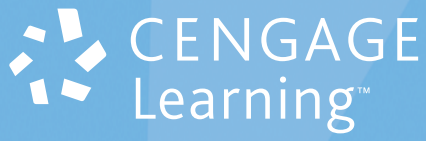
NEW CENTURY MATHS 2ND EDITION

NSW STAGES 5.2/5.3

ADVANCED

YEAR 9

Klaas Bootsma
David Badger
Sarah Hamper
Series editor: Robert Yen



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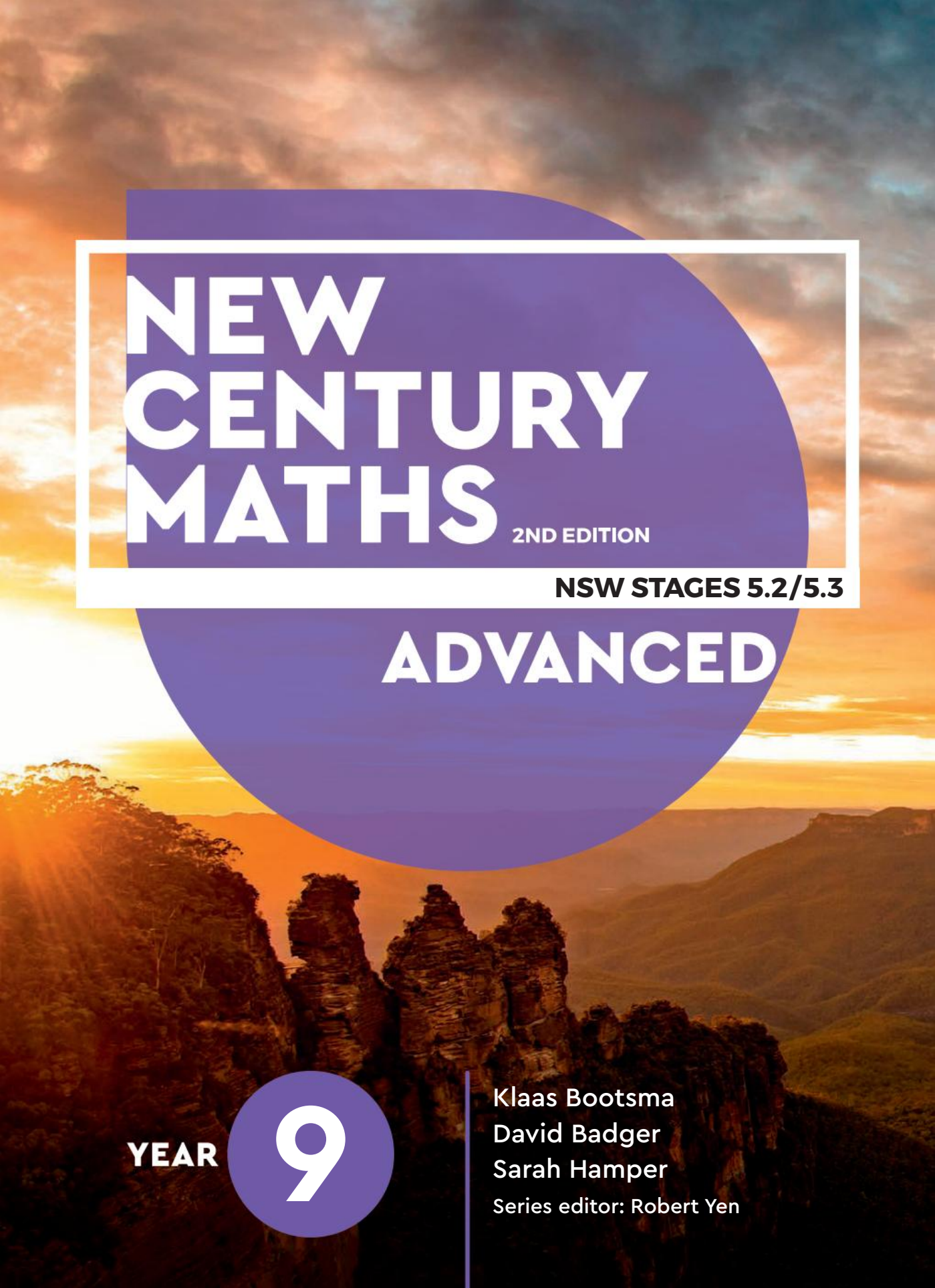

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Sarah Hamper
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PREFACE

Teachers, it's time to declutter and start teaching maths again. You asked and we listened.

New Century Maths 7–10 has been refreshed for the 2020s classroom, focusing on core skills to discourage 'teaching by syllabus dot points', featuring explicit grading of exercise questions, more 'flipped classroom' video tutorials, more applications and problem-solving questions, and worked solutions to every question.

In schools for over 25 years, *New Century Maths* is carefully mapped to the NSW syllabus and built on solid pedagogical foundations that integrate into every chapter practical classroom activities, engaging investigations, problem-solving, reasoning, communicating, reflecting, summarising, extension, revision, mental calculation, technology, numeracy and literacy.

We publish 2 levels of mathematics books for Years 9 and 10 (Stage 5):

- *New Century Maths,* covering NSW Stages 5.1/5.2
- *New Century Maths Advanced,* covering NSW Stages 5.1/5.2/5.3

This book, *New Century Maths 9 Advanced,* has been designed for Year 9 students progressing along Stages 5.2 or 5.3 of the continuum.

The *NelsonNet* student and teacher websites contain worksheets, video tutorials, topic tests, worked solutions and much more. We have provided an abundance of resources for teachers to plan and teach for a variety of pathways. *New Century Maths* is clear, concise, fresh and smart. We have designed this series to be user-friendly and uncomplicated so that teachers and students everywhere can pick it up and use straight away. So let's get started.

About the authors

Klaas Bootsma was head teacher of mathematics at Ambarvale High School and has taught at Lurnea and Grantham high schools. He was a senior HSC marker and has worked on the HSC Advice Line. Klaas also co-wrote *New Century Maths 11–12 Mathematics Standard 2.*

David Badger was principal of Toongabbie Christian School, deputy principal at Mt Annan Christian College and head teacher of mathematics at Eagle Vale High School. He has been involved in HSC marking and has worked on the HSC Advice Line.

Sarah Hamper teaches at Abbotsleigh School in Wahroonga and has taught at Meriden and Tara Anglican schools. She has an interest in gifted and talented (GAT) students and girls education. Sarah also co-wrote *New Century Maths 11–12 Mathematics Standard 2.*

Series editor **Robert Yen** taught at Hurlstone Agricultural High School. He co-wrote *New Century Maths 11-12 Mathematics Standard 2,* co-edited *Reflections,* the MANSW journal, and now works for Cengage as the mathematics publisher.

Contributing authors

Megan Boltze, Robert Yen and **Deborah Smith** wrote many of the *NelsonNet* worksheets and topic tests.

John Drake, Katie Jackson, Joanne Magner, Scott Smith and **Robert Yen** created the video tutorials.

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= NSW ONLY, NOT AUSTRALIAN CURRICULUM

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NSW syllabus

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NUMBER AND ALGEBRA				
Financial mathematics (Stages 5.1, 5.2)	3	Numeracy and calculation	2	Interest and depreciation
	8	Earning money		
Indices (Stages 5.1, 5.2)	5	Indices	5	Products and factors
Linear relationships (Stages 5.1, 5.2, 5.3)	11	Coordinate geometry and graphs	3	Graphing lines
Non-linear relationships (Stages 5.1, 5.2, 5.3)	11	Coordinate geometry and graphs	3	Graphing lines
			8	Graphing curves
			11	Quadratic equations and the parabola
Ratios and rates (Stages 5.2, 5.3)	3	Numeracy and calculation	8	Graphing curves
	11	Coordinate geometry and graphs		
Algebraic techniques (Stages 5.2, 5.3)	1	Products and factors	5	Products and factors
Equations (Stages 5.2, 5.3)	7	Equations	7	Equations and logarithms
			10	Simultaneous equations
			11	Quadratic equations and the parabola
Surds and indices (Stage 5.3)	2	Surds and Pythagoras' theorem	1	Products and factors
	5	Indices		
Polynomials (Stage 5.3)			14	Polynomials
Logarithms (Stage 5.3)			7	Equations and logarithms
Functions and other graphs (Stage 5.3)			16	Functions
MEASUREMENT AND GEOMETRY				
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Numbers of any magnitude (Stage 5.1)	5	Indices		
	10	Surface area and volume		
Right-angled triangles (Trigonometry) (Stages 5.1, 5.2)	2	Surds and Pythagoras' theorem	9	Further trigonometry
	4	Trigonometry		
Properties of geometrical figures (Stages 5.1, 5.2, 5.3)	6	Geometry	4	Surface area and volume
	13	Congruent and similar figures	13	Geometry
Volume (Stages 5.2, 5.3)	10	Surface area and volume	4	Surface area and volume

Strand and substrand	*New Century Maths 9 Advanced* chapter		*New Century Maths 10 Advanced* chapter	
Trigonometry and Pythagoras' theorem (Stage 5.3)			9	Further trigonometry
Circle geometry (Stage 5.3)			15	Circle geometry
STATISTICS AND PROBABILITY				
Single variable data analysis (Stages 5.1, 5.2, 5.3)	9	Analysing data	6	Comparing data
Double variable data analysis (Stage 5.2)			6	Comparing data
Probability (Stages 5.1, 5.2)	12	Probability	12	Probability

Australian curriculum

Australian CURRICULUM

Strand and substrand	*New Century Maths 9 Advanced* chapter		*New Century Maths 10 Advanced* chapter	
NUMBER AND ALGEBRA				
Real numbers	2	Surds and Pythagoras' theorem	1	Surds
	3	Numeracy and calculation	7	Graphing curves
	5	Indices	8	Equations and logarithms
	11	Coordinate geometry and graphs		
Money and financial mathematics	3	Numeracy and calculation	2	Interest and depreciation
Patterns and algebra	1	Algebra	5	Products and factors
	5	Indices	8	Equations and logarithms
	7	Equations	14	Polynomials
Linear and non-linear relationships	7	Equations	3	Graphing lines
	11	Coordinate geometry and graphs	5	Products and factors
			7	Graphing curves
			8	Equations and logarithms
			10	Simultaneous equations
			11	Quadratic equations and the parabola
			14	Polynomials
MEASUREMENT AND GEOMETRY				
Using units of measurement	10	Surface area and volume	4	Surface area and volume
Geometric reasoning	6	Geometry	11	Geometry
	13	Congruent and similar figures	15	Circle geometry
Pythagoras and trigonometry	2	Surds and Pythagoras' theorem	9	Further trigonometry
	4	Trigonometry		
STATISTICS AND PROBABILITY				
Chance	12	Probability	12	Probability
Data representation and interpretation	9	Analysing data	6	Comparing data

Year 9 content descriptions

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Content description	*New Century Maths 9 Advanced* chapter	
NUMBER AND ALGEBRA		
Real numbers		
ACMNA208: Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems	11	Coordinate geometry and graphs
ACMNA209: Apply index laws to numerical expressions with integer indices	5	Indices
ACMNA210: Express numbers in scientific notation	5	Indices
YEAR 10A ACMNA264: Define rational and irrational numbers and perform operations with surds and fractional indices	2	Surds and Pythagoras' theorem
	5	Indices
Money and financial mathematics		
ACMNA211: Solve problems involving simple interest	3	Numeracy and calculation
Patterns and algebra		
ACMNA212: Extend and apply the index laws to variables, using positive integer indices and the zero index	5	Indices
ACMNA213: Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate	1	Products and factors
YEAR 10 ACMNA233: Expand binomial products and factorise monic quadratic expressions using a variety of strategies	1	Products and factors
YEAR 10 ACMNA234: Substitute values into formulas to determine an unknown **YEAR 10 ACMNA235:** Solve problems involving linear equations, including those derived from formulas	7	Equations
YEAR 10A ACMNA269: Factorise monic and non-monic quadratic expressions and solve a wide range of quadratic equations derived from a variety of contexts	1	Products and factors
Linear and non-linear relationships		
ACMNA214: Find the distance between two points located on a Cartesian plane using a range of strategies, including graphing software	11	Coordinate geometry and graphs
ACMNA215: Sketch linear graphs using the coordinates of two points and solve linear equations	7 11	Equations Coordinate geometry and graphs
ACMNA294: Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software	11	Coordinate geometry and graphs
ACMNA296: Graph simple non-linear relations with and without the use of digital technologies and solve simple related equations	11	Coordinate geometry and graphs

Content description	*New Century Maths 9 Advanced* chapter	
MEASUREMENT AND GEOMETRY		
Using units of measurement		
ACMMG216: Calculate the areas of composite shapes	10	Surface area and volume
ACMMG217: Calculate the surface area and volume of cylinders and solve related problems	10	Surface area and volume
ACMMG218: Solve problems involving the surface area and volume of right prisms	10	Surface area and volume
ACMMG219: Investigate very small and very large time scales and intervals	10	Surface area and volume
Geometric reasoning		
ACMMG220: Use the enlargement transformation to explain similarity and develop the conditions for triangles to be similar	13	Congruent and similar figures
ACMMG221: Solve problems using ratio and scale factors in similar figures	13	Congruent and similar figures
YEAR 10 ACMMG243: Formulate proofs involving congruent triangles and angle properties **YEAR 10 ACMMG244:** Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes	13	Congruent and similar figures
Pythagoras and trigonometry		
ACMMG222: Investigate Pythagoras' theorem and its application to solving simple problems involving right-angled triangles	2	Surds and Pythagoras' theorem
ACMMG223: Use similarity to investigate the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles	4	Trigonometry
ACMMG224: Apply trigonometry to solve right-angled triangle problems	4	Trigonometry
YEAR 10 ACMMG245: Solve right-angled triangle problems including those involving direction and angles of elevation and depression	4	Trigonometry
STATISTICS AND PROBABILITY		
Chance		
ACMSP225: List all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Assign probabilities to outcomes and determine probabilities for events	12	Probability
ACMSP226: Calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or'	12	Probability
ACMSP227: Investigate reports of surveys in digital media and elsewhere for information on how data were obtained to estimate population means and medians	12	Probability
Data representation and interpretation		
ACMSP228: Identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly from secondary sources	9	Analysing data
ACMSP282: Construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including 'skewed', 'symmetric' and 'bi-modal'	9	Analysing data
ACMSP283: Compare data displays using mean, median and range to describe and interpret numerical data sets in terms of location (centre) and spread	9	Analysing data

ABOUT THIS BOOK

Coverage of the syllabus

- *New Century Maths 9 Advanced* covers both the NSW syllabus and the Australian curriculum, as shown by the table of contents and syllabus grids on the previous pages.
- This book contains Stages 5.1, 5.2 and 5.2 content. Stage 5.3 content is marked by * and coded in blue, while NSW-only content that is not Australian curriculum is marked by #.
- Each chapter begins with a **chapter outline** that includes the Working Mathematically proficiencies covered in each section.

Chapter outline

	Working mathematically				
2.01 Pythagoras' theorem	U	F			
2.02 Surds and irrational numbers	U	F		R	C
2.03 Simplifying surds*	U	F		R	
2.04 Adding and subtracting surds*	U	F		R	
2.05 Multiplying and dividing surds*	U	F		R	
2.06 Finding a shorter side	U	F	PS	R	
2.07 Mixed problems	U	F	PS		C
2.08 Testing for right-angled triangles	U	F		R	C
2.09 Pythagorean triads	U	F		R	C

*STAGE 5.3

U = UNDERSTANDING

Understanding is 'knowing and relating' maths. It is more than just learning facts. It's deep understanding, seeing how mathematical content is interconnected, knowing 'why' as well as 'how'.

F = FLUENCY

Fluency is 'applying' maths. It is being able to use mathematics competently and effectively. When you are fluent in a language, you have mastered it so that you can improvise and confidently use the correct word or phrase. Fluency in maths is choosing an appropriate skill, method or formula to use at the right place and time.

PS = PROBLEM SOLVING

Problem solving is 'modelling and investigating' with maths. It involves interpreting a rich, elaborate problem, selecting an appropriate strategy or model, solving the problem, then evaluating, communicating and justifying the solution.

R = REASONING

Reasoning is 'generalising and proving' with maths, using higher-order thinking to connect specific facts to general principles, using algebra, logic, proof and justification.

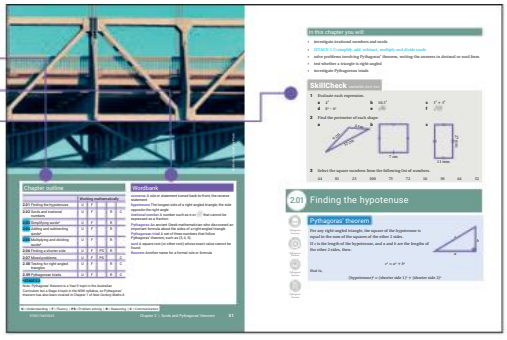
C = COMMUNICATING

Communicating is 'describing and explaining' maths, representing mathematical theory and solutions in words, algebraic symbols, special notations, diagrams, graphs and tables.

- **Understanding** and **Fluency** can be found in every exercise and activity, while **Problem solving**, **Reasoning** and **Communicating** are found in the **Investigations**, **Technology**, **Mental skills**, **Language of Maths** and **Topic summary** activities, and explicitly labelled in every exercise (see 'In each chapter' next page).

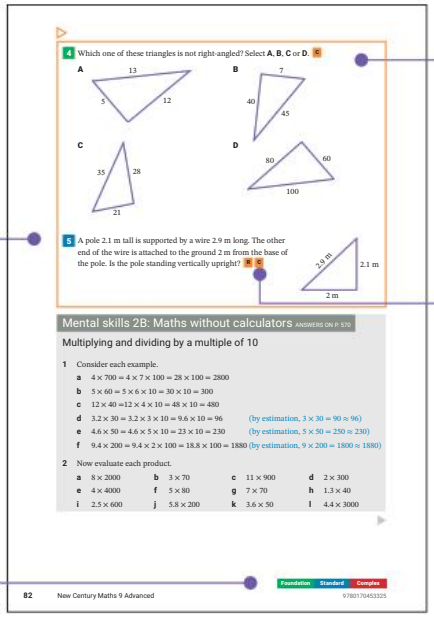
At the beginning of each chapter

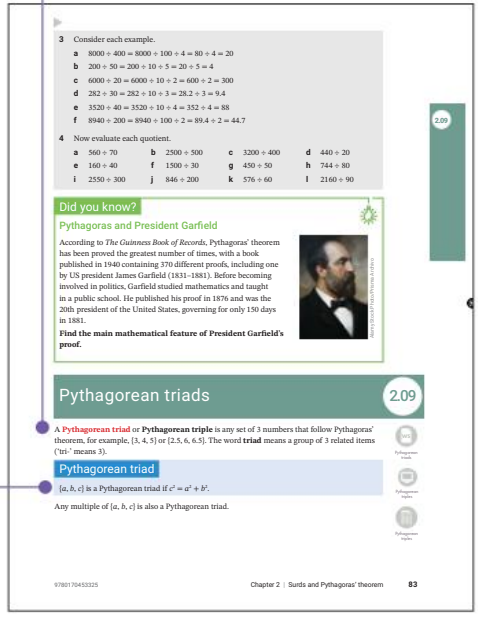
- A **Chapter outline** and **Wordbank** chapter glossary (a full glossary at the back of the book)
- **SkillCheck** reviews prerequisite skills and knowledge for the chapter.



In each chapter

- Important facts and formulas are highlighted in a shaded box.
- Glossary terms are printed in **red**
- Graded exercises are linked to worked examples and include multiple-choice questions, exam-style problems and realistic applications.

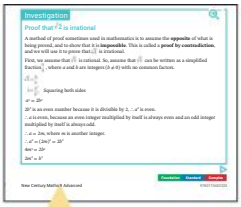




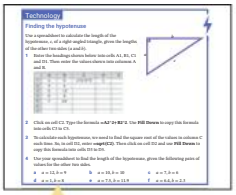
- The exercise questions are colour-coded for level of difficulty:

Foundation | Standard | Complex

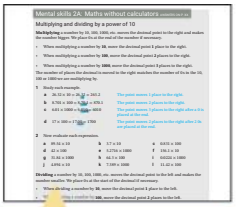
- Specific questions are also labelled **PS** for **Problem solving**, **R** for **Reasoning** and **C** for **Communicating.**



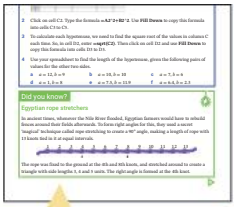
Investigations explore the syllabus in more detail, through group work, discovery and modelling activities.



Technology promotes ICT in the classroom, using spreadsheets, dynamic geometry software and the Internet.



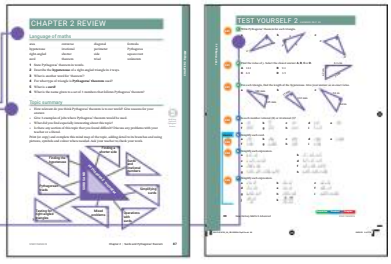
Mental skills reinforce mental calculation strategies ('calculator-free maths')



Did you know? contains interesting facts and applications of the mathematics learned in the chapter.

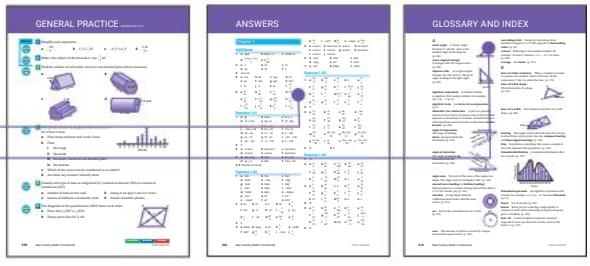
At the end of each chapter

- **Power plus** is an extension/challenge exercise.
- **Language of maths** has a chapter word list and literacy questions.
- **Topic summary** has a mind map activity with downloadable solutions.
- **Test Yourself** contains chapter revision linked to the relevant exercise set.
- **Practice sets** after every 3-4 chapters



At the end of the book

- **General practice** exercise
- **Answers** (worked solutions are on the teacher website).
- **Glossary and index.**



NelsonNet student website

Visit the *NelsonNet* student website, **www.nelsonnet.com.au** to access 140+ worksheets, 80+ videos and more:

Worksheets and **puzzle sheets**

Skillsheets of examples and exercises

Video tutorials: worked examples explained by 'flipped classroom' teachers

Technology worksheets

Chapter quizzes: interactive and self-marking

NelsonNet teacher website

Teachers can access the *NelsonNet* teacher website, also at **www.nelsonnet.com.au**, for:

- A **teaching program**
- **Topic tests**
- **Worked solutions** to every question
- **Quiz generator and questionbank (Cognero)**
- **Chapter PDFs**

Note: Complimentary access to these resources is only available to teachers who use this book as a core educational resource in their classroom. Contact your Cengage Education Consultant for information about access codes and conditions.

NelsonNetBook

NelsonNetBook is the web-based interactive version of this book found on *NelsonNet*.

- To each page of NelsonNetBook you can add notes, voice and sound bites, highlighting, weblinks and bookmarks
- **Zoom** and **Search** functions
- Chapters can be customised for different groups of students

New Century Maths 7-10 workbooks

Each *New Century Maths* title has a companion workbook for students to write in, sold separately, containing 200 pages of worksheets, puzzle sheets, *StartUp* topic assignments and weekly homework assignments. Handy for homework, class assessment, practice, revision, relief classes or 'catch-up' lessons.

New Century Maths / Maths in Focus 7–12 series

MATHEMATICAL VERBS

A glossary of 'doing words' commonly found in mathematics problems

analyse: study in detail the parts of a situation
bisect: cut in half
calculate: *see* **evaluate**
classify, identify: state the type, category or feature of an item or situation
comment: express an observation or opinion about a result
complete: fill in detail to make a statement, diagram or table
compare: show how two or more things are similar or different
construct: draw an accurate diagram
convert: change from one form to another, for example, from a fraction to a decimal, or from kilograms to grams
decrease: make smaller
describe: state the features of a situation
estimate: make an educated guess for a number, measurement or solution, to find roughly or approximately.
evaluate, calculate: find the value of a numerical expression, for example, 3×8^2 or $4x + 1$ when $x = 5$
expand: remove brackets in an algebraic expression by multiplying, for example, expanding $3(2y + 1)$ gives $6y + 3$
explain: describe why or how
factorise: take out the highest common facor (HCF) of an expression and insert brackets, for example, factorising $5x - 20$ gives $5(x - 4)$. The opposite of **expand.**
give reasons: show the rules or thinking used when solving a problem. See also **justify.**
graph: display on a number line, number plane or statistical graph.
hence find/prove: find an answer or prove a result using previous answers or information supplied
identify: *see* **classify.**
increase: make larger
interpret: find meaning in an answer or result
justify: give reasons or evidence to support your argument or conclusion. See also **give reasons.**
measure: determine the size of something, for example, use a ruler to determine the length of a pen.
prove that: *see* **show that**
recall: remember and state.
reduce (a fraction) to its lowest terms: see **simplify (a fraction)**.
round (a number): find the nearest approximation of a number. For example, 4.3 rounded to the nearest whole number is 4, \$12.9598 rounded to the nearest cent is \$12.96, 0.166 66 rounded to 3 decimal places is 0.167.
show that, prove: (in questions where the answer is given) use calculation, procedure or reasoning to prove that an answer or result is true
show working: show the steps you used to find the answer
simplify: give a result in its most basic, shortest, neatest form, for example, simplifying a ratio or algebraic expression
simplify (a fraction): reduce the numerator and denominator of a fraction by dividing by their highest common factor (HCF), for example, $\frac{16}{20}$ simplified is $\frac{4}{5}$.
simplify (a ratio or rate): reduce the terms or units of a ratio or rate by dividing by their highest common factor (HCF), for example, 10 : 4 simplified is 5 : 2.
sketch: draw a rough diagram that shows the general shape or ideas, less accurate than **construct**
solve: find the value(s) of an unknown variable in an equation or inequality
state: *see* **write.**
substitute: replace a variable by a number and evaluate
verify: check that a solution or result is correct, usually by substituting back into the equation or referring back to the problem
write correct to: See **round (a number).**
write, state: give the answer, formula or result without showing any working or explanation (This usually means that the answer can be found mentally, or in one step)

SYMBOLS AND ABBREVIATIONS

$=$	is equal to
$\neq$	is not equal to
$\approx$	is approximately equal to
$<$	is less than
$>$	is greater than
$\leq$	is less than or equal to
$\geq$	is greater than or equal to
( )	parentheses, round brackets
[ ]	(square) brackets
{ }	braces
$\pm$	plus or minus
-3	negative 3
π	pi = 3.14159
$0.\dot{1}5\dot{2}$	the recurring decimal 0.152152...
$^\circ$	degree
$42^\circ 17' 54''$	42 degrees, 17 minutes, 54 seconds
$\angle A$	angle A
$\triangle ABC$	triangle ABC
S 37° W	a compass bearing
217°T	a true bearing
$\parallel$	is parallel to
$\perp$	is perpendicular to
$\equiv$	is congruent to
$\|\|\|$	is similar to
$\therefore$	therefore
x^2	x squared, $x \times x$
x^3	x cubed, $x \times x \times x$

$\sqrt{\ }$	square root, radical sign
$\sqrt[3]{\ }$	cube root
$P(E)$	the probability of event E occurring
$P(\overline{E})$	the probability of event E not occurring
LHS	left-hand side
RHS	right-hand side
%	percentage
p.a.	per annum (per year)
sin	sine ratio
cos	cosine ratio
tan	tangent ratio
$\overline{x}$	the mean
σ	the standard deviation
Σ	the sum of
Q_1	first quartile or lower quartile
Q_2	median (second quartile)
Q_3	third quartile or upper quartile
IQR	interquartile range
α	alpha
θ	theta
μ	micro-, mu
m	gradient

1

NUMBER AND ALGEBRA

PRODUCTS AND FACTORS

Algebra uses symbols and formulas to describe number patterns and relationships in our natural and physical world. Algebra has helped people to unlock some of the secrets of the universe, such as showing that mass and energy are related to one another by the equation $E = mc^2$. Algebra is used to better understand climate change, population growth and economic growth. Algebraic modelling is used to predict the behaviour of bushfires, which may help save lives and reduce property damage.

Shutterstock.com/Karl Hofman

Chapter outline

	Working mathematically				
1.01 Adding and subtracting terms	U	F		R	
1.02 Multiplying and dividing terms	U	F			
1.03 Adding and subtracting algebraic fractions	U	F			
1.04 Multiplying and dividing algebraic fractions	U	F			
1.05 Expanding expressions	U	F			
1.06 Factorising expressions	U	F			
1.07 Expanding binomial products	U	F		R	C
1.08 Perfect squares*	U	F		R	C
1.09 Difference of 2 squares*	U	F		R	C
1.10 Mixed expansions*	U	F		R	C
1.11 Factorising special binomial products*	U	F			
1.12 Factorising quadratic expressions	U	F		R	
1.13 Factorising quadratic expressions of the form $ax^2 + bx + c$*	U	F		R	
1.14 Mixed factorisations*	U	F		R	
1.15 Factorising algebraic fractions*	U	F		R	

*** STAGE 5.3**

Wordbank

binomial An algebraic expression that consists of 2 terms, for example, $4a + 9$, $3 - y$, $x^2 - 4x$

binomial product An algebraic expression showing 2 or more binomials multiplied together, for example, $(x + 9)(3x - 4)$.

expand To rewrite an expression such as $5(2k - 6)$ without grouping symbols; for example, $5(2k - 6)$ expands to $10k - 30$

factorise To rewrite an expression with grouping symbols, by taking out the highest common factor; factorising is the opposite of expanding; for example, $9r^2 + 36r$ factorised is $9r(r + 4)$

highest common factor (HCF) The largest term that is a factor of 2 or more terms, for example, the HCF of $9r^2 + 36r$ is $9r$; also called greatest common divisor (GCD)

perfect square A square number or an algebraic expression that represents one, for example, 64, $(x + 9)^2$

quadratic expression An algebraic expression in which the highest power of the variable is 2, for example, $2x^2 + 5x - 3$ or $x^2 + 2$

quadratic trinomial A quadratic expression that consists of 3 terms, for example, $x^2 + 2x + 6$

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- add, subtract, multiply and divide algebraic expressions
- add, subtract, multiply and divide simple algebraic fractions
- expand and factorise algebraic expressions, including binomial products
- (STAGE 5.3) recognise and expand special binomial products that are perfect squares or difference of 2 squares
- factorise simple (monic) quadratic expressions
- (STAGE 5.3) factorise an expression with 4 terms by grouping in pairs
- (STAGE 5.3) factorise non-monic quadratic expressions, including those that are perfect squares or difference of 2 squares
- (STAGE 5.3) factorise and simplify algebraic fractions

SkillCheck ANSWERS ON P. 564

Algebraic expressions

Generalised arithmetic

1 Write an **algebraic expression** for each statement.

a The cost of p books at \$$Q$ each

b The selling price after a GST of \$$t$ is added to the marked price of \$40

c The number of metres in h km

d The average of m, p, v and w

e The number of dollars in y cents

2 If $f = 5$, $g = -4$ and $r = 2$, then evaluate each expression.

a $2f + g$ **b** $8 + rf$ **c** $f^2 - r^2$

d $3g^2 - 5$ **e** $r(f - g)$ **f** $\frac{5r + f}{3}$

3 If the perimeter of a rectangle with length l and width w has the formula $P = 2l + 2w$, find the perimeter of a rectangle with length 4.7 cm and width 2.5 cm.

4 Simplify each expression.

a $5n + 6n$ **b** $9h - 3h$ **c** $10pq - qp$ **d** $x^2 + 4x^2$

e $4 \times w$ **f** $8 \times 5x$ **g** $2g \times 3h$ **h** $3y \times 6y$

i $10x \div 2$ **j** $6b \div b$ **k** $\frac{16xy}{8x}$ **l** $\frac{15n^2}{25n}$

5 Evaluate each expression.

a $\frac{2}{5} + \frac{1}{3}$ **b** $\frac{3}{4} - \frac{2}{5}$ **c** $\frac{5}{8} \times \frac{1}{2}$ **d** $\frac{4}{5} \div \frac{2}{7}$

e $\frac{6}{11} \times \frac{5}{12}$ **f** $\frac{7}{8} + \frac{1}{3}$ **g** $\frac{4}{9} \div \frac{3}{10}$ **h** $\frac{5}{6} - \frac{3}{4}$

i $1\frac{3}{4} + 2\frac{5}{7}$ **j** $3\frac{1}{2} \times 2\frac{3}{5}$ **k** $2\frac{4}{9} \div 1\frac{5}{6}$ **l** $5\frac{1}{4} - 2\frac{2}{3}$

Adding and subtracting terms 1.01

Like terms are **terms** whose variables are exactly the same, for example, $3k$ and $7k$, $6ab$ and $2ba$, $9x^2$ and x^2.

We can add or subtract like terms because their variables represent the same number.

Adding and subtracting terms

Only **like terms** can be added or subtracted

For example, $2x - 5y$ *cannot* be simplified because x and y represent different numbers.

Example 1

Simplify each expression.

a $5r + 4s - 7r - 2s$ **b** $11u^2 - 5u + 3u - 7u^2$

Solution

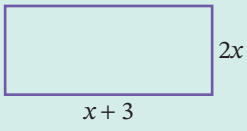
a
$$5r + 4s - 7r - 2s = 5r - 7r + 4s - 2s$$ Group the like terms
$$= -2r + 2s$$ $-2r$ and $2s$ cannot be simplified further
$$= 2s - 2r$$ Rewrite with positive term first

b
$$11u^2 - 5u + 3u - 7u^2 = 11u^2 - 7u^2 - 5u + 3u$$
$$= 4u^2 - 2u$$

Each + and – sign belongs to the term that follows it.

Example 2

Write a simplified algebraic expression for the perimeter of this rectangle.



Solution

$$\text{Perimeter} = x + 3 + 2x + x + 3 + 2x$$ Adding the lengths of the 4 sides.
$$= x + 2x + x + 2x + 3 + 3$$ Group the like terms.
$$= 6x + 6$$

EXERCISE 1.01 ANSWERS ON P. 564

Adding and subtracting terms U F R

1 Simplify each algebraic expression.

EXAMPLE 1

a	$2fg + 3fg - 4fg$	**b**	$6uvw - uvw + 4uvw$	**c**	$3t + 3 + 5t$
d	$3x + 2y + 8y$	**e**	$6g + h - 4g$	**f**	$2fg + 3f - 4fg$
g	$7p^2 - 3 - 2p^2$	**h**	$4p - q - 3q$	**i**	$-8r + 6r + 3$

Foundation | Standard | Complex

j $-3a + 4b - 7a$ **k** $4v + 3 + 2v + 7$ **l** $2y + 6x + y - 3x$

m $7p - 8q - p - q$ **n** $8 - 3w + 7 - 2w$ **o** $2n - 3 - 12n + 5$

p $8e - 3f - 2e - 4f$ **q** $2l^2 + 2l + 3l^2 - 6l$ **r** $b - 12b^2 + 6b^2 - 15b$

s $6p^3 - 5p^2 - 4p^3 + p^2$ **t** $3 - 8k - 5 - 9k$ **u** $-3 - 5d + 9 - 8d$

2 Simplify $15a^2 + 12a + 48 - 18a$. Select the correct answer **A**, **B**, **C** or **D**.

A $24a + 48$ **B** $27a^2 + 30a$

C $15a^2 - 6a + 48$ **D** $15a^2 + 30a - 48$

3 Use the substitutions $a = 2$, $b = 3$ and $c = 4$ to test whether each equation is correct or incorrect. R

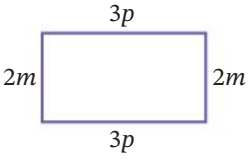
a $20c - 12c = 8c$ **b** $9b + b = 9b^2$ **c** $7a + 3b = 10 + a + b$

d $3a^2 + 5a^2 = 8a^2$ **e** $4ab + 3ab = 7ab$ **f** $12c - c = 12$

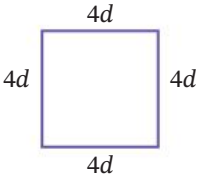
EXAMPLE 2

4 Write a simplified algebraic expression for the perimeter of each shape.

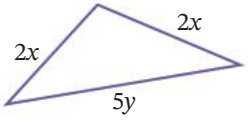
a



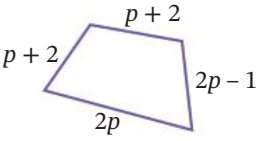
b



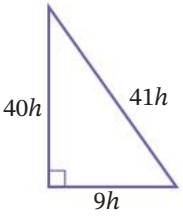
c



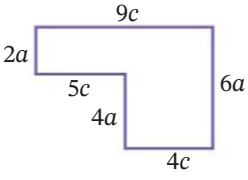
d



e



f



5 Draw a triangle and write algebraic expressions for its side lengths so that it has a perimeter of $7d + 6$. R

6 Draw a rectangle and find algebraic expressions for its length and width so that it has a perimeter of $18a - 30$. R

1.02 Multiplying and dividing terms

WS Perimeter and area

Multiplying and dividing terms

When **multiplying and dividing terms**, multiply or divide the numbers first, then the variables.

Foundation Standard Complex

9780170453325

Example 3

Simplify each expression.

a $2 \times 5n \times 4m$ **b** $3d \times 4d$ **c** $2g \times 3h \times (-5f)$

Solution

a $2 \times 5n \times 4m = 2 \times 5 \times 4 \times n \times m$
$= 40mn$

Multiply the numbers and multiply the variables
Remember to write the variables in alphabetical order

b $3d \times 4d = 3 \times 4 \times d \times d$
$= 12d^2$

$d \times d = d^2$

c $2g \times 3h \times (-5f) = 2 \times 3 \times (-5) \times g \times h \times f$
$= -30fgh$

Remember to write the variables in alphabetical order

Example 4

Simplify each expression.

a $\frac{15abc}{3ab}$ **b** $\frac{k^2m}{4km}$ **c** $27cf \div (-3fc)$ **d** $35p^2 \div 15pw$

Solution

a $\frac{15abc}{3ab} = \frac{15^5 \cancel{a}^1 \cancel{b}^1 c}{\cancel{3}^1 \cancel{a}^1 \cancel{b}^1}$
$= 5c$

Divide the numbers and divide the variables

We can cross out the ab in the **numerator** and **denominator** because $\frac{a}{a} = 1$ and $\frac{b}{b} = 1$

b $\frac{k^2m}{4km} = \frac{kkm}{4km}$
$= \frac{1}{4} \frac{k \cancel{km}^1}{\cancel{km}^1}$
$= \frac{k}{4}$

$k^2 = k \times k = kk$

Divide the numbers and divide the variables

c $27cf \div (-3fc) = \frac{27cf}{-3fc}$
$= \frac{27}{-3} \frac{\cancel{cf}^1}{\cancel{cf}^1}$
$= -9$

The variables divide by themselves to give 1

d $35p^2 \div 15pw = \frac{35p^2}{15pw}$
$= \frac{35^7}{15^3} \frac{\cancel{p}^1 p}{\cancel{p}^1 w}$
$= \frac{7p}{3w}$

15 and 35 can both be divided by 5

EXERCISE 1.02 ANSWERS ON P. 564

Multiplying and dividing terms U F

EXAMPLE 3

1 Simplify each expression.

a	$5 \times 8x$	**b**	$5n \times 3$	**c**	$b \times 5a$
d	$7m \times 6t$	**e**	$-4 \times 3w$	**f**	$2k \times (-3p)$
g	$4d \times 3e$	**h**	$6v \times 4w$	**i**	$11q \times 8q$
j	$-5a \times (-4a)$	**k**	$2y \times 3 \times 4b$	**l**	$-6w \times 3w \times 2$
m	$-9c \times (-4h) \times 2k$	**n**	$\frac{1}{3}a \times 24bc$	**o**	$-5d \times (-3e) \times (-2w)$
p	$(5n)^2$	**q**	$(-3r)^2$	**r**	$(at)^2$
s	$-15d \times \frac{2}{3}ad$	**t**	$-\frac{1}{4}ek \times (-4k)$	**u**	$(-2xy)^2$

EXAMPLE 4

2 Simplify each expression.

a	$\frac{12x}{3}$	**b**	$\frac{15y}{y}$	**c**	$\frac{36a}{4a}$
d	$18mn \div 6m$	**e**	$8vw \div 48vw$	**f**	$\frac{64abc}{16b}$
g	$\frac{18g}{-3g}$	**h**	$-25yz \div 5y$	**i**	$\frac{9p}{-45pq}$
j	$\frac{-75xyz}{-25x}$	**k**	$10r^2 \div 40r$	**l**	$-12hk^2 \div 2hk$
m	$24ak^2 \div 8a^2$	**n**	$16mwx \div 12mx$	**o**	$7gh \div 21g$

3 Simplify $\frac{9yx^2}{24xy^2}$. Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{3x^2}{8y^2}$ **B** $\frac{3x^3}{8y^3}$ **C** $\frac{3x}{8y}$ **D** $\frac{3y}{8x}$

4 Simplify each expression.

a	$2a \times 3b \times 5c$	**b**	$4p \times 3 \times 2q$	**c**	$40n^2 \div 5n$
d	$8b^2 \div 40b^2$	**e**	$6h \times 2k \times (-3)$	**f**	$-2 \times 5k \times (-4k)$
g	$\frac{6d}{12}$	**h**	$\frac{7r}{49r}$	**i**	$(-33m) \div 3$
j	$\frac{-9w}{-45w}$	**k**	$8ab \div 2bc$	**l**	$2ef \times (-3fe)$
m	$18ac^2d \div 27acd$	**n**	$-3pq \times (-2q) \times (-4)$	**o**	$\frac{24v^2w}{18v}$

5 Use the substitutions $p = 5$, $q = 4$ and $r = 3$ to test whether each equation is correct or incorrect.

a	$3 \times 4p = 12p$	**b**	$q + q = q^2$
c	$2r \times 3r = 6r^2$	**d**	$3q^2 \times 5q^2 = 15q^2$
e	$\frac{12p}{3p} = 4$	**f**	$\frac{7qr}{qr} = 7$
g	$3r \times 5q = 15qr$	**h**	$p \times q \times q = 2pq$
i	$\frac{14r^2}{7r} = 2r$	**j**	$\frac{5pq}{10p} = \frac{q}{2}$

Foundation Standard Complex

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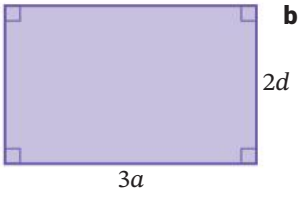
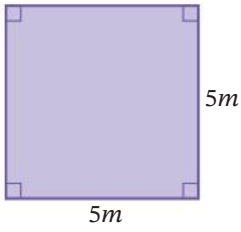
6 Write a simplified algebraic expression for the area of each shape.

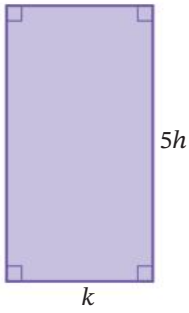
a



b



c



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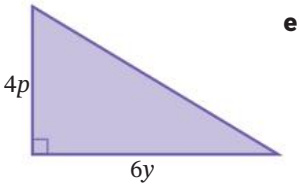

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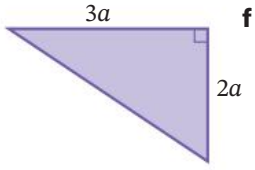
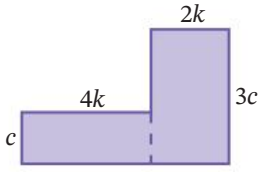
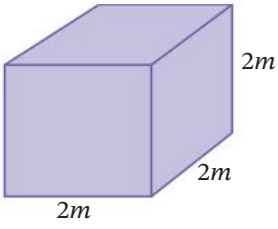


f

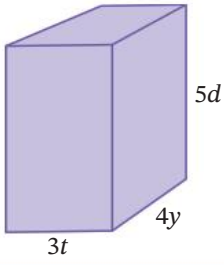


7 Find an algebraic expression for the volume of each rectangular prism.

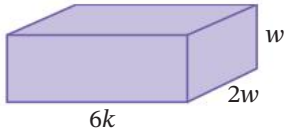
a



b



c



Adding and subtracting algebraic fractions

1.03

Adding and subtracting fractions

To **add or subtract fractions**, convert them (if needed) so that they have the same denominator, then simply add or subtract the numerators.

Example 5

Simplify each expression.

a $\frac{7r}{10} - \frac{3r}{10}$

b $\frac{k}{3} + \frac{k}{2}$

c $\frac{3h}{4} + \frac{2h}{5}$

d $\frac{5m}{6} - \frac{2m}{3}$

e $\frac{3}{4} + \frac{7a}{10}$

f $\frac{5x}{8} - \frac{2w}{3}$

Algebraic fractions

Adding and subtracting fractions

Adding and subtracting algebraic fractions

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Solution

a $\frac{7r}{10} - \frac{3r}{10} = \frac{4r}{10}$

$= \frac{2r}{5}$

b $\frac{k}{3} + \frac{k}{2} = \frac{2 \times k}{2 \times 3} + \frac{3 \times k}{3 \times 2}$ Common denominator $= 2 \times 3 = 6$

$= \frac{2k}{6} + \frac{3k}{6}$

$= \frac{5k}{6}$

c $\frac{3h}{4} + \frac{2h}{5} = \frac{5 \times 3h}{5 \times 4} + \frac{4 \times 2h}{4 \times 5}$ Common denominator $= 5 \times 4 = 20$

$= \frac{15h}{20} + \frac{8h}{20}$

$= \frac{23h}{20}$

d $\frac{5m}{6} - \frac{2m}{3} = \frac{15m}{18} - \frac{12m}{18}$

$= \frac{3m}{18}$

$= \frac{m}{6}$

or $\frac{5m}{6} - \frac{2m}{3} = \frac{5m}{6} - \frac{4m}{6}$ The lowest common denominator is 6

$= \frac{m}{6}$

e $\frac{3}{4} + \frac{7a}{10} = \frac{15}{20} + \frac{14a}{20}$

$= \frac{15 + 14a}{20}$

f $\frac{5x}{8} - \frac{2w}{3} = \frac{15x}{24} - \frac{16w}{24}$

$= \frac{15x - 16w}{24}$

EXERCISE 1.03 ANSWERS ON P. 564

Adding and subtracting algebraic fractions U F

EXAMPLE 5

1 Simplify each expression.

a $\frac{w}{4} + \frac{2w}{4}$ **b** $\frac{4k}{8} + \frac{7k}{8}$ **c** $\frac{7m}{10} - \frac{2m}{10}$ **d** $\frac{x}{3} + \frac{2x}{3}$

e $\frac{4}{q} + \frac{5}{q}$ **f** $\frac{5}{3d} - \frac{2}{3d}$ **g** $\frac{4r}{w} + \frac{r}{w}$ **h** $\frac{5}{c} - \frac{2}{c}$

i $\frac{4t}{3} + \frac{s}{3}$ **j** $\frac{11y}{2h} - \frac{7y}{2h}$ **k** $\frac{6}{a} - \frac{5}{a}$ **l** $\frac{8}{5e} + \frac{7}{5e}$

m $\frac{p}{z} + \frac{3}{z}$ **n** $\frac{5u}{8g} - \frac{3u}{8g}$ **o** $\frac{4}{9f} - \frac{1}{9f}$ **p** $\frac{7e}{10} - \frac{3e}{10}$

q $\frac{5a}{6} + \frac{5a}{6}$ **r** $\frac{5}{2d} + \frac{3}{2d}$ **s** $\frac{7}{5k} + \frac{13}{5k}$ **t** $\frac{12}{4a} - \frac{8}{4a}$

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2 Simplify the expression $\frac{9}{4k} - \frac{3}{4k}$. Select **A**, **B**, **C** or **D**.

A $\frac{6}{4k}$ **B** $\frac{3}{4k}$ **C** $\frac{3}{2k}$ **D** $\frac{6}{k}$

3 Simplify each expression.

a $\frac{x}{3} + \frac{x}{4}$ **b** $\frac{s}{3} - \frac{s}{7}$ **c** $\frac{h}{5} + \frac{h}{3}$ **d** $\frac{m}{7} - \frac{m}{2}$

e $\frac{w}{4} + \frac{w}{5}$ **f** $\frac{5t}{4} - \frac{2t}{5}$ **g** $\frac{2p}{5} + \frac{p}{3}$ **h** $\frac{5r}{2} - \frac{5r}{3}$

i $\frac{5y}{6} + \frac{2y}{3}$ **j** $\frac{3m}{4} - \frac{5m}{12}$ **k** $\frac{5a}{9} - \frac{a}{6}$ **l** $\frac{3y}{10} + \frac{7y}{15}$

m $\frac{3c}{2} - \frac{c}{5}$ **n** $\frac{2d}{11} - \frac{r}{3}$ **o** $\frac{3h}{5} + \frac{2d}{3}$ **p** $\frac{5}{6} + \frac{4w}{5}$

q $\frac{2a}{7} + \frac{3}{4}$ **r** $\frac{4e}{7} - \frac{2e}{5}$ **s** $\frac{3m}{2} - \frac{5n}{11}$ **t** $\frac{2k}{5} + \frac{m}{6}$

4 Simplify the expression $\frac{5m}{12} - \frac{3m}{8}$. Select **A**, **B**, **C** or **D**.

A $\frac{2m}{4}$ **B** $\frac{m}{24}$ **C** $\frac{m}{2}$ **D** $\frac{3m}{20}$

5 Simplify the expression $\frac{4h}{9} + \frac{2d}{5}$. Select **A**, **B**, **C** or **D**.

A $\frac{36h+10d}{45}$ **B** $\frac{4h+2d}{14}$ **C** $\frac{20h+18d}{45}$ **D** $\frac{9h+18d}{14}$

Multiplying and dividing algebraic fractions

1.04

Multiplying and dividing algebraic fractions

- To **multiply fractions**, cancel any common factors, then multiply the numerators and denominators separately
- To **divide by a fraction** $\frac{a}{b}$, multiply by its **reciprocal** $\frac{b}{a}$

WS Algebraic fractions

Algebraic fractions puzzle

Algebraic fractions

Upside-down fractions

Example 6

Simplify each **product**.

a $\frac{3}{d} \times \frac{4}{c}$ **b** $\frac{4}{k} \times \frac{3k}{16}$ **c** $\frac{3x}{4} \times \frac{2x}{9}$

Solution

a $\frac{3}{d} \times \frac{4}{c} = \frac{3 \times 4}{d \times c}$

$= \frac{12}{dc}$

b $\frac{4}{k} \times \frac{3k}{16} = \frac{\cancel{4}^{1}}{\cancel{k}_{1}} \times \frac{\cancel{3k}^{3}}{\cancel{16}_{4}}$

$= \frac{3}{4}$

c $\frac{3x}{4} \times \frac{2x}{9} = \frac{\cancel{3x}^{1}}{\cancel{4}_{2}} \times \frac{\cancel{2x}^{1}}{\cancel{9}_{3}}$

$= \frac{x^2}{6}$

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Algebraic fractions

Example 7

Simplify each quotient.

a $\frac{3}{h} \div \frac{4}{k}$ **b** $\frac{xy}{5} \div \frac{3x}{25}$

Solution

a $\frac{3}{h} \div \frac{4}{k} = \frac{3}{h} \times \frac{k}{4}$ The reciprocal of $\frac{4}{k}$ is $\frac{k}{4}$.

$= \frac{3k}{4h}$

b $\frac{xy}{5} \div \frac{3x}{25} = \frac{{}^{1}\cancel{xy}}{\cancel{5}_{1}} \times \frac{\cancel{25}^{5}}{\cancel{3x}_{1}}$

$= \frac{5y}{3}$

EXERCISE 1.04 ANSWERS ON P. 564

Multiplying and dividing algebraic fractions U F

EXAMPLE 6

1 Simplify the product $\frac{3x}{5} \times \frac{w}{6}$. Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{xw}{10}$ **B** $\frac{3xw}{11}$ **C** $\frac{3xw}{30}$ **D** $\frac{18x+5w}{30}$

EXAMPLE 7

2 Simplify each product.

a $\frac{w}{3} \times \frac{1}{2}$ **b** $\frac{s}{5} \times \frac{t}{4}$ **c** $\frac{3}{h} \times \frac{5}{6k}$

d $\frac{4}{m} \times \frac{3}{n}$ **e** $\frac{l}{3} \times \frac{5}{f}$ **f** $\frac{1}{v} \times \frac{2}{3v}$

g $\frac{2}{x} \times \frac{3}{x}$ **h** $\frac{a}{b} \times \frac{c}{d}$ **i** $\frac{d}{e} \times \frac{e}{g}$

j $\frac{4ad}{9} \times \frac{d}{16a}$ **k** $\frac{5p}{4} \times \frac{8}{15p}$ **l** $\frac{4}{ak} \times \frac{3a}{5k}$

m $\frac{2}{3} \times \frac{9u}{10}$ **n** $\frac{u}{3} \times \frac{3}{u}$ **o** $\frac{3z}{r} \times \frac{2r}{9dz}$

3 Simplify each quotient.

a $\frac{r}{2} \div \frac{r}{5}$ **b** $\frac{m}{6} \div \frac{n}{3}$ **c** $\frac{h}{2} \div \frac{h}{8}$

d $\frac{q}{4} \div \frac{3}{4}$ **e** $\frac{3y}{5} \div \frac{2y}{d}$ **f** $\frac{4t}{9} \div \frac{3t}{5}$

g $\frac{3}{e} \div \frac{5}{e}$ **h** $\frac{3}{2a} \div \frac{b}{6a}$ **i** $\frac{5m}{n} \div \frac{2m}{3n}$

j $\frac{h}{k} \div \frac{k}{h}$ **k** $\frac{8w}{3x} \div \frac{2w}{9x}$ **l** $\frac{3s}{4} \div \frac{6s}{11}$

m $\frac{t}{3} \div \frac{3t}{5u}$ **n** $\frac{3e}{7g} \div \frac{e}{14g}$ **o** $\frac{xh}{5} \div \frac{3h}{15}$

4 Simplify the quotient $\frac{3}{4m} \div \frac{9n}{8m}$. Select **A**, **B**, **C** or **D**.

A $\frac{3n}{2}$ **B** $\frac{32m^2}{27n}$ **C** $\frac{36m}{24n}$ **D** $\frac{2}{3n}$

5 Simplify each expression.

a $\frac{3p}{2} \div \frac{p^2}{4}$ **b** $\frac{2w}{7} \div \frac{5w}{6}$ **c** $\frac{8y}{5} \times \frac{3}{32y^2}$

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d $\frac{5}{2g} \div \frac{2}{g}$ **e** $\frac{xy}{z} \times \frac{5}{y}$ **f** $\frac{a}{b} \times \frac{c}{b} \times \frac{b}{3a}$

g $5 \div \frac{3}{5b}$ **h** $\frac{4t}{9} \div 3$ **i** $\frac{5mn}{2d} \times \frac{4d}{n} \times \frac{1}{15mn}$

j $5p \div \frac{7}{10pt}$ **k** $\frac{s}{3} \div \frac{5s}{2} \times \frac{3s}{7}$ **l** $\frac{7}{h} \times \frac{1}{42h}$

m $\frac{5ty}{k} \times \frac{5ky}{t}$ **n** $\frac{6}{r} \times \frac{5r}{9} \div \frac{15}{yh}$ **o** $\frac{6cf}{q} \div \frac{5}{qf} \div \frac{10c}{a}$

6 Simplify the expression $\frac{4ab}{5} \div \frac{a}{2b} \div \frac{8b}{15}$. Select **A**, **B**, **C** or **D**.

A 3 **B** $3b$ **C** $\frac{3b}{5}$ **D** $\frac{3b}{a}$

1.04

Mental skills 1A: Maths without calculators ANSWERS ON P. 565

Multiplying and dividing by 5, 15, 25 and 50

It is easier to multiply or divide a number by 10 than by 5. So whenever we multiply or divide a number by 5, we can double the 5 (to make 10) and then adjust the first number.

1 Study each example.

a **To multiply by 5**, halve the number, then multiply by 10.

$18 \times 5 = 18 \times \frac{1}{2} \times 10$ (or $9 \times 2 \times 10$)

$= 9 \times 10$

$= 90$

b **To multiply by 50**, halve the number, then multiply by 100.

$26 \times 50 = 26 \times \frac{1}{2} \times 100$ (or $13 \times 2 \times 100$)

$= 13 \times 100$

$= 1300$

c **To multiply by 25**, quarter the number, then multiply by 100.

$44 \times 25 = 44 \times \frac{1}{4} \times 100$ (or $11 \times 4 \times 25$)

$= 11 \times 100$

$= 1100$

d **To multiply by 15**, halve the number, then multiply by 30.

$8 \times 15 = 8 \times \frac{1}{2} \times 30$ (or $4 \times 2 \times 15$)

$= 4 \times 30$

$= 120$

e **To divide by 5**, divide by 10 and double the answer. We do this because there are two 5s in every 10.

$140 \div 5 = 140 \div 10 \times 2$

$= 14 \times 2$

$= 28$

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f **To divide by 50**, divide by 100 and double the answer. This is because there are two 50s in every 100.

$400 \div 50 = 400 \div 100 \times 2$

$= 4 \times 2$

$= 8$

g **To divide by 25**, divide by 100 and multiply the answer by 4. This is because there are four 25s in every 100.

$600 \div 25 = 600 \div 100 \times 4$

$= 6 \times 4$

$= 24$

h **To divide by 15**, divide by 30 and double the answer. This is because there are two 15s in every 30.

$240 \div 15 = 240 \div 30 \times 2$

$= 8 \times 2$

$= 16$

2 Now evaluate each expression.

a	32×5	**b**	14×5	**c**	48×5	**d**	18×50
e	52×50	**f**	36×25	**g**	28×5	**h**	12×25
i	12×15	**j**	22×35	**k**	$90 \div 5$	**l**	$170 \div 5$
m	$230 \div 5$	**n**	$1300 \div 50$	**o**	$900 \div 50$	**p**	$300 \div 25$
q	$1000 \div 25$	**r**	$360 \div 45$	**s**	$210 \div 15$	**t**	$360 \div 15$

1.05 Expanding expressions

Algebra using diagrams

Expanding and simplifying expressions

The distributive law for expanding an expression

Multiply each term inside the brackets by the term outside

$$a(b + c) = ab + ac$$

$$a(b - c) = ab - ac$$

Example 8

Expand each expression.

a $4(5 + d)$ **b** $2w(3w + 10)$

Solution

a $4(5 + d) = 4 \times 5 + 4 \times d$

$= 20 + 4d$

multiply each term inside the brackets by the term outside

$a(b + c) = ab + ac$

b $2w(3w + 10) = 2w \times 3w + 2w \times 10$
$= 6w^2 + 20w$

Example 9

Expand each expression.

a $3g(h - 2)$ **b** $r(10 - 4r)$

Solution

a $3g(h - 2) = 3g \times h - 3g \times 2$
$= 3gh - 6g$

multiply each term inside the brackets by the term outside

$a(b - c) = ab - ac$

b $r(10 - 4r) = r \times 10 - r \times 4r$
$= 10r - 4r^2$

Example 10

Expand each expression.

a $-(y + 4)$ **b** $-3(2m - 7)$

Solution

a $-(y + 4) = -1(y + 4)$ $-( )$ is the same as $-1 \times ( )$
$= -y + (-4)$
$= -y - 4$

b $-3(2m - 7) = -3 \times 2m - (-3) \times 7$
$= -6m - (-21)$
$= -6m + 21$

Example 11

Expand and simplify by collecting like terms.

a $4(3p + 2) - 5p$ **b** $5(2e - 3) - 4(1 - 5e)$

c $7(n - 3) + n(n - 1)$

Solution

a $4(3p + 2) - 5p = 4 \times 3p + 4 \times 2 - 5p$ Expanding
$= 12p + 8 - 5p$
$= 12p - 5p + 8$ Collecting like terms to simplify
$= 7p + 8$

b $5(2e-3)-4(1-5e) = 5\times 2e - 5\times 3 - 4\times 1 - (-4)\times 5e$ Expanding

$= 10e - 15 - 4 - (-20e)$

$= 10e - 19 + 20e$

$= 30e - 19$ Collecting like terms to simplify

c $7(n-3)+n(n-1) = 7n - 21 + n^2 - n$

$= n^2 + 7n - n - 21$ It's conventional to place n^2 first

$= n^2 + 6n - 21$ Collecting like terms to simplify

EXERCISE 1.05 ANSWERS ON P. 565

Expanding expressions U F

EXAMPLE 8

1 Expand each expression.

a $4(h+6)$ **b** $5(3+t)$ **c** $3(2b+7)$

d $5(8+3x)$ **e** $x(x+9)$ **f** $p(5+p)$

g $3r(2r+1)$ **h** $4y(1+3y)$ **i** $10e(2e+4f)$

EXAMPLE 9

2 Expand each expression.

a $3(t-2)$ **b** $7(5-d)$ **c** $8(8g-3)$

d $w(w-7)$ **e** $a(a-1)$ **f** $6h(3h-1)$

g $4x(3x-1)$ **h** $5x(2x-4y)$ **i** $3a(4b-7a)$

3 Expand $-5u(8+2u)$. Select the correct answer **A**, **B**, **C** or **D**.

A $3u - 3u^2$ **B** $-40u + 10u^2$ **C** $-40u - 10u^2$ **D** $-40u - 3u^2$

4 Use the substitution $x = 5$ to test whether each equation is correct or incorrect.

a $3(x+3) = 3x + 6$ **b** $x(4-2x) = 4x - 2x^2$

EXAMPLE 10

5 Expand each expression.

a $-(a-5)$ **b** $-(a+5)$ **c** $-2(x+6)$

d $-2(x-6)$ **e** $-(11+w)$ **f** $-(11-w)$

g $-y(y+9)$ **h** $-x(8-m)$ **i** $-4t(t-8)$

6 Expand $-k(3k-7)$. Select **A**, **B**, **C** or **D**.

A $-2k^2 - 8k$ **B** $-3k^2 + 7k$ **C** $-3k^2 - 7k$ **D** $-3k + 7k^2$

EXAMPLE 11

7 Expand $3(2-7y)+5y$. Select **A**, **B**, **C** or **D**.

A $6 - 26y$ **B** $6 + 16y$ **C** $6 - 5y$ **D** $6 - 16y$

8 Expand and simplify each expression.

a $5(3m+2)+4m$ **b** $3(1-5e)+6e$

c $4w - 2(5+2w)$ **d** $8 - 5(2x-7)$

e $5(2a+3)+4(a+7)$ **f** $3(2d+3)-5(d+4)$

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g $7(3g - 1) + 4(2g - 3)$ **h** $4(3 - 4w) - 2(w - 5)$

i $5(6c - 3) - 3(4 - 3c)$ **j** $t(t + 4) + 3(t + 4)$

k $4(3 + h) + h(7 - 2h)$ **l** $6(2e - 1) - (5 - 3e)$

m $3x(2x + 5) + 4(2x + 5)$ **n** $v(2v + 3) - 6(v + 1)$

o $3(1 - 2w) - w(2 - w)$ **p** $2y(3y - 7) - 5(3y - 7)$

9 Expand $3(1 - 4k) - 2(3k + 7)$. Select **A**, **B**, **C** or **D**.

A $17 - 18k$ **B** $11 - 6k$ **C** $-11 - 18k$ **D** $-17 + 6k$

Factorising expressions 1.06

The highest common factor (HCF)

The **highest common factor (HCF)** or **greatest common divisor (GCD)** of 2 or more terms is the largest term that is a factor of all of the terms. To find the HCF of algebraic terms:

- find the HCF of the numbers
- find the HCF of the variables
- multiply them together

HCF by factor trees

Factorising using diagrams

Factorising puzzle

Example 12

Find the highest common factor (HCF) of each pair of terms.

a $20x^2$ and $15xy$ **b** 16 and $12b$

Solution

Find the HCF of the numbers and the HCF of the variables. Their product is the HCF of the expression.

a The HCF of 20 and 15 is 5.
The HCF of x^2 and xy is x, since it is the 'largest' common part of x^2 and xy.
$\therefore$ The HCF of $20x^2$ and $15xy = 5 \times x = 5x$

b The HCF of 16 and 12 is 4.
There is **no** HCF of the variables because b is in $12b$ but not in 16.
$\therefore$ The HCF of 16 and $12b$ is 4.

Factorising algebraic expressions

When $4(2y + 5)$ is **expanded**, the answer is $8y + 20$.

Factorising is the reverse of expanding. Factorisation breaks an expression into factors. To factorise $8y + 20$, we take out the greatest common divisor and **insert** brackets. The result is $4(2y + 5)$. The factors are 4 and $(2y + 5)$.

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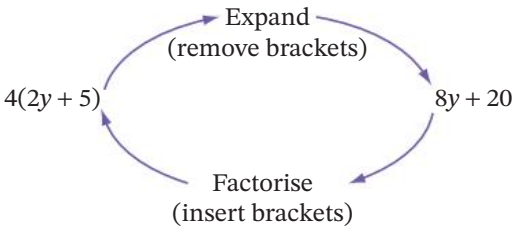

Factorising an expression

- Find the HCF of the terms and write it outside the brackets
- Divide each term by the HCF and write the answers inside the brackets

$$ab + ac = a(b + c)$$
$$ab - ac = a(b - c)$$

- To check that the factorised answer is correct, expand it

Factorising expressions

Example 13

Factorise each expression.

a $8y + 16$ **b** $25b^2 - 20ab$ **c** $v(4 + w) + 2(4 + w)$

Solution

a The HCF of $8y$ and 16 is 8.

$\therefore 8y + 16 = 8 \times y + 8 \times 2$ Rewrite the expression using the HCF 8

$= 8(y + 2)$ Write the HCF at the front of the brackets

b The HCF of $25b^2$ and $20ab$ is $5b$.

$\therefore 25b^2 - 20ab = 5b \times 5b - 5b \times 4a$ Rewrite the expression using the HCF $5b$

$= 5b(5b - 4a)$ Write the HCF at the front of the brackets

c The HCF of $v(4 + w)$ and $2(4 + w)$ is $(4 + w)$.

$\therefore v(4 + w) + 2(4 + w) = (4 + w) \times v + (4 + w) \times 2$

$= (4 + w)(v + 2)$

Factorising with negative terms

Example 14

Factorise each expression.

a $-x^2 + 3x$ **b** $-a - ab$

Solution

When factorising expressions that begin with a **negative** term, we use the 'negative' HCF.

a The highest 'negative' common factor of $-x^2$ and $3x$ is $-x$.

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$\therefore -x^2 + 3x = (-x) \times x + (-x) \times (-3)$ $\quad (-x) \times (-3) = +3x$

$= (-x)[x + (-3)]$

$= -x(x - 3)$ Check your answer by expanding.

b The highest 'negative' common factor of $-a$ and ab is $-a$.

$\therefore -a - ab = -a \times 1 + (-a) \times b$ $\quad (-a) \times b = -ab$

$= -a(1 + b)$

Factorising with negative terms

- Find the 'negative' HCF of the terms and write it outside the brackets
- Divide each term by the HCF and write the answers inside the brackets

$$-ab - ac = -a(b + c)$$

$$-ab + ac = -a(b - c)$$

- To check that the factorised answer is correct, expand it

EXERCISE 1.06 ANSWERS ON P. 565

Factorising expressions U F

1 Find the HCF of each pair of terms.

EXAMPLE 12

a 12, $6y$ **b** $8a^2$, 24 **c** $20b$, 15

d 6, $9p^2$ **e** $24mn$, m **f** c, c^2

g $18pq$, $12p^2$ **h** $24w^2$, $16w$ **i** $3(x - 5)$, $x(x - 5)$

2 Copy and complete each factorisation.

a $18p^2 + 24 = 6(___ + ___)$ **b** $9m^2 + 3m = 3m(___ + ___)$

c $20d - 30d^2 = 10d(___ - ___)$ **d** $6x^2y - 8xy = 2xy(___ - ___)$

e $x(x - 2) + 5(x - 2) = (x - 2)(___ + ___)$ **f** $n(3 + n) - 3(3 + n) = (3 + n)(___ - ___)$

g $h(h + 4) - 2(h + 4) = (___ + ___)(h - 2)$ **h** $v(1 - w) + (1 - w) = (___ + ___)(v + 1)$

3 Factorise each expression.

EXAMPLE 13

a $24x + 30$ **b** $36 + 27a$ **c** $16g^2 - 64$

d $xy + y$ **e** $mn - 3n$ **f** $6p + pq$

g $x^2 + x$ **h** $2y - y^2$ **i** $3d^2 + 6d$

j $16r^2 - 12r$ **k** $6t^2 + 27t$ **l** $36p^2 - 108p$

m $12x^2y - 16x$ **n** $18p^2 + 16pr$ **o** $4m^2n - 4mn^2$

p $14abc + 21bc$ **q** $28vw - 21v^2w^2$ **r** $45rt + 54r^2t$

s $36pq^2r - 144pr$ **t** $48x^2y + 64xy^2$ **u** $75g^3h^2 - 125gh$

v $25abc + 10a^2b$ **w** $54my^3 + 45m^2y^2$ **x** $21r^3u^2 - 35ru^3$

4 Factorise completely $24xy^2 - 16xyw$. Select the correct answer **A**, **B**, **C** or **D**.

A $8xy(3y - 2w)$ **B** $2x(12y^2 - 8yw)$

C $4xy(6y - 4w)$ **D** $8xy^2(3 - 2w)$

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5 Factorise each expression.

a $a(a-3)+6(a-3)$ **b** $t(8+t)-3(8+t)$

c $b(b+5)-2(b+5)$ **d** $x(2-y)-6(2-y)$

e $5(a-7)+b(a-7)$ **f** $y(a+3)+4(a+3)$

g $3p(g+5)-2(g+5)$ **h** $5(2-3m)+4n(2-3m)$

i $r(8+r)+(8+r)$ **j** $(y-6)-y(y-6)$

k $3m(2w+7)-(2w+7)$ **l** $3c(c+3)+5(c+3)$

EXAMPLE 14

6 Factorise each expression using the 'negative' HCF.

a $-4q-8$ **b** $-9u+18$ **c** $-3g+6$

d $-18a+12$ **e** $-n-1$ **f** $-n+1$

g $-y^2-9y$ **h** $-6t+10t^2$ **i** $-3a^2-6ab$

j $-ap+aq$ **k** $-20e^2-22e$ **l** $-9m+3m^2$

7 Factorise $-10kr+4rn$. Select **A**, **B**, **C** or **D**.

A $-2r(5k-4n)$ **B** $-2r(5k-2n)$ **C** $-5r(2k-2n)$ **D** $-2r(5k+2n)$

8 Factorise each expression.

a $\frac{1}{2}p+\frac{1}{2}q$ **b** $7g+9g^2$

c $\frac{1}{4}x^2-\frac{1}{4}xy$ **d** $8a+12y+4$

e $20xy-10x^2+5y$ **f** t^2m+tm^2+tm

g $7y(m+2)-p(m+2)$ **h** $-10p^2-10pq$

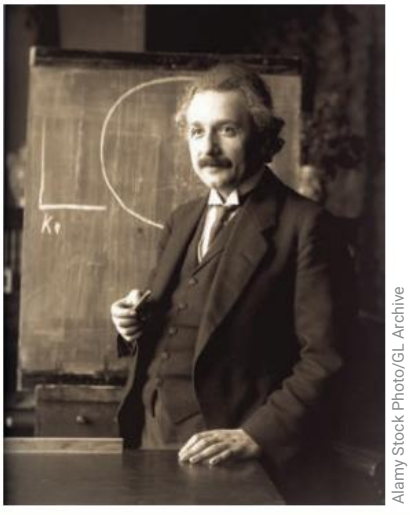
i $5a(x-y)+10a^2xy-10a^2y^2$

Did you know?

Einstein's formula

In 1905, when the German scientist Albert Einstein was only 26, he proposed a new theory of physics for small particles of matter (such as atoms) moving at very high speeds. He proposed that matter (mass) could be converted into large amounts of energy, describing this in his Special Relativity formula $E=mc^2$, where E stands for energy, m stands for mass and c stands for the speed of light (3 billion metres per second). Einstein's theory revolutionised conventional laws of physics and led to the development of nuclear energy.

Find out why Einstein renounced his German citizenship and became an American in 1940.



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Expanding binomial products

1.07

$(x + 5)$ and $(x - 1)$ are called **binomial expressions** because they each have exactly 2 terms.

$(x + 5)(x - 1)$ is called a **binomial product** because it is a product of 2 binomial expressions.

binomial = '2 terms'

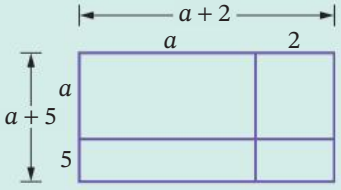
Example 15

Expand each binomial product using an area diagram.

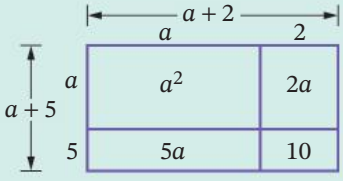
a $(a + 2)(a + 5)$ **b** $(n + 4)(n - 3)$

Solution

a Draw an area diagram (rectangle) with length $(a + 2)$ and width $(a + 5)$ and divide the diagram into 4 smaller rectangles.



Find the area of each smaller rectangle.

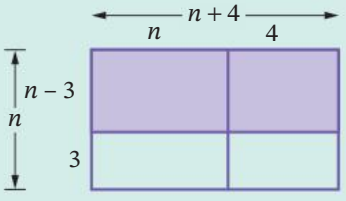


Expand $(a + 2)(a + 5)$ by adding the areas of the 4 rectangles.

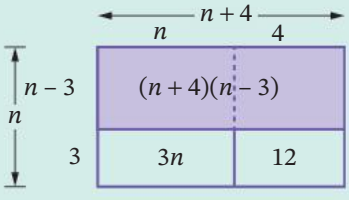
$(a + 2)(a + 5) = a^2 + 2a + 5a + 10$ Expanding

$= a^2 + 7a + 10$ Simplifying by collecting like terms

b Draw an area diagram with length $(n + 4)$ and width $(n - 3)$, then increase the width to n by adding 3. This creates 4 rectangles.



The 2 shaded upper rectangles together give the product $(n + 4)(n - 3)$. To find its area, we subtract the areas of the 2 lower rectangles from the area of the whole rectangle.



$(n + 4)(n - 3) = n(n + 4) - 3n - 12$ Whole rectangle – 2 lower rectangles

$= n^2 + 4n - 3n - 12$ Expanding

$= n^2 + n - 12$ Simplifying by collecting like terms

When expanding a binomial product algebraically, each term in the first binomial is multiplied by each term in the second binomial to give **4 terms**, which are collected and simplified.

Example 16

Expanding binomial products

Expand each binomial product.

a $(x + 5)(x + 9)$ **b** $(k + 3)(k - 7)$ **c** $(7 - m)(4 + m)$ **d** $(a - 6)^2$

Solution

a $(x + 5)(x + 9) = x(x + 9) + 5(x + 9)$ — Each term in $(x + 5)$ is multiplied by $(x + 9)$
$= x^2 + 9x + 5x + 45$ — Expanding to make 4 terms
$= x^2 + 14x + 45$ — Adding $9x$ and $5x$

b $(k + 3)(k - 7) = k(k - 7) + 3(k - 7)$ — Each term in $(k + 3)$ is multiplied by $(k - 7)$
$= k^2 - 7k + 3k - 21$ — Expanding to make 4 terms
$= k^2 - 4k - 21$ — Adding $-7k$ and $3k$

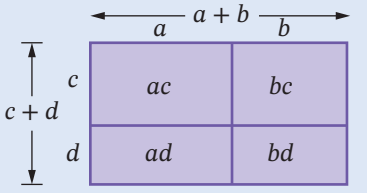
c $(7 - m)(4 + m) = 7(4 + m) - m(4 + m)$ — Each term in $(7 - m)$ is multiplied by $(4 + m)$
$= 28 + 7m - 4m - m^2$ — Expanding to make 4 terms
$= 28 + 3m - m^2$ — Adding $7m$ and $-4m$

d $(a - 6)^2 = (a - 6)(a - 6)$
$= a(a - 6) - 6(a - 6)$ — Each term in $(a - 6)$ is multiplied by $(a - 6)$
$= a^2 - 6a - 6a + 36$ — Expanding to make 4 terms
$= a^2 - 12a + 36$ — Adding $-6a$ and $-6a$

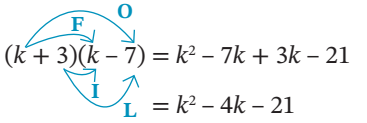
The distributive law for expanding a binomial product

Multiply each term in the first binomial by each term in the second binomial.

$$(a + b)(c + d) = ac + ad + bc + bd$$



One way of remembering which pairs of terms to multiply together in a binomial product is called the **FOIL method**, as shown below.



- **F** means multiply the **first** terms: $k \times k = k^2$
- **O** means multiply the **outside** terms: $k \times (-7) = -7k$
- **I** means multiply the **inside** terms: $3 \times k = 3k$
- **L** means multiply the **last** terms: $3 \times (-7) = -21$

Example 17

Expand each binomial product.

a $(x - 6)(4x + 2)$ **b** $(3t - 1)(2t - 5)$ **c** $(2x + 5)(3y - 4)$

Solution

a $(x - 6)(4x + 2) = x(4x + 2) - 6(4x + 2)$
$= 4x^2 + 2x - 24x - 12$ Expanding
$= 4x^2 - 22x - 12$ Simplifying

b $(3t - 1)(2t - 5) = 3t(2t - 5) - 1(2t - 5)$
$= 6t^2 - 15t - 2t + 5$ Expanding
$= 6t^2 - 17t + 5$ Simplifying

c $(2x + 5)(3y - 4) = 2x(3y - 4) + 5(3y - 4)$
$= 6xy - 8x + 15y - 20$ Expanding

Expanding binomial products

Expanding binomial products

1.07

EXERCISE 1.07 ANSWERS ON P. 565

Expanding binomial products UFRC

EXAMPLE 15

1 Expand each binomial product by copying and completing the area diagram.

a $(x + 10)(x + 8)$

	x	10
x	---	---
8	---	---

b $(4h + 1)(3h + 7)$

	$4h$	1
$3h$	---	---
7	---	---

2 Expand each binomial product by copying and completing the area diagram. (Find the shaded area.)

a $(x + 9)(x - 6)$

	x	9
$x - 6$		
6	---	---

b $(n - 5)(n - 7)$

	$n - 5$	5
$n - 7$	---	---
7	---	---

3 Draw an area diagram for each binomial product and use it to expand the product.

a $(2y + 5)(y + 3)$ **b** $(w - 5)(w + 3)$ **c** $(p - 3)(p - 5)$

EXAMPLE 16

4 Expand each binomial product algebraically.

a $(n + 6)(n + 3)$ **b** $(t + 6)(t + 5)$ **c** $(p + 10)(p - 10)$

d $(x - 8)(x + 3)$ **e** $(b - 2)(9 + b)$ **f** $(u - 8)(u - 7)$

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g $(15 - r)(r + 1)$ **h** $(a - 10)(a - 9)$ **i** $(5 - c)(3 - c)$

j $(t - 1)(t + 2)$ **k** $(y - 4)(y + 10)$ **l** $(n - 9)(11 + n)$

m $(e - 2)^2$ **n** $(5 + w)^2$ **o** $(g + 11)^2$

5 Expand $(b + 7)^2$. Select the correct answer **A**, **B**, **C** or **D**.

A $b^2 + 49$ **B** $b^2 + 49b$

C $b^2 + 7b + 49$ **D** $b^2 + 14b + 49$

6 Expand $(x + 8)(3 - x)$. Select **A**, **B**, **C** or **D**.

A $24 - 5x - x^2$ **B** $-x^2 + 24$

C $x^2 + 5x - 24$ **D** $-x^2 + 5x + 24$

EXAMPLE 17

7 Expand each binomial product.

a $(2x + 5)(x + 3)$ **b** $(3e + 2)(4e + 5)$ **c** $(10 + 3p)(p - 1)$

d $(7d - 2)(7d - 2)$ **e** $(2f - 2)(3f + 5)$ **f** $(4m - 5)(5 + 3m)$

g $(3 - 4h)(2 + 5h)$ **h** $(4p - 5)(2p - 7)$ **i** $(2m - 3)(4 - 5m)$

j $(3t + 5)(2t - 1)$ **k** $(5y - 5)(5y + 5)$ **l** $(7 - 2a)(2a - 7)$

m $(3k + 8)^2$ **n** $(4c - 5)^2$ **o** $(2d + 9)(3x - 1)$

p $(5 - 2g)(h + 3)$ **q** $(7 - 4w)^2$ **r** $(3n - 5)(4n + 9)$

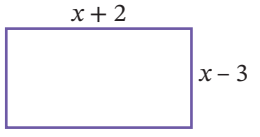
8 Expand $(3h + 8)(2h - 5)$. Select **A**, **B**, **C** or **D**.

A $6h^2 - 31h - 40$ **B** $6h^2 - h + 40$

C $6h^2 + 31h - 40$ **D** $6h^2 + h - 40$

9 Write a simplified algebraic expression for the area of each shape.

a



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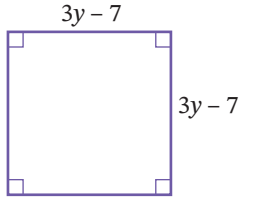

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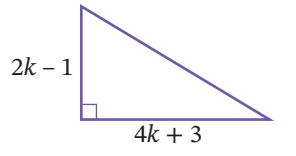

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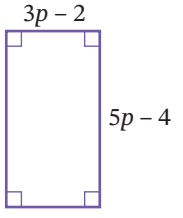

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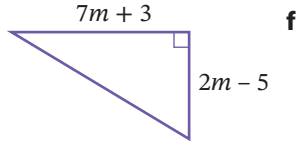

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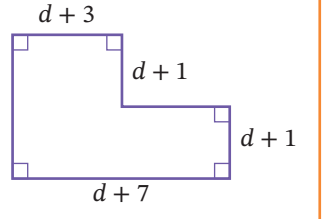

10 A rectangular barbecue plate has a length of 100 cm and a width of 75 cm. The length and width are both increased by x cm. R C

a Write an expression for the new length of the plate in cm.

b Write an expression for the new width of the plate in cm.

c Hence find a simplified expression for the new area of the plate in cm^2.

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d By how much has the area of the plate increased?

e If $x = 0.1$, find the increase in the area of the plate.

11 A family room in a house is to be extended. The room is 4 m long and 3 m wide. The length is to be increased by x m and the width by y m. R C

a Write down expressions for the new length and width in metres.

b Write down a binomial expression for the new area of the room in square metres.

c Expand and simplify your expression for the area.

d By how much has the area of the room increased?

12 Prove that: R C

a $(a - b)^2 = (b - a)^2$

b $(a + b)(a - b) = a^2 - b^2$

c $(a - b)^2 = a^2 - 2ab + b^2$

Investigation

Expanding perfect squares

There is a special pattern when you expand a binomial by itself, for example, $(y + 5)(y + 5)$.

This is called the square of a binomial, or a **perfect square**.

1 **a** Expand and simplify the perfect square $(y + 5)^2 = (y + 5)(y + 5)$.

b How many terms are there in your answer?

c The terms in the binomial $(y + 5)$ are y and 5. The first term of your *answer* in part **a** is y^2, which is the square of y. How are the other terms in the answer related to y and 5?

2 Expand and simplify each perfect square.

a $(k + 3)^2$ **b** $(m + 7)^2$ **c** $(p + 2)^2$

3 For each of the expansions in question **2**:

a how many terms are there?

b describe how each is related to the 2 terms of the perfect square

c compare and discuss your results with other students

4 Expand and simplify each perfect square.

a $(t - 1)^2$ **b** $(g - 6)^2$ **c** $(d - 5)^2$

5 For each of the expansions in question **4**:

a how many terms are there?

b describe how each is related to the 2 terms of the perfect square

c compare and discuss your results with other students

Foundation Standard Complex

6 Describe the expansion of the perfect square (first term + second term)^2 by copying and completing this statement:

The square of a binomial is equal to the square of the first term plus double the product of the 2 terms plus _____

7 **a** Find a formula for $(a + b)^2$.

b Check this formula by expanding $(a + b)^2$ using an area diagram.

c Find a formula for $(a - b)^2$.

d Check this formula by expanding $(a - b)^2$ using an area diagram.

1.08 Perfect squares

STAGE 5.3

Factorominoes

16, 49, v^2 and $(y + 5)^2$ are called **perfect squares** because they are square numbers.

Perfect squares

The formulas for expanding the perfect square of a binomial are:

$$(a + b)^2 = a^2 + 2ab + b^2$$
$$(a - b)^2 = a^2 - 2ab + b^2$$

Proof:

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b)\\ &= a(a + b) + b(a + b)\\ &= a^2 + ab + ba + b^2\\ &= a^2 + 2ab + b^2\end{aligned}$$

$$\begin{aligned}(a - b)^2 &= (a - b)(a - b)\\ &= a(a - b) - b(a - b)\\ &= a^2 - ab - ba + b^2\\ &= a^2 - 2ab + b^2\end{aligned}$$

Example 18

Copy and complete the expansion of each perfect square.

a $(x + 4)^2 = x^2 + ____ + 16$

b $(y - 6)^2 = y^2 - 12y + ____$

c $(5g + 9)^2 = ____ + 90g + 81$

d $(3d - 5)^2 = 9d^2 - ____ + 25$

Solution

a In the expansion,

$_____ = 2 \times x \times 4$ Doubling the product of the 2 terms

$= 8x$

$\therefore (x + 4)^2 = x^2 + 8x + 16$

b In the expansion,

$_____ = 6^2$ The second term squared

$= 36$

$\therefore (y - 6)^2 = y^2 - 12y + 36$

c $_____ = (5g)^2$ — The first term squared
$= 25g^2$
$\therefore (5g + 9)^2 = 25g^2 + 90g + 81$

d $_____ = 2 \times 3d \times 5$ — Doubling the product of the 2 terms
$= 30d$
$\therefore (3d - 5)^2 = 9d^2 - 30d + 25$

Example 19

Expand each perfect square.

a $(n - 5)^2$ **b** $(k + 4)^2$ **c** $(3y - 8)^2$

Solution

a $(n - 5)^2 = n^2 - 2 \times n \times 5 + 5^2$ — 1st term squared – double product + 2nd term squared
$= n^2 - 10n + 25$

b $(k + 4)^2 = k^2 + 2 \times k \times 4 + 4^2$
$= k^2 + 8k + 16$

c $(3y - 8)^2 = (3y)^2 - 2 \times 3y \times 8 + 8^2$
$= 9y^2 - 48y + 64$

EXERCISE 1.08 ANSWERS ON P. 565

Perfect squares U F R C

1 Copy and complete the expansion of each perfect square.

a $(x + 10)^2 = x^2 + _____ + 100$ **b** $(m - 8)^2 = _____ - 16m + 64$
c $(p - t)^2 = p^2 - 2pt + _____$ **d** $(h + 4)^2 = h^2 _____ + 16$
e $(k - 9)^2 = k^2 _____ + 81$ **f** $(8 + 5f)^2 = 64_____ + 25f^2$
g $(2d + 3)^2 = _____ + _____ + 9$ **h** $(6a + 1)^2 = _____ + 12a + _____$

EXAMPLE 18

2 Expand each perfect square.

a $(m + 9)^2$ **b** $(u + 3)^2$ **c** $(y - 6)^2$
d $(8 + k)^2$ **e** $(5 - h)^2$ **f** $(7 + k)^2$
g $(f + 20)^2$ **h** $(q - 11)^2$ **i** $(10 + t)^2$
j $(x - w)^2$ **k** $(a + g)^2$ **l** $(2m - 3)^2$
m $(5x - 6)^2$ **n** $(9a + 2)^2$ **o** $(3e - 4)^2$
p $(5 + 7b)^2$ **q** $(4 - 5p)^2$ **r** $(11 - 2c)^2$
s $(10g + 3)^2$ **t** $(3k + 11)^2$ **u** $(5 + 2v)^2$

EXAMPLE 19

3 Expand each perfect square.

a $(7h + 2k)^2$ **b** $(8a - 3y)^2$ **c** $(xy + z)^2$
d $\left(1 + \frac{1}{y}\right)^2$ **e** $\left(k - \frac{1}{k}\right)^2$ **f** $\left(w + \frac{3}{w}\right)^2$

Foundation Standard Complex

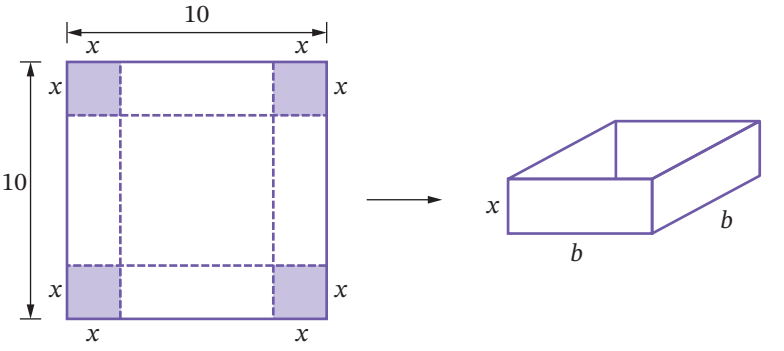
STAGE 5.3

4 Use the expansion of a perfect square to evaluate each square number without using a calculator.

a $21^2 = (20 + 1)^2$ **b** $45^2 = (40 + 5)^2$ **c** $29^2 = (30 - 1)^2$

d $59^2 = (60 - 1)^2$ **e** $102^2 = (100 + 2)^2$ **f** $98^2 = (100 - 2)^2$

5 A 10 cm square sheet of cardboard has a square of dimension x cm cut from each corner. It is then folded to form a square-based prism. R C



a Why is the length of the square base (b) equal to $(10 - 2x)$ cm?

b Find the area of the square base.

c Find the area of one side-face of the prism.

d Hence show that the surface area of the prism is $(100 - 4x^2)$ cm^2.

Investigation

Squaring a number ending in 5

Study this mental shortcut for squaring a number ending in 5:

- to evaluate 35^2, calculate $3 \times 4 = 12$, append '25' to the end: $35^2 = 1225$
- to evaluate 75^2, calculate $7 \times 8 = 56$, append '25' to the end: $75^2 = 5625$
- to evaluate 105^2, calculate $10 \times 11 = 110$, append '25' to the end: $105^2 = 11\,025$

Let n stand for the tens digit of the number ending in 5 being squared.

Expand $(10n + 5)^2$ and investigate why the above method works.

Investigation

Expanding sums by differences

There is also special pattern when you multiply the sum of 2 terms by the difference of those 2 terms, for example, $(x + 9)(x - 9)$.

1 **a** Expand and simplify $(x + 9)(x - 9)$.

b How many terms are there in your answer?

c How are the terms in the answer related to x and 9?

Foundation Standard Complex

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2 Expand and simplify each 'sum by difference'.

a $(k + 2)(k - 2)$

b $(m + 5)(m - 5)$

c $(p - 6)(p + 6)$

3 Describe the expansion of (first term + second term)(first term – second term) by copying and completing this statement:

The product of a 'sum by difference' *is equal to the square of the first term* _____

_____

4 **a** Find a formula for $(a + b)(a - b)$.

b Check this formula by expanding $(a + b)(a - b)$ using an area diagram.

Difference of 2 squares 1.09

Difference of 2 squares

$$(a + b)(a - b) = a^2 - b^2$$

The answer is called the **difference of 2 squares**.

Proof:

$$\begin{aligned}(a - b)(a + b) &= a(a + b) - b(a + b)\\ &= a^2 + ab - ba - b^2\\ &= a^2 - b^2\end{aligned}$$

When the sum of 2 terms is multiplied by their difference, the answer is the square of the first term minus the square of the second term (the difference of 2 squares).

STAGE 5.3

WS Special products

Difference of 2 squares

Enough time

Special binomial products

Example 20

Expand each expression.

a $(d + 3)(d - 3)$

b $(2 + r)(2 - r)$

c $(7x + 2)(7x - 2)$

d $(4k - 5p)(4k + 5p)$

Solution

a $\begin{aligned}(d + 3)(d - 3) &= d^2 - 3^2\\ &= d^2 - 9\end{aligned}$

b $\begin{aligned}(2 + r)(2 - r) &= 2^2 - r^2\\ &= 4 - r^2\end{aligned}$

c $\begin{aligned}(7x + 2)(7x - 2) &= (7x)^2 - 2^2\\ &= 49x^2 - 4\end{aligned}$

d $\begin{aligned}(4k - 5p)(4k + 5p) &= (4k)^2 - (5p)^2\\ &= 16k^2 - 25p^2\end{aligned}$

STAGE 5.3

EXERCISE 1.09 ANSWERS ON P. 566

Difference of 2 squares U F R C

EXAMPLE 20

1 Expand each expression.

a $(m+5)(m-5)$ **b** $(c-10)(c+10)$ **c** $(a+12)(a-12)$

d $(6-y)(6+y)$ **e** $(8-m)(8+m)$ **f** $(p+1)(p-1)$

g $(5+e)(5-e)$ **h** $(v+11)(v-11)$ **i** $(w-3)(w+3)$

j $(x-10)(x+10)$ **k** $(q+7)(q-7)$ **l** $(9-g)(9+g)$

m $(b-2)(b+2)$ **n** $(15-r)(15+r)$ **o** $(d+13)(d-13)$

2 Expand each expression.

a $(2h-3)(2h+3)$ **b** $(5r+4)(5r-4)$ **c** $(5b+8)(5b-8)$

d $(4p-7)(4p+7)$ **e** $(3-8k)(3+8k)$ **f** $(7x-5)(7x+5)$

g $(2+9m)(2-9m)$ **h** $(9k-4l)(9k+4l)$ **i** $(7n+8m)(7n-8m)$

j $(4g-5h)(4g+5h)$ **k** $(7u+3w)(7u-3w)$ **l** $(11a+3b)(11a-3b)$

m $\left(a+\frac{1}{a}\right)\left(a-\frac{1}{a}\right)$ **n** $\left(\frac{w}{3}-2\right)\left(\frac{w}{3}+2\right)$ **o** $\left(1-\frac{1}{r}\right)\left(1+\frac{1}{r}\right)$

3 Natalie's age is p years. R C

a What was Natalie's age last year?

b What will Natalie's age be next year?

c Write an expression for (Natalie's age last year) × (Natalie's age next year).

d If (Natalie's age last year) × (Natalie's age next year) is equal to 48, what is Natalie's age?

4 By expressing 31×29 as $(30+1)(30-1)$, use the difference of 2 squares to find the value of 31×29. R C

5 Use the method of question 4 to evaluate each expression. R C

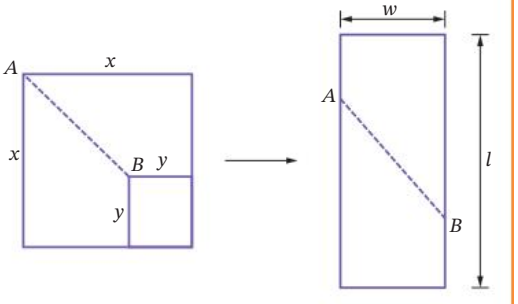
a 21×19 **b** 51×49 **c** 89×91 **d** 78×82

6 Use 'sum by difference' to evaluate each expression.

a $15^2 - 14^2$ **b** $24^2 - 23^2$ **c** $65^2 - 35^2$

d $101^2 - 99^2$ **e** $23^2 - 17^2$ **f** $50^2 - 48^2$

7 A square sheet of paper of length x cm has a smaller square of length y cm cut from one corner. It is then cut on the diagonal AB and rearranged to form a rectangle as shown. R C



a Find an expression for:

i the length (l) of the rectangle

ii the width (w) of the rectangle

iii the area of the rectangle

b The area of the rectangle should be the same as the area of the square on the left minus the area of the smaller square. Find an expression for this area. What does this prove?

Foundation Standard Complex

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Mixed expansions 1.10

STAGE 5.3

1.10

Expanding binomials

WS Algebra 6

Example 21

Expand and simplify each expression.

a $(4r + 5)(1 - 2r)$ **b** $(7 + 9x)^2$

c $(3d - 10)(3d + 10)$ **d** $(a + 6)(a - 6) + (a + 12)(a + 3)$

e $(m - 2)^2 - (m - 2)(m + 2)$

Solution

a $(4r + 5)(1 - 2r) = 4r(1 - 2r) + 5(1 - 2r)$
$= 4r - 8r^2 + 5 - 10r$
$= -8r^2 - 6r + 5$

b $(7 + 9x)^2 = 7^2 + 2 \times 7 \times 9x + (9x)^2$
$= 49 + 126x + 81x^2$
$= 81x^2 + 126x + 49$

c $(3d - 10)(3d + 10) = (3d)^2 - 10^2$
$= 9d^2 - 100$

d $(a + 6)(a - 6) + (a + 12)(a + 3) = a^2 - 36 + a^2 + 3a + 12a + 36$
$= 2a^2 + 15a$

e $(m - 2)^2 - (m - 2)(m + 2) = m^2 - 4m + 4 - (m^2 - 4)$
$= m^2 - 4m + 4 - m^2 + 4$
$= -4m + 8$

EXERCISE 1.10 ANSWERS ON P. 566

Mixed expansions U F R C

EXAMPLE 21

1 Expand and simplify each expression.

a $(2m - 1)(2m + 1)$ **b** $(y + 4)(5y - 3)$ **c** $(2k - 7)^2$

d $(d + 9)^2$ **e** $(2e - 1)(e + 1)$ **f** $(5a + 4)(5a - 4)$

g $(2 - p)(p - 2)$ **h** $(10 - 6y)(10 + 6y)$ **i** $(h - 3m)^2$

j $(2x - 3)(y + 3)$ **k** $(11a - 4b)(11a + 4b)$ **l** $\left(u - \frac{1}{u}\right)^2$

2 Expand and simplify each expression. R C

a $(m - 5)(m + 5) + 25$ **b** $6y + (y - 3)^2 + 9$

c $(3x + 1)(2 - x) + 2x + 4$ **d** $(d + 4)^2 - 8d + 5$

e $16 + (4k - 8)(4k + 8)$ **f** $(x - y)^2 - (x + y)^2$

g $20a - (a - 2)(a - 5) + a^2$ **h** $2(f - 2)(f + 2)$

i $(2h + 3)^2 - (2h - 3)(2h + 3)$ **j** $7xy - (2x - 3)(y + 3)$

Foundation Standard Complex

9780170453325

STAGE 5.3

3 Expand and simplify each expression. R C

a $(8a - 1)(8a + 1) - 4a^2 + 1$

b $(n + 1)^2 + 2n + 3$

c $3(4 - u)(4 + u) + (u - 12)(u + 4)$

d $(2m - n)^2 + (2m + n)^2$

e $(x - 2)(x + 3) - (x - 2)(x + 2)$

f $2(b - 1)^2 - (2b - 1)^2$

g $(y + 1)^2 + (y + 2)^2 + (y + 3)^2$

h $(x - 3)(x + 3) + (x + 3)^2 + (x - 3)^2$

i $(5n + 3)(5n - 3) + (3n - 5)(3n + 5)$

j $2(a - b)(a + b) - (a + b)^2 - (a - b)^2$

Mental skills 1B: Maths without calculators ANSWERS ON P. 566

Multiplying by 9, 11, 99 and 101

We can use expansion when multiplying by a number near 10 or near 100.

1 Study each example.

a $25 \times 11 = 25 \times (10 + 1)$
$= 25 \times 10 + 25 \times 1$
$= 250 + 25$
$= 275$

b $14 \times 9 = 14 \times (10 - 1)$
$= 14 \times 10 - 14 \times 1$
$= 140 - 14$
$= 126$

c $32 \times 12 = 32 \times (10 + 2)$
$= 32 \times 10 + 32 \times 2$
$= 320 + 64$
$= 384$

d $7 \times 99 = 7 \times (100 - 1)$
$= 7 \times 100 - 7 \times 1$
$= 700 - 7$
$= 693$

e $27 \times 101 = 27 \times (100 + 1)$
$= 27 \times 100 + 27 \times 1$
$= 2700 + 27$
$= 2727$

f $18 \times 8 = 18 \times (10 - 2)$
$= 18 \times 10 - 18 \times 2$
$= 180 - 36$
$= 144$

2 Now evaluate each product.

a	16×11	**b**	33×11	**c**	29×9	**d**	45×9
e	62×11	**f**	7×101	**g**	18×101	**h**	36×99
i	19×8	**j**	45×12	**k**	21×102	**l**	6×98
m	32×9	**m**	7×99	**o**	39×101	**p**	71×12

Factorising special binomial products 1.11

STAGE 5.3

1.11

Factorising by grouping in pairs

Grouping

An algebraic expression with 4 terms can often be factorised in pairs, that is, 2 terms at a time, to make a binomial product.

Example 22

Factorise each expression.

a $3ac + 2bd + 2bc + 3ad$ **b** $4km + 6mn - 6kp - 9np$

c $10xw - 6yw - 10xt + 6yt$

Solution

a
$$\begin{aligned} 3ac + 2bd + 2bc + 3ad &= 3ac + 3ad + 2bd + 2bc && \text{Grouping into pairs for factorising} \\ &= 3a(c + d) + 2b(d + c) && \text{Factorising each pair} \\ &= (c + d)(3a + 2b) && \text{Factorising again} \end{aligned}$$

b
$$\begin{aligned} 4km + 6mn - 6kp - 9np &= 2m(2k + 3n) - 3p(2k + 3n) && \text{Factorising each pair} \\ &= (2k + 3n)(2m - 3p) && \text{Factorising again} \end{aligned}$$

c
$$\begin{aligned} 10xw - 6yw - 10xt + 6yt &= 2(5xw - 3yw - 5xt + 3yt) && \text{Factorising all terms first} \\ &= 2(5xw - 5xt - 3yw + 3yt) && \text{Grouping into pairs for factorising} \\ &= 2[5x(w - t) - 3y(w - t)] && \text{Factorising each pair} \\ &= 2(w - t)(5x - 3y) && \text{Factorising again} \end{aligned}$$

Factorising the difference of 2 squares

Difference of 2 perfect squares

We have learned the formula $(a + b)(a - b) = a^2 - b^2$.

If we use this rule in reverse, then the factors of $a^2 - b^2$ are $(a + b)$ and $(a - b)$.

Factorising the difference of 2 squares

$$a^2 - b^2 = (a + b)(a - b)$$

Example 23

Difference of 2 squares

Factorise each expression.

a $x^2 - 4$ **b** $9 - 16b^2$ **c** $20d^2 - 5a^2$ **d** $y^3 - y$

Solution

a
$$\begin{aligned} x^2 - 4 &= x^2 - 2^2 \\ &= (x + 2)(x - 2) \end{aligned}$$

b
$$\begin{aligned} 9 - 16b^2 &= 3^2 - (4b)^2 \\ &= (3 + 4b)(3 - 4b) \end{aligned}$$

c
$$\begin{aligned} 20d^2 - 5a^2 &= 5(4d^2 - a^2) \\ &= 5[(2d)^2 - a^2] \\ &= 5(2d + a)(2d - a) \end{aligned}$$

d
$$\begin{aligned} y^3 - y &= y(y^2 - 1) \\ &= y(y + 1)(y - 1) \end{aligned}$$

STAGE 5.3

EXERCISE 1.11 ANSWERS ON P. 566

Factorising special binomial products U F

EXAMPLE 22

1 Factorise each expression.

a $4ab + 5bc + 4ad + 5cd$

b $2xy - 5wy + 2xt - 5wt$

c $9ac + 6bc + 12ad + 8bd$

d $10x^2 + 30 + x^3 + 3x$

e $3a^2 + 3ab + 3ac + 3bc$

f $6rt - 18wt + 6rp - 18wp$

g $14e - 21 + 2de - 3d$

h $hk - h^2 - 2k + 2h$

i $3mn - 6m + pn - 2p$

j $9p^2 - 27 + qp^2 - 3q$

k $fg - fh - 10g + 10h$

l $9kl - 12ml + 9kn - 12mn$

m $2p - 2c - p^2 + pc$

n $l^3 + lm^2 - 3l^2 - 3m^2$

o $a(x + 1) + y(x + 1) - ka - ky$

p $p(a - b) - 2q(a - b) + 3qp - 6q^2$

EXAMPLE 23

2 Factorise each expression.

a $w^2 - 9$

b $y^2 - 36$

c $k^2 - 1$

d $m^2 - 121$

e $p^2 - 64$

f $c^2 - 100$

g $4e^2 - f^2$

h $a^2 - 9b^2$

i $16y^2 - 1$

j $4 - b^2$

k $25 - e^2$

l $1 - 16x^2$

m $k^2 - u^2$

n $49 - 16m^2$

o $b^2 - 121d^2$

p $36c^2 - 25k^2$

q $16 - 81h^2$

r $25a^2 - 64m^2$

s $100 - 49n^2$

t $121p^2 - 144q^2$

u $\frac{1}{4} - 25c^2$

v $4w^2 - \frac{1}{9}$

w $64h^2 - 2\frac{1}{4}$

x $1 - m^2n^2$

3 Factorise each expression.

a $2a^2 - 2b^2$

b $7k^2 - 28$

c $3 - 75u^2$

d $x^3 - 49x$

e $k - 16k^3$

f $50q^2 - 2$

g $3d^2 - 12v^2$

h $5t^5 - 125t^3$

i $2a^2b^2 - 2$

j $x^2y^2 - x^2w^2$

k $192f^2 - 108g^2$

l $45d^2 - \frac{5}{4}$

m $2x^2 - 8a^2$

n $100 - 25w^2$

o $1\frac{1}{4} - 80e^2$

p $9c^2 - 6\frac{1}{4}$

4 Factorise each expression.

a $\frac{p^2}{16} - \frac{b^2}{25}$

b $x^2 - \frac{1}{9}$

c $\frac{v^2}{64} - \frac{u^2}{81}$

d $\frac{2y^2}{9} - \frac{2m^2}{121}$

e $\frac{16a^2}{49} - \frac{25b^2}{4}$

f $g^4 - 81$

g $100 - n^4$

h $(x + y)^2 - x^2$

i $(p - 2q)^2 - (2p + q)^2$

j $\frac{x^2}{4} - \frac{y^2}{36}$

k $(a + b)^2 - (a - b)^2$

l $25 - (4 - x)^2$

Foundation Standard Complex

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Investigation

Factorising quadratic expressions

1 **a** Show that $(x + 3)(x + 5) = x^2 + 8x + 15$.

b The quadratic expression $x^2 + 8x + 15$ has 3 terms. The coefficient of x is 8, the number in front of the x. How are the 3 and 5 in $(x + 3)(x + 5)$ related to the 8?

c The constant term in $x^2 + 8x + 15$ is 15, the number with no x at the end. How are the 3 and 5 related to the 15?

2 **a** Expand $(x + 9)(x + 2)$.

b What is the coefficient of x? How are 9 and 2 related to it?

c What is the constant term? How are 9 and 2 related to it?

3 **a** Expand $(x + 8)(x - 3)$.

b What is the coefficient of x? How are 8 and -3 related to it?

c What is the negative constant term? How are 8 and -3 related to it?

4 **a** Expand $(x - 4)(x - 1)$.

b What is the negative coefficient of x? How are -4 and -1 related to it?

c What is the positive constant term? How are -4 and -1 related to it?

5 In the expansion of any binomial product, how are the coefficient of x and the constant term related to the numbers in the binomials?

6 Copy and complete:

a $(x + ___)(x + ___) = x^2 + 5x + 4$ **b** $(x + ___)(x + ___) = x^2 + 8x + 15$

c $(x + ___)(x + ___) = x^2 + 7x + 12$ **d** $(x + ___)(x - ___) = x^2 - 4x - 32$

e $(x + ___)(x - ___) = x^2 + 2x - 3$ **f** $(x - ___)(x - ___) = x^2 - 9x + 20$

Factorising quadratic expressions 1.12

A quadratic expression such as $x^2 - 5x + 7$ is called a **trinomial** because it has 3 terms.
Other examples of quadratic trinomials are $x^2 + x - 15$, $2x^2 - 3x + 9$ and $-4x^2 + 9x + 20$.

The expansion of $(x + 2)(x + 4)$ is $x^2 + 6x + 8$.

$\therefore$ The factorisation of $x^2 + 6x + 8$ is $(x + 2)(x + 4)$.

When factorising a quadratic trinomial such as $x^2 + 6x + 8$:

- each factor must have an x term to give x^2
- $2 + 4 = 6$, which is the **coefficient** of x, the number in front of the x
- $2 \times 4 = 8$, which is the **constant term** with no x.

$x^2 + 6x + 8 = (x + 2)(x + 4)$

$x^2 + 6x + 8 = (x + 2)(x + 4)$

$x^2 + 6x + 8 = (x + 2)(x + 4)$

Perfect squares

Factorising trinomials

Factorising trinomials

Factorising quadratic expressions 1

Example 24

Factorise $a^2 + 7a + 12$.

Solution

Find the 2 numbers that have a sum of 7 and a product of 12.
It is best to test numbers that have a product of 12 and then check if their sum equals 7:
The correct numbers are 3 and 4.
$\therefore a^2 + 7a + 12 = (a + 3)(a + 4)$

Pair of numbers	Product	Sum
6, 2	6 × 2 = 12	6 + 2 = 8
3, 4	3 × 4 = 12	3 + 4 = 7

Factorising quadratic expressions 1

Example 25

Factorise $x^2 + x - 6$.

Solution

Find 2 numbers with a sum of 1 and a product of –6.
Test numbers that have a product of –6 and check their sums:
The correct numbers are 3 and –2.
$\therefore x^2 + x - 6 = (x + 3)(x - 2)$

Pair of numbers	Product	Sum
+6, −1	+6 × (−1) = −6	+6 + (−1) = 5
+2, −3	+2 × (−3) = −6	2 + (−3) = −1
+3, −2	+3 × (−2) = −6	+3 + (−2) = 1

Factorising quadratic trinomials of the form $x^2 + bx + c$

- Find 2 numbers that have a sum of b and a product of c
- Use these 2 numbers to write a binomial product of the form $(x ___)(x ___)$

Factorising quadratic expressions 1

Example 26

Factorise each quadratic expression.

a $a^2 - 2a - 15$ **b** $y^2 - 6y + 8$ **c** $x^2 - 2x + 1$

Solution

a $a^2 - 2a - 15$

Find 2 numbers that have a product of –15 and a sum of –2. Since the product is negative, one of the numbers must be negative. They are –5 and +3.

$\therefore a^2 - 2a - 15 = (a - 5)(a + 3)$

b $y^2 - 6y + 8$

Product = 8, sum = −6.

Since the sum is negative, one of the numbers must be negative, but since the product is positive, *both* of the numbers must be negative. They are −2 and −4.

$\therefore y^2 - 6y + 8 = (y - 2)(y - 4)$

c $x^2 - 2x + 1$

Product = 1, sum = −2. Since the sum is negative, one of the numbers must be negative, but since the product is positive, *both* of the numbers must be negative.

They are −1 and −1.

$\therefore x^2 - 2x + 1 = (x - 1)(x - 1)$

$= (x - 1)^2$ *Note:* $(x - 1)^2$ is a perfect square.

Example 27

Factorise each quadratic trinomial.

a $3g^2 + 12g - 36$ **b** $48 - 8p - p^2$

Solution

a $3g^2 + 12g - 36 = 3(g^2 + 4g - 12)$ Taking out the HCF of 3 first

$= 3(g - 2)(g + 6)$ Product = –12, sum = 4

b $48 - 8p - p^2 = -p^2 - 8p + 48$ Rearranging the terms to make the p^2 term first

$= -1(p^2 + 8p - 48)$ Taking out a common factor of –1

$= -(p + 12)(p - 4)$ Product = –48, sum = 8

EXERCISE 1.12 ANSWERS ON P. 566

Factorising quadratic expressions U F R

1 Find 2 numbers whose: R

a product is 6 and their sum is −7
b product is −12 and their sum is 1
c product is −15 and their sum is −2
d product is 12 and their sum is 7
e product is 20 and their sum is −9
f product is −14 and their sum is 5
g product is −10 and their sum is 3
h product is−25 and their sum is 0
i product is −2 and their sum is −1
j product is −18 and their sum is −7

2 Factorise each quadratic expression. R EXAMPLE 24

a $m^2 + 7m + 12$ **b** $x^2 + 12x + 35$ **c** $k^2 + 5k + 4$
d $p^2 + 7p + 10$ **e** $w^2 + 9w + 20$ **f** $b^2 + 6b + 5$
g $e^2 + 5e + 6$ **h** $r^2 + 4r + 4$ **i** $n^2 + 11n + 10$
j $a^2 + 11a + 30$ **k** $d^2 + 10d + 24$ **l** $y^2 + 15y + 44$

3 Factorise each quadratic expression. R EXAMPLE 25

a $p^2 - 5p + 4$ **b** $v^2 - 11v + 30$ **c** $y^2 - 7y + 12$
d $x^2 - 12x + 20$ **e** $a^2 - 9a + 18$ **f** $e^2 - 8e + 12$
g $c^2 - 14c + 48$ **h** $w^2 - 5w + 6$ **i** $d^2 - 18d + 77$
j $u^2 - 3u + 2$ **k** $n^2 - 8n + 16$ **l** $b^2 - 15b + 56$

4 Factorise each quadratic expression. R EXAMPLE 26

a $x^2 + 3x - 4$ **b** $g^2 + 5g - 24$ **c** $y^2 - 2y - 3$
d $r^2 - 5r - 14$ **e** $m^2 + 2m - 15$ **f** $a^2 + a - 2$
g $m^2 - 9m + 20$ **h** $h^2 - 3h - 4$ **i** $d^2 - 6d + 9$
j $w^2 + 4w - 12$ **k** $a^2 - 4a - 12$ **l** $k^2 + 5k - 14$
m $c^2 - 7c - 18$ **n** $e^2 - 6e - 27$ **o** $u^2 + 6u - 55$
p $p^2 + 9p - 36$ **q** $x^2 + 9x - 52$ **r** $q^2 - 9q - 70$

5 Factorise each expression, given it is a perfect square. R

a	$m^2 + 4m + 4$	**b**	$p^2 + 20p + 100$	**c**	$a^2 - 10a + 25$
d	$y^2 - 14y + 49$	**e**	$w^2 + 24w + 144$	**f**	$a^2 - 3a + 2\frac{1}{4}$

EXAMPLE 27

6 Factorise each quadratic expression. Look for the highest common factor first. R

a	$3m^2 + 9m + 6$	**b**	$2y^2 + 2y - 4$	**c**	$5t^2 - 10t - 400$
d	$5e^4 + 25e^3 - 120e^2$	**e**	$x^3 - x^2 - 110x$	**f**	$4b^2 - 4b - 168$
g	$4w^2 + 4w - 48$	**h**	$3a^3 - 9a^2 - 12a$	**i**	$2e^2 + 18e + 40$
j	$24 - 5t - t^2$	**k**	$42 + u - u^2$	**l**	$28 + 3x - x^2$
m	$12 - b - b^2$	**n**	$7k - 12 - k^2$	**o**	$12x - 35 - x^2$

7 Factorise each quadratic expression. R

a	$h^2 - 3h - 18$	**b**	$18 - 7t - t^2$	**c**	$w^2 + 8w + 7$
d	$k^2 - 12k - 45$	**e**	$v^2 + 8v - 20$	**f**	$3c^2 + 30c + 75$
g	$q^2 - 6q + 5$	**h**	$a^2 - 4a + 3$	**i**	$6x^2 + 36x - 96$
j	$x^2 - 16x + 64$	**k**	$8u^2 - 24u - 32$	**l**	$b^2 + 11b + 30$
m	$y^2 - 12y + 36$	**n**	$5r^2 - 5r - 10$	**o**	$4l^2 - 8l - 32$
p	$g^2 - 24g + 80$	**q**	$108 - 6d - 2d^2$	**r**	$26 + 11n - n^2$

STAGE 5.3

Investigation

Factorising quadratic trinomials by grouping in pairs

1 We can factorise the quadratic trinomial $6y^2 + 19y + 15$ by rewriting it as $6y^2 + 10y + 9y + 15$, splitting up the middle term $19y$.

a Factorise $6y^2 + 10y + 9y + 15$ by grouping in pairs.

b Factorise $6y^2 + 9y + 10y + 15$ by grouping in pairs. Is your answer the same as that for part **a**?

c In $6y^2 + 19y + 15$, 6 is the coefficient of y^2 while 15 is the constant term. Find the product of 6 and 15 and the product of 10 and 9. What do you notice?

2 **a** Factorise $2c^2 + 13c + 21$ by rewriting it as $2c^2 + 6c + 7c + 21$.

b Factorise $2c^2 + 13c + 21$ by rewriting it as $2c^2 + 7c + 6c + 21$.

c Find the product of 2 and 21 and the product of 6 and 7. What do you notice?

1.13 Factorising quadratic expressions of the form $ax^2 + bx + c$

STAGE 5.3

Trinominoes

We will now factorise quadratic trinomials such as $6x^2 + 19x + 15$, where x^2 has a coefficient that is not a common factor. We do this by splitting the middle term into 2 terms and then factorise by grouping in pairs.

Foundation Standard Complex

Factorising quadratic expressions (Advanced)

Example 28

Factorise $3x^2 + 8x + 4$.

Solution

We need to split up the middle term $8x$.
Find 2 numbers that have a product of 12 and a sum of 8.

$3x^2 + 8x + 4 = 3x^2 + 8x + 4$

sum of 8

product of $3 \times 4 = 12$

The 2 numbers are $+6$ and $+2$, so we will split $8x$ into $6x$ and $2x$.

$\therefore 3x^2 + 8x + 4 = 3x^2 + 6x + 2x + 4$

$= 3x(x + 2) + 2(x + 2)$ Factorising by grouping in pairs

$= (x + 2)(3x + 2)$ Factorising again

Factorising quadratic trinomials of the form $ax^2 + bx + c$

- Find 2 numbers that have a sum of b and a product of ac
- Use these 2 numbers to split the middle term bx into 2 terms
- Factorise by grouping in pairs

Example 29

Factorising quadratic expressions 2

Factorise each quadratic expression.

a $5k^2 - 12k + 4$ **b** $9m^2 - 9m - 4$ **c** $6t^2 + t - 12$

Solution

a $5k^2 - 12k + 4$

$5 \times 4 = 20$

Find 2 numbers that have a product of 20 and a sum of -12.

$-5 \times (-4) = 20$; $-5 + (-4) = -9$ ✗

$-10 \times (-2) = 20$; $-10 + (-2) = -12$ ✓

Since the sum is negative and the product is positive, both of the numbers must be negative.

They are -10 and -2.

Split $-12k$ into $-10k$ and $-2k$.

$5k^2 - 12k + 4 = 5k^2 - 10k - 2k + 4$

$= 5k(k - 2) - 2(k - 2)$ Factorising by grouping in pairs

$= (k - 2)(5k - 2)$

b $9m^2 - 9m - 4$

$9 \times (-4) = -36$

Find 2 numbers with a product of -36 and a sum of -9.

Since the product is negative, one of the numbers must be negative.

$12 \times (-3) = -36$; $12 + (-3) = 9$ ✗

$-12 \times 3 = -36$, $-12 + 3 = -9$ ✓

They are –12 and 3.

$9m^2 - 9m - 4 = 9m^2 - 12m + 3m - 4$
$= 3m(3m - 4) + 1(3m - 4)$
$= (3m - 4)(3m + 1)$

c $6t^2 + t - 12$

$6 \times (-12) = -72$.

Find 2 numbers with a product of –72 and a sum of 1.

$8 \times (-9) = -72$; $8 + (-9) = -1$ ✗

$-8 \times 9 = -72$; $-8 + 9 = 1$ ✓

They are –8 and +9.

$6t^2 + t - 12 = 6t^2 - 8t + 9t - 12$
$= 2t(3t - 4) + 3(3t - 4)$
$= (3t - 4)(2t + 3)$

Example 30

Factorise each quadratic expression.

a $18a^2 - 18a - 8$ **b** $10 - 7x - 12x^2$

Solution

a $18a^2 - 18a - 8 = 2(9a^2 - 9a - 4)$ Taking out the HCF of 2 first
$= 2(9a^2 - 12a + 3a - 4)$ Product = –36, sum = –9
$= 2[3a(3a - 4) + 1(3a - 4)]$
$= 2(3a - 4)(3a + 1)$

b $10 - 7x - 12x^2 = -12x^2 - 7x + 10$ Rearranging the terms to make the x^2 term first
$= -(12x^2 + 7x - 10)$ Taking out a common factor of –1
$= -(12x^2 + 15x - 8x - 10)$ Product = –120, sum = –7
$= -[3x(4x + 5) - 2(4x + 5)]$
$= -(4x + 5)(3x - 2)$

EXERCISE 1.13 ANSWERS ON P. 567

Factorising quadratic expressions of the form $ax^2 + bx + c$ UFR

EXAMPLE 28

1 Factorise each quadratic expression. R

a $2x^2 + 11x + 5$ **b** $4m^2 + 13m + 3$ **c** $5y^2 + 17y + 6$
d $6u^2 + 19u + 10$ **e** $2w^2 + 31w + 15$ **f** $4e^2 + 15e + 9$
g $8f^2 + 14f + 3$ **h** $3d^2 + 5d + 2$ **i** $2b^2 + 9b + 7$
j $5y^2 + 16y + 11$ **k** $8g^2 + 26g + 15$ **l** $6a^2 + 23a + 21$

EXAMPLE 29

2 Factorise each quadratic expression. R

a $2y^2 - 11y + 12$ **b** $10k^2 - 19k + 6$ **c** $6e^2 - 13e + 6$
d $4b^2 - 13b + 3$ **e** $6w^2 - 23w + 15$ **f** $12f^2 - 25f + 12$
g $15m^2 - 26m + 8$ **h** $9x^2 - 12x + 4$ **i** $2a^2 - 23a + 45$
j $12y^2 - 35y + 8$ **k** $8d^2 - 34d + 21$ **l** $4h^2 - 36h + 81$

Foundation Standard Complex

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STAGE 5.3

3 Factorise each quadratic expression. R

a $5y^2 - 6y - 11$	**b** $4d^2 - d - 5$	**c** $2m^2 - 3m - 9$
d $3t^2 - t - 30$	**e** $6h^2 - h - 7$	**f** $2y^2 - 5y - 12$
g $8a^2 - 2a - 3$	**h** $15u^2 - 7u - 4$	**i** $9c^2 - 12c - 5$
j $6c^2 - 7c - 24$	**k** $20n^2 - 27n - 14$	**l** $12x^2 - 7x - 10$

4 Factorise each quadratic expression. R

a $5m^2 + 2m - 7$	**b** $6g^2 + g - 12$	**c** $3p^2 + 4p - 4$
d $7w^2 + 6w - 1$	**e** $5y^2 + 14y - 3$	**f** $3n^2 + 10n - 8$
g $4b^2 + 9b - 9$	**h** $8m^2 + 10m - 3$	**i** $3x^2 + 2x - 16$
j $8u^2 + 26u - 15$	**k** $24e^2 + 31e - 15$	**l** $20h^2 + 11h - 42$

5 Factorise each expression given it is a perfect square. R

a $81w^2 - 180w + 100$	**b** $4y^2 + 8y + 4$	**c** $25h^2 - 40h + 16$

6 Factorise each quadratic expression by first taking out a common factor. R

a $6y^2 + 10y - 4$	**b** $6g^2 + 15g - 36$	**c** $24e^2 - 28e - 12$
d $8a^2 - 10a - 12$	**e** $12u^2 + 20u - 8$	**f** $-25q^2 - 5q + 6$
g $-12m^2 + 14m - 4$	**h** $20 - h - 12h^2$	**i** $18 + 48c + 24c^2$
j $15 + 9w - 6w^2$	**k** $12d^2 + 2d - 30$	**l** $22x - 12 - 6x^2$

7 Factorise each quadratic expression. R

a $2a^2 + 5a + 3$	**b** $12m^2 - 32m + 5$	**c** $4x^2 + 11x - 3$
d $7w^2 - 8w + 1$	**e** $4h^2 - 7h - 15$	**f** $8x^2 - 2x - 3$
g $5r^2 + 26r + 5$	**h** $2d^2 - 15d + 7$	**i** $6n^2 - 7n - 3$
j $8 - 6m - 9m^2$	**k** $3 - 2c - 5c^2$	**l** $15g^2 + 19g + 6$
m $15 + 14q - 8q^2$	**n** $3x^2 - 13x + 14$	**o** $16 - 8d - 3d^2$
p $42 - 10y - 12y^2$	**q** $28d^2 - 44d - 24$	**r** $100k^2 + 80k + 16$

1.14

EXAMPLE 30

Mixed factorisations

1.14

Factorisation strategies

STAGE 5.3

- Look for any common factors and factorise first
- If there are 2 terms, try factorising using the difference of 2 squares
- If there are 3 terms, try factorising as a quadratic trinomial
- If there are 4 terms, try factorising by grouping in pairs

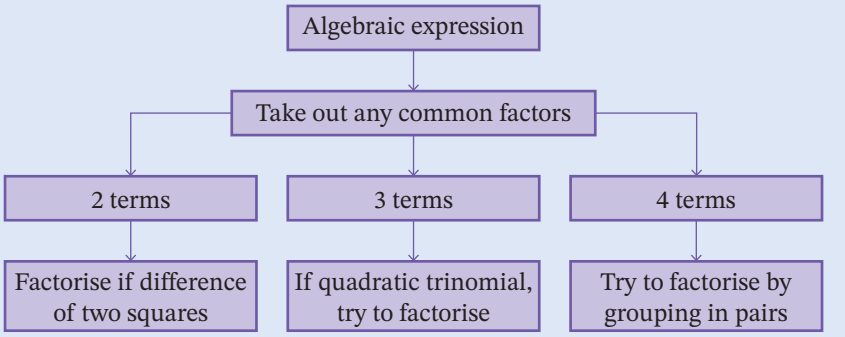
Factorising puzzle (Advanced)

Foundation | Standard | Complex

STAGE 5.3

Factorominoes

Products and factors squaresaw



Example 31

Factorise each quadratic expression.

a $3a^2 - 27$

b $5a^2 + 100$

c $20b^2 - 52b + 24$

d $d^3 - d^2 - d + 1$

Solution

a $3a^2 - 27 = 3(a^2 - 9)$ — Taking out the HCF of 3 first

$= 3(a + 3)(a - 3)$ — Difference of 2 squares

b $5a^2 + 100 = 5(a^2 + 20)$ — 2 terms but not a difference of 2 squares

c $20b^2 - 52b + 24 = 4(5b^2 - 13b + 6)$

$= 4(5b^2 - 10b - 3b + 6)$

$= 4[5b(b - 2) - 3(b - 2)]$

$= 4(b - 2)(5b - 3)$

d $d^3 - d^2 - d + 1 = d^2(d - 1) - 1(d - 1)$ — Factorising by grouping in pairs

$= (d - 1)(d^2 - 1)$

$= (d - 1)(d + 1)(d - 1)$ — Difference of 2 squares

$= (d - 1)^2(d + 1)$

EXERCISE 1.14 ANSWERS ON P. 567

Mixed factorisations U F R

EXAMPLE 31

1 Factorise each expression. R

a $m^2 - 16m + 64$

b $3d^2 - 3d$

c $3w^2 - 4w - 15$

d $3k - 15 - 5h + hk$

e $25y^2 - 64$

f $100f^2 - 64$

g $q^2 + 3q - 3pq$

h $3 + 2g - g^2$

i $24n^2 + 44n - 40$

j $25r^2 - 1$

k $b^3 + b^2 + b + 1$

l $4x^2 - 20x + 25$

m $4 - d - 5d^2$

n $a^3 - a^2 - a + 1$

o $8 - 2v^2$

p $mn^2 + mnp + 3mn + 3mp$

q $2w^2 - 24w + 72$

r $36h^2 + 12h + 1$

Foundation Standard Complex

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2 Factorise each expression. R

a	$15r^2 - 31rt - 24t^2$	**b**	$4d^2 + 4d + 1$	**c**	$9g^2 - 36k^2$
d	$e^3 - 3e^2 - 10e$	**e**	$5(p+q)^2 - 125(p-q)^2$	**f**	$28x^2 - 7$
g	$a^2 - b^2 + 4a - 4b$	**h**	$c^3 - 2c^2 - 4c + 8$	**i**	$6a^2 + 13a - 5$
j	$y^2 - 3y + 5y - 35$	**k**	$18p^2 + 24p + 8$	**l**	$1 - 2a - 24a^2$
m	$9x^2 - 27x + 18x - 54$	**n**	$2a^2b - 6ab - 3a + 9$	**o**	$2a^2 + 12a + 18$
p	$25u^2 - 10u + 1$	**q**	$4k^2 - 5k - 21$	**r**	$48 - 3w^2$
s	$3 - 27c^2$	**t**	$k^3 + 4k^2 - 16k - 64$	**u**	$5y^3 - 10y^2 + 15y$
v	$m^3n - 4mn$	**w**	$8 - 2a^2$	**x**	$32c^2 - 40c - 12$

3 Factorise each expression. R

a	$12e^2 + 13e - 35$	**b**	$10y^2 + 19ky - 15k^2$	**c**	$16h^4 - 81$
d	$w^2 - (6-w)^2$	**e**	$d^3 - 16d + 4 - d$	**f**	$u^2 - 8u + 16 - k^2$

STAGE 5.3

1.15

Factorising algebraic fractions 1.15

Example 32

STAGE 5.3

WS Simplifying algebraic fractions

Simplify each expression.

a $\dfrac{10a+25b}{5}$ **b** $\dfrac{9y^2-16}{6y+8}$ **c** $\dfrac{x^2+x}{-4x-4}$ **d** $\dfrac{t^2-3t+2}{3t^2-5t-2}$

Solution

a $\dfrac{10a+25b}{5} = \dfrac{5(2a+5b)}{5}$ Factorise, then cancel.

$= 2a + 5b$

b $\dfrac{9y^2-16}{6y+8} = \dfrac{(3y+4)(3y-4)}{2(3y+4)}$

$= \dfrac{(3y+4)(3y-4)}{2(3y+4)}$

$= \dfrac{3y-4}{2}$

c $\dfrac{x^2+x}{-4x-4} = \dfrac{x(x+1)}{-4(x+1)}$

$= \dfrac{x(x+1)}{-4(x+1)}$

$= -\dfrac{x}{4}$

d $\dfrac{t^2-3t+2}{3t^2-5t-2} = \dfrac{(t-2)(t-1)}{(t-2)(3t+1)}$

$= \dfrac{(t-2)(t-1)}{(t-2)(3t+1)}$

$= \dfrac{t-1}{3t+1}$

Foundation Standard Complex

STAGE 5.3

Example 33

Simplify each expression.

a $\dfrac{4}{x^2+x}+\dfrac{3}{x^2-1}$

b $\dfrac{3}{m^2+2m-3}-\dfrac{2}{m^2+2m-15}$

Solution

a
$$\frac{4}{x^2+x}+\frac{3}{x^2-1}=\frac{4}{x(x+1)}+\frac{3}{(x+1)(x-1)}$$

Factorising denominators

$$=\frac{4(x-1)}{x(x+1)(x-1)}+\frac{3x}{x(x+1)(x-1)}$$

Using common denominators

$$=\frac{4x-4+3x}{x(x+1)(x-1)}$$
$$=\frac{7x-4}{x(x+1)(x-1)}$$

b
$$\frac{3}{m^2+2m-3}-\frac{2}{m^2-2m-15}$$
$$=\frac{3}{(m+3)(m-1)}-\frac{2}{(m+3)(m-5)}$$

Factorising denominators

$$=\frac{3(m-5)}{(m+3)(m-1)(m-5)}-\frac{2(m-1)}{(m+3)(m-1)(m-5)}$$

Using common denominators

$$=\frac{3(m-5)-2(m-1)}{(m+3)(m-1)(m-5)}$$
$$=\frac{3m-15-2m+2}{(m+3)(m-1)(m-5)}$$
$$=\frac{m-13}{(m+3)(m-1)(m-5)}$$

Example 34

Simplify each expression.

a $\dfrac{3m-6}{4}\times\dfrac{8m}{m^2-2m}$

b $\dfrac{d^2+3d+2}{d^2-9}\div\dfrac{d^2+d}{3d+9}$

Solution

a
$$\frac{3m-6}{4}\times\frac{8m}{m^2-2m}=\frac{3(m-2)}{4}\times\frac{8m}{m(m-2)}$$
$$=\frac{3\cancel{(m-2)}}{\cancel{4}_{1}}\times\frac{\overset{2}{\cancel{8}}\,\cancel{m}}{\cancel{m(m-2)}}$$
$$=6$$

b
$$\frac{d^2+3d+2}{d^2-9}\div\frac{d^2+d}{3d+9}=\frac{d^2+3d+2}{d^2-9}\times\frac{3d+9}{d^2+d}$$
$$=\frac{(d+2)(d+1)}{(d+3)(d-3)}\times\frac{3(d+3)}{d(d+1)}$$
$$=\frac{(d+2)\cancel{(d+1)}}{\cancel{(d+3)}(d-3)}\times\frac{3\cancel{(d+3)}}{d\cancel{(d+1)}}$$
$$=\frac{3(d+2)}{d(d-3)}$$

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EXERCISE 1.15 ANSWERS ON P. 568

Factorising algebraic fractions U F R

EXAMPLE 32

1 Simplify each expression. R

a $\frac{3x+3y}{3}$ **b** $\frac{5}{10t-10r}$ **c** $\frac{ab-ac}{a^2}$

d $\frac{y-1}{1-y}$ **e** $\frac{w^2-16}{w+4}$ **f** $\frac{5d-5w}{d^2-w^2}$

g $\frac{(k+5)^2}{k^2-25}$ **h** $\frac{6c^2-6}{2c+2}$ **i** $\frac{am-an+m-n}{m^2-n^2}$

j $\frac{y^2+9y+20}{2y+10}$ **k** $\frac{k^2-3k-4}{k^2-16}$ **l** $\frac{16a^2-25c^2}{4a^2-9ac+5c^2}$

m $\frac{x^2+4x+4}{x^2-x-6}$ **n** $\frac{1-c-2c^2}{3c^2+2c-1}$ **o** $\frac{ap+4p-2a-8}{2p^2-8}$

p $\frac{c^2-9c+20}{cu-4+c-4u}$ **q** $\frac{h^2+3h+2}{2h^2+4h}$ **r** $\frac{8e^2-14e-15}{4e^3-25e}$

EXAMPLE 33

2 Simplify each expression. R

a $\frac{5}{m(m+1)}+\frac{2}{(m+1)(m+2)}$ **b** $\frac{6}{(w+5)(w+3)}-\frac{4}{w(w+3)}$

c $\frac{3}{(b+2)(b-1)}+\frac{1}{(b-1)(b-3)}$ **d** $\frac{2}{k^2+k}-\frac{3}{k^2-1}$

e $\frac{5}{4h+4}+\frac{3}{h^2+h}$ **f** $\frac{3}{d^2+3d+2}-\frac{4}{d+2}$

g $\frac{3}{r^2-36}-\frac{5}{4r+24}$ **h** $\frac{3}{d^2+2d}+\frac{d}{d^2-4}$

i $\frac{5}{k^2-3k-4}-\frac{k}{k^2-1}$ **j** $\frac{2}{q^2-1}+\frac{3}{q+1}$

k $\frac{4}{a^2-8a+15}+\frac{3}{a^2+4a-21}$ **l** $\frac{2}{y^2+5y+6}-\frac{7}{y^2-y-12}$

m $\frac{5}{e^2+3e+2}-\frac{8}{e^2+9e+14}$ **n** $\frac{3}{6h^2-h-2}+\frac{4}{12h^2-23h+10}$

3 Simplify each expression. R

a $\frac{3m+9}{2}\times\frac{4m}{m+3}$ **b** $\frac{5d-10}{3d-9}\times\frac{5d-15}{8d-16}$ **c** $\frac{4}{e+2}\times\frac{e^2+2e}{8e}$

d $\frac{3k+6}{5}\times\frac{10k}{k+2}$ **e** $\frac{5h}{3h+9}\times\frac{6h+18}{h^2+h}$ **f** $\frac{4}{a^2-b^2}\times\frac{3a+3b}{8}$

g $\frac{r+t}{t^2-r^2}\times\frac{r^2-rt}{5r+5t}$ **h** $\frac{20m+16}{7m-7}\times\frac{7m}{5m+4}$ **i** $\frac{p^2+2p+1}{p^2-1}\times\frac{4p-4}{p^2+p}$

j $\frac{y+2}{5y}\div\frac{7y+14}{15y}$ **k** $\frac{5}{x^2-4}\div\frac{15}{2x+4}$ **l** $\frac{4n+8}{n+5}\div\frac{6n+12}{5n+25}$

m $\frac{d^2+d}{d+3}\div\frac{6d}{d^2-9}$ **n** $\frac{1}{f^2-6f+9}\div\frac{4}{f^2-9}$ **o** $\frac{3f+6}{f^2+f-6}\div\frac{f^2-2f-8}{f^2-f-12}$

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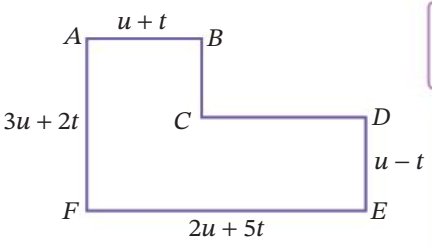
Power plus ANSWERS ON P.568

1 For this composite shape, write an expression for:



- **a** the length of CD
- **b** the length of BC
- **c** its perimeter
- **d** its area

2 Find the average of $6r$, $2r + 8$, $r - 5$, $2r$, $r + 7$ and $3r + 8$.

3 A rectangular garden has its longer sides each 5 m longer than its shorter sides. If its longer sides each have length y m, then write a simplified expression for:

- **a** the area of the garden
- **b** the perimeter of the garden

4 How many hours does it take a car to travel a distance of $20t^2$ kilometres at an average speed of $4t$ km/h?

5 Expand and simplify each expression.

- **a** $(a + b)(a + b + c)$
- **b** $(x - 2)(x + y - 3)$
- **c** $(p + 3)(p - 4)(p - 5)$
- **d** $(x - 1)(y - 1)(w - 1)$
- **e** $(ad - 3r)^2$
- **f** $(x^3 + 2)^2$

6 Factorise each expression.

- **a** $n^2 + 4mn + 4m^2$
- **b** $x^2 - 2xy + y^2$
- **c** $25x^2 - 40xy + 16y^2$
- **d** $5a^2 - 30ab + 45b^2$
- **e** $c^4 + 2c^2 + 1$
- **f** $8 - 7k^2 - k^4$
- **g** $x^4 - 1$
- **h** $(a + b)^2 - c^2$
- **i** $(p + q)^2 - (p - q)^2$

7 If $x^2 + bx + c = (x + p)(x + q)$, what are the signs of p and q if:

- **a** b and c are both positive?
- **b** b and c are both negative?
- **c** b is positive and c is negative?
- **d** b is negative and c is positive?

8 Simplify each expression.

- **a** $\frac{x-1}{4} + \frac{4x+2}{10}$
- **b** $\frac{2x+5}{6} + \frac{x-4}{3}$
- **c** $\frac{x}{3} - \frac{x+1}{5}$

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CHAPTER 1 REVIEW

WS Algebra 2

Qz Algebra

Language of maths

area diagram	binomial	binomial product	coefficient
common factor	constant term	difference of 2 squares	distributive law
expand	expression	factor	factorise
formula	grouping in pairs	highest common factor (HCF)	like terms
perfect square	pronumeral	product	quadratic trinomial
simplify	sum	variable	

1 Why is it possible to add or subtract terms like $6n$ and $4n$ but not $7x$ and $2y$?

2 In the expression $2x^2 - 3x + 6$, what is:

a the constant term? **b** the coefficient of x^2?

3 In what type of algebraic expression is the highest power of the variable equal to 2?

4 Explain what **product** means.

5 In algebra, what is the opposite of **expand?**

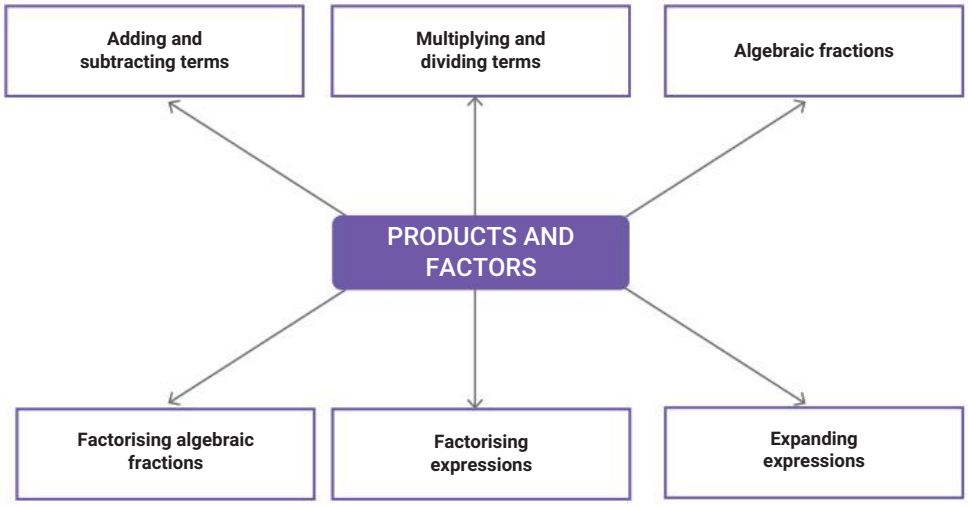
6 Complete: A binomial is an algebraic expression with ____ _____.

Topic summary

WS Mind map: Products and factors

- Using your own words, write down the new things you have learnt.
- What parts of this topic did you like?
- What parts of this topic did you find difficult or you did not understand?
- Make a list of the skills you have learnt in this topic, such as substituting and expanding.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



TEST YOURSELF 1

ANSWERS ON P.568

1.01 **1** Simplify each expression.

a $9y + 7y$ **b** $4ab - 7ab$ **c** $3x^2 + 5x^2$

d $7p + 2q - 5p$ **e** $4w - 5 + 2w$ **f** $8x^2 - 13x - 5x^2$

g $2m + 3n + 5m + 7n$ **h** $5u + 7 - 2u - 3$ **i** $12d^2 + 5d - 7d + 4$

1.02 **2** Simplify each expression.

a $9 \times 2n$ **b** $g \times 3h$ **c** $a \times d$

d $(-3) \times 2r$ **e** $12y^2 \div 3y$ **f** $2xy \div 8y$

g $20pq \div (-5q)$ **h** $15g^2 \div 25g$ **i** $36md^2 \div 27md$

1.03 **3** Simplify each expression.

a $\frac{h}{6} + \frac{4h}{6}$ **b** $\frac{11k}{8} - \frac{7k}{8}$ **c** $\frac{7}{m} - \frac{5}{m}$

d $\frac{w}{4} - \frac{2w}{5}$ **e** $\frac{4k}{3} - \frac{k}{2}$ **f** $\frac{5}{2d} - \frac{4}{d}$

g $\frac{d}{5k} - \frac{3}{k}$ **h** $\frac{8}{m+1} - \frac{3}{m+1}$ **i** $\frac{2y}{5} + \frac{3d}{2}$

1.04 **4** Simplify each expression.

a $\frac{5}{m} \times \frac{4}{i}$ **b** $\frac{h}{k} \times \frac{3}{k}$ **c** $\frac{14}{3y} \times \frac{9y}{2d}$

d $\frac{r}{4} \div \frac{r}{5}$ **e** $\frac{5}{v} \div \frac{2}{v}$ **f** $\frac{a}{b} \div \frac{a}{b}$

g $\frac{m}{5} \div \frac{5m}{10d}$ **h** $\frac{4}{x} \div \frac{12}{xp}$ **i** $\frac{ad}{6} \times \frac{5a}{2} \div \frac{ad}{8}$

j $\frac{3c}{4d} \div \frac{9c}{16d} \div \frac{6}{8}$ **k** $\frac{15a}{4b} \div \frac{4}{3a^2b} \times \frac{a}{bc}$ **l** $\frac{5y}{8} \div \frac{y}{4} \times \frac{15a}{2y}$

1.05 **5** Expand and simplify each expression.

a $5(r - 2)$ **b** $a(a - 10)$ **c** $3y(2x - 5y)$

d $3p(4p - 7q)$ **e** $-6n(n - 9)$ **f** $-6t(t + 7)$

g $b(b + 1) + 5(b + 1)$ **h** $3x(2x - 7) - 5(2x - 7)$ **i** $12 - 6(2x - 3)$

j $3a - 2(1 + 5a)$ **k** $-5(k + 3) - 6k$ **l** $4(3 - 2y) - 2(4y + 5)$

1.06 **6** Factorise each expression.

a $4m - 12$ **b** $xy + xp$ **c** $-3g - 30$

d $16w^2 + 24w$ **e** $-8q + 16$ **f** $15k - 10hk^2$

g $3(y - 6) + y(y - 6)$ **h** $3x(2x - 1) - 2(2x - 1)$ **i** $p(p + 6) - 2(p + 6)$

1.07 **7** Expand each binomial product.

a $(a + 2)(a + 3)$ **b** $(y + 3)(y - 7)$ **c** $(t - 3)(t + 8)$

d $(h - 5)(h - 4)$ **e** $(4 - g)(3 + g)$ **f** $(3p + 1)(2p + 3)$

g $(5x + 1)(4x - 3)$ **h** $(4r - 5)(3r + 4)$ **i** $(5 - 3q)(2q - 1)$

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8 Expand each perfect square. 1.08

a $(n+5)^2$ **b** $(u-3)^2$ **c** $(9+x)^2$ **d** $(6-k)^2$

e $(3y+1)^2$ **f** $(2x-5)^2$ **g** $(6-7b)^2$ **h** $(1-4h)^2$

9 Expand each expression. 1.09

a $(n+6)(n-6)$ **b** $(10+m)(10-m)$ **c** $(3-8w)(3+8w)$

10 Expand and simplify each expression. 1.10

a $(2x+5)(x-1)+(x+3)^2$ **b** $(y+4)^2-(y-1)(y+2)$

c $(3a+1)(3a-1)-(3a+1)^2$ **d** $(2p-3q)^2+(2p+3q)^2$

11 Factorise each expression. 1.11

a $5mn-3n+10m-6$ **b** $ab-a-b+1$ **c** $x^2y+xy^3-5x-5y^2$

d $10p^2+30+p^3+3p$ **e** w^2-9 **f** m^2-n^2

g $36-r^2$ **h** $16a^2-9$ **i** $81x^2-y^2$

j $49a^2-25b^2$ **k** $3q^2-27$ **l** $50m^2-32$

m $24-150d^2$ **n** $4d^2+3de-3e^2-4de$ **o** $15h^2-6hk+2k^2-5kh$

12 Factorise each expression. 1.12

a $m^2-11m-12$ **b** y^2-4y+3 **c** $h^2-8h-65$

d $p^2+6p-27$ **e** $3n^2+9n+6$ **f** $4y^2+4y-8$

g $18+3x-x^2$ **h** $60-5b-5b^2$ **i** $11k-4-6k^2$

13 Factorise each expression. 1.13

a $6x^2+x-2$ **b** $5p^2-7p-6$ **c** $8w^2-14w+3$

d $3e^2+12e+12$ **e** $6y^2+15y-36$ **f** $24a^2-28a-12$

g $36x-27x^2-12$ **h** $52k-30-16k^2$ **i** $12g^2-46g+14$

14 Factorise each expression. 1.14

a $64y^2-25$ **b** $x^2-16x+64$ **c** $16r-48$

d $3g-15-5h+hg$ **e** a^3-a^2-a+1 **f** $3b^3-3b$

g $4y^2-5y-21$ **h** $18q^2+24q+8$ **i** $1-2p-24p^2$

15 Simplify each expression. 1.15

a $\frac{9y^2-25}{12y-20}$ **b** $\frac{2x^2+3x-5}{6x^2+19x+10}$ **c** $\frac{3w^2-75}{3w^2-30w+75}$

16 Simplify each expression. 1.15

a $\frac{4}{x^2+x}-\frac{3}{x^2-1}$ **b** $\frac{4}{x^2-9}+\frac{2}{3x+9}$

c $\frac{k}{2k+6}+\frac{5}{k^2-9}$ **d** $\frac{1}{(x-3)(x+2)}+\frac{4}{(x+2)(x+3)}$

e $\frac{4d}{d^2-3d+2}-\frac{3}{d^2-2d}$ **f** $\frac{3}{m^2-2m-3}-\frac{2}{m^2+5m+4}$

g $\frac{3d-6}{d+3}\times\frac{3d+9}{5d-10}$ **h** $\frac{y^2-5y+6}{y^3-4y}\times\frac{2y^2-8}{y^2-2y}$

i $\frac{4}{d^2-4}\div\frac{8}{d+2}$ **j** $\frac{m}{m^2+m}\div\frac{4}{5m+5}$

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2

NUMBER AND ALGEBRA, MEASUREMENT AND GEOMETRY

SURDS AND PYTHAGORAS' THEOREM

Pythagoras was an ancient Greek mathematician who lived in the 5th century BCE. The theorem (or rule) which carries his name was well-known before his time, but Pythagoras may have been the first to prove it. Over 300 proofs for the theorem are known today. Pythagoras' theorem is the most famous mathematical formula, and it is still used today in architecture, engineering, surveying and astronomy.

Chapter outline

	Working mathematically				
2.01 Finding the hypotenuse	U	F			
2.02 Surds and irrational numbers	U	F		R	C
2.03 Simplifying surds*	U	F		R	
2.04 Adding and subtracting surds*	U	F		R	
2.05 Multiplying and dividing surds*	U	F		R	
2.06 Finding a shorter side	U	F	PS	R	
2.07 Mixed problems	U	F	PS		C
2.08 Testing for right-angled triangles	U	F		R	C
2.09 Pythagorean triads	U	F		R	C

***STAGE 5.3**

Note: Pythagoras' theorem is a Year 9 topic in the Australian Curriculum but a Stage 4 topic in the NSW syllabus, so Pythagoras' theorem has also been covered in Chapter 1 of *New Century Maths 8*.

Wordbank

converse A rule or statement turned back-to-front; the reverse statement

hypotenuse The longest side of a right-angled triangle; the side opposite the right angle

irrational number A number such as π or $\sqrt{2}$ that cannot be expressed as a fraction

Pythagoras An ancient Greek mathematician who discovered an important formula about the sides of a right-angled triangle

Pythagorean triad A set of three numbers that follow Pythagoras' theorem, such as {3, 4, 5}.

surd A square root (or other root) whose exact value cannot be found

theorem Another name for a formal rule or formula

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- investigate irrational numbers and surds
- (STAGE 5.3) simplify, add, subtract, multiply and divide surds
- solve problems involving Pythagoras' theorem, writing the answers in decimal or surd form
- test whether a triangle is right-angled
- investigate Pythagorean triads

SkillCheck ANSWERS ON P. 569

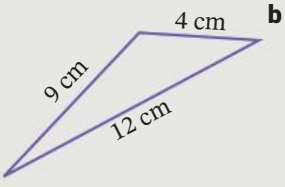
1 Evaluate each expression.

a 4^2 **b** 10.3^2 **c** $3^2 + 5^2$

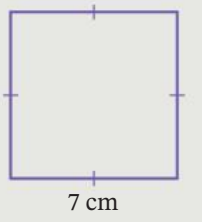
d $8^2 - 6^2$ **e** $\sqrt{49}$ **f** $\sqrt{121}$

2 Find the perimeter of each shape.

a



b



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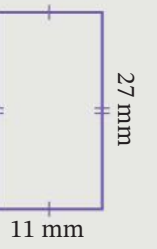


3 Select the square numbers from the following list of numbers.

44 81 25 100 75 72 16 50 64 32

2.01 Finding the hypotenuse

Pythagoras' discovery

Pythagoras' theorem

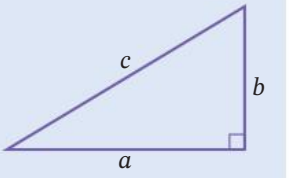
Pythagoras' theorem

Pythagoras' theorem

Pythagoras' theorem

For any right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other 2 sides.

If c is the length of the hypotenuse, and a and b are the lengths of the other 2 sides, then:



$$c^2 = a^2 + b^2$$

that is,

$$(\text{hypotenuse})^2 = (\text{shorter side 1})^2 + (\text{shorter side 2})^2$$

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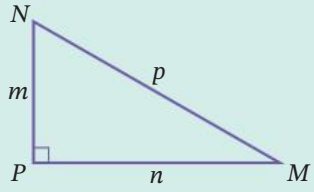
Example 1

Write Pythagoras' theorem for this triangle.



Solution

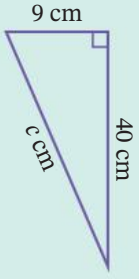
p is the hypotenuse, so $p^2 = m^2 + n^2$

OR

NM is the hypotenuse, so $NM^2 = NP^2 + PM^2$

Example 2

Find the value of c in this triangle.



Solution

We want to find the length of the hypotenuse.

Using Pythagoras' theorem:

$c^2 = 9^2 + 40^2$

$= 1681$

$c = \sqrt{1681}$ Use the square root to find c

$= 41$

An answer of $c = 41$ looks reasonable because:

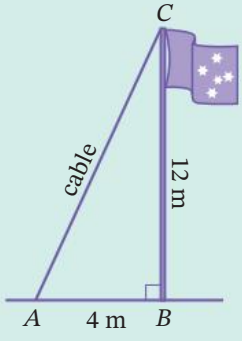
- the hypotenuse is the longest side
- from the diagram, the hypotenuse looks a little longer than the side that is 40 cm

Example 3

Find the length of the cable supporting this flagpole:

a as a surd

b correct to one decimal place.



Solution

a $AC^2 = 12^2 + 4^2$

$= 160$

$AC = \sqrt{160}$ m This is the answer as a surd (in $\sqrt{\ }$ form).

b $AC = \sqrt{160}$ From part **a**

$= 12.6491...$

≈ 12.6 m Rounded to one decimal place

Example 4

In $\triangle PQR$, $\angle P = 90°$, $PQ = 25$ cm and $PR = 32$ cm. Sketch the triangle and find the length of the hypotenuse, correct to one decimal place.

Solution

$p^2 = 32^2 + 25^2$

$= 1649$

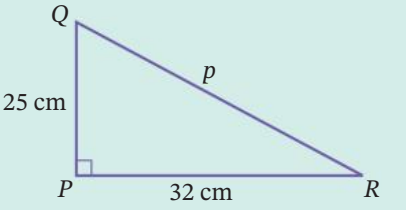
p is the side opposite $\angle P$ and is also the hypotenuse

$p = \sqrt{1649}$

$= 40.607\,881\,01...$

≈ 40.6

$\therefore$ The length of the hypotenuse is 40.6 cm.



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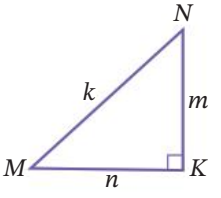
EXERCISE 2.01 ANSWERS ON P. 569

Finding the hypotenuse U F PS R C

EXAMPLE 1

1 Write Pythagoras' theorem for each triangle. C

a



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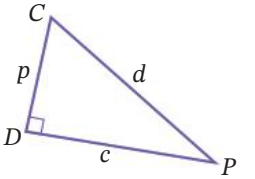
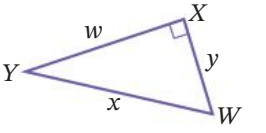


c



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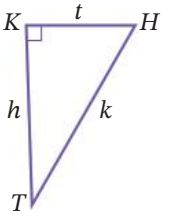

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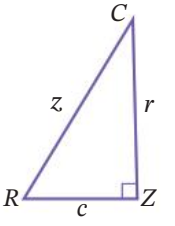

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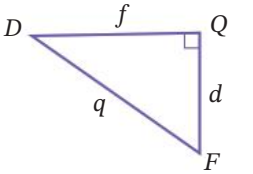

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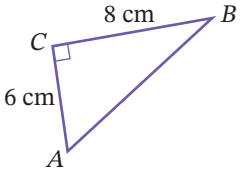
EXAMPLE 2

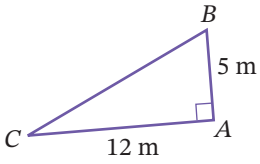
2.01

2 Find the length of the hypotenuse in each triangle.

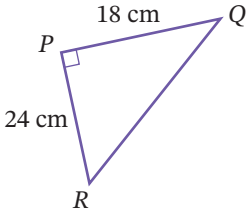
a



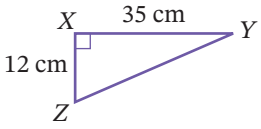
b



c



d



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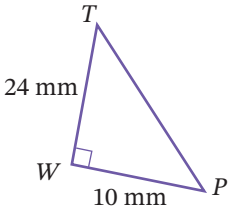

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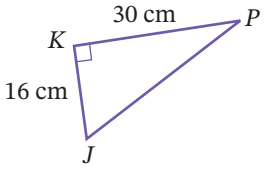

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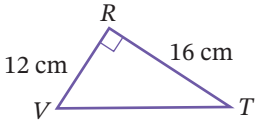

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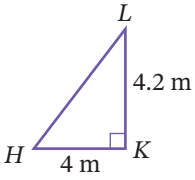

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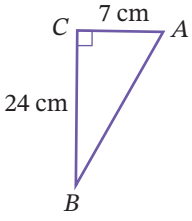

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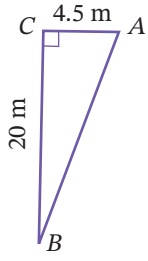

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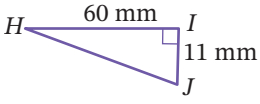

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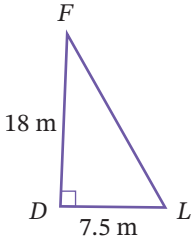
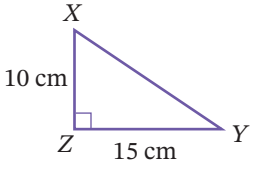


EXAMPLE 3

3 Find the length of the hypotenuse in each triangle, as a surd.

a



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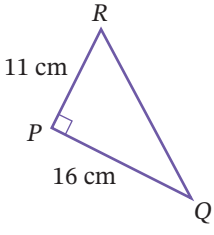

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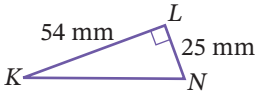

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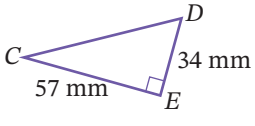

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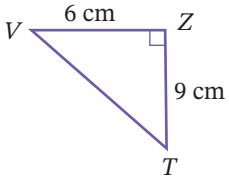

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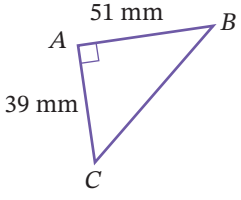

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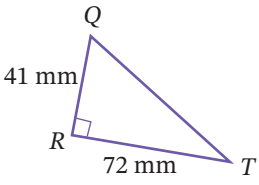
4 Find the length of the hypotenuse in each triangle, correct to one decimal place.

a



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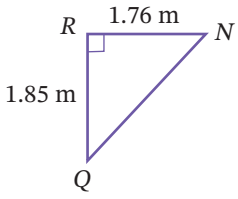

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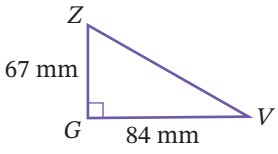

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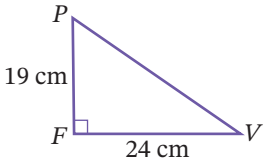

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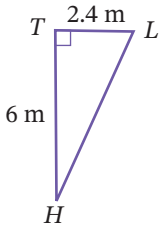

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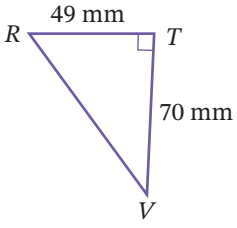
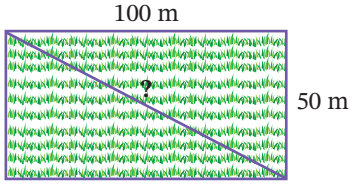


5 A rectangular field is 100 m long and 50 m wide. How far is it from one corner to the opposite corner, along the diagonal? Select the correct answer **A**, **B**, **C** or **D**.



A 150 m **B** 111.8 m

C 100.2 m **D** 98.3 m

6 A firefighter places a ladder on a window 4.5 m above the ground. If the foot of the ladder is 1.6 m from the wall, how long is the ladder, correct to one decimal place?

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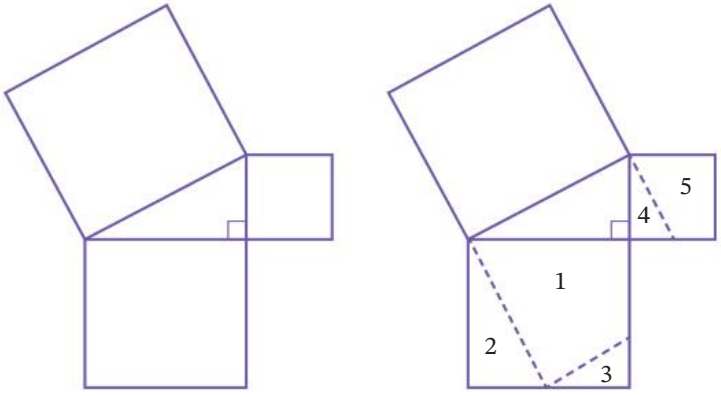
7 Pythagoras' theorem can be demonstrated visually by dissection (cutting up shapes). PS R

WS Pythagoras 1

a On a sheet of paper, construct a right-angled triangle and draw squares on all 3 sides, as shown in the diagram on the left. Alternatively, download the worksheet 'Pythagoras 1'.



b In the 2 smaller squares, draw lines perpendicular to the hypotenuse as shown, and in the bottom square, draw a line parallel to the hypotenuse as shown in the diagram on the right.

c Number the regions 1, 2, 3, 4, 5 as shown.

d Cut out the 5 pieces and rearrange them to form the square on the hypotenuse.

e Does the area of the 2 smaller squares equal the area of the larger square?

f How does this 'prove' Pythagoras' theorem?

g Repeat this question on another right-angled triangle.

8 **a** In $\triangle ABC$, $\angle ABC = 90°$, $AB = 39$ cm and $BC = 57$ cm. Find AC, correct to one decimal place.

EXAMPLE 4

b In $\triangle MPQ$, $\angle PQM = 90°$, $QM = 2.4$ m and $PQ = 3.7$ m. Find PM as a surd.

c In $\triangle RVJ$, $\angle J = 90°$, $JV = 12.7$ cm and $JR = 42$ mm. Find RV, correct to the nearest millimetre.

d In $\triangle EGB$, $EG = EB = 127$ mm and $\angle GEB = 90°$. Find the length of BG, correct to one decimal place.

e In $\triangle VZX$, $\angle V = 90°$, $VX = 547$ cm and $VZ = 11.6$ m. Find ZX in metres, correct to the nearest 0.1 m.

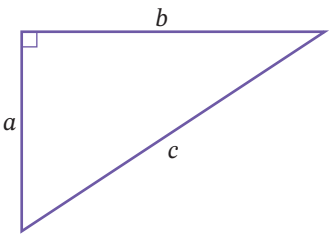
f In $\triangle PQR$, $\angle RPQ = 90°$, $PQ = 2350$ mm and $PR = 5.8$ m. Find QR in metres, correct to 2 decimal places.

Spreadsheets

Technology

Finding the hypotenuse

Use a spreadsheet to calculate the length of the hypotenuse, c, of a right-angled triangle, given the lengths of the other 2 sides (a and b).



1 Enter the headings shown below into cells A1, B1, C1 and D1. Then enter the values shown into columns A and B.

	A	B	C	D	E
1	a	b	a^2+b^2	c	
2	3	4			
3	5	12			
4	6	8			
5	7	24			
6					
7					

2 Click on cell C2. Type the formula **=A2^2+B2^2**. Use **Fill Down** to copy this formula into cells C3 to C5.

3 To calculate each hypotenuse, we need to find the square root of the values in column C each time. So, in cell D2, enter **=sqrt(C2)**. Then click on cell D2 and use **Fill Down** to copy this formula into cells D3 to D5.

4 Use your spreadsheet to find the length of the hypotenuse, given the following pairs of values for the other 2 sides.

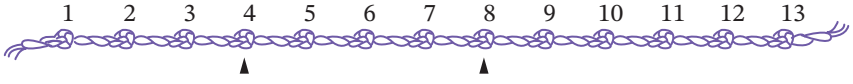
a $a = 12, b = 9$ **b** $a = 10, b = 10$ **c** $a = 7, b = 6$

d $a = 1, b = 8$ **e** $a = 7.5, b = 11.9$ **f** $a = 6.4, b = 2.3$

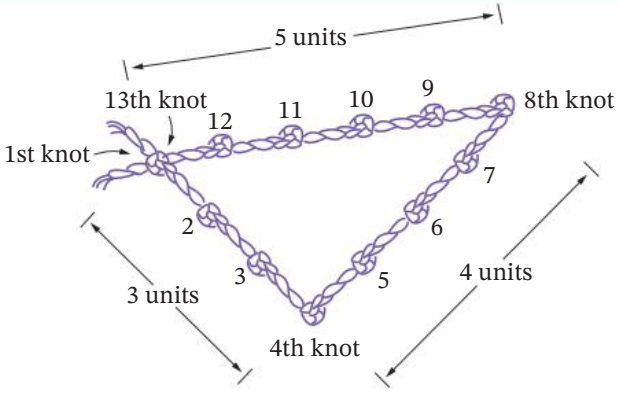
Did you know?

Egyptian rope stretchers

In ancient times, whenever the Nile River flooded, Egyptian farmers would have to rebuild fences around their fields afterwards. To form right angles for this, they used a secret 'magical' technique called rope stretching to create a 90° angle, making a length of rope with 13 knots tied in it at equal intervals.



The rope was fixed to the ground at the 4th and 8th knots, and stretched around to create a triangle with side lengths 3, 4 and 5 units. The right angle is formed at the 4th knot.



The builders of the great pyramids also used rope stretching, but could not understand why this 'trick' produced a right angle. Pythagoras studied with the rope stretchers and eventually worked it out.

Make and stretch a knotted rope to create a right angle using a 3-4-5 triangle.

Surds and irrational numbers 2.02

- $\sqrt{25} = 5$ because $5^2 = 25$ 'the square root of 25'
- $\sqrt{81} = 9$ because $9^2 = 81$ 'the square root of 81'

Most square roots do not give exact answers like the ones above.

For example, $\sqrt{7} = 2.645751311\ldots \approx 2.6$. Such roots are called **surds**.

A surd is a square root ($\sqrt{\ }$), cube root ($\sqrt[3]{\ }$), or any type of root whose exact decimal or fraction value cannot be found. As a decimal, its digits run endlessly *without repeating* (like π), so they are neither terminating nor recurring decimals.

Rational numbers such as fractions, decimals and percentages, can be expressed in the form $\frac{a}{b}$, where a and b are integers (and $b \neq 0$), but surds are **irrational numbers** because they cannot be expressed in this form.

Example 5

Select the surds from this list of square roots:

$$\sqrt{72} \quad \sqrt{121} \quad \sqrt{64} \quad \sqrt{90} \quad \sqrt{28}$$

Solution

$\sqrt{72} = 8.4852...$

$\sqrt{121} = 11$

$\sqrt{64} = 8$

$\sqrt{90} = 9.4868...$

$\sqrt{28} = 5.2915...$

so the surds are $\sqrt{72}$, $\sqrt{90}$ and $\sqrt{28}$.

$\sqrt{72}$ is called an '*exact* value' while 8.4852 is called its '*approximate* value'

Example 6

Is each number rational or irrational?

a $4\frac{2}{5}$ **b** $\sqrt[3]{-8}$ **c** $-\sqrt{7}$ **d** $0.\dot{6}$ **e** 5π

Solution

a $4\frac{2}{5} = \frac{22}{5}$ which is in the form of a fraction $\frac{a}{b}$

$\therefore 4\frac{2}{5}$ is a rational number.

b $\sqrt[3]{-8} = -2$ which can be written as $\frac{-2}{1}$

$\therefore \sqrt[3]{-8}$ is a rational number.

c $-\sqrt{7} = -2.645\,751\,311\ldots$ The digits run endlessly without repeating

$\therefore -\sqrt{7}$ is an irrational number.

d $0.\dot{6} = 0.666\ldots$ which is a recurring decimal

$= \frac{2}{3}$ which is a fraction

$\therefore 0.\dot{6}$ is a rational number.

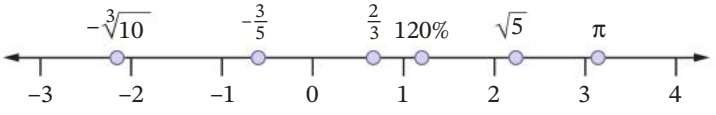
e $5\pi = 15.707\,963\,27...$ The digits run endlessly without repeating

$\therefore 5\pi$ is an irrational number.

STAGE 5.3

Surds on a number line

The rational and irrational numbers together make up the **real numbers**. Any real number can be represented by a point on the number line.



$-\sqrt[3]{10} = -2.1544\ldots$	irrational (surd)
$-\frac{3}{5} = -0.6$	rational (fraction)
$\frac{2}{3} = 0.6666\ldots$	rational (fraction)
$120\% = 1.2$	rational (percentage)
$\sqrt{5} = 2.2360\ldots$	irrational (surd)
$\pi = 3.1415...$	irrational (pi)

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WS Surds on the number line

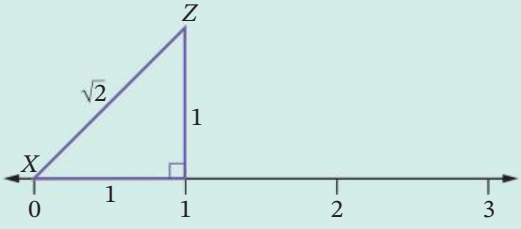
Example 7

Use a pair of compasses and Pythagoras' theorem to estimate the value of $\sqrt{2}$ on a number line.

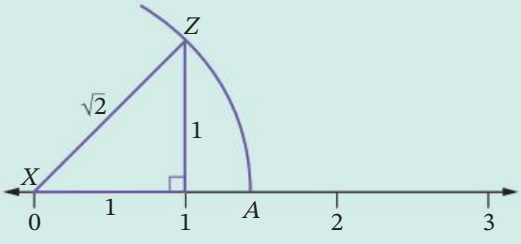
Solution

Step 1: Using a scale of 1 unit to 2 cm, draw a number line as shown.

Step 2: Construct a right-angled triangle on the number line with base length and height 1 unit as shown. By Pythagoras' theorem, show that $XZ = \sqrt{2}$ units.



Step 3: With 0 as centre, use compasses with radius XZ ($\sqrt{2}$) to draw an arc to meet the number line at A as shown. The point A represents the value of $\sqrt{2}$ and should be approximately 1.4142...



EXERCISE 2.02 ANSWERS ON P. 569

Surds and irrational numbers UFRC

EXAMPLE 5

1 Which one of the following is a surd? Select the correct answer **A**, **B**, **C** or **D**. C

A $\sqrt{9}$ **B** $\sqrt{225}$ **C** $\sqrt{160}$ **D** $\sqrt{81}$

2 Which one of the following is NOT a surd? Select **A**, **B**, **C** or **D**. C

A $\sqrt{77}$ **B** $\sqrt{144}$ **C** $\sqrt{18}$ **D** $\sqrt{200}$

3 Select the surds from the following list of square roots. C

$\sqrt{32}$ $\sqrt{33}$ $\sqrt{289}$ $\sqrt{81}$ $\sqrt{4.9}$ $\sqrt{52}$ $\sqrt{121}$ $\sqrt{144}$ $\sqrt{196}$ $\sqrt{200}$

EXAMPLE 6

4 Is each number rational (R) or irrational (I)? R C

a $5.\dot{6}$ **b** $\sqrt{8}$ **c** $\sqrt{4}$ **d** $3\frac{1}{7}$

e $\sqrt[3]{27}$ **f** $1.3\dot{5}$ **g** $\sqrt[3]{-64}$ **h** $27\frac{1}{2}\%$

i $\sqrt{5\times10^3}$ **j** $\frac{3}{11}$ **k** $\frac{\sqrt{50}}{3}$ **l** $\sqrt{\sqrt{4}}$

5 Arrange each set of numbers in descending order.

a $1\frac{4}{7}, \sqrt{2}, \frac{\pi}{2}$ **b** $\sqrt[3]{20}, 2.\dot{6}, 2\frac{7}{9}$

STAGE 5.3

EXAMPLE 7

6 Use the method from Example **7** to estimate the value of $\sqrt{2}$ on a number line.

7 **a** Use the method from Example **7** to estimate the value of $\sqrt{5}$ on a number line by constructing a right-angled triangle with base length 2 units and height 1 unit.

b Use a similar method to estimate the following surds on a number line.

i $\sqrt{10}$ **ii** $\sqrt{17}$

8 Without using a calculator, determine between which 2 consecutive numbers the value of $\sqrt{40}$ lies. Select **A**, **B**, **C** or **D**. R

A 4 and 5 **B** 5 and 6 **C** 6 and 7 **D** 7 and 8

9 Is the statement $\sqrt{x^2} = x$ always true? R

10 Is the statement $\left(\sqrt{x}\right)^2 = x$ always true? R

Investigation

Proof that $\sqrt{2}$ is irrational

A method of proof sometimes used in mathematics is to assume the **opposite** of what is being proved, and to show that it is **impossible**. This is called a **proof by contradiction**, and we will use it to prove that $\sqrt{2}$ is irrational.

First, we assume that $\sqrt{2}$ is rational. So, assume that $\sqrt{2}$ can be written as a simplified fraction $\frac{a}{b}$, where a and b are integers ($b \neq 0$) with no common factors.

$\sqrt{2} = \frac{a}{b}$

$2 = \frac{a^2}{b^2}$ Squaring both sides

$a^2 = 2b^2$

$2b^2$ is an even number because it is divisible by 2, $\therefore a^2$ is even.

$\therefore a$ is even, because an even integer multiplied by itself is always even and an odd integer multiplied by itself is always odd.

$\therefore a = 2m$, where m is another integer.

$\therefore a^2 = (2m)^2 = 2b^2$

$4m^2 = 2b^2$

$2m^2 = b^2$

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$b^2 = 2m^2$

$\therefore b^2$ is even

$\therefore b$ is even.

But a and b can't **both** be even because this contradicts the assumption that a and b have no common factors. Therefore, the assumption that $\sqrt{2}$ is rational is false, so $\sqrt{2}$ must be irrational.

1 Use proof by contradiction to show that these surds are irrational:

a $\sqrt{3}$ **b** $\sqrt{5}$

2 Compare your proofs with those of other students.

Did you know?

The Pythagoreans

Sometimes when applying Pythagoras' theorem, lengths are found that cannot be expressed as exact rational numbers. Pythagoras encountered this when calculating the diagonal of a square of side length 1 unit.

The Pythagoreans were a group of men who were Pythagoras' followers in ancient Greece. They were the first to study the properties of whole numbers and aimed to explain nature, the universe — in fact everything — in terms of whole numbers. Apparently, they were so upset about the discovery of surds that they tried to keep it a secret. Hippasus, one of the Pythagoreans, was drowned for revealing the secret to outsiders.

π is an irrational number but it is not a surd. Why? Another such number is e. Investigate the numbers π and e and the meaning of transcendental numbers.

Simplifying surds 2.03

The square of any real number is always positive (except for $0^2 = 0$), so it is not possible to give the square root of a *negative* number.

The radical symbol $(\sqrt{\ })$ stands for the *positive* square root of a number, for example $\sqrt{4} = 2$ (not -2).

The square root of x

For $x < 0$ (negative), $\sqrt{x}$ is undefined.

For $x = 0$, $\sqrt{x} = 0$.

For $x > 0$ (positive), $\sqrt{x}$ is the positive square root of x.

For $x \geq 0$, $\left(\sqrt{x}\right)^2 = \sqrt{x} \times \sqrt{x} = x$ and $\sqrt{x^2} = x$.

STAGE 5.3

Simplifying surds

Simplifying surds quiz

Simplifying surds quiz

STAGE 5.3

Example 8

Simplify each expression.

a $\left(\sqrt{7}\right)^2$ **b** $\left(3\sqrt{5}\right)^2$ **c** $\left(-2\sqrt{3}\right)^2$

Solution

a $\left(\sqrt{7}\right)^2 = 7$

b $(3\sqrt{5})^2 = 3\sqrt{5} \times 3\sqrt{5}$ $3\sqrt{5}$ means $3 \times \sqrt{5}$

$= 3^2 \times (\sqrt{5})^2$

$= 9 \times 5$

$= 45$

c $(-2\sqrt{3})^2 = (-2)^2 \times (\sqrt{3})^2$

$= 4 \times 3$

$= 12$

The square root of a product

For $x > 0$ and $y > 0$:

$$\sqrt{xy} = \sqrt{x} \times \sqrt{y}$$

A surd $\sqrt{n}$ can be simplified if n can be divided into 2 factors, where one of them is a square number such as 4, 9, 16, 25, 36, 49, ...

Simplifying surds 1

Simplifying surds 2

Example 9

Simplify each surd.

a $\sqrt{50}$ **b** $\sqrt{432}$ **c** $4\sqrt{12}$ **d** $\frac{\sqrt{288}}{3}$

Solution

a $\sqrt{50} = \sqrt{25} \times \sqrt{2}$ 25 is a square number

$= 5 \times \sqrt{2}$

$= 5\sqrt{2}$

b **Method 1**

$\sqrt{432} = \sqrt{4} \times \sqrt{108}$

$= 2 \times \sqrt{108}$

$= 2 \times \sqrt{36} \times \sqrt{3}$

$= 2 \times 6 \times \sqrt{3}$

$= 12 \times \sqrt{3}$

$= 12\sqrt{3}$

Method 2

$\sqrt{432} = \sqrt{144} \times \sqrt{3}$

$= 12 \times \sqrt{3}$

$= 12\sqrt{3}$

Method 1 involves simplifying surds *twice* ($\sqrt{432}$ and $\sqrt{108}$). Method 2 shows that when simplifying surds, it is more efficient to first look for the highest square factor possible.

c $4\sqrt{12} = 4 \times \sqrt{4} \times \sqrt{3}$
$= 4 \times 2 \times \sqrt{3}$
$= 8\sqrt{3}$

d $\frac{\sqrt{288}}{3} = \frac{\sqrt{144} \times \sqrt{2}}{3}$
$= \frac{12\sqrt{2}}{3}$
$= \frac{12^{4}\sqrt{2}}{3_{1}}$
$= 4\sqrt{2}$

EXERCISE 2.03 ANSWERS ON P. 569

Simplifying surds U F R

EXAMPLE 8

1 Simplify each expression.

a $\left(\sqrt{2}\right)^2$ **b** $\left(\sqrt{5}\right)^2$ **c** $\left(3\sqrt{3}\right)^2$ **d** $\left(5\sqrt{10}\right)^2$

e $\left(\sqrt{0.09}\right)^2$ **f** $\left(-2\sqrt{7}\right)^2$ **g** $\left(-3\sqrt{5}\right)^2$ **h** $\left(-5\sqrt{2}\right)^2$

EXAMPLE 9

2 Simplify each surd.

a $\sqrt{8}$ **b** $\sqrt{27}$ **c** $\sqrt{24}$ **d** $\sqrt{54}$

e $\sqrt{243}$ **f** $\sqrt{45}$ **g** $\sqrt{48}$ **h** $\sqrt{200}$

i $\sqrt{96}$ **j** $\sqrt{63}$ **k** $\sqrt{288}$ **l** $\sqrt{108}$

m $\sqrt{75}$ **n** $\sqrt{147}$ **o** $\sqrt{32}$ **p** $\sqrt{242}$

q $\sqrt{162}$ **r** $\sqrt{245}$ **s** $\sqrt{125}$ **t** $\sqrt{512}$

3 Simplify each expression.

a $5\sqrt{50}$ **b** $3\sqrt{8}$ **c** $4\sqrt{27}$ **d** $8\sqrt{98}$

e $\frac{\sqrt{40}}{2}$ **f** $\frac{\sqrt{243}}{9}$ **g** $\frac{\sqrt{28}}{6}$ **h** $3\sqrt{24}$

i $9\sqrt{68}$ **j** $\frac{\sqrt{3125}}{10}$ **k** $\frac{1}{2}\sqrt{72}$ **l** $\frac{3}{4}\sqrt{48}$

m $10\sqrt{160}$ **n** $3\sqrt{75}$ **o** $7\sqrt{68}$ **p** $\frac{\sqrt{52}}{6}$

4 Decide whether each statement is true (T) or false (F). R

a $3\sqrt{7} = \sqrt{21}$ **b** $\sqrt{12} = 6$

c $\left(\sqrt{9.4}\right)^2 = 9.4$ **d** $\sqrt{75} = 5\sqrt{3}$

e $\sqrt{3} \approx 1.7$ **f** The exact value of $\sqrt{10}$ is 3.162 277 8

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Mental skills 2A: Maths without calculators ANSWERS ON P. 569

Multiplying and dividing by a power of 10

Multiplying a number by 10, 100, 1000, etc. moves the decimal point to the right and makes the number bigger. We place 0s at the end of the number if necessary.

- When multiplying a number by **10**, move the decimal point **1** place to the right.
- When multiplying a number by **100**, move the decimal point **2** places to the right.
- When multiplying a number by **1000**, move the decimal point **3** places to the right.

The number of places the decimal is moved to the right matches the number of 0s in the 10, 100 or 1000 we are multiplying by.

1 Study each example.

a	$26.32 \times 10 = 26.32 = 263.2$	The point moves 1 place to the right.
b	$8.701 \times 100 = 8.70\,1 = 870.1$	The point moves 2 places to the right.
c	$6.01 \times 1000 = 6.010 = 6010$	The point moves 3 places to the right after a 0 is placed at the end.
d	$17 \times 100 = 17.00 = 1700$	The point moves 2 places to the right after 2 0s are placed at the end.

2 Now evaluate each expression.

a	89.54×10	**b**	3.7×10	**c**	0.831×100
d	42×100	**e**	5.2716×1000	**f**	156.1×10
g	31.84×1000	**h**	64.3×100	**i**	0.0224×1000
j	4.894×10	**k**	7.389×1000	**l**	11.42×100

Dividing a number by 10, 100, 1000, etc. moves the decimal point to the left and makes the number smaller. We place 0s at the start of the decimal if necessary.

- When dividing a number by **10**, move the decimal point **1** place to the left.
- When dividing a number by **100**, move the decimal point **2** places to the left.
- When dividing a number by **1000**, move the decimal point **3** places to the left.

3 Study each example.

a	$145.66 \div 10 = 145.66 = 14.566$	The point moves 1 place to the left.
b	$2.357 \div 100 = 002.357 = 0.023\,57$	The point moves 2 places to the left after 2 0s are inserted at the start.
c	$14.9 \div 1000 = 0014.9 = 0.0149$	The point moves 3 places to the left after 2 0s are inserted at the start.
d	$45 \div 100 = 045. = 0.45$	The point moves 2 places to the left after 1 zero is inserted at the start.

4 Now evaluate each expression.

a	$733.4 \div 10$	**b**	$9.4 \div 10$	**c**	$652 \div 100$	**d**	$10.4 \div 100$
e	$704 \div 1000$	**f**	$198.5 \div 100$	**g**	$2 \div 100$	**h**	$4159 \div 1000$
i	$123 \div 10$	**j**	$0.758 \div 100$	**k**	$8.49 \div 100$	**l**	$25.1 \div 1000$

Investigation

A formula for calculating square roots

Calculators and computers use a formula repeatedly to give an approximate decimal answer for the square root. The formula for calculating $\sqrt{M}$ is $x_{n+1} = \frac{1}{2}\left(x_n + \frac{M}{x_n}\right)$, where x_0 is the first guess and each calculation of the formula gives a better approximation than the last one.

For example, to calculate $\sqrt{500}$ using a first guess of $x_0 = 20$:

$$x_1 = \frac{1}{2}\left(20 + \frac{500}{20}\right) = 22.5$$

$$x_2 = \frac{1}{2}\left(22.5 + \frac{500}{22.5}\right) = 22.361\,111\,11$$

$$x_3 = \frac{1}{2}\left(22.36 + \frac{500}{22.36}\right) = 22.360\,679\,78$$

This process can continue endlessly, with the accuracy increasing each time. This is called an **iterative** formula or process, where iterative means 'repeated'.

$\therefore \sqrt{500} \approx 22.36$, correct to 2 decimal places.

1 **a** Estimate $\sqrt{55}$.

b Use the formula to evaluate $\sqrt{55}$ to 3 decimal places, using the iterative formula.

c Evaluate $\sqrt{55}$ on your calculator to check your answer.

2 **a** Estimate $\sqrt{700}$.

b Use the formula to evaluate $\sqrt{700}$ to 4 decimal places.

c Evaluate $\sqrt{700}$ on your calculator to check your answer.

3 Design a spreadsheet that uses the formula repeatedly to calculate square roots.

Adding and subtracting surds

2.04

STAGE 5.3

Just as you can only add or subtract 'like terms' in algebra, you can only add or subtract 'like surds'.

Example 10

Simplify each expression.

a $4\sqrt{2} + 5\sqrt{2}$ **b** $7\sqrt{3} - 2\sqrt{3}$ **c** $5\sqrt{2} - 3\sqrt{3} + \sqrt{2}$

d $\sqrt{50} + \sqrt{32}$ **e** $\sqrt{8} - \sqrt{27} + \sqrt{18}$ **f** $5\sqrt{20} - 3\sqrt{125}$

Solution

a $4\sqrt{2} + 5\sqrt{2} = 9\sqrt{2}$

b $7\sqrt{3} - 2\sqrt{3} = 5\sqrt{3}$

c $5\sqrt{2} - 3\sqrt{3} + \sqrt{2} = 6\sqrt{2} - 3\sqrt{3}$

d $\sqrt{50}+\sqrt{32}=\sqrt{25}\sqrt{2}+\sqrt{16}\sqrt{2}$
$=5\sqrt{2}+4\sqrt{2}$
$=9\sqrt{2}$

Simplifying each surd

e $\sqrt{8}-\sqrt{27}+\sqrt{18}=\sqrt{4}\sqrt{2}-\sqrt{9}\sqrt{3}+\sqrt{9}\sqrt{2}$
$=2\sqrt{2}-3\sqrt{3}+3\sqrt{2}$
$=5\sqrt{2}-3\sqrt{3}$

f $5\sqrt{20}-3\sqrt{125}=5\sqrt{4}\sqrt{5}-3\sqrt{25}\sqrt{5}$
$=5\times 2\sqrt{5}-3\times 5\sqrt{5}$
$=10\sqrt{5}-15\sqrt{5}$
$=-5\sqrt{5}$

EXERCISE 2.04 ANSWERS ON P. 569

Adding and subtracting surds U F

EXAMPLE 10

1 Simplify each vexpression.

a $5\sqrt{7}+2\sqrt{7}$
b $3\sqrt{2}-8\sqrt{2}$
c $7\sqrt{5}-\sqrt{5}$
d $\sqrt{5}+3\sqrt{5}$
e $5\sqrt{17}-5\sqrt{17}$
f $3\sqrt{10}-2\sqrt{10}$
g $4\sqrt{15}-3\sqrt{15}+7\sqrt{15}$
h $5\sqrt{6}-2\sqrt{6}-4\sqrt{6}$
i $3\sqrt{3}+4\sqrt{3}-5\sqrt{3}$
j $4\sqrt{5}+7\sqrt{5}-\sqrt{5}$
k $8\sqrt{10}-5\sqrt{10}+3\sqrt{10}$
l $10\sqrt{3}-3\sqrt{3}-12\sqrt{3}$

2 Simplify each expression.

a $3\sqrt{5}-8+2\sqrt{5}$
b $11\sqrt{10}+3\sqrt{2}+2\sqrt{10}$
c $-4\sqrt{3}+5\sqrt{2}-5\sqrt{3}$
d $3\sqrt{15}+3\sqrt{2}+4\sqrt{15}+5\sqrt{2}$
e $\sqrt{7}-3\sqrt{5}-4\sqrt{7}+\sqrt{5}$
f $4\sqrt{6}-3\sqrt{3}-2\sqrt{6}-5\sqrt{3}$
g $10\sqrt{11}-5\sqrt{3}+3\sqrt{11}+4\sqrt{3}$
h $\sqrt{13}+8\sqrt{7}-7\sqrt{13}+3\sqrt{7}$
i $2\sqrt{5}-3\sqrt{7}-2\sqrt{5}-3\sqrt{7}$
j $4\sqrt{10}-3\sqrt{5}-4\sqrt{10}$

3 For each expression, select the correct simplified answer **A**, **B**, **C** or **D**.

a $\sqrt{3}+\sqrt{12}$
A $5\sqrt{3}$ **B** $\sqrt{15}$ **C** $2\sqrt{6}$ **D** $3\sqrt{3}$

b $4\sqrt{5}-2\sqrt{125}$
A $-6\sqrt{5}$ **B** $\sqrt{5}$ **C** $-\sqrt{45}$ **D** $-46\sqrt{5}$

4 Simplify each expression.

a $\sqrt{8}+\sqrt{32}$
b $\sqrt{108}-\sqrt{27}$
c $\sqrt{20}-\sqrt{80}$
d $\sqrt{28}-\sqrt{63}$
e $3\sqrt{6}+\sqrt{24}$
f $2\sqrt{5}+\sqrt{125}$
g $\sqrt{40}-\sqrt{90}$
h $5\sqrt{11}+\sqrt{99}$
i $3\sqrt{2}+\sqrt{18}$
j $\sqrt{27}+5\sqrt{3}$
k $\sqrt{200}-7\sqrt{2}$
l $\sqrt{50}+\sqrt{32}$
m $5\sqrt{3}+2\sqrt{27}$
n $3\sqrt{20}-\sqrt{245}$
o $7\sqrt{12}-5\sqrt{48}$
p $4\sqrt{27}+2\sqrt{243}$
q $3\sqrt{63}-2\sqrt{28}$
r $2\sqrt{98}+3\sqrt{162}$
s $-5\sqrt{6}+2\sqrt{150}$
t $4\sqrt{50}+3\sqrt{18}$
u $5\sqrt{27}-6\sqrt{75}$
v $3\sqrt{112}-2\sqrt{252}$
w $\sqrt{32}+\sqrt{8}+\sqrt{12}$
x $\sqrt{27}+\sqrt{54}+\sqrt{243}$
y $\sqrt{98}-3\sqrt{20}-2\sqrt{8}$
z $3\sqrt{96}-2\sqrt{150}+\sqrt{24}$

Foundation Standard Complex

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Multiplying and dividing surds

2.05

STAGE 5.3

The square root of products and quotients

For $x > 0$ and $y > 0$:

$$\sqrt{xy} = \sqrt{x} \times \sqrt{y}$$

$$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$$

Example 11

Simplify each expression.

a $\sqrt{3} \times \sqrt{5}$ **b** $\sqrt{10} \times \sqrt{6}$ **c** $3\sqrt{7} \times 5\sqrt{7}$

d $5\sqrt{27} \times 3\sqrt{6}$ **e** $\sqrt{54} \div (-\sqrt{2})$ **f** $\frac{15\sqrt{32}}{5\sqrt{8}}$

Solution

a $\sqrt{3} \times \sqrt{5} = \sqrt{15}$

b $\sqrt{10} \times \sqrt{6} = \sqrt{60}$
$= \sqrt{4} \times \sqrt{15}$
$= 2\sqrt{15}$

c $3\sqrt{7} \times 5\sqrt{7} = 3 \times 5 \times \sqrt{7} \times \sqrt{7}$
$= 15 \times 7$
$= 105$

d $5\sqrt{27} \times 3\sqrt{6} = 5 \times 3 \times \sqrt{27} \times \sqrt{6}$
$= 15\sqrt{162}$
$= 15 \times \sqrt{81} \times \sqrt{2}$
$= 15 \times 9\sqrt{2}$
$= 135\sqrt{2}$

e $\sqrt{54} \div (-\sqrt{2}) = -\frac{\sqrt{54}}{\sqrt{2}}$
$= -\sqrt{27}$
$= -\sqrt{9} \times \sqrt{3}$
$= -3\sqrt{3}$

f $\frac{15\sqrt{32}}{5\sqrt{8}} = 3\sqrt{4}$
$= 3 \times 2$
$= 6$

Example 12

Simplify $\frac{5\sqrt{2} \times 4\sqrt{12}}{10\sqrt{8}}$.

Solution

$\frac{5\sqrt{2} \times 4\sqrt{12}}{10\sqrt{8}} = \frac{20\sqrt{24}}{10\sqrt{8}}$
$= 2\sqrt{3}$

STAGE 5.3

EXERCISE 2.05 ANSWERS ON P. 570

Multiplying and dividing surds U F

EXAMPLE 11

1 Simplify each expression.

a $\sqrt{7} \times \sqrt{2}$ **b** $-\sqrt{5} \times \sqrt{7}$ **c** $\sqrt{6} \times \sqrt{8}$

d $\sqrt{12} \times \sqrt{3}$ **e** $\sqrt{10} \times (-\sqrt{5})$ **f** $3\sqrt{3} \times 5\sqrt{3}$

g $5\sqrt{10} \times 3\sqrt{3}$ **h** $-2\sqrt{7} \times 5\sqrt{3}$ **i** $7\sqrt{5} \times 4\sqrt{5}$

j $2\sqrt{3} \times (-5\sqrt{6})$ **k** $4\sqrt{3} \times \sqrt{27}$ **l** $-3\sqrt{5} \times 4\sqrt{10}$

m $-7\sqrt{2} \times 4\sqrt{8}$ **n** $\sqrt{18} \times 8\sqrt{3}$ **o** $10\sqrt{2} \times 2\sqrt{8}$

p $3\sqrt{18} \times 5\sqrt{12}$ **q** $3\sqrt{44} \times (-2\sqrt{99})$ **r** $5\sqrt{8} \times 4\sqrt{40}$

s $8\sqrt{3} \times 3\sqrt{54}$ **t** $-8\sqrt{32} \times \sqrt{27}$ **u** $\sqrt{90} \times \sqrt{72}$

v $-5\sqrt{20} \times 3\sqrt{8}$ **w** $7\sqrt{18} \times 3\sqrt{24}$ **x** $3\sqrt{48} \times 2\sqrt{42}$

2 Simplify each expression.

a $\sqrt{15} \div \sqrt{3}$ **b** $\sqrt{18} \div (-\sqrt{6})$ **c** $\dfrac{6\sqrt{48}}{2\sqrt{8}}$

d $10\sqrt{54} \div 5\sqrt{27}$ **e** $-3\sqrt{98} \div 6\sqrt{14}$ **f** $\dfrac{7\sqrt{18}}{\sqrt{2}}$

g $2\sqrt{24} \div 4\sqrt{6}$ **h** $\dfrac{\sqrt{128}}{\sqrt{2}}$ **i** $15\sqrt{18} \div 3\sqrt{6}$

j $\dfrac{-20\sqrt{10}}{-4\sqrt{5}}$ **k** $36\sqrt{24} \div 9\sqrt{8}$ **l** $16\sqrt{30} \div 8\sqrt{5}$

m $12\sqrt{14} \div 6$ **n** $\dfrac{3\sqrt{2}}{-12}$ **o** $\sqrt{80} \div 4\sqrt{5}$

p $5\sqrt{60} \div \sqrt{15}$ **q** $6\sqrt{8} \div 3\sqrt{2}$ **r** $\dfrac{-42\sqrt{54}}{6\sqrt{3}}$

s $12\sqrt{63} \div 3\sqrt{7}$ **t** $\dfrac{8\sqrt{50}}{2\sqrt{200}}$ **u** $6\sqrt{3} \div \sqrt{243}$

EXAMPLE 12

3 Simplify each expression.

a $\dfrac{3\sqrt{5} \times 4\sqrt{2}}{3\sqrt{40}}$ **b** $\dfrac{3\sqrt{12} \times 8\sqrt{6}}{4\sqrt{27}}$

c $\dfrac{5\sqrt{8} \times 2\sqrt{90}}{10\sqrt{24}}$ **d** $\dfrac{4\sqrt{5}}{2\sqrt{15} \times 5\sqrt{27}}$

e $\dfrac{10\sqrt{686} \times 3\sqrt{12}}{5\sqrt{28} \times \sqrt{18}}$ **f** $\dfrac{8\sqrt{80} \times 3\sqrt{2}}{4\sqrt{5} \times 6\sqrt{8}}$

Foundation Standard Complex

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Finding a shorter side

2.06

Pythagoras' theorem can also be used to find the length of a shorter side of a right-angled triangle, if the hypotenuse and the other side are known.

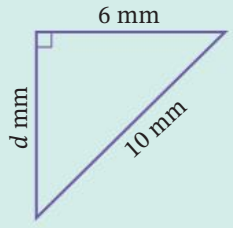
Pythagoras' theorem 2

Pythagoras' theorem

Pythagoras' theorem

Example 13

Find the value of d in this triangle.



Solution

We want to find the length of a shorter side.

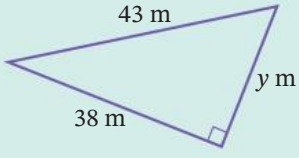
Using Pythagoras' theorem:

$10^2 = d^2 + 6^2$
$100 = d^2 + 36$
$d^2 + 36 = 100$
$d^2 = 100 - 36$
$= 64$
$d = \sqrt{64}$
$= 8$

From the diagram, a length of 8 mm looks reasonable because it must be shorter than the hypotenuse, which is 10 mm.

Example 14

Find the value of y as a surd for this triangle.



Pythagoras' theorem 1

Solution

$43^2 = y^2 + 38^2$
$1849 = y^2 + 1444$
$y^2 + 1444 = 1849$
$y^2 = 1849 - 1444$
$= 405$
$y = \sqrt{405}$
$= 9\sqrt{5}$ Leave the answer as a surd

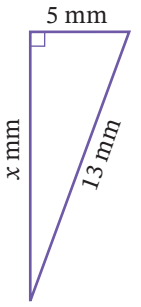
EXERCISE 2.06 ANSWERS ON P. 570

Finding a shorter side U F PS R

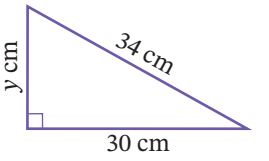
EXAMPLE 13

1 Find the value of the variable in each triangle.

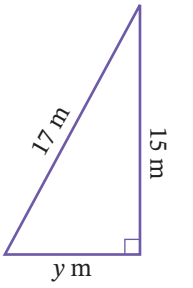
a



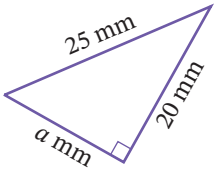
b



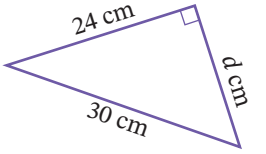
c



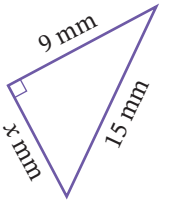
d



e

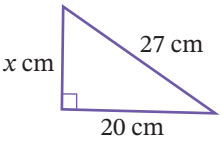


f

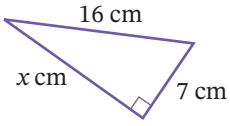


2 Find the value of the variable in each triangle, correct to one decimal place.

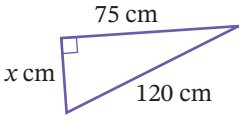
a



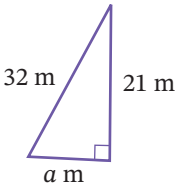
b



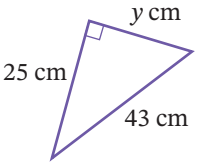
c



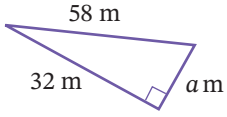
d



e



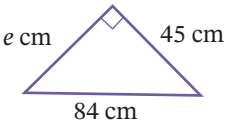
f



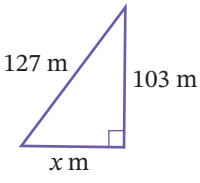
EXAMPLE 14

3 Find the value of the variable in each triangle as a surd.

a



b



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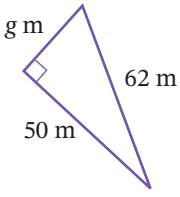
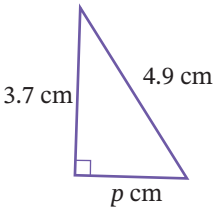


d



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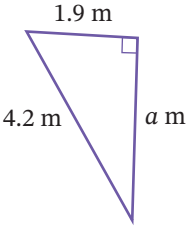
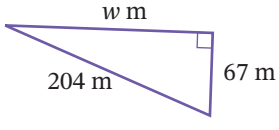


f



Foundation | Standard | Complex

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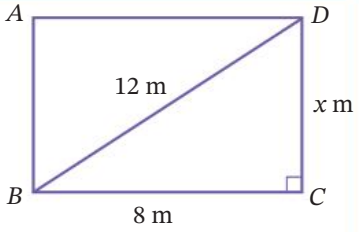
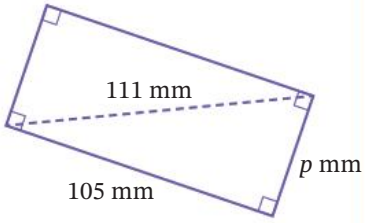
4 Find the value of x in this rectangle.
Select the correct answer **A**, **B**, **C** or **D**.



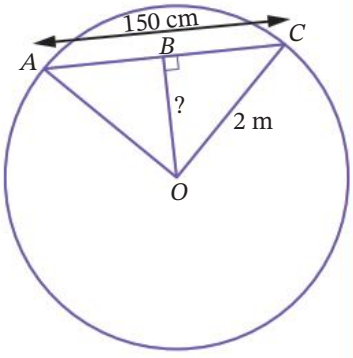
A $\sqrt{20}$
B $\sqrt{80}$
C $\sqrt{136}$
D $\sqrt{208}$

5 Find the value of p in this rectangle.
Select **A**, **B**, **C** or **D**.



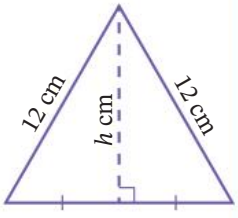
A 40
B 36
C 32
D 28

6 In this diagram, O is the centre of a circle.
A perpendicular line is drawn from O to B such that $OB \perp AC$ and $AB = BC$. Calculate the length of OB.
Select **A**, **B**, **C** or **D**. PS R



A 1.3 m
B 1.8 m
C 1.9 m
D 3.4 m

7 An equilateral triangle has sides of length 12 cm.
Find the perpendicular height, h cm, of the triangle, correct to 2 decimal places. PS R

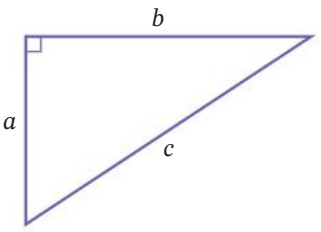


8 A square has a diagonal of length 30 cm. What is the length of each side of the square, correct to the nearest millimetre? PS R

Technology

Finding a shorter side

Use a spreadsheet to calculate the length of a shorter side, b, of a right-angled triangle, given the length of the hypotenuse and the other side (c and a).



1 Enter the headings shown below into cells A1, B1, C1 and D1. Then enter the values shown into columns A and B.

	A	B	C	D
1	c	a	c^2 - a^2	b
2	25	20		
3	37	12		
4	18	16		
5	11	7		
6				

2 Click on cell C2. Type the formula **=A2^2-B2^2**. Use **Fill Down** to copy this formula into cells C3 to C5.

3 To calculate the values of b, we need to find the square root of each of the values in column C. So, in cell D2, enter **=sqrt(C2)**. Then click on cell D2 and use **Fill Down** to copy this formula into cells D3 to D5.

4 Use your spreadsheet to find the length of the unknown shorter side, given the following pairs of values for the hypotenuse and other side.

a $c = 61, a = 11$ **b** $c = 87, a = 60$ **c** $c = 35, a = 22$

d $c = 9.2, a = 2.7$ **e** $c = 158, a = 63$ **f** $c = 24.5, a = 12.8$

2.07 Mixed problems

WS Applications of Pythagoras' theorem

Pythagoras 1

Pythagoras 2

Pythagorean two-step problems

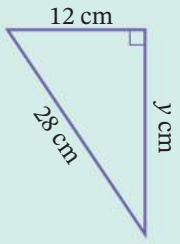
In this section, you will need to decide whether to find the hypotenuse or a shorter side, using Pythagoras' theorem.

When using Pythagoras' theorem to solve problems, it is useful to follow these steps.

- Read the problem carefully
- Draw a diagram involving a right-angled triangle and label any given information
- Choose a variable to represent the length or distance you want to find
- Use Pythagoras' theorem to find the value of the variable
- Answer the question

Example 15

Find the value of y as a surd.



Solution

y is the length of a shorter side.

$$28^2 = y^2 + 12^2$$
$$784 = y^2 + 144$$
$$y^2 + 144 = 784$$
$$y^2 = 784 - 144$$
$$= 640$$
$$y = \sqrt{640}$$
$$= 8\sqrt{10}$$

Example 16

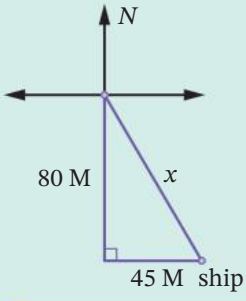
A ship sails 80 nautical miles south and then 45 nautical miles east. How far is it from its starting point, correct to one decimal place?

Solution

Let x be the distance the ship is from the starting point.

$$x^2 = 80^2 + 45^2$$
$$= 8425$$
$$x = \sqrt{8425}$$
$$= 91.7877...$$
$$\approx 91.8\text{M}$$

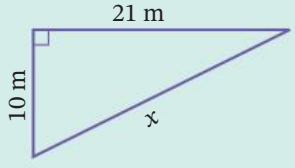
From the diagram, this looks like a reasonable answer



M is the symbol for nautical miles

Example 17

Find the perimeter of this triangle, correct to one decimal place.



Solution

Let x be the length of the hypotenuse.

$$x^2 = 10^2 + 21^2$$
$$= 541$$
$$x = \sqrt{541}$$
$$\approx 23.3 \text{ m}$$

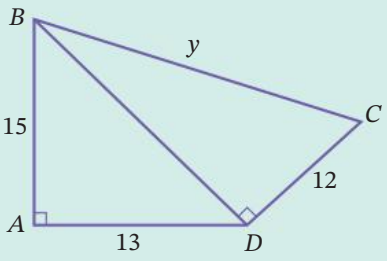
Perimeter $\approx 10 + 21 + 23.3$
$= 54.3$ m

2.07

Pythagoras' theorem problems

Example 18

Find the value of y, correct to 2 decimal places.



Solution

We need to find BD first.

In $\triangle ABD$,

$BD^2 = 15^2 + 13^2$

$= 394$

$BD = \sqrt{394}$ Leave BD as a surd for further working

In $\triangle BCD$,

$y^2 = \left(\sqrt{394}\right)^2 + 12^2$ $\left(\sqrt{394}\right)^2 = 394$

$= 538$

$y = \sqrt{538}$

$= 23.1948...$

≈ 23.19 From the diagram, this looks like a reasonable answer.

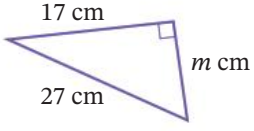
EXERCISE 2.07 ANSWERS ON P. 570

Mixed problems U F PS R

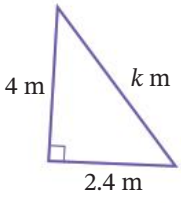
EXAMPLE 15

1 Find the value of the variable in each triangle, correct to one decimal place.

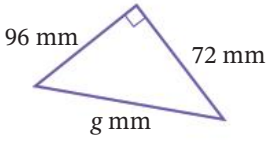
a



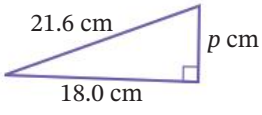
b



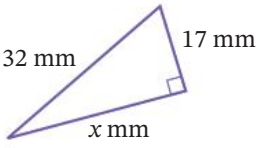
c



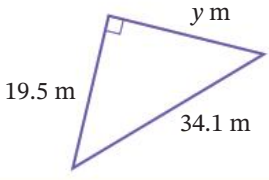
d



e



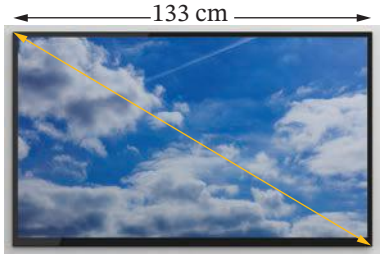
f



2 A ladder 5 m long is leaning against a wall. If the base of the ladder is 2 m from the bottom of the wall, how far does the ladder reach up the wall? Answer correct to 2 decimal places. PS

iStock.com/bmcent1

3 The size of a TV screen is described by the length of its diagonal. If a TV screen is 133 cm wide and 75 cm high, what is the size of the screen? Answer to the nearest centimetre.



Shutterstock.com/rawf8

4 Cooper wanted to find the length XY of the lake shown. He placed a marker at Z so that $\angle YXZ = 90°$. He measured XZ to be 450 m long and ZY to be 780 m long. What is the length of the lake? Answer correct to one decimal place.

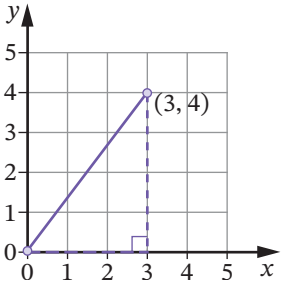
Shutterstock.com/Jamen Percy

5 A ship sails 70 nautical miles west and then 60 nautical miles north. How far is it from its starting point, correct to one decimal place? PS

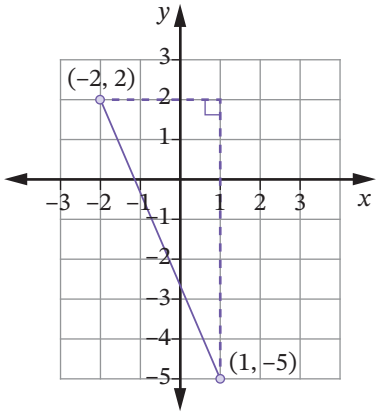
EXAMPLE 16

6 Use Pythagoras' theorem to calculate the distance between the 2 points on each number plane. In part **b**, write the answer as a surd. R

a



b

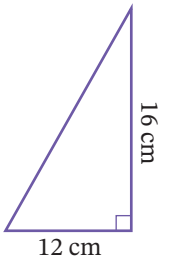


Foundation | Standard | Complex

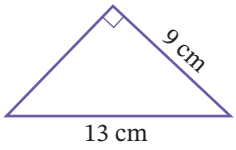
EXAMPLE 17

7 Calculate the perimeter of each shape, correct to one decimal place where necessary.

a

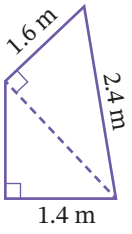


b



c

d



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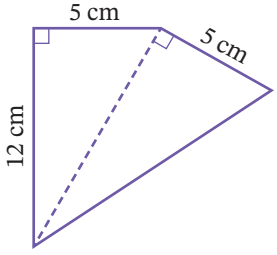

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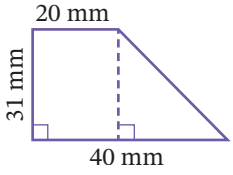
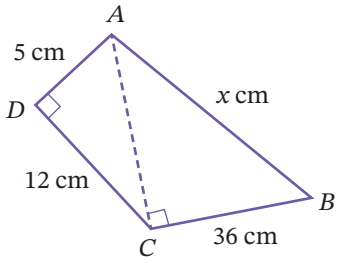
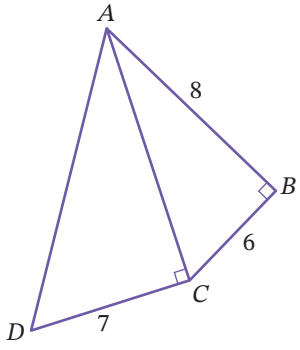


EXAMPLE 18

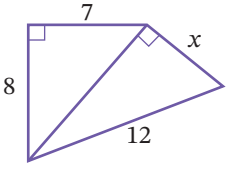
8 **a** Find the value of x, correct to 1 decimal place. R



b Find the length of AD, correct to 2 decimal places. R



c Calculate the length of x as a surd. R



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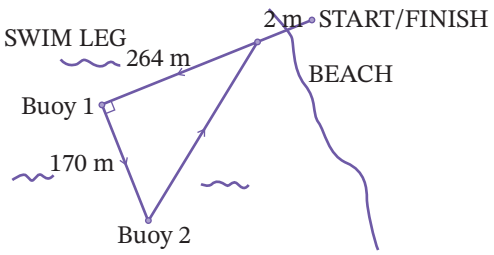
9 The swim course of a triathlon race has the shape of a right-angled triangle joined by a 2-metre link to the beach as shown. Calculate the total distance covered in this course (starting and finishing on the beach) PS

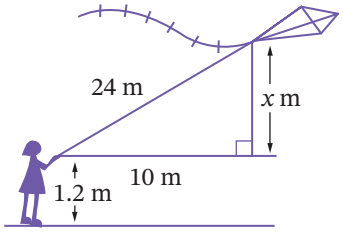


10 A kite is attached to a 24 m piece of string. The rope is held 1.2 m above the ground and covers a horizontal distance of 10 m.

Find:

a the value of x, correct to one decimal place

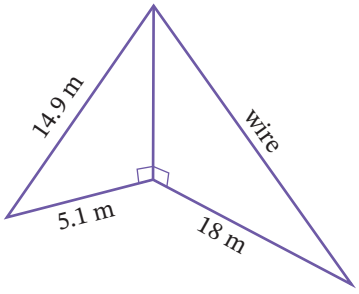
b the height of the kite above the ground, correct to the nearest metre.



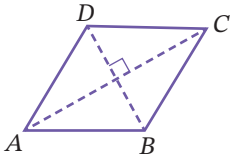
11 The diagonal of a rectangle is 200 mm in length. One of the side lengths of the rectangle is 167 mm. Find the length of the other side, correct to the nearest millimetre. PS

12 To keep a power pole vertically upright, it is supported by 2 wires as shown.

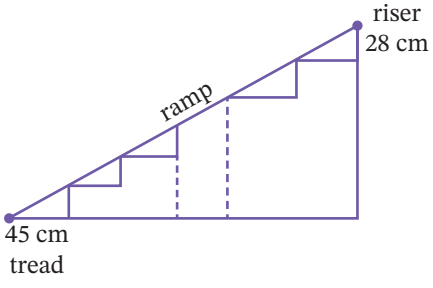
Find, correct to one decimal place, the length of the second wire. R



13 $DCBA$ is a rhombus with diagonals $AC = 18$ cm and $DB = 12$ cm. Find the perimeter of the rhombus, correct to one decimal place. PS R



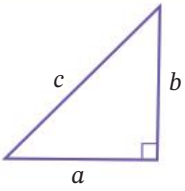
14 A set of 24 steps is to be replaced by a ramp as shown. Each step has a tread of 45 cm and a riser of 28 cm. Find the length of the ramp in metres. PS R



2.08 Testing for right-angled triangles

WS
Testing for right-angled triangles

Pythagoras' theorem says that if a right-angled triangle has sides of length a, b, and c, then $c^2 = a^2 + b^2$.



The reverse of this is also true:

If any triangle has sides of length a, b, and c that follow the formula $c^2 = a^2 + b^2$, then the triangle must be right-angled.

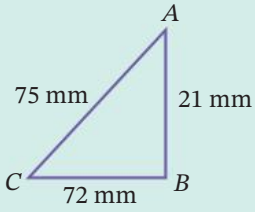
The right angle is always the angle that is opposite the hypotenuse.

This is called the **converse** (or opposite) of Pythagoras' theorem, because it is the 'back-to-front' version of the theorem.

Testing for right-angled triangles

Example 19

Test whether $\triangle ABC$ is right-angled.



Solution

$75^2 = 5625$	Square the longest side
$21^2 + 72^2 = 5625$	Square the 2 shorter sides, then add
$\therefore 75^2 = 21^2 + 72^2$	The sides of this triangle follow $c^2 = a^2 + b^2$
$\therefore \triangle ABC$ is right-angled.	The right angle is $\angle B$

Example 20

Rahul constructed a triangle with sides of length 37 cm, 12 cm and 40 cm. Show that these measurements do not form a right-angled triangle.

Solution

$40^2 = 1600$	Square the longest side
$12^2 + 37^2 = 1513 \neq 1600$	Square the 2 shorter sides, then add
$\therefore 40^2 \neq 12^2 + 37^2$	The sides of this triangle do not follow $c^2 = a^2 + b^2$

$\therefore$ The triangle is not right-angled.

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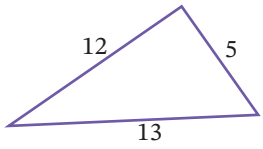
EXERCISE 2.08 ANSWERS ON P. 570

Testing for right-angled triangles U F R C

EXAMPLE 19

1 Test whether each triangle is right-angled. C

a



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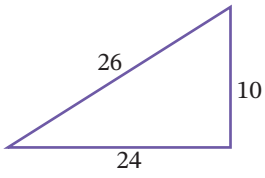

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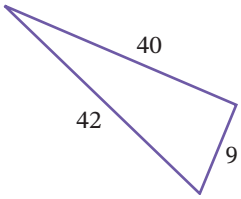

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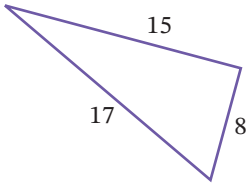

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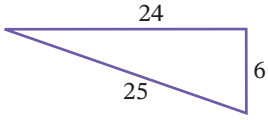

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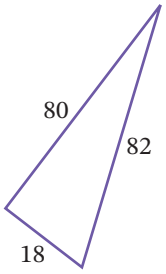

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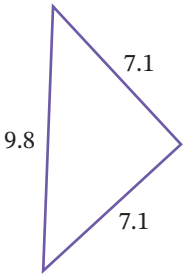

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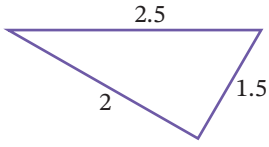

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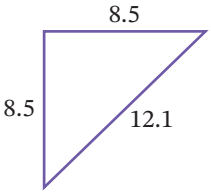

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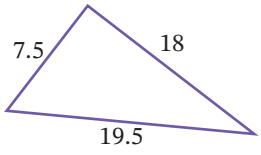

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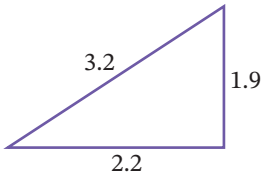

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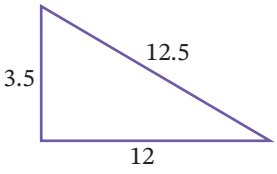


EXAMPLE 20

2 Paba constructed a triangle with sides of length 17 cm, 21 cm and 30 cm. Show that these measurements do *not* form a right-angled triangle. C

3 Which set of measurements would make a right-angled triangle? Select the correct answer **A**, **B**, **C** or **D**. C

A 2 cm, 3 cm, 4 cm

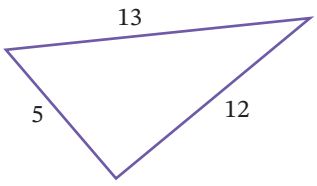
B 5 mm, 10 mm, 15 mm

C 12 cm, 16 cm, 20 cm

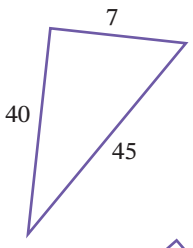
D 7 m, 24 m, 31 m

4 Which one of these triangles is not right-angled? Select **A**, **B**, **C** or **D**. C

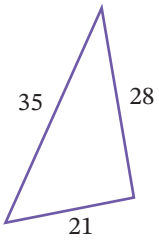
A



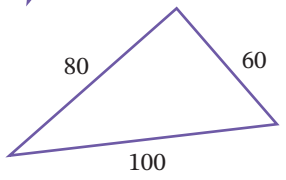
B



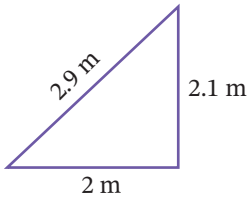
C



D



5 A pole 2.1 m tall is supported by a wire 2.9 m long. The other end of the wire is attached to the ground 2 m from the base of the pole. Is the pole standing vertically upright? R C



Mental skills 2B: Maths without calculators ANSWERS ON P. 570

Multiplying and dividing by a multiple of 10

1 Consider each example.

a $4 \times 700 = 4 \times 7 \times 100 = 28 \times 100 = 2800$

b $5 \times 60 = 5 \times 6 \times 10 = 30 \times 10 = 300$

c $12 \times 40 = 12 \times 4 \times 10 = 48 \times 10 = 480$

d $3.2 \times 30 = 3.2 \times 3 \times 10 = 9.6 \times 10 = 96$ (by estimation, $3 \times 30 = 90 \approx 96$)

e $4.6 \times 50 = 4.6 \times 5 \times 10 = 23 \times 10 = 230$ (by estimation, $5 \times 50 = 250 \approx 230$)

f $9.4 \times 200 = 9.4 \times 2 \times 100 = 18.8 \times 100 = 1880$ (by estimation, $9 \times 200 = 1800 \approx 1880$)

2 Now evaluate each product.

a	8×2000	**b**	3×70	**c**	11×900	**d**	2×300
e	4×4000	**f**	5×80	**g**	7×70	**h**	1.3×40
i	2.5×600	**j**	5.8×200	**k**	3.6×50	**l**	4.4×3000

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3 Consider each example.

a $8000 \div 400 = 8000 \div 100 \div 4 = 80 \div 4 = 20$

b $200 \div 50 = 200 \div 10 \div 5 = 20 \div 5 = 4$

c $6000 \div 20 = 6000 \div 10 \div 2 = 600 \div 2 = 300$

d $282 \div 30 = 282 \div 10 \div 3 = 28.2 \div 3 = 9.4$

e $3520 \div 40 = 3520 \div 10 \div 4 = 352 \div 4 = 88$

f $8940 \div 200 = 8940 \div 100 \div 2 = 89.4 \div 2 = 44.7$

4 Now evaluate each quotient.

a	$560 \div 70$	**b**	$2500 \div 500$	**c**	$3200 \div 400$	**d**	$440 \div 20$
e	$160 \div 40$	**f**	$1500 \div 30$	**g**	$450 \div 50$	**h**	$744 \div 80$
i	$2550 \div 300$	**j**	$846 \div 200$	**k**	$576 \div 60$	**l**	$2160 \div 90$

Did you know?

Pythagoras and President Garfield

According to *The Guinness Book of Records*, Pythagoras' theorem has been proved the greatest number of times, with a book published in 1940 containing 370 different proofs, including one by US president James Garfield (1831–1881). Before becoming involved in politics, Garfield studied mathematics and taught in a public school. He published his proof in 1876 and was the 20th president of the United States, governing for only 150 days in 1881.

Find the main mathematical feature of President Garfield's proof.

Alamy Stock Photo/Prisma Archivo

Pythagorean triads

2.09

A **Pythagorean triad** or **Pythagorean triple** is any set of 3 numbers that follow Pythagoras' theorem, for example, {3, 4, 5} or {2.5, 6, 6.5}. The word **triad** means a group of 3 related items ('tri-' means 3).

Pythagorean triad

$\{a, b, c\}$ is a Pythagorean triad if $c^2 = a^2 + b^2$.

Any multiple of $\{a, b, c\}$ is also a Pythagorean triad.

Pythagorean triads

Pythagorean triples

Pythagorean triples

Example 21

Test whether {5, 12, 13} is a Pythagorean triad.

Solution

$13^2 = 169$ — Squaring the largest number

$5^2 + 12^2 = 169$ — Squaring the 2 smaller numbers and adding them

$\therefore 13^2 = 5^2 + 12^2$ — These 3 numbers follow Pythagoras' theorem

$\therefore$ {5, 12, 13} is a Pythagorean triad.

Example 22

{3, 4, 5} is a Pythagorean triad. Create other Pythagorean triads by multiplying (3, 4, 5) by:

a 2 **b** 9 **c** $\frac{1}{2}$

Solution

a $2 \times \{3, 4, 5\} = \{6, 8, 10\}$

Checking: $10^2 = 100$

$6^2 + 8^2 = 100$

$\therefore 10^2 = 6^2 + 8^2$

$\therefore$ {6, 8, 10} is a Pythagorean triad.

b $9 \times \{3, 4, 5\} = \{27, 36, 45\}$

Checking: $45^2 = 2025$

$27^2 + 36^2 = 2025$

$\therefore 45^2 = 27^2 + 36^2$

$\therefore$ {27, 36, 45} is a Pythagorean triad.

c $\frac{1}{2} \times \{3, 4, 5\} = \{1.5, 2, 2.5\}$

Checking: $2.5^2 = 6.25$

$1.5^2 + 2^2 = 6.25$

$\therefore 2.5^2 = 1.5^2 + 2^2$

$\therefore$ {1.5, 2, 2.5} is a Pythagorean triad.

EXERCISE 2.09 ANSWERS ON P. 570

Pythagorean triads U F R C

EXAMPLE 21

1 Test whether each triad is a Pythagorean triad.

a {8, 15, 17} **b** {10, 24, 26} **c** {30, 40, 50}

d {5, 7, 9} **e** {9, 40, 41} **f** {4, 5, 9}

g {11, 60, 61} **h** {7, 24, 25} **i** {15, 114, 115}

2 Which of the following is a Pythagorean triad? Select the correct answer **A**, **B**, **C** or **D**.

A {4, 6, 8} **B** {5, 10, 12} **C** {6, 7, 10} **D** {20, 48, 52}

3 Use the spreadsheet you created in **Technology: Finding the hypotenuse** on page 58 to check your answers to questions **1** and **2**.

EXAMPLE 22

2.09

4 For each Pythagorean triad, create another Pythagorean triad by multiplying each number in the triad by:

i a whole number **ii** a fraction **iii** a decimal.

a {5, 12, 13} **b** {8, 15, 17} **c** {30, 40, 50} **d** {7, 24, 25}

Check that each answer follows Pythagoras' theorem.

5 Pythagoras developed formulas for finding Pythagorean triads $\{a, b, c\}$. If one number in the triad is a, the formulas for the other 2 numbers are $b = \frac{1}{2}(a^2 - 1)$ and $c = \frac{1}{2}(a^2 + 1)$.

a If $a = 5$, use the formulas to find the values of b and c.

b Show that $\{a, b, c\}$ from part **a** is a Pythagorean triad.

6 Use the formulas to find Pythagorean triads for each value of a.

a $a = 7$ **b** $a = 11$ **c** $a = 15$ **d** $a = 4$

e $a = 9$ **f** $a = 19$ **g** $a = 10$ **h** $a = 51$

7 Prove that if $b = \frac{1}{2}(a^2 - 1)$ and $c = \frac{1}{2}(a^2 + 1)$, then $c^2 = a^2 + b^2$ is true. R C

8 There are many other formulas for creating Pythagorean triads. Use the Internet to research some of them.

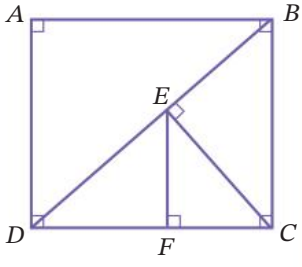
Power plus ANSWERS ON P. 570

1 Simplify each expression as an exact value.

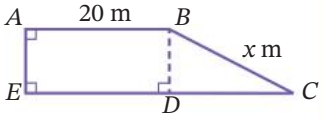
a $\left(\sqrt{2}\right)^3$ **b** $\left(\sqrt{2}\right)^4$ **c** $\left(-\sqrt{3}\right)^4$

d $\left(-\sqrt{3}\right)^3$ **e** $\left(2\sqrt{5}\right)^3$ **f** $\left(-3\sqrt{6}\right)^4$

2 Name the 5 hypotenuses on this diagram.



3 The rectangle $ABDE$ in the diagram has an area of 160 m^2. Find the value of x as a surd if $\triangle BDC$ is an isosceles triangle.



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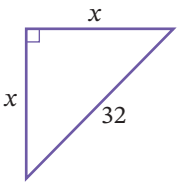
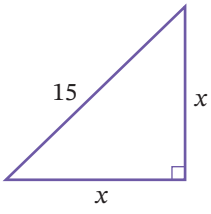
4 Find the value of x in each triangle, correct to one decimal place.

a



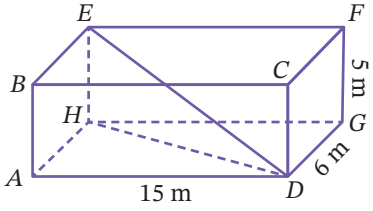
b



5 For this rectangular prism, find, correct to one decimal place, the length of diagonal:

a HD

b DE

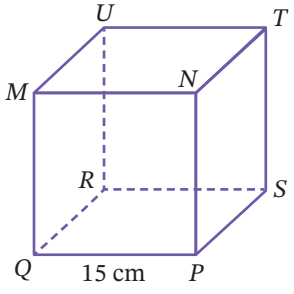


6 Find, correct to one decimal place, the length of the interval joining points $A(-2, 1)$ and $B(3, -2)$ on the number plane.

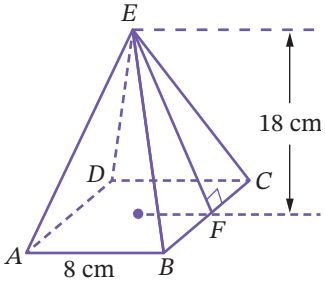
7 For this cube, find, correct to 2 decimal places, the length of diagonal:

a QS

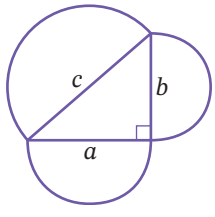
b QT



8 For this square pyramid, find, as a surd, the slant height EF.



9 Use Pythagoras' theorem to prove that for a right-angled triangle, the area of the semicircle constructed on the hypotenuse is equal to the areas of the semicircles constructed on the other 2 sides.



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CHAPTER 2 REVIEW

Language of maths

area	converse	diagonal	formula
hypotenuse	irrational	perimeter	Pythagoras
right-angled	shorter	side	square root
surd	theorem	triad	unknown

1 State Pythagoras' theorem in words.

2 Describe the **hypotenuse** of a right-angled triangle in 2 ways.

3 What is another word for 'theorem'?

4 For what type of triangle is **Pythagoras' theorem** used?

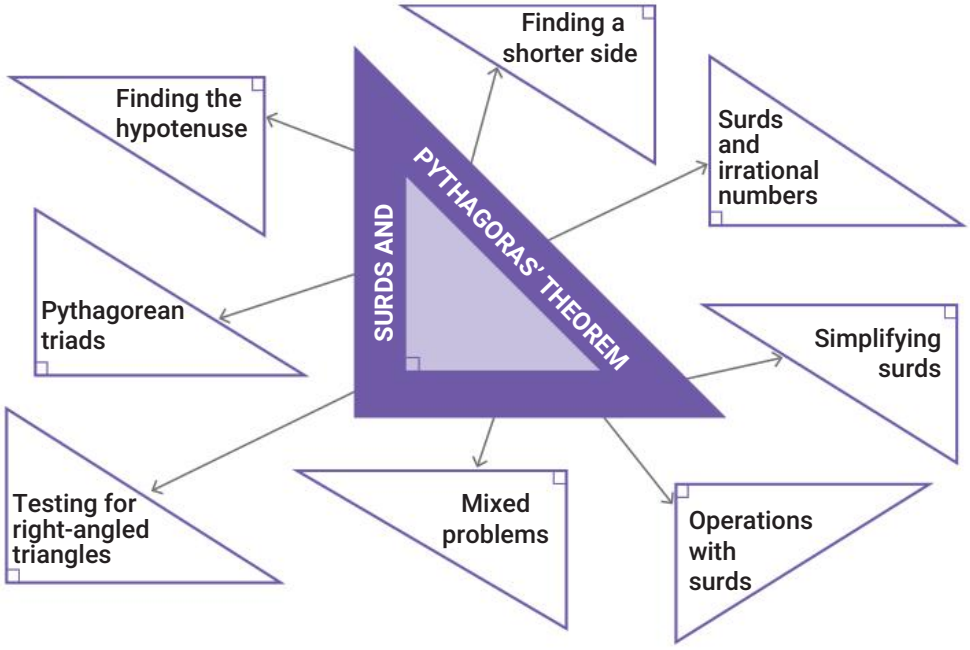
5 What is a **surd**?

6 What is the name given to a set of 3 numbers that follows Pythagoras' theorem?

Topic summary

- How relevant do you think Pythagoras' theorem is to our world? Give reasons for your answer.
- Give 3 examples of jobs where Pythagoras' theorem would be used.
- What did you find especially interesting about this topic?
- Is there any section of this topic that you found difficult? Discuss any problems with your teacher or a friend.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



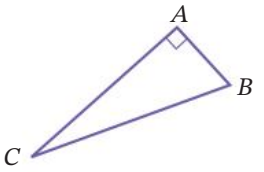
WS
Mind map: Surds and Pythagoras' theorem

TEST YOURSELF 2 ANSWERS ON P. 570

2.01

1 Write Pythagoras' theorem for each triangle.

a



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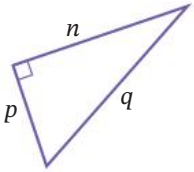

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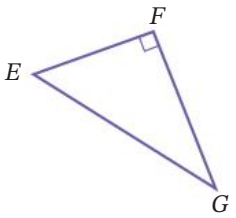
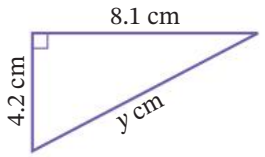


2.01

2 Find the value of y. Select the closest answer **A**, **B**, **C** or **D**.

A 12.3 **B** 9.1

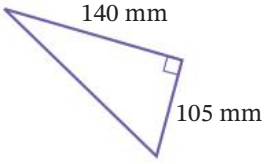
C 6.9 **D** 3.5



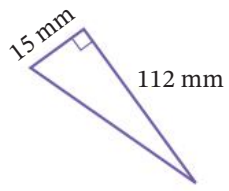
2.01

3 For each triangle, find the length of the hypotenuse. Give your answer as an exact value.

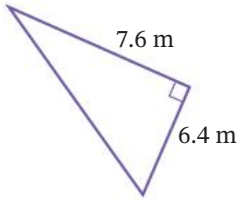
a



b



c



2.02

4 Is each number rational (R) or irrational (I)?

a $\sqrt{8}$ **b** $\frac{22}{7}$ **c** $0.5\dot{7}$ **d** $3\sqrt{5}$ **e** $\sqrt{81}$

f $\sqrt[3]{125}$ **g** $\sqrt[3]{-8}$ **h** $0.\dot{1}2\dot{3}$ **i** $5+\sqrt{3}$

STAGE 5.3

2.03

5 Simplify each surd.

a $\sqrt{72}$ **b** $\sqrt{98}$ **c** $\sqrt{275}$ **d** $\sqrt{128}$ **e** $3\sqrt{150}$

f $7\sqrt{28}$ **g** $4\sqrt{288}$ **h** $5\sqrt{45}$ **i** $7\sqrt{48}$

2.04

6 Simplify each expression.

a $8\sqrt{5}-\sqrt{5}$ **b** $\sqrt{7}-3\sqrt{2}+2\sqrt{7}$

c $6+\sqrt{5}-4+5\sqrt{5}$ **d** $7\sqrt{7}-\sqrt{7}-3\sqrt{7}$

e $\sqrt{200}+\sqrt{18}$ **f** $3\sqrt{5}+\sqrt{50}-2\sqrt{125}$

g $7\sqrt{32}-\sqrt{27}-2\sqrt{98}+4\sqrt{75}$ **h** $4\sqrt{45}-3\sqrt{63}+5\sqrt{80}$

2.05

7 Simplify each expression.

a $\sqrt{3}\times\sqrt{7}$ **b** $\sqrt{8}\times\sqrt{5}$ **c** $\sqrt{6}\times\sqrt{8}$

d $\sqrt{5}\times\sqrt{11}$ **e** $\sqrt{8}\times\sqrt{3}$ **f** $\sqrt{98}\div\sqrt{7}$

g $8\sqrt{42}\div 2\sqrt{7}$ **h** $\sqrt{125}\div 5\sqrt{5}$ **i** $4\sqrt{147}\div 12\sqrt{98}$

j $\frac{\sqrt{18}\times\sqrt{3}}{\sqrt{12}}$ **k** $\frac{\sqrt{6}\times\sqrt{24}}{\sqrt{27}\times 2\sqrt{3}}$

Foundation Standard Complex

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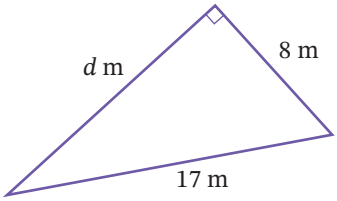
8 Find the value of d. Select **A**, **B**, **C** or **D**. 2.06

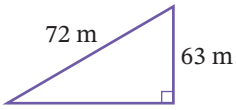
A 12 m **B** 15 m

C 16.8 m **D** 18.9 m

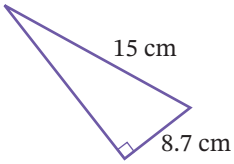


9 For each triangle, find the length of the unknown side, correct to one decimal place. 2.06

a



b



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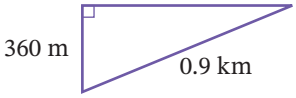


10 In $\triangle DEF$, $\angle F = 90°$, $DF = 84$ cm, and $EF = 1.45$ m. Find DE, correct to the nearest centimetre. 2.07

11 Julie walks 6 km due east from a starting point P, while Luka walks 4 km due south from P. 2.07

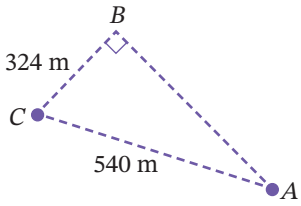
a Draw a diagram showing this information.

b How far are Julie and Luka apart? Give your answer correct to one decimal place.

12 On a cross-country ski course, checkpoints C and A are 540 m apart and checkpoints C and B are 324 m apart. Find the distance: 2.07

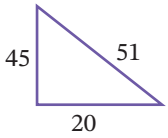
a between checkpoints A and B

b skied in one lap of the course in kilometres.

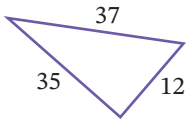


13 Test whether each triangle is right-angled. 2.08

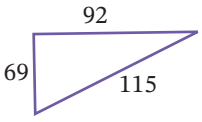
a



b



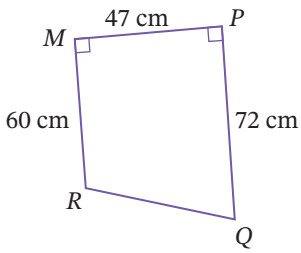
c



14 Test whether each triad is Pythagorean. 2.09

a {7, 24, 25} **b** {5, 7, 10} **c** {20, 21, 29} **d** {1.1, 6, 6.1}

15 Find the length of QR in this trapezium, correct to one decimal place. 2.07



3

NUMBER AND ALGEBRA

NUMERACY AND CALCULATION

Numbers are an important part of our lives. We work with numbers when we shop for the best deal and when we borrow or invest money. Calculations are made when using recipes to cook and in the design and construction of houses. Whenever numbers are used to help us solve problems in our lives, calculations will need to be made.

Chapter outline

	Working mathematically				
3.01 Recurring decimals	U	F		R	C
3.02 Converting recurring decimals to fractions*	U	F	PS	R	C
3.03 Applying percentages	U	F	PS	R	C
3.04 Profit, discounts and GST	U	F	PS	R	
3.05 Simple interest	U	F	PS	R	
3.06 Ratios and rates	U	F	PS	R	
3.07 Converting rates	U	F	PS	R	

*STAGE 5.3

Wordbank

cost price The price an item costs the retailer

GST Goods and services tax, a 10% tax added to the original price of an item or service

loss The amount lost when selling an item at a lower price

per annum (p.a.) Per year

principal An amount of money invested or borrowed, on which interest is given or charged

profit The amount made when selling an item at a higher price

recurring decimal A decimal with one or more digits that repeat endlessly, for example, $0.1\dot{6} = 0.1666\ldots$

unitary method A method of finding a quantity by finding one part or 1% first

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- convert between fractions and decimals
- (STAGE 5.3) convert recurring decimals to fractions
- solve problems involving percentages, ratios and rates
- solve problems involving profit, loss, discounts, GST and simple interest
- convert between units for rates, for example, kilometres per hour to metres per second

Integers

Integers using diagrams

Decimals

WS Where's the point?

SkillCheck ANSWERS ON P. 571

1 Evaluate each expression.

a $7 \times (-8)$ **b** $-120 \div (-6)$ **c** $7 - (-15)$

d $-10 \times 5 \times (-2)$ **e** $\frac{27}{-3}$ **f** $20 - 5 \times 8$

g $(-3)^2 + (-2)^2$ **h** $(-10 + 7) \times 4$ **i** $-24 \div 6 \times (-5)$

j $\frac{-100}{-5} \times 4$ **k** $-8 - (10 - 15)$ **l** $(-4)^3$

m $6 - (-30) \div 6$ **n** $40 + 8 \times (-9)$ **o** $(-5)^2 + 4 \times (-7)$

p $24 \div (-6) + (-5) \times 3$ **q** $12 - (4 \times 8)$ **r** $64 \div (-4)^2$

2 Write each set of decimals in **ascending order**.

a 0.7, 0.707, 7.007, 0.7007 **b** 0.202, 0.22, 0.02202, 0.002

3 Evaluate each expression.

a $0.45 \times 0.2 \times 0.1$ **b** $12.6 + 0.4 \times 7$ **c** $36.9 \div 3 \times 0.02$

4 Robyn had to calculate $36.024 \div 0.08$ without using a calculator. Which of these calculations will give her the same answer? Select **A**, **B**, **C** or **D**.

A $36\,024 \div 8$ **B** $36.024 \div 8$ **C** $3602.4 \div 8$ **D** $0.360\,24 \div 8$

5 Evaluate each expression.

a $\frac{2}{3} + \frac{4}{7}$ **b** $\frac{3}{5} + \frac{1}{2}$ **c** $\frac{5}{9} - \frac{1}{4}$ **d** $\frac{9}{10} - \frac{2}{3}$

e $\frac{5}{7} \times \frac{3}{4}$ **f** $\frac{1}{2} \times \frac{3}{5}$ **g** $6 \times \frac{3}{4}$ **h** $\frac{2}{3} \times 7$

i $\frac{3}{4} \div \frac{1}{2}$ **j** $\frac{2}{3} \div \frac{4}{5}$ **k** $\frac{3}{7} \div 5$ **l** $\frac{6}{7} \div \frac{2}{3}$

6 Express each number as a percentage.

a $\frac{1}{4}$ **b** 0.3 **c** 0.07 **d** $\frac{3}{5}$

7 Arrange $\frac{3}{5}$, 5%, 0.5 and $\frac{5}{9}$ in ascending order.

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Recurring decimals 3.01

Recurring decimals have a pattern of numbers that repeat or recur. We use dots to indicate the digits that repeat. Here are some examples:

0.4444... $= 0.\dot{4}$

0.588 88... $= 0.5\dot{8}$

0.636 363... $= 0.\dot{6}\dot{3}$

0.415 415 415... $= 0.\dot{4}1\dot{5}$

Terminating decimals do not repeat (terminating means 'stopping'). Here are some examples:

0.6 0.875 0.057

Some decimals are neither recurring nor terminating. They have an infinite number of digits but there is no pattern. These are **irrational numbers**. Here are some examples:

$\pi = 3.141\ 592\ 653...$ $\sqrt{3} = 1.732\ 050\ 808...$

To convert a fraction to a decimal, divide its **numerator** by its **denominator.**

Terminating and recurring decimals

3.01

Example 1

Convert each fraction to a decimal and state whether it is recurring or terminating.

a $\frac{5}{6}$ **b** $\frac{11}{40}$

Solution

a $\frac{5}{6} = 5 \div 6$

$= 0.8333...$

$= 0.8\dot{3}$ (a recurring decimal)

b $\frac{11}{40} = 11 \div 40$

$= 0.275$ (a terminating decimal)

EXERCISE 3.01 ANSWERS ON P. 571

Recurring decimals U F R C

1 Write each recurring decimal using dot notation. C

a 0.166 666... **b** 0.273 273 27... **c** 0.888 88... **d** 0.209 99...

2 Show the repeating pattern for each recurring decimal. C

a $0.\dot{1}09\dot{8}$ **b** $0.\dot{1}\dot{7}$ **c** $0.2\dot{5}$ **d** $8.3\dot{1}\dot{8}$

3 Convert each fraction to a decimal and state whether it is recurring or terminating. C

EXAMPLE 1

a $\frac{7}{8}$ **b** $\frac{3}{11}$ **c** $4\frac{8}{9}$ **d** $\frac{13}{5}$ **e** $\frac{19}{25}$

f $\frac{2}{3}$ **g** $\frac{13}{20}$ **h** $1\frac{7}{12}$ **i** $8\frac{13}{40}$ **j** $\frac{17}{24}$

4 **a** Convert $\frac{1}{3}$ to a recurring decimal. C

b Use your answer to part **a** to determine the value of: R

i $0.\dot{6}$ **ii** $0.\dot{9}$

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3.02 Converting recurring decimals to fractions

STAGE 5.3

A recurring decimal can be converted to a fraction using a non-calculator method.

Example 2

Convert each recurring decimal to a simple fraction.

a $0.1\dot{7}$ **b** $0.\dot{5}\dot{4}$

Solution

a Let $x = 0.1\dot{7}$

$= 0.177777...$

$\therefore 10x = 1.77777...$ One digit is recurring, so multiply by 10

$\therefore 10x - x = 1.77777... - 0.177777...$ To delete the recurring digits

$9x = 1.6$

$x = \frac{1.6}{9}$

$= \frac{16}{90}$ Multiplying numerator and denominator by 10

$= \frac{8}{45}$

Since $x = 0.1\dot{7}$, $0.1\dot{7} = \frac{8}{45}$

This answer can be checked by converting $\frac{8}{45}$ to a decimal on a calculator:

$\frac{8}{45} = 0.177777...$

On some calculators, you can enter many digits of a recurring decimal and it will convert it into a fraction. For example:

0.17777777777777777777777 = S⇔D.

The S⇔D key converts between standard fractions and decimals.

b Let $x = 0.\dot{5}\dot{4}$

$= 0.545454...$

$\therefore 100x = 54.545454...$ 2 digits are recurring, so multiply by 100

$\therefore 100x - x = 54.545454 .... - 0.545454...$ To delete the recurring digits

$99x = 54$

$x = \frac{54}{99}$

$= \frac{6}{11}$

Since $x = 0.\dot{5}\dot{4}$, $0.\dot{5}\dot{4} = \frac{6}{11}$

Checking:

$\frac{6}{11} = 0.545454 ....$

Or converting by calculator:

0.545454545454545454 = S⇔D.

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EXERCISE 3.02 ANSWERS ON P. 571

STAGE 5.3

Converting recurring decimals to fractions U F PS R C

EXAMPLE 2

1 Convert each recurring decimal to a simple fraction. C

a $0.\dot{2}$ **b** $0.\dot{8}$ **c** $0.1\dot{5}$ **d** $0.3\dot{8}$

e $0.4\dot{6}$ **f** $0.\dot{6}\dot{5}$ **g** $0.\dot{3}\dot{1}$ **h** $0.6\dot{2}\dot{4}$

i $0.58\dot{3}$ **j** $0.\dot{8}3\dot{2}$ **k** $0.0\dot{3}$ **l** $0.7\dot{5}$

2 **a** Convert $0.4\dot{5}$ to a simple fraction.

b Use your answer from part **a** to write $0.\dot{9}\dot{0}$ as a simple fraction. R C

3 **a** Convert $0.6\dot{3}\dot{4}$ to a simple fraction.

b Use your answer from part **a** to write $3.6\dot{1}\dot{5}$ as a simple fraction. PS R C

4 **a** Convert $2.04\dot{5}$ to a simple fraction.

b Use your answer from part **a** to write $7.08\dot{5}$ as a simple fraction. PS R C

3.03

Applying percentages 3.03

Example 3

Find:

a 12% of \$90 **b** $5\frac{1}{2}\%$ of \$540

Solution

a $12\% \times \$90 = \frac{12}{100} \times \$90$ or $0.12 \times \$90$

$= \$10.80$

Entering the decimal 0.12 for 12% on a calculator is faster

b $5\frac{1}{2}\%$ of $\$540 = 5\frac{1}{2} \div 100 \times \$540$ or $0.055 \times \$540 = \$29.70$

Example 4

a Increase \$200 by 7%. **b** Decrease \$150 by 12%.

Solution

a Increase $= 7\% \times \$200$

$= \$14$

$\therefore$ New amount $= \$200 + \$14$

$= \$214$

$100\% + 7\% = 107\%$

or New amount $= 107\%$ of \$200

$= 1.07 \times \$200$

$= \$214$

b Decrease $= 12\% \times \$150$

$= 18$

$\therefore$ New amount $= \$150 - \$18$

$= \$132$

$100\% - 12\% = 88\%$

or New amount $= 88\%$ of \$150

$= 0.88 \times \$150$

$= \$132$

Fractions, decimals and percentages

Mental percentages

Working with percentages

Percentages to fractions

Percentage increase and decrease

Percentages of an amount

Fractions to percentages

Foundation Standard Complex

Percentages, fractions and decimals

Percentages, fractions and decimals

Converting marks to percentages

Expressing quantities as percentages

To **express one quantity as a percentage of another**, write the appropriate fraction and then change it to a percentage:

$$\text{Percentage} = \frac{\text{amount}}{\text{whole amount}} \times 100\%$$

or Percentage = amount ÷ whole amount × 100%

Example 5

a Jamila scores 43 out of 60 in a History test. What is her percentage mark to the nearest whole number?

b What percentage (correct to one decimal place) is 35 s of 5 min?

Solution

a $\text{Percentage mark} = \frac{43}{60} \times 100\%$

$= 71.666...\%$

$\approx 72\%$

Use $\text{Percentage} = \frac{\text{amount}}{\text{whole amount}} \times 100\%$

b $\text{Percentage} = \frac{35\text{ s}}{5\text{ min}} \times 100\%$

$= \frac{35\text{ s}}{300\text{ s}} \times 100\%$

$= 11.666...\ \%$

$\approx 11.7\%$

Change minutes to seconds so that the same units are being used.

5 min = 5 × 60 s = 300 s

The **unitary method** can be used to find the whole amount (100%) when part of the amount is known. 'Unit' means 'one' and the unitary method involves finding 1% first.

The unitary method

The unitary method

Given a percentage of an amount, to find the whole amount:

1 find 1% of the amount by dividing by the known percentage

2 multiply by 100 to find the whole amount (100%).

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Example 6

This week, Julio received a bonus of $86, which was 9% of his weekly wage. Calculate Julio's weekly wage, correct to the nearest dollar.

Solution

9% of the wage = $86

∴ 1% of the wage = $86 ÷ 9

= $9.555...

∴ Julio's wage = $9.55... × 100 — 100% of the wage.

= $955.555...

≈ $956

EXERCISE 3.03 ANSWERS ON P. 571

Applying percentages U F PS R C

EXAMPLE 3

1 Find:

a 15% of $85 **b** 120% of 60 kg **c** 45% of 16 m

d 2.4% of $350 **e** $5\frac{1}{4}\%$ of $80 **f** 12% of 95 kg

g $2\frac{1}{2}\%$ of $870 **h** 150% of 16 L **i** 3.75% of $2850

j $33\frac{1}{3}\%$ of $6540 **k** 6.25% of 25 t **l** 0.4% of $3720

2 A total of 84 600 people attended the State of Origin match between NSW and Queensland in Sydney. If 71% of the crowd supported NSW:

a what percentage supported Queensland?

b how many people supported Queensland?

Newspix/Brett Costello

3 980 students attend St Klaas' College. If 19% of them are in Year 9, calculate the number of Year 9 students.

4 Buzz Coffee advertises an extra 20% of coffee for the same price. If the normal amount is 700 g, how much extra coffee will you get for the same price?

5 Jakub's weight is now 125% of what it was a year ago. If his weight then was 85 kg, what is his weight now?

EXAMPLE 4

6 Increase:

a $455 by 10% **b** $80 by 15.5% **c** $680 by 200%

d 80 kg by 2.5% **e** 35 m by 30% **f** 120 L by 140%

7 Decrease:

a $830 by 35% **b** 120 min by 5% **c** 90 L by 12%

d 6.8 m by 25% **e** 130 kg by 18% **f** $96.50 by 10%

8 Matt buys a bike for $860 and sells it, making a profit of 5%.

a How much profit did Matt make?

b For what price did Matt sell the bike?

9 T-shirts priced at $38 are reduced by 45% at an end-of-summer sale. What is the sale price of the T-shirts?

10 Due to severe storms in summer, the price of fresh fruit and vegetables increased by 34%. How much will broccoli cost if its original price was $4.99/kg?

11 Lucy's weekly wage of $895 increased by 5.5%. Find her new weekly wage.

12 A car dealer is offering new year sales. What will you pay for a used car marked at $16 500 if a **discount** of 12% is given?

EXAMPLE 5

13 Express each proportion as a percentage, correct to one decimal place if necessary. C

a 55 out of 80 **b** 80 out of 120 **c** 45 out of 70

d 15 out of 18 **e** $8 out of $12 **f** $9 out of $60

g 35 s out of 4 min **h** 15 min out of 2 h **i** 340 mm out of 2 m

j 3 hours out of 1 day **k** 85 cm out of 5 m **l** 140c out of $10

14 Selina achieved the following marks in two tests: PS R C

English: 32 out of 40

Mathematics: 47 out of 60

Convert each mark to a percentage to decide which is the better mark.

15 Travis earns $840 per week, of which $280 goes to rent. Find what percentage of Travis' wage goes to rent, correct to one decimal place.

16 Bozena earns $480 per week while she is completing Year 12 at school. If she spends $95 of it on petrol, what percentage of her wage is this, correct to one decimal place?

17 The nutritional information on a food label is shown. C

a What percentage of a 100 g serve is:

i protein?

ii carbohydrate?

b For a 100 g serve, what percentage of the total fat is saturated fat?

	Per 100 g
Energy	2105 kJ
Total Fat (of which is Saturated fat)	25 g 16 g
Carbohydrate (of which is sugars)	62 g 38 g
Protein	5 g
Salt	1 g

EXAMPLE 6

18 In the Netball World Cup final, Australia scored 15 goals in the second quarter. If this represented 29.4% of the goals Australia scored in the match, how many goals did Australia score in total? PS R

Foundation Standard Complex

19 A jeweller charges $105 for valuing a diamond ring. If this charge represents 1.25% of the value of the ring, find this value correct to the nearest dollar.

20 A can of energy food drink contains 20% more for the same price as the standard can. If the extra amounts to 150 g, how much does the standard can contain?

21 Last year, Ben paid 28% of his salary in tax. If he paid $23 423 in tax, what was Ben's salary? Answer to the nearest dollar.

22 Indigo spent 65% of her money on rent and 50% of the remainder on food. What percentage of her money did she have left?

23 Increase $6580 by 6% and then decrease the result by 10%. Is this the same as decreasing $6580 by 4%? Show working to justify your answer. PS R

24 An amount is increased by 10% and then decreased by 10%. What is the overall percentage change? PS R

25 An amount is decreased by 10% and then increased by 10%. What is the overall percentage change? PS R

Did you know?

Blood: it takes all types

The blood groups in Australia are as follows.

O		A		B		AB	
+	−	+	−	+	−	+	−
40%	9%	31%	7%	8%	2%	2%	1%

1 If Australia's population is 26 million, how many people would be in each blood group?

2 Do you know what your blood group is? How would you find out?

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Profit, discounts and GST 3.04

Profit and loss

Profit and loss

- When an item is sold for **more** than it costs, a **profit** is made.
- When an item is sold for **less** than it costs, a **loss** is made.
- The percentage profit and loss are usually calculated as a percentage of the **cost price**.

WS Profit and loss

Percentage profit or loss

Foundation Standard Complex

Example 7

The online store *Jackson* buys notebook computers for $550 each and sells them for $680. Find:

a the profit on each computer

b the percentage profit on the cost price, correct to one decimal place.

Solution

a Profit = $680 – $550
= $130

b Percentage profit $= \frac{\text{profit}}{\text{cost price}} \times 100\%$
$= \frac{130}{550} \times 100\%$
= 23.6363...
≈ 23.6%

Discounts

Discounts and GST

Example 8

At *Jeans Minus*' end-of-season sale, jeans are discounted by 25%. What is the discount price of a pair of jeans marked at $148?

Solution

Method 1

Discount = 25% of $148
= $37
Discount price = $148 – $37
= $111

Method 2

Discount price = 75% of $148
= $111

Discounts and GST

Example 9

A home cinema costs $865 after a 24% discount. What was its original price, rounded to the nearest dollar?

Solution

Sale price = $865, discount = 24%
∴ 76% of the original price = $865 — 100% – 24% = 76%
$1\% = \frac{\$865}{76}$ — Using the unitary method
= $11.381 578...
∴ Original price = $11.381 578... × 100 — To find 100%
= $1138.157...
≈ $1138

GST

Goods and Services Tax (GST) is a tax paid to the government on most goods (items) and services that we purchase. In Australia, GST is charged at 10% of the original price and is generally included in the marked price of the good or service.

Example 10

The **selling price** of a lounge suite is $2695 with GST included. How much of this price is the GST?

Solution

Selling price = 100% + 10% Adding 10% GST

= 110%

110% of the original price = $2695

10% GST = $2695 ÷ 11

= $245

EXERCISE 3.04 ANSWERS ON P. 571

Profit, discounts and GST U F PS R

In this exercise, express all percentages correct to one decimal place and all prices to the nearest cent.

EXAMPLE 7

1 A home theatre system with a cost price of $650 is sold for $1250. Find:

a the profit **b** the percentage profit on the cost price.

2 A motorbike originally priced at $16 000 is sold for $14 500. Find:

a the loss **b** the percentage loss

3 Alla bought an old car for $3000, spent another $1500 on repairs and then sold it for $6200. Find:

a the total amount that Alla spent on the car

b the profit made when she sold the car

c her percentage profit.

4 A house was bought for $850 000. A year later, it was sold for $720 000.

Calculate:

a the loss **b** the percentage loss

5 Towels with a cost price of $25 each are sold for $35. Calculate the percentage profit.

6 A car costing $12 000 is sold for $8000. Calculate the percentage loss.

7 *Home Necessities* store buys cutlery sets for $120 that are then sold for $230. Calculate the percentage profit.

EXAMPLE 8

8 GIA supermarket reduces the price of some items by 35%. Find the discount price of the following items given their original prices.

a 250 g butter, $2.40 **b** 2 L milk, $3

c 1 kg cheese, $11 **d** 24 cans of soft drink, $15.99

9 Calculate the selling price of a computer marked at $760, now discounted by 15%. Select the correct answer **A**, **B**, **C** or **D**.

A $114 **B** $660 **C** $646 **D** $636

10 The price of a punnet of strawberries marked at $4.95 was reduced by 25%. Calculate its discounted price. Select **A**, **B**, **C** or **D**.

A $4.74 **B** $3.74 **C** $3.71 **D** $1.25

11 At a book sale, all books are discounted by 35%. What is the discount price of a book marked at $25.50?

12 A factory-seconds washing machine has its price reduced from $980 to $750. What is the percentage discount?

13 A TV priced at $1599 is sold for $1250. Find the discount as a percentage of the marked price.

14 Calculate the percentage discount for each item.

a

b

EXAMPLE 9

15 A jacket discounted by 40% after Christmas sells for $84. What was the original price of the jacket?

16 A bedroom suite reduced by 15% is sold for $2801. What was its original price? Answer to the nearest dollar.

17 Running shoes are sold to make a profit of 85%. If they sold for $164.65, what was their cost price? Select **A**, **B**, **C** or **D**.

A $140 **B** $89 **C** $76 **D** $65

18 A plane fare from Brisbane to Sydney has been discounted by 45%. If Amy paid $159 for a discounted ticket, what was the full price, to the nearest dollar?

19 For each item, calculate the GST payable and the final price.

a a car priced at $20 900 **b** a home cinema priced at $1810

c a theatre ticket priced at $145 **d** plumber's fees of $180

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EXAMPLE 10

3.04

20 Given the selling price of each item with GST included, calculate the GST and the original price, correct to the nearest cent. PS R

a haircut: $46.20
b car repairs: $739.20
c photo frame: $31.35
d pair of jeans: $65.89
e refrigerator: $924
f piano lessons: $198 per term

21 A discount warehouse has a 30% discount on all computers with a further 5% off the discount price if you pay in cash. Carlos wants to buy a computer priced at $1899. PS R

a What is the discount price of the computer?
b Carlos pays by cash. How much will he need to pay now, to the nearest 5 cents?
c What is the total discount Carlos received?
d Calculate his total percentage discount (on the original cost).

22 Justine buys 20 boxes of paper at $29.95 a box. PS R

a What is the cost of the 20 boxes of paper?
b Justine receives a business discount of 15%. How much will she need to pay?
c Justine pays by cash and receives a further discount of 5%. How much will she now pay, to the nearest 5 cents?
d Find the amount Justine would pay if she received a single discount of 20% on the cost of the paper.
e Is the single discount of 20% the same as the two successive discounts of 15% and 5%? Give reasons for your answer.

Mental skills 3A: Maths without calculators ANSWERS ON P. 572

Squaring a number ending in 5, 1 or 9

The square of a number ending in 5 always ends in 25.
For example, $35^2 = 1225$ and $105^2 = 11\,025$.
A mental calculation trick requires three easy steps:

- delete the 5 from the number
- multiply the remaining number by the next consecutive number
- write '25' at the end of the product.

1 Study each example.

a To calculate 35^2:
- deleting the 5 from 35 leaves 3
- multiply 3 by the next consecutive number: $3 \times 4 = 12$
- write '25'at the end: 1225

$35^2 = 1225$

b To calculate 105^2:
- deleting the 5 from 105 leaves 10
- multiply 10 by the next consecutive number: $10 \times 11 = 110$
- write 25 at the end: 11 025

$105^2 = 11\,025$

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2 Now calculate each square number.

a	25^2	**b**	55^2	**c**	45^2	**d**	65^2	**e**	75^2	**f**	2.5^2
g	4.5^2	**h**	205^2	**i**	6.5^2	**j**	7.5^2	**k**	275^2	**l**	345^2

The square of a number ending in 1 always ends in 1.

For example, $41^2 = 1681$ and $71^2 = 5041$.

A mental calculation trick requires three steps.

- Round the number to the nearest 10 (by subtracting 1) to make a new number
- Square the new number
- Add to your answer the new number and its next consecutive number

3 Study each example.

a To calculate 41^2:

- Round 41 down to 40.
- Square 40: $40^2 = 1600$
- Add 40 and 41: $1600 + 40 + 41 = 1681$

$41^2 = 1681$

b To calculate 71^2:

- $70^2 = 4900$ Rounding down and squaring
- $4900 + 70 + 71 = 5041$

$71^2 = 5041$

4 Now calculate each square number.

a	21^2	**b**	101^2	**c**	31^2	**d**	91^2	**e**	5.1^2
f	81^2	**g**	61^2	**h**	201^2	**i**	2.1^2	**j**	9.1^2

The square of a number ending in 9 also ends in 1.

For example, $29^2 = 841$ and $99^2 = 9801$.

A mental calculation trick requires three steps:

- round the number to the nearest 10 (by adding 1) to make a new number
- square the new number
- subtract from your answer the new number and its *previous* consecutive number

5 Study each example.

a To calculate 29^2:

- round up to 30
- square 30: $30^2 = 900$
- subtract 30 and 29:

$900 - 30 - 29 = 841$

$29^2 = 841$

b To calculate 99^2:

- round up to 100
- square 100: $100^2 = 10\,000$
- $10\,000 - 100 - 99 = 9801$

$99^2 = 9801$

6 Now calculate each square number.

a	59^2	**b**	69^2	**c**	89^2	**d**	19^2	**e**	109^2
f	5.9^2	**g**	6.9^2	**h**	119^2	**i**	39^2	**j**	8.9^2

Simple interest 3.05

Banks, credit unions and other financial institutions reward investors by paying them interest on their savings or investments. Conversely, they charge borrowers by making them pay interest on their loans.

WS Simple interest

- The original amount of money invested or borrowed is called the **principal**.
- **Interest** is calculated as a percentage of the principal.
- This percentage is called the **interest rate**, usually written as a rate **per annum** ('per year'), abbreviated 'p.a.'
- **Simple interest** (or **flat rate interest**) is interest calculated simply on the original principal.

Example 11

Find the simple interest earned on an investment of \$18 000 at 3.5% p.a. for 5 years.

Solution

Principal = \$18 000, interest rate = 3.5% p.a., term = 5 years.

Term means the amount of time the investment or loan has occurred

Interest = $3.5\% \times \$18\,000 \times 5$ Over 5 years.

$= \$3150$

The simple interest formula

$I = Prn$, where:

I is the simple interest

P is the principal

r is the interest rate per year, expressed as a decimal

n is the number of years

Applying this to Example **11** above,

$P = \$18\,000$, $r = 3.5\% = 0.035$, $n = 5$ years

$I = Prn$

$= \$18\,000 \times 0.035 \times 5$

$= \$3150$

Simple interest

Example 12

Find the simple interest on:

a $8620 at 2.4% p.a. for 7 months

b $5600 at 6.25% p.a. for 220 days

Solution

a $P = \$8620$, $r = 2.4\% = 0.024$, $n = 7$ months $= \frac{7}{12}$ years *n must be in years*

$I = Prn$

$= \$8620 \times 0.024 \times \frac{7}{12}$

$= \$120.68$

b $P = \$5600$, $r = 6.25\% = 0.0625$, $n = 220$ days $= \frac{220}{365}$ years

$I = Prn$

$= \$5600 \times 0.0625 \times \frac{220}{365}$

$= \$210.9589...$

$\approx \$210.96$ *rounded to the nearest cent*

Simple interest

Example 13

After 2 years, an investment of $1560 has earned $87.36 in simple interest.

What is the annual interest rate?

Solution

$I = \$87.36$, $P = \$1560$, $n = 2$ years

$I = Prn$

$\$87.36 = \$1560 \times r \times 2$

$\$87.36 = \$3120r$

$r = \frac{\$87.36}{\$3120}$

$= 0.028$

$= 2.8\%$

$\therefore$ Annual interest rate $= 2.8\%$

Example 14

For how long will $12 000 need to be invested to earn $1350 if the flat rate of interest is 4.5% p.a.?

Solution

$I = \$1350$, $P = \$12\,000$, $n = 4.5\% = 0.045$

$I = Prn$

$1350 = 12\,000 \times 0.045 \times n$

$1350 = 540n$

$n = \frac{1350}{540}$

$= 2.5$ years

EXERCISE 3.05 ANSWERS ON P. 572

Simple interest U F PS R

In this exercise, give all money answers correct to the nearest cent.

EXAMPLE 11

1 Calculate the simple interest earned on each investment.

a $18 000 at 7% p.a. for 5 years

b $5280 at 2.85% p.a. for 2 years

c $3000 at 5.5% p.a. for 4 years

d $9450 at $2\frac{1}{2}$% p.a. for 3 years

e $6800 at 6.2% p.a. for 6 years

f $12 500 at 7.3% p.a. for 2 years

3.05

2 Steffi invests $17 000 at 6.45% p.a. simple interest for 4 years. To what final value will her investment grow?

EXAMPLE 12

3 Calculate the simple interest for each situation.

a $2100 invested at 6% p.a. for 9 months

b $3450 invested for 5 months at 3.25% p.a.

c $10 000 invested at 6.2% p.a. for 150 days

d $4000 borrowed for 14 months at 6.55% p.a.

e $280 000 borrowed at 8.3% p.a. for 30 weeks

f $18 400 borrowed for 160 days at 14.7% p.a.

EXAMPLE 13

4 Find the annual interest rate if:

a an investment of $3000 earns $756 simple interest after 4 years

b an investment of $17 630 earns $2715.02 simple interest after 2 years

c a flat rate loan of $9000 is charged $2115 after 5 years.

EXAMPLE 14

5 For how long will $12 350 need to be invested to earn $1778.40 if the flat rate of interest is 2.4% p.a.? PS R

6 How many months will it take for an investment of $22 000 to earn $12 650 in interest if the interest rate is 6.9% p.a.?

7 How many days will it take for $15 600 to earn $436.97 in interest if the flat rate of interest is 7.1%?

8 William took 2 years to pay off a flat rate loan of $7000. His total loan repayments amounted to $8400. Calculate:

a the interest charged

b the interest rate

9 Simone and Jeff borrowed $4000 to build a garden in their backyard. They repaid the loan at $128 per month for 4 years. PS

a Calculate the total amount they paid for the garden.

b How much interest did they pay on the loan?

c Calculate the flat rate of interest charged.

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3.06 Ratios and rates

Ratios

Rates exercise

A **ratio** compares quantities of the same type measured in the same units.
It is written in the form '$a : b$' (pronounced 'a to b'), where a and b are numbers.

Simplifying ratios

To **simplify a ratio**, keep dividing each term by the same number, preferably a large number such as their highest common factor (HCF), until each term is as small as possible.

Example 15

Simplify each ratio.

a 36 : 48 : 18

b 425 mL to 5 L

Solution

a $36 : 48 : 18 = \frac{36}{6} = \frac{48}{6} = \frac{18}{6}$ — Dividing all terms by 6, their HCF.

$= 6 : 8 : 3$

b $425 \text{ mL} : 5 \text{ L} = 425 \text{ mL} : 5000 \text{ mL}$ — Expressing both ratios in the same units.

$= 425 : 5000$

$= \frac{425}{25} : \frac{5000}{25}$ — Dividing both terms by 25, their HCF.

$= 17 : 200$

Ratio problems

Ratio problems can be solved using **equivalent ratios** or the **unitary method**.
The unitary method requires finding one part first.

Example 16

During summer, the ratio of jeans to shorts sold at a beachside store was 3 : 10.
If 240 pairs of jeans were sold, how many pairs of shorts were sold?

Solution

Method 1: Equivalent ratios

Jeans : shorts = 3 : 10 = 240 : _____

Since $3 \times 80 = 240$,

Number of shorts $= 10 \times 80 = 800$

Method 2: Unitary method

Jeans : shorts = 3 : 10

Since 240 jeans were sold, 3 parts = 240

$\therefore$ 1 part $= 240 \div 3 = 80$ — Finding 1 part first.

$\therefore$ Number of shorts $= 80 \times 10 = 800$ — Finding 10 parts.

Dividing a quantity in a given ratio

- Find the total number of parts
- If using the **unitary method**, find the size of one part, then multiply to calculate each share
- If using the **fraction method**, calculate fractions of the quantity to find each share
- Check that the shares add up to the original quantity

3.06

Dividing a quantity in a given ratio

Example 17

Divide a cash prize of $420 between Elyse and Henry in the ratio 3 : 4.

Solution

Method 1: Unitary method

Total number of parts = 3 + 4 = 7

∴ 7 parts = $420

1 part = $420 ÷ 7 = $60

∴ Elyse's share = 3 × $60 = $180

Henry' share = 4 × $60 = $240

Checking: $180 + $240 = $420

Method 2: Fraction method

Total number of parts = 3 + 4 = 7

∴ Elyse's share = $\frac{3}{7} \times \$420 = \$180$

Henry' share = $\frac{4}{7} \times \$420 = \$240$

Checking: $180 + $240 = $420

Simplifying rates

A **rate** compares 2 quantities of different types measured in different units.

It measures how one quantity changes with another quantity.

It is written in the form 'a/b' (pronounced 'a per b'), where a and b are units of measurement.

Example 18

Write each statement as a rate.

a Travelling 600 km in 12 hours.

b A phone call costs $6.45 for 15 minutes

Solution

a 600 km in 12 h = 600 ÷ 12
= 50 km/h

b $6.45 for 15 min = $6.45 ÷ 15
= $0.43/min

Rate problems

Rate problems

Problems involving rates can be solved usually by multiplying or dividing.
The following strategy may help.

- Write the units of the rate x/y as a fraction: $\frac{x}{y}$
- To find the quantity in the numerator, x, **multiply** by the rate
- To find the quantity in the denominator, y, **divide** by the rate

Example 19

A car travels at 92 km/h.

a How far will it travel in $4\frac{1}{2}$ hours?

b How long will it take to travel 800 km? Answer correct to one decimal place.

Solution

a Write the units of the rate as a fraction: km/h $= \frac{\text{km}}{\text{h}}$

To find the distance (km), multiply by the rate.

Distance $= 4\frac{1}{2} \times 92$

$= 414$ km

b To find the time (h), divide by the rate.

Time $= 800 \div 92$

$= 8.6956...$

≈ 8.7 h

EXERCISE 3.06 ANSWERS ON P.572

Ratios and rates U F PS R

EXAMPLE 15

1 Simplify each ratio.

a	48 : 28	**b**	15 : 36	**c**	125 : 60	**d**	84 : 56
e	124 : 62	**f**	$\frac{3}{5} : \frac{1}{4}$	**g**	0.68 : 0.24	**h**	1.25 : 4.5
i	$5 : \frac{4}{5}$	**j**	$\frac{5}{8} : \frac{2}{3}$	**k**	4.8 : 0.64	**l**	15 : 18 : 6
m	27 : 36 : 9	**n**	120 : 20 : 80	**o**	$\frac{5}{6} : 3$	**p**	$2\frac{1}{2} : 5$

2 Simplify each ratio.

a	75c : \$4	**b**	5 kg : 800 g	**c**	2 hours : 45 min
d	600 mm : 2 m	**e**	8 hours : 1 day	**f**	8 mm : 4 cm
g	300 mL : 2 L	**h**	\$3.50 : 25c	**i**	6 min : 45 s

EXAMPLE 16

3 The ratio of Ken's age to his daughter Jess' age is 10 : 3. If Ken is 30, How old is Jess?

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4 Bella and Suri invest in a business in the ratio 3 : 5. If the larger investment is $520 000, find the amount of the smaller investment.

5 A survey of car buyers found that they purchased cars with colours in the ratio of white : grey : blue : other colour = 8 : 5 : 11 : 10.

A car dealer ordered 20 cars in colours other than white, grey and blue. How many white cars should be ordered?

3.06

EXAMPLE 17

6 A concrete mixture of gravel to sand to cement of 4 : 3 : 1 is needed for strong foundations. How much of each is needed to make 40 cubic metres of concrete? PS R

7 At a busy intersection involving a red-light camera, the ratio of cars running a red light to cars stopping was 1 : 79. If 90 000 cars passed through the intersection last week, how many ran the red light?

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8 Shahid and Bridie won $60 000 in a lottery. If they share the prize in the ratio 23 : 27, how much does each person receive?

9 An alloy consists of nickel and copper in the ratio 4 : 7. If the alloy weighs 3.41 kg, how much copper was used? Select the correct answer **A**, **B**, **C** or **D**.

A 0.21 kg **B** 0.85 kg **C** 1.24 kg **D** 2.17 kg

EXAMPLE 18

10 Write each statement as a rate using the units in brackets. Round answers to one decimal place if needed.

a 540 km in 8 h (km/h)
b 5 kg for $42 ($/kg)
c 2800 words in 50 min (words/min)
d 14 L for 170 km (km/L)
e 200 m in 26 s (m/s)
f 5 kg of seed for 120 m^2 (g/m^2)
g 40 days to walk 900 km (km/day)
h $215 for 3 hours ($/h)

EXAMPLE 19

11 A truck maintains an average speed of 65 km/h. Calculate how far it will travel:

a in $2\frac{1}{2}$ h
b from 6:30 a.m. to 3:00 p.m.

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12 Walid earns $220 for an 8-hour day. PS R

a Express this pay as an hourly rate ($/h).

b How much would he earn in a 40-hour week?

c How many days will it take him to earn $1760?

13 5 kg of lamb loin chops cost $79.95.

a Express this as a rate ($/kg).

b How much would 3 kg cost?

c How many kilograms (correct to one decimal place) can be bought for $40?

14 James' heart beats at 78 beats/min.

a How many times will it beat in one hour?

b How long will it take to beat 1000 times? Answer to the nearest second.

15 Gold jewellery is classified according to its gold content. The ratio of gold to other metals is given in carats. Pure gold is 24 carats, so 10-carat gold has gold mixed with other metals in the ratio 10 : 14. PS R

a Write the ratio of gold to other metals in:

i a 9-carat bracelet **ii** an 18-carat ring

b Nami purchased a 14-carat gold necklace with a mass of 50 g. How much gold is in the necklace, correct to one decimal place?

16 A utility van has a fuel consumption of 11.6 L/100 km. Calculate how much fuel it will use for a trip of:

a 640 km **b** 58 km **c** 140 km

17 Calculate the fuel consumption, in L/100 km, of a Toyota Prius hybrid that uses 30.6 L to travel 900 km.

18 Birth rates are given as births/1000 people. A city's birth rate is 16 births/1000 people. If there were 520 births in the city last year, what was its population?

19 David claims that 5 kg of Greenie grass seed will plant an area of 1200 m^2. How much grass seed is needed for an area of 60 m^2? Select **A**, **B**, **C** or **D**.

A 0.25 kg **B** 0.4 kg **C** 1.44 kg **D** 4 kg

20 **a** Indonesia has a population of 264 million and an area of 1 919 440 km^2. Calculate its population density in persons/km^2, correct to one decimal place.

b Australia has a much smaller population of 24.6 million but a much larger area of 7 692 000 km^2. Calculate its population density, correct to one decimal place.

c If Australia was as densely populated as Indonesia, what would its population be? Answer to the nearest hundred.

21 A large pump takes 4 minutes to fill a tank. A smaller pump takes 10 minutes to fill the same tank. How long, in minutes and seconds, will it take to fill the tank, if both pumps are turned on at the same time?

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Did you know?

The beat of life

The heart of an unfit person works harder whether at rest, during activity or even while recovering from activity. A fit person's heart copes with activity better and its beat returns to normal sooner.

	Heartbeats per minute		
	Fit	Unfit	Difference
Before activity	65	80	15
During activity	95	135	40
3 minutes after	80	115	35
6 minutes after	70	100	30
9 minutes after	65	95	30

What patterns do you notice in the differences in heart rates between fit and unfit persons?

Alamy Stock Photo/MBI

Converting rates 3.07

It is often necessary to convert rates from one set of units to another, for example, from kilometres/hour to metres/second, or from millilitres/minute to litres/day.

Example 20

Convert:

a 80 L/kg to L/g **b** 5 m/s to km/h **c** 2 mL/min to L/day

Solution

a $80 \text{ L/kg} = \frac{80}{1000} \text{ L/g}$ — 1 kg = 1000 g

$= 0.08 \text{ L/g}$

b $5 \text{ m/s} = 5 \times 3600 \text{ m/h}$ — 1 h = 3600 s

$= 18\,000 \text{ m/h}$

$= \frac{18\,000}{1000} \text{ km/h}$ — 1 km = 1000 m

$= 18 \text{ km/h}$

c $2 \text{ mL/min} = 2 \times 60 \text{ mL/h}$ — 1 h = 60 min

$= 120 \text{ mL/h}$

$= 120 \times 24 \text{ mL/day}$ — 1 day = 24 h

$= 2880 \text{ mL/day}$

$= \frac{2880}{1000} \text{ L/day}$ — 1 L = 1000 mL

$= 2.88 \text{ L/day}$

EXERCISE 3.07 ANSWERS ON P.572

Converting rates U F PS R

EXAMPLE 20

1 Convert:

a	5 m/s to m/h	**b**	30 m/s to m/h	**c**	\$10/kg to \$/g
d	20 mL/min to mL/h	**e**	40 mL/min to mL/h	**f**	5 mL/s to mL/h
g	5 kg/m^2 to kg/ha	**h**	2.5 tonnes/day to kg/day	**i**	\$750/week to \$/year
j	0.5 km/min to km/h	**k**	15 sheep/h to sheep/day	**l**	\$60/day to \$/week

2 Convert (round to one decimal place if needed):

a	5 m/s to km/h	**b**	35 cm/s to m/h	**c**	10 cents/g to \$/kg
d	40 mL/min to L/h	**e**	40 L/min to mL/s	**f**	5 mL/s to L/day
g	25 g/m^2 to kg/ha	**h**	2.5 tonnes/h to kg/day	**i**	110 km/h to m/s
j	5 kg/m to g/cm	**k**	8 km/L to m/mL	**l**	70c/min to \$/h
m	45 mm/s to km/h	**n**	0.03 g/mL to kg/kL	**o**	0.45 kL/h to mL/s

3 Convert each speed into kilometres per hour. Round your answers to two decimal places.

a An African cheetah was measured running at 27 m/s.

b A German peregrine falcon dived at 97 m/s.

c A Tanzanian snake can travel at a speed of 3.3 m/s.

d A sailfish off the coast of Florida was estimated to swim 30 m/s.

e A racing cyclist rode at 23 m/s.

4 If Nate's reaction time is 0.9 seconds, how far will his car travel if its speed is: PS R

a	60 km/h?	**b**	80 km/h?	**c**	110 km/h?

Mental skills 3B: Maths without calculators ANSWERS ON P. 572

Estimating square roots

We can use the square numbers to estimate square roots.

n	1	2	3	4	5	6	7	8	9	10	11	12
n^2	1	4	9	16	25	36	49	64	81	100	121	144

1 Study each example.

a Estimate $\sqrt{7}$.

7 is between the square numbers 4 and 9 (2^2 and 3^2 respectively).

Because 7 is closer to 9, $\sqrt{7}$ is closer to 3.

An estimate is $\sqrt{7} \approx 2.6$. (Actual answer = 2.645...)

b Estimate $\sqrt{55}$.

55 is between the square numbers 49 and 64 (7^2 and 8^2 respectively).

Since 55 is closer to 49, $\sqrt{55}$ is closer to 7.

An estimate is $\sqrt{55} \approx 7.4$. (Actual answer = 7.416...)

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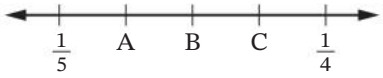
2 Now estimate each square root correct to one decimal place.

a $\sqrt{8}$	**b** $\sqrt{18}$	**c** $\sqrt{28}$	**d** $\sqrt{111}$
e $\sqrt{80}$	**f** $\sqrt{31}$	**g** $\sqrt{12}$	**h** $\sqrt{65}$
i $\sqrt{75}$	**j** $\sqrt{29}$	**k** $\sqrt{40}$	**l** $\sqrt{126}$

Power plus ANSWERS ON P. 572

1 What fractions do A, B and C represent on this number line if the intervals between the fractions are equal?



2 Deepti sold a car for $14 700, making a loss of 30% on its original price. Find the original price.

3 A store marks up the price of a lounge suite by 80%. What percentage discount would bring the price of the lounge suite back to its original price?

4 Ineke went to a supermarket and bought 5 kg of potatoes. If 3 kg of potatoes cost $4.99, how much change would Ineke receive from $20?

5 2 L of water is to be shared equally between 3 people.

a Calculate how much water each person would receive (correct to 3 decimal places).

b Explain why 3 decimal places would be too accurate for practical purposes.

6 Melinda bought a pair of shoes that were reduced by $\frac{1}{3}$. If the shoes sold for $72, what was their original price?

7 After training for 6 months, Owen improved his time for running 100 m by 8%. If his new time is 10.2 seconds, what was his old time? Answer correct to one decimal place.

8 At a local football game, the ratio of men to children was 5 : 8 and the ratio of women to children was 7 : 10. What is the ratio of men to women?

9 Hannah, Nicola and Olivia share an amount of money in the ratio 2 : 3 : 5. If Nicola gives $35 to Hannah, both of them will have an equal amount of money. How much money will Olivia receive?

10 James and Jordan each drove their cars to the beach. James drove his car at a speed of 40 km/h for half the distance and at 60 km/h for the other half. Jordan drove at 50 km/h for the entire distance. Who arrived first, James or Jordan? Give reasons for your answer.

11 **a** How long will it take for an investment of $5000 to double if it is invested at 5% p.a. simple interest?

b How long will it take for an investment of $10 000 to double at the same interest rate?

CHAPTER 3 REVIEW

Language of maths

convert	cost price	decimal	decrease
discount	flat rate interest	fraction	GST
increase	interest rate	irrational	loss
per	per annum (p.a.)	percentage	principal
profit	rate	ratio	recurring
selling price	simple interest	terminating	unitary method

1 What is another name for **flat rate interest**?

2 What is the product of a fraction and its reciprocal?

3 Find a non-mathematical meaning for:

a reciprocal **b** recurring **c** terminating.

4 What is the difference between the selling price and the cost price called when the selling price is higher?

5 What does 'annum' mean, as in 'per annum'?

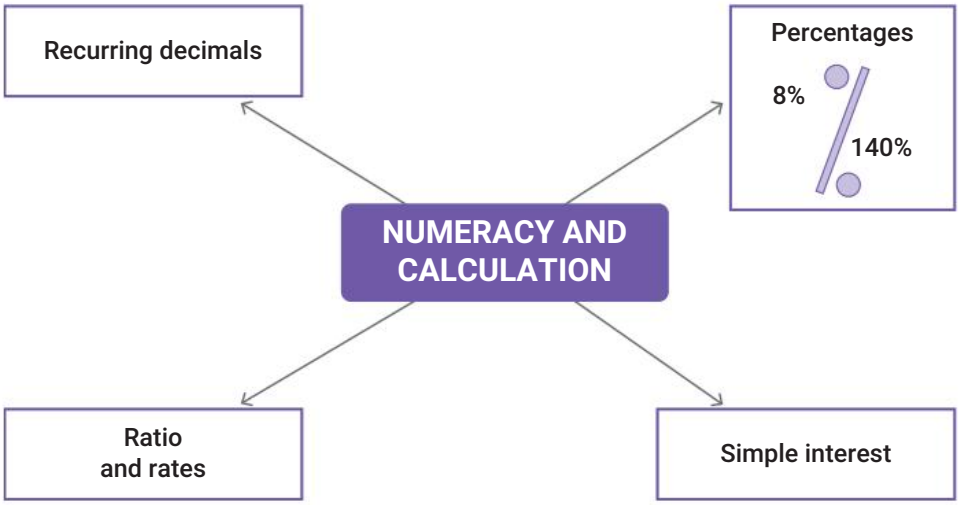
6 Which method involves finding the value of one part when given the value of the whole amount or several parts?

Topic summary

Write 10 questions (with solutions) that could be used in a test for this chapter. Include some questions that you have found difficult to answer.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.

WS
Mind map: Numeracy and calculation (Advanced)



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TEST YOURSELF 3

ANSWERS ON P. 572

STAGE 5.3

1 Convert $\frac{4}{11}$ to a recurring decimal. 3.01

2 Convert each recurring decimal to a simple fraction. 3.02

a $0.\dot{8}$ **b** $0.\dot{2}\dot{7}$ **c** $0.1\dot{6}$ **d** $2.\dot{6}4\dot{2}$

3 Find: 3.03

a 42% of $580 **b** $8\frac{1}{4}\%$ of 90 kg **c** 2.45% of $358

4 **a** Increase 120 L by 35%. **b** Decrease $25 by 12.5% 3.03

5 What percentage is (correct to one decimal place): 3.03

a 5 minutes of 1 hour? **b** 75c out of $4.50?

6 Calculate the selling price of a smartphone marked at $485 and discounted by 15%. Select the correct answer **A**, **B**, **C** or **D**. 3.04

A $72.75 **B** $470 **C** $412.20 **D** $412.25

7 A block of land was purchased for $250 000. Four years later, it was sold for $630 000. Calculate: 3.04

a the profit **b** the profit as a percentage of cost price.

8 In a recent test cricket match, Steve made 286 runs in both innings, which represented 37% of the team total. How many runs did the team make in the match? 3.04

9 At a sale, Garrett and Kay bought a cutlery set, saving $150 on the original price. If this discount represented 55% of the price, what was the price of the cutlery? 3.04

10 Calculate, to the nearest cent, the simple interest on: 3.05

a $8050 invested for 5 years at 4.8% p.a.

b $3890 borrowed for 7 months at 2.4% p.a.

11 Calculate the annual simple interest rate if an investment of $7000 earns $1820 in interest over 2 years. 3.05

12 Simplify each ratio. 3.06

a 200 : 450 **b** 35 mm : 2 m **c** 5 h : 45 min

13 Bill and Ben win $800 000 in Lotto. If they agree to share the prize in the ratio of 15 : 17, how much will each person receive? 3.06

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3.06

14 A truck travels 581 km in 7 hours.

- **a** Calculate its average speed in km/h.
- **b** How far does the truck travel in 4 hours?
- **c** How long will it take to travel 1000 km? Answer correct to one decimal place.

3.06

15 A farmer spreads fertiliser at the rate of 15 kg/ha. Calculate:

- **a** the amount of fertiliser needed to cover an area of 26 000 m^2.
- **b** the area, in hectares, which will be covered by 450 kg of fertiliser.

3.07

16 Convert:

- **a** 0.8 mL/s to mL/h
- **b** 140 g/m^2 to kg/m^2
- **c** 30 km/h to m/s
- **d** 345.6 m/day to cm/min

3.06

17 The ratio of Zoe's savings to Lucy's savings was 9 : 4. After Zoe spent $81, the ratio became 3 : 2. What was the value of Lucy's savings?

PRACTICE SET 1

ANSWERS ON P. 572

1 Convert each fraction to a decimal and state whether it is recurring or terminating. 3.01

a $\frac{5}{8}$ **b** $\frac{7}{13}$ **c** $\frac{8}{9}$ **d** $4\frac{3}{4}$

2 Simplify each expression. 1.01

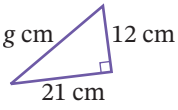
a $7 + 5m - 9$ **b** $8p - 3m + 12p + 4m$ **c** $5n^2 - 3n - 2n^2 - 7n$

3 Select the surds from this list of numbers. 2.03

$\sqrt{98}$ $\sqrt{9}$ $\sqrt{225}$ $\sqrt{160}$ $\sqrt{36}$ $\sqrt{0.25}$ $\sqrt{52}$

4 Find, correct to one decimal place, the value of each variable. 2.01

a



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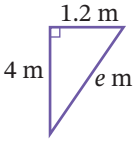

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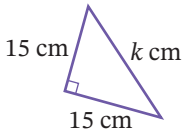


5 Convert each recurring decimal to a simple fraction. STAGE 5.3 3.02

a $0.\dot{4}\dot{2}$ **b** $0.2\dot{6}$ **c** $0.\dot{1}8\dot{0}$ **d** $3.2\dot{1}\dot{6}$

6 Simplify each surd. 2.04

a $\sqrt{75}$ **b** $\left(3\sqrt{7}\right)^2$ **c** $2\sqrt{28}$ **d** $\sqrt{162}$

7 Expand each expression. 1.09

a $(5y - 7)(5y + 7)$ **b** $(2g + 3m)(2g - 3m)$ **c** $\left(4u - \frac{1}{u}\right)\left(4u + \frac{1}{u}\right)$

8 Expand each expression. 1.07

a $(p + 4)(p - 8)$ **b** $(2n + 1)(3n + 5)$ **c** $(7 - y)(7 + y)$

9 Expand each expression. STAGE 5.3 1.10

a $(5a + 3)^2$ **b** $(3r + 4)(3r - 4)$ **c** $(x - 3)(x + 3) - (x - 3)^2$

10 Simplify each expression. 2.05

a $10\sqrt{2} - 3\sqrt{5} + 3\sqrt{2}$ **b** $\sqrt{18} - \sqrt{8}$ **c** $\sqrt{50} + 5\sqrt{32}$

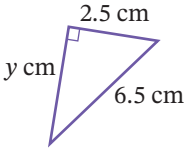
11 Simplify each expression. 2.06

a $\sqrt{3} \times \sqrt{18}$ **b** $3\sqrt{2} \times 5\sqrt{2}$ **c** $4\sqrt{5} \times \left(-\sqrt{8}\right)$

d $\sqrt{27} \div \sqrt{3}$ **e** $-8\sqrt{15} \div 2\sqrt{5}$ **f** $\frac{-7\sqrt{12}}{\sqrt{48}}$

12 Find the value of each variable. 2.02

a



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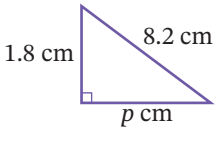

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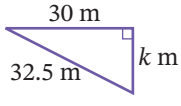

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1.06

13 Which expression is $16mn + 24m^2$ completely factorised? Select the correct answer **A**, **B**, **C** or **D**.

A $4(4n + 6m)$ **B** $8m(2n + 3m)$ **C** $4m(4n + 6m)$ **D** $m(16n + 24m)$

1.06

14 Factorise each expression.

a $3p - 15$ **b** $5(m + 4) - t(m + 4)$

c $-8x^2y - 12xy$ **d** $2ky^2 - 3ky - 2y + 3$

STAGE 5.3

1.11

15 Factorise each expression.

a $36y^2 - 49x^2$ **b** $m^2 - 5mn + 3m - 15n$

c $8m^2 - 50$ **d** $2ky^2 - 3ky - 2y + 3$

1.08

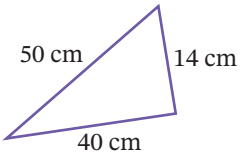
16 Expand each perfect square.

a $(3m - 5)^2$ **b** $(2y + 9)^2$ **c** $\left(h - \frac{4}{h}\right)^2$ **d** $(4p + 3w)^2$

2.08

17 Test whether each triangle is right-angled.

a



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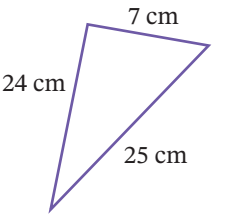

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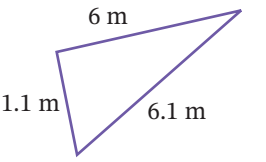

3.03

18 Calculate:

a 170% of \$60 **b** 14% of \$250 **c** $3\frac{1}{2}$% of \$670

3.06

19 Paba bought 55 L of petrol at 189.9 cents per litre.

a How much did Paba pay (to the nearest cent)?

b How many litres can be bought for \$60? Answer to one decimal place.

STAGE 5.3

1.12

20 Factorise each expression.

a $y^2 - 2y - 63$ **b** $k^2 - 7k + 10$ **c** $w^2 + 5w - 14$ **d** $m^2 - 17m + 72$

1.13

21 Factorise each expression.

a $5k^2 + 14k - 3$ **b** $3y^2 + y - 24$ **c** $4h^2 - 7h - 15$ **d** $8w^2 - 38w + 35$

e $6y^2 + 25y + 14$ **f** $12m^2 - 31m + 20$ **g** $48a^2 + 22a - 15$ **h** $36x^2 + 73x + 35$

1.15

22 Simplify each expression.

a $\frac{7}{7k - 21}$ **b** $\frac{x + 3}{x^2 - 9}$ **c** $\frac{5y + 25}{30}$ **d** $\frac{d^2 - 49}{d^2 + 2d - 35}$

1.15

23 Simplify each expression.

a $\frac{n + 1}{n^2 - n} - \frac{2}{n^2 - 1}$ **b** $\frac{5y + 20}{9} \times \frac{3y}{y + 4}$ **c** $\frac{k^2 + 4k}{k - 3} \div \frac{4k}{k^2 - 4k + 3}$

d $\frac{w^2 + 5w - 24}{w^2 - 7w + 10} \times \frac{w^2 - 4}{w^2 + 5w + 6}$ **e** $\frac{4}{k^2 - 4k - 5} + \frac{2}{k^2 - 11k + 30}$

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STAGE 5.3

24 Factorise each expression.

a $25 - 9y^2 - 30 + 18y$ **b** $28m^2 - 5m - 12$ **c** $4k^4 - 64$

2.04

25 Simplify each expression.

a $2a^2 \times 4a$ **b** $5 \times 4p \times 2pd$ **c** $9b^2d \times 3bd$ **d** $\frac{3}{4}xw^2 \times (-12xw)$

1.02

26 Simplify each expression.

a $\frac{21mn}{7n}$ **b** $36x \div 12y$ **c** $\frac{w^2v^3}{2vw}$ **d** $35a^3d^2 \div 15a^2d^4$

1.02

27 A dishwasher costing $525 is sold for $699.95. Calculate the percentage profit on the cost price, correct to one decimal place.

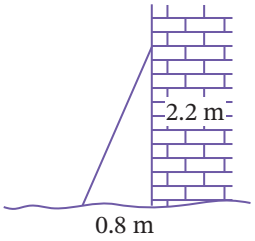
3.04

28 Simplify:

a 160 : 96 **b** 3 h : 45 min **c** 1275 words in 15 minutes (as words/min)

3.06

29 The foot of a ladder is placed 0.8 m from the base of a wall. If the ladder reaches 2.2 m up the wall, calculate, correct to one decimal place, the length of the ladder.



1.07

30 **a** Increase 85 kg by 30%

b Decrease $6.40 by 18%

3.03

31 Expand each expression.

a $8(5 - x)$ **b** $3h(2 - 7h)$ **c** $9c(2d - 3c)$

1.05

32 At a sale, the price of a TV was reduced by 30%. Find the original price of the TV if its sale price is $339.50.

3.04

33 Expand and simplify each expression.

a $2(5w - 3) + 4(1 + 3w)$ **b** $3(1 - 4k) - 2(2k + 1)$ **c** $4(3t - 4) - 7(1 - 2t)$

1.05

34 Divide $450 between Klaas, David and Sarah in the ratio 3 : 5 : 7.

3.06

35 The speed of a spacecraft leaving the Earth's gravitational pull must reach 11.2 km/s. Express this speed in km/h.

3.07

36 Simplify each expression.

a $\frac{9d}{11} - \frac{3d}{11}$ **b** $\frac{7k}{8} + \frac{11k}{8}$ **c** $\frac{3m}{5} + \frac{2m}{3}$ **d** $\frac{2y}{7} - \frac{3h}{4}$

1.03

37 Simplify each expression.

a $\frac{a}{6} \div \frac{m}{5}$ **b** $\frac{2d}{w} \times \frac{5w^2}{6m}$ **c** $\frac{3x}{5} \div \frac{9x^2}{10}$ **d** $\frac{a}{4m} \times \frac{2d}{5y} \div \frac{2y}{3m}$

1.04

38 A rectangle has side lengths of 5 cm and 7 cm. Find the length of a diagonal as a surd.

1.07

39 A discount of $75 represents 4% of the marked price. What is the marked price?

3.05

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4

MEASUREMENT AND GEOMETRY

TRIGONOMETRY

In the second century BCE, the Greek astronomer Hipparchus could calculate distances to the moon and the Sun and he was the first scientist to chart the positions of over 850 stars. How was he able to achieve this over 2100 years ago? Hipparchus started a branch of mathematics called **trigonometry**, meaning 'triangle measure', which uses angles, triangles and circles to calculate lengths and distances that cannot be physically measured. Trigonometry is used widely today in engineering, surveying, navigation, astronomy, electronics and construction.

Chapter outline

	Working mathematically				
4.01 The sides of a right-angled triangle	U	F			C
4.02 The trigonometric ratios	U	F		R	C
4.03 Similar right-angled triangles	U	F		R	C
4.04 Trigonometry on a calculator	U	F			
4.05 Finding an unknown side	U	F	PS		
4.06 Finding more unknown sides	U	F	PS		
4.07 Finding an unknown angle	U	F	PS		
4.08 Angles of elevation and depression	U	F	PS		
4.09 Bearings	U	F		R	C
4.10 Problems involving bearings	U	F	PS	R	C

Wordbank

adjacent side In a right-angled triangle, the side that is next to a given angle and pointing to the right angle

angle of depression The angle of looking **down**, measured from the horizontal

angle of elevation The angle of looking **up**, measured from the horizontal

bearing The angle used to show the direction of one location from a given point

minute (') A unit for measuring angle size, $\frac{1}{60}$ of a degree

opposite side In a right-angled triangle, the side that is facing a given angle and not one of its arms

theta (θ) A letter of the Greek alphabet used as a variable for angles

trigonometric ratio The ratio of 2 sides in a right-angled triangle, for example, sine is the ratio of the opposite side to the hypotenuse

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- investigate the sine, cosine and tangent ratios in similar right-angled triangles
- use trigonometry to solve problems involving right-angled triangles
- find unknown sides and angles in right-angled triangles, where the angle is measured in degrees and minutes
- solve trigonometry problems involving bearings and angles of elevation and depression

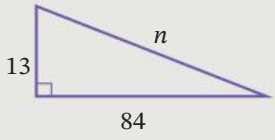
SkillCheck ANSWERS ON P. 573

1 Solve each equation.

a $\frac{x}{5}=7$ **b** $\frac{h}{4}=8.3$ **c** $\frac{45}{y}=9$

2 For each triangle, find the value of n.

a



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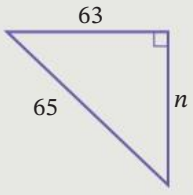

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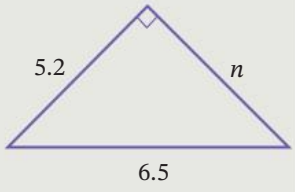


3 Round each time to the nearest hour.

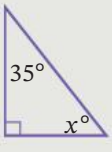
a 8 h 18 min **b** 3 h 45 min **c** 1 h 30 min

4 Convert each time to hours and minutes.

a 4.7 h **b** 2.25 h **c** 6.85 h

5 Find the value of each variable.

a



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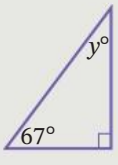

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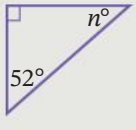

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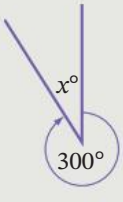

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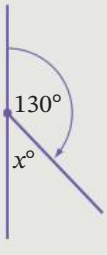

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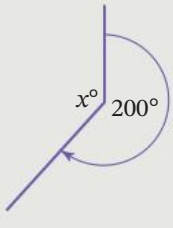

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The sides of a right-angled triangle 4.01

The 3 sides of a **right-angled triangle** have special names. These names depend on the position of a given angle in the triangle.

- The **hypotenuse** is the longest side and is always opposite the right angle
- The **opposite side** directly faces the given angle
- The **adjacent side** runs from the given angle to the right angle

We have already learnt about the hypotenuse in Pythagoras' theorem.

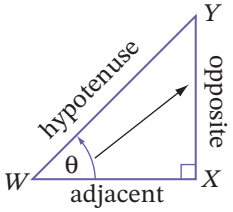
Adjacent means 'next to'.

In this diagram, the marked angle $\angle W$ has also been labelled with the Greek letter, θ, ('theta').

The hypotenuse is *WY*.

The opposite side is *XY*.

The adjacent side is *WX*.

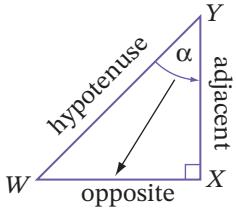


In this second diagram, the marked angle $\angle Y$ has also been labelled with the Greek letter α ('alpha'), so the opposite and adjacent sides are different.

The hypotenuse is *WY*.

The opposite side is *WX*.

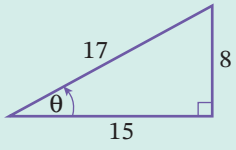
The adjacent side is *XY*.



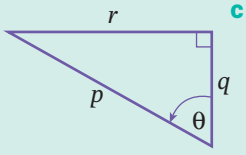
Example 1

For each triangle, name the hypotenuse, opposite and adjacent sides for angle θ.

a



b



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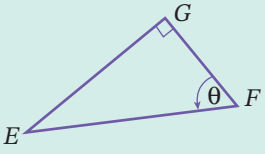


Solution

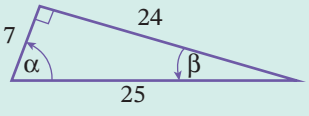
a Hypotenuse is 17.
Opposite side is 8.
Adjacent side is 15.

b Hypotenuse is *p*
Opposite side is *r*.
Adjacent side is *q*.

c Hypotenuse is *EF*.
Opposite side is *EG*.
Adjacent side is *FG*.

Example 2

For angles α and β, name the adjacent side.



Solution

For α, the adjacent side is 7.

For β, the adjacent side is 24.

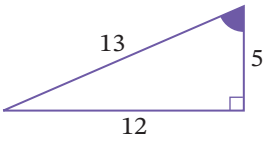
EXERCISE 4.01 ANSWERS ON P.573

The sides of a right-angled triangle U F C

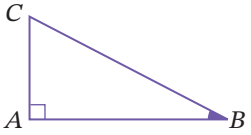
EXAMPLE 1

1 For the marked angle in each triangle, name the hypotenuse, opposite and adjacent sides. C

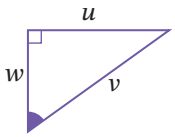
a



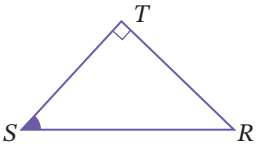
b



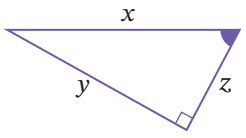
c



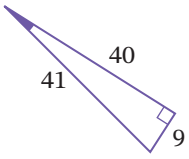
d



e



f



2 For $\triangle LKM$, name the angle:

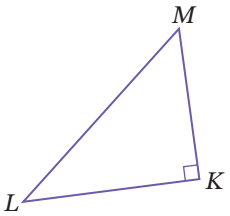
a opposite the hypotenuse

b opposite side KM

c opposite side LK

d adjacent to side KM

e adjacent to side LK.



3 Which one of these statements is **false** about $\triangle TSU$?

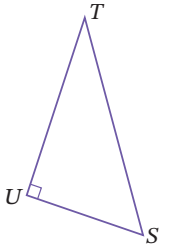
Select the correct answer **A**, **B**, **C** or **D**.

A The adjacent side to $\angle S$ is US.

B The adjacent side to $\angle T$ is TU.

C The hypotenuse is UT.

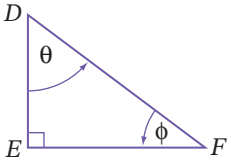
D The opposite side to $\angle T$ is US.



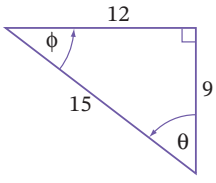
EXAMPLE 2

4 For each triangle, find the opposite sides for angles θ ('theta') and ϕ ('phi').

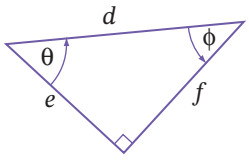
a



b

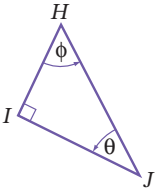


c

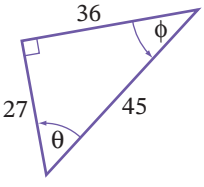


5 For each triangle, find the adjacent sides for angles θ and ϕ.

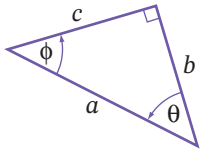
a



b



c



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6 Which side of a right-angled triangle is fixed and does not depend on a given angle? Select **A**, **B**, **C** or **D**.

A adjacent **B** hypotenuse **C** opposite **D** shortest

7 Given each description of a right-angled triangle, sketch the triangle with the correctly-labelled vertices and angle. C

a $\triangle ABC$ has hypotenuse AB and side AC opposite angle θ

b $\triangle XYZ$ has hypotenuse YZ and side XZ adjacent to angle α

c $\triangle PRQ$ has side RQ opposite $\angle P$ and adjacent to $\angle R$

d $\triangle DEF$ is right-angled at E, with the opposite and adjacent sides of $\angle D$ equal

Did you know?

The Greek alpha-bet

Here are 8 letters (in lower-case and capitals) from the Greek alphabet:

α, A alpha β, B beta γ, Γ gamma δ, $\triangle$ delta

θ, Θ theta π, Π pi σ, Σ sigma ω, Ω omega

Greek letters are commonly used as variables, particularly in geometry and trigonometry.

1 **Find how many letters there are in the Greek alphabet, and name each one.**

2 **Compare the Greek alphabet with our Roman alphabet.**

3 **Can you see where the word alphabet comes from? Explain how it originated.**

The trigonometric ratios 4.02

There are 3 special fractions called **trigonometric ratios** that relate the lengths of 2 sides of a right-angled triangle: **sine**, **cosine** and **tangent**.

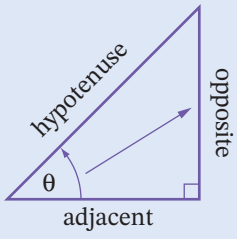
Trigonometry match-up

Trigonometry values

Trigonometry values

The trigonometric ratios

Ratio	Abbreviation	Meaning
sine	sin	$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$
cosine	cos	$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
tangent	tan	$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$



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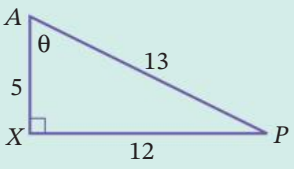
Example 3

In $\triangle AXP$, find $\sin\theta$, $\cos\theta$ and $\tan\theta$.



Solution

For angle θ, opposite = 12, adjacent = 5, hypotenuse = 13.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{12}{13} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{5}{13}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{12}{5}$$

A useful **mnemonic** (memory aid) for remembering the 3 ratios is to look at the initials of the words in the ratios:

$$\sin = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\text{O}}{\text{H}} \qquad \text{SOH}$$

$$\cos = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\text{A}}{\text{H}} \qquad \text{CAH}$$

$$\tan = \frac{\text{opposite}}{\text{adjacent}} = \frac{\text{O}}{\text{A}} \qquad \text{TOA}$$

If you remember **SOH-CAH-TOA** (pronounced 'so-car-toe-ah'), then you can remember the ratios for sin, cos and tan. Some students also learn a phrase where the first letter of each word follows the SOH-CAH-TOA sequence, for example, 'Sun Over Head Caused A Huge Tan On Arms'. Find your own mnemonic for the 3 ratios.

The trigonometric ratios

Example 4

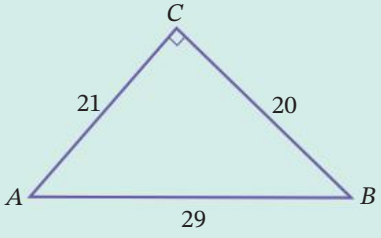
For the triangle shown, find:

a $\sin A$

b $\cos B$

c $\tan A$

d $\sin B$



Solution

For angle A, opposite = 20, adjacent = 21, hypotenuse = 29

For angle B, opposite = 21, adjacent = 20, hypotenuse = 29

a $\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{20}{29}$

b $\cos B = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{20}{29}$

c $\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{20}{21}$

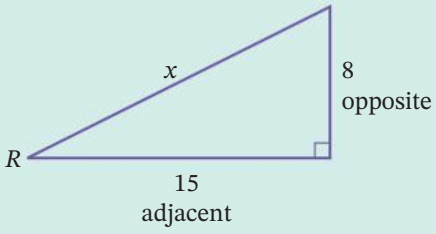
d $\sin B = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{21}{29}$

Example 5

If $\tan R = \frac{8}{15}$, find the value of sin R and cos R.

Solution

Since $\tan R = \frac{\text{opposite}}{\text{adjacent}} = \frac{8}{15}$, draw a right-angled triangle that has an angle R with opposite side 8 and adjacent side 15. Let x be the length of the hypotenuse.



Find x using Pythagoras' theorem.

$x^2 = 8^2 + 15^2$

$= 289$

$x = \sqrt{289}$

$= 17$

$\therefore \sin R = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{8}{17}$

$\cos R = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{15}{17}$

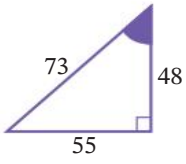
EXERCISE 4.02 ANSWERS ON P. 573

The trigonometric ratios U F R C

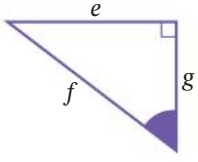
1 For each marked angle, find the sine, cosine and tangent ratios. C

EXAMPLE 3

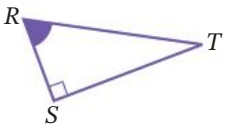
a



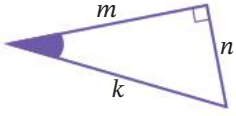
b



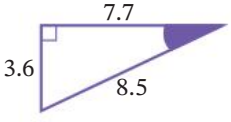
c



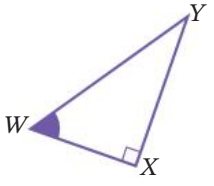
d



e



f

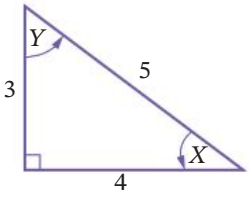


2 For the triangle shown, find:

EXAMPLE 4

a cos X **b** tan Y

c sin X **d** sin Y



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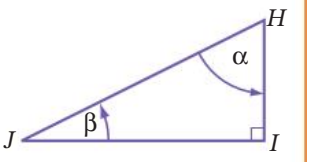
3 Complete each statement below with the correct angle (α or β).

a $\sin\square = \frac{IJ}{HJ}$ **b** $\sin\square = \frac{HI}{HJ}$ **c** $\cos\square = \frac{IJ}{HJ}$

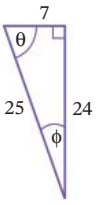
d $\cos\square = \frac{HI}{HJ}$ **e** $\tan\square = \frac{IJ}{HI}$ **f** $\tan\square = \frac{HI}{IJ}$



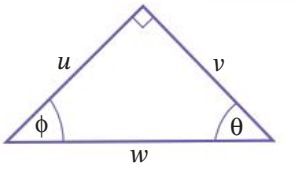
4 For each triangle below, find:

i $\tan\theta$ **ii** $\cos\theta$ **iii** $\cos\phi$ **iv** $\tan\phi$ ϕ = 'phi'

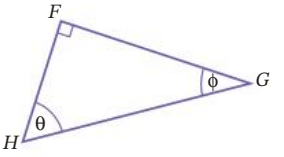
a



b

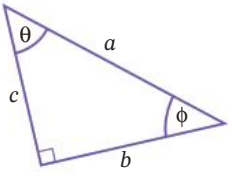


c

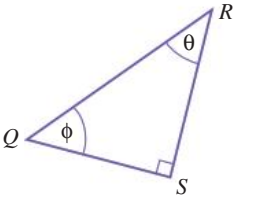


d

e



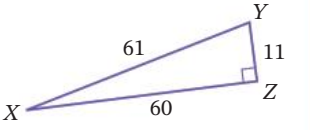
f



5 For each fraction, write a correct trigonometric ratio involving angle X or Y in the triangle. C

a $\frac{60}{11}$ **b** $\frac{11}{60}$

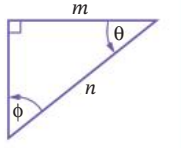
c $\frac{11}{61}$ **d** $\frac{60}{61}$



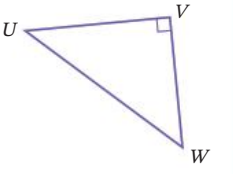
6 Which ratio is equal to $\frac{p}{m}$? Select the correct answer **A**, **B**, **C** or **D**.

A $\cos\theta$ **B** $\cos\phi$

C $\tan\theta$ **D** $\tan\phi$



7 Which statement is true for this triangle?
Select **A**, **B**, **C** or **D**. R C



A $\sin U = \cos W$ **B** $\tan U = \sin W$

C $\cos U = \tan W$ **D** $\tan U = \tan W$

EXAMPLE 5

8 Sketch a right-angled triangle for each trigonometric ratio, then use Pythagoras' theorem to find the length of the unknown side and the other 2 trigonometric ratios for the same angle.

a $\tan A = \frac{5}{12}$ **b** $\sin B = \frac{3}{5}$ **c** $\cos X = \frac{9}{41}$ **d** $\sin Y = \frac{7}{25}$

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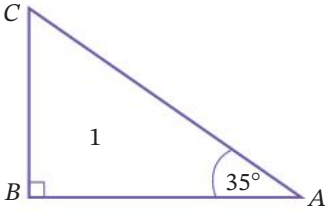
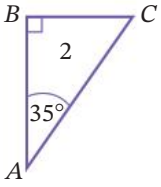
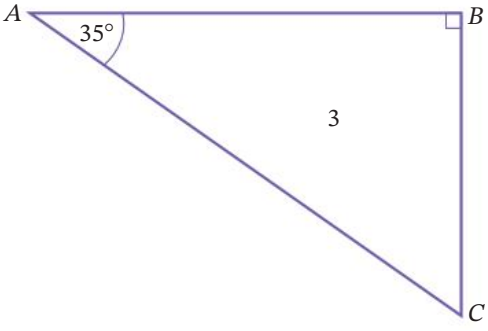
Similar right-angled triangles

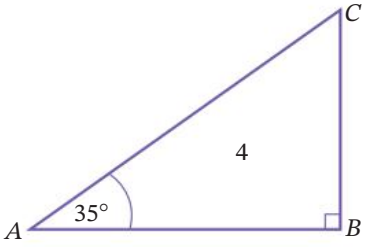
4.03

In each right-angled triangle below, $\angle A = 35°$.









Furthermore, because $\angle B = 90°$, $\angle C = 180° - 90° - 35° = 55°$ because of the angle sum of a triangle.

These 4 triangles are called **similar triangles** because they have the same shape but are not the same size. In fact, they are enlargements or reductions of one another.

Example 6

For each triangle above, measure the length of each side (correct to the nearest mm) and then calculate $\sin A$, $\cos A$ and $\tan A$ as decimals (correct to 2 decimal places). Write your results in the table below.

	Side length (mm)			Trigonometric ratio		
	BC (opp)	AB (adj)	AC (hyp)	$\sin A = \frac{BC}{AC}$	$\cos A = \frac{AB}{AC}$	$\tan A = \frac{BC}{AB}$
1						
2						
3						
4						

Solution

	Side length (mm)			Trigonometric ratio		
	BC (opp)	AB (adj)	AC (hyp)	$\sin A = \frac{BC}{AC}$	$\cos A = \frac{AB}{AC}$	$\tan A = \frac{BC}{AB}$
1	25	36	44	$25 \div 44 \approx 0.57$	$36 \div 44 \approx 0.82$	$25 \div 36 \approx 0.69$
2	14	20	24	0.58	0.83	0.70
3	40	57	69	0.59	0.83	0.70
4	29	41	50	0.58	0.82	0.71

Similar triangles have matching equal angles. We will learn more about them in Chapter 13, *Congruent and similar figures*.

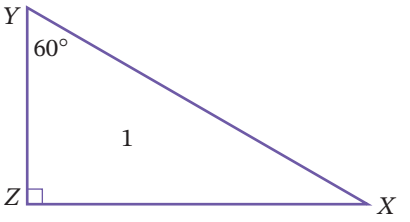
Note: See **Technology: Similar right-angled triangles** after Exercise 4.03 for a dynamic geometry activity based on this example.

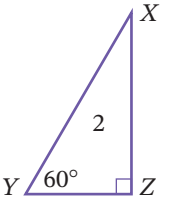
EXERCISE 4.03 ANSWERS ON P. 574

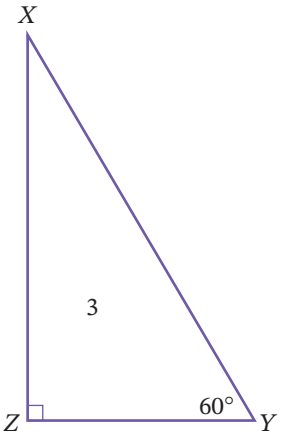
Similar right-angled triangles U F R C

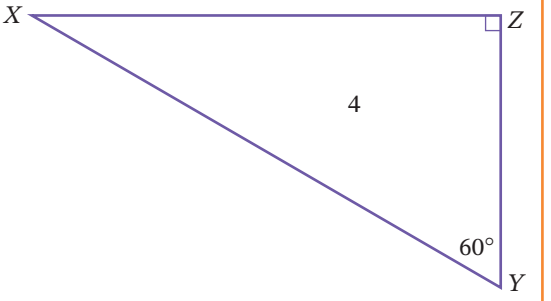
EXAMPLE 6

1 a For each similar right-angled triangle below, measure the length of each side (correct to the nearest mm) and then calculate sin Y, cos Y and tan Y as decimals (correct to 2 decimal places). Copy and complete the table. R C.









	Side length (mm)			Trigonometric ratio		
	XZ (opp)	ZY (adj)	XY (hyp)	$\sin Y = \frac{XZ}{XY}$	$\cos Y = \frac{ZY}{XY}$	$\tan Y = \frac{XZ}{ZY}$
1						
2						
3						
4						

b What do you notice about the value of sin Y for all 4 similar right-angled triangles?

c Use your calculator to evaluate sin 60° by pressing sin 60 =.
What do you notice about your answer?

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d What do you notice about the value of cos Y for all 4 similar triangles?

e Use your calculator to evaluate cos 60°. What do you notice about your answer?

f What do you notice about the value of tan Y for all 4 similar triangles?

g Use your calculator to evaluate tan 60°. What do you notice about your answer?

2 **a** Draw 4 similar right-angled triangles that have an angle of 48°, measure the length of each side (correct to the nearest mm) and then calculate sin 48°, cos 48° and tan 48° as decimals (correct to 2 decimal places). Copy and complete the table. R C

	Side length (mm)			Trigonometric ratio		
	Opp	Adj	Hyp	$\sin 48° = \frac{\text{opp}}{\text{hyp}}$	$\cos 48° = \frac{\text{adj}}{\text{hyp}}$	$\tan 48° = \frac{\text{opp}}{\text{adj}}$
1						
2						
3						
4						

b Examine the value of sin 48° for all 4 similar triangles, then evaluate sin 48° on a calculator. What do you notice?

c Examine the values of cos 48°, then evaluate cos 48° on a calculator.

d Examine the values of tan 48°, then evaluate tan 48° on a calculator.

3 For each trigonometric ratio, draw a large right-angled triangle with the given angle, then by measurement and calculation, find the value of the ratio, correct to 3 decimal places. Compare your answer to the calculator's answer. R C

a tan 55° **b** cos 39° **c** sin 67° **d** cos 21°

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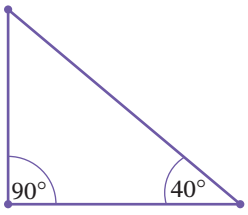
Technology

Similar right-angled triangles

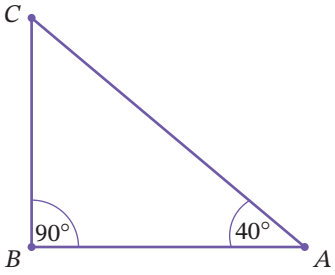
In this activity use dynamic geometry software to measure and calculate trigonometric ratios.



1 **a** Before you start, set the measuring tool to measure angles to degrees.

b By constructing intervals, draw 2 similar right-angled triangles. Then use the measuring tool to measure the right angle and another angle (as shown right).

c Now keeping the triangle right-angled, move any of the other 2 vertices so that one angle is 40°.



2 Draw 2 more similar right-angled triangles with a 40° angle.

3 Label the vertices of each triangle as A, B and C.

Make sure $\angle A = 40°$ and $\angle B = 90°$.

4 **a** Copy this table.

	Side length (mm)			Trigonometric ratio		
	BC (opp)	AB (adj)	AC (hyp)	$\sin A = \frac{BC}{AC}$	$\cos A = \frac{AB}{AC}$	$\tan A = \frac{BC}{AB}$
1						
2						
3						

b Use the measurement tool to measure the sides of each triangle and write them in the table.

c Then, for each triangle, calculate the trigonometric ratios, correct to 2 decimal places.

d What do you notice about the values of each ratio for all of the triangles?

Trigonometry on a calculator 4.04

In the previous section, we discovered that for any particular angle, the sine, cosine and tangent ratios stay constant (the same) for all right-angled triangles with that angle. For example, $\sin 35° \approx 0.57$ always, no matter what size the similar right-angled triangle.

Trigonometry squaresaw

Trigono-metric calculations

Constancy of the sine, cosine and tangent ratios

For any given angle, the values of the sine, cosine and tangent ratios are constant.

This means that the value of a trigonometric ratio can be easily found on a calculator rather than through constructing and measuring triangles.

Angles are measured in **degrees**, but one degree can be subdivided into 60 **minutes**.

One minute can be further subdivided into 60 **seconds**. The abbreviations for minutes and seconds are shown below.

Degrees and minutes

$1° = 60'$ (1 degree = 60 minutes)

$1' = 60''$ (1 minute = 60 seconds)

For example, an angle size of $48° 35' 56''$ is 48 degrees, 35 minutes and 56 seconds, about halfway between 48° and 49°.

To enter degrees, minutes and seconds into a scientific calculator, use the °’” or D°M’S (Degrees-Minutes-Seconds) key.

Trigonometry on a calculator

Example 7

Evaluate each expression correct to 2 decimal places.

a $\sin 46°$

b $\tan 57.4$

c $4 \cos 20°$

d $68.3 \sin 38°25'$

e $\dfrac{23}{\cos 18°50'}$

Solution

Make sure that your calculator is in the degrees mode (D or DEG) or your answer will be incorrect.

a $\sin 46° = 0.71933...$
≈ 0.72

On calculator: sin 46 =

b $\tan 57.4° = 1.56365...$

This angle is 57.4° (a decimal), not 57°4′.

≈ 1.56

On calculator: tan 57.4 =

c $4 \cos 20° = 3.75877...$

This means $4 \times \cos 20°$.

≈ 3.76

On calculator: 4 cos 20 =

d $68.3 \sin 38°25′ = 42.43996...$
≈ 42.44

On calculator: 68.3 sin 38 °’” 25 °’” =

e $\dfrac{23}{\cos 18°50′} = 24.30103...$
≈ 24.30

On calculator: 23 ÷ cos 18 °’” 50 °’” =

Rounding angle sizes

When rounding an angle to the nearest degree or minute, use 30 as the halfway mark.

Example 8

Round each angle correct to the nearest degree.

a 73° 27′ **b** 9° 41′ **c** 42° 30′ **d** 128° 29′ 47″

Solution

a $73°27′ \approx 73°$ — $27′ < 30′$, so round down.

b $9°41′ \approx 10°$ — $41′ \geq 30′$, so round up.

c $42° 30′ \approx 43°$ — $30′ \geq 30′$, so round up.

d $128° 29′ 47″ \approx 128°$ — $29′ < 30′$, so round down.

Example 9

Round each angle correct to the nearest minute.

a 33°53′30″ **b** 44°15′40″

Solution

a $33°53′ 30″ \approx 33°54′$ — $30″ \geq 30″$, so round up.

b $44°15′40″ \approx 44°16″$ — $40″ \geq 30″$, so round up.

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Example 10

Convert each angle size to degrees and minutes, correct to the nearest minute.

a 82.5° **b** 60.81°

Solution

a $82.5° = 82° 30′$ On a calculator: 82.5 = °′″

b $60.81° = 60° 48′ 36″$
$\approx 60° 49′$ On a calculator: 60.81 = °′″

EXERCISE 4.04 ANSWERS ON P.574

Trigonometry on a calculator U F

1 Evaluate each expression, correct to 2 decimal places. EXAMPLE 7

a tan 84° **b** cos 15° **c** tan 47° **d** sin 33°
e sin 77° **f** cos 60.1° **g** tan 39.55° **h** cos 18°
i 8 tan 75° **j** 14 sin 56° **k** 12 ÷ tan 20° **l** $\frac{7}{\sin 43°}$
m 50 × sin 70° 34′ **n** 66.2 cos 81° 42′ **o** 18.53 sin 11° 8′ **p** $\frac{27}{\cos 35°22′}$
q $\frac{44.5}{\tan 65°58′}$ **r** $\frac{200}{\sin 54°47′}$ **s** 24.1 ÷ tan 63° 2′ **t** $\frac{15.7}{\cos 21°8′}$

2 Round each angle size correct to the nearest degree. EXAMPLE 8

a 27° 54′ **b** 40° 30′ **c** 19° 18′ **d** 33° 7′ 25″
e 33° 41′ 5″ **f** 56.4° **g** 29.75° **h** 44° 18′

3 Round each angle size correct to the nearest minute. EXAMPLE 9

a 68° 39′ 42″ **b** 54° 22′ 21″ **c** 24° 46′ 30″ **d** 18° 30′ 27″
e 9° 10′ 55″ **f** 47° 59′ 9.5″ **g** 3° 45′ 35″ **h** 57° 10′ 29″

4 Convert each angle size to degrees and minutes, correct to the nearest minute. EXAMPLE 10

a 55.5° **b** 14.15° **c** 72.38° **d** 33.77°
e 66.41° **f** 7.875° **g** 28.123° **h** 31.046°
i 34.45° **j** 71.087° **k** 5.4829° **l** 69.4545°

5 By guess-and-checking with your calculator, find the angle size, θ (to the nearest degree), that gives each value.

a $\sin θ = 0.7880$ **b** $\tan θ = 0.2493$ **c** $\tan θ = 1.2799$
d $\cos θ = 0.5$ **e** $\sin θ = 0.5446$ **f** $\cos θ = 0.8829$
g $\tan θ = 0.7265$ **h** $\sin θ = 0.9998$

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Mental skills 4: Maths without calculators ANSWERS ON P. 574

Estimating answers

A quick way of estimating an answer is to round each number in the calculation.

1 Study each example.

a $631 + 280 + 51 + 43 + 96$
$\approx 600 + 300 + 50 + 40 + 100$
$= (600 + 300 + 100) + (50 + 40)$
$= 1000 + 90$
$= 1090$ (Actual answer = 1101)

b $55 + 132 - 34 + 17 - 78$
$\approx 60 + 130 - 30 + 20 - 80$
$= (60 + 20 - 80) + (130 - 30)$
$= 0 + 100$
$= 100$ (Actual answer = 92)

c $78 \times 7 \approx 80 \times 7$
$= 560$ (Actual answer = 546)

d $510 \div 24 \approx 500 \div 20$
$= 50 \div 2$
$= 25$ (Actual answer = 21.25)

2 Now estimate each answer.

a $27 + 11 + 87 + 142 + 64$

b $55 + 34 - 22 - 46 + 136$

c $684 + 903$

d $35 + 81 + 110 + 22 + 7$

e $517 - 96$

f $210 - 38 - 71 + 151 - 49$

g $766 - 353$

h 367×2

i 83×81

j 984×16

k $828 \div 3$

l $507 \div 7$

3 Study each example involving decimals.

a $20.91 - 11.3 + 2.5$
$\approx 21 - 11 + 3$
$= 13$ (Exact answer = 12.11)

b $4.78 \times 19.2 \approx 5 \times 20$
$= 100$ (Exact answer = 91.776)

c $75.13 \div 8.4 \approx 75 \div 8$
$< 80 \div 8$
< 10
≈ 9 (Exact answer = 8.944...)

d $\dfrac{37.6 + 9.3}{41.2 - 12.7} \approx \dfrac{38 + 9}{40 - 13}$
$= \dfrac{47}{27}$
$\approx \dfrac{50}{30}$
≈ 1.6 (Exact answer = 1.645…)

4 Now estimate each answer.

a $3.75 + 9.381 + 4.6 + 10.5$

b $14.807 + 6.6 - 7.22$

c 18.47×9.61

d 4.27×97.6

e $\dfrac{11.07 + 18.4}{12.2}$

f $\dfrac{38.18}{17.2 - 9.6}$

g $54.75 - 18.6 - 14.4$

h $\dfrac{18.46 \times 4.9}{39.72 - 15.2}$

i $62.13 \div 10.7$

j $(4.89)^2$

Did you know?

Degrees, minutes and seconds

We use base 10 systems in number, measurement and currency: there are 100 centimetres in a metre, 1000 grams in a kilogram and 100 cents in a dollar. So why are there 360° in a revolution, 60 minutes in a degree and 60 seconds in a minute?

In 2000 BCE, the Babylonians used a base 60 or sexagesimal system of numeration, because 60 is a rounder, more convenient number than 10. This is because 60 has more factors and is divisible by 3, 4 and 6. Furthermore, $6 \times 60 = 360$, which was the Babylonian approximation for the number of days in a year, so that each day the Earth would travel 1° around the Sun.

As measuring devices and calculations required greater precision, each degree was subdivided into 60 equal parts called minutes, and these were further divided into 60 parts called seconds. This level of accuracy is essential in navigation and mapping.

1 **A minute is a 'small' part of a degree. Investigate how an alternative meaning (and pronunciation) of 'minute' is 'tiny'.**

2 **A second is the 'second' subdivision of a degree. Explain how there are 2 different meanings of 'second'.**

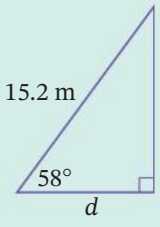
Finding an unknown side 4.05

Since the trigonometric ratio of any angle is a constant number, we can use it to calculate the length of an unknown side in a right-angled triangle if one other side is known. We need to select the correct ratio that links the given angle to the unknown side and known side.

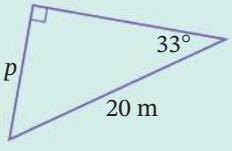
Example 11

Find the value of each variable, correct to 2 decimal places.

a



b



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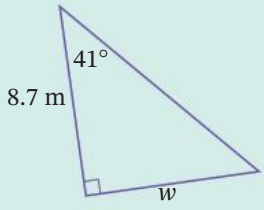


Finding an unknown side
Trigonometry

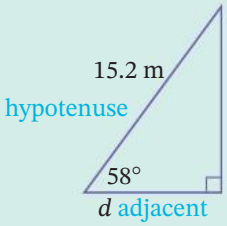
Solution

a SOH, CAH OR TOA?

The marked sides related to 58° are the adjacent side (A) and the hypotenuse (H), so use cos.

$$\cos 58° = \frac{\text{adjacent}}{\text{hypotenuse}}$$

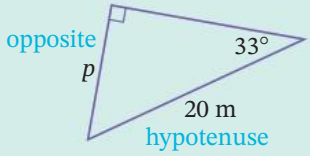
$$= \frac{d}{15.2}$$



$15.2 \times \cos 58° = 15.2 \times \frac{d}{15.2}$ Multiplying both sides by 15.2

$15.2 \cos 58° = d$
$d = 15.2 \cos 58°$
$= 8.05477...$
$= 8.05$
From the diagram, a length of 8.05 m looks reasonable.

b SOH, CAH OR TOA?
The marked sides related to 33° are the opposite side (O) and the hypotenuse (H), so use sin.



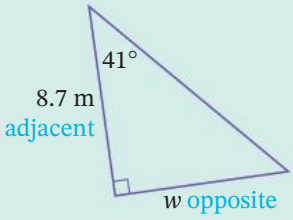
$\sin 33° = \frac{\text{opposite}}{\text{hypotenuse}}$

$= \frac{p}{20}$

$20 \times \sin 33° = 20 \times \frac{p}{20}$

$20 \sin 33° = p$
$p = 20 \sin 33°$
$= 10.8927...$
$= 10.89$
From the diagram, a length of 10.89 m looks reasonable.

c SOH, CAH OR TOA?
The marked sides related to 41° are the opposite side (O) and the adjacent side (A), so use tan.



$\tan 41° = \frac{\text{opposite}}{\text{adjacent}}$

$= \frac{w}{8.7}$

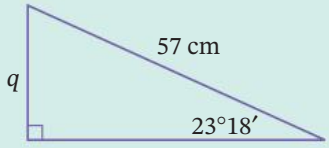
$w = 8.7 \tan 41°$
$= 7.5627...$
$= 7.56$
From the diagram, a length of 7.56 m looks reasonable.

Finding an unknown side in a right-angled triangle

1 Identify the 2 labelled sides and decide whether to use sin, cos or tan
2 Write an equation using the ratio, the given angle and the variable
3 Solve the equation to find the value of the variable

Example 12

Find the value of q, correct to the nearest centimetre.



Solution

q is opposite 23° 18′, 57 cm is the hypotenuse, so use sin.

$$\sin 23° 18' = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 23°18' = \frac{q}{57}$$

$q = 57 \sin 23° 18'$

$= 22.5460...$

$= 23$ cm

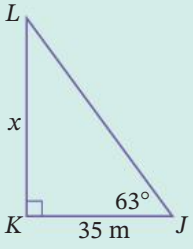
From the diagram, a length of 23 cm looks reasonable.

Example 13

$\triangle JKL$ is right-angled at K, $JK = 35$ m and $\angle J = 63°$. Find the length of LK, correct to the nearest metre.

Solution

Draw a diagram.



Let the length of LK be x.

x is opposite, 35 m is adjacent, so use tan.

$$\tan 63° = \frac{x}{35}$$

$x = 35 \tan 63°$

$= 68.6913...$

$LK \approx 69$ m

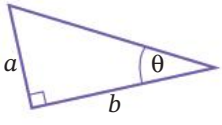
From the diagram, a length of 69 m looks reasonable.

EXERCISE 4.05 ANSWERS ON P. 574

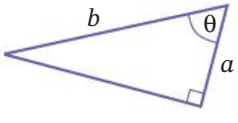
Finding an unknown side U F PS

1 For each triangle, which trigonometric ratio ($\sin\theta$, $\cos\theta$ or $\tan\theta$) is equal to $\frac{a}{b}$?

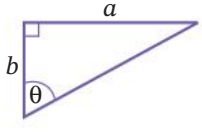
a



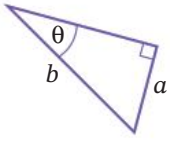
b



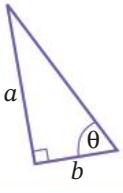
c



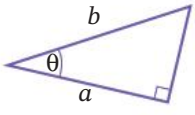
d



e



f



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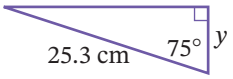
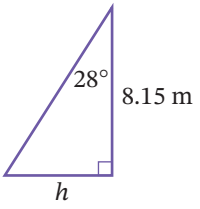
EXAMPLE 11

2 Find the value of the variable in each triangle, correct to 2 decimal places.

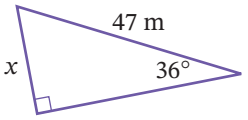
a



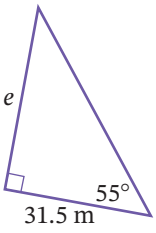
b



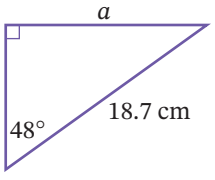
c



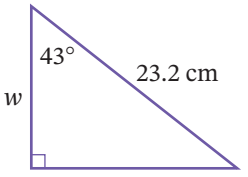
d



e



f



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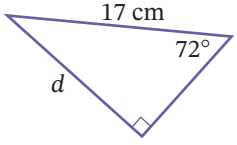

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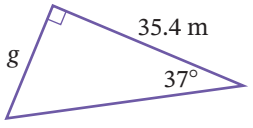

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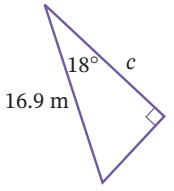
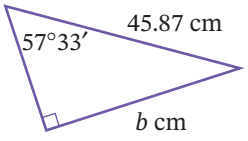


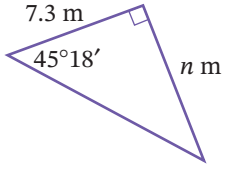
EXAMPLE 12

3 Find the value of the variable in each triangle, correct to 2 decimal places.

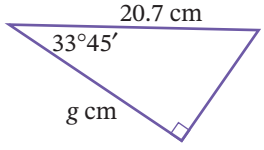
a



b



c



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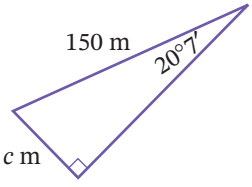
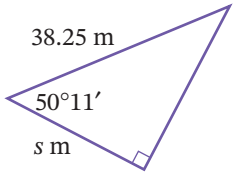


e



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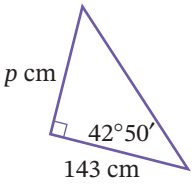
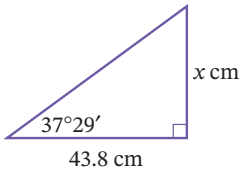


g



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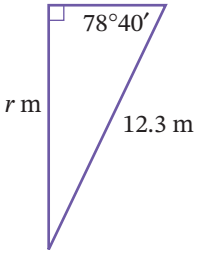

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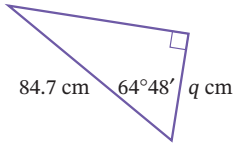
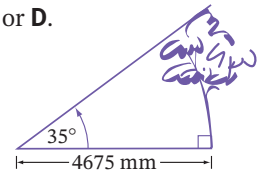


4 What is the height of this tree? Select the correct answer **A**, **B**, **C** or **D**.

A 2.68 m **B** 3.27 m

C 3.83 m **D** 6.68 m



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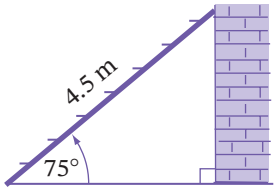
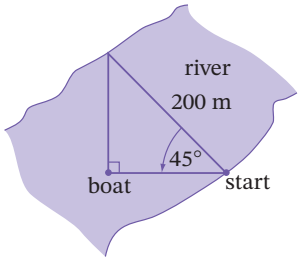
5 Find each length or distance, correct to one decimal place. PS

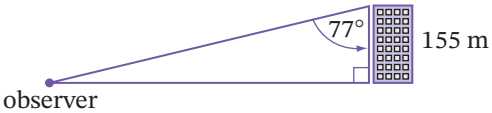
a How high the ladder reaches up the wall.



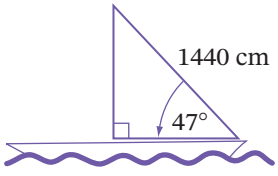
b The distance between the boat and the start.



c The distance from the observer to the base of the building.



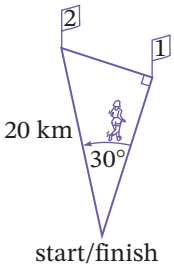
d The height of the boat's mast.



e The distance between:

i checkpoints 1 and 2

ii checkpoint 1 and the start.



6 $\triangle ABC$ is right-angled at B, $AC = 14.8$ m and $\angle C = 56°$. Find the length of side AB, correct to one decimal place.

7 $\triangle MNR$ is right-angled at M, $MN = 19$ cm and $\angle N = 27°$. Find the length of MR, correct to the nearest centimetre.

8 In $\triangle XYW$, $\angle X = 90°$, $\angle Y = 43.7°$ and $WY = 8.34$ m. Find the length of XW, correct to 2 decimal places.

9 $\triangle AHK$ is right-angled at K, $\angle H = 76°$ and $AH = 13.9$ m. Find the length of HK, correct to one decimal place.

10 A tree casts a shadow 20 m long. If the Sun's rays meet the ground at 25°, find the height of the tree, correct to the nearest cm. PS

11 A 6 m ladder is placed against a pole. If the ladder makes an angle of 17° with the pole, how far up the pole does the ladder reach? Answer to the nearest mm. PS

12 A boat is anchored by a rope 5.5 m long. If the anchor rope makes an angle of 23° with the vertical, calculate the depth of the water (correct to one decimal place). PS

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13 A rectangular gate has a diagonal brace that makes an angle of 60° with the bottom of the gate. If the length of the diagonal brace is 1860 mm, calculate the height of the gate. Select **A**, **B**, **C** or **D**. PS

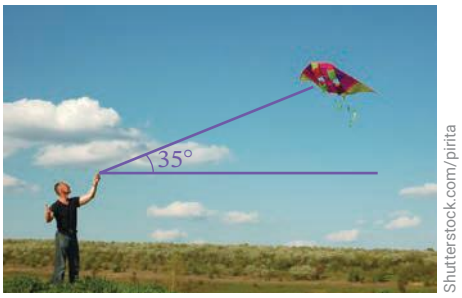
A 2148 mm **B** 930 mm **C** 1611 mm **D** 3221 mm

EXAMPLE 13

14 Tayyab is flying a kite that is attached to a string 155 m long. The string makes an angle of 35° to the horizontal. How high, to the nearest metre, is the kite above Tayyab's hand? Select **A**, **B**, **C** or **D**. PS

A 78 m **B** 89 m

C 109 m **D** 127 m



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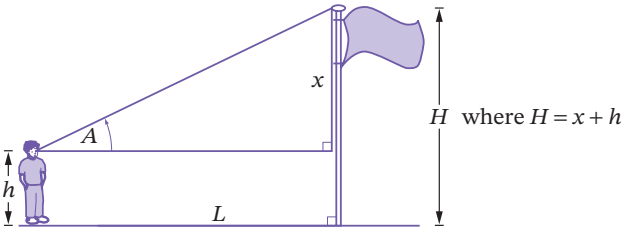
Investigation: Calculating the height of an object

You will need: tape measure or trundle wheel, a clinometer (or protractor) to measure the angle.

Trigonometry can be used to find the heights of buildings, flagpoles and trees without actually measuring them. This can be done by measuring the distance along the ground from the base of the object to a person. The person then measures the angle to the top of the object. For example, the height of a flagpole can be calculated using the set-up shown in the diagram.



- h is the eye height of the person who measures the angle, A, to the top of the flagpole
- L is the distance the person is from the base of the flagpole
- x is the height of the flagpole above the person's eye height
- $H = x + h$ is the height of the flagpole above the ground.

1 Select a tall object outside to measure.

2 Work with a partner to measure (in cm) the distance, L, along the ground, the height, h, of the person, and the angle (in degrees) to the top of the object. Copy the table below and record your information in the first row.

Distance, L (cm)	Angle, $A°$	Height of person, h (cm)	Calculated height, x cm	Height of flagpole, H cm

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3 Use the tan ratio to calculate the value of x, correct to the nearest whole number.

4 Hence find H, the height of the kite, correct to the nearest centimetre. Write your answers in the table.

5 Repeat the measurements and calculations three more times from different positions, with different persons measuring the angle. This will help to improve the accuracy of your results and minimise errors. Write your results in the table.

6 Did you find similar values for H? Do they seem reasonable for the height of the object?

7 Calculate the average value for H.

Finding more unknown sides 4.06

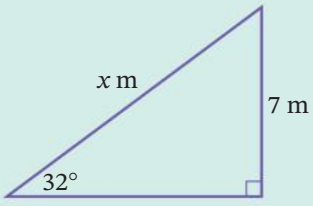
In the following examples, the unknown appears in the **denominator** of the equation.

Trigonometry equations 1

Finding the hypotenuse

Example 14

Find the value of x, correct to one decimal place.



Solution

7 m is the opposite side, x m is the hypotenuse, so use sin.

$$\sin 32° = \frac{7}{x}$$

Note that the variable x appears in the denominator of the equation

$$\sin 32° \times x = \frac{7}{x} \times x$$ Multiply both sides by x.

$$x \sin 32° = 7$$

$$\frac{x \sin 32°}{\sin 32°} = \frac{7}{\sin 32°}$$ Divide both sides by sin 32°

$$x = \frac{7}{\sin 32°}$$

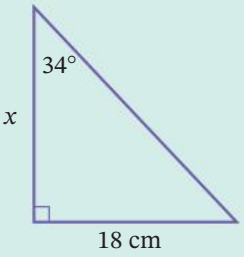
$$= 13.2095...$$

$$\approx 13.2$$

Note that when the unknown appears in the denominator of an equation, it can swap positions with the trigonometric ratio, so that $\sin 32° = \frac{7}{x}$ becomes $x = \frac{7}{\sin 32°}$.

Example 15

Find the length of x, correct to 2 decimal places.



Solution

18 cm is the opposite side, x is the adjacent side, so use tan.

$\tan 34° = \frac{18}{x}$ — x appears in the denominator

$x = \frac{18}{\tan 34°}$ — Swap the position of x with tan 34°

$= 26.6860...$

≈ 26.69 cm

Alternative method

To avoid having x in the denominator, we could use tan with the 3rd angle of the triangle.

3rd angle $= 180° - 90° - 34° = 56°$

$\tan 56° = \frac{x}{18}$

$x = 18 \tan 56°$

$= 26.6860...$

≈ 26.69 cm

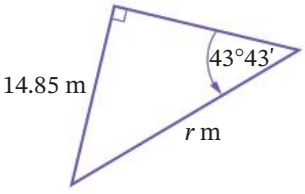
EXERCISE 4.06 ANSWERS ON P.574

Finding more unknown sides U F PS

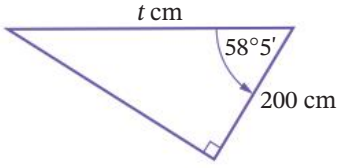
EXAMPLE 14

1 Find the value of each variable, correct to one decimal place.

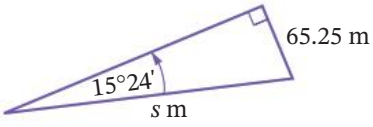
a



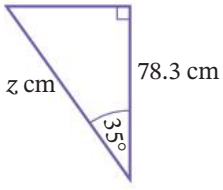
b



c



d

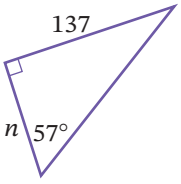


2 $\triangle XYZ$ is right-angled at Z, $ZY = 230$ mm and $\angle Y = 45°$. Find the length of XY, correct to the nearest millimetre.

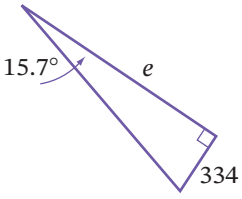
3 In $\triangle KLW$, $\angle L = 90°$, $KL = 12$ m and $\angle W = 75.2°$. Find KW, correct to one decimal place.

4 Find the value of each variable, correct to 2 decimal places.

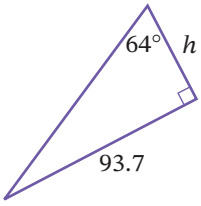
a



b



c



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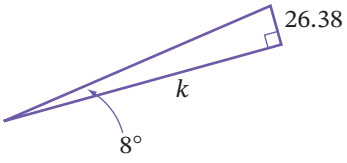


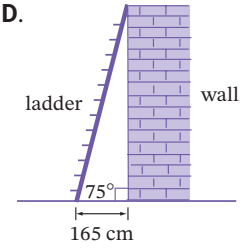
5 $\triangle CDE$ is right-angled at D, $\angle E = 36°$ and $CD = 5$ m. Find the length of side DE, correct to 2 decimal places.

EXAMPLE 15

6 Find the length of this ladder. Select the correct answer **A**, **B**, **C** or **D**.

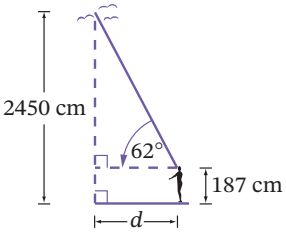
A 159 cm **B** 171 cm

C 243 cm **D** 638 cm

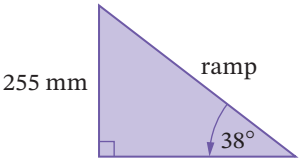


7 Find each length or distance, correct to one decimal place. PS

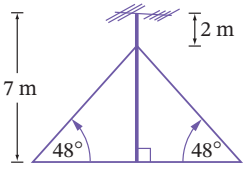
a How far the person is from being directly under the birds.



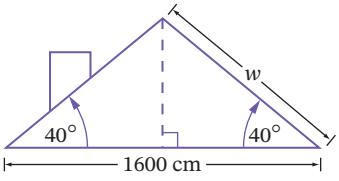
b The length of the ramp.



c The length of the support wire.



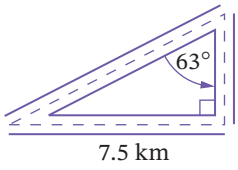
d The slant height of the roof.



e The length of:

i the shortest road

ii the longest road.



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EXAMPLE 15

8 A ladder rests against a wall. The foot of the ladder is 355 cm from the wall and makes an angle of 63° with the ground. How long (to the nearest cm) is the ladder? PS

9 A supporting wire is attached to the top of a flagpole. The wire meets the ground at an angle of 51° and the flagpole is 15 m high. How far, to the nearest 0.1 m, from the base of the flagpole is the wire anchored to the ground? PS

10 A glider is flying at an altitude (height) of 1.5 km. To land, it descends at an angle of 18° to the ground. How far, to the nearest 0.1 km, must the glider travel before landing? PS

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11 A shooter aims directly at a target, but just before firing, the rifle is lifted 1° off target. The shot misses the target by 67 mm. How far is the shooter standing from the target? Select **A**, **B**, **C** or **D**. PS

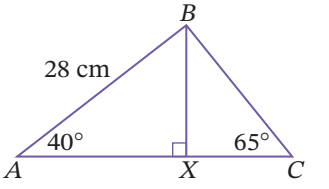
A 1169 mm **B** 3838 mm **C** 3839 mm **D** 6701 mm

12 A hot air balloon is anchored to the ground by a rope. When it drifts 20 m sideways, it makes an angle of 75° with the ground. How long is the rope (to one decimal place)? PS

13 For $\triangle ABC$, $\angle A = 40°$, $\angle C = 65°$, $AB = 28$ cm and BX is perpendicular to AC. Calculate, correct to one decimal place, the length of:

a BX

b CX



Investigation

Finding an angle, given a trigonometric ratio

You will need: a ruler, compasses and a calculator.

1 Copy and complete this table, calculating each ratio as a decimal, correct to 3 decimal places.

θ	sin θ	cos θ	tan θ
0°			
15°			
30°			
45°			
60°			
75°			
90°			undefined

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2 Can you work out why there is no answer for tan 90°?

3 What are the minimum and maximum values of sin θ?

4 What are the minimum and maximum values of cos θ?

5 Is there a pattern between the values of sin θ and cos θ?

6 Check the value of sin 30° by constructing a right-angled triangle with one angle that is 30°, measuring the opposite side and hypotenuse and dividing them.

7 Check the value of tan 45° by constructing a right-angled triangle with one angle that is 45°, measuring the opposite and adjacent sides and dividing them.

8 Use the table to estimate each trigonometric ratio and check your estimate using a calculator.

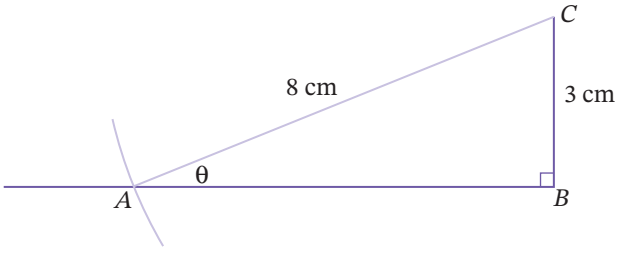
a sin 80° **b** cos 34° **c** tan 55°

9 If $\sin\theta = \frac{3}{8}$, find the value of the unknown angle θ, correct to the nearest degree:

a by using the table and estimating (change $\frac{3}{8}$ to a decimal first)

b using a calculator to guess-and-check

c constructing a right-angled triangle with one angle θ, opposite side 3 cm and hypotenuse 8 cm, then measuring the size of θ.



10 If $\cos\theta = \frac{2}{5}$, find the value of the unknown angle θ, correct to the nearest degree:

a by using the table and estimating (change $\frac{2}{5}$ to a decimal first)

b using a calculator to guess-and-check

c constructing a right-angled triangle with one angle θ, adjacent side 2 cm and hypotenuse 5 cm, then measuring the size of θ.

11 If $\tan\theta = \frac{7}{10}$, find the value of the unknown angle θ, correct to the nearest degree:

a by using the table and estimating (change $\frac{7}{10}$ to a decimal first)

b using a calculator to guess-and-check

c constructing a right-angled triangle with one angle θ, opposite side 7 cm and adjacent side 10 cm, then measuring the size of θ.

4.07 Finding an unknown angle

Trigonometry problems

Trigonometry squaresaw

Trigonometry: Finding angles

Trigonometry equations 2

Trigonometry

A scientific calculator can be used to evaluate a trigonometric ratio such as sin 38°, but it can also be used to find an unknown angle, θ, if the trigonometric ratio of the angle is known, for example, if sin θ = 0.9063.

An unknown angle can be found using the sin^{-1}, cos^{-1} and tan^{-1} keys on the calculator. These are called the **inverse sin**, **inverse cos** and **inverse tan** functions, found by pressing the SHIFT or 2nd F key before the sin, cos or tan key.

Example 16

a If sin θ = 0.9063, find angle θ, correct to the nearest degree.

b If tan X = 3.754, find angle X, correct to the nearest minute.

c If cos $\alpha = \frac{4}{7}$, find angle α, correct to the nearest degree.

Solution

a sin θ = 0.9063

θ = 64.9989...°

≈ 65°

On a calculator: SHIFT sin 0.9063 =

b tan X = 3.754

X = 75.0837 ...°

= 75° 5′ 1.62″

≈ 75° 5′

SHIFT tan 3.754 =

°’” or D°M’S

c cos $\alpha = \frac{4}{7}$

α = 55.1500...°

≈ 55

SHIFT cos 4 ÷ 7 =

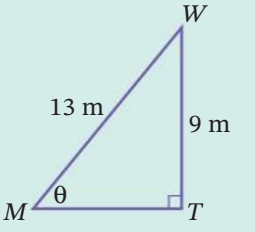
Note: or $\frac{a}{b/c}$ is the fraction key.

Example 17

Finding an unknown angle

Trigonometry

Find the size of angle θ, correct to the nearest degree.



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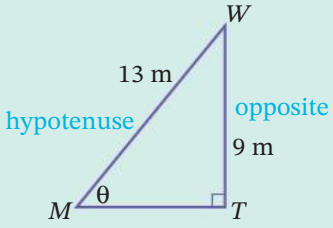
Solution

SOH, CAH or TOA?
The known sides for θ are the opposite (O) side and the hypotenuse (H), so use sin.

$\sin\theta = \frac{9}{13}$

$\theta = 43.8130...°$ SHIFT sin 9 ÷ 13 =
$\approx 44°$

From the diagram, an angle size of 44° looks reasonable.



Finding an unknown angle in a right-angled triangle

1 Identify the 2 known sides and decide whether to use the sin, cos or tan ratio.

2 Write an equation using the ratio, the angle and the 2 sides as a fraction.

3 Use the calculator's inverse trigonometric function to find the size of the angle.

Example 18

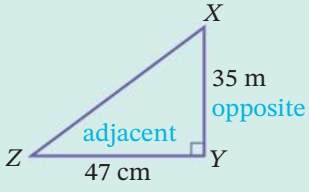
$\triangle XYZ$ is right-angled at Y, with $XY = 35$ cm and $YZ = 47$ cm. Find $\angle Z$, correct to the nearest degree.

Solution

Sketch a diagram.
SOH, CAH or TOA?
The known sides to $\angle Z$ are the opposite (O) and the adjacent (A), so use tan.

$\tan Z = \frac{35}{47}$

$Z = 36.6743...°$
$\approx 37°$
From the diagram, an angle size of 37° looks reasonable.



SHIFT tan 35 ÷ 47 =

EXERCISE 4.07 ANSWERS ON P. 575

Finding an unknown angle U F P S

EXAMPLE 16

1 Find the size of angle θ, correct to the nearest degree.

a $\cos\theta = 0.76$ **b** $\tan\theta = 2.0532$ **c** $\sin\theta = \frac{\sqrt{3}}{2}$

d $\tan\theta = 6$ **e** $\sin\theta = \frac{7}{8}$ **f** $\cos\theta = \frac{13}{15}$

g $\sin\theta = \frac{1}{10}$ **h** $\cos\theta = \frac{1}{\sqrt{2}}$ **i** $\tan\theta = \sqrt{3}$

j $\cos\theta = 0.1352$ **k** $\tan\theta = 8.836$ **l** $\sin\theta = \frac{1}{4}$

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2 Find the size of angle A, correct to the nearest minute.

a $\tan A = \frac{15}{7}$ **b** $\sin A = 0.815$ **c** $\cos A = \frac{4}{5}$

d $\cos A = 0.9387$ **e** $\tan A = \frac{19}{20}$ **f** $\cos A = \frac{3}{10}$

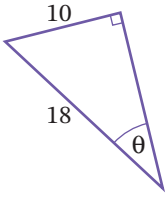
g $\sin A = \frac{5}{11}$ **h** $\sin A = 0.88$ **i** $\tan A = 15.07$

j $\cos A = \frac{1}{7}$ **k** $\tan A = \frac{\sqrt{2}}{2}$ **l** $\sin A = \frac{7}{9}$

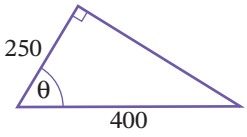
EXAMPLE 17

3 Find θ, correct to the nearest degree.

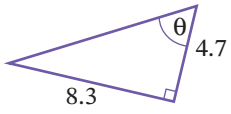
a



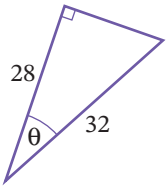
b



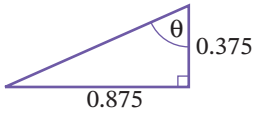
c



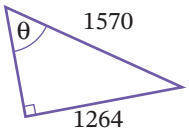
d



e



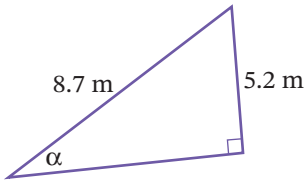
f



4 Find the size of angle α, correct to the nearest minute. Select the correct answer **A**, **B**, **C** or **D**.

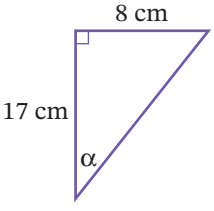
A 30° 52′ **B** 30° 53′

C 36° 42′ **D** 36° 43′

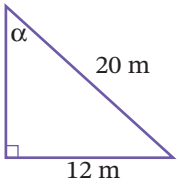


5 Find the size of angle α, correct to the nearest minute.

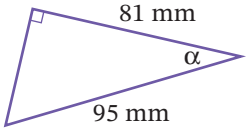
a



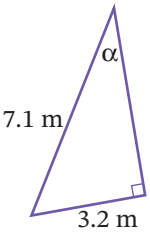
b



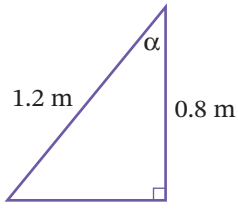
c



d



e



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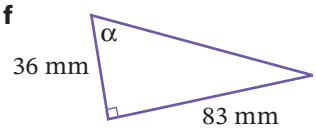

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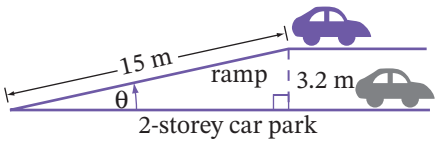
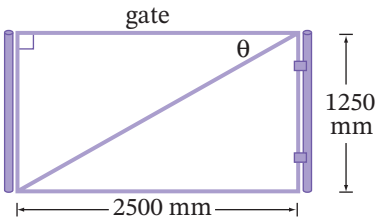
6 Find the size of angle θ, correct to the nearest degree.

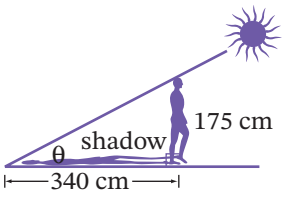
a



b



c



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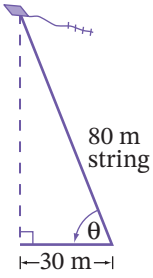
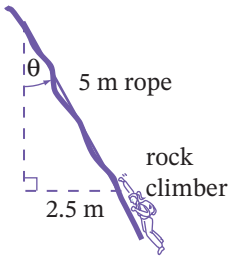


e



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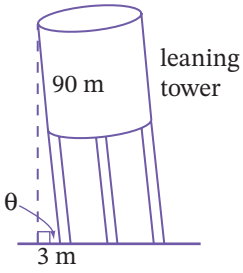

EXAMPLE 18

7 In $\triangle XYW$, $\angle X = 90°$, $XY = 8$ cm and $XW = 10$ cm. Find $\angle W$, correct to the nearest degree.

8 $\triangle HTM$ is right-angled at T, $HM = 45$ m and $MT = 35$ m. Find $\angle M$, correct to one decimal place.

9 In $\triangle FGH$, $\angle G = 90°$, $GH = 3.7$ m and $FH = 19.5$ m. Find the size of angle F, correct to the nearest minute.

10 In $\triangle CDE$, $\angle C = 90°$, $CD = 54$ mm and $CE = 42$ mm. Find $\angle E$, correct to the nearest minute.

For questions **11** to **17**, write answers correct to the nearest degree.

11 A stretch of freeway rises 55 m for every 300 m travelled along the road. Find the angle at which the road is inclined to the horizontal. PS

12 A ladder 20 m long is placed against a building. If the ladder reaches 16 m up the building, find the ladder's angle of inclination to the building. PS

13 An aircraft is descending in a straight line to an airport. At a height of 1270 m, it is 1500 m horizontally from the airport. Find its angle of descent to the horizontal. Select **A**, **B**, **C** or **D**. PS

A 32° **B** 40° **C** 50° **D** 58°

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14 A tree 8.5 m high casts a shadow 3 m long. What is the angle of the Sun from the ground? PS

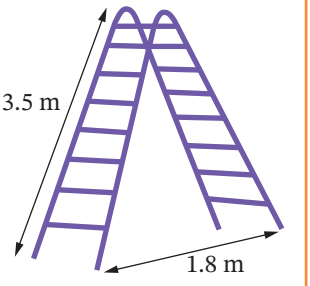
15 At a resort, an artificial beach slopes down at a steady angle. After walking 8.5 m down the slope from the water's edge, the water has a depth of 1.6 m. At what angle is the beach inclined to the horizontal? PS

16 A pile of wheat is in the shape of a cone that has a diameter of 35 m and measures 27 m up the slope to the apex. Calculate the angle of repose of the wheat (the angle the sloping side makes with the horizontal base). PS

17 A ship is anchored in water 40 m deep by a 65 m anchor chain. Find the angle at which the chain is inclined to the sea floor. PS

18 The arms of this fold out ladder are 3.5 m long. When fully spread, the feet of the ladder are 1.8 m apart.

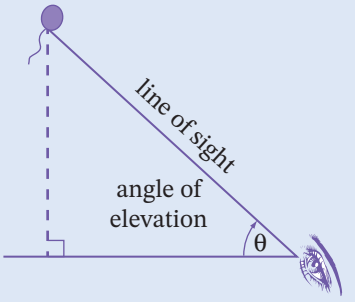
Calculate, correct to the nearest minute, the angle between the arms of the ladder when fully spread.



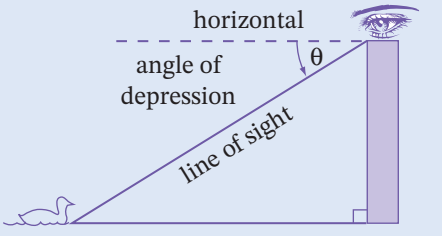
4.08 Angles of elevation and depression

Angles of elevation and depression

The **angle of elevation** is the angle of looking **up,** measured from the horizontal.



The **angle of depression** is the angle of looking **down,** measured from the horizontal.



Problems involving angles of elevation and depression usually require the tan ratio in their solutions.

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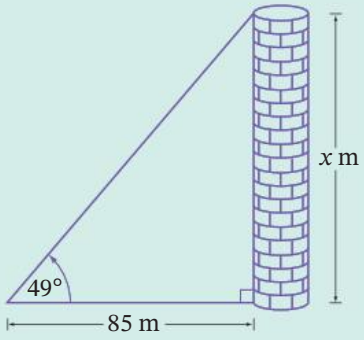
An instrument for measuring an angle of elevation or depression is a **clinometer**. It is like a protractor with a sighting tube attached.

Getty Images/VW Pics

4.08

Example 19

The angle of elevation of a chimney is 49° at a distance of 85 m from its base. Find the height of the chimney, to the nearest metre.



Solution

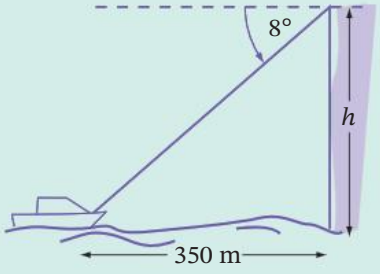
Let the height be x metres.

$$\tan 49° = \frac{x}{85}$$
$$x = 85 \tan 49°$$
$$= 97.7813 \ldots$$
$$\approx 98$$

The height of the chimney is 98 m.

Example 20

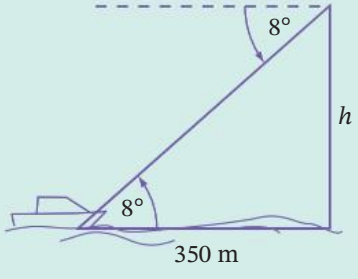
The angle of depression of a boat from the top of a cliff is 8°. If the boat is 350 m from the base of the cliff, calculate the height of the cliff, correct to the nearest metre.



Angles of elevation and depression

Solution

By alternate angles on parallel lines, the angle of elevation of the top of the cliff from the boat is also 8°.



$$\tan 8° = \frac{h}{350}$$
$$h = 350 \tan 8°$$
$$= 49.1892...$$
$$\approx 49$$

The height of the cliff is 49 m.

Alternative method:

The third angle in the triangle (adjacent to the angle of depression) = 90° – 8° = 82°.

$$\tan 82° = \frac{350}{h}$$

$$h = \frac{350}{\tan 82°}$$

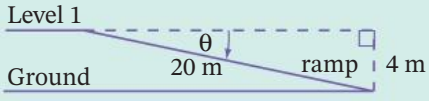
$$= 49.1892 \ldots$$

$$\approx 49$$

The height of the cliff is 49 m.

Example 21

The ramp from one level to the next in a car park is 20 m long and drops 4 m. Find the angle of depression of the ramp, to the nearest degree.



Solution

$$\sin\theta = \frac{4}{20}$$

$$\theta = 11.5369 \ldots°$$

$$\approx 12°$$

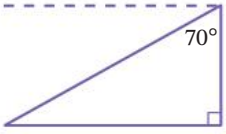
The angle of depression of the ramp is 12°.

EXERCISE 4.08 ANSWERS ON P. 575

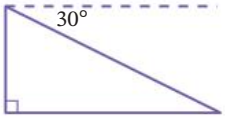
Angles of elevation and depression U F PS

1 Copy each diagram, mark the angle of elevation θ and find its size.

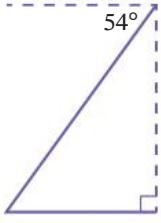
a



b

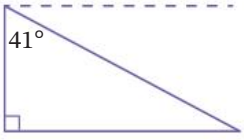


c

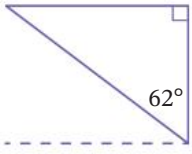


2 Copy each diagram, mark the angle of depression θ and find its size.

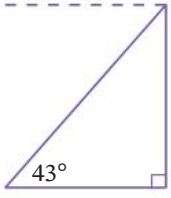
a



b

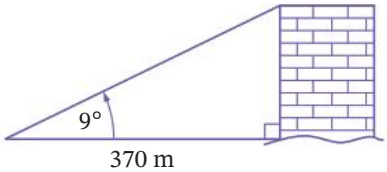


c



EXAMPLE 19

3 Emily stands 370 m from the base of a building. Using a clinometer, she finds that the angle of elevation of the top is 9°. Find the height of the building, to the nearest metre.



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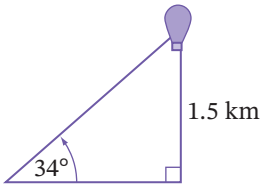
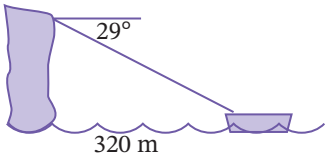
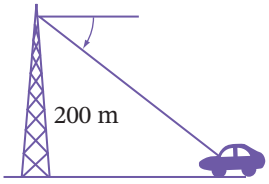
4 The angle of elevation of a weather balloon at a height of 1.5 km is 34°. How far (to the nearest metre) is the observer from being directly under the balloon?



5 A raft is 320 m from the base of a cliff. The angle of depression of the raft from the top of the cliff is 29°. Find the height of the cliff, to the nearest metre. PS



6 From the top of a 200 m tower, the angle of depression of a car is 48°. How far is the car from the foot of the tower? PS



7 A 275 m radio mast is 1.7 km from a school. Find, correct to the nearest degree, the angle of elevation of the top of the mast from the school. PS

EXAMPLE 21

8 A monument 24 m high casts a shadow 20 m long. Calculate, correct to the nearest degree, the angle of elevation of the sun at this time of day. PS

9 In a concert hall, Ethan is sitting 20 m from the stage by line of sight. He is also 5 m above the level of the stage. At what angle of depression is the stage? Answer to the nearest minute. PS

10 A plane is 340 m directly above one end of a 1000 m runway. Find, correct to the nearest minute, the angle of depression to the far end of the runway. PS

Shutterstock.com/ verzellenberg

11 An observer 174 cm tall is standing 11.6 m from the base of a flagpole. The angle of elevation to the top of the flagpole is 43°. How high is the flagpole, to the nearest cm? PS

12 A flagpole is mounted on top of a tall building. At a distance of 250 m from the base of the building, the angles of elevation of the bottom and top of the flagpole are 38° and 40° respectively. Calculate the height of the flagpole, correct to one decimal place. PS

13 A news helicopter hovers at a height of 500 m. The angles of depression of a fire moving in the direction of the helicopter are first 10° and then 15°. How far (to the nearest metre) has the fire moved between the 2 observations? PS

EXAMPLE 20

14 The angle of elevation to the bottom of a transmission tower on a hill from an observer 1.8 km away from the base of the hill is 5°. The angle of elevation to the top of the tower from the observer is 6.8°. Find the height of the tower to the nearest metre. PS

4.08

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4.09 Bearings

NSW map bearings

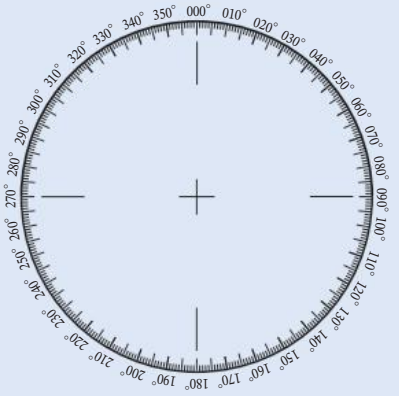
16 points of the compass

Bearings match-up

Bearings are used in navigation. A **bearing** is an angle measurement used to describe precisely the direction of one location from a given reference point.

Three-figure bearings

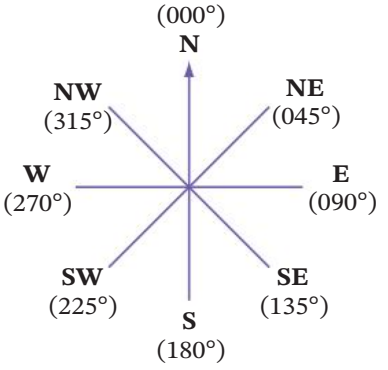
Three-figure bearings, also called **true bearings**, use angles from 000° to 360° to show the amount of turning measured **clockwise from north 000°**. Note that the angles are always written with 3 digits.



True bearings

This **compass rose** shows the three-figure bearings of 8 points on the compass. A bearing of due east is 090°, while a compass direction of southwest (SW) is 225°.

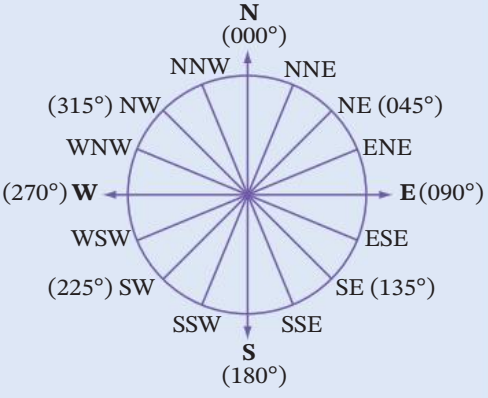
- Bearings from 000º to 090º are in the NE quadrant
- Bearings from 090º to 180º are in the SE quadrant
- Bearings from 180º to 270º are in the SW quadrant
- Bearings from 270º to 360º are in the NW quadrant



Compass bearings

Compass bearings refer to the 16 points of a mariner's compass.

N = north	NE = northeast
E = east	SE = southeast
S = south	SW = southwest
W = west	NW = northwest
NNE = north-northeast	ENE = east-northeast
ESE = east-southeast	SSE = south-southeast
SSW = south-southwest	WSW = west-southwest
WNW = west-northwest	NNW = north-northwest



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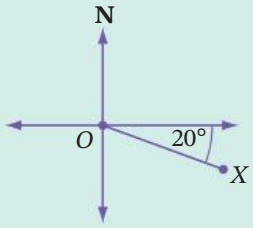
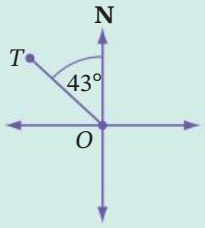
Example 22

Write the three-figure bearing of each point from O.

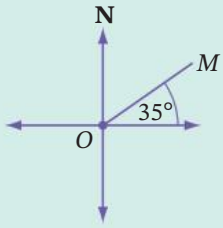
a



b



c



Solution

a Bearing of X from O is $90° + 20° = 110°$.

b Bearing of T from O is $360° - 43° = 317°$.

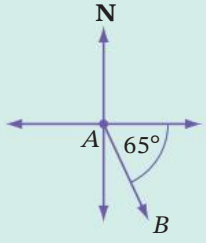
c Bearing of M from O is $90° - 35° = 055°$ Must be written as a 3-digit angle

Example 23

Sketch point B on a compass rose if B has a bearing of 155° from A.

Solution

Draw the compass rose on the point where the bearing is measured from.
155° is between 90° and 180°, so B is in the southeast (SE) quadrant.
$155° - 90° = 65°$, so B is 65° from east (E).

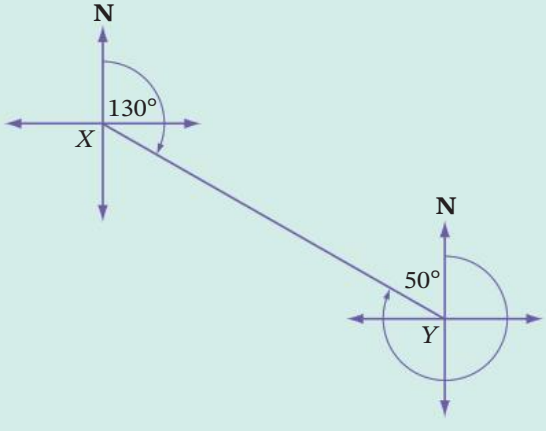


Example 24

The bearing of Y from X is 130°. What is the bearing of X from Y?

Solution

Sketch the bearing of Y from X.
On the same diagram, draw a compass rose at Y and find $\angle NYX$.
$\angle NYX = 50°$ (co-interior angles, $NX \parallel NY$)
$\therefore$ Bearing of X from $Y = 360° - 50°$
$= 310°$



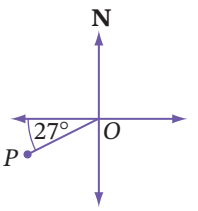
EXERCISE 4.09 ANSWERS ON P. 575

Bearings U F R C

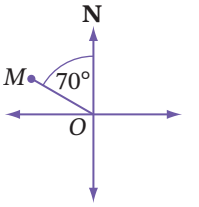
EXAMPLE 22

1 Write the three-figure bearing of each point from O. C

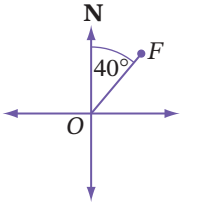
a



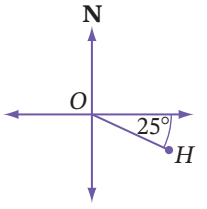
b



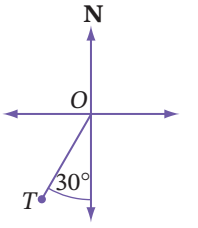
c



d



e



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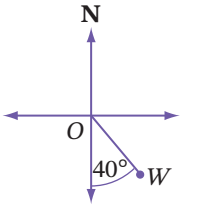

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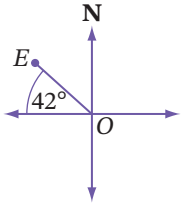

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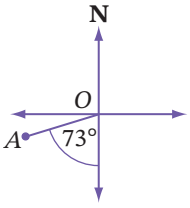
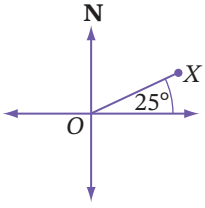
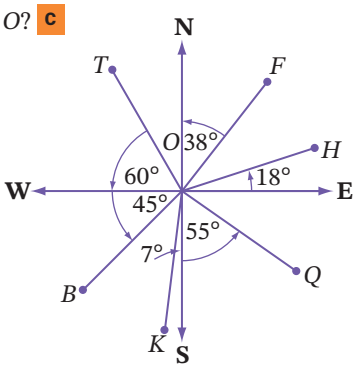


i



2 What is the bearing of each direction or point from O? C

a	**N**	**b**	**E**	**c**	**S**
d	**W**	**e**	F	**f**	Q
g	T	**h**	B		
i	H	**j**	K		



3 What is the compass direction shown by point B in question **2**? C

4 Find the three-figure bearing of each compass bearing. C

a	SW	**b**	E	**c**	NE	**d**	ESE
e	NNW	**f**	W	**g**	SE	**h**	WSW

5 Sketch each bearing on a compass rose.

a	220°	**b**	060°	**c**	260°	**d**	125°
e	350°	**f**	267°	**g**	171°	**h**	032°

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6 **a** What is the compass direction halfway between northwest and west?

b What is the three-figure bearing of this compass direction? C

7 Sketch P on a compass rose if P has a bearing of:

a 132° from T **b** 260° from M **c** 335° from X **d** 010° from K

8 If the bearing of P from A is 060°, what is the bearing of A from P? R

9 The bearing of T from Y is 100°. What is the bearing of Y from T? R

10 What is the (smallest) angle between:

a S and SW? **b** NE and SE? **c** E and NW?

d NE and SSW? **e** E and SSW? **f** SW and WNW?

11 The compass bearing of H from M is WNW. Find the compass bearing of M from H. R C

12 Draw a diagram for each situation described. C

a A plane flies on a bearing of 280° for 150 km and then another 250 km on a bearing of 080°.

b A cyclist travels 15 km due east and then 20 km on a SW bearing.

13 For this diagram, find the bearing of: C

a Y from Z

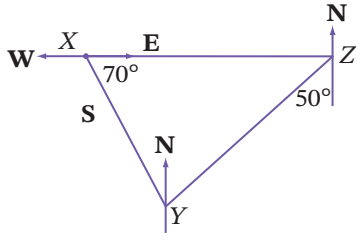
b X from Z

c Y from X

d X from Y

e Z from Y

f Z from X



EXAMPLE 23

EXAMPLE 24

4.09

Investigation

Compass walks

You need: a directional compass and a tape measure or trundle wheel.

This activity can also be done in the classroom using scale drawings, graph paper, a ruler and a protractor.

A triangular walk

1 Starting at A, walk due east for 3 m to B.

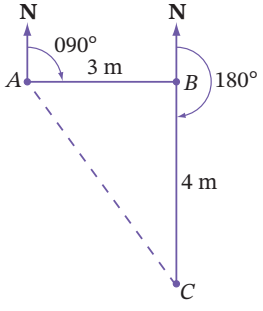
2 From B, walk due south for 4 m to C.

3 How far is C from A?

4 What is the bearing of:

a A from C?

b C from A?



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A square walk

1. Starting at P, walk at a bearing of 045° for 8 m to Q.
2. From Q, walk at a bearing of 315° for 8 m to R.
3. From R, walk at a bearing of 225° for 8 m to S.
4. How far is S from P?
5. What is the bearing of:
   - **a** P from S?
   - **b** S from P?

A pentagonal walk

1. Starting at U, walk at a bearing of 130° for 4 m to V.
2. From V, walk at a bearing of 40° for 7 m to W.
3. From W, walk at a bearing of 320° for 4.8 m to X.
4. From X, walk at a bearing of 270° for 4.5 m to Y.
5. How far is Y from U?
6. What is the bearing of:
   - **a** U from Y?
   - **b** Y from U?

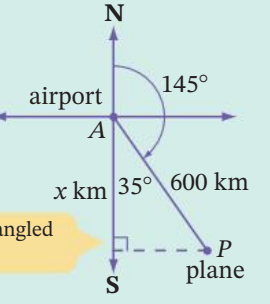
4.10 Problems involving bearings

Bearings

Trigonometry 2

Example 25

A plane leaves an airport and flies 600 km on a bearing of 145°. How far south of the airport is the plane (to the nearest km)?



To find x, create a right-angled triangle.

Solution

Let x km = distance south

$\therefore \angle SAP = 180° - 145°$ (angles on a straight line)

$= 35°$

$\cos 35° = \frac{x}{600}$

$x = 600 \cos 35°$

$= 491.4912 \ldots$

≈ 491

The plane is 491 km south of the airport.

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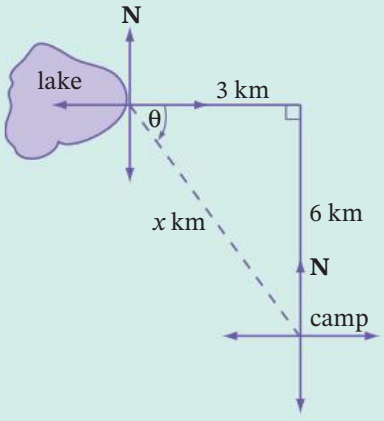
Example 26

From camp, Sumaiyaa walks due north for 6 km, then 3 km due west to a lake.

a How far is Sumaiyaa from the camp, correct to the nearest 0.1 km?

b What is the bearing of the camp from the lake (to the nearest minute)?



Solution

a Let x = distance from camp.

$x^2 = 6^2 + 3^2$ by Pythagoras' theorem

$= 45$

$x = \sqrt{45}$

$= 6.708...$

≈ 6.7 km

Sumaiyaa is 6.7 km from the camp.

b Note angle θ in the diagram.

$\tan\theta = \frac{6}{3}$

$\theta = 63.434...°$

$= 63° 26' 5.82"$

$\approx 63° 26'$

Bearing of camp from lake

$= 90° + 63° 26'$

$= 153° 26'$

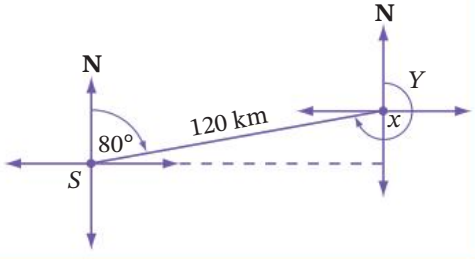
EXERCISE 4.10 ANSWERS ON P. 576

Problems involving bearings U F PS C

1 A yacht leaves Sydney and sails 120 km on a bearing of 080°.

a How far north (to the nearest km) of Sydney is the yacht?

b What is the bearing of Sydney from the yacht?



4.10

EXAMPLE 25

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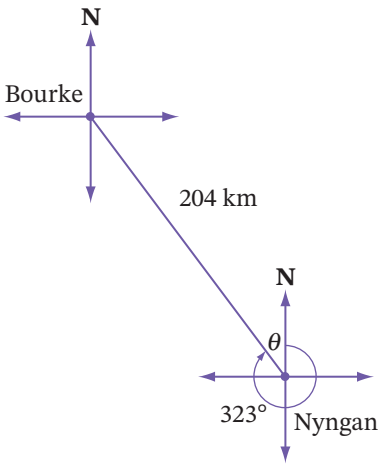
2 Declan leaves Nyngan and drives 204 km to Bourke. The bearing of Bourke from Nyngan is 323°.

a Find the value of θ.

b How far north (to the nearest km) of Nyngan is Bourke?

c What is the bearing of Nyngan from Bourke?



3 The distance 'as the crow flies' from Sydney to Wollongong is 69 km. If the bearing of Wollongong from Sydney is 205°, calculate:

a how far south Wollongong is from Sydney (correct to one decimal place)

b how far east Sydney is from Wollongong (correct to one decimal place)

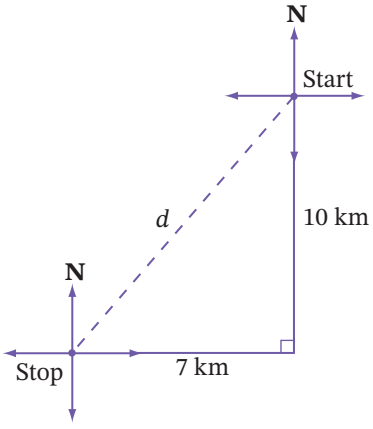
c the bearing of Sydney from Wollongong.

EXAMPLE 26

4 Jana cycles 10 km due south, then 7 km due west.

a How far (correct to one decimal place) is Jana from her starting point?

b What is her three-figure bearing from the starting point, correct to the nearest degree?



5 A triathlete cycles 20 km on a SSE bearing to the finish line.

a How far (to the nearest km) has the triathlete travelled in a southerly direction?

b What is the compass bearing of the starting point from the finish line? PS C

6 A hiking group walks from Sandy Flats to Black Ridge (a distance of 20.9 km) in the direction 078°. They then turn and hike due south to Rivers End, then due west back to Sandy Flats. How far have they hiked altogether (to the nearest 0.1 km)? PS C

7 A triangular orienteering run starts at Alpha and passes through the checkpoints of Bravo and Charlie before finishing at Alpha. Bravo is 8.5 km due east of Alpha, and Charlie is 10.5 km due south of Bravo. PS C

a Calculate, correct to 3 decimal places, the distance from Charlie to Alpha.

b Find the three-figure bearing of Alpha from Charlie to the nearest degree.

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8 A plane takes off from Darwin at 10:15 a.m. and flies on a bearing of 150° at 700 km/h. PS C

a How far (to the nearest km) due south of Darwin is the plane at 1:45 p.m.?

b What is the bearing (correct to the nearest degree) of Darwin from the plane?

9 A fishing trawler sails 30 km from port on a bearing of 120° until it reaches a submerged reef. How far (to the nearest km) is the port: PS C

a north of the reef? **b** west of the reef?

10 2 racing pigeons are set free at the same time. The first bird flies on a course of 040° while the second bird flies on a course of 130°. PS C

a The first bird flies 200 km until it is due north of the second bird. Find their distance apart, correct to 2 decimal places.

b How far has the second bird flown?

11 2 horse riders start from the same stable. The rider of the black horse goes due west for 5.5 km and stops. The rider of the chestnut horse travels in a direction of 303° until he is due north of the black horse. How far did the rider of the chestnut horse travel? Answer correct to 2 significant figures. PS C

12 2 ships leave from the same port. One ship travels on a bearing of 157° at 20 knots. The second ship travels on a bearing of 247° at 35 knots. (1 knot is a speed of 1 nautical mile per hour.) PS C

a How far apart are the ships after 8 hours, to the nearest nautical mile?

b Calculate the bearing of the second ship from the first, to the nearest minute.

13 A plane flies 1200 km on a NE compass bearing and then changes direction to a bearing of NW and flies for a distance of 450 km. PS C

a How far, correct to the nearest km, is the plane from its starting point?

b What is the bearing of the plane from its starting point, to the nearest minute?

14 A fishing boat leaves Sydney Harbour on a compass bearing of ESE and after 15 nautical miles, changes direction to a bearing of SSW and travels a further 10 km.

Find the bearing of Sydney from the boat's final position, to the nearest minute. PS C

Power plus ANSWERS ON P. 576

1 **a** Copy and complete each pair of trigonometric ratios, correct to 3 decimal places.

i $\sin 20° =$ _____, $\cos 70° =$ _____ **ii** $\sin 47° =$ _____, $\cos 43° =$ _____

iii $\sin 55° =$ _____, $\cos 35° =$ _____ **iv** $\sin 85° =$ _____, $\cos 5° =$ _____

b What do you notice about each pair of answers in part **a**?

c What do you notice about each pair of angles in part **a**?

d If $\cos 30° \approx 0.8660$ and $\sin \theta \approx 0.8660$, what is the value of θ?

e Copy and complete each equation.

i $\sin 75° = \cos$ ____ **ii** ____ $80° = \cos 10°$

iii $\cos$ ____ $= \sin 72°$ **iv** $\sin 30° =$ ____ $60°$

v $\cos 65° = \sin$ ____ **vi** $\sin$ ____ $= \cos 58°$

f Copy and complete this general rule: $\sin x = \cos$ (_____).

g Use a right-angled triangle with one angle x and sides a, b and c to prove that the above rule is true.

2 A plane is flying at an angle of 15° inclined to the horizontal.

a How far will the plane have to travel along its line of flight to increase its altitude (height) by 500 m?

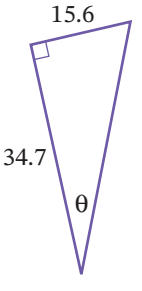
b At what angle must the plane climb to achieve an increase in altitude of 500 m in half the distance needed at an angle of 15°?

3 If $\sin 30° = \frac{1}{2}$, find, as a surd, the value of:

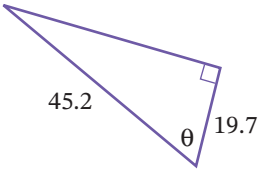
a $\cos 30°$ **b** $\tan 30°$

4 Find angle θ, correct to the nearest second.

a



b



5 By drawing an appropriate triangle, prove that:

a $\tan 45° = 1$ **b** $\sin 45° = \frac{1}{\sqrt{2}}$ **c** $\cos 45° = \frac{1}{\sqrt{2}}$

6 Sketch a right-angled triangle with side lengths a, b and c, where c is the hypotenuse. Let θ be one of the acute angles. Prove that $(\sin \theta)^2 + (\cos \theta)^2 = 1$.

CHAPTER 4 REVIEW

Trigonometry

Language of maths

adjacent	angle of depression	angle of elevation	bearing
clinometer	compass bearing	cosine (cos)	degree (°)
denominator	horizontal	hypotenuse	inverse ($^{-1}$)
minute (′)	opposite	right-angled	second (″)
sine (sin)	tangent (tan)	theta (θ)	three-figure bearing
trigonometry	trigonometric ratio	unknown	vertical

1 When measuring angle size, what is a **second** and what is its symbol?

2 What word means 'next to'?

3 Which side of a right-angled triangle is fixed and does not depend on the position of an angle?

4 What are the first 2 letters of the Greek alphabet?

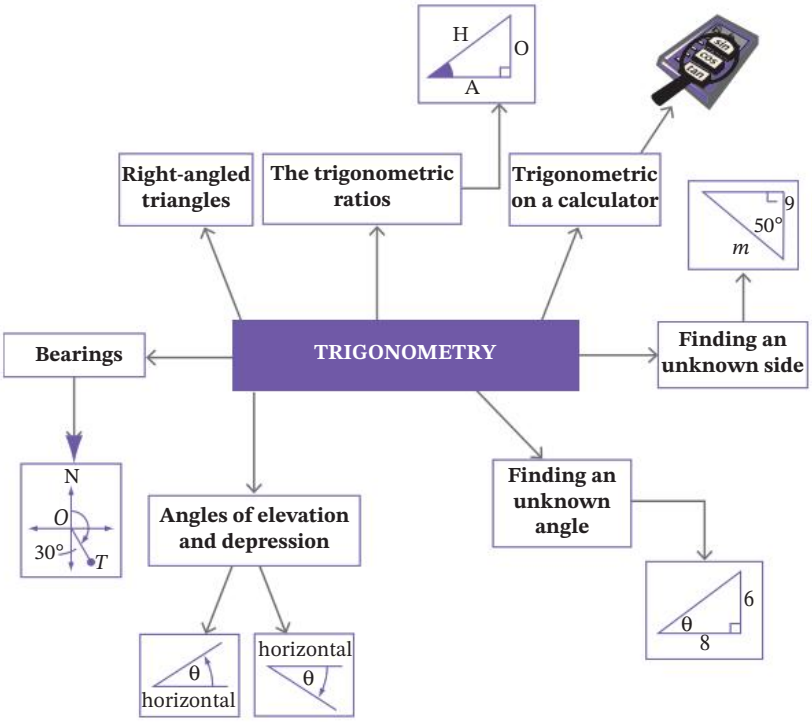
5 The word **minute** has an alternative pronunciation and meaning. What is its alternative meaning?

6 What does **inverse** mean and how is it used in trigonometry?

Topic summary

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.

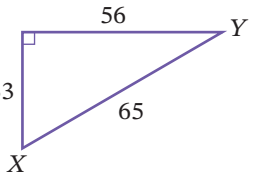
Mind map: Trigonometry (Advanced)



TEST YOURSELF 4

ANSWERS ON P. 576

4.02 **1** For this triangle, write as a fraction:



a $\sin Y$ **b** $\tan Y$

c $\sin X$ **d** $\cos X$

4.02 **2** If $\sin \alpha = \frac{36}{85}$, find the values of $\cos \alpha$ and $\tan \alpha$ as fractions. (Draw a diagram.)

4.04 **3** Round each angle to the nearest degree.

a 64° 27′ **b** 25° 43′ **c** 12° 8′ 50″

4.04 **4** Round each angle to the nearest minute.

a 50° 19′ 26″ **b** 31° 55′ 55″ **c** 64° 18′ 30″

4.04 **5** Evaluate each expression, correct to 4 decimal places.

a $\cos 32°$ **b** $\sin 50° 9'$ **c** $\tan 8° 45'$

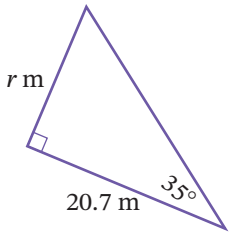
d $200 \tan 18°$ **e** $14 \sin 87° 40'$ **f** $\frac{13}{\cos 18° 27'}$

4.04 **6** Convert each angle size to degrees and minutes, correct to the nearest minute.

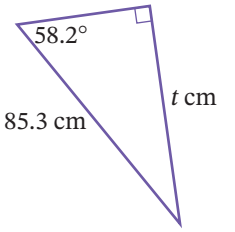
a 45.8° **b** 33.175° **c** 5.346°

4.05 **7** Find the value of each variable, correct to 2 decimal places.

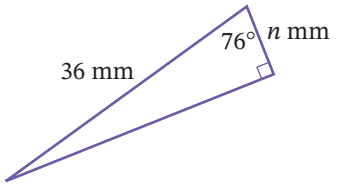
a



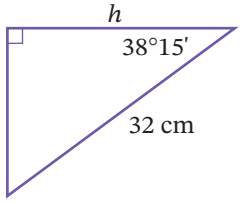
b



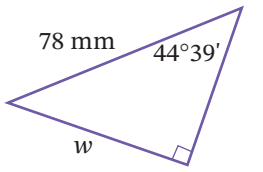
c



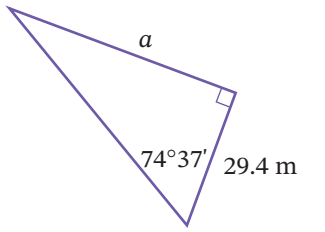
d



e

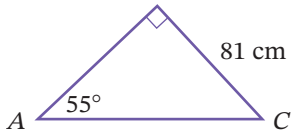


f



4.06 **8** For each triangle, find the length of side AC, correct to one decimal place.

a



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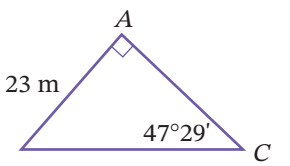

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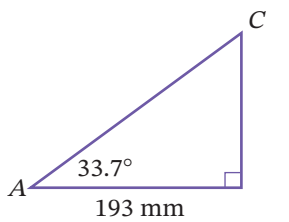

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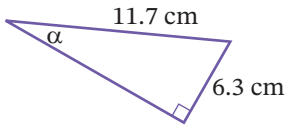
9 Find θ, correct to the nearest minute. 4.07

a $\tan\theta = 2.57$ **b** $\cos\theta = \frac{4}{7}$ **c** $\sin\theta = \frac{1.5}{1.6}$

10 Find the size of angle α, correct to the nearest degree. 4.07

a



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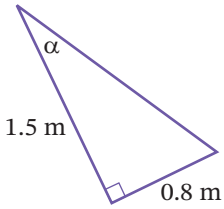

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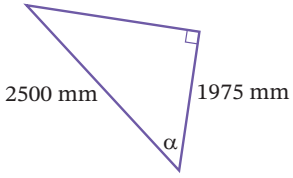
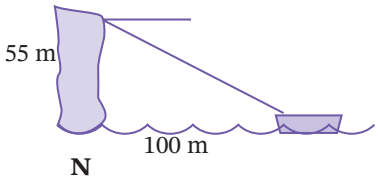


11 In $\triangle AEC$, $\angle C = 90°$, $CE = 3.9$ m and $AE = 4.2$ m. Find $\angle A$, correct to the nearest minute. 4.07

12 The angle of elevation of a tower roof is 26° at a point 400 m from its base. Find the height of the tower correct to the nearest metre. 4.08

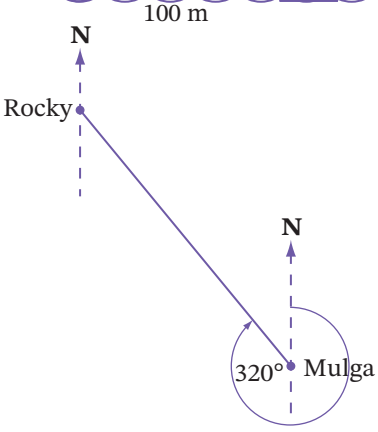
13 Find the angle of depression (correct to the nearest degree) of a boat that is 100 m from the base of a 55 m cliff. 4.08



14 What is the bearing of: 4.09

a Rocky from Mulga?

b Mulga from Rocky?



15 Two planes leave an airport at the same time. The first travels on a bearing of 063° at 500 km/h. The second travels on a bearing of 153° at 400 km/h. 4.10

a How far apart are the planes after 2 hours, to the nearest km?

b Calculate, correct to the nearest degree, the bearing of the first plane from the second plane.

16 A boat sails from Perth on a compass bearing of SSW for 12 nautical miles and then changes direction to a bearing of WNW and sails a further 42 nautical miles. 4.10

a How far is the boat from Perth?

b What is the bearing of the boat from Perth?

c On what three-figure bearing will it have to sail on its way directly back to Perth?

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5

NUMBER AND ALGEBRA

INDICES

The speed of light is about 300 000 000 metres per second. In one year, light travels approximately 9 460 000 000 000 km and this distance is called a light year. light from the stars travels for many years before it is seen on Earth. The Milky Way contains over 100 billion stars and the approximate diameter of our solar system is 10 000 000 000 km. On a smaller scale, the mass of a human cell is about 0.000 000 000 001 kg. Powers or indices provide a way to work easily with very large and very small numbers.

Chapter outline

	Working mathematically				
5.01 Multiplying and dividing terms with the same base	U	F		R	
5.02 Power of a power	U	F		R	
5.03 Powers of products and quotients	U	F		R	
5.04 The zero index	U	F		R	
5.05 Negative indices	U	F		R	C
5.06 Fractional indices $\frac{1}{n}$*	U	F		R	C
5.07 Fractional indices $\frac{m}{n}$*	U	F		R	C
5.08 Summary of the index laws	U	F		R	C
5.09 Significant figures	U	F		R	C
5.10 Scientific notation	U	F		R	C
5.11 Scientific notation on a calculator	U	F	PS	R	C

*STAGE 5.3

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

Wordbank

base A number that is raised to a power, meaning it is multiplied by itself repeatedly, for example, in 2^5, the base is 2.

index laws Rules for simplifying algebraic expressions involving powers of the same base, for example, $a^m \div a^n = a^{m-n}$.

index notation A way of writing repeated multiplication using indices (powers), in the form a^n, for example, $2 \times 2 \times 2 \times 2 \times 2$ in index notation is 2^5.

negative power A power that is a negative number, as in the term 3^{-2}.

power (or index or exponent) The number of times a base appears in a repeated multiplication, for example, in 2^5, the power is 5.

scientific notation A shorter way of writing very large or very small numbers using powers of 10. For example, 9 460 000 000 000 in scientific notation is 9.46×10^{12}.

significant figures Meaningful digits in a numeral that tell 'how many'. For example, 28 000 000 has 2 significant figures: 2 and 8.

Powers, indices and exponents

In this chapter you will:

- apply index laws to numerical and algebraic expressions with integer indices
- interpret and use zero and negative integer indices
- (STAGE 5.3) interpret and use fractional indices
- round numbers to a significant figures
- interpret, write and order numbers in scientific notation
- interpret and use scientific notation on a calculator
- solve problems involving scientific notation

SkillCheck ANSWERS ON P. 576

Powers review

Indices

1 For each term:

i state the **base**

ii state the index

iii write the expression in words.

a 3^5 **b** 4^8 **c** h^5 **d** 7^y

2 Evaluate each product.

a 5×5 **b** $7 \times 7 \times 7$ **c** $10 \times 10 \times 10 \times 10$

d $2 \times 2 \times 2 \times 2 \times 2 \times 2$ **e** $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$ **f** 9^2

g 3^4 **h** 6^3 **i** 4^4

j $\left(\frac{1}{2}\right)^3$ **k** $\left(\frac{3}{4}\right)^2$ **l** 5^3

3 Express each repeated multiplication in index notation.

a $2 \times 2 \times 2 \times 2 \times 2$ **b** $3 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7$

c $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 8 \times 8$ **d** $\frac{3}{7} \times \frac{3}{7} \times \frac{3}{7} \times \frac{3}{7}$

e $10 \times x \times x \times x \times x \times x \times x$ **f** $6 \times 6 \times 6 \times k \times k$

g $x \times y \times x \times x \times y \times x$ **h** $a \times b \times b \times b \times a$

i $5 \times n \times 5 \times n \times n$ **j** $q \times p \times q \times p \times q \times q$

4 Write each term in expanded form.

a 9^3 **b** 7^2 **c** d^5 **d** k^2 **e** $\left(\frac{4}{5}\right)^3$

5 Evaluate each expression.

a $4^2 \times 4^3$ **b** $10^6 \div 10^2$ **c** $(3^3)^2$ **d** 6^0

e 9^1 **f** $5^5 \times 5$ **g** $2^4 \div 2$ **h** $(-8)^2$

6 For each equation, find the missing power.

a $8 = 2^{\square}$ **b** $81 = 3^{\square}$ **c** $216 = 6^{\square}$ **d** $144 = 12^{\square}$

e $4096 = 4^{\square}$ **f** $2401 = 7^{\square}$ **g** $64 = 2^{\square}$ **h** $625 = 5^{\square}$

Investigation

Multiplying and dividing terms with powers

1 Write each expression in expanded form, then evaluate it.

a **i** $2^2 \times 2^3$ **ii** 2^5

b **i** $3^4 \times 3^3$ **ii** 3^7

c **i** $4^3 \times 4^3$ **ii** 4^6

d **i** $5^5 \times 5^3$ **ii** 5^8

2 What do you notice about each pair of answers in question **1**?

3 Is it true that $2^4 \times 2^6 = 2^{10}$? Give a reason for your answer.

4 Determine whether each equation is true (T) or false (F). Justify your answer.

a $2^5 \times 2^5 = 2^{10}$ **b** $6^3 \times 6^7 = 6^{21}$

c $4^3 \times 4^9 = 4^{27}$ **d** $3^5 \times 3^{10} = 3^{15}$

5 Write in words and as a formula the rule for multiplying a^m and a^n, 2 terms with the same base.

6 Use the rule to copy and complete each equation.

a $5^4 \times 5^2 = 5 \ldots$ **b** $4^5 \times 4^3 = 4 \ldots$ **c** $10^5 \times 10^7 = \ldots$

d $9^3 \times 9^2 = \ldots$ **e** $n^3 \times n^8 = \ldots$ **f** $p^3 \times p^7 = \ldots$

7 Use a calculator to evaluate each expression.

a **i** $3^6 \div 3^3$ **ii** 3^3

b **i** $2^8 \div 2^6$ **ii** 2^2

c **i** $5^8 \div 5^3$ **ii** 5^5

d **i** $10^8 \div 10^4$ **ii** 10^4

8 What do you notice about each pair of answers in question **7**?

9 Is it true that $4^8 \div 4^6 = 4^2$? Give a reason for your answer.

10 Use a calculator to determine whether each equation is true (T) or false (F).

a $3^{10} \div 3^6 = 3^4$ **b** $4^8 \div 4^2 = 4^4$

c $2^{12} \div 2^3 = 2^4$ **d** $6^{10} \div 6^5 = 6^5$

11 Write in words and as a formula the rule for dividing a^m and a^n, 2 terms with the same base.

12 Use the rule to copy and complete each equation.

a $2^6 \div 2^3 = 2^{\cdots}$ **b** $10^8 \div 10^6 = 10^{\cdots}$ **c** $3^7 \div 3^2 =$

d $4^{11} \div 4^6 = \ldots$ **e** $x^8 \div x^5 = \ldots$ **f** $g^{12} \div g^{10} = \ldots$

5.01 Multiplying and dividing terms with the same base

Simplifying with the index laws

Consider $5^4 \times 5^3 = (5 \times 5 \times 5 \times 5) \times (5 \times 5 \times 5)$

$= 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

$= 5^7$

$\therefore 5^4 \times 5^3 = 5^{4+3}$

$= 5^7$

Multiplying terms with the same base

When multiplying terms with the same base, add the powers:

$$a^m \times a^n = a^{m+n}$$

The rule above is called an **index law**. **Index** is another name for **power**. The plural of index is **indices** (pronounced 'in-de-sees').

Proof:

$$a^m \times a^n = \underbrace{a \times a \times \cdots \times a}_{m \text{ factors}} \times \underbrace{a \times a \times \cdots \times a}_{n \text{ factors}}$$

$$= \underbrace{a \times a \times \cdots \times a}_{(m+n) \text{ factors}}$$

$$= a^{m+n}$$

Example 1

Simplify each expression, writing the answer in **index notation**.

a $8^4 \times 8^5$ **b** 10×10^3 **c** $d^3 \times d^5$

d $4m^2 \times 3m^6$ **e** $3r^2t \times 6rt^3$

Solution

a $8^4 \times 8^5 = 8^{4+5}$

$= 8^9$

b $10 \times 10^3 = 10^1 \times 10^3$

$= 10^{1+3}$

$= 10^4$

c $d^3 \times d^5 = d^{3+5}$

$= d^8$

d $4m^2 \times 3m^6 = (4 \times 3) \times (m^2 \times m^6)$

$= 12m^{2+6}$ Multiply the numbers and add the indices

$= 12m^8$

e $3r^2t \times 6rt^3 = 3r^2t^1 \times 6r^1t^3$ $3r^2t = 3r^2t^1$ and $6rt^3 = 6r^1t^3$

$= (3 \times 6) \times (r^2 \times r^1) \times (t^1 \times t^3)$

$= 18r^{2+1}t^{1+3}$ Multiply the numbers and add the indices

$= 18r^3t^4$

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Consider $5^6 \div 5^4 = \dfrac{5^6}{5^4}$

$= \dfrac{\cancel{5}\times\cancel{5}\times\cancel{5}\times\cancel{5}\times 5\times 5}{\cancel{5}\times\cancel{5}\times\cancel{5}\times\cancel{5}}$

$= 5\times 5$

$= 5^2$

$\therefore 5^6 \div 5^4 = 5^{6-4}$

$= 5^2$

Dividing terms with the same base

When dividing terms with the same base, subtract the powers:

$$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$$

This is another **index law**.

Proof:

$a^m \div a^n = \dfrac{a^m}{a^n}$

$= \dfrac{\cancel{a}\times\cancel{a}\times a\times a\times a\times\cdots\times a}{\cancel{a}\times\cancel{a}\times a\times\cdots\times a}$ $\left(\dfrac{m \text{ factors}}{n \text{ factors}}\right)$

$= a\times a\times \ldots \times a$ $(m-n)$ factors

$= a^{m-n}$

Example 2

Simplify each expression, writing the answer in index notation.

a $8^5 \div 8^3$ **b** $\dfrac{10^8}{10}$ **c** $d^{20} \div d^4$

d $20w^{10} \div 5w^2$ **e** $\dfrac{8x^3y^7}{24x^2y}$

Solution

a $8^5 \div 8^3 = 8^{5-3}$

$= 8^2$

b $\dfrac{10^8}{10} = \dfrac{10^8}{10^1}$

$= 10^{8-1}$

$= 10^7$

c $d^{20} \div d^4 = d^{20-4}$

$= d^{16}$

d $20w^{10} \div 5w^2 = \dfrac{\cancel{20}^{4}\, w^{10}}{\cancel{5}^{1}\, w^2}$

$= 4w^{10-2}$

$= 4w^8$

Divide the numbers and subtract the indices

e $\dfrac{8x^3y^7}{24x^2y^1} = \dfrac{\cancel{8}^{1}x^{3-2}y^{7-1}}{\cancel{24}_{3}}$

$= \dfrac{xy^6}{3}$

Divide the numbers and subtract the indices

Example 3

Simplify each expression, writing the answer in index notation.

a $5^3 \times 5^7 \times 5^4$ **b** $2^7 \times 2^3 \div 2^5$ **c** $m^7 \times m^6 \div m^4$

d $3e^5 \times 2e^4 \times 5e^2$ **e** $a^{12} \div a^4 \div a^5$ **f** $12y^4 \times 4y^3 \div 20y^5$

Solution

a $5^3 \times 5^7 \times 5^4 = 5^{3+7+4}$
$= 5^{14}$

b $2^7 \times 2^3 \div 2^5 = 2^{10} \div 2^5$
$= 2^5$

c $m^7 \times m^6 \div m^4 = m^{13} \div m^4$
$= m^9$

d $3e^5 \times 2e^4 \times 5e^2 = (3 \times 2 \times 5) \times (e^5 \times e^4 \times e^2)$
$= 30e^{11}$

e $a^{12} \div a^4 \div a^5 = a^{12-4-5}$
$= a^3$

f $12y^4 \times 4y^3 \div 20y^5 = \dfrac{12y^4 \times 4y^3}{20y^5}$
$= \dfrac{48y^{7-5}}{20}$
$= \dfrac{12y^2}{5}$

Divide the numbers and subtract indices

EXERCISE 5.01 ANSWERS ON P. 576

Multiplying and dividing terms with the same base U F R

EXAMPLE 1

1 Which expression is equal to $5^{12} \times 5^3$? Select the correct answer **A**, **B**, **C** or **D**.

A 5^9 **B** 5^{15} **C** 25^{15} **D** 25^{36}

2 Simplify each expression, writing the answer in index notation. R

a $10^3 \times 10^2$ **b** 2×2^4 **c** $3^2 \times 3^5$

d $7^4 \times 7$ **e** $8 \times 8^3 \times 8^4$ **f** $5^4 \times 5 \times 5^4$

g $6 \times 6^2 \times 6^3 \times 6^4$ **h** $4^4 \times 4^4 \times 4^4$ **i** $3^4 \times 3^0 \times 3^7$

j $x \times x^4$ **k** $g^4 \times g^4$ **l** $w^7 \times w$

m $b^3 \times b^{10}$ **n** $p^{10} \times p^{10}$ **o** $r \times r$

p $y^5 \times y^7$ **q** $m^{15} \times m^4$ **r** $n^8 \times n^2$

3 Which expression is equal to $10^4 \times 10$? Select **A**, **B**, **C** or **D**.

A 100^5 **B** 100^4 **C** 10^4 **D** 10^5

EXAMPLE 2

4 Which expression is equal to $5^{12} \div 5^3$? Select **A**, **B**, **C** or **D**.

A 5^4 **B** 5^9 **C** 1^4 **D** 1^9

5 Simplify each expression, writing the answer in index notation. R

a $10^7 \div 10^5$ **b** $8^5 \div 8$ **c** $20^{15} \div 20^5$ **d** $\dfrac{5^8}{5^2}$

e $\dfrac{9^{12}}{9^3}$ **f** $\dfrac{2^{27}}{2^3}$ **g** $7^4 \div 7^3$ **h** $\dfrac{2^{20}}{2}$

Foundation Standard Complex

i $11^4 \div 11^4$ **j** $p^{15} \div p^{10}$ **k** $n^7 \div n$ **l** $w^{24} \div w^6$

m $\frac{h^{20}}{h^4}$ **n** $\frac{y^8}{y^2}$ **o** $\frac{a^{12}}{a^4}$ **p** $b^{16} \div b^{15}$

q $\frac{w^{25}}{w}$ **r** $m^{16} \div m^{16}$

6 Which expression is equal to $10^4 \div 10$? Select **A**, **B**, **C** or **D**.

A 10^4 **B** 1^4 **C** 1^3 **D** 10^3

7 Which expression is equal to $6^7 \times 6^2 \times 6^4$? Select **A**, **B**, **C** or **D**.

A 216^{13} **B** 6^{56} **C** 6^{13} **D** 216^{56}

8 Which expression is equal to $2^7 \div 2^2 \times 2^4$?

A 2^9 **B** 2^{13} **C** 8^{13} **D** 4^9

9 Simplify each expression. R

a $3p^2 \times 2p^5$ **b** $4y^{10} \times 3y^2$ **c** $6m \times 3m^8$

d $h^3 \times 5h^8$ **e** $3q \times 8q^8$ **f** $2a^2 \times 5a^5$

g $5n^8t \times 6n^8t^4$ **h** $2ab^3 \times 15ab$ **i** $3e^4g^3 \times e^6g^2$

j $8p^4m^5 \times 4p^3m^5$ **k** $16qr^8 \times 3q^7$ **l** $9u^3v \times 6uv^2w^8$

10 Simplify each expression. R

a $10y^{15} \div 5y^3$ **b** $20w^9 \div 4w^3$ **c** $24r^8 \div 3r$

d $\frac{30x^4}{x^3}$ **e** $\frac{10m^{10}}{2m}$ **f** $\frac{4g^{12}}{8g^6}$

g $14d^4h^{10} \div 7hd^2$ **h** $15x^6y^8 \div 15xy^4$ **i** $6e^{25}d^{40} \div 18e^5d^4$

j $\frac{12q^5t^4}{16q^4t^3}$ **k** $\frac{45a^{10}b^8}{5a^5}$ **l** $\frac{36pq^3r^5}{24qr}$

11 Which expression is equal to $5d^7 \times 6d^2 \div 15d^4$? Select **A**, **B**, **C** or **D**.

A $2d^{13}$ **B** $2d^5$ **C** $15d^5$ **D** $5d^5$

12 Simplify each expression, writing the answer in index notation. R

a $3^3 \times 3^5 \times 3^2$ **b** $10^{15} \div 10^7 \div 10^2$ **c** $7^6 \times 7^8 \div 7^{10}$

d $8^9 \div 8^8 \times 8^4$ **e** $a^5 \times a^5 \times a^4$ **f** $m^3 \times m^8 \div m^{10}$

g $w^9 \div w^6 \times w^{10}$ **h** $x \times x^7 \div x^4$ **i** $p^{12} \div p \div p^6$

j $2c^5 \times 5c^8 \times 3c$ **k** $6y^4 \times 4y^8 \div 8y^4$ **l** $5d^2 \times 12d^4 \times 2d^5$

m $8q^7 \times 3q^3 \div 12q^8$ **n** $24g^{11} \div 8g^8 \div g$ **o** $6h^{10} \div h^8 \times 5h^{10}$

p $3v^5 \times 6v^4 \div 12v^6$ **q** $8b^6 \times 2b \div 10b^5$ **r** $24k^5 \div 6k^2 \div 2k^2$

13 Which expression is equal to $4p^3y^5 \times 5p^7y^3 \div 8p^6y^6$? Select **A**, **B**, **C** or **D**.

A $160p^4y^2$ **B** $\frac{20p^4y}{8}$ **C** $\frac{5p^4y}{2}$ **D** $\frac{5p^4y^2}{2}$

14 Simplify each expression, writing the answer in index notation.

a $a^{-3} \times a^3$ **b** $y \times y^{-2}$ **c** $n^{\frac{1}{2}} \times n^{\frac{1}{2}}$

d $h^{-1} \times h^{-4}$ **e** $b^4 \div b^4$ **f** $\frac{r^3}{r^5}$

5.01

EXAMPLE 3

Foundation Standard Complex

Investigation

Powers of powers

1 Write each expression in expanded form, then evaluate it.

a **i** $(2^3)^2$ **ii** 2^6

b **i** $(3^4)^3$ **ii** 3^{12}

c **i** $(5^2)^3$ **ii** 5^6

d **i** $(2^5)^4$ **ii** 2^{20}

2 What do you notice about each pair of answers in question **1**?

3 Is it true that: $(2^7)^3 = 2^{21}$? Give a reason for your answer.

4 Determine whether each equation is true (T) or false (F). Justify your answer.

a $(3^5)^3 = 3^{15}$ **b** $(2^3)^2 = 2^5$ **c** $(2^{10})^4 = 2^{14}$

d $(4^2)^5 = 4^{10}$ **e** $(3^3)^6 = 3^{18}$ **f** $(5^2)^4 = 5^6$

5 Write in words and as a formula the rule for raising a^m to a power of n, that is, $(a^m)^n$.

6 Use the rule to copy and complete each equation.

a $(3^7)^2 = 3^{\cdots}$ **b** $(5^2)^6 = 5^{\cdots}$ **c** $(4^5)^2 = 4^{\cdots}$

d $(a^3)^4 = a^{\cdots}$ **e** $(8^3)^7 = \ldots$ **f** $(k^4)^6 = \ldots$

5.02 Power of a power

Simplifying with the index laws

Indices puzzle

Consider $(5^3)^4 = 5^3 \times 5^3 \times 5^3 \times 5^3$

$= (5 \times 5 \times 5) \times (5 \times 5 \times 5) \times (5 \times 5 \times 5) \times (5 \times 5 \times 5)$

$= 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

$= 5^{12}$

$\therefore (5^3)^4 = 5^{3 \times 4}$

$= 5^{12}$

Power of a power

When raising a term with a power to another power, multiply the powers:

$$(a^m)^n = a^{m \times n}$$

Proof:

$$(a^m)^n = \underbrace{a^m \times a^m \times \cdots \times a^m}_{n \text{ factors}}$$

$$= \underbrace{\underbrace{a \times a \times \cdots \times a}_{m \text{ factors}} \times \underbrace{a \times a \times \cdots \times a}_{m \text{ factors}} \times \cdots \times \underbrace{a \times a \times \cdots \times a}_{m \text{ factors}}}_{n \text{ lots of } m \text{ factors}}$$

$$= a^{m \times n}$$

Example 4

Simplify each expression, writing the answer in index notation.

a $(8^5)^2$ **b** $(d^3)^5$ **c** $(2g)^4$

d $(5v^4)^3$ **e** $(-n)^6$ **f** $(-3t^4)^3$

Solution

a $(8^5)^2 = 8^{5\times 2}$
$= 8^{10}$

b $(d^3)^5 = d^{3\times 5}$
$= d^{15}$

c $(2g)^4 = 2^4 \times g^4$
$= 16g^4$

d $(5v^4)^3 = 5^3 \times (v^4)^3$
$= 125 \times v^{4\times 3}$
$= 125v^{12}$

e $(-n)^6 = (-1)^6 \times n^6$
$= 1 \times n^6$
$= n^6$

f $(-3t^4)^3 = (-3)^3 \times (t^4)^3$
$= -27 \times t^{4\times 3}$
$= -27t^{12}$

EXERCISE 5.02 ANSWERS ON P. 577

Power of a power U F R

EXAMPLE 4

1 Which expression is equal to $(10^3)^3$? Select the correct answer **A**, **B**, **C** or **D**.

A 30^3 **B** 1000^9 **C** 10^9 **D** 10^6

2 Simplify each expression, writing the answer in index notation. R

a $(4^3)^2$ **b** $(5^2)^8$ **c** $(3^3)^4$ **d** $(2^7)^4$ **e** $(2^1)^2$ **f** $(9)^3$

g $(10^0)^2$ **h** $(6^4)^5$ **i** $(5^3)^5$ **j** $(e^2)^4$ **k** $(t^5)^5$ **l** $(y^3)^7$

m $(c^1)^5$ **n** $(m^7)^5$ **o** $(y^4)^4$ **p** $(h^0)^6$ **q** $(q^6)^3$ **r** $(w^4)^1$

s $(2x)^{10}$ **t** $(5n^3)^8$ **u** $(4d^3)^3$ **v** $(-k^5)^9$ **w** $(-d^3)^4$ **x** $(2a^8)^8$

3 Which expression is equal to $(-3)^5$? Select **A**, **B**, **C** or **D**.

A -3^6 **B** -3^5 **C** 3^5 **D** -15

4 Simplify each expression. R

a $(2d^3)^4$ **b** $(5m^3)^2$ **c** $(4y^5)^2$ **d** $(3x^2)^4$

e $(5u^6)^5$ **f** $(2w^5)^3$ **g** $(10d^5)^4$ **h** $(3e)^3$

i $(2b^4)^1$ **j** $(6d^6)^2$ **k** $(3f^4)^5$ **l** $(2c^3)^{10}$

m $(-2r)^4$ **n** $(-5t)^3$ **o** $(-3m^3)^2$ **p** $(-y^3)^{12}$

q $(-x)^3$ **r** $(-m^3)^{10}$ **s** $(-4w^5)^4$ **t** $(-3f)^5$

u $(-3p^2)^3$ **v** $(-3h^5)^4$ **w** $(-10k)^2$ **x** $(-8y^3)^1$

5 Which expression is equal to $(-4m^3)^3$? Select **A**, **B**, **C** or **D**.

A $-64m^9$ **B** $-64m^6$ **C** $64m^6$ **D** $64m^9$

6 Simplify each expression. R

a $\left(d^{-1}\right)^3$ **b** $\left(x^2\right)^{-2}$ **c** $\left(w^{\frac{1}{2}}\right)^{\frac{1}{2}}$ **d** $\left(z^{-2}\right)^{-1}$ **e** $\left(u^{\frac{1}{2}}\right)^2$ **f** $\left(p^{-1}\right)^{-1}$

5.03 Powers of products and quotients

Simplifying with the index laws

Consider $(2 \times 5)^3 = (2 \times 5) \times (2 \times 5) \times (2 \times 5)$
$= 2 \times 2 \times 2 \times 5 \times 5 \times 5$
$= 2^3 \times 5^3$
$\therefore (2 \times 5)^3 = 2^3 \times 5^3$

Powers of products

When raising a product of terms to a power, raise each term to that power:

$$(ab)^n = a^n b^n$$

Proof:

$$(ab)^n = \underbrace{ab \times ab \times \cdots \times ab}_{n \text{ factors}}$$
$$= \underbrace{a \times a \times \cdots \times a}_{n \text{ factors}} \times \underbrace{b \times b \times \cdots \times b}_{n \text{ factors}}$$
$$= a^n b^n$$

Example 5

Simplify each expression.

a $(-2gh^2)^5$ **b** $(p^3q^4)^2$ **c** $(a^5x^2y)^3$

Solution

a $(-2gh^2)^5 = (-2)^5 \times g^5 \times (h^2)^5$
$= -32 \times g^5 \times h^{2 \times 5}$
$= -32g^5h^{10}$

b $(p^3q^4)^2 = (p^3)^2 \times (q^4)^2$
$= p^{3 \times 2} \times q^{4 \times 2}$
$= p^6q^8$

c $(a^5x^2y)^3 = (a^5)^3 \times (x^2)^3 \times y^3$
$= a^{5 \times 3} \times x^{2 \times 3} \times y^3$
$= a^{15}x^6y^3$

Consider $\left(\frac{5}{8}\right)^6 = \frac{5}{8} \times \frac{5}{8} \times \frac{5}{8} \times \frac{5}{8} \times \frac{5}{8} \times \frac{5}{8}$
$= \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5}{8 \times 8 \times 8 \times 8 \times 8 \times 8}$
$= \frac{5^6}{8^6}$
$\therefore \left(\frac{5}{8}\right)^6 = \frac{5^6}{8^6}$

Powers of quotients

When raising a quotient of terms to a power, raise each term to that power:

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Proof:

$$\left(\frac{a}{b}\right)^n = \underbrace{\frac{a}{b}\times\frac{a}{b}\times\cdots\times\frac{a}{b}}_{n \text{ factors}}$$

$$= \frac{a\times a\times\cdots\times a\ (n \text{ factors})}{b\times b\times\cdots\times b\ (n \text{ factors})}$$

$$= \frac{a^n}{b^n}$$

Example 6

Simplify each expression.

a $\left(\frac{7c}{d}\right)^2$ **b** $\left(\frac{4k^2}{5}\right)^3$ **c** $\left(\frac{6m^5p^3}{m^2p}\right)^4$ **d** $\frac{2x^4y^8}{(xy^2)^3}$

Solution

a
$$\left(\frac{7c}{d}\right)^2 = \frac{(7c)^2}{d^2}$$
$$= \frac{7^2c^2}{d^2}$$
$$= \frac{49c^2}{d^2}$$

b
$$\left(\frac{4k^2}{5}\right)^3 = \frac{(4k^2)^3}{5^3}$$
$$= \frac{4^3(k^2)^3}{125}$$
$$= \frac{64k^6}{125}$$

c
$$\left(\frac{6m^5p^3}{m^2p}\right)^4 = \frac{(6m^5p^3)^4}{(m^2p)^4}$$
$$= \frac{6^4m^{5\times4}p^{3\times4}}{m^{2\times4}p^4}$$
$$= \frac{1296m^{20}p^{12}}{m^8p^4}$$
$$= 1296m^{20-8}p^{12-4}$$
$$= 1296m^{12}p^8$$

OR
$$\left(\frac{6m^5p^3}{m^2p}\right)^4 = (6m^{5-2}p^{3-1})^4$$
$$= (6m^3p^2)^4$$
$$= 6^4m^{3\times4}p^{2\times4}$$
$$= 1296m^{12}p^8$$

d
$$\frac{2x^4y^8}{(xy^2)^3} = \frac{2x^4y^8}{x^3(y^2)^3}$$
$$= \frac{2x^4y^8}{x^3y^6}$$
$$= 2x^{4-3}y^{8-6}$$
$$= 2xy^2$$

EXERCISE 5.03 ANSWERS ON P. 577

Powers of products and quotients U F R

1 Which expression is equal to $(-4 \times 5)^2$? Select the correct answer **A**, **B**, **C** or **D**.

EXAMPLE 5

A 16×25 **B** -16×25 **C** -8×10 **D** 8×10

2 Simplify each expression. R

a $(ab)^3$ **b** $(x^2y)^5$ **c** $(l^3m^5)^6$ **d** $(6dp^2)^4$

e $(-8k^4y^5)^2$ **f** $(3m^2n)^5$ **g** $(ek^3)^3$ **h** $(-w^3x^4)^7$

i $(-8d^3y^5)^2$ **j** $(4b^2c^3)^4$ **k** $(-3a^3d)^3$ **l** $(2p^2q^3)^4$

Foundation Standard Complex

3 Express each of the following in simplest form. R

a $(x^2yw^3)^2$ **b** $(h^2y^3m^5)^4$ **c** $(3d^7k^2p^3)^2$

d $(5a^3q^3r^4)^3$ **e** $(2d^4ew^3)^5$ **f** $(km^3p^2x^6)^4$

EXAMPLE 6

4 Which expression is equal to $\left(-\frac{3}{4}\right)^3$? Select **A**, **B**, **C** or **D**.

A $-\frac{9}{12}$ **B** $\frac{9}{12}$ **C** $\frac{27}{64}$ **D** $-\frac{27}{64}$

5 Simplify each expression. R

a $\left(\frac{6}{7}\right)^2$ **b** $\left(\frac{m}{2}\right)^3$ **c** $\left(\frac{5}{x}\right)^4$ **d** $\left(\frac{2n^5}{p}\right)^8$

e $\left(\frac{w^2}{t^3}\right)^5$ **f** $\left(\frac{m^2}{4n}\right)^4$ **g** $\left(-\frac{2}{3}\right)^4$ **h** $\left(-\frac{5h}{6}\right)^3$

i $\left(\frac{7k^4}{10}\right)^2$ **j** $\left(\frac{3r}{t^2}\right)^2$ **k** $\left(\frac{a^2b}{d^5}\right)^4$ **l** $\left(-\frac{2}{3c^2}\right)^5$

6 Simplify each expression. R

a $(2x^{10}y^{15})^3 \times 5x^2y^3$ **b** $(2x^{10}y^{15} \times 5x^2y^3)^3$ **c** $18q^5r^8 \div (3qr^2)^2$

d $(18q^5r^8 \div 3qr^2)^2$ **e** $\left(\frac{3a^5x^6}{ax}\right)^3$ **f** $\frac{3a^5x^6}{(ax)^4}$

g $(4p^3h^{10})^2 \times 2p^2h^9$ **h** $(4p^3h^{10})^2 \div 2p^2h^9$ **i** $(4p^3h^{10} \times 2p^2h^9)^2$

j $\left(\frac{2a^4d^5}{a^3d^2}\right)^3$ **k** $\left(\frac{7e^4y^2}{e^2y}\right)^2$ **l** $\frac{5x^7y^{10}}{(x^2y^3)^3}$

m $\frac{6h^9k^{12}}{(2h^2k^3)^3}$ **n** $(3m^5w^3)^4 \div 6m^9w^5$ **o** $\left(\frac{3c^5d^2}{c^2d}\right)^4$

Investigation

The power of 0

What is the value of a number raised to a power of 0, for example, 2^0?

1 Copy and complete each table of decreasing powers. Notice the pattern in your answers.

a

Power of 2	Number
2^5	32
2^4	16
2^3	
2^2	
2^1	
2^0	

b

Power of 3	Number
3^5	243
3^4	
3^3	
3^2	
3^1	
3^0	

Foundation Standard Complex

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2 Simplify each expression in index notation.

a	$3^4 \times 3^0$	**b**	$5^2 \times 5^0$	**c**	$2^0 \times 2^7$
d	$7^0 \times 7^3$	**e**	$4^5 \times 4^0$	**f**	$5^0 \times 5^7$
g	$2^5 \div 2^0$	**h**	$3^5 \div 3^0$	**i**	$4^2 \div 4^0$
j	$9^3 \div 9^0$	**k**	$5^6 \div 5^0$	**l**	$8^4 \div 8^0$

3 Any number will remain unchanged when multiplied by what?

4 Any number will remain unchanged when divided by what?

5 What is the answer when any number is raised to the power of 0, that is, a^0? Justify your answer.

The zero index

5.04

Index laws

Consider $5^3 \div 5^3 = \frac{5^3}{5^3}$

$= 1$ Any number divided by itself equals 1.

But also $5^3 \div 5^3 = 5^{3-3}$

$= 5^0$

$\therefore 5^0 = 1$.

Power of 0

Any number raised to the power of 0 is equal to 1.

$$a^0 = 1$$

Proof:

$a^m \div a^m = 1$ Any number divided by itself equals 1.

But also $a^m \div a^m = a^{m-m}$

$= a^0$

$\therefore\ a^0 = 1$.

Example 7

Simplify each expression.

a	11^0	**b**	$(-8)^0$	**c**	g^0
d	$(3r)^0$	**e**	$3r^0$	**f**	-8^0

Solution

a $11^0 = 1$

b $(-8)^0 = 1$

c $g^0 = 1$

d $(3r)^0 = 1$

e $3r^0 = 3 \times r^0$
$= 3 \times 1$
$= 3$

f $-8^0 = -1 \times 8^0$
$= -1 \times 1$
$= -1$

EXERCISE 5.04 ANSWERS ON P. 577

The zero index U F R

EXAMPLE 7

1 Simplify each expression. R

a	2^0	**b**	$(-2)^0$	**c**	-2^0	**d**	$(-m)^0$
e	$-m^0$	**f**	$(4a)^0$	**g**	$\left(\frac{2}{3}\right)^0$	**h**	$7x^0$
i	-1000^0	**j**	$(p+3)^0$	**k**	$\left(\frac{p}{3}\right)^0$	**l**	$-2b^0$
m	$(9k)^0$	**n**	$(x^2y)^0$	**o**	$(xyw)^0$	**p**	$(-ab)^0$
q	$(6r)^0$	**r**	$-(6r)^0$	**s**	$-6r^0$	**t**	$6(-r)^0$
u	$(cd)^0$	**v**	$-(7x^2)^0$	**w**	$-3(a^2b^3)^0$	**x**	$(-5v^5w^4)^0$

2 Simplify each expression. R

a	$7^0 + 2^0$	**b**	$12^0 - 5^0$	**c**	$2m^0 + (2m)^0$
d	$5w^0 - (5w)^0$	**e**	$(3a)^0 + 6a^0$	**f**	$(8a)^0 - 6x^0$
g	$(5y)^0 - 4$	**h**	$(5y)^0 - 4^0$	**i**	$3^0 \times 5^0$
j	$3^2 \times 5^0$	**k**	$\left(\frac{3}{4}\right)^0 + \frac{3}{4}y^0$	**l**	$\left(\frac{1}{3}\right)^0 + \left(\frac{1}{3}d\right)^0$
m	$2w^0 \times 3p^0$	**n**	$12u^0 \div 3$	**o**	$(5d^0)^3$
p	$8b^0 - (3b^0)^2$	**q**	$\frac{12p^0}{(2p)^0}$	**r**	$6n^3 \div 2n^3$
s	$\frac{12q^5}{36q^5}$	**t**	$(3x^3)^3 \div x^9$	**u**	$(5h^4x^3)^0 - 4(h^3x)^0$
v	$4m^0 + 7$	**w**	$9c^0 + 2d^0$	**x**	$3 - 2(8h)^0$

Mental skills 5: Maths without calculators ANSWERS ON P. 577

Adding or multiplying in any order

Numbers can be added or multiplied in any order. We can use this property to make our calculations simpler.

1 Study each example.

a $19 + 5 + 5 + 1 = (19 + 1) + (5 + 5)$
$= 20 + 10$
$= 30$

b $13 + 8 + 20 + 27 + 80 = (13 + 27) + (20 + 80) + 8$
$= 40 + 100 + 8$
$= 148$

c $2 \times 36 \times 5 = (2 \times 5) \times 36$
$= 10 \times 36$
$= 360$

d $25 \times 11 \times 4 \times 7 = (25 \times 4) \times (11 \times 7)$
$= 100 \times 77$
$= 7700$

2 Now evaluate each sum.

a $45 + 16 + 45 + 4 + 7$ **b** $38 + 600 + 50 + 12 + 40$

c $18 + 91 + 9 + 20$ **d** $75 + 33 + 7 + 25$

e $24 + 16 + 80 + 44 + 10$ **f** $56 + 5 + 20 + 15 + 4$

g $100 + 36 + 200 + 10 + 90$ **h** $54 + 27 + 9 + 16 + 3$

i $70 + 50 + 30 + 25 + 25$ **j** $32 + 120 + 40 + 80 + 40$

3 Now evaluate each product.

a $8 \times 4 \times 5$ **b** $50 \times 7 \times 2$ **c** $3 \times 5 \times 6$

d $5 \times 11 \times 40$ **e** $12 \times 2 \times 3$ **f** $2 \times 4 \times 25 \times 8$

g $3 \times 20 \times 7 \times 5$ **h** $6 \times 8 \times 5 \times 2$ **i** $2 \times 3 \times 2 \times 11$

Investigation

Negative powers

What is the value of a number raised to a negative power, for example, 2^{-1} or 2^{-2}?

1 Copy and complete each table showing decreasing powers. Notice the pattern in your answers.

a

Power of 2	Number
2^3	8
2^2	4
2^1	
2^0	
2^{-1}	
2^{-2}	
2^{-3}	

b

Power of 10	Number
10^3	1000
10^2	
10^1	
10^0	
10^{-1}	
10^{-2}	
10^{-3}	

2 Copy and complete this table showing decreasing powers in expanded form. Notice the pattern in your answers.

a

Power of 5	Expanded form
3^3	$3 \times 3 \times 3$
3^2	3×3
3^1	3
3^0	1
3^{-1}	$\frac{1}{3}$
3^{-2}	$\frac{1}{3 \times 3} = \frac{1}{3^2}$
3^{-3}	$\frac{1}{3 \times 3 \times 3} = \frac{1}{3^3}$
3^{-4}	
3^{-5}	

b

Power of 5	Expanded form
5^3	$5 \times 5 \times 5$
5^2	
5^1	
5^0	
5^{-1}	
5^{-2}	
5^{-3}	
5^{-4}	
5^{-5}	

3 If $3^{-2} = \frac{1}{3^2}$ and $5^{-3} = \frac{1}{5^3}$, then write each negative power in a similar way.

a 4^{-1} **b** 7^{-4} **c** 2^{-6}

4 Simplify each expression in index notation.

a $10^4 \div 10^7$ **b** $2^3 \div 2^8$ **c** $3^4 \div 3^5$

d $5^2 \div 5^8$ **e** $a^4 \div a^6$ **f** $a \div a^4$

5 Consider $\frac{10^4}{10^7} = \frac{\cancel{10} \times \cancel{10} \times \cancel{10} \times \cancel{10}}{\cancel{10} \times \cancel{10} \times \cancel{10} \times \cancel{10} \times 10 \times 10 \times 10}$

$= \frac{1}{10 \times 10 \times 10}$

$= \frac{1}{10^3}$

But also $\frac{10^4}{10^7} = 10^{4-7} = 10^{-3}$

$\therefore 10^{-3} = \frac{1}{10^3}$

Use the method above to show that:

a $\frac{2^3}{2^8} = 2^{-5} = \frac{1}{2^5}$ **b** $\frac{3^4}{3^5} = 3^{-1} = \frac{1}{3}$

c $\frac{5^2}{5^8} = 5^{-2} = \frac{1}{5^6}$ **d** $\frac{a^4}{a^6} = a^{-2} = \frac{1}{a^2}$

6 Write in words and as a formula the rule for raising a to a negative power $-n$, that is, a^{-n}.

Technology

Negative powers

In this activity we will discover the pattern for negative powers. We will consider base values from 2 to 10 shown in column A and indices (powers) –1, –2 and –3 shown in row 1. On a spreadsheet, the symbol for power is ^ (called a **carat,** press **SHIFT 6**) . For example, 3^{-1} is entered as **3^–1.**

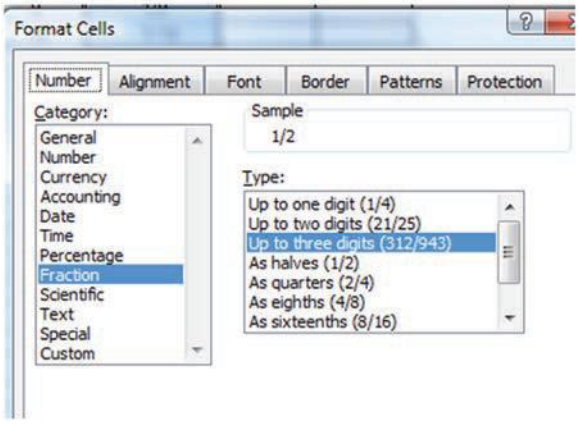
In this activity we will discover the pattern for negative powers. We will consider base values from 2 to 10 shown in column A and indices (powers) –1, –2 and –3 shown in row 1. On a spreadsheet, the symbol for power is ^ (called a **carat,** press **SHIFT 6**) . For example, 3^{-1} is entered as **3^–1.**

1 Create a spreadsheet as shown.

	A	B	C	D
1	**Index**	-1	-2	-3
2				
3	**Base**			
4	2			
5	3			
6	4			
7	5			
8	6			
9	7			
10	8			
11	9			
12	10			
13				

2 We will first examine the power of –1. In cell B4, enter **=A4^$B$1** to calculate 2^{-1}. $B$1 is an **absolute cell reference**, which ensures that the cell does not change when a formula is copied. This means that in column B, the power will always refer to cell B1 (–1) only. **Fill Down** from cell B4 to B12.

3 Use **Format cells** to set column B decimals to **Fraction** and **Up to three digits**.



4 Compare your answers in column B with the original values in column A. Can you describe the pattern when a base is raised to a power of –1?

5 Now consider powers of –2. Adapt steps from 1 to 3 for **column C**. Use **Fill Down** from cell C4 to C12.

	A	B	C	D
1	**Index**	-1	-2	-3
2				
3	**Base**			
4	2	1/2	=A4^$C$1	
5	3	1/3		
6	4	1/4		
7	5	1/5		

6 Compare your answers in column C with the original values in column A. Can you describe the pattern when a base is raised to a power of –2?

7 Now consider powers of –3. Adapt steps for **column D**. In cell D4, enter the formula **=A4^$D$1**.

Note: D12's fraction is missing as it has 4 digits in the denominator, which the spreadsheet doesn't allow for. Can you figure out what the fraction should be?

8 Compare your answers in column D with the original values in column A. Can you describe the pattern when a base is raised to a power of –3?

9 Write a rule for negative powers, given the answers you have found in this activity. Discuss with other students in your class.

5.05 Negative indices

WS Power calculations

Consider

$2^0 \div 2^3 = \frac{2^0}{2^3}$

$= \frac{1}{2^3}$

But also

$2^0 \div 2^3 = 2^{0-3}$

$= 2^{-3}$

$\therefore 2^{-3} = \frac{1}{2^3}$

Negative powers

A number raised to a negative power gives a fraction (with a numerator of 1):

$$a^{-n} = \frac{1}{a^n}$$

Proof:

$a^0 \div a^n = \frac{a^0}{a^n}$

$= \frac{1}{a^n}$

But also

$a^0 \div a^n = a^{0-n}$

$= a^{-n}$

$\therefore a^{-n} = \frac{1}{a^n}$

Negative indices

Example 8

Simplify each expression using a positive index (power).

a 5^{-3} **b** $3n^{-2}$ **c** $(3n)^{-2}$ **d** $p^{-2}q^3$

Solution

a $5^{-3} = \frac{1}{5^3}$

b $3n^{-2} = 3 \times n^{-2}$

$= \frac{3}{1} \times \frac{1}{n^2}$

$= \frac{3}{n^2}$

c $(3n)^{-2} = \frac{1}{(3n)^2}$

$= \frac{1}{9n^2}$

d $p^{-2}q^3 = \frac{1}{p^2} \times q^3$

$= \frac{q^3}{p^2}$

Example 9

Write each expression with a negative index.

a $\frac{1}{c^3}$ **b** $\frac{5}{3k^2}$

Solution

a $\frac{1}{c^3} = c^{-3}$

b $\frac{5}{3k^2} = \frac{5}{3} \times \frac{1}{k^2}$

$= \frac{5}{3}k^{-2}$

Power of −1

Consider $9^{-1} = \frac{1}{9}$

$\frac{1}{9}$ is the **reciprocal** of 9.

Consider $\left(\frac{2}{3}\right)^{-1} = \frac{1}{\left(\frac{2}{3}\right)}$

$= 1 \div \frac{2}{3}$

$= 1 \times \frac{3}{2}$

$= \frac{3}{2}$

$= 1\frac{1}{2}$

Power of −1

A number **raised to a power of −1** gives its **reciprocal**.

$$a^{-1} = \frac{1}{a}$$

$$\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$$

Example 10

Simplify each expression.

a $\left(\frac{4}{3}\right)^{-1}$ **b** $\left(\frac{y}{5}\right)^{-1}$

Solution

a $\left(\frac{4}{3}\right)^{-1} = \frac{3}{4}$ **b** $\left(\frac{y}{5}\right)^{-1} = \frac{5}{y}$

Negative powers of quotients

Consider $\left(\frac{4}{5}\right)^{-2} = \frac{1}{\left(\frac{4}{5}\right)^{2}}$

$= \frac{1}{\frac{16}{25}}$

$= 1 \div \frac{16}{25}$

$= 1 \times \frac{25}{16}$

$= \frac{25}{16}$

$= \frac{5^2}{4^2}$

$= \left(\frac{5}{4}\right)^{2}$

Negative powers of quotients

A number **raised to a power of $-n$** gives its reciprocal raised to the power of n.

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n} = \frac{b^n}{a^n}$$

Proof:

$$\left(\frac{a}{b}\right)^{-n} = \frac{1}{\left(\frac{a}{b}\right)^{n}}$$

$$= \frac{1}{\frac{a^n}{b^n}}$$

$$= \frac{b^n}{a^n}$$

$$= \left(\frac{b}{a}\right)^{n}$$

Example 11

Simplify each expression.

a $\left(\frac{4}{3}\right)^{-3}$ **b** $\left(2\frac{1}{2}\right)^{-2}$ **c** $\left(\frac{3a}{b^4}\right)^{-2}$

Solution

a $\left(\frac{4}{3}\right)^{-3} = \left(\frac{3}{4}\right)^{3}$

$= \frac{27}{64}$

Take the reciprocal, then cube.

b $\left(2\frac{1}{2}\right)^{-2} = \left(\frac{5}{2}\right)^{-2}$

$= \left(\frac{2}{5}\right)^{2}$

$= \frac{4}{25}$

c $\left(\frac{3a}{b^4}\right)^{-2} = \left(\frac{b^4}{3a}\right)^{2}$

$= \frac{b^8}{9a^2}$

EXERCISE 5.05 ANSWERS ON P. 577

Negative indices U F R C

EXAMPLE 8

1 Simplify each expression using a positive index. R C

a 6^{-2} **b** 5^{-7} **c** 3^{-1} **d** 10^{-2}

e g^{-5} **f** z^{-1} **g** n^{-3} **h** t^{-2}

i a^{-4} **j** 5^{-3} **k** y^{-d} **l** r^{-m}

2 Evaluate each expression, giving your answers in fraction form. R C

a 3^{-2} **b** 5^{-4} **c** 6^{-1} **d** 7^{-2}

e 25^{-1} **f** 2^{-7} **g** 4^{-3} **h** 10^{-6}

i 2^{-10} **j** 3^{-3} **k** 6^{-2} **l** 9^{-4}

3 Write each expression using a negative index. R C

EXAMPLE 9

a $\frac{1}{n^2}$ b $\frac{1}{n}$ c $\frac{1}{8^3}$ d $\frac{1}{8}$

e $\frac{1}{10^5}$ f $\frac{2}{a^4}$ g $\frac{1}{3}$ h $\frac{-1}{b}$

i $\frac{6}{a}$ j $\frac{4}{t^2}$ k $\frac{-2}{w^5}$ l $\frac{5}{d^3}$

4 Simplify each expression using positive indices. R C

5.05

a $5h^{-1}$ b $2b^{-5}$ c $3e^{-3}$ d $4n^{-2}$

e pb^{-2} f r^2s^{-4} g $w^{-2}y$ h $d^{-3}y^3$

i $(2m)^{-1}$ j $(xy)^{-1}$ k $(4h)^{-2}$ l $(5k)^{-3}$

m $3m^3p^{-2}$ n $15k^{-1}w^{-4}$ o $12x^{-2}y^{-3}$ p $12x^{-2}y^3$

q $(3h)^{-2}$ r $(4k)^{-3}$ s $(2c)^{-4}$ t $(8y)^{-1}$

u $4pq^{-3}$ v $4p^{-1}q^{-3}$ w vm^{-2} x $v^{-1}m^{-2}$

5 Simplify each expression. R C

EXAMPLE 10

a $\left(\frac{2}{7}\right)^{-1}$ b $\left(\frac{8}{5}\right)^{-1}$ c $\left(\frac{9}{10}\right)^{-1}$ d $\left(\frac{3}{2}\right)^{-1}$

e $\left(-\frac{3}{4}\right)^{-1}$ f $\left(\frac{5}{2}\right)^{-1}$ g $\left(\frac{x}{3}\right)^{-1}$ h $\left(\frac{5}{a}\right)^{-1}$

i $\left(-\frac{m}{2}\right)^{-1}$ j $\left(\frac{5r}{4}\right)^{-1}$ k $\left(\frac{2}{3z}\right)^{-1}$ l $\left(\frac{1}{v}\right)^{-1}$

6 Simplify each expression. R C

STAGE 5.3

EXAMPLE 11

a $\left(\frac{3}{4}\right)^{-2}$ b $\left(\frac{2}{3}\right)^{-3}$ c $\left(-\frac{1}{10}\right)^{-6}$ d $\left(\frac{5}{2}\right)^{-3}$

e $\left(-\frac{4}{3}\right)^{-5}$ f $\left(\frac{5}{4}\right)^{-4}$ g $\left(2\frac{1}{4}\right)^{-2}$ h $\left(1\frac{2}{5}\right)^{-3}$

i $\left(\frac{k}{3}\right)^{-2}$ j $\left(-\frac{3}{x}\right)^{-1}$ k $\left(\frac{a^2}{5}\right)^{-4}$ l $\left(\frac{4}{3g^3}\right)^{-2}$

m $\left(\frac{2d}{5t}\right)^{-2}$ n $\left(\frac{h^2}{m^3}\right)^{-5}$ o $\left(-\frac{5k^7}{3p^3}\right)^{-2}$ p $\left(-\frac{3c^3}{4a^2}\right)^{-3}$

q $\left(\frac{5k}{2m^2n^4}\right)^{-2}$ r $\left(-\frac{ac^3d^2}{3}\right)^{-4}$ s $\left(\frac{4y^5w^2}{7k^3}\right)^{-3}$ t $\left(\frac{2g^3h^7}{5x^4y}\right)^{-2}$

Foundation Standard Complex

5.06 Fractional indices $\frac{1}{n}$

STAGE 5.3

We know now the meaning of zero and negative indices, that is, a^0, a^{-1} and a^{-n}.

What is the meaning of fractional indices, that is, $a^{\frac{1}{2}}$ and $a^{\frac{1}{n}}$?

Consider $\left(25^{\frac{1}{2}}\right)^2 = 25^{\frac{1}{2}\times 2}$ Power of a power

$= 25^1$

$= 25$

but $\left(\sqrt{25}\right)^2 = 25$

$\therefore 25^{\frac{1}{2}} = \sqrt{25} = 5$

Power of $\frac{1}{2}$

Any number raised to the power of $\frac{1}{2}$ is the square root of that number:

$$a^{\frac{1}{2}} = \sqrt{a}$$

Proof:

$\left(a^{\frac{1}{2}}\right)^2 = a^{\frac{1}{2}\times 2}$

$= a^1$

$= a$

But $\left(\sqrt{a}\right)^2 = a$

$\therefore a^{\frac{1}{2}} = \sqrt{a}$

Now consider $\left(27^{\frac{1}{3}}\right)^3 = 27^{\frac{1}{3}\times 3}$ Power of a power

$= 27^1$

$= 27$

but $\left(\sqrt[3]{27}\right)^3 = 27$

$\therefore 27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

Power of $\frac{1}{3}$

Any number raised to the power of $\frac{1}{3}$ is the cube root of that number:

$$a^{\frac{1}{3}} = \sqrt[3]{a}$$

Proof:

$$\left(a^{\frac{1}{3}}\right)^3 = a^{\frac{1}{3}\times 3}$$
$$= a^1$$
$$= a$$

But $\left(\sqrt[3]{a}\right)^3 = a$

$$\therefore a^{\frac{1}{3}} = \sqrt[3]{a}$$

Now consider $\left(32^{\frac{1}{5}}\right)^5 = 32^{\frac{1}{5}\times 5}$ Power of a power

$$= 32^1$$
$$= 32$$

If $\left(32^{\frac{1}{5}}\right)^5 = 32$, then $32^{\frac{1}{5}}$ is called the 5th root of 32, written $\sqrt[5]{32}$.

$$2^5 = 32$$

$$\therefore 32^{\frac{1}{5}} = \sqrt[5]{32} = 2$$

On a calculator, the nth root key is $\sqrt[x]{\square}$ or $\sqrt[\square]{\square}$, found by pressing the SHIFT or 2nd F key before pressing $x^{\square}$ or y^x respectively.

Fractional powers $\frac{1}{n}$

Any number raised to the power of $\frac{1}{n}$ is the nth root of that number:

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

Proof:

$$\left(a^{\frac{1}{n}}\right)^n = a^{\frac{1}{n}\times n}$$
$$= a^1$$
$$= a$$

but $\left(\sqrt[n]{a}\right)^n = a$

$$\therefore a^{\frac{1}{n}} = \sqrt[n]{a}$$

STAGE 5.3

Example 12

Evaluate each expression.

a $900^{\frac{1}{2}}$ **b** $125^{\frac{1}{3}}$ **c** $1024^{\frac{1}{10}}$

Solution

a $900^{\frac{1}{2}} = \sqrt{900}$
$= 30$

b $125^{\frac{1}{3}} = \sqrt[3]{125}$
$= 5$

c $1024^{\frac{1}{10}} = \sqrt[10]{1024}$
$= 2$

Enter on calculator: 10 $\sqrt{\square}$ 1024 =
because $2^{10} = 1024$

Example 13

Write each expression using a fractional index.

a $\sqrt{8}$ **b** $\sqrt[3]{39}$ **c** $\sqrt[4]{n}$ **d** $\sqrt[7]{ab}$ **e** $\sqrt[5]{12y}$

Solution

a $\sqrt{8} = 8^{\frac{1}{2}}$

b $\sqrt[3]{39} = 39^{\frac{1}{3}}$

c $\sqrt[4]{n} = n^{\frac{1}{4}}$

d $\sqrt[7]{ab} = (ab)^{\frac{1}{7}}$

e $\sqrt[5]{12y} = (12y)^{\frac{1}{5}}$

EXERCISE 5.06 ANSWERS ON P. 578

Fractional indices $\frac{1}{n}$ U F R C

EXAMPLE 12

1 Evaluate each expression.

a $25^{\frac{1}{2}}$ **b** $343^{\frac{1}{3}}$ **c** $625^{\frac{1}{2}}$ **d** $1000^{\frac{1}{3}}$

e $32^{\frac{1}{5}}$ **f** $(-0.027)^{\frac{1}{3}}$ **g** $(0.04)^{\frac{1}{2}}$ **h** $64^{\frac{1}{3}}$

i $(-8)^{\frac{1}{3}}$ **j** $(-729)^{\frac{1}{3}}$ **k** $256^{\frac{1}{8}}$ **l** $3125^{\frac{1}{5}}$

2 Write each expression using a radical (root) sign. R C

a $10^{\frac{1}{2}}$ **b** $17^{\frac{1}{3}}$ **c** $g^{\frac{1}{2}}$ **d** $m^{\frac{1}{4}}$

e $(8r)^{\frac{1}{2}}$ **f** $6h^{\frac{1}{3}}$ **g** $(3m^4)^{\frac{1}{6}}$ **h** $hk^{\frac{1}{2}}$

i $(xy^3)^{\frac{1}{7}}$ **j** $(3u^2w^5)^{\frac{1}{4}}$ **k** $d^{\frac{1}{2}}e$ **l** $15ab^{\frac{1}{3}}$

EXAMPLE 13

3 Write each expression using a fractional index. R C

a $\sqrt{5}$ **b** $\sqrt[3]{49}$ **c** $\sqrt{20}$ **d** $\sqrt[5]{400}$

e $\sqrt[6]{54}$ **f** $\sqrt[5]{80}$ **g** $\sqrt{a}$ **h** $\sqrt[3]{q}$

i $\sqrt[7]{h}$ **j** $\sqrt[6]{3w}$ **k** $\sqrt{4xy^3}$ **l** $\sqrt[5]{2m^2p^3}$

STAGE 5.3

4 Evaluate each expression correct to 2 decimal places.

a $20^{\frac{1}{3}}$ **b** $215^{\frac{1}{2}}$ **c** $\sqrt[3]{36}$ **d** $\sqrt{201}$

e $(-24)^{\frac{1}{3}}$ **f** $\sqrt[4]{1111}$ **g** $\sqrt[5]{-47}$ **h** $\sqrt[6]{0.008}$

5 Simplify each expression. R

a $b^{\frac{1}{2}} \times b^{\frac{1}{2}}$ **b** $e^{\frac{1}{3}} \times e^{\frac{1}{3}} \times e^{\frac{1}{3}}$ **c** $y \times y^{\frac{1}{5}}$ **d** $m^{\frac{3}{5}} \times m^{\frac{2}{5}}$

e $2y^{\frac{1}{3}} \times 5y^{\frac{2}{3}}$ **f** $\left(-6a^{\frac{3}{2}}\right)^2$ **g** $\left(n^{12}m^4\right)^{\frac{1}{4}}$ **h** $\left(16a^2b^6\right)^{\frac{1}{2}}$

i $\left(8v^6w^9\right)^{\frac{1}{3}}$ **j** $40a^{\frac{1}{10}} \div 8a^{\frac{1}{10}}$ **k** $35x \div 5x^{\frac{1}{3}}$ **l** $36y^{\frac{1}{4}} \div 4y$

m $4m^2 \div 12m^{\frac{1}{2}}$ **n** $12k^{\frac{1}{3}} \div 15k^{\frac{1}{3}}$ **o** $\left(-2d^{\frac{1}{3}}\right)^9$ **p** $5c^{\frac{1}{3}} \times 4c^{\frac{1}{6}}$

Fractional indices $\frac{m}{n}$

5.07

STAGE 5.3

What is the meaning of fractional indices such as $a^{\frac{2}{3}}$ and $a^{\frac{3}{2}}$?

Consider $32^{\frac{3}{5}} = \left(32^{\frac{1}{5}}\right)^3$

$= \left(\sqrt[5]{32}\right)^3$

$= 2^3$

$= 8$

or consider $32^{\frac{3}{5}} = \left(32^3\right)^{\frac{1}{5}}$

$= \sqrt[5]{32^3}$

$= \sqrt[5]{32\,768}$

$= 8$

Fractional powers $\frac{m}{n}$

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m \text{ or } \sqrt[n]{a^m}$$

Proof:

$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m$ or $\left(a^m\right)^{\frac{1}{n}}$

$= \left(\sqrt[n]{a}\right)^m$ or $\sqrt[n]{a^m}$

Note: Taking the root first often makes the calculation simpler

Foundation Standard Complex

STAGE 5.3

Example 14

Evaluate each expression.

a $8^{\frac{2}{3}}$ **b** $81^{\frac{5}{4}}$ **c** $64^{-\frac{1}{3}}$ **d** $16^{-\frac{3}{4}}$

Solution

a $8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2$
$= 2^2$
$= 4$

b $81^{\frac{5}{4}} = \left(\sqrt[4]{81}\right)^5$
$= 3^5$
$= 243$

c $64^{-\frac{1}{3}} = \frac{1}{64^{\frac{1}{3}}}$
$= \frac{1}{\sqrt[3]{64}}$
$= \frac{1}{4}$

d $16^{-\frac{3}{4}} = \frac{1}{16^{\frac{3}{4}}}$
$= \frac{1}{\left(\sqrt[4]{16}\right)^3}$
$= \frac{1}{2^3}$
$= \frac{1}{8}$

Example 15

Evaluate $300^{\frac{3}{5}}$, correct to 2 decimal places.

Solution

$300^{\frac{3}{5}} = 30.638\,870\,63...$
≈ 30.64

Enter on calculator: 300 $x^{\blacksquare}$ 3 $\frac{\blacksquare}{\blacksquare}$ 5 =

Example 16

Write each expression using a fractional index.

a $\sqrt[4]{p^3}$ **b** $\sqrt{b^7}$ **c** $\frac{1}{\sqrt[5]{q^2}}$

Solution

a $\sqrt[4]{p^3} = \left(p^3\right)^{\frac{1}{4}}$
$= p^{\frac{3}{4}}$

b $\sqrt{b^7} = \left(b^7\right)^{\frac{1}{2}}$
$= b^{\frac{7}{2}}$

c $\frac{1}{\sqrt[5]{q^2}} = \frac{1}{\left(q^2\right)^{\frac{1}{5}}}$
$= \frac{1}{q^{\frac{2}{5}}}$
$= q^{-\frac{2}{5}}$

Example 17

Simplify each expression.

a $\sqrt[4]{\left(16r^2\right)^3}$ **b** $\left(\sqrt[3]{27k}\right)^2$ **c** $\left(32a^5\right)^{-\frac{3}{5}}$

Solution

a $\sqrt[4]{\left(16r^2\right)^3} = \left(16r^2\right)^{\frac{3}{4}}$
$= 16^{\frac{3}{4}} r^{2\times\frac{3}{4}}$
$= 8r^{\frac{3}{2}}$

b $\left(\sqrt[3]{27k}\right)^2 = (27k)^{\frac{2}{3}}$
$= 27^{\frac{2}{3}} k^{\frac{2}{3}}$
$= 9k^{\frac{2}{3}}$

c $\left(32a^5\right)^{-\frac{3}{5}} = \frac{1}{\left(32a^5\right)^{\frac{3}{5}}}$
$= \frac{1}{32^{\frac{3}{5}} a^{5\times\frac{3}{5}}}$
$= \frac{1}{8a^3}$

EXERCISE 5.07 ANSWERS ON P. 578

Fractional indices $\frac{m}{n}$ UFRC

1 Evaluate each expression. R C

EXAMPLE 14

a $4^{\frac{3}{2}}$ **b** $8^{\frac{5}{3}}$ **c** $128^{\frac{5}{7}}$

d $27^{\frac{2}{3}}$ **e** $1024^{\frac{3}{5}}$ **f** $64^{\frac{4}{3}}$

g $32^{\frac{3}{5}}$ **h** $81^{\frac{3}{4}}$ **i** $1000^{\frac{2}{3}}$

j $125^{\frac{2}{3}}$ **k** $8^{-\frac{1}{3}}$ **l** $81^{-\frac{1}{4}}$

m $25^{-\frac{3}{2}}$ **n** $36^{-\frac{3}{2}}$ **o** $256^{-\frac{3}{4}}$

p $3125^{-\frac{4}{5}}$ **q** $1024^{-\frac{4}{5}}$ **r** $400^{-\frac{3}{2}}$

s $128^{-\frac{4}{7}}$ **t** $1024^{-\frac{7}{10}}$

2 Evaluate each expression correct to 2 decimal places.

EXAMPLE 15

a $15^{\frac{3}{4}}$ **b** $8^{-\frac{7}{5}}$ **c** $50^{\frac{5}{4}}$

d $6^{-\frac{2}{3}}$ **e** $100^{-\frac{3}{4}}$ **f** $16^{\frac{3}{5}}$

g $12^{-\frac{3}{2}}$ **h** $179^{\frac{2}{5}}$

3 Write each expression using a fractional index. R C

EXAMPLE 16

a $\sqrt[7]{g^2}$ **b** $\sqrt{e^5}$ **c** $\sqrt[6]{x^{18}}$

d $\frac{1}{\sqrt[4]{y^{16}}}$ **e** $\sqrt[3]{m^5}$ **f** $\sqrt[5]{m^3}$

g $\frac{1}{\sqrt[4]{n^3}}$ **h** $\frac{1}{\sqrt[3]{5h^4}}$

4 Simplify each expression. R C

EXAMPLE 17

a $(16n^4)^{\frac{3}{4}}$ **b** $(8w)^{\frac{2}{3}}$ **c** $\sqrt[5]{(32d^{10})^3}$

d $\left(\sqrt{64m^8}\right)^3$ **e** $\left(\sqrt[4]{81r^4}\right)^5$ **f** $\sqrt[4]{(81h^{12})^3}$

g $\frac{1}{\left(\sqrt[3]{8s^6}\right)^4}$ **h** $\left(\frac{1}{\sqrt[5]{1024x^{15}}}\right)^2$ **i** $(125b^6)^{-\frac{2}{3}}$

j $(625t^2)^{-\frac{3}{4}}$ **k** $(49p^4q^{10})^{\frac{3}{2}}$ **l** $(1000x^3y^6)^{-\frac{2}{3}}$

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5.08 Summary of the index laws

Indices squaresaw (Advanced)

The index laws

$a^m \times a^n = a^{m+n}$ $\quad a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$ $\quad (a^m)^n = a^{m \times n}$

$(ab)^n = a^n b^n$ $\quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $\quad a^0 = 1$

$a^{-1} = \frac{1}{a}$ $\quad a^{-n} = \frac{1}{a^n}$ $\quad \left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$ $\quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{-n} = \frac{b^n}{a^n}$

STAGE 5.3

$a^{\frac{1}{2}} = \sqrt{a}$ $\quad a^{\frac{1}{3}} = \sqrt[3]{a}$

$a^{\frac{1}{n}} = \sqrt[n]{a}$ $\quad a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

EXERCISE 5.08 ANSWERS ON P. 578

Summary of the index laws U F R C

1 Simplify each expression. R

a $a^4 \times a^3$ **b** $t^8 \times t$ **c** $n^8 \div n^2$ **d** $p^3 \div p$

e $(w^2)^4$ **f** $(g^3)^6$ **g** $2b^2 \times 3b^5$ **h** $4d^7 \times 5d^6$

i $\frac{30c^{12}}{5c^8}$ **j** $(5b^4)^4$ **k** $24m^6 \div 8m^4$ **l** $(3a)^2$

2 Evaluate each expression. R

a 4^0 **b** $(-9)^0$ **c** 7×2^0 **d** $(5 \times 3)^0$

e $(-2)^3$ **f** $(-3)^2$ **g** $(5^2)^4$ **h** $11^4 \times 11^3$

i $(9^2)^0$ **j** $3^5 \div 3^2$ **k** $8^2 \div 8^5$ **l** $10^3 \div 10^3$

m $5^2 \div 5^0$ **n** $10^{-2} \div 10^2$ **o** $\left(\frac{1}{2}\right)^0$ **p** $8^{-5} \times 8^5$

3 Evaluate each expression, giving your answers in fraction form. R C

a 5^{-2} **b** 3^{-5} **c** 20^{-1} **d** 10^{-3}

STAGE 5.3

4 Evaluate each expression. R C

a $16^{\frac{1}{2}}$ **b** $27^{\frac{1}{3}} + 4^0$ **c** $25^{\frac{5}{2}}$ **d** $8^{\frac{1}{3}} + 4^{\frac{1}{2}}$

e $\left(8^2\right)^{\frac{1}{3}}$ **f** $\left(9^3\right)^{\frac{1}{2}}$ **g** $81^{\frac{3}{4}}$ **h** $(-32)^{\frac{3}{5}}$

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5 Simplify each expression. R

a $(3mn^3)^2$ **b** $8a^2w^2 \times 5a^3w^7$ **c** $(4a^2b^5)^4$ **d** $\frac{20p^3q^8}{5p^2q^6}$

e $\left(\frac{4}{5}\right)^3$ **f** $6c^2d^0$ **g** $\frac{48u^5v^4}{16uv}$ **h** $\left(\frac{3x}{10}\right)^2$

i $(-4n^2t)^3$ **j** $\left(-\frac{2}{3}\right)^4$ **k** $\frac{7x^2y^6}{35x^5y^3}$ **l** $\left(\frac{p^5}{9y}\right)^2$

m $(2p^3q^2)^5$ **n** $\left(\frac{1}{7n}\right)^3$ **o** $-2n^0$ **p** $\frac{(a^2b)^4 \times a^3}{b^5}$

6 Simplify each expression using a positive index. R C

a 8^{-7} **b** 3^{-5} **c** y^{-1} **d** x^{-3}

e $(5b)^{-2}$ **f** $5b^{-2}$ **g** $(ab)^{-1}$ **h** ab^{-1}

i $11t^{-3}$ **j** $(11t)^{-3}$ **k** p^3q^{-5} **l** mw^{-3}

m $8u^{-3}v^{-4}$ **n** $-2r^6y^{-5}$ **o** $10e^{-1}f^3$ **p** $\frac{1}{2}k^{-4}n^7$

7 Simplify each expression. R

a $\left(\frac{7}{4}\right)^{-1}$ **b** $\left(\frac{5}{2}\right)^{-1}$ **c** $\left(\frac{2}{3}\right)^{-1}$ **d** $\left(\frac{1}{7}\right)^{-1}$

e $\left(\frac{r}{8}\right)^{-1}$ **f** $\left(\frac{1}{10p}\right)^{-1}$ **g** $\left(\frac{6y}{w}\right)^{-1}$ **h** $\left(\frac{2}{5a}\right)^{-1}$

8 Write each expression using a negative index. R C

a $\frac{1}{4^3}$ **b** $\frac{1}{2}$ **c** $\frac{1}{10^4}$ **d** $\frac{1}{9^2}$

e $\frac{1}{k}$ **f** $\frac{1}{3m}$ **g** $\frac{12}{y^4}$ **h** $\frac{-4}{x^7}$

i $\frac{5}{2p^3}$ **j** $\frac{-2}{5a^6}$ **k** $\frac{1}{8w^4}$ **l** $\frac{-1}{6h^2}$

9 Simplify each expression, expressing your answer with a positive index. R C

a $q^5 \times q^{-2}$ **b** $d^{-3} \times d^7$ **c** $m^{-6} \div m^5$ **d** $y \div y^{-1}$

e $5g^3 \times 6g^{-1}$ **f** $8a^{-2} \times 3a^3$ **g** $7x^{-2} \times 4x$ **h** $\frac{64p^{-1}}{16p^2}$

i $48q \div 3q^{-2}$ **j** $\frac{5d^3}{10d^{-1}}$ **k** $2(b^{-1})^4$ **l** $(3h)^{-2}$

10 Write each expression using a fractional index. R C

a $\sqrt{5}$ **b** $\sqrt[3]{d}$ **c** $\sqrt{3y}$ **d** $\sqrt[4]{10}$

e $\left(\sqrt[3]{p}\right)^2$ **f** $\sqrt{(xy)^5}$ **g** $\sqrt[4]{(5a)^3}$ **h** $\left(\sqrt{\frac{y}{6}}\right)^5$

11 Simplify each expression. R

a $\left(\frac{4}{5a}\right)^{-3}$ **b** $\left(8c^3\right)^{\frac{2}{3}}$ **c** $\left(\frac{10}{7m}\right)^{-2}$ **d** $\left(25w^5\right)^{\frac{5}{2}}$

e $\left(\frac{49}{d^2}\right)^{\frac{3}{2}}$ **f** $\left(\frac{a^3b^9}{c^6}\right)^{\frac{2}{3}}$ **g** $\sqrt[4]{625m^6}$ **h** $\left(\frac{64}{y^3}\right)^{-\frac{2}{3}}$

i $\sqrt[5]{32m^{10}}$ **j** $\left(\frac{1}{3g^2}\right)^{-2}$ **k** $\left(16x^8\right)^{\frac{5}{4}}$ **l** $\left(\frac{2a^3}{c^2}\right)^4$

5.08

STAGE 5.3

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5.09 Significant figures

WS Significant figures

A way of rounding a number is to give the most relevant or important digits of the number. For example, a crowd of 47 321 people can be written as 47 000, which is rounded to the nearest thousand, or to 2 **significant figures**.

The first significant figure in a number is the first **non-zero** digit.

Number	First significant digit	Number of significant digits
47 321	4	5
47 000	4	2
0.000 159 2	1	4
0.000 2	2	1

- When rounding to significant figures, start counting from the first digit that is not 0.
- If it is a large number, you may need to insert 0s at the end as placeholders.
- 0s at the end of a whole number or at the beginning of a decimal are **not significant:** they are necessary placeholders.
- 0s *between* significant figures or at the end of a decimal *are* significant. For example, the significant figures are shown in bold in this table.

Number	First significant digit	Number of significant digits
809 000	8	3
0.0**20 70**	2	4

Example 18

State the number of significant figures in each number.

a 63.70 **b** 0.003 05 **c** 7600

Solution

a The 0 after 7 is significant.

∴ 63.70 has 4 significant figures.

b The first significant figure is 3, and the 0 between 3 and 5 is significant.

∴ 0.003 05 has 3 significant figures.

c The 0s after 6 are not significant.

∴ 7600 has 2 significant figures.

Example 19

Round each number to 3 significant figures.

a 56.357 **b** 9.249 **c** 548 307

Solution

a **56.3**57 ≈ 56.4 **b** **9.24**9 ≈ 9.25 **c** **548** 307 ≈ 548 000

The 0s in part **c** are not significant, but they are placeholders that are necessary for showing the place values of the 5, 4 and 8.

Significant figures

Example 20

Write each number correct to one significant figure.

a 0.007 39 **b** 0.025 **c** 0.963

Solution

a 0.00**7** 39 ≈ 0.007 **b** 0.0**2**5 ≈ 0.03 **c** 0.**9**63 ≈ 1

The 0s at the beginning of a decimal are not significant: they are placeholders.

Significant figures

EXERCISE 5.09 ANSWERS ON P. 579

Significant figures U F R C

1 State the number of significant figures in each number. R C

EXAMPLE 18

a 457 **b** 0.23 **c** 15 000
d 4.0004 **e** 0.0005 **f** 5000
g 0.002 07 **h** 89 072 **i** 0.040
j 76 000 000 **k** 0.000 328 **l** 169.320

2 Round each number to 3 significant figures. C

EXAMPLE 19

a 37.609 **b** 9435 **c** 168.39
d 2.813 **e** 15.99 **f** 60 522
g 1 769 000 **h** 385 764 **i** 10.2717

3 Write each number correct to 2 significant figures. C

EXAMPLE 20

a 0.0637 **b** 0.903 **c** 0.084 55
d 0.000 158 **e** 0.007 625 **f** 0.038 71
g 0.2795 **h** 0.018 944 **i** 0.3145

4 What is 45 067 853 rounded to 3 significant figures? Select the correct answer **A**, **B**, **C** or **D**.

A 45 167 853 **B** 45 100 000 **C** 45 067 900 **D** 45 070 000

5 What is 0.005 605 0 rounded to 2 significant figures? Select **A**, **B**, **C** or **D**.

A 0.01 **B** 0.010 000 0 **C** 0.0056 **D** 0.005 600 0

Foundation Standard Complex

6 Round each number to one significant figure.

a	9.478	**b**	57.12	**c**	0.0367
d	0.007 66	**e**	0.5067	**f**	10 675
g	1856.78	**h**	0.000 28	**i**	56 239 400

7 A company makes a profit of $35 754 125. C

a Round the profit to the nearest million and state the number of significant figures in the answer.

b Round the profit to the nearest 10 million and state the number of significant figures in the answer.

8 Australia's population in 2019 was 25 498 000. To how many significant figures has this number been written? C

9 A total of 21 558 people attended a football match. Express this number to 3 significant figures. C

10 The radius of a hydrogen atom is 0.000 000 000 104 m. To how many significant figures has this number been written? Select **A**, **B**, **C** or **D**.

A 12 **B** 9 **C** 3 **D** 2

11 Evaluate each expression, correct to the number of significant figures shown in the brackets.

a $45.6 \times 8.7 - 2.75 \times 78.32$ [2]

b $15.5 - 9.87 \div 0.24 + 8.43 \times 2.4$ [1]

c $(63.73 - 27.89) \div 5.82$ [3]

d $\dfrac{63.25 + 76.03}{55.89 - 89.24}$ [4]

e $\dfrac{9.732 + 2.765}{12.27 \times 15.8}$ [1]

f $78.91 \div (23.6 + 94.7)$ [2]

g $\dfrac{1}{0.941} + \dfrac{253}{0.0076}$ [3]

h $\sqrt{84.3} \times 0.0715$ [4]

Did you know?

Big numbers

The table below lists the names of some big numbers and their meanings.

According to the Guinness Book of Records, the largest number for which there is an accepted name is the centillion, first recorded in 1852. It is equal to 10^{303}.

What is the special name for the number:

a 10^{100}? **b** $10^{(10^{100})}$?

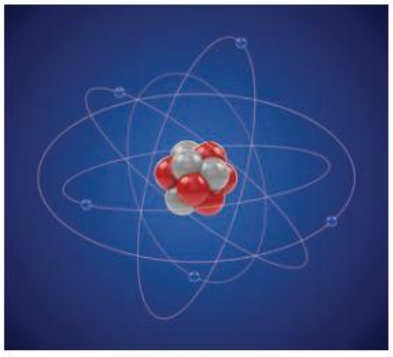
Name	Numeral
million	$10^6 = 1\,000\,000$
billion	$10^9 = 1\,000\,000\,000$
trillion	10^{12}
quadrillion	10^{15}
quintillion	10^{18}
sextillion	10^{21}
septillion	10^{24}
octillion	10^{27}
nonillion	10^{30}
decillion	10^{33}

Foundation Standard Complex

Scientific notation

5.10

Scientific notation is a short way of writing very large or very small numbers using powers of 10. It was invented in the early 20th century when scientists needed to describe very large values, such as astronomical distances, and very small values, such as the masses of atoms.



iStock.com/farakos

WS Scientific notation 1

WS Scientific notation 2

Scientific notation

Scientific notation

Scientific notation

Numbers written in **scientific notation** are expressed in the form

$$m \times 10^n$$

where m is a number between 1 and 10 and n is an integer.

Scientific notation

Example 21

Express each number in scientific notation.

a 764 000 000 000 **b** 6000 **c** 0.0008 **d** 0.000 000 472

Solution

a Use the significant figures in the number to write a value between 1 and 10: 7.64

Count how many places the decimal point moves to the right to make 764 000 000 000.

764 000 000 000

11 places

or count the number of places after the first significant figure, 7

$\therefore 764\,000\,000\,000 = 7.64 \times 10^{11}$

b Use the significant figures in the number to write a value between 1 and 10: 6

Count how many places the decimal point moves to the right to make 6000.

6000

3 places

or count the number of places after the first significant figure, 6

$\therefore 6000 = 6 \times 10^3$

c Use the significant figures in the number to write a value between 1 and 10: 8

Count how many places the decimal point moves to the left to make 0.0008.

0.0008 (or count the number of decimal places to the first significant figure, 8)

4 places

$\therefore 0.0008 = 8 \times 10^{-4}$

Note that small numbers are written with *negative* powers of 10.

d Use the significant figures in the number to write a value between 1 and 10: 4.72

Count the number of places the decimal point moves to the left to make 0.000 000 472.

0.000 000 472 (or count the number of decimal places to the first significant figure, 4)

7 places

$\therefore 0.000\,000\,472 = 4.72 \times 10^{-7}$

Example 22

Express each number in decimal form.

a 2.7×10^4 **b** 3.56×10^{-2}

Solution

a $2.7 \times 10^4 = 2.7000$ Move the decimal point 4 places to the right.

$= 27\,000$

b $3.56 \times 10^{-2} = 0.0356$ Move the decimal point 2 places to the left.

$= 0.0356$

Example 23

a Which number is the larger: 3.65×10^{12} or 8.1×10^{12}?

b Write these numbers in **ascending order**: 4.3×10^6, 2.8×10^7, 1.9×10^7

Solution

To compare numbers in scientific notation, first compare the powers of 10.

If the powers of 10 are the same, then compare the decimal parts.

a The powers of 10 are the same. Compare the decimal parts: $8.1 > 3.65$.

$\therefore$ The larger number is 8.1×10^{12}

b Compare the powers of 10: $10^6 < 10^7$.

Then compare the 2 numbers with 10^7: $1.9 < 2.8$.

$\therefore$ The numbers in ascending order are 4.3×10^6, 1.9×10^7, 2.8×10^7.

EXERCISE 5.10 ANSWERS ON P. 579

Scientific notation U F R C

1 Express each number in scientific notation. R C

EXAMPLE 21

a	2400	**b**	786 000	**c**	55 000 000
d	95	**e**	7.8	**f**	348 000 000
g	59 670	**h**	15	**i**	3 000 000 000
j	80	**k**	763	**l**	10
m	0.035	**n**	0.000 076	**o**	0.8
p	0.0713	**q**	0.000 003	**r**	0.913
s	0.000 007 146	**t**	0.009	**u**	0.000 000 1
v	0.000 89	**w**	0.000 000 078	**x**	0.1

2 Express the value shown in each measurement in scientific notation. C

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a The world's largest mammal is the blue whale, which can weigh up to 130 000 kg.

b The diameter of an oxygen molecule is 0.000 000 29 cm.

c The thickness of a human hair is 0.000 08 m.

d Light travels at a speed of 300 000 000 m/s.

e The nearest star to Earth, excluding the Sun, is Alpha Centauri, which is 40 000 000 000 000 km away.

f The thickness of a typical piece of paper is 0.000 12 m.

g The small intestine of an adult is approximately 610 cm long.

h The diameter of a hydrogen atom is 0.000 000 000 106 m.

i The diameter of our galaxy, the Milky Way, is 770 000 000 000 000 000 000 m.

j A microsecond means 0.000 001 s.

k The Andromeda Galaxy is the most remote body visible to the naked eye, at a distance of 2 200 000 light years away.

3 Express each number in decimal form. R C

EXAMPLE 22

a	6×10^{5}	**b**	7.1×10^{3}	**c**	3.02×10^{8}
d	3.14×10^{0}	**e**	6×10^{-5}	**f**	7.1×10^{-3}
g	3.02×10^{-8}	**h**	5.9×10^{-10}	**i**	1.1×10^{12}
j	4×10^{-4}	**k**	5×10^{3}	**l**	4.76×10^{-4}
m	8.03×10^{-1}	**n**	6.32×10^{4}	**o**	1.6×10^{-2}
p	2.2×10^{-7}	**q**	9.0×10^{6}	**r**	1.11×10^{-1}

EXAMPLE 23

4 For each pair of numbers, write the larger one. R

a 3×10^5 or 4×10^5

b 8.4×10^5 or 2.7×10^6

c 8.4×10^0 or 1.3×10^7

d 3.6×10^{-7} or 6.3×10^{-7}

e 9.3×10^9 or 7.6×10^9

f 3.5×10^{-6} or 9.3×10^2

g 3.04×10^0 or 3.04×10^{-4}

h 4.5×10^{-5} or 3.7×10^{-7}

i 2×10^{-15} or 2×10^{-17}

j 6.23×10^{-5} or 9.7×10^{-5}

5 Write each set of numbers in ascending order. R

a 3.8×10^9, 7.3×10^9, 5.5×10^9

b 2.2×10^{-4}, 5.8×10^{-6}, 7×10^{-4}

c 3.5×10^0, 5.3×10^2, 4.9×10^2

6 Write each set of numbers in **descending order**. R

a 6×10^5, 2.9×10^2, 8×10^2

b 1.2×10^{-9}, 6.3×10^2, 8.1×10^{-4}

c 4.1×10^{-1}, 9.5×10^{-1}, 6.4×10^{-3}

5.11 Scientific notation on a calculator

WS Scientific notation problems

Scientific notation: accomplishing great things

To enter a number in scientific notation on a calculator, use the $\times 10^x$ or EXP key.

Example 24

Evaluate each expression using scientific notation.

a $(4.25 \times 10^7) \times (8.2 \times 10^6)$

b $(1.08 \times 10^{-15}) \div (3 \times 10^{11})$

c $(4.9 \times 10^7)^2$

Solution

a Enter 4.25 $\times 10^x$ 7 $\times$ 8.2 $\times 10^x$ 6 =

$(4.25 \times 10^7) \times (8.2 \times 10^6) = 3.485 \times 10^{14}$

Note that with scientific notation on a calculator, there is no need to enter brackets ( ) around the numbers.

b Enter 1.08 $\times 10^x$ $-$ 15 $\div$ 3 $\times 10^x$ 11 =

$(1.08 \times 10^{-15}) \div (3 \times 10^{11}) = 3.6 \times 10^{-27}$

c Enter 4.9 $\times 10^x$ 7 x^2 =

$(4.9 \times 10^7)^2 = 2.401 \times 10^{15}$

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Example 25

Estimate the value of each expression in scientific notation, then evaluate it correct to 3 significant figures.

a $\frac{9.2 \times 10^9}{2.7 \times 10^5}$ **b** $(8.5 \times 10^4) \times (6.3 \times 10^7)$ **c** $(6.08 \times 10^5)^3$

Solution

Estimate

a
$$\frac{9.2 \times 10^9}{2.7 \times 10^5} \approx \frac{9 \times 10^9}{3 \times 10^5}$$
$$= \frac{9}{3} \times \frac{10^9}{10^5}$$
$$= 3 \times 10^4$$

b
$$(8.5 \times 10^4) \times (6.3 \times 10^7) \approx (9 \times 10^4) \times (6 \times 10^7)$$
$$= (9 \times 6) \times (10^4 \times 10^7)$$
$$= 54 \times 10^{11}$$
$$= 5.4 \times 10 \times 10^{11}$$
$$= 5.4 \times 10^{12}$$

c
$$(6.08 \times 10^5)^3 \approx (6 \times 10^5)^3$$
$$= 6^3 \times (10^5)^3$$
$$= 216 \times 10^{15}$$
$$= 2.16 \times 10^2 \times 10^{15}$$
$$= 2.16 \times 10^{17}$$

Calculated answer

$$\frac{9.2 \times 10^9}{2.7 \times 10^5} = 34\,074.074\,07$$
$$\approx 34\,000$$
$$= 3.40 \times 10^4$$

$$(8.5 \times 10^4) \times (6.3 \times 10^7) = 5.355 \times 10^{12}$$
$$\approx 5.36 \times 10^{12}$$

$$(6.08 \times 10^5)^3 = 2.24755... \times 10^{17}$$
$$\approx 2.25 \times 10^{17}$$

5.11

EXERCISE 5.11 ANSWERS ON P. 579

Scientific notation on a calculator U F PS R C

1 Evaluate each expression using scientific notation. EXAMPLE 24

a $(2 \times 10^3) \times (3 \times 10^5)$ **b** $(8 \times 10^7) \div (4 \times 10^2)$

c $(2 \times 10^5)^3$ **d** $\sqrt{9 \times 10^{12}}$

e $(4 \times 10^7) \times (6 \times 10^8)$ **f** $(1 \times 10^8) \div (2 \times 10^3)$

g $(4 \times 10^3)^5$ **h** $24.08 \div (8 \times 10^6)$

i $\sqrt{3.969 \times 10^{19}}$ **j** $(2 \times 10^5)^{-2}$

k $\sqrt[3]{8 \times 10^{-9}}$ **l** $\frac{7.62 \times 10^9}{2 \times 10^{-4}}$

2 Estimate the value of each expression in scientific notation, then evaluate correct to 3 significant figures. R EXAMPLE 25

a $(5.7 \times 10^3) \times (2.3 \times 10^5)$ **b** $(8 \times 10^5) \times (3.7 \times 10^7)$

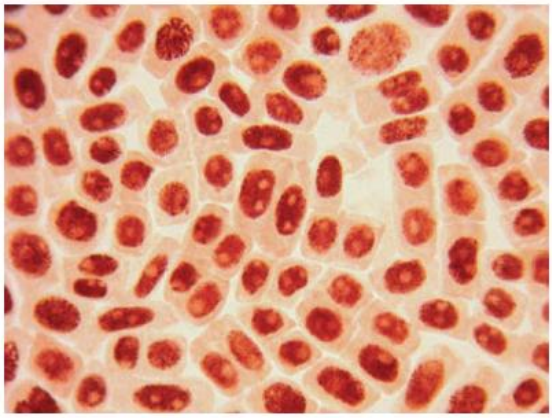
c $(9.1 \times 10^{20}) \div (3.2 \times 10^5)$ **d** $(1.2 \times 10^8)^2$

e $(7.13 \times 10^{10}) \times (9.8 \times 10^8)$ **f** $(1.9 \times 10^{11}) \div (2.1 \times 10^7)$

g $(5.85 \times 10^4)^3$ **h** $(6 \times 10^{12}) \div (2.8 \times 10^3)$

Foundation Standard Complex

3 The human body consists of approximately 6×10^9 cells, and each cell consists of 6.3×10^9 atoms. Roughly how many atoms are there in a human body? PS R



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4 *The Complete Sherlock Holmes* is a book that is 4.5 cm thick and contains 2000 pages. Find the thickness, in *millimetres*, of one page in scientific notation. PS R

5 Evaluate each expression in scientific notation, correct to 2 significant figures.

a $(7.4 \times 10^{30}) - (3.59 \times 10^{29})$

b $(1.076 \times 10^{17}) + (2.3 \times 10^{16})$

c $\sqrt{6.6 \times 10^{27}}$

d $(7.5 \times 10^{23}) \div (3.3 \times 10^{-13})$

e $(8.17 \times 10^{16})^3$

f $\sqrt{2.69 \times 10^{45}}$

g $(7.05 \times 10^3) \div (3.9 \times 10^7)$

h $\dfrac{(5.6 \times 10^4) \times (3.9 \times 10^5)}{(2.3 \times 10^7)}$

i 1595×1959

j 5^{20}

k 801^{-1}

l 3^{-10}

m 9^9

n $(0.7)^{-5}$

6 The Earth is 1.50×10^8 km from the Sun and the speed of light is 3×10^5 km/s. How long does it take for light to travel from the Sun to Earth? Express your answer in: PS R C

a seconds

b minutes and seconds

7 The Sun burns 6 million tonnes of hydrogen a second. Calculate to 2 significant figures how many tonnes of hydrogen it burns in a year (that is, 365.25 days). PS R

8 Sound travels at approximately 330 metres per second. If Mach 1 is the speed of sound, how fast is Mach 5? Convert your answer to kilometres per second to 2 significant figures. PS R C

9 The distance that light travels in one year is called a light year. If the speed of light is approximately 3×10^5 km per second, how far to 2 significant figures does light travel in a leap year? PS R

Foundation Standard Complex

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10 A thunderstorm is occurring 30 km from where you are standing. Use the speed of light (3×10^5 km per second) and the speed of sound (330 metres per second) to calculate in seconds how long it takes: PS R C

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a the light from the lightning to reach you

b the sound from the thunder to reach you.

11 **a** What is the largest number that can be displayed on your calculator?

b What is the smallest number that can be displayed? PS R C

Investigation

A lifetime of heartbeats

How many times does your heart beat in an average lifetime of 80 years?

1 Work in pairs and copy this table.

Name	Trial 1	Trial 2	Average beats per minute

2 Use 2 fingers to measure your pulse. Have your partner time you for a minute. Do this twice, record your results in the table and find the average.

3 Repeat Step **2** for your partner.

4 Calculate how many times your heart (and your partner's heart) beats in the following periods. Write your answers in scientific notation, correct to 2 significant figures.

a an hour **b** a day **c** a week

d a year (use 365.25 days) **e** an average lifetime of 80 years

Did you know?

Hairy numbers

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Straight hair
• round follicle

Alamy Stock Photo/Stephen Flint

Wavy hair
⬬ oval follicle

Shutterstock.com/Cookie Studio

Curly hair
▬ flat follicle

There are about 110 000 hairs on your head. Each hair grows at the rate of about 1.3×10^{-3} cm per hour. A single hair lasts about 6 years. Every day you lose between 30 and 60 hairs. Each hair grows from a small depression in the skin called a follicle (a gland). After the hair falls out, the follicle rests for about 3 to 4 months before the next hair starts growing. Hair follicles are either oval, flat or round in shape. How straight, wavy or curly your hair is depends on the shape of your hair follicles.

How many hairs are on all the heads in China if its population is approximately 1.445×10^9? Answer in both scientific notation and decimal notation.

WS Binary number system

Power plus ANSWERS ON P. 579

1 Write each number in scientific notation.

a 438.2×10^9 **b** 0.52×10^{-7} **c** 0.0004×10^{12}

d 2013×10^{-3} **e** 57.8 thousand **f** 57.8 thousandths

g 6.7 millionths **h** 3.2 billion **i** 3.2 billionths

2 Evaluate each expression.

a $\sqrt{\sqrt{81}}$ **b** $\sqrt{\sqrt{625}}$ **c** $\sqrt{\sqrt{\sqrt{256}}}$

d $\sqrt{\sqrt[5]{1024}}$ **e** $\sqrt{\sqrt{\sqrt{6561}}}$ **f** $\sqrt[4]{\sqrt{256}}$

g $\sqrt{\sqrt[3]{1000000}}$ **h** $\sqrt[3]{\sqrt{1000000}}$

3 Find values of a, m and n so that each equation is true.

a $a^{\frac{m}{n}} = 2$ **b** $a^{\frac{m}{n}} = 3$ **c** $a^{\frac{m}{n}} = 64$ **d** $a^{\frac{m}{n}} = 125$

4 Find values of a and b that solve $a^b = b^a$. What if a and b are different?

5 The terms in the pattern 3, 5, 17, 257, 65 537, ... can all be generated by a simple method, using only the numbers 1 and 2.

a What is this method?

b What is the next number in the sequence?

CHAPTER 5 REVIEW

Language of maths

Qz Numbers and indices

ascending	base	descending	estimate
expanded form	exponent	fractional power	index
index laws	index notation	indices	negative power
product	power	quotient	reciprocal
significant figures	scientific notation	term	zero power

1 What does a power of $\frac{1}{2}$ mean?

2 Which 2 words from the list mean 'power'?

3 What is the $\times 10^x$ or EXP key on a calculator used for?

4 What is the index law for dividing terms with the same base?

5 Which digits in 0.006 701 are **significant figures**?

6 What power is associated with the **reciprocal** of a term or number?

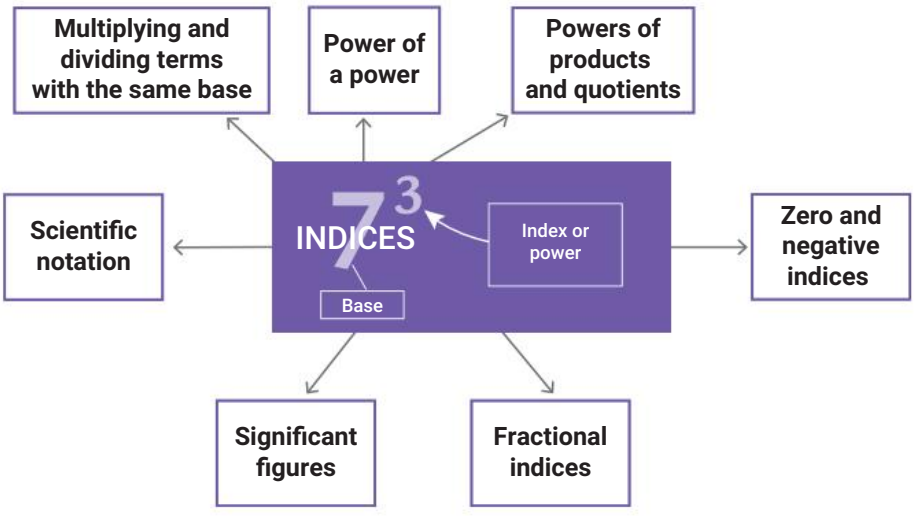
7 What type of numbers when written in **scientific notation** have **negative** powers of 10?

Topic summary

- What was this topic about? What was the main theme?
- What content was new and what was revision?
- What are the index laws?
- Write 10 questions (with solutions) that could be used in a test for this chapter.
- Include some questions that you have found difficult to answer.
- List the sections of work in this chapter that you did not understand. Follow up this work with a friend or your teacher.

WS Mind map: Indices (Advanced)

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



TEST YOURSELF 5

ANSWERS ON P. 580

5.01

1 Simplify each expression, writing the answer in index notation.

a $10^3 \times 10^7$ **b** $4^{20} \div 4^4$ **c** $a^{12} \div a^2$

d $h^8 \times h^2$ **e** $3n^3 \times 4n$ **f** $10d^{15} \div 5d^3$

g $20m^9 \div 4m$ **h** $3v^4w^2 \times 2v^3w^5$ **i** $5x^5y^2 \times 3xy$

j $\frac{24t^8h^8}{3t^4h^2}$ **k** $\frac{p^6q^{10}}{p^2q^2}$ **l** $\frac{100a^2b^4}{5ab^2}$

5.02

2 Simplify each expression, writing the answer in index notation.

a $(2^2)^3$ **b** $(k^5)^5$ **c** $(x^3)^4$

d $(2y^3)^{10}$ **e** $(5t^2)^2$ **f** $(10g)^3$

g $(-2)^5$ **h** $(-2k)^5$ **i** $(-5m^3)^2$

5.03

3 Simplify each expression.

a $(ab^2)^4$ **b** $(5x^3y^2)^2$ **c** $(4t^2)^3$

d $(-4h^2g)^3$ **e** $\left(\frac{a}{7}\right)^4$ **f** $(2pqr)^5$

g $\left(\frac{3m}{2}\right)^5$ **h** $(-3np^2)^4$ **i** $\left(\frac{2a^7}{b}\right)^4$

j $(4t^4u^5)^3 \times 8t^2u$ **k** $\left(\frac{b^8y^6}{8b^2y}\right)^3$ **l** $45c^6d^8 \div (3cd^2)^2$

5.04

4 Simplify each expression.

a 7^0 **b** $(-23)^0$ **c** e^0

d $(-k)^0$ **e** $-m^0$ **f** $8g^0h$

g $(5wy)^0$ **h** $\left(\frac{2p^5}{3}\right)^0$ **i** $\frac{2p^0}{3}$

j $15u^2 \div 20u^0$ **k** $(13a)^0 - 13a^0$ **l** $4^0 \times 5 \times (6x)^0$

5.05

5 Simplify each expression using a positive index.

a 8^{-3} **b** 19^{-2} **c** x^{-1} **d** p^{-5}

e $(4m)^{-1}$ **f** $(3g^2)^{-3}$ **g** $\left(\frac{2}{b}\right)^{-4}$ **h** $5b^{-1}$

i $-2x^{-4}$ **j** $\left(\frac{3}{5a^2c}\right)^{-1}$ **k** $c^{-4}d^2$ **l** $\left(\frac{100}{9}\right)^{-1}$

5.05

6 Write each expression using a negative index.

a $\frac{1}{10^3}$ **b** $\frac{1}{r^5}$ **c** $\frac{1}{kg}$ **d** $\frac{3}{b}$

STAGE 5.3

5.05

7 Simplify $\left(-\frac{8}{3x}\right)^{-2}$ using a positive index.

5.06

8 Write each expression using a radical (root) sign.

a $q^{\frac{1}{3}}$ **b** $u^{\frac{1}{2}}$ **c** $(2q)^{\frac{1}{3}}$ **d** $(ar)^{\frac{1}{2}}$

5.07

9 Evaluate each expression.

a $64^{\frac{2}{3}}$ **b** $(-32)^{\frac{3}{5}}$ **c** $36^{-\frac{3}{2}}$

Foundation Standard Complex

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10 Simplify each expression. 5.07

a $\left(125d^{15}\right)^{\frac{4}{3}}$ **b** $\left(16y^{20}\right)^{\frac{1}{4}}$ **c** $\left(32x^{8}\right)^{\frac{2}{5}}$ **d** $\sqrt[3]{-64n^{12}}$

e $\left(\sqrt[5]{32k^{10}}\right)^{3}$ **f** $\left(\frac{1}{\sqrt{4g}}\right)^{5}$ **g** $\left(\sqrt[4]{81a^{2}}\right)^{6}$ **h** $\left(\frac{\left(2\sqrt[3]{8y}\right)^{4}}{10}\right)^{-2}$

11 Simplify each expression using a positive index. 5.08

a $\left(\frac{3}{2m}\right)^{-2}$ **b** $3q^{-5} \times 2q^{3}$ **c** $\left(36w^{2}\right)^{\frac{3}{2}}$ **d** $24k^{3} \div 8k^{-5}$

e $\sqrt[3]{125y^{9}}$ **f** $\left(\frac{5d}{3k}\right)^{-3}$ **g** $36p^{-4} \div 27p^{-5}$ **h** $\left(\frac{8}{m^{3}}\right)^{-\frac{2}{3}}$

i $4a^{2}b^{-5} \times 8ab^{4}$ **j** $\left(2d^{-2}\right)^{-3}$ **k** $\left(\frac{16}{81w^{8}}\right)^{-\frac{1}{4}}$ **l** $\sqrt[5]{32g^{5}h^{15}}$

m $\left(\frac{3x^{2}}{u^{5}}\right)^{4}$ **n** $\left(\frac{8n^{6}}{27c^{9}}\right)^{-\frac{2}{3}}$ **o** $8h^{3} \div 12h^{8}$ **p** $\frac{(2c^{4}e^{3})^{4}}{\sqrt[3]{c^{3}e^{6}}}$

12 Round each value correct to the number of significant figures shown in the brackets. 5.09

a 8.5678 [2] **b** 15 712 [3] **c** 476 [1]

d 0.007 126 6 [4] **e** 0.9045 [3] **f** 301 378 [2]

g 4805.28 [3] **h** 0.000 87 [1] **i** 67 000 000 [1]

13 Express each number in scientific notation. 5.10

a 37 000 **b** 0.61 **c** 250 000

d 0.000 49 **e** 13 **f** 0.000 000 000 08

14 Express each number in decimal form. 5.10

a 8.1×10^{5} **b** 6×10^{7} **c** 5.182×10^{0}

d 2.03×10^{-4} **e** 3×10^{-8} **f** 9.475×10^{-2}

15 Write these numbers in ascending order: 3×10^{3}, 9.1×10^{-8}, 2.4×10^{3}. 5.10

16 Evaluate each expression using scientific notation. 5.11

a $(3.65 \times 10^{12}) \times (7.4 \times 10^{8})$ **b** $(1.47 \times 10^{10}) \div (3.72 \times 10^{4})$

c $(8.41 \times 10^{5})^{3}$ **d** $\sqrt{6.25 \times 10^{-9}}$

17 Estimate the value of each expression in scientific notation, then evaluate correct to 2 significant figures. 5.11

a $(8.9 \times 10^{9}) \times (1.1 \times 10^{7})$ **b** $(9.3 \times 10^{15}) \div (4 \times 10^{2})$

c $(3.1 \times 10^{4})^{2}$

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6

MEASUREMENT AND GEOMETRY

GEOMETRY

'Geometry' comes from the Greek word *geometria*, which means 'land measuring'. The principles and ideas of geometry are evident everywhere – in road signs, buildings, bridges and patterns for tiles and wallpaper. Many examples of geometric patterns can be seen in architecture, such as in this ground view of an apartment building in Berlin, Germany.

Chapter outline

	Working mathematically				
6.01 Angle geometry	U	F		R	C
6.02 Triangle geometry	U	F		R	C
6.03 Quadrilateral geometry	U	F		R	C
6.04 Angle sum of a polygon#	U	F		R	C
6.05 Exterior angle sum of a convex polygon#	U	F		R	

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

Wordbank

angle sum The total of the sizes of all the angles in a shape, such as a triangle

bisect To cut in half

convex polygon A polygon whose vertices all point outwards

diagonal An interval joining 2 non-adjacent vertices of a shape

exterior angle of a triangle An 'outside' angle of a triangle formed after extending one of the sides of the triangle

polygon A plane shape with straight sides

regular polygon A polygon with all angles equal and all sides equal, such as a square

supplementary angles Angles that add up to 180°

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

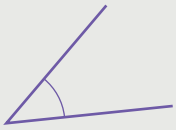
- solve geometrical problems involving angles and parallel lines
- classify triangles, quadrilaterals and polygons
- solve geometrical problems involving the angle sum of a triangle and quadrilateral and the exterior angle of a triangle
- solve geometrical problems involving the angle sum of a polygon and the exterior angle sum of a complex polygon

SkillCheck ANSWERS ON P. 580

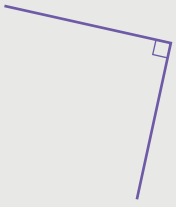
Types of angles

1 Classify each angle as acute, obtuse, reflex, right, straight or a revolution.

a



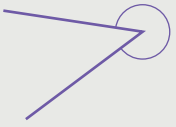
b



c

d

e



f

g

h

2 What type of angle is 95°? Select the correct answer **A**, **B**, **C** or **D**.

A acute **B** a revolution **C** reflex **D** obtuse

3 Which of the following is a reflex angle? Select **A**, **B**, **C** or **D**.

A 70° **B** 270° **C** 115° **D** 360°

4 An angle of 270° is which of the following? Select **A**, **B**, **C** or **D**.

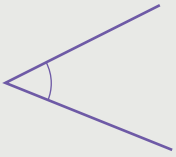
A an obtuse angle **B** a right angle

C equal to a complete turn **D** equal to 3 right angles

Measuring angles

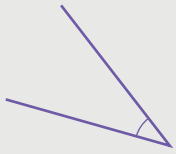
5 Use a protractor to measure the size of each angle.

a

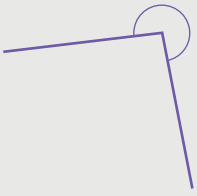


b

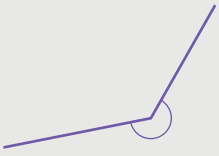
c



d



e



f

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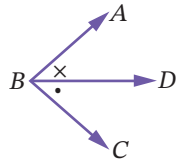
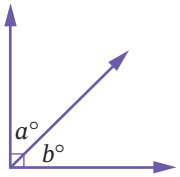
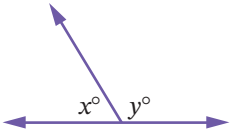
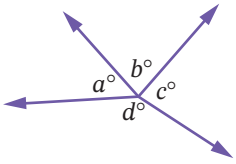
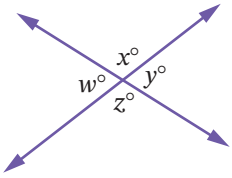
Angle geometry

6.01

Angle facts

Adjacent angles	Complementary angles	Angles in a right angle
Angles next to each other, sharing a common arm. $\angle ABD$ and $\angle DBC$ are adjacent.  	2 angles that have a sum of 90°, for example, 35° and 55°.	Are complementary. $a + b = 90$ 
Angles on a straight line	**Angles at a point**	**Vertically opposite angles**
Are supplementary. $x + y = 180$  	(In a revolution) Add up to 360°. $a + b + c + d = 360$  	Are equal. $w = y$ and $x = z$ 

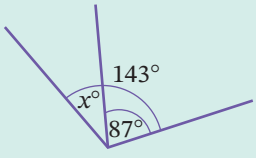
Supplementary angles

2 angles that have a sum of 180°, for example, 140° and 40°.

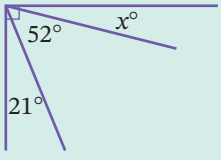
Example 1

Find the value of each variable, giving reasons.

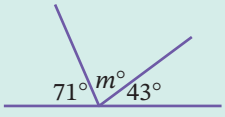
a



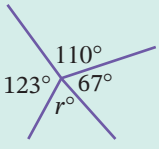
b



c



d



Solution

a $x + 87 = 143$ (adjacent angles)

$x = 143 - 87$

$= 56$

b $x + 52 + 21 = 90$ (angles in a right angle)

$x = 90 - 52 - 21$

$= 17$

c $m + 71 + 43 = 180$

(angles on a straight line)

$m = 180 - 71 - 43$

$= 66$

d $r + 123 + 110 + 67 = 360$

(angles at a point)

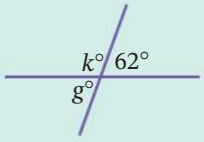
$r + 300 = 360$

$r = 60$

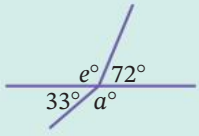
Example 2

Find the value of each variable, giving reasons.

a



b



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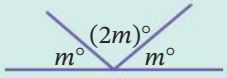
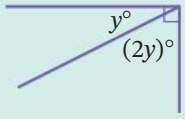


d



Solution

a
$g = 62$ (vertically opposite angles)
$k + 62 = 180$ (angles on a straight line)
$k = 118$

b
$e + 72 = 180$ (angles on a straight line)
$e = 108$
$a + 33 = 180$ (angles on a straight line)
$a = 147$

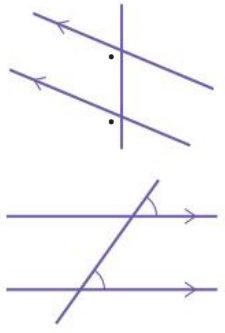
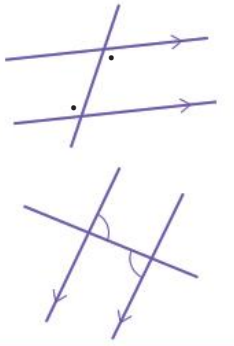
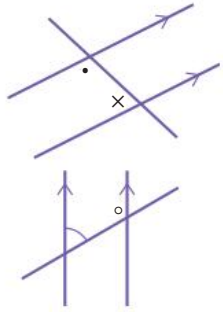
c
$m + 2m + m = 180$ (angles on a straight line)
$4m = 180$
$m = \frac{180}{4}$
$= 45$

d
$y + 2y = 90$ (angles in a right angle)
$3y = 90$
$y = 30$

Angles and parallel lines

Angles on parallel lines

When **parallel lines** are crossed by another line (called a **transversal**), special pairs of angles are formed.

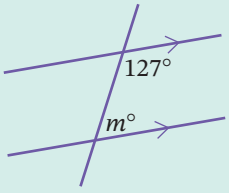
Corresponding angles	Alternate angles	Co-interior angles
Corresponding angles on parallel lines are equal.	Alternate angles on parallel lines are equal.	Co-interior angles on parallel lines are supplementary (add to 180°).
		

- Corresponding angles are in 'matching' positions on the *same* side of the transversal: 'corresponding' means 'matching'
- Alternate angles are between the parallel lines on *opposite* sides of the transversal: 'alternate' means 'going back and forth in turns'
- Co-interior angles are between the parallel lines on the *same* side of the transversal: 'co-interior' means 'together inside'

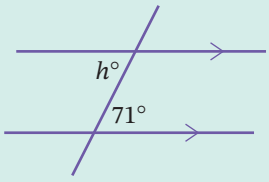
Example 3

Find the value of each variable, giving reasons.

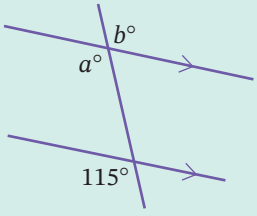
a



b



c



Solution

a $m + 127 = 180$ (co-interior angles on parallel lines)

$m = 53$

b $h = 71$ (alternate angles on parallel lines)

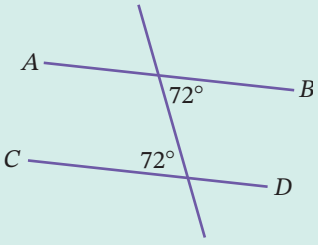
c $a = 115$ (corresponding angles on parallel lines)

$b = 115$ (vertically opposite angles)

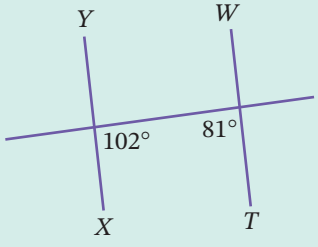
Example 4

Test whether each labelled pair of lines are parallel.

a



b



Solution

a Alternate angles (72°) are equal.

$\therefore AB \parallel CD$.

'$\therefore AB \parallel CD$' means 'Therefore line AB is parallel to line CD'

b Co-interior angles 102° and 81° are not supplementary ($102 + 81 = 183$).

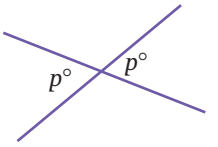
$\therefore XY$ is not parallel to TW.

EXERCISE 6.01 ANSWERS ON P. 580

Angle geometry U F R C

1 Which diagram shows complementary angles? Select the correct answer **A**, **B**, **C** or **D**.

A



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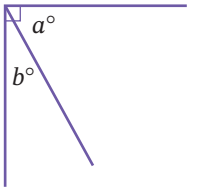
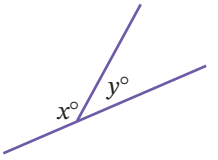
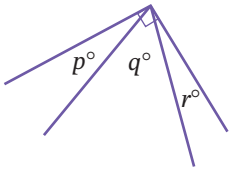


C



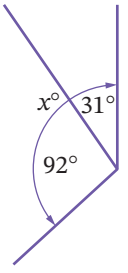
D



EXAMPLE 1

2 Find the value of each variable, giving reasons. R C

a



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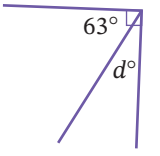

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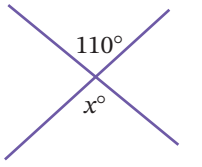

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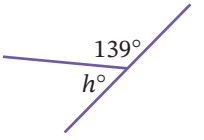

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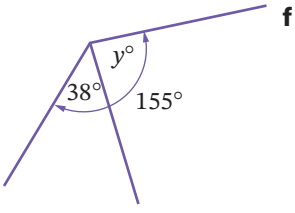

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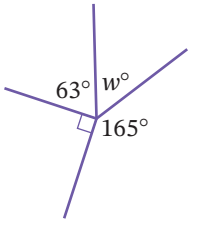

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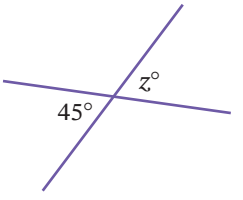

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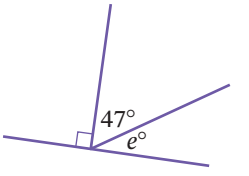

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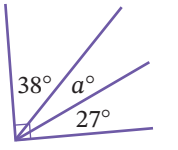

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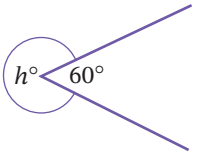

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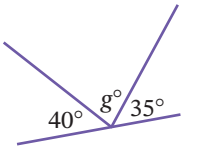

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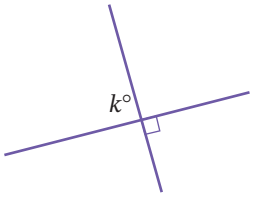


Foundation Standard Complex

9780170453325

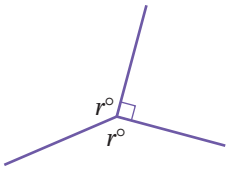
3 Which pair of angles are supplementary? Select **A**, **B**, **C** or **D**.

A 14°, 76° **B** 45°, 135° **C** 120°, 80° **D** 74°, 96°

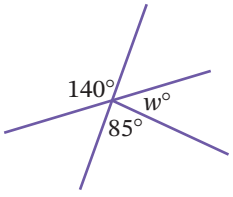
EXAMPLE 2

4 Find the value of each variable, giving reasons. R C

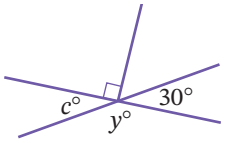
a



b



c



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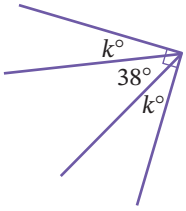

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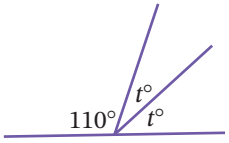

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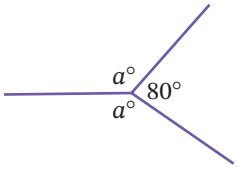
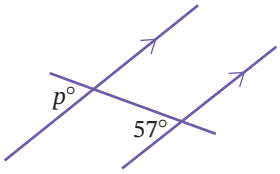


EXAMPLE 3

5 Find the value of each variable, giving reasons. R C

a



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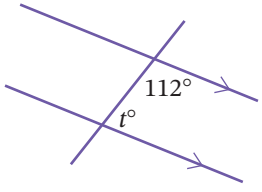

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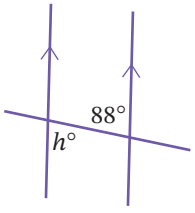

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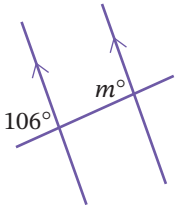

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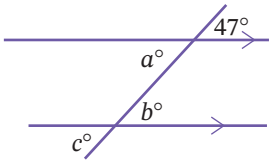

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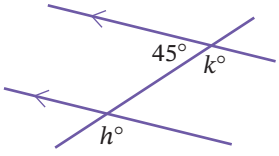

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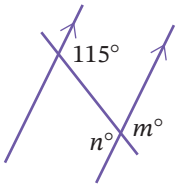

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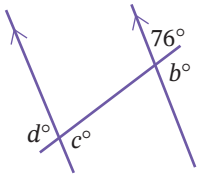

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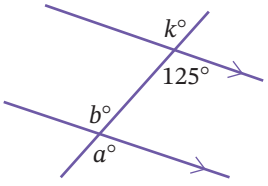

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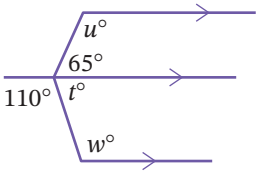

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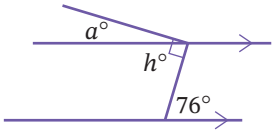

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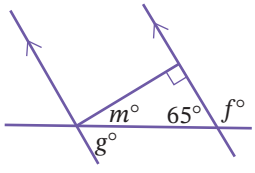
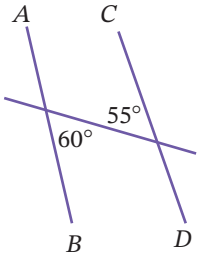


EXAMPLE 4

6 Test whether each labelled pair of lines are parallel. Give reasons for each answer. R C

a



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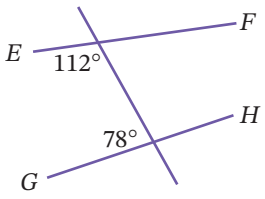

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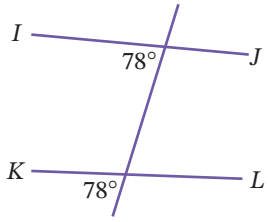
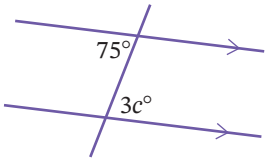


7 Find the value of each variable. R C

a



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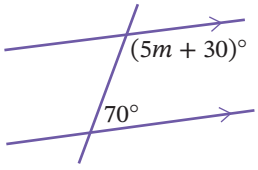

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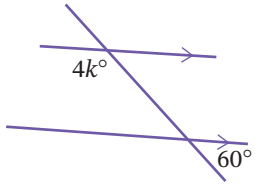

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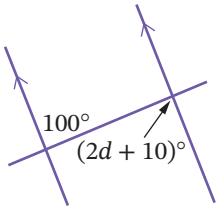

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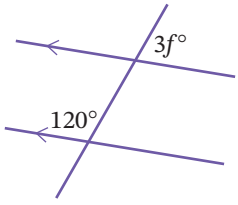

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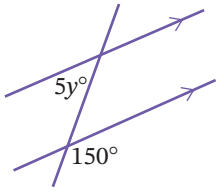


Did you know?

The Indian Pacific railway

The Indian Pacific train runs between Sydney and Perth and the rail journey is one of the longest in the world, covering 4352 km in 65 hours (3 days). The 478 km section of track across the Nullarbor Plain in South Australia forms the longest pair of straight parallel lines in the world.

Alamy Stock Photo/Duncan Sharrocks

1 **Why is the train called the Indian Pacific?**

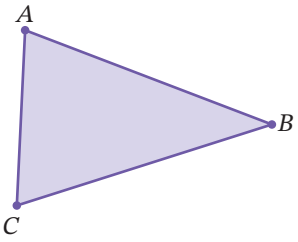
2 **What is its average speed?**

Technology

Constructing triangles

In this activity you will use dynamic geometry software to draw triangles.

1 Construct any triangle $\triangle ABC$. Now drag/move the vertices to change the triangle.



2 Measure the lengths of the triangle's sides and the sizes of its angles.

3 Change the triangle you have constructed to create:

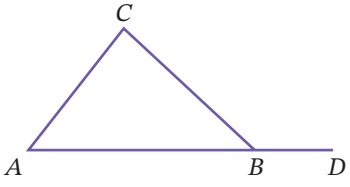
a an equilateral triangle **b** an isosceles triangle.

4 Construct other triangles and measure their sides and angles. Swap your measurements with other students in your class and try to reproduce their triangles using geometry software.

Technology

Exterior angle of a triangle

1 Draw $\triangle ABC$ and **produce** (extend) the side AB to D, as shown in the diagram.



The side AB has been **produced** to D.

$\angle CBD$ is an **exterior** angle of $\triangle BAC$.

2 Measure the size of $\angle CBD$, $\angle ACB$ and $\angle CAB$.

3 What is $\angle ACB + \angle CAB$? Compare your answer to $\angle CBD$. What do you notice?

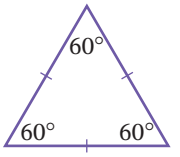
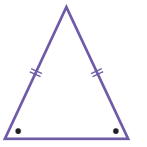
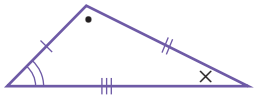
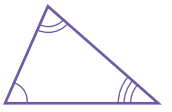
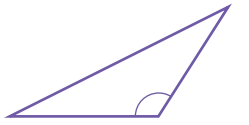
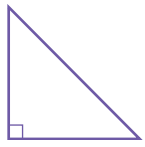
4 Now construct $\triangle PQR$ with side PQ produced to point S.

5 Measure the exterior angle $\angle RQS$ and then measure $\angle QPR$ and $\angle PRQ$. What do you notice? (The angles $\angle QPR$ and $\angle PRQ$ are the **interior opposite angles** to angle $\angle RQS$.)

6 Manipulate your triangle. What do you observe?

6.02 Triangle geometry

Triangles can be classified in 2 ways: by their sides or by their angles.

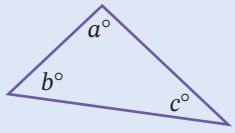
Classifying by sides		
Equilateral triangle	**Isosceles triangle**	**Scalene triangle**
		
3 equal sides (Also 3 equal angles, each 60°)	2 equal sides (Also 2 equal angles, opposite the equal sides)	No equal sides (Also no equal angles)
Classifying by angles		
Acute-angled triangle	**Obtuse-angled triangle**	**Right-angled triangle**
		
3 acute angles (less than 90°)	One obtuse angle (between 90° and 180°)	One right angle (90°)

Angle sum of a triangle

Angle sum of a triangle

The **angle sum** of a triangle is 180°.

$a + b + c = 180$



Proof:

Consider any triangle PQR with angles $a°$, $b°$ and $c°$.

Construct a line through P parallel to QR.

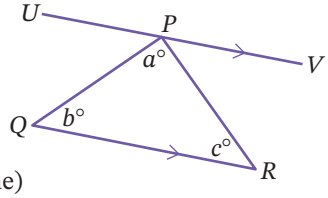
$\therefore \angle UPQ = b°$ (alternate angles, $UV \parallel QR$)

and $\angle VPR = c°$ (alternate angles, $UV \parallel QR$)

But $\angle UPQ + \angle QPR + \angle VPR = 180°$ (angles on a straight line)

$\therefore a° + b° + c° = 180°$

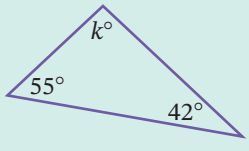
$\therefore$ The sum of the angles of a triangle is 180°.



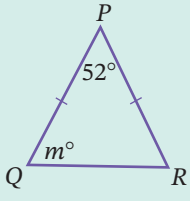
Example 5

Find the value of each variable, giving reasons.

a



b



Solution

a $k + 55 + 42 = 180$ (angle sum of a triangle)

$k = 180 - 55 - 42$

$= 83$

b $\angle R = \angle Q = m°$ ($\triangle PQR$ is isosceles)

$m + m + 52 = 180$ (angle sum of a triangle)

$2m + 52 = 180$

$2m = 128$

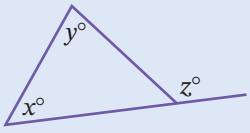
$m = 64$

The exterior angle of a triangle

Exterior angle of a triangle

The **exterior angle** of a triangle is equal to the sum of the 2 interior opposite angles.

$z = x + y$

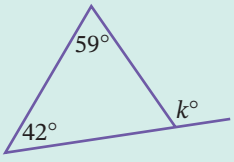


Example 6

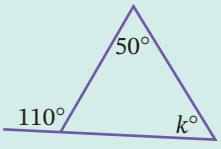
Exterior angle of a triangle

Find the value of k in each diagram.

a



b



Solution

a $k = 42 + 59$ (exterior angle of triangle)

$= 101$

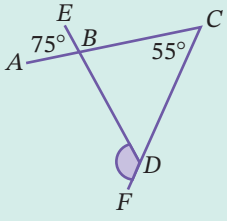
b $k + 50 = 110$ (exterior angle of triangle)

$k = 60$

Example 7

Exterior angle of a triangle

Find the size of the marked angle $\angle BDF$.



Solution

$\angle DBC = \angle ABE = 75°$ (vertically opposite angles)

$\angle BDF = 75° + 55°$ (exterior angle of $\triangle BCD$)

$= 130°$

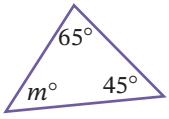
EXERCISE 6.02 ANSWERS ON P. 581

Triangle geometry U F R C

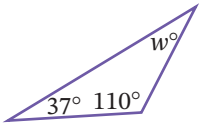
EXAMPLE 5

1 Find the value of each variable, giving reasons. R C

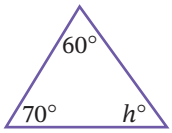
a



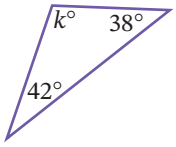
b



c



d



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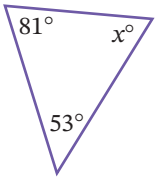

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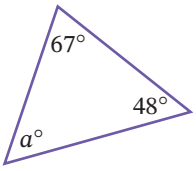
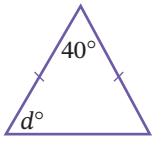
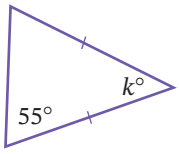


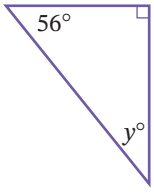
g



h



i



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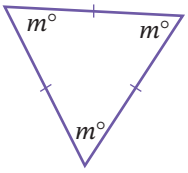
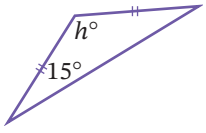


k



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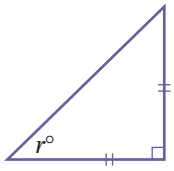


2 What is the size of each angle in an equilateral triangle? Select **A**, **B**, **C** or **D**.

A 30° **B** 45° **C** 60° **D** 90°

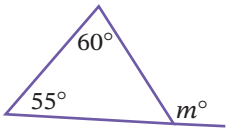
3 One angle of an isosceles triangle is equal to 80°. Which 2 of the following could be the sizes of the other angles? Select 2 of **A**, **B**, **C** or **D**. R

A 80° and 20° **B** 40° and 60° **C** 40° and 40° **D** 50° and 50°

EXAMPLE 6

4 Find the value of m. R C

a



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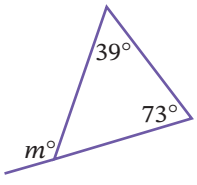
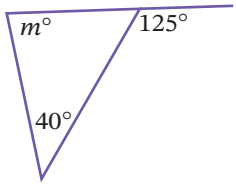


c



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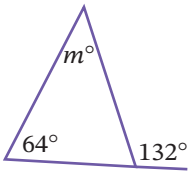

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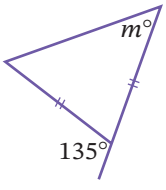

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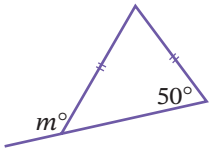
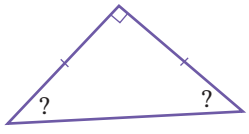


5 What is the size of the unknown angles in this triangle? Select **A**, **B**, **C** or **D**. R



A 30° **B** 45°

C 60° **D** 90°

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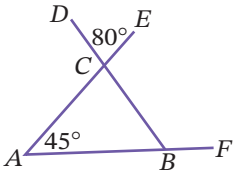
6 One angle of an obtuse-angled triangle is 50°. Which of the following could be the sizes of the other angles? Select **A**, **B**, **C** or **D**. R

A 80° and 50° **B** 100° and 30° **C** 65° and 65° **D** 45° and 95°

7 Find the size of $\angle CBF$ in each diagram. R C

a



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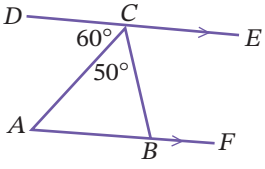

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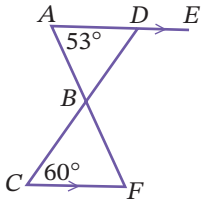

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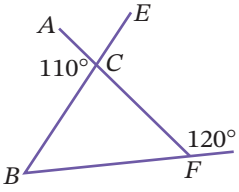

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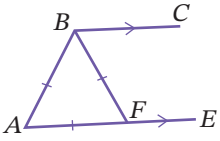

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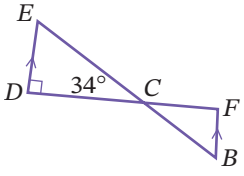
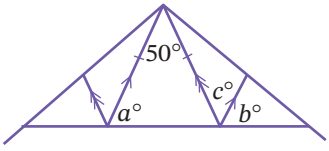
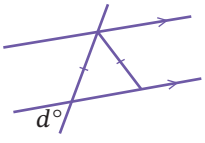


8 An isosceles triangle has a side length of 6 cm and one of its angles equal to 40°. Draw all possible shapes of the triangle. R

9 The diagram shows the shape of a roof truss. Find the value of each variable. R

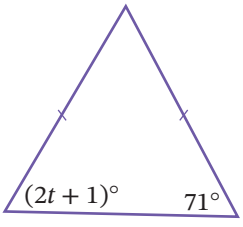


10 Copy this diagram and mark all angles equal to the angle marked $d°$. R



11 Find the value of each variable. R C

a



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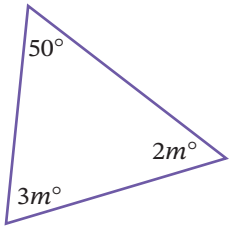

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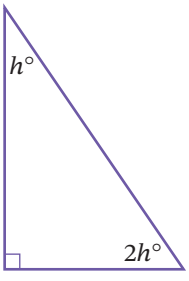
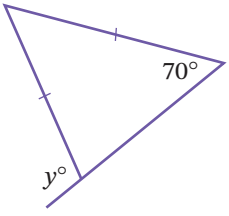


d



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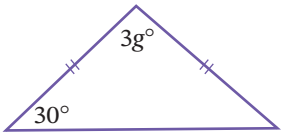

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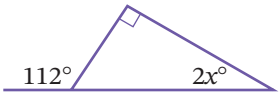

EXAMPLE 7

6.02

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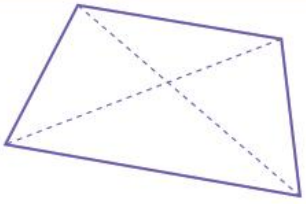
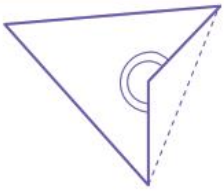
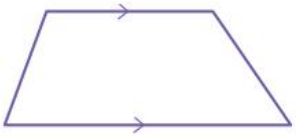
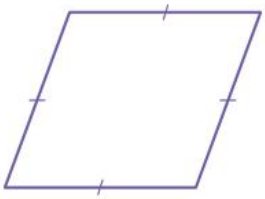
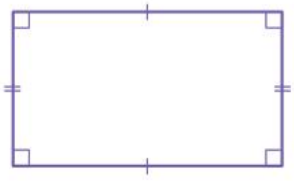
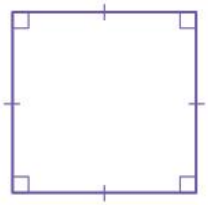
6.03 Quadrilateral geometry

Classifying quadrilaterals
Properties of triangles and quadrilaterals
Deductive geometry
Diagonal properties of quadrilaterals
Shapes and angles review
Naming shapes
Mixed angle problems
Parallel lines
Triangles and quadrilaterals
Angles and shapes

A **quadrilateral** is any shape with 4 straight sides. A quadrilateral may be either **convex** or **non-convex** (**concave**).

Convex quadrilateral	Non-convex quadrilateral
	
• All **vertices** (corners) point outwards. • All **diagonals** lie within the shape. • All angles are less than 180°.	• One vertex points inwards. • One diagonal lies outside the shape. • One angle is more than 180° (reflex angle).

There are 6 special types of quadrilaterals.

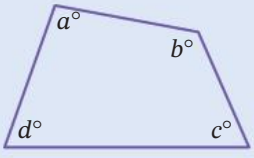
Trapezium	Kite	Parallelogram
		
One pair of parallel sides	2 pairs of equal adjacent sides 'Adjacent' means 'next to each other'.	2 pairs of parallel sides
Rhombus	**Rectangle**	**Square**
		
4 equal sides	4 right angles	4 equal sides and 4 right angles

Angle sum of a quadrilateral

The **angle sum of a quadrilateral** is 360°.

$a + b + c + d = 360$

This property is true for both convex and non-convex quadrilaterals.



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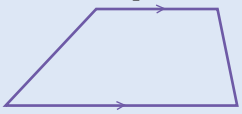
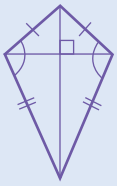
Properties of quadrilaterals

Trapezium



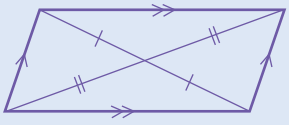
- One pair of opposite sides parallel
- No axes of symmetry

Kite



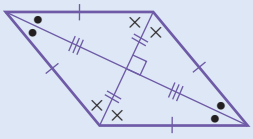
- 2 pairs of equal adjacent sides
- One pair of opposite angles equal
- One axis of symmetry
- Diagonals intersect at right angles

Parallelogram



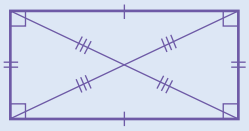
- Opposite sides are parallel and equal
- Opposite angles are equal
- No axes of symmetry
- Diagonals bisect each other

Rhombus



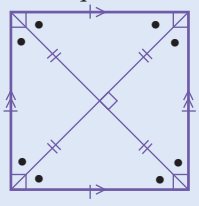
- All sides are equal
- 2 axes of symmetry
- A special type of parallelogram
- Diagonals bisect each other at right angles
- Diagonals bisect the angles of the rhombus

Rectangle



- All angles are 90° (right angles)
- 2 axes of symmetry
- A special type of parallelogram
- Diagonals are equal (in length)
- Diagonals bisect each other

Square

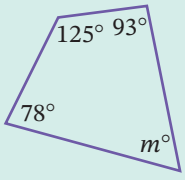


- All sides are equal
- All angles are 90° (right angles)
- 4 axes of symmetry
- A special type of rhombus and rectangle
- Diagonals are equal
- Diagonals bisect each other at right angles
- Diagonals bisect the angles of the square

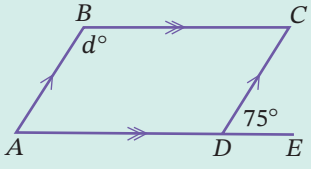
Example 8

Find the value of each variable.

a



b



Solution

a $m + 78 + 125 + 93 = 360$ (angle sum of a quadrilateral)

$m = 360 - 78 - 125 - 93$

$= 64$

b $\angle CDA = 180° - 75°$ (angles on a straight line)

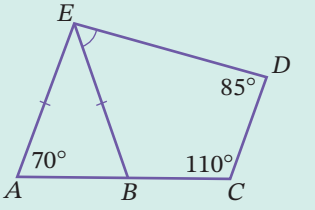
$= 105°$

$d = 105$ (opposite angles of a parallelogram)

Can you see another method for finding d?

Example 9

Find the size of $\angle BED$.



Solution

$\angle ABE = 70°$ (equal angles in isosceles $\triangle ABE$)

$\angle EBC = 180° - 70°$ (angles on a straight line)

$= 110°$

$\angle BED = 360° - 85° - 110° - 110°$ (angle sum of quadrilateral $BCDE$)

$= 55°$

Can you see another method for finding $\angle BED$?

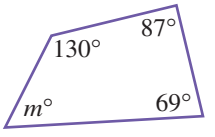
EXERCISE 6.03 ANSWERS ON P. 581

Quadrilateral geometry U F R C

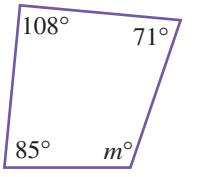
EXAMPLE 8

1 Find the value of m in each diagram. R

a



b



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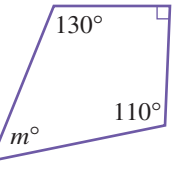

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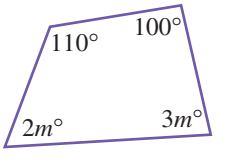

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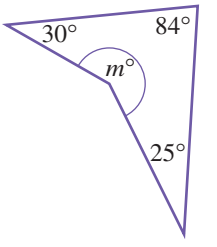

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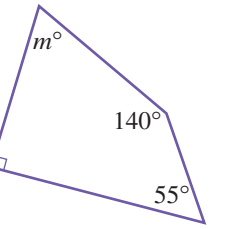

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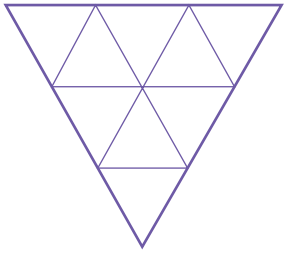
2 Copy and complete this table. R C

Property	Trapezium	Kite	Parallelogram	Rhombus	Rectangle	Square
One pair of opposite sides parallel						
Opposite sides parallel						
Opposite sides equal			✓			
All sides equal						
2 pairs of adjacent sides equal						
Diagonals equal					✓	
Diagonals bisect each other						
Diagonals meet at right angles						
Diagonals bisect the angles of the shape						
Opposite angles equal						
One pair of opposite angles equal						
All angles 90°						
Axes of symmetry	0					

3 Refer to the table you completed in question **2**, and name all quadrilaterals that have: R C

a no axes of symmetry
b one pair of parallel sides
c 4 equal sides
d equal diagonals
e opposite sides equal
f 4 axes of symmetry
g adjacent sides equal
h one axis of symmetry
i opposite sides parallel
j all angles measuring 90°
k 2 axes of symmetry
l diagonals which bisect each other
m opposite angles equal
n diagonals meeting at 90°

4 Each triangle in this diagram is an equilateral triangle. R



a Name the different types of quadrilaterals that you can find in the diagram.
b How many of each type of quadrilateral are there in the diagram?
c How many triangles are there? (It may help to copy the diagram so that you can draw on it using coloured pencils.)

5 Which statements are *always* true? R

A A rhombus is a parallelogram.

B The diagonals of a parallelogram meet at right angles.

C A square is a rhombus.

D A parallelogram is a quadrilateral with a pair of opposite sides equal and parallel.

E A square is a rectangle.

F The diagonals of an isosceles trapezium bisect each other.

G The opposite angles of a rhombus are equal.

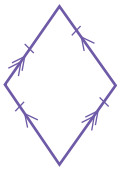
H The diagonals of a rhombus are equal and bisect each other at right angles.

I A rectangle is a parallelogram.

6 Name each quadrilateral using the properties marked on it. R

a

b



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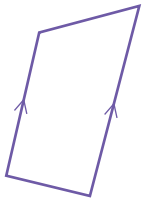

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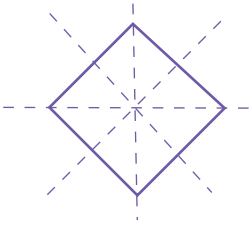

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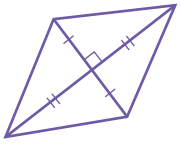

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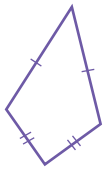

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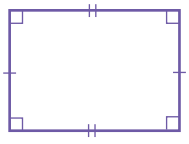

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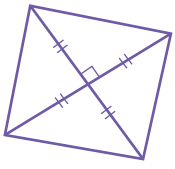

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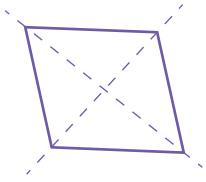
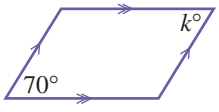


7 Find the value of each variable, giving reasons. R C

a



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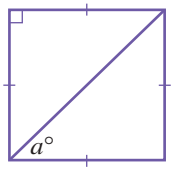

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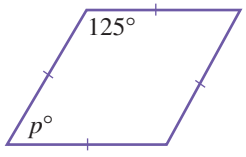

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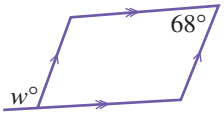

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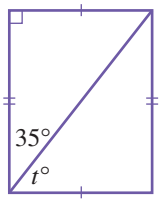

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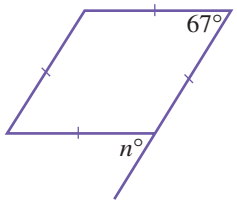

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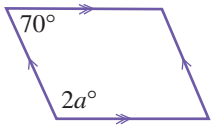

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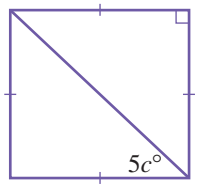

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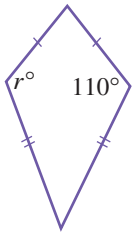
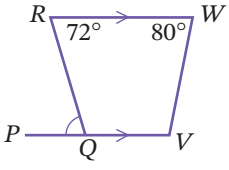


8 Find the size of $\angle PQR$ in each diagram, giving reasons. R C

EXAMPLE 9

6.03

a



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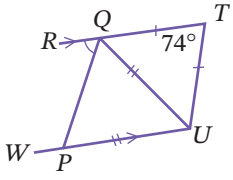

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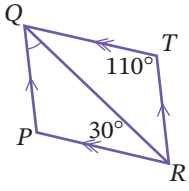

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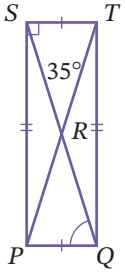

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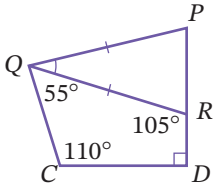

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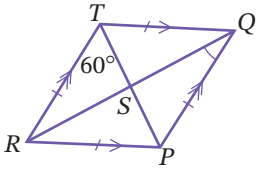


9 Use geometrical instruments or dynamic geometry software to construct each quadrilateral.

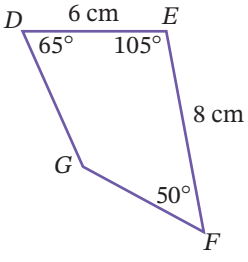
a Parallelogram $TUVW$ with sides $TU = 30$ mm, $UV = 60$ mm and $\angle TUV = 113°$.

b Rhombus with side length 5 cm.

c Convex quadrilateral $WXYZ$ with $WX = 65$ mm, $WZ = 40$ mm, $\angle ZWX = 54°$, $\angle WZY = 114°$ and $XY = 24$ mm.

d Non-convex quadrilateral with one side 4.5 cm and one angle 200°.

e



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Did you know?

Geometry in art

Geometry has been used in artwork for centuries.

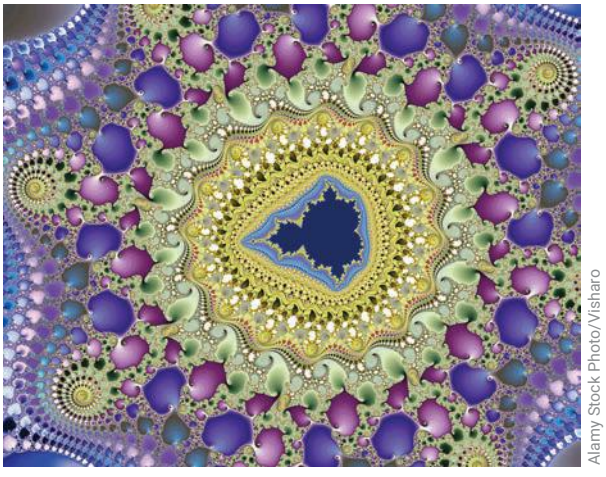
Tessellations can be used to create artwork, such as the regular hexagons used in the tessellation shown.

Indian and Islamic artworks show incredible intricacy, detail and colour within the geometric images used.

iStock.com/sharrocks

Fractals use geometric formulas to create infinitely-occurring images, so they are usually created using computer software. A famous fractal is the Mandelbrot set, shown here.

1. Design a tessellation or find examples of geometry in art.
2. Investigate fractals such the Mandelbrot set and the Koch snowflake, and their history.



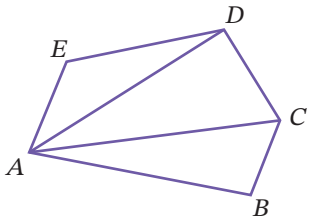
Alamy Stock Photo/Visharo

Investigation

Angle sum of a polygon

To find the angle sum of a pentagon *ABCDE* (5 sides), follow this reasoning.

From one vertex of the pentagon, draw all the diagonals. The diagonals from vertex *A* have divided the pentagon into 3 triangles.



$\therefore$ Angle sum of a pentagon = angle sum of the 3 triangles

$= 3 \times 180°$

$= 540°$

1 **a** Draw a hexagon (6 sides) and from one vertex draw all the diagonals.

b How many diagonals are there?

c How many triangles did you form?

d Hence find the angle sum of a hexagon.

2 Repeat the procedure to find the angle sum of:

a an octagon (8 sides) **b** a decagon (10 sides)

3 Copy and complete this table.

Polygon	Number of sides	Number of triangles	Angle sum
triangle	3	1	180°
quadrilateral	4	2	
pentagon	5		
hexagon			
octagon			
decagon			

4 Copy and complete this pattern: The number of triangles formed is always 2 _____ than the number of _____ of the polygon.

5 **a** Using your own words, describe the rule for finding the angle sum of a polygon.

b What is the angle sum of a polygon with 20 sides?

c For a polygon with n sides, write a formula for the sum of its angles. Discuss your result with other students.

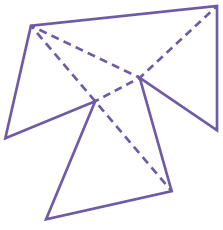
6 **a** Draw a non-convex octagon.

b Divide the polygon into triangles as shown.

c How many triangles have been formed?

d Find the angle sum of this non-convex octagon.

e Does your rule for the angle sum of a polygon also apply to the non-convex polygon?



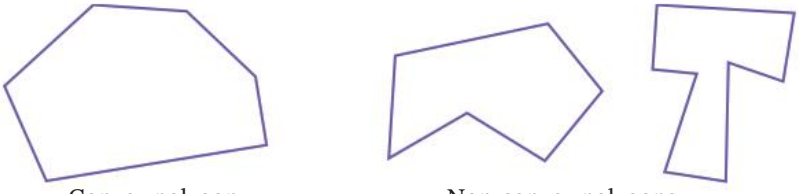
6.04 Angle sum of a polygon#

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

WS Angle sum of a polygon

A **polygon** is a general name for any shape with straight sides. The word is derived from the Greek, meaning 'many angles'. Shapes with curved sides, such as circles, ellipses and semicircles, are *not* polygons.

A polygon may be either convex or **non-convex** (**concave**).



Convex polygon | Non-convex polygons

In a convex polygon, all vertices point outwards, all diagonals lie within the shape and all angles are less than 180°.

In a non-convex polygon, some vertices point inwards, some diagonals lie outside the shape and some angles are more than 180° (reflex angles).

A polygon's name is determined by its number of sides.

Name	Number of sides
Pentagon	5
Hexagon	6
Heptagon	7
Octagon	8
Nonagon	9
Decagon	10
Undecagon	11
Dodecagon	12

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The images above show the Pentagon building in the USA, a 50 p coin from the UK, and a stop sign in Hong Kong, China.

Angle sum of a polygon

The **angle sum of a polygon** with n sides is given by the formula $A = 180(n - 2)°$.

This formula applies to both convex and non-convex polygons.

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Example 10

Find the angle sum of a nonagon (9 sides).

Solution

Substitute $n = 9$ into $A = 180(n - 2)°$.

Angle sum $= 180(9 - 2)°$

$= (180 \times 7)°$

$= 1260°$

Example 11

Find the number of sides of a polygon that has an angle sum of 720°.

Solution

$180(n - 2) = 720$ $\quad A = 720$

$180n - 360 = 720$

$180n = 1080$

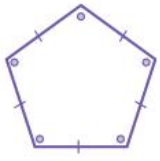
$n = \frac{1080}{180}$

$= 6$

∴ The polygon has 6 sides (hexagon).

Regular polygons

A **regular polygon** has all angles equal and all sides equal. For example, a regular pentagon has 5 equal sides and 5 equal angles. A square is a regular polygon but a rhombus is not.



WS Equal angles

Angle in a regular polygon

The size of each angle in a **regular polygon** with n sides is

$$\frac{\text{Angle sum}}{\text{No. of sides}} = \frac{180(n - 2)°}{n}$$

Example 12

Find the size of one angle in a regular hexagon.

Solution

A hexagon has 6 sides ($n = 6$).

Size of one angle $= \frac{180(6 - 2)°}{6}$

$= \frac{(180 \times 4)°}{6}$

$= 120°$

Each angle in a regular hexagon is 120°.

EXERCISE 6.04 ANSWERS ON P. 582

Angle sum of a polygon U F R C

1 How many sides has:

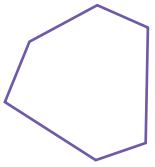
a a hexagon? **b** a quadrilateral? **c** a nonagon?

d a decagon? **e** a heptagon? **f** a pentagon?

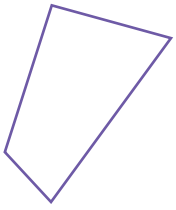
g a dodecagon? **h** an octagon? **i** an undecagon?

2 Name each polygon. R C

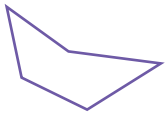
a



b

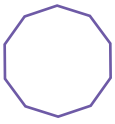


c

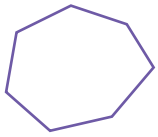


d

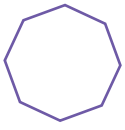
e



f



g



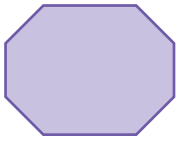
h

3 Which polygons from question **2** are: R C

a convex? **b** regular? **c** concave?

4 Which shape is a regular octagon? Select **A**, **B**, **C** or **D**.

A



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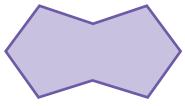
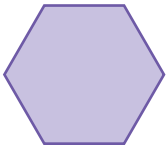
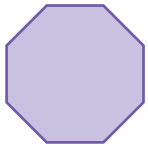


C



D



5 What is the more common name for: R C

a a regular triangle? **b** a regular quadrilateral?

EXAMPLE 10

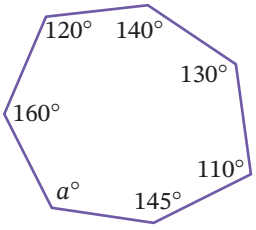
6 Find the angle sum of a polygon with:

a 5 sides **b** 8 sides **c** 15 sides **d** 12 sides

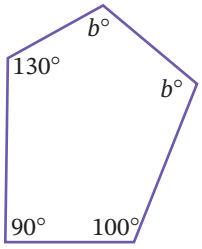
e 7 sides **f** 10 sides **g** 20 sides **h** 11 sides

7 Find the value of each variable. R

a



b



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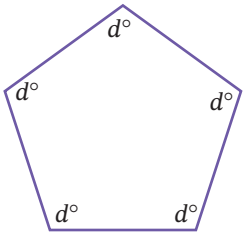

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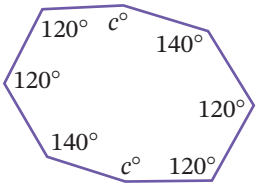

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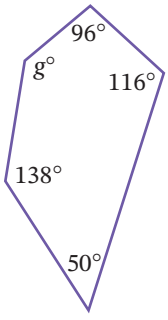

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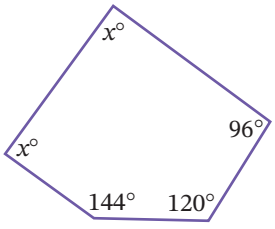

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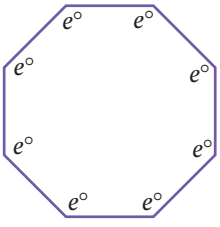
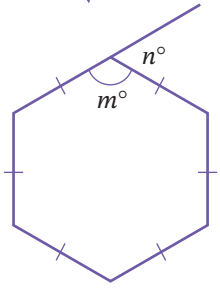
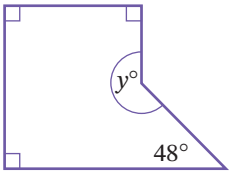


h



i



8 Find the number of sides of a polygon that has an angle sum of: R

a 2160° **b** 1620° **c** 3960° **d** 2700°

EXAMPLE 11

9 Which polygon has an angle sum of 1440°? Select **A**, **B**, **C** or **D**. R

A pentagon **B** decagon **C** nonagon **D** octagon

10 Find the size of one angle in a regular: R

EXAMPLE 12

a octagon **b** decagon **c** dodecagon **d** hexagon

11 The angle sum of a regular polygon is 6840°. R

a How many sides does the polygon have?

b Find the size of each angle.

12 How many sides does a regular polygon have if each of its angles is: R

a 165°? **b** 170°? **c** 144°?

Mental skills 6: Maths without calculators ANSWERS ON P. 582

Dividing decimals

To divide one decimal by another, first move the decimal points in both decimals the same number of places to the right so that the second decimal is a whole number.

1 Study each example.

a $0.24 \div 0.06 = 24 \div 6 = 4$

b $0.45 \div 0.5 = 4.5 \div 5 = 0.9$

c $0.006 \div 0.3 = 0.06 \div 3 = 0.02$

d $27 \div 0.9 = 270 \div 9 = 30$

e $1.6 \div 0.4 = 16 \div 4 = 4$

f $5.6 \div 0.07 = 560 \div 7 = 80$

2 Now evaluate each quotient.

a $0.25 \div 0.5$ **b** $63 \div 0.7$ **c** $3.2 \div 0.4$ **d** $0.18 \div 0.2$

e $2.7 \div 0.03$ **f** $0.042 \div 0.06$ **g** $4 \div 0.5$ **h** $1.2 \div 0.04$

i $0.072 \div 0.9$ **j** $0.35 \div 0.1$ **k** $0.28 \div 0.07$ **l** $0.033 \div 0.11$

3 Study each example.

Given that $112 \div 14 = 8$, evaluate each expression.

a
$$\begin{aligned} 112 \div 1.4 &= 112.0 \div 1.4 \\ &= 1120 \div 14 \\ &= 112 \times 10 \div 14 \\ &= 112 \div 14 \times 10 \\ &= 8 \times 10 \\ &= 80 \end{aligned}$$

Estimate: $112 \div 1.4 \approx 112 \div 1 = 112$

b
$$\begin{aligned} 0.112 \div 0.14 &= 0.112 \div 0.14 \\ &= 11.2 \div 14 \\ &= 112 \div 10 \div 14 \\ &= 112 \div 14 \div 10 \\ &= 8 \div 10 \\ &= 0.8 \end{aligned}$$

Estimate: $0.112 \div 0.14 \approx 0.1 \div 0.1 = 1$

c
$$\begin{aligned} 1120 \div 1.4 &= 11\,200 \div 14 \\ &= 112 \times 100 \div 14 \\ &= 112 \div 14 \times 100 \\ &= 8 \times 100 \\ &= 800 \end{aligned}$$

Estimate: $1120 \div 1.4 \approx 1120 \div 1 = 1120$

d
$$\begin{aligned} 1.12 \div 14 &= 112 \div 100 \div 14 \\ &= 112 \div 14 \div 100 \\ &= 8 \div 100 \\ &= 0.08 \end{aligned}$$

Estimate: $1.12 \div 14 \approx 1.12 \div 10 = 0.112$

4 Now given that $368 \div 23 = 16$, evaluate each quotient.

a $36.8 \div 2.3$ **b** $368 \div 2.3$ **c** $3.68 \div 2.3$ **d** $0.368 \div 0.23$

e $36.8 \div 23$ **f** $3.68 \div 0.23$ **g** $36.8 \div 0.23$ **h** $0.368 \div 2.3$

I $0.368 \div 23$ **j** $3.68 \div 0.023$ **k** $3.68 \div 23$ **l** $0.368 \div 230$

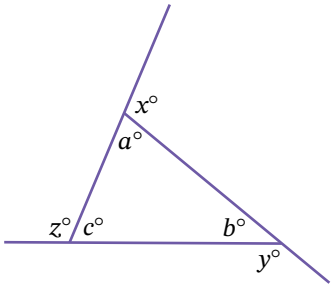
Investigation

Exterior angle sum of a convex polygon

1 **a** Draw a triangle and extend each side to show the exterior angles of the triangle as shown.

Exterior angles are $x°$, $y°$ and $z°$.

Interior angles are $a°$, $b°$ and $c°$.



b Use a protractor to measure angles $x°$, $y°$ and $z°$.

c Find the sum of the exterior angles.

d Looking at the diagram, what must be the value of $a° + x° + b° + y° + c° + z°$?

e But what do we know about the value of $a° + b° + c°$?

f Therefore, what must be the value of $x° + y° + z°$?

2 Repeat this procedure for the exterior angles of a convex quadrilateral. What is the sum of the exterior angles of a convex quadrilateral?

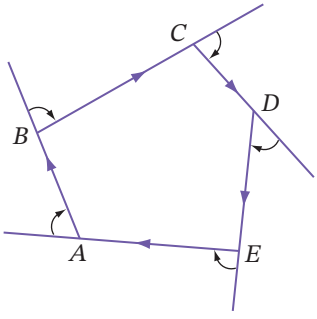
3 Repeat the procedure for a convex pentagon and a convex hexagon. What do you notice about the sum of the exterior angles of those polygons?

4 **a** Draw any convex polygon and extend the sides. Label the vertices of your polygon A, B, C, etc.

b Start at A and move around the polygon, turning in the direction indicated at each vertex.

c Continue until you return to A, facing the same way you started. What must be the sum of the turns in any round trip of a convex polygon?

d Test whether this rule works for a non-convex polygon.



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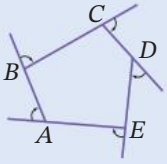
6.05 Exterior angle sum of a convex polygon#

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

WS Angles in polygons

Exterior angle sum of a convex polygon

The **sum of the exterior angles** of a convex polygon is 360°.

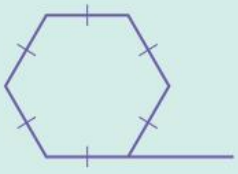


Example 13

For a regular hexagon, find the size of:

a each exterior angle

b each (interior) angle.



Solution

a Sum of exterior angles = 360°

One exterior angle = 360° ÷ 6

= 60°

b Each angle = 180° − 60° (angles on a straight line)

= 120°

OR: Each angle = $\frac{180(6-2)^\circ}{6}$

= 120°

Example 14

Find the number of sides of a regular polygon if:

a each exterior angle is 12°

b each (interior) angle is 160°.

Solution

a Sum of exterior angles = 360°

Number of exterior angles = 360 ÷ 12

= 30

∴ The regular polygon has 30 sides.

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b Exterior angle $= 180° - 160°$ (angles on a straight line)

$= 20°$

Sum of exterior angles $= 360°$

Number of exterior angles $= 360 \div 20$

$= 18$

$\therefore$ The regular polygon has 18 sides.

OR: $\frac{180(n-2)°}{n} = 160°$

$180(n - 2) = 160n$

$180n - 360 = 160n$

$20n - 360 = 0$

$20n = 360$

$n = \frac{360}{20} = 18$

$\therefore$ The regular polygon has 18 sides.

EXERCISE 6.05 ANSWERS ON P. 582

Exterior angle sum of a convex polygon U F R

EXAMPLE 13

1 Find the size of each exterior angle of a regular:

a octagon **b** decagon

c 15-sided polygon **d** nonagon

2 Find the size of each angle of a regular: R

a pentagon **b** dodecagon

c nonagon **d** 16-sided polygon

EXAMPLE 14

3 Find the number of sides of a regular polygon if each exterior angle is:

a 24° **b** 36° **c** 40°

d 10° **e** 18° **f** 60°

4 Find the number of sides of a regular polygon if each angle is: R

a 150° **b** 175° **c** 162°

d 140° **e** 108° **f** 176°

Power plus ANSWERS ON P. 582

1 How many diagonals has:

a a quadrilateral? **b** an octagon? **c** a dodecagon?

2 Name all quadrilaterals whose diagonals:

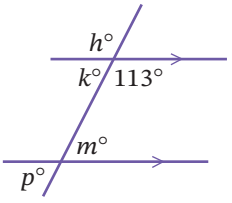
a bisect each other at right angles **b** bisect each other

c intersect at right angles **d** have equal length

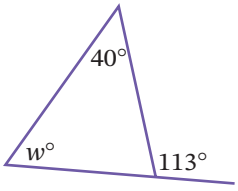
e bisect the angles of the quadrilateral **f** are equal and bisect each other

3 Find the value of each variable, giving reasons.

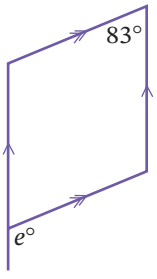
a



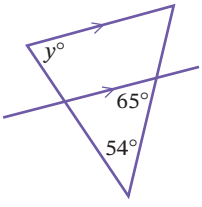
b



c



d



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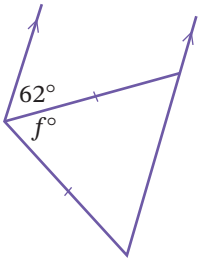

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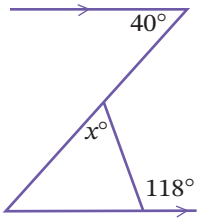

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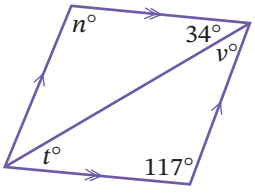

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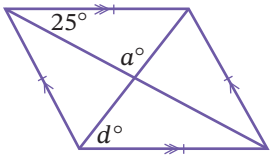

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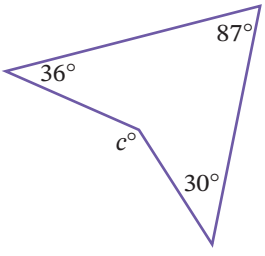

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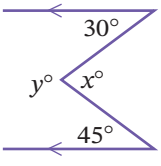

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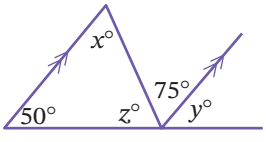

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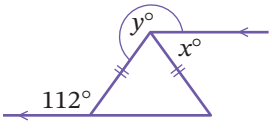

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CHAPTER 6 REVIEW

Language of maths

Qz Shapes and angles

adjacent	alternate	angle sum	bisect
co-interior	convex	corresponding	equilateral
exterior angle	interior	isosceles	kite
parallelogram	polygon	quadrilateral	rectangle
regular polygon	rhombus	right angle	square
supplementary	trapezium	vertex	vertically opposite

1 What shape has 2 pairs of equal adjacent sides?

2 Name one property of a convex quadrilateral.

3 What type of angle is created when one side of a triangle is extended?

4 What is the sum of the exterior angles of any polygon?

5 What is a **regular polygon?** Are all regular polygons also **convex**?

6 Copy and complete: The exterior angle of a triangle is equal to the _____ of the _____ _____ _____ angles.

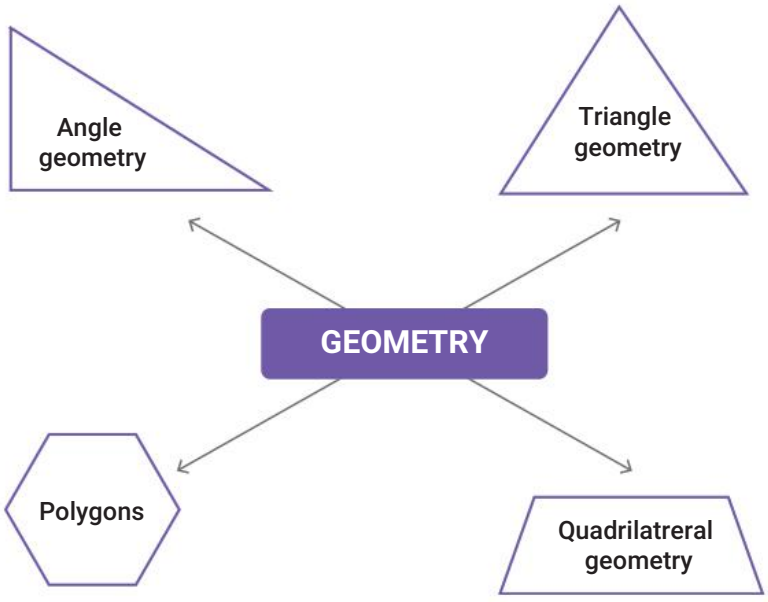
Topic summary

- Write 3 ideas from this topic that were new to you.
- Summarise what you know about the angle sum of a triangle, quadrilateral and polygon.
- Name the types of triangles, choose one and list all of its properties.
- Name the 6 special quadrilaterals, choose one and list all of its properties.

WS Geometry summary poster

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.

WS Mind map: Geometry



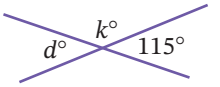
TEST YOURSELF 6

ANSWERS ON P. 583

6.01

1 Find the value of each variable, giving reasons.

a



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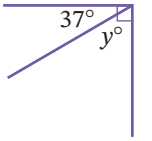

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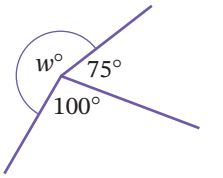

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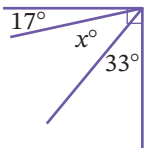

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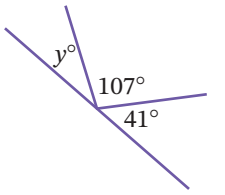

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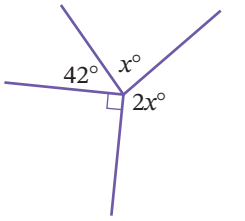
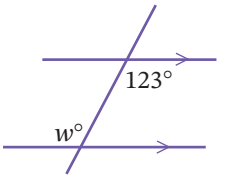


6.01

2 Find the value of w in each diagram, giving reasons.

a



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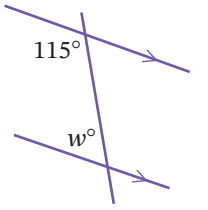

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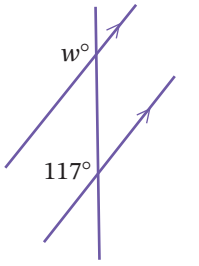

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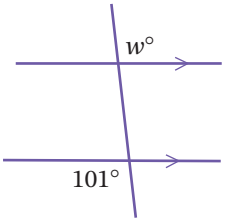

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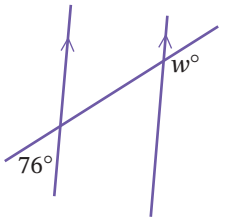

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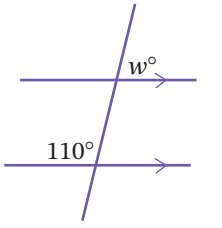
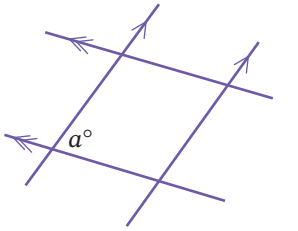


6.01

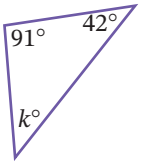
3 Copy the diagram shown and mark all angles equal to $a°$.



6.02

4 Find the value of each variable, giving reasons.

a



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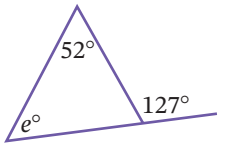

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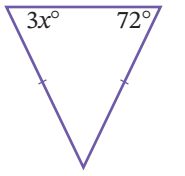

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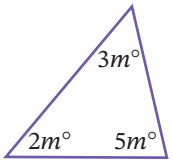
d



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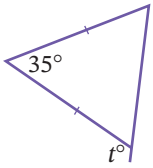

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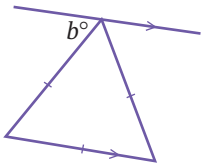

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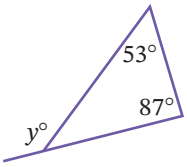

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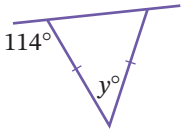

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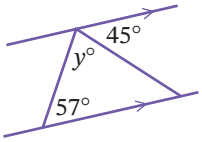
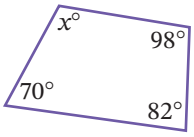


5 What are the sizes of the angles in: 6.02

a an equilateral triangle? **b** a right-angled isosceles triangle?

6 Find the value of each variable, giving reasons. 6.03

a



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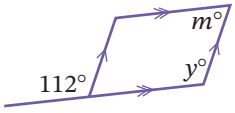

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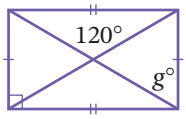

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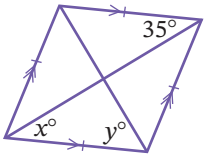

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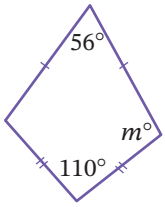

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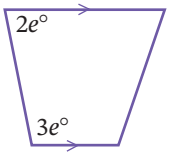

7 Name all quadrilaterals that have: 6.03

a both pairs of opposite sides parallel **b** 2 equal diagonals

c all sides equal **d** diagonals that bisect each other.

8 Draw: 6.04

a a regular pentagon **b** a non-regular pentagon **c** a concave pentagon

9 a Show that the angle sum of a decagon is 1440°. 6.04

b Find the size of one angle in a regular nonagon.

10 a Find the size of each exterior angle in a regular hexagon. 6.05

b If each angle in a regular polygon is 150°, how many sides does it have?

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7

NUMBER AND ALGEBRA

EQUATIONS

Equation-solving has been recorded as far back as 1500 BCE. It was first used in ancient Babylon and Egypt and was brought to Europe from India by the Arabs during the 9th century. The word 'algebra' comes from the Arabic word *al-jabr*, meaning restoration, the process of performing the same operation on both sides of an equation to solve the equation.

Chapter outline

	Working mathematically				
7.01 Equations with variables on both sides	U	F		R	
7.02 Equations with brackets	U	F		R	C
7.03 Equation problems	U	F	PS	R	C
7.04 Equations with algebraic fractions	U	F		R	
7.05 Simple quadratic equations $ax^2 = c$	U	F		R	C
7.06 Simple cubic equations $ax^3 = c$*#	U	F		R	C
7.07 Equations and formulas	U	F	PS	R	
7.08 Changing the subject of a formula*#	U	F		R	C

*** STAGE 5.3**

NSW ONLY, NOT AUSTRALIAN CURRICULUM

Wordbank

cubic equation An equation involving a variable cubed (power of 3), such as $2x^3 = 250$.

equation A mathematical statement that 2 quantities are equal, involving algebraic expressions and an equals sign (=)

formula A rule written as an algebraic equation, using variables.

inverse operation An opposite used in solving an equation, for example, the inverse operation of multiplying is dividing

linear equation An equation involving a variable that is not raised to a power, such as $2x + 9 = 17$.

quadratic equation An equation involving a variable squared (power of 2), such as $3x^2 - 6 = 69$.

solution The answer to an equation or problem, the correct value(s) of the variable that makes an equation true

solve (an equation) To find the value of an unknown variable in an equation

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- solve linear equations and problems involving equations
- solve linear equations involving simple algebraic fractions
- (STAGE 5.3) solve linear equations involving complex algebraic fractions
- solve simple quadratic equations of the form $ax^2 = c$
- (STAGE 5.3) solve simple cubic equations of the form $ax^3 = c$
- use formulas to solve problems
- (STAGE 5.3) change the subject of a formula

Solving equations

Backtracking

Solving equations by balancing

Solving equations by backtracking

Solving equations using diagrams

SkillCheck ANSWERS ON P. 583

1 Solve each **equation**. Use substitution to check your **solutions**.

a $3x - 10 = 5$ **b** $4 + 2y = 21$ **c** $12y + 5 = 23$ **d** $\frac{m}{4} - 6 = 2$

e $\frac{x-5}{4} = 7$ **f** $\frac{5r}{3} = 10$ **g** $\frac{-2x}{5} = 3$ **h** $11 - 2a = 17$

i $20 + 4d = -6$ **j** $\frac{w}{2} + 3 = 4$ **k** $\frac{a}{6} - 1 = 7$ **l** $6 - \frac{n}{3} = 4$

2 Using n to represent 'the number', write an expression for each of these statements.

a The product of the number and 7.

b The square of the number.

c 5 times the sum of the number and 8.

d The number decreased by 20.

e The product of 6 and the number, decreased by nine.

f If the number is even, the *next* even number.

7.01 Equations with variables on both sides

Equations with unknowns on both sides

Equations with variables on both sides

Equations with variables on both sides

For **equations with variables on both sides**, perform operations on both sides to move:

- all the variables onto one side of the equation
- all the numbers onto the other side of the equation.

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Example 1

Solve each equation.

a $7x + 7 = 2x + 2$ **b** $9 - 6y = 10 - 2y$

Solution

a

$7x + 7 = 2x + 2$

$7x - 2x + 7 = 2x - 2x + 2$ Subtracting $2x$ from both sides

$5x + 7 = 2$

$5x + 7 - 7 = 2 - 7$ Subtracting 7 from both sides

$5x = -5$

$\frac{5x}{5} = \frac{-5}{5}$ Dividing both sides by 5

$x = -1$

Check:

LHS $= 7 \times (-1) + 7 = 0$

RHS $= 2 \times (-1) + 2 = 0$

LHS = RHS.

LHS = left-hand side, RHS = right-hand side.

b

$9 - 6y = 10 - 2y$

$9 - 6y + 2y = 10 - 2y + 2y$ Adding $2y$ to both sides

$9 - 4y = 10$

$9 - 4y - 9 = 10 - 9$ Subtracting 9 from both sides

$-4y = 1$

$\frac{-4y}{-4} = \frac{1}{-4}$ Dividing both sides by (-4)

$y = -\frac{1}{4}$

Check:

LHS $= 9 - 6 \times \left(-\frac{1}{4}\right) = 10\frac{1}{2}$

RHS $= 10 - 2 \times \left(-\frac{1}{4}\right) = 10\frac{1}{2}$

LHS = RHS.

7.01

EXERCISE 7.01 ANSWERS ON P. 583

Equations with variables on both sides U F R

1 Solve each equation and check your solutions. R

EXAMPLE 1

a $5w + 3 = 2w + 21$ **b** $2q - 10 = q - 4$ **c** $13x + 1 = 8x + 26$

d $12n + 3 = 5n - 11$ **e** $8y - 10 = 10y - 30$ **f** $3m - 2 = 10 - 3m$

g $9 - 2a = a - 9$ **h** $9 - 2x = 18 + 7x$ **i** $12y + 6 = 6 + 9y$

j $-12 - 10u = -20 - 18u$ **k** $15 - 7x = 22 - 3x$ **l** $10 - 6x = -15 - 11x$

2 For each equation, select the correct solution **A**, **B**, **C** or **D**.

a $6x - 1 = 2x + 11$

A $x = 12$ **B** $x = 3$ **C** $x = 0$ **D** $x = 2.5$

b $11 - 4p = 2p + 2$

A $p = 6.5$ **B** $p = 2$ **C** $p = 1.5$ **D** $p = 3$

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3 Solve each equation. R

a $7w + 15 = w + 3$ **b** $10 - 3t = 16 + t$ **c** $4a + 2 = 10 - 4a$

d $50 + 7y = 20 - 3y$ **e** $8y - 2 = 10y + 1$ **f** $9y + 3 = 9 - y$

g $9 - t = 7t - 2$ **h** $5y + 2 = 17 - y$ **i** $25 - 12k = 15 - 6k$

4 Solve $-3n - 8 = -7n - 12$. Select **A**, **B**, **C** or **D**.

A $n = 5$ **B** $n = 2$ **C** $n = -1$ **D** $n = -0.4$

Did you know?

The outer planets

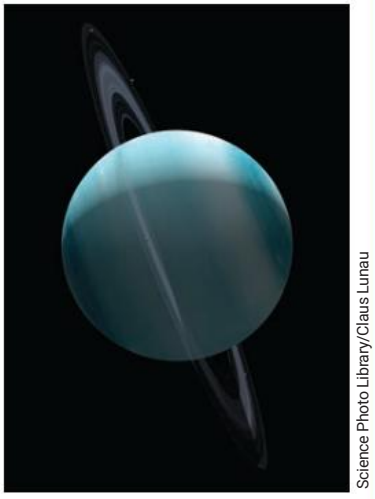
In 1781, British astronomer William Herschel discovered the planet Uranus. At that time, it was the farthest planet known in our solar system. However, astronomers found that Uranus' orbit around the Sun did not follow the expected path.

Working separately, mathematicians John Couch Adams of England and Urbain Leverrier of France both predicted that this different orbit was caused by an unknown planet. They calculated the position of this undiscovered planet using a number of equations.

In 1846, a German astronomer called Johann Galle located this planet and named it Neptune.

The dwarf planet, Pluto, was discovered in a similar manner.

How long ago was Pluto discovered?



Science Photo Library/Claus Lunau

7.02 Equations with brackets

WS Equations 2

WS Checking solutions

Equations

Equations with brackets

For **equations with brackets (grouping symbols)**, expand the expressions and then solve as usual.

Example 2

Solve each equation.

a $3(a + 7) = 6$

b $9(m - 5) = 7(m + 1)$

c $10y - 3(2y - 5) = 6(8 - 3y)$

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Solution

a $3(a + 7) = 6$

$3a + 21 = 6$

Can you think of another way to solve this equation without expanding?

Expanding the expression to make it a 2-step equation

$3a + 21 - 21 = 6 - 21$ Subtract 21 from both sides

$3a = -15$

$\frac{3a}{3} = \frac{-15}{3}$ Divide both sides by 3

$a = -5$ Check: $3(-5 + 7) = 3 \times 2 = 6$

b $9(m - 5) = 7(m + 1)$

$9m - 45 = 7m + 7$ Expanding brackets on both sides

$9m - 7m - 45 = 7m - 7m + 7$ Subtracting $7m$ from both sides

$2m - 45 = 7$

$2m - 45 + 45 = 7 + 45$ Adding 45 to both sides

$2m = 52$

$\frac{2m}{2} = \frac{52}{2}$ Dividing both sides by 2

$m = 26$

Check:

LHS $= 9 \times (26 - 5) = 9 \times 21 = 189$

RHS $= 7 \times (26 + 1) = 7 \times 27 = 189$

LHS = RHS.

c $10y - 3(2y - 5) = 6(8 - 3y)$

$10y - 6y + 15 = 48 - 18y$ Expanding brackets on both sides

$4y + 15 = 48 - 18y$ Collecting like terms

$4y + 18y + 15 = 48 - 18y + 18y$ Adding $18y$ to both sides

$22y + 15 = 48$

$22y + 15 - 15 = 48 - 15$ Subtracting 15 from both sides

$22y = 33$

$\frac{22y}{22} = \frac{33}{22}$ Dividing both sides by 22

$y = 1\frac{1}{2}$

EXERCISE 7.02 ANSWERS ON P. 584

Equations with brackets UFRC

1 Solve each equation. R

a	$2(m + 3) = 8$	**b**	$3(x + 1) = 9$	**c**	$5(y - 2) = 15y$
d	$35 = 7(k + 1)$	**e**	$4(3 - a) = 16$	**f**	$11 = 9(1 + 2p)$
g	$3h = 4(h + 6)$	**h**	$6(m - 10) = -6$	**i**	$8u = 11(u - 3)$
j	$27 = 7(2y + 1)$	**k**	$5(2 + 3p) = -8$	**l**	$22x = 9(4x - 3)$

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2 In which line has an error been made in solving $5(x - 3) = 25$? Select **A**, **B**, **C** or **D**.

$5(x - 3) = 25$

$5x - 8 = 25$ Line 1

$5x - 8 + 8 = 25 + 8$ Line 2

$5x = 33$ Line 3

$x = \frac{33}{5}$ Line 4

$= 6\frac{3}{5}$

A Line 1 **B** Line 2 **C** Line 3 **D** Line 4

3 Show that $k = 5$ is the solution to $12(k - 1) = 48$. R C

4 Show that $a = 6$ is the solution to $10 + a = 2(2 + a)$. R C

5 Solve each equation. R

a $8(m + 2) = 5(m + 5)$ **b** $2(y - 3) = 4(y - 5)$

c $3(2 + x) = 4(1 + x)$ **d** $5(p + 2) = 3(6 + p)$

e $5n + 6 = 2(2n + 1)$ **f** $2(4 - 3x) = 4(7 - 3x)$

g $4(3w - 1) = 5(4 + 3w)$ **h** $-2(x + 1) = 16 - 5x$

i $-8y - 5 = 5(2y - 3)$

6 Show that the solution to $5(2m - 2) = 6(m + 1)$ is $m = 4$. R C

7 Solve each equation. R

a $5(m + 6) + 10 = 3(m + 2) + 20$ **b** $3(y + 2) - 10 = 2(y - 1) + 5$

c $7y + 2(y + 5) = 4(y - 10)$ **d** $3x + 4(5 + x) = 6(2 + x) + 20$

e $5y + 2(y - 3) = 4y + 2(2y + 10)$ **f** $11 - 2(5 + y) = 4(3 + y) - 1$

g $8 - 3(1 - m) = 5(m + 3) + 4$ **h** $12 - 7(2y - 5) = 6 - 15(2 - 5y)$

7.03 Equation problems

WS Angle problems with algebra

Writing and solving equations

Steps in solving word problems using equations

1 Read the problem carefully and determine what needs to be found: 'What is the question?'

2 Use a variable to represent the unknown quantity.

3 Write the problem as an equation.

4 Solve the equation.

5 Answer the problem.

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Example 3

When three-quarters of a number is decreased by 8, the result is 46. What is the number?

Solution

Let the number be x.

$\frac{3x}{4} - 8 = 46$ Translating from words to algebra

$\frac{3x}{4} - 8 + 8 = 46 + 8$ Adding 8 to both sides

$\frac{3x}{4} = 54$

$\frac{3x}{4} \times 4 = 54 \times 4$ Multiplying both sides by 4

$3x = 216$

$\frac{3x}{3} = \frac{216}{3}$ Dividing both sides by 3

$x = 72$ Check: $\frac{3 \times 72}{4} - 8 = 54 - 8 = 46$

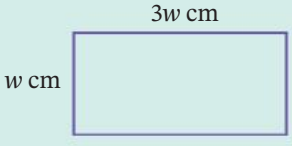
The number is 72.

Example 4

A rectangle is 3 times as long as it is wide. If its perimeter is 60 cm, find its dimensions.

Solution

Let the width of the rectangle be w cm. Then the length is $3w$ cm.



The perimeter is $w + 3w + w + 3w$ and this is given as 60.

$w + 3w + w + 3w = 60$

$8w = 60$

$w = 7.5$

$\therefore$ The width of the rectangle is 7.5 cm and the length is $3 \times 7.5 = 22.5$ cm.

Check: The perimeter of a rectangle with dimensions 7.5 cm and 22.5 cm is

$7.5 + 22.5 + 7.5 + 22.5 = 60$ cm.

Example 5

The sum of 3 consecutive numbers is 150.
Find the numbers.

Consecutive numbers follow each other in order, such as 7, 8, 9.

Solution

Let the first number be x.

The next number is $x + 1$ and the third number is $x + 2$.

Their sum is $x + (x + 1) + (x + 2)$ and this is given as 150.

$$x + x + 1 + x + 2 = 150$$
$$3x + 3 = 150$$
$$3x = 147$$
$$x = \frac{147}{3}$$
$$= 49$$

$\therefore$ The consecutive numbers are 49, 50 and 51.
Check: $49 + 50 + 51 = 150$.

Equation problems

Applying linear equations

Example 6

Ryan is 6 years older than his sister Kelsey. Their mother is 3 times Ryan's age.

a If the sum of the 3 ages is 59, write a simplified equation to find Ryan's age.

b Solve the equation and find each person's age.

Shutterstock.com/Syda Productions

Solution

a Let x = Ryan's age.

Kelsey's age is $x - 6$. Kelsey is 6 years younger than Ryan

The mother's age is $3x$.
$x + (x - 6) + 3x = 59$

$5x - 6 = 59$ Simplify the equation

b $5x - 6 = 59$
$5x = 65$
$x = 13$

Ryan is 13 years old.
Kelsey is $13 - 6 = 7$ years old.

Their mother is $3 \times 13 = 39$ years old.

Check: $13 + 7 + 39 = 59$

EXERCISE 7.03 ANSWERS ON P. 584

Equation problems U F PS R C

1 When 7 is subtracted from 4 times a certain number, the answer is 37. What is the number? PS R C

2 If 15 more than a number is 3 more than double the number, what is the number?

3 Two-thirds of a number is 16. What is the number?

4 When two-fifths of a number is increased by 15, the result is 27. What is the number?

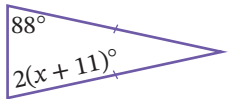
5 A rectangle is 4 times as long as it is wide. The perimeter of the rectangle is 100 cm. Find the dimensions of the rectangle. PS R C

6 The length of a rectangle is 7 cm longer than its width.

a Let w be the width of the rectangle. Write an equation for w if the perimeter of the rectangle is 94 cm.

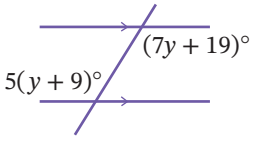
b Solve the equation and find the dimensions of the rectangle.

7 Find the value of x in this triangle.

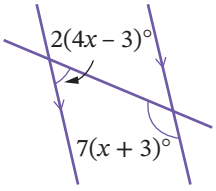


8 **a** Find the value of y.

b What is the size of each alternate angle?



9 Calculate the size of each marked angle.



10 The sum of 2 consecutive numbers is 87. Find the numbers. PS R C

11 The sum of 3 consecutive numbers is 87. Find the numbers.

12 The sum of 3 consecutive even integers is 168. What are the 3 integers?

13 The sum of 3 consecutive *odd* integers is 75. Find the integers.

14 Dean's father, Franco, is 5 times Dean's age. Dean is 8 years older than his sister, Helen. The sum of all their ages is 62 years. How old is each person? (*Hint:* Let Dean's age be x.) PS R C

15 Bill is 3 times as old as his daughter, Rebekah, who is 6 years younger than her brother, Tyrone. How old is Rebekah if the sum of their 3 ages is 76 years?

EXAMPLE 3

EXAMPLE 4

EXAMPLE 5

EXAMPLE 6

7.03

Foundation | Standard | Complex

16 In my money box I have only $1 and $2 coins. I have 240 coins in total, worth $318. How many $2 coins do I have in the money box?

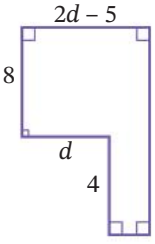
17 Aerin bought 4 ice creams and received $2.80 change from his $20 note. How much did each ice cream cost?

18 Ashvin is twice as tall as his little sister, Sureti, and 30 cm shorter than his father. Their combined height is 3.8 m. Find the height of each person.

19 The length of a rectangle is 9.5 cm longer than its width. The perimeter of the rectangle is 87 cm. Find the dimensions of the rectangle.

20 Janine is 6 years younger than her husband, Vila, who is 3 times the age of their son Brett. Brett is 5 years older than his sister Amanda. The sum of all their ages is 125 years. How old is each person?

21 A house is constructed in the shape shown below. If the house has a floor area of 54m^2, what is the value of d? PS R C



22 Ashleigh only collects 20-cent and 50-cent coins. Last week she counted 420 coins to the value of $156. How many 50-cent coins did Ashleigh have?

Mental skills 7A: Maths without calculators ANSWERS ON P. 584

Fraction of a quantity

Learn these commonly-used fractions and their decimal equivalents.

Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{2}{5}$
Decimal	0.5	0.25	0.125	0.75	0.2	0.1	0.05	0.4

Now we will use them to find a fraction or decimal of a quantity.

1 Study each example.

a $\frac{1}{4} \times 72 = 72 \div 4$
$= 18$

b $\frac{2}{5} \times 40 = (\frac{1}{5} \times 40) \times 2$
$= 8 \times 2$
$= 16$

c $\frac{3}{4} \times 32 = (\frac{1}{4} \times 32) \times 3$
$= 8 \times 3$
$= 24$

d $0.5 \times 66 = \frac{1}{2} \times 66$
$= 33$

e $0.05 \times 80 = \frac{1}{20} \times 80$
$= 4$

f $0.125 \times 56 = \frac{1}{8} \times 56$
$= 7$

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2 Now simplify each expression.

a	$\frac{1}{2} \times 28$	**b**	$\frac{1}{4} \times 36$	**c**	$\frac{1}{10} \times 70$	**d**	$\frac{1}{8} \times 64$
e	$\frac{1}{5} \times 15$	**f**	$\frac{1}{10} \times 80$	**g**	$\frac{2}{5} \times 25$	**h**	$\frac{1}{20} \times 100$
i	$\frac{3}{4} \times 44$	**j**	$\frac{1}{8} \times 40$	**k**	0.25×60	**l**	0.4×45
m	0.1×260	**n**	0.125×48	**o**	0.75×48	**p**	0.05×120
q	0.2×70	**r**	0.5×320	**s**	0.25×56	**t**	0.125×16

Equations with algebraic fractions 7.04

Example 7

Solve each equation.

a $\frac{x+11}{4} + 9 = -3$

b $\frac{a-2}{4} = \frac{2a+5}{3}$

Solution

a $\frac{x+11}{4} + 9 = -3$

$\frac{x+11}{4} + 9 - 9 = -3 - 9$

$\frac{x+11}{4} = -12$

$\frac{x+11}{4} \times 4 = -12 \times 4$

$x + 11 = -48$

$x + 11 - 11 = -48 - 11$

$x = -59$

b $\frac{a-2}{4} = \frac{2a+5}{3}$

For equations where all terms are fractions, multiply both sides by a common multiple of the denominators to remove the fractions.

The **lowest common multiple (LCM)** of 3 and 4 is 12, so multiply both sides by 12.

$12 \times \frac{(a-2)}{4} = 12 \times \frac{(2a+5)}{3}$

$12^{3} \times \frac{(a-2)}{4_{1}} = 12^{4} \times \frac{(2a+5)}{3_{1}}$

$3(a - 2) = 4(2a + 5)$

$3a - 6 = 8a + 20$

$-5a - 6 = 20$

$-5a = 26$

$a = \frac{26}{-5}$

$a = -5\frac{1}{5}$

Equations 3

Equations code puzzle

Solving equations with fractions

Equations with algebraic fractions

Example 8

Solve each equation.

a $\frac{k}{3}+\frac{k}{7}=4$

b $\frac{2m}{3}-\frac{m}{2}=2$

Solution

a Multiply both sides by 21, the LCM of 3 and 7.

$$21\left(\frac{k}{3}+\frac{k}{7}\right)=21\times 4$$

$$21\times\frac{k}{3}+21\times\frac{k}{7}=84$$
$$7\cancel{21}\times\frac{k}{\cancel{3}_1}+\cancel{21}^3\times\frac{k}{\cancel{7}_1}=84$$
$$7k+3k=84$$
$$10k=84$$
$$k=\frac{84}{10}$$
$$k=8\frac{2}{5}$$

b Multiply both sides by 6, the LCM of 3 and 2.

$$6\left(\frac{2m}{3}-\frac{m}{2}\right)=6\times 2$$

$$\cancel{6}^2\times\frac{2m}{\cancel{3}_1}-\cancel{6}^3\times\frac{m}{\cancel{2}_1}=12$$
$$4m-3m=12$$
$$m=12$$

STAGE 5.3

Equations with algebraic fractions

Example 9

Solve each equation.

a $\frac{2n+1}{3}-\frac{3n-2}{2}=-5$

b $\frac{a+5}{6}+\frac{3a}{4}=5$

Solution

a Multiply both sides by 6, the LCM of 3 and 2.

$$6\left(\frac{2n+1}{3}-\frac{3n-2}{2}\right)=6\times(-5)$$
$$\cancel{6}^2\left(\frac{2n+1}{\cancel{3}_1}\right)-\cancel{6}^3\left(\frac{3n-2}{\cancel{2}_1}\right)=-30$$
$$2(2n+1)-3(3n-2)=-30$$
$$4n+2-9n+6=-30$$
$$-5n+8=-30$$
$$-5n=-38$$
$$n=\frac{-38}{-5}$$
$$=7\frac{3}{5}$$

b Multiply both sides by 12, the LCM of 6 and 4.

$$12\left(\frac{a+5}{6}+\frac{3a}{4}\right)=12\times 5$$
$$\cancel{12}^2\left(\frac{a+5}{\cancel{6}_1}\right)+\cancel{12}^3\left(\frac{3a}{\cancel{4}_1}\right)=60$$
$$2(a+5)+3(3a)=60$$
$$2a+10+9a=60$$
$$11a+10=60$$
$$11a=50$$
$$a=\frac{50}{11}$$
$$=4\frac{6}{11}$$

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EXERCISE 7.04 ANSWERS ON P. 584

Equations with algebraic fractions U F R

EXAMPLE 7

1 Solve each equation. R

a $\frac{a+1}{4}+5=15$

b $\frac{x+4}{5}-10=2$

c $\frac{x-5}{5}-1=4$

d $\frac{y-2}{5}-2=6$

e $\frac{p+1}{3}-11=8$

f $5+\frac{4y+6}{9}=15$

g $1+\frac{10x-2}{2}=-10$

h $\frac{3y+5}{4}+9=-1$

i $\frac{12-x}{3}-1=6$

j $\frac{15-d}{7}+10=0$

k $\frac{3-2w}{11}+4=5$

l $\frac{6-4p}{9}-10=-6$

2 Solve each equation. R

a $\frac{x+5}{4}=\frac{x+4}{5}$

b $\frac{y+8}{3}=\frac{y-2}{2}$

c $\frac{2p-1}{4}=\frac{p+5}{3}$

d $\frac{2w+5}{3}=\frac{3w+1}{5}$

e $\frac{8x-4}{3}=\frac{4x+5}{2}$

f $\frac{m-6}{4}=\frac{2m-3}{10}$

g $\frac{2y+4}{3}=\frac{5y-2}{5}$

h $\frac{4x-2}{6}=\frac{x+7}{4}$

i $\frac{12-2x}{4}=\frac{9+x}{3}$

j $\frac{8-3w}{5}=\frac{2w+1}{4}$

k $\frac{7-5x}{2}=\frac{1-9x}{4}$

l $\frac{8-x}{2}=\frac{2x+1}{3}$

EXAMPLE 8

3 Solve $\frac{x}{3}+\frac{x}{2}=1$. Select the correct answer **A**, **B**, **C** or **D**. R

A $x=1\frac{1}{5}$ **B** $x=1\frac{1}{2}$ **C** $x=1$ **D** $x=2$

4 Solve each equation. R

a $\frac{d}{4}+\frac{d}{3}=14$

b $\frac{k}{2}-\frac{k}{3}=3$

c $\frac{m}{3}+\frac{m}{2}=10$

d $\frac{x}{8}+\frac{x}{3}=10$

e $\frac{p}{3}-\frac{p}{5}=4$

f $\frac{k}{4}-\frac{k}{5}=1$

g $\frac{4k}{3}-\frac{k}{5}=34$

h $\frac{3m}{5}+\frac{m}{2}=11$

i $\frac{4a}{5}+\frac{2a}{3}=10$

STAGE 5.3

EXAMPLE 9

5 What is the solution to $\frac{p-1}{5}-\frac{p+3}{2}=4$? Select **A**, **B**, **C** or **D**. R

A $p=-12$ **B** $p=-19$ **C** $p=14$ **D** $p=3$

6 Solve each equation. R

a $\frac{x}{7}+\frac{x-1}{4}=0$ **b** $\frac{2p}{3}-\frac{p+1}{6}=2$

c $\frac{m+2}{2}+\frac{m+1}{3}=12$ **d** $\frac{c-2}{5}-\frac{c-3}{2}=4$

e $\frac{2y-1}{4}+\frac{y+1}{2}=6$ **f** $\frac{x-1}{3}+\frac{x-4}{6}=-5$

g $\frac{7+2p}{5}-\frac{1-p}{2}=-1$ **h** $\frac{6y-1}{4}-\frac{y+2}{3}=8$

i $\frac{a+5}{4}+3=\frac{2a}{5}$ **j** $\frac{3x-1}{4}=2x+\frac{5}{6}$

k $\frac{12u+3}{7}=2u-\frac{u+4}{6}$ **l** $\frac{m-6}{5}+2m=\frac{3m-1}{4}$

Investigation

Solving $x^2 = c$

1 $x^2 = 25$ has 2 solutions. What are they?

2 What are the possible solutions for each of the following?

a $x^2 = 9$ **b** $x^2 = 49$ **c** $x^2 = 100$

3 What is the inverse operation of 'squaring'?

4 Study this example:

$x^2 = 81$

$x = \pm\sqrt{81}$ which means $\sqrt{81}$ or $-\sqrt{81}$

$x = \pm 9$ which means 9 or –9

Check: When $x = 9$, $x^2 = 9^2 = 81$

When $x = -9$, $x^2 = (-9)^2 = 81$

Now use the same method to solve each equation and check your answers.

a $m^2 = 1$ **b** $k^2 = 64$

5 How many solutions does each quadratic equation have?

a $m^2 = 1$ **b** $k^2 = 64$ **c** $x^2 = 81$

6 Do the following quadratic equations have solutions? (Give reasons for your answers.)

a $w^2 = -1$ **b** $y^2 = -64$ **c** $h^2 = -81$

7 Write an example of a quadratic equation that has only *one* solution.

Simple quadratic equations $ax^2 = c$ 7.05

An equation in which the highest power of the variable is 2 is called a **quadratic equation**, for example, $x^2 = 5$, $3m^2 + 7 = 10$, $d^2 - 4 = 0$ and $4t^2 - 3t = 8$.

The quadratic equation $x^2 = c$

The quadratic equation $x^2 = c$ (where c is a positive number) has 2 solutions:

$$x = \pm\sqrt{c} \text{ (which means } x = \sqrt{c} \text{ and } x = -\sqrt{c}).$$

Example 10

Solve each quadratic equation.

a $y^2 = 16$ **b** $p^2 = 65$ **c** $5a^2 = 245$

Solution

a $y^2 = 16$

$y = \pm\sqrt{16}$ Finding the square root of both sides

$= \pm 4$

65 is not a square number, so leave the answer as a surd.

b $p^2 = 65$

$p = \pm\sqrt{65}$ Finding the square root of both sides

c $5a^2 = 245$

$a^2 = \frac{245}{5}$ Dividing both sides by 5

$a^2 = 49$

$a = \pm\sqrt{49}$

$= \pm 7$

Example 11

Solve each quadratic equation, writing the solution correct to one decimal place.

a $4x^2 = 500$ **b** $\frac{3h^2}{5} = 26$

Solution

a $4x^2 = 500$

$x^2 = \frac{500}{4}$

$x^2 = 125$

$x = \pm\sqrt{125}$

$= \pm 11.1803...$

$\approx \pm 11.2$

b $\frac{3h^2}{5} = 26$

$3h^2 = 26 \times 5$

$3h^2 = 130$

$h^2 = \frac{130}{3}$

$h^2 = 43\frac{1}{3}$

$h = \pm\sqrt{43\frac{1}{3}}$

$= \pm 6.5828...$

$\approx \pm 6.6$

EXERCISE 7.05 ANSWERS ON P. 584

Simple quadratic equations $ax^2 = c$ U F R C

EXAMPLE 10

1 Solve each quadratic equation, writing the solutions as surds if necessary.

a $m^2 = 144$ **b** $x^2 = 400$ **c** $y^2 = 225$ **d** $k^2 = 59$

e $y^2 = 10$ **f** $w^2 = 36$ **g** $8x^2 = 200$ **h** $9t^2 = 81$

i $\frac{a^2}{2} = 8$ **j** $5k^2 = 40$ **k** $3w^2 = 30$ **l** $2d^2 = 288$

m $\frac{k^2}{2} = 8$ **n** $\frac{w^2}{10} = 7$ **o** $4x^2 = 1$ **p** $\frac{m^2}{3} = 27$

q $8y^2 = 40$ **r** $2p^2 + 3 = 21$ **s** $3k^2 = 48$ **t** $\frac{y^2}{5} - 2 = 9$

u $6x^2 = 42$

EXAMPLE 11

2 Solve each equation, writing the solution correct to one decimal place.

a $m^2 = 20$ **b** $b^2 = 17$ **c** $v^2 = 6$ **d** $2p^2 = 35$

e $9k^2 = 63$ **f** $\frac{x^2}{5} = 8$ **g** $\frac{k^2}{16} = 6$ **h** $\frac{7u^2}{10} = 2$

i $6y^2 = 84$ **j** $\frac{3w^2}{4} = 20$ **k** $a^2 + 11 = 28$ **l** $2y^2 - 14 = 63$

3 Explain why the quadratic equation $k^2 + 25 = 0$ has no solutions. R C

4 State which of these quadratic equations has no solutions. Give reasons. R C

a $x^2 = -9$ **b** $2k^2 + 5 = 9$ **c** $3m^2 + 8 = 4$

d $\frac{9w^2}{2} - 1 = 1$ **e** $4 + \frac{d^2}{3} = 8$ **f** $\frac{5a^2}{2} + 3 = 2$

STAGE 5.3

Investigation

Solving $x^3 = c$

1 $x^3 = 27$ has only one solution. What is it?

2 What is the solution to each equation?

a $x^3 = 125$ **b** $x^3 = 64$ **c** $x^3 = -8$

3 What is the inverse operation of 'cubing'?

4 Study this example:

$x^3 = -27$

$x = \sqrt[3]{-27}$

$x = -3$

The cube root of a negative number is also negative.

Check: When $x = -3$, $x^3 = (-3)^3 = -27$

Now use the same method to solve each equation and check your answers.

a $r^3 = -1000$ **b** $u^3 = 216$

5 How many solutions does each cubic equation have?

a $n^3 = 1$ **b** $t^3 = -343$ **c** $x^3 = -1$

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Simple cubic equations $ax^3 = c$#

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7.06

STAGE 5.3

An equation in which the highest power of the variable is 3 is called a **cubic equation**, for example, $x^3 = 12$, $2m^3 + 1 = 25$, $d^3 - 14 = 4$ and $x^3 - 3x^2 + 5x + 4 = 0$.

The cubic equation $x^3 = c$

The cubic equation $x^3 = c$ has one solution:

$$x = \sqrt[3]{c}$$

Example 12

Solve each cubic equation.

a $n^3 = 729$ **b** $d^3 = 40$ **c** $-3y^3 = 1029$

Solution

a
$n^3 = 729$
$n = \sqrt[3]{729}$
$= 9$

Finding the cube root of both sides

b
$d^3 = 40$
$d = \sqrt[3]{40}$

Finding the cube root of both sides.
40 is not a cube number so leave the answer as a surd.

c
$-3y^3 = 1029$
$y^3 = \frac{1029}{-3}$
$y^3 = -343$
$y = \sqrt[3]{-343}$
$= -7$

Dividing both sides by -3

Example 13

Solve each cubic equation, writing the solution correct to one decimal place.

a $2h^3 = -18$ **b** $\frac{z^3}{7} = 11$

Solution

a
$2h^3 = -18$
$h^3 = \frac{-18}{2}$
$h^3 = -9$
$h = \sqrt[3]{-9}$
$= -2.0800\ldots$
≈ -2.1

b
$\frac{z^3}{7} = 11$
$z^3 = 11 \times 7$
$z^3 = 77$
$z = \sqrt[3]{77}$
$= 4.2543\ldots$
≈ 4.3

STAGE 5.3

EXERCISE 7.06 ANSWERS ON P. 584

Simple cubic equations $ax^2 = c$ U F R C

EXAMPLE 12

1 Solve each cubic equation, writing the solutions as surds if necessary.

a $r^3 = 1$

b $k^3 = 216$

c $d^3 = -27$

d $x^3 = -45$

e $w^3 = 100$

f $f^3 = 64$

g $4m^3 = 32$

h $2t^3 = -250$

i $\frac{c^3}{2} = 108$

j $-3k^3 = 192$

k $7a^3 = 105$

l $\frac{2p^3}{9} = -4$

m $4q^3 = 665.5$

n $\frac{n^3}{5} = -345.6$

o $8s^3 = 150$

p $\frac{e^3}{8} = -1$

q $-6y^3 = 40$

r $2v^3 - 10 = 1014$

s $\frac{4b^3}{5} = 12$

t $\frac{z^3}{6} + 4 = -8$

u $-4x^3 = -144$

EXAMPLE 13

2 Solve each equation, writing the solution correct to one decimal place.

a $c^3 = 47$

b $g^3 = 151$

c $y^3 = -450$

d $-8p^3 = 728$

e $3u^3 = 245$

f $\frac{x^3}{5} = -11$

g $-\frac{h^3}{20} = 25$

h $\frac{11a^3}{7} = 80$

i $-6d^3 = 186$

j $\frac{5v^3}{8} = -27$

k $a^3 - 45 = 220$

l $4j^3 + 72 = -166$

3 **a** Does a cubic equation of the form $x^3 = c$ always have a solution?

b When is the solution to $x^3 = c$ positive?

c When does $x^3 = c$ have 2 solutions? R C

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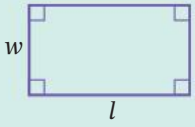
Equations and formulas 7.07

A **formula** is an algebraic equation that shows a relationship between variables. For example, the formula for the area of a circle is $A = \pi r^2$, where A is the area and r is the radius of the circle (π is a constant). Because the formula is for the area, A is called the **subject** of the formula and it is the variable on its own on the left side of the '=' sign.

Example 14

The formula for the perimeter (P) of a rectangle of length l and width w is given by

$$P = 2(l + w).$$



Use the formula to find:

a the perimeter of a rectangle with length 20 cm and width 9 cm

b the width of a rectangle if its length is 12 m and its perimeter is 70 m

c the length of a rectangle if its width is 42 cm and its perimeter is 1.8 m.

Solution

a $l = 20, w = 9$:

$P = 2(l + w)$

$= 2(20 + 9)$

$= 2 \times 29$

$= 58$

$\therefore$ The perimeter is 58 cm.

b $l = 12, P = 70$:

$P = 2(l + w)$

$70 = 2(12 + w)$

$70 = 24 + 2w$

$46 = 2w$

$w = \frac{46}{2}$

$= 23$

$\therefore$ The width is 23 cm.

c $w = 42, P = 1.8 \text{ m} = 180 \text{ cm}$ since w is given in cm

$P = 2(l + w)$

$180 = 2(l + 42)$

$180 = 2l + 84$

$96 = 2l$

$l = \frac{96}{2}$

$= 48$

$\therefore$ The length is 48 cm.

Example 15

The cost of hiring a sound system for a party is $250 plus $75 per hour. The cost can be represented by the formula $C = 250 + 75h$, where C is the total cost (in dollars), and h is the number of hours.

a Find the cost of hiring the sound system for 4 hours.

b A family is willing to spend $750 for hiring the sound system. What is the maximum number of *whole* rental hours that the family can afford?

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Solution

a $h = 4$:

$C = 250 + 75h$

$= 250 + 75 \times 4$

$= 550$

$\therefore$ The cost is $550.

b $C = 750$:

$750 = 250 + 75h$

$500 = 75h$

$h = \frac{500}{75}$

$= 6\frac{2}{3}$

$\therefore$ The maximum number of whole hours is 6.

EXERCISE 7.07 ANSWERS ON P. 584

Equations and formulas U F PS R

1 Given the formula $y = 5x + b$, find:

a y if $x = 5$ and $b = 3$

b y if $x = -1$ and $b = 16$

c b if $y = 40$ and $x = 3$

d b if $y = -6$ and $x = -1$

e x if $y = 27$ and $b = 12$

f x if $y = 64$ and $b = -16$

EXAMPLE 14

2 A temperature in degrees Celsius (°C) can be converted to degrees Fahrenheit (°F) using the formula $F = \frac{9C}{5} + 32$. Convert each temperature to °F.

a 35°C **b** −10°C **c** 16°C

3 Use the formula in question **2** to convert each temperature to °C, correct to one decimal place. PS R

a 100°F **b** −45°F **c** 78°F

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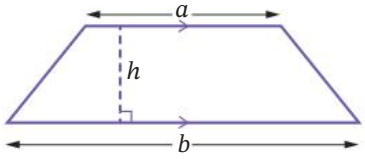
4 The formula $A = \frac{1}{2}(a + b)h$ is used to find the area of a trapezium, where A is the area, a and b are the lengths of the parallel sides, and h is the perpendicular height between them.



Use the formula to find:

a the area of a trapezium with height 6 cm and parallel sides of length 9 cm and 15 cm

b the height of a trapezium if its area is 420 cm^2 and it has parallel sides of length 22 cm and 20 cm

c the length of one side of a trapezium if its parallel side is 20.5 m, its area is 318 m^2 and its height is 12 m.

7.07

EXAMPLE 15

5 Find the value of S in the formula $M = \frac{kS}{5}$ if $M = 12.6$ and $k = 3.15$. Select **A**, **B**, **C** or **D**.

A 60 **B** 7.938 **C** 20 **D** 0.8

6 The cost C (in dollars) of hiring a limousine is given by the formula $C = 2500 + 150h$, where h is the number of hours of hire. Find: PS R

a the cost of hiring a limousine for 4 hours

b the cost of hiring a limousine for 2 days

c the number of hours for which you could hire a limousine for $4000

d the maximum number of whole hours for which a limousine could be hired at a cost of $5250

7 The profit, $\$P$, made by an online store selling games is given by $P = 35x - 750$, where x represents the number of games sold. Find: PS R

a the profit made when 230 games are sold

b the number of games sold if the profit is $18 010.

8 A catering company charges $\$C$ for a function with P people using the formula $C = 1500 + 75.5P$. PS R

a How much does the company charge for a function with 10 guests?

b Find the cost of catering for a group of 60 people.

c Diane has $10 000 to spend on catering for her wedding reception. What is the maximum number of people she can invite?

9 The perimeter of a rectangle is given by the formula $P = 2(l + w)$, where l is the rectangle's length and w is its width. What is the length of a rectangle with a width of 4 and a perimeter of 52? Select **A**, **B**, **C** or **D**.

A 9 **B** 12 **C** 18 **D** 22

10 The temperature T (in °C) of a hot liquid as it cools is given by the formula $T = 100 - 17.5h$, where h is the number of hours it has been cooling. Find: PS R

a the temperature of the liquid after 2 hours

b the temperature of the liquid after 30 minutes

c the number of hours it takes for the temperature of the liquid to reach 30°C.

11 Archeologists use the formula $H = 2.52t + 75.8$ to estimate the height, H cm, of a man when the tibia (shin bone) length, t cm, is measurable. PS R

a A male tibia 42 cm long was found intact. Estimate the height of the male, correct to the nearest centimetre.

b Estimate, to the nearest centimetre, the length of the tibia of a male whose height is 174 cm.

Mental skills 7B: Maths without calculators ANSWERS ON P. 584

Percentage of a quantity

Learn these commonly-used percentages and their fraction equivalents.

Percentage	50%	25%	12.5%	75%	20%	10%	$33\frac{1}{3}\%$	$66\frac{2}{3}\%$
Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{3}$	$\frac{2}{3}$

Now we will use them to find a percentage of a quantity.

1 Study each example.

a $20\% \times 25 = \frac{1}{5} \times 25$
$= 5$

b $50\% \times 120 = \frac{1}{2} \times 120$
$= 60$

c $12.5\% \times 32 = \frac{1}{8} \times 32$
$= 4$

d $75\% \times 56 = \frac{3}{4} \times 60$
$= (\frac{1}{4} \times 60) \times 3$
$= 15 \times 3$
$= 45$

e $33\frac{1}{3}\% \times 27 = \frac{1}{3} \times 27$
$= 9$

f $66\frac{2}{3}\% \times 60 = \frac{2}{3} \times 60$
$= (\frac{1}{3} \times 60) \times 2$
$= 20 \times 2$
$= 40$

2 Now simplify each expression.

a $25\% \times 44$	**b** $33\frac{1}{3}\% \times 120$	**c** $20\% \times 35$	**d** $66\frac{2}{3}\% \times 36$
e $10\% \times 230$	**f** $12\frac{1}{2}\% \times 48$	**g** $50\% \times 86$	**h** $20\% \times 400$
i $75\% \times 24$	**j** $33\frac{1}{3}\% \times 45$	**k** $25\% \times 160$	**l** $10\% \times 650$
m $12.5\% \times 88$	**n** $66\frac{2}{3}\% \times 21$	**o** $20\% \times 60$	**p** $75\% \times 180$

Changing the subject of a formula#

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7.08

In the formula $v = u + at$, v is the **subject** of the formula. When the formula is rearranged so that one of the other variables becomes the subject, the process is called **changing the subject of the formula**. To change the subject of a formula, use the same rules as for solving an equation. The answer is not a number but an algebraic expression (another formula).

STAGE 5.3

Formulas squaresaw

Changing the subject of a formula

Changing the subject of a formula

Example 16

a For the formula $v = u + at$, change the subject to a

b Make b the subject of the formula $x = \sqrt{b^2 - 4ac}$.

c Given $P = \dfrac{m-1}{m+1}$, change the subject of the formula to m.

Solution

a To make a the subject of the formula, solve it like an equation for a.

$v = u + at$

$u + at = v$ — Swapping sides so the new subject a appears on the LHS

$at = v - u$ — Subtracting u from both sides

$a = \dfrac{v-u}{t}$ — Dividing both sides by t

b $x = \sqrt{b^2 - 4ac}$

$\sqrt{b^2 - 4ac} = x$ — Swapping sides

$b^2 - 4ac = x^2$ — Squaring both sides

$b^2 = x^2 + 4ac$ — Adding $4ac$ to both sides

$b = \pm\sqrt{x^2 + 4ac}$ — Taking the positive and negative square root of both sides

c $P = \dfrac{m-1}{m+1}$

$\dfrac{m-1}{m+1} = P$ — Swapping sides

$m - 1 = P(m+1)$ — Multiplying both sides by $m + 1$

$= Pm + P$ — Expanding

$m - Pm = P + 1$ — Moving the m-terms to the LHS, the 1 to the RHS

$m(1 - P) = P + 1$ — Factorise m from the LHS

$m = \dfrac{P+1}{1-P}$ — Dividing both sides by $1 - P$

STAGE 5.3

EXERCISE 7.08 ANSWERS ON P. 585

Changing the subject of the formula U F R C

EXAMPLE 16

1 Change the subject of each formula to x. R C

a $d = x + c$ **b** $y = mx + c$ **c** $p = ax - y$

d $y = \frac{x}{m} + p$ **e** $k = mrx$ **f** $v = \frac{m}{x}$

g $A = \frac{x+y}{2}$ **h** $c = ax + by$ **i** $A = \frac{1}{2}h(x+y)$

2 The volume of a pyramid has the formula $V = \frac{1}{3}Ah$, where A is the area of the base and h is the perpendicular height. Change the subject of the formula to A.
Select the correct answer **A**, **B**, **C** or **D**. R C

A $A = \frac{1}{3}Vh$ **B** $A = \frac{3}{Vh}$ **C** $A = 3Vh$ **D** $A = \frac{3V}{h}$

3 Make y the subject of each formula. R C

a $m = an + ay$ **b** $x = y^2$ **c** $k = \frac{x}{y}$

d $Q = P + \sqrt{y}$ **e** $x^2 + y^2 = r^2$ **f** $M = ny^2$

g $m = \frac{r}{\sqrt{y}}$ **h** $b = c^2 - 2ay$ **i** $t = \sqrt{y - x}$

4 Change the subject of each formula to the variable shown. R C

a $V = lbh;\ h = ?$ **b** $A = \pi r^2;\ r = ?$ **c** $K = \frac{1}{2}mv^2;\ v = ?$

d $F = \frac{9C}{5} + 32;\ C = ?$ **e** $s = ut + \frac{1}{2}at^2;\ a = ?$ **f** $A = \frac{h}{2}(x+y);\ y = ?$

5 What is the correct formula for $p = m(n + x)$ if x is the subject?
Select **A**, **B**, **C** or **D**. R C

A $x = p - mn$ **B** $x = \frac{p}{mn}$ **C** $x = \frac{p}{m} - n$ **D** $x = \frac{p}{n} - m$

6 The cost ($\$C$) of a hire car is given by $C = 80 + 4.2d$, where d is the distance travelled in kilometres. R C

a Find the cost of hiring the car for a journey of 50 km.

b Make d the subject of the formula.

c Find the number of whole kilometres that can be travelled in the hire car for $402.

7 The angle sum of a shape with n sides is $A°$, where $A = 180(n - 2)$. R C

a Use the formula to find the angle sum of a shape with 7 sides.

b Make n the subject of the formula.

c If the angle sum of a polygon is 1440°, how many sides does it have?

8 The body-mass index of an adult is given by the formula $B = \frac{m}{h^2}$, where m is the adult's mass in kilograms and h is their height in metres. R C

a Change the subject of the formula to h.

b Hence find, correct to 2 decimal places, the height of a person with a body mass index of 25 and a mass of 60 kg.

9 Make r the subject of the formula $\frac{1}{x} = \frac{1}{r} + \frac{1}{s}$. R C

10 Solve each equation for a. R C

a $\frac{a}{5} = ap + q$ **b** $ar = 3(a+b)$ **c** $Da = D - 2a$

d $p = \frac{ax}{a+y}$ **e** $k = \frac{1-a}{1+a}$ **f** $M(a+b) = N(a-b)$

Power plus ANSWERS ON P. 585

1 Solve each equation.

a $\frac{5}{2y} = 14$ **b** $\frac{7}{r} = \frac{1}{r-3}$ **c** $\frac{y^2+8}{12} = 4\frac{2}{3}$

d $4m^2 - 3 = m^2 + 72$ **e** $\frac{3(m+5)}{4} - \frac{2(m-1)}{3} = m - 10 + \frac{m-6}{2}$

2 Given $W = \frac{X-Y}{X+Y}$, find:

a W when $X = 15$ and $Y = 10$ **b** W when $X = 6$ and $Y = -12$

c X when $W = 25$ and $Y = 6$ **d** Y when $W = 5$ and $X = 1$

3 Karlie is a farmer who raises pigs and chickens. From a total of 42 animals, she can count 116 legs. Write an equation and solve it to find how many chickens she has.

4 Joe is twice as old as his daughter, Wendi. 10 years ago, he was 3 times as old as her. Write an equation and solve it to calculate how old Wendi is now.

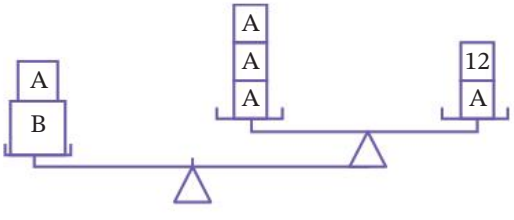
5 Consider $x^2 + y^2 = 4$.

a Explain why the smallest value for x is -2 and the largest value for x is 2.

b Are there any restrictions on the values that y can take? Explain why.

c By making y the subject, show that $y = \pm\sqrt{4 - x^2}$.

6 Find the value of B in the balance scale below.



7 Solve each cubic equation.

a $x^2 = x$ **b** $x^2 = x + 6$ **c** $x^3 = x$

d $x^3 = -x^2$ **e** $x^3 = 6x^2 - 9x$

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CHAPTER 7 REVIEW

Language of maths

backtracking method	balancing method	brackets
consecutive	cube root	equation
expand	flowchart	formula
inverse operation	LHS	lowest common multiple (LCM)
linear equation	quadratic equation	RHS
solution	solve	square root
subject	substitute	surd
undoing	unknown	variable

1 Which method of solving equations involves using inverse operations on both sides of the equation?

2 What is the **subject** of the formula $A = \frac{1}{2}(a + b)h$?

3 What name is given to numbers that follow each other in order, such as 9, 10, 11?

4 Write an example of:

a a quadratic equation **b** a linear equation

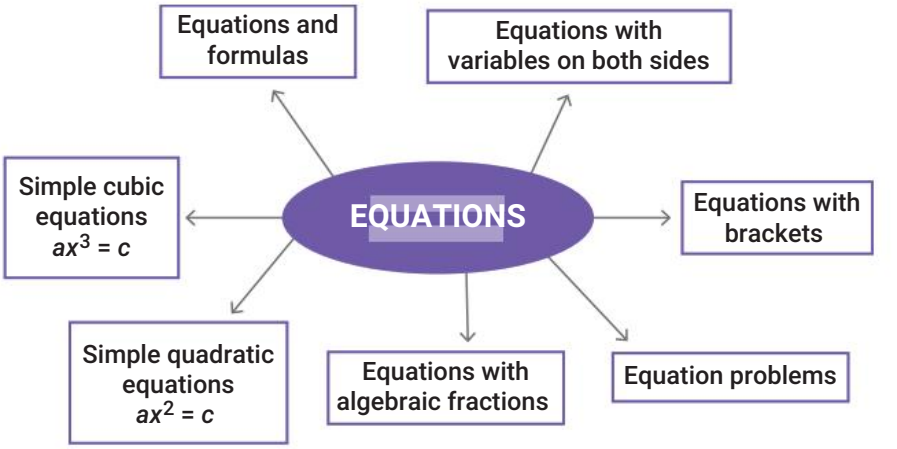
5 How many solutions does a linear equation have?

6 What does **LHS** stand for?

Topic summary

WS
Mind map: Equations (Advanced)

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



TEST YOURSELF 7

ANSWERS ON P. 585

1 Solve each equation. Check your solutions. 7.01

a $3x + 4 = 2x + 7$ **b** $5n - 3 = 2n - 15$ **c** $2d - 8 = -5d - 71$

d $4t = 12 - 4t$ **e** $8j - 17 = 10j$ **f** $6 - 3q = 8 - q$

2 Write an equation with x on both sides that has the solution $x = 3$. 7.01

3 Solve each equation. 7.02

a $2(w - 5) = 4$ **b** $3(1 + 4n) = 15$ **c** $5(1 - 3p) = 20$

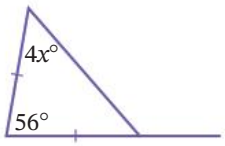
d $2(3 + x) = 5(x + 1)$ **e** $3(1 - y) = 4(2 - y)$ **f** $2(3 - 4x) = -(2x + 3)$

g $18 - 3a = 5a - 2(a - 6)$ **h** $11g + 2(g + 3) = 5(2g + 10)$

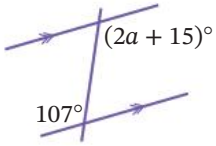
4 The length of a rectangle is 6 cm longer than it is wide. The perimeter of the rectangle is 76 cm. Find the dimensions of the rectangle. 7.03

5 Find the value of each variable. 7.03

a



b



6 If 6 more than a number equals 5 more than double the number, what is the number? 7.03

7 Solve each equation. 7.04

a $\frac{d-4}{2} + 9 = 4$ **b** $\frac{8n+6}{3} = \frac{6n+5}{2}$ **c** $\frac{2p}{3} - \frac{p}{2} = 4$

8 Solve each quadratic equation. 7.05

a $d^2 = 64$ **b** $8p^2 = 288$ **c** $3z^2 = 105$

STAGE 5.3

9 Solve each equation. 7.04

a $\frac{x-1}{5} + \frac{x-4}{2} = 6$ **b** $\frac{7n+3}{5} - \frac{5n+4}{3} = 2$ **c** $\frac{2y-1}{4} + \frac{y+1}{2} = 6$

10 Solve each cubic equation. 7.06

a $x^3 = 1331$ **b** $-4h^3 = 864$ **c** $\frac{t^3}{2} = 62.5$

11 The body mass index (BMI) of an adult is $B = \frac{M}{h^2}$, where M is the mass in kilograms and h is the height in metres. 7.07

a Find the BMI of Ned who is 1.85 m tall and has a mass of 72 kg.

b Find the mass of a person with a BMI of 24, who is 2.1 m tall.

12 The cost, C, in dollars, of hiring a taxi is $C = 5 + 2.4d$, where d is the distance travelled in kilometres. Find: 7.07

a the cost of a taxi trip if the distance travelled is 15 km

b the distance travelled if the cost of a taxi trip was \$78.20.

STAGE 5.3

13 Make w the subject of each formula. 7.08

a $a = kw + v$ **b** $p = m(t - w)$ **c** $x = \frac{1}{w} + y$

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PRACTICE SET 2 ANSWERS ON P. 585

5.09 **1** State the number of significant figures in each number.

a 1203 **b** 0.008 **c** 1.092 **d** 15 700

5.01 **2** Simplify each expression.

a $y^9 \times y$ **b** $3g^7 \times 4g^8$ **c** $2p^3q \times 9p^2q^2$

d $\frac{k^{18}}{k^6}$ **e** $\frac{x^{11}}{x^7}$ **f** $h^{10} \div h$

g $54k^7 \div 6k^3$ **h** $\frac{h^3y^7}{hy^4}$ **i** $12m^5n^8 \div 18mn^7$

5.04 **3** Simplify each expression.

a $6y^0$ **b** $4^0 \times 3^0$ **c** $(4k)^0$ **d** $x \times (3y)^0$ **e** $2q^0 - (2q)^0$

4.09 **4** Sketch on a compass rose point A if it has a bearing of:

a 260° from point M **b** 304° from point W.

5.05 **5** Evaluate each expression, giving answers in fraction form.

a 2^{-1} **b** 5^{-3} **c** 10^{-2}

6.03 **6** **a** Calculate the size of one angle in a regular hexagon.

b Name the polygon that has an angle sum of 1440°.

STAGE 5.3

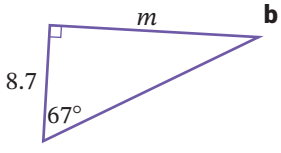
5.05 **7** Simplify each expression.

a $\left(\frac{1}{2}\right)^{-1}$ **b** $\left(\frac{8}{3}\right)^{-2}$ **c** $\left(\frac{4d}{5}\right)^{-2}$ **d** $\left(\frac{3}{10x}\right)^{-3}$

7.08 **8** Make y the subject of the formula $T = a + py^2$.

4.05 **9** Calculate the value of each variable, correct to 2 decimal places.

a



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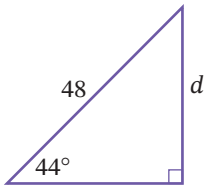

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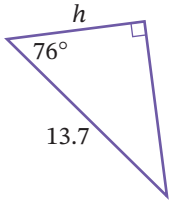


5.02 **10** Simplify each expression, giving the answers in index notation.

a $(2^3)^5$ **b** $(4^2)^6$ **c** $(3p^7)^2$ **d** $(2w^5)^3$

5.10 **11** Express each number in decimal form.

a 1.54×10^7 **b** 3.2×10^{-3} **c** 9.837×10^{-1} **d** 2.4×10^4

4.10 **12** A plane flies on a bearing of SW for 300 km. How far south (correct to the nearest kilometre) from its starting point has the plane flown?

13 Find θ, correct to the nearest minute if: 4.07

a $\cos\theta = 0.21$ **b** $\sin\theta = \frac{9}{10}$ **c** $\tan\theta = 1.5$

14 Write each expression using a negative index. 5.05

a $\frac{1}{n}$ **b** $\frac{1}{2^3}$ **c** $\frac{1}{k^5}$ **d** $\frac{7}{y^2}$

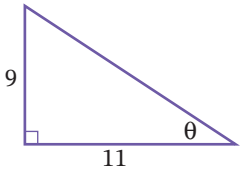
15 Write 7 280 000 in scientific notation. 5.10

16 Evaluate each expression, giving your answers in scientific notation correct to 2 significant figures. 5.11

a $(7.08 \times 10^7) \times (3.8 \times 10^8)$ **b** $(4.65 \times 10^{-3}) \div (3.9 \times 10^7)$

17 Find θ, correct to the nearest degree. 4.07

a



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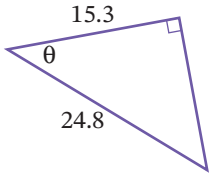

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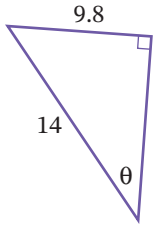
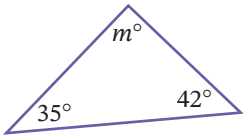


18 Find the value of each variable, giving reasons. 6.01

a



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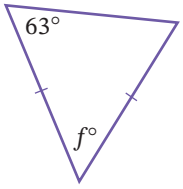

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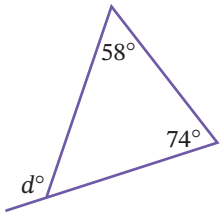


19 Solve $12 - 5k = 2k - 9$. Select the correct answer **A**, **B**, **C** or **D**. 7.01

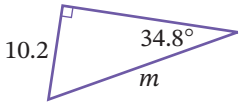
A $k = 3$ **B** $k = 1$ **C** $k = \frac{3}{7}$ **D** $k = 7$

20 Select the larger number for each pair of numbers. 5.10

a 5.3×10^9 or 8.6×10^8 **b** 3.73×10^5 or 3.4×10^5

21 Find the value of each variable, correct to one decimal place. 4.06

a



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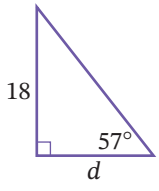

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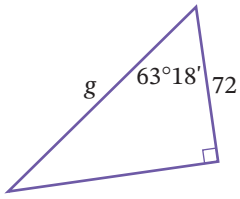

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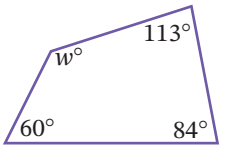
6.02

22 Find the value of w in each quadrilateral.

a



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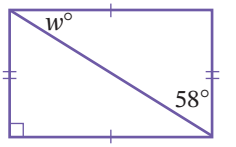

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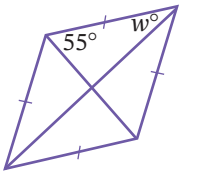


STAGE 5.3

5.07

23 Write each expression using a fractional index.

a $\sqrt{13}$ **b** $\sqrt[3]{20}$ **c** $\sqrt[5]{3p}$ **d** $\left(\sqrt{x}\right)^3$

7.07

24 The volume of useable timber (in cubic metres) in a tree is calculated using the formula:

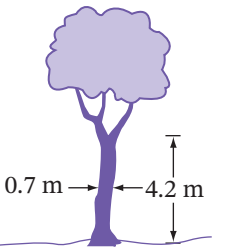
$$v = 0.5hd^2 + 10$$

where: d = the diameter of the tree

h = the height to the first branch

Calculate the volume of useable timber in this tree.

Select **A, B, C** or **D.**



A 16.2 m^3 **B** 11.5 m^3 **C** 11.0 m^3 **D** 18.8 m^3

STAGE 5.3

4.06

25 Evaluate each expression.

a $25^{\frac{3}{2}}$ **b** $4^{-\frac{5}{2}}$ **c** $\left(\sqrt[4]{16}\right)^3$ **d** $\sqrt[3]{27^2}$

6.04

26 How many sides has a regular polygon if:

a each exterior angle is 15°? **b** each interior angle is 135°?

5.03

27 Simplify each expression.

a $(2m^3n^2)^4$ **b** $\left(-\frac{2h}{5}\right)^2$ **c** $(7rt^2)^3$

7.04

28 Solve each equation.

a $\frac{3k-1}{2} = \frac{5k+1}{3}$ **b** $\frac{h}{5} + \frac{h}{2} = 6$

5.09

29 Round:

a 43 759 to 2 significant figures

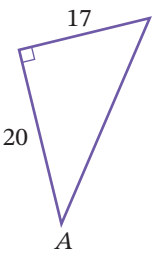
b 10.7614 to 3 significant figures

c 0.000 038 9 to one significant figure

4.07

30 Find the size of angle A, correct to the nearest degree.

a



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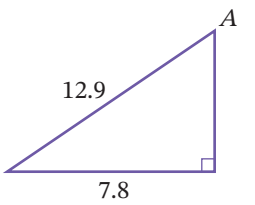

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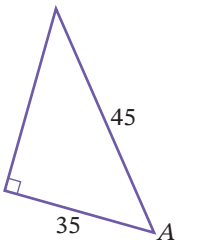

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31 A formula for cooking roast beef is $T = 40M + 25$, where T is the cooking time in minutes and M is the mass of the beef in kilograms. 7.07

a How long will it take to cook a 5 kg piece of beef?

b What mass of beef has a cooking time of $2\frac{1}{2}$ hours?

32 A 3.5 m ladder leans against a wall, making an angle of 50° with the ground. 4.05

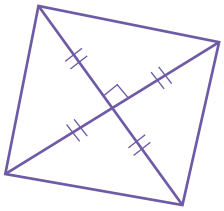
a Draw a diagram showing this information.

b Calculate, correct to the nearest 0.1 m:

i how far up the wall the ladder reaches

ii how far the foot of the ladder is from the base of the wall

33 What type of quadrilateral is shown here? 6.02



34 Solve each equation. 7.02

a $3(p - 3) = 24$ **b** $3(4y - 1) + 5 = 2(9 - 2y)$

35 **a** The sum of 4 consecutive numbers is 82. Find the 4 numbers. 7.03

b Sean collected \$1 and \$2 coins until he had 275 coins, having a total value of \$402. How many \$2 coins did Sean have?

36 Amit is at ground level 72 metres from the base of an apartment building. The angle of elevation to the top of the building is 35°. Find the height of the building, correct to the nearest metre. 4.08

37 Solve each equation. 7.05

a $5m^2 + 1 = 81$ **b** $\frac{x^2}{3} = 12$

38 Solve each equation. STAGE 5.3 7.06

a $2r^3 = 1024$ **b** $\frac{y^3}{6} = -121.5$

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8

NUMBER AND ALGEBRA

EARNING MONEY

An Australian person working full-time works an average of 35.7 hours per week. The median weekly pay was $1238 for individuals and $1701 for households in 2019. What is it now, and how much does your household earn each week?

Shutterstock.com/IR Stone

Chapter outline

	Working mathematically				
8.01 Wages and salaries#	U	F	PS	R	
8.02 Overtime pay#	U	F	PS	R	
8.03 Commission, piecework and leave loading#	U	F	PS		
8.04 Income tax#	U	F	PS		
8.05 PAYG tax and net pay#	U	F	PS	R	C

This Stage 5 (Years 9–10) topic of the NSW syllabus is not part of the Australian curriculum.

Wordbank

allowable deduction A part of a person's yearly income that is not taxed, such as work-related expenses and donations to charities

annual leave loading Extra payment to a worker based on 17.5% of 4 weeks annual leave

income tax A tax paid to the government based on the size of a person's income

net pay Pay received after deductions from gross pay; 'take-home' pay

overtime Time worked beyond normal working hours, such as at night or on weekends, at a higher rate of pay

time-and-a-half Overtime pay that is calculated at 1.5 times the normal pay rate

salary A fixed yearly amount of money that is paid weekly, fortnightly or monthly, not dependent on the number of hours worked

wage An amount of money paid to people for work, calculated on the number of hours worked

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- calculate wages, salaries, overtime, commission and piecework
- calculate weekly, fortnightly, monthly and yearly incomes
- calculate annual leave loading
- use tables to calculate income tax and PAYG instalments
- calculate net pay from gross pay

SkillCheck ANSWERS ON P. 586

1 Write 8.2% as a decimal.

2 Find 15% of $15.

3 How many:

a days in one year?
b weeks in one year?
c days in 4 weeks?
d months in 3 years?
e days in a fortnight?
f weeks in a quarter of a year?
g months in 5 years?
h fortnights in one year?

4 Round each amount to the nearest cent.

a $7.6831
b $21.4209
c $524.7373
d $92.3557
e $3085.6382
f $106.3446

5 Evaluate (correct to the nearest cent where necessary):

a $38 \times \$12.87 \times 1.5$
b $\$75\,430 \div 12$
c $0.42 \times \$7500$
d $0.13 \times \$9217$
e $\$4758 \times 0.073 \times 2$
f $0.47 \times \$31\,726$
g $\$69\,870 \div 52.18$
h $\$2150 \times 1.09$
i $\$3276 \times 0.175 \times 4$

6 Evaluate $\$3572 + 0.325 \times (\$43\,210 - \$37\,000)$.

8.01 Wages and salaries

People who work usually earn a wage or a salary.

A **wage** is calculated by the number of hours worked and is usually paid weekly. People such as supermarket cashiers, electricians and gardeners earn a wage. Wage earners can earn more income by working extra hours (overtime).

A **salary** is a fixed annual amount, paid weekly, fortnightly or monthly. People such as architects, software designers and the police earn a salary. Salary earners do not earn overtime pay, but can receive benefits such as a company car, expense account, shares in the company or paid medical expenses.

Example 1

Peter's first job pays him \$16.75 per hour. If he works 40 hours each week, calculate his weekly income.

Solution

Weekly income $= 40 \times \$16.75$

$= \$670$

Units of time

1 year = 12 months

1 fortnight = 2 weeks

1 year = 52 weeks for wage earners

1 year = 52.18 weeks for salary earners

To convert a salary to a weekly amount, divide by 52.18 because

1 year $= 365\frac{1}{4}$ days $= 365\frac{1}{4} \div 7$ weeks ≈ 52.18 weeks.

Example 2

Greta earns a salary of \$72 600 p.a.
How much (correct to the nearest cent) does she earn:

'per annum' (p.a.) = per year'

a each week? **b** each fortnight? **c** each month?

Solution

a Weekly income $= \$72\,600 \div 52.18$

$= \$1391.3376...$

$\approx \$1391.34$

b Fortnightly income $= 2 \times \$1391.34$ 1 fortnight = 2 weeks

$= \$2782.68$

c Monthly income $= \$72\,600 \div 12$ 1 year = 12 months

$= \$6050$

Example 3

Sarah earns a salary of \$62 500 and works 38 hours per week. Anh earns \$1087 per week for working 35 hours. Who has the higher hourly rate of pay?

Solution

Sarah's hourly pay rate $= \$62500 \div 52.18 \div 38$

$= \$31.5204....$

$\approx \$31.52$

Anh's hourly pay rate $= \$1087 \div 35$

$= \$31.0571...$

$\approx \$31.06$

Sarah has the higher hourly rate of pay.

EXERCISE 8.01 ANSWERS ON P. 586

Wages and salaries U F PS R

Express all answers correct to the nearest cent where necessary.

EXAMPLE 1

1 Find the weekly wage for each person.

a Alex, who earns $26.24 per hour and works for 40 hours.

b Bella, who earns $34.60 per hour and works for 38.5 hours.

c Christy, who works 8 hours a day, Monday to Friday, and is paid $22.38 per hour.

d Dilshan, who works Monday to Wednesday, 6 hours per day, and is paid $17.85 per hour.

2 What is Kim's annual salary if she earns $1980 per week?

3 If Sunil earns $6875 per month, what is his annual salary? Select **A**, **B**, **C** or **D**.

A $357 500 **B** $178 750 **C** $89 375 **D** $82 500

4 Rhys earns $3038.47 per fortnight. How much does he earn in a year?

EXAMPLE 2

5 Kelly earns a salary of $158 340 p.a. How much does she earn:

a each week? **b** each fortnight? **c** each month?

6 Mitch earns $12.40 per hour before midnight and $18.60 per hour after midnight for each shift. Calculate his income for working each of these shifts:

a 7:00 p.m. to 3:00 a.m. **b** 9:00 p.m. to 5:30 a.m. **c** 10:30 p.m. to 6:00 a.m.

7 Danica earns $18.47 an hour and works for 38 hours each week. Trent earns $19.16 per hour for his 35 hours of work each week. Who earns more per week and by how much? PS R

EXAMPLE 3

8 Abdul considers 2 jobs, one with a salary of $97 640 p.a. and the other with a fortnightly income of $3865. Calculate the weekly income for each job to determine which pays more. PS R

9 Who earns the higher hourly rate of pay for each pair of workers, and by how much? PS R

a Mick worked 40 hours for $2584 pay.
Marisol worked 38 hours for $2501.60 pay.

b Phil worked 30 hours for $1441 pay.
Paula worked 27 hours for $1384.75 pay.

c Faith worked Monday to Friday, 8.5 hours per day, for $1092.50 pay.
Fabian worked Monday to Thursday, 9 hours per day, for $1015.60 pay.

d Ranjan earned $92 760 p.a., working 9:00 a.m. to 5:30 p.m., five days per week.
Rhonda earned $98 620 p.a., working 6:00 a.m. to 6:00 p.m., Monday to Thursday.

10 A printer earns $64 per hour, working from 8:00 a.m. to 4:30 p.m., Monday to Friday. Find how much he earns:

a each day **b** each week **c** each month.

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11 Ziad earns $10 640 per month. Calculate his:

a annual salary **b** weekly income

12 Priya earns a salary of $88 720 p.a. and works 42 hours per week. Find how much Priya earns:

a per month **b** per week **c** per hour

13 Liam works 9 hours per day from Tuesday to Friday. He is paid $27.84 per hour. He also works on Saturday for 6 hours at a special rate of $41.76 per hour. How much did Liam earn for the week? PS

14 Charlie works from 7:30 a.m. to 3 p.m. Monday to Wednesday and from 8 am to 4 p.m. Thursday and Friday. His hourly pay rate is $27.38. Calculate Charlie's weekly wage.

15 Radka packs shelves at a supermarket and is paid $28.50 per hour on weekdays and $42.75 per hour on Saturdays. She worked the following hours last week. PS R

Day	Hours worked
Monday	6:30 a.m. – 2:00 pm
Tuesday	8:00 a.m. – 4:00 p.m.
Wednesday	11:00 a.m. – 4:30 p.m.
Friday	9:30 a.m. – 3:30 p.m.
Saturday	10:00 a.m. – 4:00 p.m.

What was Radka's total income for the week?

16 Jack is a café manager and works the following hours in a week:

Day	Hours worked
Monday	6:30 a.m. – 2:30 p.m.
Tuesday	7:00 a.m. – 4:00 p.m.
Thursday	11:30 a.m. – 8:30 p.m.
Friday	7:30 a.m. – 3:30 p.m.
Saturday	10:30 a.m. – 5:00 p.m.

He is paid at the following rates:

Day	Rate of pay
Monday to Friday	$24.22 per hour
Thursday after 4:00 p.m. and all day Saturday	$36.33 per hour

What is Jack's total income for the week? PS R

17 Andrew works from 8:00 a.m. to 4:00 p.m. from Monday to Wednesday and from 7:30 a.m. to 5:30 p.m. on Thursday and Friday. If his weekly wage is $1274, find his hourly rate of pay. PS R

18 Lidija's monthly income is $7600. Convert this amount to a fortnightly income.

Investigation

Positions vacant

1 **a** Find 5 job advertisements from a newspaper or employment website.

b For each advertisement, write:

- a brief job description, including whether the job is full-time or part-time, permanent or casual
- whether a salary or wage is paid, and how much
- whether there are any other payments, incentives, or benefits that may come with the job.

2 This job advertisement is for a courier.

COURIERS

Full-time Permanent Positions

Baulkham Hills, Lidcombe, Pymble, Five Dock, Wahroonga, Dural and Hornsby

Part-time Permanent Positions

Thomson – 21 hours per week; shifts start at approximately 6:30am
Wetherill Park – 20 hours per week; shifts start at approximately 7:30am

Fixed-term Positions

PART-TIME

Regents Park – until 14 June 2022; 20 hours per week; shifts start at approximately 6:00am
Thomson – until 10 June 2022; 24 hours per week; shifts start at approximately 6:30am

FULL-TIME

Regents Park – until 10 August 2022
North Bank – until 19 June 2022
Camperdown
Blacktown – until 22 October 2022
Lane Cove – until 31 May 2022

All full-time positions are from Monday to Friday, 36.75 hours per week. Shifts start at approximately 6:00am.

Essential Conditions/Requirements:
• Must hold a current unrestricted motorcycle licence

a How many hours would a full-time permanent employee be expected to work in a week?

b Explain what a 'fixed-term' position is.

c The advertisement does not give wage rates. Can you give a reason for this?

3 According to this advertisement, how much would a glasscutter earn in one year if he is expected to work 35 hours per week?

Glasscutters
$27 p/h

Exp. Cutters req. in Gladstone for immediate start. Must be able to cut up to 10 mm, be familiar with cutting machinery and have a minimum 2 years exp.

Overtime pay

8.02

Wage earners receive **overtime** pay when they work more than their standard number of hours.

Overtime pay

The two most common rates of overtime pay are:

- **time-and-a-half** = 1.5 × normal hourly rate
- **double time** = 2 × normal hourly rate

Example 4

Siki works 35 hours at $28.94 per hour and 4 hours overtime at time-and-a-half. Calculate his total earnings.

Solution

Normal pay = 35 × $28.94 = $1012.90

Overtime pay = 4 × $28.94 × 1.5 = $173.64

Total earnings = $1012.90 + $173.64 = $1186.54

Alternative method:

Equivalent number of normal hours = 35 + (4 × 1.5) = 41

Total earnings = 41 × $28.94

= $1186.54

Example 5

Sinead earns time-and-a-half on Saturdays and double time on Sundays. She works 38 hours from Monday to Friday, 6 hours on Saturday and 4 hours on Sunday. Calculate Sinead's total earnings if her normal rate of pay is $24.50 per hour.

Solution

Normal pay (Monday to Friday) = 38 × $24.50

= $931

Saturday pay = 6 × $24.50 × 1.5

= $220.50

Sunday pay = 4 × $24.50 × 2

= $196

Total pay = $931 + $220.50 + $196

= $1347.50

Example 6

Carrie and Vinh are cadet hamburger chefs. Last week, Carrie worked for 40 hours and Vinh worked for 43 hours. Calculate their respective total earnings based on the following award.

Award for cadet hamburger chefs	
Normal rate = $15.80 per hour	
Normal rate	For first 38 hours worked
Time-and-a-half	For the next 3 hours
Double time	For each additional hour after that

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Solution

For Carrie (40 hours):

Normal pay $= 38 \times \$15.80 = \$600.40$ — First 38 hours

Time-and-a-half pay $= 2 \times \$15.80 \times 1.5 = \$47.40$ — Remaining 2 hours

Total earnings $= \$600.40 + \$47.40 = \$647.80$

For Vinh (43 hours):

Normal pay $= \$600.40$ — First 38 hours

Time-and-a-half pay $= 3 \times \$15.80 \times 1.5 = \$71.10$ — Next 3 hours

Double time pay $= 2 \times \$15.80 \times 2 = \$63.20$ — Remaining 2 hours

Total earnings $= \$600.40 + \$71.10 + \$63.20$
$= \$734.70$

Example 7

Last week Rani worked her normal 38 hours, then 4 hours at time-and-a-half and 5 hours at double time. She was paid $1009.90 for the week. Find her hourly rate of pay.

Solution

Equivalent number of normal hours $= 38 + (4 \times 1.5) + (5 \times 2)$
$= 54$

Hourly rate of pay $= \$1009.90 \div 54$
$= \$18.70$

EXERCISE 8.02 ANSWERS ON P. 586

Overtime U F PS R

1 The hourly rate of pay for workers in a factory is $28.65. Calculate the total earnings for each employee. PS

EXAMPLE 4

a Soo-jin works 28 hours at normal rates.

b Jemima works 33.5 hours at normal rates.

c Ethan works 38 hours at normal rates and 3 hours at time-and-a-half.

d Stavros works 35 hours at normal rates and 5.5 hours at time-and-a-half.

e Lana works 37 hours at normal rates, 6 hours at time-and-a-half and 4 hours at double-time.

f Marco works 40 hours at normal rates, 5 hours at time-and-a-half and 4 hours at double-time.

2 Pavel works 20 hours at normal rates with an hourly pay rate of $25.46 and 4 hours at time-and-a-half. Which calculation could be used to find his weekly wage? Select the correct answer **A**, **B**, **C** or **D**.

A $25.46 × 24

B $25.46 × 20 + $12.73 × 24 × 1.5

C $25.46 × 20 + $25.46 × 4 × 1.5

D $25.46 × 24 × 1.5

3 Josie works 38 hours at $26.87 per hour and 3 hours overtime at time-and-a-half. Calculate her total earnings.

4 Wayne is a store assistant manager who works 40 hours at $37.32 per hour, 5 hours overtime at time-and-a-half and 3.5 hours double-time. Calculate his total earnings.

5 Pancake House pays employees normal rates for Monday to Friday, time-and-a-half for Saturday and double-time for Sunday. Mia works 36 hours Monday to Friday, 5.5 hours on Saturday and 4 hours on Sunday. Calculate her total earnings if her normal rate of pay is $21.40 per hour. PS

EXAMPLE 5

6 Anu works at a mobile phone store. She is paid normal rates for shifts worked on Monday to Friday, time-and-a-half for Thursday nights from 5 p.m. onwards, and double-time on Saturdays. In a single week, Anu worked 37 normal hours and on Thursday night she worked from 5 p.m. to 8 p.m. On Saturday, she worked from 9:30 a.m. until 2:30 p.m. If her normal pay rate is $23.74 per hour, what is Anu's total wage for the week?

7 Holly works from 9 a.m. until 4 p.m. each day on Monday, Tuesday and Wednesday at a normal rate of $19.60 per hour. She works Thursday nights from 4:30 p.m. to 9:30 p.m. at time-and-a-half and Sundays from 8:00 a.m. until 4:00 p.m. at double-time. Which expression is used to calculate her weekly wage? Select **A**, **B**, **C** or **D**.

A $19.60 × 34 + $19.60 × 1.5 × 4 + $19.60 × 12 × 2

B $19.60 × 21 + $19.60 × 1.5 × 5 + $19.60 × 8 × 2

C $19.60 × 7 + $19.60 × 1.5 × 6 + $19.60 × 8 × 2

D $19.60 × (13 + 1.5 × 11 + 2 × 12)

EXAMPLE 6

8 *Elegant Jewellery* pays its staff at the following rates:

Weekly Award Normal rate = $23.70/h	
Normal	**For the first 35 hours**
Time-and-a-half	For the next 5 hours
Double-time	For each hour after that

Calculate the weekly wages for each employee. PS R

a Grant, who works 33 hours
b Tashi, who works 36 hours
c Kylie, who works 40 hours
d Mannix, who works 44.5 hours
e Noah, who works 38.5 hours
f Jennifer, who works 42 hours

9

Nuts and bolts machinists' enterprise agreement	
Normal rate	Applies Monday to Friday
Time-and-a-half	For any hours worked on Saturday
Double time	For any hours worked on Sunday

Calculate the total earnings for each employee. PS R

			Hours worked		
	Name	**Hourly rate**	**Mon–Fri**	**Sat**	**Sun**
a	Alan Douglas	$19.60	36	4	8
b	Ava Hawkes	$27.30	38	0	6
c	Seetha Mehta	$28.60	38	4	2.5
d	Tarek Mifsud	$25.00	40	8	0
e	Ivan Vitsic	$38.00	34	4	6
f	John Hackett	$26.40	36	4.5	6

EXAMPLE 7

10 Calculate the normal hourly rate of pay for each person. PS R

a Sarah earns $1262.40 by working 36 hours at normal rates and 4 hours at double time.
b Fiona earns $1469.60 by working 37 hours at normal rates, 3 hours at time-and-a-half and 4 hours at double time.
c Thierry earns $340 by working 6 hours at normal rates and 4 hours at double time.
d Sana earns $263.80 by working 4 hours at time-and-a-half and 3 hours at double time.
e Phillip earns $1127.60 by working 30 hours at normal rates, 5 hours at time-and-a-half and 3 hours at double time.
f Tara earns $126.50 by working 7 hours at normal rates and 1.5 hours at time-and-a-half.

Technology

Calculating incomes

In this activity, we will use a spreadsheet to calculate the incomes of employees at J-mart department store. The award for all workers is as follows.

- Normal rate for up to 36 hours worked
- Time-and-a-half for the next 4 hours worked
- Double-time for any hours worked after that

1 Create a spreadsheet as shown below, then for each employee enter the correct number of hours for columns D, E and F.

Name	Hourly Pay	Hours Worked	Normal	Time-and-a-half	Double time	TOTAL PAY
R Jones	$42.85	38				
A Watson	$39.40	34				
Q Tran	$35.67	40				
L Djokovic	$23.90	42				
C Lam	$22.96	39				
J Hampen	$20.76	41.5				
L Ngo	$20.28	43				
K Barnett	$19.22	45.5				

2 In cell G1, enter the heading **Total Pay.**

3 To calculate R Jones' total pay, in cell G2 enter the formula **B2*D2+B2*1.5*E2+B2*2*F2**

4 **Fill Down** the formula from G2 to G9. Use **Format Cells** to change the values in column G to **Currency.**

Did you know?

Labour Day

Alamy Stock Photo/picturelibrary

Each year, Australians have a public holiday on a Monday called Labour Day (Eight-Hours Day in Tasmania, May Day in the Northern Territory). This day celebrates the significant improvement of wages and working conditions in the 19th century.

In the early 1800s, the Australian worker was working up to 14 hours per day, 6 days per week, often under difficult conditions. Sick leave or holidays did not exist, and employees could be sacked immediately without reason.

In 1856, Victorian workers won the right to limit their working time to 8 hours per day. This was a world first, allowing workers more time in the day for recreation, reading, education and family, dividing the day into '8 hours labour, 8 hours recreation, 8 hours rest'. The Eight-hour Day was adopted in all Australian states by the 1920s. There is an Eight Hour monument that stands in central Melbourne with the numbers '888' featured at the top.

Labour Day is celebrated on a different day for each state in Australia. Find out when these are.

8.03 Commission, piecework and leave loading

WS
Earning money

Percentage calculations

Some workers are not paid by the amount of time they work, but by the number of items they sell, make or process.

Commission is earned by salespeople or agents of a business or company. It is calculated as a percentage of the value of items sold or income made. A fixed amount called a **retainer** may also be paid. Real estate agents and actors' agents earn a commission.

Example 8

Nathan sells cars and is paid a weekly retainer of $750 plus 7.5% of the value of cars he sells. Calculate his income if he sells cars worth a total value of $44 000 this week.

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Solution

Nathan's income $= \$750 + 7.5\% \times \$44\,000$

$= \$4050$

Example 9

Corrina is a real estate agent and is paid according to the following commission rates.

- 2% on the first $100 000 of property sold
- 1.5% on any value thereafter.

Calculate Corrina's commission for selling an apartment for $580 000.

Solution

Commission on 1st $100 000 $= 2\% \times \$100\,000$

$= \$2000$

Remaining amount $= \$580\,000 - \$100\,000$

$= \$480\,000$

Commission on remaining amount $= 1.5\% \times 480\,000$

$= \$7200$

Corrina's commission $= \$2000 + \$7200$

$= \$9200$

Piecework is earned according to the number of items made or tasks completed. Fruit pickers and dressmakers earn money this way.

Example 10

Sue earns $18 750 for each garden she designs. How many gardens must Sue design to earn over $100 000?

Solution

Number of gardens $= \$100\,000 \div 18\,750$

$= 5.333...$

≈ 6 Round up

Sue must design at least 6 gardens to earn over $100 000.

A **bonus** is extra money paid to employees who produce work of high quality or quantity, or for meeting an important goal or deadline. Some businesses reward their employees with a Christmas bonus for a productive year's work.

Example 11

After a successful year, a software company gave each employee a 2.5% bonus on their annual income. Find:

a the bonus received by Shaina, who earns $5324.50 per week

b the annual income of Kahil, whose bonus amounted to $4175.

Solution

a Shaina's annual income $= 52 \times \$5324.50$

$= \$276\,874$

Shaina's bonus $= 2.5\%$ of $\$276\,874$

$= \$6921.85$

b 2.5% of Kahil's income $= \$4175$

1% of Kahil's income $= \$4175 \div 2.5$ Using the unitary method.

$= \$1670$

$\therefore$ Kahil's annual income $= \$1670 \times 100$ 100%

$= \$167\,000$

Annual leave loading

Annual leave loading or **holiday loading** is extra pay given during annual leave (holidays). It is paid at the rate of 17.5% of 4 weeks' normal pay.

Example 12

For her Christmas holidays, Katrina received 4 weeks normal pay plus 17.5% annual leave loading for the 4 weeks. If Katrina's annual salary is $91 485, find:

a her normal weekly pay

b her leave loading

c her total pay for the 4-week holiday.

Solution

a Weekly income $= \$91\,485 \div 52.18$
$= \$1753.2579...$
$\approx \$1753.26$

b Leave loading $= 17.5\% \times \$1753.26 \times 4$
$= \$1227.282$
$\approx \$1227.28$

c Total holiday pay $= 4 \times \$1753.26 + \$1227.28$
$= \$8240.32$

EXERCISE 8.03 ANSWERS ON P. 586

Commission, piecework and leave loading U F PS

EXAMPLE 8

1 Anita is paid a commission of 3% on the value of kitchenware she sells. How much will Anita earn if she sells goods to the value of $24 560 in one week?

2 Dan is paid a commission of 5.5% of the value of the book sales he makes in excess of $5000. In one week, Dan sells books to the value of $37 427. Calculate his income for that week.

'in excess of' means to go over or above

3 Karen is a real estate agent who is paid a monthly retainer of $800 and a commission of 1.5% of the value of properties sold. Her sales in January were $1 780 400. Calculate Karen's income for January.

4 Consider these 2 salespeople and their method of payment.

- Perry: $1250 retainer and 2.5% of the value of holidays sold.
- Amy: no retainer and 7.5% of the value of holidays sold.

Calculate the income earned by Perry and Amy if they each sell holidays worth $54 000.

5 Lara earns 2.5% commission on the sale of thoroughbred horses. Last year she sold horses worth $2 750 000. What is Lara's commission? Select the correct answer **A**, **B**, **C** or **D**.

A $6 875 000 **B** $1 100 000

C $68 750 **D** $11 000

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6 A real estate agency pays the following rates of commission.

- 5% on the first $500 000
- 2.5% on the next $250 000
- 1.25% on any value thereafter.

Calculate the commission earned by selling property valued at:

a $500 000 **b** $750 000 **c** $1 350 000

EXAMPLE 9

7 Xanthe is paid 5% of the value of used cars she sells and 2.5% of the value of new cars. What commission is earned by selling new cars worth $108 400 and used cars worth $36 200?

8 Calculate the income for each amount of piecework.

a assembling 450 boxes at 75 cents per box
b delivering 2300 pamphlets at 10 cents per pamphlet
c assembling 95 prams at $5.85 per pram
d polishing 16 cars at $19.75 per car
e mowing nine lawns at $45 per lawn
f hemming 320 pairs of trousers at $1.50 per pair
g polishing 18 shoes at $2.50 per shoe
h laying 30.75 m^2 of paving at $24 per m^2

EXAMPLE 10

9 Fiona earns 85 cents for each toy she assembles. PS

a How much does she earn for assembling:
i 146 toys? **ii** 203 toys?
b Last week Fiona earned $459. How many toys did she assemble?
c Find the number of toys that Fiona must assemble to earn (at least) the following amounts:
i $200 **ii** $620

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10 Ricky earns $7.50 for each car he washes. Calculate his income if he washes 16 cars.

11 A publishing house gives all employees a 4.75% annual bonus for a profitable year. Calculate the annual bonus for each income.

a $92 400 p.a. **b** $165 600 p.a. **c** $1795 per week
d $994 per week **e** $8750 per month **f** $4156 per fortnight

EXAMPLE 11

12 Frank works 40 hours a week and is paid $27.40 per hour. Calculate:

a Frank's weekly income
b the annual leave loading of 17.5% on 4 weeks pay
c Frank's total pay for the four-week holiday.

EXAMPLE 12

13 Astra is the manager of a computer megastore. Her salary is $252 000 p.a. Calculate:

a Astra's fortnightly pay
b the holiday loading Astra would receive when she takes her 4 weeks' annual leave
c her total holiday pay.

8.03

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14 A process worker is paid $1.15 per article assembled, up to 500 articles, then $1.65 per article assembled after that. Calculate the total income earned for assembling each number of articles.

a 346 **b** 506 **c** 1406

15 Kevin works at the SuperGlo Hand Car Wash. He earns $7.50 per car for a wash only and $12.50 per car for a wash and polish. His job sheet for last week is shown. PS

Day	Wash only (number of cars)	Wash and polish (number of cars)
Monday	11	4
Tuesday	12	8
Wednesday	7	10
Thursday	16	0
Friday	18	13
Saturday	10	12

Calculate Kevin's:

a total earnings for the week

b average daily earnings

c average hourly rate of pay if he worked for 8.5 hours each day.

16 Zlatko, a construction supervisor, has signed a contract worth $970 000 to ensure that a factory is built by June 30. He will receive a bonus of 0.5% of his contract for each day the job is completed ahead of time. How much will Zlatko earn if the factory is completed on June 22? PS

17 Valentine, a storeman, works from 8:00 a.m. to 4:00 p.m., Monday to Friday. Valentine's rate of pay is $25.80 per hour. Calculate:

a his weekly wage **b** his holiday pay for 4 weeks (including leave loading).

8.04 Income tax

WS Income tax tables

WS Tax return form

WS Income tax and Medicare levy

Income tax

Anyone who earns more than $18 200 in a **financial year** (1st July to 30th June the following year) must pay **income tax** to the federal government. Income includes money earned from a job or interest earned from investments. The government uses the money paid in tax to fund public services such as schools, hospitals, roads and the defence forces.

Not all of a person's income is taxed. If we use some of our income for work-related expenses or to donate money to charities, these amounts are called **allowable deductions** (or **tax deductions**) and are not taxed. Other examples of allowable deductions are tools of trade, uniforms, car-related expenses, subscriptions to professional organisations and journals.

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Taxable income

Income tax is calculated on a person's **taxable income,** which is the **gross income** (total earnings) less all allowable deductions, **rounded down to the nearest dollar**.

Taxable income = gross income – allowable deductions

The more a person earns, the higher the rate of tax to be paid. The table below shows how income tax is calculated.

Tax rates for Australian residents	
Taxable income	**Tax on this income**
0 – $18 200	Nil
$18 201 – $37 000	19c for each $1 over $18 200
$37 001 – $90 000	$3572 plus 32.5c for each $1 over $37 000
$90 001 – $180 000	$20 797 plus 37c for each $1 over $90 000
$180 001 and over	$54 097 plus 45c for each $1 over $180 000
Source: © Commonwealth of Australia	

Example 13

Harry earned $65 660 last financial year and has allowable deductions of $727 in work-related car expenses and $259 in donations to charities.

a Calculate his taxable income.

b Use the tax table to calculate the income tax Harry must pay.

Solution

a Taxable income = $65 660 – $727 – $259
= $64 674

b According to the table, a taxable income of $64 674 is in the $37 001 – $90 000 tax bracket.

Income tax = $3572 + 0.325 × ($64 674 – $37 000)
= $12 566.05

'32.5c for each $1' means 32.5% or 0.325'

Example 14

Alice is an accountant who earned $148 642 one financial year. She also earned $532.51 in interest from her bank savings. Alice has allowable deductions of $287.20 for office supplies and $75.88 in books. Calculate:

a Alice's taxable income

b the amount of income tax payable

Solution

a Taxable income = $148 642 + $532.51 – $287.20 – $75.88
= $148 811.43
≈ $148 811 Rounded **down** to the nearest dollar

b According to the table, a taxable income of $148 811 is in the $90 001 – $180 000 tax bracket.

Tax payable = $20 797 + 0.37 × ($148 811 – $90 000)
= $42 557.07

EXERCISE 8.04 ANSWERS ON P. 586

Income tax U F PS

In this exercise, use the tax table on the previous page to calculate the income tax when needed.

1 Jake works as a plumber and earned $93 219 this year. He is entitled to these deductions: tools $1527.40, training courses $735.26 and office supplies $321.55. What is Jake's taxable income, rounded down to the nearest dollar?

EXAMPLE 13

2 Raina earns $75 660 in a year and has allowable deductions of $986.40.

a Calculate her taxable income, rounded down to the nearest dollar.

b Calculate the income tax Raina must pay.

EXAMPLE 14

3 Ali had a yearly income of $125 157 and earned $3240 in dividends from his shares. He also had allowable deductions of $540 for car expenses and $332 in the depreciation of his photocopier. Calculate: PS

a Ali's taxable income **b** the amount of tax payable.

4 Kadin earns $2145 per fortnight and has allowable annual deductions of $337. Calculate his income tax for the financial year. Select **A**, **B**, **C** or **D**.

A $3412.40 **B** $6824.60 **C** $9562.73 **D** $18 015.73

5 Joemon is a dive instructor and earns $67 300 per year. He has allowable deductions of $1126 for diving equipment and $912 for a wetsuit. Calculate:

a his taxable income

b his income tax.

Shutterstock.com/littlesam

6 Cassie earns $1956.55 per week and claims deductions of $655.30 in travel expenses, $276 depreciation of computer equipment and $155 membership to a professional association. She also donated $32 each month to a charity. Calculate Cassie's: PS

a taxable income **b** income tax payable

7 Jamie is a swimming pool manager with a yearly income of $85 958. He also earned $327.58 in interest from his bank savings account. Jamie also had allowable deductions of $134 and $55 in donations to charity. Calculate Jamie's: PS

a gross annual income **b** taxable income **c** the amount of tax payable

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8 Addison is a casual dance teacher who earns $32.50 per hour. She works 12 hours a week for 35 weeks of the year. How much should she pay in income tax? Select **A**, **B**, **C** or **D**.

A $4717.95 **B** $3572 **C** $669.94 **D** no tax paid

9 Sefu earns $295 160 p.a. His allowable deductions are donations to charity of $3045, uniforms costing $595 and car-related work expenses worth $890. Calculate the income tax payable. PS

10 Eerin earns $2846.87 per fortnight and claims deductions of $1955.30 in travel expenses, $572 depreciation of camera equipment, $180.20 membership to a professional association and $150 donated to a charity. Calculate Eerin's income tax payable. PS

Technology

Online income tax calculators

The Australian Taxation Office (ATO) website www.ato.gov.au has an online calculator for income tax. Visit the website and search 'Simple Tax Calculator' to find the income tax calculator for individuals.

1. Enter the taxable income $63 000 as '63000' (no spaces).
2. Select the current financial year.
3. Select 'Resident for full year' and click 'Next'.
4. The estimated tax payable will be shown on a new screen.

Repeat for at least 2 more taxable incomes. Compare the amount of tax payable with the results of other students in your class.

Mental skills 8: Maths without calculators ANSWERS ON P. 586

Percentage increase and decrease

The fraction equivalents of commonly-used percentages can help us when we need to increase or decrease a number by a percentage.

Percentage	1%	5%	10%	$12\frac{1}{2}\%$	20%	25%	$33\frac{1}{3}\%$	50%
Fraction	$\frac{1}{100}$	$\frac{1}{20}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$

1 Study each example.

a Increase 360 by 25%.

$$25\% \text{ of } 360 = \frac{1}{4} \times 360$$
$$= 360 \div 4$$
$$= 90$$
$$360 + 90 = 450$$

b Increase $80 by 5%.

$$5\% \text{ of } \$80 = \frac{1}{20} \times \$80$$
$$= \$80 \div 20$$
$$= \$4$$
$$\$80 + \$4 = \$84$$

or

$$10\% \text{ of } \$80 = \$8$$
$$\therefore 5\% \text{ of } \$80 = \$8 \div 2$$
$$= \$4$$

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2 Now increase:

a	\$340 by 20%	**b**	66 by 50%	**c**	150 by $33\frac{1}{3}\%$	**d**	\$400 by 1%
e	640 by 5%	**f**	\$72 by $12\frac{1}{2}\%$	**g**	\$470 by 10%	**h**	180 by 25%
i	420 by $33\frac{1}{3}\%$	**j**	\$80 by 5%	**k**	\$280 by 25%	**l**	70 by 20%

3 Study each example.

a Decrease \$225 by $33\frac{1}{3}\%$.

$33\frac{1}{3}\%$ of \$225 $= \frac{1}{3} \times \$225$

$= \$225 \div 3$

$= \$75$

\$225 – \$75 = \$150

b Decrease \$70 by 15%.

10% of $\$70 = \frac{1}{10}\$70 = \$7$

$\therefore 5\%$ of $\$70 = \frac{1}{2} \times \$7 = \$3.50$

$\therefore 15\%$ of $\$70 = (10\% \times \$70) + (5\% \times \$70)$

$= \$7 + \$3.50$

$= \$10.50$

\$70 – \$10.50 = \$59.50

4 Now decrease:

a	440 by 25%	**b**	\$300 by 20%	**c**	2400 by $33\frac{1}{3}\%$	**d**	\$500 by 15%
e	\$250 by 10%	**f**	\$120 by 50%	**g**	\$72 by $12\frac{1}{2}\%$	**h**	80 by 5%
i	\$85 by 20%	**j**	\$3800 by 1%	**k**	\$440 by 15%	**l**	\$150 by $33\frac{1}{3}\%$

8.05 PAYG tax and net pay

WS Pay day

To avoid paying income tax as a huge sum at the end of the financial year, your employer takes out an estimate of the tax from your pay every payday. This is called **PAYG (Pay As You Go) tax**. The total amount of PAYG tax paid over the year is usually more than the actual income tax payable, so at the end of the financial year you will receive the difference as a **tax refund**. However, if the PAYG tax paid is less than the income tax payable, you will have a **tax debt** and have to pay more.

Example 15

Employers use a PAYG tax table to determine how much tax to deduct from their employees' pays.

While he was at university, Li worked casually in a factory outlet store and earned \$196 per week for 8 weeks.

Weekly earnings (\$)	PAYG tax withheld (\$)
182–185	40
186–189	41
190–194	42
195–198	43
199–202	44

a Calculate Li's gross income for the 8 weeks.

b How much PAYG tax was deducted from his pay over the 8 weeks?

Solution

a Gross income = $196 × 8 = $1568

b In the table, $196 falls in the $195–$198 range.
Weekly PAYG tax = $43
Total PAYG tax = $43 × 8 = $344

Gross and net pay

Gross pay is the total amount a person earns or receives each payday.

Most income earners have a variety of deductions made against their gross pay before they receive it, including PAYG tax, superannuation contributions, union fees and health fund payments. The amount of income left after the deductions is called **net pay**.

Net pay

Net pay = gross pay – tax – other deductions

Example 16

Paula earns a gross pay of $700.70 per fortnight. Her deductions are for PAYG tax, $45.60 for private health insurance and $35 for superannuation.

a Use the PAYG tax table to find Paula's PAYG tax per fortnight.

Fortnightly earnings ($)	PAYG tax withheld ($)
672–678	152
680–686	154
688–694	156
696–704	158
706–712	160

b Calculate Paula's net pay.

c What are Paula's total deductions as a percentage of her gross income (correct to one decimal place)?

Solution

a In the table, $700.70 falls in the $696–$704 range.
Fortnightly PAYG tax = $158

b Net pay = $700.70 − $158 − $45.60 − $35
= $462.10

c Total deductions = $158 + $45.60 + $35
= $238.60

$$\text{Deductions percentage} = \frac{\$238.60}{\$700.70} \times 100\%$$
$$= 34.0516...\%$$
$$\approx 34.1\%$$

EXERCISE 8.05 ANSWERS ON P. 587

PAYG tax and net pay U F PS R C

Use the table from Example **15** on page 300 when calculating PAYG tax in questions **1** to **6**.

EXAMPLE 15

1 Xavier earned $200 per week for 16 weeks in a casual job as a party clown.

a Calculate his gross income.

b How much PAYG tax was deducted from his pay in total?

2 Rose is a part-time dental nurse. She is paid $192 to work half a day per week.

a Calculate Rose's gross income over 42 weeks.

b Find the total amount she paid in PAYG tax.

3 Pushkara earned $184 per week over the summer holidays working as a casual at the local plant nursery. How much did he pay in tax for the 6 weeks that he worked?

EXAMPLE 16

4 Sonja is a part-time music teacher who earns $195 per week. PS

a How much tax will she pay per week?

b What percentage of her gross pay will she pay in tax? Answer correct to 1 decimal place.

c Calculate her net pay per week.

5 In the holidays Phoebe earns $201 per week, while Jemima earns $191 per week. If they both work for 12 weeks in these jobs, who pays more in tax and by how much?

6 Huan wants to work during his school holidays over Christmas. He has been offered 2 different jobs for 6 weeks: PS R C

- Cinema: Earn $368 per fortnight (fortnightly tax of $80)
- Electronics store: Earn $190 per week

Over 6 weeks, which job offers the higher net pay? Show all working to justify your answer. PS R C

7 Goran earns $698 per fortnight. He also pays $19.20 in union fees and $84.90 in superannuation per fortnight.

a Use the table from Example **16** on page ## to calculate how much PAYG tax he pays per fortnight.

b Calculate Goran's fortnightly net pay.

c What percentage (correct to one decimal place) of his gross pay is made up of deductions?

8 Caleb paid $154 in tax for one fortnight's work at the local cinema. Use the fortnightly tax table to determine how much he could have possibly earned. Select **A**, **B**, **C** or **D**.

A $672 **B** $679 **C** $685 **D** $690

9 Cooper has a gross income of $1125.86 per week. His deductions are $219.48 tax, $64.35 for private health insurance and $30 for superannuation. Calculate Cooper's:

a net income

b total deductions as a percentage of his gross income (correct to one decimal place).

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10 Stefan earns a gross income of $864.25 per week. His deductions are $141.94 tax and $51.33 for private health insurance. Calculate:

- **a** Stefan's net income
- **b** the percentage (to one decimal place) of Stefan's gross income that is paid in tax.

11 Copy and complete each pay slip. PS R

a

Employee: Janice Hall		Hourly pay rate: $28.80	
Hours worked		**Deductions**	
Normal	38	Tax: $215	Other: $124.60
Time-and-a-half	0	Gross weekly income	
Double time	0	Total deductions	
		Net weekly income	

b

Employee: Amber Menzies		Hourly pay rate: $32.50	
Hours worked		**Deductions**	
Normal	39	Tax: $275	Other: $229.10
Time-and-a-half	4	Gross weekly income	
Double time	1	Total deductions	
		Net weekly income	

12 Jodie earns a gross fortnightly pay of $1027.60. Apart from tax, she has the following deductions: superannuation ($26.80), health insurance ($21.60), union fees ($5.20) and savings plan ($65).

- **a** If Jodie's net fortnightly pay is $657.30, find how much tax is taken out.
- **b** Express the tax she pays as a percentage of her gross income (to one decimal place).

13 Lina earns a salary of $76 800 p.a. Each fortnight she has deductions of $186.50 for family health insurance and $65.12 for superannuation taken from her gross income. She also pays tax at the rate of 32.5 cents in the dollar. Calculate: PS R

- **a** Lina's fortnightly gross income
- **b** the amount of tax Lina pays per fortnight
- **c** Lina's total deductions each fortnight
- **d** Lina's net income for the fortnight.

14 Balun earns a salary of $128 900 p.a. Each month he has deductions of $43.26 for union fees and he pays $341.05 per month for health insurance taken from his gross income. He also pays $36 432 tax for the year. Calculate:

- **a** Balun's annual union fees
- **b** his annual health insurance payments
- **c** Balun's annual net income
- **d** the percentage of Balun's gross income that is paid in tax (correct to one decimal place).

Technology

Online PAYG tax calculator

Visit the Australian Tax Office website **www.ato.gov.au/taxtables** to use the online PAYG tax calculator or to download the PAYG tax tables. It contains the PAYG tax amounts for weekly, fortnightly and monthly incomes. Find the PAYG tax payable and net pay for a gross pay of:

a $505 weekly **b** $1466 fortnightly **c** $2730 monthly

Did you know?

The Medicare levy

The Medicare levy is a tax that helps fund Australia's public health system, known as Medicare. The federal government introduced Medicare in 1984. The Medicare levy is currently 2% of your taxable income and is additional to the income tax you pay. The PAYG amount your employer takes from your salary or wages includes an amount to cover the Medicare levy. The actual Medicare levy is calculated by the Australian Taxation Office when you lodge your income tax return.

Use the online Medicare levy calculator at www.ato.gov.au to check that it calculates 2% of taxable income.

Power plus ANSWERS ON P. 587

1. In one year, tennis player Roger Federer earned $136.7 million. How much is this per minute? Assume 365 days in a year.
2. Danielle is paid $2500 per month. On average, how much does she earn in 4 hours, if she works 9 hours per day, 5 days per week, for 48 weeks a year?
3. A fitness instructor conducts a class with 12 participants who pay $18 each. The class lasts for 45 minutes. What is the instructor's average hourly income?
4. Felicity and Rose have the same net pay. Felicity has $187.60 deducted from her fortnightly gross pay of $514. Rose, who is paid weekly, has $64.20 deducted from her gross pay each week. Calculate the difference between Felicity's and Rose's gross fortnightly pays.
5. Mirko earns $63 for working from 7:30 a.m. to 9:15 a.m. At this rate, how much will he earn for a 38-hour week?
6. Anya earned $903 for working 36 hours at normal rates and 4 hours at time-and-a-half. Find her hourly rate of pay.
7. When Bhavin works 6 hours overtime at time-and-a-half, his gross income increases by $202.50. Find his weekly income for working 38 hours at normal rates.
8. One year, Robert received a holiday loading of $589.05, which was 17.5% of 4 weeks' pay. What was his normal weekly pay?
9. If Ingrid paid income tax of $34 561 last financial year, what was her taxable income?
10. Maddy earns $2420 for a 40-hour week. How long, in hours and minutes, will Maddy need to work overtime at time-and-a-half so her overtime is equivalent to her annual leave loading?

CHAPTER 8 REVIEW

Money and business

Language of maths

allowable deduction	annual leave loading	bonus	commission
deduction	double time	financial year	fortnight
gross pay	holiday loading	income tax	net pay
overtime	PAYG tax	per annum	piecework
retainer	salary	taxable income	time-and-a-half
wage			

1 What is the difference between a **wage** and a **salary?**

2 What is an **allowable deduction?** Give an example of one.

3 What earnings are calculated by 17.5% of 4 weeks of normal pay?

4 What earnings are calculated as a percentage of the value of items sold?

5 What does **PAYG** stand for?

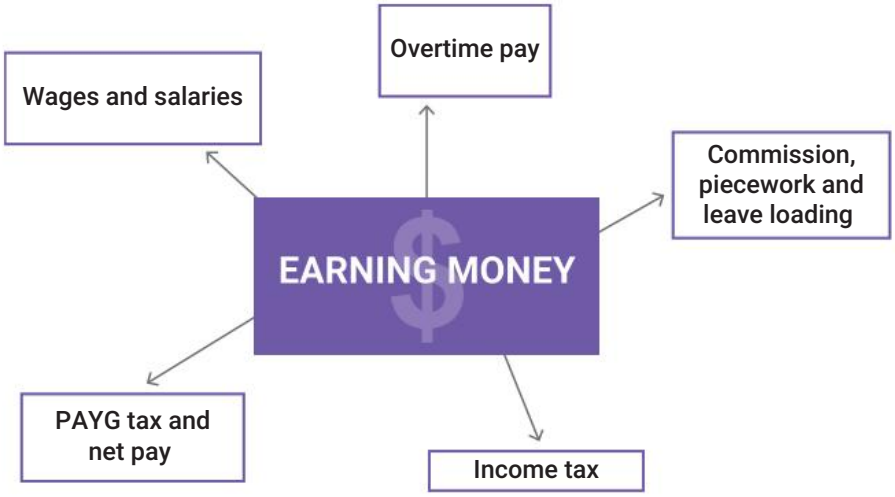
6 What is a **financial year?**

Topic summary

- Write 10 questions (with solutions) that could be used in a test for this chapter
- Swap your questions with another student and check their solutions against yours
- Write down any part of this topic that you did not understand and discuss it with your teacher

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.

Mind map: Earning money



TEST YOURSELF 8 ANSWERS ON P. 587

8.01 **1** Milan has an annual salary of $87 410. How much would he be paid:

a per week? **b** per fortnight? **c** per month?

8.01 **2** Diane is paid $23.65 per hour and works a 37-hour week. Find her weekly wage.

8.02 **3** Riley works 38 hours at $37.38 per hour and 5 hours overtime at time-and-a-half. Calculate his total earnings.

8.02 **4** Liam works at a warehouse under the following weekly award:

Award schedule: Normal rate is $24.75 per hour	
Normal rate	For 0 to 38 hours worked
Time-and-a-half	For the next 3 hours worked
Double time	For each hour worked after that

Calculate his wage for working:

a 38 hours **b** 44 hours.

8.03 **5** Nikky is a bookseller who earns 4% commission on the value of books she sells. She is also paid a monthly retainer of $900. If Nicole's sales for March were $82 500, find her total income for March.

8.03 **6** Sam is paid commission on a sliding scale of 3% on the first $50 000 of sales and 1.5% on all sales after that. Calculate Sam's income if he sells goods worth $80 000.

8.03 **7** Calculate the piecework pay for:

a delivering 1500 pamphlets at 50 cents per pamphlet

b washing 45 cars at $27.50 per car

c laying 35.5 m^2 of tiles at $35 per m^2.

8.03 **8** A jockey earns $500 per race and 4.5% of the value of prize money his horses win. Last week, Dan had 12 rides and his horses won prize money worth $120 000. How much did Dan earn?

8.03 **9** Zoe earns $1786 per week and is about to begin her annual holidays. Along with her 4 weeks pay, she also receives a 17.5% annual holiday loading on the 4 weeks. Find Zoe's total holiday pay.

8.04 **10** Emily is a department store manager who earned a gross salary last financial year of $187 542 and had the following allowable deductions: $378 uniform cleaning costs, $218 training course costs and $1492 travel expenses. Calculate:

a Emily's taxable income

b the amount of income tax Emily should pay using the table on page 297

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11 Raj earns $1492.44 per fortnight and pays PAYG tax, $33.15 for life insurance, $20.16 in union fees and $48.17 in superannuation.

8.05

a Use the PAYG table to find Raj's PAYG tax paid per fortnight.

Fortnightly earnings ($)	PAYG tax withheld ($)
1470–1475	174
1476–1481	176
1482–1485	178
1486–1491	180
1492–1497	182

b Calculate Raj's fortnightly net pay.

c Calculate, correct to one decimal place, the percentage of Raj's gross pay that was paid in tax.

12 For the pay slip below, calculate the:

a gross income **b** total deductions **c** net income.

8.05

Employee: Shan Zhou Hours worked		Hourly pay rate: $22.80 Deductions	
Normal	37	Tax: $275.10	Other: $84.50
Time-and-a-half	4	Gross weekly income	
Double time	2	Total deductions	
		Net weekly income	

9

STATISTICS AND PROBABILITY

ANALYSING DATA

Statistical data are used by governments and businesses to make decisions about health and welfare, the population, production, tourism and the environment. Data collected from a census, through market research (such as online surveys, questionnaires, street polls) can lead to advertising campaigns for health issues and road safety, improved customer service and new products. However, before any action can be taken, the data must be carefully analysed in order to avoid misinterpretation and to check it for relevance and reliability.

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Chapter outline

	Working mathematically				
9.01 The mean, median, mode and range	U	F		R	C
9.02 Histograms and stem-and-leaf plots	U	F		R	C
9.03 The shape of a distribution	U	F		R	C
9.04 Comparing data sets	U	F		R	C
9.05 Sampling and types of data	U	F		R	C
9.06 Bias and questionnaires	U	F	PS	R	C

Wordbank

bias Something unwanted that causes a sample to not truly represent the population

cluster Values in a data set that are close or 'bunched' together

frequency histogram A special column graph that shows the frequencies of numerical data

outlier An extreme value that is very different from the other values in a set of data

population All of the items being studied, the entire group

random sample A sample in which every item in the population has an equal chance of being selected

skewed distribution A distribution in which most of the values are clustered at one end, creating a 'tail' at the other end

symmetrical distribution A distribution in which all values are distributed equally on both sides of the centre, its shape having line symmetry

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

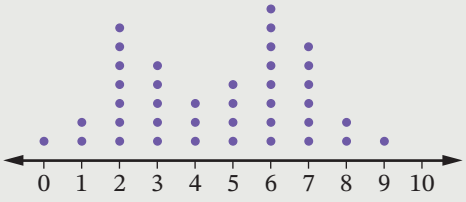
Statistics

In this chapter you will:

- calculate and interpret the mean, median, mode and range for a set of data
- investigate the effect of an outlier on the mean and median
- construct histograms and stem-and-leaf plots
- describe the shape of a statistical distribution by identifying skewness, symmetry and modality
- compare 2 or more sets of data by analysing their graphs and statistics
- investigate sampling and types of data
- collect and analyse data involving numerical and categorical variables
- (STAGE 5.2) evaluate statistics found in the media and detect bias in reports of surveys

SkillCheck ANSWERS ON P. 587

1 This dot plot shows the number of apps purchased by students last month.



Number of apps purchased

a How many students were surveyed?

b What was the most frequent number of apps purchased?

c What percentage (correct to one decimal place) of students purchased over 5 apps?

2 This stem-and-leaf plot shows the ages of people at a café during 8:00–8:30 a.m. on a Saturday morning.

Stem	Leaf
1	4 6 7 9 9
2	0 4 5 7 8 8
3	1 5 5 6 6 6 7 8 9
4	2 3 3 5 7
5	0 0 1 4 5 6 8 8
6	3 5 6 7 7

a How many people were in the café?

b How many of them were aged in their 20s?

c What was the most common age?

d What was the youngest age?

e What percentage (correct to one decimal place) of people were 45 or older?

3 The length of a queue at a supermarket checkout was recorded every 5 minutes for 3 hours.

4	3	6	5	1	4	7	5	4	1	4	4
4	3	6	3	1	3	4	2	4	5	1	3
5	4	2	3	5	2	5	2	6	5	1	3

a Arrange the data in a frequency table.

b Draw a frequency histogram and a frequency polygon.

c What was the most common number of people in the queue?

d How many times was the length of the queue longer than 3 people?

The mean, median, mode and range 9.01

The **mean**, **median** and **mode** are called **measures of location**, **measures of central tendency** or **averages**, because they indicate a central value of a set of **data**.

The **range** is called a measure of spread because it indicates how widely the values are spread in a set of data.

WS Collecting and describing data
Statistical measures
The mode, median and mean
Mode, median and mean
Statistical measures calculator
Statistical measures calculator

Measures of location and spread

Mean, $\bar{x} = \dfrac{\text{sum of data values}}{\text{number of data values}}$

$= \dfrac{\Sigma x}{n}$

where Σx means 'the sum of the values' and n is the total number of values.

Σ is the Greek letter **sigma**, and stands for 'the sum of'.

The **mode** is the value (or values) that occurs most often.

When the data values are arranged in order, the **median** is:

- the middle value, for an odd number of values
- the average of the 2 middle values, for an even number of values.

Range = highest value – lowest value

An **outlier** (pronounced 'out-ly-er') is an extremely high or low value that is separated from the other values in the data set.

Comparing the mean, median and mode

The **mean** is usually the best measure of location because it takes into account every data value.

If there are outliers in the set of data, then the mean may be affected by these extreme values and will not accurately represent all of the values. In this case, the **median** is the best measure because it is not affected by outliers.

The **mode** is useful when the most common value is important, or when the data is categorical (such as make of car or hair colour). With categorical data, it is not possible to have a mean or median.

Example 1

The maximum daily temperatures (in °C) in Sydney for a fortnight in March were:

34	22	32	26	23	24	24
26	31	26	27	28	28	27

Find:

a the mean

b the median

c the mode

d the range

Solution

a $\text{Mean} = \frac{34+22+32+\cdots+28+27}{14}$

$= \frac{378}{14}$

$= 27$

b Arranging the values from lowest to highest: the middle values

22 23 24 24 26 26 **26 27** 27 28 28 31 32 34

There is an even number of values (14), so we have 2 middle values, the 7th and 8th values 26 and 27.

$\text{Median} = \frac{26+27}{2}$

$= 26.5$

c Mode = 26 The most common value, occurring 3 times

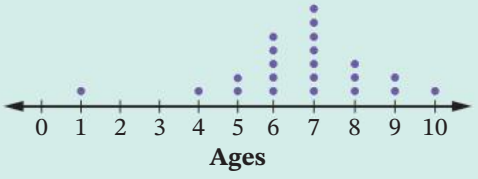
d Range = 34 – 22 Highest value – lowest value

$= 12$

Analysing dot plots

Example 2

This **dot plot** shows the ages of the children at Soshanna's birthday party.



Find:

a the mean (correct to one decimal place)

b the median

c the mode

d the range

e the outlier

Solution

a $\text{Mean} = \frac{1+4+5+5+\cdots+10}{22}$

$= 6.636363...$

≈ 6.6

b There are 22 values, so the median is the average of the 11th and 12th values. Counting dots from the left and upwards, these are 7 and 7.

$\text{Median} = \frac{7+7}{2} = 7$

c Mode = 7 The value with the most dots

d Range = 10 – 1 = 9

e Outlier = 1 This value is much different from the rest

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The mean from a calculator's statistics mode

Scientific and graphics calculators have a statistics mode (SD or STAT). Follow the instructions in the table below to calculate the mean of the data from Example **1** using your calculator's statistics mode.

Operation	Casio	Sharp
Start statistics mode.	MODE STAT 1-VAR	MODE STAT =
Clear the statistical memory.	SHIFT 1 Edit, Del-A	2nd F DEL
Enter data.	SHIFT 1 Data to get table 34 = 22 = , etc. to enter in column AC to leave table	34 M+ 22 M+, etc.
Calculate the mean. ($\bar{x}$ = 27:0714)	SHIFT 1 Var $\bar{x}$ =	RCL $\bar{x}$
Check the number of values. (n = 14)	SHIFT 1 Var n =	RCL n
Return to normal (COMP) mode.	MODE COMP	MODE 0

The mean from a frequency table

For a large set of data, we can use a **frequency distribution table** to organise the data. By adding an fx column, the mean can be easily found.

Score (x)	Frequency (f)
3	2
4	5
5	10
6	36
7	25
8	13
9	4
10	3

Example 3

The scores of Year 9 students giving a speech in English (out of 10) are shown in the frequency table above.

a How many students were there?

b Use an fx column to calculate the mean score.

c Find the mode.

d What is the range?

Solution

a The total sum of the Frequency column is 98.

There were 98 students.

WS The mean and fx tables

Statistics from a frequency table

b

Mark (x)	Frequency (f)	fx
3	2	6
4	5	20
5	10	50
6	36	216
7	25	175
8	13	104
9	4	36
10	3	30
	$\sum f = 98$	$\sum fx = 637$

fx means $f \times x$, which multiplies each score by its frequency.

$\sum f$ is the sum of the frequency column (that is, the total number of scores).

$\sum fx$ is the sum of the fx column (that is, the sum of all scores).

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$$\text{Mean} = \frac{637}{98}$$

$$= 6.5$$

Sum of all scores divided by the number of scores

c Mode = 6

Score with the highest frequency

d Range = 10 − 3 = 7

Mean, mode and range for data in a frequency table

- **Mean**, $\bar{x} = \frac{\text{sum of } fx}{\text{sum of } f} = \frac{\sum fx}{\sum f}$
- The **mode** is the value(s) with the highest frequency, f
- The **range** is the difference between the last and first values in the score (x) column

This table shows calculator instructions for finding the mean of data presented in a frequency table. It uses the table of data from Example **3**.

Operation	Casio	Sharp
Start statistics mode.	MODE STAT 1-VAR SHIFT MODE scroll down to STAT Frequency? ON	MODE STAT =
Clear the statistical memory.	SHIFT 1 Edit, Del-A	2nd F DEL
Enter data.	SHIFT 1 Data to get table 3 = 4 =, etc. to enter in x column 2 = 5 =, etc. to enter in FREQ column AC to leave table	3 2nd F STO 2 M+ 4 2nd F STO 5 M+, etc.
Calculate the mean. ($\bar{x}$ = 6.5)	SHIFT 1 Var $\bar{x}$ =	RCL $\bar{x}$
Check the number of values. (n = 98)	SHIFT 1 Var n =	RCL n
Return to normal (COMP) mode.	MODE COMP	MODE 0

Cumulative frequency and the median

To find the median of data presented in a frequency table, add a **cumulative frequency** column, which is a progressive total of the frequency column.

Example 4

Find the median of the Year 9 student speech scores from Example **3**.

Solution

The cumulative frequency column is a running total of the frequencies.

$2 + 5 = 7$

$7 + 10 = 17$, etc.

There are 98 scores, so the 2 middle values are the 49th and 50th scores. Using the cumulative frequency column, the 17th score is a 5 and the 53rd score is 6, so the 49th and 50th scores must both be 6s.

$\text{Median} = \frac{6+6}{2} = 6$

Mark (x)	Frequency (f)	Cumulative frequency (cf)
3	2	2
4	5	7
5	10	17
6	36	53
7	25	78
8	13	91
9	4	95
10	3	98
	$\Sigma f = 98$	

9.01

EXERCISE 9.01 ANSWERS ON P. 587

The mean, median, mode and range U F R C

Round all means to one decimal place where necessary, unless otherwise specified.

1 For each set of data, find:

i the mean **ii** the median **iii** the mode **iv** the range

EXAMPLE 1

a	23	20	12	15	28	19	15	14	18		
b	5	3	8	7	1	6	7	7	5	15	12
c	32	45	12	45	38	32					
d	8.8	8.2	8.8	8.3	8.5	8.9	9.2	8.8			
e	−5	0	−2	4	6	−2	5	−1	−2	0	

2 The maximum and minimum daily temperatures (in °C) over a week at Point Perpendicular on the NSW south coast are listed.

Maximum	23	24	22	23	22	23	23
Minimum	17	17	15	16	15	15	14

a Calculate the mean daily maximum and mean daily minimum temperature.

b What was the difference between the 2 means?

c Find the mode of daily maximum and minimum temperatures.

d What was the difference between the 2 modes?

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3 The monthly rainfall (in mm) in Cairns is shown in the table.

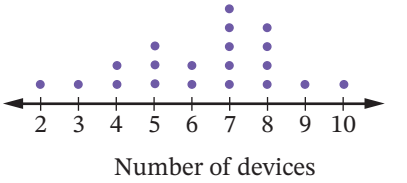
Month	J	F	M	A	M	J	J	A	S	O	N	D
Rainfall	413	435	442	191	94	49	28	27	36	38	90	175

a What is the range? **b** Find the mean (correct to the nearest mm).

c Find the median.

EXAMPLE 2

4 A number of homes were surveyed on how many digital devices they had, with the results shown in the dot plot.



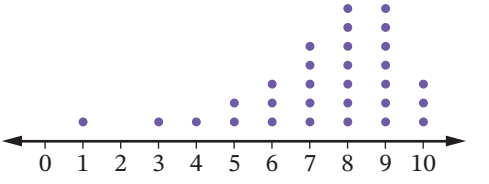
Number of devices

Find:

a the mean **b** the median

c the mode **d** the range

5 This dot plot shows the results of a maths quick quiz (out of 10). R C



Quiz results

a Are there any outliers? (Give reasons)

b What is the mode?

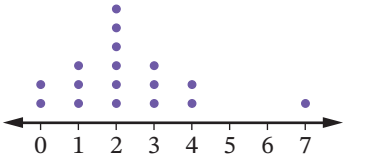
c Find the mean.

d Find the median.

e What percentage of results (correct to one decimal place) were above:

i the mean? **ii** the median?

6 This dot plot shows the number of goals scored by a soccer team during the season.



Number of goals

a Are there any outliers?

b Find the mean and median:

i without the outlier

ii with the outlier

c Which measure (mean or median) is affected by the outlier?

7 The salaries of 10 people working at a software company are:

$80 000	$105 000	$70 000	$93 000	$78 500
$180 000	$90 000	$72 000	$93 000	$101 000 R C

a Which salary is an outlier?

b Find the mean and median:

i without the outlier **ii** with the outlier

c Which measure, the mean or the median, represents the salaries better?

d Find the mode and range:

i without the outlier **ii** with the outlier

e Which measure, the mode or the range, is affected by the outlier?

8 Use your calculator in the statistics mode to find the mean of each set of data.

a 5 7 8 3 6 8 5 4
8 3 8 10 6 8 4 9

b 13.5 14.2 11.9 15.7 18.4 17.6 12.0

c 56 72 84 57 49 65 59
79 80 76 57 66 68

9 Jay played in a golf tournament. His average score for the 4 rounds was 71. If the scores in his first 3 rounds were 72, 66 and 70, what was his score in the 4th round? R

10 Tasnuba scored a mean of 74% for 5 maths tests that she completed. R

a What was the total number of marks Tasnuba scored for the 5 tests?

b Tasnuba did a 6th test and her mean test mark increased to 77%.
What is the total that she scored for the 6 tests?

c What mark did she achieve in the last test?

11 Alice's average mark for 4 Science exams is 68%. Can Alice score enough marks in her 5th exam so that her average mark increases to 75%? Give reasons. R C

12 Copy and complete each frequency table, then calculate the mean, correct to 2 decimal places.

a

Score, x	Frequency, f	fx
2	5	
3	3	
4	7	
5	6	
6	4	
	Σf =	Σfx =

b

x	f	fx
22	3	
23	8	
24	7	
25	9	
26	5	
27	4	
	Σf =	Σfx =

13 Use your calculator in the statistics mode to find the mean of each frequency table.

a

Score, x	Frequency, f
32	5
33	8
34	12
35	9
36	7
37	4

b

x	f
10	12
11	18
12	27
13	37
14	23
15	15
16	9

EXAMPLE 4

14 For each frequency table, add a cumulative frequency column and find:

i the median **ii** the mode **iii** the range

a

x	f
2	2
3	5
4	11
5	7
6	10
7	9
8	6

b

x	f
17	3
18	8
19	20
20	15
21	11
22	7

EXAMPLE 4

15 The number of seedlings that have sprouted in each punnet at a nursery are shown:

10	5	7	7	9	8	8	11	7	8	5	8	4	6	4
12	11	9	4	6	6	10	7	6	9	10	9	7	8	8
8	7	4	7	7	11	10	9	8	9	9	8	8	8	9

a Complete a frequency table and find the median.

b What is the mode?

c Find the range.

d Calculate the mean.

Investigation

The effect of outliers

1 Find the mean, median, mode and range of each data set.

a 1 2 2 2 3 4 5 5 5

b 1 2 2 2 3 4 5 5 6

c 1 2 2 2 3 4 5 5 8

d 1 2 2 2 3 4 5 5 17

2 The 4 sets of data in question **1** are the same, apart from the last value in each set. If one value is changed, what is the effect on:

a the mean? **b** the median? **c** the mode? **d** the range?

3 Consider these 2 sets of data:

A	1	10	11	14	15	12	13	11		
B	3	4	2	6	3	14	6	3	5	4

a Find the mean, median and range of each set of data.

b Identify the outlier for each set.

c Find the mean, median and range of each set of data, excluding the outlier.

d What effect does the size of the outlier have on:

i the mean? **ii** the median? **iii** the range?

e Why is the median not affected by the size of the outlier?

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Technology

Most valuable player

A team of basketball players would like to compare the number of points each player has scored per game over 10 games.

1 Enter the following data into a spreadsheet.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	NAME	Game 1	Game 2	Game 3	Game 4	Game 5	Game 6	Game 7	Game 8	Game 9	Game 10	Total	Mean	Mode	Median	Range
2	Sirisha	5	8	2	10	6	8	4	4	6	10					
3	Colin	4	8	6	7	7	9	10	6	5	8					
4	Eloise	12	8	8	16	4	5	7	10	11	8					
5	Diana	4	3	5	2	6	8	4	3	3	6					
6	Tran	10	14	7	8	12	16	10	10	8	6					
7	Michelle	12	4	6	10	9	9	5	16	8	12					
8	Anish	4	7	6	8	7	5	3	8	9	2					
9	Jamar	14	12	8	7	10	9	5	12	15	8					
10	Total															
11	Mean															

2 In B10, enter **=sum(B2:B9)** to calculate the **total score** for Game 1. **Fill Right** to repeat this step for cells C10 to K10.

3 In L2, enter **=sum(B2:K2)** to calculate the **total score** for Sirisha for the 10 games. **Fill Down** to repeat this step for cells L3 to L9.

4 In B11, enter **=average(B2:B9)** to calculate the **mean** score per player for Game 1. **Fill Right** from cell B11 across to K11.

5 In M2, enter **=average(B2:K2)** to calculate the **mean** score for Sirisha for the 10 games. **Fill Down** into cells M3 to M9.

6 In N2, enter **=mode(B2:K2)** to calculate the **mode** score for Sirisha. **Fill Down** to cell N9.

7 In column O, use the formula **=median(B2:K2)** and **Fill Down**.

8 There is no formula for the **range**, but we can create one using the formulas for the **maximum** and **minimum** values. In P2, enter **=max(B2:K2)–min(B2:K2)** for Sirisha's range, then **Fill Down**.

9 Now answer these questions, writing each answer in your spreadsheet in the given cell [ ].

- **a** Which player had the best mean score? [A13]
- **b** Which player was the most inconsistent? Why? [A14]
- **c** Which of the statistics is the most useful for describing the players' overall contribution to team score? [A15]
- **d** Who do you think was the MVP (Most Valuable Player)? Why? [A16]

9.02 Histograms and stem-and-leaf plots

WS Analysing data

WS Stem-and-leaf plots

Highway elephants

Frequency histograms and polygons

A **frequency histogram** is a column graph of numerical data where the columns stand together without gaps between them.

A **frequency polygon** is a line graph that is constructed by joining the midpoints of the tops of the columns of a frequency histogram, starting and ending on the horizontal axis. It is called a 'polygon' because the graph and the horizontal axis together make a shape with straight sides.

Example 5

45 students were surveyed on the number of hours of homework they did in the past week. The results were as follows:

3	7	5	8	8	7	7	4	8	7	9	8	8	5	7
4	2	5	3	8	6	6	7	9	6	5	4	9	8	4
2	3	6	7	8	2	8	8	4	5	6	8	7	2	5

a Arrange the data in a frequency table.

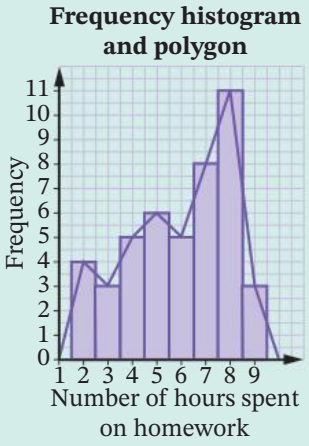
b Draw a frequency histogram and polygon for this data.

Solution

a

No. of hours, x	Tally	Frequency, f
2	IIII	4
3	III	3
4	𝍸	5
5	𝍸 I	6
6	𝍸	5
7	𝍸 III	8
8	𝍸 𝍸 I	11
9	III	3
	Total	45

b



Note that each column of the histogram is centred on each value marked on the horizontal axis, so there is a half-column gap at the start between the Frequency axis and the first column.

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Stem-and-leaf plots

A **stem-and-leaf plot** lists the values of a data set, usually in order. It shows:

- any **clusters** where values are grouped or bunched together
- any **outliers**
- how the values are spread out

9.02

Example 6

This stem-and-leaf plot shows the pulse rate (in heartbeats per minute) of 30 people in a shopping centre.

The stem is formed by the 10s digit of the values.

Stem	Leaf
5	1 5 8
6	1 3 4 4 5 6 8 8 9
7	0 1 2 3 3 4 4 5 7 8 8 9
8	0 3 4 6 7
9	0

The leaves are formed by the units digits of the values.

The data shown by this row is 80, 83, 84, 86 and 87.

a Are there any outliers?

b Is there any clustering of data?

c Find the mean, correct to one decimal place.

d What is the median?

e What is the mode?

f What is the range?

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Solution

a There are no outliers as no values are 'separate' from the main body of data.

b The values are bunched in the 60s and 70s, so we say that there is clustering in the 60s and 70s.

c $\text{Mean} = \frac{51+55+58+\cdots+87+90}{30}$

$= \frac{2156}{30}$

$= 71.8666...$

≈ 71.9

d The 30 data values in the stem-and-leaf plot are in order, so the median is the average of the 15th and 16th values, 72 and 73 (counting from top to bottom, left to right).

$\text{Median} = \frac{72+73}{2} = 72.5$

e Modes = 64, 68, 73, 74, 78 Each occurs 2 times.

f The lowest value is 51 and the highest is 90.

Range = 90 – 51

= 39

Back-to-back stem-and-leaf plots

Back-to-back stem-and-leaf plots

When 2 related sets of data are compared, we can use back-to-back stem-and-leaf plots.

Example 7

Buzz batteries and *Eternal* batteries are to be compared by testing 15 batteries of each brand until they fail. The lengths of the battery lives are recorded to the nearest hour as shown.

Buzz	8	12	22	18	23	20	11	15	18	8	9	19	26	30	31
Eternal	14	16	24	32	35	36	37	28	31	30	24	26	19	35	32

a Arrange the data into a back-to-back stem-and-leaf plot to determine which battery is better.

b Describe any clusters in each distribution.

c Compare the median of each brand.

d Compare the mean of each brand.

e Which brand of battery is better? Justify your answer.

Solution

a

Buzz				Eternal							
		9	8	8	0						
9	8	8	5	2	1	4	6	9			
		6	3	2	0	2	4	4	6	8	
			1	0	3	0	1	2	2	5	5	6	7

b Buzz is clustered in the 10–20 hours' life, Eternal is clustered in the 30s hours of life.

c Median for Buzz = 18 — The 8th value, counting from top to bottom, right to left

Median for Eternal = 30 — The 8th value, counting from top to bottom, left to right

Eternal has the higher median of 30 hours.

d Mean for Buzz = 18

Mean for Eternal = 27.933...

Eternal has the higher mean of approximately 27.9 hours.

e From looking at the statistics and the stem-and-leaf plot, Eternal is better as its median and mean are higher.

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EXERCISE 9.02 ANSWERS ON P. 588

Histograms and stem-and-leaf plots UFRC

EXAMPLE 5

9.02

1 The results of an Art project (out of 10) for 30 students are shown.

7	9	3	5	6	6	7	5	2	8
7	8	6	5	5	4	3	6	6	7
8	10	7	5	8	9	8	5	4	5

a Arrange the data into a frequency distribution table.

b Draw a frequency histogram and polygon.

c Find the mean, correct to one decimal place.

d Find the median.

e Find the mode.

f Find the range.

g Are there any outliers and is there any clustering? C

2 A sample of people were surveyed on the number of children in their families.

1	2	6	5	3	3	2	1	7	4	4	4	7	2
3	5	3	1	2	1	1	2	7	4	2	2	3	6
3	6	5	3	5	7	9	8	3	2	3	4		

a Arrange the data into a frequency table and construct a frequency histogram and polygon.

b Find the mode and the range.

c Are there any outliers and is there any clustering?

d Find the median.

e Find the mean, correct to one decimal place.

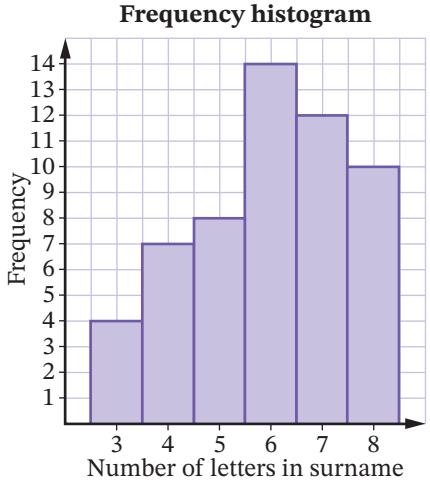
3 The numbers of letters in the surnames of 55 students are shown in this frequency histogram.

a What is the mode? How is this shown on the frequency histogram?

b Find the mean, correct to 2 decimal places.

c Find the median.

d Find the range.



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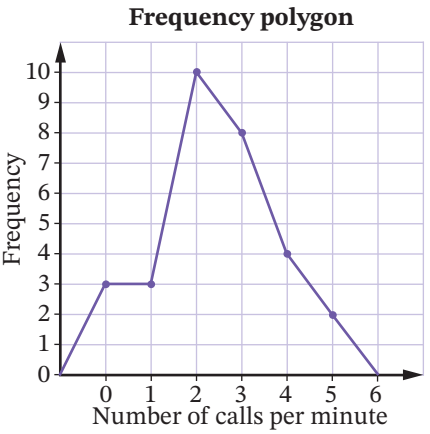
4 This frequency polygon shows the number of calls made per minute to a customer support line over a 30-minute period. R C



- **a** How many calls were made altogether?
- **b** How many times were there 3 or more calls received per minute?
- **c** What is the mode? How is this shown on the frequency polygon?
- **d** Find the range.
- **e** Find the median.
- **f** Find the mean, correct to 2 decimal places.
- **g** Is there any clustering of data?

EXAMPLE 6

5 This unordered stem-and-leaf plot shows the marks of a class of students in a Science test.

Stem	Leaf
4	0 1 5
5	3 1 5 6 8
6	7 7 8 6 4 3 5 7
7	9 1 7 4 2
8	8 2 7
9	2 0 4

- **a** Construct an ordered stem-and-leaf plot.
- **b** Find the range.
- **c** Are there any outliers or clusters?
- **d** How many students are there in the class?
- **e** Find the mean, correct to 2 decimal places.
- **f** Find the median.

6 This stem-and-leaf plot shows the points scored by a basketball team in different matches.

Stem	Leaf
3	0 7
4	1 1 3 5
5	4 6 6 8 9
6	3 5 8 8 8 9
7	2 4 5 5
8	0 2

- **a** Find the range.
- **b** What is the mode?
- **c** Find the mean, correct to one decimal place.
- **d** Find the median.

7 When 30 students ran 100 m, their times (in seconds) were recorded.

13.5	15.0	12.1	12.5	17.6	13.7	15.7	18.7	13.1	11.8
12.6	15.7	14.4	15.4	14.9	15.0	18.3	16.3	15.4	15.1
11.9	16.1	14.3	14.5	16.3	14.6	14.9	15.0	16.1	13.5

Stem	Leaf
11	8 9
12	
13	
14	
15	
16	
17	
18	

- **a** Copy and complete this stem-and-leaf plot.
- **b** Are there any:
  - **i** outliers?
  - **ii** clusters?
- **c** How many students ran the 100 m in less than 13.7 seconds?
- **d** What percentage of students took more than 14 seconds to run the 100 m?
- **e** What percentage (to one decimal place) of students ran times that were below:
  - **i** the mean
  - **ii** the median?

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8 This ordered stem-and-leaf plot shows the marks obtained by students in a History exam.

Stem	Leaf
2	□
3	0 2 3 4
4	1 5 7 8
5	2 4 □ 6 9 9
6	0 2 3 3 6 8 8 7
7	2 5 6 7 7
8	2 5

a One of the marks in the 50s is missing. What could this missing mark be?

b The entry for the lowest mark is missing. If the range of the marks is 63, find the lowest mark. R

c Find the median.

9 This back-to-back stem-and-leaf plot shows the number of goals scored per match by 2 netball teams during a season.

Rockets		Blues
7 5 4 3	1	7
8 7 6 5 4 2	2	0 5 6 8
7 6 4 3	3	4 5 7 7 8 9
5 4 0	4	2 3 7 8
2	5	1 3 7

a How many times did each team score more than 30 goals?

b Find the highest number of goals scored by each team.

c Which team had the greater range?

d Which team performed better? Give reasons. R C

e Which team had the greater median?

10 The number of hamburgers made by Robyn and Anthony at lunchtime over 14 days was: R

Robyn	37	23	33	35	17	42	37	54	45	42	37	27	36	41
Anthony	35	28	29	36	43	37	55	42	47	51	36	34	35	39

a Construct an ordered back-to-back stem-and-leaf plot.

b Who made more hamburgers?

c On how many days did Robyn make more hamburgers than Anthony?

d Are there any outliers in the figures for either Robyn or Anthony?

e Is there any clustering of figures for either person?

f Which person had the higher mean?

11 A sample of commuters were surveyed on how far (in kilometres) they live from the closest railway station. R C

1.5	4.8	6.5	1.1	3.3	3.4	3.5	4.5	4.5	12.5	3.1	3.6
3.8	4.0	4.2	2.3	2.5	1.9	4.7	5.5	2.7	3.2	5.2	

a Arrange the data in a stemplot.

b Are there any outliers? Give reasons.

c Are there any clusters?

d Find the mean, correct to 2 decimal places.

e Find the median.

f Which is the better measure of location, the mean or median? Give reasons.

12 The results for a writing task for 20 girls and 20 boys are shown.

Girls:	18	48	56	59	67	73	76	94	76	78
	82	39	45	59	60	85	80	74	51	36
Boys:	50	88	76	72	10	28	40	95	74	63
	67	55	32	29	16	75	26	82	57	50

a Construct a back-to-back stem-and-leaf plot for the data.

b What was the highest mark? Was it scored by a girl or boy?

c What is the median for:

i girls **ii** boys?

d What percentage of boys scored more than 60?

e What percentage of girls scored more than 60?

f Find the mean mark for:

i girls **ii** boys.

g Is there a there a significant difference between the median mark and the mean mark for:

i girls? **ii** boys?

Technology

Histograms

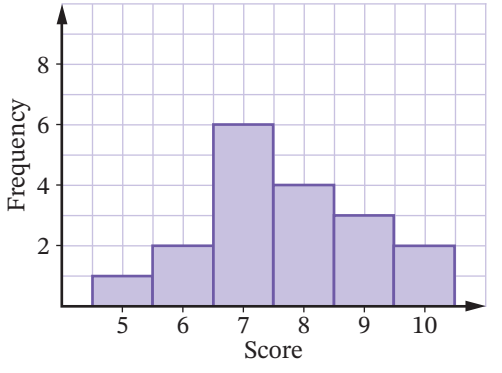
In this activity, we will use statistical, spreadsheet or dynamic geometry software to construct a histogram of players' scores (out of 10) in a trivia quiz.

Score	Frequency
5	1
6	2
7	6
8	4
9	3
10	2

1 Enter the data shown.

2 Your histogram should also have a heading and the axes labelled as shown below.

Players Scores (out of 10) in a Trivia Quiz



3 Remember, the data represents one-variable statistics. In the menu, select **1VAR** (or similar) to show calculations such as mean, median and mode.

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The shape of a distribution

9.03

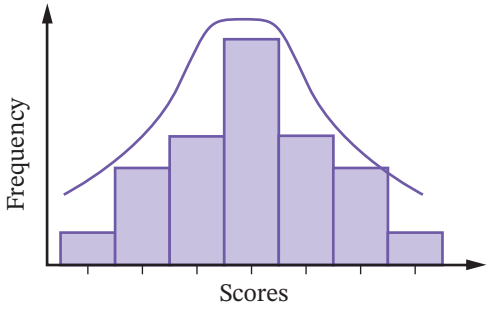
Statistical match-up

A **statistical distribution** is the way the values of a data set are arranged, especially when graphed. When looking at histograms, dot plots and stem-and-leaf plots, an overall pattern can be seen from the **shape of a distribution**.

The shape of a statistical distribution shows how the data is spread and can be seen by drawing a curve around the graph or display.

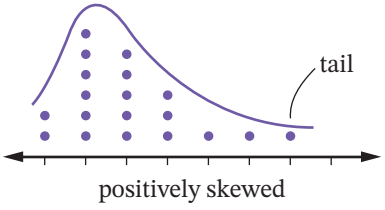
A distribution is **symmetrical** if the data is evenly spread or balanced about the centre.

For example, the frequency histogram and stem-and-leaf plot shown below are both symmetrical in shape.

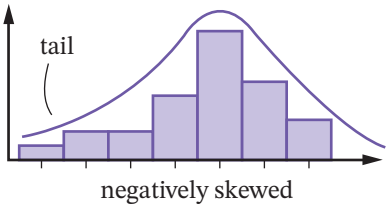


Stem	Leaf
2	6
3	3 7 9
4	5 8
5	0 2 3 3 7 8
6	0 4
7	6 7 8
8	9

A distribution is **skewed** if most of the data is bunched or clustered at one 'end' of the distribution and the other end has a 'tail', as shown on this dot plot and frequency histogram.



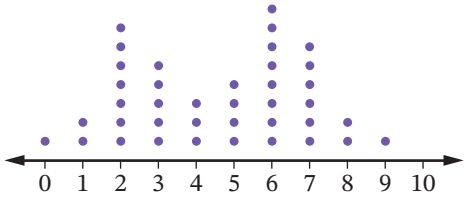
A distribution is **positively skewed** if its tail points to the right.



A distribution is **negatively skewed** if its tail points to the left.

A distribution is **bimodal** if it has 2 peaks. The higher peak is the mode, while the other peak indicates another data value that has a high frequency.

For example, this dot plot has peaks at 2 and 6 so it is bimodal. The mode, however, is 6.



The shape of a frequency distribution

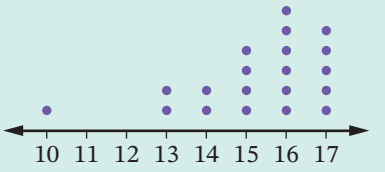
Example 8

For each statistical distribution:

i describe the shape

ii identify any outliers and clusters.

a



b

Stem	Leaf
3	0 1 2
4	1 3 4 4 5 6
5	0 4 5 7 8
6	3 7 8
7	0 1
8	4
9	8

Solution

a **i** The shape is negatively skewed (tail points towards the lower values).

ii 10 is an outlier and clustering occurs at 15, 16 and 17.

b **i** The shape is positively skewed (tail points towards the higher values).

ii 84 and 98 are outliers and clustering occurs in the 40s and 50s.

EXERCISE 9.03 ANSWERS ON P. 589

The shape of a distribution U F R C

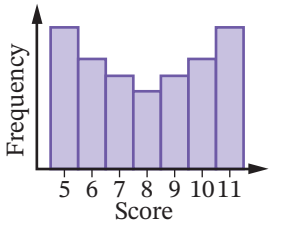
EXAMPLE 8

1 For each statistical distribution:

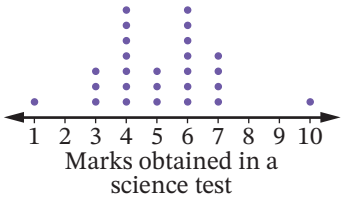
i describe the shape

ii identify any outliers and clusters. C

a



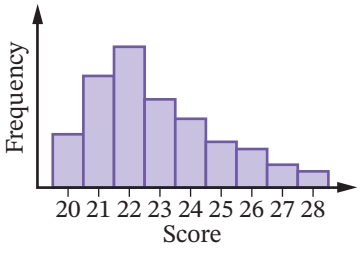
b



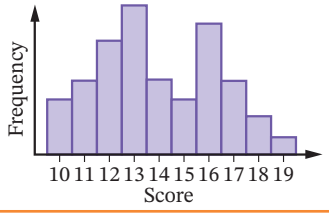
c

Stem	Leaf
2	0 2
3	1 4
4	2 5 6
5	2 3 6 7 8
6	0 4 4 9 9 9
7	1 2 4 5 5 6 7 8 8
8	2 4 5 6 7 7 8
9	0 2 2

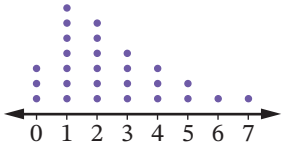
d



e



f



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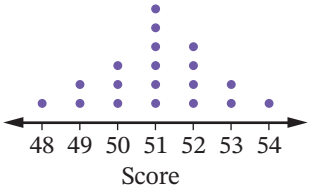
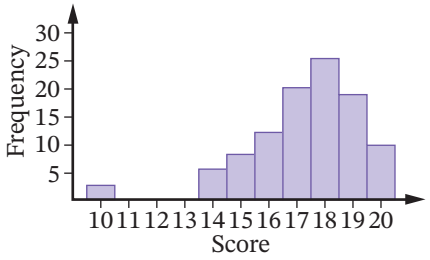
g



h



2 The results of a Mathematics test for a Year 9 class are shown in the stem-and-leaf plot. R C

Stem	Leaf
1	8
2	3 5 8 8
3	2 4 6 7 8 8 9
4	0 4 5 5 7 7
5	3 6 7 8
6	0 4 4
7	2 5
8	1 2
9	2

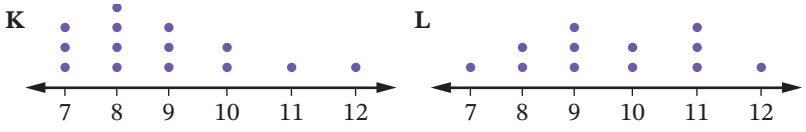
a How many students are in the class?
b Where does the clustering occur?
c Describe the shape of the data.
d Give a possible reason for the shape of the distribution.
e Find the mean, median and mode.

3 The number of children in the families of students at Eastleigh High School are shown below. C

4 4 7 3 3 6 8 6 3 3 4 2 9 5 1 2 3 3 4
7 2 7 2 2 3 5 3 1 4 1 4 1 2 5 6 3 5 2

a Arrange the data into a frequency table and construct a frequency histogram.
b Are there any outliers?
c Describe the shape of the distribution.
d Where does clustering occur?
e Find the mode, the mean (to one decimal place) and the median and show their position on the histogram.

4 Which statement is true about the 2 sets of data? Select the correct answer **A**, **B**, **C** or **D**.



A K is positively skewed and L is symmetrical.
B K is negatively skewed and L is bimodal.
C L has two peaks and K is positively skewed.
D K is positively skewed and the median of L is 9.

5 The results of an entrance exam to select apprentices for an energy supplier are shown in the stem-and-leaf plot. R C

Stem	Leaf
9	2 3 5 6 7 8
8	3 3 6 7
7	2 4 8
6	0 3 4 4
5	3 5 6 6 8 9
4	
3	
2	6

a Describe the shape of the distribution, excluding the outlier.
b What is the mode?
c Find the median.
d Find the mean, correct to one decimal place.
e Find the range.
f Is the range a good indicator of the spread of scores? Give reasons.

6 The average monthly maximum temperatures for Sydney and Melbourne are shown. R C

Month	J	F	M	A	M	J	J	A	S	O	N	D
Sydney	26	26	25	22	19	17	16	18	20	22	24	26
Melbourne	26	26	24	20	17	14	13	15	17	20	22	24

a **i** Display the data for Sydney and Melbourne on dot plots.

ii For each city, describe the shape of the dot plot.

b Find the mean (to 2 decimal places), median, mode and range for each city.

c Which city is generally warmer? Give reasons for your answer.

9.04 Comparing data sets

WS Comparing word lengths

WS Investigating young drivers

Population polygons

World climate

World climate

Example 9

This back-to-back stem-and-leaf plot shows the results of a Year 9 class in a Health exam.

Girls		Boys
2	2	3 4 8
8 3 1	3	0 3 5 9 9
6 2	4	1 3 7 8 8 8 9 9
7 3 0	5	0 2 3 4 5 5 8 9 9
8 7 7 6 2 1	6	0 1 1 4 7 8
9 8 8 7 5 4 4 3	7	0 2 6 7
9 9 8 7 7 5 3 2 2 0	8	1 3 8
8 5 4 0	9	1 6

a Find the mean (to one decimal place) for:

i the girls **ii** the boys **iii** the whole group.

b Find the median for:

i the girls **ii** the boys

c Where are the scores clustered for:

i the girls? **ii** the boys?

d Compare the shapes of the 2 data sets.

e Which group, girls or boys, performed better? Justify your answer.

Solution

a i Mean for girls $= \frac{2600}{37}$
≈ 70.3

ii Mean $= \frac{2234}{40}$
≈ 55.9

iii Mean for group $= \frac{4834}{77}$
≈ 62.8

b i Median for girls $= 75$ — 19th score

ii Median for boys $= \frac{54+55}{2}$
$= 54.5$ — Average of the 20th and 21st scores

c i The girls' scores are clustered at 70s and 80s.

ii The boys' scores are clustered at 40s and 50s.

d The scores for the girls are negatively skewed, while the scores for the boys are positively skewed. The scores for the girls are generally higher than the scores for the boys.

e The girls performed better, which can be seen from their mean of 70.3 being greater than the mean of 55.8 for the boys. Also, the median for the girls was 75, which was higher than the median of 54.5 for the boys.

EXERCISE 9.04 ANSWERS ON P. 589

Comparing data sets U F R C

1 The times (in seconds) taken for swimming 100 metres by Brooke and Ryan are shown. R C

Brooke:	59.71	59.20	60.21	60.13	59.69	59.50	60.41	59.33	58.95	58.70
Ryan:	55.60	56.20	57.50	56.70	57.15	57.65	58.10	59.70	60.15	56.65

a Find the mean for each swimmer.

b Find the median for each swimmer.

c Find the range for each swimmer.

d Which swimmer is faster? Give reasons.

e Which swimmer is more consistent? Give reasons.

2 The maximum daily temperatures in Sydney and Adelaide for a 15-day period in March are shown. R C

Sydney	34.3	21.9	32.3	30.6	21.2	22.0	25.1	29.2
	31.5	29.2	25.9	30.6	32.1	27.3	24.7	
Adelaide	22.0	22.4	24.3	24.3	26.2	34.6	34.9	23.2
	22.1	21.6	26.8	32.0	28.3	27.1	29.0	

a Find the range for each city.

b Find, correct to 2 decimal places, the mean for each city.

c Find the median for each city.

d On how many days was Sydney warmer than Adelaide?

e Which city was warmer? Give reasons.

Foundation | Standard | Complex

EXAMPLE 9

3 A class was given a topic test on trigonometry. The results were disappointing and, after some more work on the topic, the class was given a second test on trigonometry. The results of the 2 tests are shown in this back-to-back stem-and-leaf plot. R C

Test 1		Test 2
1	9	0 9
5 2	8	4 8 8
4 3 2	7	0 1 5 5 8 8
7 4 4 1 0	6	0 3 4 4 7 7 7 8
9 9 9 6 5 3 2 2	5	3 4 4 9 9
9 8 7 4 3 0	4	5 7 8
4 2	3	

a Find the median for each test.

b Find, correct to one decimal place, the mean for each test.

c Find the range for each test.

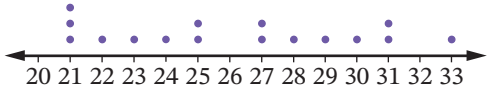
d Describe the shape of each test.

e Has significant improvement been made from Test 1 to Test 2? Give reasons.

4 The dot plots show the temperatures (in °C) at 3 p.m. for Logan City, Queensland and Wollongong, NSW for 16 days in Autumn. R C

Logan City

Wollongong



a Which city's temperatures are more skewed?

b Find, correct to 2 decimal places, the mean for each city.

c Find the range for each city.

d Which city had a more consistent temperature? Give reasons.

e Find the median for each city.

f Which city had cooler temperatures? Give reasons.

AAP Image/DARREN ENGLAND

Foundation Standard Complex

5 The reaction times (in seconds) of a group of men and women were measured. R C

Men:	38	29	37	28	30	32	31	53	31	26
	49	47	33	55	29	44	32	48	43	29
Women:	51	34	40	35	45	40	36	48	57	39
	42	35	51	54	36	37	37	39	43	45

a Arrange the 2 sets of data in a back-to-back stem-and-leaf plot.

b Would dot plots be suitable for displaying the 2 sets of data? Give reasons.

c Are the mean and median the same or different for:

i men? **ii** women?

d Find the mode for each group.

e Describe the shape of each distribution. Are there any outliers or clusters?

f Which group has the faster reaction times?

g Which group has the greater spread of reaction times?

6 Use the Internet to find the gold medal winning times for the 100 metre race for men and women for each Olympic Games from 1960 to 2021. R C

a Copy and complete this table by entering the winning times in seconds.

	1960	1964	1968	1972	1976	1980	1984	1988	1992	1996	2000	2004	2008	2012	2016	2021
Men																
Women																

b Calculate the mean winning time (correct to 2 decimal places) for:

i men **ii** women.

c Find the median for:

i men **ii** women.

d Where are the data values mainly clustered for:

i men? **ii** women?

e Identify any outliers for:

i men **ii** women.

f Compare the shape of the 2 data sets.

g Did either group show a significant improvement in the winning time for the period 1960–2021? Identify any contributing factors.

Alamy Stock Photo/PCN Photography

9.04

Technology

Comparing relative humidities

Relative humidity is the amount of moisture in the atmosphere, measured as a percentage of water vapour per area at a certain temperature. It is reported more often in summer. When presented together with a high temperature, it shows that a day has been particularly warm and 'muggy'. In this activity, we will compare the relative humidity in Adelaide at 9 a.m. and again at 3 p.m. each day for the first fortnight in September 2018 and 2019.

1 Enter the data into a spreadsheet (as shown).

	A	B	C	D	E	F	G
1	Sept 2018	Relative Humidity 9am Adelaide 2018 (%)	Relative Humidity 3pm Adelaide 2018 (%)		Sept 2019	Relative Humidity 9am Adelaide 2019 (%)	Relative Humidity 3pm Adelaide 2019 (%)
2	1	92	55		1	79	44
3	2	62	62		2	39	26
4	3	57	35		3	97	57
5	4	38	27		4	92	71
6	5	23	22		5	82	57
7	6	76	53		6	70	62
8	7	70	40		7	74	55
9	8	54	38		8	78	69
10	9	54	53		9	53	49
11	10	63	20		10	53	38
12	11	17	56		11	33	22
13	12	62	52		12	47	60
14	13	65	39		13	91	65
15	14	35	31		14	58	35

2 Select **Scatter graph with smooth lines and markers** to graph this data on one set of axes. Give the graph a title and label the axes.

3 In B16, C16, F16 and G16, enter formulas to calculate the **average (mean)** relative humidity at 9 a.m. and 3 p.m. for each fortnight.

5 In B17, C17, F17 and G17, enter formulas to calculate the **maximum** relative humidity.

6 In B18, C18, F18 and G18, enter formulas to calculate the **minimum** relative humidity.

7 In B19, C19, F19 and G19, enter formulas to calculate the **range**.

8 In B20, C20, F20 and G20, enter formulas to calculate the **median**.

9 Compare your graphs as well as your statistical values. What do they suggest about the relative humidity each year? Is there a year and time of day showing a more consistent relative humidity? Justify your answer.

10 Visit the Bureau of Meteorology website **www.bom.gov.au** and select **Daily Weather Observations** to find the relative humidity for the past fortnight in your area. Compare the data with this time last year.

- **a** Can you see any patterns?
- **b** Is the relative humidity consistent?
- **c** Are there any outliers?
- **d** Analyse the data and describe any similarities or differences you can see.

Mental skills 9: Maths without calculators ANSWERS ON P. 590

Finding a percentage of a multiple of 10

We can use the unitary method to calculate a percentage of a multiple of 10. To do this, we first find 1% of the number by dividing it by 100. This is easily done by moving the decimal point two places to the left.

1 Study each example.

a $1\% \times 300 = 3.00$
$= 3$

b $4\% \times \$500$
$1\% \times \$500 = \$5$
$\therefore 4\% \times \$500 = 4 \times \$5$
$= \$20$

c $9\% \times 40$
$1\% \times 40 = 0.4$
$\therefore 9\% \times 40 = 9 \times 0.4$
$= 3.6$

d $12\% \times \$700$
$1\% \times \$700 = \$7$
$\therefore 12\% \times \$700 = 12 \times \$7$
$= \$84$

2 Now evaluate each expression.

a $1\% \times 800$ **b** $1\% \times 30$ **c** $3\% \times \$400$
d $4\% \times 20$ **e** $8\% \times \$600$ **f** $3\% \times 40$
g $7\% \times 80$ **h** $2\% \times \$900$ **i** $6\% \times \$50$
j $8\% \times \$90$ **k** $11\% \times 700$ **l** $13\% \times 200$

3 Study each example.

a $4\% \times 320$
$1\% \times 320 = 3.2$
$\therefore 4\% \times 320 = 4 \times 3.2$
$= 12.8$

b $1.5\% \times \$200$
$1\% \times \$200 = \$2$
$\therefore 1.5\% \times \$200 = 1.5 \times \$2$
$= \$3$

c $3\% \times 250$
$1\% \times 250 = 2.5$
$\therefore 3\% \times 250 = 3 \times 2.5$
$= 7.5$

d $4.5\% \times \$800$
$1\% \times \$800 = \$8$
$\therefore 4.5\% \times \$800 = 4.5 \times \$8$
$= \$36$

4 Now evaluate each expression.

a $5\% \times 250$ **b** $1.5\% \times \$300$ **c** $2\% \times \$540$
d $9\% \times 110$ **e** $2.5\% \times \$600$ **f** $3\% \times 160$
g $4\% \times \$750$ **h** $4.5\% \times \$2000$ **i** $3.5\% \times \$400$
j $6\% \times \$120$ **k** $3\% \times 190$ **l** $1.5\% \times \$700$

9.05 Sampling and types of data

Data match-up

When the marketing department of a large company wishes to launch a new product, it collects information about consumer tastes. Governments collect information about the population so that they can provide for transport, housing, health and education needs. A school may need to collect information regarding school uniforms or food to be sold in the school canteen.

Sample vs census

In statistics, the **population** refers to all the items under investigation or being studied. For example, if the life of AAA batteries is being tested, then the population is all the different brands or types of AAA batteries.

A **census** is a survey of the entire population. If the population is too large or difficult to survey, then a **sample** of the population is selected instead. The sample must be representative of the population, by being:

- **random:** every item in the population should have an equal chance of being selected for the sample
- **unbiased:** there should not be any factors that may influence the sample and prevent it from being a true indicator of the population
- **large:** the bigger the sample, the more representative it will be.

Example 10

Determine whether a census or a sample should be used for each investigation.

a Finding the number of primary school children in Victoria.

b Determining people's opinions on public transport.

c Finding the most streamed TV series in Australia.

Solution

a Census, because accurate figures are needed and these figures would be recorded.

b Sample, because it would be too expensive and time-consuming to survey all people.

c Census, because data can be collected about the number of accounts with a provider that streamed a certain TV series.

Types of data

There are 2 types of data: categorical and numerical.

Categorical data are usually words and can be grouped into categories, such as a person's hair colour or cultural background.

Numerical data (or quantitative data) are numbers describing things that can be counted or measured, like the number of goals scored or a person's height. Numerical data is either discrete or continuous.

- **Discrete data** are counted or measured and can only take on separate, distinct values, with 'gaps' or 'jumps', for example, the number of girls in a family is discrete because it could be 0, 1, 2, etc, but not 2.3 or $1\frac{2}{5}$.
- **Continuous data** are measured on a smooth scale with no 'gaps' or 'jumps' between values, e.g., temperature during the day is continuous because it can be 28°C, 29°C, 29.5°C, 30.1°, etc.

Example 11

Classify whether each type of data is categorical or numerical. If numerical, then classify whether it is discrete or continuous.

a School report grades (for example, A, B, C).

b The noise level at a concert.

c People's shoe sizes.

d Marital status (for example, single, married).

Solution

a	School report grades is categorical data.	Involves words or categories.
b	Noise level is continuous numerical data.	Takes on a smooth scale of values.
c	People's shoe sizes is discrete numerical data.	Takes on separate values.
d	Marital status is categorical data.	Involves words or categories.

EXERCISE 9.05 ANSWERS ON P. 590

Sampling and types of data U F R C

EXAMPLE 10

1 Determine whether a census or a sample should be used for each investigation. R

a Finding the number of people in Adelaide who were born overseas.

b Testing the quality of running shoes.

c Determining whether Australia needs a new flag.

d Determining the number of people in Darwin who earn $60 000–$70 000 per year.

e Determining the average family size in Australia.

2 In Australia, a national census was held in 2011, 2016 and 2021.

a How often is a census held in Australia?

b When will the next census be held?

3 The table shows the number of students in each year at Southwest High.

Year	7	8	9	10	11	12
Students	148	151	160	153	132	128

a What is the school population?

b What percentage (to one decimal place) of the school population is formed by:

i Year 8? **ii** Year 11?

c A sample of 100 students is to be surveyed on the amount of time that they spend on the Internet. How many students should be selected from:

i Year 8? **ii** Year 11?

4 Students are surveyed about how they travel to school. The first 30 students arriving at school were selected for surveying. Is this a representative sample to use? Give reasons. R C

EXAMPLE 11

5 People were asked how often they eat take-away food. A sample of 100 people was surveyed at a shopping centre. Is this a representative sample to use? Give reasons. R C

6 Classify each sample of data as either categorical or numerical. R C

a The number of people absent from school today.

b The distance travelled by a car on a trip.

c The amount of petrol bought when filling up the car.

d The different types of pies available at the supermarket.

e The blood pressure of a patient in hospital.

f The amount of time spent on devices during the week.

g The time for swimming 100 m.

h The country of birth of people living in Australia.

i The computer games played by students.

j The age you turn this year.

7 Students are asked to rate a movie on a scale of 1 to 5 stars. What type of data is this? Select **A**, **B**, **C** or **D**.

A continuous **B** categorical **C** discrete **D** random

8 Classify whether each type of numerical data is discrete or continuous. R C

a The number of people watching a cricket match.

b The temperature in Perth over a 24-hour period.

c The amount of time spent doing homework.

d The speed of a car passing through an intersection.

e The age you turn this year.

f The results in a maths test.

g The amount of rainfall in a month.

h The number of dogs at the RSPCA.

Did you know?

Australia's first census

The first census in Australia was conducted in November 1828. Before this, statistical information was collected by musters on the same day each year. Worried about the accuracy of the figures, Governor Darling and the Legislative Council passed an Act requiring a census to be held at regular intervals. Each citizen needed to disclose his or her name, age, marital status, number of children, the name of the ship on which they arrived, the amount of land owned and the number of cattle and other stock owned. Collectors were employed to gather this information by personal interview.

The first census showed that the total non-Aboriginal population was 36 598 and nearly 30% of that population lived in Sydney.

How many national censuses have there been in Australia?

Technology

Random numbers

Random numbers can be used to select people or items for a sample. A calculator or spreadsheet can be set up to generate a random number from 1 to 100, for example, and that number can be matched to the name of a person or item from a list. For a scientific calculator:

Casio	Sharp
Enter the formula RanInt#(1,100) as shown: ALPHA . 1 SHIFT) 100) =	2nd F 7 (RANDOM), select 3 R-INT, + 1 =

This generates a random number from 1 to 100. Press = more times to generate more random numbers.

On a **spreadsheet,** type **=INT(RAND()*100+1)** into a cell and press the ENTER key, then the F9 key repeatedly for more random numbers.

1. Use your calculator or spreadsheet to generate 12 random numbers from 1 to 100.
2. How many students are there in your class?
3. Use your calculator or spreadsheet to generate 12 random numbers from 1 to the number of students in your class.
4. Use a class roll to number the students in your class from 1 onwards. Suppose you need to choose a random sample of 10 students from your class for a survey on homework. Use your calculator or spreadsheet to generate 10 random numbers to select the 10 students for your survey.
5. Choose 4 students from each Year 9 maths class to answer some questions. Devise several different methods to ensure that your selection is random. Prepare a short class talk about your methods.

Investigation

Australian Bureau of Statistics

The Australian Bureau of Statistics (ABS) is the official organisation in charge of collecting data for government departments. The data collected covers many areas, from the height of children in Australia, the alcohol consumption in the country to the number of Internet subscribers and Internet providers. The ABS conducts a census of the entire Australian population every 5 years.

Visit the ABS website www.abs.gov.au to answer the following questions.

1. a What is a population clock?
   b What is the current predicted population of Australia?
   c On average, another baby is born in Australia after how many minutes?
   d On average, another person dies in Australia after how many minutes?
   e A new migrant arrives in Australia after how many minutes?
   f Australia's population increases by one after how many minutes?
2. Go to Australian Demographic Statistics and find the populations of the states and territories.
   a What percentage of Australia's population lives in:
      i NSW ii ACT?
   b Which state had the greatest percentage increase over the previous year?

3 Find the population pyramid for Australia's population.
   a What is a population pyramid?
   b What percentage of Australian males are aged 15–19?
   c What is the modal age?
   d How many people are aged 100 years and over?
   e Which group, males or females, have the higher life expectancy? Justify your answer.

4 a Explain why Australia's population is said to be ageing.
   b What is the median age of Australia's population?
   c Find the median ages of the people for all states and territories and display them in a table. Which has the:
      i highest median age? ii lowest median age?

5 Use the ABS website to find the number of people that live in your local area.

9.06 Bias and questionnaires

Bias in sampling

When taking a sample, it is important that each item in the population has an equal chance of being chosen to be surveyed. This is called **random sampling**. To find a sample of students to survey about school uniform, we could place students' names in a box, mix them up, and then choose 100 names without looking. Alternatively, we could use a computer or calculator to generate random numbers to select students.

A sample that is not random is called a **biased sample**, and is not truly representative of the population. For example, if you survey the first 100 students to arrive at school today, or the first 100 students on the school roll, then not all students in the school have an equal chance of being selected. Your sample would not represent students who arrive at school later or whose names begin with letters at the end of the alphabet.

Example 12

A random sample of students is to be selected for their views on a new school uniform. Will each of the following sampling methods provide a sample that is representative or biased? If biased, explain why.

a Randomly selecting one student from each class
b Selecting 20 students from your mathematics class
c Selecting the students from the school's netball teams
d Randomly selecting students from the school roll by rolling a die and counting down the roll.

Solution

a	Representative.	Every student in the school has an equal chance.
b	Biased.	Because your mathematics class does not represent the other Year groups (or even the other Year 9 classes) at your school.
c	Biased.	Because the netball teams do not represent the whole school; it will also have mainly female students.
d	Representative.	Every student in the school has an equal chance.

Designing a questionnaire

Questionnaires are often used to collect information. They may ask for details about a person's opinions and attitudes, their lifestyle and behaviours, such as their recreational activities, TV viewing habits and types of foods eaten.

When designing a questionnaire, it is important that the questions asked are clear, fair and not biased.

- Decide what questions you want answered before you begin.
- Write the questions in plain English so that they are easily understood: make sure they are not vague, too open-ended or have a 'double meaning'.
- Ask for only one piece of information per question.
- Do not allow for too many different answers, provide choices if possible:
  - a tick or a cross beside each response
  - 'Yes', 'No', 'Don't know' or 'Other' responses
  - a rating on a scale
  - a one-word response
  - a sentence
- Determine how you are going to present and use the data you collect.
- Do not try to influence a particular answer unfairly (this is also called bias).
- Do not ask for information that may be too personal, such as phone numbers.

Example 13

Explain what is wrong with each survey question and suggest an alternative.

a 'How old are you?'

b 'How much pocket money do you receive each week?'

Solution

a The question is not precise enough. Possible answers could be:

15 15 years, 4 months 14 years 10 months and 3 days $16\frac{1}{2}$

A better question might be:

'What age do you turn this year?' or 'What is your date of birth?'

14☐ 15☐ 16☐ ...../...../..... (dd/mm/yy)

b The question is too general and responses could be:

Nothing $5/day $40/week Whatever I need

A better question could be:

'Circle the amount of pocket money you receive in a normal week:'

$5 or less $6 to $10 $11 to $15 more than $15

EXERCISE 9.06 ANSWERS ON P. 590

Bias and questionnaires U F PS R C

EXAMPLE 12

1 When collecting data, it is important to have an unbiased sample. Write, in your own words, what is meant by an 'unbiased sample'. C

2 List the possible areas of bias in each sampling method. R C

a A survey about the amount of time spent doing homework, taken at a football match.

b A survey about violence in sport, taken at a sporting event.

c A survey about the use of alternative fuels, taken at a car show.

d A survey about types of music listened to, taken at a rock concert.

3 A market research company uses a telephone survey to determine the most popular news program on TV. List some of the advantages and disadvantages of using this type of survey to collect data. C

4 In order to determine what type of music should be played on the school radio station, 20 students were given a questionnaire. PS R C

a Is the sample suitable for obtaining the required information? Give reasons.

b List some of the advantages of using a questionnaire to obtain the data.

c List some of the disadvantages of using this method.

d What other methods could be used to find out the type of music that should be played?

5 You need to determine how many students at your school have part-time jobs. Which of the following methods would be most suitable? Give reasons. C

A You ask 15 students in your class at school.

B You randomly select 15 students from each Year level at your school.

C You ask all students at your school to fill in a questionnaire and return them to you.

EXAMPLE 13

6 For each survey question, explain what is wrong with the question and suggest a better question. R C

a For how much time do you study?

b What is your favourite sport?

c What is your income?

d What do you have for lunch each day?

e What do you do in the holidays?

f How often do you exercise?

g How much time do you spend on social media websites?

h How many text messages do you send in one day?

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7 Select one topic for a questionnaire. PS R C

- Amount of TV watched at home.
- How students in Year 9 spend their time on a Sunday.
- The amount of time spent on homework during the week.
- The amount of time spent on the internet.
- Another topic of your choice.

a Design the questionnaire.
b Select a sample of students to answer it.
c Collect and present the information.
d Did the questions obtain the required data?
e Do you need to change any of the questions?
f Are there any questions you forgot to ask?
g Are there any questions you would omit next time?

8 Describe how you would collect data about each topic. PS C

a The most popular fast food eaten by students at your school.
b The average daily amount of money spent by students at your school canteen.
c The most popular TV series watched by students at your school.
d The most popular Australian artist listened to by students at your school.
e The average number of girls in a classroom in NSW.
f The most commonly used brand of toothpaste.

9.06

Investigation

Media reports of surveys

We live in a world of 24-hour news, often quoting the results of surveys. To detect bias in news reports, we need to always consider where the news item comes from and what types of samples the statistics are based on. Is the information supplied by a reporter, the police, a company executive, a government official or an opinionated blogger? Are the statistics based on a small sample, large sample, unrepresentative sample, or a phone or Internet poll of volunteers? Each may have a particular bias that may influence how the story is reported.

Find a report of a statistical survey from a newspaper, magazine or the Internet and investigate the following questions.

a Where did the article come from?
b Who wrote the article and what claims does it make about the population?
c Does it show any bias?
d What type of sampling was used? Was it representative?
e How many people were questioned?
f When was the survey conducted?
g Was there any bias in the sample?
h Was there any bias in the survey questions?

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Investigation

Year 9 student survey

The survey below is designed to collect data about Year 9 students. Questions can be changed or more can be added, but remember that the data is only useful if good questions are asked in a controlled way.

1 What is your gender? ☐ Male ☐ Female

2 What is your age (as of last birthday)? _____ years

3 How tall are you? _____ cm

4 How many people are in your household? _____ people

5 What is your position in your household (1 = oldest)? _____

6 How many pets do you have? _____ pets

7 How far from school do you live (to nearest 0.1 km)? _____ km

8 How did you travel to school today?

☐ Car ☐ Bus ☐ Walk ☐ Train

9 How long did the journey take? _____ minutes

10 On a scale of 0–10, rate your enjoyment of school (10 = most). _____

11 Which subject do you like best?

☐ Maths ☐ Science ☐ English

☐ Art ☐ PE ☐ History ☐ Other

12 Which subject do you study most?

☐ Maths ☐ Science ☐ English

☐ Art ☐ PE ☐ History ☐ Other

13 How much home study do you do each week? _____ hours

14 How many hours do you study maths each week? _____ hours

15 On a scale of 0–10, rate your enjoyment of maths (10 = most) _____

16 How many hours each week do you play sport out of school? _____ hours

17 How many sports do you play out of school? _____ sports

18 Which sport do you prefer to watch?

☐ Football ☐ Soccer ☐ Netball

☐ Tennis ☐ Surfing ☐ Other

19 What is your favourite TV station? _____

20 What is your favourite radio station? _____

Steps in conducting the survey

1. Finalise the questions and print the survey sheets.
2. Decide how many students you want to respond to the survey.
3. Decide how you will select the sample of students.
4. Decide whether the survey will be handed to selected students or if an interviewer will record responses.
5. Decide whether you will interpret the questions/responses or not.
6. Ask the selected students to complete the survey.
7. Collect all the sheets and label the first sheet 1, the second sheet 2 and so on.
8. Decide on the code you will use for each response, then code the responses and enter the results in a table, either on a spreadsheet or in your workbook.
9. Decide what to do if a question is not answered or the answer appears to be a mistake.

Power plus ANSWERS ON P. 591

1 The number of yoghurts sold daily in a school canteen were:

25	28	22	35	26	36	33	48	42	37	25	25	37	44	19	40	23
38	42	32	28	21	36	22	39	46	28	45	28	41	43	26	17	28

a Complete a frequency table by grouping the data into class intervals like this:

Class interval	Frequency
16–21	
22–27	
28–33	
34–39	
40–45	
46–51	

b Draw a frequency histogram and describe the shape, identifying any outliers or clusters.

c What is the modal class?

d Add a cumulative frequency column to your frequency table and find the median class.

2 When would it be impractical to use a stem-and-leaf plot?

3 Seeds were sown in 40 large boxes. The number of seedlings that sprouted per box were:

14	23	18	18	17	28	28	33	4	24
17	27	11	19	13	18	18	7	25	16
14	3	31	4	5	12	23	26	11	17
19	20	24	7	30	16	25	8	19	20

a Organise the data into a stem-and-leaf plot using intervals of:

i 10 for the stem **ii** 5 for the stem (0^0 for 0–4, 0^5 for 5–9, 1^0 for 10–14, ...)

b Describe the shape of both plots, identifying outliers and clusters.

c Which plot gives a better indication of how the seedlings have grown? Explain your answer.

Statistics

CHAPTER 9 REVIEW

Language of maths

bias	categorical data	census
cluster	continuous	cumulative frequency
discrete	distribution	dot plot
histogram	mean	measure of location/central tendency
median	mode	negatively skewed
numerical data	outlier	polygon
population	positively skewed	sample
stem-and-leaf plot	symmetrical	

1 What is the collective name for the mean, median and mode?

2 Is a person's arm length an example of discrete data or continuous data?

3 What is the meaning of 'skewed'?

4 Explain the difference between a *population* and a *sample*.

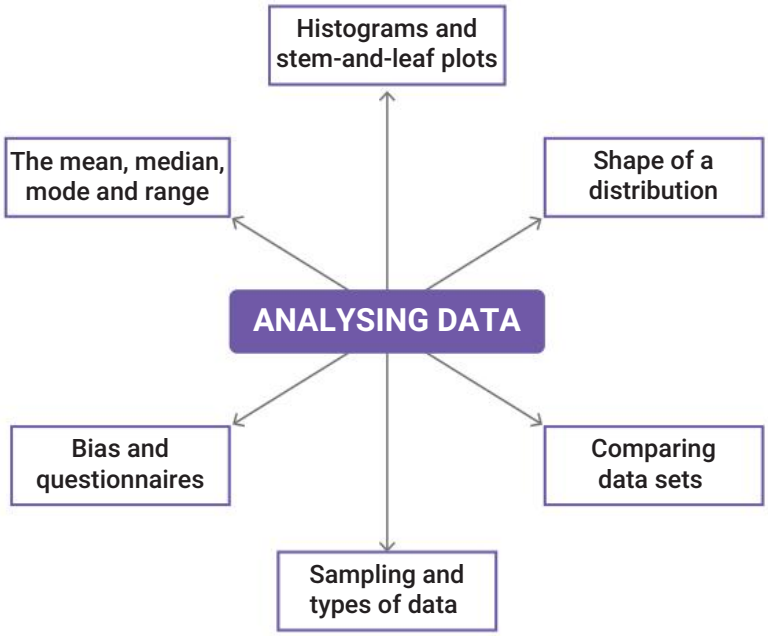
5 What name is given to an extreme value that is very different from the other values in a data set?

6 Name one way of avoiding bias when selecting a sample.

Topic summary

Mind map: Analysing data

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



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TEST YOURSELF 9

ANSWERS ON P. 591

9.01

1 Find the mean, median, mode and range of this set of data.

8 6 9 4 10 8 5 5 8

2 A real estate agent sold 10 properties during one week. The prices of these properties are:

$420 000	$485 000	$379 000	$295 000	$1 750 000
$515 500	$315 500	$408 000	$368 000	$315 000

a Find the mean price of the properties.

b Find the median price of the properties.

c Compare the mean price with the median price. Which is the better measure of central tendency? Give reasons.

d When data on sales of properties are given, the median price is used, not the mean. Explain why.

9.02

3 These percentage marks were scored by 30 students in a Maths test.

45	67	78	54	22	38	58	64	73	89
84	70	78	62	56	59	42	47	86	90
64	63	72	77	43	85	75	71	70	83

a Construct an ordered stem-and-leaf plot for the data.

b Find the mean, median, mode and range of the marks.

c Are there any outliers?

d Compare the mean and median. Which is more accurate? Give reasons.

4 A group of people were surveyed on the number of brothers and sisters they had. The results are displayed on the histogram.

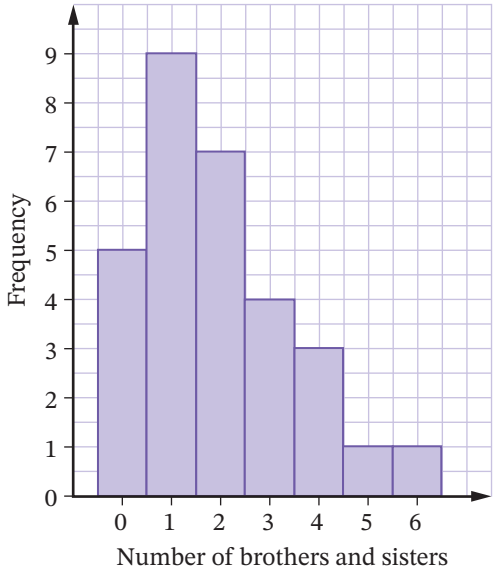
a What is the range of this data?

b What is the mode?

c Find the mean, correct to 2 decimal places.

d What is the median?

e Which is the best average?



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5 24 patients were chosen at random from 2 medical centres. The waiting times (in minutes) for each patient are listed below.

Northbridge:	7	16	28	27	30	30	47	9	9	15	27	11
	38	35	42	53	23	22	45	32	37	37	18	16
Westvale:	44	13	7	9	25	26	37	35	8	18	18	15
	22	24	7	5	24	24	33	37	24	15	36	43

a Display the information in a back-to-back stem-and-leaf plot.

b Find the mean, median and range of both sets of data.

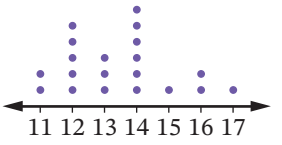
c Identify any outliers or clusters.

d Are there significant differences between the 2 sets of data? Explain your answer.

9.03

6 For each distribution, describe the shape and identify any outliers or clusters.

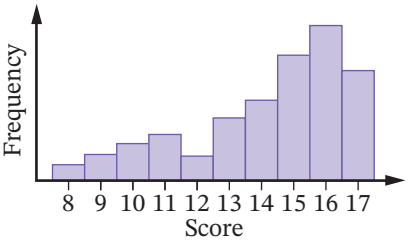
a



b

Stem	Leaf
8	7 8
9	1 3 4 4
10	0 2 7 8 8 9
11	1 2 2
12	0 7

c



9.04

7 The heights (in centimetres) of students in a Year 9 class are:

Girls:	160	153	155	156	142	163	162
	148	150	150	131	138	147	145
Boys:	150	165	164	178	170	171	164
	148	154	176	134	180	176	175

a Arrange the data in a back-to-back stem-and-leaf plot.

b Describe the shape of the display for:

i the girls **ii** the boys.

c Are there any outliers or clusters for either group?

d Find the mean (correct to 2 decimal places) and median for both groups.

e Which measure is the better indicator of the measure of central tendency?

f Which group is shorter?

g Which group has the greater spread of heights?

8 Should a sample or a census be used to investigate each topic? 9.05

- **a** The most popular car colour
- **b** Whether energy-saving light globes really use less electricity
- **c** The number of retired people living on the Gold Coast
- **d** People's views on which nation will win the soccer World Cup

9 Classify each type of data as categorical or numerical. If numerical, then classify it as discrete or continuous.

- **a** The temperature in Alice Springs.
- **b** The number of views of a YouTube video.
- **c** Suburb or town of home.
- **d** The make of car passing through a motorway toll collection point.
- **e** The probability of rain today.

10 Explain why each of the following samples is biased. 9.06

- **a** A sample of online gamers were surveyed to investigate teenagers' views on road safety.
- **b** Customers in a café can give feedback about their meal by filling in a survey and posting it to the café.
- **c** At a factory, a sample of executives were surveyed about safety in the workplace.
- **d** A sample of audience members at a movie premiere were surveyed about their views on the film.
- **e** Classic Gold FM radio listeners were asked to vote for the best song of the year.

11 Explain what is wrong with each survey question and suggest an alternative.

- **a** Are you happy?
- **b** How did you enjoy your meal at our restaurant?
- **c** Australian soldiers have fought under our country's flag. Do you think that it should be changed?

10

MEASUREMENT AND GEOMETRY

SURFACE AREA AND VOLUME

The Sydney Opera House was designed by Danish architect Jørn Utzon, with construction taking place between 1959 to 1973 at a cost of $102 million. Its shell design represents a ship under full sail, with each concrete shell being segments of a sphere of radius 75.2 metres. The shells are covered by 1 056 006 white and cream 120 mm square tiles. The Sydney Opera House stands on 1.3 hectares of land, is 183 metres long and 120 metres wide at its widest point.

Chapter outline

	Working mathematically				
10.01 Metric units	U	F	PS	R	C
10.02 Limits of accuracy of measuring instruments#	U			R	C
10.03 Perimeters and areas of composite shapes	U	F	PS	R	
10.04 Areas of quadrilaterals	U	F	PS	R	
10.05 Circumferences and areas of circular shapes	U	F	PS	R	
10.06 Surface area of a prism	U	F	PS	R	
10.07 Surface area of a cylinder	U	F	PS	R	
10.08 Volumes of prisms and cylinders	U	F	PS	R	C
10.09 Volumes of pyramids and cones*	U	F		R	

*STAGE 5.3

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

Wordbank

capacity The amount of fluid (liquid or gas) in a container

cross-section A 'slice' of a solid, taken across the solid rather than along it

limits of accuracy How accurate a measured value using a measuring instrument can be, equal to half of a unit on the instrument's scale

mega- (abbreviated M) A metric prefix meaning one million

micro- (abbreviated μ) A metric prefix meaning one-millionth

prism A solid shape with identical cross-sections and ends with straight sides

sector A region of a circle cut off by 2 radii, shaped like a slice of pizza

surface area The total area of all the faces of a solid shape

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

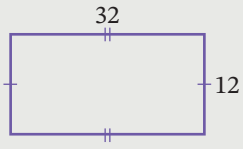
- learn the metric prefixes for very small and very large units of measurement
- convert between units of digital memory, such as megabytes and gigabytes
- calculate the limits of accuracy of measuring instruments
- calculate the perimeters and areas of triangles, quadrilaterals, circles, sectors and composite shapes
- calculate the surface areas, volumes and capacities of right prisms and cylinders
- (STAGE 5.3) calculate volumes of pyramids and cones

SkillCheck ANSWERS ON P.592

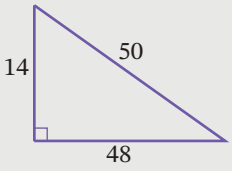
1 The measurements shown on the shapes below are in centimetres. For each shape, find (correct to one decimal place if necessary):

i its perimeter **ii** its area

a



b



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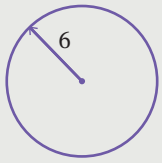

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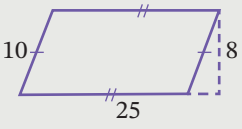
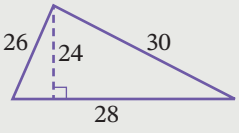
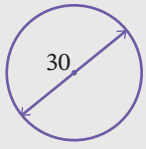


e



f



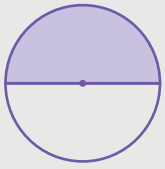
2 Convert:

a 34 m to mm **b** 925 cm to m **c** 1500 m to km

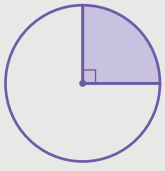
d 3750 mL to L **e** 38 cm to mm **f** 7.5 L to mL

3 What fraction of each circle is shaded?

a



b



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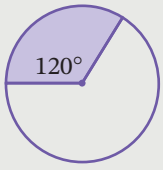

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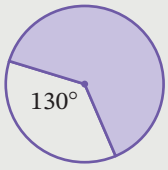
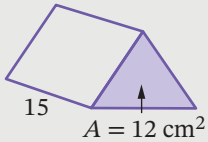
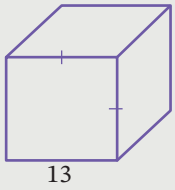


4 Calculate the volume of each prism (all units are in cm).

a



b



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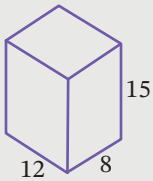

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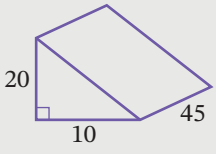

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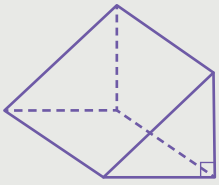
5 For each prism:

i count the number of faces

ii name the shapes of the different faces

iii name the prism.

a



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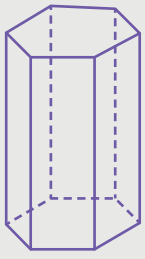

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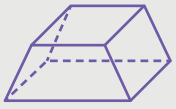
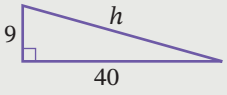
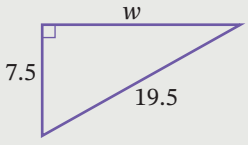


6 Find the value of each variable.

a



b



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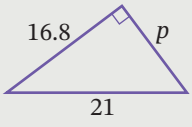
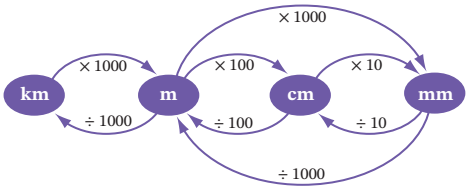
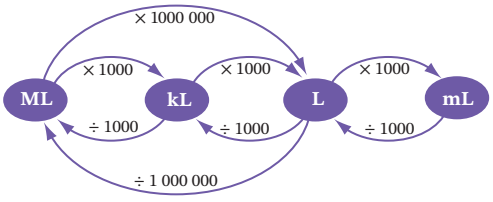
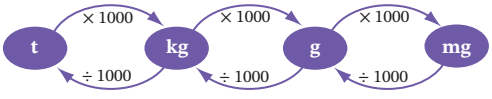
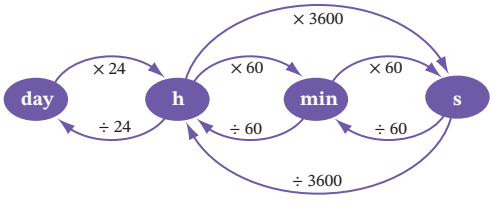


10.01

Metric units

10.01

Length	Capacity
1 cm = 10 mm 1 m = 100 cm = 1000 mm 1 km = 1000 m 	1 L = 1000 mL 1 kL = 1000 L 1 ML = 1000 kL = 1000 000 L 
Mass	**Time**
1 g = 1000 mg 1 kg = 1000 g 1 t = 1000 kg 	1 min = 60 s 1 h = 60 min = 3600 s 1 day = 24 h 

Other than for time, units of the metric system have prefixes based on powers of 10.

- **kilo-** means 1000 (thousand) or 10^3 — 1 kilogram = 1000 grams
- **mega-** means 1 000 000 (million) or 10^6 — 1 megalitre = 1 000 000 litres
- **centi-** means $\frac{1}{100}$ (hundredth) or 10^{-2} — 1 centimetre = $\frac{1}{100}$ metre
- **milli-** means $\frac{1}{1000}$ (thousandth) or 10^{-3} — 1 millilitre = $\frac{1}{1000}$ litre

Example 1

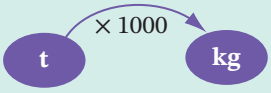
Convert:

a 5.8 t to kg **b** 16 km to cm **c** 23 700 mL to L **d** 1500 min to h

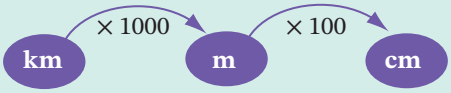
Solution

a $5.8 \text{ t} = 5.8 \times 1000 \text{ kg}$
$= 5800 \text{ kg}$

To convert from large to small units, multiply (×).

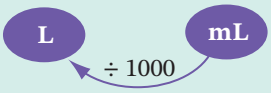


b $16 \text{ km} = 16 \times 1000 \times 100 \text{ cm}$
$= 1\,600\,000 \text{ cm}$

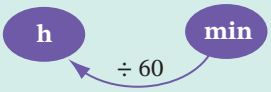


c $23\,700 \text{ mL} = 23\,700 \div 1000 \text{ L}$
$= 23.7 \text{ L}$

To convert from small to large units, divide (÷).



d $1500 \text{ min} = 1500 \div 60 \text{ h}$
$= 25 \text{ h}$



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9780170453325

Metric prefixes

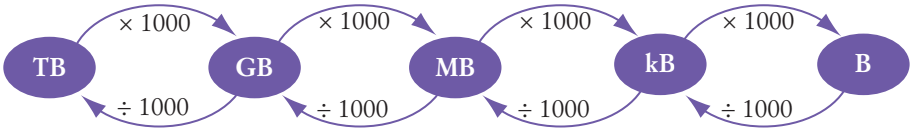
This table shows a more detailed list of metric prefixes.

Prefix (abbreviation)	Meaning	Example
pico (p)	10^{-12}	
nano (n)	$10^{-9} = \frac{1}{1000000000}$	nanosecond (ns) = billionth of a second
micro (μ)	$10^{-6} = \frac{1}{1000000}$	microgram (μg) = millionth of a gram
milli (m)	$10^{-3} = \frac{1}{1000}$	millibar (mbar) = thousandth of a bar (air pressure)
centi (c)	$10^{-2} = \frac{1}{100}$	centimetre (cm) = hundredth of a metre
deci (d)	$10^{-1} = \frac{1}{10}$	decibel (dB) = tenth of a bel (sound level)
deca (da)	$10^{1} = 10$	decametre (dam) = ten metres
hecto (h)	$10^{2} = 100$	hectopascal (hPa) = hundred pascals (air pressure)
kilo (k)	$10^{3} = 1000$	kilojoule (kJ) = thousand joules (energy)
mega (M)	$10^{6} = 1\,000\,000$	megahertz (MHz) = million hertz (frequency)
giga (G)	10^{9}	gigalitre (GL) = billion litres (water)
tera (T)	10^{12}	terawatt (TW) = trillion watts (power)

Digital memory

Metric prefixes are used in units of digital (computer) memory. A **byte**, abbreviated B, is the amount of memory used to store one character, such as @, g, #, 7 or ?.

Unit	Relationship	Size
kilobyte (kB)	1 kB = 1000 B	About half a page of text.
megabyte (MB)	1 MB = 1000 kB	A million bytes, about the size of a novel.
gigabyte (GB)	1 GB = 1000 MB	A billion bytes, about 500 photos or 7.5 hours of video.
terabyte (TB)	1 TB = 1000 GB	A trillion bytes, about the size of all of the books in a large library.



Some examples of digital memory sizes:

E-mail (without attachment): 75 kB

Photo: 2 MB

Music file: 4 MB

CD: 750 MB

Streaming a video: 790 MB per hour

DVD: 4.7 GB

Blu-ray disc: 25 GB

Online game: 40 to 300 MB per hour

Example 2

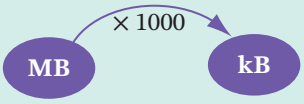
Convert:

a 3.2 MB to kB **b** 147 000 MB to GB **c** 2 TB to GB

Solution

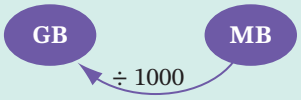
a $3.2 \text{ MB} = 3.2 \times 1000 \text{ kB}$
$= 3200 \text{ kB}$

To convert from large to small units, multiply (×)

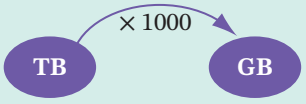


b $147\,000 \text{ MB} = 147\,000 \div 1000 \text{ GB}$
$= 147 \text{ GB}$

To convert from small to large units, divide (÷)



c $2.6 \text{ TB} = 2.6 \times 1000 \text{ GB}$
$= 2600 \text{ GB}$



Very small and very large units

Some very small and very large units of length, distance and time also use metric prefixes.

Length and distance

- One **nanometre** (nm) $= \frac{1}{1\,000\,000\,000}$ m: your fingernails grown 1 nm every second, the diameter of a helium atom is 0.1 nm
- One **micrometre** (μm) $= \frac{1}{1\,000\,000}$ m: used to measure diameters of bacteria
- One **astronomical unit** (AU) = 149 597 871 km: the average distance between the Earth and the Sun, used to measure distances between planets in our solar system
- One **light year** (ly) $= 9.46 \times 10^{12}$ km: the distance light travels in a year, for measuring distances between stars
- One **parsec** (pc) $= 3.0857 \times 10^{13}$ km or 3.26 ly: for measuring distances between stars, the nearest star *Proxima Centauri* is 1.30 parsecs away

Time

- One **nanosecond** (ns) $= \frac{1}{1\,000\,000\,000}$ s: the time it takes electricity to travel along a 30 cm wire
- One **microsecond** (μs) $= \frac{1}{1\,000\,000}$ s
- One **millisecond** (ms) $= \frac{1}{1\,000}$ s: the duration of the flash on a camera
- One **decade** = 10 years
- One **century** = 100 years
- One **millennium** = 1000 years
- One **mega-annum** (Ma) = 1 000 000 years: used in geology and paleontology, the *Tyrannosaurus Rex* dinosaur lived 65 Ma ago
- One **giga-annum** (Ga) = 1 000 000 000 years: the Earth formed 4.57 Ga ago

Example 3

Convert:

a 0.73 s to μs

b 25 610 000 km to AU

c 4.57 Ma to years

d 1 920 000 nm to m

Solution

a $0.73 \text{ s} = 0.73 \times 1\,000\,000 \text{ μs}$
$= 730\,000 \text{ μs}$

To convert from large to small units, multiply (×)

b $25\,610\,000 \text{ km} = 25\,610\,000 \div 149\,597\,871 \text{ AU}$
$= 0.17119...$
$\approx 0.17 \text{ AU}$

To convert from small to large units, divide (÷)

c $4.57 \text{ Ma} = 4.57 \times 1\,000\,000 \text{ years}$
$= 4\,570\,000 \text{ years}$

d $1\,920\,000 \text{ nm} = 1\,920\,000 \div 1\,000\,000\,000 \text{ m}$
$= 1.92 \times 10^{-3} \text{ m}$
$= 0.001\,92 \text{ m}$

10.01

EXERCISE 10.01 ANSWERS ON P. 592

Metric units U F PS R C

1 What metric unit would you use for each measurement? R C

a Width of a computer screen

b Mass of a plane

c Distance between 2 towns

d Capacity of a car's fuel tank

e Amount of water in a pool

f Time taken to fly overseas

2 Convert: R

EXAMPLE 1

a 750 g to mg

b 36 kL to L

c 67 kg to g

d 1.5 L to mL

e 6.7 cm to mm

f 3.5 km to m

g 9.4 t to kg

h 3.75 g to mg

i 4.25 m to cm

j 0.2 kg to g

k 26.2 L to mL

l 0.3 km to m

m 3 kL to mL

n 4 days to min

o 15 hr to s

p 2.7 km to cm

q 5 kg to mg

r 0.7 m to mm

s 29.375 t to g

t 18.14 km to mm

u 3.27 g to mg

3 How many mL in 18.9 kL? Select the correct answer **A**, **B**, **C** or **D**.

A 1 890 000 **B** 18 900 000 **C** 189 000 **D** 18 900

4 Select the most appropriate answer **A**, **B** or **C** for each measurement.

a height of a door

A 20 cm **B** 200 cm **C** 20 m

b capacity of a soft drink can

A 37.5 mL **B** 375 mL **C** 3.5 L

c time to run 100 metres

A 12.3 s **B** 34.3 s **C** 50.3 s

d height of Sydney Tower

A 30.5 m **B** 3050 m **C** 305 m

e mass of a family car

A 100 kg **B** 1000 kg **C** 10 000 kg

5 Convert:

a	8000 L to kL	**b**	90 mm to cm	**c**	180 s to min
d	240 min to h	**e**	375 mL to L	**f**	14 700 kg to t
g	8600 mg to g	**h**	750 cm to m	**i**	223 mm to cm
j	800 g to kg	**k**	125 L to kL	**l**	3.5 kg to t
m	17 000 000 mg to kg	**n**	7200 min to days	**o**	45 000 000 g to t
p	144 000 s to h	**q**	86 400 s to days	**r**	2500 cm to km
s	75 000 mL to kL	**t**	34 700 mm to m	**u**	628 L to ML

6 How many km in 408 m? Select the correct answer **A**, **B**, **C** or **D**.

A 4.08 **B** 0.408 **C** 0.0408 **D** 0.004 08

7 The capacity of Sydney Harbour is approximately a quarter of Warragamba Dam's capacity. If the capacity of Warragamba dam is 2013 GL, what is the capacity of Sydney Harbour in litres?

EXAMPLE 2

8 Convert: R C

a	4.8 MB to kB	**b**	2.1 TB to GB	**c**	8.5 GB to MB
d	10.8 MB to B	**e**	2910 B to kB	**f**	740 MB to GB
g	1050 kB to MB	**h**	5900 GB to TB	**i**	0.94 kB to MB
j	57 GB to MB	**k**	0.43 GB to kB	**l**	21 TB to GB
m	7.8 kB to B	**n**	2180 MB to GB	**o**	42 000 B to MB
p	940 MB to kB	**q**	0.64 TB to kB	**r**	278 MB to GB

9 Mala made a 3-minute phone call to her friend in Rockhampton using the Internet at 3.3 MB per minute. How many kB did she use?

10 Ritu made a video call lasting $1\frac{1}{4}$ hours. If it used 3MB/min, how much data did the call take?

11 Calculate the number of MP3 files of average size 5 MB that can be stored on a 64 GB MP3 player. PS R

12 Convert, correct to 2 significant figures: R C

a	3.5 AU to km	**b**	6000 ms to s	**c**	7.5 centuries to years
d	12 000 000 nm to m	**e**	0.64 Gm to m	**f**	2.1 ly to km
g	0.07 s to ms	**h**	0.25 Ma to years	**i**	4 800 000 μs to s
j	5 290 000 km to Mm	**k**	0.2 pc to km	**l**	6 mm to μm
m	0.94 Ma to millennia	**n**	849 years to centuries	**o**	8×10^{14} km to ly
p	0.000 008 cm to nm	**q**	20 ly to pc	**r**	156 millennia to Ma

EXAMPLE 3

10.01

Did you know?

Is 1 MB = 1000 or 1024 kB?

Computers represent all information as a series of 0s and 1s. These binary digits, called bits (b), give their name to the smallest unit of computer memory. Computer memory is measured in bytes, where one byte is equal to 8 bits. A byte (B) is the amount of memory needed to store one character such as #, d, D, 8 or $. For example, the letter 'd' is represented by the byte 01100100, while 'D' is the byte 01000100.

On most storage devices such as USB drives, hard drives and flash drives, 1 MB = 1000 kB. Internet and phone companies use the same. Originally, however, computer programmers used 1 MB = 2^{10} kB = 1024 kB because that was actually how computer hardware stored data. This table shows the original conversions.

Unit	Meaning	Number of bytes
byte (B)	1 B = 8 b	1 B
kilobyte (kB)	1 kB = 1024 B	1024 B or 2^{10} B (approx. 1000)
megabyte (MB)	1 MB = 1024 kB	1024 × 1024 = 1 048 576 B or 2^{20} B (approx. one million)
gigabyte (GB)	1 GB = 1024 MB	1024^3 = 1 073 741 824 B or 2^{30} B (approx. one billion)
terabyte (TB)	1 TB = 1024 GB	1024^4 = 1 099 511 627 776 B or 2^{40} B (approx. one trillion)

Find the byte (code) for '8' and '$'.

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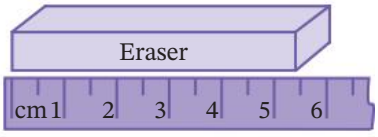
Foundation Standard Complex

10.02 Limits of accuracy of measuring instruments#

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

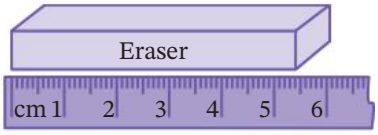
WS Accuracy in measurement

All measurements are only approximations.
No measurement is ever **exact**.



For example, we might measure this eraser to be **5.5 cm** long, because the scale on the ruler measures to the nearest 0.5 cm.

If we used a ruler with a more precise scale, such as 0.1 cm markings, then we may find the length to be **5.4 cm**.

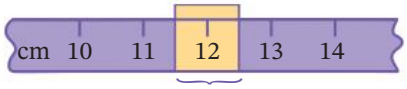


If we used an instrument such as a micrometer that measured to the nearest 0.01 cm, we may find that the length is **5.41 cm**.

Notice that we can always find a more accurate measurement using a more precise measuring instrument. So all measurements are approximations.

A measuring instrument such as a ruler has **limits of accuracy** due to its level of **precision**. 'Accuracy' means how close a measured value is to the true value, while 'precision' means how fine the scale is on the measuring instrument.

For example, this ruler is marked in centimetres, so any length measured with it can only be given to the nearest centimetre.



Any measurement in the shaded region should be recorded as 12 cm. The measured length is 12 cm, but the actual length is 12 ± 0.5 cm, meaning '12 centimetres, give or take 0.5 centimetres' or '11.5 to 12.5 cm'.

The **limits of accuracy** of this ruler are **$\pm$0.5 cm** or 'plus or minus half a centimetre'.

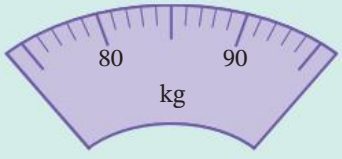
Limits of accuracy of measuring instruments

The **limits of accuracy** of a measuring instrument are ± 0.5 of the unit shown on the instrument's scale.

Example 4

Find the limits of accuracy for each measuring scale.

a



b

Solution

a The size of one unit on the scale is 1 kg.

The limits of accuracy are $\pm 0.5 \times 1 \text{ kg} = \pm 0.5 \text{ kg}$

b The size of one unit on the scale is 5 mL.

The limits of accuracy are $\pm 0.5 \times 5 \text{ mL} = \pm 2.5 \text{ mL}$.

EXERCISE 10.02 ANSWERS ON P. 593

Limits of accuracy for measuring instruments U R C

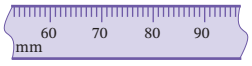
EXAMPLE 4

10.02

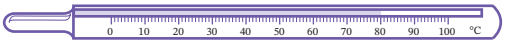
1 For each measuring instrument, state: R C

i the size of one unit on the scale **ii** its limits of accuracy.

a

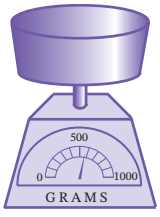


b

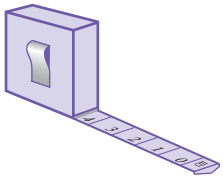


c

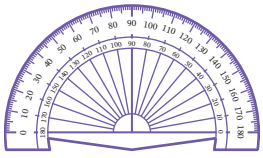
d



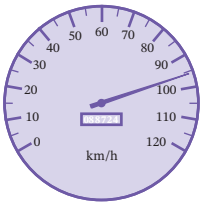
e



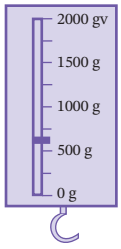
f



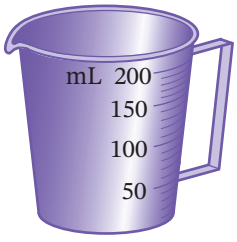
g



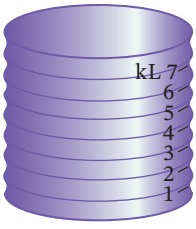
h



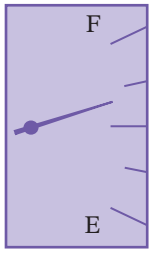
i



j



k



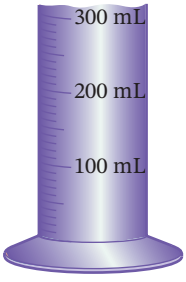
2 What are the limits of accuracy for this measuring cylinder?
Select the correct answer **A**, **B**, **C** or **D**.

A ± 0.5 mL

B ± 10 mL

C ± 20 mL

D ± 5 mL



3 Measuring with a ruler, Gemma correctly gives a measurement as 26.5 to 27.5 cm. R C

a What is the size of a unit on the ruler?

b What are the limits of accuracy of this measurement?

c Using a ruler marked in millimetres, Emily gives the same measurement as 26.8 cm. What are the limits of accuracy of this measurement?

d What is the range within which this measurement must lie?

Foundation Standard Complex

10.03 Perimeters and areas of composite shapes

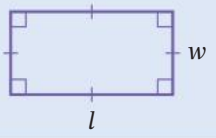
What is area?

The **perimeter** of a shape is the distance around the shape, the sum of the lengths of the sides of the shape.

The **area** of a shape is the amount of surface covered by the shape, measured in **square units**.

Areas of rectangles and triangles

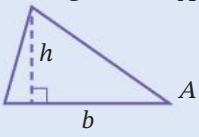
Rectangle



$A = \text{length} \times \text{width}$

$A = lw$

Triangle



$A = \frac{1}{2} \times \text{base} \times \text{perpendicular height}$

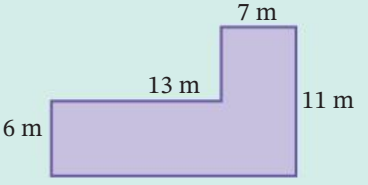
$A = \frac{1}{2}bh$

Example 5

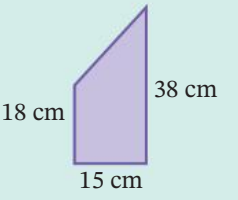
For each **composite shape**, find:

i its perimeter **ii** its area.

a

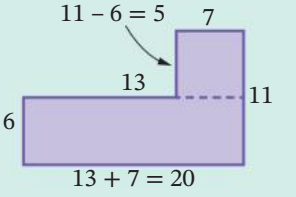


b



Solution

a i Find any unknown lengths on the shape.



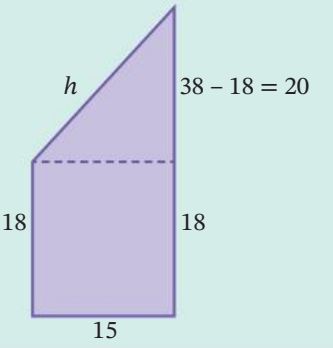
Perimeter $= 6 + 13 + 5 + 7 + 11 + 20$

$= 62$ m

ii Area $= (7 \times 5) + (20 \times 6)$ Sum of areas of 2 rectangles

$= 155$ m^2

b i Divide the shape into a rectangle and a triangle, and find the unknown length, h, using Pythagoras' theorem.



$h^2 = 15^2 + 20^2$

$= 625$

$h = \sqrt{625}$

$= 25$ cm

Perimeter $= 18 + 25 + 38 + 15$

$= 96$ cm

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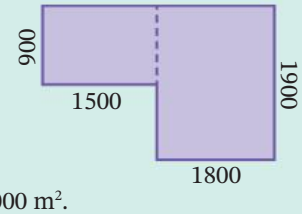
ii Area $= \left(\frac{1}{2} \times 15 \times 20\right) + (15 \times 18)$ Area of triangle + Area of rectangle

$= 420 \text{ cm}^2$

Example 6

This is the plan of a farm up for sale.

All measurements are in metres.



a Find the area of the farm.

b Express the area in square kilometres.

c Find the area of the property in **hectares**, where 1 ha = 10 000 m².

Solution

a Area $= 900 \times 1500 + 1900 \times 1800$

$= 4\,770\,000 \text{ m}^2$

b Since 1 km = 1000 m, $1 \text{ km}^2 = 1000^2 \text{ m}^2 = 1\,000\,000 \text{ m}^2$

$\therefore$ Area $= 4\,770\,000 \div 1\,000\,000$

$= 4.77 \text{ km}^2$

c $1 \text{ ha} = 10\,000 \text{ m}^2$

$\therefore$ Area $= 4\,770\,000 \div 10\,000$

$= 477 \text{ ha}$

EXERCISE 10.03 ANSWERS ON P. 593

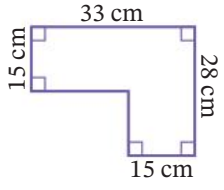
Perimeters and areas of composite shapes U F PS R

1 For each composite shape, find: PS R

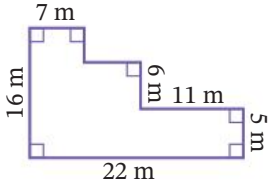
i its perimeter **ii** its area.

EXAMPLE 5

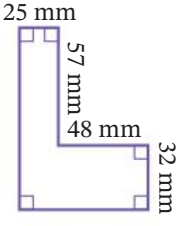
a



b



c



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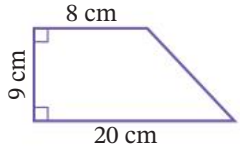
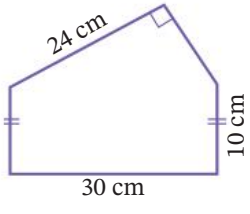


e



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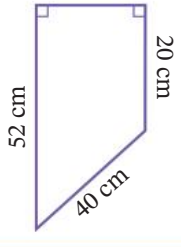

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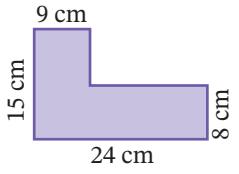
2 The length of a rectangular block of land is 112 m and its perimeter is 334 m.
How wide is the block? Select the answer **A**, **B**, **C** or **D**. R

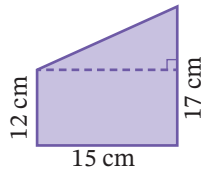
A 222 m **B** 110 m **C** 55 m **D** 224 m

3 Find the area of each composite shape. PS R

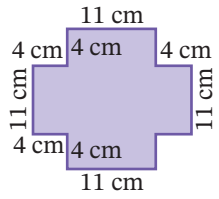
a



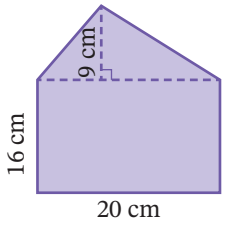
b



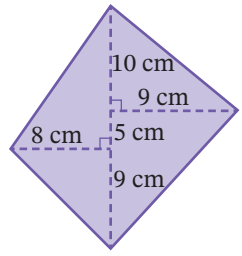
c



d



e



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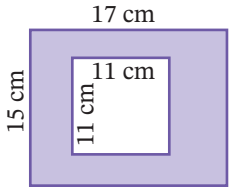
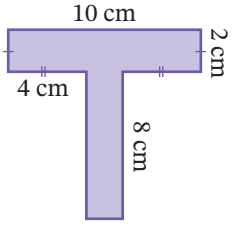
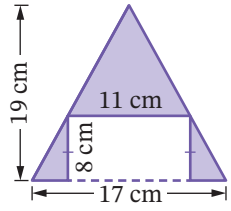


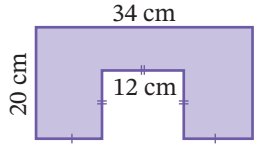
g



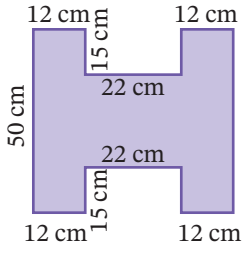
h



i



j



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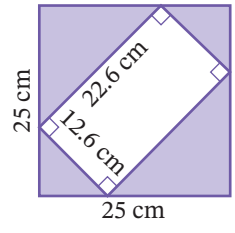
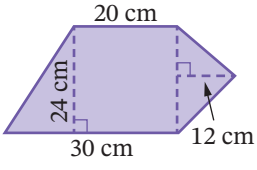


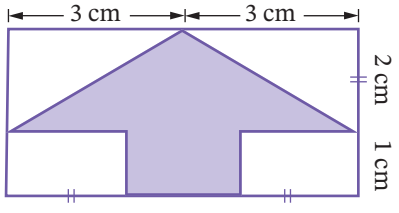
l



4 What is the area of the shaded arrow?
Select the correct answer **A**, **B**, **C** or **D**.

A 14 cm^2 **B** 8 cm^2

C 9 cm^2 **D** 12 cm^2



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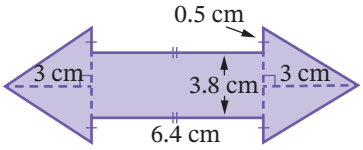
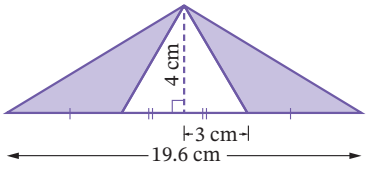
5 Find each shaded area. PS R

a



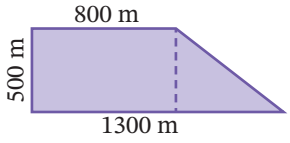
b



6 **a** Find the area of the large block of land shown in the diagram (in m^2).

b Express the area of the land in:

i km^2 **ii** hectares



7 The area of a large rectangular property is 8.6 ha. Calculate the width if its length is 400 m. Select the correct answer **A**, **B**, **C** or **D**.

A 21.5 m **B** 215 m **C** 2150 m **D** 0.0215 km

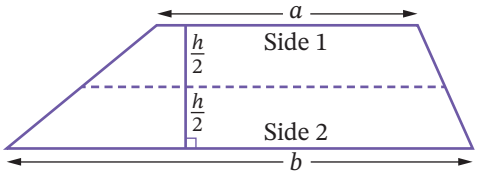
10.03

EXAMPLE 6

Investigation

Area of a trapezium, kite and rhombus

1 Draw 2 copies of this trapezium onto a sheet of paper. Draw a dotted line, parallel to the sides labelled a and b, halfway down the height of each trapezium and label the sides as shown.



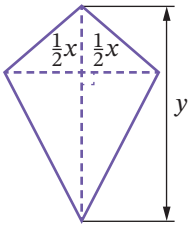
2 Cut out one trapezium and cut along the dotted line to make 2 smaller trapeziums. Join the pieces to make a long parallelogram.

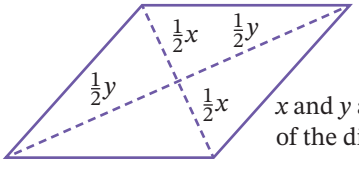
3 By measuring the height and base of the parallelogram to the nearest millimetre, find its area.

4 By comparing the measurements of the parallelogram to the measurements of the original trapezium, suggest a general formula for finding the area of any trapezium. Check your answer with your teacher.

5 Cut out the original trapezium and paste it and the parallelogram into your book.

6 Draw 2 copies of the kite and rhombus on a sheet of paper. Draw diagonals of length x and y and label them as shown. Notice that they cross at right angles. For the kite, one diagonal (x) is bisected by the other (y), while for the rhombus, *both* diagonals bisect each other.





x and y are the lengths of the diagonals

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7 Cut out one of your kites and rhombuses and cut along their diagonals to make 4 triangles each. Join each set of triangles to make a rectangle.

8 By measuring the sides of each rectangle to the nearest millimetre, find the area of each one.

9 By comparing the measurements of the kite to the measurements of the rectangle formed from it, suggest a general formula for finding the area of any kite. Check your answer with your teacher.

10 Is a rhombus a special type of kite?

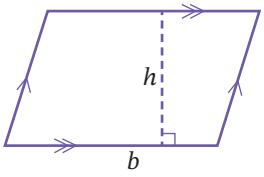
11 Suggest a general formula for finding the area of any rhombus. Check your answer with your teacher.

12 Cut out the original kite and rhombus and paste all shapes into your book.

10.04 Areas of quadrilaterals

Areas of special quadrilaterals

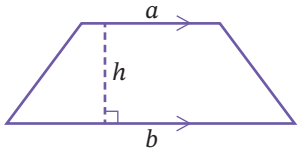
Parallelogram



Area = base × perpendicular height

$A = bh$

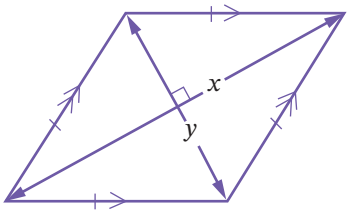
Trapezium



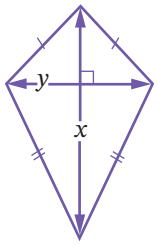
Area $= \frac{1}{2} \times$ (sum of parallel sides) × perpendicular height

$A = \frac{1}{2}(a + b)h$

Rhombus



Kite



Area $= \frac{1}{2} \times$ (product of diagonals)

$A = \frac{1}{2}xy$

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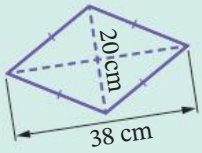
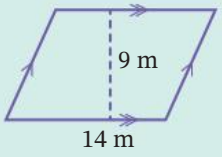
Example 7

Find the area of each quadrilateral.

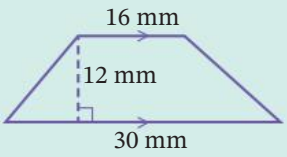
a



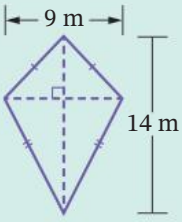
b



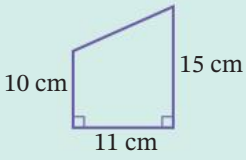
c



d



e



Solution

a Area of rhombus $= \frac{1}{2} \times 38 \times 20$ $\qquad A = \frac{1}{2}xy$

$= 380 \text{ cm}^2$

b Area of parallelogram $= 14 \times 9$ $\qquad A = bh$

$= 126 \text{ m}^2$

c Area of trapezium $= \frac{1}{2} \times (16 + 30) \times 12$ $\qquad A = \frac{1}{2}(a + b)h$

$= 276 \text{ mm}^2$

d Area of kite $= \frac{1}{2} \times 9 \times 14$ $\qquad A = \frac{1}{2}xy$

$= 63 \text{ m}^2$

e Area of trapezium $= \frac{1}{2} \times (10 + 15) \times 11$ $\qquad A = \frac{1}{2}(a + b)h$

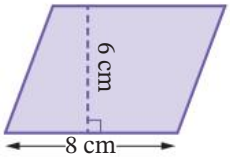
$= 137.5 \text{ cm}^2$

EXERCISE 10.04 ANSWERS ON P. 593

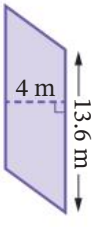
Areas of quadrilaterals U F PS R

1 Find the shaded area of each shape.

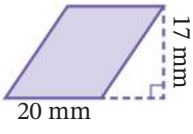
a



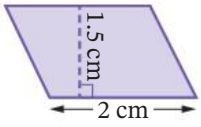
b



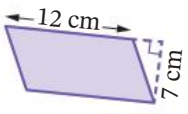
c



d



e



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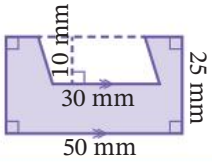
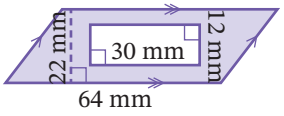


Foundation | Standard | Complex

EXAMPLE 7

g



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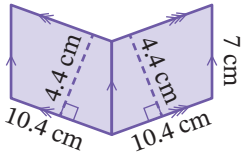

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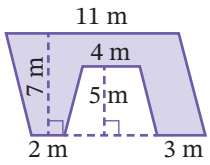
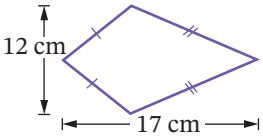


2 Find the area of each shape.

a



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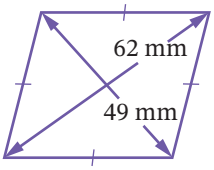

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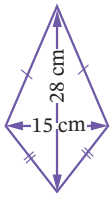

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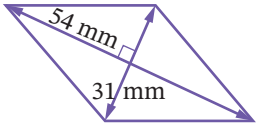

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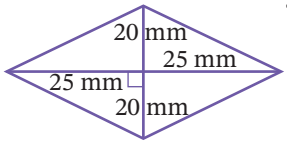

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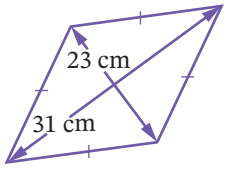

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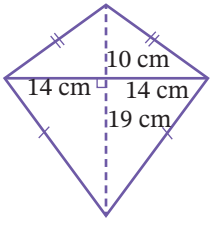

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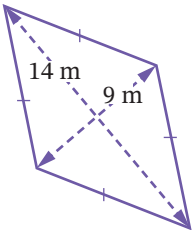

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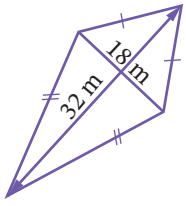
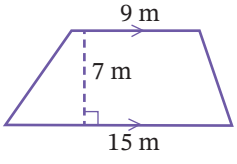


3 Find the area of each shape.

a



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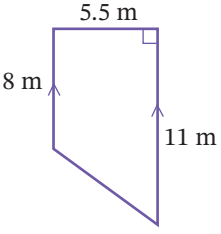

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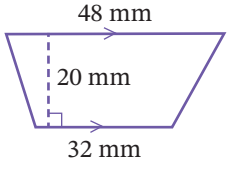

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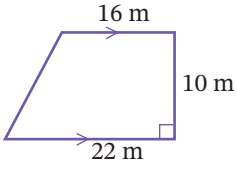

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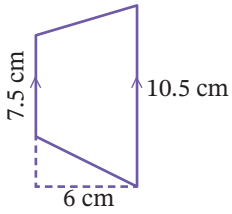

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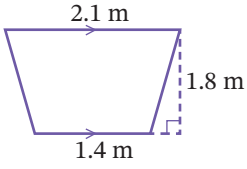

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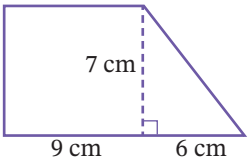

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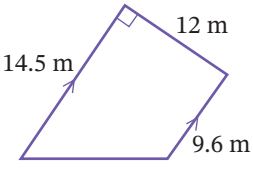

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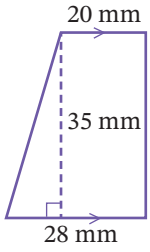

Foundation Standard Complex

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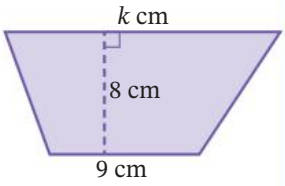
4 The area of the trapezium is 96 cm². What is the length of the missing side? Select the correct answer **A**, **B**, **C** or **D**.

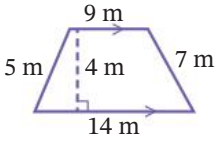


A 10 cm **B** 15 cm

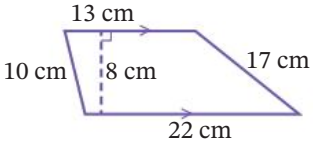
C 17 cm **D** 12 cm

5 Find the area of each shape.

a



b



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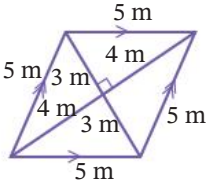

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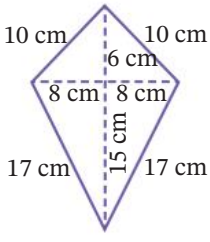

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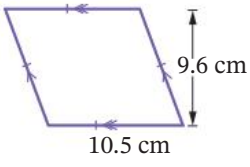

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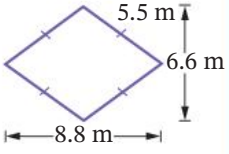
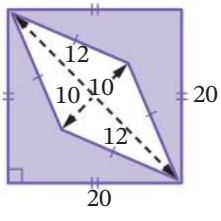


6 Calculate each shaded area (all measurements are in metres). PS R

a



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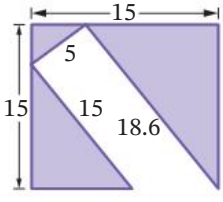

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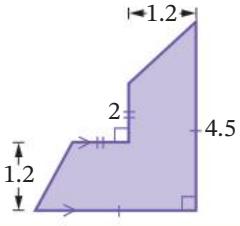


Mental skills 10A: Maths without calculators ANSWERS ON P. 593

Finding 10%, 20% and 5%

To find 10% or $\frac{1}{10}$ of a number, simply divide the number by 10 by moving the decimal point one place to the left.

1 Study each example.

a $10\% \times 150 = 15\overset{\curvearrowleft}{0}. = 15$

b $10\% \times \$1256.80 = \$125\overset{\curvearrowleft}{6}.8 = \$125.68$

c $10\% \times 4917 = 491\overset{\curvearrowleft}{7}. = 491.7$

d $10\% \times \$48.55 = \$4\overset{\curvearrowleft}{8}.55 = \$4.885$

2 Now find 10% of each amount.

a	190	**b**	$75	**c**	875	**d**	$202
e	$37.60	**f**	400	**g**	$9.25	**h**	896
i	$2700	**j**	$3.80	**k**	$1527.60	**l**	$72.50
m	3154	**n**	$10.70	**o**	426	**p**	$24 317.60

20% is 10% doubled, so to find 20% of a number, first find 10% then double it.

3 Study each example.

a
$20\% \times 700$
$10\% \times 700 = 70$
$\therefore 20\% \times 700 = 70 \times 2$
$= 140$

b
$20\% \times \$876$
$10\% \times \$876 = \$87.60$
$\therefore 20\% \times \$876 = \$87.60 \times 2$
$= \$175.20$

c
$20\% \times 325$
$10\% \times 325 = 32.5$
$\therefore 20\% \times 325 = 32.5 \times 2$
$= 65$

d
$20\% \times \$38.50$
$10\% \times \$38.50 = \$3.85$
$\therefore 20\% \times \$38.50 = \$3.85 \times 2$
$= \$7.70$

4 Now find 20% of each amount.

a	50	**b**	620	**c**	$2450	**d**	$8.60
e	72	**f**	$12 700	**g**	390	**h**	$5.80
i	$45	**j**	$84	**k**	$4600	**l**	320

5% is half of 10%, so to find 5% of a number, first find 10% then divide it by 2.

5 Study each example.

a
$5\% \times 180$
$10\% \times 180 = 18$
$\therefore 5\% \times 180 = 18 \div 2$
$= 9$

b
$5\% \times \$76$
$10\% \times \$76 = \$7.60$
$\therefore 5\% \times \$76 = \$7.60 \div 2$
$= \$3.80$

c
$5\% \times 120$
$10\% \times 120 = 12$
$\therefore 5\% \times 120 = 12 \div 2$
$= 6$

d
$5\% \times \$142.20$
$10\% \times \$142.20 = \$14.22$
$\therefore 5\% \times \$142.20 = \$14.22 \div 2$
$= \$7.11$

6 Now find 5% of each amount.

a	2000	**b**	$12	**c**	50	**d**	$27
e	$36.80	**f**	$84	**g**	800	**h**	130
i	$9.60	**j**	$138	**k**	$72	**l**	840

Circumferences and areas of circular shapes

10.05

The **circumference** of a circle is found by multiplying its **diameter** by a special number called **pi** (pronounced 'pie'), represented by the Greek letter π.

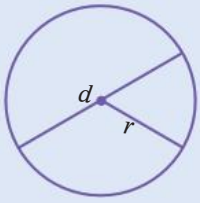
The **area** of a circle is found by multiplying π by the **radius squared.**

Circumference of a circle

The **circumference** (**perimeter**) **of a circle** is:

$C = \pi \times \text{diameter}$ or $C = 2 \times \pi \times \text{radius}$

$C = \pi d$ or $C = 2\pi r$



Area of a circle

The **area of a circle** is:

$A = \pi \times (\text{radius})^2$

$A = \pi r^2$

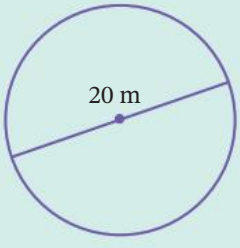
$\pi = 3.141\ 592\ 653\ 897\ 93...$ and can be found by pressing the π key on your calculator. It has no exact decimal or fraction value, so like a surd such as $\sqrt{2}$, it is an **irrational number**.

Example 8

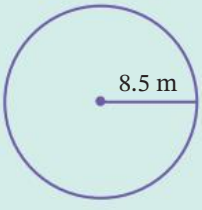
For each circle, calculate correct to 2 decimal places:

i its circumference **ii** its area.

a



b



Solution

a i $C = \pi \times 20$ $C = \pi d$

$= 62.8318...$

≈ 62.83 m

ii $A = \pi \times 10^2$ $A = \pi r^2$

$= 314.1592...$

≈ 314.16 m^2

b i $C = 2 \times \pi \times 8.5$ $C = 2\pi r$

$= 53.4070...$

≈ 53.41 m

ii $A = \pi \times 8.5^2$ $A = \pi r^2$

$= 226.9800...$

≈ 226.98 m^2

Perimeter and area of a sector

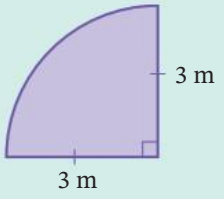
Example 9

For each sector, calculate correct to one decimal place:

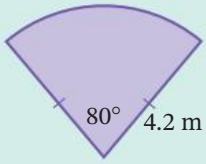
A **sector** is a fraction of a circle 'cut' along 2 radii, like a pizza slice.

i its perimeter **ii** its area.

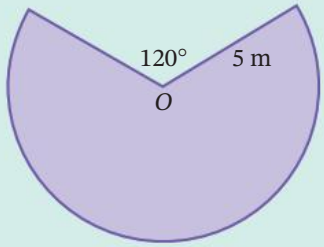
a



b



c



Solution

a i This sector is a **quadrant**, a quarter of a circle. $\frac{1}{4} \times$ circumference + radius + radius

$$\text{Perimeter} = \frac{1}{4} \times 2 \times \pi \times 3 + 3 + 3$$
$$= 10.71238...$$
$$= 10.7 \text{ m}$$

ii $\text{Area} = \frac{1}{4} \times \pi \times 3^2$ $\frac{1}{4} \times$ area of circle

$$= 7.06858...$$
$$= 7.1 \text{ m}^2$$

b i $\text{Perimeter} = \frac{80}{360} \times 2 \times \pi \times 4.2 + 4.2 + 4.2$ $\frac{80}{360} \times$ circumference + radius + radius

$$= 14.26430...$$
$$= 14.3 \text{ m}$$

There are 360° in a circle, but a sector is a fraction of a circle.

ii $\text{Area} = \frac{80}{360} \times \pi \times 4.2^2$ $\frac{80}{360} \times$ area of circle

$$= 12.31504...$$
$$= 12.3 \text{ m}^2$$

c i Sector angle $= 360° - 120°$

$$= 240°$$

$$\text{Perimeter} = \frac{240}{360} \times 2 \times \pi \times 5 + 5 + 5$$
$$= 30.94395...$$
$$= 30.9 \text{ m}$$

ii $\text{Area of sector} = \frac{240}{360} \times \pi \times 5^2$

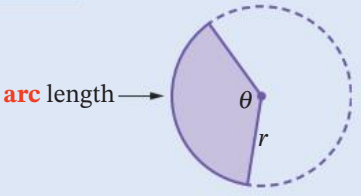
$$= 52.35987...$$
$$= 52.4 \text{ m}^2$$

Arc length, perimeter and area of a sector

For a sector with angle θ:

- Arc length $= \frac{\theta}{360} \times 2\pi r$
- Perimeter of the sector $= \frac{\theta}{360} \times 2\pi r + r + r$

  (arc length + radius + radius)
- Area of the sector $= \frac{\theta}{360} \times \pi r^2$



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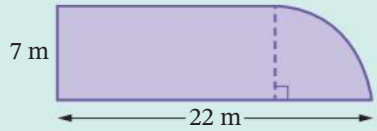
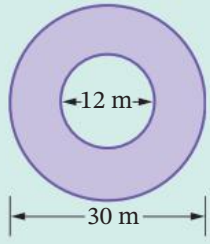
Example 10

Find, correct to one decimal place, the area of each shape.

a



b



Solution

a The shape is made up of a rectangle and a quadrant.

Radius of quadrant $= 7$ m

Length of rectangle $= 22 - 7$

$= 15$ m

Area of shape = area of rectangle + quadrant

$= 15 \times 7 + \frac{1}{4} \times \pi \times 7^2$

$= 143.4845...$

≈ 143.5 m^2

b This ring shape is called an **annulus**, it is the area enclosed by 2 circles with the same centre.

Radius of large circle $= \frac{1}{2} \times 30$ m

$= 15$ m

Radius of small circle $= \frac{1}{2} \times 12$ m

$= 6$ m

Shaded area = large circle – small circle

$= \pi \times 15^2 - \pi \times 6^2$

$= 593.7610...$

≈ 593.8 m^2

EXERCISE 10.05 ANSWERS ON P. 593

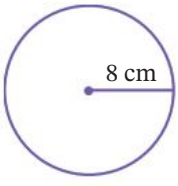
Circumferences and areas of circular shapes U F PS R

1 For each circle, calculate correct to 2 decimal places:

EXAMPLE 8

i its circumference **ii** its area.

a



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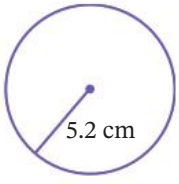

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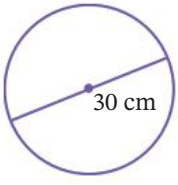

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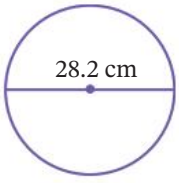
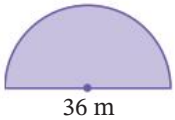


2 For each sector, calculate correct to one decimal place:

EXAMPLE 9

i its perimeter **ii** its area.

a



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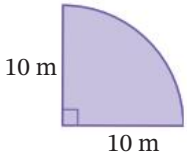

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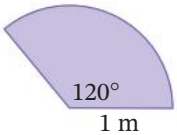

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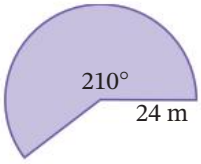

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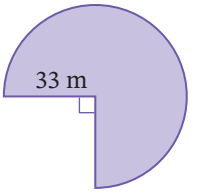
e



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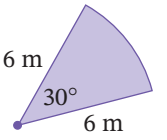

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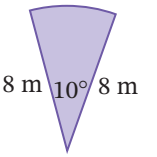

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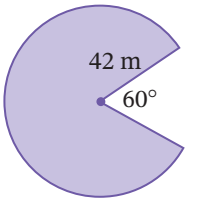
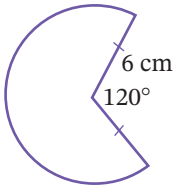


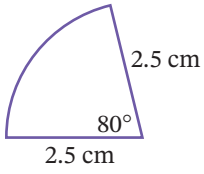
3 Find the perimeter of this sector. Select the correct answer **A**, **B**, **C** or **D**.



A 12.6 cm **B** 18.3 cm

C 24.6 cm **D** 37.1 cm

4 Find the area of this sector. Select the correct answer **A**, **B**, **C** or **D**.

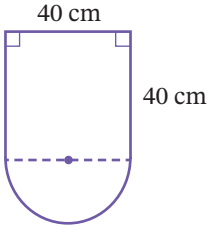


A 19.6 cm^2 **B** 15.3 cm^2

C 8.5 cm^2 **D** 4.4 cm^2

5 Calculate the perimeter of each shape, correct to 2 decimal places.

a



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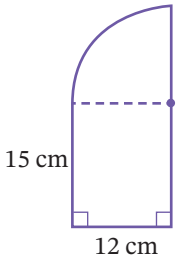

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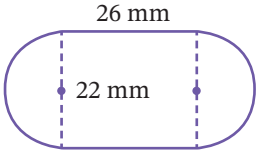

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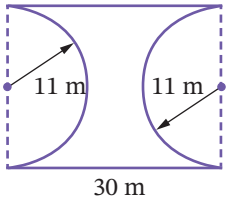

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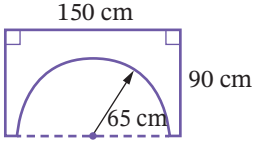

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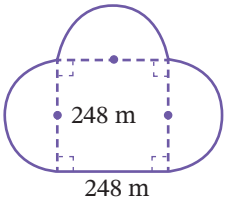

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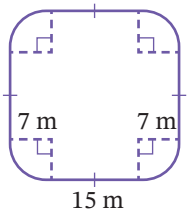

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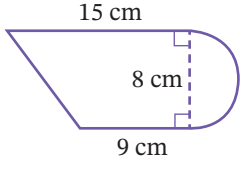

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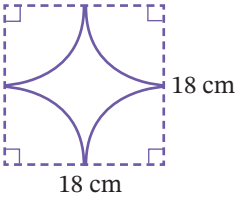
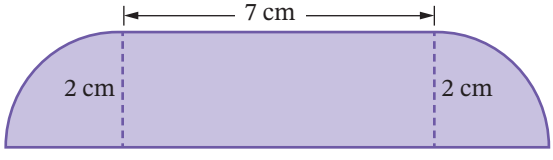


EXAMPLE 10

6 Calculate the area of the shape. Select the correct answer **A**, **B**, **C** or **D**.



A 26.57 cm^2 **B** 20.28 cm^2 **C** 24.28 cm^2 **D** 28.28 cm^2

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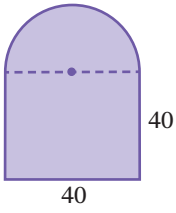
7 Calculate the area of each shape, correct to the nearest square metre.
All measurements are in metres.

a



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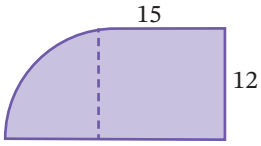

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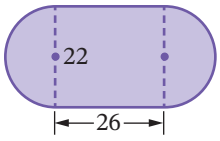

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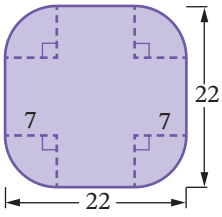

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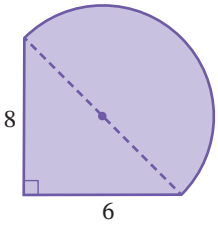

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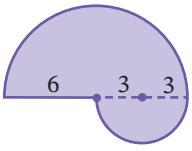

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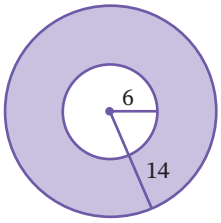

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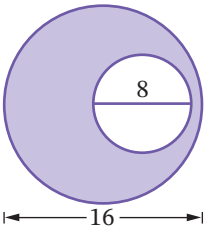

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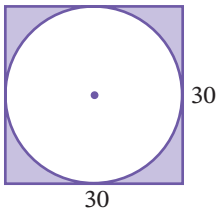

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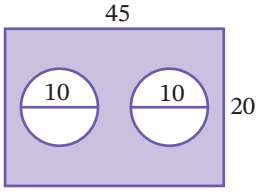

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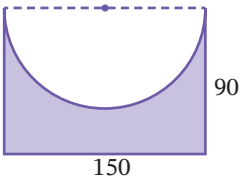

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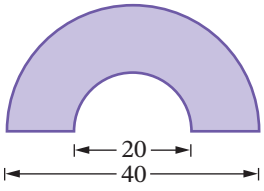

8 A circular playing field has a radius of 80 m. A rectangular cricket pitch measuring 25 m by 2 m is placed in the middle. The field, excluding the pitch, is to be fertilised. PS R

a Calculate, to the nearest square metre, the area to be fertilised.

b How much will this cost if the fertiliser is $19.95 per 100 square metres? Give your answer to the nearest dollar.

9 The diameter of the Earth is 12 756 km.

a Find the circumference of the Earth to the nearest kilometre.

b The International Space Station orbits the Earth at an average height of 400 km above the Earth's surface. Find the distance it travels in one orbit of the Earth.

Shutterstock.com/Dima Zel

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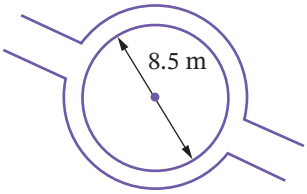
10 A circular plate of diameter 2 m has 250 holes of diameter 10 cm drilled in it. What is the remaining area of the plate? Answer to the nearest 0.1 m^2.

11 A circular pond of diameter 10 m is surrounded by a path one metre wide. PS R

a Calculate the area of the path, correct to 2 decimal places.

b If pavers are $165 per square metre laid, what is the cost of the path?

12 A circular patch of grass has a diameter of 8.5 m. How much further does Arya walk if she walks around the border instead of directly across it? Give your answer correct to the nearest 0.1 m. PS R



13 A new tractor tyre has a diameter of 120 cm, while a worn tyre has a diameter of 115 cm.

a Calculate the difference in circumference between a new and a worn tyre, correct to 3 decimal places.

b Over 1000 revolutions, how much further (to the nearest metre) will a new tyre travel compared to a worn tyre?

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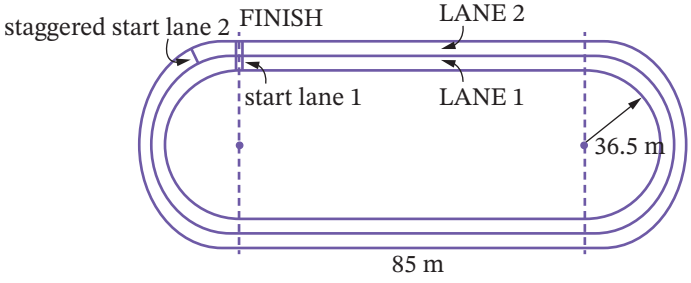
14 A square courtyard measuring 5 m by 5 m has a semi-circular area added to each side. PS R

a Calculate the area of the semi-circular additions, correct to the nearest square metre.

b By what percentage has the area of the courtyard increased?

(This can be calculated as $\frac{\text{increase in area}}{\text{original area}} \times 100\%$)

15 The measurements of a running track are shown below. Each lane is 1 m wide and the athletes run along the insides of the lanes. PS R



a Izabella runs one lap in lane 1. What distance does she cover, correct to one decimal place?

b Noah runs one lap in lane 2. What distance does he cover, correct to one decimal place?

c By how much should the start line be staggered in lane 2 (from lane 1) so that Izabella and Noah run the same distance in one lap?

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16 The wheels of a bicycle have a diameter of 66 cm.

a How far (correct to 2 decimal places) will the bicycle move after one rotation of the wheel?

b How many rotations of the wheel are needed to cover a distance of 5 km?

Did you know?

A piece of pi

$\pi = 3.141\ 592\ 653\ 589\ 793\ 238\ 462\ 643\ 383\ 279\ 502\ 884\ 197\ 169\ 399\ 375\ 105\ 820...$

Because π is an irrational number, its decimal digits run endlessly without repeating. Over history, mathematicians and scientists (and now computer scientists) have tried to calculate more accurate values of π, using more sophisticated formulas and calculation techniques. The ancient Greeks, Romans and Chinese first estimated π as 3. This value is also mentioned in the Bible. Since the first computer, the ENIAC, was invented in 1949, much progress has been made. Supercomputers have been used to calculate more decimal places of π and, in 2019, Emma Haruka Iwao (Japan), working for Google, calculated π to over 31.4 trillion places. In 2015, Suresh Sharma (India) memorised π to 70 030 decimal places. It took him 17 hours 14 minutes to recite it.

Research the modern history of π and find out about recent calculations of its value.

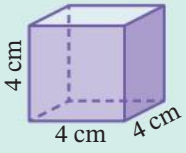
Surface area of a prism 10.06

The **surface area** of a solid is the total area of all the faces of the solid. To calculate the surface area of a solid, find the area of each face and add the areas together.

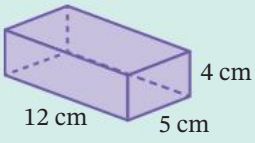
Example 11

Calculate the surface area of each prism.

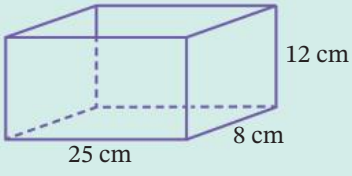
a cube



b rectangular prism



c open rectangular prism



Solution

a A cube has 6 identical faces in the shape of a square. 6 × area of a square

Surface area $= 6 \times 4^2$

$= 96\ \text{cm}^2$

WS Surface area
WS Nets of solids
Solid shapes
Surface area
Surface area calculator
Surface area calculator

b A rectangular prism has 6 faces that are not all the same. However, opposite faces, such as the top and bottom, are the same.

Surface area = 2 top and bottom faces + 2 end faces + 2 side faces

$= (2 \times 5 \times 12) + (2 \times 5 \times 4) + (2 \times 12 \times 4)$

$= 256 \text{ cm}^2$

For the surface area of an open solid, we only count the *external* surfaces, not the internal ones (so that we don't count each surface twice).

c The open rectangular prism has no top so it has 5 faces that are not all the same.

Surface area = Bottom face + 2 front and back faces + 2 side faces

$= (25 \times 8) + (2 \times 25 \times 12) + (2 \times 8 \times 12)$

$= 992 \text{ cm}^2$

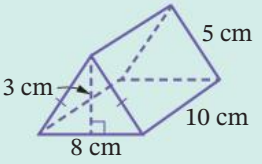
Example 12

Surface area of a prism

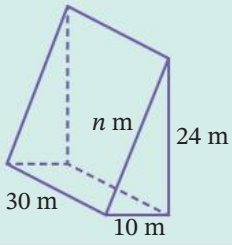
Surface area of prisms

Calculate the surface area of each triangular prism.

a

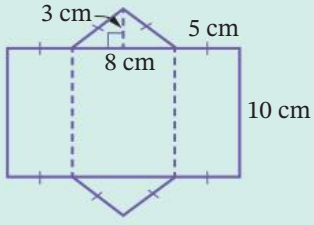


b



Solution

a This triangular prism has 5 faces: 2 identical triangles (front and back), 2 identical rectangles (sides) and another rectangle (bottom). Drawing the net of the prism may help you see this.



Surface area $= (2 \times \frac{1}{2} \times 8 \times 3) + (2 \times 10 \times 5) + (8 \times 10)$

$= 204 \text{ cm}^2$

Front and back faces + side faces + bottom face

b This triangular prism has 5 faces: 2 identical triangles (front and back), 3 different rectangles.

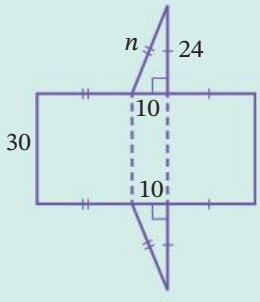
To calculate the area of the left face, we need to know the value of n, which can be found using Pythagoras' theorem.

$n^2 = 10^2 + 24^2$

$= 676$

$n = \sqrt{676}$

$= 26$



Surface area $= (2 \times \frac{1}{2} \times 10 \times 24) + (30 \times 26)$

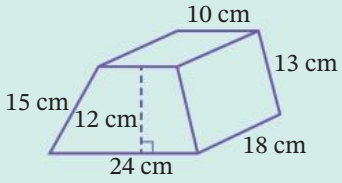
$+ (30 \times 10) + (30 \times 24)$

$= 2040 \text{ m}^2$

Front and back + left + bottom + right

Example 13

Calculate the surface area of this trapezoidal prism.

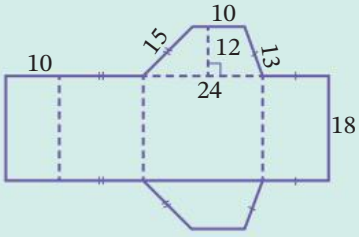


Surface area of a prism

10.06

Solution

This trapezoidal prism has 6 faces: 2 identical trapeziums (front and back), 4 different rectangles.



Area of each trapezium $= \frac{1}{2} \times (10 + 24) \times 12$

$= 204 \text{ cm}^2$

Surface area $= (2 \times 204) + (18 \times 10)$ 2 trapeziums + 4 rectangles

$+ (18 \times 15) + (18 \times 24)$

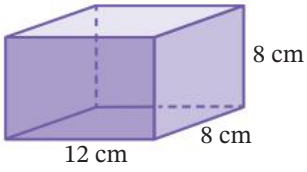
$+ (18 \times 13)$

$= 1524 \text{ cm}^2$

EXERCISE 10.06 ANSWERS ON P. 594

Surface area of a prism U F PS R

1 What is the surface area of this rectangular prism? Select the correct answer **A**, **B**, **C** or **D**.



EXAMPLE 11

A 320 cm^2 **B** 352 cm^2

C 512 cm^2 **D** 768 cm^2

2 Draw a net of each solid.

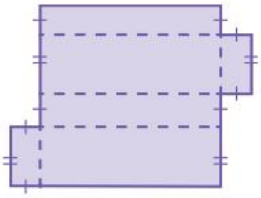
a cube **b** rectangular prism **c** triangular prism **d** cylinder

3 Match each net of a solid to the name of the solid.

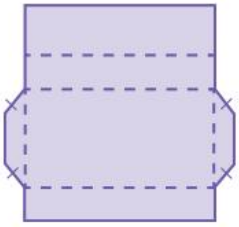
triangular prism cube rectangular prism

square pyramid cylinder trapezoidal prism

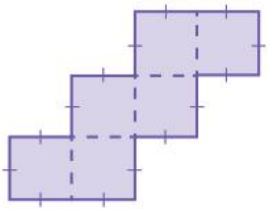
a



b



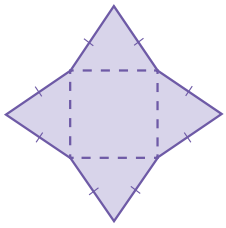
c



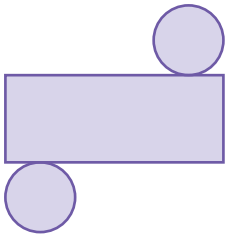
Foundation Standard Complex

d

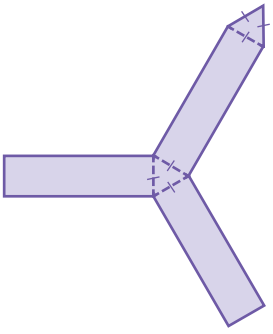
e



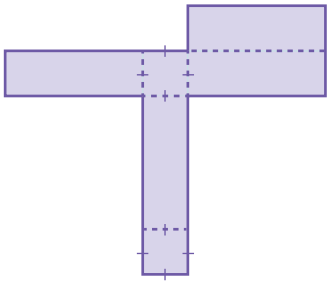
f



g



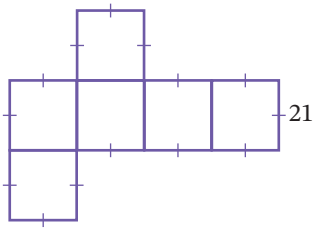
h



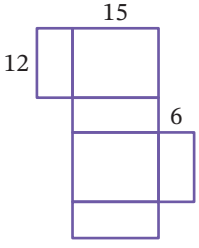
EXAMPLE 12

4 Name the prism that each net represents, then calculate the surface area of the prism. All lengths are in metres.

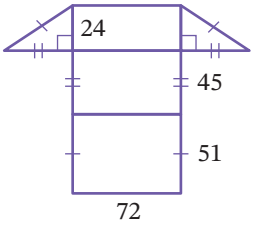
a



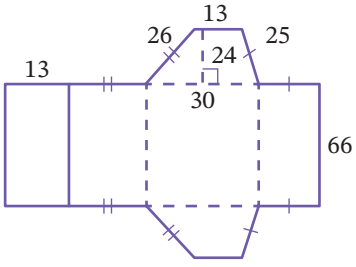
b



c

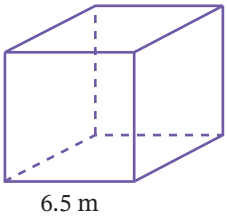


d

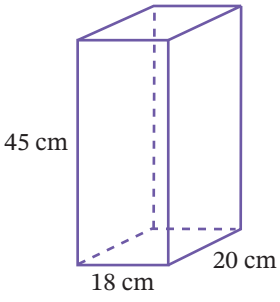


5 Calculate the surface area of each prism. PS R

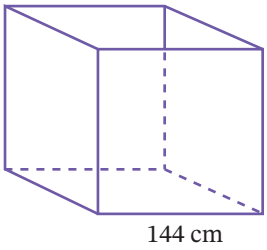
a cube



b rectangular prism



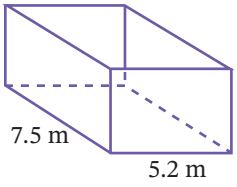
c open cube (no top)



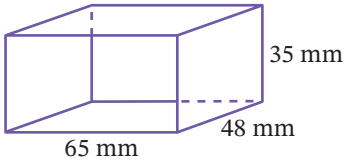
Foundation Standard Complex

9780170453325

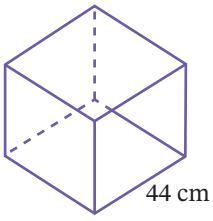
d open rectangular prism



e rectangular prism, open front and back

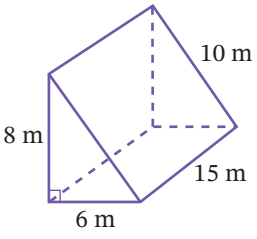


f cube, open one end

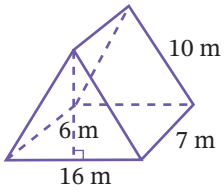


6 Find the surface area of each triangular prism. PS R

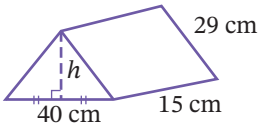
a



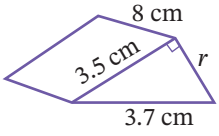
b



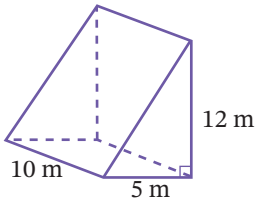
c



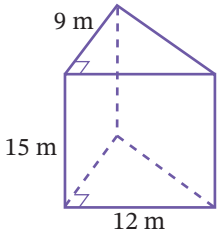
d



e



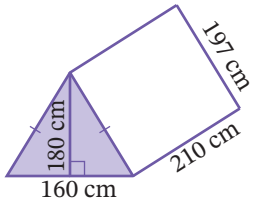
f



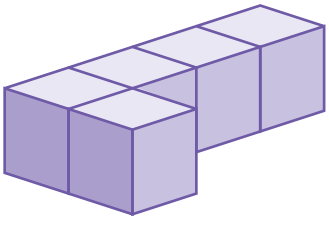
7 Yasmin made this tent. PS R

a Find the surface area of the tent, including the floor.

b Yasmin bought material at $12 per square metre. How much did it cost her for the material to make the tent?



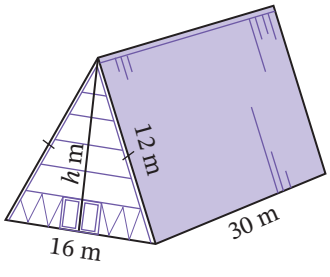
8 5 cubes of side length 1 cm are joined to make this solid. What is the surface area of the solid? Select the correct answer **A**, **B**, **C** or **D**.



A 17 cm^2 **B** 18 cm^2

C 5 cm^2 **D** 22 cm^2

9 An art gallery has a width of 16 metres, a length of 30 metres and a slant height of 12 metres, as shown in the diagram. PS R



a Find the perpendicular height, h, of the building, correct to one decimal place.

b Find the area of the front wall.

c Find the surface area of the art gallery, without the floor.

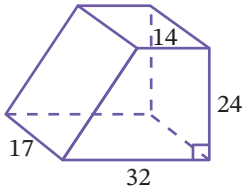
10.06

Foundation Standard Complex

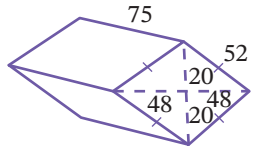
EXAMPLE 13

10 Calculate the surface area of each prism. Measurements are in centimetres. PS R

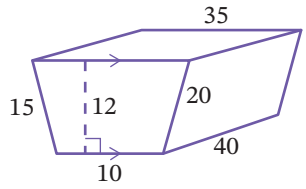
a



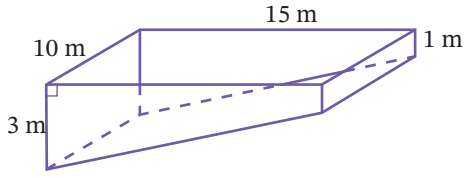
b



c



11 This swimming pool is 15 m long and 10 m wide. The depth of the water ranges from 1 m to 3 m. PS R



Calculate, correct to 2 decimal places:

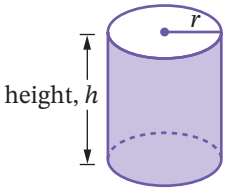
a the area of the floor of the pool

b the total surface area of the pool.

Investigation

Surface area of a cylinder

Collect 5 different cylindrical cans with paper labels.



1 Copy this table with rows for Cans 1 to 5 and complete it as you perform the following measurements and calculations.

	Circular ends					Curved surface			Surface area
	Height	Diameter	Radius	Area	Circumference	Length	Width	Area	
Can 1									
Can 2									

2 Measure the height and diameter of the first can.

3 Calculate the radius, area and circumference of one circular end, correct to 2 decimal places.

4 Cut the label off the first can and lay it out flat. What shape is this curved surface?

5 Measure the length, width and area of the curved surface (label).

6 Calculate the surface area of the can by adding the areas of the 2 circular ends and the area of the label.

7 Why is the height of each can and the width of its label the same?

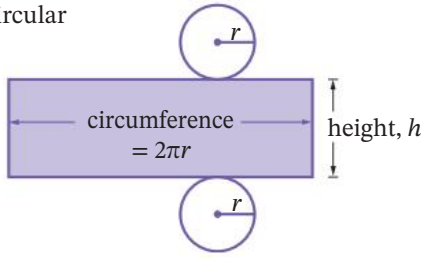
8 Why is the circumference of each circular end and the length of the label the same?

9 Repeat Steps **2** to **6** for the other 4 cans.

Surface area of a cylinder

10.07

A closed **cylinder** has 3 faces made up of 2 circles (the circular ends) and a rectangle (the curved surface). The length of the rectangle is the circumference of the circular end, while the width of the rectangle is the height of the cylinder.



Surface area of a cylinder = area of 2 circles
+ area of rectangle

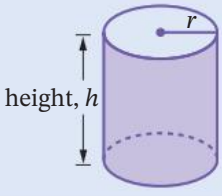
$$SA = 2 \times \pi r^2 + 2\pi r \times h$$
$$= 2\pi r^2 + 2\pi rh$$

Surface area of a closed cylinder

$SA = 2\pi r^2 + 2\pi rh$

where r = radius of circular base

h = perpendicular height



The area of the 2 circular ends = $2\pi r^2$ and the area of the curved surface = $2\pi rh$.

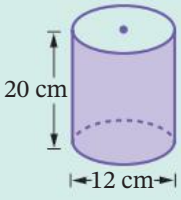
Example 14

Find, correct to one decimal place, the surface area of a cylinder with diameter 12 cm and height 20 cm.

Solution

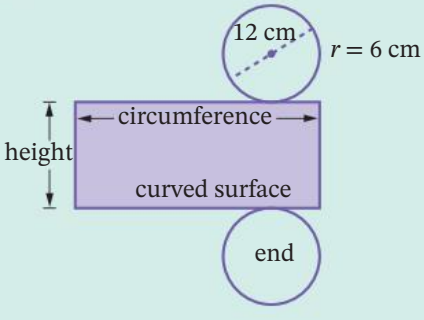
Radius = $\frac{1}{2} \times 12$ cm = 6 cm

$\frac{1}{2}$ of diameter



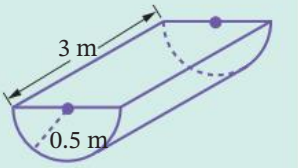
Surface area = area of 2 ends + area of the curved surface

$$SA = 2\pi r^2 + 2\pi rh$$
$$= 2 \times \pi \times 6^2 + 2 \times \pi \times 6 \times 20$$
$$= 980.1769...$$
$$\approx 980.2 \text{ cm}^2$$



Example 15

Find, correct to 2 decimal places, the surface area of this open half-cylinder with radius 0.5 m and height 3 m.



Note: For the surface area of an open solid, we only count the external surfaces, not the internal ones (so that we don't count each surface twice).

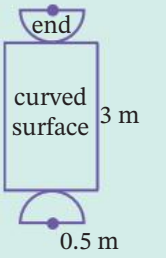
Solution

Surface area = 2 semicircle ends + $\frac{1}{2}$ × curved surface

$$SA = 2 \times (\tfrac{1}{2} \times \pi \times 0.5^2) + \tfrac{1}{2} \times (2 \times \pi \times 0.5 \times 3)$$

$$= 5.49778...$$

$$\approx 5.50\,\text{cm}^2$$



EXERCISE 10.07 ANSWERS ON P. 594

Surface area of a cylinder U F PS R

EXAMPLE 14

1 Calculate, correct to 2 decimal places, the surface area of a cylinder with:

a radius 7 m, height 10 m

b diameter 28 cm, height 15 cm

c diameter 6.2 m, height 7.5 m

d radius 0.8 m, height 2.35 m

2 Find, correct to one decimal place, the curved surface area of a cylinder with:

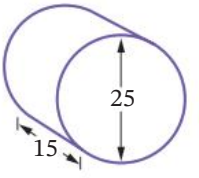
a diameter 9 cm, height 32 cm

b radius 85 mm, height 16 mm

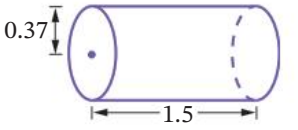
EXAMPLE 15

3 Calculate, correct to one decimal place, the surface area of each solid. All lengths shown are in metres. PS R

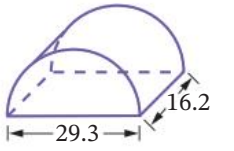
a closed cylinder



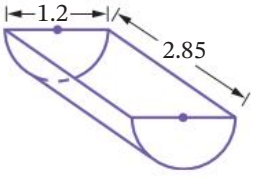
b cylinder with one open end



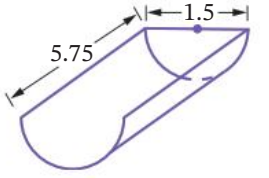
c closed half-cylinder



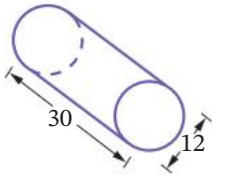
d half-cylinder with open top



e half-cylinder with open top, one end open



f cylinder open at both ends



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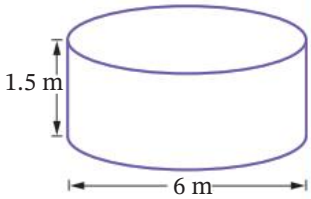
4 A swimming pool is in the shape of a cylinder 1.5 m deep and 6 m in diameter. The inside of the pool is to be repainted, including the floor. Find: PS

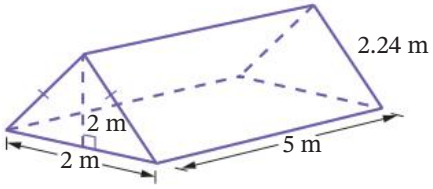


a the area to be repainted, correct to one decimal place

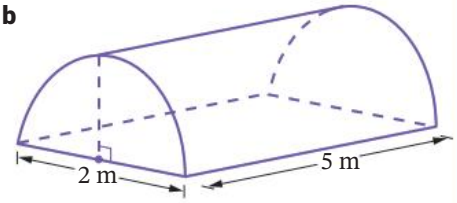
b the number of whole litres of paint needed if coverage is 9 m^2 per litre.

5 Which tent has the greater surface area?

a

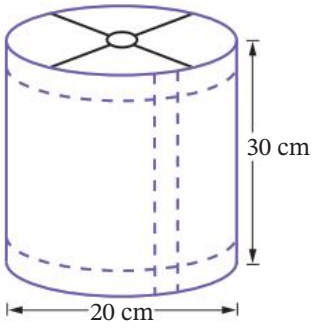


b



(*Note*: the floor is included for both tents)

6 This lampshade is to be covered. Find the area of material needed, correct to one decimal place, if 10% extra is allowed for seams and overlaps. Note: The lampshade is a cylinder open at both ends. PS



10.08

Volumes of prisms and cylinders 10.08

The **volume** of a solid is the amount of space it takes up. Volume is measured in **cubic units**, for example, cubic metres (m^3) or cubic centimetres (cm^3).

The **capacity** of a container is the amount of fluid (liquid or gas) it holds, measured in millilitres (mL), litres (L), kilolitres (kL) and megalitres (ML).

WS Volume and capacity

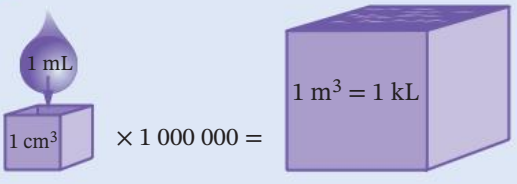
What is volume?

Formula matching game

Volume and capacity

1 cm^3 contains 1 mL

1 m^3 contains 1000 L or 1 kL



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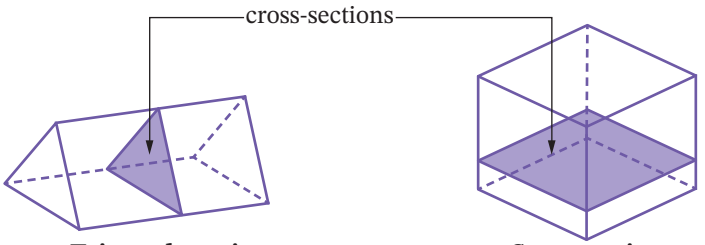
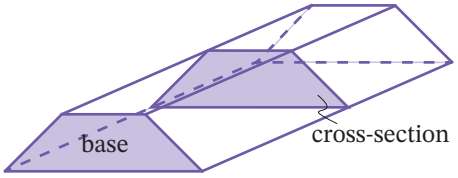
Volume of a prism

A **cross-section** of a solid is a 'slice' of the solid cut *across* it, parallel to its end faces, rather than along it. A **prism** has the same (uniform) cross-section along its length, and each cross-section is a **polygon** (with straight sides).



Triangular prism **Square prism**

An end face of a prism is called its **base**. Prisms take their names from their base and cross-section. For example, this prism is a **trapezoidal prism** because its base and cross-sections are trapeziums.



Trapezoidal prism

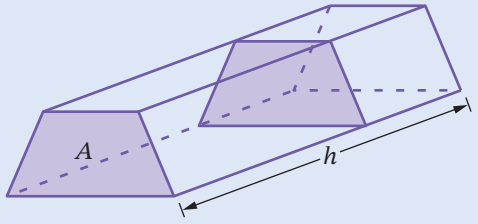
Because a prism is made up of identical cross-sections, its volume can be calculated by multiplying the **area of its base** by its **perpendicular height** (the length or depth of the prism).

Volume of a prism

$V = Ah$

where A = area of base

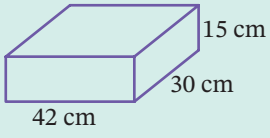
h = perpendicular height



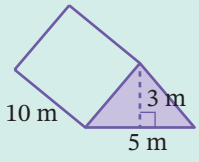
Example 16

Find the volume of each prism.

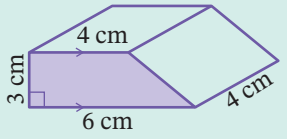
a



b



c



Volumes of prisms and cylinders

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Solution

a $V = 42 \times 30 \times 15$
$= 18\,900 \text{ cm}^3$
For a rectangular prism, volume = length × width × height ($V = lwh$)

b $A = \frac{1}{2} \times 5 \times 3$
$= 7.5$
Area of a triangle.
$V = 7.5 \times 10$
$= 75 \text{ m}^3$
$V = Ah$, where height $h = 10$

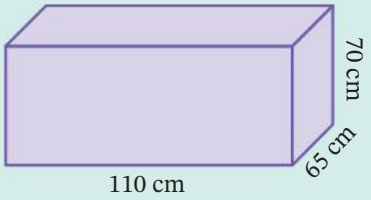
c $A = \frac{1}{2} \times (4 + 6) \times 3$
$= 15 \text{ cm}^2$
Area of a trapezium
$V = 15 \times 4$
$= 60 \text{ cm}^3$
$V = Ah$, where height $h = 4$

Example 17

A storage box in the shape of a rectangular prism is 110 cm long, 65 cm wide and 70 cm high.

a Find the volume of the box in cm^3.

b What is the volume in m^3?



Solution

a $V = 110 \times 65 \times 70$
$= 500\,500 \text{ cm}^3$

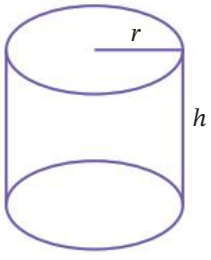
b $1 \text{ m} = 100 \text{ cm}$, so $1 \text{ m}^3 = 100^3 \text{ cm}^3 = 1\,000\,000 \text{ cm}^3$
$\therefore V = 500\,500 \div 1\,000\,000$
$= 0.5005 \text{ m}^3$

Alternatively, you could convert 110 cm, 65 cm and 70 cm to metres, then multiply them together for the volume.

Volume of a cylinder

A cylinder is like a 'circular prism' because its cross-sections are identical circles. Because of this, we can also use $V = Ah$ to find the volume of a cylinder. But for a circle, $A = \pi r^2$, so:

Volume $= Ah$
$V = \pi r^2 \times h$
$= \pi r^2 h$



Volume of a cylinder

$V = \pi r^2 h$

where r = radius of circular base
h = perpendicular height

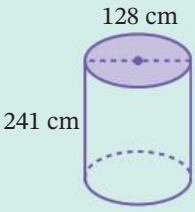
Volumes of prisms and cylinders

Example 18

For this cylinder, calculate:

a its volume, correct to the nearest cm^3

b its capacity in kL, correct to 1 decimal place.



Solution

a Radius $= \frac{1}{2} \times 128$ cm $\quad$ $\frac{1}{2}$ of diameter

$= 64$ cm

$V = \pi \times 64^2 \times 241$ $\quad$ $V = \pi r^2 h$

$= 3\,101\,179.206...$

$\approx 3\,101\,179\ cm^3$

b Capacity $= 3\,101\,179$ mL $\quad$ $1\ cm^3 = 1$ mL

$= (3\,101\,179 \div 1000 \div 1000)$ kL

$= 3.101\,179$ kL

≈ 3.1 kL

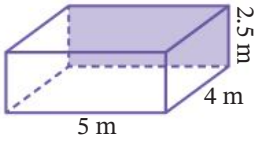
EXERCISE 10.08 ANSWERS ON P. 594

Volumes of prisms and cylinders U F PS R C

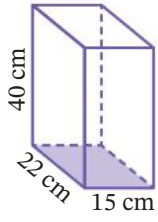
EXAMPLE 16

1 Find the volume of each rectangular prism using the formula $V = lwh$.

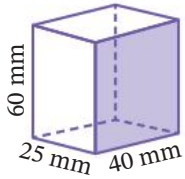
a



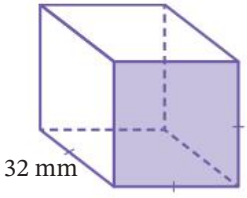
b



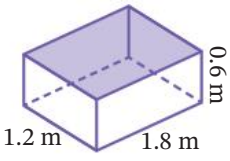
c



d



e



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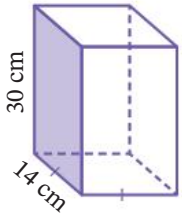

EXAMPLE 17

2 The volume of a cube is 125 000 cm^3. Convert this to m^3. Select the correct answer **A**, **B**, **C** or **D**.

A 1250 m^3 **B** 12.5 m^3 **C** 1.25 m^3 **D** 0.125 m^3

3 The volume of a small jewellery box is 160 000 mm^3. Which expression gives its volume in cm^3? Select the correct answer **A**, **B**, **C** or **D**.

A $160\,000 \div 10^3$ **B** $160\,000 \div 100$

C $160\,000 \div 100^2$ **D** $160\,000 \div 10^2$

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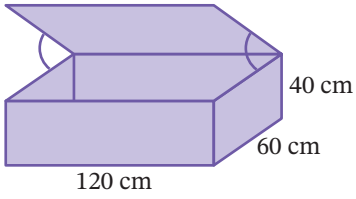
4 Find the volume of this toolbox in cubic metres by: PS R C

a calculating the volume in cm^3, then converting to m^3

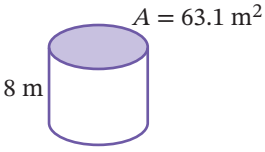
b converting each length to metres first, then calculating the volume.

Which method is easier?

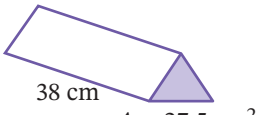


5 Find the volume of each solid.

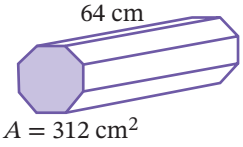
a



b

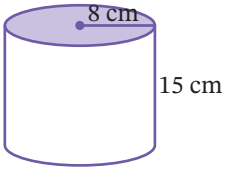


c

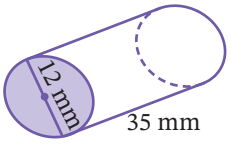


6 Find, correct to nearest whole unit, the volume of each cylinder.

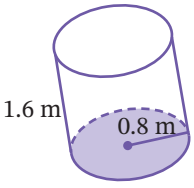
a



b



c



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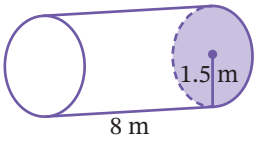

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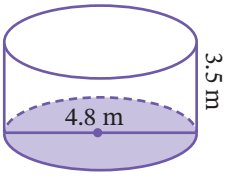

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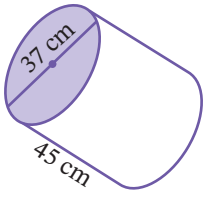
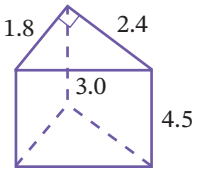
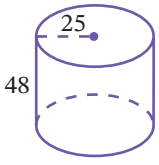


7 Calculate, correct to one decimal place, the volume of each solid. All lengths are in metres.

a



b



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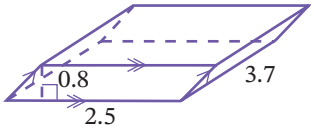
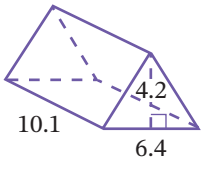


d



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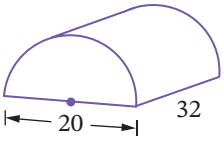

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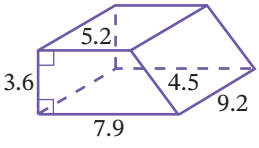

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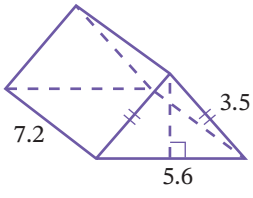
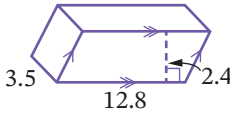


h



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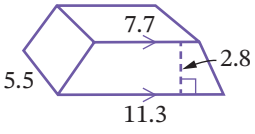


10.08

EXAMPLE 18

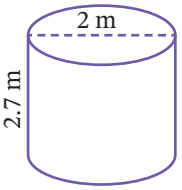
Foundation Standard Complex

8 What is the capacity of this cylinder in litres?

Select the closest answer **A**, **B**, **C** or **D**.

A 33 900 L **B** 8482 L

C 6280 L **D** 16 964 L



9 A triangular prism has base length 15 cm, height 8 cm and volume 630 cm^3. What is its length? R

10 A fish tank that is 55 cm long, 24 cm wide and 22 cm high is filled to 4 cm below the top. Calculate the amount of water in the tank in litres.

11 A wedding cake with 3 tiers rests on a table. Each tier is 6 cm high. The layers have radii of 20 cm, 15 cm and 10 cm respectively. Find the total volume of the cake, correct to the nearest cm^3.

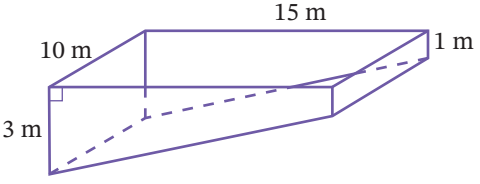
Shutterstock.com/John Wollwerth

12 Calculate the volume of the skip bin, correct to the nearest m^3.

Shutterstock.com/Stephen Rees

13 This swimming pool is 15 m long and 10 m wide. The depth of the water ranges from 1 m to 3 m.

Calculate the capacity of this pool in kilolitres.



14 Robyn and Anthony are planning to make a raised vegetable garden. If the dimensions of the garden bed are 7.5 m × 9 m × 60 cm, calculate how much soil they will need to buy.

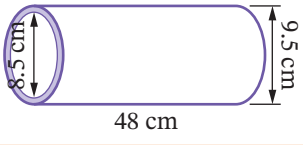
Select the correct answer **A**, **B**, **C** or **D**.

A 405 m^3 **B** 40.5 m^3

C 4050 cm^3 **D** 4050 m^3

Shutterstock.com/Alison Hancock

15 This metal pipe has an inner diameter of 8.5 cm and an outer diameter of 9.5 cm. Calculate, correct to 2 decimal places, the volume of metal needed to make the pipe. PS R



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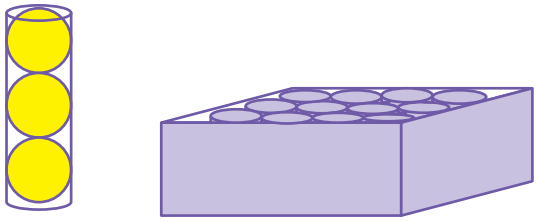
16 A cube has a volume of 2150 mm^3. What is its side length, correct to the nearest millimetre? R

17 The official diameter of a tennis ball is defined as 6.54 cm to 6.86 cm. PS R C



a Find the average diameter of a tennis ball.

b The tennis balls are packed in cylinders as shown. Using the larger diameter, calculate the radius and height of one cylinder.

c **i** To find the volume of the cylinder, its radius and height are rounded *up* to one decimal place. Explain why, and write the radius and height rounded up.

ii Hence, calculate the volume of a cylinder, correct to one decimal place.

d For packaging, 12 cylinders are put in a cardboard box, as shown above.

i What are the dimensions of the box?

ii Calculate the volume of the box.

e Calculate the volume of the box not taken up by the cylinders.

f What percentage (correct to one decimal place) of the box is not taken up by the cylinders?

10.08

Investigation

Volume vs surface area

You will need: At least 20 centicubes

1 Copy the following table.

Length	Width	Height	Volume	Surface area
			8	
			8	
			⋮	

2 Build as many rectangular prisms as you can with a volume of 8 cubes, but with different dimensions. *Note*: $1 \times 1 \times 8$ is the same as $1 \times 8 \times 1$ and $8 \times 1 \times 1$.

3 Record your dimensions in the table and calculate the surface area of each prism.

4 What are the dimensions of the solid that has the smallest surface area?

5 What name do we give to this solid?

6 Repeat the experiment for a volume of:

a 12 cubes **b** 20 cubes.

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Mental skills 10B: Maths without calculators ANSWERS ON P. 594

Finding 15%, $2\frac{1}{2}$%, 25% and $12\frac{1}{2}$%

- To find 10% or $\frac{1}{10}$ of a number, divide by 10.
- To find 5% of a number, find 10% first, then halve it (since 5% is half of 10%).
- So to find 15% of a number, find 10% and 5% of the number separately, then add the answers together.

1 Study each example.

a $15\% \times 80 = (10\% \times 80) + (5\% \times 80)$
$= 8 + 4$
$= 12$

b $15\% \times \$170 = (10\% \times \$170) + (5\% \times \$170)$
$= \$17 + \$8.50$
$= \$25.50$

c $15\% \times 3600 = (10\% \times 3600) + (5\% \times 3600)$
$= 360 + 180$
$= 540$

d $15\% \times \$28 = (10\% \times \$28) + (5\% \times \$28)$
$= \$2.80 + \$1.40$
$= \$4.20$

2 Now find 15% of each amount.

a	120	**b**	\$840	**c**	260	**d**	\$202
e	\$50	**f**	72	**g**	\$180	**h**	400
i	\$1600	**j**	\$22	**k**	6000	**l**	\$350

To find $2\frac{1}{2}$% of a number, first find 5%, then halve it.

3 Study each example.

a $2\frac{1}{2}\% \times 600$
$10\% \times 600 = 60$
$5\% \times 600 = \frac{1}{2} \times 60 = 30$
$2\frac{1}{2}\% \times 600 = \frac{1}{2} \times 30 = 15$

b $2\frac{1}{2}\% \times \$820$
$10\% \times \$820 = \$82$
$5\% \times \$820 = \frac{1}{2} \times 82 = \$41$
$2\frac{1}{2}\% \times \$820 = \frac{1}{2} \times \$41 = \$20.50$

4 Now find $2\frac{1}{2}$% of each amount.

a	400	**b**	6640	**c**	\$2000	**d**	\$880
e	1500	**f**	\$232	**g**	5400	**h**	\$904

To find 25% of a number, halve the number twice as $25\% = \frac{1}{4}$.

5 Study each example.

a $25\% \times 700$
$50\% \times 700 = \frac{1}{2} \times 700 = 350$
$25\% \times 700 = \frac{1}{2} \times 350 = 175$

b $25\% \times \$86$
$50\% \times \$86 = \frac{1}{2} \times \$86 = \$43$
$\therefore 25\% \times \$86 = \frac{1}{2} \times \$43 = \$21.50$

 9780170453325

6 Now find 25% of each of each amount.

a	2000	**b**	\$80	**c**	18	**d**	\$25
e	\$324	**f**	\$140	**g**	66	**h**	298
i	\$780	**j**	\$1700	**k**	\$126	**l**	1160

To find $12\frac{1}{2}\%$ of a number, find 25% first, then halve it.
In other words, halve 3 times because $12\frac{1}{2}\% = \frac{1}{8}$.

7 Study each example.

a $12\frac{1}{2}\% \times 400$

$50\% \times 400 = \frac{1}{2} \times 400 = 200$

$25\% \times 400 = \frac{1}{2} \times 200 = 100$

$12\frac{1}{2}\% \times 400 = \frac{1}{2} \times 100 = 50$

b $12\frac{1}{2}\% \times \$144$

$50\% \times \$144 = \frac{1}{2} \times \$144 = \$72$

$25\% \times \$144 = \frac{1}{2} \times \$72 = \$36$

$12\frac{1}{2}\% \times \$144 = \frac{1}{2} \times \$36 = \$18$

8 Now find $12\frac{1}{2}\%$ of each amount.

a	1280	**b**	\$12	**c**	60	**d**	\$260
e	\$540	**f**	\$250	**g**	304	**h**	1360

10.08

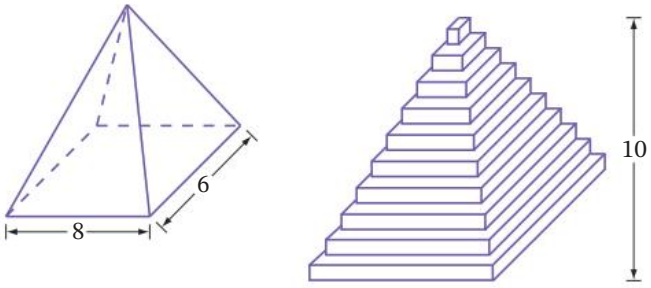
Technology

Approximating the volume of a pyramid

Measuring pyramids

Approximating the volume of a cone

We can approximate the volume of a pyramid by dividing it into of layers of prisms as shown below.



We will create a spreadsheet that approximates the volume of a rectangular pyramid with a base of length 8 units and width 6 units, and a perpendicular height of 10 units.

The volume of each layer can be easily calculated using $V = lwh$. Finding the sum of the layers will then give an approximation of the volume of the pyramid.

Let n be the number of layers. Then the height of each layer is $\frac{10}{n}$.

The length and width of each layer decreases from 8 units and 6 units by a constant amount of $\frac{8}{n}$ and $\frac{6}{n}$ respectively from layer to layer.

1 Create this spreadsheet for calculating the volue if each layer and the sum of the volumes.

	A	B	C	D	E	F
1				Number of layers		
2						
3	Height	Length	Width	Thickness of layer	Volume of layer	Sum of volumes
4	10	8	6	=$A$4/$D$2	=B4*C4*D4	=E4
5		=B4-$B4/$D$2	=C4-$C$4/$D$2			=E5+F4
6						
⋮						
13						

2 **a** To divide the volume of the pyramid into 10 layers, enter **10** in cell D2.

b Copy each formula down to row 13.

c Explain the results in cells E13 and F13.

d How accurate was your result in F13? Explain.

e Print out your spreadsheet.

3 **a** To divide the pyramid into 40 layers to calculate a better approximation, enter **40** in cell D2 and copy each formula down to row 43.

b In one or 2 sentences, compare your volume approximation in F43 with the previous approximation in F13.

4 **a** Enter each of these values in cell D2, copy the formulas down to the appropriate row and write down the approximation for the volume of the pyramid.

i 100 (copy down to row 104)

ii 200 (copy down to row 204)

iii 400 (copy down to row 404)

b Use the formula $V = \frac{1}{3}Ah$ to calculate the exact volume of the pyramid.

c Write a brief report about your results in parts **a** and **b**.

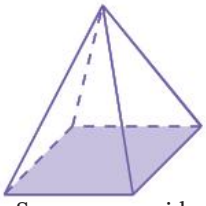
Volumes of pyramids and cones 10.09

STAGE 5.3

WS Back-to-front problems (Advanced)

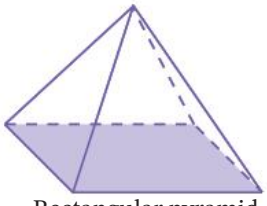
10.09

A pyramid is named by the shape of its base.



Square pyramid

Triangular pyramid



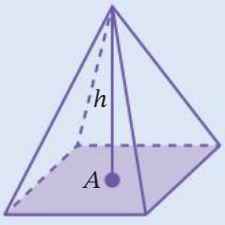
Rectangular pyramid

Volume of a pyramid

$V = \frac{1}{3}Ah$

where A = area of base

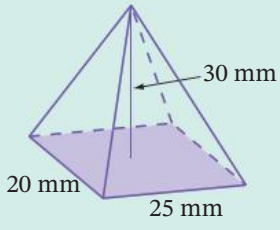
h = perpendicular height



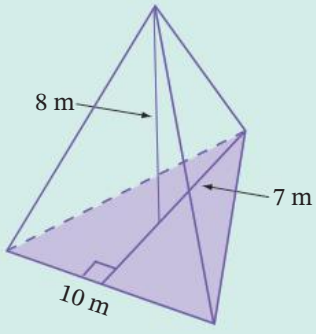
Example 19

Find the volume of each pyramid.

a



b



Solution

a $A = 25 \times 20$ — Area of rectangular base

$= 500 \text{ mm}^2$

$V = \frac{1}{3} \times 500 \times 30$ — $V = \frac{1}{3}Ah$, where height $h = 30$

$= 5000 \text{ mm}^3$

b $A = \frac{1}{2} \times 10 \times 7$ — Area of triangular base

$= 35 \text{ mm}^2$

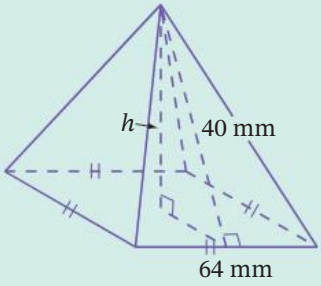
$V = \frac{1}{3} \times 35 \times 8$ — $V = \frac{1}{3}Ah$, where height $h = 8$

$= 93\frac{1}{3} \text{ m}^3$

STAGE 5.3

Example 20

Find the capacity (to the nearest millilitre) of a square pyramid with base edge 64 mm and slant height 40 mm.



Solution

First find h, the perpendicular height of the pyramid.

$h^2 = 40^2 - 32^2$ — Using Pythagoras' theorem
$= 576$
$h = \sqrt{576}$
$= 24$ mm

$A = 64^2$ — Area of square base
$= 4096$ mm^2

$V = \frac{1}{3} \times 4096 \times 24$ — $V = \frac{1}{3}Ah$, where height $h = 24$
$= 32\,768$ mm^3
$= 32.768$ cm^3 — 1 cm^3 = 1000 mm^3
Capacity $= 32.768$ mL — 1 cm^3 = 1 mL
≈ 33 mL

Example 21

The volume of a square pyramid is 100 cm^3. If its height is 12 cm, calculate the length of its base.

Solution

Let the length of the square base of the pyramid be x cm.

$V = \frac{1}{3}Ah$
$100 = \frac{1}{3} \times x^2 \times 12$
$100 = 4x^2$
$x^2 = \frac{100}{4}$
$= 25$
$x = \sqrt{25}$
$= 5$
$\therefore$ Length of base = 5 cm.

Volume of a cone

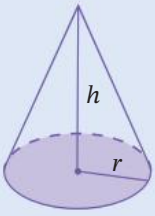
A cone is like a 'circular pyramid' so:

$$\text{Volume} = \frac{1}{3}Ah$$
$$V = \frac{1}{3} \times \pi r^2 \times h$$
$$= \frac{1}{3}\pi r^2 h$$

Volume of a cone

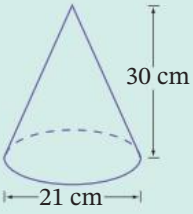
$V = \frac{1}{3}\pi r^2 h$

where r = radius of circular base and h = perpendicular height



Example 22

Calculate the volume of the cone, correct to the nearest cm^3.



Solution

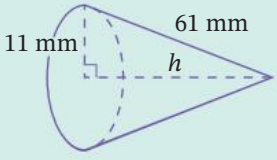
$V = \frac{1}{3} \times \pi \times 10.5^2 \times 30$ $V = \frac{1}{3}\pi r^2 h$, where $r = \frac{1}{2} \times 21 = 10.5$

$= 3463.6059...$

$\approx 3464 \text{ cm}^3$

Example 23

A cone has a slant edge of 61 mm and a base radius of 11 mm. Find its volume, correct to one decimal place.



Solution

First, find the height, h.

$h^2 = 61^2 - 11^2$ Using Pythagoras' theorem

$= 3600$

$h = \sqrt{3600}$

$= 60$

$V = \frac{1}{3} \times \pi \times 11^2 \times 60$ $V = \frac{1}{3}\pi r^2 h$

$= 7602.6542...$

$\approx 7602.7 \text{ mm}^2$

STAGE 5.3

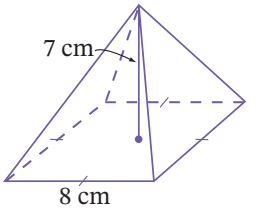
EXERCISE 10.09 ANSWERS ON P. 594

Volumes of pyramids and cones U F R

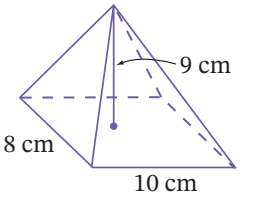
EXAMPLE 19

1 Calculate the volume of each pyramid (correct to one decimal place where necessary).

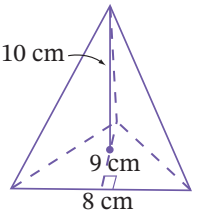
a



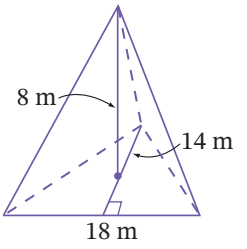
b



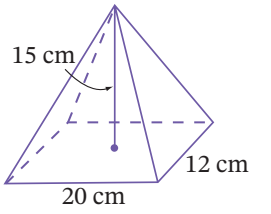
c



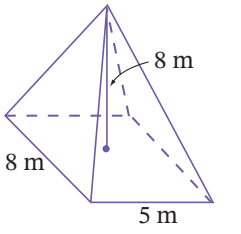
d



e



f



2 This grain hopper is in the shape of an inverted square pyramid.

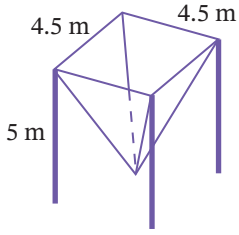
What is its capacity? Select the correct answer **A**, **B**, **C** or **D**.

A 15 m^3

B 33.75 m^3

C 50.625 m^3

D 101.25 m^3

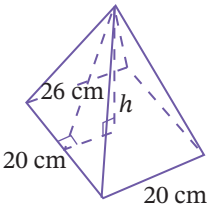


EXAMPLE 20

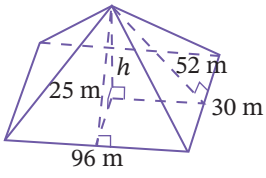
3 For each pyramid, find correct to 2 decimal places:

i its perpendicular height, h **ii** its volume **iii** its capacity.

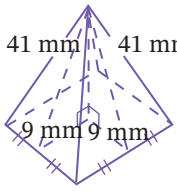
a



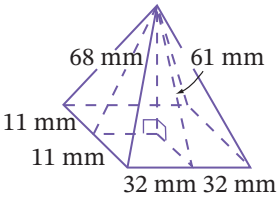
b



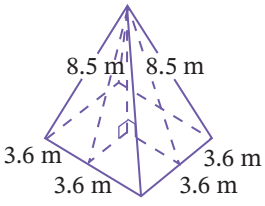
c



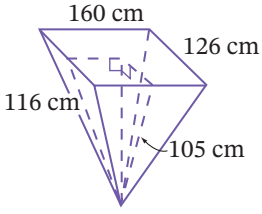
d



e



f



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4 The great pyramid of Khufu (or Cheops) in Egypt was built on a square base with side lengths approximately 230 m.

Shutterstock.com/Jeremy Red

a Find the volume in cubic metres if the original height of the pyramid was 147 m.

b There are an estimated 2.3 million stone blocks in the pyramid. Calculate the average volume of each block.

5 The area of the base of a pyramid is 40 m^2. If its volume is 360 m^3, calculate its perpendicular height. R

EXAMPLE 21

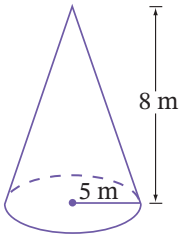
6 The volume of a square pyramid toy is 1620 mm^3. If the length of its base is 8 mm, calculate, correct to the nearest millimetre, the height of the pyramid. R

7 A square pyramid has a volume of 80 cm^3 and a height of 10 cm. Calculate, correct to one decimal place, the length of the base of the pyramid. R

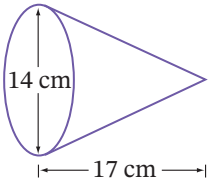
EXAMPLE 22

8 Calculate the volume of each cone, correct to one decimal place.

a



b



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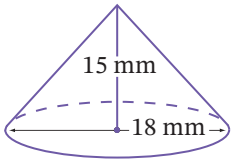

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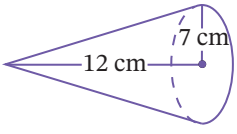

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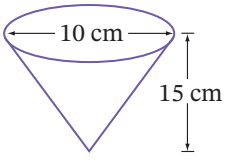

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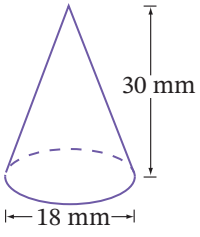
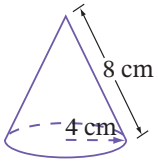


EXAMPLE 23

9 Calculate the volume of each cone, correct to one decimal place.

a



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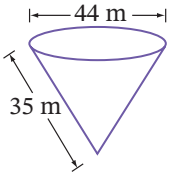

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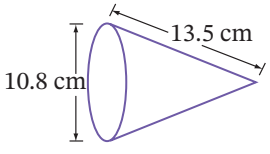

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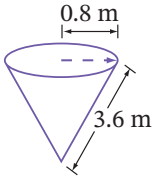

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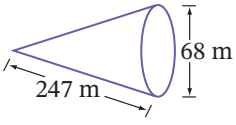

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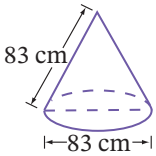

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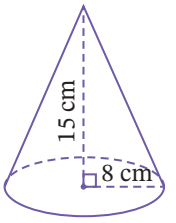
STAGE 5.3

10 Calculate the volume of each cone, leaving your answer in terms of π.

a



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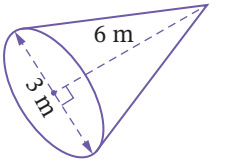

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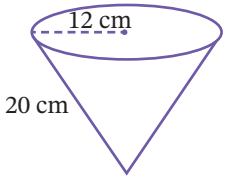


11 A cone has a volume of 1468 cm^3 and a base radius of 12 cm. Find its height, correct to one decimal place. R

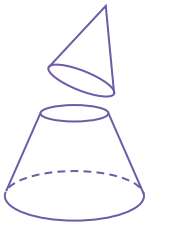
12 A cone has a volume of 820 m^3 and a perpendicular height of 10 m. Find its radius, correct to one decimal place. R

13 A cone has a volume of 150 m^3. If the height and radius of the cone are equal in length, calculate the radius of the cone. Give your answer to 2 decimal places. R

14 A cone has a base radius of 20 cm and a perpendicular height of 48 cm.

a Find the volume of the cone.

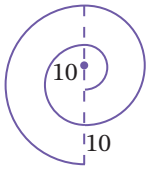
b The top of the cone is removed at half its height. What percentage of the cone remains? R



Power plus ANSWERS ON P. 595

1 A spiral is formed from 4 semicircles as shown. The diameter of the smallest semicircle is 10 cm, and the semicircles are 10 cm apart. Find the total length of the spiral:

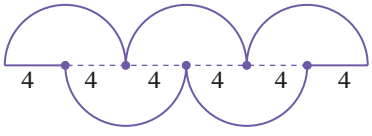
a correct to 2 decimal places **b** in terms of π.



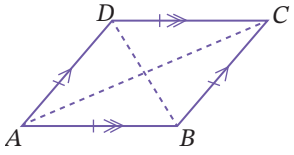
2 In this pattern, all lengths are in metres. Calculate, correct to 2 decimal places:

a its perimeter

b its area



3 A rhombus has diagonals of length 24 cm and 18 cm. Find its perimeter.



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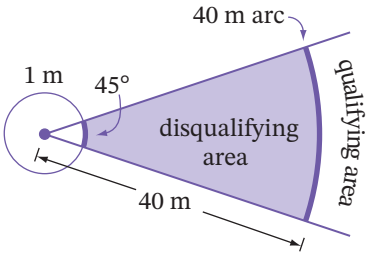
4 To qualify for the next round of the discus trials, Bronte must throw the discus beyond the 40 m arc. Find, correct to the nearest square metre, the shaded disqualifying area of the sector, given that the small circle has a radius of 1 m.

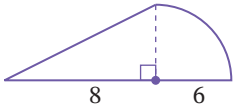


5 For each composite shape calculate, correct to one decimal place:

i its area **ii** its perimeter.

All measurements are in centimetres.

a



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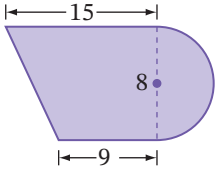

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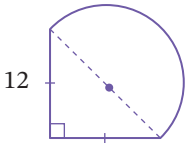
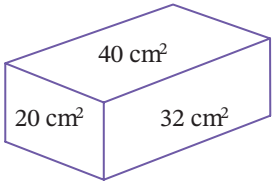
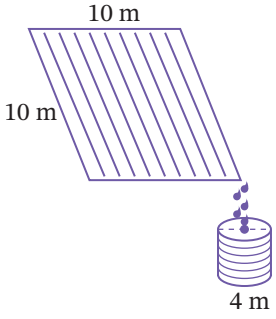


6 The 3 faces of a rectangular prism have areas as shown. Calculate the volume of the prism.



7 A cube opened at one end has an external surface area of 1125 cm^2. Find its volume.

8 A 10 m flat square roof drains into a cylindrical rainwater tank with a diameter of 4 m. If 5 mm of rain falls on the roof, by how much (to the nearest mm) does the level of water in the tank rise?



9 Calculate, correct to one decimal place, the volume of a tetrahedron with a side length of 24 cm.

Measurement

Surface area and volume crossword (Advanced)

CHAPTER 10 REVIEW

Language of maths

arc length	base	capacity	circumference
cross-section	curved surface	diagonal	diameter
giga-	kilobyte	limits of accuracy	mega-
megabyte	micro- (μ)	net	perimeter
perpendicular height	pi (π)	prefix	radius/radii
rectangular prism	sector	surface area	triangular prism

1 Draw a circle and label on it:

a a sector **b** a diameter **c** an arc length

2 In the formula $V = Ah$, explain what V, A and h stand for.

3 Which metric prefix means 'one-millionth'?

4 Write the definition of a **prism** using the word **cross-section**.

5 How do you find the **surface area** of a solid?

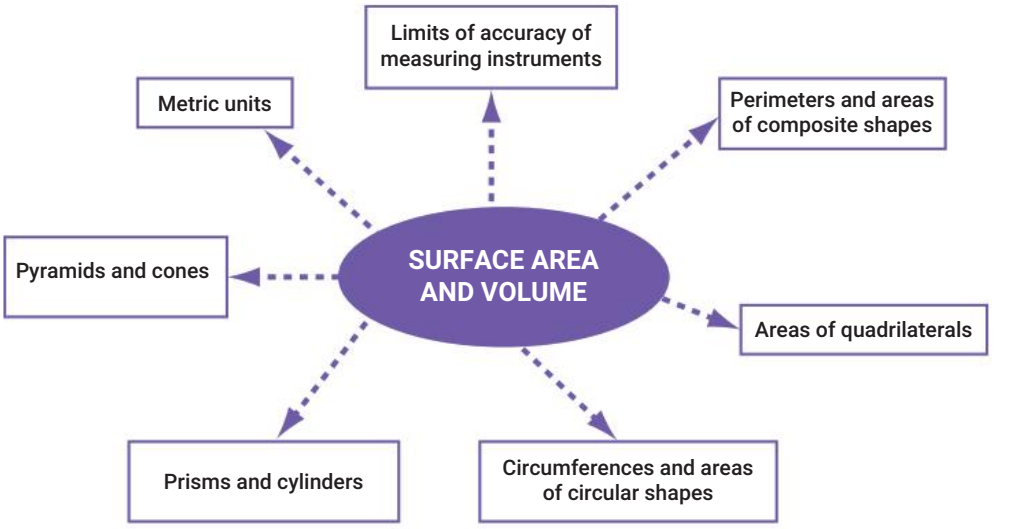
6 Name a solid that has a **curved surface**.

Mind map: Surface area and volume (Advanced)

Topic summary

- Write 10 questions (with solutions) that could be used in a test for this chapter. Include some questions that you have found difficult to answer.
- Swap your questions with another student and check their solutions against yours.
- List the sections of work in this chapter that you did not understand. Follow up this work with your friend or teacher.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



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TEST YOURSELF 10

ANSWERS ON P. 595

1 Convert: 10.01

a	8 km to m	**b**	15 min to s	**c**	0.7 m to mm
d	250 000 g to kg	**e**	8.4 GB to MB	**f**	9.5 t to kg
g	300 min to h	**h**	34 000 B to kB	**i**	125 kL to L
j	2500 cm to km	**k**	17 000 000 kg to t	**l**	8900 GB to TB
m	10 500 ms to s	**n**	9.1 millennia to years	**o**	2.6 Mm to m
p	3 ly to km	**q**	0.000 000 4 s to μs	**r**	75 000 000 μm to m

2 For each measuring instrument, state: 10.02

i the size of one unit on the scale **ii** its limits of accuracy.

a

Shutterstock.com/DG-Studio

b

iStock.com/AlonzoDesign

c

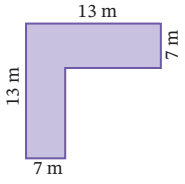
iStock.com/AndrewScherbackov

d

Shutterstock.com/Uros Jonic

3 Calculate the perimeter of each shape. 10.03

a



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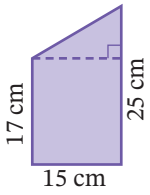

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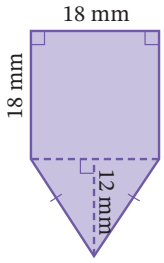

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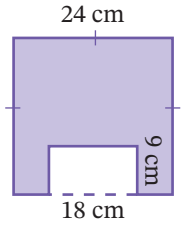
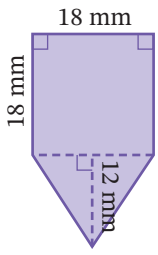


4 Calculate the area of each shape. 10.03

a



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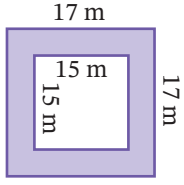

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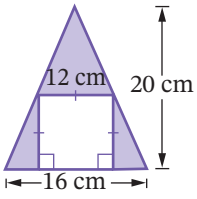

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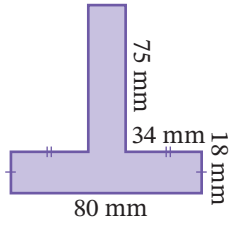

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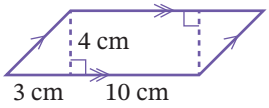
10.04

5 Calculate the area of each quadrilateral.

a



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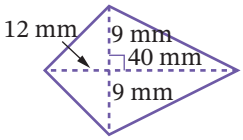

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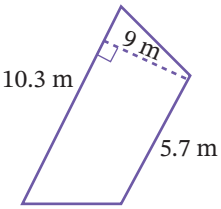

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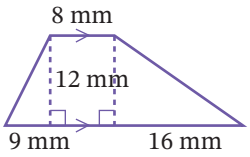

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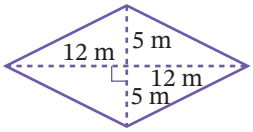

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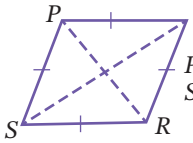


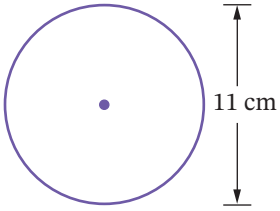
$PR = 20$ cm
$SQ = 26$ cm

10.05

6 For each shape calculate, correct to one decimal place:

i its perimeter **ii** its area.

a



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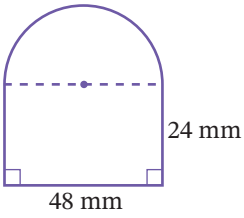

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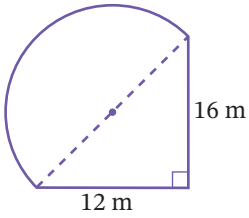

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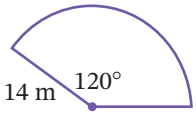

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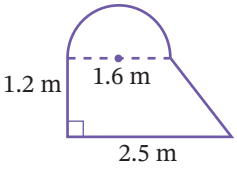

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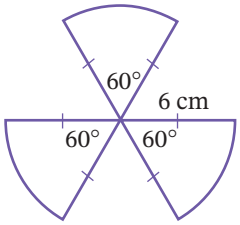
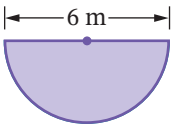


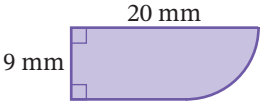
10.05

7 Calculate, correct to 2 decimal places, the area of each shape.

a



b



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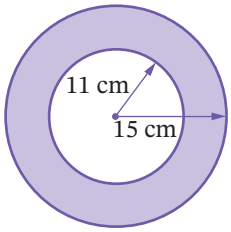
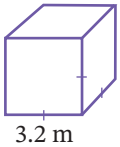


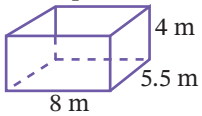
10.06

8 Calculate the surface area of each prism.

a closed cube



b open rectangular prism



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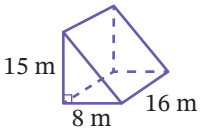

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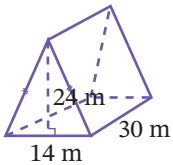

10.08

9 Calculate the volume of each prism above.

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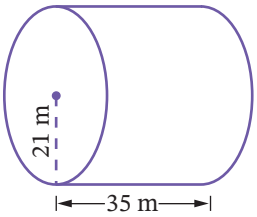
10 Calculate, correct to 2 decimal places, the surface area of each cylinder. 10.07

a



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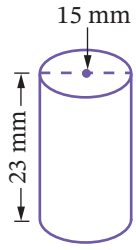

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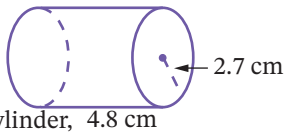
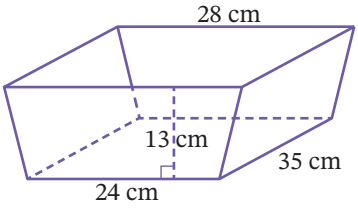


Cylinder, open at one end, 4.8 cm

11 Calculate, correct to 2 decimal places, the volume of each cylinder above. 10.08

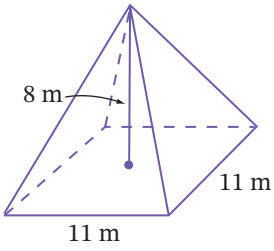
12 A rectangular fish tank that is 75 cm long by 55 cm wide by 35 cm deep is filled to 5 cm from the top. How many litres of water is in the tank? 10.08

13 Calculate the volume of this lunchbox. 10.08



14 Find the volume of each pyramid. STAGE 5.3 10.09

a



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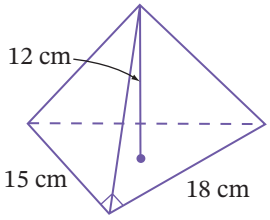

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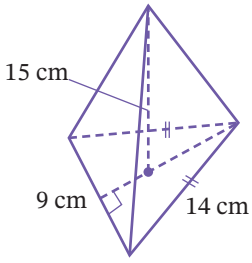
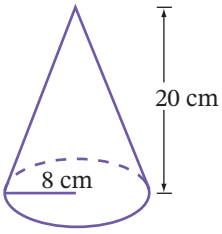


15 Find, correct to one decimal place, the volume of each cone. 10.09

a



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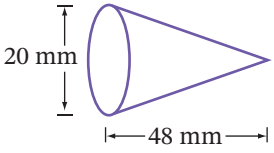

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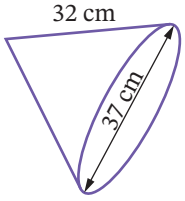

16 A cone has a volume of 115 cm^3 and a base radius of 3.2 cm. Find, correct to one decimal place, the height of the cone. 10.09

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PRACTICE SET 3 ANSWERS ON P. 595

10.01

1 Copy and complete each conversion.

a 750 min = _____ h

b 1350 mL = _____ L

c 512 MB = _____ GB

d 3.005 kg = _____ g

e 2.7 km = _____ m

f 0.25 MB = _____ kB

8.03

2 Mason is paid $3.94 for each toy he assembles.

a How much does he earn for assembling 140 toys?

b How many toys did Mason assemble if he earned $1260.80?

8.04

3 Katrina's annual salary is $85 000 and she earns another $4800 from investments. Katrina's work-related tax deductions amount to $2380. Calculate Katrina's:

a taxable income

b income tax payable using the table below.

Taxable income	Tax on this income
0–$18 200	Nil
$18 201–$37 000	19% for each $1 over $18 200
$37 001–$90 000	$3572 plus 32.5% for each $1 over $37 000
$90 001–$180 000	$20 797 plus 37% for each $1 over $90 000
$180 01 and over	$54 097 plus 45% for each $1 over $180 000

9.02

4 The shoe sizes of 30 Year 9 students are:

6	10	7	6	8	6	7	7	6	5
5	6	5	8	10	11	9	8	9	5
6	7	9	8	7	6	5	6	7	8

a Arrange the data in a frequency table.

b Draw a frequency histogram and polygon of this data.

c What percentage (correct to one decimal place) of students had shoe sizes less than 8?

d Find the mode and the range.

e Find the median.

f Calculate the mean, correct to one decimal place.

8.01

5 For a salary of $89 760 p.a., calculate, correct to the nearest cent, the:

a weekly income **b** fortnightly income **c** monthly income.

8.03

6 Bhavin works 38 hours a week and is paid $43.50 per hour. Calculate:

a Bhavin's weekly income

b the holiday loading of $17\frac{1}{2}$ % on 4 weeks' pay

c Bhavin's fortnightly pay when he receives his holiday loading.

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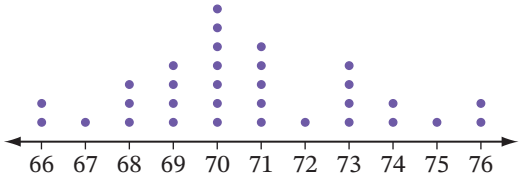
7 For each set of data, find: 9.01

i the mode **ii** the mean (correct to one decimal place)

iii the median **iv** the range

a 47 46 50 51 48 48 52 46 49

b



8 Classify whether each type of data is categorical or numerical. If numerical, then classify whether it is discrete or continuous. 9.05

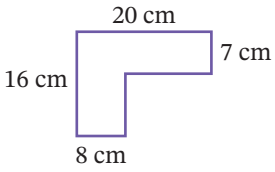
a the age Year 9 students turn this year **b** colour of cars in a car park

c travelling time between home and work **d** amount of rainfall

9 For each composite shape, find: 10.03

i its perimeter **ii** its area

a



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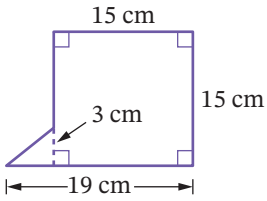


10 Nadia's gross pay for a fortnight is $3038. She pays $724.50 in tax and her other deductions amount to $285. 8.05

a Calculate Nadia's net pay.

b Express her net pay as a percentage (correct to one decimal place) of her gross pay.

11 For the data in this stem-and-leaf plot, which statement is true? Select the correct answer **A**, **B**, **C** or **D**. 9.02

Stem	Leaf
6	0 1 5
7	3 4 4 8 9
8	0 1 2 2 5 6 6 6
9	3 4 7
10	6

A The mean is 82.

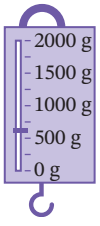
B The mode is 6.

C The range is 46.

D The median is 81.

12 Sefa is paid $27.40 per hour. Calculate his earnings for working 38 hours at normal time, 8 hours at time-and-a-half and 4 hours at double time. 8.02

13 What are the limits of accuracy for these measuring scales? 10.02



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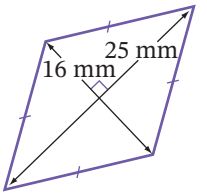
10.04

14 Find the area of each quadrilateral.

a



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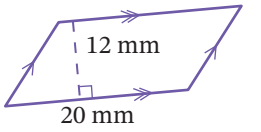
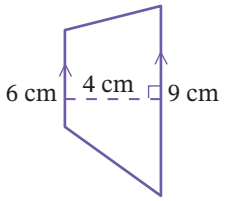


c



8.03

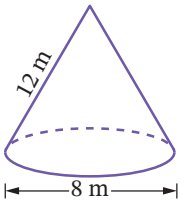
15 Thiago earns a commission of 2.75% on every property he sells. Calculate his commission for selling an apartment for $635 200.

STAGE 5.3

10.09

16 Calculate the volume of each solid (correct to one decimal place where necessary).

a



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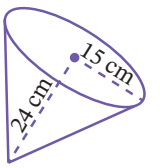

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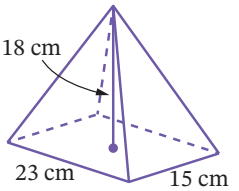
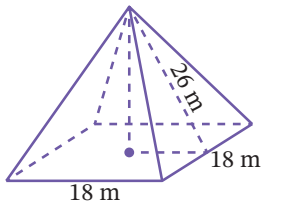


d



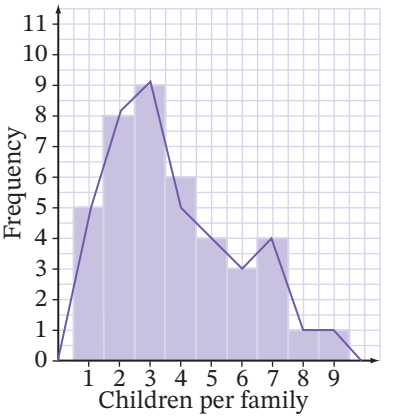
9.03

17 A sample of students was surveyed on the number of children in their families, and the results were graphed on a frequency histogram and polygon.

a Describe the shape of the distribution.

b Where does clustering occur?

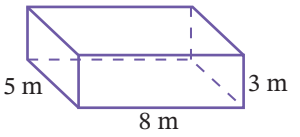
c Find the mode and range.



10.06

18 Find the surface area of each prism.

a



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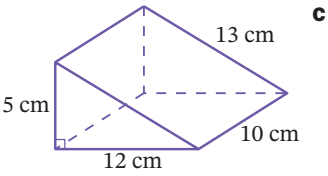

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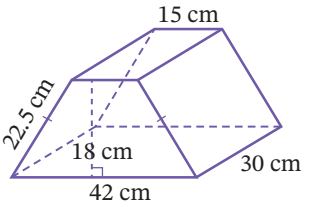

10.08

19 Find the volume of each prism above.

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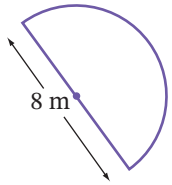
20 For each circular shape, calculate, correct to one decimal place: 10.05

i its perimeter **ii** its area.

a



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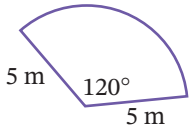

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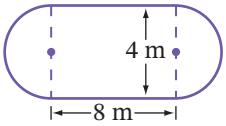


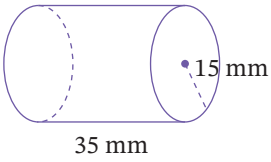
21 In a survey, a sample of people at a park were asked 'Are you employed?'. 9.06

a Why might this sample of people be biased?

b What is wrong with the survey question?

c Suggest a better-worded question.

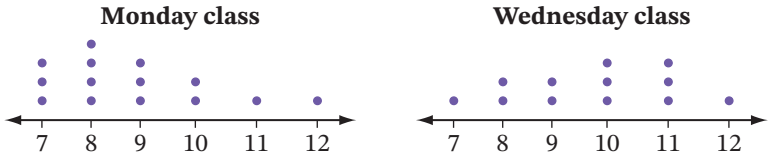
22 Find, correct to 3 significant figures, the volume of this cylinder. 10.08



23 Find, correct to 3 significant figures, the surface area of the cylinder above it is open at one end. 10.07

24 Brooke is on a salary and earns $7850 per month. Calculate Brooke's fortnightly income. 8.01

25 These dot plots show the ages of students in 2 Taekwondo classes. 9.04



a Describe the shape of the age distribution for each class.

b Find the mean for each class.

c Find the median for each class.

d Which class had more consistent ages?

e Which class had more older students?

26 A cube has a surface area of 614 cm^2. Find its length, correct to one decimal place. 10.06

STAGE 5.3

27 The volume of a square pyramid is 2000 cm^3. Calculate, correct to one decimal place, the length of the base of the pyramid if the perpendicular height is 25 cm. 10.06

28 Celine earns $1490.40 by working 38 hours at normal rates, 4 hours at time-and-a-half and 1 hour at double time. What is Celine's normal rate of pay? 8.02

29 Rabibah's mean mark for 4 science tests was 81%. After her 5th test, her mean test mark was 84%. What mark did Rabibah score in the 5th test? 9.01

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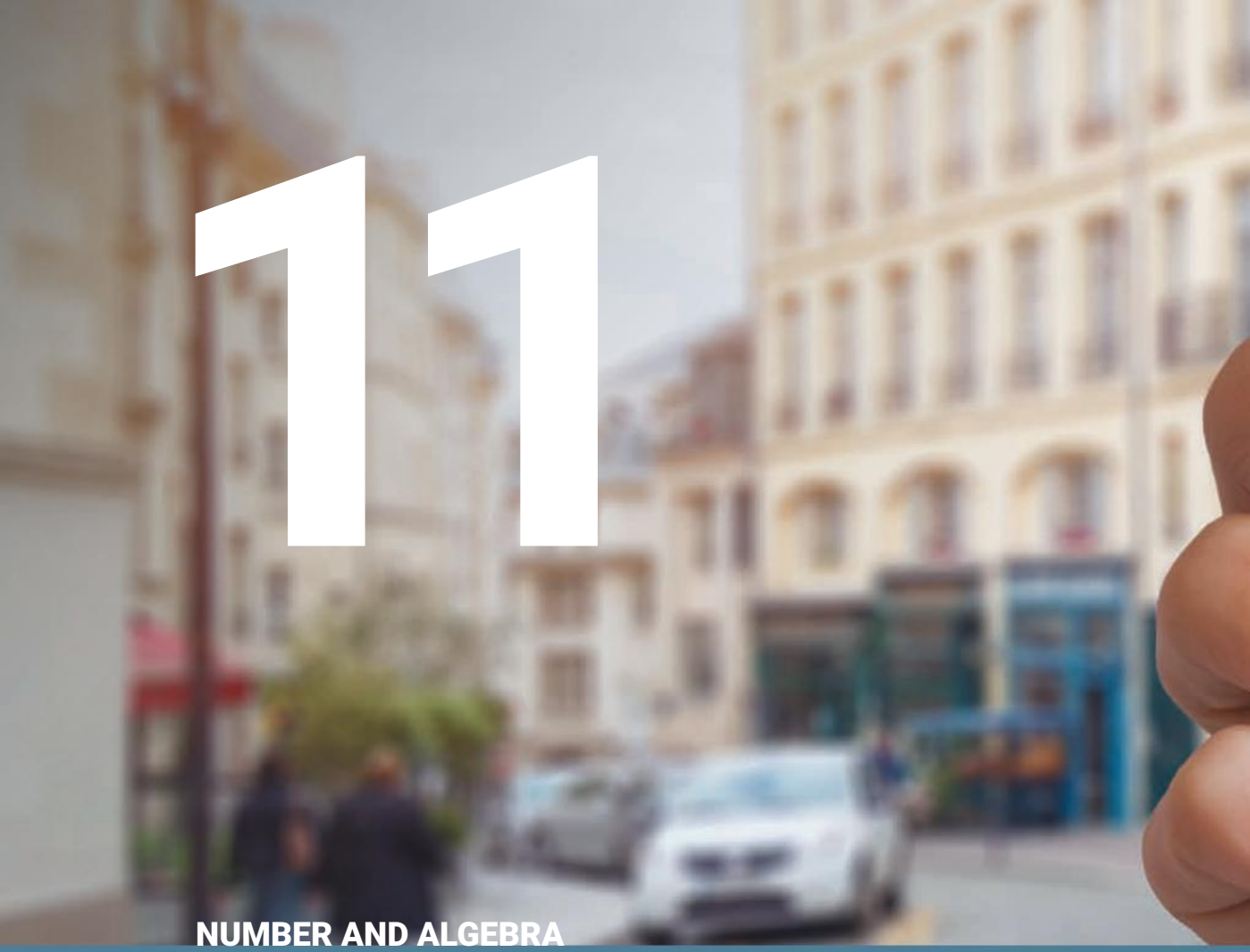
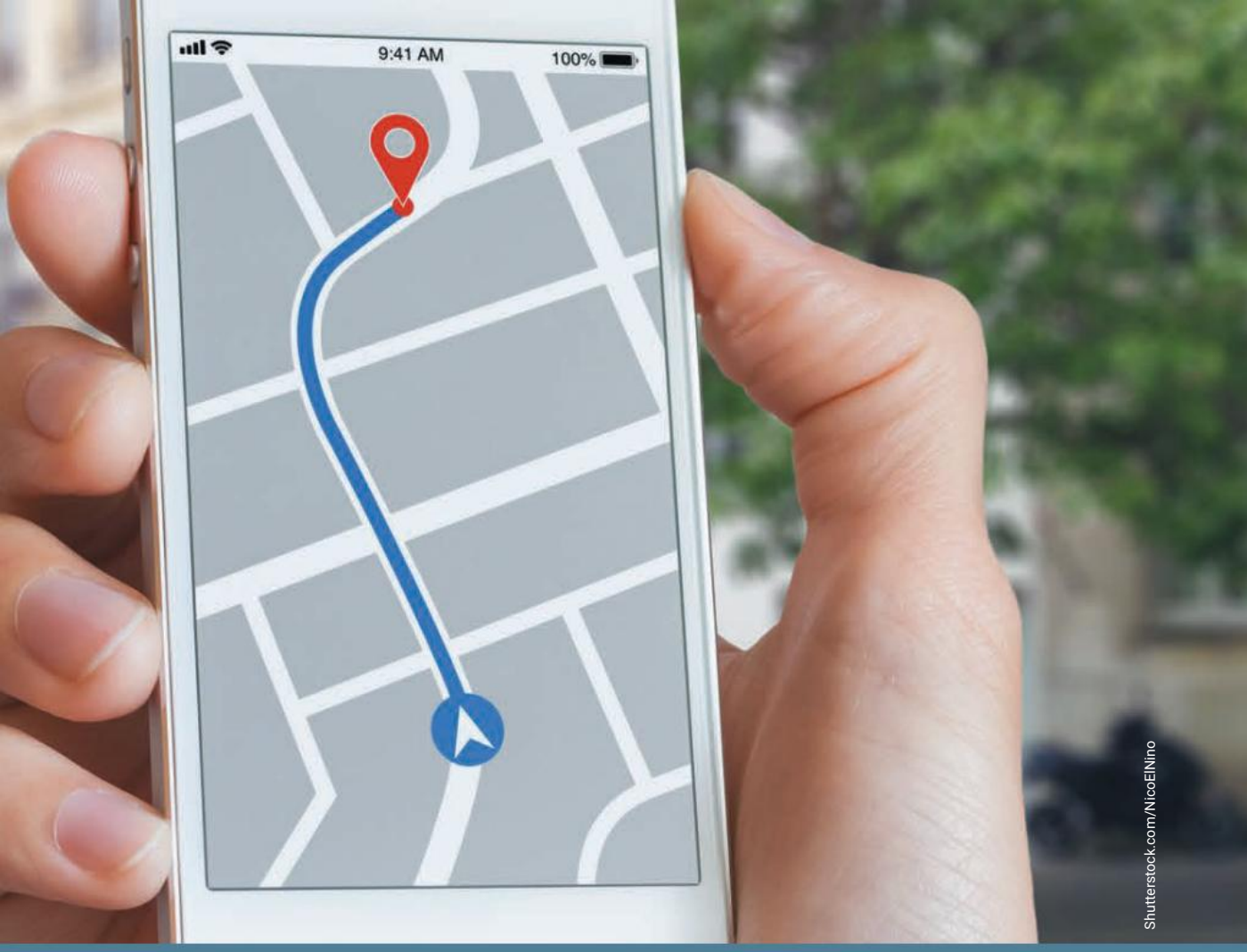


11

NUMBER AND ALGEBRA

COORDINATE GEOMETRY AND GRAPHS

The use of a coordinate system allows us to monitor the location of objects on Earth or in space. Whether reading street maps, using global positioning systems or managing road traffic flow, coordinates provide an accurate method for indicating position. Coordinates have important applications in many areas including astronomy, architecture, engineering and art design.



Chapter outline

	Working mathematically				
11.01 The length of an interval	U	F		R	C
11.02 The midpoint of an interval	U	F		R	C
11.03 The gradient of a line	U	F		R	C
11.04 Graphing linear equations	U	F		R	C
11.05 The gradient-intercept equation $y = mx + c$	U	F			
11.06 Finding the equation of a line	U	F			
11.07 Solving linear equations graphically	U	F			
11.08 Direct proportion	U	F	PS	R	
11.09 Graphing quadratic equations	U	F		R	C
11.10 Graphing circles	U	F			

***STAGE 5.3:** The distance, midpoint and gradient formulas in 11.01, 11.02 and 11.03 respectively, graphing lines by finding intercepts in 11.04.

Wordbank

direct proportion A relationship between 2 variables in which one variable is a constant multiple of the other, for example, if $y = 8.5x$, then y is directly proportional to x

gradient The steepness of a line or interval, measured by the fraction $\frac{\text{rise}}{\text{run}}$

interval A part of a line with a definite length

linear equation An equation whose graph is a straight line

midpoint The point in the middle of an interval or halfway between 2 given points

parabola A U-shaped curve that is the graph of a quadratic equation

quadratic equation An equation involving a variable squared (power of 2), such as $y = 3x^2 - 6$, whose graph is a curve called a parabola

***y*-intercept** The y-value at which a graph cuts the y-axis

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

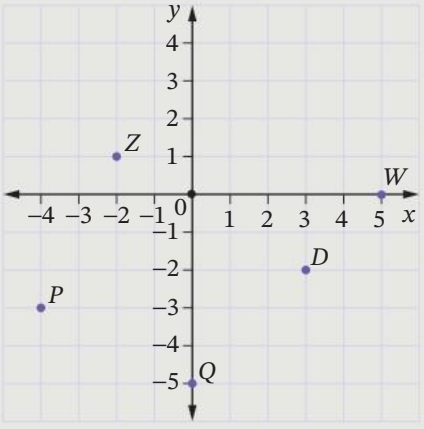
- find the distance between 2 points on a number plane
- find the midpoint and gradient of an interval on a number plane
- (STAGE 5.3) use the distance, midpoint and gradient formulas for an interval on the number plane
- graph a line on a number plane given the coordinates of 2 points on the line
- graph and solve problems involving direct proportion
- test whether a point lies on a line
- use the gradient–intercept equation of a straight line $y = mx + c$
- (STAGE 5.3) graph a linear equation by first finding its x- and y-intercepts
- find the equation of a line from its graph
- solve linear equations graphically
- graph quadratic equations of the form $y = ax^2$ and $y = ax^2 + c$
- graph circles with equations of the form $x^2 + y^2 = r^2$

SkillCheck ANSWERS ON P. 596

WS Tables of values

The number plane

1 Write the coordinates of points Z, D, Q, W and P, shown on this number plane.



2 Complete each table of values using the given equation.

a $y = -2x + 1$

x	−2	−1	0	1	2
y					

b $y = \frac{x}{2} + 1$

x	−2	−1	0	1	2
y					

3 Given the equation $y = 5 - 2x$, find:

a y when $x = 0$ **b** y when $x = 3$

c x when $y = 0$ **d** x when $y = 7$

4 Calculate the average of:

a 1 and 3 **b** −2 and 6 **c** 5 and 12

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The length of an interval

11.01

A line is usually drawn with arrows at both ends to show that it runs infinitely in both directions. However, an **interval** or line segment is part of a line, with endpoints on both sides and a definite length. The interval joining points P and Q is called the interval PQ. We can calculate the length of an interval (or the distance between 2 points) on a **number plane** if we know the coordinates of its endpoints.

WS Length of an interval

11.01

Length of a line segment

Example 1

Find the distance between the points:

a $P(-4, -2)$ and $Q(2, 6)$

b $A(-2, 5)$ and $B(2, -2)$

Solution

a With P and Q plotted on a number plane, construct a right-angled triangle PTQ such that PQ is the hypotenuse.

$PT = 6$ units, $TQ = 8$ units.

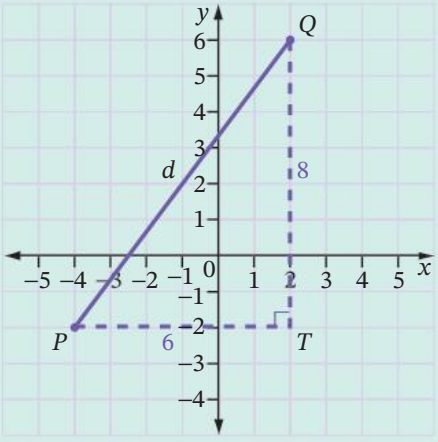
Let the length of PQ be d, so by Pythagoras' theorem:

$d^2 = 6^2 + 8^2$

$= 100$

$d = \sqrt{100} = 10$

The distance between the points P and Q is 10 units.



b By plotting A and B and constructing $\triangle ABC$, we have:

$AB^2 = 7^2 + 4^2$

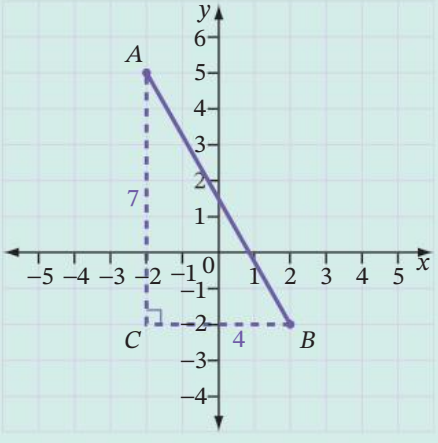
$= 65$

$AB = \sqrt{65}$

$= 8.0622...$

≈ 8.06 units

We can write the distance AB in **surd** form, $\sqrt{65}$ units as an exact answer, or we can approximate (round) it as a decimal, 8.06 units.



Example 2

a Plot the points $A(0, 6)$, $B(5, 6)$, $C(5, 2)$ and $D(-4, 2)$ on a number plane and join them to make the quadrilateral $ABCD$.

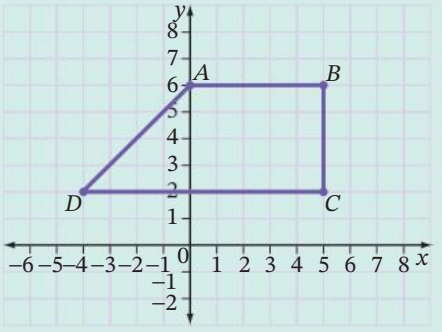
b What type of quadrilateral is $ABCD$?

c Find the exact length of AD.

d Hence find the perimeter of $ABCD$, correct to 2 decimal places.

Solution

a



b $ABCD$ is a trapezium.

c $AD^2 = 4^2 + 4^2$ By Pythagoras' theorem.

$= 32$

$AD = \sqrt{32}$ units In exact surd form.

d By counting grid squares, $AB = 5$, $BC = 4$, $CD = 9$.

Perimeter of $ABCD = 5 + 4 + 9 + \sqrt{32}$

$= 23.656...$

≈ 23.66 units

STAGE 5.3

The distance formula

The method for finding the length of an interval can be summarised by a formula.

The is called the **distance formula** because it is used to calculate the distance (d) between any 2 points $P(x_1, y_1)$ and $Q(x_2, y_2)$, in other words, the length of the interval PQ.

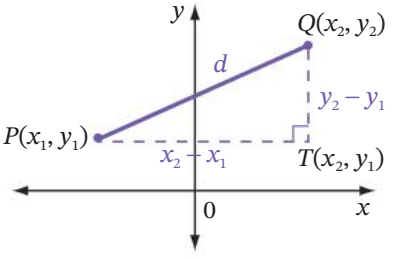
Length of $PT = x_2 - x_1$

Length of $TQ = y_2 - y_1$

By Pythagoras' theorem:

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$\therefore d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



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The distance formula

STAGE 5.2

The distance (d) between the 2 points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Group the x-coordinates together, group the y-coordinates together.

11.01

The length of an interval

Distance, midpoint and gradient formulas

Example 3

Find the length of the interval joining the points:

a $A(2, 6)$ and $B(4, 7)$, correct to one decimal place

b $P(-3, 5)$ and $Q(9, -11)$

Solution

a For $A(2, 6)$ and $B(4, 7)$:

$x_1 = 2, y_1 = 6, x_2 = 4, y_2 = 7$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4-2)^2 + (7-6)^2}$$
$$= \sqrt{2^2 + 1^2}$$
$$= \sqrt{5}$$
$$= 2.2360...$$
$$\approx 2.2 \text{ units}$$

Group the x-coordinates together, group the y-coordinates together.

b For $P(-3, 5)$ and $Q(9, -11)$:

$x_1 = -3, y_1 = 5, x_2 = 9, y_2 = -11$

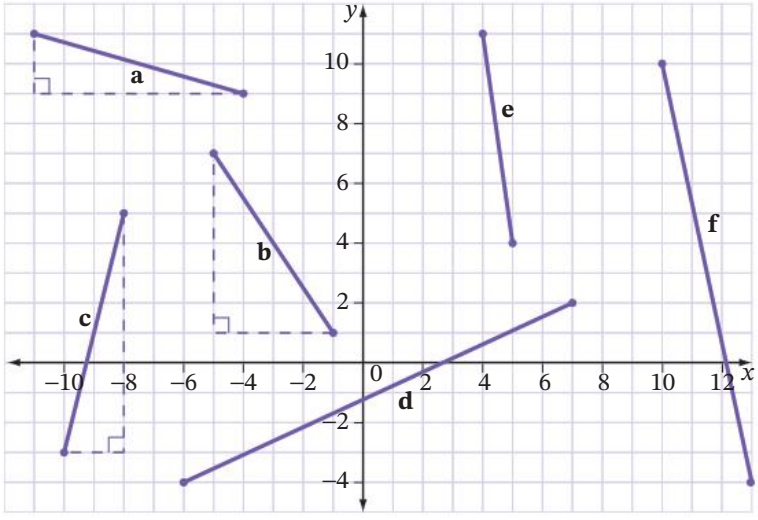
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{[9-(-3)]^2 + (-11-5)^2}$$
$$= \sqrt{12^2 + (-16)^2}$$
$$= \sqrt{400}$$
$$= 20 \text{ units}$$

EXERCISE 11.01 ANSWERS ON P. 596

The length of an interval U F R C

EXAMPLE 1

1 Calculate the length of each interval shown on this number plane. Express your answers correct to one decimal place, where necessary.



Foundation | Standard | Complex

STAGE 5.3

EXAMPLE 3

EXAMPLE 2

2 Plot each pair of points on a number plane and construct a right-angled triangle to find the distance between the 2 points. Express your answer:

i as a surd **ii** correct to one decimal place.

a $A(7, 6)$ and $B(1, 9)$ **b** $P(0, -5)$ and $Q(6, 4)$

c $G(9, -2)$ and $H(1, 6)$ **d** $W(-5, -4)$ and $X(-3, 4)$

3 Use the distance formula to check your answers to question **2**.

4 What is the exact distance between the points $Y(6, -3)$ and $Z(-2, 4)$? Select **A**, **B**, **C** or **D**.

A $\sqrt{113}$ units **B** $\sqrt{65}$ units **C** $\sqrt{57}$ units **D** $\sqrt{15}$ units

5 A triangle is formed by joining the points $A(5, 2)$, $B(-2, 3)$ and $C(1, 7)$. Find, correct to 3 decimal places:

a the length AB **b** the length BC

c the length AC **d** the perimeter of $\triangle ABC$

6 The vertices of triangle RST are $R(-1, -1)$, $S(1, 3)$ and $T(3, 1)$.

a Find the exact length of each side of the triangle.

b What type of triangle is $\triangle ABC$? R C

7 A quadrilateral is formed by joining the points $M(-3, 5)$, $N(5, 5)$, $P(8, 1)$ and $T(-3, 1)$.

a What type of quadrilateral is $MNPT$? R C

b Find the perimeter of $MNPT$.

c Find the area of $MNPT$.

8 The vertices of a quadrilateral are $H(1, 6)$, $I(7, 2)$, $J(3, -4)$ and $K(-3, 0)$.

a What type of quadrilateral is $HIJK$? R C

b Find the exact length of each side of the quadrilateral.

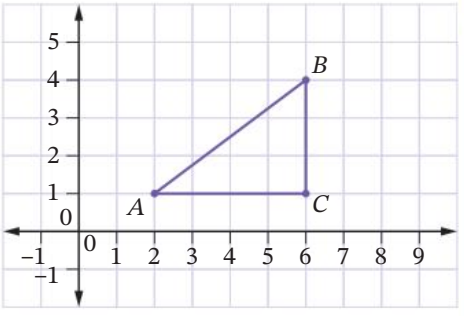
c Find its perimeter, correct to one decimal place.

d Calculate its area.

Technology

The length of an interval

In this activity, you will use dynamic geometry software to apply Pythagoras' theorem to find the length of an interval.



1 Insert a grid on the number plane, with x-values: 0 to 14 and y-values: 0 to 8.

2 Plot the points $A(2, 1)$, $B(6, 4)$ and $C(6, 1)$ on the grid. Join AB, BC and AC.

3 Find the lengths of AC and BC.

Foundation Standard Complex

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4 Calculate $(AC)^2 + (BC)^2$ and $\sqrt{(AC)^2 + (BC)^2}$.

5 Now find the length of AB.

6 Compare answer **5** with answer **4**. What do you notice?

7 Move the number plane until the axes are centred in the middle of the screen. [Approx: x: –10 to 10 and y: –10 to 10]

8 Plot each pair of points and calculate the distance between them by repeating all the steps above.

a $X(-4, 2)$ and $Y(4, -4)$ **b** $P(-7, 5)$ and $Q(-2, -7)$

Did you know?

René Descartes

Alamy Stock Photo/IanDagnall Computing

The number plane is also called the Cartesian plane, named after its inventor, the French philosopher and mathematician René Descartes (1596–1650). In an age when Shakespeare and Galileo had made contributions to drama and science respectively, Descartes showed that many relationships and formulas could be represented by simple graphs. In 1637, he made a great contribution to mathematics when he published a book linking algebra and geometry for the first time.

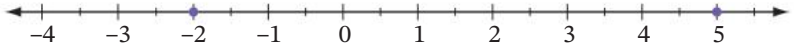
As a philosopher, the big question that Descartes spent many years thinking about was whether he, or the world, really existed. He finally concluded that he did exist, using a brilliant argument summarised by his famous quote.

Complete his famous quote: "I _____, therefore _____"

Investigation

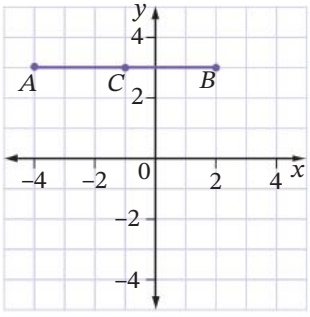
The midpoint of an interval

1 a What number is halfway between –2 and 5 on this number line?



b What is the average of –2 and 5?

2 a What are the coordinates of A and B on this number plane?



b What is the average of the x-values of A and B?

c What is the average of the y-values of A and B?

d What are the coordinates of C, the midpoint of AB?

e What do you notice about your answers for parts **b, c** and **d**?

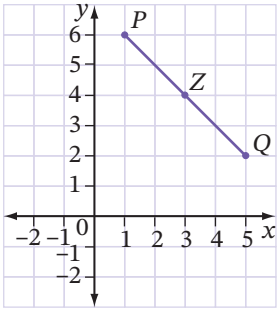
3 **a** What are the coordinates of P and Q?

b What is the average of the x-values of P and Q?

c What is the average of the y-values of P and Q?

d What are the coordinates of Z, the midpoint of PQ?

e How do the coordinates of Z relate to your answers for parts **b** and **c**?



4 Describe how to find the midpoint of any interval on a number plane given the coordinates of its endpoints.

Technology

The midpoint of an interval

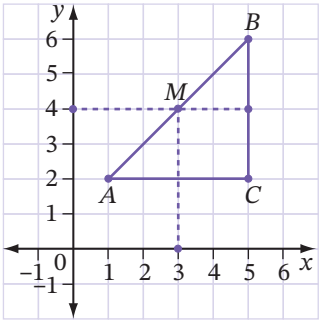
1 Use dynamic geometry software to create a number plane with x-values: 0 to 14 and y-values: 0 to 8.

2 Plot the points $A(1, 2)$, $B(5, 6)$ and $C(5, 2)$ and join AB, BC and AC.

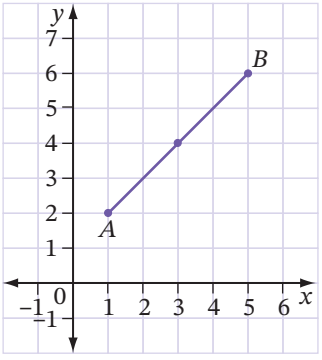
3 What is the average x-value on AC?

4 What is the average y-value on BC?

5 Label the midpoint of AB, M (as shown below). What are its coordinates?



6 Redraw interval AB. Select the midpoint and label it M. Does it have the same coordinates as you found in step **5**?



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The midpoint of an interval

11.02

The **midpoint of an interval** is the point in the middle of an interval or halfway between 2 given points.

Finding midpoints

11.02

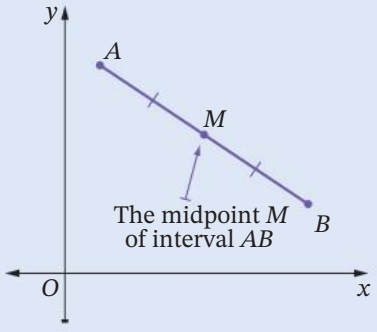
Finding midpoints

The midpoint of an interval

To find the midpoint of an interval joining 2 points A and B:

- calculate the average of the x-coordinates of A and B
- calculate the average of the y-coordinates of A and B

These 2 values give the coordinates of the midpoint.



Example 4

Find the midpoint of the interval joining:

a $A(5, -2)$ and $B(1, 6)$

b $P(-2, 7)$ and $Q(4, 0)$

Solution

a The x-coordinates are 5 and 1.

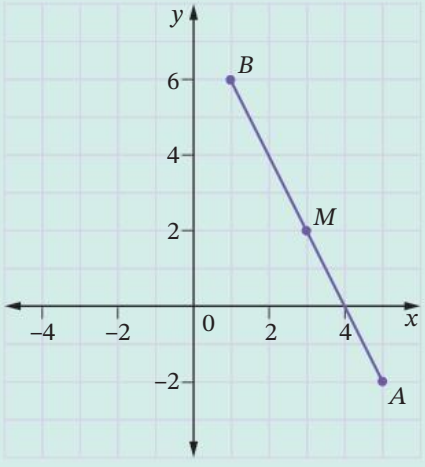
The average of 5 and $1 = \frac{5+1}{2} = 3$.

The y-coordinates are -2 and 6.

The average of -2 and $6 = \frac{-2+6}{2} = 2$

$\therefore$ The midpoint is the point $(3, 2)$.

The diagram shows that this is true.



b The x-coordinates are -2 and 4.

The average of -2 and $4 = \frac{-2+4}{2} = 1$

The y-coordinates are 7 and 0.

The average of 7 and $0 = \frac{7+0}{2} = 3\frac{1}{2}$

$\therefore$ The midpoint is the point $\left(1, 3\frac{1}{2}\right)$.

STAGE 5.3

The midpoint formula

The method for finding the midpoint of an interval can be summarised by a formula.

Distance, midpoint and gradient formulas

Midpoint and distance between 2 points

Midpoint and distance between 2 points

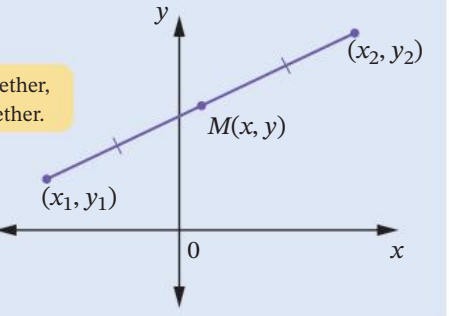
The midpoint formula

The midpoint, M, of the interval joining 2 points (x_1, y_1) and (x_2, y_2) has coordinates:

$$M(x, y) \equiv \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Group the x-coordinates together, group the y-coordinates together.

We've used '$\equiv$' ('is identical to') rather than '$=$' because we are referring to points (x, y), not numbers.



Example 5

Use the midpoint formula to find the midpoint of the interval joining $P(12, -8)$ and $Q(7, -2)$.

Solution

For $P(12, -8)$ and $Q(7, -2)$:

$x_1 = 12, y_1 = -8, x_2 = 7, y_2 = -2$

$$\text{Midpoint, } M \equiv \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\equiv \left(\frac{12 + 7}{2}, \frac{-8 + (-2)}{2} \right)$$

$$\equiv \left(\frac{19}{2}, \frac{-10}{2} \right)$$

$$\equiv \left(9\frac{1}{2}, -5 \right)$$

Group the x-coordinates together, group the y-coordinates together.

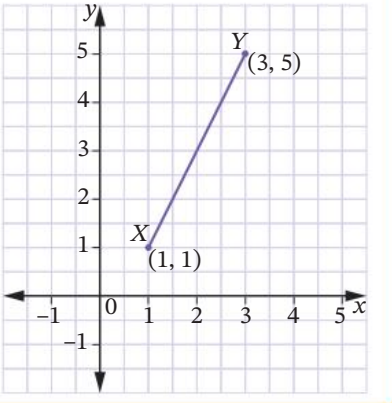
EXERCISE 11.02 ANSWERS ON P. 596

The midpoint of an interval U F R C

1 What is the midpoint of interval XY?
Select the correct answer **A**, **B**, **C** or **D**.

A (2, 3) **B** (2, 4)

C (3, 2) **D** (3, 3)



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2 On a number plane, plot and join each pair of points, then find and plot the midpoint of each interval.

a (1, 3) and (7, 5) **b** (0, 2) and (10, 4) **c** (−3, 3) and (5, −1)

EXAMPLE 4, 5

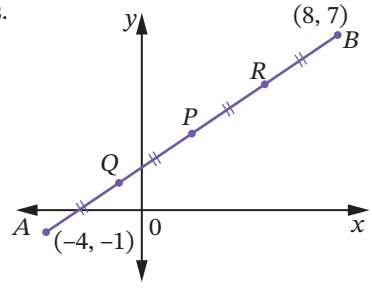
3 Find the midpoint of the interval joining:

a (10, 10) and (6, 4) **b** (2, 7) and (12, 9) **c** (0, 5) and (9, 3)
d (2, 2) and (2, 4) **e** (−2, 6) and (10, 3) **f** (9, −1) and (5, 2)
g (−8, 11) and (9, −4) **h** (−6, −1) and (0, 0) **i** (3, −3) and (−5, −10)
j (6, −1) and (−8, −1) **k** (5, −6) and (9, −10) **l** (−8, −7) and (−3, −5)

11.02

4 The interval AB has been subdivided into 4 equal parts. Find the coordinates of:

a P
b Q
c R



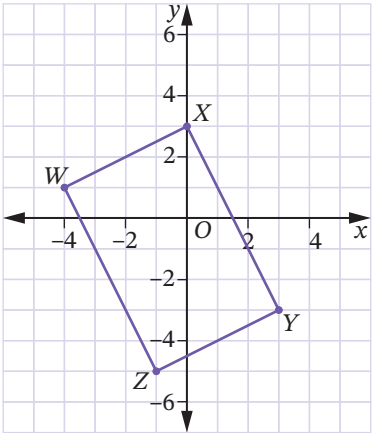
5 What is the midpoint of $K(6, 9)$ and $L(-2, 7)$? Select **A**, **B**, **C** or **D**.

A (4, 16) **B** (−4, 2) **C** (−2, 1) **D** (2, 8)

6 $WXYZ$ is a rectangle.

a Find the midpoint of:
- **i** diagonal WY
- **ii** diagonal XZ.

b What do you notice about the 2 midpoints in part **a**? Which property for the diagonals of a rectangle does this show? R C

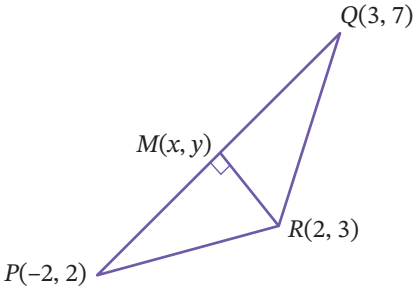


7 The diagram shows an isosceles triangle, PQR.

a Find M, the midpoint of PQ.

b Find, as a surd, the length of:
- **i** PQ **ii** MR **iii** QR

c Calculate, correct to one decimal place
- **i** the area of $\triangle PQR$
- **ii** the perimeter of $\triangle PQR$.



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8 The vertices of a square are $K(1, 6)$, $L(7, 2)$, $M(3, -4)$ and $P(-3, 0)$.

- **a** Find the length of the diagonal KM.
- **b** Find the length of the diagonal LP.
- **c** Are the diagonals equal in length?
- **d** Find the coordinates of the midpoint of KM.
- **e** Find the coordinates of the midpoint of LP.
- **f** Do the diagonals bisect each other? Explain.

11.03 The gradient of a line

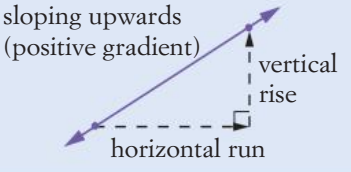
WS
A page of lines

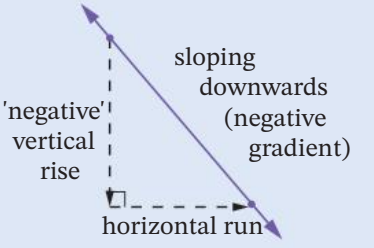
The **gradient** of a line (or interval) is a measure of how steep the line is. It is represented by the variable m and is the ratio (fraction) of the vertical **rise** to the horizontal **run** between any 2 points on a line.

The gradient of a line or interval

$$m = \frac{\text{vertical rise}}{\text{horizontal run}} = \frac{\text{rise}}{\text{run}}$$

- A line **sloping upwards** has a **positive rise** and a **positive gradient**.
- A line **sloping downwards** has a **negative rise** and a **negative gradient**.
- The **run** is always **positive**.

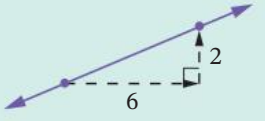




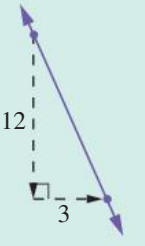
Example 6

Calculate the gradient of each line.

a



b



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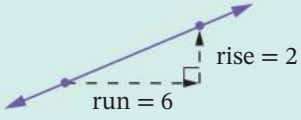
Solution

a The line has a positive rise (↑).

$m = \frac{2}{6}$ $\quad m = \frac{\text{rise}}{\text{run}}$

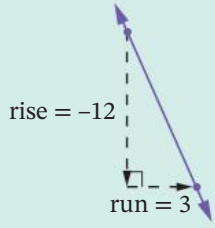
$= \frac{1}{3}$



b The line has a negative rise or a drop (↓).

$m = \frac{-12}{3}$ $\quad m = \frac{\text{rise}}{\text{run}}$

$= -4$



Example 7

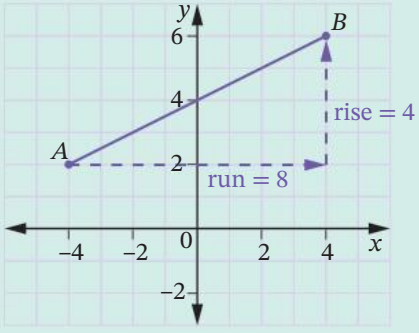
Calculate the gradient of the line going through:

a $A(-4, 2)$ and $B(4, 6)$ **b** $C(-2, 4)$ and $D(2, -4)$.

Solution

Form a right-angled triangle with the interval as the hypotenuse, as shown.

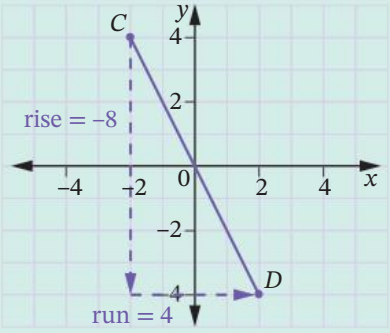
a



$m = \frac{4}{8}$

$= \frac{1}{2}$

b



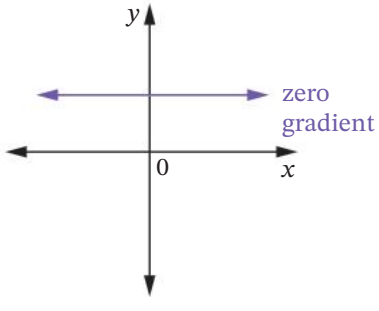
$m = \frac{-8}{4}$

$= -2$

The size and sign of the gradient

The **size** of the gradient shows how quickly the line is going up or down: the higher the number, the steeper the line. The **sign** of the gradient shows whether the line is sloping upwards (positive) or downwards (negative) as you look from left to right.

A **horizontal (flat) line** has **zero gradient**.



11.03

The gradient of a line

Gradient (slope) of a line

Gradient between two points

Find the gradient 1

STAGE 5.3

Distance, midpoint and gradient formulas

Intervals match-up

WS Gradient, midpoint and distance

The gradient formula

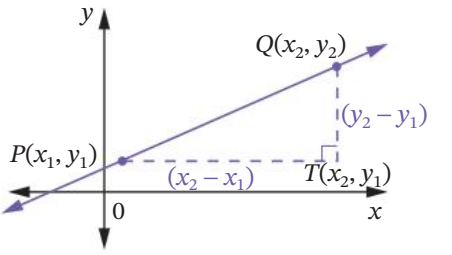
The method for finding the gradient of an interval can be summarised by a formula.

Consider the line passing through the points (x_1, y_1) and (x_2, y_2).

Vertical rise = difference in y-coordinates $= y_2 - y_1$

Horizontal run = difference in x-coordinates $= x_2 - x_1$

Gradient, $m = \dfrac{\text{difference in } y}{\text{difference in } x} = \dfrac{y_2 - y_1}{x_2 - x_1}$



The gradient formula

The gradient (m) of a line or interval joining 2 points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Group the y-coordinates together **on top**, group the x-coordinates together **below**.

Example 8

Use the gradient formula to calculate the gradient of the line going through $A(-4, 2)$ and $B(4, 6)$ (from Example **7 a**).

Solution

For $A(-4, 2)$ and $B(4, 6)$:

$x_1 = -4, y_1 = 2, x_2 = 4, y_2 = 6$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{6 - 2}{4 - (-4)}$$
$$= \frac{4}{8}$$
$$= \frac{1}{2}$$

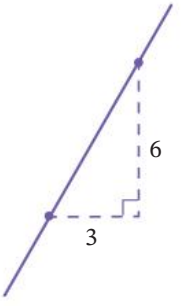
Group the y-coordinates together **on top**, group the x-coordinates together **below**.

EXERCISE 11.03 ANSWERS ON P. 596

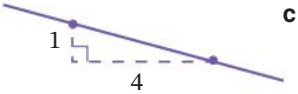
The gradient of a line U F R C

1 State whether the gradient of each line is positive (+) or negative (–).

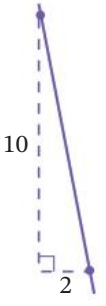
a



b



c



Foundation Standard Complex

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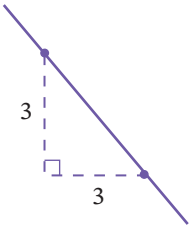
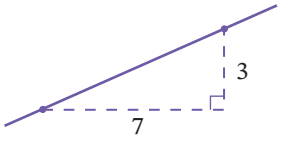
d

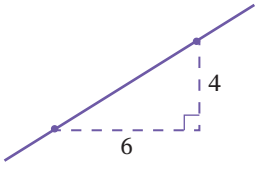


e

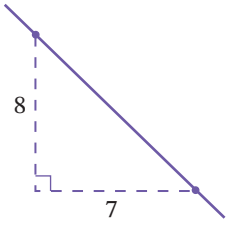


f

g



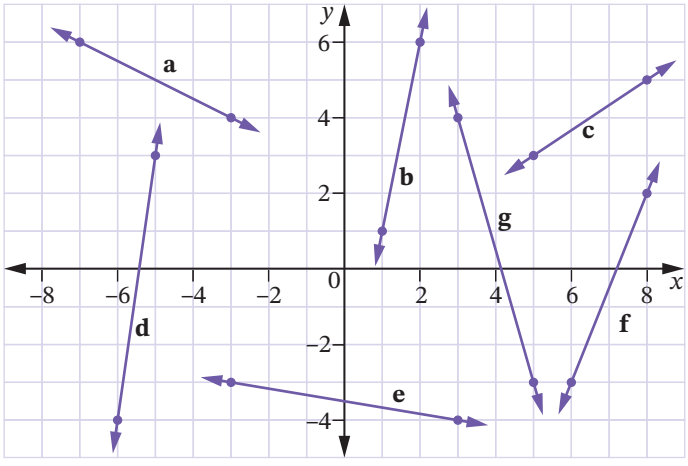
h



i

2 Calculate the gradient of each line in question **1**.

3 Calculate the gradient of each interval.



4 Which pair of points form an interval that has a gradient of 2?
Select the correct answer **A**, **B**, **C** or **D**.

A (0, 2), (2, 0) **B** (−5, 2), (5, 7) **C** (−2, −2), (1, 4) **D** (−6, −8), (−2, −6)

5 Calculate the gradient of the interval joining each pair of points, either by plotting the points on a number plane or by using the gradient formula.

a $A(1, 4)$ and $B(3, 6)$ **b** $Z(4, 7)$ and $T(1, 6)$ **c** $M(4, -2)$ and $D(3, 3)$

d $A(-2, -1)$ and $P(4, -4)$ **e** $D(0, -1)$ and $F(3, -1)$ **f** $H(9, -2)$ and $L(-1, 3)$

g $K(4, 9)$ and $M(-4, -5)$ **h** $Z(-8, -6)$ and $T(4, 0)$ **i** $F(0, 4)$ and $B(3, 4)$

EXAMPLE 6

EXAMPLE 7

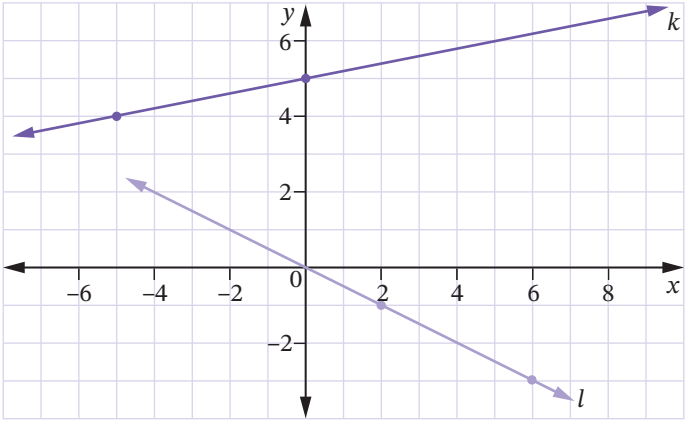
EXAMPLE 8

Foundation Standard Complex

6 Which pair of points form an interval that has a gradient of –1? Select **A**, **B**, **C** or **D**.

A (1, 2), (2, 1)
B (–5, 6), (6, 7)
C (2, 2), (3, –1)
D (–5, –10), (4, –11)

7 Find the gradient of the lines labelled k and l.



8 Which pair of points lie on a line with a gradient of 0? Select **A**, **B**, **C** or **D**.

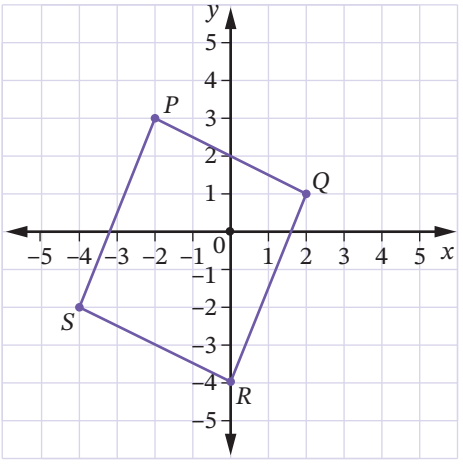
A (3, 3), (–3, –3)
B (–5, 7), (5, 7)
C (1, –6), (1, 4)
D (0, 9), (9, 0)

9 $PQRS$ is a quadrilateral. R C

a Find the gradient of each side of $PQRS$.

b Which sides have the same gradient? What does that mean about those sides?

c What type of quadrilateral is $PQRS$?



10 A triangle is formed by joining $P(-8, -2)$, $Q(-2, 10)$ and $R(4, 6)$. T is the midpoint of PQ, and Z is the midpoint of QR. R C

a Draw a diagram showing all the given information.

b Find the coordinates of T and Z.

c Calculate the gradient of interval TZ and side PR. What do you notice?

d Calculate the length of interval TZ and side PR. What do you notice?

Did you know?

Stairs, ramps and hills

- Some stairs are easier to climb than others: the gradient of stairs should not be more than 5 in 6 $\left(\frac{5}{6}\right)$.
- A ramp for walking should not have a gradient higher than $\frac{3}{10}$ and the gradient of a wheelchair ramp should be between $\frac{1}{14}$ and $\frac{1}{8}$.
- The maximum slope of a hill for parking your car safely is $\frac{2}{9}$.
- The steepest street in the world is Baldwin Street in Dunedin, New Zealand, with one section having a gradient of 35% or $\frac{7}{20}$.

Would it be safe to park your car on Bulli Pass in Wollongong, NSW, which has a slope of $\frac{1}{6}$?

Shutterstock.com/Deyan Denchev

Baldwin St, the steepest street in the world, in Dunedin, New Zealand.

Technology

The gradient of collinear points

In this activity, you will use dynamic geometry software to investigate collinear points.

1 Create a number plane with x-values: 0 to 30 and y-values: 0 to 16.

2 Plot the points $A(5, 2)$, $B(9, 5)$ and $C(17, 11)$. Create intervals AB and BC.

3 To check if the 3 points are collinear, select **Slope** for interval AB, and again for interval BC. Is the slope (gradient) the same for AB and BC? If so, these points are called **collinear points**. What do you think **collinear points** are? Discuss this with other students and explain it in your own words.

4 Now test whether each set of points are collinear.

a (0, 3), (2, 4) and (6, 6)

b (8, 0), (−8, 0) and (−6, −10)

c (1, 8), (2, 9) and (4, 10)

d (−2, 8), (−1, 6) and (0, 1)

e (2, 7), (4, 10), (5, 13) and (6, 17)

f (−14, 11), (−8, 9), (1, 6) and (16, 1)

11.04 Graphing linear equations

Graphing linear equations

Graphing linear equations

A page of lines

A page of number planes

Number plane grid paper

A relationship between 2 variables such as x and y whose graph is a **straight line** is called a **linear relationship**. The expression of that relationship as an algebraic formula, such as $y = x + 2$, is called a **linear equation**.

Example 9

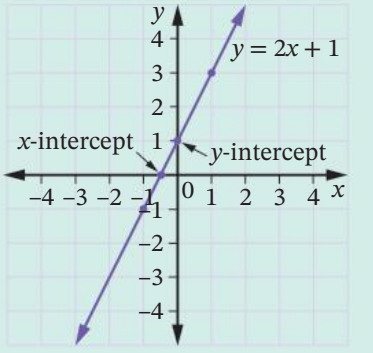
Graph $y = 2x + 1$ on a number plane.

Solution

Complete a table of values. Choose x-values close to 0 for easy calculation and graphing.

x	−1	0	1
y	−1	1	3

Graph the table of values on a number plane. Rule a straight line through the points, place arrows at each end, and label the line with its equation.



Note:

- the ***x*-intercept** of the line is $-\frac{1}{2}$: this is the x-value where the line cuts the ***x*-axis**
- the ***y*-intercept** of the line is 1: this is the y-value where the line cuts the ***y*-axis**
- every point on the line follows the linear equation $y = 2x + 1$: for example, (–4, –7), (–2, –3) and (2, 5) lie on the line and also follow the rule $y = 2x + 1$
- there are an infinite number of points that follow the rule: arrows on both ends of the line indicate that it has infinite length.

STAGE 5.3

Using *x*- and *y*-intercepts to graph lines

We can also graph a linear equation by finding its x- and y-intercepts first.

Since any point on the x-axis has a y-coordinate of 0, we can substitute $y = 0$ into the equation to find the x-intercept.

Similarly, any point on the y-axis has an x-coordinate of 0, so we can substitute $x = 0$ into the equation to find the y-intercept.

Using *x*- and *y*-intercepts to graph lines

- To find the x-intercept, substitute $y = 0$ and solve the equation.
- To find the y-intercept, substitute $x = 0$ and solve the equation.

Example 10

Find the x- and y-intercepts for $y = -4x + 8$ to draw its graph.

Solution

For x-intercept, $y = 0$:

$0 = -4x + 8$

$4x = 8$

$x = 2$

The x-intercept is 2.

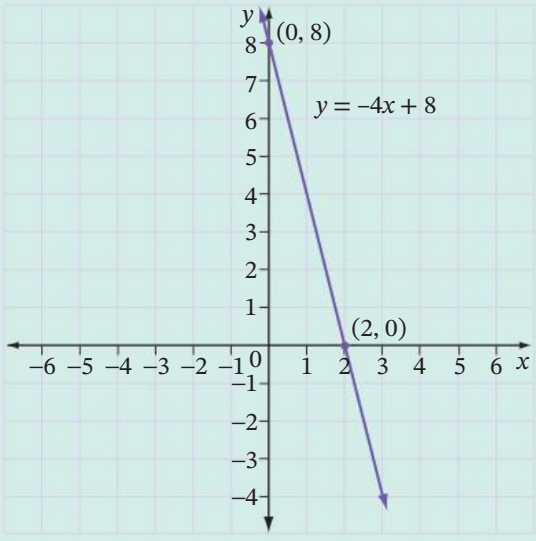
For y-intercept, $x = 0$:

$y = -4 \times 0 + 8$

$= 8$

The y-intercept is 8.

Plot both intercepts on the axes, draw a line through the 2 points and label the line with its equation.



Testing if a point lies on a line

A point lies on a line if its (x, y) coordinates satisfy the equation of the line.

- Separate the equation into LHS (left-hand side) and RHS (right-hand side)
- Substitute the coordinates of the point into both sides
- If LHS = RHS, the point satisfies the equation and lies on the line
- If LHS ≠ RHS, the point does not lie on the line.

WS Straight-line equations

Example 11

Test whether each point lies on the line $y = 2x + 3$.

a (3, 9) **b** (8, −4)

Solution

a Substitute $x = 3$ and $y = 9$ into $y = 2x + 3$.

LHS $= y$ RHS $= 2x + 3$

$= 9$ $= 2 \times 3 + 3$

$= 9$

LHS = RHS, so (3, 9) lies on the line.

b Substitute $x = 8$ and $y = -4$ into $y = 2x + 3$.

LHS $= y$ RHS $= 2x + 3$

$= -4$ $= 2 \times 8 + 3$

$= 19$

LHS ≠ RHS, so (8, −4) does not lie on the line.

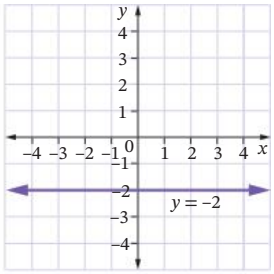
Testing if a point lies on a line

Horizontal and vertical lines

A **horizontal line** is flat and runs across (sideways), parallel to the x-axis.

Every point on the horizontal line has y-coordinate -2, and the y-intercept is -2.

x	-1	0	1	2
y	-2	-2	-2	-2

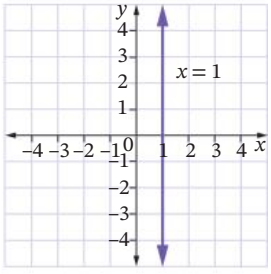


y is always equal to -2, so the equation of this horizontal line is $y = -2$.

A **vertical line** runs up and down, parallel to the y-axis and at right angles to a horizontal line.

Every point on the vertical line has x-coordinate 1, and the x-intercept is 1.

x	1	1	1	1
y	0	1	2	3



x is always equal to 1, so the equation of this vertical line is $x = 1$.

Horizontal and vertical lines

The **equation of a horizontal line** is of the form $\boldsymbol{y = c}$ (where c is a constant number).

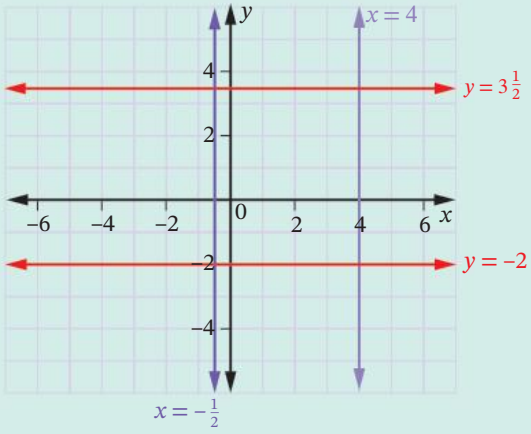
The **equation of a vertical line** is of the form $\boldsymbol{x = c}$ (where c is a constant number).

Example 12

Graph each line on the same number plane.

a $x = 4$ **b** $y = -2$ **c** $x = -\frac{1}{2}$ **d** $y = 3\frac{1}{2}$

Solution



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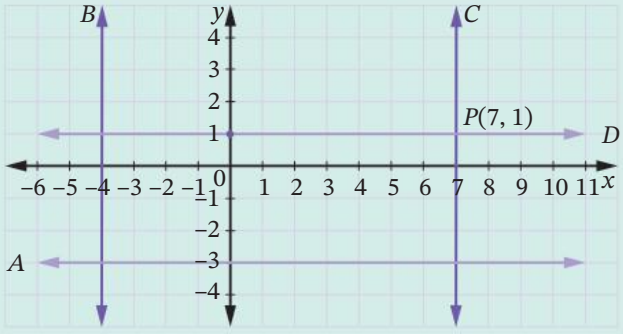
Example 13

Find the equations of lines A, B, C and D.



Solution

- Line A is horizontal and has a y-intercept of -3, so its equation is $y = -3$.
- Line B is vertical and has an x-intercept of -4, so its equation is $x = -4$.
- Line C is vertical and goes through $(7, 1)$, so its equation is $x = 7$.
- Line D is horizontal and goes through $(7, 1)$, so its equation is $y = 1$.

EXERCISE 11.04 ANSWERS ON P. 596

Graphing linear equations UFRC

1 Graph each linear equation on a number plane, and write:

i its x-intercept **ii** its y-intercept.

EXAMPLE 9

a $y = x - 3$ **b** $y = 2x + 2$ **c** $y = -x + 5$

d $y = 3x$ **e** $y = -x$ **f** $y = 10 - 4x$

g $y = \frac{x+2}{2}$ **h** $y = 2x + 1$ **i** $y = \frac{x}{3} - 2$

j $y = \frac{3x}{4}$ **k** $x + y = 4$ **l** $x - y = 2$

2 What is the x-intercept of the graph of $y = 6 - 3x$? Select the correct answer **A**, **B**, **C** or **D**.

A -3 **B** 2 **C** 3 **D** 6

3 Graph each linear equation after finding its x- and y-intercepts.

STAGE 5.3
EXAMPLE 10

a $y = 3x - 6$ **b** $y = \frac{6-x}{2}$ **c** $x + y = 7$

d $x - y = 6$ **e** $y = \frac{2x}{3} + 1$ **f** $3x - 4y = 12$

g $x + y = 4$ **h** $2x - y = 5$ **i** $\frac{x}{2} + \frac{y}{3} = 1$

Foundation Standard Complex

EXAMPLE 11

4 Test whether the point (3, –1) lies on each line:

a $y = 2x - 7$ **b** $y = -2x + 4$ **c** $x - y = 4$

d $x + 2y = 1$ **e** $y = -3x + 8$ **f** $x - 3y = 0$

5 Which of these points lies on the line $x - y = -2$? Select **A**, **B**, **C** or **D**.

A (1, 1) **B** (–1, –1) **C** (0, 2) **D** (–2, 4)

EXAMPLE 12

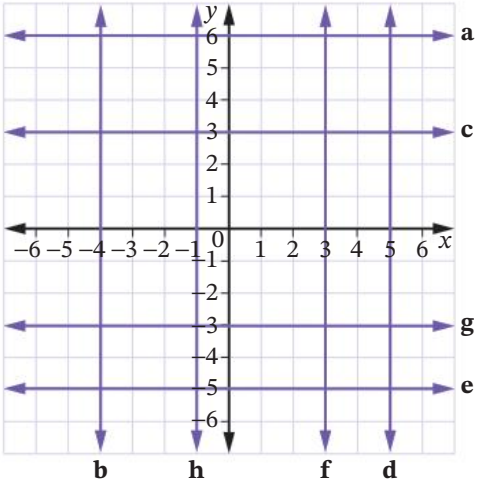
6 Graph each set of lines on a number plane.

a $x = 2, x = -3$ **b** $y = 6, y = -2$

c $x = 2\frac{1}{2}, x = 0, y = -5$ **d** $y = 1.5, y = 3, x = -2$

EXAMPLE 13

7 Write down the equation of each line shown.



8 **a** Show that the point (3, 2) lies on both the lines $x = 2y - 1$ and $x + y = 5$. R C

b So at what point do the lines $x = 2y - 1$ and $x + y = 5$ cross?

9 Show that the lines $y = -3x + 6$ and $y = -2x$ intersect at the point (6, –12). R C

10 Find the equation of each line described. R C

a A horizontal line that passes through the y-axis at 6.

b A vertical line that goes through (–5, 2).

c Each point on the line is 2 units above the x-axis.

d Each point on the line is 5 units to the left of the y-axis.

e The line is parallel to the x-axis and passes through the point (0, –2).

f A horizontal line that goes through (4, –1).

g The line goes through the points (–1, 8) and (–1, 2).

11 What is the equation of: R C

a the y-axis? **b** the x-axis?

Technology

Graphing linear equations

In this activity, we will use dynamic geometry software to graph a linear equation.

1 Create a number plane with axes: x: –10 to 10 and y: –10 to 10.

2 Enter the following linear equations in column A of the spreadsheet.

A1: $y = 3x + 1$ A2: $y = 3x$ A3: $y = 2x + 4$

A4: $y = -2x - 5$ A5: $y = x + 2$ A6: $y = -x - 1$

3 Read the gradient of each line by selecting the **Slope tool**. Select a line and you will see the right-angled triangle showing $\frac{\text{rise}}{\text{run}}$.

4 Enter each linear equation and find the gradient of each line.

a $y = x + 1$ **b** $y = -2x - 4$ **c** $y = 6x + 3$

d $y = 3x$ **e** $y = -8x$ **f** $y = -4x - 6$

Mental skills 11: Maths without calculators ANSWERS ON P. 599

Divisibility tests

How can you tell if a number is divisible by 2? Look at its last digit. If that digit is 2, 4, 6, 8 or 0, then the number is divisible by 2 (that is, it is even).

How can you tell if a number is divisible by 5? If its last digit is 0 or 5, then the number is divisible by 5.

These are examples of **divisibility tests**—rules for checking whether a number is divisible by a certain number. The table below shows some common divisibility tests.

A number is divisible by:	if:
2	its last digit is 2, 4, 6, 8 or 0
3	the sum of its digits is divisible by 3
4	its last 2 digits form a number divisible by 4
5	its last digit is 0 or 5
6	it is even and the sum of its digits is divisible by 3
9	the sum of its digits is divisible by 9
10	its last digit is 0

1 Study each example.

a Test whether 748 is divisible by 2, 3 or 4.

- Last digit is 8 (even). ∴ 748 is divisible by 2.
- Sum of digits = 7 + 4 + 8 = 19, which is not divisible by 3. ∴ 748 is not divisible by 3.
- 48 is divisible by 4. ∴ 748 is divisible by 4 (748 ÷ 4 = 187).

b Test whether 261 is divisible by 5 or 9.

- Last digit is 1, not 0 or 5. ∴ 261 is not divisible by 5.
- $2 + 6 + 1 = 9$, which is divisible by 9. ∴ 261 is divisible by 9 ($261 \div 9 = 29$).

c Test whether 570 is divisible by 4, 6 or 10.

- 70 is not divisible by 4. ∴ 570 is not divisible by 4.
- 570 is even and $5 + 7 + 0 = 12$, which is divisible by 3. ∴ 570 is divisible by 6 ($570 \div 6 = 95$).
- Last digit is 0. ∴ 570 is divisible by 10 ($570 \div 10 = 57$).

2 Test whether each number is divisible by 2, 3, 5 or 6.

a	250	**b**	189	**c**	78	**d**	465
e	1024	**f**	840	**g**	715	**h**	627

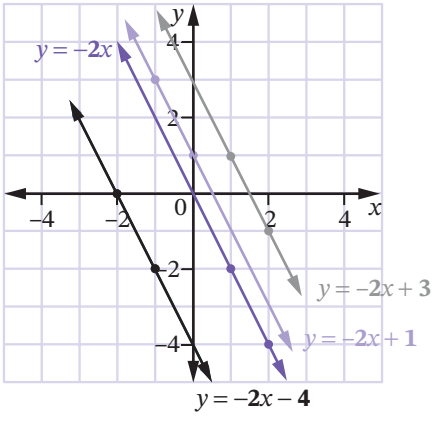
3 Test whether each number is divisible by 4, 9 or 10.

a	144	**b**	280	**c**	522	**d**	4170
e	936	**f**	726	**g**	342	**h**	5580

Investigation

Comparing gradients and *y*-intercepts

1 This number plane shows the graphs of 4 linear equations.



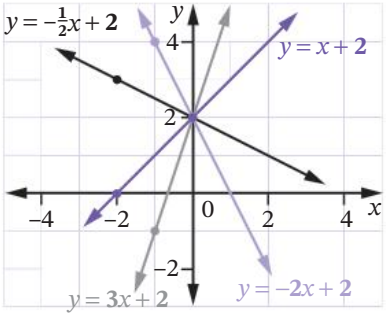
a Copy and complete this table by finding the gradient and *y*-intercept of each line.

Equation	Gradient	*y*-intercept
$y = -2x - 4$	−2	−4
$y = -2x$		
$y = -2x + 1$		
$y = -2x + 3$		

b How are the 4 lines:

i similar? **ii** different?

2 This number plane shows the graphs of another 4 linear equations.



a Copy and complete this table.

Equation	Gradient	*y*-intercept
$y = -2x + 2$		
$y = -\frac{1}{2}x + 2$		
$y = x + 2$		
$y = 3x + 2$		

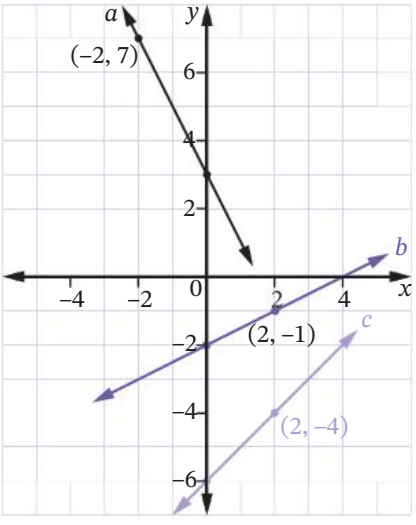
b How are the 4 lines:

i similar? **ii** different?

3 Copy and complete this table for the 3 lines graphed.

a

Equation	Gradient	*y*-intercept
$y = -2x + 3$		
$y = \frac{1}{2}x - 2$		
$y = x - 6$		



b Explain how the values for the gradient and *y*-intercept of each line relate to its equation.

c What is the equation of a line that has a gradient of 4 and a *y*-intercept of –1?

d What is the gradient and *y*-intercept of the line with equation $y = -x + 7$?

11.05 The gradient–intercept equation $y = mx + c$

Equation of a line

The gradient–intercept form of a linear equation

The equation of a straight line is $y = mx + c$, where m is the gradient and c is the y-intercept.

For this reason, $y = mx + c$ is also called the **gradient–intercept form** of a linear equation.

Example 14

Find the gradient and y-intercept of the line with equation:

a $y = 4x - 7$ **b** $y = \frac{1}{2}x + 3$ **c** $y = 3 - 5x$ **d** $y = \frac{3x}{4} - 1$

Drawing straight lines: $y = mx + c$

Solution

a $y = 4x - 7$ is in the form $y = mx + c$ $\therefore$ Gradient $m = 4$ and y-intercept $c = -7$

b For $y = \frac{1}{2}x + 3$, gradient $m = \frac{1}{2}$ and y-intercept $c = 3$

c $y = 3 - 5x$ can be rewritten as $y = -5x + 3$ $\therefore$ Gradient $m = -5$ and y-intercept $c = 3$

d For $y = \frac{3x}{4} - 1$, gradient $m = \frac{3}{4}$ and y-intercept $c = -1$

Example 15

Write the equation of the line that has a gradient of -4 and a y-intercept of -6.

Solution

$m = -4$, $c = -6$

$\therefore$ The equation of the line is $y = -4x - 6$. Using $y = mx + c$.

Graphing $y = mx + c$

Example 16

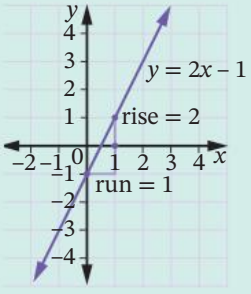
Graph each linear equation by finding the gradient and y-intercept first.

a $y = 2x - 1$ **b** $y = -2x + 5$ **c** $y = \frac{3}{4}x - 2$

Solution

a $y = 2x - 1$ has a gradient of 2 and a y-intercept of -1.

- Plot the y-intercept -1 on the y-axis.
- Make a gradient of 2 by moving **across** 1 unit (**run**) and **up** 2 units (**rise**) and marking the point at (1, 1).
- Rule a line through this point and the y-intercept.
- Label the line with the equation $y = 2x - 1$



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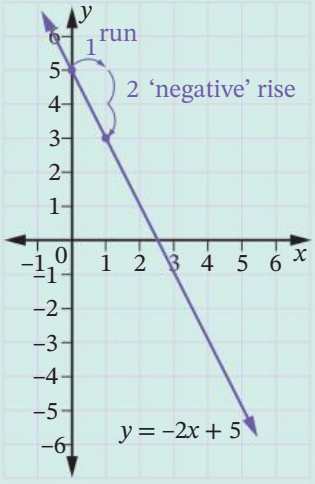
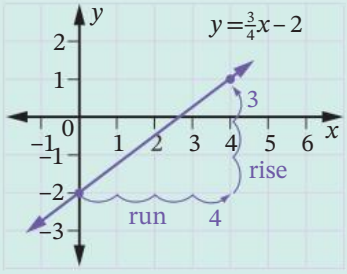
b $y = -2x + 5$ has a gradient of -2 and a y-intercept of 5.

- Plot the y-intercept 5 on the y-axis.
- Make a gradient of -2 by moving **across** 1 unit (**run**) and **down** 2 units (**'negative' rise**) and marking the point at (1, 3).
- Rule a line through this point and the y-intercept.



c $y = \frac{3}{4}x - 2$ has a gradient of $\frac{3}{4}$ and a y-intercept of -2.

- Plot the y-intercept -2 on the y-axis.
- Make a gradient of $\frac{3}{4}$ by moving **across** 4 units (**run**) and **up** 3 units (**rise**) and marking the point at (4, 1).
- Rule a line through this point and the y-intercept.



EXERCISE 11.05 ANSWERS ON P. 599

The gradient–intercept equation $y = mx + c$ U F

1 Find the gradient and y-intercept of the line with equation:

a $y = 3x - 2$ **b** $y = 2x + 7$ **c** $y = x + 4$ **d** $y = -x - 9$

e $y = \frac{3x}{4} + 6$ **f** $y = x$ **g** $y = \frac{x}{2} - 11$ **h** $y = \frac{2}{3}x + 6$

i $y = 8 - 2x$ **j** $y = 5 - x$ **k** $y = \frac{x}{4}$ **l** $y = -3x + 11$

m $y = 5$ **n** $y = \frac{2}{5}x + 1$ **o** $y = x - \frac{7}{2}$ **p** $y = \frac{3x}{4} + 1$

EXAMPLE 14

2 Which equation has a gradient of 5? Select the correct answer **A**, **B**, **C** or **D**.

A $y = x + 5$ **B** $y = 5 - x$ **C** $y = 5$ **D** $y = 5x$

3 Write the equation of the line with:

a a gradient of 2 and a y-intercept of -1 **b** a gradient of $\frac{3}{4}$ and a y-intercept of 2

c a gradient of -3 and a y-intercept of 5 **d** a gradient of $-\frac{2}{5}$ and a y-intercept of 3

EXAMPLE 15

4 Which equation has a y-intercept of -1? Select **A**, **B**, **C** or **D**.

A $y = 1 - x$ **B** $y = 1 + x$ **C** $y = x - 1$ **D** $y = -x$

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EXAMPLE 16

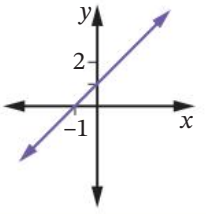
5 Graph each linear equation by finding the gradient and y-intercept first.

a $y = 2x + 1$ **b** $y = x - 4$ **c** $y = 3x + 2$ **d** $y = -4x$

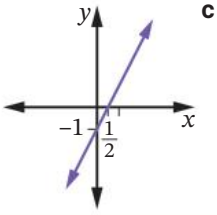
e $y = -3x - 3$ **f** $y = \frac{x}{2} + 2$ **g** $y = 8 - \frac{3x}{4}$ **h** $y = \frac{5x}{2} - 1$

6 Which diagram shows the graph of $y = 2x - 1$? Select **A**, **B**, **C** or **D**.

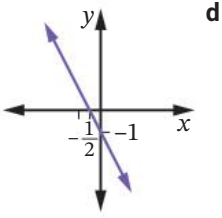
a



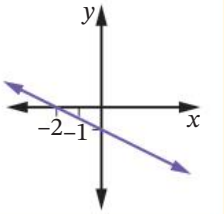
b



c



d



11.06 Finding the equation of a line

Finding the equation of a line

Finding the equation of a line

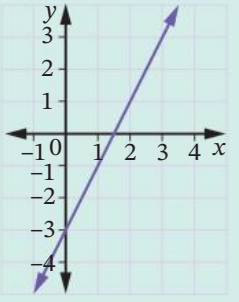
A page of lines

Straight-line equations

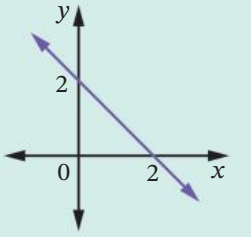
Example 17

Find the equation of each line.

a



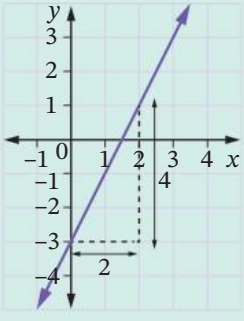
b



Solution

a Select 2 points on the line to find the gradient, say (0, −3) and (2, 1).

Gradient $m = \frac{\text{rise}}{\text{run}} = \frac{4}{2} = 2$



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y-intercept $c = -3$ — from graph

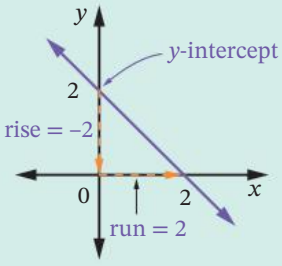
$\therefore$ The equation of the line is $y = 2x - 3$ — $y = mx + c$

We can check that this equation is correct for another point, say (3, 3).

When $x = 3$, $y = 2 \times 3 - 3 = 3$

b Find the gradient of the line passing through (0, 2) and (2, 0).

Gradient $m = \frac{\text{rise}}{\text{run}} = \frac{-2}{2} = -1$



y-intercept $c = 2$ — from graph

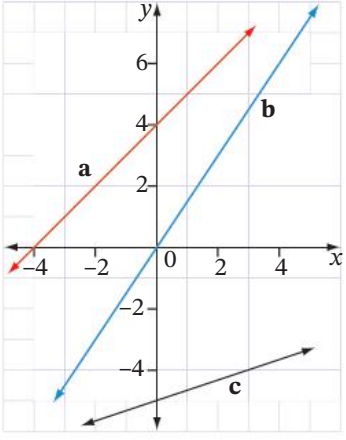
$\therefore$ The equation of the line is $y = -x + 2$ — $y = mx + c$

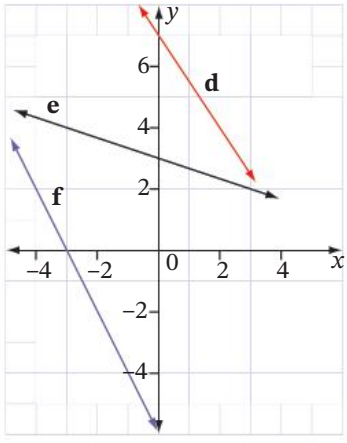
EXERCISE 11.06 ANSWERS ON P. 600

Finding the equation of a line U F

EXAMPLE 17

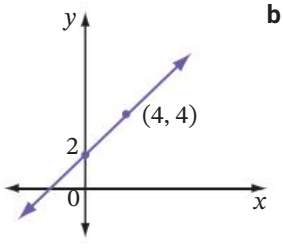
1 Find the equation of each line.



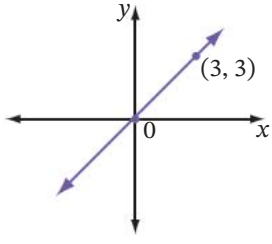


2 Find the equation of each line.

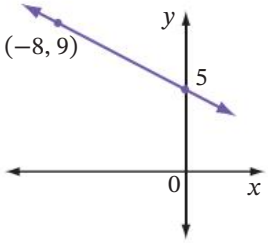
a



b



c



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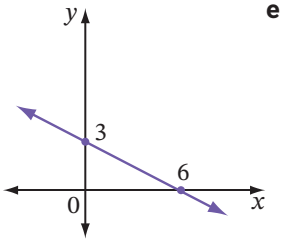
d



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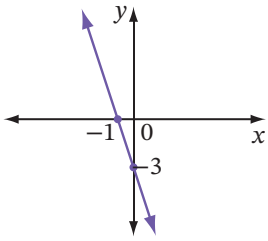

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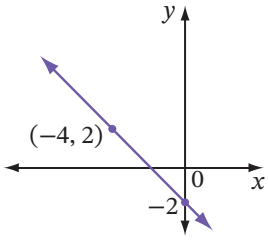
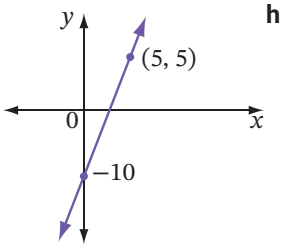


g



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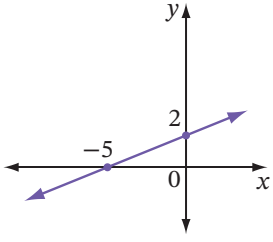
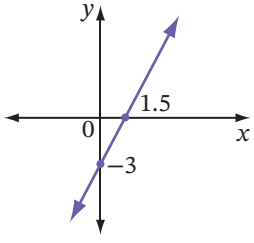


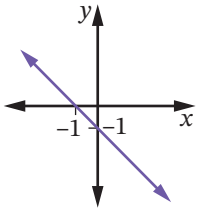
i



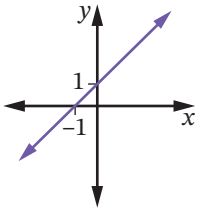
3 Match each linear equation to its graph.

a $y = x - 1$ **b** $y = x + 1$ **c** $y = -x - 1$ **d** $y = -x + 1$

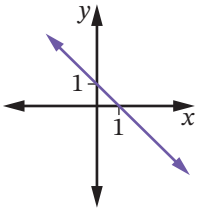
A



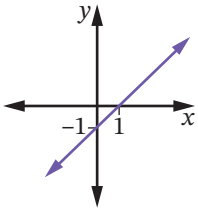
B

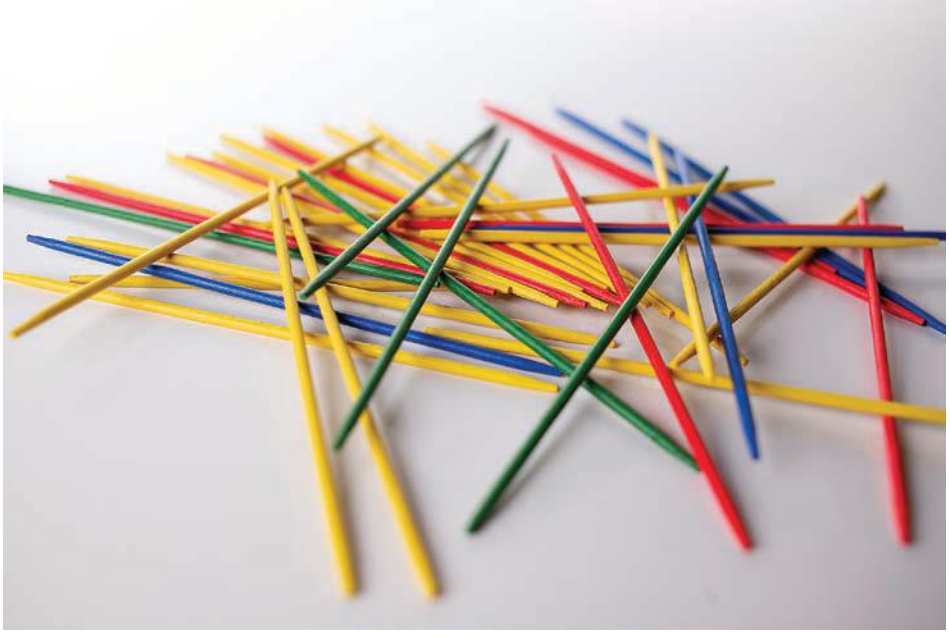


C



D





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Solving linear equations graphically 11.07

In Chapter 7, *Equations*, we solved equations **algebraically**. We can also solve an equation **graphically** by first graphing it on the number plane.

Example 18

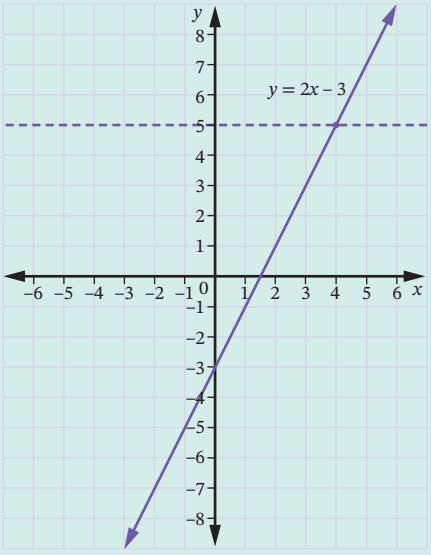
Solve the equation $2x - 3 = 5$ graphically.

Solution

Graph $y = 2x - 3$ (the LHS of the equation) on a number plane.

For every point on the line, the y-coordinate is the value of $2x - 3$. So to solve $2x - 3 = 5$, we need to find the point whose y-coordinate is 5.

Draw a dotted horizontal line at $y = 5$ and read off the coordinates of the point on $y = 2x - 3$ crossed by this line.



This point is (4, 5), which means the solution to $2x - 3 = 5$ is $x = 4$.

Check: $2 \times 4 - 3 = 5$

EXERCISE 11.07 ANSWERS ON P. 600

Solving linear equations graphically U F

This exercise is best completed using graphing technology such as dynamic geometry software or graphing websites.

1 Use the graph in Example **13** above to solve each equation. Check your solutions.

a $2x - 3 = 3$ **b** $2x - 3 = -7$ **c** $2x - 3 = 0$

2 Graph $y = 2x + 1$ and use it to solve each equation graphically.

a $2x + 1 = 7$ **b** $2x + 1 = 10$ **c** $2x + 1 = -5$

3 Solve each equation in question **2** algebraically.

EXAMPLE 18

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4 At which point do the graphs of $y = 2x + 3$ and $y = 1$ intersect? Select the correct answer **A**, **B**, **C** or **D**.

A (2, 1) **B** (–1, 1) **C** (4, –2) **D** (8, –4)

5 Graph $y = 3x - 2$ and use it to solve each equation graphically.

a $3x - 2 = 7$ **b** $3x - 2 = -11$ **c** $3x - 2 = -2$

6 Graph $y = -x + 1$ and use it to solve each equation graphically.

a $-x + 1 = -3$ **b** $-x + 1 = 4$ **c** $-x + 1 = -1$

7 Graph $y = -2x - 1$ and use it to solve each equation graphically.

a $-2x - 1 = -3$ **b** $-2x - 1 = 0$ **c** $-2x - 1 = 3$

8 Graph $y = \frac{1}{2}x + 3$ and use it to solve each equation graphically.

a $\frac{1}{2}x + 3 = 5$ **b** $\frac{1}{2}x + 3 = 1$ **c** $\frac{1}{2}x + 3 = 2$

11.08 Direct proportion

Ann-Marie noticed a relationship between the amount of petrol used by her car and the distance travelled by the car, as shown in the table of values.

Amount of petrol used (L)	10	14	22	32	36	42
Distance travelled (km)	135	189	297	432	486	567

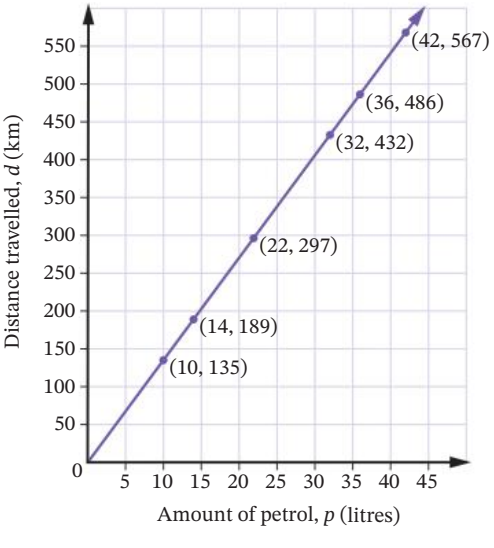
When she graphed these values, she found a linear relationship.

If p stands for the amount of petrol used and d stands for the distance travelled, then it can be shown that d is directly related to p by the formula $d = 13.5p$. This means that the distance travelled is a constant multiple of the amount of petrol used.

This is an example of **direct proportion** (or **direct variation**).

Two variables are **directly proportional** to each other if one variable is a constant multiple of the other, and when one variable changes, the other one changes by the same factor.

Distance travelled and petrol used



Foundation Standard Complex

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Direct proportion

If y is **directly proportional** to x, then $y = kx$, where k is a constant (number) called the **constant of proportionality** or **constant of variation**.

- A direct linear relationship exists between x and y.
- If x increases (or decreases), y increases (or decreases).
- If x is doubled (or halved), y is doubled (or halved).
- Another way of saying 'y is directly proportional to x' is y **varies directly** with x'.
- A graph for direct proportion is a straight line going through (0, 0) with gradient k.

Example 19

The distance (d), in metres, travelled by a car is directly proportional to the number of rotations (r) of its tyres. After 540 rotations, a distance of 950 m is travelled.

a What distance (to the nearest metre) will be travelled after 800 rotations?

b How many full rotations will be needed to cover 360 m?

Solution

a d is directly proportional to r.

$\therefore d = kr$ Forming a proportion equation

To find k, substitute the information given for r and d.

When $r = 540$ and $d = 950$,

$950 = k \times 540$ Finding k.

$$k = \frac{950}{540}$$
$$= 1.759...$$

$\therefore d = 1.759... r$ Rewriting the equation $d = kr$

When $r = 800$, Solving the problem

$d = 1.759... \times 800$

$= 1407.4074...$

≈ 1407 m

After 800 rotations, the distance travelled will be 1407 m.

b When $d = 360$, Solving the problem

$360 = 1.759... r$

$$r = \frac{360}{1.759...}$$
$$= 204.661...$$
≈ 205 rotations Rounding up for full rotations

For a distance of 360 m, there will be 205 rotations.

Solving direct proportion problems

1 Identify the 2 variables (say x and y) and form a proportion equation, $y = kx$

2 Substitute values for x and y to find k, the constant of proportionality

3 Rewrite $y = kx$ using the value of k

4 Substitute a value for x or y into $y = kx$ to solve the problem.

EXERCISE 11.08 ANSWERS ON P. 601

Direct proportion **U F PS R**

EXAMPLE 19

1 The stretch, S cm, of a spring is directly proportional to the mass, M kg, pulling on it.

Mass, M (kg)	0	8	12	20	24	40
Stretch, S (cm)	0	5	7.5	12.5	15	25

a Graph the linear relationship between S and M.

b The equation of the line is $S = kM$. Use the table of values to find the value of k as a decimal, then rewrite the equation for S.

c Use the equation to find the stretch caused by a mass of 36 kg.

d What is the mass that will cause a stretch of 40 cm?

2 B is directly proportional to h. If when $h = 5$, $B = 16$, what is the value of k in $B = kh$? Select the correct answer **A**, **B**, **C** or **D**.

A 80 **B** 11 **C** 3.2 **D** 0.3125

3 Y is directly proportional to X. If when $X = 15$, $Y = 75$, find the constant of proportionality, then use the proportion equation to find Y when $X = 23$.

4 The mass, M, in grams of a chemical is directly proportional to its volume, V, in cm^3. **PS** **R**

a Write the formula $M = kV$, where k is a decimal, given that $M = 140$ when $V = 80$.

b Use the formula to calculate the mass of 406 cm^3 of the chemical.

c Calculate (correct to 1 decimal place) the volume of the chemical if the mass is:

i 28 g **ii** 0.36 kg

5 Find the linear formula for n in terms of m for this table of values.

m	3	6	9	12	15
n	7.5	15	22.5	30	37.5

6 p varies directly with q. If $q = 7$ when $p = 42$, find p when $q = 18$.

7 The distance, D, travelled by Pete, a marathon runner, varies directly with time, t. Pete runs 8.25 km in 44 minutes.

a How far will he run in one hour?

b How long will it take him to run 24 km?

8 For an object that is cooling, the drop in temperature varies directly with time.

If the temperature drops 8°C in 5 minutes, how long would it take to drop 10°C?
Select **A**, **B**, **C** or **D**.

A 6.25 min **B** 7 min **C** 12.8 min **D** 16 min

9 The amount of interest, I, earned on an investment account varies directly with the size of the deposit, D. PS R

a If Hannah earns $35 interest on an investment of $1400, write an equation relating I to D.

b How much will she earn on an investment of $2500?

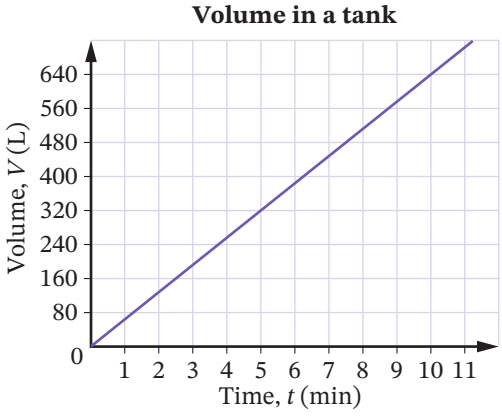
c If Hannah doubles the size of her investment in **b**, how much will she earn in interest?

10 The volume V (in litres) of water in a tank being filled is proportional to the time taken t (in minutes), as shown on the graph.

a Find the formula for V.

b Calculate the amount of water in the tank after 8 minutes.

c Calculate how long it takes to pump 960 L of water.



11 The weight of an astronaut on Mars is proportional to his weight on Earth. A 72 kg astronaut weighs 27.4 kg on Mars. PS R

a Calculate how much a 60 kg astronaut weighs on Mars, correct to 1 decimal place.

b If an astronaut weighs 32 kg on Mars, calculate his weight on Earth, correct to 1 decimal place.

12 The pressure experienced by a scuba diver is directly proportional to her depth under the water. The pressure at 25 m is 147 kilopascals (kPa). PS R

a What pressure is experienced at 40 m?

b At what depth is the pressure 833 kPa?

Investigation

Graphing $y = x^2$ and $y = ax^2$

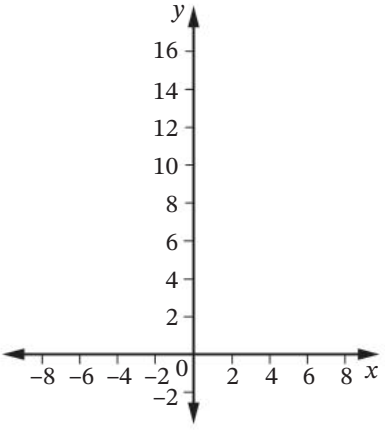
You will need: 1 cm grid paper. This activity can also be completed using graphing software.

1 Copy and complete this table of values for $y = x^2$.

x	−4	−3	−2	−1	−0.5	0	0.5	1	2	3	4
y											

Foundation Standard Complex

2 Draw this number plane on grid paper.



3 Plot the points from the table of values onto the number plane.

4 Draw a smooth curve through all the points and label the graph $y = x^2$.

5 Compare your graph with those of other students.

6 List as many features as you can about the graph of $y = x^2$ that you have drawn. Compare your list with those of other students.

7 Copy and complete this table of values for $y = 2x^2$.

x	−3	−2	−1	0	1	2	3
y							

8 Graph $y = 2x^2$ on the same number plane as $y = x^2$ and label it.

9 How is the graph of $y = 2x^2$ different from the graph of $y = x^2$? Give an explanation for the difference.

10 On the same set of axes, draw these graphs:

a $y = x^2$ **b** $y = \frac{1}{2}x^2$ **c** $y = \frac{1}{3}x^2$

11 What do you notice about the graphs you have drawn? Give an explanation.

12 Compare your observations and explanation with those of other students.

Investigation

Graphing $y = ax^2 + c$

You will need: 1 cm grid paper. This activity can also be completed using graphing software.

1 Graph $y = x^2$ on a number plane for values of x from −3 to 3.

2 Copy and complete this table of values for $y = x^2 + 2$

x	−3	−2	−1	0	1	2	3
y							

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3 Graph $y = x^2 + 2$ on the same number plane as $y = x^2$ and label it.

4 What do the graphs of $y = x^2$ and $y = x^2 + 2$ have in common?

5 How are the 2 graphs different?

6 Describe how the graph of $y = x^2 + 2$ could be drawn using the graph of $y = x^2$.

7 Graph each equation on the same number plane. Label your graphs clearly.

a $y = x^2 - 1$ **b** $y = x^2 + 4$ **c** $y = x^2 - 3$

8 Write a sentence to describe the effect of the constant term (which is added to, or subtracted from, x^2) on the shape and position of the parabola $y = x^2$.

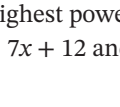
9 Graph each equation on the same number plane. Label your graphs clearly.

a $y = -x^2$ **b** $y = -x^2 + 2$ **c** $y = -x^2 - 3$ **d** $y = -x^2 - 1$

10 What do you notice about the shape and position of each of the graphs in question **9**? Compare your results with those of other students.

Graphing quadratic equations 11.09

In Chapter 7, *Equations*, we learnt that an equation in which the highest power of the variable is 2 is called a **quadratic equation**, for example, $y = 2x^2 - 5$, $y = x^2 + 7x + 12$ and $y = -5x^2$.



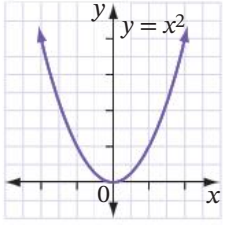
Graphing parabolas

Curve sketcher

Curve sketcher

The graph of $y = x^2$

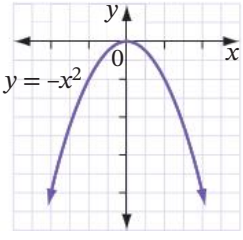
The graph of $y = x^2$ is a smooth U-shaped curve called a **parabola**.



- The axis of symmetry, called the **axis of the parabola**, is the y-axis.
- The **vertex** or **turning point** is (0, 0).
- Because x^2 is always positive or zero, the minimum value of the parabola is 0.
- The parabola's shape is **concave up** (looks like a smile ☺).

The graph of $y = -x^2$

The graph of $y = -x^2$ is the parabola $y = x^2$ reflected in the x-axis, because $-x^2$ is the 'negative' of x^2.

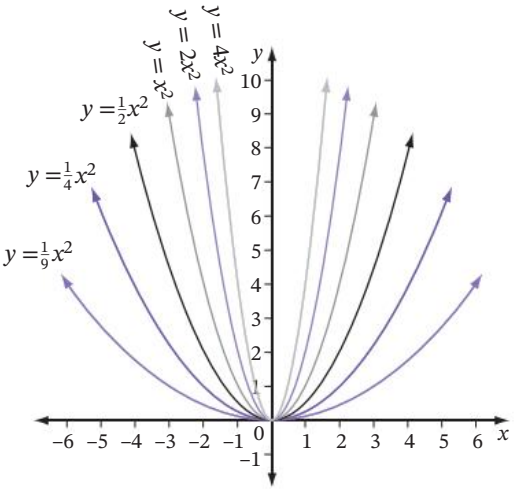


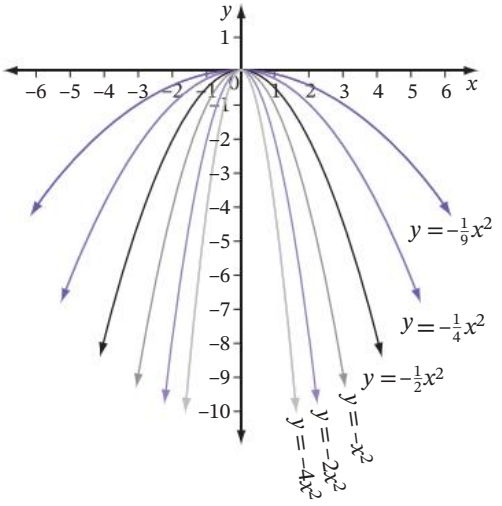
- The **axis of the parabola** is the y-axis.
- The **vertex** or **turning point** is (0, 0).
- Because $-x^2$ is always negative or zero, the maximum value of the parabola is 0.
- The parabola's shape is **concave down** (looks like a frown ☹).

The graph of $y = ax^2$

For the graph of a quadratic equation in the form $y = ax^2$, where a is a constant (number), the size of a affects whether the parabola is 'wide' or 'narrow'.

As the size of a increases, the parabola becomes 'narrower' and as the size of a decreases, the parabola 'widens'. If a is negative, then the parabola is concave down.





Example 20

a Graph $y = -x^2$ and $y = -2x^2$ on the same set of axes.

b What is the vertex of each parabola?

c Are the parabolas concave up or concave down?

d At what point do the 2 parabolas meet?

e Which parabola is 'wider'?

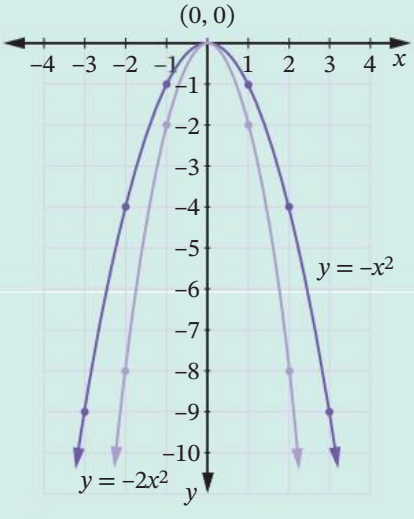
Solution

a $y = -x^2$

x	−3	−2	−1	0	1	2	3
y	−9	−4	−1	0	−1	−4	−9

$y = -2x^2$

x	−3	−2	−1	0	1	2	3
y	−18	−8	−2	0	−2	−8	−18



b The vertex of both parabolas is (0, 0).

c The parabolas are both concave down.

d The parabolas meet at (0, 0).

e The parabola $y = -x^2$ is wider.

The graph of $y = ax^2 + c$

For the graph of a quadratic equation in the form $y = ax^2 + c$, where a and c are constants, the effect of c is to move the parabola up or down from the **origin**. Also, c is the y-intercept of the parabola.

Example 21

a Graph $y = x^2$, $y = x^2 + 2$, $y = -x^2 - 1$ and $y = -x^2 - 4$ on the same set of axes.

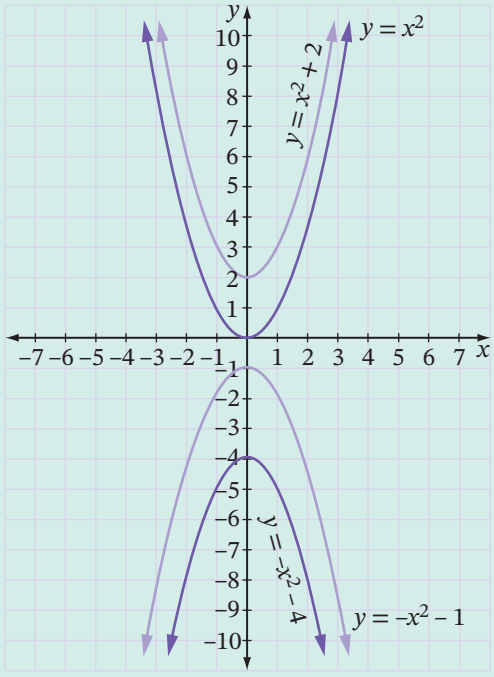
b Compare each parabola with the parabola $y = x^2$.

c What is the vertex of each parabola?

d Is each parabola concave up or concave down?

Solution

a



b $y = x^2 + 2$ is $y = x^2$ shifted up 2 units.

$y = -x^2 - 1$ is $y = x^2$ turned upside down and shifted down 1 unit.

$y = -x^2 - 4$ is $y = x^2$ turned upside down and shifted down 4 units.

c $y = x^2$ has vertex $(0, 0)$.

$y = x^2 + 2$ has vertex $(0, 2)$.

$y = -x^2 - 1$ has vertex $(0, -1)$.

$y = -x^2 - 4$ has vertex $(0, -4)$.

d $y = x^2$ and $y = x^2 + 2$ are concave up.

$y = -x^2 - 1$ and $y = -x^2 - 4$ are concave down.

EXERCISE 11.09 ANSWERS ON P. 601

Graphing quadratic equations U F R C

The questions in this exercise may also be completed using graphing software.

EXAMPLE 20

1 **a** Graph $y = 2x^2$ on a number plane after copying and completing this table of values.

x	−2	−1	0	1	2
y					

b Graph $y = \frac{1}{2}x^2$ on the same number plane after copying and completing this table of values.

x	−2	−1	0	1	2
y					

c Compare both parabolas. What features are the same? What is different? R C

2 Which statement is correct about the parabola $y = x^2$? Select **A**, **B**, **C** or **D**.

A It has a y-intercept of 1.
B It is symmetrical about the x-axis.
C It is concave down.
D Its vertex is (0, 0).

3 **a** Graph $y = -3x^2$ and $y = -0.1x^2$ on the same number plane after copying and completing this table of values for each equation. R C

x	−2	−1	0	1	2
y					

b Compare both parabolas. What features are the same? What is different?

4 Graph each pair of parabolas on the same set of axes and state: R C

i the vertex of each parabola
ii whether each parabola is concave up or concave down
iii which parabola is 'wider'
iv the point at which the 2 parabolas meet.

a $y = x^2$ and $y = \frac{1}{4}x^2$
b $y = 2x^2$ and $y = -2x^2$
c $y = -3x^2$ and $y = -\frac{1}{2}x^2$
d $y = 5x^2$ and $y = 0.2x^2$

5 Which statement is correct about the parabola $y = -6x^2$? Select **A**, **B**, **C** or **D**.

A It has a y-intercept of −6.
B It passes through (−1, 6).
C It is concave down.
D Its vertex is (0, −6).

6 Graph each pair of parabolas on the same set of axes and state: R C

EXAMPLE 21

i the vertex of each parabola

ii whether each parabola is concave up or concave down

iii which parabola is 'wider'

iv the y-intercept of each parabola.

a $y = x^2 + 1$ and $y = x^2 + 3$

b $y = -x^2 - 2$ and $y = 2x^2 + 4$

c $y = -5x^2 - 2$ and $y = -2x^2 - 1$

d $y = -3x^2 - 4$ and $y = 2x^2 + 2$

7 Which statement is correct about the parabola for $y = x^2 + 2$? Select **A**, **B**, **C** or **D**.

A It has a maximum value of 2.

B It is symmetrical about the x-axis.

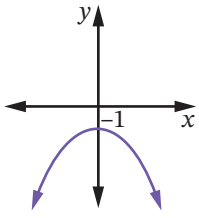
C It is concave up.

D Its vertex is $(2, 0)$.

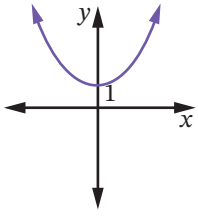
8 Match each quadratic equation to its parabola.

a $y = x^2 - 1$ **b** $y = x^2 + 1$ **c** $y = -x^2 - 1$ **d** $y = -x^2 + 1$

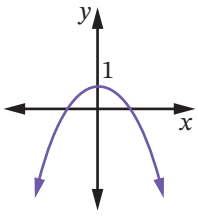
A



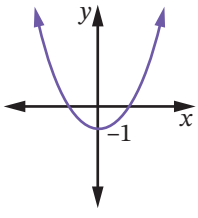
B



C



D



11.09

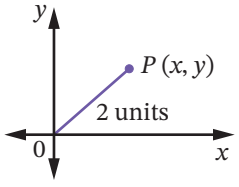
Graphing circles

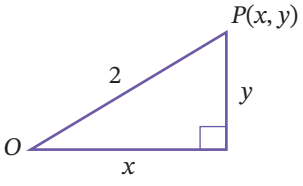
11.10

A circle is a completely round shape. Every point on a circle is exactly the same distance from its centre. We can use this fact to find the equation of a circle with a radius of 2 units.

Let $P(x, y)$ be any point so that the distance $OP = 2$ units and O is the origin.

If we plotted every possible position of P, we would graph a circle centred at O with a radius of 2. We can use Pythagoras' theorem to find the equation of this circle by drawing a right-angled triangle with OP as the hypotenuse.





Since P has coordinates (x, y), the triangle must have a base length of x and a height of y, so by Pythagoras' theorem:

$x^2 + y^2 = 2^2 = 4$

$\therefore$ The equation of a circle with centre $(0, 0)$ and radius 2 is $x^2 + y^2 = 4$.

This can be generalised for a circle of any radius.

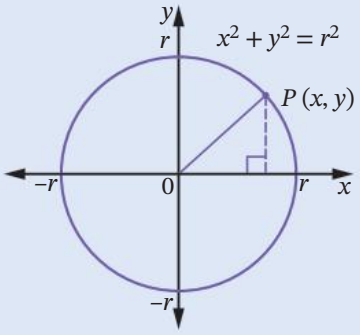
Curve sketcher

Curve sketcher

Foundation Standard Complex

Equation of a circle with centre (0, 0)

The **equation of a circle** with centre (0, 0) and radius r units is $x^2 + y^2 = r^2$.



We can use this formula to find the centre and radius of any circle.

Example 22

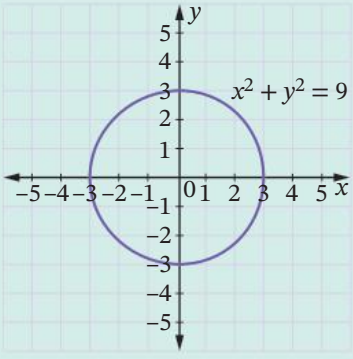
Find the centre and radius of the circle $x^2 + y^2 = 9$ and sketch its graph.

Solution

The centre is (0, 0). The radius is r, where $r^2 = 9$.

$r = \sqrt{9}$

$\therefore$ Radius, $r = 3$ units



Example 23

Find the equation of this circle.

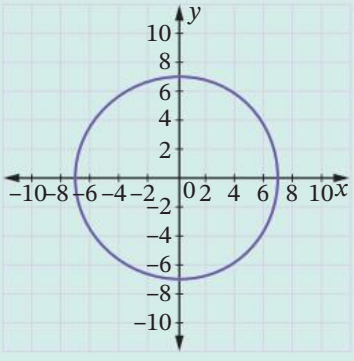
Solution

The circle has a radius of 7 units and the centre is (0, 0).

The equation of the circle is

$x^2 + y^2 = 7^2$

$x^2 + y^2 = 49$



EXERCISE 11.10 ANSWERS ON P. 603

Graphing circles U F

The questions in this exercise may also be completed using graphing software.

EXAMPLE 22

1 Find the centre and radius of the circle given by each equation and sketch its graph.

a $x^2 + y^2 = 4$ **b** $x^2 + y^2 = 1$ **c** $x^2 + y^2 = 81$

d $x^2 + y^2 = 100$ **e** $x^2 + y^2 = 25$ **f** $x^2 + y^2 = \frac{1}{4}$

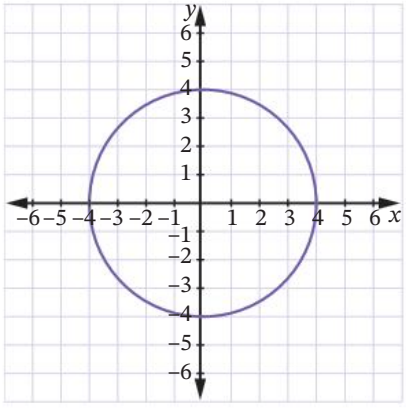
2 What is the equation of a circle with centre (0, 0) and radius 6 units?
Select the correct answer **A**, **B**, **C** or **D**.

A $x^2 + y^2 = 12$ **B** $x^2 + y^2 = 6$ **C** $x^2 + y^2 = 8$ **D** $x^2 + y^2 = 36$

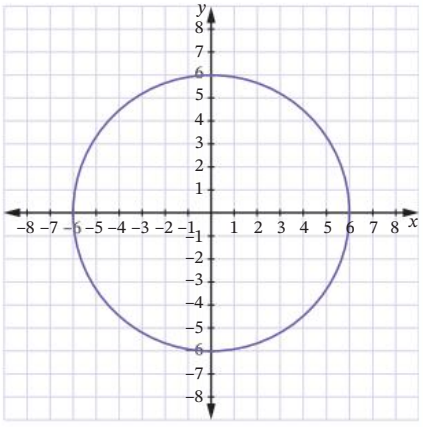
EXAMPLE 23

3 Find the equation of each circle shown.

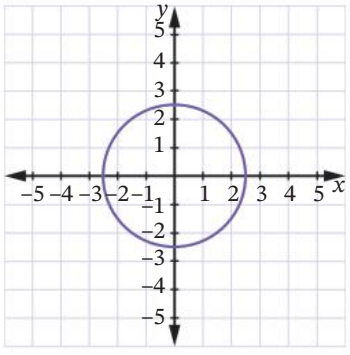
a



b



c



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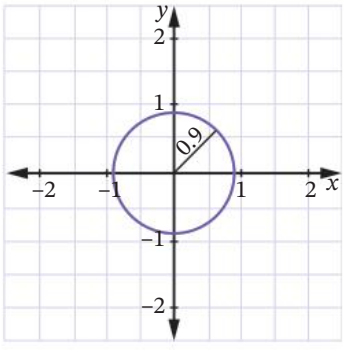


4 What is the equation of a circle with centre (0, 0) and radius 1.3 units? Select **A**, **B**, **C** or **D**.

A $x^2 + y^2 = 2.6$ **B** $x^2 + y^2 = 1.69$ **C** $x^2 + y^2 = 1.72$ **D** $x^2 + y^2 = 3.69$

5 What is the radius of a circle with equation $x^2 + y^2 = 144$? Select **A**, **B**, **C** or **D**.

A 12 units **B** 14 units **C** 36 units **D** 72 units

6 What is the equation of a circle with centre (0, 0) and diameter 16 units?

Foundation | Standard | Complex

7 What is the radius of a circle with equation $x^2 + y^2 = \frac{9}{4}$? Select **A**, **B**, **C** or **D**.

A $2\frac{1}{4}$ units **B** $1\frac{1}{2}$ units **C** $\frac{3}{4}$ units **D** $\frac{7}{8}$ units

8 What is the diameter of the circle whose equation is $x^2 + y^2 = 196$?

9 **a** What is the radius of the circle whose equation is $x^2 + y^2 = 12$?

b Sketch the graph of $x^2 + y^2 = 12$

Finding coordinates for given segment lengths

Power plus ANSWERS ON P. 603

1 $ABCD$ is a parallelogram. The coordinates of A, B and C are $(-1, 4)$, $(4, 6)$ and $(2, 7)$ respectively. Find the coordinates of D.

2 A line drawn through points $A(0, -2)$ and $B(3, 0)$ also goes through point C with x-coordinate 9. Find:

a the equation of the line

b the y-coordinate of C.

3 $M(1, 1)$ is the midpoint of the interval XY. If the coordinates of X are $(-3, -2)$, what are the coordinates of Y?

4 A circle is drawn with its centre at $C(2, 3)$. The point $A(-1, -5)$ lies on the circumference.

a Calculate the radius of the circle.

b The point P is 10 units from C. Explain why P lies outside the circle.

5 The points of $R(2, 19)$, $S(-3, 7)$ and $T(10, 7)$ are joined to form a triangle. Show that the triangle is isosceles.

6 The point $(2, 1)$ is a vertex of a square. $M(5, 1)$ is the point of intersection between the diagonals of the square. Find the coordinates of the other 3 vertices of the square.

CHAPTER 11 REVIEW

Language of maths

axis/axes	constant of proportionality	direct proportion
distance	exact	formula
gradient	horizontal	interval
length	linear equation	midpoint
number plane	parabola	quadratic equation
radius	rise	run
surd	varies directly	vertex
vertical	x–intercept	y–intercept

1 What is the name given to the graph of:

a a quadratic equation? **b** a linear equation?

2 Is the line $y = 3$ horizontal or vertical?

3 What is another name for the steepness of a line?

4 What is the value at which a line crosses the y-axis called?

5 What are the coordinates of the **vertex** of the parabola with equation $y = -x^2$?

6 If $y = kx$, what is the constant of proportionality?

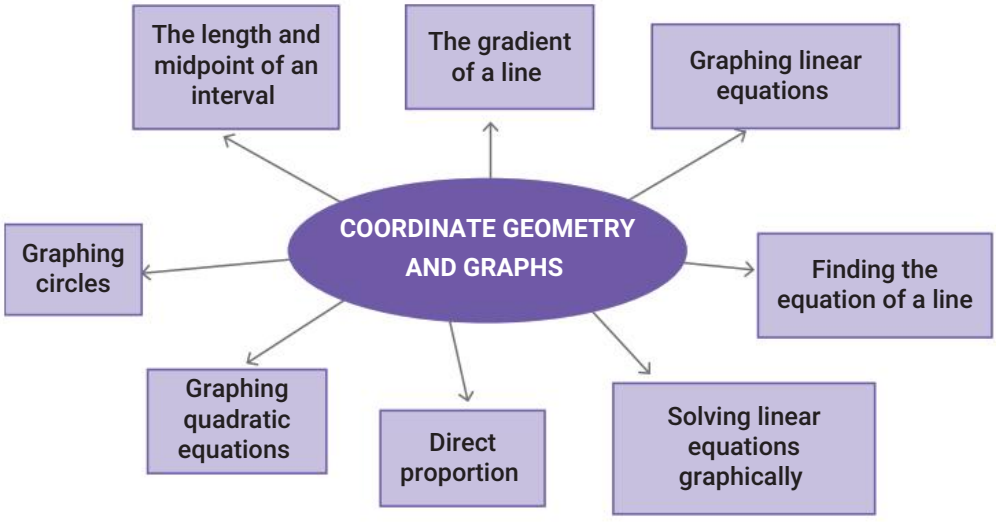
7 When the length of an interval needs to be given as an **exact** length, what does this mean?

8 What is the equation of the x-axis?

Topic summary

- In your own words, describe what a gradient is.
- What parts of this topic did you find difficult?

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



WS
Mind map: Coordinate geometry and graphs (Advanced)

TEST YOURSELF 11 ANSWERS ON P. 603

11.01 **1** Find the length of the interval joining the points $H(7, 2)$ and $T(-2, 6)$. Express your answer:

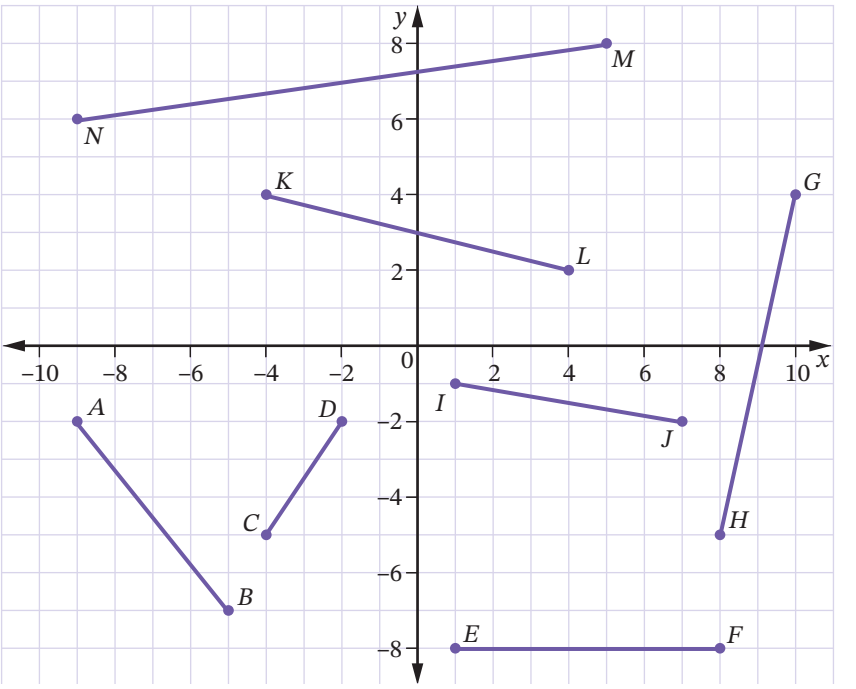
a as a surd **b** correct to one decimal place.

11.02 **2** Find the coordinates of the midpoint of the interval joining:

a (8, 4) and (−2, 10) **b** (−1, −1) and (−7, 9) **c** (3, −6) and (−5, 10)

11.03 **3** **a** Which intervals below have a gradient that is:

i positive? **ii** negative? **iii** 0?



b Find the gradient of:

i interval NM **ii** interval AB **iii** interval KL

c Find the midpoint of interval HG.

d Find the exact length of interval CD.

11.03 **4** Find the gradient of the interval joining:

a (1, 9) and (3, 5) **b** (6, 0) and (8, −4) **c** (−1, −1) and (5, −3)

11.04 **5** Graph $y = -3x + 6$ on a number plane and write:

a its x-intercept **b** its y-intercept.

11.04 **6** Test whether the point (4, −2) lies on the line with equation:

a $y = x - 6$ **b** $y = -2x + 6$ **c** $x - 3y = 10$

Foundation Standard Complex

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7 **a** Graph and label each line on the same number plane: 11.04

$x = -4, \quad x = 3, \quad y = -6, \quad y = 2$

b What is the area of the rectangle enclosed by these lines?

8 Write the equation of a line that is: 11.04

a horizontal with a y-intercept of 4

b parallel to the y-axis and passes through $(-1, 7)$

c the x-axis

9 Find the gradient and y-intercept of the line with equation: 11.05

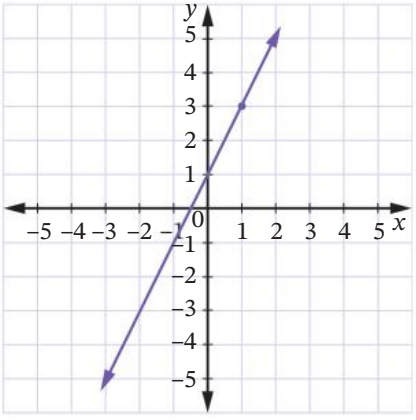
a $y = 3x + 1$ **b** $y = 8 - x$ **c** $y = \frac{x}{5} + 4$ **d** $y = -6x$

10 Graph each linear equation by finding the gradient and y-intercept first. 11.05

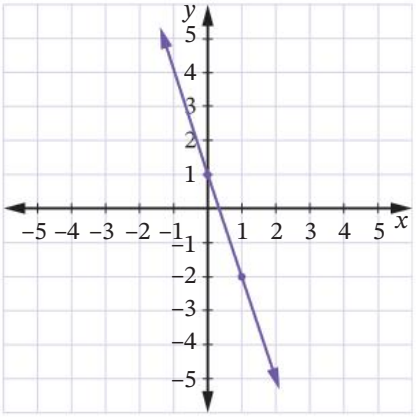
a $y = -3x - 1$ **b** $y = \frac{x}{2} + 6$

11 Find the equation of each line. 11.06

a



b



12 Solve the equation $3x - 4 = 5$ graphically. 11.07

13 During a thunderstorm, the time, T seconds, between seeing lightning and hearing thunder is directly proportional to the distance, d kilometres, of the observer from the centre of the storm. Thunder is heard 6 seconds after the lightning is seen when a storm is 35 km away. 11.08

a Find a formula for T in terms of d.

b Calculate, correct to the nearest kilometre, the distance of a storm when thunder is heard 10 seconds after the lightning.

14 Graph $y = x^2 + 1$ and $y = -2x^2$ on the same set of axes and state: 11.09

a the vertex of each parabola

b whether each parabola is concave up or concave down

c which parabola is 'wider'.

15 Which equation below represents a concave up parabola with vertex $(0, 0)$? Select **A**, **B**, **C** or **D**. 11.09

A $y = x^2 - 1$ **B** $y = 2x^2$ **C** $y = -x^2$ **D** $y = 3x^2 + 2$

16 Find the centre and radius of the circle with equation $x^2 + y^2 = 16$ and sketch its graph. 11.10

Foundation | Standard | Complex

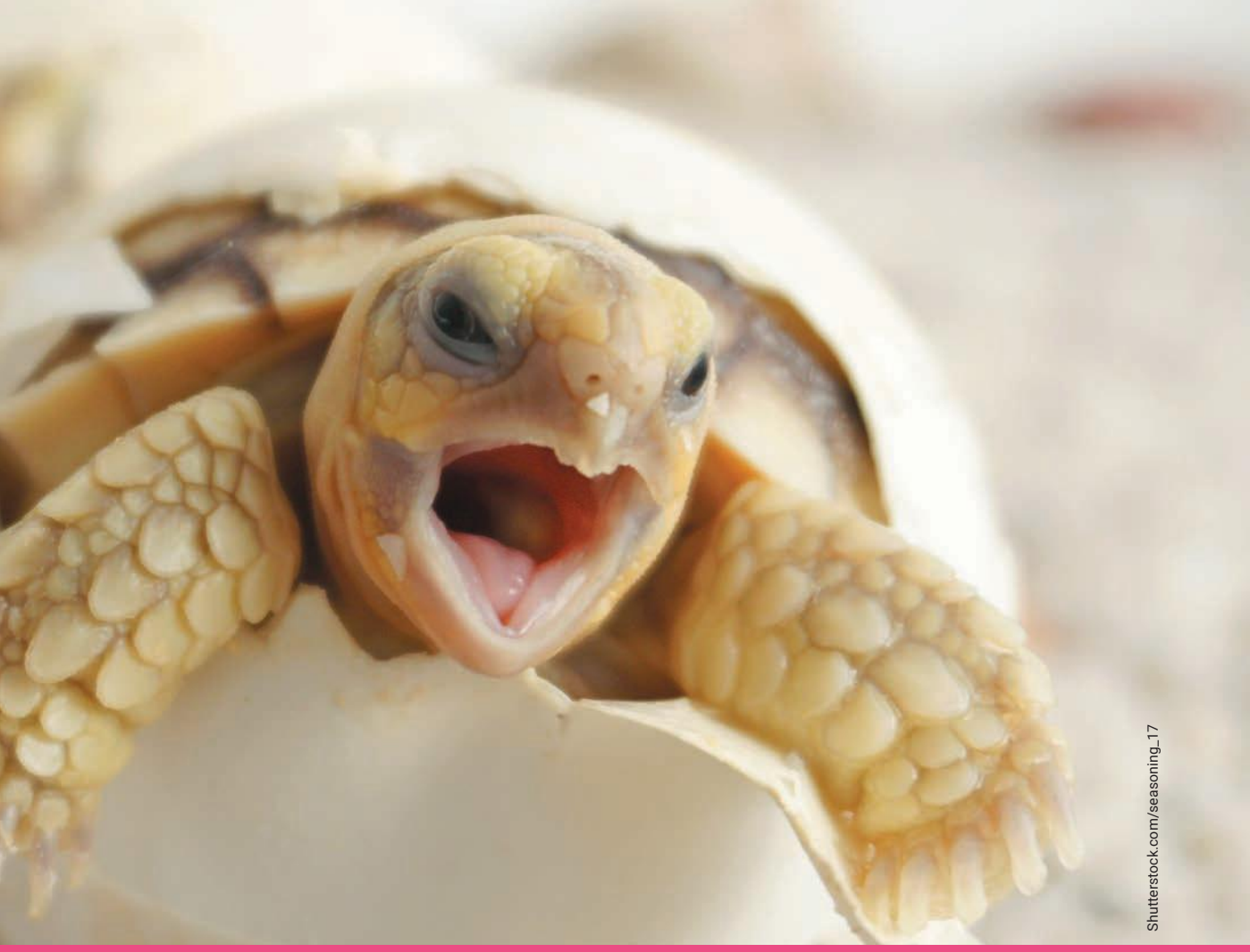
12

STATISTICS AND PROBABILITY

PROBABILITY

Probability, the study of chance, was first used in France to solve problems in gambling in the 1600s. In 1654, the writer and gambler, the Chevalier de Méré, asked mathematician Blaise Pascal many questions involving gambling, including how many rolls of a pair of dice were required to have a better than 50% chance of rolling a double 6. Pascal wrote to another mathematician, Pierre de Fermat, about the problems, and the correspondence between them is regarded to be the beginning of probability theory.

Probability is now used widely in many areas, including determining the survival rate of newborn turtles, predicting the sales of a new version of smartphone and forecasting the weather for a week ahead.



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Chapter outline

	Working mathematically				
12.01 Probability	U	F	PS	R	C
12.02 Relative frequency	U	F		R	C
12.03 Venn diagrams	U	F		R	C
12.04 Two-way tables	U	F		R	C
12.05 Tree diagrams	U	F	PS	R	C

Wordbank

complementary event The 'opposite' event, for example the complementary event to selecting an Ace from a deck of cards is selecting a card that is not an Ace

relative frequency The frequency of an event over repeated trials as a fraction of the total number of trials

tree diagram A diagram that uses branches to list all the possible outcomes in a multi-step chance experiment

trial One go or run of a repeated probability experiment, for example, one roll of a die

two-step experiment A chance experiment with 2 steps or stages, such as rolling a pair of dice

two-way table A way of grouping items into 2 overlapping categories, such as gender and the ability to drive a car

Venn diagram A diagram of circles (usually overlapping) for grouping items into categories

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

Probability

In this chapter you will:

- calculate the probabilities and relative frequencies of events, including complementary events
- describe compound events using terminology such as 'and', exclusive 'or', inclusive 'or' and 'at least'
- use Venn diagrams and two-way tables to represent sample spaces and compound events to solve probability problems
- use tree diagrams and tables (arrays) to represent the sample space of two-step chance experiments, with and without replacement, to solve probability problems

SkillCheck ANSWERS ON P. 604

1 A bag contains 3 green lollies, 4 red lollies, 5 white and 8 yellow lollies.

a What fraction of the lollies are red?

b What percentage of the lollies are yellow?

c What percentage of the lollies are not yellow?

d What fraction of the lollies are green or yellow?

2 If a die is rolled, which one of the following events is most likely?
Select the correct answer **A**, **B**, **C** or **D**.

A an even number **B** a number greater than 2

C a number greater than 3 **D** a number greater than 4

3 A card is drawn at random from a deck of cards. What is the probability that it is:

a a 7? **b** a heart card? **c** a black card?

4 Match each description (**a** to **f**) below with the correct set of all whole numbers (**A** to **F**) that meet that description.

a between 3 and 10 **b** **at least** 1 **c** more than 4

d at most 3 **e** 5 or less **f** not more than 4

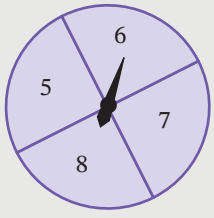
A {1, 2, 3, 4, 5} **B** {4, 5, 6, 7, 8, 9} **C** {1, 2, 3}

D {1, 2, 3, 4} **E** {1, 2, 3, ...} **F** {5, 6, 7, ...}

5 A die is rolled. Find the probability that the number that comes up is:

a 1 **b** more than 2 **c** odd **d** composite

6 **a** Write the sample space for this spinner.



b Find, as a percentage, the probability that the number spun is:

i 5 **ii** at least 5

iii 5 or less **iv** more than 5

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Probability

12.01

The language of probability

WS Games of chance

12.01

A **chance experiment** is an activity or process that involves chance, for example, rolling a die or tossing a coin.

An **outcome** is the result of a chance experiment. For example, when rolling a die, there are 6 possible outcomes – the numbers 1 to 6.

The **sample space** is the set of all possible outcomes of an experiment. When rolling a die, the sample space is {1, 2, 3, 4, 5 and 6}.

An **event** is one or more outcomes of an experiment. When rolling a die, one event may be rolling an odd number, which consists of the 3 outcomes 1, 3 and 5.

A **trial** is one go or run of an experiment, for example, one roll of the die.

In a **random** experiment, every possible outcome has an equal (fair) chance of occurring.

Probability of an event

$P(E)$ means 'the **probability** of an event, E (occurring)'. If all possible outcomes are **equally likely**, then:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

or $$P(E) = \frac{\text{number of outcomes matching } E}{\text{number of outcomes in the sample space}}$$

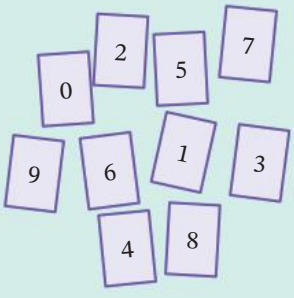
A **favourable outcome** is one of the outcomes in the event that you want, whose probability you are calculating.

Example 1

A card is chosen at random from a set of cards numbered 0 to 9.

Find the probability of selecting:

a the card numbered 8
b a number less than 4
c a number 5 or more



Solution

There are 10 outcomes in the sample space.

a $P(8) = \frac{1}{10}$ — 1 chance in 10

b Out of the 10 numbers, there are 4 numbers less than 4: {0, 1, 2, 3}.

$P(\text{less than } 4) = \frac{4}{10}$ — 4 chances in 10

$= \frac{2}{5}$

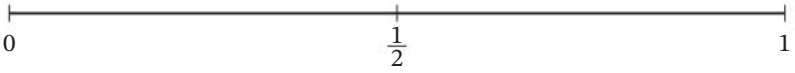
c Out of the 10 numbers, there are 5 numbers that are 5 or more: {5, 6, 7, 8, 9}.

$P(5 \text{ or more}) = \frac{5}{10}$ 5 chances in 10

$= \frac{1}{2}$

The range of probability

The probability of an event can be written as a fraction, decimal or percentage. Probability is measured on a scale from 0 to 1, or as a percentage from 0% to 100%. If an event has a probability of 0, it is **impossible** and it will not occur. If an event has a probability of 1, it is **certain** and it *must* occur.



impossible event

(For example, rolling a 7 on a die)

even chance

(For example, tossing tails on a coin)

certain event

(For example, the Sun will rise in the east)

Complementary events

Complementary events

Complementary events are 2 events that together make up all the possible outcomes, such as a head and a tail when tossing a coin. The **complement** of an event E is made up of those outcomes in the sample space that are 'not E', usually written as $\bar{E}$. Because an event and its complement covers all possible outcomes in the sample space, the sum of their probabilities must equal 1.

Complementary events

$P(\text{event}) + P(\text{complementary event}) = 1$

$P(E) + P(\bar{E}) = 1$

$P(\bar{E}) = 1 - P(E)$, where $\bar{E}$ is the complement of E.

Example 2

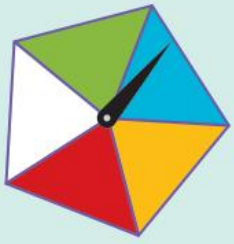
a List the sample space when this spinner is spun.

b Find $P(\text{red})$.

c What is the complementary event to the spinner landing on red?

d List the outcomes in the complementary event.

e Find $P(\text{not red})$.



Solution

a Sample space = {red, yellow, blue, green, white}

b $P(\text{red}) = \frac{1}{5}$

c The complementary event to landing on red is landing on a colour other than red.

d The outcomes in the complementary event are {yellow, blue, green, white}.

e $P(\text{not red}) = 1 - P(\text{red})$

$$= 1 - \frac{1}{5}$$

$$= \frac{4}{5}$$

EXERCISE 12.01 ANSWERS ON P. 604

Probability U F PS R C

12.01

EXAMPLE 1

1 A letter is drawn at random from the alphabet. PS R

a How many outcomes are there in the sample space?

b Are all the outcomes equally likely?

c What is the probability of choosing a vowel?

d What is the probability of choosing a letter from the word PROBABILITY?

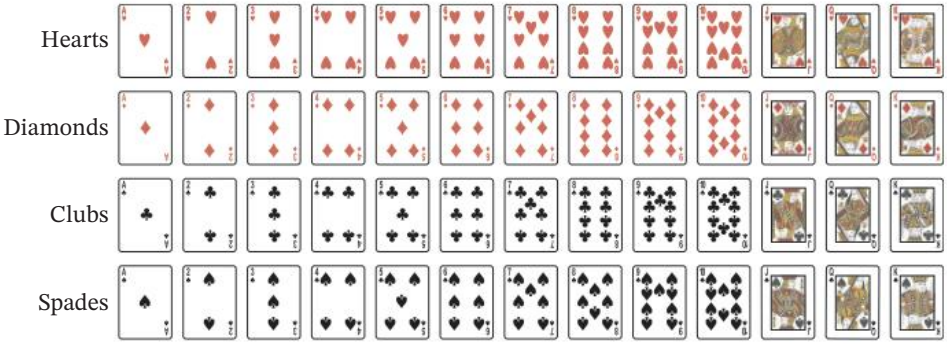
2 This spinner is spun. What is the decimal probability that the needle points to:

a yellow?

b black?

c red?

3 A normal pack of playing cards is divided into 4 suits as shown. PS R



A card is selected at random. Find the probability that it is:

a a black card

b an ace

c an even-numbered card

d a picture card

4 A die is rolled. What is the probability of rolling a number that is: PS R

a greater than 2?

b prime?

c a factor of 6?

d less than 4?

e greater than 6?

f less than 7?

5 A bag contains 15 red jelly beans and 20 blue jelly beans. One jelly bean is drawn at random from the bag. What is the probability it will be:

a blue?

b yellow?

c red or blue?

6 A student is selected at random from this group: Robert, Samantha, James, Joshua, Hayley, Jasmine, Caitlin, Dianne, Sarah. What is the probability of selecting a student: PS R

a who is male?
b who is female?
c whose name begins with J?
d whose name has one syllable?
e whose name contains an E?

7 A bag contains 14 black pens and 9 red pens. One pen is drawn at random from the bag. Calculate, as a percentage, the probability that it is:

a black **b** red **c** black or red **d** blue

8 A die with faces marked 1, 1, 2, 3, 4 and 5 is rolled. Find the probability of rolling:

a 1
b 5
c an odd number
d a number greater than 1

9 A bag contains 35 blue badges, 45 green badges and 20 yellow badges. A badge is drawn at random. Calculate, as a percentage, the probability that it is:

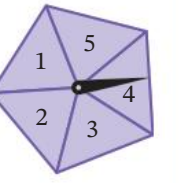
a blue
b green
c not green
d neither blue nor yellow

10 For each event, write the complementary event. R C

a Selecting a consonant from the alphabet
b The weather being sunny tomorrow
c A traffic light showing red
d Selecting a spade from a normal deck of cards
e Hitting bullseye on a dartboard
f Selecting a blue ball from a bag containing blue, red and yellow balls

EXAMPLE 2

11 This spinner is spun. R C



a List the sample space.
b Find $P(4)$.
c What is the complementary event to spinning a 4?
d List the outcomes in the complementary event.
e If $P(\overline{4})$ means $P(\text{not } 4)$, find $P(\overline{4})$.

12 Dilone buys a ticket in a raffle in which 50 tickets are sold and there is only one prize.

a What is Dilone's chance of winning the prize? Write your answer as a:
i fraction **ii** decimal **iii** percentage

b Calculate the probability of Dilone not winning the prize. Write your answer as a:
i fraction **ii** decimal **iii** percentage

13 Tracey buys tickets in a raffle in which 200 tickets are sold and there is only one prize. She has a 40% chance of winning the prize. PS R

a What is the probability that Tracey will not win the prize?

b How many tickets did Tracey buy?

14 Tamin bought a ticket in a raffle in which 500 tickets were sold. If there are 5 prizes, what is the probability that Tamin does not win a prize? Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{1}{100}$ **B** 49% **C** $\frac{499}{500}$ **D** 0.99

15 The probability of rain on Tuesday is 0.35. What is the probability that it will not rain?

16 What is the probability that a student chosen at random from your school was not born in a month beginning with the letter M?

17 Stephanie has a 76% chance of hitting the bullseye when playing darts. What is the probability that she misses the bullseye? Select **A**, **B**, **C** or **D**.

A 24% **B** 26% **C** 34% **D** 76%

18 What is the probability that a mobile phone number selected at random does not begin with a 0?

12.01

Did you know?

The history of playing cards

Bridgeman Images/DeAgostini Picture Library/A. Dagli Orti

The earliest European cards were Tarot packs, which contained either 78 or 97 cards, and were used to play a game called Tarocco. The smaller 52-card pack may have become popular for practical reasons because cards were originally drawn and painted by hand. The names and symbols on cards have changed many times but in the 14th century a pack was introduced with the symbols for hearts, spades, diamonds and clubs, the symbols which were eventually adopted by most Western countries. The names for the symbols still differ from country to country.

The pictures used for the Kings, Queens and Jacks have undergone some changes. The 4 Kings originally represented Charlemagne (hearts), the emperor of most of Europe until 814 CE, David (spades), the Hebrew king mentioned in the Bible, Julius Caesar (diamonds) of ancient Rome and Alexander the Great (clubs) of ancient Greece.

Investigate who the original Queens and Jacks were.

Foundation Standard Complex

12.02 Relative frequency

Experimental probability

Coins probability

Dice probability

Probability review

Relative frequencies

Final 6 investigation

Probability calculated using the formula:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

is more specifically called the **theoretical probability**.

We can also determine probability based on the results of a trial that has been repeated many times, such as testing the effectiveness of 100 light globes, or rely on past statistics, such as weather patterns or the ages of drivers having car accidents. This type of probability is called **experimental probability** or **observed probability**, which is based on **relative frequency**, the number of times an event occurred as a fraction of the total frequency of outcomes.

Experimental probability

$$P(E) = \frac{\text{number of times the event happened}}{\text{total number of trials}}$$

$$\text{or } P(E) = \frac{\text{frequency of } E}{\text{total frequency}}$$

Expected frequency is the expected number of times an event will occur over repeated trials.

Expected frequency

Expected frequency = theoretical probability × number of trials.

Example 3

Connor tossed a coin 100 times and recorded the results in a table.

Outcome	Frequency
Head	46
Tail	54
	100

a Calculate as a decimal:

- **i** the experimental probability of tossing a head
- **ii** the theoretical probability of tossing a head.

b Are the experimental and theoretical probabilities similar?

c For 100 tosses, what is the expected frequency of heads based on the theoretical probability? How does this compare with the observed frequency?

Solution

a i Experimental probability: $P(H) = \frac{46}{100} = 0.46$

ii Theoretical probability: $P(H) = \frac{1}{2} = 0.5$

b By comparing the decimals for the 2 answers, we see that the experimental and theoretical probabilities are similar.

c Expected frequency of heads $= 0.5 \times 100$ probability × number of trials

$= 50$

From the table, the observed frequency of heads = 46, which is close to 50.

Note: When calculating experimental probability, the results will not always be consistent with what is expected, unless a very large number of trials are performed.

Example 4

Briana rolled a die 200 times and recorded her results.

Outcome	Frequency
1	38
2	40
3	29
4	24
5	35
6	34

a Find the experimental probability of rolling:

- **i** an even number
- **ii** an even number or a number greater than 2
- **iii** an even number and a number greater than 2
- **iv** an even number not greater than 2.

b Calculate the probability of rolling a 5 or 6:

- **i** as an experimental probability
- **ii** as a theoretical probability.

c What is the expected frequency of rolling a 5 or 6 in 200 rolls of a die? How does this compare with Briana's observed frequency?

Solution

a i Rolls of even numbers $= 40 + 24 + 34$ — Frequencies of 2, 4, 6.

$= 98$

Experimental $P(\text{even}) = \frac{98}{200} = \frac{49}{100}$

ii Rolls of even numbers or numbers greater than 2 $= 40 + 24 + 34 + 29 + 35$ — Frequencies of 2, 4, 6, 3, 5.

$= 162$

Experimental $P(\text{even or} > 2) = \frac{162}{200} = \frac{81}{100}$

iii Rolls of even numbers greater than 2 — Frequencies of 4 and 6.

$= 24 + 34$

$= 58$

Experimental $P(\text{even and} > 2) = \frac{58}{200} = \frac{29}{100}$

iv Rolls of even numbers not greater than 2 $= 40$ — Frequency of 2.

Experimental $P(\text{even and not} > 2) = \frac{40}{200} = \frac{1}{5}$

b i Rolls of 5 or 6 $= 35 + 34$ — Frequencies of 5 and 6.

$= 69$

Experimental $P(5 \text{ or } 6) = \frac{69}{200}$

ii Theoretical $P(5 \text{ or } 6) = \frac{2}{6} = \frac{1}{3}$ — 2 chances out of 6.

c Expected frequency of 5 or 6 $= \frac{1}{3} \times 200$ — probability × number of trials

$= 66.666...$

≈ 67

From the table, the observed frequency $= 35 + 34 = 69$, which is close to 67.

EXERCISE 12.02 ANSWERS ON P. 605

Relative frequency U F R C

EXAMPLE 3

1 Fatima spun this spinner 200 times and recorded the results in the table. R C

Event	Frequency
Red	90
Green	36
Blue	74
	200

a Calculate, as a decimal, the experimental probability that the needle points to:

i red **ii** green

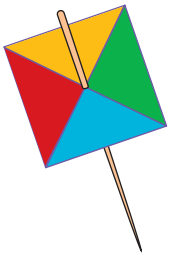
b Calculate, as a decimal, the theoretical probability that the needle points to:

i red **ii** green

c Are the experimental and theoretical probabilities similar?

d For 200 spins, what is the expected frequency of red or blue based on the theoretical probability? How does this compare with the observed frequency?

2 Use cardboard and a toothpick to make a square spinner that has 4 colours as shown in the diagram below. Spin the spinner 50 times and record your results in a table similar to the one shown. R C



Event	Tally	Frequency
Red		
Blue		
Green		
Yellow		

a Calculate, as a percentage, the relative frequency of spinning:

i red **ii** blue

iii a colour that is not yellow **iv** blue or green

b Calculate, as a percentage, the theoretical probability of spinning blue. How does this compare with the relative frequency?

c For 50 spins, what is the expected frequency that blue should come up? How does this compare with the observed frequency?

3 A die is rolled. R C

Event	Tally	Frequency
1		
2		
3		
4		
5		
6		
		100

a Predict the number of times a 2 would appear if the die was rolled 100 times.

b Roll a die 100 times and record your results in a table similar to the one shown.

c What is the relative frequency of rolling a 2?

d How does your predicted frequency compare with the observed frequency?

e What is the relative frequency of rolling an even number?

4 Place 4 black counters, 5 red counters and 1 yellow counter in a bag.
Select a counter at random, record the colour and return the counter to the bag.
Run this experiment 50 times. R C

Event	Tally	Frequency
Black		
Red		
Yellow		

a What is the experimental probability of selecting:

i a black counter?

ii a red counter?

iii a yellow counter?

b What is the theoretical probability of selecting:

i a black counter?

ii a red counter?

iii a yellow counter?

c Compare your experimental probabilities to the theoretical probabilities.

d Repeat the experiment another 50 times and find the relative frequency of each event. Are the results different to the results found in part **a**?

5 A die was repeatedly rolled and the results are shown in the table.

Outcome	Frequency
1	90
2	84
3	92
4	76
5	83
6	75

a What was the total number of trials?

b Find the experimental probability (as a decimal) of rolling:

i an even number

ii a number greater than 3

iii a 3 or a 5

iv a number greater than 2 and even

vii a number greater than 3 or odd.

c Calculate the probability of rolling a number not less than 3:

i as an experimental probability

ii as a theoretical probability.

d How do the 2 probabilities in part **c** compare?

EXAMPLE 4

6 Students at a high school were asked how they travelled to school. The results are presented in the table. Find the probability (correct to 2 decimal places) that a student chosen at random will:

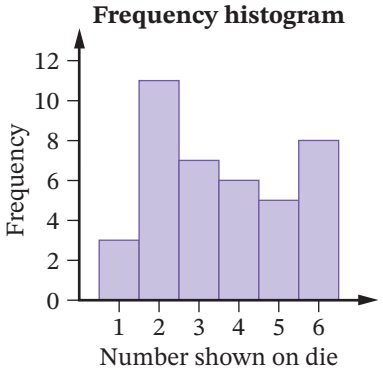
Outcome	Frequency
Walk	186
Bus	295
Car	103
Bicycle	74

a walk **b** be driven

c catch a bus **d** not ride a bicycle

e be driven or catch a bus.

7 A die was rolled many times and the results are shown in the frequency histogram.



a What is the total number of trials?

b Find the experimental probability of rolling:

i 2

ii 5

iii an even number or a number greater than 3.

iv a number greater than 4 and odd.

8 The table shows the number of babies born per day at a hospital during the month of January. On any particular day, find the probability of the birth of exactly:

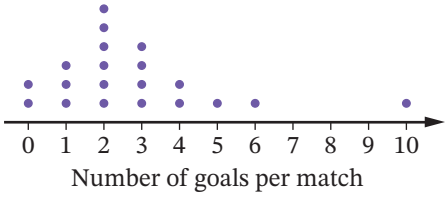
Number of babies per day	Frequency
0	1
1	4
2	7
3	10
4	5
5	4

a 1 baby **b** 3 babies

c 1 or 2 babies **d** **at least** 2 babies

e at most 1 baby **f** 4 or 5 babies.

9 This dot plot shows the number of goals per match scored by a soccer team during the season.



For a match selected from the season at random, find the probability of the team scoring:

a 2 goals **b** 4 goals per game **c** fewer than 2 goals

d 2 or 3 goals **e** at least 3 goals **f** not more than 4 goals

10 A card is drawn randomly from a normal pack of cards to see if it is an Ace. This trial is run 120 times, with each card being replaced in the deck before the next card is drawn. The results are shown in the table. R C

Outcome	Frequency
Ace	11
Not an Ace	109

a Calculate, correct to 3 decimal places:

i the experimental probability of selecting an Ace

ii the theoretical probability of selecting and Ace.

b Are the experimental and theoretical probabilities similar?

c If a card is drawn 300 times, what is the expected frequency of an Ace being drawn? Select **A**, **B**, **C** or **D**.

A 27 **B** 28 **C** 23 **D** 6

12.02

Technology

Throwing a coin

WS Coins probability

In this activity, a spreadsheet is used to simulate the throwing of a coin. The computer can quickly generate either a 1 (representing heads) or a 2 (tails).

Shutterstock.com/Yeti studio

1 In cell A1, enter the label 'Tossing a coin'.

2 In cell A2, enter the formula **=INT(RAND()*2+1)**, press Enter and either 1 or 2 will appear in the cell.

3 Select cell A2 and **Fill Down** to cell A11 to simulate the tossing of a coin 10 times.

4 Copy the table below and in the first blank row record your results for Trials 1–10 (the numbers of heads and tails in your first 10 'tosses').

Trial	Number of heads	Number of tails
1–10		
11–20		
21–30		
31–40		
41–50		
51–60		
61–70		
71–80		
81–90		
91–100		
Total		

5 Simulate another 10 tosses of the coin by making the spreadsheet generate another set of random numbers. Do this by placing the cursor between the tops of columns A and B (so that it turns into a 'double arrow') and clicking.

6 Enter the results for Trials 11–20 in your table.

7 Repeat 8 more times so that you have 100 tosses of the coin recorded in the table.

8 **a** For 100 tosses of a coin, what is the expected frequency of heads based on the theoretical probability?

b Compare this with the actual frequency.

9 Calculate, as a decimal, the theoretical and experimental probabilities of tossing heads.

10 Compare your results with those of other students in your class. Briefly explain any differences and why they may have occurred.

11 Combine the results of students in your class to calculate the experimental probability of throwing heads, as a decimal. How does this compare with the theoretical probability?

WS Dice probability

Technology

Rolling dice

In this activity, we will use a scientific calculator to simulate the rolling of a die.

1 Copy the table shown.

Outcome	Tally	Frequency	Experimental probability
1			
2			
3			
4			
5			
6			

2

Casio	Sharp
Enter the formula RanInt#(1,6) as shown: ALPHA · 1 SHIFT) 6) =	2nd F 7 (RANDOM), select 1 R-DICE, =

This generates a random number from 1 to 6. Press = more times to generate more random numbers. Alternatively, on a spreadsheet, type **=INT(RAND()*6+1)** into a cell and press the Enter key, then the F9 key repeatedly for more random numbers.

3 Run the simulation 50 times and record the results in the table.

4 Calculate the theoretical probability for each number being tossed on the die.

5 Compare the theoretical probability and experimental probability for each number. Are there similarities or differences? How could the experimental probabilities be more accurate? Explain.

6 Copy this table for rolling 2 dice and finding their sum.

Sum	Tally	Frequency	Experimental probability
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			

7 On a scientific calculator, rolling 2 dice is achieved by adding the result of rolling one die twice.

Casio	Sharp
Enter the formula RanInt#(1,6) + RanInt#(1,6) as shown below: ALPHA · 1 SHIFT) 6) + ALPHA · 1 SHIFT) 6) =	Enter the formula R.DICE + R.DICE as shown below: 2nd F 7 (RANDOM), select 1 R-DICE, + 2nd F 7 (RANDOM), select 1 R-DICE, =

This generates the sum of 2 random numbers from 1 to 6.

Press = more times to generate more sums.

8 Run the simulation 50 times and record the results in the table.

9 Which sum is the most likely to occur?

10 Which sum is the least likely to occur? Why?

11 Which sum is theoretically the:

a most likely? **b** least likely?

Why?

12 Compare your answers to questions **9**, **10** and **11** and then compare them with the answers of other students in your class. Write a few sentences explaining your observations.

13 If you ran 50 more simulations, would you expect your results to be similar or different to your first 50 results? Explain.

12.03 Venn diagrams

Venn diagrams

Venn diagrams

Probability review

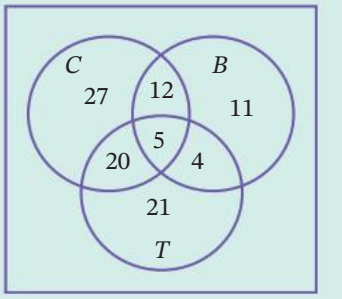
Venn diagrams matching activity

Venn diagrams

A **Venn diagram** is a diagram of circles (usually overlapping) for grouping items into categories. A rectangle represents the whole group, while the circles represent categories. Items common to 2 or more categories are placed in the intersection (overlapping region) of the circles. The Venn diagram was invented in 1880 by English mathematician and priest, John Venn (1834–1923).

Example 5

This Venn diagram shows the results of a survey on how people travel to work (car (C), bus (B) or train (T)).



a How many people participated in the survey?

b How many people used all 3 methods: car, bus *and* train? Give an explanation on how this might happen.

c Calculate the probability of selecting a person from this survey who travels by:

i car only or bus only **ii** car and train

iii car and train only **iv** bus but not train.

d A person is chosen at random from the people who travel by car or train only (but not bus). What is the probability that the person travels by car only?

e If 110 people actually participated in the survey, what extra information would be needed on the diagram?

Solution

a Number of people $= 27 + 12 + 5 + 20 + 21 + 4 + 11$

$= 100$

b 5 people travelled by car, bus and train.

The region where the 3 circles intersect

They may have driven to the station, caught a train and then a bus to the workplace, or used different methods on different days.

c **i** People travelling by car only or bus only $= 27 + 11$

$= 38$

The regions in C and B that don't overlap

$P(\text{car only or bus only}) = \frac{38}{100}$

'Car only or bus only' is an example of a **compound event**.

$= \frac{19}{50}$

9780170453325

ii People travelling by car and train $= 20 + 5$
$= 25$

The region where C and T intersect

$P(\text{car and train}) = \frac{25}{100}$

'Car and train' is another example of a **compound event**.

$= \frac{1}{4}$

iii People travelling by car and train only $= 20$

Where C and T intersect, but excluding B

$P(\text{car and train only}) = \frac{20}{100}$

$= \frac{1}{5}$

iv People travelling by bus but not train
$= 11 + 12$
$= 23$

The circle B excluding T

$P(\text{bus but not train}) = \frac{23}{100}$

d People travelling by car or train only $= 27 + 20 + 21$
$= 68$

People travelling by car only $= 27$.

$P(\text{car only}) = \frac{27}{68}$

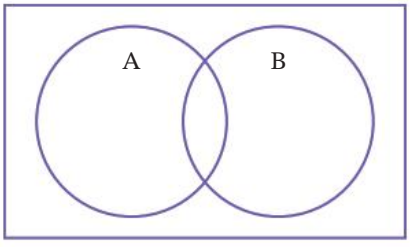
e $110 - 100 = 10$. This means that there were 10 people who did not use a car, bus or train. A '10' would need to be placed in the rectangle, but outside of the 3 circles.

When we combine 2 or more simple events, we get a **compound event**. In the above example, 'car and train' and 'bus but not train' are compound events.

'And' vs 'or'

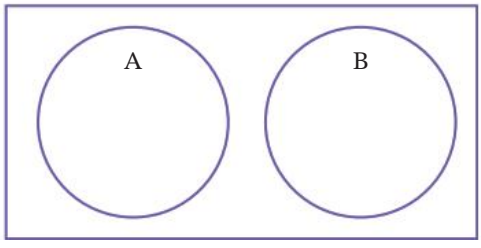
For 2 categories or events A and B, the compound event **'A and B'** means to have both of them occurring together. For example, 'to drive a car' **and** 'to ride a bus' means to do both things.

If A and B are **overlapping**, the compound event **'A or B'** means to have A or B or both. For example, 'to drive a car' **or** 'to ride a bus' means to drive a car only, or to ride a bus only, or to do both. In this case, 'A or B' actually **includes** 'A and B' so this is an example of an **inclusive** 'or'.



Overlapping events: 'A or B' means A or B or both

If A and B are **mutually exclusive**, this means that they are **not overlapping** and on a Venn diagram they appear as 2 separate circles. For mutually exclusive categories or events, the phrase **'A or B'** means to have A only or B only (but not both). For example, 'male' **or** 'female' means to be male, or female, but not both. In this case, 'A or B' **excludes** 'A and B' so this is an example of an **exclusive** 'or'.



Mutually exclusive events: 'A or B' means A or B but not both

Venn diagrams

Example 6

A survey of 110 students at Redford College showed that 61 students had dark hair, 42 students had brown eyes and 20 students had dark hair and brown eyes.

Shutterstock.com/michaeljung

Alamy Stock Photo/blickwinkel

Shutterstock.com/Dorota Zietek

a Represent this information on a Venn diagram.

b How many students had brown eyes or dark hair but not both?

c What is the probability of randomly selecting from this group a student:

i with dark hair?

ii with dark hair and brown eyes?

iii with dark hair or brown eyes?

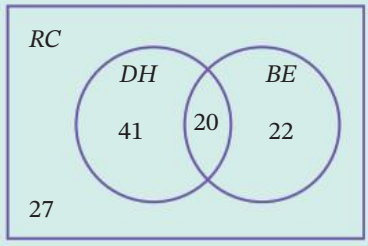
iv with neither dark hair nor brown eyes?

Solution

a RC = students at Redford College

DH = students with dark hair

BE = students with brown eyes



When completing a Venn diagram, always begin with the intersection.

There are 20 students with dark hair and brown eyes.

$\therefore$ Students with dark hair only $= 61 - 20$

$= 41$

$\therefore$ Students with brown eyes but not dark hair $= 42 - 20$

$= 22$

$\therefore$ (Remaining) Students with neither dark hair nor brown eyes $= 110 - 41 - 20 - 22$

$= 27.$

b Number of students with brown eyes or dark hair only $= 22 + 41$

$= 63$

c **i** Students with dark hair $= 41 + 20$

$= 61$

$P(\text{dark hair}) = \frac{61}{110}$

ii $P(\text{dark hair and brown eyes}) = \frac{20}{110}$

$= \frac{2}{11}$

'Dark hair and brown eyes' is a compound event.

iii Number of students with dark hair or brown eyes $= 41 + 20 + 22$

$= 83$

$P(\text{dark hair or brown eyes}) = \frac{83}{110}$

iv $P(\text{neither dark hair nor brown eyes}) = \frac{27}{110}$

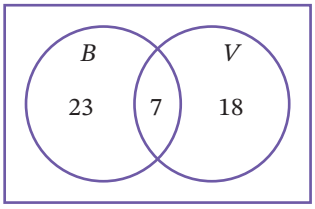
EXERCISE 12.03 ANSWERS ON P. 605

Venn diagrams U F R C

EXAMPLE 5

12.03

1 The Venn diagram shows the number of students who play either basketball (B) or volleyball (V). R



a Find the total number of students who play basketball or volleyball.

b Find the probability of randomly selecting a student who plays:

i basketball **ii** volleyball.

c What is the probability of selecting a student who plays:

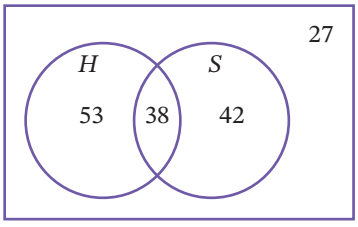
i basketball or volleyball? **ii** basketball and volleyball?

iii basketball but not volleyball? **iv** neither basketball nor volleyball?

d Of the students who play volleyball, find the probability of selecting a student who also plays basketball. Select the correct answer **A**, **B**, **C** or **D**.

A 0.28 **B** 0.375 **C** 0.72 **D** 0.92

2 The Venn diagram shows the number of Year 9 students who achieved Credit grades in their Science (S) and History (H) exams. R



a How many students are in Year 9?

b Find the probability of selecting a student who achieved a Credit in:

i History **ii** Science

iii History and Science **iv** History or Science

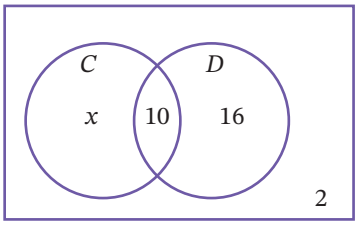
v History or Science but not both **vi** History only

vii Science only

c What is the percentage probability of selecting a student who did not achieve a Credit in both History and Science?

d Out of the students who achieved a Credit in History, what is the probability that the student also achieved a Credit in Science?

3 40 people were asked whether they preferred watching movies at the cinema (C) or on a device (D) at home. The results are shown in the Venn diagram. R C



a Describe the category that contains the pronumeral x in the diagram.

b Find the value of x.

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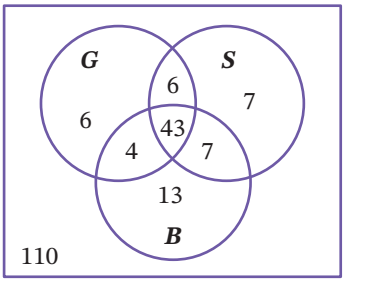
c Calculate the decimal probability of randomly selecting a person who likes:

- **i** the cinema and a device
- **ii** a device only
- **iii** the cinema or a device
- **iv** neither the cinema nor a device.

4 This Venn diagram describes the number of countries that won gold, silver, bronze or no medals at the 2016 Olympic Games in Rio de Janeiro, Brazil. R C



a How many countries competed at the 2016 games? Select **A**, **B**, **C** or **D**.

A 86 **B** 183

C 189 **D** 196

b How many countries did not win a medal? How is this shown on the Venn diagram?

c What is the probability of randomly selecting a country that:

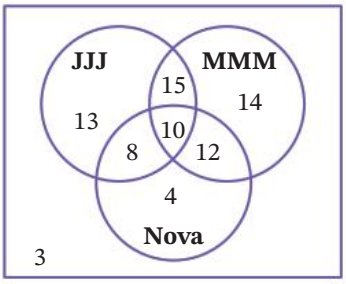
- **i** won only gold medals?
- **ii** won gold or silver medals, but not both?
- **iii** won gold, silver and bronze medals?
- **iv** won gold and bronze only?
- **v** won gold only or silver only or bronze only?
- **vi** did not win a gold medal?
- **vii** did not win a gold or silver medal?

d Out of the countries that won medals, what is the probability of selecting a country that:

i won gold medals? **ii** won silver but not bronze?

Getty Images/AFP/Christophe Simon

5 A group of teachers were asked what radio stations they listened to and the results are shown in the Venn diagram. R



a How many teachers were surveyed?

b What is the probability of selecting a teacher who did not listen to the 3 stations?

c What is the probability of selecting a teacher who listens to:

i JJJ? **ii** MMM? **iii** Nova?

d What is the probability of selecting a teacher who listens to at least 2 of the stations? Select **A**, **B**, **C** or **D**.

A $\frac{50}{79}$ **B** $\frac{45}{79}$ **C** $\frac{35}{79}$ **D** $\frac{30}{79}$

e Find the probability of selecting a teacher who listens to:

i JJJ or MMM but not both **ii** all 3 stations

iii MMM only

6 In a drama group of 30 people, 18 people dance (D), 20 people sing (S) and 8 people dance and sing. R C

EXAMPLE 6

a Show this information on a Venn diagram.

b Find the probability of selecting a person who:

i dances and sings

ii sings but does not dance

iii dances but does not sing

iv dances or sings

v dances or sings but not both.

7 A class of 30 students were given the option of playing an indoor sport (S) or watching a movie (M) when regular sport was cancelled due to bad weather. 15 students wanted to play an indoor sport and 24 students wanted to watch a movie. R C

a How many students indicated that they would do either?

b Copy and complete a Venn diagram using the information given.

c For this Venn diagram, there are no values outside the circles. Explain why.

d What is the probability of selecting a student who prefers to:

i play an indoor sport or watch a movie, but not both?

ii watch a movie but not play an indoor sport?

iii play an indoor sport but not watch a movie?

12.03

8 80 families living in Mt Robinson were surveyed on whether they had a dog (D) or a cat (C) as a pet. 12 families had no pets, 35 had a dog and 47 had a cat. R C

Shutterstock.com/Chendongshan

a Draw Venn diagram showing the information given.

b How many families had:

i a dog and a cat?

ii a dog but not a cat?

iii at least a dog or a cat?

iv a dog or a cat, but not both?

c Find the decimal probability of selecting a family with:

i a dog only

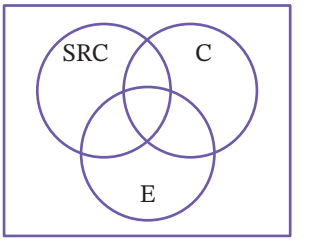
ii either a dog or a cat, but not both

iii at most one pet

iv at least one pet.

d Given that a family has a pet, what is the probability that it is a cat?

9 At Nelson River High School, the Student Representative Council (SRC) has 25 members, the Chess club has 20 members and the Environment club has 33 members. 5 students belong to all 3 groups, while 13 members belong to the SRC and no other club. 4 members of the Environment club are members of the SRC, but not the Chess club, while 10 members belong to both the Environment and Chess clubs. There are 8 members of the Chess club who are members of the SRC. R C



a Copy and complete the Venn diagram.

b How many students belong to the Environment club only?

c How many students belong to the Chess club and no others?

d Find the probability of selecting a student who belongs to exactly 2 clubs.

e What is the probability of a student belonging to at least 2 clubs?

f Find the probability of selecting a student who belongs to:

i the Environment club and the SRC, but not the Chess club

ii only one club

iii the SRC or the Chess club, but not the Environment club

iv at most 2 clubs

g Is it necessary to include the rectangle in this Venn diagram? Give reasons.

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Two-way tables

12.04

A **two-way table** is another way of grouping items into overlapping categories, especially when there are many overlaps that cannot easily be represented by Venn diagrams.

WS Two-way tables

WS Two-way probability tables

Example 7

This table categorises the heights and ages of 200 Year 9 students.

	Short	Tall
Aged 14	45	25
Aged 15	90	40

a How many students are:

i aged 15 and tall?

ii aged 14 or short?

b What fraction of students are neither tall nor aged 15?

c What is the probability of selecting a student at random who is:

i aged 14 and tall?

ii aged 15 but not tall?

iii tall or aged 14, but not both?

d If a short student is selected at random, what is the probability that he or she is aged 14?

Solution

a i Students aged 15 and tall = 40

ii Students aged 14 or short = 45 + 25 + 90

= 160

b Students who are neither tall nor 15 = 45 The short 14-year-olds

$\therefore$ Fraction of students neither tall nor 15 $= \frac{45}{200} = \frac{9}{40}$

c i $P(\text{tall and 14}) = \frac{25}{200}$ 'Tall and 14' is a compound event.

$= \frac{1}{8}$

ii $P(\text{15 but not tall}) = \frac{90}{200}$

$= \frac{9}{20}$

iii Students tall or aged 14, but not both = 40 + 45

= 85

$P(\text{tall or 14, but not both}) = \frac{85}{200} = \frac{17}{40}$

d Short students = 90 + 45 = 135

Students who are short and 14 = 45

$P(\text{short student who is 14}) = \frac{45}{135} = \frac{1}{3}$

EXERCISE 12.04 ANSWERS ON P. 606

Two-way tables U F R C

EXAMPLE 7

1 Households were surveyed at random to see whether or not they had dishwashers and clothes dryers. The results are in the table. R C

	Dishwasher	No dishwasher
Clothes dryer	75	108
No clothes dryer	49	68

a How many households were surveyed?

b How many households had a clothes dryer and a dishwasher?

c How many households had neither?

d What is the probability of selecting a household at random that has:

i a dishwasher?

ii a dishwasher and a dryer?

iii a dishwasher or a clothes dryer, but not both?

iv at least one of them?

2 People at a train station were asked whether they ate breakfast or lunch that day. R C

	Ate lunch	No lunch
Ate breakfast	35	26
No breakfast	54	15

a How many people were surveyed?

b What percentage of people ate lunch?

c Find the probability of picking a person at random who ate:

i breakfast

ii lunch

iii breakfast but not lunch

iv breakfast or lunch

v breakfast and lunch

vi breakfast or lunch, but not both

d What is the probability of picking a person who has eaten lunch given that the person ate breakfast? Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{26}{61}$ **B** $\frac{35}{61}$ **C** $\frac{35}{89}$ **D** $\frac{89}{130}$

e What is the probability of picking a person who has not eaten lunch given that the person did not eat breakfast?

3 People at a shopping mall were asked if they watched football on TV. R C

	Football	No football
Male	85	27
Female	36	92

a How many people were surveyed?

b How many people:

i watched football?

ii did not watch football?

c What percentage of people surveyed were females?

d What is the probability of randomly selecting a person who did not watch football and was male?

e Given that a person watched football, what is the probability that the person was female?

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4 The results comparing gender and handedness of students at a primary school are shown in the table. R C

	Male	Female
Left-handed	14	18
Right-handed	86	65

a How many students are at the school?

b What is the probability of selecting a student at random who is:

i male and left-handed?

ii male or left-handed?

c Find the probability, expressed as a percentage, of randomly selecting a right-handed female.

d Given that a student is a male, what is the probability that he is right-handed?

e What is the probability of selecting a female or a right-handed student?

5 After the 2019 Federal election, the members (politicians) of the House of Representatives in the Australian Parliament were as shown. R C

	Government	Opposition
Male	62	43
Female	15	31

a How many members of parliament (MPs) were there in the House of Representatives?

b Find the percentage probability of randomly selecting an MP who is:

i male

ii female and in Opposition

iii in the Government or female

iv male and in Opposition.

c Find the probability of selecting an MP who is male and in Government.

d If a female MP is selected at random, what is the probability that she is in Government?

Getty Images/Tracey Nearmy

6 Students at Cengage Anglican College were surveyed on whether they played video games. The results are shown in the table. R C

	Plays video games	Does not play video games
Female	178	267
Male	310	210

a How many students attended the college?

b Find the percentage probability that a student selected at random from the college:

i plays video games and is female

ii doesn't play video games

iii is male and doesn't play video games

iv is female and doesn't play video games.

c If a female student is selected at random, what is the probability that she plays video games?

d If a male student is selected at random, what is the probability that he plays video games?

7 A group of retired people was surveyed on whether they exercised regularly. R C

	Exercise	Don't exercise
Men	65	106
Women	74	75

a How many people were surveyed?

b What percentage of people exercised?

c What is the probability of selecting a person at random who doesn't exercise regularly?

d Of the women who were surveyed, what percentage exercised regularly?

e If a man was selected randomly from this survey, what is the probability that he exercises regularly? Select **A**, **B**, **C** or **D**.

A 38.0% **B** 62.0% **C** 20.3% **D** 53.4%

Investigation

Using two-way tables

Work in groups of 3 or 4.

Investigate 2 characteristics, for example, gender and exercise, gender and eating habits, eating habits and exercise, playing sport and going to the movies, eye colour and hair colour.

Conduct a survey to obtain the information required, then summarise your results in a two-way table similar to this one:

	Eat regular meals	Don't eat regular meals
Play sport		
Don't play sport		

Note any patterns in your data. This may include answering questions such as 'What percentage of ...' or 'What is the probability of picking a person who ...'.

Mental skills 12: Maths without calculators ANSWERS ON P. 606

The unitary method with percentages

The unitary method is used when you are only given a *percentage* of an amount and you need to find the amount. It is called the unitary method because we find 1% of the amount first, then multiply that by 100 to find the whole (100%).

1 Study each example.

a If 8% of a number is 24, what is the number?

8% of the number = 24

$\therefore$ 1% of the number = 24 ÷ 8 = 3

$\therefore$ 100% of the number = 3 × 100 = 300

The number is 300. *Check*: 8% × 300 = 24

b If 15% of an amount is \$90, what is the whole amount?

15% of the amount = \$90

$\therefore$ 1% of the amount = \$90 ÷ 15 = \$6

$\therefore$ 100% of the amount = \$6 × 100 = \$600

The amount is \$600. *Check*: 15% × \$600 = \$90

2 Find the whole amount if:

a 5% of the amount is \$35

b 11% of the amount is \$88

c 20% of the amount is 80

d 6% of the amount is 42

e 90% of the amount is \$270

f 15% of the amount is \$60

g 40% of the amount is 100

h 120% of the amount of \$360

i 25% of the amount is \$75

j 8% of the amount is 40

Did you know?

Lotto

iStock.com/askhamdesign

Lotto is a game of chance in which 6 balls are selected from a barrel of balls numbered 1 to 45. Players who correctly predict the 6 numbered balls share a first prize of at least 4 million dollars. The original Lotto game introduced in 1979 used 40 balls, and by increasing to 44 balls in 1989, the chance of winning first prize dropped from 1 in 3 838 380 to 1 in 7 059 052. The addition of 4 extra balls made Lotto almost twice as difficult to win. Another ball was added in 2004, making the chance of winning first prize even slimmer.

Research the probability of winning Lotto today now that it uses 45 balls.

Investigation

Are the Lotto numbers equally likely?

In each Lotto game, 8 numbers are selected at random from the barrel: 6 main numbers, plus 2 supplementary numbers. Between January 2015 and October 2019, there were 2000 Lotto numbers drawn on the Saturday game. The frequency of each number (from 1 to 45) were as follows:

1	53	10	50	19	36	28	36	37	38
2	37	11	53	20	45	29	49	38	47
3	39	12	41	21	43	30	47	39	47
4	41	13	44	22	53	31	42	40	35
5	53	14	41	23	43	32	48	41	39
6	44	15	47	24	47	33	45	42	45
7	46	16	51	25	39	34	45	43	35
8	41	17	41	26	52	35	39	44	45
9	40	18	57	27	52	36	46	45	43

a If 2000 numbers were drawn over the 5 years, and each number from 1 to 45 was equally likely, how often would you expect each number to come up?

b How many of the frequencies from the above table are equal to this expected number?

c Which Lotto number came up the:

i most often? **ii** least often?

d Do you think the Lotto numbers are equally likely, or is the Lotto draw biased? Give a reason for your answer.

Source: www.thelott.com

12.05 Tree diagrams

Tables and tree diagrams

Probability review

Combined events

A **two-step experiment** or **two-stage experiment** is a chance experiment that has 2 parts or stages, for example:

- rolling 2 dice
- tossing a coin and die together
- drawing 2 prizes in a raffle
- observing the weather each day over a weekend.

The outcome of the second step or stage may or may not be affected by the outcome of the first step. The sample space for a two-step experiment can be displayed using lists, tables or tree diagrams.

Example 8

2 coins are tossed together.

a Find all the outcomes in the sample space:

i by simply listing them **ii** using a table.

b What is the probability of tossing:

i a head and a tail? **ii** 2 tails? **iii** at least one head?

Solution

a Let H = heads and T = tails.

i The possible outcomes are {HH, HT, TH, TT}.

Finding the 4 outcomes by listing

ii

		1st coin	
		H	T
2nd coin	H	HH	TH
	T	HT	TT

Using a table ensures that all outcomes are counted.

b **i** $P(\text{a head and a tail}) = P(\text{HT or TH}) = \frac{2}{4} = \frac{1}{2}$ 2 chances out of 4

ii $P(\text{2 tails}) = P(\text{TT}) = \frac{1}{4}$ 1 chance out of 4

iii $P(\text{at least one head}) = P(\text{1 or 2 heads})$ 'At least one' means '1 or more'

$= P(\text{HT or TH or HH})$ 3 chances out of 4

$= \frac{3}{4}$

Example 9

While staying at a hotel, John-Paul can choose one item from each course of a breakfast menu.

1st course	2nd course
Cereal (C)	Bacon and eggs (B)
Raisin toast (R)	Pancakes (P)
Watermelon (W)	Sausages and hash browns (S)
Yoghurt (Y)	

a Use a table to list all the different 2-course breakfasts available.

b If one of these combinations is chosen at random, find the probability that it includes:

i watermelon but not pancakes **ii** yoghurt or pancakes

iii yoghurt and pancakes.

'Watermelon, not pancakes' is a compound event.

Solution

a

		1st course			
		C	R	W	Y
2nd course	B	CB	RB	WB	YB
	P	CP	RP	WP	YP
	S	CS	RS	WS	YS

$4 \times 3 = 12$ different breakfasts possible.

b i $P(\text{watermelon, not pancakes}) = \frac{2}{12}$ WB or WS

$= \frac{1}{6}$

ii $P(\text{yoghurt or pancakes}) = \frac{6}{12}$ column Y + row P

$= \frac{1}{2}$

iii $P(\text{yoghurt and pancakes}) = \frac{1}{12}$ YP

WS Tree diagrams

Tree diagrams

Tree diagrams

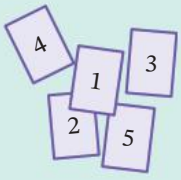
Tree diagrams

Tree diagrams

A **tree diagram** lists all the possible outcomes of each stage. Branches stretch out to show the possible pathways of outcomes at each step or stage. An outcomes column at the end of the diagram lists the sample space.

Example 10

From a set of cards numbered 1 to 5, 2 cards are drawn at random (without replacement) to make a 2-digit number.



a Show all outcomes in the sample space using a tree diagram.

b What is the probability of making a number:

i less than 32? **ii** divisible by 3?

Solution

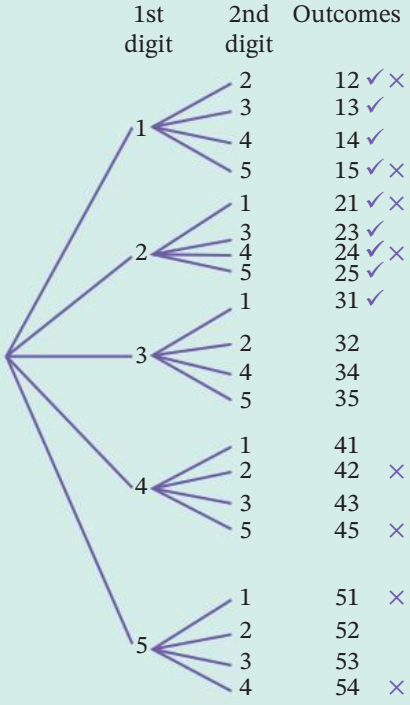
a This experiment has 2 steps: the 1st digit and the 2nd digit.

For the 1st digit, there are 5 possible digits: 1, 2, 3, 4 or 5.

For the 2nd digit, there are only 4 possible digits left because the 1st digit cannot be used again ('without replacement').

There are $5 \times 4 = 20$ possible 2-digit numbers in the sample space.

Using a tree diagram ensures that all outcomes are counted.



b i There are 9 numbers that are less than 32 (ticked in the tree diagram).

$\therefore P(\text{number} < 32) = \frac{9}{20}$.

ii There are 8 numbers that are divisible by 3 (crossed in the tree diagram).

$\therefore P(\text{number divisible by 3}) = \frac{8}{20}$

$= \frac{2}{5}$.

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EXERCISE 12.05 ANSWERS ON P. 606

Tree diagrams U F PS R C

EXAMPLE 8

1 Two people out of Ben, Anita, William, Lachlan and Noreen are to be selected for the positions of president and secretary in a tennis club. PS R C

a List the possible pairings for the positions of president and secretary.

b What is the probability of the committee consisting of:

i Ben and Anita?

ii 2 males?

iii a female president?

iv at least one female?

2 The digits 5, 6, 8 and 9 are written on cards and put in a bag. 2 cards are drawn at random to make a 2-digit number. PS R C

a Make a list of all possible outcomes.

b How many 2-digit numbers are possible?

c Find as a decimal (**iv** to 2 decimal places) the probability that the number is:

i even

ii greater than 60

iii between 60 and 90

iv divisible by 3

EXAMPLE 9

3 A coin and a die are tossed together. PS R C

a Copy and complete the table to display the sample space.

		Die					
		1	2	3	4	5	6
Coin	H						
	T						

b Find the probability of tossing:

i a head and 4

ii a tail and an even number

iii a tail and a number less than 5.

4 Two dice are rolled and the sum of the numbers is calculated. PS R C

a Copy and complete this table to show all possible sums.

		1st die					
		1	2	3	4	5	6
2nd die	1						
	2					7	
	3						
	4		6				
	5						
	6						

b Find the probability of rolling a sum

i of 3

ii of 7

iii of at least 6

iv less than 10

v that is even

vi between 3 and 7.

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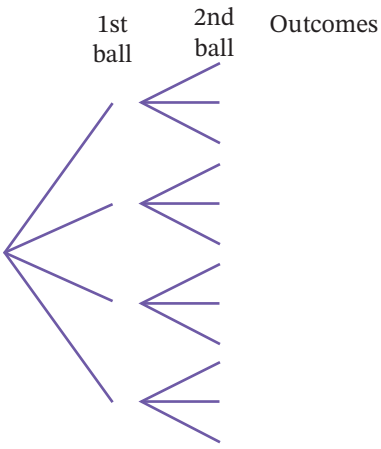
EXAMPLE 10

5 A bag contains 2 blue balls, 1 yellow ball and 1 green ball. 2 balls are drawn out of the bag at random. PS R C



a Copy and complete the tree diagram to list the sample space.

b Find the probability of drawing:

i a blue and green ball, in that order

ii a yellow ball and a green ball, in any order

iii a blue ball and a yellow ball

iv at least one blue ball.

6 The numbers 3, 4, 5 and 8 are written on separate cards and placed in a bag. 2 cards are drawn at random, one after the other, to form a 2-digit number. PS R C

a Use a tree diagram to show all possible outcomes if the cards are drawn *with replacement*, that is, the same card can be used twice.

b Find the probability that the 2-digit number formed:

i ends in 4

ii is an odd number

iii is less than 45

iv begins with 3 and is even.

7 The weather this weekend will be either fine or rainy each day, with each outcome being equally likely. PS R C

a Draw a tree diagram to show the possible outcomes for Saturday and Sunday.

b What is the probability that:

i it is fine on both days?

ii Saturday is fine and it rains on Sunday?

iii it rains on at least one day of the weekend?

8 Two cards are chosen at random with replacement from a set of 4 cards consisting of the 2 red Kings and 2 red Queens from a normal deck of cards. Calculate the probability that the cards are: PS R C

a both Kings

b a King and a Queen

c a King and Queen of the same suit

d both Queens

e at least one King

f no Queens

9 Draw a table for the sample space when 2 dice are rolled. Find the probability of rolling: PS R C

a 2 numbers the same

b a 3 and a 6 (in any order)

c at least one 2

d a 4 or a 5 on only one of the dice.

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10 While staying at a hotel, Phoebe can choose one item from each course of a breakfast menu. PS R C

1st course	2nd course
Muesli (M)	Bacon and eggs (B)
Raisin toast (R)	Pancakes (P)
Watermelon (W)	Sausages and hash browns (S)
Yoghurt (Y)	

a Find all the different 2-course breakfasts available:

i by listing them **ii** using a tree diagram.

b If one of these combinations is chosen at random, find the probability that it includes raisin toast or bacon and eggs.

11 From a set of cards numbered 1 to 5, 2 cards are drawn at random to make a 2-digit number. PS R C

a Show all outcomes in the sample space:

i by simply listing them **ii** using a table.

b What is the probability of making a number:

i that is odd? **ii** greater than 30?

12.05

Power plus ANSWERS ON P. 607

1 Jenny needs to roll a 6 with a die to start moving on a board game.
What is the probability that Jenny will start the game:

a on the first roll of the die?

b on the second roll?

c on the third roll?

2 Consider a family with 4 children. Use a tree diagram to list the 16 possible orders of boys and girls in the family. Find the probability of the family having:

a 3 boys and 1 girl

b no boys

c 2 boys and 2 girls

d at least 1 girl

e at most one girl

f more than 2 boys.

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3 A bag contains 5 yellow marbles and 4 red marbles. Two draws are made *without replacement.* Calculate the probability of drawing:

a 2 yellow marbles

b a yellow marble first and a red marble second

c a red marble and a yellow marble in any order

d at most one yellow marble.

4 Set theory can be used to describe different groups and represent calculations using special notation.

If we consider the set $S = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$ that represents all the even numbers from 1 to 20, then $n(S) = 10$, where $n(S)$ represents the *number* of elements in set S.

a Let $E = \{3, 6, 9, 12, 15, 18\}$, which represents the multiples of 3 from 1 to 20.

Find $n(E)$.

b **i** $S \cup E$ represents the **union** of the 2 sets. It consists of all of the elements that are in S or in E or in both S and E.

Copy and complete: $S \cup E = \{ \quad ... \quad \}$

ii $S \cap E$ represents the **intersection** of the 2 sets. It consists of the elements that are common to both S and E.

Copy and complete: $S \cap E = \{ \quad ... \quad \}$

c Given F represents the set of multiples of 5 from 1 to 20, copy and complete:

i $F = \{ ... \}$

ii $n(F) =$ ____

iii $S \cap F = \{ ... \}$

CHAPTER 12 REVIEW

Language of maths

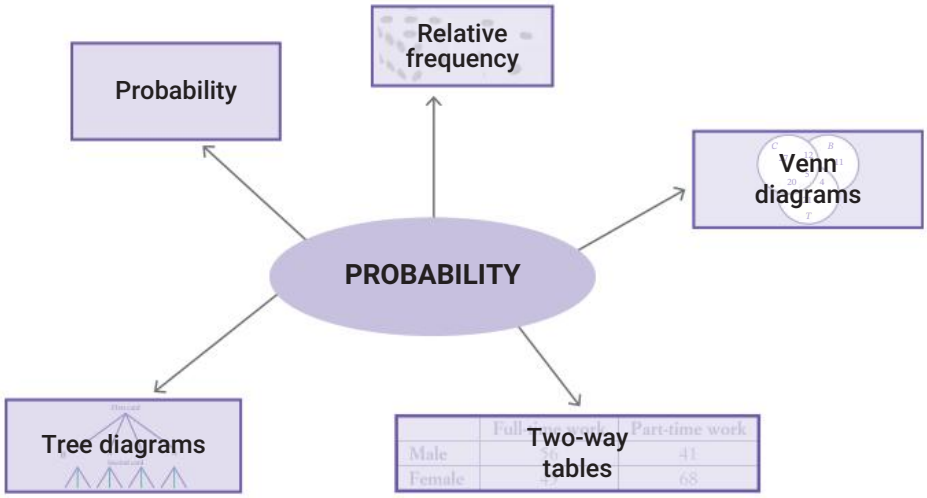
at least	at most	chance experiment
complementary event	compound event	die/dice
even chance	event	expected frequency
experimental probability	favourable	frequency
mutually exclusive events	observed frequency	random
relative frequency	replacement	sample space
theoretical probability	tree diagram	trial
two-step experiment	two-way table	Venn diagram

1. Which term from the above list is used to calculate **experimental probability**?
2. What is the meaning of **relative frequency**?
3. What happens to the experimental probability of an event as the number of trials becomes very large?
4. Which term describes the set of all possible outcomes of an experiment?
5. Name the event that is **complementary** to winning first prize in Lotto.
6. What is drawn using 2 or more circles, often overlapping?

Topic summary

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.

WS
Mind map: Probability



TEST YOURSELF 12 ANSWERS ON P. 607

12.01

1 A jar contains only red and yellow marbles. The probability of selecting a red marble from the jar is 40%.

a What is the probability of selecting a yellow marble?

b If there are 40 marbles in the bag, how many red marbles are there?

12.01

2 A card is drawn randomly from a normal deck of cards. Find the probability that it is:

a a red card

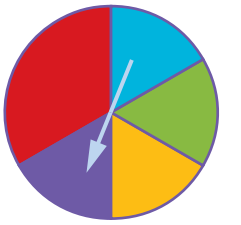
b a King or a Queen card

c a card with a number greater than 6

12.02

3 For the spinner shown, the red sector is twice as large as each of the other sectors. The spinner was spun 200 times and the results are shown in the table.

Outcome	Frequency
Blue	29
Green	35
Yellow	30
Purple	34
Red	72



a Find the experimental probability (as a decimal) that the arrow lands on:

i red **ii** blue or green **iii** not purple **iv** green.

b Find the theoretical probability (as a decimal) that the arrow lands on:

i red **ii** blue or green **iii** not purple **iv** green.

c Are the experimental probabilities from part **a** similar to the theoretical probabilities from part **b**?

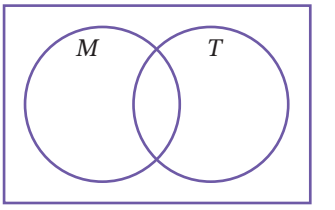
12.03

d If the spinner is spun 500 times, calculate the expected frequency of spinning red using the theoretical probability. Select the correct answer **A**, **B**, **C** or **D**.

A 100 **B** 85 **C** 167 **D** 200

12.04

4 30 teenagers were surveyed on whether they preferred to listen to music (M) or watch TV (T). 21 students preferred music and 18 liked TV, while 4 students did not like either.



a How many students preferred both music and TV?

b Copy and complete the Venn diagram.

c Find the probability of randomly selecting a teenager who likes:

i music but not TV **ii** TV

iii music or TV, but not both **iv** both or neither music nor TV.

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5 A group of men and women was surveyed on whether they worked full-time or part-time. 12.04

	Full-time work	Part-time work
Male	56	41
Female	45	68

a How many people were surveyed?

b Find the probability (as a percentage, correct to one decimal place) of selecting a person who:

i works full-time

ii is female and works part-time

iii does not work full-time

iv works part-time or is female.

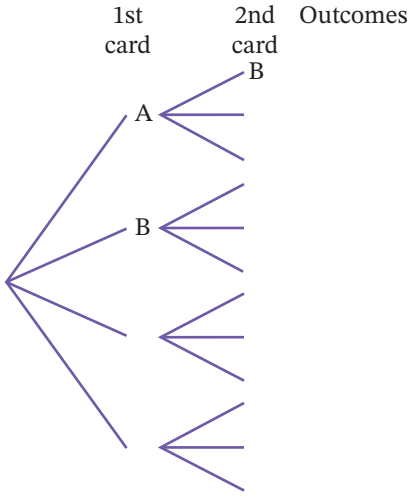
c If a part-time worker is selected at random, what is the probability that the person is female?

6 Four cards labelled with the letters A, B, C and D are placed in a bag. 12.05

2 cards are drawn from the bag at random without replacement.

a List all of the possible outcomes.

b Copy and complete the tree diagram.



c Use the tree diagram to find the probability of drawing 2 cards with:

i the same letter

ii the letters A and B

iii the letters A or B or both

iv no letter D.

7 Two dice are rolled and the difference between the numbers is calculated. 12.06

Die 2 \ Die 1	1	2	3	4	5	6
1						
2						
3						3
4						
5				1		
6		4				

6 − 3 = 3

5 − 4 = 1

6 − 2 = 4

a Copy and complete the table to show the sample space.

b Find the probability of rolling a difference of:

i 0 **ii** at least 2 **ii** 4 or 5.

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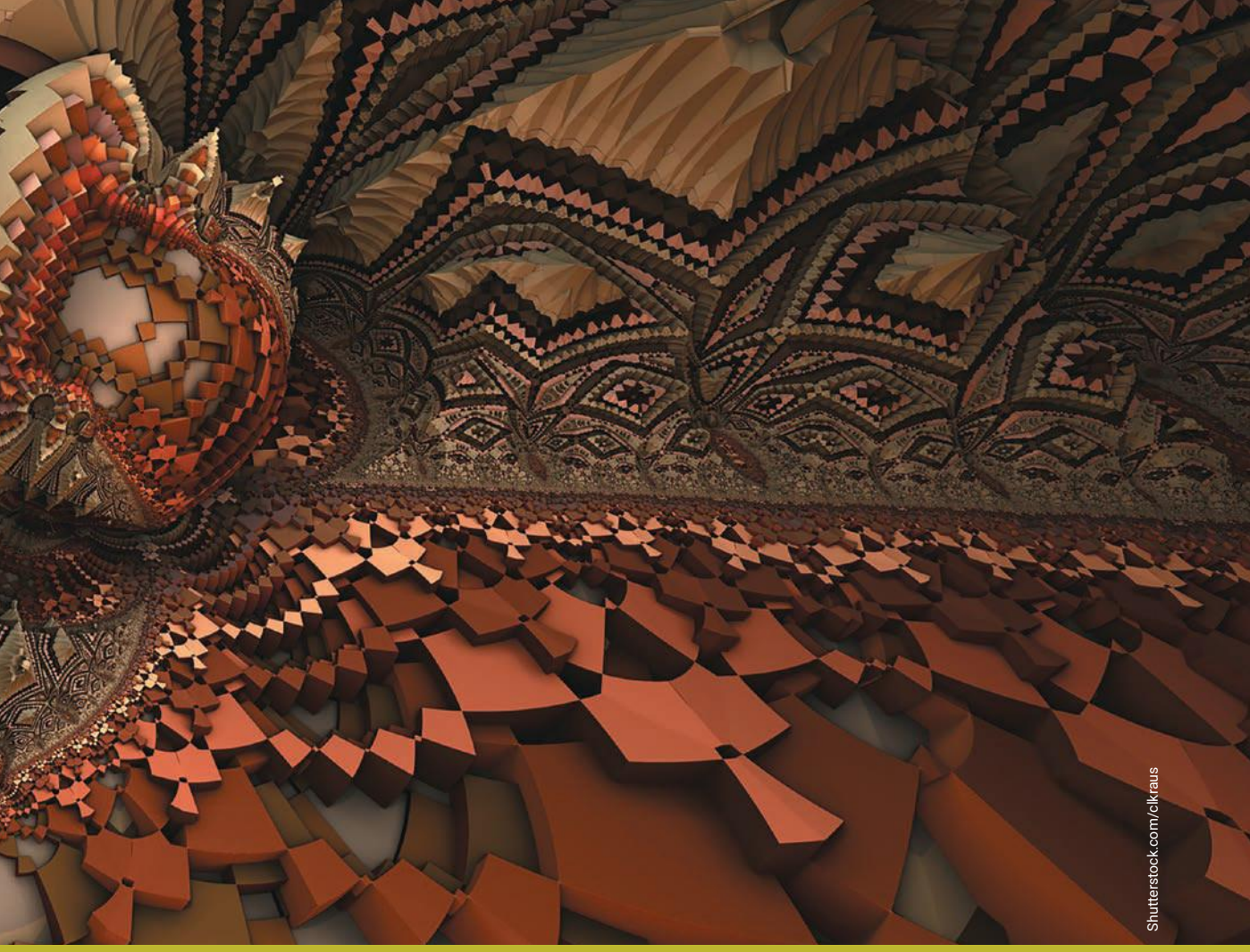
13

MEASUREMENT AND GEOMETRY

CONGRUENT AND SIMILAR FIGURES

MC (Maurits Cornelis) Escher (1898–1972) was a Dutch graphic artist who used congruent figures to create tessellations (repeated 'tile' patterns). Although he had no mathematical training, Escher was talented in visualising and creating complex geometrical patterns, especially those involving 'impossible' shapes and structures.



Chapter outline

	Working mathematically				
13.01 Tests for congruent triangles	U	F		R	C
13.02 Congruent triangle proofs	U	F		R	C
13.03 Using congruence to prove geometrical properties	U	F		R	C
13.04 Similar figures	U	F		R	C
13.05 Properties of similar figures	U	F	PS	R	
13.06 Scale diagrams	U	F	PS	R	
13.07 Areas of similar figures*	U	F		R	
13.08 Tests for similar triangles	U	F		R	C
13.09 Similar triangle proofs*	U	F		R	C

*STAGE 5.3

Wordbank

congruence test One of 4 tests for proving that triangles are congruent: SSS, SAS, AAS and RHS

congruent Identical; exactly the same (symbol: ≡)

enlargement An increase in the size of a shape

image A transformed shape after it has been enlarged or reduced

included angle The angle between 2 given sides of a shape

scale factor The amount by which a shape has been enlarged or reduced, equal to $\frac{\text{image length}}{\text{original length}}$

similar To have the same shape but not necessarily the same size, an enlargement or reduction (symbol: |||)

superimpose To place one figure on top of another figure so that sides and angles match

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

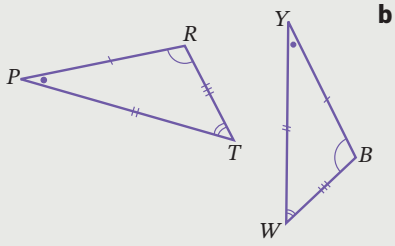
- identify and use congruent and similar figures
- investigate the properties of congruent and similar figures
- write formal proofs for congruent triangles
- investigate properties of quadrilaterals using congruent triangles and solve related geometry problems
- formulate proofs involving congruent triangles and angle properties
- use scale factors to find unknown sides in similar figures
- measure and calculate using scale diagrams
- (STAGE 5.3) investigate ratio of areas of similar figures
- identify and use the 4 tests for similar triangles
- (STAGE 5.3) write formal proofs for similar triangles

SkillCheck ANSWERS ON P. 608

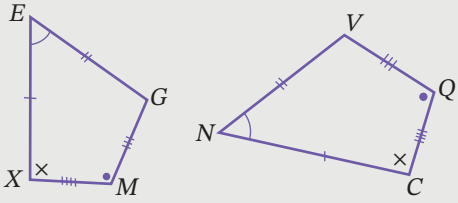
1 For each pair of congruent figures, list all pairs of:

i equal angles **ii** equal sides

a



b



Technology

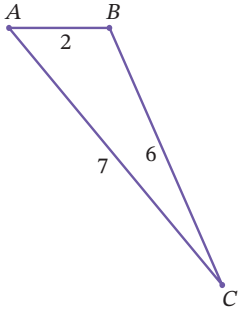
Tests for congruent triangles

Use dynamic geometry software to test whether 2 triangles are congruent. It is best to do this activity in small groups.

Given 3 sides: SSS

1 Construct a triangle with side lengths 2 cm, 6 cm and 7 cm.

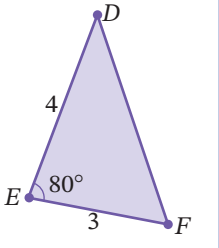
2 Can you construct a **different** triangle with the same 3 side lengths? What do you notice about the second triangle? Is it congruent (exactly the same shape and size) to the first triangle?



Given 2 sides and the included angle: SAS

1 Construct a triangle with sides of length 3 cm and 4 cm, and the angle between them to be exactly 80°.

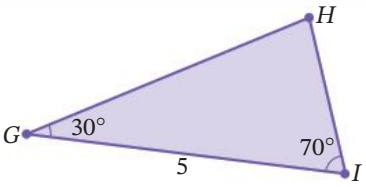
2 Can you construct a different triangle with the same 2 sides and included angle of 80°? Or is it congruent to the first triangle?



Given 2 angles and a side: AAS

1 Construct a triangle with a side length of 5 units between 2 angles of size 30° and 70°.

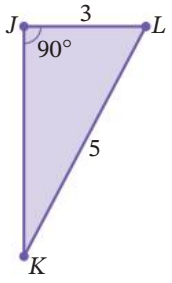
2 Can you construct a different triangle with the same 2 angles and a matching side of 5 cm? Or is it congruent to the first triangle?



Given a right angle, hypotenuse and side: RHS

1 Construct a right-angled triangle with one of the shorter sides being 3 cm and the hypotenuse 5 cm.

2 Can you construct a different triangle with the same 2 sides and right angle? Or is it congruent to the first triangle?

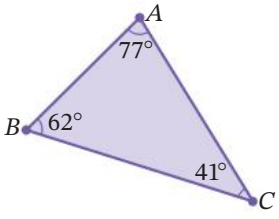


Given 3 angles: AAA

1 Construct a triangle with angles 77°, 62° and 41°.

2 Can you construct a different triangle with the 3 angles? Or is it congruent to the first triangle?

3 Are all triangles with angles 77°, 62° and 41° congruent?



Investigation

Congruent triangles

Work in groups of 3 or 4 to complete this investigation.

You will need: paper, a ruler, a pencil, a protractor, compasses and scissors.

1 **a** Individually, construct a triangle with side lengths 5 cm, 7 cm and 9 cm.

b Use your protractor to measure the angles of the triangle.

c Compare your results with those of the students in your group. Are all your group's triangles congruent?

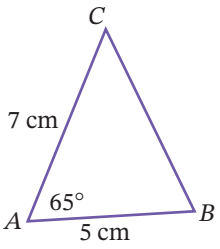
d Use your results to decide whether each statement is true or false.

i *If 3 sides of one triangle are respectively equal to the 3 sides of another triangle, then the matching angles of the triangles are equal.*

ii *If all sides of one triangle are equal to all sides of another triangle, then the 2 triangles are congruent.*

WS Congruent or different triangles?

2 **a** Construct $\triangle ABC$ so that $AB = 5$ cm, $AC = 7$ cm and $\angle A = 65°$.



Since $\angle A$ lies between AB and AC, $\angle A$ is called the **included angle**.

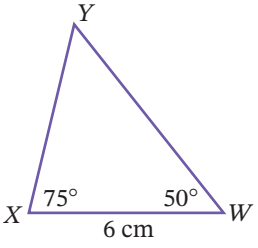
b Measure the third side of the triangle, and the sizes of the other 2 angles.

c Compare your results with those of others in your group.

d Use your results to decide whether this statement is true or false.

If 2 sides and the included angle of one triangle are respectively equal to 2 sides and the included angle of another triangle, then the 2 triangles are congruent.

3 **a** Construct $\triangle XWY$ with $XW = 6$ cm, $\angle X = 75°$ and $\angle W = 50°$.



b Measure $\angle Y$ and the other 2 sides of the triangle.

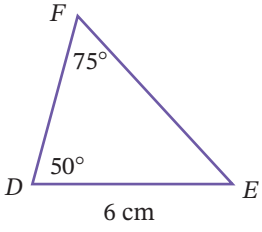
c Compare your results with those of your group members.

d Now construct $\triangle DEF$ with $DE = 6$ cm, $\angle D = 50°$ and $\angle F = 75°$.

e Are $\triangle XWY$ and $\triangle DEF$ congruent? Give reasons for your answer.

f Use your results to decide whether the following statement is true or false.

If 2 angles and one side of one triangle are respectively equal to 2 angles and the matching side of another triangle, then the 2 triangles are congruent.



4 **a** Construct $\triangle GHK$ with $\angle K = 90°$, $GH = 8$ cm, $GK = 5$ cm.

b What is another name for the side GH?

c Measure the other side and angles of the triangle and compare your results with those of other members in your group. Are their triangles congruent to your triangle?

d Use your results to decide whether the following statement is true or false:

If the hypotenuse and a second side of one right-angled triangle are respectively equal to the hypotenuse and a second side of another right-angled triangle, then the triangles are congruent.

5 **a** Draw a triangle with angles 40°, 63° and 77°.

b Compare your triangle with those of other members of your group. Are the triangles congruent? Give reasons.

6 Use your results to determine what information is needed to prove that 2 triangles are congruent.

Tests for congruent triangles 13.01

Congruent figures

2 figures are **congruent** if they are identical in shape and size. For **congruent figures**:

- **matching sides** are equal
- **matching angles** are equal

Matching sides of congruent figures are **corresponding sides** that are in the same position.

Matching angles of congruent figures are **corresponding angles** that are in the same position.

There are 4 sets of conditions that can be used to determine if 2 triangles are congruent. These are called the **tests for congruent triangles** or **congruence tests**.

Congruence tests

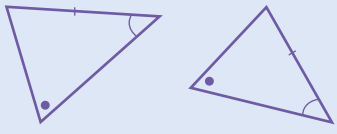
There are 4 tests for congruent triangles: **SSS**, **SAS**, **AAS** or **RHS**.

2 triangles are congruent if:

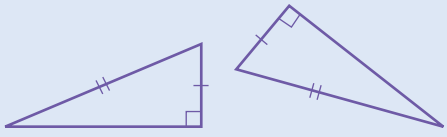
- the 3 sides of one triangle are respectively equal to the 3 sides of the other triangle **(SSS rule)**

- 2 sides and the **included angle** of one triangle are respectively equal to 2 sides and the **included** angle of the other triangle **(SAS rule)**

- 2 angles and one side of one triangle are respectively equal to 2 angles and the matching side of the other triangle **(AAS rule)**



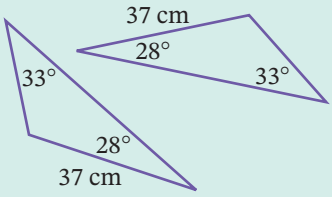
- they are right-angled and the hypotenuse and another side of one triangle are respectively equal to the hypotenuse and another side of the other triangle **(RHS rule)**.



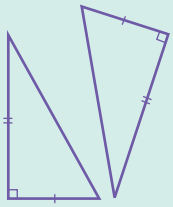
Example 1

Which congruence test (SSS, SAS, AAS or RHS) can be used to prove that each pair of triangles are congruent?

a



b



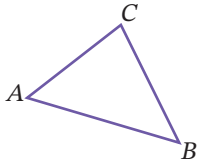
Solution

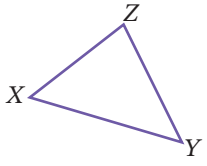
a 2 angles and one side of one triangle are equal to 2 matching angles and one matching side of the other triangle.
$\therefore$ The congruence test is AAS.

b 2 sides and the included angle of one triangle are equal to 2 matching sides and the included angle of the other triangle.
$\therefore$ The congruence test is SAS.

The congruence symbol $\equiv$

The symbol for 'is congruent to' is a special equals sign, written as '$\equiv$' (which also means 'is identical to'). The 2 triangles here are congruent, so we can write $\triangle \boldsymbol{ABC} \equiv \triangle \boldsymbol{XYZ}$, which is read: 'triangle ABC is congruent to triangle XYZ'.



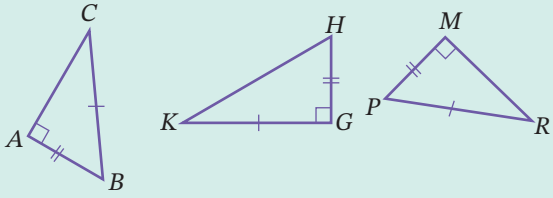


When using this notation, we must make sure that the vertices (and angles) of the congruent figures are written in matching order:

$\triangle ABC \equiv \triangle XYZ$ means $\angle A = \angle X$, $\angle B = \angle Y$, $\angle C = \angle Z$

Example 2

a Decide which 2 of these triangles are congruent and state the congruence test used.



b Use the correct notation to write a congruency statement relating the 2 triangles.

Solution

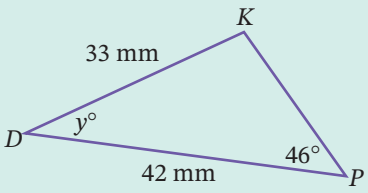
a The 3 right-angled triangles all have 2 equal sides, but BC and PR are both hypotenuses while KG is not.
$\therefore \triangle CAB$ and $\triangle MPR$ are congruent by the RHS rule.

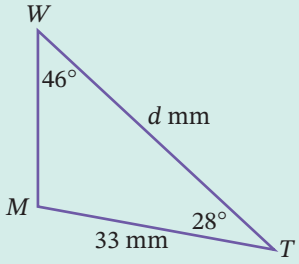
b $\angle A$ matches with $\angle M$, $\angle B$ matches with $\angle P$, $\angle C$ matches with $\angle R$.
$\therefore \triangle ABC \equiv \triangle MPR$

Matching order of vertices

Example 3

If the 2 triangles are congruent, find the value of each variable.





Solution

Since $\triangle DPK \equiv \triangle TWM$,

$DP = TW$ Matching sides of congruent triangles

$\therefore d = 42$

$\angle T = \angle D$ Matching angles of congruent triangles

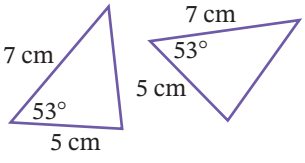
$\therefore y = 28$

EXERCISE 13.01 ANSWERS ON P. 608

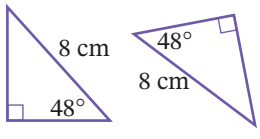
Tests for congruent triangles U F R C

1 Which congruence test (SSS, SAS, AAS or RHS) can be used to prove that each pair of triangles are congruent? R C

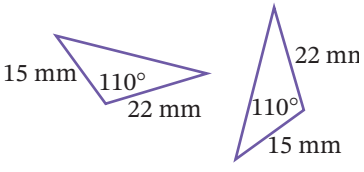
a



b



c



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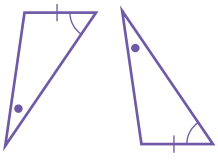

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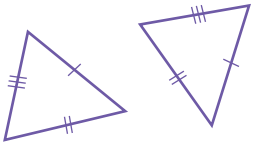

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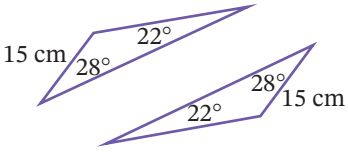
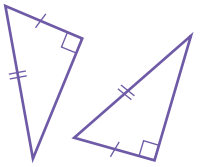


Foundation Standard Complex

g



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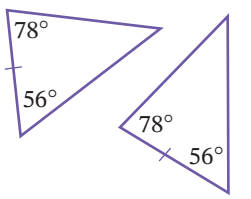

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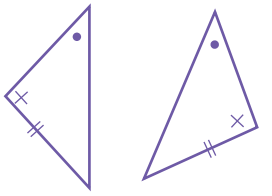

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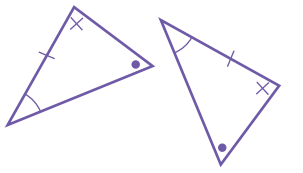

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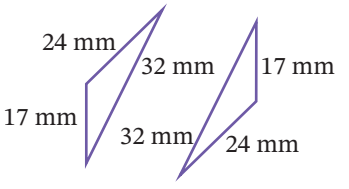

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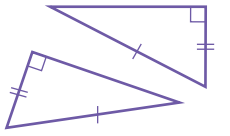
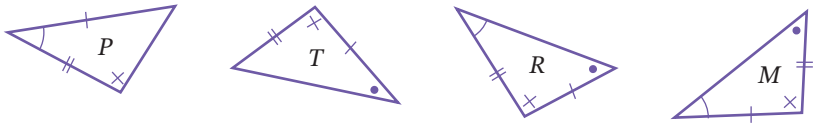


EXAMPLE 1

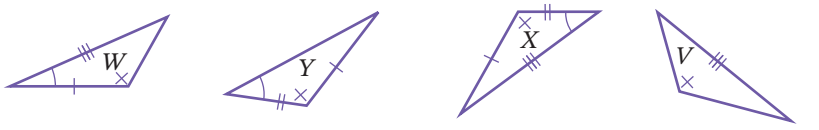
2 Find the 2 congruent triangles in each set. Select the correct answer **A**, **B**, **C** or **D**.

a



A P and T **B** T and R **C** M and R **D** M and P

b



A W and Y **B** Y and X **C** X and V **D** W and V

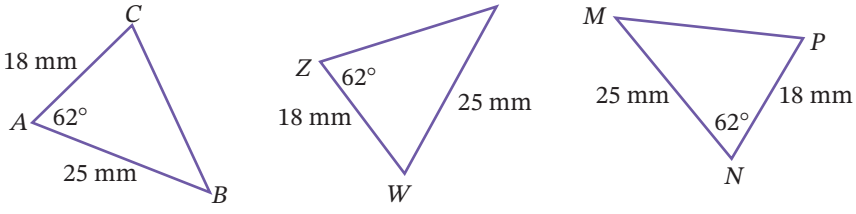
EXAMPLE 2

3 For each set of triangles: R C

i decide which 2 are congruent and state the congruence test used

ii use the correct notation to write a congruency statement relating those 2 triangles.

a



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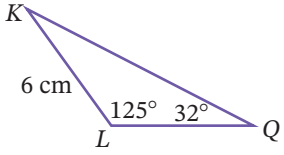
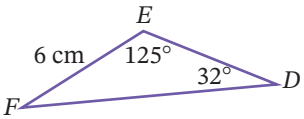
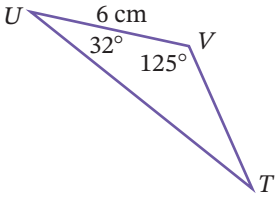
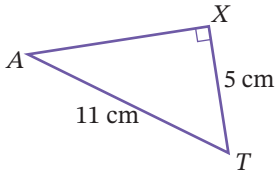
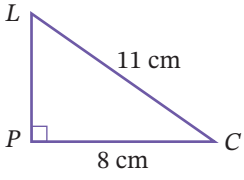


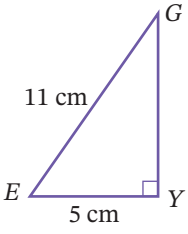




c







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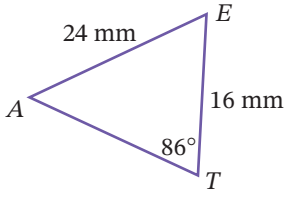

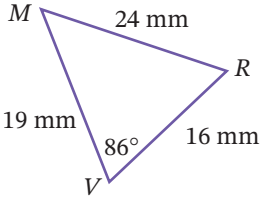

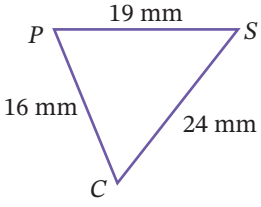

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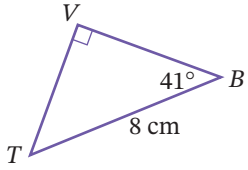

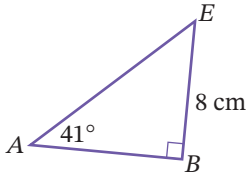

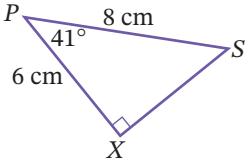

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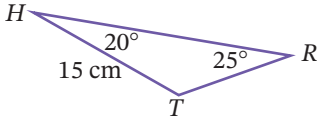

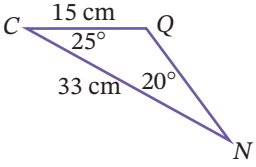

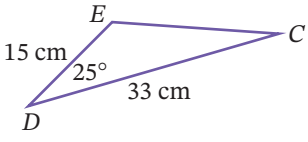

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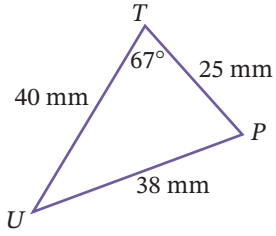

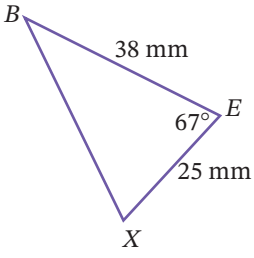

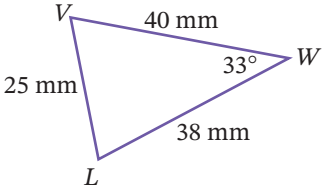

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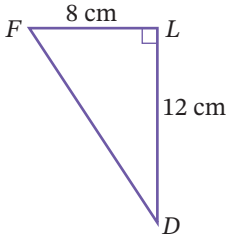

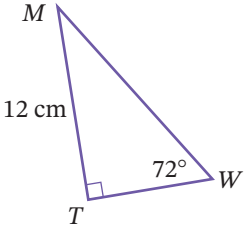

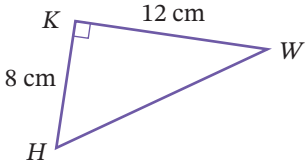

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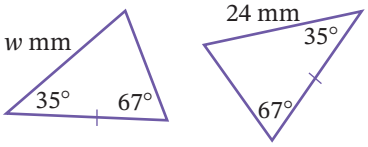
EXAMPLE 3

4 Find the value of the variable in each pair of congruent triangles. R

a



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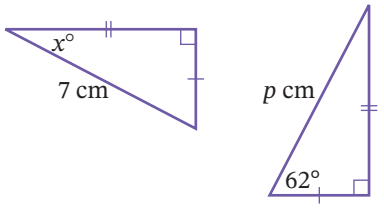

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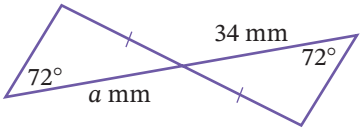

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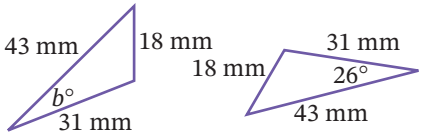

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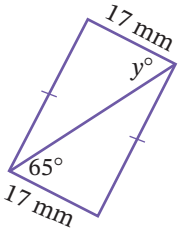

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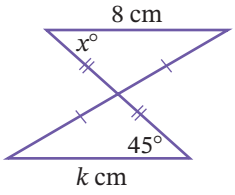
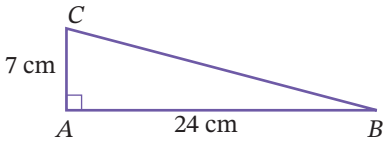
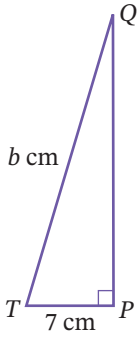


5 $\triangle ABC \equiv \triangle TPQ$. R

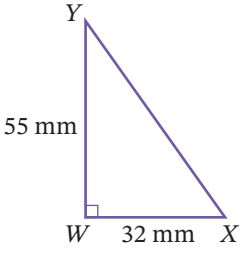
a Use Pythagoras' theorem to find BC.

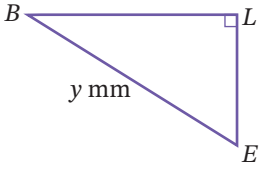
b Find the value of b.





6 If $\triangle YXW \equiv \triangle BEL$, find y correct to one decimal place. R





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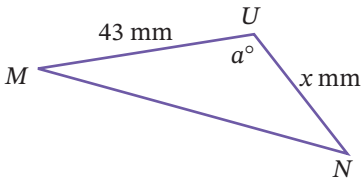
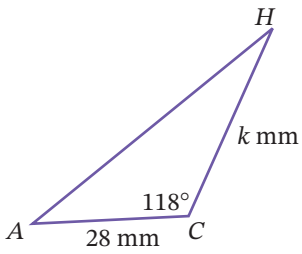
7 If $\triangle NUM \equiv \triangle ACH$, find the value of each variable. R



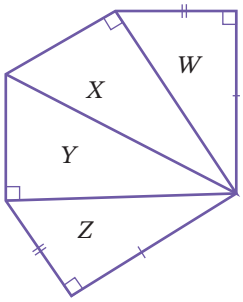


8 The diagram shows 4 right-angled triangles joined together. State whether each pair of triangles are congruent, and if so, state the test used. R C

a W and X **b** W and Y

c W and Z **d** X and Y

e X and Z **f** Y and Z



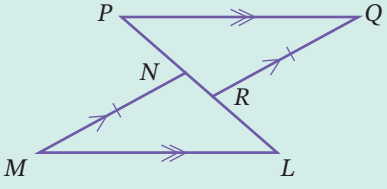
13.02

Congruent triangle proofs 13.02

To formally prove that 2 triangles are congruent, there is a specific format using one of the 4 congruence tests SSS, SAS, AAS or RHS.

Example 4

In the diagram, $PQ \parallel LM$, $QR \parallel MN$ and $QR = MN$.
Prove that $\triangle PQR \equiv \triangle LMN$.



Solution

In $\triangle PQR$ and $\triangle LMN$:	Identifying the triangles in matching order of vertices
$QR = MN$ (given)	Stating each part of the congruence test, giving reasons
$\angle P = \angle L$ (alternate angles, $PQ \parallel LM$)	
$\angle QRP = \angle MNL$ (alternate angles, $QR \parallel MN$)	
$\therefore \triangle PQR \equiv \triangle LMN$ (AAS)	Concluding the congruence proof, stating the test used

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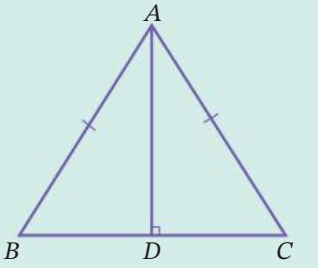
Example 5

In $\triangle ABC$, $AB = AC$ and $AD \perp BC$.
Prove that $\triangle ABD \equiv \triangle ACD$ and hence that AD bisects $\angle BAC$.



Solution

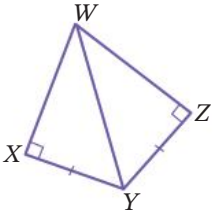
In $\triangle ABD$ and $\triangle ACD$:
$AB = AC$ (given)
AD is common.
$\angle ADB = \angle ADC = 90°$ ($AD \perp BC$)
$\therefore \triangle ABD \equiv \triangle ACD$ (RHS)
$\therefore \angle BAD = \angle CAD$ (matching angles of congruent triangles)
$\therefore AD$ bisects $\angle BAC$.

EXERCISE 13.02 ANSWERS ON P. 608

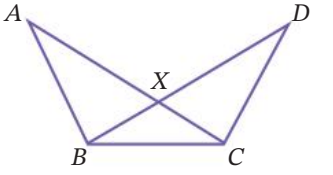
Congruent triangle proofs U F R C

EXAMPLE 4

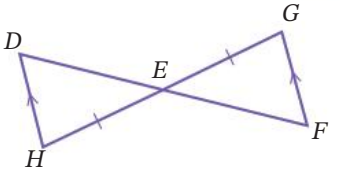
1 Prove that $\triangle WXY \equiv \triangle WZY$. R C



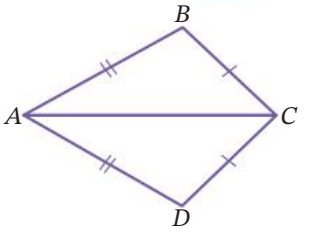
2 If $\angle ABC = \angle DCB$ and $AB = DC$, prove that $\triangle ABC \equiv \triangle DCB$. R C



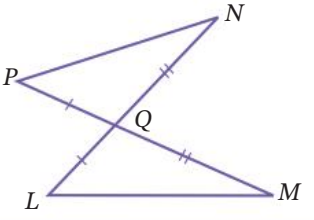
3 In the diagram, $EG = EH$ and $DH \parallel FG$. Show that $\triangle DEH \equiv \triangle FEG$. R C



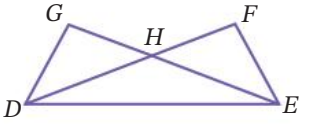
4 For this kite $ABCD$, prove that $\triangle ABC \equiv \triangle ADC$. R C



5 If $PQ = LQ$ and $NQ = MQ$, prove that $\triangle PQN \equiv \triangle LQM$. R C



6 If $\angle DEG = \angle EDF$ and $GE = FD$, prove that $\triangle DEG \equiv \triangle EDF$. R C



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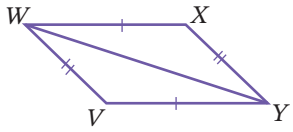
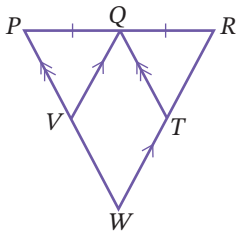
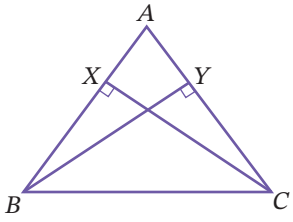
7 Prove that $\triangle WXY \equiv \triangle YVW$. R C



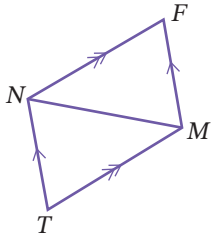
8 $PW \parallel QT$, $RW \parallel QV$ and $PQ = QR$.
Prove that $\triangle PVQ \equiv \triangle QTR$. R C



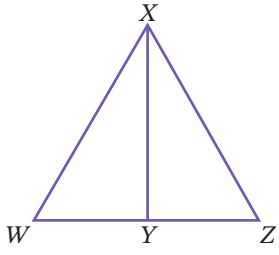
9 If $CX \perp AB$, $BY \perp AC$ and $XC = YB$, prove that $\triangle BCX \equiv \triangle CBY$. R C



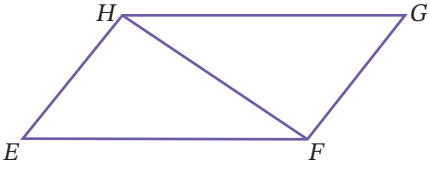
10 Prove that $\triangle FNM \equiv \triangle TMN$. R C



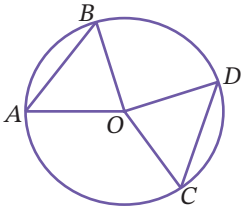
11 $XW = XZ$ in this isosceles triangle and Y is the midpoint of WZ.
Prove that $\triangle WYX \equiv \triangle ZYX$. R C



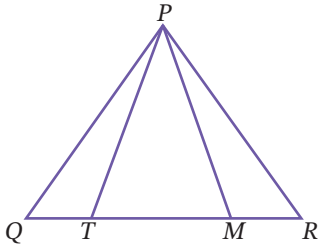
12 The opposite sides are equal for this quadrilateral.
Prove that $\triangle EFH \equiv \triangle GHF$. R C



13 O is the centre of the circle and $AB = CD$.
Prove that $\triangle AOB \equiv \triangle COD$. R C

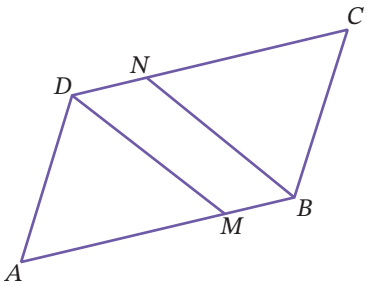


14 $\angle Q = \angle R$ in this isosceles triangle.
If $QT = RM$, prove that $\triangle PQT \equiv \triangle PRM$. R C

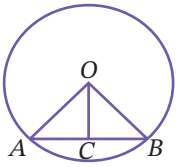


13.02

15 $ABCD$ is a parallelogram and $DN = BM$. Prove that $\triangle AMD \equiv \triangle CNB$. R C

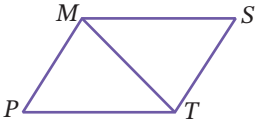


16 O is the centre of the circle and $OC \perp AB$.
Prove that $\triangle OAC \equiv \triangle OBC$. R C



EXAMPLE 5

17 In the quadrilateral $MSTP$, opposite sides are equal. Prove that: R C

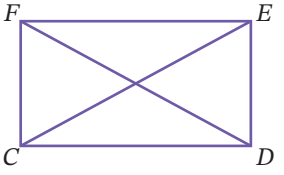


a $\triangle TPM \equiv \triangle MST$

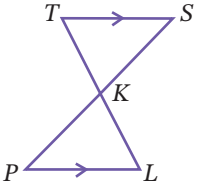
b $\angle PMT = \angle STM$

c $PM \parallel ST$

18 $CDEF$ is a rectangle. Prove that $\triangle FED \equiv \triangle EFC$ and that the diagonals of the rectangle are equal in length. R C

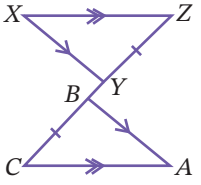


19 $TS \parallel PL$ and K is the midpoint of TL.



Prove that $\triangle TSK \equiv \triangle LPK$. R C

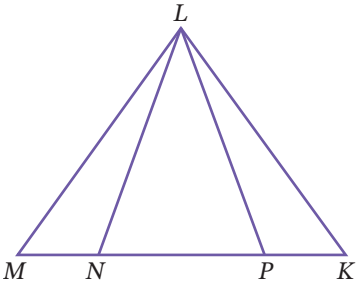
20 $AC \parallel XZ$, $XY \parallel AB$ and $BC = YZ$. R C



a Prove that $\angle XYZ = \angle ABC$.

b Hence prove that $\triangle XYZ \equiv \triangle ABC$.

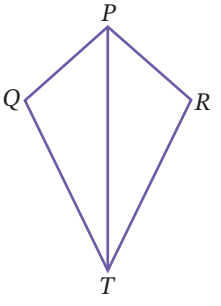
21 $LM = LK$ in this isosceles triangle LMK, and $MN = KP$. Prove that: R C



a $\triangle LMN \equiv \triangle LKP$

b $\triangle LNP$ is isosceles.

22 For this kite, prove that: R C



a $\triangle PQT \equiv \triangle PRT$

b $\angle Q = \angle R$

c PT bisects $\angle QTR$.

Foundation Standard Complex

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Using congruence to prove geometrical properties

13.03

Properties of triangles and quadrilaterals can be proved using the congruence tests.

Congruent triangles proofs

13.03

WS Proving properties of quadrilaterals

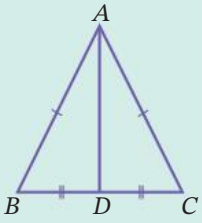
Example 6

$\triangle ABC$ is an isosceles triangle. AD bisects BC.

a Prove that $\triangle ABD \equiv \triangle ACD$.

b Hence prove that in an isosceles triangle, the angles opposite the equal sides are equal.

c Prove that $AD \perp BC$.



Solution

a In $\triangle ABD$ and $\triangle ACD$:

$AB = AC$ ($\triangle ABC$ is isosceles)

AD is common.

$BD = CD$ (AD bisects BC)

$\therefore \triangle ABD \equiv \triangle ACD$ (SSS)

b $\therefore \angle B = \angle C$ (matching angles of congruent triangles)

$\therefore$ The angles opposite the equal sides in an isosceles triangle are equal.

c $\angle ADB = \angle ADC$ (matching angles of congruent triangles)

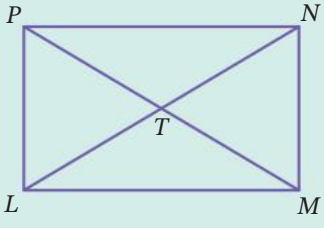
$\therefore \angle ADB = \angle ADC = 90°$ (angles on a straight line)

$\therefore AD \perp BC$.

Example 7

a If $LMNP$ is a rectangle, prove that $\triangle PNT \equiv \triangle MLT$.

b Prove that the diagonals of a rectangle bisect each other.



Solution

a In $\triangle PNT$ and $\triangle MLT$:

$PN = ML$ (opposite sides of a rectangle)

$\angle PNT = \angle MLT$ (alternate angles on parallel lines, $PN \parallel ML$)

$\angle PTN = \angle MTL$ (vertically opposite angles)

$\therefore \triangle PNT = \triangle MLT$ (AAS)

b $\therefore PT = MT$ and $NT = LT$ (matching sides of congruent triangles)

$\therefore T$ is the midpoint of the diagonals LN and MP.

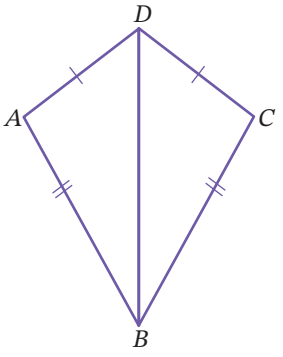
$\therefore$ The diagonals of a rectangle bisect each other.

EXERCISE 13.03 ANSWERS ON P. 609

Using congruence to prove geometrical properties U F R C

EXAMPLE 6

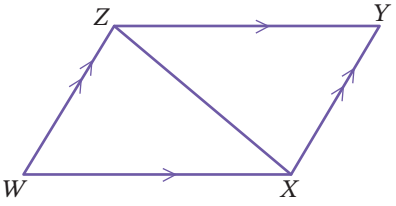
1 *ABCD* is a kite so adjacent sides are equal. R C



- **a** Prove that $\triangle ABD \equiv \triangle CBD$.
- **b** Prove that $\angle A = \angle C$.
- **c** Prove that diagonal *DB* bisects $\angle ADC$ and $\angle ABC$.
- **d** Copy the diagram and draw the other diagonal *AC*, intersecting *DB* at point *X*.
- **e** Prove that $\triangle DAX \equiv \triangle DCX$.
- **f** Prove that diagonal *DB* bisects diagonal *AC*.
- **g** Prove that $DB \perp AC$.

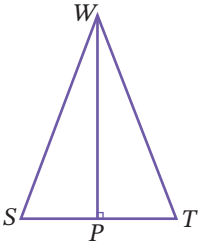
EXAMPLE 7

2 *WXYZ* is a parallelogram with opposite sides parallel. R C



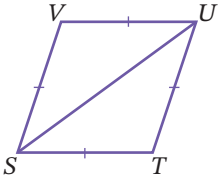
- **a** Copy the diagram into your book.
- **b** Prove that $\triangle WXZ \equiv \triangle YZX$.
- **c** Prove that $\angle W = \angle Y$.
- **d** Draw the other diagonal *WY* and prove that $\triangle WXY \equiv \triangle YZW$.
- **e** Prove that $\angle WXY = \angle YZW$.
- **f** What angle property of a parallelogram does this prove?

3 In the diagram, $\angle S = \angle T$ and $WP \perp ST$. R C



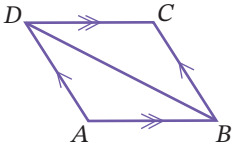
- **a** Prove that $\triangle SPW \equiv \triangle TPW$.
- **b** Prove that the sides opposite equal angles in a triangle are equal.

4 *STUV* is a rhombus, so all sides are equal. R C



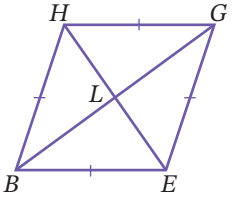
- **a** Prove that $\triangle VUS \equiv \triangle TUS$.
- **b** Prove that the diagonal *US* bisects the angles $\angle VUT$ and $\angle VST$.

5 *ABCD* is a parallelogram with opposite sides parallel. R C



- **a** Prove that $\triangle ABD \equiv \triangle CDB$.
- **b** Prove that opposite sides of a parallelogram are equal.

6 *BEGH* is a rhombus (a parallelogram with equal sides) whose diagonals *BG* and *EH* intersect at *L*. R C



- **a** Prove that $\triangle BEL \equiv \triangle GHL$.
- **b** Prove that the diagonals of a rhombus bisect each other.
- **c** $\triangle BEH$ is isosceles, so which 2 angles are equal?
- **d** Hence prove that $\triangle BEL \equiv \triangle BHL$.
- **e** Hence prove that the diagonals of a rhombus cross at right angles.

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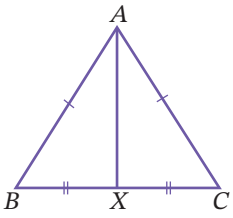
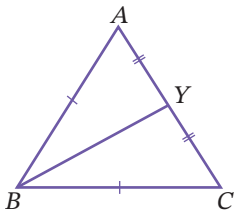
7 $\triangle ABC$ is an equilateral triangle ($AB = BC = AC$). X is the midpoint of BC. R C



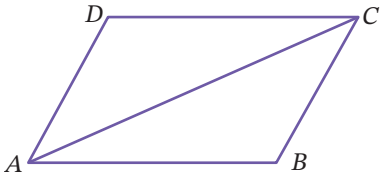


a Prove that $\triangle ABX \equiv \triangle ACX$ and $\angle B = \angle C$.

b In the second diagram, $\triangle ABC$ is redrawn so that Y is the midpoint of AC.
Prove that $\triangle BAY \equiv \triangle BCY$ and $\angle A = \angle C$.

c Calculate the sizes of the 3 angles of $\triangle ABC$.

8 $ABCD$ is a quadrilateral in which opposite sides are equal. R C

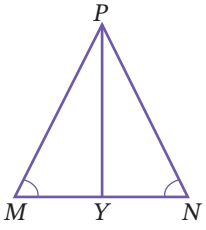


a Prove that $\triangle ACD \equiv \triangle ACB$.

b Prove that $AB \parallel CD$ and $AD \parallel BC$.

c What type of quadrilateral is $ABCD$?

9 In $\triangle PMN$, $\angle M = \angle N$ and YP bisects $\angle MPN$. R C

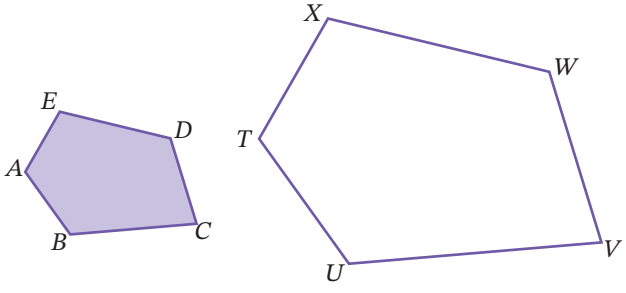


a Prove that $\triangle PMY \equiv \triangle PNY$.

b Prove that $MY = NY$ and $PY \perp MN$.

Investigation

Same shape, different size



Foundation | Standard | Complex

1 **a** Consider the pentagons *ABCDE* and *TUVWX*. Do they look similar?

b What do you think is meant by the word 'similar'?

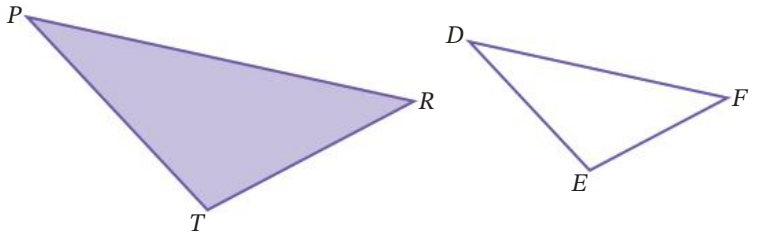
c Copy and complete this table by measuring each angle and side in the 2 'similar' pentagons.

Angles		Sides (in mm)	
$A =$	$T =$	$AB =$	$TU =$
$B =$	$U =$	$BC =$	$UV =$
$C =$	$V =$	$CD =$	$VW =$
$D =$	$W =$	$DE =$	$WX =$
$E =$	$X =$	$AE =$	$TX =$

d What do you notice about the sizes of the matching angles?

e What do you notice about the sizes of the matching sides?

2 $\triangle PTR$ has been reduced to produce the similar triangle $\triangle DEF$.



a List the 3 pairs of matching angles and measure the size of each one.

b List the 3 pairs of matching sides and measure the length of each one.

3 Copy and complete:

a For similar figures, m_____ angles are e_____.

b Similar figures have the s_____ shape, but not the same s_____.

13.04 Similar figures

WS A page of similar figures

WS Enlargements and reductions

WS Enlarging a logo

Cartoon enlargement

Similar figures have the same shape but are not necessarily the same size.

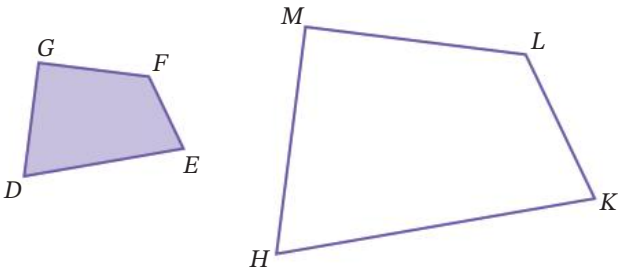
When a photograph is enlarged or reduced, all parts of the picture remain the same shape, but all lengths and distances will be respectively larger or smaller than the corresponding lengths and distances on the original.

iStock.com/aluxum

When a figure is enlarged or reduced, a **similar figure** is created. The original figure is called the **original**, while the enlarged or reduced figure is called the **image**.

The scale factor

The **scale factor** describes by how much a figure has been enlarged or reduced. For example, if the quadrilateral *DEFG* is enlarged by doubling each side, the scale factor is 2.



Original Image

$HK = 2 \times DE$

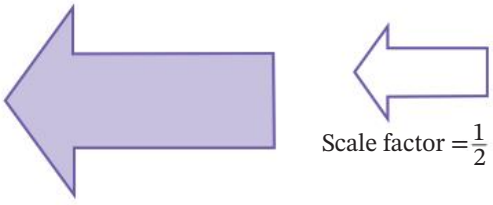
$KL = 2 \times EF$

$LM = 2 \times FG$

$HM = 2 \times DG$

$\therefore$ Scale factor $= 2$

In a similar manner, if the arrow figure is reduced by halving each side, the scale factor is $\frac{1}{2}$.



Original Image

Scale factor

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$

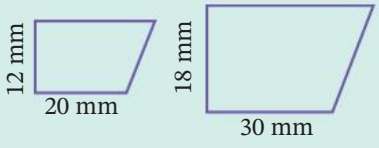
- If the scale factor is greater than 1, then the image is an enlargement.
- If the scale factor is between 0 and 1, then the image is a reduction.

Example 8

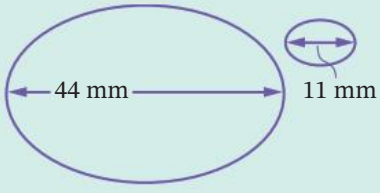
Find the scale factor for each pair of similar figures.

In all questions, assume the left figure is the original and the right figure the image.

a



b



Solution

a Scale factor $= \frac{30}{20}$ (or $\frac{18}{12}$) $= 1\frac{1}{2}$ $\quad \frac{\text{image length}}{\text{original length}}$

b Scale factor $= \frac{11}{44} = \frac{1}{4}$ $\quad \frac{\text{image length}}{\text{original length}}$

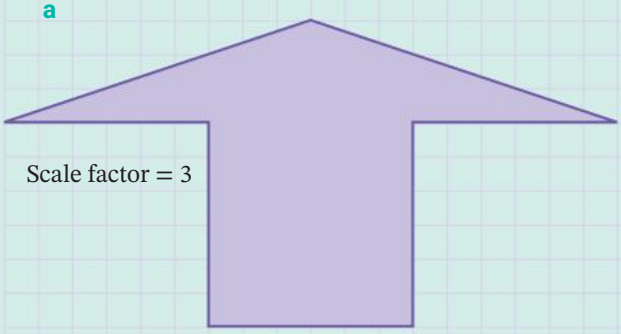
Example 9

Construct the image of the figure shown if the scale factor is:

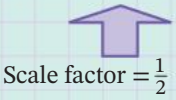
a 3 **b** $\frac{1}{2}$

Solution

a



b



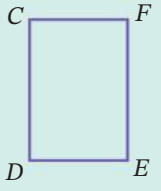
The similarity symbol |||

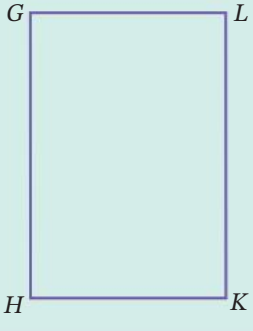
The symbol for 'is similar to' is '|||'. As with congruence notation, we must make sure that the vertices (or angles) of the similar figures are written in matching order.

Example 10

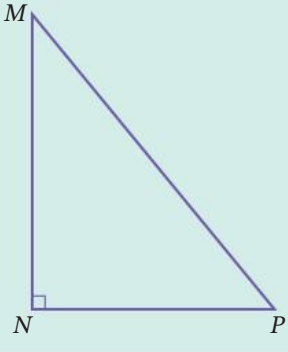
Test whether each pair of figures are similar by measuring them.

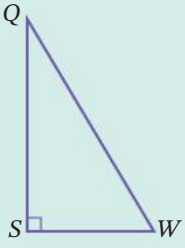
a





b





Solution

a The rectangle $GHKL$ is an enlargement of rectangle $CDEF$.

$$\text{Scale factor} = \frac{GH}{CD} = \frac{38}{19} = 2$$

$$\frac{HK}{DE} = \frac{26}{13} = 2$$

$$\therefore \frac{GH}{CD} = \frac{HK}{DE}$$

$\therefore CDEF \,|||\, GHKL$ Matching order of vertices.

b $\triangle QWS$ is not a reduction of $\triangle MPN$.

$$\frac{QS}{MN} = \frac{28}{40} = \frac{7}{10}$$

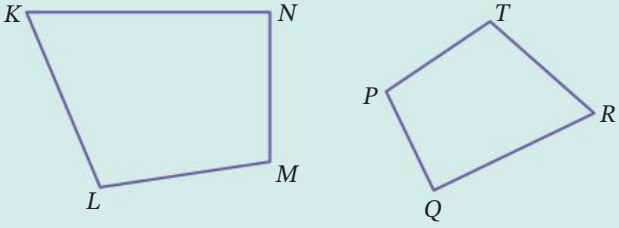
$$\frac{SW}{NP} = \frac{16}{32} = \frac{1}{2}$$

$$\frac{QS}{MN} \neq \frac{SW}{NP}$$

$\therefore \triangle MNP$ is not similar to $\triangle QSW$.

Example 11

The 2 quadrilaterals $KLMN$ and $PQRT$ are similar.



a List all pairs of matching sides and matching angles.

b Use the correct notation to write a similarity statement relating these 2 quadrilaterals.

Solution

a By rotating the figure $KLMN$, its shape can be matched with $PQRT$.

The pairs of matching sides are:	The pairs of matching angles are:
KN and QR	$\angle K$ and $\angle R$
MN and PQ	$\angle N$ and $\angle Q$
ML and PT	$\angle M$ and $\angle P$
LK and TR.	$\angle L$ and $\angle T$.

b $\angle K$ matches with $\angle R$, $\angle L$ matches with $\angle T$, $\angle M$ matches with $\angle P$, $\angle N$ matches with $\angle Q$.

$\therefore KLMN \,|||\, RTPQ$ Matching order of vertices

EXERCISE 13.04 ANSWERS ON P. 611

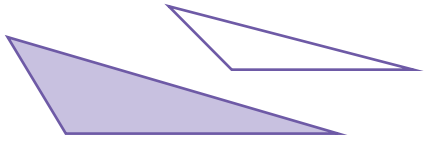
Similar figures UFRC

EXAMPLE 8

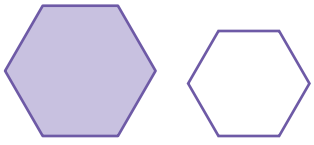
1 By measurement, find the scale factor for each pair of similar figures.

a

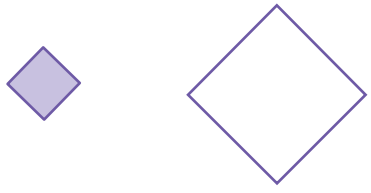
b



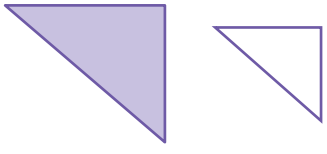
c



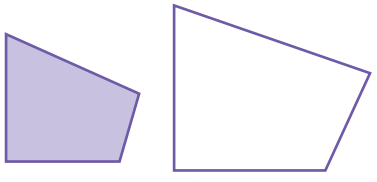
d



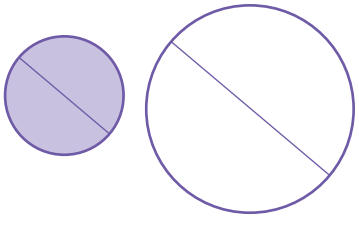
e



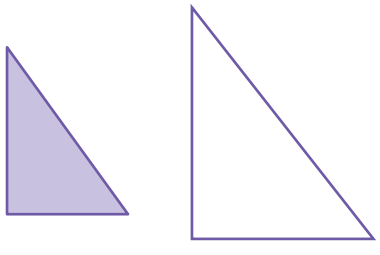
f



g



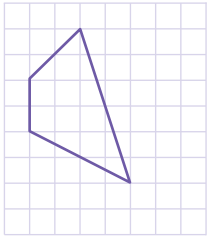
h



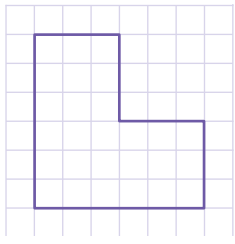
EXAMPLE 9

2 Copy each figure onto graph paper and draw its image using the given scale factor.

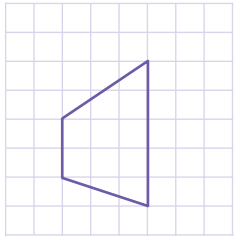
a Scale factor $= 2$



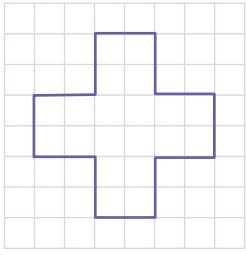
b Scale factor $= \frac{1}{3}$



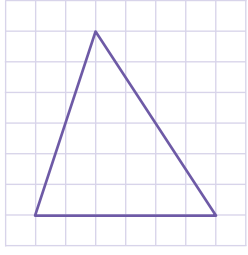
c Scale factor $= 2$



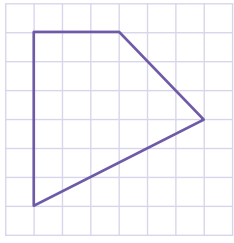
d Scale factor $= 3$



e Scale factor $= \frac{1}{2}$



f Scale factor $= \frac{2}{3}$



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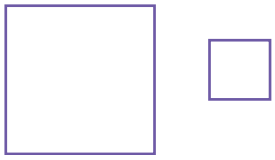
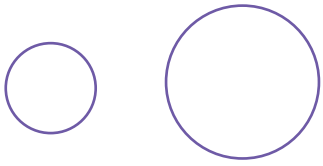
3 Are each pair of figures similar?

EXAMPLE 10

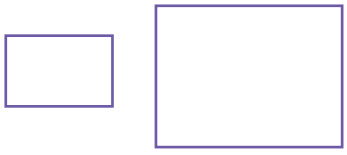
a



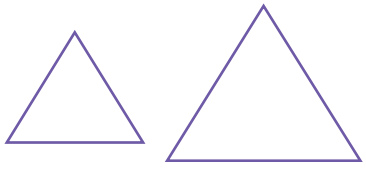
b



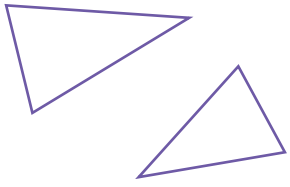
c



d

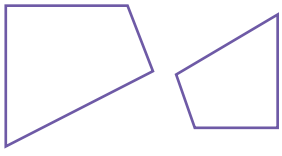


e



f

g



h

4 For each pair of similar figures: R C

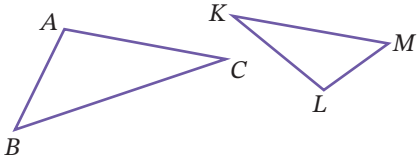
EXAMPLE 11

i list all pairs of matching angles

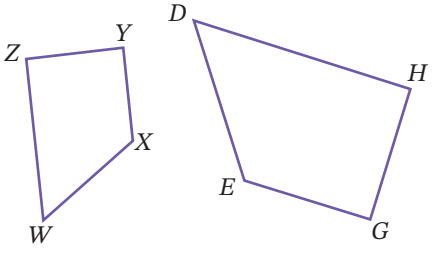
ii list all pairs of matching sides

iii use the correct notation to write a similarity statement relating them.

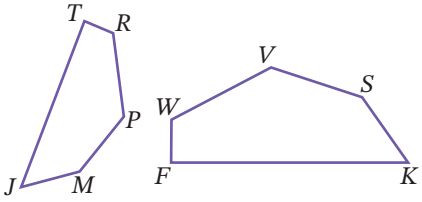
a



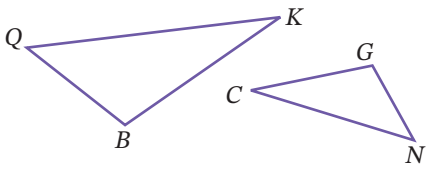
b



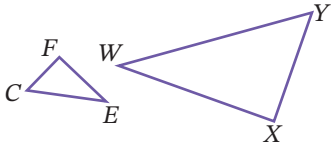
c



d



e



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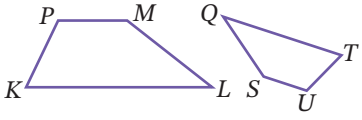

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Mental skills 13: Maths without calculators ANSWERS ON P. 611

Simplifying fractions and ratios

When simplifying a fraction or a ratio, look for a common factor to divide into both the numerator and the denominator, preferably the highest common factor (HCF).
Remember the divisibility tests:

A number is divisible by ...	if ...
2	it is even (its last digit is 2, 4, 6, 8 or 0)
3	the sum of its digits is divisible by 3
4	its last 2 digits form a number divisible by 4
5	its last digit is 0 or 5
6	it is even and the sum of its digits is divisible by 3
9	the sum of its digits is divisible by 9
10	its last digit is 0

1 Study each example.

a Simplify $\frac{27}{45}$.

$\frac{27^{9}}{45_{15}} = \frac{9}{15}$ dividing numerator and denominator by 3

$\frac{9^{3}}{15_{5}} = \frac{3}{5}$ dividing numerator and denominator by 3 again

Note: This fraction could be simplified in one step if you divided by 9, the highest common factor (HCF) of 27 and 45.

b Simplify $\frac{160}{400}$.

$\frac{160}{400} = \frac{16}{40}$ dividing numerator and denominator by 10

$\frac{16^{2}}{40_{5}} = \frac{2}{5}$ dividing numerator and denominator by 8

Note: This fraction could be simplified in one step if you divided by 80, the HCF of 160 and 400.

c Simplify 24 : 36.

$24 : 36 = 24^{4} : 36^{6} = 4 : 6$ dividing both terms by 6

$4^{2} : 6^{3} = 2 : 3$ dividing both terms by 2

Note: This fraction could be simplified in one step if you divided by 12, the HCF of 160 and 400.

d Simplify 135 : 90.

$135 : 90 = 135^{27} : 90^{18} = 27 : 18$ dividing both terms by 5

$27^{3} : 18^{2} = 3 : 2$ dividing both terms by 9

e Calculate $\frac{3}{8} \times \frac{2}{15}$ in simplest form.

$\frac{3}{8} \times \frac{2}{15} = \frac{3}{4\cancel{8}} \times \frac{\cancel{2}^{1}}{15}$ dividing 2 and 8 by 2

$= \frac{\cancel{3}^{1}}{4} \times \frac{2}{\cancel{15}_{5}}$ dividing 3 and 15 by 3

$= \frac{1}{20}$

f What fraction is 36 minutes of 1 hour?

$\frac{36}{1\text{ h}} = \frac{36\text{ min}}{60\text{ min}} = \frac{3}{5}$

2 Now simplify each fraction or ratio.

a $\frac{10}{15}$	**b** $\frac{16}{20}$	**c** $\frac{30}{42}$	**d** $\frac{8}{16}$
e $\frac{20}{80}$	**f** $\frac{6}{36}$	**g** $\frac{20}{24}$	**h** $\frac{12}{30}$
i 20 : 36	**j** 25 : 45	**k** 18 : 40	**l** 28 : 35
m 27 : 21	**n** 16 : 12	**o** $\frac{5}{6} \times \frac{18}{25}$	**p** $\frac{12}{50} \times \frac{10}{21}$

3 Express each as a simplified fraction.

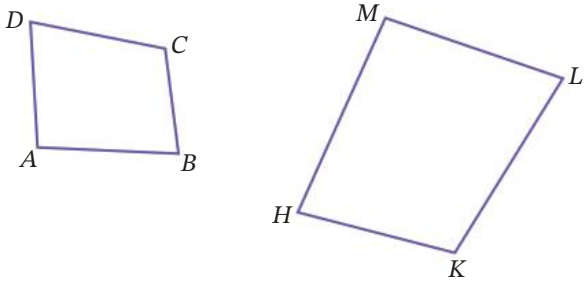
a 425 g of 1 kg	**b** 8 months of 1 year	**c** 64 cm of 1 m
d 750 mL of 3 L	**e** 10 hours of 2 days	**f** 80c of $10

13.04

Investigation

Properties of similar figures

1 The quadrilaterals *ABCD* and *HKLM* are similar.



a Measure the sizes of each pair of matching angles. What do you notice?

b Measure the lengths of each pair of matching sides.

c Calculate each ratio of matching sides, as a decimal correct to one decimal place.

i $\frac{MH}{AB}$ **ii** $\frac{HK}{BC}$ **iii** $\frac{KL}{CD}$ **iv** $\frac{ML}{AD}$

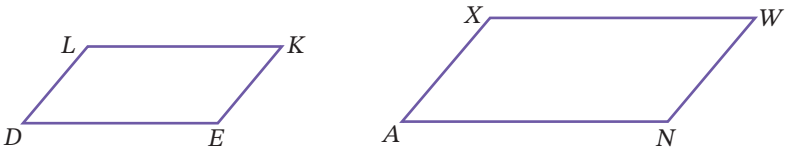
What do you notice?

d Copy and complete.

i The matching a_____ of similar figures are e_____.

ii Matching s_____ of similar figures are in the same r_____.

2 Consider the parallelograms *DEKL* and *ANWX*.



a Measure the sizes of each pair of matching angles. Are they equal?

b Measure the lengths of each pair of matching sides.

c Calculate each ratio of matching sides, as a decimal correct to one decimal place:

i $\frac{AN}{DE}$ **ii** $\frac{AX}{DL}$

Are they equal?

d *DEKL* is not similar to *ANWX*. Explain why.

13.05 Properties of similar figures

Finding sides in similar triangles

Similar triangles

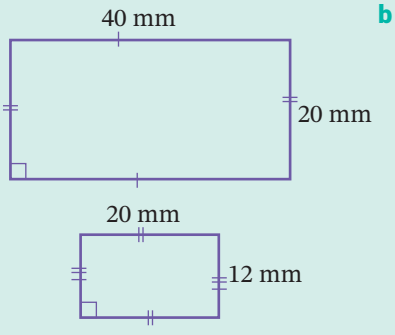
Properties of similar figures

- **Matching angles** are equal
- **Matching sides** are in the same ratio

Example 12

Test whether each pair of figures are similar.

a



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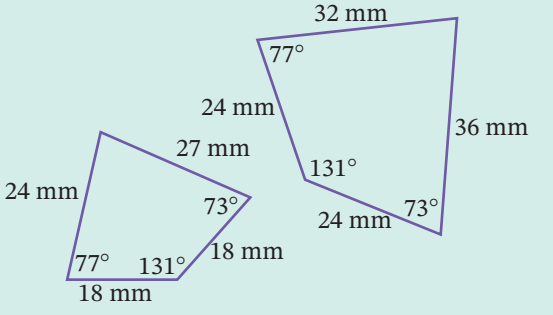

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Solution

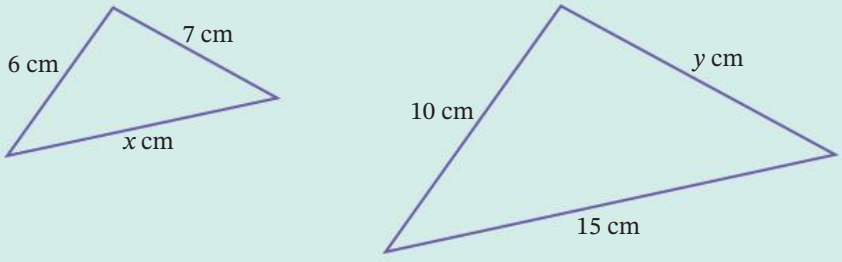
a For the 2 rectangles, matching angles are equal (90°), but the ratios of the matching sides are not equal $\left(\frac{20}{40}=\frac{1}{2}\text{ but }\frac{12}{20}=\frac{3}{5}\right)$.
∴ The rectangles are not similar.

b For the 2 quadrilaterals, matching angles are equal and the ratios of matching sides are equal $\left(\frac{32}{24}=\frac{36}{27}=\frac{24}{18}=\frac{4}{3}\right)$.
∴ The quadrilaterals are similar.

13.05

Example 13

These 2 triangles are similar. Find the value of x and y.



Finding an unknown side in similar figures

Solution

Since the triangles are similar, the ratios of matching sides are equal.

$\frac{x}{15}=\frac{6}{10}$

$x=\frac{6}{10}\times 15$

$=9$

$\frac{y}{7}=\frac{10}{6}$

$y=\frac{10}{6}\times 7$

$=11\frac{2}{3}$

Alternative method:

Scale factor $=\frac{10}{6}=\frac{5}{3}$

$x=15\div\frac{5}{3}$

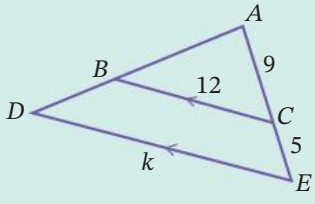
$=9$

$y=7\times\frac{5}{3}$

$=11\frac{2}{3}$

Example 14

$\triangle ABC \,|||\, \triangle ADE$. Find the value of k.



Nested similar triangles

Solution

$\frac{DE}{BC} = \frac{AE}{AC}$ Ratios of matching sides are equal.

$\frac{k}{12} = \frac{14}{9}$ $AE = 9 + 5 = 14$

$k = \frac{14}{9} \times 12$

$= 18\frac{2}{3}$

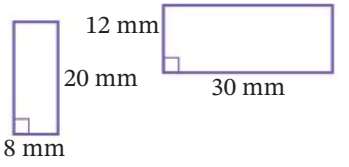
EXERCISE 13.05 ANSWERS ON P. 611

Properties of similar figures U F PS R

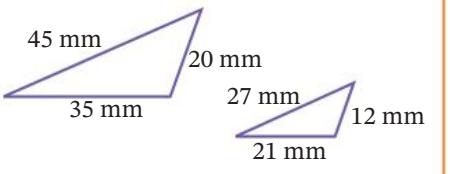
EXAMPLE 12

1 Test whether each pair of figures are similar. R

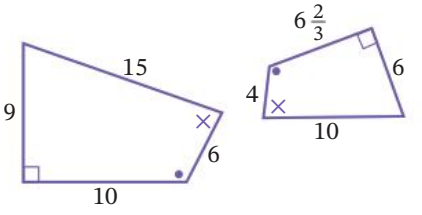
a



b



c



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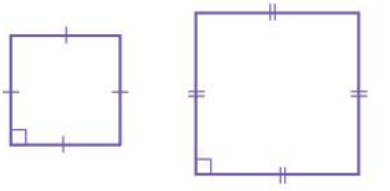
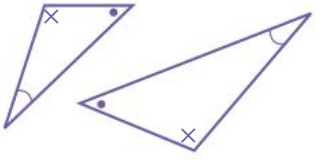
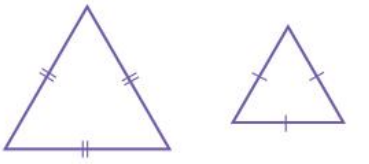


e



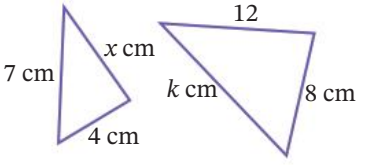
f



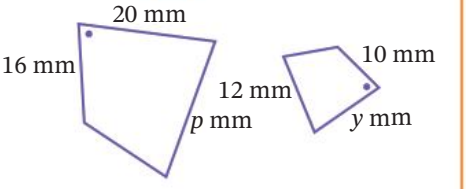
EXAMPLE 13

2 Find the value of each variable in each pair of similar figures. R

a



b



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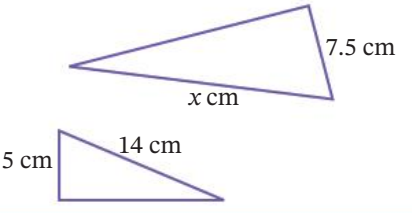

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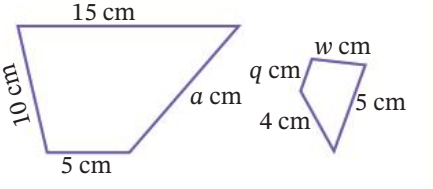

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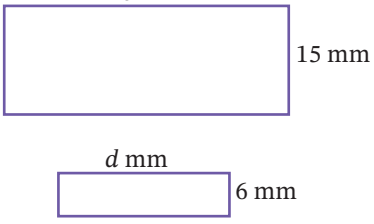
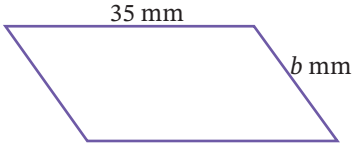
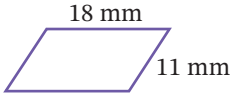
e

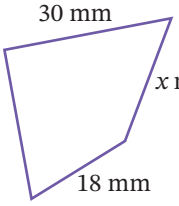


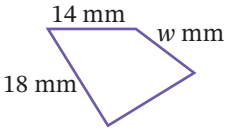
f



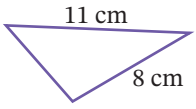


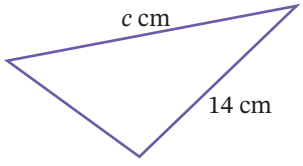
g





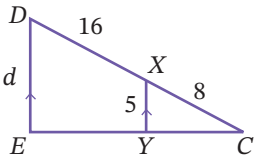
h



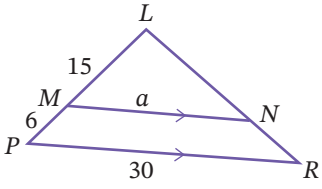


3 $\triangle CED \,|||\, \triangle CYX$. Find the value of d. R

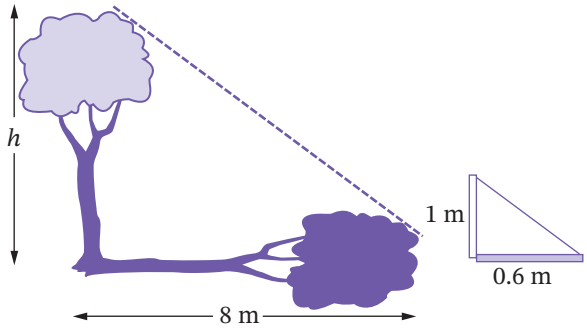
EXAMPLE 14



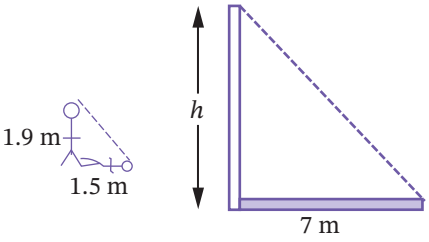
4 $\triangle LMN \,|||\, \triangle LPR$. Find the value of a. R



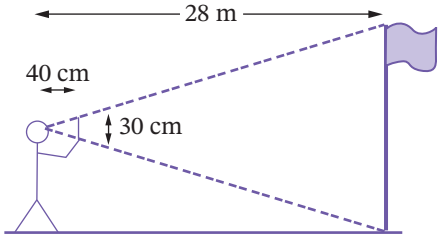
5 A metre stick casts a shadow of length 0.6 m when a tree casts a shadow of 8 m. Find the height h of the tree. R



6 Ayon, who is 1.9 m tall, casts a 1.5 m long shadow. At the same time, a light pole casts a shadow 7 m long. What is the height of the light pole, correct to one decimal place? R

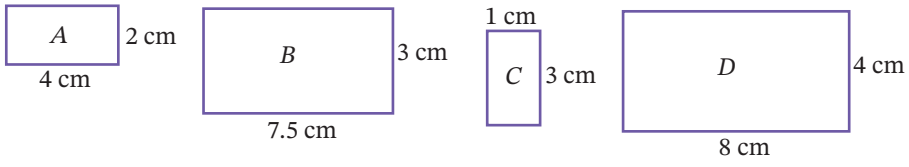


7 Emily holds up a small 40 cm long stick from her eye so that it appears to be the same height as the flagpole. She is standing 28 m from the flagpole and the stick is 30 cm long. Find the height of the flagpole. PS R

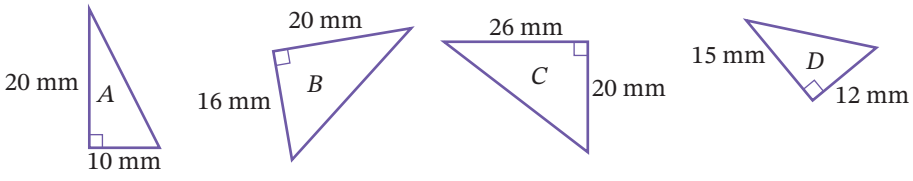


8 For each set of figures, find the pair of similar figures and the scale factor between them. R

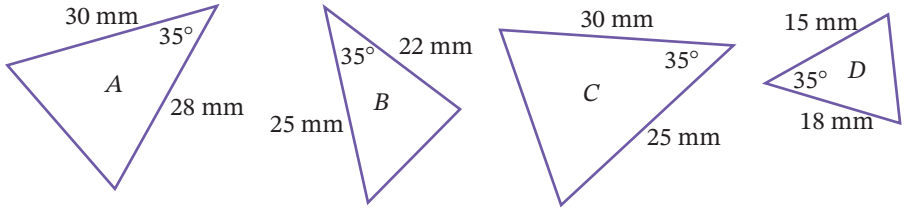
a



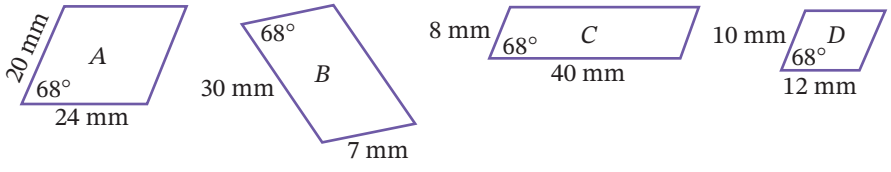
b



c



d



Investigation

Are all shapes similar?

Work in groups of 3 or 4 to complete this activity.

You will need: a ruler, a pencil, scissors.

1 **a** Draw 2 equilateral triangles, using a different side length for each triangle.

b Are the 2 equilateral triangles similar? Give reasons.

c Compare your triangles with those of other members of your group. Are all equilateral triangles similar? Give reasons.

2 **a** Draw a pair of similar isosceles triangles.

b Are all isosceles triangles similar? Give reasons.

3 **a** Draw a pair of rectangles that are similar.

b Draw a pair of rectangles that are *not* similar.

c When are rectangles similar? Discuss your answer with other students.

4 State whether the following statements are true or false. Give reasons for your answers. (Making a few sketches will help.)

a All parallelograms are always similar.

b All circles are always similar.

c All squares are always similar.

d All rhombuses are always similar.

Scale diagrams 13.06

A **scale diagram** accurately represents a larger or smaller object such that the lengths and distances on it are in the same ratio as the real lengths and distances. This means that the scale diagram is either an enlargement or reduction of the real object. The scale factor used is called the **scale** of the diagram.

A scale may be represented in different ways.

- 1 cm = 10 m or 1 cm to 10 m (a pair of corresponding measurements)
- 1 : 50 (a ratio)
-

(a line drawn to scale)

WS Problems involving scale drawings
Converting map scales to ratios
Converting map scales to ratios
Scale drawings

Scaled length

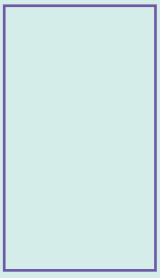
The scale on a scale diagram is written as a ratio of **scaled length : real length**, where **scaled length** is the length on the diagram.

For example, a scale of 1 : 100 means that the real lengths are 100 times larger than the lengths on the diagram.

Example 15

A scale diagram of a rectangular block of land has been drawn. What scale has been used?

Shutterstock.com/jax10289



Solution

By measurement, the length and width of the scale drawing are 35 mm and 20 mm.

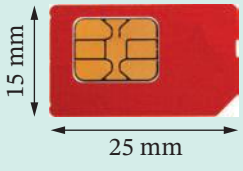
Scale = 35 mm : 35 m (or 20 mm : 20 m) scaled length : real length

= 35 mm : 35 000 mm (or 20 mm : 20 000 m) 1 m = 1000 mm

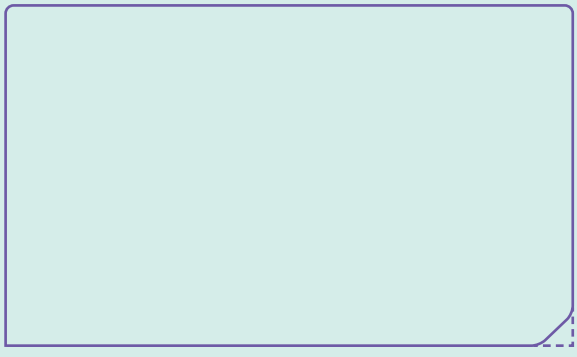
= 1 : 1000

Example 16

A SIM card for a mobile phone has a length of 25 mm and a width of 15 mm. A scale diagram of the SIM card is shown below.



Shutterstock.com/
GeniusKp



By measurement, determine what scale has been used.

Solution

By measurement, the length and width of the scale diagram are 75 mm and 45 mm respectively.

Scale = 75 mm : 25 mm (or 45 mm : 15 mm) scaled length : real length

= 3 : 1

Example 17

The scale on a map is 1 : 500 000. If the distance from Campbelltown to Sydney on the scale drawing is 9 cm, calculate the actual distance from Campbelltown to Sydney.

Solution

Scaled distance = 9 cm

Actual distance = $9 \times 500\,000$ cm

$= 4\,500\,000$ cm

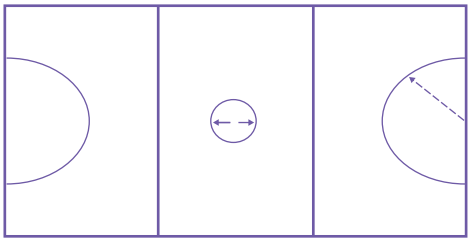
$= 45\,000$ m $\quad$ 1 m = 100 cm

$= 45$ km $\quad$ 1 km = 1000 m

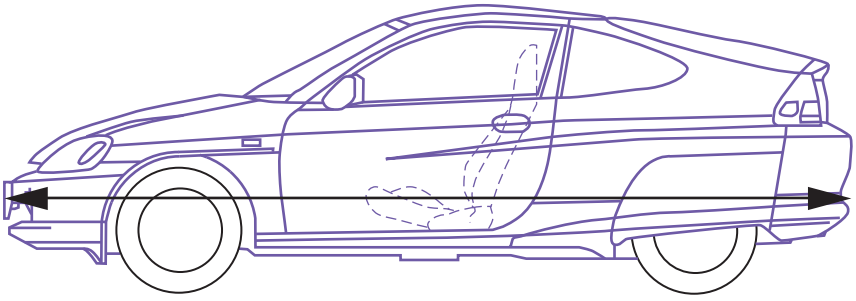
EXERCISE 13.06 ANSWERS ON P. 612

Scale diagrams **U F R**

1 If the length of a netball court is 30.5 m, what scale has been used on this diagram?



2 What scale has been used on this diagram of a car if its actual length is 3.92 m?

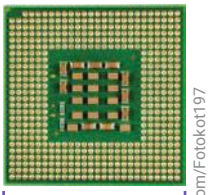


EXAMPLE 16

3 The picture of the Giant Brown Bull Ant has been enlarged. If its actual length is 25 mm, what scale has been used?

Nature Picture Library/Kim Taylor

4 A scale image of a microchip is shown. Determine the scale that has been used if its real length is 8 mm.



iStock.com/Fotokot197

5 The Q1 Tower on the Gold Coast, Queensland, is 322 m high and is Australia's tallest building. What scale has been used in this photo if the scaled height is 40 mm?

Shutterstock.com/Steven Bostock

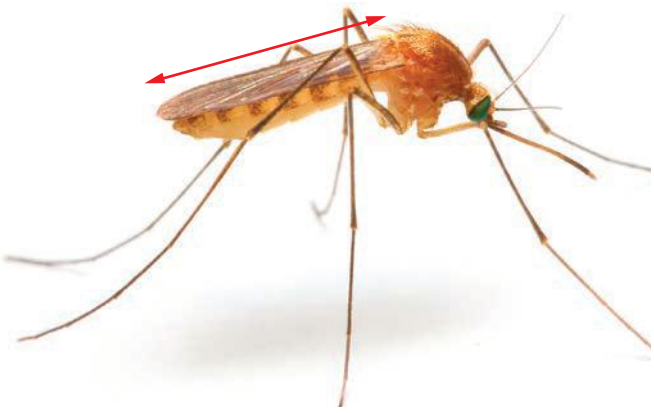
6 This photo of the Sydney Harbour Bridge has a scale of 1 : 9000. By measurement and calculation, find: R

a the height of the top of the bridge above the water

Shutterstock.com/ EerikSandstrom

b the length of the arch span of the bridge.

7 This is a photograph of a mosquito, at a scale of 15 : 1. Calculate, to the nearest 0.1 mm, the length of the wing. R



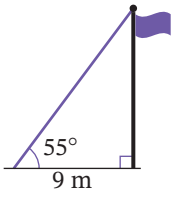
Shutterstock.com/Kletr

8 This box of cereal is 16 cm long and 22 cm tall. Construct a scale drawing of the front of the box, using a scale of 1 : 5.

Shutterstock.com/Nor Gal

9 **a** Construct a scale drawing of this flagpole using a scale of 1 : 300.

b Use the scale to calculate the actual height of the flagpole. R

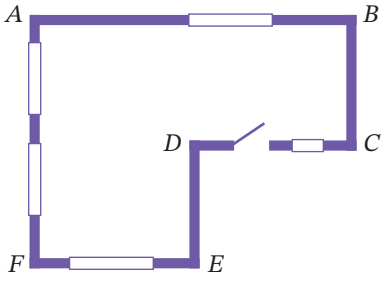


10 The scale on a map of Australia is 1 : 10 000 000. If the distance from Perth to Adelaide on the map is 21.2 cm, calculate the actual distance in kilometres. PS R

EXAMPLE 17

11 This scale plan of a holiday house has a scale of 1 : 200. Find the actual (internal) length of: PS R

a AF **b** FE



Foundation | Standard | Complex

12 Use the scale on this map to find, to the nearest kilometre, the distance from: R

a Nowra to Batemans Bay

b Bega to Moruya

c Mollymook/Ulladulla to Mogo

d Nowra to Eden.

Investigation

Floor plans

Construct a scale diagram of your bedroom or classroom.

Measure the dimensions of the room and each piece of furniture in it. Remember to measure the location of immovable objects such as windows, doors, wardrobes and whiteboards. Include a list of your measurements and the scale used. You may also do an Internet search of architectural symbols (bed, lounge, sink etc.) and include these in your room, where applicable.

Did you know?

Australia's Big Things

Unique to Australia are a number of tourist attractions known as the 'Bigs' or 'Biggies':

- the Big Merino (Goulburn, NSW)
- the Big Banana (Coffs Harbour, NSW)
- the Big Golden Guitar (Tamworth, NSW)
- the Big Pineapple (Woombye, Qld)
- the Big Orange (Berri, SA)
- the Big Crocodile (Wyndham, WA).
- the Big Captain Cook (Cairns, Qld)
- the Big Mango (Bowen, Qld)
- the Big Rocking Horse (Gumeracha, SA)
- the Giant Koala (Dadsells Bridge, Vic)

The Big Merino is 15 metres high. If the height of an average ordinary merino sheep is 90 cm, work out the scale of the Big Merino to a real sheep.

iStock.com/kokkai

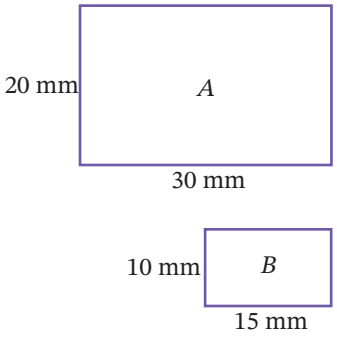
Investigation

Areas of similar figures

1 **a** The 2 rectangles A and B are similar. Explain why.

b What is the ratio of their matching sides (A to B)?

c What is the ratio of their areas (A to B)?

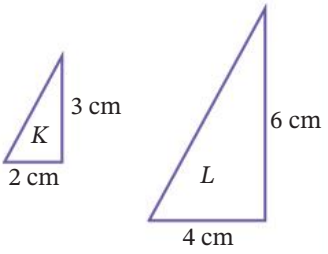


13.06

Area of similar shapes

Area of similar shapes

2 Triangles K and L are similar.



- **a** What is the ratio of their matching sides (K to L)?
- **b** Find the areas of both triangles.
- **c** What is the ratio of their areas (K to L)?
- **d** Copy and complete: When you double the lengths of a figure, the area is increased _____ times.

3 **a** What is the area of a rectangle with length 5 cm and width 4 cm?

- **b** The length and width of the rectangle are both multiplied by 3. What is the area of the enlarged rectangle?
- **c** What is the ratio of the matching sides of the 2 rectangles from parts **a** and **b**?
- **d** What is the ratio of the areas of the 2 rectangles?
- **e** Copy and complete: When you triple the lengths of a figure, the area is increased _____ times.

4 **a** What is the area of a circle with radius 2 cm?

- **b** The radius of this circle is multiplied by 4.
- **c** What is the area of the enlarged circle?
- **d** What is the ratio of the areas of the 2 circles?
- **e** Copy and complete: When you multiply the lengths of a figure by 4, the area is increased _____ times.

5 If the sides of a square are multiplied by 5, by how many times is the area increased?

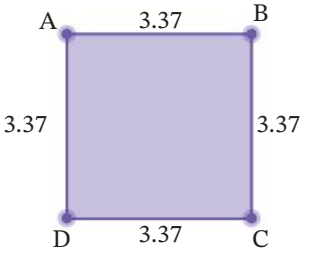
6 If the sides of a figure are multiplied by k, by how many times is the area increased?

Technology

Areas of similar figures

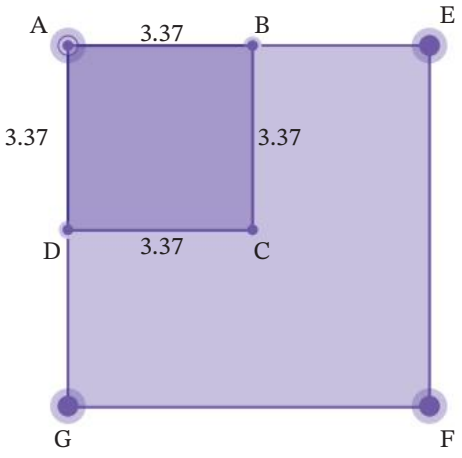
For this activity, use dynamic geometry software to draw similar figures and compare their areas.

1 Construct a square. An example is shown below.



2 Calculate the area of the square you have constructed in question 1.

3 Dilate (enlarge) the square by a scale factor of 2.



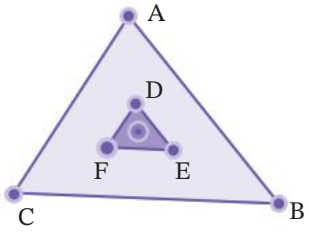
4 Use the tools to:

- **a** measure the length of each side
- **b** calculate the area of the enlarged square.
- **c** What is the ratio of the sides $DC : GF$?
- **d** What is the ratio of the areas of the squares $ABCD : AEFG$?
- **e** Compare the 2 ratios, **c** and **d**. How are they related?

5 **a** Draw a rectangle with length 3 cm and width 1.5 cm. What is the area of the rectangle?

- **b** Dilate the rectangle by a scale factor of 2 to enlarge the rectangle.
- **c** Find the ratios of
  - **i** sides $DC : GF$
  - **ii** sides $BC : EF$
  - **iii** areas of rectangles $ABCD : AEFG$.
- **d** Compare the ratio of the sides to the ratio of the areas. How are they related?

6 **a** Draw a triangle with $AB = 8$ cm. It should look like triangle ABC shown below.



b Dilate (reduce) the triangle by a scale factor of $\frac{1}{4}$ to reduce the triangle. Dilation can also be done by creating a point inside triangle ABC (it doesn't have to be one of the vertices, as shown in the diagram above).

c Find the ratios of:

i sides $AB : DE$ **ii** areas of triangles $ABC : DEF$

d Compare the ratio of the sides to the ratio of the areas. How are they related?

7 **a** Draw any polygon.

b Select **dilate** and either enlarge or reduce the polygon.

c Find the ratios of:

i matching sides of the similar shapes

ii the areas of the similar shapes

d Compare the ratio of the sides to the ratio of the areas. How are they related?

13.07 Areas of similar figures

STAGE 5.3

Areas of similar figures

If the matching sides of 2 similar figures are in the ratio $1 : k$, then their areas are in the ratio $1 : k^2$.

If the matching sides are in the ratio $m : n$, then their areas are in the ratio $m^2 : n^2$.

Example 18

a 2 similar triangles have sides in the ratio 1 : 5. Find the ratio of their areas.

b 2 similar parallelograms have sides in the ratio 3 : 1. Find the ratio of their areas.

Solution

a Ratio of sides $= 1 : 5$

Ratio of areas $= 1 : 5^2$ $1 : k^2$

$= 1 : 25$

b Ratio of sides $= 3 : 1$

Ratio of areas $= 3^2 : 1$

$= 9 : 1$

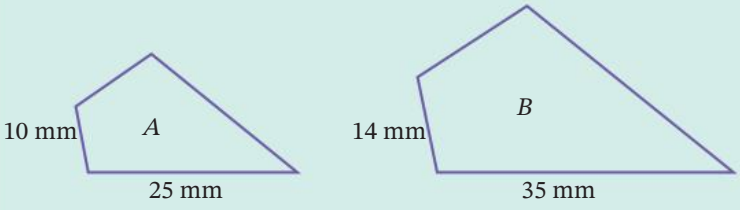
STAGE 5.3

Areas of similar figures

13.07

Example 19

What is the ratio of the areas of the similar quadrilaterals shown?



Solution

$$\text{Ratio of matching sides } (A \text{ to } B) = 25 : 35 \quad \text{or } 10 : 14$$
$$= 5 : 7$$
$$\text{Ratio of areas } (A \text{ to } B) = 5^2 : 7^2$$
$$= 25 : 49$$

Example 20

Two similar trapeziums have their areas in the ratio 25 : 16.
Find the ratio of the lengths of the matching sides.

Solution

Ratios of areas $= m^2 : n^2 = 25 : 16$

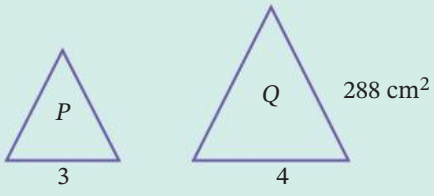
$\therefore$ Ratio of sides $= m : n = \sqrt{25} : \sqrt{16} = 5 : 4$

Example 21

Two similar triangles P and Q have matching sides in the ratio 3 : 4.
If the area of the larger triangle is 288 cm^2, find the area of the smaller triangle.

Solution

Drawing a diagram of the problem is useful.



Ratio of matching sides $= 3 : 4$

Ratio of areas $= 3^2 : 4^2 = 9 : 16$

$$\therefore \frac{\text{Area of } P}{288} = \frac{9}{16}$$

$$\text{Area of } P = \frac{9}{16} \times 288$$
$$= 162 \text{ cm}^2$$

The area of the smaller triangle is 162 cm^2.

STAGE 5.3

EXERCISE 13.07 ANSWERS ON P. 612

Areas of similar figures UFR

EXAMPLE 18

1 2 similar rectangles have sides in the ratio 1 : 4. Find the ratio of their areas.

2 2 similar circles have radii in the ratio 1 : 2.5. Find the ratio of their areas.

3 The ratio of the matching sides for pairs of similar figures are as shown. Find the ratio of the areas for each pair.

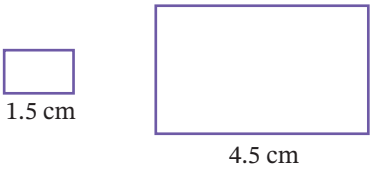
a 10 : 1 **b** 5.5 : 1 **c** 3 : 4 **d** 5 : 2

e 4 : 7 **f** 1 : 10 **g** 8 : 5 **h** 2 : 9

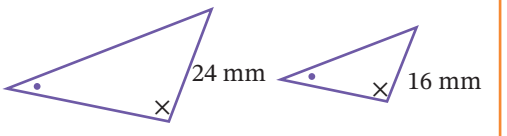
EXAMPLE 19

4 Find the ratio of the areas for each pair of similar figures.

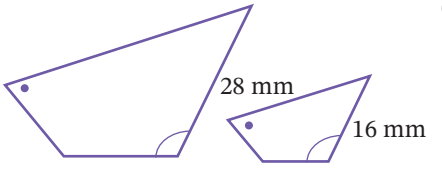
a



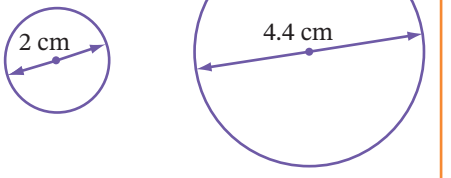
b



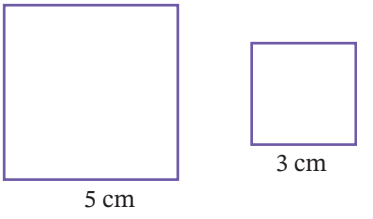
c



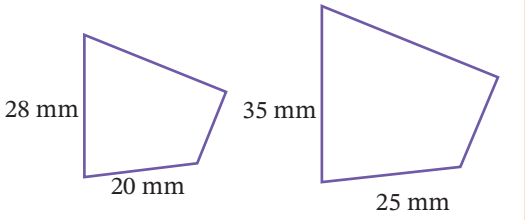
d



e



f



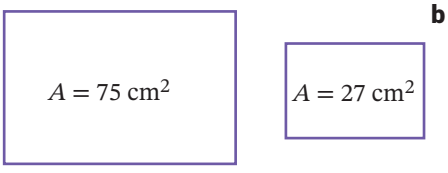
EXAMPLE 20

5 For each ratio between the areas of a pair of similar figures, find the ratio of the lengths of the matching sides.

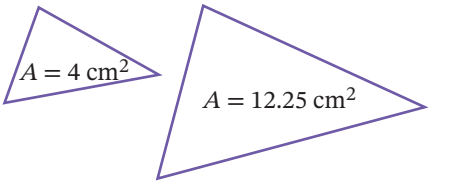
a 64 : 49 **b** 36 : 1 **c** 9 : 400 **d** 1 : 20.25

6 Find the ratio of the length of matching sides for each pair of similar figures.

a



b



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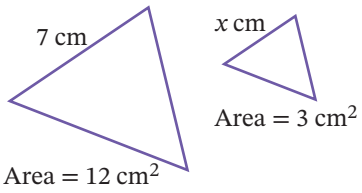
7 Find the value of x if these 2 triangles are similar.



8 2 similar triangles have matching sides in the ratio 4 : 1. If the area of the smaller triangle is 192 cm², find the area of the larger triangle. R

9 2 squares have sides in the ratio 3 : 4. If the area of the larger square is 288 cm², find the area of the smaller square. R

10 2 similar quadrilaterals have matching sides in the ratio 8 : 5. If the area of the smaller quadrilateral is 400 mm², find the area of the larger quadrilateral. R

11 2 circles have radii in the ratio 3 : 7. Find the area of the smaller circle if the area of the larger circle is 154 m². R

12 2 similar rectangles have areas in the ratio 36 : 121. If the width of the smaller rectangle is 84 mm, find the width of the larger rectangle. R

13 2 similar triangles have areas in the ratio 4 : 9. If the length of the base of the smaller triangle is 15 mm, find the length of the base of the larger triangle. R

14 2 squares have areas in the ratio 49 : 25. The length of the larger square is 17.5 cm. Find the length of the smaller square. R

15 If the radius of a circle is tripled, how has its area changed? R

16 If the area of a square is reduced by a factor of 4, how have the sides changed? R

17 The area of an equilateral triangle is decreased to $\frac{9}{16}$ of its original amount. How does its new area compare to its original area? R

18 A rectangle is reduced using a scale factor of $\frac{1}{2}$. If the area of the rectangle was 200 cm², what is the area of the reduced rectangle? R

Investigation

Similar triangles

Work in groups of 3 or 4 to complete this investigation.

You will need: paper, a ruler, a pencil, a protractor, compasses and scissors.

1 **a** Draw $\triangle ABC$ with $AB = 25$ mm, $AC = 35$ mm and $BC = 20$ mm.

b How many different triangles can be drawn with sides of 25 mm, 35 mm and 20 mm? Is this triangle unique?

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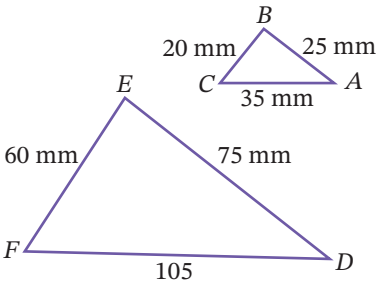
c Draw $\triangle DEF$ with $DE = 75$ mm, $DF = 105$ mm and $EF = 60$ mm.

d Are the matching sides of the triangles in the same ratio?

e Use your protractor to measure the angles of both triangles. Are matching angles equal?

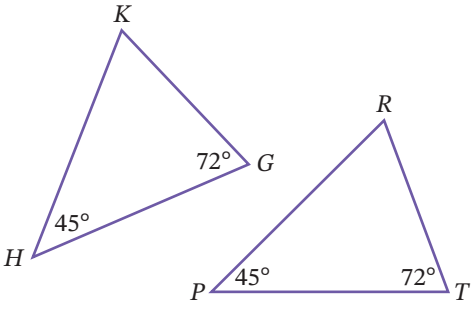
f Is $\triangle ABC$ similar to $\triangle DEF$?



2 **a** Draw $\triangle GHK$ with $\angle G = 72°$ and $\angle H = 45°$.

b Draw $\triangle TPR$ with $\angle T = 72°$ and $\angle P = 45°$. Is $\triangle GHK \,|||\, \triangle TPR$?

c Compare your triangles with those of other students. Are they all similar?

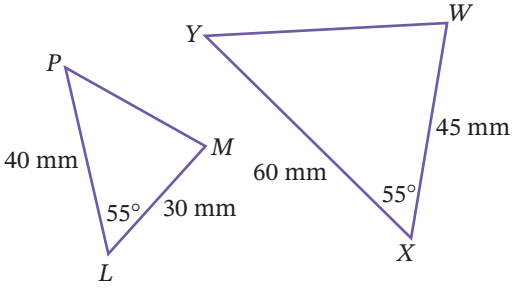


3 **a** Draw $\triangle LMP$ with $LM = 30$ mm, $LP = 40$ mm and $\angle L = 55°$ (the included angle).

b Draw $\triangle XWY$ with $XW = 45$ mm, $XY = 60$ mm and $\angle X = 55°$.

c Find the ratio of 2 pairs of matching sides $\frac{LM}{XW}$ and $\frac{LP}{XY}$. Are they equal?

d Are the 2 triangles similar? Explain.

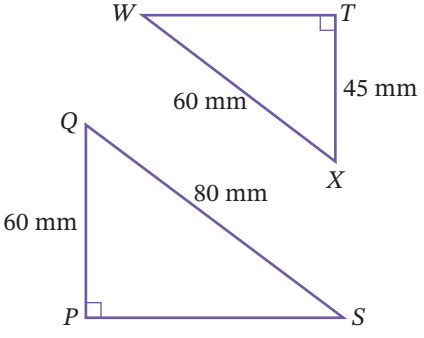


4 **a** Construct $\triangle PQS$ with $PQ = 60$ mm, $\angle P = 90°$ and $QS = 80$ mm (the hypotenuse).

b Construct $\triangle TXW$ with $TX = 45$ mm, $\angle T = 90°$ and $XW = 60$ mm.

c Calculate the ratios of matching sides $\frac{PQ}{TX}$ and $\frac{QS}{XW}$.

d Are the triangles similar? Explain.



5 **a** Compare and discuss your results with other students.

b What tests are necessary for 2 triangles to be similar?

c How do these conditions compare to the conditions for congruent triangles?

Tests for similar triangles

13.08

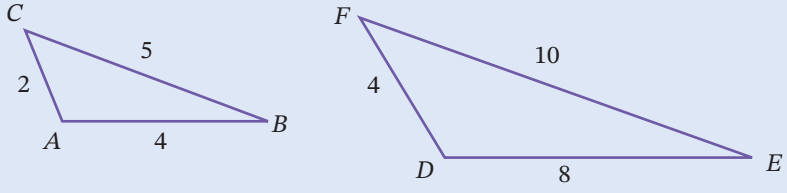
There are 4 sets of conditions that can be used to determine if 2 triangles are similar.
These are called the **tests for similar triangles** or **similarity tests**.

Similarity tests

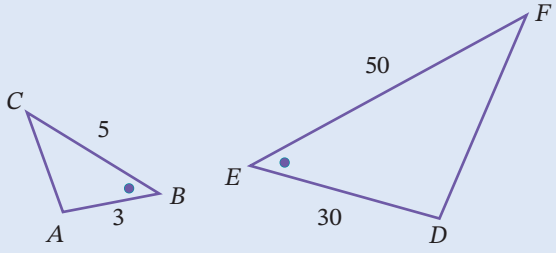
There are 4 tests for similar triangles.

Two triangles are similar if:

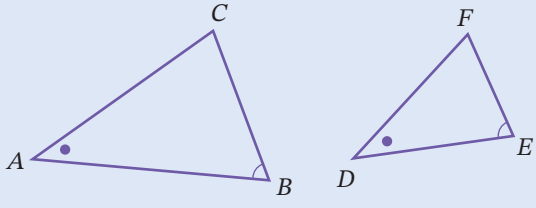
- the 3 sides of one triangle are proportional to the 3 sides of the other triangle **('SSS')**



- 2 sides of one triangle are proportional to 2 sides of the other triangle, and the **included** angles are equal **('SAS')**

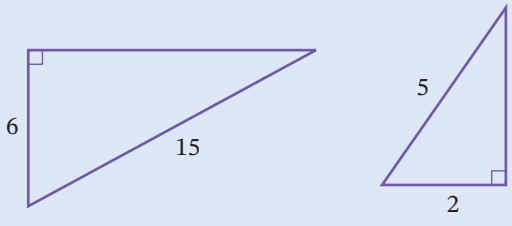


- 2 angles of one triangle are equal to 2 angles of the other triangle **('AA' or 'equiangular')**



Equiangular means 'equal angles'

- they are right-angled and the hypotenuse and a second side of one triangle are proportional to the hypotenuse and a second side of the other triangle **('RHS').**

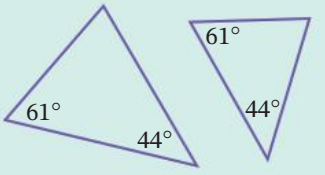


Tests for similar triangles

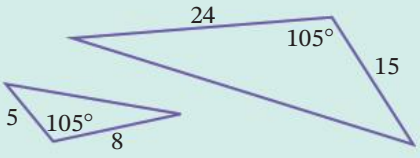
Example 22

Which test can be used to prove that each pair of triangles are similar?

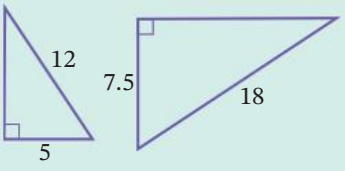
a



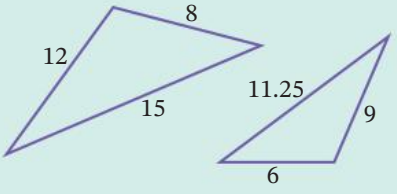
b



c



d



Solution

a 2 pairs of angles are equal, or equiangular ('AA').

b 2 pairs of matching sides are in the same ratio $\frac{15}{5}=\frac{24}{8}=3$ and the included angles in both triangles are 105° ('SAS').

c In both right-angled triangles, the pairs of hypotenuses and second sides are in the same ratio $\frac{7.5}{5}=\frac{18}{12}=\frac{3}{2}$ ('RHS').

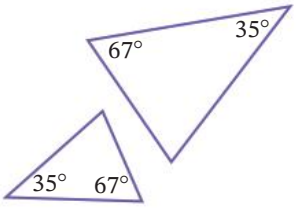
d All 3 pairs of matching sides are in the same ratio $\frac{11.25}{15}=\frac{9}{12}=\frac{6}{8}=\frac{3}{4}$ ('SSS').

EXERCISE 13.08 ANSWERS ON P. 612

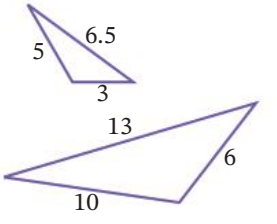
Tests for similar triangles U F R C

1 Which test can be used to prove that each pair of triangles are similar? R C

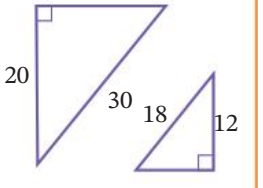
a



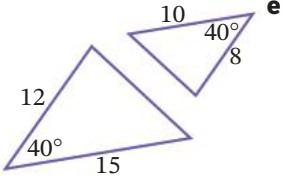
b



c



d



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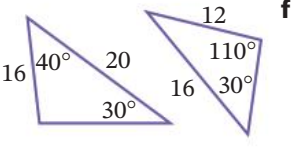

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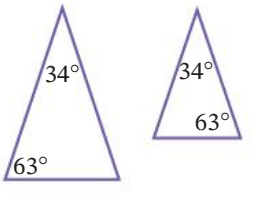

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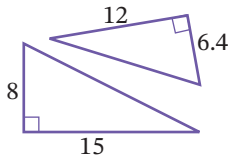
g



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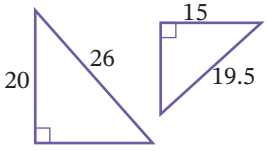
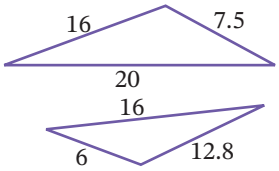
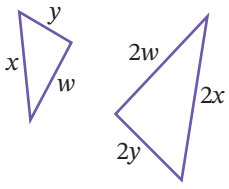


i



j



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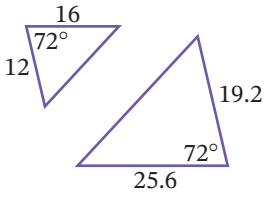

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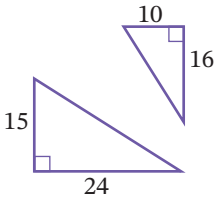
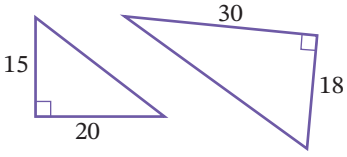
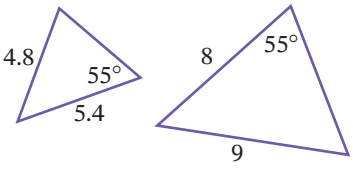


2 Which pair of triangles are similar? Select the correct answer **A**, **B**, **C** or **D**. R

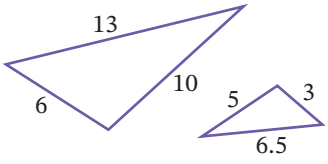
A



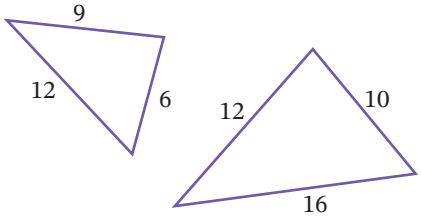
B



C

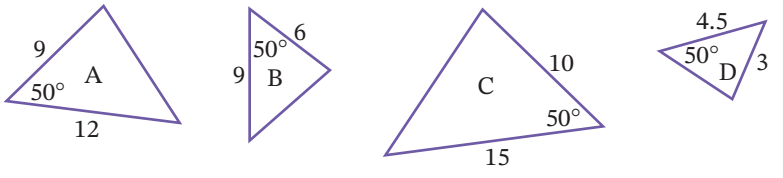


D

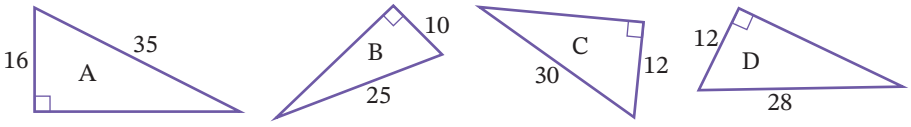


3 For each set of triangles, find the pair of similar triangles. R

a



b



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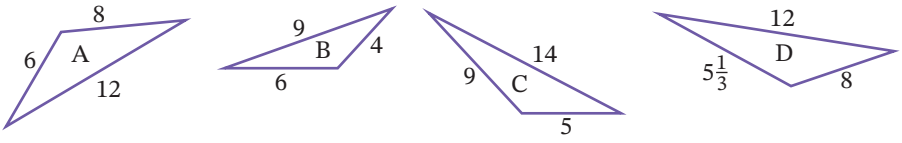
c



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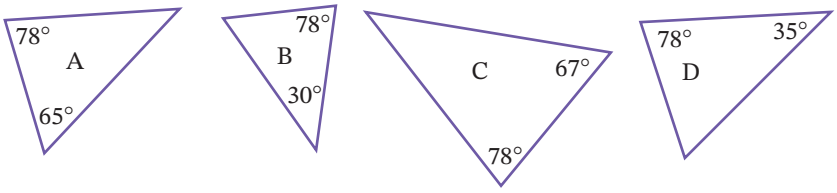
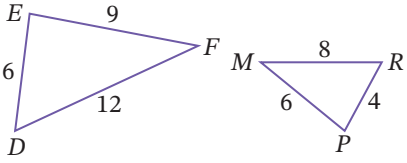


4 Use the correct notation to write a similarity statement relating each pair of similar triangles. R C

a



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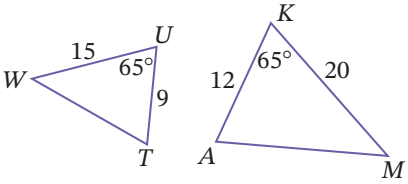

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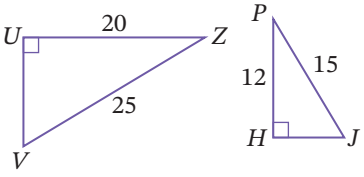

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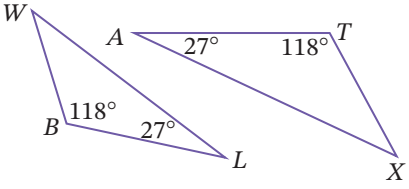

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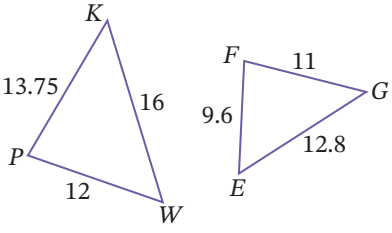

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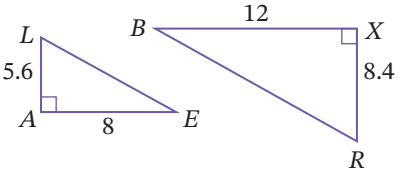


5 Similar triangles have 3 pairs of matching angles. So why is the similarity test AA and not AAA? R C

Similar triangle proofs

13.09

To formally prove that 2 triangles are similar, we use a specific format that involves applying one of the 4 similarity tests SSS, SAS, AAA or RHS.

STAGE 5.3

WS Congruent and similar triangle proofs

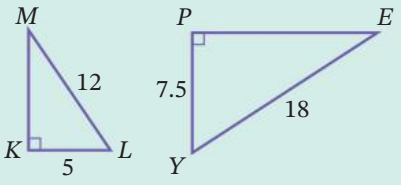
WS Complete the proofs

13.09

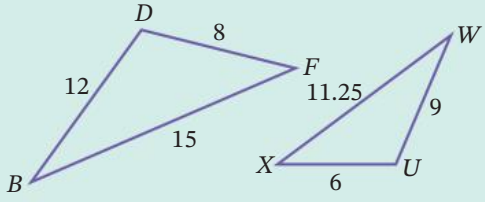
Example 23

Prove that each pair of triangles are similar.

a



b



Solution

a In $\triangle MKL$ and $\triangle EPY$:

$$\frac{ML}{EY} = \frac{12}{18} = \frac{2}{3}$$

$$\frac{KL}{PY} = \frac{5}{7.5} = \frac{2}{3}$$

$$\therefore \frac{ML}{EY} = \frac{KL}{PY}$$

$\angle K = \angle P = 90°$

$\therefore \triangle MKL \,|||\, \triangle EPY$ (hypotenuses and another pair of matching sides of right-angled triangles in proportion, or 'RHS')

b In $\triangle DFB$ and $\triangle UXW$:

$$\frac{BF}{WX} = \frac{15}{11.25} = \frac{4}{3}$$

$$\frac{BD}{WU} = \frac{12}{9} = \frac{4}{3}$$

$$\frac{DF}{UX} = \frac{8}{6} = \frac{4}{3}$$

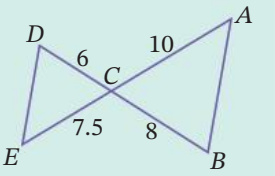
$$\therefore \frac{BF}{WX} = \frac{BD}{WU} = \frac{DF}{UX}$$

$\therefore \triangle DFB \,|||\, \triangle UXW$ (3 pairs of matching sides in proportion, or 'SSS')

STAGE 5.3

Example 24

Prove that $\triangle ABC \,|||\, \triangle EDC$.



Solution

In $\triangle ABC$ and $\triangle EDC$:

$$\frac{AC}{EC} = \frac{10}{7.5} = \frac{4}{3}$$

$$\frac{BC}{DC} = \frac{8}{6} = \frac{4}{3}$$

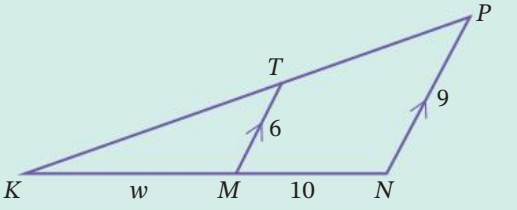
$$\therefore \frac{AC}{EC} = \frac{BC}{DC}$$

$\angle ACB = \angle DCE$ (vertically opposite angles)

$\therefore \triangle ABC \,|||\, \triangle EDC$ (2 pairs of matching sides in proportion and the included angles equal, or 'SAS')

Example 25

Prove that $\triangle KMT \,|||\, \triangle KNP$ and hence find the value of w.



Solution

In $\triangle KMT$ and $\triangle KNP$:

$\angle K$ is common.

$\angle KMT = \angle KNP$ (corresponding angles, $MT \,||\, NP$)

$\therefore \triangle KMT \,|||\, \triangle KNP$ (equiangular, or 'AA')

$$\therefore \frac{w}{w+10} = \frac{6}{9}$$ (matching sides of similar triangles)

$$9w = 6(w+10)$$
$$= 6w + 60$$
$$3w = 60$$
$$w = 20$$

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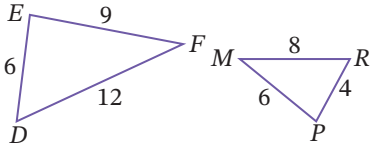
EXERCISE 13.09 ANSWERS ON P. 612

Similar triangle proofs UFRC

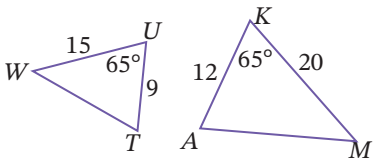
EXAMPLE 23

1 Prove that each pair of triangles are similar. R C

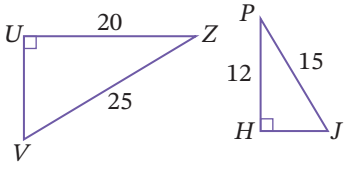
a



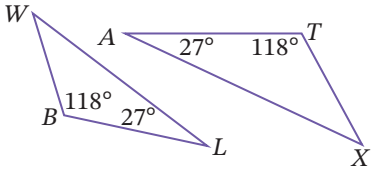
b



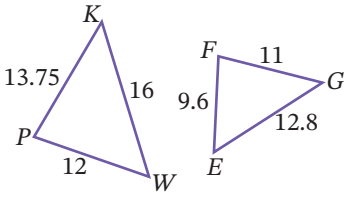
c



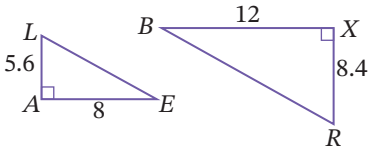
d



e



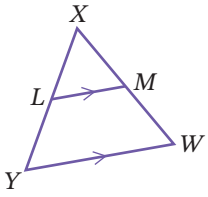
f



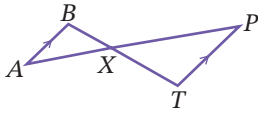
EXAMPLE 24

2 Prove that each pair of triangles are similar. R C

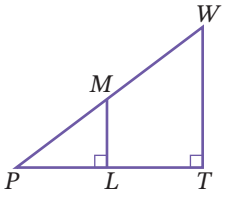
a $\triangle XYW \mid\mid\mid \triangle XLM$.



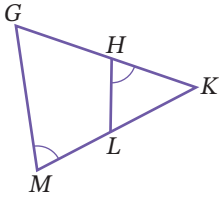
b $\triangle ABX \mid\mid\mid \triangle PTX$.



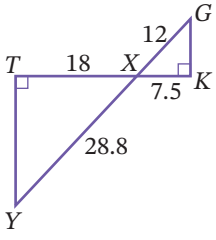
c $\triangle PLM \mid\mid\mid \triangle PTW$.



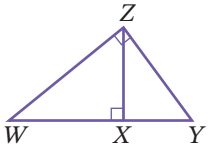
d $\triangle GKM \mid\mid\mid \triangle LKH$.



e $\triangle YTX \mid\mid\mid \triangle GKX$



f $\triangle WZY \mid\mid\mid \triangle WXZ$



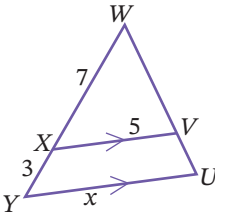
Foundation Standard Complex

STAGE 5.3

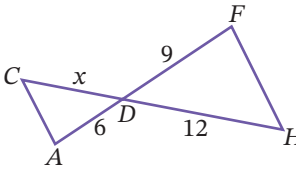
EXAMPLE 25

3 Find the value of x in each diagram after proving that 2 triangles are similar. R C

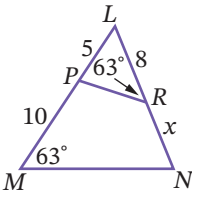
a



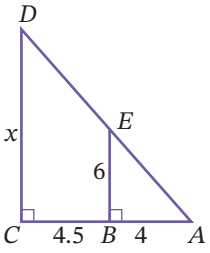
b



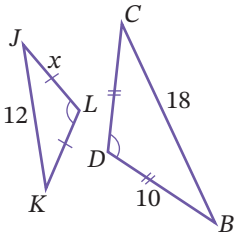
c



d



e



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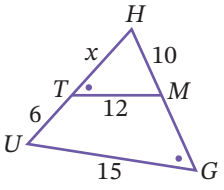
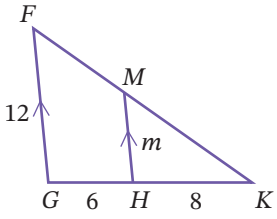
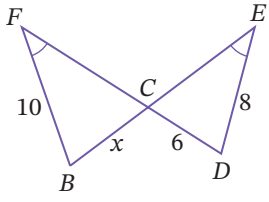


4 **a** Prove $\triangle KGF \,|||\, \triangle KHM$. R C

b Find the value of m.

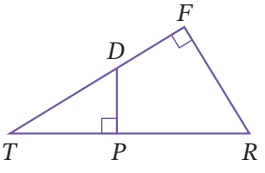


5 Prove $\triangle BCF \,|||\, \triangle DCE$ and hence find the value of x. R C



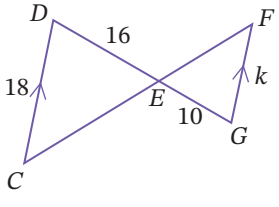
6 **a** Prove that $\triangle TFR \,|||\, \triangle TPD$. R C

b If $FR = 9$ cm, $DP = 5$ cm and $TD = 8$ cm, find the length of TR.



7 **a** Prove that $\triangle CDE \,|||\, \triangle FGE$. R C

b Find the value of k.



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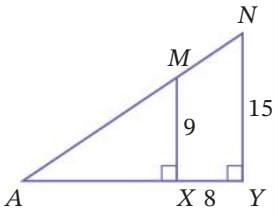
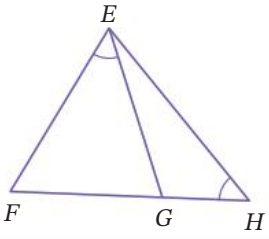
8 Prove $\triangle AXM \,|||\, \triangle AYN$ and hence find the length of AX. R C



9 **a** Prove that $\triangle FGE \,|||\, \triangle EFH$. R C

b If $FG = 16$ cm and $EF = 20$ cm, find the length of FH.



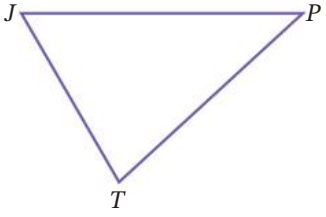
STAGE 5.3

13.09

Power plus ANSWERS ON P. 613

1 Construct $\triangle JPT$ by:

- **a** using compasses and the lengths of the 3 sides
- **b** using the length of one side and the size of the 2 angles
- **c** using the length of 2 sides and the size of one angle.



WS Investigating paper sizes

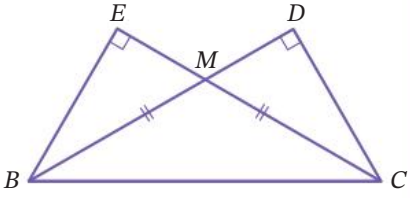
2 **a** List the information given in this diagram.

b Why is $\angle BME = \angle CMD$?

c Show that $\triangle BME \equiv \triangle CMD$.

d Why is $EM = DM$?

e Why is $\angle EBM = \angle DCM$?

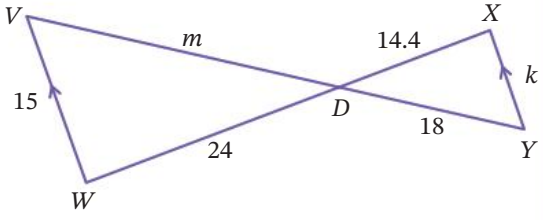


3 **a** What information is shown in the diagram?

b Why is $\angle V = \angle Y$ and $\angle W = \angle X$?

c Show that $\triangle VWD \,|||\, \triangle XYD$.

d Find the values of k and m.



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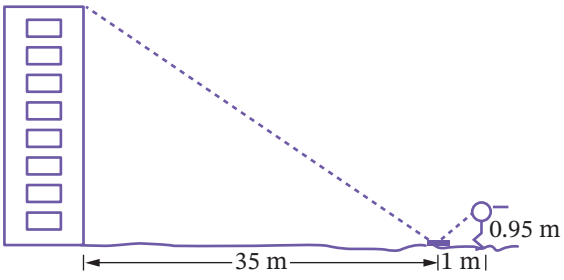
4 Alexa places a mirror on the ground. She stands 1 m from the mirror and then bobs down until she can see the top of a building in the mirror. Her eyes are 95 cm above ground level when she can see the building. If the mirror is 35 m from the base of the building, what is the height of the building?



5 Is each statement true or false? Give reasons.

- **a** All equilateral triangles are similar.
- **b** All isosceles triangles are similar.
- **c** All circles are similar.
- **d** All rectangles are similar.
- **e** All squares are similar.
- **f** All regular pentagons are similar.

CHAPTER 13 REVIEW

Language of maths

AAS	bisect	congruence test	congruent (≡)
diagonal	enlargement	hypotenuse	image
included angle	matching	original	parallel
proportional	ratio	reduction	RHS
SAS	scale	scale diagram	scale factor
similar (|||)	similarity test	SSS	vertex/vertices

1 When an original figure is enlarged or reduced, what is the new figure called?

2 What is the angle between 2 given sides of a triangle called?

3 What is the symbol and meaning of 'is congruent to'?

4 What is the symbol and meaning of 'is similar to'?

5 Which word from the above list is another name for 'corresponding'?

6 What happens to a shape that is changed by a scale factor of 3?

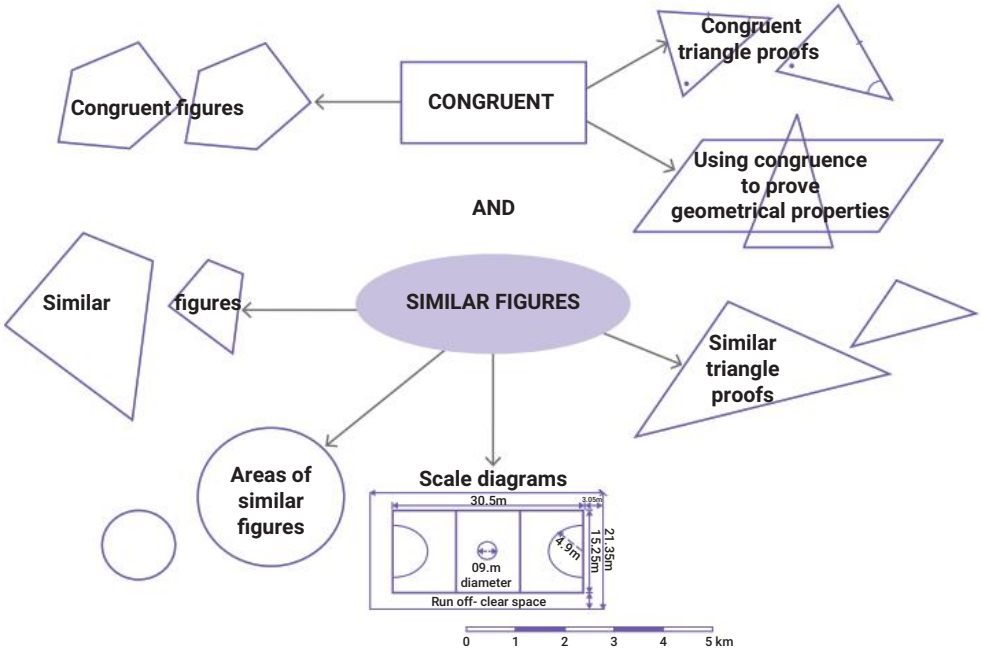
7 What are the 4 tests for congruent triangles?

8 What does RHS stand for:

a in congruent triangles? **b** in solving equations?

Topic summary

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Geometry crossword

Congruence and similarity review

Congruence and similarity

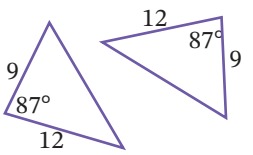
Mind map: Congruent and similar figures (Advanced)

TEST YOURSELF 13 ANSWERS ON P. 613

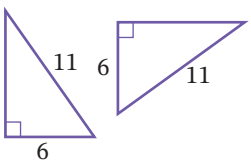
13.01

1 Which congruency test (SSS, SAS, AAS or RHS) can be used to prove that each pair of triangles are congruent?

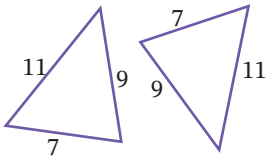
a



b

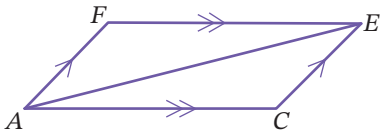


c



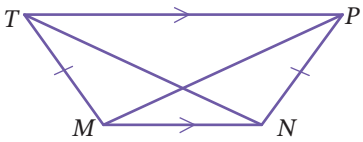
13.02

2 If $ACEF$ is a parallelogram, prove that $\triangle ACE \equiv \triangle EFA$.



13.02

3 If $MN \parallel TP$, $\angle MNP = \angle NMT$ and $TM = PN$, prove that $\triangle MNP \equiv \triangle NMT$.

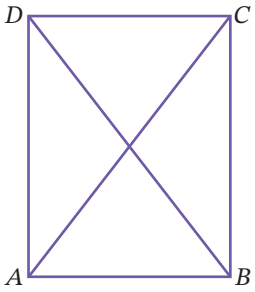


13.03

4 $ABCD$ is a rectangle.

a Prove that $\triangle ABD \equiv \triangle BAC$.

b Hence prove that the diagonals of a rectangle are equal in length.

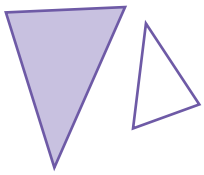


13.04

5 By measurement, find the scale factor for each pair of similar figures.

a

b



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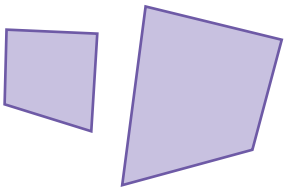
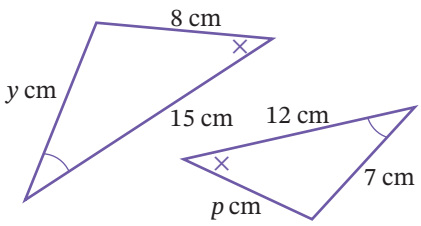


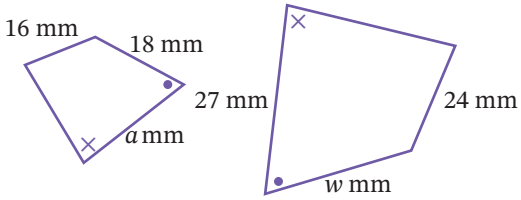
13.05

6 Find the value of each variable in each pair of similar figures.

a



b



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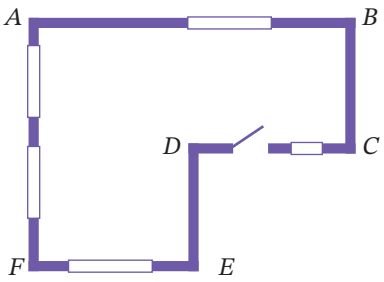
7 This floor plan of a holiday house is drawn to a scale of 1: 200. What is the actual (internal) length of:

a AB?

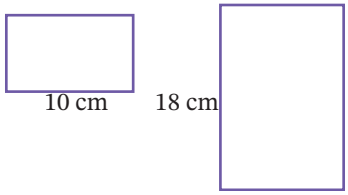
b CD?



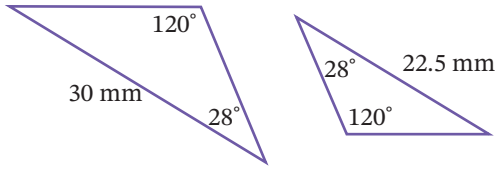
13.06

8 Find the ratio of the areas for each pair of similar figures.

a



b



STAGE 5.3

13.07

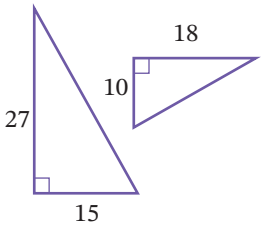
13.07

9 Two similar trapeziums have areas in the ratio 25 : 4. If the perpendicular height of the smaller trapezium is 40 cm, find the perpendicular height of the larger trapezium.

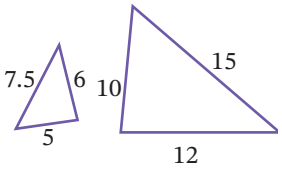
10 Which test can be used to prove that each pair of triangles is similar?

13.08

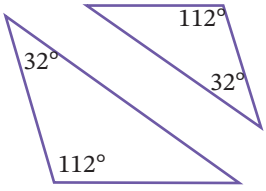
a



b

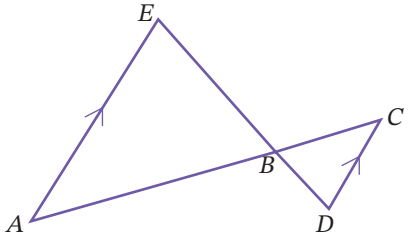


c



11 **a** Prove that $\triangle ABE \,|||\, \triangle CBD$.

b If $DC = 8$ cm, $AE = 14$ cm and $CB = 7$ cm, find the length of AB.



STAGE 5.3

13.09

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PRACTICE SET 4 ANSWERS ON P. 614

11.04

1 Test whether the point (2, 3) lies on each line.

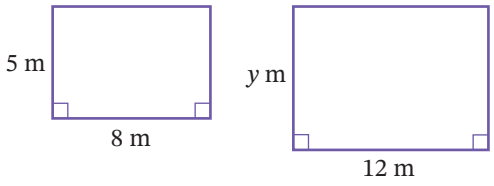
a $2x + y = 4$ **b** $y = 2x - 1$ **c** $x + y = 5$

13.05

2 The rectangle on the left has been enlarged to make the rectangle on the right.

a What is the scale factor?

b Find the value of y.



12.01

3 The probability of Eva hitting a bullseye on a target is 0.2. Find the probability of Eva missing the bullseye.

12.04

4 A class was surveyed on whether its students could swim. The results are shown.

	Can swim	Cannot swim
Boys	13	2
Girls	9	3

a Find the probability of randomly selecting a student who is:

i a boy **ii** a boy or someone who cannot swim

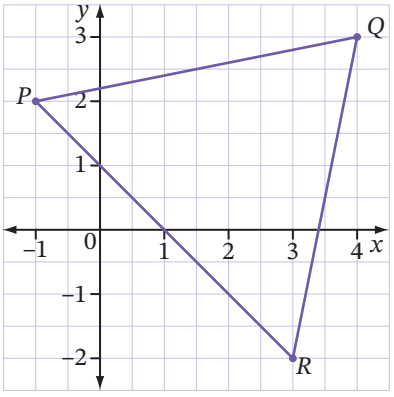
iii a girl who cannot swim **iv** a girl or someone who can swim, but not both

b If a swimmer is selected at random, what is the probability that it is a boy?

13.06

5 The scale on a tourist map of Sydney is 2 cm = 500 m. If the scaled distance from Circular Quay station to the Opera House is 2.5 cm, what is the actual distance?

Questions **6** to **9** refer to this triangle that has vertices $P(-1, 2)$, $Q(4, 3)$ and $R(3, -2)$.



11.01

6 Find the length of:

a side PQ, as a surd

b side PR, correct to one decimal place

11.01

7 Show that $\triangle PQR$ is isosceles.

11.02

8 Find the coordinates of the midpoint of side PR.

11.03

9 Find the gradient of:

a PQ

b PR

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10 Graph $y = 4 - 2x$ on a number plane and write: 11.04

a its x-intercept

b its y-intercept.

11 Daniel rolled a biased die 90 times and the number 6 came up 24 times. 12.02

a What is the experimental probability of rolling a 6?

b If the same die was rolled 400 times, what is the expected frequency of rolling a 6?

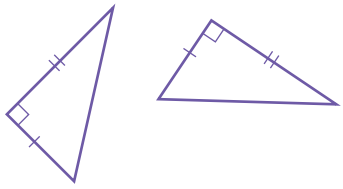
12 **a** Sketch the graph of $y = x^2 - 2$. 11.09

b Write the coordinates of the vertex.

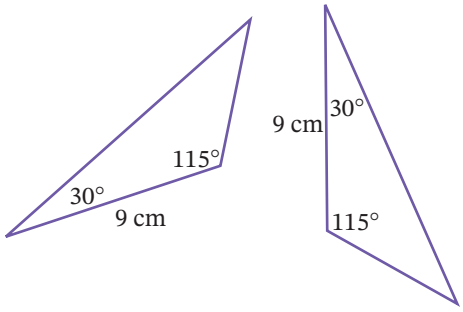
c Is the graph concave up or concave down?

13 Which congruence test (SSS, SAS, AAS or RHS) can be used to prove that each pair of triangles are congruent? 13.02

a



b



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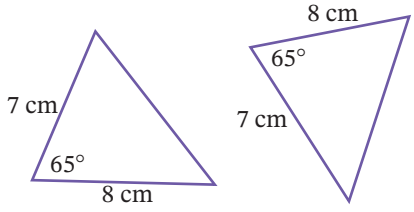

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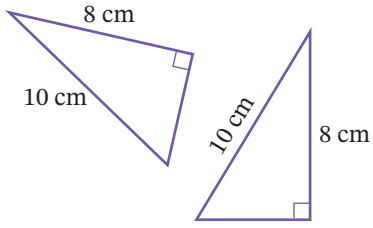

14 What is the equation of the horizontal line passing through (–2, 4)? 11.04

15 The numbers 1 to 30 are written on cards and the cards are placed in a bag. If one card is drawn at random from the bag, what is the probability that it is: 12.01

a a multiple of 5?

b greater than 10?

16 **a** Find the gradient and y-intercept of the line with equation $y = \frac{1}{2}x + 3$. 11.05

b Graph a line with gradient 2 and y-intercept –5 and write its equation.

17 The increase in pressure experienced by a scuba diver is directly proportional to the depth under the water. The increase in pressure at depth 40 m is 235 kPa (kilopascals). 11.08

a What increase in pressure is experienced at a depth of 32 m?

b At what depth, to the nearest metre, is the increase in pressure 735 kPa?

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12.05

18 2 dice are rolled together. Copy and complete the table to show the sample space.

		1st die					
		1	2	3	4	5	6
2nd die	1						
	2					5, 2	
	3						
	4		2, 4				
	5						
	6						

Find the probability of rolling:

a a 3 and a 6 (in any order)

b at least one 2

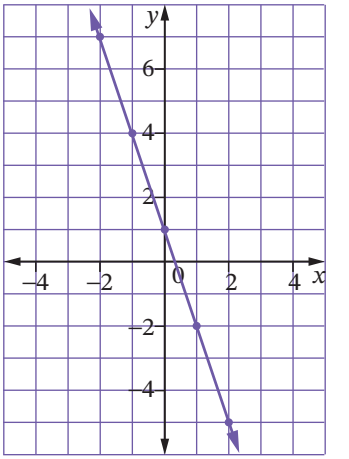
c 2 numbers the same

d a 4 or a 5 but not both

11.06

19 **a** Find the gradient of this line.

b Find the equation of this line.

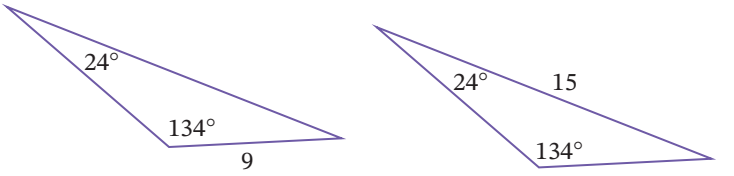


11.10

20 What is the equation of a circle with centre (0, 0) and radius 4 units?

13.07

21 Which test can be used to prove that this pair of triangles are similar?



11.10

22 Match each equation to the correct description of its graph.

a $y = x^2 + 9$

b $y = x + 9$

c $x^2 + y^2 = 9$

A straight line

B circle

C parabola

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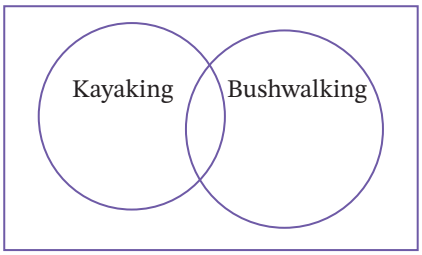
23 40 students at a school camp could select kayaking or bushwalking as an activity, or both. A total of 35 students chose kayaking, 20 chose bushwalking, including some who chose both. There weren't any students who chose neither.



12.03

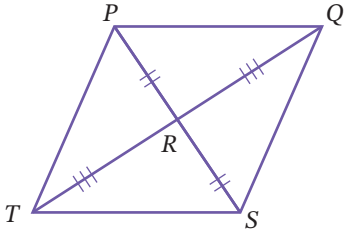
a Copy and complete the Venn diagram representing these students.

b What is the probability that a student selected at random:

i chose kayaking and bushwalking?

ii chose kayaking or bushwalking but not both?

24 The diagonals of this rhombus divide the rhombus into 4 congruent triangles.



13.03

a Prove that $\triangle PRQ \equiv \triangle PRT$.

b Hence find the size of $\angle PRQ$.

c Copy and complete: The diagonals of a rhombus _____ each other at _____ _____.

25 Two similar rectangles have their areas in the ratio 1 : 7.84. If the length of the larger rectangle is 5.04 m, calculate the length of the smaller rectangle.

STAGE 5.3

13.07

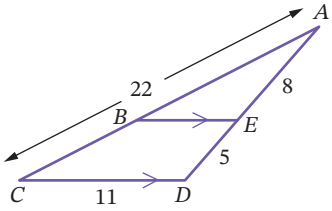
13.09

26 **a** Prove that $\triangle ABE \,|||\, \triangle ACD$.

b Find the length of:

i BE

ii BC



GENERAL PRACTICE

ANSWERS ON P. 614

STAGE 5.3

CHAPTER 2 CHAPTER 7 CHAPTER 10

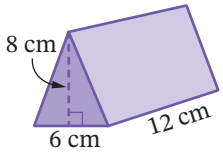
1 Simplify each expression.

a $\frac{\sqrt{125}}{5}$ **b** $7\sqrt{3}+\sqrt{12}$ **c** $-5\sqrt{7}\times 4\sqrt{7}$ **d** $\frac{9\sqrt{56}}{\sqrt{7}}$

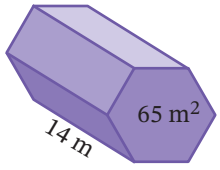
2 Make t the subject of the formula $k=my-\frac{1}{2}mt^2$.

3 Find the volume of each solid, correct to one decimal place where necessary.

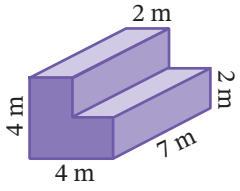
a



b



c



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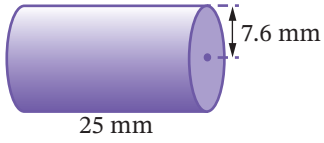
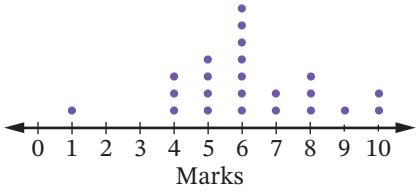


CHAPTER 9

4 This dot plot shows the project results (out of 10) for a Year 9 class.



a How many students were in the class?

b Find:

i the range

ii the mode

iii the mean, correct to one decimal place

iv the median

c Which of the scores can be considered as an outlier?

d Are there any clusters? Identify them.

CHAPTER 9

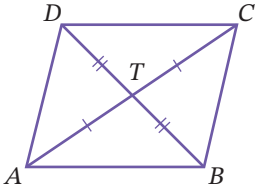
5 Classify each type of data as categorical (C), numerical discrete (ND) or numerical continuous (NC).

a number of taxis on the road **b** rating of an app (1 star to 5 stars)

c masses of babies in a maternity ward **d** brands of mobile phones

CHAPTER 13

6 The diagonals of the quadrilateral $ABCD$ bisect each other.



a Prove that $\triangle DTC \equiv \triangle BTA$.

b Hence prove that $DC \parallel AB$.

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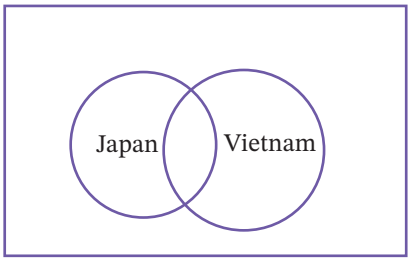
7 A survey was held to find whether a sample of 30 Australians had travelled to Japan or Vietnam. The results are shown below:

CHAPTER 12

Japan 9 Vietnam 18 Japan and Vietnam 5

a Copy and complete the Venn diagram to show this information.



b What is the probability that a person chosen at random from this sample:

i has not been to either country?

ii has been to Japan or Vietnam, but not both?

iii has been to Vietnam but not Japan?

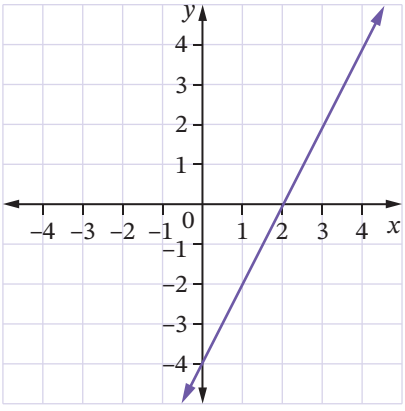
8 Convert:

CHAPTER 3

a 40 m/s to km/h **b** 10 g/s to kg/min

9 Find the equation of this line.

CHAPTER 11



10 Jotika earns a salary of $82 000 p.a. How much (correct to the nearest cent) did she earn:

CHAPTER 8

a per week? **b** per month?

11 In $\triangle LKM$, $\angle M = 90°$, $MK = 6$ m and $KL = 9$ m.

CHAPTER 4

a Sketch a diagram of $\triangle LKM$.

b Find the length of ML, correct to 2 decimal places.

STAGE 5.3 CHAPTER 1 CHAPTER 5

12 Factorise each expression.

a $y^2 - 4y + 2y - 8$ **b** $m^2 - 8m + 15$ **c** $5m^2 - 245$

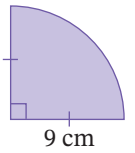
13 What is the value of $8^{-\frac{5}{3}}$? Select the correct answer **A**, **B**, **C** or **D**.

A $-13\frac{1}{3}$ **B** -32 **C** $\frac{1}{32}$ **D** $\frac{3}{40}$

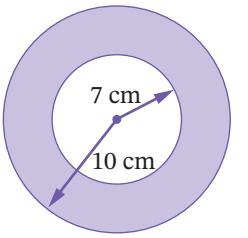
CHAPTER 10

14 Find each shaded area, correct to one decimal place.

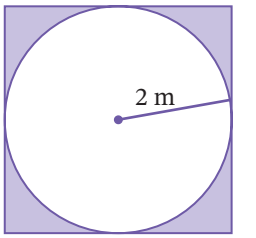
a



b



c



CHAPTER 8

15 Simok earns $21.45 per hour. How much does he earn for working 38 hours at normal rates and 6 hours at time-and-a-half?

CHAPTER 5

16 Simplify each expression.

a $k^7 \times k^5$ **b** $d^{10} \div d^5$ **c** $3p^2q \times 5pq^3$

d $20hk^5 \div 5hk$ **e** $\left(\frac{2r}{5}\right)^3$ **f** $5c^0$

CHAPTER 9

17 **a** Complete a frequency table for this data, including an 'fx' column.

53	50	50	52	47	49	49	50	48	50
52	49	47	51	51	52	50	53	50	52
50	48	52	49	50	49	51	45	52	50
51	52	51	50	52	50	51	52	49	51

b Find the mean of this data.

c Show this data in a frequency histogram and polygon.

d Find the mode.

e Find the median.

CHAPTER 1

18 Expand each expression.

a $3(1 - 2y)$ **b** $x(2x - 7)$ **c** $-4m(1 - m)$ **d** $(5x + 7)(x - 4)$

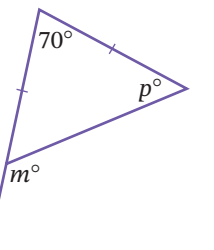
STAGE 5.3 CHAPTER 7

19 Simplify each expression.

a $\left(\frac{5a}{3}\right)^{-3}$ **b** $(16x)^{\frac{1}{4}}$ **c** $\sqrt[3]{-8p}$ **d** $r^{\frac{2}{3}}$

CHAPTER 6

20 What are the values of m and p in this diagram?
Select **A**, **B**, **C** or **D**.



A $m = 110, p = 70$

B $m = 125, p = 70$

C $m = 110, p = 55$

D $m = 125, p = 55$

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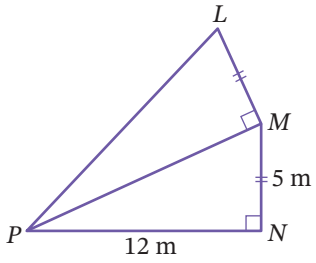
21 For the quadrilateral *LMNP*, calculate its:

a perimeter, correct to 2 decimal places

b area



CHAPTER 10

22 The number of goals scored by Damien's hockey team in 18 games were:

1	3	1	3	4	1	3	3	2
1	1	1	2	2	3	3	3	4

If 5 goals were scored in the tenth game, which statistical measure will not change?
Select **A**, **B**, **C** or **D**.

A the range **B** the mean **C** the median **D** the mode

CHAPTER 9

23 Tahlia is paid a weekly commission of 4.3% on the value of her sales of perfume over $1500. Calculate her commission if her sales during one week totalled $19 200.

CHAPTER 8

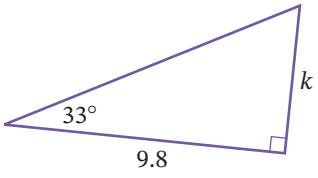
24 Factorise each expression.

a $2y + 8$ **b** $5a - 15ab$ **c** $-8ab + 2b$

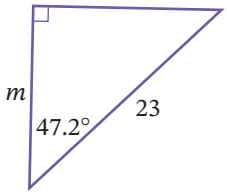
CHAPTER 1

25 Find, correct to 2 decimal places, the value of each variable.

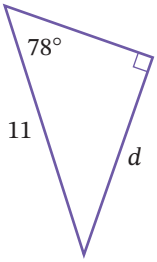
a



b



c



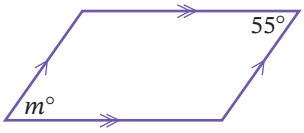
CHAPTER 4

26 Romesh buys a car for $11 000 and sells it for $5200 2 years later. Calculate the percentage loss, correct to one decimal place.

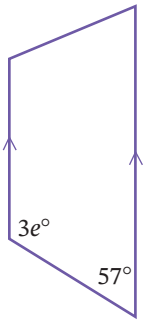
CHAPTER 3

27 Find the value of each variable, giving reasons.

a



b



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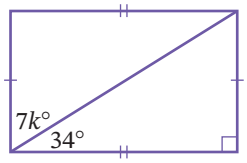

CHAPTER 6

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CHAPTER 11

28 What is the equation of the horizontal line that passes through the point $(-2, 5)$?

CHAPTER 11

29 For the interval joining the points $(5, 3)$ and $(-4, 2)$, find:

a its midpoint **b** its length in exact form **c** its gradient

CHAPTER 13

30 The scale of a plan is 1 : 2000. What distance, in centimetres, on the plan would represent a real distance of 50 m?

CHAPTER 6

31 Name the most general shape that matches each description.

a A quadrilateral with equal diagonals.

b A triangle with 3 equal sides.

c A quadrilateral with opposite sides equal and 2 axes of symmetry.

d A triangle with 2 equal angles.

CHAPTER 9

32 The median of the data shown in this stem-and-leaf plot is 154.

Stem	Leaf
13	2 3
14	0 5 6 7
15	1 □ 6 8 8 9
16	3 4 4 5

a What is the missing value shown by □?

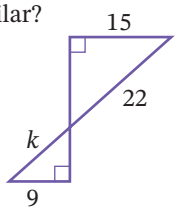
b Find the range.

c Describe the shape of the distribution.

CHAPTER 13

33 **a** Which similarity test can be used to prove that these 2 triangles are similar?

b Find the value of k.



CHAPTER 11

34 **a** Graph each equation on a separate number plane.

i $y = 2x - 1$ **ii** $y = -x^2 + 3$ **iii** $x^2 + y^2 = 4$

b What is the gradient and y-intercept of the graph of $y = 2x - 1$?

c What are the coordinates of the vertex of the graph of $y = -x^2 + 3$?

CHAPTER 3

35 Calculate the simple interest on \$8750 invested at 5.5% for 4 years.

CHAPTER 3

36 The cost of a computer, including 10% GST, is \$968. What is the amount of the GST?

CHAPTER 11

37 Which point lies on the line $y = 2x - 5$? Select **A**, **B**, **C** or **D**.

A $(0, -3)$ **B** $(1, -3)$ **C** $(-3, 0)$ **D** $(-3, 1)$

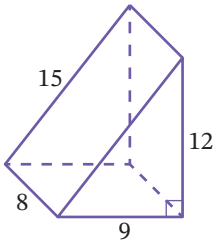
Foundation Standard Complex

9780170453325

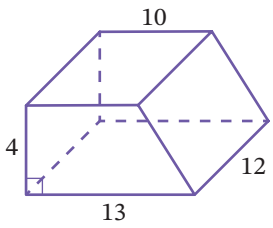
38 Find the surface area of each solid, correct to 3 significant figures where necessary. All measurements are in centimetres.

CHAPTER 10

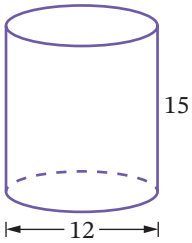
a



b



c



39 Solve each equation.

CHAPTER 7

a $\frac{m}{2}-\frac{m}{7}=3$ **b** $\frac{3k-1}{5}=\frac{2k+1}{4}$ **c** $4(3y-1)+6=2(2y+7)$

d $3x^2=27$ **e** $-6(12-x)=48$ **f** $\frac{3a-8}{2}=7$

40 Express each number in scientific notation.

CHAPTER 5

a 714 000 **b** 0.000 35 **c** 15 763 000

41 Evaluate each expression in scientific notation, correct to 2 significant figures.

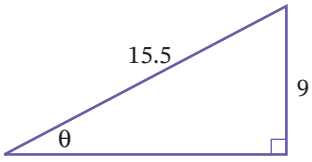
CHAPTER 5

a $(4.89\times10^7)^2$ **b** $\frac{9.3\times10^{-3}}{2.2\times10^8}$

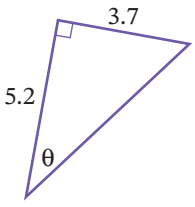
42 Find the value of θ, correct to the nearest minute.

CHAPTER 4

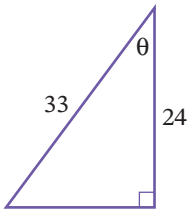
a



b



c



43 A die was repeatedly rolled and the results are shown in this table.

CHAPTER 12

Outcome	Frequency
1	40
2	35
3	53
4	45
5	50
6	47

a Find, correct to 2 decimal places, the relative frequency of rolling:

i a 1

ii a 4

iii an even number

iv a number less than 5

v neither a 2 nor a 6.

b Based on the theoretical probability, what is the expected frequency of rolling a number less than 5? How does the observed frequency compare with the expected frequency?

44 **a** What is the angle sum of a regular octagon?

CHAPTER 6

b What is the size of one angle in a regular octagon?

c What is the size of one exterior angle in a regular octagon?

d What is the sum of the exterior angles of any polygon?

Foundation Standard Complex

ANSWERS

Chapter 1

SkillCheck

1 **a** $\$pQ$ **b** $\$(40 + t)$ **c** $1000h$ m
d $\frac{m+p+v+w}{4}$ **e** $\$\frac{y}{100}$

2 **a** 6 **b** 18 **c** 21
d 43 **e** 18 **f** 5

3 14.4 cm

4 **a** $11n$ **b** $6h$ **c** $9pq$ **d** $5x^2$
e $4w$ **f** $40x$ **g** $6gh$ **h** $18y^2$
i $5x$ **j** 6 **k** $2y$ **l** $\frac{3n}{5}$

5 **a** $\frac{11}{15}$ **b** $\frac{7}{20}$ **c** $\frac{5}{16}$ **d** $2\frac{4}{5}$
e $\frac{5}{22}$ **f** $1\frac{5}{24}$ **g** $1\frac{13}{27}$ **h** $\frac{1}{12}$
i $4\frac{13}{28}$ **j** $9\frac{1}{10}$ **k** $1\frac{1}{3}$ **l** $2\frac{7}{12}$

Exercise 1.01

1 **a** fg **b** $9uvw$ **c** $8t + 3$
d $3x + 10y$ **e** $2g + h$ **f** $3f - 2fg$
g $5p^2 - 3$ **h** $4p - 4q$ **i** $-2r + 3$
j $-10a + 4b$ **k** $6v + 10$ **l** $3x + 3y$
m $6p - 9q$ **n** $15 - 5w$ **o** $2 - 10n$
p $6e - 7f$ **q** $5l^2 - 4l$ **r** $-6b^2 - 14b$
s $2p^3 - 4p^2$ **t** $-2 - 17k$ **u** $6 - 13d$

2 C

3 **a** correct **b** incorrect **c** incorrect
d correct **e** correct **f** incorrect

4 **a** $4m + 6p$ **b** $16d$ **c** $4x + 5y$
d $6p + 3$ **e** $90h$ **f** $12a + 18c$

5, 6 Teacher to check

Exercise 1.02

1 **a** $40x$ **b** $15n$ **c** $5ab$
d $42mt$ **e** $-12w$ **f** $-6kp$
g $12ed$ **h** $24vw$ **i** $88q^2$
j $20a^2$ **k** $24by$ **l** $-36w^2$
m $72chk$ **n** $8abc$ **o** $-30dew$
p $25n^2$ **q** $9r^2$ **r** a^2t^2
s $-10ad^2$ **t** ek^2 **u** $4x^2y^2$

2 **a** $4x$ **b** 15 **c** 9 **d** $3n$ **e** $\frac{1}{6}$
f $4ac$ **g** -6 **h** $-5z$ **i** $-\frac{1}{5q}$ **j** $3yz$
k $\frac{r}{4}$ **l** $-6k$ **m** $\frac{3k^2}{a}$ **n** $\frac{4w}{3}$ **o** $\frac{h}{3}$

3 C

4 **a** $30abc$ **b** $24pq$ **c** $8n$ **d** $\frac{1}{5}$ **e** $-36hk$
f $40k^2$ **g** $\frac{d}{2}$ **h** $\frac{1}{7}$ **i** $-11m$ **j** $\frac{1}{5}$
k $\frac{4a}{c}$ **l** $-6e^2f^2$ **m** $\frac{2c}{3}$ **n** $-24pq^2$ **o** $\frac{4vw}{3}$

5 **a** correct **b** incorrect **c** correct **d** incorrect
e correct **f** correct **g** correct **h** incorrect
i correct **j** correct

6 **a** $6ad$ **b** $25m^2$ **c** $5hk$
d $12py$ **e** $3a^2$ **f** $10ck$

7 **a** $8m^3$ **b** $60dty$ **c** $12kw^2$

Exercise 1.03

1 **a** $\frac{3w}{4}$ **b** $\frac{11k}{8}$ **c** $\frac{m}{2}$ **d** x **e** $\frac{9}{q}$
f $\frac{1}{d}$ **g** $\frac{5r}{w}$ **h** $\frac{3}{c}$ **i** $\frac{4t+s}{3}$ **j** $\frac{2y}{h}$
k $\frac{1}{a}$ **l** $\frac{3}{e}$ **m** $\frac{p+3}{z}$ **n** $\frac{u}{4g}$ **o** $\frac{1}{3f}$
p $\frac{2e}{5}$ **q** $\frac{5a}{3}$ **r** $\frac{4}{d}$ **s** $\frac{4}{k}$ **t** $\frac{1}{a}$

2 C

3 **a** $\frac{7x}{12}$ **b** $\frac{4s}{21}$ **c** $\frac{8h}{15}$ **d** $-\frac{5m}{14}$
e $\frac{9w}{20}$ **f** $\frac{17t}{20}$ **g** $\frac{11p}{15}$ **h** $\frac{5r}{6}$
i $\frac{3y}{2}$ **j** $\frac{m}{3}$ **k** $\frac{7a}{18}$ **l** $\frac{23y}{30}$
m $\frac{13c}{10}$ **n** $\frac{6d-11r}{33}$ **o** $\frac{9h+10d}{15}$ **p** $\frac{25+24w}{30}$
q $\frac{21+8a}{28}$ **r** $\frac{6e}{35}$ **s** $\frac{33m-10n}{22}$ **t** $\frac{12k+5m}{30}$

4 B **5** C

Exercise 1.04

1 A

2 **a** $\frac{w}{6}$ **b** $\frac{st}{20}$ **c** $\frac{5}{2hk}$ **d** $\frac{12}{mn}$ **e** $\frac{5l}{3f}$
f $\frac{2}{3v^2}$ **g** $\frac{6}{x^2}$ **h** $\frac{ac}{bd}$ **i** $\frac{d}{g}$ **j** $\frac{d^2}{36}$
k $\frac{2}{3}$ **l** $\frac{12}{5k^2}$ **m** $\frac{3u}{5}$ **n** 1 **o** $\frac{2}{3d}$

3 **a** $\frac{5}{2}$ **b** $\frac{m}{2n}$ **c** 4 **d** $\frac{q}{3}$ **e** $\frac{3d}{10}$
f $\frac{20}{27}$ **g** $\frac{3}{5}$ **h** $\frac{9}{b}$ **i** $\frac{15}{2}$ **j** $\frac{h^2}{k^2}$
k 12 **l** $\frac{11}{8}$ **m** $\frac{5u}{9}$ **n** 6 **o** x

4 D

5 **a** $\frac{6}{p}$ **b** $\frac{12}{35}$ **c** $\frac{3}{20y}$ **d** $\frac{5}{4}$ **e** $\frac{5x}{z}$
f $\frac{c}{3b}$ **g** $\frac{25b}{3}$ **h** $\frac{4t}{27}$ **i** $\frac{2}{3n}$ **j** $\frac{50p^2t}{7}$
k $\frac{2s}{35}$ **l** $\frac{1}{6h^2}$ **m** $25y^2$ **n** $\frac{2yh}{9}$ **o** $\frac{3af^2}{25}$

6 B

Mental skills 1A

2 **a** 160 **b** 70 **c** 240 **d** 900 **e** 2600
f 900 **g** 140 **h** 300 **i** 180 **j** 770
k 18 **l** 34 **m** 46 **n** 26 **o** 18
p 12 **q** 40 **r** 8 **s** 14 **t** 24

Exercise 1.05

1 **a** $4h + 24$ **b** $15 + 5t$ **c** $6b + 21$
d $40 + 15x$ **e** $x^2 + 9x$ **f** $5p + p^2$
g $6r^2 + 3r$ **h** $4y + 12y^2$ **i** $20e^2 + 40ef$
2 **a** $3t - 6$ **b** $35 - 7d$ **c** $64g - 24$
d $w^2 - 7w$ **e** $a^2 - a$ **f** $18h^2 - 6h$
g $12x^2 - 4x$ **h** $10x^2 - 20xy$ **i** $12ab - 21a^2$
3 C
4 **a** incorrect **b** correct
5 **a** $-a + 5$ **b** $-a - 5$ **c** $-2x - 12$
d $-2x + 12$ **e** $-11 - w$ **f** $-11 + w$
g $-y^2 - 9y$ **h** $-8x + xm$ **i** $-4t^{2+} 32t$
6 B **7** D
8 **a** $19m + 10$ **b** $3 - 9e$
c -10 **d** $43 - 10x$
e $14a + 43$ **f** $d - 11$
g $29g - 19$ **h** $22 - 18w$
i $39c - 27$ **j** $t^2 + 7t + 12$
k $12 + 11h - 2h^2$ **l** $15e - 11$
m $6x^2 + 23x + 20$ **n** $2v^2 - 3v - 6$
o $w^2 - 8w + 3$ **p** $6y^2 - 29y + 35$
9 C

Exercise 1.06

1 **a** 6 **b** 8 **c** 5 **d** 3 **e** m
f c **g** $6p$ **h** $8w$ **i** $(x - 5)$
2 **a** $6(3p^2 + 4)$ **b** $3m(3m + 1)$
c $10d(2 - 3d)$ **d** $2xy(3x - 4)$
e $(x - 2)(x + 5)$ **f** $(3 + n)(n - 3)$
g $(h + 4)(h - 2)$ **h** $(1 - w)(v + 1)$
3 **a** $6(4x + 5)$ **b** $9(4 + 3a)$ **c** $16(g^2 - 4)$
d $y(x + 1)$ **e** $n(m - 3)$ **f** $p(6 + q)$
g $x(x + 1)$ **h** $y(2 - y)$ **i** $3d(d + 2)$
j $4r(4r - 3)$ **k** $3t(2t + 9)$ **i** $36p(p - 3)$
m $4x(3xy - 4)$ **n** $2p(9p + 8r)$ **o** $4mn(m - n)$
p $7bc(2a + 3)$ **q** $7vw(4 - 3vw)$ **r** $9rt(5 + 6r)$
s $36pr(q^2 - 4)$ **t** $16xy(3x + 4y)$ **u** $25gh(3g^2h - 5)$
v $5ab(5c + 2a)$ **w** $9my^2(6y + 5m)$
x $7ru^2(3r^2 - 5u)$
4 A
5 **a** $(a - 3)(a + 6)$ **b** $(8 + t)(t - 3)$
c $(b + 5)(b - 2)$ **d** $(2 - y)(x - 6)$
e $(a - 7)(5 + b)$ **f** $(a + 3)(y + 4)$
g $(g + 5)(3p - 2)$ **h** $(2 - 3m)(5 + 4n)$
i $(8 + r)(r + 1)$ **j** $(y - 6)(1 - y)$
k $(2w + 7)(3m - 1)$ **l** $(c + 3)(3c + 5)$
6 **a** $-4(q + 2)$ **b** $-9(u - 2)$ **c** $-3(g - 2)$
d $-6(3a - 2)$ **e** $-(n + 1)$ **f** $-(n - 1)$
g $-y(y + 9)$ **h** $-2t(3 - 5t)$ **i** $-3a(a + 2b)$
j $-a(p - q)$ **k** $-2e(10e + 11)$ **l** $-3m(3 - m)$
7 B
8 **a** $\frac{1}{2}(p+q)$ **b** $g(7 + 9g)$ **c** $\frac{1}{4}x(x-y)$
d $4(2a + 3y + 1)$ **e** $5(4xy - 2x^2 + y)$
f $tm(t + m + 1)$ **g** $(m + 2)(7y - p)$
h $-10p(p + q)$ **i** $5a(1 + 2ay)(x - y)$

Exercise 1.07

1 **a** $x^2 + 18x + 80$ **b** $12h^2 + 31h + 7$
2 **a** $x^2 + 3x - 54$ **b** $n^2 - 12n + 35$
3 **a** $2y^2 + 11y + 15$ **b** $w^2 - 2w - 15$
c $p^2 - 8p - 15$
4 **a** $n^2 + 9n + 18$ **b** $t^2 + 11t + 30$ **c** $p^2 - 100$
d $x^2 - 5x - 24$ **e** $b^2 + 7b - 18$ **f** $u^2 - 15u + 56$
g $15 + 14r - r^2$ **h** $a^2 - 19a + 90$ **i** $c^2 - 8c + 15$
j $t^2 + t - 2$ **k** $y^2 + 6y - 40$ **l** $n^2 + 2n - 99$
m $e^2 - 4e + 4$ **n** $25 + 10w + w^2$
o $g^2 + 22g + 121$
5 D **6** A
7 **a** $2x^2 + 11x + 15$ **b** $12e^2 + 23e + 10$
c $3p^2 + 7p - 10$ **d** $49d^2 - 28d + 4$
e $6f^2 + 4f - 10$ **f** $12m^2 + 5m - 25$
g $6 + 7h - 20h^2$ **h** $8p^2 - 38p + 35$
i $-10m^2 + 23m - 12$ **j** $6t^2 + 7t - 5$
k $25y^2 - 25$ **l** $-4a^2 + 28a - 49$
m $9k^2 + 48k + 64$ **n** $16c^2 - 40c + 25$
o $6dx - 2d + 27x - 9$ **p** $5h + 15 - 2gh - 6g$
q $49 - 56w + 16w^2$ **r** $12n^2 + 7n - 45$
8 D
9 **a** $x^2 - x - 6$ **b** $9y^2 - 42y + 49$
c $4k^2 + k - 1.5$ **d** $15p^2 - 22p + 8$
e $7m^2 - 14.5m - 7.5$ **f** $2d^2 + 12d + 10$
10 **a** $100 + x$ **b** $75 + x$
c $A = x^2 + 175x + 7500$ cm^2
d $x^2 + 175x$ **e** 17.51 cm^2
11 **a** $l = 4 + x, w = 3 + y$
b $A = (4 + x)(3 + y)$
c $A = 12 + 4y + 3x + xy$ m^2
d $3x + 4y + xy$
12 Teacher to check

Exercise 1.08

1 **a** $20x$ **b** m^2 **c** t^2 **d** $+ 8h$
e $- 18k$ **f** $+ 80f$ **g** $4d^2 + 12d$ **h** $36a^2, 1$
2 **a** $m^2 + 18m + 81$ **b** $u^2 + 6u + 9$
c $y^2 - 12y + 36$ **d** $64 + 16k + k^2$
e $25 - 10h + h^2$ **f** $49 + 14k + k^2$
g $f^2 + 40f + 400$ **h** $q^2 - 22q + 121$
i $100 + 20t + t^2$ **j** $x^2 - 2xw + w^2$
k $a^2 + 2ag + g^2$ **l** $4m^2 - 12m + 9$
m $25x^2 - 60x + 36$ **n** $81a^2 + 36a + 4$
o $9e^2 - 24e + 16$ **p** $25 + 70b + 49b^2$

ANSWERS

q $16 - 40p + 25p^2$ **r** $121 - 44c + 4c^2$

s $100g^2 + 60g + 9$ **t** $9k^2 + 66k + 121$

u $25 + 20v + 4v^2$

3 **a** $49h^2 + 28hk + 4k^2$ **b** $64a^2 - 48ay + 9y^2$

c $x^2y^2 + 2xyz + z^2$ **d** $1 + \frac{2}{y} + \frac{1}{y^2}$

e $k^2 - 2 + \frac{1}{k^2}$ **f** $\frac{9}{w^2} + 6 + w^2$

4 **a** 441 **b** 2025 **c** 841

d 3481 **e** 10 404 **f** 9604

5 **a** 2 squares of length x cm have been cut from the ends, so the length of the base is $(10 - 2x)$ cm.

b $100 - 40x + 4x^2$ **c** $10x - 2x^2$

e $10^2 - 4 \times x^2 = 100 - 4x^2$

Exercise 1.09

1 **a** $m^2 - 25$ **b** $c^2 - 100$ **c** $a^2 - 144$

d $36 - y^2$ **e** $64 - m^2$ **f** $p^2 - 1$

g $25 - e^2$ **h** $v^2 - 121$ **i** $w^2 - 9$

j $x^2 - 100$ **k** $q^2 - 49$ **l** $81 - g^2$

m $b^2 - 4$ **n** $225 - r^2$ **o** $d^2 - 169$

2 **a** $4h^2 - 9$ **b** $25r^2 - 16$ **c** $25b^2 - 64$

d $16p^2 - 49$ **e** $9 - 64k^2$ **f** $49x^2 - 25$

g $4 - 81m^2$ **h** $81k^2 - 16l^2$ **i** $49n^2 - 64m^2$

j $16g^2 - 25h^2$ **k** $49u^2 - 9w^2$ **l** $121a^2 - 9b^2$

m $a^2 - \frac{1}{a^2}$ **n** $\frac{w^2}{9} - 4$ **o** $1 - \frac{1}{r^2}$

3 **a** $p - 1$ **b** $p + 1$ **c** $p^2 - 1$ **d** 7

4 899

5 **a** 399 **b** 2499 **c** 8099 **d** 6396

6 **a** 29 **b** 47 **c** 3000

d 400 **e** 240 **f** 196

7 **a** **i** $l = x + y$ **ii** $w = x - y$ **iii** $x^2 - y^2$

b $(x + y)(x - y) = x^2 - y^2$

Exercise 1.10

1 **a** $4m^2 - 1$ **b** $5y^2 + 17y - 12$

c $4k^2 - 28k + 49$ **d** $d^2 + 18d + 81$

e $2e^2 + e - 1$ **f** $25a^2 - 16$

g $-p^2 + 4p - 4$ **h** $100 - 36y^2$

i $h^2 - 6hm + 9m^2$ **j** $2xy + 6x - 3y - 9$

k $121a^2 - 16b^2$ **l** $u^2 - 2 + \frac{1}{u^2}$

2 **a** m^2 **b** $y^2 + 18$ **c** $-3x^2 + 7x + 6$

d $d^2 + 21$ **e** $16k^2 - 48$ **f** $-4xy$

g $27a - 10$ **h** $2f^2 - 8$ **i** $12h + 18$

j $5xy - 6x + 3y + 9$

3 **a** $60a^2$ **b** $n^2 + 4n + 4$ **c** $-2u^2 - 8u$

d $8m^2 + 2n^2$ **e** $x - 2$ **f** $1 - 2b^2$

g $3y^2 + 12y + 14$ **h** $3x^2 + 9$ **i** $34n^2 - 34$

j $-4b^2$

Mental skills 1B

2 **a** 176 **b** 363 **c** 261 **d** 405

e 682 **f** 707 **g** 1818 **h** 3564

i 152 **j** 540 **k** 2142 **l** 588

m 288 **n** 693 **o** 3939 **p** 852

Exercise 1.11

1 **a** $(b + d)(4a + 5c)$ **b** $(y + t)(2x - 5w)$

c $(3c + 4d)(3a + 2b)$ **d** $(10 + x)(x^2 + 3)$

e $3(a + b)(a + c)$ **f** $6(t + p)(r - 3w)$

g $(7 + d)(2e - 3)$ **h** $(h - 2)(k - h)$

i $(3m + p)(n - 2)$ **j** $(9 + q)(p^2 - 3)$

k $(f - 10)(g - h)$ **l** $3(l + n)(3k - 4m)$

m $(2 - p)(p - c)$ **n** $(l - 3)(l^2 + m^2)$

o $(a + y)(x + 1 - k)$ **p** $(a - b + 3q)(p - 2q)$

2 **a** $(w + 3)(w - 3)$ **b** $(y + 6)(y - 6)$

c $(k + 1)(k - 1)$ **d** $(m + 11)(m - 11)$

e $(p + 8)(p - 8)$ **f** $(c + 10)(c - 10)$

g $(2e + f)(2e - f)$ **h** $(a + 3b)(a - 3b)$

i $(4y + 1)(4y - 1)$ **j** $(2 + b)(2 - b)$

k $(5 + e)(5 - e)$ **l** $(1 + 4x)(1 - 4x)$

m $(k + u)(k - u)$ **n** $(7 + 4m)(7 - 4m)$

o $(b + 11d)(b - 11d)$ **p** $(6c + 5k)(6c - 5k)$

q $(4 + 9h)(4 - 9h)$ **r** $(5a + 8m)(5a - 8m)$

s $(10 + 7n)(10 - 7n)$ **t** $(11p + 12q)(11p - 12q)$

u $\left(\frac{1}{2} + 5c\right)\left(\frac{1}{2} - 5c\right)$ **v** $\left(2w + \frac{1}{3}\right)\left(2w - \frac{1}{3}\right)$

w $\left(8h + \frac{3}{2}\right)\left(8h - \frac{3}{2}\right)$ **x** $(1 + mn)(1 - mn)$

3 **a** $2(a + b)(a - b)$ **b** $7(k + 2)(k - 2)$

c $3(1 + 5u)(1 - 5u)$ **d** $x(x + 7)(x - 7)$

e $k(1 + 4k)(1 - 4k)$ **f** $2(5q + 1)(5q - 1)$

g $3(d + 2v)(d - 2v)$ **h** $5t^3(t + 5)(t - 5)$

i $2(ab + 1)(ab - 1)$ **j** $x^2(y + w)(y - w)$

k $12(4f + 3g)(4f - 3g)$

l $5\left(3d + \frac{1}{2}\right)\left(3d - \frac{1}{2}\right)$ or $\frac{5}{4}(6d + 1)(6d - 1)$

m $2(x + 2a)(x - 2a)$ **n** $25(2 + w)(2 - w)$

o $5\left(\frac{1}{2} + 4e\right)\left(\frac{1}{2} - 4e\right)$ or $\frac{5}{4}(1 + 8e)(1 - 8e)$

p $\left(3c + 2\frac{1}{2}\right)\left(3c - 2\frac{1}{2}\right)$ or $\frac{1}{4}(6c + 5)(6c - 5)$

4 **a** $\left(\frac{p}{4} + \frac{b}{5}\right)\left(\frac{p}{4} - \frac{b}{5}\right)$ **b** $\left(x + \frac{1}{3}\right)\left(x - \frac{1}{3}\right)$

c $\left(\frac{v}{8} + \frac{u}{9}\right)\left(\frac{v}{8} - \frac{u}{9}\right)$ **d** $2\left(\frac{y}{3} - \frac{m}{11}\right)\left(\frac{y}{3} + \frac{m}{11}\right)$

e $\left(\frac{4a}{7} + \frac{5b}{2}\right)\left(\frac{4a}{7} - \frac{5b}{2}\right)$ **f** $(g + 3)(g - 3)(g^2 + 9)$

g $(10 + n^2)(10 - n^2)$

h $y(2x + y)$ **i** $-(p + 3q)(3p - q)$

j $\left(\frac{x}{2} + \frac{y}{6}\right)\left(\frac{x}{2} - \frac{y}{6}\right)$ **k** $4ab$

l $(1 + x)(9 - x)$

Exercise 1.12

1 **a** $-1, -6$ **b** $-3, 4$ **c** $-5, 3$

d $3, 4$ **e** $-5, -4$ **f** $-2, 7$

g $-2, 5$ **h** $-5, 5$ **i** $-2, 1$

j $-9, 2$

2 **a** $(m + 3)(m + 4)$ **b** $(x + 7)(x + 5)$

c $(k + 4)(k + 1)$ **d** $(p + 5)(p + 2)$

e $(w+5)(w+4)$ **f** $(b+1)(b+5)$
g $(e+2)(e+3)$ **h** $(r+2)^2$
i $(n+10)(n+1)$ **j** $(a+6)(a+5)$
k $(d+4)(d+6)$ **l** $(y+4)(y+11)$

3 **a** $(p-4)(p-1)$ **b** $(v-5)(v-6)$
c $(y-4)(y-3)$ **d** $(x-10)(x-2)$
e $(a-3)(a-6)$ **f** $(e-6)(e-2)$
g $(c-8)(c-6)$ **h** $(w-2)(w-3)$
i $(d-11)(d-7)$ **j** $(u-2)(u-1)$
k $(n-4)(n-4)$ **l** $(b-7)(b-8)$

4 **a** $(x+4)(x-1)$ **b** $(g+8)(g-3)$
c $(y+1)(y-3)$ **d** $(r-7)(r+2)$
e $(m+5)(m-3)$ **f** $(a+2)(a-1)$
g $(m-4)(m-5)$ **h** $(h-4)(h+1)$
i $(d-3)^2$ **j** $(w+6)(w-2)$
k $(a-6)(a+2)$ **l** $(k-2)(k+7)$
m $(c-9)(c+2)$ **n** $(e-9)(e+3)$
o $(u-5)(u+11)$ **p** $(p+12)(p-3)$
q $(x-4)(x+13)$ **r** $(q-14)(q+5)$

5 **a** $(m+2)^2$ **b** $(p+10)^2$ **c** $(a-5)^2$
d $(y-7)^2$ **e** $(w+12)^2$ **f** $\left(a-1\frac{1}{2}\right)^2$

6 **a** $3(m+1)(m+2)$ **b** $2(y+2)(y-1)$
c $5(t-10)(t+8)$ **d** $5e^2(e+8)(e-3)$
e $x(x-11)(x+10)$ **f** $4(b-7)(b+6)$
g $4(w+4)(w-3)$ **h** $3a(a-4)(a+1)$
i $2(e+5)(e+4)$ **j** $-(t+8)(t-3)$
k $-(u-7)(u+6)$ **l** $-(x-7)(x+4)$
m $-(b+4)(b-3)$ **n** $-(k-3)(k-4)$
o $-(x-5)(x-7)$

7 **a** $(h-6)(h+3)$ **b** $-(t+9)(t-2)$
c $(w+1)(w+7)$ **d** $(k+3)(k-15)$
e $(v+10)(v-2)$ **f** $3(c+5)^2$
g $(q-1)(q-5)$ **h** $(a-3)(a-1)$
i $6(x+8)(x-2)$ **j** $(x-8)^2$
k $8(u-4)(u+1)$ **l** $(b+5)(b+6)$
m $(y-6)^2$ **n** $5(r-2)(r+1)$
o $4(l-4)(l+2)$ **p** $(g-20)(g-4)$
q $-2(d-6)(d+9)$ **r** $-(n+2)(n-13)$

Exercise 1.13

1 **a** $(x+5)(2x+1)$ **b** $(m+3)(4m+1)$
c $(y+3)(5y+2)$ **d** $(2u+5)(3u+2)$
e $(w+15)(2w+1)$ **f** $(e+3)(4e+3)$
g $(2f+3)(4f+1)$ **h** $(d+1)(3d+2)$
i $(b+1)(2b+7)$ **j** $(y+1)(5y+11)$
k $(4g+3)(2g+5)$ **l** $(3a+7)(2a+3)$

2 **a** $(y-4)(2y-3)$ **b** $(2k-3)(5k-2)$
c $(2e-3)(3e-2)$ **d** $(b-3)(4b-1)$
e $(w-3)(6w-5)$ **f** $(3f-4)(4f-3)$
g $(3m-4)(5m-2)$ **h** $(3x-2)^2$
i $(2a-5)(a-9)$ **j** $(3y-8)(4y-1)$
k $(4d-3)(2d-7)$ **l** $(2h-9)^2$

3 **a** $(y+1)(5y-11)$ **b** $(4d-5)(d+1)$
c $(2m+3)(m-3)$ **d** $(3t-10)(t+3)$
e $(6h-7)(h+1)$ **f** $(y-4)(2y+3)$
g $(2a+1)(4a-3)$ **h** $(5u-4)(3u+1)$
i $(3c+1)(3c-5)$ **j** $(3c-8)(2c+3)$
k $(4n-7)(5n+2)$ **l** $(4x-5)(3x+2)$

4 **a** $(5m+7)(m-1)$ **b** $(3g-4)(2g+3)$
c $(3p-2)(p+2)$ **d** $(7w-1)(w+1)$
e $(5y-1)(y+3)$ **f** $(3n-2)(n+4)$
g $(4b-3)(b+3)$ **h** $(4m-1)(2m+3)$
i $(3x+8)(x-2)$ **j** $(4u+15)(2u-1)$
k $(3e+5)(8e-3)$ **l** $(4h+7)(5h-6)$

5 **a** $(9w-10)^2$ **b** $4(y+1)^2$
c $(5h-4)^2$

6 **a** $2(y+2)(3y-1)$ **b** $3(g+4)(2g-3)$
c $4(2e-3)(3e+1)$ **d** $2(a-2)(4a+3)$
e $4(u+2)(3u-1)$ **f** $-(5q+3)(5q-2)$
g $-2(2m-1)(3m-2)$ **h** $-(3h+4)(4h-5)$
i $6(2c+3)(2c+1)$ **j** $-3(w+1)(2w-5)$
k $2(2d-3)(3d+5)$ **l** $-2(x-3)(3x-2)$

7 **a** $(a+1)(2a+3)$ **b** $(2m-5)(6m-1)$
c $(4x-1)(x+3)$ **d** $(w-1)(7w-1)$
e $(h-3)(4h+5)$ **f** $(4x-3)(2x+1)$
g $(r+5)(5r+1)$ **h** $(d-7)(2d-1)$
i $(3n+1)(2n-3)$ **j** $-(3m-2)(3m+4)$
k $-(5c-3)(c+1)$ **l** $(3g+2)(5g+3)$
m $-(4q+3)(2q-5)$ **n** $(x-2)(3x-7)$
o $-(3d-4)(d+4)$ **p** $-2(3y+7)(2y-3)$
q $4(d-2)(7d+3)$ **r** $4(5k+2)^2$

Exercise 1.14

1 **a** $(m-8)^2$ **b** $3d(d-1)$
c $(w-3)(3w+5)$ **d** $(3+h)(k-5)$
e $(5y+8)(5y-8)$ **f** $4(5f+4)(5f-4)$
g $q(q+3-3p)$ **h** $(3-g)(g+1)$
i $4(2n+5)(3n-2)$ **j** $(5r+1)(5r-1)$
k $(b^2+1)(b+1)$ **l** $(2x-5)^2$
m $-(5d-4)(d+1)$ **n** $(a-1)^2(a+1)$
o $2(2+v)(2-v)$ **p** $m(n+3)(n+p)$
q $2(w-6)^2$ **r** $(6h+1)^2$

2 **a** $(3r-8t)(5r+3t)$ **b** $(2d+1)^2$
c $9(g+2k)(g-2k)$ **d** $e(e-5)(e+2)$
e $-20(2p-3q)(3p-2q)$ **f** $7(2x+1)(2x-1)$
g $(a-b)(a+b+4)$ **h** $(c-2)^2(c+2)$
i $(3a-1)(2a+5)$ **j** $(y+7)(y-5)$
k $2(3p+2)^2$ **l** $-(6a-1)(4a+1)$
m $9(x+2)(x-3)$ **n** $(a-3)(2ab-3)$
o $2(a+3)^2$ **p** $(5u-1)^2$
q $(k-3)(4k+7)$ **r** $3(4+w)(4-w)$
s $3(1+3c)(1-3c)$ **t** $(k+4)^2(k-4)$
u $5y(y^2-2y+3)$ **v** $mn(m+2)(m-2)$
w $-2(a+2)(a-2)$ **x** $4(2c-3)(4c+1)$

3 **a** $(3e+7)(4e-5)$ **b** $(2y+5k)(5y-3k)$
c $(2h-3)(2h+3)(4h^2+9)$
d $12(w-3)$ **e** $(d-4)(d^2+4d-1)$
f $(u-4-k)(u-4+k)$

Exercise 1.15

1 **a** $x+y$ **b** $\frac{1}{2(t-r)}$ **c** $\frac{b-c}{a}$
d -1 **e** $w-4$ **f** $\frac{5}{d+w}$
g $\frac{k+5}{k-5}$ **h** $3(c-1)$ **i** $\frac{a+1}{m+n}$
j $\frac{y+4}{2}$ **k** $\frac{k+1}{k+4}$ **l** $\frac{4a+5c}{a-c}$
m $\frac{x+2}{x-3}$ **n** $\frac{1-2c}{3c-1}$ **o** $\frac{a+4}{2(p+2)}$
p $\frac{c-5}{u+1}$ **q** $\frac{h+1}{2h}$ **r** $\frac{4e+3}{e(2e+5)}$

2 **a** $\frac{7m+10}{m(m+1)(m+2)}$ **b** $\frac{2w-20}{w(w+3)(w+5)}$
c $\frac{4b-7}{(b-1)(b+2)(b-3)}$ **d** $\frac{-k-2}{k(k+1)(k-1)}$
e $\frac{5h+12}{4h(h+1)}$ **f** $\frac{-4d-1}{(d+2)(d+1)}$
g $\frac{42-5r}{4(r+6)(r-6)}$ **h** $\frac{d^2+3d-6}{d(d+2)(d-2)}$
i $\frac{-k^2+9k-5}{(k+1)(k-1)(k-4)}$ **j** $\frac{3q-1}{(q+1)(q-1)}$
k $\frac{7a+13}{(a-3)(a-5)(a+7)}$ **l** $\frac{-5y-22}{(y+3)(y+2)(y-4)}$
m $\frac{-3e+27}{(e+2)(e+1)(e+7)}$ **n** $\frac{20h-11}{(3h-2)(2h+1)(4h-5)}$

3 **a** $6m$ **b** $1\frac{1}{24}$ **c** $\frac{1}{2}$
d $6k$ **e** $\frac{10}{h+1}$ **f** $\frac{3}{2(a-b)}$
g $\frac{-r}{5(r+t)}$ **h** $\frac{4m}{m-1}$ **i** $\frac{4}{p}$
j $\frac{3}{7}$ **k** $\frac{2}{3(x-2)}$ **l** $3\frac{1}{3}$
m $\frac{(d+1)(d-3)}{6}$ **n** $\frac{f+3}{4(f-3)}$ **o** $\frac{3}{f-2}$

Power plus

1 **a** $u+4t$ **b** $2u+3t$
c $10u+14t$ **d** $4u^2+8ut-2t^2$
2 $\frac{5r+6}{2}$
3 **a** y^2-5y **b** $4y-10$
4 $5t$ hours
5 **a** $a^2+b^2+2ab+ac+bc$
b $x^2+xy-5x-2y+6$
c $p^3-6p^2-7p+60$
d $xyw-xy-yw-xw+x+y+w-1$
e $a^2d^2-6rad+9r^2$
f x^6+4x^3+4
6 **a** $(n+2m)^2$ **b** $(x-y)^2$
c $(5x-4y)^2$ **d** $5(a-3b)^2$
e $(c^2+1)^2$ **f** $-(k^2+8)(k+1)(k-1)$
g $(x^2+1)(x+1)(x-1)$
h $(a+b-c)(a+b+c)$
i $4pq$
7 **a** both positive
b the larger of p, q is negative; the smaller of p, q is positive
c the larger of p, q is positive; the smaller of p, q is negative
d both negative
8 **a** $\frac{13x-1}{20}$ **b** $\frac{4x-3}{6}$ **c** $\frac{2x-3}{15}$

Test yourself 1

1 **a** $16y$ **b** $-3ab$ **c** $8x^2$
d $2p+2q$ **e** $6w-5$ **f** $3x^2-13x$
g $7m+10n$ **h** $3u+4$ **i** $12d^2-2d+4$
2 **a** $18n$ **b** $3gh$ **c** ad **d** $-6c$
e $4y$ **f** $\frac{x}{4}$ **g** $-4p$ **h** $\frac{3g}{5}$
i $\frac{4d}{3}$
3 **a** $\frac{5h}{6}$ **b** $\frac{k}{2}$ **c** $\frac{2}{m}$
d $-\frac{3w}{20}$ **e** $\frac{5k}{6}$ **f** $\frac{-3}{2d}$
g $\frac{d-15}{5k}$ **h** $\frac{5}{m+1}$ **i** $\frac{4y+15d}{10}$
4 **a** $\frac{20}{im}$ **b** $\frac{3h}{k^2}$ **c** $\frac{21}{d}$ **d** $\frac{5}{4}$
e $\frac{5}{2}$ **f** 1 **g** $\frac{2d}{5}$ **h** $\frac{p}{3}$
i $\frac{10a}{3}$ **j** $\frac{16}{9}$ **k** $\frac{45a^4}{16bc}$ **l** $\frac{75a}{4y}$
5 **a** $5r-10$ **b** a^2-10a **c** $6xy-15y^2$
d $12p^2-21pq$ **e** $-6n^2+54n$ **f** $-6t^2-42t$
g b^2+6b+5 **h** $6x^2-31x+35$ **i** $30-12x$
j $-7a-2$ **k** $-11k-15$ **l** $2-16y$
6 **a** $4(m-3)$ **b** $x(y+p)$
c $-3(g+10)$ **d** $8w(2w+3)$
e $-8(q-2)$ **f** $5k(3-2hk)$
g $(y-6)(3+y)$ **h** $(2x-1)(3x-2)$
i $(p+6)(p-2)$
7 **a** a^2+5a+6 **b** $y^2-4y-21$
c $t^2+5t-24$ **d** $h^2-9h+20$
e $12+g-g^2$ **f** $6p^2+11p+3$
g $20x^2-11x-3$ **h** $12r^2+r-20$
i $-6q^2+13q-5$
8 **a** $n^2+10n+25$ **b** u^2-6u+9
c $81+18x+x^2$ **d** $36-12k+k^2$
e $9y^2+6y+1$ **f** $4x^2-20x+25$
g $36-84b+49b^2$ **h** $1-8h+16h^2$
9 **a** n^2-36 **b** $100-m^2$ **c** $9-64w^2$
10 **a** $3x^2+9x+4$ **b** $7y+18$ **c** $-6a-2$
d $8p^2+18q^2$
11 **a** $(5m-3)(n+2)$ **b** $(b-1)(a-1)$
c $(x+y^2)(xy-5)$ **d** $(p^2+3)(10+p)$
e $(w+3)(w-3)$ **f** $(m+n)(m-n)$
g $(6+r)(6-r)$ **h** $(4a+3)(4a-3)$
i $(9x+y)(9x-y)$ **j** $(7a+5b)(7a-5b)$
k $3(q+3)(q-3)$ **l** $2(5m+4)(5m-4)$
m $6(2+5d)(2-5d)$ **n** $(d-e)(4d+3e)$
o $(5h-2k)(3h-k)$

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12 **a** $(m-12)(m+1)$ **b** $(y-3)(y-1)$
c $(h-13)(h+5)$ **d** $(p+9)(p-3)$
e $3(n+1)(n+2)$ **f** $4(y+2)(y-1)$
g $-(x-6)(x+3)$ **h** $-5(b+4)(b-3)$
i $-(2k-1)(3k-4)$

13 **a** $(3x+2)(2x-1)$ **b** $(5p+3)(p-2)$
c $(4w-1)(2w-3)$ **d** $3(e+2)^2$
e $3(2y-3)(y+4)$ **f** $4(3a+1)(2a-3)$
g $-3(3x-2)^2$ **h** $-2(4k-3)(2k-5)$
i $2(3g-1)(2g-7)$

14 **a** $(8y+5)(8y-5)$ **b** $(x-8)^2$
c $16(r-3)$ **d** $(g-5)(3+h)$
e $(a-1)^2(a+1)$ **f** $3b(b+1)(b-1)$
g $(4y+7)(y-3)$ **h** $2(3q+2)^2$
i $-(6p-1)(4p+1)$

15 **a** $\frac{3y+5}{4}$ **b** $\frac{x-1}{3x+2}$ **c** $\frac{w+5}{w-5}$

16 **a** $\frac{x-4}{x(x+1)(x-1)}$ **b** $\frac{2}{3(x-3)}$
c $\frac{k^2-3k+10}{2(k+3)(k-3)}$ **d** $\frac{5x-9}{(x+2)(x+3)(x-3)}$
e $\frac{4d^2-3d+3}{d(d-1)(d-2)}$ **f** $\frac{m+18}{(m+1)(m+4)(m-3)}$
g $\frac{9}{5}$ **h** $\frac{2(y-3)}{y^2}$ **i** $\frac{1}{2(d-2)}$ **j** $\frac{5}{4}$

Chapter 2

SkillCheck

1 **a** 16 **b** 106.09 **c** 34
d 28 **e** 7 **f** 11

2 **a** 25 cm **b** 28 cm **c** 76 mm

3 81, 25, 100, 16, 64

Exercise 2.01

1 **a** $k^2=m^2+n^2$ **b** $d^2=p^2+c^2$ **c** $x^2=w^2+y^2$
d $k^2=h^2+t^2$ **e** $z^2=c^2+r^2$ **f** $q^2=f^2+d^2$

2 **a** 10 cm **b** 13 m **c** 30 cm
d 37 cm **e** 26 mm **f** 34 cm
g 20 cm **h** 5.8 m **i** 25 cm
j 20.5 m **k** 61 mm **l** 19.5 m

3 **a** $\sqrt{325}$ cm **b** $\sqrt{377}$ cm **c** $\sqrt{3541}$ mm
d $\sqrt{4405}$ mm **e** $\sqrt{117}$ cm **f** $\sqrt{4122}$ mm

4 **a** 82.9 mm **b** 2.6 m **c** 107.4 mm
d 30.6 cm **e** 6.5 m **f** 85.4 mm

5 B **6** 4.8 m

7 **e** Yes
f The area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other 2 sides of the triangle. The area of any square is its length squared. So this proves Pythagoras' theorem: the square of the hypotenuse in a right-angled triangle is equal to the sum of the squares of the other 2 sides.

8 **a** 69.1 cm **b** $\sqrt{19.45}$ m **c** 134 mm
d 179.6 mm **e** 12.8 m **f** 6.26 m

Exercise 2.02

1 C **2** B

3 $\sqrt{32}$, $\sqrt{33}$, $\sqrt{4.9}$, $\sqrt{52}$, $\sqrt{200}$

4 **a** R **b** I **c** R **d** R
e R **f** R **g** R **h** R
i I **j** R **k** I **l** I

5 **a** $1\frac{4}{7}, \frac{\pi}{2}, \sqrt{2}$ **b** $2\frac{7}{9}, \sqrt[3]{20}, 2.\dot{6}$

6, 7 Teacher to check, $\sqrt{5}\approx2.24$, $\sqrt{10}\approx3.16$, $\sqrt{17}\approx4.12$

8 C **9** No, only if $x \geq 0$

10 Yes, as $\sqrt{x}$ is defined for $x \geq 0$

Exercise 2.03

1 **a** 2 **b** 5 **c** 27 **d** 250
e 0.09 **f** 28 **g** 45 **h** 50

2 **a** $2\sqrt{2}$ **b** $3\sqrt{3}$ **c** $2\sqrt{6}$ **d** $3\sqrt{6}$
e $9\sqrt{3}$ **f** $3\sqrt{5}$ **g** $4\sqrt{3}$ **h** $10\sqrt{2}$
i $4\sqrt{6}$ **j** $3\sqrt{7}$ **k** $12\sqrt{2}$ **l** $6\sqrt{3}$
m $5\sqrt{3}$ **n** $7\sqrt{3}$ **o** $4\sqrt{2}$ **p** $11\sqrt{2}$
q $9\sqrt{2}$ **r** $7\sqrt{5}$ **s** $5\sqrt{5}$ **t** $16\sqrt{2}$

3 **a** $25\sqrt{2}$ **b** $6\sqrt{2}$ **c** $12\sqrt{3}$ **d** $56\sqrt{2}$
e $\sqrt{10}$ **f** $\sqrt{3}$ **g** $\frac{\sqrt{7}}{3}$ **h** $6\sqrt{6}$
i $18\sqrt{17}$ **j** $\frac{5\sqrt{5}}{2}$ **k** $3\sqrt{2}$ **l** $3\sqrt{3}$
m $40\sqrt{10}$ **n** $15\sqrt{3}$ **o** $14\sqrt{17}$ **p** $\frac{\sqrt{13}}{3}$

4 **a** F **b** F **c** T
d T **e** T **f** F

Mental skills 2A

2 **a** 895.4 **b** 37 **c** 83.1 **d** 4200
e 5271.6 **f** 1561 **g** 31 840 **h** 6430
i 22.4 **j** 48.94 **k** 7389 **l** 1142

4 **a** 73.34 **b** 0.94 **c** 6.52 **d** 0.104
e 0.704 **f** 1.985 **g** 0.02 **h** 4.159
i 12.3 **j** 0.007 58 **k** 0.0849 **l** 0.0251

Exercise 2.04

1 **a** $7\sqrt{7}$ **b** $-5\sqrt{2}$ **c** $6\sqrt{5}$ **d** $4\sqrt{5}$
e 0 **f** $\sqrt{10}$ **g** $8\sqrt{15}$ **h** $-\sqrt{6}$
i $2\sqrt{3}$ **j** $10\sqrt{5}$ **k** $6\sqrt{10}$ **l** $-5\sqrt{3}$

2 **a** $5\sqrt{5}-8$ **b** $13\sqrt{10}+3\sqrt{2}$ **c** $-9\sqrt{3}+5\sqrt{2}$
d $7\sqrt{15}+8\sqrt{12}$ **e** $-3\sqrt{7}-2\sqrt{5}$ **f** $2\sqrt{6}-8\sqrt{3}$
g $13\sqrt{11}-\sqrt{3}$ **h** $-6\sqrt{13}+11\sqrt{7}$ **i** $-6\sqrt{7}$
j $-3\sqrt{5}$

3 **a** D **b** A

4 **a** $6\sqrt{2}$ **b** $3\sqrt{3}$ **c** $-2\sqrt{5}$
d $-\sqrt{7}$ **e** $5\sqrt{6}$ **f** $7\sqrt{5}$
g $-\sqrt{10}$ **h** $8\sqrt{11}$ **i** $6\sqrt{2}$
j $8\sqrt{3}$ **k** $3\sqrt{2}$ **l** $9\sqrt{2}$
m $11\sqrt{3}$ **n** $-\sqrt{5}$ **o** $-6\sqrt{3}$
p $30\sqrt{3}$ **q** $5\sqrt{7}$ **r** $41\sqrt{2}$
s $5\sqrt{6}$ **t** $29\sqrt{2}$ **u** $-15\sqrt{3}$
v 0 **w** $6\sqrt{2}+2\sqrt{3}$ **x** $12\sqrt{3}+3\sqrt{6}$
y $3\sqrt{2}-6\sqrt{5}$ **z** $4\sqrt{6}$

Exercise 2.05

1 **a** $\sqrt{14}$ **b** $-\sqrt{35}$ **c** $4\sqrt{3}$ **d** 6
e $-5\sqrt{2}$ **f** 45 **g** $15\sqrt{30}$ **h** $-10\sqrt{21}$
i 140 **j** $-30\sqrt{2}$ **k** 36 **l** $-60\sqrt{2}$
m -112 **n** $24\sqrt{6}$ **o** 80 **p** $90\sqrt{6}$
q -396 **r** $160\sqrt{5}$ **s** $216\sqrt{2}$ **t** $-96\sqrt{6}$
u $36\sqrt{5}$ **v** $-60\sqrt{10}$ **w** $252\sqrt{3}$ **x** $72\sqrt{14}$

2 **a** $\sqrt{5}$ **b** $-\sqrt{3}$ **c** $3\sqrt{6}$
d $2\sqrt{2}$ **e** $-\frac{\sqrt{7}}{2}$ or $-\frac{1}{2}\sqrt{7}$ **f** 21
g 1 **h** 8 **i** $5\sqrt{3}$
j $5\sqrt{2}$ **k** $4\sqrt{3}$ **l** $2\sqrt{6}$
m $2\sqrt{14}$ **n** $-\frac{\sqrt{2}}{4}$ or $-\frac{1}{4}\sqrt{2}$ **o** 1
p 10 **q** 4 **r** $-21\sqrt{2}$
s 12 **t** 2 **u** $\frac{2}{3}$

3 **a** 2 **b** $4\sqrt{6}$ **c** $\sqrt{30}$
d $\frac{2}{45}$ **e** $14\sqrt{3}$ **f** 2

Exercise 2.06

1 **a** 12 mm **b** 16 cm **c** 8 m
d 15 mm **e** 18 cm **f** 12 mm

2 **a** 18.1 cm **b** 14.4 cm **c** 93.7 cm
d 24.1 m **e** 35.0 cm **f** 48.4 m

3 **a** $\sqrt{5031}$ cm **b** $\sqrt{5520}$ m or $4\sqrt{345}$ m
c $\sqrt{1344}$ m or $8\sqrt{21}$ m **d** $\sqrt{10.32}$ cm
e $\sqrt{14.03}$ m **f** $\sqrt{37127}$ m

4 B **5** B **6** C
7 10.39 cm **8** 212 mm

Exercise 2.07

1 **a** 21.0 cm **b** 4.7 m **c** 120.0 mm
d 11.9 cm **e** 27.1 mm **f** 28.0 m

2 4.58 m **3** 153 cm **4** 637.1 m **5** 92.2 M

6 **a** 5 **b** $\sqrt{58}$

7 **a** 48 cm **b** 31.4 cm **c** 170 cm
d 6.5 m **e** 35.9 cm **f** 127.9 mm

8 **a** 38.3 cm **b** 12.21 **c** $\sqrt{31}$

9 752 m

10 **a** 21.8 m **b** 23 m

11 110 mm **12** 22.8 m

13 43.3 cm **14** 12.72 m

Exercise 2.08

1 **a** Yes **b** Yes **c** No **d** Yes
e No **f** Yes **g** No **h** Yes
i No **j** Yes **k** No **l** Yes

2 $30^2 = 900$
$17^2 + 21^2 = 730 \neq 900$
$\therefore$ not a right-angled triangle

3 C **4** B

5 The pole is vertically upright because {2.9, 2.1, 2} follows Pythagoras' theorem.

Mental skills 2B

2 **a** 16 000 **b** 210 **c** 9900 **d** 600
e 16 000 **f** 400 **g** 490 **h** 52
i 1500 **j** 1160 **k** 180 **l** 13 200

4 **a** 8 **b** 5 **c** 8 **d** 22
e 4 **f** 50 **g** 9 **h** 9.3
i 8.5 **j** 4.23 **k** 9.6 **l** 24

Exercise 2.09

1 **a** Yes **b** Yes **c** Yes
d No **e** Yes **f** No
g Yes **h** Yes **i** No

2 D **3, 4** Teacher to check.

5 **a** $b = 12, c = 13$
b $\sqrt{12^2 + 5^2} = \sqrt{169} = 13$
$\therefore$ {5, 12, 13} is a triad.

6 **a** {7, 24, 25} **b** {11, 60, 61}
c {15, 112, 113} **d** {4, 7.5, 8.5}
e {9, 40, 41} **f** {19, 180, 181}
g {10, 49.5, 50.5} **h** {51, 1300, 1301}

7 Proof: see worked solutions

Power plus

1 **a** $2\sqrt{2}$ **b** 4 **c** 9
d $-3\sqrt{3}$ **e** $40\sqrt{5}$ **f** 2916

2 *BD*, *BC*, *CE*, *DE*, *CD*

3 $\sqrt{128}$ m or $8\sqrt{2}$ m

4 **a** 22.6 **b** 10.6

5 **a** 16.2 m **b** 16.9 m

6 5.8 units

7 **a** 21.21 cm **b** 25.98 cm

8 $\sqrt{340} = 2\sqrt{85}$ cm

9 Proof: see worked solutions.

Test yourself 2

1 **a** $(AB)^2 + (AC)^2 = (BC)^2$ **b** $n^2 + p^2 = q^2$
c $(EF)^2 + (FG)^2 = (EG)^2$

2 B

3 **a** 175 mm **b** 113 mm **c** $\sqrt{98.72}$ m

4 **a** I **b** R **c** R **d** I **e** R
f R **g** R **h** R **i** I

5 **a** $6\sqrt{2}$ **b** $7\sqrt{2}$ **c** $5\sqrt{11}$
d $8\sqrt{2}$ **e** $15\sqrt{6}$ **f** $14\sqrt{7}$
g $48\sqrt{2}$ **h** $15\sqrt{5}$ **i** $28\sqrt{3}$

6 **a** $7\sqrt{5}$ **b** $3\sqrt{7} - 3\sqrt{2}$ **c** $2 + 6\sqrt{5}$
d $3\sqrt{7}$ **e** $13\sqrt{2}$ **f** $5\sqrt{2} - 7\sqrt{5}$
g $14\sqrt{2} + 17\sqrt{3}$ **h** $32\sqrt{5} - 9\sqrt{7}$

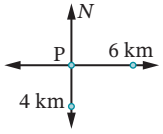
7 **a** $\sqrt{21}$ **b** $2\sqrt{10}$ **c** $4\sqrt{3}$ **d** $\sqrt{55}$
e $2\sqrt{6}$ **f** $\sqrt{14}$ **g** $4\sqrt{6}$ **h** 1
i $\frac{\sqrt{6}}{6}$ or $\frac{\sqrt{3}}{3\sqrt{2}}$ **j** $\frac{3\sqrt{2}}{2}$ or $\frac{3}{\sqrt{2}}$ **k** $\frac{2}{3}$

8 B

9 **a** 34.9 m **b** 12.2 cm **c** 824.9 m

10 168 cm

11 **a**



b 7.2 km

12 **a** 432 m **b** 1.296 km

13 **a** No **b** Yes **c** Yes

14 **a** Yes **b** No **c** Yes **d** Yes

15 48.5 cm

Chapter 3

SkillCheck

1 **a** 56 **b** 20 **c** 22 **d** 100
e −9 **f** −20 **g** 13 **h** −12
i 20 **j** 80 **k** −3 **l** −64
m 11 **n** −32 **o** −3 **p** −19
q −20 **r** 4

2 **a** 0.7, 0.7007, 0.707, 7.007
b 0.002, 0.022 02, 0.202, 0.22

3 **a** 0.009 **b** 15.4 **c** 0.246

4 C

5 **a** $1\frac{5}{21}$ **b** $1\frac{1}{10}$ **c** $\frac{11}{36}$ **d** $\frac{7}{30}$
e $\frac{15}{28}$ **f** $\frac{3}{10}$ **g** $4\frac{1}{2}$ **h** $4\frac{2}{3}$
i $1\frac{1}{2}$ **j** $\frac{5}{6}$ **k** $\frac{3}{35}$ **l** $1\frac{2}{7}$

6 **a** 25% **b** 30% **c** 7% **d** 60%

7 5%, 0.5, $\frac{5}{9}$, $\frac{3}{5}$

Exercise 3.01

1 **a** $0.1\dot{6}$ **b** $0.\dot{2}7\dot{3}$ **c** $0.\dot{8}$ **d** $0.20\dot{9}$

2 **a** 0.10981098..... **b** 0.17171717....
c 0.25555.... **d** 8.318318....

3 **a** 0.875 T **b** $0.2\dot{7}$ R **c** $4.\dot{8}$ R
d 2.6 T **e** 0.76 T **f** $0.6\dot{}$ R
g 0.65 T **h** $1.58\dot{3}$ R **i** 8.325 T
j $0.708\dot{3}$ R

4 **a** $0.\dot{3}$ **b** **i** $\frac{2}{3}$ **ii** 1

Exercise 3.02

1 **a** $\frac{2}{9}$ **b** $\frac{8}{9}$ **c** $\frac{7}{45}$ **d** $\frac{7}{18}$
e $\frac{7}{15}$ **f** $\frac{65}{99}$ **g** $\frac{31}{99}$ **h** $\frac{103}{165}$
i $\frac{7}{12}$ **j** $\frac{832}{999}$ **k** $\frac{1}{30}$ **l** $\frac{25}{33}$

2 **a** $\frac{5}{11}$ **b** $\frac{10}{11}$

3 **a** $\frac{634}{999}$ **b** $3\frac{205}{333}$

4 **a** $2\frac{1}{22}$ **b** $7\frac{17}{198}$

Exercise 3.03

1 **a** \$12.75 **b** 72 kg **c** 7.2 m **d** \$8.40
e \$4.20 **f** 11.4 kg **g** \$21.75 **h** 24 L
i \$106.88 **j** \$2180 **k** 1.5625 t **l** \$14.88

2 **a** 29% **b** 24 534

3 186 **4** 140 g **5** 106.25 kg

6 **a** \$500.50 **b** \$92.40 **c** \$2040
d 82 kg **e** 45.5 m **f** 288 L

7 **a** \$539.50 **b** 114 min **c** 79.2 L
d 5.1 m **e** 106.6 kg **f** \$86.85

8 **a** \$43 **b** \$903

9 \$20.90 **10** \$6.69

11 \$944.23 **12** \$14 520

13 **a** 68.8% **b** 66.7% **c** 64.3% **d** 83.3%
e 66.7% **f** 15% **g** 14.6% **h** 12.5%
i 17% **j** 12.5% **k** 17% **l** 14%

14 English 80%, Maths 78%. English is the better mark.

15 33.3% **16** 19.8%

17 **a** **i** 5% **ii** 62%
b 64%

18 51 goals

19 \$8400 **20** 750 g

21 \$83 654 **22** 17.5%

23 no, $1.06 \times 0.9 \times 6580 = \$6277.32$ and $0.96 \times 6580 = 6316.80$

24 decrease of 1% **26** decrease of 1%

Exercise 3.04

1 **a** \$600 **b** 92.3%

2 **a** \$1500 **b** 9.4%

3 **a** \$4500 **b** \$1700 **c** 37.8%

4 **a** \$130 000 **b** 15.3%

5 40% **6** $33\frac{1}{3}$% **7** $91\frac{2}{3}$%

8 **a** \$1.56 **b** \$1.95 **c** \$7.15 **d** \$10.39

9 C **10** C **11** \$16.58

12 23.5% **13** 21.8%

14 **a** 31.3% **b** 48.7%

15 \$140 **16** \$3295 **17** B **18** \$289

19 **a** \$2090, \$22 990 **b** \$181, \$1991
c \$14.50, \$159.50 **d** \$18, \$198

20 **a** \$4.20, \$42 **b** \$67.20, \$672 **c** \$2.85, \$28.50
d \$5.99, \$59.90 **e** \$84, \$840 **f** \$18, \$180

21 **a** \$1329.30 **b** \$1262.85 **c** \$636.15 **d** 33.5%

22 **a** \$599 **b** \$509.15 **c** \$483.70 **d** \$479.20
e No, the 15% discount followed by the 5% discount is the same as a 19.25% discount.

ANSWERS

Mental skills 3A

2 **a** 625 **b** 3025 **c** 2025 **d** 4225
e 5625 **f** 6.25 **g** 20.25 **h** 42 025
i 42.25 **j** 56.25 **k** 75 625 **l** 119 025

4 **a** 441 **b** 10 201 **c** 961 **d** 8281
e 26.01 **f** 6561 **g** 3721 **h** 40 401
i 4.41 **j** 82.81

6 **a** 3481 **b** 4761 **c** 7921 **d** 361
e 11 881 **f** 34.81 **g** 47.61 **h** 14 161
i 1521 **j** 79.21

Exercise 3.05

1 **a** \$6300 **b** \$300.96 **c** \$660
d \$708.75 **e** \$2529.60 **f** \$1825

2 \$21 386

3 **a** \$94.50 **b** \$46.72 **c** \$254.79
d \$305.67 **e** \$13 407.69 **f** \$1185.67

4 **a** 6.3% p.a. **b** 7.7% p.a. **c** 4.7% p.a.

5 6 years **6** 100 months **7** 144 days

8 **a** \$1400 **b** 10% p.a.

9 **a** \$6144 **b** \$2144 **c** 13.4% p.a.

Exercise 3.06

1 **a** 12 : 7 **b** 5 : 12 **c** 25 : 12 **d** 3 : 2
e 2 : 1 **f** 12 : 5 **g** 17 : 6 **h** 5 : 18
i 25 : 4 **j** 15 : 16 **k** 15 : 2 **l** 5 : 6 : 2
m 3 : 4 : 1 **n** 6 : 1 : 4 **o** 5 : 18 **p** 1 : 2

2 **a** 3 : 16 **b** 25 : 4 **c** 8 : 3
d 3 : 10 **e** 1 : 3 **f** 1 : 5
g 3 : 20 **h** 14 : 1 **i** 8 : 1

3 9 **4** \$312 000 **5** 16

6 20 m^3 gravel, 15 m^3 sand, 5 m^3 cement **7** 1125

8 Shahid \$27 600, Bridie \$32 400 **9** D

10 **a** 67.5 km/h **b** \$8.40/kg **c** 56 words/min
d 12.1 km/L **e** 7.7 m/s **f** 41.7 g/m^2
g 22.5 km/day **h** \$71.67/h

11 **a** 162.5 km **b** 552.5 km

12 **a** \$27.50/h **b** \$1100 **c** 8 days

13 **a** \$15.99/kg **b** \$47.97 **c** 2.5 kg

14 **a** 4680 beats/ hour **b** 12 min 49 seconds

15 **a** **i** 3 : 5 **ii** 3 : 1 **b** 29.2 g

16 **a** 74.24 L **b** 6.728 L **c** 16.24 L

17 3.4 L/100 km **18** 32 500 **19** A

20 **a** 137.5 persons/km^2 **b** 3.2 people/km^2
c 1 057 650 000

21 2 min 51 s

Exercise 3.07

1 **a** 18 000 m/h **b** 108 000 m/h **c** \$0.01/g
d 1200 mL/h **e** 2400 mL/h **f** 18 000 mL/h
g 50 000 kg/ha **h** 2500 kg/day **i** \$39 000/year
j 30 km/h **k** 360 sheep/day **l** \$420/week

2 **a** 18 km/h **b** 1260 m/h **c** \$100/kg
d 2.4 L/h **e** 666.7 mL/s **f** 432 L/day
g 250 kg/ha **h** 60 000 kg/day **i** 30.6 m/s
j 50 g/cm **k** 8 m/mL **l** \$42/h
m 0.162 km/h **n** 30 kg/kL **o** 125 mL/s

3 **a** 97.2 km/h **b** 349.2 km/h **c** 11.88 km/h
d 108 km/h **e** 82.8 km/h

4 **a** 15 m **b** 20 m **c** 27.5 m

Mental skills 3B

2 **a** 2.8 **b** 4.2 **c** 5.3 **d** 10.5
e 8.9 **f** 5.6 **g** 3.5 **h** 8.1
i 8.7 **j** 5.4 **k** 6.3 **l** 11.2

Power plus

1 $A = \frac{17}{80}, B = \frac{9}{40}, C = \frac{19}{80}$

2 \$21 000 **3** $44.\dot{4}\%$ **4** \$11.68

5 **a** 0.667 L
b 0.001 L = 1 mL, and measuring cups do not measure to the nearest mL.

6 \$108 **7** 11.1 s

8 25 : 28 **9** \$350

10 Jordan arrives first. Jordan takes $\frac{D}{50}$ h and James takes $\frac{D}{48}$ h, where D is the distance travelled.

11 **a** 20 years **b** 20 years

Test yourself 3

1 $0.\dot{3}\dot{6}$

2 **a** $\frac{8}{9}$ **b** $\frac{3}{11}$ **c** $\frac{1}{6}$ **d** $2\frac{214}{333}$

3 **a** \$243.60 **b** 7.425 kg **c** \$8.77

4 **a** 162 L **b** \$21.88

5 **a** 8.3% **b** 16.7%

6 D

7 **a** \$380 000 **b** 152%

8 773 runs **9** \$272.73

10 **a** \$1932 **b** \$54.46

11 13%

12 **a** 4 : 9 **b** 7 : 400 **c** 20 : 3

13 Bill: \$375 000, Ben: \$425 000

14 **a** 83 km/h **b** 332 km **c** 12 h

15 **a** 39 kg **b** 30 ha

16 **a** 2880 mL/h **b** 0.14 kg/m^2
c $8\frac{1}{3}$ m/s **d** 24 cm/min

17 \$108

Practice set 1

1 **a** 0.625, terminating **b** $0.\dot{5}3846\dot{1}$, recurring
c $0.\dot{8}$, recurring **d** 4.75, terminating

2 **a** $5m - 2$ **b** $20p + m$ **c** $3n^2 - 10n$

3 $\sqrt{98}$, $\sqrt{160}$, $\sqrt{52}$

4 **a** $g = 24.2$ **b** $e = 4.2$ **c** $k = 21.2$

5 **a** $\frac{14}{33}$ **b** $\frac{4}{15}$ **c** $\frac{20}{111}$ **d** $3\frac{8}{37}$

9780170453325

6 **a** $5\sqrt{3}$ **b** 63 **c** $4\sqrt{7}$ **d** $9\sqrt{2}$

7 **a** $25y^2 - 49$ **b** $4g^2 - 9m^2$ **c** $16u^2 - \frac{1}{u^2}$

8 **a** $p^2 - 4p - 32$ **b** $6n^2 + 13n + 5$ **c** $49 - y^2$

9 **a** $25a^2 + 30a + 9$ **b** $9r^2 - 16$
c $6x - 18$

10 **a** $13\sqrt{2} - 3\sqrt{5}$ **b** $\sqrt{2}$ **c** $25\sqrt{2}$

11 **a** $3\sqrt{6}$ **b** 30 **c** $-8\sqrt{10}$
d 3 **e** $-4\sqrt{3}$ **f** $-\frac{7}{2}\left(\text{or} -3\frac{1}{2}\right)$

12 **a** $y = 6$ **b** $p = 8$ **c** $k = 12.5$

13 B

14 **a** $3(p - 5)$ **b** $(m + 4)(5 - t)$
c $-4xy(2x + 3)$ **d** $(2y - 3)(ky - 1)$

15 **a** $(6y + 7x)(6y - 7x)$ **b** $(m - 5n)(m + 3)$
c $2(2m + 5)(2m - 5)$ **d** $(2y - 3)(ky - 1)$

16 **a** $9m^2 - 30m + 25$ **b** $4y^2 + 36y + 81$
c $h^2 - 8 + \frac{16}{h^2}$ **d** $16p^2 + 24pw + 9w^2$

17 **a** No, $50^2 \neq 40^2 + 14^2$ **b** Yes, $25^2 = 7^2 + 24^2$
c Yes, $6.1^2 = 1.1^2 + 6^2$

18 **a** \$102 **b** \$35 **c** \$23.45

19 **a** \$104.45 **b** 31.6 L

20 **a** $(y - 9)(y + 7)$ **b** $(k - 2)(k - 5)$
c $(w + 7)(w - 2)$ **d** $(m - 9)(m - 8)$

21 **a** $(k + 3)(5k - 1)$ **b** $(y + 3)(3y - 8)$
c $(h - 3)(4h + 5)$ **d** $(2w - 7)(4w - 5)$
e $(3y + 2)(2y + 7)$ **f** $(3m - 4)(4m - 5)$
g $(8a - 3)(6a + 5)$ **h** $(9x + 7)(4x + 5)$

22 **a** $\frac{1}{k-3}$ **b** $\frac{1}{x-3}$ **c** $\frac{y+5}{6}$ **d** $\frac{d-7}{d-5}$

23 **a** $\frac{n^2+1}{n(n+1)(n-1)}$ **b** $\frac{5y}{3}$ **c** $\frac{(k+4)(k-1)}{4}$
d $\frac{(w+8)(w-3)}{(w-5)(w+3)}$ **e** $\frac{6k-22}{(k-5)(k+1)(k-6)}$

24 **a** $(5 - 3y)(3y - 1)$
b $(7m + 4)(4m - 3)$
c $4(k - 2)(k + 2)(k^2 + 4)$

25 **a** $8a^3$ **b** $40p^2d$ **c** $27b^3d^2$ **d** $-9x^2w^3$

26 **a** $3m$ **b** $\frac{3x}{y}$ **c** $\frac{wv^2}{2}$ **d** $\frac{7a}{3d^2}$

27 33.3%

28 **a** 5 : 3 **b** 4 : 1
c 85 words/min

29 2.3 m

30 **a** 110.5 kg **b** \$5.25

31 **a** $40 - 8x$ **b** $6h - 21h^2$ **c** $18cd - 27c^2$

32 \$485

33 **a** $22w - 2$ **b** $1 - 16k$ **c** $26t - 23$

34 Klaas \$90, David \$150, Sarah \$210

35 40 320 km/h

36 **a** $\frac{6d}{11}$ **b** $\frac{9k}{4}$ **c** $\frac{19m}{15}$ **d** $\frac{8y-21h}{28}$

37 **a** $\frac{5a}{6m}$ **b** $\frac{5dw}{3m}$ **c** $\frac{2}{3x}$ **d** $\frac{3ad}{20y^2}$

38 **a** $\sqrt{74}$ cm **39** \$1875

Chapter 4

SkillCheck

1 **a** 35 **b** 33.2 **c** 5

2 **a** 85 **b** 16 **c** 3.9

3 **a** 8 h **b** 4 h **c** 2 h

4 **a** 4h 42min **b** 2h 15min **c** 6h 51min

5 **a** $x = 55$ **b** $y = 23$ **c** $n = 38$
d $x = 60$ **e** $x = 50$ **f** $x = 160$

Exercise 4.01

1 **a** hyp = 13, opp = 12, adj = 5
b hyp = BC, opp = AC, adj = AB
c hyp = v, opp = u, adj = w
d hyp = SR, opp = TR, adj = ST
e hyp = x, opp = y, adj = z
f hyp = 41, opp = 9, adj = 40

2 **a** $\angle K$ **b** $\angle L$ **c** $\angle M$ **d** $\angle M$ **e** $\angle L$

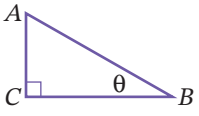
3 C

4 **a** $\angle\theta$ EF, $\angle\phi$ DE **b** $\angle\theta$ 12, $\angle\phi$ 9
c $\angle\theta$ f, $\angle\phi$ e

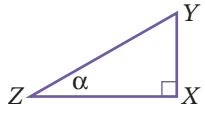
5 **a** $\angle\theta$ IJ, $\angle\phi$ HI **b** $\angle\theta$ 27, $\angle\phi$ 36
c $\angle\theta$ b, $\angle\phi$ c

6 B

7 **a**



b



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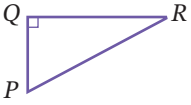
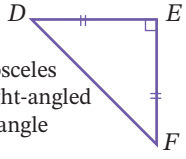


d



isosceles
right-angled
triangle

Exercise 4.02

1 **a** $\sin = \frac{55}{73}$ $\cos = \frac{48}{73}$ $\tan = \frac{55}{48}$

b $\sin = \frac{e}{f}$ $\cos = \frac{g}{f}$ $\tan = \frac{e}{g}$

c $\sin R = \frac{ST}{RT}$ $\cos R = \frac{RS}{RT}$ $\tan R = \frac{ST}{RS}$

d $\sin = \frac{n}{k}$ $\cos = \frac{m}{k}$ $\tan = \frac{n}{m}$

e $\sin = \frac{3.6}{8.5}$ $\cos = \frac{7.7}{8.5}$ $\tan = \frac{3.6}{7.7}$

f $\sin W = \frac{XY}{WY}$ $\cos W = \frac{WX}{WY}$ $\tan W = \frac{XY}{WX}$

2 **a** $\frac{4}{5}$ **b** $\frac{4}{3}$ **c** $\frac{3}{5}$ **d** $\frac{4}{5}$

ANSWERS

3 **a** α **b** β **c** β
d α **e** α **f** β

4 **a** **i** $\frac{24}{7}$ **ii** $\frac{7}{25}$ **iii** $\frac{24}{25}$ **iv** $\frac{7}{24}$

b **i** $\frac{u}{v}$ **ii** $\frac{v}{w}$ **iii** $\frac{u}{w}$ **iv** $\frac{v}{u}$

c **i** $\frac{FG}{FH}$ **ii** $\frac{FH}{HG}$ **iii** $\frac{FG}{HG}$ **iv** $\frac{FH}{FG}$

d **i** $\frac{84}{13}$ **ii** $\frac{13}{85}$ **iii** $\frac{84}{85}$ **iv** $\frac{13}{84}$

e **i** $\frac{b}{c}$ **ii** $\frac{c}{a}$ **iii** $\frac{b}{a}$ **iv** $\frac{c}{b}$

f **i** $\frac{QS}{RS}$ **ii** $\frac{RS}{QR}$ **iii** $\frac{QS}{QR}$ **iv** $\frac{RS}{QS}$

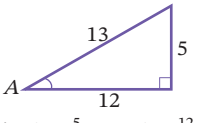
5 **a** $\tan Y = \frac{60}{11}$ **b** $\tan X = \frac{11}{60}$

c $\sin X = \frac{11}{61}$ or $\cos Y = \frac{11}{61}$

d $\cos X = \frac{60}{61}$ or $\sin Y = \frac{60}{61}$

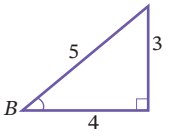
6 C **7** A

8 **a**



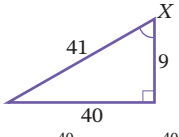
$\sin A = \frac{5}{13}$, $\cos A = \frac{12}{13}$

b



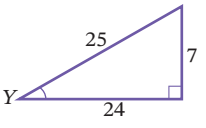
$\cos B = \frac{4}{5}$, $\tan B = \frac{3}{4}$

c



$\sin X = \frac{40}{41}$, $\tan X = \frac{40}{9}$

d



$\cos Y = \frac{24}{25}$, $\tan Y = \frac{7}{24}$

Exercise 4.03

1 **a**

	Side length (mm)			Trigonometric ratio		
	XZ (opp)	*ZY* (adj)	*XY* (hyp)	$\sin Y = \frac{XZ}{XY}$	$\cos Y = \frac{ZY}{XY}$	$\tan Y = \frac{XZ}{ZY}$
1	45	26	52	0.87	0.50	1.73
2	24	14	28	0.86	0.50	1.71
3	51	30	59	0.86	0.51	1.70
4	62	36	72	0.86	0.50	1.72

b All are approximately the same (0.9).

c The table answer is an approximation of the calculator answer.

d All are approximately the same (0.5).

e The table answer is an approximation of the calculator answer.

f All are approximately the same (1.7).

g The table answer is an approximation of the calculator answer.

2 **a** Teacher to check.

b, c, d The table answer is an approximation of the calculator answer.

3 Teacher to check.

Exercise 4.04

1 **a** 9.51 **b** 0.97 **c** 1.07 **d** 0.54
e 0.97 **f** 0.50 **g** 0.83 **h** 0.95
i 29.86 **j** 11.61 **k** 32.97 **l** 10.26
m 47.15 **n** 9.56 **o** 3.58 **p** 33.11
q 19.84 **r** 244.80 **s** 12.26 **t** 16.83

2 **a** 28° **b** 41° **c** 19° **d** 33°
e 34° **f** 56° **g** 30° **h** 44°

3 **a** 68° 40′ **b** 54° 22′ **c** 24° 47′ **d** 18° 30′
e 9° 11′ **f** 47° 59′ **g** 3° 46′ **h** 57° 10′

4 **a** 55° 30′ **b** 14° 9′ **c** 72° 23′ **d** 33° 46′
e 66° 25′ **f** 7° 53′ **g** 28° 7′ **h** 31° 3′
i 34° 27′ **j** 71° 5′ **k** 5° 29′ **l** 69° 27′

5 **a** 52° **b** 14° **c** 52° **d** 60°
e 33° **f** 28° **g** 36° **h** 89°

Mental skills 4

Other answers possible as these are estimates.

2 **a** 330 **b** 160 **c** 1580 **d** 260
e 420 **f** 200 **g** 410 **h** 740
i 6600 **j** 16 000 **k** 280 **l** 71

4 **a** 29 **b** 15 **c** 180 **d** 400 **e** 3
f 4 **g** 20 **h** 5 **i** 6 **j** 25

Exercise 4.05

1 **a** $\tan\theta$ **b** $\cos\theta$ **c** $\tan\theta$
d $\sin\theta$ **e** $\tan\theta$ **f** $\cos\theta$

2 **a** 6.55 **b** 4.33 **c** 27.63
d 44.99 **e** 13.90 **f** 16.97
g 16.17 **h** 26.68 **i** 16.07

3 **a** 38.71 **b** 7.38 **c** 17.21
d 51.59 **e** 24.49 **f** 132.57
g 33.59 **h** 12.06 **i** 36.06

4 B

5 **a** 4.3 m **b** 141.4 m **c** 671.4 m
d 1053.1 cm **e** **i** 10 km **ii** 17.3 km

6 12.3 m **7** 10 cm **8** 5.76 m

9 3.4 m **10** 933 cm

11 5738 mm **12** 5.1 m

13 C **14** B

Exercise 4.06

1 **a** 21.5 **b** 378.3 **c** 245.7 **d** 95.6

2 325 mm **3** 12.4 m

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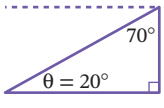
4 **a** 88.97 **b** 1188.24 **c** 45.70 **d** 187.70
5 6.88 m **6** D
7 **a** 1203.3 cm **b** 414.2 mm **c** 6.7 m
d 1044.3 cm
e **i** 3.8 km **ii** 8.4 km
8 782 cm **9** 12.1 m **10** 4.9 km
11 B **12** 77.3 m
13 **a** 18.0 cm **b** 8.4 cm

Exercise 4.07

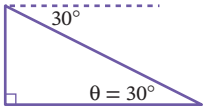
1 **a** 41° **b** 64° **c** 60° **d** 81°
e 61° **f** 30° **g** 6° **h** 45°
i 60° **j** 82° **k** 84° **l** 14°
2 **a** 64° 59′ **b** 54° 35′ **c** 36° 52′ **d** 20° 10′
e 43° 32′ **f** 72° 33′ **g** 27° 2′ **h** 61° 39′
i 86° 12′ **j** 81° 47′ **k** 35° 16′ **l** 51° 3′
3 **a** 34° **b** 51° **c** 60°
d 29° **e** 67° **f** 54°
4 C
5 **a** 25° 12′ **b** 36° 52′ **c** 31° 30′
d 26° 47′ **e** 48° 11′ **f** 66° 33′
6 **a** 12° **b** 27° **c** 27°
d 68° **e** 30° **f** 88°
7 39° **8** 38.9° **9** 10°56′
10 52° 8′
11 11° **12** 37° **13** B **14** 71°
15 11° **16** 50° **17** 38° **18** 29°48′

Exercise 4.08

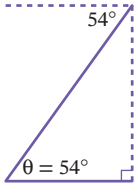
1 **a**



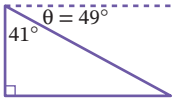
b



c



2 **a**



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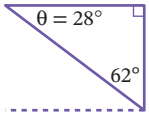

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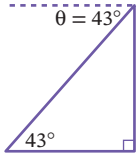
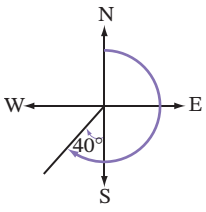


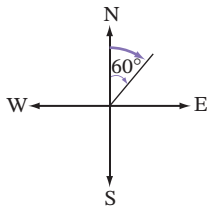
3 59 m **4** 2224 m or 2.224 km
5 177 m **6** 180 m **7** 9°
8 50° **9** 14° 2′ **10** 18° 47′
11 1256 cm **12** 14.5 m
13 970 m **14** 57 m

Exercise 4.09

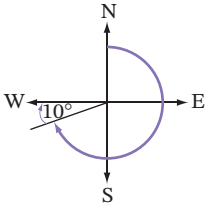
1 **a** 243° **b** 290° **c** 040° **d** 115° **e** 210°
f 140° **g** 312° **h** 253° **i** 065°
2 **a** 000° **b** 090° **c** 180° **d** 270° **e** 038°
f 125° **g** 330° **h** 225° **i** 072° **j** 187°
3 SW
4 **a** 225° **b** 090° **c** 045° **d** 112.5° **e** 337.5°
f 270° **g** 135° **h** 247.5°
5 **a**



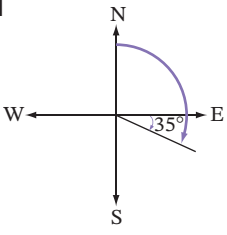
b



c



d



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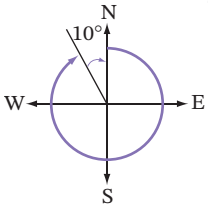
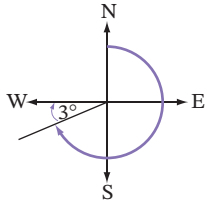
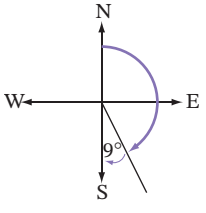


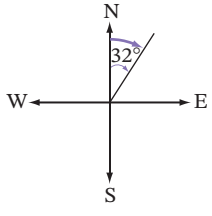
f



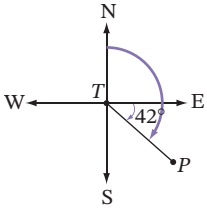
g



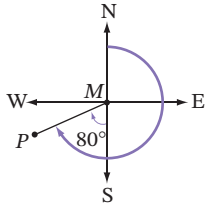
h



6 **a** WNW **b** 292.5°
7 **a**



b



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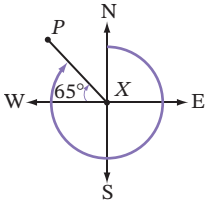
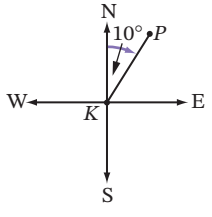


d



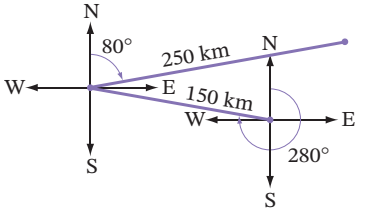
ANSWERS

8 240° **9** 280°

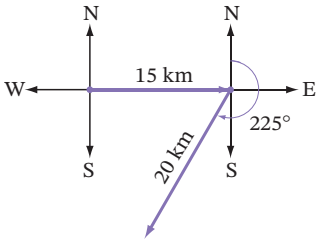
10 **a** 45° **b** 90° **c** 135°
d 157.5° **e** 112.5° **f** 67.5°

11 ESE

12 **a**



b



13 **a** 230° **b** 270° **c** 160°
d 340° **e** 050° **f** 090°

Exercise 4.10

1 **a** 21 km **b** 260°

2 **a** 37° **b** 163 km **c** 143°

3 **a** 62.5 km **b** 29.2 km **c** 025°

4 **a** 12.2 km **b** 215°

5 **a** 18 km **b** NNW

6 45.7 km

7 **a** 13.509 km **b** 321°

8 **a** 2122 km **b** 330°

9 **a** 15 km **b** 26 km

10 **a** 261.08 km **b** 167.82 km

11 6.6 km

12 **a** 322 nautical miles **b** 276° 45′

13 **a** 1282 km **b** 024° 27′

14 326° 11′

Power plus

1 **a** **i** 0.342, 0.342 **ii** 0.731, 0.731
iii 0.819, 0.819 **iv** 0.996, 0.996

b Each pair of trignometric ratios has the same value

c The pairs of angles are complementary (add to 90°)

d 60°

e **i** 25° **ii** sin **iii** 18°
iv cos **v** 25° **vi** 32°

f $\sin x = \cos(90° - x)$ **g** Teacher to check

2 **a** 1932 m **b** 31°

3 **a** $\frac{\sqrt{3}}{2}$ **b** $\frac{1}{\sqrt{3}}$

4 **a** 24° 12′ 26″ **b** 64° 9′ 41″

5, 6 Teacher to check

Test yourself 4

1 **a** $\frac{33}{65}$ **b** $\frac{33}{56}$ **c** $\frac{56}{65}$ **d** $\frac{33}{65}$

2 $\cos\alpha = \frac{77}{85}$; $\tan\alpha = \frac{36}{77}$

3 **a** 64° **b** 26° **c** 12°

4 **a** 50° 19′ **b** 31° 56′ **c** 64° 19′

5 **a** 0.8480 **b** 0.7677 **c** 0.1539
d 64.9839 **e** 13.9884 **f** 13.7044

6 **a** 45°48′ **b** 33°11′ **c** 5°21′

7 **a** 14.49 **b** 72.50 **c** 8.71
d 25.13 cm **e** 54.82 mm **f** 106.86 m

8 **a** 98.9 cm **b** 21.1 m **c** 232.0 mm

9 **a** 68°44′ **b** 55°9′ **c** 69° 38′

10 **a** 33° **b** 28° **c** 38°

11 68°13 ′ **12** 195 m **13** 29°

14 **a** 320° **b** 140°

15 **a** 1281 km **b** 024°

16 **a** 44 nautical miles **b** 276.6° **c** 096.6°

Chapter 5

SkillCheck

1 **a** **i** 3 **ii** 5 **iii** three to the power of five
b **i** 4 **ii** 8 **iii** four to the power of eight
c **i** h **ii** 5 **iii** h to the power of five
d **i** 7 **ii** y **iii** seven to the power of y

2 **a** 25 **b** 343 **c** 10 000 **d** 64
e $\frac{8}{27}$ **f** 81 **g** 81 **h** 216
i 256 **j** $\frac{1}{8}$ **k** $\frac{9}{16}$ **l** 125

3 **a** 2^5 **b** $3^4 \times 7^3$ **c** $5^6 \times 8^2$
d $\frac{3^4}{7^4}$ or $\left(\frac{3}{7}\right)^4$ **e** $10^1 \times x^5$ **f** $6^3 \times k^2$
g $x^3 \times y^2$ **h** $a^2 \times b^3$ **i** $5^2 \times n^3$
j $p^2 \times q^4$

4 **a** $9 \times 9 \times 9$ **b** 7×7
c $d \times d \times d \times d \times d$ **d** $k \times k$
e $\frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}$

5 **a** 1024 **b** 10 000 **c** 729 **d** 1
e 9 **f** 15 625 **g** 8 **h** 64

6 **a** 3 **b** 4 **c** 3 **d** 2
e 6 **f** 4 **g** 6 **h** 4

Exercise 5.01

1 B

2 **a** 10^5 **b** 2^5 **c** 3^7 **d** 7^5
e 8^8 **f** 5^9 **g** 6^{10} **h** 4^{12}
i 3^{11} **j** x^5 **k** g^8 **l** w^8
m b^{13} **n** p^{20} **o** r^2 **p** y^{12}
q m^{19} **r** n^{10}

3 D **4** B

5 **a** 10^2 **b** 8^4 **c** 20^{10} **d** 5^6 **e** 9^9
f 2^{24} **g** 7^1 **h** 2^{19} **i** 11^0 **j** p^5

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k n^6 **l** w^{18} **m** h^{16} **n** y^6 **o** a^8
p b^1 **q** w^{24} **r** m^0

6 D **7** C **8** A

9 **a** $6p^7$ **b** $12y^{12}$ **c** $18m^9$
d $5h^{11}$ **e** $24q^9$ **f** $10a^7$
g $30n^{16}t^5$ **h** $30a^2b^4$ **i** $3e^{10}g^5$
j $32p^7m^{10}$ **k** $48q^8r^8$ **l** $54u^4v^3w^8$

10 **a** $2y^{12}$ **b** $5w^6$ **c** $8r^7$
d $30x$ **e** $5m^9$ **f** $\frac{g^6}{2}$
g $2d^2h^9$ **h** x^5y^4 **i** $\frac{e^{20}d^{36}}{3}$
j $\frac{3qt}{4}$ **k** $9a^5b^8$ **l** $\frac{3pq^2r^4}{2}$

11 B

12 **a** 3^{10} **b** 10^6 **c** 7^4 **d** 8^5 **e** a^{14}
f m **g** w^{13} **h** x^4 **i** p^5 **j** $30c^{14}$
k $3y^8$ **l** $120d^{11}$ **m** $2q^2$ **n** $3g^2$ **o** $30h^{12}$
p $\frac{3v^3}{2}$ **q** $\frac{8b^2}{5}$ **r** $2k$

13 D

14 **a** a^0 **b** y^{-1} **c** n^1 or n
d h^{-5} **e** b^0 **f** r^{-2}

Exercise 5.02

1 C

2 **a** 4^6 **b** 5^{16} **c** 3^{12} **d** 2^{28}
e 2^2 **f** 9^3 **g** 10^0 **h** 6^{20}
i 5^{15} **j** e^8 **k** t^{25} **l** y^{21}
m c^5 **n** m^{35} **o** y^{16} **p** h^0
q q^{18} **r** w^4 **s** $1024x^{10}$ **t** $390\,625n^{24}$
u $64d^9$ **v** $-k^{45}$ **w** d^{12} **x** $256a^{64}$

3 B

4 **a** $16d^{12}$ **b** $25m^6$ **c** $16y^{10}$ **d** $81x^8$
e $3125u^{30}$ **f** $8w^{15}$ **g** $10\,000d^{20}$ **h** $27e^3$
i $2b^4$ **j** $36d^{12}$ **k** $243f^{20}$ **l** $1024c^{30}$
m $16r^4$ **n** $-125t^3$ **o** $9m^6$ **p** y^{36}
q $-x^3$ **r** m^{30} **s** $256w^{20}$ **t** $-243f^5$
u $-27p^6$ **v** $81h^{20}$ **w** $100k^2$ **x** $-8y^3$

5 A

6 **a** d^{-3} **b** x^{-4} **c** $w^{\frac{1}{4}}$ **d** z^2
e u^1 or u **f** p^1 or p

Exercise 5.03

1 A

2 **a** a^3b^3 **b** $x^{10}y^5$ **c** $l^{18}m^{30}$
d $1296d^4p^8$ **e** $64k^8y^{10}$ **f** $243m^{10}n^5$
g e^3k^9 **h** $-w^{21}x^{28}$ **i** $64d^6y^{10}$
j $256b^8c^{12}$ **k** $-27a^9d^3$ **l** $16p^8q^{12}$

3 **a** $x^4y^2w^6$ **b** $h^8y^{12}m^{20}$ **c** $9d^{14}k^4p^6$
d $125a^9q^9r^{12}$ **e** $32d^{20}e^5w^{15}$ **f** $k^4m^{12}p^8x^{24}$

4 D

5 **a** $\frac{36}{49}$ **b** $\frac{m^3}{8}$ **c** $\frac{625}{x^4}$ **d** $\frac{256n^{40}}{p^8}$
e $\frac{w^{10}}{t^{15}}$ **f** $\frac{m^8}{256n^4}$ **g** $\frac{16}{81}$ **h** $-\frac{125h^3}{216}$
i $\frac{49k^8}{100}$ **j** $\frac{9r^2}{t^4}$ **k** $\frac{a^8b^4}{d^{20}}$ **l** $-\frac{32}{243c^{10}}$

6 **a** $40x^{32}y^{48}$ **b** $1000x^{36}y^{54}$ **c** $2q^3r^4$
d $36q^8r^{12}$ **e** $27a^{12}x^{15}$ **f** $3ax^2$
g $32p^8h^{29}$ **h** $8p^4h^{11}$ **i** $64p^{10}h^{38}$
j $8a^3d^9$ **k** $49e^4y^2$ **l** $5xy$
m $\frac{3h^3k^3}{4}$ **n** $\frac{27m^{11}w^7}{2}$ **o** $81c^{12}d^4$

Exercise 5.04

1 **a** 1 **b** 1 **c** −1 **d** 1
e −1 **f** 1 **g** 1 **h** 7
i −1 **j** 1 **k** 1 **l** −2
m 1 **n** 1 **o** 1 **p** 1
q 1 **r** −1 **s** −6 **t** 6
u 1 **v** −1 **w** −3 **x** 1

2 **a** 2 **b** 0 **c** 3 **d** 4
e 7 **f** −5 **g** −3 **h** 0
i 1 **j** 9 **k** $1\frac{3}{4}$ **l** 2
m 6 **n** 4 **o** 125 **p** −1
q 12 **r** 3 **s** $\frac{1}{3}$ **t** 27
u −3 **v** 11 **w** 11 **x** 1

Mental skills 5

2 **a** 117 **b** 740 **c** 138 **d** 140 **e** 174
f 100 **g** 436 **h** 109 **i** 200 **j** 312

3 **a** 160 **b** 700 **c** 90
d 2200 **e** 72 **f** 1600
g 2100 **h** 480 **i** 132

Exercise 5.05

1 **a** $\frac{1}{6^2}$ **b** $\frac{1}{5^7}$ **c** $\frac{1}{3^1}$ **d** $\frac{1}{10^2}$
e $\frac{1}{g^5}$ **f** $\frac{1}{z^1}$ **g** $\frac{1}{n^3}$ **h** $\frac{1}{t^2}$
i $\frac{1}{a^4}$ **j** $\frac{1}{5^3}$ **k** $\frac{1}{y^d}$ **l** $\frac{1}{r^m}$

2 **a** $\frac{1}{9}$ **b** $\frac{1}{625}$ **c** $\frac{1}{6}$ **d** $\frac{1}{49}$
e $\frac{1}{25}$ **f** $\frac{1}{128}$ **g** $\frac{1}{64}$ **h** $\frac{1}{1000\,000}$
i $\frac{1}{1024}$ **j** $\frac{1}{27}$ **k** $\frac{1}{36}$ **l** $\frac{1}{6561}$

3 **a** n^{-2} **b** n^{-1} **c** 8^{-3} **d** 8^{-1}
e 10^{-5} **f** $2a^{-4}$ **g** 3^{-1} **h** $-b^{-1}$
i $6a^{-1}$ **j** $4t^{-2}$ **k** $-2w^{-5}$ **l** $5d^{-3}$

4 **a** $\frac{5}{h}$ **b** $\frac{2}{b^5}$ **c** $\frac{3}{e^3}$ **d** $\frac{4}{n^2}$
e $\frac{p}{b^2}$ **f** $\frac{r^2}{s^4}$ **g** $\frac{y}{w^2}$ **h** $\frac{y^3}{d^3}$
i $\frac{1}{2m}$ **j** $\frac{1}{xy}$ **k** $\frac{1}{16h^2}$ **l** $\frac{1}{125k^3}$
m $\frac{3m^3}{p^2}$ **n** $\frac{15}{kw^4}$ **o** $\frac{12}{x^2y^3}$ **p** $\frac{12y^3}{x^2}$

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q $\frac{1}{9h^2}$ **r** $\frac{1}{64k^3}$ **s** $\frac{1}{16c^4}$ **t** $\frac{1}{8y}$

u $\frac{4p}{q^3}$ **v** $\frac{4}{pq^3}$ **w** $\frac{v}{m^2}$ **x** $\frac{1}{vm^2}$

5 **a** $\frac{7}{2}$ **b** $\frac{5}{8}$ **c** $\frac{10}{9}$ **d** $\frac{2}{3}$

e $-\frac{4}{3}$ **f** $\frac{2}{5}$ **g** $\frac{3}{x}$ **h** $\frac{a}{5}$

i $-\frac{2}{m}$ **j** $\frac{4}{5r}$ **k** $\frac{3z}{2}$ **l** v

6 **a** $\frac{16}{9}$ **b** $\frac{27}{8}$ **c** 1 000 000 **d** $\frac{8}{125}$

e $-\frac{243}{1024}$ **f** $\frac{256}{625}$ **g** $\frac{16}{81}$ **h** $\frac{125}{343}$

i $\frac{9}{k^2}$ **j** $-\frac{x}{3}$ **k** $\frac{625}{a^8}$ **l** $\frac{9g^6}{16}$

m $\frac{25t^2}{4d^2}$ **n** $\frac{m^{15}}{h^{10}}$ **o** $\frac{9p^6}{25k^{14}}$ **p** $-\frac{64a^6}{27c^9}$

q $\frac{4m^4n^8}{25k^2}$ **r** $\frac{81}{a^4c^{12}d^8}$ **s** $\frac{343k^9}{64y^{15}w^6}$ **t** $\frac{25x^8y^2}{4g^6h^{14}}$

Exercise 5.06

1 **a** 5 **b** 7 **c** 25 **d** 10

e 2 **f** −0.3 **g** 0.2 **h** 4

i −2 **j** −9 **k** 2 **l** 5

2 **a** $\sqrt{10}$ **b** $\sqrt[3]{17}$ **c** $\sqrt{g}$

d $\sqrt[4]{m}$ **e** $\sqrt{8r}$ or $2\sqrt{2r}$ **f** $6\sqrt[3]{h}$

g $\sqrt[6]{3m^4}$ **h** $h\sqrt{k}$ **i** $\sqrt[7]{xy^3}$

j $\sqrt[4]{3u^2w^5}$ **k** $e\sqrt{d}$ **l** $15a\sqrt[3]{b}$

3 **a** $5^{\frac{1}{2}}$ **b** $49^{\frac{1}{3}}$ **c** $20^{\frac{1}{2}}$

d $400^{\frac{1}{5}}$ **e** $54^{\frac{1}{6}}$ **f** $80^{\frac{1}{5}}$

g $a^{\frac{1}{2}}$ **h** $q^{\frac{1}{3}}$ **i** $h^{\frac{1}{7}}$

j $(3w)^{\frac{1}{6}}$ **k** $(4xy^3)^{\frac{1}{2}}$ or $2\left(xy^3\right)^{\frac{1}{2}}$

l $(2m^2p^3)^{\frac{1}{5}}$

4 **a** 2.71 **b** 14.66 **c** 3.30 **d** 14.18

e −2.88 **f** 5.77 **g** −2.16 **h** 0.45

5 **a** b **b** e **c** $y^{\frac{6}{5}}$ **d** m

e $10y$ **f** $36a^3$ **g** n^3m **h** $4ab^3$

i $2v^2w^3$ **j** 5 **k** $7x^{\frac{2}{3}}$ **l** $9y^{-\frac{3}{4}}$

m $\frac{m^{\frac{3}{2}}}{3}$ or $\frac{1}{3}m^{\frac{3}{2}}$ **n** $\frac{4}{5}$ **o** $-512d^3$

p $20c^{\frac{1}{2}}$

Exercise 5.07

1 **a** 8 **b** 32 **c** 32 **d** 9

e 64 **f** 256 **g** 8 **h** 27

i 100 **j** 25 **k** $\frac{1}{2}$ **l** $\frac{1}{3}$

m $\frac{1}{125}$ **n** $\frac{1}{216}$ **o** $\frac{1}{64}$ **p** $\frac{1}{625}$

q $\frac{1}{256}$ **r** $\frac{1}{8000}$ **s** $\frac{1}{16}$ **t** $\frac{1}{128}$

2 **a** 7.62 **b** 0.05 **c** 132.96 **d** 0.30

e 0.03 **f** 5.28 **g** 0.02 **h** 7.96

3 **a** $g^{\frac{2}{7}}$ **b** $e^{\frac{5}{2}}$ **c** x^3 **d** y^{-4}

e $m^{\frac{5}{3}}$ **f** $m^{\frac{3}{5}}$ **g** $n^{-\frac{3}{4}}$ **h** $(5h^4)^{-\frac{1}{3}}$

4 **a** $8n^3$ **b** $4\sqrt[3]{w^2}$ **c** $8d^6$ **d** $512m^{12}$

e $243r^5$ **f** $27h^9$ **g** $\frac{1}{16s^8}$ **h** $\frac{1}{16x^6}$

i $\frac{1}{25b^4}$ **j** $\frac{1}{125\sqrt{t^3}}$ **k** $343p^6q^{15}$ **l** $\frac{1}{100x^2y^4}$

Exercise 5.08

1 **a** a^7 **b** t^9 **c** n^6 **d** p^2

e w^8 **f** g^{18} **g** $6b^7$ **h** $20d^{13}$

i $6c^4$ **j** $625b^{16}$ **k** $3m^2$ **l** $9a^2$

2 **a** 1 **b** 1 **c** 7 **d** 1

e −8 **f** 9 **g** 390 625 **h** 19 487 171

i 1 **j** 27 **k** $\frac{1}{512}$ **l** 1

m 25 **n** $\frac{1}{10000}$ **o** 1 **p** 1

3 **a** $\frac{1}{25}$ **b** $\frac{1}{243}$ **c** $\frac{1}{20}$ **d** $\frac{1}{1000}$

4 **a** 4 **b** 4 **c** 3125 **d** 4

e 4 **f** 27 **g** 27 **h** −8

5 **a** $9m^2n^6$ **b** $40a^5w^9$ **c** $256a^8b^{20}$ **d** $4pq^2$

e $\frac{64}{125}$ **f** $6c^2$ **g** $3u^4v^3$ **h** $\frac{9x^2}{100}$

i $-64n^6t^3$ **j** $\frac{16}{81}$ **k** $\frac{y^3}{5x^3}$ **l** $\frac{p^{10}}{81y^2}$

m $32p^{15}q^{10}$ **n** $\frac{1}{343n^3}$ **o** −2 **p** $\frac{a^{11}}{b}$

6 **a** $\frac{1}{8^7}$ **b** $\frac{1}{3^5}$ **c** $\frac{1}{y}$ **d** $\frac{1}{x^3}$

e $\frac{1}{25b^2}$ **f** $\frac{5}{b^2}$ **g** $\frac{1}{ab}$ **h** $\frac{a}{b}$

i $\frac{11}{t^3}$ **j** $\frac{1}{1331t^3}$ **k** $\frac{p^3}{q^5}$ **l** $\frac{m}{w^3}$

m $\frac{8}{u^3v^4}$ **n** $\frac{-2r^6}{y^5}$ **o** $\frac{10f^3}{e}$ **p** $\frac{n^7}{2k^4}$

7 **a** $\frac{4}{7}$ **b** $\frac{2}{5}$ **c** $\frac{3}{2}$ **d** 7

e $\frac{8}{r}$ **f** $10p$ **g** $\frac{w}{6y}$ **h** $\frac{5a}{2}$

8 **a** 4^{-3} **b** 2^{-1} **c** 10^{-4} **d** 9^{-2}

e k^{-1} **f** $(3m)^{-1}$ **g** $12y^{-4}$ **h** $-4x^{-7}$

i $\frac{5p^{-3}}{2}$ **j** $\frac{-2a^{-6}}{5}$ **k** $\frac{w^{-4}}{8}$ **l** $-\frac{h^{-2}}{6}$

9 **a** q^3 **b** d^4 **c** $\frac{1}{m^{11}}$ **d** y^2

e $30g^2$ **f** $24a$ **g** $\frac{28}{x}$ **h** $\frac{4}{p^3}$

i $16q^3$ **j** $\frac{d^4}{2}$ **k** $\frac{2}{b^4}$ **l** $\frac{1}{9h^2}$

10 **a** $5^{\frac{1}{2}}$ **b** $d^{\frac{1}{3}}$ **c** $(3y)^{\frac{1}{2}}$ **d** $10^{\frac{1}{4}}$

e $p^{\frac{2}{3}}$ **f** $(xy)^{\frac{5}{2}}$ **g** $(5a)^{\frac{3}{4}}$ **h** $\left(\frac{y}{6}\right)^{\frac{5}{2}}$

11 **a** $\frac{125a^3}{64}$ **b** $4c^2$ **c** $\frac{49m^2}{100}$ **d** $3125w^{\frac{25}{2}}$

e $\frac{343}{d^3}$ **f** $\frac{a^2b^6}{c^4}$ **g** $5m^{\frac{3}{2}}$ **h** $\frac{y^2}{16}$

i $2m^2$ **j** $9g^4$ **k** $32x^{10}$ **l** $\frac{16a^{12}}{c^8}$

Exercise 5.09

1 **a** 3 **b** 2 **c** 2 **d** 5

e 1 **f** 1 **g** 3 **h** 5

i 2 **j** 2 **k** 3 **l** 6

2 **a** 37.6 **b** 9440 **c** 168

d 2.81 **e** 16.0 **f** 60 500

g 1 770 000 **h** 386 000 **i** 10.3

3 **a** 0.064 **b** 0.90 **c** 0.085

d 0.000 16 **e** 0.0076 **f** 0.039

g 0.28 **h** 0.019 **i** 0.31

4 B **5** C

6 **a** 9 **b** 60 **c** 0.04

d 0.008 **e** 0.5 **f** 10 000

g 2000 **h** 0.0003 **i** 60 000 000

7 **a** \$36 000 000, 2 **b** \$40 000 000, 1

8 5 **9** 21 600 **10** C

11 **a** 180 **b** −5 **c** 6.16 **d** −4.176

e 0.06 **f** 0.67 **g** 33 300 **h** 0.6565

Exercise 5.10

1 **a** 2.4×10^3 **b** 7.86×10^5 **c** 5.5×10^7

d 9.5×10^1 **e** 7.8×10^0 **f** 3.48×10^8

g 5.967×10^4 **h** 1.5×10^1 **i** 3×10^9

j 8×10^1 **k** 7.63×10^2 **l** 1×10^1

m 3.5×10^{-2} **n** 7.6×10^{-5} **o** 8×10^{-1}

p 7.13×10^{-2} **q** 3×10^{-6} **r** 9.13×10^{-1}

s 7.146×10^{-6} **t** 9×10^{-3} **u** 1×10^{-7}

t 8.9×10^{-4} **w** 7.8×10^{-8} **x** 1×10^{-1}

2 **a** 1.3×10^5 **b** 2.9×10^{-7} **c** 8×10^{-5}

d 3×10^8 **e** 4×10^{13} **f** 1.2×10^{-4}

g 6.1×10^2 **h** 1.06×10^{-10} **i** 7.7×10^{20}

j 1×10^{-6} **k** 2.2×10^6

3 **a** 600 000 **b** 7100

c 302 000 000 **d** 3.14

e 0.000 06 **f** 0.0071

g 0.000 000 030 2 **h** 0.000 000 000 59

i 1 100 000 000 000 **j** 0.0004

k 5000 **l** 0.000 476

m 0.803 **n** 63 200

o 0.016 **p** 0.000 000 22

q 9 000 000 **r** 0.111

4 **a** 4×10^5 **b** 2.7×10^6 **c** 1.3×10^7

d 6.3×10^{-7} **e** 9.3×10^9 **f** 9.3×10^2

g 3.04×10^0 **h** 4.5×10^{-5} **i** 2×10^{-15}

j 9.7×10^{-5}

5 **a** 3.8×10^9, 5.5×10^9, 7.3×10^9

b 5.8×10^{-6}, 2.2×10^{-4}, 7×10^{-4}

c 3.5×10^0, 4.9×10^2, 5.3×10^2

6 **a** 6×10^5, 8×10^2, 2.9×10^2

b 6.3×10^2, 8.1×10^{-4}, 1.2×10^{-9}

c 9.5×10^{-1}, 4.1×10^{-1}, 6.4×10^{-3}

Exercise 5.11

1 **a** 6×10^8 **b** 2×10^5 **c** 8×10^{15}

d 3×10^6 **e** 2.4×10^{16} **f** 5×10^4

g 1.024×10^{18} **h** 3.01×10^{-6} **i** 6.3×10^9

j 2.5×10^{-11} **k** 2×10^{-3} **l** 3.81×10^{13}

2 **a** 1.31×10^9 **b** 2.96×10^{13} **c** 2.84×10^{15}

d 1.44×10^{16} **e** 6.99×10^{19} **f** 9.05×10^3

g 2.00×10^{14} **h** 2.14×10^9

3 3.78×10^{19} atoms **4** 2.25×10^{-2} mm

5 **a** 7.0×10^{30} **b** 1.3×10^{17} **c** 8.1×10^{13}

d 2.3×10^{36} **e** 5.5×10^{50} **f** 5.2×10^{22}

g 1.8×10^{-4} **h** 9.5×10^2 **i** 3.1×10^6

j 9.5×10^{13} **k** 1.2×10^{-3} **l** 1.7×10^{-5}

m 3.9×10^8 **n** 5.9×10^0

6 **a** 500 s **b** 8 mins 20 s

7 1.9×10^{14} t **8** 1.7×10^0 km/s **9** 9.5×10^{12} km

10 **a** 0.000 1 s **b** 91 s

11 **a** $9.999\,999\,999 \times 10^{99}$

b $1.000\,000\,000 \times 10^{-99}$

Power plus

1 **a** 4.382×10^{11} **b** 5.2×10^{-8} **c** 4×10^8

d 2.013×10^0 **e** 5.78×10^4 **f** 5.78×10^{-2}

g 6.7×10^{-6} **h** 3.2×10^9 **i** 3.2×10^{-9}

2 **a** 3 **b** 5 **c** 2 **d** 2

e 3 **f** 2 **g** 10 **h** 10

3 Different answers possible.

	a	m	n	
a	4	1	2	$4^{\frac{1}{2}} = 2$
b	9	1	2	$9^{\frac{1}{2}} = 3$
c	16	3	2	$16^{\frac{3}{2}} = 64$
d	625	3	4	$625^{\frac{3}{4}} = 125$

4 If $a = b$, there are an infinite number of solutions, otherwise $a = 2$, $b = 4$

5 **a** $2^n + 1$ where $n = 1, 2, 4, 8, \ldots$ (doubling the power each time), or $2^{2^n} + 1$ for $n = 1, 2, 3, \ldots$

b 4 294 967 297

Test yourself 5

1 **a** 10^{10} **b** 4^{16} **c** a^{10} **d** h^{10}
e $12n^4$ **f** $2d^{12}$ **g** $5m^8$ **h** $6v^7w^7$
i $15x^6y^3$ **j** $8t^4h^6$ **k** p^4q^8 **l** $20ab^2$

2 **a** 2^6 **b** k^{25} **c** x^4 **d** $1024y^{30}$
e $25t^4$ **f** $1000g^3$ **g** -32 **h** $-32k^5$
i $25m^6$

3 **a** a^4b^8 **b** $25x^6y^4$ **c** $64t^6$ **d** $-64h^6g^3$
e $\frac{a^4}{2401}$ **f** $32p^5q^5r^5$ **g** $\frac{243m^5}{32}$ **h** $81n^4p^8$
i $\frac{16a^{28}}{b^4}$ **j** $512t^{14}u^{16}$ **k** $\frac{b^{18}y^{15}}{512}$ **l** $5c^4d^4$

4 **a** 1 **b** 1 **c** 1 **d** 1
e -1 **f** $8h$ **g** 1 **h** 1
i $\frac{2}{3}$ **j** $\frac{3u^2}{4}$ **k** -12 **l** 5

5 **a** $\frac{1}{8^3}$ **b** $\frac{1}{19^2}$ **c** $\frac{1}{x}$ **d** $\frac{1}{p^5}$
e $\frac{1}{4m}$ **f** $\frac{1}{27g^6}$ **g** $\frac{b^4}{16}$ **h** $\frac{5}{b}$
i $\frac{-2}{x^4}$ **j** $\frac{5a^2c}{3}$ **k** $\frac{d^2}{c^4}$ **l** $\frac{9}{100}$

6 **a** 10^{-3} **b** r^{-5} **c** $(kg)^{-1}$ **d** $3b^{-1}$

7 $\frac{9x^2}{64}$

8 **a** $\sqrt[3]{q}$ **b** $\sqrt{u}$ **c** $\sqrt[3]{2q}$ **d** $\sqrt{ar}$

9 **a** 16 **b** -8 **c** $\frac{1}{216}$

10 **a** $625d^{20}$ **b** $2y^5$ **c** $4x^{\frac{16}{5}}$ **d** $-4n^4$
e $8k^6$ **f** $\frac{1}{32g^{\frac{5}{2}}}$ **g** $729a^3$ **h** $\frac{25}{16\,384y^{\frac{8}{3}}}$

11 **a** $\frac{4m^2}{9}$ **b** $\frac{6}{q^2}$ **c** $216w^3$ **d** $3k^8$
e $5y^3$ **f** $\frac{27k^3}{125d^3}$ **g** $\frac{4p}{3}$ **h** $\frac{m^2}{64}$
i $\frac{32a^3}{b}$ **j** $\frac{d^6}{8}$ **k** $\frac{3w^2}{2}$ **l** $2gh^3$
m $\frac{81x^8}{u^{20}}$ **n** $\frac{9c^6}{4n^4}$ **o** $\frac{2}{3h^5}$ **p** $16c^{15}e^{10}$

12 **a** 8.6 **b** 15 700 **c** 500
d 0.007 127 **e** 0.904 **f** 300 000
g 4810 **h** 0.0009 **i** 70 000 000

13 **a** 3.7×10^4 **b** 6.1×10^{-1} **c** 2.5×10^5
d 4.9×10^{-4} **e** 1.3×10^1 **f** 8×10^{-11}

14 **a** 810 000 **b** 60 000 000 **c** 5.182
d 0.000 203 **e** 0.000 000 03 **f** 0.094 75

15 9.1×10^{-8}, 2.4×10^3, 3×10^3

16 **a** 2.701×10^{31} **b** 4×10^5
c 1.25×10^{17} **d** 2.5×10^{-4}

17 **a** 9.8×10^{16} **b** 2.3×10^{13} **c** 9.6×10^8

Chapter 6

SkillCheck

1 **a** acute **b** right **c** reflex
d revolution **e** reflex **f** obtuse
g reflex **h** straight

2 D **3** B **4** D

5 **a** 49° **b** 127° **c** 36°
d 266° **e** 229° **f** 30°

Exercise 6.01

1 B

2 **a** $x = 123$ (adjacent angles)
b $d = 27$ (angles in a right angle)
c $x = 110$ (vertically opposite angles)
d $h = 41$ (angles on a straight line)
e $y = 117$ (adjacent angles)
f $w = 42$ (angles at a point)
g $z = 45$ (vertically opposite angles)
h $e = 43$ (angles on a straight line)
i $a = 25$ (angles in a right angle)
j $h = 300$ (angles at a point)
k $g = 105$ (angles on a straight line)
l $k = 90$ (vertically opposite angles)

3 B

4 **a** $r = 135$ (angles at a point)
b $w = 55$ (vertically opposite angles)
c $c = 30$ (vertically opposite angles), $y = 150$ (angles on a straight line)
d $k = 26$ (angles in a right angle)
e $t = 35$ (angles on a straight line)
f $a = 140$ (angles at a point)

5 Other reasons possible:
a $p = 57$ (corresponding angles on parallel lines)
b $t = 68$ (co-interior angles on parallel lines)
c $h = 88$ (alternate angles on parallel lines)
d $m = 106$ (corresponding angles on parallel lines)
e $a = 47$ (vertically opposite angles), $b = 47$ (corresponding angles on parallel lines), $c = 47$ (vertically opposite angles)
f $k = 135$ (angles on a straight line), $h = 135$ (corresponding angles on parallel lines)
g $m = 115$ (corresponding angles on parallel lines), $n = 115$ (vertically opposite angles)
h $b = 104$ (angles on a straight line), $c = 104$ (corresponding angles on parallel lines), $d = 104$ (vertically opposite angles)
i $k = 125$ (vertically opposite angles), $a = 125$ (corresponding angles on parallel lines), $b = 125$ (vertically opposite angles)
j $u = 115$ (co-interior angles on parallel lines), $t = 70$ (angles on a straight line), $w = 110$ (alternate angles on parallel lines)

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k $h = 76$ (alternate angles on parallel lines), $a = 14$ (angles in a right angle)

l $f = 115$ (angles on a straight line), $g = 65$ (alternate angles on parallel lines), $m = 25$ (co-interior angles on parallel lines)

6 **a** AB is not parallel to CD since alternate angles are not equal.

b EF is not parallel to GH since co-interior angles are not supplementary.

c IJ is parallel to KL since corresponding angles are equal.

7 **a** $c = 25$ **b** $m = 16$ **c** $k = 30$
d $d = 45$ **e** $f = 20$ **f** $y = 6$

Exercise 6.02

1 **a** $m = 70$ **b** $w = 33$ **c** $h = 50$
d $k = 100$ **e** $x = 46$ **f** $a = 65$
g $d = 70$ **h** $k = 70$ **i** $y = 34$
j $m = 60$ **k** $h = 150$ **l** $r = 45$

Reasons: **a-f**, **i** (angle sum of a triangle), **g**, **h**, **k**, **l** (isosceles triangle, angle sum of a triangle), **j** (equilateral triangle)

2 C **3** A, D

4 **a** 115 **b** 112 **c** 85
d 68 **e** 67.5 **f** 130

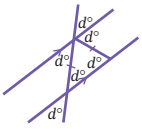
5 B **6** B

7 **a** 125° **b** 110° **c** 67°
d 50° **e** 60° **f** 56°

8 Teacher to check: 4 possible triangles, depending on whether 6 cm is one of the equal sides, and 40° is one of the equal angles. See worked solutions.

9 $a = 65$, $c = 50$, $b = 65$

10



11 **a** $t = 35$ **b** $m = 26$ **c** $h = 30$
d $y = 110$ **e** $g = 40$ **f** $x = 11$

Exercise 6.03

1 **a** $m = 74$ **b** $m = 96$ **c** $m = 30$
d $m = 30$ **e** $m = 221$ **f** $m = 75$

2

Properties	Trapezium	Kite	Parallelogram	Rhombus	Rectangle	Square
One pair of opposite sides parallel	✓					
Opposite sides parallel			✓	✓	✓	✓
Opposite sides equal			✓	✓	✓	✓
All sides equal				✓		✓
2 pairs of adjacent sides equal		✓				
Diagonals equal					✓	✓
Diagonals bisect each other			✓	✓	✓	✓
Diagonals meet at right angles		✓		✓		✓
Diagonals bisect the angles of the shape				✓		✓
Opposite angles equal			✓	✓	✓	✓
One pair of opposite angles equal		✓				
All angles 90°					✓	✓
Axes of symmetry	0	1	0	2	2	4

3 **a** trapezium, parallelogram
b trapezium
c square, rhombus
d rectangle, square
e parallelogram, rectangle, square, rhombus
f square
g rhombus, square
h kite
i parallelogram, rectangle, square, rhombus
j rectangle, square
k rectangle, rhombus
l parallelogram, rectangle, square, rhombus
m parallelogram, rectangle, square, rhombus
n rhombus, kite, square

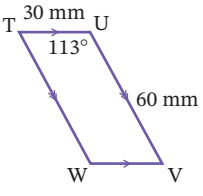
4 **a** rhombus, parallelogram, trapezium
b 9 rhombuses, 6 parallelograms, 18 trapeziums
c 13

5 A, C, D, E, G, I

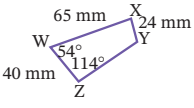
6 **a** parallelogram **b** rhombus
c trapezium **d** square
e rhombus **f** kite
g rectangle **h** square
i rhombus

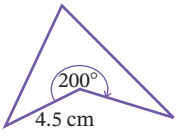
7 **a** $k = 70$ (opposite angles of parallelogram are equal)
b $a = 45$ (angles of square bisected by diagonals)
c $p = 55$ (co-interior angles, opposite sides of rhombus parallel)
d $w = 112$ (opposite angles of parallelogram are equal, then angles on a straight line)
e $t = 55$ (angles of a rectangle equal 90°)
f $n = 67$ (corresponding angles, opposite sides of a rhombus are parallel)
g $a = 55$ (co-interior angles)
h $c = 9$ (diagonals bisect angles of a square)
i $r = 110$ (angles between unequal sides of a kite are equal)

8 **a** $\angle PQR = 72°$ (alternate angles, $RW \parallel PV$)
b $\angle TQU = \angle TUQ = 53°$ (angles in an isosceles triangle)
$\angle QUP = 53°$ (alternate angles, $QT \parallel PU$)
$\angle UPQ = \angle PQU = 63.5°$ (angles in an isosceles triangle)
$\therefore \angle PQR = 63.5°$ (alternate angles, $RQ \parallel PU$)
c $(30° + \angle QRT) + 110° = 180°$ (co-interior angles, $QT \parallel PR$)
$\angle QRT = 40°$
$\therefore \angle PQR = 40°$ (alternate angles, $PQ \parallel RT$)
d $\angle PRQ = 35°$ (vertically opposite angles)
$\therefore \angle PQR = 72.5°$ (angles in isosceles $\triangle PQR$, diagonals of rectangle are equal and bisect each other)
e $\angle QRP = 75°$ (angle sum of straight line)
$\angle QPR = 75°$ (angles in isosceles triangle)
$\therefore \angle PQR = 30°$ (angle sum of triangle)
f $\angle TPQ = 60°$ (alternate angles, $RT \parallel PQ$)
$\therefore \angle PQR = 30°$ (angle sum of triangle, diagonals of rhombus meet at 90°)

9 sample answers
a


b

c


d


Exercise 6.04

1 **a** 6 **b** 4 **c** 9 **d** 10 **e** 7
f 5 **g** 12 **h** 8 **i** 11

2 **a** hexagon **b** quadrilateral
c pentagon **d** triangle
e regular decagon **f** heptagon
g regular octagon **h** nonagon

3 **a** **a**, **b**, **d**, **e**, **f**, **g** **b** **e**, **g** **c** **c**, **h**

4 D

5 **a** equilateral triangle **b** square

6 **a** 540° **b** 1080° **c** 2340° **d** 1800°
e 900° **f** 1440° **g** 3240° **h** 1620°

7 **a** $a = 95$ **b** $b = 110$
c $d = 108$ **d** $c = 160$
e $g = 140$ **f** $x = 90$
g $e = 135$ **h** $m = 120$, $n = 60$
i $y = 222$

8 **a** 14 **b** 11 **c** 24 **d** 17

9 B

10 **a** 135° **b** 144° **c** 150° **d** 120°

11 **a** 40 **b** 171°

12 **a** 24 **b** 36 **c** 10

Mental skills 6

2 **a** 0.5 **b** 90 **c** 8 **d** 0.9
e 90 **f** 0.7 **g** 8 **h** 30
i 0.08 **j** 3.5 **k** 4 **l** 0.3

4 **a** 16 **b** 160 **c** 1.6 **d** 1.6
e 1.6 **f** 16 **g** 160 **h** 0.16
i 0.016 **j** 160 **k** 0.16 **l** 0.0016

Exercise 6.05

1 **a** 45° **b** 36° **c** 24° **d** 40°

2 **a** 108° **b** 150° **c** 140° **d** 157.5°

3 **a** 15 **b** 10 **c** 9
d 36 **e** 20 **f** 6

4 **a** 12 **b** 72 **c** 20
d 9 **e** 5 **f** 90

Power plus

1 **a** 2 **b** 20 **c** 54

2 **a** square, rhombus
b parallelogram, rhombus, rectangle, square
c kite, rhombus, square
d rectangle, square
e rhombus, square
f square, rectangle

3 Some reasons are abbreviated here.
Other reasons are also possible.
a $h = 113$ (vertically opposite angles)
$m = 67$ (co-interior angles on parallel lines)
$p = 67$ (vertically opposite angles)
$k = 67$ (corresponding angles)
b $w = 73$ (exterior angle of triangle is equal to the sum of the two opposite interior angles)
c $e = 97$ (co-interior angles on parallel lines, then alternate angles on parallel lines)
d $y = 61$ (angle sum of a triangle, then corresponding angles on parallel lines)

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e $f = 56$ (alternate angles on parallel lines, then angle sum of isosceles triangle)

f $x = 78$ (alternate angles, then exterior angle of triangle result)

g $t = 34$ (alternate angles on parallel lines)
$n = 117$ (opposite angles of parallelogram)
$v = 29$ (angle sum of a triangle)

h $a = 90$ (diagonals of rhombus meet at right angles)
$d = 65$ (alternate angles on parallel lines, then angle sum of a triangle)

i $c = 153$ (angle sum of quadrilateral, then angles at a point)

j $x = 75$ (construct a line parallel to the other lines, then sum of 2 alternate angles)
$y = 285$ (angles at a point)

k $x = 75$ (alternate angles)
$y = 50$ (corresponding angles)
$z = 55$ (either angle sum of a triangle or angles on a straight line)

l $x = 68$ (angles on a straight line, isosceles triangle, then alternate angles)
$y = 248$ (alternate angles, angles at a point)

Test yourself 6

1 **a** $d = 115$ (vertically opposite angles), $k = 65$ (angles on a straight line)

b $y = 53$ (angles in a right angle)

c $w = 185$ (angles at a point)

d $x = 40$ (angles in a right angle)

e $y = 32$ (angles on a straight line)

f $x = 76$ (angles at a point)

2 Some answers are abbreviated to show the 2 reasons required to reach the final answer.

a $w = 123$ (alternate angles on parallel lines)

b $w = 65$ (co-interior angles on parallel lines)

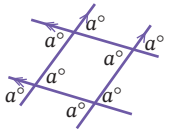
c $w = 117$ (corresponding angles on parallel lines)

d $w = 101$ (corresponding angles on parallel lines, vertically opposite angles)

e $w = 104$ (corresponding angles on parallel lines, angles on a straight line)

f $w = 70$ (co-interior angles on parallel lines, vertically opposite angles, other reasons possible)

3



4 **a** $k = 47$ (angle sum of triangle)

b $e = 75$ (exterior angle of a triangle)

c $x = 24$ (equal angles in an isosceles triangle)

d $m = 18$ (angle sum of triangle)

e $t = 107.5$ (equal angles in an isosceles triangle, exterior angle of a triangle)

f $b = 60$ (equilateral triangle , alternate angles on parallel lines)

g $y = 140$ (exterior angle of a triangle)

h $y = 48$ (angles on a straight line, isosceles triangle)

i $y = 78$ (co-interior angles on parallel lines)

5 **a** all 60° **b** 90°, 45°, 45°

6 **a** $x = 110$ (angle sum of quadrilateral)

b $y = 112$ (corresponding angles on parallel lines), $m = 68$ (co-interior angles on parallel lines)

c $g = 60$ (angles on a straight line, diagonals of a rectangle bisect each other, isosceles triangle)

d $x = 35$ (alternate angles on parallel lines)
$y = 55$ (diagonals of a rhombus bisect at right angles, angle sum of a triangle)

e $m = 97$ (one pair of opposite angles equal in a kite, angle sum of a quadrilateral)

f $e = 36$ (co-interior angles on parallel lines)

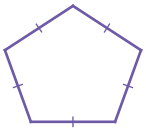
7 **a** parallelogram, rectangle, square, rhombus

b square, rectangle

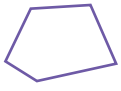
c square, rhombus

d square, rectangle, rhombus, parallelogram

8 **a**



b



c

9 **a** $(10 - 2) \times 180 = 1440°$

b 140°

10 **a** 60° **b** 12

Chapter 7

SkillCheck

1 **a** $x = 5$ **b** $y = 8\frac{1}{2}$ **c** $y = 1\frac{1}{2}$ **d** $m = 32$

e $x = 33$ **f** $r = 6$ **g** $x = -7\frac{1}{2}$ **h** $a = -3$

i $d = -6\frac{1}{2}$ **j** $w = 2$ **k** $a = 48$ **l** $n = 6$

2 **a** $7n$ **b** n^2 **c** $5(n + 8)$

d $n - 20$ **e** $6n - 9$ **f** $n + 2$

Exercise 7.01

1 **a** $w = 6$ **b** $q = 6$ **c** $x = 5$

d $n = -2$ **e** $y = 10$ **f** $m = 2$

g $a = 6$ **h** $x = -1$ **i** $y = 0$

j $u = -1$ **k** $x = -1\frac{3}{4}$ **l** $x = -5$

2 **a** B **b** C

3 **a** $w = -2$ **b** $t = -1\frac{1}{2}$ **c** $a = 1$

d $y = -3$ **e** $y = -1\frac{1}{2}$ **f** $y = \frac{3}{5}$

g $t = 1\frac{3}{8}$ **h** $y = 2\frac{1}{2}$ **i** $k = 1\frac{2}{3}$

4 C

Exercise 7.02

1 **a** $m = 1$ **b** $x = 2$ **c** $y = -1$ **d** $k = 4$
e $a = -1$ **f** $p = \frac{1}{9}$ **g** $h = -24$ **h** $m = 9$
i $u = 11$ **j** $y = 1\frac{3}{7}$ **k** $p = -1\frac{1}{5}$ **l** $x = 1\frac{13}{14}$
2 A
3 LHS = RHS = 48
4 LHS = RHS = 16
5 **a** $m = 3$ **b** $y = 7$ **c** $x = 2$
d $p = 4$ **e** $n = -4$ **f** $x = 3\frac{1}{3}$
g $w = -8$ **h** $x = 6$ **i** $y = \frac{5}{9}$
6 LHS = RHS = 30
7 **a** $m = -7$ **b** $y = 7$ **c** $y = -10$ **d** $x = 12$
e $y = -26$ **f** $y = -1\frac{2}{3}$ **g** $m = -7$ **h** $y = -\frac{71}{89}$

Exercise 7.03

1 11 **2** 12 **3** 24
4 30 **5** 40 cm × 10 cm
6 **a** $4w + 14 = 94$ **b** 27 cm × 20 cm
7 $x = 33$
8 **a** $y = 13$ **b** 110°
9 82°, 98° **10** 43, 44 **11** 28, 29, 30
12 54, 56, 58 **13** 23, 25, 27
14 Dean is 10, Franco is 50, Helen is 2.
15 Rebekah is 14.
16 78 **17** $4.30
18 Ashvin: 140 cm, Sureti: 70 cm, father: 170 cm.
19 26.5 cm × 17 cm
20 Brett is 17, Vila is 51, Amanda is 12, Janine is 45.
21 $d = 5.7$ m **22** 240

Mental skills 7A

2 **a** 14 **b** 9 **c** 7 **d** 8 **e** 3
f 8 **g** 10 **h** 5 **i** 33 **j** 5
k 15 **l** 18 **m** 26 **n** 6 **o** 36
p 6 **q** 14 **r** 160 **s** 14 **t** 2

Exercise 7.04

1 **a** $a = 39$ **b** $x = 56$ **c** $x = 30$ **d** $y = 42$
e $p = 56$ **f** $y = 21$ **g** $x = -2$ **h** $y = -15$
i $x = -9$ **j** $d = 85$ **k** $w = -4$ **l** $p = -7\frac{1}{2}$
2 **a** $x = -9$ **b** $y = 22$ **c** $p = 11\frac{1}{2}$ **d** $w = -22$
e $x = 5\frac{3}{4}$ **f** $m = 24$ **g** $y = 5\frac{1}{5}$ **h** $x = 5$
i $x = 0$ **j** $w = 1\frac{5}{22}$ **k** $x = 13$ **l** $x = 3\frac{1}{7}$
3 A
4 **a** $d = 24$ **b** $k = 18$ **c** $m = 12$
d $x = 21\frac{9}{11}$ **e** $p = 30$ **f** $k = 20$
g $k = 30$ **h** $m = 10$ **i** $a = 6\frac{9}{11}$
5 B
6 **a** $x = \frac{7}{11}$ **b** $p = 4\frac{1}{3}$ **c** $m = 12\frac{4}{5}$
d $c = -9\frac{2}{3}$ **e** $y = 5\frac{3}{4}$ **f** $x = -8$
g $p = -2\frac{1}{9}$ **h** $y = 7\frac{9}{14}$ **i** $a = 28\frac{1}{3}$
j $x = -\frac{13}{15}$ **k** $u = 9\frac{1}{5}$ **l** $m = \frac{19}{29}$

Exercise 7.05

1 **a** $m = \pm 12$ **b** $x = \pm 20$ **c** $y = \pm 15$
d $k = \pm\sqrt{59}$ **e** $y = \pm\sqrt{10}$ **f** $w = \pm 6$
g $x = \pm 5$ **h** $t = \pm 3$ **i** $a = \pm 4$
j $k = \pm\sqrt{8}$ **k** $w = \pm\sqrt{10}$ **l** $d = \pm 12$
m $k = \pm 4$ **n** $w = \pm\sqrt{70}$ **o** $x = \pm\frac{1}{2}$
p $m = \pm 9$ **q** $y = \pm\sqrt{5}$ **r** $p = \pm 3$
s $k = \pm 4$ **t** $y = \pm\sqrt{55}$ **u** $x = \pm\sqrt{7}$
2 **a** $m = \pm 4.5$ **b** $b = \pm 4.1$ **c** $v = \pm 2.4$
d $p = \pm 4.2$ **e** $k = \pm 2.6$ **f** $x = \pm 6.3$
g $k = \pm 9.8$ **h** $u = \pm 1.7$ **i** $y = \pm 3.7$
j $w = \pm 5.2$ **k** $a = \pm 4.1$ **l** $y = \pm 6.2$
3 $\sqrt{-25}$ doesn't exist for real numbers; there isn't a real number whose square is −25.
4 **a**, **c** and **f**. They all involve square roots of negative numbers.

Exercise 7.06

1 **a** $r = 1$ **b** $k = 6$ **c** $d = -3$
d $x = \sqrt[3]{-45}$ **e** $w = \sqrt[3]{100}$ **f** $f = 4$
g $m = 2$ **h** $t = -5$ **i** $c = 6$
j $k = -4$ **k** $a = \sqrt[3]{15}$ **l** $p = \sqrt[3]{-18}$
m $q = 5.5$ **n** $n = -12$ **o** $s = \sqrt[3]{18.75}$
p $e = -2$ **q** $y = \sqrt[3]{-6\frac{2}{3}}$ **r** $v = 8$
s $b = \sqrt[3]{15}$ **t** $z = \sqrt[3]{-72}$ **u** $x = \sqrt[3]{36}$
2 **a** $c \approx 3.6$ **b** $g \approx 5.3$ **c** $y \approx -7.7$ **d** $p \approx -4.5$
e $u \approx 4.3$ **f** $x \approx -3.8$ **g** $h \approx -7.9$ **h** $a \approx 3.7$
i $d \approx -3.1$ **j** $v = -3.5$ **k** $a \approx 6.4$ **l** $j \approx -3.9$
3 **a** Yes **b** When c is positive.
c Never; it always has exactly one solution.

Exercise 7.07

1 **a** $y = 28$ **b** $y = 11$ **c** $b = 25$
d $b = -1$ **e** $x = 3$ **f** $x = 16$
2 **a** 95°F **b** 14°F **c** 60.8°F
3 **a** 37.8°C **b** −42.8°C **c** 25.6°C
4 **a** $A = 72$ cm² **b** $h = 20$ cm **c** $p = 32.5$ m
5 C
6 **a** $3100 **b** $9700
c 10 hours **d** 18 hours
7 **a** $7300 **b** 536
8 **a** $2255 **b** $6030 **c** 112 people
9 D
10 **a** 65°C **b** 91.25°C **c** 4 hours
11 **a** 182 cm **b** 39 cm

Mental skills 7B

2 **a** 11 **b** 40 **c** 7 **d** 24
e 23 **f** 6 **g** 43 **h** 80

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i 18 **j** 15 **k** 40 **l** 65
m 11 **n** 14 **o** 12 **p** 135

Exercise 7.08

1 **a** $x = d - c$ **b** $x = \frac{y-c}{m}$
c $x = \frac{p+y}{a}$ **d** $x = my - mp$ or $m(y - p)$
e $x = \frac{k}{mr}$ **f** $x = \frac{m}{V}$
g $x = 2A - y$ **h** $x = \frac{c-by}{a}$
i $x = \frac{2A}{h} - y$

2 D

3 **a** $y = \frac{m-an}{a}$ or $\frac{m}{a} - n$ **b** $y = \pm\sqrt{x}$
c $y = \frac{x}{k}$ **d** $y = (Q - P)^2$
e $y = \pm\sqrt{r^2 - x^2}$ **f** $y = \pm\sqrt{\frac{M}{n}}$
g $y = \frac{r^2}{m^2}$ **h** $= y = \frac{c^2-b}{2a}$
i $y = t^2 + x$

4 **a** $h = \frac{V}{lb}$ **b** $r = \pm\sqrt{\frac{A}{\pi}}$ **c** $V = \pm\sqrt{\frac{2K}{m}}$
d $C = \frac{5}{9}(F - 32)$ **e** $a = \frac{2(s-ut)}{t^2}$ **f** $y = \frac{2A}{h} - x$

5 C

6 **a** \$290 **b** $d = \frac{C-80}{4.2}$ **c** 76 km

7 **a** 900° **b** $n = \frac{A}{180} + 2$ or $\frac{A+360}{180}$
c 10

8 **a** $h = \sqrt{\frac{m}{B}}$ (h is positive) **b** 1.55

9 $r = \frac{sx}{s-x}$

10 **a** $a = \frac{5q}{1-5p}$ **b** $a = \frac{3b}{r-3}$ **c** $a = \frac{D}{D+2}$
d $a = \frac{-py}{p-x}$ or $\frac{py}{x-p}$ **e** $a = \frac{1-k}{k+1}$
f $a = \frac{-b(M+N)}{M-N}$ or $\frac{b(M+N)}{N-M}$

Power plus

1 **a** $y = \frac{5}{28}$ **b** $r = 3\frac{1}{2}$
c $y = \pm\sqrt{48}$ or $\pm 4\sqrt{3}$
d $m = \pm 5$ **e** $m = 12\frac{5}{17}$

2 **a** $W = \frac{1}{5}$ **b** $W = -3$ **c** $X = -6\frac{1}{2}$ **d** $Y = -\frac{2}{3}$

3 26 **4** Wendi is 20

5 **a** If $x^2 + y^2 = 4$, then x^2, which is always positive or zero, must be from 0 and 4. So the smallest value of x must be $-\sqrt{4} = -2$ and its largest value must be $\sqrt{4} = 2$.
b If $x^2 + y^2 = 4$, then y^2 must also be from 0 and 4. So y must be from -2 and 2.

6 $B = 30$

7 **a** $x = 0, 1$ **b** $x = -2, 3$ **c** $x = -1, 0, 1$
d $x = -1, 0$ **e** $x = 0, 3, 3$

Test yourself 7

1 **a** $x = 3$ **b** $n = -4$ **c** $d = -9$
d $t = 1\frac{1}{2}$ **e** $j = -8\frac{1}{2}$ **f** $q = -1$

2 Teacher to check.

3 **a** $w = 7$ **b** $n = 1$ **c** $p = -1$
d $x = \frac{1}{3}$ **e** $y = 5$ **f** $x = 1\frac{1}{2}$
g $a = 1$ **h** $g = 14\frac{2}{3}$

4 22 cm × 16 cm

5 **a** $x = 15\frac{1}{2}$ **b** $a = 46$

6 1

7 **a** $d = -6$ **b** $n = -1\frac{1}{2}$ **c** $p = 24$

8 **a** $d = \pm 8$ **b** $p = \pm 6$ **c** $z = \pm\sqrt{35}$

9 **a** $x = 11\frac{5}{7}$ **b** $n = -10\frac{1}{4}$ **c** $y = 5\frac{3}{4}$

10 **a** $x = 11$ **b** $h = -6$ **c** $t = 5$

11 **a** BMI = 21 **b** 105.84 kg

12 **a** \$41 **b** $d = 30.5$ km

13 **a** $w = \frac{a-v}{k}$ **b** $w = t - \frac{p}{m}$ **c** $w = \frac{1}{x-y}$

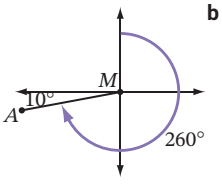
Practice set 2

1 **a** 4 **b** 1 **c** 4 **d** 3

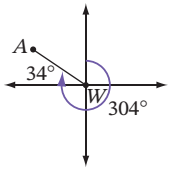
2 **a** y^{10} **b** $12g^{15}$ **c** $18p^5q^3$ **d** k^{12}
e x^4 **f** h^9 **g** $9k^4$ **h** h^2y^3
i $\frac{2m^4n}{3}$

3 **a** 6 **b** 1 **c** 1 **d** x **e** 1

4 **a**



b



5 **a** $\frac{1}{2}$ **b** $\frac{1}{125}$ **c** $\frac{1}{100}$

6 **a** 120° **b** decagon

7 **a** 2 **b** $\frac{9}{64}$ **c** $\frac{25}{16d^2}$ **d** $\frac{1000x^3}{27}$

8 $y = \pm\sqrt{\frac{T-a}{p}}$

9 **a** 20.50 **b** 33.34 **c** 3.31

10 **a** 2^{15} **b** 4^{12} **c** 3^2p^{14}
d 2^3w^{15}

11 **a** 15 400 000 **b** 0.0032 **c** 0.9837
d 24 000

12 212 km

13 **a** 77° 53′ **b** 64° 9′ **c** 56° 19′

14 **a** n^{-1} **b** 2^{-3} **c** k^{-5} **d** $7y^{-2}$

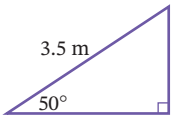
15 7.28×10^6

16 **a** 2.7×10^{16} **b** 1.2×10^{-10}

17 **a** 39° **b** 52° **c** 44°

ANSWERS

18 **a** $m = 103$ (angle sum of triangle)
b $f = 54$ (equal angles of isosceles triangle, angle sum of triangle)
c $d = 132$ (exterior angle of triangle)
19 A
20 **a** 5.3×10^9 **b** 3.73×10^5
21 **a** $m = 17.9$ **b** $d = 11.7$ **c** $g = 160.2$
22 **a** $w = 103$ **b** $w = 32$ **c** $w = 35$
23 **a** $13^{\frac{1}{2}}$ **b** $20^{\frac{1}{3}}$ **c** $(3p)^{\frac{1}{5}}$ **d** $x^{\frac{3}{2}}$
24 C
25 **a** 125 **b** $\frac{1}{32}$ **c** 8 **d** 9
26 **a** 24 **b** 8
27 **a** $16m^{12}n^8$ **b** $\frac{4h^2}{25}$ **c** $343r^3t^6$
28 **a** $k = -5$ **b** $h = 8\frac{4}{7}$
29 **a** 44 000 **b** 10.8 **c** 0.000 04
30 **a** 40° **b** 37° **c** 39°
31 **a** 225 min or 3h 45 min **b** 3.125 kg
32 **a**



b **i** 2.7 m **ii** 2.2 m
33 square
34 **a** $p = 11$ **b** $y = 1$
35 **a** 19, 20, 21, 22 **b** 127 $2 coins
36 50 m
37 **a** $m = \pm 4$ **b** $x = \pm 6$
38 **a** $r = 8$ **b** $y = -9$

Chapter 8

SkillCheck

1 0.082 **2** $2.25
3 **a** 365 **b** 52 **c** 28 **d** 36
e 14 **f** 13 **g** 60 **h** 26
4 **a** $7.68 **b** $21.42 **c** $524.74
d $92.36 **e** $3085.64 **f** $106.34
5 **a** $733.59 **b** $6285.83 **c** $3150
d $1198.21 **e** $694.67 **f** $14 911.22
g $1339.02 **h** $2343.50 **i** $2293.20
6 $5590.25

Exercise 8.01

1 **a** $1049.60 **b** $1332.10
c $895.20 **d** $321.30
2 $103 316.40 **3** D **4** $79 000.22
5 **a** $3034.50 **b** $6068.99 **c** $13 195
6 **a** $117.80 **b** $139.50 **c** $130.20
7 Danica, by $31.26.
8 Fortnightly income pays more
9 **a** Marisol by $1.23/h
b Paula by $3.26/h
c Fabian by by $2.50/h
d Ranjan by $2.46/h
10 **a** $544 **b** $2720 **c** $11 786.67
11 **a** $127 680 **b** $2446.91
12 **a** $7393.33 **b** $1700.27 **c** $40.48
13 $1252.80 **14** $1054.13
15 $1026 **16** $1114.12
17 $28.95/h **18** $3495.59

Exercise 8.02

1 **a** $802.20 **b** $959.78 **c** $1217.63
d $1239.11 **e** $1547.10 **f** $1590.08
2 C **3** $1141.98 **4** $2033.94
5 $1118.15 **6** $1222.61 **7** B
8 **a** $782.10 **b** $865.05 **c** $1007.25
d $1220.55 **e** $953.93 **f** $1102.05
9 **a** $1136.80 **b** $1365 **c** $1401.40
d $1300 **e** $1976 **f** $1445.40
10 **a** $28.69/h **b** $29.69/h **c** $24.29/h
d $21.98/h **e** $25.92/h **f** $13.68/h

Exercise 8.03

1 $736.80 **2** $1783.49 **3** $27 506
4 Perry: $2600, Amy: $4050
5 C
6 **a** $25 000 **b** $31 250 **c** $38 750
7 $4520
8 **a** $337.50 **b** $230 **c** $555.75 **d** $316
e $405 **f** $480 **g** $45 **h** $738
9 **a** **i** $124.10 **ii** $172.55
b 540
c **i** 236 **ii** 730
10 $120
11 **a** $4389 **b** $7866 **c** $4433.65
d $2455.18 **e** $4987.50 **f** $5132.66
12 **a** $1096 **b** $767.20 **c** $5151.20
13 **a** $9658.87 **b** $3380.60 **c** $22 698.34
14 **a** $397.90 **b** $584.90 **c** $2069.90
15 **a** $1142.50 **b** $190.42 **c** $22.40/h
16 $38 800
17 **a** $1032 **b** $4850.40

Exercise 8.04

1 $90 634
2 **a** $74 673 **b** $15 815.73
3 **a** $127 525 **b** $34 681.25
4 C
5 **a** $65 262 **b** $12 757.15
6 **a** $100 270 **b** $24 596.90
7 **a** $86 285.58 **b** $86 096 **c** $19 528.20
8 D **9** $103 880.50 **10** $14 674.33

Mental skills 8

2 **a** $408 **b** 99 **c** 200
d $404 **e** 672 **f** $81

g $517 **h** 225 **i** 560
j $84 **k** $350 **l** 84

4 **a** 330 **b** $240 **c** 1600 **d** $425
e $225 **f** $60 **g** $63 **h** 76
i $68 **j** $3762 **k** $374 **l** $100

Exercise 8.05

1 **a** $3200 **b** $704
2 **a** $8064 **b** $1764
3 $240
4 **a** $43 **b** 22.1% **c** $152
5 Phoebe, by $24.
6 Electronics Store offers higher net pay by $24 over the 6 weeks.
7 **a** $158 **b** $435.90 **c** 37.6%
8 C
9 **a** $812.03 **b** 27.9%
10 **a** $670.98 **b** 16.4%
11 **a** gross weekly income = $1094.40,
total deductions = $339.60,
net weekly income = $754.80
b gross weekly income = $1527.50,
total deductions = $504.10,
net weekly income = $1023.40
12 **a** $251.70 **b** 24.5%
13 **a** $2943.66 **b** $956.69
c $1208.31 **d** $1735.35
14 **a** $519.12 **b** $4092.60
c $87 856.28 **d** 28.3%

Power plus

1 $260.08/min **2** $55.56 **3** $288/h
4 Felicity: $514; Rose: $454.80;
Difference: $59.20
5 $1368 **6** $21.50/h **7** $855
8 $841.50 **9** $127 200 **10** 18 h 40 min

Test yourself 8

1 **a** $1675.16 **b** $3350.33 **c** $7284.17
2 $875.05 **3** $1700.79
4 **a** $940.50 **b** $1200.38
5 $4200 **6** $1950
7 **a** $750 **b** $1237.50 **c** $1242.50
8 $11 400 **9** $8394.20
10 **a** $185 454 **b** $56 551.30
11 **a** $182 **b** $1208.96 **c** 12.2%
12 **a** $1071.60 **b** $359.60 **c** $712

Chapter 9

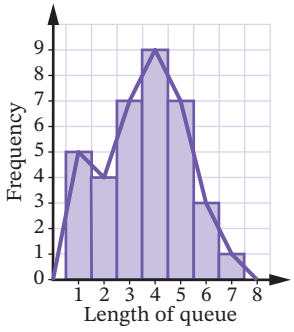
SkillCheck

1 **a** 39 **b** 6 **c** 43.6%
2 **a** 38 **b** 6 **c** 36 **d** 14 **e** 39.5%

3 **a**

Length of queue	Frequency
1	5
2	4
3	7
4	9
5	7
6	3
7	1

b



c 4 **d** 20

Exercise 9.01

1 **a** **i** 18.2 **ii** 18 **iii** 15 **iv** 16
b **i** 6.9 **ii** 7 **iii** 7 **iv** 14
c **i** 34 **ii** 35 **iii** 32, 45 **iv** 33
d **i** 8.7 **ii** 8.8 **iii** 8.8 **iv** 1
e **i** 0.3 **ii** −0.5 **iii** −2 **iv** 11
2 **a** max $\bar{x}$ = 22.9°C, min $\bar{x}$ = 15.6°C
b 7.3°
c max mode = 23°C, min mode = 15°C
d 8°
3 **a** 415 mm **b** 168 mm **c** 92 mm
4 **a** 6.3 **b** 7 **c** 7 **d** 8
5 **a** 1 is an outlier as it is away from the main body of data.
b 8, 9 **c** 7.3 **d** 8
e **i** 56.7% **ii** 33.3%
6 **a** Yes, 7
b **i** $\bar{x}$ = 2, median = 2
ii $\bar{x}$ = 2.3, median = 2
c The mean.
7 **a** $180 000
b **i** $\bar{x}$ = $86 944, median = $90 000
ii $\bar{x}$ = $96 250, median = $91 500
c The median, as it is only $1500 higher with the outlier included, while the mean is $9306 higher.
d **i** mode = $93 000, range = $35 000
ii mode = $93 000, range = $110 000
e Range
8 **a** 6.4 **b** 14.8 **c** 66.8
9 76
10 **a** 370 **b** 462 **c** 92
11 No, as she needs to score 103%.

ANSWERS

12 **a** 10, 9, 28, 30, 24
$\Sigma f = 25$, $\Sigma fx = 101$, $\bar{x} = 4.04$

b 66, 184, 168, 225, 130, 108
$\Sigma f = 36$, $\Sigma fx = 881$, $\bar{x} = 24.47$

13 **a** 34.4 **b** 12.9

14 **a** 2, 7, 18, 25, 35, 44, 50
i 5.5 **ii** 4 **iii** 6

b 3, 11, 31, 46, 57, 64
i 20 **ii** 19 **iii** 5

15 **a**

Number of seedlings per punnet	Frequency
4	4
5	2
6	4
7	8
8	11
9	8
10	4
11	3
12	1

Median = 8

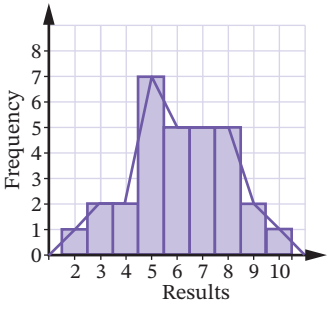
b 8 **c** 8 **d** 7.8

Exercise 9.02

1 **a**

Results (Score)	Frequency
2	1
3	2
4	2
5	7
6	5
7	5
8	5
9	2
10	1

b

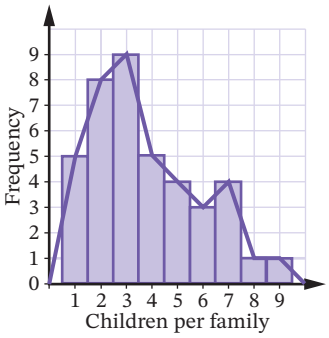


c 6.1 **d** 6 **e** 5 **f** 8

g No outliers but clustering at 5–8.

2 **a**

Children per family	Frequency
1	5
2	8
3	9
4	5
5	4
6	3
7	4
8	1
9	1



b mode = 3, range = 8

c no outliers, clustered near 2–3 children

d 3 **e** 3.8

3 **a** 6 It's the highest column.

b 5.96 **c** 6 **d** 5

4 **a** 73 **b** 14

c 2 It's the score with the greatest frequency.

d 5 **e** 2 **f** 2.43

g clustering at 2–3 calls per minute

5 **a**

Stem	Leaf
4	0 1 5
5	1 3 5 6 8
6	3 4 5 6 7 7 7 8
7	1 2 4 7 9
8	2 7 8
9	0 2 4

b 54 **c** Clustering occurs in the 60s.

d 27 **e** 67.85 **f** 67

6 **a** 52 **b** 68 **c** 60.0 **d** 63

7 **a**

Stem	Leaf
11	8 9
12	1 5 6
13	1 5 5 7
14	3 4 5 6 9 9
15	0 0 0 1 4 4 7 7
16	1 1 3 3
17	6
18	3 7

b **i** no outliers

ii clustering occurs near 14–15 seconds

c 8 **d** 70%

e **i** 43.3% **ii** 50%

8 **a** 54, 55 or 56 **b** 22 **c** 59.5

9 **a** Rockets: 8, Blues: 13

b Rockets: 52, Blues: 57

c The Blues, 40.

d The Blues performed better as they had more games in which they scored more than 30 points.

e The Blues' median was higher – 37.5 compared to Rockets' median of 27.5.

10 **a**

Robyn	Stem	Anthony
7	1	
7 3	2	8 9
7 7 7 6 5 3	3	4 5 5 6 6 7 9
5 2 2 1	4	2 3 7
4	5	1 5

9780170453325

b Anthony (547) made more than Robyn (506).

c 7 **d** Robyn, 54

e Clustering for both is in the 30's for both.

f Anthony

11 a

Stem	Leaf
1	1 5 9
2	3 5 7
3	1 2 3 4 5 6 8
4	0 2 5 5 7 8
5	2 5
6	5
12	5

b Yes, 12.5 – this value is away from the main body of data.

c Clustering occurs in the 3–4 km distances.

d 4.01 **e** 3.6

f The median is the better as it is not affected by the outlier of 12.5 km.

12 a

Girls	Stem	Boys
8	1	0 6
	2	6 8 9
9 6	3	2
8 5	4	0
9 9 6 1	5	0 0 5 7
7 0	6	3 7
8 6 6 4 3	7	2 4 5 6
5 2 0	8	2 8
4	9	5

b 95, boy

c **i** 63.5 **ii** 56

d 45% **e** 50%

f **i** 62.8 **ii** 54.25

g **i** no **ii** no

Exercise 9.03

1 a **i** symmetrical and bimodal

ii no outliers, some clustering near 5, 11

b **i** symmetrical and bimodal

ii 10 is an outlier, clustering near 4, 6

c **i** negatively skewed

ii no outliers, clustering in the 70s and 80s

d **i** positively skewed

ii no outlier, clustering near 21, 22

e **i** not symmetrical, bimodal

ii no outliers, some clustering 12–13, 16

f **i** positively skewed

ii no outliers, clustering near 1, 2

g **i** symmetrical

ii no outliers, some clustering near 51

h **i** negatively skewed

ii 10 is an outlier, clustering at 17–19

2 a 30 **b** in the 30s and 40s

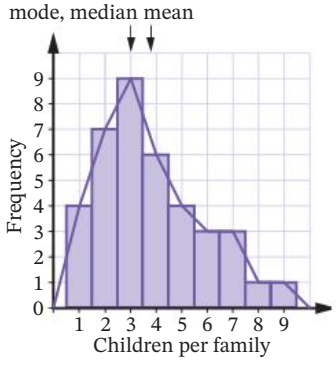
c positively skewed

d Test was too hard or students did not prepare for the test.

e $\bar{x} = 48.6$, median = 45, mode is 28, 38, 45, 47, 64

3 a

Children per family	Frequency
1	4
2	7
3	9
4	6
5	4
6	3
7	3
8	1
9	1



b No **c** positively skewed

d 2 to 4

e mode = 3, median = 3, $\bar{x} = 3.8$

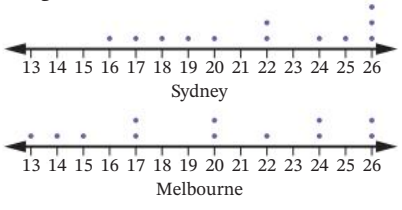
4 C

5 a symmetrical **b** 56, 64, 83

c 76 **d** 72.8 **e** 72

f No, because the range has been affected by the outlier of 26. Without the outlier, the range is 45.

6 a **i**



ii Both are slightly negatively-skewed.

b Sydney: $\bar{x} = 21.75$, median = 22, mode = 26, range = 10

Melbourne: $\bar{x} = 19.83$, median = 20, mode(s) = 17, 20, 24, 26, range = 13

c Sydney is generally warmer – higher mean, median and lower range.

Exercise 9.04

1 a Brooke: 59.58 s Ryan: 57.54 s

b Brooke: 59.60 s Ryan: 57.33 s

c Brooke: 1.71 s Ryan: 4.55 s

d Ryan, his mean time is lower.

e Brooke, as her range is 1.71 s compared to Ryan's range of 4.55 s.

2 a Sydney: 13.1°C Adelaide: 13.3°C

b Sydney: 27.9°C Adelaide: 26.6°C

c Sydney: 29.2°C Adelaide: 26.2°C

d 8 days

e Sydney. Its mean and median were higher.

3 **a** Test 1: 53 Test 2: 64

b Test 1: 58.3 Test 2: 68.0

c Test 1: 59 Test 2: 54

d Test 1 is positively skewed. Test 2 is slightly positively skewed.

e Yes, the mean and median have increased significantly.

4 **a** Logan City

b Logan City: 27.25°C Wollongong: 26.13°C

c Logan City: 11°C Wollongong: 12°C

d Logan City, slightly lower range and temperatures clustered from 26°C to 28°C.

e Logan City: 27°C Wollongong: 26°C

f Wollongong, its mean and median are slightly lower and it had a temperature of 26°C or more on only 8 days compared to Logan City's 14 days.

5 **a**

Men	Stem	Women
9 9 9 8 6	2	
8 7 3 2 2 1 1 0	3	4 5 5 6 6 7 7 9 9
9 8 7 4 3	4	0 0 2 3 5 5 8
5 3	5	1 1 4 7

b No, the range is too high.

c **i** Different ($\bar{x} = 37.2$, median = 32.5).

ii Different ($\bar{x} = 42.2$, median = 40), but the difference is less.

d Men: 29 Women: 35, 36, 37, 39, 40, 45, 51

e Both are positively skewed. Clustering occurs in 30s for both.

f Men, as their mean and median are less than that for women.

g Men. Their range is 29 compared to a range of 23 for women.

6 Teacher to check.

Mental skills 9

2 **a** 8 **b** 0.3 **c** $12 **d** 0.8

e $48 **f** 1.2 **g** 5.6 **h** $18

i $3 **j** $7.20 **k** 77 **l** 26

4 **a** 12.5 **b** $4.50 **c** $10.80 **d** 9.9

e $15 **f** 4.8 **g** $30 **h** $90

i $14 **j** $7.20 **k** 5.7 **l** $10.50

Exercise 9.05

1 **a** census **b** sample **c** sample

d census **e** census

2 **a** every 5 years

b 2021, 2026, 2031, ... – every year ending in a 1 or 6

3 **a** 872

b **i** 17.3% **ii** 15.1%

c **i** 17 **ii** 15

4 No, as these students may live close to school or they may be early because they come by private transport.

5 No, as they only come from one shopping centre.

6 **a** N **b** N **c** N **d** C **e** N

f N **g** N **h** C **i** C **j** N

7 C

8 **a** D **b** C **c** C **d** C

e D **f** D **g** C **h** D

Exercise 9.06

1 Teacher to check.

2 **a** People at a football match would not be thinking and/or interested in the amount of time spent on homework.

b Many sports are/have body contact to some degree. Spectators may be less likely to complain about violence.

c Most of the cars at the car show will use petrol.

d People at a rock concert will usually favour rock music.

3 Advantages – high accessibility as most people have a telephone
– good quality control as questions are asked the same way
– anonymity of people being surveyed
– data can be processed quickly

Disadvantages – time taken for survey is limited so questions are restricted to short answer type ones
– people who screen calls may not answer when called
– if the survey is about a product, participants cannot see, taste or feel the product

4 **a** Depends on the number of students in the school. For a large school, more students need to be surveyed.

b Can be given out at random to students in a school, may only take a short time to compete.

c May take too much time to complete, some of the questions may require a lengthy response.

d Personal interview – students are asked at random at recess or lunch.

5 C as all students in the school are asked. Answer A means only year 9 students are asked and Answer B would only provide an estimate.

6 **a** The question is not precise enough. A better question might be:

'Circle the amount of time you spend studying per week:

0 – less than 2 hours 2 – less than 4 hours

4 hours or more '

b The question is too general. A better question could be:

'Select your favourite sport from the list below' and include 'Other' in the list.

c The question is too general. A better question could be:

'What is your income?

A $15 000 – <$25 000

B $25 000 – < $40 000

C $40 000 – < $60 000 .....'

d, e The question is too general. A better question could be:

'Select your favourite food/activity from the list below' and include 'Other' in the list.

f, g The question is not precise enough. A better question might be:

'Circle the amount of time you spend exercising/on social media per day or week:

0 – less than 1 hours
1 hour – less than 2 hours
2 hours or less than 4 hours
more than 4 hours '

h The question is too general. A better one might be:

'How many text messages do you send in one day?

❑ 0 – 10 ❑ 11 – 20

❑ 21 – 40 ❑ more than 40

7 Teacher to check.

8 a Students from each year group are chosen at random.

b Choose students at random as they leave the canteen.

c, d Choose students from each year group.

e Randomly select some classes from your school or from different schools.

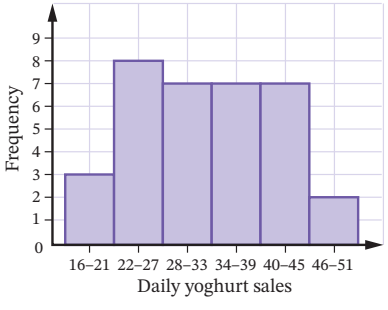
f Telephone survey or personal interview in a shopping centre.

Power plus

1 a

Class interval	Frequency	Cumulative frequency
16–21	3	3
22–27	8	11
28–33	7	18
34–39	7	25
40–45	7	32
46–51	2	34

b



Shape is nearly symmetrical. No outliers or clusters.

c 22–27 **d** 28–33

2 If the range is too large.

3 a i

Stem	Seedlings
0	3 4 4 5 7 7 8
1	1 1 2 3 4 4 6 6 7 7 7 8 8 8 8 9 9 9
2	0 0 3 3 4 4 5 5 6 7 8 8
3	0 1 3

ii

Stem	Seedlings
0^0	3 4 4
0^5	5 7 7 8
1^0	1 1 2 3 4 4
1^5	6 6 7 7 7 8 8 8 8 9 9 9
2^0	0 0 3 3 4 4
2^5	5 5 6 7 8 8
3^0	0 1 3

b i Positively skewed, no outliers, clustered in the 10s stem.

ii Symmetrical, no outliers, clustered in 15–19 stem.

c Plot **ii** gives the better indication. It gives a better breakdown of how many seedlings grow per box (modal group is 15–19).

Test yourself 9

1 $\bar{x} = 7$, median = 8, mode = 8, range = 6

2 a $525 100 **b** $393 500

c The median is the better measure as it is not affected by the high price of $1 750 000.

If $1 750 000 is excluded, the mean is $389 000.

d Because the mean is affected by either very high or very low prices (outliers).

3 a

Stem	Leaf
2	2
3	8
4	2 3 5 7
5	4 6 8 9
6	2 3 4 4 7
7	0 0 1 2 3 5 7 8 8
8	3 4 5 6 9
9	0

b $\bar{x} = 65.5$, median = 68.5, modes = 64, 70 and 78, range = 68

c Yes, 22

d The median, as the mean is affected by the outlier 22.

4 a 6 **b** 1 **c** 1.93 **d** 2

e The mean as there are no outliers.

5 a

Northbridge	Stem	Westvale
9 9 7	0	5 7 7 8 9
8 6 6 5 1	1	3 5 5 8 8
8 7 7 3 2	2	2 4 4 4 4 5 6
8 7 7 5 2 0 0	3	3 5 6 7 7
7 5 2	4	3 4
3	5	

b Northbridge: 27.25 min, 27.5 min, 46 min

Westvale: 22.875 min, 24 min, 39 min

c No outliers, some clustering for Northbridge in the 30s and for Westvale in the 20s.

d The waiting times for Westvale are less than for Northbridge and more consistent.

ANSWERS

6 **a** positively skewed, bimodal, no outliers, clustering at 12, 14.

b symmetrical, no outliers, clustering near 100 – 109.

c negatively skewed, no outliers, clustering near 15 – 17.

7 **a**

Girls	Stem	Boys
8 1	13	4
8 7 5 2	14	8
6 5 3 0 0	15	0 4
3 2 0	16	4 4 5
	17	0 1 5 6 6 8
	18	0

b **i** Nearly symmetrical

ii negatively skewed

c No outliers or clusters for girls, 134 is an outlier for the boys, clustering in the 170s for boys.

d Girls: 150, 150;
Boys: 164.64, 167.5

e For the boys, the median is the better measure. For the girls, both measures are good.

f Girls

g Boys. Their range is 46 cm compared to the range for girls, which is 32 cm.

8 **a** sample **b** sample **c** census **d** sample

9 **a** numerical, continuous

b numerical, discrete

c categorical

d categorical

e numerical, continuous

10 **a** Not all teens necessarily play computer games.

b Only those customers interested in completing the survey are sampled.

c Need to sample workers as well, not only management who may have a biased view.

d People at a premiere are not the usual moviegoers.

e It is restricted to one radio station playing a specific type of music.

11 **a** The question is too general and many responses could be given. Instead, give choices such as always, most of the time, sometimes, half the time, not often, rarely, never.

b The question is too general. Give choices such as excellent, good, satisfactory, poor.

c The question encourages bias towards saying yes because people have fought for the flag. A better question might be 'Do you think Australia's flag should be changed?' with a Yes/No response and then a choice of reasons such as:

❑ Australia is independent of the United Kingdom

❑ Australia is a multicultural nation

❑ Australia is 'part' of Asia

❑ Other

Chapter 10

SkillCheck

1 **a** **i** 88 cm **ii** 384 cm^2

b **i** 112 cm **ii** 336 cm^2

c **i** 37.7 cm **ii** 113.10 cm^2

d **i** 70 cm **ii** 200 cm^2

e **i** 84 cm **ii** 336 cm^2

f **i** 94.2 cm **ii** 706.9 cm^2

2 **a** 34 000 mm **b** 9.25 m **c** 1.5 km

d 3.75 L **e** 380 mm **f** 7500 mL

3 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{1}{3}$ **d** $\frac{23}{36}$

4 **a** 180 cm^3 **b** 2197 cm^3

c 1440 cm^3 **d** 4500 cm^3

5 **a** **i** 5

ii 3 rectangles and 2 triangles

iii Triangular prism

b **i** 8

ii 6 rectangles and 2 hexagons

iii Hexagonal prism

c **i** 6

ii 4 rectangles and 2 trapeziums

iii Trapezoidal prism

6 **a** 41 **b** 18 **c** 12.6

Exercise 10.01

1 **a** cm **b** t **c** km

d L **e** kL **f** hours

2 **a** 750 000 mg **b** 36 000 L

c 67 000 g **d** 1500 mL

e 67 mm **f** 3500 m

g 9400 kg **h** 3750 mg

i 425 cm **j** 200 g

k 26 200 mL **l** 300 m

m 3 000 000 mL **n** 5760 min

o 54 000 s **p** 270 000 cm

q 5 000 000 mg **r** 700 mm

s 29 375 000 g **t** 18 140 000 mm

u 3270 mg

3 B

4 **a** B **b** B **c** A **d** C **e** B

5 **a** 8 kL **b** 9 cm **c** 3 min **d** 4 h

e 0.375 L **f** 14.7 t **g** 8.6 g **h** 7.5 m

i 22.3 cm **j** 0.8 kg **k** 0.125 kL **l** 0.0035 t

m 17 kg **n** 5 days **o** 45 t **p** 40 h

q 1 day **r** 0.025 km **s** 0.075 kL **t** 34.7 m

u 0.000 628 ML

6 B

7 503 250 000 000 L or 5.0325×10^{11} L

8 **a** 4800 kB **b** 2100 GB **c** 8500 MB

d 10 800 000 B **e** 2.91 kB **f** 0.74 GB

g 1.05 MB **h** 5.9 TB **i** 0.000 94 MB

j 57 000 MB **k** 430 000 kB **l** 21 000 GB

9780170453325

m 7800 B **n** 2.18 GB **o** 0.042 MB
p 940 000 kB **q** 640 000 000 kB **r** 0.278 GB

9 9900 kB **10** 225 MB **11** 12 800 files

12 **a** 520 000 000 ≈ 5.2×10^8 km
b 6 s **c** 750 y
d 0.012 m **e** 6.4×10^8 m
f 2.0×10^{13} km **g** 70 ms
h 250 000 years **i** 4.8 s
j 5290 Mm **k** 6.2×10^{12} km
l 6000 μm **m** 940 millennia
n 8.5 centuries **o** 85 ly
p 80 nm **q** 6.1 pc
r 0.16 Ma

Exercise 10.02

1 **a** **i** 1 mm **ii** ±0.5 mm
b **i** 1°C **ii** ±0.5°C
c **i** 1 min **ii** ±0.5 min
d **i** 100 g **ii** ±50 g
e **i** 1 cm **ii** ±0.5 cm
f **i** 1° **ii** ±0.5°
g **i** 5 km/h **ii** ±2.5 km/h
h **i** 250 g **ii** ±125 g
i **i** 10 mL **ii** ±5 mL
j **i** 1 kL **ii** ±0.5 kL
k **i** $\frac{1}{4}$ tank **ii** $\pm\frac{1}{8}$ tank

2 D

3 **a** 1 cm **b** ±0.5cm
c ±0.5 mm **d** 26.75 to 26.85 mm

Exercise 10.03

1 **a** **i** 122 cm **ii** 690 cm^2
b **i** 76 m **ii** 211 m^2
c **i** 324 mm **ii** 3761 mm^2
d **i** 52 cm **ii** 126 cm^2
e **i** 92 cm **ii** 516 cm^2
f **i** 136 cm **ii** 864 cm^2

2 C

3 **a** 255 cm^2 **b** 217.5 cm^2 **c** 297 cm^2
d 410 cm^2 **e** 204 cm^2 **f** 134 cm^2
g 36 cm^2 **h** 73.5 cm^2 **i** 536 cm^2
j 1640 cm^2 **k** 340.24 cm^2 **l** 744 cm^2

4 B

5 **a** 38.72 cm^2 **b** 27.2 cm^2

6 **a** 525 000 m^2
b **i** 0.525 km^2 **ii** 52.5 ha

7 B

Exercise 10.04

1 **a** 48 cm^2 **b** 54.4 m^2 **c** 340 mm^2
d 3 cm^2 **e** 84 cm^2 **f** 950 mm^2
g 1048 mm^2 **h** 91.52 cm^2 **i** 52 m^2

2 **a** 102 cm^2 **b** 1519 mm^2 **c** 210 cm^2
d 837 mm^2 **e** 1000 mm^2 **f** 356.5 cm^2
g 406 cm^2 **h** 63 m^2 **i** 288 m^2

3 **a** 84 m^2 **b** 52.25 m^2 **c** 800 mm^2
d 190 m^2 **e** 54 cm^2 **f** 3.15 m^2
g 84 cm^2 **h** 144.6 m^2 **i** 840 mm^2

4 B

5 **a** 46 m^2 **b** 140 cm^2 **c** 24 m^2
d 168 cm^2 **e** 100.8 cm^2 **f** 29.04 m^2

6 **a** 160 m^2 **b** 141 m^2 **c** 7.8 m^2

Mental skills 10A

2 **a** 19 **b** $7.50 **c** 87.5 **d** $20.20
e $3.76 **f** 40 **g** $0.925 **h** 89.6
i $270 **j** $0.38 **k** $152.76 **l** $7.25
m 315.4 **n** $1.07 **o** 42.6 **p** $2431.76

4 **a** 10 **b** 124 **c** $490 **d** $1.72
e 14.4 **f** $2540 **g** 78 **h** $1.16
i $9 **j** $16.80 **k** $920 **l** 64

6 **a** 100 **b** $0.60 **c** 2.5 **d** $1.35
e $1.84 **f** $4.20 **g** 40 **h** 6.5
i $0.48 **j** $6.90 **k** $3.60 **l** 42

Exercise 10.05

1 **a** **i** 50.27 cm **ii** 201.06 cm^2
b **i** 32.67 cm **ii** 84.95 cm^2
c **i** 94.25 cm **ii** 706.86 cm^2
d **i** 88.59 cm **ii** 624.58 cm^2

2 **a** **i** 92.5 m **ii** 508.9 m^2
b **i** 35.7 m **ii** 78.5 m^2
c **i** 4.1 m **ii** 1.0 m^2
d **i** 136.0 m **ii** 1055.6 m^2
e **i** 221.5 m **ii** 2565.9 m^2
f **i** 15.1 m **ii** 9.4 m^2
g **i** 17.4 m **ii** 5.6 m^2
h **i** 303.9 m **ii** 4618.1 m^2

3 D **4** D

5 **a** 182.83 cm **b** 72.85 cm **c** 121.12 mm
d 129.12 m **e** 554.20 cm **f** 1416.67 m
g 103.98 m **h** 46.57 cm **i** 56.55 cm

6 B

7 **a** 2228 m^2 **b** 293 m^2 **c** 952 m^2 **d** 442 m^2
e 63 m^2 **f** 71 m^2 **g** 503 m^2 **h** 151 m^2
i 193 m^2 **j** 743 m^2 **k** 4664 m^2 **l** 471 m^2

8 **a** 20 056 m^2 **b** $4001

9 **a** 40 074 km **b** 42 587 km

10 1.2 m^2

11 **a** 34.56 m^2 **b** $5702.40

12 4.9 m

13 **a** 15.708 cm **b** 157 m

14 **a** 39 m^2 **b** 157% increase

15 **a** Izabella 399.3 m **b** Noah 405.6 m
c 6.3 m

16 **a** 207.35 cm **b** 2411

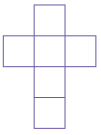
ANSWERS

Exercise 10.06

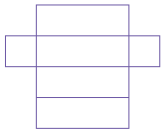
1 C

2 Other answers possible.

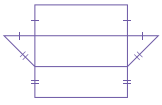
a



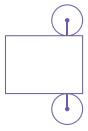
b



c



d



3 **a** rectangular prism **b** trapezoidal prism
c cube **d** triangular prism
e square pyramid **f** cylinder
g triangular prism **h** rectangular prism

4 **a** cube, $A = 2646\ m^2$
b rectangular prism, $A = 684\ m^2$
c right triangular prism, $A = 9720\ m^2$
d trapezoidal prism, $A = 7236\ m^2$

5 **a** $253.5\ m^2$ **b** $4140\ cm^2$ **c** $103\ 680\ cm^2$
d $125.36\ cm^2$ **e** $9600\ mm^2$ **f** $9680\ cm^2$

6 **a** $408\ m^2$ **b** $348\ m^2$ **c** $2310\ cm^2$
d $71.4\ cm^2$ **e** $360\ m^2$ **f** $648\ m^2$

7 **a** $145\ 140\ cm^2$ **b** \$174.17

8 D

9 **a** $h = 8.9$ m **b** $71.2\ m^2$ **c** $862.4\ m^2$

10 **a** $2804\ cm^2$ **b** $19\ 440\ cm^2$ **c** $3740\ cm^2$

11 **a** $151.33\ m^2$ **b** $251.33\ m^2$

Exercise 10.07

1 **a** $747.70\ m^2$ **b** $2550.97\ cm^2$
c $206.47\ m^2$ **d** $15.83\ m^2$

2 **a** $904.8\ cm^2$ **b** $8545.1\ mm^2$

3 **a** $2159\ .8\ m^2$ **b** $3.9\ m^2$ **c** $1894.5\ m^2$
d $6.5\ m^2$ **e** $14.4\ m^2$ **f** $1131.0\ m^2$

4 **a** $A = 56.5\ m^2$ **b** 7 L of paint

5 The triangular prism tent (a) has $7.6\ m^2$ more surface area.

6 $2073.5\ cm^2$

Exercise 10.08

1 **a** $50\ m^3$ **b** $13\ 200\ cm^3$ **c** $60\ 000\ mm^3$
d $32\ 768\ mm^3$ **e** $1.296\ m^3$ **f** $5880\ cm^3$

2 D **3** A

4 **a** $288\ 000\ cm^3 = 288\ 000 \div 100^3\ m^3 = 0.288\ m^3$
b $l = 1.2$ m, $b = 0.6$ m, $h = 0.4$ m,
$V = 1.2 \times 0.6 \times 0.4 = 0.288\ m^3$
It is easier to convert each measurement to the required units before doing the calculation.

5 **a** $504.8\ m^3$ **b** $1045\ cm^3$ **c** $19\ 968\ cm^3$

6 **a** $3016\ cm^3$ **b** $3958\ mm^3$ **c** $3\ m^3$
d $57\ m^3$ **e** $63\ m^3$ **f** $48\ 384\ cm^3$

7 **a** $9.7\ m^3$ **b** $94\ 247.8\ m^3$ **c** $7.4\ m^3$
d $135.7\ m^3$ **e** $5026.5\ m^3$ **f** $216.9\ m^3$
g $42.3\ m^3$ **h** $107.5\ m^3$ **i** $146.3\ m^3$

8 B **9** 10.5 m **10** 23.76 L

11 $13\ 666\ cm^3$ **12** $6\ m^3$ **13** 300 kL

14 B **15** $678.58\ cm^3$ **16** 13 mm

17 **a** 6.7 cm
b radius = 3.43 cm, height = 20.58 cm
c **i** If the radius and height were rounded down to 3.4 cm and 20.5 cm instead, the balls would not fit in the container.
Radius = 3.5 cm, height = 20.6 cm.
ii $792.8\ cm^3$
d **i** length = 28 cm, width = 21 cm, height = 20.6 cm (after rounding up)
ii $V = 12\ 112.8\ cm^3$
e $2599.2\ cm^3$ **f** 21.5%

Mental skills 10B

2 **a** 18 **b** \$126 **c** 39 **d** \$30.30
e \$7.50 **f** 10.8 **g** \$27 **h** 60
i \$240 **j** \$3.30 **k** 900 **l** \$52.50

4 **a** 10 **b** 166 **c** \$50 **d** \$22
e 37.5 **f** \$5.80 **g** 135 **h** \$22.60

6 **a** 500 **b** \$20 **c** 4.5 **d** \$6.25
e 81 **f** \$35 **g** 16.5 **h** 74.5
i \$195 **j** \$425 **k** \$31.50 **l** 290

8 **a** 160 **b** \$1.50 **c** 7.5 **d** \$32.50
e \$67.50 **f** \$31.25 **g** 38 **h** 170

Exercise 10.09

1 **a** $149.3\ cm^3$ **b** $240\ cm^3$ **c** $120\ cm^3$
d $336\ m^3$ **e** $1200\ cm^3$ **f** $106.7\ m^3$

2 B

3 **a** **i** 24 cm **ii** $3200\ cm^3$
iii 3200 mL or 3.2 L
b **i** 20 m **ii** $19\ 200\ m^3$ **iii** 19 200 kL
c **i** 40 mm **ii** $4320\ mm^3$ **iii** 4.32 mL
d **i** 60 mm **ii** $28\ 160\ mm^3$ **iii** 28.16 mL
e **i** 7.7 m **ii** $133.056\ m^3$ **iii** 133.056 kL
f **i** 84 cm **ii** $564\ 480\ cm^3$
iii 564 480 mL or 564.48 L

4 **a** $2\ 592\ 100\ m^3$ **b** $1.127\ m^3$

5 27 m **6** 76 mm **7** 4.9 cm

8 **a** $209\ m^3$ **b** $872\ cm^3$ **c** $1272\ mm^3$
d $616\ cm^3$ **e** $393\ cm^3$ **f** $2545\ mm^3$

9 **a** $116.1\ cm^3$ **b** $13\ 796.9\ m^3$ **c** $377.8\ cm^3$
d $2.4\ m^3$ **e** $296\ 162.1\ m^3$ **f** $129\ 638.4\ cm^3$

10 **a** 320π **b** 4.5π **c** 768π

11 9.7 cm **12** 8.8 m **13** 5.23 m

14 **a** $20\ 106.2\ cm^3$ **b** 87.5%

Power plus

1 **a** 157.08 cm **b** 50π cm
2 **a** 70.83 m **b** 125.66 m^2
3 60 cm **4** 628 m^2
5 **a** **i** $A = 52.3\ cm^2$ **ii** $P = 33.4$ cm
b **i** $A = 121.1\ cm^2$ **ii** $P = 46.6$ cm
c **i** $A = 185.1\ cm^2$ **ii** $P = 50.7$ cm
6 160 cm^3 **7** 3375 cm^3 **8** 40 mm
9 1629.2 cm^3

Test yourself 10

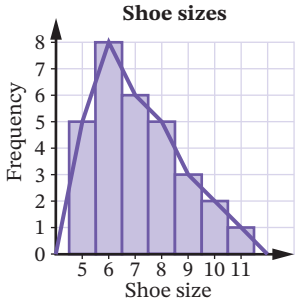
1 **a** 8000 m **b** 900 s **c** 700 mm
d 250 kg **e** 8400 MB **f** 9500 kg
g 5 h **h** 34 kB **i** 125 000 L
j 0.025 km **k** 17 000 t **l** 8.9 TB
m 10.5 s **n** 9100 years **o** 2 600 000 m
p 2.838×10^{13} km **q** 0.4 μs
r 75 m
2 **a** **i** 1 mm **ii** ±0.5 mm
b **i** 1° **ii** ±0.5°
c **i** 1 min **ii** ±0.5 min
d **i** 5 km/h **ii** ±2.5 km/h
3 **a** 52 m **b** 74 cm **c** 84 mm **d** 114 cm
4 **a** 432 mm^2 **b** 64 m^2 **c** 16 cm^2 **d** 2340 mm^2
5 **a** 52 cm^2 **b** 468 mm^2 **c** 72 m^2
d 246 mm^2 **e** 120 m^2 **f** 260 cm^2
6 **a** **i** $P = 34.6$ cm **ii** $A = 95.0\ cm^2$
b **i** $P = 171.4$ mm **ii** $A = 2056.8\ mm^2$
c **i** $P = 59.4$ m **ii** $A = 253.1\ m^2$
d **i** $P = 57.3$ m **ii** $A = 205.3\ m^2$
e **i** $P = 7.7$ m **ii** $A = 3.5\ m^2$
f **i** 54.8 cm **ii** 56.5 cm^2
7 **a** 14.14 m^2 **b** 162.62 mm^2 **c** 326.73 cm^2
8 **a** 61.44 m^2 **b** 152 m^2 **c** 760 m^2 **d** 2256 m^2
9 **a** 32.768 m^3 **b** 176 m^3 **c** 960 m^3 **d** 5040 m^3
10 **a** 7389.03 m^2 **b** 1437.28 mm^2 **c** 104.33 cm^2
11 **a** 48 490.48 m^3 **b** 4064.44 mm^3 **c** 109.93 cm^3
12 123.75 L **13** 11 830 cm^3
14 **a** 323 m^3 **b** 540 cm^3 **c** 298 cm^3
15 **a** 1340.4 cm^3 **b** 5026.5 mm^3 **c** 9358.0 cm^3
16 10.7 cm

Practice set 3

1 **a** 12.5 h **b** 1.35 L **c** 0.512 GB
d 3005 g **e** 2700 m **f** 250 kB
2 **a** \$551.60 **b** 320
3 **a** \$87 420 **b** \$19 958.50
4 **a**

Shoe size	Frequency
5	5
6	8
7	6
8	5
9	3
10	2
11	1

b



c 63.3% **d** mode = 6, range = 6
e 7 **f** 7.1
5 **a** \$1720.20 **b** \$3440.40 **c** \$7480
6 **a** \$1653 **b** \$1157.10 **c** \$3884.55
7 **a** **i** 46, 48 **ii** 48.6 **iii** 48 **iv** 6
b **i** 70 **ii** 70.7 **iii** 70 **iv** 10
8 **a** numerical, discrete
b categorical
c numerical, continuous
d numerical, continuous
9 **a** Perimeter = 72 cm, Area = 212 cm^2
b Perimeter = 66 cm, Area = 231 cm^2
10 **a** \$2028.50 **b** 66.8%
11 C **12** \$1589.20 **13** ±125 g
14 **a** 200 mm^2 **b** 240 mm^2 **c** 30 cm^2
15 \$17 468
16 **a** 189.6 m^3 **b** 5654.9 cm^3
c 2070 cm^3 **d** 2634.4 m^3
17 **a** Positively skewed **b** Near 2, 3
c mode = 3, range = 8
18 **a** 158 m^2 **b** 360 cm^2 **c** 4086 cm^2
19 **a** 120 m^3 **b** 300 cm^3 **c** 15 390 cm^3
20 **a** **i** 20.6 m **ii** 25.1 m^2
b **i** 20.5 m **ii** 26.2 m^2
c **i** 28.6 m **ii** 44.6 m^2
21 **a** 'People at a park' does not represent everybody, as employed people are less likely to be at the park.
b Not enough information obtained as responses will just be Yes or No.
c Which of the following applies to you?
- ❏ going to school/TAFE/University
- ❏ working part time
- ❏ working full time
- ❏ retired
- ❏ carer for children and/or other person
- ❏ on unemployment benefits

(Other answers possible)
22 24 700 mm^3 **23** 4010 mm^2 **24** \$3610.58
25 **a** Monday: positively skewed
Wednesday: negatively skewed
b Monday: 8.79 Wednesday: 9.67
c Monday: 8.5 Wednesday: 10
d Monday
e Wednesday

ANSWERS

26 10.1 cm **27** 15.5 cm **28** \$32.40
29 96

Chapter 11

SkillCheck

1 $Z(-2, 1)$ $D(3, -2)$ $Q(0, -5)$ $W(5, 0)$ $P(-4, -3)$

2 **a**

x	−2	−1	0	1	2
y	5	3	1	−1	−3

b

x	−2	−1	0	1	2
y	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2

3 **a** $y = 5$ **b** $y = -1$ **c** $x = 2\frac{1}{2}$ **d** $x = -1$

4 **a** 2 **b** 2 **c** 8.5

Exercise 11.01

1 **a** 7.3 **b** 7.2 **c** 8.2
d 14.3 **e** 7.1 **f** 14.3

2 **a** **i** $\sqrt{45}$ or $3\sqrt{5}$ **ii** 6.7
b **i** $\sqrt{117}$ or $3\sqrt{13}$ **ii** 10.8
c **i** $\sqrt{128}$ or $8\sqrt{2}$ **ii** 11.3
d **i** $\sqrt{68}$ or $2\sqrt{17}$ **ii** 8.2

4 A

5 **a** 7.071 **b** 5 **c** 6.403 **d** 18.474

6 **a** $RS = \sqrt{20}, ST = \sqrt{8}, RT = \sqrt{20}$
b isosceles triangle

7 **a** trapezium **b** $P = 28$ units
c $A = 38$ units^2

8 **a** square **b** $\sqrt{52}$ or $2\sqrt{13}$ units
c 28.8 units **d** 52 units^2

Exercise 11.02

1 A

2 **a** (4, 4) **b** (5, 3) **c** (1, 1)

3 **a** (8, 7) **b** (7, 8) **c** $(4\frac{1}{2}, 4)$
d (2, 3) **e** $(4, 4\frac{1}{2})$ **f** $(7, \frac{1}{2})$
g $(\frac{1}{2}, 3\frac{1}{2})$ **h** $(-3, -\frac{1}{2})$ **i** $(-1, -6\frac{1}{2})$
j (−1, −1) **k** (7, −8) **l** $(-5\frac{1}{2}, -6)$

4 **a** $P(2, 3)$ **b** $Q(-1, 1)$ **c** $R(5, 5)$

5 D

6 **a** **i** $(-\frac{1}{2}, -1)$ **ii** $(-\frac{1}{2}, -1)$
b They are the same point. The diagonals bisect each other.

7 **a** $M(\frac{1}{2}, 4\frac{1}{2})$
b **i** $\sqrt{50}$ or $5\sqrt{2}$ **ii** $\sqrt{4.5}$ **iii** $\sqrt{17}$
c **i** 7.5 units^2 **ii** $P = 15.3$ units

8 **a** $2\sqrt{26}$ units **b** $2\sqrt{26}$ units **c** Yes
d (2, 1) **e** (2, 1)
f Yes, their midpoints are the same.

Exercise 11.03

1 **a** + **b** − **c** − **d** − **e** +
f + **g** + **h** − **i** +

2 **a** 2 **b** $-\frac{1}{4}$ **c** −5 **d** −1 **e** $\frac{3}{7}$
f $\frac{7}{3}$ **g** $\frac{2}{3}$ **h** $-\frac{8}{7}$ **i** $\frac{2}{5}$

3 **a** $-\frac{1}{2}$ **b** 5 **c** $\frac{2}{3}$ **d** 7
e $-\frac{1}{6}$ **f** $\frac{5}{2}$ **g** $-\frac{7}{2}$

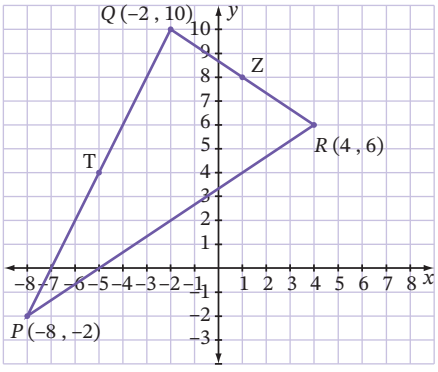
4 C

5 **a** 1 **b** $\frac{1}{3}$ **c** −5 **d** $-\frac{1}{2}$ **e** 0
f $-\frac{1}{2}$ **g** $\frac{7}{4}$ **h** $\frac{1}{2}$ **i** 0

6 A **7** $k: \frac{1}{5}$, $l: -\frac{1}{2}$ **8** B

9 **a** $m_{PQ} = -\frac{1}{2}, m_{QR} = \frac{5}{2}, m_{RS} = -\frac{1}{2}, m_{PS} = \frac{5}{2}$
b QR and PS, PQ and RS. They are parallel.
c parallelogram

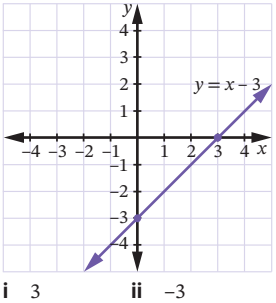
10 **a**



b $T(-5, 4), Z(1, 8)$
c Both gradients are $\frac{2}{3}$.
d $TZ = 2\sqrt{13}$ units, $PR = 4\sqrt{13}$ units, PR is double the length of TZ.

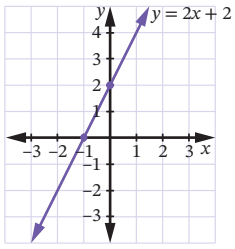
Exercise 11.04

1 **a**



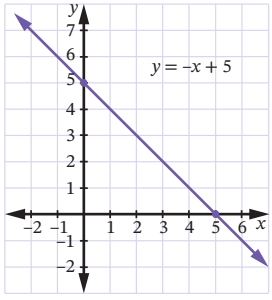
i 3 **ii** −3

b



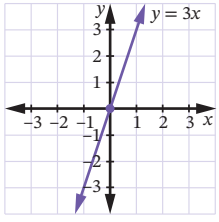
i −1 **ii** 2

c



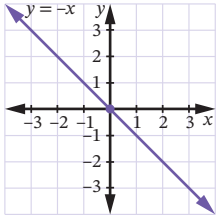
i 5 **ii** 5

d



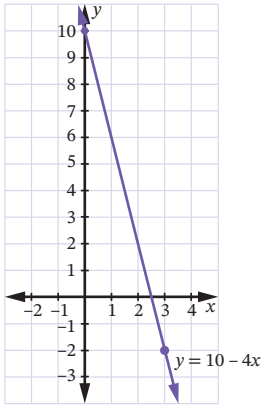
i 0 **ii** 0

e



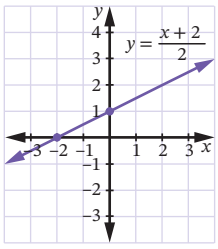
i 0 **ii** 0

f



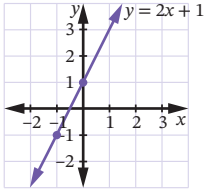
i $2\frac{1}{2}$ **ii** 10

g



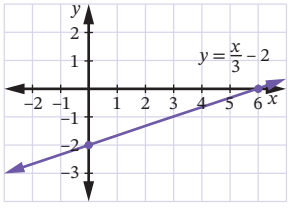
i −2 **ii** 1

h



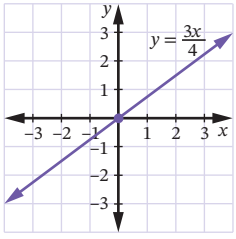
i $-\frac{1}{2}$ **ii** 1

i



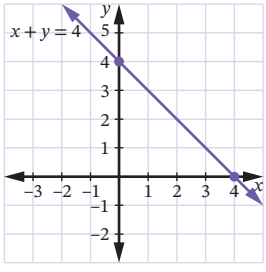
i 6 **ii** −2

j



i 0 **ii** 0

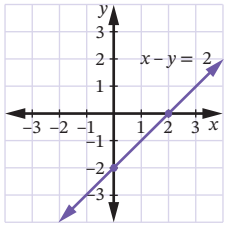
k



i 4 **ii** 4

ANSWERS

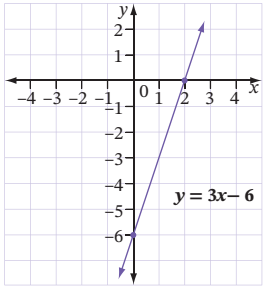
l



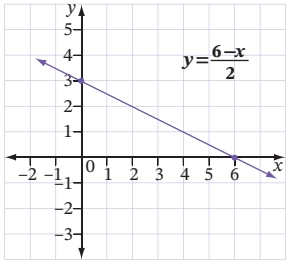
i 2 **ii** −2

2 B

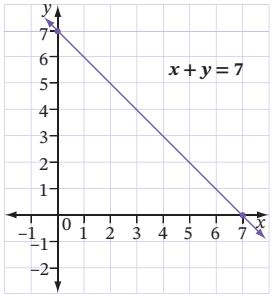
3 **a**



b



c



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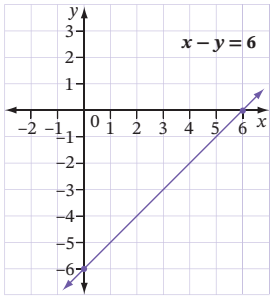
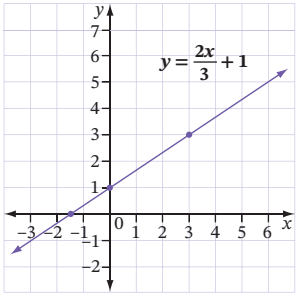


e



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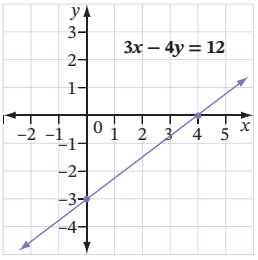
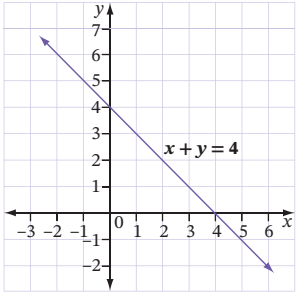


g



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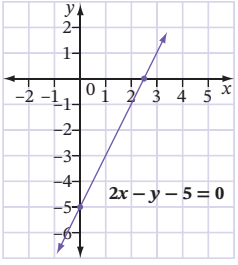
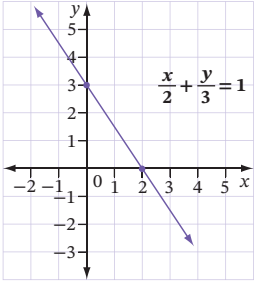


i

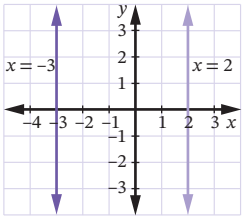


4 **a** yes **b** no **c** yes
d yes **e** yes **f** no

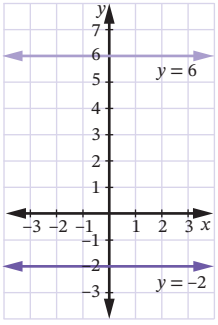
9780170453325

5 C

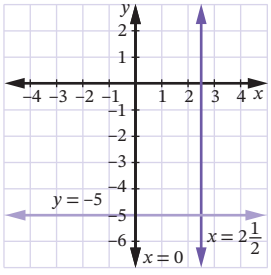
6 **a**



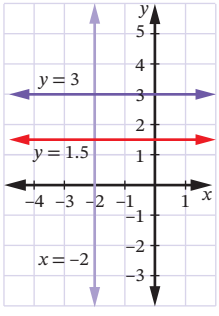
b



c



d



7 **a** $y = 6$ **b** $x = -4$ **c** $y = 3$ **d** $x = 5$
e $y = -5$ **f** $x = 3$ **g** $y = -3$ **h** $x = -1$

8 **a** LHS = RHS = 3 for $x = 2y - 1$; LHS = RHS = 5 for $x + y = 5$
b (3, 2)

9 LHS = RHS = −12 for $y = -3x + 6$; LHS = RHS = −12 for $y = -2x$

10 **a** $y = 6$ **b** $x = -5$ **c** $y = 2$ **d** $x = -5$
e $y = -2$ **f** $y = -1$ **g** $x = -1$

11 **a** $x = 0$ **b** $y = 0$

Mental skills 11

2 **a** 2, 5 **b** 3 **c** 2, 3, 6 **d** 3, 5
e 2 **f** 2, 3, 5, 6 **g** 5 **h** 3

3 **a** 4, 9 **b** 4, 10 **c** 9 **d** 10
e 4, 9 **f** None **g** 9 **h** 4, 9, 10

Exercise 11.05

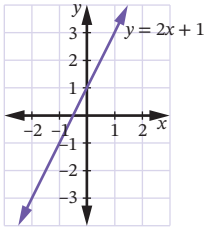
1 **a** $m = 3, c = -2$ **b** $m = 2, c = 7$
c $m = 1, c = 4$ **d** $m = -1, c = -9$
e $m = \frac{3}{4}, c = 6$ **f** $m = 1, c = 0$
g $m = \frac{1}{2}, c = -11$ **h** $m = \frac{2}{3}, c = 6$
i $m = -2, c = 8$ **j** $m = -1, c = 5$
k $m = \frac{1}{4}, c = 0$ **l** $m = -3, c = 11$
m $m = 0, c = 5$ **n** $m = \frac{2}{5}, c = 1$
o $m = 1, c = -\frac{7}{2}$ **p** $m = \frac{3}{4}, c = 1$

2 D

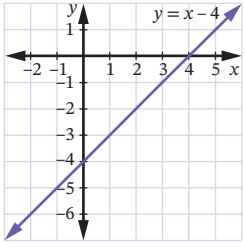
3 **a** $y = 2x - 1$ **b** $y = \frac{3}{4}x + 2$
c $y = -3x + 5$ **d** $y = -\frac{2}{5}x + 3$

4 C

5 **a**



b



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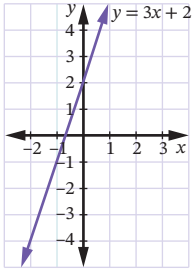
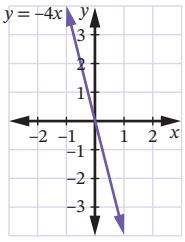
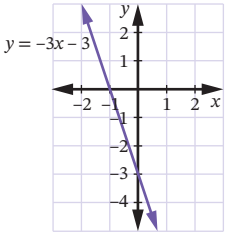


d



e



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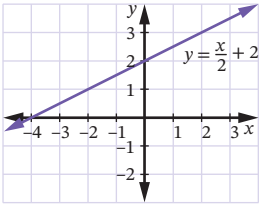
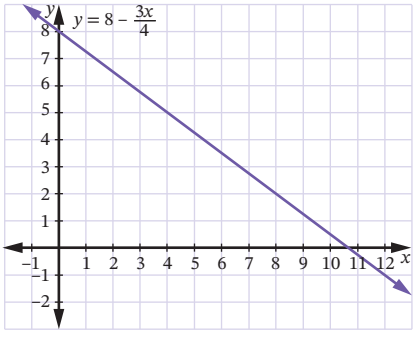
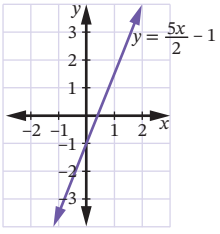


g



h



6 B

Exercise 11.06

1 **a** $y = x + 4$ **b** $y = \frac{3x}{2}$ **c** $y = \frac{x}{3} - 5$

d $y = \frac{-3x}{2} + 7$ **e** $y = -\frac{x}{3} + 3$ **f** $y = -2x - 6$

2 **a** $y = \frac{x}{2} + 2$ **b** $y = x$ **c** $y = -\frac{x}{2} + 5$

d $y = -\frac{x}{2} + 3$ **e** $y = -3x - 3$ **f** $y = -x - 2$

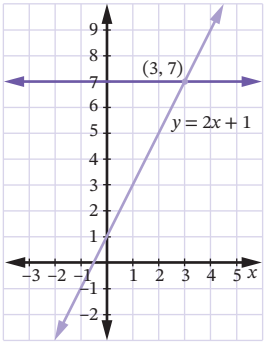
g $y = 3x - 10$ **h** $y = \frac{2x}{5} + 2$ **i** $y = 2x - 3$

3 **a** D **b** B **c** A **d** C

Exercise 11.07

1 **a** $x = 3$ **b** $x = -2$ **c** $x = 1\frac{1}{2}$

2 **a**



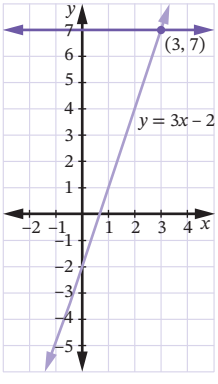
$x = 3$

b $x = 4.5$ **c** $x = -3$

3 **a** $x = 3$ **b** $x = 4\frac{1}{2}$ **c** $x = -3$

4 B

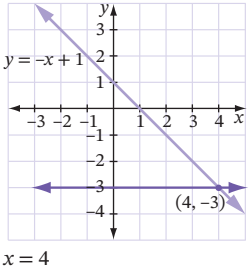
5 **a**



$x = 3$

b $x = -3$ **c** $x = 0$

6 **a**



$x = 4$

b $x = -3$ **c** $x = 2$

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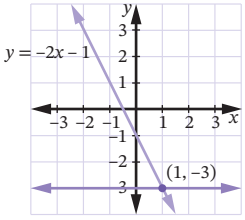
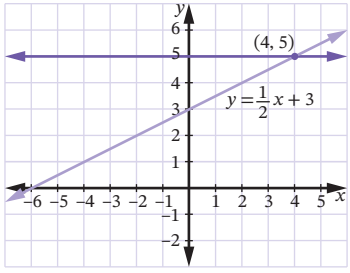
7 **a**



$x = 1$

b $x = -0.5$ **c** $x = -2$

8 **a**

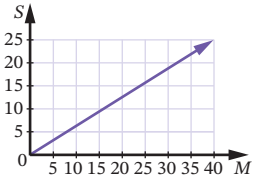


$x = 4$

b $x = -4$ **c** $x = -2$

Exercise 11.08

1 **a**



b $k = 0.625$, $S = 0.625\ M$

c 22.5 cm

d 64 kg

2 C **3** $k = 5$, $Y = 115$

4 **a** $M = 1.75V$ **b** 710.5 g

c **i** 16 cm³ **ii** 205.7 cm³

5 $n = 2.5m$ **6** $p = 108$

7 **a** 11.25 km **b** 2 h 8 min

8 A

9 **a** $I = 0.025D$ **b** \$62.50 **c** \$125

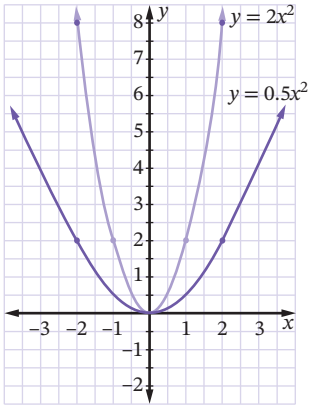
10 **a** $V = 64\ t$ **b** 512 L **c** 15 min

11 **a** 22.8 kg **b** 84.1 kg

12 **a** 235.2 kPa **b** 141.7 m

Exercise 11.09

1 **a**, **b**

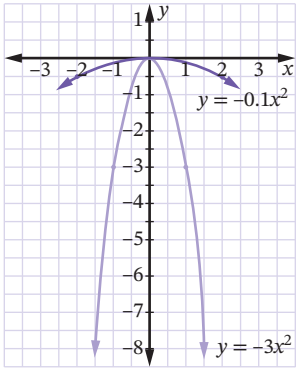


c Same: both concave up, vertex (0, 0)

Different: $y = 2x^2$ is narrower than $y = \frac{1}{2}x^2$

2 D

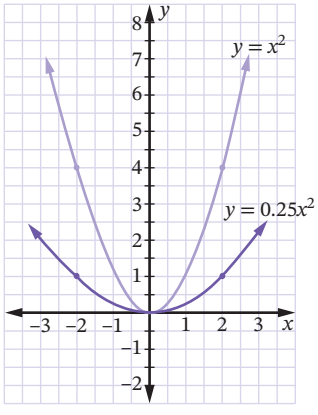
3 **a**



b Same: concave down, vertex (0, 0)

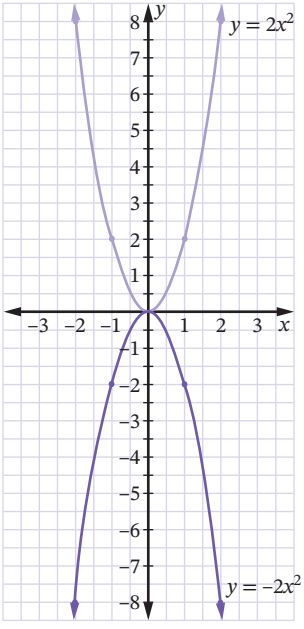
Different: $y = -3x^2$ narrower than $y = -0.1x^2$

4 **a** **i** vertex (0, 0)



ii Both concave up. **iii** $y = \frac{1}{4}x^2$ **iv** (0, 0)

b **i** vertex (0, 0)

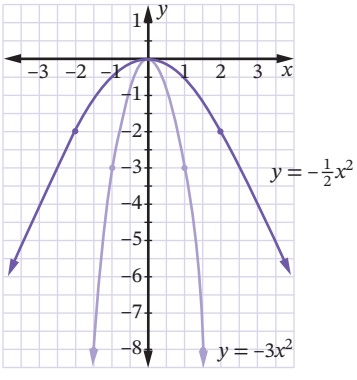


ii $y = 2x^2$ concave up, $y = -2x^2$ concave down

iii same width

iv (0, 0)

c **i** vertex (0, 0)

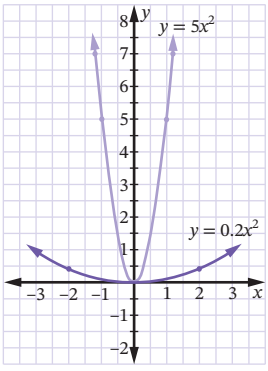


ii both concave down

iii $y = -\frac{1}{2}x^2$ wider

iv (0, 0)

d **i** vertex (0, 0)



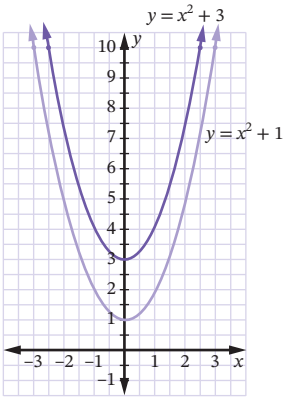
ii both concave up

iii $y = 0.2x^2$ is wider

iv (0, 0)

5 C

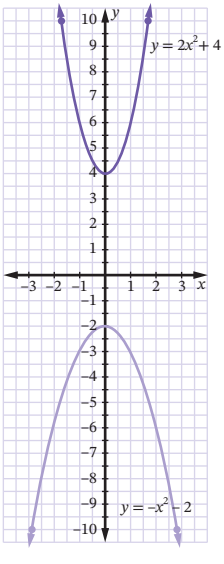
6 **a** **i** $y = x^2 + 1$, vertex (0, 1); $y = x^2 + 3$, vertex (0, 3)



ii both concave up **iii** both same width

iv $y = x^2 + 1$, y-int: 1; $y = x^2 + 3$, y-int: 3

b **i** $y = -x^2 - 2$, vertex (0, −2); $y = 2x^2 + 4$, vertex (0, 4)



ii $y = 2x^2 + 4$ concave up;
$y = -x^2 - 2$ concave down

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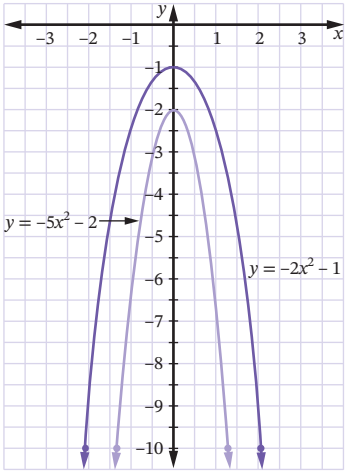
iii $y = -x^2 - 2$

iv $y = -x^2 - 2$, y-int: -2; $y = 2x^2 + 4$, y-int: 4

c i



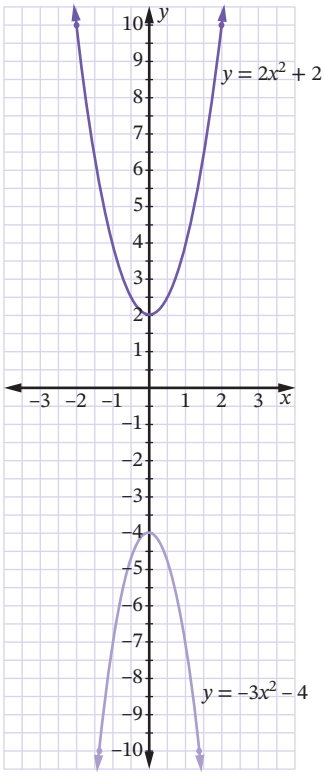
$y = -5x^2 - 2$, vertex $(0, -2)$; $y = -2x^2 - 1$, vertex $(0, -1)$

ii Both concave down.

iii $y = -2x^2 - 1$

iv $y = -5x^2 - 2$, y-int: -2; $y = -2x^2 - 1$, y-int: -1

d i $y = -3x^2 - 4$, vertex $(0, -4)$; $y = 2x^2 + 2$, vertex $(0, 2)$



ii $y = 2x^2 + 2$ concave up; $y = -3x^2 - 4$ concave down

iii $y = 2x^2 + 2$

iv $y = 2x^2 + 2$, y-int: 2; $y = -3x^2 - 4$, y-int: -4

7 C

8 **a** D **b** B **c** A **d** C

Exercise 11.10

1 **a** centre $(0, 0)$, $r = 2$ **b** centre $(0, 0)$, $r = 1$

c centre $(0, 0)$, $r = 9$ **d** centre $(0, 0)$, $r = 10$

e centre $(0, 0)$, $r = 5$ **f** centre $(0, 0)$, $r = \frac{1}{2}$

2 D

3 **a** $x^2 + y^2 = 16$ **b** $x^2 + y^2 = 36$

c $x^2 + y^2 = 6.25$ **d** $x^2 + y^2 = 0.81$

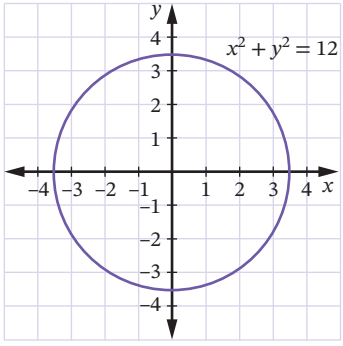
4 B **5** A

6 $x^2 + y^2 = 64$

7 B **8** 28 units

9 **a** $\sqrt{12}$ or $2\sqrt{3}$

b



Power plus

1 $D(-3, 5)$

2 **a** $y = \frac{2}{3}x - 2$ **b** 4

3 $Y(5, 4)$

4 **a** $r = \sqrt{73}$

b $r = \sqrt{73} \approx 8.54$; $10 > 8.54$, $\therefore P$ lies outside the circle as its distance from the centre C is greater than the radius of the circle.

5 $RS = 13$, $ST = 13$, $RT = \sqrt{208}$ $\therefore RS = ST = 13$ units
$\therefore$ isosceles triangle

6 $(5, 4)$, $(8, 1)$, $(5, -2)$

Test yourself 11

1 **a** $\sqrt{97}$ **b** 9.8

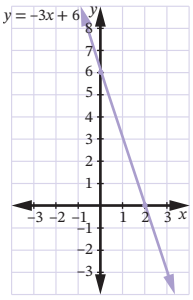
2 **a** $(3, 7)$ **b** $(-4, 4)$ **c** $(-1, 2)$

3 **a** **i** MN, CD, GH **ii** KL, IJ, AB **iii** EF

b **i** $\frac{1}{7}$ **ii** $-\frac{5}{4}$ **iii** $-\frac{1}{4}$

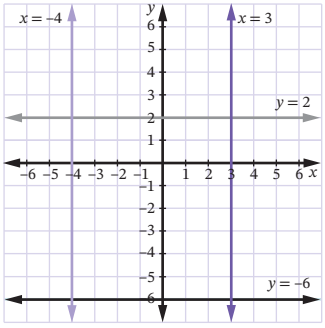
c $(9, -\frac{1}{2})$ **d** $\sqrt{13}$

4 **a** $m = -2$ **b** $m = -2$ **c** $m = -\frac{1}{3}$

5 

a 2 **b** 6

6 **a** yes **b** yes **c** yes

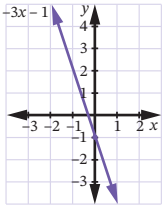
7 **a** 

b 56 units^2

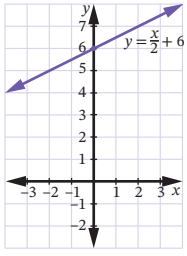
8 **a** $y = 4$ **b** $x = -1$ **c** $y = 0$

9 **a** $m = 3, c = 1$ **b** $m = -1, c = 8$

c $m = \frac{1}{5}, c = 4$ **d** $m = -6, c = 0$

10 **a** 

$m = -3, c = -1$

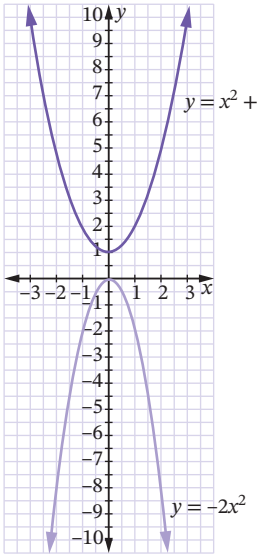
b 

$m = \frac{1}{2}, c = 6$

11 **a** $y = 2x + 1$ **b** $y = -3x + 1$

12 $x = 3$

13 **a** $T = \frac{6}{35}d$ **b** $d = 58$ km

14 

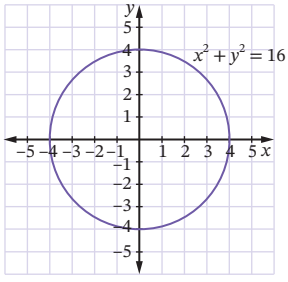
a $y = x^2 + 1$, vertex (0, 1): $y = -2x^2$, vertex (0, 0)

b $y = x^2 + 1$ concave up; $y = -2x^2$ concave down

c $y = x^2 + 1$

15 B

16 centre (0, 0), radius 4



Chapter 12

SkillCheck

1 **a** $\frac{1}{5}$ **b** 40% **c** 60% **d** $\frac{11}{20}$

2 B

3 **a** $\frac{1}{13}$ **b** $\frac{1}{4}$ **c** $\frac{1}{2}$

4 **a** B **b** E **c** F

d C **e** A **f** D

5 **a** $\frac{1}{6}$ **b** $\frac{2}{3}$ **c** $\frac{1}{2}$ **d** $\frac{1}{3}$

6 **a** {5, 6, 7, 8}

b **i** 25% **ii** 100% **iii** 25% **iv** 75%

Exercise 12.01

1 **a** 26 **b** yes **c** $\frac{5}{26}$ **d** $\frac{9}{26}$

2 **a** 0.375 **b** 0.25 **c** 0.25

3 **a** $\frac{1}{2}$ **b** $\frac{1}{13}$ **c** $\frac{5}{13}$ **d** $\frac{3}{13}$

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4 **a** $\frac{2}{3}$ **b** $\frac{1}{2}$ **c** $\frac{2}{3}$
d $\frac{1}{2}$ **e** 0 **f** 1

5 **a** $\frac{4}{7}$ **b** 0 **c** 1

6 **a** $\frac{1}{3}$ **b** $\frac{2}{3}$ **c** $\frac{1}{3}$
d $\frac{1}{9}$ **e** $\frac{5}{9}$

7 **a** 60.9% **b** 39.1% **c** 100% **d** 0 %

8 **a** $\frac{1}{3}$ **b** $\frac{1}{6}$ **c** $\frac{2}{3}$ **d** $\frac{2}{3}$

9 **a** 35% **b** 45% **c** 55% **d** 45%

10 **a** selecting a vowel
b weather not being sunny or weather being overcast
c traffic light showing amber or green
d selecting a heart, diamond or club
e not hitting the bullseye
f selecting a red or yellow ball

11 **a** {1, 2, 3, 4, 5} **b** $\frac{1}{5}$
c Spinning a number other than 4 (spinning a 1, 2, 3 or 5).
d {1, 2, 3, 5} **e** $\frac{4}{5}$

12 **a** **i** $\frac{1}{50}$ **ii** 0.02 **iii** 2%
b **i** $\frac{49}{50}$ **ii** 0.98 **iii** 98%

13 **a** 60% **b** 80

14 D **15** 0.65

16 $\frac{5}{6}$ **17** A

18 0 as all mobile numbers begin with 0.

Exercise 12.02

1 **a** **i** 0.45 **ii** 0.18
b **i** 0.4 **ii** 0.2
c They are similar but not equal.
d 160, which is close to the observed frequency of 164.

2 **a** Teacher to check **b** 25%, teacher to check.
c 12 or 13, teacher to check.

3 **a** 16 or 17 **b–e** Teacher to check.

4 **a** Teacher to check.
b **i** 0.4 **ii** 0.5 **iii** 0.1
c, d Teacher to check.

5 **a** 500
b **i** 0.47 **ii** 0.468 **iii** 0.35
iv 0.302 **v** 0.832
c **i** 0.652 **ii** 0.667
d The probabilities are close.

6 **a** 0.28 **b** 0.16 **c** 0.45 **d** 0.89 **e** 0.60

7 **a** 40
b **i** $\frac{11}{40}$ **ii** $\frac{1}{8}$ **iii** $\frac{3}{4}$ **iv** $\frac{1}{8}$

8 **a** $\frac{4}{31}$ **b** $\frac{10}{31}$ **c** $\frac{11}{31}$
d $\frac{26}{31}$ **e** $\frac{5}{31}$ **f** $\frac{9}{31}$

9 **a** $\frac{3}{10}$ **b** $\frac{1}{10}$ **c** $\frac{1}{4}$
d $\frac{1}{2}$ **e** $\frac{9}{20}$ **f** $\frac{17}{20}$

10 **a** **i** 0.092 **ii** 0.077
b They are similar. The experimental probability is just higher.
c C

Exercise 12.03

1 **a** 48
b **i** $\frac{5}{8}$ **ii** $\frac{25}{48}$
c **i** 1 **ii** $\frac{7}{48}$ **iii** $\frac{23}{48}$ **iv** 0
d A

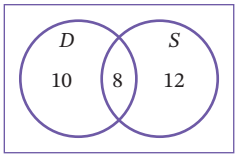
2 **a** 160
b **i** $\frac{91}{160}$ **ii** $\frac{1}{2}$ **iii** $\frac{19}{80}$ **iv** $\frac{133}{160}$
v $\frac{19}{32}$ **vi** $\frac{53}{160}$ **vii** $\frac{21}{80}$
c 76.25% **d** $\frac{38}{91}$

3 **a** People who preferred watching movies only at the cinema.
b 12
c **i** 0.25 **ii** 0.4 **iii** 0.95 **iv** 0.05

4 **a** D
b 110 – inside the rectangle but outside of the circles.
c **i** $\frac{3}{98}$ **ii** $\frac{6}{49}$ **iii** $\frac{43}{196}$ **iv** $\frac{1}{49}$
v $\frac{13}{98}$ **vi** $\frac{137}{196}$ **vii** $\frac{123}{196}$
d **i** $\frac{59}{86}$ **ii** $\frac{13}{86}$

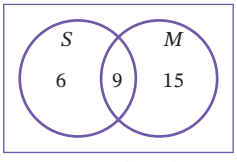
5 **a** 79 **b** $\frac{3}{79}$
c **i** $\frac{46}{79}$ **ii** $\frac{51}{79}$ **iii** $\frac{34}{79}$
d B
e **i** $\frac{47}{79}$ **ii** $\frac{10}{79}$ **iii** $\frac{14}{79}$

6 **a**



b **i** $\frac{4}{15}$ **ii** $\frac{2}{5}$ **iii** $\frac{1}{3}$ **iv** 1 **v** $\frac{11}{15}$

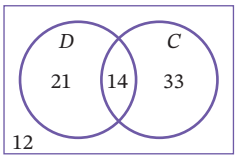
7 **a** 9
b



c Because neither playing sport nor watching a movie was not an option.
d **i** $\frac{7}{10}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{5}$

ANSWERS

8 a

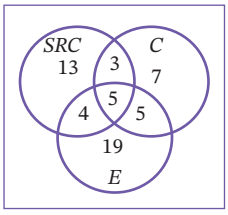


b i 14 **ii** 21 **iii** 68 **iv** 54

c i 0.2625 **ii** 0.675 **iii** 0.825 **iv** 0.85

d $\frac{47}{68}$

9 a



b 19 **c** 7 **d** $\frac{3}{14}$ **e** $\frac{17}{56}$

f i $\frac{1}{14}$ **ii** $\frac{39}{56}$ **iii** $\frac{23}{56}$ **iv** $\frac{51}{56}$

g No, because information is only given about numbers belonging to the clubs and not about the numbers attending the school.

Exercise 12.04

1 a 300 **b** 75 **c** 68

d i $\frac{31}{75}$ **ii** $\frac{1}{4}$ **iii** $\frac{157}{300}$ **iv** $\frac{58}{75}$

2 a 130 **b** 68.5%

c i $\frac{61}{130}$ **ii** $\frac{89}{130}$ **iii** $\frac{1}{5}$

iv $\frac{23}{26}$ **v** $\frac{7}{26}$ **vi** $\frac{8}{13}$

d B **e** $\frac{5}{23}$

3 a 240 **b i** 121 **ii** 119

c $53.\dot{3}\%$ **d** $\frac{9}{80}$ **e** $\frac{36}{121}$

4 a 183 **b i** $\frac{14}{183}$ **ii** $\frac{118}{183}$

c 35.5% **d** $\frac{43}{50}$ **e** $\frac{169}{183}$

5 a 151

b i 69.5% **ii** 20.5% **iii** 71.5% **iv** 28.5%

c $\frac{62}{151} = 41.1\%$ **d** $\frac{15}{46} = 32.6\%$

6 a 965

b i 18.4% **ii** 49.4% **iii** 21.8% **iv** 27.7%

c $\frac{2}{5}$ **d** $\frac{31}{52}$

7 a 320 **b** 43.4% **c** $56.6\% = \frac{181}{320}$

d 49.7% **e** A

Mental skills 12

2 a \$700 **b** \$800 **c** 400 **d** 700

e \$300 **f** \$400 **g** 250 **h** \$300

i \$300 **j** 500

Exercise 12.05

1 a BA, BW, BL, BN, AB, AW, AL, AN, WB, WA, WL, WN, LB, LA, LW, LN, NB, NA, NW, NL

b i $\frac{2}{20} = \frac{1}{10}$ **ii** $\frac{6}{20} = \frac{3}{10}$ **iii** $\frac{8}{20} = \frac{2}{5}$ **iv** $\frac{14}{20} = \frac{7}{10}$

2 a 56, 58, 59, 65, 68, 69, 85, 86, 89, 95, 96, 98

b 12

c i 0.5 **ii** 0.75 **iii** 0.5 **iv** 0.17

3 a

		Die					
		1	2	3	4	5	6
Coin	H	H1	H2	H3	H4	H5	H6
	T	T1	T2	T3	T4	T5	T6

b i $\frac{1}{12}$ **ii** $\frac{3}{12} = \frac{1}{4}$ **iii** $\frac{4}{12} = \frac{1}{3}$

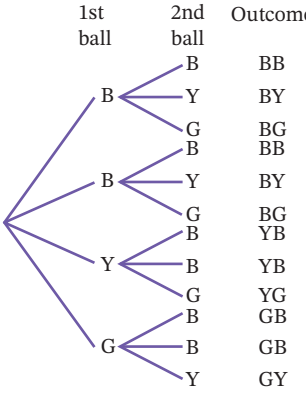
4 a

2nd die \ 1st die	+	1	2	3	4	5	6
	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

b i $\frac{2}{36} = \frac{1}{18}$ **ii** $\frac{6}{36} = \frac{1}{6}$ **iii** $\frac{26}{36} = \frac{13}{18}$

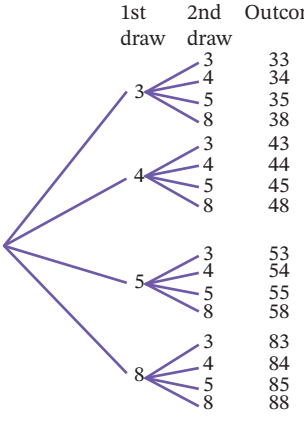
iv $\frac{30}{36} = \frac{5}{6}$ **v** $\frac{18}{36} = \frac{1}{2}$ **vi** $\frac{12}{36} = \frac{1}{3}$

5 a



b i $\frac{2}{12} = \frac{1}{6}$ **ii** $\frac{2}{12} = \frac{1}{6}$ **iii** $\frac{4}{12} = \frac{1}{3}$ **iv** $\frac{10}{12} = \frac{5}{6}$

6 a



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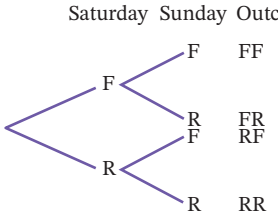
b **i** $\frac{4}{16}=\frac{1}{4}$ **ii** $\frac{8}{16}=\frac{1}{2}$ **iii** $\frac{6}{16}=\frac{3}{8}$ **iv** $\frac{2}{16}=\frac{1}{8}$

7 **a**



b **i** $\frac{1}{4}$ **ii** $\frac{1}{4}$ **iii** $\frac{3}{4}$

8 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$

d $\frac{1}{4}$ **e** $\frac{3}{4}$ **f** $\frac{1}{4}$

		1st card			
		K♥	K♦	Q♥	Q♦
2nd card	K♥	K♥ K♥	K♦ K♥	Q♥ K♥	Q♦ K♥
	K♦	K♥ K♦	K♦ K♦	Q♥ K♦	Q♦ K♦
	Q♥	K♥ Q♥	K♦ Q♥	Q♥ Q♥	Q♦ Q♥
	Q♦	K♥ Q♦	K♦ Q♦	Q♥ Q♦	Q♦ Q♦

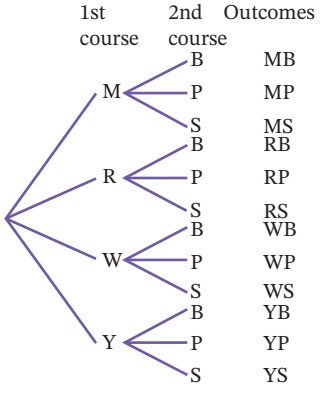
9

		1st die					
		1	2	3	4	5	6
2nd die	1	1, 1	2, 1	3, 1	4, 1	5, 1	6, 1
	2	1, 2	2, 2	3, 2	4, 2	5, 2	6, 2
	3	1, 3	2, 3	3, 3	4, 6	5, 3	6, 3
	4	1, 4	2, 4	3, 4	4, 4	5, 4	6, 4
	5	1, 5	2, 5	3, 5	4, 5	5, 5	6, 5
	6	1, 6	2, 6	3, 6	4, 6	5, 6	6, 6

a $\frac{1}{6}$ **b** $\frac{1}{18}$ **c** $\frac{11}{36}$ **d** $\frac{4}{9}$

10 **a** **i** MB, MP, MS, RB, RP, RS, WB, WP, WS, YB, YP, YS

ii



b $\frac{1}{2}$

11 **a** **i** 12, 13, 14, 15, 21, 23, 24, 25, 31, 32, 34, 35, 41, 42, 43, 45, 51, 52, 53, 54

ii

		1st digit				
		1	2	3	4	5
2nd digit	1		21	31	41	51
	2	12		32	42	52
	3	13	23		43	53
	4	14	24	34		54
	5	15	25	35	45	

b **i** $\frac{3}{5}$ **ii** $\frac{3}{5}$

Power plus

1 **a** $\frac{1}{6}$ **b** $\frac{5}{36}$ **c** $\frac{25}{216}$

2 Teacher to check the tree diagram.

a $\frac{1}{4}$ **b** $\frac{1}{16}$ **c** $\frac{3}{8}$

d $\frac{15}{16}$ **e** $\frac{5}{16}$ **f** $\frac{5}{16}$

3 **a** $\frac{5}{18}$ **b** $\frac{5}{18}$ **c** $\frac{5}{9}$ **d** $\frac{13}{18}$

4 **a** 6

b **i** $S \cup E = \{2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20\}$

ii $S \cap E = \{6, 12, 18\}$

c **i** $F = \{5, 10, 15, 20\}$

ii $n(F) = 4$ **ii** $S \cap F = \{10, 20\}$

Test yourself 12

1 **a** 60% **b** 16

2 **a** $\frac{1}{2}$ **b** $\frac{2}{13}$ **c** $\frac{4}{13}$

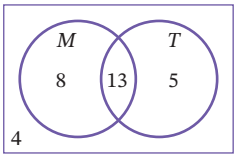
3 **a** **i** 0.36 **ii** 0.32 **iii** 0.83 **iv** 0.175

b **i** $0.\dot{3}$ **ii** $0.\dot{3}$ **iii** $0.8\dot{3}$ **iv** $0.1\dot{6}$

c They are similar but not equal. **d** C

4 **a** 13

b



c **i** $\frac{4}{15}$ **ii** $\frac{3}{5}$ **iii** $\frac{13}{30}$ **iv** $\frac{17}{30}$

5 **a** 210

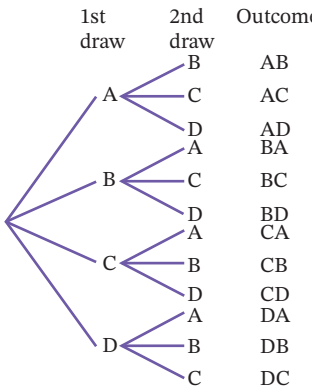
b **i** 48.1% **ii** 32.4% **iii** 51.9% **iv** 73.3%

c **i** $\frac{68}{109} = 62.4\%$

6 **a** AB, AC, AD, BA, BC, BD, CA, CB, CD, DA, DB, DC

ANSWERS

b



c **i** 0 **ii** $\frac{1}{6}$ **iii** $\frac{5}{6}$ **iv** $\frac{1}{2}$

7 **a**

Die 1 / Die 2	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

b **i** $\frac{1}{6}$ **ii** $\frac{5}{9}$ **iii** $\frac{1}{6}$

Chapter 13

SkillCheck

1 **a** **i** $\angle P$ and $\angle Y$, $\angle R$ and $\angle B$, $\angle T$ and $\angle W$

ii PT and YW, PR and YB, RT and BW

b **i** $\angle E$ and $\angle N$, $\angle G$ and $\angle V$, $\angle M$ and $\angle Q$, $\angle X$ and $\angle C$

ii EX and NC, XM and CQ, MG and QV, GE and VN

Exercise 13.01

1 **a** SAS **b** AAS **c** SAS **d** AAS
e SSS **f** AAS **g** RHS **h** AAS
i AAS **j** AAS **k** SSS **l** RHS

2 **a** B **b** B

3 **a** $\triangle ABC \equiv \triangle NMP$ (SAS)
b $\triangle KLQ \equiv \triangle FED$ (AAS)
c $\triangle AXT \equiv \triangle GYE$ (RHS)
d $\triangle MVR \equiv \triangle SPC$ (SSS)
e $\triangle BTV \equiv \triangle PSX$ (AAS)
f $\triangle CNQ \equiv \triangle DCE$ (SAS)
g $\triangle PTU \equiv \triangle LVW$ (SSS)
h $\triangle FLD \equiv \triangle HKW$ (SAS)

4 **a** $w = 24$ **b** $x = 28$, $p = 7$ **c** $a = 34$
d $b = 26$ **e** $y = 65$
f $x = 45$, $k = 8$

5 **a** 25 cm **b** $b = 25$

6 63.6

7 $a = 118$, $k = 43$, $x = 28$

8 **a** No **b** No **c** Yes, SAS
d Yes, RHS **e** No **f** No

Exercise 13.02

1 In $\triangle WXY$ and $\triangle WZY$
$\angle X = \angle Z = 90°$ (given)
$XY = ZY$ (given)
WY is common
$\therefore \triangle WXY \equiv \triangle WZY$ (RHS)

2 In $\triangle ABC$ and $\triangle DCB$
$\angle ABC = \angle DCB$ (given)
$AB = DC$ (given)
BC is common
$\therefore \triangle ABC \equiv \triangle DCB$ (SAS)

3 In $\triangle DEH$ and $\triangle FEG$
$\angle D = \angle F$ (alternate angles, $HD \parallel GF$)
$\angle H = \angle G$ (alternate angles, $HD \parallel GF$)
$EG = EH$ (given)
$\therefore \triangle DEH \equiv \triangle FEG$ (AAS)

4 In $\triangle ABC$ and $\triangle ADC$
$BC = DC$ (given)
$AB = AD$ (given)
AC is common
$\therefore \triangle ABC \equiv \triangle ADC$ (SSS)

5 In $\triangle PQN$ and $\triangle LQM$
$\angle PQN = \angle LQM$ (vertically opposite angles)
$PQ = LQ$ (given)
$NQ = MQ$ (given)
$\therefore \triangle PQN \equiv \triangle LQM$ (SAS)

6 In $\triangle DEG$ and $\triangle EDF$
$\angle DEG = \angle EDF$ (given)
$GE = FD$ (given)
ED is common
$\therefore \triangle DEG \equiv \triangle EDF$ (SAS)

7 In $\triangle WXY$ and $\triangle YVW$
$XY = WV$ (given)
$WX = VY$ (given)
WY is common
$\therefore \triangle WXY \equiv \triangle YVW$ (SSS)

8 In $\triangle PVQ$ and $\triangle QTR$
$\angle VPQ = \angle TQR$ (corresponding angles, $PW \parallel QT$)
$\angle VQP = \angle TRQ$ (corresponding angles, $RW \parallel VQ$)
$PQ = QR$ (given)
$\therefore \triangle PVQ \equiv \triangle QTR$ (AAS)

9 In $\triangle BCX$ and $\triangle CBY$
$\angle BXC = \angle CYB$ ($CX \perp AB$, $BY \perp AC$)
$XC = YB$ (given)
BC is common
$\therefore \triangle BCX \equiv \triangle CBY$ (RHS)

10 In $\triangle FNM$ and $\triangle TMN$
$\angle FNM = \angle TMN$ (alternate angles, $FN \parallel TM$)
$\angle FMN = \angle TNM$ (alternate angles, $TN \parallel FM$)
NM is common
$\therefore \triangle FNM \equiv \triangle TMN$ (AAS)

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11 In $\triangle WYX$ and $\triangle ZYX$
$XW = XZ$ (given)
$WY = ZY$ (Y is the midpoint of WZ)
XY is common
$\therefore \triangle WYX \equiv \triangle ZYX$ (SSS)

12 In $\triangle EFH$ and $\triangle GHF$
$EF = GH$ (given)
$EH = GF$ (given)
FH is common
$\therefore \triangle EFH \equiv \triangle GHF$ (SSS)

13 In $\triangle AOB$ and $\triangle COD$
$OA = OC$ (equal radii)
$OB = OD$ (equal radii)
$AB = CD$ (given)
$\therefore \triangle AOB \equiv \triangle COD$ (SSS)

14 In $\triangle PQT$ and $\triangle PRM$
$\angle Q = \angle R$ (given)
$PQ = PR$ (sides opposite equal angles of isosceles triangle)
$QT = RM$ (given)
$\therefore \triangle PQT \equiv \triangle PRM$ (SAS)

15 In $\triangle AMD$ and $\triangle CNB$
$\angle A = \angle C$ (opposite angles of a parallelogram are equal)
$AD = CB$ (opposite sides of a parallelogram are equal)
$AM = AB - BM$
But $AB = CD$ (opposite sides of a parallelogram) and $BM = DN$ (given)
$\therefore AM = CD - DN$
$= CN$
$\therefore \triangle AMD \equiv \triangle CNB$ (SAS)

16 In $\triangle OAC$ and $\triangle OBC$
$OA = OB$ (equal radii)
OC is common
$\angle ACO = \angle BCO = 90°$ ($OC \perp AB$)
$\therefore \triangle OAC \equiv \triangle OBC$ (RHS)

17 a In $\triangle TPM$ and $\triangle MST$
$PT = SM$ (given)
$PM = ST$ (given)
MT is common
$\therefore \triangle TPM \equiv \triangle MST$ (SSS)

b $\angle PMT = \angle STM$ (matching angles of congruent triangles)

c $PM \parallel ST$ (alternate angles proved equal in **b**)

18 In $\triangle FED$ and $\triangle EFC$
$\angle FED = \angle EFC = 90°$ (angles in a rectangle)
$ED = FC$ (opposite sides of a rectangle are equal)
FE is common
$\therefore \triangle FED \equiv \triangle EFC$ (SAS)
$\therefore FD = EC$ (matching sides of congruent triangles)

19 In $\triangle TSK$ and $\triangle LPK$
$\angle T = \angle L$ (alternate angles on parallel lines, $TS \parallel PL$)
$\angle S = \angle P$ (alternate angles on parallel lines, $TS \parallel PL$)
$TK = LK$ (K is the midpoint of TL)
$\therefore \triangle TSK \equiv \triangle LPK$ (AAS)

20 a $\angle XYB = \angle ABY$ (alternate angles, $XY \parallel AB$)
$\angle XYZ = 180° - \angle XYB$ (angles on a straight line)
$= 180° - \angle ABY$ ($\angle XYB = \angle ABY$)
$= \angle ABC$

b In $\triangle XYZ$ and $\triangle ABC$
$\angle XYZ = \angle ABC$ (proved in **a**)
$\angle XZY = \angle ACB$ (alternate angles, $XZ \parallel AC$)
$ZY = CB$ (given)
$\therefore \triangle XYZ \equiv \triangle ABC$ (AAS)

21 a In $\triangle LMN$ and $\triangle LKP$
$LM = LK$ (given)
$MN = KP$ (given)
$\angle M = \angle K$ (angles opposite equal sides of isosceles triangle LMK)
$\therefore \triangle LMN \equiv \triangle LKP$ (SAS)

b $LN = LP$ (matching sides of congruent triangles)
$\therefore \triangle LNP$ is isosceles (two sides equal)

22 a In $\triangle PQT$ and $\triangle PRT$
$PQ = PR$ (equal adjacent sides of a kite)
$TQ = TR$ (equal adjacent sides of a kite)
PT is common
$\therefore \triangle PQT \equiv \triangle PRT$ (SSS)

b $\angle Q = \angle R$ (matching angles of congruent triangles)

c $\angle QTP = \angle RTP$ (matching angles of congruent triangles)
$\therefore PT$ bisects $\angle QTR$

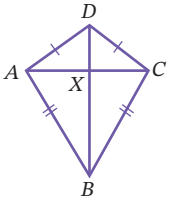
Exercise 13.03

1 a In $\triangle ABD$ and $\triangle CBD$
$AB = CB$ (given)
$AD = CD$ (given)
BD is common
$\therefore \triangle ABD \equiv \triangle CBD$ (SSS)

b $\angle A = \angle C$ (matching angles of congruent triangles)

c $\angle ADB = \angle CDB$ (matching angles of congruent triangles)
$\angle ABD = \angle CBD$ (matching angles of congruent triangles)
$\therefore DB$ bisects $\angle ADC$ and $\angle ABC$

d



e In $\triangle DAX$ and $\triangle DCX$
$AD = CD$ (given)
DX is common
$\angle ADX = \angle CDX$ (Matching angles of congruent triangles proved in **a**)
$\therefore \triangle ADX \equiv \triangle CDX$ (SAS)

f $AX = CX$ (matching sides of congruent triangles)
$\therefore X$ is midpoint of AC

g $\angle AXD = \angle CXD$ (matching angles of congruent triangles) and $\angle AXD + \angle CXD = 180°$ (angles on a straight line)
$\therefore \angle AXD = \angle CXD = 90°$
$\therefore DX \perp AC$
$\therefore BD \perp AC$

2 b In $\triangle WXZ$ and $\triangle YZX$
$\angle WXZ = \angle YZX$ (alternate angles, $WX \parallel YZ$)
$\angle WZX = \angle YXZ$ (alternate angles, $WZ \parallel YX$)
XZ is common
$\therefore \triangle WXZ \equiv \triangle YZX$ (AAS)

c $\angle W = \angle Y$ (matching angles of congruent triangles)

d In $\triangle WXY$ and $\triangle YZW$
$\angle XYW = \angle ZWY$ (alternate angles, $XY \parallel ZW$)
$\angle XWY = \angle ZYW$ (alternate angles, $WX \parallel YZ$)
WY is common
$\therefore \triangle WXY \equiv \triangle YZW$ (AAS)

e $\angle WXY = \angle YZW$ (matching angles of congruent triangles)

f Opposite angles of a parallelogram are equal.

3 a In $\triangle SPW$ and $\triangle TPW$
$\angle S = \angle T$ (given)
WP is common
$\angle WPS = \angle WPT = 90°$ ($WP \perp ST$)
$\therefore \triangle SPW \equiv \triangle TPW$ (AAS)

b $SW = TW$ (matching sides of congruent triangles)
$\therefore$ Sides opposite equal angles in a triangle are equal (SW is opposite to $\angle T$, TW is opposite to $\angle S$)

4 a In $\triangle VUS$ and $\triangle TUS$
$VU = TU$ (given)
$VS = TS$ (given)
SU is common
$\therefore \triangle VUS \equiv \triangle TUS$ (SSS)

b $\angle VUS = \angle TUS$ (matching angles of congruent triangles)
$\angle VSU = \angle TSU$ (matching angles of congruent triangles)
$\therefore US$ bisects $\angle VUT$ and $\angle VST$.

5 a In $\triangle ABD$ and $\triangle CDB$
$\angle ABD = \angle CDB$ (alternate angles, $AB \parallel CD$)
$\angle ADB = \angle CBD$ (alternate angles, $AD \parallel CB$)
BD is common
$\therefore \triangle ABD \equiv \triangle CDB$ (AAS)

b $AD = CB$ (matching sides of congruent triangles)
$AB = CD$ (matching sides of congruent triangles)
$\therefore$ opposite sides of a parallelogram are equal.

6 a In $\triangle BEL$ and $\triangle GHL$
$\angle BEL = \angle GHL$ (alternate angles, $BE \parallel GH$)
$\angle BLE = \angle GLH$ (vertically opposite angle are equal)
$BE = GH$ (given)
$\therefore \triangle BEL \equiv \triangle GHL$ (AAS)

b $BL = GL$ (matching sides of congruent triangles)
$EL = HL$ (matching sides of congruent triangles)
$\therefore$ diagonals of a rhombus bisect each other.

c $\angle BEH = \angle BHE$ (angles opposite equal sides in isosceles triangle)
$\therefore \angle BEL = \angle BHL$

d In $\triangle BEL$ and $\triangle BHL$
$\angle BEL = \angle BHL$ (proven in **c**)
$BE = BH$ (given)
$EL = HL$ (proven in **b**)
$\therefore \triangle BEL \equiv \triangle BHL$ (SAS)

e $\angle BLE = \angle BLH$ (matching angles of congruent triangles) and $\angle BLE + \angle BLH = 180°$ (angles on a straight line)
$\therefore \angle BLE = \angle BLH = 90°$
$\therefore$ The diagonals of a rhombus at right angles

7 a In $\triangle ABX$ and $\triangle ACX$
$AB = AC$ (given)
$BX = CX$ (X is the midpoint of BC)
AX is common
$\therefore \triangle ABX \equiv \triangle ACX$ (SSS)
$\angle B = \angle C$ (matching angles of congruent triangles)

b In $\triangle BAY$ and $\triangle BCY$
$BA = BC$ (given)
$AY = CY$ (Y is the midpoint of AC)
BY is common
$\therefore \triangle BAY \equiv \triangle BCY$ (SSS)
$\angle A = \angle C$ (matching angles of congruent triangles)

c Since $\angle B = \angle C$, $\angle A = \angle C$,
$\angle A = \angle B = \angle C$
$\angle A + \angle B + \angle C = 180°$ (angle sum of a triangle)
$\therefore \angle A = \angle B = \angle C = 60°$

8 a In $\triangle ACD$ and $\triangle CAB$
$AD = CB$ (given)
$AB = CD$ (given)
AC is common
$\therefore \triangle ACD \equiv \triangle CAB$ (SSS)

b $\angle DAC = \angle BCA$ (matching angles of congruent triangles)
$\therefore AD \parallel BC$ (alternate angles are equal)
$\angle DCA = \angle BAC$ (matching angles of congruent triangles)
$\therefore AB \parallel CD$ (alternate angles are equal)

c $ABCD$ is a parallelogram

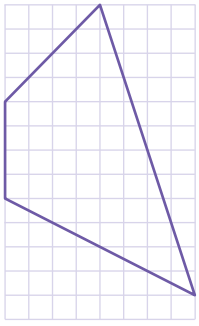
9 a In $\triangle PMY$ and $\triangle PNY$
$\angle M = \angle N$ (given)
$\angle MPY = \angle NPY$ (PY bisects $\angle MPN$)
PY is common
$\therefore \triangle PMY \equiv \triangle PNY$ (AAS)

b $MY = NY$ (matching sides of congruent triangles)
$\angle PYM = \angle PYN$ (matching angles of congruent triangles) and $\angle PYM + \angle PYN = 180°$ (angles on a line)
$\therefore \angle PYM = \angle PYN = 90°$
$\therefore PY \perp MN$

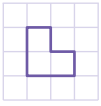
Exercise 13.04

1 **a** 2 **b** $\frac{2}{3}$ **c** $\frac{4}{5}$ **d** $\frac{8}{3}$
e $\frac{2}{3}$ **f** $\frac{4}{3}$ **g** $\frac{9}{5}$ **h** $\frac{3}{2}$

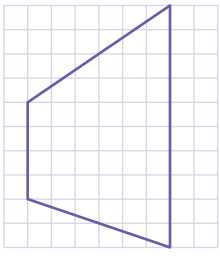
2 **a**



b



c



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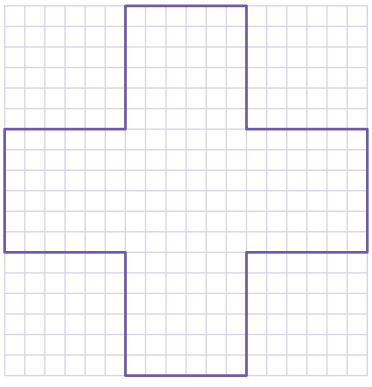
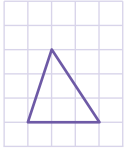
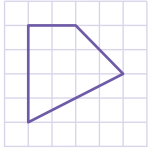


e



f



3 **a** No **b** Yes **c** No **d** Yes
e No **f** Yes **g** No **h** Yes

4 **a** **i** $\angle A$ and $\angle L$, $\angle B$ and $\angle M$, $\angle C$ and $\angle K$
ii AC and LK, AB and LM, BC and MK
iii $\triangle ACB \,|||\, \triangle LKM$
b **i** $\angle W$ and $\angle D$, $\angle X$ and $\angle E$, $\angle Y$ and $\angle G$, $\angle Z$ and $\angle H$
ii WX and DE, XY and EG, YZ and GH, WZ and DH
iii $ZYXW \,|||\, HGED$
c **i** $\angle T$ and $\angle F$, $\angle R$ and $\angle W$, $\angle P$ and $\angle V$, $\angle M$ and $\angle S$, $\angle J$ and $\angle K$
ii TR and FW, RP and WV, PM and VS, MJ and SK, TJ and FK
iii $TRPMJ \,|||\, FWVSK$
d **i** $\angle Q$ and $\angle N$, $\angle K$ and $\angle C$, $\angle B$ and $\angle G$
ii QB and NG, BK and GC, KQ and CN
iii $\triangle QKB \,|||\, \triangle NCG$
e **i** $\angle F$ and $\angle X$, $\angle E$ and $\angle W$, $\angle C$ and $\angle Y$
ii FE and XW, CE and YW, CF and YX
iii $\triangle FEC \,|||\, \triangle XWY$
f **i** $\angle K$ and $\angle T$, $\angle L$ and $\angle Q$, $\angle M$ and $\angle S$, $\angle P$ and $\angle U$
ii KL and TQ, KP and TU, PM and US, LM and QS
iii $PMLK \,|||\, USQT$

Mental skills 13

2 **a** $\frac{2}{3}$ **b** $\frac{4}{5}$ **c** $\frac{5}{7}$ **d** $\frac{1}{2}$
e $\frac{1}{4}$ **f** $\frac{1}{6}$ **g** $\frac{5}{6}$ **h** $\frac{2}{5}$
i 5 : 9 **j** 5 : 9 **k** 9 : 20 **l** 4 : 5
m 9 : 7 **n** 4 : 3 **o** $\frac{3}{5}$ **p** $\frac{4}{35}$

3 **a** $\frac{17}{40}$ **b** $\frac{2}{3}$ **c** $\frac{16}{25}$ **d** $\frac{1}{4}$
e $\frac{5}{24}$ **f** $\frac{2}{}$

Exercise 13.05

1 **a** Yes $(\frac{8}{12}=\frac{20}{30}=\frac{2}{3})$
b Yes $(\frac{12}{20}=\frac{27}{45}=\frac{21}{35}=\frac{3}{5})$
c Yes $(\frac{4}{6}=\frac{6}{9}=\frac{6\frac{2}{3}}{10}=\frac{10}{15}=\frac{2}{3})$
d Yes, all squares are similar.
e Yes, matching angles are equal.
f Yes, all equilateral triangle are similar.

2 **a** $k = 14, x = 6$
b $y = 12\frac{1}{2}, p = 19\frac{1}{5}$ **c** $x = 21$
d $a = 12, q = 1\frac{2}{3}, w = 3\frac{1}{3}$ **e** $d = 16$
f $b = 21\frac{7}{18}$ **g** $w = 10\frac{4}{5}, x = 23\frac{1}{3}$
h $c = 19\frac{1}{4}$

3 $d = 15$ **4** $a = 21\frac{3}{7}$

5 $13\frac{1}{3}$ m **6** 8.9 m **7** 21 m

8 **a** A and D, scale factor $= 2$
b B and D, scale factor $= \frac{3}{4}$
c C and D, scale factor $= \frac{3}{5}$
d A and D, scale factor $= \frac{1}{2}$

ANSWERS

Exercise 13.06

1 1 : 500 **2** 1 : 35 **3** 2 : 1
4 3 : 1 **5** 1 : 8050
6 **a** 135 m **b** 378 m
7 2.1 mm
8 Scaled dimensions: 3.2 cm long by 4.4 cm tall
9 **a** Teacher to check: base 3 cm
b approx. 13 m
10 2120 km **11** **a** 6.2 m **b** 4 m
12 **a** 100 km **b** 80 km **c** 54 km **d** 224 km

Exercise 13.07

1 1 : 16 **2** 1 : 6.25 or 4 : 25
3 **a** 100 : 1 **b** 30.25 : 1 or 121 : 4
c 9 : 16 **d** 25 : 4 **e** 16 : 49
f 1 : 100 **g** 64 : 25 **h** 4 : 81
4 **a** 1 : 9 **b** 9 : 4 **c** 49 : 16
d 25 : 121 **e** 25 : 9 **f** 16 : 25
5 **a** 8 : 7 **b** 6 : 1 **c** 3 : 20 **d** 1 : 4.5
6 **a** 5 : 3 **b** 4 : 7
7 3.5 **8** 3072 cm^2 **9** 162 cm^2
10 1024 mm^2 **11** $28\frac{2}{7}$ m^2 **12** 154 mm
13 22.5 mm **14** 12.5 cm
15 Increased 9 times
16 reduced by a factor of 2
17 9 : 16 **18** 50 cm^2

Exercise 13.08

1 **a** AA **b** SSS **c** RHS **d** SAS
e SAS **f** AA **g** SAS **h** RHS
i SSS **j** SSS **k** SAS **l** SAS
2 C
3 **a** B and C (SAS) **b** B and C (RHS)
c B and D (SSS) **d** C and D (AA)
4 **a** $\triangle EFD \,|||\, \triangle PMR$ (SSS)
b $\triangle UTW \,|||\, \triangle KAM$ (SAS)
c $\triangle UZV \,|||\, \triangle HPJ$ (RHS)
d $\triangle WLB \,|||\, \triangle XAT$ (AA)
e $\triangle KWP \,|||\, \triangle GEF$ (SSS)
f $\triangle LEA \,|||\, \triangle RBX$ (SAS)
5 In 2 triangles, if 2 pairs of matching angles are equal, then the remaining pair of matching angles must also be equal, due to the angle sum of all triangles being 180°

Exercise 13.09

1 **a** In $\triangle DEF$ and $\triangle RPM$

$\frac{DE}{RP} = \frac{6}{4} = \frac{3}{2}$

$\frac{EF}{PM} = \frac{9}{6} = \frac{3}{2}$

$\frac{FD}{MR} = \frac{12}{8} = \frac{3}{2}$

$\therefore \triangle DEF \,|||\, \triangle RPM$ (3 pairs of matching sides in proportion, or 'SSS')

b In $\triangle WUT$ and $\triangle MKA$

$\frac{WU}{MK} = \frac{15}{20} = \frac{3}{4}$

$\frac{UT}{KA} = \frac{9}{12} = \frac{3}{4}$

$\angle U = \angle K = 65°$

$\therefore \triangle WUT \,|||\, \triangle MKA$ (2 pairs of matching sides in proportion and the included angles equal, or 'SAS')

c In $\triangle UVZ$ and $\triangle HJP$

$\frac{UZ}{HP} = \frac{20}{12} = \frac{5}{3}$

$\frac{VZ}{JP} = \frac{25}{15} = \frac{5}{3}$

$\angle U = \angle H = 90°$

$\therefore \triangle UVZ \,|||\, \triangle HJP$ (hypotenuses and another pair of matching sides of right-angled triangles in proportion, or 'RHS')

d In $\triangle WBL$ and $\triangle XTA$

$\angle B = \angle T = 118°$

$\angle L = \angle A = 27°$

$\therefore \triangle WBL \,|||\, \triangle XTA$ (equiangular, or 'AA')

e In $\triangle KPW$ and $\triangle GFE$

$\frac{KP}{GF} = \frac{13.75}{11} = \frac{5}{4}$

$\frac{PW}{FE} = \frac{12}{9.6} = \frac{5}{4}$

$\frac{WK}{EG} = \frac{16}{12.8} = \frac{5}{4}$

$\therefore \triangle KPW \,|||\, \triangle GFE$ (3 pairs of matching sides in proportion, or 'SSS')

f In $\triangle LAE$ and $\triangle RXB$

$\frac{LA}{RX} = \frac{5.6}{8.4} = \frac{2}{3}$

$\frac{AE}{XB} = \frac{8}{12} = \frac{2}{3}$

$\angle A = \angle X = 90°$

$\therefore \triangle LAE \,|||\, \triangle RXB$ (2 pairs of matching sides in proportion and the included angles equal, or 'SAS')

2 **a** In $\triangle XLM$ and $\triangle XYW$

$\angle X$ is common

$\angle XLM = \angle XYW$ (corresponding angles, $LM \,||\, YW$)

$\therefore \triangle XLM \,|||\, \triangle XYW$ (equiangular, or 'AA')

b In $\triangle ABX$ and $\triangle PTX$

$\angle A = \angle P$ (alternate angles, $AB \,||\, PT$)

$\angle B = \angle T$ (alternate angles, $AB \,||\, PT$)

$\therefore \triangle ABX \,|||\, \triangle PTX$ (equiangular, or 'AA')

c In $\triangle PLM$ and $\triangle PTW$

$\angle P$ is common

$\angle PLM = \angle PTW = 90°$ (given)

$\therefore \triangle PLM \,|||\, \triangle PTW$ (equiangular, or 'AA')

d In $\triangle KHL$ and $\triangle KMG$

$\angle K$ is common

$\angle KHL = \angle KMG$ (given)

$\therefore \triangle KHL \,|||\, \triangle KMG$ (equiangular, or 'AA')

e In $\triangle TXY$ and $\triangle KXG$

$\angle T = \angle K = 90°$ (given)

$\angle TXY = \angle KXG$ (vertically opposite angles)

$\therefore \triangle TXY \,|||\, \triangle KXG$ (equiangular, or 'AA')

f In $\triangle WZY$ and $\triangle WXZ$
$\angle W$ is common
$\angle WZY = \angle WXZ = 90°$ (given)
$\therefore \triangle WZY \,|||\, \triangle WXZ$ (equiangular, or 'AA')

3 **a** $x = 7\frac{1}{7}$ **b** $x = 8$ **c** $x = 1\frac{3}{8}$
d $x = 12\frac{3}{4}$ **e** $x = 6\frac{2}{3}$ **f** $x = 6\frac{1}{2}$

4 **a** In $\triangle KGF$ and $\triangle KHM$
$\angle K$ is common
$\angle KGF = \angle KHM$ (corresponding angles on parallel lines, $GF \parallel HM$)
$\therefore \triangle KGF \,|||\, \triangle KHM$ (equiangular, or 'AA')
b $m = 6\frac{6}{7}$

5 In $\triangle BCF$ and $\triangle DCE$
$\angle F = \angle E$ (given)
$\angle BCF = \angle DCE$ (vertically opposite angles)
$\therefore \triangle BCF \,|||\, \triangle DCE$ (equiangular, or 'AA')
$x = 7\frac{1}{2}$

6 **a** In $\triangle TFR$ and $\triangle TPD$
$\angle T$ is common
$\angle TFR = \angle TPD = 90°$ (given)
$\therefore \triangle TFR \,|||\, \triangle TPD$ (equiangular, or 'AA')
b $TR = 14.4$ cm

7 **a** In $\triangle CDE$ and $\triangle FGE$
$\angle C = \angle F$ (alternate angles on parallel lines, $CD \parallel FG$)
$\angle D = \angle G$ (alternate angles on parallel lines, $CD \parallel FG$)
$\therefore \triangle CDE \,|||\, \triangle FGE$ (equiangular, or 'AA')
b $k = 11\frac{1}{4}$ or 11.25

8 In $\triangle AXM$ and $\triangle AYN$,
$\angle A$ is common
$\angle AXM = \angle AYN = 90°$ (given)
$\therefore \triangle AXM \,|||\, \triangle AYN$ (equiangular, or 'AA')
$AX = 12$

9 **a** In $\triangle FGE$ and $\triangle EFH$
$\angle F$ is common
$\angle FEG = \angle EHF$ (given)
$\therefore \triangle FGE \,|||\, \triangle EFH$ (equiangular, or 'AA')
b $FH = 25$ cm

Power plus

1 Teacher to check.

2 **a** $\angle E = \angle D = 90°$, $BM = CM$
b Vertically opposite angles are equal.
c $\angle E = \angle D = 90°$
$\angle BME = \angle CMD$ (see part **b**)
$BM = CM$
$\therefore \triangle BME \equiv \triangle CMD$ (AAS)
d Matching sides of congruent triangles.
e Matching angles of congruent triangles.

3 **a** $VW \parallel XY$
b Alternate angles are equal, $VW \parallel XY$
c $\angle V = \angle Y$ (proved in **b**)
$\angle W = \angle X$ (proved in **b**)
$\therefore \triangle VWD \,|||\, \triangle XYD$ (AA)
d $k = 9$, $m = 30$

4 33.25 m

5 **a** true **b** false **c** true
d false **e** true **f** true

Test yourself 13

1 **a** SAS **b** RHS **c** SSS

2 In $\triangle ACE$ and $\triangle EFA$
$\angle AEC = \angle EAF$ (alternate angles, $AF \parallel EC$)
$\angle CAE = \angle FEA$ (alternate angles, $AC \parallel EF$)
AE is common
$\therefore \triangle AEC \equiv \triangle EFA$ (AAS)

3 In $\triangle MNP$ and $\triangle NMT$
$NP = MT$ (given)
$\angle MNP = \angle NMT$ (given)
MN is common
$\therefore \triangle MNP \equiv \triangle NMT$ (SAS)

4 In $\triangle ABD$ and $\triangle BAC$
a $\angle BAD = \angle ABC = 90°$ (angles in a rectangle)
$AD = BC$ (opposite sides of a rectangle)
AB is common
$\therefore \triangle ABD \equiv \triangle BAC$ (SAS)
b $\therefore BD = AC$ (matching sides of congruent triangles)
$\therefore$ the diagonals of a rectangle are equal.

5 **a** 2 **b** $\frac{3}{5}$ **c** 1.5

6 **a** $y = 8.75$, $p = 6.4$ **b** $a = 18$, $w = 27$

7 **a** 8.2 m **b** 4.2 m

8 **a** 25 : 81 **b** 16 : 9

9 100 cm

10 **a** 2 pairs of matching sides in proportion and the included angles equal, or 'SAS'
b 3 pairs of matching sides in proportion, or 'SSS'
c equiangular, or 'AA'

11 **a** In $\triangle ABE$ and $\triangle CBD$
$\angle A = \angle C$ (alternate angles, $AE \parallel CD$)
$\angle E = \angle D$ (alternate angles, $AE \parallel CD$)
$\therefore \triangle ABE \,|||\, \triangle CBD$ (equiangular, or 'AA')
b 12.25 cm

Practice set 4

1 **a** no **b** yes **c** yes

2 **a** $\frac{3}{2}$ **b** 7.5 m

3 0.8

4 **a** **i** $\frac{5}{9}$ **ii** $\frac{2}{3}$ **iii** $\frac{1}{9}$ **iv** $\frac{16}{27}$
b $\frac{13}{22}$

5 625 m

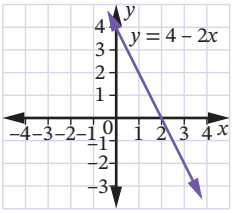
6 **a** $\sqrt{26}$ units **b** 5.7 units

7 $PQ = \sqrt{26} = QR$ Since 2 sides are equal it is an isosceles triangle

8 (1, 0)

9 **a** $\frac{1}{5}$ **b** -1

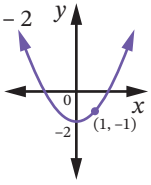
10



a 2 **b** 4

11 **a** $\frac{4}{15}$ **b** 107

12 **a**



b (0, –2) **c** concave up

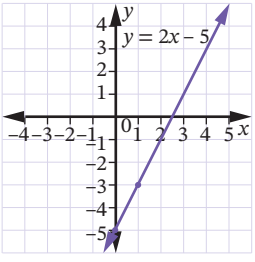
13 **a** SAS **b** AAS
c SAS **d** RHS

14 $y = 4$

15 **a** $\frac{1}{5}$ **b** $\frac{2}{3}$

16 **a** gradient $\frac{1}{2}$, y-intercept 3

b $y = 2x - 5$



17 **a** 188 kPa **b** 125 m

18

		1st die					
		1	2	3	4	5	6
2nd die	1	1, 1	2, 1	3, 1	4, 1	5, 1	6, 1
	2	1, 2	2, 2	3, 2	4, 2	5, 2	6, 2
	3	1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
	4	1, 4	2, 4	3, 4	4, 4	5, 4	6, 4
	5	1, 5	2, 5	3, 5	4, 5	5, 5	6, 5
	6	1, 6	2, 6	3, 6	4, 6	5, 6	6, 6

a $\frac{1}{18}$ **b** $\frac{11}{36}$ **c** $\frac{1}{6}$ **d** $\frac{1}{2}$

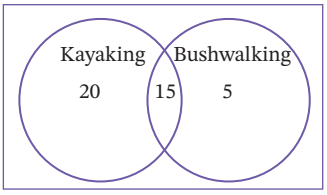
19 **a** -3 **b** $y = -3x + 1$

20 $x^2 + y^2 = 16$

21 equiangular or AA

22 **a** C **b** A **c** B

23 **a**



b **i** $\frac{3}{8}$ **ii** $\frac{5}{8}$

24 **a** Teacher to check (SSS)
b 90° **c** bisect, right angles

25 1.8 m

26 **a** Teacher to check (equiangular or AA)
b **i** $6\frac{10}{13}$ **ii** $8\frac{6}{13}$

General practice

1 **a** $\sqrt{5}$ **b** $9\sqrt{3}$ **c** -140 **d** $18\sqrt{2}$

2 $t = \pm\sqrt{\frac{2my - 2k}{m}}$ or $t = \pm\sqrt{2y - \frac{2k}{m}}$

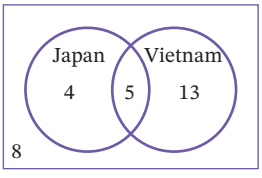
3 **a** 288 cm^3 **b** 910 m^3 **c** 84 m^3
d 4536.5 mm^3

4 **a** 23
b **i** 9 **ii** 6 **iii** 6.2 **iv** 6
c 1 **d** Yes, around 4 to 6.

5 **a** ND **b** ND **c** NC **d** C

6 **a** In $\triangle DTC$ and $\triangle BTA$
$TD = TB$ (given)
$TC = TA$ (given)
$\angle DTC = \angle ATB$ (vertically opposite angles equal)
$\therefore \triangle DTC \equiv \triangle BTA$ (SAS)
b $\angle CDT = \angle ABT$ (matching angles in congruent triangles)
$\therefore DC \parallel AB$ (alternate angles on parallel lines equal)

7 **a**



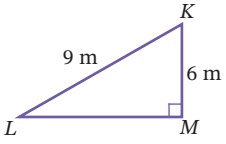
b **i** $\frac{4}{15}$ **ii** $\frac{17}{30}$ **iii** $\frac{13}{30}$

8 **a** 144 km/h **b** 0.6 kg/min

9 $y = 2x - 4$

10 **a** \$1571.48 **b** \$6833.33

11 **a**



b 6.71 m

9780170453325

12 **a** $(y-4)(y+2)$ **b** $(m-3)(m-5)$
c $5(m+7)(m-7)$

13 C

14 **a** 63.6 cm^2 **b** 160.2 cm^2 **c** 3.4 m^2

15 $1008.15

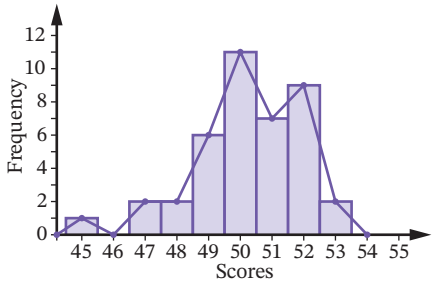
16 **a** k^{12} **b** d^5 **c** $15p^3q^4$
d $4k^4$ **e** $\frac{8r^3}{125}$ **f** 5

17 **a**

Score	Tally	f	fx
45	|	1	45
47	||	2	94
48	||	2	96
49	𝍸 |	6	294
50	𝍸 𝍸 |	11	550
51	𝍸 ||	7	357
52	𝍸 |||	9	468
53	||	2	106
	Total	40	2010

b 50.25

c



d 50 **e** 50

18 **a** $3-6y$ **b** $2x^2-7x$
c $-4m+4m^2$ **d** $5x^2-13x-28$

19 **a** $\frac{27}{125a^3}$ **b** $2\sqrt[4]{x}$ **c** $-2\sqrt[3]{p}$
d $\left(\sqrt[3]{r}\right)^2$ or $\sqrt[3]{r^2}$

20 D

21 **a** 35.93 m **b** 62.5 m^2

22 D **23** $761.10

24 **a** $2(y+4)$ **b** $5a(1-3b)$ **c** $-2b(4a-1)$

25 **a** 6.36 **b** 15.63 **c** 10.76

26 52.7%

27 **a** $m = 55$ (opposite angles of parallelogram)
b $e = 41$ (co-interior angles on parallel lines)
c $k = 8$ (angles in a right angle)

28 $y = 5$

29 **a** $\left(\frac{1}{2}, 2\frac{1}{2}\right)$ **b** $\sqrt{82}$ **c** $\frac{1}{9}$

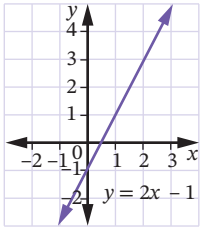
30 2.5 cm

31 **a** rectangle **b** equilateral triangle
c rectangle **d** isosceles triangle

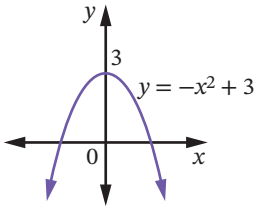
32 **a** 2 **b** 33
c negatively skewed

33 **a** equiangular (AA) **b** $k=13\frac{1}{5}$

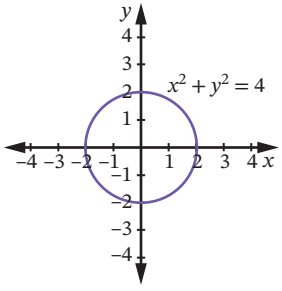
34 **a** **i**



ii



iii



b gradient 2, y-intercept -1
c (0, 3)

35 $1925 **36** $88 **37** B

38 **a** 396 cm^2 **b** 476 cm^2 **c** 792 cm^2

39 **a** $m=8\frac{2}{5}$ **b** $k=4\frac{1}{2}$ **c** $y=1\frac{1}{2}$
d $x=\pm3$ **e** $x=20$ **f** $a=7\frac{1}{3}$

40 **a** 7.14×10^5 **b** 3.5×10^{-4} **c** 1.5763×10^7

41 **a** 2.4×10^{15} **b** 4.2×10^{-11}

42 **a** 35° 30′ **b** 35° 26′ **c** 43° 21′

43 **a** **i** 0.15 **ii** 0.17 **iii** 0.47
iv 0.64 **v** 0.70
b 180; the observed frequency of 173 is close to this.

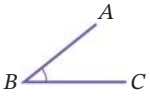
44 **a** 1080° **b** 135° **c** 45° **d** 360°

ANSWERS

GLOSSARY AND INDEX

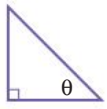
A

acute angle A 'sharp' angle between 0° and 90°, such as the marked angle in the diagram. (p. 224)



acute-angled triangle A triangle with all 3 angles acute. (p. 224)

adjacent side In a right-angled triangle, the side 'next to' the given angle, leading to the right angle. (p. 125)



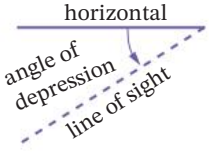
adjacent side (to θ)

algebraic expression A number written in algebraic form using variables, for example, $2xy + 4y - 5$. (p. 4)

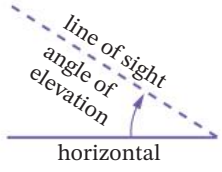
algebraic term See **term (of an expression)**. (p. 5)

allowable (tax) deduction A part of a person's annual income that is not taxed, such as work-related expenses or donations to charities. All deductions are subtracted from annual income to determine **taxable income**. (p. 296)

angle of depression The angle of looking **down**, measured from the horizontal. (p. 154)



angle of elevation The angle of looking **up**, measured from the horizontal. (p. 154)



angle sum The total of the sizes of the angles in a shape. The angle sum of a triangle is 180°. (p. 224)

annual leave loading (or **holiday loading**) Extra payment to a worker during annual leave that is 17.5% of 4 weeks' pay. (p. 293)

annulus A ring shape between 2 different-sized circles with the same centre. (p. 373)

arc Part of the circumference of a circle. (p. 372)

area The amount of surface covered by a shape, measured in square units. (p. 362)

ascending order Going up, increasing, from smallest to largest (1-2-3). The opposite of **descending order**. (p. 92)

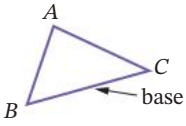
at least Referring to the smallest number, for example, 'at least 2' means 2, 3, 4, ..., or '2 or more'. (p. 460)

average See **mean**. (p. 311)

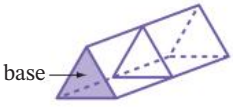
B

base (in index notation) When a number is raised to a power, the number raised is the base. In the expression 3^5, the 3 is called the base. (p. 172)

base (of a flat shape) The bottom side of a shape. (p. 362)



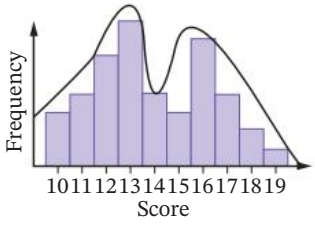
base (of a solid) The bottom or end face of a solid shape. (p. 386)



bearing The angle used to show the direction of one location from a given point. See also **compass bearing** and **three-figure bearing**. (p. 158)

bias In statistics, something that causes a sample to not truly represent the population. (p. 340)

bimodal distribution A statistical distribution that has 2 peaks. (p. 327)



binomial expression An algebraic expression with 2 terms, for example, $x + 9$, $2y - 12$. See also **trinomial**. (p. 21)

bisect To cut in half. (p. 229)

bonus Extra pay for achieving a high quality or volume of work, such as meeting an important quota, goal or deadline. (p. 293)

byte (B) A unit of digital (computer) memory required to store one character of data, such as the letter 'a'. (p. 355)

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C

capacity The amount of material (usually liquid) that a container can hold, measured in millilitres (mL), litres (L), kilolitres (kL) and megalitres (ML). See also **volume**. (p. 385)

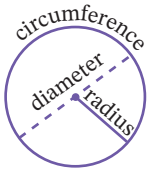
Cartesian plane Another name for **number plane**. (p. 413)

categorical data Data that can be classified into categories, such as hair colour, favourite radio station or postcode. Data that is not **numerical**. (p. 336)

census A survey of the entire **population** of people or items, not just a survey of a **sample**. (p. 336)

chance experiment An activity or process that involves chance, for example, rolling a die or tossing a coin. (p. 461)

circumference The perimeter of a circle. $C = \pi d$ or $C = 2\pi r$, where C is the circumference, π is pi, d is the diameter and r is the radius. (p. 371)



cluster A group of data values that are bunched or close together. (p. 321)

coefficient The number in front of a variable in an algebraic term. For example, the coefficient of x in $2x - 5$ is 2. (p. 35)

commission Pay earned by salespeople and agents, calculated as a percentage of the value of items sold or income made. (p. 292)

compass bearing A bearing that describes one of the 16 points of a mariner's compass, for example, north-northwest (NNW). See also **bearing** and **three-figure bearing**. (p. 158)

compass rose A cross-shaped diagram that shows the direction of north. See also **compass bearing**. (p. 158)

complementary event All the outcomes that are *not* the event; the 'opposite' event. For example, the complementary event to rolling 1 on a die is rolling a number that is not 1. (p. 462)

composite shape A shape made up of 2 or more basic shapes. (p. 362)

compound event A chance event that is a combination of 2 or more simple events, for example, 'female or left-handed'. (p. 475)

congruent Identical, exactly the same. The symbol '≡' means 'is congruent to' or 'is identical to'. (p. 501)

congruence test One of 4 tests for proving that 2 triangles are congruent: SSS, SAS, AAS and RHS. (p. 501)

consecutive numbers Any series of integers that follow each other in order, for example, 8, 9 and 10. (p. 256)

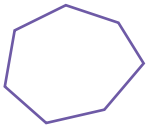
constant of proportionality See **direct proportion**. (p. 443)

constant term The term in an algebraic expression that is a number only, with no variable. For example, the constant term in $x^2 - 4x + 6$ is 6. (p. 35)

continuous data Numerical data that can be measured on a smooth scale without any gaps, and can take on a full range of values, such as the height of people. Continuous data is measured on a scale without 'gaps', unlike **discrete data**. (p. 336)

converse The 'reverse' of a rule, the rule written 'back-to-front' or 'turned around'. The converse of Pythagoras' theorem is that if the square of the longest side of a triangle is equal to the sum of the squares of the other 2 sides, then the triangle is right-angled. (p. 80)

convex polygon A polygon whose vertices all point outwards. All diagonals lie within the shape, and all angles are less than 180°. (pp. 228, 236)



Convex

Non-convex

cosine A ratio in a right-angled triangle:

$$\cos\theta = \frac{\text{side adjacent to } \theta}{\text{hypotenuse}}$$

where θ is an angle. See also **sine** and **tangent**. (p. 127)

cost price The price it costs a retailer (shop) to buy an item (for reselling). The cost price is usually less than the **selling price**. (p. 99)

cross-section A 'slice' of a solid cut across it rather than along it, as shaded in the diagram. (p. 386)

cubic equation An equation in which the highest power of the variable is 3, that is, a variable cubed, for example, $x^3 = 12$. (p. 265)

cumulative frequency A progressive or running total of frequencies, the sum of frequencies of a particular data value and all values below it. (p. 315)

cylinder A can-shaped solid with ends that are circles. (p. 383)

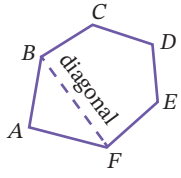
D

data Information, a collection of facts. (p. 311)

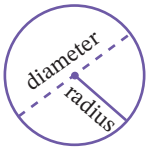
denominator The number below the line in a fraction. The denominator of $\frac{2}{3}$ is 3. (pp. 7, 93)

descending order Going down, decreasing, from largest to smallest (3-2-1). The opposite of **ascending order**. (p. 206)

diagonal An interval joining 2 non-adjacent vertices of a shape. (p. 228)



diameter An interval joining 2 points on the circumference and passing through the centre of a circle, and the length of that interval. The diameter is double the **radius**. (p. 371)



difference of 2 squares An algebraic expression of the form $a^2 - b^2$, that can be factorised into $(a + b)(a - b)$. For example, $x^2 - 25 = (x + 5)(x - 5)$. (p. 29)

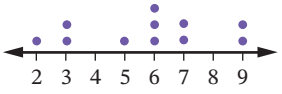
direct proportion (or **direct variation**) A relationship between 2 variables, where one variable is a constant multiple of the other, that is, $y = kx$, where k is a constant called the **constant of proportionality**. For example, if $y = 8.5x$, then y is directly proportional to x. (p. 442)

discount The saving made that is the difference between the original price of an item and the reduced price. (pp. 98, 100)

discrete data Numerical data that are counted or measured and only take on distinct, separate values, such as the number of children in a family (0, 1, 2, ...). Discrete data has a scale with 'gaps' or jumps, unlike **continuous data**. (p. 336)

distributive law A law of arithmetic that says you can multiply by a number by splitting it into the sum or difference of 2 other numbers. For example, $27 \times 12 = 27 \times (10 + 2) = 27 \times 10 + 27 \times 2$. Algebraically, $a(b + c) = ab + ac$ for any 3 numbers a, b and c. (p. 14)

dot plot A graph that uses dots above a number line to represent the frequencies of data values. (p. 312)



double time Overtime pay that is calculated at 2 times the normal pay rate. (p. 287)

E

equation A mathematical statement that 2 quantities are equal. For example, $8 + 2 = 10$ or $3b - 7 = 5$. (p. 250)

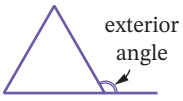
equilateral triangle A triangle with all 3 sides equal. (p. 224)

expected frequency The expected number of times an event will occur over repeated trials, calculated by multiplying the probability of the event by the number of trials. (p. 466)

experimental probability An estimate of theoretical probability; the **relative frequency** of an event in repeated trials of an experiment, found using the formula:

$$P(E) = \frac{\text{frequency of } E}{\text{total frequency}} \text{ (p. 466)}$$

exterior angle An 'outside' angle of a shape created by extending one of the sides of the shape. (p. 225)



event In probability, a result involving one or more outcomes. For example, when rolling a die, the event 'rolling an even number' contains the 3 outcomes {2, 4, 6}. (p. 461)

F

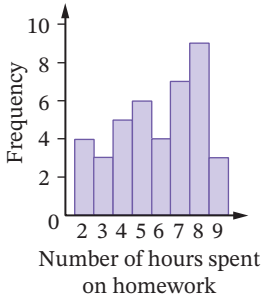
factor or divisor (**of a number**) A value that divides evenly into a given number. For example, the factors of 15 are 1, 3, 5 and 15. (p. 17)

formula (plural: **formulas** or **formulae**) A rule written as an algebraic equation, using variables. The formula for the area of a triangle is $A = \frac{1}{2}bh$. (p. 267)

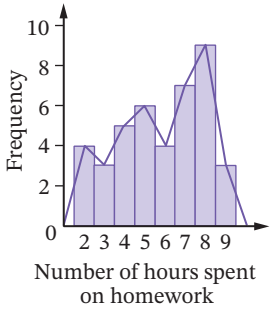
fraction A number written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. (p. 93)

frequency The number of times a value appears in a set of data, or the number of times an event occurs in repeated trials of a probability experiment. (p. 313)

frequency histogram A column graph that shows the frequencies of numerical data. There are no spaces between the columns. (p. 320)



frequency polygon A line graph that shows the frequencies of numerical data. It can be made by joining the midpoints of the tops of the columns of a histogram. (p. 320)



frequency (**distribution**) **table** A table listing the frequency of each value in a set of data, with columns for Score (x), Frequency (f) and sometimes Tally and fx. (p. 313)

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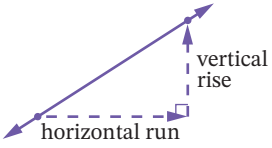
G

giga- A prefix meaning one billion (1 000 000 000), represented by the letter G. (p. 355)

gigabyte (GB) A unit of digital memory equal to one billion bytes. (p. 355)

goods and services tax (GST) A 10% tax added to the original price of an item or service. (p. 101)

gradient The steepness of a line or interval, measured by the fraction $\frac{\text{rise}}{\text{run}}$. (p. 422)



gradient–intercept form (of a linear equation) The equation of a straight line $y = mx + c$, where m is the **gradient** and c is the ***y*-intercept.** (p. 436)

greatest common divisor (GCD) See **highest common factor**. (p. 17)

gross pay Pay received before tax and other deductions are taken out. (p. 301)

GST See **goods and services tax**. (p. 101)

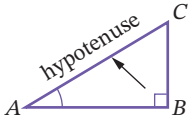
H

hectare A large metric unit of area, equivalent to 10 000 m^2 or to the area of a square 100 m × 100 m. (p. 363)

highest common factor (HCF) or **greatest common divisor (GCD)** The largest factor shared by 2 or more numbers or algebraic terms. For example, the HCF of 36 and 8 is 4 and the HCF of $6xy$ and $12y^2$ is $6y$. (p. 17)

horizontal Going across, sideways, flat. (p. 430)

hypotenuse The longest side of a right-angled triangle, opposite the right angle. (p. 125)

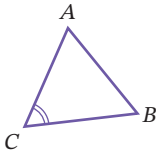


I

image A transformed shape after it has been enlarged or reduced. (p. 515)

improper fraction A fraction whose numerator is greater than or equal to its denominator, such as $\frac{7}{4}$.

included angle The angle between 2 given sides of a shape. For example, the included angle for sides AC and CB in this triangle is $\angle C$. (p. 501)



income tax A tax paid to the government based on the size of a person's income. (p. 296)

index (plural: **indices**) See **power**. (p. 174)

index notation A way of writing repeated multiplication using indices (powers), in the form a^n. Index notation for $2 \times 2 \times 2 \times 2$ is 2^4. (p. 174)

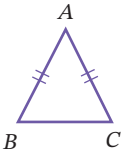
index law An algebraic rule for simplifying expressions involving powers of the same base, for example, $a^m \times a^n = a^{m+n}$. (p. 174)

interval A line segment (section of a line) with a definite length, such as AB shown. (p. 413)

inverse operation The opposite or reverse operation, used when solving equations. For example, the inverse operation to adding is subtracting, the inverse operation to dividing is multiplying. (p. 150)

irrational number A number such as π or $\sqrt{2}$ that cannot be expressed as a fraction (rational number). In decimal form, its digits run endlessly without repeating. (pp. 59, 93, 371) See also **rational number** and **real number**.

isosceles triangle A triangle with 2 equal sides. (p. 224)

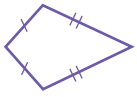


K

kilo- A prefix meaning one thousand (1000), represented by the symbol k. For example, one kilogram is 1000 grams. (p. 354)

kilobyte (kB) A unit of digital memory equal to 1000 bytes. (p. 355)

kite A quadrilateral with 2 pairs of equal adjacent sides. (p. 228)



L

LHS The left-hand side (of an equation). (p. 251)

like terms Algebraic terms that have exactly the same variables. For example, $5xy$ and $2xy$ are like terms, $3xy$ and $4x^2$ are not like terms. (p. 5)

limits of accuracy (of a measuring instrument) A measure of the precision of an instrument, calculated as ±0.5 of the smallest unit on the instrument. (p. 360)

linear equation A formula whose graph is a straight line, or an equation involving a variable that is not raised to a power, such as $2x + 9 = 17$. (p. 428)

loss The money lost when selling an item at a lower price, when the selling price is less than the cost price. The opposite of **profit**. (p. 99)

lowest common multiple (LCM) The smallest multiple that is shared by two or more numbers. For example, the LCM of 4 and 10 is 20. (p. 259)

M

mean The average of a set of data, represented by $\bar{x}$, calculated by dividing the sum of the data values by the number of values. (p. 311)

measure of location or **measure of central tendency** An average, middle or typical value of a set of data. The 3 measures of location are the **mean**, **median** and **mode.** (p. 311)

median The middle value when the values of a data set are arranged in order. If the number of values is even, then the median is the average of the 2 middle values. (p. 311)

mega- A prefix meaning one million (1 000 000), represented by the letter M. A megalitre is one million litres. (p. 354)

megabyte (**MB**) A unit of digital memory equal to 1 000 000 bytes. (p. 355)

micro- A prefix meaning one-millionth, represented by the Greek letter μ. One microsecond is one-millionth of a second. (p. 355)

midpoint The point in the middle of an interval or halfway between 2 given points. (p. 419)

milli- A prefix meaning one-thousandth $\left(\frac{1}{1000}\right)$, represented by the letter m. One millimetre is one-thousandth of a metre. (p. 354)

minute (**symbol ′**) A measure of angle size. $\frac{1}{60}$ of a degree. $1° = 60′$. (p. 135)

mode The most common or frequent value(s) in a set of data. (p. 311)

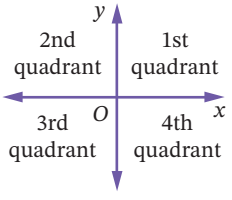
mutually exclusive events Events or categories that have no items in common. (p. 475)

N

nano- A prefix meaning one-billionth, represented by the letter n. One nanosecond is one-billionth of a second. (p. 355)

net pay Pay received after deductions from gross pay; 'take-home' pay. (p. 301)

number plane or **Cartesian plane** A coordinate grid system based on 2 number lines that cross at right angles: a horizontal line called the x-axis and a vertical line called the y-axis. (p. 413)

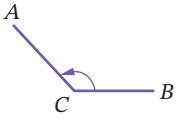


numerator The number above the line in a fraction. The numerator of $\frac{2}{3}$ is 2. (pp. 7, 93)

numerical data Data that can be measured or counted, such as a person's height or number of goals scored. Data that is not **categorical**. (p. 336)

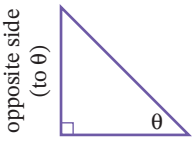
O

obtuse angle A 'wide' angle greater than 90° but less than 180°. (p. 224)

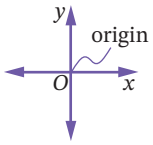


obtuse-angled triangle A triangle with one obtuse angle (between 90° and 180°). (p. 224)

opposite side In a right-angled triangle, the side directly facing the given angle. (p. 125)



origin The point $O(0, 0)$ at the centre of the number plane, where the x-axis and y-axis cross. (p. 449)



outcome In probability, a result of a situation or experiment. For example, when rolling a die, one possible outcome is rolling a 4. (p. 461)

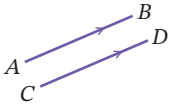
outlier An extreme data value that is much different from the other values in a set. (p. 311)

overtime Time worked beyond normal working hours, such as at night or on weekends, at a higher rate of pay. (p. 287)

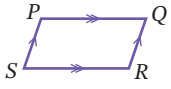
P

parabola A U-shaped curve that is the graph of a quadratic equation, such as $y = x^2$. (p. 447)

parallel lines Lines that point in the same direction and do not intersect. $AB \parallel CD$ means 'AB is parallel to CD'. (p. 218)



parallelogram A quadrilateral in which the opposite sides are parallel. (p. 228)



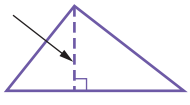
PAYG (**Pay As You Go**) **tax** Income tax deducted from your pay each payday by your employer. (p. 300)

per annum (**p.a.**) Per year. (p. 105)

perfect square A square number or an algebraic expression that represents one, for example, 64, $(x + 9)^2$, $(a - b)^2$. (p. 26)

perimeter The distance around the outside of a shape; the sum of the lengths of its sides. (p. 362)

perpendicular height The height of a shape, measured at right angles to its base. (p. 362)



pi (π) A special number, approximately 3.1416, used in calculating circle measurements. The circumference of any circle is found by multiplying π by the diameter of the circle. (p. 371)

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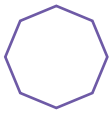
piecework Earnings based on the number of items processed, made or delivered, paid at a rate per item rather than on the number of hours worked. (p. 293)

polygon Any flat shape made up of straight sides. (p. 236)

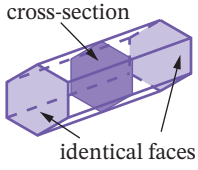


population In statistics, all of the items being studied, the entire group. (p. 336)

power (**or index or exponent**) The number of times a base is multiplied by itself.
In 2^5, the power is 5. (p. 174)

principal An amount of money invested or borrowed, on which interest is calculated. (p. 105)

prism A solid shape with identical cross-sections with straight sides, such as the hexagonal prism below. (p. 386)



probability The chance of an event occurring, measured as a fraction, decimal or percentage between 0 and 1. (p. 461)

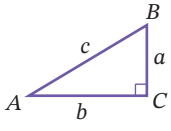
product The result of multiplication. The product of 7 and 3 is 21. (p. 11)

profit The amount made when selling an item at a higher price, when the selling price is more than the cost price. The opposite of **loss**. (p. 99)

pronumeral Another name for **variable**. (p. 5)

proportion See **direct proportion**. (p. 442)

Pythagoras' theorem The formula $c^2 = a^2 + b^2$ for a right-angled triangle, where c is the length of the hypotenuse and a and b are the lengths of the other 2 shorter sides. (p. 52)



Pythagorean triad A set of 3 numbers that follow Pythagoras' theorem, such as {3, 4, 5}. (p. 83)

Q

quadrant (**of a circle**) A sector that is a quarter of a circle, containing a right angle. (p. 372)

quadrant (of a number plane) See **number plane**.

quadratic equation An equation in which the highest power of the variable is 2, that is, a variable squared, for example, $3x^2 - 6 = 69$, or an equation such as $y = 3x^2 - 6$ whose graph is a **parabola**. (pp. 263, 447)

quadratic trinomial See **trinomial**. (p. 35)

quadrilateral Any polygon with 4 sides. (p. 228)

R

radius (plural: **radii**) An interval joining the centre of a circle to the circumference, and the length of that interval. The radius is half of the **diameter**. (p. 371)

random In probability, describing a situation where every possible outcome has an equal chance, or is equally likely. (p. 461)

random sampling In statistics, selecting a sample in which every person or item in the population has an equal chance of being selected. A sample should be random to be truly representative of the population. (p. 340)

range In a set of data, the difference between the highest and lowest values. (p. 311)

rate A relationship between 2 quantities measured in different units. For example, a speed of 107 km/h compares distance travelled (in kilometres) with time (in hours). (p. 109)

ratio A relationship between quantities measured in the same units. For example, the ratio of 3 teachers to 40 students is 3 : 40 (read '3 to 40'). (p. 108)

rational number A number that can be written as a fraction in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. See also **irrational number** and **real number**. (p. 59)

real number A **rational** or **irrational number**, that can be ordered on a number line. (p. 60)

reciprocal The product of any number and its reciprocal is 1. The reciprocal of any number is found by first writing the number as a fraction and then swapping the numerator with the denominator. The reciprocal of 5 is $\frac{1}{5}$ and the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$. (pp. 11, 189)

rectangle A quadrilateral with 4 right angles. (p. 228)

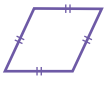
recurring or **repeating decimal** A decimal with one or more digits that repeat endlessly. For example 0.1666..., abbreviated as $0.1\dot{6}$. (p. 93)

regular polygon A polygon that has all sides equal and all angles equal. For example, this regular pentagon has 5 equal sides and 5 equal angles. (p. 237)

relative frequency The number of times an event or data value occurred, written as a fraction of the total number of events or values. See also **experimental probability**. (p. 466)

retainer A fixed amount paid to a salesperson before **commission** is added. (p. 292)

rhombus A quadrilateral with 4 equal sides. (p. 228)



RHS The right-hand side (of an equation). (p. 251)

right angle A 90° angle, a 'square corner', a quarter-turn. (p. 52)

right-angled triangle A triangle with one 90° angle. (pp. 52, 224)

rise Short for 'vertical rise', the change in vertical position between 2 points on a line or interval, the number of units 'going up', used with the **run** to calculate the **gradient** of a line or interval. See **gradient** for diagram. (p. 422)

run Short for 'horizontal run', the change in horizontal position between 2 points on a line or interval, the number of units 'going right', used with the **rise** to calculate the **gradient** of a line or interval. See **gradient** for diagram. (p. 422)

S

salary A fixed annual amount of money that is paid weekly, fortnightly or monthly, not dependent on the number of hours worked. (p. 282)

sample In statistics, a group of people or items selected from a population for study. (p. 336)

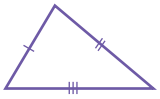
sample space In a probability situation, the set of all possible outcomes. (p. 461)

scale diagram A diagram that is in proportion to the object it represents, such as a house plan or map, that is drawn according to a scale (ratio). (p. 527)

scale factor The amount by which a shape has been enlarged or reduced, equal to $\frac{\text{image length}}{\text{original length}}$ (p. 515)

scaled length The length on a scale drawing, map or plan, not the actual length. (p. 527)

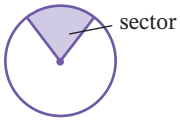
scalene triangle A triangle with no equal sides. (p. 224)



scientific notation A shorter way of writing very large or very small numbers using powers of 10, for example, 9 460 000 000 000 in scientific notation is 9.46×10^{12}. (p. 203)

second (″) A measure of angle size. $\frac{1}{60}$ of a minute. $1' = 60''$. (p. 135)

sector A region of a circle cut off by 2 radii. (p. 372)



selling price The price at which an item is sold by the retailer (shop). The shop buys the item for the **cost price**. (p. 101)

shape of a distribution The way the data in a frequency distribution is spread, can be **symmetrical**, **positively skewed** or **negatively skewed**. (p. 327)

significant figures The meaningful digits in a number that show its level of accuracy, the first non-zero digits, for example, 31 487 000 has 5 significant figures. (p. 200)

similar To have the same shape but not necessarily the same size, an enlargement or reduction. The symbol '|||' means 'is similar to'. (p. 131, 514)

similarity test One of 4 tests for proving that 2 triangles are similar. (p. 541)

simple interest Interest that is calculated as a percentage of the original principal. (p. 105)

sine A ratio in a right-angled triangle:

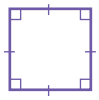
$$\sin\theta = \frac{\text{side opposite to } \theta}{\text{hypotenuse}}$$

where θ is an angle. See also **cosine** and **tangent**. (p. 127)

skewed distribution A statistical distribution in which most of the data values are clustered at one end, creating a 'tail' at the other end. The tail determines whether the skew is positive or negative. See also **symmetrical distribution**. (p. 327)

solution In algebra, the value(s) that makes an equation or inequality true. (p. 250)

square A quadrilateral with 4 equal sides and 4 right angles. (p. 228)



stem-and-leaf plot A 'number graph' that lists all the data values in groups. Each value is split into a 'stem' and a 'leaf'. This stem-and-leaf plot shows 12 test marks, from 42 to 82. (p. 321)

Stem	Leaf
4	2 5
5	0 2 8
6	6 7
7	3 5 7 7
8	2

Key: 5|8 stands for 58

subject of a formula The variable for which a formula is written, the variable on the left-hand-side of a formula. For example, the subject of the formula $A = \frac{1}{2}bh$ is A. (p. 267, 271)

supplementary angles 2 angles whose sum is 180°, for example, the angles 70° and 110°. (p. 217)

surd A square root (or other root) whose exact value cannot be found because it is **irrational**, such as $\sqrt{10}$ or $\sqrt[3]{7}$. (p. 59)

surface area The total area of all the faces of a solid shape. (p. 377)

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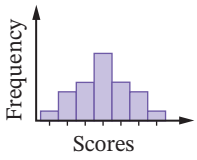
symmetrical distribution A statistical distribution in which all data values are distributed equally on both sides of the centre, its shape having line symmetry. See also **skewed distribution**. (p. 327)



T

tangent A ratio in a right-angled triangle:

$$\tan\theta = \frac{\text{side opposite to } \theta}{\text{side adjacent to } \theta}$$

where θ is an angle. See also **sine** and **cosine**. (p. 127)

tax deduction See **allowable deduction**. (p. 296)

taxable income The part of a person's income that is taxed, equal to annual income minus allowable deductions. (p. 297)

terabyte (**TB**) A unit of digital memory equal to one trillion bytes. (p. 355)

terminating decimal A decimal whose digits do not repeat endlessly, for example, 0.125. (p. 93)

term (**of an expression**) A part of an algebraic expression. For example, $b^2 + 6b - 9$ has 3 terms: b^2, $6b$ and -9. (p. 5)

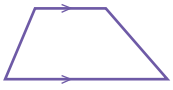
theoretical probability Probability calculated using the formula:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$ (p. 466)

three-figure bearing or **true bearing** A bearing that uses three-digit angles from 000° to 360° to show the amount of turning measured clockwise from north. See also **bearing** or **compass bearing**. (p. 158)

time-and-a-half Overtime pay that is calculated at 1.5 times the normal pay rate. (p. 287)

trapezium A quadrilateral with one pair of opposite sides parallel. (p. 228)



tree diagram A diagram of branches for listing all of the possible outcomes in a multi-step chance experiment. (p. 488)

trial One go or run of a repeated probability experiment, for example, one roll of a die. (p. 461)

trinomial An algebraic expression with 3 terms, for example, $3x + 2y - 5$. In a **quadratic** trinomial such as $x^2 + 4x + 6$, the highest power of the pronumeral is 2. See also **binomial** and **quadratic expression**. (p. 35)

two-step experiment (or **two-stage experiment**) A chance experiment with 2 steps or stages, such as rolling a pair of dice. (p. 486)

two-way table A table that shows the number of items belonging to overlapping categories. (p.481)

	Can swim	Cannot swim
Boys	13	2
Girls	9	3

U

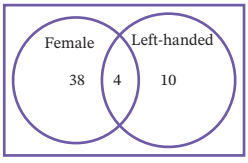
unitary method A method for finding a quantity by finding the size of one part or 1% first. (pp. 96, 108)

V

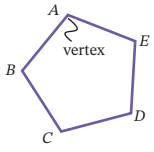
variable A symbol, usually a letter of the alphabet, that stands for a number. Also called a **pronumeral** or **unknown**. (p. 5)

variation See **direct proportion**. (p. 442)

Venn diagram. A diagram of circles (usually overlapping) for grouping items into categories. (p. 474)



vertex (plural: **vertices**) A corner of a shape, angle or curve. (p. 228)



vertical Going up and down, at a right angle to the **horizontal**. (p. 430)

volume The amount of space taken up by a solid object, measured in cubic units. (p. 385)

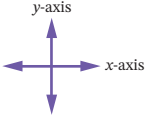
W

wage An amount of money paid to people for work, calculated on the number of hours worked. (p. 282)

X

***x*-axis** The horizontal axis of a number plane (running across). (p. 428)

***x*-intercept** The *x*-value at which a graph cuts the *x*-axis. (p. 428)



Y

***y*-axis** The vertical axis of a number plane (running up and down, see diagram above). (p. 428)

***y*-intercept** The *y*-value at which a line cuts the *y*-axis. (p. 428)

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