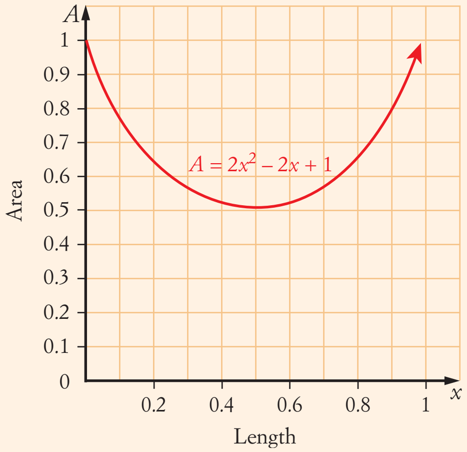
a About 9 m (8.7 m).
b About 34 m (33.8 m).
c About 71 km/h (70.7 km/h).

19 a On **b** Inside **c** Outside
d Find the distance to the centre of the circle and compare it with the radius.

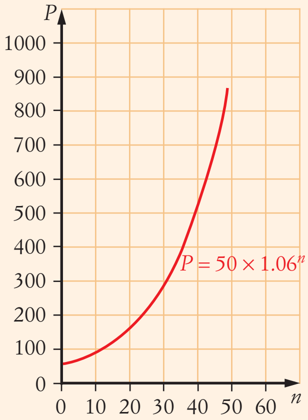
20 a $ER = 1 - x$ and $ER = FS = GP = DQ$
b Use Pythagoras' theorem to show that the side of the smaller square measures $\sqrt{2x^2 - 2x + 1}$.
c

Area of Smaller square



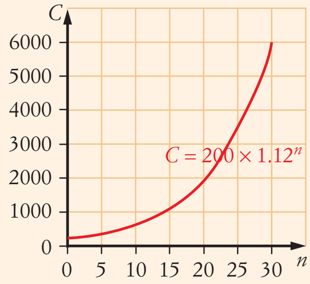
d $x = 0.5$ unit, $A = 0.5$ unit^2

21 a 50
b About 101.
c About 287.
d



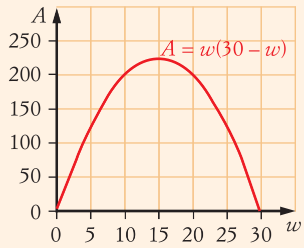
e About 39.5 months.

22 a 200 **b** About 621. **c** About 1929.
d



e About 24 weeks.

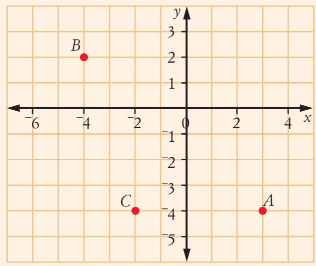
23 a $l = 30 - w$
b $A = w(30 - w)$
c



d 15 m by 15 m

Chapter review

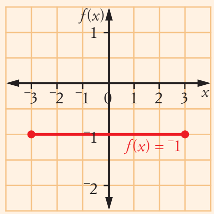
1



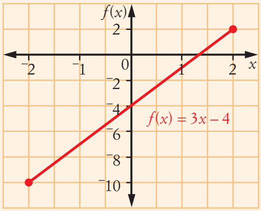
2 a $^{-}\frac{3}{2}$ **b** 0 **c** 1 **d** $^{-}4$

3 a iii **b** iv **c** i **d** ii

4

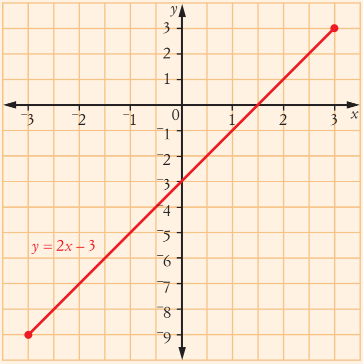


5



6 **a** Linear **b** Linear
c Not linear **d** Not linear

7



x-intercept = (1.5, 0), y-intercept = $(0, {}^{-}3)$.

8 Not on the line.

9 $y = 7$

10 **a** ii **b** i **c** iv **d** iii

11 13 units

12 $(\frac{1}{2}, {}^{-}2)$

13 $^{-}2$

14 $y = 3x - 1$ or $3x - y - 1 = 0$

15 Gradient = $\frac{1}{2}$, y-intercept = (0, 2)

16 $y = 3x + 11$, $3x - y + 11 = 0$

17 $2x + 3y - 5 = 0$

18 $y = {}^{-}x + 1$, $x + y - 1 = 0$

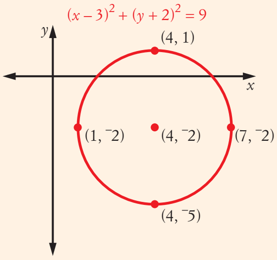
19 Perpendicular

20 $y = 3x - 2$ or $3x - y - 2 = 0$

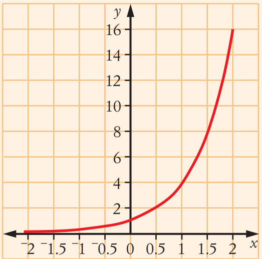
21 $3x - y - 2 = 0$ (parallel), $x + 3y + 16 = 0$ (perpendicular)

22 $(x + 1)^2 + (y - 3)^2 = 16$

23

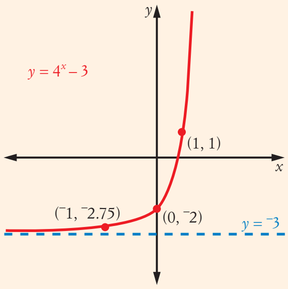


24 **a**

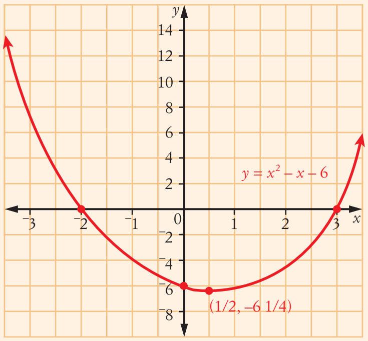


b 3
c 1.3

25



26 **a**



b Minimum

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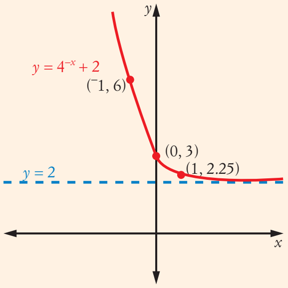
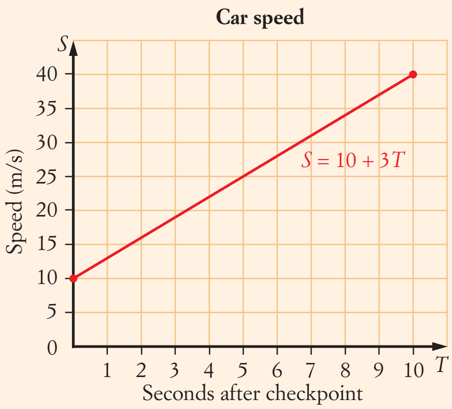
27



28 $B = (2, 1)$

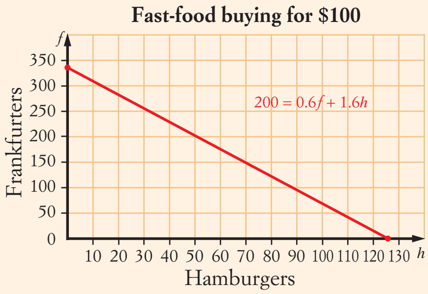
29 a



b After about 6.7 seconds.
c About 22 m/s.

30 a $200 = 0.6f + 1.6h$
b

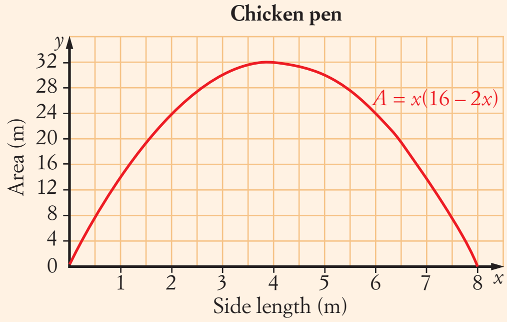
h	0	25	50	75	100	125
f	333	267	200	133	67	0



c 146 frankfurters.

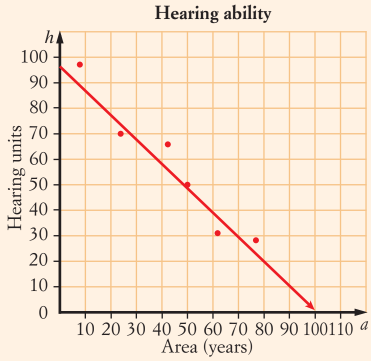
31 a $A = x(16 - 2x)$

b



c 32 m^2 **d** 4 m by 8 m

32 a



b $h = -a + 100$
c About 70 hearing units.
d About age 80.

Chapter 6

Exercise 6.1

1 a 0.15 **b** 3000 letters

2 a Brisbane 0.29, Sydney 0.417, Melbourne 0.513, Adelaide 0.497, Perth 0.503, Darwin 0.02, Hobart 0.483.
b About 47. **c** About 65.

3 a 0.121 **b** 0.879
c About 48 pearls. **d** About 3314 oysters.

4 a 0.2 **b** 0.5

5 a $\frac{1}{5} = 0.2$ **b** $\frac{8}{15} \approx 0.533$

6 a Cricket **b** $\frac{3}{10} = 0.3$ **c** About 40 times.

7 a $\frac{1}{4} = 0.25$ **b** 4 **c** $\frac{1}{13} \approx 0.077$
d $\frac{3}{13} \approx 0.231$ **e** $\frac{1}{13} \approx 0.077$ **f** $\frac{9}{13} \approx 0.692$
g ≈ 14

8 a nearly 0 **b** in between **c** in between
d in between **e** nearly 1 **f** nearly 0

9 **a** {SuncapZinc: RP, RY, YP, YY, BP, BY}
b $\frac{1}{6} \approx 0.167$ (yellow)

10 18.49%

11 **a** 0.14 **b** About 6 bulls-eyes.
c About 179 arrows.

12 **a** 0.000 82 **b** About 71 40-year-old men.
c \$536.67 **d** \$683.33
e It allows about 80% for costs and profit, so it is reasonable.
f It allows only about 48% for costs and profit so it is quite good.

13 **a** {G1G2, G1G3, G1Y1, G1Y2, G2G3, G2Y1, G2Y2, G3Y1, G3Y2, Y1Y2}
b Unmatched socks **c** $\frac{3}{10} = 0.3$ **d** $\frac{1}{10} = 0.1$

14 **a** 56.83% **b** 3.63% **c** 0.39%

15 The calculation is done by working out where the centre of the coin lands. Since the table is divided into the squares, this includes the whole area of the table. If the squares didn't go right to the edge it would make a difference. If you were allowed another go if your coin fell off, it would increase your chance of winning. For example, if the table was only 5 cm square, your probability would go up to about 33.6%, but for larger tables it would not go up as much.

Exercise 6.2

1 $\frac{1}{4}$

2 $\frac{9}{25} = 0.36$

3 $\frac{1}{8}$

4 $\frac{1}{16}$

5 $\frac{1}{216}$

6 **a** $\frac{1}{16}$ **b** $\frac{1}{64}$

7 $\frac{12}{25} = 0.48$

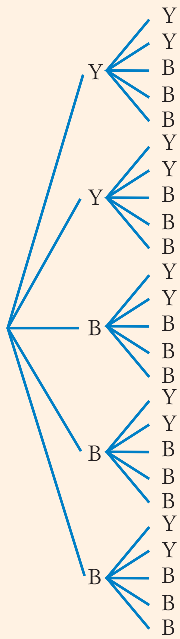
8 $\frac{10}{13}$

9

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10

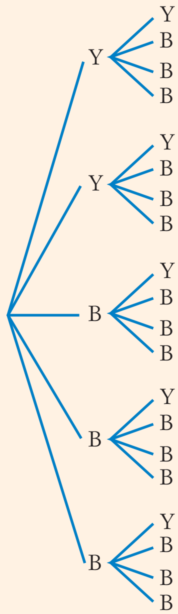
a $\frac{1}{12}$ **b** $\frac{3}{8}$ **c** $\frac{1}{24}$ **d** $\frac{5}{12}$
e $\frac{1}{6}$ **f** $\frac{2}{3}$ **g** $\frac{3}{24}$ **h** $\frac{17}{24}$

10 **a**



b $\frac{4}{25} = 0.16$ **c** $\frac{9}{25} = 0.36$ **d** $\frac{12}{25} = 0.48$

11 **a**



b $\frac{1}{10} = 0.1$ **c** $\frac{3}{10} = 0.3$ **d** $\frac{3}{5} = 0.6$

12 **a** **i** $\frac{1}{3} \approx 0.333$
ii $\frac{2}{3} \approx 0.667$
iii $\frac{5}{9} \approx 0.556$

b **i** $\frac{1}{5} = 0.2$ **ii** $\frac{4}{5} = 0.8$ **iii** $\frac{3}{5} = 0.6$

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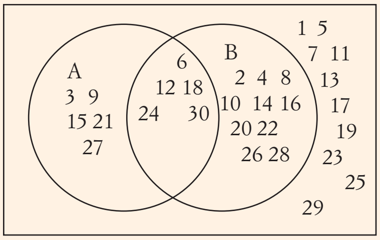
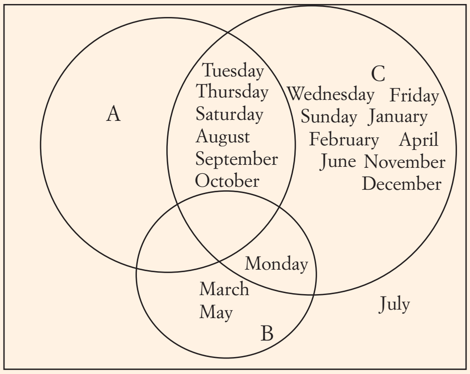
13



a $\frac{1}{6} \approx 0.167$ **b** $\frac{2}{3} \approx 0.667$ **c** $\frac{1}{2} = 0.5$

14 a

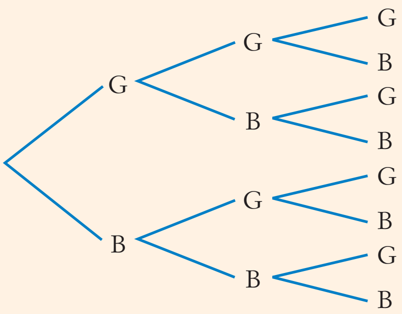


b $\frac{6}{19}$ **c** $\frac{15}{19}$ **d** 0

e $\frac{9}{19}$ **f** $\frac{1}{19}$ **g** $\frac{18}{19}$

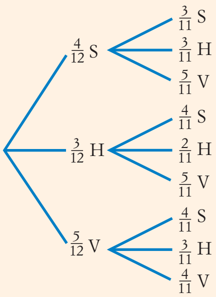
h $\frac{10}{19}$ **i** $\frac{1}{19}$ **j** $\frac{3}{19}$

15 a



b $\frac{3}{8}$ **c** $\frac{4}{8} = \frac{1}{2}$ **d** $\frac{1}{8}$

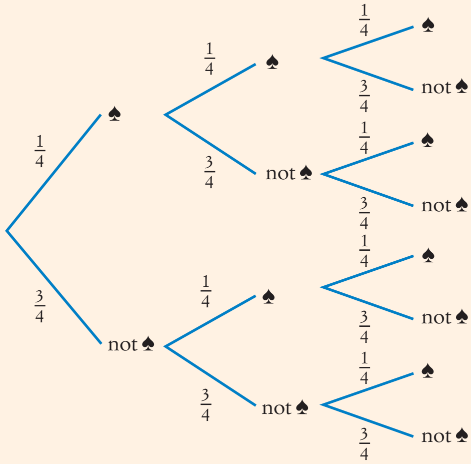
16 a



b $\frac{1}{22}$ **c** $\frac{1}{11}$ **d** $\frac{5}{33}$ **e** $\frac{2}{11}$

f $\frac{5}{22}$ **g** $\frac{10}{33}$ **h** $\frac{19}{66}$ **i** $\frac{47}{66}$

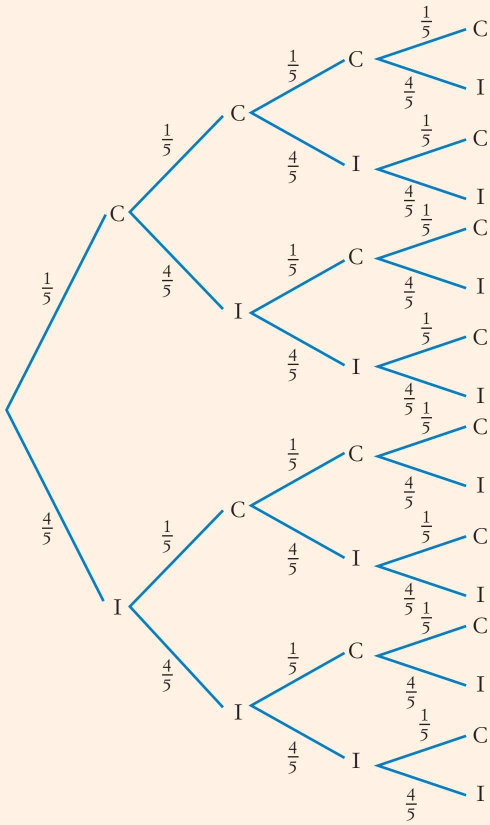
17 a



b $P(3S) = \frac{1}{64} = 0.015625$

c $P(0S \text{ or } 1S) = \frac{27}{32} = 0.84375$

18 a



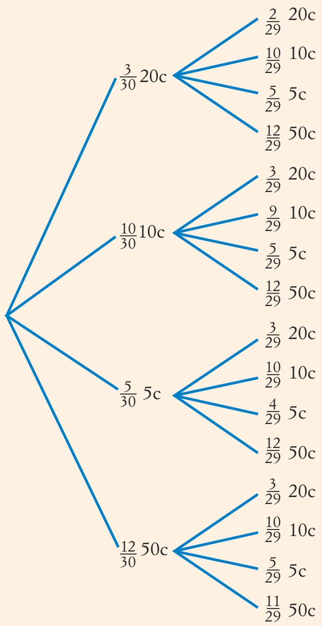
b $P(4C) = \frac{1}{625} = 0.0016$

c $P(2C \text{ or } 3C \text{ or } 4C) = \frac{113}{625} = 0.1808$

d About 9 people.

19 a $\frac{13}{150} \approx 0.08667$ **b** $\frac{19}{50} = 0.38$ **c** $\frac{31}{50} = 0.62$

20 a



b $\frac{124}{435} \approx 0.2851$ **c** $\frac{311}{435} \approx 0.7149$ **d** $\frac{94}{145} \approx 0.6483$

e $\frac{28}{145} \approx 0.1931$ **f** $\frac{12}{145} \approx 0.08276$ **g** $\frac{22}{29} \approx 0.7586$

h $\frac{98}{145} \approx 0.6759$ **i** $\frac{21}{29} \approx 0.7241$ **j** $\frac{7}{29} \approx 0.2414$

21 a $\frac{1}{4} = 0.25$ **b** $\frac{1}{16} = 0.0625$ **c** $\frac{3}{4} = 0.75$
d $\frac{7}{16} = 0.4375$ **e** About 100 times.

22 a $\frac{1}{2} = 0.5$ **b** $\frac{5}{27} \approx 0.1852$
c Non-smokers have a 31% lower probability of having lots of colds.

23 a $\frac{32}{147} \approx 0.2177$ **b** $\frac{17}{43} \approx 0.3721$
c The table doesn't show no accidents, so we cannot say. However, it appears that of those who do have accidents, younger drivers have more.

24 a About 36%. **b** About 81%.
c Non-indigenous people are about 45% more likely to live to 65, so it appears that the health of Aboriginal and Torres Strait Islanders is much poorer than other Australians.

25 a $P(0) = \frac{9}{16} = 0.5625$, $P(1) = \frac{3}{8} = 0.375$, $P(2) = \frac{1}{16} = 0.0625$
b About 56 people, 38 people and 6 people respectively.

26 $\frac{2}{3} \approx 0.667$

27 a $\frac{1}{17} = 0.0588$ **b** $\frac{4}{17} = 0.235$ **c** $\frac{13}{17} = 0.765$
d $\frac{15}{34} = 0.441$ **e** About 94.

28 0.352

29 a $\frac{33}{66\,640} \approx 0.000\,495\,2$ **b** $\frac{33}{16\,660} \approx 0.001\,981$

30 a $\frac{1}{10} = 0.1$ **b** $\frac{11}{20} = 0.55$ **c** $\frac{1}{5} = 0.2$
d $\frac{3}{5} = 0.6$ **e** $\frac{7}{10} = 0.7$

31 There are 20 planks that are longer than 4.5 m (L) out of 60 planks. On the branch of the simplified tree diagram going L-L-L, the fractions are $\frac{20}{60}$, $\frac{19}{59}$ and $\frac{18}{58}$. The probability of getting 3 lengths longer than 4.5 m is thus $P(3 > 4.5) = \frac{20 \times 19 \times 18}{60 \times 59 \times 58} = \frac{57}{1711} \approx 0.03\,331$.

32 There are 4 leaky bags (L) and 16 good bags (G) out of 20 bags altogether. On the branch of the simplified tree diagram going G-G-G, the fractions are $\frac{16}{20}$, $\frac{15}{19}$ and $\frac{14}{18}$. The probability of getting 3 good bags is thus $P(3G) = \frac{16 \times 15 \times 14}{20 \times 19 \times 18} = \frac{28}{57} \approx 0.4912$.

33 You can draw a tree diagram with the first stage as the rotor, the second as the bearings and the third as the case. At each stage you can get a bad component (B) or a good component (G). The fitting of one component to another are independent events, so the probabilities are multiplied. Only the G-G-G branch gives a motor that is not a lemon. It has a probability of $0.97 \times 0.98 \times 0.99 \approx 0.9411$. the probability of getting a lemon is thus about 5.89%. You cannot just add the percentages because there will be overlap between the poor fittings.

Exercise 6.3

1 a $\frac{1}{3} \approx 0.3333$ **b** $\frac{2}{11} \approx 0.1818$ **c** $\frac{1}{5} = 0.2$
d $\frac{1}{6} \approx 0.1667$ **e** $\frac{1}{6} \approx 0.1667$ **f** $\frac{1}{3} \approx 0.3333$
g $\frac{1}{2} = 0.5$ **h** $\frac{2}{11} \approx 0.1818$

2 a $\frac{1}{19} \approx 0.05\,263$ **b** $\frac{5}{19} \approx 0.2632$ **c** $\frac{3}{19} \approx 0.1579$
d $\frac{2}{19} \approx 0.1053$ **e** $\frac{4}{19} \approx 0.2105$ **f** $\frac{4}{19} \approx 0.2105$
g $\frac{4}{19} \approx 0.2105$ **h** $\frac{3}{19} \approx 0.1579$ **i** $\frac{2}{19} \approx 0.1053$
j $\frac{4}{19} \approx 0.2105$

3 a $\frac{2}{9} \approx 0.2222$ **b** $\frac{1}{9} \approx 0.1111$ **c** $\frac{1}{9} \approx 0.1111$
d $\frac{1}{9} \approx 0.1111$ **e** $\frac{1}{6} \approx 0.1667$ **f** $\frac{1}{6} \approx 0.1667$
g $\frac{2}{9} \approx 0.2222$ **h** $\frac{2}{9} \approx 0.2222$ **i** $\frac{1}{171} \approx 0.005\,848$
j $\frac{2}{57} \approx 0.03\,509$

4 $\frac{1}{12} \approx 0.083\,33$

5 $\frac{1}{6} \approx 0.1667$

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6 a $\frac{11}{81} \approx 0.1358$ **b** $\frac{11}{24} \approx 0.4583$ **c** $\frac{1}{2} = 0.5$
d $\frac{3}{16} = 0.1875$ **e** $\frac{3}{71} \approx 0.042\ 25$ **f** $\frac{3}{56} \approx 0.053\ 57$
g $\frac{19}{27} \approx 0.7037$ **h** $\frac{19}{82} \approx 0.2317$ **i** $\frac{41}{71} \approx 0.5775$
j $\frac{41}{64} \approx 0.6406$

7 $P(\text{2K and no better}) = 2 \times \frac{3}{49} \times \frac{44}{48} = \frac{11}{98} \approx 0.1122$; $P(\text{3A and no better}) = 6 \times \frac{2}{49} \times \frac{47}{48} \times \frac{44}{47} = \frac{11}{49} \approx 0.2245$; so you would be better off discarding the king.

8 a $\frac{17}{27} \approx 0.6296$ **b** $\frac{94}{127} \approx 0.7402$ **c** $\frac{14}{71} \approx 0.1972$

d While most people with black hair have brown eyes and nearly three-quarters of people with blond hair have blue eyes, they could have any eye colour, and only about 1 in 5 people with red hair have green eyes. Green eyes are uncommon anyway.

e $P(\text{black hair} | \text{brown eyes}) = \frac{17}{55} \approx 0.3091$, so it is an inaccurate stereotype.

9 The correct answer is $\frac{2}{11}$. Instead of using the number of ways of getting a total of 4 from the restricted sample space, the student used the number from the whole sample space.

10 The correct answer is $\frac{37}{138}$. All three students used the number of students with blue eyes in the restricted sample space correctly, but used the wrong sample space in the formula. The first student used 'blue eyes' as the sample space, the second used the whole sample and the third used the number of students with brown hair and brown eyes.

Chapter review

1 $\frac{1}{8} = 0.125$

2 a 0.16 **b** 0.36 **c** 0.48

3 a $\frac{4}{51} \approx 0.078\ 43$ **b** $\frac{16}{51} \approx 0.3137$ **c** $\frac{4}{25} = 0.16$
d $\frac{3}{5} = 0.6$ **e** $\frac{19}{25} = 0.76$ **f** 14

4 $\frac{16}{25} = 0.64$

5 a $\frac{1}{22} \approx 0.045\ 45$ **b** $\frac{1}{11} \approx 0.090\ 91$
c $\frac{5}{33} \approx 0.1515$ **d** $\frac{47}{66} \approx 0.7121$

6 a $\frac{1}{3} \approx 0.3333$ **b** $\frac{2}{3} \approx 0.6667$ **c** About 22.

7 $\frac{5}{9} \approx 0.5556$

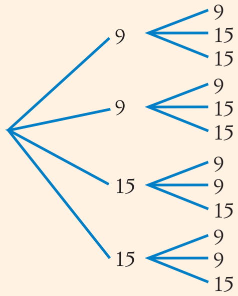
8 Answers may vary.
a 0.05 **b** 0 **c** 1 **d** 0.7

9

	1	2	3	4	5
1	2	3	4	5	6
2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10

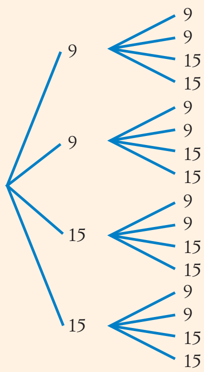
a $\frac{1}{5} = 0.2$ **b** $\frac{9}{25} = 0.36$ **c** $\frac{2}{25} = 0.08$
d $\frac{12}{25} = 0.48$ **e** $\frac{16}{25} = 0.64$ **f** $\frac{3}{25} = 0.12$
g $\frac{3}{25} = 0.12$ **h** $\frac{16}{25} = 0.64$

10 a



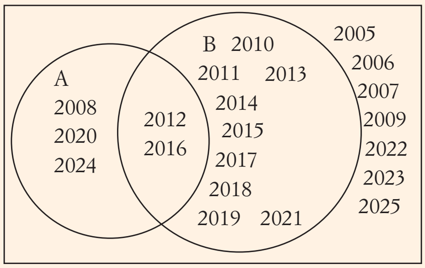
b $\frac{1}{6} \approx 0.1667$ **c** $\frac{2}{3} \approx 0.6667$

11 a



b $\frac{1}{4} = 0.25$ **c** $\frac{1}{2} = 0.5$

12 a

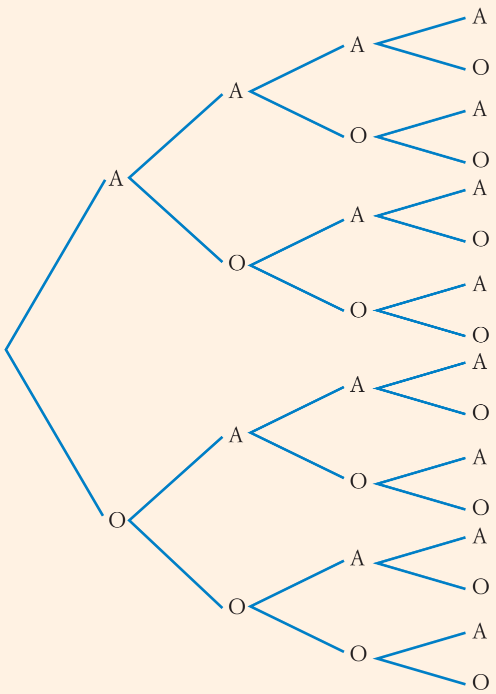


b $\frac{5}{21} \approx 0.2381$ **c** $\frac{11}{21} \approx 0.5238$ **d** $\frac{2}{21} \approx 0.095\ 24$
e $\frac{2}{3} \approx 0.6667$ **f** $\frac{1}{7} \approx 0.1429$

13 a {B1B2, B1R1, B1R2, B1B, B2R1, B2R2, B2B, R1R2, R1B, R2B}

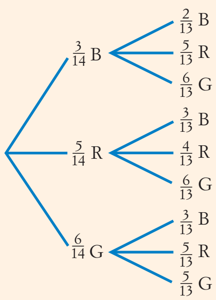
b $\frac{2}{5} = 0.4$

14 a



b $\frac{3}{16} = 0.1875$ **c** $\frac{11}{16} = 0.6875$ **d** $\frac{1}{16} = 0.0625$

15 a



b $\frac{10}{91} \approx 0.1099$ **c** $\frac{15}{182} \approx 0.082\ 42$ **d** $\frac{15}{91} \approx 0.1648$

e $\frac{30}{91} \approx 0.3297$ **f** $\frac{9}{13} \approx 0.6923$

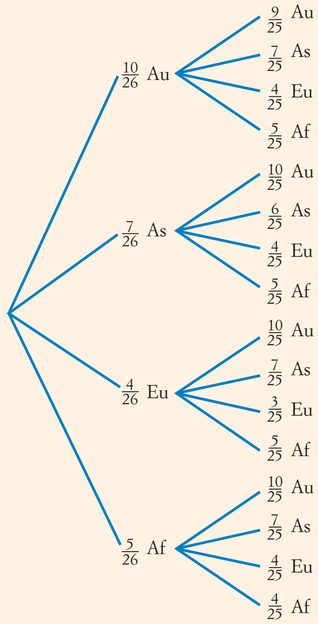
16 a $\frac{4}{11} \approx 0.3636$ **b** $\frac{3}{5} = 0.6$

c $\frac{1}{6} \approx 0.1667$ **d** $\frac{3}{22} \approx 0.1364$

17 $\frac{8}{9} \approx 0.8889$

18 a $\frac{23}{80} = 0.2875$ **b** $\frac{33}{160} = 0.206\ 25$

19 a



b $\frac{21}{325} \approx 0.064\ 62$ **c** $\frac{243}{325} \approx 0.7477$

d $\frac{4}{65} \approx 0.061\ 54$ **e** $\frac{37}{130} \approx 0.2846$

20 a $\frac{7}{30} \approx 0.2333$ **b** About 27. **c** About 133.

21 a $\frac{29}{44} \approx 0.6591$ **b** $\frac{3}{5} = 0.6$

c Considering the sample size, there is no real difference.

22 a {T1T2, T1T3, T1T4, T1F1, T1F2, T2T3, T2T4, T2F1, T2F2, T3T4, T3F1, T3F2, T4F1, T4F2, F1F2}

b $\frac{7}{15} \approx 0.4667$

23 a $\frac{1}{200} = 0.005$ **b** $\frac{3}{25} = 0.12$

c $\frac{3}{200} = 0.015$ **d** $\frac{2}{25} = 0.08$

24 $\frac{2}{3} \approx 0.6667$

25 a $\frac{36}{67} \approx 0.5373$ **b** $\frac{36}{107} \approx 0.3364$

c $\frac{5}{8} = 0.625$ **d** 0.4627 of students who work make >\$100 compared to 0.375 of students who received gifts

26 $\frac{3}{20} = 0.15$

27 a $\frac{1}{8} = 0.125$ **b** $\frac{1}{3} \approx 0.3333$

c $\frac{5}{24} \approx 0.2083$ **d** $\frac{19}{24} \approx 0.7917$

28 Of the 55 recordings, 14 are netball or basketball. A simplified tree diagram for picking 3 recordings, basketball/netball (BN) or other (O) will have a

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branch that goes BN-BN-BN that has probabilities $\frac{14}{55}$, $\frac{13}{54}$ and $\frac{12}{53}$. The probability of getting all three basketball or netball is thus $\frac{14\times13\times12}{55\times54\times53} = \frac{364}{26\,235} \approx$ 0.013 87. It could also be done using conditional probability.

29 The student has not taken into account the order in which the heads and tails appear. When the order is taken into account, there are 16 ways the coins could fall, and only one of them has 4 heads, so the correct probability is $\frac{1}{16} = 0.0625$.

30 There are actually only 11 ways that one of the dice could show 4, because in one case they are both showing 4. Of the 11 ways they could show 4, only two, 3 and 4 or 4 and 3, have a total of 7. Thus the probability is $\frac{2}{11} \approx 0.1818$ or about 18%.

31 The person probably looked at the total number of people who got \$10–19, which was 16 out of the 40 who got pocket money to get 40% instead of using the restricted sample space consisting only of the people who got pocket money, so the correct numerator is actually only 11 to give $\frac{11}{40} = 27.5\%$.

32 The answer to question **17** can be calculated as the part of a single square where the centre of the circle overlaps the lines of the square. This is all but a square 1 cm by 1 cm in the middle of the square. However, if the piece was not allowed to be partly off the board, there would be part of the outside squares where it would not count. This would be the outside 1 cm of these squares. The board is 24 cm by 24 cm, so the part where the piece was allowed would be $22 \times 22\text{ cm}^2 = 484\text{ cm}^2$. All 64 squares would have 1 cm^2 where the pieces do not overlap, so the probability would be $\frac{64}{484} = \frac{105}{121} \approx 0.8678$ which is a little less than in the original case.

33 The class member worked out the probability using the number intending to do Physics in the whole sample space instead of restricting the sample space to the 10 intending to take Chemistry. Of the 10 intending to do Chemistry, there are 5 intending to do take Physics, so the probability is actually 50%.

Chapter 7

Exercise 7.1

1 a $(x+y)(x-y)$ **b** $(4+a)(4-a)$
c $(8+b)(8-b)$ **d** $(w+7)(w-7)$
e $(p+q)(p-q)$ **f** $(a+5)(a-5)$
g $(c+9)(c-9)$ **h** $(x+11)(x-11)$
i $(6+d)(6-d)$ **j** $(10+m)(10-m)$

2 a $(m+n)^2$ **b** $(x-y)^2$
c $(a+2)^2$ **d** $(w+3)^2$
e $(h-k)^2$ **f** $(b-5)^2$
g $(n+6)^2$ **h** $(m-6)^2$
i $(x-7)^2$ **j** $(k+9)^2$

3 a $(a+m)(a-m)$ **b** $(x+1)(x-1)$
c $(c+3)(c-3)$ **d** $(y+4)(y-4)$
e $(5+w)(5-w)$ **f** $(7+a)(7-a)$
g $(b+10)(b-10)$ **h** $(g+9)(g-9)$
i $(13+n)(13-n)$ **j** $(x+8)(x-8)$

4 a $(a+b)^2$ **b** $(m-n)^2$
c $(h-3)^2$ **d** $(x+10)^2$
e $(p+5)^2$ **f** $(g-6)^2$
g $(x-7)^2$ **h** $(h+3)^2$
i $(5-n)^2$ **j** $(p+4)^2$
k $(3-x)^2$ **l** $(y+9)^2$

5 a $(b+m)(6+a)$ **b** $(y-g)(7+d)$
c $(b-a)(5+c)$ **d** $(m+q)(m+n)$
e $(b+a^2)(a-3)$ **f** $(m^2-3)(2m+1)$
g $(a+b)(x+y)$ **h** $(x+a)(y-z)$
i $(d-p)(a+b)$

6 a $(2a+5b)(2a-5b)$ **b** $(4x+3y)(4x-3y)$
c $(7m+4n)(7m-4n)$ **d** $(3a+8b)(3a-8b)$
e $(2x+11y)(2x-11y)$ **f** $(6x+5y)(6x-5y)$
g $(12m+5n)(12m-5n)$ **h** $(8p+7q)(8p-7q)$
i $(9a+4b)(9a-4b)$

7 a $(2x-1)^2$ **b** $(3x+1)^2$
c $(4g-1)^2$ **d** $(2a-b)^2$
e $(3p+q)^2$ **f** $(5x-y)^2$
g $(x+8b)^2$ **h** $(b+11c)^2$
i $(a-9b)^2$ **j** $(a-8b)^2$
k $(3h+k)^2$ **l** $(4g+1)^2$
m $(2m-n)^2$ **n** $(4p+q)^2$
o $(5x-1)^2$ **p** $(7y-4)^2$

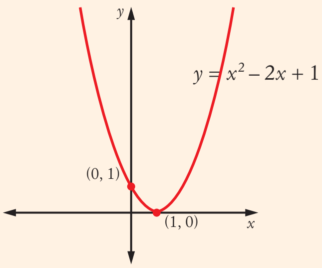
8 a $(2a+3)^2$ **b** $(3x-5)^2$
c $(4m-3)^2$ **d** $(3n+2)^2$
e $(5k-4)^2$ **f** $(2a-7)^2$
g $4(m-3)^2$ **h** $(9x+2)^2$
i $(4a-5)^2$ **j** $(8m+7)^2$

9 a $(y+1)(y-1)(2x-1)$ **b** $(m^2n-1)(n-1)$
c $(x+z)(h+g+j)$ **d** $(d-c)(a-f-g)$
e $(3+a+b)(a-b)$ **f** $(x-4)(4x+1)$
g $(a+4)(c+d)$ **h** $(xy-4)(x+3)$
i $(a+1)(ab-1)$ **j** $2(3a^2-2)(a-2)$

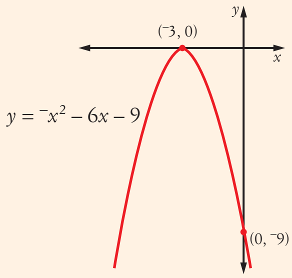
10 a $(a-4)(a-2)$ **b** $(a+b-3)(a+b+3)$
c $(x-11)(x-3)$ **d** $(5-2m)(2m-1)$

e $(9-4a)(4a-5)$ **f** $(x-8)(x+2)$
g $(11x+4)(x-16)$ **h** $(11a-1)(5a+17)$
i $(11-8x)(20x-31)$ **j** $-3y(2x+y)$
k $3b(2a-b)$

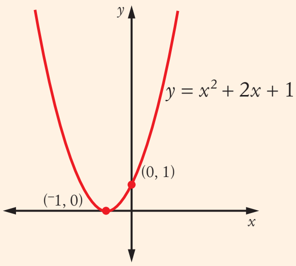
11 a



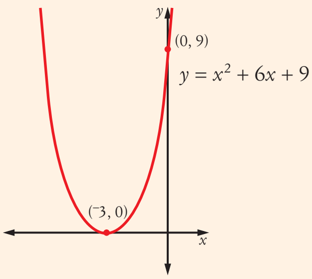
b



c



d



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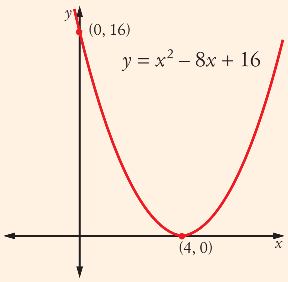

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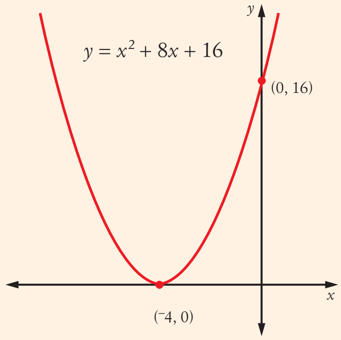

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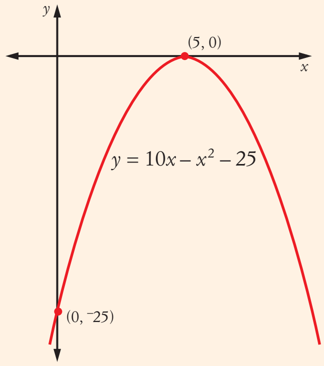

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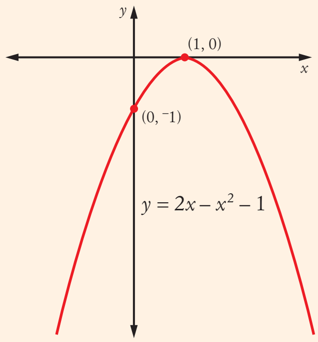

12 $A = \pi r_1^2 - \pi r_2^2 = \pi(r_1 - r_2)(r_1 + r_2) = \pi d_1 d_2$

13 a b^2 m^2 **b** a^2 m^2
c $(b^2 - a^2)$ m^2 **d** $(b+a)(b-a)$
e 17 400 m^2; easier to calculate with the factorised expression.

14 Proof.

Exercise 7.2

1 a $(a+4)(a+1)$ **b** $(x+5)(x+1)$
c $(y+2)(y+5)$ **d** $(m+1)(m+9)$
e $(p-4)(p-1)$ **f** $(b-3)(b-4)$
g $(a+1)(a-4)$ **h** $(x+2)(x-4)$
i $(y-3)(y+4)$ **j** $(a-2)(a+10)$

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2 a $(g+2)(g+6)$ **b** $(h+3)(h+2)$
c $(k+7)(k+6)$ **d** $(x+2)(x+3)$
e $(t+6)(t+1)$ **f** $(z+5)(z+6)$
g $(t+6)(t+9)$ **h** $(y+4)(y+3)$
i $(a+7)(a+4)$ **j** $(w+5)(w+7)$

3 a C **b** A **c** D
d D **e** C

4 a 3 and 5 **b** $^-9$ and 3
c $^-8$ and 5 **d** 8 and 4
e $^-9$ and 4 **f** $^-8$ and $^-8$
g 5 and $^-3$ **h** $^-9$ and 6

5 a $(2x+7)(x-1)$ **b** $(3a-2)(2a+3)$
c $(4x+3)(3x-5)$ **d** $(5y+6)(y-4)$
e $(7x+3)(2x+1)$ **f** $(3x-1)^2$
g $(5a-2)(4a+3)$ **h** $(5x+1)(2x-7)$
i $(6a-1)(a-3)$ **j** $(4x+3)(2x-9)$

6 a $(a+1)(a+3)$ **b** $(x+1)(x+8)$
c $(y+1)(y+6)$ **d** $(m+2)(m+3)$
e $(x+4)(x+2)$ **f** $(p+4)(p+5)$
g $(p+2)(p+8)$ **h** $(g+5)(g+3)$
i $(x+6)(x+4)$ **j** $(x+10)(x+3)$
k $(a+10)(a+4)$ **l** $(m+14)(m+3)$
m $(x-2)(x-1)$ **n** $(a-3)(a-1)$
o $(y-2)(y-3)$ **p** $(x-9)(x-1)$
q $(y-2)(y-5)$ **r** $(a-2)(a-8)$
s $(x-3)(x-7)$ **t** $(w-2)(w-9)$
u $(a-3)(a-8)$ **v** $(a-2)(a-16)$
w $(y-15)(y-1)$ **x** $(b-7)(b-5)$

7 a $(b-3)(b+2)$ **b** $(a+4)(a-5)$
c $(x+3)(x-4)$ **d** $(y+6)(y-7)$
e $(c-3)(c+1)$ **f** $(a+2)(a-4)$
g $(n-5)(n+3)$ **h** $(m-8)(m+1)$
i $(a-5)(a+2)$ **j** $(x-5)(x+8)$
k $(y-6)(y+2)$ **l** $(b-9)(b+3)$
m $(p+6)(p-1)$ **n** $(k-1)(k+4)$
o $(a-2)(a+3)$ **p** $(c+5)(c-4)$
q $(x+8)(x-5)$ **r** $(y-1)(y+6)$
s $(a-2)(a+6)$ **t** $(b-1)(b+7)$
u $(k-2)(k+8)$ **v** $(p-4)(p+8)$
w $(a-5)(a+7)$ **x** $(x-4)(x+9)$

8 a $(e-5n)(e-9n)$ **b** $(w-7t)(w+15t)$
c $(a-5g)(a-3g)$ **d** $(b-9v)(b+12v)$
e $(p-e)(p+4e)$ **f** $(c+3m)(c-11m)$
g $(z+m)(z-11m)$ **h** $(y+11n)(y-5n)$
i $(g+13e)(g-6e)$ **j** $(g-9p)(g+11p)$
k $(h+7w)(h+14w)$ **l** $(z+15q)(z+14q)$
m $(z-15u)(z+4u)$ **n** $(x+7z)(x+12z)$
o $(q-4n)(q+7n)$ **p** $(p+r)(p-3r)$
q $(x-6v)(x-8v)$ **r** $(k+3p)(k+p)$
s $(h+9z)(h-12z)$ **t** $(c-10p)(c-13p)$
u $(z-10r)(z+4r)$ **v** $(t+3p)(t-9p)$
w $(g-2q)(g+12q)$ **x** $(k-12m)(k-2m)$

9 a $(2x+3)(x+3)$ **b** $(2a+5)(a+2)$
c $(2x+3)(x+5)$ **d** $(3m+2)(m+4)$
e $(3x+2)(x+6)$ **f** $(3b+4)(b+3)$
g $(5x+2)(x+4)$ **h** $(5c+9)(c+1)$
i $(2x+3)^2$ **j** $(4b+1)(b+3)$
k $(5p+2)(p+1)$ **l** $(5h+1)(h+2)$
m $(2a+1)(3a+2)$ **n** $(6x+1)(x+2)$
o $(3y+2)(y+2)$ **p** $(3x+4)(2x+5)$
q $(3n+2)(4n+7)$ **r** $(4x+3)(2x+3)$
s $(4y+5)(3y+5)$ **t** $(4w+3)^2$
u $(5x+6)(3x+2)$ **v** $(8r+3)(4r+5)$
w $2(3a+4)(2a+1)$ **x** $(7x+4)(6x+7)$

10 a $(2x+1)(x-5)$ **b** $(5a+4)(a-1)$
c $(2x+3)(x-5)$ **d** $(2m+1)(m-1)$
e $(3x+1)(x-1)$ **f** $(6b+1)(b-1)$
g $(2x+1)(x-3)$ **h** $(2h+1)(h-2)$
i $(6y+1)(y-2)$ **j** $(5x+1)(x-2)$
k $(9n+1)(n-2)$ **l** $(10x+1)(x-2)$
m $(2a+1)(a-4)$ **n** $(x+2)(7x-1)$
o $(10y-1)(y+2)$ **p** $(4x-1)(x+4)$
q $(5x-1)(x+3)$ **r** $(2b-1)(b+3)$
s $(3x-1)(x+3)$ **t** $(4w-1)(w+3)$
u $(5x-1)(x+4)$ **v** $(6r-1)(r+4)$
w $(2x-1)(x+4)$ **x** $(2a-1)(a+5)$

11 a $(5x-1)(x+8)$ **b** $(3a+2)(a-4)$
c $(7b+5)(b-3)$ **d** $(x+2)(5x-3)$
e $(3y+1)(y+6)$ **f** $(5x-9)(x-3)$
g $(7f-5)(f-7)$ **h** $(11k-1)(k-8)$
i $(2a+7)(6a-5)$ **j** $(4y+7)(5y-12)$
k $(6x+11)(4x+11)$ **l** $(2d+7)(3d-11)$
m $(2x+5)(x-2)$ **n** $(5m+4)(6m-11)$
o $(3x+1)(2x-5)$ **p** $(2c+5)(3c-4)$
q $(3x+1)^2$ **r** $(3a+4)(6a-1)$
s $(2p+5)(6p-5)$ **t** $(3x+2)(5x-1)$
u $(4a+1)(2a-3)$ **v** $(7x+1)(x-4)$
w $(n+4)(9n-1)$ **x** $(3x+11)(2x-7)$

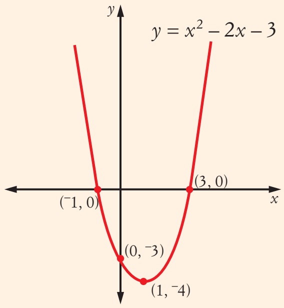
12 a $(9z-8n)(3z+4n)$ **b** $(9b+14g)(3b+2g)$
c $(6p+e)(14p-5e)$ **d** $(5v+11k)(v-9k)$
e $(10b+11n)(4b+5n)$ **f** $(3m-2p)(5m+4p)$
g $(7x+11u)(14x+5u)$ **h** $(6p-11n)(12p-11n)$
i $(8x+7g)(6x-11g)$ **j** $(2v-15n)(15v-2n)$
k $(6m-5h)(4m-15h)$ **l** $(3z-4m)(15z+m)$
m $(8b+9h)(14b+3h)$ **n** $(9y+8a)(7y-10a)$
o $(8m-12r)(3m-2r)$ **p** $(8n+a)(13n-10a)$

q $(13u + 15g)(11u - 6g)$ **r** $(11e - 14u)(10e + 9u)$
s $(15x - 14p)(12x + p)$ **t** $(13t - 7c)(15t + c)$
u $(6r - k)(6r - 11k)$ **v** $(3x - b)(x + 14b)$
w $(9z - 13n)(11z - 3n)$ **x** $(8t + 3p)(t + 3p)$

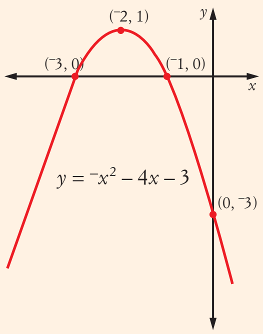
13 Four values: $^{-}7$, $^{-}5$, 5, 7

14 Six values: $^{-}11$, $^{-}4$, $^{-}1$, 1, 4, 11

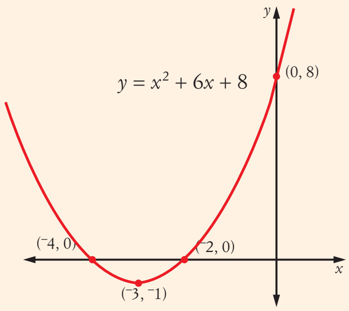
15 **a**



b



c



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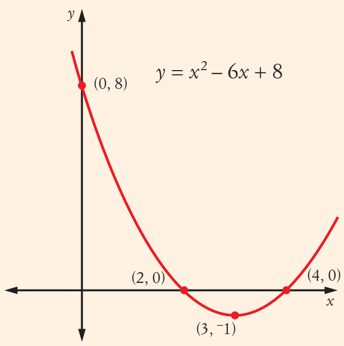

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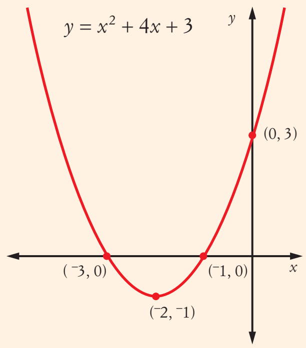

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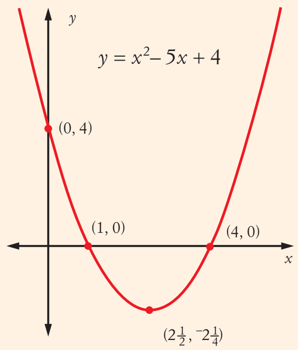

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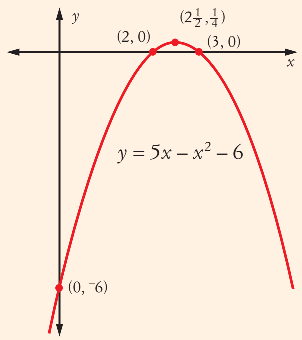
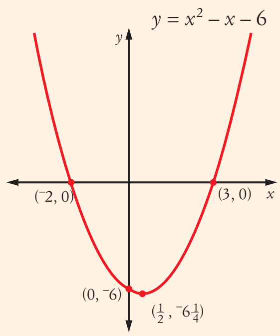


h



16 Nine values: 2, 6, 12, 20, 30, 42, 56, 72, 90

17 Nine values: 3, 8, 15, 24, 35, 48, 63, 80, 99

18 Six values: 117, 126, 145, 155, 176, 187

19 None, because they always occur in positive and negative pairs.

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Exercise 7.3

1 a $3(x + 2y)(x - 2y)$ **b** $5(a + 3b)(a - 3b)$
c $8(m + 2n)(m - 2n)$ **d** $3ab(a + 3b)(a - 3b)$
e $6xy(x + 2y)(x - 2y)$ **f** $7pq(p + 3q)(p - 3q)$
g $7n(2n + 1)(2n - 1)$ **h** $3x(3x + 1)(3x - 1)$
i $4h(2h + 1)(2h - 1)$ **j** $2(3x + 1)(3x - 1)$

2 a $2(n - 4k)(n - 2k)$ **b** $7(n - 9y)(n - 4y)$
c $5(q - 7t)(q - 6t)$ **d** $4(u + 4t)(u - 3t)$
e $3(w + 7m)^2$ **f** $8(n - 4w)(n + 7w)$
g $7(z - 2w)(z - 4w)$ **h** $6(a - 5c)(a - 8c)$
i $3(t - 4h)(t + 6h)$ **j** $8(a - 5m)(a + 3m)$

3 a $8(2w - 7c)(3w + 7c)$ **b** $8(2m - 9x)(8m - 3x)$
c $3(4q - 3m)(q - 3m)$ **d** $2(3c + 7m)(7c - 4m)$
e $2(2p - 9r)(4p - 7r)$ **f** $7(7a - 6m)(a + 5m)$
g $9(2n + 3q)^2$ **h** $7(2c + 5v)(4c - 7v)$
i $2(8t + w)(t - 9w)$ **j** $4(6h + 5q)(9h - 7q)$

4 a $(a - 4)(a - 2)$ **b** $(a + b - 3)(a + b + 3)$
c $(x - 11)(x - 3)$ **d** $8(5 - 2m)(2m - 1)$
e $9(9 - 4a)(4a - 5)$ **f** $5(x - 8)(x + 2)$
g $(11x + 4)(x - 16)$ **h** $(11a - 1)(5a + 17)$
i $(11 - 8x)(20x - 31)$ **j** $^{-}9y(2x + y)$
k $6b(2a - b)$ **l** $(a^2 + b^2)(a + b)(a - b)$
m $(x^2 + 4)(x + 3)(x - 3)$ **n** $(a^2 + 7)(a + 3)(a - 3)$
o $5(4a^2 + n^2)(2a + n)(2a - n)$

5 a $(5a^2 + 4)(a + 1)$ **b** $(b^2 + 2)(b - 5)$
c $(m^2 + 3)(5m - 1)$ **d** $(a - 3)(a + b + 3)$
e $(x - 2y)(3x + 6y + 1)$ **f** $(6 + a^2)(1 + a)$
g $(x + y - a)(x + y + a)$ **h** $(y - a - 3)(y + a + 3)$
i $(3x - p + 1)(3x + p - 1)$
j $(6k + 1 + 5h)(6k + 1 - 5h)$

6 a $(2xy - x + y)(2xy + x - y)$
b $(3ab - x - p)(3ab + x + p)$
c $(x + y - z)(x + y + z)$
d $(m - n - p)(m - n + p)$
e $(z - x - y)(z + x + y)$
f $(a - b - 2)(a - b + 2)$
g $(p + q - 5)(p + q + 5)$
h $(a - b - 2)(a - b + 2)$
i $(m - 3 - 4k)(m - 3 + 4k)$
j $(m - 5 - 4c)(m - 5 + 4c)$
k $(r - s - 4)(r + s + 4)$
l $(a - 2c + 3m)(a + 2c - 3m)$
m $(4x - y - z)(4x - y + z)$
n $(a^2 - 5 - b)(a^2 - 5 + b)$

7 a $4(a - 3)(a - 4)$ **b** $10(x + 3)(x + 2)$
c $5(2m + 5)(2m - 5)$ **d** $3a^2(2a + 5)(2a - 5)$
e $8(m + 2np)(m - 2np)$ **f** $(a - b + 4)(a - b + 1)$
g $(a + 3b)(3a + b)$ **h** $(x^2 + 5)(x + 2)(x - 2)$
i $a(3y + 1)(2y + 1)$ **j** $16(2 - 3y)(2 + y)$
k $2y^2(3xy + 1)(2xy - 3)$ **l** $(8 - 3h)(5 - h)$
m $(x^2 - 2)(9x^2 + 16)$

8 a 441 **b** 361 **c** 961
d 841 **e** 10 201

Chapter review

1 a $(m + p)(m - p)$ **b** $(x - 3)^2$ **c** $(g + 5)^2$

2 a $(r + 4)(r - 4)$ **b** $(h + 6)^2$ **c** $(k - 7)^2$

3 a $(x + 2y)(3x - y)$ **b** $(b + c)(a + d)$
c $(x - 3y)(x - 6z)$

4 a $(m + 4)(m + 3)$ **b** $(x - 8)(x + 2)$

5 a $(k + 2)(k + 8)$ **b** $(g + 4)(g + 7)$

6 a $(5g + 6t)(5g - 6t)$ **b** $(4u - 7)^2$ **c** $(3m + n)^2$

7 a $(m - 1)(7m^2 + 4a)$ **b** $(6 - m + n)(6 + m - n)$

8 a $(x - 6)(x + 7)$ **b** $(p - 5)(p + 3)$
c $(k - 8)(k - 2)$ **d** $(a + 4)(a + 9)$

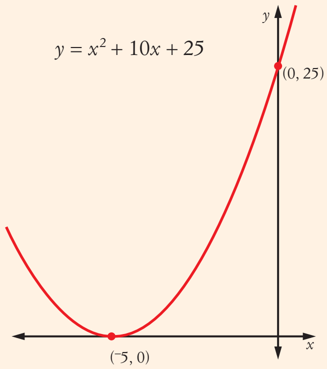
9 a $(y + 2k)(y + 9k)$ **b** $(m + z)(m + 7z)$
c $(n - 11z)(n + 12z)$

10 a $5x(x - 4y)(x + 4y)$ **b** $2(m + 7a)(m - 4a)$
c $3(q + 8t)(q - 9t)$ **d** $7(g - 6m)(g + 4m)$

11 a $(g - 10p)(g - 3p)$ **b** $(8u - 7g)(6u + 7g)$
c $(3t + 7b)(5t + 9b)$

12 a $(2u + 5)(5u + 8)$ **b** $(4b + 3)(7b - 3)$
c $(8a + 3)(7a - 9)$ **d** $(4a - 7)(a - 7)$

13 a



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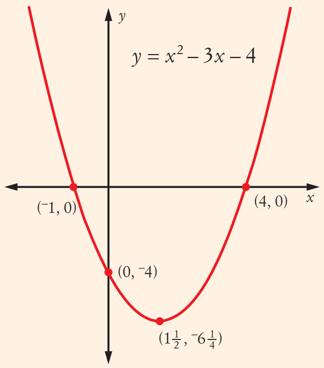


14 Six: ⁻13, ⁻8, ⁻7, 7, 8, 13

15 Six: ⁻19, ⁻8, ⁻1, 1, 8, 19

16 a $^-7(3a + 2)(3a + 8)$ **b** $(x - 2)(3x + 6 + y)$
c $3x^2y^2(x^2 - 4y + 5xy^2)$ **d** $(4p^2 - 9q)(4p^2 + 9q)$

17 Proof.

18 Eight: 4, 10, 18, 28, 40, 54, 70, 88

19 Two: 152 and 168

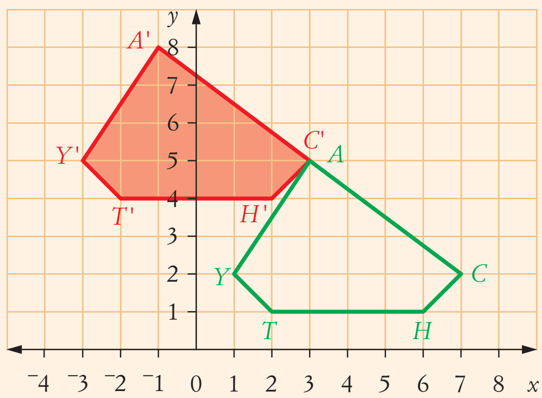
Chapter 8

Exercise 8.1

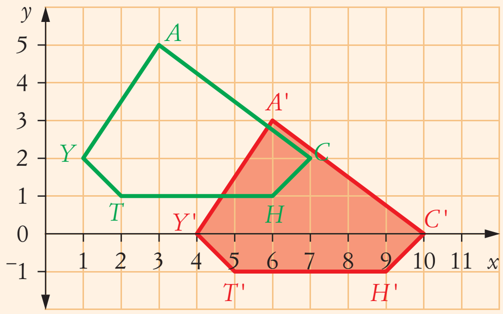
1 a $B(1, 2) \to B'(0, 1)$, $I(2, 3) \to I'(2, 3)$, $R(2, 2) \to R'(2, 1)$, $D(6, 3) \to D'(10, 3)$, $Y(3, 1) \to Y'(4, {}^-1)$
b $B(1, 2) \to B'(1, 2)$, $I(2, 3) \to I'(2, 1)$, $R(2, 2) \to R'(1, 1)$, $D(6, 3) \to D'(2, {}^-3)$, $Y(3, 1) \to Y'(0, 0)$
c $B(1, 2) \to B'(2, {}^-1)$, $I(2, 3) \to I'(3, 0)$, $R(2, 2) \to R'(3, {}^-1)$, $D(6, 3) \to D'(7, 0)$, $Y(3, 1) \to Y'(4, {}^-2)$
d $B(1, 2) \to B'(1, 4)$, $I(2, 3) \to I'(2, 3)$, $R(2, 2) \to R'(2, 4)$, $D(6, 3) \to D'(6, 3)$, $Y(3, 1) \to Y'(3, 5)$
e $B(1, 2) \to B'(7, 2)$, $I(2, 3) \to I'(6, 3)$, $R(2, 2) \to R'(6, 2)$, $D(6, 3) \to D'(2, 3)$, $Y(3, 1) \to Y'(5, 1)$

2 a *THISWO*, regular hexagon
b *ULDBT*, pentagon
c *HEGLIB*, concave hexagon
d *VNGWEHA*, concave heptagon
e *VPSI*, concave quadrilateral
f *NEDFORTIM*, concave nonagon
g *EANDT*, pentagon
h *DTI*, acute-angled scalene triangle
i *MEA*, acute-angled equilateral triangle
j *GAIN*, rectangle
k *SNCE*, parallelogram
l *VOLDEMNT*, octagon
m *MET*, acute-angled scalene triangle
n *OUR*, right-angled isosceles triangle
o *HER*, obtuse-angled scalene triangle
p *OHAR*, trapezium
q *POTERBYWK*, concave nonagon
r *ROWLING*, heptagon
s *ANDI*, quadrilateral

3 a



b



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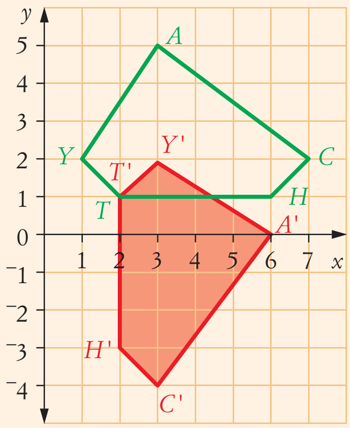

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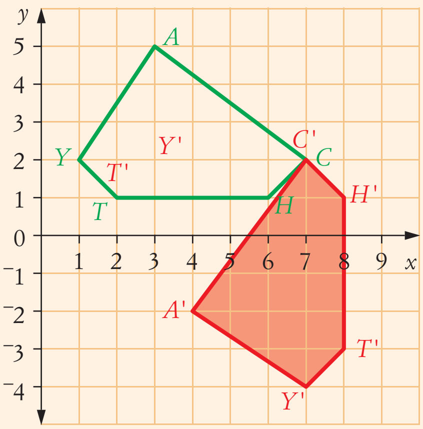
d



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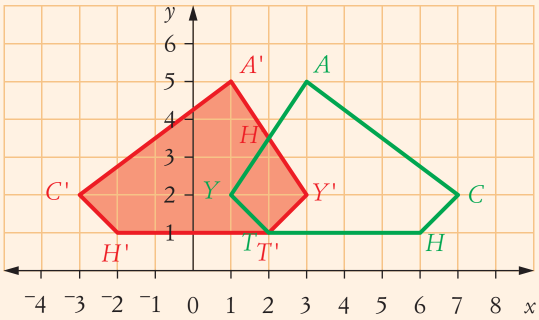

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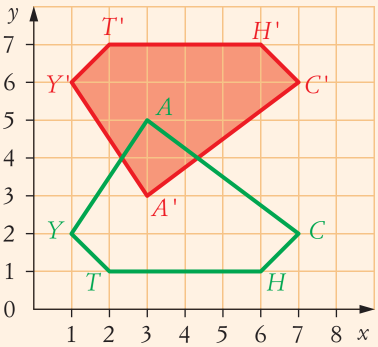

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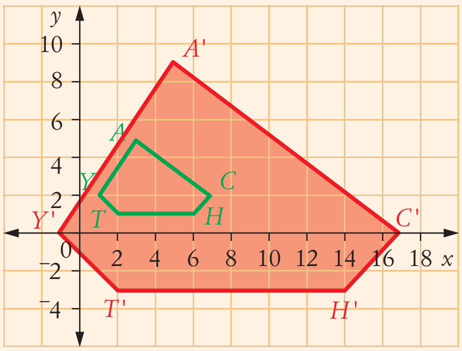
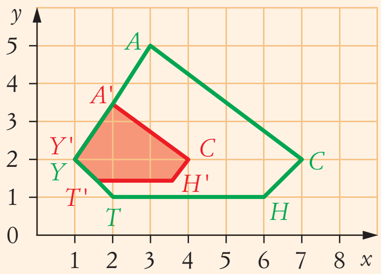


h



4 a Translation 10 units to the right and 3 units up.
b Reflection in the y-axis.
c Translation 8 units to the right and 6 units down.
d Rotation 270° clockwise about $B(^{-}4, 5)$.
e Magnification of 2 with centre $B(^{-}4, 5)$.
f Reflection in the line $y = {}^{-}1$.

5 a Magnification with factor 2 and centre (2, 3).
b Rotation 90° clockwise around (1, 2).
c Translation 1 right and 3 down.
d Reflection in line $y = 3$.
e Reflection in line $x = 4$.

6 a $P'(0, 1)$, $Q'(3, 4)$, $R'(4, 1)$ and $S'(1, 0)$
b $P'(3, 6)$, $Q'(6, 9)$, $R'(7, 6)$ and $S'(4, 5)$
c $P'(5, 3)$, $Q'(2, 6)$, $R'(1, 3)$ and $S'(4, 2)$
d $P'(^{-}1, {}^{-}1)$, $Q'(4, {}^{-}4)$, $R'(5, {}^{-}1)$ and $S'(2, 0)$
e $P'(5, 1)$, $Q'(2, {}^{-}2)$, $R'(1, 1)$ and $S'(4, 2)$
f $P'(1, 3)$, $Q'(^{-}2, 6)$, $R'(1, 7)$ and $S'(2, 4)$
g $P'(^{-}1, 1)$, $Q'(5, 7)$, $R'(7, 1)$ and $S'(1, {}^{-}1)$
h $P'\left(1\frac{1}{2}, 3\frac{1}{2}\right)$, $Q'(3, 5)$, $R'\left(3\frac{1}{2}, 3\frac{1}{2}\right)$ and $S'(2, 3)$

7 a equilateral triangle **b** parallelogram
c rectangle **d** trapezium
e kite **f** trapezium
g square **h** equilateral triangle
i rhombus **j** isosceles triangle
k trapezium **l** scalene triangle

8 a T **b** T **c** T **d** F
e T **f** F **g** T **h** F
i T **j** F **k** T **l** T
m F **n** F

9 $A(^{-}3, 1)$, $B(1, {}^{-}4)$, $C(7, {}^{-}3)$, $D(7, 3)$ and $E(3, 4)$

10 a Rotation 90° clockwise about (1, 4).
b Translation 8 units to the right and 5 units up.
c Translation 4 units to the left and 8 units down.
d Reflection in the y-axis.
e Reflection in the x-axis.
f Reflection in the line $y = {}^{-}5$.
g Reflection in the line $x = 6$.
h Magnification of 3 with centre $Y(^{-}3, 4)$.

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11 $ABCD$: Rotation 270° clockwise about $A(4, 4)$ followed by a reflection in the x-axis.
XYZ: Reflection in the y-axis followed by a translation 8 units to the left and 6 units down.
$PQRS$: Rotation 180° clockwise about $R(^-2, ^-9)$ followed by a magnification of 2 with centre $Q(^-2, ^-11)$.

Exercise 8.2

1 Congruent: **a**, **d** and **f**.
Similar (but not congruent), with scale factors: **b** $\times$ 2, **e** $\times \frac{1}{2}$, **i** $\times \frac{1}{3}$.

Neither (different): **c**, **g**, **h**.

2 **a**, **c**, **d**, **e** and **h** are similar.

3 **a** $\frac{1}{2}$ **b** $2\frac{2}{3}$ **c** 1.8 **d** $\frac{1}{2}$

4 **a** SSS **b** AAS **c** AAS
d SAS **e** SSS **f** SAS
g AAS **h** RHS **i** AAS

5 Congruent: $ELOT \equiv BODY$, $JON \equiv SID$
Similar, with scale factors: $PUSH \mathbin{|||} LEDG$, $\times \frac{1}{2}$

6 **a** 1.2 **b** Not similar
c $\frac{2}{3}$ **d** Not similar

7 **a**, **b**, **c** and **e** are similar.

8 **a** Not similar **b** $\triangle PQR \mathbin{|||} \triangle JMK$
c $\triangle MCN \mathbin{|||} \triangle CDN$ **d** $\triangle PQS \mathbin{|||} \triangle QSR$ (SSS)

9 **a** Teacher to check. **b** Teacher to check.
c Yes **d** Yes
e $\triangle ABC \mathbin{|||} \triangle HIQ$. Ratio of corresponding sides is equal and corresponding angles are equal.

10 Congruent: **a**, AAS; **b**, RHS; **c**, SSS; **e**, SAS; **f**, SSS; **g**, RHS.
Not necessarily congruent: **d**
Not congruent: **h**, **i**, **j**

11 **a** $\triangle CJQ \mathbin{|||} \triangle MWL$ (AAA)
b $\triangle BIZ$ not similar to $\triangle NPV$
c $\triangle BRT \mathbin{|||} \triangle EAN$ (SSA)
d $\triangle APS \mathbin{|||} \triangle MKR$ (SSS)
e $\triangle CPW \mathbin{|||} \triangle MLE$ (SAS)
f $\triangle AMP \mathbin{|||} \triangle QSD$ (AAA)
g $\triangle PRZ$ not similar to $\triangle DKJ$
h $\triangle AMW \mathbin{|||} \triangle SEL$ (RHS)
i $WVR \mathbin{|||} PEN$ (SSS)

12 $A \equiv L$, AAS; $C \equiv I$, SSS; $D \equiv G$, SSS; $E \equiv H$, SAS; $F \equiv K$, RHS. Others are not necessarily congruent.

13 $A \mathbin{|||} E$ (AAA), $B \mathbin{|||} L$ (SAS), $C \mathbin{|||} F$ (SSS), $G \mathbin{|||} I$ (RHS), D, H, J, K not necessarily similar to any others.

14 **a** T **b** T **c** F **d** T **e** T
f F **g** F **h** T **i** F **j** F
k F **l** T **m** T

15 **a** 125 cm **b** 146 cm **c** 14 cm

16 **a** 1.25 cm **b** 1.53 cm

17 35 cm

18 **a** $d = 22.4$ **b** $p = 22.4$ **c** $w = 14$
d $v = 48$ **e** $b = 10$ **f** $c = 33.6$

19 **a** $h = 6$ **b** $k = 10$ **c** $m = 56$ **d** $n = 2.5$

20 **a** $w = 50°, z = 45°, x = 85°$
b $w = 9, x = 4, y = 5\frac{1}{3}$
c $m = 113°, n = 77°, p = 80°$
d $m = 8, n = 44, p = 28$

21 35.6

22 26.7

23 **a** $x = 28°, y = 15$ cm **b** $n = 112°, m = 4.2$ cm
c $a = 21°, b = 12$ mm **d** $p = 29$ mm, $q = 43$ mm
e $s = 23$ cm, $t = 28$ cm **f** $k = 63°, j = 3.8$ cm

24 29.5 m

25 5.6 cm

26 170 cm

27 13.5 m

Exercise 8.3

1 **a** Teacher to check. **b** 2 **c** 3 **d** 540°

2 **a** 900° **b** 1800° **c** 2880°
d 3420° **e** 4140° **f** 5040°

3 **a** 11 **b** 13 **c** 9
d 16 **e** 22 **f** 25

4 **a** $\angle DOG = 62°$, opposite angles of parallelogram 

So $a = 62°$, vertically opposite 
b $\angle SAN = 69°$, supplementary 
So $b = 60°$, angle sum of quadrilateral 

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c $\angle DCA = 42°$, vertically opposite
So $c = 113°$, exterior angle of triangle

d $\angle TSV = 68°$, isosceles triangle
So $d = 112°$, supplementary

e $\angle UER = 139°$, exterior angle of
So $e = 72°$, exterior angle of $\angle UTE$

f $\angle PET = 28°$, isosceles triangle and angle sum of $\angle EPT$
So $\angle SET = 62°$, complementary
So $f = 62°$, isosceles triangle

5 a $\angle ASO = 106°$, angles at a point
So $a = 74°$, cointerior

b $\angle ONY = 64°$, complementary
So $b = 64°$, alternate

c $\angle ARL = 76°$, corresponding
So $c = 35°$, angle sum of triangle

d $\angle GEH = 68°$, exterior angle of triangle
So $d = 68°$, alternate

e $\angle LSR = 68°$, isosceles triangle *and* angle sum of triangle
So $e = 68°$, corresponding

f $\angle HEG = 55°$, cointerior
So $f = 55°$, vertically opposite
$\angle OHE = 125°$, opposite angles of parallelogram
So $\angle WHY = 125°$, vertically opposite
So $g = 46°$, angle sum of quadrilateral $YHWS$

6 a $\angle MTE = 37°$, isosceles triangle and angle sum of triangle
So $a = 37°$, vertically opposite

b $\angle SGL = 64°$, complementary
So $b = 118°$, angle sum of quadrilateral $SGLA$

c $\angle OYS = 44°$, corresponding
So $c = 95°$, exterior angle of triangle

d $\angle OFT = 78°$, opposite angles of parallelogram
So $d = 168°$, exterior angle of triangle

e $\angle NKS = 70°$, isosceles triangle *and* exterior angle of triangle
So $\angle LKS = 110°$, supplementary
So $e = 70°$, angle sum of quadrilateral $KLES$

7 $x = 20°$, vertically opposite
$y = 15°$, complementary
$\theta = 51°$, angles at a point

8 $a = 62°$, exterior angle of triangle
$b = 13°$, angle sum of quadrilateral

9 $a = 12°$, alternate
$b = 19°$, cointerior
$c = 11°$, corresponding

10 $a = 41°$, vertically opposite
$b = 18°$, isosceles triangle *and* angle sum of triangle
$c = 17°$, exterior angle of triangle
$d = 15°$, opposite angles of parallelogram
$e = 6°$, alternate
$f = 59°$, angle sum of quadrilateral

11 a 167.14° **b** 135° **c** 150° **d** 172°

12 a 9 **b** 12 **c** 72 **d** 24

13 $PS = QR$ (given)
$SR = RS$ (common side)
$\angle PSR = \angle QRS$ (both equal 90°)
So $\triangle PSR \equiv \triangle QRS$ (SAS)
So $SQ = RP$ (corresponding sides of congruent triangles)

14 $\angle ZWY = \angle XYW$ (alternate)
$\angle ZYW = \angle XWY$ (alternate)
$WY = YW$ (common side)
So $\triangle ZWY \equiv \triangle XYW$ (AAS)
So $WZ = YX$ (corresponding sides of congruent triangles)
and $WX = YZ$ (corresponding sides of congruent triangles)

15 We know that $\triangle ZWY \equiv \triangle XYW$ (AAS)
$\angle ZWY = \angle XYW$ (corresponding angles of congruent triangles)
$\angle ZYW = \angle XWY$ (corresponding angles of congruent triangles)
So $\angle ZWY + \angle XWY = \angle XYW + \angle ZYW$
So $\angle ZWX = \angle XYZ$
and $\angle WZY = \angle YXW$ (corresponding angles of congruent triangles). QED

16 $AB = CD$ (given)
$BC = DA$ (given)
$DB = BD$ (common side)
So $\triangle ABD \equiv \triangle CDB$ (SSS)
So $\angle ABD = \angle CDB$ (corresponding angles of congruent triangles)
and $\angle CDB = \angle DBC$ (isosceles triangles)
So $\angle ABD = \angle DBC$. QED

Other letters could be used for each of the following proofs.

17 Consider $\triangle WXZ$ and $\triangle XYZ$

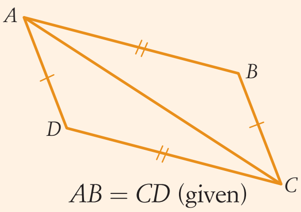
$\angle WXZ = 90° - \angle XWZ$	(angles of a triangle add to 180°)
$\angle XYZ = 90° - \angle XWZ$	(angles of a triangle add to 180°)
$\angle WXZ = \angle XYZ$	(both equal $90° - \angle XWZ$)
$\angle WZX = \angle XZY$	(both equal 90°)
$\triangle WXZ \mid\mid\mid \triangle XYZ$	(All angles are equal)

Use a similar approach to show $\triangle WXZ \mid\mid\mid \triangle WXY$
Hence $\triangle WXZ \mid\mid\mid \triangle XYZ \mid\mid\mid \triangle WXY$

18

$PA = RB$	(given)
$\angle QPA = \angle QRB$	(both equal = 90°)
$QP = QR$	(both are sides of a square)
$\triangle QPA \equiv \triangle QRB$	(SAS)
Hence $QA = QB$	(corresponding sides of congruent triangles)

19



$AB = CD$ (given)
$BC = DA$ (given)
$CA = AC$ (common side)
So $\triangle ABC \equiv \triangle CDA$ (SSS)
So $\angle CAB = \angle ACD$
So $AB \parallel CD$ (alternate)
Similarly, $\angle BCA = \angle DAC$ and $BC \parallel AD$.
So $ABCD$ is a parallelogram. QED

20

$EF = GH$ (given)
$\angle EFH = \angle GHF$ (alternate)
$FH = HF$ (common side)
So $\triangle EFH \equiv \triangle GHF$ (SAS)
So $\angle EHF = \angle GFH$
So $EH \parallel GF$ (alternate)
So $EFGH$ is a parallelogram. QED

Chapter review

1 a $P(4, 1) \to P'(6, 5)$, $G(1, 3) \to G'(4, 2)$, $M(5, 5) \to M'(2, 6)$, $N(3, 3) \to N'(4, 4)$
b $P(4, 1) \to P'(^-2, 1)$, $G(1, 3) \to G'(1, 3)$, $M(5, 5) \to M'(^-3, 5)$, $N(3, 3) \to N'(^-1, 3)$

2 a *FATSL*, regular pentagon
b *UGSARE*, concave hexagon
c *APRESTI*, heptagon
d *NTH*, right-angled/scalene triangle
e *EGAR*, kite
f *DENT*, parallelogram
g *ODAY*, trapezium

3 a Neither (different)
b Similar (but not congruent)
c Congruent

4 a SAS **b** SAS **c** ASA

5 a Rotation 90° anticlockwise around (3, 4).
b Reflection in the line $x = 1$.

6 a $M'(^-1, 8)$ **b** $M'(4, 2)$

7 B and C are congruent. A and F are similar, with scale factor 2.

8 a $\triangle NPQ \mid\mid\mid \triangle EFC$. Scale factor = 2.

9 b Scale factor 3. Not similar.

10 1080°

11 a $\angle SEA = 46°$, complementary
So $a = 50°$, angle sum of triangle
b $\angle HCE = 87°$, angle sum of quadrilateral
So $b = 93°$, supplementary
c $\angle TPS = 38°$, angle sum of triangle
So $c = 38°$, vertically opposite
d $\angle YBS = 36°$, isosceles triangle *or* angle sum of triangle
So $d = 36°$, corresponding

12 a F **b** T **c** T

13 a Congruent, SSS. **b** Congruent, ASA.
c Not necessarily congruent.

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14 $A \equiv E$, RHS; $B \equiv F$, ASA; C and D are not necessarily congruent.

15 a Congruent to B (SAS).
b Congruent to C (ASA).

16 a 1 : 30 **b** About 45 cm. **c** About 87 cm.

17 a About 1.9 mm. **b** About 2.9 mm.
c About 6.2 mm.

18 a $x = 4, y = 12$ **b** $m = 9.6, n = 5.625$

19 5

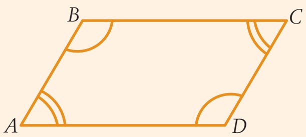
20 144°

21 $a = 60°$, Exterior angle of triangle
$b = 40°$, corresponding

22 $AB = AD$ (given)
$BC = DC$ (given)
$CA = CA$ (same side)
So $\triangle ABC$ $\triangle ABC \equiv \triangle ADC$ (SSS).
Thus $\angle B = \angle D$.
So the angles between the unequal sides in a kite are equal. QED

23 21 m

24



$\angle A + \angle B + \angle C + \angle D = 360°$ (angle sum of quadrilateral)
But $\angle A = \angle C$ and $\angle B = \angle D$ (given).
So $\angle A + \angle B + \angle A + \angle B = 360°$
$\angle A + \angle B = 180°$
So $BC \parallel AD$ (cointerior angles).
Similarly, $\angle A + \angle D = 180°$ and $AB \parallel DC$.
So $ABCD$ is a parallelogram. QED

Chapter 9

Exercise 9.1

1 a \$212.40 **b** \$1176 **c** \$6120
d \$175.87 **e** \$3616.08

2 a 1.08 **b** 1.064 **c** 1.038
d 1.124 **e** 1.096

3 a 1.006 75 **b** 1.001 25 **c** 1.014 75
d 1.0105 **e** 1.0035

4 a 1872% **b** 1115% **c** 2555%
d 390% **e** 2509%

5 a 35.01% **b** About 17.9%.

6 a \$4214.78, \$1214.78 **b** \$6666.40, \$1766.40
c \$7946.18, \$1746.18 **d** \$11 058.87, \$2658.87
e \$18 008.43, \$5708.43

7 a \$4195.88, \$195.88 **b** \$3911.40, \$111.40
c \$4897.74, \$297.74 **d** \$9025.16, \$625.16
e \$7513.71, \$313.71

8 a \$6211.48, \$1211.48 **b** \$9736.82, \$2736.82
c \$19 099.34, \$6799.34 **d** \$7342.58, \$1142.58
e \$5679.26, \$759.26

9 a \$12 157.89, \$4157.89 **b** \$5254.35, \$1254.35
c \$10 139.88, \$3139.88 **d** \$12 048.23, \$2848.23
e \$7119.33, \$1379.33

10 a \$845.60 **b** \$148.87 **c** About 37.3%.

11 \$3153.40, 42.4%

12 \$5077.30, 25.2%

13 7%, just over 10 years; 10%, just over 7 years; 14%, just over 5 years; 20%, 4 years, so a little more than 70 divided by the interest rate.

14 Quarterly and monthly rests make a big difference, but increased frequency makes little difference.

Rate	5	10	15	20
daily	1051.27	1105.2	1161.8	1221.34
weekly	1051.25	1105.1	1161.58	1220.93
fortnightly	1051.22	1105	1161.33	1220.47
monthly	1051.16	1104.7	1160.75	1219.39
quarterly	1050.95	1103.8	1158.65	1215.51
annually	1050	1100	1150	1200

Exercise 9.2

1 \$12.02

2 \$44.30

3 \$33.44

4 \$8532.40

5 \$12 548.80

6 \$10 980.38

7 \$521.78

8 $912.33

9 $407.34

10 $589.05

11 $1248

12 $8636.13

13 $7935.11

14 $7751.72

15 $17 016.34

16 $11 951.83

17 5 years

18 6 years 8 months (or 7 years)

19 About 7 years 9 months (or 8 years).

20 About 4 years 11 months (5 years).

21 About 4 years 11.5 months (5 years).

22 About 12 years 4 months (13 years).

23 Calculate the compound interest and the simple rate that gives the required interest to get about 13.5%.

24 Choose a figure such as $100, find the growth factor and thus the compound interest and the simple rate that gives that interest to get about 12.6%.

25 Find the ratio of the amount from simple interest to the principal for some amount, say $100. Then find the 12th root for the number of rests to get the growth factor for a quarter. Subtract 1 and multiply by 4 and 100 to change to a nominal percentage of about 12.58%.

26 Find the ratio of the amount from simple interest to the principal for some amount, say $100. Then find the 60th root for the number of rests to get the growth factor for a month. Subtract 1 and multiply by 12 and 100 to work out the nominal percentage: 12.91%.

Exercise 9.3

1 **a** $4485 **b** $17 485 **c** $485.69

2 **a** $7560 **b** $25 060 **c** $240.96

3 **a** $2550.60 **b** $13 450.60 **c** $560.44

4 **a** $4080, $670 **b** $8366.40, $320.08
c $4400.40, $151.93 **d** $13 904, $748
e $8295, $308.91

5 **a** $7.0678 **b** $6.3207 **c** $6.8789
d $7.3899 **e** $7.3376

6 **a** $2348.03 **b** $3547.15 **c** $2162.15
d $3459.56 **e** $5435.80

7 **a** $525 290.80 **b** $584 145.00 **c** $228 916.00
d $725 441.60 **e** $1 096 888.00

8 **a** $508 **b** About 2464%.

9 **a** $645 **b** About 1764%.
c $910 **d** About 2721%.

10 **a** $595 **b** About 840%.
c $715 **d** About 1062%.

11 **a** $412.32 **b** 43.2%
c $25.88 **d** 132.4%

12 **a** $\dfrac{\text{Half the total}}{\text{Days in billing period}}$ **b** $1207.82
c 59.1% **d** $3353.60
e $3556.11 **f** 80.5%

13 **a** $6.89 **b** 136.3%

14 $3251.56, $2025.12

15 **a** $352 786.94 **b** $705.24 **c** $164 748

16 **a** $375 000 **b** $417 000 **c** $448 000

17 **a** $285 000 **b** $328 000 **c** $378 000

18 $444 570.90, $3285

19 If you are really strong-willed, choose A because it won't cost anything, otherwise choose B for the low cost.

20 Card A would cost $187.50, Card B $130, Card C $225 and Card D $347.50, so B is best for her.

Chapter review

1 **a** $395.20 **b** $535.30

2 **a** 1.075 **b** 1.097 **c** 1.038

3 **a** 1.004 25 **b** 1.0015

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4 a 2106% **b** 1174%

5 27%

6 \$25.23

7 \$15 394.58

8 a \$7820 **b** \$24 820 **c** \$517.08

9 a \$1732.5, \$485.14 **b** \$14 508, \$327

10 a \$10 077.70, \$2077.70 **b** \$10 239.64, \$2339.64

11 \$12 704.98, \$404.98

12 \$448.20

13 \$475.20

14 \$24 834.79

15 \$7.7182

16 \$2243.85, \$452 786

17 \$8875.30, \$2635.30

18 \$40 342.48, \$16 342.48

19 \$17 350.32

20 a 22.6% **b** 28.1%

21 5 years (4.76)

22 a \$480 **b** 2433%
c \$725 **d** 3084%

23 8 years (7.67)

24 a \$427.39 **b** 35.7%
c \$207.92 **d** 206.5%

25 \$2093.98, \$2368.46

26 a \$322 000 **b** \$356 000 **c** \$379 000

27 Use constant multiplication of \$100 by the growth factor and count how many times it takes to exceed \$300 to get 12 years.

28 Work out 12% compound interest on \$100 over 4 years, divide by 4 and work out the rate to get 14.34%.

29 Find the ratio of the amount from simple interest to the principal for some amount, say \$100. Then find the 24th root for the number of rests to get the growth factor for a month. Subtract 1 and multiply by 12 and 100 to work out the nominal percentage: 19.77%.

30 Assuming that she always paid on time, Card A would cost \$408, Card B \$318, Card C \$369 and Card D \$305. Card D would be best for her.

Chapter 10

Exercise 10.1

1 a c **b** r **c** AB
d FD **e** f **f** PR

2 a $c^2 = a^2 + b^2$ **b** $r^2 = p^2 + q^2$
c $AB^2 = AC^2 + CB^2$ **d** $FD^2 = FE^2 + ED^2$
e $f^2 = g^2 + k^2$ **f** $PR^2 = PQ^2 + QR^2$

3 a 27.2 m **b** 103.4cm **c** 17.8 km
d 8.2 cm **e** 18.6 mm

4 a $e \approx 35.1$ m **b** $m \approx 24.7$ cm **c** $d \approx 36.2$ m
d $y = 30$ mm **e** $n \approx 82.5$ km **f** $r \approx 38.3$mm
g $k \approx 11.1$ cm **h** $x \approx 15.6$m **i** $q \approx 14.7$ km

5 a $w = 300$ m **b** $m = 30$ cm **c** $q = 20$ cm
d $b = 48$ mm **e** $r = 120$km

6 a, **b**, **c**, and **f** are right-angled triangles.

7 a 2360.1 mm **b** 3.0 m **c** 55.2 cm
d 644.0 cm **e** 132.9 mm **f** 3.5 km

8 15.2 m

9 4.2 m

10 11.4 m

11 5.7 cm

12 57.8 km

13 6.8 m

Exercise 10.2

1 a i BC **ii** AC **iii** AB

b i QR **ii** PQ **iii** PR

c i c **ii** b **iii** a

d i x **ii** z **iii** y

e i XZ **ii** ZY **iii** XY

f i e **ii** d **iii** f

2 a $\sin\theta = \frac{BC}{AB}$, $\cos\theta = \frac{AC}{AB}$, $\tan\theta = \frac{BC}{AC}$

b $\sin\theta = \frac{QR}{PR}$, $\cos\theta = \frac{PQ}{PR}$, $\tan\theta = \frac{QR}{PQ}$

c $\sin\theta = \frac{c}{a}$, $\cos\theta = \frac{b}{a}$, $\tan\theta = \frac{c}{b}$

d $\sin\theta = \frac{x}{y}$, $\cos\theta = \frac{z}{y}$, $\tan\theta = \frac{x}{z}$

e $\sin\theta = \frac{XZ}{XY}$, $\cos\theta = \frac{ZY}{XY}$, $\tan\theta = \frac{XZ}{ZY}$

f $\sin\theta = \frac{e}{f}$, $\cos\theta = \frac{d}{f}$, $\tan\theta = \frac{e}{d}$

3

Triangle	$\sin\theta$	$\cos\theta$	$\tan\theta$
ABC	$\frac{BC}{AC}$	$\frac{AB}{AC}$	$\frac{BC}{AB}$
PQR	$\frac{QR}{PR}$	$\frac{PQ}{PR}$	$\frac{QR}{PQ}$
XYZ	$\frac{YZ}{XY}$	$\frac{XZ}{XY}$	$\frac{YZ}{XZ}$
KLM	$\frac{LM}{KM}$	$\frac{KL}{KM}$	$\frac{LM}{KL}$
UVW	$\frac{UV}{UW}$	$\frac{VW}{UW}$	$\frac{UV}{VW}$

4 a $\frac{3}{5}$ **b** $\frac{3}{5}$ **c** $\frac{4}{3}$
d $\frac{4}{5}$ **e** $\frac{3}{4}$ **f** $\frac{4}{5}$

5 a $\sin\theta$ **b** $\cos\theta$ **c** $\tan\theta$

6 a $\sin x = \frac{7}{25}$ **b** $\tan y = \frac{24}{7}$ **c** $\cos y = \frac{7}{25}$
d $\sin y = \frac{24}{25}$ **e** $\cos x = \frac{24}{25}$ **f** $\tan x = \frac{7}{24}$

7 a 0.342 **b** 1 **c** 0.174 **d** 0.934
e 1 **f** 0.158 **g** 0.306 **h** 0.545
i 0.225 **j** 0.017 **k** 0.577 **l** 0.866

8 a 0.342 **b** 0.743 **c** 0.292 **d** 0.707
e 1 **f** 0.839 **g** 0.985 **h** 0
i 0.707 **j** 0.574

9 a 76° **b** 27° **c** 81° **d** 44°
e 9° **f** 26° **g** 7° **h** 47°
i 2° **j** 35° **k** 29° **l** 49°

10 a 76° **b** 35° **c** 83°
d 60° **e** 9° **f** 47°
g 15° **h** 24° **i** 2°

11 a 1 **b** 0.5 **c** 0.563

12 a 45° **b** 30° **c** 56°

13 a i $\sin\alpha = \frac{4}{5}$, $\cos\alpha = \frac{3}{5}$, $\tan\alpha = \frac{4}{3}$
ii $\sin\beta = \frac{3}{5}$, $\cos\beta = \frac{4}{5}$, $\tan\beta = \frac{3}{4}$
iii $\alpha \approx 53°$, $\beta \approx 37°$
b i $\sin\alpha = \frac{15}{17}$, $\cos\alpha = \frac{8}{17}$, $\tan\alpha = \frac{15}{8}$
ii $\sin\beta = \frac{8}{17}$, $\cos\beta = \frac{15}{17}$, $\tan\beta = \frac{8}{15}$
iii $\alpha \approx 62°$, $\beta \approx 28°$
c i $\sin\alpha = \frac{12}{13}$, $\cos\alpha = \frac{5}{13}$, $\tan\alpha = \frac{12}{5}$
ii $\sin\beta = \frac{5}{13}$, $\cos\beta = \frac{12}{13}$, $\tan\beta = \frac{5}{12}$
iii $\alpha \approx 67°$, $\beta \approx 23°$
d i $\sin\alpha = \frac{7}{\sqrt{58}}$, $\cos\alpha = \frac{3}{\sqrt{58}}$, $\tan\alpha = \frac{7}{3}$
ii $\sin\beta = \frac{3}{\sqrt{58}}$, $\cos\beta = \frac{7}{\sqrt{58}}$, $\tan\beta = \frac{3}{7}$
iii $\alpha \approx 67°$, $\beta \approx 23°$
e i $\sin\alpha = \frac{7}{25}$, $\cos\alpha = \frac{24}{25}$, $\tan\alpha = \frac{7}{24}$
ii $\sin\beta = \frac{24}{25}$, $\cos\beta = \frac{7}{25}$, $\tan\beta = \frac{24}{7}$
iii $\alpha \approx 16°$, $\beta \approx 74°$
f i $\sin\alpha = \frac{1}{\sqrt{5}}$, $\cos\alpha = \frac{2}{\sqrt{5}}$, $\tan\alpha = \frac{1}{2}$
ii $\sin\beta = \frac{2}{\sqrt{5}}$, $\cos\beta = \frac{1}{\sqrt{5}}$, $\tan\beta = 2$
iii $\alpha \approx 27°$, $\beta \approx 63°$

14 a 10 **b** 5.130 **c** 18
d 54.164 **e** 18.852 **f** 38.704

15 a 20.0 **b** 28.9 **c** 45.3
d 20.6 **e** 55.4 **f** 84.9

16 a $a \approx 23.1$ **b** $b \approx 86.6$ **c** $d \approx 12.7$
d $y \approx 125.4$ **e** $x \approx 15.0$ **f** $n \approx 4.5$

17 a $a \approx 31.4$ **b** $d \approx 77.7$ **c** $m \approx 70.4$
d $n \approx 36.8$ **e** $y \approx 68.0$ **f** $x \approx 225.2$

18 a $x \approx 8.1$ cm **b** $y \approx 15.9$ m **c** $k \approx 16.7$ cm
d $m \approx 20.6$ mm **e** $p \approx 12.3$ km **f** $z \approx 24.1$ cm

19 a $a \approx 17.5$ cm **b** $x \approx 11.5$ cm **c** $b = 17.7$ cm

20 a 34.99 cm **b** 57.84 mm **c** 139.17 cm

21 a i $x = 9, y = 12, r = 15$
ii $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$
b i $x = {}^{-}12, y = {}^{-}9, r = 15$
ii $\sin\theta = {}^{-}\frac{3}{5}$, $\cos\theta = {}^{-}\frac{4}{5}$, $\tan\theta = \frac{3}{4}$
c i $x = {}^{-}24, y = 7, r = 25$
ii $\sin\theta = \frac{7}{25}$, $\cos\theta = {}^{-}\frac{24}{25}$, $\tan\theta = {}^{-}\frac{7}{24}$

22 a i $x = 8, y = 15$ **ii** $r = 17$
iii $\sin\theta = \frac{15}{17}$, $\cos\theta = \frac{8}{17}$, $\tan\theta = \frac{15}{8}$
b i $x = {}^{-}8, y = {}^{-}6$ **ii** $r = 10$
iii $\sin\theta = {}^{-}\frac{3}{5}$, $\cos\theta = {}^{-}\frac{4}{5}$, $\tan\theta = \frac{3}{4}$
c i $x = 12, y = {}^{-}5$ **ii** $r = 13$
iii $\sin\theta = {}^{-}\frac{5}{13}$, $\cos\theta = \frac{12}{15}$, $\tan\theta = {}^{-}\frac{5}{12}$

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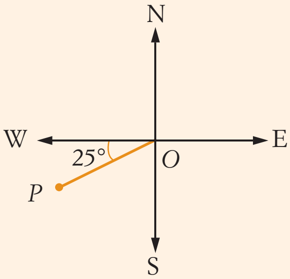
23 **a** 0 **b** 0 **c** 0 **d** 1
e 0 **f** 0 **g** Not defined **h** $^{-}1$
i 0 **j** 1

24 **a** 0.87 **b** 0.5 **c** 0.87 **d** 0.98
e 1 **f** 0.17 **g** 0.17 **h** 0.94
i 0.58 **j** 5.67

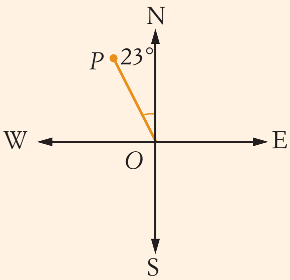
25 **a** 070° **b** 280° **c** 115°
d 248° **e** 207° **f** 143°

26 **a** 250° **b** 100° **c** 295°
d 068° **e** 027° **f** 323°

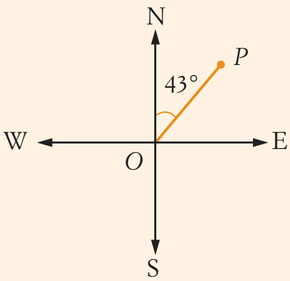
27 **a**



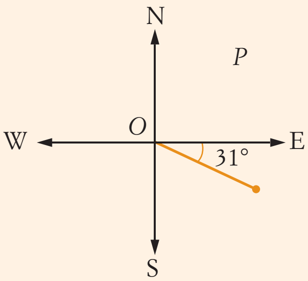
b



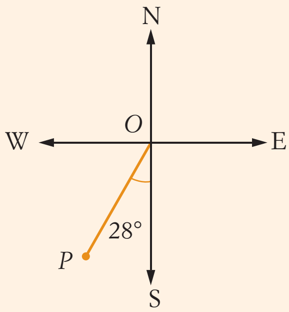
c



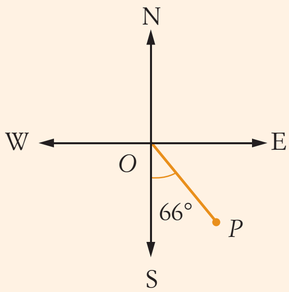
d



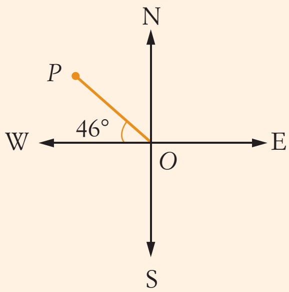
e



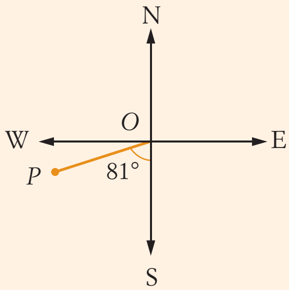
f



g



h



28 **a** 5 units due N, then 7 units E.
b 8 units W 45° S, then 15 units E, then 7 units E 70° S.
c 18 units on bearing 060°, then 11 units on bearing 150°.

29 **a** **i** $\frac{4}{5}$ **ii** 53°
b **i** $\frac{12}{13}$ **ii** 67°

c **i** $\frac{24}{25}$ **ii** $16° + 90° = 106°$

d **i** $\frac{-3}{5}$ **ii** $53° + 270° = 323°$

e **i** $\frac{5}{13}$ **ii** $23°$

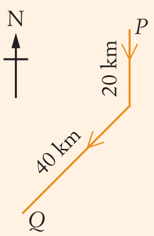
f **i** $\frac{-4}{5}$ **ii** $53° + 180° = 233°$

g **i** $\frac{\sqrt{3}}{2}$ **ii** $60°$

h **i** $\frac{\sqrt{5}}{3}$ **ii** $48°$

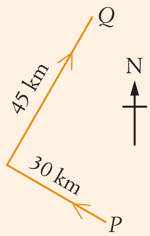
30 $\alpha \approx 23°$, $\theta \approx 67°$

31 **a** Drawn half actual size. Yours should be twice this size.



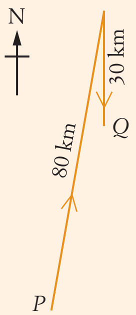
Scale 1 cm : 20 km

b Drawn half actual size. Yours should be twice this size.



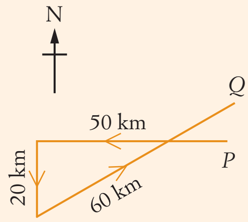
Scale 1 cm : 20 km

c Drawn half actual size. Yours should be twice this size.



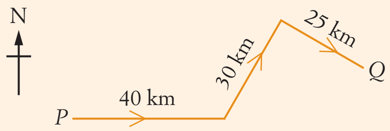
Scale 1 cm : 20 km

d Drawn half actual size. Yours should be twice this size.



Scale 1 cm : 20 km

e Drawn half actual size. Yours should be twice this size.



Scale 1 cm : 20 km

32 **a** **i** 210° **ii** 56 km

b **i** 356° **ii** 54 km

c **i** 016° **ii** 51 km

d **i** 011° **ii** 10 km

e **i** 080° **ii** 78 km

33 **a** **i** $\frac{3}{5}$ **ii** $\frac{4}{5}$ **iii** 1

b **i** $\frac{\sqrt{3}}{2}$ **ii** $\frac{1}{2}$ **iii** 1

c **i** $\frac{5}{13}$ **ii** $\frac{12}{13}$ **iii** 1

34 **a** 393.3 m **b** 338°

35 **a** 25.1 km **b** 208.6°

36 **a** 283 km **b** 307°

Exercise 10.3

1 **a** $w \approx 17.26$ m **b** $m \approx 4.08$ m **c** $x \approx 126.59$ m

2 **a** 44.5 m **b** 71.2 m

3 58.5 m

4 **a** 12 800 m **b** 51.2 km/h

5 4.8 m

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6 a $\theta \approx 59°$ **b** $239°$

7 6.9 km

8 $105°$

9 a 50 km **b** 86.6 km **c** 287.9 km

10 453.2 km

11 a $x \approx 2.94$ m **b** $a \approx 3.22$ m **c** $m \approx 1.85$ m

12 202.4 m

13 68.8 m

14 $24°$

15 a $x \approx 9.9$ cm, $y \approx 18.8$ cm
b $x \approx 22.8$ cm, $y \approx 21.2$ cm
c $x \approx 6.5$ mm, $y \approx 7.5$ mm
d $x \approx 15.3$ m, $y \approx 30.6$ m

16 18.91 m

17 a 12.04 m (south), 9 m (west)
b $44°54'$ (south), $53°8'$ (west)
c 15.03 m

18 1396 m

19 a 401 m **b** $233°16'$

20 a 2.32 m **b** $139\ \text{m}^2$

Chapter review

1 a XY **b** $XY^2 = YZ^2 + XZ^2$

2 a XZ **b** YZ

3 a $\sin\theta = \frac{ZX}{YX}$ **b** $\cos\theta = \frac{ZY}{YX}$ **c** $\tan\theta = \frac{ZX}{ZY}$

4 a 1.664 **b** $\theta \approx 34°$

5 a 0.515 **b** $26°$ **c** 0.6

6 $x = 8$ cm

7 a $\tan\alpha = \frac{4}{3}$ **b** $\alpha \approx 53°$

8 $\alpha \approx 28°$, $\beta \approx 62°$

9 a $a \approx 21.8$ **b** $b \approx 9.4$

10 a $x \approx 16.3$ m **b** $y° \approx 35.3°$

11 a $m \approx 9.3$ km **b** $w \approx 30.2$ mm

12 a $\frac{3}{5}$ **b** $\frac{^-4}{5}$ **c** $143°$

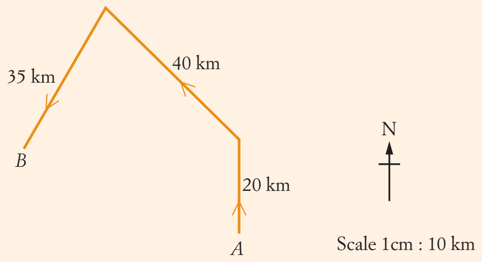
13 a $\frac{^-2}{\sqrt{8}}$ or $\frac{^-1}{\sqrt{2}}$ **b** 1 **c** $117°$ **d** $225°$

14 a 1 **b** 0 **c** Not defined **d** 0

15 P 070°, Q 300°

16 Obtuse-angled

17



18 a $292°$ **b** 49 km

19 a 1 **b** $\sqrt{3}$ **c** $30°$
d $\frac{^-\sqrt{3}}{2}$ **e** $\frac{^-1}{2}$ **f** $\sqrt{3}$
g $\frac{^-\sqrt{3}}{2}$ **h** $\frac{1}{2}$ **i** $^-\sqrt{3}$

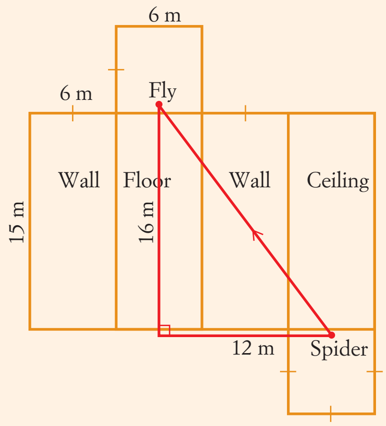
20 About 226 cm.

21 About 46.2 m.

22 15.9 m

23 About 40 m.

24 20 m



25 125.7 m (The Chevron Renaissance).

Chapter 11

Exercise 11.1

1 a $a = {}^-20.1$ **b** $b = {}^-1.5$ **c** $m = {}^-20$
d $h = {}^-20$ **e** $k = {}^-56$ **f** $y = {}^-13.5$

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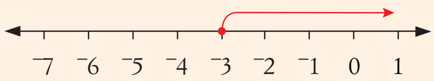
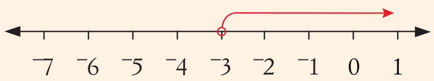
g $x = {}^{-}120$ **h** $h = 5$ **i** $n = {}^{-}5$
j $x = {}^{-}3$ **k** $m = {}^{-}2$ **l** $n = {}^{-}2\frac{1}{2}$
m $n = {}^{-}35$ **n** $d = {}^{-}2.5$ **o** $a = {}^{-}11.5$
p $a = {}^{-}1$ **q** $e = {}^{-}50$ **r** $r = {}^{-}1.2$
s $m = {}^{-}1.9$ **t** $d = {}^{-}10$

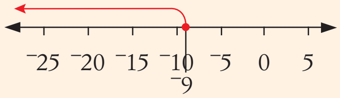
2 a $r \geq {}^{-}3$



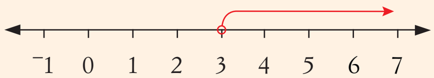
b $r > {}^{-}3$



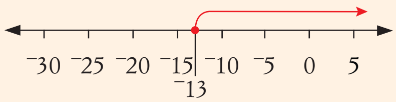
c $b \leq {}^{-}9$



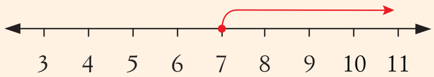
d $v > 3$



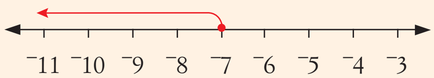
e $g \geq {}^{-}13$



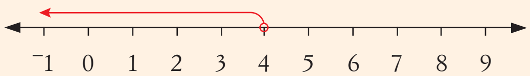
f $h \geq 7$



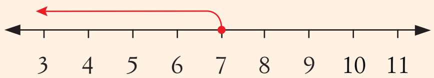
g $w \leq {}^{-}7$



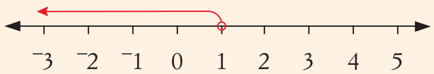
h $m < 4$



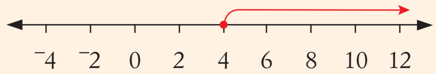
i $x \leq 7$



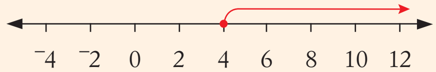
j $e < 1$



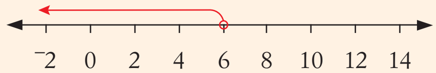
3 a $a \geq 4$



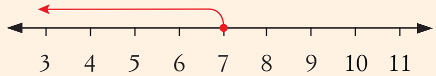
b $m \geq 4$



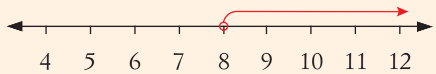
c $w < 6$



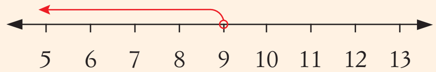
d $m \leq 7$



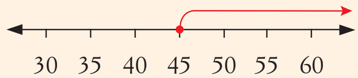
e $q > 8$



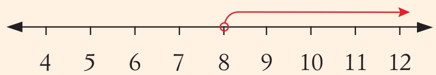
f $e < 9$



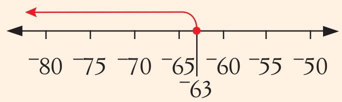
g $r \geq 45$



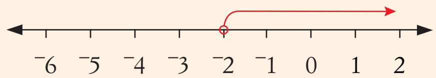
h $h > 8$



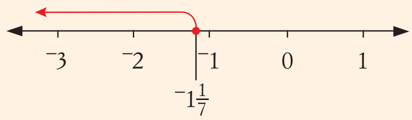
i $v \leq {}^{-}63$



j $e > {}^{-}2$



4 a $b \leq {}^{-}1\frac{1}{7}$



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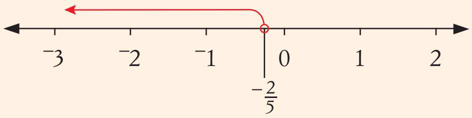
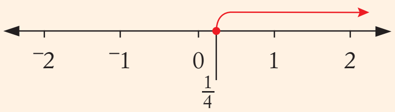
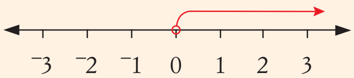
b $x < ^{-}\frac{2}{5}$



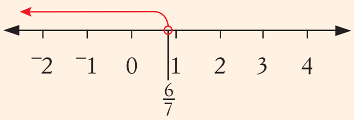
c $e \geq \frac{1}{4}$



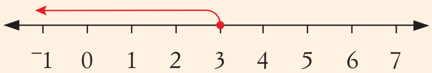
d $q > 0$



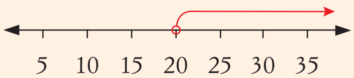
e $z < \frac{6}{7}$



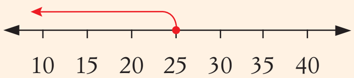
f $h \leq 3$



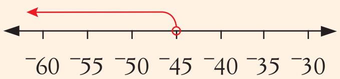
g $e > 20$



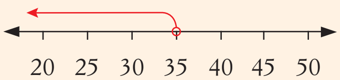
h $g \leq 25$



i $z < ^{-}45$



j $t < 35$



5 a $a = ^{-}3$ **b** $b = ^{-}3$ **c** $m = ^{-}5$
d $n = ^{-}7$ **e** $k = ^{-}5$ **f** $m = ^{-}9$
g $r = ^{-}12$ **h** $h = ^{-}7$ **i** $w = ^{-}40$
j $c = ^{-}8$ **k** $b = ^{-}3$ **l** $d = ^{-}6$

6 a $a = ^{-}9$ **b** $m = ^{-}5$ **c** $a = ^{-}2$
d $m = ^{-}2$ **e** $x = ^{-}15$ **f** $g = ^{-}2$
g $k = ^{-}6$ **h** $w = ^{-}4$ **i** $x = \frac{1}{2}$
j $a = ^{-}3$ **k** $m = ^{-}1\frac{2}{3}$ **l** $x = ^{-}2$
m $r = ^{-}3$ **n** $g = ^{-}10$ **o** $w = ^{-}2$
p $m = ^{-}5$ **q** $w = ^{-}2\frac{2}{3}$ **r** $x = ^{-}4$
s $d = ^{-}16$ **t** $b = ^{-}4$ **u** $n = ^{-}4$

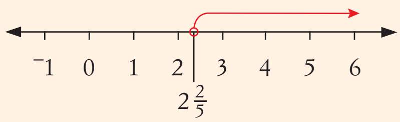
7 a $a = ^{-}13$ **b** $b = ^{-}4$ **c** $n = ^{-}7$
d $m = ^{-}1$ **e** $y = ^{-}6$ **f** $x = ^{-}10$
g $d = 5$ **h** $a = ^{-}2$ **i** $w = ^{-}3$
j $k = ^{-}\frac{1}{3}$ **k** $x = ^{-}1$ **l** $w = 4$
m $x = ^{-}1$ **n** $a = ^{-}4$ **o** $k = 3\frac{1}{4}$
p $b = 12$ **q** $y = 8$ **r** $m = ^{-}6\frac{1}{2}$
s $n = 14$ **t** $x = 4\frac{1}{2}$ **u** $a = 0$

8 a $a = 2$ **b** $b = 3$ **c** $m = ^{-}5$
d $k = 10$ **e** $y = 5$ **f** $x = 2.5$
g $d = ^{-}2$ **h** $w = ^{-}2\frac{1}{4}$ **i** $n = 13$
j $x = 3$

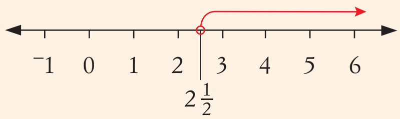
9 a $x = 16$ **b** $a = 14$ **c** $b = 12$
d $d = 5$ **e** $a = 12$ **f** $x = ^{-}20$
g $b = 14$ **h** $m = ^{-}15$ **i** $n = ^{-}15$
j $k = 5$ **k** $d = 30$ **l** $x = ^{-}2\frac{1}{4}$

10 a $a = 6$ **b** $b = 19$ **c** $h = ^{-}1$
d $k = 9$ **e** $n = ^{-}7\frac{1}{3}$ **f** $b = 9$

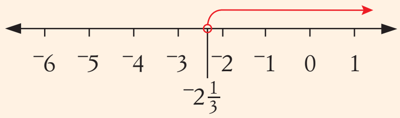
11 a $n > 2\frac{2}{5}$



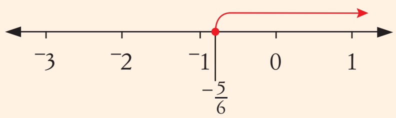
b $p > 2\frac{1}{2}$



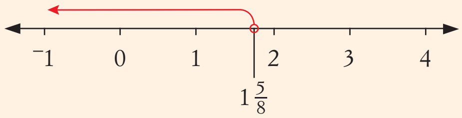
c $c > ^{-}2\frac{1}{3}$



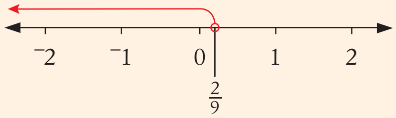
d $e \geq ^{-}\frac{5}{6}$



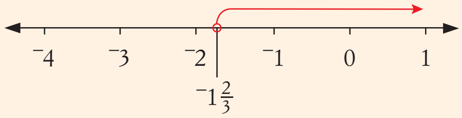
e $k < 1\frac{5}{8}$



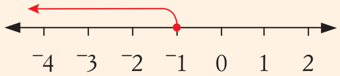
f $y < \frac{2}{9}$



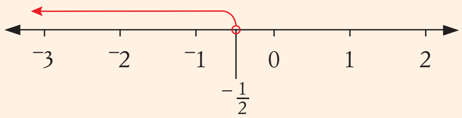
g $c > {}^{-}1\frac{2}{3}$



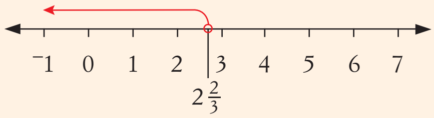
h $k \leq {}^{-}1$



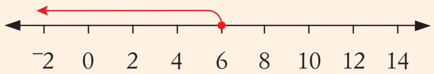
i $z < {}^{-}\frac{1}{2}$



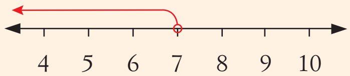
j $u < 2\frac{2}{3}$



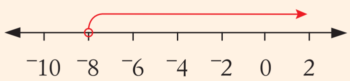
12 a $q \leq 6$



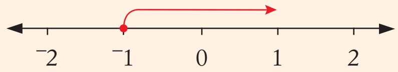
b $b < 7$



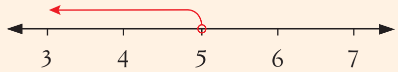
c $u > {}^{-}8$



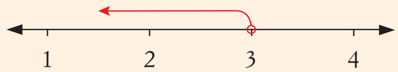
d $e \geq {}^{-}1$



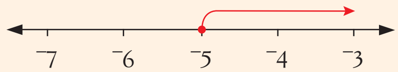
e $p < 5$



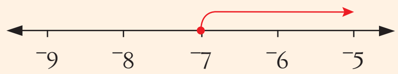
f $k < 3$



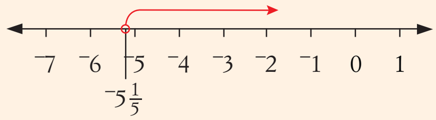
g $b \geq {}^{-}5$



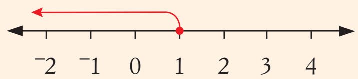
h $w \geq {}^{-}7$



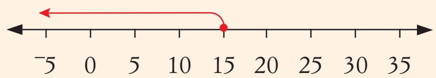
i $c > {}^{-}5\frac{1}{5}$



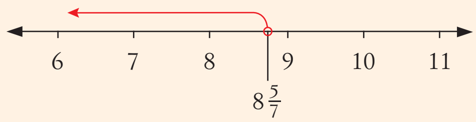
j $a \leq 1$



k $g \leq 15$



l $g < 8\frac{5}{7}$



13 a $a = 1$ **b** $b = 2\frac{2}{3}$ **c** $p = 2$
d $b = 3$ **e** $m = 5\frac{1}{2}$ **f** $c = 4$
g $g = 1$ **h** $n = \frac{2}{9}$ **i** $x = 18\frac{2}{3}$

14 2.5 m **15** 4 km/h **16** 100 coins.

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17 25 m by 25 m

18 29, 31 and 33

19 a 115.5 °F
b $x = {}^-40$, possible in many places in the US, including California, but not in Australia.

20 9 L to make 10 L of solution.

21 $12 \times \frac{n}{5} + 8 \times \frac{4n}{5} \leq 200$, he can get up to 22 plants.

22 $8.4w \leq 29.5$, about 3.5 kg.

23 $\frac{7A}{3} + 32 \leq 300$, about 114 m^2 (38 tubes).

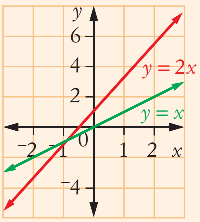
24 $\frac{c-120}{45} \geq \frac{450}{70}$, \$435 (\$409.29, but there can only be a whole number of blades).

25–28 Proofs, teacher to check.

Exercise 11.2

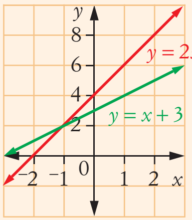
1 a $x = 2$ and $y = 4$
b $x = 1$ and $y = 4$

2 a



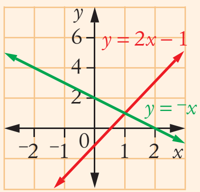
Simultaneous solution = ($^-1$, $^-1$).

b



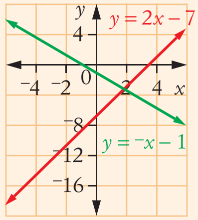
Simultaneous solution = ($^-1$, 2).

c



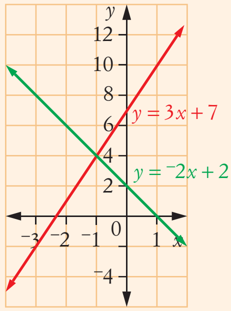
Simultaneous solution = (1, 1).

d



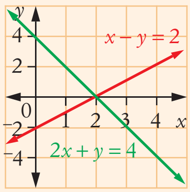
Simultaneous solution = (2, $^-3$).

e



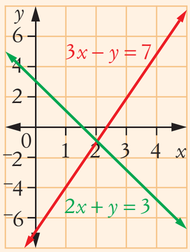
Simultaneous solution = ($^-1$, 4).

3 a



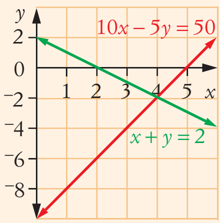
Simultaneous solution = (2, 0).

b



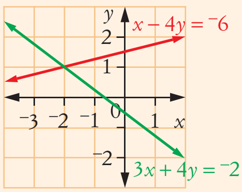
Simultaneous solution = (2, $^-1$).

c



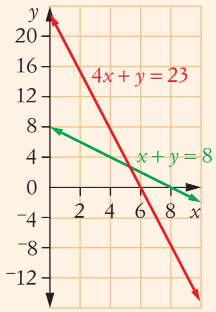
Simultaneous solution = (4, $^-2$).

d



Simultaneous solution = ($^-2$, 1).

e



Simultaneous solution = (5, 3).

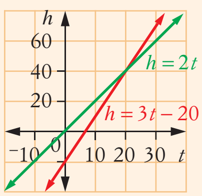
4 **a** $x = 5, y = 2$ **b** $x = 6, y = 18$
c $a = 2, b = 1$ **d** $a = 5, b = {}^-7$
e $x = 0, y = 3$ **f** $c = 8, d = 3$
g $c = 5, d = 4$ **h** $c = 4, d = 1\frac{1}{2}$
i $x = 2, y = {}^-1$

5 **a** $x = 3, y = 2$ **b** $x = 5, y = 2$
c $m = 3, n = 1$ **d** $a = 3, b = 2$
e $m = 1, n = {}^-2$ **f** $x = 3, y = {}^-3$
g $x = 4, y = {}^-3$ **h** $a = {}^-2, b = 1$
i $m = {}^-2, n = 1$

6 **a** $x = 4, y = {}^-2$ **b** $a = 3, b = 3$
c $m = 2, n = {}^-3$ **d** $x = 2, y = 4$
e $m = 2, n = {}^-3$ **f** $c = 4, d = 2$
g $x = 4, y = 1$ **h** $a = 5, b = {}^-1$
i $x = 4, y = {}^-2$

7 Adult's ticket \$7, child's ticket \$2.

8



Same height after 20 seconds.

9 18 m

10 Sam 50 bricks, Peta 200 bricks.

11 Strawberries \$2/tray, cherries \$3/tray.

12 $a = 40$ cm, $b = 10$ cm

13 2 CDs

14 Jackie 22/h, Patrick 16/h.

Exercise 11.3

1 **a** $x = {}^-2$ or $x = 2$ **b** $a = {}^-7$ or $a = 7$
c $b = {}^-11$ or $b = 11$ **d** $h = {}^-5$ or $h = 5$
e $x = {}^-9$ or $x = 9$ **f** $a = {}^-2$ or $a = 2$
g $x = {}^-3$ or $x = 3$ **h** $b = {}^-5$ or $b = 5$
i $x = {}^-5$ or $x = 5$ **j** $k = {}^-\frac{3}{7}$ or $k = \frac{3}{7}$
k $n = {}^-2\frac{1}{4}$ or $n = 2\frac{1}{4}$ **l** $m = {}^-\frac{3}{4}$ or $m = \frac{3}{4}$

2 **a** $z = {}^-5$ or $z = 1$ **b** $h = 5$ or $h = 15$
c $e = {}^-4$ or $e = 14$ **d** $k = {}^-3$ or $k = 17$
e $c = {}^-2$ or $c = 18$ **f** $q = {}^-7$ or $q = 1$
g $q = {}^-10$ or $q = {}^-4$ **h** $b = {}^-3$ or $b = 21$
i $y = {}^-1$ or $y = 23$ **j** $m = 0$ or $m = 18$

3 **a** $x = 0$ or $x = {}^-1$ **b** $x = 0$ or $x = {}^-2$
c $x = 3$ or $x = {}^-1$ **d** $f = 4$ or $f = {}^-9$
e $x = 10$ or $x = {}^-3$ **f** $n = 4$ or $n = {}^-3$
g $b = 0$ or $b = {}^-8$ **h** $x = 3$
i $m = {}^-1\frac{1}{2}$ or $m = 4$ **j** $k = {}^-1\frac{1}{2}$ or $k = 3$
k $w = {}^-1\frac{2}{5}$ or $w = 1\frac{1}{4}$ **l** $x = \frac{^-1}{10}$ or $x = 2$

4 **a** $v = 3$ or $v = {}^-1$ **b** $a = 8$ or $a = 9$
c $r = 6$ or $r = {}^-5$ **d** $n = {}^-5$ or $n = {}^-7$
e $w = {}^-5$ or $w = 3$ **f** $y = 3$ or $y = {}^-10$
g $k = 13$ or $k = 10$ **h** $g = 4$ or $g = 5$
i $c = {}^-11$ or $c = 15$ **j** $n = \frac{^-5}{6}$ or $n = {}^-1\frac{1}{5}$
k $b = 1\frac{1}{5}$ or $b = \frac{^-1}{3}$ **l** $m = \frac{9}{4}$ or $m = \frac{9}{8}$
m $x = 2\frac{2}{3}$ or $x = \frac{^-4}{9}$

5 **a** $x = {}^-4$ or $x = {}^-5$ **b** $a = {}^-6$ or $a = {}^-2$
c $x = 7$ or $x = 1$ **d** $m = 7$ or $m = {}^-4$
e $n = 8$ or $n = {}^-5$ **f** $a = {}^-9$ or $a = 7$
g $m = {}^-8$ or $m = 3$ **h** $d = 9$
i $a = {}^-4$ or $a = {}^-7$ **j** $x = 0$ or $x = {}^-1$
k $x = {}^-3$ or $x = 0$ **l** $a = 0$ or $a = 2$
m $d = 0$ or $d = {}^-2$ **n** $y = 0$ or $y = 4$
o $x = 0$ or $x = 6$ **p** $x = {}^-3$ or $x = {}^-2$
q $a = 4$ or $a = {}^-5$ **r** $x = 9$ or $x = {}^-5$
s $b = 1$ or $b = 7$ **t** $h = 5$ or $h = {}^-7$
u $a = 6$ or $a = {}^-1$ **v** $u = 2$ or $u = 3$
w $b = 0$ or $b = \frac{1}{4}$ **x** $d = 0$ or $d = \frac{1}{3}$
y $n = 0$ or $n = \frac{2}{5}$

6 **a** $x = 1$ or $x = \frac{1}{2}$ **b** $x = {}^-1$ or $x = \frac{1}{3}$
c $x = {}^-2$ or $x = \frac{1}{4}$ **d** $x = 3$ or $x = \frac{2}{5}$
e $x = 2$ or $x = {}^-1\frac{1}{3}$ **f** $x = 5$ or $x = 1\frac{1}{2}$

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g $x = {}^{-}4$ or $x = {}^{-}2\frac{1}{2}$ **h** $x = 4$ or $x = {}^{-}1\frac{1}{2}$
i $x = 5$ or $x = {}^{-}2\frac{1}{3}$ **j** $x = 1$ or $x = {}^{-}\frac{2}{5}$
k $x = {}^{-}1$ or $x = {}^{-}1\frac{3}{4}$ **l** $x = 2\frac{1}{2}$ or $x = {}^{-}4$

7 a $x = {}^{-}1$ or $x = \frac{1}{2}$ **b** $x = {}^{-}1$ or $x = {}^{-}\frac{2}{3}$
c $x = 3$ or $x = 1\frac{2}{3}$ **d** $x = 3$ or $x = \frac{2}{3}$
e $c = {}^{-}3$ or $c = 5\frac{1}{2}$ **f** $a = {}^{-}3$ or $a = 2\frac{1}{3}$
g $a = {}^{-}3$ or $a = {}^{-}\frac{4}{7}$ **h** $h = {}^{-}3\frac{2}{3}$ or $h = {}^{-}\frac{1}{3}$
i $x = 2$ or $x = {}^{-}\frac{3}{5}$ **j** $a = 1$ or $a = \frac{1}{6}$
k $m = \frac{3}{4}$ or $m = 2\frac{1}{2}$ **l** $m = \frac{1}{3}$ or $m = {}^{-}1\frac{1}{2}$
m $h = {}^{-}\frac{5}{6}$ or $h = 1\frac{1}{2}$ **n** $m = \frac{3}{7}$ or $m = 3\frac{1}{2}$
o $x = \frac{2}{3}$ **p** $m = \frac{1}{2}$ or $m = 1\frac{2}{3}$
q $c = 6$ or $c = {}^{-}1\frac{1}{4}$ **r** $b = {}^{-}6$ or $b = \frac{3}{4}$
s $a = {}^{-}\frac{1}{2}$ or $a = 1\frac{1}{2}$ **t** $b = {}^{-}7$ or $b = \frac{1}{7}$
u $m = {}^{-}2$ or $m = 2\frac{1}{2}$ **v** $g = 7$ or $g = {}^{-}1\frac{1}{2}$
w $c = 1\frac{1}{2}$ or $c = 4$

8 a $x = {}^{-}2 + \sqrt{3}$ or $x = {}^{-}2 - \sqrt{3}$
b $x = \dfrac{7 - \sqrt{41}}{2}$ or $x = \dfrac{7 + \sqrt{41}}{2}$
c $x = {}^{-}3 - 2\sqrt{2}$ or $x = {}^{-}3 + 2\sqrt{2}$
d $x = 3 - \sqrt{6}$ or $x = 3 + \sqrt{6}$
e $x = \dfrac{2 - \sqrt{14}}{2}$ or $x = \dfrac{2 + \sqrt{14}}{2}$
f $x = \dfrac{{}^{-}3 - \sqrt{7}}{2}$ or $x = \dfrac{{}^{-}3 + \sqrt{7}}{2}$
g $x = \dfrac{{}^{-}3 - \sqrt{3}}{3}$ or $x = \dfrac{{}^{-}3 + \sqrt{3}}{3}$
h $x = \dfrac{1 - \sqrt{17}}{8}$ or $x = \dfrac{1 + \sqrt{17}}{8}$
i $x = \dfrac{5 - \sqrt{10}}{5}$ or $x = \dfrac{5 + \sqrt{10}}{5}$
j $x = \dfrac{3 - \sqrt{41}}{8}$ or $x = \dfrac{3 + \sqrt{41}}{8}$
k $x = \dfrac{{}^{-}2 - \sqrt{6}}{2}$ or $x = \dfrac{{}^{-}2 + \sqrt{6}}{2}$
l $x = \dfrac{{}^{-}1 - \sqrt{6}}{5}$ or $x = \dfrac{{}^{-}1 + \sqrt{6}}{5}$

9 a $x \approx {}^{-}0.38$ or $x \approx {}^{-}2.62$ **b** $a \approx 6.85$ or $a \approx 0.15$
c $b \approx {}^{-}0.70$ or $b \approx {}^{-}4.30$ **d** $w \approx 2.76$ or $w \approx 0.24$
e $n \approx 0.22$ or $n \approx {}^{-}2.22$ **f** $x \approx 0.72$ or $x \approx 2.78$
g $x \approx 2.56$ or $x \approx {}^{-}1.56$ **h** $x \approx 0.78$ or $x \approx {}^{-}1.28$
i $n \approx 1.12$ or $n \approx {}^{-}1.79$ **j** $w \approx {}^{-}0.74$ or $w \approx {}^{-}2.\ 26$
k $c = 2$ or $c = {}^{-}\frac{1}{3}$

10 a $x = 0$ or $x = {}^{-}1$ **b** $x = {}^{-}3$ or $x = {}^{-}5$
c $y = 5$ or $y = {}^{-}2$ **d** $x = 0$ or $x = 17$
e $b = 2$ or $b = {}^{-}12$ **f** $x = 0$ or $x = 1\frac{2}{3}$
g $w = 1$ or $w = {}^{-}4\frac{1}{4}$ **h** $x = {}^{-}1$ or $x = 8\frac{1}{2}$
i $m = 5\frac{1}{2}$ or $m = 9\frac{1}{2}$ **j** $x = {}^{-}2\frac{1}{4}$ or $x = {}^{-}1\frac{1}{2}$

11 a 7 **b** 10 **c** 10 or ${}^{-}9$

12 6 m by 2 m

13 7 cm

14 $x = 10$ cm, so the dimensions are 10 cm, 24 cm, 26 cm.

15 6 cm by 20 cm by 2 cm

16 a 9 integers **b** 12 integers **c** 15 integers

17 $1\frac{3}{5}$ seconds after it was thrown.

18 2.52 cm by 47.57 cm by 10 cm

19 a $(x + 3)^2 = {}^{-}1$
b The square root of ${}^{-}1$ cannot be done on the number line.
c $({}^{-}3, 1)$

20 $\left(x + \dfrac{b}{2}\right)^2 = \dfrac{b^2}{4} - c = \dfrac{b^2 - 4c}{4}$, which gives

$x = \dfrac{-b}{2} \pm \sqrt{\dfrac{b^2 - 4c}{4}} = \dfrac{-b \pm \sqrt{b^2 - 4c}}{2}$, which is

the quadratic formula with $a = 1$.

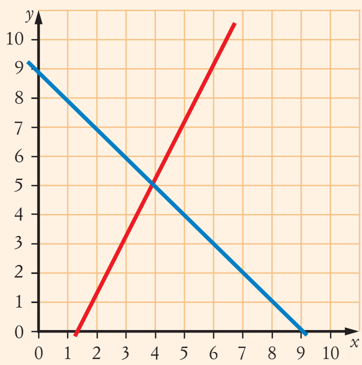
21 Proof, teacher to check.

22 If $b^2 < 4ac$, then the formula contains the square root of a negative number, which has no place on the real number line.

Chapter review

1 a $m = {}^{-}5$ **b** $g = {}^{-}6$ **c** $p = {}^{-}11$ **d** $d = {}^{-}4$

2



Solution is (4, 5), or $x = 4$, $y = 5$.

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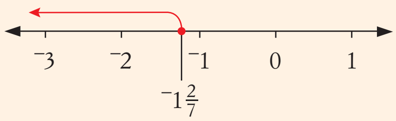
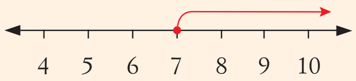
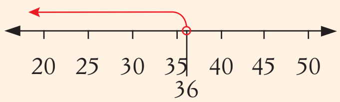
3 a $y \le {}^{-}1\frac{2}{7}$



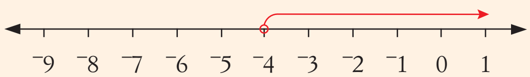
b $q \ge 7$



c $q < 36$



d $n > {}^{-}4$



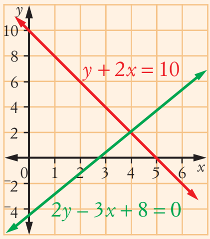
4 a $y = {}^{-}3$ or $y = 3$ **b** $t = {}^{-}4$ or $t = 4$
c $g = {}^{-}\frac{4}{5}$ or $g = \frac{4}{5}$

5 $v = {}^{-}19$ or $v = 5$

6 a $x = 0$ or $x = {}^{-}5$ **b** $a = 4$ or $a = {}^{-}7$
c $m = {}^{-}\frac{2}{3}$ or $m = 2\frac{1}{2}$

7 a $z = 28$ **b** $t = {}^{-}6$

8

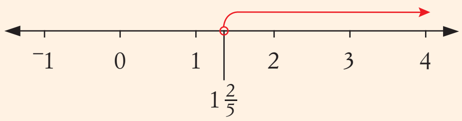


Solution is $x = 4$ and $y = 2$.

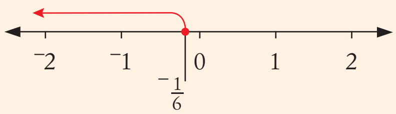
9 a $b = {}^{-}3$ **b** $c = 9$ **c** $b = 4$

10 a $k = 1\frac{1}{5}$ **b** $c = 6$ **c** $g = 2\frac{1}{2}$

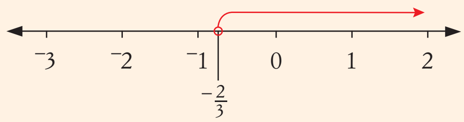
11 a $x > 1\frac{2}{5}$



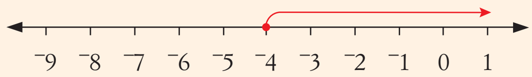
b $h \le {}^{-}\frac{1}{6}$



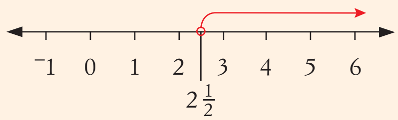
12 a $x > {}^{-}\frac{2}{3}$



b $z \ge {}^{-}4$



c $x > 2\frac{1}{2}$



13 $a = {}^{-}2$ and $b = 3$

14 $u = {}^{-}4$ and $w = 7$

15 a $y = {}^{-}7$ or $y = {}^{-}3$ **b** $a = {}^{-}10$ or $a = 2$
c $b = 1\frac{1}{5}$ or $b = {}^{-}\frac{1}{3}$

16 a $x = 0$ or $x = 5$ **b** $w = 6$ or $w = 7$
c $y = 2$ or $y = {}^{-}9$ **d** $c = {}^{-}2$ or $c = 8$
e $h = 1\frac{1}{4}$ or $h = {}^{-}\frac{5}{7}$ **f** $7g = {}^{-}4$ or $4g = 1$

17 $m = {}^{-}2$ or $m = \frac{1}{4}$

18 $x = \dfrac{{}^{-}7 - \sqrt{41}}{4} \approx {}^{-}3.3507$ or

$x = \dfrac{{}^{-}7 + \sqrt{41}}{4} \approx {}^{-}0.1492$

19 $a = \frac{1}{2}$

20 a $a = 3$ and $b = {}^{-}1$ **b** $x = 6\frac{1}{9}$ and $y = \frac{1}{9}$

21 2.5 m by 2 m

22 $2s + s = 78$. Liam is 52 and his son is 26.

23 7 m by 13 m

24 Woman is 30 years old, daughter is 10 years old.

25 $d = \frac{1}{4}$ and $g = \frac{1}{2}$

26 $x = 4$ or $x = 7\frac{1}{3}$

27 4 seconds, the negative 2 seconds is false and represents a time before it was thrown.

28 $(x + 2)^2 = {}^{-}5$, the square root of a negative number has no place on the number line.

29 Proof, teacher to check.

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Word	Definition
adjacent side	In a right-angled triangle, the side joining the right angle and nominated angle. *See also* hypotenuse, opposite side.
allied (angles)	Inside angles on the same side of a transversal over two lines. *See also* co-interior angles.
alternate (angles)	Inside opposite (or outside opposite) angles on a transversal over two lines.
angle of depression	Angle between the horizontal and the line of sight *down* to an object.
angle of elevation	Angle between the horizontal and the line of sight *up* to an object.
asymptote	A straight line on a graph that is approached by a curve.
bearing	A three-digit number showing the angle in degrees clockwise from north to the direction of an object.
bias	Data or method that is likely to give results generally different to the population. *See also* fair.
bimodal	Having two modes; two scores with equal highest frequency.
binomial products, binomial brackets	Product of two brackets, such as $(2x + 3)(4x - 1)$.
bivariate	Data set where each item has two measurement variables. *See also* univariate.
cardinal number	The numbers 0, 1, 2, 3, . . . etc.; the number of elements in a finite set.
Cartesian plane	A flat surface with horizontal and vertical axes intersecting at 0.
categorical (data)	Data that can be put into categories, such as names of colours.
certain	Will always happen.
closure, closed	An operation closed for a set always gives another element of the set when performed on element(s) of the set.
coefficient	Number in front of a variable; number that is multiplied by a variable.
co-interior (angles)	Inside angles on the same side of a transversal over two lines. *See also* allied angles.
common factor	Number or algebraic term that is a factor of each of some given numbers or terms.
commutative (law)	Addition (or multiplication) of two numbers is the same for either number order.
complementary (angles)	Adjacent angles that together form a right angle (90°).
complementary (events)	Two separate events that together make the entire sample space.
completion of the square	Procedure that changes a quadratic expression to the form $a(x + b)^2 + c$.

Glossary

Word	Definition
compound interest	Interest calculated on the current balance of a loan or investment. *See also* reducing interest.
conditional probability	The probability $P(A\|B)$ that A occurs, assuming that B occurs.
congruent (shapes), **congruence**	Shapes that are the same. They may be exactly superimposed, possibly by turning one over.
constant	A number, particularly a term of an expression that is just a number.
continuous (data)	Data that can have any values, such as heights.
converse	For logic or mathematical theorems, the reverse statement.
coordinates	The numbers showing position in a Cartesian plane; e.g. (3, 5).
corresponding (angles)	Angles on the same side of a transversal and the same sides of lines it crosses.
cosine, **cos**	The adjacent side divided by the hypotenuse for a nominated angle in a right-angled triangle. *See also* sine, tangent.
counter example	An example that shows that something is not true.
cross method	Factorisation of a quadratic using a cross to show possible factors.
cross-section	The 2D shape on a flat surface made by cutting through a solid (3D) shape.
cumulative frequency	The progressive total of frequencies of ordered numeric data.
curved surface area	The area of the curved face of a cone, cylinder, or sphere.
deductive geometry	Geometry worked out using logical reasoning from assumptions (axioms).
degree	Angle measure such that a complete revolution is 360 degrees, written as 360°. *See also* minute, second.
dependent events	Events where the probability of one is affected by the occurrence of the other. *See also* independent.
discrete (data)	Data that can only have particular values, such as shoe sizes.
distributive (law)	The product of the sum (or difference) of two numbers with another number is the same as the sum (or difference) of the products of that number with each of the two numbers; $a(b \pm c) = ab \pm ac$.
dummy variable	Variable substituted for an expression to simplify a process.
element	A basic probability outcome; '5' on a die. *See also* sample point.
elimination method	Solution of simultaneous equations by elimination of variables.
empty set	*See* null set.
equivalent ratios	Equal ratios.
event (probability)	An individual outcome or collection of outcomes.
expected frequency	The number of times an event is expected from a particular number of trials.

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Word	Definition
exponent	The raised number of a power, such as the 4 in 5^4. *See also* index.
exponential function	A function with a variable in the exponent such as $y = 5 \times 2^x$.
expression	Variables and/or numbers connected by operations.
exterior (angle of a triangle)	Angle on the outside of a triangle made by extending one side.
extrapolation	Estimation of a value outside the range of known values. *See also* interpolation.
factorise	Use the distributive law to change an expression into factors, include a bracketed factor.
fair	A sample/question/method that does not favour particular results and will be truly representative of the population. *See also* bias.
favourable outcome	The particular outcome being considered.
five number summary	The lowest, first quartile, median, third quartile and highest values of a frequency distribution.
flat-rate interest	Interest calculated on the initial value of a loan or investment. *See also* simple interest.
formula	An equation that gives one quantity in terms of others. The quantity on its own is called the subject.
function	A rule that changes a number (input) to a new one (output).
given	For a statement P in mathematical logic, given P means 'assuming that P is true'.
gradient	Property of a straight line obtained by dividing the vertical distance between two points (rise) by the horizontal (run). *See also* slope.
grouping factorisation	The creation of a common factor of separate groups of an expression.
growth factor	Factor that is multiplied by a value to produce the next value, especially when growth is a fixed percentage over time such as compound interest.
hectare	The area of a square 100 m by 100 m; 10 000 m^2.
hypotenuse	The long side of a right-angled triangle (opposite the right angle).
impossible	Will never happen.
index	The raised number of a power, such as the 4 in 5^4; plural *indices*. *See also* exponent.
inequality	An equation where, instead of two equal sides, one side may be smaller, equal to or bigger than the other.
instalment	Equal payments used to repay a loan at fixed intervals, such as monthly instalments.
intercept	Position where a line intersects an axis.

Word	Definition
interest	Charge made for the use of money (obtained as a loan). The amount of the loan is called the principal.
interest rate	Interest stated as a yearly percentage of the amount borrowed.
interpolation	Estimation of a value within the range of known values. *See also* extrapolation.
interquartile range (IQR)	IQR = third quartile − first quartile for a frequency distribution.
inverse operations	Operations that do the reverse of each other; × and ÷, etc.
intersection (sets)	The set that contains the elements of both of two given sets.
irrational number	A number that cannot be expressed as a ratio of integers such as $\sqrt{2}$.
LHS	Left-hand side.
line of best fit	A (usually straight) line drawn as closely as possible to the points on a graph.
linear (rule, equation, function, inequality)	A rule, equation, function or inequality whose graph is a straight line.
mantissa	The value part of a number in scientific notation: The mantissa of 3.52×10^4 is '3.52'.
mean	The arithmetic average, $\bar{x}$ = total ÷ number; for 3, 6, 12, $\bar{x} = 21 \div 3 = 7$.
median	The central value, by count, of ordered statistical data.
midpoint	The point halfway between two given points.
minute (angle)	$\frac{1}{60}$ of a degree. *See also* second.
mode	The value that occurs most often in statistical data; the score with the highest frequency.
monic quadratic	Quadratic expression or equation where the coefficient of x^2 is 1.
negative skew	A greater spread of a frequency distribution below the median (to the left) than above the median.
nominal	Categorical data with no natural order, such as the names of countries.
nominal interest rate	Interest rate as an annual rate, especially when rests are not annual.
non-compliant data	Data that is obviously wrong, such as someone's age being 164.
non-monic quadratic	Quadratic expression or equation where the coefficient of x^2 is not 1.
null set	Set with no elements, shown as {} or ø. *See also* empty set.
numeric, numerical (data)	Data that are numbers.
opposite side	In a right-angled triangle, the short side not joining the right angle and opposite the nominated angle. *See also* adjacent side, hypotenuse.
ordinal	Categorical data having a natural order, such as places in a race.
outlier	A data value well outside the range of most numerical data.

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Word	Definition
parabola	A mathematical relation such as $y = 3x^2 + 2$. Its graph is a symmetric U-shape with sides curving away from each other.
perfect square	Square of an integer or expression that is the square of a linear expression. *See also* square of a sum, square of a difference.
polygon	Closed 2D figure with straight sides.
population	The entire group that could be relevant to a survey.
positive skew	A greater spread of a frequency distribution above the median (to the right) than below the median.
power	A number multiplied by itself, written as 5^4, say, for $5 \times 5 \times 5 \times 5$.
principal	The initial amount of a loan.
prism	Polyhedron where every cross-section parallel to one face is identical to that face.
probability	The chance of an event; experimental or theoretical probability.
proof	A series of mathematical statements that logically show that a statement is true. *See also* QED, RTP, theorem.
pyramid	Polyhedron such that cross-sections parallel to one face are similar and reduce evenly in size to a vertex called the apex opposite this face.
Pythagoras' theorem	Relationship between the sides of a right-angled triangle attributed to Pythagoras (569?–475? BCE).
Pythagorean triple (triad)	Three positive integers a, b, c such that $a^2 + b^2 = c^2$, that could be sides of a right-angled triangle.
QED	Quod erat demonstrandum; Latin phrase at the end of a formal proof to say it is finished. *See also* proof, RTP, theorem.
quadratic	Expression, equation or function with a squared term, of the form $ax^2 + bx + c\ (= 0)$.
quartile	Any of 3 points that divide an ordered data set into 4 equal groups. The highest element may be called the 4th quartile.
range	The difference between the highest and lowest scores in a set of numeric data.
reducing interest	Interest calculated on the current balance of a loan. *See also* compound interest.
relative frequency	The frequency of an outcome divided by the total frequency.
rest	Time between regular calculations of compound interest.
rise	The vertical distance between two points. *See also* gradient.
RHS	Right-hand side.
root (1)	Reverse of raising to a power. A square root, cube root, fourth root, etc. *See also* surd.
root (2)	The solution of an equation.

Word	Definition
RTP	Required to prove; statement of a theorem before its formal proof. *See also* proof, QED, theorem.
run	The horizontal distance between two points. *See also* gradient.
sample	Part of a population (usually used for a survey).
sample point	A basic probability outcome; '5' on a die. *See also* element.
sample space	List of all the simple outcomes in a situation. *See also* event space.
satisfy	Make true; (3, 7) satisfies $y = 2x + 1$.
scale factor	The ratio of the sizes of an enlargement or reduction of a shape.
scatter plot	Graph with points representing the values of bivariate data as coordinates.
scientific notation	Notation used to write a value as a number between 1 and 10, multiplied by a power of 10.
second (angle)	$\frac{1}{60}$ of a minute = $\frac{1}{3600}$ of a degree.
significant figures	The (number) of digits in the mantissa when a number is written in scientific notation.
similar	Shapes that are enlargements or reductions of each other.
simple interest	Interest charged as a percentage of the initial loan or investment, multiplied by the period. *See also* flat-rate interest.
simplification, simplify	Multiply out brackets and/or collect like terms.
simultaneous equations	A system with two (or more) equations considered at the same time.
simultaneous solution	A solution that satisfies all the simultaneous equations.
sine, sin	The opposite side divided by the hypotenuse for a nominated angle in a right-angled triangle. *See also* cosine, tangent.
skewed (distribution)	An asymmetrical set of scores with a unequal number of scores above and below the mean.
skewed to the left	A greater spread of a frequency distribution to the left of the median (below) than to the right of the median.
skewed to the right	A greater spread of a frequency distribution to the right of the median (above) than to the left of the median.
slant height	The distance from the edge of the base of a cone to the apex.
slope	Property of a straight line obtained by dividing the vertical distance between two points (rise) by the horizontal (run). *See also* gradient.
solution	The answer to a problem, particularly an equation.
solve	Find the answer to a problem, particularly an equation.
spread	Measure of the differences between the values of some data.
square (number)	The product of a number with itself.

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Word	Definition
square of a difference	An expression of the form $(x - a)^2 = x^2 - 2ax + a^2$. *See also* perfect square.
square of a sum	An expression of the form $(x + a)^2 = x^2 + 2ax + a^2$. *See also* perfect square.
standard form	Agreed format of an expression, equation, etc., such as $ax + by + c = 0$ or $ax^2 + bx + c = 0$.
stem-and-leaf plot	Display with high place values arranged vertically and the other place values arranged horizontally.
subject	The variable on its own (on the left) that is calculated from others in a formula.
supplementary (angles)	Adjacent angles that form a straight line; their sum is 180°.
surd (1)	The symbol $\sqrt{\ }$ used to show a root (1). *See also* root (1).
surd (2)	Root of a rational number (especially an integer) that is not rational.
surface area	The area of a surface, particularly the total area of the faces of a solid shape.
tangent, tan	The opposite side divided by the adjacent side for a nominated angle in a right-angled triangle. *See also* cosine, sine.
term (1)	One of the values of a number pattern; 6 is the second term of 3, 6, 9, 12, 15, . . .
term (2)	Part of an expression separated from other parts by addition or subtraction.
theorem	Precise mathematical statement capable of formal proof consisting of the parts RTP, proof, QED.
transformation	Change in a shape: a rotation, reflection, enlargement, reduction or translation.
transpose	Change a formula to make a particular variable the subject.
transversal	A line that crosses two or more others.
tree diagram	Diagram with many branches showing the outcomes in a multi-stage situation.
trigonometric ratio	Commonly, any of $\cos \theta$, $\sin \theta$ or $\tan \theta$ for an angle θ.
trinomial	Quadratic expression in unfactorised form. *See also* quadratic.
turning point	Point on a curved line such as a parabola where it changes direction.
two-way table	A table that lists one set of outcomes horizontally and the other set vertically.
union	The set that contains all the elements in two given sets.
unit circle	Circle of radius 1, especially as used for trigonometric ratios.
unitary method	Method using the value of a unit quantity to calculate a result.

Word	Definition
univariate	Data set where each item has only one measurement variable. *See also* bivariate.
universal set	Set of all elements under consideration in a particular context. *See also* Venn diagram.
variable	A symbol, particularly a letter, which stands for a number.
Venn diagram	A diagram that shows the elements of events (or sets) in overlapping circles within a rectangle representing the universal set.
vertex	The intersection of two sides of a polygon or three or more faces of a solid shape; plural *vertices*.
vertically opposite (angles)	Opposite angles at the intersection of two lines; they are not adjacent.
***x*-axis**	The horizontal axis of a Cartesian plane.
***x*-coordinate**	The first coordinate of a point in the Cartesian plane; the 4 in (4, 3).
***x*-intercept**	The point where a line crosses the *x*-axis, or the distance from this to the origin.
***y*-axis**	The vertical axis of a Cartesian plane.
***y*-coordinate**	The second coordinate of a point in the Cartesian plane; the 3 in (4, 3).
***y*-intercept**	The point where a line crosses the *y*-axis, or the distance from this to the origin.
zero	Value that makes a function equal to zero. An *x*-intercept of a line, especially a curved line.
zero product rule	The rule that if the product of two numbers is zero, at least one must be zero.

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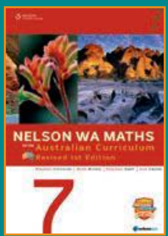

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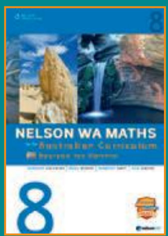

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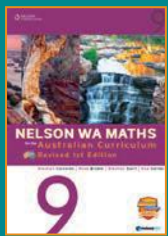

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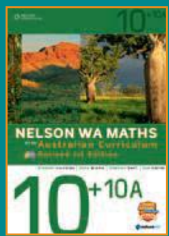

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