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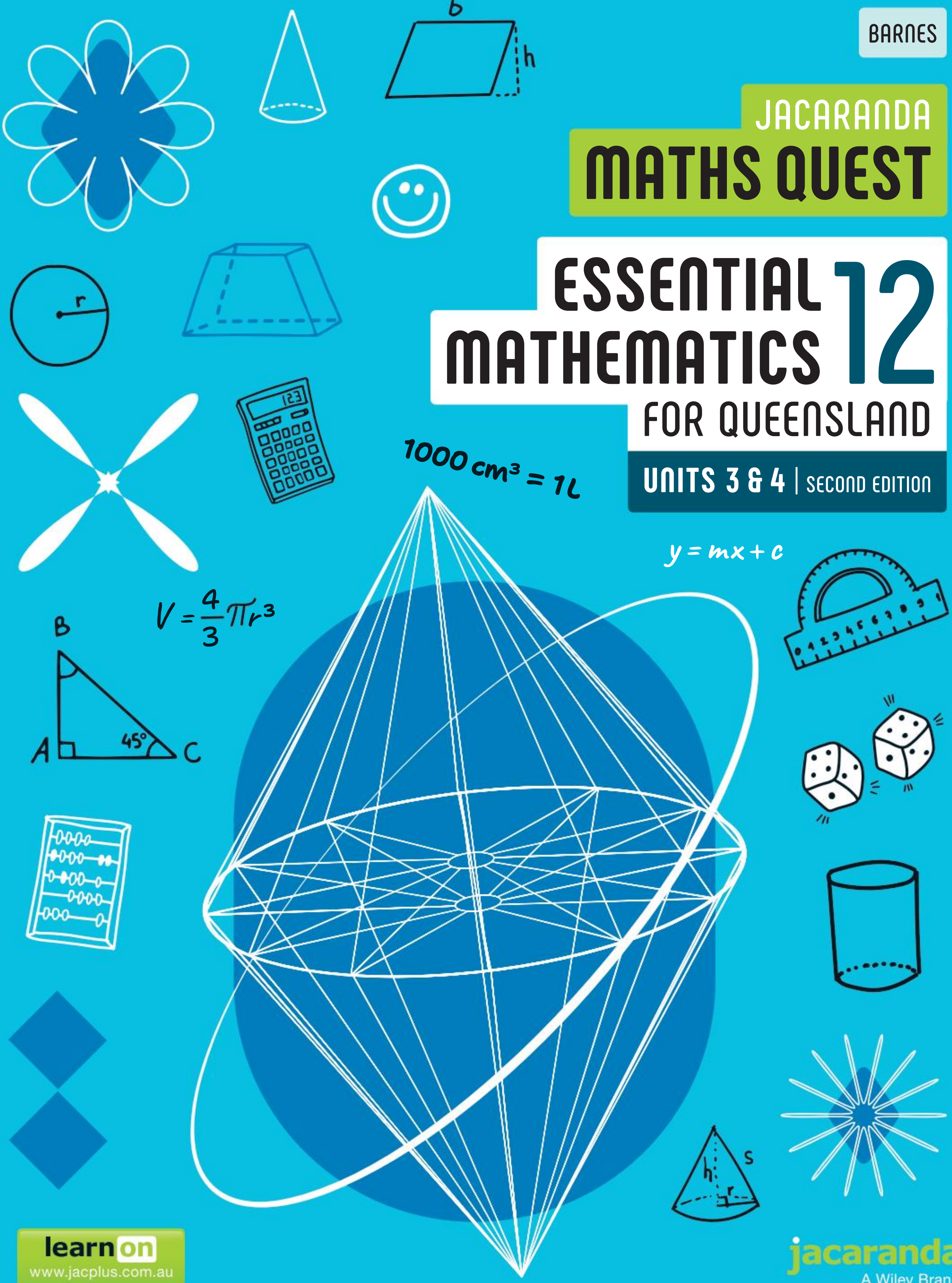
MATHS QUEST

ESSENTIAL MATHEMATICS 12
FOR QUEENSLAND

UNITS 3 & 4 | SECOND EDITION



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MARK BARNES

CONTRIBUTING AUTHORS

Arrian Hannebach | Paul Johansen | Emilia Sinton

REVIEWED BY

Carl Harbinson | Annabelle Jones | Kamlesh Kumar | Craig Otto
Tanya Summers | Fiona Swan | Taylor Richards

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This suite of resources may include references to (including names, images, footage or voices of) people of Aboriginal and/or Torres Strait Islander heritage who are deceased. These images and references have been included to help Australian students from all cultural backgrounds develop a better understanding of Aboriginal and Torres Strait Islander Peoples' history, culture and lived experience.

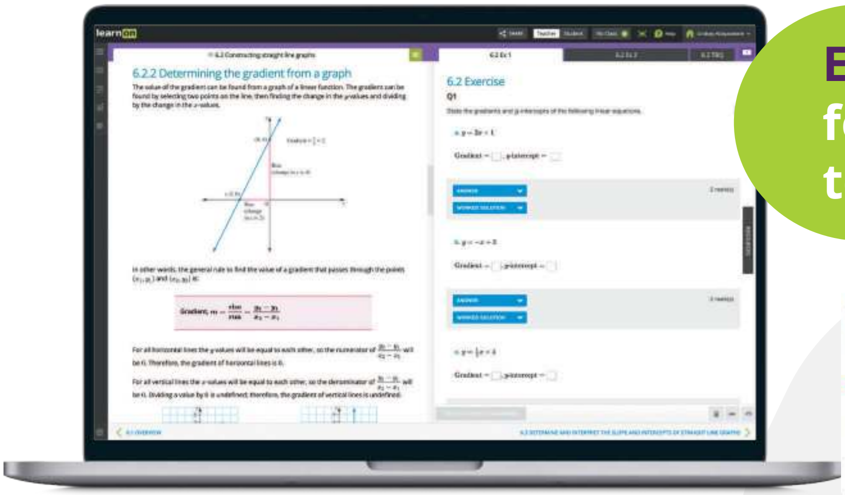
It is strongly recommended that teachers examine resources on topics related to Aboriginal and/or Torres Strait Islander Cultures and Peoples to assess their suitability for their own specific class and school context. It is also recommended that teachers know and follow the guidelines laid down by the relevant educational authorities and local Elders or community advisors regarding content about all First Nations Peoples.

All activities in this resource have been written with the safety of both teacher and student in mind. Some, however, involve physical activity or the use of equipment or tools. **All due care should be taken when performing such activities**. To the maximum extent permitted by law, the authors and publisher disclaim all responsibility and liability for any injury or loss that may be sustained when completing activities described in this resource.

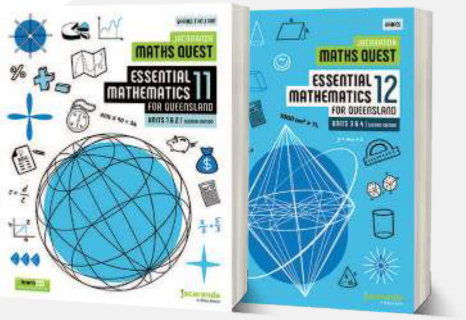
The publisher acknowledges ongoing discussions related to gender-based population data. At the time of publishing, there was insufficient data available to allow for the meaningful analysis of trends and patterns to broaden our discussion of demographics beyond male and female gender identification.

Contents

Learning with learnON



Everything you need for your students to succeed



JACARANDA MATHS QUEST

ESSENTIAL MATHEMATICS 12

UNITS 3 AND 4 FOR QUEENSLAND
SECOND EDITION

Developed by expert teachers for students

Tried, tested and trusted. The completely revised and updated second edition of *Jacaranda Maths Quest 12 Essential Mathematics Units 3 & 4 for Queensland* continues to focus on helping teachers achieve learning success for every student — ensuring no student is left behind and no student is held back.

Because both *what* and *how* students learn matter

Learning is personal

Whether students need a challenge or a helping hand, you'll find what you need to create engaging lessons.

Whether in class or at home, students can get unstuck and progress! Scaffolded lessons with detailed worked examples and difficult concepts are supported by teacher-led video eLessons. Automatically marked, differentiated question sets are all supported by detailed worked solutions. And brand-new quick quiz questions support in-depth skill acquisition in every lesson.

Learning is effortful

Learning happens when students push themselves. With learnON, Australia's most powerful online learning platform, students can challenge themselves, build confidence and ultimately achieve success.

Learning is rewarding

Through real-time results data, students can track and monitor their own progress and easily identify areas of strength and weakness.

And for teachers, Learning Analytics provide valuable insights to support student growth and drive informed intervention strategies.

Learn online with Australia's most

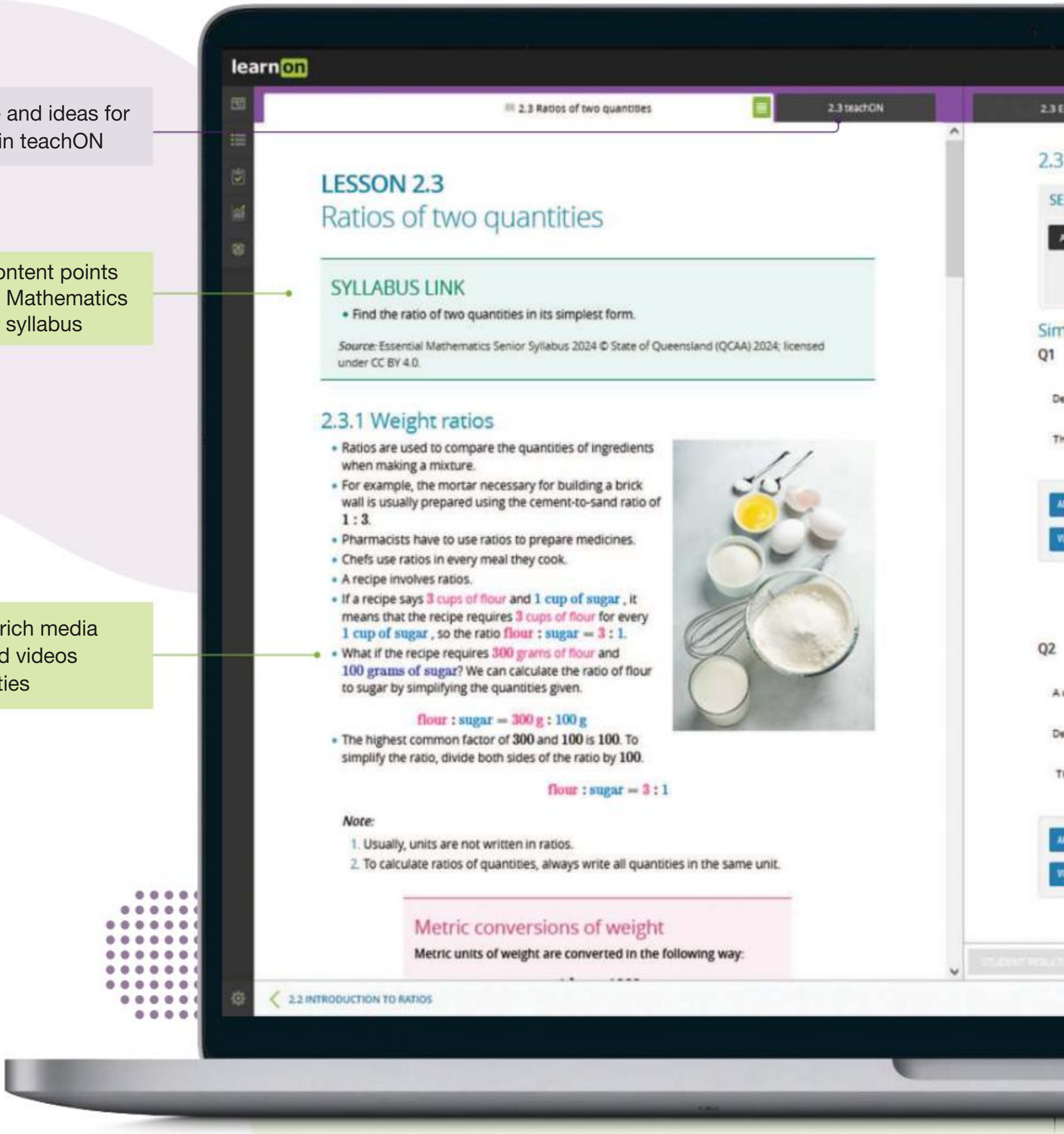
Everything you need for each of your lessons in one simple view

- **Trusted, curriculum-aligned theory**
- **Engaging, rich multimedia**
- **All the teacher support resources you need**
- **Deep insights into progress**
- **Immediate feedback for students**
- **Create custom assignments in just a few clicks.**

Practical teaching advice and ideas for each lesson provided in teachON

Each lesson linked to content points from the QCAA Essential Mathematics 2025 Applied senior syllabus

Reading content and rich media including embedded videos and interactivities



LESSON 2.3
Ratios of two quantities

SYLLABUS LINK

- Find the ratio of two quantities in its simplest form.

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2.3.1 Weight ratios

- Ratios are used to compare the quantities of ingredients when making a mixture.
- For example, the mortar necessary for building a brick wall is usually prepared using the cement-to-sand ratio of $1:3$.
- Pharmacists have to use ratios to prepare medicines.
- Chefs use ratios in every meal they cook.
- A recipe involves ratios.
- If a recipe says 3 cups of flour and 1 cup of sugar, it means that the recipe requires 3 cups of flour for every 1 cup of sugar, so the ratio flour : sugar = 3 : 1.
- What if the recipe requires 300 grams of flour and 100 grams of sugar? We can calculate the ratio of flour to sugar by simplifying the quantities given.

$$\text{flour} : \text{sugar} = 300\text{ g} : 100\text{ g}$$

- The highest common factor of 300 and 100 is 100. To simplify the ratio, divide both sides of the ratio by 100.

$$\text{flour} : \text{sugar} = 3 : 1$$

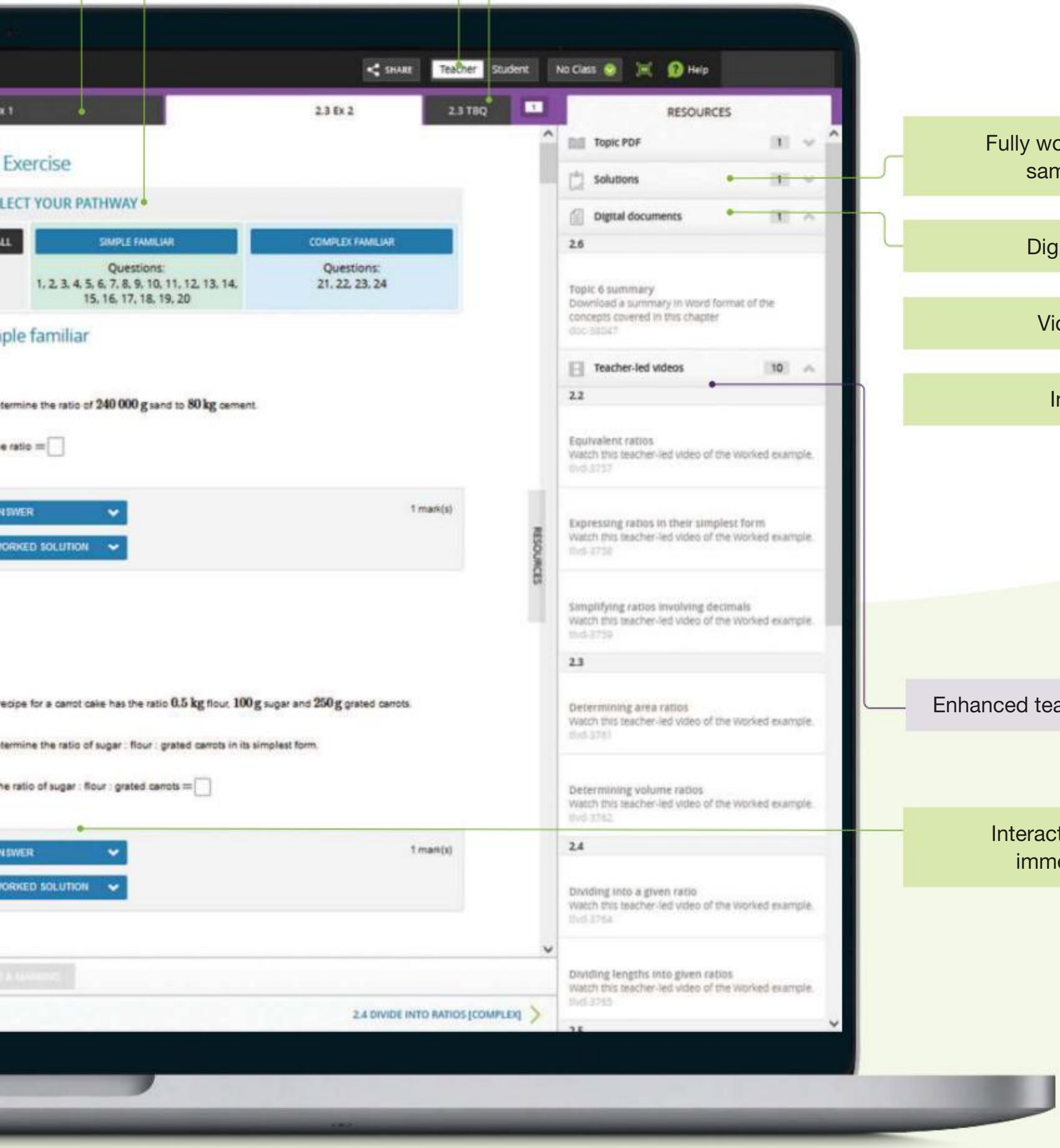
Note:

1. Usually, units are not written in ratios.
2. To calculate ratios of quantities, always write all quantities in the same unit.

Metric conversions of weight

Metric units of weight are converted in the following way:

powerful learning tool, learnON



Exercise

LECT YOUR PATHWAY

SIMPLE FAMILIAR
Questions:
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20

COMPLEX FAMILIAR
Questions:
21, 22, 23, 24

ple familiar

termine the ratio of $240\,000$ g sand to 80 kg cement.

e ratio = ☐

1 mark(s)

ecipe for a carrot cake has the ratio 0.5 kg flour, 100 g sugar and 250 g grated carrots.

termine the ratio of sugar : flour : grated carrots in its simplest form.

he ratio of sugar : flour : grated carrots = ☐

1 mark(s)

2.4 DIVIDE INTO RATIOS [COMPLEX]

Topic PDF 1

Solutions 1

Digital documents 1

2.6

Topic 6 summary
Download a summary in Word format of the concepts covered in this chapter
doc-38047

Teacher-led videos 10

2.2

Equivalent ratios
Watch this teacher-led video of the worked example.
tlvd-3757

Expressing ratios in their simplest form
Watch this teacher-led video of the worked example.
tlvd-3758

Simplifying ratios involving decimals
Watch this teacher-led video of the worked example.
tlvd-3759

2.3

Determining area ratios
Watch this teacher-led video of the worked example.
tlvd-3761

Determining volume ratios
Watch this teacher-led video of the worked example.
tlvd-3762

2.4

Dividing into a given ratio
Watch this teacher-led video of the worked example.
tlvd-3764

Dividing lengths into given ratios
Watch this teacher-led video of the worked example.
tlvd-3765

Quick quiz questions

Differentiated question sets

Teacher and student views

Textbook questions

Fully worked solutions and sample responses

Digital documents

Video eLessons

Interactivities

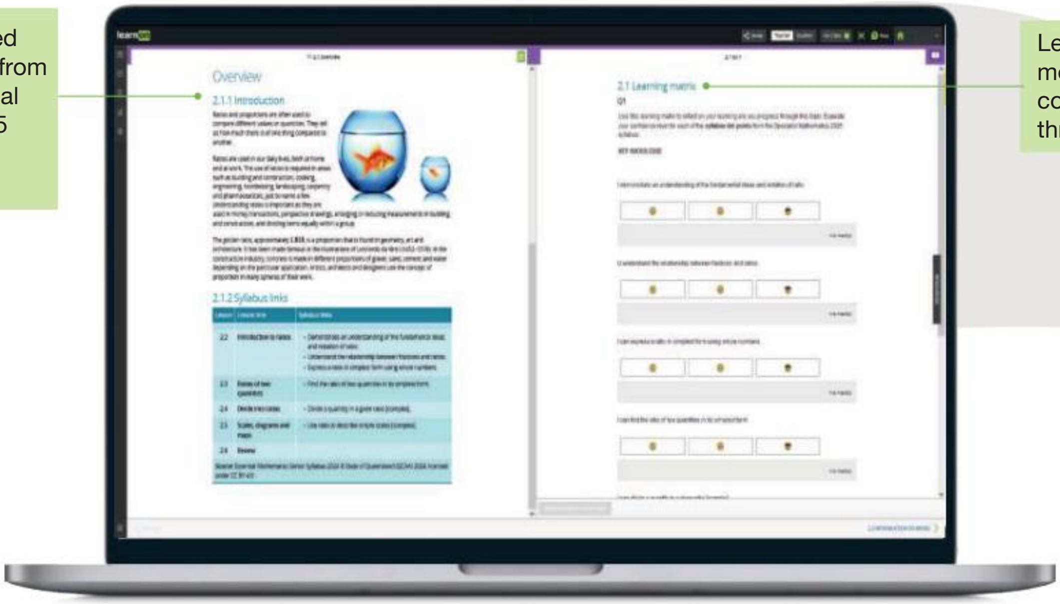
Enhanced teaching support resources

Interactive questions with immediate feedback

Online, these new editions are the complete package

Trusted Jacaranda theory, plus tools to support teaching and make learning more engaging, personalised and visible.

Each topic is linked to content points from the QCAA Essential Mathematics 2025 Applied senior syllabus.



Learning matrix to monitor student's confidence level throughout topics

4.2 Time series and trend lines

LEARNING INTENTION

At the end of this subtopic you should be able to:

- identify key qualitative features of a time series plot including trend (using smoothing if necessary), seasonality, irregular fluctuations and outliers, and interpret these in the context of the data.

4.2.1 Types of time series

In this topic, we consider cases where the x-variable is time. Time goes up in even increments such as hours, to see how some quantity varies with time.

For example, a company may wish to record its daily sales figures over a 10-day period.

Time	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10
Sales ($)	5200	5600	6100	6200	7000	7100	7500	7700	7700	8000

We could also make a graph of this time series as shown.

As can be seen from this graph, there seems to be a trend upwards — clearly, this company is increasing its revenues.

In time series data, trend, seasonal, cyclic and irregular fluctuations are important features to be observed.

Trend

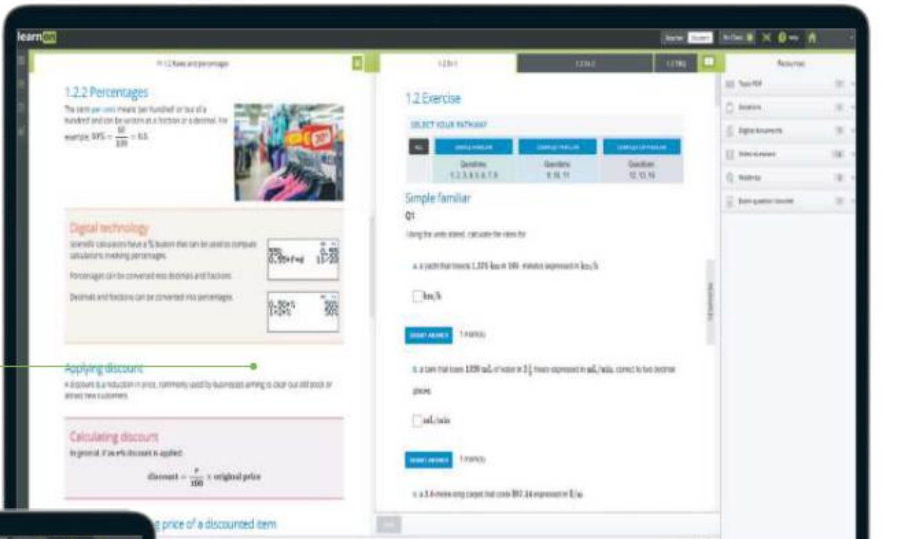
When there is long-term increase or decrease, it is said to be a **trend**. Trend is not always linear.

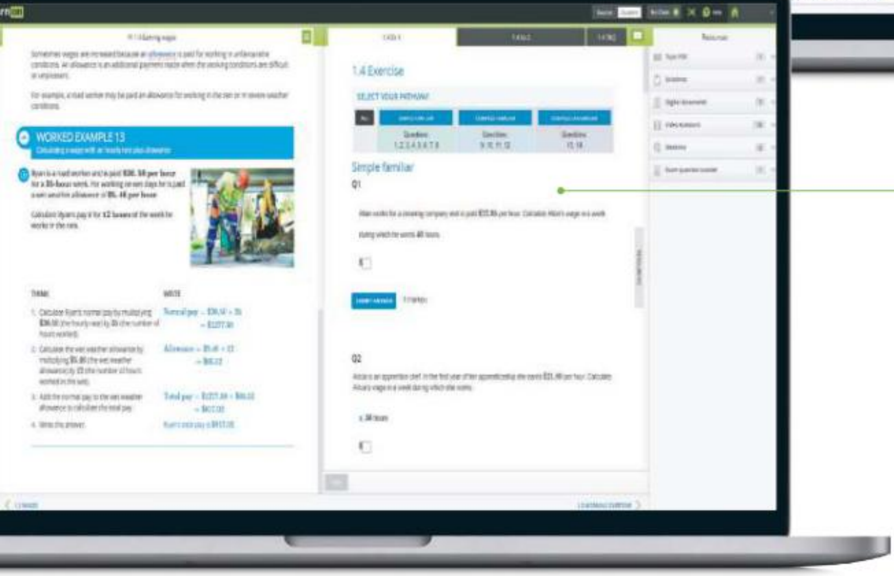
onResources link to targeted digital resources including video eLessons and fully worked solutions.

Tables and images break down content, allowing students to understand complex concepts.

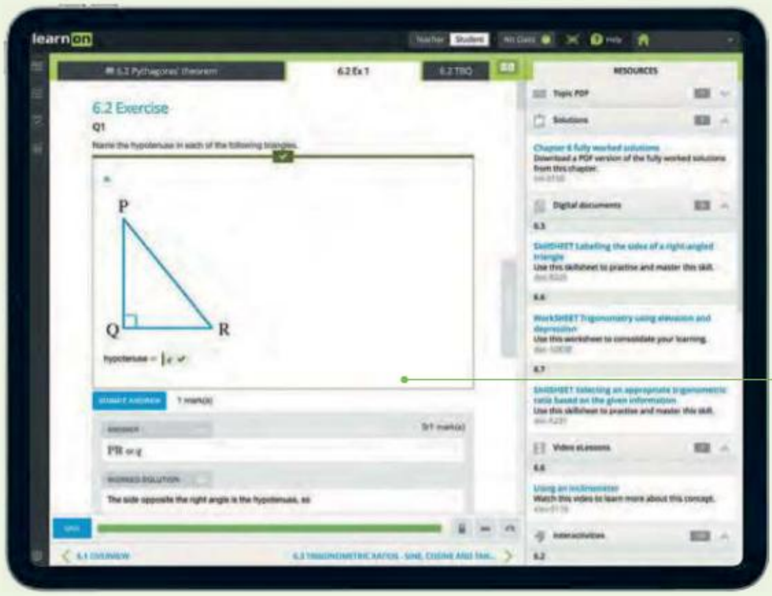
Interactive glossary terms help develop and support mathematical literacy.

Pink highlight boxes summarise key information.





Worked examples, supported by teacher-led videos, break down the process of answering questions using a think/write format.



- Online and offline question sets contain practice questions, with exemplary responses.
- Every question has immediate, corrective feedback to help students to overcome misconceptions as they occur and to study independently — in class and at home.

The new jacTUTOR

An AI tutor to build confidence and get unstuck

With jacTUTOR, every student can have the help of a personal tutor when they get stuck, in class or at home. This purpose-built tool sits safely within your favourite Jacaranda resource, so guidance will always be specific to that lesson, question and curriculum.

A personal tutor for every student

Now every student can get the help they need, when they need it. jacTUTOR uses AI to create a fair and level playing field for all students.

Get guidance, not the answer

jacTUTOR doesn't just give the answer away. Students are given prompts to help understand what they should be asking to get to a correct answer.

Combat anxiety

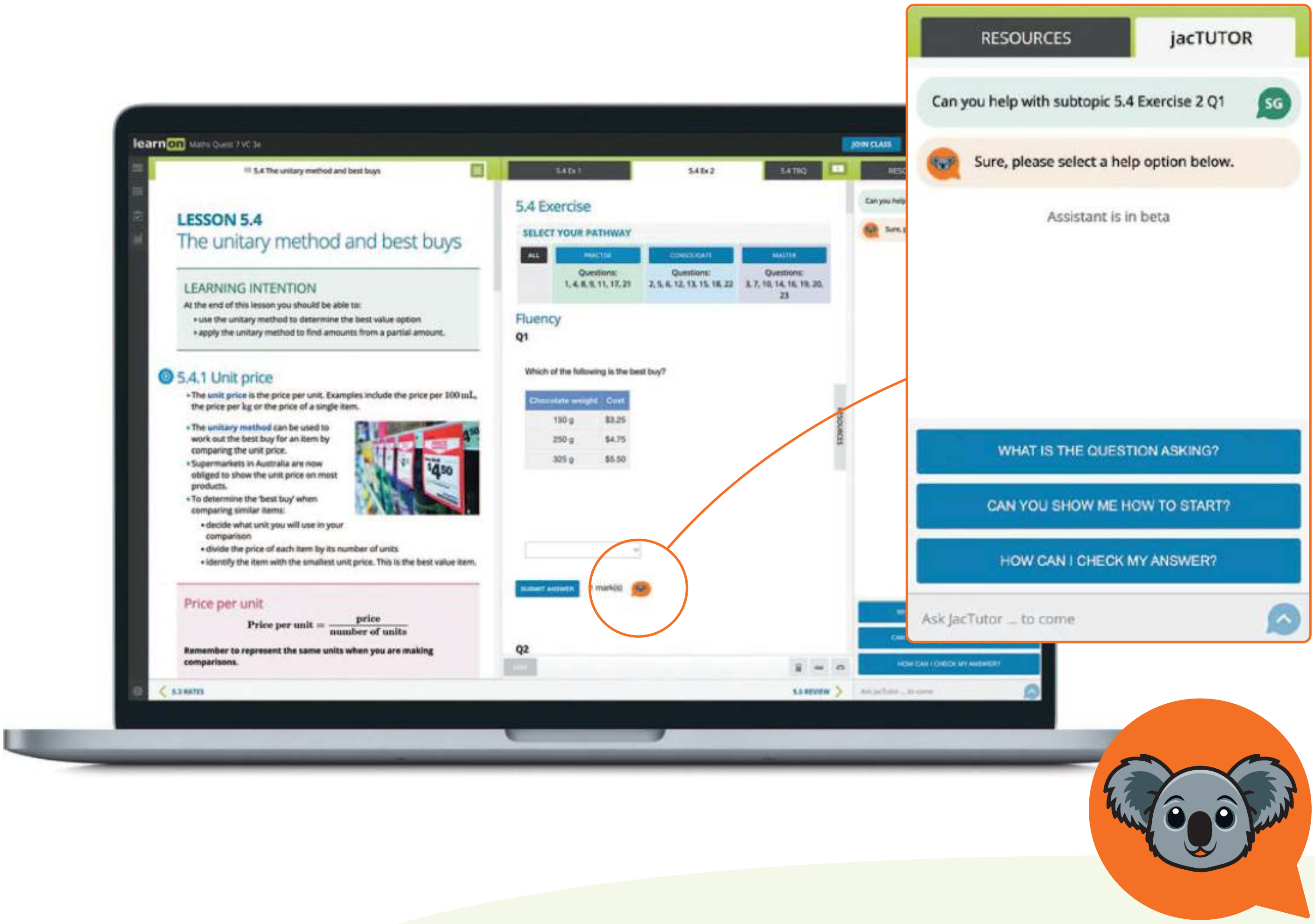
Afraid to raise a hand or ask more questions? With jacTUTOR, students will find a safe space to ask questions, get clarification and try again.

Consistency over confusion

There are so many ways to learn. To avoid confusion, jacTUTOR is built on the same trusted method used in the Jacaranda text.

A safe space

To keep students safe, any concerning or inappropriate comments are automatically flagged and sent to their teacher.



LESSON 5.4
The unitary method and best buys

LEARNING INTENTION

At the end of this lesson you should be able to:

- use the unitary method to determine the best value option
- apply the unitary method to find amounts from a partial amount.

5.4.1 Unit price

- The unit price is the price per unit. Examples include the price per 100 mL, the price per kg or the price of a single item.
- The unitary method can be used to work out the best buy for an item by comparing the unit price.
- Supermarkets in Australia are now obliged to show the unit price on most products.
- To determine the 'best buy' when comparing similar items:
  - decide what unit you will use in your comparison
  - divide the price of each item by its number of units
  - identify the item with the smallest unit price. This is the best value item.

Price per unit

$$\text{Price per unit} = \frac{\text{price}}{\text{number of units}}$$

Remember to represent the same units when you are making comparisons.

5.4 Exercise

SELECT YOUR PATHWAY

ALL	PRACTISE	CONSOLIDATE	MASTER
	Questions: 1, 4, 8, 9, 11, 17, 21	Questions: 2, 5, 6, 12, 13, 15, 18, 22	Questions: 3, 7, 10, 14, 16, 19, 20, 23

Fluency

Q1

Which of the following is the best buy?

Chocolate weight	Cost
150 g	$3.25
250 g	$4.75
325 g	$5.50

Q2

Can you help with subtopic 5.4 Exercise 2 Q1

Sure, please select a help option below.

Assistant is in beta

WHAT IS THE QUESTION ASKING?

CAN YOU SHOW ME HOW TO START?

HOW CAN I CHECK MY ANSWER?

Ask JacTutor ... to come

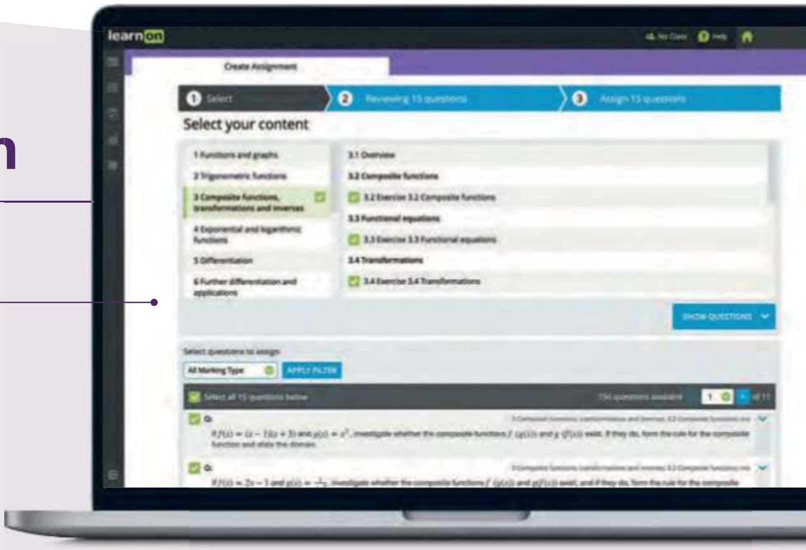
Teaching with learnON

Enhanced teacher support resources, including:

- work programs and curriculum grids
- teaching advice and additional activities
- quarantined topic tests (with solutions)
- Unit reviews
- Quarantined PSMTs and examinations
- Custom exam-builder with question differentiation (SF/CF/CU) question filters

Customise and assign

A testmaker enables you to create custom tests from the complete bank of thousands of questions.



Reports and results

Data analytics and instant reports provide data-driven insights into performance across the entire course.

Show students (and their parents or carers) their own assessment data in fine detail. You can filter their results to identify areas of strength and weakness.

Acknowledgements

The authors and publisher would like to thank the following copyright holders, organisations and individuals for their assistance and for permission to reproduce copyright material in this book.

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UNIT

3 Measurement, scales and chance

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UNITS 3 & 4 FUNDAMENTAL TOPIC Calculations

1 Calculations

LESSON SEQUENCE

Fully worked solutions for this chapter are available online.

on Resources

Solutions	Solutions — Chapter 1
Digital documents	Learning matrix — Chapter 1 Quick quizzes — Chapter 1 Chapter summary — Chapter 1

LESSON
1.1 Overview

1.1.1 Introduction

Being able to add, subtract, multiply and divide integers is important in many parts of everyday life. It helps with budgeting and knowing what you can and can’t afford to buy — or how much of something to buy. It also helps to keep you safe; for example, being able to read the integers on the speedometer helps you stay within the speed limit. It is also useful for understanding temperatures, weights and measures when you’re cooking or you need to store food safely.

In finance, positive numbers are used to represent the amount of money in someone’s bank account, while negative numbers are used to represent how much money someone owes (e.g. how much they have to pay back after borrowing money or taking out a loan from a bank).

These calculations are the building blocks in mathematics and will be covered in this chapter as well as across all of Units 3 & 4.

1.1.2 Syllabus links

Lesson	Lesson title	Syllabus links
1.2	**Number operations**	○ Solve practical problems requiring basic number operations.
1.3	**Order of operations and reasonableness**	○ Apply arithmetic operations according to their correct order. ○ Ascertain the reasonableness of answers to arithmetic calculations.
1.4	**Estimation**	○ Use leading-digit approximation to obtain estimates of calculations. ○ Check results of calculations for accuracy. ○ Round up or round down numbers to the required number of decimal places.
1.5	**Decimal significance**	○ Recognise the significance of place value after the decimal point. ○ Evaluate decimal fractions to the required number of decimal places. ○ Round up or round down numbers to the required number of decimal places.
1.6	**Calculator skills**	○ Use a calculator for multi-step calculations. ○ Check results of calculations for accuracy. ○ Ascertain the reasonableness of answers to arithmetic calculations.
1.7	**Approximation**	○ Apply approximation strategies for calculations.

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This chapter only contains Simple familiar questions from the Fundamental topic Calculations in the syllabus.

LESSON
1.2 Number operations

SYLLABUS LINKS

- Solve practical problems requiring basic number operations.

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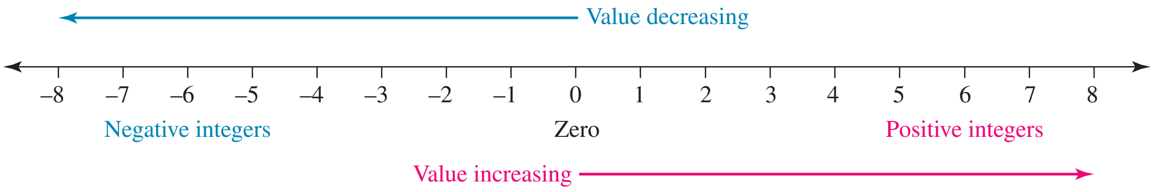
1.2.1 Addition and subtraction of integers using a number line

- An **integer** is a whole number. It can be positive (2, 4, 89, 1035) or negative (−2, −4, −89, −1035).
- Zero is also an integer because it is a whole number.



- The positive numbers do not usually have a symbol to show that they are positive (+2 is the same as 2).
- The negative numbers must include the negative sign to show that they are negative (−2, −27).
- A whole number is a number that does not include any fractions or parts of a number.
  For example, the following are *not* whole numbers or integers:
  - Fractions $\left(\frac{1}{2}, -12\frac{3}{4}, 5\frac{1}{2}\right)$ because they are or include parts of a whole number
  - Numbers that continue after a decimal point (0.5, −12.75, 5.5)

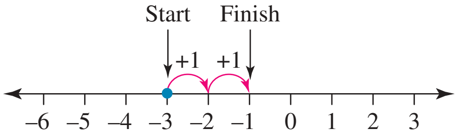
Addition and subtraction using a number line

Start at the first number.

To add a:
- **positive integer, move to the right**
- **negative integer, move to the left.**

To subtract a:
- **positive integer, move to the left**
- **negative integer, move to the right.**

- For example, to show (−3) + (+2) on a number line, start at the first number (−3) and place a pointer on the number line at −3.
  To add the positive integer (+2), move 2 places to the right.



Therefore, using the number line, we can see that (−3) + (+2) = −1.

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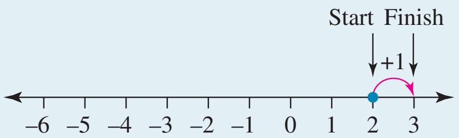
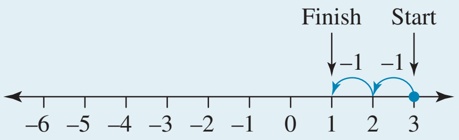
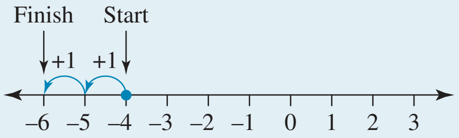
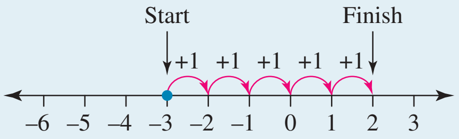
WORKED EXAMPLE 1 Solving addition and subtraction of integers using a number line

Use a number line showing intervals from −6 to 3 to calculate each of the following.

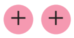
a. $2+(+1)$ **b.** $3+(-2)$ **c.** $-4-(+2)$ **d.** $-3-(-5)$

THINK	WRITE
a. 1. Start at 2 and move 1 unit to the right, as you are adding a positive number.	**a.** 
2. Write the answer.	$2+(+1)=3$
b. 1. Start at 3 and move 2 units to the left, as you are adding a negative number.	**b.** 
2. Write the answer.	$3+(-2)=1$
c. 1. Start at −4 and move 2 units to the left, as you are subtracting a positive number.	**c.** 
2. Write the answer.	$-4-(+2)=-6$
d. 1. Start at −3 and move 5 units to the right, as you are subtracting a negative number.	**d.** 
2. Write the answer.	$-3-(-5)=2$

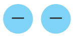
1.2.2 Addition and subtraction of integers using symbols

int-3922
int-3924

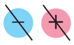
- Another way to remember the rules for adding and subtracting integers is to imagine positive and negative symbols (or counters) representing each of the numbers in an equation.
- A pink positive symbol represents +1 and a blue negative symbol represents −1.
- When calculating:
  - 2 is written as:



  - −2 is written as:



  - positive and negative symbols cancel each other out: $-1+1=0$



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WORKED EXAMPLE 2 Solving addition and subtraction of integers using symbols

Calculate the value of each of the following.

a. $2+2$

b. $-2+2$

c. $-2-(3)$

d. $-4-(-2)$

THINK	WRITE
a. 1. Set the equation up using symbols.	**a.** $2+2=\oplus\ \oplus + \oplus\ \oplus$
2. Add all the positives.	4 positives $=+4$
3. Write the answer.	$2+2=4$
b. 1. Set the equation up using symbols.	**b.** $-2+2=\ominus\ \ominus + \oplus\ \oplus$
2. Cancel any pairs of positives and negatives.	$\cancel{\ominus}\ \cancel{\ominus} + \cancel{\oplus}\ \cancel{\oplus}$
3. Count how many symbols are left.	Zero symbols remain.
4. Write the answer.	$-2+2=0$
c. 1. Set the equation up using symbols.	**c.** $-2-(3)=\ominus\ \ominus - \oplus\ \oplus\ \oplus$
2. When subtracting positive integers, change the sign of the symbols and then add them.	$\ominus\ \ominus + \ominus\ \ominus\ \ominus$
3. Count all the negatives.	5 negatives $=-5$
4. Write the answer.	$-2-(3)=-5$
d. 1. Set the equation up using symbols.	**d.** $-4-(-2)=\ominus\ \ominus\ \ominus\ \ominus - \ominus\ \ominus$
2. When subtracting negative integers, change the sign of the symbols and then add them.	$\ominus\ \ominus\ \ominus\ \ominus + \oplus\ \oplus$
3. Cancel any pairs of positive and negative symbols.	$\cancel{\ominus}\ \cancel{\ominus}\ \ominus\ \ominus + \cancel{\oplus}\ \cancel{\oplus}$
4. Count how many positives or negatives remain.	Two negatives remain.
5. Write the answer.	$-4-(-2)=-2$

1.2.3 Addition and subtraction of integers by applying rules

- Follow the rules shown in this table to add and subtract positive and negative numbers.

Rules for addition and subtraction of integers

Rule		Example
Adding a positive integer is the same as adding.	$+\,+=+$	$+2+(+5)=+7$
Adding a negative integer is the same as subtracting.	$+\,-=-$	$+2+(-5)=-3$
Subtracting a positive integer is the same as subtracting.	$-\,+=-$	$+2-(+5)=-3$
Subtracting a negative integer is the same as adding.	$-\,-=+$	$+2-(-5)=7$

tlvd-3551

WORKED EXAMPLE 3 Solving addition and subtraction of integers by applying rules

Calculate the value of each of the following.

a. $2+2$ **b.** $2+(-2)$ **c.** $2-(+3)$ **d.** $-4-(-2)$

THINK	WRITE
a. 1. Adding a positive integer is the same as adding.	**a.** $2+2$
2. Apply the rule $+ + = +$ and calculate the value.	$2+2=4$
b. 1. Adding a negative integer is the same as subtracting.	**b.** $2+(-2)$
2. Apply the rule $+ - = -$ and calculate the value.	$2+(-2)=2-2$
3. Write the answer.	$2-2=0$
c. 1. Subtracting a positive integer is the same as subtracting.	**c.** $2-(+3)$
2. Apply the rule $+ - = -$ and calculate the value.	$2-(+3)=2-3$
3. Write the answer.	$2-3=-1$
d. 1. Subtracting a negative integer is the same as adding.	**d.** $-4-(-2)$
2. Apply the rule $- - = +$ and calculate the value.	$-4--2=-4+2$
3. Write the answer.	$-4+2=-2$

1.2.4 Multiplying or dividing integers

- When multiplying integers, the following rules apply.

Rules for the multiplication of integers

- **When multiplying two integers with the same sign, the answer is positive.**

$$+ \times + = +$$
$$- \times - = +$$

- **When multiplying two integers with different signs, the answer is negative.**

$$+ \times - = -$$
$$- \times + = -$$

WORKED EXAMPLE 4 Solving integer multiplication by applying rules

Evaluate each of the following.

a. $(-4) \times +3$ **b.** $(-7) \times (-6)$

THINK	WRITE
a. The two numbers have different signs, so the answer is negative.	**a.** $(-4)\times(+3)$ $=-12$
b. The two numbers have the same sign, so the answer is positive.	**b.** $(-7)\times(-6)$ $=42$ (or $+42$)

- Division is the inverse (opposite) operation of multiplication.
- We can use the multiplication facts for integers to work out the division facts for integers.

Multiplication fact	Division fact	Multiplication pattern	Division pattern
$4 \times 5 = 20$	$20 \div 5 = 4$ and $20 \div 4 = 5$	positive × positive = positive	$\frac{\text{positive}}{\text{positive}} = \text{positive}$
$(-4) \times (-5) = 20$	$(-20) \div (-4) = 5$	negative × negative = positive	$\frac{\text{negative}}{\text{negative}} = \text{positive}$
$(-4) \times 5 = -20$	$(-20) \div 5 = -4$ and $20 \div (-5) = -4$	negative × positive = negative and positive × negative = negative	$\frac{\text{negative}}{\text{positive}} = \text{negative}$ and $\frac{\text{positive}}{\text{negative}} = \text{negative}$

Determining the sign of the answer when dividing integers

- **When dividing two integers with the same sign, the answer is positive.**

$$+ \div + = +$$
$$- \div - = +$$

- **When dividing two integers with different signs, the answer is negative.**

$$+ \div - = -$$
$$- \div + = -$$

- Remember that division statements can be written as fractions and then simplified.
  For example:

$$\begin{aligned}(-12) \div (-4) &= \frac{-12}{-4} \\ &= \frac{12 \times \cancel{-1}}{4 \times \cancel{-1}} \\ &= 3\end{aligned}$$

WORKED EXAMPLE 5 Solving division of integers

Evaluate each of the following.

a. $(-48) \div 6$

b. $\frac{-54}{-9}$

c. $144 \div (-6)$

d. $(-240) \div (-16)$

THINK	WRITE
a. The two numbers have different signs, so the answer is negative.	**a.** $(-48) \div 6$ $= -8$

b. The two numbers have the same sign, so the answer is positive.	**b.** $\frac{-54}{-9} = \frac{(-1)\times 54}{(-1)\times 9}$ $= \frac{54}{9}$ $= 6$
c. 1. Complete the division as if both numbers were positive numbers.	**c.** $\begin{array}{r} 24 \\ 6\overline{)14^{2}4} \end{array}$
2. Determine the sign of the answer. The two numbers have different signs, so the answer is negative.	$144 \div (-6) = -24$
d. 1. Complete the division as if both numbers were positive numbers.	**d.** $\begin{array}{r} 15 \\ 16\overline{)24^{8}0} \end{array}$
2. Determine the sign of the answer. The two numbers have the same sign, so the answer is positive.	$(-240) \div (-16) = 15$

1.2.5 Solving practical problems

- We constantly solve practical problems using basic number operations without realising that we are doing it.
- You do this when you calculate what time you need to leave home to get to school on time or how much money you need to purchase food from the school canteen.

WORKED EXAMPLE 6 Solving practical numerical problems

A USB stick can hold 512 MB of data. If 386 MB of the USB stick is already filled, determine how much space is left. Check your result for accuracy.

THINK	WRITE
1. Determine the total storage space.	Space available = 512 MB
2. Identify how much space has been used.	Space used = 386 MB
3. The space left is the difference between the total space and the space used.	Space left = 512 − 386 = 126 MB
4. Check the answer for accuracy.	Space available = space used + space left LHS = RHS
5. Substitute in each value.	512 MB = 386 MB + 126 MB 512 MB = 512 MB
6. Write the answer.	The space left on the USB stick is 126 MB.

WORKED EXAMPLE 7 Solving real-world problems using calculations

Nathan has a part-time job that pays $15.50 per hour. Nathan gets paid time and a half for hours worked on Saturdays and double time for hours worked on Sundays. Determine how much Nathan gets paid in a week in which they work 5 h on Friday, 4 h on Saturday and 5.5 h on Sunday.

THINK	WRITE
1. Calculate the amount Nathan earned on Friday.	$\text{Money earned Friday} = 5 \times \$15.50$ $= \$77.50$
2. Calculate the amount Nathan earned on Saturday.	$\text{Money earned Saturday} = 4 \times \$15.50 \times 1.5$ $= \$93.00$
3. Calculate the amount Nathan earned on Sunday.	$\text{Money earned Sunday} = 5.5 \times \$15.50 \times 2$ $= \$170.50$
4. Calculate Nathan's weekly pay by adding the amounts earned on Friday, Saturday and Sunday.	$\text{Total pay} = \$77.50 + \$93.00 + \$170.50$ $= \$341.00$
5. Write Nathan's weekly pay.	Nathan earned $341.00 that week.

Exercise 1.2 Number operations

learnon

1.2 Quick quiz on

1.2 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22	N/A	N/A

Simple familiar

1. Select the integers from the following numbers.

$$5, -2, \frac{3}{4}, 212, 12.3, -2.5, -33, -2\frac{1}{2}$$

2. **WE1** Calculate each of the following using the number line method.

a. (−5) + 2
b. (−6) + 1
c. 5 + (−3)
d. (−8) + (−3)
e. 21 + (+9)
f. 18 + (−5)

3. **WE2** Calculate each of the following using the cancelling signs method.

a. $3 + (+1)$
b. $3 + (-2)$
c. $4 - (+5)$
d. $2 + (-2)$
e. $3 - (-2)$
f. $(-1) - (-3)$

4. **WE3** Calculate the value of each of the following using rules.

a. $(-7) + (-3)$
b. $8 - (-7)$
c. $(-23) + (+15)$
d. $(-18) - (-17)$
e. $26 - (-13)$
f. $(-72) - (-26)$

5. Calculate the value of each of the following using the method of your choice.

a. $(-37) + (-12)$
b. $42 - (+7)$
c. $(-14) - (+18)$
d. $(-27) - (-15)$
e. $37 - (+12)$
f. $135 - (-37)$

6. Calculate the value of each of the following using the method of your choice.

a. $12 + (-4) + (+6)$
b. $28 + (-7) - (-10)$
c. $(-15) + (+5) - (+8)$
d. $28 - (+15) - (+4)$
e. $18 - (-12) + (-5)$
f. $(-42) - (-21) - (-21)$

7. **WE4** Evaluate each of the following.

a. $(-3) \times 5$
b. $3 \times (-7)$
c. $(-6) \times (-5)$
d. $2 \times (-10)$
e. $(-8) \times (-5)$
f. $(-7) \times 8$

8. Evaluate each of the following.

a. $(-10) \times 25$
b. $(-125) \times 10$
c. $(-6) \times 9$
d. $(+9) \times (-7)$
e. $(-11) \times (-7)$
f. $250 \times (-2)$

9. Use an appropriate method to evaluate the following.

a. $(-2) \times 3 \times (-5) \times (-10)$
b. $6 \times (-1) \times 5 \times (-2) \times 1$
c. $8 \times (-3) \times (-1) \times (-2) \times 4$
d. $(-3) \times 5 \times (-2) \times (-1) \times (-1) \times (-1)$
e. $(-5) \times 6 \times (-2) \times (-2)$
f. $6 \times (-1) \times 3 \times (-2)$

10. **WE5** Evaluate each of the following.

a. $(-54) \div 9$
b. $10 \div (-2)$
c. $(-8) \div (-2)$
d. $(-5) \div (-1)$
e. $99 \div (-11)$
f. $0 \div (-9)$

11. Evaluate each of the following.

a. $42 \div (-3)$
b. $(-130) \div 5$
c. $(-56) \div (-8)$
d. $(+88) \div (-4)$
e. $(-66) \div (-6)$
f. $168 \div (-8)$

12. Evaluate each of the following.

a. $184 \div (-8)$
b. $(-189) \div 9$
c. $(-161) \div (-7)$
d. $(-132) \div (-2)$
e. $(-204) \div 6$
f. $1080 \div (-9)$

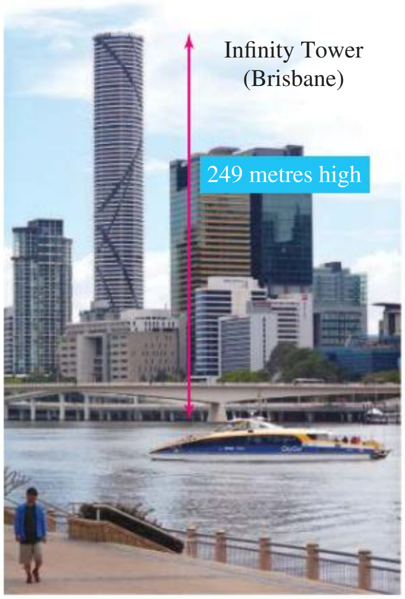
13. Evaluate each of the following.

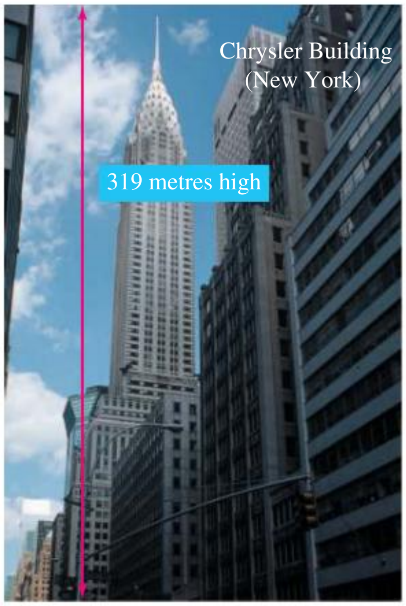
a. $216 \div (-12)$
b. $(-345) \div 15$
c. $(-1536) \div (-24)$
d. $(-1764) \div (-49)$
e. $4096 \div 64$
f. $(-2695) \div 55$

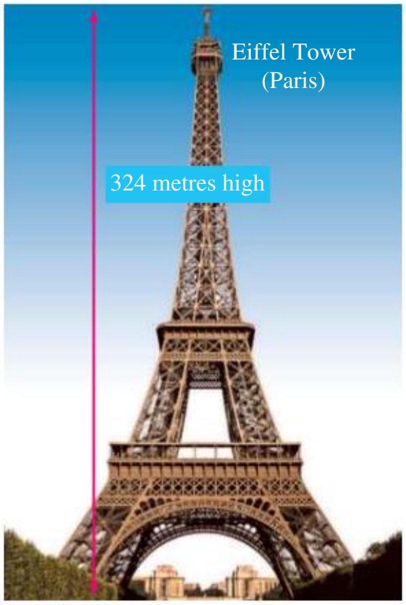
14. **WE6** The monthly data allowance included in your mobile phone plan is 15 GB. If you have already used 12 GB of data this month, determine how much data you have left to use for the rest of the month. Check your result for accuracy.

15. A UHD television was priced at $8999. It has a sale sign on it that reads: ‘Take a further $1950 off the marked price.’ Determine the sale price of the television. Check your result for accuracy.

16. The photographs show three tall structures.







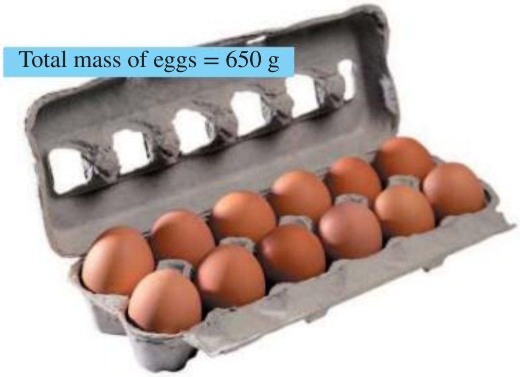
a. Determine how much taller the Eiffel Tower is than the Chrysler Building.
b. Calculate the height difference between the Infinity Tower and the Eiffel Tower.
c. Determine the difference in height between the Chrysler Building and the Infinity Tower.
d. Explain how you obtained your answers to parts a, b and c.

17. This sign shows the distances to a number of locations in the Northern Territory and Queensland.

a. Determine how much further Tobermorey is from Harts Range.
b. Calculate the distance between Boulia and Tobermorey.

18. **WE7** Sarah has a part-time job that pays $18.50 per hour. Sarah gets paid time and a half for hours worked on Saturdays and double time for hours worked on Sundays. Use your calculator to calculate how much Sarah gets paid in a week in which she works 4 h on Friday, 5 h and 30 min on Saturday and 8 h on Sunday.

19. Assuming that all the eggs in the photograph are the same size, determine the mass of each egg.



20. Bryce decides to cook meat pies for lunch. He takes the pies out of the freezer that is set at −18 °C to defrost on the bench where the room temperature is 21 °C. He sets the oven to 180 °C to cook the pies.

- **a.** Identify the difference between the temperature of the freezer and the room temperature.
  (*Hint:* difference = largest number − smallest number)
- **b.** Calculate the difference between the room temperature and the temperature of the oven.
- **c.** Calculate the difference of the freezer temperature and the temperature of the oven.

21. This photograph shows a mobile phone and its associated costs.

Voice calls:
47c per 30 s
Text messages:
15c per message
Picture messages:
50c per message

- **a.** Determine the cost of a 1-min call on this phone.
- **b.** Determine the cost of a 35-min call on this phone.
- **c.** If you use the phone to make five 3-min calls every day, calculate how much the calls will cost you for a year. (Assume 365 d in a standard year.)
- **d.** Determine the cost of sending 20 text messages from this phone.
- **e.** Calculate the difference in cost between sending 100 text messages and sending 50 picture messages using this phone.

22. Consider the digits 2, 3, 4 and 5.

- **a.** Construct the largest possible number using these digits.
- **b.** Construct the smallest possible number using these digits.
- **c.** Calculate the difference between the numbers from parts **a** and **b**.

Fully worked solutions for this chapter are available online.

LESSON
1.3 Order of operations and reasonableness

SYLLABUS LINKS

- Apply arithmetic operations according to their correct order.
- Ascertain the reasonableness of answers to arithmetic calculations.

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1.3.1 Order of operations

- The **order of operations** is a set of rules that determines the order in which mathematical operations are to be performed.
- The acronym BIDMAS is one example used to remember the correct order in which to complete operations within an equation.

Order of operations

The order of operations rules can be remembered as BIDMAS:

B Brackets	**I** Indices	**D** Divide	**M** Multiply	**A** Add	**S** Subtract
( )	$\sqrt{x}$ x^2	÷ or ×		+ or −	

Order	Operations	What does it mean?
First	Brackets	Calculate any parts of the expression that are shown in brackets. For example: $(3+1)\times4=4\times4$
Second	Indices	Multiply or divide out any indices. Indices are: • powers (3^2); a number raised to a power is multiplied by itself $3^2 = 3\times3$ $= 9$ $3^3 = 3\times3\times3$ $= 27$ • roots ($\sqrt{9}$); a root is the opposite of a power $\sqrt{9}=3$ $\sqrt[3]{27}=3$

(continued)

(continued)

Order	Operations	What does it mean?
Third	**D**ivision and **M**ultiplication	Calculate any parts of the expression that involve division or multiplication. If the expression contains both multiplication and division, start from the left and work across to the right. For example: $3 \times 4 + 36 \div 6 = 12 + 6$ $= 18$
Last	**A**ddition and **S**ubtraction	Calculate any parts of the expression that involve addition or subtraction. If the expression contains both addition and subtraction, start from the left and work across to the right. For example (remember division comes before addition or subtraction): $4 + 24 \div 4 - 2 = 4 + 6 - 2$ $= 10 - 2$ $= 8$

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WORKED EXAMPLE 8 Applying the order of operations

Calculate the value of each of the following.

a. $\mathbf{23 - 6 \times 4}$

b. $\mathbf{(12 - 8) + 5^2 - \left(10 + 2^2\right)}$

c. $\mathbf{\dfrac{3(4+8) + 4}{4 + 2(3^2 - 1)}}$

THINK	WRITE
a. 1. Apply BIDMAS to determine the first step (in this case, multiplication).	**a.** $23 - 6 \times 4$ $= 23 - 24$
2. Complete the next step in the calculation (in this case, subtraction) and write the answer.	$= -1$
b. 1. Apply BIDMAS to determine the first step (perform the calculations in brackets first, then remove the brackets).	**b.** $(12 - 8) + 5^2 - \left(10 + 2^2\right)$ $= 4 + 5^2 - (10 + 4)$ $= 4 + 5^2 - 14$
2. Complete the next steps in the calculation (work out the powers, then carry out addition and subtraction from left to right).	$= 4 + 25 - 14$ $= 29 - 14$
3. Complete the final step and write the answer.	$= 15$
c. 1. Apply BIDMAS to determine the first step (brackets).	**c.** $\dfrac{3(4+8)+4}{4+2(3^2-1)} = \dfrac{3 \times 12 + 4}{4 + 2(9 - 1)}$
2. Complete the next step in the calculation (multiplication).	$= \dfrac{3 \times 12 + 4}{4 + 2 \times 8}$ $= \dfrac{36 + 4}{4 + 16}$
3. Perform the calculations on the numerator and denominator separately.	$= \dfrac{40}{20}$
4. Complete the division and write the answer.	$= 2$

WORKED EXAMPLE 9 Applying the order of operations to real-world problems

Sean buys 4 oranges at \$0.70 each and 10 pears at \$0.55 each.

a. Calculate the price Sean paid for the oranges and pears.

b. Calculate the average price Sean paid for a piece of fruit.

THINK	WRITE
a. 1. Calculate the total amount of money Sean spent on fruit.	**a.** $(4 \times \$0.70) + (10 \times \$0.55) = \$2.80 + \$5.50$ $= \$8.30$
2. Write the answer.	The total amount Sean spent on fruit is \$8.30.
b. 1. Count the total number of pieces of fruit that Sean purchased.	**b.** 4 oranges + 10 pears = 14 pieces of fruit
2. The average price Sean paid for a piece of fruit is equal to the total cost divided by the total number of pieces of fruit.	$\text{Average price} = \frac{\text{total cost}}{\text{total number of pieces}}$ $= \frac{\$8.30}{14}$ $= \$0.59$
3. Write the answer.	The average price Sean paid for a piece of fruit is \$0.59.

1.3.2 Reasonableness

- When calculating mathematical answers, it is always important to understand the question so you have an idea of what a reasonable answer would be.
- Checking the reasonableness of answers can help you see possible mistakes in your working.

WORKED EXAMPLE 10 Evaluating the reasonableness of an answer

Cathy goes shopping to purchase an outfit for a hike. She buys a \$110 pair of waterproof pants and a \$180 jacket, and gets \$55 off the combined price by purchasing the two together. She also buys a pair of boots for \$120 and a pair of socks for \$15.
Calculate how much Cathy spent in total and check your answer for reasonableness.

THINK	WRITE
1. Read the question carefully to understand what it is about.	The question asks you to add up Cathy's total shopping bill including the discount.

2. Have an idea of what sort of answer you expect.	Cathy purchased three items, each over $100 dollars, so the answer should be over $300.
3. Write a mathematical expression to calculate the total amount that Cathy spent, including the discount.	$\text{Amount spent} = \$110 + \$180 - \$55 + \$120 + \$15$ $= \$370$
4. Check the answer for reasonableness.	This answer is above $300, so it seems reasonable.

Digital technology

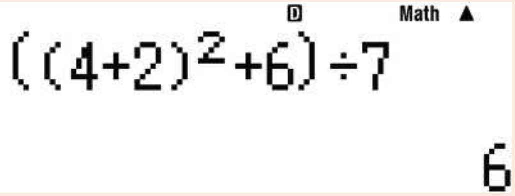
Using a calculator to evaluate expressions involving the order of operations

Many scientific calculators can input and evaluate expressions in one line.

Consider the expression $\left[(4+2)^2+6\right]\div 7$.

To determine its value on a calculator, follow the steps below.

- Input the expression $\left((4+2)^2+6\right)\div 7$ in one continuous line into your calculator using only round brackets.
- Press = or ENTER to evaluate.



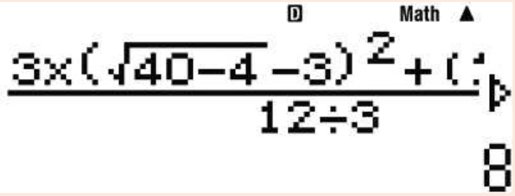
$((4+2)^2+6)\div 7$

6

Now consider the expression $\dfrac{3\times\left(\sqrt{40-4}-3\right)^2+(10\div 2)}{12\div 3}$.

To determine its value on a calculator, follow the steps below.

- Press the fraction template button. This will set up two entry boxes — one for the numerator and one for the denominator.
- Input $3\times\left(\sqrt{40-4}-3\right)^2+(10\div 2)$ into the entry box for the numerator.
- Input $12\div 3$ into the entry box for the denominator.
- Press = or ENTER to evaluate.



$\dfrac{3\text{x}(\sqrt{40-4}-3)^2+(}{12\div 3}$

8

Exercise 1.3 Order of operations and reasonableness

1.3 Quick quiz on

1.3 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14	N/A	N/A

These questions are even better in jacPLUS!

- Receive immediate feedback
- Access sample responses
- Track results and progress

Find all this and MORE in jacPLUS

Simple familiar

1. **WE8** Calculate the values of the following.

   a. $(-4)\times 2+1$
   b. $8\div(2-4)+4$
   c. $9\times(8-3)$
   d. $(-3)-40\div 8+2$
   e. $4+12\times(-5)$
   f. $(-5)\times 12+2$

2. Calculate the values of the following.

   a. $12-6\div 3$
   b. $45\div\big(27\div(-3)\big)$
   c. $(17-+7)\div(-5)$
   d. $-12+8\times 7$
   e. $100\div\big((-50)\div(-2)\big)+10$
   f. $9+\dfrac{24}{-6}\times 3$

3. Calculate the values of the following.

   a. $(-7)+4\times -4$
   b. $\big((-63)\div(-7)\big)\times(-3)+(-2)$
   c. $(-5)^2-3\times(-5)$
   d. $(-6)\times(-8)-\big(3+(-6)^2\big)$
   e. $52\div\big((-9)-4\big)-8$
   f. $-6-64\div(-16)+8$

4. **WE9** Pat buys 5 bananas at \$0.30 each and 8 apples at \$0.25 each.

   a. Calculate the price Pat paid for the bananas and apples.
   b. Calculate the average price Pat paid for a piece of fruit (rounded to the nearest cent).

5. Carrol purchases chocolate for her daughter's party. She buys 6 packets of Mars bars at \$4.85 each and 8 packets of Freddo Frogs at \$3.55 each. Calculate the average price that Carrol paid for each packet of chocolate.

6. **WE10** Fred went to a sports store to get some clothes for the gym. He purchased:
   - a tracksuit for \$120
   - shoes for \$150
   - shorts for \$30
   - a singlet top for \$25.

Given that he got a \$35 discount, calculate the amount that he spent at the sports store. Check your answer for reasonableness.

7. Erika goes shopping for some party food. She buys:
- 6 packets of chips at \$2.50 each
- 2 boxes of cola at \$8.50 each
- 5 packets of biscuits at \$1 each
- 3 dips at \$2 each.

Calculate the amount Erika spent in total, if she got a \$2 discount. Check your answer for reasonableness.

8. Bob went to the warehouse to get some building supplies.

He bought:
- three 4 m-long pieces of timber at \$4.50 per metre
- 2 packets of nails at \$5.50 each
- 3 tubes of liquid nails at \$4 each.

Because Bob is a regular customer, he received a \$15 discount off his total purchase.

Calculate the amount Bob spent in total. Check your answer for reasonableness.

9. Insert one set of brackets in the appropriate place to make each of these statements true.

a. $12 - 8 \div 4 = 1$
b. $4 + 8 \times 5 - 4 \times 5 = 40$
c. $3 + 4 \times 9 - 3 = 27$
d. $3 \times 10 - 2 \div 4 + 4 = 10$
e. $10 \div 5 + 5 \times 9 \times 9 = 81$
f. $18 - 3 \times 3 \div 5 = 9$

10. For a birthday party, you buy two packets of paper plates at \$2 each, three bags of chips at \$4 each and three boxes of party pies at \$5 each.

a. Explain how the order of operations helps you calculate the correct total cost of these items.
b. Devise an equation to show the operations required to calculate the total cost.
c. Calculate the total cost.

11. Taiki headed north on a bike ride, initially travelling 25 km. He then turned around and travelled 15 km south before stopping for a drink.

After his drink, Taiki continued to ride south for another 20 km, before again turning around and travelling north for a further 25 km.

a. Calculate how many kilometres Taiki covered on his ride.
b. Determine how far north he finished from where he started.

12. Students were given the following question to evaluate.

$$4 + 8 \div (-2)^2 - 7 \times 2$$

a. A number of different answers were obtained, including -8, -12 and -17. Determine which one of these is correct.
b. Using only brackets, change the question in two ways so that the other answers are correct.

13. Calculate the number required to make the following equation true:

$$13 \times (15 - 1) = 180 + \square$$

14. Calculate the number required to make the following equation true:

$$\left(10^2 + 12 \div 3\right) \div (-8) = \square$$

Fully worked solutions for this chapter are available online.

LESSON
1.4 Estimation

SYLLABUS LINKS

- Use leading-digit approximation to obtain estimates of calculations.
- Check results of calculations for accuracy.
- Round up or round down numbers to the required number of decimal places.

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1.4.1 Rounding

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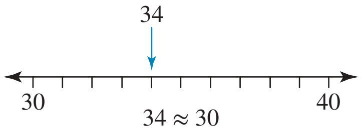
- Numbers can be rounded to different degrees of accuracy.
  For example, they can be rounded to 1, 2, 3 or more decimal places. The more decimal places a number has, the more accurate it is. However, more decimal places are not always necessary or relevant.

Rounding to the nearest 10

- To round to the nearest 10, think about which multiple of 10 the number is closest to.
  For example, 34 rounded to the nearest 10 is 30, because 34 is closer to 30 than 40.

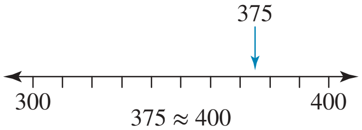


$34 \approx 30$

(***Note:*** The ≈ symbol represents ‘is approximately equal to’.)

Rounding to the nearest 100

- To round to the nearest 100, think about which multiple of 100 the number is closest to.
  For example, 375 rounded to the nearest 100 is 400, because 375 is closer to 400 than 300.

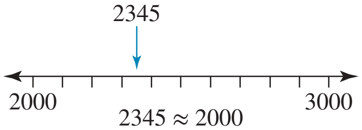


$375 \approx 400$

Rounding to the first digit

- When rounding to the leading (first) digit, the second digit needs to be looked at.
  - If the second digit is 0, 1, 2, 3 or 4, the first digit stays the same and all the following digits are replaced with zeros.
  - If the second digit is 5, 6, 7, 8 or 9, the first digit is increased by 1 (rounded up) and all the following digits are replaced with zeros.

  For example, if 2345 is rounded to the first digit, the result is 2000 because the second digit in the number is 3, so all digits following the first digit (in this case, 2) are replaced with 0.



$2345 \approx 2000$

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WORKED EXAMPLE 11 Applying rounding to integers

Round each of the following as directed.

a. 563 to the nearest 10
b. 12 786 to the nearest 100
c. 7523 to the leading digit

THINK	**WRITE**
a. 1. To round to the nearest 10, look at the units place value.	**a.** 563
2. Is the units place value less than 5? Since 3 is less than 5, we round down. Replace 3 with 0 and write the answer.	≈ 560
b. 1. To round to the nearest 100, look at the tens place value.	**b.** 12 7**8**6
2. Is the tens place value greater than or equal to 5? Since 8 is greater than or equal to 5, we round up. Increase 7 by 1 to 8 and replace the following digits with zeros.	12 786 $\downarrow\downarrow\downarrow$ 800
3. Write the answer.	$12\,786 \approx 12\,800$
c. 1. When rounding to the leading digit, look at the second digit.	**c.** 7523
2. Since 5 is greater than or equal to 5, we round up. Increase the first digit (7) by 1 and replace the remaining digits with zeros.	7523 $\downarrow\downarrow\downarrow\downarrow$ 8000
3. Write the answer.	$7523 \approx 8000$

1.4.2 Estimation

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- An **estimate** is not the same as a guess, because it is based on information.
- Estimation is useful for checking that your answer is reasonable when performing calculations.
- This can help avoid simple mistakes, like typing an incorrect number into a calculator.

WORKED EXAMPLE 12 Solving problems of estimating by rounding numbers to the leading (first) digit

Estimate $66\,123 \times 749$ by rounding each number to the leading (first) digit and then completing the calculation.

THINK	**WRITE**
1. Round the first number (**66** 123) by looking at the second digit. Since the second digit (6) is greater than or equal to 5, increase the leading digit (6) by 1 and replace the remaining digits with zeros.	**66** $123 \approx 70\,000$
2. Round the second number (**7**49) by looking at the second digit. Since the second digit (4) is less than 5, leave the leading digit and replace the remaining digits with zeros.	7**4**9 ≈ 700
3. Multiply the rounded numbers by multiplying the leading digits ($7 \times 7 = 49$) and then adding the number of zeros ($4 + 2 = 6$).	$70\,000 \times 700$ $7 \times 7 = 49$ $70\,000 \times 700 = 49\,000\,000$
4. Write the estimated answer and compare to the actual answer.	$66\,123 \times 749 \approx 49\,000\,000$ This compares well with the actual answer of 49 526 127, found using a calculator.

1.4.3 Checking an answer for accuracy

- When performing calculations, you have to be very careful not to make careless or common mistakes.
- It is helpful to be able to identify mistakes to make sure you don't also make common calculation errors.

WORKED EXAMPLE 13 Testing an answer for accuracy

Check the following answers for accuracy. If the answer is not accurate, identify and explain where the mistake was made.

a. $2 + 5 \times 2 = 14$ **b.** $12 - (6 \times 2) = 12$ **c.** $\frac{12 + 12}{12} = 12$

THINK	WRITE
a. Using BIDMAS, the multiplication needs to be done first.	**a.** $2 + 5 \times 2 = 14$ $2 + 10 = 14$ $12 \neq 14$
This answer is not accurate. Ask yourself how they got 14 as an answer.	They would get 14 if they did the calculation from left to right without following BIDMAS. $2 + 5 \times 2 = 14$ $7 \times 2 = 14$ $14 = 14$ This is not correct mathematics, but it identifies the error made.
b. Using BIDMAS, the brackets need to be worked out first.	**b.** $12 - (6 \times 2) = 12$ $12 - (12) = 12$ $0 \neq 12$
This answer is not accurate. Ask yourself how they got 12 as an answer.	They would get 12 if they did the calculation from left to right and ignored the brackets. $12 - (6 \times 2) = 12$ $12 - 6 \times 2 = 12$ $6 \times 2 = 12$ $12 = 12$ This is not correct mathematics, but it identifies the error made.
c. You should calculate the numerator first.	**c.** $\frac{12 + 12}{12} = 12$ $\frac{24}{12} = 12$ $2 \neq 12$
This answer is not accurate. Ask yourself how they got 12 as an answer.	One possible error could be that they cancelled 12 from the numerator with 12 from the denominator. $\frac{12 + 12}{12} = 12$ $\frac{\cancel{12} + 12}{\cancel{12}} = 12$ $\frac{12}{1} = 12$ $12 = 12$ This is not correct mathematics, but it identifies the error made.

Exercise 1.4 Estimation

1.4 Quick quiz on

1.4 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20	N/A	N/A

Simple familiar

1. **WE11** Round each of the following as directed.
   - a. 934 to the nearest 10
   - b. 12 963 to the nearest 100
   - c. 85 945 to the leading digit
2. Round the following.
   - a. 347 to the nearest 10
   - b. 86 557 to the nearest 100
   - c. 65 321 to the leading digit
3. Round each of the following numbers to the nearest 10.
   - a. 47
   - b. 82
   - c. 129
   - d. 162
   - e. 250
   - f. 2463
   - g. 4836
   - h. 7
4. Round each of the following numbers to the nearest 100.
   - a. 43
   - b. 87
   - c. 142
   - d. 177
   - e. 3285
   - f. 56 346
   - g. 86 621
   - h. 213 951
5. Round each of the following numbers to the nearest 1000.
   - a. 512
   - b. 3250
   - c. 1324
   - d. 6300
   - e. 7500
   - f. 13 487
   - g. 435 721
   - h. 728 433
6. Round each of the following numbers to the leading digit.
   - a. 12
   - b. 23
   - c. 45
   - d. 153
   - e. 1388
   - f. 16 845
   - g. 23 598
   - h. 492 385
7. **WE12** Estimate $74\,852 \times 489$ by rounding each number to its leading digit.
8. Estimate $87\,342 \div 449$ by rounding each number to its leading digit.
9. Estimate the answer to the following expressions by rounding each number to the leading digit and then completing the calculation.
   - a. $482 + 867$
   - b. $123 + 758$
   - c. $1671 - 945$
   - d. $2932 - 1455$
   - e. 88×543
   - f. 57×2632
   - g. $69\,523 \div 1333$
   - h. $3600 \div 856$

10. In your own words, explain how to round using leading digit approximation.

11. Emily purchased a number of items at the supermarket that cost \$1.75, \$5.99, \$3.45, \$5.65, \$8.95, \$2.35 and \$7.45. She was worried she didn't have enough money, so she used leading digit approximation to do a quick check of how much her purchases would cost.
Determine what approximation she came up with.

12. **WE13** Check the following answers for accuracy. If the answer is not accurate, identify and explain where the mistake was made.
   a. $15 - 3 \times 4 = 48$
   b. $13 + (2 \times 2) = 30$
   c. $3^2 - 2 = 1$

13. Check the following answers for accuracy. If the answer is not accurate, identify and explain where the mistake was made.
   a. $27 - 3 \times 2 + 5 = 53$
   b. $6 \times 7 + (2 \times 2) = 52$
   c. $2^3 - 2 = 2$

14. Shae went to the shop to purchase two steaks for dinner. One steak was \$12.50 and the other steak was \$8.50. The difference in the prices of the two steaks is \$21. Determine whether this is accurate, and if not, identify the error.

15. A bank teller was counting \$10 notes. They counted 100 \$10 notes and said they had \$10 000 worth of money. Determine whether this is accurate, and if not, identify the error.

16. Give three examples of situations where it is suitable to use an estimate or a rounded value instead of an exact value.

17. The crowds at the Melbourne Cricket Ground for each of the 5 d of the Boxing Day Test were: 88 214, 64 934, 55 349, 47 567 and 38 431.
Using leading digit approximation, determine the estimated total crowd over the 5 d of the test.

18. The crowds at the Melbourne Cricket Ground for each of the 5 d of the Boxing Day Test were: 88 214, 64 934, 55 349, 47 567 and 38 431.
Determine the difference between the leading digit estimate and the actual crowd over the 5 d.

19. A car's GPS estimates that a family's trip for Christmas lunch is going to take 45 min.
   a. Determine if the trip is likely to take exactly 45 min. Explain your answer.
   b. Explain how this time is estimated by the GPS.

20. Rhonda went to the shop to buy a new school uniform. She decided she needed a new blazer, a new winter dress and a new shirt costing \$167, \$89 and \$55, respectively.
Determine the difference between her actual total and a leading digit estimate.

Fully worked solutions for this chapter are available online.

LESSON
1.5 Decimal significance

SYLLABUS LINKS

- Recognise the significance of place value after the decimal point.
- Evaluate decimal fractions to the required number of decimal places.
- Round up or round down numbers to the required number of decimal places.

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1.5.1 Whole numbers and decimal parts

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- Each position in a number has its own place value.

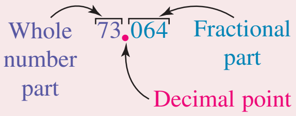
Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Units
100 000	10 000	1000	100	10	1

- Each place to the left of another position has a value that is 10 times larger.
- Each place to the right of another position has a value that is $\frac{1}{10}$ of the previous position.
- The position of a digit in a number gives the value of the digit.
- The following table shows the value of the digit 3 in some numbers.

Number	Value of 3 in the number
132	3 tens or 30
3217	3 thousands or 3000
4103	3 units (ones) or 3

Decimal numbers

In a decimal number, the whole number part and the fractional part are separated by a decimal point.



- A place value table can be extended to include decimal place values. It can be continued to an infinite number of **decimal places**.

Thousands	Hundreds	Tens	Units	.	Tenths	Hundredths	Thousandths	Ten thousandths
1000	100	10	1	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10\,000}$

- The following table shows the value of the digit 3 in some decimal numbers.

Number	Value of 3 in the number
14.32	3 tenths or $\frac{3}{10}$
106.013	3 thousandths or $\frac{3}{1000}$
0.000 03	3 hundred thousandths or $\frac{3}{100\,000}$

- The number of decimal places in a decimal is the number of digits after the decimal point.
- The number 73.064 has 3 decimal places. The zero (0) in 73.064 means that there are no tenths.
- The zero must be written to hold the place value; otherwise the number would be written as 73.64, which does not have the same value.

Expanded notation

A number can be written in expanded notation by adding the values of each digit.

The number 76.204 can be written in expanded notation as:

$$(7 \times 10) + (6 \times 1) + \left(2 \times \frac{1}{10}\right) + \left(4 \times \frac{1}{1000}\right)$$

WORKED EXAMPLE 14 Identifying the place value

Write the value of the 3 in each of the following decimal numbers.

a. **59.378** **b.** **1.0003** **c.** **79.737**

THINK	WRITE
a. **1.** Identify at which place after the decimal point the 3 lies.	**a.** 59.**3**78
2. The 3 is the first number after the decimal point, which represents tenths.	$\frac{3}{10}$
b. **1.** Identify at which place after the decimal point the 3 lies.	**b.** 1.000**3**
2. The 3 is the fourth number after the decimal point, which represents tens of thousandths.	$\frac{3}{10\,000}$
c. **1.** Identify at which place after the decimal point the 3 lies.	**c.** 79.7**3**7
2. The 3 is the second number after the decimal point, which represents hundredths.	$\frac{3}{100}$

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1.5.2 Rounding

- When rounding decimals, look at the first digit after the number of decimal places required.

Rules for rounding

- **If the first digit after the number of decimal places required is 0, 1, 2, 3 or 4, write the number without any change.**
- **If the first digit after the number of decimal places required is 5, 6, 7, 8 or 9, add 1 to the digit in the last required decimal place.**

- We can use the symbol ≈ to represent approximation when rounding has occurred, as shown in Worked example 15.
- ***Note:*** If you need to add 1 to the last decimal place and the digit in this position is 9, the result is 10. Zero is put in the last required place and 1 is added to the digit in the next place to the left.
  For example, 0.298 rounded to 2 decimal places is 0.30.

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WORKED EXAMPLE 15 Applying rounding to decimal numbers to a given number of decimal places

Round the following to the number of decimal places shown in the brackets.

a. **7.562 432 (2)**
b. **27.875 327 (2)**
c. **35.324 971 (4)**
d. **0.129 91 (3)**

THINK	WRITE
a. **1.** Write the number and underline the required decimal place.	**a.** $7.5\underline{6}2\,432$
2. Circle the next digit and round according to the rule. ***Note:*** Since the circled digit is less than 5, we leave the number as it is.	$= 7.5\underline{6}\textcircled{2}432$ ≈ 7.56
b. **1.** Write the number and underline the required decimal place.	**b.** $27.8\underline{7}5\,327$
2. Circle the next digit and round according to the rule. ***Note:*** Since the circled digit is greater than or equal to 5, add 1 to the last decimal place that is being kept.	$= 27.8\underline{7}\textcircled{5}327$ ≈ 27.88
c. **1.** Write the number and underline the required decimal place.	**c.** $35.324\,\underline{9}71$
2. Circle the next digit and round according to the rule. ***Note:*** Since the circled digit is greater than 5, add 1 to the last decimal place that is being kept. As 1 is being added to 9, write 0 in the last place and add 1 to the previous digit.	$= 35.324\,\underline{9}\textcircled{7}1$ ≈ 35.3250
d. **1.** Write the number and underline the required decimal place.	**d.** $0.12\underline{9}\,91$
2. Circle the next digit and round according to the rule. ***Note:*** Since the circled digit is greater than 5, add 1 to the last decimal place that is being kept. As 1 is being added to 9, write 0 in the last place and add 1 to the previous digit.	$= 0.12\underline{9}\textcircled{9}1$ ≈ 0.130

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1.5.3 Fractions as decimals

- A fraction can be expressed as a decimal by dividing the numerator by the denominator. For example, $\frac{3}{8} = 3 \div 8 = 0.375$
- If a decimal does not terminate and has decimal numbers repeating in a pattern, it is called a **recurring decimal**.
- A recurring decimal with only one repeating digit can be written by placing a dot above the digit.
- When there is more than one repeating digit, the recurring decimal can be written by placing a dot over the first and last digits of the repeating part, or by placing a bar above the entire repeating pattern.

  $\frac{1}{3} = 1 \div 3 = 0.333\,333\,33\ldots = 0.\dot{3}$

  $\frac{3}{7} = 3 \div 7 = 0.428\,574\,285\,7\ldots = 0.\dot{4}28\,5\dot{7}$ or $0.\overline{428\,57}$

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WORKED EXAMPLE 16 Applying the conversion of fractions to decimals

Change the following fractions into decimals.

a. $\frac{2}{5}$

b. $\frac{1}{8}$

THINK	WRITE
a. 1. Set out the question as for division of whole numbers, adding a decimal point and the required number of zeros.	**a.** $5\overline{)2.0}$
2. Divide, writing the answer with the decimal points aligned.	$\begin{array}{r} 0.4 \\ 5\overline{)2.0} \end{array}$
3. Write the answer.	$\frac{2}{5} = 0.4$
b. 1. Set out the question as for division of whole numbers, adding a decimal point and the required number of zeros. *Note:* $\frac{1}{8} = 1 \div 8$	**b.** $\begin{array}{r} 0.125 \\ 8\overline{)1.000} \end{array}$
2. Divide, writing the answer with the decimal point exactly in line with the decimal point in the question, and write the answer.	$\frac{1}{8} = 0.125$

- By knowing the decimal equivalent of any fraction, it is possible to determine the equivalent of any multiple of that fraction.

Exercise 1.5 Decimal significance

1.5 Quick quiz on

1.5 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18	N/A	N/A

Simple familiar

1. **WE14** Identify the value of 7 in each of the following decimals.
   - a. 45.871
   - b. 81.710
   - c. 33.007
2. Identify the value of 9 in each of the following decimals.
   - a. 21.090
   - b. 0.009
   - c. 47.1059
3. Identify the value of 7 in the following decimals.
   - a. 34.075
   - b. 1.7459
   - c. 0.1007
   - d. 1945.27
   - e. 450.07
   - f. 2.7
   - g. 0.007
   - h. 2.170
4. Determine the number of decimal places in each of the following numbers.
   - a. 2.195
   - b. 394.7
   - c. 104.25
   - d. 0.0003
   - e. 1997.4
   - f. 125.333
   - g. 69.15
   - h. 2.6936
5. Determine whether the following statements are True or False.
   - a. $0.125 = 0.521$
   - b. $0.025 = 0.0250$
   - c. $45.0005 = 45.005$
   - d. $3.333 = 3.333\,000$
   - e. $6.666 = 6.6666$
   - f. $20.123 = 2.123$
6. **WE15** Express each of the following rounded to 3 decimal places.
   - a. 59.123 45
   - b. 72.216 81
7. Express each of the following rounded to 2 decimal places.
   - a. 2.069 14
   - b. 0.7962
8. Express each of the following rounded to 3 decimal places.
   - a. 39.184 23
   - b. 3.232 323 23
   - c. 99.4358
   - d. 125.294 61
   - e. 100.0035
   - f. 0.0479
9. Express each of the following rounded to 2 decimal places.
   - a. 58.123
   - b. 23.345
   - c. 71.097
   - d. 0.8686
   - e. 30.9999
   - f. 125.8929
10. **WE16** Express the following fractions as decimals.
    - a. $\frac{3}{4}$
    - b. $\frac{2}{3}$
11. Express the following as decimals to 3 decimal places.
    - a. $\frac{7}{8}$
    - b. $\frac{1}{7}$
12. Express the following fractions as decimals.
    - a. $\frac{3}{10}$
    - b. $\frac{3}{5}$
    - c. $\frac{3}{12}$
    - d. $\frac{7}{25}$
    - e. $\frac{33}{100}$
    - f. $\frac{45}{50}$

13. Express the following fractions as recurring decimals.

a. $\frac{1}{3}$ **b.** $\frac{5}{9}$ **c.** $\frac{1}{12}$ **d.** $\frac{4}{7}$ **e.** $\frac{3}{13}$ **f.** $\frac{1}{21}$

14. Determine which packet in the photograph contains more mince.

15. Narelle is environmentally conscious and has fitted water tanks to her house to water the garden and flush the toilets. She checks the amount of rainfall each week to see how much water the tanks have collected. In the month of March, the rainfall readings were 12.48, 8.82, 27.51 and 44.73 mm. Round the total rainfall for March to 1 decimal place.

16. The Nguyen family decided to install solar panels on the roof to reduce their electricity bill. They compared the prices per kWh of three different suppliers to see how much they would save by using their panels. The prices quoted by companies A, B and C per kWh were 26.78, $25\frac{3}{4}$ and $25\frac{7}{9}$ cents, respectively.

a. List the companies from cheapest to most expensive.

b. Express the fractions as decimal numbers rounded to 1 decimal place.

17. Using the table below, explain whether the total height of the five players is accurate to 9.91 m.

Player	Height (m)
Kobe Bryant	1.98
Kevin Durant	2.06
Russell Westbrook	1.91
Carmelo Anthony	2.02
LeBron James	2.03

18. The following table shows items that were purchased at the supermarket and their price.

Item	Price
Salmon	\$4.99
Chicken fillets	\$7.35
Shampoo	\$8.95
Ham	\$4.42
Milk	\$4.29
Bread	\$3.60

Explain, using mathematical reasoning, whether the total cost of \$34 is accurate.

Fully worked solutions for this chapter are available online.

LESSON
1.6 Calculator skills

SYLLABUS LINKS

- Use a calculator for multi-step calculations.
- Check results of calculations for accuracy.
- Ascertain the reasonableness of answers to arithmetic calculations.

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1.6.1 Using a calculator

- Your calculator can complete many different mathematical operations that are difficult to calculate in your head or with pen and paper. This includes dealing with decimals and fractions.
- Your calculator will always give the correct answer, but you have to input the information correctly. To make sure you get the correct answer, it's always good to estimate your answer to check for **reasonableness** and **accuracy**.
- You will notice that your calculator automatically follows the correct order of operations (BIDMAS). ***Note:*** The keys used may vary between different calculators.

WORKED EXAMPLE 17 Calculating values of expressions using a calculator

Calculate the value of the following expressions using your calculator.

a. $140 + 23 \times 8$ **b.** $7(28 + 43) - 18$ **c.** $283 - 13.78 \times 11.35$

THINK	WRITE
a. 1. Estimate your answer using leading digit approximation.	**a.** $140 + 23 \times 8$ $\approx 100 + 20 \times 10$ ≈ 300
2. Enter the exact equation into your calculator. Check the reasonableness of your estimate.	$140 + 23 \times 8 = 324$

b. **1.** Estimate your answer using leading digit approximation.	**b.** $7(28+43)-18$ $\approx 7(30+40)-20$ $\approx 7(70)-20$ $\approx 490-20$ ≈ 470
2. Enter the exact expression into your calculator. Check the reasonableness of your estimate.	$7(28+43)-18=479$
c. **1.** Estimate your answer using leading digit approximation.	**c.** $283-13.78\times 11.35$ $\approx 300-10\times 10$ $\approx 300-100$ ≈ 200
2. Enter the exact expression into your calculator. Check the reasonableness of your estimate.	$283-13.78\times 11.35=126.597$

- When using your calculator, first estimate the result to compare with the answer from the calculator.
- Always check that what you type into the calculator matches the question.

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WORKED EXAMPLE 18 Evaluating expressions using a calculator and applying rounding

Evaluate the following expressions using your calculator, rounding your answer to 4 decimal places.

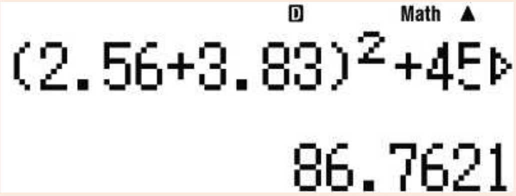
a. $\mathbf{(2.56+3.83)^2+45.93}$ **b.** $\mathbf{\sqrt{4.39-2.51}+(3.96)^2}$ **c.** $\mathbf{\frac{4}{7}+\frac{5}{9}+5}$

THINK	WRITE
a. **1.** Estimate your answer using leading digit approximation.	**a.** $(2.56+3.83)^2+45.93$ $\approx(3+4)^2+50$ $\approx 7^2+50$ $\approx 49+50$ ≈ 99
2. Enter into the calculator. Use the x^2 key to raise the brackets to the power of 2. Compare with your estimate to check for reasonableness.	$(2.56+3.83)^2+45.93$ $=86.7621$
b. **1.** Estimate your answer using leading digit approximation.	**b.** $\sqrt{4.39-2.51}+(3.96)^2$ $\approx\sqrt{4-3}+4^2$ $\approx\sqrt{1}+16$ $\approx 1+16$ ≈ 17
2. Enter into the calculator. Use the $\sqrt{\ }$ and x^2 keys. Make sure the square root covers both numbers. Compare with your estimate to check for reasonableness.	$\sqrt{4.39-2.51}+(3.96)^2$ $=17.0527$

c. 1. Estimate your answer using leading digit approximation.	**c.** $\frac{4}{7}+\frac{5}{9}+5$ $\approx 1+1+5$ ≈ 7
2. Enter into the calculator. Use the fraction key to enter each fraction. Compare with your estimate to check for reasonableness.	$\frac{4}{7}+\frac{5}{9}+5$ $=6.1270$

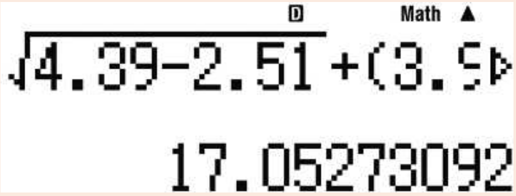
Digital technology

To evaluate the expressions in Worked example 18a using a calculator, enter $(2.56+3.83)^2+45.93$ into the calculator, by using the x^2 key to raise the brackets to the power of 2.



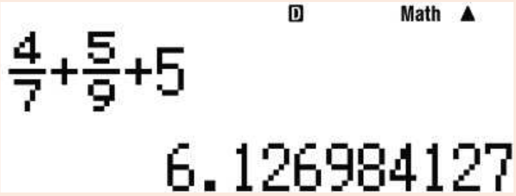
```
(2.56+3.83)²+45▶
             86.7621
```

In 18b, enter the expression $\sqrt{4.39-2.51}+(3.96)^2$ into the calculator, by using the $\sqrt{\ }$ and x^2 keys. Make sure the square root covers both numbers.



```
√4.39-2.51 +(3.9▶
         17.05273092
```

In 18c, enter the expression $\frac{4}{7}+\frac{5}{9}+5$ into the calculator, by using the fraction key to enter each fraction. Use the S ⟷ D button to convert from fraction to decimal form.



$\frac{4}{7}+\frac{5}{9}+5$

```
         6.126984127
```

Exercise 1.6 Calculator skills

learnon

1.6 Quick quiz on

1.6 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	N/A	N/A

Simple familiar

1. **WE17** Calculate the value of each of the following expressions using your calculator.
   a. $269 + 12 \times 16$ **b.** $9(78 + 61) - 45$ **c.** $506.34 - 39.23 \times 17.04$

2. Calculate the value of each of the following expressions using your calculator.
   a. $497 - 13 \times 24 \div 6$ **b.** $13(194 - 62) + 4 \times 8$
   c. $459.38 \div 18.5 \times 17.04 - (34.96 + 45.03)$

3. Evaluate the following expressions using your calculator.
   a. $3 + 6 \times 7$ **b.** $47 + 8 \times 6$ **c.** $285 + 21 \times 16$ **d.** $2859 + 178 \times 79$

4. Evaluate the following expressions using your calculator.
   a. $12 - 5 \times 2$ **b.** $68 - 4 \times 9$ **c.** $385 - 16 \times 9$ **d.** $1743 - 29 \times 45$

5. Evaluate the following expressions using your calculator.
   a. $4(6 + 24) - 58$ **b.** $5(23.5 - 18.3) + 23$
   c. $2.56(89.43 - 45.23) - 92.45$ **d.** $6\,(45.89 - 32.78) - 3(65.89 - 59.32)$

6. **WE18** Evaluate the following expressions using your calculator, rounding your answer to 4 decimal places.
   a. $(5.89 + 2.16)^2 + 67.99$ **b.** $\sqrt{8.77 - 3.81} + (5.23)^2$ **c.** $\frac{3}{5} + \frac{7}{8} + 10$

7. Evaluate the following expressions using your calculator, rounding your answer to 4 decimal places.
   a. $\left(\frac{7}{12}\right)^2 - \frac{3}{4}$ **b.** $\sqrt{3.56 + 8.28} - \sqrt{5.29 - 3.14}$ **c.** $\left(\frac{4}{5}\right)^2 + \sqrt{\frac{7}{12}}$

8. Evaluate the following expressions using your calculator, rounding your answers to 4 decimal places.
   a. $16.9 + 5.2^2 \div 4.3$ **b.** $9.3^2 \div 4.5$ **c.** $(3.7 + 5.9)^2 - 15.5$ **d.** $\frac{(7.2)^2}{4.2}$

9. Evaluate the following expressions using your calculator, rounding your answers to 4 decimal places.
   a. $\sqrt{5.67 + 8.34}$ **b.** $2.5 \times \sqrt{8.64} - 2.5$
   c. $12.8 + \sqrt{3.5 \times 5.8} \times 1.2$ **d.** $\frac{\sqrt{4.7 - 3.6}}{5}$

10. Evaluate the following expressions to 2 decimal places using your calculator.
    a. $\frac{1}{2} + \frac{1}{4}$ **b.** $\frac{3}{7} - \frac{1}{9}$ **c.** $\frac{18}{13} - \frac{2}{5} \times \frac{2}{7}$ **d.** $15 - \frac{7}{5} \times \frac{5}{3} + 7$

11. Cathy is 7 years older than Marie, and Marie is twice the age of her younger brother Fergus. Determine Cathy's age if Fergus is 3 years old.

12. After a football party night, some empty soft drink cans were left and had to be collected for recycling. Of the cans found, 7 were half full, 6 were one-quarter full and the remaining 8 cans were completely empty. Use your calculator to find out how many equivalent full cans were wasted.

Fully worked solutions for this chapter are available online.

LESSON
1.7 Approximation

SYLLABUS LINKS

- Apply approximation strategies for calculations.

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1.7.1 Approximating calculations

- Approximating or estimating an answer is useful when an accurate answer is not necessary.
- You can approximate the size of a crowd at a sporting event based on an estimate of the fraction of seats filled. This approach can be applied to a variety of situations, such as approximating the amount of water in a water tank.
- When approximating size, it is important to use an appropriate unit; e.g. for the distance you drive, you would use 24 km instead of 2 400 000 cm.
- Approximations can also be made by using past history to indicate what might happen in the future.

WORKED EXAMPLE 19 Solving approximation by carrying out relevant calculations

Approximate the size of the crowd on the first day of the cricket test at the Melbourne Cricket Ground, if it is estimated to be at $\frac{8}{10}$ of its capacity of 95 000.

THINK	WRITE
1. Write down the information given in the question.	Estimate of crowd $= \frac{8}{10}$ of capacity
2. Calculate the fraction of the total capacity.	$\frac{8}{10}$ of 95 000 $= \frac{8}{10} \times 95\,000$ $= 0.8 \times 95\,000$ $= 76\,000$
3. Write the answer.	The approximate crowd is 76 000 people.

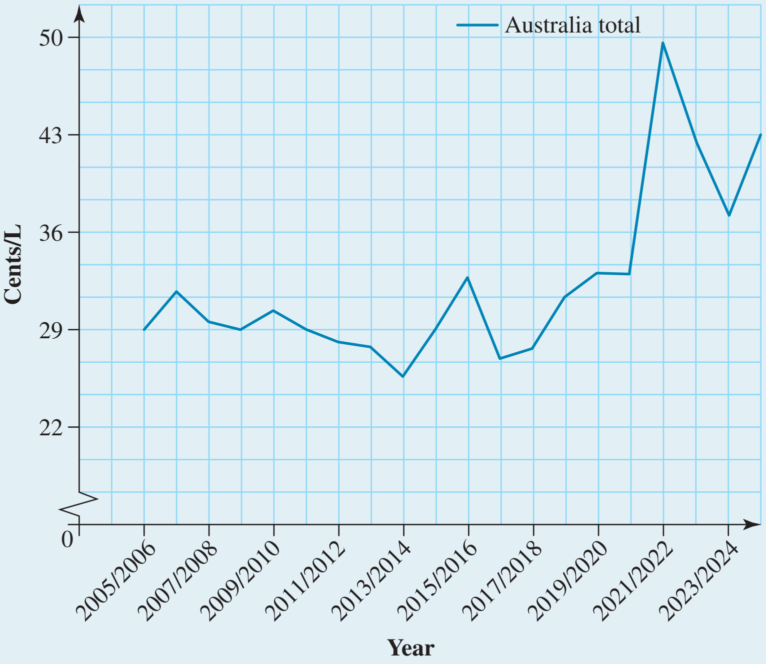
1.7.2 Approximation from graphs

- When approximating from graphs:
  - if the data is shown, read from the graph, noting the scale on the axes
  - if the data is not shown, follow the trend (behaviour of the graph) to make an estimate of future or past values.

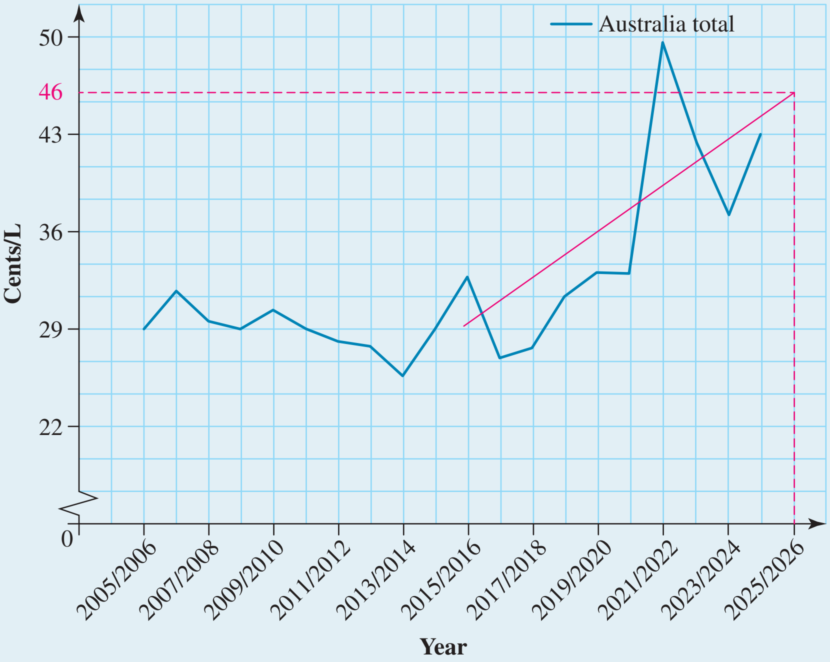
tlvd-3668

WORKED EXAMPLE 20 Applying approximation to values from graphs

The following graph shows the farmgate price of milk in cents/L. Approximate the price in 2025/2026.



THINK	**WRITE/DRAW**
1. Check what each axis represents.	Vertical axis: cents/L Horizontal axis: years
2. Look for the trend that the graph has followed in previous years. Draw a line to represent this trend and extend it to the required period of 2023/2024.	The graph has been on an upward trend for the past 8 years.
3. Read the answer from the graph.	The price in 2025/2026 is approximately 46 cents/L.



Exercise 1.7 Approximation

learnon

1.7 Quick quiz on	1.7 Exercise

These questions are even better in jacPLUS!
- Receive immediate feedback
- Access sample responses
- Track results and progress

Find all this and MORE in jacPLUS

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16	N/A	N/A

Simple familiar

1. **WE19** Approximate the number of people at a Bledisloe Cup match at Stadium Australia in Sydney if the crowd was estimated at $\frac{7}{10}$ of its capacity of 83 500.

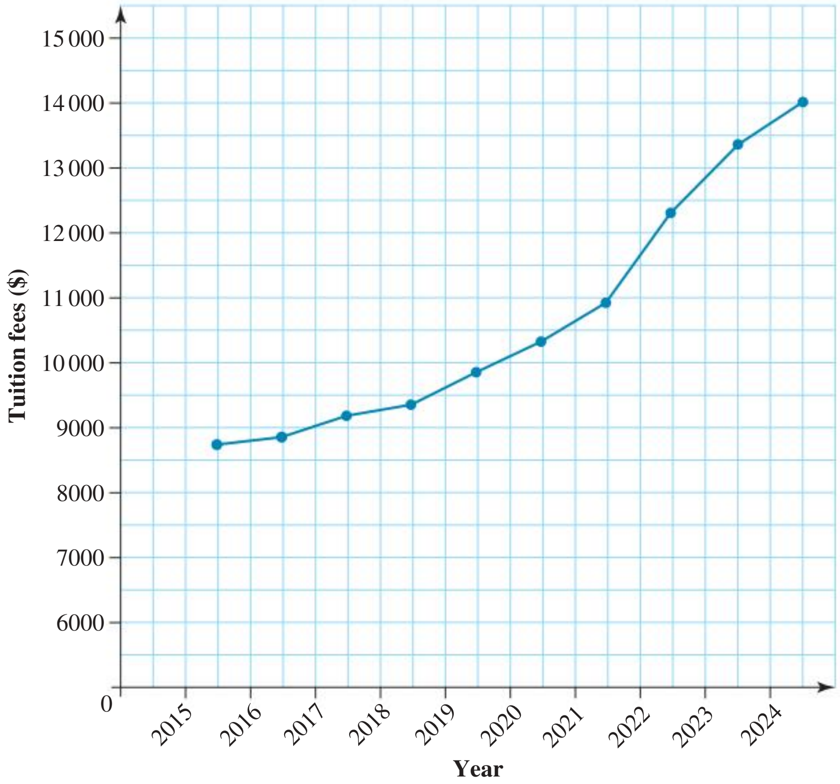
2. When full, a pool holds 51 000 L of water. Approximate the number of litres in the pool when it is estimated to be 90% full.

3. Select the most appropriate unit from the options below for each of the following measurements.
centimetres (cm), metres (m), kilometres (km), millilitres (mL), litres (L), grams (g), kilograms (kg)
   - a. The length of a football field
   - b. The weight of a packet of cheese
   - c. The volume of soft drink in a can
   - d. The distance you travel to go on a holiday
   - e. The size of an OLED TV
   - f. The weight of a Rugby League player

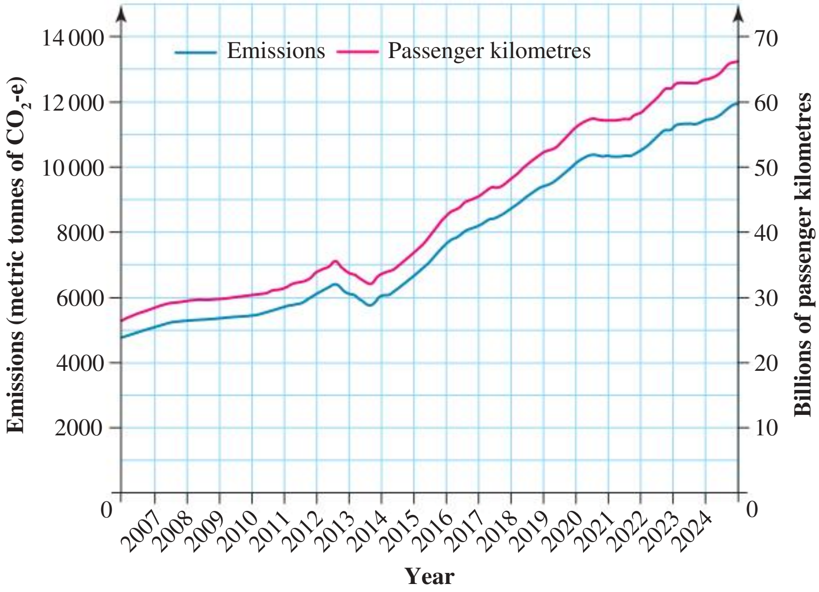
4. Determine the best approximation of the volume of milk in the container shown.

5. Determine the best approximation of the weight of the tub of butter shown.

6. **WE20** From the graph of the cost of tuition fees for one school, predict the approximate tuition fees in 2025.



7. Use the following graph to approximate the volume of domestic air travel emissions in 2025.



8. If the Sydney Olympics swimming venue was $\frac{9}{10}$ full and had a capacity of 17 000, determine the approximate crowd on this night.

9. The capacity at Sydney's Qudos Bank Arena, where the Sydney Kings play, is 18 200. Determine the approximate crowd at the game shown in the picture.

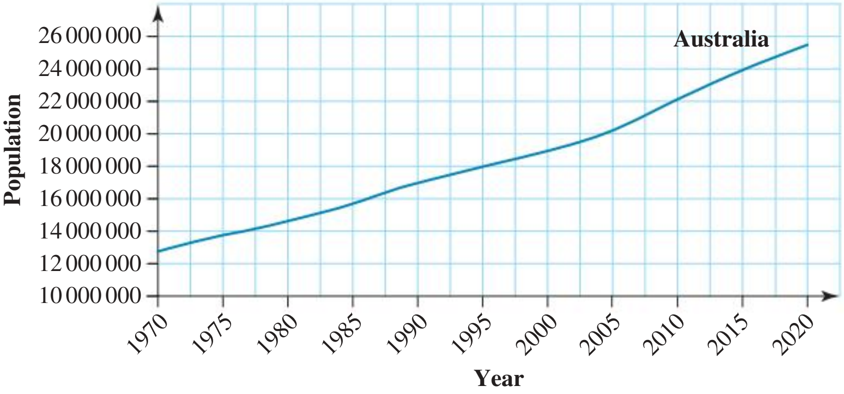
10. The capacity of Rod Laver Arena is 14 820. Approximate the crowd at this match from the Australian Open.

11. The picture shown is of Tiger Woods at the US Masters. Comment on why it is difficult to estimate the size of the Masters crowd on this day by looking at the picture.

12. Explain in your own words the difference between an estimate and a guess.

13. Describe a way to approximate how many students there are at your school. Investigate the actual number and compare this to your estimate. Explain how you could improve your initial estimate.

14. From the graph of the population of Australia since 1970, approximate how many people will live in Australia in 2030.



15. Use the graph of the value of the Australian dollar against the US dollar to approximate the value of the Australian dollar in January 2025.

16. From the graph of Aboriginal and Torres Strait Islander population, predict the approximate population in:

a. 2025

b. 2030.

Fully worked solutions for this chapter are available online.

LESSON
1.8 Review

1.8.1 Summary

Hey students! Now that it's time to revise this chapter, go online to:

- **Access the chapter summary**
- **Review your results**
- **Watch teacher-led videos**
- **Practise questions with immediate feedback**

Find all this and MORE in jacPLUS

1.8 Exercise

learnon

1.8 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20	N/A	N/A

These questions are even better in jacPLUS!

- Receive immediate feedback
- Access sample responses
- Track results and progress

Find all this and MORE in jacPLUS

Simple familiar

1. Calculate the value of the expression $17 + 3 \times 7$.
2. Calculate the value of the expression $-3(5 - -10) + 50$.
3. Estimate the number when 47 321 is rounded to the leading digit.
4. If three purchases were made with the values \$7.34, \$18.05 and \$2.69, use leading digit estimation on the three purchases to calculate the total purchase.
5. Using your calculator, calculate the value of $\sqrt{6.78 + 19.83} + 3.91$ to 2 decimal places.
6. In the following calculation, 20 is not the correct answer. Determine the most likely error in the calculation.
$$58 - 38 + (5 \times 2) = 20$$
7. Estimate the number 37.395 851 rounded to 4 decimal places.
8. Determine what fraction the number 7.9 is most accurately written as.
9. Determine the approximate volume of Coca-Cola in the can shown.

10. Calculate the following.

a. $-18 - -25$

b. -8×6

c. $36 \div 9 \times 8 - 12$

d. $-4(-4 + -6) - 33$

11. Use leading digit approximation to calculate the following.

a. 49×821

b. $1396 + 183$

c. $\dfrac{7563}{676}$

d. $17 \times 873 + 47$

12. Use your calculator to evaluate the following to 2 decimal places.

a. $\sqrt{3(5.94 - 1.48)}$

b. $(5.74)^2 - \dfrac{5}{8}$

c. $\dfrac{6}{9} + \dfrac{2}{5} \times \dfrac{3}{7}$

d. $\sqrt{(3.25)^2} + 1.5$

13. Round the following fractions to 2 decimal places.

a. $\dfrac{1}{8}$

b. $\dfrac{37}{50}$

c. $\dfrac{6}{25}$

d. $\dfrac{2}{3}$

14. Round the following to 3 decimal places.

a. 0.5555

b. 1.036 39

c. 33.1047

d. 10.9999

15. Approximate each of the following.

a. A sportsground estimated to be $\dfrac{3}{4}$ full, with a capacity of 50 000 people.

b. A water tank estimated to be 80% full, with a capacity of 7500 L.

16. Copy and complete the table shown.

- In the column headed ‘Estimate’, round each number to the leading digit.
- In the column headed ‘Estimated answer’, calculate the answer.
- In the column headed ‘Prediction’, assess whether the actual answer will be higher or lower than your estimate.
- Use a calculator to work out the actual answer and record it in the column headed ‘Calculation’ to determine whether it is higher or lower than your estimate.

	Question	**Estimate**	**Estimated answer**	**Prediction**	**Calculation**
Example	4200/230	$4000 \div 200$	20	Lower	1.7391 (higher)
a.	$487 + 962$				
b.	$33\,041 + 82\,629$				
c.	$184\,029 + 723\,419$				
d.	$93\,261 - 37\,381$				
e.	$321 - 194$				
f.	$468\,011 - 171\,962$				
g.	36×198				
h.	$623 \times 12\,671$				
i.	$29\,486 \times 39$				
j.	$31\,690 \div 963$				
k.	$63\,003 \div 2590$				
l.	$69\,241 \div 1297$				

17. A student is selling tickets for their school's theatre production of *South Pacific*. So far they have sold 439 tickets for Thursday night's performance, 529 for Friday night's and 587 for Saturday night's. The tickets cost $9.80 for adults and $4.90 for students.

a. Round the figures to the first digit to estimate the number of tickets the student has sold so far.

b. If approximately half the tickets sold were adult tickets and the other half were student tickets, estimate how much money has been received so far by rounding the cost of the tickets to the first digit.

18. Yi Rong is saving for her end-of-year holiday with her school friends. Her parents said if she did work around the house they would help contribute to the holiday expenses. They agreed to pay her $25 a week for doing the dishes and $15 a week for doing the washing, and as an extra incentive they said they would double her pay if she did each of the jobs for the next 4 weeks without missing a day.

Assuming Yi Rong didn't miss a day, calculate how much money she earned in the 4 weeks.

19. Bob the builder intends to build a new deck out the back of his house so he can entertain friends. He wants to get a rough idea of how much it is going to cost him in materials so he can see if he can afford to go ahead with the job. The hardware store gave him prices for the materials he needs, as shown.

Material	Price
Timber	$789
Ready-mix bags	$32.50
Nails	$67.25
Decking stain	$77.95
General equipment	$65.45

a. Estimate the rough price of materials if rounding each price to the leading digit.

b. Estimate the rough price if rounding each price to the nearest dollar.

c. Identify which of the two rounded prices is greater. Explain why this is so.

20. A roller coaster converts its stored potential energy (PE) to kinetic energy (KE) as it picks up speed on its descent.

Given $m = 750$, $g = 9.8$, $h = 15$ and $v = 15$, calculate KE and PE, using $\text{KE} = \frac{1}{2}mv^2$ and $\text{PE} = mgh$.

Fully worked solutions for this chapter are available online.

Answers

Chapter 1 Calculations

1.2 Number operations

1.2 Exercise

1. 5, −2, 212, −33

2. a. −3 **b.** −5 **c.** 2
d. −11 **e.** 30 **f.** 13

3. a. 4 **b.** 1 **c.** −1
d. 0 **e.** 5 **f.** 2

4. a. −10 **b.** 15 **c.** −8
d. −1 **e.** 39 **f.** −46

5. a. −49 **b.** 35 **c.** −32
d. −12 **e.** 25 **f.** 172

6. a. 14 **b.** 31 **c.** −18
d. 9 **e.** 25 **f.** 0

7. a. −15 **b.** −21 **c.** 30
d. −20 **e.** 40 **f.** −56

8. a. −250 **b.** −1250 **c.** −54
d. −63 **e.** 77 **f.** −500

9. a. −300 **b.** 60 **c.** −192
d. −30 **e.** −120 **f.** 36

10. a. −6 **b.** −5 **c.** 4
d. 5 **e.** −9 **f.** 0

11. a. −14 **b.** −26 **c.** 7
d. −22 **e.** 11 **f.** −21

12. a. −23 **b.** −21 **c.** 23
d. 66 **e.** −34 **f.** −120

13. a. −18 **b.** −23 **c.** 64
d. 36 **e.** 64 **f.** −49

14. 3 GB

15. $7049

16. a. 5 m
b. 75 m
c. 70 m
d. To calculate the difference, subtract the height of the smaller building from the height of the taller building.

17. a. 351 km **b.** 247 km

18. $522.63

19. $54\frac{1}{6}$ g

20. a. 39 °C **b.** 159 °C **c.** 198 °C

21. a. $0.94 **b.** $32.90 **c.** $5146.50
d. $3 **e.** $10

22. a. 5432 **b.** 2345 **c.** 3087

1.3 Order of operations and reasonableness

1.3 Exercise

1. a. −7 **b.** 0 **c.** 45
d. −6 **e.** −56 **f.** −58

2. a. 10 **b.** −5 **c.** −2
d. 44 **e.** 14 **f.** −3

3. a. −23 **b.** −29 **c.** 40
d. 9 **e.** −12 **f.** 6

4. a. $3.50 **b.** $0.27

5. $4.11

6. $290

7. $41
Answer is reasonable since roughly chips ≈ $10, Coke ≈ $20, biscuits ≈ $5 and dips ≈ $5.
This totals $40, which reduces to $38 after the discount.
This value is close to $41, so it is reasonable.

8. $62
Answer is reasonable since roughly timber ≈ $60, nails ≈ $10 and liquid nails ≈ $10.
This totals $80, which reduces to $65 after the discount.
This value is close to $62, so it is reasonable.

9. a. $(12-8)\div 4=1$
b. $(4+8)\times 5-4\times 5=40$
c. $3+4\times(9-3)=27$
d. $3\times(10-2)\div 4+4=10$
e. $10\div(5+5)\times 9\times 9=81$
f. $(18-3)\times 3\div 5=9$

10. a. Use multiplication first to calculate the cost of each type of food, and then add these costs together to calculate the total cost of all the food.
b. Cost = (2 × $2) + (3 × $4) + (3 × $5)
c. $31

11. a. 85 km
b. 15 km north

12. a. −8
b. $4+8\div-(2)^2-7\times 2=-12$

$(4+8)\div-(2)^2-7\times 2=-17$

13. 2

14. −13

1.4 Estimation

1.4 Exercise

1. a. 930 **b.** 13 000 **c.** 90 000

2. a. 350 **b.** 86 600 **c.** 70 000

3. a. 50 **b.** 80 **c.** 130 **d.** 160
e. 250 **f.** 2460 **g.** 4840 **h.** 10

4. a. 0 **b.** 100 **c.** 100 **d.** 200
e. 3300 **f.** 56 300 **g.** 86 600 **h.** 214 000

5. a. 1000 **b.** 3000 **c.** 1000
d. 6000 **e.** 8000 **f.** 13 000
g. 436 000 **h.** 728 000

6. a. 10 **b.** 20 **c.** 50
d. 200 **e.** 1000 **f.** 20 000
g. 20 000 **h.** 500 000

7. 35 000 000

8. 225

9. **a.** 1400 **b.** 900 **c.** 1100 **d.** 2000
e. 45 000 **f.** 180 000 **g.** 70 **h.** $\frac{40}{9}$

10. Since this is leading digit approximation, you look at the second digit. If it is 5 or greater, you increase the leading digit by 1 and replace the rest of the digits with zeros. If the second digit is less than 5, replace it and all of the other digits with zeros.

11. \$35

12. **a.** Sample responses can be found in the worked solutions in the online resources.
b. Sample responses can be found in the worked solutions in the online resources.
c. Sample responses can be found in the worked solutions in the online resources.

13. **a.** Sample responses can be found in the worked solutions in the online resources.
b. Sample responses can be found in the worked solutions in the online resources.
c. Sample responses can be found in the worked solutions in the online resources.

14. Sample responses can be found in the worked solutions in the online resources.

15. Sample responses can be found in the worked solutions in the online resources.

16. Adding the cost of groceries when shopping.
Calculating the cost of petrol needed for a trip.
Determining the size of a crowd.

17. 300 000

18. Actual crowd = 294 495
Estimated crowd ≈ 300 000
Difference = 5505

19. **a.** It could be correct, but the GPS cannot predict traffic or red lights on the way, so it is only an estimate.
b. By calculating the distance of the trip and the speed limits on the roads to come up with an estimate for time

20. \$39

1.5 Decimal significance

1.5 Exercise

1. **a.** $\frac{7}{100}$ **b.** $\frac{7}{10}$ **c.** $\frac{7}{1000}$

2. **a.** $\frac{9}{100}$ **b.** $\frac{9}{1000}$ **c.** $\frac{9}{10\,000}$

3. **a.** $\frac{7}{100}$ **b.** $\frac{7}{10}$ **c.** $\frac{7}{10\,000}$ **d.** $\frac{7}{100}$
e. $\frac{7}{100}$ **f.** $\frac{7}{10}$ **g.** $\frac{7}{1000}$ **h.** $\frac{7}{100}$

4. **a.** 2.195 → 3 decimal places
b. 394.7 → 1 decimal place
c. 104.25 → 2 decimal places
d. 0.0003 → 4 decimal places
e. 1997.4 → 1 decimal place
f. 125.333 → 3 decimal places
g. 69.15 → 2 decimal places
h. 2.6936 → 4 decimal places

5. **a.** False **b.** True **c.** False
d. True **e.** False **f.** False

6. **a.** 59.123 **b.** 72.217

7. **a.** 2.07 **b.** 0.80

8. **a.** 39.184 **b.** 3.232 **c.** 99.436
d. 125.295 **e.** 100.004 **f.** 0.048

9. **a.** 58.12 **b.** 23.35 **c.** 71.10
d. 0.87 **e.** 31.00 **f.** 125.89

10. **a.** 0.75 **b.** $0.\dot{6}$

11. **a.** 0.875 **b.** 0.143

12. **a.** 0.3 **b.** 0.6 **c.** 0.25
d. 0.28 **e.** 0.33 **f.** 0.9

13. **a.** $0.\dot{3}$ **b.** $0.\dot{5}$ **c.** $0.08\dot{3}$
d. $0.\overline{571428}$ **e.** $0.\overline{230769}$ **f.** $0.\overline{047619}$

14. The one on the right, since 0.630 g > 0.584 g

15. 93.5 mm

16. **a.** Company B: $25\frac{3}{4}$ (25.75); Company C: $25\frac{7}{9}$ $\left(25.\dot{7}\right)$;
Company A: 26.78

b. $26.78 \approx 26.8$

$$25\frac{7}{9} = 25.\dot{7}$$

$$\approx 25.8$$

$$25\frac{3}{4} = 25.75$$

$$\approx 25.8$$

17. Sample responses can be found in the worked solutions in the online resources.

18. Sample responses can be found in the worked solutions in the online resources.

1.6 Calculator skills

1.6 Exercise

1. **a.** 461 **b.** 1206 **c.** −162.1392

2. **a.** 445 **b.** 1748 **c.** 343.136227

3. **a.** 45 **b.** 95 **c.** 621 **d.** 16 921

4. **a.** 2 **b.** 32 **c.** 241 **d.** 438

5. **a.** 62 **b.** 49 **c.** 20.702 **d.** 58.95

6. **a.** 132.7925 **b.** 29.5800 **c.** 11.4750

7. **a.** −0.4097 **b.** 1.9746 **c.** 1.4038

8. **a.** 23.1884 **b.** 19.2200 **c.** 76.6600
d. 12.3429

9. **a.** 3.7430 **b.** 4.8485 **c.** 18.2067
d. 0.2098

10. **a.** 0.75 **b.** 0.32 **c.** 1.27
d. 19.67

11. Cathy is 13 years old.

12. 5 cans

1.7 Approximation

1.7 Exercise

1. 58 450
2. 45 900 L
3. a. m b. g c. mL
   d. km e. cm f. kg
4. 2 L
5. 500 g
6. $14 500
7. 14 000 metric tonnes of CO_2-e
8. 15 300
9. Around 17 000
10. Around 11 115
11. Even though the crowd behind Tiger Woods looks large, we don't know the capacity of the golf course. Even if we did, the picture doesn't show us the crowd at the other holes. There might not be many people watching the other groups because they are not as popular as Tiger Woods.
12. A guess doesn't really have any thought or logic behind it, but an estimate does.
13. Estimate the size of each year level and add them up. This could be improved by finding how many home-room or tutor groups there are at each year level, estimating the number of students in each and adding them all up.
14. Estimate: 28 000 000
15. Estimate: $0.70
16. a. Estimate: 750 000
    b. Estimate: 820 000

1.8 Review

1.8 Exercise

1. 38
2. 5
3. 50 000
4. $30
5. 9.07
6. Sample responses can be found in the worked solutions in the online resources.
7. 37.3959
8. $\frac{79}{10}$
9. 375 mL
10. a. 7 b. −48 c. 20 d. 7
11. a. 40 000 b. 1200 c. $\frac{80}{7}$ d. 18 050
12. a. 3.66 b. 32.32 c. 0.84 d. 4.75
13. a. 0.13 b. 0.74 c. 0.24 d. 0.67
14. a. 0.556 b. 1.036 c. 33.105
    d. 11.000
15. a. 37 500 people
    b. Between 5000 and 7000 L (6000 L)
16.

	Estimate	Estimated answer	Actual answer
a.	$500 + 1000$	1500	1449
b.	$33\,000 + 80\,000$	110 000	115 670
c.	$200\,000 + 700\,000$	900 000	907 448
d.	$90\,000 - 40\,000$	50 000	55 880
e.	$300 - 200$	100	127
f.	$500\,000 - 200\,000$	300 000	296 049
g.	40×200	8000	7128
h.	$600 \times 10\,000$	6 000 000	7 894 033
i.	$30\,000 \times 40$	1 200 000	1 149 954
j.	$30\,000 \div 1000$	30	32.9076
k.	$60\,000 \div 3000$	20	24.325 483
l.	$70\,000 \div 1000$	70	53.3855

17. a. 1500 tickets b. $11 250
18. $320
19. a. Around $1050
    b. $1032
    c. The price in part a is higher since four of the five values were rounded up using first digit rounding.
20. PE = 110 250 J
    KE = 84 375 J

UNIT 3 Measurement, scales and chance
TOPIC 1 Measurement

2 Geometry

LESSON SEQUENCE

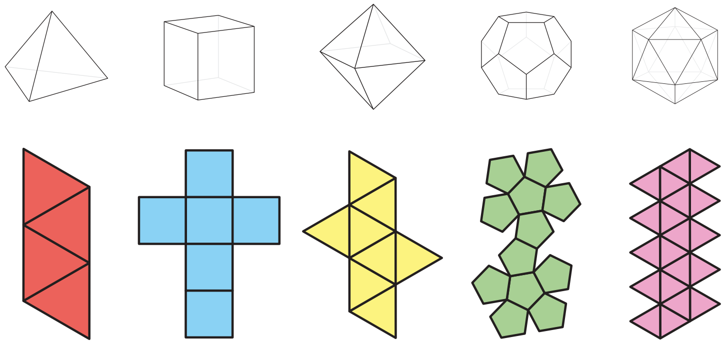
Fully worked solutions for this chapter are available online.

on Resources

Solutions	Solutions — Chapter 2
Digital documents	Learning matrix — Chapter 2 Quick quizzes — Chapter 2 Chapter summary — Chapter 2

LESSON
2.1 Overview

2.1.1 Introduction



Understanding the properties of polygons, prisms and pyramids is key to mastering geometry and its real-world applications. Polygons, such as triangles and squares, are defined by their sides, vertices and angles. Exploring these shapes helps us understand three-dimensional objects like prisms and pyramids, which have edges, vertices and flat faces. Recognising these properties and interpreting two-dimensional representations, such as nets, is essential for visualising how 3-dimensional shapes are formed.

These concepts are crucial in many careers. Architects, engineers and designers use them to create structures and models, while 3-dimensional modelling in technology relies on these principles for video games, simulations and virtual environments. Even in everyday life, understanding geometry helps with interpreting maps and designing products.

2.1.2 Syllabus links

Lesson	Lesson title	Syllabus links
2.2	**Review of angles**	○ Recognise the properties of standard polygons, e.g. number of vertices, straight edges and angles.
2.3	**Two-dimensional geometric shapes**	○ Recognise the properties of standard polygons, e.g. number of vertices, straight edges and angles.
2.4	**Three-dimensional objects**	○ Recognise the properties of prisms and pyramids, e.g. number of vertices, straight edges and flat faces.
2.5	**Two-dimensional representations of three-dimensional objects [complex]**	○ Interpret different forms of two-dimensional representations of three-dimensional objects, including nets of prisms and pyramids [complex].

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LESSON
2.2 Review of angles

SYLLABUS LINKS

- Recognise the properties of standard polygons, e.g. number of vertices, straight edges and angles.

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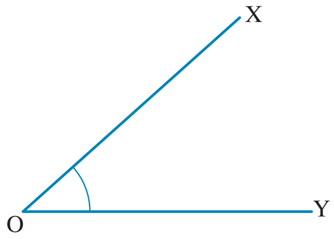
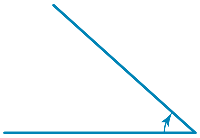
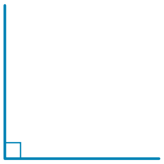
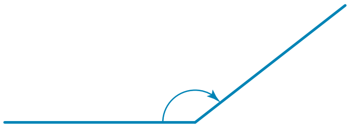
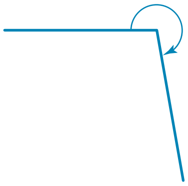
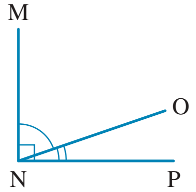
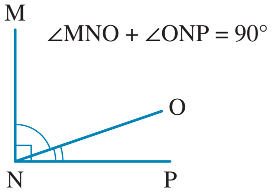
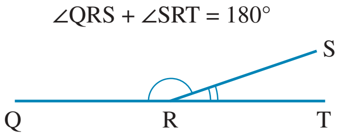
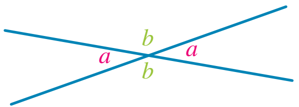
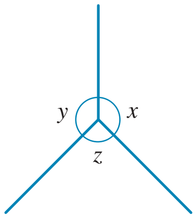
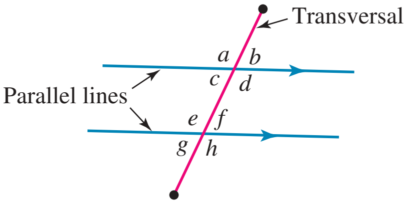
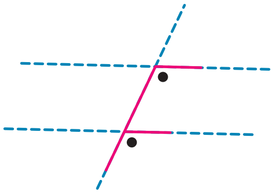
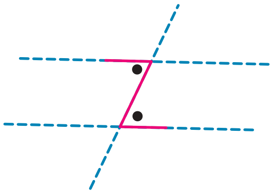
2.2.1 Angles and angle properties

- **Angles** are formed at the point of **intersection** between two straight lines.
- A common way to name an angle is to use three letters: two letters to represent the arms of the angle, and a third letter to represent its vertex.
- Instead of writing the word *angle*, we use the symbol ∠.
- In the diagram at right, the angle at O can be written as ∠XOY, ∠YOX or simply ∠O.
- The letter representing the vertex is always placed in the middle (between the two letters representing the arms).

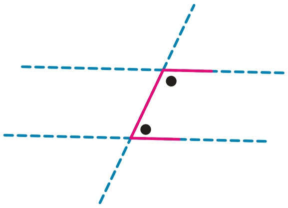


Angle types	**Diagram**
- An **acute angle** is greater than 0°, but less than 90°.	
- A **right angle** is an angle that equals exactly 90°.	
- An **obtuse angle** is greater than 90° but less than 180°.	
- A **straight angle** equals exactly 180°.	
- A **reflex angle** is greater than 180° but less than 360°.	
- A **revolution** or a **perigon** is an angle that equals exactly 360° (a full circle).	

Angle property	Diagram
• **Adjacent angles** share a common side (arm) and vertex. • ∠MNO and ∠ONP are adjacent angles as they share the arm $\overline{\text{NO}}$.	
• **Complementary angles** add to 90°. • ∠MNO and ∠ONP are complementary angles.	 Complementary angles
• **Supplementary angles** add to 180°. • ∠QRS and ∠SRT are supplementary angles.	 Supplementary angles
int-3968 • **Vertically opposite angles** are formed when two lines intersect. • The intersection creates two sets of equal-sized angles.	 Vertically opposite angles
int-6157 • Angles around a point share a common vertex and add to 360°. • In the example shown, $x + y + z = 360°$.	 $x + y + z = 360°$
int-3972 • **Parallel** lines never intersect; the distance between them is constant along their lengths. • Parallel lines are marked with identical arrowheads. • A straight line that crosses two parallel lines is called a **transversal**. • A transversal creates three sets of angle pairs, as follows.	

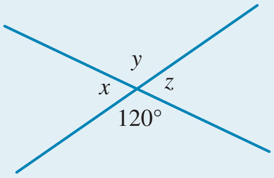
Angle pair	Positions	Relationship
int-3969 **Corresponding angles (F angles)**		The same size. For example, in the parallel lines preceding this table: $a = e;\ b = f;\ d = h;\ c = g$.
int-3971 **Alternate angles (Z angles)**		The same size. For example, in the parallel lines preceding this table: $c = f; d = e$.

int-3970

Angle pair	Positions	Relationship
Co-interior angles (C angles)		Supplementary. For example, in the parallel lines preceding this table: $c + e = 180°$; $d + f = 180°$.

WORKED EXAMPLE 1 Using angle properties 1

Calculate the values of the pronumerals in the diagram.



THINK	WRITE
1. The angles y and 120° are vertically opposite, so they are equal.	$y = 120°$
2. The angle z and the angle marked 120° form a straight line, so they are supplementary angles (i.e. they add to 180°). Subtract 120° from 180° to find z.	$z = 180° - 120°$ $= 60°$
3. The angles x and z are vertically opposite, so they are equal.	$x = 60°$

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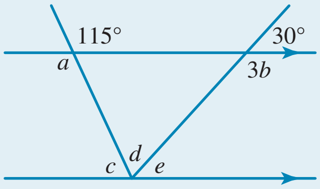
WORKED EXAMPLE 2 Using angle properties 2

Calculate the values of the pronumerals in the diagram.



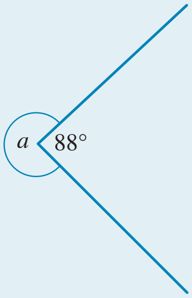
THINK	WRITE
1. Angle a is vertically opposite to 115°, so a is equal to 115°.	$a = 115°$
2. $3b$ and 30° are supplementary (add to 180°). Write an equation to represent this.	$3b + 30° = 180°$
3. Solve the equation for b by subtracting 30° from both sides, then dividing both sides by 3.	$3b + 30° - 30° = 180° - 30°$ $3b = 150°$ $\frac{3b}{3} = \frac{150°}{3}$ $b = 50°$
4. a and c are co-interior, so they add to 180°. Write an equation to represent this.	$a + c = 180°$

5. Substitute $a = 115°$ into the equation and solve for c by subtracting 115° from both sides.	$115° + c = 180°$ $115° + c - 115° = 180° - 115°$ $c = 65°$
6. e and 30° are corresponding and hence equal.	$e = 30°$
7. c, d and e form a straight line, so they add to 180°. Write an equation to represent this.	$c + d + e = 180°$
8. Substitute $c = 65°$ and $e = 30°$. Solve for d by adding 65° and 30°, then subtracting 95° from both sides.	$65° + d + 30° = 180°$ $d + 95° = 180°$ $d + 95° - 95° = 180° - 95°$ $d = 85°$
9. Write your answer	$a = 115°, b = 50°, c = 65°, d = 85°, e = 30°$

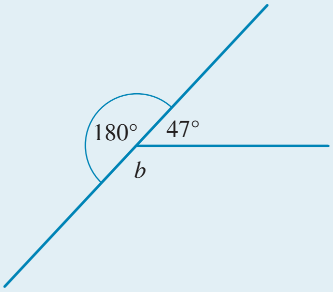
WORKED EXAMPLE 3 Using angle properties 3

Calculate the value of the pronumeral in each of the following.

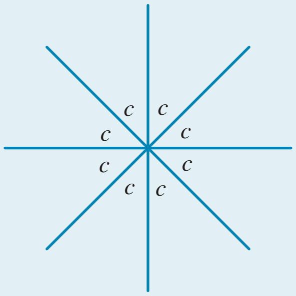
a.



b.



c.

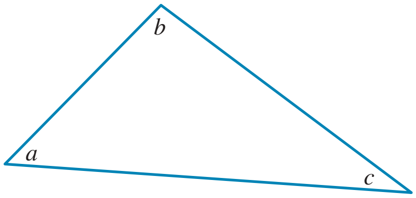


THINK	WRITE
a. 1. These angles form a circle, so they must add to 360°. Write an equation to represent this.	**a.** $a + 88° = 360°$
2. Subtract 88° from 360° to determine the value of a.	$a + 88° - 88° = 360° - 88°$ $a = 272°$
b. 1. These angles form a circle, so they must add to 360°. Write an equation to represent this.	**b.** $b + 180° + 47° = 360°$
2. Add 180° and 47° to simplify the left side of the equation.	$b + 227° = 360°$
3. Subtract 180° and 47° from both sides to determine the value of b.	$b = 360° - 227°$ $b = 133°$
c. 1. These angles form a circle, so they must add to 360°. Write an equation to represent this.	**c.** $c + c + c + c + c + c + c + c = 360°$
2. There are eight equal angles in this diagram. Simplify the equation by collecting like terms.	$8c = 360°$
3. Divide both sides by 8 to determine the value of c.	$\frac{8c}{8} = \frac{360°}{8}$ $c = 45°$

2.2.2 Interior angles of a triangle

int-3965

- In the triangle shown, $a+b+c=180°$, where a, b and c represent the three interior angles of the triangle.



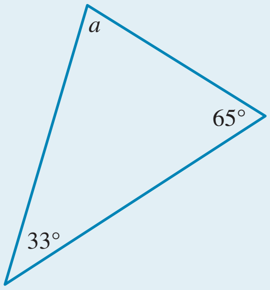
Sum of interior angles of a triangle

The sum of the interior angles of a triangle is 180°.

$$a+b+c=180°$$

WORKED EXAMPLE 4 Using the sum of interior angles of a triangle

Calculate the value of the pronumeral in the triangle shown.



THINK

1. The sum of the three interior angles is 180°. Write this as an equation.
2. Simplify the equation by adding 33° and 65°.
3. Solve the equation for a by subtracting 98° from both sides of the equation.

WRITE

$a+33°+65° = 180°$

$a+98° = 180°$

$a+98°-98° = 180°-98°$

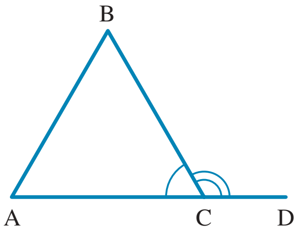
$a = 82°$

2.2.3 Exterior angles of a triangle

int-3966

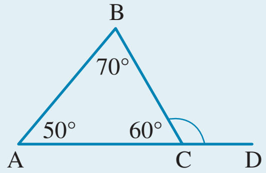
- If one edge of a triangle continues past the vertex, the angle formed is called an **exterior angle**.
- The interior angle and the exterior angle adjacent (next) to it are supplementary angles; they add to 180°.
- For example, the triangle formed by the house shown:

$$\angle ACB+\angle BCD = 180°$$
$$\angle ACB = 180°-\angle BCD$$
$$\angle BCD = 180°-\angle ACB$$



WORKED EXAMPLE 5 Using angle properties in a triangle

Determine the value of the exterior angle $\angle BCD$ in the diagram shown.



THINK	WRITE
1. The sum of the supplementary angles, $\angle ACB$ and $\angle BCD$, is 180°.	$\angle ACB + \angle BCD = 180°$
2. $\angle ACB$ is 60°. Subtract this value from 180° to determine the value of $\angle BCD$.	$\begin{aligned}\angle BCD &= 180° - \angle ACB \\ &= 180° - 60° \\ &= 120°\end{aligned}$

- The sum of the interior angles of a triangle is 180°, and there are 180° in a straight angle. Therefore, we can establish the following rule.

Exterior angle of a triangle

An exterior angle of a triangle is equal to the sum of the two opposite interior angles.

- In Worked example 5, the size of the exterior angle $\angle BCD$ could have been calculated by adding the two opposite interior angles, $\angle BAC$ and $\angle ABC$:

$$\begin{aligned}\angle BCD &= \angle BAC + \angle ABC \\ &= 50° + 70° \\ &= 120°\end{aligned}$$

Exercise 2.2 Review of angles

learnon

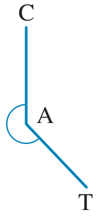
2.2 Quick quiz on

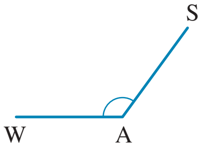
2.2 Exercise

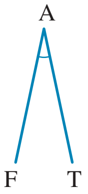
Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15	N/A	N/A

Simple familiar

1. Consider the angles shown.
   a. Name each angle using:
      i. three letters
      ii. a single letter.
   b. Name the *type* of each angle.





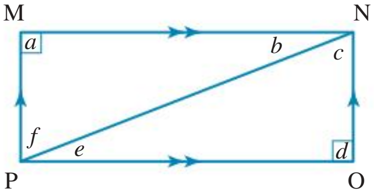


2. Complete the following table by placing the listed angles in the appropriate places.

Acute angles	Obtuse angles	Right angles	Reflex angles

23°	92°	122°	154°	179°	5°	90°	110°	45°	55°	270°	14°	160°
100°	78°	160°	69°	89°	190°	145°	300°	359°	80°	2°	92°	181°

3. Consider the diagram shown.



a. Name the six interior angles in three-letter format.

b. Name the type of angle for each of the following:

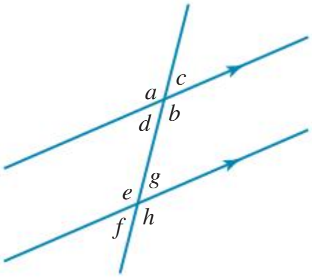
i. *a*
ii. *b*
iii. *c*
iv. *d*
v. *e*
vi. *f*

c. List the angle pairs that are complementary.

d. List the angle pairs that are alternate.

e. If $e = 30°$, determine the values of the remaining pronumerals.

4. Complete the following sentences using the diagram shown.

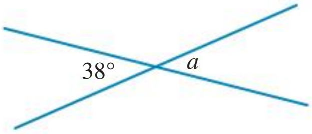


a. Angle *a* is vertically opposite angle _____.
b. Angle *a* is a corresponding angle with angle _____.
c. Angle *b* is co-interior with angle _____.
d. Angle *g* is alternate to angle _____.
e. Angle *h* is vertically opposite angle _____.
f. Angle *b* is a corresponding angle with angle _____.

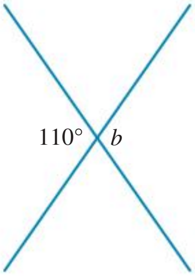
5. a. Sketch two angles, XOY and VOW, so that they are vertically opposite each other.
b. From your angles in part **a**, list another pair of vertically opposite angles.

6. **WE1** Calculate the values of the pronumerals in each of the following.

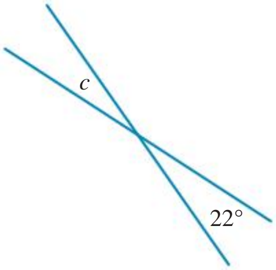
a.



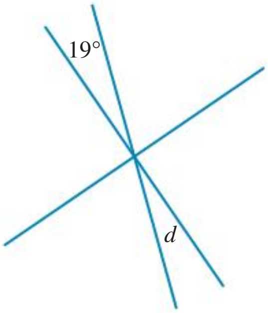
b.



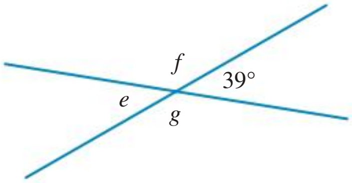
c.



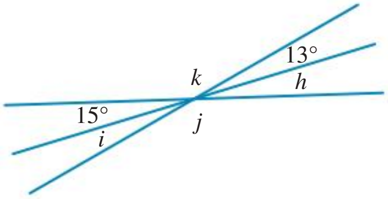
d.



e.

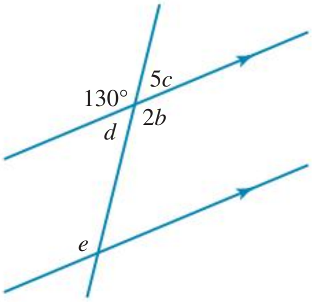


f.

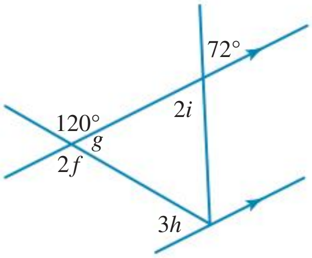


7. **WE2** Calculate the values of the pronumerals in the following diagrams.

a.

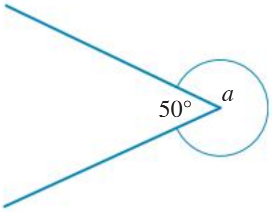


b.



8. **WE3** Calculate the value of the pronumeral in each of the following.

a.



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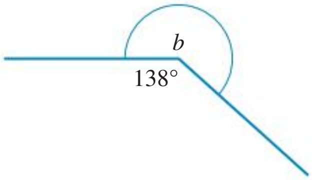

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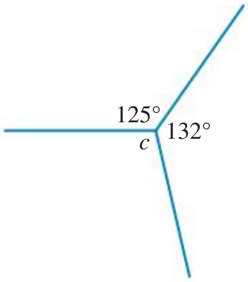
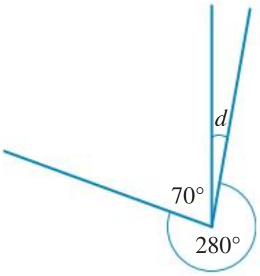
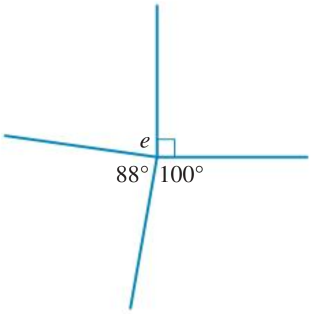


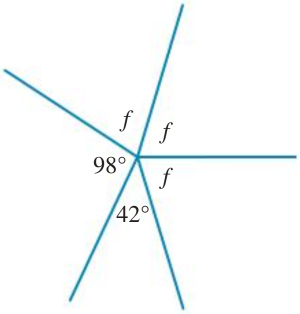
d.



e.

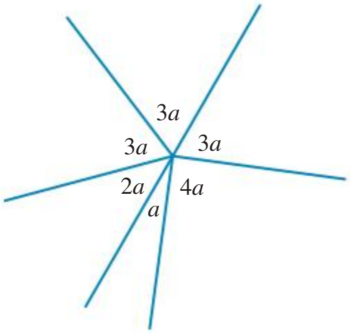


f.

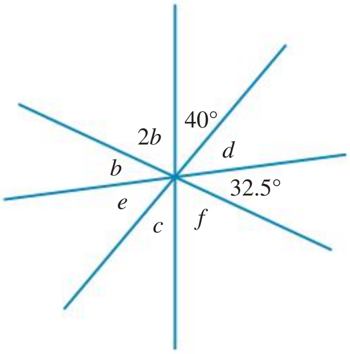


9. Calculate the value of the pronumeral(s) in each of the following.

a.

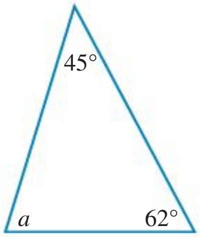


b.

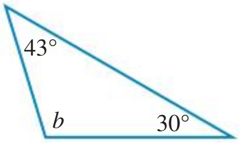


10. **WE4** Calculate the value of the pronumeral in each of the following triangles.

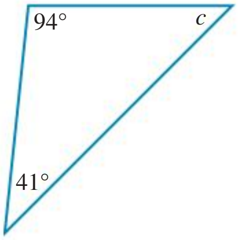
a.



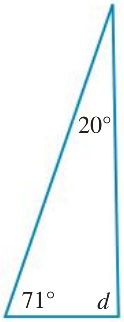
b.



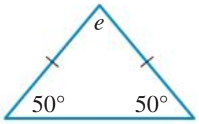
c.



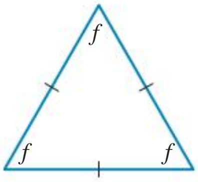
d.



e.

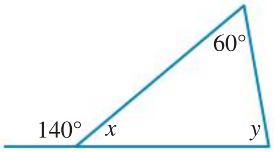


f.

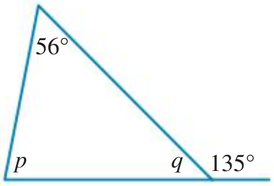


11. **WE5** Calculate the value of the pronumerals in each of the following diagrams.

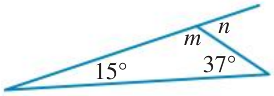
a.



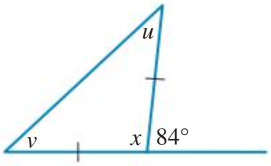
b.



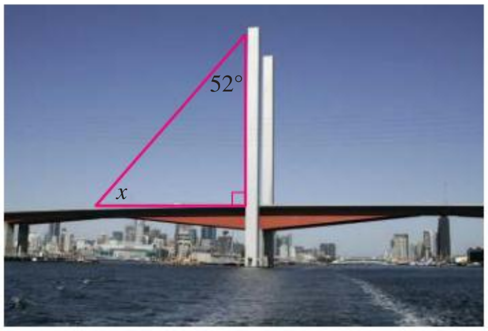
c.



d.



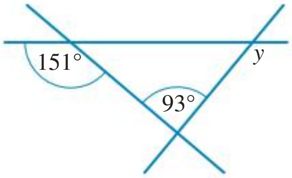
12. Calculate the value of the pronumeral in the photograph.



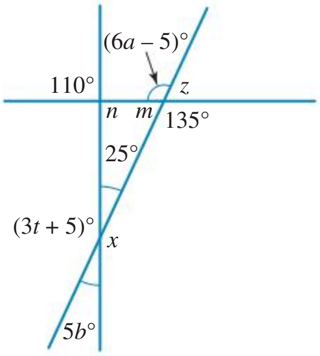
13. A long ladder is placed against the side of a building. The angle that the ladder makes with the ground is 55.5°.

a. Sketch a diagram of the ladder against the building and label it.
b. Identify the type of the triangle created by the ladder, the building and the ground.
c. Calculate the exterior angle that the ladder makes with the ground. Show this on your diagram.

14. Determine the value of the unknown pronumeral in the diagram.



15. Calculate the value of the pronumerals in the diagram.



Fully worked solutions for this chapter are available online.

LESSON
2.3 Two-dimensional geometric shapes

SYLLABUS LINKS

- Recognise the properties of standard polygons, e.g. number of vertices, straight edges and angles.

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- Two-dimensional shapes are recognised by their property of having length and width only and no depth.
- This means 2-dimensional shapes can be easily drawn on paper.

2.3.1 Polygons

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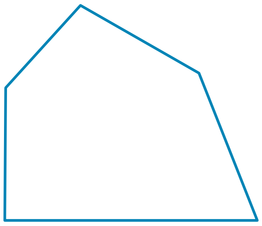
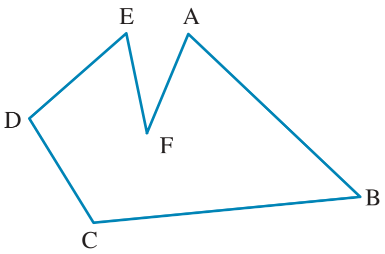
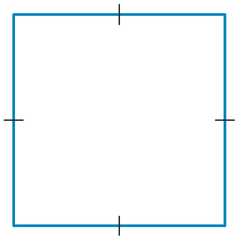
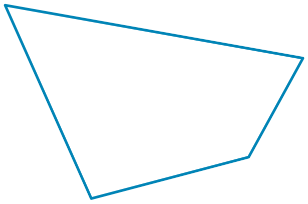
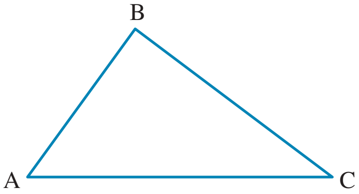
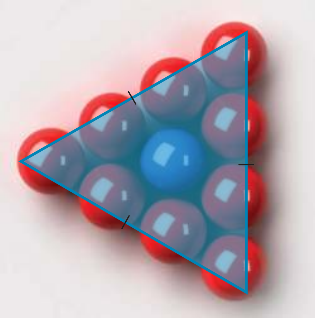
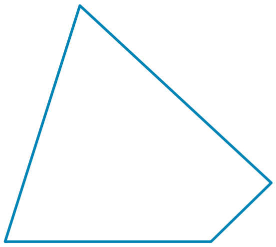
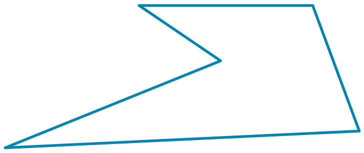
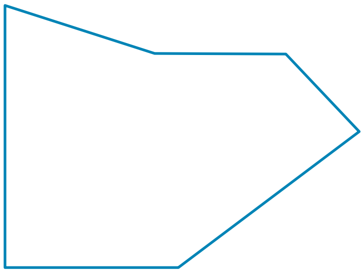
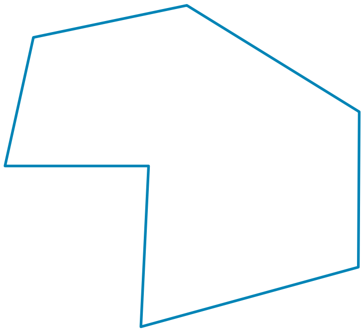
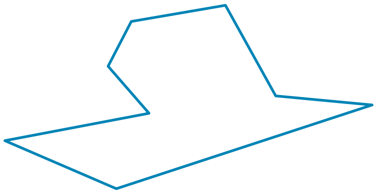
- A **polygon** is a closed two-dimensional shape made of straight lines.
- It has at least three sides, and each **side** meets at a **vertex**.
- The number of sides is always equal to the number of vertices and gives the polygon its name.
- The vertices of a polygon are labelled using uppercase (capital) letters.

Type of polygon	Definition	Diagram
Convex polygon	• A **convex polygon** has all interior angles less than 180°. • All its vertices point outside the polygon as shown in the diagram.	
Concave polygon	• A **concave polygon** has at least one reflex angle. • A reflex angle points inwards, inside the shape. • Extending the arms of a reflex angle cuts through the polygon.	
Regular polygon	• A **regular polygon** has equal sides and equal angles.	
Irregular polygon	• An **irregular polygon** is a shape in which not all sides and angles are equal.	

Sum of interior angles in a polygon

The sum of the internal angles of a polygon = $(n-2) \times 180°$ where n = number of sides.

Polygon	Number of sides and vertices	Regular polygon	Sum of angles in the polygon	Size of one angle in the regular polygon
Triangle  	3	Equilateral triangle 	180°	60°
Quadrilateral 	4	Square 	360°	90°
Pentagon 	5	Regular pentagon 	540°	108°

Polygon	Number of sides and vertices	Regular polygon	Sum of angles in the polygon	Size of one angle in the regular polygon
Hexagon 	6	Regular hexagon 	720°	120°
Heptagon 	7	Regular heptagon 	900°	$128\frac{4}{7}°$
Octagon 	8	Regular octagon 	1080°	135°

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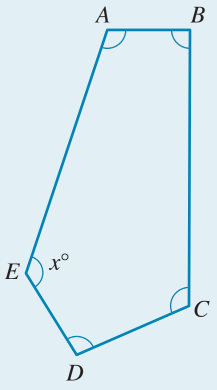
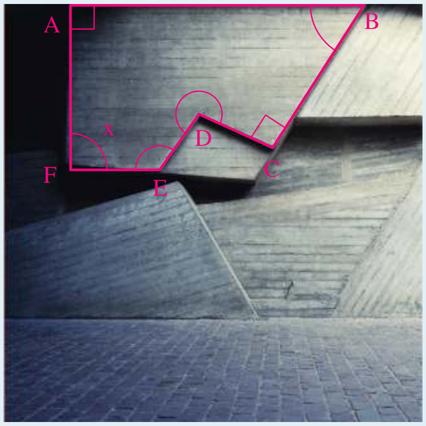
WORKED EXAMPLE 6 Using the properties of standard polygons

Name the polygons shown in the diagram, with mathematical reasons, and calculate the sizes of the unknown angles.

a. $\angle A = 120°, \angle B = 90°, \angle C = 132°$ **and** $\angle D = 90°$



b. $\angle A = \angle C = 90°, \angle B = 63°, \angle D = 270°$ **and** $\angle E = 115°$



THINK	**WRITE**
a. 1. Identify the name of the polygon.	**a.** The polygon is an irregular convex pentagon because: • it has five sides and five vertices (pentagon) • the angles and side lengths are not equal in size (irregular) • all vertices point outwards and interior angles are not >180°.
2. Determine the sum of the interior angles of a polygon with 5 sides.	$\begin{aligned}\text{Angle sum} &= (n-2) \times 180° \\ &= (5-2) \times 180° \\ &= 3 \times 180° \\ &= 540°\end{aligned}$

3. Write an equation connecting all the angles of the polygon.

$$\angle A + \angle B + \angle C + \angle D + \angle E = 540°$$

4. Substitute any known values.

$$120° + 90° + 132° + 90° + x° = 540°$$

5. Solve the equation to determine the value of the unknown angle.

$$\begin{aligned} 432° + x° &= 540° \\ x° &= 540° - 432° \\ x° &= 108° \\ \therefore \angle E &= 108° \end{aligned}$$

b. 1. Identify the name of the polygon.

b. The polygon is an irregular concave hexagon because:

- it has six sides and six vertices (hexagon)
- the angles and side lengths are not equal in size (irregular)
- one of the vertices points inside the polygon.

2. Determine the sum of the interior angles of a polygon with 6 sides.

$$\begin{aligned} \text{Angle sum} &= (n-2) \times 180° \\ &= (6-2) \times 180° \\ &= 4 \times 180° \\ &= 720° \end{aligned}$$

3. Write an equation connecting all the angles of the polygon.

$$\angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 720°$$

4. Substitute any known values.

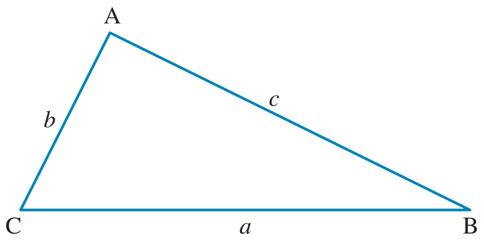
$$90° + 63° + 90° + 270° + 115° + \angle F = 720°$$

5. Solve the equation to determine the value of the unknown angle.

$$\begin{aligned} 628° + \angle F &= 720° \\ \angle F &= 720° - 628° \\ \therefore \angle F &= 92° \end{aligned}$$

2.3.2 Triangles

- A **triangle** is a three-sided polygon.
- The name of the triangle in the diagram shown is ΔABC.



- Alternative notations for this triangle are ΔBCA and ΔCAB.

Types of triangles

- The types of triangles can be identified using either the lengths of the sides (being equal or unequal) or by the angles within the triangle.

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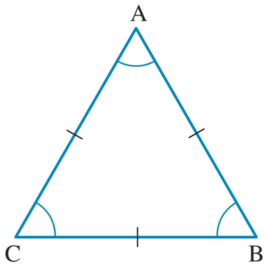
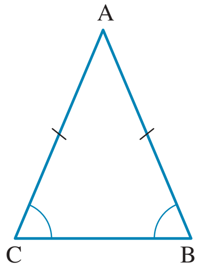
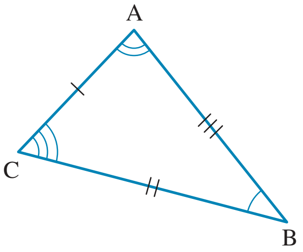
Type of triangle	Diagram	Properties
Equilateral		AB = BC = AC $\angle A = \angle B = \angle C = 60°$
Isosceles		AB = AC $\angle C = \angle B$
Scalene		Sides of different lengths and angles are different sizes.

Triangles are classified according to their angles as follows.

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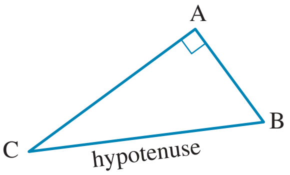
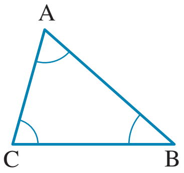
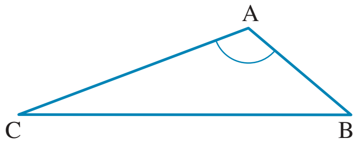
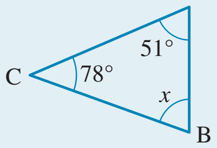
Type of triangle	Diagram	Properties
Right-angled		$\angle A = 90°$ The side opposite to the right angle is called the **hypotenuse**.
Acute-angled		All angles are acute angles (less than 90°). $\angle A < 90°$ $\angle B < 90°$ $\angle C < 90°$
Obtuse-angled		There is one obtuse angle (greater than 90°). $90° < \angle A < 180°$ $\angle B < 90°$ $\angle C < 90°$

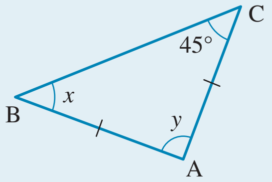
WORKED EXAMPLE 7 Classifying triangles and determining unknown angles

Classify the following triangles by first calculating the size of the unknown angles. Provide reasons for your answer.

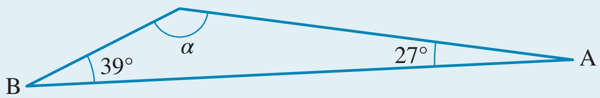
a.



b.



c.



THINK	**WRITE**
a. 1. Write an equation connecting the three angles.	**a.** $\angle A + \angle B + \angle C = 180^\circ$
2. Substitute any known values.	$51^\circ + x + 78^\circ = 180^\circ$
3. Solve the equation to determine the value of the unknown angle.	$129^\circ + x = 180^\circ$ $129^\circ - 129^\circ + x = 180^\circ - 129^\circ$ $x = 51^\circ$
4. Identify the type of triangle, clearly stating the reasons.	ΔABC is an acute isosceles triangle because $\angle A = \angle B = 51^\circ$ (isosceles). $\angle A, \angle B$ and $\angle C$ are $< 90^\circ$ (acute)
b. 1. Look for any sides and angles that are equal, and hence determine the value of *x*.	**b.** $AB = AC$ => the opposite angles to AB and AC are equal in size $=> \angle C = \angle B = 45^\circ$ $=> x = 45^\circ$
2. Write an equation connecting the three angles.	$\angle A + \angle B + \angle C = 180^\circ$
3. Substitute any known values.	$y + 45^\circ + 45^\circ = 180^\circ$
4. Solve the equation to determine the value of *y*.	$y + 90^\circ = 180^\circ$ $y + 90^\circ - 90^\circ = 180^\circ - 90^\circ$ $y = 90^\circ$
5. Identify the type of triangle, clearly stating the reasons.	ΔABC is a right-angled isosceles triangle because $\angle B = \angle C = 45^\circ$ (isosceles) $\angle A = 90^\circ$ (right-angled).
c. 1. Write an equation connecting the three angles.	**c.** $\angle A + \angle B + \angle C = 180^\circ$
2. Substitute any known values.	$27^\circ + 39^\circ + \alpha = 180^\circ$

3. Solve the equation to determine the value of the unknown angle.	$66° + \alpha = 180°$ $66° - 66° + \alpha = 180° - 66°$ $\alpha = 114°$
4. Identify the type of triangle, clearly stating the reasons.	ΔABC is an obtuse scalene triangle because $\angle A \neq \angle B \neq \angle C$ (scalene). $\angle C$ is an obtuse angle as $90° < 114° < 180°$ (obtuse).

2.3.3 Quadrilaterals

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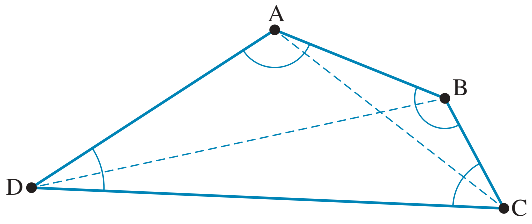
- A **quadrilateral** is a four-sided polygon.
- The name of the quadrilateral in the diagram shown is ABCD.
- Other notations are possible if the letters are read in order in either a clockwise or an anticlockwise direction: BCDA, CDAB or DABC.
- Every quadrilateral has two diagonals.
- A **diagonal** is a line segment which connects two opposite angles of a quadrilateral such as AC and BD in the diagram.



Sum of interior angles in a quadrilateral

The sum of the internal angles of a quadrilateral ($n = 4$) is $(n - 2) \times 180° = 360°$.

The sum of the interior angles of any quadrilateral is 360°.

$$\angle A + \angle B + \angle C + \angle D = 360°$$

Types of quadrilaterals

Quadrilaterals are classified according to their specific properties, as follows.

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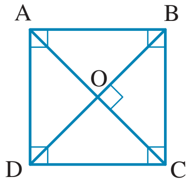
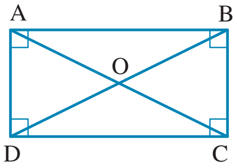
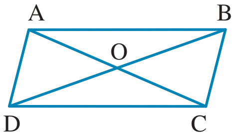
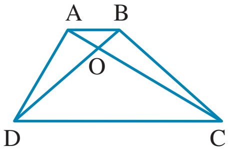
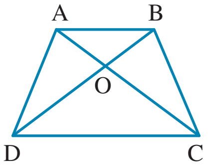
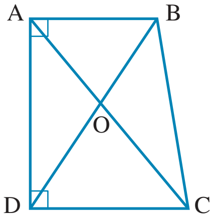
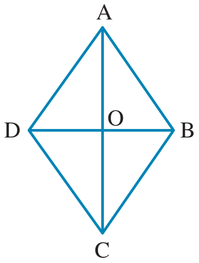
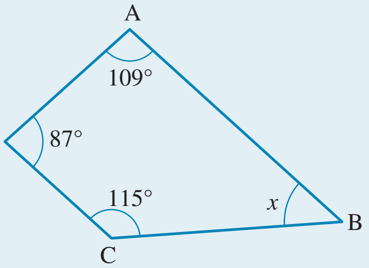
Quadrilateral	Sides	Angles	Diagonals
Square 	AB = BC = AD = CD AB || DC and BC || AD The symbol || means ‘parallel to’.	$\angle A = \angle B = \angle C = \angle D$ $= 90°$	AC ⊥ BD AC = BD AO = OC BO = OD The symbol ⊥ means ‘perpendicular (forms a right angle) to’.

Quadrilateral	Sides	Angles	Diagonals
Rectangle 	AB $\underset{=}{\parallel}$ DC and BC $\underset{=}{\parallel}$ AD The symbol $\underset{=}{\parallel}$ means 'parallel and equal to'.	$\angle A = \angle B = \angle C = \angle D$ $= 90°$	AC = BD AO = OC BO = OD
Parallelogram 	AB $\underset{=}{\parallel}$ DC and BC $\underset{=}{\parallel}$ AD	Opposite angles are equal. $\angle A = \angle C$ $\angle B = \angle D$ Consecutive angles are supplementary. $\angle A + \angle B = 180°$ $\angle B + \angle C = 180°$ $\angle C + \angle D = 180°$ $\angle D + \angle A = 180°$	AO = OC BO = OD
Trapezium 	AB||CD	All different	No characteristics
Isosceles trapezium 	AB||CD AD = BC	$\angle A = \angle B$ $\angle C = \angle D$ Opposite angles are supplementary. $\angle B + \angle D = 180°$ $\angle A + \angle C = 180°$	AC = BD
Right-angled trapezium 	AB||CD	$\angle A = \angle D = 90°$ $\angle A + \angle D = 180°$ $\angle B + \angle C = 180°$	No characteristics
Rhombus 	AB||DC and BC||AD AB = BC = CD = AD	Opposite angles are equal. Consecutive angles are supplementary.	$AC \perp BD$ AO = OC BO = OD

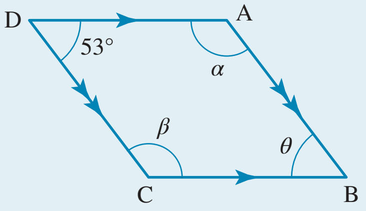
WORKED EXAMPLE 8 Calculating unknown angles in quadrilaterals

Calculate the sizes of the unknown angles in the following quadrilaterals.

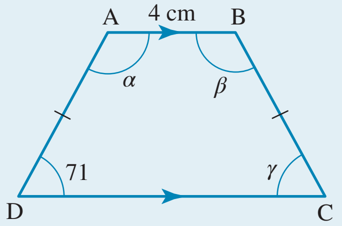
a.



b.



c.



THINK	WRITE
a. 1. Write an equation connecting the four angles.	**a.** $\angle A + \angle B + \angle C + \angle D = 360°$
2. Substitute any known values.	$109° + x + 115° + 87° = 360°$
3. Solve the equation to determine the size of the unknown angle.	$x + 311° = 360°$ $x + 311° - 311° = 360° - 311°$ $x = 49°$
b. 1. Identify the relationship between angles B and D, and hence calculate the value of θ.	**b.** $\angle B = \angle D = 53°$ => $\theta = 53°$
2. Write an equation stating the relationship between angles A and D and solve the equation to determine the value of α and β.	$\angle A + \angle D = 180°$ $\alpha + 53° = 180°$ $\alpha + 53° - 53° = 180° - 53°$ $\alpha = 127°$ => $\beta = \alpha = 127°$
c. 1. Identify the relationship between angles C and D, and hence determine the value of γ.	**c.** $\angle C = \angle D = 71°$ => $\gamma = 71°$
2. Write an equation stating the relationship between angles A and D and solve the equation to determine the value of α and β.	$\angle A + \angle D = 180°$ $\alpha + 71° = 180°$ $\alpha + 71° - 71° = 180° - 71°$ => $\alpha = 109°$ => $\beta = \alpha = 109°$

Exercise 2.3 Two-dimensional geometric shapes

2.3 Quick quiz on | **2.3 Exercise**

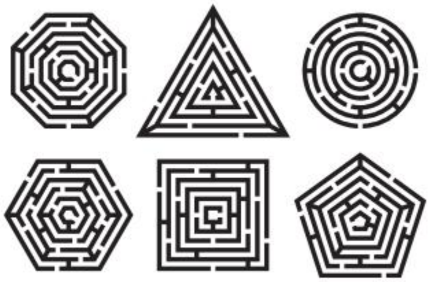
Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14	N/A	N/A

These questions are even better in jacPLUS!
- Receive immediate feedback
- Access sample responses
- Track results and progress

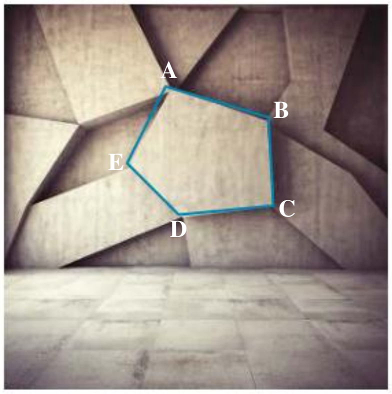
Find all this and MORE in jacPLUS

Simple familiar

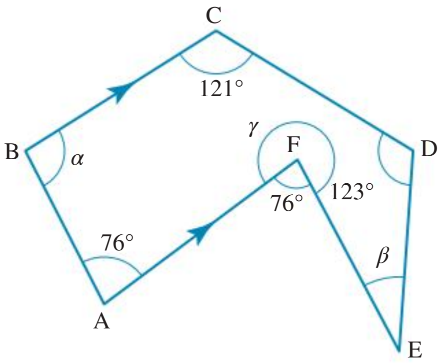
1. Classify the shapes of the mazes in the diagram shown.



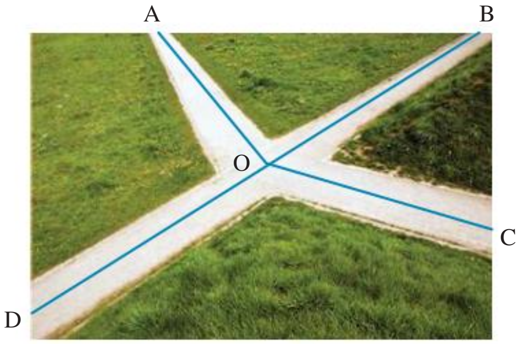
2. **WE6** Name the polygon shown in the diagram, with mathematical reasons, and calculate the size of the unknown angle if ∠A = 97°, ∠B = 118°, ∠C = 92° and ∠D = 124°.



3. Calculate the size of the unknown angles of the polygon ABCDEF and identify the name of the polygon.

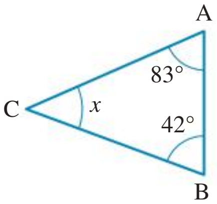


4. Calculate the size of ∠DOC if ∠AOB = ∠BOC = 75° and ∠AOD = 110°.

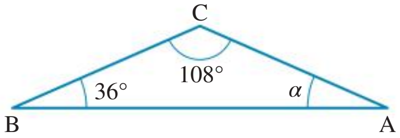


5. **WE7** Classify the following triangles by first calculating the sizes of the unknown angles. Provide reasons for your answers.

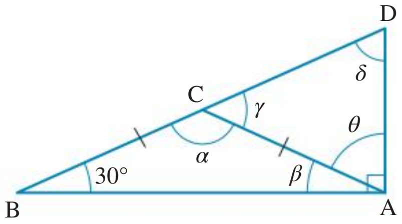
a.



b.



6. Calculate the size of each unknown angle, providing reasons for your answers. Use this information to classify the triangles ΔABC, ΔABD and ΔACD.



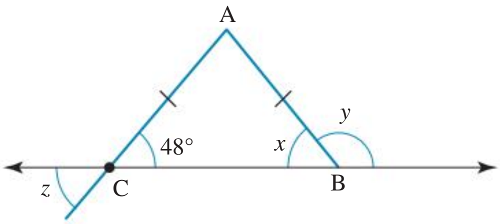
7. Consider the diagram shown.

a. Classify triangle ΔABC.

b. Determine the value of x.

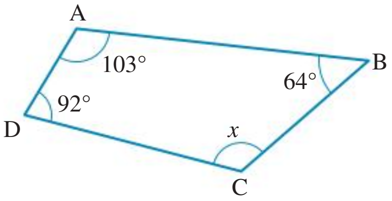
c. Calculate the value of y.

d. Determine the value of z.



8. **WE8** Calculate the sizes of the unknown angles in the following quadrilaterals.

a.



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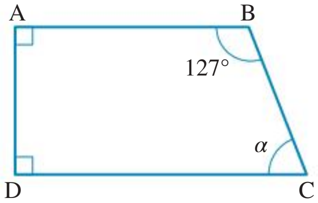
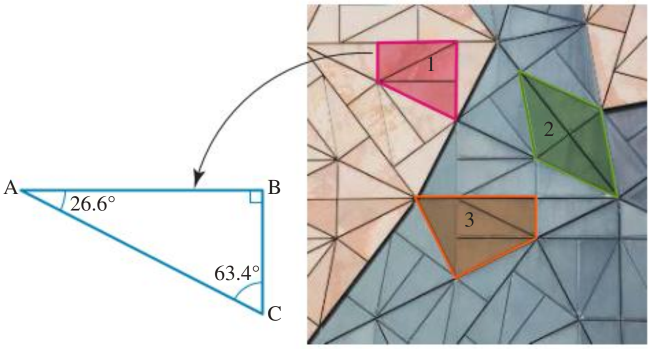
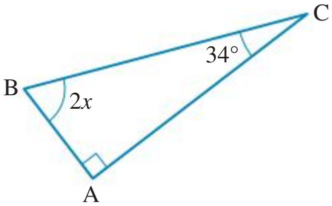


9. The triangles that form the façade of a building are right-angled triangles, with the two acute angles of approximate sizes 26.6° and 63.4°. Calculate the internal angles of each polygon and hence identify their names.

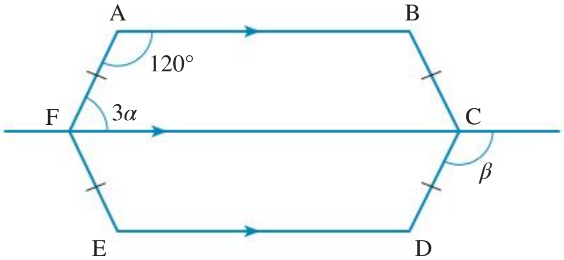


10. Calculate the value of x and the size of $\angle ABC$.



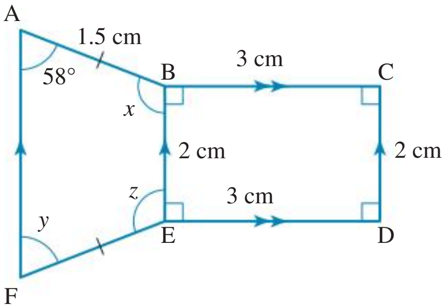
11. Consider the diagram shown. Trapezium ABCF is identical to trapezium CDEF.

a. Calculate the value of α and the size of $\angle AFE$.
b. Identify the type of the quadrilaterals ABCF and FCDE.
c. Identify the type of the polygon ABCDEF.
d. Calculate the value of β.



12. Consider the polygons shown in the diagram.

a. Identify the type of the quadrilateral ABEF and provide reasons.
b. Identify the type of the quadrilateral BCDE and provide reasons.
c. Calculate the value of the unknown pronumerals.
d. Calculate the size of the reflex angle $\angle FED$.



13. A parallelogram has vertices A, B, C, D, where AB || CD. If $\angle ABC = 120°$, determine the acute angle of the parallelogram.

14. An irregular pentagon has three known angles of magnitudes 50°, 70° and 80°. If the fourth angle is 10° less than the fifth angle, calculate the magnitude of the unknown angles.

Fully worked solutions for this chapter are available online.

LESSON

2.4 Three-dimensional objects

SYLLABUS LINKS

- Recognise the properties of prisms and pyramids, e.g. number of vertices, straight edges and flat faces.

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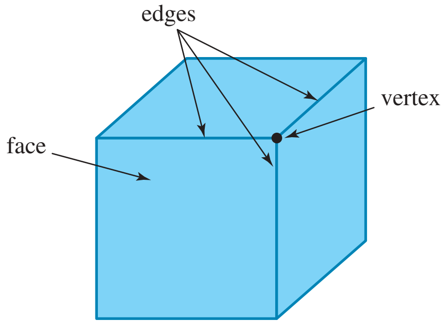
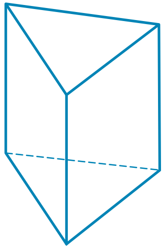
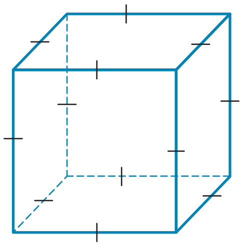
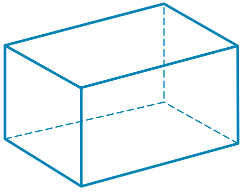


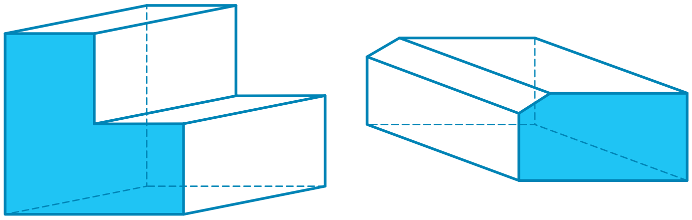
- Three-dimensional objects are recognised by their property of having length, width and depth (or height).
- A three-dimensional object with flat surfaces and straight edges enclosing a space is called a **polyhedron** (the plural is **polyhedra** or **polyhedrons**).
- They are also called **geometric solids** or **Platonic solids**.
- Each flat surface of a polyhedron is called a **face**.
- Each face is a polygon.
- Two faces of a polyhedron intersect to form an **edge**.
- Three faces intersect to form a **vertex** (plural vertices).
- Two basic types of polyhedrons are prisms and pyramids.

2.4.1 Prisms

- **Prisms** are solid shapes with identical opposite polygonal ends that are joined by straight edges.
- They are three-dimensional objects that can be cut into identical slices, called cross-sections.
- Some common prisms and their properties are as follows.

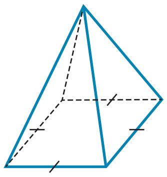
Polyhedron	Shape of the base	Number of faces	Number of edges	Number of vertices
Triangular prism 	Triangle	5	9	6
Cube 	Square	6	12	8
Rectangular prism 	Rectangle	6	12	8

- The following solids are also prisms. This time, though, the two bases are not basic polygons; they are composite shapes.

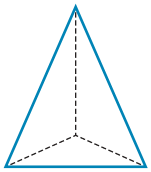


2.4.2 Pyramids

- A **pyramid** is a three-dimensional shape with a polygonal base and a single apex (point) that is not in the same plane as the base.
- A pyramid has one vertex at the apex, plus the vertices of the base. So, a square pyramid has 5 vertices — 4 at the base and 1 at the apex.
- A pyramid has triangular faces that meet at the apex. The number of triangular faces is equal to the number of edges on the base.
- A prism has two parallel, identical bases, while a pyramid has only one base with the sides meeting at a single point (the apex).
- Prisms are uniform in shape with a consistent cross-section, while pyramids 'narrow' as you move towards the apex.



Square-based pyramid

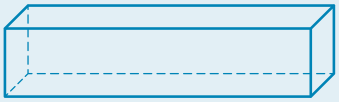


Triangle-based pyramid

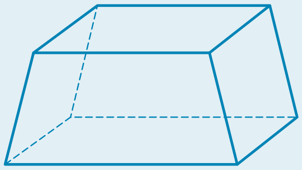
WORKED EXAMPLE 9 Recognising the properties of prisms and pyramids

Identify the name, the number of faces, vertices and edges of each of the following shapes.

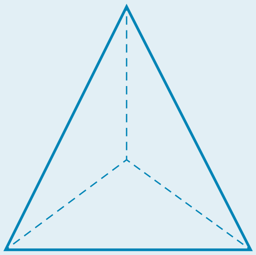
a.



b.



c.



THINK	WRITE
a. 1. Determine the shape of the base, identify whether it is a prism or a pyramid, and identify its name.	**a.** The base of the shape is a rectangle. The shape has a consistent cross-section and identical base on the opposite end. This is a rectangular prism.
2. Identify the number of faces.	6 faces
3. Identify the number of vertices.	8 vertices
4. Identify the number of edges.	12 edges

▶

b. 1. Determine the shape of the base, identify whether it is a prism or a pyramid, and identify its name.	**b.** The base of the shape is a trapezium. The shape has a consistent cross-section and identical base on the opposite end. This is a trapezoidal prism, or a quadrilateral prism.
2. Identify the number of faces.	6 faces
3. Identify the number of vertices.	8 vertices
4. Identify the number of edges.	12 edges
c. 1. Determine the shape of the base, identify whether it is a prism or a pyramid, and identify its name.	**c.** The base of the shape is a triangle. The shape has a single base, and the edges meet at a single apex. This is a triangular pyramid.
2. Identify the number of faces.	4 faces
3. Identify the number of vertices.	4 vertices
4. Identify the number of edges.	6 edges

Exercise 2.4 Three-dimensional objects

2.4 Quick quiz on

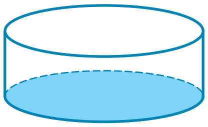
2.4 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14	N/A	N/A

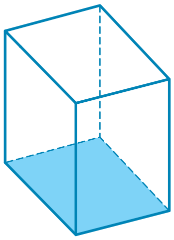
Simple familiar

1. Identify the three-dimensional shapes and decide whether they are prisms.

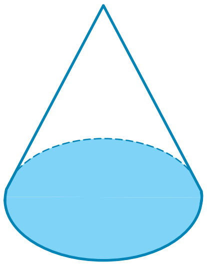
a.



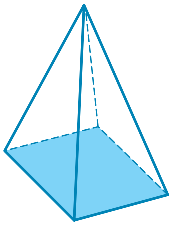
b.



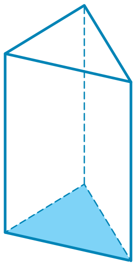
c.



d.

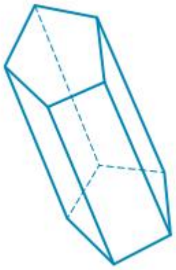


e.



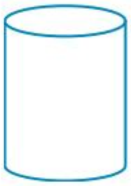
2. For each of the following three-dimensional shapes, identify whether they are polyhedrons.

a.

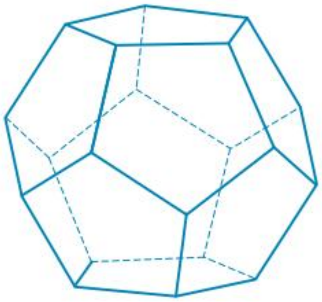


b.

c.

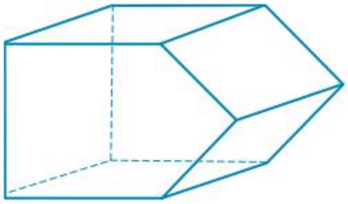


d.

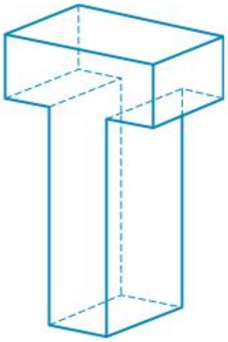


3. Identify the number of faces, edges and vertices of each object.

a.



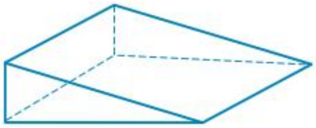
b.



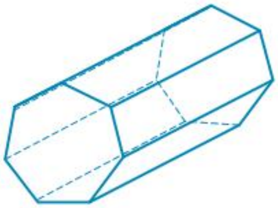
4. **WE9** The prism shown has edges of equal length. Identify its name, and its number of faces, vertices and edges.

5. **WE9** Identify the name, and the number of faces, vertices and edges of each of the following shapes.

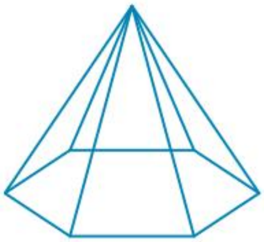
a.



b.



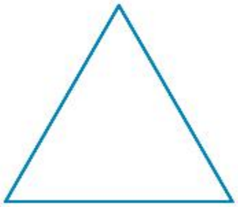
c.



6. Sketch a prism using each of the following shapes as the base.

a.

b.



7. Sketch a prism using each of the following shapes as the base.

a.

b.

c.

8. Observe the following objects and identify the three-dimensional shape that most accurately matches it.

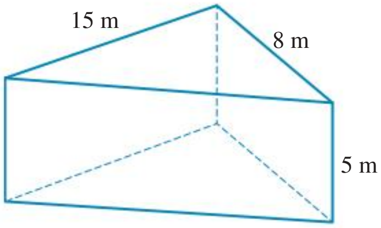
a.

b.

c.

d.

9. A toybox is displayed in an enclosed cabinet in the shape of a triangular-based prism, as shown.



a. Identify how many vertices the cabinet has.
b. Identify how many faces the cabinet has.

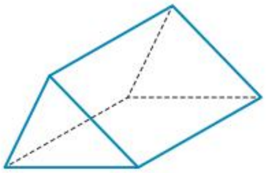
10. Identify the missing words and or numbers that accurately complete these sentences.

a. A square-based pyramid has 5 _____ and ___ vertices.
b. A rectangular prism has _____ edges and 6 _____.
c. A pentagonal prism has _____edges and _____ vertices.

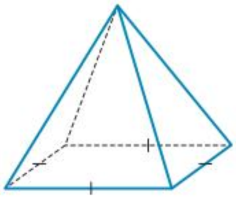
11. A prism has 8 faces, 12 vertices and 18 edges. Determine the type of the prism.

12. Classify each of the following solids as either a prism or pyramid.

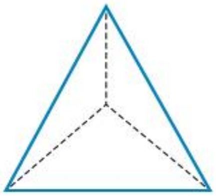
a.



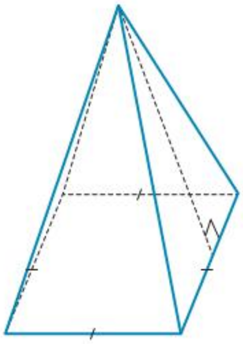
b.



c.

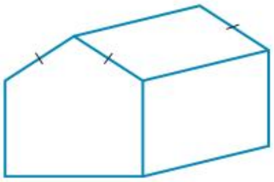


13. Use the solid below to answer the following questions.



a. Identify the type of solid shown.
b. Identify the number of faces.
c. Identify the number of vertices.
d. Identify the number of edges.

14. Identify the number of faces, edges and vertices in the canvas tent shown below.



Fully worked solutions for this chapter are available online.

LESSON
2.5 Two-dimensional representations of three-dimensional objects [complex]

SYLLABUS LINKS

- Interpret different forms of two-dimensional representations of three-dimensional objects, including nets of prisms and pyramids [complex].

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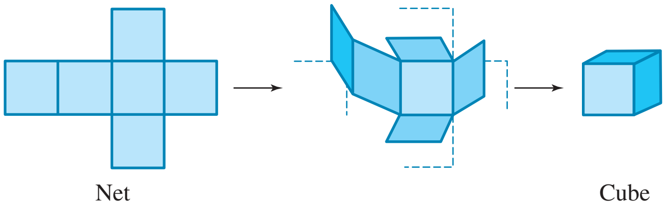
2.5.1 Nets of three-dimensional objects

- Prisms and pyramids can be drawn as a two-dimensional representation.
- This shape is called a **net**.
- In other words, a net is a two-dimensional figure that can be folded to form a three-dimensional object.

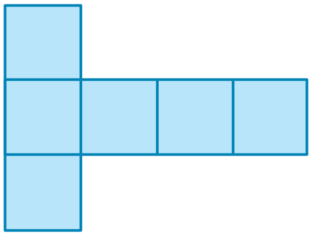
2.5.2 Nets of prisms

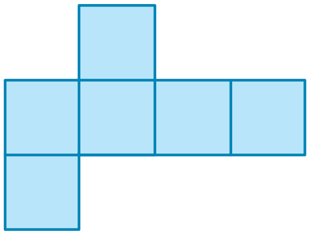
Net of a cube

- The diagram displays the net of a cube. Notice that when folded, the faces do not overlap.



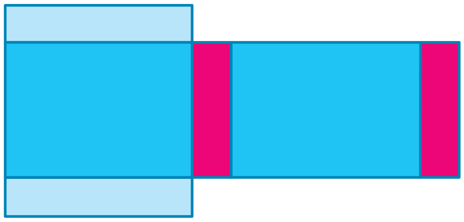
- The following nets are also nets of a cube.

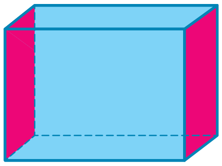




Net of a rectangular prism

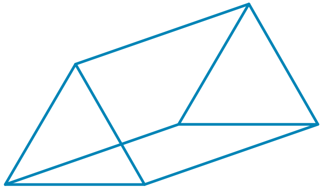
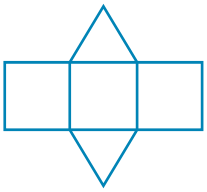
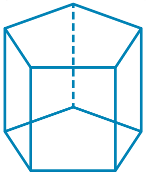
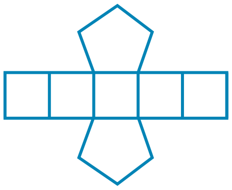
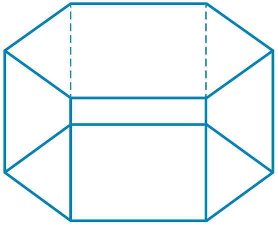
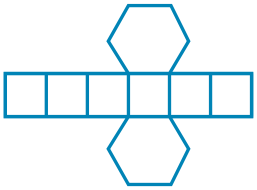
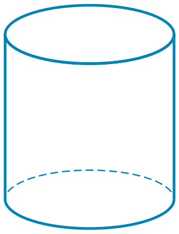
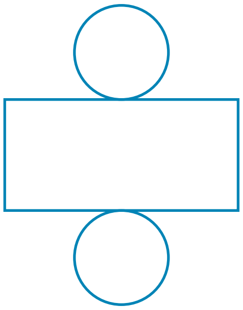
- The following net can be folded to form a rectangular prism.
- Notice that the top and the bottom faces of the rectangular prism are on either side of the net.
- These two faces cannot be drawn on the same side of the net.
- The left and right faces alternate with the back and front faces.





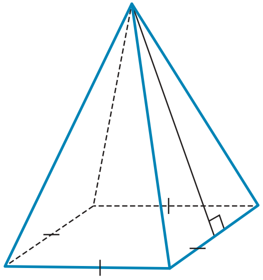
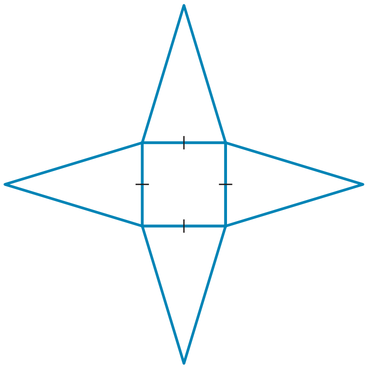
Nets of other prisms

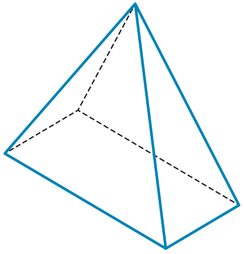
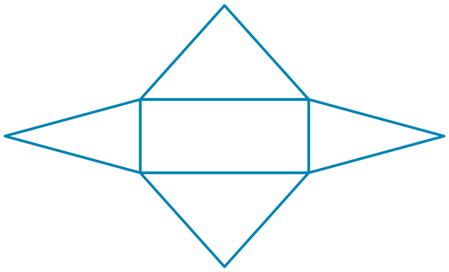
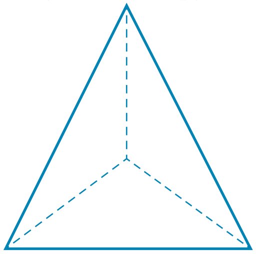
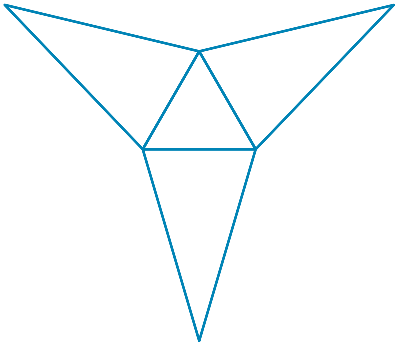
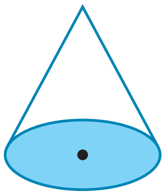
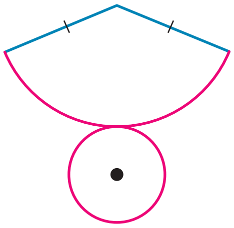
- A net can be drawn for any type of prism.
- Below are some examples:

Prism	Net
Triangular-based prism 	
Pentagonal-based prism 	
Hexagonal-based prism 	
Cylinder (circle-based prism) 	

2.5.3 Nets of pyramids

- The nets of some pyramids, including a cone, are drawn in the table below.

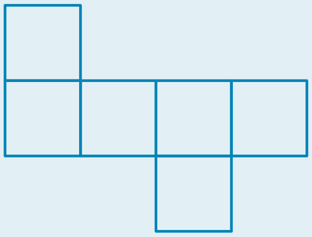
Pyramid	Net
Square-based pyramid 	

Pyramid	Net
Rectangular-based pyramid 	
Triangular-based pyramid 	
Cone 	

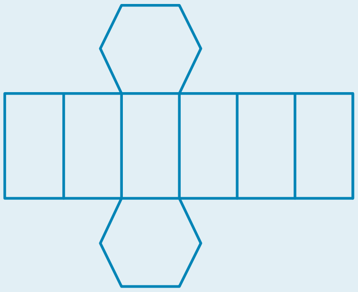
WORKED EXAMPLE 10 Identifying the nets of prisms and pyramids

Identify the name of the three-dimensional object that can be formed by folding each of the following nets.

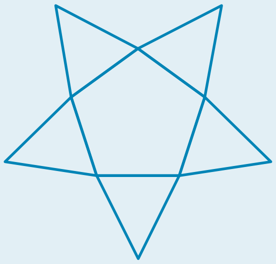
a.



b.



c.



THINK	**WRITE**
a. 1. Identify any features of the two-dimensional shapes.	**a.** All the shapes are equal-sized squares.
2. Count the number of faces and check for any possible overlaps or missing faces.	There are 6 faces. There are no overlaps. There are no missing faces.
3. Identify the name of the three-dimensional object.	The object is a cube.
b. 1. Identify any features of the two-dimensional shapes.	**b.** There are six identical rectangles and two identical regular hexagons.
2. Count the number of faces and check for any possible overlaps or missing faces.	There are 8 faces. There are no overlaps. There are no missing faces.
3. Identify the name of the base and the name of the three-dimensional object.	The base is a regular hexagon. The object is a hexagonal prism.
c. 1. Identify any features of the two-dimensional shapes.	**c.** All triangles are the same size and fold up to an apex. The base is a regular pentagon.
2. Count the number of faces and check for any possible overlaps or missing faces.	There are 6 faces and no missing faces or overlaps.
3. Identify the name of the three-dimensional object.	The object is a pentagonal pyramid.

2.5.4 Three-dimensional objects and perspective drawing

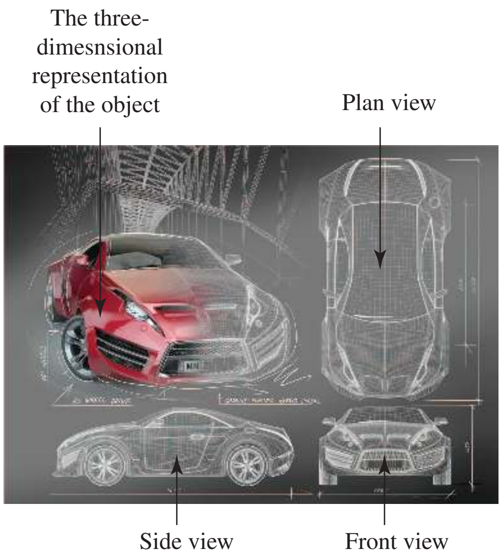
- Designers, engineers and architects use two-dimensional drawings to represent three-dimensional objects.
- It is important to understand how to construct, read and interpret such diagrams. Three-dimensional objects can be represented by an **isometric drawing**.

Plans and views

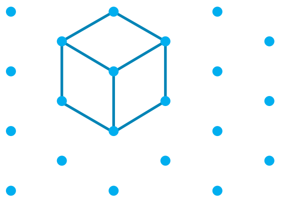
int-4149

In the diagram shown there are four views of a car:

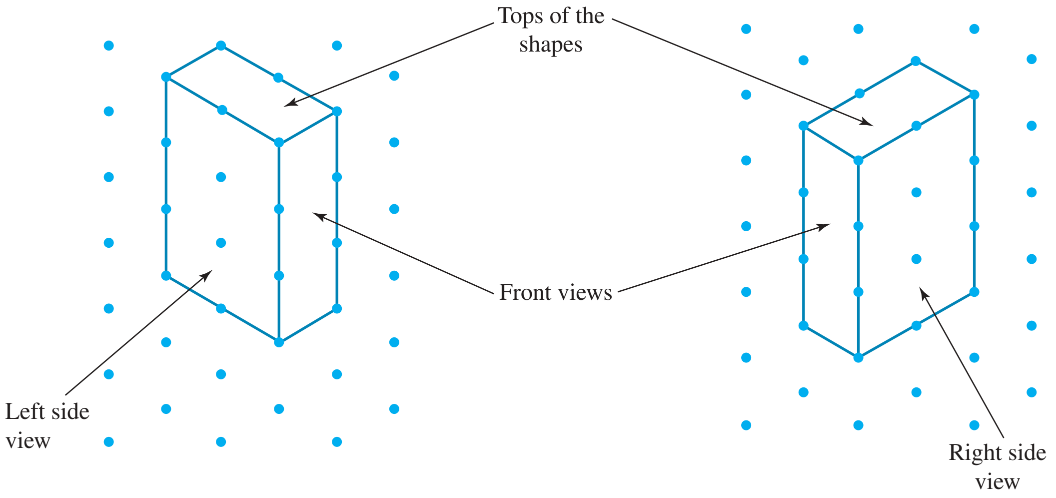
- the **plan view** is the diagram of the object as seen looking straight at the object from above
- the **side view**, also called **side elevation**, is the diagram of the object as seen looking straight at the object from one side, left or right
- the **front view**, also called **front elevation**, is the diagram of the object as seen looking straight at the object from the front
- the three-dimensional representation of the object.



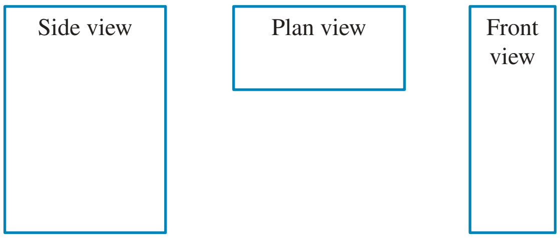
Isometric drawing



- Isometric drawings are created on **isometric paper**, which uses dots or lines forming equilateral triangles.
- This paper helps maintain parallel lines of objects, while right angles appear as acute or obtuse angles.
- The simplest 3-dimensional object, the cube, is shown on the right as an isometric representation.
- Different perspectives, such as left and right views, are demonstrated in the diagrams on the right for a rectangular prism.
- Isometric paper is essential for accurately representing three-dimensional objects.



- The three views of the rectangular prism are shown. Notice that the angles are 90° angles although they have different measurements on the isometric drawing.



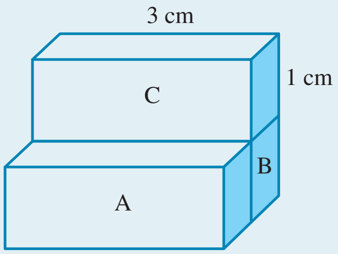
int-4151

WORKED EXAMPLE 11 Sketching three-dimensional objects

The three-dimensional object shown in the diagram is made of three rectangular prisms. Each rectangular prism is 3 cm long and has a square base with side length of 1 cm.

a. Sketch the object on isometric paper.

b. Sketch the front, side and plan views of the object.

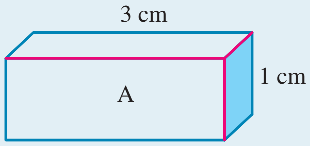


THINK

WRITE

a. 1. Sketch the first rectangular prism by first drawing the corner at the front, considering the lengths of its dimensions.

a.

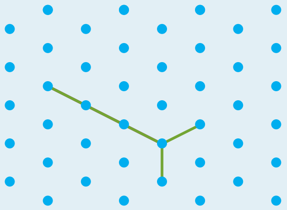


Rectangular prism A has:

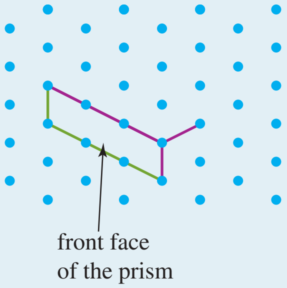
- width = 1 unit
- height = 1 unit
- length = 3 units.

Note: The height and the width are equal in length so they are represented by lines of equal lengths.

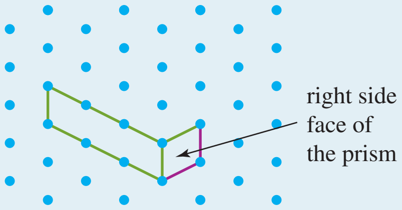
The length is three times longer than the other dimensions. The proportions between the sides of the original object have to be maintained.



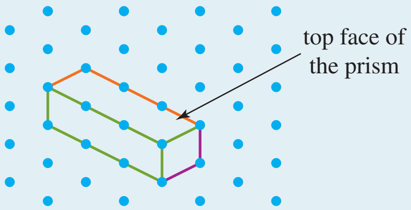
2. Complete the front face of the prism by drawing parallel lines to the existing lines.



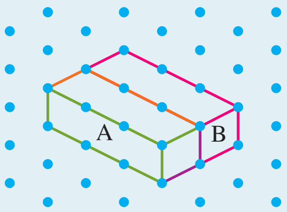
3. Complete the right side of the prism by drawing parallel lines to the existing lines.



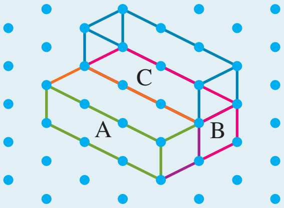
4. Complete the top side of the rectangle by drawing parallel lines to the existing lines.



5. Sketch the second prism by drawing parallel lines to the existing lines.

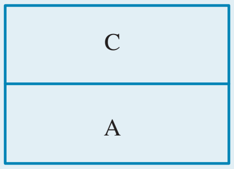


6. Sketch the third prism by drawing parallel lines to the existing lines.

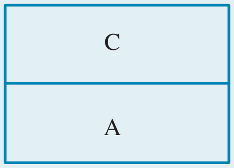


b. 1. Sketch the plan view. Looking at the object from above, only the tops of prisms A and C can be seen.

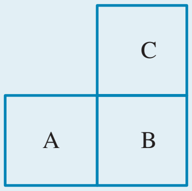
b.



2. Sketch the front view. Looking at the object from the front, only the front faces of prisms A and C can be seen.



3. Sketch the side view. Looking at the object from the side, the sides of all three prisms can be seen.



Exercise 2.5 Two-dimensional representations of three-dimensional objects [complex]

learn**on**

2.5 Quick quiz on

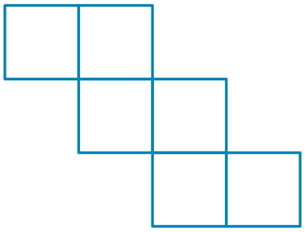
2.5 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
N/A	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15	N/A

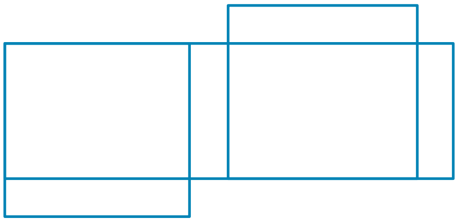
Complex familiar

1. **WE10ab** Identify the name of the three-dimensional object that can be formed by folding each of the following nets.

a.

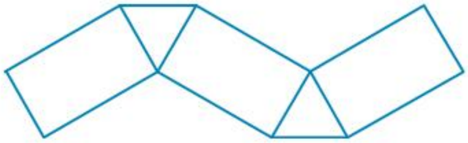


b.

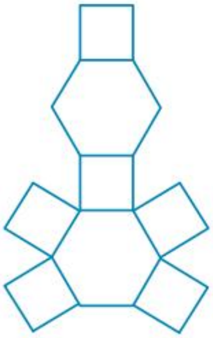


2. Determine the shape that can be constructed using each of the following nets.

a.

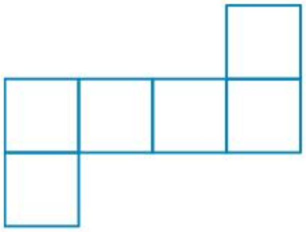


b.

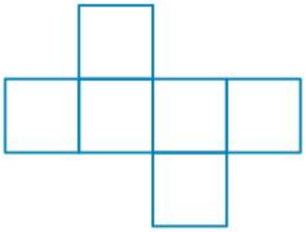


3. The following nets are of a cube. Determine whether the statement is True or False for each net.

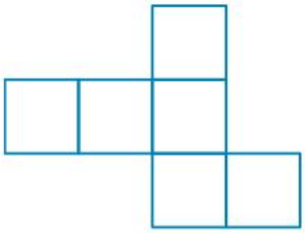
a.



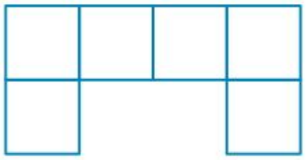
b.



c.

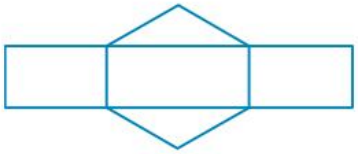


d.

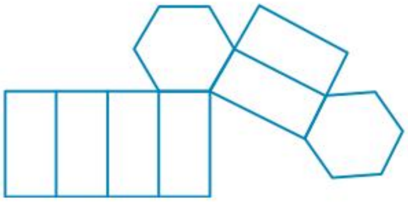


4. Identify the three-dimensional object that can be formed by folding each of the following nets.

a.

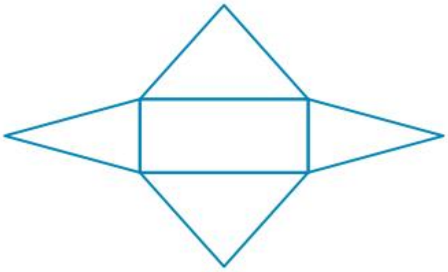


b.

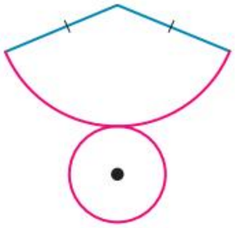


5. **WE10c** Identify the name of the three-dimensional object that can be formed by folding each of the following nets.

a.

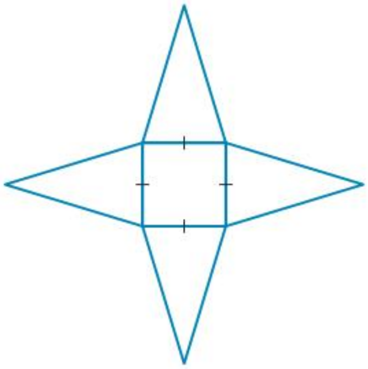


b.

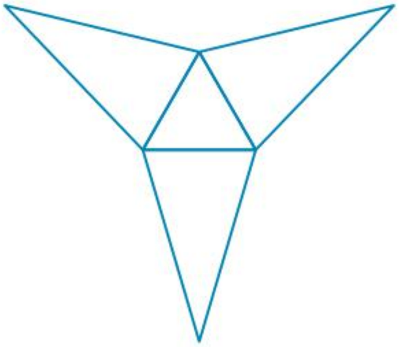


6. Identify the name of the three-dimensional object that can be formed by folding each of the following nets.

a



b

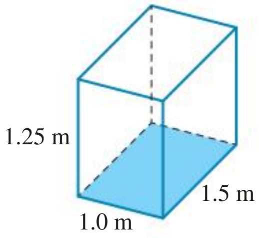


7. Describe the type and number of shapes that exist in the net of a hexagonal-based prism.

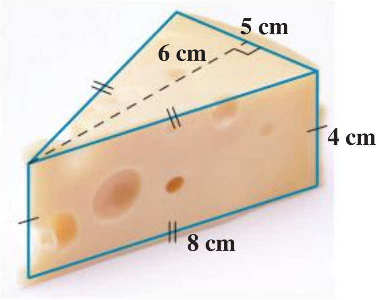
8. Describe the type and number of shapes that exist in the net of a prism with a cross-section that is an equilateral triangle.

9. Describe the type and number of shapes that exist in the net of a square-based prism.

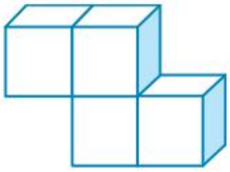
10. Draw the net of the rectangular prism shown.



11. Sketch the net of the following solid.

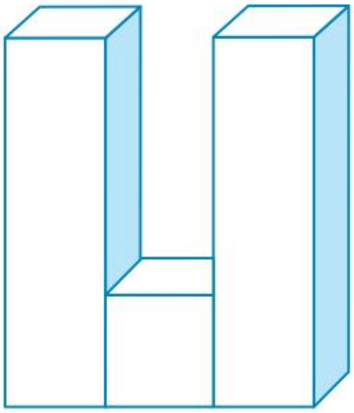


12. **WE11** The three-dimensional object shown in the diagram is made of four cubes of side length 1 cm.



a. Sketch the object on isometric paper.
b. Sketch the front, side and plan views of the object.

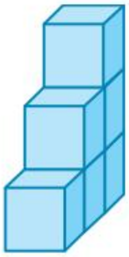
13. Consider the three-dimensional object shown in the diagram.



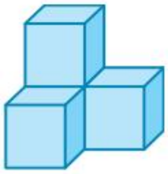
a. Sketch the object on isometric paper.
b. Sketch the front, side and plan views of the object.

14. Sketch an isometric diagram of each of the following three-dimensional objects.

a.

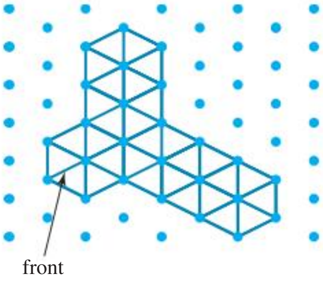


b.

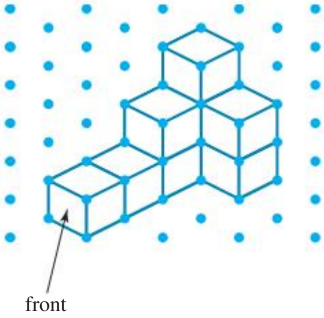


15. Sketch the front view, side view and plan view of the following isometric drawings.

a.



b.



Fully worked solutions for this chapter are available online.

LESSON
2.6 Review

2.6.1 Summary

Hey students! Now that it's time to revise this chapter, go online to:

- Access the chapter summary
- Review your results
- Watch teacher-led videos
- Practise questions with immediate feedback

Find all this and MORE in jacPLUS

2.6 Exercise

learnon

2.6 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14	15, 16, 17	18, 19, 20

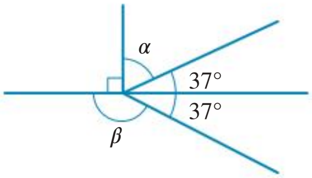
These questions are even better in jacPLUS!

- Receive immediate feedback
- Access sample responses
- Track results and progress

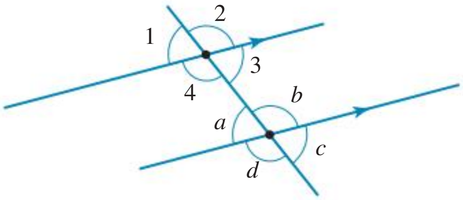
Find all this and MORE in jacPLUS

Simple familiar

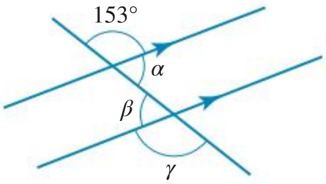
1. Calculate the size of angles α and β respectively.



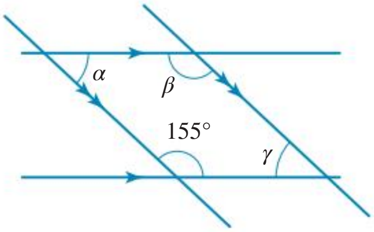
2. Identify the angle that is co-interior with angle b.



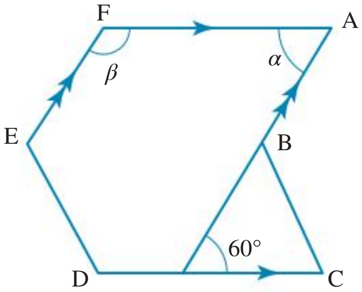
3. Determine the sizes of the unknown angles in the diagram.



4. Determine the values of the pronumerals in the diagram.

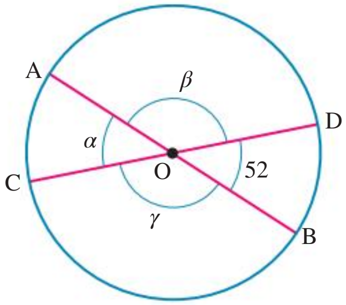


5. Determine the values of the pronumerals in the diagram.



6. Using the properties of angles and circles, determine the value of the following measurements if CD = 8 cm.

a. α
b. β
c. γ
d. AB
e. OD



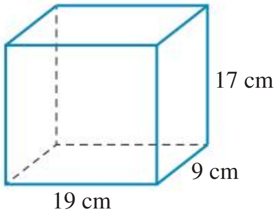
7. Complete the following description of a quadrilateral.
A quadrilateral is a four-sided __________. The sum of all interior angles is ______ degrees.

8. Identify the type of plane shape that matches the given description.

a. This shape has 5 sides, but they all have different lengths, and its internal angles are not identical.
b. This shape has 3 sides that all have the same length. Its internal angles have the exact same size (magnitude).

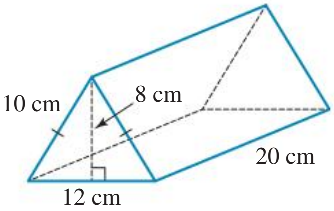
9. An apprentice is building a cabinet with the dimensions shown.

a. Identify the name of the three-dimensional solid.
b. Identify the number of vertices in the cabinet.
c. Identify the number of faces in the cabinet.
d. Identify the number of edges in the cabinet.

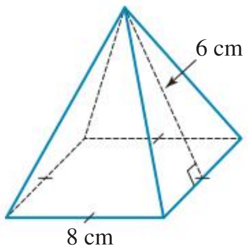


10. The design of a solid is shown below.

a. Identify the name of the three-dimensional solid.
b. Identify the number of vertices in the solid.
c. Identify the number of faces in the solid.
d. Identify the number of edges in the solid.



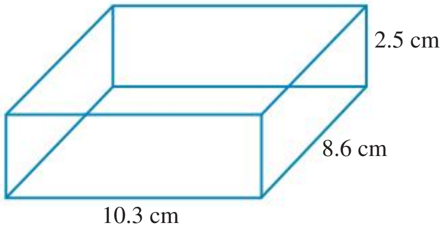
11. Consider the following three-dimensional shape.



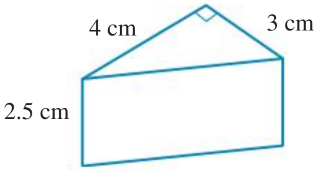
a. Identify the name of the three-dimensional solid.
b. Identify the number of vertices.
c. Identify the number of faces.
d. Identify the number of edges.

12. A solid wooden block has the following dimensions.

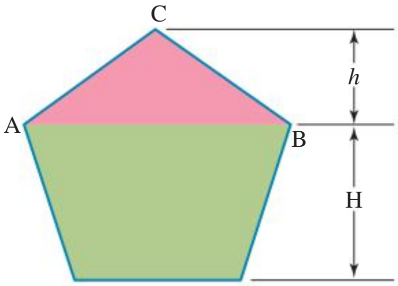
a. Identify the name of the three-dimensional solid.
b. Identify how many faces and vertices the solid has.



13. Identify the name, and the number of faces, vertices, and edges of the following prism.



14. A courtyard is in the form of a regular pentagon as shown in the diagram. The side length of the pentagon is 7.5 m.

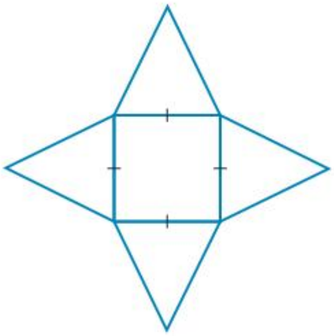


a. Calculate the size of the angle ABC given the sum of the interior angles in a pentagon is 540°.
b. A small brick fence is to be built around four sides of the courtyard. Calculate the total cost of the fence if one brick costs $0.95 and 25 bricks are used to build 1 m of fence.

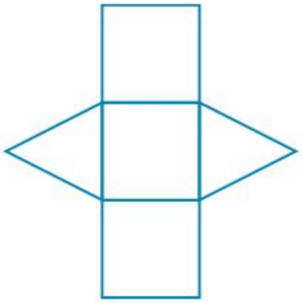
Complex familiar

15. Identify the solids from the nets below.

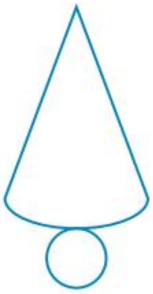
a.



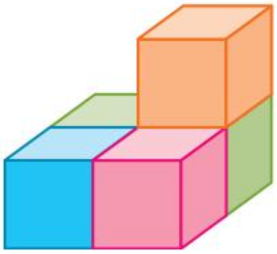
b.



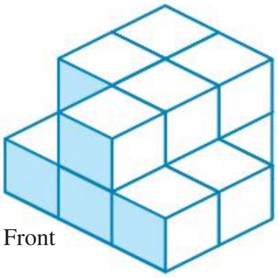
c.



16. Create a plan-view sketch of the children's blocks shown.

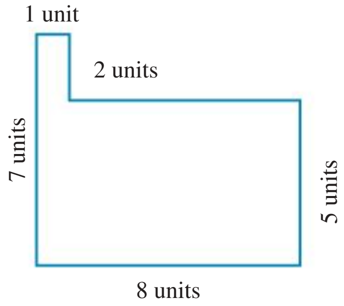


17. Draw the front, right side and top view of the following object.

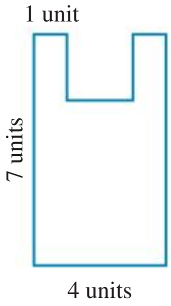


Complex unfamiliar

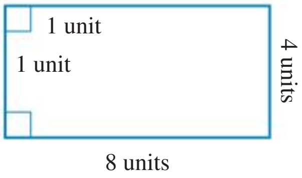
18. Using isometric paper, draw a building with the following right side, front and top views.



Right side

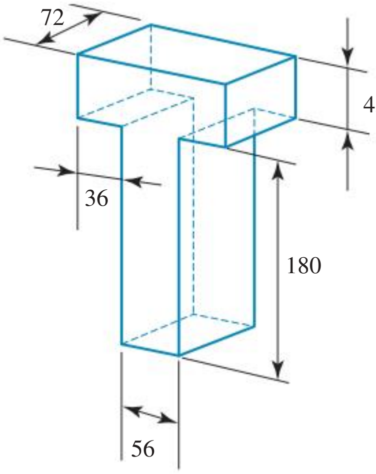


Front

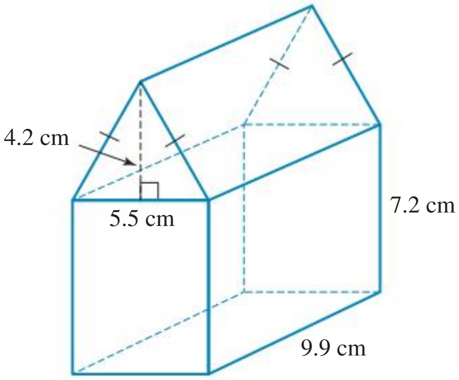


Top

19. Construct a clearly labelled plan view, front view and side view of the letter T shown in the diagram.



20. A student is designing a doll house with the following dimensions.



Sketch a net of the doll house and determine how many more vertices there are compared to faces.

Fully worked solutions for this chapter are available online.

Answers

Chapter 2 Geometry

2.2 Review of angles

2.2 Exercise

1. a. i. $\angle CAT$ or $\angle TAC$ **ii.** $\angle A$
$\angle WAS$ or $\angle SAW$ $\angle A$
$\angle FAT$ or $\angle TAF$ $\angle A$

b. Reflex
Obtuse
Acute

2. See the table at the bottom of the page.*

3. a. $\angle NMP$, $\angle MNP$, $\angle MPN$, $\angle NOP$, $\angle OPN$, $\angle ONP$

b. i. Right angle **ii.** Acute angle
iii. Acute angle **iv.** Right angle
v. Acute angle **vi.** Acute angle

c. b and c, e and f

d. b and e, c and f

e. $a = 90°$, $b = 30°$, $c = 60°$, $d = 90°$, $f = 60°$

4. a. b **b.** e **c.** g
d. d **e.** e **f.** h

5. a. Sample responses can be found in the worked solutions in the online resources.

b. $\angle XOV$ and $\angle YOW$

6. a. $a = 38°$

b. $b = 110°$

c. $c = 22°$

d. $d = 19°$

e. $e = 39°$, $g = 141°$, $f = 141°$

f. $i = 13°$, $h = 15°$, $k = 152°$, $j = 152°$

7. a. $b = 65°$, $c = 10°$, $d = 50°$, $e = 130°$

b. $f = 60°$, $g = 60°$, $i = 36°$, $h = 20°$

8. a. $a = 310°$ **b.** $b = 222°$ **c.** $c = 103°$
d. $d = 10°$ **e.** $e = 82°$ **f.** $f = 73.3°$

9. a. $a = 22.5°$

b. $c = 40°$, $b = 32.5°$, $d = 42.5°$, $e = 42.5°$, $f = 65°$

10. a. 73° **b.** 107° **c.** 45°
d. 89° **e.** 80° **f.** 60°

11. a. $x = 40°$
$y = 80°$

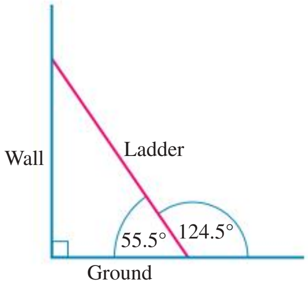
b. $q = 45°$
$p = 79°$

c. $m = 128°$
$n = 52°$

d. $x = 96°$
$v = 42°$
$u = 42°$

12. $x = 38°$

13. a.



b. A scalene triangle

c. 124.5°

14. $y = 122°$

15. $a = 23.3°$
$b = 5°$
$z = 45°$
$m = 45°$
$n = 110°$
$x = 155°$
$t = 50°$

2.3 Two-dimensional geometric shapes

2.3 Exercise

1. Top row: regular octagon, equilateral triangle, circle
Second row: regular hexagon, square, regular pentagon

2. $\angle E = 109°$
ABCDE is an irregular pentagon.

3. $\alpha = 104°$
$\gamma = 284°$
$\beta = 12°$
ABCDEF is an irregular hexagon.

4. $\angle DOC = 100°$

5. a. $x = 55°$
Acute scalene triangle

b. $\alpha = 36°$
Since two angles are equal, this is an obtuse isosceles triangle.

6. ΔABC is an isosceles triangle. ΔABD is a right-angled triangle. ΔACD is an equilateral triangle.
$\beta = 30°$, $\alpha = 120°$, $\gamma = 60°$, $\theta = 60°$, $\delta = 60°$

7. a. Isosceles triangle **b.** $x = 48°$
c. $y = 132°$ **d.** $z = 48°$

8. a. $x = 101°$ **b.** $\alpha = 53°$

---

*2.

Acute angles	Obtuse angles	Right angles	Reflex angles
2°, 5°, 14°, 23°, 45°, 55°, 69°, 78°, 80°, 89°	92°, 100°, 110°, 122°, 145°, 154°, 160°, 160°, 179°	90°	181°, 190°, 192°, 270°, 300°, 359°

9. Polygon 1: trapezium
Angle 1: 90°
Angle 2: 90°
Angle 3: 63.4°
Angle 4: 116.6°
Polygon 2: rhombus
Angle 1: 53.2°
Angle 2: 53.2°
Angle 3: 126.8°
Angle 4: 126.8°
Polygon 3: quadrilateral
Angle 1: 63.4°
Angle 2: 90°
Angle 3: 116.6°
Angle 4: 90°

10. $x = 28°$
$\angle ABC = 56°$

11. **a.** $\alpha = 20°$
$\angle AFE = 120°$
b. Isosceles trapezium
c. Hexagon
d. $\beta = 120°$

12. **a.** ABEF is an isosceles trapezium, $AF \parallel BE$, $AB = EF$
b. BCDE is a rectangle because $BC \underset{=}{\parallel} DE$, $BE \underset{=}{\parallel} CD$
c. $x = 122°$, $y = 58°$, $z = 122°$
d. $\angle FED = 148°$

13. 60°

14. 165°, 175°

2.4 Three-dimensional objects

2.4 Exercise

1. **a.** Cylinder; not a prism
b. Rectangular prism; a prism
c. Cone; not a prism
d. Square-based pyramid; not a prism
e. Triangular prism; a prism

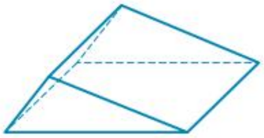
2. **a.** Polyhedron **b.** Polyhedron
c. Not a polyhedron **d.** Polyhedron

3. **a.** 7 faces, 15 edges, 10 vertices
b. 10 faces, 24 edges, 16 vertices

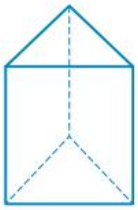
4. Cube: 6 faces, 8 vertices, 12 edges

5. **a.** Triangular prism: 5 faces, 6 vertices, 9 edges
b. Heptagonal prism: 9 faces, 14 vertices, 21 edges
c. Hexagonal pyramid: 7 faces, 7 vertices, 12 edges

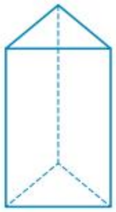
6. **a.**



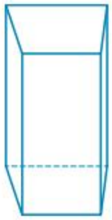
b.



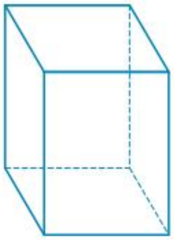
7. **a.**



b.



c.



8. **a.** Rectangular prism **b.** Square-based pyramid
c. Triangular-based prism **d.** Cylinder

9. **a.** 6 vertices **b.** 5 faces

10. **a.** faces, five **b.** twelve, faces **c.** fifteen, ten

11. Hexagonal-based prism

12. **a.** Prism **b.** Pyramid **c.** Pyramid

13. **a.** Square-based pyramid **b.** 5
c. 5 **d.** 8

14. 7 faces, 15 edges, 10 vertices

2.5 Two-dimensional representations of three-dimensional objects [complex]

2.5 Exercise

1. **a.** Cube **b.** Rectangular prism

2. **a.** Triangular prism
b. Hexagonal prism

3. **a.** True **b.** True
c. True **d.** False

4. **a.** Triangular prism
b. Hexagonal prism

5. **a.** Rectangular-based pyramid
b. Cone

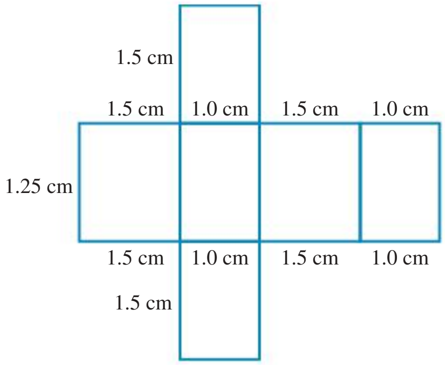
6. **a.** Square-based pyramid
b. Triangular-based pyramid

7. Two hexagons, six rectangles, and eight shapes in total

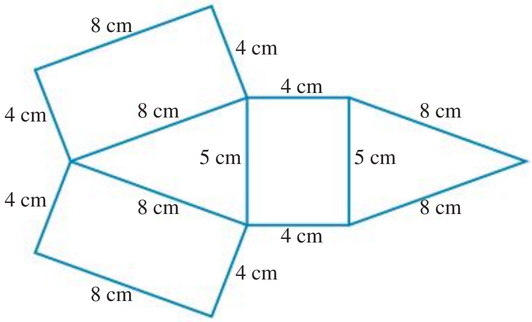
8. Two triangles, three rectangles, and five shapes in total

9. Two squares, four rectangles, and six shapes in total

10.

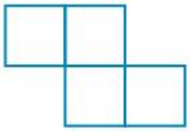


11.



12. a.

b.



Front view

Side view

Plan view

13. a.

b.

Front view

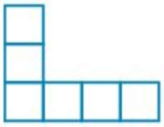
Side view

Plan view

14. a.

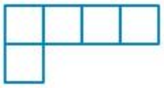
b.

15. a.



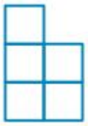
Front view

Side view

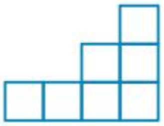


Plan view

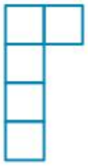
b.



Front view



Side view



Plan view

2.6 Review

2.6 Exercise

1. $\alpha + 37° = 90°$

$\alpha = 53°$

$\beta + 37° = 180°$

$\beta = 143°$

2. 3

3. $\alpha = 27°$ {supplementary with 153°},
$\beta = 27°$ {alternative with α},
$\gamma = 153°$ {supplementary with β}

4. $\alpha = 25°$ {co-interior with 155°},
$\beta = 155°$ {co-interior with α},
$\gamma = 25°$ {co-interior with β}

5. $\alpha = 60°$

$\alpha + \beta = 180°$

$60° + \beta = 180°$

$\beta = 120°$

6. **a.** 52 **b.** 128 **c.** 128
d. 8 cm **e.** 4 cm

7. Polygon; 360

8. **a.** Irregular pentagon

b. Equilateral triangle

9. **a.** Rectangular prism

b. 8

c. 6

d. 12

10. **a.** Triangular-based prism

b. 6

c. 5

d. 9

11. **a.** Square-based pyramid

b. 5

c. 5

d. 8

12. **a.** Rectangular prism **b.** 6 faces, 8 vertices

13. Triangular-based prism. It contains 8 faces, 6 vertices and 9 edges.

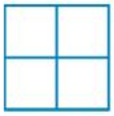
14. **a.** 36° **b.** \$712.50

15. **a.** Square-based prism

b. Triangular-based prism

c. Cone

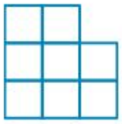
16.



17.

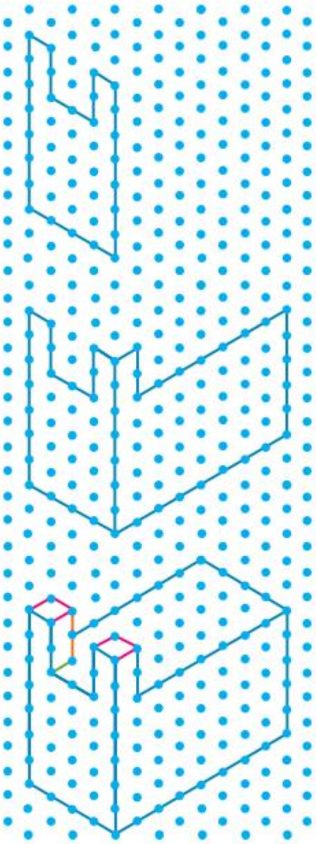
Front view

Right view

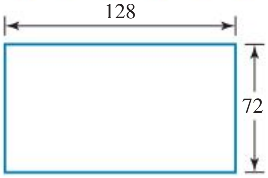


Top view

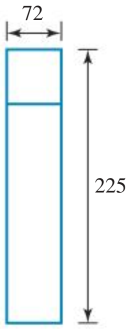
18.



19.

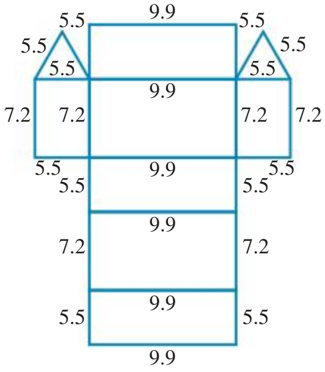


Plan view



Side view

20.



1 more vertice than face.

UNIT 3 Measurement, scales and chance
TOPIC 1 Measurement

3 Linear and area measure

LESSON SEQUENCE

Fully worked solutions for this chapter are available online.

on Resources

Solutions	Solutions — Chapter 3
Digital documents	Learning matrix — Chapter 3 Quick quizzes — Chapter 3 Chapter summary — Chapter 3

LESSON
3.1 Overview

Hey students! Bring these pages to life online

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Engage with interactivities

Answer questions and check results

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3.1.1 Introduction

Understanding and mastering the use of metric units of length and area, their conversions and their appropriate applications are crucial skills in many fields. Skills in estimating lengths and areas, and calculating perimeters and areas of various shapes — from standard polygons to complex composite shapes — are fundamental to disciplines such as construction, architecture and engineering. These skills enable accurate material estimation, cost calculation and project planning.

Furthermore, knowing how to compute the surface areas of prisms, cylinders, pyramids, cones and spheres is essential for designing and assessing structures and objects. In everyday life, these skills aid in tasks ranging from home improvement projects to understanding space utilisation. Mastery of these mathematical techniques not only enhances problem-solving abilities but also builds a foundation for more advanced studies and professional applications, fostering confidence and precision in tackling complex real-world challenges.

3.1.2 Syllabus links

Lesson	Lesson title	Syllabus links
3.2	**Units of length and estimation**	○ Estimate lengths. ○ Use metric units of length (millimetres, centimetres, metres, kilometres), their abbreviations $(\text{mm}, \text{cm}, \text{m}, \text{km})$, conversions between them, and appropriate levels of accuracy and choice of units.
3.3	**Perimeters of familiar and composite shapes**	○ Calculate perimeters of standard polygons, circles and arc lengths. • circle: $C = 2\pi r$ where C is circumference and r is radius • arc length: $l = \frac{\theta}{180}\pi r$ where l is arc length, θ is central angle and r is radius. ○ Calculate perimeters of composite shapes [complex].
3.4	**Units of area and estimation**	○ Estimate areas of different shapes. ○ Use metric units of area (square millimetres, square centimetres, square metres, square kilometres, hectares), their abbreviations $(\text{mm}^2,\ \text{cm}^2,\ \text{m}^2,\ \text{km}^2,\ \text{ha})$, conversions between them and appropriate choices of units.
3.5	**Areas of standard shapes**	○ Calculate areas of standard shapes, triangles, parallelograms and circles. • triangle: $A = \frac{1}{2}bh$ where b is base length and h is perpendicular height • parallelogram: $A = bh$ where b is base length and h is perpendicular height • circle: $A = \pi r^2$ where r is radius. ○ Calculate areas of trapeziums and sectors [complex]. • trapezium: $A = \frac{1}{2}(a + b)h$ where a and b are parallel lengths and h is perpendicular height • sector: $A = \frac{\theta}{360}\pi r^2$ where θ is central angle and r is radius.
3.6	**Areas of composite figures [complex]**	○ Calculate areas of composite figures by decomposing them into standard shapes [complex].
3.7	**Surface areas of prisms and cylinders [complex]**	○ Calculate surface areas of prisms and cylinders [complex]. • cylinder: $S = 2\pi rh + 2\pi r^2$ where r is radius and h is perpendicular height.
3.8	**Surface areas of pyramids and cones [complex]**	○ Calculate surface areas of pyramids and cones [complex]. • cone: $S = \pi rs + \pi r^2$ where r is radius and s is slant height.
3.9	**Surface areas of composite shapes and spheres [complex]**	○ Calculate surface areas of composite shapes and spheres [complex]. • sphere: $S = 4\pi r^2$ where r is radius.

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LESSON
3.2 Units of length and estimation

SYLLABUS LINKS

- Estimate lengths.
- Use metric units of length (millimetres, centimetres, metres, kilometres), their abbreviations (mm, cm, m, km), conversions between them, and appropriate levels of accuracy and choice of units.

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3.2.1 Units of length

- Units of length are used to describe the dimensions of an object, such as its length, width and height, or the distance between two points.
- The metric system we use is a base 10 system, with its base unit being the metre.
- All other units of length in the metric system are related to the metre by powers of 10.
- The most common of these are the millimetre (mm), centimetre (cm), metre (m) and kilometre (km).
- They are related in the following way.

Length conversions

10 millimetres = 1 centimetre

100 centimetres = 1 metre

1000 metres = 1 kilometre

Millimetre (mm)	Centimetre (cm)	Metre (m)	Kilometre (km)
			
1 mm is approximately the width of a microchip pin.	1 cm is approximately the width of a person's finger.	1 m is approximately the length of an adult stride.	1 km is approximately the length of Surfers Paradise Beach.

- When measuring lengths, you should use an appropriate unit, which is one that gives a reasonable value (not too large and not too small).

WORKED EXAMPLE 1 Estimating lengths using appropriate units

Select the most suitable metric unit of length to measure the following.

a. Width of a school desk
b. Length of Queensland's Bruce Highway
c. Height of a Year 12 student
d. Width of an ant

THINK	WRITE
a. The width of a school desk would be up to 2 m depending on the shape. For more accuracy, it would most suitable to be measured in centimetres.	**a.** Centimetres (cm)
b. The length of Queensland's Bruce Highway is about 1700 km long. It would be most suitable to be measured in kilometres.	**b.** Kilometres (km)
c. The height of a Year 12 student would be between 140 and 180 cm on average. It would be most suitable to be measured in centimetres.	**c.** Centimetres (cm)
d. The width of an ant is between 1 and 3 mm. It would be most suitable to be measured in millimetres.	**d.** Millimetres (mm)

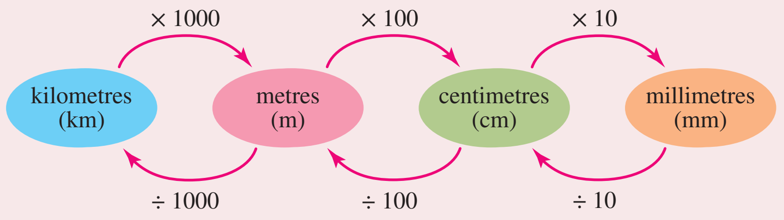
3.2.2 Converting between units of length

int-4011

- The relationship between the metric units of length can be used to convert a measurement from one unit to another.

Converting metric units of length

The following diagram shows how to convert between metric units.



WORKED EXAMPLE 2 Converting between units of length

Convert the following lengths to the units shown.

a. **0.739 km = _____ m**
b. **53 250 mm = _____ m**

THINK	WRITE
a. **1.** To convert from kilometre to metre, multiply by 1000.	**a.** $0.739 \times 1000 = 739$
2. When multiplying by 1000, move the decimal point three places to the right. Make sure the new unit is stated.	Thus: 0.739 km = 739 m
b. **1.** To convert from millimetre to metre, divide by 1000.	**b.** $53\,250 \div 1000 = 53.25$
2. When dividing by 1000, move the decimal point three places to the left. Make sure the new unit is stated.	Thus: 53 250 mm = 53.25 m

WORKED EXAMPLE 3 Adding different units of length

John needs to replace two sections of fencing of his yard. One section measures 7 m 36 cm and the other measures 5 m 45 cm.
Calculate the length of fencing he needs to purchase, in appropriate units of length.

THINK	WRITE
1. Convert both values to metres by dividing the centimetre part of the measurement by 100 and adding it to the metre part of the measurement.	$7\text{ m }36\text{ cm} = 7 + \frac{36}{100}$ $= 7 + 0.36$ $= 7.36\text{ m}$ $5\text{ m }45\text{ cm} = 5 + \frac{45}{100}$ $= 5 + 0.45$ $= 5.45\text{ m}$
2. Add the values in metres.	$7.36 + 5.45 = 12.81$
3. Write the answer with the correct units.	John needs to purchase 12.81 m of fencing.

Exercise 3.2 Units of length and estimation

learnon

3.2 Quick quiz on

3.2 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14	N/A	N/A

Simple familiar

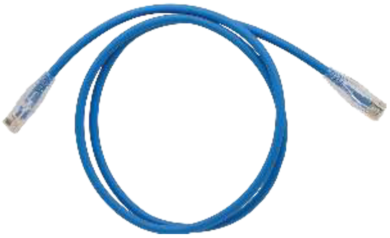
1. **WE1** Select the most suitable metric unit of length to measure the following distances.

a. Height of the tallest building

b. Size of the television

c. The length of the Brisbane River

d. The width of a cable connector



2. **WE2** Convert the following lengths to the units shown.

a. 0.283 km = ______ m
b. 520 cm = ______ m
c. 0.418 m = ______ cm
d. 450 m = ______ km
e. $5\frac{1}{2}$ km = ______ m
f. 78 459 mm = ______ m
g. 0.0378 km = ______ mm
h. 6500 cm = ______ km

3. Convert 375 cm to millimetres.

4. Convert 95.6 km to metres.

5. Convert 875 mm to metres.

6. Convert the following lengths into the units shown.

a. 0.27 km = ______ m
b. 73 500 mm = ______ m
c. 257 mm = ______ cm
d. 1.3 km = ______ cm

7. **WE3** Calculate the value of the following.

a. 5 m 67 cm + 9 m 51 cm
b. 2 m 88 cm + 3 m 20 cm

8. Explain why using millimetres is not a good choice to measure the distance between Brisbane and Sydney.

9. The combined height of two students is 3.62 m. If one student is 1.85 m tall, calculate the height of the second student in centimetres.

10. Arrange each of the following in ascending order.

a. 0.15 km, 135 m, 2400 cm
b. 25 cm, 120 mm, 0.5 m
c. 9 m, 10 000 mm, 0.45 km
d. 32 000 cm, 1200 m, 1 km

11. Calculate the value of each of the following in centimetres.

a. 15 mm + 5 mm
b. 1.5 m + 40 mm
c. 995 mm + 1.2 m
d. 5.67 cm + 1156 mm − 0.25 m

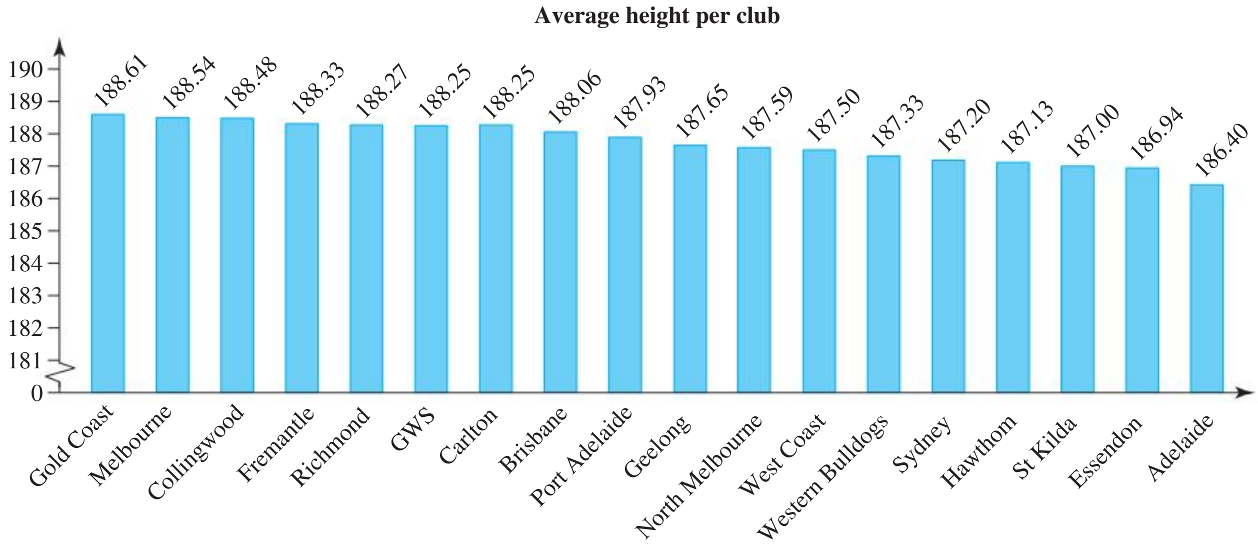
12. Calculate the value of each of the following.

a. 15 cm 15 mm + 27 cm 86 mm
b. 3 km 65 m + 79 km 38 m
c. 66 cm 15 mm − 26 cm 5 mm
d. 125 m 49 cm − 37 m 79 cm

13. The heights of Melbourne's tallest building, the Eureka Tower, and Queensland's tallest building, the Q1 building, measure 0.297 km and 322.5 m respectively.

- **a.** State the tallest building.
- **b.** Calculate the height difference, in metres, between the buildings.
- **c.** Calculate the sum of the heights of both buildings, in metres.

14. The following graph shows the average height of the players in each of the AFL clubs in centimetres.



Determine:

- **a.** the average height of Collingwood players, in metres.
- **b.** the difference between the average height of Carlton and Sydney players, in centimetres.
- **c.** the difference between the tallest and smallest teams' average height, in centimetres.
- **d.** the League's average height.
- **e.** West Coast's average height in millimetres.
- **f.** two clubs whose difference in average height is 5 mm.

Fully worked solutions for this chapter are available online.

LESSON
3.3 Perimeters of familiar and composite shapes

SYLLABUS LINKS

- Calculate perimeters of standard polygons, circles and arc lengths.
  - circle: $C = 2\pi r$ where C is circumference and r is radius
  - arc length: $l = \frac{\theta}{180}\pi r$ where l is arc length, θ is central angle and r is radius.
- Calculate perimeters of composite shapes [complex].

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3.3.1 Perimeter

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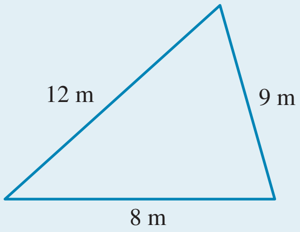
- The **perimeter** is the distance around the outside of an enclosed shape.
- To calculate the perimeter of a shape, change all lengths to the same units and add them together.

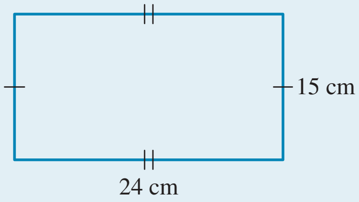
WORKED EXAMPLE 4 Calculating perimeters of standard polygons

Calculate the perimeter of the following shapes.

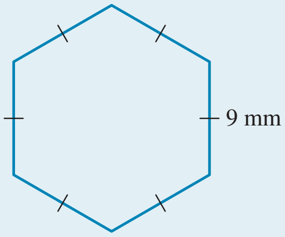
a.



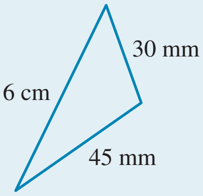
b.



c.



d.



THINK	WRITE
a. 1. Add the distance along each edge of the triangle to calculate the perimeter.	**a.** $P = 12 + 9 + 8$ $= 29$
2. Write your answer.	The perimeter is 29 m.
b. 1. Add the distance along each edge of the rectangle to calculate the perimeter.	**b.** $P = 24 + 15 + 24 + 15$ $= 78$
2. Write your answer.	The perimeter is 78 cm.
c. 1. Add the distance along each edge of the hexagon to calculate the perimeter.	**c.** $P = 9 + 9 + 9 + 9 + 9 + 9$ $= 54$
2. Write your answer.	The perimeter is 54 mm.

THINK	WRITE
d. **1.** Convert all lengths into the same units.	**d.** 6 cm = 60 mm
2. Add the distance along each edge of the triangle in millimetres to calculate the perimeter.	$P = 60 + 30 + 45$ $= 135$
3. Write your answer.	The perimeter is 135 mm.

WORKED EXAMPLE 5 Determining perimeters of real-life objects

Determine the perimeter of the following.
a. **A square table of length 1.8 m**
b. **A rectangular yard 26 m by 30 m**
c. **An octagonal-shaped board game with side lengths of 42 cm**

THINK	WRITE
a. **1.** Add the distance along each edge of the square to calculate the perimeter.	**a.** $P = 1.8 + 1.8 + 1.8 + 1.8$ $= 7.2$
2. Write your answer.	The perimeter is 7.2 m.
b. **1.** Add the distance along each edge of the rectangle to calculate the perimeter.	**b.** $P = 26 + 26 + 30 + 30$ $= 112$
2. Write your answer.	The perimeter is 112 m.
c. **1.** Add the distance along each edge of the octagon to calculate the perimeter.	**c.** $P = 42 + 42 + 42 + 42 + 42 + 42 + 42 + 42$ $= 336$
2. Write your answer.	The perimeter is 336 cm.

3.3.2 Circumference

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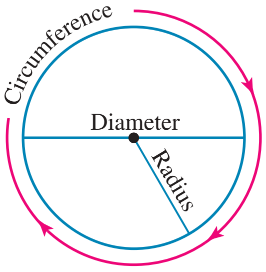
- The **circumference** (C) is the term used for the perimeter of a circle.
- The **diameter** (D) of a circle is the distance from one side of the circle to the other, passing through the centre.
- The **radius** (r) of a circle is the distance from the centre of the circle to the outside and it is half the distance of the diameter.



Circumference of a circle

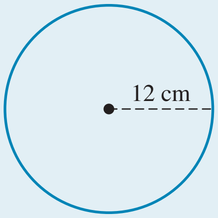
The circumference of a circle is given by the formula:

$$C = 2\pi r$$
or
$$C = \pi D$$

The diameter is twice the length of the radius. That is, $D = 2r$ **or** $r = \frac{D}{2}$**.**

WORKED EXAMPLE 6 Calculating circumference

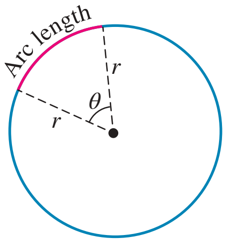
Calculate the circumference of the following circle, correct to two decimal places.



THINK	WRITE
1. Use the formula for the circumference of a circle in terms of the radius.	$C = 2\pi r$
2. Substitute the radius into the equation and solve for C using the π key on your calculator.	$C = 2 \times \pi \times 12$ $= 75.398\,223\,69$ $\approx 75.40\text{ cm}$
3. Write the answer to two decimal places.	The circumference approximately is 75.40 cm.

3.3.3 Arc length

- An **arc** is part of the circumference of a circle.
- To calculate the **arc length**, we use the circumference of the whole circle and multiply it by the angle (as a fraction) that it forms at the centre of the circle.



Arc length

The arc length can be calculated by the following formula:

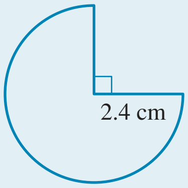
$$\begin{aligned} l &= \frac{\theta}{360} \times 2\pi r \\ &= \frac{\theta}{180}\pi r \end{aligned}$$

where:
***l* is arc length**
***θ* is central angle (in degrees)**
***r* is radius.**

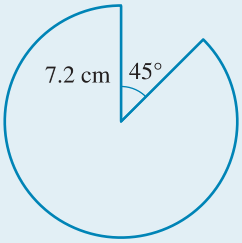
WORKED EXAMPLE 7 Determining arc length

Calculate the arc length of the each of the following shapes, correct to two decimal places.

a.



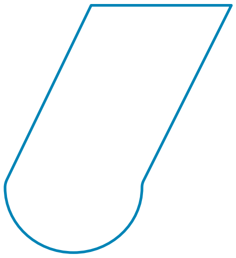
b.



THINK	WRITE
a. 1. Determine the angle the ends of the arc make with the centre.	**a.** $\theta = 360° - 90°$ $= 270°$
2. Substitute the values for the radius and the angle into the formula for the arc length.	$\text{Arc length} = \dfrac{\theta}{180} \times \pi r$ $= \dfrac{270}{180} \times \pi \times 2.4$ $\approx 11.3097...$
3. Write your answer.	Arc length = 11.31 cm
b. 1. Determine the angle the ends of the arc make with the centre.	**b.** $\theta = 360° - 45°$ $= 315°$
2. Substitute the values for the radius and the angle into the formula for the arc length.	$\text{Arc length} = \dfrac{\theta}{180} \times \pi r$ $= \dfrac{315}{180} \times \pi \times 7.2$ $\approx 39.5840...$
3. Write your answer.	Arc length = 39.58 cm

3.3.4 Perimeter of composite shapes [complex]

- A **composite shape** is a figure made by combining two or more simple shapes, such as rectangles, triangles and circles, into one larger shape.

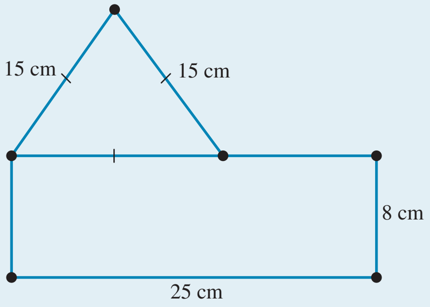


- The perimeter of a composite shape is the total distance around the outside of a shape.

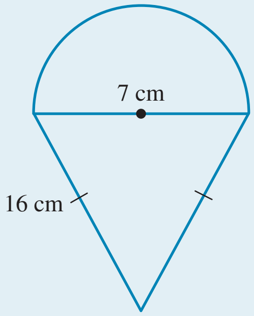
WORKED EXAMPLE 8 Calculating perimeters of composite shapes

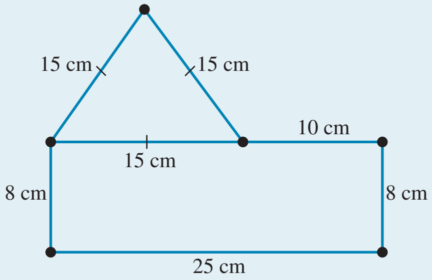
Calculate the perimeter of the following shapes, correct to two decimal places.

a.



b.



THINK	WRITE
a. 1. Determine all of the outside lengths and label them.	**a.** Since the triangle is equilateral, all sides are the same length. Therefore, the length of top of the rectangle is 15 + ___ to equal the total length of the rectangle, which is 25. Therefore, the missing length is 10. The height of the rectangle is 8 cm on both sides. 
2. Determine the perimeter of the composite shape.	$15 + 15 + 10 + 8 + 25 + 8 = 81$
3. Write your answer.	The perimeter of the shape is 81 cm.
b. 1. Determine the different shapes that make the composite shape.	**b.** The shape is made up of a semi-circle and two lengths of 16 cm.
2. Calculate the arc length of a semi-circle. Recall that $r = \frac{D}{2}$.	$\text{Arc length} = \frac{\theta}{180}\pi r$ $= \frac{180}{180} \times \pi \times \frac{7}{2}$ $= 10.995\,574\text{ cm}$
3. Calculate the perimeter of the shape by adding all three sides.	$P = 10.995\,574 + 16 + 16$ $= 42.995\,574\text{ cm}$ $\approx 43.00\text{ cm}$
4. Write your answer.	The perimeter is 43.00 cm.

Exercise 3.3 Perimeters of familiar and composite shapes

3.3 Quick quiz on

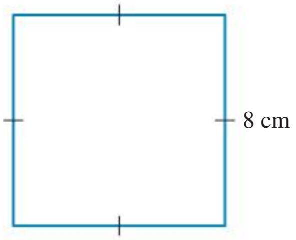
3.3 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14	15, 16, 17	18

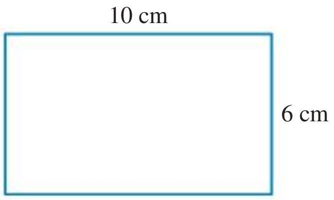
Simple familiar

1. Calculate the perimeter of the following rectangles.

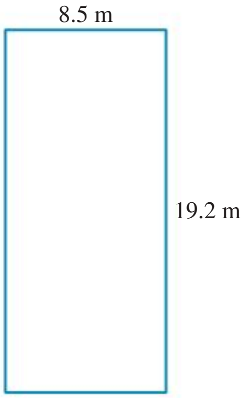
a.



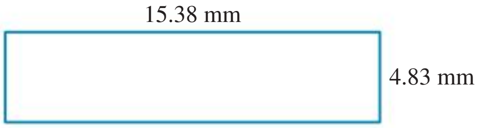
b.



c.

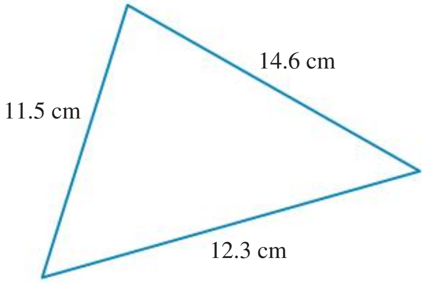


d.

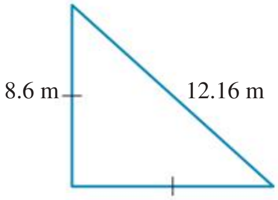


2. Calculate the perimeter of the following triangles.

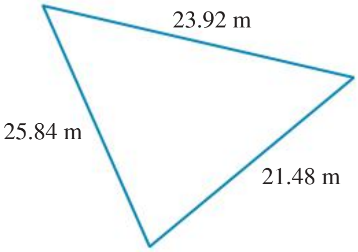
a.



b.



c.



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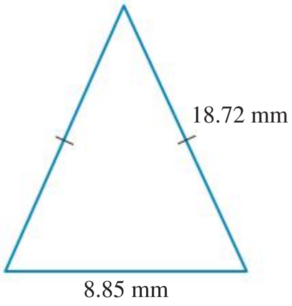
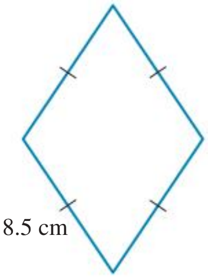
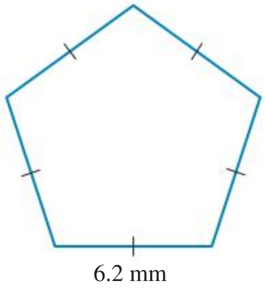


3. Calculate the perimeter of the following polygons.

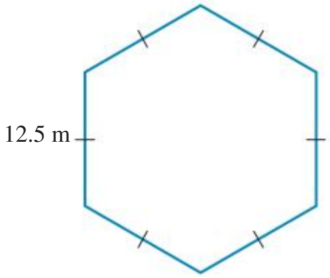
a.



b.

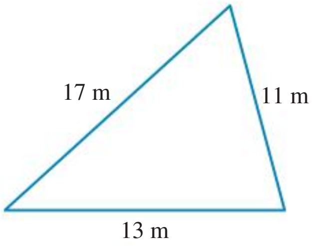


c.

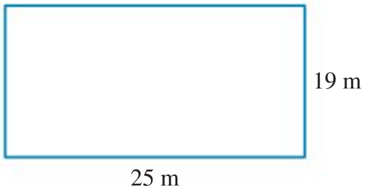


4. **WE4** Calculate the perimeter of the following shapes.

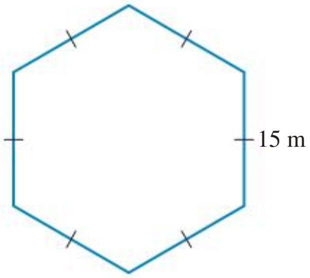
a.



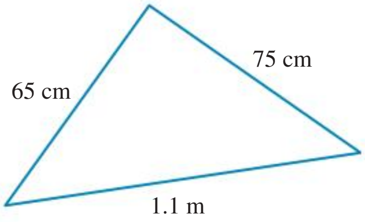
b.



c.



d.



5. **WE5** Determine the perimeter of the following.

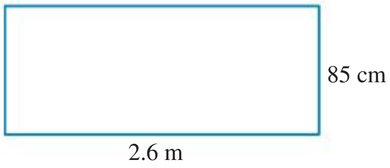
a. A square table of length 1.2 m

b. A rectangular yard 23 m by 37 m

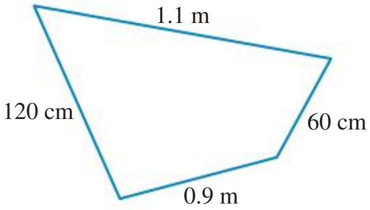
c. An octagon of side length 25 cm

6. Calculate the perimeter of the following shapes.

a.



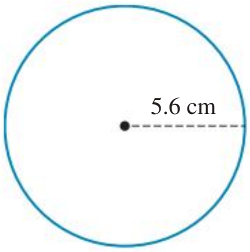
b.



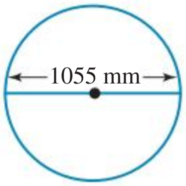
7. Calculate the perimeter of a triangle with side lengths of 7 cm, 8 cm and 10 cm respectively.

8. **WE6** Calculate the circumference of the following circles, correct to two decimal places.

a.

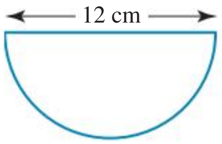


b.

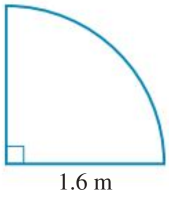


9. **WE7** Calculate the arc length of the following shapes, correct to two decimal places.

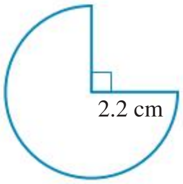
a.



b.

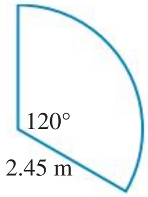


c.

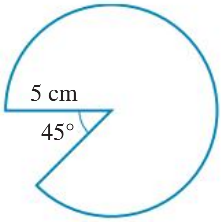


10. Calculate the arc length of the following shapes, correct to two decimal places.

a.

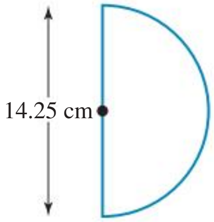


b.

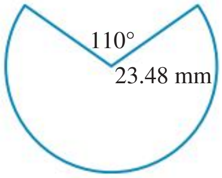


11. Calculate the arc length of the following shapes, correct to two decimal places.

a.

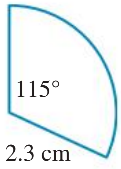


b.

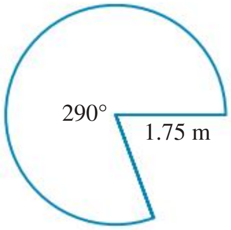


12. Calculate the arc length of the following shapes, correct to two decimal places.

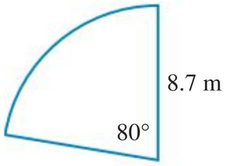
a.



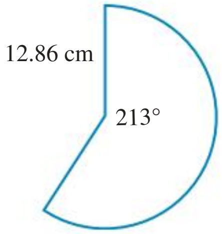
b.



c.



d.



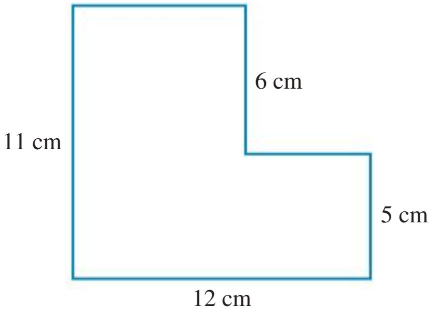
13. Calculate the radius (in centimetres) of a bicycle wheel that travels 1 m in one revolution.

14. The minute hand on a clock is 22 cm in length. Calculate the distance travelled by the tip of the hand in 3 h.

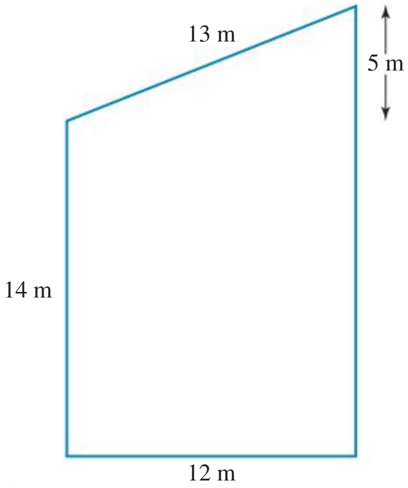
Complex familiar

15. **WE8** Calculate the perimeter of the following shapes, correct to two decimal places where necessary.

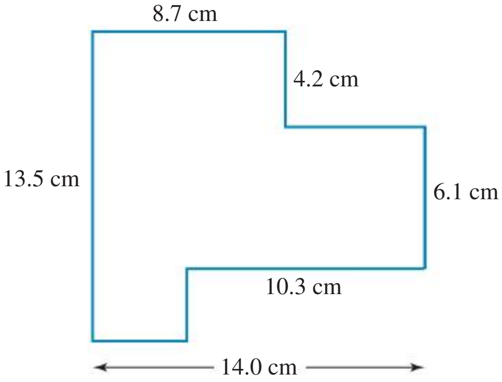
a.



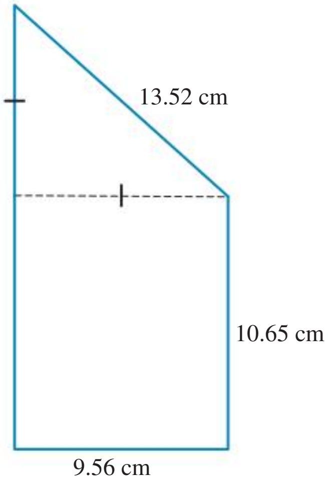
b.



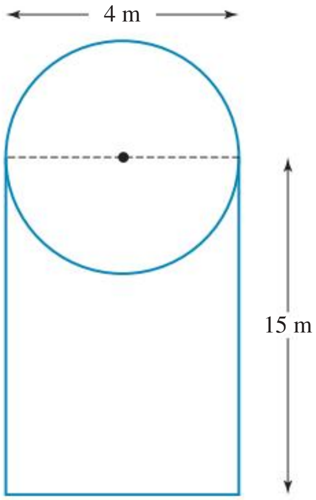
c.



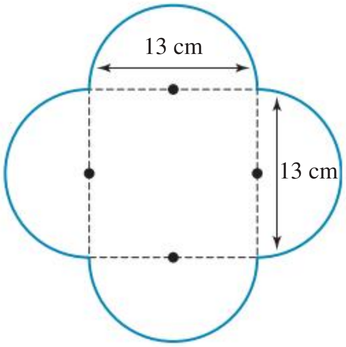
d.



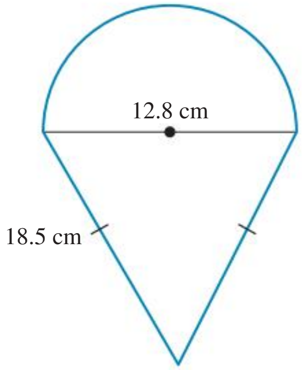
e.



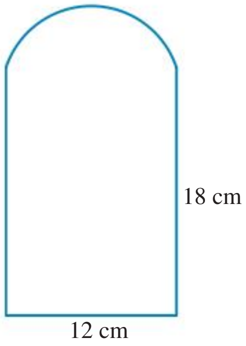
f.



16. Calculate the perimeter of the following shape, correct to two decimal places.

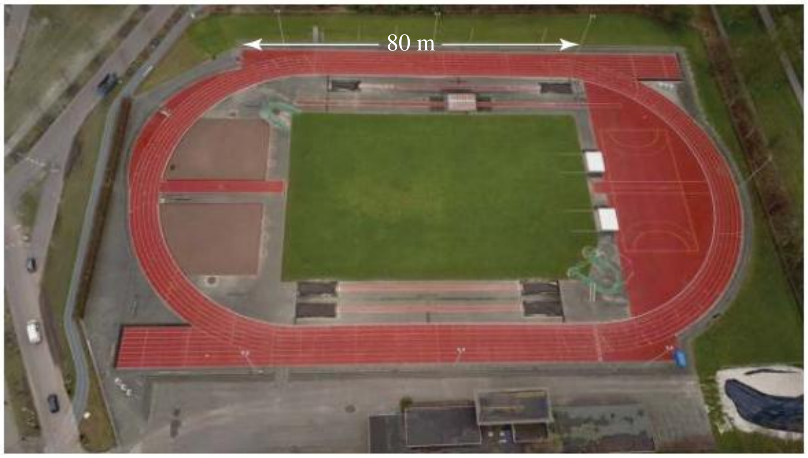


17. Calculate the amount of edge tape a cabinet-maker needs for this cabinet door, given it is made up of a rectangle and semi-circle.



Complex unfamiliar

18. The straight sections of a running track are each 80 m long. The two curved sections at either end are semi-circles of equal size. The total length of the inside lane (Lane 1) is exactly 400 m, and each lane is 1 m wide. Assuming that each runner stays in the centre of their lane, calculate how much further ahead the runner in Lane 2 must start so that both runners complete exactly 400 m. Give your answer correct to two decimal places.



Fully worked solutions for this chapter are available online.

LESSON
3.4 Units of area and estimation

SYLLABUS LINKS

- Estimate areas of different shapes.
- Use metric units of area (square millimetres, square centimetres, square metres, square kilometres, hectares), their abbreviations (mm^2, cm^2, m^2, km^2, ha), conversions between them and appropriate choices of units.

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3.4.1 Conversion of area units

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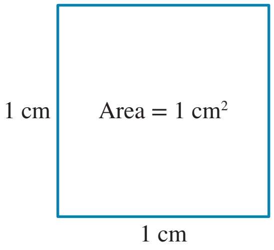
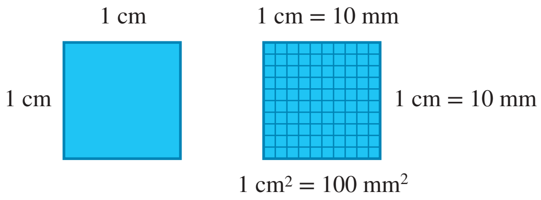


- The **area** of a two-dimensional shape is the amount of space enclosed by the shape.
- A square centimetre is the amount of space enclosed by a square with a length 1 cm and a width 1 cm.
- Some common metric units of area and their abbreviations are:
  - square kilometres (km^2)
  - square metres (m^2)
  - square centimetres (cm^2)
  - square millimetres (mm^2)
- When converting area units you need to take into account that the shape is two-dimensional, thus squared units. To convert area units:
  1. determine the factor of conversion of the units of length (i.e. × 10 or ÷ 100)
  2. then square the conversion.
- Consider, for example, converting 1 cm^2 to mm^2.

  To convert from cm to mm, multiply by 10. Since area is two-dimensional we need to square the conversion, so multiply by $10^2 = 100$.

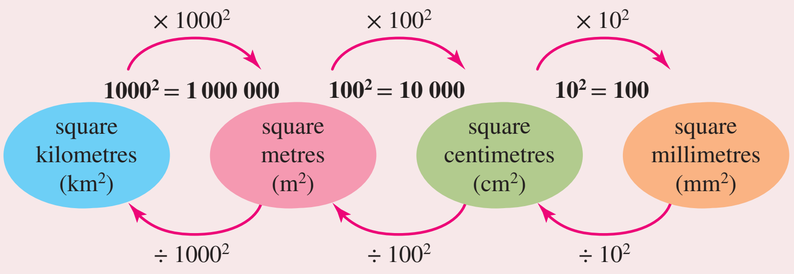
  Therefore, $1\,cm^2 = 1 \times 100\,mm^2 = 100\,mm^2$.

1 cm² = 100 mm²



Converting metric units of area

The following diagram shows how to convert between metric units of area.



- Large parcels of land such as farmland are often measured in **hectares (ha)**. One hectare is the space enclosed by a square with side lengths equal to 100 m.

Hectares (ha)

$$1\text{ ha} = 10\,000\text{ m}^2$$
$$100\text{ ha} = 1\text{ km}^2$$

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int-4016

WORKED EXAMPLE 9 Converting units of area 1

Convert the following units of area.

a. **56 cm^2 = _______ mm^2** **b.** **78 700 cm^2 = _______ m^2** **c.** **591 ha = _______ km^2**

THINK	WRITE
a. 1. To convert from cm^2 to mm^2, multiply by 10^2.	**a.** $56\text{ cm}^2 = 56 \times 10^2\text{ mm}^2$
2. Complete the calculation and write your answer.	$56\text{ cm}^2 = 56 \times 100\text{ mm}^2$ $56\text{ cm}^2 = 5600\text{ mm}^2$
b. 1. To convert from cm^2 to m^2, divide by 100^2.	**b.** $78\,700\text{ cm}^2 = 78\,700 \div 100^2\text{ m}^2$
2. Complete the calculation and write your answer.	$78\,700\text{ cm}^2 = 78\,700 \div 10\,000\text{ m}^2$ $78\,700\text{ cm}^2 = 7.87\text{ m}^2$
c. 1. To convert from ha to km^2, divide by 100.	**c.** $591\text{ ha} = 591 \div 100\text{ km}^2$
2. Complete the calculation and write your answer.	$591\text{ ha} = 5.91\text{ km}^2$

3.4.2 Area of a rectangle

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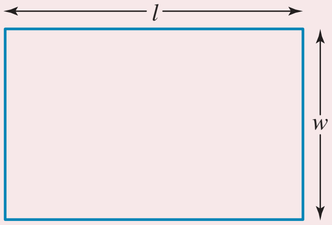
- The area of a rectangle can by calculated by multiplying its length by its width.

Area of a rectangle

$$A = l \times w$$

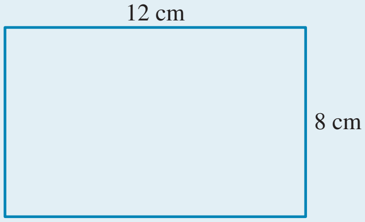
where:
l = length
w = width



WORKED EXAMPLE 10 Converting units of area 2

Calculate the area of the following shape in:

a. **cm^2**
b. **m^2.**



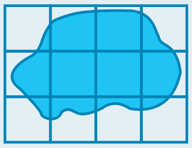
THINK	WRITE
a. **1.** The shape is a rectangle, so use the area of a rectangle formula.	**a.** $A = l \times w$
2. Substitute in $l = 12$ cm and $w = 8$ cm.	$A = 12 \times 8$ $= 96$
3. Write your answer.	Area $= 96\,\text{cm}^2$
b. **1.** To convert cm^2 to m^2, divide by 100^2.	**b.** $96\,\text{cm}^2 = 96 \div 100^2\,\text{m}^2$ $= 0.0096\,\text{m}^2$
2. Write your answer with the correct units.	Area $= 0.0096\,\text{m}^2$

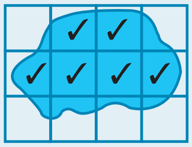
3.4.3 Estimation of area

- Follow the steps below as a guide to estimate the area of a shape when a square is not completely filled by the object:
  - If more than half of the square is covered, count it in the area.
  - If less than half of the square is covered, don't count it in the area.

WORKED EXAMPLE 11 Estimating area

Estimate the area of the following shape, giving your answer in units of squares.



THINK	WRITE
1. Place a tick in each square that is more than half full.	
2. Count the number of squares that contain a tick. Write your answer.	The area is approximately 6 square units.

Exercise 3.4 Units of area and estimation

learnon

3.4 Quick quiz on	3.4 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18	N/A	N/A

These questions are even better in jacPLUS!
- Receive immediate feedback
- Access sample responses
- Track results and progress

Find all this and MORE in jacPLUS

Simple familiar

1. Name the most appropriate unit for measuring each of the following.

a. Area of a floor rug
b. Area of a fingerprint
c. Area of a page from a novel
d. Area of a painted wall in the lounge room
e. Area of the surface of Uluru
f. Area of a 5-cent coin

2. State the metric unit of area you would use for measuring each of the following.

a. The palm of your hand
b. A suburban house block
c. A dairy farm
d. Your fingernail
e. Australia
f. Your local football ground

3. Provide two examples (that have not already been mentioned) of area that could be measured in:

a. mm^2 **b.** cm^2 **c.** m^2 **d.** km^2

4. **WE9** Convert the following units of area.

a. $34\ \text{cm}^2 = \_\_\_\_\_\_\_\ \text{mm}^2$ **b.** $14\,250\ \text{cm}^2 = \_\_\_\_\_\_\_\ \text{m}^2$ **c.** $592\ \text{ha} = \_\_\_\_\_\_\_\ \text{km}^2$

5. Convert the following units of area.

a. $0.18\ \text{m}^2 = \_\_\_\_\_\_\_\ \text{cm}^2$ **b.** $0.0798\ \text{km}^2 = \_\_\_\_\_\_\_\ \text{m}^2$ **c.** $374\,300\ \text{m}^2 = \_\_\_\_\_\_\_\ \text{ha}$

6. Convert each of the following measurements to the units shown in the brackets.

a. $2.3\ \text{cm}^2\ (\text{mm}^2)$ **b.** $2.57\ \text{m}^2\ (\text{cm}^2)$ **c.** $470\ \text{mm}^2\ (\text{cm}^2)$
d. $27\,000\ \text{m}^2\ (\text{km}^2)$ **e.** $87\,500\ \text{m}^2\ (\text{ha})$ **f.** $17\,000\ \text{cm}^2\ (\text{m}^2)$

7. Convert each of the following measurements to the units shown in the brackets.

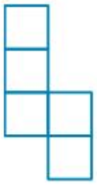
a. $51\,200\ \text{m}^2\ (\text{ha})$ **b.** $2.85\ \text{ha}\ (\text{m}^2)$ **c.** $8380\ \text{ha}\ (\text{km}^2)$
d. $12.6\ \text{km}^2\ (\text{ha})$ **e.** $0.23\ \text{ha}\ (\text{m}^2)$ **f.** $623\,450\ \text{m}^2\ (\text{ha})$

8. Convert each of the following measurements to the units shown in the brackets.

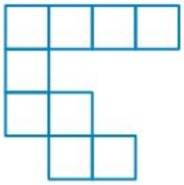
a. $0.048\ \text{m}^2\ (\text{mm}^2)$ **b.** $0.0012\ \text{km}^2\ (\text{cm}^2)$ **c.** $300\,800\ \text{mm}^2\ (\text{m}^2)$ **d.** $4478\ \text{cm}^2\ (\text{km}^2)$

9. If each square is 1 cm^2, determine the area of the following shapes.

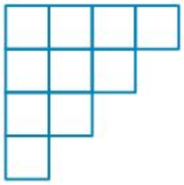
a.



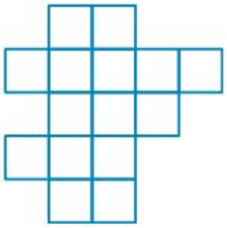
b.



c.

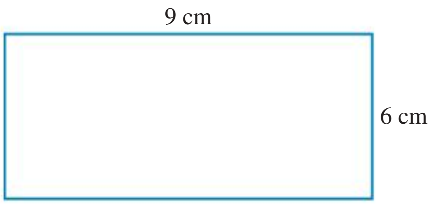


d.

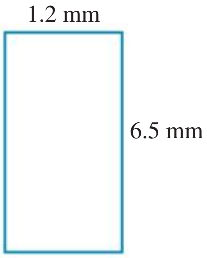


10. Calculate the area of the following shapes.

a.



b.



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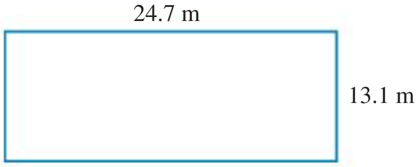
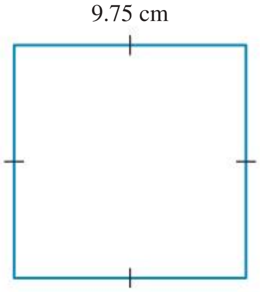


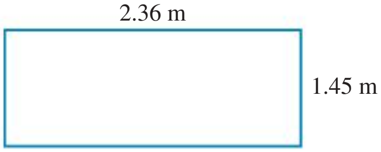
d.



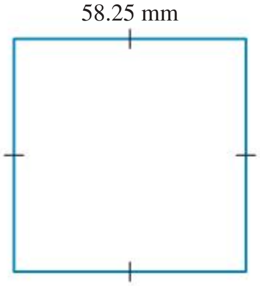
11. **WE10** Calculate the area of the following shapes in the specified units.

- **i.** cm^2
- **ii.** m^2

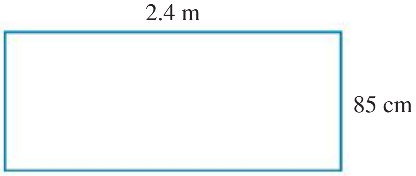
a.



b.



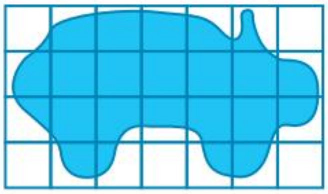
c.



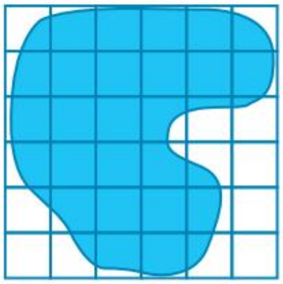
d.

12. **WE11** Estimate the area of each of the following shapes (giving your answer in units of squares).

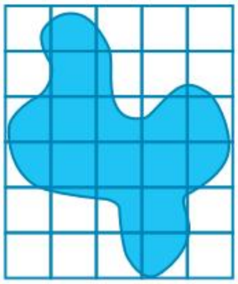
a.



b.



c.



13. Estimate the area of the banner held up by the club members. Use the squares shown in the diagram to help determine the area from one end to the other, where each square represents 1 m^2.

14. Three different classrooms have respective areas of 3.75 m^2, 46 000 cm^2 and 0.000 008 3 km^2. Calculate the combined area of the classrooms in square metres.

15. If a cricket pitch has an area of 60.36 m^2 and it is 3 m wide, calculate the length of the cricket pitch.

16. A rectangular photograph measuring 8 cm by 10 cm is enlarged so that its dimensions are doubled.
   a. Calculate the area of the original photograph.
   b. Calculate the dimensions of the new photograph.
   c. Calculate the area of the new photograph.
   d. Describe the relationship between the areas of the original and the enlarged photographs by completing the following sentence:
   If the side lengths of the rectangle are doubled, its area is _______________.

17. A rectangular flower bed measures 25 m by 14 m. A gravel path 2 m wide surrounds it.
   a. Draw a diagram representing the flower bed and path.
   b. Calculate the area of the flower bed.
   c. Calculate the area of the gravel path.
   d. If gravel costs $7.50 per square metre, calculate how much it will cost to cover the path.

18. A wall in a house is to be painted. The wall is rectangular and measures 9.5 m by 3.2 m. The wall has two rectangular windows, 2.4 m by 80 cm.
   a. Calculate the area of the wall to be painted in square metres.
   b. If the wall needs three coats, calculate the amount of coverage of paint that is required to paint the wall, in square metres.
   c. If 1 litre of paint covers 5 m^2, calculate the amount of paint required, to the nearest litre.
   d. If the paint costs $15.75 per litre, calculate the total cost to complete the painting.

Fully worked solutions for this chapter are available online.

LESSON
3.5 Areas of standard shapes

SYLLABUS LINKS

- Calculate areas of standard shapes, triangles, parallelograms and circles.
    - triangle: $A = \frac{1}{2}bh$ where b is base length and h is perpendicular height
    - parallelogram: $A = bh$ where b is base length and h is perpendicular height
    - circle: $A = \pi r^2$ where r is radius.
- Calculate areas of trapeziums and sectors [complex].
    - trapezium: $A = \frac{1}{2}(a + b)h$ where a and b are parallel lengths and h is perpendicular height
    - sector: $A = \frac{\theta}{360}\pi r^2$ where θ is central angle and r is radius.

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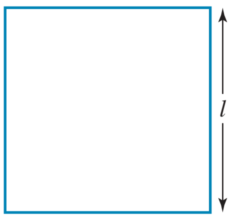
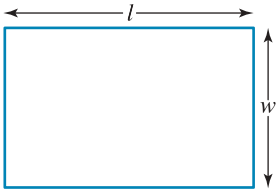
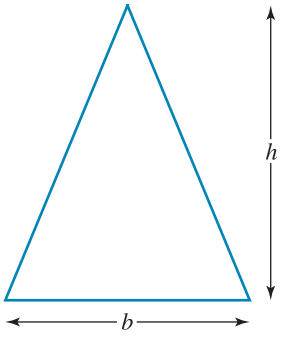
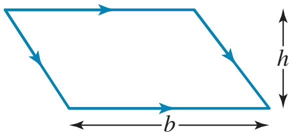
3.5.1 Area of standard two-dimensional shapes

int-3786

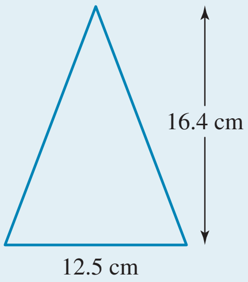
- The area of two-dimensional shapes can be calculated using the following formulae.

Shape	Diagram	Area formula
Square		$A_{\text{Square}} = l \times l$ $= l^2$
Rectangle		$A_{\text{Rectangle}} = l \times w$
Triangle		$A_{\text{Triangle}} = \frac{1}{2}bh$
Parallelogram		$A_{\text{Parallelogram}} = bh$

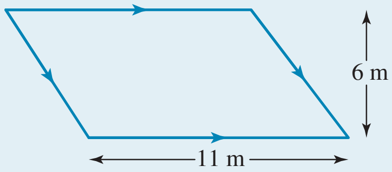
WORKED EXAMPLE 12 Calculating area of triangles and parallelograms

Calculate the area of the following shapes.

a.



b.



THINK	WRITE
a. 1. Identify the shape as a triangle and use the area of a triangle formula.	**a.** $A_{\text{Triangle}} = \frac{1}{2} \times b \times h$
2. Substitute $b = 12.5$ cm and $h = 16.4$ cm.	$A_{\text{Triangle}} = \frac{1}{2} \times 12.5 \times 16.4$ $= 102.5\ \text{cm}^2$
3. Write your answer with the correct units.	The area is $102.5\ \text{cm}^2$.
b. 1. Identify the shape as a parallelogram and apply the area of a parallelogram formula.	**b.** $A_{\text{Parallelogram}} = bh$
2. Substitute $b = 11$ m and $h = 6$ m.	$A_{\text{Parallelogram}} = 11 \times 6$ $= 66\ \text{m}^2$
3. Write your answer with the correct units.	The area is $66\ \text{m}^2$.

3.5.2 Area of a trapezium [complex]

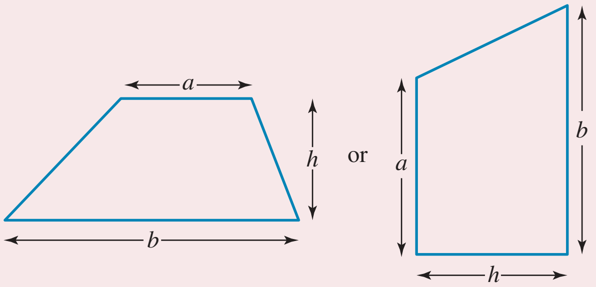
int-3789
int-3790
int-4442

- The area of a trapezium can be calculated by the following formula:

Area of a trapezium

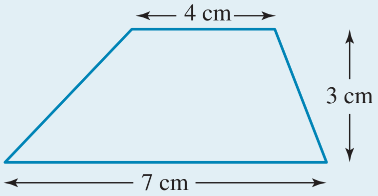
$$A_{\text{Trapezium}} = \frac{1}{2}(a+b)h$$

where:
a **and** ***b*** **are parallel lengths**
h **is the perpendicular height.**



WORKED EXAMPLE 13 Calculating the area of a trapezium

Calculate the area of the following shape.



THINK	**WRITE**
1. Identify the shape as a trapezium and use the area of a trapezium formula.	$A_{\text{Trapezium}} = \frac{1}{2}(a+b)h$
2. Substitute $a = 4$ cm, $b = 7$ cm and $h = 3$ cm.	$A_{\text{Trapezium}} = \frac{1}{2}(4+7) \times 3$ $= \frac{1}{2}(11) \times 3$ $= \frac{33}{2}$ $= 16.5\,\text{cm}^2$
3. Write your answer with the correct units.	The area is $16.5\,\text{cm}^2$.

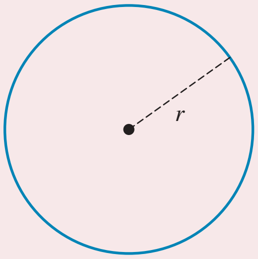
3.5.3 Area of a circle

int-3788

- The area of a circle can be calculated using the following formula.

Area of a circle

$$A_{\text{Circle}} = \pi r^2$$



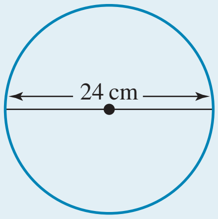
where:

r is the radius.

Note: $r = \frac{D}{2}$.

WORKED EXAMPLE 14 Calculating the area of a circle

Calculate the area of the following circle, correct to two decimal places.



THINK	WRITE
1. Write the formula for the area of a circle.	$A_{\text{Circle}} = \pi r^2$
2. Determine the radius, given the diameter is 24 cm.	The radius is half the length of the diameter, so: $r = \frac{24}{2}$ $= 12\text{ cm}$
3. Substitute $r = 12$ into the formula for the area of a circle and evaluate the expression for the area.	$A_{\text{Circle}} = \pi \times 12^2$ $= 144\pi$ $\approx 452.39\text{ cm}^2$

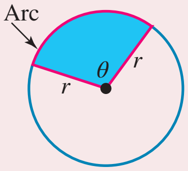
3.5.4 Area of a sector [complex]

int-6076

- The area of a sector can be calculated by finding the area of the circle and multiplying it by the fraction of the angle that the sector forms at the centre of the circle.

Area of a sector

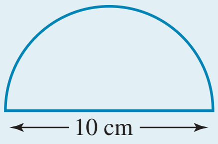
$$\textbf{Area of a sector} = \frac{\theta}{360} \times \pi r^2$$



where:
***r* is the radius**
θ is the central angle that the sector makes with the centre of the circle, in degrees.

WORKED EXAMPLE 15 Calculating the area of a semi-circle

Calculate the area of the following semi-circle, correct to two decimal places.



THINK	**WRITE**
1. Write the formula for the area of a circle.	$A = \pi r^2$
2. Determine the radius of the circle.	$r = \frac{10\,\text{cm}}{2}$ $= 5\,\text{cm}$
3. Substitute $r = 5$ cm into the formula for the area of a circle and evaluate the expression for the area.	$A = \pi \times 5^2$ $= 78.54\,\text{cm}^2$
4. Divide the answer by two since a semi-circle is half of a circle.	$A = \frac{78.54\,\text{cm}^2}{2}$ $= 39.27\,\text{cm}^2$
5. Write your answer with the correct units.	The area of the semi-circle is 39.27 cm^2.

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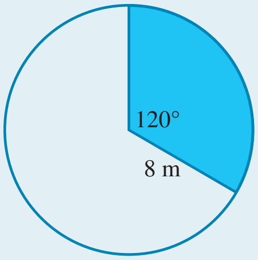
WORKED EXAMPLE 16 Calculating the area of a sector

Calculate the area of the shaded region, correct to two decimal places.



THINK	**WRITE**
1. Write the formula for the area of a sector.	Area of a sector $= \frac{\theta}{360} \times \pi r^2$
2. Substitute $r = 8$ cm and $\theta = 120°$.	Area of a sector $= \frac{120}{360} \times \pi \times 8^2$ $= 67.020\,643$ $\approx 67.02\,\text{cm}^2$
3. Write your answer with the correct units.	The shaded area is 67.02 cm^2.

Exercise 3.5 Areas of standard shapes

3.5 Quick quiz

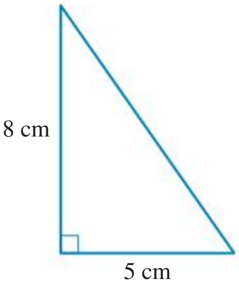
3.5 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9	10, 11, 12, 13, 14, 15	16

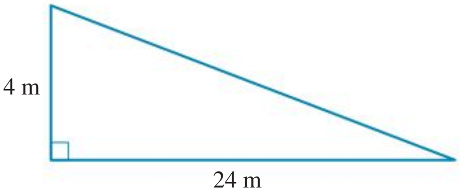
Simple familiar

1. Calculate the area of the following triangles.

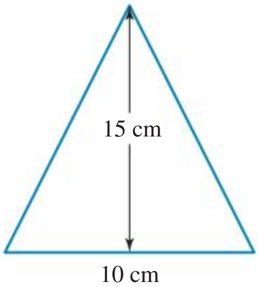
a.



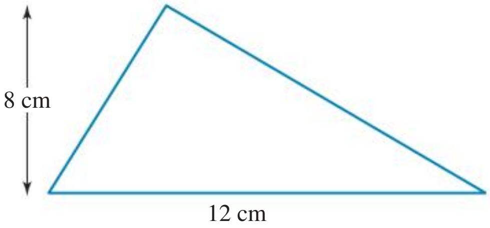
b.



c.



d.

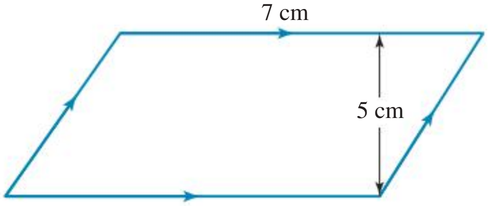


2. Calculate the area of a triangle with base length 12 cm and vertical height 12 cm.

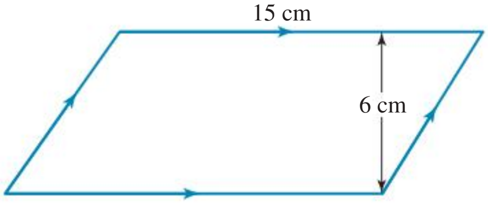
3. Calculate the area of a parallelogram of length 25 cm and vertical height 5 cm.

4. Calculate the area of the following parallelograms.

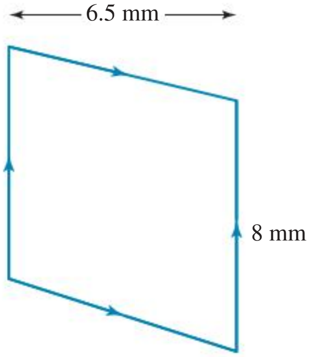
a.



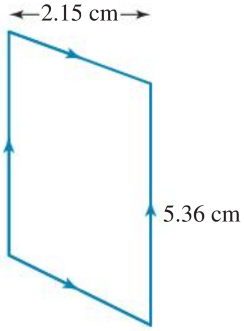
b.



c.

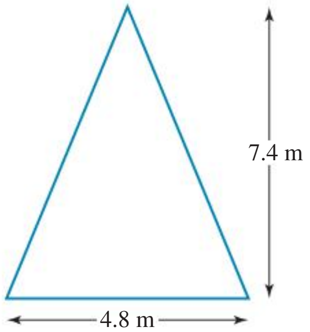


d.

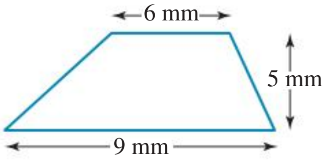


5. **WE12** Calculate the area of the following shapes.

a.

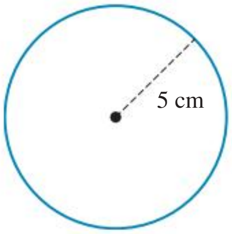


b.

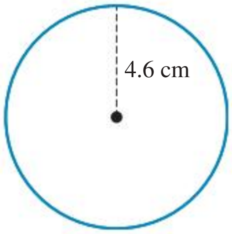


6. **WE14** Calculate the area of the following circles, correct to two decimal places.

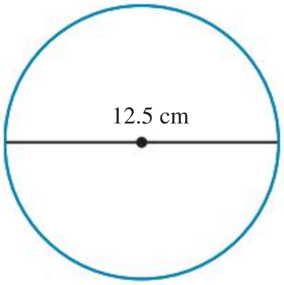
a.



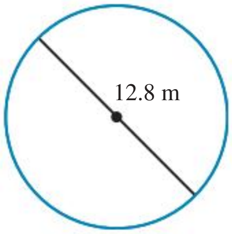
b.



c.



d.



7. The area of a circle is equal to 113.14 mm^2. Calculate the length of the unknown radius.

8. Circular pizza trays come in three different sizes: small, medium and large.

a. Calculate the area of each tray if the diameters are 20 cm, 30 cm and 40 cm respectively.

b. Calculate the area of material that must be ordered to make 50 trays.

9. A rugby pitch is rectangular and measures 100 m in length and 68 m wide.

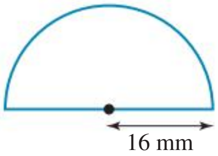
a. Calculate the area of the pitch.

b. If the pitch was laid with instant turf, with each sheet measuring 4 m by 50 cm, calculate how many sheets of turf would be required to cover the pitch.

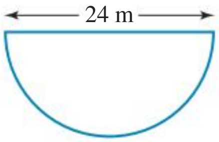
c. If each sheet costs $10.50, calculate the cost to cover the pitch.

Complex familiar

10. **WE15** Calculate the area of the following semi-circle, correct to two decimal places.

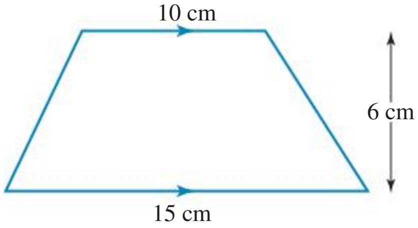


11. Calculate the area of the following semi-circle, correct to two decimal places.

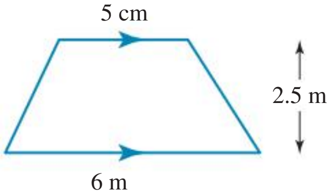


12. **WE13** Calculate the area of the following trapeziums.

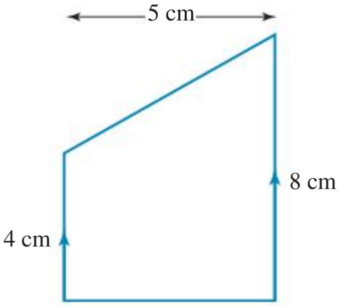
a.



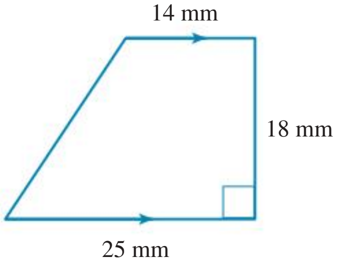
b.



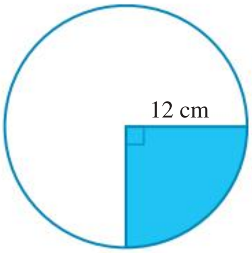
c.



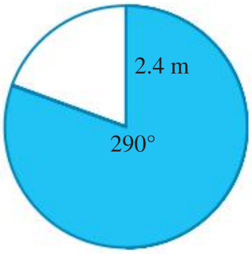
d.



13. **WE16** Calculate the area of the shaded region, correct to two decimal places.

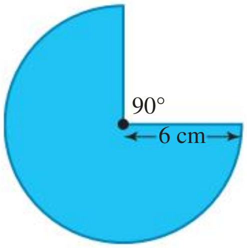


14. Calculate the area of the shaded region, correct to two decimal places.

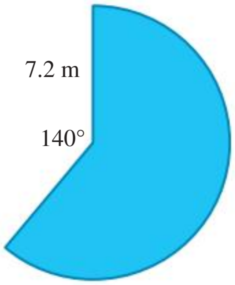


15. Calculate the area of the shaded region, correct to two decimal places.

a.



b.



Complex unfamiliar

16. A garden sprinkler is positioned at the centre of a circular garden. The sprinkler sprays water over a section of the garden, covering an area of 50 m^2. The sprinkler reaches up to 6 m from where it is placed. Determine the angle at which the sprinkler sprays the water, correct to two decimal places.

Fully worked solutions for this chapter are available online.

LESSON
3.6 Areas of composite figures [complex]

SYLLABUS LINKS

- Calculate areas of composite figures by decomposing them into standard shapes [complex].

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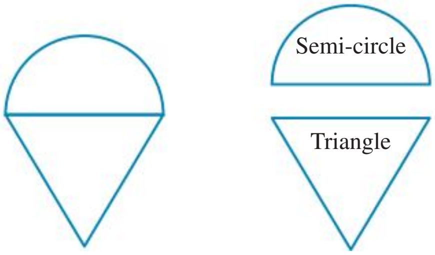
3.6.1 Sum of areas

- A composite figure can be divided into two or more sections, each of which is a smaller regular shape with a known area formula.
- The shape shown below is a combination of a semi-circle and a triangle.



- To find the area of a composite figure:
  - divide the shape into smaller figures that have a known area formula
  - calculate the area of each smaller figure
  - calculate the area of the composite figure by *adding* the area of the smaller figures.

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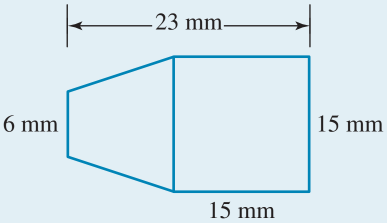
WORKED EXAMPLE 17 Calculating the area of composite figures by adding areas

Calculate the area of the following composite figure.



THINK

1. This composite figure is made up of a square and a trapezium. Add the areas of both to find the total area.

$A_{\text{Total}} = A_{\text{Trapezium}} + A_{\text{Square}}$

2. Calculate the area of the square.

$$\begin{aligned} A_{\text{Square}} &= l \times l \\ &= 15 \times 15 \\ &= 225\,\text{mm}^2 \end{aligned}$$

3. Calculate the area of the trapezium, where the height is $23 - 15 = 8$ mm.

$$\begin{aligned} A_{\text{Trapezium}} &= \frac{1}{2}(a+b)\,h \\ &= \frac{1}{2}(6+15) \times 8 \\ &= 84\,\text{mm}^2 \end{aligned}$$

4. Calculate the total area of the figure by adding the two areas.

$$\begin{aligned} A_{\text{Total}} &= A_{\text{Trapezium}} + A_{\text{Square}} \\ &= 84 + 225 \\ &= 309\,\text{mm}^2 \end{aligned}$$

5. Write your answer with the correct units.

The area of the figure is 309 mm^2.

WRITE

3.6.2 Difference between areas

- Not all composite figures are made by adding a number of smaller shapes together.
- Some shapes are made by subtracting one shape from another.

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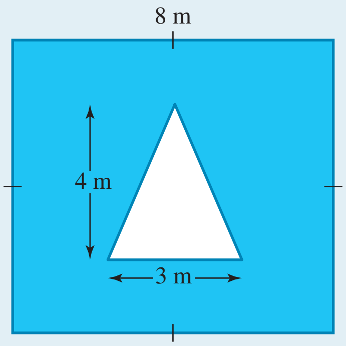
WORKED EXAMPLE 18 Calculating the area of composite figures by subtracting areas

Calculate the area of the shaded region.



THINK	WRITE
1. This composite shape is a square with a triangle cut out. Find the shaded area by subtracting the area of the triangle from the area of the square.	$A_{\text{Shaded}} = A_{\text{Square}} - A_{\text{Triangle}}$
2. Calculate the area of the square.	$A_{\text{Square}} = l \times l$ $= 8 \times 8$ $= 64\text{ m}^2$
3. Calculate the area of the triangle.	$A_{\text{Triangle}} = \frac{1}{2}bh$ $= \frac{1}{2} \times 3 \times 4$ $= 6\text{ m}^2$
4. Calculate the shaded area by subtracting the area of the triangle from the area of the square.	$A_{\text{Shaded}} = A_{\text{Square}} - A_{\text{Triangle}}$ $= 64 - 6$ $= 58\text{ m}^2$
5. Write your answer with the correct units.	The shaded area is 58 m^2.

Exercise 3.6 Areas of composite figures [complex]

3.6 Quick quiz on

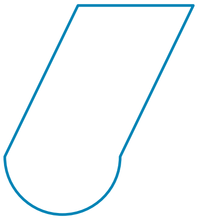
3.6 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
N/A	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	13, 14, 15

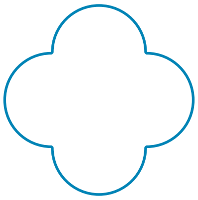
Complex familiar

1. Name the smaller shapes that make the following composite shapes.

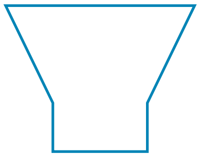
a.



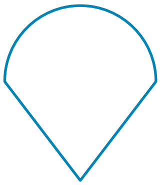
b.



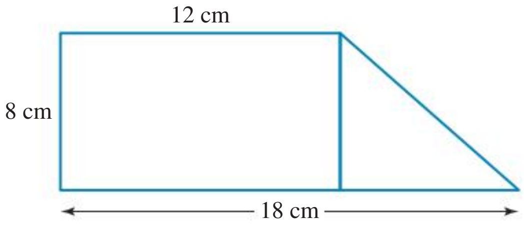
c.



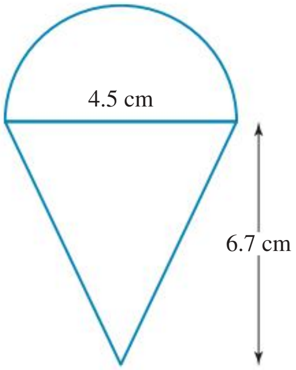
d.



2. **WE17** Calculate the area of the following composite figure.

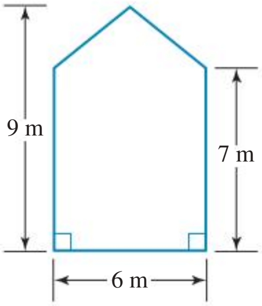


3. Calculate the area of the following shape, correct to two decimal places.

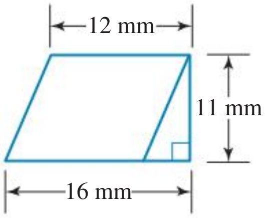


4. Calculate the area of the following composite shapes.

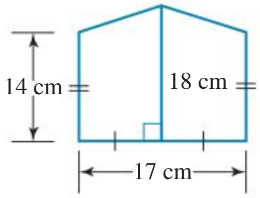
a.



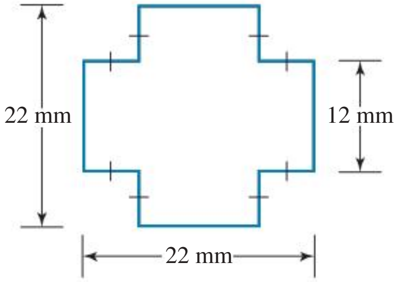
b.



c.

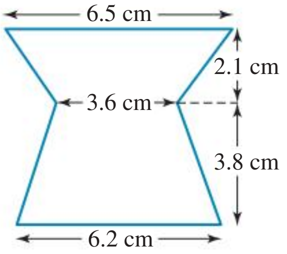


d.

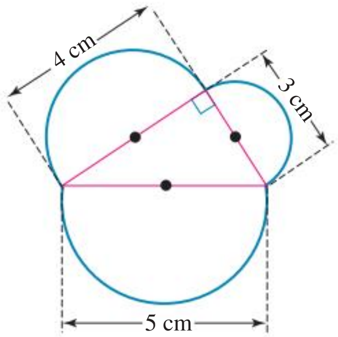


5. Calculate the area of the following composite shapes.

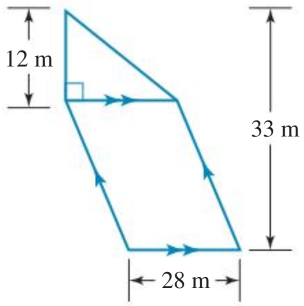
a.



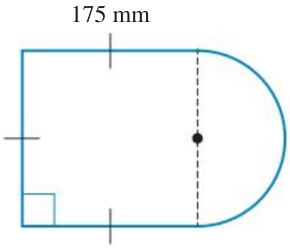
b.



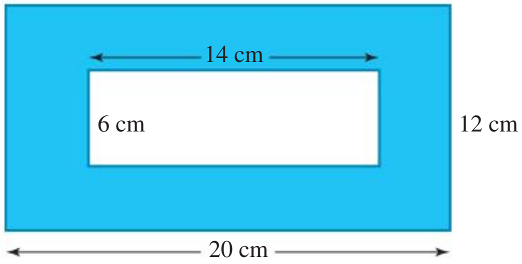
c.



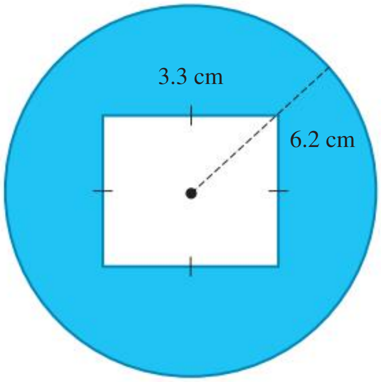
d.



6. **WE18** Calculate the area of the shaded region.

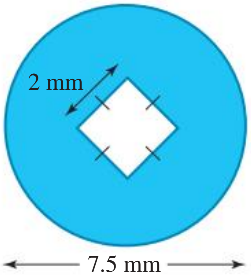


7. Calculate the area of the shaded region, correct to two decimal places.

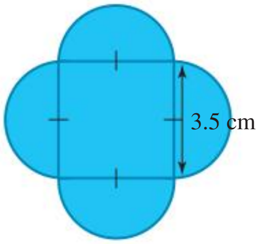


8. Calculate the shaded area of the following shapes.

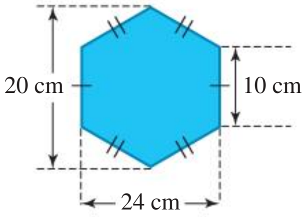
a.



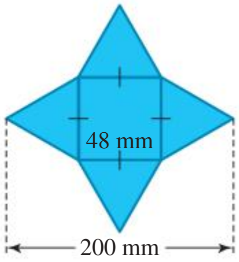
b.



9. a. Calculate the area of the hexagon shown by dividing it into two trapeziums.
b. Calculate the area of the hexagon by dividing it into a rectangle and two triangles.
c. Compare your answers from parts **a** and **b**.

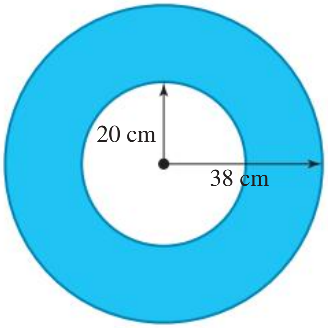


10. A triangular pyramid can be constructed from the net shown. Calculate the total area of the net.

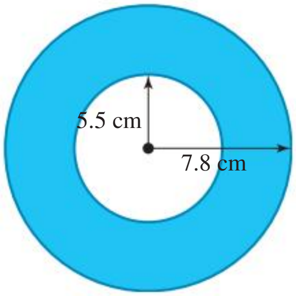


11. Calculate the area of the shaded region of the following shapes, correct to two decimal places.

a.

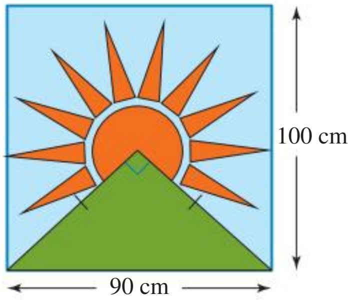


b.



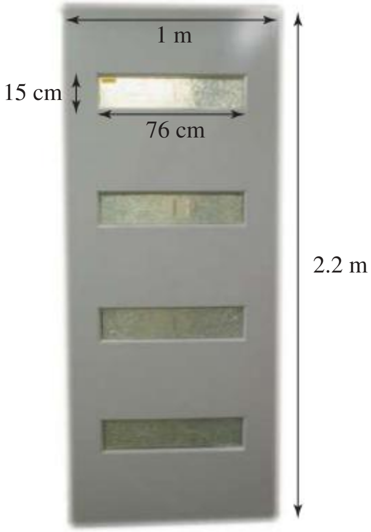
12. The leadlight panel shown depicts a sunrise over the mountains. The mountain is represented by a green triangle 45 cm high. The orange sun is represented by a section of a circle with an 18 cm radius. There are 10 orange sunrays in the shape of isosceles triangles with a base of 3 cm and a height of 12 cm, and the sky is blue. Calculate the area of the leadlight panel made of:

a. green glass
b. orange glass
c. blue glass.

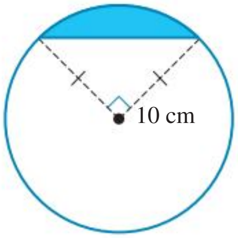


Complex unfamiliar

13. The door shown is 1 m wide, 2.2 m high and has four identical glass panels, each measuring 76 cm by 15 cm.
The door is to be painted on both sides and requires two coats of paint. Paint is sold in 1-l tins at \$24.95 per litre, and each litre covers 8 m^2 of the surface. Calculate the total cost of painting the door.



14. Calculate the area of the shaded region shown.



15. A farmer is designing a new vegetable garden that consists of three sections:
- a rectangular carrot patch that is 8 m long and 5 m wide
- a triangular lettuce patch with a base of 6 m and a height of 4 m
- a semi-circular herb garden with a diameter of 6 m.

The farmer wants to cover the entire garden with nutrient-rich soil. If the soil costs \$3 per square metre, calculate how much the farmer will spend on soil.

Fully worked solutions for this chapter are available online.

LESSON
3.7 Surface areas of prisms and cylinders [complex]

SYLLABUS LINKS

- Calculate surface areas of prisms and cylinders [complex].
  - cylinder: $S = 2\pi rh + 2\pi r^2$ where r is radius and h is perpendicular height.

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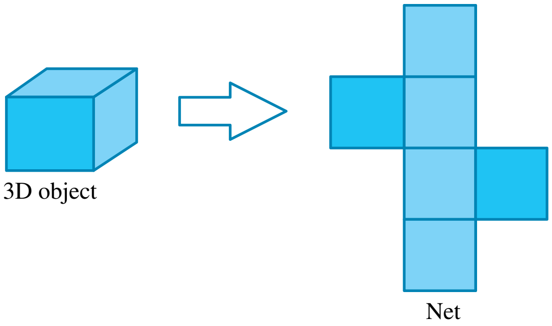
3.7.1 Total surface area

- The **total surface area** (TSA) of a 3-dimensional (3D) object is the total area of each outer face of that object.
- The total surface area of a 3D object can be calculated by finding the area of each individual face and adding them together.
- Drawing a net of a solid can help you determine the individual shapes of the faces that make up the solid.
- As with area, total surface area is measured using squared units (mm^2, cm^2, m^2 or km^2).

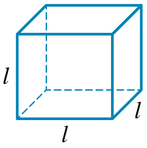
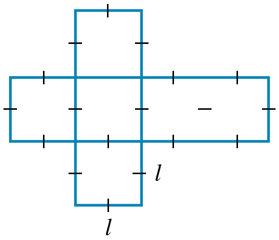
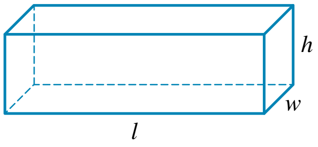
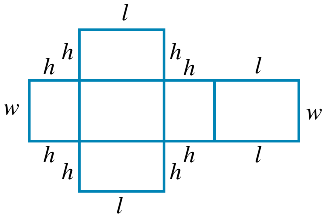
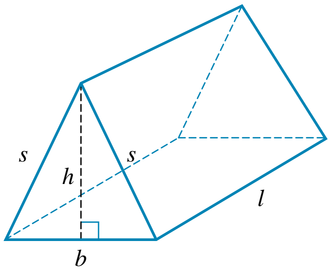
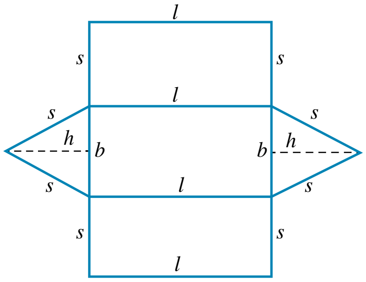


3.7.2 Total surface area of prisms

int-6079
int-2771
int-4531

- A prism is a 3D shape with identical opposite ends joined by straight edges.
- The total surface area of some common 3D objects can be calculated using appropriate formulae.

eles-1909

Prism	Net	Total surface area (TSA) formula
Cube 		$\text{TSA} = 6l^2$
Rectangular prism 		$\text{TSA} = 2(lh + lw + wh)$
Triangular prism 		$\text{TSA} = bh + 2ls + bl$ Note: 2 × area of a triangle $\left(\frac{1}{2}bh\right) = bh$

Total surface area of prisms

$$TSA_{\text{Cube}} = 6l^2$$

$$TSA_{\text{Rectangular prism}} = 2(lh + lw + wh)$$

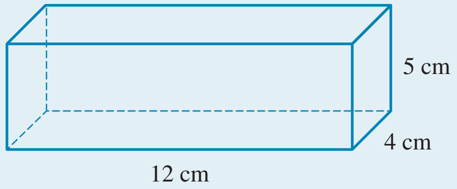
$$TSA_{\text{Triangular prism}} = bh + 2ls + bl$$

tlvd-12158

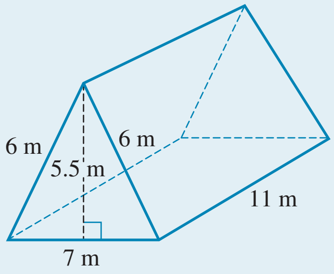
WORKED EXAMPLE 19 Calculating the total surface area of prisms

Calculate the total surface of the following prisms.

a.



b.



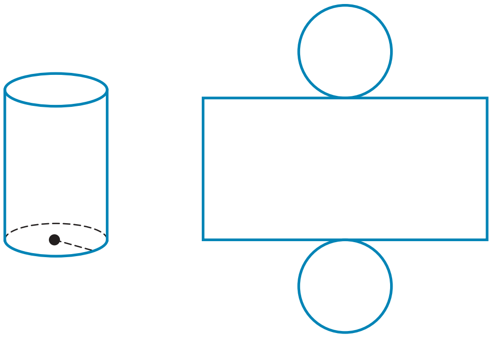
THINK	**WRITE**
a. 1. Identify the object as a rectangular prism, and write the appropriate total surface area (TSA) formula.	**a.** $TSA_{\text{Rectangular prism}} = 2(lh + lw + wh)$
2. Identify the dimensions of the object.	$l = 12\text{ cm}, w = 4\text{ cm}$ and $h = 5\text{ cm}$
3. Substitute the length, width and height values into the TSA formula and calculate the TSA of the rectangular prism.	$TSA_{\text{Rectangular prism}} = 2(12 \times 5 + 12 \times 4 + 4 \times 5)$ $= 2(60 + 48 + 20)$ $= 2(128)$ $= 256$
4. Write your answer with the correct units.	The total surface area of the rectangular prism is 256 cm^2.
b. 1. Identify the object as a triangular prism, and write the appropriate TSA formula.	**b.** $TSA_{\text{Triangular Prism}} = bh + 2ls + bl$
2. Identify the dimensions of the object.	$b = 7\text{ m}, s = 6\text{ m}, h = 5.5\text{ m}$ and $l = 11\text{ m}$
3. Substitute the values into the TSA formula and calculate the TSA of the triangular prism.	$TSA_{\text{Triangular Prism}} = 7 \times 5.5 + 2 \times 11 \times 6 + 7 \times 11$ $= 247.5\text{ m}^2$
4. Write your answer with the correct units.	The total surface area of the triangular prism is 247.5 m^2 .

3.7.3 Total surface area of a cylinder

int-4532

int-6080

- Drawing the net of a cylinder can help determine the formula for the total surface area.



- A cylinder is made up of two circles and a curved surface (which when rolled out and flattened becomes a rectangle).

Total surface area of a cylinder

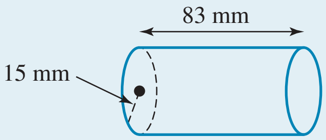
The formula for the total surface area of a cylinder is:

$$\mathbf{TSA_{Cylinder} = 2\pi rh + 2\pi r^2}$$

tlvd-12159

WORKED EXAMPLE 20 Calculating the total surface area of cylinders

Calculate the total surface area of the following cylinder, correct to two decimal places.



THINK	WRITE
1. Identify the object as a cylinder, and write the appropriate total surface area (TSA) formula.	$TSA_{Cylinder} = 2\pi rh + 2\pi r^2$
2. Identify the radius and the height of the cylinder.	$r = 15$ mm, $h = 83$ mm
3. Substitute the value of the radius and height into the TSA formula and calculate the TSA of the cylinder.	$TSA_{Cylinder} = 2\times\pi\times15\times83 + 2\times\pi\times15^2$ $= 2490\pi + 450\pi$ $= 2940\pi$ $\approx 9236.282\,401\,55$
4. Write your answer using the correct units.	The total surface area of the cylinder is 9236.28 mm^2.

Exercise 3.7 Surface areas of prisms and cylinders [complex]

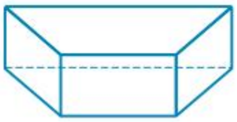
3.7 Quick quiz | **3.7 Exercise**

Simple familiar	Complex familiar	Complex unfamiliar
1	2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13	14, 15

Simple familiar

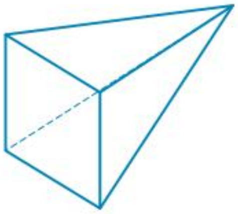
1. Identify the shapes below that are prisms.

a.

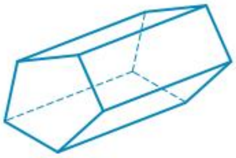


b.

c.



d.



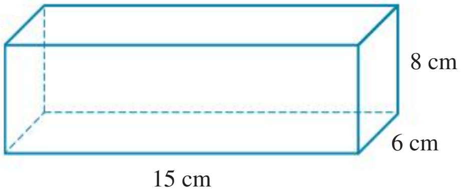
Complex familiar

2. Calculate the total surface area of a cube with the following side lengths.

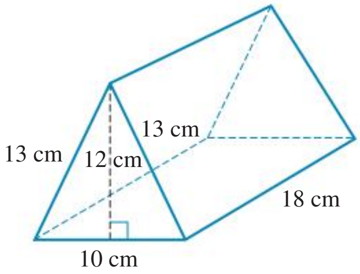
a. 4 cm **b.** 12.5 mm **c.** 0.85 m **d.** $7\frac{3}{4}$ cm

3. **WE19** Calculate the total surface area of the following prisms.

a.

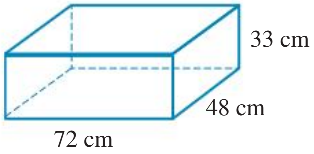


b.

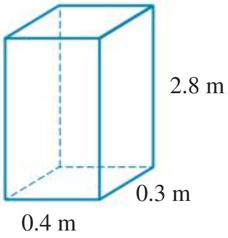


4. Calculate the total surface area of the following rectangular prisms.

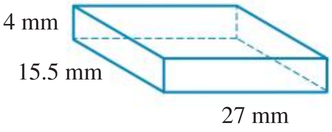
a.



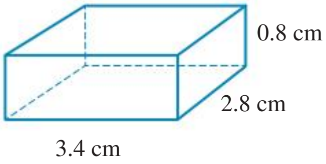
b.



c.

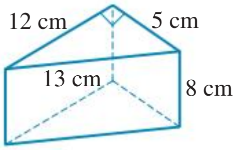


d.

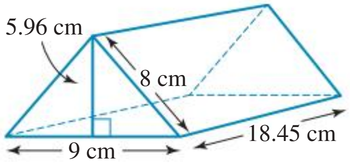


5. For each of the following triangular prisms, calculate:
   - i. the area of the triangular face
   - ii. the area of each of the three rectangular faces
   - iii. the total surface area.

a.

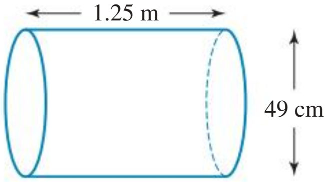


b.

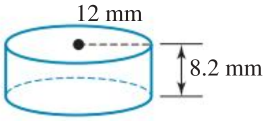


6. **WE20** Calculate the total surface area of the following cylinders, correct to two decimal places.

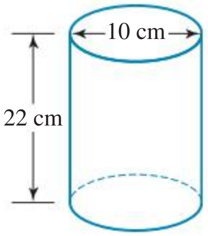
a.



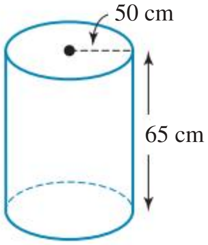
b.



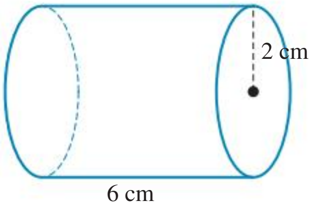
c.



d.



7. Calculate the total surface area of the following cylinder, correct to two decimal places.



8. Calculate the side length of a cube with a total surface area of 384 m^2.

9. Calculate the total surface area of the tin of tennis balls shown, correct to two decimal places, in:
   - a. mm^2
   - b. cm^2.

10. Calculate the surface area of a cylinder with radius of 9 m and a height of 12.5 m, correct to one decimal place.

11. A cylindrical water tank has a radius of 4.2 m and a height of 7.8 m. Calculate the total surface area of the tank, including the top and bottom surfaces, to the nearest whole number.

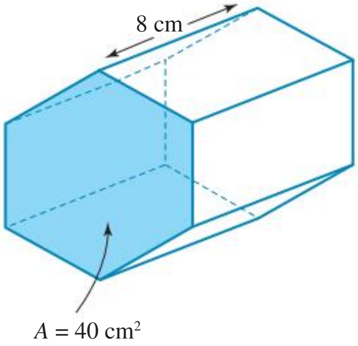
12. A storage container is shaped like a triangular prism. The triangular cross-section has a base of 5 m, a height of 4 m and a slant height of 6 m. The prism is 8 m long. Calculate the total surface area, including all faces.

13. A tent is in the shape of a triangular prism. The triangular cross-section has a base of 6 m, a height of 5 m and a slant height of 7 m. The length of the tent is 10 m. Calculate the total surface area of the tent, including all faces.

Complex unfamiliar

14. An outdoor storage unit is in the shape of a triangular prism. The triangular cross-section has a base of 9 m, a height of 6 m and a slant height of 10 m. The unit is covered with paint on all its faces, except for one of the triangular ends, which remains unpainted. The length of the prism is 12 m.
The cost of painting the triangular face is \$5 per square metre, and the cost of painting the rectangular faces is \$3 per square metre. Calculate the total cost of painting the storage unit.

15. A hexagonal-based prism is shown below. The sides of the hexagon are 3.924 cm. Based on this information, calculate the surface area of the prism, correct to two decimal places.



Fully worked solutions for this chapter are available online.

LESSON
3.8 Surface areas of pyramids and cones [complex]

SYLLABUS LINKS

- Calculate surface areas of pyramids and cones [complex].
  - cone: $S = \pi rs + \pi r^2$ where r is radius and s is slant height.

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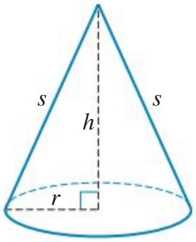
3.8.1 Total surface area of a cone

- The total surface area of a cone, when the radius r and the slant height s are known, can be found using the formula shown.
- The slant height is the measurement along the side of the cone, not the vertical height h of the cone.



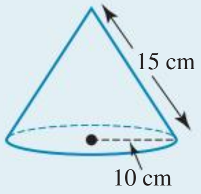
Total surface area of a cone

$$\text{TSA}_{\text{Cone}} = \pi rs + \pi r^2$$

tlvd-12160

WORKED EXAMPLE 21 Calculating the surface area of a cone

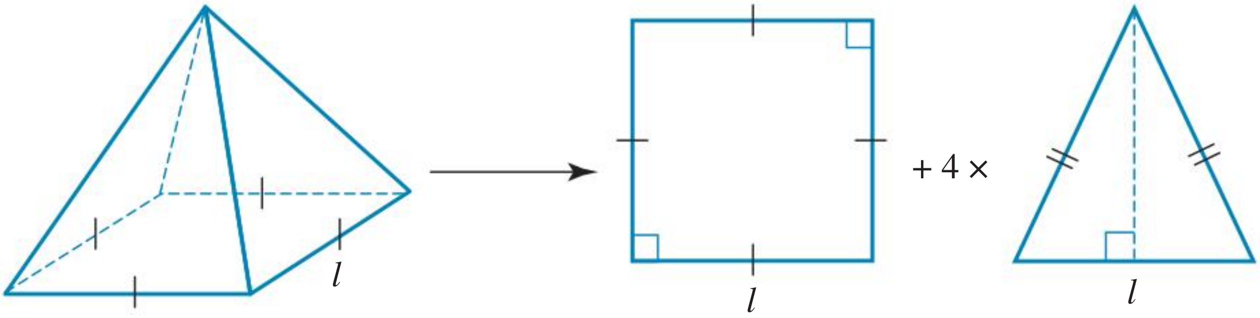
Calculate the total surface area of the cone shown, correct to two decimal places.



THINK	WRITE
1. Identify the shape as a cone and write the appropriate total surface area (TSA) formula.	$\text{TSA}_{\text{Cone}} = \pi rs + \pi r^2$
2. Identify the dimensions of the object.	$r = 10\text{ cm}$ and $s = 15\text{ cm}$
3. Substitute the values for the radius and the slant height into the TSA formula and calculate the TSA of the cone.	$\text{TSA}_{\text{Cone}} = \pi \times 10 \times 15 + \pi \times 10^2$ $= 150\pi + 100\pi$ $= 250\pi$ $= 785.398\,163\,4$ ≈ 785.40
4. Write your answer with the correct units.	The total surface area of the cone is 785.40 cm^2.

3.8.2 Total surface area of pyramids

- The total surface area of a pyramid can be calculated by adding the area of the base to the area of each of its sides.
- Taking the square-based pyramid as an example, its total surface area can be found as shown.



Total surface area of a square-based pyramid

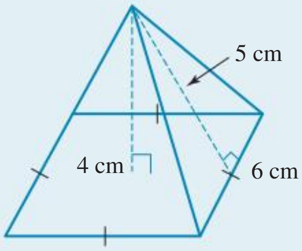
$$\text{TSA}_{\text{Square-based pyramid}} = A_{\text{Base}} + 4 \times A_{\text{Sides}}$$

- This formula can be adapted for rectangular-based pyramids and triangular-based pyramids.

tlvd-12161

WORKED EXAMPLE 22 Calculating the surface area of a pyramid

Calculate the surface of the following square-based pyramid.



THINK	**WRITE**
1. Calculate the area of the square base.	$A = l^2$ $= 6^2$ $= 36\,\text{cm}^2$
2. Calculate the area of a triangular side. Note that each side is identical and the height of each triangular side is 5 cm.	$A = \frac{1}{2}bh$ $= \frac{1}{2} \times 6 \times 5$ $= 15\,\text{cm}^2$
3. Calculate the total surface area. (*Note:* There are four identical triangular sides.)	$\text{TSA} = 36 + 4 \times 15$ $= 96\,\text{cm}^2$

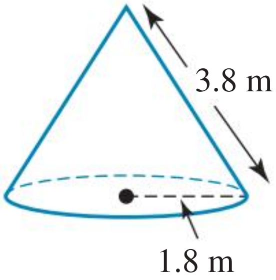
Exercise 3.8 Surface areas of pyramids and cones [complex]

learnon

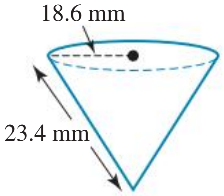
3.8 Quick quiz on	3.8 Exercise	
Simple familiar	**Complex familiar**	**Complex unfamiliar**
N/A	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11	12, 13, 14, 15, 16

Complex familiar

1. **WE21** Calculate the total surface area of the following cone, correct to two decimal places.

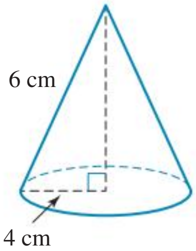


2. Calculate the total surface area of the following shape, correct to two decimal places.

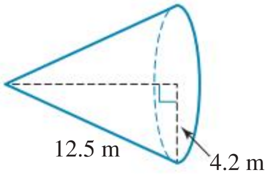


3. Calculate the total surface area of the following shapes, correct to two decimal places.

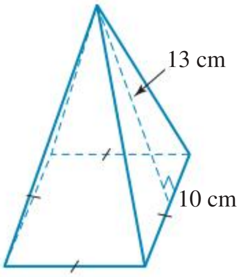
a.



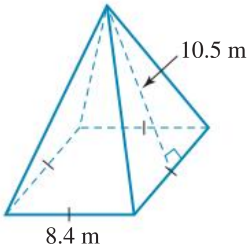
b.



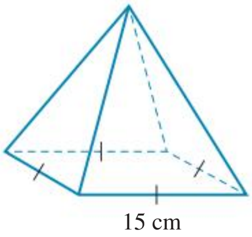
4. **WE24** Calculate the total surface of the square pyramid.



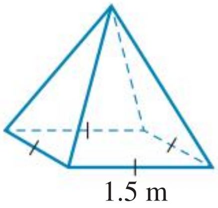
5. Calculate the total surface area of the square pyramid.



6. The following square-based triangular prism has four equal triangular faces. The perpendicular height of the triangle face is 9.37 cm. Calculate the total surface area.



7. The following square-based triangular prism has four equal triangular faces. The perpendicular height of the triangular face is 2.38 m. Calculate the total surface area.



8. A cone has a radius of 2 cm and a slant height of 5 cm. Calculate the total surface area of the cone, correct to two decimal places.
9. A cone has a radius of 4 cm and a slant height of 6 cm. Calculate the total surface area of the cone.
10. A square-based prism has a square base with a side length of 5 cm. Each triangular face has a perpendicular height of 7 cm. Calculate the total surface area of the prism.
11. A square-based triangular prism has four equal triangular faces. The perpendicular height of the triangular face is 6.8 m, with sides of the square being 3.2 m. Calculate the total surface area.

Complex unfamiliar

12. A square-based pyramid contains triangles with a perpendicular height of x cm. The side lengths of the square are 12 cm, and the total surface area is 340 cm^2. Calculate the length of x, correct to two decimal places.
13. Meleena is comparing two model square-based pyramids. The first pyramid has triangles with a perpendicular height of 12 cm, and the square base is 16 cm^2. The second pyramid has triangles with the same perpendicular height; however, the sides of the square base are 15 per cent greater. Determine the difference between the surface area of both pyramids.
14. The surface area of a cone is 650 m^2. The radius is 10 m; however, the slant height is unknown. Calculate the slant height, correct to two decimal places.
15. Two cone designs need to be compared in terms of surface area. Cone 1 has a slant height of 16 cm and a radius of 7 cm. In Cone 2 the values are swapped, so it has a slant height of 7 cm and a radius of 16 cm. Determine which design produces the greater surface area.
16. A glamping company, BushLife, wants to commission fully enclosed conical tents with a base diameter of 5 m and a slant height of 6 m. The manufacturer must calculate the fabric required for each tent, considering both the curved surface area and the circular base. If BushLife supplies the manufacturer with a 1000-m roll of 4-m wide fabric, determine how many tents can be produced.

Fully worked solutions for this chapter are available online.

LESSON
3.9 Surface areas of composite shapes and spheres [complex]

SYLLABUS LINKS

- Calculate surface areas of composite shapes and spheres [complex].
  - sphere: $S = 4\pi r^2$ where r is radius.

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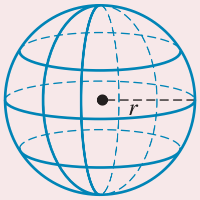
3.9.1 Total surface area of a sphere

- A sphere is a round solid figure, such as a tennis ball, soccer ball or basketball.

Total surface area of a sphere

The formula for the total surface area of a sphere is:

$$\mathbf{TSA_{Sphere} = 4\pi r^2}$$



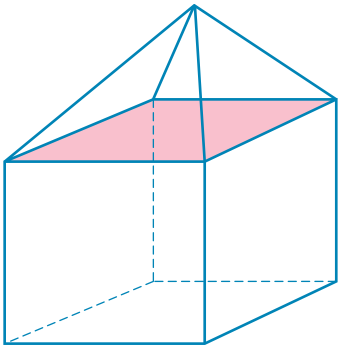
WORKED EXAMPLE 23 Calculating the area of a sphere

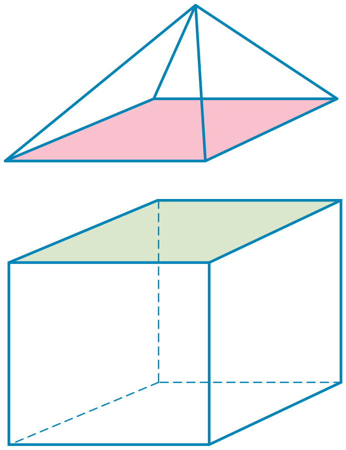
Calculate the total surface area of a basketball with a radius of 15 cm, correct to two decimal places.

THINK	WRITE
1. Identify the basketball as a sphere, and write the appropriate total surface area (TSA) formula.	$\text{TSA}_{\text{Sphere}} = 4\pi r^2$
2. Identify the radius of the sphere.	$r = 15\text{ cm}$
3. Substitute the value of the radius into the TSA formula and calculate the TSA of the basketball.	$\text{TSA}_{\text{Sphere}} = 4 \times \pi \times 15^2$ $= 4 \times \pi \times 225$ $= 900\pi$ $= 2827.433\,388$ $\approx 2827.43\text{ cm}^2$
4. Write your answer with the correct units.	The TSA of the basketball is 2827.43 cm^2.

3.9.2 Surface area of composite shapes

- Like the area of combined 2-dimensional (2D) shapes, some 3D shapes can be made by combining two or more regular 3D objects.
- In the example below, we can see the 3D object can be separated into a cube and a square-based pyramid as shown.



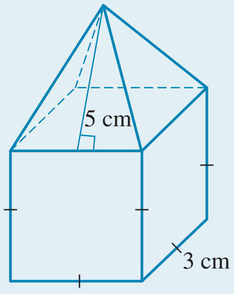


- *Note:*
  - When two 3D shapes are combined, adding the total surface area of the two individual shapes is generally not correct.
  - Referring to the cube and the square-based pyramid above, the top of the cube (green) and the base of the square-based pyramid (pink) sit on top of each other and are therefore no longer an outside surface.
  - Remember that the total surface area is the area of all the outside surfaces (i.e. the surfaces that can be touched).

tlvd-12162

WORKED EXAMPLE 24 Calculating the surface area of composite shapes

Calculate the total surface area of the following shape.



THINK	**WRITE**
1. The total surface area of the shape is made up of five faces of the cube and four sides of the pyramid.	$\text{TSA} = 5 \times A_{\text{Cube sides}} + 4 \times A_{\text{Pyramid sides}}$
2. Calculate the area of one cube face.	$A_{\text{Cube side}} = l^2$ $= 3 \times 3$ $= 9\,\text{cm}^2$

3. Calculate the area of one pyramid face.

$$\begin{aligned} A_{\text{Pyramid sides}} &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 3 \times 5 \\ &= 7.5\text{ cm}^2 \end{aligned}$$

4. Calculate the total surface area of the shape.

$$\begin{aligned} \text{TSA} &= 5 \times A_{\text{Cube sides}} + 4 \times A_{\text{Pyramid sides}} \\ &= 5 \times 9 + 4 \times 7.5 \\ &= 45 + 30 \\ &= 75 \end{aligned}$$

5. Write your answer with the correct units.

The total surface area of the shape is 75 cm^2.

Exercise 3.9 Surface areas of composite shapes and spheres [complex]

learnon

3.9 Quick quiz on

3.9 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
N/A	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13	14, 15, 16

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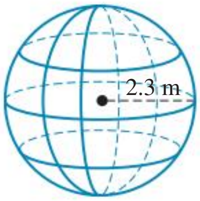
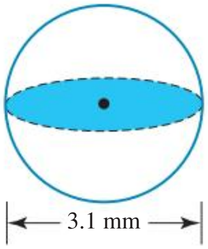
Complex familiar

1. **WE23** Calculate the total surface area of the following sphere, correct to two decimal places.

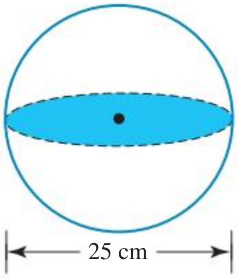


2. Calculate the total surface area of the following shapes, correct to two decimal places.

a.



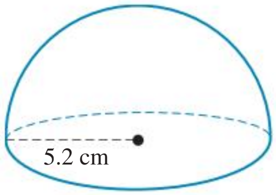
b.



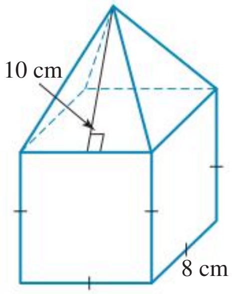
3. Calculate the total surface area of a tennis ball with a diameter of 6 cm, correct to two decimal places.

4. A sphere has a total surface area of 125 cm^2. Calculate the radius of the sphere correct to two decimal places.

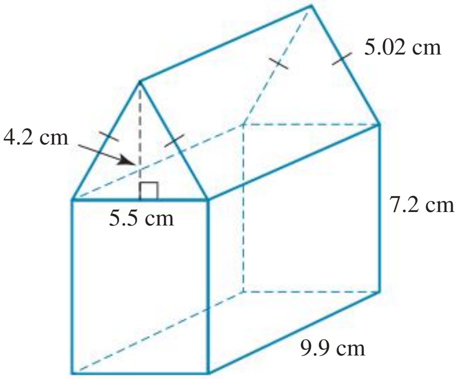
5. Calculate the surface area of the following shape, correct to two decimal places.



6. **WE24** Calculate the total surface area of the following shape.



7. Calculate the total surface area of the shape shown, correct to one decimal place.



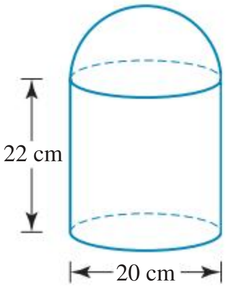
8. A 22 mm cube has a square-based pyramid on top of it, where the triangular faces have a 25.75 mm perpendicular height. Calculate the total surface area of the shape, correct to two decimal places.

9. Calculate the total surface area of a hemisphere of radius 6 cm that sits on a cylinder of radius 6 cm and height of 12 cm.

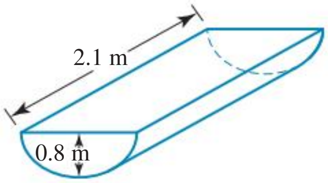
10. Calculate the total surface area of two cones joined together at their circular bases if they both have a diameter of 24 cm and a slant length of 18 cm.

11. Calculate the total surface area of the following shapes, correct to two decimal places.

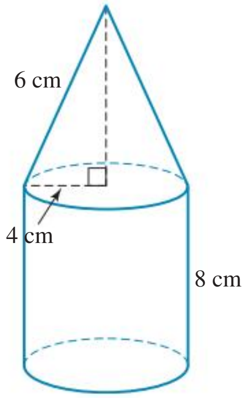
a.



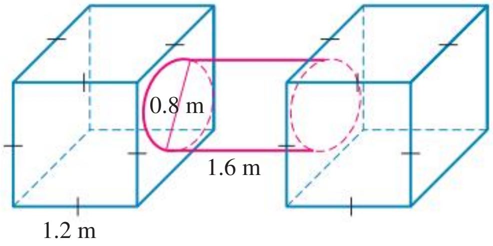
b.



12. Calculate the total surface area of the following shape, correct to two decimal places.



13. The drawing shown is a part of a child's playground toy. Calculate the total surface area, correct to two decimal places.



Complex unfamiliar

14. Calculate the total surface area of a cube with side length 15 cm that has a hole drilled through it with a diameter of 4 cm. Round the answer correct to two decimal places.

15. A large cylindrical concrete tank needs to be painted inside and outside (excluding the outside bottom). The cost of the paint per square metre is $17.50. If the height and the outer diameter of the tank are both 5 m and the inner diameter and height are both 4.5 m, calculate the total cost to paint the concrete tank, to the nearest dollar.

16. A caterer needs to create a three-tiered cake for a wedding. The diameters of the three cake tins they have are 30 cm, 25 cm and 20 cm. If each layer is 10 cm tall, calculate the total surface area that needs to be iced and decorated, correct to one decimal place.

Fully worked solutions for this chapter are available online.

LESSON
3.10 Review

3.10.1 Summary

Hey students! Now that it's time to revise this chapter, go online to:

- Access the chapter summary
- Review your results
- Watch teacher-led videos
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3.10 Exercise

learnon

3.10 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6	7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18	19, 20

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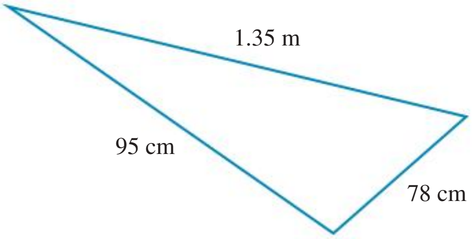
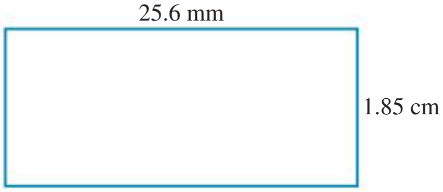
Simple familiar

1. Calculate the perimeter of the following shape.

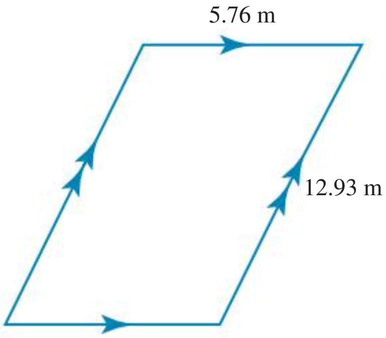


2. Calculate the area of the following shapes.

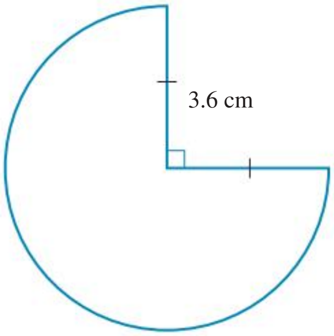
a.



b.



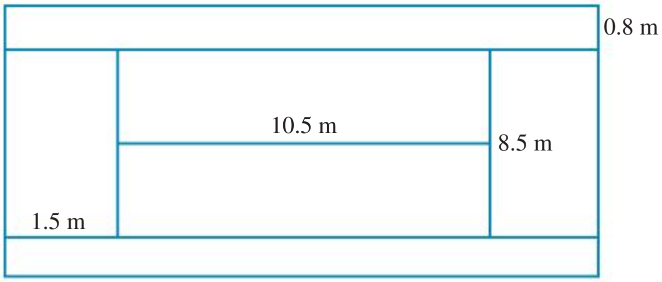
3. Calculate the arc length of the following shape, correct to two decimal places.



4. Select the most appropriate unit (mm^2, cm^2, m^2 or km^2) for measuring each of the following areas.
   - a. 5-cent coin
   - b. School oval
   - c. The Great Barrier Reef
   - d. A pencil case

5. Convert the following metric units of length.
   - a. 5 km = _________ m
   - b. 30 000 mm = ________ m
   - c. 6 m = _________ cm
   - d. 400 000 000 cm = ___________ km

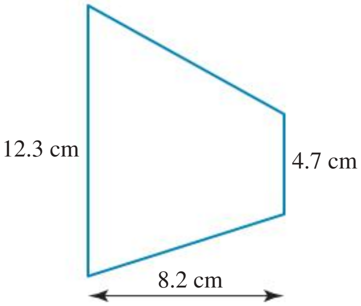
6. Tennis is played on a rectangular court, with dimensions as shown.



   - a. Determine what length of tape would be required to mark all the lines shown on the court.
   - b. Determine the area of the tennis court.

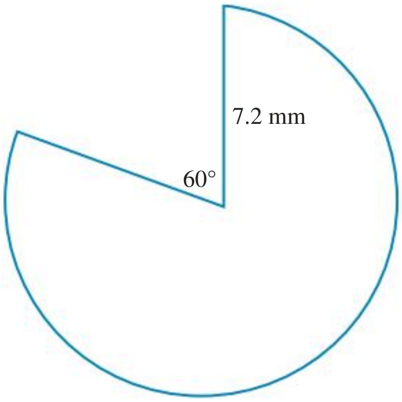
Complex familiar

7. Calculate the area of the following shape.

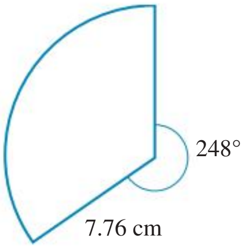


8. Calculate the area of the following shapes, correct to two decimal places.

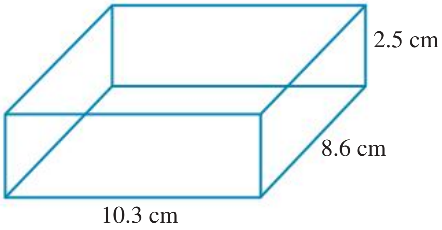
a.



b.



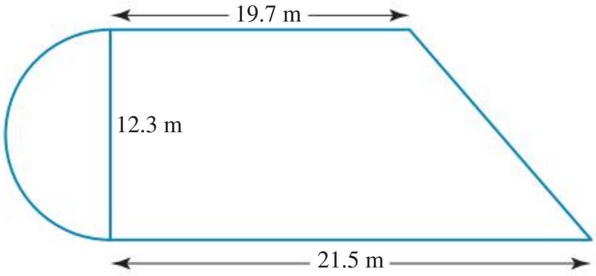
9. Calculate the total surface area of the following rectangular prism.



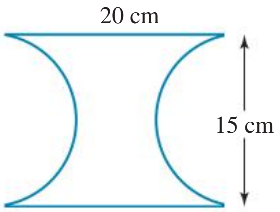
10. Calculate the total surface area of a sphere with a radius of 32 cm.

11. Calculate the area of the following shapes, correct to two decimal places.

a.

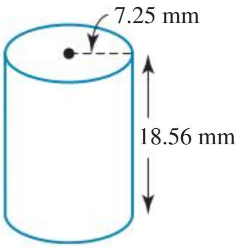


b.

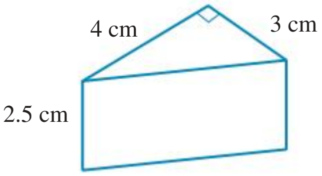


12. Calculate the total surface area of the following shapes, correct to two decimal places.

a.

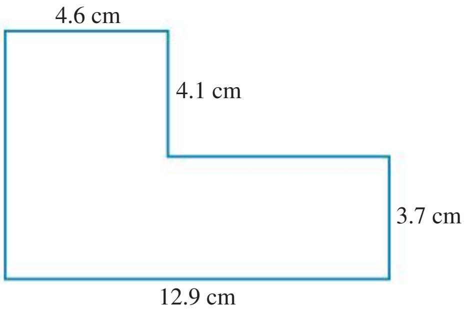


b.

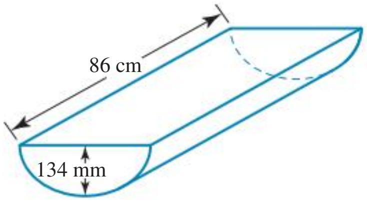


13. For the given shape:

a. calculate the perimeter
b. calculate the area.



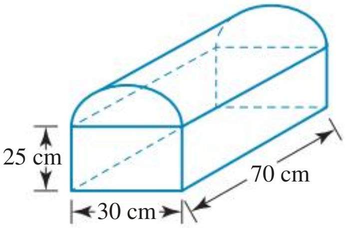
14. Determine the total surface area of the following shape, correct to two decimal places.



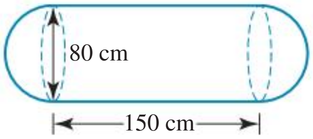
15. A cube of length 18 cm has a hole with a 6 cm diameter drilled through its centre. Calculate the total surface area of the new shape.

16. Calculate the total surface area of the following shapes, correct to one decimal place.

a.



b.



17. A rectangular garden bed has a length of 15.35 m and a width of 9.52 m. An 85-cm wide path is built on the outside of the garden bed, reducing the size of the garden bed.

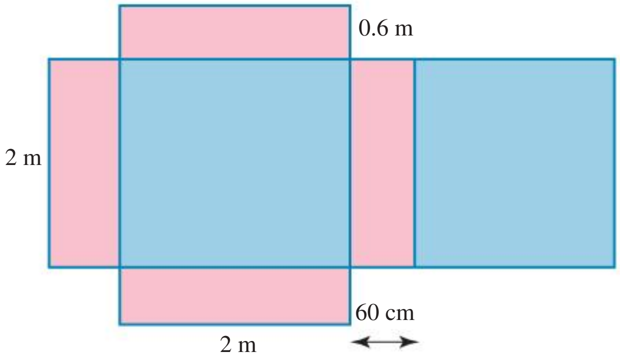
a. Calculate the perimeter of the garden bed before the path was built.
b. Calculate the perimeter of the garden bed after the path is built.
c. Determine the area of the garden bed once the path is built.

18. A tin contains four tennis balls, each with a diameter of 6.3 cm. The tin has a height of 27.2 cm and a diameter of 73 mm.

a. Determine the circumference of the lid.
b. Calculate the total surface area of the tin.
c. Calculate the total surface area of the four tennis balls.
d. Determine the difference between the total surface area of the four tennis balls and surface area of the tin.

Complex unfamiliar

19. If a rectangular prism can be made from the following net, determine if the total surface area of the prism is a greater numerical value than the perimeter.



20. An Olympic swimming pool is 50 m long, 25 m wide and 1.8 m deep.
When Haylee begins her swimming training the pool is completely full of water; however, it is leaking at a rate of 920 l/min. She completes one lap every minute. Determine the percentage volume of the pool that has leaked when Haylee has swum 2.5 km.

Fully worked solutions for this chapter are available online.

Answers

Chapter 3 Linear and area measure

3.2 Units of length and estimation

3.2 Exercise

1. a. Metres b. Centimetres
c. Kilometres d. Millimetres

2. a. 283 m b. 5.2 m c. 41.8 cm
d. 0.45 km e. 5500 m f. 78.459 m
g. 37 800 mm h. 0.065 km

3. 3750 mm

4. 95 600 m

5. 0.875 m

6. a. 270 m b. 73.5 m
c. 25.7 cm d. 130 000 cm

7. a. 15.18 m b. 6.08 m

8. Since Brisbane to Sydney is a large distance, you would use a large unit such as kilometres.

9. 177 cm

10. a. 2400 cm, 135 m, 0.15 km
b. 120 mm, 25 cm, 0.5 m
c. 9 m, 10 000 mm, 0.45 km
d. 32 000 cm, 1 km, 1200 m

11. a. 2.0 cm b. 154 cm
c. 219.5 cm d. 96.27 cm

12. a. 52 cm 1 mm or 52.1 cm b. 82 km 103 m or 82.103 km
c. 41 cm d. 87.7 m

13. a. The Q1 b. 25.5 m c. 619.5 m

14. a. 1.88 m
b. 1.05 cm
c. 2.21 cm
d. 187.75 cm
e. 1875 mm
f. West Coast (1875 mm) and St Kilda (1870 mm)

3.3 Perimeters of familiar and composite shapes

3.3 Exercise

1. a. 32 cm b. 32 cm
c. 55.4 m d. 40.42 mm

2. a. 38.4 cm b. 29.36 m
c. 71.24 m d. 46.29 mm

3. a. 34 cm b. 31 mm c. 75 m

4. a. 41 m b. 88 m
c. 90 m d. 250 cm

5. a. 4.8 m b. 120 m c. 200 cm

6. a. 690 cm b. 3.8 m

7. 25 cm

8. a. 35.19 cm b. 3314.38 mm

9. a. 18.85 cm b. 2.51 m c. 10.37 cm

10. a. 5.13 m b. 27.49 cm

11. a. 44.77 cm b. 45.08 mm

12. a. 4.62 cm b. 8.86 m
c. 12.15 m d. 47.81 cm

13. 15.92 cm

14. 414.69 cm

15. a. 46 cm b. 58 m c. 55 cm
d. 53.94 cm e. 40.28 m f. 81.68 cm

16. 57.11 cm

17. 66.8 cm

18. 6.30 m

3.4 Units of area and estimation

3.4 Exercise

1. a. m^2 b. mm^2 c. cm^2
d. m^2 e. km^2 f. mm^2

2. a. cm^2 b. m^2 c. ha
d. mm^2 e. km^2 f. m^2

3. There are many possible answers. Some examples are given.
a. A computer chip or face of a small diamond
b. A fridge magnet or dinner plate
c. A block of land or surface of a concrete slab
d. Tasmania or Kakadu National Park

4. a. 3400 mm^2 b. 1.4250 m^2 c. 5.92 km^2

5. a. 1800 cm^2 b. 79 800 m^2 c. 37.43 ha

6. a. 230 mm^2 b. 25 700 cm^2 c. 4.7 cm^2
d. 0.027 km^2 e. 8.75 ha f. 1.7 m^2

7. a. 5.12 ha b. 28 500 m^2 c. 83.8 km^2
d. 1260 ha e. 2300 m^2 f. 62.345 ha

8. a. 48 000 mm^2 b. 12 000 000 cm^2
c. 0.3008 m^2 d. 0.000 000 447 8 km^2

9. a. 5 squares = 5 cm^2 b. 9 squares = 9 cm^2
c. 10 squares = 10 cm^2 d. 15 squares = 15 cm^2

10. a. 54 cm^2 b. 7.8 mm^2
c. 323.57 m^2 d. 95.0625 cm^2

11. a. i. 34 220 cm^2
ii 3.422 m^2
b. i. 33.93 cm^2
ii 0.003 393 m^2
c. i. 20 400 cm^2
ii 2.04 m^2
d. i. 3375 cm^2
ii 0.3375 m^2

12. a. 18 squares b. 25 squares c. 15 squares

13. 108 m^2

14. 16.65 m^2

15. 20.12 m

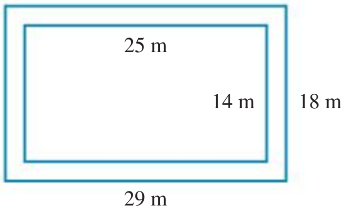
16. a. 80 cm^2

b. Width = 16 cm
Length = 20 cm

c. 320 cm^2

d. Multiplied by 4 or quadrupled

17. a.



b. 350 m^2

c. 172 m^2

d. $1290

18. a. 26.56 m^2 **b.** 79.68 m^2 **c.** 16 litres **d.** $252

3.5 Areas of standard shapes

3.5 Exercise

1. a. 20 cm^2 **b.** 48 m^2 **c.** 75 cm^2 **d.** 48 cm^2

2. 72 cm^2

3. 125 cm^2

4. a. 35 cm^2 **b.** 90 cm^2
c. 52 mm^2 **d.** 11.524 cm^2

5. a. 17.76 m^2 **b.** 37.5 mm^2

6. a. 78.54 cm^2 **b.** 66.48 cm^2
c. 122.72 cm^2 **d.** 128.68 m^2

7. 6 mm

8. a. Small = 314.16 cm^2

Medium = 706.86 cm^2

Large = 1256.64 cm^2

b. 113 883 cm^2

9. a. 6800 m^2 **b.** 3400
c. $35 700

10. 402.12 mm^2

11. 226.19 m^2

12. a. 75 cm^2 **b.** 13.75 m^2
c. 30 cm^2 **d.** 351 mm^2

13. 113.10 cm^2

14. 14.58 m^2

15. a. 84.82 cm^2 **b.** 99.53 m^2

16. $\theta = 159.15°$

3.6 Areas of composite figures [complex]

3.6 Exercise

1. a. Parallelogram and semi-circle

b. Square and four semi-circles

c. Trapezium and rectangle

d. Triangle and semi-circle

2. 120 cm^2

3. 23.03 cm^2

4. a. 48 m^2 **b.** 154 mm^2
c. 272 cm^2 **d.** 384 mm^2

5. a. 29.225 cm^2 **b.** 25.63 cm^2
c. 756 m^2 **d.** 42 651.41 mm^2

6. 156 cm^2

7. 109.87 cm^2

8. a. 40.18 mm^2 **b.** 31.49 cm^2

9. a. 360 cm^2

b. 360 cm^2

c. They are the same as you would expect.

10. 9600 mm^2

11. a. 3279.82 cm^2 **b.** 96.10 cm^2

12. a. 2025 cm^2 **b.** 943.41 cm^2 **c.** 6031.59 cm^2

13. $24.95

14. 28.54 cm^2

15. $198.42

3.7 Surface areas of prisms and cylinders [complex]

3.7 Exercise

1. a. Prism **b.** Not a prism
c. Not a prism **d.** Prism

2. a. 96 cm^2 **b.** 937.5 mm^2
c. 4.335 m^2 **d.** 360.375 cm^2

3. a. 516 cm^2 **b.** 768 cm^2

4. a. 14 832 cm^2 **b.** 4.16 m^2
c. 1177 mm^2 **d.** 28.96 cm^2

5. a. i. 30 cm^2

ii. 104 cm^2, 40 cm^2, 96 cm^2

iii. 300 cm^2

b. i. 26.82 cm^2

ii. 147.6 cm^2, 147.6 cm^2, 166.05 cm^2

iii. 514.89 cm^2

6. a. 2.30 m^2 **b.** 1523.04 mm^2
c. 848.23 cm^2 **d.** 36 128.32 cm^2

7. 100.53 cm^2

8. 8 m

9. a. 72 452.98 mm^2 **b.** 724.5298 cm^2

10. 1215.8 m^2

11. 317 m^2

12. 156 m^2

13. 230 m^2

14. $1179

15. 268.35 cm^2

3.8 Surface areas of pyramids and cones [complex]

3.8 Exercise

1. 31.67 m^2

2. 2454.21 mm^2

3. a. 125.66 cm^2 b. 220.35 m^2
4. 360 cm^2
5. 246.96 cm^2
6. 506.1 cm^2
7. 9.39 m^2
8. 43.98 cm^2
9. 125.66 cm^2
10. 95 cm^2
11. 53.76 m^2
12. 8.17 cm
13. 19.56 cm^2
14. 10.69 m
15. Cone 2 has the greater surface area.
16. 59 tents

3.9 Surface areas of composite shapes and spheres [complex]

3.9 Exercise

1. 66.48 m^2
2. a. 30.19 mm^2 b. 1963.50 cm^2
3. 113.10 cm^2
4. 3.15 cm
5. 254.85 cm^2
6. 480 cm^2
7. 344.3 cm^2
8. 3552.95 mm^2
9. 791.68 cm^2
10. 1357.17 cm^2
11. a. 2324.78 cm^2 b. 10.65 m^2
12. 326.73 cm^2
13. 20.30 m^2
14. 1513.36 cm^2
15. \$3388
16. 3063.1 cm^2

3.10 Review

3.10 Exercise

1. 308 cm
2. a. 473.6 mm^2 b. 76.48 m^2
3. 16.96 cm
4. a. mm^2 b. m^2
   c. km^2 d. cm^2
5. a. 5000 m b. 30 m
   c. 600 cm d. 4000 km
6. a. 101.7 m b. 136.35 m^2
7. 69.7 cm^2
8. a. 135.72 mm^2 b. 58.86 cm^2
9. 271.66 cm^2
10. 12 867.96 cm^2
11. a. 312.79 m^2 b. 123.29 cm^2
12. a. 1175.73 cm^2 b. 42 cm^2
13. a. 41.4 cm b. 66.59 cm^2
14. 0.65 m^2
15. 2226.74 cm^2
16. a. 11 105.53 cm^2 b. 57 805.30 cm^2
17. a. 49.74 m b. 42.94 m c. 106.743 m^2
18. a. 22.93 cm b. 707.50 cm^2
    c. 498.76 cm^2 d. 208.74 cm^2
19. The perimeter is a greater numerical value.
20. 2 per cent has leaked.

UNIT 3 Measurement, scales and chance
TOPIC 1 Measurement

4 Volume, capacity, time and mass

LESSON SEQUENCE

Fully worked solutions for this chapter are available online.

on Resources

Solutions	Solutions — Chapter 4
Digital documents	Learning matrix — Chapter 4 Quick quizzes — Chapter 4 Chapter summary — Chapter 4

LESSON
4.1 Overview

Hey students! Bring these pages to life online

Watch videos

Engage with interactivities

Answer questions and check results

Find all this and MORE in jacPLUS

4.1.1 Introduction

Understanding volume, capacity, time and mass is essential for students, as these concepts form the foundation of many real-world applications and advanced studies. In the sub-topic of volume and capacity, students learn to estimate and calculate the volumes and capacities of various objects. This includes understanding how to determine the volumes of different shapes such as prisms, cylinders, pyramids, cones and spheres.

Learning these calculations helps students develop critical thinking and problem-solving skills. Estimating and calculating volumes and capacities are not only vital in mathematics, but also in fields like engineering, architecture and environmental science. By mastering these concepts, students gain the ability to tackle complex problems, understand the physical world better and prepare for future academic and career opportunities.

4.1.2 Syllabus links

Lesson	Lesson title	Syllabus links
4.2	**Units of volume and capacity**	○ Estimate volumes and capacities of various objects. ○ Use metric units of volume (cubic millimetres, cubic centimetres, cubic metres, cubic kilometres), their abbreviations (mm^3, cm^3, m^3, km^3), conversions between them and appropriate choices of units ○ Understand and use the relationship between volume and capacity, recognising that $1\ \mathrm{cm}^3 = 1\ \mathrm{mL}$ (millilitre), $1000\ \mathrm{cm}^3 = 1\ \mathrm{L}$ (litre), $1\ \mathrm{m}^3 = 1\ \mathrm{kL}$ (kilolitre), $1000\ \mathrm{kL} = 1\ \mathrm{ML}$ (megalitre).
4.3	**Volumes and capacities of prisms and cylinders**	○ Calculate volumes and capacities of prisms and cylinders. • prism: $V = Ah$ where A is base area and h is perpendicular height • cylinder: $V = \pi r^2 h$ where r is radius and h is perpendicular height.
4.4	**Volumes and capacities of pyramids and cones**	○ Calculate volumes and capacities of pyramids and cones. • pyramid: $V = \frac{1}{3}Ah$ where A is base area and h is perpendicular height • cone: $V = \frac{1}{3}\pi r^2 h$ where r is radius and h is perpendicular height.
4.5	**Volumes and capacities of composite shapes and spheres [complex]**	○ Calculate volumes and capacities of composite shapes and spheres [complex]. • sphere: $V = \frac{4}{3}\pi r^3$ where r is radius.
4.6	**Units of time**	○ Use units of time and convert between fractional, decimal and digital representations.
4.7	**Units of mass**	○ Use metric units of mass (milligrams, grams, kilograms, tonnes), their abbreviations (mg, g, kg, t), conversions between them and appropriate choices of units.

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LESSON
4.2 Units of volume and capacity

SYLLABUS LINKS

- Estimate volumes and capacities of various objects.
- Use metric units of volume (cubic millimetres, cubic centimetres, cubic metres, cubic kilometres), their abbreviations (mm^3, cm^3, m^3, km^3), conversions between them and appropriate choices of units
- Understand and use the relationship between volume and capacity, recognising that $1\ cm^3 = 1$ mL (millilitre), $1000\ cm^3 = 1$ L (litre), $1\ m^3 = 1$ kL (kilolitre), 1000 kL $= 1$ ML (megalitre).

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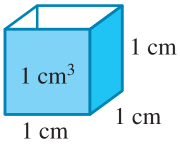
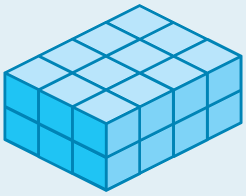
4.2.1 Metric units of volume

- The **volume** of a 3-dimensional object is the amount of space that can be occupied inside it. This can be measured by counting the number of cube-shaped units that will fit into the space that the object occupies.
- Volume can be used to describe the size of a pool or the space inside a room as examples.
- The units commonly used to describe the volume of an object are the cubic millimetre (mm^3), cubic centimetre (cm^3) and cubic metre (m^3). One cubic centimetre is the space occupied by a cube with side lengths equal to 1 cm.
- One cubic metre is the space occupied by a cube with side lengths equal to 1 m; however, something with a volume of one cubic metre can take many different shapes.



WORKED EXAMPLE 1 Estimate volume

Determine how many cubic centimetres are in the solid shape shown below, given each small cube represents 1 cm³.



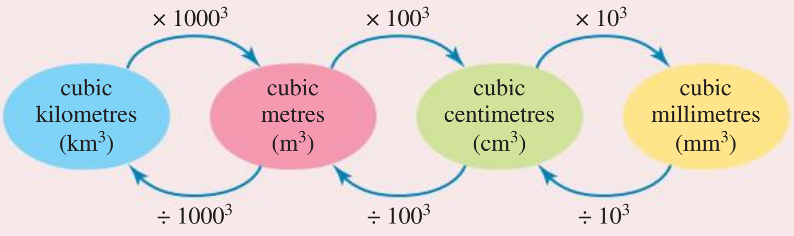
THINK	WRITE
1. Count the number of cubes. There are 12 cubes in each layer, and there are two layers.	Number $= 2 \times 12$ $= 24$ cubes
2. Each cube has a volume of $1\ cm^3$. Express the volume in terms of cm^3.	Volume $= 24\ cm^3$

4.2.2 Converting metric units of volume

- Converting units of volume means raising the conversion factor to the power of 3 because volume measures three dimensions: length, width and height.
- For example, when you convert from cubic centimetres to cubic metres, you multiply the conversion factor (100 cm = 1 m) for each dimension. So, (100^3) equals 1 000 000 cubic centimetres per cubic metre.

Converting units of volume

Metric units of volume can be converted according to the diagram below.



WORKED EXAMPLE 2 Converting units of volume

Complete the following unit conversions.

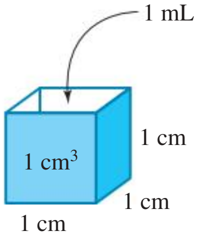
a. **$50\ cm^3 =$ _____ mm^3** **b.** **$4\ m^3 =$ _____ mm^3**

c. **$3\,000\,000\,000\ m^3 =$ _____ km^3**

THINK	WRITE
a. To convert cm^3 to mm^3, you need to multiply by 10^3.	**a.** $50 \times 10^3 = 50\,000\ mm^3$
b. **1.** To convert m^3 to mm^3, you need to first convert to cm^3 by multiplying by 100^3.	**b.** $4 \times 100^3 = 4\,000\,000\ cm^3$
2. To convert cm^3 to mm^3, you need to multiply by 10^3.	$4\,000\,000 \times 10^3 = 4\,000\,000\,000\ mm^3$ $4\ m^3 = 4\,000\,000\,000\ mm^3$
c. To convert m^3 to km^3, you need to divide by 1000^3.	**c.** $3\,000\,000\,000 \div 1000^3 = 3\ km^3$

4.2.3 Capacity

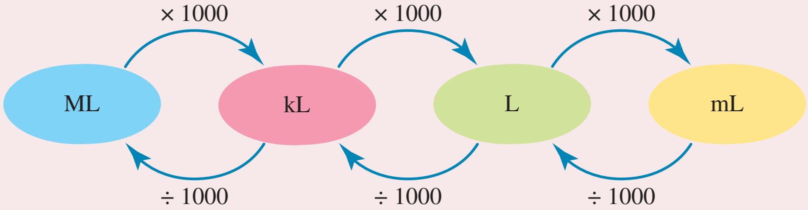
- If a 3-dimensional object is hollow, it can hold another substance. In this case the volume is also referred to as **capacity**.
- Capacity is usually used as a measure of the volume of liquid a container can hold.
- The units to describe capacity include the millilitre (mL), litre (L), kilolitre (kL) and megalitre (ML). For example, 1 L occupies a space of 1000 cm^3.



Converting units of capacity

Units of capacity are related as shown below.

1 L = 1000 mL
1 kL = 1000 L
1 ML = 1 000 000 L



WORKED EXAMPLE 3 Converting units of capacity

Complete the following unit conversions.
a. **6 L = _____ mL**
b. **4500 L = _____ kL**
c. **500 000 L = _____ ML**

THINK	WRITE
a. To convert L to mL, you need to multiply by 1000.	a. $6 \times 1000 = 6000$ mL
b. To convert L to kL, you need to divide by 1000.	b. $4500 \div 1000 = 4.5$ kL
c. 1. First you need to convert L to kL by dividing by 1000.	c. $500\,000 \div 1000 = 500$ kL
2. To convert kL to ML, you need to divide by 1000.	$500 \div 1000 = 0.5$ ML

4.2.4 Relationship between volume and capacity

- When calculating the capacity of an object, it is sometimes useful to calculate the volume of the object first, then convert to units of capacity.
- The metric unit, 1 cm^3, is defined as having a capacity of 1 mL. Therefore, volume and capacity units are related as shown below.

Relationship between volume and capacity

$$1\text{ cm}^3 = 1\text{ mL (millilitre)}$$
$$1000\text{ cm}^3 = 1000\text{ mL} = 1\text{ L (litre)}$$
$$1\text{ m}^3 = 1000\text{ L} = 1\text{ kL (kilolitre)}$$
$$1000\text{ kL} = 1\text{ ML (megalitre)}$$

WORKED EXAMPLE 4 Unit conversions

Complete the following unit conversions.

a. **70 mL = _______ $\mathbf{cm^3}$** **b.** **50 000 kL = _______ ML** **c.** **0.382 L = _______ $\mathbf{cm^3}$**

THINK	WRITE
a. There is 1 cm^3 in each 1 mL.	**a.** 1 mL = 1 cm^3 Therefore: 70 mL = 70 cm^3
b. There are 1000 kL in 1 ML, so to convert kilolitres to megalitres divide by 1000.	**b.** $50\,000\,\text{kL} = (50\,000 \div 1000)\ \text{ML}$ $= 50\,\text{ML}$
c. **1.** There are 1000 mL in 1 L, so to convert litres to millilitres multiply by 1000.	**c.** $0.382\,\text{L} = (0.382 \times 1000)\ \text{mL}$ $= 382\,\text{mL}$
2. Since 1 mL = 1 cm^3.	382 mL = 382 cm^3

Exercise 4.2 Units of volume and capacity

learn**on**

4.2 Quick quiz **on** | **4.2 Exercise**

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16	N/A	N/A

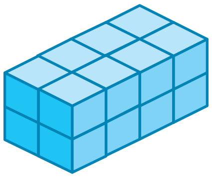
Simple familiar

1. State the appropriate unit (mm^3, cm^3, m^3) to measure the following volumes.

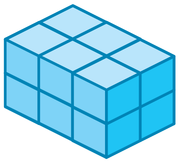
a. The volume of a Big M milk carton
b. The volume of a diamond on a ring
c. The volume of water in a swimming pool
d. The volume of a basketball

2. **WE1** Determine how many cubic centimetres are in the following solid shapes, given each cube represents 1 cm^3.

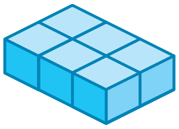
a.



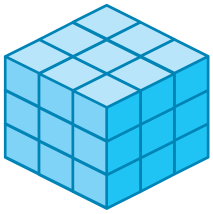
b.



c.

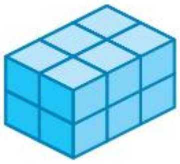


d.

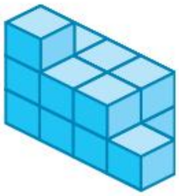


3. Determine how many cubic centimetres are in the following solid shapes.

a. (Each cube represents 2 cm^3.)

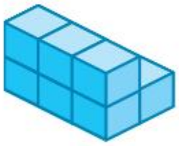


b. (Each cube represents 2 m^3.)

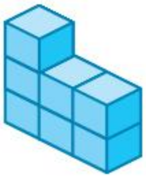


4. Determine the volume of the following shapes, given each cube represents 1 cm^3.

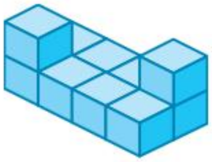
a.



b.



c.



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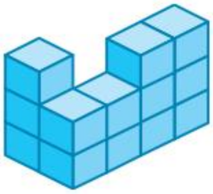
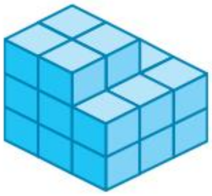
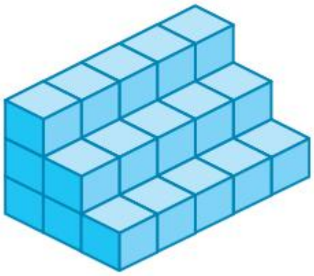


5. Determine the volume of the following shapes, given each cube represents 4 cm^3.

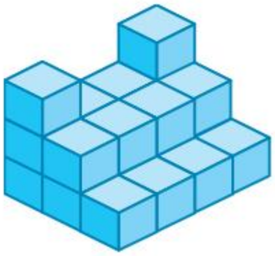
a.



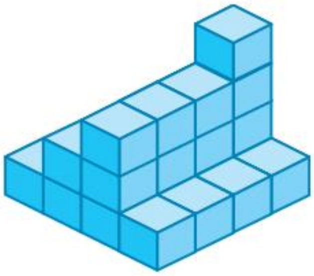
b.



c.



d.



6. State the difference between capacity and volume.

7. Estimate the volume and capacity of each of the following.

a. A bath
b. An Olympic-sized swimming pool
c. Five cans of Coke
d. Your classroom

8. State whether the following measures are a measure of volume or capacity.

a. 35 mL **b.** 120 m^3 **c.** 1.2 cm^3 **d.** 1750 kL **e.** 432 mm^3 **f.** 97.37 L

9. **WE2** Complete the following unit conversions.

a. 24 cm^3 = ______ mm^3
b. 2 m^3 = ______ mm^3
c. 40 000 000 000 m^3 = ______ km^3
d. 600 000 cm^3 = ______ m^3
e. 3200 mm^3 = ______ cm^3
f. 5 600 000 000 mm^3 = ______ m^3

10. **WE3** Complete the following unit conversions.

a. 12 L = __________ mL
b. 3125 L = __________ ML
c. 397 mL = __________ L
d. 0.0078 L = __________ mL
e. 4893 mL = __________ L
f. 36.97 L = __________ ML

11. **WE4** Complete the following unit conversions.

a. 372 cm^3 = __________ mL
b. 1630 L = __________ cm^3
c. 3.4 L = __________ cm^3
d. 0.38 mL = __________ cm^3
e. 163 L = __________ cm^3
f. 49.28 cm^3 = __________ mL

12. Complete the following unit conversions.

a. 578 mL = ________ L
b. 750 mL = ________ L
c. 0.429 L = ________ mL

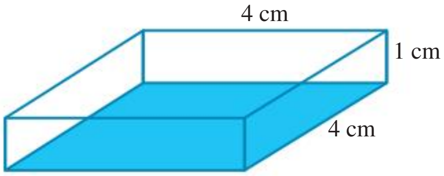
13. Complete the following unit conversions.

a. 47 cm^3 = ________ mL
b. 3594 cm^3= ________ L
c. 0.0042 L = ________ cm^3

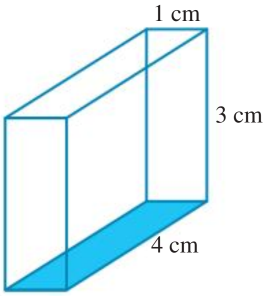
14. If the following shapes were broken up into 1 cm cubes:

i. Determine how many 1 cm cubes fit inside the shape.
ii. Calculate the volume of the shape in cm^3.
iii. Determine the shape's full capacity in millilitres.

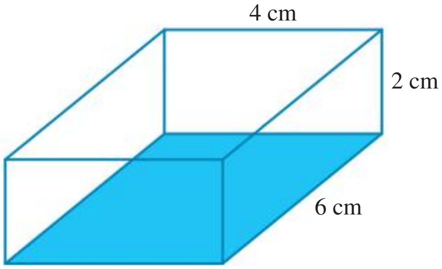
a.



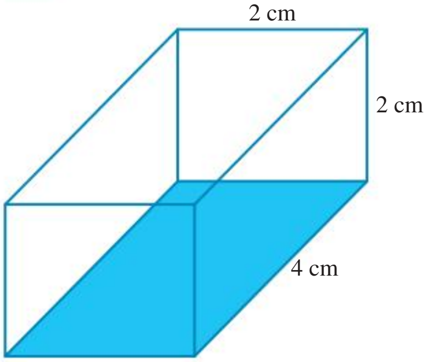
b.



c.



d.



15. A rectangular swimming pool is 12 m long, 4 m wide and 1.5 m deep.

a. Calculate how many 1 cm^3 would fit in the pool.
b. Determine the volume of the pool.
c. Determine the capacity of the pool, in litres, if it is full.
d. Determine the capacity of the pool if it is 80% full.

16. The milk container shown has dimensions of 10 cm × 5 cm × 20 cm.

a. Determine how many 1 cm^3 would fit in the milk container.
b. Calculate the volume of the container.
c. Determine the capacity of the container in litres.
d. Determine, in millilitres, how much milk is in the milk carton if it is $\frac{3}{4}$ full.

Fully worked solutions for this chapter are available online.

LESSON

4.3 Volumes and capacities of prisms and cylinders

SYLLABUS LINKS

- Calculate volumes and capacities of prisms and cylinders.
  - prism: $V = Ah$ where A is base area and h is perpendicular height
  - cylinder: $V = \pi r^2 h$ where r is radius and h is perpendicular height.

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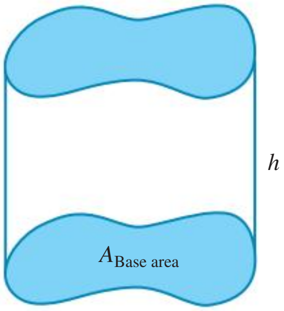
4.3.1 Volume of prisms

int-3794

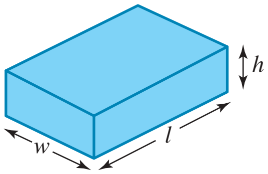
- A prism has the same cross-sectional area along its length, thus it has the same two end faces. The cross-sectional area is shaded in blue in the diagram shown.
- The volume of a prism is calculated by multiplying the cross-sectional area by its height.



Volume of a prism

$$V_{\text{Prism}} = A_{\text{Base area}} \times h$$

Volume of a rectangular prism



- The volume (V) of a rectangular prism follows the general rule of all prisms, hence multiply the cross-sectional area by its height.
  Taking the bottom face as the cross-sectional area, then:

$$V = A_{\text{Base area}} \times h$$
$$V = (l \times w) \times h$$

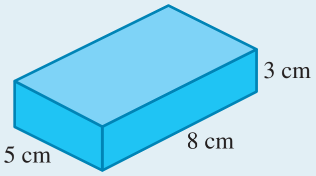
- This is the same as multiplying the three dimensions of the prism (length, width and height).

Volume of a rectangular prism

$$\mathbf{V_{\text{Rectangular prism}} = \text{length} \times \text{width} \times \text{height}}$$
$$\mathbf{V_{\text{Rectangular prism}} = l \times w \times h}$$

WORKED EXAMPLE 5 Volume of a rectangular prism

Determine the volume of the rectangular prism shown.



THINK	WRITE
1. Write the volume of a rectangular prism formula.	$V = l \times w \times h$
2. Substitute the values length = 8 cm, width = 5 cm and height = 3 cm into the formula.	$V = 8 \times 5 \times 3$
3. Calculate the volume and state the units.	$V = 120\,\text{cm}^3$

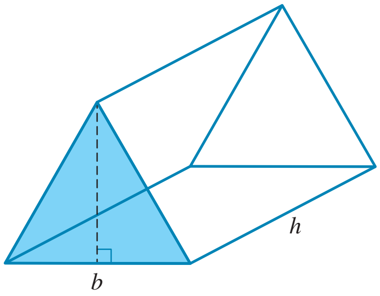
Volume of a triangular prism

The volume of a triangular prism can also be calculated by multiplying the cross-sectional area of the triangular face by its height.

Volume of a triangular prism

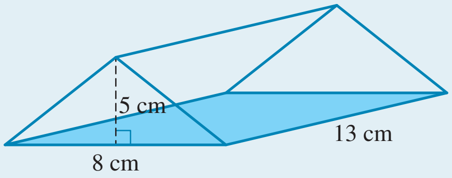
$$\mathbf{V_{\text{Triangular prism}} = A_{\text{Cross-sectional}} \times h}$$
$$\mathbf{= \text{area of triangle} \times h}$$

where the area of a triangle is $A = \frac{1}{2} \times$ base length × height of the triangle



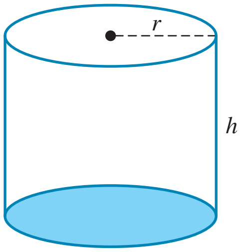
WORKED EXAMPLE 6 Volume of a triangular prism

Determine the volume of the triangular prism shown.



THINK	WRITE
1. Write the volume of a triangular prism formula.	$V = \text{area of triangle} \times h$
2. Recall that the area of a triangle is $A = \frac{1}{2} \times \text{base} \times \text{height}$. Be careful not to confuse the height of the triangle with the height of the entire object.	$V = \left(\frac{1}{2} \times 8 \times 5\right) \times 13$
3. Calculate the volume and state the units.	$V = 260\,\text{cm}^3$

4.3.2 Volume of a cylinder



- While the cylinder is not technically classed as a prism, its volume can be calculated in the same way as a prism since its cross-sectional area (the circle) is along its length.
- Its volume is calculated by:

$$\begin{aligned} V &= A_{\text{Cross-sectional}} \times \text{perpendicular height} \\ V &= A_{\text{Circle}} \times h \\ &= \pi r^2 \times h \\ &= \pi r^2 h \end{aligned}$$

Volume of a cylinder

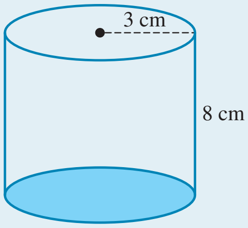
$$V_{\text{Cylinder}} = \pi r^2 h$$

where:

r = radius
h = perpendicular height

WORKED EXAMPLE 7 Volume of a cylinder

Determine the volume of the cylinder shown to two decimal places.



THINK	WRITE
1. Write the volume of a cylinder formula.	$V = \pi r^2 h$
2. Substitute the values radius = 3 cm and height = 8 cm into the formula.	$V = \pi \times 3^2 \times 8$ $= \pi \times 9 \times 8$
3. Calculate the volume and state the units.	$V = 226.19\text{ cm}^3$

Exercise 4.3 Volumes and capacities of prisms and cylinders

learnon

4.3 Quick quiz on

4.3 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16	N/A	N/A

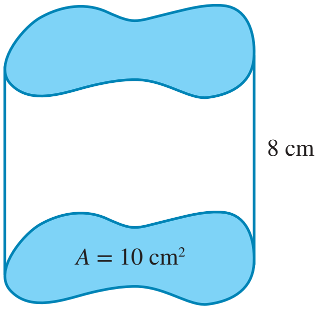
These questions are even better in jacPLUS!
- Receive immediate feedback
- Access sample responses
- Track results and progress

Find all this and MORE in jacPLUS

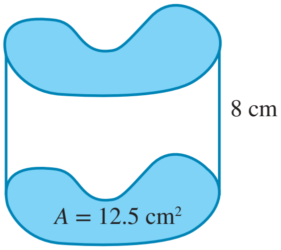
Simple familiar

1. Determine the volume of the following prisms.

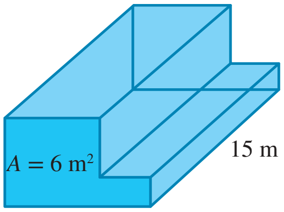
a.



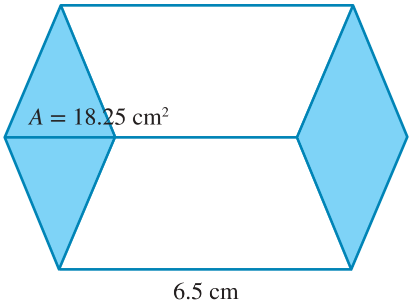
b.



c.

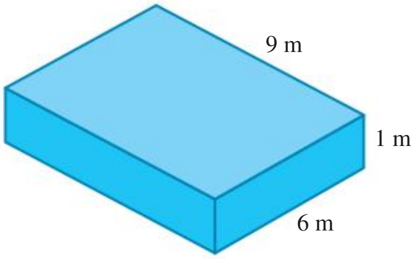


d.

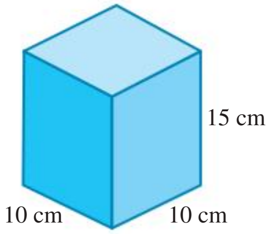


2. **WE5** Calculate the volume of the following rectangular prisms.

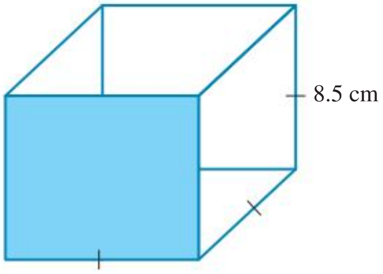
a.



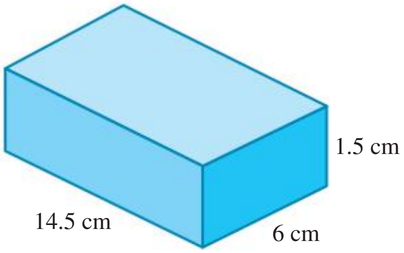
b.



c.

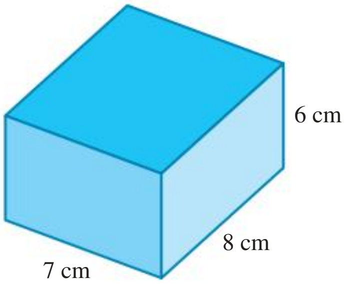


d.

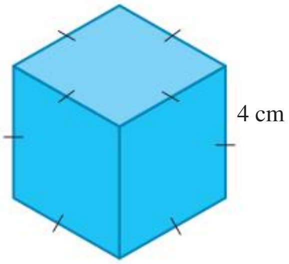


3. Calculate the volume of the following prisms.

a.

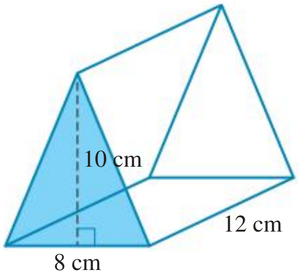


b.

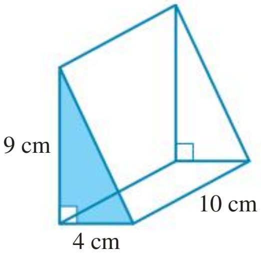


4. **WE6** Calculate the volume of the following triangular prisms.

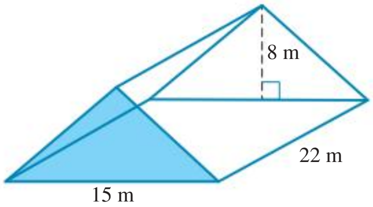
a.



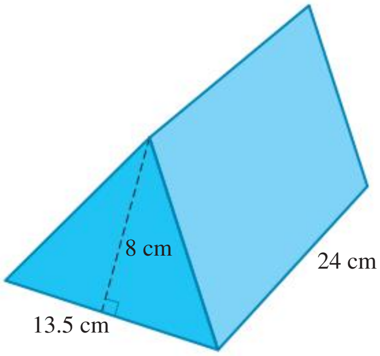
b.



c.

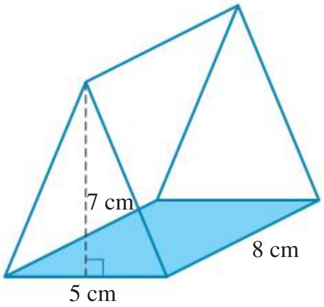


d.

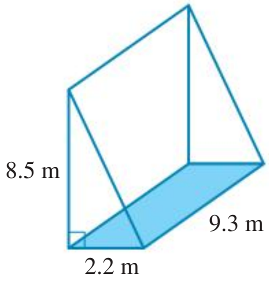


5. Calculate the volume of the following triangular prisms.

a.

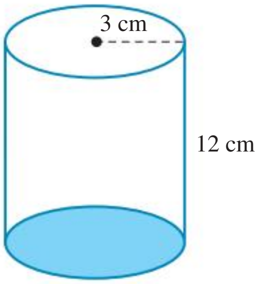


b.

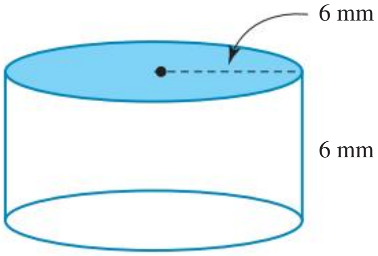


6. **WE7** Calculate the volume of the following cylinders correct to two decimal places.

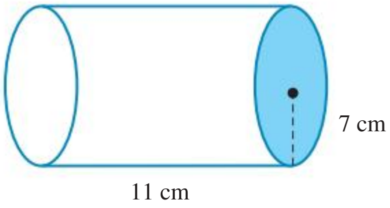
a.



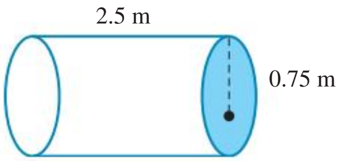
b.



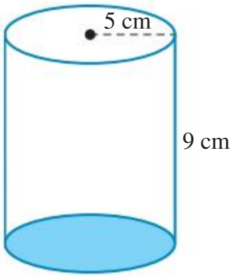
c.



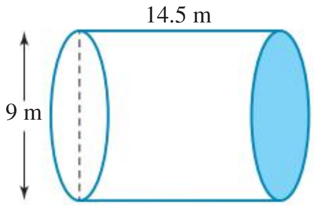
d.



7. Calculate the volume of the cylinder to two decimal places.

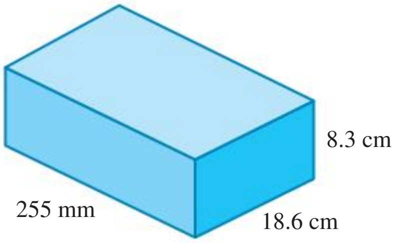


8. Calculate the volume of the following shape to two decimal places.

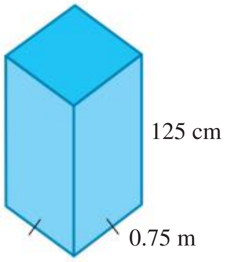


9. Calculate the volume of the following shapes correct to two decimal places.

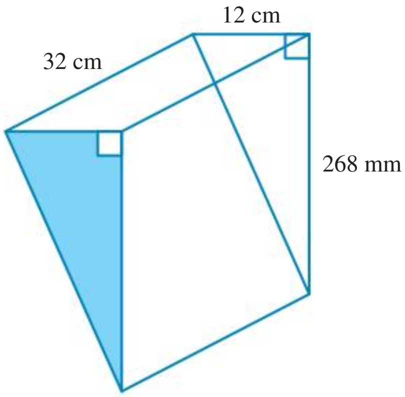
a.



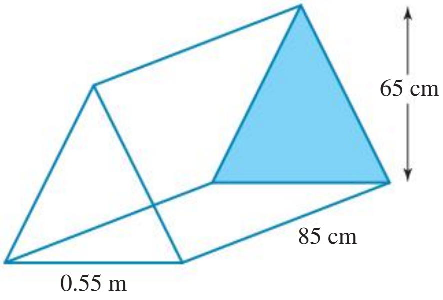
b.



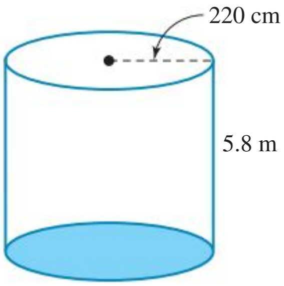
c.



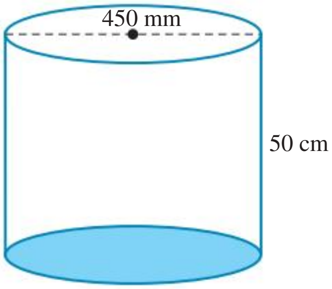
d.



e.



f.



10. A pool is made in the shape of an open-top rectangular prism of length 12 m, width 5 m and depth 1.85 m. Calculate the total volume of the pool in cubic metres, and how many litres it holds when full.

11. A cylindrical water tank has a diameter of 2.4 m and vertical height of 2.6 m. Calculate the volume of the water tank, to three decimal places, and determine its capacity when full.

12. A tent is made in the shape of a triangular prism. If the tent has a vertical height of 2.2 m, a width of 3.6 m and a length of 4.2 m, calculate the volume of the inside of the tent.

13. A cylinder has a volume of 245 cm^3 and a height of 6 cm. Determine the length of the radius of the cylinder to two decimal places.

14. A garden water feature has a base in the shape of a rectangular prism with dimensions length = 2 m, width = 1.5 m and height = 40 cm. Calculate the following.
   a. The volume of the base of the water feature.
   b. The volume of water, in m^3, in the base if it is filled to a height of 30 cm.
   c. The capacity of the water feature base when filled to a height of 25 cm in litres.

15. A road bike is packed to be sent interstate in a rectangular prism-shaped box with dimensions 2 m × 85 cm × 40 cm. If the bike takes up 30% of the volume of the box, answer the following.
   a. Determine the volume of the box in m^3.
   b. Determine the volume taken up by the road bike.
   c. After the bike was placed inside the box, the rest of the box was filled with packing foam. Determine the volume of packing foam that was used.

16. A rectangular shed has dimensions of 5.5 m long by 5.5 m wide and 2.5 m high.

   a. Determine the volume of the shed.
   b. If the shed is extended in length by 2 m, determine its new volume.
   c. If the width of the shed is extended (length remains 5.5 m) so that the volume of the shed is now 100 m^3, determine the new width of the shed.

Fully worked solutions for this chapter are available online.

LESSON
4.4 Volumes and capacities of pyramids and cones

SYLLABUS LINKS

- Calculate volumes and capacities of pyramids and cones.
  - pyramid: $V = \frac{1}{3}Ah$ where A is base area and h is perpendicular height
  - cone: $V = \frac{1}{3}\pi r^2 h$ where r is radius and h is perpendicular height.

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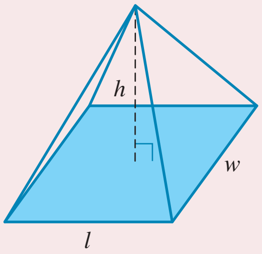
4.4.1 Volume of pyramids

- A pyramid has a base and triangular-shaped sides that meet at the top.
- A pyramid can have any polygon as the base, with the most common bases being squares, rectangles and triangles.
- These are the pyramids we will look at in this chapter. The general rule for the volume of a pyramid is as follows.

Volume of a pyramid

$$V_{\text{Pyramid}} = \frac{1}{3} \times \text{area of base} \times \text{height}$$

$$V_{\text{Pyramid}} = \frac{1}{3}Ah$$



- This general formula can be adapted to square-, rectangular- and triangular-based pyramids.

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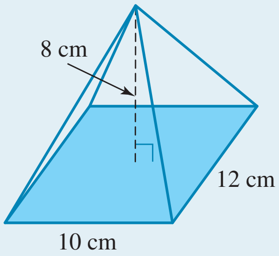
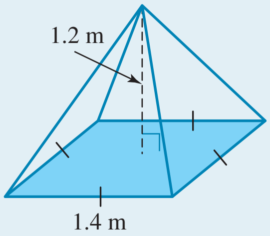
WORKED EXAMPLE 8 Volume of pyramids

Calculate the volume of the following pyramids.

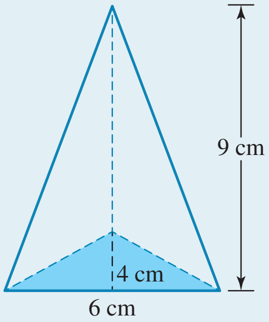
a.



b.

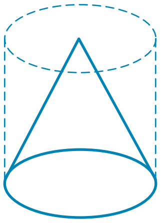


c.



THINK	WRITE
a. 1. Write the formula for the volume of a rectangular-based pyramid.	**a.** $V_{\text{Rectangular pyramid}} = \frac{1}{3} \times l \times w \times h$
2. Substitute the values length = 12 cm, width = 10 cm and height = 8 cm into the formula.	$V = \frac{1}{3} \times 12 \times 10 \times 8$
3. Calculate the volume and state the units.	$V = 320\text{ cm}^3$
b. 1. Write the formula of the volume of a square-based pyramid.	**b.** $V_{\text{Square pyramid}} = \frac{1}{3} \times l^2 \times h$
2. Substitute the values length = 1.4 cm and height = 1.2 cm into the formula.	$V = \frac{1}{3} \times 1.4^2 \times 1.2$
3. Calculate the volume and state the units.	$V = 0.784\text{ m}^3$
c. 1. Write the formula for the volume of a triangular-based pyramid.	**c.** $V_{\text{Triangular pyramid}} = \frac{1}{3}Ah$ $V = \frac{1}{3} \times \text{area of base} \times \text{height of pyramid}$
2. Calculate the area of the base by substituting the values.	$\text{Area of base} = \frac{1}{2}bh$ $= \frac{1}{2} \times 6 \times 4$ $= 12\text{ cm}^2$
3. Substitute the base area into the volume of a triangular-based pyramid formula.	$V = \frac{1}{3} \times 12 \times 9$
4. Calculate the volume and state the units.	$V = 36\text{ cm}^2$

4.4.2 Volume of a cone



- A cone is a pyramid with a circular base.
- Therefore, a cone has a similar formula for volume to a pyramid.

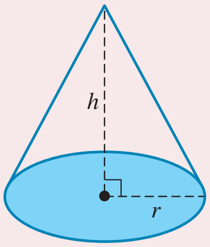
$$V_{\text{Pyramid}} = \frac{1}{3} \times \text{area of base} \times \text{height}$$

$$V_{\text{Pyramid}} = \frac{1}{3}Ah$$

- Since the area of the base of a cone is a circle, the area would be represented as πr^2.

Volume of a cone

$$V_{\text{Cone}} = \frac{1}{3}\pi r^2 h$$

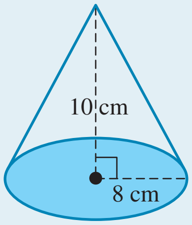


where:
***r* is the radius**
***h* is the perpendicular height**

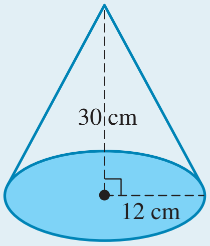
WORKED EXAMPLE 9 Calculating the volume of cones

Calculate the volume of each of the following cones, correct to two decimal places.

a.



b.



THINK	WRITE
a. 1. Write the formula for the volume of a cone.	**a.** $V = \frac{1}{3}\pi r^2 h$
2. Identify the values of *r* and *h*.	$r = 8$ $h = 10$
3. Substitute and evaluate, stating the units.	$V = \frac{1}{3} \times \pi \times 8^2 \times 10$ $= 670.21 \text{ cm}^3$
b. 1. Write the formula for volume of a cone.	**b.** $V = \frac{1}{3}\pi r^2 h$
2. Identify the values of *r* and *h*.	$r = 12$ $h = 30$
3. Substitute and evaluate, stating the units.	$V = \frac{1}{3} \times \pi \times 12^2 \times 30$ $= 4523.89 \text{ cm}^3$

Exercise 4.4 Volumes and capacities of pyramids and cones

learnon

4.4 Quick quiz on

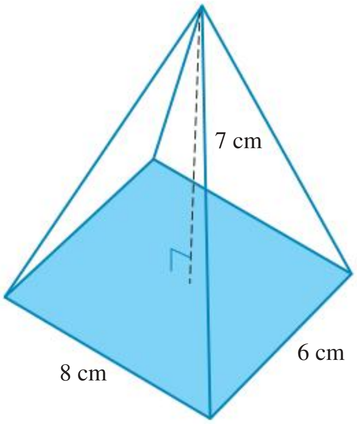
4.4 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16	N/A	N/A

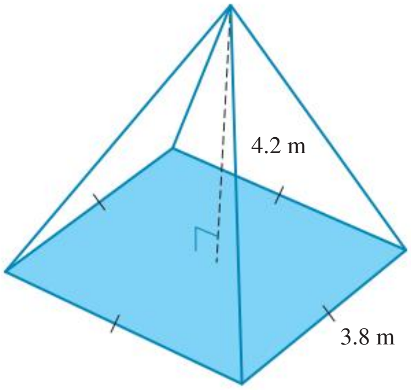
Simple familiar

1. Calculate the volume of the following rectangular-based pyramids, to two decimal places if appropriate.
   - a. Base length = 4 cm, base width = 5 cm and height = 6 cm
   - b. Base length = 12 cm, base width = 7 cm and height = 14 cm
   - c. Base length = base width = 6.5 cm and height = 10.2 cm
   - d. Base length = base width = 15.5 cm and height = 20.5 cm
   - e. Base length = 2.8 m, base width = 4.2 m and height = 6.9 m
   - f. Base length = 1.8 m, base width = 80 cm and height = 1.5 m

2. **WE8a** Calculate the volume of the pyramid shown.

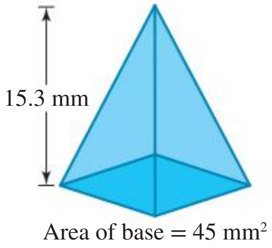


3. **WE8b** Calculate the volume of the pyramid shown.

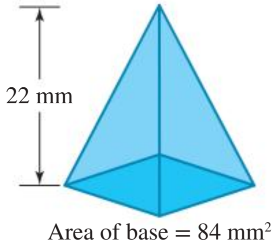


4. Calculate the volume of the following pyramids.

a.

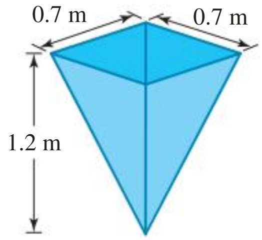


b.

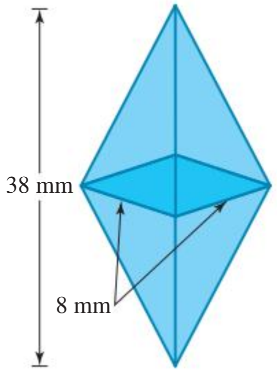


5. Calculate the volume of the following pyramids.

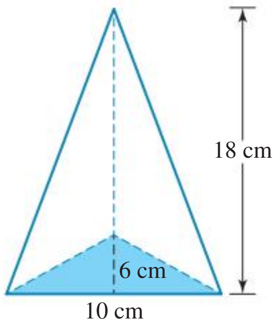
a.



b.

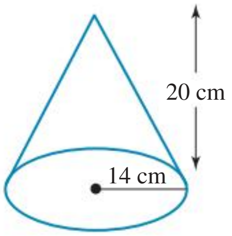


6. **WE8c** Calculate the volume of the pyramid shown.

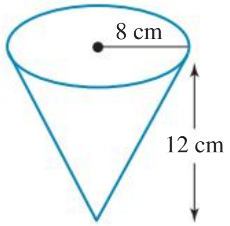


7. **WE9** Calculate the volume of each of the following cones, correct to two decimal places.

a.

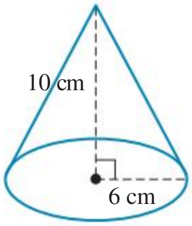


b.

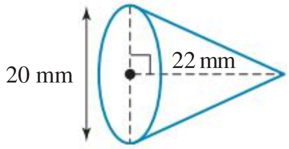


8. Calculate the volume of each of the following cones, correct to one decimal place

a.

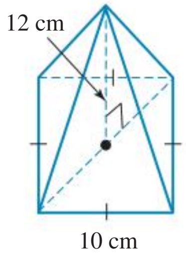


b.

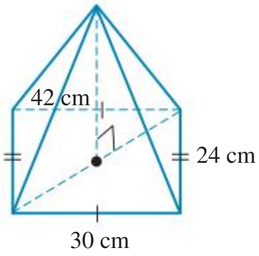


9. Calculate the volume of each of the following pyramids.

a.



b.



10. Calculate the volume of a square-based pyramid with a base length of 8 cm and a height of 6 cm.

11. Calculate the volume of a square-based pyramid with a base length and height of 9.6 m each.

12. Calculate the volume of a cone that has a radius of 4 m and a height of 6 m, correct to two decimal places.

13. Calculate the volume of a cone that has a diameter of 14 cm and a height of 9 cm, correct to two decimal places.

14. If a square-based pyramid has a height of 12 cm and a volume of 128 cm^3. Calculate the length of the square base, correct to two decimal places.

15. If a square-based pyramid has a volume of 375 mm^3 and a length of base 15 mm, determine the height of the pyramid.

16. If a triangular-based pyramid has a volume of 80 m^3, a base length of 6 m and a base height of 8 m, determine the height of the pyramid.

Fully worked solutions for this chapter are available online.

LESSON

4.5 Volumes and capacities of composite shapes and spheres [complex]

SYLLABUS LINKS

- Calculate volumes and capacities of composite shapes and spheres [complex].
  - sphere: $V = \frac{4}{3}\pi r^3$ where r is radius.

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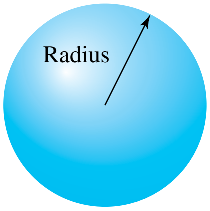
4.5.1 Volume of spheres

- A sphere is a 3-dimensional circular shape, such as a ball.
- The volume of a sphere can be calculated by:



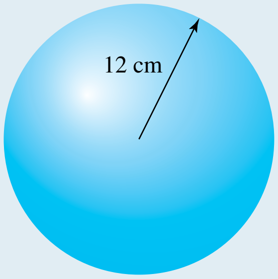
Volume of a sphere

$$V_{\text{Sphere}} = \frac{4}{3}\pi r^3$$

where: r = radius

WORKED EXAMPLE 10 Calculating the volume of a sphere

Calculate the volume of the following sphere.



THINK	WRITE
1. Write the volume of a sphere formula.	$V_{\text{Sphere}} = \frac{4}{3}\pi r^3$
2. Substitute $r = 12$ cm into the formula.	$V = \frac{4}{3} \times \pi \times 12^3$
3. Calculate the volume and state the units.	$V = 7238.23\text{ cm}^3$

4.5.2 Volume of hemispheres

- A hemisphere is half of a sphere.
- The volume of a hemisphere is therefore half the volume of a sphere.

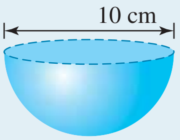
$$V = \frac{1}{2} \times \text{volume of a sphere}$$

$$V = \frac{1}{2} \times \frac{4}{3}\pi r^3$$

$$V = \frac{2}{3}\pi r^3$$

WORKED EXAMPLE 11 Calculating the volume of a hemisphere

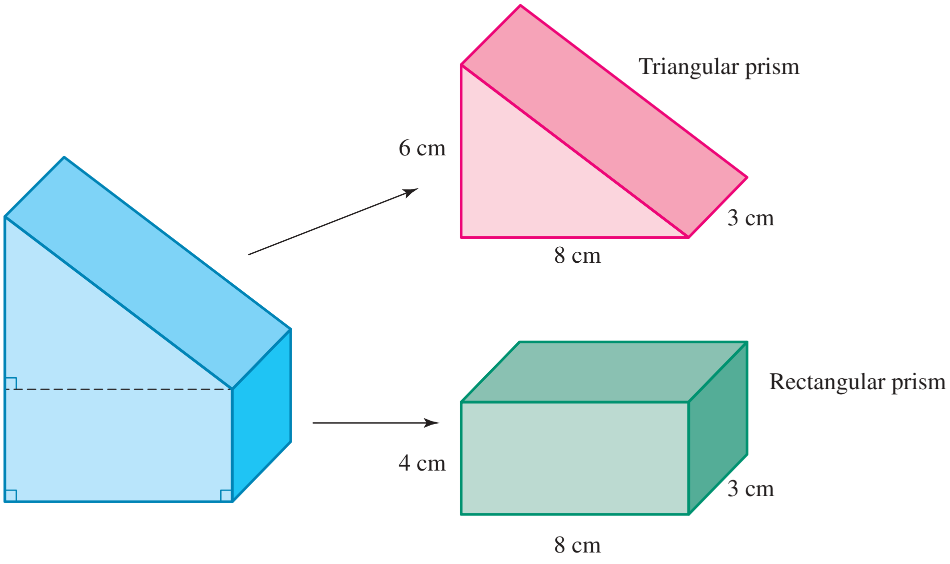
Calculate the volume of the following hemisphere, correct to two decimal places.



THINK	WRITE
1. Write the volume of a hemisphere formula.	$V = \frac{2}{3}\pi r^3$
2. Substitute $r = 10$ cm into the formula.	$V = \frac{2}{3} \times \pi \times 5^3$
3. Calculate the volume and round your answer to two decimal places.	$V = 261.7993...$ $= 261.80 \text{ cm}^3$

4.5.3 Volumes of composite solids

- A composite solid is a combination of a number of solids.
- Calculate the volume of each solid separately.
- Sum these volumes to give the volume of the composite solid.



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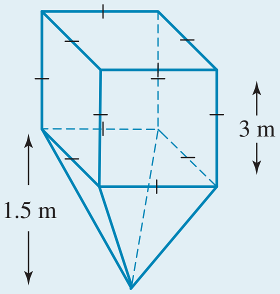
WORKED EXAMPLE 12 Calculating the volume of a composite solid

Calculate the volume of the composite solid shown.



THINK	WRITE
1. The given solid is a composite figure, made up of a cube and a square-based pyramid.	$V = \text{volume of cube} + \text{volume of pyramid}$
2. Calculate the volume of the cube.	$V_{\text{Cube}} = l^3 \text{ where } l = 3$ $= 3^3$ $= 27\text{ m}^3$
3. Write the formula for the volume of a square-based pyramid.	$V_{\text{Square-based pyramid}} = \frac{1}{3}Ah$
4. Calculate the area of the square base.	$A = l^2$ $= 3^2$ $= 9\text{ m}^2$
5. Identify the value of *h*.	$h = 1.5$
6. Substitute and evaluate the volume of the pyramid.	$V_{\text{Square-based pyramid}} = \frac{1}{3} \times 9 \times 1.5$ $= 4.5\text{ m}^3$
7. Calculate the total volume by adding the volume of the cube and pyramid.	$V = 27 + 4.5$ $= 31.5\text{ m}^3$

Exercise 4.5 Volumes and capacities of composite shapes and spheres [complex]

4.5 Quick quiz

4.5 Exercise

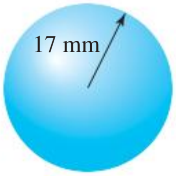
Simple familiar	Complex familiar	Complex unfamiliar
N/A	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	13, 14, 15

Complex familiar

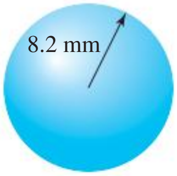
1. Calculate the volume of the following spheres to two decimal places.
   - a. Radius of 5 cm
   - b. Radius of 12 cm
   - c. Diameter of 1.8 m
   - d. Diameter of 27 cm
   - e. Radius of 24.8 cm
   - f. Diameter of 32.5 m

2. **WE10** Calculate the volumes of the following spheres.

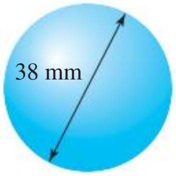
a.



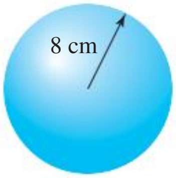
b.



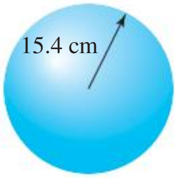
c.



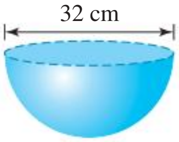
d.



e.

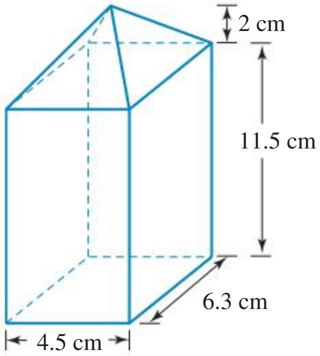


3. **WE11** Calculate the volume of the following hemisphere.



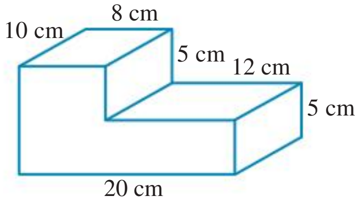
4. A spherical container with a radius of 15 cm is filled with water.
   - a. Calculate the volume of the sphere, correct to two decimal places.
   - b. Determine the total capacity of the sphere, in litres, correct to two decimal places.

5. **WE12** Calculate the volume of the shape shown, correct to two decimal places.

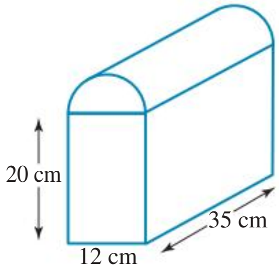


6. Calculate the volume of each of the following composite solids, correct to two decimal places where appropriate.

a.

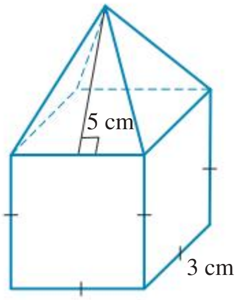


b.

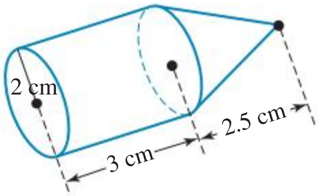


7. Calculate the volume of each of the following composite solids, correct to two decimal places where appropriate.

a.

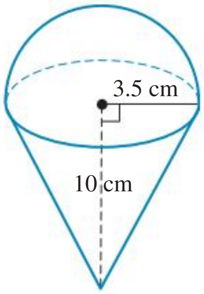


b.

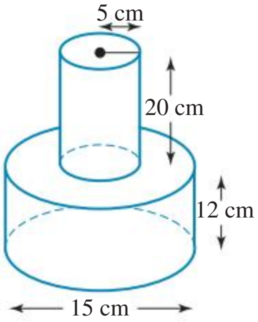


8. Calculate the volume of each of the following composite solids, correct to two decimal places where appropriate.

a.

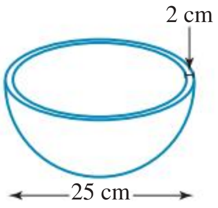


b.

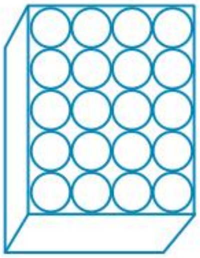


9. A hemisphere of diameter 24 cm sits perfectly on top of a cylinder of height 46 cm. Calculate the volume of the composite shape correct to two decimal places.

10. A hemispherical bowl has a thickness of 2 cm and an outer diameter of 25 cm. If the bowl is filled with water, determine the capacity of the bowl, in liters, to two decimal places.

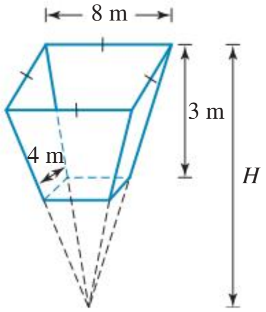


11. Tennis balls of diameter 8 cm are packed in a box measuring 40 cm × 32 cm × 10 cm, as shown.



- **a.** Estimate the volume of one tennis ball.
  (*Hint:* Use π as approximately equal to 3.)
- **b.** Estimate the volume of all the tennis balls in the box.
- **c.** Estimate the volume of the box.
- **d.** Estimate the unfilled volume of the box.
- **e.** Determine, correct to two decimal places, how much space is left unfilled.
- **f.** Compare your estimate of the unfilled volume to your answer in part **e**.

12. A swimming pool is being constructed so that it is the upper part of an inverted square-based pyramid.



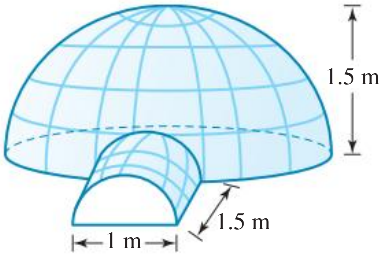
- **a.** Calculate H.
- **b.** Calculate the volume of the pool.
- **c.** When digging the pool out, the dirt will be carted away in a 6 m^3 capacity small truck. Determine how many truck loads will be required to take the dirt away.
- **d.** Determine how many litres of water are required to fill this pool.
- **e.** Determine how deep the pool is when it is half-filled.

Complex Unfamiliar

13. A monument in the shape of a rectangular pyramid (base length of 10 cm, base width of 6 cm, height of 8 cm), a spherical glass ball (diameter of 17 cm) and a conical piece of glassware (radius of 14 cm, height of 10 cm) are packed in a rectangular prism of dimensions 30 cm × 25 cm × 20 cm. The extra space in the box is filled up by packing material.
Determine, correct to two decimal places, the volume of packing material that is required.

14. The Hastings' family house has a rectangular roof with dimensions 17 m × 10 m providing water to three cylindrical water tanks, each with a radius of 1.25 m and a height of 2.1 m.
Show that approximately 182 mm of rain must fall on the roof to fill the tanks.

15. Nathaniel and Annie are going to the snow for survival camp. They plan to construct an igloo, consisting of an entrance and a hemispherical living section, as shown.



Nathaniel and Annie are asked to redraw their plans and increase the height of the livable region (hemispherical structure) so that the total volume (including the entrance) is doubled.
Determine what the new height of the hemisphere must be to achieve this so that the total volume (including the entrance) is doubled. Write your answer in metres correct to two decimal places.

Fully worked solutions for this chapter are available online.

LESSON
4.6 Units of time

SYLLABUS LINKS

- Use units of time and convert between fractional, decimal and digital representations.

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4.6.1 Time

- Time is divided into units including seconds, minutes, hours, days, weeks, months and years.
- A day is divided into two 12-h periods:

Midnight to midday	am	ante meridiem (before noon)
Midday to midnight	pm	post meridiem (after noon)

- Time in hours and minutes can either be given by the 12-h or 24-h clock system.
- The 24-h clock is commonly used in areas like military, aviation, navigation, tourism, meteorology, astronomy, computing, logistics, emergency services and hospitals, to avoid any confusion about whether it is am or pm time.
- Twenty four-hour time always uses four digits to represent the time. For example, 1500 hours or 15:00 hours is used for 3:00 pm, and midnight can be shown as 0000 (00:00) hours or 2400 (24:00) hours.

Time conversions

Converting from a 12-h to a 24-h clock:

- **Starting from the first hour of the day (12:00 am or midnight to 12:59 am), subtract 12 h.**
  For example:
  12:00 am = 0:00
  12:15 am = 0:15
- **From 1:00 am to 12:59 pm, the hours and minutes remain the same.**
  For example:
  9:00 am = 9:00
  12:59 pm = 12:59
- **For times between 1:00 pm and 11:59 pm, add 12 h.**
  For example:
  4:25 pm = 16:25
  11:59 pm = 23:59

Converting from a 24-h to a 12-h clock:

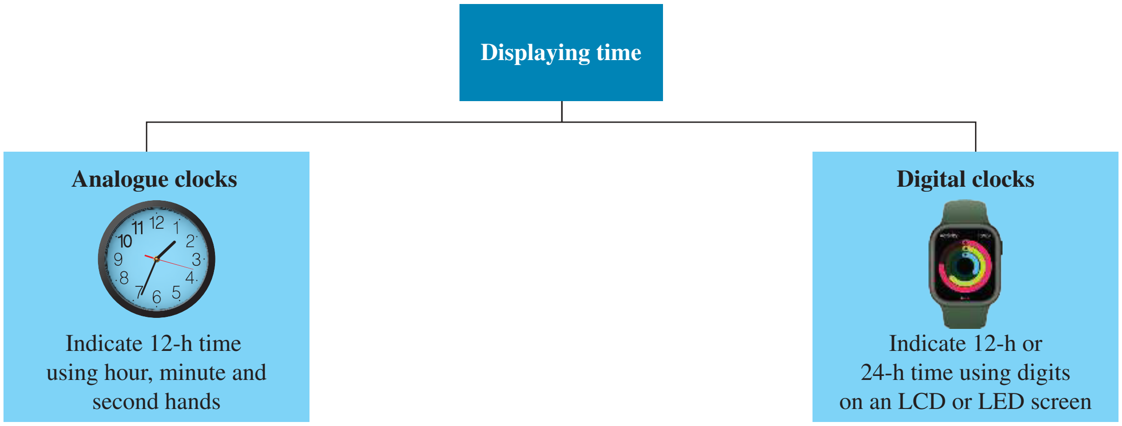
- **Starting from the first hour of the day (0:00/midnight to 0:59), add 12 h and 'am' to the time.**
  For example:
  0:20 = 12:20 am
  0:45 = 12:45 am
- **From 1:00 to 11:59, simply add 'am' to the time.**
  For example:
  3:25 = 3:25 am
  10:30 = 10:30 am
- **For times between 12:00 and 12:59, simply add 'pm' to the time.**
  For example:
  12:15 = 12:15 pm
  12:55 = 12:55 pm
- **For times between 13:00 and 23:59, subtract 12 h and add 'pm' to the time.**
  For example:
  17:45 = 5:45 pm
  22:30 = 10:30 pm

Time is divided into units. There are:

- **60 s in 1 min**
- **60 min in 1 h**
- **24 h in 1 day**
- **7 d in 1 week**
- **2 weeks in 1 fortnight**
- **about 4 weeks in 1 month**
- **12 months in 1 year**
- **about 365 d in 1 year**
- **10 years in 1 decade**
- **100 years in 1 century**
- **1000 years in 1 millennium.**

4.6.2 Displaying time

Time can be shown in 12-h time or 24-h time.



Displaying time

Analogue clocks

Indicate 12-h time using hour, minute and second hands

Digital clocks

Indicate 12-h or 24-h time using digits on an LCD or LED screen

Time shown in writing	Time on an analogue clock	Time on a digital clock	
		12-h time	24-h time
Three o'clock in the morning			
Twenty-five minutes past 1 in the afternoon			
Ten minutes to 7 in the evening			

WORKED EXAMPLE 13 Convert between 12- and 24-hour time

Convert the following times to the times shown in the brackets.

a. **0800 (12-h time)**
b. **16:45 (12-h time)**
c. **1:33 pm (24-h time)**
d. **3:21 am (24-h time)**

THINK	WRITE
a. Since the 24-h time is less than 1200, the time is am.	0800 = 8:00 am
b. Since the 24-h time is greater than 12:00, the time is pm.	$16{:}45 = 12{:}00 + 4{:}45$ $= 4{:}45$ pm
c. Since it is pm, it is 1:33 after 12:00.	$1{:}33$ pm $= 1{:}33 + 12{:}00$ $= 13{:}33$
d. Since it is am, it is a straight conversion to 24-h time.	3:21 am = 03:21

4.6.3 Converting between fractional and decimal time

In many real-life situations, time can also be represented in fractional or decimal form.

WORKED EXAMPLE 14 Converting fractional and decimal time

Write the following times in hours, minutes and seconds.

a. **5.6 h**

b. $\mathbf{3\frac{3}{8}}$ **h**

THINK			WRITE
a.	**1.**	There are 5 h and 0.6 of an hour in 5.6 h.	$5.6 = 5\text{ h} + 0.6$ of an hour
	2.	There are 60 min in an hour, so calculate 0.6 of 60 min.	$5.6 = 5\text{ h} + 0.6 \times 60$ $= 5\text{ h} + 36\text{ min}$
	3.	Alternatively, you can use the degree, minutes and seconds function on your calculator. Type 5.6, press = then press °’’”.	 D Math ▲ 5.6 5°36’0”
	4.	Write the answer.	5.6 h is the same as 5 h and 36 min.
b.	**1.**	There are 3 h and $\frac{3}{8}$ of an hour in $3\frac{3}{8}$ h.	$3\frac{3}{8} = 3\text{ h} + \frac{3}{8}$ of an hour
	2.	There are 60 min in an hour, so calculate $\frac{3}{8}$ of 60 min.	$3\frac{3}{8} = 3\text{ h} + \frac{3}{8} \times 60\text{ min}$ $= 3\text{ h} + 22.5\text{ min}$
	3.	There are 60 s in a minute so calculate 0.5 of 60 s.	$3\frac{3}{8} = 3\text{ h} + 22\text{ min} + 0.5 \times 60\text{ s}$ $= 3\text{ h} + 22\text{ min} + 30\text{ s}$
	4.	Alternatively, you can use the degree, minutes and seconds function on your calculator. Type $3\frac{3}{8}$, press = then press °’’”.	 D Math ▲ $3\frac{3}{8}$ 3°22’30”
	5.	Write the answer.	$3\frac{3}{8}$ h is equal to 3 h, 22 min and 30 s.

tlvd-12165

WORKED EXAMPLE 15 Applications of time conversions

The movie shown runs for 142 min.

a. **Determine the length of the movie in hours correct to two decimal places.**
b. **Determine the length of the movie in hours as a fractional time.**
c. **Determine the length of the movie in hours and minutes.**

THINK	**WRITE**
a. **1.** There are 60 min in 1 h, so divide 142 by 60.	$142 \div 60 = 2.366\,666\,67$
2. Round to two decimal places.	$2.366\,666\,67 \approx 2.37$ h
3. Write the answer.	The movie goes for 2.37 h.
b. **1.** There are 60 min in 1 h, so divide 142 by 60.	$\frac{142}{60} = 2\frac{22}{60}$ $= 2\frac{11}{30}$ h
2. Write the answer.	The movie goes for $2\frac{11}{30}$ h.
c. **1.** There are 60 min in 1 h, so divide 142 by 60.	$142 \div 60 = 2.366\,666\,67$
2. Two hours is 120 min ($2 \times 60 = 120$). Subtract 120 from 142 to find how many min over 2 h the movie goes for.	$142 - 120 = 22$
3. Write the answer.	The movie goes for 2 h and 22 min.

Exercise 4.6 Units of time

learnon

4.6 Quick quiz on	4.6 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15	N/A	N/A

These questions are even better in jacPLUS!
- Receive immediate feedback
- Access sample responses
- Track results and progress

Find all this and MORE in jacPLUS

Simple familiar

1. **WE13** Convert the following times to the times shown in the brackets.
   a. 0500 (12-h time)
   b. 18:15 (12-h time)
   c. 2:48 pm (24-h time)
   d. 4:05 am (24-h time)
2. Convert the following times to 24-h time.
   a. 10:43 pm
   b. 11:03 pm
3. Convert the following times to 12-h time.
   a. 13:28
   b. 21:58
4. **WE14** Convert the following times into hours, minutes and seconds.
   a. 6.2 h
   b. $8\frac{5}{8}$ h
5. Convert the following times into hours, minutes and seconds.
   a. 7.138 h
   b. $5\frac{7}{9}$ h
6. Express the following times as 24-h times.
   a. 10:33 am
   b. 5:17 am
   c. 3:50 pm
   d. 11:30 pm
7. **WE15** A concert at the Gabba lasted for 124 min.

   a. Determine the length of the concert in hours, correct to two decimal places.
   b. Determine the length of the concert in hours as a fractional time.
   c. Determine the length of the concert in hours, minutes and seconds.

8. A Rugby League game is played over two 40-min halves. The game also had 5 min of injury time in the first half and another 4 min in the second half.

a. Determine the length of the entire game in hours to two decimal places.
b. Determine the length of the entire game as a fractional time.
c. Determine the length of the entire game in hours, minutes and seconds.

9. Express the following 24-h times as digital am or pm times.

a. 15:31 b. 21:44 c. 04:25

10. Express the following 24-h times as digital am or pm times.

a. 10:51 b. 07:07 c. 22:25

11. Express twelve o'clock at night in 24-h time and digital time.

12. Express each of the following time periods in minutes.

a. 4 h b. $7\frac{1}{4}$ h c. 2 d

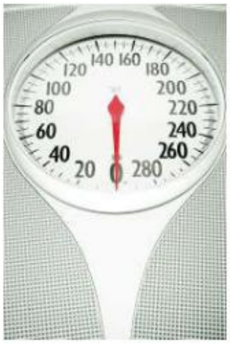
13. Express each of the following times into minutes.

a. $\frac{1}{4}$ h b. 2 h and 35 min c. $7\frac{3}{4}$ h

14. Determine if 2 h and 35 min and 2.35 h represent the same length of time. Explain your answer.

15. Express the following time periods in hours, minutes and seconds.

a. 220 min b. 315 min
c. 3.45 h d. 133.865 min



Fully worked solutions for this chapter are available online.

4.7 Units of mass

SYLLABUS LINKS

- Use metric units of mass (milligrams, grams, kilograms, tonnes), their abbreviations (mg, g, kg, t), conversions between them and appropriate choices of units.

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4.7.1 Units of mass

- **Mass** describes how much matter makes up an object, and its standard unit of measurement is the kilogram (kg).
- The mass of an object can be measured in milligrams (mg), grams (g), kilograms (kg) and tonnes (t). Just as with length and area, it is important to use the appropriate unit to measure the mass of an object.

Milligrams (mg)

- Milligrams are used to measure the mass of very light objects, such as food nutrients like sodium or minerals, or active ingredients in medications like paracetamol.

Gram (g)

- There are 1000 milligrams in 1 gram. Grams are commonly used when measuring non-liquid cooking ingredients. The mass of 1 cup of flour is about 150 g.

Kilograms (kg)

- There are 1000 grams in 1 kilogram. Kilograms are commonly used to measure larger masses such as the mass of a person.

Metric tonnes (t)

- There are 1000 kilograms in 1 tonne. Tonnes are used to measure the mass of very large objects such as a truck or boat.

4.7.2 Conversion between units of mass

- The relationship between the units of mass can be used to change a measurement from one unit to another.

$$\begin{aligned} 1000\,\text{mg} &= 1\,\text{g} \\ 1000\,\text{g} &= 1\,\text{kg} \\ 1000\,\text{kg} &= 1\,\text{t} \end{aligned}$$

Converting between units of mass

The diagram shown can be used to assist in the conversion between different units of mass.

$$\begin{tikzcd}
\text{t} \arrow[r, bend left, "\times 1000"] & \text{kg} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \text{g} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \text{mg} \arrow[l, bend left, "\div 1000"]
\end{tikzcd}$$

WORKED EXAMPLE 16 Converting units of mass

Convert the following mass measurements to the units specified.

a. **15.86 kg to grams**

b. **13 650 mg to kilograms**

c. **0.0071 t to kilograms**

THINK	WRITE
a. To convert from kilograms to grams we need to multiply by 1000.	**a.** $15.86\text{ kg} = (15.86 \times 1000)\text{ g}$ $= 15\,860\text{ g}$
b. **1.** To convert from milligrams to kilograms we need to divide by 1000 to convert to grams first.	**b.** $13\,650\text{ mg} = (13\,650 \div 1000)\text{ g}$ $= 13.65\text{ g}$
2. Once converted to grams we then divide by 1000 again to convert to kilograms.	$13.65\text{ g} = (13.65 \div 1000)\text{ kg}$ $= 0.013\,65\text{ kg}$
c. To convert from metric tonnes to kilograms we need to multiply by 1000.	**c.** $0.0071\text{ t} = (0.0071 \times 1000)\text{ kg}$ $= 7.1\text{ kg}$

Exercise 4.7 Units of mass

learnon

4.7 Quick quiz on

4.7 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15	N/A	N/A

Simple familiar

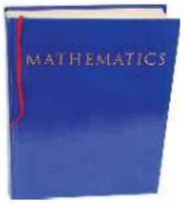
1. Determine the unit of mass that would be best used to describe the mass of each of the following.

a. A full suitcase checked in for a flight
b. A grass seed
c. A family car
d. A calculator
e. A bag of apples
f. A brick

2. Estimate the mass of each of the following objects.

a. A football

b. A maths textbook



c. An iPad

d. A computer printer

e. Your pencil

f. A can of soft drink

3. Estimate the mass of the following items.

a.

b.

c.

d.

4. **WE16** Convert the following mass measurements to the units specified.

a. 18.9 kg to grams **b.** 76 490 mg to kilograms **c.** 0.083 t to kilograms

5. Convert the following mass measurements to the units specified.

a. 717 mg to grams **b.** 3867 g to metric tonnes **c.** 0.0084 t to grams

6. Change the following mass measurements to the units specified.

a. 2.4 g to milligrams
b. 46.7 t to kilograms
c. 2510 mg to gram

7. Change the following mass measurements to the units specified.

a. 82 g to kilograms
b. 0.03 kg to grams
c. 7893 g to milligrams

8. Change the following mass measurements to the units specified.

a. 25 781 mg to kilograms
b. $\frac{7}{8}$ kg to milligrams
c. 384.2 kg to metric tonnes

9. Change the following mass measurements to the units specified.

a. 0.075 kg to milligrams
b. 0.000 004 5 t to grams
c. 33 456 mg to kilograms

10. For Noah’s party, eight blocks of the chocolate shown were purchased. The mass of the chocolate is shown on the block.

- **a.** Calculate the total mass of the eight blocks of chocolate.
- **b.** State what the term **net** means.

11. A family of five (two parents and three children) is going on a holiday to Disneyland.

The allowable limit for their luggage is 23 kg per person. They pack light, with each parent taking 18 kg of luggage and each of the three children taking 12 kg. They hope to do some shopping during their holiday. Calculate the maximum mass of their shopping so that they don’t exceed the luggage allowance.

12. Christiana decided to go shopping to get some fruit and vegetables for the weekend. Complete the table shown to calculate her total cost to the nearest cent.

Item	Cost per kg	Cost
1.5 kg of bananas	$3.50	1.5 × $3.50 = $5.25
3.4 kg of apples	$3.95	
1.2 kg of carrots	$2.80	
2.3 kg of grapes	$7.90	
Total cost		

13. If the mass of Rob is 94.4 kg and his son Jack is 67.8 kg, calculate the difference in their masses.

14. A group of contestants entered a weight-loss competition. Given the information in the table, complete the table and answer the following questions.

Name	Original mass (kg)	Loss	New mass (kg)
Matthew	112.3	16.7 kg	
Jack	133.8	24% of his mass	
Sarah	93.6	$\frac{1}{6}$ of her mass	
Jane	88.3	Jane now weighs $\frac{7}{9}$ of her original mass	
Paul	105.6	13.9 kg	
Kelly	96.1	17.5% of her mass	

a. Determine who lost the most mass and how much they lost.
b. Determine who lost the least mass and how much they lost.
c. List the contestants in descending order of their new mass.
d. Calculate the total mass loss of the six contestants.

15. The mass of an object is constant, but its weight changes depending on gravity. To calculate the weight (W) of an object, you multiply its mass (m) by the value of the acceleration due to gravity (g). On the Moon the acceleration due to gravity is about $\frac{1}{6}$ of that on Earth (which is 9.8 m s^{-2}).

The method for calculating the weight of a 100 kg object on the Moon, measured in newtons (N), is shown below.

$$\begin{aligned} W &= m \times g \\ &= 100\,\text{kg} \times \left(\frac{1}{6} \times 9.8\,\text{m s}^{-2}\right) \\ &\approx 163.33\,\text{N} \end{aligned}$$

Note: Weight is different from mass, as weight describes the gravitational force acting on an object, and is measured in newtons (N).
Use this information to calculate the weight of each of the following objects on the Moon.

a. A 4500 kg boat
b. A 0.56 kg bottle
c. A 76.25 kg person
d. A 331 g iPad mini

Fully worked solutions for this chapter are available online.

LESSON
4.8 Review

4.8.1 Summary

Hey students! Now that it's time to revise this chapter, go online to:

- Access the chapter summary
- Review your results
- Watch teacher-led videos
- Practise questions with immediate feedback

Find all this and MORE in jacPLUS

4.8 Exercise

learnon

4.8 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15	16, 17, 18, 19	20

These questions are even better in jacPLUS!

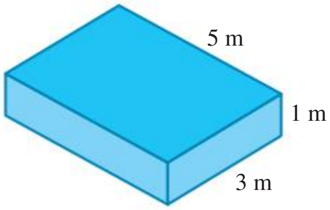
- Receive immediate feedback
- Access sample responses
- Track results and progress

Find all this and MORE in jacPLUS

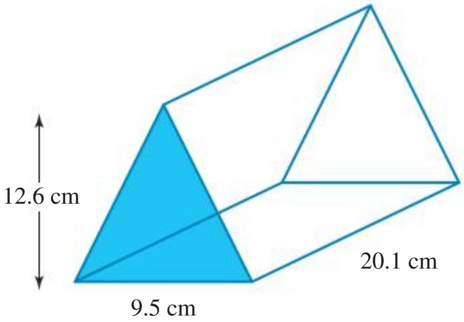
Simple familiar

1. Convert 259 g into kilograms.
2. If Jack buys seven 250-g packets of chips for a group of friends who are coming around to watch a movie, determine the total mass of the chips.

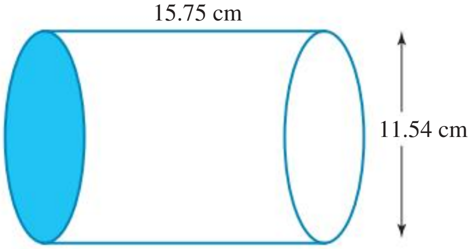
3. Determine the capacity of a 465 cm^3 jug in litres.
4. Determine the number of cubes with side length 1 cm that can fit into a larger cube with side length 4 cm.
5. Calculate the volume of the shape shown.



6. Calculate the volume of the triangular prism shown to two decimal places.



7. Calculate the volume of the cylinder shown to one decimal place.



8. Calculate the volume of a rectangular-based pyramid with length 2.5 m, width 1.8 m and vertical height 1.5 m.

9. If a square-based pyramid has a volume of 12.4 cm^3 and a vertical height of 2.2 cm, calculate the length of the pyramid, correct to 1 decimal place.

10. The Luxor in Las Vegas is known for its square-based pyramid and bright light. The Luxor pyramid is 111 m high and 90 m wide.

Calculate the volume of the Luxor Pyramid to two decimal places.

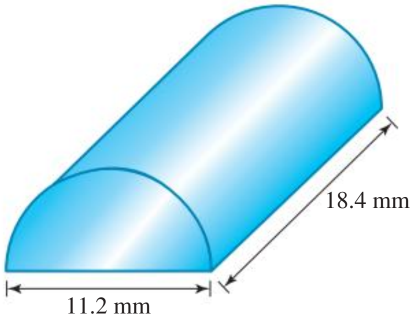
11. Write the following in hours, minutes and seconds.

a. 5.125 h

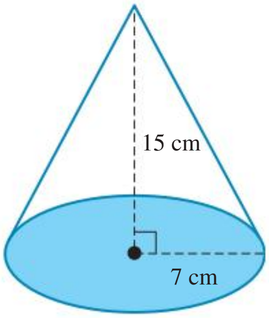
b. $3\frac{4}{5}$ h

12. Calculate the volume of each of the following objects, correct to two decimal places.

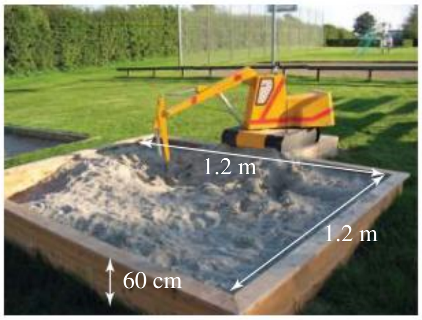
a.



b.



13. A sandpit has the dimensions as shown.



a. Calculate how much sand is required to fill the sandpit.

b. If the sand came in bags shaped as rectangular prisms with dimensions 45 cm × 30 cm × 10 cm, determine how many bags would be required to fill the sandpit.

c. If the bags of sand cost $7.50 each, calculate how much it would cost to fill the sand pit.

d. The digger's scoop is in the shape of a rectangular-based pyramid with base length of 20 cm, base width of 18 cm and height of 15 cm. Assuming the scoop was full each time, determine how many scoops would be required to empty the full sandpit.

14. A block of cheese is in the shape of a triangular prism.

The area of the triangular face is 2.3 cm^2 and the length of the prism is 9.7 cm. If the price of cheese is \$0.53 per cm^3, calculate the price of the block.

15. A fish tank has dimensions 90 cm × 40 cm × 60 cm.

Calculate the amount of water that would be needed to:

a. three-quarters-fill the tank, in cm^3

b. three-quarters-fill the tank, in litres

c. fill 90% of the tank, in cm^3

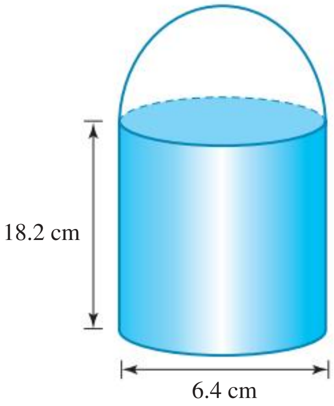
d. fill 90% of the tank, in litres.

Complex familiar

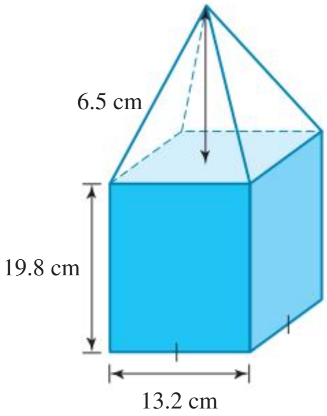
16. Calculate the volume, in cubic metres, of a disco ball that has a diameter of 55 cm, to three decimal places.

17. Calculate the volume of each of the following objects.

a.

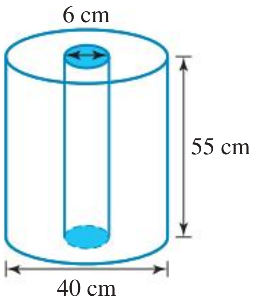


b.

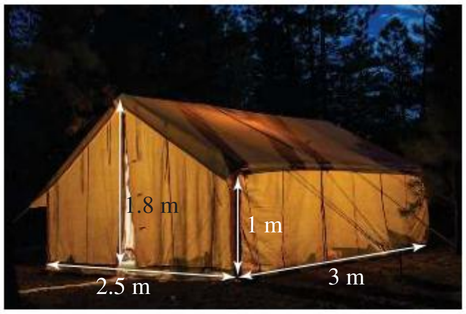


18. Calculate the volume of water that can completely fill the inside of the washing machine barrel shown in the diagram, to four decimal places, in:

a. cm^3
b. m^3
c. litres.



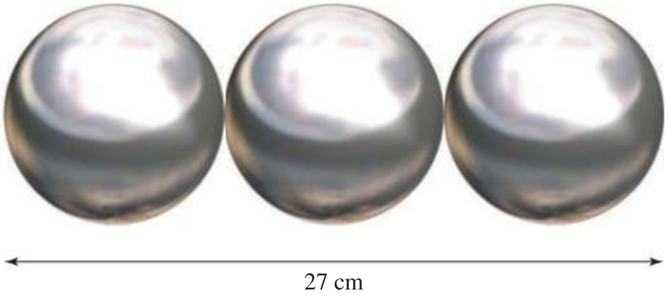
19. Callum's Camping Company uses tents as shown for outback adventures.



a. Calculate the amount of material required to make each of the tents, assuming no base is required.
b. Calculate the room in each of the tents, in m^3.
c. If there was a requirement that a tent provided a minimum of 3 m^3 per person, determine how many people can sleep in the tent shown.
d. The side wall of the tents can come off so that two tents can join together into one tent. Determine how many people can fit in the double tent given the previous requirement.

Complex unfamiliar

20. The sculpture shown is to be packaged into a rectangular or cylindrical cardboard box. Use calculations to justify if you would choose a rectangular or cylindrical package box.



Fully worked solutions for this chapter are available online.

Answers

Chapter 4 Volume, capacity, time and mass

4.2 Units of volume and capacity

Exercise 4.2

1. a. cm^3 **b.** mm^3 **c.** m^3 **d.** cm^3

2. a. 16 cm^3 **b.** 12 cm^3 **c.** 6 cm^3 **d.** 27 cm^3

3. a. 24 cm^3 **b.** 28 m^3

4. a. 9 cm^3 **b.** 7 cm^3 **c.** 10 cm^3 **d.** 13 cm^3

5. a. 88 cm^3 **b.** 120 cm^3 **c.** 88 cm^3 **d.** 116 cm^3

6. Volume is the amount of space taken up by the object, while capacity is the measure of an object's ability to hold a substance.

7. a. 300 000 cm^3 and 300 L **b.** 2500 m^3 and 2.5 ML
c. 1875 mL and 1.875 L **d.** 160 m^3 and 160 kL

8. a. Capacity **b.** Volume **c.** Volume
d. Capacity **e.** Volume **f.** Capacity

9. a. 24 000 mm^3 **b.** 2 000 000 000 mm^3
c. 40 km^3 **d.** 0.6 m^3
e. 3.2 cm^3 **f.** 5.6 m^3

10. a. 12 000 mL **b.** 0.003 125 ML **c.** 0.397 L
d. 7.8 mL **e.** 4.893 L **f.** 0.000 036 97 ML

11. a. 372 mL **b.** 1 630 000 cm^3 **c.** 3400 cm^3
d. 0.38 cm^3 **e.** 163 000 cm^3 **f.** 49.28 mL

12. a. 0.578 L **b.** 0.75 L **c.** 429 mL

13. a. 47 mL **b.** 3.594 L **c.** 4.2 cm^3

14. a. i. 16 cubes **ii.** 16 cm^3 **iii.** 16 mL
b. i. 12 cubes **ii.** 12 cm^3 **iii.** 12 mL
c. i. 48 cubes **ii.** 48 cm^3 **iii.** 48 mL
d. i. 16 cubes **ii.** 16 cm^3 **iii.** 16 mL

15. a. 72 000 000 cubes **b.** 72 000 000 cm^3
c. 72 000 L **d.** 57 600 L

16. a. 1000 cubes **b.** 1000 cm^3
c. 1 L **d.** 750 mL

4.3 Volumes and capacities of prisms and cylinders

Exercise 4.3

1. a. 80 cm^3 **b.** 100 cm^3 **c.** 90 m^3 **d.** 118.625 cm^3

2. a. 54 m^3 **b.** 1500 cm^3
c. 614.125 cm^3 **d.** 130.5 cm^3

3. a. 336 cm^3 **b.** 64 cm^3

4. a. 480 cm^3 **b.** 180 cm^3 **c.** 1320 m^3 **d.** 1296 cm^3

5. a. 140 cm^3 **b.** 86.955 m^3

6. a. 339.29 cm^3 **b.** 678.58 mm^3
c. 1693.32 cm^3 **d.** 4.42 m^3

7. 706.86 cm^3

8. 922.45 m^3

9. a. 3936.69 cm^3 **b.** 0.70 m^3 **c.** 5145.6 cm^3
d. 0.15 m^3 **e.** 88.19 m^3 **f.** 79 521.56 cm^3

10. 111 m^3, 111 000 L

11. 11.762 m^3, 11.762 kL

12. 16.632 m^3

13. 3.61 cm

14. a. 1.2 m^3 **b.** 0.9 m^3 **c.** 750 L

15. a. 0.68 m^3 **b.** 0.204 m^3 **c.** 0.476 m^3

16. a. 75.625 m^3 **b.** 103.125 m^3 **c.** 7.27 m

4.4 Volumes and capacities of pyramids and cones

Exercise 4.4

1. a. 40 cm^3 **b.** 392 cm^3 **c.** 143.65 cm^3
d. 1641.71 cm^3 **e.** 27.05 m^3 **f.** 0.72 m^3

2. 112 cm^3

3. 20.216 m^3

4. a. 229.5 mm^3 **b.** 616 mm^3

5. a. 0.196 m^3 **b.** 810.67 mm^3

6. 180 cm^3

7. a. 4105.01 cm^3 **b.** 804.25 cm^3

8. a. 377.0 cm^3 **b.** 2303.8 mm^3

9. a. 400 cm^3 **b.** 10 080 cm^3

10. 128 cm^3

11. 294.912 m^3

12. 100.53 m^3

13. 461.81 cm^3

14. 5.66 cm

15. 5 mm

16. 10 m

4.5 Volumes and capacities of composite shapes and spheres [complex]

Exercise 4.5

1. a. 523.60 cm^3 **b.** 7238.23 cm^3
c. 3.05 m^3 **d.** 10 306.00 cm^3
e. 63 891.58 cm^3 **f.** 17 974.16 m^3

2. a. 20 579.53 mm^3 **b.** 2309.56 mm^3
c. 28 730.91 mm^3 **d.** 2144.66 cm^3
e. 15 298.57 cm^3

3. 8578.64 cm^3

4. a. 14 137.17 cm^3 **b.** 14.14 L

5. 344.93 cm^3

6. a. 1400 cm^3 **b.** 10 379.20 cm^3

7. a. 41.31 cm^3 **b.** 48.17 cm^3

8. a. 218.08 cm^3 **b.** 3691.37 cm^3

9. 24 429.02 cm^3

10. 2.42 L

11. **a.** 256 cm^3 **b.** 5120 cm^3 **c.** 12 000 cm^3
d. 6880 cm^3 **e.** 7438.35 cm^3 **f.** 558.35 cm^3

12. **a.** $H = 6$ m **b.** 112 m^3 **c.** 19 truck loads
d. 112 000 L **e.** 1.95 m from floor

13. 10215.05 cm^3

14. Sample response is available in the online worked solutions.

15. 1.94 m

4.6 Units of time

Exercise 4.6

1. **a.** 5:00 am **b.** 6:15 pm **c.** 14:48 **d.** 04:05

2. **a.** 22:43 **b.** 23:03

3. **a.** 1:28 pm **b.** 9:58 pm

4. **a.** 6 h and 12 min
b. 8 h and 37 min and 30 s

5. **a.** 7 h and 8 min and 16.8 s
b. 5 h and 46 min and 40 s

6. **a.** 10:33 **b.** 05:17 **c.** 15:50 **d.** 23:30

7. **a.** 2.07 h
b. $2\frac{1}{15}$ h
c. 2 h and 42 min and 0 s

8. **a.** 1.48 h
b. $1\frac{29}{60}$ h
c. 1 h and 29 min and 0 s

9. **a.** 3:31 pm **b.** 9:44 pm **c.** 4:25 am

10. **a.** 10:51 am **b.** 7:07 am **c.** 10:25 pm

11. 12 o'clock at night = 24:00 = 12:00 am = 00:00

12. **a.** 240 min **b.** 435 min **c.** 2880 min

13. **a.** 15 min **b.** 155 min **c.** 465 min

14. 2 h and 35 min is 14 min longer than 2.35 h.

$$\begin{aligned}
\text{2 h and 35 min} &= 2 \times 60 + 35 \\
&= 155 \text{ min} \\
2.35 \text{ h} &= 2.35 \times 60 \text{ min} \\
&= 141 \text{ min} \\
\text{Difference} &= 155 - 141 \\
&= 14 \text{ min} \\
&= \text{Therefore 2 h and 35 is} \\
&\quad\ \text{14 min longer then 2.35 h.}
\end{aligned}$$

15. **a.** 3 h and 40 min and 0 s
b. 15 h and 15 min and 0 s
c. 3 h and 27 min and 0 s
d. 2 h and 13 min and 51.9 s

4.7 Units of mass

Exercise 4.7

1. **a.** kilogram **b.** milligram **c.** tonne
d. gram **e.** kilogram **f.** kilogram

2. **a.** 400 g **b.** 1 kg **c.** 500 g
d. 2000 g **e.** 5 g **f.** 300 g

3. **a.** 2 kg **b.** 250 g **c.** 750 g **d.** 380 g

4. **a.** 18 900 g **b.** 0.076 490 mg **c.** 83 kg

5. **a.** 0.717 g **b.** 0.003 867 t **c.** 8400 g

6. **a.** 2400 mg **b.** 46 700 kg **c.** 2.510 g

7. **a.** 0.082 kg **b.** 30 g **c.** 7 893 000 mg

8. **a.** 0.025 781 kg **b.** 875 000 mg **c.** 0.3842 t

9. **a.** 75 000 mg **b.** 4.5 g **c.** 0.033 456 kg

10. **a.** 2000 g
b. Net means the weight of the chocolate not including the packaging.

11. 43 kg

12.

Item	Cost per kg	Cost
1.5 kg of bananas	\$3.50	\$5.25
3.4 kg of apples	\$3.95	\$13.43
1.2 kg of carrots	\$2.80	\$3.36
2.3 kg of grapes	\$7.90	\$18.17
Total cost		\$40.21

13. 26.6 kg

14.

Name	Original mass (kg)	Loss	New mass (kg)
Matthew	112.3	16.7 kg	95.6 kg
Jack	133.8	24% of his mass	101.688 kg
Sarah	93.6	$\frac{1}{6}$ of her mass	78 kg
Jane	88.3	Jane now weighs $\frac{7}{9}$ of her original mass	68.68 kg
Paul	105.6	13.9 kg	91.7 kg
Kelly	96.1	17.5% of her mass	79.28 kg

a. Jack lost 32.112 kg
b. Paul lost 13.9 kg
c. Jack, Matthew, Paul, Kelly, Sarah, Jane
d. 114.752 kg

15. **a.** 7350 N **b.** 0.91 N **c.** 124.54 N **d.** 0.541 N

4.8 Review

Exercise 4.8

1. 0.259 kg

2. 1.75 kg

3. 0.465 L

4. 64

5. 15 m^3

6. 1202.99 cm^3

7. 1647.3 cm^3

8. 2.25 m^3

9. 4.1 cm

10. 299 700.00 m^3

11. a. 5 h, 7 min and 30 s
    b. 3 h and 48 min

12. a. 906.39 mm^2 b. 769.69 cm^3

13. a. 0.864 m^3 b. 64 bags c. \$480 d. 480 scoops

14. $V = 22.31$ cm^3
    Cost = \$11.82

15. a. 162 000 cm^3 b. 162 L
    c. 194 400 cm^3 d. 194.4 L

16. 0.087 m^3

17. a. 654.1 cm^3 b. 3827.47 cm^3

18. a. 67 559.9500 cm^3 b. 0.0676 m^3
    c. 67.5600 L

19. a. 21.90 m^2
    b. 10.5 m^3
    c. Three people can fit in the tent under these requirements.
    d. People = 7

20. Since the cylindrical box only has 572.56 cm^3 of empty space compared to the rectangular box having 1041.89 cm^3, the cylindrical box would be more tightly packed and less filling would be required.

UNIT 3 Measurement, scales and chance
TOPIC 2 Scales, plans and models

5 Interpreting and creating scale drawings

LESSON SEQUENCE

Fully worked solutions for this chapter are available online.

on Resources

Solutions	Solutions — Chapter 5
Digital documents	Learning matrix — Chapter 5 Quick quizzes — Chapter 5 Chapter summary — Chapter 5

LESSON
5.1 Overview

5.1.1 Introduction

This chapter helps you understand how to interpret common symbols and abbreviations in scale drawings. By learning to find actual measurements from these drawings, including lengths, perimeters and areas, you'll gain skills directly applicable in real-life scenarios. You'll develop the ability to estimate and compare quantities, materials and costs using actual measurements from scale drawings. This skill is particularly useful in construction, architecture and interior design, where precise calculations are vital.

Understanding and applying drawing conventions, such as scales in ratio, clear indications of dimensions and proper labelling, will enhance your ability to communicate complex information effectively. Additionally, you'll learn to construct scale drawings both by hand and using software packages, providing a comprehensive understanding of traditional and modern methods. By mastering these skills, you'll be equipped to tackle real-world challenges, from designing a living space to planning large projects such as your own house.

5.1.2 Syllabus links

Lesson	Lesson title	Syllabus links
5.2	**Reading and interpreting scale drawings**	○ Interpret commonly used symbols and abbreviations in scale drawings. ○ Find actual measurements from scale drawings, including lengths, perimeters and areas.
5.3	**Calculating, costing and comparing using measurements from scale drawings**	○ Find actual measurements from scale drawings, including lengths, perimeters and areas. ○ Estimate and compare quantities, materials and costs using actual measurements from scale drawings [complex].
5.4	**Creating scale drawings [complex]**	○ Understand and apply drawing conventions of scale drawings, including scales in ratio, clear indications of dimensions and clear labelling [complex]. ○ Construct scale drawings by hand and by using software packages [complex].

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LESSON
5.2 Reading and interpreting scale drawings

SYLLABUS LINKS

- Interpret commonly used symbols and abbreviations in scale drawings.
- Find actual measurements from scale drawings, including lengths, perimeters and areas.

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5.2.1 Scales review

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- A **scale** is a ratio of the dimension on a drawing to the actual dimension.

Scale

Scale = dimension of drawing : actual dimension

- The sign ':' is read as 'to'.
- Scales are usually written with no units. If a scale is given in two different units, the larger unit has to be converted into the smaller unit.
- A **scale factor** is the ratio of two corresponding dimensions in two similar shapes. The scale factor of $\frac{1}{2}$ or a scale (ratio) of 1 : 2 means that 1 unit on the drawing represents 2 units in actual size. The unit can be millimetre, centimetre, metre or kilometre.

Scale factor

$$\textbf{Scale factor} = \frac{\textbf{dimension of drawing}}{\textbf{actual dimension}}$$

WORKED EXAMPLE 1 Determining scale factor

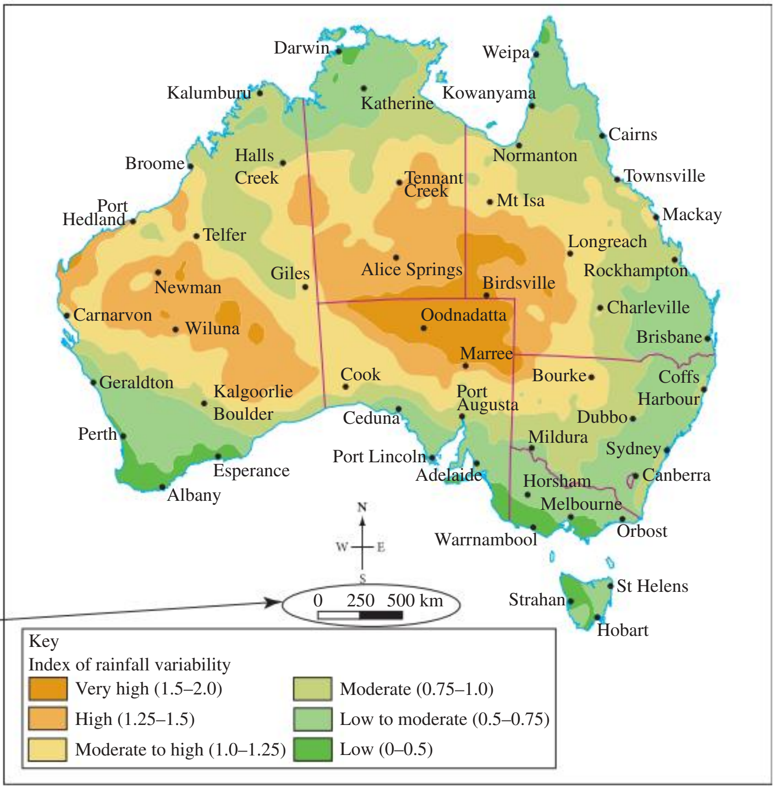
Determine the scale and the scale factor of a drawing where 8 cm on the diagram represents 4 km in reality.

THINK	WRITE
1. Convert the larger unit into the smaller unit.	$4\text{ km} = 4 \times 1000 \times 100$ $= 400\,000\text{ cm}$
2. Write the scale of the drawing with the same units.	$8\text{ cm} : 400\,000\text{ cm}$ $\Rightarrow 8 : 400\,000$

3. Simplify the scale by dividing both sides of the ratio by the highest common factor. We divide both sides of the ratio by 8 as 8 is the highest common factor of both 8 and 400 000. State the scale.	1 : 50 000
4. State the scale factor of the drawing.	The scale factor is $\frac{1}{50\,000}$.

5.2.2 Maps and scales

- Maps are always drawn at a smaller scale. A map has its scale written or drawn using a diagram.



Scale of the map

Source: MAPgraphics Pty Ltd, Brisbane

WORKED EXAMPLE 2 Determining scales of maps

State the scale of a map as a ratio using the graphical scale shown, where the length of each partition is 1 cm.

THINK	WRITE
1. Measure the length of each partition in the diagram.	Each partition is 1 cm.
2. Determine the length that each partition represents in reality.	Each partition represents 500 km.
3. Write the scale as a ratio.	1 cm : 500 km

4. Convert the larger unit into the smaller unit. $500\text{ km} = 500 \times 1000 \times 100$
$= 50\,000\,000\text{ cm}$

5. Write the scale as a ratio in the same units. $1\text{ cm} : 50\,000\,000\text{ cm}$

6. State the scale of the map in ratio form. $1 : 50\,000\,000$

5.2.3 Lengths

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- We can rearrange the scale factor formula to determine the actual dimension.

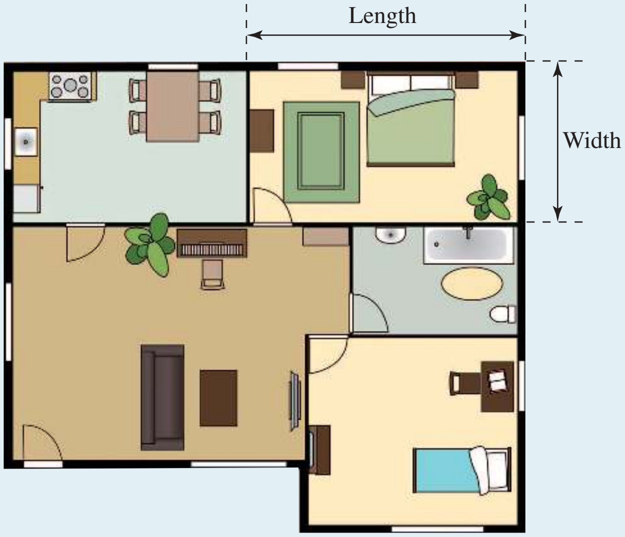
Actual dimension

$$\text{Actual dimension} = \frac{\text{dimension on the drawing}}{\text{scale factor}}$$

- To calculate the actual dimensions, we measure the dimensions on the diagram and then divide these by the scale factor.

WORKED EXAMPLE 3 Using scales to find length and width

A scale of 1 : 200 was used for the diagram of the bedroom shown. Calculate both the length and the width of the master bedroom, given the dimensions on the plan are length = 2.5 cm and width = 1.5 cm.



THINK	WRITE
1. Measure the length and the width of the room in the diagram.	Length = 2.5 cm Width = 1.5 cm

2. Write down the scale of the drawing and state the scale factor.	1 : 200. This means that every 1 cm on the drawing represents 200 cm of the actual dimension. The scale factor is $\frac{1}{200}$.
3. Divide both dimensions by the scale factor.	$\begin{aligned}\text{Length of the bedroom} &= 2.5 \div \frac{1}{200}\\ &= 2.5 \times 200\\ &= 500\text{ cm}\\ &= 5\text{ m}\end{aligned}$ $\begin{aligned}\text{Width of the bedroom} &= 1.5 \div \frac{1}{200}\\ &= 1.5 \times 200\\ &= 300\text{ cm}\\ &= 3\text{ m}\end{aligned}$

Exercise 5.2 Reading and interpreting scale drawings

learnon

5.2 Quick quiz on

5.2 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15	N/A	N/A

These questions are even better in jacPLUS!

- Receive immediate feedback
- Access sample responses
- Track results and progress

Find all this and MORE in jacPLUS

Simple familiar

1. **WE1** Determine the scale and the scale factor of a drawing where 150 mm on the diagram represents 45 m in reality.

2. If the scale of a diagram is 3.9 cm : 520 km, write the scale using same units.

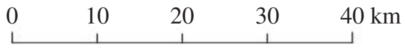
3. If the scale of a diagram is 1 : 450, determine the actual length represented by a length of 3 cm on the diagram.

4. Determine the scale and the scale factor of a drawing where:
   a. 7 cm represents 3.5 km in reality
   b. 16 mm represents 6.4 m in reality.

5. Calculate the scale and scale factor given the following.
   a. 5 cm represents 185 m in reality.
   b. 12 mm represents 19.2 m in reality.

6. Determine the scale and scale factor given the following.
   a. 12 cm represents 144 m in reality.
   b. 6 mm represents 3.6 km in reality.

7. **WE2** State the scale of a map as a ratio using the graphical scale shown, where the length of each partition is 1 cm.

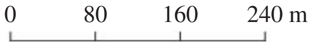


8. State the scale of a map as a ratio using the graphical scale shown, where the length of each partition is 1 cm.

9. State the scale of a map as a ratio using the graphical scale shown.

   a. (Each partition is 1.5 cm.)

   b. (Each partition is 1 cm.)



10. State the scale of a map as a ratio using the graphical scale shown, given each partition is 2 cm.

11. **WE3** A scale of 1 : 300 was used for the diagram of the house shown. Calculate both the height and width, in metres, of the garage door, given the dimensions on the plan are height = 0.8 cm and width = 2 cm.

12. A house plan is drawn to a scale of 1 : 1000.

   a. Calculate the actual length of the house if it is represented by 3.2 cm on the plan.

   b. The width of the house is 17 m. Calculate the width of the house on the plan.

13. A house plan is drawn to a scale of 1 : 500.

   a. Calculate the actual length of the lounge room if it is represented by 0.6 cm on the plan.

   b. The width of the bedroom is 5.2 m. Calculate the width of the bedroom on the house plan.

14. Calculate the dimensions of the carpet shown, given that its diagram was drawn using a scale of 1 : 75 and its length and width on the diagram are 3 cm and 2 cm respectively.

15. A 15 m fence is represented by a straight line 4.5 cm long on a drawing. Determine the scale of the drawing.

Fully worked solutions for this chapter are available online.

LESSON
5.3 Calculating, costing and comparing using measurements from scale drawings

SYLLABUS LINKS

- Find actual measurements from scale drawings, including lengths, perimeters and areas.
- Estimate and compare quantities, materials and costs using actual measurements from scale drawings [complex].

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- All diagrams, maps or plans are drawn using a given scale.
- This scale is used to convert the lengths in the diagram to the actual lengths.

5.3.1 Perimeters

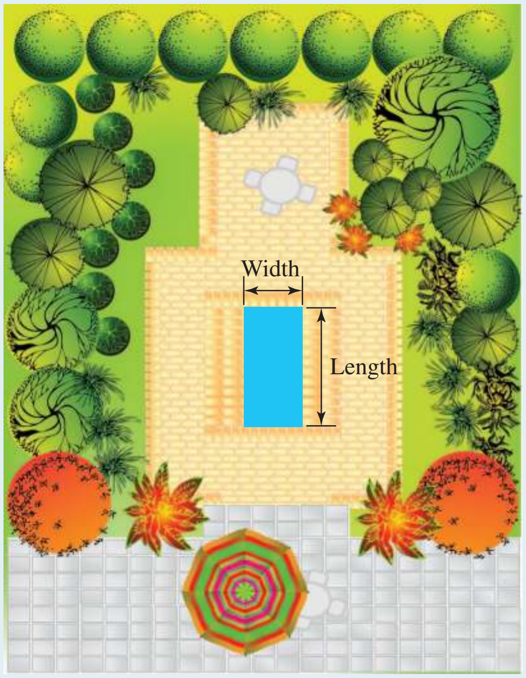
- Calculating the actual perimeters of shapes from scale drawings involves first calculating the lengths required. This procedure was described in the previous section.
- Once the lengths of the sides of the shape are known, the perimeter can be calculated by adding up all the sides of the shape.

WORKED EXAMPLE 4 Using scale to find perimeter

Given a scale of 1 : 1000 for the diagram of the backyard shown, calculate:

a. the actual length and the width of the pool, given the diagram measurements are 2.4 cm by 1.2 cm

b. the actual perimeter of the pool.



THINK	WRITE
a. 1. Measure the length and the width of the pool on the diagram.	**a.** Length = 2.4 cm Width = 1.2 cm
2. Write down the scale of the drawing and state the scale factor.	1 : 1000. This means that every 1 cm on the drawing represents 1000 cm of the actual dimension. The scale factor is $\frac{1}{1000}$.
3. Divide both dimensions by the scale factor.	$\text{Length of the pool} = 2.4 \div \frac{1}{1000} = 2.4 \times 1000 = 2400\text{ cm} = 24\text{ m}$ $\text{Width of the pool} = 1.2 \div \frac{1}{1000} = 1.2 \times 1000 = 1200\text{ cm} = 12\text{ m}$
b. 1. Calculate the perimeter of the pool.	**b.** $\text{Perimeter of the pool} = 2 \times \text{length} + 2 \times \text{width} = 2 \times 24 + 2 \times 12 = 72\text{ m}$

5.3.2 Areas

- Areas of surfaces from scale drawings can be calculated in a similar way as the perimeter. The dimensions of the shape have to be calculated first.
- Once the dimensions of the shape are known, the area can be calculated by using an appropriate formula.

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WORKED EXAMPLE 5 Using scale to find area

Given a scale of 1:15 for the diagram of the tile shown, calculate:

a. the actual length and the width of the tile, given the diagram measurements are 2 cm by 2 cm

b. the actual area of the tile

c. the actual area covered by 20 tiles.

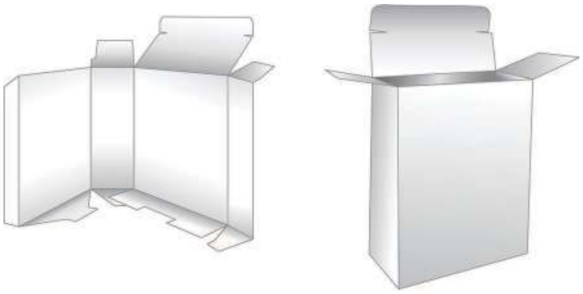
THINK	WRITE
a. 1. Measure the length and the width of the shape.	**a.** Length = 2 cm Width = 2 cm
2. Write down the scale of the drawing and state the scale factor.	1 : 15. This means that every 1 cm on the drawing represents 15 cm of the actual dimension. The scale factor is $\frac{1}{15}$.
3. Divide both dimensions by the scale factor.	$\text{Length of the tile} = 2 \div \frac{1}{15} = 2 \times 15 = 30\text{ cm}$ $\text{Width of the tile} = 2 \div \frac{1}{15} = 2 \times 15 = 30\text{ cm}$
b. 1. Calculate the area of the tile using the formula for the area of a square. Use $A = l^2$.	**b.** $\text{Area of one tile} = \text{length}^2 = 30^2 = 900\text{ cm}^2$
c. 1. Calculate the area required by multiplying the area of one tile by 20. *Note:* Recall the conversion from square centimetre to square metre is to divide by 100^2.	**c.** $\text{Total area} = \text{Area of one tile} \times 20 = 900 \times 20 = 18\,000\text{ cm}^2 = 18\,000 \div 100^2 = 1.8\text{ m}^2$

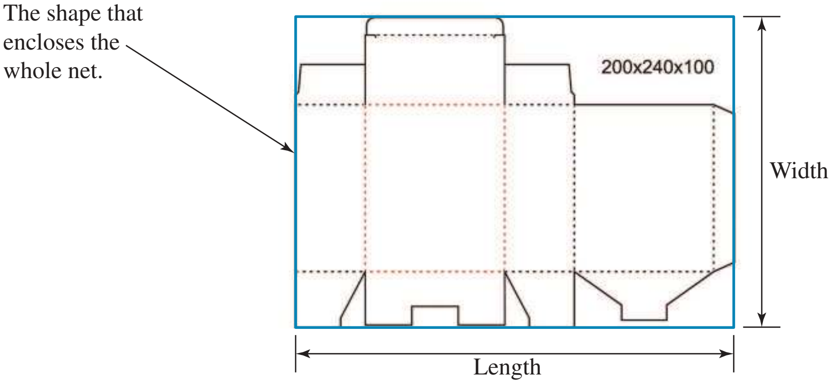
5.3.3 Quantities, materials and costs [complex]

- Scaled diagrams are used in many areas of work in order to estimate elements such as production costs and quantities of materials required.

Materials

- There are many products that are usually placed in packaging materials when purchased or delivered. Boxes are cut out of cardboard or other materials using a template.
- The dimensions of the package are determined by using a net of the 3-dimensional shape as shown in the diagram.





- The required area can be estimated by calculating the area of the rectangle that encloses the whole net.

Costs

- To calculate the cost of the materials used, we use the cost per metre or square metre, known as the cost per unit.

Cost of materials

$$\textbf{Cost required} = \frac{\textbf{area of material}}{\textbf{area per squared unit}} \times \textbf{cost per unit}$$

- If the area of the packaging is 520 cm^2 and the cost of the material is \$1.20 per square metre, then the cost for packaging is as follows.

$$\begin{aligned}\text{Cost required} &= \frac{\text{area of material}}{\text{area per squared unit}} \times \text{cost per unit}\\ &= \frac{520\ \text{cm}^2}{1\ \text{m}^2} \times 1.20\end{aligned}$$

Note: The two area units have to be the same: $1\ \text{m}^2 = 10\,000\ \text{cm}^2$.

$$\begin{aligned}&= \frac{520}{10\,000} \times 1.20\\ &= \$0.0624\end{aligned}$$

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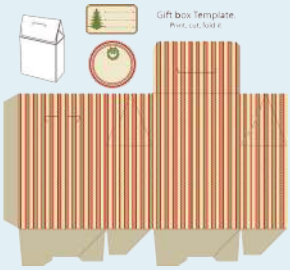
WORKED EXAMPLE 6 Using scale to estimate area and determine cost

Consider the gift box shown and the template required to make it. The scale of the diagram is 1 : 15, and the piece the template is cut from has measurements of 6 cm by 4 cm.

a. Calculate the width and the height of the packaging template.

b. Estimate the total area of material required.

c. If the cost of materials is \$0.25 per square metre, calculate how much it would cost to make this gift box.



THINK	WRITE
a. 1. Measure the width and the height of the template.	**a.** Width = 6 cm Height = 4 cm
2. Write down the scale of the drawing and state the scale factor.	1 : 15. This means that every 1 cm on the drawing represents 15 cm of the actual dimension. The scale factor is $\frac{1}{15}$.
3. Divide both dimensions by the scale factor.	$\text{Width of template} = 6 \div \frac{1}{15}$ $= 6 \times 15$ $= 90\text{ cm}$ $\text{Height of template} = 4 \div \frac{1}{15}$ $= 4 \times 15$ $= 60\text{ cm}$
b. 1. Estimate the area of the box template using the formula for the area of a rectangle.	**b.** $\text{Area of template} = \text{width} \times \text{height}$ $= 90 \times 60$ $= 5400\text{ cm}^2$
c. 1. Calculate the cost of the material.	**c.** Cost is \$0.25 per m^2 or $10\,000\text{ cm}^2$ $\text{Cost required} = \frac{\text{Area of material}}{\text{Area per squared unit}} \times \text{Cost per unit}$ $= \frac{5400}{10\,000} \times 0.25$ $= \$0.135$ ≈ 0.14
2. State the answer.	It will cost \$0.14 or 14 cents to make the gift box.

Painting

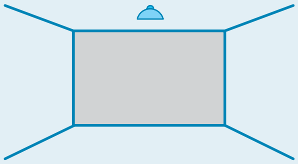
- Painting a wall requires some calculations. Three pieces of information are required to calculate the amount of paint needed:
  - the area of the surface to be painted
  - the amount of paint per square metre
  - the cost of the paint per litre.

- The following formulas are useful.

$$\text{Amount of paint required} = \text{area to be painted} \times \text{amount of paint per square metre}$$

$$\text{Total cost} = \text{amount of paint required} \times \text{cost of paint per litre}$$

WORKED EXAMPLE 7 Using scale to determine area and cost



Indra is going to paint a feature wall in her lounge room. She has a diagram of the room drawn at a scale of 1 : 100 with a length of 4 cm and height 2.5 cm. Calculate:

a. the length and the height of the wall

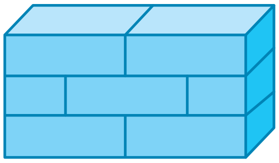
b. the area of the wall

c. the amount of paint Indra has to buy if the amount of paint needed is 3 L per m^2

d. the total cost of the paint if the price is $7.60 per litre.

THINK	WRITE
a. 1. Measure the length and the height of the shape.	**a.** Length = 4 cm Height = 2.5 cm
2. Write down the scale of the drawing and state the scale factor.	1 : 100. This means that every 1 cm on the drawing represents 100 cm of the actual dimension. The scale factor is $\frac{1}{100}$.
3. Divide both dimensions by the scale factor.	$\text{Length of the wall} = 4 \div \frac{1}{100} = 4 \times 100 = 400\text{ cm} = 4\text{ m}$ $\text{Height of the wall} = 2.5 \div \frac{1}{100} = 2.5 \times 100 = 250\text{ cm} = 2.5\text{ m}$
b. Calculate the area of the wall using the formula for the area of a rectangle.	**b.** $\text{Area wall} = \text{length} \times \text{width (or height)} = 4 \times 2.5 = 10\text{ m}^2$
c. Calculate the amount of paint required.	**c.** $\text{Amount of paint required} = \text{area to be painted} \times \text{amount of paint per square metre} = 10 \times 3\text{ L} = 30\text{ L}$
d. Calculate the cost of the paint.	**d.** $\text{Total cost} = \text{amount of paint required} \times \text{cost of paint per litre} = 30 \times 7.60 = \$228$

Bricklaying



- Bricks are frequently used as a basic product in the construction of houses or fences.
- Bricklayers have to calculate the number of bricks required to build a wall by considering the length, width and height of the brick and the dimensions of the wall.
- The amount is an estimated value, as calculating the number of bricks doesn't generally take into account the mortar that is added to the structure of the wall.

tlvd-12168

WORKED EXAMPLE 8 Estimating cost

Michael is going to build a brick fence with the dimensions shown in the diagram. The scale factor of the drawing is $\frac{1}{50}$.

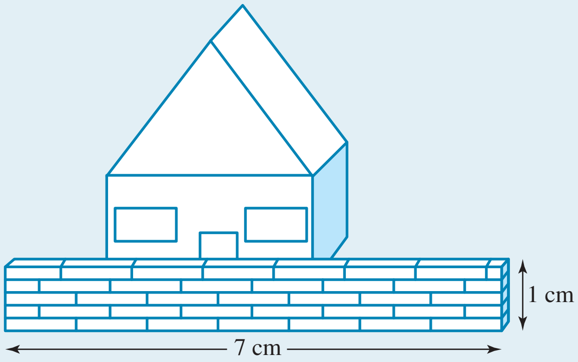
Calculate:

a. **the length and the height of the fence, given it measures 7 cm by 1 cm in the diagram**

b. **the area of the front side of the fence**

c. **the number of bricks required if the dimensions of the exposed side of one brick are 20 cm and 10 cm**

d. **the cost of building the fence if the price of one brick is $1.20.**



THINK	WRITE
a. 1. Measure the length and the height of the fence.	**a.** Length = 7 cm Height = 1 cm
2. Write down the scale factor.	The scale factor is $\frac{1}{50}$.
3. Divide both dimensions by the scale factor.	$\text{Length of the fence} = 7 \div \frac{1}{50} = 7 \times 50 = 350\text{ cm}$ $\text{Height of the fence} = 1 \div \frac{1}{50} = 1 \times 50 = 50\text{ cm}$
b. Calculate the area of the front face of the fence using the formula for the area of a rectangle.	**b.** $\text{Number of bricks} = \frac{17\,500}{200} = 87.5\text{ bricks} = 88\text{ bricks}$
c. 1. Calculate the area of the exposed face of one brick.	**c.** $\text{Area of the exposed face of the brick} = 20 \times 10 = 200\text{ cm}^2$

2. Calculate the number of bricks required. *Note:* Round up as you don't usually buy half a brick.

$$\text{Number of bricks} = \frac{17\,500}{200}$$
$$= 87.5 \text{ bricks}$$
$$= 88 \text{ bricks}$$

d. 1. Calculate the cost.

d. Cost = number of bricks × cost of one brick
$= 88 \times 1.20$
$= \$105.60$

Exercise 5.3 Calculating, costing and comparing using measurements from scale drawings

learnon

5.3 Quick quiz on

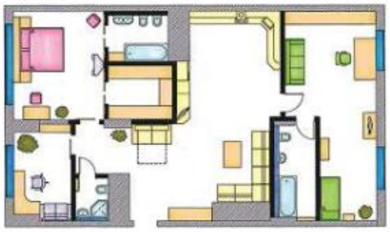
5.3 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6	7, 8, 9, 10, 11, 12, 13	14, 15

Note: Not all diagrams shown are drawn to scale of measurements given.

Simple Familiar

1. **WE4** Given a scale of 1 : 250 for the floor plan of the apartment shown, calculate:
   a. the actual length and the width of the apartment, given the diagram measurements are 5 cm by 3 cm
   b. the actual perimeter of the apartment.



2. If the scale of a diagram is 1 : 800, calculate, to the nearest metre, the radius and the circumference of a Ferris wheel, given the radius measures 1.5 cm on the diagram.

3. **WE5** Given a scale of 1 : 40 for the window shown, calculate:

   a. the actual width and height of each piece of glass needed to cover the window, given that the large piece of glass measures 1.5 cm by 2.5 cm on the diagram, and the width of the small piece of glass measures 0.75 cm
   b. the actual areas of the two glass pieces
   c. the total area covered by glass.

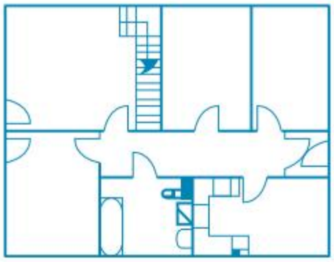
4. If the scale of the diagram shown is 1 : 20, calculate, to the nearest centimetre, the radius and the area of the clock face shown, given the radius measures 1.5 cm on the diagram.

5. **WE6** Consider the mailing box shown and the template required to make it. The scale of the diagram is 1 : 8, and the diagram measurements are 3.5 cm by 1.5 cm.

   a. Calculate the length and the width of the packaging template.
   b. Estimate the total area of material required.
   c. If the cost of materials is $0.85 per square metre, calculate how much it would cost to make this DVD mailing box.

6. The scale of the house plan shown is 1 : 200. Calculate:

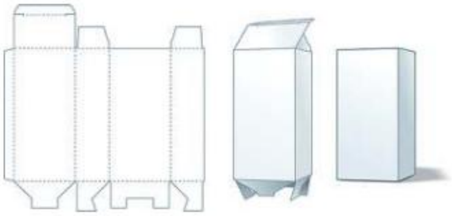
   a. the length of the bathroom, given it measures 1.2 cm long on the plan
   b. the width of the hallway, given it is 0.6 cm wide on the plan
   c. the perimeter of the kitchen, given it measures 1 cm by 2 cm on the plan
   d. the floor area of the house, given it measures 4.5 cm by 3.5 cm on the plan.



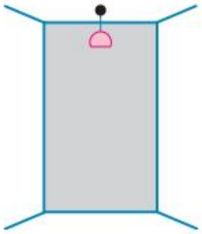
Complex familiar

7. Consider the milk box shown and the template required to make it. The scale of the diagram is 1 : 7.

   a. Calculate the length and the width of the packaging template, given the dimensions measure 3 cm by 3 cm on the diagram.
   b. Estimate the total area of material required.
   c. If the cost of materials is $0.45 per square metre, calculate how much it would cost to make this milk box.



8. **WE7** Ilia was asked by his friend to paint a bedroom wall. His friend gave him a diagram of the bedroom drawn at a scale of 1 : 100. If the length and height of the wall on the diagram are 1.5 cm and 2.5 cm respectively, calculate:

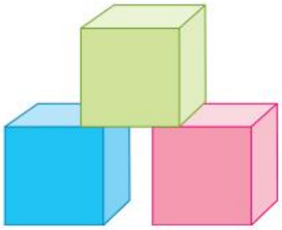


a. the length and the height of the wall
b. the area of the wall
c. the amount of paint Ilia has to buy if the amount of paint needed is 2.6 L per m^2
d. the total cost of the paint if the price is $12.80 per litre.

9. Jonathan wants to paint a wall with dimensions 5.6 m long and 2.4 m high.

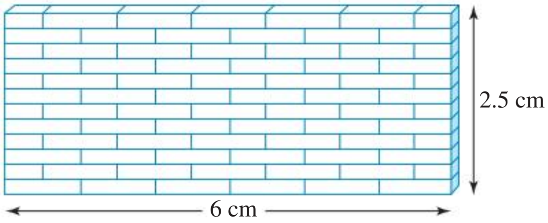
a. Calculate how many litres of paint are required if 1.5 L of paint covers 1 m^2.
b. Calculate the cost of the paint if the price per litre is $15.70.

10. The three identical wooden cubes shown are pictured at a scale of 1 : 20. Calculate:



a. the length of their sides, given they measure 1.3 cm on the diagram
b. the area of each face
c. the total area to be painted for the three cubes (state the answer in square metres correct to 1 decimal place)
d. the amount of paint required if 2 L of paint is needed per square metres
e. the total cost of the paint if the price is $9.25 per litre.

11. **WE8** Consider a brick fence with the dimensions shown in the diagram. The scale factor of the brick wall shown is $\frac{1}{100}$. Calculate:



a. the length and the height of the fence, given that it measures 6 cm by 2.5 cm on the diagram
b. the area of the front side of the fence
c. the number of bricks required if the dimensions of the exposed side of one brick are 20 cm by 5 cm
d. the cost of the fence if the price of one brick is $0.75.

12. Sky wants to pave a garden path with bricks. The garden path is shown at a scale of 1 : 150. Calculate:

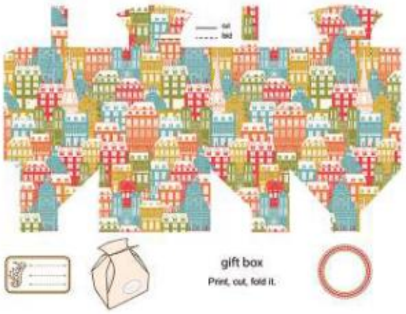
a. the actual dimensions of the garden path, given the diagram measures 8.5 cm by 0.75 cm
b. the area of the garden path
c. the number of bricks required if the bricks used are squares with side length 24 cm
d. the cost of paving the garden path if the price of one brick is $1.65.

13. A brick has length 215 mm and height 65 mm. A bricklayer is building a wall 6 m long and 7.5 m high. Calculate:

a. the number of bricks required
b. the total cost for building the wall if one brick costs $1.95.

Complex unfamiliar

14. Consider the gift box shown and the template required to make it.
The scale of the diagram is 1 : 8.
Estimate the total area of the material required, given it measures 8 cm by 5 cm on the diagram, and hence estimate the cost to make the gift box, given the material costs \$2.15 per square metre.



15. If the scale of the diagram shown is 1 : 12, given the radius measures 1.5 cm on the diagram and the cost of the material is \$50 per square metre, compare the cost of the material to make the circular chopping board to the cost of the wasted material if the board is cut from the smallest square piece of material possible.

Fully worked solutions for this chapter are available online.

LESSON
5.4 Creating scale drawings [complex]

SYLLABUS LINKS

- Understand and apply drawing conventions of scale drawings, including scales in ratio, clear indications of dimensions and clear labelling [complex].
- Construct scale drawings by hand and by using software packages [complex].

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5.4.1 Calculating the dimensions of a drawing

- If the scale factor is calculated by

$$\text{scale factor} = \frac{\text{dimension on the drawing}}{\text{actual dimension}}$$

then the dimensions of the drawing can be calculated by the following formula.

Calculating the dimensions of a drawing

$$\textbf{Dimension of drawing} = \textbf{scale factor} \times \textbf{actual dimension}$$

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WORKED EXAMPLE 9 Calculating dimensions for scale drawings

Calculate the dimensions required to create a scale drawing of a shipping container 12 m long, 2.5 m wide and 3 m high using a scale of 1 : 100.

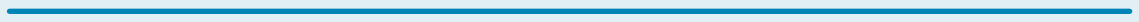
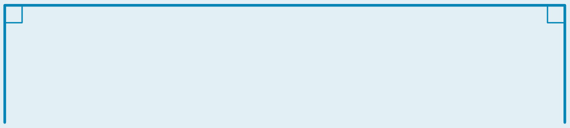
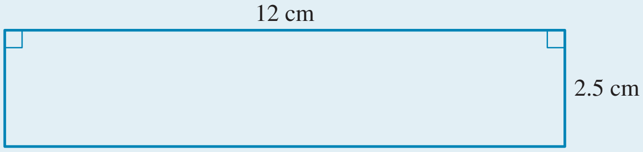
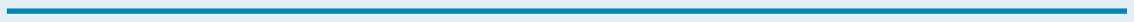
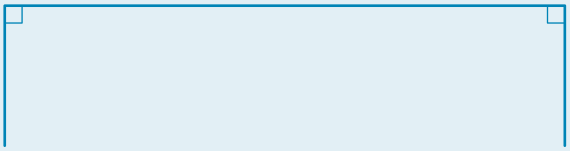
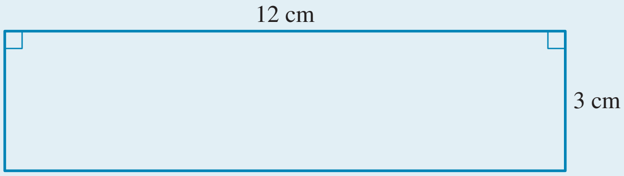
THINK	WRITE
1. Convert all dimensions to an appropriate drawing unit.	The dimensions of the diagram are going to be smaller than the actual dimensions of the shipping container. Convert m to cm: $12\text{ m} = 1200\text{ cm}$ $2.5\text{ m} = 250\text{ cm}$ $3\text{ m} = 300\text{ cm}$
2. Write the scale as a scale factor.	$1 : 100 = \frac{1}{100}$
3. Calculate the length of the object on the scale drawing.	$\frac{\text{Dimension on the drawing}}{\text{Actual dimension}} = \frac{1}{100}$ $\text{Length on the drawing} = \text{scale factor} \times \text{actual dimension}$ $= \frac{1}{100} \times 1200$ $= \frac{1200}{100}$ $= 12\text{ cm}$
4. Calculate the width of the object on the scale drawing.	$\text{Width on the drawing} = \text{scale factor} \times \text{actual dimension}$ $= \frac{1}{100} \times 250$ $= \frac{250}{100}$ $= 2.5\text{ cm}$
5. Calculate the height of the object on the scale drawing.	$\text{Height on the drawing} = \text{scale factor} \times \text{actual dimension}$ $= \frac{1}{100} \times 300$ $= \frac{300}{100}$ $= 3\text{ cm}$

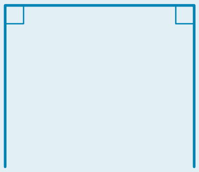
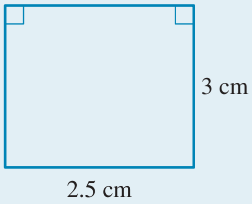
5.4.2 Constructing scale drawings

- Scale drawings are drawn on graph paper or plain paper using a pencil and a ruler.
- Scale drawings can also be constructed using various software packages.
- Accuracy is very important when constructing these drawings.

WORKED EXAMPLE 10 Constructing scale drawings

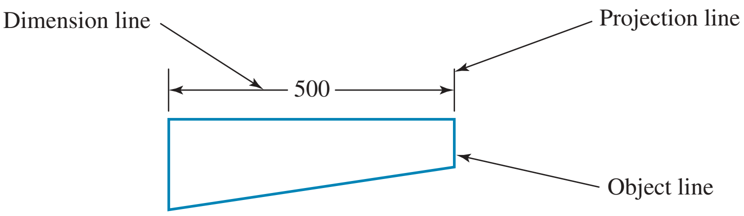
Construct a plan view, side view and front view scale drawing of the shipping container from Worked example 9.

THINK	WRITE/DRAW
1. State the drawing dimensions of the plan view of the object.	The drawing dimensions of the plan view of the shipping container are: Length = 12 cm Width = 2.5 cm
2. Draw the plan view of the object. Recall that a plan view is a diagram of the object from above.	To draw the length of the object, construct a horizontal line 12 cm long.  Construct two 90° angles on both sides of the line segment. Ensure the two vertical lines are 2.5 cm long.  Connect the bottom ends of the vertical lines to complete the rectangle. 
3. State the drawing dimensions of the side view of the object.	The drawing dimensions of the side view of the shipping container are: Length = 12 cm Height = 3 cm
4. Draw the side view of the object. Recall that the side view is a diagram of the object looking at the object from one side.	To draw the length of the object, construct a horizontal line 12 cm long.  Construct two 90° angles on both sides of the line segment. Ensure the two vertical lines are 3 cm long.  Connect the bottom ends of the vertical lines to complete the rectangle. 

5. State the drawing dimensions of the front view of the object.	The drawing dimensions of the front view of the shipping container are: Width = 2.5 cm Height = 3 cm
6. Draw the front view of the object. Recall that the front view is a diagram of an object looking straight at the object from the front.	We are going to draw the length of the object, which is the width of the shipping container. Draw a horizontal line 2.5 cm long.  Construct two 90° angles on both sides of the line segment. Ensure the two vertical lines are 3 cm long.  Connect the bottom ends of the vertical lines to complete the rectangle. 

5.4.3 Labelling scale drawings

- Scale diagrams are labelled using the actual dimensions of the object.
- The labels of a scale drawing have to be clear and easily read.
- The line style and its thickness indicate different details of the drawing.
  - A *thick continuous line* represents a visible line, a contour of a shape or an object.
  - A *thick dashed line* represents a hidden line, a hidden contour of a shape or an object.
  - *Thin continuous lines* are used to mark the dimensions of the shape or object.
- Dimension lines are thin continuous lines showing the dimension of a line. They have arrowheads at both ends.
- Projection lines are thin continuous lines drawn perpendicular to the measurement shown. These lines do not touch the object.

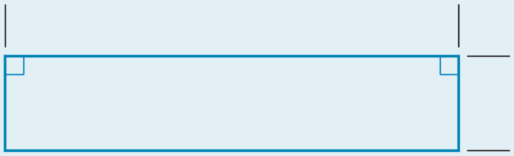
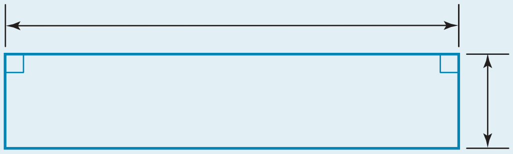
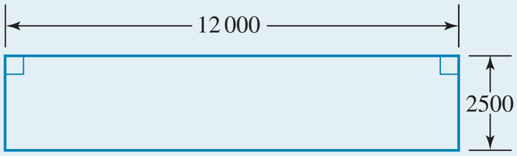
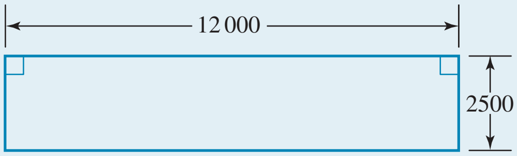
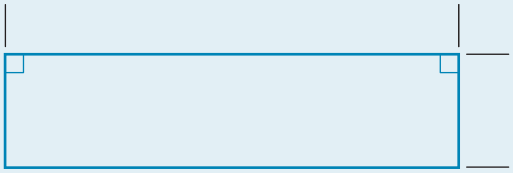
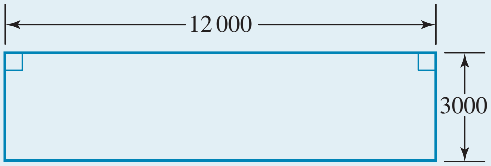


All dimensions in mm
Scale 1 : 100

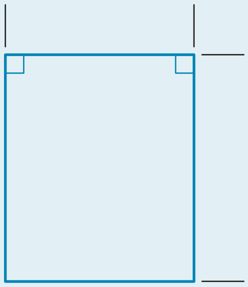
- All dimensions on a drawing must be given in millimetres; however, 'mm' is not to be written on the drawing. This can be written near the scale as shown in the diagram above.
- Dimensions must be written in the middle of the dimension line.

WORKED EXAMPLE 11 Sketching scale drawings

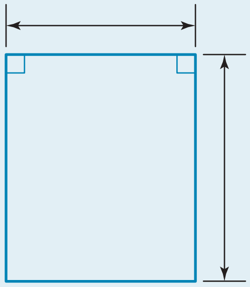
Label the three scale drawings of the shipping container from Worked example 10.

THINK	WRITE/DRAW
1. Draw the projection lines of the object in the plan view drawing using thin continuous lines.	
2. Draw the dimension lines of the object in the plan view drawing using thin continuous lines with arrowheads at both ends.	
3. Label the drawing.	The actual dimensions of the plan view of the shipping container are 12 m long and 2.5 m wide. *Note:* The dimensions have to be written in mm. 12 m = 12 000 mm and 2.5 m = 2500 mm 
4. Draw the projection lines of the object in the side view drawing using thin continuous lines.	
5. Draw the dimension lines of the object in the side view drawing using thin continuous lines with arrowheads at both ends.	 *Note:* The arrowheads of the dimension line for the height are directed from the outside of the projection line. This happens because the space between the projection lines is too small.
6. Label the drawing.	The actual dimensions of the side view of the shipping container are 12 m long and 3 m high. *Note:* The dimensions have to be written in mm. 12 m = 12 000 mm and 3 m = 3000 mm 

7. Draw the projection lines of the object in the front view drawing using thin continuous lines.



8. Draw the dimension lines of the object in the front view drawing using thin continuous lines with arrowheads at both ends.

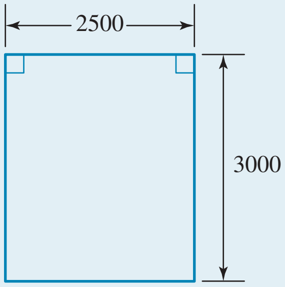


9. Label the drawing.

The actual dimensions of the front view of the shipping container are 2.5 m wide and 3 m high.

Note: The dimensions have to be written in mm.

2.5 m = 2500 mm and 3 m = 3000 mm



ACTIVITY: Constructing scale drawings using software

- Measure each of the rooms (including the hallways) in your house.
- Sketch a rough plan of your house on a piece of paper, including your measurements.
- Research a software package that can be used to draw scaled diagrams.
- Use this software package to construct a scaled diagram/plan of your house.

Exercise 5.4 Creating scale drawings [complex]

learnon

5.4 Quick quiz on

5.4 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
N/A	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11	12, 13

Complex familiar

1. **WE9** Calculate the dimensions required to create a scale drawing of a classroom 17 m long and 12 m wide to a scale of 1 : 100.

2. a. Calculate the dimensions required to create a scale drawing of a door 1.5 m wide, 2.4 m tall and 9 cm thick to a scale of 1 : 30.
   b. State the measurements that are to be written on a scale drawing.
   c. State whether the projection lines are parallel or perpendicular to the measurement shown.

3. The drawings described are constructed to a scale of 1 : 50.
   a. Calculate the height, on the drawing, of a tree 3.6 m tall.
   b. Calculate the dimensions, on the drawing, of a carpet 2.8 m by 1.4 m.
   c. Calculate the height, on the drawing, of a student 1.8 m tall.

4. The scale drawing of a house plan is constructed using a scale of 1 : 150.
   a. Determine the length of the hallway (in cm) on the plan if the actual hallway is 9 m long.
   b. Determine the dimensions of the actual kitchen if on the plans it measures 24 mm by 36 mm.

5. a. **WE10** Construct a plan view, front view and side view scale drawing of a skip with drawing dimensions 5 cm long, 2 cm wide and 1.5 cm deep.
   b. **WE11** Label the three scale drawings of the skip from part **a**, given that the actual dimensions of the skip are 2.5 m long, 1 m wide and 0.75 m deep.

6. Construct a plan view, front view and side view scale drawing of a rectangular lunch box with drawing dimensions 5.3 cm long, 2.7 cm wide and 0.9 cm deep.

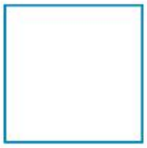
7. Calculate the dimensions to construct a scale drawing of the following shapes to the scale stated in brackets.
   a. A square of side length 1.7 cm (2 : 1)
   b. A rectangular pool table with dimensions 4.8 m by 4.0 m (1 : 80)
   c. A circular table with diameter 1.4 m (1 : 200)

8. Draw a house floor plan with actual dimensions 27.6 m by 18.3 m drawn to a scale of 1 : 600.

9. Construct and label a scale drawing of the floor of a bedroom 3.5 m long and 2.5 m wide drawn to a scale of 1 : 100.

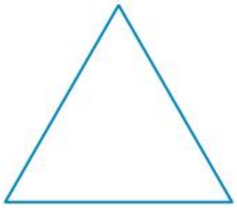
10. Construct and label a scale drawing of a window frame 1.8 m wide and 1.4 m tall drawn to a scale of 1 : 40.

11. Draw and label the projection and dimension lines on the following diagrams.
   a. Square with side length 6.1 m



   b. Rectangle 430 cm long and 210 cm wide

   c. Equilateral triangle with side length 49.7 cm and height 43.0 cm



Complex unfamiliar

12. A sculptor plans a sculpture by first creating a scale drawing. If the sculptor charged \$3900 for the actual sculpture, determine how much the sculptor charged (to the nearest dollar) per metre of the actual sculpture if the scale drawing was 35 cm tall using a scale of 1 : 25.

13. A painter is commissioned to draw a picture on the side of a building. They first draw a sketch with dimensions 45 cm by 35 cm using a scale of 1 : 12. The painter charges \$250 per square metre painted. Determine the cost to get the picture on the building painted.

Fully worked solutions for this chapter are available online.

LESSON
5.5 Review

5.5.1 Summary

Hey students! Now that it's time to revise this chapter, go online to:

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5.5 Exercise

5.5 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11	12, 13, 14, 15, 16, 17, 18	19, 20

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Simple familiar

1. If the scale of a diagram is 5 cm : 100 km, convert the scale using same units.

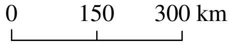
2. The scale factor of a diagram is 1 : 250. Determine the actual length of 7.5 cm on the diagram.

3. Calculate the scale and the scale factor of a drawing where:

a. 7 cm represents 4.9 km in reality

b. 8 mm represents 6.4 m in reality.

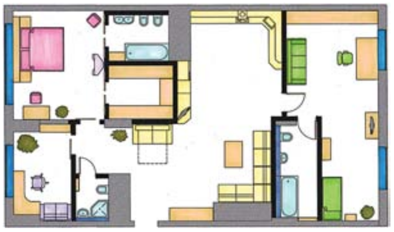
4. The graphical scale on a map is shown. State the scale of the map as a ratio, if each partition measures 2.5 cm.



5. Given a scale of 1 : 300 for the floor plan of the apartment shown, calculate:

a. the length and the width of the apartment, given it measures 6 cm by 4 cm on the diagram

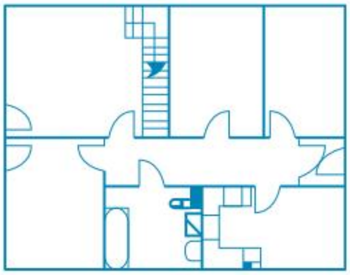
b. the perimeter of the apartment.



6. A sculptor plans a sculpture by first creating a scale drawing. Determine the height of the sculpture on a drawing if its actual height is intended to be 6.3 m and the scale factor is $\frac{1}{20}$.

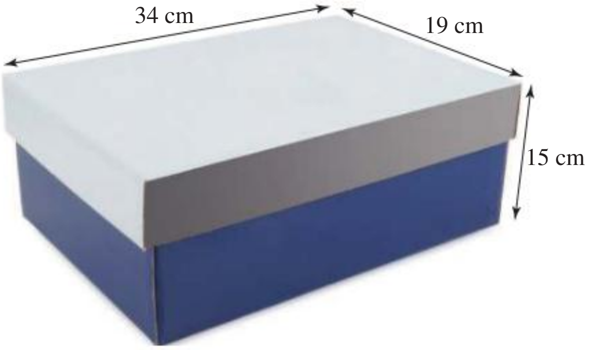
7. On a map with a scale 1 : 2 500 000, the distance between two towns is 3.9 cm. Determine the actual distance between the two towns in kilometres.

8. The scale of the house plan shown is 1 : 250. Calculate:



- **a.** the dimensions of the top left room, given it measures 1.8 cm by 1.7 cm on the plan
- **b.** the width of the hallway, given it is 0.6 cm wide on the plan
- **c.** the perimeter of the kitchen, given it measures 1 cm by 1 cm on the plan
- **d.** the floor area of the house, given it measures 4.5 cm by 3.5 cm on the plan.

9. A shoe box is 34 cm long, 19 cm wide and 15 cm high, as shown in the diagram. If a scale of 1 : 2 is used, determine the dimensions of the side view.



10. A scale drawing of a rectangle of length 4.5 cm and width 1.5 cm is to be constructed using a scale 1 : 3. Determine the length and width of the rectangle on the drawing.

11. A house plan is drawn to a scale of 1 : 500.

- **a.** Determine the actual length of the lounge room if it is represented by 0.6 cm on the plan.
- **b.** The width of the room is 5.2 m. Determine the width of the room on the house plan.

Complex familiar

12. The radius of a circular mirror on a plan measures 75 mm. If the scale of the diagram is 1 : 50, calculate, to the nearest centimetre, the radius and the area of the mirror.

13. An engineer is creating a scale drawing of a swimming pool. The actual pool dimensions are 15 m by 10 m. The scale drawing uses a scale of 1 : 400.

a. Calculate the measurements of the scale drawing.
b. Draw a scale drawing of the pool.
c. If the cost of constructing the pool is $220 per square metre of the actual pool, determine the total construction cost for the entire pool.

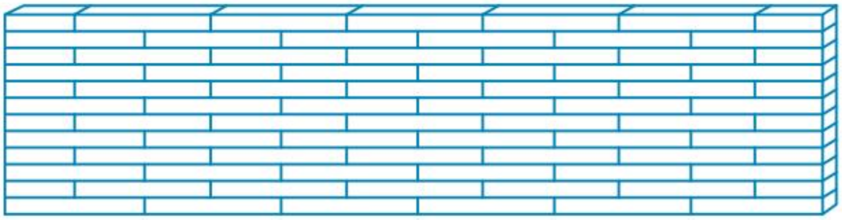
14. A brick has length 210 mm and height 55 mm. A bricklayer is building a wall 10 m long and 8 m high. Calculate:

a. the number of bricks required
b. the total cost for building the wall if one brick costs $2.15.

15. Sonia is to paint a feature wall in her lounge room. The plan uses a scale of 1 : 100 and it measures 8.5 cm by 24 mm on the plan. Calculate:

a. the length and the width of the wall
b. the area of the wall
c. the amount of paint Sonia has to buy if the amount of paint needed is 2.4 L per m^2
d. the total cost of the paint if the price is $16.75 per litre.

16. The scale factor of the brick wall shown is $\frac{1}{250}$.



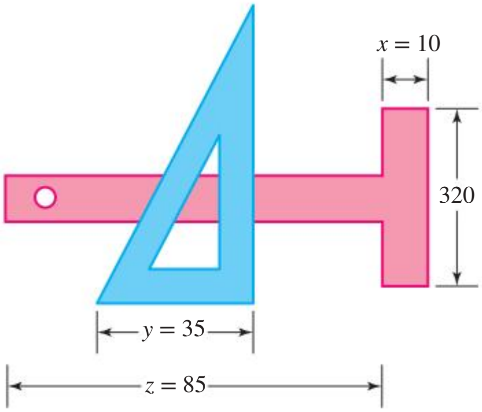
Calculate:

a. the length and the width of the wall, given it measures 10 cm by 2.5 cm on the diagram
b. the area of the front side of the wall
c. the number of bricks required if the dimensions of the exposed side of one brick are 20 cm by 5 cm
d. the cost of the wall if the price of one brick is $0.65.

17. Calculate the total cost of the paint required to paint an area of 68 m^2 at \$16.90 per litre if 2 L of paint are required per square metre.

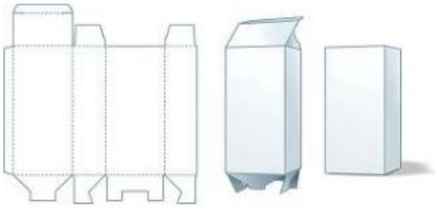
18. The objects in the diagram shown are drawing instruments.

- **a.** Calculate the scale factor of the drawing, given the length that is 32 mm measures 24 mm on the diagram.
- **b.** Calculate the actual measurements of the unknown lengths x, y and z, given they measure 6 mm, 21 mm and 51 mm respectively on the diagram.
- **c.** Determine the lengths of x, y and z on a drawing with a scale of 5 : 4.
- **d.** Construct a scale drawing using the new drawing dimensions from part **c**. Clearly label all measurements.



Complex unfamiliar

19. Consider the juice container shown and the template required to make it. The scale of the diagram is 1 : 5.
The juice is sold in crates of 36. If the packaging template measure 4 cm by 4 cm on the diagram. Given the package material costs \$0.55 per square metre, estimate the cost of packaging material for one crate.



20. Shae wants to pave a path towards her front door with bricks. The path is shown at a scale of 1 : 100.

Calculate the cost of the paving given it measures 10.5 cm by 0.75 cm on the diagram and the price per brick is \$1.85.

Fully worked solutions for this chapter are available online.

Answers

Chapter 5 Interpreting and creating scale drawings

5.2 Reading and interpreting scale drawings

5.2 Exercise

1. 1 : 300, $\frac{1}{300}$

2. 3 : 40 000 000

3. 1350 cm

4. a. 1 : 50 000, $\frac{1}{50\,000}$ **b.** 1 : 400, $\frac{1}{400}$

5. a. 1 : 3700, $\frac{1}{3700}$ **b.** 1 : 1600, $\frac{1}{1600}$

6. a. 1 : 1200, $\frac{1}{1200}$ **b.** 1 : 600 000, $\frac{1}{600\,000}$

7. 1 : 1 000 000

8. 1 : 100 000

9. a. 1 : 10 000 000 **b.** 1 : 8000

10. 1 : 500 000

11. Height = 2.4 m
Width = 6 m

12. a. 32 m **b.** 1.7 cm

13. a. 3 m **b.** 1.04 cm

14. Length = 2.25 m
Width = 1.5 m

15. 3 : 1000

5.3 Calculating, costing and comparing using measurements from scale drawings

5.3 Exercise

1. a. Width = 7.5 m
Length = 12.5 m

b. 40 m

2. Radius = 12 m
Circumference = 75 m

3. a. Height = 1 m
Big glass width = 0.6 m
Small glass width = 0.3 m

b. Area big glass = 0.6 m^2
Area of small glass = 0.3 m^2

c. 0.9 m^2

4. Radius = 30 cm and area = 2827 cm^2

5. a. Length = 28 cm
Width = 12 cm

b. 336 cm^2

c. 3 cents each

6. a. Length = 2.4 m **b.** Width = 1.2 m
c. Perimeter = 12 m **d.** Area = 63 m^2

7. a. Length = Width = 21 cm

b. 441 cm^2

c. 2 cents each

8. a. Length = 1.5 m
Height = 2.5 m

b. 3.75 m^2

c. 9.75 L

d. $124.80

9. a. 20.16 L **b.** $316.51

10. a. Width = Length = 0.26 m

b. 0.0676 m^2

c. 1.2 m^2

d. 2.4 L

e. $22.2

11. a. 6 m, 2.5 m **b.** 15 m^2
c. 1500 bricks **d.** $1125

12. a. 12.75 m, 1.125 m **b.** 14.34 m^2
c. 249 bricks **d.** $410.85

13. a. 3221 bricks **b.** $6280.95

14. $0.55

15. Board area = 0.1018 m^3
Board cost = $5.09
Waste area = 0.0278 m^3
Waste cost = $1.39
The ratio of used material to wasted material is 3.66 : 1.

5.4 Creating scale drawings [complex]

5.4 Exercise

1. Length = 17 cm
Width = 12 cm

2. a. Width = 5 cm, height = 8 cm, depth = 0.3 cm

b. Actual measurements

c. Perpendicular to the measurement shown

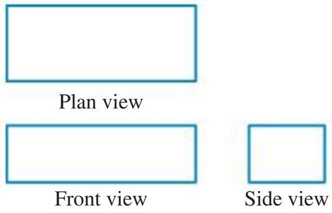
3. a. Height = 7.2 cm

b. 5.6 cm, 2.8 cm

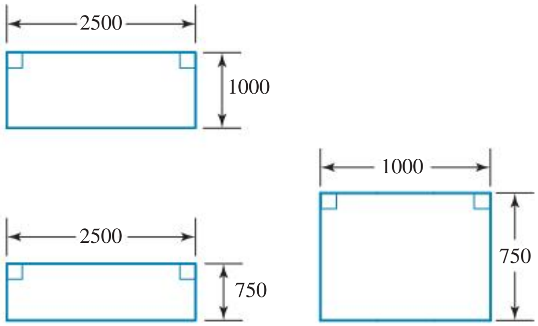
c. Height = 3.6 cm

4. a. 6 cm **b.** 3.6 m by 5.4 m

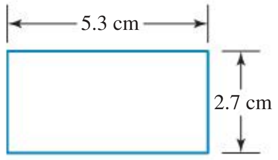
5. a.



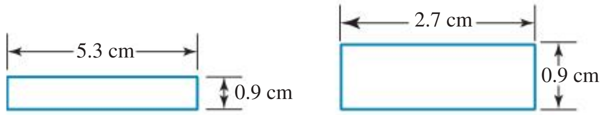
b.



6.



Plan view

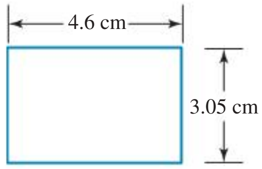


Front view

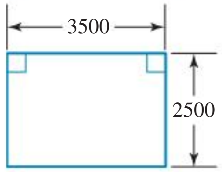
Side view

7. a. 3.4 cm **b.** 6 cm, 5 cm **c.** 0.7 cm

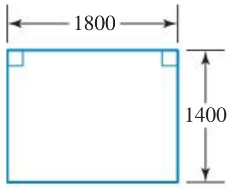
8.



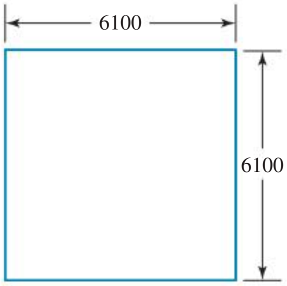
9.



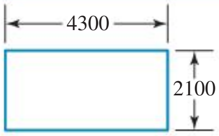
10.



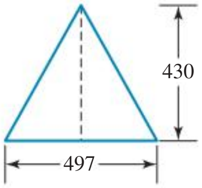
11. a.



b.



c.



12. \$446/m

13. \$5670

5.5 Review

5.5 Exercise

1. 1 : 2 000 000

2. 1875 cm

3. a. 1 : 70 000, $\frac{1}{70\,000}$ **b.** 1 : 800, $\frac{1}{800}$

4. 1 : 60 00 000

5. a. 18 m, 12 m **b.** 60 m

6. 31.5 cm

7. 97.5 km

8. a. Length = 4.5 m
Width = 4.25 m

b. 1.5 m

c. Length = 2.5 m
Width = 2.5 m
Perimeter = 10 m

d. Length = 11.25 m
Width = 8.75 m
Area = 98.44 m^2

9. 9.5 cm by 7.5 cm

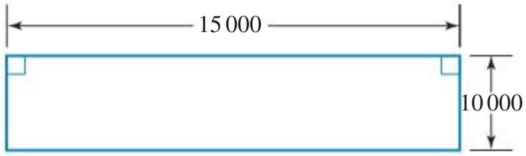
10. Length = 1.5 cm
Width = 0.5 cm

11. a. 3 m **b.** 1.04 cm

12. Radius = 375 cm
Area = 441 786 cm^2

13. a. 3.75 cm by 2.5 cm

b.



c. \$33 000

14. a. 6927 bricks **b.** \$14 893.05

15. a. Length = 8.5 m
Width = 2.4 m

b. 20.4 m^2

c. 48.96 L

d. \$820.08

16. a. 25 m, 6.25 m

b. 156.25 m^2

c. 15 625 bricks

d. \$10 156.25

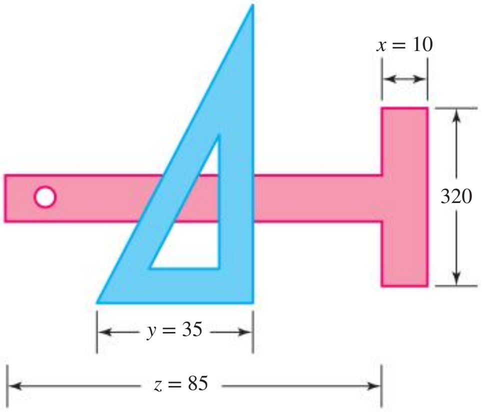
17. \$2298.40

18. **a.** $3:4$

b. $x = 8\text{ mm},\ y = 28\text{ mm},\ z = 68\text{ mm}$

c. $x = 6.4\text{ mm},\ y = 22.4\text{ mm},\ z = 40.8\text{ mm}$

d.



19. \$0.79

20. \$233.10

UNIT 3 Measurement, scales and chance
TOPIC 3 Scales, plans and models

6 Right-angled triangles

LEARNING SEQUENCE

Fully worked solutions for this chapter are available online.

on Resources

Solutions	Solutions — Chapter 6
Digital documents	Learning matrix — Chapter 6 Quick quizzes — Chapter 6 Chapter summary — Chapter 6

LESSON
6.1 Overview

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6.1.1 Introduction

In this chapter, we explore fundamental geometry and trigonometry concepts that are essential for solving problems involving right-angled triangles. The chapter begins with Pythagoras' theorem, a crucial tool for determining side lengths in right-angled triangles. This theorem has extensive practical applications, such as measuring distances and heights. We then delve into the trigonometric ratios of sine, cosine and tangent, which are key to finding unknown angles and sides, equipping you with skills applicable in fields such as engineering, carpentry and architecture.

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Finally, we examine the concepts of angles of elevation and depression, which help solve real-world problems such as calculating the height of a building or the slope of a ramp. By mastering these principles, you will be well-prepared for advanced studies and practical applications in various aspects of everyday life.

6.1.2 Syllabus links

Lesson	Lesson title	Syllabus links
6.2	**Pythagoras' theorem**	○ Apply Pythagoras' theorem to solve problems for all side lengths where c is length of hypotenuse and a and b are lengths of the perpendicular sides, using the formulas: • $c^2 = a^2 + b^2$ • $a^2 = c^2 - b^2$ • $b^2 = c^2 - a^2$
6.3	**Trigonometric ratios — sine, cosine and tangent**	○ Revision of trigonometric ratios
6.4	**Calculating unknown side lengths of right-angled triangles [complex]**	○ Apply the cosine, sine and tangent ratios to find unknown angles (θ) and sides [complex]. • $\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$ • $\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$ • $\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$
6.5	**Calculating unknown angles in right-angled triangles [complex]**	○ Apply the cosine, sine and tangent ratios to find unknown angles $\left(\theta\right)$ and sides [complex]. • $\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$ • $\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$ • $\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$
6.6	**Angles of elevation and depression [complex]**	○ Use the concepts of angle of elevation and angle of depression to solve practical problems [complex].

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LESSON
6.2 Pythagoras' theorem

SYLLABUS LINKS

- Apply Pythagoras' theorem to solve problems for all side lengths where c is length of hypotenuse and a and b are lengths of the perpendicular sides, using the formulas:
  - $c^2 = a^2 + b^2$
  - $a^2 = c^2 - b^2$
  - $b^2 = c^2 - a^2$

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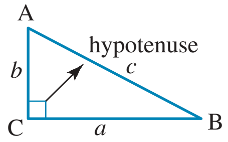
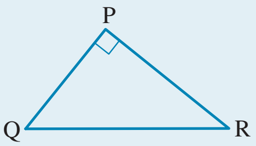
6.2.1 Identifying the hypotenuse

- Pythagoras' theorem allows us to calculate the length of a side of a right-angled triangle if we know the lengths of the other two sides.
- Consider triangle ABC shown. AB is the **hypotenuse** (the longest side). It is opposite the right angle.
- Note that the sides of a triangle can be named in either of two ways.
  1. A side can be named by the two capital letters given to the vertices at each end. This is what has been done in the figure shown to name the hypotenuse AB.
  2. We can also name a side by using the lower-case letter of the opposite vertex. In the figure shown, we could have named the hypotenuse 'c'.



WORKED EXAMPLE 1 Identifying the hypotenuse

Name the hypotenuse in the triangle shown.



THINK	WRITE
1. Identify the hypotenuse by locating the longest side.	The longest side is opposite the right angle, between vertices Q and R, and opposite vertex P.
2. Name the hypotenuse.	The hypotenuse is QR or p.

6.2.2 The formula for Pythagoras' theorem

- Consider the right-angled triangle ABC with sides 3, 4 and 5 cm.
- Squares have been constructed on each of the sides. The area of each square has been calculated by squaring the side length, and indicated inside the square.
- Note that the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.

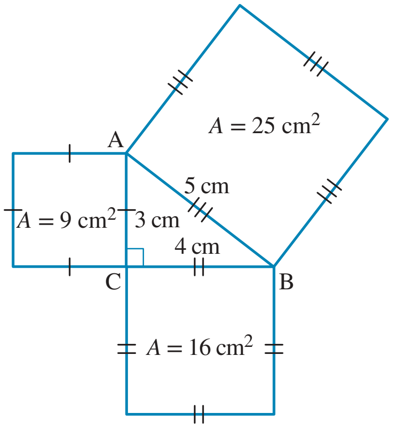
$$25\,\text{cm}^2 = 16\,\text{cm}^2 + 9\,\text{cm}^2$$

Alternatively: $(5\,\text{cm})^2 = (4\,\text{cm})^2 + (3\,\text{cm})^2$

Which means: $\text{hypotenuse}^2 = \text{base}^2 + \text{height}^2$

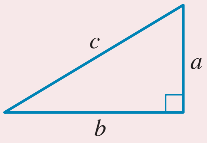
$$c^2 = a^2 + b^2$$

This result is known as Pythagoras' theorem.



Pythagoras’ theorem

In any right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the two shorter sides. That is,



$$c^2 = a^2 + b^2$$

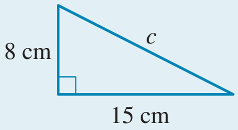
where *a* and *b* are the two shorter sides and *c* is the hypotenuse.

- This is the formula used to find the length of the hypotenuse in a right-angled triangle when we are given the lengths of the two shorter sides.

int-3844

WORKED EXAMPLE 2 Determining the length of hypotenuse

Determine the length of the hypotenuse in the triangle shown.



THINK	WRITE
1. This is a right-angled triangle, so we can use Pythagoras’ theorem. Write the formula for Pythagoras’ theorem.	$c^2 = a^2 + b^2$
2. Substitute the lengths of the shorter sides.	$c^2 = 15^2 + 8^2$
3. Evaluate the expression for c^2 by squaring 15 and 8, then completing the addition.	$= 225 + 64$ $= 289$
4. Calculate the value of c by taking the square root of both sides of the equation.	$\sqrt{c^2} = \sqrt{289}$ $c = \sqrt{289}$ $= 17\text{ cm}$
5. State the answer.	The length of the hypotenuse is 17 cm.

- In Worked example 2, the answer is a whole number because we can find $\sqrt{289}$ exactly.
- In most examples this will not be possible. In such cases, we are asked to write the answer correct to a given number of decimal places.

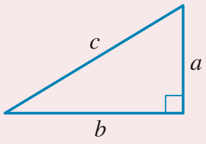
6.2.3 Determining the shorter side length

- By rearranging Pythagoras' theorem, we can write the formula to find the length of a shorter side of a right-angled triangle.

Pythagoras' theorem — determining the shorter side lengths

To determine the length of a shorter side of a right-angled triangle:

- $a^2 = c^2 - b^2$
- $b^2 = c^2 - a^2$

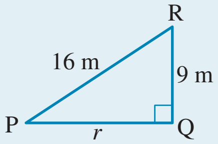


- The method of solving this type of question is the same as in the previous example, except that here we use subtraction instead of addition.
- For this reason, it is important to look at each question carefully to determine whether you are finding the length of the hypotenuse or one of the shorter sides.

int-3845

WORKED EXAMPLE 3 Determining the shorter side of a right-angled triangle

Calculate the length of side PQ in triangle PQR, correct to one decimal place.



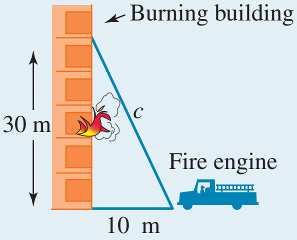
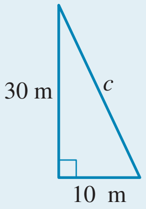
THINK	WRITE
1. This is a right-angled triangle, so we can use Pythagoras' theorem. Write the formula for Pythagoras' theorem when finding the length of a shorter side.	$b^2 = c^2 - a^2$
2. Substitute the lengths of the known sides. Place the unknown pronumeral on the left-hand side of the equation.	$r^2 = 16^2 - 9^2$
3. Evaluate the expression for r^2.	$= 256 - 81$ $= 175$
4. Calculate the answer by taking the square root of both sides of the equation and rounding the result to two decimal places.	$\sqrt{r^2} = \sqrt{175}$ $r = \sqrt{175}$ $\approx 13.2\text{ m}$
5. State the answer.	The length of PQ is 13.2 m

6.2.4 Problem solving with Pythagoras' theorem

- Pythagoras' theorem can be used to solve practical problems.
- In these cases, it is necessary to draw a diagram that will help you decide the appropriate method for finding a solution.
- The diagram simply needs to represent the triangle; it does not need to show details of the situation described.

WORKED EXAMPLE 4 Using Pythagoras' theorem to solve practical problems

The fire brigade attends a blaze in a tall building. They need to rescue a person from the sixth floor of the building, which is 30 m above ground level. Their ladder is 32 m long and must be at least 10 m from the foot of the building. Can the ladder be used to reach the people needing rescue?

THINK	WRITE
1. Define a pronumeral to represent the unknown distance in this problem.	For the fire brigade's ladder to be long enough, the straight-line distance from a point on the ground 10 m from the base of the building to the window 30 m above the ground must be less than 32 m. Let c represent the unknown distance.
2. Draw a diagram showing all important information.	 
3. The triangle is right-angled, so Pythagoras' theorem can be used. Write the rule for Pythagoras' theorem when finding the hypotenuse.	$c^2 = a^2 + b^2$
4. Substitute the lengths of the known sides.	$c^2 = 10^2 + 30^2$
5. Evaluate the expression on the right-hand side of the equation.	$= 100 + 900$ $= 1000$
6. Calculate the answer by taking the square root of both sides of the equation.	$\sqrt{c^2} = \sqrt{1000}$ $c = \sqrt{1000}$ ≈ 31.62 m
7. Answer the question.	The ladder will be long enough to make the rescue, since it is 32 m long.

Exercise 6.2 Pythagoras' theorem

6.2 Quick quiz on

6.2 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15	N/A	N/A

These questions are even better in jacPLUS!

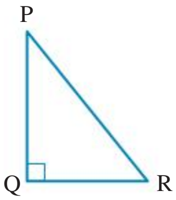
- Receive immediate feedback
- Access sample responses
- Track results and progress

Find all this and MORE in jacPLUS

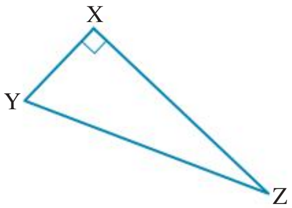
Simple familiar

1. **WE1** Identify the hypotenuse in each of the following triangles.

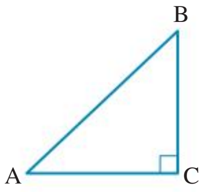
a.



b.

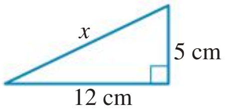


c.

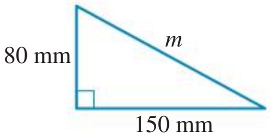


2. **WE2** Determine the length of the hypotenuse in each of the following triangles.

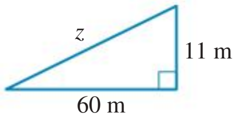
a.



b.

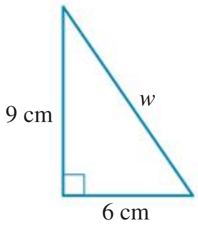


c.

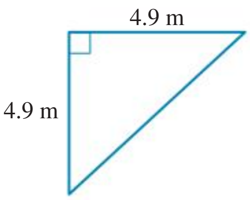


3. In each of the following, determine the length of the hypotenuse, correct to two decimal places.

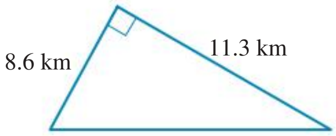
a.



b.

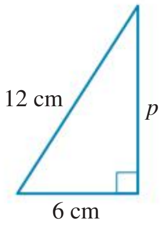


c.

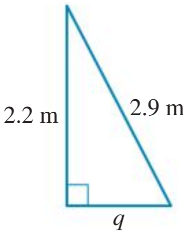


4. **WE3** Calculate the length of the unknown shorter side in each right-angled triangle, correct to one decimal place.

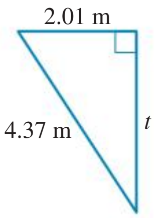
a.



b.

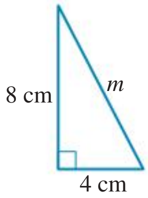


c.

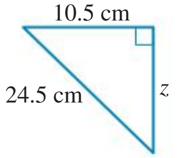


5. In each of the following right-angled triangles, determine the length of the side marked with a pronumeral, correct to one decimal place.

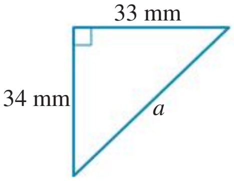
a.



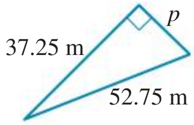
b.



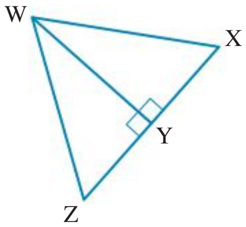
c.



d.



6. Identify the hypotenuse in triangle WXY



7. Calculate the length of the hypotenuse to two decimal places of a right-angled triangle in which the two shorter sides are 11 and 18 cm.

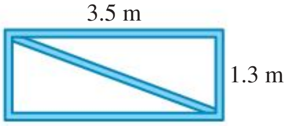
8. Calculate the length of the third side of a right-angled triangle to two decimal places if it has a hypotenuse of 9 m and another side length of 7 m.

9. Determine the unknown side length to one decimal place of a right-angled triangle with a hypotenuse of 12.5 cm and another side of length 8.6 cm.

10. **WE4** A television antenna is 12 m high. To support it, wires are attached to the ground 5 m from the foot of the antenna. Determine the length of each wire.

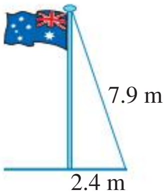
11. Susie needs to clean the guttering on her roof. She places her ladder 1.2 m back from the edge of the guttering that is 3 m above the ground. Determine how long Susie's ladder will need to be (correct to two decimal places).

12. A rectangular gate is 3.5 m long and 1.3 m wide. The gate is to be strengthened by a diagonal brace as shown. Determine how long the brace should be (correct to two decimal places).



13. A 2.5-m ladder leans against a brick wall. The foot of the ladder is 1.2 m from the foot of the wall. Determine how high up the wall the ladder will reach (correct to one decimal place).

14. Determine the height of the flagpole shown, correct to one decimal place.



15. A sheet of plaster leans against a wall, reaching 1.1 m vertically up the wall and 0.6 m along the horizontal ground from the base of the wall. Determine the length of the plaster to two decimal places.

Fully worked solutions for this chapter are available online.

LESSON
6.3 Trigonometric ratios — sine, cosine and tangent

SYLLABUS LINKS

- Revision of trigonometric ratios

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6.3.1 Trigonometry

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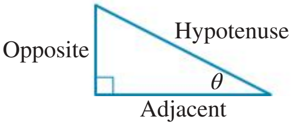


- Trigonometryalso allows us to work with the angles of triangles; that is, deal with relationships between angles and sides of right-angled triangles.
- For example, trigonometry enables us to find the length of a side if we are given the length of another side and the size of an angle.
- So that we are clear about which lines and angles we are describing, we need to identify the given angle and name the shorter sides with reference to it.
- For this reason, we label the sides ‘opposite’ and ‘adjacent’ — that is, the sides **opposite** and **adjacent** to the given angle.
- The diagram shows this relationship between the sides and the angle, θ.

6.3.2 The sine ratio

- Trigonometry uses the ratios of side lengths to calculate the lengths of sides and the size of angles.
- For all right-angled triangles that contain the same angles, the ratio of the length of the opposite side to the length of the hypotenuse will remain the same, regardless of the size of the triangle.

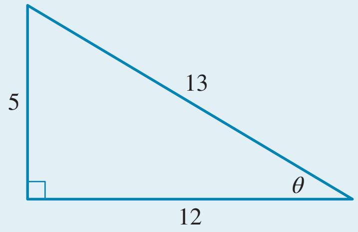
The sine ratio

The formula for the sine ratio (abbreviated 'sin') is:

$$\sin(\theta) = \frac{\text{opposite side}}{\text{hypotenuse}}$$

WORKED EXAMPLE 5 Identifying the sine ratio

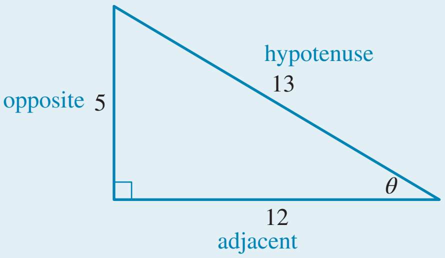
Identify an expression for the sine ratio of θ for the triangle shown.



THINK

1. Label the sides of the triangle as opposite, adjacent and hypotenuse according to the position of the angle θ.

WRITE



2. Write the general form of the sine ratio.

$\sin(\theta) = \dfrac{\text{opposite side}}{\text{hypotenuse}}$

3. Substitute the lengths of the opposite side and the hypotenuse.

$\sin(\theta) = \dfrac{5}{13}$

- Calculators require a particular sequence of button presses in order to perform this calculation. This sequence may differ for different calculators.
- For all calculations in trigonometry you will need to make sure that your calculator is in **DEGREES MODE**. Check the set-up on your calculator to ensure that this is the case.
- The value of the sine ratio for any angle can be found using the sin function on the calculator.

WORKED EXAMPLE 6 Determining sine values using a calculator

Using your calculator, determine the values of the following, correct to three decimal places.

a. $\sin(57°)$

b. $9\sin(45°)$

c. $\dfrac{18}{\sin(44°)}$

d. $9.6\sin(26°12')$

THINK	WRITE
a. With a scientific calculator, press sin and enter 57, then press =.	**a.** $\sin(57°) = 0.839$
b. Enter 9, press × and sin, enter 45, then press =.	**b.** $9\sin(45°) = 6.364$
c. Enter 18, press ÷ and sin, enter 44, then press =.	**c.** $\dfrac{18}{\sin(44°)} = 25.912$
d. Enter 9.6, press × and sin, enter 26, press DMS, enter 12, press DMS, then press =.	**d.** $9.6\sin(26°12') = 4.238$

Note: Check the sequence of button presses required by your calculator.

6.3.3 The cosine ratio

- A second trigonometric ratio is the cosine ratio. This ratio compares the length of the adjacent side and the hypotenuse.

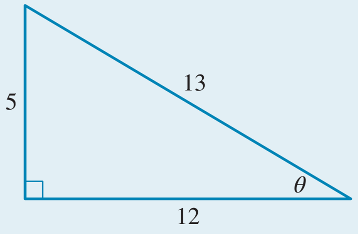
The cosine ratio

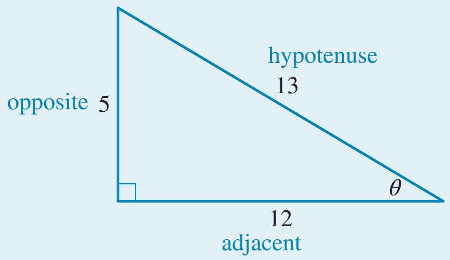
The cosine ratio (abbreviated 'cos') is found using the formula:

$$\cos(\theta) = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

WORKED EXAMPLE 7 Identifying the cosine ratio

Identify an expression for the cosine ratio of θ for the triangle shown.



THINK	WRITE
1. Label the sides of the triangle as opposite, adjacent and hypotenuse according to the position of the angle θ.	
2. Write the general form of the cosine ratio.	$\cos(\theta) = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$
3. Substitute the lengths of the adjacent side and the hypotenuse.	$\cos(\theta) = \dfrac{12}{13}$

- To calculate the cosine ratio for a given angle on your calculator, use the cos function.

WORKED EXAMPLE 8 Determining cosine values using a calculator

Using your calculator, determine the values of the following, correct to three decimal places.

a. $\cos(27°)$ **b.** $6\cos(55°)$ **c.** $\dfrac{21.3}{\cos(74°)}$ **d.** $\dfrac{4.5}{\cos(82°46')}$

THINK	WRITE
a. With a scientific calculator, press cos and enter 27, then press =.	**a.** $\cos(27°) = 0.891$
b. Enter 6, press × and cos, enter 55, then press =.	**b.** $6\cos(55°) = 3.441$
c. Enter 21.3, press ÷ and cos, enter 74, then press =.	**c.** $\dfrac{21.3}{\cos(74°)} = 77.275$
d. Enter 4.5, press ÷ and cos, enter 82, press DMS, enter 46, press DMS, then press =.	**d.** $\dfrac{4.5}{\cos(82°46')} = 35.740$

Note: Check the sequence of button presses required by your calculator.

6.3.4 The tangent ratio

- The ratio of the opposite side to the adjacent side is called the **tangent ratio** (abbreviated 'tan'). This ratio is fixed for any particular angle.

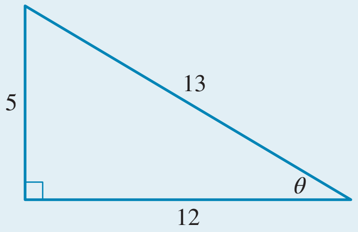
The tangent ratio

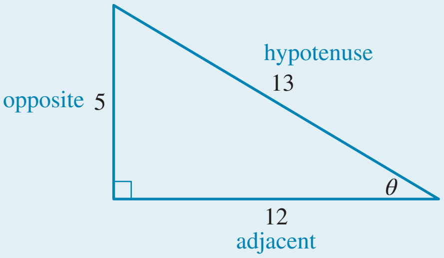
The tangent ratio for any angle, θ, can be found using:

$$\tan(\theta) = \frac{\text{opposite side}}{\text{adjacent side}}$$

WORKED EXAMPLE 9 Identifying the tangent ratio

Identify an expression for the tangent ratio of θ for the triangle shown.



THINK	WRITE
1. Label the sides of the triangle as opposite, adjacent and hypotenuse according to the position of the angle θ.	
2. Write the general form of the tangent ratio.	$\tan(\theta) = \dfrac{\text{opposite side}}{\text{adjacent side}}$
3. Substitute the lengths of the opposite and adjacent sides.	$\tan(\theta) = \dfrac{5}{12}$

- We can find an accurate value for the tangent ratio of any angle using a calculator.

WORKED EXAMPLE 10 Determining tangent values using a calculator

Using your calculator, determine the values of the following, correct to three decimal places.

a. $\tan(60°)$

b. $15\tan(75°)$

c. $\dfrac{8}{\tan(69°)}$

d. $\tan(49°32')$

THINK	WRITE
a. With a scientific calculator, press tan and enter 60, then press =.	**a.** $\tan(60°) = 1.732$
b. Enter 15, press × and tan, enter 75, then press =.	**b.** $15\tan(75°) = 55.981$
c. Enter 8, press ÷ and tan, enter 69, then press =.	**c.** $\dfrac{8}{\tan(69°)} = 3.071$
d. Press tan, enter 49, press DMS, enter 32, press DMS, then press =.	**d.** $\tan(49°32') = 1.172$

Notes: Some calculators require that the angle size be entered before the trigonometric functions.

The symbol ′ represents minutes, which are a measure of part of a degree.

6.3.5 Inverse trigonometric functions

- If we are given the sine, cosine or tangent of an angle, we can calculate the size of that angle using a calculator.
- We do this using the inverse trigonometric functions. On most calculators these are the second function of the sin, cos and tan functions and are denoted $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$.

WORKED EXAMPLE 11 Calculating angles using the inverse trigonometric functions

Calculate the value of θ correct to the nearest degree, given that $\sin(\theta) = 0.738$.

THINK	**WRITE**
1. With a scientific calculator, press **2nd F** **[$\sin^{-1}$]** (or press **SHIFT** **sin**) and enter 0.738, then press **=**.	$\sin^{-1}0.738 = 47.56...$
2. Round your answer to the nearest degree.	$\theta = 48°$

Note: Check the sequence of button presses required by your calculator.

- Problems sometimes measure angles in degrees (°), minutes (′) and seconds (″), or require answers to be written in the form of degrees, minutes and seconds.
- On scientific calculators, you will use the **DMS** (Degrees, Minutes, Seconds) function or the **° ’ ”** function.
- If you are using a graphics calculator, the ‘angle’ function provides this facility.
- To round an angle to the nearest minute, you need to look at the number of seconds. If there are 30 or more seconds, you will need to round the number of minutes up by 1.

WORKED EXAMPLE 12 Calculating angles in degrees and minutes

Given that $\tan(\theta) = 1.647$, calculate the value of θ correct to the nearest minute.

THINK	**WRITE**
1. With a scientific calculator, press **2nd F** **[$\tan^{-1}$]** (or press **SHIFT** **tan**) and enter 1.647, then press **=**.	$\tan^{-1}(1.647) = 58.73...$
2. Convert your answer to degrees and minutes by pressing **DMS**.	$\theta = 58°44'$

Note: Check the sequence of button presses required by your calculator.

Exercise 6.3 Trigonometric ratios — sine, cosine and tangent

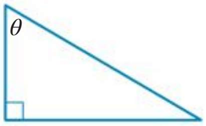
6.3 Quick quiz **6.3 Exercise**

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15	N/A	N/A

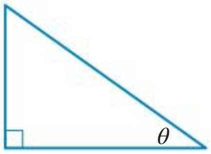
Simple familiar

1. Label the sides of the following right-angled triangles using the words hypotenuse, adjacent and opposite.

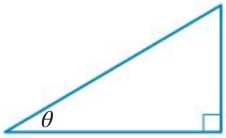
a.



b.

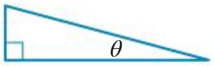


c.

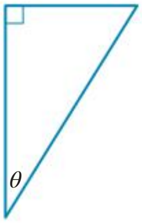


d.

e.



f.

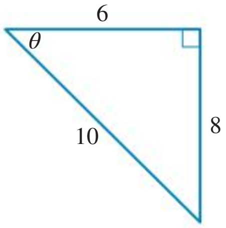


2. Write an expression for each of the following ratios for the angle θ.

a. **WE5** sine

b. **WE7** cosine

c. **WE9** tangent



3. **WE6** Using your calculator, determine the values of the following, correct to three decimal places.

a. $\sin(37^\circ)$ **b.** $9.3\sin(13^\circ)$ **c.** $\dfrac{14.5}{\sin(72^\circ)}$ **d.** $\dfrac{48}{\sin(67^\circ 40')}$

4. **WE8** Using your calculator, determine the values of the following, correct to three decimal places.

a. $\cos(45^\circ)$ **b.** $0.25\cos(9^\circ)$ **c.** $\dfrac{6}{\cos(24^\circ)}$ **d.** $5.9\cos(2^\circ 3')$

5. **WE10** Using your calculator, determine the values of the following, correct to three decimal places.

a. $\tan(57^\circ)$ **b.** $9\tan(63^\circ)$ **c.** $\dfrac{8.6}{\tan(12^\circ)}$ **d.** $\tan(33^\circ 19')$

6. Calculate the value of each of the following, correct to three decimal places if necessary.

a. $\sin(30^\circ)$ **b.** $\cos(15^\circ)$ **c.** $\tan(45^\circ)$

d. $48\tan(85^\circ)$ **e.** $128\cos(60^\circ)$ **f.** $9.35\sin(8^\circ)$

7. Calculate the value of each of the following, correct to three decimal places if necessary.

a. $\dfrac{4.5}{\cos(32^\circ)}$ **b.** $\dfrac{0.5}{\tan(20^\circ)}$ **c.** $\dfrac{15}{\sin(72^\circ)}$

8. Calculate the value of each of the following, correct to two decimal places.

a. $\sin(24^\circ 38')$
b. $\tan(57^\circ 21')$
c. $\cos(84^\circ 40')$
d. $9\cos(55^\circ 30')$
e. $4.9\sin(35^\circ 50')$
f. $2.39\tan(8^\circ 59')$

9. Calculate the value of each of the following, correct to two decimal places.

a. $\dfrac{19}{\tan(67^\circ 45')}$
b. $\dfrac{49.6}{\cos(47^\circ 25')}$
c. $\dfrac{0.84}{\sin(75^\circ 5')}$

10. **WE11** Calculate the value of θ correct to the nearest degree, given that $\sin(\theta) = 0.167$.

11. Determine θ correct to the nearest degree, given that:

a. $\sin(\theta) = 0.698$
b. $\cos(\theta) = 0.173$
c. $\tan(\theta) = 1.517$.

12. **WE12** Given that $\cos(\theta) = 0.058$, calculate the value of θ correct to the nearest minute.

13. Calculate the value of θ correct to the nearest minute, given that:

a. $\tan(\theta) = 0.931$
b. $\cos(\theta) = 0.854$
c. $\sin(\theta) = 0.277$.

14. Calculate the value of θ correct to two decimal places, given that:

a. $\cos(\theta) = 0.458$
b. $\sin(\theta) = 0.458$
c. $\tan(\theta) = 0.458$

15. Calculate the value of θ correct to the nearest degree, given that:

a. $\tan(\theta) = 1.784$
b. $\sin(\theta) = 0.25$
c. $\cos(\theta) = 0.888$

Fully worked solutions for this chapter are available online.

LESSON
6.4 Calculating unknown side lengths of right-angled triangles [complex]

SYLLABUS LINKS

- Apply the cosine, sine and tangent ratios to find unknown angles (θ) and sides [complex].
  - $\cos(\theta) = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
  - $\sin(\theta) = \dfrac{\text{opposite}}{\text{hypotenuse}}$
  - $\tan(\theta) = \dfrac{\text{opposite}}{\text{adjacent}}$

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6.4.1 Using trigonometric ratios to calculate unknown side lengths

- We can use the trigonometric ratios to find the length of one side of a right-angled triangle if we know the length of another side and an angle.
- We need to be able to look at a problem and then decide if the solution can be determined using the sine, cosine or tangent ratio. To do this we need to examine the three formulas.

$$\sin(\theta) = \frac{\text{opposite side}}{\text{hypotenuse}}$$

- The sine ratio is used when finding the opposite side or the hypotenuse when given the length of the other.

$$\cos(\theta) = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

- The cosine ratio is for problems where we are finding the adjacent side or the hypotenuse and are given the length of the other.

$$\tan(\theta) = \frac{\text{opposite side}}{\text{adjacent side}}$$

- We use the tangent ratio when we are finding either the opposite or adjacent side and are given the length of the other.

Remembering trigonometric ratios

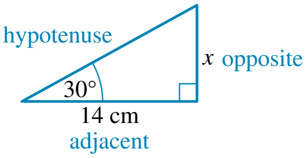
To remember the trigonometric ratios more easily, we can use this acronym:

SOHCAHTOA

We pronounce this acronym as 'Sock ca toe ahh'. The initials of the acronym represent the three trigonometric formulas.

$$\begin{pmatrix}\text{S}\\\text{O}\\\text{H}\end{pmatrix} \sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \begin{pmatrix}\text{C}\\\text{A}\\\text{H}\end{pmatrix} \cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \begin{pmatrix}\text{T}\\\text{O}\\\text{A}\end{pmatrix} \tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

- Consider the triangle shown.

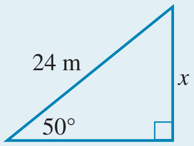


- In this triangle we are asked to find the length of the opposite side and have been given the length of the adjacent side.
- We know that the tangent ratio uses the opposite and adjacent sides: $\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$. In this example, $\tan(30°) = \frac{x}{14}$. We can set up an equation that can be solved to determine the value of x.

$$\begin{aligned} \tan(\theta) &= \frac{\text{opposite}}{\text{adjacent}} \\ \tan(30°) &= \frac{x}{14} \\ 14 \times \tan(30°) &= \frac{\cancel{14}x}{\cancel{14}} && \text{Multiply both sides by 14} \\ x &= 14\tan(30°) && \text{Enter } 14\ \tan(30°) \text{ into your calculator} \\ &= 8.083\text{ cm} \end{aligned}$$

WORKED EXAMPLE 13 Using the sine ratio to find length

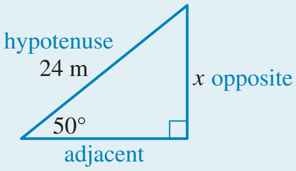
Use the sine ratio to determine the length of the side marked x, correct to two decimal places.



THINK

WRITE

1. Label the sides of the triangle opposite, adjacent and hypotenuse.



2. x is the opposite side and 24 m is the hypotenuse; therefore, use the sine ratio.

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

3. Substitute the values of the known angle and the known side length: $\theta = 50°$ and hypotenuse = 24.

$$\sin(50°) = \frac{x}{24}$$

4. Make x the subject of the equation by multiplying both sides of the equation by 24.

$$24 \times \sin(50°) = \frac{\cancel{24}x}{\cancel{24}}$$

$$x = 24\ \sin(50°)$$

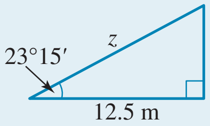
5. Use your calculator to evaluate the expression for x.

$$= 18.39\text{ m}$$

- Care needs to be taken at the substitution stage.
- If, after substitution, the unknown side is in the denominator, the final step is a division.

WORKED EXAMPLE 14 Using cosine to find length

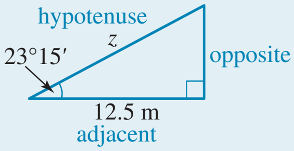
Use the cosine ratio to determine the length of the side marked z in the triangle shown, correct to two decimal places.



THINK

WRITE

1. Label the sides of the triangle opposite, adjacent and hypotenuse.



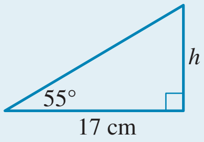
2. Choose the cosine ratio because we are finding the hypotenuse and have been given the adjacent side. Write the cosine ratio.

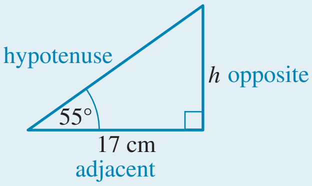
$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

3. Substitute the values of the known angle and the known side length: $\theta = 23^\circ 15'$ and adjacent = 12.5.	$\cos(23^\circ 15') = \frac{12.5}{z}$
4. Multiply both sides of the equation by z to remove z from the denominator.	$z \times \cos(23^\circ 15') = \frac{\cancel{z} \times 12.5}{\cancel{z}}$ $z\cos(23^\circ 15') = 12.5$
5. Divide both sides of the equation by $\cos(23^\circ 15')$ to make z the subject.	$\frac{z\,\cancel{\cos(23^\circ 15')}}{\cancel{\cos(23^\circ 15')}} = \frac{12.5}{\cos(23^\circ 15')}$ $z = \frac{12.5}{\cos(23^\circ 15')}$
6. Use your calculator to evaluate the expression for z.	$z = 13.60$ m

WORKED EXAMPLE 15 Using the tan ratio to determine side lengths

Use the tangent ratio to determine the value of *h* in the triangle shown, correct to two decimal places.



THINK	WRITE
1. Label the sides of the triangle opposite, adjacent and hypotenuse.	
2. Write the tangent ratio.	$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$
3. Substitute the values of the known angle and the known side length: $\theta = 55^\circ$ and adjacent = 17.	$\tan(55^\circ) = \frac{h}{17}$
4. Make h the subject of the equation by multiplying both sides of the equation by 17.	$17 \times \tan(55^\circ) = \frac{\cancel{17}h}{\cancel{17}}$ $h = 17\ \tan(55^\circ)$
5. Use your calculator to evaluate the expression for h.	$= 24.28$ cm

6.4.2 Solving practical problems involving unknown side lengths

- Trigonometry is used to solve many practical problems.
- In these cases, it is necessary to draw a diagram to represent the problem and then use trigonometry to solve the problem.
- With written problems that require you to draw the diagram, it is necessary to give the answer in words.

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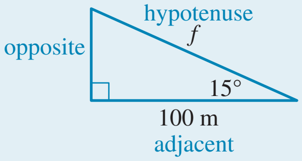
WORKED EXAMPLE 16 Using trigonometry to solve practical problems

A flying fox is used in an army training camp. The flying fox is supported by a cable that runs from the top of a cliff face to a point on the ground 100 m from the base of the cliff. The cable makes a 15° angle with the horizontal. Determine the length of the cable used to support the flying fox, correct to one decimal place.

THINK	WRITE
1. Draw a diagram and show all important information.	
2. Label the sides of the triangle opposite, adjacent and hypotenuse.	
3. Choose the cosine ratio to solve this problem because we are finding the hypotenuse and have been given the adjacent side.	
4. Write the cosine ratio.	$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$
5. Substitute the values of the known angle and the known side length: $\theta = 15°$ and adjacent $= 100$.	$\cos(15°) = \frac{100}{f}$
6. Make f the subject of the equation by multiplying both sides by f and then dividing both sides by $\cos(15°)$.	$f \times \cos(15°) = \frac{100\cancel{f}}{\cancel{f}}$ $f\cos(15°) = 100$ $\frac{f\cancel{\cos(15°)}}{\cancel{\cos(15°)}} = \frac{100}{\cos(15°)}$ $f = \frac{100}{\cos(15°)}$
7. Use your calculator to evaluate the expression for f.	$f = 103.5$ m
8. Answer the question.	The cable is approximately 103.5 m long.

Exercise 6.4 Calculating unknown side lengths of right-angled triangles [complex]

learnon

6.4 Quick quiz on

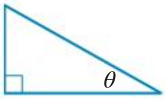
6.4 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1	2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16	17, 18, 19

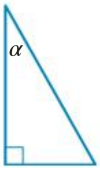
Simple familiar

1. Label the sides of each of the following triangles with respect to the angle marked with the pronumeral.

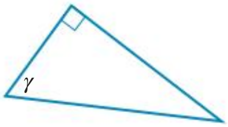
a.



b.

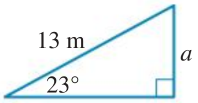


c.

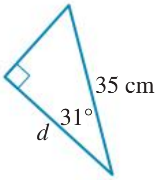


Complex familiar

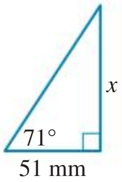
2. **WE13** Use the sine ratio to determine the length of the side marked a (correct to two decimal places).



3. **WE14** Use the cosine ratio to determine the length of the side marked d (correct to nearest whole number).

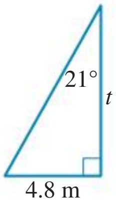


4. **WE15** Use the tangent ratio to determine the length of the side marked x (correct to one decimal place).

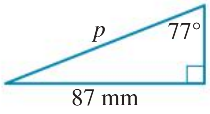


5. Determine the length of the side marked with the pronumeral, correct to one decimal place.

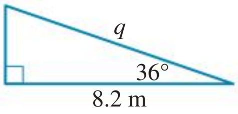
a.



b.

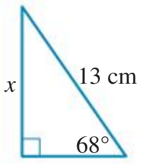


c.

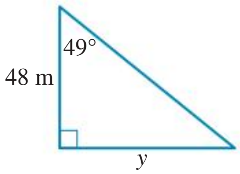


6. Calculate the length of the side marked with the pronumeral in each of the following (correct to one decimal place).

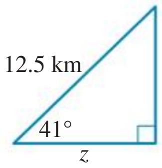
a.



b.

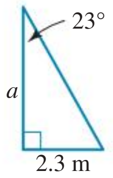


c.

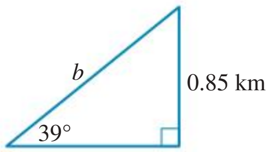


7. Calculate the length of the side marked with the pronumeral in each of the following (correct to one decimal place).

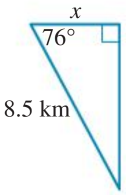
a.



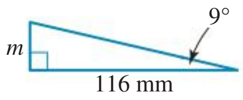
b.



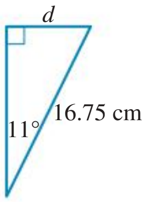
c.



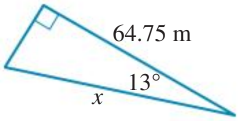
d.



e.

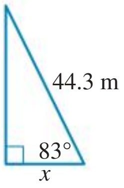


f.

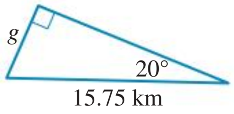


8. Calculate the length of the side marked with the pronumeral in each of the following (correct to one decimal place).

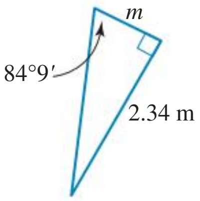
a.



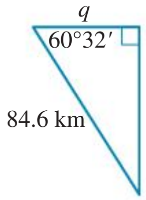
b.



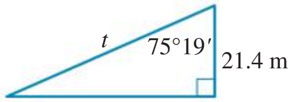
c.



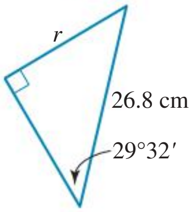
d.



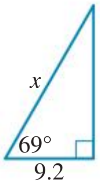
e.



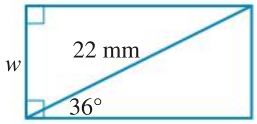
f.



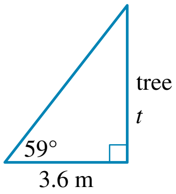
9. Determine the equation that would solve for x (do not solve for x).



10. Calculate the length w.

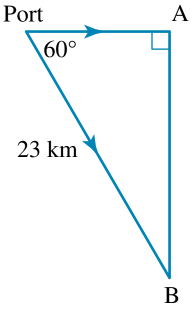


11. **WE16** Calculate the height of the tree in the following diagram, correct to the nearest metre.



12. A 10-m ladder just reaches to the top of a wall when it is leaning at 65° to the ground. Determine how far the base of the ladder is from the foot of the wall (correct to one decimal place).

13. The diagram shows the paths of two ships, A and B, after they have left a port. If ship B sends a distress signal, determine how far ship A must sail to give assistance (to the nearest kilometre).



14. A rectangular sign 13.5 m wide has a diagonal brace that makes a 24° angle with the horizontal.

a. Draw a diagram of this situation.
b. Calculate the height of the sign, correct to the nearest metre.

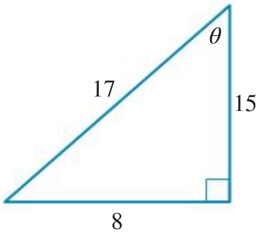
15. A wooden gate has a diagonal brace built in for support. The gate stands 1.4 m high and the diagonal makes a 60° angle with the horizontal.

a. Draw a diagram of the gate.
b. Calculate the length that the diagonal brace needs to be (correct to one decimal place).

16. The wire support for a flagpole makes a 70° angle with the ground. If the support is 3.3 m from the base of the flagpole, calculate the length of the wire support (correct to two decimal places).

Complex unfamiliar

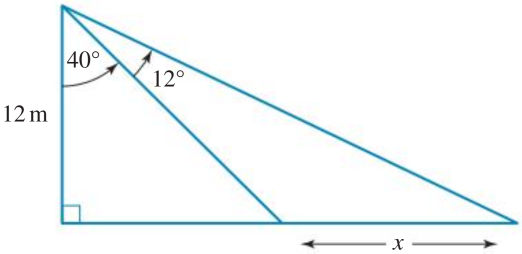
17. Determine the trigonometric function used to produce the smallest fraction.



18. A ship drops anchor vertically with an anchor line 60 m long. After 1 h the anchor line makes a 15° angle with the vertical.

Determine the speed at which the ship has drifted, correct to one decimal place.

19. Determine the distance x in the following diagram to two decimal places.



Fully worked solutions for this chapter are available online.

LESSON
6.5 Calculating unknown angles in right-angled triangles [complex]

SYLLABUS LINKS

- Apply the cosine, sine and tangent ratios to find unknown angles (θ) and sides [complex]
  - $\cos(\theta) = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
  - $\sin(\theta) = \dfrac{\text{opposite}}{\text{hypotenuse}}$
  - $\tan(\theta) = \dfrac{\text{opposite}}{\text{adjacent}}$

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6.5.1 Using trigonometric ratios to calculate unknown angles

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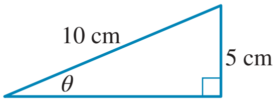
- We are also able to use trigonometry to find the sizes of angles when we have been given two side lengths.
- We can use the inverse trigonometric ratios $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ to find an unknown angle in a right-angled triangle.

Inverse trigonometric ratios

$$\textbf{If } \sin(\theta) = a\textbf{, then } \sin^{-1}(a) = \theta.$$
$$\textbf{If } \cos(\theta) = a\textbf{, then } \cos^{-1}(a) = \theta.$$
$$\textbf{If } \tan(\theta) = a\textbf{, then } \tan^{-1}(a) = \theta.$$



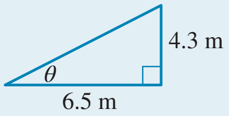
- Consider the right-angled triangle shown.
- We want to find the size of the angle marked θ.
- Using the formula $\sin(\theta) = \dfrac{\text{opposite}}{\text{hypotenuse}}$, we know that in this triangle:

$$\begin{aligned}\sin(\theta) &= \frac{5}{10}\\ &= \frac{1}{2}\\ &= 0.5\end{aligned}$$

- We then take the inverse sin of both sides of the equation and calculate $\sin^{-1}(0.5)$ to find that $\theta = 30°$.
- As with all trigonometry, it is important that you have your calculator set to degrees mode.

WORKED EXAMPLE 17 Calculating the angle to the nearest degree

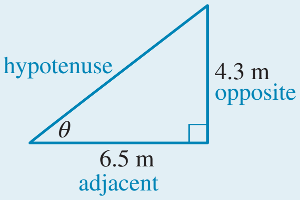
Determine the size of angle θ in the triangle shown, correct to the nearest degree.



THINK

WRITE

1. Label the sides of the triangle.



2. Choose the tangent ratio to solve the problem as we are given the opposite and adjacent side lengths. Write the tangent ratio.

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

3. Substitute the values of the known side lengths: opposite = 4.3 and adjacent = 6.5.

$$\tan(\theta) = \frac{4.3}{6.5}$$

4. Make θ the subject of the equation by taking the inverse tan of both sides.

$$\theta = \tan^{-1}\left(\frac{4.3}{6.5}\right)$$

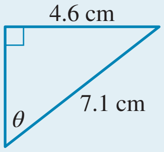
5. Use your calculator to evaluate the expression for θ.

$$\theta = 33°$$

- In many cases, we will need to calculate the size of an angle, correct to the nearest minute.
- The same method for finding the solution is used; however, you will need to use your calculator to convert to degrees and minutes.

WORKED EXAMPLE 18 Calculate the angle to the nearest minute

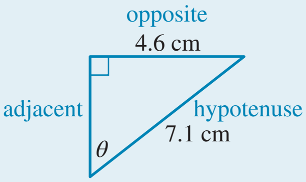
Calculate the size of the angle θ in the triangle, correct to the nearest minute.



THINK

WRITE

1. Label the sides of the triangle.



2. Choose the sine ratio to solve this problem as we are given the opposite side and the hypotenuse. Write the sine ratio.	$\sin(\theta) = \dfrac{\text{opposite}}{\text{hypotenuse}}$
3. Substitute the values of the known side lengths: opposite = 4.6 and hypotenuse = 7.1.	$\sin(\theta) = \dfrac{4.6}{7.1}$
4. Make θ the subject of the equation by taking the inverse sin of both sides.	$\theta = \sin^{-1}\left(\dfrac{4.6}{7.1}\right)$
5. Use your calculator to evaluate the expression for θ and convert your answer to degrees and minutes.	$\theta = 40°23'$

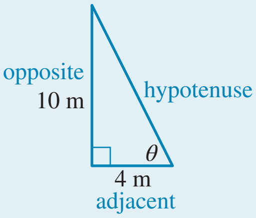
6.5.2 Solving practical problems involving unknown angles

- The same methods can be used to solve problems requiring an unknown angle to be found.
- As with finding sides, we set the question up by drawing a diagram of the situation.

tlvd-12171

WORKED EXAMPLE 19 Solving practical problems involving angles

A ladder leans against a wall. The foot of the ladder is 4 m from the base of the wall and the ladder reaches 10 m up the wall. Calculate the angle that the ladder makes with the ground, correct to the nearest minute.

THINK	WRITE
1. Draw a diagram showing all important information and label the sides.	
2. Choose the tangent ratio to solve this problem as we are given the opposite and adjacent sides. Write the tangent ratio.	$\tan(\theta) = \dfrac{\text{opposite}}{\text{adjacent}}$
3. Substitute the values of the known side lengths: opposite = 10 and adjacent = 4.	$\tan(\theta) = \dfrac{10}{4}$
4. Make θ the subject of the equation by taking the inverse tan of both sides.	$\theta = \tan^{-1}\left(\dfrac{10}{4}\right)$
5. Use your calculator to evaluate the expression for θ.	$= 68°12'$
6. Answer the question.	The ladder makes an angle of 68°12′ with the ground.

Exercise 6.5 Calculating unknown angles in right-angled triangles [complex]

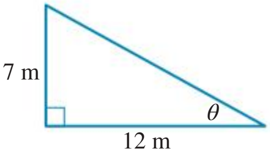
6.5 Quick quiz on

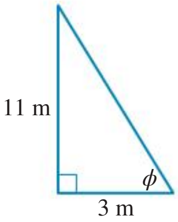
6.5 Exercise

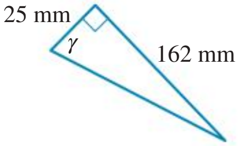
Simple familiar	Complex familiar	Complex unfamiliar
N/A	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13	14, 15

Complex familiar

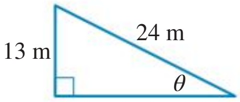
1. **WE17** Use the tangent ratio to determine the size of the angle marked with the pronumeral in each of the following, correct to the nearest degree.

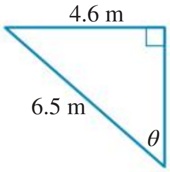
a. 

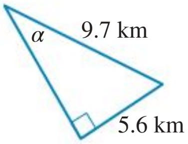
b. 

c. 

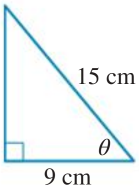
2. **WE18** Use the sine ratio to determine the size of the angle marked with the pronumeral in each of the following, correct to the nearest minute.

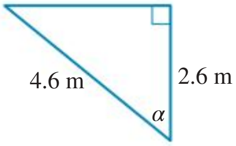
a. 

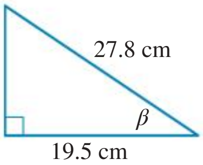
b. 

c. 

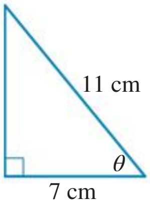
3. Use the cosine ratio to determine the size of the angle marked with the pronumeral in each of the following, correct to the nearest minute.

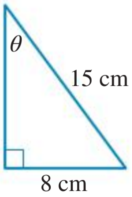
a. 

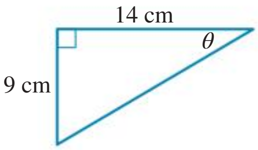
b. 

c. 

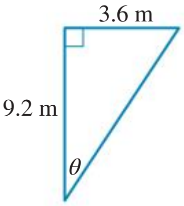
4. Calculate the size of the angle marked θ in the following triangles, correct to the nearest degree.

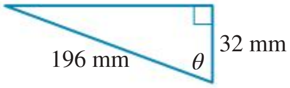
a. 

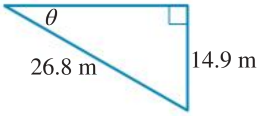
b. 

c. 

5. Calculate the size of the angle marked θ in the following triangles, correct to the nearest degree.

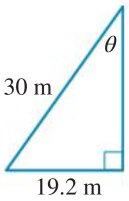
a. 

b. 

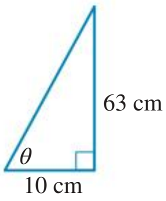
c. 

6. In each of the following, calculate the size of the angle marked θ, correct to the nearest minute.

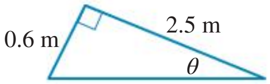
a.



b.

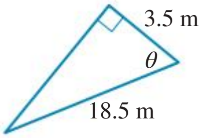


c.

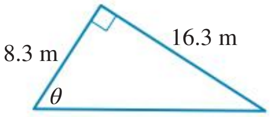


7. In each of the following, calculate the size of the angle marked θ, correct to the nearest minute.

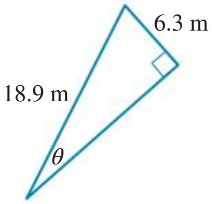
a.



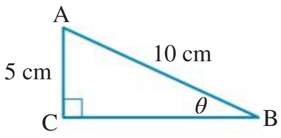
b.



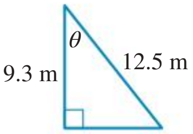
c.



8. Determine the unknown angle.

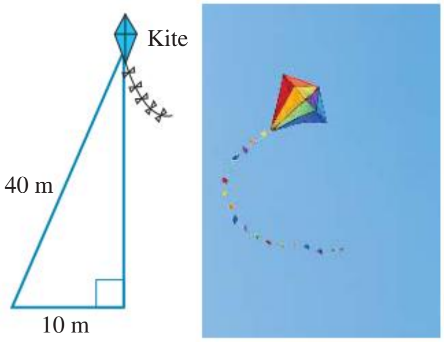


9. Determine the correct value of θ the nearest minute.

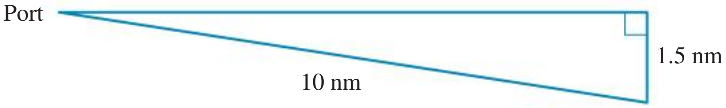


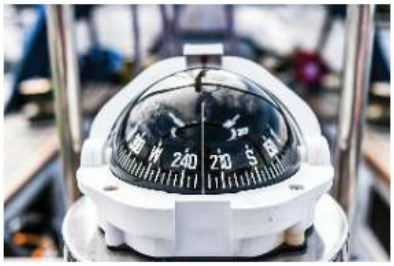
10. **WE19** A 10-m ladder leans against a wall 6 m high. Calculate the angle that the ladder makes with the horizontal, correct to the nearest degree.

11. A kite is flying on a 40-m string. The kite is flying 10 m away from the vertical as shown in the figure. Calculate the angle the string of the kite makes with the horizontal, correct to the nearest minute.

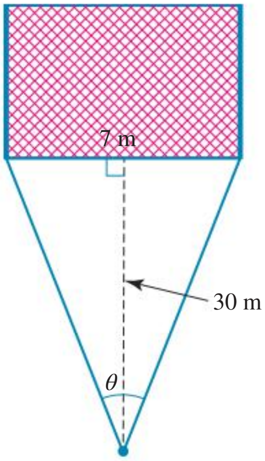


12. A ship's compass shows a course due east of the port from which it sails. After sailing 10 nautical miles, it is found that the ship is 1.5 nautical miles off course as shown in the figure.
Determine the error in the compass reading, correct to the nearest minute.





13. The diagram shows a footballer's shot at goal.
By dividing the isosceles triangle in half to form a right-angled triangle, calculate, to the nearest degree, the angle within which the footballer must kick to get the ball to go between the posts.



Complex unfamiliar

14. A golfer hits a ball 250 m, but 20 m off centre. Calculate the angle at which the ball deviated from a straight line, correct to the nearest minute.

15. A cricketer bowls a ball to a batsman from 20 m away, and the batsman hits it to a fielder 48 m away. From where the ball was bowled to where it was caught, the ball deviated 2.4 m from a straight line shot back to the bowler. Calculate the angle the ball deviated from where it was bowled, correct to one decimal place.

Fully worked solutions for this chapter are available online.

LESSON
6.6 Angles of elevation and depression [complex]

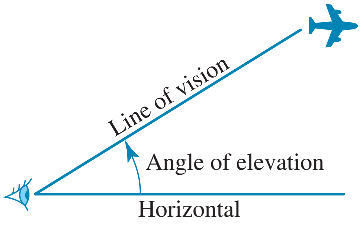
SYLLABUS LINKS

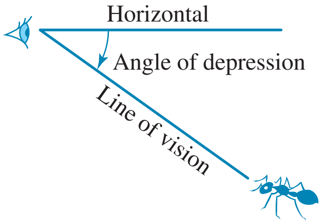
- Use the concepts of angle of elevation and angle of depression to solve practical problems [complex].

6.6.1 Angles of elevation and depression

int-6047

- The **angle of elevation** is measured upwards from the **horizontal** and refers to the angle at which we need to look up to see an object.
- Similarly, the **angle of depression** is the angle at which we need to look down from the horizontal to see an object.
- We can use the angles of elevation and depression to calculate the heights and distances of objects that would otherwise be difficult to measure.

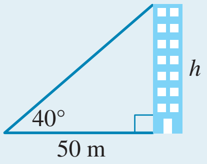


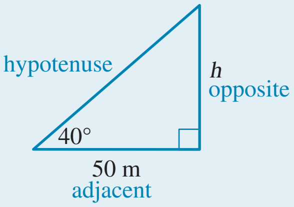


tlvd-12172

WORKED EXAMPLE 20 Solving problems involving angles of elevation

From a point 50 m from the foot of a building, the angle of elevation to the top of the building is measured as 40°. Calculate the height, *h*, of the building, correct to the nearest metre.



THINK	**WRITE**
1. Label the sides of the triangle opposite, adjacent and hypotenuse.	

2. Choose the tangent ratio because we are finding the opposite side and have been given the adjacent side.

3. Write the tangent ratio.

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

4. Substitute the values of the known side and angle: $\theta = 40°$ and adjacent $= 50$.

$$\tan(40°) = \frac{h}{50}$$

5. Make h the subject of the equation by multiplying both sides by 50.

$$50 \times \tan(40°) = \frac{\cancel{50}h}{\cancel{50}}$$
$$h = 50\ \tan(40°)$$

6. Use your calculator to evaluate the expression for h.

$h = 42$ m

7. Answer the question.

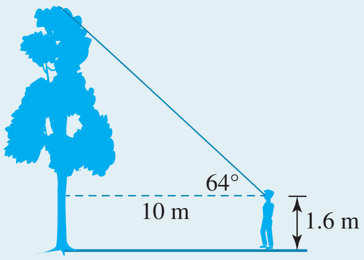
The height of the building is approximately 42 m.

- In practical situations, the angle of elevation is measured using a clinometer (or inclinometer).
- Therefore, the angle of elevation is measured from a person's height at eye level.
- For this reason, the height at eye level must be added to the calculated answer.

tlvd-12173

WORKED EXAMPLE 21 Solving problems involving angles of elevation from eye level

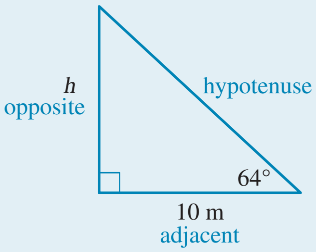
Bryan measures the angle of elevation to the top of a tree as 64° from a point 10 m from the foot of the tree. If the height of Bryan's eyes is 1.6 m above the ground, calculate the height of the tree, correct to one decimal place.



THINK

1. Label the sides opposite, adjacent and hypotenuse.

WRITE



2. Choose the tangent ratio because we are finding the opposite side and have been given the adjacent side.

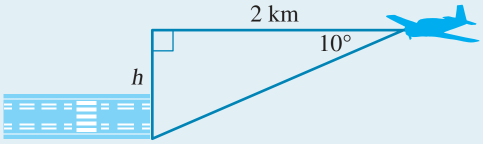
3. Write the tangent ratio.	$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$
4. Substitute the values of the known side and angle: $\theta = 64°$ and adjacent $= 10$.	$\tan(64°) = \frac{h}{10}$
5. Make h the subject of the equation by multiplying both sides by 10.	$10 \times \tan(64°) = \frac{\cancel{10}h}{\cancel{10}}$ $h = 10\ \tan(64°)$
6. Use your calculator to evaluate the expression for h.	$h = 20.5$ m
7. The height of the tree is equal to the sum of h and the eye height.	Height of tree $= 20.5 + 1.6$ $= 22.1$
8. Answer the question.	The height of the tree is approximately 22.1 m.

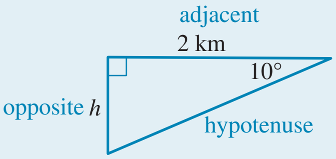
- A similar method for finding the solution is used for problems that involve an angle of depression.

tlvd-12174

WORKED EXAMPLE 22 Solving problems involving angles of depression

When an aeroplane in flight is 2 km from a runway, the angle of depression to the runway is 10°. Calculate the altitude of the aeroplane, correct to the nearest metre.



THINK	WRITE
1. Label the sides of the triangle opposite, adjacent and hypotenuse.	
2. Choose the tangent ratio, because we are finding the opposite side and have been given the adjacent side.	
3. Write the tangent ratio.	$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$
4. Substitute the values of the known side and angle, converting 2 km to 2000 m: $\theta = 10°$ and adjacent $= 2000$.	$\tan(10°) = \frac{h}{2000}$

5. Make h the subject of the equation by multiplying both sides by 2000.	$2000 \times \tan(10°) = \frac{\cancel{2000}h}{\cancel{2000}}$ $h = 2000\ \tan(10°)$
6. Use your calculator to evaluate the expression for h.	$h = 353$ m
7. Answer the question.	The altitude of the aeroplane is approximately 353 m.

- Angles of elevation and depression can also be calculated by using known measurements.
- This is done by drawing a right-angled triangle to represent a situation.

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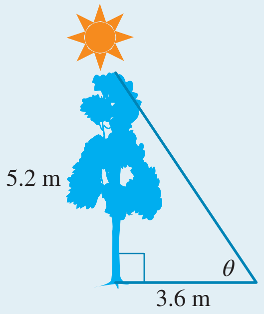
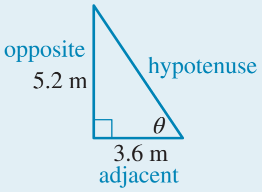
WORKED EXAMPLE 23 Calculating angles involving angles of elevation

A 5.2 -m tree casts a 3.6 -m shadow. Calculate the angle of elevation of the sun, correct to the nearest degree.



THINK	**WRITE**
1. Label the sides opposite, adjacent and hypotenuse.	
2. Choose the tangent ratio because we are given the opposite and adjacent sides.	
3. Write the tangent.	$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$
4. Substitute the values of the known side lengths: opposite = 5.2 and adjacent = 3.6.	$\tan(\theta) = \frac{5.2}{3.6}$
5. Make θ the subject of the equation by taking the inverse tan of both sides.	$\theta = \tan^{-1}\frac{5.2}{3.6}$
6. Use your calculator to evaluate the expression for θ.	$\theta = 55°$
7. Answer the question.	The angle of elevation of the sun is approximately 55°.

Exercise 6.6 Angles of elevation and depression [complex]

learnon

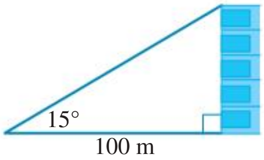
6.6 Quick quiz on

6.6 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
N/A	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11	12, 13, 14, 15

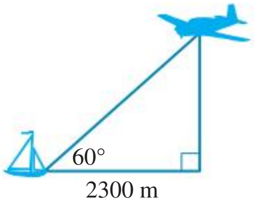
Complex familiar

1. **WE20** From a point 100 m from the foot of a building, the angle of elevation to the top of the building is 15°.



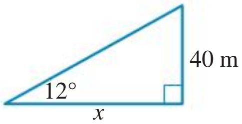
Calculate the height of the building, correct to one decimal place.

2. From a ship the angle of elevation to an aeroplane is 60°. The aeroplane is located at a horizontal distance of 2300 m away from the ship.



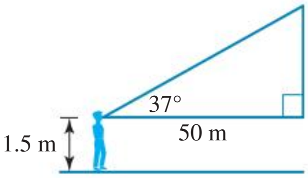
Calculate the altitude of the aeroplane, correct to the nearest metre.

3. From a point out to sea, a ship sights the top of a lighthouse at an angle of elevation of 12°. It is known that the top of the lighthouse is 40 m above sea level.



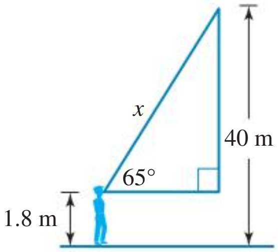
Calculate the horizontal distance of the ship from the lighthouse, correct to the nearest 10 m.

4. **WE21** From a point 50 m from the foot of a building, Rod sights the top of a building at an angle of elevation of 37°.



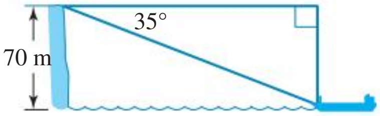
Given that Rod's eyes are 1.5 m above the ground, calculate the height of the building, correct to one decimal place.

5. Richard is flying a kite and sights the kite at an angle of elevation of 65°. The altitude of the kite is 40 m and Richard's eyes are at a height of 1.8 m.



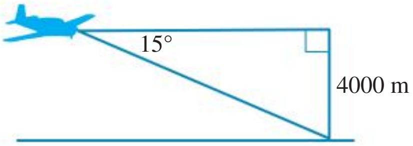
Calculate the length of string the kite is flying on, correct to one decimal place.

6. From the top of a cliff, 70 m above sea level, a ship is spotted out to sea at an angle of depression of 35°.



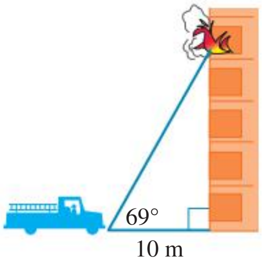
Calculate the distance of the ship from shore, to the nearest metre.

7. **WE22** From an aeroplane flying at an altitude of 4000 m, the runway is sighted at an angle of depression of 15°.

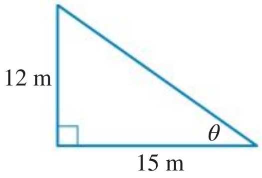


Calculate the horizontal distance of the aeroplane from the runway, correct to the nearest kilometre.

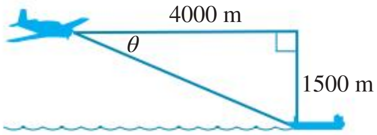
8. There is a fire on the fifth floor of a building. The closest a fire truck can get to the building is 10 m. The angle of elevation from this point to where people need to be rescued is 69°. If the fire truck has a 30-m ladder, determine whether the ladder can be used to make the rescue.



9. **WE23** A 12-m high building casts a shadow 15 m long. Calculate the angle of elevation of the sun, to the nearest degree.



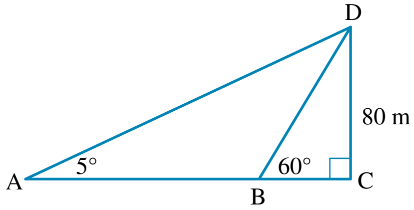
10. An aeroplane, which is at an altitude of 1500 m, is 4000 m from a ship in a horizontal direction, as shown. Calculate the angle of depression from the aeroplane to the ship, to the nearest degree.



11. The angle of elevation to the top of a tower is 12° from a point 400 m from the foot of the tower.
   a. Draw a diagram of this situation.
   b. Calculate the height of the tower, correct to one decimal place.
   c. Calculate the angle of elevation to the top of the tower, from a point 100 m from the foot of the tower.

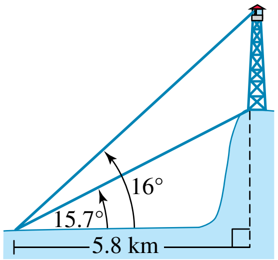
Complex unfamiliar

12. From a navy vessel, a beacon that is 80 m above sea level was sighted at an angle of elevation of 5°. The vessel sailed towards the beacon and 30 min later the beacon was at an angle of elevation of 60°.



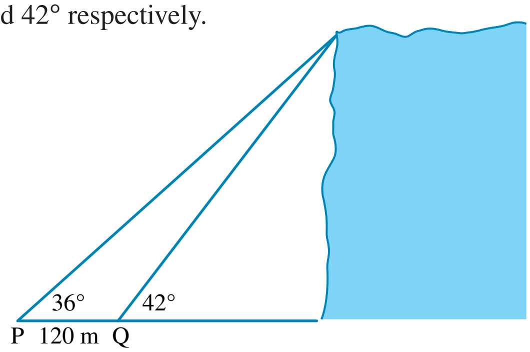
Calculate the distance that the vessel sailed in the 30 min between the two readings.

13. A lookout tower has been erected on top of a cliff. At a distance of 5.8 km from the foot of the cliff, the angle of elevation to the base of the tower is 15.7° and the angle to the observation deck at the top of the tower is 16°, as shown in the figure below.



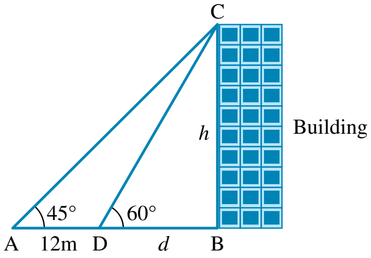
Determine how high from the top of the cliff the observation deck is, to the nearest metre.

14. P and Q are two points an a horizontal line that are 120 m apart. The angles of elevation from P and Q to the top of a mountain are 36° and 42° respectively.



Determine the height of the mountain, in metres, correct to one decimal place.

15. At a certain distance away, the angle of elevation to the top of a building is 60°. From 12 m further back the angle of elevation is 45°.



Determine the height of the building to one decimal place.

Fully worked solutions for this chapter are available online.

LESSON
6.7 Review

6.7.1 Summary

Hey students! Now that it's time to revise this chapter, go online to:

- Access the chapter summary
- Review your results
- Watch teacher-led videos
- Practise questions with immediate feedback

Find all this and MORE in jacPLUS

6.7 Exercise

learnon

6.7 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6	7, 8, 9, 10, 11, 12, 13, 14, 15, 16,	17, 18, 19, 20

These questions are even better in jacPLUS!

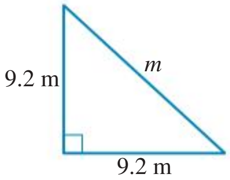
- Receive immediate feedback
- Access sample responses
- Track results and progress

Find all this and MORE in jacPLUS

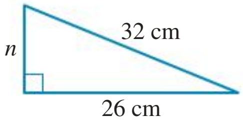
Simple familiar

1. Determine the length of the side marked with a pronumeral in each of the triangles shown. In each case write your answer correct to two decimal places.

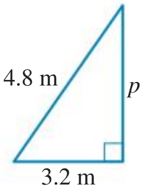
a.



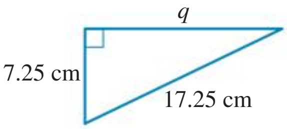
b.



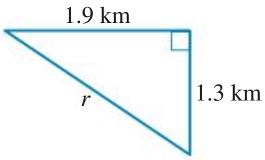
c.



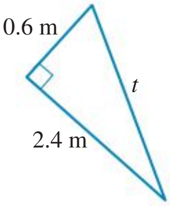
d.



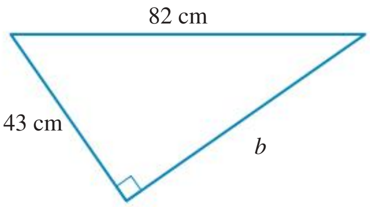
e.



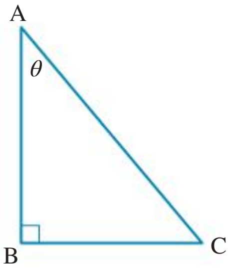
f.



2. Determine the length of the third side in this triangle



3. State the names of sides AB, BC and AC from the triangle shown.



4. Calculate each of the following, correct to four decimal places.

a. $\sin(46°)$ **b.** $\tan(76°42')$ **c.** $4.9\cos(56°)$

d. $8.9\sin(67°3')$ **e.** $\dfrac{5.69}{\cos(75°)}$ **f.** $\dfrac{2.5}{\tan(9°55')}$

5. Calculate θ, correct to the nearest degree, given that:

a. $\cos(\theta) = 0.5874$ **b.** $\tan(\theta) = 1.23$ **c.** $\sin(\theta) = 0.8$.

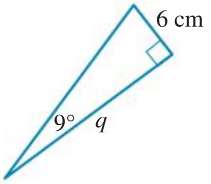
6. Calculate θ, correct to the nearest minute, given that:

a. $\cos(\theta) = 0.199$ **b.** $\tan(\theta) = 0.5$ **c.** $\sin(\theta) = 0.257$.

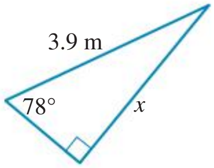
Complex familiar

7. Determine the length of each side marked with a pronumeral, correct to one decimal place.

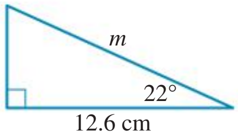
a.



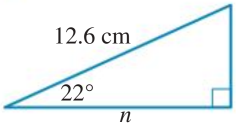
b.



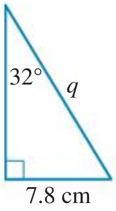
c.



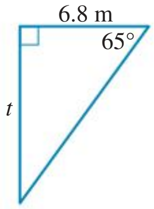
d.



e.

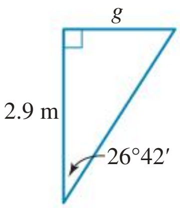


f.

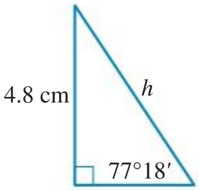


8. Determine the length of each side marked with a pronumeral, correct to one decimal place.

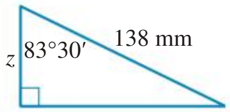
a.



b.



c.



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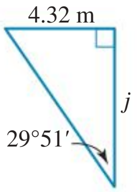

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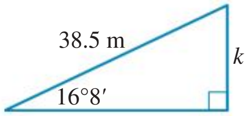
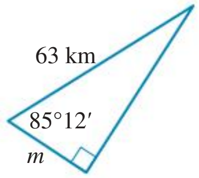
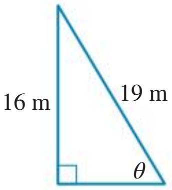


f.

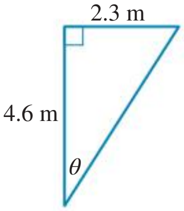


9. Determine the size of the angle marked θ in each of the following, giving your answer correct to the nearest degree.

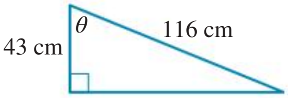
a.



b.

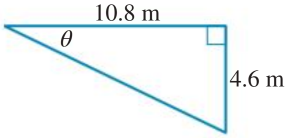


c.

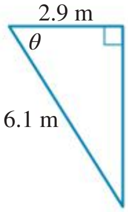


10. Determine the size of the angle marked θ in each of the following, giving your answer correct to the nearest minute.

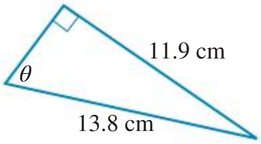
a.



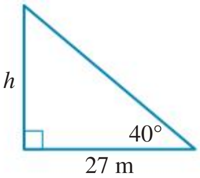
b.



c.

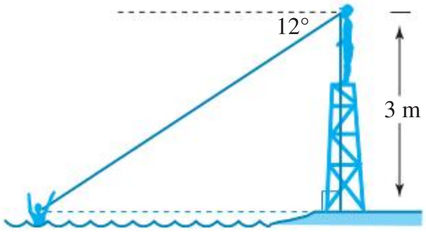


11. The top of a building is sighted at an angle of elevation of 40° when an observer is 27 m back from the base.



Calculate the height of the building, correct to the nearest metre.

12. A lifesaver standing on his tower 3 m above the ground spots a swimmer experiencing difficulty. The angle of depression of the swimmer from the lifesaver is 12°. Determine how far the swimmer is from the lifesaver's tower. (Give your answer correct to two decimal places.)



13. To travel between the towns of Bolong and Molong, you need to travel west along a road for 45 km, then north along another road for another 87 km.
Calculate the straight-line distance between the two towns.

14. A rope is 80 m long and runs from a cliff top to the ground, 45 m from the base of the cliff. Calculate the height of the cliff, to the nearest metre.

15. A fire is burning in a building and people need to be rescued. The fire brigade's ladder must reach a height of 60 m and must be angled at 70° to the horizontal. Determine how long the ladder must be to complete the rescue.

16. A rope that is used to support a flagpole makes an angle of 70° with the ground. If the rope is tied down 3.1 m from the foot of the flagpole, calculate the height of the flagpole, correct to one decimal place.

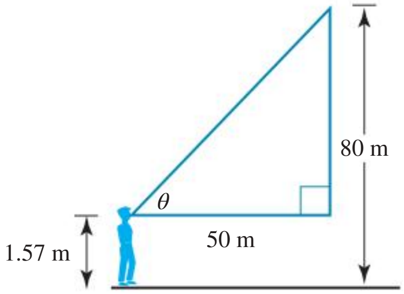
Complex unfamiliar

17. A dirt track runs off a road at an angle of 34° to the road. If you travel for 4.5 km along the dirt track, determine the shortest direct distance back to the road (correct to one decimal place).

18. A child flies a kite on an 80-m string. He holds the end of the string 1.5 m above the ground, and the kite reaches a height of 51.5 m above the ground in a strong wind. Calculate the angle the string makes with the horizontal.

19. There is 50 m of line on a fishing reel. When all the line is out, the bait sits on the bed of a lake and has drifted 20 m from the boat. Calculate the angle that the fishing line makes with the vertical.

20. Hakam stands 50 m back from the foot of an 80-m telephone tower. Hakam's eyes are at a height of 1.57 m. Calculate the difference between the angle of elevation from the ground and from Hakam's eyes.



Fully worked solutions for this chapter are available online.

Answers

Chapter 6 Right-angled triangles

6.2 Pythagoras' theorem

6.2 Exercise

1. a. PR or q b. YZ or x c. AB or c

2. a. 13 cm b. 170 mm c. 61 m

3. a. 10.82 cm b. 6.93 m c. 14.20 km

4. a. 10.4 cm b. 1.9 m c. 3.9 m

5. a. 8.9 cm b. 22.1 cm c. 47.4 mm
   d. 37.3 m

6. WX

7. 21.10 cm

8. 5.66 m

9. 9.1 cm

10. 13 m

11. 3.23 m

12. 3.73 m

13. 2.2 m

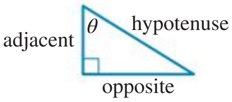
14. 7.5 m

15. 1.25 m

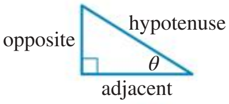
6.3 Trigonometric ratios — sine, cosine and tangent

6.3 Exercise

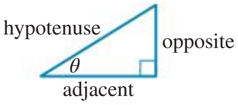
1. a.



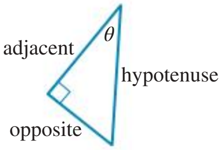
b.



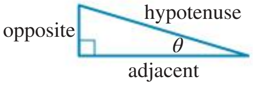
c.



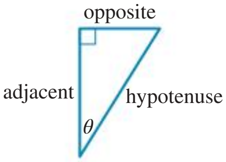
d.



e.



f.



2. a. $\sin(\theta) = \frac{8}{10}$ b. $\cos(\theta) = \frac{6}{10}$ c. $\tan(\theta) = \frac{8}{6}$

3. a. 0.602 b. 2.092 c. 15.246 d. 51.893

4. a. 0.707 b. 0.247 c. 6.568 d. 5.896

5. a. 1.540 b. 17.663 c. 40.460 d. 0.657

6. a. 0.5 b. 0.966 c. 1
   d. 548.643 e. 64 f. 1.301

7. a. 5.306 b. 1.374 c. 15.772

8. a. 0.42 b. 1.56 c. 0.09
   d. 5.10 e. 2.87 f. 0.38

9. a. 7.77 b. 73.30 c. 0.87

10. 10°

11. a. 44° b. 80° c. 57°

12. 86°40′

13. a. 42°57′ b. 31°21′ c. 16°5′

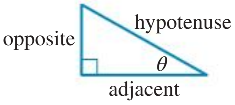
14. a. 62.74° b. 27.26° c. 24.61°

15. a. 61° b. 14° c. 27°

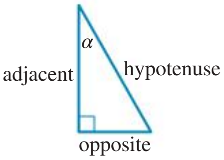
6.4 Calculating unknown side lengths of right-angled triangles [complex]

6.4 Exercise

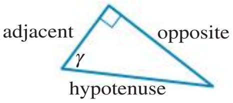
1. a.



b.



c.



2. 5.08 m

3. 30 cm

4. 148.1 mm

5. a. 12.5 m b. 89.3 mm c. 10.1 m

6. a. 12.1 cm b. 55.2 m c. 9.4 km

7. a. 5.4 m b. 1.4 km c. 2.1 km d. 18.4 mm
   e. 3.2 cm f. 66.5 m

8. a. 5.4 m b. 5.4 km c. 0.2 m d. 41.6 km
   e. 84.4 m f. 13.2 cm

9. $x = \frac{9.2}{(\cos(69))}$

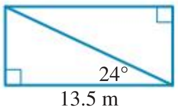
10. 12.93 m

11. 6 m

12. 4.2 m

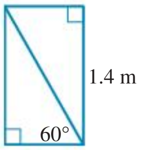
13. 20 km

14. a.



b. 6 m

15. a.



b. 1.6 m

16. 9.65 m

17. $\sin(\theta) = \dfrac{8}{17}$

18. $15.53\ \mathrm{mh}^{-1}$

19. 5.29 m

6.5 Calculating unknown angles in right-angled triangles [complex]

6.5 Exercise

1. a. 30° **b.** 75° **c.** 81°

2. a. 32°48′ **b.** 45°3 **c.** 35°16′

3. a. 53°8′ **b.** 55°35′ **c.** 45°27′

4. a. 50° **b.** 32° **c.** 33°

5. a. 21° **b.** 81° **c.** 34°

6. a. 39°48′ **b.** 80°59′ **c.** 13°30′

7. a. 79°6′ **b.** 63°1′ **c.** 19°28′

8. 30°

9. 41°56′

10. 37°

11. 75°31′

12. 8°38′

13. 13°

14. 4°35′

15. 2.9°

6.6 Angles of elevation and depression [complex]

6.6 Exercise

1. 26.8 m

2. 3984 m

3. 190 m

4. 39.2 m

5. 42.1 m

6. 100 m

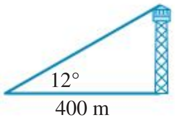
7. 15 km

8. 28 m, so a 30-m ladder can be used.

9. 39°

10. 21°

11. a.



b. 85 m

c. 40°

12. 868 m

13. 0.033 km or 33 m

14. 451.5 m

15. 28.4 m

6.7 Review

6.7 Exercise

1. a. 13.01 m **b.** 18.65 cm **c.** 3.58 m
d. 15.65 cm **e.** 2.30 km **f.** 2.47 m

2. 69.82 cm

3. AB = Adjacent
BC = Opposite
AC = Hypotenuse

4. a. 0.7193 **b.** 4.2303 **c.** 2.7400
d. 8.1955 **e.** 21.9845 **f.** 14.2998

5. a. 54° **b.** 51° **c.** 53°

6. a. 78°31′ **b.** 26°34′ **c.** 14°54′

7. a. 37.9 cm **b.** 3.8 m **c.** 13.6 cm
d. 11.7 cm **e.** 14.7 cm **f.** 14.6 m

8. a. 1.5 m **b.** 4.9 cm **c.** 15.6 mm
d. 7.5 m **e.** 10.7 m **f.** 5.3 km

9. a. 57° **b.** 27° **c.** 68°

10. a. 23°4′ **b.** 61°37′ **c.** 59°35′

11. 23 m

12. 14.11 m

13. 98 km

14. 66 m

15. 63.9 m

16. 8.5 m

17. 2.5 km

18. 39°

19. 24°

20. 0.51°

UNIT 3 Measurement, scales and chance
TOPIC 3 Probability and relative frequencies

7 Simulations and simple probabilities

LESSON SEQUENCE

Fully worked solutions for this chapter are available online.

on Resources

Solutions	Solutions — Chapter 7
Digital documents	Learning matrix — Chapter 7 Quick quizzes — Chapter 7 Chapter summary — Chapter 7

LESSON
7.1 Overview

7.1.1 Introduction

Probability is a crucial part of our daily lives, playing a significant role in decision-making and predicting outcomes. By learning to express probabilities using fractions, decimals, ratios and percentages, you will gain a clear understanding of likelihoods. Constructing and using sample spaces for experiments will help determine possible outcomes.

This chapter delves into practical applications of probability in fields such as finance, science, engineering and everyday problem-solving. For instance, predicting weather patterns, calculating risks in investments and even strategies for games of chance rely on probability concepts. Through modelling and solving probability experiments, performing simulations with technology, and recognising the variation in repeated chance events, you will understand the real-world relevance of probability.

7.1.2 Syllabus links

Lesson	Lesson title	Syllabus links
7.2	**Theoretical probability and sample spaces**	○ Express probabilities formally using fractions, decimals, ratios and percentages. ○ Construct a sample space for an experiment. ○ Use a sample space to determine the probability of outcomes for an experiment.
7.3	**Multi-stage experiments (arrays and tree diagrams)**	○ Use arrays and tree diagrams to determine the outcomes and the probabilities for experiments. ○ Model and solve problems involving probability experiments.
7.4	**Relative frequency and simulations**	○ Perform simulations of probability experiments using technology. ○ Recognise that the repetition of chance events is likely to produce different results. ○ Identify relative frequency as probability. ○ Identify factors that could complicate the simulation of real-world events [complex]. ○ Model and solve problems involving probability experiments.

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LESSON
7.2 Theoretical probability and sample spaces

SYLLABUS LINKS

- Express probabilities formally using fractions, decimals, ratios and percentages.
- Construct a sample space for an experiment.
- Use a sample space to determine the probability of outcomes for an experiment.

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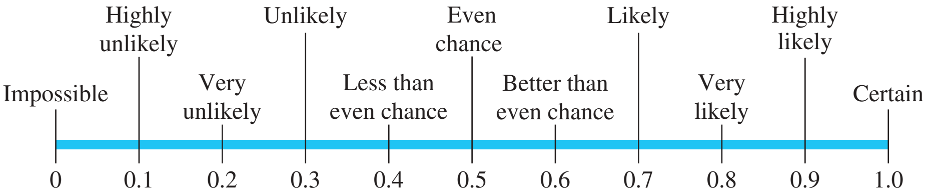
7.2.1 The probability scale

- Probability is a measure of the likely occurrence of an event.
- The probability of an event occurring is measured with a scale ranging from and including 0 (**impossible** event) to 1 (**certain** event).
- Probability can be written as a decimal, fraction, ratio or a percentage.
  For example, if the probability is an even chance, this can be written as: 0.5, $\frac{1}{2}$, 1 : 2 or 50%. The fractions and ratios should always be simplified where possible.
- The probability of an event (A) occurring can be denoted by P(A).
- That means that $0 \le \text{P}(\text{A}) \le 1$, with $\text{P}(\text{A}) = 0$ if A is an impossibility and $\text{P}(\text{A}) = 1$ if A is a certainty.



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WORKED EXAMPLE 1 Converting probabilities

Convert the following probabilities into the form shown in the brackets.

a. 0.55 (fraction and percentage)

b. 3 : 4 (fraction and percentage)

c. $\frac{3}{8}$ (ratio and percentage)

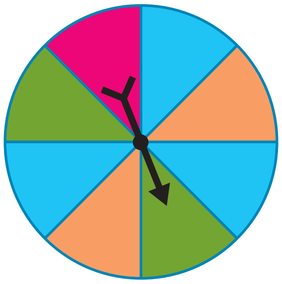
d. $\frac{5}{8}$ (ratio and decimal)

e. 70% (ratio and decimal)

THINK	WRITE
a. 1. To convert from decimal to fraction, multiply the numerator and denominator by 100 and simplify.	**a.** $0.55 = \frac{0.55}{1}$ $= \frac{0.55 \times 100}{1 \times 100}$ $= \frac{55}{100}$ $= \frac{11}{20}$
2. To convert from decimal to percentage, multiply by 100.	$0.55 \times 100 = 55\%$

b. **1.** To convert from ratio to fraction, use $a:b=\frac{a}{b}$.	**b.** $3:4=\frac{3}{4}$
2. To convert from ratio to percentage, multiply the fraction by 100.	$\frac{3}{4}\times 100=75\%$
c. **1.** To convert from fraction to ratio, use $a:b=\frac{a}{b}$.	**c.** $\frac{3}{8}=3:8$
2. To convert from fraction to percentage, multiply the fraction by 100.	$\frac{3}{8}\times 100=37.5\%$
d. **1.** To convert from fraction to ratio, use $a:b=\frac{a}{b}$.	**d.** $\frac{5}{8}=5:8$
2. To convert from fraction to decimal, calculate the fraction in decimal form.	$\frac{5}{8}=0:625$
e. **1.** To convert from percentage to ratio, firstly convert to fraction and then use $a:b=\frac{a}{b}$.	**e.** $70\% = \frac{70}{100}$ $= \frac{7}{10}$ $= 7:10$
2. To convert from percentage to decimal, divide by 100.	$70\%=\frac{70}{100}=0.7$

7.2.2 Sample spaces



- An **outcome** is a particular result of an experiment.
- A **sample space** is a list of all possible outcomes.
- If an outcome appears more than once, it is counted only once and written once in the sample space.
- A sample space is often listed as a set of outcomes. For example, the sample space for the following spinner is {pink, blue, green, orange}; the sample space includes all of the colours on the spinner.

WORKED EXAMPLE 2 Determining sample space

For each of the following experiments, identify the sample space to show the possible outcomes.

a. A coin is flipped.

b. An arrow is shot at a target where there is a blue area, a white area, a red area and a black area.

c. A circular spinner with seven sectors that are labelled 1 to 7 is spun.

THINK	**WRITE**
a. A coin has two sides: head and tail. List each possible outcome.	**a.** When a coin is flipped, the two possible outcomes are head and tail. Therefore, the sample space is {head, tail}.
b. There are four possible areas that the arrow could hit on the target: blue, white, red and black. List each possible outcome.	**b.** When an arrow is shot at the target, the four possible outcomes are blue, white, red and black. Therefore, the sample space is {blue, white, red, black}.

c. There are seven different sectors on the spinner so there are seven possible outcomes. List each possible outcome.	**c.** When the circular spinner is spun, the seven possible outcomes are 1, 2, 3, 4, 5, 6 and 7. Therefore, the sample space is $\{1, 2, 3, 4, 5, 6, 7\}$.

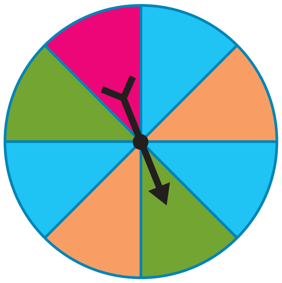
7.2.3 Calculating theoretical probability

- A **favourable outcome** is an outcome that you want or are looking for.
- The theoretical probability of an event can be calculated using the following probability formula.

Calculating theoretical probability

$$\mathbf{P(event)} = \frac{\textbf{number of favourable outcomes}}{\textbf{total number of outcomes}}$$

- Probabilities can be expressed as fractions, decimal ratios or percentages.
- In this chapter, unless otherwise specified, we will express probabilities as fractions.
- **Equally likely outcomes** have the same chance of occurring.
- For the spinner shown, P(blue) $= \frac{3}{8}$ because there are three favourable outcomes (blue sectors) out of eight possible outcomes (total number of equal sectors on the spinner).
- Orange and green are equally likely outcomes as they both occupy two of the eight sectors on the spinner. P(orange) = P(green) $= \frac{2}{8}$ or $\frac{1}{4}$.
- Pink is the least likely outcome as it occupies only one of the eight sectors on the spinner. P(pink) $= \frac{1}{8}$. Pink has the smallest value for its theoretical probability.



WORKED EXAMPLE 3 Calculating the probability of an event

A standard six-sided die is rolled. Determine the probability of rolling:

a. **a 3**

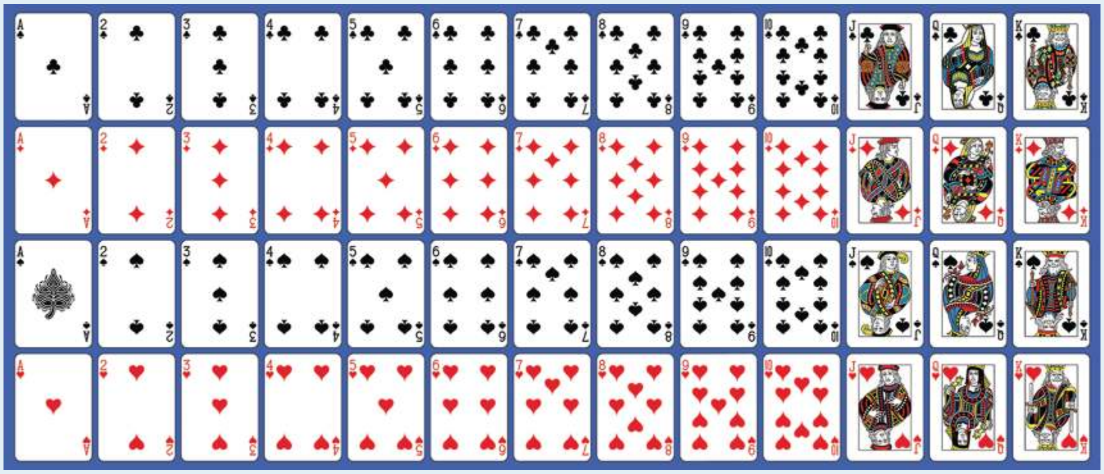
b. **an odd number.**

THINK	WRITE
a. **1.** The number 3 occurs once on a die.	**a.** Number of favourable outcomes = 1
2. Possible outcomes are 1, 2, 3, 4, 5 and 6.	Number of outcomes = 6
3. Calculate the probability of rolling a 3.	$P(3) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$ $= \frac{1}{6}$
4. Write your answer.	The probability of rolling a 3 is $\frac{1}{6}$.
b. **1.** The odd numbers are 1, 3 and 5. There are six numbers on a die.	**b.** Number of favourable outcomes = 3 Number of outcomes = 6

2. Calculate the probability of rolling an odd number. Write the fraction in simplest form.	$\text{P(odd number)} = \dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$ $= \dfrac{3}{6}$
3. Write your answer.	The probability of rolling an odd number is $\dfrac{1}{2}$.

WORKED EXAMPLE 4 Calculating the probability of an event 2

A normal pack of 52 playing cards is well shuffled and one card is drawn.



Determine the probability of drawing:

a. **a club**

b. **an ace**

c. **a red queen.**

THINK	WRITE
a. **1.** There are 13 clubs in a deck of cards. **2.** There are 52 cards in the deck, making 52 possible outcomes.	**a.** Number of favourable outcomes = 13 Number of outcomes = 52
3. Calculate the probability of drawing a club, making sure to simplify the fraction.	$\text{P(club)} = \dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$ $= \dfrac{13}{52}$ $= \dfrac{1}{4}$
4. Write your answer.	The probability of drawing a club is $\dfrac{1}{4}$.
b. **1.** There are four aces in a deck of cards. **2.** There are 52 cards in the deck, making 52 possible outcomes.	**b.** Number of favourable outcomes = 4 Number of outcomes = 52

3. Calculate the probability of drawing an ace. Write the fraction in simplest form.

$$\begin{aligned}\text{P(ace)} &= \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}\\ &= \frac{4}{52}\\ &= \frac{1}{13}\end{aligned}$$

4. Write your answer.

The probability of drawing an ace is $\frac{1}{13}$.

c. 1. There are two red queens in a deck of cards.

2. There are 52 cards in the deck, making 52 possible outcomes.

c. Number of favourable outcomes = 2

Number of outcomes = 52

3. Calculate the probability of drawing a red queen. Write the fraction in simplest form.

$$\begin{aligned}\text{P(red queen)} &= \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}\\ &= \frac{2}{52}\\ &= \frac{1}{26}\end{aligned}$$

4. Write your answer.

The probability of drawing a red queen is $\frac{1}{26}$.

7.2.4 The complement of an event

- The probability of the **complement** of an event is the probability that the event discussed will not happen. For example, if an event is 'it will rain today', then the complement of this is 'it will not rain today'.
- The other way to interpret the complement is to recognise it as the opposite of the event.
- If the probability of an event is denoted by P(A), then the probability of the complement is denoted by P(A′).
- The sum of the probability of an event and its complement is equal to 1.

$$P(A) + P(A') = 1$$

- This equation can be transposed (rearranged) so it is expressed in terms of the complement.

The complement of an event

$$\mathbf{P(A') = 1 - P(A)}$$

For example, if the probability of rain today P(A) is $\frac{7}{10}$, then the probability of no rain today P(A′) is:

$$\begin{aligned}P(A') &= 1 - \frac{7}{10}\\ &= \frac{3}{10}\end{aligned}$$

This means there is a 30% chance that it will not rain.

WORKED EXAMPLE 5 Calculating the complement of an event

Eli's Maths teacher told him that he had a 70% chance of passing the final exam, based on Eli's prior exam scores.

a. Identify the complement of this event.

b. Calculate the probability of the complement of this event. Give your answer as a percentage.

THINK	WRITE
a. The complement is if the discussed event will not happen.	**a.** The complement to Eli passing the exam is not passing the exam.
b. 1. Use the formula to find the complement.	**b.** $\text{P(passing)} = 70\% = \frac{7}{10}$ $\text{P(not passing)} = 1 - \text{P(passing)}$ $= 1 - \frac{7}{10}$ $= \frac{3}{10}$
2. Convert the answer into a percentage by multiplying by 100.	$\frac{3}{10} \times 100 = \frac{300}{10}$ $= 30\%$
3. Write your answer.	Eli has a 30% chance of not passing the final exam.

Exercise 7.2 Theoretical probability and sample spaces

learnon

7.2 Quick quiz on

7.2 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18	N/A	N/A

Note: Unless otherwise stated, express probabilities as fractions.

Simple familiar

1. **WE1** Convert the following probabilities to fractions.

   a. 0.6 b. 1 : 4 c. 35%

2. **WE1** Convert the following probabilities to decimals.

   a. 2 : 5 b. 87% c. $\frac{13}{50}$

3. **WE1** Convert the following probabilities to ratios.

   a. 0.2 b. $\frac{2}{7}$ c. 60%

4. **WE1** Convert the following probabilities to percentages. Round to 1 decimal place where necessary.

a. 0.93 b. 1 : 20 c. $\frac{1}{12}$

5. Match the words with one of the numbers between 0 and 1 that are given. Choose the number depending on what sort of chance the word indicates, between *impossible* and *certain*. You may use a number more than once. The numbers to choose from are 1, 0.75, 0.5, 0.25 and 0.

a. Certain
b. Likely
c. Unlikely
d. Probable
e. Improbable
f. Definite
g. Impossible
h. Slim chance
i. Sure thing
j. Doubtful
k. Fifty–fifty
l. More than likely

6. List two events that have a probability of:

a. 0 b. 0.5 c. 1 d. 0.75 e. 0.25.

7. **WE2** For each of the following experiments, list the sample space to show the total possible outcomes.

a. A standard six-sided die is rolled.
b. A marble is randomly selected from a bag containing 4 green, 2 yellow and 3 blue marbles.
c. A letter is selected from the letters of the alphabet.

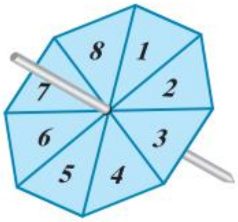
8. **WE3** A standard six-sided die is rolled. Determine the probability of rolling:

a. a 5
b. a number less than 3
c. an even number.

9. The letters of the word *MATHEMATICS* are each written on a small piece of cardboard and placed in a bag. If one card is selected from the bag, calculate the probability that it is:

a. a vowel b. a consonant c. the letter M d. the letter C.

10. A spinner of eight equally likely numbers is spun. The spinner is numbered 1–8.



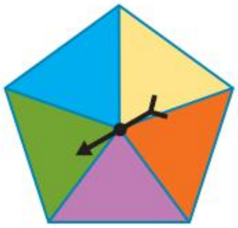
a. List the sample space.
b. Determine the probability, expressed as a percentage, of obtaining:
   i. an odd number
   ii. a number that is less than 6
   iii. the number 9
   iv. a number that is at most 8.

11. For each of the following spinners:

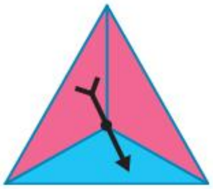
i. determine if there is an equal chance of landing on each colour
ii. list all the possible outcomes
iii. determine the probability of each outcome.

a.

b.

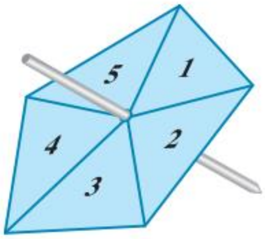


c.



12. All the jelly beans in the photograph are placed in a bag for a simple probability experiment.
    a. Determine which colour of jelly bean is most likely to be selected from the bag and justify your response.
    b. Identify the colour of jelly bean that is least likely to be selected from the bag and justify your reasoning.
    c. Calculate the probability of selecting each colour of jelly bean from the bag.

13. Determine the probability of obtaining a number less than 5 on the spinner shown.



14. **WE4** A normal pack of 52 playing cards is well shuffled and one card is drawn. Calculate the probability of drawing:
    a. a heart
    b. a red card
    c. an ace of diamonds
    d. a picture card (jack, queen or king).

15. Draw spinners with the following probabilities.
    a. $P(\text{blue}) = \frac{1}{3}$ and $P(\text{white}) = \frac{2}{3}$
    b. $P(\text{blue}) = \frac{1}{2}, P(\text{white}) = \frac{1}{4}, P(\text{green}) = \frac{1}{8}$ and $P(\text{red}) = \frac{1}{8}$
    c. $P(\text{blue}) = 0.75$ and $P(\text{white}) = 0.25$

16. Determine the complement event of each of the following events.
    a. Owning a dog
    b. Flipping a coin and getting a tail
    c. Sleeping
    d. Attending school
    e. Passing your learner license knowledge test
    f. Owning a mobile phone

17. **WE5** Bol's Physics teacher told him that he had a 20% chance of passing the final exam, based on Bol's prior exam scores.
    a. Identify the complement of this event.
    b. Determine the probability of the complement of this event. Give your answer as a percentage.

18. If two dice are rolled, determine the probabilities that the sum of the rolled numbers is:
    a. 5
    b. not 5
    c. 6 or 7
    d. not 6 or 7
    e. 2
    f. not 2.

Fully worked solutions for this chapter are available online.

LESSON
7.3 Multi-stage experiments (arrays and tree diagrams)

SYLLABUS LINKS

- Use arrays and tree diagrams to determine the outcomes and the probabilities for experiments.
- Model and solve problems involving probability experiments.

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7.3.1 Arrays

- When two events occur to form a single result, an **array** can be used to display the sample space.
- Consider an experiment in which a red die and a blue die are rolled, and the two numbers that are rolled by each die are recorded. The array shows the sample space for that experiment. There are 36 possible outcomes in this sample space.

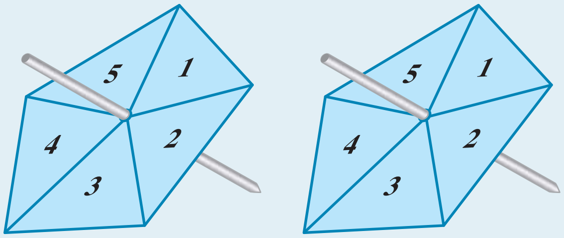
		Blue die					
		1	**2**	**3**	**4**	**5**	**6**
Red die	**1**	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

- An array is also useful when there are two events, but a different number of outcomes for each event. For example, the array shows the sample space for an experiment where a coin is flipped and a die is rolled.

		Die					
		1	**2**	**3**	**4**	**5**	**6**
Coin	**H**	(H, 1)	(H, 2)	(H, 3)	(H, 4)	(H, 5)	(H, 6)
	T	(T, 1)	(T, 2)	(T, 3)	(T, 4)	(T, 5)	(T, 6)

WORKED EXAMPLE 6 Using an array to determine the probability of multi-stage experiments

The two spinners shown are spun and the numbers that are chosen on each spinner are recorded. Determine the probability of spinning at least one 1.



THINK

WRITE

1. Draw the array for the sample space. Highlight the outcomes that have at least one 1, as shown in green. There are 9 outcomes with at least one 1.

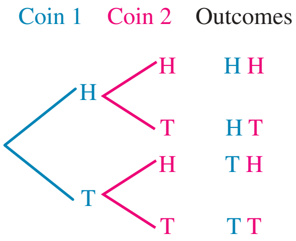
		Spinner 2				
		1	**2**	**3**	**4**	**5**
Spinner 1	**1**	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)

2. Calculate the probability using the formula. There are 25 outcomes in the sample space.

$$\begin{aligned}\text{P(at least one 1)} &= \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}\\ &= \frac{9}{25}\end{aligned}$$

7.3.2 Tree diagrams

- **Tree diagrams** are used to list all possible outcomes of two or more events.
- The branches show the possible links between one outcome and the next outcome.
- The tree diagram shown represents the sample space for flipping two coins.
- When all outcomes are equally likely, probabilities can be calculated from tree diagrams. For example, in the tree diagram shown, the probability of obtaining one tail and one head can be calculated as:



$$\begin{aligned}\text{P(one H and one T)} &= \frac{\text{number of outcomes with one H and one T}}{\text{total number of outcomes}}\\ &= \frac{2}{4}\\ &= \frac{1}{2}\end{aligned}$$

- Each event is treated as if it were a separate event, even if events occur at the same time. For example, the example tree diagram applies to experiments where:
  - two coins are flipped together
  - the same coin is flipped twice.

WORKED EXAMPLE 7 Using tree diagrams to determine the probability of multi-stage experiments

The uniform committee at school is deciding on a new colour combination for the school uniform. The two colour choices for the school pants, shirt and jumper are red and yellow.

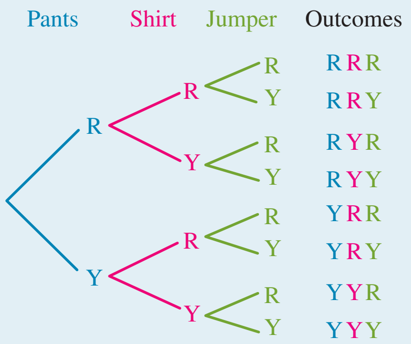
a. Use a tree diagram to show all the possible combinations.
b. Calculate the probability that the uniform will consist of only one colour.
c. Calculate the probability that a red jumper will be part of the uniform.
d. Calculate the probability that the uniform will be red pants, a yellow shirt and a red jumper.

THINK

WRITE

a. The three events are the colour of the pants, the colour of the shirt and the colour of the jumper. There are two choices for each of these.

a.



b. For the uniform to consist of only one colour, all pieces need to be red or all pieces need to be yellow. There is 1 outcome with all red and 1 outcome with all yellow. There are 8 outcomes altogether.

b. $$\begin{aligned}\text{P(all one colour)} &= \frac{2}{8}\\ &= \frac{1}{4}\end{aligned}$$

c. Four of the outcomes have a red jumper.

c. $$\begin{aligned}\text{P(red jumper)} &= \frac{4}{8}\\ &= \frac{1}{2}\end{aligned}$$

d. Calculate the probability of each option and multiply the probabilities together.

d. $$\text{P(red pants)} = \frac{1}{2}$$

$$\text{P(yellow shirt)} = \frac{1}{2}$$

$$\text{P(red jumper)} = \frac{1}{2}$$

$$\begin{aligned}\text{P(RYR)} &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\\ &= \frac{1}{8}\end{aligned}$$

Exercise 7.3 Multi-stage experiments (arrays and tree diagrams)

learnon

7.3 Quick quiz on	7.3 Exercise

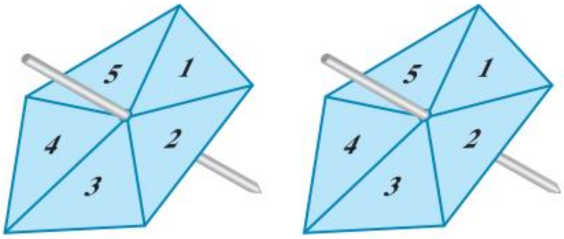
Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20	N/A	N/A

Note: Unless otherwise stated, express probabilities as fractions.

Simple familiar

1. A spinner is divided into five sections (red, blue, green, yellow and orange). A six-sided die is also rolled. Draw the array for one spin of the spinner and one roll of the die.

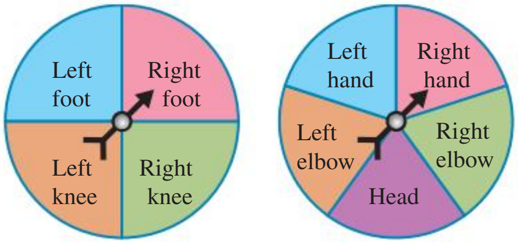
2. **WE6** The two spinners shown are spun and the numbers that are chosen on each spinner are recorded.



Calculate the probability of spinning exactly one 5.
You must show your working using an array.

3. A six-sided die and a coin are tossed at the same time. List the sample space.

4. In the game of Pretzel, the two spinners shown are spun and each contestant places the two body parts spun in contact and holds them there until the next two body parts are spun. If you fall over, you are out. The winner is the last person standing.



Determine the probability, expressed as a percentage, that the contestants will need to place:

a. their right foot on their head
b. their left elbow on a knee
c. their head on a foot
d. their left hand on a body part lower than their belly button.

5. A restaurant has the menu shown.
   a. Use a tree diagram to show how many different combinations are possible for a meal with one selection at each course.
   b. If a person selects a dinner at random, determine the probability that:
      i. they have soup as the entrée
      ii. they have spring rolls and curry
      iii. they don't have ice-cream for dessert
      iv. they have soup and pasta but not ice-cream.

Entrée

Pumpkin soup
Mini spring rolls

Main course

Pasta bolognaise
Grilled fish
Beef curry

Dessert

Lemon pie
Ice-cream

6. **WE7** The uniform committee at school is deciding on a new colour combination for the school uniform.
   The two colour choices for the school hat, bag and hoodie are blue and white.
   a. Use a tree diagram to show all the possible combinations.
   b. Calculate the probability that the uniform will consist of only one colour.
   c. Calculate the probability that a white hat will be part of the uniform.
   d. Calculate the probability that a white hat, blue bag and blue hoodie will be part of the uniform.

7. For a fundraiser, your class is selling single-scoop ice-cream cones at lunchtime. There is a choice of vanilla and chocolate ice-cream, topped with chocolate curls, sprinkles, melted chocolate or no topping at all.
   a. Draw a tree diagram to show all the possible combinations of ice-cream and topping that you could have from the stall.
   b. If a person chooses an ice-cream at random, calculate the probability that:
      i. both the ice-cream and the topping are chocolate
      ii. it is a vanilla ice-cream with sprinkles
      iii. there is no topping on the ice-cream.

8. In your drawers at home, there are two white T-shirts, a green T-shirt and a red T-shirt. There is also a pair of black pants and a pair of khaki pants.

   a. Draw a tree diagram to show all the possible combinations of T-shirts and pants that you could wear.
   b. If you get dressed in the dark and put on one T-shirt and one pair of pants, calculate the probability that you put on the red T-shirt and khaki pants.

9. In the last Science test, your friend guessed the answers to three true/false questions.
   a. Use a tree diagram to show all the different answer combinations for the three questions.
   b. Calculate the probability that your friend:
      i. got all three answers correct
      ii. got exactly two correct answers
      iii. got no correct answers.

10. Assume that the chance of a baby being a boy or a girl is the same.

   a. Calculate the probability that a family with three children has:
      i. all boys
      ii. two girls and one boy
      iii. three children of the same gender
      iv. at least one girl
      v. two children of the same gender.
   b. If the family was expecting another baby, determine the probability that the new baby will be a boy.
   c. If the family already has three boys, determine the probability that the new baby will be a boy.
   d. If the family has three girls, determine the probability that the new baby will be a boy.
   e. Determine the likelihood that the children in the family are three girls, as shown in the photo.

11. A fair coin is flipped three times. Calculate the probability of obtaining:
   a. at least two heads or at least two tails
   b. exactly two tails.

12. A fair coin is flipped three times. Determine the probability of obtaining at most 1 tail.

13. A spinner of seven equally likely numbers (numbered 1–7) is spun twice and the two numbers are added. Calculate the probability of obtaining a total of:

a. 2 or 14
b. 9
c. at least 12.

14. Two dice are rolled and the product of the two numbers is found.

Calculate the probability that the product of the two numbers is:

a. an odd number
b. a prime number
c. more than 1
d. at most 36.

15. A fair die is rolled and a fair coin flipped. Calculate the probability of obtaining:

a. an even number and a head
b. a tail from the coin
c. a prime number from the die
d. a number less than 5 and a head.

16. a. Use an array to display the sample space for an experiment where two dice are rolled and the sum of the two numbers appearing is found.
b. List any patterns you found in the sample space.
c. Copy and complete the following table.

Sum of two dice	2	3	4	5	6	7	8	9	10	11	12
Probability											

d. List any patterns that you notice in the probabilities you found in the table.

17. In a new game, two dice are rolled and the difference between the two dice is noted. If the difference is larger than 2, you win; otherwise you lose.

a. Use an array to determine the sample space for the game.
b. Calculate the probability of winning (having a difference greater than 2).
c. Calculate the probability of losing (having a difference of 2 or less).
d. A fair game is one in which the chances of winning are the same as the chances of losing. Justify whether this game is fair or not.

18. The game rock, paper, scissors is sometimes used to make a decision between two people. It uses the three different hand signs shown in the photo. On the count of 3, each player displays one of the hand signs. If both players display the same hand sign then the game is a tie. The photos show all other possible results.

Rock breaks scissors

Rock wins

Paper covers rock

Paper wins

Scissors cut paper

Scissors wins

a. Draw an array to show all the possible combinations for a single game of rock, paper, scissors.

		Person 2		
		Rock	**Paper**	**Scissors**
Person 1	**Rock**			
	Paper			
	Scissors			

b. Calculate the probability that:

- **i.** rock wins
- **ii.** paper wins
- **iii.** scissors wins.

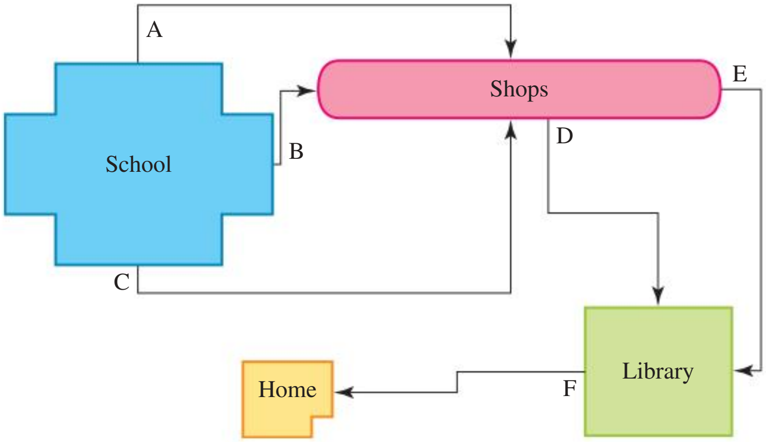
c. Decide whether rock, paper, scissors is a fair game or not. Justify your answer.

d. A friend suggested that you add dynamite to the game with the following rules.

- Dynamite blows up the rock and wins.
- Dynamite has its fuse cut by scissors and loses.
- Dynamite sets the paper on fire and wins.

  - **i.** Modify your array in part **a** to include dynamite.
  - **ii.** Analyse whether each hand sign has an equal chance of winning and justify your response.

19. There are three different ways to go from school to the shops. There are two different ways to go from the shops to the library. There is only one way to go from the library to home.



This afternoon you need to travel from school to home via the shops and the library.

a. Use a tree diagram to calculate the number of different routes you could use on your journey. (*Hint:* Use the letters A, B and C to represent the different routes from school to the shops and D and E to represent the different routes from the shops to the library. Use F to represent the route from the library to home.)

b. Evaluate whether the number of outcomes would change if the last leg of the journey from the library to home was omitted, and justify your response.

20. A die is rolled and 2 coins are flipped. Display the sample space using an appropriate method.

Fully worked solutions for this chapter are available online.

LESSON
7.4 Relative frequency and simulations

SYLLABUS LINKS

- Perform simulations of probability experiments using technology.
- Recognise that the repetition of chance events is likely to produce different results.
- Identify relative frequency as probability.
- Identify factors that could complicate the simulation of real-world events [complex].
- Model and solve problems involving probability experiments.

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7.4.1 Relative frequency

- Often in real life it is not possible to calculate a theoretical probability for an event. In these situations, an experiment is carried out.
- An experiment that is repeated a number of times is called a **trial** and becomes part of a larger experiment. For example, a die is rolled and the number is recorded. This is an experiment.
- If the experiment is repeated another 49 times, the first roll of the die is now referred to as the first trial in an experiment where the die is rolled 50 times.

- A **successful trial** is a trial where the result is the outcome that you wanted.
- The **relative frequency**, sometimes called the empirical or experimental probability, of an event is calculated using the following formula:

Relative frequency

$$\text{relative frequency} = \frac{\text{number of successful trials}}{\text{total number of trials}}$$

- The relative frequency of an event for a very large number of trials gives an indication of the value of the theoretical probability.
- As the number of trials increases, the relative frequency of an event will gradually become closer in value to the theoretical probability. This is known as the 'law of large numbers'.
- If the number of trials is large enough, the relative frequency can be used as an estimate of probability.

7.4.2 Simulations

- Sometimes it is not possible to conduct trials of a real experiment because it is too expensive, too difficult or impracticable.
- In situations like this, outcomes of events can be modelled using devices such as spinners, dice or coins to simulate or represent what happens in real life.
- These simulated experiments are called **simulations**.

tlvd-12108

WORKED EXAMPLE 8 Simulations of probability experiments

To simulate whether a baby is born male or female, a coin is flipped. If the coin lands heads up, the baby is a boy. If the coin lands tails up, the baby is a girl. Student 1 flips a coin 10 times and obtains 3 heads and 7 tails, while Student 2 flips a coin 100 times and obtains 43 heads and 57 tails.

a. Determine the relative frequency of females in both cases. Give your answers as decimals.

b. Compare the results from the two simulations and determine which is a better estimate of the true probability.

THINK	WRITE
a. 1. A baby born female is a successful trial. For Student 1, in the simulation, females are tails. There were 7 tails, so 7 females were born and there were 7 successful trials out of 10 trials. Write the formula for relative frequency and substitute the results.	**a.** Student 1: $\text{Relative frequency} = \frac{\text{number of successful trials}}{\text{total number of trials}}$ $= \frac{7}{10}$ $= 0.7$
2. For Student 2, there were 57 tails, so 57 females were born and there were 57 successful trials out of 100 trials. Write the formula for relative frequency and substitute the results.	Student 2: $\text{Relative frequency} = \frac{\text{number of successful trials}}{\text{total number of trials}}$ $= \frac{57}{100}$ $= 0.57$

b. 1. Compare the results of the two simulations. Theoretically, there should be an equal number of males and females born, so we would expect the results to be close to 0.5.

2. Give your answer.

b.

Student	Relative frequency
1	0.7
2	0.57

The true probability is likely to be close to 0.5. Student 2 used 90 more trials than Student 1 and therefore 0.57 is likely to be closer to the theoretical probability than 0.7.

- As you can see from Worked example 8, the greater the number of trials, the better the estimate of the theoretical probability achieved.

7.4.3 Using technology to perform simulations

- Simulating experiments using manual devices such as dice and spinners can take a lot of time.
- A more efficient method of collecting results is to use a list of randomly generated numbers.
- **Random number generators** can generate a series of numbers between two given values — for example, decimals between 0 and 0.9999.
- The following table shows some Excel formulas that generate random numbers.

Formula	Output
=RAND()	A random decimal from 0 to 0.9999
=INT(6*RAND()+1)	Integers between 1 and 6 that can be used to simulate rolling a die. Both of these formulas can be modified to generate the type of numbers that you require.
=RANDBETWEEN(1,6)	Generates random numbers between 1 and 6.

tlvd-12109

WORKED EXAMPLE 9 Simulations of probability experiments using technology

a. Use a random number generator to simulate the number of chocolate chips in 50 biscuits, with a maximum of 80 chocolate chips in each.

b. Calculate the probability that there will be more than 30 chocolate chips in a randomly chosen biscuit.

THINK

a. Use a random number generator to generate 50 numbers between 0 and 80. Use either of the following formulas in Excel:
=INT(80*RAND()+1) or
=RANDBETWEEN(1,80)

WRITE

a.

8	55	8	62	80	50	42	80	57	39
15	7	64	73	47	12	74	74	16	42
41	22	50	33	68	72	64	16	6	72
70	72	52	48	14	22	59	48	65	34
67	62	72	59	10	30	13	7	40	18

b. 1. Count the number of biscuits with more than 30 chocolate chips. There are 34 in the example shown.

b.

8	55	8	62	80	50	42	80	57	39
15	7	64	73	47	12	74	74	16	42
41	22	50	33	68	72	64	16	6	34
70	72	52	48	14	22	59	48	65	34
67	62	72	59	10	30	13	7	40	18

2. The probability is the number of successful outcomes (34) divided by the number of trials (50).

$$\text{P(more than 30 chocolate chips)} = \frac{34}{50}$$

$$= \frac{17}{25}$$

7.4.4 Using relative frequencies: polls

Statisticians use frequency tables to visually compare the attitudes of different groups, typically showing results from random samples of large populations or complete data from small populations.

- When calculating relative frequencies, six steps are followed.
  1. Gather the required statistics.
  2. Determine the total number of people polled.
  3. Determine each group's attitude in relation to the total number of people polled and write in fractional form. For example:
     - Of the 100 people polled, 30 liked the caravan park $= \frac{30}{100}$.
     - Of the 100 people polled, 70 did not like the caravan park $= \frac{70}{100}$.
  4. Convert the fraction to a decimal.
     $$\frac{30}{100} = 0.3$$
  5. Convert the decimal to a percentage.
     $$0.3 = 30\%$$
  6. Check that all relative frequencies add to 100%.
     Note: When the decimals or percentages are rounded off, the figure may be slightly below or above 100%.

WORKED EXAMPLE 10 Using relative frequencies

In a recent election, there were four candidates. During a telephone poll conducted a week before the election, 500 randomly selected voters were asked to indicate their preferences for the four candidates. The results are shown in the table.

Candidate	**1**	**2**	**3**	**4**
Number of voters who preferred this candidate	**115**	**168**	**145**	**72**

Estimate each candidate's probability of winning the election, giving your answers as decimals and percentages.

THINK

1. The probability of a candidate winning is the number of votes they received divided by the total number of votes.

WRITE

$$\begin{aligned} \text{P(candidate 1 wins)} &= \frac{115}{500} \\ &= 0.230 \\ &= 23\% \\ \text{P(candidate 2 wins)} &= \frac{168}{500} \\ &= 0.336 \\ &= 33.6\% \\ \text{P(candidate 3 wins)} &= \frac{145}{500} \\ &= 0.290 \\ &= 29\% \\ \text{P(candidate 4 wins)} &= \frac{72}{500} \\ &= 0.144 \\ &= 14.4\% \end{aligned}$$

2. The closer the relative frequency is to 1 (or 100%), the higher the probability of success.

Candidate 2 is the favourite to win.

7.4.5 Calculating the number of successful trials

- Sometimes, a relative frequency is known and may be used to calculate the number of individuals who responded a certain way, or the number of successful trials.

Calculating successful trials

number of successful trials = relative frequency × total number of trials

WORKED EXAMPLE 11 Calculating successful trials

In 2017, the Australian Marriage Law Postal Survey was conducted to gauge support for legalising same-sex marriage in Australia.
The table shows the breakdown of votes per state as a decimal frequency.

State/territory	Yes	No
New South Wales	0.578	0.422
Victoria	0.649	0.351
Queensland	0.607	0.393
South Australia	0.625	0.375
Western Australia	0.637	0.363
Northern Territory	0.606	0.394
Australian Capital Territory	0.740	0.260

5000 individuals from Queensland were randomly selected and asked for their vote.

a. **Estimate the number of people who will state they answered ‘yes’.**
b. **Estimate the number of people who will state they answered ‘no’.**

THINK	WRITE
a. 1. Look on the chart and identify the row with Queensland and those who stated ‘yes’.	**a.** 0.607
2. Multiply 5000 (number of individuals) by the result from Step 1. The answer represents the number of Queenslanders who answered ‘yes’.	$0.607 \times 5000 = 3035$
3. Since a fractional result is not valid in this context, round to the nearest whole number.	Approximately 3035 of the individuals answered ‘yes’.
b. 1. The number of Queenslanders was 5000. We know 3035 voted ‘yes’, so we subtract the answer to part **a** from 5000.	**b.** $5000 - 3035 = 1965$
2. Write the answer.	Approximately 1965 of the 5000 individuals answered ‘no’.

7.4.6 Factors that could complicate the simulation of real-world events [complex]

- Simulations are used to reproduce real-life situations when it is not feasible to implement the actual test.
- The reasons for not conducting an actual test can include financial or safety constraints, or just the overall complexity of running the test.
- Most simulations are conducted using computers and various other mechanical devices. For example, airbag manufacturers run airbag tests using cars, but instead of using real humans, they use mannequins.
- These simulations are fairly accurate, but they have their limitations.
- One problem with the airbag simulation is the difficulty of modelling the interaction of the airbag with the seatbelt.
- There are numerous other factors that can affect the success of the airbag in an accident. Some of these factors are easy to simulate, but others are not yet known or are too difficult to simulate.

WORKED EXAMPLE 12 Factors that complicate simulations [complex]

Six runners are to compete in a race.

a. Describe how a simulation could be used to determine the winner of the race.

b. Evaluate any limitations in your answer.

THINK	WRITE
a. There are six runners, so six numbers are needed to represent them. A die is the obvious choice. Assign numbers **1** to **6** to each runner.	**a.** Each runner could be assigned a number on a die, which could be rolled to determine the winner of the race. When the die is rolled, it simulates a race. The number that lands uppermost is the winner of the race.
b. In this simulation, each runner has an equally likely chance of winning each game.	**b.** The answer depends on each runner having an equally likely chance of winning the race. This is rarely the case.

Exercise 7.4 Relative frequency and simulations

learnon

7.4 Quick quiz on

7.4 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16	17, 18, 19, 20	N/A

Note: Unless otherwise stated, express probabilities as fractions.

Simple familiar

1. **WE8** To simulate whether or not the weather will be suitable for sailing, a coin is flipped. If the coin lands heads up, the weather is perfect. If the coin lands tails up, the weather is not suitable.
   Mandy flips a coin 20 times and obtains 13 heads and 7 tails. Sophia flips a coin 100 times and obtains 47 heads and 53 tails.
   a. Determine the relative frequency of perfect weather days in both cases.
   b. Compare the results from the two simulations and determine which is a better estimate of the true probability of the experiment.

2. A box of 50 batteries was tested and three were found to be flat.
   a. Determine the relative frequency of a flat battery.
   b. Predict the number of flat batteries in a box of 500 based on the relative frequency.

3. The results of a class experiment that involved rolling a standard six-sided die 300 times are shown in the table.

Number on die	1	2	3	4	5	6
Number of times rolled	42	50	61	37	52	58

   Use the results to calculate the experimental probability of rolling:
   a. the number 5
   b. an odd number
   c. an even number.

4. In his last 20 games, a basketball player sank 17 free throws and missed 11. Estimate the probability he will sink his next free throw.

5. A student rolls a pair of dice 10 times per trial and records how many times the total is 7. They repeat the experiment multiple times, with the following results:

$$1, 5, 3, 2, 1, 0, 2, 3, 4, 3, 2, 1, 0, 1, 2, 6, 2, 0, 1, 2$$

Using the data, determine the estimated probability of getting a total of 7 when two dice are rolled.

6. **a.** **WE9** Use a random number generator to simulate the number of walnuts in 40 different carrot cakes, with a maximum of 60 walnuts in each cake.

b. Calculate the probability that there will be more than 40 walnuts in a randomly chosen cake.

7. The gender of babies in a set of triplets is simulated by flipping 3 coins. If a coin lands tails up, the baby is a boy. If a coin lands heads up, the baby is a girl.
In the simulation, the trial is repeated 40 times and the following results show the number of heads obtained in each trial.

$$\begin{array}{l} 0, 3, 2, 1, 1, 0, 1, 2, 1, 0, \\ 1, 0, 2, 0, 1, 0, 1, 2, 3, 2, \\ 1, 3, 0, 2, 1, 2, 0, 3, 1, 3, \\ 0, 1, 0, 1, 3, 2, 2, 1, 2, 1 \end{array}$$

a. Calculate the probability, as a percentage, that exactly one of the babies in a set of triplets is female.

b. Calculate the probability, as a percentage, that more than one of the babies in the set of triplets is female.

8. **WE10** A school canteen asked 150 random Year 12 students to indicate their favourite food before they asked the entire school to place their vote. The food with the most votes was going to be made half price for a week.
The results from the Year 12 students are shown in the table.

Meal	Tally
Hamburger	45
Fish and chips	31
Macaroni and cheese	30
Lamb souvlaki	25
BBQ pork ribs	19

Calculate the probability of each food's popularity in the overall vote, giving your answer as a decimal.

9. Explain whether using a random number generator to select computers on the assembly line would be an appropriate method for simulating the probability of laptops having a fault. Justify your answer.

10. A random number generator was used to select how many students will receive a free lunch on any given day. The maximum number of students who can receive a free lunch is 10 and the minimum is 0.
    a. If the test is run for 30 d, determine the theoretical probability that:
        i. 8 or more students are selected on any given day
        ii. 0 students are selected on any given day
        iii. 10 students are selected on any given day.
    b. Use a random number generator to simulate the free-lunch program over 30 d, and compare the results to those from part a.
    c. Compare the results from the random number generator with your predictions and explain why they may or may not be the same.

11. Determine if the following simulations are easy or difficult to undertake. If your response is 'difficult', explain why.
    a. Predicting how a helicopter pilot will react in an emergency
    b. Estimating how many times you will win when spinning a roulette wheel
    c. Predicting how many kilometres a new car will travel before any major mechanical problems
    d. Estimating how many car accidents you will have in your lifetime
    e. Estimating the number of defective mobile phones that one manufacturer will produce over 2 months.

12. **WE12** In 2024, a survey was conducted to gauge support for changing the age of gaining a provisional licence from a minimum of 17 years old to a minimum of 16 years old.

The table shows the breakdown of votes by age as a decimal frequency.

Age	Yes	No
16–20	0.871	0.129
21–25	0.764	0.236
26–30	0.641	0.359
31–35	0.443	0.557
36–40	0.398	0.602
41–45	0.287	0.713
46–50	0.213	0.787
51–55	0.122	0.878

2800 individuals aged 16–20 were randomly selected and asked for their vote.
    a. Estimate how many people will state they answered 'yes'.
    b. Estimate how many people will state they answered 'no'.

13. **a.** Five thousand new domestic university students are selected to take part in a study. Use the most recent data in the chart to predict how many students will not come back after their first year of university for the 4-year period of 2025–2028.

b. Evaluate whether your estimate in part **a** will be higher or lower than the actual percentage, based on the domestic trend of students not returning after the first year, and justify your response.

Completion rates of higher education students — cohort analysis, 2005–2014				
Year	**Completed (in any year)**	**Still enrolled at the end of the 4-year cohort period**	**Re-enrolled, but dropped out**	**Never came back after the first year**
2005	47.3%	30.2%	11.5%	11.0%
2006	46.7%	32.7%	10.6%	10.0%
2007	45.9%	33.8%	10.4%	9.9%
2008	46.7%	33.5%	10.9%	8.9%
2009	46.1%	34.3%	11.1%	8.5%
2010	45.1%	34.7%	11.2%	9.0%
2011	45.0%	34.5%	11.7%	8.8%

Source: Department of Education and Training, *Completion Rates of Higher Education Students — Cohort Analysis*, 2005–2014.

14. The results of a coin-flipping experiment are shown.

Outcome	Frequency
Heads	38
Tails	62
Total	100

a. Determine the relative frequencies, expressed as a percentage, of:

i. heads **ii.** tails.

b. Assess whether you would expect the same results if the experiment were repeated, and justify your answer.

c. Predict what will happen to the relative frequencies of the outcomes if the experiment is repeated 100 times, and justify your response.

15. The latest promotion by a chocolate bar manufacturer claims ‘every third one is free’. If there is a special message inside the wrapper, you get a refund on the purchase price. The manufacturer has put the special message in every third wrapper. When the chocolate bars are packaged into boxes, they are mixed randomly. Use technology or a die (using 3 and 6 to represent the free chocolate bar) to run your simulation.

a. Conduct 50 trial purchases of chocolate bars.

b. Calculate the relative frequency of receiving a free chocolate bar.

c. Compare your results with those of the students around you.

d. Explain whether other students obtained different results and why this might have happened.

e. Estimate how many free chocolate bars you would expect to receive in 100 trials.

16. a. Calculate how many times, to the nearest whole number, you would expect to roll a 4 if you rolled a fair die:
   i. 10 times
   ii. 60 times
   iii. 1000 times.

   b. Evaluate whether you would get the exact number predicted if you actually carried out these experiments, and justify your answer.

Complex familiar

17. **WE9** There are 24 drivers lining up at the start of a Formula One race.
   a. Describe how simulation could be used to determine the winner.
   b. Evaluate any limitations in your answer.

18. Young children were playing an Olympic game, where they rolled an eight-sided die to simulate the placing of the Olympic sprint races. Explain if this simulation can be directly related to the actual Olympic events.

19. A simulation of the ages of a class of Year 12 students using random numbers is shown.

16	17	16	17	18	18
16	17	16	16	16	17
16	17	17	17	17	18
18	17	18	17	16	17

   a. Calculate the relative frequency of each age.
   b. Conduct the same simulation using your random number generator.
   c. Compare the results of the two simulations and comment on any differences or similarities.

20. A player suspects that a die is loaded (unfair), so she rolls the die 600 times to find out. The results of the experiment are shown in the table.

Number on die	1	2	3	4	5	6
Number of times rolled	63	90	96	118	95	138

   a. Predict the number of times each number should appear if the die were a fair die.
   b. Explain whether you think the die is loaded.

Fully worked solutions for this chapter are available online.

LESSON
7.5 Review

7.5.1 Summary

Hey students! Now that it's time to revise this chapter, go online to:

- Access the chapter summary
- Review your results
- Watch teacher-led videos
- Practise questions with immediate feedback

Find all this and MORE in jacPLUS

7.5 Exercise

learnon

7.5 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18	19	20

These questions are even better in jacPLUS!
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Note: Unless otherwise stated, express probabilities as fractions.

Simple familiar

1. Identify the sample space for the following scenarios.
  - **a.** Rolling an eight-sided die numbered 1–8
  - **b.** Drawing an ‘a’ from a bag containing the letters of the alphabet
  - **c.** Drawing the winning ticket in a raffle with 100 tickets sold

2. Identify which of the following events are ‘most likely’ and ‘very unlikely’.
  - **a.** A cyclone in Victoria
  - **b.** The temperature rising above 30 °C on a summer day in Darwin
  - **c.** Snow in Cairns
  - **d.** Getting struck by lightning during a storm in Brisbane

3. Construct three events that meet the following criteria.
  - **a.** Less than even chance of occurring
  - **b.** Greater than even chance of occurring

4. Two fair dice are rolled and the product of the numbers is recorded.
  - **a.** Construct a table to illustrate the sample space. List the sample space for the experiment.
  - **b.** Determine the most likely outcome.
  - **c.** Determine the least likely outcome.

5. Two dice are rolled and the difference of the numbers is recorded.
  - **a.** Construct a table to illustrate the sample space. List the sample space for the experiment.
  - **b.** Determine the most likely outcome.
  - **c.** Determine the least likely outcome.

6. a. Calculate the probability of rolling a 3 on a six-sided die.
   b. Identify the complement of the event in part a.
   c. Determine the probability of the complement occurring.

7. a. Calculate the probability that the two numbers you roll are the same when tossing two fair dice.
   b. Determine the probability of the complement of this event.
   c. Explain whether it would be easier to calculate the probability of the complement before calculating the original event.

8. The following table shows what people did with their old mobile phones in Australia in 2024.

Kept it	Gave it away	Recycled it	Sold it	Lost it
45%	28%	14%	9%	4%

What is the probability of *not* selling or *not* losing your mobile phone in Australia? Give your answer as a percentage.

9. In the game of backgammon, two dice are rolled together. The resulting numbers can be used separately in two separate moves or their total can be used for one move.

   a. List the sample space of rolling two dice simultaneously.
   b. Calculate the probability of 'getting a 4 on a die or a total of 4 on both dice'.
   c. Calculate the probability of *not* 'getting a 4 on a die or a total of 4 on both dice'.
   d. Calculate the probability of 'getting a 4 or getting a 5 on a die or a total of 5 on both dice'.

10. A soft-drink chemist is trying to create a new drink with three different ingredients.
The chemist wants to experiment with the mixing order of the three key ingredients.

   a. Construct a tree diagram to show the sample space.
   b. Determine how many combinations are possible.

11. A car dealership offers these package options on one of its new models.
   - Interior: leather or suede
   - Transmission: manual or automatic
   - Front seats: bucket, captain or regular

   a. Determine the number of different configurations of options available.
   b. Calculate the theoretical probability that a random customer chooses a bucket seat in their package.

12. A fair coin is flipped and then an eight-sided die is rolled.

   a. Identify all the possible outcomes in an array.
   b. Determine the probability of getting a head and an 8.
   c. Calculate the probability of getting an even number or a head.

13. At a fair, people were asked to taste and state if they liked two new brands of soft drink: brand A and brand B.

a. Represent all the possible outcomes using a tree diagram.
b. Determine the probability (assuming that there is a 50% chance of liking or disliking the soft drink) expressed as a percentage, that a randomly selected person liked:

- **i.** brand A but not brand B
- **ii.** brand A and brand B
- **iii.** brand A.

14. a. Estimate the number of times, to the nearest whole number, you would expect to roll a 6 if you rolled a fair die:

- **i.** 10 times
- **ii.** 150 times
- **iii.** 5000 times

b. Justify whether you would get the exact number predicted if you actually conducted the experiments in part **a**.

15. You are about to sit a Mathematics examination that contains 40 multiple-choice questions. You didn't have time to study so you are going to choose the answers completely randomly. There are four choices for each question.
Describe how you could use random numbers to select your answers for you.

16. Jose has to call his girlfriend, but he cannot remember her full number. He knows the first eight digits, but can't recall the final two digits.

a. Calculate the probability that he correctly guesses either one of the two final digits (but not both).
b. Calculate the probability that he correctly guesses both of the two final digits.
c. Calculate the probability that he incorrectly guesses both of the final two digits.

17. Sonya knows her credit card number for online shopping. However, she has forgotten the three-digit CVV number on the back of the card, and wants to purchase a dress before the sale ends.

a. Calculate the probability she correctly guesses all three digits.
b. Calculate the probability she incorrectly guesses all three digits.
c. Determine the probability that she correctly guesses at least one digit.
d. Evaluate the probability of her correctly guessing all the numbers now that she knows her CVV number does not have any repeated digits.

18. There are five different possible hair colours: red, brunette, brown, grey and blonde. Assume that each hair colour is equally likely. If there are five people in a room, what is the probability that at least two have the same coloured hair? Give your answer as a percentage correct to 2 decimal places. (*Hint:* Use the complement to solve this problem.)

Complex familiar

19. The spinner shown was spun 50 times and the outcome each time was recorded in the table.

Segment	Blue	Green	Pink
Tally	17	23	10

a. List the sample space.

b. Given the experimental results, determine the relative frequency for each segment.

c. The theoretical probability of the spinner landing on any particular segment with one spin is $\frac{1}{3}$.

Analyse how the experiment could be changed to provide a better estimate of the true probabilities.

Complex unfamiliar

20. A friend offers you a chance to roll a single die three times, and if you roll a 3 on any one of the rolls he will pay for your lunch. Otherwise, you will pay for his lunch. Should you play his game? Explain your answer.

Fully worked solutions for this chapter are available online.

Answers

Chapter 7 Simulations and simple probabilities

7.2 Theoretical probability and sample spaces

7.2 Exercise

1. a. $\frac{3}{5}$ **b.** $\frac{1}{4}$ **c.** $\frac{7}{20}$

2. a. 0.4 **b.** 0.87 **c.** 0.26

3. a. 1 : 5 **b.** 2 : 7 **c.** 3 : 5

4. a. 93% **b.** 5% **c.** 8.3%

5. a. 1 **b.** 0.75 **c.** 0.25
d. 0.75 **e.** 0.25 **f.** 1
g. 0 **h.** 0.25 **i.** 1
j. 0.25 **k.** 0.5 **l.** 0.75

6. Answers will vary. Sample responses can be found in the worked solutions in the online resources.

7. a. $\{1, 2, 3, 4, 5, 6\}$
b. {green, yellow, blue}
c. {a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z}

8. a. $\frac{1}{6}$ **b.** $\frac{1}{3}$ **c.** $\frac{1}{2}$

9. a. $\frac{4}{11}$ **b.** $\frac{7}{11}$ **c.** $\frac{2}{11}$
d. $\frac{1}{11}$

10. a. $\{1, 2, 3, 4, 5, 6, 7, 8\}$
b. i. 50%
ii. 62.5%
iii. 0%
iv. 100%

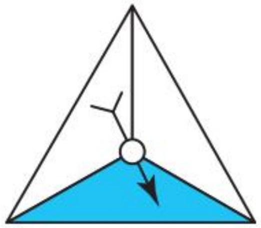
11. a. i. No, the colours are not represented equally.
ii. Blue, green, yellow, pink
iii. $P(\text{blue}) = \frac{1}{3}$, $P(\text{green}) = \frac{1}{6}$, $P(\text{pink}) = \frac{1}{3}$, $P(\text{yellow}) = \frac{1}{6}$
b. i. Yes, as there are equal numbers of the same colour.
ii. Blue, green, yellow, purple, orange
iii. $P(\text{blue}) = \frac{1}{5}$, $P(\text{green}) = \frac{1}{5}$, $P(\text{purple}) = \frac{1}{5}$, $P(\text{yellow}) = \frac{1}{5}$, $P(\text{orange}) = \frac{1}{5}$
c. i. No, the colours are not represented equally.
ii. Pink, blue
iii. $P(\text{blue}) = \frac{1}{3}$, $P(\text{pink}) = \frac{2}{3}$

12. a. Blue. There are more blue jelly beans (6) than beans of any other colour.
b. Yellow. There are fewer yellow jelly beans (2) than beans of any other colour.
c. $P(\text{blue}) = \frac{6}{17}$, $P(\text{green}) = \frac{5}{17}$, $P(\text{red}) = \frac{4}{17}$, $P(\text{yellow}) = \frac{2}{17}$

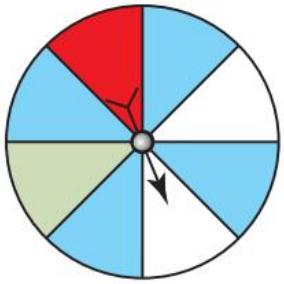
13. $\frac{4}{5}$

14. a. $\frac{1}{4}$ **b.** $\frac{1}{2}$ **c.** $\frac{1}{52}$ **d.** $\frac{3}{13}$

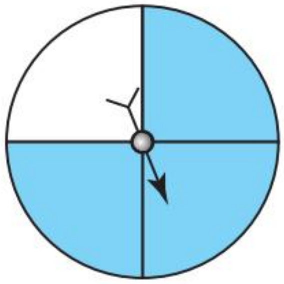
15. a.



b.



c.



16. a. Not owning a dog
b. Flipping a coin and getting a head
c. Being awake
d. Not attending school
e. Failing your learner knowledge test
f. Not owning a mobile phone

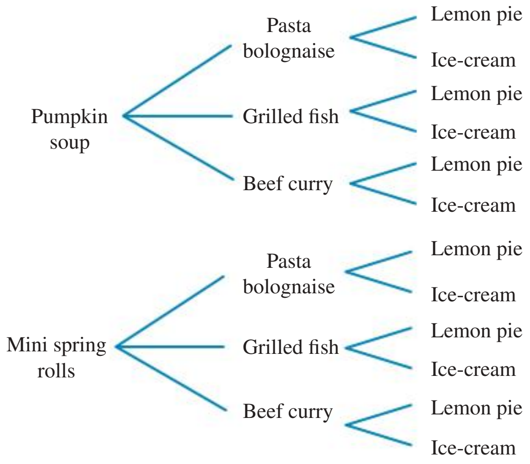
17. a. Failing the final exam
b. 80%

18. a. $\frac{1}{9}$ **b.** $\frac{8}{9}$ **c.** $\frac{11}{36}$
d. $\frac{25}{36}$ **e.** $\frac{1}{36}$ **f.** $\frac{35}{36}$

7.3 Multi-stage experiments (arrays and tree diagrams)

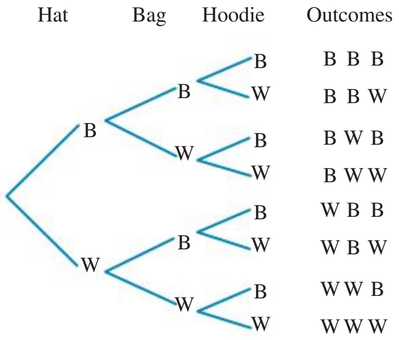
7.3 Exercise

1. See the table at the bottom of the page.*
2. See the table at the bottom of the page.*
3. {(1,H), (2,H), (3,H), (4,H), (5,H), (6,H), (1,T), (2,T), (3,T), (4,T), (5,T), (6,T)}
4. a. 5% b. 10% c. 10%
   d. 20%
5. a.



   b. i. $\frac{1}{2}$ ii. $\frac{1}{6}$ iii. $\frac{1}{2}$ iv. $\frac{1}{12}$

6. a.



   b. $\frac{1}{4}$

   c. $\frac{1}{2}$

   d. $\frac{1}{8}$

---

*1.

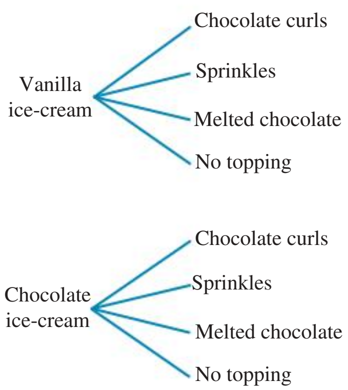
		Die					
		1	2	3	4	5	6
Spinner	Red	(R, 1)	(R, 2)	(R, 3)	(R, 4)	(R, 5)	(R, 6)
	Blue	(B, 1)	(B, 2)	(B, 3)	(B, 4)	(B, 5)	(B, 6)
	Green	(G, 1)	(G, 2)	(G, 3)	(G, 4)	(G, 5)	(G, 6)
	Yellow	(Y, 1)	(Y, 2)	(Y, 3)	(Y, 4)	(Y, 5)	(Y, 6)
	Orange	(O, 1)	(O, 2)	(O, 3)	(O, 4)	(O, 5)	(O, 6)

*2.

		Spinner 2				
		1	2	3	4	5
Spinner 1	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)

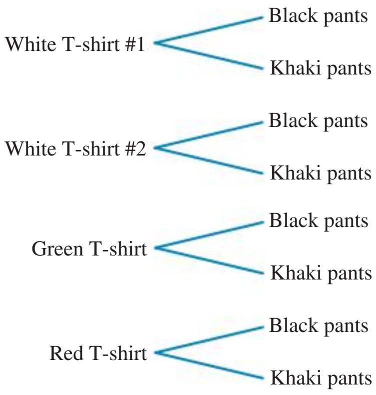
P(exactly one 5) $= \frac{8}{25}$

7. a.



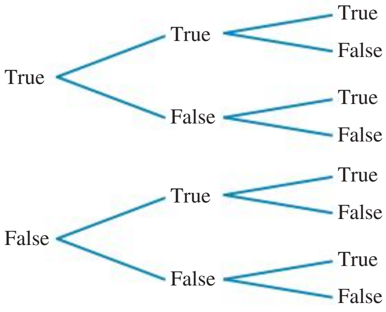
b. i. $\frac{1}{4}$ **ii.** $\frac{1}{8}$ **iii.** $\frac{1}{4}$

8. a.



b. $\frac{1}{8}$

9. a.



b. i. $\frac{1}{8}$ **ii.** $\frac{3}{8}$ **iii.** $\frac{1}{8}$

10. a. i. $\frac{1}{8}$ **ii.** $\frac{3}{8}$ **iii.** $\frac{1}{4}$ **iv.** $\frac{7}{8}$ **v.** $\frac{3}{4}$

b. $\frac{1}{2}$

c. $\frac{1}{2}$

d. $\frac{1}{2}$

e. $\frac{1}{8}$

11. a. 1 **b.** $\frac{3}{8}$

12. $\frac{1}{2}$

13. a. $\frac{2}{49}$ **b.** $\frac{6}{49}$ **c.** $\frac{6}{49}$

14. a. $\frac{1}{4}$ **b.** $\frac{1}{6}$ **c.** $\frac{35}{36}$ **d.** 1

15. a. $\frac{1}{4}$ **b.** $\frac{1}{2}$ **c.** $\frac{1}{2}$ **d.** $\frac{1}{3}$

16. a.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b. The diagonals in the sample space are repeating numbers. Also, 7 is the outcome that shows up the most, and each outcome further away from 7 has a reduced chance of occurring.

c. See the table at the bottom of the page.*

d. The probabilities on either side of the 7 decrease at the same rate.

17. a.

–	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

b. $\frac{1}{3}$

c. $\frac{2}{3}$

d. No

***16. c.**

Sum of two dice	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

18. a.

		Person 2		
		Rock	**Paper**	**Scissors**
Person 1	**Rock**	RR	RP	RS
	Paper	PR	PP	PS
	Scissors	SR	SP	SS

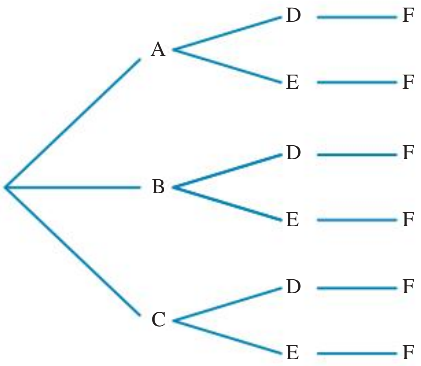
b. i. $\frac{2}{9}$ **ii.** $\frac{2}{9}$ **iii.** $\frac{2}{9}$

c. Yes

d. i. See the table at the bottom of the page.*

ii. No

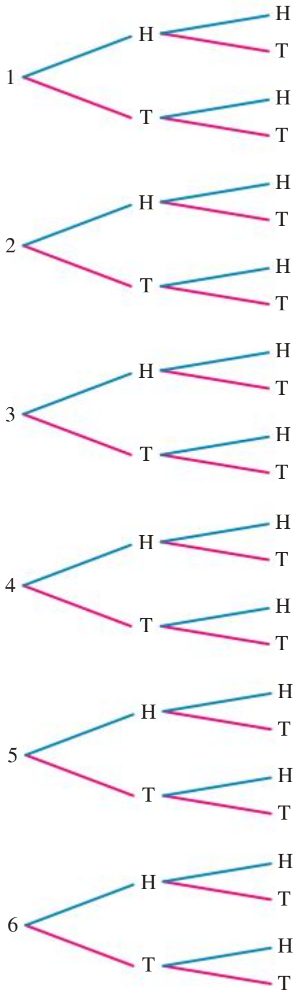
19. a.



6 routes

b. No. There are still six paths, even if we omit home from the journey. Since there is only one choice to go from the library to home, the outcomes do not change.

20.



7.4 Relative frequency and simulations

7.4 Exercise

1. a. Mandy's simulation: $\frac{13}{20}$

Sophia's simulation: $\frac{47}{100}$

b. Therefore, 0.47 is a better estimate than 0.65 for the true probability of a perfect weather day for sailing. Sophia's simulation is likely to be more reliable as there is a higher number of coin tosses.

---

*18. d. i.

		Person 2			
		Rock	**Paper**	**Scissors**	**Dynamite**
Person 1	**Rock**	RR	RP	RS	RD
	Paper	PR	PP	PS	PD
	Scissors	SR	SP	SS	SD
	Dynamite	DR	DP	DS	DD

2. a. $\frac{3}{50}$ **b.** 30 batteries

3. a. $\frac{13}{75}$ **b.** $\frac{31}{60}$ **c.** $\frac{29}{60}$

4. $\frac{17}{28}$

5. $\frac{41}{200}$

6. Answers will vary. Sample responses can be found in the worked solutions in the online resources. You can use =RANDBETWEEEN(1, 60) in Excel to generate a random number.

7. a. 35% **b.** 40%

8. a. $\text{P(fish and chips)} = \frac{45}{150} = 0.30$

b. $\text{P(fish and chips)} = \frac{31}{150} = 0.21$

c. $\text{P(macaroni and cheese)} = \frac{30}{150} = 0.20$

d. $\text{P(lamb souvlaki)} = \frac{25}{150} = 0.17$

e. $\text{P(BBQ pork ribs)} = \frac{19}{150} = 0.13$

9. Yes. It is too expensive to check every computer for every possible fault. A random number generator will allow the manufacturer to gain insight into the probability of a computer having a fault Assuming that a large enough sample of computers are tested.

10. a. i. $\frac{3}{11}$ **ii.** $\frac{1}{11}$ **iii.** $\frac{1}{11}$

b. Answers will vary. Sample responses can be found in the worked solutions in the online resources.

c. Answers will vary. Accuracy can be improved by using the random number generator to test more days. As more data is obtained, the results will become closer and closer to the theoretical probability.

11. Answers will vary. Sample responses can be found in the worked solutions in the online resources.

12. a. 2439 **b.** 361

13. a. 440 domestic students (using most recent data)

b. Based on the trend observed from 2005 to 2011, the number of students who never came back after the first year has steadily decreased from 11.0% to 8.8%. Hence, the estimate in part **a** may be too high.

14. a. i. 38% **ii.** 62%

b. No. If the experiment was repeated, it is unlikely that the results would be the same as or similar to the previous experiment.

c. According to the law of large numbers, as the number of trials increases, the gap between the experimental probability and the theoretical probability decreases.

15. a–c. Answers will vary. Sample responses can be found in the worked solutions in the online resources.

d. Yes. Since it is a random experiment, and only 50 trials are executed, everyone should have different results.

e. Approximately 33

16. a. i. 2 **ii.** 10 **iii.** 167

b. It is unlikely that you would achieve the theoretical probability during an experiment. However, the more times the experiment is repeated, the closer the results will be to theoretical probability. To obtain the theoretical probability consistently would require that the experiment was run an infinite amount of times.

17. a. Each driver could be assigned a number from 1 to 24. A random number generator could then be used to simulate the race. The number that is generated is the winner of the race.

b. The answer depends on each driver having an equally likely chance of winning the race. This is rarely the case.

18. Since the 8-sided dice has an equal chance on landing on any of the 8 options, that would mean in reality each athlete had an equal chance of winning the race. This is a limitation to using this simulation since not all athletes have an equal chance of winning the race.

19. a. $P(16) = \frac{1}{3}$

$P(17) = \frac{11}{24}$

$P(18) = \frac{5}{24}$

b. Answers will vary. Sample responses can be found in the worked solutions in the online resource.

c. Answers will vary. Sample responses can be found in the worked solutions in the online resource.

20. a. 100 times

b. There are not enough trials to decide whether the die is loaded, although the trend suggests that it might be. There is a large difference between the frequencies of 1 and 6. (The numbers 1 and 6 are on opposite sides of a standard die, so a weight on the 1 side would tend to make 6 show up more often.)

7.5 Review

7.5 Exercise

1. a. $\{1, 2, 3, 4, 5, 6, 7, 8\}$

b. $\{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$

c. {winning ticket, losing ticket}

2. a. Very unlikely **b.** Most likely

c. Very unlikely **d.** Very unlikely

3. Answers will vary. Sample responses can be found in the worked solutions in the online resources.

4. a.

×	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

Sample space is
$\{1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 30, 36\}$

b. 6 or 12

c. 1, 9, 16, 25 or 36

5. a.

–	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

Sample space is $\{0, 1, 2, 3, 4, 5\}$

b. 1

c. 5

6. a. $\frac{1}{6}$

b. The complement is the set $\{1, 2, 4, 5, 6\}$. or not rolling a 3.

c. $\frac{5}{6}$

7. a. $\frac{1}{6}$

b. $\frac{5}{6}$

c. No. The probability of the original event has only one successful outcome, the probability of the complement event has five successful outcomes, these probabilities would then have to be added.

8. 87%

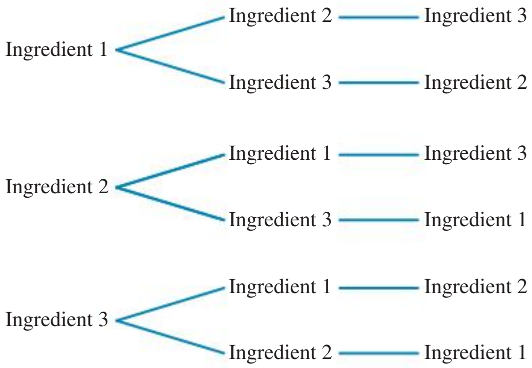
9. a. See the table at the bottom of the page.*

b. $\frac{7}{18}$

c. $\frac{11}{18}$

d. $\frac{2}{3}$

10. a.



b. 6

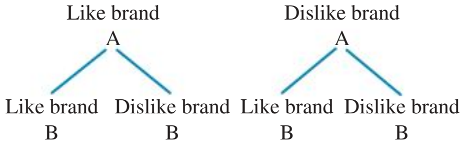
11. a. 12 **b.** $\frac{1}{3}$

12. a. See the table at the bottom of the page.*

b. $\frac{1}{16}$

c. $\frac{3}{4}$

13. a.



b. i. 25% **ii.** 25% **iii.** 50%

14. a. i. 1 **ii.** 25 **iii.** 833

b. No, these are estimated numbers calculated from the theoretical probabilities. As the experiment is carried out more times, the experimental probabilities will approach the theoretical probabilities. There is a slight chance in any experiment that you may obtain the theoretical probability, but it is unlikely.

***9. a.**

		Die 2					
		1	**2**	**3**	**4**	**5**	**6**
Die 1	**1**	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

***12. a.**

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
H	H1	H2	H3	H4	H5	H6	H7	H8
T	T1	T2	T3	T4	T5	T6	T7	T8

15. You could run your random number generator using the numbers 1–4. These numbers would represent A, B, C and D respectively in the multiple-choice questions. You would run this simulation 40 times.

16. a. $\frac{9}{50}$ **b.** $\frac{1}{100}$ **c.** $\frac{81}{100}$

17. a. $\frac{1}{1000}$ **b.** $\frac{999}{1000}$ **c.** $\frac{3}{10}$ **d.** $\frac{1}{720}$

18. 96.16%

19. a. Blue, green, pink

b. $\text{P(blue)} = \frac{17}{50}$

$\text{P(green)} = \frac{23}{50}$

$\text{P(pink)} = \frac{1}{5}$

c. Run the experiment more times (e.g. 10 000 times).

20. No. The theoretical probability of rolling a 3 in three rolls is only $\frac{91}{216}$.

UNIT

4 Graphs, data and loans

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UNIT 4 Graphs, data and loans
TOPIC 1 Bivariate graphs

8 Cartesian plane

LEARNING SEQUENCE

Fully worked solutions for this chapter are available online.

on Resources

Solutions	Solutions — Chapter 8
Digital documents	Learning matrix — Chapter 8 Quick quizzes — Chapter 8 Chapter summary — Chapter 8

LESSON
8.1 Overview

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8.1.1 Introduction

Understanding the Cartesian plane is fundamental in mathematics and has practical applications in our daily lives and various professions. By learning to identify and plot points on the Cartesian plane, you gain familiarity with Cartesian coordinates in two dimensions, which is crucial for fields such as engineering, computer science and architecture. Additionally, generating tables of values for linear functions helps you interpret and organise data, a skill valuable in business analytics, economics and scientific research.

Finally, graphing linear functions from tables of values, both manually and using graphing software, ensures you understand these concepts thoroughly and equips you with the tools needed in today's technology-driven world. These skills make you more adept at solving real-world problems and prepare you for various technical and analytical careers.

8.1.2 Syllabus links

Lesson	Lesson title	Syllabus links
8.2	**Plotting points on a Cartesian plane**	○ Demonstrate familiarity with Cartesian coordinates in two dimensions by identifying and plotting points on the Cartesian plane.
8.3	**Generating tables of values for linear functions**	○ Generate a table of values for a given linear function, including for negative values of x.
8.4	**Graphing linear functions**	○ Graph a linear function from a table of values with pencil and paper and with graphing software.

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LESSON
8.2 Plotting points on a Cartesian plane

SYLLABUS LINKS

- Demonstrate familiarity with Cartesian coordinates in two dimensions by identifying and plotting points on the Cartesian plane.

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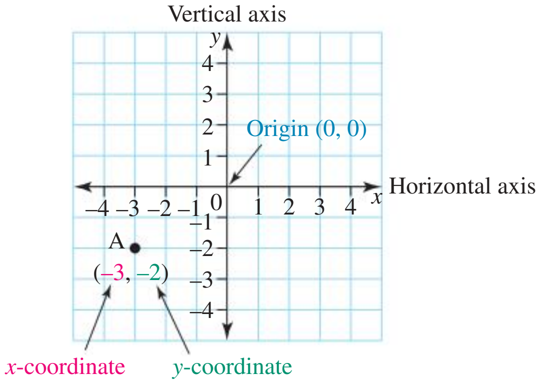
8.2.1 The Cartesian plane

- The **Cartesian plane** is created by drawing a horizontal number line (the x-axis) and a vertical number line (the y-axis).
- The point where the two axes meet is called the **origin**. The origin is at the point $(0, 0)$.
- The position of any point on the Cartesian plane can be located using **coordinates**.

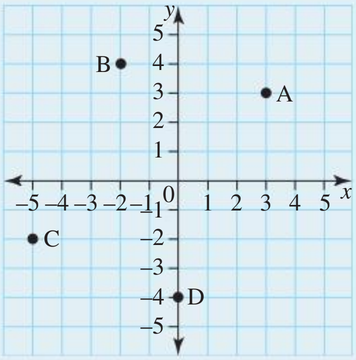


- When writing coordinates for a point, the value of the x-coordinate is written first and the value of the y-coordinate is written second.
- The x- and y-coordinates are separated by a comma and enclosed in round brackets.
- The x-coordinate describes how far across from the origin a point is located, and the y-coordinate describes how far up (or down) from the origin a point is located.
- Point A on the diagram, positioned at $x = -3$ and $y = -2$, can be described as the point $(-3, -2)$.

int-3831

WORKED EXAMPLE 1 Identifying points on the Cartesian plane

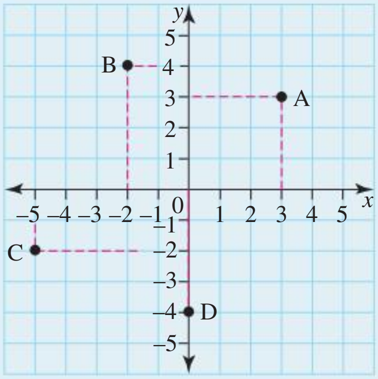
Identify the coordinates for the points labelled A, B, C and D on the Cartesian plane shown.



THINK

WRITE

1. The point marked A is above the 3 on the x-axis and level with the 3 on the y-axis.



Point A = (3, 3)

2. The point marked B is above the −2 on the x-axis and level with the 4 on the y-axis.

Point B = (−2, 4)

3. The point marked C is below the −5 on the x-axis and level with the −2 on the y-axis.

Point C = (−5, −2)

4. The point marked D is below the 0 on the x-axis and level with the −4 on the y-axis.

Point D = (0, −4)

int-4136

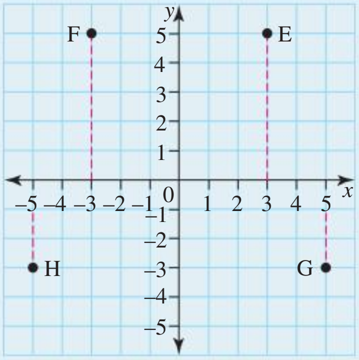
WORKED EXAMPLE 2 Plotting coordinates on the Cartesian plane

Plot the coordinates E = (3, 5), F = (−3, 5), G = (5, −3) and H = (−5, −3) on a single Cartesian plane.

THINK

WRITE

1. Draw a Cartesian plane that extends from −5 to 5 on both the x- and the y-axis.
2. Point E is at 3 on the x-axis and 5 on the y-axis. From the origin, move across 3 units to the right and up 5 units. Draw a point and label it with the letter E.
3. Point F is at −3 on the x-axis and 5 on the y-axis. From the origin, move across 3 units to the left and up 5 units. Draw a point and label it with the letter F.
4. Point G is at 5 on the x-axis and −3 on the y-axis. From the origin, move across 5 units to the right and down 3 units. Draw a point and label it with the letter G.
5. Point H is at −5 on the x-axis and −3 on the y-axis. From the origin, move across 5 units to the left and down 3 units. Draw a point and label it with the letter H.



ACTIVITY: Drawing pictures using the Cartesian plane

1. Draw a Cartesian plane from −10 to 10 on the x-axis and −10 to 15 on the y-axis. Plot and join the following points to uncover the picture.

START (6, 7) (7.5, 9) (5, 9) (4.5, 12) (2, 11) (0, 13) (–1.5, 10) (–5, 11) (–5, 8) (–8, 6) (–6, 4) (–8, 2) (–6, 1) (–7, –2) (–4, –1.5) (–4, –3.5) (–1.5, –3) (–2, –4) (–4, –7) (–5, –8) STOP

START (–2, –9) (–1, –7) (1, –8) (3, –8) (4, –7.5) (5, –10) STOP

START (4, –7.5) (3.5, –6) (3.5, –4) (4, –3) (5, –2.5) (5, –2) (4, –1.5) (4, –1) (5, 0.5) (7, 1) (8, 2) (8, 2.5) (6.5, 3) STOP

START (4, –2.5) (2, –3) (0.5, –3) (0, –2) (1, –1) (2, –0.5) (3, 0) (7, 1) STOP

START (6, 2.5) (6.5, 3) (6.5, 4) (6, 4) (4, 4) STOP

START (6.5, 4) (7, 5) (7, 6) (6, 7) (5, 7.5) (4, 7) (3, 6) (2, 6) (1, 6) (0, 5) (–1, 4) (0, 2) (1.5, 1.5) (3, 2) (4, 3) (4, 4) (4, 5) (3, 6) STOP

START (1, –1) (5, 0) STOP

DOTS AT (1, 3) AND (5, 5)

LINES (–1, 4) TO (–2, 4.5), (0, 5) TO (–0.5, 6), (1, 6) TO (0.5, 7), (2, 6) TO (2, 7), (4, 7) TO (3.5, 8), (5, 7.5) TO (5, 8.5), (6, 7) TO (6.5, 8), (6.5, 6.5) TO (7, 7)

2. a. On your own Cartesian plane, plot your own picture and then create a list of coordinates and instructions to match.
b. Ask another student to complete your picture on a blank Cartesian plane.

Exercise 8.2 Plotting points on a Cartesian plane

learnon

8.2 Quick quiz on

8.2 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14	N/A	N/A

These questions are even better in jacPLUS!

- Receive immediate feedback
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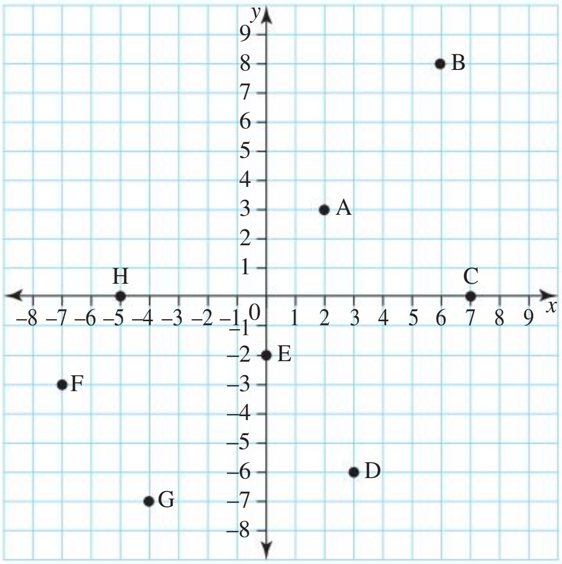
Simple familiar

1. **WE1** State the coordinates for the points labelled A to H on the Cartesian plane shown.



2. **WE2** Plot the following points on a single Cartesian plane.

a. A (1, 3)
b. B (2, 3)
c. C (2, 5)
d. D (5, 0)

3. Plot the following points on a Cartesian plane.

a. A (0, –5)
b. B (–2, –5)
c. C (–3, 0)
d. D (–4, 1)

4. Plot the following points on an appropriately scaled Cartesian plane.

a. A$\left(\frac{1}{2}, 3\right)$ **b.** B$\left(2\frac{1}{2}, -4\right)$ **c.** C$\left(-3\frac{1}{2}, 3\frac{1}{3}\right)$ **d.** D$\left(0, 2\frac{1}{2}\right)$

5. Plot the following points on an appropriately scaled Cartesian plane.

a. A$(0.8, 2.6)$ **b.** B$(2.5, -2.5)$ **c.** C$(-1.2, 3)$ **d.** D$(-1.6, -2.8)$

6. Determine the point that moves vertically up 3 units from the origin and then 5 units horizontally left on a Cartesian plane.

7. Determine the point that has moved -6 units on the x-axis and -7 units on the y-axis, from the point $(3, 5)$ on a Cartesian plane.

8. Describe how the point $(-1, 4)$ has moved from the origin on a Cartesian plane.

9. Determine the point that is described by 3 on the x-axis and -4 on the y-axis of a Cartesian pane.

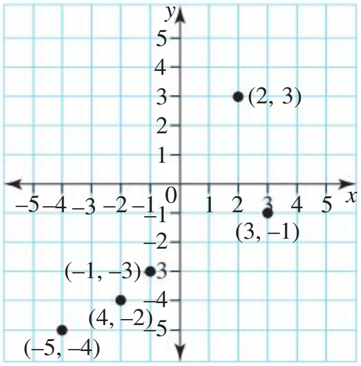
10. Determine where a point finishes if it:

- starts from the origin
- moves 5 units left
- moves 3 units down
- moves 3 units right
- moves 1 unit up.

11. Determine where a point finishes if it:

- starts at (2, –1)
- moves 4 units down
- moves 3 units right
- moves 2 units left
- moves 1 unit up.

12. Given the points plotted, correct the mistakes and explain what has been done incorrectly.



13. Your house is located 6 km north and 3 km west of the city centre.

a. Represent this on a Cartesian plane, assuming that the city centre is located at the origin.

b. Your friend's house is 10 km south and 1 km west of your house.
Represent your friend's house on the Cartesian plane.

c. Your school is 2 km north and 5 km east of your friend's house.
Represent your school on the Cartesian plane.

d. How far north/south and east/west is your school from your house?

14. You are given the following coordinates:
$(0, 0)$, $(5, 4)$, $(8, 4)$, $(14, 9)$, $(22, 0)$.

- **a.** Draw these points on a Cartesian plane.
- **b.** These points are used to describe a journey, which begins at your house. The y-axis represents distance in kilometres from your house and the x-axis represents the time taken for the journey in minutes. Make up a story that describes this journey in detail.

Fully worked solutions for this chapter are available online.

LESSON

8.3 Generating tables of values for linear functions

SYLLABUS LINKS

- Generate a table of values for a given linear function, including for negative values of x.

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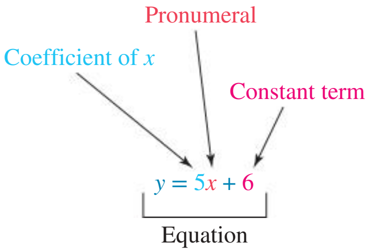
8.3.1 Algebra review

- Algebra is a type of language used in mathematics that consists of **pronumerals** (letters or groups of letters) that are used to represent unknown numbers.
- Pronumerals can also be used to describe **variables** (varying values). Some common words used in linear functions are shown in the diagram.
  - **Term:** A group of letters and/or numbers. Terms are separated by an addition or subtraction sign. For example, $5x$ is a term.
  - **Pronumeral:** The letter part of a term. For example, x is the pronumeral of the term $5x$.
  - **Coefficient:** The number multiplying (out the front) the pronumeral. For example, 5 is the coefficient of x in the term $5x$.
  - **Constant term:** The term that does not have a pronumeral. For example, 6 is the constant term as there is no letter attached to it.
  - **Equation:** A mathematical statement containing a left- and right-hand side separated by an equals sign. For example, $y = 5x + 6$ is an equation.



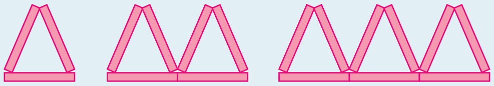
8.3.2 Patterns and rules

- A table of values can be used to investigate the relationship between two variables.

tlvd-12176

WORKED EXAMPLE 3 Using patterns to construct a rule

For the set of shapes shown:



a. **construct a table to show the relationship between the number of triangles in each figure and the number of matchsticks used to construct it**

b. **devise a rule in words that describes the pattern relating the number of triangles in each figure and the number of matchsticks used to construct it**

c. **use your rule to determine the number of matchsticks required to construct a figure made up of 20 such triangles.**

THINK

a. 1. Identify the two variables that are being investigated.

2. Three matches were needed for 1 triangle, 6 matches for 2 triangles and 9 matches for 3 triangles.
Construct a table of values displaying the relationship between the number of triangles and the number of matchsticks required.

b. 1. Look for a pattern between the number of triangles and number of matchsticks. Each triangle requires 3 matchsticks.

c. 1. Apply the rule 'the number of matchsticks equals 3 times the number of triangles' to determine the number of matchsticks required for 20 triangles.

WRITE

a. The variables are:
- the number of triangles in the shape
- the number of matchsticks used to construct the shape.

Number of triangles	1	2	3
Number of matchsticks	3	6	9

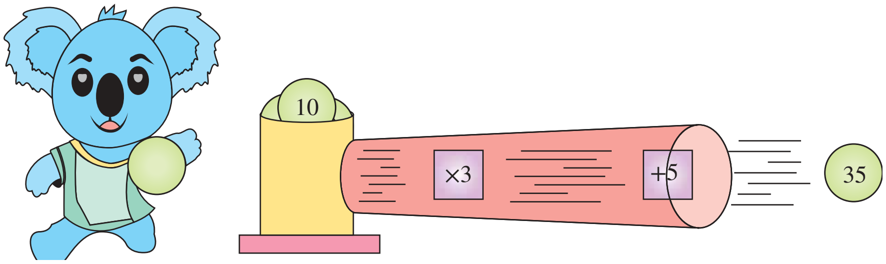
b. The number of matchsticks equals 3 times the number of triangles.

c. $20 \times 3 = 60$
To construct 20 triangles, 60 matchsticks would be required.

8.3.3 Input and output tables

int-4001

- A rule can be used to produce a set of output numbers from a set of input numbers.
- We can think of this as a machine that takes any input number and changes it according to a given rule to produce an output number.
- The rule can involve more than one step; for example, 'multiply each input number by 3 and then add 5'.



WORKED EXAMPLE 4 Using rules to calculate outputs

Complete the table using the given rules to calculate the correct output numbers.

Input	**4**	**10**	**38**	**144**
Output				

a. **Rule: Subtract 3 from each input number.**
b. **Rule: Divide each input number by 2.**

THINK	WRITE
a. 1. Take the first input number (4) and apply the rule; that is, subtract 3.	a. $4 - 3 = 1$
2. Apply the rule to each of the other input numbers.	$10 - 3 = 7$ $38 - 3 = 35$ $144 - 3 = 141$
3. Enter these output values in the table under their corresponding input value.	

Input	4	10	38	144
Output	1	7	35	141

THINK	WRITE
b. 1. Take the first input number (4) and apply the rule; that is, divide by 2.	b. $4 \div 2 = 2$
2. Apply the rule to each of the other input numbers.	$10 \div 2 = 5$ $38 \div 2 = 19$ $144 \div 2 = 72$
3. Enter these output values in the table under their corresponding input value.	

Input	4	10	38	144
Output	2	5	19	72

8.3.4 Substitution

int-4003

- A **formula** or **rule** that describes the relationship between the input and output values is often expressed as an equation.
- In Worked example 4a, the rule 'subtract 3 from each input number' could be written as the following equation.

$$\text{Output} = \text{input} - 3$$

- Pronumerals are usually used to represent the variables 'input' and 'output'.
- If x is used to represent the input values and y is used to represent the output values, then the equation

$$\text{Output} = \text{input} - 3$$

can be rewritten as follows.

$$y = x - 3$$

- When a variable in a formula is replaced by a number, we say that the number is **substituted** into the formula.

For example, if we wanted to determine the output number (y) for the rule $y = x - 3$ when the input number (x) is 2, we would substitute 2 into the formula for x as follows.

$$y = x - 3$$
$$y = 2 - 3$$
$$y = -1$$

When x is equal to 2, y is equal to -1.

WORKED EXAMPLE 5 Substitution

If $y = 5x - 4$, substitute the given value of x into the formula to find the value of y in each case.

a. $x = 2$

b. $x = 6$

THINK	WRITE
a. 1. Write the formula.	**a.** $y = 5x - 4$
2. Substitute 2 for x in the formula.	$y = 5 \times 2 - 4$
3. Evaluate the expression for y and write the answer.	$y = 10 - 4$ $y = 6$
b. 1. Write the formula.	**b.** $y = 5x - 4$
2. Substitute 6 for x in the formula.	$y = 5 \times 6 - 4$
3. Evaluate the expression for y and write the answer.	$y = 30 - 4$ $y = 26$

- The equation or expression will not always be given to you. You may need to create the equation from the worded problem.

WORKED EXAMPLE 6 Creating expressions

If y represents any number, create expressions for:

a. **3 times the number**

b. **7 less than the number**

c. **12 more than the number.**

THINK	WRITE
a. 1. Think about which operation is used.	**a.** 'Times' means multiplication.
2. When multiplying we don't show the multiplication sign. Write the number first, then the pronumeral.	$3 \times y = 3y$
b. 1. Think about which operation is used.	**b.** 'Less than' means subtraction.
2. Since less than, take 7 away from y.	$y - 7$
c. 1. Think about which operation is used.	**c.** 'More than' means addition.
2. Since more than, add 12 to y.	$y + 12$

- The pronumeral may not be specified. In this case, you will need to choose an appropriate pronumeral that relates well to the question.
- For example, you may use C to represent cost.

8.3.5 Creating a table of values

- A table of values can be created by substituting multiple input values into an equation and determining the output value that corresponds to each input value.

WORKED EXAMPLE 7 Generating a table of values

Generate a table of values for the rule $y = 7 - 4x$ for x-values of $-2, -1, 0, 1, 2, 3$.

THINK

WRITE

1. Draw a table with a row for the x values and a row for the y-values. Write the given x-values in the first row.

x	−2	−1	0	1	2	3
y						

2. Substitute the first x-value, −2, into the rule for x and solve for y.

$$\begin{aligned} y &= 7 - 4x \\ &= 7 - 4 \times -2 \\ &= 7 + 8 \end{aligned}$$

x	−2	−1	0	1	2	3
y	−15					

3. Determine the y-value when $x = -1$.

When $x = -1$:

$$\begin{aligned} y &= 7 - 4x \\ &= 7 - 4 \times -1 \\ &= 11 \end{aligned}$$

4. Substitute the remaining x-values into the rule and determine their corresponding y-values.

When $x = 0$:

$$\begin{aligned} y &= 7 - 4x \\ &= 7 - 4 \times 0 \\ &= 7 \end{aligned}$$

When $x = 1$:

$$\begin{aligned} y &= 7 - 4x \\ &= 7 - 4 \times 1 \\ &= 7 - 4 \\ &= 3 \end{aligned}$$

When $x = 2$:

$$\begin{aligned} y &= 7 - 4x \\ &= 7 - 4 \times 2 \\ &= 7 - 8 \\ &= -1 \end{aligned}$$

When $x = 3$:

$$\begin{aligned} y &= 7 - 4x \\ &= 7 - 4 \times 3 \\ &= 7 - 12 \\ &= -5 \end{aligned}$$

5. Write the y-values in the table underneath their corresponding x-values.

x	−2	−1	0	1	2	3
y	15	11	7	3	−1	−5

Exercise 8.3 Generating tables of values for linear functions

8.3 Quick quiz on

8.3 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16	N/A	N/A

These questions are even better in jacPLUS!

- Receive immediate feedback
- Access sample responses
- Track results and progress

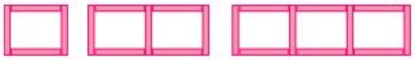
Find all this and MORE in jacPLUS

Simple familiar

1. **WE3** For each of the sets of shapes shown:

- **i.** construct a table to show the relationship between the number of shapes in each figure and the number of matchsticks used to construct it
- **ii.** devise a rule in words that describes the pattern relating the number of shapes in each figure and the number of matchsticks used to construct it
- **iii.** use your rule to determine the number of matchsticks required to construct a figure made up of 20 such shapes.

a.



b.

2. For each of the sets of shapes:

- **i.** construct a table to show the relationship between the number of shapes in each figure and the number of matchsticks used to construct it
- **ii.** devise a rule in words that describes the pattern relating the number of shapes in each figure and the number of matchsticks used to construct it.

a.

b.

3. **WE4** Copy and complete the following tables using the rule given in each case to calculate the correct output numbers.

a. Subtract 4 from each input number.

Input	4	5	6	10	14
Output					

b. Add 12 to each input number.

Input	1	2	6	13	107
Output					

c. Multiply each input number by itself.

Input	0	1	5	8	10
Output					

d. Multiply each input number by itself, then add 4.

Input	0	1	4	7	12
Output					

4. Copy and complete the following tables. Use the rule given in each case to calculate the correct output numbers.

a. Multiply each input number by 2.

Input	1	10	51	60	144
Output					

b. Divide each input number by 3.

Input	3	12	21	66	141
Output					

c. Add 2 to each input number.

Input	1	3	27			
Output				4	55	193

d. Add 20 to each input number.

Input	3	25	56			
Output				20	94	1773

5. **WE5** If $y = 2x + 7$, substitute the given value of x into the formula to find the value of y in each case.

a. $x = 3$

b. $x = 6$

6. If $y = -3x - 2$, substitute the given value of x into the formula to find the value of y in each case.

a. $x = 1$

b. $x = -3$

7. **WE6** If Y represents any number, write expressions for:

a. 7 times the number
b. 13 less than the number
c. 5 more than the number.

8. If M represents any number, write expressions for:

a. 3 times the number
b. 7 less than the number
c. 12 more than the number.

9. If a is the number of lollies in a jar, write the algebraic expression that describes the number of lollies left in the jar in the following situations.

a. Eight lollies were eaten.
b. Half of the lollies were eaten.
c. Twenty-three lollies were taken, 18 were eaten and the rest were put back in the jar.
d. Thirty-three lollies were eaten.
e. State the assumption you have made with your answer in part d.

In questions 10 and 11, D represents an unknown number.

10. Write an expression for one less than the number.

11. Write an expression for three more than double the number.

12. **WE7** Generate a table of values for each of the following rules given the specified x-values.

a. $y = -x + 3$ for $x = -4, -2, 0, 2$

b. $y = -4x + 18$ for $x = -8, -4, 0, 4$

13. Complete the tables given the following rules.

a. $y = 5x - 4$

x	0	1	2	3
y				

b. $y = -2x + 16$

x	3	6	9	12
y				

14. Complete the tables given the following rules.

a. $y = 4x + 10$

x	−4	−2	0	2
y				

b. $y = -15x + 35$

x	−8	−4	0	4
y				

15. Complete the tables given the following rules.

a. $y = -\frac{x}{2} + 4$

x	−4	−2	0	2
y				

b. $y = \frac{x}{2} - 2$

x	−8	−4	0	4
y				

16. Generate a table of values for each of the following rules given the specified x-values.

a. $y = -\frac{2x}{5} - 1$ for $x = -10, -5, 0, 5$

b. $y = -\frac{x}{3} + 3$ for $x = -6, -3, 0, 9$

Fully worked solutions for this chapter are available online.

LESSON
8.4 Graphing linear functions

SYLLABUS LINKS

- Graph a linear function from a table of values with pencil and paper and with graphing software.

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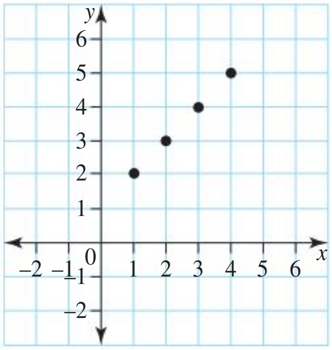
8.4.1 Plotting linear graphs

int-4465

- A set of points may be such that the x- and y-coordinates of each point are connected by the same rule.
- For example, each of the points $(1, 2)$, $(2, 3)$, $(3, 4)$, $(4, 5)$ has the same relationship (rule) between their x- and y-values; namely, the y-coordinate of every point is one more than its x-coordinate.
- If the set of points where x- and y-coordinates are connected by a certain rule is plotted on the Cartesian plane, the points will form a pattern.
- If the pattern formed by the set of points is a straight line, we refer to it as a linear pattern. For example, the points $(1, 2)$, $(2, 3)$, $(3, 4)$, $(4, 5)$, when plotted, will form a linear pattern as shown.
- A set of points with their coordinates can be presented in the form of a table. For example, the points $(-2, 4)$, $(-1, 2)$, $(0, 0)$, $(1, -2)$, $(2, -4)$ can be presented as shown.



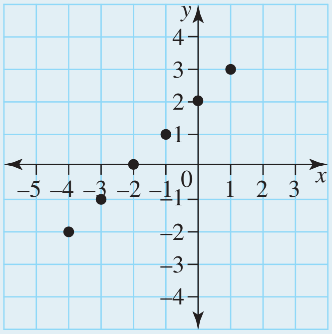
x	−2	−1	0	1	2
y	4	2	0	−2	−4

WORKED EXAMPLE 8 Plotting points on a Cartesian plane

a. Plot the points in the following table on a Cartesian plane.

x	−4	−3	−2	−1	0	1
y	−2	−1	0	1	2	3

b. Determine whether the points form a linear graph. If they do, state the next point in the pattern.

THINK	WRITE
a. 1. Look at the x- and y-values of the points and draw a Cartesian plane. **2.** Plot each point.	**a.** 
b. 1. Look at the position of the points and answer the question.	**b.** Yes, the points form a linear graph.
2. Study the pattern and answer the question. The pattern shows that the x-values increase by 1, and the y-values increase by 1.	The next point in the pattern is (2, 4).

- To plot a linear graph with the equation given, follow these steps.
  1. Create a table of values first.
     - Draw a table.
     - Select some x-values.
     - Substitute the selected x-values into the rule to find corresponding y-values.
  2. Draw a Cartesian plane.
  3. Plot the points from the table and join them with the straight line.
  4. Label the graph.

WORKED EXAMPLE 9 Completing a table of values and plotting them on a Cartesian plane

Draw a table of values, plot the graph of $y = 2x + 1$ and label the line.

THINK	WRITE
1. Write the rule.	$y = 2x + 1$
2. Draw a table and choose simple x-values.	

x	−2	−1	0	1	2
y					

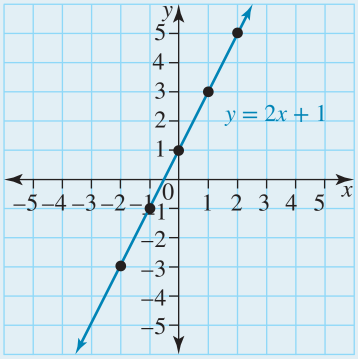
3. Use the rule to find each y-value and enter them in the table.

When $x = -2$, $y = 2 \times -2 + 1 = -3$.
When $x = -1$, $y = 2 \times -1 + 1 = -1$.
When $x = 0$, $y = 2 \times 0 + 1 = 1$.
When $x = 1$, $y = 2 \times 1 + 1 = 3$.
When $x = 2$, $y = 2 \times 2 + 1 = 5$.

x	−2	−1	0	1	2
y	−3	−1	1	3	5

4. Draw a Cartesian plane and plot the points.

5. Join the points to form a straight line and label the graph.



- We only need to know the coordinates of two points that lie on the line to sketch a linear graph.
- The two points can be obtained by substituting any two x-values into the rule to obtain the corresponding y-values.
- Alternatively, there are two other convenient techniques used to sketch straight lines: (a) the intercept method and (b) the gradient and y-intercept method. These are discussed below.

8.4.2 Sketching linear graphs from intercepts

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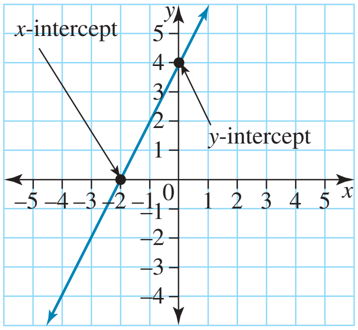
int-3840

- The intercept method involves finding the coordinates of both the x- and the y-intercepts, and plotting them.
- At the x-intercept, $y = 0$. Therefore, to obtain the value of the x-intercept, substitute $y = 0$ into the equation and solve the equation for x.
- At the y-intercept, $x = 0$. Therefore, to obtain the value of the y-intercept, substitute $x = 0$ into the equation and evaluate.
- To sketch the graph, plot both intercepts on the Cartesian axes and join them with the straight line. Label the line.



WORKED EXAMPLE 10 Sketching linear graphs from intercepts

For the linear function $y = 2x + 4$:

a. determine the x- and y-intercepts

b. sketch the graph of the function.

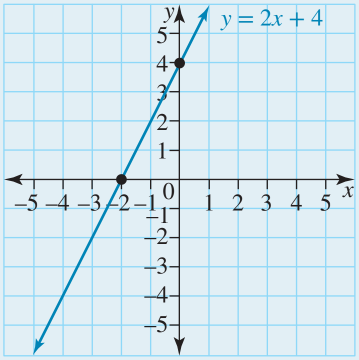
THINK	WRITE
a. 1. The y-intercept is found by substituting $x = 0$ into the equation and solving for y.	**a.** $y = 2 \times 0 + 4$ $= 0 + 4$ $= 4$ y-intercept: $(0, 4)$

2. The x-intercept is found by substituting $y = 0$ into the equation and solving for x. Subtract 4 from both sides of the equation. Divide both sides by 2.

$0 = 2x + 4$
$2x = -4$
$x = -2$
x-intercept: $(-2, 0)$

b. 1. Plot the x- and y-intercepts on a suitably scaled Cartesian plane. Join the two intercepts with a straight line, extending the line in both directions. Write the equation of the line.

b.



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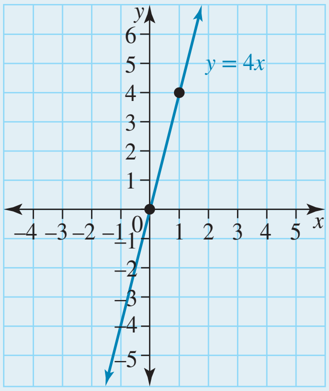
WORKED EXAMPLE 11 Sketching linear graphs using two points

Sketch the graph given by the equation $y = 4x$.

THINK	WRITE/DRAW
1. The y-intercept is found by substituting $x = 0$ into the equation and solving for y.	$y = 4x$ $y = 4 \times 0$ $y = 0$ The y-intercept is the point $(0, 0)$.
2. The x-intercept is found by substituting $y = 0$ into the equation and solving for x.	$y = 4x$ $0 = 4x$ $0 = x$ The x-intercept is the point $(0, 0)$.
3. As both intercepts are at the origin, we have only one unique coordinate so far. Choose any other x-value, say $x = 1$, and substitute this into the original equation to find a second point.	$y = 4x$ $= 4 \times 1$ $= 4$ Another point is $(1, 4)$.
4. Plot these two points on a suitably scaled Cartesian plane. Join the two points with a straight line, extending the line in both directions. Write the equation of the line.	

8.4.3 The slope

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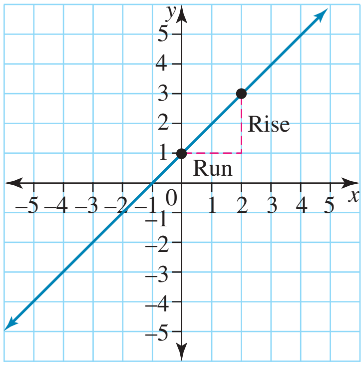
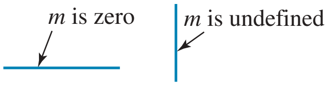


- **Slope (gradient)** gives a measure of steepness. For example, the slope of a road going up the hill would inform us of the steepness of the slope of the hill.
- The slope of a straight line shows the change in vertical height as the horizontal distance increases by one unit.
- The slope of a straight line is denoted by the letter *m* and is given by the formula: $m = \frac{\text{rise}}{\text{run}}$, where *rise* is the vertical distance between any two points on the line and *run* is the horizontal distance between the same two points.

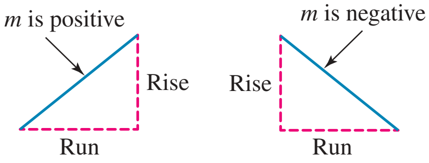
Slope

$$\textbf{Slope } (m) = \frac{\textbf{rise}}{\textbf{run}}$$

- To find the slope of a straight line on a Cartesian plane, draw a right-angled triangle anywhere along the line as shown in the diagram and use it to measure the rise and run.
- The value of the slope is given by the ratio $\frac{\text{rise}}{\text{run}}$. For example, in the diagram, rise = 2 and run = 2, so the gradient $m = \frac{2}{2} = 1$.
- The slope of a straight line is constant; that is, it will be the same when measured anywhere along the line.
- Since the slope is the measure of steepness, the greater the value of the gradient, the steeper the line.
- If the line is horizontal, there is no slope; hence, the value of the slope is zero. If the line is vertical, we say that its slope is **infinite** or **undefined**.



- If the line slopes upwards from left to right (that is, it *rises*), the slope is positive. If it slopes downwards from left to right (that is, it *falls*), the slope is negative.

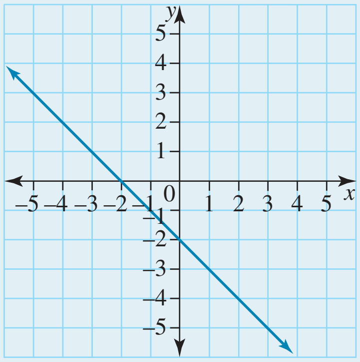


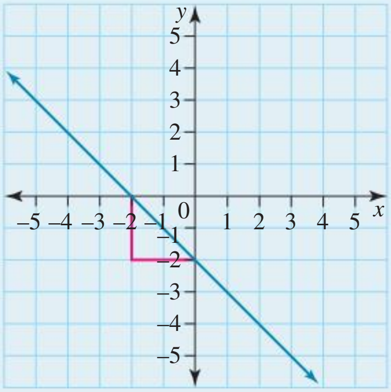
WORKED EXAMPLE 12 Determining the slope and *y*-intercept from a linear graph

Determine:

a. the slope, *m*

b. the *y*-intercept, *c*, of the linear graph shown.



THINK	**WRITE**
a. 1. Choose two convenient points on the line and draw a triangle to find the rise and the run.	**a.**  rise = −2, run = 2
2. Calculate the slope. In this case, the rise is negative since the line slopes down from left to right.	Slope : $m = \frac{\text{rise}}{\text{run}} = \frac{-2}{2}$ $= -1$
b. The *y*-intercept is where the graph crosses the *y*-axis, and *c* is the *y*-value at this point.	**b.** $c = -2$ *y*-intercept: (0, −2)

8.4.4 Sketching linear graphs using the slope and *y*-intercept

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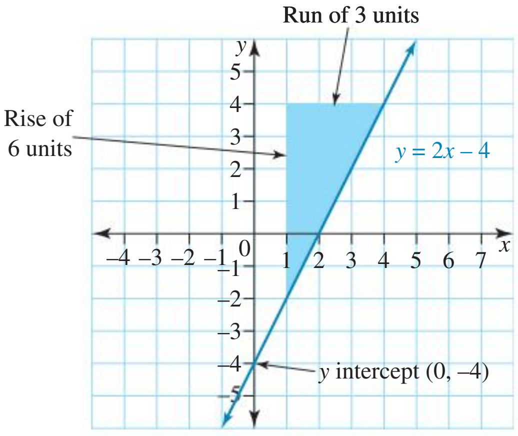
- The graph of the linear equation $y = 2x - 4$ is shown.
- The slope of the line is 2.

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{6}{3}$$
$$= 2$$



- The *y*-coordinate of the *y*-intercept is −4.
- The equation of a straight line contains the values of both the slope and the *y*-coordinate of the *y*-intercept.

$$y = 2x - 4$$

gradient: coefficient of the *x* term

y-intercept: the constant

Slope–intercept form

When an equation is written in the form

$$y = mx + c$$

m is the slope (gradient) and c is the y-intercept.

- When constructing linear graphs using the gradient and y-intercept, follow these steps.
  1. Mark the y-intercept on the y-axis.
  2. Another point is found by using the gradient. From the y-intercept, move horizontally by the run value, then vertically by the rise value. For example, for a gradient of 3, the run is 1 and the 'rise' is 3. If the gradient is −3, the rise will be down by 3.
  3. Connect the two points with a straight line.
  4. Remember to show arrows on the ends of the line and to name the graph.

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WORKED EXAMPLE 13 Sketching linear graphs using the gradient and y-intercept

For each of the linear functions below:

i. state the gradient and y-intercept

ii. sketch the graph of the function.

a. $y = 4x - 11$ **b.** $y = -4x$

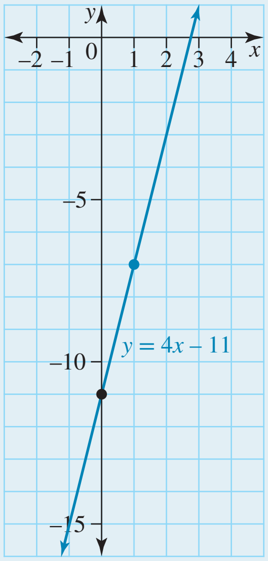
THINK

a. i. Determine the value of the gradient and the coordinates of the y-intercept. The gradient is given by the coefficient of x and the y-coordinate of the y-intercept is given by the constant in the equation.

ii.
- Construct a set of axes and mark the position of the y-intercept.
- The y-intercept is −11, as shown in black.
- The gradient is 4, so $\frac{\text{rise}}{\text{run}} = \frac{4}{1}$.
- From the y-intercept, move 4 units up and 1 unit to the right, then mark a second point, as shown in pink.
- The two points can now be connected with a straight line.
- Write the equation next to the line.

WRITE

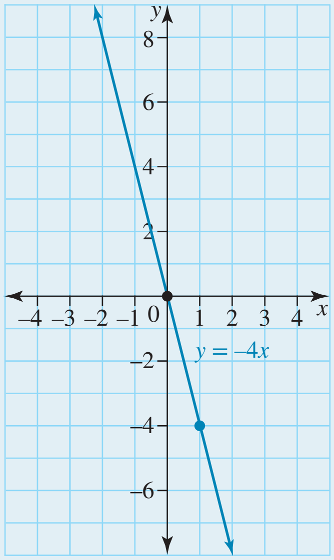
a. For $y = 4x - 11$, gradient = 4 and y-intercept = −11.



b. i. Determine the value of the gradient and the coordinates of the y-intercept.
The gradient is given by the coefficient of x and the y-coordinate of the y-intercept is given by the constant in the equation.

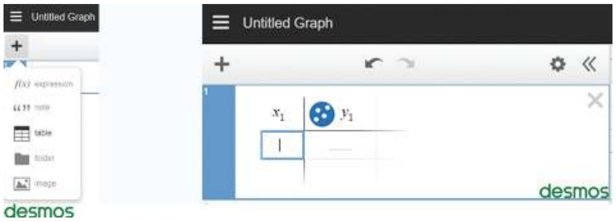
b. For $y = -4x$ or $(y = -4x + 0)$, gradient $= -4$ and y-intercept $= 0$.

ii.
- Construct a set of axes.
- Mark in the position of the y-intercept at 0, as shown in black. The gradient is −4, so $\frac{\text{rise}}{\text{run}} = \frac{-4}{1}$.
- From the y-intercept, move 4 units down and 1 unit to the right, then mark a second point, as shown in pink.
- The two points can now be connected with a straight line to form the graph.
- Write the equation next to the line.

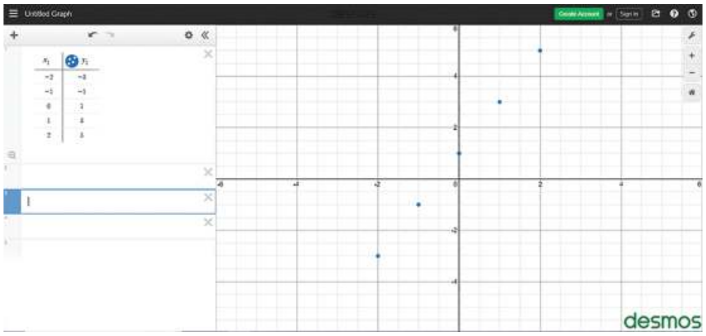


8.4.5 Sketching linear graphs using technology

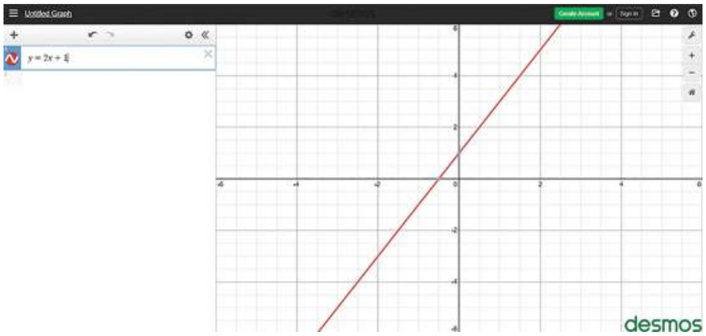
- Although it is important to understand the processes behind sketching a straight line using a pencil and paper method, it is also vital to engage in the use of graphing software packages to produce graphs.
- There are many freely available graphing software packages on the internet. Some common ones are Desmos, GeoGebra and Excel. You can also use a graphics calculator.
- In Worked example 9, we are asked to plot the graph of $y = 2x + 1$ by constructing a table of values.
- Using a graphing software package such as Desmos, we could answer this question in two ways:
  (a) by plotting all the points from the table
  (b) by graphing the line without the points.
- For option (a), after we have constructed the table of values for the rule, we can input the table of values directly into the software using the ‘table’ feature.



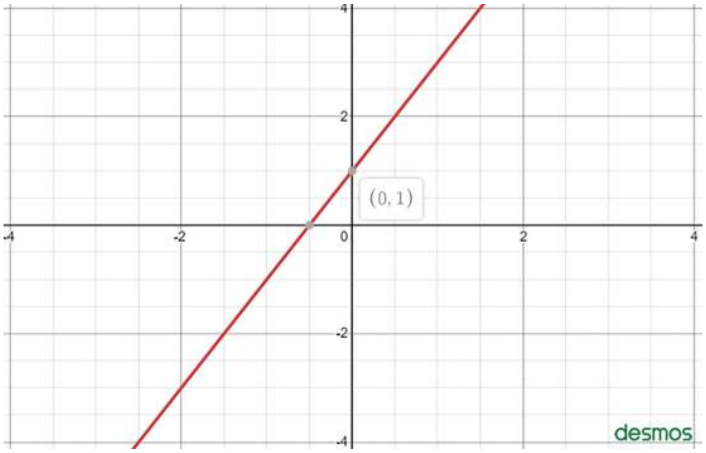
- Input the calculated values into the table. (Take note that these tables are set out slightly differently to those you have seen in this section.) The software will automatically plot the points.



- For option (b), we could simply type $y = 2x + 1$ into the program and it will produce the graph.


$y = 2x + 1$

- One handy feature of this program is that once the line is produced, a dot is placed where the graph crosses the *x*- and *y*-axes.
- You can hover the cursor over these dots to display the coordinates of the *x*- and *y*-intercepts.



ACTIVITY: Graphing linear functions with graphing software

Experiment with using various graphing software to graph linear functions. Desmos is used in the example above, but you could also use GeoGebra, Excel, a graphics calculator or other software.

1. Create a table of values.
2. Use graphing software to determine if the relationship in your table of values forms a linear (straight) graph.
3. Determine what happens when you change your *x* and *y* variables. What happens to the slope?
4. Determine how you know which coordinate gives you the *y*-intercept.

Exercise 8.4 Graphing linear functions

8.4 Quick quiz

8.4 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16	N/A	N/A

Simple familiar

1. **WE8** For each of the following: i) Plot the following points on a Cartesian plane. ii) Determine whether the points form a linear graph. If they do, state the next point in the pattern.

a. $(-3, -3)\ (-2, -1)\ (-1, 1)\ (0, 3)\ (1, 5)\ (2, 7)\ (3, 9)$

b. $(-3, -5)\ (-2, -3)\ (-1, 0)\ (0, 1)\ (1, 4)\ (2, 5)\ (3, 7)$

c.

x	-2	-1	0	1	2
y	3	-1	-2	-3	-4

d.

x	-2	-1	0	1	2
y	-6	-3	0	3	6

2. **WE9** For each of the following rules: i) Complete the following tables of values, ii) plot the points on a Cartesian plane, and join them to make a linear graph. Label the graphs with the rules.

a. Rule: $y = x + 3$

x	-2	-1	0	1	2
y	1	2	3		

b. Rule: $y = x - 5$

x	-2	-1	0	1	2
y		-6			-3

c. Rule: $y = 5x$

x	-2	-1	0	1	2
y	-10		0		

d. Rule: $y = 2x + 4$

x	-2	-1	0	1	2
y	0			6	

e. Rule: $y = -3x + 2$

x	-2	-1	0	1	2
y		5			-4

3. **WE10** For each of the linear functions below:

i. determine the x- and y-intercepts
ii. sketch the graph of the function.

a. $y = 3x + 6$ **b.** $y = 3x - 9$ **c.** $y = 3x + 12$

4. For each of the straight-line graphs below:

i. determine the x- and y-intercepts
ii. sketch the graph of the function.

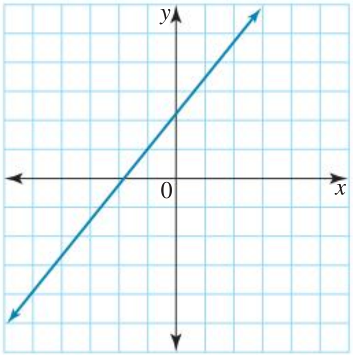
a. $y = 4x + 2$ **b.** $y = 2x - 5$ **c.** $y = -2x + 1$

5. **WE11** Sketch the graphs given by the following linear equations on a single Cartesian plane.

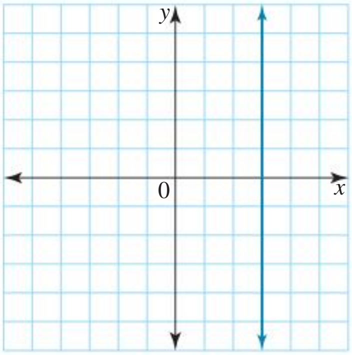
a. $y = x$ **b.** $y = 3x$ **c.** $y = -3x$

6. State whether each of the following lines has a positive, negative, zero or undefined slope.

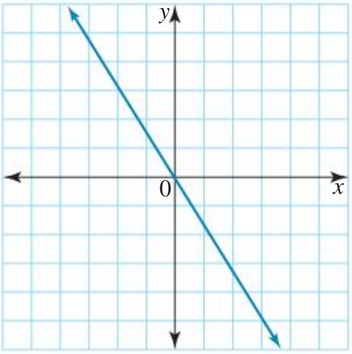
a.



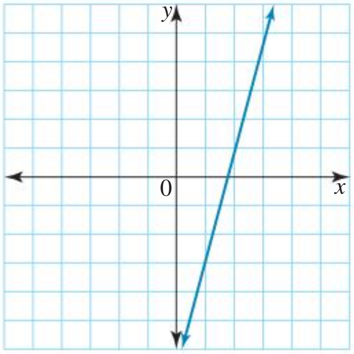
b.



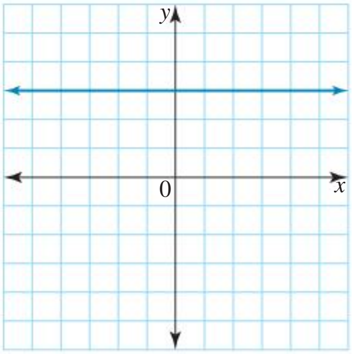
c.



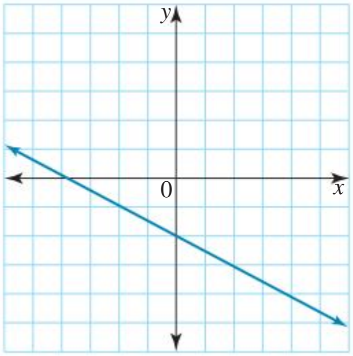
d.



e.



f.

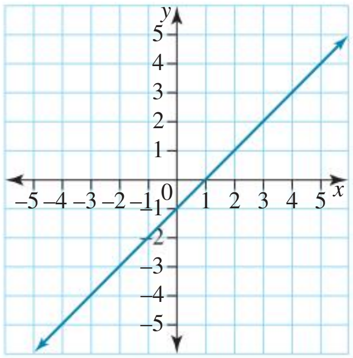


7. **WE12** For each of the following linear graphs, determine

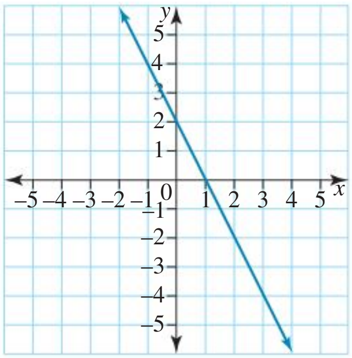
i. the slope, m

ii. the y-intercept, c.

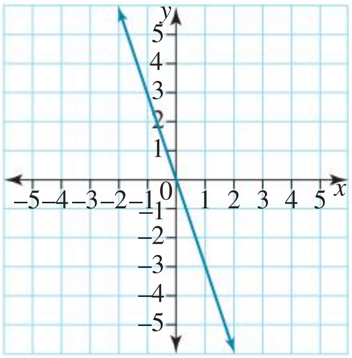
a.



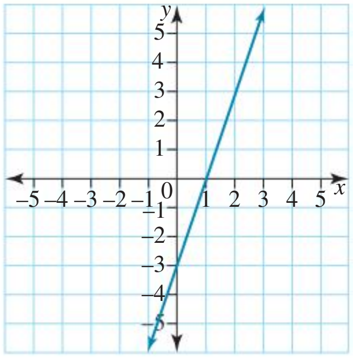
b.



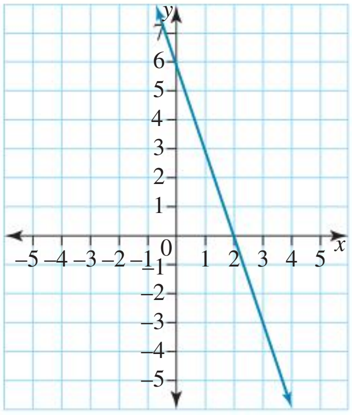
c.



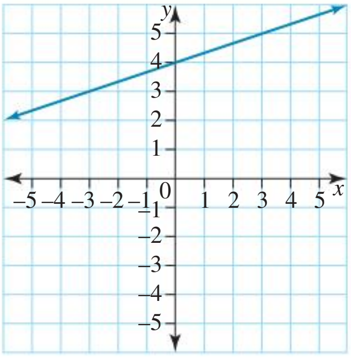
d.



e.



f.



8. **WE13** For each of the linear functions below:

i. state the slope and the y-intercept
ii. sketch the graph of the function.

a. $y = 2x + 4$
b. $y = 2x + 6$
c. $y = 2x - 6$
d. $y = x - 6$
e. $y = x + 4$
f. $y = 4x - 8$

9. For each of the linear functions below:

i. state the slope and the y-intercept
ii. sketch the straight line graph.

a. $y = 2x + 5$
b. $y = -4x - 3$
c. $y = -3x + 5$
d. $y = \frac{1}{2}x - 3$
e. $y = \frac{-3}{2}x + 3$
f. $y = \frac{4}{5}x$

10. Using the slope and y-intercept, sketch the following linear graphs on the same set of axes.

Slope	y-intercept
3	2
4	−3
−2	0

11. a. Sketch the graph of the linear function with a slope of $\frac{1}{2}$ and y-intercept of 4.
b. State the equation of this straight line graph.

12. a. Sketch the graph of the linear function with a slope of $\frac{-2}{3}$ and a y-intercept of -5.
b. State the equation of this straight line graph.

13. Sketch the following linear equations on the same set of axes.

a. $y = 2x + 12$
b. $y = -\frac{1}{2}x + 6$
c. $y = x - 1.5$

14. a. Sketch the linear equation $y = -\frac{5}{7}x - \frac{3}{4}$:

i. using the y-intercept and the slope
ii. using the x- and y-intercepts.

b. Compare and contrast the methods and generate a list of advantages and disadvantages for each method. Identify which method you think is best, and explain why.

15. a. Sketch each of the following straight lines on the same set of axes.

i. $y = \frac{5}{2}x + 5$
ii. $y = -\frac{5}{2}x + 5$

b. Determine the x-intercepts for these straight lines.
c. Identify the shape that is formed by the three axis intercepts.
d. Find the area enclosed by the axis intercepts.

16. a. Using an appropriate method, sketch each of the following straight lines on the same set of axes.

i. $y = 4x + 4$
ii. $y = 4x - 6$
iii. $x + y = 4$
iv. $x + y + 1 = 0$

b. Identify the shape that is formed by these four lines.
c. Write down the coordinates of the vertices (corners) of this shape.

Fully worked solutions for this chapter are available online.

LESSON
8.5 Review

8.5.1 Summary

Hey students! Now that it's time to revise this chapter, go online to:

- Access the chapter summary
- Review your results
- Watch teacher-led videos
- Practise questions with immediate feedback

Find all this and MORE in jacPLUS ▶

8.5 Exercise

learnon

8.5 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20	N/A	N/A

These questions are even better in jacPLUS!
- Receive immediate feedback
- Access sample responses
- Track results and progress

Find all this and MORE in jacPLUS ▶

Simple familiar

1. Describe the shift the point (7, −1) has undergone from the origin of a Cartesian plane.
2. State the point described by 4 on the y-axis and −3 on the x-axis.
3. Determine the point that has moved 3 units left and 2 unit up from the point (1, 1).
4. Complete the following table given the rule $y = 5x - 4$.

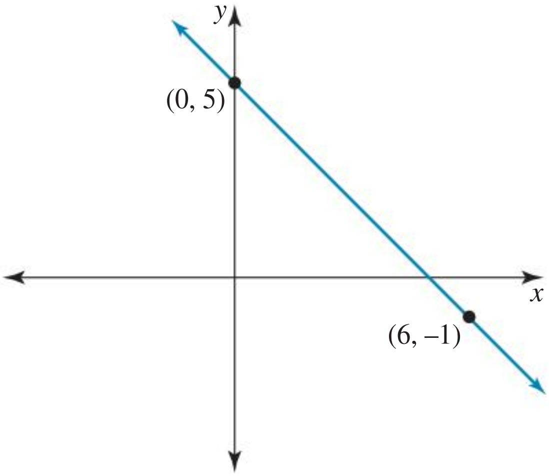
x	−2	0	2	5
y				

5. Complete the following table given the rule $y = -3x + 15$.

x	−4	−1	2	6
y				

6. For the rule $y = 3x - 1$, determine the value of y when $x = 2$.
7. Determine the slope of the linear rule $y = 4 - 6x$.
8. Calculate the x-intercept of the graph with the rule $2y - x + 6 = 0$.
9. Calculate the y-intercept of the graph with the rule $2y - x + 6 = 0$.

10. Determine the slope of the line shown.



11. Determine the rule for a line with slope -4 and y-intercept $= 8$.

12. If the point $(3, m)$ lies on the line $2y - x = 15$, determine the value of m.

13. For each of the following linear graphs, determine:

- **i.** the slope
- **ii.** the y-intercept.

a. $y = 8x - 3$
b. $y = 5 - 9x$
c. $2x + y - 6 = 0$
d. $4x - 2y = 0$
e. $y = \frac{2x - 1}{3}$

14. Determine the linear equation on which the table of values shown is modelled.

x	0	2	4	10
y	4	10	16	34

15. For the following equations, use the slope–intercept method to sketch linear graphs.

a. $y = -x + 5$
b. $y = 4x - 2.5$
c. $y = \frac{2}{3}x - 1$
d. $y = 3 - \frac{5}{4}x$

16. For the following equations, use the x- and y-intercept method to sketch linear graphs.

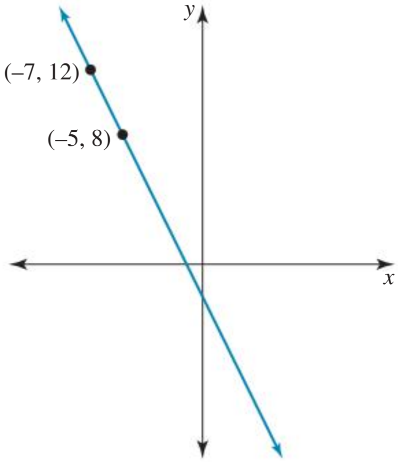
a. $y = -6x + 25$
b. $y = 20x + 45$
c. $2y + x = -5$
d. $4y + x - 2.5 = 0$

17. If $y = 2x + 1$, and the point $(n, 7)$ lies on the line, determine the value of n.

18. Complete the table given the following rule. $3x - 2y - 2 = 0$

x	0	2	4	8
y				

19. The graph shows a straight line that passes through the points $(-7,\ 12)$ and $(-5,\ 8)$.



Determine the coordinates of the point where the line crosses the x-axis.

20. Louise owes her friend Sula \$400 and agrees to pay her back \$15 per week.

- **a.** State a linear rule that demonstrates this reducing debt schedule and sketch the graph.
- **b.** Determine how many weeks it takes her to repay the debt.
- **c.** Determine how much she owes after 15 weeks.
- **d.** Determine after how many repayments Louise will have \$85 of debt remaining.

Fully worked solutions for this chapter are available online.

Answers

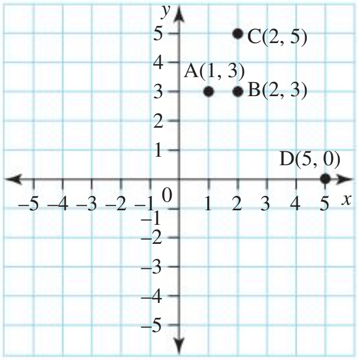
8 Cartesian plane

8.2 Plotting points on a Cartesian plane

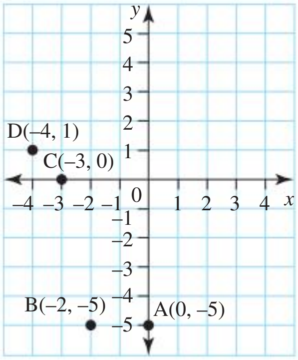
8.2 Exercise

1. a. $(2, 3)$ **b.** $(6, 8)$ **c.** $(7, 0)$
d. $(3, -6)$ **e.** $(0, -2)$ **f.** $(-7, -3)$
g. $(-4, -7)$ **h.** $(-5, 0)$

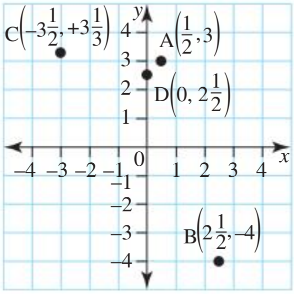
2.



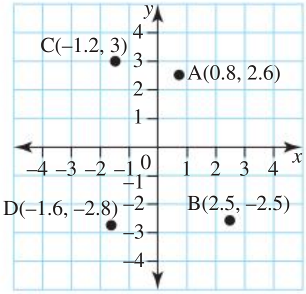
3.



4.



5.



6. $(-5, 3)$

7. $(-3, -2)$

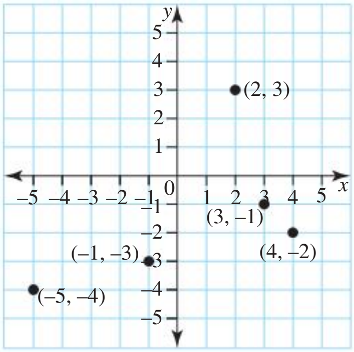
8. -1 means it has moved -1 unit on the x-axis (left).
4 means it has moved 4 units on the y-axis (up).

9. $(3, -4)$

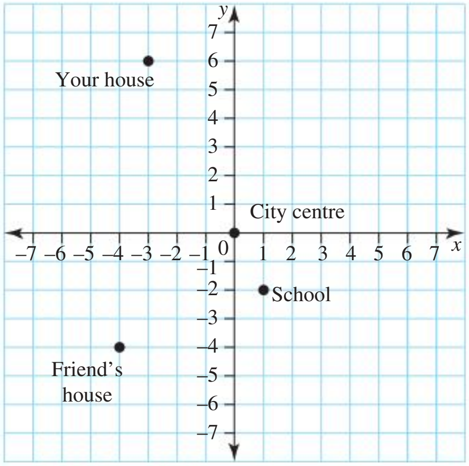
10. $(-2, -2)$

11. $(3, -4)$

12. The point $(4, -2)$ is in the wrong position; it is in the $(-2, -4)$ position.
Also, the point $(-5, -4)$ is in the wrong position; it is in the $(-4, -5)$ position.
The correct plot is:

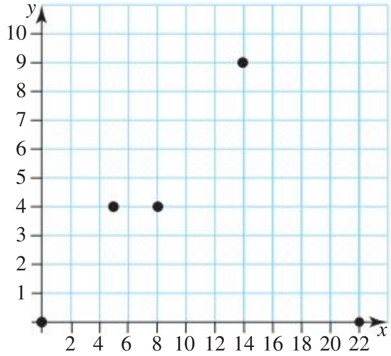


13. a, b, c.



d. Your school is 4 km east and 8 km south from your house.

14. a.



b. Sample responses can be found in the Worked solutions in the online resources.

8.3 Generating tables of values for linear functions

8.3 Exercise

1. a. i.

Number of squares	1	2	3
Number of matchsticks	4	7	10

ii. The number of matchsticks equals three times the number of squares and then add one.

iii. 61

b. i.

Number of triangles	1	2	3
Number of matchsticks	3	5	7

ii. The number of matchsticks equals two times the number of triangles and then add one.

iii. 41

2. a. i.

Number of shapes	1	2	3
Number of matchsticks	6	11	16

ii. The number of matchsticks is equal to five times the number of shapes and then add one.

b. i.

Number of shapes	1	2	3
Number of matchsticks	4	7	10

ii. The number of matchsticks equals three times the number of shapes and then add one.

3. a.

Input	4	5	6	10	14
Output	0	1	2	6	10

b.

Input	1	2	6	13	107
Output	13	14	18	25	119

c.

Input	0	1	5	8	10
Output	0	1	25	64	100

d.

Input	0	1	4	7	12
Output	4	5	20	53	148

4. a.

Input	1	10	51	60	144
Output	2	20	102	120	288

b.

Input	3	12	21	66	141
Output	1	4	7	22	47

c.

Input	1	3	27	2	53	191
Output	3	5	29	4	55	193

d.

Input	3	25	56	0	74	1753
Output	23	45	76	20	94	1773

5. a. 13 **b.** 19

6. a. -5 **b.** 7

7. a. $7Y$ **b.** $Y-13$ **c.** $Y+5$

8. a. $3M$ **b.** $M-7$ **c.** $M+12$

9. a. $a-8$

b. $\frac{1}{2}\times a=\frac{a}{2}$

c. $a-18$

d. $a-33$

e. That there were at least 33 lollies in the jar and the lollies that were eaten have only come from the jar we are referring to.

10. $D-1$

11. $2D+3$

12. a.

x	-4	-2	0	2
y	7	5	3	1

b.

x	-8	-4	0	4
y	50	34	18	2

13. a.

x	0	1	2	3
y	-4	1	6	11

b.

x	3	6	9	12
y	10	4	-2	-8

14. a.

x	-4	-2	0	2
y	-6	2	10	18

b.

x	-8	-4	0	4
y	155	95	35	-25

15. a.

x	-4	-2	0	2
y	6	5	4	3

b.

x	-8	-4	0	4
y	-6	-4	-2	0

16. a.

x	-10	-5	0	5
y	3	1	-1	-3

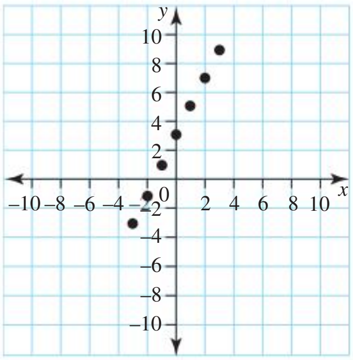
b.

x	-6	-3	0	9
y	5	4	3	0

8.4 Graphing linear functions

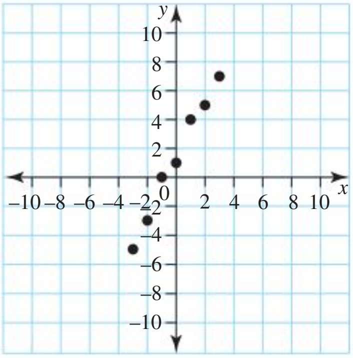
8.4 Exercise

1. a.



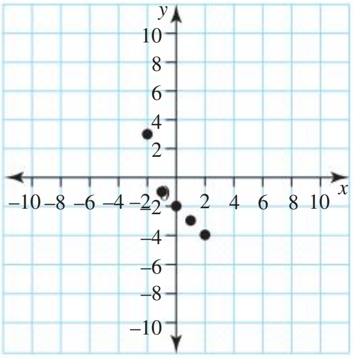
Linear, (4, 11)

b.



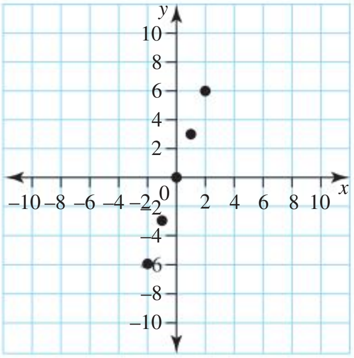
Not linear

c.



Not linear

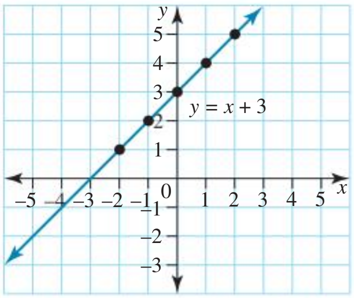
d.



Linear, (3, 9)

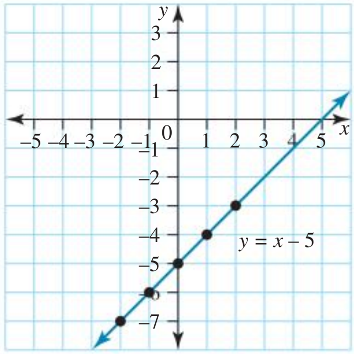
2. a.

x	−2	−1	0	1	2
y	1	2	3	4	5



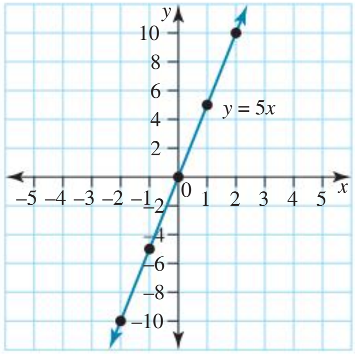
b.

x	−2	−1	0	1	2
y	−7	−6	−5	−4	−3



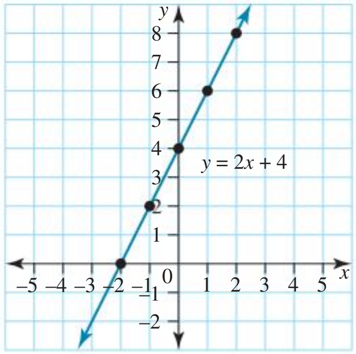
c.

x	−2	−1	0	1	2
y	−10	−5	0	5	10



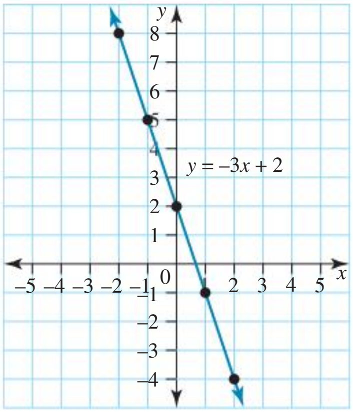
d.

x	−2	−1	0	1	2
y	0	2	4	6	8

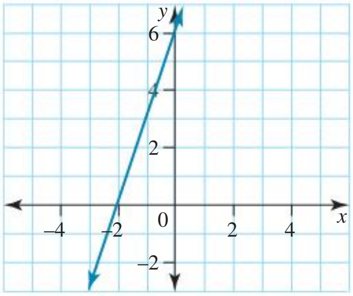


e.

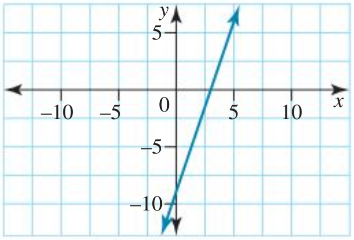
x	-2	-1	0	1	2
y	8	5	2	-1	-4



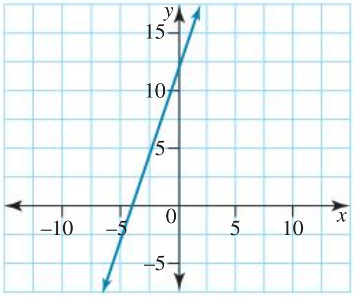
3. a. x-intercept: $(-2, 0)$
y-intercept: $(0, 6)$



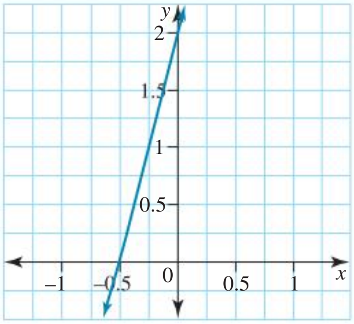
b. x-intercept: $(3, 0)$
y-intercept: $(0, -9)$



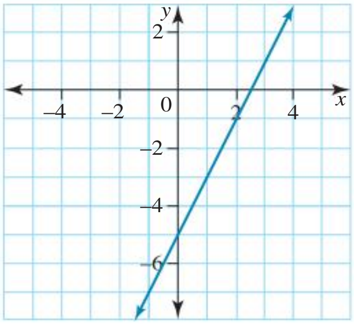
c. x-intercept: $(-4, 0)$
y-intercept: $(0, 12)$



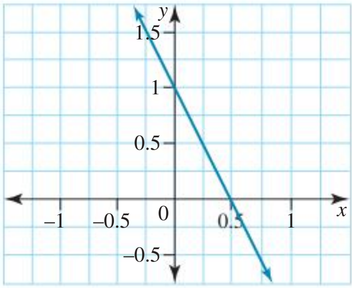
4. a. x-intercept: $(-0.5, 0)$
y-intercept: $(0, 2)$



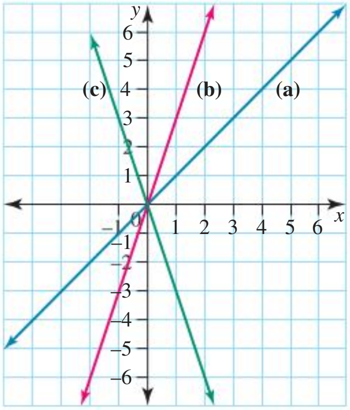
b. x-intercept: $(2.5, 0)$
y-intercept: $(0, -5)$



c. x-intercept: $(0.5, 0)$
y-intercept: $(0, 1)$



5.



6. a. Positive **b.** Undefined **c.** Negative
d. Positive **e.** Zero **f.** Negative

7. a. i. $m = 1$ **ii.** $c = -1$

b. i. $m = -2$ **ii.** $c = 2$

c. i. $m = -3$ **ii.** $c = 0$

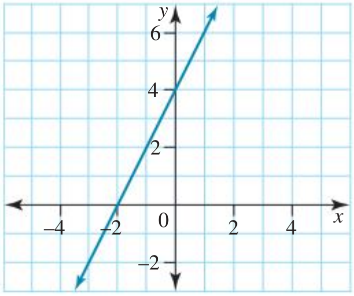
d. i. $m = 3$ **ii.** $c = -3$

e. i. $m = -3$ **ii.** $c = 6$

f. i. $m = \frac{1}{3}$ **ii.** $c = 4$

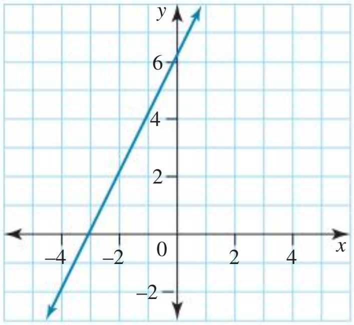
8. a. i. Slope $= 2$; y-intercept $= (0, 4)$

ii.



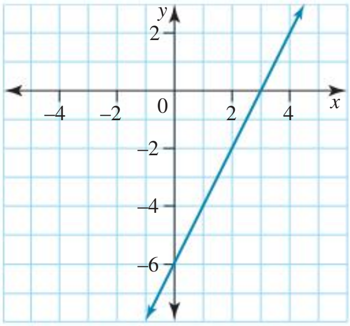
b. i. Slope $= 2$; y-intercept $= (0, 6)$

ii.



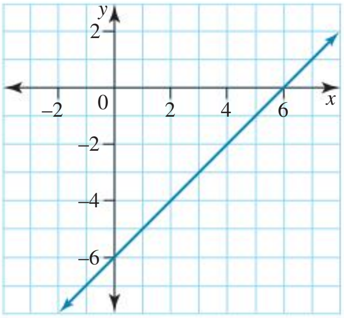
c. i. Slope $= 2$; y-intercept $= (0, -6)$

ii.



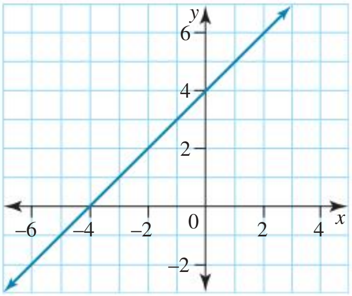
d. i. Slope $= 1$; y-intercept $= (0, -6)$

ii.



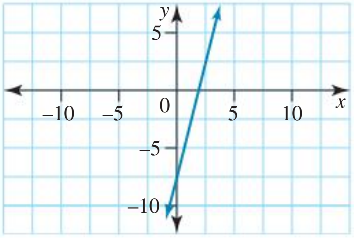
e. i. Slope $= 1$; y-intercept $= (0, 4)$

ii.



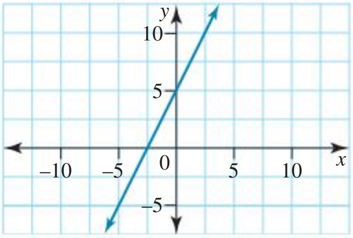
f. i. Slope $= 4$; y-intercept $= (0, -8)$

ii.



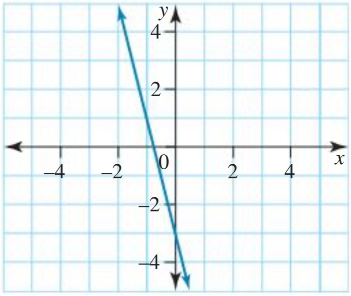
9. a. i. Slope $= 2$; y-intercept $= (0, 5)$

ii.



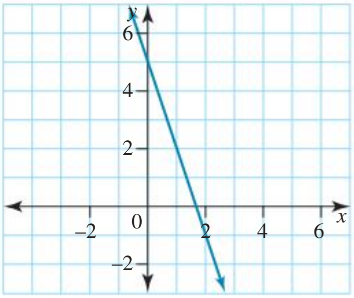
b. i. Slope $= -4$; y-intercept $= (0, -3)$

ii.



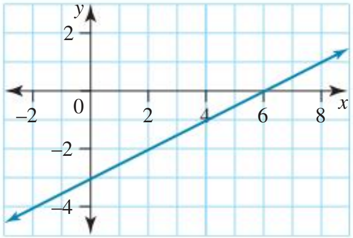
c. i. Slope $= -3$; y-intercept $= (0, 5)$

ii.



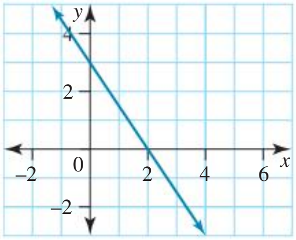
d. **i.** Slope $= \frac{1}{2}$; y-intercept $= (0, -3)$

ii.



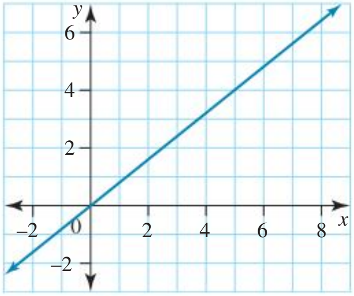
e. **i.** Slope $= \frac{-3}{2}$; y-intercept $= (0,\ 3)$

ii.

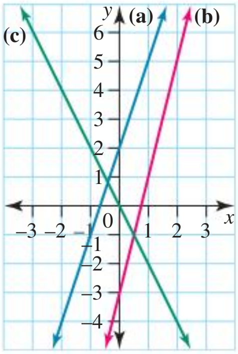


f. **i.** Slope $= \frac{4}{5}$; y-intercept $= (0,\ 0)$

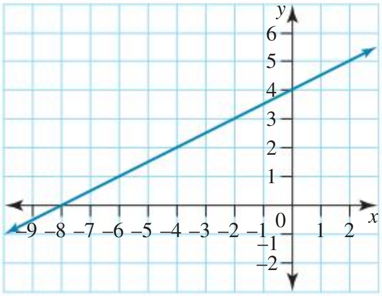
ii.



10.

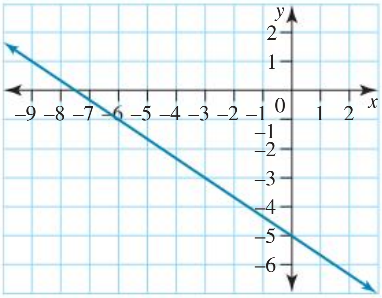


11. **a.**



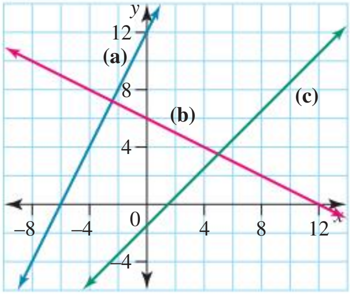
b. $y = \frac{1}{2}x + 4$

12. **a.**

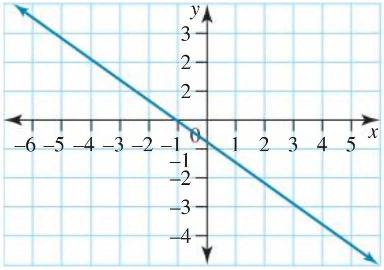


b. $y = \frac{-2}{3}x - 5$

13.

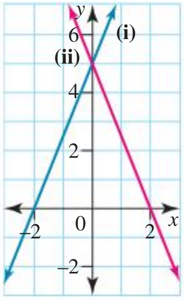


14. **a.**



b. Sample responses can be found in the Worked solutions in the online resources.

15. **a.**



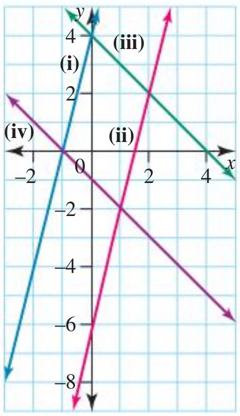
b. **i.** $(-2, 0)$

ii. $(2, 0)$

c. Isosceles triangle

d. 10 units^2

16. a.



b. Parallelogram

c. $(-1, 0), (0, 4), (2, 2)$ and $(1, -2)$

8.5 Review

8.5 Exercise

1. $+7$ units on the x-axis.
-1 unit on the y-axis.

2. $(-3, 4)$

3. $(-2, 3)$

4.

x	−2	0	2	5
y	−14	−4	6	21

5.

x	−4	−1	2	6
y	27	18	9	−3

6. 5

7. $m = -6$

8. $x = 6$

9. $y = -3$

10. $m = -1$

11. $y = -4x + 8$

12. $m = 9$

13. a. $m = 8$ and y-intercept $= -3$

b. $m = -9$ and y-intercept $= 5$

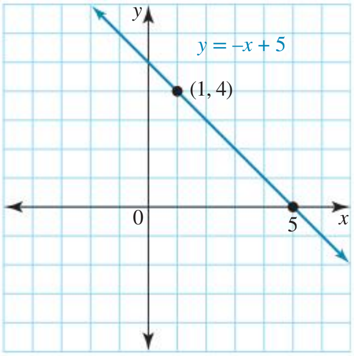
c. $m = -2$ and y-intercept $= 6$

d. $m = 2$ and y-intercept $= 0$

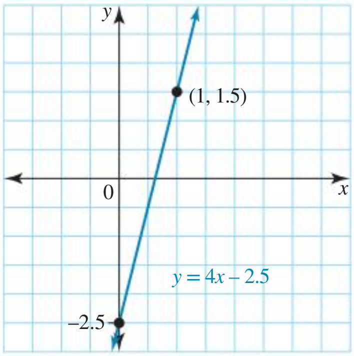
e. $m = \frac{2}{3}$ and y-intercept $= -\frac{1}{3}$

14. $y = 3x + 4$

15. a.



b.



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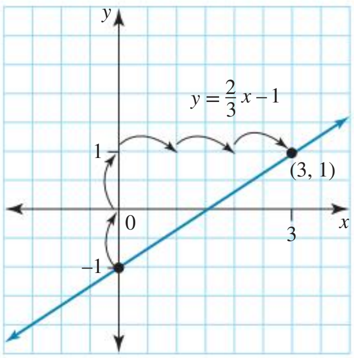

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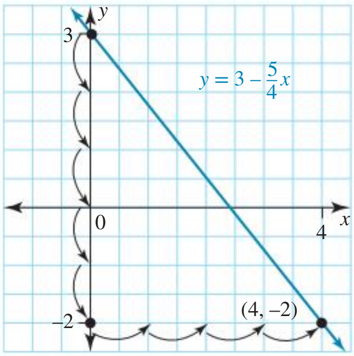
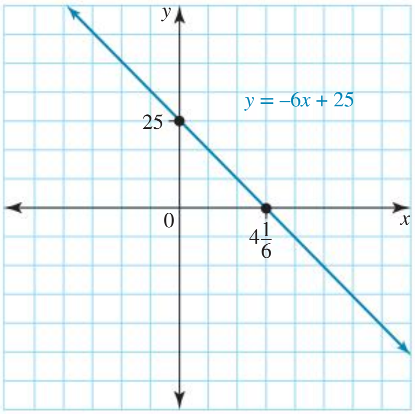


16. a.



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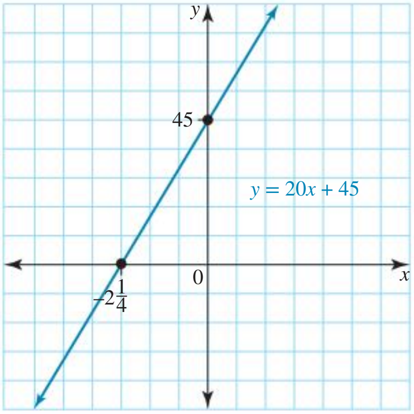
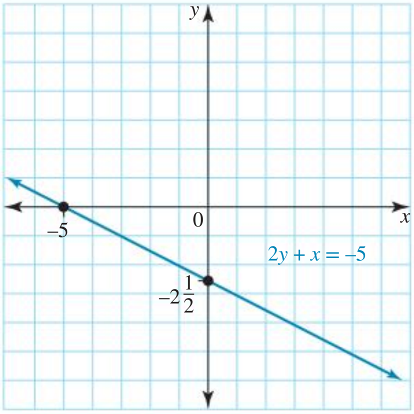
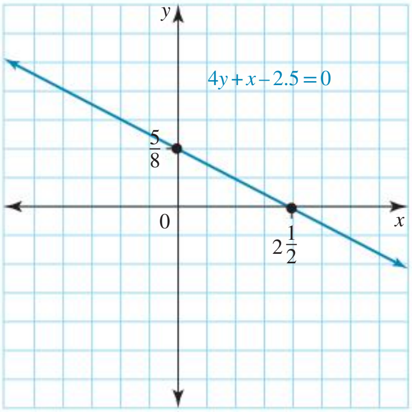


c.



d.



17. $n = 3$

18.

x	0	2	4	8
y	-1	2	5	11

19. $(-1, 0)$

20. a. y: the amount owed

x: the number of weeks $y = 400 - 15x$

b. $x = 26.67$ weeks

$x \approx 27$ weeks

c. $y = \$175$

d. $x = 21$ weeks

UNIT 4 Graphs, data and loans
TOPIC 1 Bivariate graphs

9

Bivariate scatterplots and lines of best fit

LESSON SEQUENCE

Fully worked solutions for this chapter are available online.

on Resources

Solutions	Solutions — Chapter 9
Digital documents	Learning matrix — Chapter 9 Quick quizzes — Chapter 9 Chapter summary — Chapter 9

LESSON
9.1 Overview

9.1.1 Introduction

Understanding scatterplots and the line of best fit is essential for anyone looking to delve into data analysis, a skill increasingly relevant in our data-driven world. Constructing scatterplots from given data sets allows us to visually interpret relationships between two variables, revealing patterns that can inform decision-making in fields such as economics, health sciences and environmental studies. Recognizing the direction, form and strength of associations between variables helps in predicting outcomes and identifying trends, critical for making informed choices.

In real-life contexts, these skills are invaluable. For example, in business, they can inform marketing strategies by identifying the correlation between advertising spend and sales revenue. In health, they can help establish the effectiveness of a new treatment by analyzing patient recovery data.

9.1.2 Syllabus links

Lesson	Lesson title	Syllabus links
9.2	**Interpreting scatterplots**	○ Construct a scatterplot using a given data set. ○ Describe the patterns and features of bivariate data. ○ Describe the association between two numerical variables in terms of direction (positive/negative), form (linear/non-linear) and strength (strong/moderate/weak). ○ Identify dependent and independent variables.
9.3	**The line of best fit**	○ Identify dependent and independent variables. ○ Draw a line of best fit by eye. ○ Use technology to determine the equation of the line of best fit in the form $y = mx + c$ where m is slope (gradient) and c is y-intercept [complex]. ○ Interpret the effect of the parameters *m* and *c* from the equation of the line of best fit in the form $y = mx + c$ [complex].
9.4	**Making predictions using a line of best fit [complex]**	○ Use the line of best fit to make predictions, both by interpolation and extrapolation [complex]. ○ Recognise the dangers of extrapolation [complex].
9.5	**Causality and correlation [complex]**	○ Use technology to calculate the correlation coefficient (an indicator of the strength of linear association) [complex]. ○ Distinguish between causality and correlation through examples [complex].

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LESSON
9.2 Interpreting scatterplots

SYLLABUS LINKS

- Construct a scatterplot using a given data set.
- Describe the patterns and features of bivariate data.
- Describe the association between two numerical variables in terms of direction (positive/negative), form (linear/non-linear) and strength (strong/moderate/weak).
- Identify dependent and independent variables.

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9.2.1 Bivariate data

- **Bivariate** data involves analysing two variables to determine their relationship. For example, you might look at how hours of study affect exam scores.
- This type of data helps in creating scatterplots to visualise the relationship and calculate the correlation coefficient to quantify the strength of their association.

Independent and dependent variables

- When examining the association between a pair of variables, we commonly have an **independent variable** and a **dependent variable**.
- The dependent variable, as the name suggests, is the one whose value depends on the other variable.
- The independent variable takes on values that do *not* depend on the value of the other variable.

Determining variables

To identify the dependent variable you will need to determine, generally speaking, which variable depends on the other.

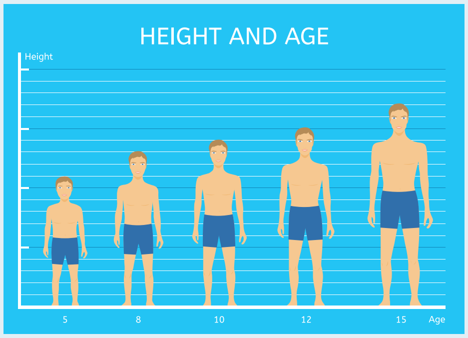
WORKED EXAMPLE 1 Identifying dependent and independent variables

For each of the following pairs of variables:

i. state which variable would be classified as the dependent variable

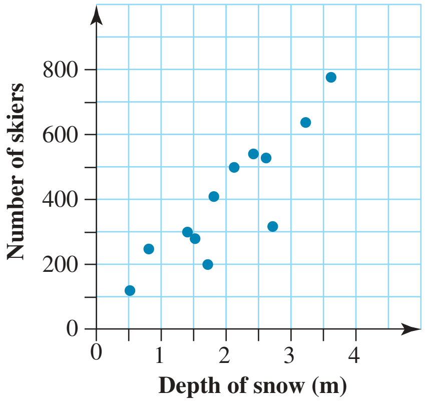
ii. state what is expected to happen to the value of the dependent variable when there is an increase in the value of the independent variable

- **a. Height and age**
- **b. Temperature and elevation**
- **c. Blood alcohol level and reaction time**
- **d. IQ and results on an academic test**
- **e. Money earned and hours worked**
- **f. Value of a car and its age**
- **g. Time taken to travel a given distance and speed of travel.**



THINK	WRITE
a. Typically, children grow taller as they get older, so a child's height depends on their age.	**a.** **i.** The dependent variable is height. **ii.** As age increases, height is expected to increase.
b. Typically, the temperature grows colder the higher above sea level the elevation, so temperature depends on elevation.	**b.** **i.** The dependent variable is temperature. **ii.** As the elevation increases, the temperature is expected to decrease.
c. Typically, a person's reaction time becomes slower as they increase their alcohol consumption, so reaction time depends on blood alcohol level.	**c.** **i.** The dependent variable is reaction time. **ii.** As blood alcohol level increases, reaction time is expected to increase.
d. Typically, a person with a higher IQ will score a higher result on an academic test, so their result depends on their IQ.	**d.** **i.** The dependent variable is the result on an academic test. **ii.** As IQ increases, results on an academic test are expected to increase.
e. Typically, a person earns more money when they work more hours, so money earned depends on hours worked.	**e.** **i.** The dependent variable is money earned. **ii.** As the number of hours worked increases, the amount of money earned is expected to increase.
f. Typically, a car is worth less the older it gets, so the value of a car depends on the age of the car.	**f.** **i.** The dependent variable is the value of a car. **ii.** As the age of a car increases, its value is expected to decrease.
g. Typically, a journey will take less time travelling at a faster speed, so travel time depends on speed.	**g.** **i.** The dependent variable is travel time. **ii.** As the speed increases, the travel time is expected to decrease.

9.2.2 Scatterplots



- Bivariate data can be arranged on a **scatterplot**.
- Each of the data points is represented by a single visible point on the graph.
- When drawing a scatterplot, it is important to choose the correct variable to assign to each of the axes.
- The convention is to place the independent variable on the x-axis and the dependent variable on the y-axis.
- In the example given, the 'Number of skiers' would be considered the dependent variable because it is likely to change depending on depth of snow.
- As 'Number of skiers' is the dependent variable, we graph it on the y-axis and the 'Depth of snow' on the x-axis.
- Notice how the scatterplot for the ski resort data shows a general upward trend. It is not a perfectly straight line, but it is still clear that a general trend or association has formed: as the depth of snow increases, so too does the number of skiers.

Constructing a scatterplot

When constructing a scatterplot, it is important to place the independent variable along the x-axis and the dependent variable along the y-axis.

WORKED EXAMPLE 2 Sketching scatterplots

The table below shows the height and mass of ten year 12 students. Display this information in the form of a scatterplot.

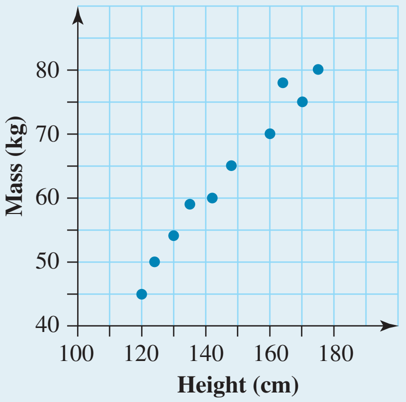
Height (cm)	120	124	130	135	142	148	160	164	170	175
Mass (kg)	45	50	54	59	60	65	70	78	75	80

THINK

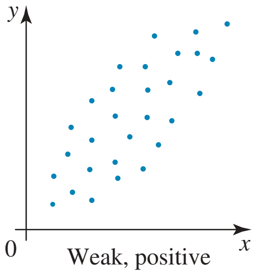
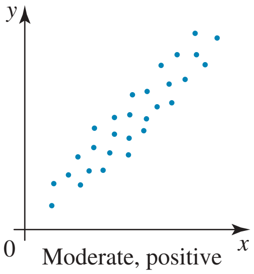
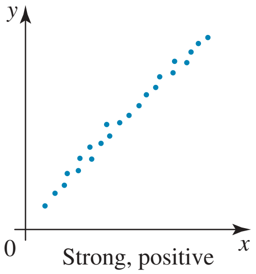
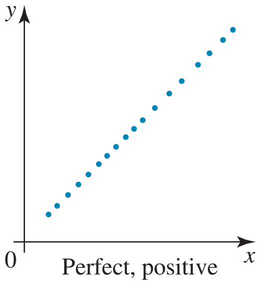
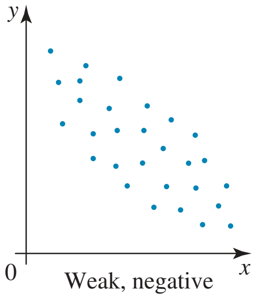
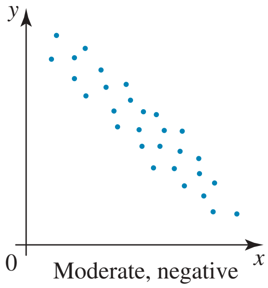
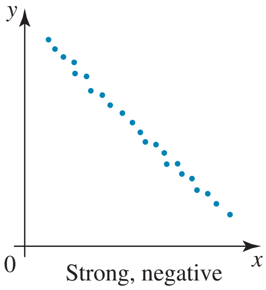
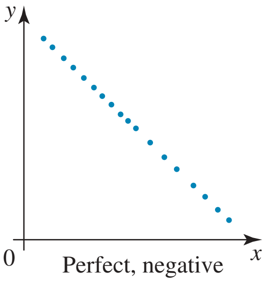
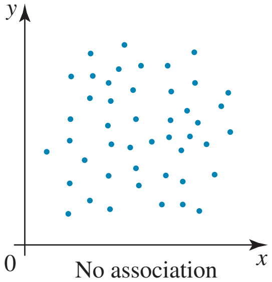
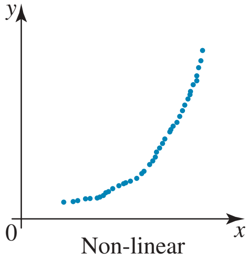
1. Determine the independent and dependent variables. Taller people tend to be heavier, so mass depends on height.
2. Show the height on the x-axis and the mass on the y-axis.
3. Plot the point given by each pair of data.

WRITE

The independent variable is height and the dependent variable is mass.



- Certain patterns tell us what kind of associations exist between the two variables. This is referred to as **correlation**.
- We look at what type of correlation exists and how strong it is.

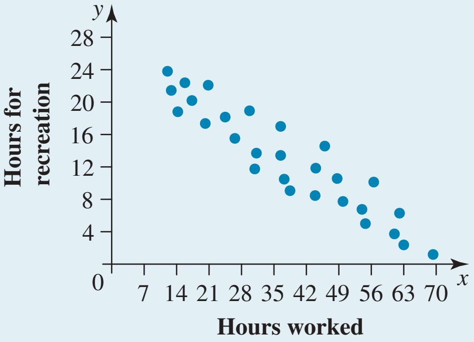
Positive linear	 Weak, positive linear association	 Moderate, positive linear association	 Strong, positive linear association	 Perfect, positive linear association
Negative linear	 Weak, negative linear association	 Moderate, negative linear association	 Strong, negative linear association	 Perfect, negative linear association
No correlation or non-linear	 No association	 Non-linear association		

- When describing the association between two variables displayed on a scatterplot, we need to comment on the following.

Direction	Positive or negative
Form	Linear or non-linear
Strength	Strong, moderate or weak

WORKED EXAMPLE 3 Using a scatterplot to determine the association between variables

The scatterplot shows the number of hours people spend at work each week and the number of hours people get to spend on recreational activities during the week. Decide whether or not an association exists between the variables and, if it does, comment on whether it is positive or negative; weak, moderate or strong; and whether or not it has a linear form.



THINK	**WRITE**
1. The points on the scatterplot are spread in a certain pattern, namely in a rough corridor from the top left to the bottom right corner. Interpret this pattern.	As the work hours increase, the recreation hours decrease.
2. The corridor is straight (i.e. it would be reasonable to fit a straight line into it).	A linear association exists.
3. The points are neither too tight nor too dispersed.	
4. The pattern resembles the central diagram in the gallery of scatterplots shown previously. Describe the relationship between the variables.	There is a moderate, negative linear association between the two variables.

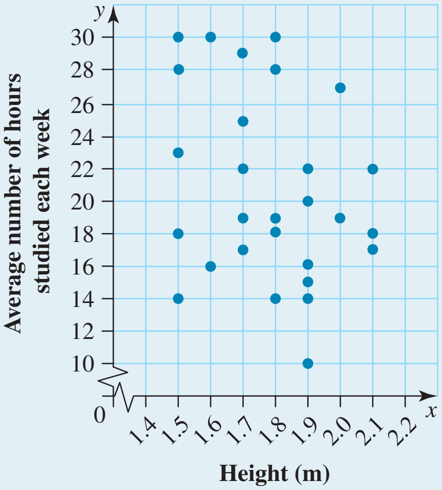
WORKED EXAMPLE 4 Determining the association between variables using a scatterplot

Data showing the average weekly number of hours studied by each student in 12B at Northbank Secondary College and the corresponding height of each student (correct to the nearest tenth of a metre) are given in the table.

Average hours of study	**18**	**16**	**22**	**27**	**15**	**28**	**18**	**20**	**10**	**28**	**25**	**18**	**19**	**17**
Height (m)	**1.5**	**1.9**	**1.7**	**2.0**	**1.9**	**1.8**	**2.1**	**1.9**	**1.9**	**1.5**	**1.7**	**1.8**	**1.8**	**2.1**

Average hours of study	**19**	**22**	**30**	**14**	**17**	**14**	**19**	**16**	**14**	**29**	**30**	**30**	**23**	**22**
Height (m)	**2.0**	**1.9**	**1.6**	**1.5**	**1.7**	**1.8**	**1.7**	**1.6**	**1.9**	**1.7**	**1.8**	**1.5**	**1.5**	**2.1**

Construct a scatterplot for the data and use it to comment on the direction, form and strength of any association between the number of hours studied and the height of the students. Take the independent variable to be height (m).

THINK	**WRITE**
1. Plot the point given by each pair of data. There should be a total of 28 points.	
2. Comment on the direction of any association.	There is no association; the points appear to be randomly placed.
3. Comment on the form of the association.	There is no form, just a random placement of points.
4. Comment on the strength of any association.	Since there is no association, strength is not relevant.
5. Draw a conclusion.	From the scatterplot, there appears to be little to no association between height and number of hours spent studying.

Exercise 9.2 Interpreting scatterplots

9.2 Quick quiz **9.2 Exercise**

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15	N/A	N/A

Simple familiar

1. **WE1** For each of the following pairs, decide which of the variables is independent and which is dependent.
   - a. Number of hours spent studying for a Mathematics test and the score on that test
   - b. Daily amount of rainfall (in mm) and daily attendance at the Botanical Gardens
   - c. Number of hours per week spent in a gym and the annual number of visits to the doctor
   - d. The amount of computer memory taken by an essay and the length of the essay (in words)

2. Determine the independent and dependent variables from the following pairs of variables.
   - a. The cost of care in a childcare centre and attendance at the childcare centre
   - b. The cost of the property (real estate) and the age of the property
   - c. The entry requirements for a certain tertiary course and the number of applications for that course
   - d. The heart rate of the runner and their running speed

3. **WE2** The table below shows the marks obtained by a group of ten students who study both History and Geography. Display this information in the form of a scatterplot, using History as the independent variable.

History	36	65	82	72	58	39	58	74	82	66
Geography	45	78	66	72	50	51	61	70	60	88

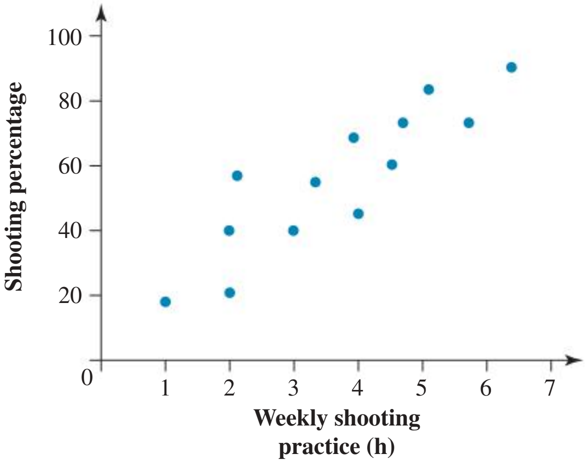
4. The table below shows the maximum temperature each day, together with the number of people who attend the cinema that day. Display the information in the form of a scatterplot.

Temperature (°C)	25	33	30	22	15	18	27	22	28	20
Number at cinema	256	184	190	312	458	401	200	357	312	423

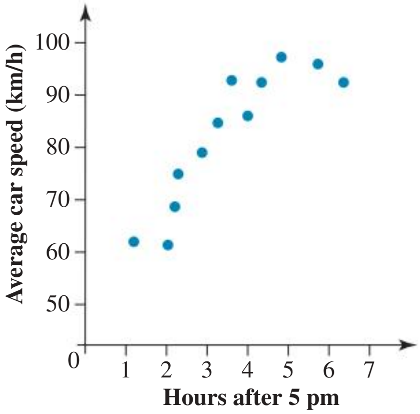
5. The table below shows the wages of 20 people and the amount of money they spend each week on entertainment. Display this information in the form of a scatterplot.

Wages ($)	370	380	500	510	395	430	535	490	495	550
Amount spent on entertainment ($)	55	85	150	75	145	100	130	115	70	150
Wages ($)	810	475	460	520	530	475	610	780	350	460
Amount spent on entertainment ($)	220	50	100	150	140	160	90	130	40	50

6. WE3 The scatterplot shown represents the number of hours of basketball practice each week and a player's shooting percentage. Decide whether or not an association exists between the variables and, if it does, comment on whether it is positive or negative; weak, moderate or strong; and whether or not it is linear form.



7. The scatterplot shows the hours after 5 pm and the average speed of cars on a freeway. Explain the direction, form and strength of the association of the two variables.



8. For each of the following pairs of variables, write down whether or not you would reasonably expect an association to exist between the pair and, if so, comment on whether it would be a positive or negative association.

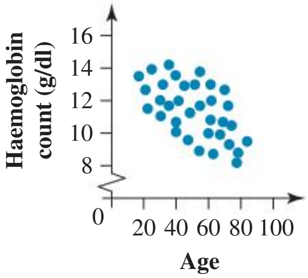
- **a.** Time spent in a supermarket and total money spent
- **b.** Income and value of car driven
- **c.** Number of children living in a house and time spent cleaning the house
- **d.** Age and number of hours of competitive sport played per week

9. For each of the following pairs of variables, suggest whether there would be a positive association, a negative association or no association between them.

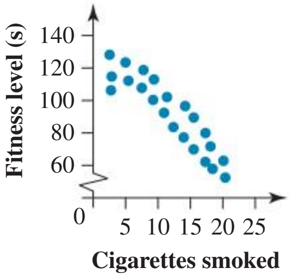
- **a.** Amount spent on petrol each week and distance travelled by car each week
- **b.** Number of hours spent in front of a computer each week and time spent playing the piano each week
- **c.** Amount spent on weekly groceries and time spent gardening each week

10. For each of the following scatterplots, describe whether or not an association exists between the variables and, if it does, comment on whether it is positive or negative, whether it is weak, moderate or strong, and whether or not it has a linear form.

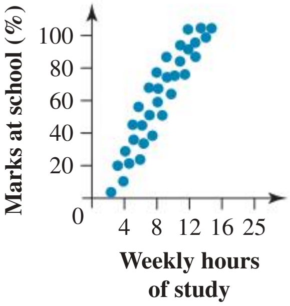
a.



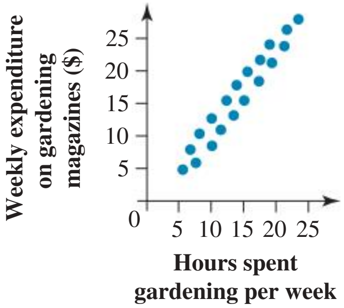
b.



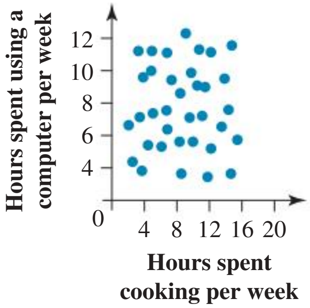
c.



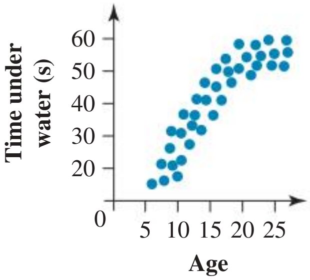
d.



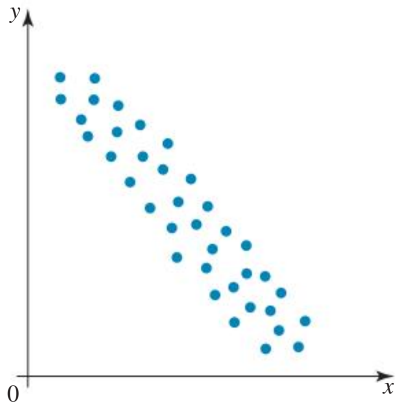
e.



f.



11. From the scatterplot shown, describe the relationship between x and y.



12. WE4 Data on the height of a person and the length of their hair is shown. Construct a scatterplot for the data and use it to comment on the direction, form and strength of any association between the height of a person and the length of their hair.

Height (cm)	158	164	184	173	194	160	198	186	166
Hair length (cm)	18	12	5	10	7	3	10	6	14

13. The following table shows data on hours spent watching television per week and age. Use the data to construct a scatterplot and use it to comment on the direction, form and strength of any association between the two variables.

Age (years)	12	25	61	42	18	21	33	15	29
TV per week (hours)	23	30	26	18	12	30	20	19	26

14. The population of a municipality (to the nearest ten thousand) and the number of primary schools in that particular municipality are given below for 11 municipalities.

Population (×1000)	110	130	130	140	150	160	170	170	180	180	190
Number of primary schools	4	4	6	5	6	8	6	7	8	9	8

Construct a scatterplot for the data and use it to comment on the direction, form and strength of any association between the population and the number of primary schools.

15. The table contains data for the time taken to do a paving job and the cost of the job. Construct a scatterplot for the data. Comment on whether an association exists between the time taken and the cost. If there is an association describe it.

Time taken (hours)	Cost of job ($)
5	1000
7	1000
5	1500
8	1200
10	2000
13	2500
15	2800
20	3200
18	2800
25	4000
23	3000

Fully worked solutions for this chapter are available online.

LESSON
9.3 The line of best fit

SYLLABUS LINKS

- Identify dependent and independent variables.
- Draw a line of best fit by eye.
- Use technology to determine the equation of the line of best fit in the form $y = mx + c$ where m is slope (gradient) and c is y-intercept [complex].
- Interpret the effect of the parameters m and c from the equation of the line of best fit in the form $y = mx + c$ [complex].

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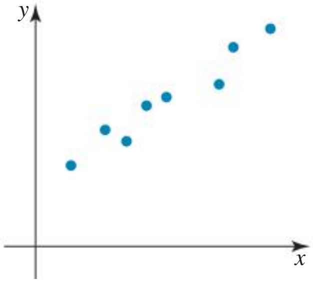
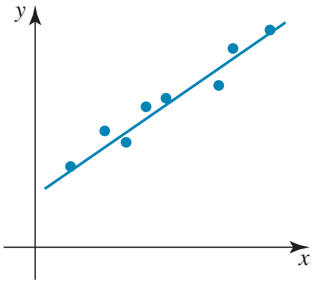
- The process of 'fitting' straight lines to bivariate data (known as **linear regression**) enables us to analyse associations between the data and possibly make predictions based on the given data set.

9.3.1 Fitting a line of best fit by eye

int-6180

- Consider the set of bivariate data points shown. In this case the x-values could be the hand lengths of a group of people, while y-values could be the lengths of their feet.
- We wish to determine a linear association between these two random variables.
- Of course, there is no single straight line which would go through all the points, so we can only *estimate* such a line.

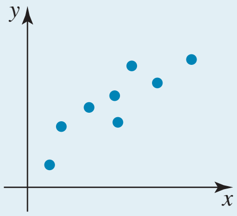




- Consider the estimate, drawn 'by eye' in the figure.
- The line extends through all data points, and most of the points are very close to this straight line, indicating a strong linear association between the two variables.
- This line was easily drawn since the points are very much part of an apparent linear association.
- Due to the strong linear association between foot length and hand length, we can conclude from this graph that there is evidence to suggest that a person's foot length is related to the length of their hand.
- *Regression analysis* is concerned with finding these lines of best fit using various methods so that the number of points above and below the lines is 'balanced'.
- The technique of balancing the number of points involves fitting a line so that there is an *equal number of points* above and below the line. For example, if there are 12 points in the data set, 6 should be above the line and 6 below it.

WORKED EXAMPLE 5 Constructing a line of best fit by eye

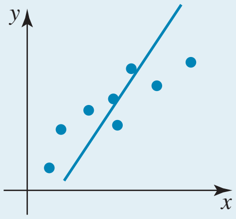
Construct the line of best fit for the data in the diagram using the equal-number-of-points method.

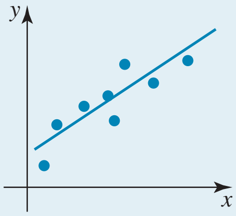


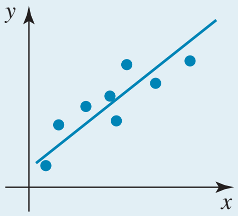
THINK

1. Note that the number of points (n) is 8.
2. Fit a line where 4 points (i.e. half n) are above the line. Using a clear plastic ruler, try to fit the best line. Make sure the line extends through all data points.
3. The first attempt shown has only 3 points below the line, where there should be 4. Make refinements.
4. The second attempt is an improvement, but the line is too close to the points above it.
5. Improve the position of the line until a better 'balance' between upper and lower points is achieved.

WRITE







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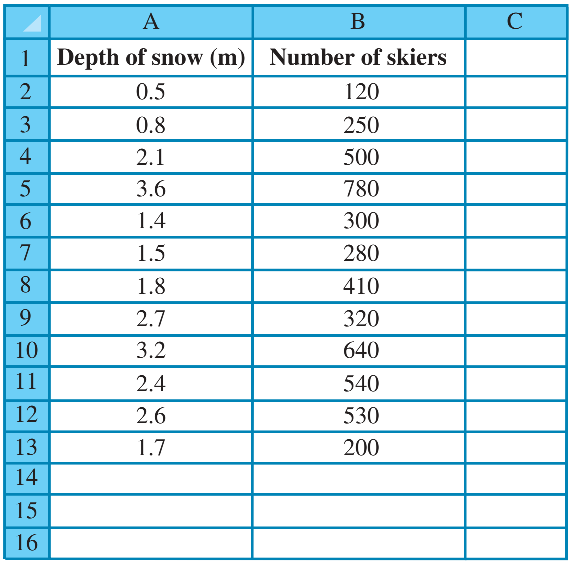
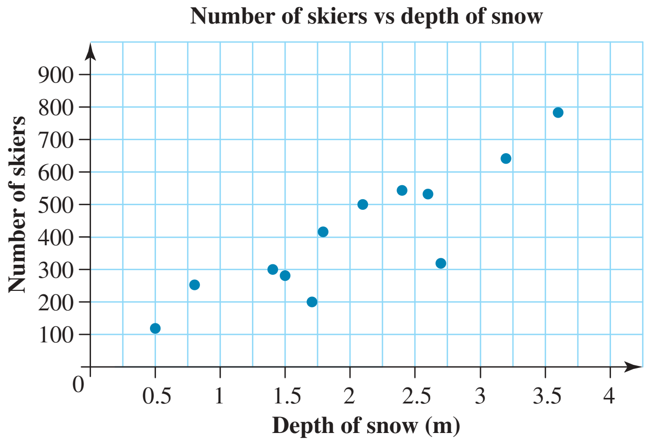
9.3.2 Use technology to determine the equation of the line of best fit [complex]

- Using a spreadsheet, the equation of the line of best fit, commonly referred to as a **regression line**, can readily be produced from two lists of data.
  Consider the example used at the beginning of the chapter involving skiers and snow depth.
  - Step 1: Enter the two lists of data into two columns in the spreadsheet.

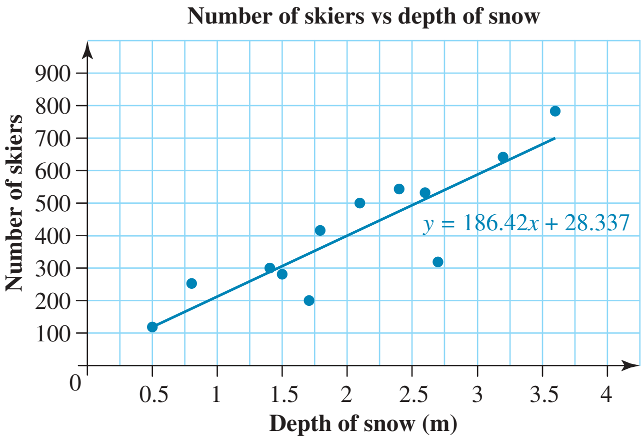


	A	B	C
1	**Depth of snow (m)**	**Number of skiers**	
2	0.5	120	
3	0.8	250	
4	2.1	500	
5	3.6	780	
6	1.4	300	
7	1.5	280	
8	1.8	410	
9	2.7	320	
10	3.2	640	
11	2.4	540	
12	2.6	530	
13	1.7	200	
14			
15			
16			

  - Step 2: Highlight the data and insert a scatterplot from the spreadsheet software's chart options.



- Step 3: Display the linear regression line (also known as a trendline) and its equation on the scatterplot.



- Step 4: Replace the variables *x* and *y* in the regression equation with the variable names:

$$\text{Number of skiers} = 186.42 \times \text{Depth of snow} + 28.337$$

- To determine the equation of a line of best fit by hand, we must determine the slope and *y*-intercept of the drawn line.

WORKED EXAMPLE 6 Using a scatterplot to draw a regression line

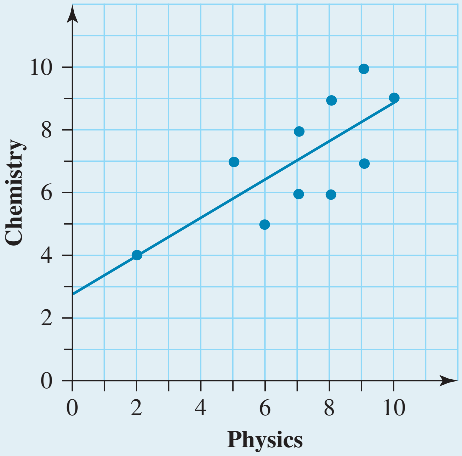
The table below shows the marks of ten students in a Physics and Chemistry quiz. Using the variable Physics as the independent variable, draw a scatterplot and show the regression line on the graph.

Physics	**2**	**5**	**6**	**7**	**7**	**8**	**8**	**9**	**9**	**10**
Chemistry	**4**	**7**	**5**	**8**	**6**	**6**	**9**	**7**	**10**	**9**

THINK

1. Plot the point corresponding to each pair of marks.
2. Add a regression line, making sure that there are an equal number of points above and below the line.

WRITE



9.3.3 Interpreting the effect of the parameters m and c from the equation $y = mx + c$ [complex]

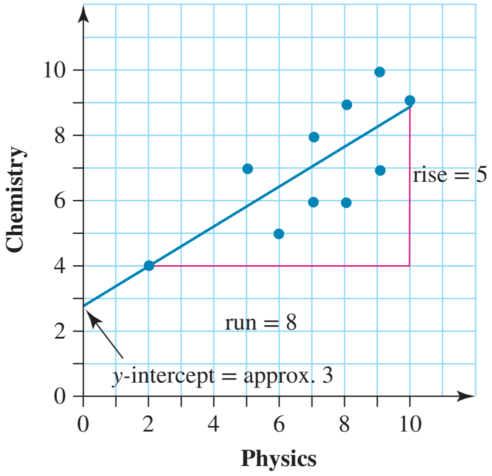
- Once the regression line has been drawn, we can determine its equation.
- The equation of a straight line has the form $y = mx + c$, where m is the slope and c is the y-intercept.
- The slope of a regression line is best found by calculating the rise and the run from two points on the line of regression. (Note that these two points are not necessarily given data points.) The slope is then found using the following formula.

The slope formula

$$\textbf{Slope } (m) = \frac{\textbf{rise}}{\textbf{run}}$$

- The y-intercept can then be seen by noting the point where the line crosses the y-axis.
- The y-intercept can only be determined from the graph if the scale on the horizontal axis starts at 0.
- Consider Worked example 6. We can see that the points (2, 4) and (10, 9) lie on the line.
- The rise between these points is 5 and the run is 8. Hence, the slope of the line is:

$$\text{slope } (m) = \frac{\text{rise}}{\text{run}}$$
$$= \frac{5}{8}$$



- From the graph, we can see that the y-intercept is approximately (0, 3).
- Substituting the value of the slope and the y-coordinate of the y-intercept into the general form of a linear equation, we can determine that the equation of the regression line is approximately $y = \frac{5}{8}x + 3$ or $y = 0.625x + 3$.
- That is, Chemistry $= \frac{5}{8} \times$ Physics + 3.
- A teacher may then be able to estimate a student's Chemistry mark by using their result in Physics.
- For example, a student who achieved a mark of 5 in Physics may get:

$$\begin{aligned}\text{Chemistry} &= \frac{5}{8} \times \text{Physics} + 3\\ &= \frac{5}{8} \times 5 + 3\\ &= 6\frac{1}{8} \text{ or } 6.125\end{aligned}$$

- The teacher would estimate a mark of approximately 6 for this student in Chemistry.

tlvd-12179

WORKED EXAMPLE 7 Determining the equation for the line of best fit

The following table shows the bus fare charged by a bus company for different distances.

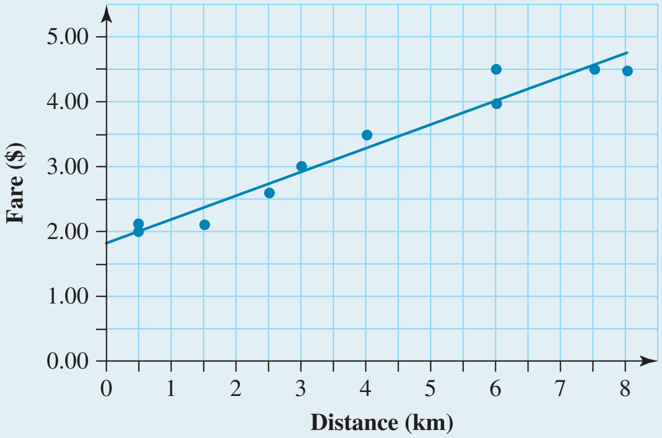
Distance (km)	Fare ($)
1.5	2.10
0.5	2.00
7.5	4.50
6	4.00
6	4.50
2.5	2.60
0.5	2.10
8	4.50
4	3.50
3	3.00

a. **Represent the data using a scatterplot and add the regression line.**

b. **Determine the slope and *y*-intercept of the regression line, and hence determine its equation manually.**

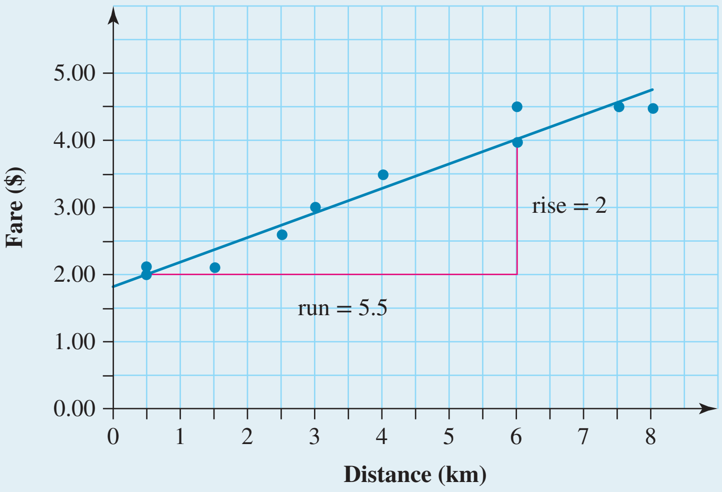
c. **Use technology to determine the equation of the regression line.**

THINK	WRITE
a. 1. The fare would generally depend on the distance travelled. Show the distance on the *x*-axis and the fare on the *y*-axis. **2.** Plot the points given by each pair. **3.** Add the regression line to the graph.	**a.**



b. 1. Take two points on the regression line and determine the slope between them by substituting in the formula $m = \frac{\text{rise}}{\text{run}}$ and simplifying. Use the points (0.5, 2) and (6, 4).

b.



Rise = 2, run = 5.5

$$\begin{aligned} m &= \frac{2}{5.5} \\ &\approx 0.36 \end{aligned}$$

2. Give an estimate for the y-intercept.

y-intercept ≈ 1.8

3. Substitute the slope and y-intercept into the formula $y = mx + c$ and state the equation.

$y = 0.36x + 1.8$

c. 1. Enter the distances into a list and the fares into a second list in a spreadsheet.

2. Follow the procedures outlined previously to determine the equation of the regression line.

c. The equation of the regression line is $y = 0.36x + 1.8$

3. Write the equation in terms of the variables.

Fare ($) = 0.36 × Distance + 1.8

tlvd-12180

WORKED EXAMPLE 8 Determining the line of best fit using two points

Determine the equation of the line of best fit that passes through the points (−2, 5) and (1, 11).

THINK	WRITE
1. To find the line of best fit, use $y = mx + c$.	$m = \frac{y_2 - y_1}{x_2 - x_1}$
2. First calculate the gradient.	$= \frac{11 - 5}{1 - (-2)}$ $= \frac{6}{1 + 2}$ $= \frac{6}{3}$ $= 2$

3. Calculate the y-intercept by substituting the gradient and any one of the given points into $y = mx + c$. We will use the point $(1, 11)$.

$$\begin{aligned} y &= mx + c \\ 11 &= 2(1) + c \\ 11 &= 2 + c \\ 11 - 2 &= 2 + c - 2 \\ 9 &= c \\ c &= 9 \end{aligned}$$

4. Substitute the gradient and y-intercept into $y = mx + c$ and write the answer.

$$y = 2x + 9$$

- Note that there may be some slight discrepancies in the calculated values when manually determining the regression line compared to using technology.

Exercise 9.3 The line of best fit

learnon

9.3 Quick quiz on

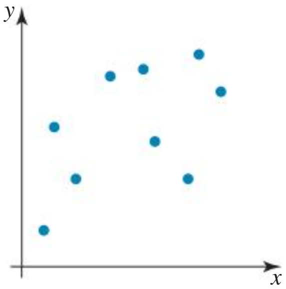
9.3 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5	6, 7, 8, 9, 10, 11, 12	13, 14, 15

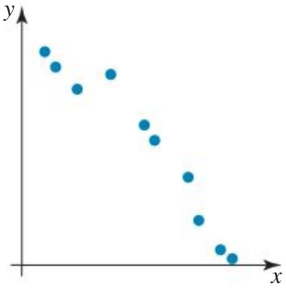
Simple familiar

Note: Technology may be used.

1. **WE5** Draw the line of best fit for the scatterplot shown using the equal-number-of-points method.

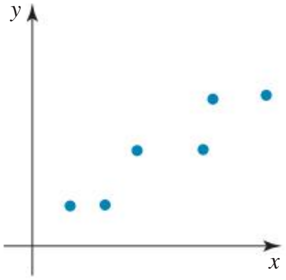


2. Draw the line of best fit for the scatterplot shown using the equal-number-of-points method.

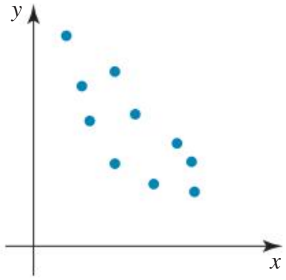


3. Fit a straight line to the data in the scatterplots using the equal-number-of-points method.

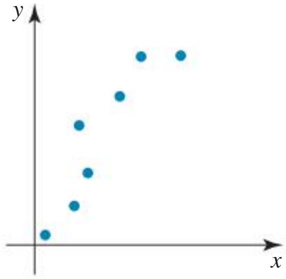
a.



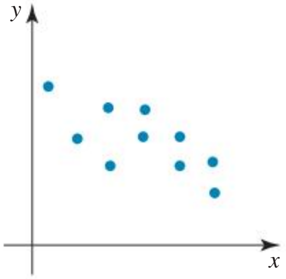
b.



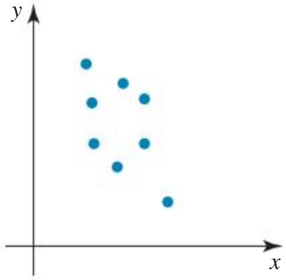
c.



d.



e.



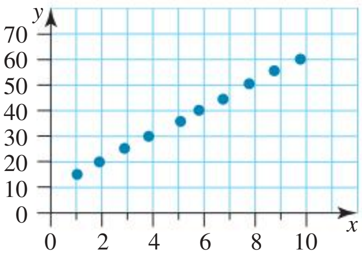
4. **WE6** The table below shows the marks achieved by a class of students in English and Maths.

English	64	75	81	63	32	56	47	59	73	64
Maths	76	62	89	56	49	57	53	72	80	50

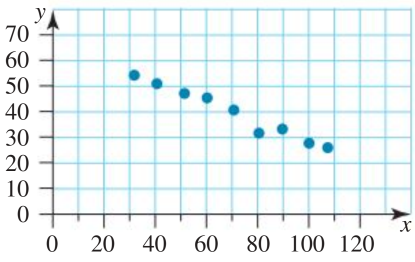
Using English as the independent variable, draw a scatterplot and on the graph show the regression line.

5. Position the line of best fit through each of the following graphs and determine the equation of each.

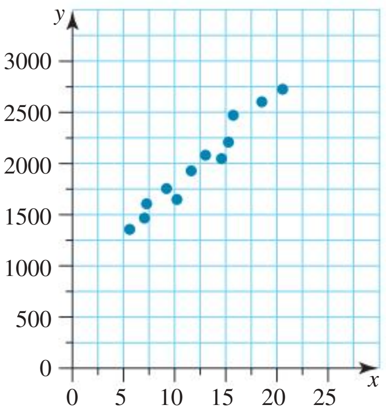
a.



b.



c.



Complex familiar

6. **WE7** In an experiment, a student measures the length of a spring (L) when different masses (M) are attached to it. Her results are shown below.

Mass (g)	Length of spring (mm)
0	220
100	225
200	231
300	235
400	242
500	246
600	250
700	254
800	259
900	264

a. Draw a scatterplot of the data and on it insert the regression line.

b. Determine the slope and y-intercept of the regression line and, hence, state the equation of the regression line. Write your equation in terms of the variables L and M.

7. **WE8** Determine the equation that represents the line of best fit that passes through the points (3, 8) and (12, 35).

8. Determine the equation of the line of best fit that passes through the points $(-3, -2)$ and $(-1, 6)$.

9. A scientist who measures the volume of a gas at different temperatures provides the following table of values.

Temperature (°C)	Volume (L)
−40	1.2
−30	1.9
−20	2.4
0	3.1
10	3.6
20	4.1
30	4.8
40	5.3
50	6.1
60	6.7

a. Draw a scatterplot of the data and on it insert the regression line.

b. Determine the equation of the regression line. Write your equation in terms of the variables: volume of gas, V, and its temperature, T.

10. A sports scientist is interested in the importance of muscle bulk to strength. He measures the biceps circumference of ten people and tests their strength by asking them to complete a lift test. His results are given in the following table.

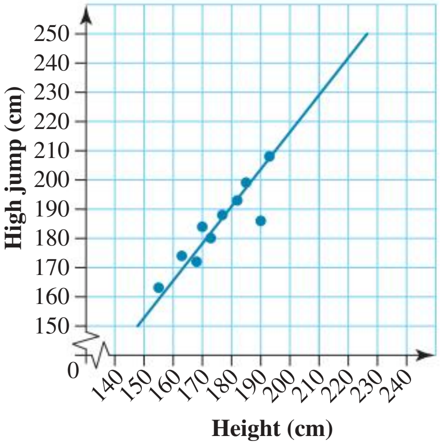
Circumference of biceps (cm)	Lift test (kg)
25	50
25	52
27	58
28	51
30	60
30	62
31	53
33	62
34	61
36	66

a. Draw a scatterplot of the data and draw the regression line.

b. Determine the line of best fit that links the ability of a person to complete a lift test, *S*, to the circumference of their biceps, *B*.

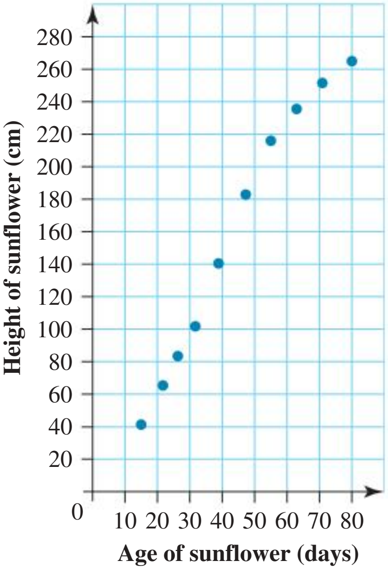
11. A sports scientist is looking at data comparing the heights of athletes and their performance in the high jump. The following table and scatterplot represent the data they have collected. A line of best fit by eye has been drawn on the scatterplot. Use technology to determine the equation of the line.

Height (cm)	168	173	155	182	170	193	177	185	163	190
High jump (cm)	172	180	163	193	184	208	188	199	174	186



12. The following table and scatterplot show the age and height of a field of sunflowers planted at different times throughout summer.

Age of sunflower (days)	63	71	15	33	80	22	55	47	26	39
Height of sunflower (cm)	237	253	41	101	264	65	218	182	82	140



a. Xavier draws a line of best fit by eye that goes through the points (10, 16) and (70, 280). Draw his line of best fit on the scatterplot and comment on his choice of line.

b. Calculate the equation of the line of best fit using the two points that Xavier selected.

c. Patricia draws a line of best fit by eye that goes through the points (10, 18) and (70, 258). Draw her line of best fit on the scatterplot and comment on her choice of line.

d. Calculate the equation of the line of best fit using the two points that Patricia selected.

e. Explain why the value of the y-intercept is not 0 in either equation.

Complex unfamiliar

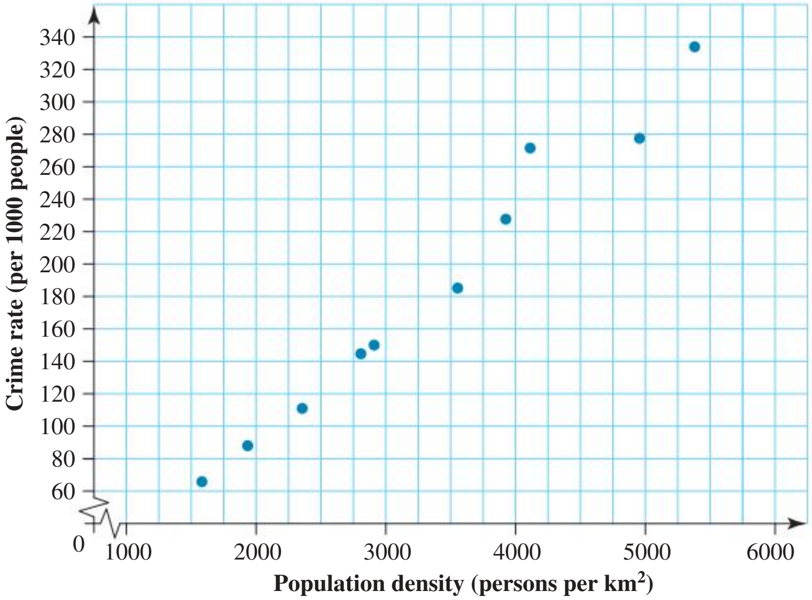
13. Nidya is analysing the following data, but a clerical error means that she only has access to two points of data: (170, 184) and (177, 188).
Comment on the similarities and differences between the two lines of best fit.

Height (cm)	168	173	155	182	170	193	177	185	163	190
High jump (cm)	172	180	163	193	184	208	188	199	174	186

14. Steven is looking at data comparing the size of different music venues across the country and the average ticket price at these venues. After plotting his data in a scatterplot, he calculates a line of best fit for his data as $y = 0.04x + 15$, where y is the average ticket price in dollars and x is the capacity of the venue.
Describe what the y-intercept represents in Steven's equation and explain whether it is a realistic value in this context.

15. A government department is analysing the population density and crime rate of different suburbs to see if there is a connection. The following table and scatterplot display the data that has been collected so far.

Population density (persons per km^2)	3525	2767	4931	3910	1572	2330	2894	4146	1968	5337
Crime rate (per 1000 people)	185	144	279	227	65	112	150	273	87	335



Determine the equation of the line of best fit and use this to help explain what the *x*-intercept means in this context and whether it is realistic. Justify your answer.

Fully worked solutions for this chapter are available online.

LESSON
9.4 Making predictions using a line of best fit [complex]

SYLLABUS LINKS

- Use the line of best fit to make predictions, both by interpolation and extrapolation [complex].
- Recognise the dangers of extrapolation [complex].

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- If a linear association exists between a pair of variables, it is useful to be able to make predictions of one variable from the other.
- The line of best fit ($y = mx + c$) can be extended and then used to make predictions. When this is done, the line is called a regression line.
- There are two methods available:

1. Algebraically
   We can find an equation for the regression line (or line of best fit), substitute the value of the known variable and calculate the value of the unknown variable. The equation of the regression line can be generated by hand or using technology from data entered in lists.
2. Graphically
   We can draw lines horizontally or vertically from the known value to the regression line, then read off the corresponding value of the unknown variable.

9.4.1 Algebraic predictions

- Once the equation of the regression line has been determined, it is possible to use this equation to make predictions by substituting the value of the given variable into it.

WORKED EXAMPLE 9 Using algebra to make a prediction

A casino records the number of people, N, playing a jackpot game and the prize money, p, for that game, and plots the results on a scatterplot. The regression line is found to have the equation.

$$N = 0.07p + 220$$

a. Determine the number of people playing when the prize money is $2500.

b. Calculate the likely prize on offer when there are 500 people playing.

THINK	WRITE
a. 1. Write the equation of the regression line.	**a.** $N = 0.07p + 220$
2. Substitute 2500 for p in the regression equation.	$N = 0.07 \times 2500 + 220$
3. Calculate N.	$= 395$
4. Give a written answer.	When the prize money is $2500, it is estimated that 395 people will be playing.
b. 1. Write the equation of the regression line.	**b.** $N = 0.07p + 220$

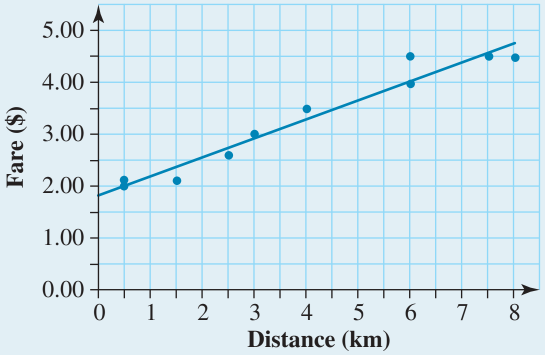
2. Substitute 500 for N in the regression equation.	$500 = 0.07p + 220$
3. Solve the equation for p by subtracting 220 from both sides, then dividing the result by 0.07.	$280 = 0.07p$ $p = \dfrac{280}{0.07}$ $p = 4000$
4. Give a written answer.	When 500 people are playing, the prize is estimated to be $4000.

9.4.2 Graphical predictions

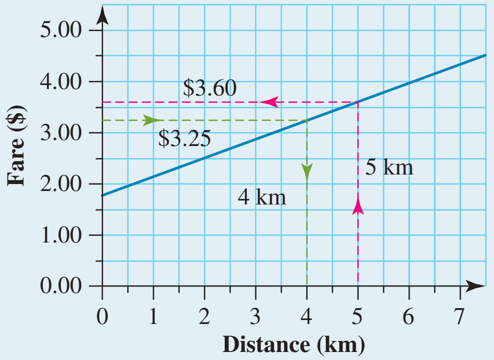
- Once the regression line has been drawn, horizontal or vertical lines can be added from the axes to the regression line to read off the value of the unknown variable.
- Remember that the values determined are only estimates, and not exact values.

WORKED EXAMPLE 10 Using a graph to make a prediction

Consider the graph drawn in Worked example 7 (section 9.3.2) representing bus fares for travelling various distances.
Use the graph to estimate:
a. the fare for a journey of 5 km
b. how far you could expect to travel for a cost of $3.25.



THINK	WRITE
a. 1. Locate the value of 5 km on the x-axis.	**a.**
2. Draw a vertical line from this point to meet the regression line. See pink line.	
3. Draw a horizontal line from this point to the y-axis. See pink line.	



THINK	WRITE
4. Read this value on the y-axis.	A distance of 5 km of bus travel is predicted to cost about $3.60.
b. 1. Locate $3.25 on the y-axis.	**b.** Refer to the graph above.
2. Draw a horizontal line from this point to the regression line. See green line.	
3. From this point, draw a vertical line to the x-axis. See green line.	
4. Read the x-value at this point.	For $3.25 you could expect to travel about 4 km.

9.4.3 Reliability of predictions

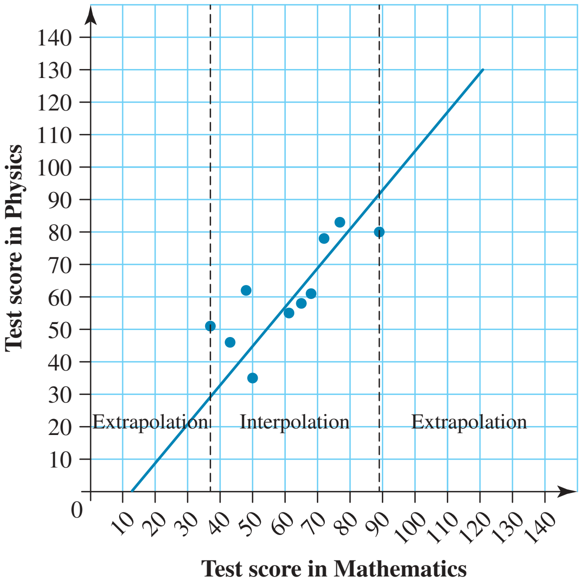
int-6181

Interpolation

- When we use **interpolation**, we are making a prediction from a line of best fit that appears within the parameters of the original data set.

Extrapolation

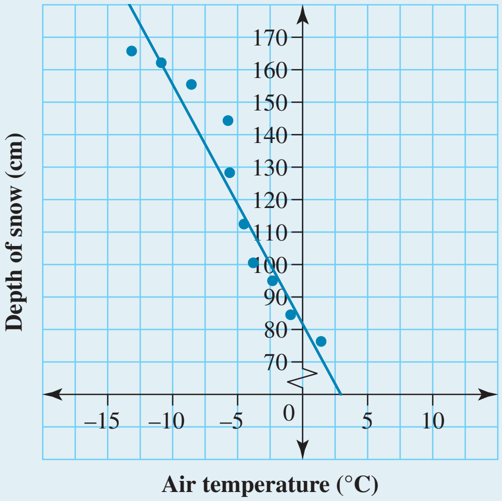
- **Extrapolation** predicts values outside the original data set using a line of best fit.
- More data points improve the reliability of the line of best fit and predictions.
- Extrapolation is usually unreliable due to the lack of supporting data in that region.
- Interpolation is generally more reliable, especially when closer to the centre of the data set.
- A strong correlation between data points improves the accuracy of predictions.



tlvd-12181

WORKED EXAMPLE 11 Estimating values by interpolation and extrapolation

The following data represents the air temperature (°C) and depth of snow (cm) at a popular ski resort.



Air temperature (°C)	−4.5	−2.3	−8.9	−11.0	−13.3	−6.2	−0.4	1.5	−3.7	−5.4
Depth of snow (cm)	111.3	95.8	155.6	162.3	166.0	144.7	84.0	77.2	100.5	129.3

The equation of the line of best fit for this data set has been calculated as $y = -7.2x + 84$.

a. **Use the equation of the line of best fit to estimate the depth of snow if the air temperature is −6.5 °C.**

b. **Use the equation of the line of best fit to estimate the depth of snow if the air temperature is 25.2 °C.**

c. **Comment on the reliability of your estimations in parts a and b.**

THINK	**WRITE**
a. 1. Rewrite the regression equation in terms of the variables.	**a.** $y = -7.2x + 84$ $\text{Depth of snow} = -7.2 \times \text{Air temperature} + 84$
2. Substitute −6.5 for the air temperature into the equation for the line of best fit.	$\text{Air temperature} = -6.5$ $\text{Depth of snow} = -7.2 \times \text{Air temperature} + 84$
3. Evaluate the expression for the depth of snow.	$= -7.2 \times -6.5 + 84$ $= 130.8$
4. Write the answer.	When the air temperature is −6.5 °C the depth of snow will be approximately 130.8 cm.
b. 1. Substitute 25.2 for the air temperature into the equation for the line of best fit.	**b.** $\text{Air temperature} = 25.2$ $\text{Depth of snow} = -7.2 \times \text{Air temperature} + 84$
2. Evaluate the expression for the depth of snow.	$= -7.2 \times 25.2 + 84$ $= -97.4$ (one decimal place)
3. Write the answer.	When the air temperature is 25.2 °C the depth of snow will be approximately −97.4 cm.
c. Relate the answers back to the original data to check their reliability.	**c.** The estimate in part **a** was made using interpolation, with the point being comfortably located within the parameters of the original data. The estimate appears to be consistent with the given data and as such is reliable.The estimate in part **b** was made using extrapolation, with the point being located well outside the parameters of the original data. This estimate is clearly unreliable, as we cannot have a negative depth of snow.

Exercise 9.4 Making predictions using a line of best fit [complex]

learnon

9.4 Quick quiz

9.4 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
N/A	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13	14, 15

These questions are even better in jacPLUS!

- Receive immediate feedback
- Access sample responses
- Track results and progress

Find all this and MORE in jacPLUS

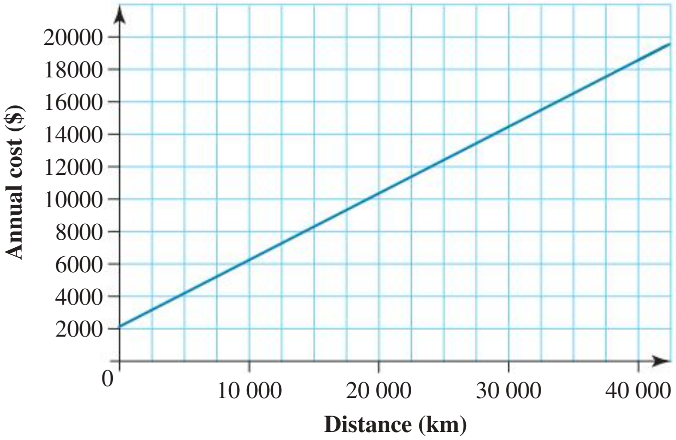
Complex familiar

1. **WE9** A taxi company adjusts its meters so that the fare is charged according to the following equation: $F = 1.2d + 3$ where F is the fare, in dollars, and d is the distance travelled, in km.
   a. Determine the fare charged for a distance of 12 km.
   b. Determine the fare charged for a distance of 4.5 km.
   c. Determine the distance that could be covered on a fare of $27.
   d. Determine the distance that could be covered on a fare of $13.20.

2. Detectives can use the equation $H = 6.1f - 5$ to estimate the height of a burglar who leaves footprints behind. (H is the height of the burglar, in cm, and f is the length of the footprint.)
   a. Determine the height of a burglar whose footprint is 27 cm in length.
   b. Determine the height of a burglar whose footprint is 30 cm in length.
   c. Determine the footprint length of a burglar of height 185 cm. (Give your answer correct to two decimal places.)
   d. Determine the footprint length of a burglar of height 152 cm. (Give your answer correct to two decimal places.)

3. A football match pie seller finds that the number of pies sold is related to the temperature of the day. The situation could be modelled by the equation $N = 870 - 23t$, where N is the number of pies sold and t is the temperature in degrees Celsius of the day.
   a. Determine the number of pies sold if the temperature was 5°C.
   b. Determine the number of pies sold if the temperature was 25 °C.
   c. Determine the likely temperature if 400 pies were sold.
   d. Determine how hot the day would have to be before the pie seller sold no pies at all.

4. **WE10** The following graph shows the average annual costs of running a car. It includes all fixed costs (registration, insurance etc.) as well as running costs (petrol, repairs etc.).



Use the graph to estimate:

a. the annual cost of running a car if it is driven 15 000 km

b. the annual cost of running a car if it is driven 1000 km

c. the likely number of kilometres driven if the annual costs were $8000

d. the likely number of kilometres driven if the annual costs were $16 000.

5. A market researcher finds that the number of people who would purchase ‘Wise-up’ (the thinking man’s deodorant) is related to its price. He provides the following table of values.

a. Draw a scatterplot of the data.

b. Draw in the line of best fit.

c. Determine an equation that represents the association between the number of cans of ‘Wise-up’ sold, N (in thousands), and its price, p.

d. Use the equation to predict the number of cans sold each week if:

   i. the price was $3.10

   ii. the price was $4.60.

e. Determine the price at which ‘Wise-up’ should be sold if the manufacturers wish to sell 80 000 cans?

f. Given that the manufacturers of ‘Wise-up’ can produce only 100 000 cans each week, determine the price at which it should be sold to maximise production.

Price ($)	**Weekly sales ($\times$ 1000)**
1.40	105
1.60	101
1.80	97
2.00	93
2.20	89
2.40	85
2.60	81
2.80	77
3.00	73
3.20	69
3.40	65

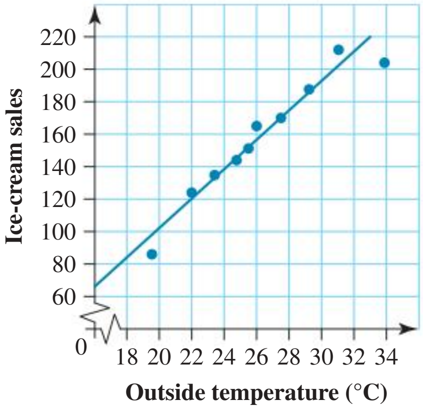
6. The following table gives the adult return air fares between some Australian cities.

City	Distance (km)	Price ($)
Melbourne–Sydney	713	580
Perth–Melbourne	2728	1490
Adelaide–Sydney	1172	790
Brisbane–Melbourne	1370	890
Hobart–Melbourne	559	520
Hobart–Adelaide	1144	820
Adelaide–Melbourne	669	570

a. Draw a scatterplot of the data and on it draw the regression line.
b. Determine an equation that represents the association between the air fare, A, and the distance travelled, d.
c. Use the equation to predict the likely air fare (to the nearest dollar) from:
   i. Sydney to the Gold Coast (671 km)
   ii. Perth to Adelaide (2125 km)
   iii. Hobart to Sydney (1024 km)
   iv. Perth to Sydney (3295 km).

7. WE11 The owner of an ice-cream parlour has collected data relating the outside temperature to ice-cream sales.

Outside temperature (°C)	23.4	27.5	26.0	31.1	33.8	22.0	19.7	24.6	25.5	29.3
Ice-cream sales	135	170	165	212	204	124	86	144	151	188



A line of best fit for this data has been calculated as: ice-cream sales = 9 × outside temperature −77.

a. Use the line of best fit to estimate ice-cream sales if the outside temperature is 27.9 °C.
b. Use the line of best fit to estimate ice-cream sales if the air temperature is 15.2 °C.
c. Comment on the reliability of your answers to parts a and b.

8. Georgio is comparing the cost and distance of various long-distance flights, and after drawing a scatterplot he creates an equation for a line of best fit to represent his data. Georgio's line of best fit is $y = 0.08x + 55$, where y is the cost of the flight and x is the distance of the flight in kilometres.

a. Estimate the cost of a flight between Melbourne and Sydney (713 km) using Georgio's equation.

b. Estimate the cost of a flight between Melbourne and Broome (3121 km) using Georgio's equation.

c. All of Georgio's data came from flights of distances between 400 and 2000 km. Comment on the suitability of using Georgio's equation for shorter and longer flights than those he analysed. Discuss other factors that might affect the cost of these flights.

9. Mariana is a scientist and is collecting data measuring lung capacity (in L) and time taken to swim 25 m (in seconds). Unfortunately a spillage in her lab causes all of her data to be erased apart from the records of a person with a lung capacity of 3.5 L completing the 25 m in 55.8 s and a person with a lung capacity of 4.8 L completing the 25 m in 33.3 s.

a. Use the remaining data to construct an equation for the line of best fit relating lung capacity (x) to the time taken to swim 25 m (y). Give any numerical values correct to two decimal places.

b. Identify what the value of the slope (m) represents in the equation.

c. Use the equation to estimate the time it takes people with the following lung capacities to swim 25 m (to one decimal place).

- **i.** 3.2 L
- **ii.** 4.4 L
- **iii.** 5.3 L

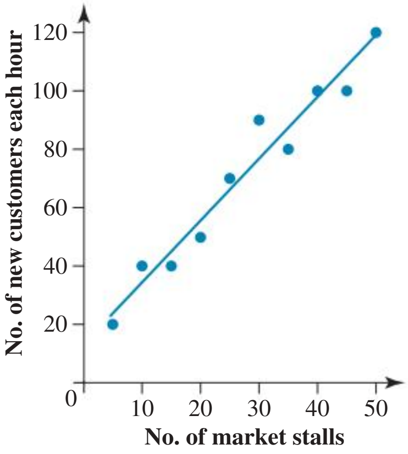
d. Comment on the reliability of creating the equation from Mariana's two remaining data points.

10. The regression line shown on the scatterplot has the equation $c = 13.33 + 2.097m$, where c is the number of new customers each hour and m is the number of market stalls.

a. Use the line of best fit to determine the number of new customers expected if there are 30 market stalls.

b. Use the formula to determine the number of market stalls required in order to expect 150 new customers.

c. Identify whether parts **a** and **b** represent interpolation or extrapolation, and determine which method is likely to produce more accurate predictions.



11. A survey of the nightly room rate for Sydney hotels and their proximity to the Sydney Harbour Bridge produced the regression equation $C = 281.92 - 50.471d$, where C is the cost of a room per night in dollars and d is the distance to the bridge in kilometres.

a. Based on this equation, calculate the cost of a hotel room 2.5 km from the bridge. Give your answer correct to the nearest cent.

b. Determine the distance of a hotel room from the bridge if the cost of the room was \$115. Give your answer correct to two decimal places.

12. An AFL coach is analysing data comparing the kicking efficiency (x) with the handball efficiency (y) of different AFL players. The data is shown in the following table.

Kicking efficiency (%)	75.3	65.6	83.1	73.9	79.0	84.7	64.4	72.4	68.7	80.2
Handball efficiency (%)	84.6	79.8	88.5	85.2	87.1	86.7	78.0	81.3	82.4	90.3

a. A line of best fit for the data goes through the points (66, 79) and (84, 90). Determine the equation for the line of best fit for this data. Give any numerical values correct to two decimal places.

b. Use the equation from part **a** and the figures for kicking efficiency to create a table for the predicted handball efficiency of the same group of players.

Kicking efficiency (%)	75.3	65.6	83.1	73.9	79.0	84.7	64.4	72.4	68.7	80.2
Predicted handball efficiency (%)										

13. Data on people's average monthly income and the amount of money they spend at restaurants was collected.

Average monthly income ($000s)	Money spent at restaurants per month ($)
2.8	150
2.5	130
3.0	220
3.1	245
2.2	100
4.0	400
3.7	380
3.8	200
4.1	600
3.5	360
2.9	175
3.6	350
2.7	185
4.2	620
3.6	395

- **a.** Draw a scatterplot of this data.
- **b.** Find the equation of the line of best fit in terms of average monthly income in thousands of dollars (I) and money spent at restaurants in dollars (R). Give values correct to one decimal place.
- **c.** Extrapolate the data to predict how much a person who earns $5000 a month might spend at restaurants each month.
- **d.** Explain why part **c** is an example of extrapolation.
- **e.** A person spent $265 eating out last month. Estimate their monthly income, giving your answer to the nearest $10. Determine whether this is an example of interpolation or extrapolation.

Complex unfamiliar

14. Use the data given in the table topredict the value of y when $x = 23$ and explain what assumptions have been made in making this prediction.

x	10	11	12	13	14	15	16	17	18	19
y	22	18	20	15	17	11	11	7	9	8

15. While camping a mathematician estimated that:

$$\text{number of mosquitoes around fire} = 10.2 + 0.5 \times \text{temperature of the fire (°C)}.$$

Determine the temperature of the fire if there were only 12 mosquitoes in the area and identify some factors that would affect the reliability of this prediction.

Fully worked solutions for this chapter are available online.

LESSON

9.5 Causality and correlation [complex]

SYLLABUS LINKS

- Use technology to calculate the correlation coefficient (an indicator of the strength of linear association) [complex].
- Distinguish between causality and correlation through examples [complex].

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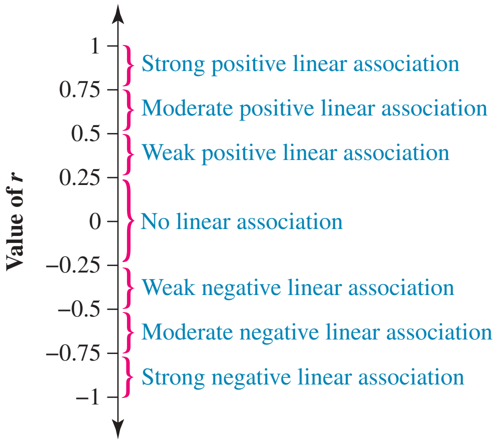
9.5.1 Correlation

int-6251

- Correlation is a measure of the strength of the linear association between two variables.
- **Pearson's correlation coefficient**, r, (also known as Pearson's product–moment correlation coefficient) is a numerical measure of strength. It can be calculated if the scatterplot indicates that a linear association is present between the variables and there are no outliers.
- Pearson's correlation coefficient, r, can have values between -1 and $+1$.
- The figure shown is a guide when using r to describe the strength of a linear association between two variables.



- Pearson's correlation coefficient can be calculated using technology.
- In Excel, the formula =PEARSON(array1,array2) can be used to calculate r. For the worked example below, the formula =PEARSON(A2:A16,B2:B16) will calculate the r value of the data set where the x-values are stored in cells A2 to A16 and the corresponding y-values are stored in cells B2 to B16.

tlvd-12182

WORKED EXAMPLE 12 Using technology to determine the correlation coefficient

Data was collected on the time it takes students to get to school and their ATAR score.

Time (min)	**12**	**35**	**19**	**42**	**33**	**31**	**25**	**46**	**45**	**40**	**14**	**44**	**39**	**31**	**22**
ATAR score	**53**	**75**	**97**	**59**	**87**	**70**	**71**	**66**	**37**	**48**	**94**	**68**	**33**	**59**	**42**

Using technology:

a. draw a scatterplot of this data

b. determine the equation of the line of best fit, writing the terms correct to two decimal places

c. determine Pearson's correlation coefficient, *r*, correct to two decimal places

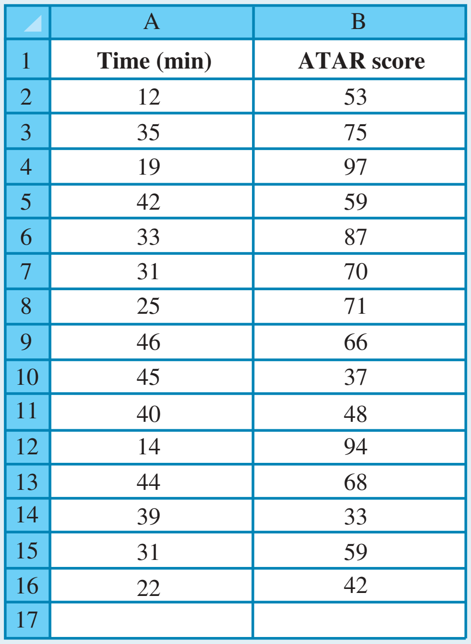
d. interpret this value.

THINK

a. 1. In a spreadsheet, enter the time data into cells A2 to A16, and the ATAR data into cells B2 to B16. You could also input the data into lists on a graphics calculator if one is available.

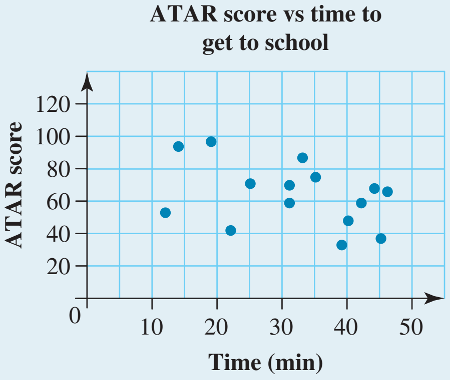
WRITE

a.



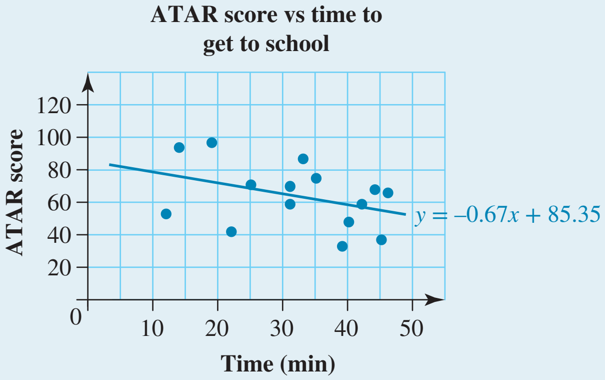
	A	B
1	**Time (min)**	**ATAR score**
2	12	53
3	35	75
4	19	97
5	42	59
6	33	87
7	31	70
8	25	71
9	46	66
10	45	37
11	40	48
12	14	94
13	44	68
14	39	33
15	31	59
16	22	42
17		

2. Highlight the two lists of data, then insert a scatterplot from the software's chart options.



b. 1. Display the line of best fit (trendline) with its equation on the scatterplot.

b.

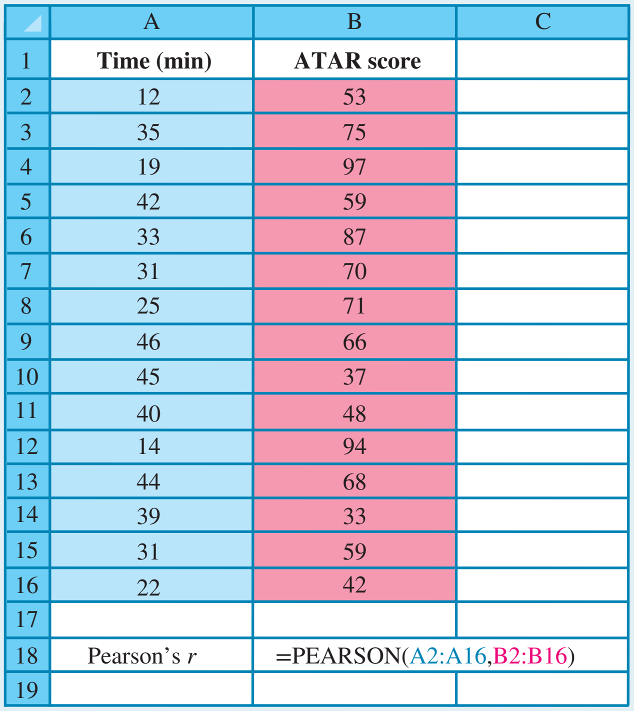


2. Rewrite the equation of the line of best fit in terms of the variables ATAR score and time travelled.

ATAR score = −0.67 × time + 85.35

c. 1. To determine the value of Pearson's correlation coefficient, r, enter the formula =PEARSON(A2:A16,B2:B16) into a blank cell and then press ENTER.

c.



	A	B	C
1	**Time (min)**	**ATAR score**	
2	12	53	
3	35	75	
4	19	97	
5	42	59	
6	33	87	
7	31	70	
8	25	71	
9	46	66	
10	45	37	
11	40	48	
12	14	94	
13	44	68	
14	39	33	
15	31	59	
16	22	42	
17			
18	Pearson's r	=PEARSON(A2:A16,B2:B16)	
19			

2. Write the value of r.

$r = -0.39$

d. Interpret the r value.

d. An r value of −0.39 implies that there is a weak negative linear association between the time taken to get to school and ATAR score.
Students who live further from school tend to have lower ATAR scores.

9.5.2 Causation

- A correlation gives only information about the strength of the association between the two variables. It tells you nothing about the cause of the association.
- **Causation** is a measure of how much change in one variable is caused by the change in the other variable.
- If two variables display an association, this demonstrates that they are correlated, but it does not necessarily mean that a change in one variable will *cause* a change in the other.

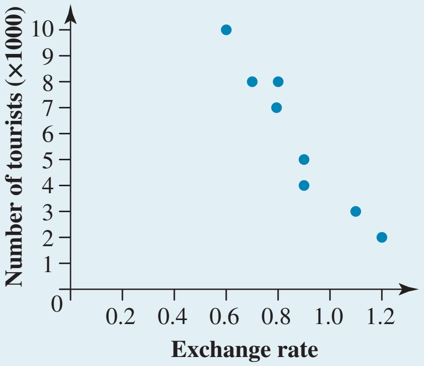
tlvd-12183

WORKED EXAMPLE 13 Understanding causation

Data showing the number of tourists visiting a small country in a month and the corresponding average monthly exchange rate for the country's currency against the American dollar are as given.

Exchange rate	1.2	1.1	0.9	0.9	0.8	0.8	0.7	0.6
Number of tourists (×1000)	2	3	4	5	7	8	8	10

a. **Construct a scatterplot for the data**
b. **Comment on the correlation between the number of tourists and the exchange rate**
c. **Calculate r.**
d. **Based on the pattern shown in the scatterplot and the value of r, would it be appropriate to conclude that the decrease in tourist numbers is caused by the increasing exchange rate?**

THINK	WRITE
a. **1.** Determine the independent and dependent variables.	**a.** Exchange rate (independent) Tourist numbers (dependent).
2. Sketch a scatterplot.	
b. A negative association can be observed.	**b.** From the scatterplot there appears to be a strong negative correlation between the number of tourists visiting the country and the exchange rate.
c. Use technology to find r.	**c.** $r = -0.96$
d. Correlation does not imply causation.	**d.** Just because $r = -0.96$, it cannot be stated that the decreasing tourist numbers are caused by the increasing exchange rate. There may be other factors that need to be considered. Further investigation is needed before any conclusions can be drawn.

Exercise 9.5 Causality and correlation [complex]

learnon

9.5 Quick quiz on

9.5 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
N/A	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13	14

Complex familiar

1. Explain the meanings of ‘correlation’ and ‘causation’ in your own words.
2. If Pearson’s correlation coefficient, r, is found to be -0.7564, describe the strength and direction of the association.
3. Describe the strength and direction of the association between two variables with a Pearson’s correlation coefficient of 0.93.
4. Two variables have a Pearson’s correlation coefficient of -0.89. Describe the direction and strength of their association.
5. Describe the strength and direction of the association between two variables when $r = -0.2$.
6. Describe the difference(s) between a Pearson’s correlation coefficient of 0.76 and -0.76.
7. Describe the difference(s) between a Pearson’s correlation coefficient of $r = 0.79$ and $r = 0.57$.
8. **WE12** Data on people’s average monthly income and the amount of money they spend at restaurants was collected.

Average monthly income ($000s)	2.8	2.5	3.0	3.1	2.2	4.0	3.7	3.8	4.1	3.5	2.9	3.6	2.7	4.2	3.6
Money spent at restaurants per month ($)	150	130	220	245	100	400	380	200	600	360	175	350	185	620	395

   a. Draw a scatterplot of this data.
   b. Determine the equation of the line of best fit.
   c. Determine Pearson’s correlation coefficient, r.
   d. Interpret this value.

9. Data on a student's marks in Geography and their marks in Music were collected.

Geography	65	80	72	61	99	54	39	66	78	89	84	73	68	57	60
Music	91	57	77	89	51	76	62	87	88	64	90	45	60	79	96

a. Draw a scatterplot of this data, using Geography as the independent variable.
b. Determine the equation of the line of best fit.
c. Determine Pearson's correlation coefficient, *r*.
d. Interpret this value.

10. Data on the daily sales of gumboots and the maximum temperature was collected.

Temperature (°C)	17	16	12	10	14	17	18	22	23	19	17	15	12	15	20
Daily sales (no. of pairs)	2	3	8	16	7	3	2	1	1	2	3	3	12	9	1

a. Draw a scatterplot of this data.
b. Determine the equation of the line of best fit.
c. Determine Pearson's correlation coefficient, *r*.
d. Interpret this values.

11. The following table outlines the cost of an annual magazine subscription along with the number of magazine issues per year.

No. of magazine issues per year	7	9	10	6	8	4	4	5	11	9	10	5	11	3	7	12	7	6	12
Subscription cost ($)	34	40	52	38	50	25	28	40	55	55	45	28	65	24	38	55	50	33	59

Use technology to determine Pearson's correlation coefficient for this data correct to four decimal places. Determine what this tells you about the strength of the linear association between the variables.

12. Use your understanding of Pearson's correlation coefficient to explain what the following results indicate.

a. $r = 0.68$
b. $r = -0.97$
c. $r = -0.1$
d. $r = 0.30$

13. **WE13** A survey asked random people for their house number and the combined age of the household members. The following data was collected:

House number	Total age of household
14	157
65	23
73	77
58	165
130	135
95	110
54	94
122	25
36	68
101	53
57	64
34	120
120	180
159	32
148	48
22	84
9	69

a. Using the house number as the independent variable, display this data in a scatterplot.
b. Comment on the correlation between house numbers and total age of household members.
c. Calculate Pearson's correlation coefficient. Give your answer correct to four decimal places.
d. Draw conclusions about the context of the value of Pearson's correlation coefficient calculated in part c.

Complex unfamiliar

14. Data showing the number of people on eight fundraising committees and the annual funds raised are given in the table.

Number of people oncommittee	3	6	4	8	5	7	3	6
Annual funds raised $(\$)$	4500	8500	6100	12 500	7200	10 000	4700	8800

Determine the value of r, and explain whether it would be appropriate to conclude that the increase in funds raised is caused by an increase in the number of people on the committee.

Fully worked solutions for this chapter are available online.

LESSON
9.6 Review

9.6.1 Summary

Hey students! Now that it's time to revise this chapter, go online to:

- **Access the chapter summary**
- **Review your results**
- **Watch teacher-led videos**
- **Practise questions with immediate feedback**

Find all this and MORE in jacPLUS

9.6 Exercise

learnon

9.6 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4	5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17	18, 19, 20

These questions are even better in jacPLUS!

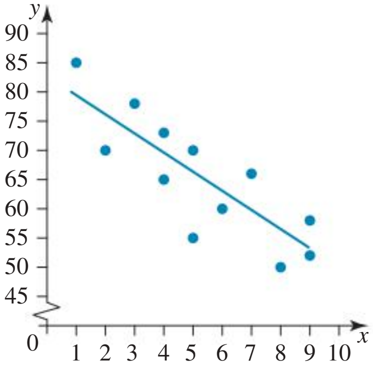
- Receive immediate feedback
- Access sample responses
- Track results and progress

Find all this and MORE in jacPLUS

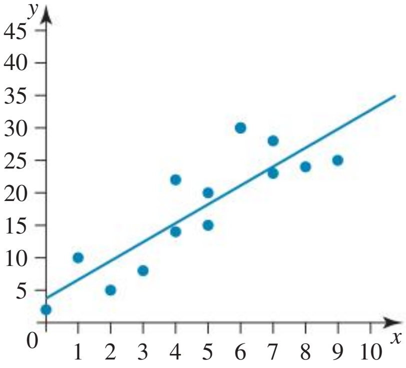
Simple familiar

1. For each of the following graphs:
   - i. justify whether the line of best fit is appropriate
   - ii. describe the type of correlation shown in the graph.

a.



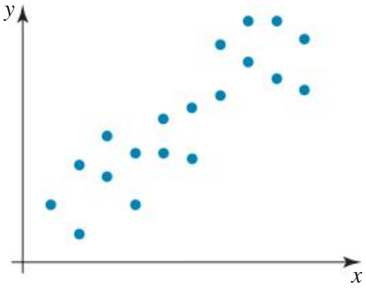
b.



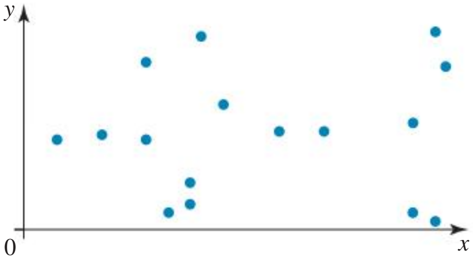
2. For each of the following graphs:

- **i.** describe the strength of correlation between the independent and dependent variables
- **ii.** draw a line of best fit where possible.

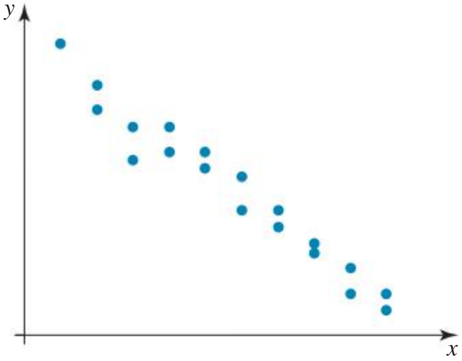
a.



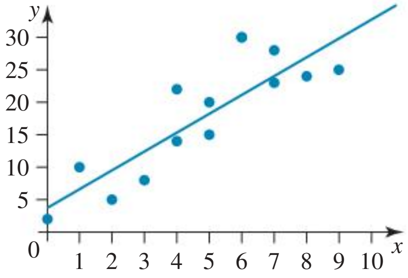
b.



c.



3. Determine the regression line equation for the graph shown.



4. Identify the independent and dependent variable for each of the following scenarios.

- **a.** In a junior Science class, students plot the time taken to boil various quantities of water.
- **b.** Extra buses are ordered to transport a number of students to the school athletics carnival.

Complex familiar

5. Eugene is selling leather bags at the local market. During the day he keeps records of his sales. The table below shows the number of bags sold over one weekend and their corresponding prices (to the nearest dollar).

Price of a bag ($)	30	35	40	45	50	55	60	65	70	75	80
Number of bags sold	10	12	8	6	4	3	4	2	2	1	1

   a. Construct a scatterplot of the data.
   b. State the type of association between the two variables.

6. The table below shows the number of questions solved by each student on a test, and the corresponding total score on that test.

Number of questions	2	0	7	10	5	2	6	3	9	4	8	3	6
Total score	22	39	69	100	56	18	60	36	87	45	84	32	63

   a. Construct a scatterplot of the data.
   b. Suggest the type of association shown in the scatterplot.

7. Ari was given a baby rabbit for his birthday. To monitor the rabbit's growth, Ari decided to measure it once a week.

The table below shows the length of the rabbit for various weeks.

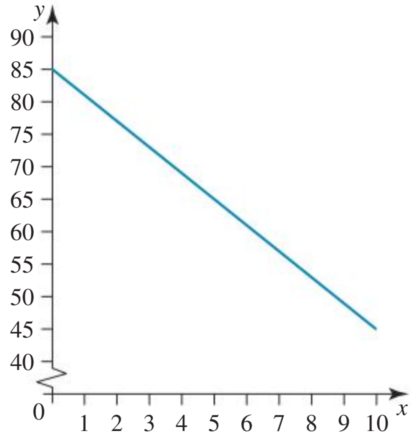
Week number, n	1	2	3	4	6	8	10	13	14	17	20
Length, l (cm)	20	21	23	24	25	30	32	35	36	37	39

Construct a scatterplot of the data and draw a line of best fit.

8. Determine what type of correlation an r-value of 0.64 indicates.

9. A gardener tracks a correlation coefficient of 0.79 between the growth rate of his trees and the amount of fertilizer used. Determine what can the gardener conclude from this result.

10. Estimate the regression equation of the linear graph shown.



11. An electrician charges a call out fee of \$150 and an hourly rate of \$80. The electrician uses the following equation to calculate his bill: $C = 80h + 150$ where C is the cost, in dollars, and h is the number of hours worked.

a. Determine the cost if the electrician worked for 3 h.
b. Determine the cost if the electrician worked for 7 h and 30 min.
c. Determine the hours worked if the cost was \$550.
d. Determine the hours worked if the cost was \$670.

12. Determine the line of best fit that passes through the points $(-2, 4)$ and $(3, -1)$.

13. Use the following data to complete this question.

x	10	9	8	7	6	5	4	3	2	1
y	6	10	4	11	13	18	15	19	21	26

a. Plot the data on a scatterplot.
b. Comment on the direction and strength of the data.
c. Determine Pearson's correlation coefficient.
d. Use your answer from part **c** to further discuss the correlation between the variables.

14. A driver tracks how many kilometres they travelled against the amount of fuel they used in litres and the data is shown in the table.

Distance travelled (km)	38	66	95	167	181	207	275	459
Fuel used (L)	3.2	5.6	7.2	12.8	14.1	16.9	21.4	37.5

a. State the dependent and independent variables.
b. Sketch a scatterplot of distance travelled against fuel used.
c. Draw a line of best fit.
d. State the equation of the line of best fit in terms of the variables. Give values correct to three decimal places.
e. Using your equation in part **d**, estimate the fuel used when travelling 50 km.
f. Explain the reliability of your answer in part **e**.

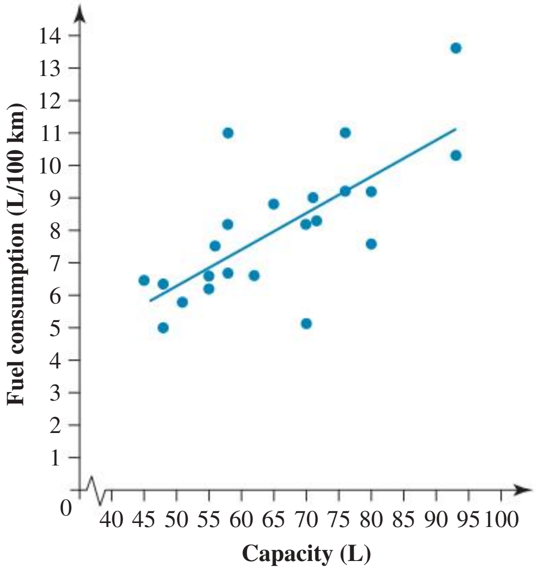
15. Data on 15 people's shoe size and the length of their hair was collected.

a. Draw a scatterplot of this data.
b. Determine the equation of the line of best fit.
c. Determine Pearson's correlation coefficient.
d. Discuss what conclusions you could draw from this data (if any).

Shoe size	Length of hair (cm)
6	9
8	14
7	12
8	1
9	7
6	8
7	5
12	22
8	15
9	8
10	18
12	4
7	5
9	9
11	3

16. An independent agency test-drove a random sample of current model vehicles and measured their fuel-tank capacity against the average fuel consumption. Along with the following scatterplot, a regression equation of $y = 0.1119x + 0.6968$ was established.

a. Identify the independent variable in this situation.
b. Rewrite the equation in terms of the independent and dependent variables.
c. It is often said that smaller vehicles are more economical. Determine correct to two decimal places the fuel consumption of a vehicle that had a 40-L fuel tank.
d. Explain whether your answer to part **c** is an example of interpolation or extrapolation.
e. Calculate, correct to the nearest whole number, the tank size of a vehicle that had a fuel consumption rate of 10.2 L per 100 km.
f. Pearson's correlation coefficient for this data is 0.516. Discuss how you can use this value to evaluate the reliability of your data.
g. List at least two other factors that could influence the data.

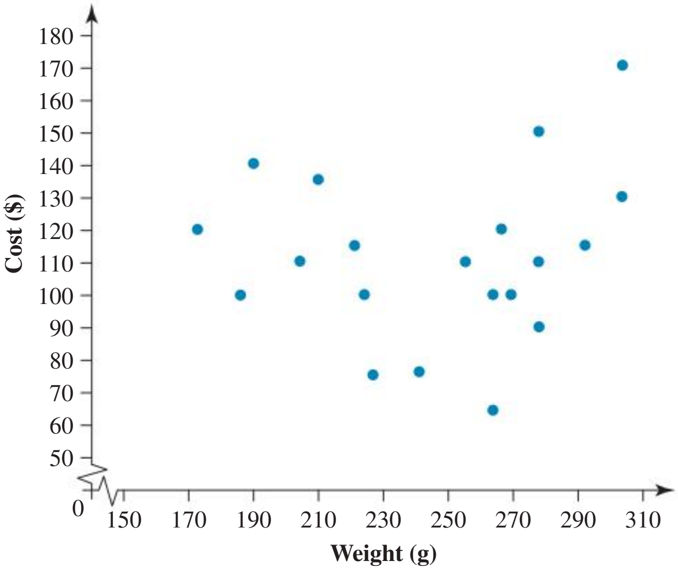


17. The weight of top-brand runners was tracked against the recommended retail price, and the results were recorded in the following scatterplot.

a. Identify the independent variable for this situation.

b. Describe the association between these two variables.

c. Identify two external factors that could explain the distribution of the data points.



Complex unfamiliar

18. For the following sample data set, predict the y-value when $x = 33$ and explain with reasons the confidence in this prediction.

x	1	5	15	25
y	7	22	64	103

19. During an interview investigating the link between the sales of healthy snack foods (functional foods) and the increasing consumer demand for these products, an advertising expert made the following comment:

> There is a correlation but it's not causation … our increasing need for healthy food and our laziness has resulted in mass innovation of functional foods.

Explain why he might have stated there is no causative link between the sales of healthy foods and laziness.

20. The Bureau of Meteorology records data such as maximum temperatures and solar exposure on a daily and monthly basis. The following data table shows the monthly average amount of solar energy measured on a horizontal surface and the monthly average maximum temperature in a particular town. (*Note:* The data values have been rounded to the nearest whole number.)

Month	**Jan.**	**Feb.**	**Mar.**	**Apr.**	**May**	**Jun.**	**Jul.**	**Aug.**	**Sep.**	**Oct.**	**Nov.**	**Dec.**
Average solar exposure (MJ)	25	21	17	11	8	6	7	10	13	18	21	24
Average max daily temp. (°C)	43	41	34	33	24	19	24	24	28	32	25	40

Using the data in the table, predict the average maximum temperature expected for a month with an average solar exposure of 3 MJ versus that expected for a month with 35 MJ. State which prediction you would determine to be the more reliable.

Fully worked solutions for this chapter are available online.

Answers

Chapter 9 Bivariate scatterplots and lines of best fit

9.2 Interpreting scatterplots

9.2 Exercise

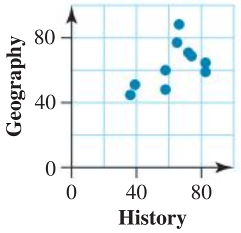
1.

	Independent	Dependent
a.	Number of hours	Test results
b.	Rainfall	Attendance
c.	Hours in gym	Visits to the doctor
d.	Length of essay	Memory taken

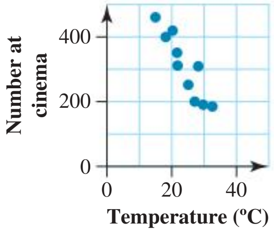
2.

	Independent	Dependent
a.	Cost of care	Attendance
b.	Age of property	Cost of property
c.	Entry requirements	Number of applicants
d.	Running speed	Heart rate

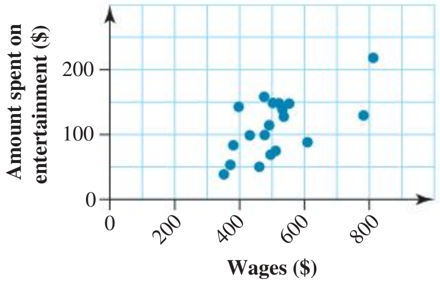
3.



4.



5.



6. Moderate positive linear association

7. Moderate positive linear association

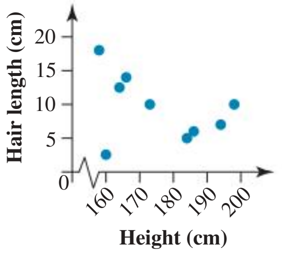
8. a. Yes — positive association
   b. Yes — positive association
   c. Yes — positive association
   d. Yes — both positive association (ages 0–18) and a negative association (ages 18–80).

9. a. Yes — positive association
   b. Yes — negative association
   c. No — no association

10. a. Weak, negative association of linear form
    b. Moderate, negative association of linear form
    c. Moderate, positive association of linear form
    d. Strong, positive association of linear form
    e. No association
    f. Non-linear moderate, positive association

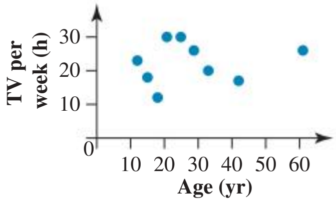
11. As the value of x increases, the value of y decreases.

12.



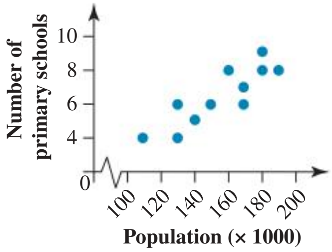
There is a linear, weak negative association between height (cm) and hair length (cm).

13.



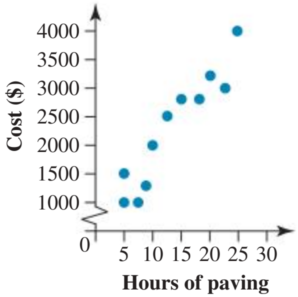
No relationship

14.



There is a linear strong/moderate positive association between population of municipality and the number of primary schools.

15.

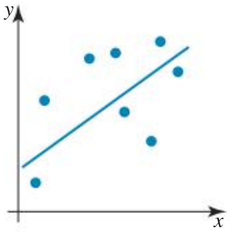


There is a trong positive association of linear form between time taken to do paving (hours) and the cost of the job ($).

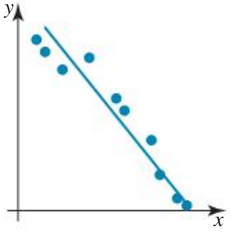
9.3 The line of best fit

9.3 Exercise

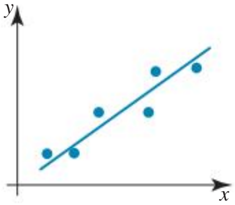
1.



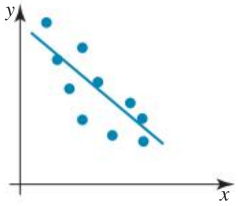
2.



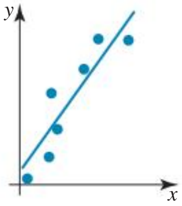
3. a.



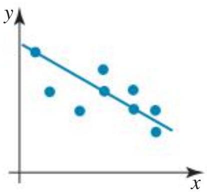
b.



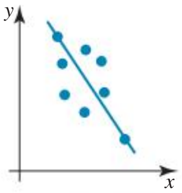
c.



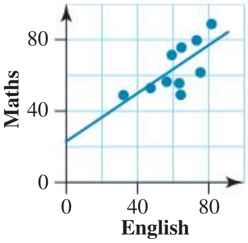
d.



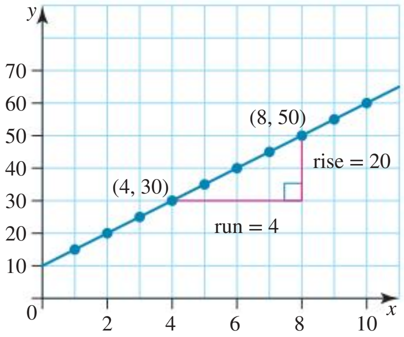
e.



4.

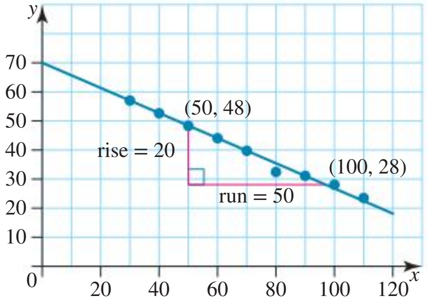


5. a.



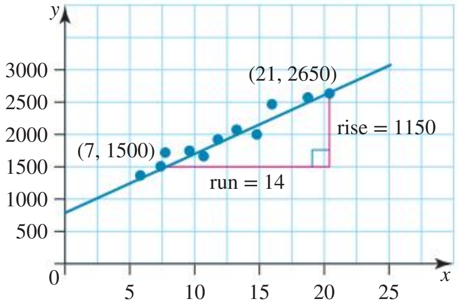
$y = 5x + 10$

b.



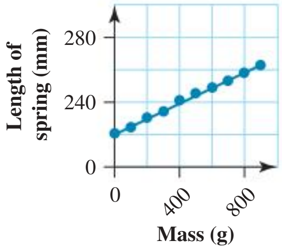
$y = -0.4x + 70$

c.



$y = 82x + 750$

6. a.

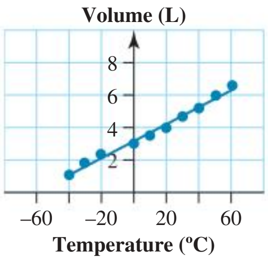


b. $L = 0.05M + 220$

7. $y = 3x - 1$

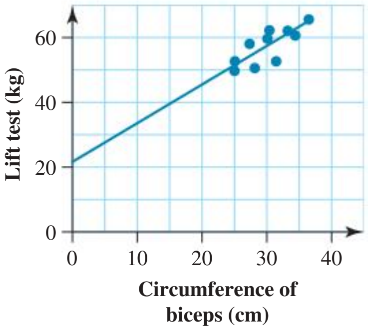
8. $y = 4x + 10$

9. a.



b. $V = 0.055T + 3.1$

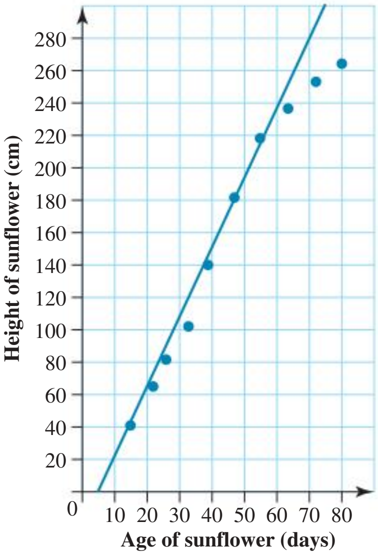
10. a.



b. $S = 1.3B + 21$

11. $y = 1.25x - 33.75$

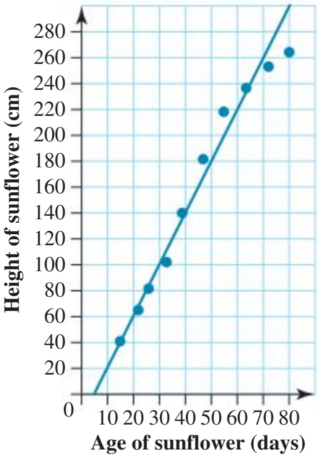
12. a.



Xavier's line is closer to the values above the line than those below it, and there are more values below the line than above it, so this is not a great line of best fit.

b. $y = 4.4x - 28$

c.



Patricia's line is more appropriate as an even amount of data points lie on either side of the line and the total distance of the points from the line appears to be minimal.

d. $y = 4x - 22$

e. The line of best fit does not approximate the height for values that appear outside the parameters of the data set and the y-intercept lies well outside these parameters.

13. Nidya's line of best fit is not a good representation of the data. In this instance having only two points of data to create the line of best fit was not sufficient. More points of data create a more accurate line of best fit.

14. The y-intercept represents the price of a ticket if a venue has no capacity.

No, this is not realistic since the smallest venues would still have some capacity.

15. Crime rate $= 0.07 \times$ population density $- 52.13$

The y-intercept represents the amount of crime in a suburb with 0 people.

No, this is not realistic since if there are 0 people in a suburb there should be no crime. Also, crime rates can't be negative.

9.4 Making predictions using a line of best fit [complex]

9.4 Exercise

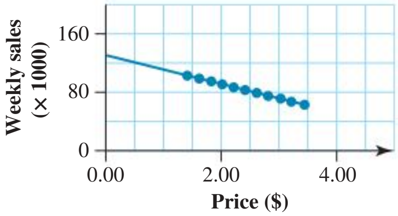
1. a. \$17.40 **b.** \$8.40 **c.** 20 km **d.** 8.5 km

2. a. 159.7 cm **b.** 178 cm **c.** 31.15 cm **d.** 25.74 cm

3. a. 755 **b.** 295 **c.** 20 °C **d.** 38 °C

4. a. \$8300 **b.** \$2500
c. 14 000 km **d.** 33 750 km

5. a–b.



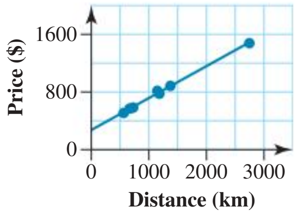
c $N = -20p + 135$

d i. 73 000 **ii.** 43 000

e \$2.75

f \$1.75

6. a.



b. $A = 0.45d + 280$

c. i. \$582 **ii.** \$1236 **iii.** \$741 **iv.** \$1763

7. a. 174

b. 60

c. The estimate in part **a** is more reliable as it was made using interpolation, it is located within the parameters of the original data set and appears consistent with the given data.

The estimate in part **b** is less reliable as it was made using extrapolation and is located well outside the parameters of the original data set.

8. a. \$112
   b. \$305
   c. All estimates outside the parameters of Georgio's original data set (400–2000 km) will be unreliable, with estimates further away from the data set being more unreliable than those closer to the data set.
   Other factors that might affect the cost of flights include air taxes, fuel costs and the choice of airlines for various flight paths.

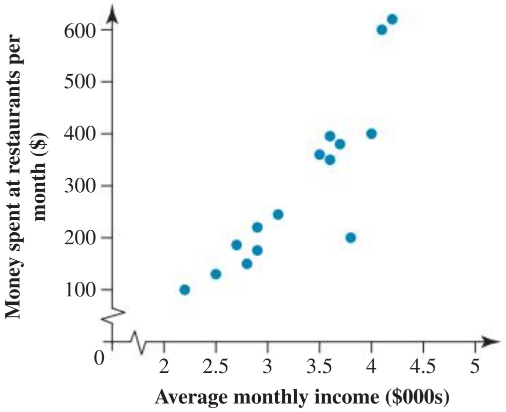
9. a. $y = -17.31x + 116.38$
   b. For each increase in 1 L of lung capacity, the swimmers will take 17.31 s less time to swim 25 m.
   c. i. 61 s   ii. 40.2 s   iii. 24.6 s
   d. As Mariana has only two data points and we have no idea of how typical these are of the data set, the equation for the line of best fit and the estimates established from it are all very unreliable.

10. a. 76
   b. 65
   c. Part **a** looks at data within the parameters of original data set range, so it is interpolation, while part **b** asks to predict data outside the parameters of the original data set (which has a range of 20–120 new customers each hour), so it is extrapolation. Extrapolation is less reliable since it makes predictions outside the given data.

11. a. \$155.74   b. 3.31 km

12. a. $y = 38.67 + 0.61x$
   b. See the table bottom of the page*

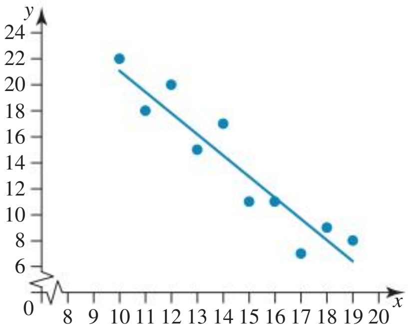
13. a.



   b. $R = -459.8 + 229.5I$
   c. \$687.70

   d. Part **c** is asking you to predict data outside of the original data set range.
   e. To the nearest \$10, \$3160. This is an example of interpolation.

14.



$y = 37.703 - 1.648x$

When $x = 23$, $y = -0.21$

It is assumed the data will continue to behave in the same manner as the data originally supplied.

15. The temperature with 12 mosquitoes is 3.6 °C . The location of the fire, air temperature, proximity to water and so on.

Exercise 9.5 Causality and correlation

9.5 Exercise

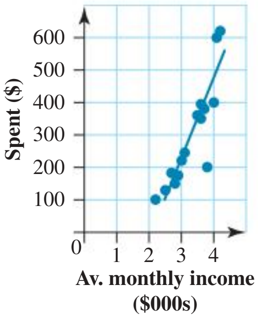
1. Correlation measures the possible association between two variables. The correlation coefficient (r) measures the strength of this association. A strong correlation does not imply causation. Causation is a measure of how much of the change in one variable is caused by the change in the other variable.
2. Strong negative association
3. Strong positive linear association
4. Strong negative linear association
5. No linear association
6. Since they are both 0.76 the strength is the same (strong). Since one is negative and one is positive, one has a negative association and the other a positive association.
7. Since 0.79 fits in the range 0.75–1.0 there is a strong positive association. Whereas 0.57 fits in the range of 0.5–0.75 so it has a moderate positive linear association. Thus, the data with the 0.75 Pearson correlation coefficient has a stronger linear association and the points fit closer to the line of best fit.

---

*12. b.

Kicking efficiency (%)	75.3	65.6	83.1	73.9	79.0	84.7	64.4	72.4	68.7	80.2
Predicted handball efficiency (%)	84.6	78.7	89.4	83.7	86.9	90.3	78.0	82.8	80.6	87.6

8. a.

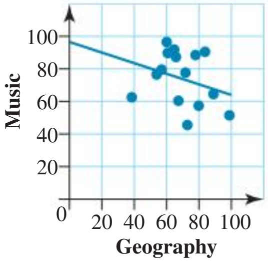


b. Money spent = 229.5 × monthly income − 460

c. $r = 0.8777$

d. There is a strong positive linear association between the average monthly income and money spent at restaurants per month.

9. a.

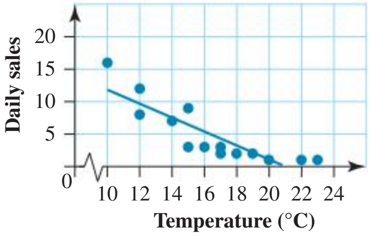


b. Music mark = −0.3 × Geography mark + 95

c. $r = -0.28$

d. There is a weak negative linear association between the Geography marks and the Music marks.

10. a.



b. Daily sales = −1.07 × temperature + 22.5

c. $r = -0.8621$

d. There seems to be a strong negative association between daily sales and temperature.

11. $r = 0.8947$, which indicates a strong positive linear association.

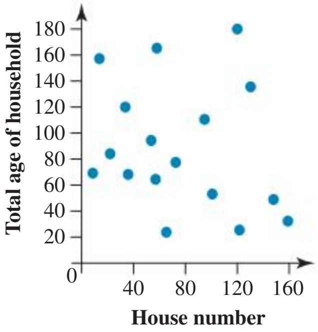
12. a. Indicates a moderate positive linear association

b. Indicates a strong negative linear association

c. Indicates a weak to no negative linear association

d. Indicates a weak positive linear association

13. a.



b. The data points appear random, indicating little to no correlation.

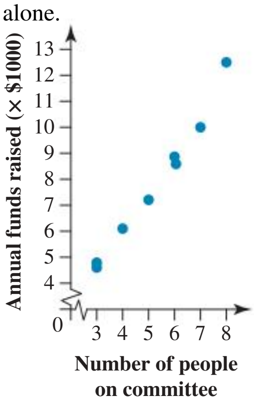
c. $r = -0.2135$

d. There is no association between the house number and the age of the household.

14. There is a strong (almost perfect) positive linear correlation between the number of people on the committee and the annual amount of funds raised.

$r = 0.99$

Causation cannot be established on a strong correlation alone.



9.6 Review

9.6 Exercise

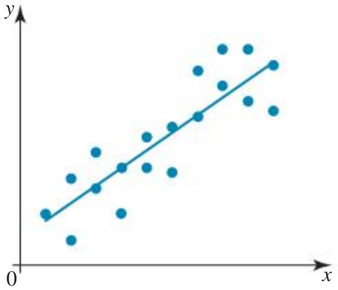
1. a. i. Yes, it is a good line of best fit since it has the points evenly distributed above and below the line, with 6 above and 6 below.

ii. Moderate negative correlation

b. i. No, this is not an appropriate line of best fit as it has more points below the line than above the line.

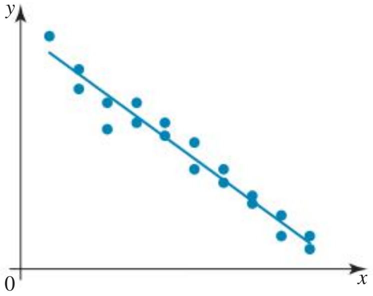
ii. Moderate positive correlation

2. a. i. Moderate positive correlation



ii.

b. i. No correlation



ii.

c. i. Strong negative correlation

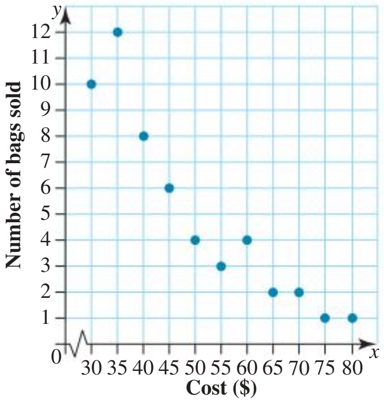
ii. No line of best fit possible.

3. $y = 2.9x + 4.2$

4. a. Independent variable = quantity of water
Dependent variable = time

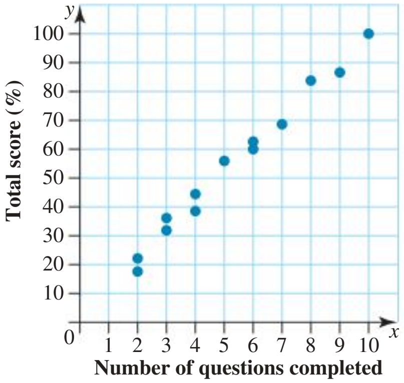
b. Independent variable = number of students
Dependent variable = number of buses required

5. a.



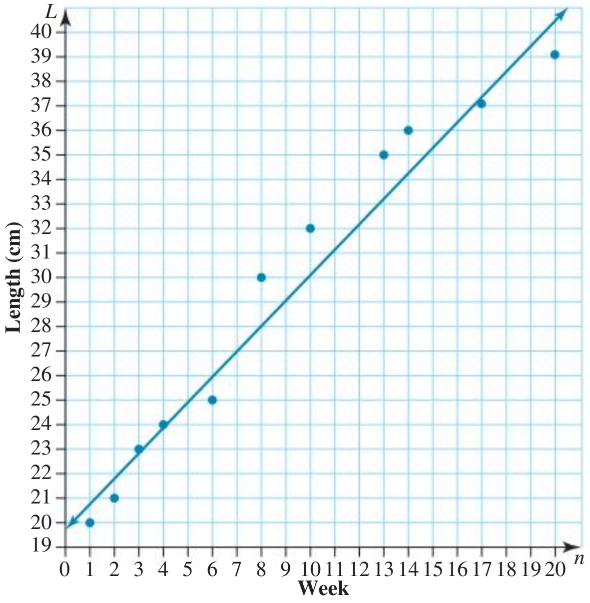
b. Negative, linear, moderate.

6. a.



b. Strong, positive, linear

7.



8. Moderate positive correlation

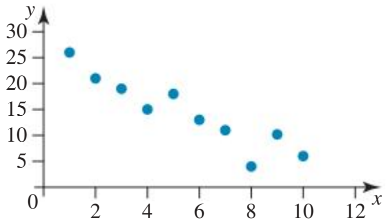
9. The growth of the trees is influenced by the amount of fertilizer used. (There is a strong positive association between the growth rate of the trees and amount of fertiliser used.)

10. $y = -4x + 85$

11. a. \$390 **b.** \$750
c. 5 h **d.** 6.5 h

12. $y = -x + 2$

13. a.



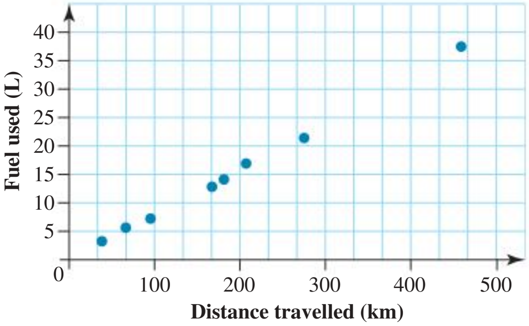
b. Strong negative correlation

c. $r = -0.9329$

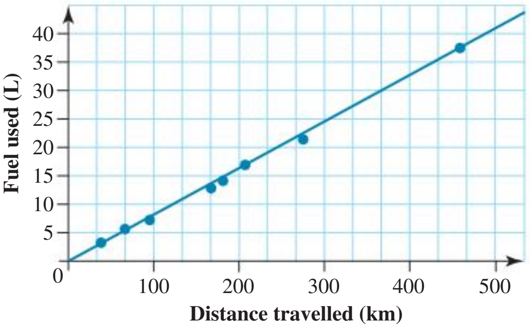
d. Pearson's correlation coefficient confirms a strong association between the two variables.

14. a. Independent variable: Distance travelled
Dependent variable: Fuel used

b.



c.

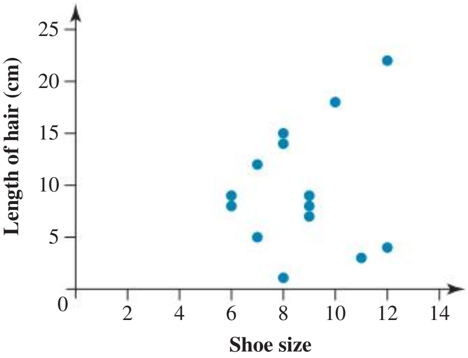


d. Fuel used (L) = 0.081 × distance travelled

e. 4.05 L

f. The estimate in part **e** is reliable since the prediction uses interpolation. This is when making a prediction inside the range of the supplied data.

15. a.



b. $y = 4.0386 + 0.6157x$

c. $r = 0.2055$

d. Based on the graph as well as Pearson's correlation coefficient, there is no association between the two variables. Therefore, no solid conclusions can be made to suggest a change in a person's shoe size will affect the length of their hair.

16. a. Fuel tank capacity

b. Average fuel consumption = 0.6968 + 0.1119 × fuel tank capacity

c. 5.17

d. Extrapolation

e. 85 L

f. This value indicates a moderate association between the variables. Therefore the data can be used; however, other factors should be considered.

g. Various possible answers, e.g. the manner in which a person drives the vehicle, weather conditions, road condition

17. a. Weight (grams)

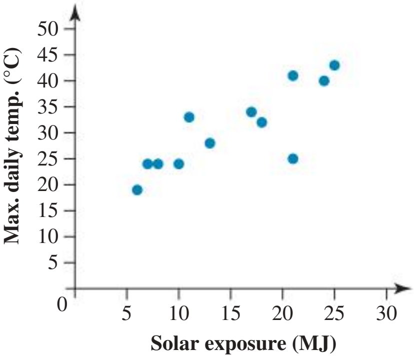
b. No correlation

c. Various possible answers, e.g. popularity of the shoe, desired profits

18. $y = 135.619$, with a Pearson's coefficient of 0.9998.

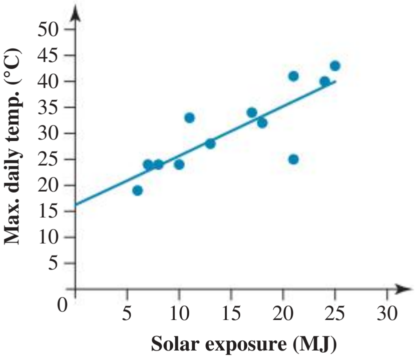
19. While there appears to be a link between the laziness of people and the increase in the sales of healthy foods, there are also many other factors, other than laziness. Based on this observation alone, the cause of an increase in healthy foods cannot be concluded to be due to laziness.

20.



$r = 0.8242$

This value indicates a strong association between the two variables.



Max daily temperature = 16.232 + 0.9515 × average solar exposure

When x-value equals 3 MJ, Max Temp equals 19 °C.
When x-value equals 35 MJ, Max Temp equals 50 °C
3 MJ is outside the data set, thus extrapolation. 35 MJ is also outside the data set, thus extrapolation. Since 3 MJ is closer to the lowest value of 6 MJ than 35 MJ with its closest data value being 25 MJ. Therefore they are both extrapolation but the 35 MJ prediction is less reliable.

UNIT 4 Graphs, data and loans
TOPIC 2 Summarising and comparing data

10 Summarising and interpreting data

LESSON SEQUENCE

Fully worked solutions for this chapter are available online.

on Resources

Solutions	Solutions — Chapter 10
Digital documents	Learning matrix — Chapter 10 Quick quizzes — Chapter 10 Chapter summary — Chapter 10

LESSON
10.1 Overview

10.1.1 Introduction

Understanding data is essential. Identifying the mode, calculating the median and mean, and interpreting quartiles, deciles and percentiles help people grasp central tendencies and data distribution, essential for managing large datasets. Professionals such as data analysts, statisticians, market researchers and financial analysts rely on these skills for informed decision-making.

Learning to calculate and interpret measures of spread like range, interquartile range and standard deviation fosters critical thinking, crucial for careers in epidemiology and policy analysis. Using everyday language to describe data and interpret graphs improves communication skills, vital in data journalism and education. Additionally, media literacy helps critically assess the accuracy and reliability of data, which is essential for policy analysts ensuring public policies are based on accurate data.

Lesson	Lesson title	Syllabus links
10.2	**Measures of central tendency and mode**	○ Identify mode from a dataset. ○ Calculate measures of central tendency: median and mean ($\bar{x}$) from a dataset of n values. • mean: $\bar{x} = \frac{\sum x}{n}$ where $\sum x$ = sum of all data values
10.3	**Measures of spread**	○ Calculate quartiles from a dataset. ○ Interpret quartiles, deciles and percentiles from a graph [complex]. ○ Use everyday language to describe spread, including spread out, dispersed, tightly packed, clusters, gaps, more/less dense regions, outliers. ○ Calculate and interpret statistical measures of spread, including the range, interquartile range and standard deviation [complex]. • range = highest score – lowest score • $\text{IQR} = Q_3 - Q_1$
10.4	**Outliers [complex]**	○ Investigate the effect of outliers on the mean and the median [complex]. ○ Use everyday language to describe spread, including spread out, dispersed, tightly packed, clusters, gaps, more/less dense regions, outliers.
10.5	**Applications of measures of centre and spread [complex]**	○ Investigate the suitability of measures of central tendency in various real-world contexts [complex]. ○ Investigate real-world examples from the media illustrating inappropriate uses of measures of central tendency and spread [complex].

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LESSON
10.2 Measures of central tendency and mode

SYLLABUS LINKS

- Identify mode from a dataset.
- Calculate measures of central tendency: median and mean ($\bar{x}$) from a dataset of n values.
  - mean: $\bar{x} = \frac{\sum x}{n}$ where $\sum x$ = sum of all data values

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10.2.1 Mean and median

eles-1905

int-4621

- Summary statistics are a way to easily interpret a set of data, particularly when dealing with a large set of data or comparing more than one dataset.
- One such **summary statistic** is the measure of central tendency.
- Recall from Unit 2 that a population is the entire set of subjects or objects being investigated and a sample is a smaller selection of the population.

The mean

int-3818

- The mean — often referred to as the average — is used to describe the balance of all values in a set of data. It is a single number used to describe the centre of a dataset; whether a population or a sample.
- It is not always a value in a dataset; for example, the mean number of dogs in an Australian household is 1.5.
- Obtaining the mean of a population is not always practicable, so the mean of a sample is often calculated instead.

Population and sample means

$$\bar{x} = \frac{\text{sum of all data values in the sample}}{\text{number of values}} = \frac{\sum x}{n}$$

The median

int-3819

- The median is used to identify the middle position of an ordered set of data. This value indicates the mid-point or halfway point of the dataset. Exactly half of the values are greater than the median, and exactly half are less than the median.
- To determine the median, arrange the values in the set of data in *ascending* order (smallest to largest).
- For an *odd* number of values, the median is the *middle* value. For example, if the data is 6, 7, 9, 10 and 15, then the median is 9.

$$\begin{array}{c} 6, 7, 9, 10, 15 \\ \uparrow \\ \text{Median} = 9 \end{array}$$

- For an *even* number of values, the median is the *mean* of the two middle values. In this case, the median may not be an actual data value. For example, if the data is 3, 5, 7, 9, 11 and 17, the median is 8.

$$3, 5, 7, \,|\, 9, 11, 17$$

$$\uparrow$$

$$\text{Median} = \frac{7+9}{2} = 8$$

- If the set of data contains n values, the position of the median when the values are arranged in numerical (ascending) order can be calculated using the following formula.

$$\text{Median position } = \frac{n+1}{2}$$

For an *even* number of values

$$\text{If, } n = 6 \text{ then } \frac{n+1}{2} = \frac{6+1}{2} = \frac{7}{2} = 3.5$$

The median is halfway between the third and fourth values.

For an *odd* number of values

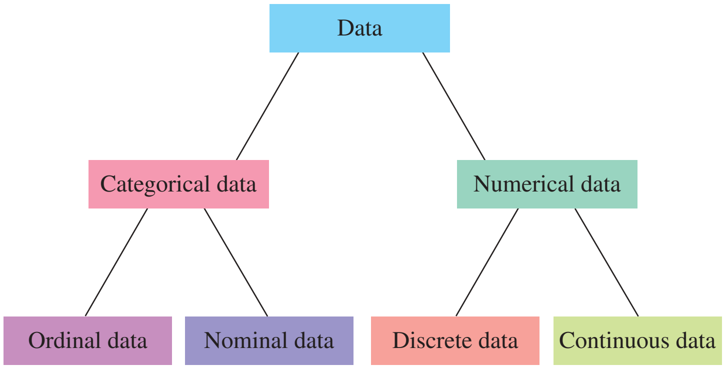
$$\text{If, } n = 5 \text{ then } \frac{n+1}{2} = \frac{5+1}{2} = \frac{6}{2} = 3$$

The median is the third value.

10.2.2 The mode

int-3820

- The **mode** is the value that occurs most in a set of data.
- The mode is not considered a measure of central tendency; it is simply one of the features of a dataset.
- The mode can be found for both numerical and categorical data. (Recall that **categorical data** are data that can be grouped or classified, and **numerical data** are data that can be counted or measured.)



- Some sets of data have no mode at all. If all of the values occur only once, the dataset is not modal.
- Datasets can also have more than one mode.
- A dataset with two modes is referred to as ‘bimodal’.
- A dataset with three modes is referred to as ‘trimodal’.
- A dataset with more than three modes is referred to as ‘multimodal’.

WORKED EXAMPLE 1 Calculating and interpreting mean, median and mode

Ten Year 11 students were asked how many hours they spent competing in sport out of school hours. The results of the survey are:

$$2, 4, 5, 6, 7, 2, 2, 0, 5, 1$$

Calculate the values of:

a. the mean **b. the median** **c. the mode**

and explain what they tell us about the data.

THINK	WRITE
a. **1.** Calculate the sum of all the values in the dataset.	**a.** Sum of all the values $= 2 + 4 + 5 + 6 + 7 + 2 + 2 + 0 + 5 + 1$ $= 34$
2. Count the number of values in the dataset (0 must be included).	Number of values = 10
3. The mean is the sum of all the values divided by the number of values.	$\text{Mean } (\bar{x}) = \dfrac{\text{sum of all the values}}{\text{number of values}}$ $= \dfrac{\sum x}{n}$ $= \dfrac{34}{10}$ $= 3.4$
4. Explain what the mean tells us about the data.	If the total number of hours spent competing in sport was shared equally between all ten students, each student would spend on average 3.4 hours competing in sport.
b. **1.** Arrange the values in the dataset in ascending order (smallest to largest).	**b.** 0, 1, 2, 2, 2, 4, 5, 5, 6, 7
2. The median is the middle value. Locate the position of the median.	There are 10 values in the set of data so the median is the $\dfrac{10 + 1}{2} = 5.5$th value (mean of the 5th and 6th values). 0, 1, 2, 2, 2, $\vert$ 4, 5, 5, 6, 7
3. Determine the median value by calculating the mean of 2 and 4.	$\text{Median} = \dfrac{2 + 4}{2}$ $= \dfrac{6}{2}$ $= 3$
4. Explain what the median tells us about the data.	On average, half of the students compete in more than 3 hours of sport and half of the students compete in less than 3 hours.
c. **1.** The mode is the most common value in the set of data.	**c.** 2, 4, 5, 6, 7, 2, 2, 0, 5, 1 Mode = 2
2. Explain what the mode tells us about the data.	The most common amount of time that students spend competing in sport is 2 hours.

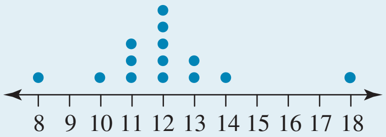
10.2.3 Measures of central tendency in dot plots and stem plots

- Data may be represented in different formats, such as a dot plot or a stem (stem-and-leaf) plot.
- In a dot plot, each data value is represented as a dot on a number line.
- In a stem-and-leaf plot, each data value is split into two components, the stem and the leaf. Data is then grouped according to its stem.

WORKED EXAMPLE 2 Determining mean from a dot plot and a stem plot

Calculate the mean of each of the following sets of data. (Give your answer to 1 decimal place.)

a.



b. Key: 12 |1 = 121

Stem	Leaf
12	1 5 9
13	4 7 9
14	8 8
15	2 8
16	3 8
17	2

THINK	WRITE
a. 1. Write the list of data represented in the dot plot.	**a.** 8, 10, 11, 11, 11, 12, 12, 12, 12, 12, 13, 13, 14, 18
2. Calculate the sum of all the values in the dataset.	Sum of all the values $= 8 + 10 + 11 + 11 + 11 + 12 + 12 + 12 + 12 + 12 + 13 + 13 + 14 + 18 = 169$
3. Count the number of values in the dataset.	Number of values = 14
4. The mean is the sum of all the values divided by the number of values.	$\text{Mean } (\bar{x}) = \dfrac{\text{sum of all the values}}{\text{number of values}}$ $= \dfrac{\sum x}{n}$ $= \dfrac{169}{14}$ ≈ 12.1
b. 1. Write the list of data represented in the stem plot.	**b.** 121, 125, 129, 134, 137, 139, 148, 148, 152, 158, 163, 168, 172

2. Calculate the sum of all the values in the dataset.

$$\begin{aligned}\text{Sum of all the values} &= 121 + 125 + 129 + 134 + 137 + 139\\ &\quad + 148 + 148 + 152 + 158 + 163\\ &\quad + 168 + 172\\ &= 1894\end{aligned}$$

3. Count the number of values in the dataset.

$$\text{Number of values} = 13$$

4. The mean is the sum of all the values divided by the number of values.

$$\begin{aligned}\text{Mean }(\bar{x}) &= \frac{\text{sum of all the values}}{\text{number of values}}\\ &= \frac{\sum x}{n}\\ &= \frac{1894}{13}\\ &\approx 145.7\end{aligned}$$

10.2.4 Measures of central tendency and mode in frequency distribution tables

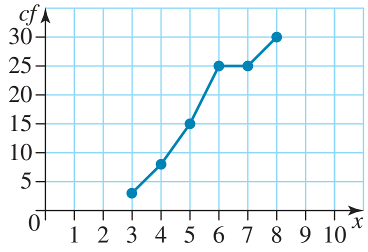
Cumulative frequency

- **Cumulative frequency (*cf*)** is the running total of frequencies in a dataset. It shows the sum of all previous frequencies up to a certain point.
- Cumulative frequency is recorded as a **cumulative frequency table**.
- The final value in a cumulative frequency table will always equal the total number of observations in the dataset.
- The cumulative frequency table given shows the number of movies watched in the last month by a group of 30 Year 9 students.

Data (x)	Frequency (f)	Cumulative frequency (cf)
3	3	3
4	5	$3 + 5 = 8$
5	7	$8 + 7 = 15$
6	10	$15 + 10 = 25$
7	0	$25 + 0 = 25$
8	5	$25 + 5 = 30$

Ogives

- Data from a cumulative frequency table can be plotted to form a **cumulative frequency curve**, which is also called an **ogive** (pronounced *oh-jive*).
- To plot an ogive for data that is in class intervals, the maximum value for the class interval is used as the value against which the cumulative frequency is plotted.
- Further uses of an ogive will be explored in the next lesson.



Cumulative frequency curve

Calculating the mean from a frequency distribution table

If data are presented in a frequency distribution table, the formula used to calculate the mean is:

$$\bar{x} = \frac{\text{sum of (frequency} \times \text{data values)}}{\text{sum of frequencies}} = \frac{\sum (f \times x)}{n}$$

- The table below shows the number of goals a player scored in 15 games.

Goals (x)	Frequency (f)	Frequency × score ($f \times x$)	Cumulative frequency (cf)
1	4	$1 \times 4 = 4$ goals	4
2	6	$2 \times 6 = 12$ goals	$4 + 6 = 10$
3	2	$3 \times 2 = 6$ goals	$10 + 2 = 12$
4	3	$4 \times 3 = 12$ goals	$12 + 3 = 15$
	$n = 15$	$\sum (f \times x) = 34$ goals	

The 8th data point occurs in here (between 4 games and 10 games); hence the median is 2 goals.

To determine the total number of goals, each data value (score) in the table is multiplied by its corresponding frequency.

All the ($f \times x$) results are added together to find the total number of goals scored.

The total sum is then divided by the total number of games to determine the **mean**:

$$\bar{x} = \frac{\sum (f \times x)}{n} = \frac{34}{15} = 2.3 \text{ goals}$$

To calculate the **median**, determine the position of each score from the cumulative frequency column.

$$\begin{aligned}\text{Median position} &= \frac{n+1}{2} \\ &= \frac{(15+1)}{2} \\ &= 8\end{aligned}$$

- The mode is the score with the *highest frequency*. In this example, the most common number of goals scored per game is 2, with a frequency of 6.

WORKED EXAMPLE 3 Determining mean, median and mode from a frequency distribution table

For the data shown in the frequency distribution table, calculate the:

a. **mean** **b.** **median** **c.** **mode.**

Score (x)	**Frequency (f)**
4	**1**
5	**2**
6	**5**
7	**4**
8	**3**
Total	**15**

THINK **WRITE**

a. 1. Construct a table with four columns titled Score (x), Frequency (f), Frequency × score ($f \times x$) and Cumulative frequency (cf).

a.

Score (x)	Frequency (f)	Frequency × score ($f \times x$)	Cumulative frequency (cf)
4	1		
5	2		
6	5		
7	4		
8	3		
Total	$n = 15$		

2. Calculate the values of the $f \times x$ column by multiplying the frequency by the score in each row.
In the first row: $f \times x = 4 \times 1$
$= 4$

3. Calculate the total sum of all the data values by adding all values in the $f \times x$ column. The total of the $f \times x$ column is 96.

Score (x)	Frequency (f)	Frequency × score ($f \times x$)	Cumulative frequency (cf)
4	1	4	
5	2	10	
6	5	30	
7	4	28	
8	3	24	
Total	$n = 15$	$\sum(f \times x)$ $= 96$	

4. Write the rule for the mean.

$$\bar{x} = \frac{\sum(f \times x)}{n}$$

5. Substitute the known values into the rule and evaluate.

$$\bar{x} = \frac{96}{15}$$
$$= 6.4$$

6. Write your answer.

The mean of the dataset is 6.4.

b. 1. Complete the cf column by calculating the running total of frequencies.

b.

Score (x)	Frequency (f)	Frequency × score ($f \times x$)	Cumulative frequency (cf)
4	1	4	1
5	2	10	$1 + 2 = 3$
6	5	30	$3 + 5 = 8$
7	4	28	$8 + 4 = 12$
8	3	24	$12 + 3 = 15$
	$n = 15$	$\sum(f \times x)$ $= 96$	

	2. Locate the position of the median using the rule $\frac{n+1}{2}$, where $n = 15$. This places the median as the 8th score.		The median is the $\frac{15+1}{2}$th or 8th score.
	3. Use the cumulative frequency column to locate the 8th score and answer the question.		The median of the dataset is 6.
c.	**1.** The mode is the score with the highest frequency.	**c.**	The score with the highest frequency is 6.
	2. Write your answer.		The mode of the dataset is 6.

10.2.5 Measures of central tendency and mode in real-world contexts

- The mean is often called the average in real-world contexts. It is used:
  - In scientific research to describe many different phenomena, from weather patterns to sports statistics.
  - To analyse the results of students in schools and universities.
- In some contexts, the mean will round to the nearest whole number to make sense. For example, a shoe shop selling a mean of 26.7 shoes per day could be interpreted as selling 26 to 27 pairs per day.
- The median is the middle value of a dataset by position, and is often used when the information being investigated has some extremely high or extremely low values in the dataset.
- The median is not greatly affected by these extreme values, as it looks at the number of data points, not their values. For this reason, it is often used in economics, such as to show median household income and median house prices.
- The mode is less commonly used in real-world contexts, but it can still be a practical and useful way of describing information.
- For example, clothing shop owners will look at the highest-selling clothing size to make sure they have the most floor stock in that particular size.

10.2.6 Grouped data

- **Class intervals** group large datasets into equal-sized categories, ensuring each value belongs to only one interval.
- Discrete data intervals are written with minimum and maximum values (e.g. 10–20), while continuous data use inequalities (e.g. $10 \leq x < 20$).
- A frequency table displays how often values appear in each interval, with the interval that has the highest frequency called the **modal class**.
- The **midpoint**, calculated as the average of the minimum and maximum values, represents the interval and is used to determine the mean of grouped data.

Calculating the mean from grouped data

$$\bar{x} = \frac{\text{sum of (midpoints} \times \text{frequency)}}{\text{sum of frequencies}} = \frac{\sum (x_{\text{mid}} \times f)}{n}$$

where:

x_{mid} is the midpoint of the class interval

f is the frequency of the class interval

n is the total number of data values

$\sum$ is the summation symbol, meaning ‘sum of’.

WORKED EXAMPLE 4 Determining mean using class intervals

A takeaway food shop is trying to improve its ordering system. The manager takes a survey of the number of hamburgers made each day for a month and collects the data shown in the frequency table. Calculate the mean number of hamburgers made each day.

Class interval	Frequency (f)
0–4	**6**
5–9	**8**
10–14	**2**
15–19	**3**
20–24	**5**

THINK

1. To calculate the mean of grouped data, a single value is needed to represent the class interval.
   To calculate the midpoint of the class intervals, calculate the mean of the maximum and minimum values for each class interval.
   - For the first class interval 0–4,
     $x_{\text{mid}} = \frac{0+4}{2} = 2$
   - For the second class interval 5–9,
     $x_{\text{mid}} = \frac{5+9}{2} = 7$

WRITE

Class interval	Frequency (f)	Midpoint (x_{mid})
0–4	6	②
5–9	8	⑦
10–14	2	12
15–19	3	17
20–24	5	22

▶

2. Add a new column to the table, $x_{\text{mid}} \times f$, and multiply the midpoint by the frequency of each class interval to calculate a value to represent the sum of all values in that class interval.
   - For the first class interval 0–4, $x_{\text{mid}} \times f = 2 \times 6 = 12$.

Class interval	Frequency (f)	Midpoint (x_{mid})	$x_{\text{mid}} \times f$
0–4	6	2	12
5–9	8	7	56
10–14	2	12	24
15–19	3	17	51
20–24	5	22	110
Totals	24		253

3. Calculate the total sum of the frequency (f) and $x_{\text{mid}} \times f$ columns.

The total of the $x_{\text{mid}} \times f$ column is 253.

The total of the frequency column is 24.

4. Substitute the total of the $x \times f$ column and the total of the frequency column into the formula for the mean of grouped data.

$$\bar{x} = \frac{\sum (x_{\text{mid}} \times f)}{n}$$
$$= \frac{253}{24}$$
$$\approx 10.54$$

5. Write your answer.

The mean number of hamburgers made per day is approximately 10.54. This means that, on average, between 10 and 11 hamburgers are made every day.

Exercise 10.2 Measures of central tendency and mode

learnon

10.2 Quick quiz on

10.2 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15	N/A	N/A

Note: Where necessary give answers correct to 1 decimal place.

Simple familiar

1. **WE1** For each of the following datasets, calculate the:
   - i. mean
   - ii. median
   - iii. mode.

   a. $3, 3, 4, 5, 5, 6, 6, 7, 8, 8, 8, 9$
   b. $12, 18, 4, 17, 5, 12, 0, 10, 12$
   c. $42, 29, 11, 28, 21$
   d. $8, 2, 5, 6, 9, 9, 7, 3, 2, 9, 3, 7, 6, 8$
   e. $5, 5, 6, 4, 8, 3, 4$
   f. $3.7, 3.5, 3.8, 3.8, 3.5$

2. For each of the following datasets, calculate the mean, median and mode.
   a. 10, 12, 21, 23, 23, 25, 44
   b. 7, 8, 10, 6, 9, 11, 4, 12, 2
   c. 50, 44, 50, 46, 50, 48
   d. 2.5, 1.4, 1.7, 2.1, 1.4, 1.8, 1.6, 1.7, 2.9

3. Determine the mean, median and mode of this dataset.

1024, 1032, 1067, 1112, 1112, 1178, 1236, 1269, 1290, 1301, 1345, 1357, 1365, 1377, 1400

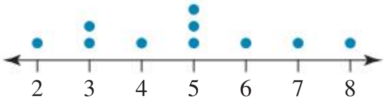
4. The number of students standing in line at the tuck shop 5 minutes after the start of lunch was recorded over a 2-week period. The results were as follows:

52, 45, 41, 42, 53, 45, 47, 32, 52, 56

Determine the mean number of students standing in line at this time for this 2-week period. Round your answer to the nearest whole number.

5. The police conducted a survey of the speed of cars down a highway. The lowest and highest speeds recorded were 91 km/h and 154 km/h respectively. The average speed was 104 km/h, and the police found that the speed most commonly recorded was 101 km/h. Half of the cars were also found to be travelling under 102 km/h. Identify the values of the mean, median and mode.

6. **WE2a** Calculate the mean of the dataset shown.



7. **WE2b** Calculate the mean of the dataset shown.

Key: 1|6 = 16 years

Stem	**Leaf**
1	5 6 7 7 8 9 9
2	1 2 4 8 8
3	0 1 1 1 5
4	2 3
5	3

8. **WE3** Calculate the mean, median and mode in each of the following frequency tables.

a.

Score (x)	Frequency (f)
1	12
2	10
3	8
4	7
5	2

b.

Score (x)	Frequency (f)
25	1
26	15
27	11
28	7
29	3

c.

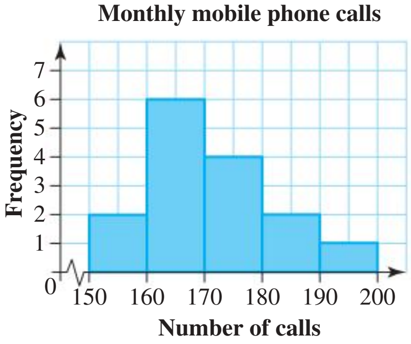
Score (x)	Frequency (f)
1.5	2
2.0	9
2.5	7
3.0	11
3.5	4

9. **WE4** A state librarian is completing a stocktake across all state libraries. They collated the number of DVDs at each library shown in the frequency table.

Number of DVDs (x)	Frequency (f)
1–15	3
16–30	9
31–45	8
46–60	11
61–75	10
76–90	14
91–105	15
106–120	18

Calculate the mean number of DVDs across each state library.

Questions 10–12 refer to the following graph.



10. Calculate the mean number of calls made on mobile phones in the month shown in the graph.

11. Determine the median class interval for the number of calls made on mobile phones in the month shown in the graph.

12. Identify the modal class for the number of calls made on mobile phones in the month shown in the graph.

13. Create a dataset that fits each of the following descriptions.

- **a.** Five data values with a mean of 3 and a mode of 3
- **b.** Five data values with a mean of 3 and a mode of 4
- **c.** Five data values with a mean of 3 and a median of 2

14. The mean length of three pieces of string is 145 cm. If they are joined together from end to end, determine their total length.

15. The median mark for a Science test was 45. No student actually achieved this result. Explain how this is possible.

Fully worked solutions for this chapter are available online.

LESSON
10.3 Measures of spread

SYLLABUS LINKS

- Calculate quartiles from a dataset.
- Interpret quartiles, deciles and percentiles from a graph [complex].
- Use everyday language to describe spread, including spread out, dispersed, tightly packed, clusters, gaps, more/less dense regions, outliers.
- Calculate and interpret statistical measures of spread, including the range, interquartile range and standard deviation [complex].
  - range = highest score − lowest score
  - IQR = $Q_3 - Q_1$

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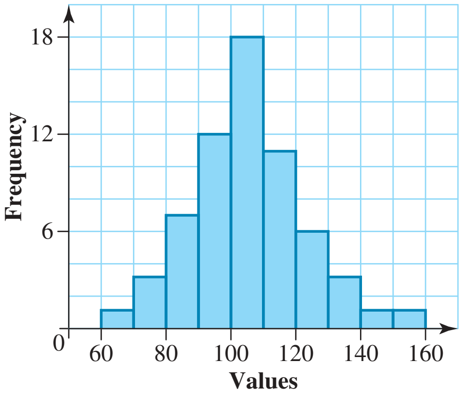
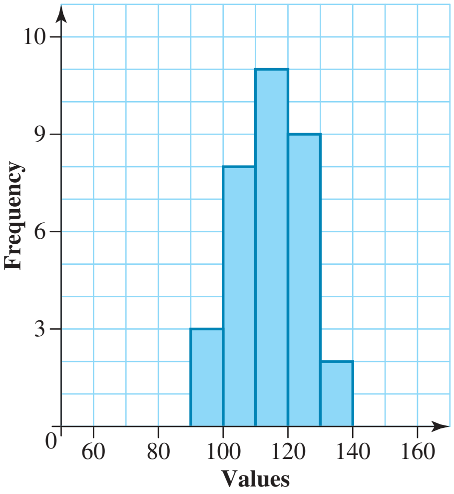
10.3.1 Measures of spread

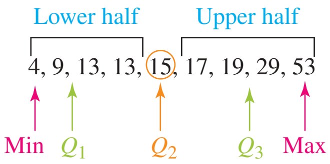
- The **spread** (or distribution) of a set of data indicates how far the data values are spread from the centre or from each other.
- There are many statistical measures of spread, including the range, interquartile range and the standard deviation.
- We can use everyday language to interpret the spread, with expressions such as *spread out*, *dispersed* or *tightly packed*.





- In comparing the two histograms, the histogram on the left has a larger spread of values compared to the histogram on the right.

10.3.2 Quartiles, deciles and percentiles



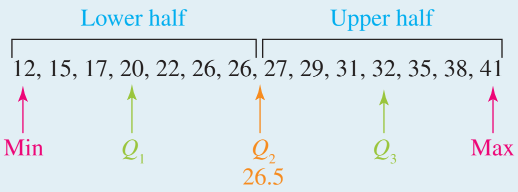
- Datasets can be split up into any given number of equal parts called **quantiles**. Quantiles are named after the number of parts that the data is divided into.
- **Quartiles** divide the data into **4** equal-sized parts.
- **Deciles** divide the data into **10** equal-sized parts.
- **Percentiles** divide the data into **100** equal-sized parts.
- The lower quartile (Q_1) is the median of the lower half of the data and the upper quartile (Q_3) is the median of the upper half of the data.

- If the median is one of the actual data values, it is not considered to be in either the upper or lower half of the data.

WORKED EXAMPLE 5 Calculating quartiles

Calculate the quartiles for the following data.

26, 32, 15, 12, 35, 27, 22, 31, 38, 20, 41, 26, 17, 29

THINK	**WRITE**
1. Write the data in ascending order.	12, 15, 17, 20, 22, 26, 26, 27, 29, 31, 32, 35, 38, 41
2. The median value will split the data into the top and bottom half. There are 14 values, so the median is at the $\frac{14\ +1}{2} = 7.5$th value.	12, 15, 17, 20, 22, 26, 26, $\vert$ 27, 29, 31, 32, 35, 38, 41 Median $= \frac{26+27}{2} = 26.5$
3. List the lower half of the data. Identify the median of the lower half of the data. This is Q_1. There are 7 values, so the median is at the $\frac{7\ +1}{2} = 4$th value.	Lower half: 12, 15, 17, 20, 22, 26, 26 $Q_1 = 20$
4. List the upper half of the data. Identify the median of the upper half of the data. This is Q_3. There are 7 values, so the median is at the $\frac{7\ +1}{2} = 4$th value.	Upper half: 27, 29, 31, 32, 35, 38, 41 $Q_3 = 32$
5. Write your answer.	 The quartiles for the dataset are 20, 26.5 and 32.

int-6174

10.3.3 Interpreting percentiles, deciles and percentiles from a graph [complex]

- Percentiles, deciles and quartiles are measures that divide a dataset into equal parts. When interpreting these from a graph, such as a cumulative frequency graph or a box plot, you can identify the values below which a certain percentage of the data falls.

Percentile	Quartile and symbol	Common name
25th percentile	First quartile, Q_1	Lower quartile
50th percentile	Second quartile, Q_2	Median
75th percentile	Third quartile, Q_3	Upper quartile
100th percentile	Fourth quartile, Q_4	Maximum

- A percentile is named after the percentage of data that lies at or below that value. For example, 60% of the data values lie at or below the 60th percentile.
- Percentiles can be read off a percentage cumulative frequency curve.

WORKED EXAMPLE 6 Interpreting percentiles and quartiles from a graph

A group of Year 11 students counted the number of skips per minute that they made with a skipping rope. The following cumulative frequency graph shows the cumulative frequency versus the number of skips.

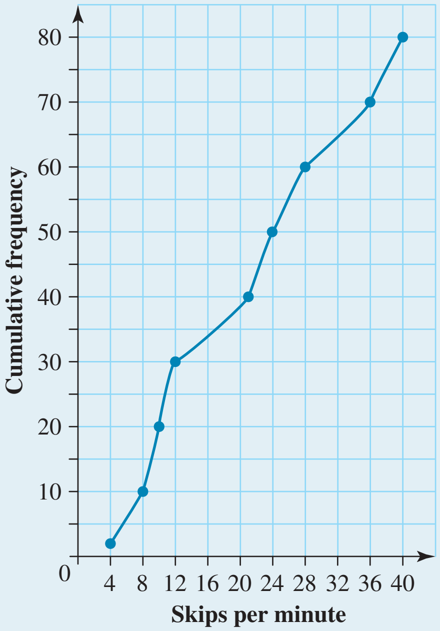
Calculate:

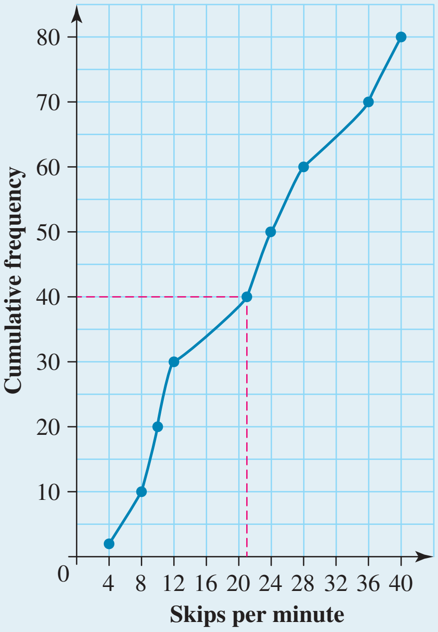
a. **the median**

b. Q_1

c. Q_3

d. **the number of skips per minute for the 40th percentile.**

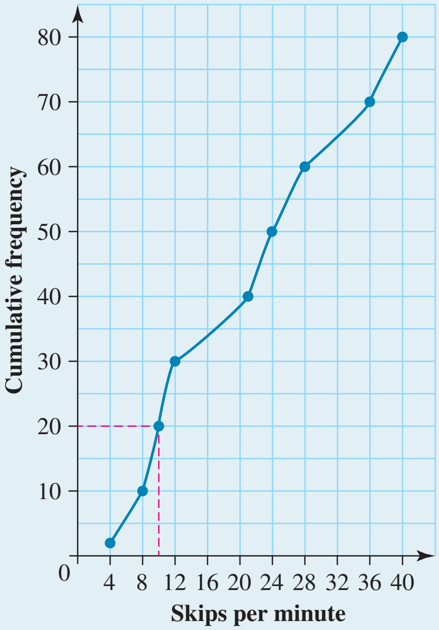


THINK	WRITE
a. **1.** Determine the total number of subjects. This is the highest vertical value on the cumulative frequency curve.	**a.** 80 students
2. The median occurs at the middle of the total cumulative frequency, so divide the total frequency by 2.	The median occurs at the $\frac{80}{2} = 40$th person.
3. Draw a line from 40 on the vertical axis until it hits the curve. Read down to the corresponding skips per minute on the horizontal axis.	
4. Write your answer.	The median number of skips per minute is approximately 21.

b. **1.** Q_1 occurs at a quarter of the total cumulative frequency, so divide the total frequency by 4.

b. Q_1 occurs at the $\frac{80}{4} = 20$th person.

2. Draw a line from 20 on the vertical axis until it hits the curve. Read down to the corresponding skips per minute on the horizontal axis.



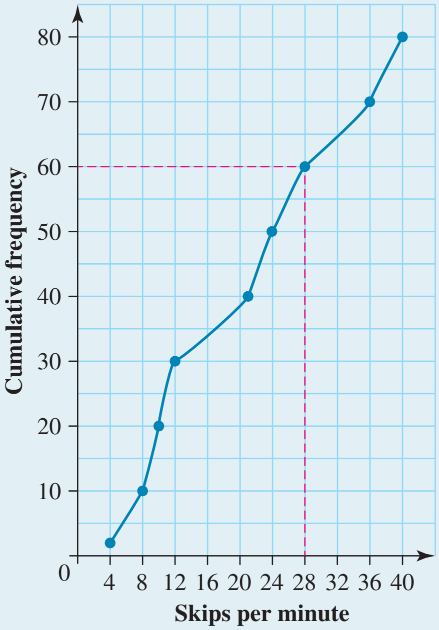
3. Write your answer.

Q_1 is at approximately 10 skips per minute.

c. **1.** Q_3 occurs at three-quarters of the total cumulative frequency, so divide the total frequency by 4 and multiply the result by 3.

c. Q_3 occurs at the $\frac{80}{4} \times 3 = 60$th person.

2. Draw a line from 60 on the vertical axis until it hits the curve. Read down to the corresponding skips per minute on the horizontal axis.



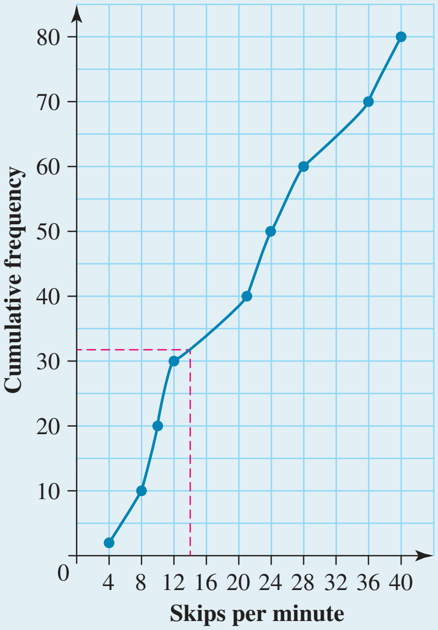
3. Answer the question.

Q_3 is at approximately 28 skips per minute.

d. 1. Find the 40th percentile of the cumulative frequency total, so multiply the total frequency by $\frac{40}{100}$.

d. $\frac{40}{100} \times 80 = 32$nd person.

2. Draw a line from 32 on the vertical axis until it hits the curve. Read down to the corresponding skips per minute on the horizontal axis.

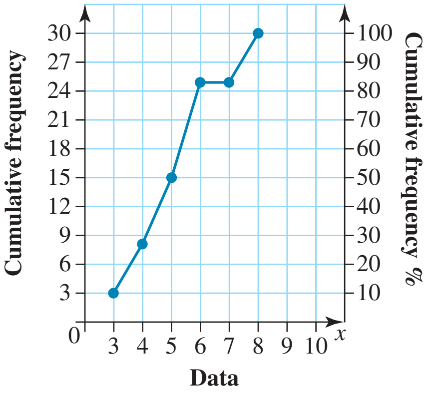


3. Write your answer.

The 40th percentile is at approximately 14 skips per minute.

- A percentage cumulative frequency curve is created by:
  - writing the cumulative frequencies as a percentage of the total number of data values
  - plotting the percentage cumulative frequencies against the maximum value for each interval.

Data (x)	**Frequency (f)**	**Cumulative frequency (cf)**	**Percentage cumulative frequency ($\%cf$)**
3	3	3	$\frac{3}{30} \times \frac{100}{1}\% = 10\%$
4	5	8	$\frac{8}{30} \times \frac{100}{1}\% = 27\%$
5	7	15	$\frac{15}{30} \times \frac{100}{1}\% = 50\%$
6	10	25	$\frac{25}{30} \times \frac{100}{1}\% = 83\%$
7	0	25	$\frac{25}{30} \times \frac{100}{1}\% = 83\%$
8	5	30	$\frac{30}{30} \times \frac{100}{1}\% = 100\%$



WORKED EXAMPLE 7 Determining and creating percentiles and quartiles graphs

The mass of eggs in three egg cartons ranges between 55 grams and 65 grams, as shown in the table.

a. Draw a percentage cumulative frequency table for the data.

b. Construct a percentage cumulative frequency curve (ogive).

c. Evaluate the 70th percentile and the lower quartile.

Mass (g)	Frequency
55–< 57	2
57–< 59	6
59–< 61	12
61–< 63	11
63–< 65	5

THINK

a. 1. Construct the cumulative frequency table by calculating the cumulative frequency for each class interval, as shown in blue.

2. Calculate the percentage cumulative frequency for each interval by dividing the cumulative frequency for each interval by the total cumulative frequency, as shown in pink.

WRITE

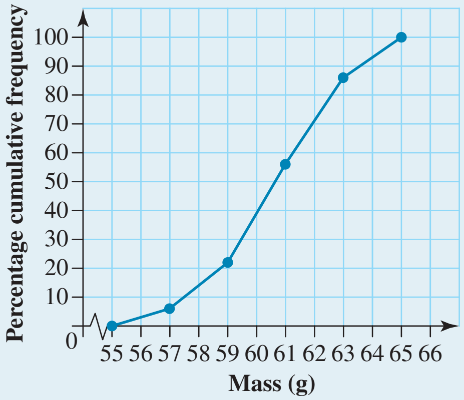
a.

Mass (g)	Frequency (f)	Cumulative frequency (cf)	Percentage cumulative frequency ($\%cf$)
55–< 57	2	2	$\frac{2}{36} \times \frac{100}{1}\% \approx 6\%$
57–< 59	6	$2 + 6 = 8$	$\frac{8}{36} \times \frac{100}{1}\% \approx 22\%$
59–< 61	12	$8 + 12 = 20$	$\frac{20}{36} \times \frac{100}{1}\% \approx 56\%$
61–< 63	11	$20 + 11 = 31$	$\frac{31}{36} \times \frac{100}{1}\% \approx 86\%$
63–65	5	$31 + 5 = 36$	$\frac{36}{36} \times \frac{100}{1}\% = 100\%$

b. 1. Plot the percentage cumulative frequency curve. For the first interval (55–< 57), plot the minimum value for the interval (55) against 0%.

2. Plot the maximum value for each interval against the percentage cumulative frequency for the interval.
For example:
For the first interval, plot 57 against 6%.
For the second interval, plot 59 against 22%.

b.

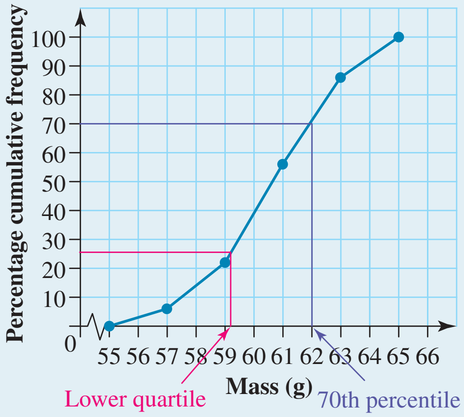


c. 1. The 70th percentile is read from the graph by following across from 70% on the vertical axis to the curve and then down to the horizontal axis, as shown in green.

2. The lower quartile is the 25th percentile and is read from the graph by following across the graph from 25% on the vertical axis to the curve and then down to the horizontal axis, as shown in pink.

3. Answer the question.

c.

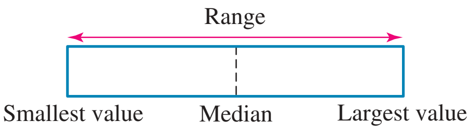


The 70th percentile is 62 grams.
The lower quartile is approximately 59.2 grams.

10.3.4 The range and interquartile range [complex]

int-3822

- The **range** is the difference between the highest and lowest values of the dataset.

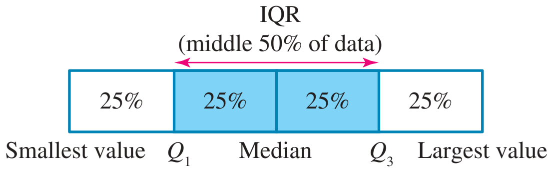


Range

Range = largest value − smallest value

int-4813

- The **interquartile range (IQR)** is the range of the middle 50% of the dataset. It measures the spread of the middle 50% of data.



Interquartile range

$$\textbf{Interquartile range (IQR)} = Q_3 - Q_1$$

WORKED EXAMPLE 8 Determining range and IQR

Calculate the range and IQR of the following dataset.

$$\mathbf{5, 12, 18, 23, 29, 34, 38, 42, 47, 50}$$

THINK	WRITE
1. Write the data in ascending order.	5, 12, 18, 23, 29, 34, 38, 42, 47, 50
2. The range is found by subtracting the smallest number from the largest number.	$\text{Range} = 50 - 5$ $= 45$
3. Determine the median (Q_2) of the dataset by finding the middle.	$5, 12, 18, 23, 29, \overset{Q_2}{\vert}\, 34, 38, 42, 47, 50$ $\text{Median } (Q_2) = \dfrac{29+34}{2}$ $= 31.5$
4. Split the dataset into two halves: lower half (Q_1) and upper half (Q_3).	$5, 12, \underset{Q_1}{\textcircled{18}}, 23, 29, \overset{Q_2}{\vert}\, 34, 38, \underset{Q_3}{\textcircled{42}}, 47, 50.$ $Q_1 = 18$ $Q_3 = 42$
5. Calculate the IQR.	$\text{IQR} = Q_3 - Q_1$ $= 42 - 18$ $= 24$

- Since the interquartile range excludes the lower and upper 25% of observations, it is generally unaffected by extreme values, making it a more useful measure of spread than the range.

10.3.5 The standard deviation [complex]

int-4814

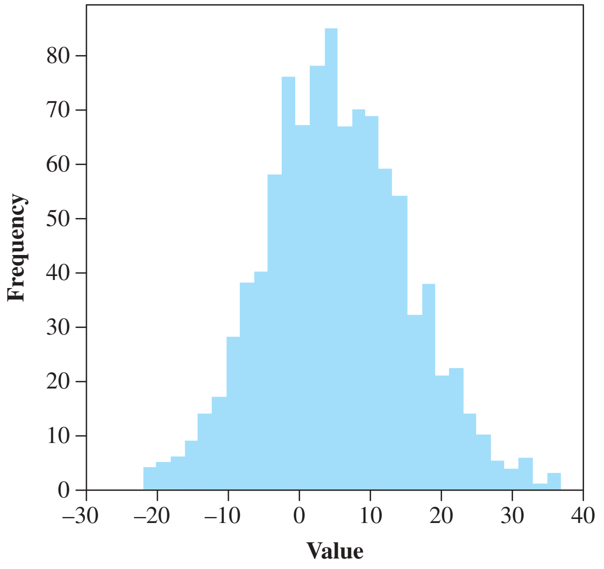
- The **standard deviation**, like the IQR, is a measure of the spread of the data.
- The standard deviation measures how far on average each number is from the mean.
- A smaller standard deviation means the numbers are close to the mean, and a larger standard deviation means they are more spread out.
- The standard deviation of a population and the standard deviation of a sample are calculated differently:

Standard deviation

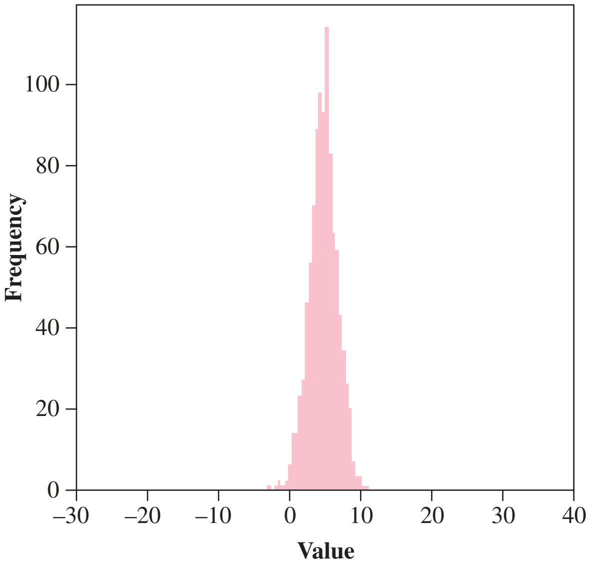
Population standard deviation: $\sigma = \sqrt{\dfrac{\sum (x-\mu)^2}{N}}$	**where:** **σ = population standard deviation** **x = a data value** **μ = the population mean** **N = the total number of population data values.**
Sample standard deviation: $s = \sqrt{\dfrac{\sum (x-\bar{x})^2}{n}}$	**where:** **s = sample standard deviation** **x = a data value** **$\bar{x}$ = the sample mean** **n = the total number of population data values.**

- These formulas use the square of the distance between data points and the mean as part of the calculation.
- Therefore, a small standard deviation shows that the data is clustered closer to the mean, whereas a large standard deviation indicates that the distribution is more spread out about the mean.

Histogram with standard deviation = 10 and mean = 5



Histogram with standard deviation = 2 and mean = 5



- The sample standard deviation is more commonly used because questions usually involve samples instead of whole populations.
- When asked to calculate the standard deviation, assume it is referring to the sample standard deviation.

WORKED EXAMPLE 9 Calculating the standard deviation without technology

The number of lollies in a sample of 8 packets is:

11, 12, 13, 14, 16, 17, 18, 19

Calculate the standard deviation correct to 2 decimal places.

THINK

WRITE

1. Calculate the mean.

$$\bar{x} = \frac{11 + 12 + 13 + 14 + 16 + 17 + 18 + 19}{8}$$
$$= \frac{120}{8}$$
$$= 15$$

2. To calculate the deviations $(x - \bar{x})$, set up a table as shown and complete by subtracting the mean from each data value.

No. of lollies (x)	$(x - \bar{x})$
11	$11 - 15 = -4$
12	-3
13	-2
14	-1
16	1
17	2
18	3
19	4
Total	

3. Add another column to the table to calculate the square of the deviations, $(x - \bar{x})^2$, by squaring each value in the second column.
Note: all of these values should be positive after multiplication.
Then sum the results: $\sum (x - \bar{x})^2$

No. of lollies (x)	$(x - \bar{x})$	$(x - \bar{x})^2$
11	$11 - 15 = -4$	16
12	-3	9
13	-2	4
14	-1	1
16	1	1
17	2	4
18	3	9
19	4	16
Total		$\sum (x - \bar{x})^2 = 60$

4. To calculate the standard deviation, divide the sum of the squares by one less than the number of data values, then take the square root of the result.

$$s = \sqrt{\frac{\sum (x-\overline{x})^2}{n-1}}$$
$$= \sqrt{\frac{60}{7}}$$
$$\approx 2.93 \text{ (correct to 2 decimal places)}$$

5. Interpret the result.

The standard deviation is 2.93, which means that the number of lollies in each pack differs from the mean by an average of 2.93.

- Calculations for standard deviation can be completed using different digital technologies, such as calculators, statistics programs or spreadsheets.
- This is generally the preferred method of calculating the standard deviation, particularly for larger datasets.

WORKED EXAMPLE 10 Calculating standard deviation with technology

The number of chocolate drops in a sample of different bags was recorded. These values were as follows.

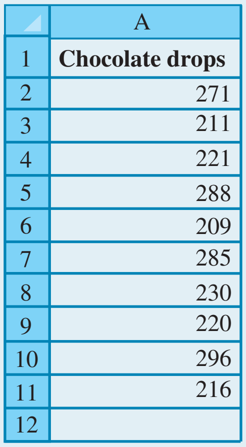
271, 211, 221, 288, 209, 285, 230, 220, 296, 216

Using technology, calculate the standard deviation, correct to 1 decimal place, of the number of chocolate drops in a bag.

THINK

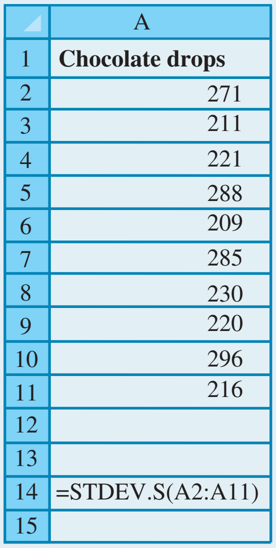
1. Enter the data into the digital technology of your choice. This example will be done in Excel.

WRITE



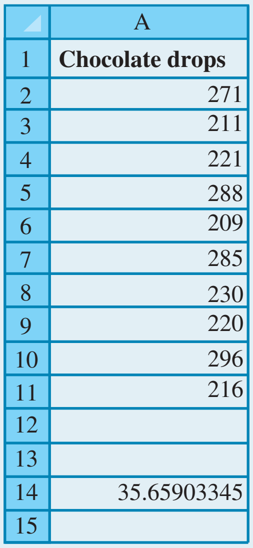
	A
1	**Chocolate drops**
2	271
3	211
4	221
5	288
6	209
7	285
8	230
9	220
10	296
11	216
12	

2. To calculate the sample standard deviation, enter the formula '=STDEV.S()' and select the values to be included in the calculation. The data starts in cell A2 and finishes in cell A11. Alternatively, in a blank cell, type '=STDEV.S(A2:A11)'.



	A
1	**Chocolate drops**
2	271
3	211
4	221
5	288
6	209
7	285
8	230
9	220
10	296
11	216
12	
13	
14	=STDEV.S(A2:A11)
15	

3. Press Enter to calculate the sample standard deviation.



	A
1	**Chocolate drops**
2	271
3	211
4	221
5	288
6	209
7	285
8	230
9	220
10	296
11	216
12	
13	
14	35.65903345
15	

4. Write the answer correct to 1 decimal place.

The standard deviation is 35.7.

DIGITAL TECHNOLOGY: Calculating standard deviation

1. **Enter Data Mode**: Press MODE and select 2 for STAT mode.
Press 1 for 1−VAR (one-variable statistics).

2. **Enter Data**: Input your data points. For each data point, enter the number and press =.

3. **Finish Data Entry**: After entering all data points, press SHIFT, then 1, to access the STAT menu, then 4 for VAR (variance).

4. **Calculate Standard Deviation**: Press 4 for sx (standard deviation). The calculator will display the standard deviation of your dataset.

```
SX
        35.65903345
```

Exercise 10.3 Measures of spread

10.3 Quick quiz on | **10.3 Exercise**

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5	6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19	20

Simple familiar

1. Put these expressions in order from the one with the smallest value to the one with the largest value: upper quartile; minimum; median; maximum; lower quartile.

2. **WE5** Calculate the quartiles for the following data.

$$18,\ 24,\ 30,\ 12,\ 28,\ 35,\ 22,\ 25,\ 33,\ 19,\ 40,\ 27,\ 16,\ 31$$

3. Calculate the quartiles for the following data.

$$136,\ 106,\ 141,\ 144,\ 183,\ 102,\ 196,\ 175,\ 158,\ 123,\ 162,\ 113,\ 118,\ 195,\ 154$$

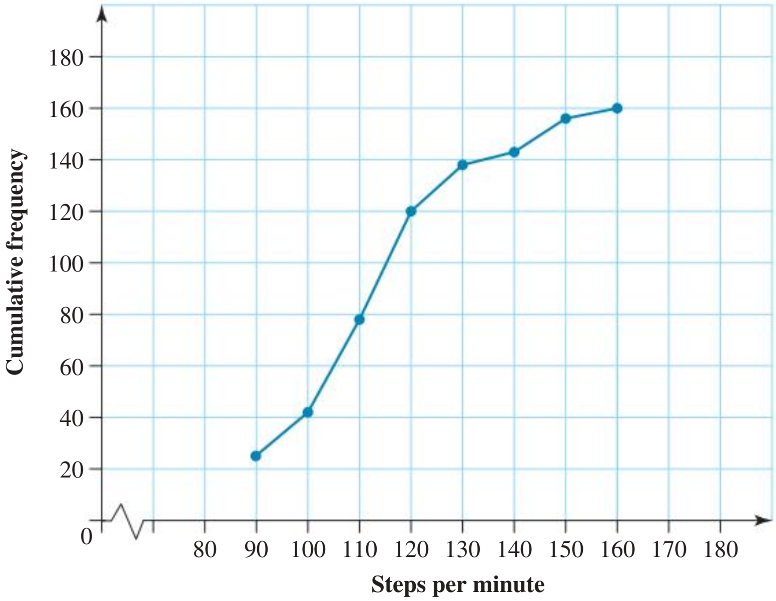
4. A group of senior students recorded their heights in centimetres. The heights are as follows:

$$171,\ 182,\ 154,\ 145,\ 175,\ 189,\ 157,\ 173,\ 186,\ 151,$$
$$200,\ 169,\ 196,\ 181,\ 160,\ 184,\ 166,\ 183,\ 141,\ 190$$

Calculate the first quartile (Q_1), the median (Q_2) and the third quartile (Q_3) for the given student heights.

The following information relates to questions 5 and 6.

The number of steps per minute runners took at a local park run event was recorded. The cumulative frequency graph is as shown.



5. **WE6abc** Calculate:
   - a. the median
   - b. Q_1
   - c. Q_3.

Complex familiar

6. **WE6d** A local running group invited all people over the 65th percentile to join the group. Determine how many steps per minute a runner would need to take to be invited to join the running group.

7. People who train for triathlons run a great distance every day. The distances (in km) that 60 athletes cover are shown in the frequency table.

Class interval	Frequency
10–15	3
15–20	7
20–25	12
25–30	18
30–35	14
35–40	6

   - a. Draw a cumulative frequency curve (ogive) for this data.
   - b. Identify how many of the distances run were less than 19 km.
   - c. Determine the median for the data.
   - d. Calculate the 30th percentile and the upper quartile.

8. **WE7** Sometimes when you go through a fast-food outlet drive-through, they don't have your order ready and you have to wait in the waiting bay for it to be brought out to you. The waiting times of 50 customers were recorded in minutes and seconds, as shown in the table.

Class intervals	Frequency
2:00–< 2:30	3
2:30–< 3:00	2
3:00–< 3:30	8
3:30–< 4:00	8
4:00–< 4:30	9
4:30–< 5:00	7
5:00–< 5:30	2
5:30–< 6:00	4
6:00–< 6:30	0
6:30–< 7:00	2
7:00–< 7:30	2
7:30–< 8:00	1
8:00–< 8:30	2

a. Draw a percentage cumulative frequency table.
b. Construct a percentage cumulative frequency curve.
c. The fast-food outlet claims that 80% of customers spend less than 4 minutes 30 seconds in the waiting bay. Explain whether you agree or disagree with this statement. Justify your response.

9. Calculate the range of each of the following sets of data.

a. 4, 6, 8, 11, 15
b. 1.7, 1.9, 2.5, 0.5, 3.1, 1.9, 1.7, 1.6, 1.2
c.

Score (x)	Frequency (f)
110	12
111	9
112	18
113	27
114	5

10. **WE8** The speed of 20 cars (in km/h) is monitored along a stretch of road that is a designated 80 km/h zone. Calculate and interpret the range and IQR of the data.

80, 82, 77, 75, 80, 80, 81, 78, 79, 78, 80, 80, 85, 70, 79, 81, 81, 80, 80, 80

11. 30 pens are randomly selected off the conveyor belt at the factory and are tested to see how long they will last (with values given in hours). Determine the range and IQR of the data and interpret what this means in context.

20, 32, 38, 22, 25, 34, 47, 31, 26, 29, 30, 36, 28, 40, 31
26, 37, 38, 32, 36, 35, 25, 29, 30, 40, 35, 38, 39, 37, 30

12. **WE9** Determine the sample standard deviation of the data (correct to 1 decimal place).

26, 32, 15, 12, 35, 27, 22, 31, 38, 20, 41, 26, 17, 29

Use the following dataset to answer questions 13–15.

171, 122, 182, 153, 167, 184, 171, 177, 189, 175, 128, 190, 135, 147, 171

13. Determine the range of the data.

14. Determine the IQR of the data.

15. Determine the sample standard deviation of the data (correct to 1 decimal place).

16. **a.** Calculate the maximum value of a dataset if its minimum value is 23 and its range is 134.
b. Calculate the minimum value of a dataset if its range is 32 and its maximum value is 101.

17. Aptitude tests are often used by companies to help them decide who to employ. An employer gave 30 potential employees an aptitude test with a total of 90 marks. The scores achieved are shown below.

67, 67, 68, 68, 68, 69, 69, 72, 72, 73
73, 74, 74, 75, 75, 77, 78, 78, 78, 79
79, 79, 81, 81, 81, 82, 83, 83, 83, 86

Only applicants who score above the 80th percentile receive an interview. Use your knowledge of percentiles to determine how many interviews the employer will have to run.

18. **WE10** The ages of patients (in years) that came into a hospital emergency room during an afternoon were recorded.

14, 1, 3, 87, 27, 42, 19, 91, 17, 73, 68, 83, 62, 29, 32, 2

Using a digital technology of your choice, calculate the sample standard deviation of the data.

19. The weights of packages (in kilograms) that were delivered to a warehouse in a single day were recorded.

15, 22, 18, 35, 27, 40, 19, 45, 21, 33, 30, 38, 25, 29, 32, 20

Using a digital technology of your choice, calculate the sample standard deviation of the data, correct to 1 decimal place.

Complex unfamiliar

20. Compare the average monthly temperatures of two Australian cities for 2024. Using your knowledge of statistics, determine which city is better to live in. Use mathematical reasoning and calculations to support your answers.

Months	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
Monthly mean (Canberra), in °C	27.1	28.1	27.4	20.8	16.8	12.9	12.5	16.7	18.6	22.3	27.5	29.4
Monthly mean (Cairns), in °C	32.6	31.9	30.7	30.4	28.0	27.4	26.5	27.5	29.2	31.0	32.3	31.6

Fully worked solutions for this chapter are available online.

LESSON
10.4 Outliers [complex]

SYLLABUS LINKS

- Investigate the effect of outliers on the mean and the median [complex].
- Use everyday language to describe spread, including spread out, dispersed, tightly packed, clusters, gaps, more/less dense regions, outliers.

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10.4.1 Calculating outliers

- Outliers are extreme values on either end of a dataset that appear very different from the rest of the data. They may be extremely large or extremely small compared to other data values in the set.

- Outliers can be calculated by considering the distance a data point is from the mean or median compared to the rest of the data.
- If the value of a data point is clearly far away from the mean or median, as well as from other data points, it may be considered to be an outlier.
- For example, in the dataset 2, 2, 4, 6, 7, 7, 100, it is reasonable to conclude that 100 is an outlier, as it is much larger than the rest of the data and is far away from the median of 5.
- It is not always obvious if a data point is an outlier.
- The IQR can be used in a calculation to determine if a data point is far enough away from the median to be considered an outlier.

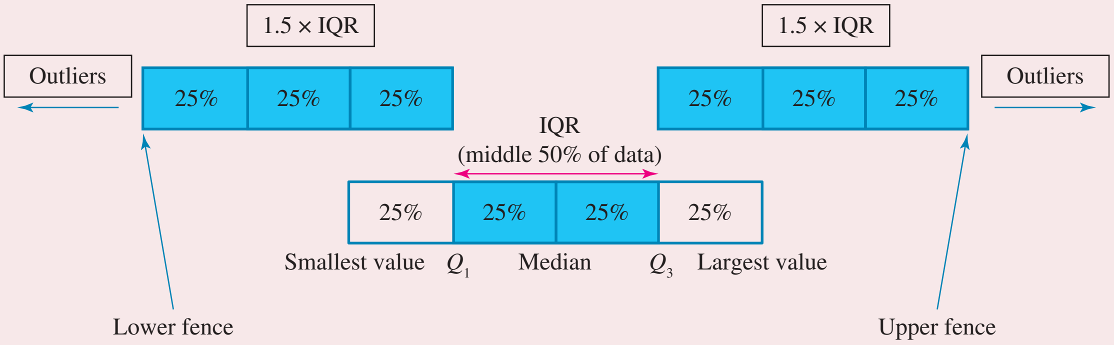
Determining outliers

To determine if a data point is an outlier, first calculate the upper and lower fences:

$$\textbf{lower fence} = Q_1 - 1.5 \times \textbf{IQR}$$

$$\textbf{upper fence} = Q_3 + 1.5 \times \textbf{IQR}$$

where: Q_1 is the lower quartile, Q_3 is the upper quartile and IQR is the interquartile range.



Any data point below the lower fence is an outlier.

Any data point above the upper fence is an outlier.

WORKED EXAMPLE 11 Determining outliers from a dataset

For the following data:

$$\mathbf{34, 41, 63, 39, 48, 41, 33, 39, 40, 34, 44}$$

a. **Calculate the Interquartile range (IQR).**
b. **Calculate if there are any outliers.**

THINK	WRITE
a. **1.** Write the data in ascending order.	33, 34, 34, 39, 39, 40, 41, 41, 44, 48, 63
2. The median value will split the data into the top and bottom half. There are 11 values, so the median is at the $\frac{11+1}{2} = 6\text{th}$ value.	33, 34, 34, 39, 39, 40, 41, 41, 44, 48, 63 Median = 40
3. List the lower half of the data. Identify the median of the lower half of the data. This is Q_1. There are 5 values, so the median is at the $\frac{5+1}{2} = 3\text{rd}$ value.	Lower half = 33, 34, 34, 39, 39, $Q_1 = 34$
4. List the upper half of the data. Identify the median of the upper half of the data. This is Q_3. There are 5 values, so the median is at the $\frac{5+1}{2} = 3\text{rd}$ value.	Upper half = 41, 41, 44, 48, 63 $Q_3 = 44$

5. Calculate the IQR.	$\text{IQR} = Q_3 - Q_1$ $= 44 - 34$ $= 10$
b. 1. Calculate the lower fence. Identify any values below the lower fence.	$\text{Lower fence} = Q_1 - 1.5 \times \text{IQR}$ $= 34 - 1.5 \times 10$ $= 19$ No values are below this point.
2. Calculate the upper fence.	$\text{Upper fence} = Q_3 + 1.5 \times \text{IQR}$ $= 44 + 1.5 \times 10$ $= 59$
3. Interpret your answer.	There is a value above this point; therefore, 63 is an outlier.

tlvd-12110

WORKED EXAMPLE 12 Determining outliers from a stem plot

Consider the following stem plot.

Key: 1|4 = 14

Stem	**Leaf**
0	**1 3**
1	**0 0 1 3 4 4 4 6 8 9 9**
2	**2 2 5 5 6 7 8 8 8 9**
3	**4 5 7 7**
4	**0 1 1 2 9**
5	
6	
7	**1**

Calculate the IQR and use this to determine whether there are any outliers.

THINK

1. To calculate the IQR, you must first calculate Q_1 and Q_3. Start by locating the median.
The median is the middle data value, so the median occurs at the $\frac{33+1}{2} = 17$th value.

WRITE

Stem	Leaf
0	1 3
1	0 0 1 3 4 4 4 6 8 9 9
	Median
2	2 2 5 5 6 7 8 8 8 9
3	4 5 7 7
4	0 1 1 2 9
5	
6	
7	1

Median = 25

2. Locate the lower quartile. Q_1 is the middle value of the lower half of the data. There are 16 data values in the lower group, so Q_1 occurs at the $\frac{16+1}{2} = 8.5$th value in the lower group.
Take the average of the 8th and 9th values.
$\frac{14+14}{2} = 14$

Stem	Leaf
0	1 3
	Q_1
1	0 0 1 3 4 4 | 4 6 8 9 9
2	2 2 5 5 6 7 8 8 8 9
3	4 5 7 7
4	0 1 1 2 9
5	
6	
7	1

$Q_1 = 14$

3. Locate the upper quartile. Q_3 is the middle value of the upper half of the data. There are 16 data values in the upper group, so Q_3 occurs at the $\frac{16+1}{2} = 8.5$th value in the upper group.
Take the average of the 8th and 9th values.
$\frac{35+37}{2} = 36$

Stem	Leaf
0	1 3
1	0 0 1 3 4 4 4 6 8 9 9
2	2 2 5 5 6 7 8 8 8 9
	Q_3
3	4 5 | 7 7
4	0 1 1 2 9
5	
6	
7	1

$Q_3 = 36$

4. Calculate the IQR.

$$\begin{aligned}\text{IQR} &= Q_3 - Q_1 \\ &= 36 - 14 \\ &= 22\end{aligned}$$

5. Calculate the lower fence.

$$\begin{aligned}\text{Lower fence} &= Q_1 - 1.5 \times \text{IQR} \\ &= 14 - 1.5 \times 22 \\ &= -19\end{aligned}$$

Identify any values below the lower fence.

No values are below this point.

6. Calculate the upper fence.

$$\begin{aligned}\text{Upper fence} &= Q_3 + 1.5 \times \text{IQR} \\ &= 36 + 1.5 \times 22 \\ &= 69\end{aligned}$$

Identify any values above the upper fence.

There is a value above this point; therefore, 71 is an outlier.

10.4.2 The effect of outliers on the mean and median

- Outliers can affect the summary statistics for a sample.
- The median is calculated using the number of data points, not the value of each data point. Therefore, outliers do not have much of an influence on the median.
- The mean involves adding all the values of the data points, so extremely high or low values can greatly affect the mean.
- If outliers are present in a dataset, the median is a better measure of central tendency, as it is less affected by outliers.

tlvd-12111

WORKED EXAMPLE 13 Investigating the effect of outliers on the mean and the median

The number of times employees in a small company took sick days per year was recorded. The results were as follows.

5, 2, 4, 3, 26, 1, 1, 6, 4, 29, 0, 2, 1

a. **Determine whether the data contains any potential outliers.**

b. **For the dataset, calculate:**

   i. **the mean**

   ii. **the median.**

c. **Exclude the outlier(s) in the dataset and calculate:**

   i. **the mean**

   ii. **the median.**

d. **Comment on the difference in results between parts b and c.**

THINK	WRITE
a. 1. Arrange the values in ascending order.	a. 0, 1, 1, 1, 2, 2, 3, 4, 4, 5, 6, 26, 29
2. Outliers are extreme values on either end of the dataset.	0, 1, 1, 1, 2, 2, 3, 4, 4, 5, 6, 26, 29 26 and 29 are possible outliers.
b. i. 1. Calculate the sum of all the values.	b. $5+2+4+3+26+1+1+6+4+29+0+2+1=84$
2. Count the number of values.	Number of values $= 13$
3. The mean is the sum of all the values divided by the number of values.	$\bar{x} = \frac{84}{13}$ $= 6.5$ The mean number of sick days employees took in a year was 6.5.
ii. 1. Arrange the values in ascending order. The median is the middle value.	0, 1, 1, 1, 2, 2, 3, 4, 4, 5, 6, 26, 29
2. There are 13 values, so the median is the $\frac{13+1}{2} = 7$th value.	0, 1, 1, 1, 2, 2, 3, 4, 4, 5, 6, 26, 29 The median number of sick days employees took in a year was 3.
c. i. 1. Write out the dataset excluding the outliers. Calculate the sum of all the values.	c. 0, 1, 1, 1, 2, 2, 3, 4, 4, 5, 6 $5+2+4+3+1+1+6+4+0+2+1=29$
2. Count the number of values.	Number of values $= 11$
3. The mean is the sum of all the values divided by the number of values.	$\bar{x} = \frac{29}{11}$ $= 2.6$ The mean number of sick days employees took in a year, when excluding outliers, was 2.6.
ii. 1. Arrange the values in ascending order. The median is the middle value.	0, 1, 1, 1, 2, 2, 3, 4, 4, 5, 6
2. There are 11 values, so the median is the $\frac{11+1}{2} = 6$th value.	0, 1, 1, 1, 2, 2, 3, 4, 4, 5, 6 The median number of sick days employees took in a year, when excluding outliers, was 2.

d.	**1.** Comment on the difference between the mean in parts **b** and **c**.	**d.** The mean sick days per year taken by employees reduced from 6.5 days to 2.6 days when outliers were excluded. Excluding the large outliers significantly decreased the mean.
	2. Comment on the difference between the median in parts **b** and **c**.	The median sick days per year taken by employees changed from 3 days to 2 days when outliers were excluded. Excluding the large outliers did not significantly change the median.
	3. Make a conclusion about the effect of outliers on the mean and median.	The outliers had a greater effect on the mean than on the median.

Exercise 10.4 Outliers [complex]

learnon

10.4 Quick quiz on | **10.4 Exercise**

Simple familiar	Complex familiar	Complex unfamiliar
N/A	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13	14

Complex familiar

1. **WE11** The following data represents car sales each month at a car yard.

$$12, 9, 15, 10, 11, 23, 14, 8, 6, 11, 13, 15$$

   a. Calculate the interquartile range (IQR).
   b. Determine whether there are any outliers.

2. **WE12** Consider the following stem plot.

Key: 1|6 = 16

Stem	Leaf
0	1
1	5
2	
3	
4	2
5	6 7
6	0 1 4 5 6
7	1 1 2 3 5 7 9
8	2 2 4 4 4 8 8
9	3 5

Calculate the IQR and use this to determine whether there are any outliers.

3. The following data represents the number of books borrowed each day from a school library:

$$18,\ 22,\ 25,\ 19,\ 21,\ 35,\ 24,\ 17,\ 15,\ 21,\ 23,\ 25$$

a. Calculate the interquartile range (IQR).
b. Identify if there are any outliers.

4. Calculate the lower and upper fence for the following data.

$$34, 41, 53, 39, 48, 41, 33, 39, 40, 34, 44$$

5. For the following data:

$$2.3, 3.1, 3.6, 1.8, 6.7, 4.4, 3.9, 2.8, 3.7, 4.0$$

a. calculate the lower and upper fence
b. determine whether there are any outliers in the dataset.

6. The number of swimmers at a pool was recorded over a fortnight. The number of swimmers was as follows.

$$56, 69, 59, 113, 9, 100, 80, 111, 94, 77, 57, 166, 101, 96$$

Determine whether any outliers exist and justify your answer with a calculations.

7. **WE13** Below is the data collected during a survey of the number of pets people own.

$$2, 1, 0, 3, 1, 1, 2, 1, 2, 3, 3, 2, 0, 2, 1, 2, 3, 4, 1, 2, 1, 1, 3, 3, 1, 3, 2, 1, 2, 2, 2, 3, 21, 1, 2, 3$$

a. Identify any possible outliers.
b. Calculate the mean and median.
c. Remove any outliers and recalculate the mean and median.
d. Comment on the difference in means and medians in parts **b** and **c**.

8. For the following set of numbers:

$$3, 5, 2, 6, 15, 1, 5, 4$$

a. Identify the possible outlier(s).
b. Determine whether the mean would increase or decrease with the outliers taken out of the dataset
c. Calculate the mean before and after taking out the outliers in the dataset

9. A teacher recorded the following scores of students in a Maths test:

$$21,\ 33,\ 44,\ 34,\ 27,\ 3,\ 29,\ 30,\ 6,\ 31,\ 25,\ 36$$

a. Identify any possible outliers.
b. Identify whether the mean would increase or decrease with the outliers taken out of the dataset.
c. Calculate the mean before and after taking out the outliers in the dataset, correct to 2 decimal places where necessary.

10. For the following set of numbers:

$$0.03, 0.05, 0.07, 0.03, 0.9, 0.03, 0.04$$

a. Identify the possible outlier(s).
b. Determine whether the median would increase or decrease with the outliers taken out of the dataset
c. Determine the median before and after taking out the outliers in the dataset

11. A researcher recorded the average temperature (in degrees Celsius) at a ski resort over 12 days:

$$-2.3, -3.6, -1.9, -2.1, 5.2, -3.9, -3.9, -2.7, 6.1, -1.7, -3.3, -2.4$$

a. Identify any possible outliers.
b. Determine whether the median would increase or decrease with the outliers taken out of the dataset.
c. Determine the median before and after taking out the outliers in the dataset.

12. The mean of a dataset was calculated to be 117. The median was calculated to be 107. The dataset has two outliers, 178 and 190. If the outliers were replaced with values of 118 and 120, determine the effect it would have on the mean and median.

13. A statistician investigated the monthly household sales in a suburb over a year. The Q_1, median and Q_3 were calculated to be 18, 27 and 35 respectively.

a. Prove that 64 houses per month is an outlier.
b. The statistician realised he made a mistake and the outlier of 64 was recorded incorrectly. The actual value was 46 houses per month. Using your answer from **a** to help you, explain what would happen to the values of Q_1, Q_3 and IQR if the data point was changed from 64 to 46 houses per month.

Complex unfamiliar

14. In minigolf, each hole has a par score, which is the expected number of strokes a skilled player should take to complete the hole. A score below par (negative score) indicates fewer strokes than expected, while a score above par (positive score) indicates more strokes than expected. A minigolfer is looking over their scores for the last 16 holes below:

Game	Score
1	−4
2	−2
3	−1
4	−2
5	−1
6	−3
7	−2
8	−4
9	−2
10	−1
11	−2
12	−4
13	−3
14	−1
15	−4
16	−1

On their 17th game, the minigolfer scores +1. The minigolfer says that this score is an outlier because they never score over par (a zero score). Evaluate the reasonableness of the minigolfer's statement. You must include mathematical reasoning to support your answer.

Fully worked solutions for this chapter are available online.

LESSON
10.5 Applications of measures of centre and spread [complex]

SYLLABUS LINKS

- Investigate the suitability of measures of central tendency in various real-world contexts [complex].
- Investigate real-world examples from the media illustrating inappropriate uses of measures of central tendency and spread [complex].

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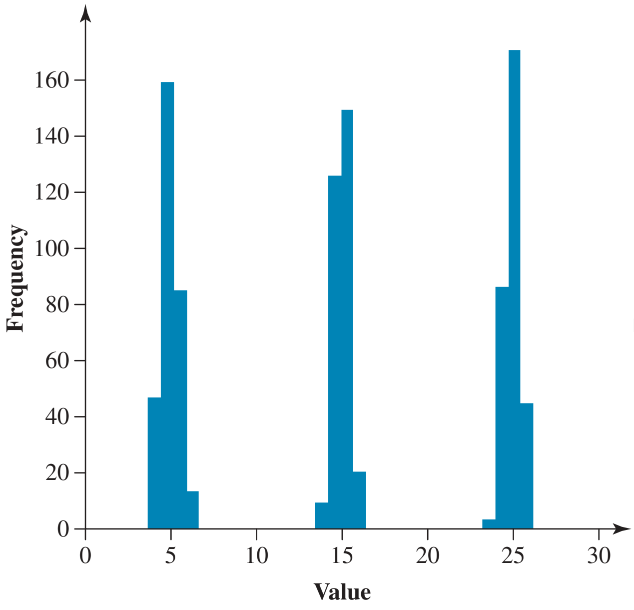
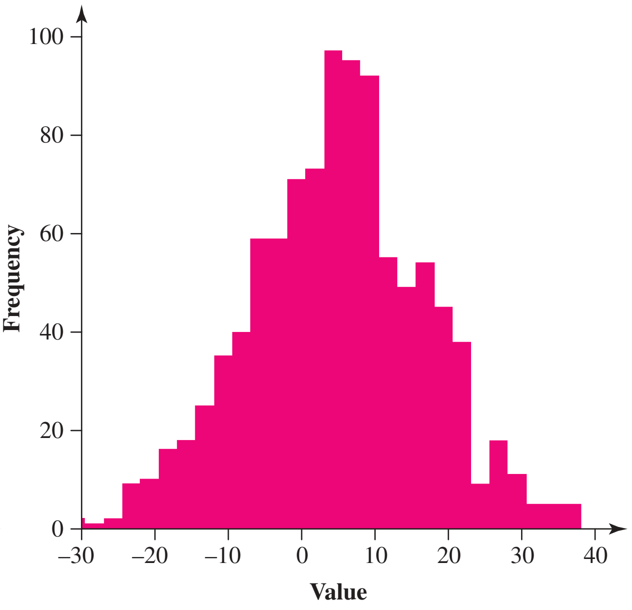
10.5.1 The suitability of measures of central tendency

- When analysing data or comparing datasets, it is common to discuss or compare measures of central tendency (mean or median) and the spread (range, IQR or standard deviation).
- We are aware that measures of central tendency give information about the centre of the distribution and that the spread provides information on how spread out (widely spread or tightly packed) the data are.
- For example, the following histograms have similar ranges and the same measure of central tendency. The graph on the left, however, has distinct gaps in the data and more/less densely populated areas. The graph on the right has a more consistent, or normal, spread of data.



Histogram with dense regions and gaps



Histogram without dense regions and gaps

- Given that there are several measures of centre and spread to select from when analysing data, it is often not necessary to calculate them all.
- The following needs to be taken into account when analysing and comparing data.
  - The mean is calculated using every data value in the set.
  - The median is the middle score of an ordered set of data, so it does not include every individual data value in its calculation.
  - The mode is the most frequently occurring data value, so it also does not include every individual data value in its calculation.

- The range is calculated by finding the difference between the maximum and minimum data values, so it includes outliers. It provides only a rough idea about the spread of the data and is inadequate in providing sufficient detail for analysis. It is useful, however, when we are interested in extreme values such as high and low tides or maximum and minimum temperatures.
- The interquartile range is the difference between the upper and lower quartiles, so it does not include every data value in its calculation, but it will overcome the problem of outliers skewing data.
- The standard deviation is calculated using every data value in the set.

Mean or median?

If the data has no outliers and is symmetric, either the mean or the median can be used as the measure of centre.

If the data is clearly skewed and/or there are outliers, it is more appropriate to use the median as the measure of centre.

Range, IQR or standard deviation?

The most appropriate measure of spread does depend on the type of data used; however, the following can be used as a guide.

If the data has no outliers and is symmetric, the standard deviation would be the preferred measure of spread.

If the data is clearly skewed and/or there are outliers, it is more appropriate to use the IQR as the preferred measure of spread.

- Although the above information can be applied in most cases, selecting the most appropriate measure of centre and spread may depend on other factors, such as wanting to include every data value in a set even if outliers are present.

tlvd-12112

WORKED EXAMPLE 14 Investigating the suitability of measures of central tendency

Below are samples of scores achieved by two students in eight Mathematics tests throughout the year.
John: 45, 62, 64, 55, 58, 51, 59, 62
Penny: 84, 37, 45, 80, 74, 44, 46, 50

a. **Determine the most appropriate measure of centre and measure of spread to compare the performance of the students.**
b. **Evaluate which student demonstrated the better overall performance across the eight tests.**
c. **Determine which student was more consistent over the eight tests.**

THINK	WRITE
a. In order to include all data values in the calculation of measures of centre and spread, calculate the mean and standard deviation.	**a.** John: $\bar{x} = 57, s = 6.41$ Penny: $\bar{x} = 57.5, s = 18.62$
b. Compare the mean for each student. The student with the higher mean performed better overall.	**b.** Penny performed slightly better on average as her mean mark was higher than John's.
c. Compare the standard deviation for each student. The student with the lower standard deviation performed more consistently.	**c.** John was the more consistent student because his standard deviation was much lower than Penny's. This means that his test results were closer to his mean score than Penny's were to hers.

10.5.2 Inappropriate uses of measures of central tendency and spread in the media

- Data can be manipulated by being selectively presented, such as using the mean instead of the median to exaggerate income levels, or omitting standard deviation to downplay variability, leading to misleading conclusions.
- The media often does this to skew the audience's interpretation of data for a variety of reasons, such as influencing public opinion, supporting a particular narrative, increasing engagement, or driving political and commercial agendas.

ACTIVITY: Investigating inappropriate uses of measures of central tendency and spread [complex]

In this activity, you will investigate real-world examples from the media that illustrate inappropriate uses of measures of central tendency (mean, median, mode) and spread (range, IQR, standard deviation).

Discuss as a class how measures of central tendency and spread can sometimes be misused or misrepresented in the media.

Form small groups and search for real-world examples from news articles, advertisements or reports where these measures are used inappropriately.

Look for misleading averages, misrepresented ranges or incorrect interpretations of data spread.

Analyse the chosen examples and identify the specific issues.

Consider questions like:
- What measure was used inappropriately?
- How does this misuse affect the interpretation of the data?
- What would be a more appropriate measure to use in this context?

Present your findings to the class.

Explain the example, the misuse and your analysis.

Discuss the importance of correctly using measures of central tendency and spread.

Talk about how these misuses can impact public perception and decision-making.

Summarise the key points from the activity.

Emphasise the importance of critical thinking when interpreting statistical information in the media.

Exercise 10.5 Applications of measures of centre and spread [complex]

learnon

10.5 Quick quiz on

10.5 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
N/A	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15	16

These questions are even better in jacPLUS!

- Receive immediate feedback
- Access sample responses
- Track results and progress

Find all this and MORE in jacPLUS

Complex familiar

1. The number of M&M's in 20 packets was recorded by a group of students.

   30, 32, 33, 35, 37, 37, 38, 38, 39, 39, 40, 40, 40, 41, 41, 41, 41, 41, 41, 50

   a. Calculate the mean number of M&M's per packet.
   b. Calculate the median number of M&M's per packet.
   c. Calculate the modal number of M&M's per packet.
   d. Based on your calculations, explain how many M&M's you would expect to find in a packet.
   e. If you were advertising M&M's, explain which summary statistic (mean, median or mode) you would use.

2. Explain how the mean, median and mode for both ungrouped and grouped data are similar and different.

3. Consider the two data sets shown.

   Dataset A: 20, 24, 29, 33, 37, 42, 51, 53, 96
   Dataset B: 20, 24, 29, 33, 37, 42, 51, 53, 66

   a. Calculate the mean and median of the two datasets.
   b. Comment on the means and medians of the datasets and explain any similarities and differences you see.
   c. Write a short statement to explain when to use the mean as a measure of central tendency and when to use the median.

4. **WE14** The Mathematics test results for two students over a year were recorded as follows.

   Student A: 49, 52, 51, 50, 54, 49, 100
   Student B: 65, 71, 64, 63, 60, 81, 0

   a. Determine the most appropriate measure of centre and spread to compare the performance of the students.
   b. Identify the student who had the better overall performance over the year.
   c. Identify the student who was more consistent over the year.

5. Create a dataset that fits the following descriptions.

   a. 8 data values with a range of 43
   b. 6 data values with a lower quartile of 5 and an upper quartile of 12
   c. 9 data values with a lower quartile of 7 and an upper quartile of 13

6. A Mathematics teacher wanted to design a question for her class. She wanted the question to have 10 data points, a lower quartile of 14 and an IQR of 20. Identify a dataset that could be used in her question.

7. The number of trees in several different parks was recorded, and the mean number of trees was calculated to be 15. The council wanted to determine whether the majority of parks contained close to 15 trees, or whether there was a large difference in the number of trees. Describe how the council could use a measure of spread to investigate this problem.

8. The scores below show the number of points scored by two AFL teams over the first 10 games of the season.

Sydney Swans	110	95	74	136	48	168	120	85	99	65
Brisbane Lions	125	112	89	111	96	113	85	90	87	92

a. Calculate the range of the scores for each team.
b. Based on the results above, determine which team is the more consistent.

9. Two machines are used to put approximately 100 Smarties into boxes. A check is made on the operation of the two machines. Ten boxes filled by each machine have the number of Smarties in them counted. The results are shown below.

Machine A: 100, 99, 99, 101, 100, 101, 100, 100, 101, 108
Machine B: 98, 104, 96, 97, 103, 96, 102, 100, 97, 104

a. Calculate the range in the number of Smarties from the first machine.
b. Calculate the range in the number of Smarties from the second machine.
c. Ralph is the quality control officer and he argues that machine A is more consistent in its distribution of Smarties. Explain why.

10. The following frequency distribution gives the prices paid by a car wrecking yard for a sample of 40 car wrecks.

Price ($)	Frequency
0 to < 500	2
500 to < 1000	4
1000 to < 1500	8
1500 to < 2000	10
2000 to < 2500	7
2500 to < 3000	6
3000 to < 3500	3

Determine the mean and standard deviation of the price paid for these wrecks.

11. The following stem plot represents the lifespan of different animals at an animal sanctuary. Determine which measure of centre is best to represent the dataset.

Key: 1|2 = 12 years

Stem	Leaf
0	3 5 9
1	2 4 6 8
2	0 1 4 5 5 7 9
3	0 2 6
4	
5	
6	0 3

12. The following dataset represents the salaries (in \$1000s) of workers at a small business.

45, 50, 55, 55, 55, 60, 65, 65, 70, 70, 75, 80, 220

a. Calculate the mean of the salaries correct to 3 decimal places.
b. Calculate the median of the salaries.
c. When it comes to negotiating salaries, the workers want to use the mean to represent the data and the management want to use the median. Explain why this might be the case.

13. A sample of crime statistics over a two-year period are shown in the following table.

Crime	Year 1	Year 2
Theft from motor vehicle	46 700	42 900
Theft from shop	19 800	20 600
Theft of motor vehicle	15 650	14 670
Theft of bicycle	4200	4660
Theft (other)	50 965	50 650

a. Calculate the interquartile range and standard deviation (correct to 1 decimal place) for both years.
b. Recalculate the interquartile range and standard deviation for both years after removing the smallest category.
c. Comment what effect removing the smallest category will have on the interquartile ranges and standard deviations.

14. Data collected on the number of daylight hours in Alice Springs is as shown.

10.3, 9.8, 9.6, 9.5, 8.5, 8.4, 9.1, 9.8, 10.0
10.0, 10.1, 10.0, 10.1, 10.1, 10.6, 8.7, 8.8, 9.0
8.0, 8.5, 10.6, 10.8, 10.5, 10.9, 8.5, 9.5, 9.3
9.0, 9.4, 10.6, 8.3, 9.3, 9.0, 10.3, 8.4, 8.9

a. Calculate the range of the data.
b. Calculate the interquartile range of the data.
c. Comment on the difference between the two measures and what this indicates.

15. A news article claims that the average salary in a small town is $63 000, leading readers to believe that most people earn an average wage. The sample data collected by the newspaper (in thousands) is as follows:

35, 38, 40, 42, 45, 50, 55, 60, 65, 200

a. Calculate the mean, median and mode of the salaries.
b. Determine which measure of central tendency best represents what a typical person earns. Justify your answer.
c. Explain how the media could mislead the public by only reporting the mean.

Complex unfamiliar

16. Two cricket players, Player A and Player B, are in the running for Player of the Series. Their scores across eight matches are recorded below:

Match	Player A	Player B
1	75	40
2	10	55
3	90	50
4	5	60
5	100	45
6	15	50
7	110	55
8	20	50

The local newspaper claims Player A should be awarded Player of the Series because they were the best overall performer. Use mathematical reasoning and your knowledge of statistics to evaluate whether this claim is accurate.

Fully worked solutions for this chapter are available online.

LESSON
10.6 Review

10.6.1 Summary

Hey students! Now that it's time to revise this chapter, go online to:

- Access the chapter summary
- Review your results
- Watch teacher-led videos
- Practise questions with immediate feedback

Find all this and MORE in jacPLUS

10.6 Exercise

learnon

10.6 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6	7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18	19, 20

These questions are even better in jacPLUS!

- Receive immediate feedback
- Access sample responses
- Track results and progress

Find all this and MORE in jacPLUS

Simple familiar

1. Calculate the mean, median and mode of each of the following datasets.

a. $4, 7, 2, 6, 9, 3, 6$

b. $11, 63, 24, 36, 25, 61, 29, 42$

c.

Score	Frequency
6	2
8	4
10	10

2. The following data was collected on the number of days people go away during the holidays.

$$2, 10, 5, 7, 9, 14, 2, 0, 6, 7, 0, 7, 14, 7, 8, 10, 12, 5, 2, 1, 16, 12, 10$$

Calculate the mean and median.

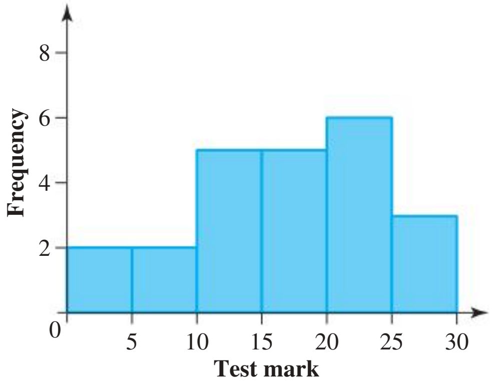
3. The screen time per day for a group of children aged between 0 and 14 is listed in the table. Calculate the mean number of minutes of screen time for 0–14-year olds.

Age (years)	Screen time/day (minutes)
0–2	45
3–5	73
6–8	98
9–11	124
12–14	142

4. Match the following terms with the descriptions below.
Mean, range, median, IQR, standard deviation, mode

- **a.** A measure of spread of the distribution that uses the highest and lowest value only
- **b.** A measure of central tendency that uses the middle value of the dataset by position
- **c.** A measure of spread that contains exactly 50% of the distribution
- **d.** A measure of central tendency that uses the sum of all data points divided by the number of data points
- **e.** A measure of spread that uses the distance from the mean in the calculation
- **f.** A summary statistic that uses the most common value

5. The test marks for a class of students are shown in the histogram and frequency distribution table.



Class interval	Frequency (f)
0–< 5	2
5–< 10	2
10–< 15	5
15–< 20	5
20–< 25	6
25–< 30	3
Total	23

- **a.** Identify which category the median test mark for the class falls into.
- **b.** Determine the mean of this data, to 1 decimal place.
- **c.** Determine the mode of this data.

6. Calculate the quartiles for the following data.

132, 98, 121, 140, 160, 109, 175, 182, 147, 124,
159, 118, 136, 154, 165, 148, 133, 155, 169, 145

Complex familiar

7. Below is a dataset showing the number of doors people have in their houses.

18, 11, 9, 14, 11, 16, 14, 16, 6, 18, 19, 9, 10, 17, 36

- **a.** Identify the minimum and maximum number of doors, and hence calculate the range.
- **b.** Calculate the mean and median.
- **c.** Identify any outliers. If there are any, remove these and recalculate the mean and median.
- **d.** Compare your answers to parts **a** and **c**.

8. The number of books borrowed by 20 students over the period of a month is monitored and the data is recorded as follows.

12, 8, 6, 10, 4, 5, 2, 7, 8, 0, 6, 4, 8, 13, 5, 3, 0, 8, 7, 9

Determine the mean, median, mode, range and IQR of the data.

9. Calculate the mean (to 2 decimal places), median, mode, range and IQR of the following data collected when the temperature of the soil around 25 germinating seedlings was recorded.

28.9, 27.4, 23.6, 25.6, 21.1, 22.9, 29.6, 25.7, 27.4, 23.6, 22.4, 24.6, 21.8,
26.4, 24.9, 25.0, 23.5, 26.1, 23.6, 25.3, 29.5, 23.5, 22.0, 27.9, 23.6

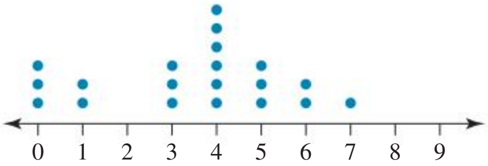
10. The following data shows the number of ice-creams sold nightly over two weeks.

4, 6, 48, 29, 39, 48, 44, 45, 39, 47, 48, 32, 31, 51

Determine whether there are any outliers. Justify your answer with appropriate calculations.

11. For the dot plot shown, calculate:

- **a.** the mean (to 2 decimal places)
- **b.** the median
- **c.** the IQR
- **d.** the standard deviation
- **e.** the range.



12. Consider the following stem plot.

Calculate:

a. the mean
b. the median
c. the IQR
d. the standard deviation.

Key: 11|2 = 112

Stem	**Leaf**
5	0 1 2
6	4 5 8 9
7	0 7
8	0 3 3
9	5 5 5 7 8
10	1 1 2 3 4 6 6
11	0 1 3 3 4 5 5 5 7 9
12	1 1 2 2 3 3 4 7
13	2 6 9

13. A survey of Year 9 students was conducted to find out the average travel time to school. The results (in minutes) were as follows.

12, 18, 17, 15, 15, 12, 14, 14, 14, 17, 10, 14, 11, 16, 14, 14,
11, 15, 10, 9, 13, 20

a. Calculate the mean, median and mode of this dataset.
b. Determine whether any outliers exist in the data.
c. Explain which measure of centre you would choose as the average travel time. Justify your answer.

14. The price of a return plane ticket from Melbourne to London from nine different airlines is as follows:

$3400, $2800, $3500, $4100, $2900, $5200, $3900, $4000, $4575

Calculate the interquartile range.

15. The test scores (out of 100) of students in a class were recorded after a recent exam.

50, 72, 85, 91, 66, 55, 80, 77, 63, 88, 92,
74, 61, 69, 79, 70

Using a digital technology of your choice, calculate the sample standard deviation of the data, correct to 1 decimal place.

16. A dataset contains the numbers 5, 6, 6 and 10. A fifth number is added to the set. If this fifth number is a whole number, determine its possible values if:

a. the mean is now equal to the median
b. the mean is now greater than the mode
c. the mean is now greater than the range.

17. A newsagent records the amount of magazines sold per day over several days.

0, 1, 1, 2, 2, 3, 3, 4, 4, 4, 5, 5, 5, 6, 6, 7, 7, 7, 7, 7, 8, 8, 8, 8, 9, 9, 10, 10, 10, 10, 10, 11, 11,
11, 11, 11, 11, 11, 12, 12, 12, 12, 13, 13, 14, 14, 15, 15, 15, 15, 16, 17, 18, 18, 18, 19, 44

Determine if 44 magazines in a day is an outlier. Justify your answer using calculations.

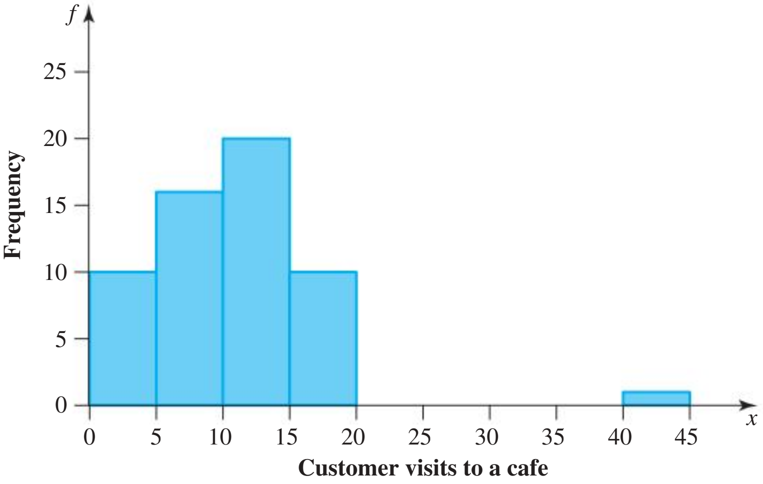
18. The following data shows the number of Indigenous and non-Indigenous women who gave birth, by state and territory, in 2024.

Status	NSW	Vic	Qld	WA	SA	Tas	ACT	NT	Aust
Indigenous	2904	838	3332	1738	607	284	107	1474	11 284
Non-Indigenous	91 958	70 328	57 665	29 022	18 994	5996	5601	2369	281 933

a. Calculate the mean births per state/territory of Australia in 2024 for both Indigenous and non-Indigenous groups. Give your answers correct to 1 decimal place.
b. Calculate the median births per state/territory of Australia in 2024 for both Indigenous and non-Indigenous groups.
c. Calculate the standard deviation (correct to 1 decimal place) and IQR for the data on births per state/territory of Australia in 2024 for both Indigenous and non-Indigenous groups.
d. Comment on the measures of centre and spread you have calculated for this data.

Complex unfamiliar

19. Kris and Loren were discussing the following histogram showing the customers visiting a cafe each day. Kris said that the best way to describe the central tendency for the number of visitors would be to look at the median. Evaluate the reasonableness of Kris's statement. Be sure to use mathematical reasoning to justify your response.



20. A real estate agent has the following sales (in thousands) over the last year.

300, 240, 350, 290, 330, 260, 270, 380, 500, 285, 250, 280, 310

The agent claims that the highest sale is an outlier and therefore the median will need to be adjusted accordingly, but insists that no other changes are necessary. Evaluate the reasonableness of the agent's changes.

Fully worked solutions for this chapter are available online.

Answers

Chapter 10 Summarising and interpreting data

10.2 Measures of central tendency and mode

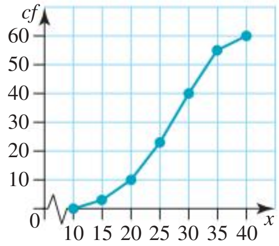
10.2 Exercise

1. **a.** **i.** 6
   ii. 6
   iii. 8
   b. **i.** 10
   ii. 12
   iii. 12
   c. **i.** 26.2
   ii. 28
   iii. There is no mode.
   d. **i.** 6
   ii. 6.5
   iii. 9
   e. **i.** 5
   ii. 5
   iii. 4, 5
   f. **i.** 3.66
   ii. 3.7
   iii. 3.5, 3.8
2. **a.** Mean: 22.6; median: 23; mode: 23
   b. Mean: 7.7; median: 8; mode: 2, 4, 6, 7, 8, 9, 10, 11, 12 (no mode)
   c. Mean: 48; median: 49; mode: 50
   d. Mean: 1.9; median: 1.7; mode: 1.4, 1.7
3. 1231, 1269, 1112
4. 47
5. Mean: 104 km/h; median: 102 km/h; mode: 101 km/h
6. 4.8
7. 27
8. **a.** Mean: 2.4; median: 2; mode: 1
   b. Mean: 26.9; median: 27; mode: 26
   c. Mean: 2.6; median: 2.5; mode: 3.0
9. 73.5
10. 171
11. Median class interval is 160–170.
12. 160–170
13. **a–c.** Sample responses can be found in the worked solutions in the online resources.
14. 435 cm
15. There must have been an even number of students so the median would be the average between the middle two test scores.

10.3 Measures of spread

10.3 Exercise

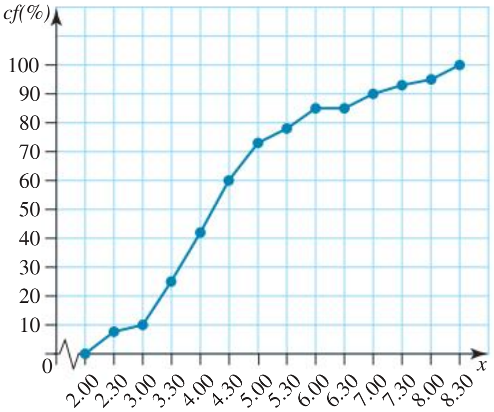
1. Minimum, lower quartile, median, upper quartile, maximum
2. $Q_1 = 19$
   $Q_2 = 26$
   $Q_3 = 31$
3. $Q_1 = 118$
   $Q_2 = 144$
   $Q_3 = 175$
4. $Q_1 = 158.5$
   $Q_2 = 174$
   $Q_3 = 185$
5. **a.** 110 **b.** 99 **c.** 120
6. more than 117 steps/minute
7. **a.**



   b. 10
   c. 27.5 km
   d. The 30th percentile is 24 km. The Upper quartile is 33.5 km.

8. a. See the table at the bottom of the page.*

b.



c. This statement is incorrect because the cumulative percentage curve shows that 80% of customers must wait over 5 minutes for their food.

9. a. 11 **b.** 2.6 **c.** 4

10. Range = 15 km/h, IQR= 2 km/h

11. Range = 27 hours, IQR = 8 hours

12. 8.6 (correct to 1 decimal place)

13. 68

14. 35

15. 22.0

16. a. 157 **b.** 69

17. 5

18. 32.0 years

19. 8.7 (correct to 1 decimal place)

20. Sample responses can be found in the worked solutions in the online resources.

10.4 Outliers [complex]

10.4 Exercise

1. a. 5 **b.** 23

2. IQR = 23; outliers are 1 and 15.

3. a. 6 **b.** 35 is an outlier.

4. 19 and 59

5. a. 1 and 5.8 **b.** 6.7

6. 166 is an outlier. (It is above the upper fence of 164.)

7. a. 21

b. Mean: 2.4; median: 2

c. Mean: 1.9; median: 2

d. Mean value was lowered; the median remained unchanged.

8. a. 15 is larger than the rest of the data.

b. By taking 15 out, which is larger than the rest of the data, the mean would decrease.

---

*8. a.

Class intervals	Frequency	Cumulative frequency (*cf*)	Percentage cumulative frequency (% *cf*)
2:00 – <2:35	3	3	6%
2:30 – <3:00	2	5	10%
3:00 – <3:30	8	13	26%
3:30 – <4:00	8	21	42%
4:00 – <4:30	9	30	60%
4:30 – <5:00	7	37	74%
5:00 – <5:30	2	39	78%
5:30 – <6:00	4	43	86%
6:00 – <6:30	0	43	86%
6:30 – <7:00	2	45	90%
7:00 – <7:30	2	47	94%
7:30 – <8:00	1	48	96%
8:00 – <8:30	2	50	100%

c. With:
Mean = 5.125
Without:
Mean = 3.71

9. a. 3 and 6 are smaller than the rest of the data.
b. Taking out 3 and 6, which are smaller than the rest of the data, would increase the mean.
c. With = 26.58; without = 31

10. a. 0.9 is larger than the rest of the data.
b. Taking out 0.9, which is large, would decrease the median.
c. With:
Median = 0.04
Without:
Median = 0.035

11. a. 5.2 and 6.1 are outliers.
b. Taking out 5.2 and 6.1, which are larger than the rest of the data, would decrease the median.
c. With = –2.3; without = –2.4

12. The mean would decrease, but the median would stay the same.

13. a. IQR = 35 − 18 = 17
Upper fence $= Q_3 + 1.5 \times \text{IQR} = 35 + 1.5 \times 17 = 60.5$
Any value above 60.5 will be an outlier, so 64 is an outlier.
b. Q_1, Q_3 and IQR would remain unchanged.

14. Answers will vary. Sample responses can be found in the worked solutions in the online resources.

10.5 Applications of measures of centre and spread [complex]

10.5 Exercise

1. a. 39
b. 40
c. 41
d. 40
e. The mode, as it gives the highest number of M&M's in a packet

2. The mean of grouped data is calculated using the midpoint of the class interval, making it an approximation. The mean of ungrouped data will give an exact value that is not an approximation. Using both grouped and ungrouped data, a single value can be obtained for the mean.
The median for grouped data will be a class interval, rather than an exact value as for ungrouped data.
The mode is calculated the same way for grouped and ungrouped data: the value with the greatest frequency.

3. a. Dataset A: mean: 42.8; median: 37
Dataset B: mean: 39.4; median: 37
b. The mean of dataset A is larger than the mean of dataset B. This is because the last value is 96 in dataset A, compared to 66 in dataset B. The medians of the two datasets are the same because each dataset has the same number of data values and all data values are the same, except the last value.
c. The mean or median can be used as good measures of central tendency when there are no outliers or extreme values. The median is best to use when there are outliers or extreme values in the data.

4. a. Student A: median = 51, IQR = 5
Student B: median = 64, IQR = 11
b. Student B
c. Student A

5. Answers will vary. Example answers are shown.
a. 2, 5, 17, 21, 29, 35, 39, 45. (Several different datasets are possible.)
b. 1, **5**, 6, 8, **12**, 14. (Several different datasets are possible, but the numbers in bold must be present in the same position.)
c. 5, 6, 8, 9, 10, 11, 12, 14, 15. (Several different datasets are possible.)

6. 5, 10, **14**, 21, 22, 23, 31, **34**, 35, 40. (Several different datasets are possible, but the numbers in bold must be present in the same position.)

7. Calculate the standard deviation. If the standard deviation is small, then the majority of parks will contain close to 15 trees.

8. a. Sydney = 120
Brisbane = 40
b. Brisbane, since the range is lower

9. a. Machine A range: 9 Smarties
b. Machine B range: 8 Smarties
c. Only 2 boxes out of 10 sampled from machine A contain less than 100 Smarties. The box with 108 Smarties causes the range to be higher than normal.

10.

Price ($)	Class centre	Frequency
0–< 500	250	2
500–< 1000	750	4
1000–< 1500	1250	8
1500–< 2000	1750	10
2000–< 2500	2250	7
2500–< 3000	2750	6
3000–< 3500	3250	3

Use the statistical function on a calculator. The sample standard deviation should be used.
Mean = \$1825, sample SD = \$797

11. The median, as the dataset has two clear outliers

12. a. \$74 231
b. \$65 000
c. It would be in the workers' interest to use a higher figure when negotiating salaries, whereas it would be in the management's interest to use a lower figure.

13. a. Year 1: interquartile range = 38 907.5, standard deviation = 20 382.8
Year 2: interquartile range = 37 110, standard deviation = 19 389.01
b. Year 1: interquartile range = 31 107.5, standard deviation = 18 123.5

Year 2: interquartile range = 29 140,
standard deviation = 17 289.2

c. Both values are reduced by a similar amount, but there is a bigger impact on the standard deviation than on the interquartile range.

14. a. 2.9

b. 1.25

c. The range is less than double the value of the interquartile range. This indicates that the data is quite tightly bunched with no outliers.

15. a. Mean = 63 000
Median = 47 500
Mode = no mode

b. The median ($47 500) better represents a typical salary because it is not skewed by the extremely high outlier ($200 000).

c. Answers will vary. Sample responses can be found in the worked solutions in the online resources.

16. Answers will vary. Sample responses can be found in the worked solutions in the online resources.

10.6 Review

10.6 Exercise

1. a. Mean: 5.3; median: 6; mode: 6

b. Mean: 36.4; median: 32.5; no mode

c. Mean: 9; median: 10; mode: 10

2. Mean: 7.2; median: 7

3. 96.4 minutes

4. a. Range

b. Median

c. IQR

d. Mean

e. Standard deviation

f. Mode

5. a. 15–20 **b.** 16.8 **c.** 20–25

6. $Q_1 = 128$
$Q_2 = 146$
$Q_3 = 159.5$

7. a. Minimum: 6; maximum: 36; range: 30

b. Mean: 14.9; median: 14

c. Possible outlier 36; mean: 13.4; median: 14

d. The median was unchanged by removing the outlier, whereas the mean decreased.

8. Mean: 6.25; median: 6.5; mode: 8; range: 13; IQR: 4

9. Mean: 25.0; median: 24.9; mode: 23.6; range: 8.5; IQR: 3.4

10. 4 is an outlier.

11. a. 3.45 **b.** 4 **c.** 3
d. 2.1 **e.** 7

12. a. 101.0 **b.** 106 **c.** 37
d. 23.5

13. a. Mean: 13.9; median: 14; mode: 14

b. 20 is an outlier.

c. Any measure could be used in this case, as they all give similar results. However, since the data contains an outlier, the median is the more appropriate measure of centre.

14. 1187.5

15. 12.5 (correct to 1 decimal place)

16. a. 3

b. Any number greater than 3

c. Any number from 4 to 12

17. 44 is above the upper fence of 23.5, so the data value is an outlier.

18. a. Indigenous mean = 1410.5
Non-Indigenous mean = 35 241.6

b. Indigenous median = 1156
Non-Indigenous median = 24 008

c. Indigenous:
standard deviation = 1193.8, IQR = 1875.5
Non-Indigenous:
standard deviation = 33 949.03, IQR = 58 198

d. The median and IQR are probably more appropriate due to the presence of potential extreme values in the data.

19. Kris. There is a possible outlier at 40–45, so the median is a better measure of central tendency, as it is less affected by outliers than the mean.

20. Answers will vary. Sample responses can be found in the worked solutions in the online resources.

UNIT 4 Graphs, data and loans
TOPIC 2 Summarising and comparing data

11 Comparing datasets

LESSON SEQUENCE

Fully worked solutions for this chapter are available online.

on Resources

Solutions	Solutions — Chapter 11
Digital documents	Learning matrix — Chapter 11 Quick quizzes — Chapter 11 Chapter summary — Chapter 11

LESSON
11.1 Overview

Hey students! Bring these pages to life online

Watch videos

Engage with interactivities

Answer questions and check results

Find all this and MORE in jacPLUS

11.1.1 Introduction

It is important to learn the content on comparing datasets for several key reasons:

- **Data summarisation:** Completing a five-number summary and constructing box plots help students summarise and visualise data efficiently. This is essential for understanding the distribution and spread of data.
- **Data comparison:** Comparing parallel box plots and back-to-back stem plots allows people to analyse and interpret differences between datasets. This skill is crucial for identifying trends and making informed decisions based on data.
- **Understanding data shapes:** Learning to compare the characteristics of histograms, such as symmetry, skewness and bimodality, helps people understand the underlying patterns and distributions in data. This is important for accurate data interpretation.
- **Analytical skills:** These activities enhance people's analytical skills by teaching them how to critically evaluate and compare different datasets. This is valuable for various real-world applications, from scientific research to business analysis.

Overall, these skills are fundamental for people to become proficient in data analysis, enabling them to make data-driven decisions and understand complex information in their future studies and careers.

11.1.2 Syllabus links

Lesson	Lesson title	Syllabus links
11.2	**Constructing box plots**	○ Complete a five-number summary for different datasets. ○ Construct a box plot using a five-number summary.
11.3	**Comparing parallel box plots [complex]**	○ Compare parallel box plots and back-to-back stem plots for different datasets [complex].
11.4	**Comparing back-to-back stem plots [complex]**	○ Compare parallel box plots and back-to-back stem plots for different datasets [complex].
11.5	**Comparing histograms [complex]**	○ Compare the characteristics of the shape of histograms using symmetry, skewness and bimodality, where applicable [complex].

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LESSON
11.2 Constructing box plots

SYLLABUS LINKS

- Complete a five-number summary for different datasets.
- Construct a box plot using a five-number summary.

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11.2.1 Five-number summary

- A **five-number summary** is a list of some key summary statistics for a given set of data:
  - lowest score
  - lower quartile (Q_1)
  - median (Q_2)
  - upper quartile (Q_3)
  - highest score.
- These key statistics allow for easy calculation and comparison of the measures of spread, range and interquartile range (IQR), along with the key measure of central tendency not affected by outliers, the median.
- Conventionally, these numbers are listed in order from smallest to largest without labels; so, for example, the five-number summary:

$$4, 15, 21, 23, 28$$

would be interpreted as lowest score 4, lower quartile $Q_1 = 15$, median 21, upper quartile $Q_3 = 23$ and highest score 28.

WORKED EXAMPLE 1 Using a five-number summary to determine the measure of spread

Consider the following five-number summary:

$$\mathbf{29, 37, 39, 44, 48}$$

Determine:

a. the median

b. the interquartile range

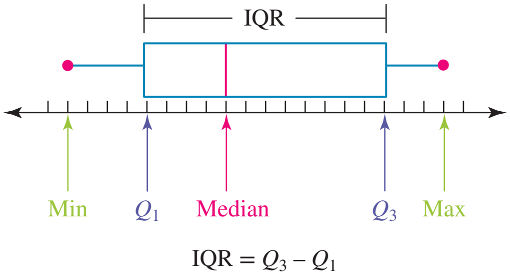
c. the range.

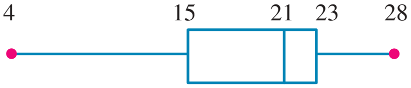
THINK	WRITE
The figures are presented in the order: lowest score, lower quartile, median, upper quartile, highest score.	Lowest $= 29$, $Q_1 = 37$, median $= 39$, $Q_3 = 44$, highest $= 48$
a. The median is the middle (here, third) number in the list.	**a.** Median $= 39$
b. The interquartile range is the difference between the upper and lower quartiles.	**b.** IQR $= Q_3 - Q_1$ $= 44 - 37$ $= 7$
c. The range is the difference between the highest score and the lowest score.	**c.** Range $=$ highest $-$ lowest $= 48 - 29$ $= 19$

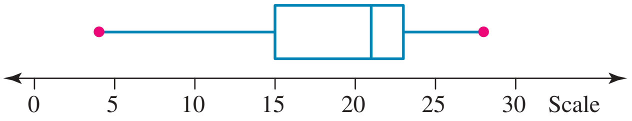
11.2.2 Box plots

int-6245

- A **box plot (or box-and-whisker plot)** is a graphical representation of the five-number summary.
- It is a powerful way to show the centre and spread of data.
- Box plots consist of a central divided box with attached 'whiskers'.
- The box spans the interquartile range.
- The median is marked by a vertical line inside the box.
- The whiskers indicate the range of scores.
- Box plots are *always drawn to scale*. They are presented either with the five-number summary figures attached as labels (below left) or, more conventionally, with a scaled axis presented alongside the box plot (below right).

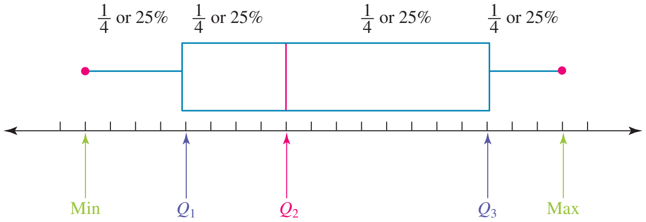




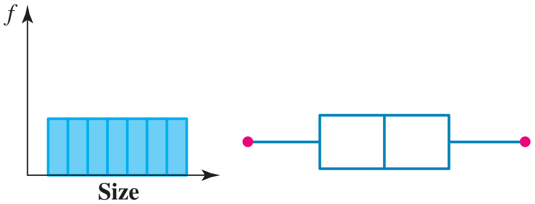


11.2.3 Interpreting a box plot

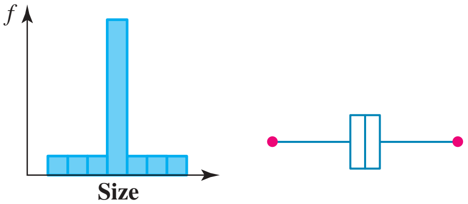
- A box plot is broken into four sections, each representing 25% of the data in that range. Using the example above:
- The reader can easily see where clustering of the data may occur.
- Box plots allow the reader to see where clustering of the data may occur and clearly shows the distribution of a dataset.



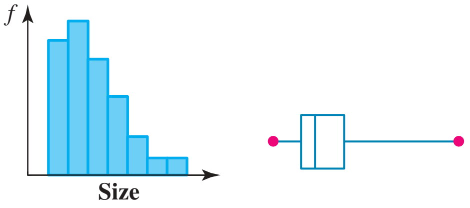
- Consider the following box plots with their matching histograms:



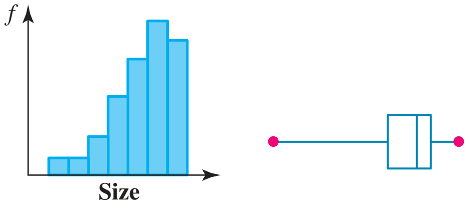
- A box with even-sized box sections and whiskers would indicate an even spread of data.



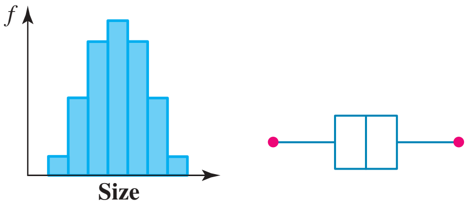
- A small box plot with relatively long whiskers indicates half of the data (between Q_1 and Q_3) is confined to a small range.
- The data could be described as clustered in the centre.



- Positively skewed data has larger amounts of data at the lower end.
- The median is left of centre, closer to Q_1.



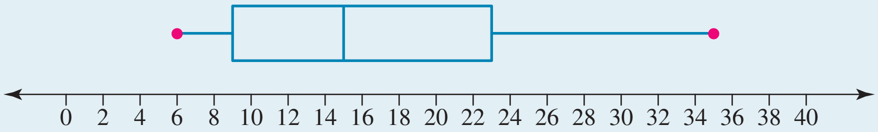
- Negatively skewed data has larger amounts of data at the higher end.
- The median is right of centre, closer to Q_3.



- Even, smaller boxes with long whiskers would indicate Data is evenly spaced around the central point

WORKED EXAMPLE 2 Interpreting a box plot

The following box plot shows the distribution of goals scored by a netball team over a season.



Determine:

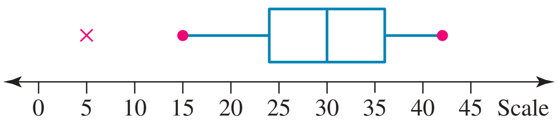
a. the team's highest score
b. the team's lowest score
c. the median goals scored by the team
d. the range of goals scored by the team
e. the interquartile range of goals scored by the team.

THINK	WRITE
a. The highest score is the whisker on the right.	**a.** The highest number of goals scored was 35.
b. The lowest score is the whisker on the left.	**b.** The lowest number of goals scored was 6.
c. The median score is the middle line in the box section of the plot.	**c.** The median number of goals scored was 15.
d. The range of goals scored is the difference between the highest and lowest score.	**d.** Range = highest score − lowest score = 35 − 6 = 29 The range is 29.
e. The interquartile range is the difference between Q_3 and Q_1 (the range of the 'box section' of the box plot).	**e.** IQR = $Q_3 - Q_1$ $= 23 - 9$ $= 14$ The interquartile range is 14.

11.2.4 Identification of outliers

int-4623

- **Outliers** can make the whiskers appear longer than they are, giving the impression that the data is far more dispersed than it actually is.
- If an outlier occurs in a set of data, it can be denoted by a small cross on the box plot. The whisker is then shortened to the next largest (or smallest) value.
- The following box plot shows that the lowest score was 5. This was an outlier, as the rest of the scores were located within the range 15 to 42.
  The second lowest score was 15. This was not an outlier, so the left-hand whisker is moved from 5 to 15.
- Recall from lesson 10.4 that extreme values (outliers) can be identified by calculating the values of the lower and upper fences. Values that lie below the lower fence or above the upper fence are outliers.



$$\text{Lower fence} = Q_1 - 1.5 \times \text{IQR}$$

$$\text{Upper fence} = Q_3 + 1.5 \times \text{IQR}$$

tlvd-12144

WORKED EXAMPLE 3 Constructing and describing a box plot

The stem plot (stem-and-leaf plot) below gives the speed of 25 cars caught by a roadside speed camera.

a. Prepare a five-number summary of the data.

b. Construct a box plot of the data (Identify any outliers).

c. Describe the distribution of the data in terms of shape, centre and spread.

Key: 8|2 = 82 km/h

8*|6 = 86 km/h

Stem	**Leaf**
8	2 2 4 4 4 4
8*	5 5 6 6 7 9 9 9
9	0 1 1 2 4
9*	5 6 9
10	0 2
10*	
11	4

THINK

a. 1. First identify the median and upper and lower quartiles.
There are 25 pieces of data. The median is the $\frac{25+1}{2}$th score — that is, the 13th score.
Q_1 is halfway between the 6th and 7th scores in the lower half of the data. Q_3 is halfway between the 6th and 7th scores in the upper half of the data.

WRITE

a. Key: 8|2 = 82 km/h

8*|6 = 86 km/h

Stem	**Leaf**
8	2 2 4 4 4 4 $\vert_{Q_1}$
8*	5 5 6 6 7 9 $\boxed{9}$ 9
9	0 1 1 2 4 $\vert_{Q_3}$
9*	5 6 9
10	0 2
10*	
11	4

$Q_1 = 84.5$ km/h
Median = 89 km/h
$Q_3 = 94.5$ km/h

2. Identify the lowest and highest data values.

Minimum = 82 km/h
Maximum = 114 km/h

3. Write the five-number summary.

Five-number summary: 82, 84.5, 89, 94.5, 114
The lowest score is 82. The lower quartile is 84.5. The median is 89. The upper quartile is 94.5. The highest score is 114.

b. 1. Identify any outliers by calculating the upper and lower fences.

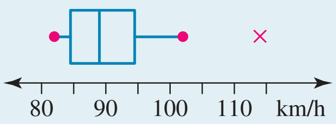
Lower fence $= Q_1 - 1.5 \times \text{IQR}$

Upper fence $= Q_3 + 1.5 \times \text{IQR}$

b. $\text{IQR} = 94.5 - 84.5$
$= 10$

$\text{Lower fence} = 84.5 - 1.5 \times 10$
$= 69.5$
$\text{Upper fence} = 94.5 + 1.5 \times 10$
$= 109.5$
114 is higher than the upper fence; hence it is an outlier.

2. To construct the box plot, start by ruling a suitable scale. Remember to include the units of measurement. The box represents the interquartile range, so it runs from 84.5 to 94.5. The median is a vertical line in the box at 89. The whiskers should extend to the lowest score (82) and the highest score that is not an outlier (102). The outlier at 114 should be indicated by a cross.



c. 1. Because an outlier is present in this set of data, the most useful measure of centre is the median, and the best measure of spread is the interquartile range. The position of the outlier and the shape of the box plot should also be commented on.

c. The distribution of car speeds is positively skewed, with an outlier at 114 km/h. The average speed, as measured by the median, is 89 km/h. The spread of speeds, as measured by the interquartile range, is 10 km/h.

Exercise 11.2 Constructing box plots

learnon

11.2 Quick quiz on | **11.2 Exercise**

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15	N/A	N/A

Simple familiar

1. **WE1** From the following five-number summary:

$$6, 11, 12, 16, 32$$

determine:

a. the median **b.** the interquartile range **c.** the range.

2. From the following five-number summary:

$$101, 119, 122, 125, 128$$

determine:

a. the median **b.** the interquartile range **c.** the range.

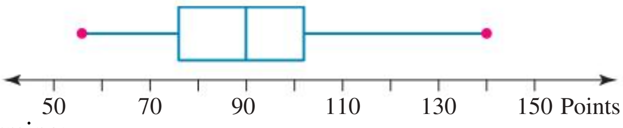
3. From the following five-number summary:

$$39.2, 46.5, 49.0, 52.3, 57.8$$

determine:

a. the median **b.** the interquartile range **c.** the range.

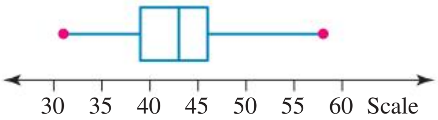
4. **WE2** The following box plot shows the distribution of final points scored by a football team over a season's roster.



Determine:

a. the team's highest score
b. the team's lowest score
c. the median points scored by the team
d. the range of points scored by the team
e. the interquartile range of points scored.

5. The following box plot below shows the distribution of data formed by counting the number of honey bears per pack in a large sample of packs.

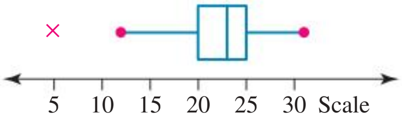


Determine:

a. the largest number of honey bears in any pack
b. the smallest number of honey bears in any pack
c. the median number of honey bears in any pack
d. the range of numbers of honey bears per pack
e. the interquartile range of honey bears per pack.

6. a. Interpret what it means if the median on a box plot is not positioned exactly halfway between the upper and lower quartiles.
b. Analyse the implications of the median being closer to the upper quartile than the lower quartile on a box plot.
c. Explain how the position of the median in a box plot provides insights into the distribution of a dataset.

Questions 7–9 refer to the following box plot.



7. Determine the median of the data.

8. Determine the interquartile range of the data.

9. Describe the spread and distribution of data in terms of the whiskers and box.

10. **WE3** The following stem plot details the age of 25 offenders who were caught during random breath testing.

Key: 1|8 = 18 years

Stem	Leaf
1	8 8 9 9 9
2	0 0 0 1 1 3 4 6 9
3	0 1 2 7
4	2 5
5	3 6 8
6	6
7	4

a. Prepare a five-number summary of the data.
b. Construct a box plot of the data.
c. Describe the distribution of the data in terms of shape, centre and spread.

11. The number of sales made each day by a salesperson is recorded over a 2-week period:

$$25, 31, 28, 43, 37, 43, 22, 45, 48, 33$$

a. Prepare a five-number summary of the data. (There is no need to draw a stem plot of the data.)
b. Construct a box plot of the data.

12. The following data show monthly rainfall in millimetres.

J	F	M	A	M	J	J	A	S	O	N	D
10	12	21	23	39	22	15	11	22	37	45	30

a. Prepare a five-number summary of the data.
b. Construct a box plot of the data.

13. The stem plot details the sale price of 30 apartments in a particular suburb.

Key: 32|4 = $324 000

Stem	Leaf
32	4 7 9
33	0 0 2 5 5
34	0 0 2 3 5 5 7 9 9
35	0 0 2 3 7 7 8
36	0 2 2 5 8
37	5

a. Prepare a five-number summary of the data.
b. Construct a box plot of the data.

14. The following data detail the number of hamburgers sold by a fast-food outlet every day over a 4-week period.

M	T	W	T	F	S	S
125	144	132	148	187	172	181
134	157	152	126	155	183	188
131	121	165	129	143	182	181
152	163	150	148	152	179	181

a. Prepare a stem plot of the data. (Use a class interval of size 10.)
b. Construct a box plot of the data.

15. The following data show the ages of 30 mothers upon the birth of their first baby.

$$22, 18, 17, 22, 24, 25,\\ 32, 19, 23, 28, 31, 19,\\ 23, 25, 23, 21, 33, 23,\\ 24, 20, 29, 18, 22, 24,\\ 20, 22, 17, 48, 18, 20$$

a. Prepare a stem plot of the data. (Use a class interval of size 5.)
b. Construct a box plot of the data. Indicate any extreme values appropriately.
c. Describe the distribution in words. Explain what the distribution indicates about the age at which mothers have their first baby.

Fully worked solutions for this chapter are available online.

LESSON
11.3 Comparing parallel box plots [complex]

SYLLABUS LINKS

- Compare parallel box plots and back-to-back stem plots for different datasets [complex].

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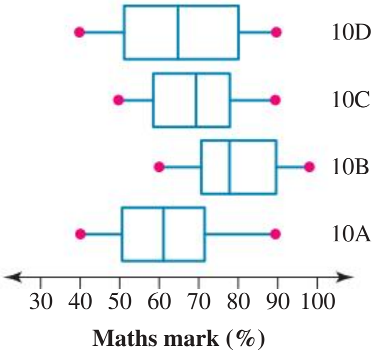
11.3.1 Parallel box plots

int-6248

- Parallel box plots are drawn next to each other using a common scale for ease of reference.
- The set of parallel box plots shown allows easy comparison of the results of four classes in a Maths test.
- When comparing parallel box plots, it is important to include a comparison of the individual features:
  - central tendency (median)
  - spread (both IQR and range)
  - outliers
  - shape of the distribution.
- Comparisons made on parallel box plots should always be in context.
  For example, to compare the above parallel box plots, comment on the median, spread, shape and outliers in terms of Maths marks for the different classes.



tlvd-12145

WORKED EXAMPLE 4 Comparing parallel box plots

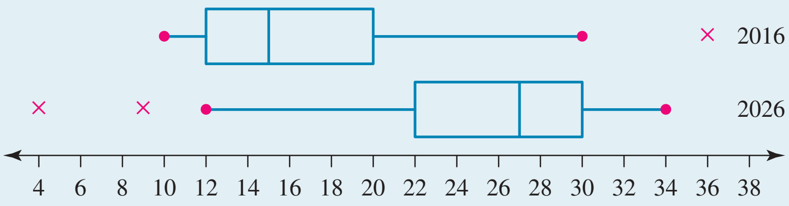
The following parallel box plots show the number of hats produced in a factory per hour over a two-day period in 2016 and 2026.

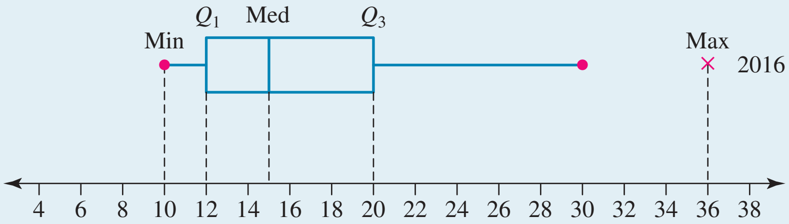
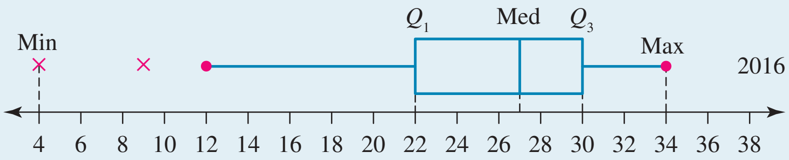
a. **Determine the five-number summary for the hourly hat production in both 2016 and 2026.**

b. **i.** **Identify the value above which the top 25% of hats produced in 2026 were distributed.**

  ii. **Identify the range of values that represent the middle 50% of hats produced in 2016.**

c. **Compare the hourly hat production for the two different years.**



THINK	WRITE
a. 1. Look at the box plot for 2016. Using the scale and the box plot, determine the values for the five-number summary.	**a.**  Five-number summary for 2016: 10, 12, 15, 20, 36
2. Look at the box plot for 2026. Using the scale and the box plot, determine the values for the five-number summary.	 Five-number summary for 2026: 4, 22, 27, 30, 34
b. i. The top 25% on a box plot occurs above the third quartile, Q_3.	**b. i.** The top 25% of hourly hats produced in 2026 occurred above 30 hats per hour.
ii. The middle 50% on a box plot is represented by the IQR, or the length of the box.	**ii.** The middle 50% of hourly hats produced in 2016 occurred between 12 and 20 hats per hour.
c. Compare the two box plots for the median, IQR, range, shape and outliers.	**c.** The distribution of hat production in 2016 was positively skewed, with an outlier at 36 hats per hour. The distribution of hat production in 2026 was negatively skewed, with outliers at 4 and 9 hats per hour. The parallel box plots show that median hourly hat production in 2016 (15 hats per hour) was lower than in 2026 (27 hats per hour). The IQR for both 2016 and 2026 was identical (8 hats per hour). The consistency of hat production was similar, as evidenced by the IQR.

Exercise 11.3 Comparing parallel box plots [complex]

learnon

11.3 Quick quiz	11.3 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4	5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16	17

Simple familiar

1. A gardener recorded how long it took to mow each lawn on the weekly route (in minutes). The dataset is shown.

32, 45, 56, 28, 19, 38, 26, 47, 54, 21, 33, 40, 17, 58, 21

a. Determine the five-number summary for the dataset.
b. Draw a box plot of the data.
c. Describe the distribution of the data in terms of shape, centre and spread.

2. The weights of 14 boxes (in kilograms) being moved from one house to another are as follows.

3, 6, 13, 15, 15, 15, 17, 17, 18, 20, 20, 22, 26, 30

a. Construct a box plot to display this dataset.
b. Describe the distribution of the data in terms of shape, centre and spread.

3. The prices of 10 mobile phones are viewed online and are as follows.

\$349, \$469, \$265, \$497, \$159, \$52, \$999, \$489, \$599, \$577

a. Construct a box plot to represent this dataset.
b. Describe the distribution of the data in terms of shape, centre and spread.

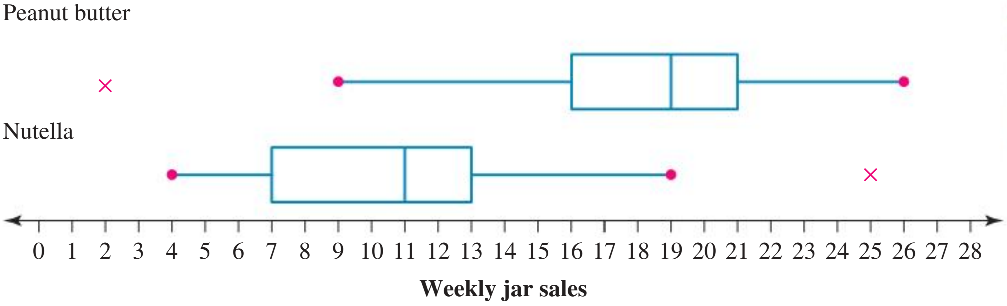
4. The speeds of 15 cars in a school zone road were as follows.

39, 51, 60, 42, 44, 38, 75, 45, 40, 52, 41, 42, 46, 41, 39

a. Construct a box plot to represent this dataset.
b. Describe the distribution of the data in terms of shape, centre and spread.

Complex familiar

5. **WE4** The following parallel box plots show the weekly sales for jars of peanut butter and Nutella in a supermarket.

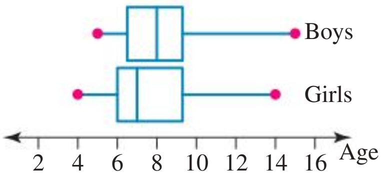


- a. Determine the five-number summary for the weekly sales of peanut butter and Nutella.
- b. i. Identify the value below which the lowest 25% of weekly peanut butter sales occur.
  - ii. Identify the range of values that defines the middle 50% of weekly Nutella sales.
- c. Compare the weekly sales for the two different products.

6. A restaurant trialled three different menus for a month each to work out which type of cuisine its patrons enjoy the most. The restaurant has asked for some help analysing the data collected.
   - a. Draw a box plot for each month of data on the same scale.
   - b. Compare the box plots to help the restaurant decide which menu will be the most successful.

	Week 1	Week 2	Week 3	Week 4
Month 1: Greek	75	62	48	50
Month 2: Italian	48	17	9	68
Month 3: Spanish	56	43	37	28

The following parallel box plots, showing the ages at which boys and girls learn to ride a 2-wheel bike, should be used to answer questions 7–9.

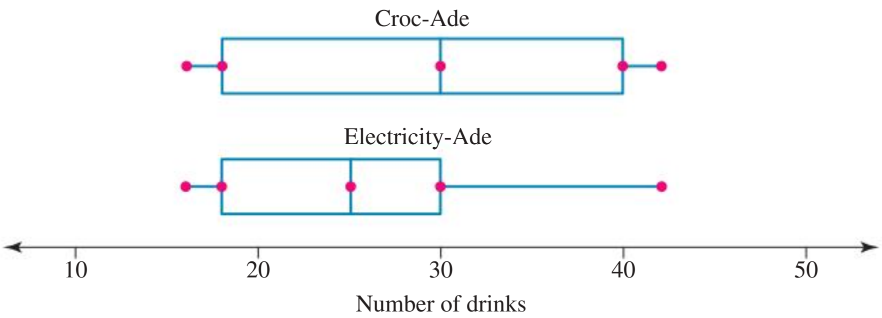


7. Looking at the parallel box plots, identify at what age the youngest girl and the oldest boy, respectively, learned to ride a two-wheel bike.

8. Identify which value is the same for both box plots.

9. Determine the difference between the medians of the box plots.

10. Two sports drink companies have made the following sales at a sporting events this year.



a. Determine the five-number summary for the sales of each sports drink.

b. **i.** Identify the value below which the lowest 75% of Electricity-Ade sales are located.

ii. Identify the values between which the middle 50% of Croc-Ade sales are located.

c. Compare the sales for each drink.

11. A study was conducted on the usage of social media among teenagers who use Instagram and TikTok. The following results are shown in minutes:

- Instagram users: 45, 50, 55, 60, 65, 70, 75
- TikTok users: 60, 65, 70, 75, 80, 85, 90

a. Construct a parallel box plot of both platforms.

b. Compare the social media usage patterns of Instagram and TikTok users.

12. Teenagers' daily song plays on two music platforms have been recorded.
The following results below are shown in minutes:

- Spotify users: 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 45
- Apple Music users: 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 50

a. Construct a parallel box plot of both platforms.

b. Compare the usage of Spotify and Apple Music.

13. The following data was collected from a company that is comparing two different compounds for sunscreen to see how long (measured in minutes) they remain effective in water before they need to be reapplied.

Compound 1	176	156	123	169	149	133	164	127	139	121	172	168
Compound 2	130	125	128	121	118	120	122	127	130	122	131	126

a. Construct parallel box plots for this dataset.

b. Compare the two compounds.

14. A comparison of Year 11 students' achievements (measured as a number out of 100) in History and English was recorded and the following results were obtained.

History	75	78	42	92	59	67	78	82	84	64	77	98
English	78	80	57	96	58	71	74	87	79	62	75	100

a. Construct parallel box plots for this dataset.

b. Analyse what the parallel box plots tell you about Year 11 students' achievements in History and English.

15. The heights of Year 11 and Year 12 students (to the nearest centimetre) are being investigated. The results of some sample data are shown.

Year 11	160	154	157	170	167	164	172	158	177	180	175	168	159	155	163	163	169	173	172	170
Year 12	160	172	185	163	177	190	183	181	176	188	168	167	166	177	173	172	179	175	174	180

a. Construct parallel box plots for this dataset.

b. Comment on what the plots tell you about the heights of Year 11 and Year 12 students.

16. The diameter of alloy rims (in inches) in a car yard are being investigated for two different ute brands. The results are shown below.

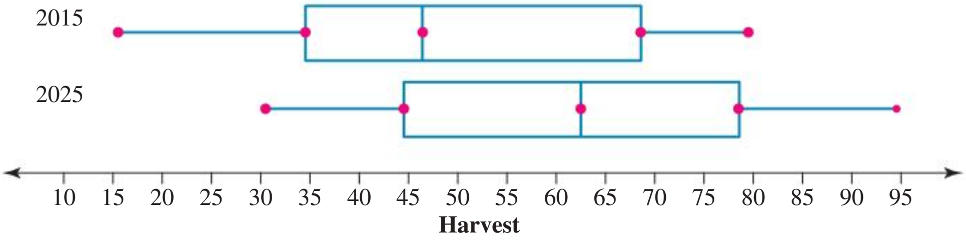
Amarok	16	16	17	16	21	20	16	20	20	21	18	16	19	18	21
Ranger	20	19	16	21	18	21	21	19	20	21	18	21	17	19	17

a. Construct parallel box plots for this data set.

b. Comment on what the plots tell you about the diameters of different brands given that a larger alloy is considered higher end.

Complex unfamiliar

17. A farmer is looking at his farm's harvest yield over the last decade. He says that it's not making any progress, but it's not going backwards, either. Its statistics are steady, with a good spread each year.



Explain whether you agree with the farmer's statement. Justify your answer.

Fully worked solutions for this chapter are available online.

LESSON
11.4 Comparing back-to-back stem plots [complex]

SYLLABUS LINKS

- Compare parallel box plots and back-to-back stem plots for different datasets [complex].

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11.4.1 Back-to-back stem plots

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- We have seen how to construct a stem plot for a single set of univariate data.
- We can extend a stem plot to display two different sets of univariate data in order to be able to make a comparison. We shall limit ourselves in this section to categorical variables with just two categories; for example, passing or failing an exam, voting yes or no.
- The two categories are used to provide two back-to-back leaves of a stem plot.

Back-to-back stem plots

A back-to-back stem plot is used to display bivariate data involving a numerical variable and a categorical variable with 2 categories.

WORKED EXAMPLE 5 Creating a back-to-back stem plot

Two classes in Grade 4 at Kingston Primary School submitted projects on the Olympic Games. The marks they obtained out of 20 are shown.

Class A's marks	**16**	**17**	**19**	**15**	**12**	**16**	**17**	**19**	**19**	**16**
Class B's marks	**14**	**15**	**16**	**13**	**12**	**13**	**14**	**13**	**15**	**14**

Display the data in the form of a back-to-back stem plot.

THINK	**WRITE**
1. Identify the highest and lowest scores in order to decide on the stems.	Highest score = 19 Lowest score = 12 Use a stem of 1, and divide into five class intervals. (Values of 12 and 13 will be included in the first class interval, values of 14 and 15 will be included in the next class interval, etc.)

2. Create an unordered stem plot first. Put Class B's scores on the left, and Class A's scores on the right.

Key: 1|2 = 12

Leaf: Class B	Stem	Leaf: Class A
3 2 3 3	1	2
4 5 4 5 4	1	5
6	1	6 7 6 7 6
	1	9 9 9

3. Now order the stem plot. The scores on the left should increase in value from right to left, while the scores on the right should increase in value from left to right.

Key: 1|2 = 12

Leaf: Class B	Stem	Leaf: Class A
3 3 3 2	1	2
5 5 4 4 4	1	5
6	1	6 6 6 7 7
	1	9 9 9

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11.4.2 Comparing back-to-back stem plots

- The back-to-back stem plot allows us to make some visual comparisons of two distributions for easy comparison to be made visually between the two distributions.
- In Worked example 5, the centre of the distribution for Class A is higher than the centre of the distribution for Class B. The spread of each of the distributions seems to be about the same.
- The marks for Class B are grouped between 12 and 15, and Class A marks are grouped between 16 and 19.
- Overall, we can conclude that Class A obtained better marks on average than Class B did.
- A more accurate way to compare each distribution is by using summary statistics of the centre and spread:
  1. the mean and the median (to measure the centre of the distributions), and
  2. the interquartile range and the standard deviation (to measure the spread of the distributions).
- The calculation of these summary statistics is very straightforward and rapid using technology.
- If the distribution of either category is skewed and/or contains outliers, then the median and interquartile range are better measures of centre and spread than the mean and the standard deviation.

tlvd-12146

WORKED EXAMPLE 6 Comparing back-to-back stem plots

The number of 'how-to-vote' cards handed out by various Australian Labor Party and Liberal Party volunteers during the course of a polling day is shown below.

Labor	**180**	**233**	**246**	**252**	**263**	**270**	**229**	**238**	**226**	**211**
	193	**202**	**210**	**222**	**257**	**247**	**234**	**226**	**214**	**204**
Liberal	**204**	**215**	**226**	**253**	**263**	**272**	**285**	**245**	**267**	**275**
	287	**273**	**266**	**233**	**244**	**250**	**261**	**272**	**280**	**279**

a. **Construct a back-to-back stem plot using the data.**

b. **Use the stem plot, together with summary statistics, to compare the distributions of the number of cards handed out by the Labor and Liberal volunteers.**

THINK	WRITE
a. 1. Identify the highest and lowest scores in order to decide on the stems.	a. The highest score is 287, and the lowest score is 180, so use stems 18 to 28.

2. Construct the stem plot.

Key: 18|0 = 180

Leaf: Labor	**Stem**	**Leaf: Liberal**
0	18	
3	19	
4 2	20	4
4 1 0	21	5
9 6 6 2	22	6
8 4 3	23	3
7 6	24	4 5
72	25	0 3
3	26	1 3 6 7
0	27	2 2 3 5 9
	28	0 5 7

b. 1. Use a form of technology to calculate the summary statistics:
the mean, the median, the standard deviation and the interquartile range for each party.
Enter each set of data as a separate list.

b. For the Labor volunteers:
Mean = 227.9
Median = 227.5
Interquartile range = 36
Standard deviation = 23.9
For the Liberal volunteers:
Mean = 257.5
Median = 264.5
Interquartile range = 29.5
Standard deviation = 23.4

2. Comment on the relationship.

From the stem plot, we see that the Labor distribution is symmetric and therefore the mean and the median are very close, whereas the Liberal distribution is negatively skewed.

Since the distribution is skewed, the median is a better indicator of the centre of the distribution than the mean.
The median number of cards handed out by Labor (228 cards) was lower than that of Liberals (265 cards).

The standard deviations were similar, as were the interquartile ranges. There was not a lot of difference in the spread of the data.
Overall, the Liberal Party volunteers handed out more ‘how-to-vote’ cards than the Labor Party volunteers.

Exercise 11.4 Comparing back-to-back stem plots [complex]

11.4 Quick quiz on

11.4 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
N/A	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	13, 14, 15, 16

Complex familiar

1. WE5 The marks (out of 50) obtained for the end-of-term test by the students in German and French classes are given below. Display the data in the form of a back-to-back stem plot.

German	20	38	45	21	30	39	41	22	27	33	30	21	25	32	37	42	26	31	25	37
French	23	25	36	46	44	39	38	24	25	42	38	34	28	31	44	30	35	48	43	34

2. The birth masses of 10 boys and 10 girls (in kilograms, to the nearest 100 grams) are recorded in the table.

Boys	3.4	5.0	4.2	3.7	4.9	3.4	3.8	4.8	3.6	4.3
Girls	3.0	2.7	3.7	3.3	4.0	3.1	2.6	3.2	3.6	3.1

Display the data using a back-to-back stem plot.

3. WE6 A nursery plants seedlings in two locations, one of which gets full sun and the other partial sun. The heights of the seedlings after 1 week are recorded in the stem plot below. Use this stem plot to identify which growing location is better.

Key 1|0 = 10 cm

Full sun	Stem	Partial sun
9 8 8 7 6 4 4 0 0	1	9
3 2 1 0 0	2	5 5 7 8 9
6	3	0 1 3 4 4 6 7 7 8

4. The following data shows the heights of students from two high school classes, in centimetres.

Class A	150	157	160	165	170	175	177	168	181	187	159	162
Class B	135	138	149	162	147	131	148	136	125	142	155	143

a. Construct a back-to-back stem plot of this data.
b. Based on this plot, determine which class would be the older class.

5. Two local cafes record the ages of their customers in the morning. Their results are recorded in the stem plot below.

Key 1|6 = 16

Cafe A	Stem	Cafe B
8 8 6	1	5 5 7 7 9 9 9
4 0	2	0 1 1 3 4 4 6 9
9 7 6	3	1 3 3 4 5 5 5 8 9
6 3 3 0	4	0 1 2 6
8 5 5 0	5	0 5 7
8 8 4 3 2	6	2
9 7 7 5 4 3 1 1	7	
1 1 0	8	

a. Determine which cafe has a larger spread of data.
b. Looking at the distribution of the data, determine which cafe attracts an older population. Justify your answer.

6. The number of delivery trucks making deliveries to a supermarket each day over a 2-week period was recorded for two neighbouring supermarkets — Supermarket A and Supermarket B. The data are as follows.

A	11	15	20	25	12	16	21	27	16	17	17	22	23	24
B	10	15	20	25	30	35	16	31	32	21	23	26	28	29

a. Construct a back-to-back stem plot using the data.
b. Use the stem plot, together with summary statistics, to compare the distributions of the number of trucks delivering to supermarkets A and B.

7. The marks out of 20 for Class A and Class B on a Science test are as follows.

Class A	12	13	14	14	15	15	16	17
Class B	10	12	13	14	14	15	17	19

a. Construct a back-to-back stem plot using the data.
b. Use the stem plot, together with summary statistics, to compare the distributions of the marks of Class A and Class B.

8. The end-of-year marks of 10 students in an English class were compared over 2 years. The marks from the first year and for the same students in the second year are shown.

First year	30	31	35	37	39	41	41	42	43	46
Second year	22	26	27	28	30	31	31	33	34	36

a. Construct a back-to-back stem plot using the data.
b. Use the stem plot, together with summary statistics, to compare the distributions of the marks obtained by the students over the 2 years.

9. The ages of participants in two different sports teams were recorded as follows:

Football	23	24	25	26	27	28	30	31
Soccer	22	25	30	31	36	37	42	46

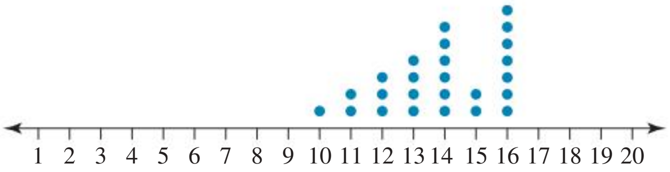
a. Construct a back-to-back stem plot using the data.
b. Use the stem plot, together with summary statistics, to compare the distributions of the ages of the football and soccer players.

10. The scores on a board game are recorded for a group of kindergarten children and for a group of children in a preparatory school.

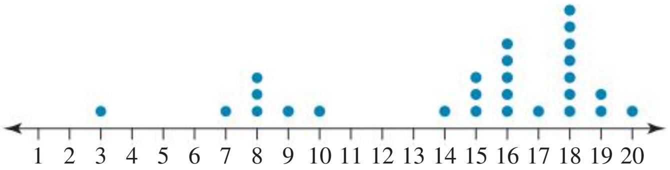
Kindergarten	3	13	14	25	28	32	36	41	47	50
Prep. school	5	12	17	25	27	32	35	44	46	52

a. Construct a back-to-back stem plot using the data.
b. Use the stem plot, together with summary statistics, to compare the distributions of the scores of the kindergarten children compared to the preparatory school children.

11. The two dot plots shown display the latest Maths test results for two Year 11 classes. The results show the marks out of 20.



Class 1



Class 2

a. Determine the number of students are in each class.
b. For each class, determine how many students scored 15 out of 20 on the test.
c. For each class, determine how many students scored more than 10 on the test.
d. Use the dot plots to describe the performance of each class on the test.

12. The following data show the ages of male and female players at a ten-pin bowling centre.

Males	20	36	16	38	32	18	19	21	25	45	29	60	31	21		
	16	38	52	43	17	28	23	23	43	17	22	23	32	34		
Females	21	23	30	16	31	46	15	17	22	17	50	34	65	25	27	19
	15	43	22	17	22	16	48	57	54	23	16	30	18	21	28	35

Draw a back-to-back stem plot of the data and compare the ages for each group.

Complex unfamiliar

13. The comparisons between the battery lives of two mobile phone brands are shown in the back-to-back stem plot below.

Key: 6|1 = 61 hours

Brand A	Stem	Brand B
8 8 7 5	0	7
9 7 4 1 0	1	0 5 5 5 7 9
2 2 2 1	2	0 2 2 6 7
8 6 4 2 0	3	0 2 4 6 8
	4	
	5	6
1	6	
	7	5

Determine which mobile phone brand has the better battery life. Explain.

14. Two internet providers were compared for their internet speeds (in Mbps). The results were recorded in the back-to-back stem plot below.

Key: 4|5 = 45 Mbps

Provider A	Stem	Provider B
5 2	4	
3	5	
8 4	6	8 9
	7	0 2 2 5 5 8
6	8	1 2
7 1	9	
8	10	
	11	
	12	
0	13	

Determine which company you should choose as your internet provider. Justify your answer using mathematical reasoning.

15. The winning times in seconds for the women's and men's 100-metre sprint in the Olympics are shown in the table. A sports commentator said that since 1928, the women have progressed faster than the men. Evaluate the reasonableness of this statement.

Year	Women's 100-m sprint	Men's 100-m sprint
1928	12.20	10.80
1932	11.90	10.30
1936	11.50	10.30
1948	11.90	10.30
1952	11.50	10.40
1956	11.50	10.50
1960	11.00	10.20
1964	11.40	10.00
1968	11.00	9.90
1972	11.07	10.14
1976	11.08	10.06
1980	11.60	10.25
1984	10.97	9.99
1988	10.54	9.92
1992	10.82	9.96
1996	10.94	9.84
2000	10.75	9.87
2004	10.93	9.85
2008	10.78	9.69

16. The following datasets show the rental price (in $) of one-bedroom apartments in two different suburbs of Townsville.

Suburb A	215	225	211	235	244	210	215	210	256	207
	200	200	242	225	231	205	240	205	235	200
Suburb B	235	245	231	232	240	280	280	270	255	275
	275	285	245	265	270	255	260	258	251	285

Your friend tells you that it doesn't matter where you find a house, the price is pretty much the same everywhere. Provide a mathematically justified response to your friend whether you support their claim or not.

Fully worked solutions for this chapter are available online.

LESSON
11.5 Comparing histograms [complex]

SYLLABUS LINKS

- Compare the characteristics of the shape of histograms using symmetry, skewness and bimodality, where applicable [complex].

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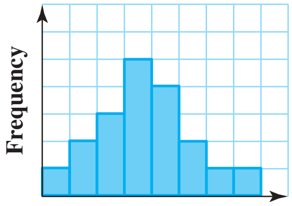
11.5.1 Frequency histograms

- A histogram is a useful way of displaying large, numerical datasets (with, say, more than 50 observations).
- The vertical axis on the histogram displays the frequency and the horizontal axis displays class intervals of the variable (e.g. height or income).
- The frequency for each class interval determines the height of the column.
- When data are given in raw form — that is, just as a list of numbers in no particular order — it is helpful to first construct a frequency table.



tlvd-12147

WORKED EXAMPLE 7 Creating a histogram with class intervals

The data show the distribution of masses (in kilograms) of 60 students in Year 7 at Northwood State High School.

45.7, 34.2, 56.3, 38.7, 52.4, 45.7, 48.2, 52.1, 58.7, 62.3,
45.8, 52.4, 60.2, 48.5, 54.3, 39.8, 36.2, 54.3, 39.7, 46.3,
45.9, 52.3, 44.2, 49.6, 48.6, 42.5, 47.2, 51.3, 43.1, 52.4,
48.2, 51.8, 53.8, 56.9, 53.7, 42.9, 46.7, 51.9, 56.2, 61.2,
48.3, 45.7, 43.5, 43.8, 58.7, 59.2, 58.7, 54.6, 43.0, 48.2,
48.4, 56.8, 57.2, 58.3, 57.6, 53.2, 53.1, 58.7, 56.3, 58.3,

a. **Complete a frequency distribution table for the data.**
b. **Construct a frequency histogram to display the data.**

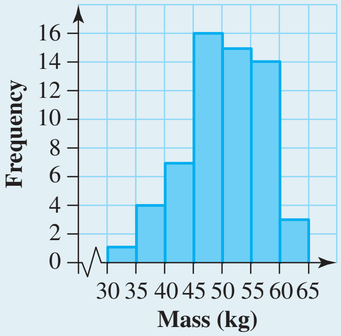
THINK

1. First construct a frequency table. The lowest data value is 34.2 and the highest is 62.3.
2. Divide the data into class intervals. If each interval was 5 kg, we would then have 7 intervals, which is a reasonable number of class intervals.
3. Count how many observations fall into each of the intervals and record these in a table.

WRITE

Class interval	Frequency
30–<35	1
35–<40	4
40–<45	7
45–<50	16
50–<55	15
55–<60	14
60–<65	3
Total	**60**

4. Check that the frequency column totals 60.
5. A histogram can be constructed. The column heights are determined by the frequency for each class interval.



WORKED EXAMPLE 8 Creating a histogram with integer values

The marks out of 20 received by 30 students for a book-review assignment are given in the frequency table.

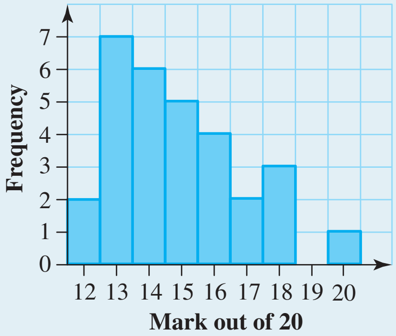
Mark	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Frequency	**2**	**7**	**6**	**5**	**4**	**2**	**3**	**0**	**1**

Display these data in the form of a histogram.

THINK

In this case we are dealing with integer values, not grouped data.
The integer values are written underneath the centre of each column. The column heights are determined by the frequency for each class interval.

WRITE

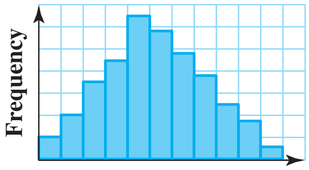


11.5.2 Describing the shape of a histogram

int-3823

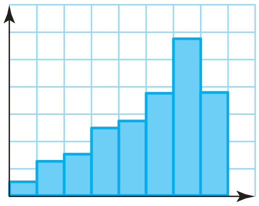
Symmetric distributions

- The data shown in the histogram to the right can be described as *symmetric*.
- There is a single peak and the data trail off on both sides of this peak in roughly the same fashion.
- The data in this case peak to the right and trail off to the left.
- The data in this case peak to the left and trail off to the right.



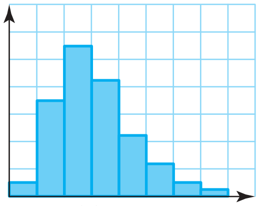
Skewed distributions

- The data in this case peak to the right and trail off to the left.



Negatively skewed distribution

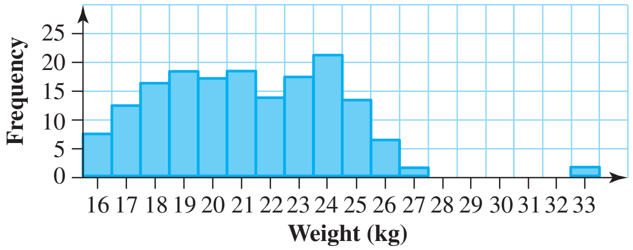
- The data in this case peak to the left and trail off to the right.



Positively skewed distribution

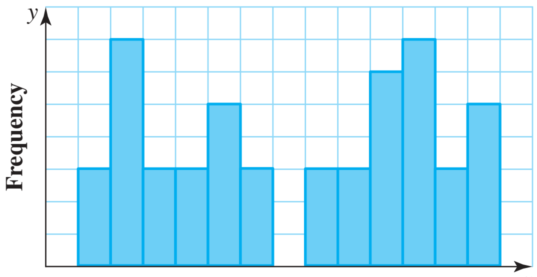
Outliers

- Possible outliers can be identified as a column that lies well away from the main body of the histogram.



Bimodality

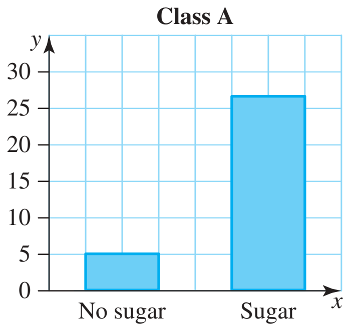
- The mode of a distribution is the data value or class interval that has the highest frequency.
- This will be the column or row on the display that is the longest.
- When there is more than one mode, the data distribution is multimodal.
- This can indicate that there may be subgroups within the distribution that may require further investigation.
- Bimodal distributions can occur when there are two distinct groups present, such as in data values that typically have clear differences between male and female measurements.

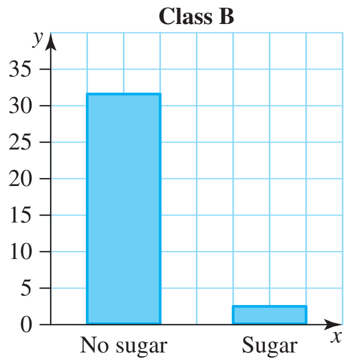


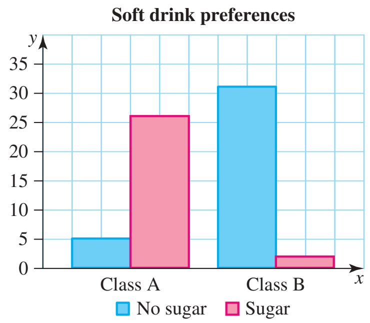
Bimodal

11.5.3 Comparing histograms

- Histograms can be drawn as side-by-side bar or column graphs or as separate graphs.
- For example, note the two separate graphs indicating preference for sugar or sugar-free soft drinks for two separate classes. The same information can also be drawn side-by-side on the right.


Class A

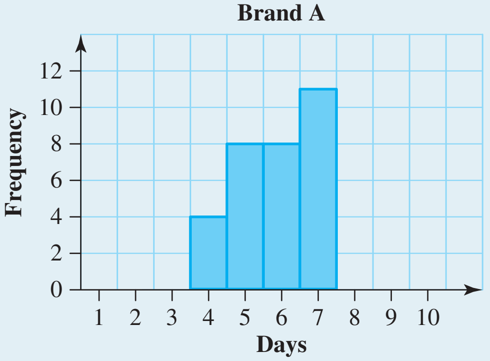

Class B

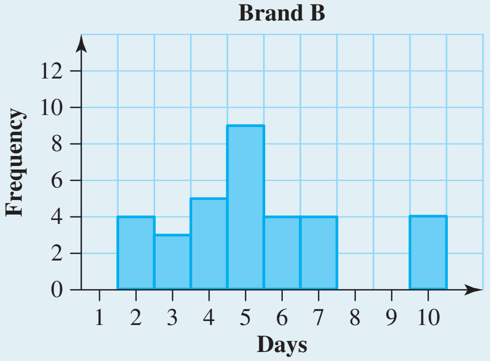

Soft drink preferences

- Either representation allows easy comparison of two or more sets of data. In this instance, it is clear that Class B preferred sugar-free while Class A preferred sugar.
- When comparing histograms, it is important include a comparison of the key characteristics of each dataset:
  - Shape (symmetry or skew)
  - Distribution and spread
  - Outliers
  - Modality
- Comparisons made should always be in context.

WORKED EXAMPLE 9 Comparing histograms

Two power tool brands have measured their battery lifespan (in days) before it needs to be recharged. Compare these two histograms to determine which brand has more consistent performance.


Brand A

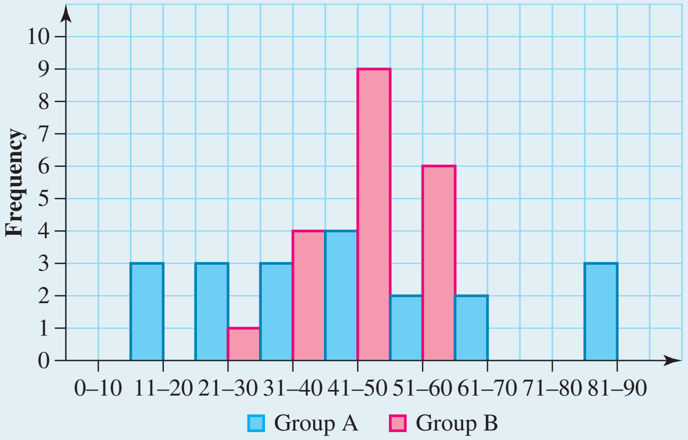

Brand B

THINK	WRITE
1. Determine the shapes and key characteristics of each histogram.	Brand A is clustered between 4 and 7 days with a modal number of 7. It appears to be negatively skewed. Brand B is much more spread out (or dispersed) between 2 and 10 days, with a modal number of 5. It appears to be symmetrical around 5, with two outliers at 10 days.

2. In context, determine which battery performs more consistently based on the characteristics listed.	Brand A appears to be more consistent, as all batteries lasted between 4 and 7 days, with the majority lasting a full work week (5 to 7 days) without needing to be recharged. Brand B performed less consistently, with a greater spread of lifespans. It did have two batteries lasting well over a week — these were likely outliers. It also had several batteries lasting only 2 or 3 days before needing to be recharged. Overall, Brand A seems to be more consistent.

WORKED EXAMPLE 10 Comparing side-by-side histograms

Below is a side-by-side histogram representing the scores of two groups of teenagers in a popular online game.



By comparing these two datasets, determine which group you would consider more consistent.

THINK	WRITE
1. Determine the shape and distribution of each dataset.	Both groups A and B appear to have a fairly symmetrical spread. Group A is more spread out, with 3 outliers in the 81–90 score range. Group B has a smaller spread and shows more consistency, with most of the data in the 30–60 range.
2. Determine which group you prefer to join.	Although Group A got some higher scores, Group B is more consistent and therefore the better group to go with.

Exercise 11.5 Comparing histograms [complex]

11.5 Quick quiz on

11.5 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5	6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18	19

These questions are even better in jacPLUS!

- Receive immediate feedback
- Access sample responses
- Track results and progress

Find all this and MORE in jacPLUS

Simple familiar

1. For each of the following sets of data:
   - i. construct a frequency table
   - ii. construct a histogram for each set of data.
   - a. 3, 4, 4, 5, 5, 6, 7, 7, 7, 8, 8, 9, 9, 10, 10, 12
   - b. 4.3, 4.5, 4.7, 4.9, 5.1, 5.3, 5.5, 5.6, 5.2, 3.6, 2.5, 4.3, 2.5, 3.7, 4.5, 6.3, 1.3
   - c. 11, 13, 15, 15, 16, 18, 20, 21, 22, 21, 18, 19, 20, 16, 18, 20, 16, 10, 23, 24, 25, 27, 28, 30, 35, 28, 27, 26, 29, 30, 31, 24, 28, 29, 20, 30, 32, 33, 29, 30, 31, 33, 34
   - d. 0.4, 0.5, 0.7, 0.8, 0.8, 0.9, 1.0, 1.1, 1.2, 1.0, 1.3, 0.4, 0.3, 0.9, 0.6

2. A class of 30 students sat for a Mathematics test. Their results out of 100 are shown below.

   68, 72, 58, 45, 69, 92, 38, 51, 70, 65, 69, 73, 52, 76, 48,
   69, 73, 41, 42, 73, 80, 50, 60, 49, 65, 94, 88, 85, 53, 60

   a. Use these results to copy and complete the frequency table.

Score	Tally	Frequency
30–39		
40–49		
50–59		
60–69		
70–79		
80–89		
90–99		

   b. Construct a histogram to display the data.

3. **WE7** A farmer measures the heights of his tomato plants. The results, in metres, are shown below.

   0.93, 1.21, 2.03, 1.40, 1.17, 1.53, 1.82, 1.77, 1.65, 0.63, 1.24,
   1.99, 0.80, 2.14, 1.53, 2.07, 1.96, 1.05, 0.94, 1.23, 1.72, 1.34,
   0.75, 1.17, 1.50, 1.41, 1.74, 1.86 1.55, 1.42, 1.52, 1.39, 1.76,
   1.67, 1.28, 1.43, 2.13

   a. Use the class intervals 0.6–<0.8, 0.8–1.0, 1.0–<1.2, ... to complete a frequency distribution table for the data.

   b. Construct a frequency histogram to display the data.

4. The following data give the times (in seconds) taken for athletes to complete a 100-m sprint.

12.2, 12.0, 11.9, 12.0, 12.6, 11.7,
11.4, 11.0, 10.9, 11.7, 11.2, 11.8,
12.2, 12.0, 12.7, 12.9, 11.3, 11.2,
12.8, 12.4, 11.7, 10.8, 13.3, 11.7,
11.6, 11.7, 12.2, 12.7, 13.0, 12.2

a. Construct a frequency distribution table for the data. Use class intervals of 0.5 seconds.

b. Construct a histogram to display the data.

5. **WE8** The marks out of 20 received by 26 students for a History assignment are given in the frequency table.

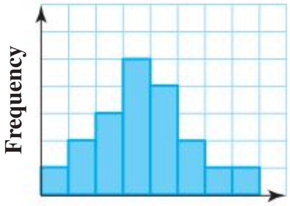
Score	Frequency
13	2
14	9
15	3
16	5
17	6
18	1

Display the data in the form of a histogram.

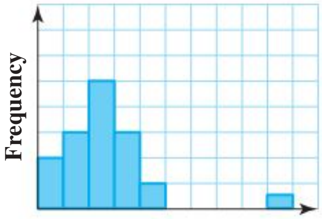
Complex familiar

6. For each of the following histograms, describe the shape of the distribution of the data and comment on the existence of any outliers.

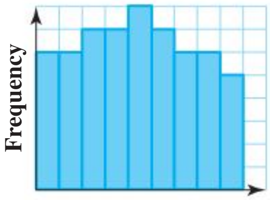
a.



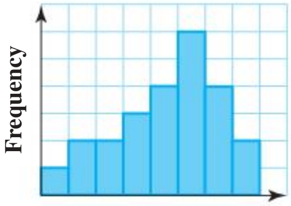
b.



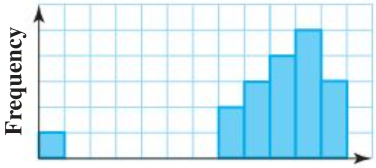
c.



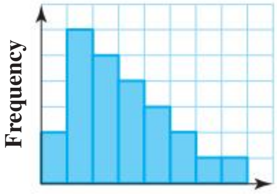
d.



e.

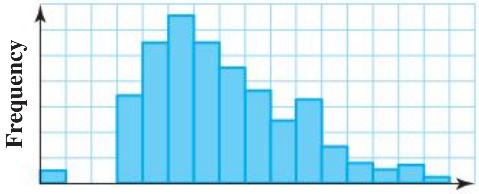


f.

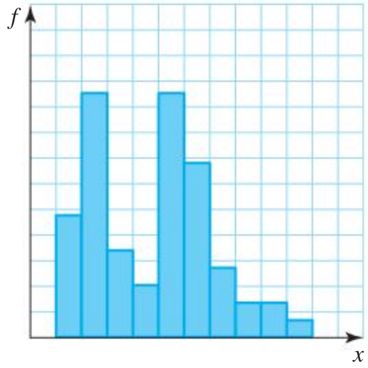


7. Describe the shape and distribution of the following histograms.

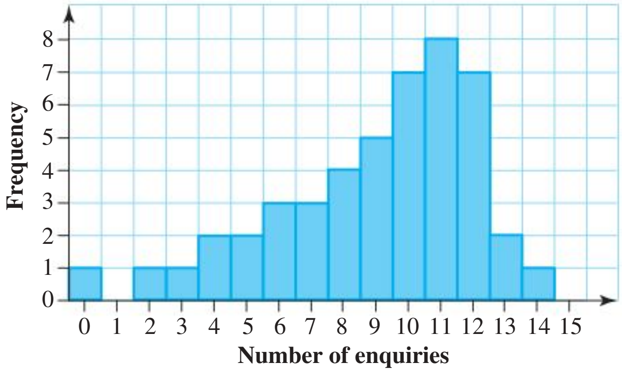
a.



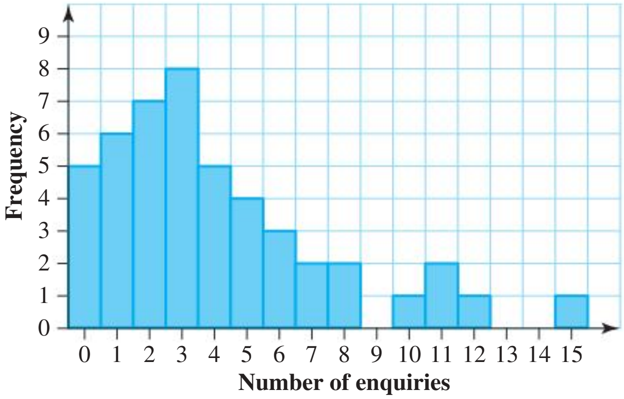
b.



8. a. The average number of product enquiries per day received by a group of small businesses who advertised in the *Yellow Pages* telephone directory is given below. Describe the shape of the distribution of the data and comment on the existence of any outliers.

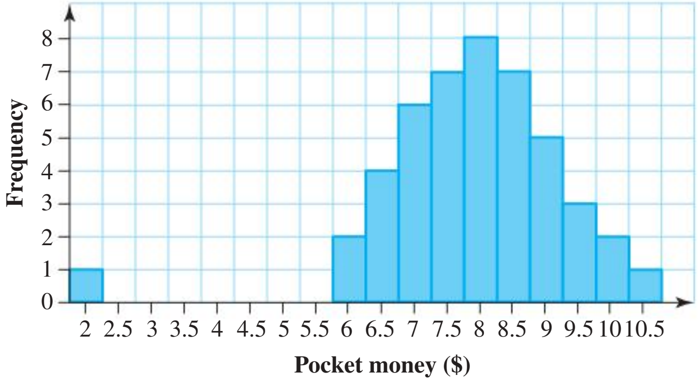


b. Describe the shape of the distribution of the same data collected from the local newspaper, and comment on the existence of any outliers.



c. Comparing these two results, determine which is the better means of advertising.

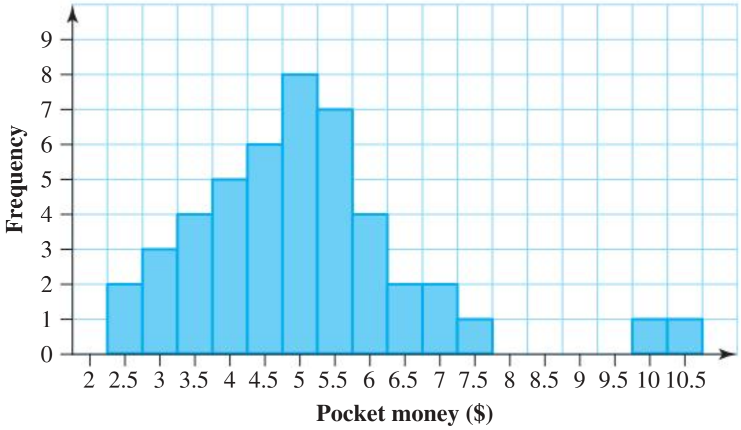
9. The amount of pocket money (to the nearest 50 cents) received each week by students in a Year 6 class is illustrated in this histogram.



a. Describe the shape of the distribution of these data and comment on the existence of any outliers.

b. Evaluate the weekly pocket money received by this group of students and justify your conclusions using evidence from the data.

c. Describe the shape of the distribution of the Year 3 data and comment on the existence of any outliers.



d. Determine what conclusions you can reach about the amount of pocket money received weekly by these groups of students.

10. For each of the frequency distributions tables below:

i. create a histogram

ii. determine whether they are symmetric, skewed or bimodal. Explain your reasoning.

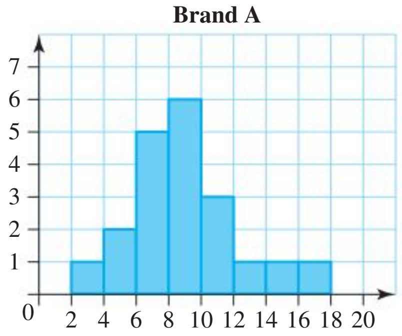
a.

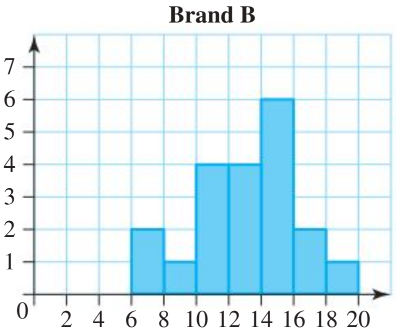
Goals	Frequency
5	6
6	8
7	5
8	0
9	1
10	6
11	9
12	7
13	5

b.

Scores	Frequency
120–130	5
130–140	8
140–150	7
150–160	11
160–170	15
170–180	14
180–190	18
190–200	19

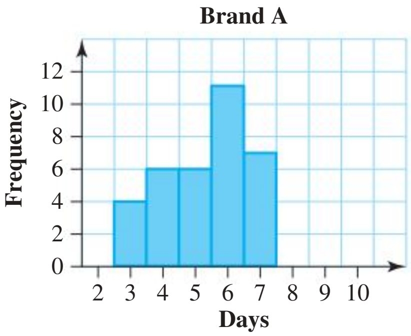
11. **WE9** Two groups of students measured the battery life of their phones over a day.

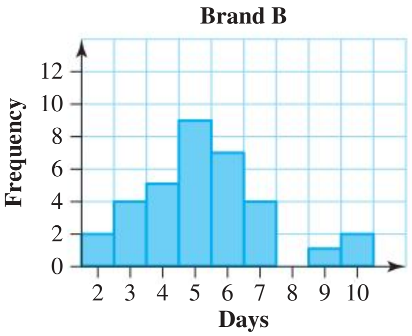




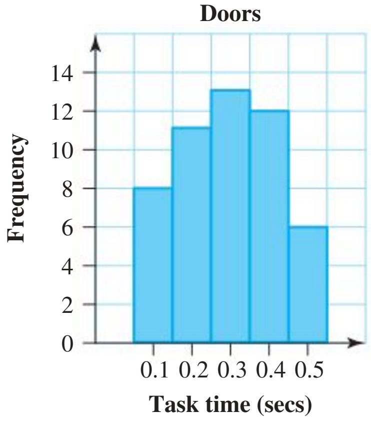
Compare these two histograms to determine which brand has more consistent performance.

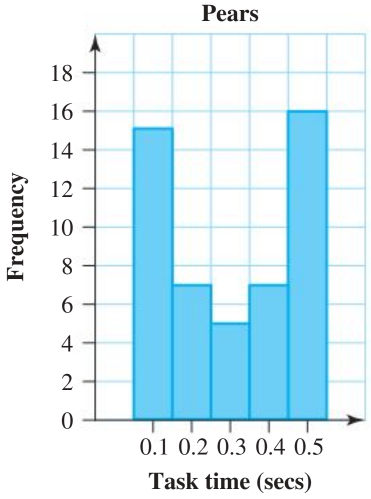
12. Two electric scooter companies have measured their battery lifespan (in days) before needing to be recharged. Compare these two histograms to determine which brand has more consistent performance. Comment on any other factors that may affect battery life.



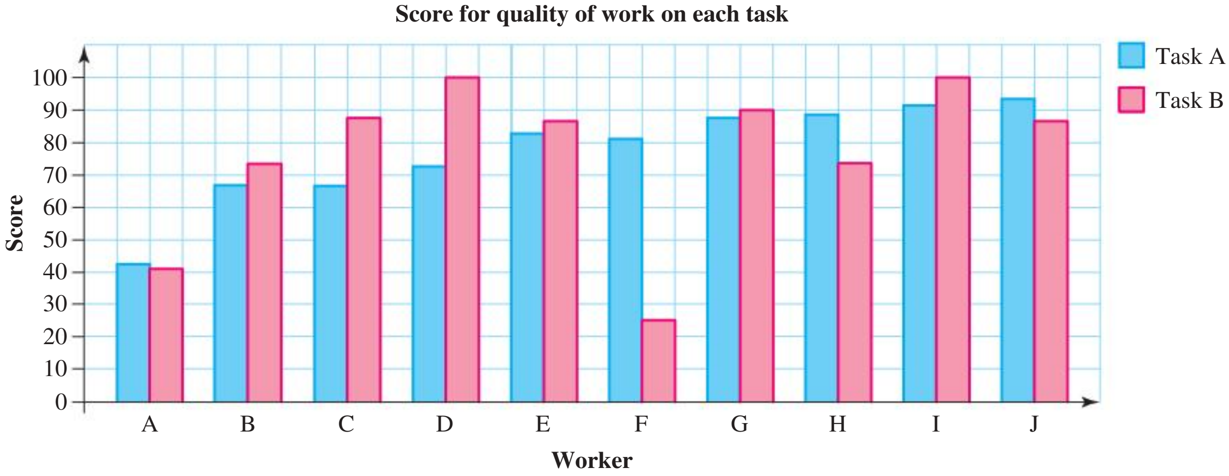


13. A computer programmer is comparing the time to complete video-editing tasks across two different computer platforms. The results are presented as histograms. Compare these two histograms to determine the best computer brand for completing video-editing tasks.



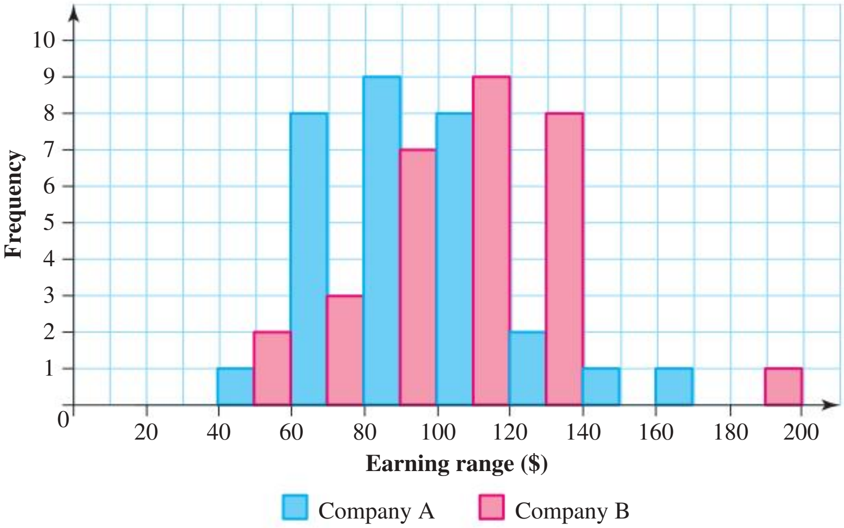


14. Ten workers were required to complete two tasks. Their supervisor observed them and gave them a score for the quality of their work on each task, where higher scores indicated better-quality work. The results are indicated in the following side-by-side bar chart.



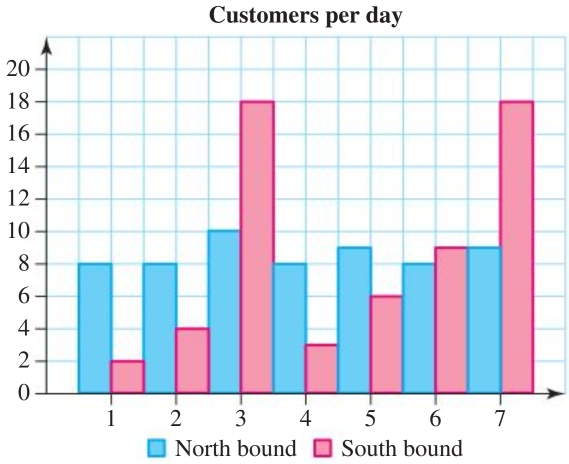
a. Identify which worker had the largest difference between scores for the two tasks.
b. Determine how many workers received a lower score for task B than task A.

15. **WE10** Employees' salaries (in $1000s) at two companies are compared using the following side-by-side histogram.



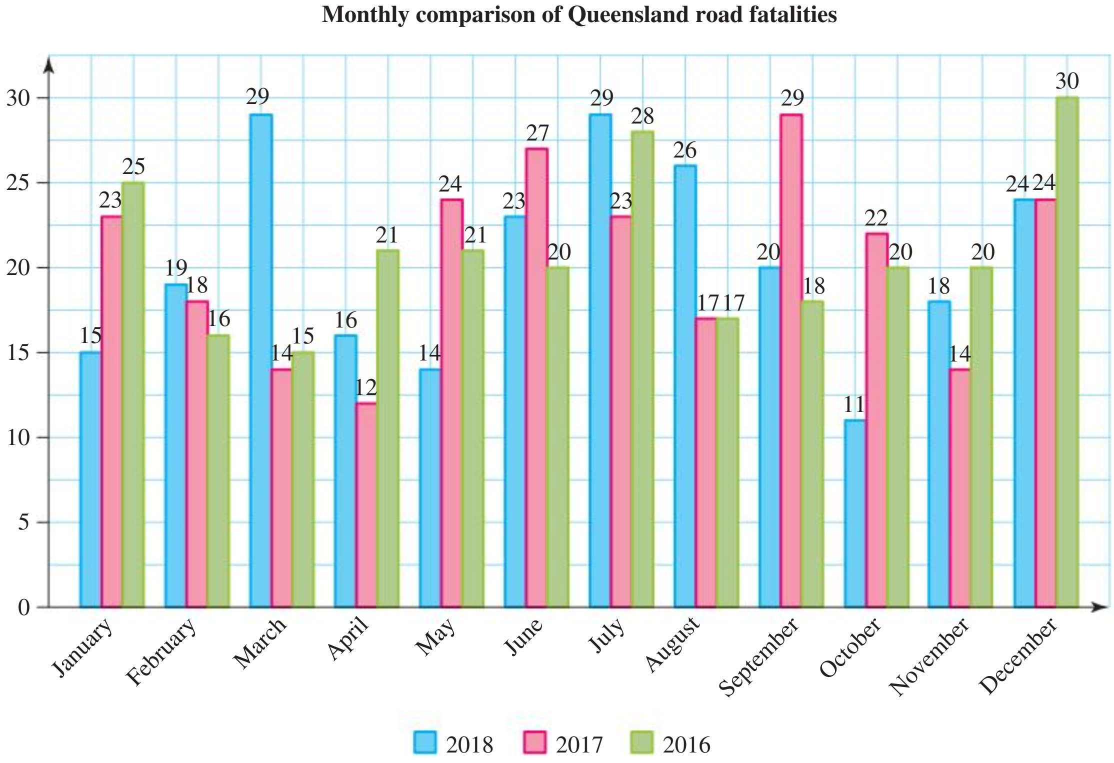
a. Determine which company has the highest paid employee.
b. Compare the distributions of Company A and Company B.
c. Determine which company you would prefer to work for. Justify your answer using mathematical reasoning.

16. Below is a side-by-side column graph representing the number of customers at a service station on each side of the highway for the past week.



By comparing these two datasets, determine which side of the highway would be better to open a diner.

17. The side-by-side bar chart shows a monthly comparison of road fatalities in Queensland.



Source: Australian Government, Department of Infrastructure, Regional Development and Cities, Australian Road Deaths Database

a. Identify the year that recorded the highest number of fatalities on Queensland roads.
b. Determine the year that had the highest number of fatalities from January to March.
c. Suggest a possible reason for the high number of fatalities in December 2016.

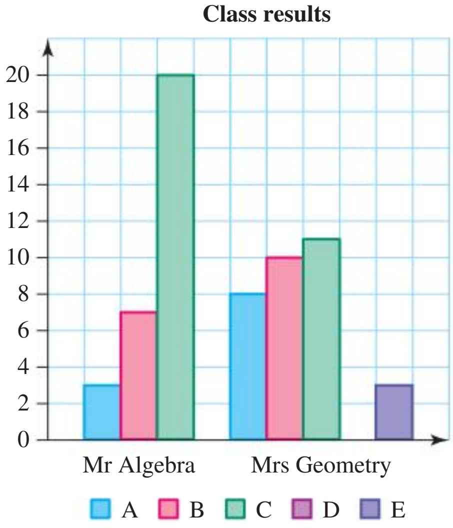
18. A coffee chain serves either skim, reduced-fat or whole milk in coffees. The coffees sold on a particular day are shown in the table below, sorted by the type of milk and the two different cafes.

	Cafes	
Type of milk	**Cafe A**	**Cafe B**
Skim	87	124
Reduced fat	55	73
Whole	112	49

a. Determine the total number of coffees sold on this day.
b. Construct a horizontal side-by-side bar chart to represent these data.
c. Calculate the percentage of Cafe A's customers who wanted skim milk in their coffees, rounding your answer to the nearest whole number.
d. Calculate the percentage of coffees sold that contained reduced-fat milk, rounding your answer to the nearest whole number..
e. Predict the percentage of the coffee chain's customers that would be from Cafe B if this sales trend continued daily, rounding your answer to the nearest whole number.

Complex unfamiliar

19. The head of a department is comparing Maths results of two classes to determine the best class in the school. Mr Algebra said his class is the best because no one failed. Evaluate the reasonableness of Mr Algebra's claim.



Fully worked solutions for this chapter are available online.

LESSON
11.6 Review

11.6.1 Summary

Hey students! Now that it's time to revise this chapter, go online to:

- Access the chapter summary
- Review your results
- Watch teacher-led videos
- Practise questions with immediate feedback

Find all this and MORE in jacPLUS

11.6 Exercise

learnon

11.6 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5	6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17	18, 19, 20

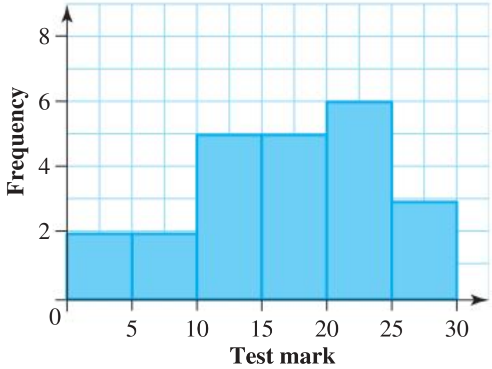
These questions are even better in jacPLUS!

- Receive immediate feedback
- Access sample responses
- Track results and progress

Find all this and MORE in jacPLUS

Simple familiar

Use this information to answer questions 1–3. The test marks for a class of students are shown in the histogram below.



1. Determine the category in which the median falls.
2. Determine the mode of this histogram.
3. Determine the range of this histogram.
4. Construct a histogram likely to correspond to this box plot.

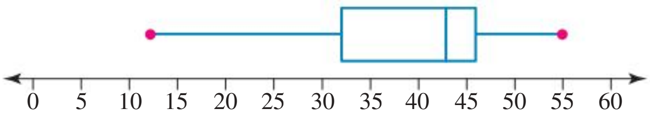
5. The number of babies born each day at a hospital over a year is tabulated and the five-number summary is as follows.

1. Minimum = 1
2. Upper quartile = 16
3. Lower quartile = 8
4. Maximum = 18
5. Median = 14

Show this information in a box plot assuming there are no outliers.

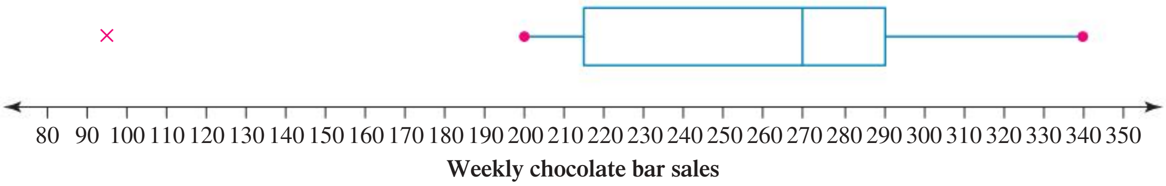
Complex familiar

6. For the box plot drawn below:



a. identify the median
b. calculate the range
c. calculate the interquartile range.

7. The monthly sales of a particular chocolate bar were recorded over the period of a year. A box plot representing these sales is as shown.



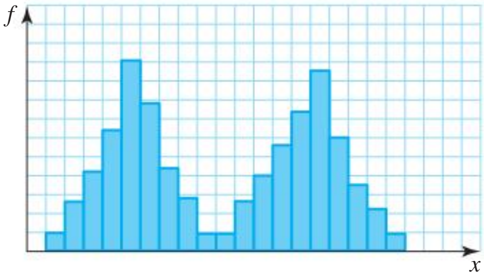
a. Determine the five-number summary for this data.
b. Calculate is the IQR.
c. Explain whether there are any outliers.

8. Two groups of students recorded the number of hours they spent studying for a Maths exam. The data (in hours) is as follows:
Group A: 2, 5, 8, 10, 12, 15, 18, 20, 22, 25
Group B: 1, 3, 5, 7, 10, 13, 17, 21, 26, 30

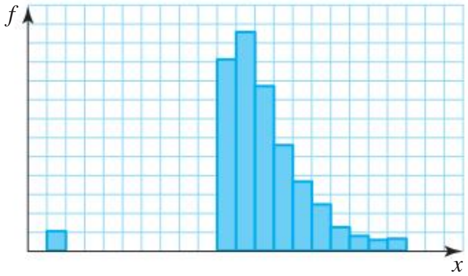
a. Calculate the five-number summary (minimum, Q_1, median, Q_3, maximum) for Group A.
b. Calculate the five-number summary for Group B.
c. Compare the spread of the two datasets using the interquartile range and explain which dataset has more variability.

9. Identify the modality and shape of each of the following graphs.

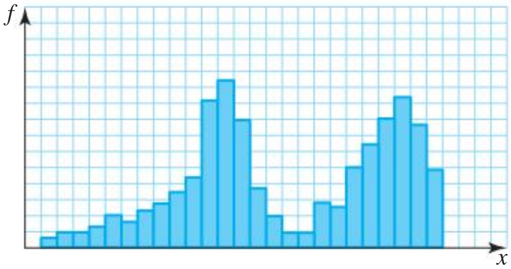
a.



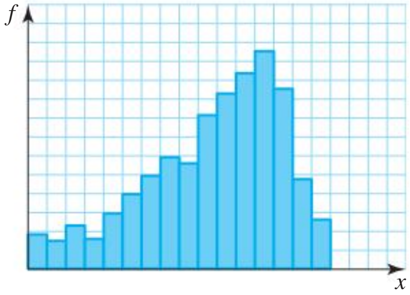
b.



c.



d.



10. Use the following two datasets to construct parallel box plots.
Dataset A: 2, 3, 6, 7, 9, 10, 1, 7, 7, 19
Dataset B: 1, 2, 3, 3, 4, 1, 2, 5, 3, 6

11. The number of goals scored by two members of a basketball team during 10 matches is as follows.

Player 1	6	8	12	9	5	15	2	1	10	24
Player 2	6	8	7	8	7	5	8	6	7	7

a. Draw parallel box plots for this data.
b. Explain which player you would rather have on your team and why.

12. The number of hours of counselling received by a group of 9 full-time firefighters and 9 volunteer firefighters after a serious bushfire is given in the table.

Full-time	2	4	3	5	2	4	6	1	3
Volunteer	8	10	11	11	12	13	13	14	15

a. Construct a back-to-back stem plot to display the data.
b. Comment on the distributions of the number of hours of counselling of the full-time firefighters and the volunteers.

13. The IQs of 8 players in 3 different football teams were recorded and are shown below.

Team A	120	105	140	116	98	105	130	102
Team B	110	104	120	109	106	95	102	100
Team C	121	115	145	130	120	114	116	123

Display the data in the form of parallel box plots.

14. A group of office workers and a group of sports instructors were asked to complete 5 minutes of exercise as part of a study of heart rates. Following the exercise, participants rested for 2 minutes before their pulse rates were measured. The results are set below in the back-to-back stem plots.

Key: 12 | 4 = 124 beats/min

Office workers	Stem	Sports instructors
	6	2 4 8 8 9
6	7	2 2 3 5 7 9
	8	2 8
5	9	6
6 7	10	8
0 2	11	
0 1 2 4 6 7 9	12	
0 0 4	13	

a. Describe the shape of each distribution.
b. Calculate the median, the interquartile range, the mode and the range for both.
c. Represent each set of data using a box plot, indicating outliers where present.
d. Calculate the mean and the standard deviation for both sets of data.
e. Using the summary statistics that you have calculated for the office workers, comment on their pulse rates. Do the same for the sports instructors. Comment on any differences between the two groups.

15. To compare two textbooks, a teacher recommends one book to one of his classes and the other book to another class. At the end of the year, the classes are both tested. The results are as follows.

Text A (25 students)							**Text B (28 students)**						
44	52	95	76	13	94	83	65	72	48	63	68	59	68
72	55	81	22	25	64	72	62	75	79	81	72	64	53
35	48	56	59	84	98	84	58	59	64	66	68	42	37
21	35	69	28				39	55	58	52	82	79	55

a. Prepare a back-to-back stem plot of the data.
b. Prepare a five-number summary for each group. (Note that the groups are different sizes.)
c. Prepare parallel box plots of the data.
d. Compare the performance of each of the classes.
e. Explain which textbook you think is better. Justify your reasoning.
f. Explain what other things you would need to take into account before drawing final conclusions.

16. The following datasets show the rental prices (in $) of one-bedroom apartments in two different suburbs of Sydney.

Suburb A	275	275	281	285	284	310	315	310	296	307
	300	300	242	295	281	305	290	305	295	300
Suburb B	235	225	231	232	240	280	300	310	295	275
	275	285	245	305	270	255	270	228	241	285

a. Draw a back-to-back stem plot to compare the datasets.
b. Compare and contrast the rental prices in the two suburbs.
c. The rental prices in a third suburb, Suburb C, were also analysed.

Suburb C	335	325	351	335	340	360	300	390	395	285
	357	385	354	305	375	345	270	358	340	365

Compare the rents in the third suburb with the rents in the other two suburbs.

17. The following table shows the AFL Grand Final statistics for a sample of players who have kicked a total of 5 or more goals from the clubs Carlton and Collingwood.

Player	Team	Kicks	Marks	Handballs	Disposals	Goals	Behinds
Alex Jesaulenko	Carlton	23	11	9	32	11	0
John Nicholls	Carlton	29	3	1	30	13	1
Wayne Johnston	Carlton	78	19	17	95	5	7
Robert Walls	Carlton	19	9	5	24	11	1
Craig Bradley	Carlton	61	11	37	98	6	2
Mark MacLure	Carlton	34	16	14	48	5	4
Stephen Kernahan	Carlton	44	26	8	52	17	5
Ken Sheldon	Carlton	36	5	12	48	5	2
Syd Jackson	Carlton	13	3	1	14	5	1
Rodney Ashman	Carlton	25	4	10	35	5	2
Greg Williams	Carlton	30	6	29	59	6	4
Alan Didak	Collingwood	46	17	24	70	6	2
Peter Moore	Collingwood	42	22	13	55	11	7
Ricky Barham	Collingwood	42	15	16	58	5	5
Travis Cloke	Collingwood	26	16	9	35	5	4
Ross Dunne	Collingwood	17	6	6	23	5	2
Craig Davis	Collingwood	27	8	8	35	6	3

a. Use the data for goals to compare the two clubs using parallel box plots.
b. Comment on what the parallel box plots indicate about the data for goals.
c. Compare the data for kicks and handballs using parallel box plots.
d. Comment on what the parallel box plots indicate about the data for kicks and handballs.

Complex unfamiliar

18. The average maximum temperature (in °C) in a small town in the Darling Downs region for two 20-year time periods is shown in the following tables.
Time period (2005–2024):

Year	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002
Temp.	22.3	22.6	21.8	22.1	22.4	22.7	22.1	22.7	23.1	23.1

Year	2003	2004	2005	2006	2007	2008	2009	2010	2011	2024
Temp.	22.7	23.4	23.4	23.1	22.7	22.1	22.9	22.6	22.6.	22.7

Time period (1893–1912):

Year	1893	1894	1895	1896	1897	1898	1899	1900	1901	1902
Temp.	20.5	20.9	21.0	20.9	21.5	21.3	21.0	20.9	21.0	21.0

Year	1903	1904	1905	1906	1907	1908	1909	1910	1911	1912
Temp.	20.7	20.9	20.9	21.5	21.4	21.3	21.1	21.4	21.3	21.4

Use a form of technology to display each set of data and analyse the differences between the two data sets.

19. The following table shows some key nutritional information about a sample of fruits and vegetables.

Food	**Calcium (mg)**	**Serve weight (g)**	**Water (%)**	**Energy (kcal)**	**Protein (g)**	**Carbohydrate (g)**
Avocado	19	173	73	305	4.0	12.0
Blackberries	46	144	86	74	1.0	18.4
Broccoli	205	180	90	53	5.3	10
Cantaloupe	29	267	90	94	2.4	22.3
Carrots	19	72	88	31	0.7	7.3
Cauliflower	17	62	92	15	1.2	2.9
Celery	14	40	95	6	0.3	1.4
Corn	2	77	70	83	2.6	19.4
Cucumber	4	28	96	4	0.2	0.8
Eggplant	10	160	92	45	1.3	10.6
Lettuce	52	163	96	21	2.1	3.8
Mango	21	207	82	135	1.1	35.2
Mushrooms	2	35	92	9	0.7	1.6
Nectarines	6	136	86	67	1.3	16.0
Peaches	4	87	88	37	0.6	9.6
Pears	19	166	84	98	0.7	25.1
Pineapple	11	155	86	76	0.6	19.2
Plums	10	95	84	55	0.5	14.4
Spinach	55	56	92	12	1.6	2.0
Strawberries	28	255	73	245	1.4	66.1

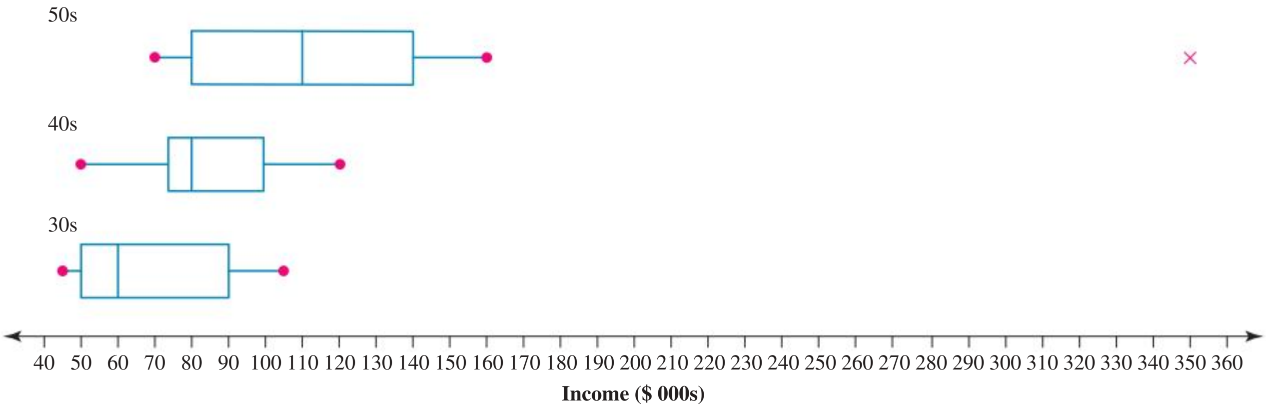
An athlete is analysing the effects of converting to a vegetarian diet. She knows she needs to consume protein, carbohydrates and fats in the ratio of 3 : 4 : 3 for optimum recovery.

Her training partner says there will be no need to supplement anymore. She will get more than enough protein on a vegetarian diet. And because most of what she eats will be water, so she will stay hydrated with ease.

Evaluate the reasonableness of her training partner's advice.

20. A large company completed a survey of the incomes of their employees. The incomes were recorded for people in their 30s, 40s and 50s. The results are summarised in the parallel box plots below.

Note: The incomes are written in $1000s; for example, 110 = $110 000 and 75 = $75 000.



John is 30 years old and currently seeking employment at the company. A position is being advertised offering 'above median' remuneration to the successful candidate. John encourages his 59-year-old mother to apply for the position stating, 'I would only make between $60 000 and $110 000, but you will easily earn between $110 000 and $350 000!'

Explain the error in John's statement.

Fully worked solutions for this chapter are available online.

Answers

Chapter 11 Comparing datasets

11.2 Constructing box plots

11.2 Exercise

1. a. 12 **b.** 5 **c.** 26

2. a. 122 **b.** 6 **c.** 27

3. a. 49 **b.** 5.8 **c.** 18.6

4. a. 140 points **b.** 56 points **c.** 90 points
d. 84 points **e.** 26 points

5. a. 58 **b.** 31 **c.** 43
d. 27 **e.** 7

6. a. The dataset is likely to be asymmetric.

b. The data are more clustered between the median and upper quartile (less spread) than between the median and the lower quartile.

c. If the median is halfway between the upper and lower quartiles, the dataset has a symmetric distribution; if it is not, then the dataset is asymmetric.

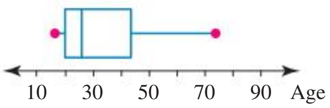
7. 23

8. 5

9. One-quarter of the scores are between 5 and 20.
Half of the scores are between 20 and 25.
It is clustered or grouped in the middle.

10. a. 18, 20, 26, 43.5, 74

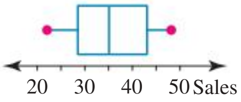
b.



c. The data is positively skewed, with no outliers. The average age of offenders, as measured by the median, is 26. The spread of ages, as measured by the range, is 56.

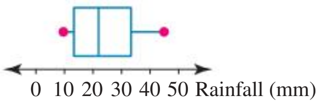
11. a. 22, 28, 35, 43, 48

b.



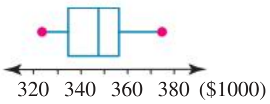
12. a. 10, 13.5, 22, 33.5, 45

b.



13. a. 324 000, 335 000, 348 000, 357 000, 375 000

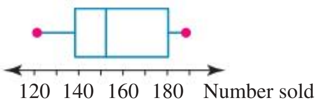
b.



14. a. Key: $12|1 = 121$

Stem	Leaf
12	1 5 6 9
13	1 2 4
14	3 4 8 8
15	0 2 2 2 5 7
16	3 5
17	2 9
18	1 1 1 2 3 7 8

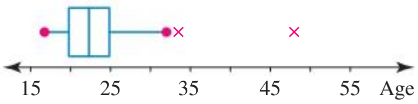
b.



15. a. Key: $1^*|7 = 17$ years

Stem	Leaf
1	7 7 8 8 8 9 9
2	0 0 0 1 2 2 2 2 3 3 3 3 4 4 4
2^*	5 5 8 9
3	1 2 3
4	8

b.



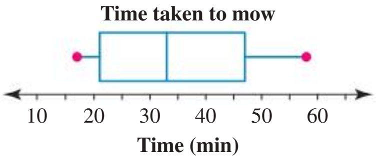
c. Data is positively skewed, with an outlier at 48. The average age to have a first baby, as measured by the median, is 22.5 years of age. 50% of mothers have their first baby between the ages of 20 and 25.

11.3 Comparing parallel box plots [complex]

11.3 Exercise

1. a. 17, 21, 33, 47, 58

b.

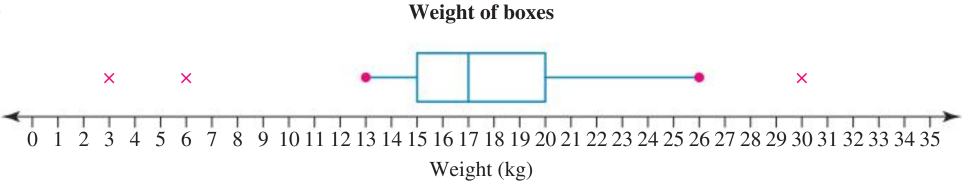


c. The distribution of time taken to mow is positively skewed, with no outliers. The average time taken, as measured by the median, is 33 min. The spread of time taken, as measured by the interquartile range, is 26 min and the range is 41 min.

2. a. See the figure at the bottom of the page.*

b. The distribution of weights is positively skewed, with outliers at 3, 6 and 30 kg.
The boxes have an IQR of 5 kg and a range of 13 kg centred around the median weight of 17 kg.

***2. a.**



3. a. See the figure at the bottom of the page.*
   b. The distribution of phone costs is widely spread out from \$50 to \$999, centred on the median price of \$479.
   Half the phones are priced between \$265 and \$577, with an IQR of \$312, indicating a slight positive skew.
4. a. See the figure at the bottom of the page.*
   b. The distribution of car speeds is positively skewed, bunched towards the lower values, with an outlier at 75 km/h.
   The median position is at 42 km/h and the lowest value is 38 km/h, with 75% of speeds being recorded at 51 km/h or less.
5. a. Peanut butter: 2, 16, 19, 21, 26; Nutella: 4, 7, 11, 13, 25
   b. i. 16
      ii. 7 and 13
   c. The distribution of peanut butter sales was negatively skewed, with an outlier at 2 jars. The distribution of Nutella sales is also negatively skewed, with an outlier at 25 jars.
   The parallel box plots show that the median weekly jar sales were much higher for peanut butter (19 jars) than for Nutella (11 jars). The IQR was approximately the same for both peanut butter and Nutella (5 and 6 jars, respectively), whereas the range was larger for peanut butter (24 jars) than for Nutella (21 jars). Overall, the average weekly jar sales for peanut butter were greater than for Nutella.
6. a. See the figure at the bottom of the page.*
   b. The distribution of Greek food was positively skewed, with no outliers. The distributions for Italian and Spanish food were approximately symmetric, with no outliers.
   The parallel box plots show that the median number of patrons that preferred the Greek cuisine in month 1 (56) was higher than the median of those that preferred Italian in month 2 (32.5) and Spanish in month 3 (40).
   The IQR was smallest in month 1 (14.5) and largest in month 2 (45). The range was also smallest in month 1 (27) compared with the range in month 2 (59) and month 3 (28).
   Overall, patrons preferred Greek food over Italian and Spanish food, as month 1 (Greek) had a larger median and smaller measures of spread compared to the other two months. The Greek menu will therefore be the most successful.
7. 4 and 15
8. Q_3
9. 1 year
10. a.

	Croc-Ade	Electricity-Ade
Min	**16**	**16**
Q_1	**18**	**18**
Median	**30**	**25**
Q_2	**40**	**30**
Max	**42**	**42**

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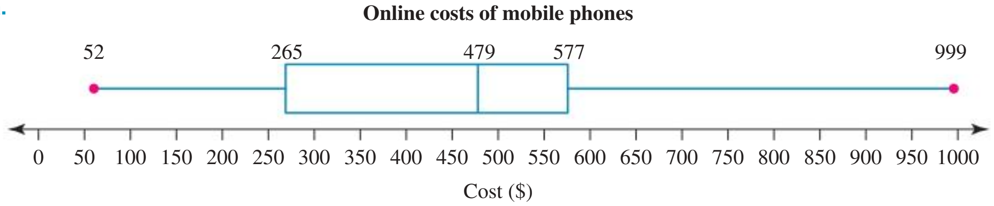
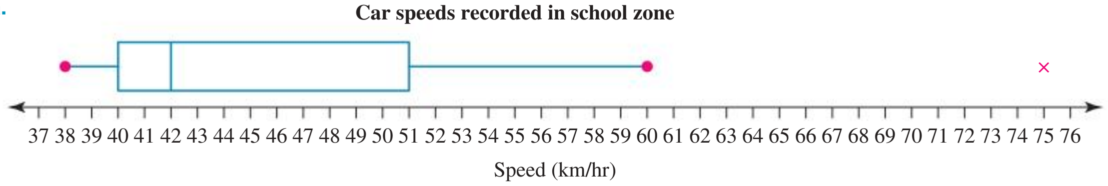
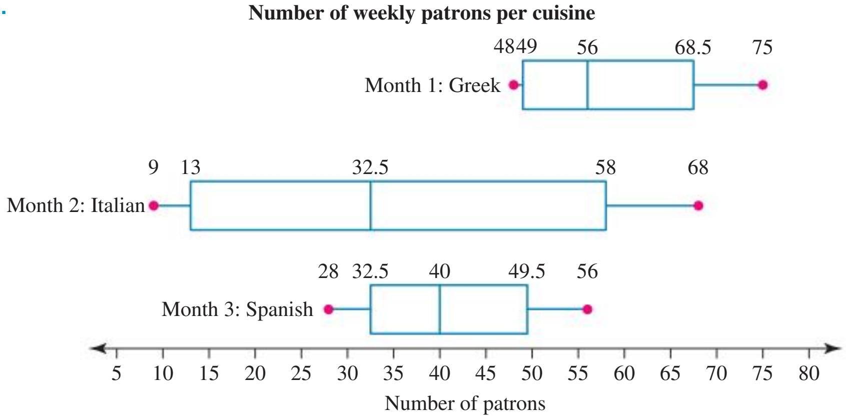
*3. a.



*4. a.



*6. a.



b. **i.** 30

ii. 18 and 40

c. The distribution of Electricity-Ade sales was negatively skewed, with 75% of sales below 30. The distribution of Croc-Ade sales is symmetrical around the median of 30, and has very short whiskers.
The parallel box plots show that the median sales were much higher for Croc-Ade (30) than for Electricity-Ade (25).
The range was the same for both drinks at 26, whereas the IQR was larger for Croc-Ade (22 compared to 12).

11. **a.** See the figure at the bottom of the page.*

b. TikTok generally has a higher usage.

12. **a.** See the figure at the bottom of the page.*

b. Similar distributions; both contain 1 outlier. In general, Spotify usage is slightly higher.

13. **a.** See the figure at the bottom of the page.*

b. The distribution for compound 1 was slightly negatively skewed, with no outliers. The distribution for compound 2 was approximately symmetric, with no outliers. The parallel box plots show that the median minutes of effectiveness of the compounds in water was much higher for compound 1 (152.5 minutes) than for compound 2 (125.5 minutes). The IQR and ranges were also much higher for compound 1 (38.5 minutes and 55 minutes, respectively), showing more variation in minutes of effectiveness than compound 2 (7.5 minutes and 13 minutes, respectively). Overall, compound 1 was more effective in water before needing to be reapplied than compound 2. However, the consistency of compound 2 gives the user a better indication of when they will have to reapply.

14. **a.** See the figure at the bottom of the page.*

b. The distribution for Year 11 students' achievements in History were negatively skewed, with no outliers, whereas the results for English were slightly negatively skewed, with no outliers. The median achievement for History (77.5) was higher than that of English (76.5). The range of achievements was larger for History (56) than English (43), indicating the results for English were more consistent than for History.

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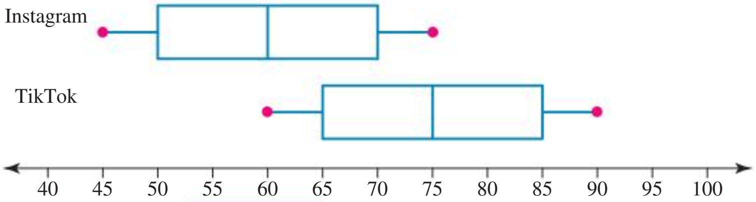
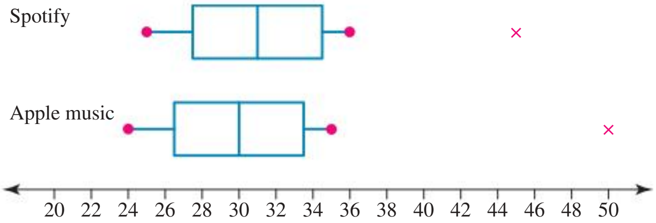
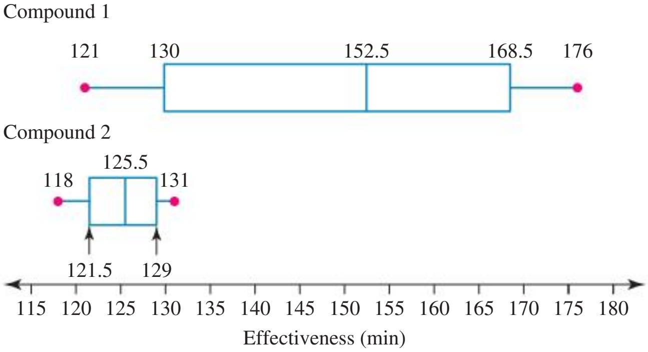
*11. a.



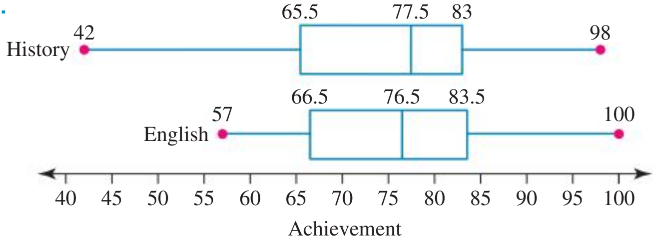
*12. a.



*13. a.



*14. a.



15. a. See the figure at the bottom of the page.*

b. The distribution of Year 11 heights was negatively skewed, with no outliers. The distribution of Year 12 heights was approximately symmetric, with no outliers. The parallel box plots show that the median student height was greater for Year 12 students (175.5 cm) than for Year 11 students (167.5 cm). The IQR and range for both Year 11 (12.5 cm and 26 cm, respectively) and Year 12 (10.5 cm and 30 cm, respectively) students was approximately the same, showing that the spread of heights in a year level are roughly equal. Overall, height increased with year level.

16. a. See the figure at the bottom of the page.*

b. Answers may vary. Sample responses can be found in the worked solutions in the online resources.

17. Responses will vary based on statistical measure used to support justification. Response should include reference to key features and statistics of the box plots.
To be correct, response must indicate that the farmer's statement is incorrect and include calculation of IQR and range to show where the error has occurred.

11.4 Comparing back-to-back stem plots [complex]

11.4 Exercise

1. Key: $2|3 = 23$

$2^*|5 = 25$

German	Stem	French
2 1 1 0	2	3 4
7 6 5 5	2^*	5 5 8
3 2 1 0 0	3	0 1 4 4
9 8 7 7	3^*	5 6 8 8 9
2 1	4	2 3 4 4
5	4^*	6 8

2. Key: $2|3 = 2.3$ kg

$2^*|7 = 2.7$ kg

Boys	Stem	Girls
	2^*	6 7
4 4	3	0 1 1 2 3
8 7 6	3^*	6 7
3 2	4	0
9 8	4^*	
0	5	

3. Partial sun is the better growing location.

4. a. Key: $15|0 = 150$ cm

Class A	Stem	Class B
	12	5
	13	1 5 6 8
	14	2 3 7 8 9
9 7 0	15	5
8 5 2 0	16	2
7 5 0	17	
7 1	18	

b. Students in Class A are older because they are taller.

5. a. Cafe A has the larger spread of data, with a range of 65 years.

b. Cafe A: it has a greater spread of data and more customers from an older age group.

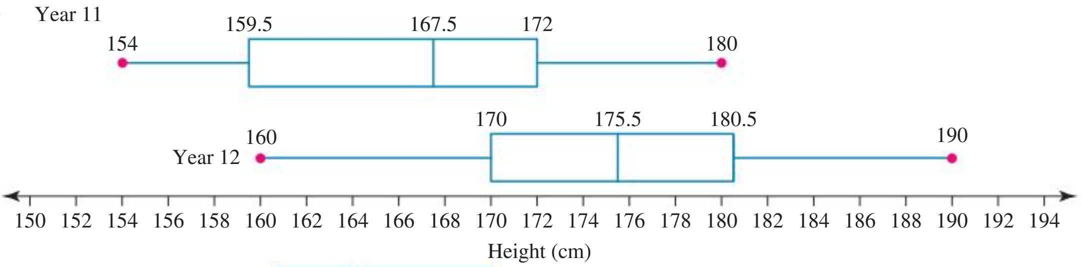
6. a. Key: $2|3 = 23$ trucks

$2^*|5 = 25$ trucks

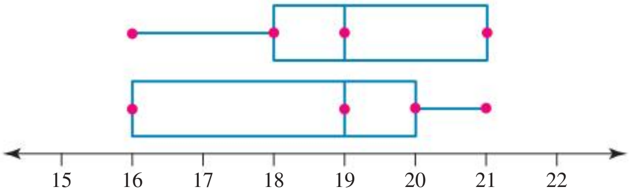
A	Stem	B
2 1	1	0
7 7 6 6 5	1^*	5 6
4 3 2 1 0	2	0 1 3
7 5	2^*	5 6 8 9
	3	0 1 2
	3^*	5

---

*15. a.



*16. a.



b. Statistical analysis

Supermarket A	Supermarket B
Mean = 19 $s_x = 4.9$	Mean = 24.4 $s_x = 7.2$
Min = 11 $Q_1 = 16$	Min = 10 $Q_1 = 20$
Median = 18.5 $Q_3 = 23$	Median = 25.5 $Q_3 = 30$
Max = 27 IQR = 7	Max = 35 IQR = 10

For Supermarket A, the mean is 19, the median is 18.5, the standard deviation is 4.9 and the interquartile range is 7. The distribution is symmetric.
For Supermarket B, the mean is 24.4, the median is 25.5, the standard deviation is 7.2 and the interquartile range is 10. The distribution is slightly negatively skewed. The centre and spread of the distribution of supermarket B is higher than that of supermarket A.
There is greater variation in the number of trucks arriving at supermarket B.

7. a. Key: $1|7 = 17$ marks

Class A	Stem	Class B
	1	0
3 2	1	2 3
5 5 4 4	1	4 4 5
7 6	1	7
	1	9

b. Statistical analysis

Class A	Class B
Mean = 14.5 $s_x = 1.6$	Mean = 14.25 $s_x = 2.8$
Min = 12 $Q_1 = 13.5$	Min = 10 $Q_1 = 12.5$
Median = 14.5 $Q_3 = 15.5$	Median = 14 $Q_3 = 16$
Max = 17 IQR = 2	Max = 19 IQR = 3.5

For the marks of the Class A, the mean is 14.5, the median is 14.5, the standard deviation is 1.6 and the interquartile range is 2.
The distribution is symmetric.
For the marks of the Class B, the mean is 14.25, the median is 14, the standard deviation is 2.8 and the interquartile range is 3.5.
The distribution is symmetric.
The centre of each distribution is about the same.

8. a. Key: $2|4 = 24$ marks

$2^*|6 = 25$ marks

First year	Stem	Second year
	2	2
	2*	6 7 8
1 0	3	0 1 1 3 4
9 7 5	3*	6
3 2 1 1	4	
6	4*	

b. Statistical analysis

First year	Second year
Mean = 38.5 $s_x = 5.2$	Mean = 29.8 $s_x = 4.2$
Min = 30 $Q_1 = 35$	Min = 22 $Q_1 = 27$
Median = 40 $Q_3 = 42$	Median = 30.5 $Q_3 = 33$
Max = 46 IQR = 7	Max = 36 IQR = 6

The distributions of marks for the first year and for the second year are each symmetric.
For the first-year marks, the mean is 38.5, the median is 40, the standard deviation is 5.2 and the interquartile range is 7. The distribution is symmetric.
For the second-year marks, the mean is 29.8, the median is 30.5, the standard deviation is 4.2 and the interquartile range is 6.
The spread of each of the distributions is much the same, but the centre of each distribution is quite different, with the centre of the second-year distribution quite a lot lower. The work may have become a lot harder!

9. a. Key: $2|3 = 23$ years old

$2^*|5 = 25$ years old

Football	Stem	Soccer
4 3	2	2
8 7 6 5	2*	5
1 0	3	0 1
	3*	6 7
	4	2
	4*	6

b. Statistical analysis

Football	Soccer
Mean = 26.75 $s_x = 2.8$	Mean = 33.6 $s_x = 8.2$
Min = 23 $Q_1 = 24.5$	Min = 22 $Q_1 = 27.5$
Median = 26.5 $Q_3 = 29$	Median = 33.5 $Q_3 = 39.5$
Max = 31 IQR = 4.5	Max = 46 IQR = 12

For the distribution of the football players, the mean is 26.75, the median is 26.5, the standard deviation is 2.8 and the interquartile range is 4.5.

For the distribution of the soccer players, the mean is 33.6, the median is 33.5, the standard deviation is 8.2 and the interquartile range is 12.

The centre of the distributions is very different: it is much higher for the soccer players. The spread of the ages of the football players who attend the fitness class is very small but very large for soccer players. Older soccer players are more likely to the attend fitness classes than older football players.

10. a. Key: $3|2 = 32$

Kindergarten	Stem	Prep. school
3	0	5
4 3	1	2 7
8 5	2	5 7
6 2	3	2 5
7 1	4	4 6
0	5	2

b. Statistical analysis

Kindergarten	Prep. school
Mean = 28.9 $s_x = 15.4$	Mean = 29.5 $s_x = 15.3$
Min = 3 $Q_1 = 14$	Min = 5 $Q_1 = 17$
Median = 30 $Q_3 = 41$	Median = 29.5 $Q_3 = 44$
Max = 50 IQR = 27	Max = 52 IQR = 27

For the distribution of scores of the kindergarten children, the mean is 28.9, the median is 30, the standard deviation is 15.4 and the interquartile range is 27.

For the distribution of scores for the prep. school children, the mean is 29.5, the median is 29.5, the standard deviation is 15.3 and the interquartile range is 27.

The distributions are very similar. There is not a lot of difference between the way the kindergarten children and the prep. school children scored.

11. a. Class 1:25; Class 2:27

b. Class 1:2; Class 2:3

c. Class 1:24; Class 2:20

d. Class 1's results are clustered tightly, with a range of 6 from 10 to 16 marks out of 20. Class 2's results are more widely spread, with a range of 17 from 3 to 20. 11 of 27 students scored higher than any member of Class 1.

12. Key: $1|6 = 16$ years

Male	Stem	Female
987766	1	5566677789
9853332110	2	1122233578
8864221	3	00145
533	4	368
2	5	047
0	6	5

13. Both Brand A and B's battery life is clustered around 10–38 hours. Brand A had more batteries with a life of less than 10 hours (4 batteries) in comparison to Brand B (1 battery). The median value of Brand A was 21 hours in comparison to Brand B, which was 22 hours. From this comparison, Brand B has the better battery life span.

14. Provider B would be the better provider, because it offers a more stable connection. The consistent performance, as evidenced by the small range of speeds, means you can expect a reliable connection every time.

15. Key: $9|69 = 9.69$

Female	Stem	Male
	9	69
	9	84 85 87 90 92 96 99
	10	00 06 14 20 25 30 30 30 40
97 94 93 82 78 75 54	10	50 80
40 08 07 00 00	11	
90 90 60 50 50 50	11	
20	12	

There is not a large difference within each gender: females have gained 1.42 seconds and males have gained 1.11 seconds. Based on these numbers, the commentator's statement is reasonable, as women have improved their times slightly more than men (approx. 0.31 seconds).

But there is a large difference between the overall speeds of the two genders, as can be seen in the stem-and-leaf plot. Males are consistently faster than females.

16. Key: $23|1 = \$231$

Suburb A	Stem	Suburb B
7 5 5 0 0 0	20	
5 5 1 0 0	21	
5 5	22	
5 5 1	23	1 2 5
4 2 0	24	0 5 5
6	25	1 5 5 8
	26	0 5
	27	0 0 5 5
	28	0 0 5 5

Suburb A has a lower median rent ($215) compared to suburb B ($259). Suburb A has a positively skewed distribution for median rental price whereas Suburb B has an even distribution. The range of rental price for Suburb A is $200–$256, compared to Suburb B, which is $231–$285. More than half of the proprieties in Suburb A are cheaper than the cheapest house available in Suburb B.

Overall, despite some overlap in prices, your friend's claim is incorrect — Suburb A is cheaper.

11.5 Comparing histograms [complex]

11.5 Exercise

1. i. a.

Score	Frequency
3	1
4	2
5	2
6	1
7	3
8	2
9	2
10	2
11	0
12	1

b.

Class interval	Frequency
1–< 2	1
2–< 3	2
3–< 4	2
4–< 5	6
5–< 6	5
6–< 7	1

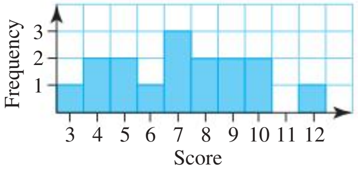
c.

Class interval	Frequency
10–< 15	3
15–< 20	9
20–< 25	10
25–< 30	10
30–< 35	10
35–< 40	1

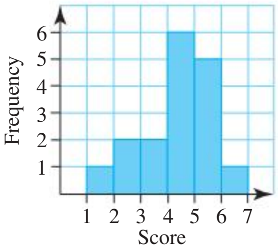
d.

Score	Frequency
0.3	1
0.4	2
0.5	1
0.6	1
0.7	1
0.8	2
0.9	2
1.0	2
1.1	1
1.2	1
1.3	1

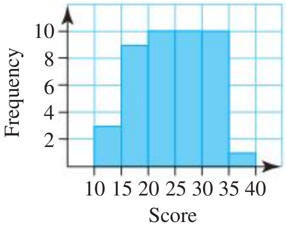
ii. a.



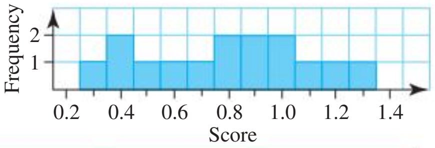
b.



c.



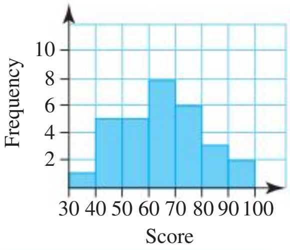
d.



2. a.

Score	Tally	Frequency
30–39	|	1
40–49	卌	5
50–59	卌	5
60–69	卌 |||	8
70–79	卌 |	6
80–89	|||	3
90–99	||	2

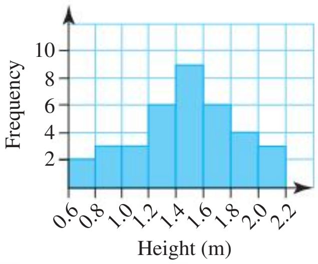
b.



3. a.

Class	Class centre	Tally	Frequency
0.6–<0.8	0.7	||	2
0.8–<1.0	0.9	|||	3
1.0–<1.2	1.1	|||	3
1.2–<1.4	1.3	卌 |	6
1.4–<1.6	1.5	卌 ||||	9
1.6–<1.8	1.7	卌 |	6
1.8–<2.0	1.9	||||	4
2.0–<2.2	2.1	|||	3

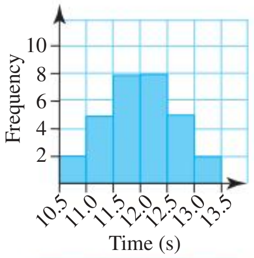
b.



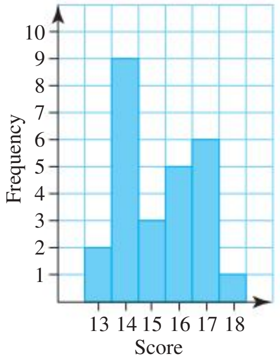
4. a.

Class	Class centre	Frequency
10.5–<11	10.7	2
11–<11.5	11.2	5
11.5–<12	11.7	8
12–<12.5	12.2	8
12.5–<13	12.7	5
13–<13.5	13.2	2

b.



5.



6. a. Symmetric, no outliers

b. Symmetric, one outlier

c. Symmetric, no outliers

d. Negatively skewed, no outliers

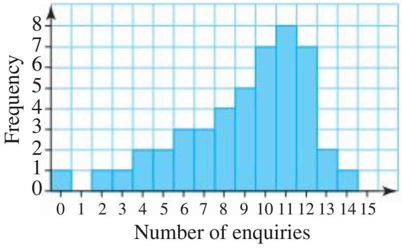
e. Negatively skewed one outlier

f. Positively skewed, no outliers

7. a. Positively skewed with one outlier

b. Bimodal and positively skewed

8. a.



The data on the *Yellow Pages* is negatively skewed, with a potential outlier of 0 enquiries.

b. The data on the local newspaper is positively skewed, centred around the mode of 3, with potential outliers between 10 and 12 and a definite outlier of 15 enquiries.

c. The Yellow Pages would be the better means of advertising.

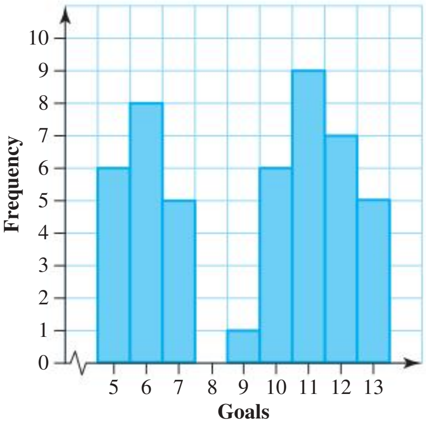
9. a. The data is symmetric, with one outlier at $2.

b. The majority of students receive about $8.00, or within the range of $7–$9 per week. There is one student, however, who receives $2.

c. The data is symmetric, with two outliers at $10 and $10.50.

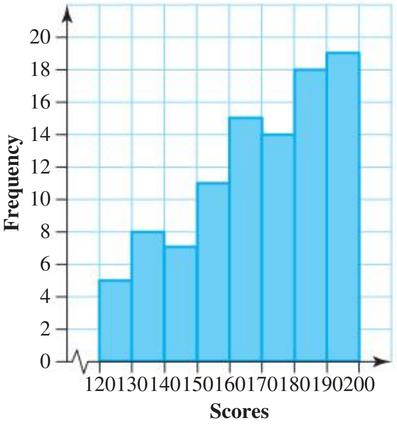
d. The majority of students in Year 6 receive about $8.00, or within the range of $7 to $9 per week. There is one student, however, who receives $2.
The majority of students in Year 3 receive about $5.00, or within the range of $3 to $6 per week. There are two students, however, who receive $10 and $10.50.
Overall, the pocket money seems to increase with age.

10. a. i.



ii. Bimodal. The data is separated into two clusters.

b. i.



ii. Negatively skewed. Most of the data is to the right.

11. Brand B performed consistently higher than Brand A.

12. Brand A data appears to be more consistent in spread, with most batteries lasting 6 to 7 days. Brand B data appears symmetrical around 5 days, with three potential outliers at 9 and 10 days. Factors affecting charge would include use; for example, if a scooter was used sparingly and for short distances, it would require charging less frequently.

13. The Doors computer has a symmetrical spread centred around 0.3 seconds. The Pears computer has the opposite spread, with a dip at 0.3 and peaks at 0.1 and 0.5 seconds. The Doors computer would be recommended, as it seems to perform consistently within the 0.2–0.4 second range, with most tasks taking less than 0.5 seconds to complete.

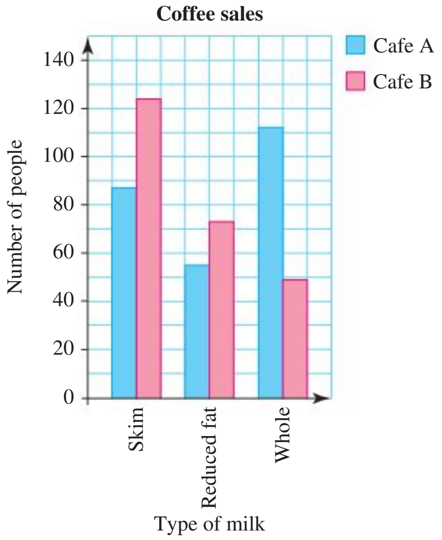
14. a. Worker F
    b. 4

15. a. Company B
    b. Company A has a positive skew, whereas Company B has a negative skew.
    c. Company B

16. Both stations had the same number of customers in the week. However, northbound was more evenly distributed and consistent. Southbound had bimodal peaks on day 3 and day 7, with more than half of customers coming in these two days. The best location for a diner would be northbound, as there would be a consistent flow of customers every day.

17. a. 2016
    b. 2018
    c. Answers will vary but could include that more people are on the roads during the Christmas break and school holidays.

18. a. 500
    b.



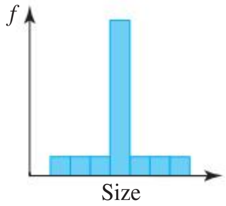
    c. 34%
    d. 26%
    e. 49%

19. Responses will vary.
Mr Algebra's claim is unreasonable. While 100% of his class did pass, the majority of his class only achieved a C (66.7%), with 10% achieving an A and 23.3% a B. The data distribution is very skewed to the lower passing grade.
In Ms Geometry's class, 26.7% achieved an A and 33.3% achieved a B (60% total of A or B). The data distribution is more evenly distributed across the grades. Ms Geometry did have one student fail, but this could be attributed to an outlier or the student being away. Therefore, the award should go to Ms Geometry's class.

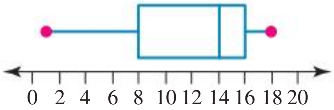
11.6 Review

11.6 Exercise

1. 15–20
2. 20–25
3. 30
4.



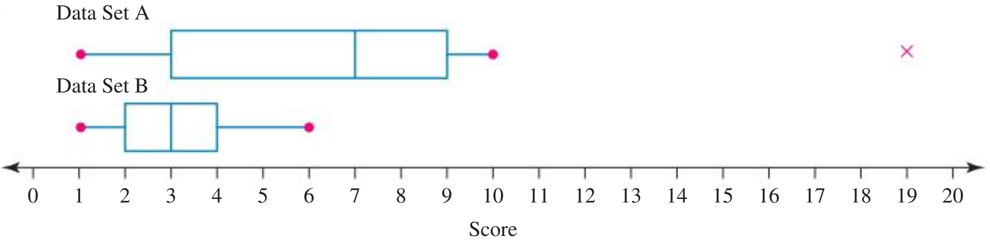
5. Min = 1, $Q_1 = 8$, $Q_2 = 14$, $Q_3 = 16$, max = 18



6. a. 43
   b. 43
   c. 14
7. a. 95, 215, 270, 290, 340
   b. 75
   c. Yes, 95
8. a. (2, 8, 13.5, 20, 25)
   b. (1, 5, 11.5, 21, 30)
   c. Group B has a larger IQR (16 compared to 12), it has a greater spread of data values, meaning more variability in study hours.
9. a. Bimodal, symmetric
   b. Unimodal, positively skewed with one outlier
   c. Bimodal, negatively skewed
   d. Unimodal, negatively skewed
10. See the figure at the bottom of the page.*

---

*10.



11. a. See the figure at the bottom of the page.*

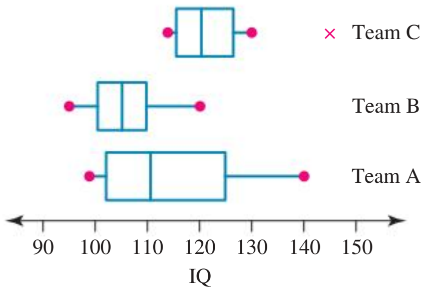
b. Player 1. Higher median number of goals scored (8.5) than Player 2 (7). Also, over 50% of the number of goals scored by Player 1 across the ten matches lie above the maximum number of goals scored by Player 2 across the 10 matches.

12. a. Key: 1|3 = 13 hours

Full-time	Stem	Volunteer
1	0	
3 3 2 2	0	
5 4 4	0	
6	0	
	0	8
	1	0 1 1
	1	2 3 3
	1	4 5
	1	
	1	

b. The full-time firefighters distribution is slightly positively skewed and the volunteers distribution is approximately symmetric.
The centre of the volunteers, distribution is much higher than that of the full-time firefighters distribution.

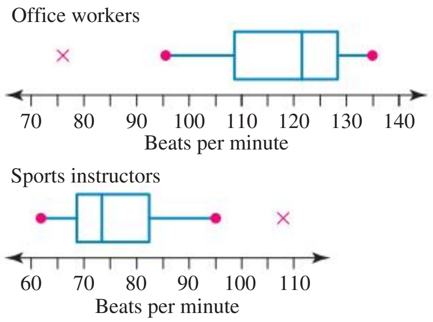
13.



14. a. The office workers distribution is negatively skewed with one outlier (76 beats/min). The sports instructors distribution is positively skewed with one outlier (108 beats/min).

b. Office workers:
Range = 58 beats/min
Median = 121.5 beats/min
IQR = 19.5 beats/min
Mode = 130 beats/min
Sports instructors:
Range = 46 beats/min
Median = 73 beats/min
IQR = 14 beats/min
Mode = 68, 72 beats/min

c.



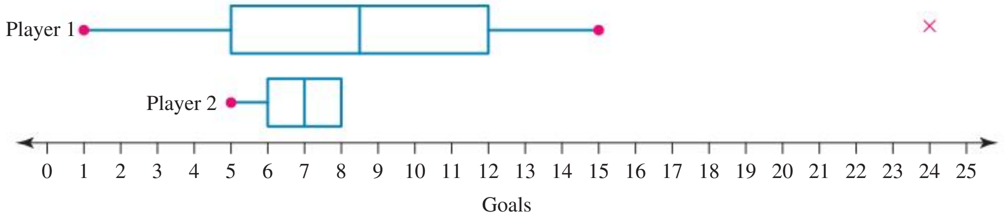
d. Office workers:
Mean = 116.8125 beats/min
Standard deviation = 15.3 beats/min
Sports instructors:
Mean = 76.87 beats/min
Standard deviation = 12.43 beats/min

e. Office workers: Pulse rates are generally very high, clustered around 120–130 beats/min. Also, there is one person whose rate was much lower than the rest. This outlier (76) produces a large range and makes the mean slightly lower than the median. As a result, the median is a more appropriate measure of the centre of the data rather than the mean.
Sports instructors: Pulse rates are generally low, clustered around 60–70 beats/min, although there are a few people with rates much higher, which makes the mean slightly higher than the median and also produces quite a large range. As a result of the skewed distribution, the median is the more appropriate measure of the centre of the data rather than the mean, although there is little difference between these values.

15. a. Key: 1|3 = 13

Text B	Stem	Text A
	1	3
	2	1 2 5 8
9 7	3	5 5
8 2	4	4 8
9 9 8 8 5 5 3 2	5	2 5 6 9
8 8 8 6 5 4 4 3 2	6	4 9
9 9 5 2 2	7	2 2 6
2 1	8	1 3 4 4
	9	4 5 8

***11. a.**



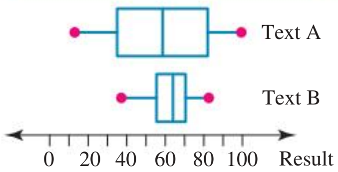
b. **Text A:** $n = 25$

Min	Q_1	Median	Q_3	Max
13	35	59	82	98

Text B: $n = 28$

Min	Q_1	Median	Q_3	Max
37	55	63.5	70	82

c.



d. Performance on test. Students who used Text A had varied results while the students who used Text B were more consistent and they had a higher median score.
Text A: range = 85, IQR = 47, median = 59
Text B: range = 45, IQR = 15, median = 63.5

e–f. Text B had a higher median mark and was more consistent/lower spread with a lower range and IQR. The other factors to take into account are amount of time studying per class, average mark using the same book per class etc.

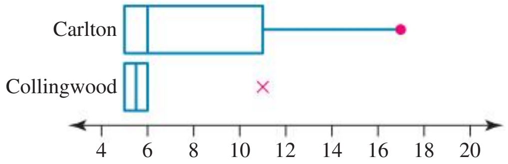
16. a. Key: 22 | 5 = 225

Suburb A	Stem	Suburb B
	22	5 8
	23	1 2 5
2	24	0 1 5
	25	5
	26	
5 5	27	0 0 5 5
5 4 1 1	28	0 0 5
6 5 5 0	29	5
7 5 5 0 0 0	30	0 5
5 0 0	31	0

b. Answers will vary. Suburb A has higher rent than Suburb B.

c. Answers will vary. Suburb C is higher than Suburb A, which is higher than Suburb B. The most expensive suburb to rent in is Suburb C. The least expensive suburb to rent a one-bedroom unit in is Suburb B.

17. a.

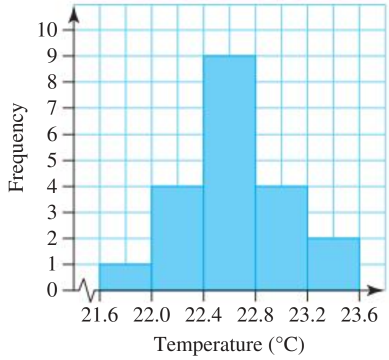


b. Goals scored in Grand Finals by this sample of players is greater but more variable among the Carlton players, as indicated by the larger range and IQR. Collingwood players are concentrated at 5 or 6, with the exception of the one upper outlier of 11.

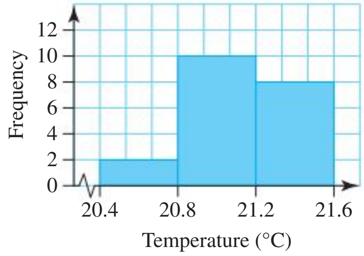
c. See the figure at the bottom of the page.*

d. Kicks for this sample of players are greater but more variable than the handballs, as indicated by the larger range and IQR. Both are positively skewed, with one upper outlier.

18. a.

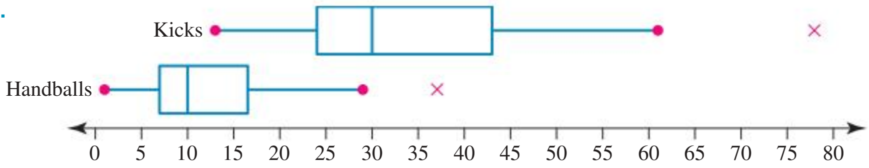


b.



c. The distribution for the interval 2005–2024 is approximately symmetrical. The distribution for the interval 1893–1912 is negatively skewed, with a smaller overall range. Both distributions are unimodal. The average temperature of 2005–2024 has more variance than the temperature of 1893–1912.

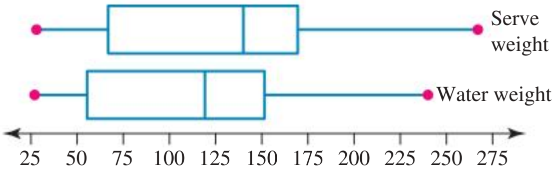
***17. c.**



19. a.

Food	Serve weight (g)	Water (%)	Water weight (g)
Avocado	173	73	126
Blackberries	144	86	124
Broccoli	180	90	162
Cantaloupe	267	90	240
Carrots	72	88	63
Cauliflower	62	92	57
Celery	40	95	38
Corn	77	70	54
Cucumber	28	96	27
Eggplant	160	92	147
Lettuce	163	96	156
Mango	207	82	170
Mushrooms	35	92	32
Nectarines	136	86	117
Peaches	87	88	77
Pears	166	84	139
Pineapple	155	86	133
Plums	95	84	80
Spinach	56	92	52
Strawberries	255	73	186

b.



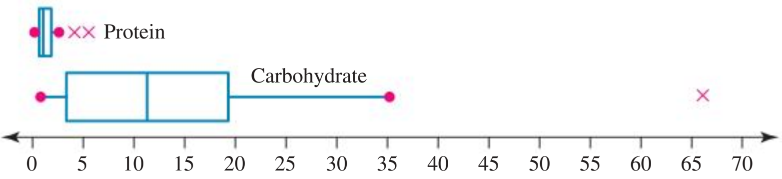
c. The box plots appear to indicate that there are only slight differences between the serve weights and water weights of the samples. The distributions are very similar in shape, with the water weights being slightly less overall.

d. See the figure at the bottom of the page.*

e. Carbohydrate for this sample of foods is much greater but more variable than protein, as indicated by the larger range and IQR. The protein amounts are all less than the Q_1 for the carbohydrate amounts, with the exception of two upper outliers for the distribution of protein amounts are slightly positively skewed compared to carbohydrate amounts which approximately symmetrical

20. Responses will vary. Responses should identify key summary statistics about each age. Example statements: John's statement doesn't take into account experience, time on the job or the position description. While the general trend indicates that salary increases with age, there is quite a spread in the salaries for all ages. This would suggest that entry-level jobs are more often filled by younger people who 'work their way up', but this is not always the case.
A person in their 40s is earning less than 75% of people in their 30s ($50 000), for example.
John has also neglected the fact that $350 000 is an outlier, no doubt attributed to a senior manager or executive. The range for a person in their 50s on an above-median salary is only $110 000 to $160 000.

***19. d.**



UNIT 4 Graphs, data and loans
TOPIC 3 Loans and compound interest

12 Compound interest

LESSON SEQUENCE

Fully worked solutions for this chapter are available online.

on Resources

Solutions	Solutions — Chapter 12
Digital documents	Learning matrix — Chapter 12 Quick quizzes — Chapter 12 Chapter summary — Chapter 12

LESSON
12.1 Overview

Hey students! Bring these pages to life online

Watch videos

Engage with interactivities

Answer questions and check results

Find all this and MORE in jacPLUS

12.1.1 Introduction

Financial literacy equips young people with essential skills to manage their money effectively. By understanding concepts like budgeting, saving and loans, young individuals can make informed decisions. This chapter compares how people can earn interest or pay interest. Interest can be simple interest or compound interest. Knowing how both work is vital to being financially literate so that you can plan for future goals like buying a car, saving for a holiday or starting a business.

12.1.2 Syllabus links

Lesson	Lesson title	Syllabus links
12.2	**Simple interest**	○ Understand the concept of simple interest where I is simple interest, P is principal, i is interest rate per period and n is total number of periods to find unknown values using the formulas: • $I = Pin$ • $P = \frac{I}{in}$ • $i = \frac{I}{Pn}$ • $n = \frac{I}{Pi}$
12.3	**Using formulas to calculate the future value**	○ Understand the concept of compound interest as a recurrence relation. ○ Calculate the future value of a compound interest loan or investment with annual periods using the formula $A = P(1 + i)^n$ where A is future value, P is principal, i is interest rate per annum and n is total number of years. ○ Calculate the total interest paid or earned for compound interest loan or investment.
12.4	**Using technology to determine the future value of compound investments or loans**	○ Use an online calculator to determine the future value of a compound interest loan or investment. ○ Use a spreadsheet to determine the future value of a compound interest loan or investment and the total interest paid or earned [complex]. ○ Investigate the effect of the principal amount, the interest rate and the number of compounding periods on the future value of a loan or investment [complex].
12.5	**Comparing simple and compound interest [complex]**	○ Compare, numerically and graphically, the growth of simple interest and compound interest loans and investments [complex]. ○ Investigate the effect of the principal, the interest rate and the number of compounding periods on the future value of a loan or investment [complex].

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LESSON
12.2 Simple interest

SYLLABUS LINKS

- Understand the concept of simple interest where I is simple interest, P is principal, i is interest rate per period and n is total number of periods to find unknown values using the formulas:
  - $I = Pin$
  - $P = \frac{I}{in}$
  - $i = \frac{I}{Pn}$
  - $n = \frac{I}{Pi}$

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12.2.1 Introducing simple interest

- When you lend money for a certain period of time (a **term deposit**) to a bank, building society or other financial institution, you expect to be rewarded by eventually getting your money back, plus an extra amount commonly known as **interest (I)**.
- Similarly, if you borrow money from any institution by taking out a loan or mortgage, you must pay back the original sum plus interest.
- The following examples deal with **simple interest** — that is, interest which is paid only on the original sum of money invested or borrowed.
- The interest earned from an investment depends on the **rate of interest**, the time period (term) for which it is invested and the amount (**principal**) invested.

Simple interest

The formula used to calculate simple interest is given by:

$$I = Pin$$

where:

I = simple interest (\$)

P = principal (\$) — the sum of money borrowed or invested

i = interest rate as a decimal per time period

n = total number of time periods — the period of time for which the sum of money is to be borrowed or invested.

- When we invest money, at the end of the time period we collect our investment (principal) and also the interest it has earned.
- The sum of the principal, P, and the interest, I, is called the *total amount* and is denoted by the symbol A.

Total amount

The formula used to calculate the total amount is given by:

$$A = P + I$$

where:

A = total amount at the end of the term ($)

P = principal ($)

I = simple interest ($).

WORKED EXAMPLE 1 Substituting into the simple interest formula

Calculate the amount of simple interest, I, earned and the total amount, A, at the end of the term, if:

a. $12 000 is invested for 5 years at 9.5% p.a.

b. $2500 is invested for 3 months at 4.5% p.a.

THINK	WRITE
a. 1. Write down the formula for simple interest.	a. $I = Pin$
2. Write down the known values of the variables. For i, a percentage to decimal conversion is needed.	$P = \$12\,000$ $i = \frac{9.5}{100}$ $= 0.095$ $n = 5$ years
3. Substitute the values into the given formula.	$I = 12\,000 \times 0.095 \times 5$
4. Evaluate.	$= 5700$
5. Answer the question and include the appropriate unit.	The amount of simple interest earned is $5700.
6. Write down the formula for the total amount.	$A = P + I$
7. Substitute the values for P and I.	$= 12\,000 + 5700$
8. Evaluate.	$= 17\,700$
9. Answer the question and include the appropriate unit.	The total amount of the investment at the end of the term is $17 700.

b. 1. Write down the formula for simple interest.	**b.** $I = Pin$
2. Write down the known values of the variables. For n, a time conversion is needed, as n is always expressed in years, so divide 3 months by 12.	$P = \$2500$ $i = \frac{4.5}{100}$ $= 0.045$ $n = 3$ months $= \frac{3}{12}$ years $= 0.25$ years
3. Substitute the values into the given formula.	$I = 2500 \times 0.045 \times 0.25$
4. Evaluate and round off the answer to 2 decimal places.	$= 28.125$ $= 28.13$
5. Answer the question and include the appropriate unit.	The amount of simple interest earned is \$28.13.
6. Write down the formula for the total amount.	$A = P + I$
7. Substitute the values for P and I.	$= 2500 + 28.13$
8. Evaluate.	$= 2528.13$
9. Answer the question and include the appropriate unit.	The total amount of the investment at the end of the term is \$2528.13.

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WORKED EXAMPLE 2 Calculating interest payments

After comparing investment options from a variety of institutions, Lynda and Jason decided to invest their \$18 000 in state government bonds at 7.75% p.a. The investment is for 5 years and the interest is paid biannually (twice per year). Calculate how much interest they will receive:

a. in every payment

b. in total.

THINK	WRITE
a. 1. Write down the formula for simple interest.	**a.** $I = Pin$
2. Write down the known values of the variables. For i, a percentage to decimal conversion is needed, as i is always in decimals. For n, a time conversion is needed, as n is always expressed in years, so divide 6 months by 12.	$P = \$18\,000$ $i = \frac{7.75}{100}$ $= 0.0775$ $n = 6$ months $= \frac{6}{12}$ years $= 0.5$ years

3. Substitute the values into the given formula.	$I = 18\,000 \times 0.0775 \times 0.5$
4. Evaluate.	$= 697.5$
5. Answer the question and include the appropriate unit.	Lynda and Jason receive \$697.50 in interest every 6 months.
b. Method 1	
1. Write down the formula for simple interest.	**b.** $I = Pin$
2. Write down the known values of the variables.	$P = \$18\,000$ $i = 0.0775$ $n = 5$ years
3. Substitute the values into the given formula.	$I = 18\,000 \times 0.0775 \times 5$
4. Evaluate.	$= 6975$
5. Answer the question and include the appropriate unit.	Lynda and Jason will receive a total of \$6975 in interest.
b. Method 2	
1. Multiply the interest received in each 6-month period by the number of 6-month periods in 5 years; that is, multiply \$697.50 by 10.	Interest obtained every 6 months = \$697.50 Number of payments to be received = 10. Total interest received = \$697.50 × 10 = \$6975
2. Answer the question.	Lynda and Jason will receive a total of \$6975 in interest.

12.2.2 Transposing the simple interest formula

- To calculate the principal, interest rate or time period of a loan, we rearrange the simple interest formula $I = Pin$.
- The following transpositions show how $I = Pin$ the equation can be transposed to make it easier to calculate the interest rate, term of the loan or the principal amount.

Simple interest formula transpositions

To calculate simple interest, use $I = Pin$.

To calculate the principal, use $P = \dfrac{I}{in}$.

To calculate the interest rate as a decimal, use $i = \dfrac{I}{Pn}$.

To calculate the number of periods in years, use $n = \dfrac{I}{Pi}$.

WORKED EXAMPLE 3 Calculating the principal, P

Calculate the principal invested for an investment offering simple interest of 9% p.a., earning $215 interest over 4 years.

THINK	**WRITE**
1. Select the transposed simple interest formula for P.	$P = \frac{I}{in}$
2. List the values of I, i and n.	$I = \$215$ $i = \frac{9}{100}$ $= 0.09$ $n = 4$ years
3. Substitute into the formula.	$P = \frac{I}{i \times n}$ $= \frac{215}{0.09 \times 4}$
4. Evaluate using a calculator.	$= \frac{215}{0.36}$ $P = 597.22$
5. Write your answer.	The amount invested was $597.22.

WORKED EXAMPLE 4 Calculating the interest, i

Calculate the interest rate offered when $720 is invested for 36 months and earns $205.20 simple interest. Express rates in % per annum.

THINK	**WRITE**
1. Select the transposed simple interest formula for i.	$i = \frac{I}{Pn}$
2. List the values of P, I and n. n must be expressed in years.	$P = \$720$ $I = \$205.20$ $n = 36$ months $= 3$ years
3. Substitute into the formula.	$i = \frac{205.20}{720 \times 3}$
4. Evaluate using a calculator.	$= 0.095$
5. An interest rate is normally displayed as a percentage. Convert this decimal to a percentage by multiplying by 100.	$0.095 \times 100 = 9.5\%$
6. Write your answer.	The interest rate offered was 9.5% per annum.

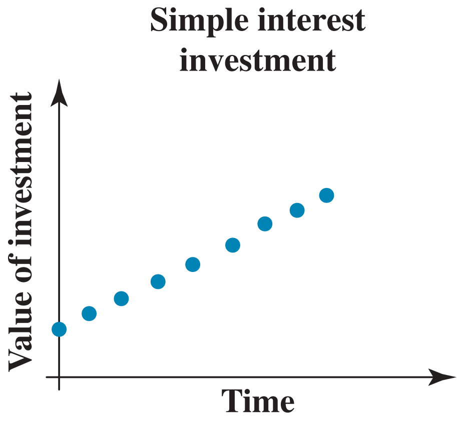
WORKED EXAMPLE 5 Calculating the time period, *n*

Calculate the period of time, to the nearest year, for an investment of $255 at a simple interest rate of 8.5% p.a. to earn $86.70 in interest.

THINK	WRITE
1. Select the transposed simple interest formula for n.	$n = \frac{I}{Pi}$
2. List the values of P, I and i.	$P = \$255$ $I = \$86.70$ $i = \frac{8.5}{100}$ $= 0.085$
3. Substitute into the formula.	$n = \frac{86.70}{255 \times 0.085}$
4. Evaluate using a calculator.	$= 4$
5. Write your answer.	The period of the investment was 4 years.

12.2.3 Simple interest as a recurrence relation

- A simple interest loan or investment can be modelled using recurrence relations, which help determine the initial principal based on the current value.



- A **recurrence relation** is a mathematical rule that we can use to generate a sequence of numbers, where each term in the sequence can be found from the previous term. It has two parts:
  1. a starting point — the value of one of the terms in the sequence
  2. a rule that can be used to generate successive terms in the sequence.
- In a simple interest loan or investment, the amount of interest paid or earned remains constant for each time period; hence, simple interest is a special case of linear growth in which the starting value is the amount borrowed or invested.

Recurrence relation for simple interest

$$A_0 = principal,\ A_{n+1} = A_n + d$$

Where:

A_0 represents the initial investment or principal

A_n represents the value of the investment after n time periods

d is the amount of simple interest earned each time period, which can be found using the formula $d = i \times A_0$

Note that in the formula $d = i \times A_0$, i represents the interest rate as a decimal per time period.

$A_n + 1$ represents the value of the investment after $n + 1$ time periods.

- When using a recurrence relation to model a simple interest problem, it may be necessary to determine the total amount of interest earned.

Total interest earned

amount of simple interest = (amount of investment) − principal

$$I = A - P$$

- It is important to clarify the differences between using a recurrence relation to model a simple interest problem and using the simple interest formula.

	Calculating the amount of interest	Calculating the total amount of the loan or investment
Simple interest formula	$I = Pin$ or $I = A - P$	$A = P + I$
Recurrence relation	$I = A_0 \times i \times n$	$A_{n+1} = A_n + d$, where $d = i \times A_0$

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WORKED EXAMPLE 6 Calculating simple interest as a recurrence relation

\$575 is invested in a simple interest account for 3 years at 4% p.a.

a. **Set up a recurrence relation to determine the value of the investment after n years.**

b. **Use the recurrence relation to determine the value of the investment at the end of each of the first 3 years.**

THINK	WRITE
a. **1.** Write the formula to calculate the amount of interest earned per period.	**a.** $d = i \times A_0$
2. Substitute the values of A_0 and i.	$d = \frac{4}{100} \times 575$ $= 23$

	3. Write the formula for the recurrence relation that models simple interest.	$A_0 =$ principal, $A_{n+1} = A_n + d$
	4. Substitute the values of A_0 and d to give the recurrence relation.	$A_0 = 575, A_{n+1} = A_n + 23$
b.	**1.** Since A_0 represents the initial amount (principal) of money invested, A_1 would represent the value of the investment after 1 year. To calculate A_1, substitute $n = 0$ into the recurrence relation.	**b.** $A_{n+1} = A_n + 23$ $A_{0+1} = A_0 + 23$ $A_1 = A_0 + 23$
	2. Substitute the value of A_0 into this equation and complete the addition.	$A_1 = 575 + 23$ $= 598$
	3. To determine the value of A_2, the value of the investment after 2 years, substitute $n = 1$ into the recurrence relation.	$A_{n+1} = A_n + 23$ $A_{1+1} = A_1 + 23$ $A_2 = A_1 + 23$
	4. Substitute the value of A_1 into this equation and complete the addition.	$A_2 = 598 + 23$ $= 621$
	5. To determine the value of A_3, the value of the investment after 3 years, substitute $n = 2$ into the recurrence relation.	$A_{n+1} = A_n + 23$ $A_{2+1} = A_2 + 23$ $A_3 = A_2 + 23$
	6. Substitute the value of A_2 into this equation and complete the addition.	$A_3 = 621 + 23$ $= 644$
	7. Write the answers.	The value of the investment after 1 year is \$598. The value of the investment after 2 years is \$621. The value of the investment after 3 years is \$644.

Determining A_0, i and n

- To calculate the principal, interest rate or time periods instead of interest, the simple interest recurrence relation must be transposed.

$$A_0 = \frac{I}{i \times n}$$

$$i = \frac{I}{A_0 \times n}$$

$$n = \frac{I}{A_0 \times i}$$

- Note that A_0 and P are interchangeable.

WORKED EXAMPLE 7 Determining the value of the initial investment (A_0)

A building society offers 8% p.a. simple interest on an investment. The investment earned $320 after 5 years. Determine how much was invested initially.

THINK	WRITE
1. Write the simple interest formula to calculate the principal.	$A_0 = \frac{I}{i \times n}$
2. Substitute values into the formula and calculate.	$A_0 = \frac{320}{\frac{8}{100} \times 5}$ $= \frac{320}{0.08 \times 5}$ $= 800$
3. Write the answer.	The principal was $800.

WORKED EXAMPLE 8 Determining the rate (i)

If $840 is invested for 24 months and the investment earns $126 in interest, calculate the yearly interest rate.

THINK	WRITE
1. Write the simple interest formula to calculate the rate.	$i = \frac{I}{A_0 \times n}$
2. Substitute values into the formula and calculate. Twenty-four months equals $\frac{24}{12}$ or 2 years.	$i = \frac{126}{840 \times \frac{24}{12}}$ $= 0.075$
3. Convert the decimal into a percentage by multiplying by 100.	$i = 0.075 \times 100$ $= 7.5$
4. Write the answer.	The yearly interest rate was 7.5%.

Exercise 12.2 Simple interest

12.2 Quick quiz on

12.2 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11,12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26	N/A	N/A

Simple familiar

1. **WE1** Calculate the amount of simple interest, I, earned and the total amount, A, at the end of the term for each of the following.
   - a. $680 for 4 years at 5% p.a.
   - b. $210 for 3 years at 9% p.a.
   - c. $415 for 5 years at 7% p.a.
   - d. $460 at 12% p.a. for 2 years
   - e. $\$1020$ at $12\frac{1}{2}\%$ p.a. for 2 years
   - f. $\$713$ at $6\frac{3}{4}\%$ p.a. for 7 years

2. Calculate the amount of simple interest, I, earned and the total amount, A, at the end of the term for each of the following.
   - a. $\$821$ at $7\frac{1}{4}\%$ p.a. for 3 years
   - b. 11.25% p.a. on $65 for 6 years
   - c. 6.15% p.a. on $21.25 for 9 years
   - d. 9.21% p.a. on $623.46 for 4 years
   - e. $13\frac{3}{4}\%$ p.a. on $\$791.35$ for 5 years

3. **WE2** Sue and Harry invested $14 500 in state government bonds at 8.65% p.a. The investment is for
10 years and the interest is paid biannually (twice per year). Calculate how much interest:
   - a. they receive with every payment
   - b. they will receive in total.

4. **WE3** For each of the following, calculate the principal invested.
   - a. Simple interest of 5% p.a., earning $307 interest over 2 years
   - b. Simple interest of 7% p.a., earning $1232 interest over 4 years
   - c. Simple interest of 8% p.a., earning $651 interest over 18 months
   - d. Simple interest of $5\frac{1}{2}\%$ p.a., earning $\$78$ interest over 6 years
   - e. Simple interest of 6.25% earning $625 interest over 4 years

5. **WE4** For each of the following, calculate the interest rate offered. Express rates in % per annum.
   - a. Loan of $10 000, with a $2000 interest charge, for 2 years
   - b. Investment of $5000, earning $1250 interest, for 4 years
   - c. Loan of $150, with a $20 interest charge, for 2 months
   - d. Investment of $1400 earning $178.50 interest, for 6 years
   - e. Investment of $\$6250$, earning $\$525$ interest, for $2\frac{1}{2}$ years

6. **WE5** For each of the following, calculate the period of time (to the nearest month) for which the principal was invested or borrowed.
   a. Investment of \$1000 at simple interest of 5% p.a., earning \$50 interest
   b. Loan of \$6000 at simple interest of 7% p.a., with an interest charge of \$630
   c. Loan of \$100 at simple interest of 24% p.a., with an interest charge of \$6
   d. Investment of \$23 000 at simple interest of $6\frac{1}{2}$% p.a., earning \$10 465 interest
   e. Loan of \$1 500 000, at simple interest of 1.5% p.a., with an interest charge of \$1875

7. **WE6** \$688 is invested in a simple interest account for 4 years at 6.25% p.a.
   a. Set up a recurrence relation to calculate the value of the investment after n years.
   b. Use the recurrence relation in part **a** to determine the value of the investment after 4 years.

8. \$728 is invested in a simple interest account for 3 years at 8.5% p.a.
   a. Set up a recurrence relation to calculate the value of the investment after n years.
   b. Use the recurrence relation in part **a** to determine the value of the investment after 3 years.

9. Marina invests \$1560 in a fixed deposit account that pays 10.4% p.a. simple interest for 18 months.
   a. Calculate how much her investment is worth after 18 months.
   b. Represent the account balance for each of the first 18 months graphically.

10. a. Determine the amount by which the investment has grown if \$770 was invested at 9.2% simple interest p.a. for 2 years.
    b. Represent the balance of the account for each of the first 12 months.

11. Determine the value of the following investments by using a recurrence relation.
    a. \$860 for 4 years at 4% p.a.
    b. \$390 for 5 years at 7% p.a.

12. **WE7** Calculate the interest charged or earned on the following loans and investments.
    a. \$1100 loaned at 11% p.a. simple interest for 21 months
    b. \$5750 invested for 2 years at 0.8% per month simple interest
    c. \$31 500 borrowed for 26 weeks at 0.15% per week simple interest
    d. \$5575 invested at 1.8% per month for 3.5 years

13. Determine the amount to which the investment has grown in each of the following cases.
    a. \$500 invested at 12% p.a. simple interest for 24 months
    b. \$680 invested for 2 years at 1.3% per month simple interest
    c. \$18 000 invested for 4 years and 6 months at 9.5% p.a. simple interest

14. Determine the interest earned on the following investments.
    a. \$690 invested at 12% p.a. simple interest for 15 months
    b. \$7500 invested for 3 years at 12% per year simple interest
    c. \$25 000 invested for 13 weeks at 5.2% p.a. simple interest
    d. \$250 invested at 21% p.a. simple interest for $2\frac{1}{2}$ years

15. Determine the amount to which each investment has grown after the investment periods shown in the following examples.

a. \$300 invested at 10% p.a. simple interest for 24 months
b. \$750 invested for 3 years at 12% p.a. simple interest
c. \$20 000 invested for 3 years and 6 months at 11% p.a. simple interest
d. \$15 invested at $6\frac{3}{4}\%$ p.a. simple interest for 2 years and 8 months
e. \$10.20 invested at $8\frac{1}{2}\%$ p.a. simple interest for 208 weeks

16. Jill and John decide to borrow money to improve their yacht, but cannot agree which loan is the better value. They would like to borrow \$2550. Jill goes to the Big-4 Bank and finds that they will lend them the money at $11\frac{1}{3}\%$ p.a. simple interest for 3 years.

John finds that the Friendly Building Society will lend the \$2550 to them at 1% per month simple interest for the 3 years.

a. Determine which institution offers the better rate over the 3 years.
b. Explain why.

17. **WE8** Lennie earned \$576 in interest when she invested in a fund paying 9.5% p.a. simple interest for 4 years. Calculate the original amount Lennie invested.

18. Lennie's sister Lisa earned \$576 interest at 9% p.a. simple interest, but she only had to invest it for 3 years. Determine Lisa's initial investment.

19. Jack Kahn put some money away for 5 years in a bank account which paid $3\frac{3}{4}\%$ p.a. simple interest. Calculate the amount Jack initially invested, given that he earned \$66 from his bank statement.

20. James needed to earn \$225. He invested \$2500 in an account earning simple interest at a rate of 4.5% p.a. Determine the number of months it will take for James to achieve his aim.

21. A building society offers 6% p.a. simple interest on an investment. The investment earned \$724 after 3 years. Calculate how much was initially invested.

22. Calculate the principal that was invested if simple interest is 9% p.a. which earns \$468 in interest over 4 years.

23. If \$4500 is invested for 4 years and the investment earns \$870 in interest, calculate the yearly simple interest rate.

24. Calculate the annual simple interest rate offered for an investment of \$600 that earns a total of \$80 in interest over 6 months.

25. Calculate the period of time for which the principal was invested, if \$3800 was invested with a simple interest of 6% p.a. and \$420 interest was earned.

26. Joel and Shae decided to borrow money to put in a basketball area and new decking at their house. They cannot agree on who they should take the loan out with. They would like to borrow \$25 000. Joel goes to Tigers Bank, which offers the money at 9.8% p.a. simple interest for 4 years. Shae finds that Twins Bank will lend the money to them at 0.9% per month simple interest for 4 years.
Evaluate which option offers the better deal over 4 years and justify your reasoning.

Fully worked solutions for this chapter are available online.

LESSON
12.3 Using formulas to calculate the future value

SYLLABUS LINKS

- Understand the concept of compound interest as a recurrence relation.
- Calculate the future value of a compound interest loan or investment with annual periods using the formula $A = P(1 + i)^n$ where A is future value, P is principal, i is interest rate per annum and n is total number of years.
- Calculate the total interest paid or earned for compound interest loan or investment.

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12.3.1 Calculating the future value

- **Compound interest** differs from simple interest, which is based only on the initial amount.
- Most loans and investments use compounding, where interest is calculated on the changing balance.
- Interest is added to the balance, and future interest is based on the new value. This results in:
  - Investments: Earning interest on interest over time.
  - Loans: Paying less interest as the balance decreases.
- The final amount (**future value**) of a compound interest investment or loan depends on the interest rate, time period and initial amount.

Future value of a compound interest investment or loan

The formula used to determine the value of a compound interest investment or loan is given by the rule:

$$A = P(1 + i)^n$$

where:

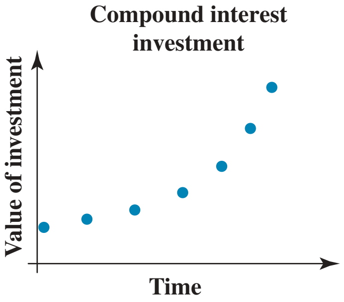
A **= the future value of the compound interest investment or loan ($)**

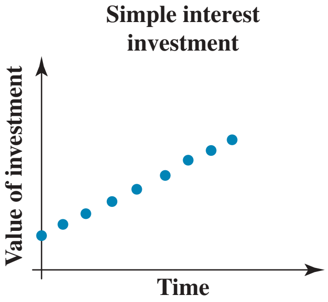
P **= principal ($) — that is, the money borrowed or invested**

i **= interest rate as a decimal per time period**

n **= total number of compounding time periods (e.g. years).**

- With compound interest, the investment grows by an increasing amount each period.
- When plotted, it forms an exponential curve, unlike the linear graph of simple interest.





12.3.2 Compound interest as a recurrence relation

- Compound interest can be calculated using a recurrence relation with a **compounding factor** (r), which determines how the investment or loan value changes each period.

Recurrence relation for value of compound interest investment or loan

The recurrence relation for the value of a compound interest investment or loan after n compounding periods is given by:

$$A_0 = \text{principal}, A_{n+1} = rA_n$$

where:

A_n is the value of the investment after n compounding periods

r is the compounding factor, with $r = (1 + i)$

i is the interest rate as a decimal per compounding period.

- Recurrence relations calculate compound interest step by step, making long-term calculations tedious.
- A formula can be derived to calculate the final amount directly.
- Next, we compare the compound interest formula and the recurrence relation formula for future value calculations.

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WORKED EXAMPLE 9 Calculating future value

\$5000 is invested for 4 years at 6.5% p.a. interest, compounded annually.

a. Generate the compound interest formula for this investment.

b. Calculate the amount in the balance after 4 years and the interest earned over this period.

THINK	WRITE
a. 1. Write the compound interest formula.	**a.** $A = P(1 + i)^n$
2. List the values of n, i and P.	$n = 4$ $i = \frac{6.5}{100} = 0.065$ $P = 5000$
3. Substitute the values into the formula.	$A = 5000(1 + 0.065)^4$
4. Complete the addition inside the brackets to simplify.	$A = 5000(1.065)^4$
b. 1. Use your calculator to evaluate5000(1.065)^4 correct to 2 decimal places. This is the amount of the investment after 4 years.	**b.** $A = 5000(1.065)^4$ $= 6432.33$
2. To determine the amount of interest earned, subtract the principal from the balance.	$I = \text{final amount} - \text{principal}$ $= 6432.33 - 5000$ $= \$1432.33$
3. Write your answer.	The amount of interest earned is \$1432.33 and the balance is \$6432.33.

Alternative method — using a recurrence relation

a. **1.** Write the recurrence relation for the value of a compound interest investment or loan.	**a.** $A_0 = \text{principal}, A_{n+1} = rA_n$
2. List the values of A_0 and r.	$A_0 = 5000$ $r = 1 + i$ $= 1 + \frac{6.5}{100}$ $= 1 + 0.065$ $= 1.065$
3. Substitute the values into the rule to write the recurrence relation.	$A_0 = 5000, A_{n+1} = 1.065A_n$
b. **1.** Calculate the value of the investment after 1 year (A_1). Substitute $n = 0$ into the recurrence relation.	**b.** $A_{0+1} = 1.065A_0$ $A_1 = 1.065 \times A_0$ $= 1.065 \times 5000$ $= 5325$
2. Calculate the value of the investment after 2 years (A_2). Substitute $n = 1$ into the recurrence relation.	$A_{1+1} = 1.065A_1$ $A_2 = 1.065 \times A_1$ $= 1.065 \times 5325$ $= 5671.125$
3. Calculate the value of the investment after 3 years (A_3). Substitute $n = 2$ into the recurrence relation.	$A_{2+1} = 1.065A_2$ $A_3 = 1.065 \times A_2$ $= 1.065 \times 5671.125$ $= 6039.748\,125$
4. Calculate the value of the investment after 4 years (A_4). Substitute $n = 3$ into the recurrence relation.	$A_{3+1} = 1.065A_3$ $A_4 = 1.065 \times A_3$ $= 1.065 \times 6039.748\,125$ $= 6432.33...$
5. Write the answer.	The amount of the investment after 4 years is \$6432.33.

Exercise 12.3 Using formulas to calculate the future value

12.3 Quick quiz on

12.3 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15	N/A	N/A

These questions are even better in jacPLUS!

- Receive immediate feedback
- Access sample responses
- Track results and progress

Find all this and MORE in jacPLUS

Simple familiar

1. **WE9** \$2500 is invested for 5 years at 7.5% p.a. compounding annually.
   a. Generate the compound interest formula for this investment.
   b. Calculate the amount in the balance after 5 years and the interest earned over this period.
2. Use the recurrence relation and $A_{n+1} = rA_n$ to calculate A_1 to A_{10} if $A_0 = \$5000$ and $i = 3.6\%$ p.a. compounded annually. Give your answers to the nearest cent.
3. Calculate the final amount A if $A_0 = \$500$, $n = 2$ years and $i = 8\%$ p.a. compounded annually.
4. Calculate the final amount A if $A_0 = \$1000$, $n = 4$ years and $i = 13\%$ p.a. compounded annually.
5. Calculate the final amount A if $A_0 = \$3600$, $n = 3$ years and $i = 7.5\%$ p.a. compounded annually.
6. Calculate the final amount A if $A_0 = \$2915$, $n = 5$ years and $i = 5.25\%$ p.a. compounded annually.
7. \$2000 is invested for 6 years at 7.5% p.a., compounded annually.
   a. Calculate the balance.
   b. Calculate the interest earned.
8. Determine the amount in the account A and interest earned I after \$6750 is invested for 7 years at 5.25% p.a. interest compounded annually.
9. Determine the interest accrued I if a principal amount of \$1000 is invested at an annual interest rate of 5%, compounded annually, for 3 years.
10. Determine the interest accrued I if Ravi invests \$2500 at an annual interest rate of 6%, compounded annually, for 5 years.
11. Determine the interest accrued I if a company deposits \$10 000 in a bank account with an annual interest rate of 4%, compounded annually, for 7 years.
12. Determine the interest accrued I if a principal of \$5000 is invested at an annual interest rate of 8%, compounded annually, for 4 years.
13. Determine the interest accrued I if James saves \$3000 at an annual interest rate of 7%, compounded annually, for 6 years.
14. Uyen saves \$1500 at an annual interest rate of 5%, compounded annually, for 10 years. Paul saves \$3600 at an annual interest rate of 7.8%, compounded annually, for 4 years. Explain who accrued the most interest and justify your answer.
15. Kirrily puts her savings into an account that earns 5.6% p.a. compounded annually. After 4 years, she has \$42 021.50. Determine how much savings Kirrily had initially, correct to the nearest dollar.
16. Siobhan saves \$2800 at an annual interest rate of 5.7%, compounded annually. Explain whether she will have earned at least \$2000 in interest after 10 years and justify your answer.

Fully worked solutions for this chapter are available online.

LESSON

12.4 Using technology to determine the future value of compound investments or loans

SYLLABUS LINKS

- Use an online calculator to determine the future value of a compound interest loan or investment.
- Use a spreadsheet to determine the future value of a compound interest loan or investment and the total interest paid or earned [complex].
- Investigate the effect of the principal amount, the interest rate and the number of compounding periods on the future value of a loan or investment [complex].

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12.4.1 Using an online calculator to determine future value

- There are many online calculators that are useful tools to help determine the future value of a loan or investment.
- Online calculators such as the Moneysmart website (https://moneysmart.gov.au/budgeting/compound-interest-calculator) allow you to substitute all the values and the online calculator can determine the future value for you.

WORKED EXAMPLE 10 Using an online calculator to determine future value

Oliver invested $\$3,200$ for 5 years at 6% p.a. interest compounded annually, with no additional deposits. Use an online compound interest calculator, such as Moneysmart, to:

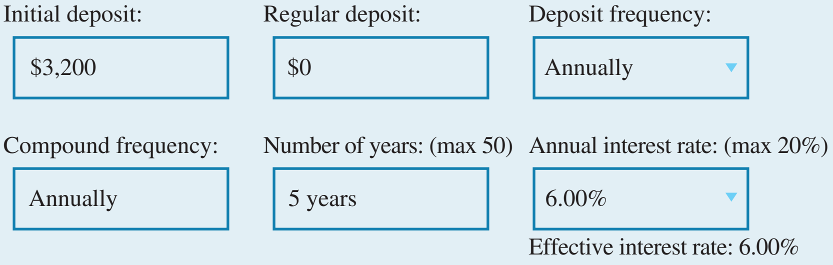
a. determine the future value of her investment and the amount of interest earned

b. determine the future value should she also include a monthly $100 deposit.

THINK

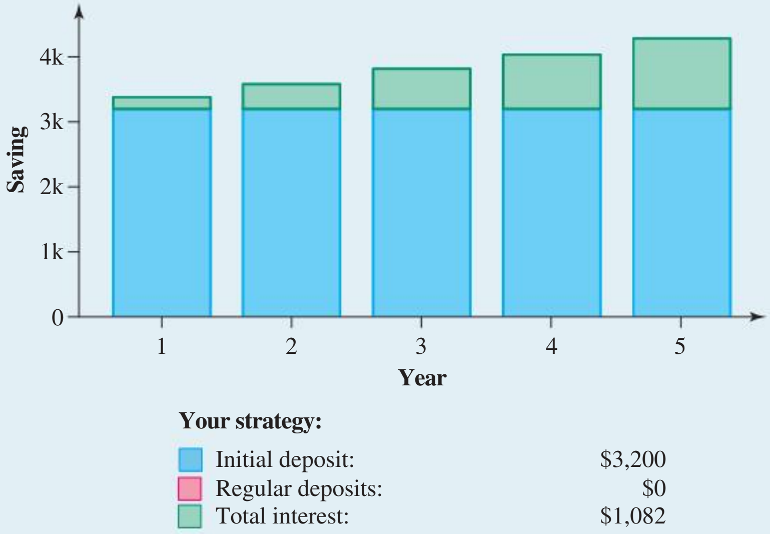
a. 1. Open an online calculator and input the known values:
Initial deposit: $3200
Regular deposit: $0
Number of years: 5 years
Interest rate: 6% p.a.
Compound frequency: annually

WRITE



Initial deposit: $3,200
Regular deposit: $0
Deposit frequency: Annually
Compound frequency: Annually
Number of years: (max 50) 5 years
Annual interest rate: (max 20%) 6.00%
Effective interest rate: 6.00%

2. Examine the results.



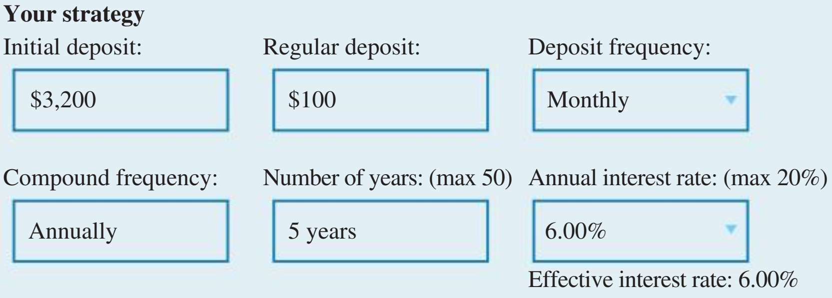
Your strategy:

🟦	Initial deposit:	$3,200
🟥	Regular deposits:	$0
🟩	Total interest:	$1,082

3. Write your answer.

The total savings (future value) is $4282, of which $3200 is the initial deposit.
Thus, the interest earned would be $1082.

b. 1. Adjust the current input to also include a regular deposit of $100 per month.



Your strategy

Initial deposit: $3,200

Regular deposit: $100

Deposit frequency: Monthly

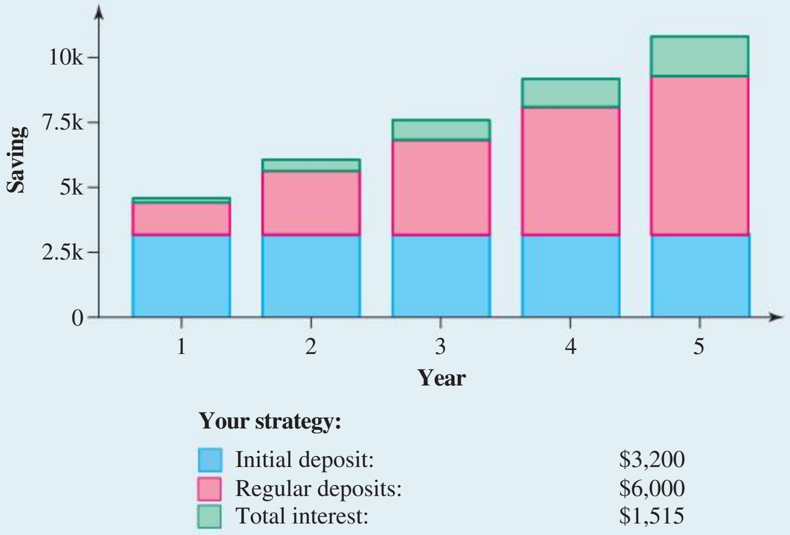
Compound frequency: Annually

Number of years: (max 50) 5 years

Annual interest rate: (max 20%) 6.00%

Effective interest rate: 6.00%

2. Examine the results.



Your strategy:

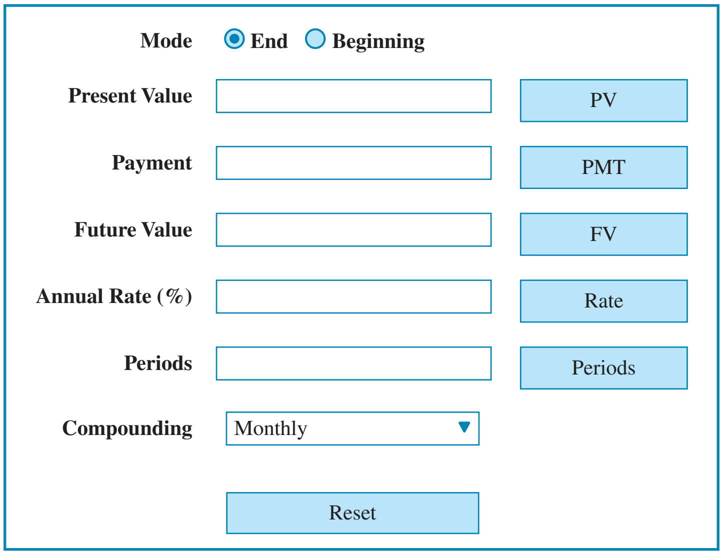
🟦	Initial deposit:	$3,200
🟥	Regular deposits:	$6,000
🟩	Total interest:	$1,515

3. Write your answer.

The total savings (future value) is $10 715, of which $3200 is the initial deposit, and $6000 will be additionally deposited.
The interest earned will be $1515.

12.4.2 Using an online calculator to investigate the effect of principal, interest rate and compounding periods on future value [complex]

- In the real world there are many types of technologies that can be used to calculate the final amount, the interest rate, the number of compounding periods, the payment or even the principal of a compound interest loan or investment.
- In this section, an online calculator involving the time value of money (TVM solver) will be used to investigate compound interest problems.



Mode ● End ○ Beginning

Compounding: Monthly

- To use a financial solver, you need to know the meaning of each of its inputs. These are defined as follows:
- *Present Value* is the current (present) value of the loan or investment.
- *Payment* is the amount paid at each time period.
- *Future Value* is the value of the investment or loan after a specified number of compounding periods.
- *Annual Rate (%)* is the annual interest rate.
- *Periods* is the total number of compounding periods.
- *Compounding* is the number of times the interest is compounded per year.
- *Mode* is used to indicate whether the interest is compounded at the end or at the beginning of the time period. Leave this set at END.

Note: If there is no mention of a payment or repayment being made, leave the cell blank.

When using a financial solver, you must be very careful in the way you enter information because it needs to know which way the money is flowing. It does this by following a *sign convention*.

Sign conventions for the flow of money

- **If you receive money, or someone owes you money, we treat this as a positive (+ve).**
- **If you pay out money or you owe someone money, we treat this as a negative (–ve).**

Analysing the effect of changing the interest rate

- When shopping for the best loan, the key factor most people consider is the interest rate.
- This section examines the impact of interest rates on the future value of a loan or investment using an online calculator.

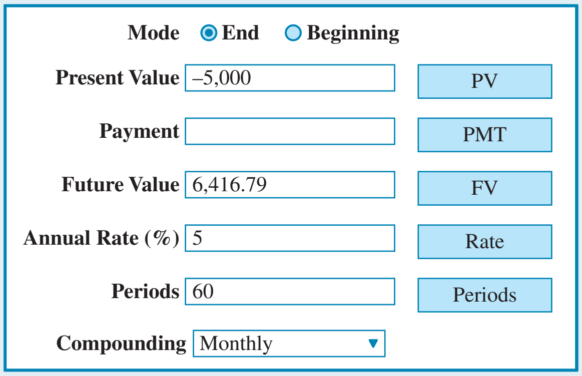
WORKED EXAMPLE 11 Investigating the effect of changing the interest rate (i)

Kasey invested \$5000 over a period of 5 years. Compare how much more she would earn by investing at 5.5% interest per annum, compounding monthly, compared to 5% interest per annum, compounding monthly.

THINK

1. Since Kasey has given the money to the bank, we input this as a negative since she no longer has possession of the money.
   Input the following values into the online calculator:
   Present Value: −5000
   Payment: leave blank (no extra money is being added to the investment each period)
   Rate: 5
   Periods: 60
   (5 years × 12 months each year)
   Compounding: Monthly
   Click on the FV button to determine the future value.

WRITE

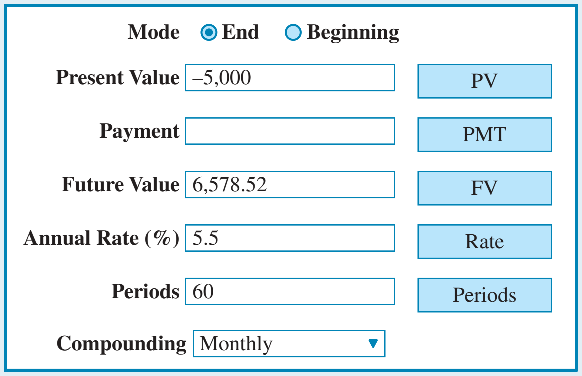


Mode ● End ○ Beginning
Present Value: –5,000
Payment:
Future Value: 6,416.79
Annual Rate (%): 5
Periods: 60
Compounding: Monthly ▾

2. Write the answer.

The future value at 5% is \$6416.79.

3. Consider the 5.5% interest investment.
   Input the following values into the online calculator:
   Present Value: −5000
   Payment: leave blank
   Rate: 5.5
   Periods: 60
   (5 years × 12 months each year)
   Compounding: Monthly
   Click on the FV button to determine the future value.



Mode ● End ○ Beginning
Present Value: –5,000
Payment:
Future Value: 6,578.52
Annual Rate (%): 5.5
Periods: 60
Compounding: Monthly ▾

4. Write the answer.

The future value at 5.5% is \$6578.52.

5. Calculate the difference by subtraction.

$$\begin{aligned}\text{The difference} &= 6578.52 - 6416.79\\ &= 161.73\end{aligned}$$

6. Write the answer.

By increasing the interest rate from 5% to 5.5%, the investment increased by \$161.73 over the 5-year period.

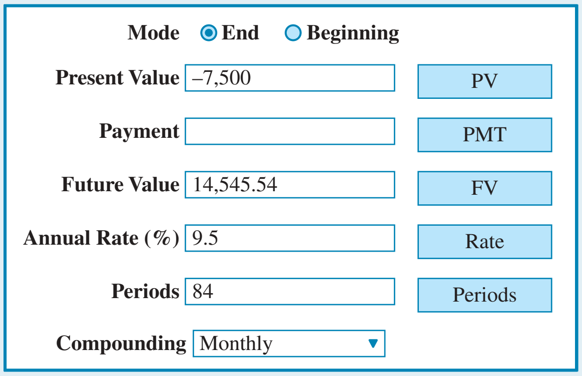
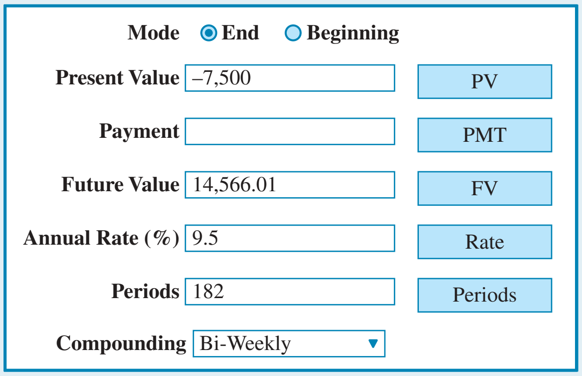
- In Worked example 11, an increase in the interest rate led to a higher future value.
- For an investment, this is favourable (benefiting Kasey), but for a \$5000 bank loan, it is generally unfavourable for the borrower.

Analysing the effect of changing the number of compounding periods

- To help us investigate the effect of the number of compounding periods, we will use an online calculator.

WORKED EXAMPLE 12 Investigating the effect of changing compound periods (n)

Hugh wants to invest his $7500 at 9.5% p.a. for a period of 7 years. He believes it is better to invest his money with more compounding periods. Calculate the additional earnings Hugh would make if his investment were compounded fortnightly instead of monthly.

THINK	WRITE
1. Consider compounding monthly. Since Hugh has given the money to the bank, we input this as a negative value. Input the following values into the online calculator: Present Value: −7500 Payment: leave blank Rate: 9.5 Periods: 84 (7 years × 12 months each year) Compounding: Monthly Click on the FV button to determine the future value.	 Mode ● End ○ Beginning Present Value –7,500 Payment Future Value 14,545.54 Annual Rate (%) 9.5 Periods 84 Compounding Monthly ▾
2. Write the answer.	The future value of the investment with monthly compounding is $14 545.54.
3. Consider compounding fortnightly. Input the following values into the online calculator: Present Value: −7500 Payment: leave blank Rate: 9.5 Periods: 182 (7 years × 26 fortnights each year) Compounding: Bi-Weekly Click on the FV button to determine the future value.	 Mode ● End ○ Beginning Present Value –7,500 Payment Future Value 14,566.01 Annual Rate (%) 9.5 Periods 182 Compounding Bi-Weekly ▾
4. Write the answer.	The future value of the investment with fortnightly compounding is $14 566.01.
5. Calculate the financial benefit of fortnightly compounding compared to monthly compounding.	Difference $= 14\,566.01 - 14\,545.54$ $= 20.47$
6. Write the answer.	Hugh would earn $20.47 more in the investment if he chooses fortnightly compounding.

- In Worked example 12, increasing the compounding frequency raised the future value, benefiting Hugh's investment.

Analysing the effect of changing the interest rate and number of compounding periods

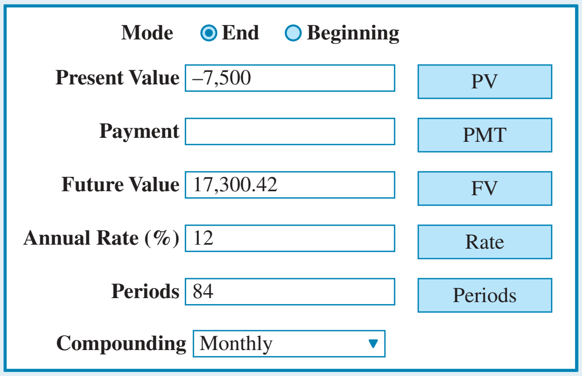
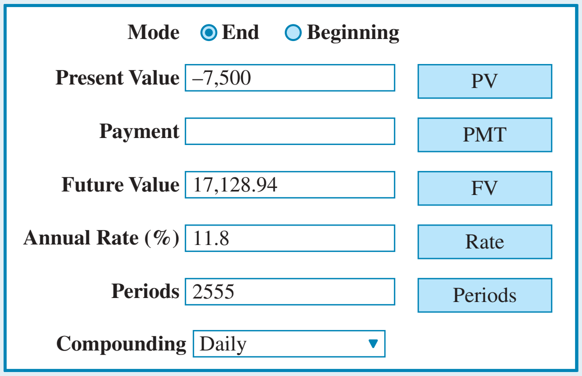
- We will now investigate the effect of changing the interest rate and the number of compounding periods simultaneously, using an online calculator.

WORKED EXAMPLE 13 Comparing two investments

Hugh now has two options to invest his $7500 for 7 years:

- **Option A: At 12% compounding monthly**
- **Option B: At 11.8% compounding daily**

Evaluate the options available to Hugh and justify your answer.

THINK	**WRITE**
1. Consider Option A. Since Hugh has given the money to the bank, we input this as a negative value. Input the following values into the online calculator: Present Value: −7500 Payment: leave blank Rate: 12 Periods: 84 (7 years × 12 months each year) Compounding: Monthly Click on the FV button to determine the future value.	 **Mode** ● **End** ○ **Beginning** **Present Value** –7,500 **Payment** **Future Value** 17,300.42 **Annual Rate (%)** 12 **Periods** 84 **Compounding** Monthly
2. Write the answer.	The future value of the investment with 12% monthly compounding is $17 300.42.
3. Consider Option B. Input the following values into the online calculator: Present Value: −7500 Payment: leave blank Rate: 11.8 Periods: 2555 (7 years × 365 each year) Compounding: Daily Click on the FV button to determine the future value.	 **Mode** ● **End** ○ **Beginning** **Present Value** –7,500 **Payment** **Future Value** 17,128.94 **Annual Rate (%)** 11.8 **Periods** 2555 **Compounding** Daily
4. Write the answer.	The future value of the investment with 11.8% daily compounding is $17 128.94. Hugh would be better off choosing Option A since his investment would be $171.48 greater after 7 years of investment.

- When both the interest rate and compounding frequency change, the effect on future value may not be clear without calculations, but generally, a lower rate yields less interest.

12.4.3 Using a spreadsheet to determine future value [complex]

- Like online calculators, spreadsheets are a useful tool to perform repetitive calculations.

WORKED EXAMPLE 14 Using a spreadsheet to determine future value

A bank offers interest of 7.5% per annum compounded yearly on investments. $5000 is invested at this bank for 10 years. Using a spreadsheet, determine the value of the investment after the 10 years.

THINK

WRITE

1. Open a spreadsheet labelling column A as ‘year, *n*’ and column B as ‘value, $’. Place these labels in row 1 as shown. In column A (starting in cell A2), input the numbers from 0 to 10. We have placed all the values of *n* (from year 0 to year 10) in column A. Column B will display the value of the investment for its corresponding year number.

◢	A	B
1	year, *n*	value, $
2	0	
3	1	
4	2	
5	3	
	⋮	
12	10	

2. Write down the formula for compound interest.

$A = P(1 + i)^n$

3. Calculate $1 + i$.

$$\begin{aligned} 1 + i &= 1 + \frac{7.5}{100} \\ &= 1 + 0.075 \\ &= 1.075 \end{aligned}$$

4. Substitute this value and the value of *P* into the compound interest formula. Input the compound interest formula into the spreadsheet.

$A = 5000(1.075)^n$

5. Use the cursor to click into cell B2. Type the following: =5000*(1.075)^A2 followed by Enter. This should automatically input the value of 5000 in cell B2.

◢	A	B
1	year, *n*	value, $
2	0	5000
3	1	

6. Copy the calculation for the next 10 years by using the fill-down function (or drag the bottom right corner down the cells). Complete column B to show 10 years.

	A	B
1	year, *n*	value, $
2	0	5 000
3	1	5 375
4	2	5 778.125
5	3	6 211.484
6	4	6 677.346
7	5	7 178.147
8	6	7 716.508
9	7	8 295.246
10	8	8 917.389
11	9	9 586.193
12	10	10 305.16

7. Answer the question.

After 10 years, the value of the investment is $10 305.16.

Exercise 12.4 Using technology to determine the future value of compound investments or loans

learnon

12.4 Quick quiz on

12.4 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3	4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15	16, 17, 18

Simple familiar

1. **WE10** Use an online calculator to determine the future value and interest earned in these annually compounding investments.

- **a.** Amina invests $1200 at an annual interest rate of 4.5% for 3 years. During this period, she makes additional payments of $100 at the end of each year.
- **b.** Li Wei deposits $3000 into a savings account with an annual interest rate of 6% for 4 years. She does not make any additional payments.
- **c.** Sarah invests $5500 at an annual interest rate of 5% for 6 years, with no additional payments.
- **d.** Harper saves $2000 in a fund with an annual interest rate of 3.5% for 8 years. They also deposit an additional $150 at the end of each year.
- **e.** Khalid invests $4500 at an annual interest rate of 7% for 5 years. He adds an additional $200 at the end of each year.

2. Use an online calculator to determine the future value and interest earned in these annually compounding investments.
   a. Zoe invests $1600 at an interest rate of 4.2% p.a. for 5 years. She makes additional payments of $120 at the end of each year.
   b. Marcus deposits $5200 into a savings account with an annual interest rate of 5.8% for 7 years. He does not make any additional payments.

3. Use an online calculator to determine the future value and interest earned in these annually compounding investments.
   a. Leo invests $3100 at an annual interest rate of 6.3% for 4 years. He makes additional payments of $180 at the end of each year.
   b. Isabella deposits $6700 into a savings account with an annual interest rate of 3.9% for 5 years. She does not make any additional payments.

Complex familiar

4. **WE11** Donna invested $3500 over a period of 4 years. Use an online calculator to determine how much more she would earn investing at 6.5% p.a. interest compounding monthly compared to 6% interest compounding monthly.

5. Mac misread his bank's advertisement of a 5.6% p.a. interest rate as 6.5% p.a. His investment was $8000 for a period of 10 years, compounded annually. Use an online calculator to determine the dollar difference between the future value he was expecting versus the future value he will accrue.

6. **WE12** Sam wants to invest his $12 400 for a period of 8 years. He believes it is better to invest his money with more compounding periods at 7.25% p.a. Use an online calculator to determine how much more Sam would earn on his investment if he had it compound fortnightly compared to annually.

7. Sasha initially invested $20 000 for a period of 5 years, at a rate of 3.5% p.a., compounding monthly. She then realised that her bank offers another option that compounds fortnightly. Determine how much more money Sasha could have earned had she chosen the fortnightly option.

8. **WE13** Tamara has two options to invest her $10 000 for 9 years:
   Option A: At 8% p.a. compounding biannually (twice per year)
   Option B: At 7.8% p.a. compounding daily
   Determine which option Tamara should take. Use an online calculator to justify your answer.

9. Katie Ann has $4800 to invest for a period of 5 years. Bank A is offering a rate of 2.15% p.a. compounded monthly, while Bank B is offering a rate of 2.65% p.a. compounded quarterly (every 3 months, four times a year). Use an online calculator to advise Katie Ann which bank she should invest with.

10. Kerry invests $100 000 at 8% p.a. for a 1-year term. For such large investments, interest is compounded daily.
    a. Calculate the daily percentage interest rate, correct to 4 decimal places.
    b. Use an online calculator to determine the compounded value of Kerry's investment on maturity.
    c. Calculate the amount of interest earned on this investment.
    d. Calculate the extra amount of interest earned, compared with the interest calculated at a simple interest rate.

11. **WE14** A bank offers 2.90% p.a. compounded yearly on investments. $25 000 is invested for 5 years. Using a spreadsheet, determine the value of the investment after the 5 years.

12. A bank offers 8% p.a. compounded yearly on investments. Assume $2000 is invested for 5 years. Using a spreadsheet, determine the value of the investment after the 5 years.

13. A bank offers compound interest on its savings account of 4% p.a. compounding yearly. A boy opens an account with $1125.

   a. Using a spreadsheet, calculate the value of the boy's investment for the first 4 years.

   b. Using your values from part a, construct a compound interest graph.

14. A sum of $1000 is invested at 10.5% p.a. interest. Using a spreadsheet, calculate the value of the investment for the following years, and types of interest.

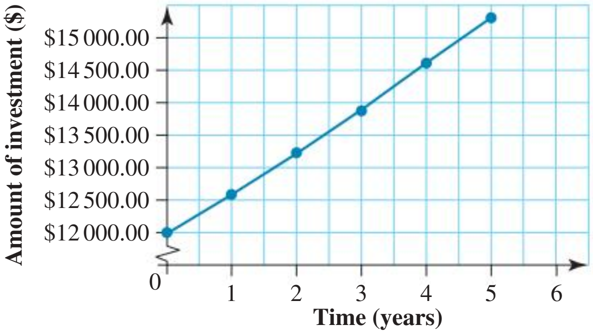
	Principal	Annual interest rate	Time (years)	Simple interest earned	Compound interest earned with yearly compounding
a.	$1000	10.5%	3		
b.	$1000	10.5%	6		
c.	$1000	10.5%	9		
d.	$1000	10.5%	12		

15. Using a spreadsheet, determine how long it would take to 'double your money' with compound interest at the following annual rates. State your answers in years. Explain whether there is a pattern to your answers.

   a. 5% b. 7% c. 10% d. 13%

Complex unfamiliar

16. A sum is invested into a bank account that is compounded annually. The graph shows the value of the investment over 5 years.
Calculate the compound interest rate for this investment.



17. Wilson has been asked to invest $15 000 in a friend's new business. His friend guarantees a return of 20% on his investment after 2 years. To invest, Wilson will have to take out a bank loan over 2 years at an annual interest rate of 4.25% compounding monthly. Use an online calculator or spreadsheet to calculate the profit (or loss) that Wilson will make on this business venture.

18. Rose's grandparents have an investment for her that grew to have a future value of $6500 over 4 years at an annual interest rate of 5%. They made no additional payments during this period. Use an online calculator or spreadsheet to calculate the initial deposit amount they invested for Rose.

Fully worked solutions for this chapter are available online.

LESSON
12.5 Comparing simple and compound interest [complex]

SYLLABUS LINKS

- Compare, numerically and graphically, the growth of simple interest and compound interest loans and investments [complex].
- Investigate the effect of the principal amount, the interest rate and the number of compounding periods on the future value of a loan or investment [complex].

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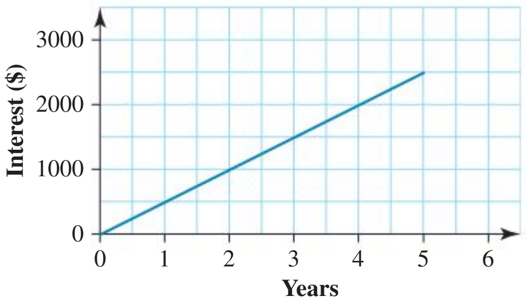
12.5.1 Graphing simple interest functions

- Suppose that we invest \$10 000 at 5% p.a. simple interest. The table below shows the amount of interest that we will receive after various lengths of time.

No. of years	1	2	3	4	5
Interest	\$500	\$1000	\$1500	\$2000	\$2500

- The amount of interest earned can be graphed by the **linear function** shown.



- The gradient of this graph is 500, which is the amount of one year's interest, or 5% of the principal.
- This means that for every 1 unit increase on the x-axis (1 year in time) the y-axis (interest) increases by 500 units (\$500).

WORKED EXAMPLE 15 Graphing simple interest functions

Leilay invests \$6000 into a savings account earning simple interest at a rate of 4% p.a.

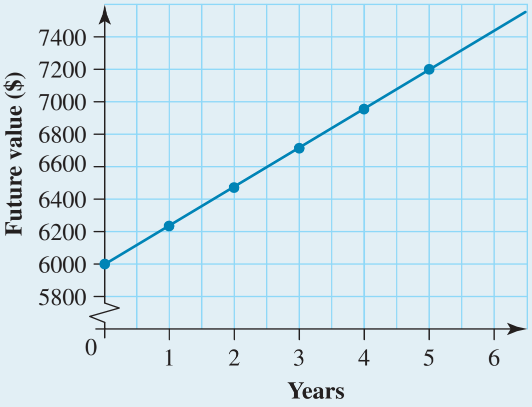
a. Complete the table below to calculate the future value of the investment at the end of each year.

No. of years	**1**	**2**	**3**	**4**	**5**
Future value					

b. Construct a graph of the future value against the number of years the money is invested.

THINK	WRITE
a. **1.** Calculate the interest earned after 1 year.	**a.** $I = Pin$ $I = 6000 \times 0.04 \times 1$ $= 240$
2. Since simple interest earns the same amount of interest each year (\$240 in this case) we can continually add this value to the previous year's amount of original principal to determine the future value after each year.	Original principal = \$6000 Future value after 1 year = \$6000 + \$240 = \$6240 Future value after 2 years = \$6240 + \$240 = \$6480 Future value after 3 years = \$6480 + \$240 = \$6720 …
3. Complete the table.	see table below
b. Construct the graph with Years on the horizontal axis and Future value (\$) on the vertical axis.	**b.** see graph below

No. of years	1	2	3	4	5
Future value	\$6240	\$6480	\$6720	\$6960	\$7200



- We are able to compare the interest that is earned by an investment at varying interest rates by graphing the interest earned at varying rates on the one axis.

WORKED EXAMPLE 16 Comparing simple interest functions graphically

Aston has \$12 000 to invest. Three different banks offer simple interest rates of 4%, 5% and 6%.

a. Complete the table below to show the interest that he could earn over 5 years.

No. of years	**1**	**2**	**3**	**4**	**5**
Interest (4%)					
Interest (5%)					
Interest (6%)					

b. Construct a graph to compare each of the three different bank offers.

THINK

a. 1. Use the simple interest formula to calculate the interest earned on $12 000 at 4% p.a. for 1, 2, 3, 4 and 5 years.

2. Use the simple interest formula to calculate the interest earned on $12 000 at 5% p.a. for 1, 2, 3, 4 and 5 years.

3. Use the simple interest formula to calculate the interest earned on $12 000 at 6% p.a. for 1, 2, 3, 4 and 5 years to complete the table.

WRITE

a. $I = Pin$

$I = 12\,000 \times 0.04 \times 1 = 480$

$I = 12\,000 \times 0.04 \times 2 = 960$

…

$I = Pin$

$I = 12\,000 \times 0.05 \times 1 = 600$

$I = 12\,000 \times 0.05 \times 2 = 1200$

…

$I = Pin$

$I = 12\,000 \times 0.06 \times 1 = 720$

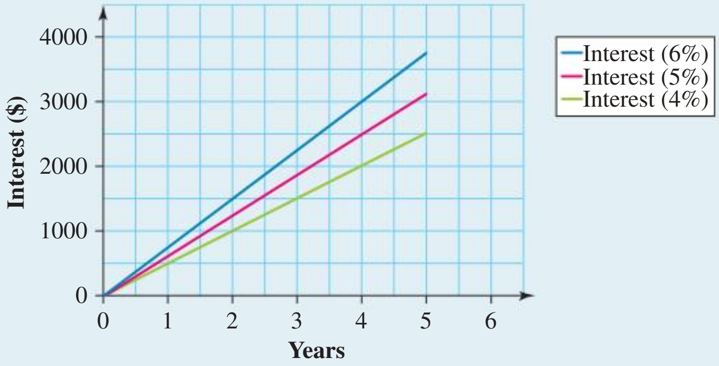
$I = 12\,000 \times 0.06 \times 2 = 1440$

…

No. of years	1	2	3	4	5
Interest (4%)	$480	$960	$1440	$1920	$2400
Interest (5%)	$600	$1200	$1800	$2400	$3000
Interest (6%)	$720	$1440	$2160	$2880	$3600

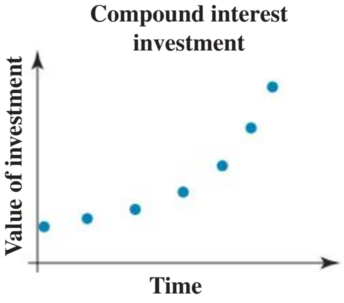
b. Draw a line graph for each investment.

b.



12.5.2 Graphing compound interest functions

- Simple interest graphs are linear since interest remains constant per period.
- Compound interest graphs are exponential, forming a smooth, progressively steeper curve as interest increases each period.
- The compound interest formula can be used to complete tables that helps us to graph a compound interest function.



WORKED EXAMPLE 17 Graphing compound interest functions

Olivia invests \$5000 at 5% p.a., with interest compounded annually.

a. Complete the table below to show the future value at the end of each year.

No. of years	**0**	**1**	**2**	**3**	**4**	**5**
Future value						

b. Construct a graph of the future value of the investment against the number of years.

THINK

a. 1. Use the compound interest formula to calculate the final amount at the end of every year for 5 years.

2. Complete the table rounding all values correct to 2 decimal places.

b. Draw a graph by drawing a smooth curve between the marked points. Note: Due to the scale used, the curve in the line isn't very prominent but it is not a straight line

WRITE

a. $A = P(1+i)^n$

$A = 5000(1+0.05)^1 = 5250$

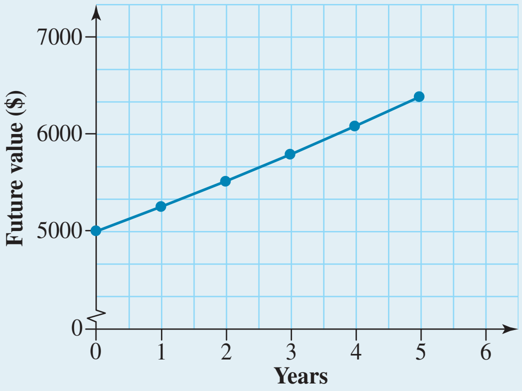
$A = 5000(1+0.05)^2 = 5512.50$

$A = 5000(1+0.05)^3 = 5788.125$

…

No. of years	0	1	2	3	4	5
Future value	\$5000	\$5250	\$5512.50	\$5788.13	\$6077.53	\$6381.41

b.



- To graph the interest earned, the principal must be subtracted from the future value of the investment. As with simple interest, such graphs can be used to compare investments.

WORKED EXAMPLE 18 Comparing compound interest functions graphically

Paul is to invest \$2000 at 5% p.a., 6% p.a. and 7% p.a., compounded annually.

a. Complete the table below to calculate the amount of interest of each investment at the end of each year.

No. of years	**1**	**2**	**3**	**4**	**5**
Interest (5%)					
Interest (6%)					
Interest (7%)					

b. Construct a graph to compare the interest earned.

THINK

WRITE

a. 1. Use the compound interest formula $A = P(1 + i)^n$ to determine the value of each investment over a 5-year period. For example, to determine the value of the 5% p.a. investment after 1 year, evaluate $A = 2000(1 + 0.05)^1$.

a.

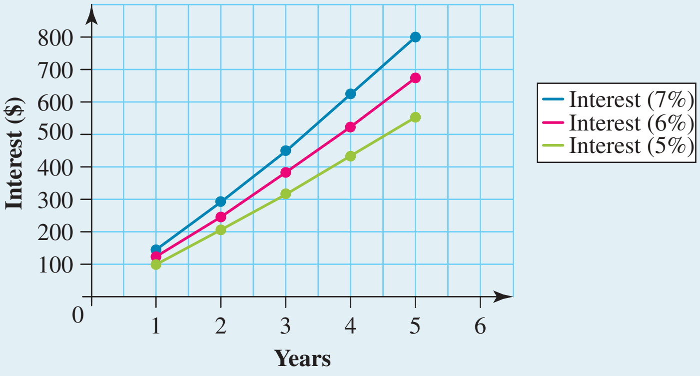
No. of years	1	2	3	4	5
Interest (5%)	\$2100	\$2205	\$2315.25	\$2431.01	\$2552.56
Interest (6%)	\$2120	\$2247.20	\$2382.03	\$2524.95	\$2676.45
Interest (7%)	\$2140	\$2289.80	\$2450.09	\$2621.59	\$2805.10

2. To calculate the amount of interest earned on for the investments at the end of each year, subtract the principal (\$2000) from all values. We do this since the formula for calculating the amount of interest is $I = A - P$.

No. of years	1	2	3	4	5
Interest (5%)	\$100	\$205	\$315.25	\$431.01	\$552.56
Interest (6%)	\$120	\$247.20	\$382.03	\$524.95	\$676.45
Interest (7%)	\$140	\$289.80	\$450.09	\$621.59	\$805.10

b. Construct each graph by joining the points with a smooth curve.

b.



12.5.3 Comparing simple and compound interest

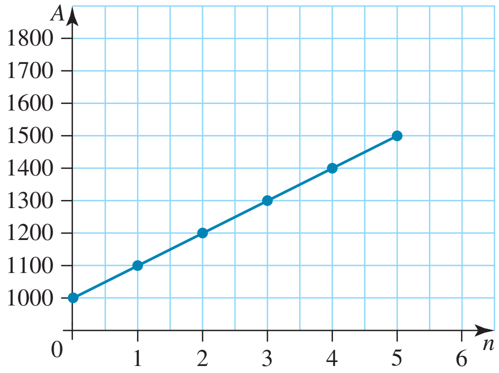
- Earlier in this chapter we have looked at simple interest (interest calculated on the principal) and compound interest (interest calculated on the principal and the interest).

- An example of the differences between simple and compound interest is shown.

Simple interest	Compound interest
Initial principal, $P = \$1000$	Initial principal, $P = \$1000$
Rate of interest, $i = 10\%$	Rate of interest, $i = 10\%$
Interest for Year 1 10% of \$1000, $I_1 = \$100$	Interest for Year 1 10% of \$1000, $I_1 = \$100$
Principal at the beginning of Year 2 $P_2 = \$1000$	Principal at the beginning of Year 2 $P_2 = \$1000 + \$100$ $= \$1100$
Interest for Year 2 10% of \$1000, $I_2 = \$100$	Interest for Year 2 10% of \$1100, $I_2 = \$110$
Principal at the beginning of Year 3 $P_3 = \$1000$	Principal at the beginning of Year 3 $P_3 = \$1100 + \$110$ $= \$1210$
Interest for Year 3 10% of \$1000, $I_3 = \$100$	Interest for Year 3 10% of \$1210, $I_3 = \$121$
Principal at the beginning of Year 4 $P_4 = \$1000$	Principal at the beginning of Year 4 $P_4 = \$1210 + \$121$ $= \$1331$
Interest for Year 4 10% of \$1000, $I_4 = \$100$	Interest for Year 4 10% of \$1331, $I_4 = \$133.10$
Principal at the beginning of Year 5 $P_5 = \$1000$	Principal at the beginning of Year 5 $P_5 = \$1331 + \$133.10$ $= \$1464.10$
Interest for Year 5 10% of \$1000, $I_5 = \$100$	Interest for Year 5 10% of \$1464.10, $I_5 = \$146.41$
The simple interest earned over a 5-year period is \$500.	The compound interest earned over a 5-year period is \$610.51.

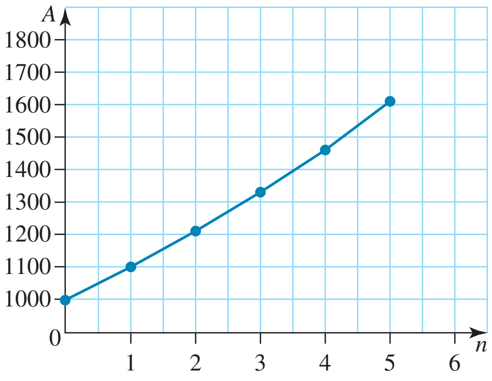
- With compound interest, both principal and interest increase each period.
- With simple interest, principal and interest remain constant.
- Over 5 years, compound interest earns \$110.51 more due to interest on interest.
- Graphing both shows simple interest grows linearly, while compound interest grows exponentially.

1. The simple interest investment is represented by a straight line.



Simple interest	
n	**A**
0	1000
1	1100
2	1200
3	1300
4	1400
5	1500

2. The compound interest investment is represented by a curve.



Compound interest	
n	***A***
0	1000
1	1100
2	1210
3	1331
4	1464.10
5	1610.51

tlvd-12151

WORKED EXAMPLE 19 Comparing simple interest and compound interest

Callum won $20 000 on a gameshow and is looking to invest the money over the next 10 years. Callum discovered two options for his investment:

- **Simple interest at 4.25% p.a.**
- **Compound interest at 3.85% p.a. compounding yearly**

a. Determine the balance of both options at the end of each year for 10 years.

b. Construct a graph of both options on the same set of axes.

c. If Callum was to withdraw his money early, determine at what point one option is better than the other.

THINK	WRITE
a. 1. Use the simple interest formula to calculate the value of the investment each year. Remember to add the principal ($20 000) to the interest each year to get the total amount.	**a.** Year 1: $I = Pin$ $I = 20\,000 \times 0.0425 \times 1$ $I = 850$ $A = P + I$ $A = 20\,000 + 850$ $A = 20\,850$ ...
2. Repeat for each year and place into a table.	

Year	Simple interest amount
0	20 000
1	20 850
2	21 700
3	22 550
4	23 400
5	24 250
6	25 100
7	25 950
8	26 800
9	27 650
10	28 500

3. Use the compound interest formula $A = P(1+i)^n$ to determine the value of the investment each year.

Year 1:
$A = P(1+i)^n$
$A = 20\,000(1+0.0385)^1$
$A = 20\,000(1.0385)$
$A = 20\,770$
…

4. Repeat for each year and place into the table.

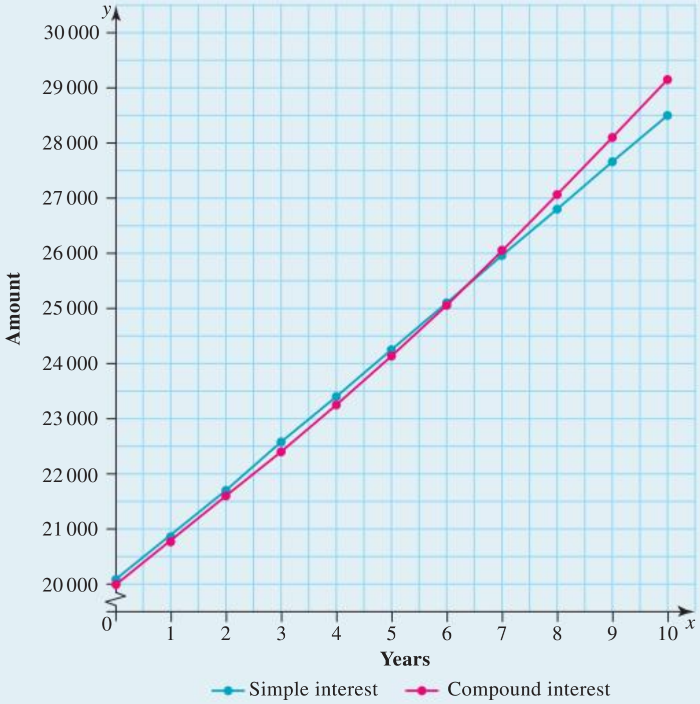
Year	Simple interest amount	Compound interest amount
0	20 000	20 000
1	20 850	20 770
2	21 700	21 570
3	22 550	22 400
4	23 400	23 262
5	24 250	24 158
6	25 100	25 088
7	25 950	26 054
8	26 800	27 057
9	27 650	28 099
10	28 500	29 181

5. Write your answer.

The balance at the end of each year (to the nearest dollar) is \$28 500 for simple interest and \$29 181 for compound interest.

b. Construct a graph for both simple interest and compound interest. Use years as the *x*-axis and amount for the *y*-axis.

b.



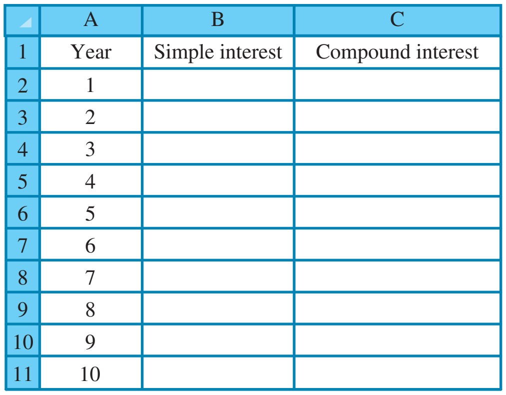
c. Determine the answer from either the table or the graph.

c. If Callum was to withdraw his money before the end of year 6, he is better off with the simple interest option. After year 6, it is better for Callum to take the compound interest option.

12.5.4 Comparing simple interest and compound interest loans and investments using technology

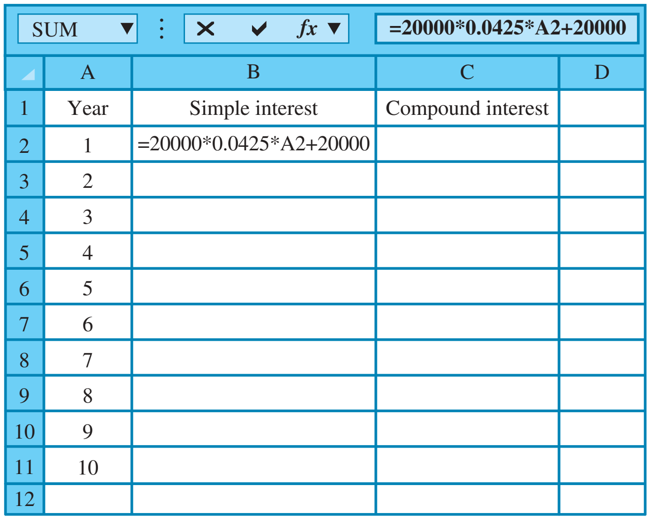
- Technology is an effective way to compare simple interest and compound interest.
- The calculations and graph from Worked example 19 could be done using technology such as a spreadsheet by following the steps below:

1. Create a table as shown:



	A	B	C
1	Year	Simple interest	Compound interest
2	1		
3	2		
4	3		
5	4		
6	5		
7	6		
8	7		
9	8		
10	9		
11	10		

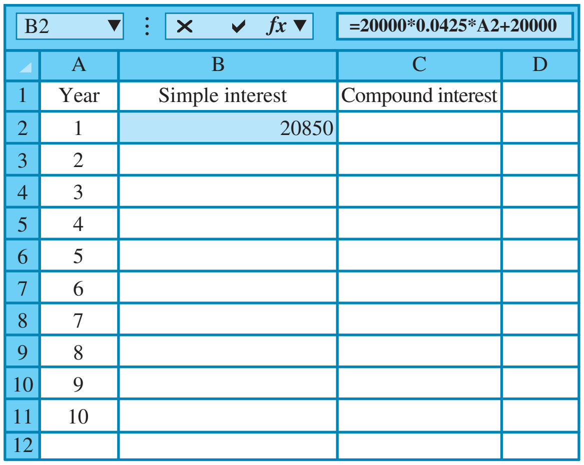
2. In cell B2, enter the simple interest formula, substituting the values required:
$= 20\,000 * 0.0425 * A2 + 20\,000$
Note that the cell A2 is used for the years and 20 000 is added to get the total amount (principal + interest).



=20000*0.0425*A2+20000

	A	B	C	D
1	Year	Simple interest	Compound interest	
2	1	=20000*0.0425*A2+20000		
3	2			
4	3			
5	4			
6	5			
7	6			
8	7			
9	8			
10	9			
11	10			
12				

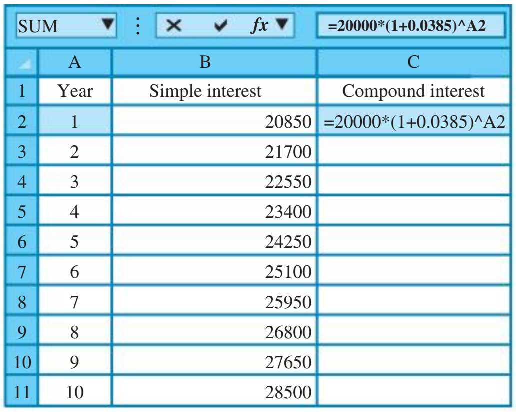
3. Select the cell B2 and double click the box in the bottom right corner to repeat the formula for each year.



=20000*0.0425*A2+20000

	A	B	C	D
1	Year	Simple interest	Compound interest	
2	1	20850		
3	2			
4	3			
5	4			
6	5			
7	6			
8	7			
9	8			
10	9			
11	10			
12				

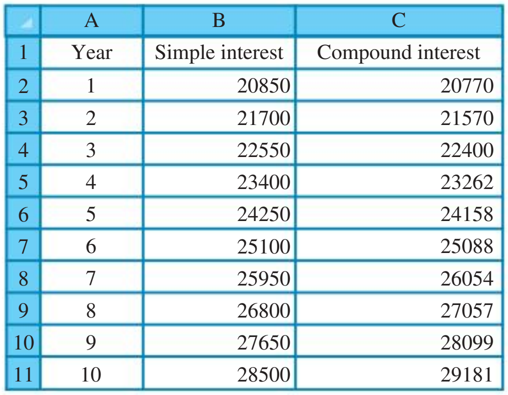
4. Repeat the same process in cell C2 using the compound interest formula, substituting the values required: $= 20000*(1 + 0.0385)\,^\wedge\,\mathrm{A2}$



SUM ▼ ⋮ ✕ ✓ *fx* ▼ =20000*(1+0.0385)^A2

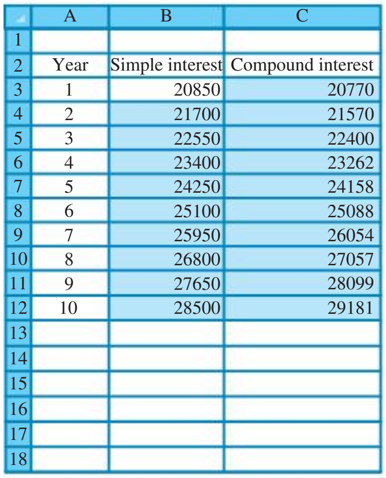
	A	B	C
1	Year	Simple interest	Compound interest
2	1	20850	=20000*(1+0.0385)^A2
3	2	21700	
4	3	22550	
5	4	23400	
6	5	24250	
7	6	25100	
8	7	25950	
9	8	26800	
10	9	27650	
11	10	28500	

5. Select the cell C2 and double click the box in the bottom right corner to repeat the formula for each year.

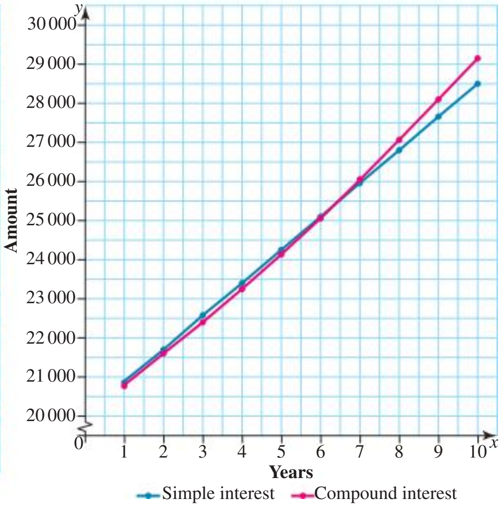


	A	B	C
1	Year	Simple interest	Compound interest
2	1	20850	20770
3	2	21700	21570
4	3	22550	22400
5	4	23400	23262
6	5	24250	24158
7	6	25100	25088
8	7	25950	26054
9	8	26800	27057
10	9	27650	28099
11	10	28500	29181

6. Select the data and insert a line graph to graph both sets of data on the same set of axes.



	A	B	C
1			
2	Year	Simple interest	Compound interest
3	1	20850	20770
4	2	21700	21570
5	3	22550	22400
6	4	23400	23262
7	5	24250	24158
8	6	25100	25088
9	7	25950	26054
10	8	26800	27057
11	9	27650	28099
12	10	28500	29181
13			
14			
15			
16			
17			
18			



7. By selecting the graph, you are able to modify the scale, labels, and so on, to suit as shown.

Exercise 12.5 Comparing simple and compound interest [complex]

learnon

12.5 Quick quiz on

12.5 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
N/A	1, 2, 3, 4, 5, 6, 7, 8, 9, 10	11, 12

Complex familiar

1. **WE15** An amount of $8000 is invested into a savings account earning simple interest at a rate of 5% p.a.

   a. Copy and complete the table below to calculate the future value of the investment at the end of each year.

No. of years	1	2	3	4	5
Future value					

   b. Draw a graph of the future value against the number of years the money is invested.

2. An investment of $2000 at 6% p.a. simple interest is calculated over 3 years.

   a. Complete the table below.

Years (n)	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**
Simple interest (I)						
Future value (A)						

   b. Use the table to construct a graph of the future value for the 3 years.

3. **WE16** Adam has $2500 to invest. Three different banks offer simple interest rates of 1.60%, 1.80% and 2% per annum.

   a. Complete the table below to show the interest that Adam could earn over 5 years.

No. of years	1	2	3	4	5
Interest (1.6%)					
Interest (1.80%)					
Interest (2.%)					

   b. Show this information in graph form.

4. **WE17** Monique invests $12 000 at 8% p.a., with interest compounded annually.

   a. Copy and complete the table below to show the future value at the end of each year (to the nearest dollar).

No. of years	1	2	3	4	5
Future value					

   b. Draw a graph of the future value of the investment against the number of years.

5. Draw a graph to represent the future value of the following investments over the first 5 years (to the nearest dollar).
   a. \$15 000 at 7% p.a. with interest compounded annually
   b. \$2000 at 10% p.a. with interest compounded annually

6. Nunez has an investment of \$2000 at 6% p.a. with interest compounding six-monthly.
   a. Complete the table below.

Years	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Future value										

   b. Use the table to construct a graph of Nunez' investment.

7. **WE18** James has \$8000 to invest at either 4% p.a., 6% p.a. or 8% p.a. compounding annually.
   a. Complete the table below to calculate the amount of interest of each investment at the end of each year.

No. of years	1	2	3	4	5
Interest (4%)					
Interest (6%)					
Interest (8%)					

   b. Draw a graph that will allow the amount of interest earned to be compared.

8. Petra has \$4000 to invest at 6% p.a.
   a. Use an online calculator to complete the table below to show the future value of the investment at the end of each year, if interest is compounded annually or six-monthly.

No. of years	1	2	3	4	5
Annually					
Six-monthly					

   b. Show this information in graphical form.

9. **WE19** Helen inherited \$50 000 and is looking to invest the money over the next 5 years to save for a house. Helen discovered two options for her investment:
   - Simple interest at 3.75% p.a.
   - Compound interest at 3.45% p.a. compounding yearly

   a. Determine the balance of both options at the end of each year for 5 years.
   b. Construct a graph of both options on the same set of axes.
   c. If Helen was to withdraw her money early, determine which option she should choose.

10. Simon invests \$4000 for 3 years at 6% p.a. simple interest. Monica also invests \$4000 for 3 years but her interest rate is 5.8% p.a., with interest compounded annually.

a. Calculate the value of Simon's investment on maturity.
b. Show that the compounded value of Monica's investment is greater than Simon's investment.
c. Explain why Monica's investment is worth more than Simon's, despite receiving a lower rate of interest.

Complex unfamiliar

11. There are two options available for a \$4500 investment.
Option 1: simple interest at 6.25% per year
Option 2: compound interest at 6% per annum, compounded annually
Analyse when Option 1 or Option 2 would be the better choice and justify your answer.

12. Zara invests \$10 000 in a savings account that offers an annual interest rate of 5% compounded annually. After 5 years, she makes a one-off withdrawal of \$4000 from the account. Construct a table showing the growth of the investment from the time of withdrawal, and determine when it reaches the value that it was at when the withdrawal was made.

Fully worked solutions for this chapter are available online.

LESSON
12.6 Review

12.6.1 Summary

Hey students! Now that it's time to revise this chapter, go online to:

- Access the chapter summary
- Review your results
- Watch teacher-led videos
- Practise questions with immediate feedback

Find all this and MORE in jacPLUS

12.6 Exercise

learnon

12.6 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15	16, 17, 18	19, 20

These questions are even better in jacPLUS!

- Receive immediate feedback
- Access sample responses
- Track results and progress

Find all this and MORE in jacPLUS

Simple familiar

1. Calculate the simple interest earned on an investment of \$5000 at 4% p.a. for 5 years.

2. Calculate the simple interest earned on each of the following investments.
   a. \$3600 at 9% p.a. for 4 years
   b. \$23 500 at 6% p.a. for 2 years
   c. \$840 at 2.5% p.a. for 2 years
   d. \$1350 at 0.2% p.a. for 18 months
   e. \$45 820 at 4.75% p.a. for $3\frac{1}{2}$ years

3. Dion invests \$32 500 paying 5.6% p.a. simple interest for 4 years.
   a. Calculate the interest earned by Dion.
   b. Calculate the total value of Dion's investment after 4 years.
   c. If the investment paid Dion in quarterly instalments, calculate the value of each interest payment.

4. Two banks pay simple interest on short-term deposits. Bank A pays 6% p.a. over 4 years and Bank B pays 6.5% p.a. for $3\frac{1}{2}$ years. Determine the difference between the two banks' final payout figures if \$5000 was invested in each account.

5. Keira invested \$360 in a bank for 3 years at 8% simple interest each year. Calculate the total amount she will receive at the end of the 3 years.

6. Philip borrowed \$7000 and intended to pay it back in 4 years. The terms of the loan indicated Philip was to pay 9% p.a. simple interest. Determine the interest Philip paid on the loan.

7. Michelle invests \$15 000 for a period of 4 years. Calculate the interest rate, given that Michelle earned a total of \$3900 in simple interest.

8. Bradley invests \$23 500 at 4.6% p.a. If he earned \$1351.25 in simple interest, calculate the length of time for which the money was invested.

9. A loan of \$5000 is taken over 5 years. The simple interest is calculated monthly. The interest bill on this loan is \$1125. Determine the simple interest rate per year on this loan.

10. Calculate the principal invested in an investment bond that will accumulate \$2015 in simple interest after 6 months at an interest rate of 6.5% p.a.

11. Calculate the simple interest rate per year on a loan of \$10 000 taken over 10 years with a total interest of \$2000.

12. An amount of \$7500 is to be invested at 6% p.a. simple interest.
    a. Copy and complete the table below to calculate the interest over 5 years.

No. of years	1	2	3	4	5
Interest					

    b. Construct a graph of the interest earned against the length of the investment.
    c. Determine and interpret the gradient of the linear graph drawn.
    d. Use your graph to determine the amount of interest that would have been earned after 10 years.

13. Vicky invests \$2400 at 5% p.a. for 3 years with interest compounded annually. Use an online calculator to determine the compounded value of the investment at the end of the term.

14. Kim invests \$7500 for a period of 5 years at 9.9% p.a. with interest compounded annually. Calculate the amount of interest Kim has earned after 5 years.

15. Dermott invests \$11 500 at 3.2% p.a. with interest compounded annually. Justify whether the total amount of interest earned on this investment will exceed \$10 000 after 20 years.

Complex familiar

16. \$20 000 is to be invested at 4% p.a. with interest compounded annually.
    a. Complete the table below to calculate the future value at the end of each year.

No. of years	1	2	3	4	5
Future value					

    b. Construct a graph of the interest earned against the length of the investment.
    c. Use your graph to estimate the future value of the investment after 10 years.

17. Yara is deciding between two investment options. Both options require an initial investment of \$2000 for a period of 5 years. Option 1 offers simple interest at an annual rate of 4.2% and Option 2 offers compound interest at an annual rate of 4.0%, compounded annually. Determine which investment will be more profitable and by how much.

18. In an Excel spreadsheet, cell **A2** contains the future value of an investment of \$3150. This investment grew over 6 years at an annual interest rate of 3.8%, compounded annually. The principal (initial investment) amount is not provided and should be entered in cell **A1**. Use a spreadsheet or otherwise to determine the initial investment.

Complex unfamiliar

19. Arjun and Leila both invested in accounts with different types of interest. After 4 years, both investments had a value of \$4320. Arjun's investment was in a simple interest account with an annual rate of 4.5%. Leila's investment was in a compound interest account with an annual rate of 4%, compounded annually. Determine who invested more initially, and by how much more.

20. Jake and Will both invest \$10 000. Jake has a simple interest account with an annual rate of 2.5%. Will's investment is a compound interest account, also with an annual rate of 2.5%, compounded annually. Both planned to invest for 10 years. At the halfway mark, Jake realised he will have a lesser future value than Will. Determine how much extra Jake should invest at the halfway mark to ensure he ends up with the same future value as Will.

Fully worked solutions for this chapter are available online.

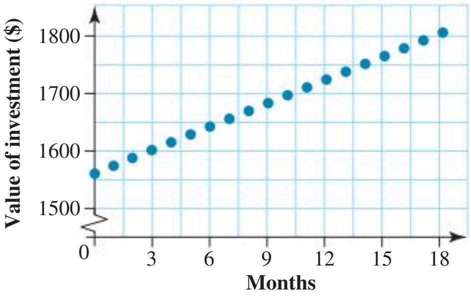
Answers

Chapter 12 Compound interest

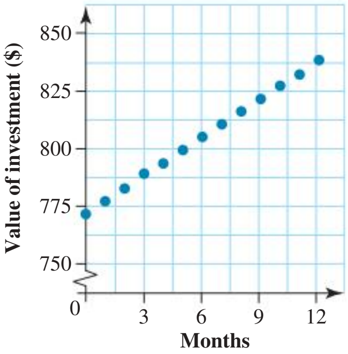
12.2 Simple interest

12.2 Exercise

1. **a.** $I = \$136.00, A = \$816$
   b. $I = \$56.70, A = \$266.70$
   c. $I = \$145.25, A = \$560.25$
   d. $I = \$110.40, A = \$570.40$
   e. $I = \$255, A = \$1275$
   f. $I = \$336.89, A = \$1049.89$
2. **a.** $I = \$178.57, A = \$999.57$
   b. $I = \$43.88, A = \$108.88$
   c. $I = \$11.76, A = \$33.01$
   d. $I = \$229.68, A = \$853.14$
   e. $I = \$544.05, A = \$1335.40$
3. **a.** \$627.13 **b.** \$12 542.50
4. **a.** \$3070 **b.** \$4400
   c. \$5425 **d.** \$236.36
   e. \$2500
5. **a.** 10% p.a. **b.** 6.25% p.a.
   c. 80% p.a. **d.** 2.125% p.a. or $2\frac{1}{8}$% p.a.
   e. 3.36% p.a.
6. **a.** 1 year **b.** 18 months
   c. 3 months **d.** 7 years
   e. 1 month
7. **a.** $A_{n+1} = A_n + 43,\ A_0 = 688$
   b. \$860
8. **a.** $A_{n+1} = A_n + 61.88,\ A_0 = 728$
   b. \$913.64
9. **a.** \$1803.36
   b.



10. **a.** \$141.68
    b.



11. **a.** \$997.60 **b.** \$526.50
12. **a.** \$211.75 charged **b.** \$1104 earned
    c. \$1228.50 charged **d.** \$4214.70 earned
13. **a.** \$620 **b.** \$892.16
    c. \$25 695
14. **a.** \$103.50 **b.** \$2700
    c. \$325 **d.** 131.25
15. **a.** \$360 **b.** \$1020
    c. \$27 700 **d.** \$17.70
    e. \$13.67
16. **a.** The Big-4 Bank offers the better rate.
    b. The Big-4 Bank charges $11\frac{1}{3}$% p.a. for a loan while.
    The Friendly Building Society charges 12%
    (12 × 1% per month).
17. \$1515.79
18. \$2133.33
19. \$352
20. 24 months
21. \$4022.22
22. \$1300
23. 4.83%
24. 26.66%
25. 1.84 years
26. Tigers Bank
    The interest on the Tigers Bank loan is \$9800 compared to Twins Bank at \$10 800, so Tigers Bank is the better option since it requires payment of \$1000 less interest at the end of the loan.

12.3 Using formulas to calculate the future value

12.3 Exercise

1. **a.** $A = 2500(1.075)^5$
   b. $A = \$3589.07,\ I = \$1089.07$
2. $A_1 = \$5180.00$ $A_2 = \$5366.48$ $A_3 = \$5559.67$
   $A_4 = \$5759.81$ $A_5 = \$5967.18$ $A_6 = \$6181.99$
   $A_7 = \$6404.55$ $A_8 = \$6635.11$ $A_9 = \$6873.97$
   $A_{10} = \$7121.44$
3. \$583.20
4. \$1630.47
5. \$4472.27
6. \$3764.86
7. **a.** \$3086.60 **b.** \$1086.60
8. $A = \$9657.36$
   $I = \$2907.36$
9. \$157.53
10. \$845.56
11. \$3159.32
12. \$1802.44
13. \$1502.19
14. Paul has accrued \$1261.58 in interest while Uyen has accrued \$943.34.

15. $32 000

16. Yes, Siobhan will have earned $2075.25 after 10 years.

12.4 Using technology to determine the future value of compound investments or loans

12.4 Exercise

1. **a.** $1683.22 and $183.22
   b. $3787.50 and $787.50
   c. $7370.55 and $1870.55
   d. $3986.91 and $786.91
   e. $7462.26 and $1962.26

2. **a.** $2618 **b.** $7716.19

3. **a.** $4749.12 **b.** $8112.46

4. Donna would earn $89.36 extra interest.

5. 6.5% FV = $15 017.10
5.6% FV = $13 795.24
The difference is $1221.86.

6. Sam would earn $422 extra on his investment.

7. Monthly FV = $23 818.86
Fortnightly FV = $23 822.12
Fortnightly Sasha would have earned $3.26 more.

8. Option A, since the final value of that investment is higher after 9 years

9. Bank A FV = $5344.24
Bank B FV = $5477.67
Bank B is better by $133.43

10. **a.** 0.0219% **b.** $108 320.72
c. $8320.72 **d.** $320.72

11.

	A	B
1	*n*, year	Value, $
2	0	25 000
3	1	25 725
4	2	26 471.03
5	3	27 238.68
6	4	28 028.61
7	5	28 841.44

After 5 years the investment is worth $28 841.44.

12.

	A	B	C
1	*t*	Value	Formulas
2	0	$2000.00	
3	1	$2160.00	= (B2 ∗ 0.08) + B2
4	2	$2332.80	
5	3	$2519.42	
6	4	$2720.98	
7	5	$2938.66	

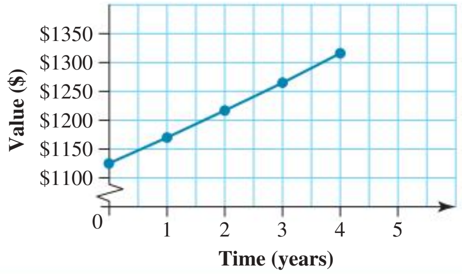
After 5 years the investment is worth $2938.66.

13. **a.**

	A	B	C
1	*t*	Value	Formulas
2	0	$1125.00	
3	1	$1170.00	= (B2 ∗ 0.04) + B2
4	2	$1216.80	
5	3	$1265.47	
6	4	$1316.09	

After 4 years the investment is worth $1316.09.

b.



14.

Principal	Annual interest rate	Time (years)	Simple interest	Compound interest
$1000	10.5%	3	$ 315.00	$ 349.23
$1000	10.5%	6	$ 630.00	$ 820.43
$1000	10.5%	9	$ 945.00	$1456.18
$1000	10.5%	12	$1260.00	$2313.96

15. **a.** At 5% compound interest, the investment is doubled in 15 years.
   b. At 7% compound interest, the investment is doubled in 11 years.
   c. At 10% compound interest, the investment is doubled in 8 years.
   d. At 13% compound interest, the investment is doubled in 6 years.

As the rate of interest increases, the length of time taken to double the investment decreases.

16. 5% p.a.

17. Wilson will make $1671.70 by investing in his friend's business.

18. $5347.57

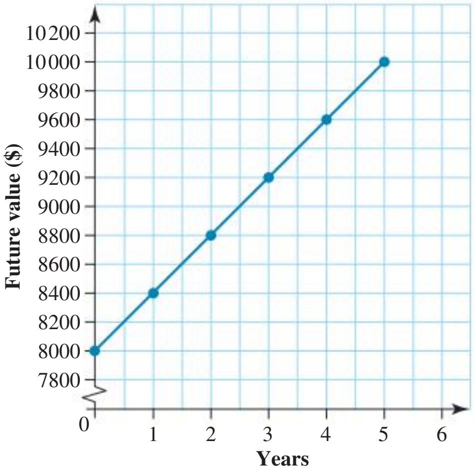
12.5 Comparing simple and compound interest [complex]

12.5 Exercise

1. a.

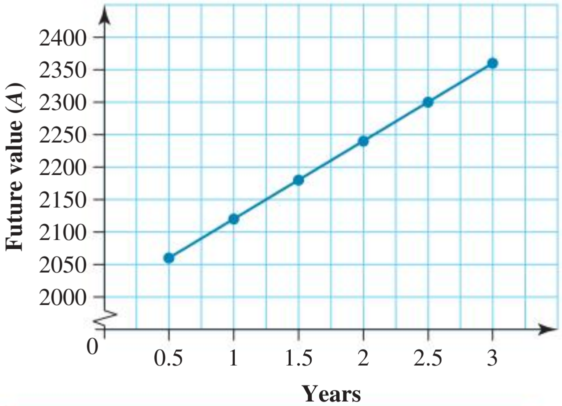
No. of years	1	2	3	4	5
Future value	$8 400	$8 800	$9 200	$9 600	$10 000

b.



2. a. See the table at the bottom of this page.*

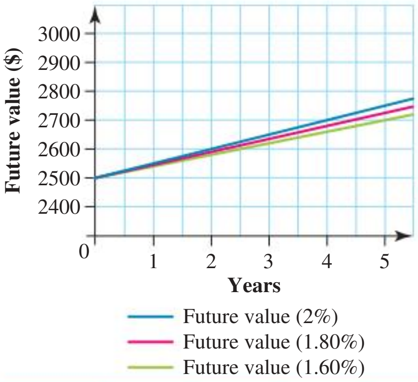
b.



3. a.

No. of years	1	2	3	4	5
Future value (1.6% p.a.)	$2540	$2580	$2620	$2660	$2700
Future value (1.8% p.a.)	$2545	$2590	$2635	$2680	$2725
Future value (2.0% p.a.)	$2550	$2600	$2650	$2700	$2750

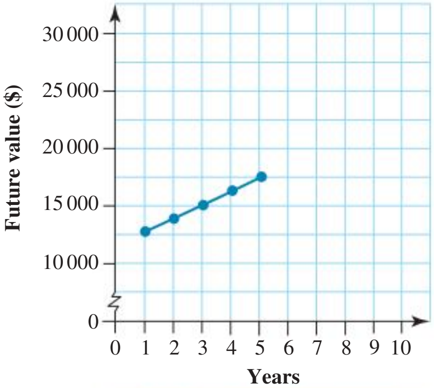
b.



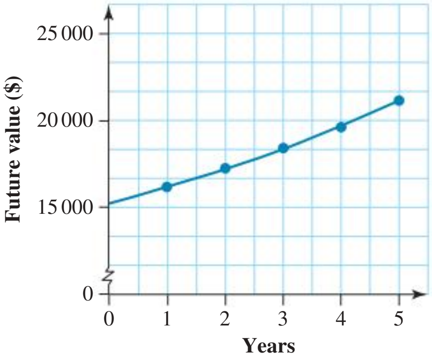
4. a.

No. of years	1	2	3	4	5
Future value	$12 960	$13 997	$15 117	$16 326	$17 632

b.



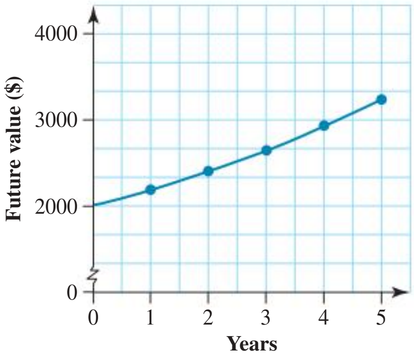
5. a.



***2a.**

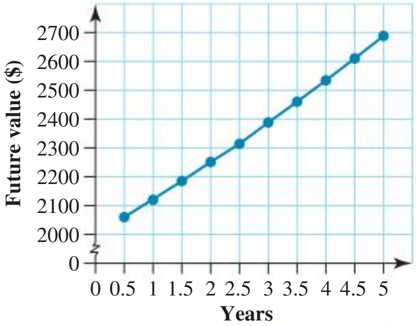
Years (*n*)	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**
Simple interest (*I*)	60	120	180	240	300	360
Future value (*A*)	2060	2120	2180	2240	2300	2360

b.



6. a. See the table at the bottom of this page.*

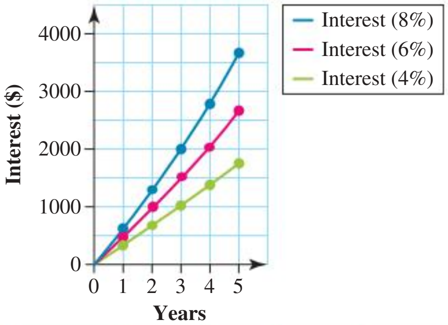
b.



7. a.

No. of years	1	2	3	4	5
Interest (4%)	$320	$653	$999	$1359	$1733
Interest (6%)	$480	$989	$1528	$2100	$2706
Interest (8%)	$640	$1331	$2078	$2884	$3755

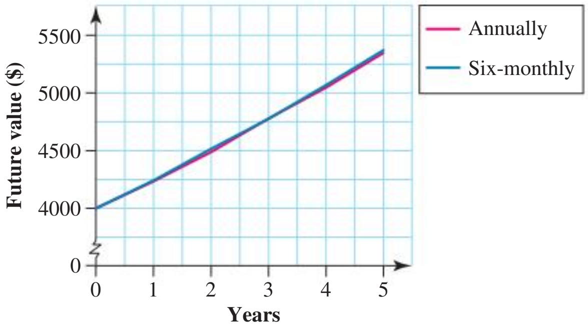
b.



8. a.

No. of years	1	2	3	4	5
Annually	$4240	$4494	$4764	$5050	$5353
Six-monthly	$4244	$4502	$4776	$5067	$5376

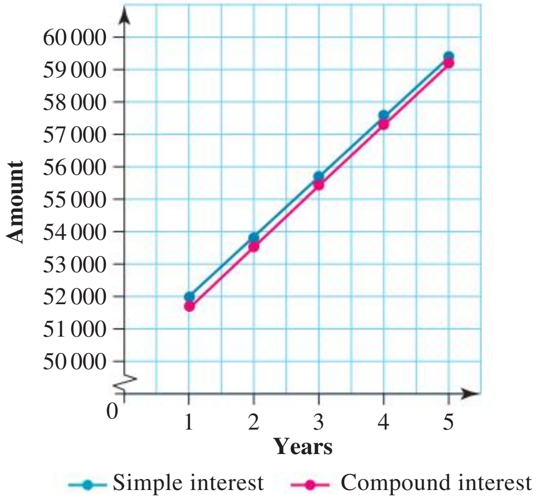
b.



9. a.

Year	Simple interest	Compound interest
0	50 000	500 000
1	51 875	51 725
2	53 750	53 510
3	55 625	55 356
4	57 500	57 265
5	59 375	59 241

b.



c. Simple interest

10. a. $4720

b. $4737.15 for Monica is greater than $4720 for Simon.

c. Compounding interest

11. After 3 years, Option 2 will become the better option with a value of $5359.57, which is more than Option 1 at $5343.75.

12. See the table at the bottom of this page.*
It will reach the amount at the end of year 13.

---

*6a.

Years	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
FV	$2060	$2122	$2185	$2251	$2319	$2388	$2460	$2534	$2610	$2688

*12.

Years (n)	5 (withdrawal)	6	7	8	9	10	11	12	13
Future value (A)	8 762.82	9 200.96	9 661.01	10 144.06	10 651.26	11 183.83	11 743.02	12 330.17	12 946.68

12.6 Review

12.6 Exercise

1. $1000

2. a. $1296 b. $2820 c. $42
d. $4.05 e. $7617.58

3. a. $7280 b. $39 780 c. $455

4. $62.50

5. $446.40

6. $2520

7. 6.5% p.a.

8. 15 months

9. 4.5%

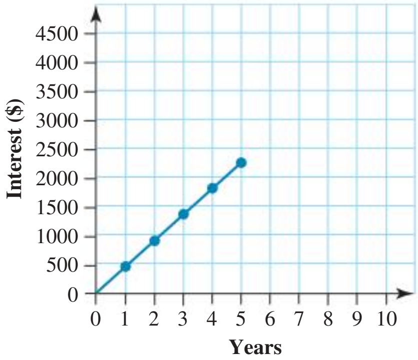
10. $62 000

11. 2%

12. a.

No. of years	1	2	3	4	5
Interest	$450	$900	$1350	$1800	$2250

b.



c. 450, which means $450 per year.

d. $4500

13. $2778.30

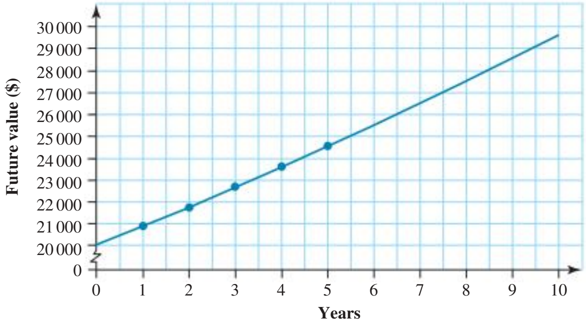
14. $4524.02

15. Yes, Dermott would have accrued $10 091.95 in interest in 20 years.

16. a.

No. of years	1	2	3	4	5
Future value	$20 800	$21 632	$22 497	$23 397	$24 333

b.



c. Approximately $29 600

17. Simple interest = $2420, compound interest = $2433.31; Compound interest is more profitable by $13.31

18. $2518.41

19. Arjun’s recurrence rate is 1.18, and he invested $3661.02. Leila invested $3692.75, which is $31.73 more.

20. Jake needs to add an extra $128.53 at the halfway point.

UNIT 4 Graphs, data and loans
TOPIC 3 Loans and compound interest

13 Reducing balance loans

LESSON SEQUENCE

Fully worked solutions for this chapter are available online.

on Resources

Solutions	Solutions — Chapter 13
Digital documents	Learning matrix — Chapter 13 Quick quizzes — Chapter 13 Chapter summary — Chapter 13

LESSON
13.1 Overview

13.1.1 Introduction

When we invest money with a financial institution, the institution pays us interest because it is using our money to lend to others. Conversely, when we borrow money from an institution, we are using the institution's money and so it charges us interest. In reducing balance loans, interest is usually charged every month by the financial institution and repayments are made by the borrower on a regular basis. The original amount of money borrowed is called the principal and the amount of money owed is called the balance of the loan.

With each payment, the principal amount decreases, which means that over time, the interest charged also reduces. This differs from simple interest loans, where interest is calculated on the original loan amount regardless of repayments made. Understanding reducing balance loans is important because they can lead to substantial interest savings over the loan term, especially with larger loans like car loans or mortgages. Knowing how these loans work allows borrowers to make informed decisions, plan repayments effectively and potentially reduce the total cost of borrowing.

13.1.2 Syllabus links

Lesson	Lesson title	Syllabus links
13.2	**Modelling reducing balance loans with technology**	○ Understand that reducing balance loans are compound interest loans with periodic repayments. ○ Use a calculator or an online calculator to model a reducing balance loan with annual repayments. ○ Use a spreadsheet to model a reducing balance loan with non-annual repayments [complex].
13.3	**The effect of repayment amounts, the interest rate and compounding periods on reducing balance loans [complex]**	○ Investigate the effect of the repayment amount, the interest rate and the number of compounding periods on the time taken to repay a loan [complex].

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LESSON
13.2 Modelling reducing balance loans with technology

SYLLABUS LINKS

- Understand that reducing balance loans are compound interest loans with periodic repayments.
- Use a calculator or an online calculator to model a reducing balance loan with annual repayments.
- Use a spreadsheet to model a reducing balance loan with non-annual repayments [complex].

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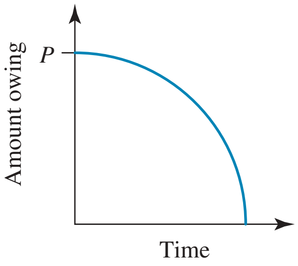
13.2.1 Understanding reducing balance loans



- **Reducing balance loans** are compound interest loans with periodic repayments.
- A bank will charge you interest for borrowing the money (the amount of interest charged mainly depends on the current interest rate and the amount still owing). The amount of interest is calculated using the compound interest formula.
- The amount of interest is then added to the balance of the loan, effectively making the balance of the loan higher.
- To reduce the balance of the loan to zero over time, a regular payment is made to reduce the amount owed.
- In order to reduce the balance, these payments must be higher than the amount of interest charged.
- The balance of the loan will increase exponentially due to the compounding interest being charged, but when regular repayments are made, the balance of the loan will reduce over time and so will the amount of compounding interest.
- If we graphed the amount owing against time for a loan, it would look like the graph shown.

WORKED EXAMPLE 1 Modelling reducing balance loans

Maddy is considering a car loan. She wishes to borrow $25 000 at a rate of 6% p.a. over 5 years. Maddy will need to make repayments of $5940 per year. Model the life of Maddy's loan.

THINK	WRITE
1. Calculate the balance of the loan after the first year of interest is included.	$A = P(1+i)^n$ $A = 25\,000(1.06)^1$ $= \$26\,500$
2. Subtract Maddy's first repayment.	Balance $= \$26\,500 - \$5940$ $= \$20\,560$
3. Calculate the balance of the loan after the second year of interest is included.	$A = P(1+i)^n$ $A = 20\,560(1.06)^1$ $= \$21\,793.60$
Subtract Maddy's second repayment.	Balance $= \$21\,793.60 - \$5940$ $= \$15\,853.60$

4. Calculate the balance of the loan after the third year of interest is included.

$A = P(1 + i)^n$

$$A = 15\,853.60(1.06)^1$$
$$= \$16\,804.82$$

5. Subtract Maddy's third repayment.

$$\text{Balance} = \$16\,804.82 - \$5940$$
$$= \$10\,864.82$$

6. Calculate the balance of the loan after the fourth year of interest is included.

$A = P(1 + i)^n$

$$A = 10\,864.82(1.06)^1$$
$$= \$11\,516.71$$

7. Subtract Maddy's fourth repayment.

$$\text{Balance} = \$11\,516.71 - \$5940$$
$$= \$5576.71$$

8. Calculate the balance of the loan after the final year of interest is included.

$A = P(1 + i)^n$

$$A = 5576.71(1.06)^1$$
$$= \$5911.31$$

The balance of the loan is $5911.31 so Maddy's final repayment is $5911.31.

13.2.2 Using online calculators to model reducing balance loan with annual repayments

- The amount owing in a reducing balance loan can be modelled using a mathematical formula (known as the annuities formula) or recurrence relations.
- In this course, we will only consider solving problems using an online financial calculator.
- There are several online calculators that can be used.
- The online calculators used in the previous chapter can also be used in this topic.
  - The Moneysmart personal loan calculator
  - TVM calculator

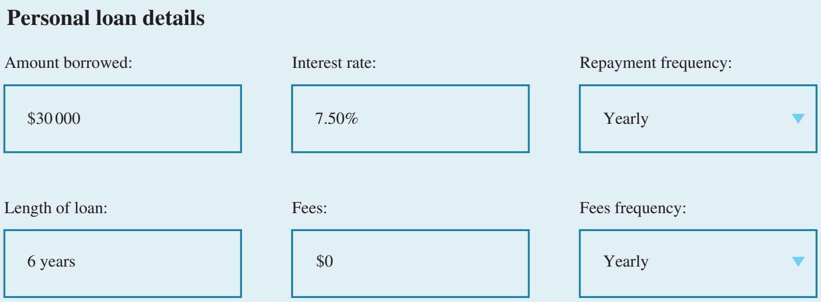
WORKED EXAMPLE 2 Using an online calculator to model a reducing balance loan with annual repayments

A small business has a loan for $30 000 at a rate of 7.5% p.a. over 6 years with no additional fees. Use an online calculator to calculate the annual repayment amount.

THINK

1. Input the known values into the online calculator:
Amount borrowed: $30 000
Interest rate: 7.50%
Repayment frequency: Yearly
Length of loan: 6 years
Fees: $0

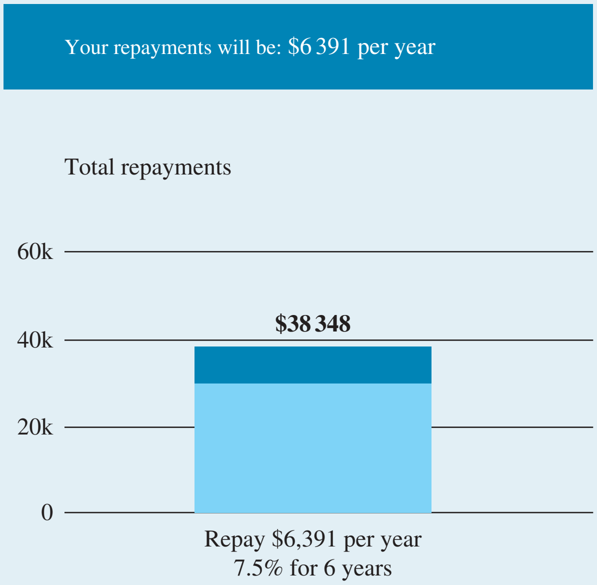
WRITE



Personal loan details

Amount borrowed: $30 000
Interest rate: 7.50%
Repayment frequency: Yearly
Length of loan: 6 years
Fees: $0
Fees frequency: Yearly

2. Examine the results.



Your repayments will be: $6 391 per year

3. Write your answer. The small business will have annual repayments of $6391.

- Recall from chapter 12, the meaning of the inputs for the TVM calculator. These are defined as follows:
  - *Present Value* is the current (present) value of the loan or investment.
  - *Payment* is the amount paid at each time period.
  - *Future Value* is the value of the investment or loan after a specified number of compounding periods.
  - *Annual Rate* is the annual interest rate.
  - *Periods* is the total number of compounding periods.
  - *Compounding* is the number of times the interest is compounded per year.
  - *Mode* is used to indicate whether the interest is compounded at the end or at the beginning of the time period. Leave this set at END.
- We also need to be mindful of sign conventions to be used for reducing balance loan calculations.

Sign conventions for online calculators when modelling reducing balance loans

- **Present value should be positive (the bank gives you money).**
- **Payment should be negative (you repay the loan by making regular payments).**
- **Future value can be negative, zero or positive:**
  - **Negative indicates you still owe the bank money.**
  - **Zero indicates the loan is fully paid out.**
  - **Positive indicates you have overpaid the loan and the bank needs to repay you some money.**

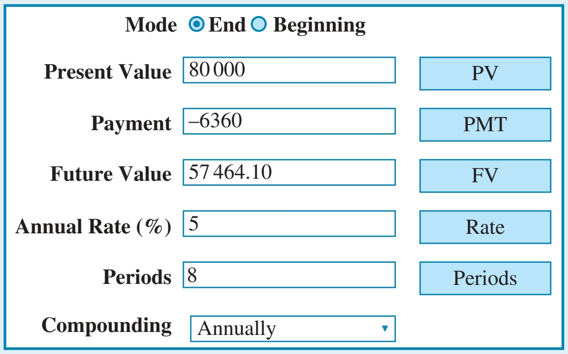
WORKED EXAMPLE 3 Using a financial calculator to determine the amount owing

A reducing balance loan of $80 000 is taken out over a 20-year period at a rate of 5% p.a. (interest charged annually) and is to be paid back with annual repayments of $6360. Determine the amount owing after 8 years.

THINK

1. Input the following values into the online calculator:
   Present Value: 80 000
   Payment: −6360
   Rate: 5
   Periods: 8
   Compounding: Annually
   To determine the future value of the loan, click the Future Value button.

WRITE



Mode ● End ○ Beginning
Present Value 80 000
Payment −6360
Future Value 57 464.10
Annual Rate (%) 5
Periods 8
Compounding Annually

2. Write the answer.

The balance remaining on the loan after 8 years is $57 464.10.

- When borrowing money, it is often wise to have an idea of the total amount of interest you will have to pay and the total cost to you to repay the loan.
- The following formulas display some important properties of a reducing balance loan.

Properties of reducing balance loans

cost of repaying the loan = the sum of all payments

total interest paid = total payments – principal

WORKED EXAMPLE 4 Using a financial calculator to explore the total time and cost of a loan

A reducing balance loan of $50 000 is to be repaid with annual repayments of $5100 at an interest rate of 6% p.a. debited annually.
Determine:

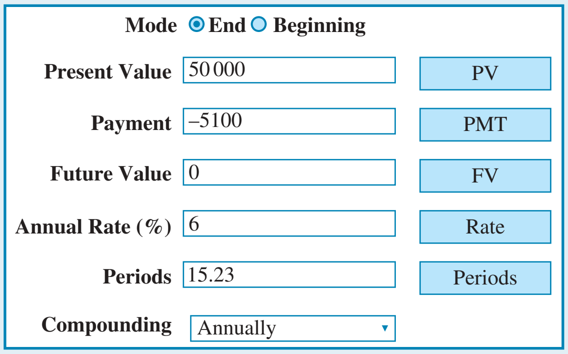
a. the number of yearly (annual) payments required to fully repay the loan
b. the total interest charged.

THINK

a. 1. If a loan is fully paid off, this implies the future value is zero.
   Input the following values into the online calculator:
   Present Value: 50 000
   Payment: −5100
   Future Value: 0
   Rate: 6
   Compounding: Annually
   To determine the number of periods needed to repay the loan, click the Periods button.

WRITE

a.



Mode ● End ○ Beginning
Present Value 50 000
Payment −5100
Future Value 0
Annual Rate (%) 6
Periods 15.23
Compounding Annually

2. Write the answer.

15.23 periods are required to fully repay repay the loan, so the loan will be repaid in the 16th year.

b. 1. Total interest = total payments – principal

b. Total interest $= (5100 \times 15.23) - 50\,000$
$= 77\,673 - 50\,000$
$= 27\,673$

2. Write the answer.

The total interest paid is $27 673.

13.2.3 Using a spreadsheet to model reducing balance loan with non-annual repayments [complex]

- To calculate the balance owing and the amount of interest each month, we need the previous month's balance.
- To determine the balance owing over many months and years, we would have to perform individual calculations for each month.
- Using technology helps us to calculate the interest and balance owing over many months and years without having to perform multiple repeated calculations.

tlvd-12152

WORKED EXAMPLE 5 Using a spreadsheet to model a reducing balance loan with non-annual repayments

A couple borrow $250 000 from a financial institution on a reducing balance loan of 4.75% p.a. for 20 years, with monthly repayments of $1620. Using a spreadsheet determine the balance after 2 years. Write your answer correct to the nearest cent.

THINK

WRITE

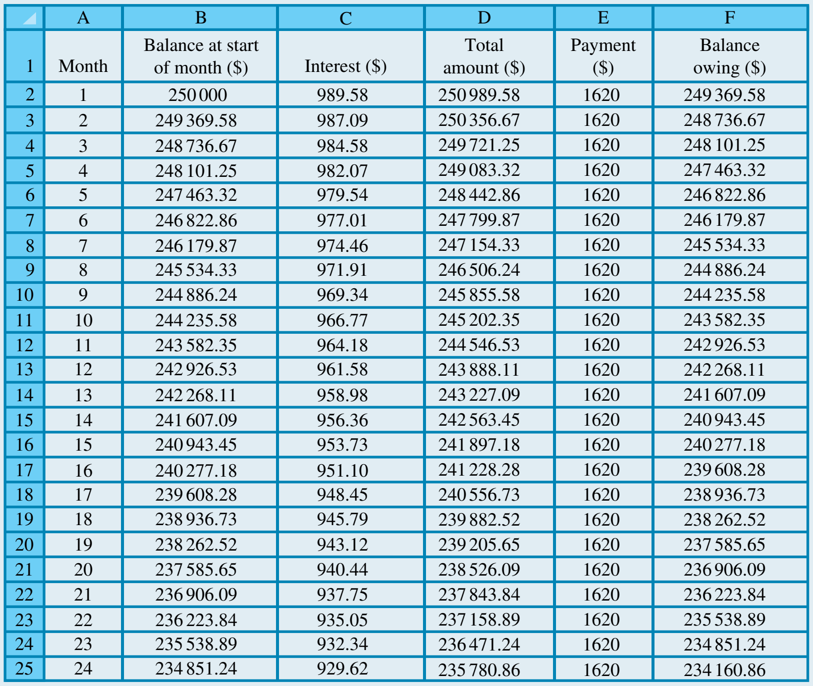
1. Set up a spreadsheet with the headings as shown.

	A	B	C	D	E	F
1	Month	Balance at start of month ($)	Interest ($)	Total amount ($)	Payment ($)	Balance owing ($)
2	1					
3	2					

2. Insert formula to calculate the interest and balance owing (as shown). Remember to round to the nearest cent.

	A	B	C	D	E	F
1	Month	Balance at start of month ($)	Interest ($)	Total amount ($)	Payment ($)	Balance owing ($)
2	1	250 000	= 0.0475/12*b2	= b2 + c2	1620	= d2 – e2
3	2	= f2				

3. Use fill down to copy the formula and complete the spreadsheet for 2 years.



	A	B	C	D	E	F
1	Month	Balance at start of month ($)	Interest ($)	Total amount ($)	Payment ($)	Balance owing ($)
2	1	250 000	989.58	250 989.58	1620	249 369.58
3	2	249 369.58	987.09	250 356.67	1620	248 736.67
4	3	248 736.67	984.58	249 721.25	1620	248 101.25
5	4	248 101.25	982.07	249 083.32	1620	247 463.32
6	5	247 463.32	979.54	248 442.86	1620	246 822.86
7	6	246 822.86	977.01	247 799.87	1620	246 179.87
8	7	246 179.87	974.46	247 154.33	1620	245 534.33
9	8	245 534.33	971.91	246 506.24	1620	244 886.24
10	9	244 886.24	969.34	245 855.58	1620	244 235.58
11	10	244 235.58	966.77	245 202.35	1620	243 582.35
12	11	243 582.35	964.18	244 546.53	1620	242 926.53
13	12	242 926.53	961.58	243 888.11	1620	242 268.11
14	13	242 268.11	958.98	243 227.09	1620	241 607.09
15	14	241 607.09	956.36	242 563.45	1620	240 943.45
16	15	240 943.45	953.73	241 897.18	1620	240 277.18
17	16	240 277.18	951.10	241 228.28	1620	239 608.28
18	17	239 608.28	948.45	240 556.73	1620	238 936.73
19	18	238 936.73	945.79	239 882.52	1620	238 262.52
20	19	238 262.52	943.12	239 205.65	1620	237 585.65
21	20	237 585.65	940.44	238 526.09	1620	236 906.09
22	21	236 906.09	937.75	237 843.84	1620	236 223.84
23	22	236 223.84	935.05	237 158.89	1620	235 538.89
24	23	235 538.89	932.34	236 471.24	1620	234 851.24
25	24	234 851.24	929.62	235 780.86	1620	234 160.86

4. Answer the question.

The balance owing after 2 years is $234 160.86.

- Using technology allows us to quickly and easily explore how changing interest rates, payment amounts and time of the loan can affect the total amount repaid.
- Lesson 13.3 will explore those effects in more detail.

Exercise 13.2 Modelling reducing balance loans with technology

learnon

13.2 Quick quiz on

13.2 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13	14, 15, 16, 17	18

These questions are even better in jacPLUS!

- Receive immediate feedback
- Access sample responses
- Track results and progress

Find all this and MORE in jacPLUS

Simple familiar

1. **WE1** A couple borrows $70 000 on a reducing balance loan for 10 years. The interest rate is 7.5% p.a. and their annual payment is $10 080 per year. For the first year, calculate, correct to the nearest cent:

a. the interest paid

b. the balance owing.

2. **WE2** Hao wants a loan for a $35 000 car. A bank will give him the loan at a rate of 8.5% p.a. over $116206.02 with no additional fees.
Use an online calculator to calculate the annual repayment amount, correct to the nearest dollar.

3. **WE3** A loan of $75 000 is taken out over a 22-year period at a rate of 4.5% (interest charged annually) and is to be paid back with annual repayments of $5512.50. Determine the amount owing after 10 years.

4. A loan of $60 000 is taken out over a 30-year period at a rate of 6% (interest charged annually) and is to be paid back with annual repayments of $4390. Determine the amount owing after 6 years.

5. Dani wants to borrow $3700 for a new sound system at 6.5% interest per annum. Determine Dani's annual repayment if the loan is paid off in 2 years.

6. Adam has borrowed $15 000 to buy a car. He agrees to pay off the loan in 4 years with annual repayments at 7.6% p.a. Calculate the annual repayment value.

7. Blake has borrowed $6000 to buy a ride-on mower. They agree to pay off the loan in 3 years with annual repayments of $2463.82. Determine the annual compounding interest rate, correct to 1 decimal place.

8. **WE4** A reducing balance loan of $24 000 is to be repaid with annual repayments of $4609.74 at an interest rate of 8% p.a.
Determine:
   a. the number of years to fully repay the loan
   b. the total interest charged.

9. A loan of $3600 is taken out with a reducing balance interest rate of 5.6% p.a., with interest debited annually. The borrower wishes to pay instalments of $1336.84 per year. Determine how long it will take to pay off the loan in full.

10. James borrowed $14 000. Interest was charged at 7.4% p.a. on the reducing balance loan and he has been paying $2381.02 each year to pay off the loan. Currently he still owes $4281.18. Determine how long ago James started the loan.

11. Stephanie borrowed $13 500. Interest on this reducing balance loan has been charged at 7.5% p.a., and she has been paying $2572.11 per year to service the loan. Currently, she has $8461.48 remaining on the loan. Determine how long ago Stephanie started the loan.

12. A loan of $16 000 is being paid off with equal annual repayments of $2258.14 and interest being charged at 10.5% p.a. Currently, the amount owed is $10 430.14.
Determine how much longer it will take to:
   a. reduce the amount owing to $6562.11
   b. repay the loan in full, correct to 2 decimal places.

13. A loan of $8500 is being repaid with equal annual repayments of $1635.89 and interest being charged at 7.4% p.a. Currently, the amount owed is $4002.91.
Determine how much longer it will take to:
   a. reduce the amount owing to $1224.43
   b. repay the loan in full, correct to 1 decimal place.

Complex familiar

14. Luke and Mike borrow $700 000 from their bank on a reducing balance loan of 6.25% p.a. for 30 years, with monthly repayments of $4100. Using a spreadsheet, determine the balance after 3 years. Write your answer correct to the nearest cent.

15. **WE5** Mara borrows $8000 from a financial institution on a reducing balance loan of 6.5% p.a. for 5 years with monthly repayments of $157.

a. Using technology, determine the balance after 2 years. Write your answer correct to the nearest cent.
b. If the payment is increased to $200, determine the balance after 2 years. Write your answer correct to the nearest cent.
c. Explain the effect that increasing the regular payment has on Mara's loan by finding how long it takes Mara to pay back the loan in full.

16. The repayment on a loan of $50 000 at 7.5% p.a. over a 15-year term is $463.51 per month.
Use a spreadsheet to calculate the following, correct to the nearest cent.

a. Calculate the interest for the first month of the loan and the balance owing at the end of the first month.
b. Calculate the amount by which the balance has reduced in the first month.
c. Calculate the interest for the second month of the loan and the balance at the end of the second month.
d. Determine how much the balance of the loan has reduced during the second month.

17. The repayments on a loan of $150 000 over a 20-year term at 9.6% p.a. are $1408.01 per month.
Use technology to complete the table shown.

Month	Principal ($)	Interest ($)	Balance owing ($)
1	150 000.00	1200.00	149 791.99
2	149 791.99		
3			
4			
5			
6			
7			
8			
9			
10			

Complex unfamiliar

18. Mortgage stress is a term used when mortgage repayments exceed 30% of your household income, making it hard to pay bills and save. Banks are unlikely to lend to borrowers if their repayments will put them into mortgage stress. A couple wishes to borrow $250 000 on a reducing balance loan compounding monthly at 5.4% p.a. with monthly payments and no fees for 20 years.
Determine their minimum household annual income to avoid mortgage stress.

Fully worked solutions for this chapter are available online.

LESSON
13.3 The effect of repayment amounts, the interest rate and compounding periods on reducing balance loans [complex]

SYLLABUS LINKS

- Investigate the effect of the repayment amount, the interest rate and the number of compounding periods on the time taken to repay a loan [complex].

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13.3.1 Frequency of repayments

- We will look at investigating the effect on the term of the loan, and on the total amount of interest charged if the frequency of the repayments is changed.

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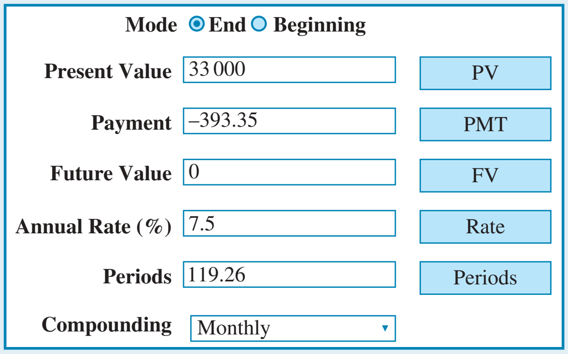
WORKED EXAMPLE 6 Using a financial calculator to compare monthly and fortnightly repayments

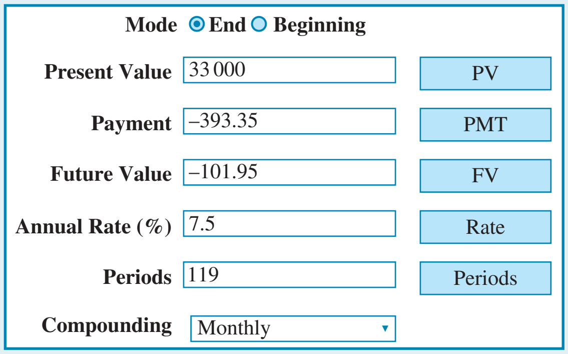
Janette borrows $33 000 to renovate her jewellery shop. The loan is charged at 7.5% p.a. interest with either fortnightly repayments of $181.55 or monthly repayments of $393.35.
Determine:
i. the term of the loan
ii. the amount still owing prior to the last payment
if Janette made repayments:
a. monthly
b. fortnightly.

THINK	WRITE
a. i. 1. Determine the number of periods using monthly compounding. Input the following values into the online calculator: Present Value: 33 000 Payment: −393.35 Future Value: 0 Rate: 7.5 Compounding: Monthly To determine the number of periods, click the Periods button.	**a. i.**  **Mode** ● **End** ○ **Beginning** **Present Value** 33 000 **Payment** –393.35 **Future Value** 0 **Annual Rate (%)** 7.5 **Periods** 119.26 **Compounding** Monthly
2. Interpret the result and convert into years by dividing by 12.	119.26 periods are required, which is 119.26 months. $\frac{119.26 \text{ months}}{12} = 9.94$ years (rounding to 10 years)
3. Write the answer.	The term of the loan is approximately 10 years.

ii. 1. To determine the amount still owing prior to the last payment, set the number of periods to 119.
Input the following values into the online calculator:
Present Value: 33 000
Payment: −393.35
Rate: 7.5
Periods: 119
Compounding: Monthly
To determine the future value, click the FV button.

2. Write the answer.

ii.



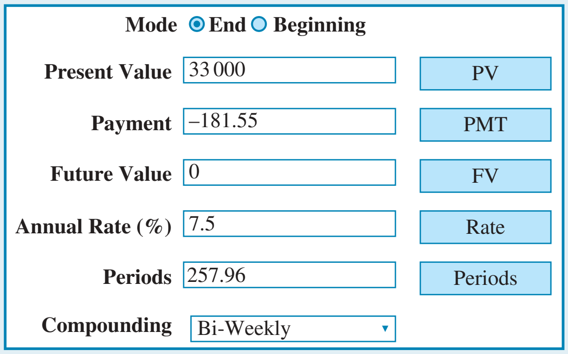
Mode ● End ○ Beginning
Present Value: 33 000
Payment: –393.35
Future Value: –101.95
Annual Rate (%): 7.5
Periods: 119
Compounding: Monthly ▾

The amount still owing prior to the last payment is \$101.95.

b. i. 1. Determine the number of periods using fortnightly compounding.
Input the following values into the online calculator:
Present Value: 33 000
Payment: −181.55
Future Value: 0
Rate: 7.5
Compounding: Bi-Weekly (fortnightly)
To determine the number of periods, click the Periods button.

2. Write the answer.

b. i.



Mode ● End ○ Beginning
Present Value: 33 000
Payment: –181.55
Future Value: 0
Annual Rate (%): 7.5
Periods: 257.96
Compounding: Bi-Weekly ▾

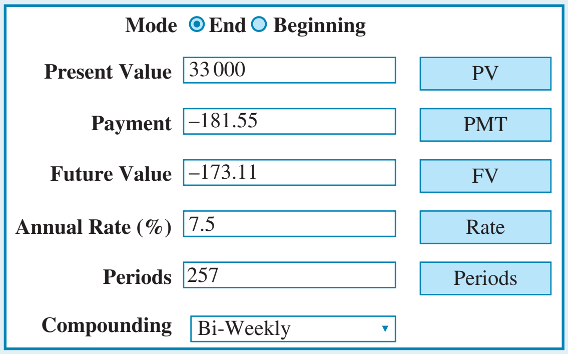
The term of the loan is 257.96 fortnights, which is 258 fortnightly repayments to cover the loan.

The loan will take $\frac{258}{26} = 9.92 \approx 10$ years to be fully paid off.

ii. 1. To find the amount still owing prior to the last payment, set the number of periods to 257.
Input the following values into the online calculator:
Present Value: 33 000
Payment: −181.55
Rate: 7.5
Periods: 257
Compounding: Bi-Weekly
To determine the future value, click the FV button.

2. Write the answer.

ii.



Mode ● End ○ Beginning
Present Value: 33 000
Payment: –181.55
Future Value: –173.11
Annual Rate (%): 7.5
Periods: 257
Compounding: Bi-Weekly ▾

The amount still owing prior to the last payment is \$173.11.

- Comparing the results from Worked example 6, it can be seen that even paying the same overall amount per year there may be a slight reduction in the term of the loan when repayments are made more regularly.
- Let us now investigate what saving in accrued interest can be made in these situations. In this situation we should consider the final (partial) payment separately because the amount of interest that it attracts is less than a complete repayment.
- The calculation of the total interest paid is now calculated as usual: total interest paid 257.96 total payments 258 principal.

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WORKED EXAMPLE 7 The effect of repayment periods on total interest

Janette's $33 000 loan at 7.5% p.a. also has the option of paying quarterly. Given the following three situations:

- **Quarterly repayments of $1180.05 for 10 years**
- **Monthly repayments of $393.35 for 119 months with $101.95 still outstanding**
- **Fortnightly repayments of $181.55 for 257 fortnights with $173.11 still outstanding**

Compare the total interest paid by Janette if she repaid her loan:

a. **quarterly** **b.** **monthly** **c.** **fortnightly.**

THINK	WRITE
a. **1.** Making quarterly payments over 10 years means that 40 (10 × 4) individual payments of $1180.05 are made. Use the formula: Total Interest = Total Payments (amount each period × number of periods) − Principal Amount	**a.** For quarterly payments: First calculate the number of periods = > 10 years × 4 periods per year = 40 Total interest = (1180.05 × 40) − 33 000 = $14 202
2. Write the answer.	The total interest paid by Janette when repaying her loan quarterly is $14 202.
b. **1.** For monthly repayments, first determine the monthly interest rate.	**b.** Rate $= \frac{7.5}{12} = 0.625\%$
2. Calculate the interest to be paid on the amount still owing ($101.95) and determine the amount of the final payment.	Interest = 0.625% of 101.95 = 0.006 25 × 101.95 = $ 0.64 Final payment = 101.95 + 0.64 = $102.59
3. To calculate the total interest paid, use the formula: Total interest = (repayment for period × number of whole periods) + (final payment amount as above) – (principal amount)	Total interest = (393.35 × 119) + 102.59 − 33 000 = 13 911.24
c. **1.** For fortnightly repayments, first determine the fortnightly interest rate.	**c.** Rate $= \frac{7.5}{26} = 0.2885\%$
2. Calculate the interest to be paid on the amount still owing ($173.11) and determine the amount of the final payment.	Interest = 0.2885% of 173.11 = 0.002 885 × 173.11 = $ 0.50 Final payment = 173.11 + 0.50 = $173.61

3. To calculate the total interest paid, use the formula:
Total interest = (repayment for period × number of whole periods) + (final payment amount as above) – (principal amount)

$$\begin{aligned}\text{Total interest} &= (181.55 \times 257) + 173.61 - 33\,000 \\ &= \$13\,831.96\end{aligned}$$

4. Compare the interest paid.

Examining all 3 scenarios, Janette would be better off making fortnightly repayments as she would pay less interest overall.

13.3.2 Increasing repayments

- Borrowers who want to pay off their loan as soon as possible can do so by increasing the repayments.
- Increasing your payments can reduce the period of your loan, while at the same time reducing the total interest you need to pay back on your loan.

WORKED EXAMPLE 8 The effect of increasing loan repayments

A reducing balance loan of $22 000 is taken over 6 years. It is to be repaid with monthly repayments at a rate of 6.4% p.a. (debited monthly).

a. Determine the repayment value.

b. Calculate the term of the loan if the repayment is increased to $429.43.

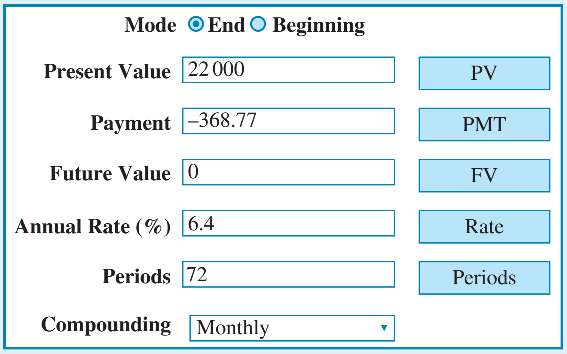
c. Calculate the total interest paid on repayments of $429.43.

THINK

WRITE

a. 1. Input the following values into the online calculator:
Present Value: 22 000
Future Value: 0
Rate: 6.4
Periods: 72
(6×12)
Compounding: Monthly
To determine the monthly payment, click the PMT button.

a.



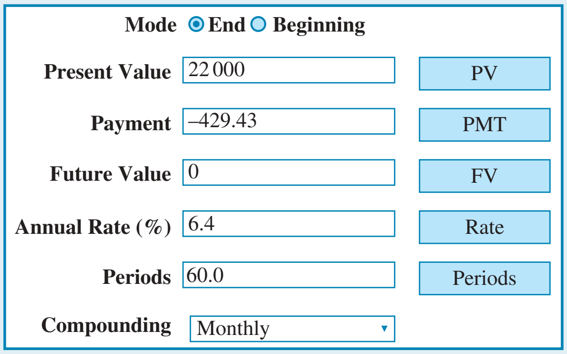
Mode ● End ○ Beginning
Present Value: 22 000
Payment: –368.77
Future Value: 0
Annual Rate (%): 6.4
Periods: 72
Compounding: Monthly

2. Write the answer.

The monthly repayment is $368.77 over 6 years.

b. 1. Using the new payment value of $429.43 we need to determine the number of periods it will take to fully repay the loan.
Input the following values into the online calculator:
Present Value: 22 000
Payment: −429.43
Future Value: 0
Rate: 6.4
Compounding: Monthly
To determine the number of periods, click the Periods button.

b.



Mode ● End ○ Beginning
Present Value: 22 000
Payment: –429.43
Future Value: 0
Annual Rate (%): 6.4
Periods: 60.0
Compounding: Monthly

Periods = 60

2. Convert the number of time periods to years by dividing by 12.

$$\frac{60}{12} = 5$$

3. Write the answer.

When making a repayment of $429.43 per month, the loan will be fully paid off after 5 years.

c. 1. Total Interest = total payments per month − principal amount

c.
$$\begin{aligned}\text{Total interest} &= (429.43 \times 60) - 22\,000\\ &= 25\,765.80 - 22\,000\\ &= \$3765.80\end{aligned}$$

2. Write the answer.

The total interest paid is $3765.80.

- Online calculators are helpful when considering a change in one circumstance over the life of the loan (for example, a change of repayment frequency or repayment amount).
- A spreadsheet can also be used to explore these changes, providing detail that a calculator may not, and allowing for adjustments to be made part way through the loan.

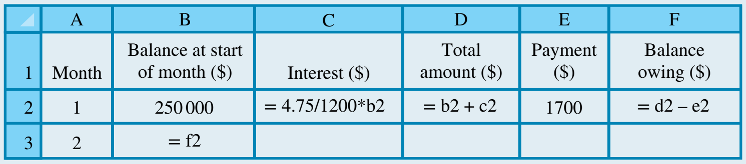
WORKED EXAMPLE 9 Using a spreadsheet to explore changes to a reducing balance loan

Use the spreadsheet from Worked example 5 to determine the balance after 2 years if the payments were increased to $1700. Hence, explain the effect that increasing the regular payment has on the loan. Write your answer correct to the nearest cent.

THINK

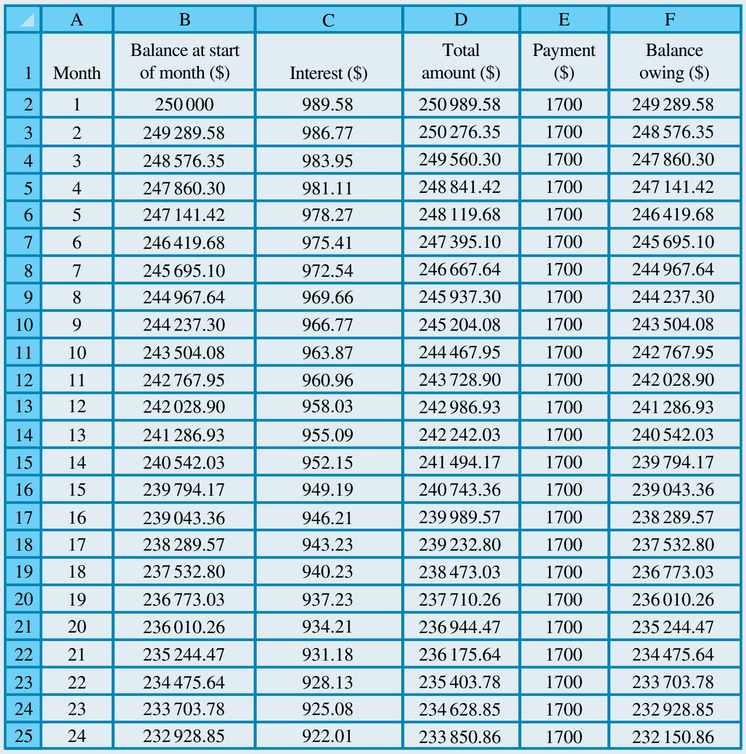
WRITE

1. Change the payment amount in column E of the spreadsheet from Worked example 5.



	A	B	C	D	E	F
1	Month	Balance at start of month ($)	Interest ($)	Total amount ($)	Payment ($)	Balance owing ($)
2	1	250 000	= 4.75/1200*b2	= b2 + c2	1700	= d2 – e2
3	2	= f2				

2. Use fill down to copy the formulas and complete the spreadsheet for 2 years.



	A	B	C	D	E	F
1	Month	Balance at start of month ($)	Interest ($)	Total amount ($)	Payment ($)	Balance owing ($)
2	1	250 000	989.58	250 989.58	1700	249 289.58
3	2	249 289.58	986.77	250 276.35	1700	248 576.35
4	3	248 576.35	983.95	249 560.30	1700	247 860.30
5	4	247 860.30	981.11	248 841.42	1700	247 141.42
6	5	247 141.42	978.27	248 119.68	1700	246 419.68
7	6	246 419.68	975.41	247 395.10	1700	245 695.10
8	7	245 695.10	972.54	246 667.64	1700	244 967.64
9	8	244 967.64	969.66	245 937.30	1700	244 237.30
10	9	244 237.30	966.77	245 204.08	1700	243 504.08
11	10	243 504.08	963.87	244 467.95	1700	242 767.95
12	11	242 767.95	960.96	243 728.90	1700	242 028.90
13	12	242 028.90	958.03	242 986.93	1700	241 286.93
14	13	241 286.93	955.09	242 242.03	1700	240 542.03
15	14	240 542.03	952.15	241 494.17	1700	239 794.17
16	15	239 794.17	949.19	240 743.36	1700	239 043.36
17	16	239 043.36	946.21	239 989.57	1700	238 289.57
18	17	238 289.57	943.23	239 232.80	1700	237 532.80
19	18	237 532.80	940.23	238 473.03	1700	236 773.03
20	19	236 773.03	937.23	237 710.26	1700	236 010.26
21	20	236 010.26	934.21	236 944.47	1700	235 244.47
22	21	235 244.47	931.18	236 175.64	1700	234 475.64
23	22	234 475.64	928.13	235 403.78	1700	233 703.78
24	23	233 703.78	925.08	234 628.85	1700	232 928.85
25	24	232 928.85	922.01	233 850.86	1700	232 150.86

3. Answer the question.	The balance owing after 2 years is \$232 150.86, which has reduced the balance owing by \$234 160.86 − \$232 150.86 = \$2010. By increasing the payment, the number of years required to pay off the loan will be reduced, saving many dollars in interest.

13.3.3 Changing the rate

- The Reserve Bank meets on the first Tuesday of every month to decide on interest rates, which then influences individual lenders in determining the rates they pass on to borrowers.
- As the meetings occur monthly, a variable interest rate can change many times over the duration of the loan.
- Some online calculators such as the Moneysmart personal loan calculator automatically consider a 2% increase in loan rates and provide you with the repayment value, should that increase occur.
- We will look at comparing loan situations by taking into account the variation of the rate.

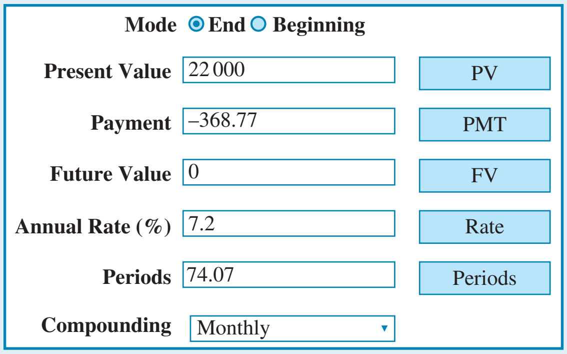
WORKED EXAMPLE 10 Using an online calculator to explore interest rate changes in a reducing balance loan

A reducing balance loan of \$22 000 is taken over 6 years. It is to be repaid with monthly repayments of \$368.77 at a rate of 6.4% p.a. (debited monthly).

a. Determine the total amount of interest paid.

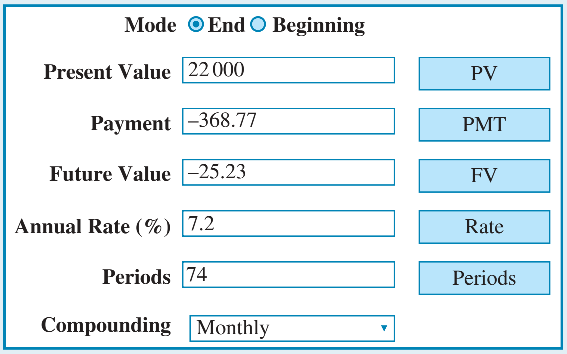
b. If instead the rate was 7.2% p.a. (adjusted monthly) and the repayments remained the same, determine:

- **i. the term of the loan**
- **ii. the total amount of interest paid.**

THINK	**WRITE**
a. 1. Calculate the amount of monthly payments	**a.** $6 \times 12 = 72$ 72 monthly payments
2. Determine the total interest paid using the formula: total interest = total payments − principal.	Total interest $= (368.77 \times 72) - 22\,000$ $= \$4551.44$
3. Write the answer.	The total interest paid over the 6 years is \$4551.44.
b. i. 1. Input the following values into the online calculator: Present Value: 22 000 Payment: −368.77 Future Value: 0 Rate: 7.2 Compounding: Monthly To determine the number of periods, click the Periods button.	**b. i.**  Mode ● End ○ Beginning Present Value 22 000 Payment −368.77 Future Value 0 Annual Rate (%) 7.2 Periods 74.07 Compounding Monthly
2. Interpret the result.	74.07 periods means that 74 full payments need to be made plus one smaller payment. Therefore 75 payments are required to repay the loan.
3. Answer the question.	The term of the loan is 75 months or 6 years and 3 months.

ii. 1. Calculate interest on the final payment by determining the loan balance after 74 payments.
Input the following values into the online calculator:
Present Value: 22 000
Payment: −368.77
Rate: 7.2
Periods: 74
Compounding: Monthly
To determine the future value, click on the FV button.

ii.



Mode ● End ○ Beginning
Present Value: 22 000
Payment: –368.77
Future Value: –25.23
Annual Rate (%): 7.2
Periods: 74
Compounding: Monthly

The balance of the loan after 74 payments is \$25.23.

2. Calculate the interest rate per month.

$$\text{Interest rate per month} = \frac{7.2}{12} = 0.6\%$$

3. Calculate the interest on the final payment and the value of the final payment.

$$\begin{aligned}\text{Interest on final payment} &= 0.6\% \text{ of } \$25.23\\ &= 0.006 \times 25.23\\ &= \$0.15\end{aligned}$$

$$\begin{aligned}\text{Final payment} &= 25.23 + 0.15\\ &= \$25.38\end{aligned}$$

4. Determine the total amount of interest paid using the formula:
total interest = total payments − principal.

$$\begin{aligned}\text{Total Interest} &= \text{total payments} + \text{final payment}\\ &\quad - \text{principal}\\ &= (\$368.77 \times 74) + 25.38 - 22\,000\\ &= \$5314.36\end{aligned}$$

- You can also use a spreadsheet to do Worked example 10. In that case, you would adjust the rate that was entered in Column C of the spreadsheet in Worked example 9.

Exercise 13.3 The effect of repayment amounts, the interest rate and compounding periods on reducing balance loans [complex]

learn**on**

13.3 Quick quiz on

13.3 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
N/A	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19	20, 21

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Complex familiar

1. **WE6** Karen borrows \$21 000. The loan is charged at 6.2% p.a. interest with either fortnightly repayments of \$108.98 or monthly repayments of \$236.13.
Determine:
  i. the term of the loan
  ii. the amount still owing prior to the last payment
if Karen made repayments:
  a. monthly
  b. fortnightly.

2. Peter borrows \$17 000. The loan is charged at 6.9% p.a. and he can afford quarterly repayments of \$871.05. This will pay the loan off in exactly 6 years.

One-third of the quarterly repayment gives the equivalent monthly repayment of \$290.35. The equivalent fortnightly repayment is \$134.01.

Determine:

i. the term of the loan
ii. the amount still owing prior to the last payment

if Peter made repayments:

a. monthly
b. fortnightly.

3. **WE7** Karen's \$21 000 loan at 6.2% p.a. also has the option of paying quarterly. Given the following three situations:

- Quarterly repayments of \$708.39 for 10 years
- Monthly repayments of \$236.13 for 119 months with \$91.16 still outstanding
- Fortnightly repayments of \$108.98 for 258 fortnights with \$35.30 still outstanding

Compare the total interest paid by Karen if she repaid her loan quarterly, monthly or fortnightly.

4. Peter's \$17 000 loan at 6.9% p.a. gave the following three situations.

- Quarterly repayments of \$871.05 for 6 years
- Monthly repayments of \$290.35 for 71 months with \$170.93 still outstanding
- Fortnightly repayments of \$134.01 for 154 fortnights with \$116.61 still outstanding

Compare the total interest paid by Peter if he repaid his loan quarterly, monthly or fortnightly.

5. **WE8** A reducing balance loan of \$16 000 is taken over 4 years. It is to be repaid with monthly repayments at a rate of 7.6% p.a. (debited monthly).

a. Determine the repayment value.
b. Determine the term of the loan if the repayment is changed to \$350.77.
c. Calculate the total interest paid with repayments of \$350.77.

6. A reducing balance loan of \$28 000 is taken over 6 years. It is to be repaid with monthly repayments at a rate of 6.2% p.a. (debited monthly).

a. Determine the repayment value.
b. Determine the term of the loan if the repayment is increased to \$543.93.
c. Calculate the total interest paid with repayments of \$543.93.

7. Tom borrowed \$22 500 for a new car and agreed to repay the loan over 12 years with quarterly repayments of \$653.90 and interest debited at 5.8% p.a. (adjusted quarterly). However, 7 years into the loan, Tom made the decision to increase his repayments to \$1220.15.

Calculate:

a. the actual term of the loan
b. the total interest paid
c. the interest saved by increasing the size of the repayments.

8. Leah borrowed \$19 500 to renovate her bathroom and agreed to repay the loan over 8 years with monthly repayments of \$258.16 and interest debited at 6.2% p.a. However, 5 years into the loan, Leah made the decision to increase her repayments to \$375.75.

Calculate:

a. the actual term of the loan
b. the total interest paid
c. the interest saved by increasing the size of the repayments.

9. Paul's reducing balance loan of $8250 is to be repaid with monthly repayments of $168.70 with interest charged at 8.25% p.a. (debited monthly).

a. If the amount owing is $5359.03, calculate how much longer it will take to:

i. reduce the amount owing to $1931.76

ii. repay the loan in full.

b. At a later time, the amount owing has dropped to $4547.42. Determine how much longer it will take to:

i. reduce the loan to $3134.18

ii. repay the loan in full.

10. Jeanie wanted to borrow $60 000 and she was offered a reducing balance loan over 15 years at 6.25% p.a. (adjusted monthly) with monthly instalments.

a. Calculate her monthly instalments.

b. Determine the term of the loan if instead the repayment was:

i. increased to $593.30

ii. decreased to $450.24.

11. **WE9** Nerida wants to buy a car. She needs to borrow $22 000. The interest rate is 10.5% p.a. compounded monthly for a personal loan. The contracted loan term is for 5 years, but she is allowed to pay it off sooner. Her current repayments are $472.87. If Nerida pays an additional $80 a month, use a spreadsheet to determine how much shorter her loan will become.

12. A couple decide to borrow $440 000 to buy an apartment. The interest rate is 5.75% p.a. compounded monthly for a home mortgage. Their current repayments are $3089.17 per month over 20 years. They want to pay off the loan quicker by paying an additional $565 per month.
Use a spreadsheet to determine how much quicker they can pay off their loan.

13. **WE10** A reducing balance loan of $13 000 is taken over 6 years. It is to be repaid with monthly repayments of $218.53 at a rate of 6.5% p.a. (debited monthly).

a. Determine the total interest paid.

b. If instead the rate was 7.0% p.a. (adjusted monthly) and the repayments remained the same, determine:

i. the term of the loan

ii. the total amount of interest paid.

14. A reducing balance loan of $19 000 is taken over 7 years. It is to be repaid with monthly repayments of $288.62 at a rate of 7.2% p.a. (debited monthly).

a. Determine the total interest paid.

b. If instead the rate was 8.0% p.a. (adjusted monthly) and the repayments remained the same, determine:

i. the term of the loan

ii. the total amount of interest paid.

15. Abby borrows $1800 to purchase new ski gear. The loan is to be paid in full over 1 year with quarterly repayments at an interest rate of 7.2% p.a.

a. Calculate the quarterly payment required.

b. Determine the amount owing on the loan after the third payment.

c. Calculate the total cost of repaying the loan.

d. Calculate the interest paid on the loan.

16. Craig borrows $2500 to purchase new golf clubs. The loan is to be paid in full over 1 year with quarterly repayments at an interest rate of 9.5% p.a.

a. Calculate the quarterly payment required.
If Craig shopped around he could get a loan at 8.8% p.a. (debited quarterly).

b. Calculate the new quarterly repayments.

c. Calculate the savings when taking the reduced rate.

17. Brigid borrows $6500 and the loan is to be paid in full over 2 years with monthly repayments at an interest rate of 10.5% p.a. (compounding monthly).

a. Calculate the monthly payment required.

b. If Brigid shopped around, she could get a loan at 9.3% p.a. (debited monthly). Calculate the new monthly repayments.

c. Calculate the savings when taking the reduced rate.

18. A $7200 loan is taken out and is to be fully paid off in 2 years with monthly repayments at a rate of 11.25% p.a.

a. Calculate the monthly payment required.

b. If the monthly repayments were increased by $40 per month, determine the effect on the term of the loan.

19. A loan of $18 000 attracts interest at 7.5% p.a. on the outstanding balance and the following three options are available.

- Half-yearly repayments of $1890.22 for 6 years
- Quarterly repayments of $945.11 for 5.75 years with $726.17 still owing
- Monthly repayments of $315.04 for 70 months with $282.68 still owing

Determine which is the best option.

Complex unfamiliar

20. The Taylors borrow $140 000 over 20 years at 9% p.a. for home renovations. The monthly minimum required repayment on this loan is $1259.62. The Taylors attempt to pay the loan off quickly by increasing their monthly payment to $1500. Determine how much the Taylors will save by increasing their monthly payment.

21. Harrison takes out a $12 000 loan for 4 years at 7.8% p.a. (debited monthly).
Calculate the interest saved by paying the loan with fortnightly repayments.

Fully worked solutions for this chapter are available online.

LESSON
13.4 Review

13.4.1 Summary

Hey students! Now that it's time to revise this chapter, go online to:

Access the chapter summary

Review your results

Watch teacher-led videos

Practise questions with immediate feedback

Find all this and MORE in jacPLUS

13.4 Exercise

learnon

13.4 Exercise

Simple familiar	Complex familiar	Complex unfamiliar
1, 2, 3, 4, 5, 6, 7, 8	9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22	23, 24, 25

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- Receive immediate feedback
- Access sample responses
- Track results and progress

Find all this and MORE in jacPLUS

Simple familiar

1. Noel took out a reducing balance loan of \$5600 with interest charged at 7.6% p.a. compounding annually. His loan is to be fully repaid in 3 years. Calculate the value of the annual repayment.

2. Calculate the annual loan repayments for each of the following loans:
   a. \$25 000 over a 10-year period at a rate of 9% p.a., compounding annually.
   b. \$45 000 over a 15-year period at a rate of 14% p.a., compounding annually.
   c. \$164 750 over a 25-year period at a rate of 15% p.a., compounding annually.
   d. \$425 000 over a 15-year period at a rate of 12% p.a., compounding annually.

3. A loan of \$75 000 is taken out over a 20-year period at a rate of 6.2% p.a. (interest debited annually) with annual repayments. Calculate the amount owing after 8 years.

4. Mr. and Mrs. Warne borrow \$125 000 to purchase a small house unit. The interest rate is 7.6% p.a. (compounding annually) over 10 years. The annual repayments are \$18 293.98.
   Determine:
   a. The first year's interest on the loan.
   b. The balance of the loan after the first year.

5. Leslie borrowed \$95 000 from a bank. Interest is charged at 8% p.a. The loan is to be repaid over 15 years with annual payments. Determine the total interest paid on this loan.

6. Fernando wants to borrow \$5200 for a new sound system at 6.4% p.a., with interest adjusted annually.
   a. Calculate Fernando's annual repayment if the loan is paid off in 4 years.
   b. Calculate the total interest charged.

7. A reducing balance loan of \$30 000 is to be repaid with annual repayments of \$4076.04 at an interest rate of 6% p.a. (debited annually). Determine:
   a. The number of years to fully repay the loan. b. The total interest charged.

8. Marco and Helena borrow $40 000 to renovate their coffee shop. Interest on the unpaid balance is charged to the loan account monthly. The $40 000 is to be fully repaid in equal monthly repayments of $481.10 for 10 years.
Determine the annual compounding rate of interest correct to 1 decimal place.

Complex familiar

9. Monique borrowed $21 000. Interest was charged at 7.8% p.a. on the reducing balance loan (adjusted monthly) and she has been paying $252.57 each month to pay off the loan. Currently she still owes $5596.37. Determine how long ago Monique started the loan.

10. Mr and Mrs Stone borrow $225 000 for their home. The interest rate is 9.6% p.a. and the term of loan is 25 years. The monthly repayment is $1989.48.

- **a.** Calculate the total repayments made on this loan.
- **b.** If Mr and Mrs Stone increase their monthly payments to $2000, the loan will be repaid in 24 years and 1 month. Calculate the amount they will save in repayments with this increase.

11. Amelia and Karin take out a $233 000 home loan at 12% p.a. over a 25-year term.

- **a.** Calculate the amount of each monthly repayment.
- **b.** After 3 years the balance on the loan has been reduced to $227 657. The interest rate then rises to 13% p.a. Calculate the new monthly repayment required to complete the loan within the existing term.

12. A $25 000 reducing balance loan is taken out over 8 years charged at 9.8% p.a. (debited fortnightly) and paid with fortnightly repayments.

- **a.** Determine the fortnightly repayments.
- **b.** Determine the term of the loan if repayments are $200.28.
- **c.** Determine the fortnightly repayments if the loan is to be paid off in 6 years.

13. A loan for $16 000 is being paid off at a rate of $312.80 per month with interest being charged at 12% p.a. (debited monthly). Currently the amount owing is $6645.19. Determine how much longer it will take to:

- **a.** reduce the amount owing to $3520.87
- **b.** repay the loan in full.

14. A reducing balance loan of $13 000 is taken over 6 years. It is to be repaid with monthly repayments at a rate of 7.6% p.a. (debited monthly).

- **a.** Calculate the value of the monthly repayment.
- **b.** Determine the term of the loan if the repayment is increased to $261.11.
- **c.** Calculate the total interest paid with repayments of $261.11.

15. Fraser borrows $27 000 to purchase a new car. The loan is charged at 8.8% p.a. and she can afford quarterly repayments of $995.68. This will pay the loan off in 10 years.
One-third of the quarterly repayment gives the equivalent monthly repayment of $331.89. The equivalent fortnightly repayment is $153.18. (Assume that the interest compounds at the same frequency that the payments are made).
Determine:

- **i.** the term of the loan
- **ii.** the amount still owing prior to the last payment

if Fraser made repayments:

- **a.** monthly
- **b.** fortnightly.

16. Kristen and Adrian borrow $150 000 for their home renovations. They have the choice of two loans.
Loan 1: At 8% p.a. interest over 25 years with fixed monthly repayments of $1157.72
Loan 2: At 8.25% p.a. interest over 25 years with minimum monthly repayments of $1182.68 and an $8 per month account management fee.
Kristen and Adrian believe they can afford to pay $1500 per month. If they do, Loan 2 will be repaid in 14 years and 2 months.
Explain which loan should Kristen and Adrian choose if they can afford to pay the extra each month.

17. Mr and Mrs Roebuck borrow $85 000 for a home loan. The interest rate is 9% p.a. and over a 20-year term the monthly repayment is $764.77.

a. Use technology to complete the table for the first year.

Month	Principal ($)	Interest ($)	Balance owing ($)
1	85 000.00	637.50	84 872.73
2	84 872.73		
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			

b. Mr and Mrs Roebuck decide to increase their monthly payment to $800. Use technology to complete the table shown.

Month	Principal ($)	Interest ($)	Balance owing ($)
1	85 000.00	637.50	84 837.50
2	84 837.50		
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			

c. Mr and Mrs Roebuck can repay the loan in 214 months by increasing their monthly payments to $800. Show that that they save $12 344.80 by increasing their monthly payments to $800.

18. A couple borrow \$100 000 on a reducing balance loan of 6.75% p.a. with monthly payments of \$1148.24. Using technology, complete the table shown and hence determine the balance owing after 5 months of payments.

Month	Principal (\$)	Interest (\$)	Balance owing (\$)
1	100 000	562.50	99 414.26
2	99 414.26		
3			
4			
5			

19. David borrows \$25 000 from a financial institution on a reducing balance loan of 7.50% p.a. for 8 years with monthly repayments of \$500.95.

a. Using technology, determine the balance after 2 years. Write your answer correct to the nearest cent.

b. If the payment is increased to \$650, determine the balance after 2 years. Write your answer correct to the nearest cent.

20. A couple borrow \$25 000 on a reducing balance loan for 8 years. The interest rate is 6.35% p.a. and their monthly payment is \$332.81 per month.

Calculate for the first month, correct to the nearest cent:

a. The interest portion of the first payment

b. the amount that the principal is reduced by

c. the balance owing.

21. A reducing balance loan of \$42 000 is taken over 8 years. It is to be repaid with monthly repayments of \$549.90 at a rate of 5.9% p.a. (debited monthly).

a. Calculate the total interest paid.

If the rate was 6.6% p.a. (adjusted monthly) instead and the repayments remained the same, calculate:

b. the term of the loan

c. the total amount of interest paid.

22. Shenna takes out a loan for \$27 000 to purchase a new car. She wants to pay the loan off in 3 years with monthly repayments. If Shenna chose a loan that charged 7.2% p.a. (debited monthly), determine the total cost of repaying the loan.

Complex unfamiliar

23. Xavier purchased a new kitchen for \$24 500. He was able to save a \$5000 deposit to reduce the size of the loan he had to take out. Xavier borrowed the remainder of the loan at 10.5% p.a. (debited fortnightly) over 4 years.

If his repayments were doubled, determine the new term of the loan.

24. Victoria has been saving to buy a city apartment. She has been able to save a \$35 000 deposit. Victoria negotiated with the real estate agent to reduce the purchase price of the apartment to \$380 000. She paid the full deposit and then took out a reducing balance loan at 5.8% p.a. (debited monthly) over 20 years. Determine how much Victoria would save if she could afford to make it a 15-year loan, compared to 20 years.

25. A couple borrow \$250 000 on a reducing balance loan at 4.85% p.a. with \$1629.24 monthly payments for 20 years. After the 10th payment, they increase their payments to \$2000. It takes them another 14 years to pay off their loan.

Show that they will save \$38 725, correct to the nearest dollar, by increasing their monthly payment to \$2000.

Fully worked solutions for this chapter are available online.

Answers

Chapter 13 Reducing balance loans

13.2 Modelling reducing balance loans with technology

13.2 Exercise

1. a. $5250 b. $65 170
2. $8882
3. $48 733.95
4. $54 489.50
5. $2032.27
6. $4488.56
7. 11.2%
8. a. 7 years b. $8268.18
9. 3 years
10. 6 years
11. 3 years
12. a. 3 years b. 6.65 years
13. a. 2 years b. 2.8 years
14. $682 067.89
15. a. $5095.12
    b. $3996.21
    c. Increasing the monthly repayments reduces the balance. Hence, the overall interest paid is reduced, in addition to the time taken to pay back the loan.
16. a. Interest = $312.50, balance owing = $49 848.99
    b. $151.01
    c. Interest = $311.56, balance owing = $49 697.04
    d. $151.95
17.

Month	Principal ($)	Interest ($)	Balance owing ($)
1	150 000.00	1200.00	149 791.99
2	149 791.99	1198.34	149 582.32
3	149 582.32	1196.66	149 370.96
4	149 370.96	1194.97	149 157.92
5	149 157.92	1193.26	148 943.18
6	148 943.18	1191.55	148 726.71
7	148 726.71	1189.81	148 508.51
8	148 508.51	1188.07	148 288.57
9	148 288.57	1186.31	148 066.87
10	148 066.87	1184.53	147 843.40

18. $81 160 per annum

13.3 The effect of repayment amounts, the interest rate and compounding periods on reducing balance loans [complex]

13.3 Exercise

1. a. i. 119.39 months, so 120 monthly payments
       ii. $91.16
   b. i. 258.32 fortnights, so 259 fortnightly repayments
       ii. $35.30
2. a. i. 71.59 months, so 72 monthly repayments
       ii. $170.93
   b. i. 154.87 fortnights, so 155 fortnightly repayments.
       ii. $116.61
3. Total interest if paid quarterly: $7335.60
   total interest if paid monthly: $7191.10
   total interest if pay fortnightly: $7152.22.
   The interest saved by paying monthly compared to quarterly is $144.50. The interest saved by paying fortnightly compared to quarterly is $183.38.
4. Total interest if paid quarterly: $3905.20
   total interest if paid monthly: $3786.76
   total interest if pay fortnightly: $3754.46.
   The interest saved by paying monthly compared to quarterly is $118.44. The interest saving by paying fortnightly compared to quarterly is $150.74.
5. a. $387.61 b. 54 months or 4.5 years
   c. $2941.58
6. a. $466.69 b. 60 months or 5 years
   c. $4635.80
7. a. 9.5 years b. $8010.70
   c. $876.50
8. a. 84 months or 7 years b. $5007.60
   c. $275.76
9. a. i. 24 months or 2 years
       ii. 35.96 months, so 36 monthly repayments, or 3 years
   b. i. 9.97 months, so 10 monthly repayments
       ii. 29.91 months, so 30 monthly repayments, or 2.5 years
10. a. $514.45
    b. i. 144 months or 12 years
        ii. 228 months or 19 years
11. Nerida will pay off her loan 10 months earlier.
12. The couple will pay off their loan 5 years earlier at 15 years.
13. a. $2734.16
    b. i. 73.28 months ii. $3013.16
14. a. $5244.08
    b. i. 86.96 months ii. $6098.14
15. a. $470.43 b. $462.11
    c. $1881.72 d. $81.72
16. a. $662.54 b. $659.75
    c. $11.16
17. a. $301.44 b. $297.85
    c. $86.16
18. a. $336.41
    b. Reduces the term of the loan to 21.18 months
19. Monthly repayments is the best option as it pays less interest.
20. $60 808.80
21. $20.88

13.4 Review

13.4 Exercise

1. $2157.32

2. a. $3895.50 b. $7326.40
   c. $25 486.73 d. $62 400.30

3. $55 108.16

4. a. $9500 b. $116 206.02

5. $71 482.15

6. a. $1 514.44 b. $857.76

7. a. 10 years b. $10 760.40

8. 7.8%

9. 96 months or 10 years

10. a. $596 844 b. $18 844

11. a. $2454.01 b. $2618.57

12. a. $173.62 b. 169 months or 6.5 years
    c. $212.26

13. a. 12 months or 1 years b. 24 months or 2 years

14. a. $225.40 b. 24 months or 5 years
    c. $2666.60

15. a. i. 124.24 months ii. $80.02
    b. i. 268.66 fortnights ii. $101.25

16. Loan 2

17. a.

Month	Principal ($)	Interest ($)	Balance owing ($)
1	85 000	637.50	84 872.73
2	84 872.73	636.54	84 744.50
3	84 744.51	635.58	84 615.31
4	84 615.32	634.61	84 485.16
5	84 485.16	633.63	84 354.03
6	84 354.03	632.65	84 221.91
7	84 221.92	631.66	84 088.81
8	84 088.81	630.66	84 954.70
9	83 954.71	629.66	84 819.59
10	83 819.06	628.64	84 683.47
11	83 683.48	627.62	84 546.33
12	83 546.33	626.59	84 408.15

b.

Month	Principal ($)	Interest ($)	Balance owing ($)
1	85 000	637.50	84 837.50
2	84 837.50	636.28	84 744.50
3	84 673.78	635.05	84 508.83
4	84 508.53	633.81	84 342.65
5	84 342.65	632.56	84 175.22
6	84 175.22	631.31	84 006.53
7	84 006.53	630.04	83 836.58
8	83 836.58	628.77	83 665.35
9	83 665.36	627.49	83 492.84
10	83 492.85	628.64	84 683.47
11	83 683.48	626.19	83 143.93
12	83 143.94	623.57	82 967.51

c. Sample responses can be found in the worked solutions in the online resources.

18.

Month	Principal ($)	Interest ($)	Balance owing ($)
1	100 000	562.50	99 414.26
2	99 414.26	559.21	98 825.23
3	98 825.23	555.89	98 232.88
4	98 232.88	552.56	97 637.20
5	97 637.20	549.21	97 038.17

19. a. $16 104.42 b. $12 257.93

20. a. $132.29 b. $200.52 c. $24 799.48

21. a. $10 790.40 b. 99.34 months c. $12 625.99

22. $30 101.40

23. 46.59 fortnights

24. $66 343.20

25. Sample responses can be found in the worked solutions in the online resources.