MATHS QUEST 8

FOR VICTORIA

Australian Curriculum edition

CATHERINE SMITH | LYN ELMS | DOUGLAS SCOTT

CONTRIBUTING AUTHORS

LEE ROLAND | ROBERT ROWLAND | ELENA IAMPOLSKY | ANITA CANN | IRENE KIROFF

KELLY WAI TSE CHOI | KELLY SHARP | ROBERT CAHN | SONJA STAMBULIC | KYLIE BOUCHER

jacaranda

A Wiley Brand

First published 2014 by
John Wiley & Sons Australia, Ltd
42 McDougall Street, Milton, Qld 4064

Typeset in 12/14pt Times LT Std

© John Wiley & Sons Australia, Ltd 2014

The moral rights of the authors have been asserted.

National Library of Australia
Cataloguing-in-Publication data

Author:	Smith, Catherine, author.
Title:	Maths quest 8 for Victoria Australian curriculum edition/Catherine Smith, Lyn Elms, Douglas Scott
ISBN:	978 0 7303 1185 0 (paperback) 978 0 7303 1188 1 (ebook)
Notes:	Includes index.
Target audience:	For secondary school age.
Subjects:	Mathematics — Study and teaching (Secondary) Mathematics — Problems, exercises, etc.
Other authors/contributors:	Elms, Lyn, author. Scott, Douglas, author.
Dewey number:	510.712

Reproduction and communication for educational purposes
The Australian *Copyright Act 1968* (the Act) allows a maximum of one chapter or 10% of the pages of this work, whichever is the greater, to be reproduced and/or communicated by any educational institution for its educational purposes provided that the educational institution (or the body that administers it) has given a remuneration notice to Copyright Agency Limited (CAL).

Reproduction and communication for other purposes
Except as permitted under the Act (for example, a fair dealing for the purposes of study, research, criticism or review), no part of this book may be reproduced, stored in a retrieval system, communicated or transmitted in any form or by any means without prior written permission. All inquiries should be made to the publisher.

Trademarks
Jacaranda, the Jacplus logo, the learnON, assessON and studyON logos, Wiley and the Wiley logo, and any related trade dress are trademarks or registered trademarks of John Wiley & Sons Inc. and/or its affiliates in the United States, Australia and in other countries, and may not be used without written permission. All other trademarks are the property of their respective owners.

Front and back cover image: © vic&dd/Shutterstock.com

Internal design images: © HTU/Shutterstock.com (swirl of lights); © Maxx-Studio/Shutterstock.com (jigsaw cube); © RedKoala/Shutterstock.com (pixel hand and arrow icons); © vic&dd/Shutterstock.com (abstract geometric background)

Illustrated by diacriTech and Wiley Composition Services

Typeset in India by diacriTech

Printed in China by
1010 Printing International Ltd

10 9 8 7 6 5 4 3 2

Contents

About eBookPLUS vii
Overview viii
Acknowledgements x

NUMERACY
Topic 1 Numeracy 1 2
1.1 Overview 2
1.2 Set A 4
1.3 Set B 9
Answers 14

NUMBER AND ALGEBRA
Topic 2 Integers 16
2.1 Overview 16
2.2 Adding and subtracting integers 18
Exercise 2.2 19
2.3 Multiplying integers 23
Exercise 2.3 24
2.4 Dividing integers 26
Exercise 2.4 27
2.5 Combined operations on integers 29
Exercise 2.5 30
2.6 Review eBookplus ONLINE ONLY 33
eBookPLUS activities 37
Answers 38

NUMBER AND ALGEBRA
Topic 3 Index laws 40
3.1 Overview 40
3.2 Review of index form 42
Exercise 3.2 43
3.3 First Index Law (multiplying numbers in index form with the same base) 45
Exercise 3.3 47
3.4 Second Index Law (dividing numbers in index form with the same base) 49
Exercise 3.4 50
3.5 Third Index Law (the power of zero) 52
Exercise 3.5 53
3.6 Fourth Index Law (raising a power to another power) 55
Exercise 3.6 56
3.7 Review eBookplus ONLINE ONLY 59
eBookPLUS activities 63
Answers 64

ProjectsPLUS ICT ACTIVITY
Attack of the killer balloons **pro-0095** 66

NUMBER AND ALGEBRA
Topic 4 Real numbers 68
4.1 Overview 68
4.2 Addition and subtraction of fractions 70
Exercise 4.2 74
4.3 Multiplication and division of fractions 76
Exercise 4.3 78
4.4 Terminating and recurring decimals 80
Exercise 4.4 83
4.5 Addition and subtraction of decimals 85
Exercise 4.5 86
4.6 Multiplication and division of decimals 88
Exercise 4.6 91
4.7 Estimation 92
Exercise 4.7 96
4.8 Review eBookplus ONLINE ONLY 99
eBookPLUS activities 103
Answers 104

NUMBER AND ALGEBRA
Topic 5 Ratios and rates 108
5.1 Overview 108
5.2 Introduction to ratios 110
Exercise 5.2 110
5.3 Simplifying ratios 114
Exercise 5.3 116
5.4 Proportion 119
Exercise 5.4 120
5.5 Comparing ratios 123
Exercise 5.5 124
5.6 Dividing in a given ratio 126
Exercise 5.6 127
5.7 Rates 129
Exercise 5.7 130
5.8 Interpreting graphs 133
Exercise 5.8 136
5.9 Review eBookplus ONLINE ONLY 139
eBookPLUS activities 143
Answers 144

NUMBER AND ALGEBRA
Topic 6 Application of percentages 148
6.1 Overview 148
6.2 Percentages, fractions and decimals 150
Exercise 6.2 152
6.3 Finding percentages of an amount 155
Exercise 6.3 156
6.4 Discount 159
Exercise 6.4 161
6.5 Profit and loss 164
Exercise 6.5 166
6.6 Goods and Services Tax (GST) 169
Exercise 6.6 170
6.7 Review eBookplus ONLINE ONLY 173
eBookPLUS activities 177
Answers 178

MEASUREMENT AND GEOMETRY
Topic 7 Congruence 180
7.1 Overview 180
7.2 Congruent figures 182
Exercise 7.2 183
7.3 Triangle constructions 186
Exercise 7.3 188
7.4 Congruent triangles 190
Exercise 7.4 193
7.5 Quadrilaterals 195
Exercise 7.5 197
7.6 Nets, polyhedra and Euler's rule 200
Exercise 7.6 202
7.7 Review eBookplus ONLINE ONLY 205
eBookPLUS activities 209
Answers 210

NUMBER AND ALGEBRA
Topic 8 Algebra 214
8.1 Overview 214
8.2 Using variables 216
Exercise 8.2 217
8.3 Substitution 221
Exercise 8.3 222
8.4 Working with brackets 223
Exercise 8.4 224
8.5 Substituting positive and negative numbers 226
Exercise 8.5 227
8.6 Number laws and variables 228
Exercise 8.6 233
8.7 Simplifying expressions 234
Exercise 8.7 236
8.8 Multiplying and dividing expressions with variables 238
Exercise 8.8 241
8.9 Expanding brackets 244
Exercise 8.9 245
8.10 Factorising 247
Exercise 8.10 249
8.11 Review eBookplus ONLINE ONLY 251
eBookPLUS activities 255
Answers 256

NUMERACY
Topic 9 Numeracy 2 260
9.1 Overview 260
9.2 Set C 262
9.3 Set D 268
Answers 274

MEASUREMENT AND GEOMETRY
Topic 10 Measurement 276
10.1 Overview 276
10.2 Perimeter 278
Exercise 10.2 281
10.3 Circumference 283
Exercise 10.3 285
10.4 Area of rectangles, triangles, parallelograms, rhombuses and kites 289
Exercise 10.4 293
10.5 Area of a circle 297
Exercise 10.5 298
10.6 Area of trapeziums 301
Exercise 10.6 302
10.7 Volume of prisms and other solids 304
Exercise 10.7 307
10.8 Time 310
Exercise 10.8 313
10.9 24-hour clock and time zones 316
Exercise 10.9 319
10.10 Review eBookplus ONLINE ONLY 323
eBookPLUS activities 327
Answers 328

NUMBER AND ALGEBRA
Topic 11 Linear equations 332
11.1 Overview 332
11.2 Identifying patterns 334
Exercise 11.2 337
11.3 Backtracking and inverse operations 338
Exercise 11.3 340
11.4 Keeping equations balanced 342
Exercise 11.4 343
11.5 Using algebra to solve problems 346
Exercise 11.5 349
11.6 Equations with the unknown on both sides 352
Exercise 11.6 355
11.7 Review eBookplus ONLINE ONLY 359
eBookPLUS activities *363*
Answers *364*

STATISTICS AND PROBABILITY
Topic 12 Representing and interpreting data 368
12.1 Overview 368
12.2 Samples and populations 370
Exercise 12.2 374
12.3 Primary and secondary data 378
Exercise 12.3 383
12.4 Organising and displaying data 386
Exercise 12.4 391
12.5 Measures of centre 395
Exercise 12.5 400
12.6 Measures of spread 404
Exercise 12.6 407
12.7 Analysing data 410
Exercise 12.7 413
12.8 Review eBookplus ONLINE ONLY 417
eBookPLUS activities *421*
Answers *422*

ProjectsPLUS ICT ACTIVITY
How to burglar-proof your bedroom **pro-0097** 430

STATISTICS AND PROBABILITY
Topic 13 Probability 432
13.1 Overview 432
13.2 Probability scale 434
Exercise 13.2 436
13.3 Experimental probability 438
Exercise 13.3 440
13.4 Sample spaces and theoretical probability 445
Exercise 13.4 448
13.5 Complementary events 453
Exercise 13.5 454
13.6 Venn diagrams 456
Exercise 13.6 460
13.7 Tree diagrams and two-way tables 463
Exercise 13.7 467
13.8 Review eBookplus ONLINE ONLY 473
eBookPLUS activities *477*
Answers *478*

NUMBER AND ALGEBRA
Topic 14 Coordinates and linear graphs 484
14.1 Overview 484
14.2 The Cartesian plane 486
Exercise 14.2 488
14.3 Linear patterns 490
Exercise 14.3 492
14.4 Plotting linear graphs 494
Exercise 14.4 496
14.5 The *y*-intercept and gradient 499
Exercise 14.5 503
14.6 Sketching linear graphs 508
Exercise 14.6 510
14.7 Solving equations graphically 511
Exercise 14.7 514
14.8 Review eBookplus ONLINE ONLY 517
eBookPLUS activities *521*
Answers *522*

MEASUREMENT AND GEOMETRY
Topic 15 Pythagoras' theorem 530
15.1 Overview 530
15.2 Right-angled triangles 532
Exercise 15.2 532
15.3 Finding the hypotenuse 534
Exercise 15.3 536
15.4 Finding a shorter side 538
Exercise 15.4 539
15.5 Working with different units 542
Exercise 15.5 543

15.6 Composite shapes 545
Exercise 15.6 547
15.7 Pythagorean triads 550
Exercise 15.7 552
15.8 Pythagoras in 3-D 553
Exercise 15.8 555
15.9 Review eBookplus ONLINE ONLY 557
eBookPLUS activities *561*
Answers *562*

NUMERACY
Topic 16 Numeracy 3 564
16.1 Overview 564
16.2 Set E 566
16.3 Set F 570
Answers *575*

NUMBER AND ALGEBRA
Topic 17 Computer programming 576
17.1 Overview eBookplus ONLINE ONLY 576
17.2 Expressions eBookplus ONLINE ONLY 578
Exercise 17.2 583
17.3 Decisions eBookplus ONLINE ONLY 586
Exercise 17.3 592
17.4 Functions eBookplus ONLINE ONLY 597
Exercise 17.4 605
17.5 Testing eBookplus ONLINE ONLY 608
Exercise 17.5 613
17.6 Debugging eBookplus ONLINE ONLY 619
Exercise 17.6 625
17.7 Review eBookplus ONLINE ONLY 633
Answers *636*

Glossary *643*

About eBookPLUS

jacaranda plus

This book features eBookPLUS: an electronic version of the entire textbook and supporting digital resources. It is available for you online at the JacarandaPLUS website (**www.jacplus.com.au**).

Using JacarandaPLUS

To access your eBookPLUS resources, simply log on to **www.jacplus.com.au** using your existing JacarandaPLUS login and enter the registration code. If you are new to JacarandaPLUS, follow the three easy steps below.

Step 1. Create a user account

The first time you use the JacarandaPLUS system, you will need to create a user account. Go to the JacarandaPLUS home page (**www.jacplus.com.au**), click on the button to create a new account and follow the instructions on screen. You can then use your nominated email address and password to log in to the JacarandaPLUS system.

Step 2. Enter your registration code

Once you have logged in, enter your unique registration code for this book, which is printed on the inside front cover of your textbook. The title of your textbook will appear in your bookshelf. Click on the link to open your eBookPLUS.

Step 3. Access your eBookPLUS resources

Your eBookPLUS and supporting resources are provided in a chapter-by-chapter format. Simply select the desired chapter from the table of contents. Digital resources are accessed within each chapter via the resources tab.

Once you have created your account, you can use the same email address and password in the future to register any JacarandaPLUS titles you own.

Using eBookPLUS references

eBookPLUS logos are used throughout the printed books to inform you that a digital resource is available to complement the content you are studying.

eBookplus

Searchlight IDs (e.g. **INT-0001**) give you instant access to digital resources. Once you are logged in, simply enter the Searchlight ID for that resource and it will open immediately.

Searchlight

Minimum requirements

JacarandaPLUS requires you to use a supported internet browser and version, otherwise you will not be able to access your resources or view all features and upgrades. The complete list of JacPLUS minimum system requirements can be found at **http://jacplus.desk.com**.

Troubleshooting

- Go to **www.jacplus.com.au** and click on the Help link.
- Visit the JacarandaPLUS Support Centre at **http://jacplus.desk.com** to access a range of step-by-step user guides, ask questions or search for information.
- Contact John Wiley & Sons Australia, Ltd.
  Email: support@jacplus.com.au
  Phone: 1800 JAC PLUS (1800 522 7587)

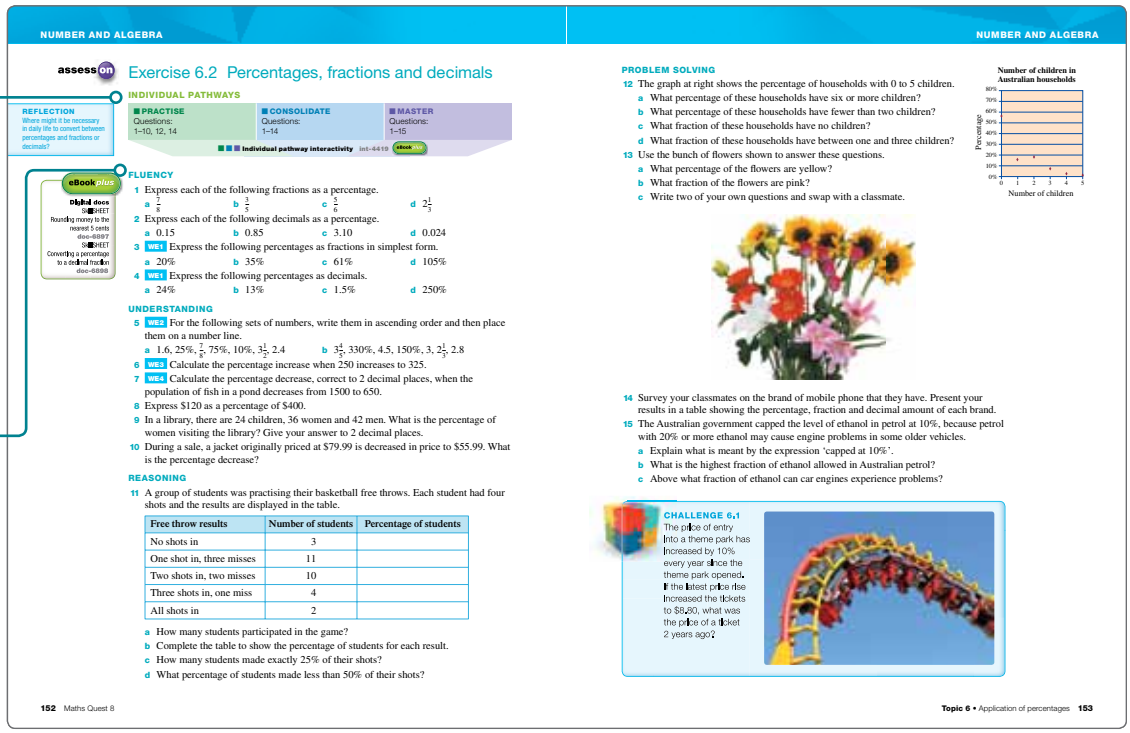
Overview

Maths Quest 8 for Victoria Australian Curriculum edition is a suite of resources designed to enrich the learning experience and improve learning outcomes for all students.

Teacher resources are available on the online eGuidePLUS.

Individual pathways cater for differentiation through carefully graded exercises in the textbook and activities on the eBookPLUS.

All exercises are organised under the working mathematically headings of **Fluency**, **Understanding**, **Reasoning** and **Problem solving**.



NUMBER AND ALGEBRA

assess on

Exercise 6.2 Percentages, fractions and decimals

INDIVIDUAL PATHWAYS

REFLECTION
Where might it be necessary in daily life to convert between percentages and fractions or decimals?

PRACTISE
Questions: 1–10, 12, 14

CONSOLIDATE
Questions: 1–14

MASTER
Questions: 1–15

Individual pathway interactivity int-4419

eBookplus
Digital docs
SkillSHEET Rounding money to the nearest 5 cents doc-6897
SkillSHEET Converting a percentage to a decimal fraction doc-6898

FLUENCY

1 Express each of the following fractions as a percentage.
a $\frac{1}{2}$ b $\frac{1}{5}$ c $\frac{5}{6}$ d $2\frac{1}{3}$

2 Express each of the following decimals as a percentage.
a 0.15 b 0.85 c 3.10 d 0.024

3 WE1 Express the following percentages as fractions in simplest form.
a 20% b 35% c 61% d 105%

4 WE1 Express the following percentages as decimals.
a 24% b 13% c 1.5% d 250%

UNDERSTANDING

5 WE2 For the following sets of numbers, write them in ascending order and then place them on a number line.
a 1.6, 25%, $\frac{2}{5}$, 75%, 10%, $3\frac{1}{2}$, 2.4 b $3\frac{4}{5}$, 330%, 4.5, 150%, 3, $2\frac{1}{5}$, 2.8

6 WE3 Calculate the percentage increase when 250 increases to 325.

7 WE4 Calculate the percentage decrease, correct to 2 decimal places, when the population of fish in a pond decreases from 1500 to 650.

8 Express $120 as a percentage of $400.

9 In a library, there are 24 children, 36 women and 42 men. What is the percentage of women visiting the library? Give your answer to 2 decimal places.

10 During a sale, a jacket originally priced at $79.99 is decreased in price to $55.99. What is the percentage decrease?

REASONING

11 A group of students was practising their basketball free throws. Each student had four shots and the results are displayed in the table.

Free throw results	Number of students	Percentage of students
No shots in	3	
One shot in, three misses	11	
Two shots in, two misses	10	
Three shots in, one miss	4	
All shots in	2	

a How many students participated in the game?
b Complete the table to show the percentage of students for each result.
c How many students made exactly 25% of their shots?
d What percentage of students made less than 50% of their shots?

152 Maths Quest 8

NUMBER AND ALGEBRA

PROBLEM SOLVING

12 The graph at right shows the percentage of households with 0 to 5 children.
a What percentage of these households have six or more children?
b What percentage of these households have fewer than two children?
c What fraction of these households have no children?
d What fraction of these households have between one and three children?

Number of children in Australian households
Percentage
Number of children

13 Use the bunch of flowers shown to answer these questions.
a What percentage of the flowers are yellow?
b What fraction of the flowers are pink?
c Write two of your own questions and swap with a classmate.

14 Survey your classmates on the brand of mobile phone that they have. Present your results in a table showing the percentage, fraction and decimal amount of each brand.

15 The Australian government capped the level of ethanol in petrol at 10%, because petrol with 20% or more ethanol may cause engine problems in some older vehicles.
a Explain what is meant by the expression 'capped at 10%'.
b What is the highest fraction of ethanol allowed in Australian petrol?
c Above what fraction of ethanol can car engines experience problems?

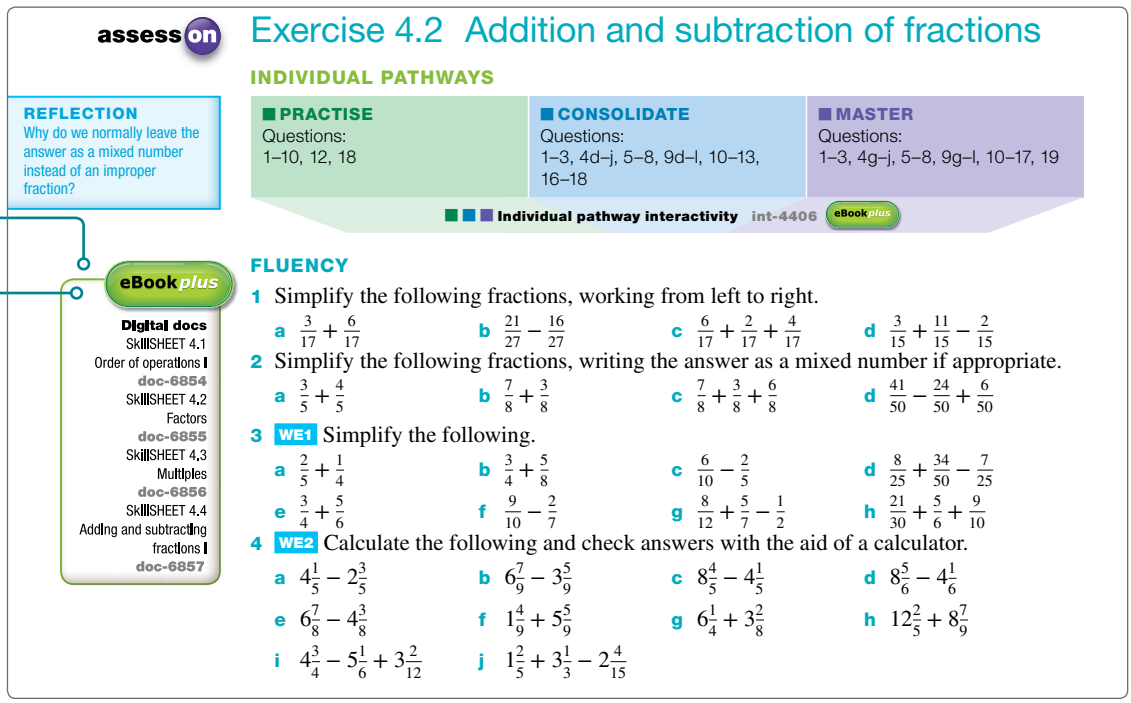
CHALLENGE 6.1
The price of entry into a theme park has increased by 10% every year since the theme park opened. If the latest price rise increased the tickets to $80, what was the price of a ticket 2 years ago?

Topic 6 • Application of percentages 153

eBookPLUS

eBookPLUS contains the entire student text in HTML format, which can be used on any device.

eBookPLUS contains hundreds of additional resources such as WorkSHEETs, SkillSHEETs, worked solutions, individual pathway activities, interactivities, eLessons, ProjectsPLUS, crosswords, word searches, Sudoku puzzles and weblinks.



assess on

Exercise 4.2 Addition and subtraction of fractions

INDIVIDUAL PATHWAYS

REFLECTION
Why do we normally leave the answer as a mixed number instead of an improper fraction?

PRACTISE
Questions: 1–10, 12, 18

CONSOLIDATE
Questions: 1–3, 4d–j, 5–8, 9d–l, 10–13, 16–18

MASTER
Questions: 1–3, 4g–j, 5–8, 9g–l, 10–17, 19

Individual pathway interactivity int-4406

eBookplus
Digital docs
SkillSHEET 4.1 Order of operations I doc-6854
SkillSHEET 4.2 Factors doc-6855
SkillSHEET 4.3 Multiples doc-6856
SkillSHEET 4.4 Adding and subtracting fractions I doc-6857

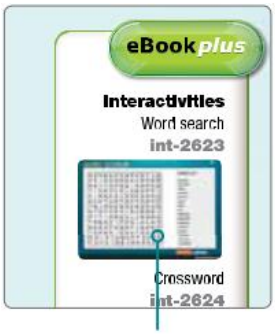
FLUENCY

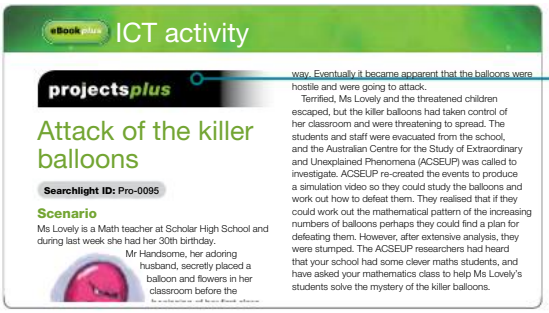
1 Simplify the following fractions, working from left to right.
a $\frac{3}{17}+\frac{6}{17}$ b $\frac{21}{27}-\frac{16}{27}$ c $\frac{6}{17}+\frac{2}{17}+\frac{4}{17}$ d $\frac{3}{15}+\frac{11}{15}-\frac{2}{15}$

2 Simplify the following fractions, writing the answer as a mixed number if appropriate.
a $\frac{3}{5}+\frac{4}{5}$ b $\frac{7}{8}+\frac{3}{8}$ c $\frac{7}{8}+\frac{3}{8}+\frac{6}{8}$ d $\frac{41}{50}-\frac{24}{50}+\frac{6}{50}$

3 WE1 Simplify the following.
a $\frac{2}{5}+\frac{1}{4}$ b $\frac{3}{4}+\frac{5}{8}$ c $\frac{6}{10}-\frac{2}{5}$ d $\frac{8}{25}+\frac{34}{50}-\frac{7}{25}$
e $\frac{3}{4}+\frac{5}{6}$ f $\frac{9}{10}-\frac{2}{7}$ g $\frac{8}{12}+\frac{5}{7}-\frac{1}{2}$ h $\frac{21}{30}+\frac{5}{6}+\frac{9}{10}$

4 WE2 Calculate the following and check answers with the aid of a calculator.
a $4\frac{1}{5}-2\frac{3}{5}$ b $6\frac{7}{9}-3\frac{5}{9}$ c $8\frac{4}{5}-4\frac{1}{5}$ d $8\frac{5}{6}-4\frac{1}{6}$
e $6\frac{7}{8}-4\frac{3}{8}$ f $1\frac{4}{9}+5\frac{5}{9}$ g $6\frac{1}{4}+3\frac{2}{8}$ h $12\frac{2}{5}+8\frac{7}{9}$
i $4\frac{3}{4}-5\frac{1}{6}+3\frac{2}{12}$ j $1\frac{2}{5}+3\frac{1}{3}-2\frac{4}{15}$





ProjectsPLUS are online projects that allow students to work collaboratively and creatively and to be monitored by the teacher.

Dynamic interactivities help students to understand and work through difficult concepts.

assessON

assessON is an online assessment tool that assesses and tracks students' readiness FOR learning, their progress AS they learn and their levels OF achievement.

assessON provides a one-to-one correspondence to the text with all questions automatically marked and instant feedback given to both students and teachers

Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

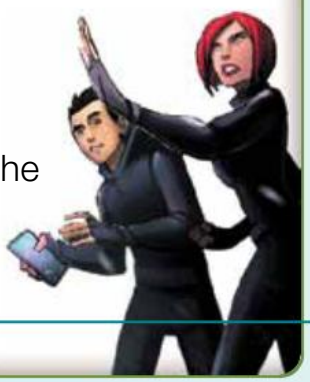
www.assesson.com.au

SpyClass

Link to SpyClass, an exciting online game combining a comic book–style story with problem-based learning in an immersive environment.

Join Jesse, Toby and Dan and help them to tackle some of the world's most dangerous criminals by using the knowledge you've gained through your study of mathematics.

www.spyclass.com.au



SpyClass is an online game combining comic book–style art with problem-based learning to allow students to hone their problem-solving skills in an exciting and immersive environment.

SpyClass directly aligns with the Maths Quest series, providing 16 missions (each directly relating to a topic from the text), each of which have a number of challenges for students to tackle at three different levels of ability.

Acknowledgements

The authors and publisher would like to thank the following copyright holders, organisations and individuals for their assistance and for permission to reproduce copyright material in this book.

Images

• 123RF.com: **5** (lower)/Sebastian Ionescu, **16**–**17**/Zhang Xiangyang, **40**–**1**/fullempty, **75**/studiovespa, **79** (lower)/burnel1, **101** (middle)/Stephen Finn, **159**/Denis Demidenko, **160** (lower)/nito500, **163** (speakers)/sergeymakarenko, **163** (tablet)/Oleksiy Mark, **165**/Konstantin Kulikov, **170** (lower)/Etienne Voss, **190**/Eugene Sergeev, **220** (upper)/luchschen, **237**/Chris Dorney, **247** (upper)/Vadym Solovyov, **272** (lower)/Thor Jorgen Udvang, **276**–**7**/Hayati Kayhan, **278**/Sadequl Hussain, **295** (upper)/Grzegorz Kula, **310** (lower)/Andrew Scherbackov, **315**/Andres Rodriguez, **320**/Michal Bednarek, **341**/vladischern, **345** (upper)/Andrei Kuzmik, **357** (upper)/kho, **384**/Jozsef Szasz-Fabian, **394** (upper)/nyul, **403**/Syda Productions, **409**/konturvid, **413**/Brian Jackson, **436**/Filip Fuxa, **498** (upper)/racorn, **507** (lower)/kzenon, **516**/Dmitriy Shironosov, **536**/potowizard, **541** (upper)/Sergey Galushko • © Banana Stock: **387**, **392** (lower) • © Corbis Digital Stock: **199** (lower) • © Digital Stock: **217** (upper), **434**, **455** • © Digital Vision: **217** (lower) • © Image Disk Photography: **158** (lower), **540** • © iStockphoto: **98** (upper)/Lifesizelmages, **169** (upper left)/pixduluxe, **169** (upper centre)/Cristian Lupu, **263** (lower)/Valentin Casarsa, **269**/organicpixel, **372**/Winston Davidian • © John Wiley & Sons Australia, Ltd: **21** (upper), **76**, **154** (upper), **309** (upper, lower), **357** (lower), **437**, **471** (upper), **498** (lower), **550**/Photos by Kari-Ann Tapp; **169** (upper right), **243**, **568**/Photos by Renee Bryon; **223**/Malcolm Cross • © PhotoAlto/Frederic Cirou: **11** (lower) • © Photodisc, Inc.: **11** (middle), **222**, **283**, **287** (left, right), **295** (middle, lower), **308** (upper, lower), **311**, **394** (lower), **395**, **402**, **415** (upper, middle), **446**, **449**, **450**, **461** (upper, lower), **468** (upper), **471** (lower), **507** (upper), **548**, **555** (upper, lower) • Public domain: **623** • Shutterstock (all images used under license from Shutterstock.com): **2**–**3**/Oleksiy Avtomonov, **4**/Lori Martin, **5** (upper)/Shyamalamuralinath, **9** (lower)/oksana2010, **9** (upper)/L_amica, **10**/Wallenrock, **11** (upper)/Steman Shots, **12** (lower)/Paul Prescott, **12** (upper)/Vaide Seskauskiene, **21** (lower)/steve estvanik, **22**/littleny, **29**/Snvv, **30**/Razmarinka, **32**/mkrol0718, **34**/Artistan, **35**/TAROKICHI, **36**/yaskii, **48**/racorn, **52**/neiromobile, **58**/Olesya Feketa, **60**/Marisha, **61**/clearviewstock, **63**/Meder Lorant, **66**–**7**/Zhiltsov Alexandr, **68**–**9**/Lightspring, **79** (upper)/Ruth Black, **83**/smikhailov, **84**/DibasUA, **87**/Timothy OLeary, **92**/Neale Cousland, **97**/Dmitriy Shironosov, **98** (lower)/Neale Cousland, **100**/ollyy, **101** (lower)/issumbosi, **101** (upper)/Volina, **102**/brux, **108**–**9**/kuleczka, **111**/Zoom Team, **112** (lower)/Monkey Business Images, **112** (upper)/Sea Wave, **113** (lower)/Maksim Shmeljov, **117**/jabiru, **119**/bikeriderlondon, **120**/Tomas Mach, **122**/vesna cvorovic, **123**/auremar, **124**/Chris Hellyar, **126**/vinzstudio, **128**/ekler, **129**/ssuaphotos, **130**/snake**3**d, **131**/Roberto Cerruti, **132**/Gusev Mikhail Evgenievich, **133**/stockshoppe, **138**/Tsekhmister, **140** (lower)/Janaka Dharmasena, **140** (upper)/Anastasios71, **141**/Marafona, **142**/onime, **148**–**9**/qvist, **154** (lower)/Bertl123, **155**/micro10x, **158** (upper)/bikeriderlondon, **160** (upper)/magicinfoto, **162**/Ingvar Bjork, **163** (bicycle)/angelo gilardelli, **163** (surfboard)/Evgenia Bolyukh, **164**/Curioso, **167** (lower)/Kamila Starzycka, **167** (upper)/Margo Harrison, **168**/dotshock, **169** (lower)/hxdbzxy, **170** (upper)/Lim Yong Hian, **174**/teena137, **175** (lower)/oknoart, **175** (upper)/Dream79, **176**/Forewer, **180**–**1**/Sviatlana St, **183**/Cienpies Design, **186**/pgaborphotos, **195**/Mariusz Szczygiel, **199** (upper)/rSnapshotPhotos, **200**/trexdruid, **203**/M. Unal Ozmen, **204**/valdis torms, **206**/anaken2012, **207**/lucadp, **208** (left)/Yayayoyo, **208** (right)/Julien Tromeur, **214**–**15**/CREATIVE DISPLAY, **217**/stevenku, **218**/Barbro Bergfeldt, **220** (lower)/Cienpies Design, **238**/ptashka, **247** (lower)/Piratka, **250**/Bohbeh, **252**/karakotsya, **254**/Odysseas Stamoglou, **255**/DPiX Center, **260**–**1**/Oleksiy Avtomonov, **262**/Vadim Kozlovsky, **263** (upper)/Anna Furman, **266** (lower)/Alvinku, **266** (upper)/OtnaYdur, **270**/Sally Scott, **272** (upper)/Ilin Sergey, **273**/Ragnarock, **282**/Krailurk Warasup, **300**/Alex Hubenov, **303**/Sally Scott, **310**/Denton Rumsey, **312**/Sergii Korolko, **313**/vita khorzhevska, **321** (lower)/Robert Cumming, **321** (upper)/Andrey Shadrin, **322**/Olinchuk, **324**/Serg Zastavkin, **325**/lzf, **326**/Danilo Sanino, **332**–**3**/Brooman, **338**/Sea Wave, **345** (lower)/graphixmania, **351** (lower)/Vitalliy, **351** (middle)/Lauren Cameo, **351** (upper)/wavebreakmedia, **356**/Goodluz, **358** (lower)/Shane White, **358** (upper)/Romiana Lee, **360**/Marcin Balcerzak, **362**/Sebastian Kaulitzki, **368**–**9**/Phil McDonald, **370**/Mitch Gunn, **375**/joephotostudio, **376**/Neale Cousland, **377** (lower)/Lonely, **377** (upper)/Sergey Nivens, **385**/Viktor1, **392** (upper)/MadTatyana, **400**/Fotokostic, **415** (lower)/hxdbzxy, **418**/Alex_F, **420**/Naomi Koster, **430** (upper left)/Monkey Business Images, **430** (upper right)/Sue Smith, **430**–**1**/iDesign, **432**–**3**/Peeradach Rattanakoses, **435**/MagSpace, **438**/jannoon028, **439**/TK Kurikawa, **443** (lower left)/Evgeny Dubinchuk, **443** (lower right)/zmkstudio, **443** (middle)/Sashkin, **443** (upper)/Sarunyu_foto, **445**/Ugorenkov Aleksandr, **447**/ronfromyork, **462**/Jack.Q, **468** (lower)/Maksym Bondarchuk, **469**/Sashkin, **470** (blue dice)/vesna cvorovic, **470** (white dice)/chrisdorney, **470** (traffic lights)/Vacclav, **474**/TeddyandMia , **476**/dedMazay, **484**–**5**/Tanawat Pontchour, **493**/Jacek Chabraszewski, **499**/abdulsatarid, **505**/Federico Rostagno, **511**/Instinia, **518**/Janfilip, **520**/Evlakhov Valeriy, **530**–**1**/ghouliirina, **541** (lower)/vasabii, **549**/Luciano Puddori, **558**/Vinogradov Illya, **560**/airdone, **564**–**5**/Oleksiy Avtomonov, **567**/Giovanni Benintende, **576**–**7**/Shai_Halud, **583**/Andrei Simonenko, **585** (upper)/Danilo Sanino, **585** (lower)/KreativKolors,

586/007NATALIIA, **592**/Dora Zett, **594**/zentilia, **596** (upper)/antishock, **596** (lower)/Laurent Renault, **605**/Olena Boiko, **606**/Anis Mardhiah, **607**/Tomacco, **608**/Involved Channel, **625**/Tom Wang • © Stockbyte: **113** (upper) • © Viewfinder Australia Photo Library: **460**

Text

• © John Wiley & Sons Australia, Ltd: **253**/Extract 'A fatal fall … or was it murder?' from *Science Alive 1* p. 9; **419**/Extract 'Who reads news?' from *English Quest 1* p. 104

Every effort has been made to trace the ownership of copyright material. Information that will enable the publisher to rectify any error or omission in subsequent editions will be welcome. In such cases, please contact the Permissions Section of John Wiley & Sons Australia, Ltd.

TOPIC 1

Numeracy 1

1.1 Overview

Why learn this?

Why should you learn to count numbers? Imagine going into a shop and asking for a dozen eggs. How could you tell how many eggs you were getting? On trust? Was your change correct? How could you be sure?

You cannot go through life without learning to accurately add, subtract, multiply and divide.

What do you know?

assessON

1 **THINK** List what you know about numeracy. Use a thinking tool such as a concept map to show your list.
2 **PAIR** Share what you know with a partner and then with a small group.
3 **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of numeracy.

Learning sequence

1.1 Overview
1.2 Set A
1.3 Set B

WATCH THIS VIDEO
NAPLAN: strategies and tips

Searchlight ID: eles-1688

assessON

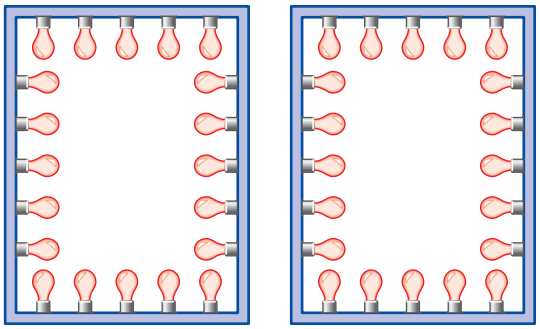
1.2 Set A

Calculator allowed

1 The original Ferris Wheel opened to the public in Chicago in 1893. The distance around the outside of the ferris wheel is approximately three times the diameter. This ferris wheel has a diameter of 76.2 m. If you were to string lights to the outside edge of the ferris wheel, what would be the approximate length of lights needed to the nearest metre?

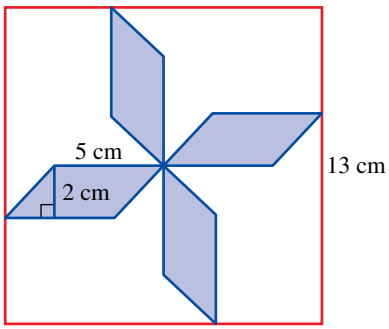
A 119 m **B** 120 m
C 200 m **D** 229 m

2 You want to string lights around two windows that measure 90 cm by 120 cm. What is the minimum length of lights needed?



A 2.5 m **B** 3.5 m
C 6.5 m **D** 8.5 m

3 This quilt design is a square with four identical parallelograms. What is the unshaded area of the quilt design?



A 65 cm^2 **B** 129 cm^2
C 149 cm^2 **D** 159 cm^2

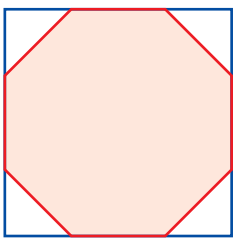
4 During the January holidays, Anna works in a café in Newcastle. She saves 75% of her earnings. If Anna earns $750, what is the best estimate of the amount of money that she saves?

A $550 **B** $300
C $150 **D** $100

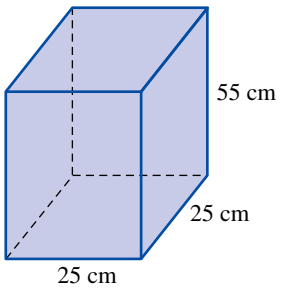
5 The measure of the interior angles of a triangle are $2x$, $6x$ and $10x$. What is the measure in degrees of the largest angle?

A 20 **B** 100
C 200 **D** 250

6 A regular octagon has an area of 100 cm^2. A square surrounds the octagon and has a perimeter of 60 cm. What is the area of one of the triangles if they are all the same size?

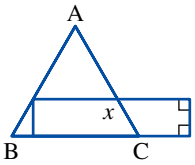


7 You have a rectangular storage box that is 55 cm high. You cut off a 5-cm strip around the top of the box. What will be the new volume of the box in cubic centimetres?



A 31 250 cm^3
B 27 500 cm^3
C 22 000 cm^3
D 20 000 cm^3

8 Triangle ABC is equilateral. What is the measure of angle x?



9 A shop reduces the price of sports shoes by 40%. The new price is $72. What was the original price of the sports shoes?

A $120
B $115.20
C $100.80
D $100

SALE 40% OFF

10 High-speed label applicators can put labels onto envelopes at a rate of 200 per second. Which of the following represents the number in a day?

A 1.728×10^5
B 1.728×10^6
C 1.728×10^7
D 1.728×10^8

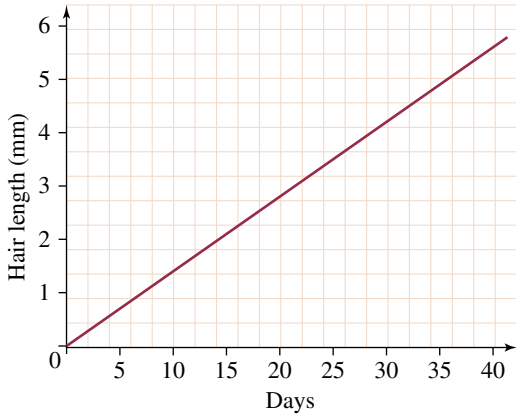
11 Alex and his six friends played a game of laser tag in which the person with the lowest final score wins. The table shows the final scores for each person except Alice.

Player	Score
Alex	151
Ben	153
Julie	149
Lee	139
Alice	
Aysha	135
Keta	143

If Alice won the game and the range was 19, what was Alice's score?

A 132
B 134
C 170
D 172

12 The following graph shows the growth rate of hair.

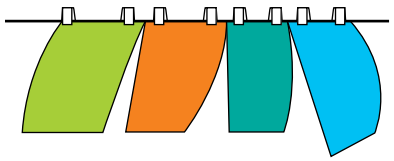


Based on the information in the graph, which figure best represents the number of millimetres that hair grows in 30 days?

A 2.4 mm **B** 3.6 mm **C** 4.2 mm **D** 5.7 mm

13 A Frisbee fits inside a cube. The top of the cube has a perimeter of 72 cm. If the Frisbee occupies 250 cm^3, how much space is left in the box?

14 You are hanging towels on a clothes line. You use two pegs per towel.



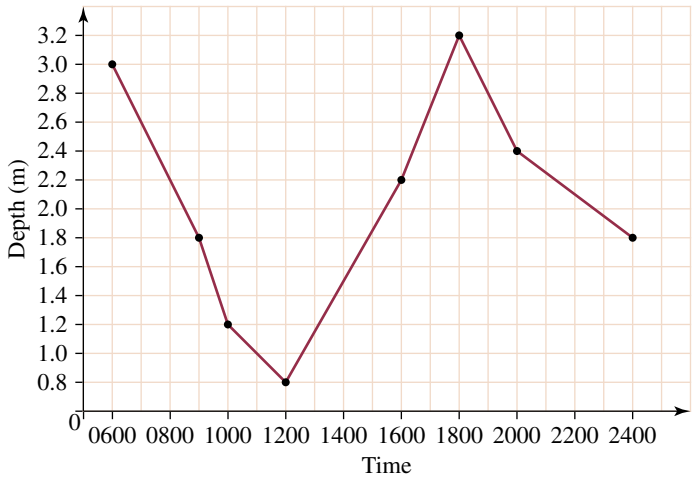
However, as you continue you realise that you will run out of pegs. Instead, you attach the end of the new towel to the old towel. In this way the towels share a peg.
If t represents the number of towels and P represents the number of pegs, which of these equations represents the number of pegs needed?

A $P = 2 \times t + 1$ **B** $P = 2 \times t + 2$ **C** $P = 3 \times t$ **D** $P = t + 1$

15 A cyclist is competing in an 80-km race. The record time for the race is 2 hours and 40 minutes. What will his speed, s (in km/h), need to be to beat this record?

A $s < 30$ **B** $s > 30$ **C** $s < 33$ **D** $s > 20$

16 The depth of the water inside Blue Lagoon Bay has been recorded over a time period as shown on the following graph.



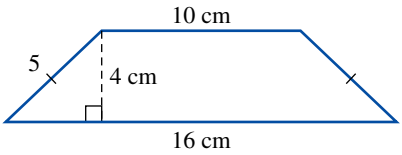
What is the depth at 10 pm?

17 Sing and Nam each buy a health bar at lunchtime from the vending machine. There are four different types to choose from: ANZAC, chocolate chip, triple fruit and yoghurt. What is the probability that they choose exactly the same type of bar?

18 A triangle with two identical sides and an angle of 119° is:

A scalene and acute **B** isosceles and acute
C isosceles and obtuse **D** isosceles and right-angled

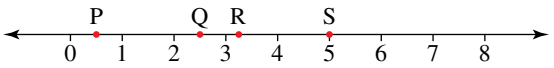
19 Alex wants to find the perimeter of the following isosceles trapezium.



Which equation could Alex use to find the perimeter of the trapezoid?

A $P = 10 + 16 + 4 + 5$ **B** $P = 10 + 16 + (2 \times 5)$
C $P = 10 + 16 + 4 + 5$ **D** $P = (10 + 16) \times 4 + 2$

20 Which point on the number line could represent the value of $\sqrt{10}$?

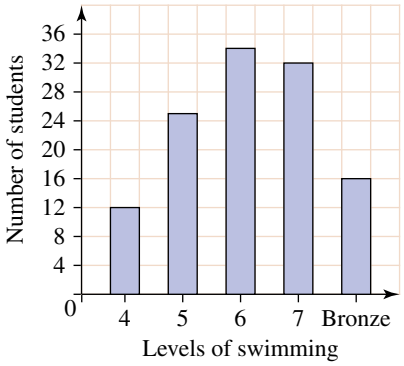


A S **B** P **C** R **D** Q

21 Lightning quickly heats the air, causing it to expand. This produces the sound of thunder. Sound travels approximately 1 km in 3 seconds. How far away is a thunderstorm when you notice a 2-second delay between the lightning and the sound of thunder?

A 1 km away **B** $\frac{1}{3}$ km away **C** $\frac{1}{2}$ km away **D** $\frac{2}{3}$ km away

22 The bar graph shows 120 students in Year 8 and their different swimming levels. What percentage of students, to the nearest whole number, have reached the bronze level?

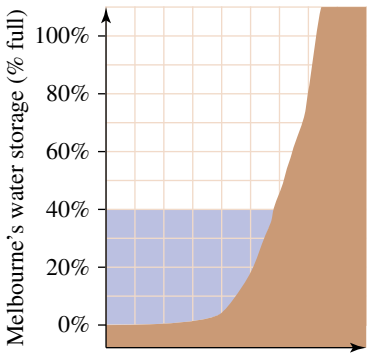


A 13% **B** 15% **C** 17% **D** 19%

23 A reservoir has a total capacity of 1 068 000 megalitres. Suppose the water is to be drained by a pump at a constant daily rate. If $\frac{9}{10}$ of the volume of the reservoir remains after the first day's pumping, how many megalitres (to the nearest megalitre) have been lost over 3 days? (Round to the nearest megalitre.)

A 772 740 **B** 320 400 **C** 289 428 **D** 1068

24 The following diagram shows the current storage level of a dam.



If the total capacity is 1 670 500 megalitres, how much water is available (to the nearest ten thousand megalitres)?

A 700 000 **B** 670 000 **C** 669 000 **D** 660 000

25 One student in the Japanese class is to make name tags, 50 mm × 50 mm square, from stiff card. The sheet of card measures 25 cm by 30 cm. What is the maximum number of name tags that can be made from one sheet?

A 10 **B** 20 **C** 30 **D** 40

26 The table shows the cost of hiring a band called The Hotshots.

The Hotshots band for hire	
Monday to Friday	$55 per hour
Saturday	$110 per hour
Acoustic Rental	$60 per booking
Deposit 20% of total cost	

A booking is made for Saturday for 4 hours, along with acoustic rental. Which of the following represents the deposit?

A $(110 \times 4 + 60) \times \frac{20}{100}$ **B** $(110 \times 4 + 60) \times \frac{100}{20}$

C $(110 + 60) \times 4 \times \frac{20}{100}$ **D** $(110 + 60) \times 4 \times \frac{100}{20}$

27 The Queenscliff Marine Centre hires a boat for 12 biologists to go diving for six days. The cost for hiring the boat for six days is usually $880. The marine centre obtains a 10% discount. What is the cost per person with the discount?

A $88 **C** $70
B $74 **D** $66

28 The manager of a cinema complex records the number of people attending the 2 pm session at the cinema from Monday to Friday.

	Monday	Tuesday	Wednesday	Thursday	Friday
Number of adults	120	170	147	160	183
Number of children	37	42	52	62	85

What is the mean number of adults attending the 2 pm session for that week?

A 120 **B** 150
C 156 **D** 160

29 Two cars begin at the same time and travel the same distance of 160 km. One car travels at 80 km per hour and the other car travels at 100 km per hour. How many minutes after the faster car will the slower car complete the journey?

A 20 minutes **B** 24 minutes
C 30 minutes **D** 36 minutes

30 You can usually stay in the sun for 9 minutes before burning. Using a sun protecting lotion with an SPF 20 rating means that you can stay in the sun for 9×20 minutes before burning. Your friend burns in 15 minutes. What sun protection factor would she need to use so that you can both stay for the same amount of time out in the sun? (Of course remember to wear a hat!)

A 8 **B** 10
C 12 **D** 15

1.3 Set B

assess on

Non-calculator

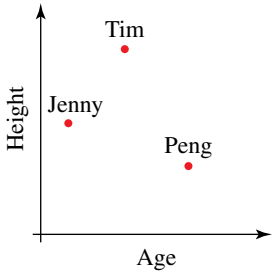
1 The height of the rectangular phone screen shown is 8 cm, and its area is 80 cm^2. The perimeter of the screen is:

A 32 cm **B** 36 cm
C 38 cm **D** 40 cm

2 The sum of $3x$ dollars and $3x$ cents, in cents, is:

A $3x + 3$ **B** $3x$
C $303x$ **D** $3x + 3x$

3 The following graph shows the ages and heights of three Year 8 students.
Which one of the following statements is true?



A Tim is the eldest and the tallest.
B Jenny is older than Peng and younger than Tim.
C Peng and Tim are the same age.
D Peng is the shortest.

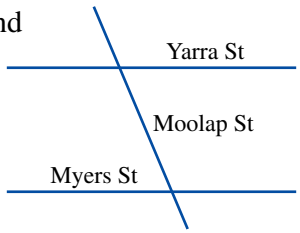
4 Two angles of a triangle are 63° and 57°. Which of the following could not be the measure of an exterior angle of the triangle?

A 110° **B** 117°
C 120° **D** 123°

5 Susan claims that the weight of her cat is at most 8 kg. What inequality represents her claim?

A $w < 8$
B $w > 8$
C $w \leq 8$
D $w \geq 8$

6 The following diagram shows two parallel streets, Yarra Street and Myers Street, intersected by Moolap St. The obtuse angle that Myers Street forms with Moolap Street is four times the measure of the acute angle that Yarra Street makes with Moolap Street. What is the measure of the acute angle at Yarra Street and Moolap Street?



A 30° **B** 36°
C 108° **D** 144°

7 Ann and Jack are playing a game where Ann gives an Input number for Jack to put in to the same expression to give an Output number.

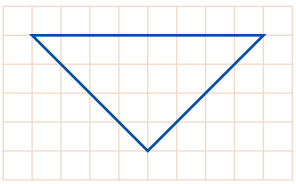
Input	1	2	3	4	?
Output	2	5	8	11	59

What is the Input number that Ann gave Jack for an Output number of 59?

A 18
B 20
C 22
D 24

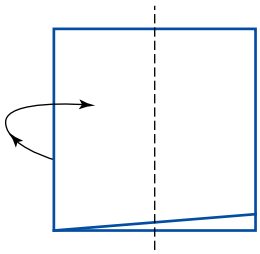
8 Five students competed in a 200-m race. Their finishing times were 47.5 s, 46.8 s, 47.3 s, 48.0 s and 48.2 s. What is their average time for running a 200-m race, correct to 2 decimal places?

9 A triangle has been drawn on 1-cm grid paper. Which statement is incorrect?



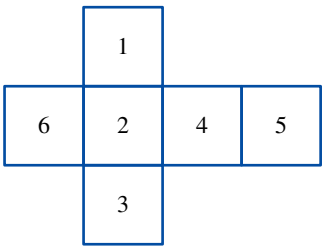
A The triangle is isosceles.
B The triangle is right-angled.
C The perimeter is 24 cm.
D The area is 16 cm^2.

10 Fold a paper square in half vertically and then cut it along the fold line. What is the ratio of the perimeter of one of the resulting two smaller rectangles to the large square?



A $\frac{1}{2}$ **B** $\frac{2}{3}$
C $\frac{3}{4}$ **D** $\frac{5}{6}$

11 Imagine that you fold the figure shown into a cube. Three faces meet at each corner. What is the largest sum of the three numbers whose faces meet at a corner?



A 15
B 14
C 13
D 12

12 Anne has to put 2 drops from an eyedropper in her eye twice a day. If the bottle of eye drops contains 20 mL and there are 8 drops in a millilitre, how many days will the bottle of eye drops last?

A 10 days **B** 20 days
C 30 days **D** 40 days

13 Two hundred Year 8 students are doing a science experiment in which 25 mL of an alkaline solution will be used by each student. How much solution is needed in total?

A 50 litres **B** 5 litres
C 0.5 litres **D** 0.05 litres

14 You have made 15 muffins for your class. You realise that this is only $\frac{3}{5}$ of the total that you need? How many more do you need to make?

A 3 **B** 5
C 9 **D** 10

15 A market gardener noted that 6 boxes of cherry tomatoes and 2 kg weighed the same amount as 5 boxes of cherry tomatoes and 4 kg. If x represents the weight of a box of cherry tomatoes in kg, which equation best represents the information?

A $8x = 9x$
B $12x = 20x$
C $6x + 2 = 5x + 4$
D $6x - 2 = 5x + 4$

16 Mulching the garden in the summertime is a great way of saving water. A bale of pea straw costs $5.00 and the delivery charge is $15.00. Olivia spent $90 in total. How many bales of pea straw did she buy for the garden?

A 18 **B** 15
C 12 **D** 9

17 In Yen's class, the ratio of the number of students who walked to school on Tuesday to the number of students who took some form of transport was 12 : 18. Which fraction is an equivalent form of this ratio?

A $\frac{2}{8}$ **B** $\frac{4}{9}$
C $\frac{2}{3}$ **D** $\frac{3}{4}$

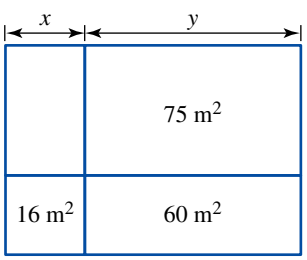
18 The Cheetahs and the Leopards are two school netball teams. The table shows the scores of their games.

	Game 1	Game 2	Game 3	Game 4	Game 5
Cheetahs	45	40	35	49	64
Leopards	28	50	27	52	63

Based on the scores in the table, which statement is true?

A The Cheetahs won 20% of the games.
B The Cheetahs won 30% of the games.
C The Leopards won 40% of the games.
D The Leopards won 60% of the games.

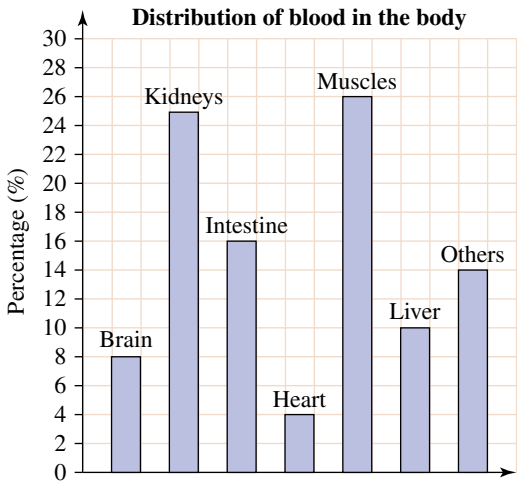
19 The following is a diagram of a proposed floor plan for an office space. The proposed plan has four areas. Three of the areas are rectangular and the fourth is a square. What is the length of y?



A 3 m **B** 5 m
C 15 m **D** 25 m

20 Adults, on average, have 5.5 litres of blood in their bodies. How many millilitres of blood are in the kidneys?

A 25
B 500
C 1000
D 1375



21 Anne and her friends decided to watch a DVD. They started it at 8.30 pm and it ran for 105 minutes. At what time did the DVD end?

A 9.30 pm
B 9.35 pm
C 10.05 pm
D 10.15 pm

22 A 900-car parking lot is divided into 3 sections. There are 330 spots in Section 1. Section 2 holds 160 more than will fit into Section 3. How many spots are in Section 3?

23 A chef assembles a cake in $\frac{2}{3}$ of an hour. If he works for $7\frac{1}{2}$ hours, how many cakes will he fully assemble?

A $10\frac{3}{4}$
B 11
C $11\frac{1}{4}$
D 12

24 A roll of material contains 6 metres of cloth. Four lengths, each of x centimetres, are cut from the cloth. What length of material (in centimetres) remains?

A $6 - 4x$
B $4x - 6$
C $600 - 4x$
D $100(6 - 4x)$

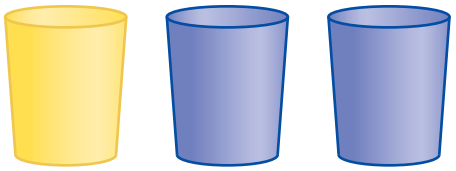
25 Two roads cross. What is the size of angle A?

26 A survey is taken of a Year 8 group to determine the length of time students spend on homework each night. What percentage of the group spend 60 minutes or more on homework each night?

Time spent in minutes	Number of pupils
0	6
15	10
30	14
40	2
50	4
60	8
70	10
80	5
90	1

27 In this diagram of yellow and blue buckets, each bucket of the same colour contains the same number of pegs. How many pegs are in each of the given coloured buckets?

Total number of pegs in 4 yellow buckets = 28 pegs



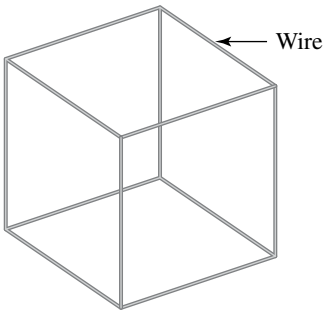
Total number of pegs in 1 yellow bucket and 2 blue buckets = 43 pegs

28 Which of these is the lightest?

A 25 000 grams
B 2.50 kilograms
C 25 000 000 milligrams
D 2.5 tonnes

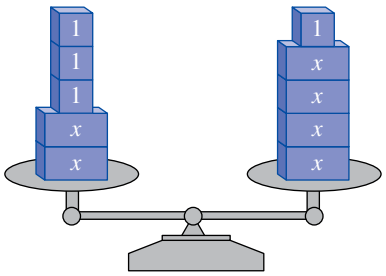
29 A piece of wire is cut into the exact number of pieces needed to create the edges of a cube. If the volume of the cube is 125 cm^3, what was the length of wire to begin with?

A 5 cm **B** 30 cm
C 50 cm **D** 60 cm



30 Given the balance, what is the value for x?

A $\frac{1}{2}$ **B** 1
C 2 **D** 4



Link to SpyClass, an exciting online game combining a comic book–style story with problem-based learning in an immersive environment.

Join Jesse, Toby and Dan and help them to tackle some of the world's most dangerous criminals by using the knowledge you've gained through your study of mathematics.

www.spyclass.com.au

ANSWERS

TOPIC 1 Numeracy 1

1.2 Set A Calculator allowed

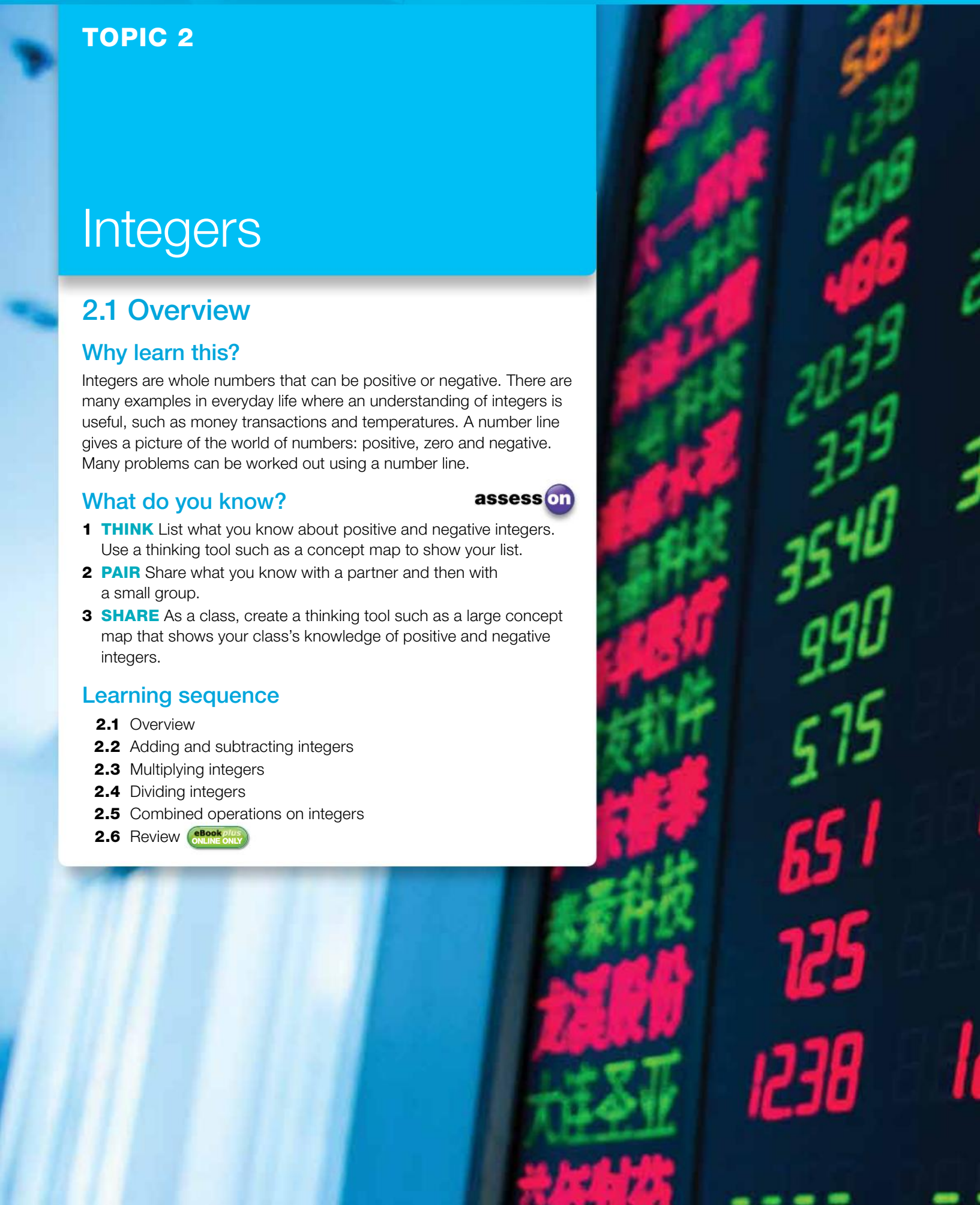
1 D **2** D **3** B **4** A
5 B **6** 31.25 cm^2 **7** A **8** 120°
9 A **10** C **11** B **12** C
13 5582 cm^3 **14** D **15** B **16** 2.1 m
17 $\frac{1}{4}$ **18** C **19** B **20** C
21 D **22** A **23** B **24** B
25 C **26** A **27** D **28** C
29 B **30** C

1.3 Set B Non-calculator

1 B **2** C **3** D **4** A
5 C **6** B **7** B **8** 47.56 s
9 C **10** C **11** B **12** D
13 B **14** D **15** C **16** B
17 C **18** C **19** C **20** D
21 D **22** 205 **23** B **24** C
25 107° **26** 40%
27 7 pegs in each yellow bucket; 18 pegs in each blue bucket.
28 B **29** D **30** B

TOPIC 2

Integers



2.1 Overview

Why learn this?

Integers are whole numbers that can be positive or negative. There are many examples in everyday life where an understanding of integers is useful, such as money transactions and temperatures. A number line gives a picture of the world of numbers: positive, zero and negative. Many problems can be worked out using a number line.

What do you know?

assessON

1 **THINK** List what you know about positive and negative integers. Use a thinking tool such as a concept map to show your list.
2 **PAIR** Share what you know with a partner and then with a small group.
3 **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of positive and negative integers.

Learning sequence

2.1 Overview
2.2 Adding and subtracting integers
2.3 Multiplying integers
2.4 Dividing integers
2.5 Combined operations on integers
2.6 Review eBook plus ONLINE ONLY

2020
335
3508
981
576
651
726
1237
603
-069
093
069
008

2.2 Adding and subtracting integers

Integers

- **Integers** are positive whole numbers, negative whole numbers and zero.
- The group of integers is often referred to as the set Z.
- $Z = \{\ldots -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$

eBookplus
eLesson
Directed numbers
eles-0040

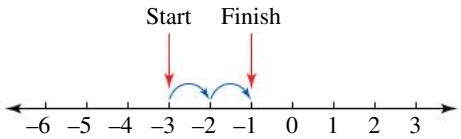
Addition of integers

- A number line can be used to add integers.
- To add a positive integer, move to the right.
- To add a negative integer, move to the left.

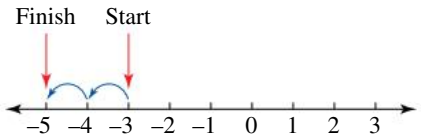
WORKED EXAMPLE 1

Calculate the value of each of the following.
a $-3 + +2$ **b** $-3 + -2$

THINK	WRITE
a 1 Start at −3 and move 2 units to the right, as this is the addition of a positive integer.	**a** $-3 + +2$



2 Write the answer.	$3 + +2 = -1$
b 1 Start at −3 and move 2 units to the left, as this is the addition of a negative integer.	**b** $-3 + -2$



2 Write the answer.	$-3 + -2 = -5$

Subtraction of integers

- Subtracting an integer gives the same result as adding its opposite.
  For example, $-3 - 5 = -3 - +5 = -3 + -5 = -8$.
  Note that +5 and −5 are opposites.
- By developing and extending a pattern, we can show that subtracting a negative has the same effect as adding a positive.
  Look at the pattern shown at right.
  It can be seen from the table that subtracting a negative is the same as adding its inverse. For example, $8 - -4 = 8 + +4 = 12$.

$8 - 3 =$	5
$8 - 2 =$	6
$8 - 1 =$	7
$8 - 0 =$	8
$8 - -1 =$	9
$8 - -2 =$	10
$8 - -3 =$	11

- In mathematics, a number without a positive or negative sign is considered to be positive. So $8 + +4$ can be written as $8 + 4$ and $-5 - +1$ can be written as $-5 - 1$.

WORKED EXAMPLE 2

Calculate the value of each of the following.
a $-7 - +1$ **b** $-2 - -3$

THINK	WRITE
a 1 Subtracting an integer gives the same result as adding its opposite.	**a** $-7 - +1$
2 Using a number line, start at -7 and move 1 unit to the left.	$= -7 + -1$
3 Write the answer.	$= -8$
b 1 Subtracting an integer gives the same result as adding its opposite.	**b** $-2 - -3$
2 Using a number line, start at -2 and move 3 units to the right.	$= -2 + +3$
3 Write the answer.	$= +1$

Exercise 2.2 Adding and subtracting integers

assessON

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–3, 4a–d, 5a–d, 6a–i, 11, 12, 17, 18	Questions: 1–6, 7a, b, 9–14, 17, 18	Questions: 3a–h, 4e–h, 5e–h, 6k–r, 7c, d, 8–11, 13–19

Individual pathway interactivity int-4397 eBookplus

REFLECTION
What strategy will you use to remember how to add and subtract integers?

FLUENCY

1 Which of the following numbers are integers?
$3, \frac{1}{2}, -4, 201, 20.1, -4.5, -62, -3\frac{2}{5}$

2 Copy and complete the following addition and subtraction number patterns by placing the correct integers in the boxes.
a 6, 4, 2, ☐, ☐, ☐
b $-5, -10, -15$, ☐, ☐, ☐
c ☐, ☐, ☐, $-1, -3, -5$
d ☐, ☐, ☐, $-2, 0, 2$

eBookplus

Digital docs
SkillSHEET
Integers on the number line
doc-6387
SkillSHEET
Adding and subtracting integers
doc-6388
SkillSHEET
Arranging numbers in order
doc-6389

3 WE1 Calculate the value of each of the following.

a $-3 + 2$ **b** $-7 + -3$ **c** $6 + -7$

d $-8 + -5$ **e** $13 + +6$ **f** $12 + -5$

g $-25 + +10$ **h** $16 + -16$

4 WE2 Calculate the value of each of the following.

a $7 - +2$ **b** $-18 - +6$ **c** $3 - +8$

d $11 - +6$ **e** $17 - -9$ **f** $-28 - -12$

g $14 - -8$ **h** $-17 - -28$

5 Calculate the value of each of the following.

a $-3 + -5$ **b** $6 - -5$ **c** $-17 + +3$

d $-14 - -13$ **e** $28 - -23$ **f** $-48 + -3$

g $-57 - -18$ **h** $-32 - -40$

6 Simplify the following.

a $-4 + -3$ **b** $-6 - +3$ **c** $5 + -2$

d $17 - +5$ **e** $-13 - -3$ **f** $10 - -3$

g $-26 + -14$ **h** $25 + -7$ **i** $32 - +5$

j $-16 - +18$ **k** $-26 - -15$ **l** $124 - -26$

m $-3 + -4 - -6$ **n** $27 + -5 - -3$ **o** $-10 + +3 - +6$

p $23 + -15 - -14$ **q** $15 - -4 + -10$ **r** $-37 - -5 - -10$

eBookplus
Interactivity
Directed number target
int-0074

UNDERSTANDING

7 Copy and complete the following tables. For the subtraction table, subtract the number on the side from the number at the top.

a

+	−8	+25	−18	32
−6	−8 + −6 = −14			
−13				
−16				
−19				

b

−	+15	−17	−27	57
+7	+15 − +7 = 8			
−6				
−9				
+12				

c

+	−11		13	
	−16			
+17		36		
		18	12	
−28				−35

d

−	+9			42
	−17			
−14			−1	
		23		
+23		−2		

8 Design your own tables for addition and subtraction of integers like those in question **7**. Fill in all answers in your tables.

9 In a kitchen, some food is stored at −18 °C in a freezer and some at 4 °C in the fridge. A roast is cooking in the oven at a temperature of 180 °C.

Before answering each of the following questions, draw a number line to show the positions of the temperatures.

a What is the difference in temperature between the food stored in the freezer and the food stored in the fridge? (*Hint:* Difference = largest value − smallest value)

b What is the difference in temperature between the food stored in the fridge and the roast cooking in the oven?

c What is the difference in temperature between the food stored in the freezer and the roast cooking in the oven?

10 Calculate the difference between the two extreme temperatures recorded at Mawson Station in Antarctica in recent times.

11 Locate the button on your calculator that allows you to enter negative numbers. Use it to answer the following.

a −458 + 157

b −5487 − 476

c −248 − −658 − −120

d −42 + 57 − −68 + −11

12 Write out these equations, filling in the missing numbers.

a $-7 + \square = 6$

b $8 - \square = 12$

c $-15 - \square = -26$

d $42 - \square + -17 = 35$

e $-7 - \square - -31 = -28$

f $\square - 13 + 21 = 79$

13 The following is a homework sheet done by a student in Year 8. Correct her work for her and give her a mark out of six. Make sure you include the correct answer if her answer is wrong.

a $-3 + -7 = -10$

b $-4 - -10 = -6$

c $-7 - 8 = 15$

d $9 - -8 + -7 = 10$

e $42 + 7 - -11 = 60$

f $-17 + 4 - 8 = 23$

REASONING

14 Evaluate and compare the following pairs of expressions.

a $-4 + 1$ and $+1 - 4$

b $-7 + 5$ and $+5 - 7$

c $-8 + 3$ and $+3 - 8$

15 What did you notice about the answers in question **14**? Use a number line to help you explain why this is the case.

PROBLEM SOLVING

16 Evaluate and compare the following pairs of expressions.

a $-2 + -5$ and $-(2 + 5)$

b $-3 + -8$ and $-(3 + 8)$

c $-7 + -6$ and $-(7 + 6)$

17 What did you notice about the answers in question **16**? Explain why this is the case.

18 Igor visited the Macau Casino. By using the following information and writing a loss as a negative number and a win as a positive number, find out how much money he had when he arrived at the casino. The currency used in the casino is the Hong Kong dollar (HKD).

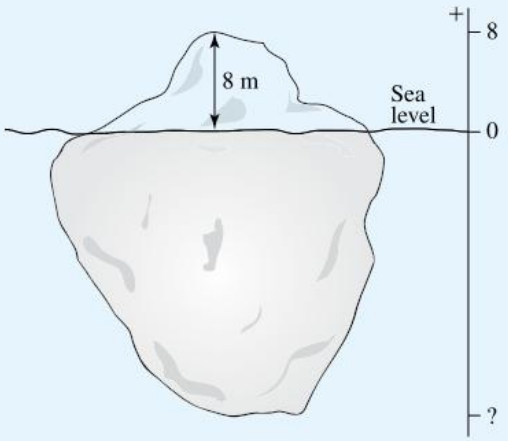
Arriving at the casino at 10 pm, Igor went straight to the baccarat table, where he won 270 HKD. The roulette wheel that he played next was a disaster, costing 340 HKD in a very short space of time.

Igor moved to the 'vingt-et-un' table where he cleaned up, making 1450 HKD after a run of winning hands.

Little did Igor know that when he sat down at the stud poker table it would signal the end of his night at the casino. He lost everything in one hand, 2750 HKD, not even leaving enough for a taxi to his hotel.

CHALLENGE 2.1

The tip of this iceberg is 8 metres above sea level. If one-tenth of its total height is above the surface, represent the depth of its lowest point as an integer.



2.3 Multiplying integers

- Patterns in the answers in multiplication tables can be used to determine the product when two directed numbers are multiplied. Consider the following patterns.

$3 \times 3 = 9$	$-3 \times 3 = -9$
$3 \times 2 = 6$	$-3 \times 2 = -6$
$3 \times 1 = 3$	$-3 \times 1 = -3$
$3 \times 0 = 0$	$-3 \times 0 = 0$
$3 \times -1 = -3$	$-3 \times -1 = 3$
$3 \times -2 = -6$	$-3 \times -2 = 6$
$3 \times -3 = -9$	$-3 \times -3 = 9$
Answers go down by 3.	Answers go up by 3.

- When multiplying two directed numbers:
  - if their signs are the same, the answer is positive
  - if their signs are different, the answer is negative.

$+ \times + = +$	$+ \times - = -$
$- \times - = +$	$- \times + = -$

WORKED EXAMPLE 3

Evaluate each of the following.

a $-3 \times +7$ **b** -8×-7

THINK

a The two numbers have different signs, so the answer is negative ($7 \times 3 = 21$).

b The two numbers have the same signs, so the answer is positive ($8 \times 7 = 56$).

WRITE

a $-3 \times +7$
$= -21$

b -8×-7
$= 56$ (or $+56$)

Powers and square roots of directed numbers

- Calculating powers of negative numbers uses the same process as calculating powers of positive numbers.
- There are two possible answers when you take the square root of a number.
  For example:

$$4^2 = 4 \times 4 \qquad (-4)^2 = (-4) \times (-4)$$
$$= 16 \qquad = 16$$

Therefore when asked to take the square root of 16, the answer could be $\pm\sqrt{16}$.
$\pm\sqrt{16} = -4$ or $+4$, which can also be written as ± 4 (positive or negative 4).

WORKED EXAMPLE 4

Evaluate each of the following.
a $(-5)^3$ **b** **The square root of 64**

THINK	WRITE
a 1 Write the question in expanded form.	**a** $(-5)^3 = (-5) \times (-5) \times (-5)$
2 Evaluate by working from left to right beginning with $-5 \times -5 = +25$.	$= +25 \times (-5)$ $= -125$
b Look for the numbers that, when squared, result in 64 ($8 \times 8 = 64$ and $-8 \times -8 = 64$).	**b** $\pm\sqrt{64} = +8$ or -8 $= \pm 8$

assess on

Exercise 2.3 Multiplying integers

REFLECTION
Can we find square roots, cube roots, fourth roots and so on for negative numbers?

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–3, 4a–c, 5a–d, 7, 8a–c, 9, 13, 15, 16	Questions: 2, 3, 4a–f, 5a–f, 7, 8a–d, 9, 10, 12, 13, 15, 16	Questions: 2, 3c–e, 4f–i, 5f–i, 6–14, 16, 17

Individual pathway interactivity int-4398 eBookplus

eBookplus
Digital doc
SkillSHEET
Multiplying integers
doc-6390

FLUENCY

1 Copy and complete the following tables.

$4 \times 4 =$ _____	$-5 \times 4 =$ _____	$-6 \times -4 =$ _____
$4 \times 3 =$ _____	$-5 \times 3 =$ _____	$-6 \times -3 =$ _____
$4 \times 2 =$ _____	$-5 \times 2 =$ _____	$-6 \times -2 =$ _____
$4 \times 1 =$ _____	$-5 \times 1 =$ _____	$-6 \times -1 =$ _____
$4 \times 0 =$ _____	$-5 \times 0 =$ _____	$-6 \times 0 =$ _____
$4 \times -1 =$ _____	$-5 \times -1 =$ _____	$-6 \times 1 =$ _____
$4 \times -2 =$ _____	$-5 \times -2 =$ _____	$-6 \times 2 =$ _____
$4 \times -3 =$ _____	$-5 \times -3 =$ _____	$-6 \times 3 =$ _____
$4 \times -4 =$ _____	$-5 \times -4 =$ _____	$-6 \times 4 =$ _____

2 WE3 Evaluate each of the following.

a -2×5	**b** 3×-8	**c** -6×-7	**d** 2×-13
e -8×-6	**f** -7×6	**g** -10×75	**h** -115×-10
i -7×9	**j** $+9 \times -8$	**k** -11×-5	**l** 150×-2

3 Use an appropriate method to evaluate the following.

a $-2 \times 5 \times -8 \times -10$ **b** $8 \times -1 \times 7 \times -2 \times 1$

c $8 \times -4 \times -1 \times -1 \times 6$ **d** $-3 \times -7 \times -2 \times -1 \times -1 \times -1$

e $-5 \times -8 \times -2 \times -2$

4 Complete the following equations.

a $7 \times ___ = -63$ **b** $-3 \times ___ = -21$ **c** $16 \times ___ = -32$

d $___ \times -3 = 36$ **e** $___ \times 7 = -42$ **f** $___ \times -9 = -72$

g $___ \times -4 = 80$ **h** $-10 \times ___ = 60$ **i** $-11 \times ___ = 121$

5 **WE4a** Evaluate each of the following.

a $(-2)^3$ **b** $(-3)^2$ **c** $(-2)^4$ **d** $(-3)^4$ **e** $(-2)^5$

f $(-4)^2$ **g** $(-5)^3$ **h** $(-4)^4$ **i** $(-5)^4$ **j** $(-6)^3$

6 Use your answers to question **5** to help complete the following statements.

a If a negative number is raised to an even power, the answer is (positive/negative).

b If a negative number is raised to an odd power, the answer is (positive/negative).

7 **WE4b** Evaluate the square root of the following numbers.

a 25 **b** 81 **c** 49 **d** 121 **e** 100

8 If $a = -2$, $b = -6$, $c = 4$ and $d = -3$, calculate the values of the following expressions.

a $a \times b \times c$ **b** $a \times -b \times -d$ **c** $b \times -c \times -d$

d $c \times -a \times -a$ **e** $d \times -(-c)$ **f** $a \times d \times b \times c^2$

UNDERSTANDING

9 For each of the following, write three possible sets of integers that can be placed in the boxes to make the equation a true statement.

a $\square \times \square \times \square = -12$ **b** $\square \times \square \times \square = 36$ **c** $\square \times \square \times \square \times = -36$

10 For each of the following, determine whether the result is a positive or negative value. You do not have to work out the value.

a $-25 \times 54 \times -47$ **b** $-56 \times -120 \times -145$ **c** $-a \times -b \times -c \times -d \times -e$

11 What happens when a number is multiplied by -1? Use some examples to illustrate your answer.

12 The notation $-(-3)$ is a short way of writing -1×-3. Write the expression represented by each of the following and then use an appropriate method to determine the answer.

a $-(-2)$ **b** $-(+3)$ **c** $-(-5)$

d $-(-(+5))$ **e** $-(-(-7))$ **f** $-(-(+4))$

REASONING

13 Explain why the answer to this question is negative.
$-2 \times -4 \times +3 \times -6 \times +4 \times +3$

14 If the answer to $(-a)^n$, $n > 0$ is negative, establish whether a is positive or negative and whether n is odd or even. Give a reasoned explanation for your answer.

PROBLEM SOLVING

15 If $a = -3$ and $b = -4$, evaluate $a^3 \times b^2$.

16 Evaluate $(-1)^n \times (-1)^{n+1}$ if:

a n is even **b** n is odd.

eBookplus
Digital doc
WorkSHEET 2.1
doc-6400

17 In a Year 12 Mathematics examination there are 30 multiple choice questions. A student scores 2 marks for a correct answer, −1 mark for an incorrect answer and zero marks for an unanswered question.

Mary scores a total of 33 marks in the multiple choice section. How could she have reached this total?

2.4 Dividing integers

- Division is the inverse operation of multiplication. We can use the multiplication facts for directed numbers to discover the division facts for directed numbers.

Multiplication fact	Division fact	Pattern
$2 \times 3 = 6$	$6 \div 3 = 2$ or $\frac{6}{3} = 2$ and $6 \div 2 = 3$ or $\frac{6}{2} = 3$	$\frac{\text{positive}}{\text{positive}} = \text{positive}$
$-2 \times -3 = 6$	$6 \div -3 = -2$ or $\frac{6}{-3} = -2$ and $6 \div -2 = -3$ or $\frac{6}{-2} = -3$	$\frac{\text{positive}}{\text{negative}} = \text{negative}$
$-2 \times 3 = -6$	$-6 \div 3 = -2$ or $\frac{-6}{3} = -2$ and $-6 \div -2 = 3$ or $\frac{-6}{-3} = 3$	$\frac{\text{negative}}{\text{positive}} = \text{negative}$ and $\frac{\text{negative}}{\text{negative}} = \text{positive}$

- When dividing two directed numbers:
  – if their signs are the same, the answer is positive
  – if their signs are different, the answer is negative.
- Remember that division statements can be written as fractions and then simplified. For example,

$$-12 \div -4 = \frac{-12}{-4}$$
$$= \frac{12 \times -\cancel{1}}{4 \times -\cancel{1}}$$
$$= 3$$

$\frac{+}{+} = +$	$\frac{+}{-} = -$
$\frac{-}{-} = +$	$\frac{-}{+} = -$

WORKED EXAMPLE 5

Evaluate each of the following.

a $-56 \div 8$ **b** $\frac{-36}{-9}$

THINK

a The two numbers have different signs, so the answer is negative.

b Cancel the common factors (−1). The two numbers have the same signs, so the answer is positive.

WRITE

a $-56 \div 8$
$= -7$

b $\frac{-36}{-9} = \frac{-1 \times 36}{-1 \times 9}$
$= \frac{36}{9}$
$= 4$

WORKED EXAMPLE 6

Evaluate the following.
a $234 \div -6$ **b** $-182 \div -14$

THINK	WRITE
a 1 Complete the division as if both numbers were positive numbers.	**a** $6\overline{)23^{5}4}$ 39
2 Determine the sign of the answer. A positive number divided by a negative number is a negative number.	$234 \div -6 = -39$
b 1 Complete the division as if both numbers were positive numbers.	**b** $14\overline{)182}$ 13 14 42 42 0
2 Determine the sign of the answer. A negative number divided by a negative number is a positive number.	$-182 \div -14 = 13$

Exercise 2.4 Dividing integers

assess on

INDIVIDUAL PATHWAYS

PRACTISE
Questions:
1, 2, 3a–f, 5, 6, 7a–c, 8a–c, 13, 16

CONSOLIDATE
Questions:
1a–h, 2, 3a–i, 5a–f, 6, 7a–d, 8, 9a, 10a, 11, 13, 15, 16

MASTER
Questions:
1g–l, 2, 3g–l, 4, 5c–h, 6, 7c–f, 8c–f, 9–12, 14, 15

REFLECTION
Can you list 4 areas in real life where directed numbers are used?

Individual pathway interactivity int-4399 eBookplus

FLUENCY

eBookplus
Digital doc
SkillSHEET
Dividing integers
doc-6391

1 WE5a Evaluate each of the following.

a $-63 \div 9$ **b** $8 \div -2$ **c** $-8 \div 2$
d $-6 \div -1$ **e** $88 \div -11$ **f** $0 \div -5$
g $48 \div -3$ **h** $-129 \div 3$ **i** $-56 \div -7$
j $+184 \div -4$ **k** $-55 \div -11$ **l** $304 \div -8$

2 WE5b Evaluate each of the following.

a $\frac{-121}{-11}$ **b** $\frac{-12}{3}$ **c** $\frac{-36}{-12}$
d $\frac{21}{-7}$ **e** $\frac{-100}{-50}$ **f** $-3 \times \frac{2}{-3}$

3 WE6 Evaluate the following.

a $960 \div -8$ **b** $-243 \div 9$ **c** $-266 \div -7$
d $-132 \div -4$ **e** $-282 \div 6$ **f** $1440 \div -9$
g $324 \div -12$ **h** $-3060 \div 17$ **i** $-6000 \div -24$
j $-2294 \div -37$ **k** $4860 \div 15$ **l** $-5876 \div -26$

UNDERSTANDING

4 Write three different division statements, each of which has an answer of -8.

5 Copy and complete the following by placing the correct integer in the box.

a $-27 \div \square = -9$ **b** $-68 \div \square = 34$ **c** $72 \div \square = -8$

d $-18 \div \square = -6$ **e** $\square \div 7 = -5$ **f** $\square \div -4 = -6$

g $-132 \div \square = 11$ **h** $-270 \div \square = 27$

6 Calculate the value of each of the following by working from left to right.

a $-30 \div 6 \div -5$ **b** $-120 \div 4 \div -5$ **c** $-800 \div -4 \div -5 \div 2$

7 If $a = -12$, $b = 3$, $c = -4$ and $d = -6$, calculate the value of each of the following expressions.

a $a \div c$ **b** $a \div b$ **c** $a \div d$

d $b \div c$ **e** $b \div d$ **f** $a \div b \div d$

8 If $a = -24$, $b = 2$, $c = -4$ and $d = -12$, calculate the value of each of the following expressions.

a $a \div b \times c$ **b** $d \times c \div b \div c$ **c** $b \div c \div d \times a$

d $c \times a \div d \div b$ **e** $a \times b \div d \div d$ **f** $a \div d \times c \div b$

9 Copy and complete the following tables:

a

×			**−6**	**+8**
			18	
−10		−40		
	10		30	
−7				−56

b

×			**−9**	
+6	30			−42
		36		
	−55		99	
		−6	−18	

10 Copy and complete the following tables. Divide the number on the top by the number on the side.

a

÷	**4**	**−10**	**12**	**−8**
−2				
7				
−3				
−10				

b

÷				**−4**
		−2		
−8	−4	3		
6			−6	
				1

REASONING

11 The answer to $\dfrac{p \times q}{2 \times -5}$ is negative. What can you deduce about p and q?

12 The answer to $\dfrac{\dfrac{(-a)^2}{(-b)^3}}{(-c)^4}$ is positive. What can you deduce about b?

PROBLEM SOLVING

13 If $a = 2$ and $b = -6$, evaluate $\dfrac{(-a)^3}{(-b)^4}$.

14 Evaluate $\dfrac{(-1)^{n+1}}{(-1)^{n+2}}$, $n > 0$, if:

a n is even

b n is odd.

15 *Acceleration* is a measure of a change in speed or velocity. If you are increasing speed, acceleration is positive; if you are decreasing speed (slowing down) acceleration is negative. (You will need to do some research to answer this question.)

a Is it possible to have positive acceleration with a negative velocity? If so, give an example.

b Is it possible to have negative acceleration with a positive velocity? If so, give an example.

16 My last five scores in a very silly computer game have been +6, −9, −15, +8 and −4. What is my average score?

2.5 Combined operations on integers

- The mathematical rules about order of operations apply when we work with directed numbers.
- BIDMAS helps us to remember the correct order in which we should perform the various operations. This means we do the brackets (B) first; then indices (I); then division (D) and multiplication (M), working from left to right; and finally addition (A) and subtraction (S), working from left to right.

WORKED EXAMPLE 7

Calculate the value of each of the following.

a $54 \div -6 + 8 \times -9 \div -4$

b $-8 \div 2 + (-2)^3$

THINK	WRITE
a 1 Write the question.	**a** $54 \div -6 + 8 \times -9 \div -4$
2 There are no brackets or powers, so, working from left to right, complete all multiplication and division, then the addition.	$= -9 + 18$
3 Write the answer.	$= 9$

b			b
	1	Write the question.	$-8 \div 2 + (-2)^3$
	2	Simplify the cubed term.	$= -8 \div 2 + -8$
	3	Complete the division.	$= -4 + -8$
	4	Write the answer.	$= -12$

assessON

Exercise 2.5 Combined operations on integers

REFLECTION
What effects do directed numbers have on order of operations?

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–3, 4a, 8	Questions: 1–3, 4a, b, 8, 9	Questions: 1–7, 9

Individual pathway interactivity int-4400 eBook plus

eBook plus
Digital doc
SkillSHEET
Order of operations and directed numbers
doc-6392

FLUENCY

1 WE7a Calculate the values of the following expressions.

a $-4 - 6 - 2$	**b** $-4 \times 2 + 1$	**c** $8 \div (2 - 4) + 1$
d $7 - (3 - 1) + 4$	**e** $6 \times (4 + 1)$	**f** $-3 - 40 \div 8 + 2$
g $-4 + 5 - 6 - 7$	**h** $-5 \times 12 + 2$	**i** $12 \div (2 - 4) - 6$
j $13 - (4 - 6) + 2$	**k** $7 \times (6 + 2)$	**l** $-6 - 36 \div 9 + 3$

2 WE7b Evaluate each of the following.

a $-7 + 6 \times -2$	**b** $-9 - 15 + 3$	**c** $(-63 \div -7) \times -3 + -2$
d $(-3)^3 - 3 \times -5$	**e** $-5 \times -7 - [5 + (-8)^2]$	**f** $[(-48 \div 8)^2 \times 36] \div -4$

3 Calculate the values of the following expressions.

a $-3 + 15 - 26 - 27$	**b** $-8 \times 11 + 12$	**c** $52 \div (-9 - 4) - 8$
d $23 - (16 - 4) + 7 - 3$	**e** $15 \times (-6 + 2)$	**f** $-6 - 64 \div -16 + 8$
g $-3 \times -4 \times -1 \times 5$	**h** $-6 \times (-13 + 5) + -4 + 2$	

UNDERSTANDING

4 A class of Year 8 students were given the following question to evaluate.

$$4 + 8 \div -(2)^2 - 7 \times 2$$

a A number of different answers were obtained, including -8, -12 and -17. Which one of these is the correct answer?

b Using only brackets, change the question in two ways so that the other two answers would be correct.

5 In a particular adventure video game, a player loses and gains points based on who or what they come in contact with during the game. See the list below right for the number of 'hit' points associated with each contact. Use the table to calculate the number of points the player has at the end of each round of the game.

Round number	Points at the start of the round	Contacts during the round	Points at the end of the round
1	100	20 gnomes, 10 goblins and 3 healing potions	
2		3 gnomes, 5 goblins, 6 orcs and 5 healing potions	
3		3 orcs, 6 trolls and a cleric	
4		5 trolls, 1 balrog and a cleric	

Character	'Hit' points
Balrog	–100
Troll	–10
Orc	–5
Goblin	–2
Gnome	–1
Healing potion	+20
Cleric	+50

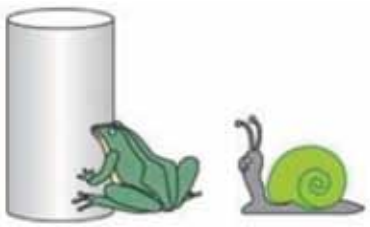
REASONING

6 The variables p and q have the same numerical value but the opposite sign; that is, one is positive and the other negative. If $-3 \times p + 4 \times q$ is positive, what can you say about p and q?

7 Test your answer to question **6** if the numerical values of p and q, written as $|p|$ and $|q|$, are both equal to 7.

PROBLEM SOLVING

8 A frog and a snail are climbing an empty vertical pipe. The snail is 30 cm from the top of the pipe while the frog is 30 cm below the snail. At the start of the first hour, the frog climbs 40 cm up the pipe and rests. The snail crawls 20 cm during this time then also rests. While they are resting, both the frog and the snail slip back down the pipe. The frog slips back 20 cm while the snail slips back 10 cm. At the start of the second hour, the frog and the snail set off again and repeat the same process of climbing and resting until they reach the top of the pipe.



a Which reaches the top of the pipe first, the frog or the snail?

b During which hour do they reach the top of the pipe?

9 A classmate is recording the weather during July for a school project and wants help calculating information. He records the following data for one week.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Max (°C)	12.2	14.5	16.7	12.8	11.3	7.2	−0.3
Min (°C)	3.0	2.1	4.6	3.2	6.4	−2.9	−6.0

Round all answers correct to 1 d.p.

a What is the difference between the lowest temperature and the highest temperature recorded during this week?

b What is the average (mean) minimum temperature?

c Your classmate tries to predict temperatures and says that the minimum temperature $= \frac{1}{4} \times$ maximum temperature. He says this is the same as dividing by −4. Explain why this may not be correct.

d He then tries to make another prediction that involves taking the square root of the maximum temperature. Why might this not be a good idea?

e On the Monday after this week, the temperature rises by 6.5 °C from Sunday's minimum temperature. It then drops by 3.2 °C overnight and rises by 8.9 °C on the Tuesday. What is the minimum temperature on the Monday night and the maximum temperature on the Tuesday?

eBookplus
Digital doc
WorkSHEET 2.2
doc-6407

f His last prediction involves subtracting 6 from the maximum temperature, then dividing by 2 to predict the minimum temperature. Calculate the predicted values for the minimum temperature for each of his three methods and discuss which method may be most accurate.

CHALLENGE 2.2

Arrange numbers in this magic square so that all rows, columns and diagonals sum to the same value. No numbers are used twice.

		2
3	−1	−5

2.6 Review

eBookplus ONLINE ONLY

www.jacplus.com.au

The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:

- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

Review questions

Download the Review questions document from the links found in your eBookPLUS.

eBookplus

Interactivities

Word search
int-2724

Crossword
int-2723

Sudoku
int-3182

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

difference	negative	product
directed number	notation	quotient
integer	positive	square root
inverse operation	power	sum

Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

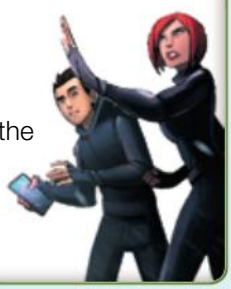
assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

www.assesson.com.au

Link to SpyClass, an exciting online game combining a comic book–style story with problem-based learning in an immersive environment.

Join Jesse, Toby and Dan and help them to tackle some of the world's most dangerous criminals by using the knowledge you've gained through your study of mathematics.

www.spyclass.com.au



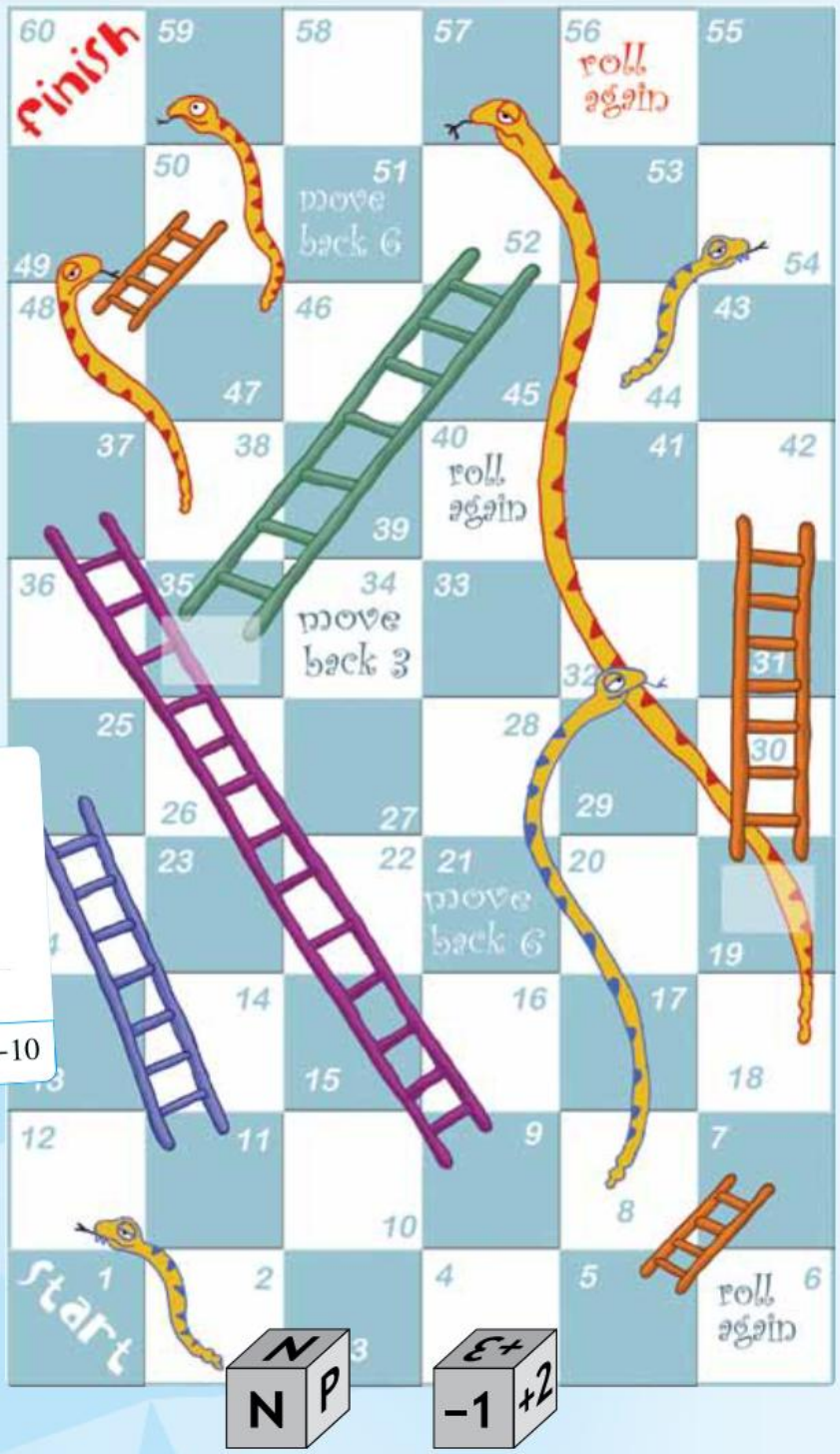
RICH TASK

Directed numbers board game

Many board games engage the players in racing each other to the end of the board. Snakes and Ladders is an example of this style of game. You can climb ladders to get to the end quicker, but sliding down a snake means you get further away from the end.

You are going to make a board game that will help you to practise addition and subtraction of directed numbers. This game is played with 2 dice and is a race to the end of the number line provided on the board. The diagram below shows part of a sample game, and can be used to explain the rules and requirements for your board game.

					10	9	8	7	6		
									5		
									4		
−2	−1	0	1	2	3						
−3											
−4	−5	−6	−7	−8	−9	−10					

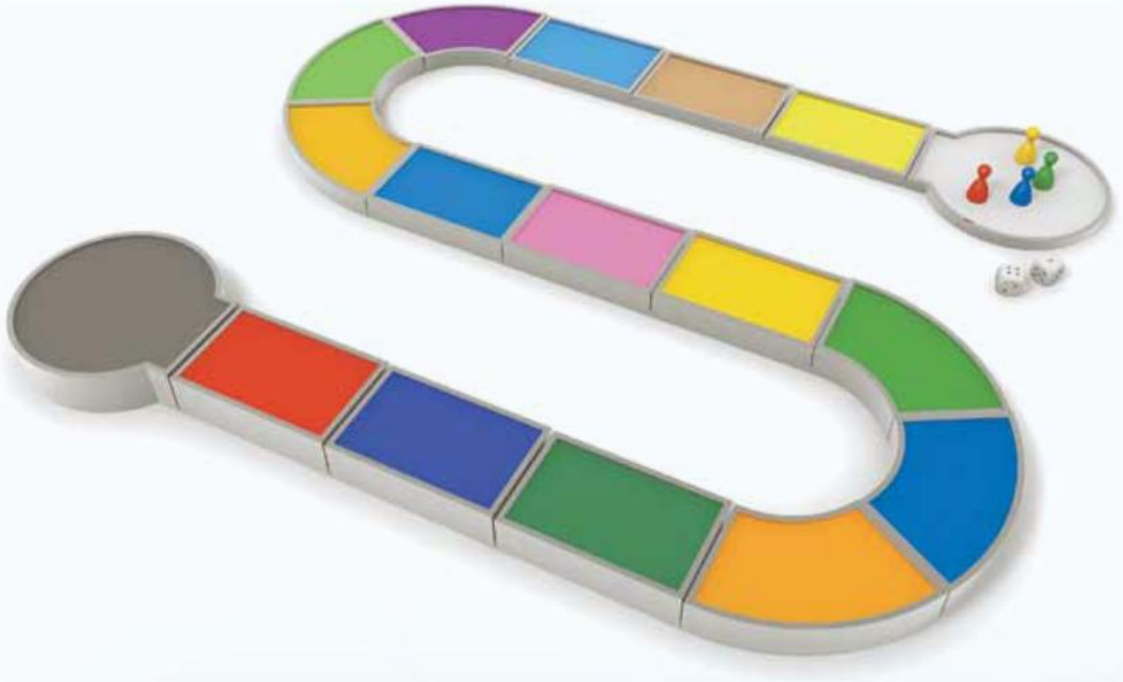


Two dice are to be used in this board game. Die 1 is labelled with 3 N's and 3 P's, and die 2 is labelled with the integers –1, –2, –3, +1, +2 and +3. (Stickers with these labels can be placed over two standard dice.) Rolling an N means you face the *negative* numbers; rolling a P means you face the *positive* numbers.

- Imagine that you are at 0, and that you roll an N and +2. N means that you face the negative numbers; +2 means that you move forward 2 places.
- If you roll a P and +1, this means that you face the positive numbers and move forward 1 place.
- If you roll a P and –2, this means that you face the positive numbers and move *backwards* 2 places.

1 For the three examples listed above, state which square you would end up in if you started at 0 each time.

2 Suppose that your first five turns at this game produced the following results on the dice.
P and +1, P and +3, P and –2, N and –1, N and +2.
If you started at 0, where were you after these five turns?



Your task is to design a board game similar to the one used in the example. The requirements for the game are listed below.

- The game is to be played with the two dice described earlier.
- Two or more players are required, taking turns to throw the dice.
- Start at 0 in the centre of the board. The race is on to get to either end of the board. The first person to reach an end is the winner.
- For 0 to be at the centre, your board will require an odd number of columns.
- At least 8 squares must have specific instructions — for example, 'go back 5 squares' or 'miss a turn'. You may even wish to include 'snakes and ladders' style obstacles.
- Use a sensible number of squares. If there are very few squares, the game will end too quickly; if there are many squares, they will be very small.

3 Work with a partner to make the two required dice and to design your board. Play with others to test it, and make necessary improvements if they are required. Be certain that all the requirements of the game are included.

4 Present your game as a package that includes instructions and a list of contents. Include examples within your instructions to help players understand how to move throughout the game.

CODE PUZZLE

Why did the teacher wear sunglasses?

Answer the questions below to find the puzzle code and my secret!

E = –6 + –8 =	T = –3 + –4 + –4 =	M = –5 + –10 =	F = –25 + +8 =
E = +11 + –9 =	T = –5 + +12 =	Q = –9 + –7 =	I = –3 + –7 =
F = +2 + –11 =	U = –8 + +5 + –10 =	R = –7 + +11 =	J = –15 + +27 =
H = +4 + –7 =	B = –2 + +5 =	S = +13 + –20 =	O = +9 + –8 =
I = +2 + –5 + +2 =	E = +1 + +8 =	T = –11 + +24 =	Q = +3 + –7 =
L = –2 + –1 + –5 =	E = –7 + –13 =	V = –24 + +5 =	S = +1 + –4 + –3 =
P = –8 + +18 =	F = –5 + +3 =	C = +20 + –5 =	T = –21 + +3 =
R = –4 + +23 =	H = –1 + +7 =	E = –12 + +7 =	T = +20 + –4 =
S = +5 + –17 =	G = –2 + +5 + +2 =	W = –6 + +6 =	X = –3 + –4 + +15 =

+6	–14	+19	+10	–13	+10	–10	+8	–6	0	+9	+4	–5	–12	+1	+3	+19	–1	+5	–3	+13

eBookplus Activities

2.2 Adding and subtracting integers

Digital docs
- SkillSHEET (doc-6387) Integers on the number line
- SkillSHEET (doc-6388) Adding and subtracting integers
- SkillSHEET (doc-6389) Arranging numbers in order

eLesson
- Directed numbers (eles-0040)

Interactivities
- Directed number target (int-0074)
- IP interactivity 2.2 (int-4397) Adding and subtracting integers

2.3 Multiplying integers

Digital docs
- SkillSHEET (doc-6390) Multiplying integers
- WorkSHEET 2.1 (doc-6400)

Interactivity
- IP interactivity 2.3 (int-4398) Multiplying integers

2.4 Dividing integers

Digital doc
- SkillSHEET (doc-6391) Dividing integers

Interactivity
- IP interactivity 2.4 (int-4399) Dividing integers

2.5 Combined operations on integers

Digital docs
- SkillSHEET (doc-6392) Order of operations and directed numbers
- WorkSHEET 2.2 (doc-6407)

Interactivity
- IP interactivity 2.5 (int-4400) Combined operations on integers

2.6 Review

Interactivities
- Word search (int-2724)
- Crossword (int-2723)
- Sudoku (int-3182)

Digital docs
- Topic summary (doc-10754)
- Concept map (doc-10767)
- Topic review (Word doc-14926, PDF doc-14927)

To access eBookPLUS activities, log on to www.jacplus.com.au

Answers

TOPIC 2 Integers

2.2 Adding and subtracting integers

1 3, −4, 201, −62

2 **a** 0, −2, −4 **b** −20, −25, −30
c 5, 3, 1 **d** −8, −6, −4

3 **a** −1 **b** −10 **c** −1 **d** −13
e 19 **f** 7 **g** −15 **h** 0

4 **a** 5 **b** −24 **c** −5 **d** 5
e 26 **f** −16 **g** 22 **h** 11

5 **a** −8 **b** 11 **c** −14 **d** −1
e 51 **f** −51 **g** −39 **h** 8

6 **a** −7 **b** −9 **c** 3 **d** 12
e −10 **f** 13 **g** −40 **h** 18
i 27 **j** −34 **k** −11 **l** 150
m −1 **n** 25 **o** −13 **p** 22
q 9 **r** −22

7 **a**

+	**−8**	**25**	**−18**	**32**
−6	−14	19	−24	26
−13	−21	12	−31	19
−16	−24	9	−34	16
−19	−27	6	−37	13

b

−	**15**	**−17**	**−27**	**57**
7	8	−24	−34	50
−6	21	−11	−21	63
−9	24	−8	−18	66
12	3	−29	−39	45

c

+	**−11**	**19**	**13**	**−7**
−5	−16	14	8	−12
17	6	36	30	10
−1	−12	18	12	−8
−28	−39	−9	−15	−35

d

−	**9**	**21**	**−15**	**42**
26	−17	−5	−41	16
−14	23	35	−1	56
−2	11	23	−13	44
23	−14	−2	−38	19

8 Check with your teacher.

9 **a** 22 °C **b** 176 °C **c** 198 °C

10 37 °C

11 **a** −301 **b** −5963 **c** 530 **d** 72

12 **a** 13 **b** −4 **c** 11 **d** −10
e 52 **f** 71

13 **a** Correct **b** Incorrect; 6 **c** Incorrect; −15
d Correct **e** Correct **f** Incorrect; −21

14 **a** −3 **b** −2 **c** −5

15 The answers are the same.

16 **a** −2 + −5 = −7; −(2 + 5) = −7
b −3 + −8 = −11; −(3 + 8) = −11
c −7 + −6 = −13; −(7 + 6) = −13

17 For each pair of expressions, the answers are the same: each number is negative, so putting brackets around the sum and then attaching a negative sign gives the same outcome.

18 1370 HKD

Challenge 2.1

−72 metres

2.3 Multiplying integers

1 $4 \times 4 = 16$ $-5 \times 4 = -20$ $-6 \times -4 = 24$
$4 \times 3 = 12$ $-5 \times 3 = -15$ $-6 \times -3 = 18$
$4 \times 2 = 8$ $-5 \times 2 = -10$ $-6 \times -2 = 12$
$4 \times 1 = 4$ $-5 \times 1 = -5$ $-6 \times -1 = 6$
$4 \times 0 = 0$ $-5 \times 0 = 0$ $-6 \times 0 = 0$
$4 \times -1 = -4$ $-5 \times -1 = 5$ $-6 \times 1 = -6$
$4 \times -2 = -8$ $-5 \times -2 = 10$ $-6 \times 2 = -12$
$4 \times -3 = -12$ $-5 \times -3 = 15$ $-6 \times 3 = -18$
$4 \times -4 = -16$ $-5 \times -4 = 20$ $-6 \times 4 = -24$

2 **a** −10 **b** −24 **c** 42 **d** −26
e 48 **f** −42 **g** −750 **h** 1150
i −63 **j** −72 **k** 55 **l** −300

3 **a** −800 **b** 112 **c** −192 **d** 42
e 160

4 **a** −9 **b** 7 **c** −2 **d** −12
e −6 **f** 8 **g** −20 **h** −6
i −11

5 **a** −8 **b** 9 **c** 16 **d** 81
e −32 **f** 16 **g** −125 **h** 256
i 625 **j** −216

6 **a** Positive **b** Negative

7 **a** ±5 **b** ±9 **c** ±7 **d** ±11
e ±10

8 **a** 48 **b** −36 **c** 72 **d** 16
e −12 **f** −576

9 Check with your teacher.

10 **a** Positive
b Negative
c Negative

11 If a positive number is multiplied by −1, the number becomes negative.
If a negative number is multiplied by −1, the number becomes positive.

12 **a** 2 **b** −3 **c** 5 **d** 5
e −7 **f** 4

13 Because there is an odd number of negatives.

14 $-a$ must be negative for the answer to be negative, thus a must be positive. The power must be odd to give a negative result, thus n is odd.

15 −432

16 **a** −1
b −1

17 One of the following: 17 correct, 1 incorrect; 18 correct, 3 incorrect; 19 correct, 5 incorrect; 20 correct, 7 incorrect; 21 correct, 9 incorrect

2.4 Dividing integers

1 **a** −7 **b** −4 **c** −4 **d** 6
e −8 **f** 0 **g** −16 **h** −43
i 8 **j** −46 **k** 5 **l** −38

2 **a** 11 **b** −4 **c** 3 **d** −3
e 2 **f** 2

3 **a** −120 **b** −27 **c** 38 **d** 33
e −47 **f** −160 **g** −27 **h** −180
i 250 **j** 62 **k** 324 **l** 226

4 Check with your teacher.

5 **a** 3 **b** −2 **c** −9 **d** 3
e −35 **f** 24 **g** −12 **h** −10

6 **a** 1 **b** 6 **c** −20

7 **a** 3 **b** −4 **c** 2 **d** $\frac{-3}{4}$
e $\frac{-1}{2}$ **f** $\frac{2}{3}$

8 **a** 48 **b** −6 **c** −1 **d** −4
e $\frac{-1}{3}$ **f** −4

9 **a**

×	−2	4	−6	8
−3	6	−12	18	−24
−10	20	−40	60	−80
−5	10	−20	30	−40
−7	14	−28	42	−56

b

×	5	−3	−9	−7
6	30	−18	−54	−42
−12	−60	36	108	84
−11	−55	33	99	77
2	10	−6	−18	−14

10 **a**

÷	4	−10	12	−8
−2	−2	5	−6	4
7	$\frac{4}{7}$	$\frac{-10}{7}$	$\frac{12}{7}$	$\frac{-8}{7}$
−3	$\frac{-4}{3}$	$\frac{10}{3}$	−4	$\frac{8}{3}$
−10	$\frac{-2}{5}$	1	$\frac{-6}{5}$	$\frac{4}{5}$

b

÷	32	−24	−36	−4
12	$\frac{8}{3}$	−2	−3	$\frac{-1}{3}$
−8	−4	3	$\frac{9}{2}$	$\frac{1}{2}$
6	$\frac{16}{3}$	−4	−6	$\frac{-2}{3}$
−4	−8	6	9	1

11 Same sign

12 Positive

13 $-\frac{1}{162}$

14 **a** −1 **b** −1

15 **a** Yes; for example, a car is getting faster while travelling towards the east.
b Yes; for example, a car is slowing down (negative acceleration) while travelling towards the west.

16 −2.8

2.5 Combined operations on integers

1 **a** −12 **b** −7 **c** −3 **d** 9
e 30 **f** −6 **g** −12 **h** −58
i −12 **j** 17 **k** 56 **l** −7

2 **a** −19 **b** −21 **c** −29 **d** −12
e −34 **f** −324

3 **a** −41 **b** −76 **c** −12 **d** 15
e −60 **f** 6 **g** −60 **h** 46

4 **a** −12
b $(4 + 8) \div -(2)^2 - 7 \times 2 = -17$
$4 + 8 \div (-(2))^2 - 7 \times 2 = -8$

5

Round number	Points at the start of the round	Contacts during the round	Points at the end of the round
1	100	20 gnomes, 10 goblins and 3 healing potions	120
2	120	3 gnomes, 5 goblins, 6 orcs and 5 healing potions	177
3	177	3 orcs, 6 trolls and a cleric	152
4	152	5 trolls, 1 balrog and a cleric	52

6 $p < 0, q > 0$

7 Check with your teacher.

8 **a** The frog and the snail get out of the pipe at exactly the same time.
b They make it out of the pipe during the second hour.

9 **a** 22.7 °C **b** 1.5 °C
c Multiplying by $\frac{1}{4}$ is not the same as dividing by $^{-}4$. However, multiplying by $\frac{1}{4}$ is the same as dividing by 4.
d This is not a good idea because the square root ($\sqrt{}$) of a number can be both positive and negative. Also, you cannot take the square root of a negative number, such as Sunday's maximum temperature.
e Monday maximum = 0.5 °C; Monday minimum = $^{-}2.7$ °C; Tuesday maximum = 6.2 °C
f

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Max (°C)	12.2	14.5	16.7	12.8	11.3	7.2	$^{-}0.3$
Min (°C)	3.0	2.1	4.6	3.2	6.4	$^{-}2.9$	$^{-}6.0$
Max × 1/4	3.1	3.6	4.2	3.2	2.8	1.8	$^{-}0.1$
√Max	3.5	3.8	4.1	3.6	3.4	2.7	
(Max − 6)/2	3.1	4.3	5.4	3.4	2.7	0.6	$^{-}3.2$

The method of multiplying the maximum temperature by $\frac{1}{4}$ gave the most accurate prediction of minimum temperature, but none of the methods is very accurate. Any prediction method should be tested over a number of weeks before deciding if it is accurate enough.

Challenge 2.2

−2	−3	2
3	−1	−5
−4	1	0

Investigation — Rich task

1 **a** −2 **b** 1 **c** −2

2 1

3 Teacher to check.

4 Teacher to check.

Code puzzle

Her pupils were so bright.

TOPIC 3

Index laws

3.1 Overview

Why learn this?

Indices (the plural of index) are the lazy mathematician's way of abbreviating operations such as multiplication and division. You do not have to use this technique if you have the time to do otherwise. For example, the lazy mathematician might write x^6, whereas you could write $x \times x \times x \times x \times x \times x$.

Why use indices at all? Because doing so will save you time and effort in more difficult calculations later. Will you use indices later in life? That depends on what work you decide to do. However, if you are contemplating any work associated with computers, cars, engines, science, design or apprenticeships, then the answer is almost certainly yes.

What do you know?

assessON

1 **THINK** List what you know about index laws. Use a thinking tool such as a concept map to show your list.
2 **PAIR** Share what you know with a partner and then with a small group.
3 **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of index laws.

Learning sequence

3.1 Overview
3.2 Review of index form
3.3 First Index Law (multiplying numbers in index form with the same base)
3.4 Second Index Law (dividing numbers in index form with the same base)
3.5 Third Index Law (the power of zero)
3.6 Fourth Index Law (raising a power to another power)
3.7 Review eBookplus ONLINE ONLY

3.2 Review of index form

- If a number or a variable is multiplied by itself several times, it can be written using short-cut notation referred to as **index form**.
- When written in index form, the number or the variable that is being multiplied is written once only. To indicate how many times it is being multiplied by itself, a small number is written above and to the right of it. For example, if number 2 is multiplied by itself 4 times, it can be written as $2 \times 2 \times 2 \times 2$ (factor form), or as 2^4 (index form).
- When written in index form, the number or variable that is being multiplied is called the *base*, while the number showing how many times it is being multiplied is called the *power*, or *index*. For example, in the number 2^4, 2 is the base and 4 is the power or index.
- When the base is multiplied by itself the number of times indicated by the power, the answer is called a *basic numeral*. For example,

$$\underset{\textit{Index form}}{2^4} \quad = \underset{\textit{Factor form}}{2 \times 2 \times 2 \times 2} = \quad \underset{\textit{Basic numeral}}{16}$$

WORKED EXAMPLE 1

State the base and power for the number 5^{14}.

THINK	WRITE
1 Write the number.	5^{14}
2 The base is the number below the power.	The base is 5.
3 The power or index is the small number just above and to the right of the base.	The power is 14.

WORKED EXAMPLE 2

Write 12^4 in factor form.

THINK	WRITE
1 Write the number.	12^4
2 The base is 12, so this is what will be multiplied.	
3 The power is 4, so this is how many times 12 should be written and multiplied.	$= 12 \times 12 \times 12 \times 12$

WORKED EXAMPLE 3

Write $2 \times 5 \times 2 \times 2 \times 5 \times 2 \times 5$ in index form.

THINK	WRITE
1 Write the problem.	$2 \times 5 \times 2 \times 2 \times 5 \times 2 \times 5$
2 Write the factors in numerical order.	$= 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5$
3 The number 2 has been written 4 times and multiplied. The number 5 has been written 3 times and multiplied.	$= 2^4 \times 5^3$

WORKED EXAMPLE 4

Write $7 \times 5^3 \times 6^5$ in factor form.

THINK	WRITE
1 Write the problem.	$7 \times 5^3 \times 6^5$
2 List the factors: 7 is written once, 5 is written 3 times and multiplied, and 6 is written 5 times and multiplied.	$= 7 \times 5 \times 5 \times 5 \times 6 \times 6 \times 6 \times 6 \times 6$

Exercise 3.2 Review of index form

assessON

INDIVIDUAL PATHWAYS

PRACTISE
Questions:
1–8, 13

CONSOLIDATE
Questions:
1–9, 11–13

MASTER
Questions:
1f–j, 2, 3e–h, 4e–h, 5, 6e–h, 7, 8a, c, f, 9–12, 14

REFLECTION
How will you remember the meaning of the base and that of the index?

Individual pathway interactivity int-4401 eBookplus

FLUENCY

eBookplus
Digital docs
SkillSHEET
Factor trees
doc-6834
SkillSHEET
Squaring numbers
doc-6835

1 WE1 State the base and power for each of the following.

a 8^4 **b** 7^{10} **c** 20^{11} **d** 19^0 **e** 78^{12}
f 3^{100} **g** m^5 **h** c^{24} **i** n^{36} **j** d^{42}

2 Write the following in index form.

a $2 \times 2 \times 2 \times 2 \times 2 \times 2$
b $4 \times 4 \times 4 \times 4$
c $x \times x \times x \times x \times x \times x$
d $9 \times 9 \times 9$
e $11 \times l \times l \times l \times l \times l \times l \times l$
f $44 \times m \times m \times m \times m \times m$

3 WE2 Write the following in factor form.

a 4^2 **b** 5^4 **c** 7^5 **d** 6^3
e 3^6 **f** n^7 **g** a^4 **h** k^{10}

4 Write each of the following as a basic numeral.

a 3^5 **b** 4^4 **c** 2^8 **d** 11^3
e 7^4 **f** 6^3 **g** 1^{10} **h** 5^4

5 a MC What does 6^3 mean?

A 6×3 **B** $6 \times 6 \times 6$ **C** $3 \times 3 \times 3 \times 3 \times 3 \times 3$
D $6 + 6 + 6$ **E** 3×6

b What does 3^5 mean?

A 3×5 **B** 5×5 **C** $3 + 3 + 3 + 3 + 3$
D $3 \times 3 \times 3 \times 3 \times 3$ **E** 5×3

6 WE3 Write each of the following in index form.

a $6 \times 2 \times 2 \times 4 \times 4 \times 4 \times 4$

b $7 \times 7 \times 7 \times 7 \times 3 \times 3 \times 3 \times 3$

c $19 \times 19 \times 19 \times 19 \times 19 \times 2 \times 2 \times 2$

d $13 \times 13 \times 4 \times 4 \times 4 \times 4$

e $66 \times p \times p \times m \times m \times m \times m \times m \times s \times s$

f $21 \times n \times n \times 3 \times i \times i \times i \times 6 \times r \times r \times r$

g $16 \times k \times e \times e \times e \times 12 \times p \times p$

h $11 \times j \times j \times j \times j \times j \times 9 \times p \times p \times l$

7 WE4 Write each of the following in factor form.

a $15f^3j^4$ **b** $7k^6s^2$ **c** $4b^3c^5$

d $19a^4mn^3$ **e** $8l^4r^2t^2$

UNDERSTANDING

8 Write each of the following numbers as a product of its prime factors, using indices.

a 64 **b** 40 **c** 36

d 400 **e** 225 **f** 2000

9 Some basic numerals (see below) are written as the product of their prime factors. Identify each of these basic numerals.

a $2^3 \times 3 \times 5$ **b** $2^2 \times 5^2$ **c** $2^3 \times 3^3$

d $2^2 \times 7 \times 11$ **e** $3^2 \times 5^2 \times 7$ **f** $2^6 \times 5^4 \times 19$

10 a Write each of the following numbers in index form with base 10.

i 10 **ii** 100 **iii** 1000 **iv** 1 000 000

b Use your knowledge of place value to rewrite each of the following basic numerals in expanded form using powers of 10. The first number has been done for you.

	Basic numeral	Expanded form
i	230	$2 \times 10^2 + 3 \times 10^1$
ii	500	
iii	470	
iv	2360	
v	1980	
vi	5430	

c Write each of the following as a basic numeral.

i $7 \times 10^4 + 5 \times 10^3$

ii $3 \times 10^4 + 6 \times 10^2$

iii $5 \times 10^6 + 2 \times 10^5 + 4 \times 10^2 + 8 \times 10^1$

REASONING

11 Explain what a^3b^4 means. Write a^3b^4 as a basic numeral in factor form as part of your explanation.

PROBLEM SOLVING

12 a, b and c are prime numbers.

- **a** Write $8 \times a \times a \times b \times b \times b \times c \times c \times c \times c$ as a product of prime factors in index form.
- **b** If $a = 2$, $b = 3$ and $c = 7$, calculate the value of the basic numeral represented by your answer in part **a**.

13
- **a** Rewrite the numbers 140 and 680 in expanded form using powers of 10.
- **b** Add the numbers in expanded form.
- **c** Convert your answer for part **b** into a basic numeral.
- **d** Check your answer for part **c** by adding 140 and 680.
- **e** Try the question again, this time with two numbers of your own. Choose numbers between 1000 and 10 000.

14
- **a** Rewrite the numbers 10 and 14 in expanded form with powers of 2 using, as appropriate, 2^3, 2^2 and 2^1.
- **b** Add the numbers in expanded form.
- **c** Convert your answer for part **b** into a basic numeral.
- **d** Check your answer for part **c** by adding 10 and 14.
- **e** Try the question again, this time with two numbers of your own. Choose numbers between 16 and 63.

3.3 First Index Law (multiplying numbers in index form with the same base)

- The numbers in index form with the same base can be multiplied together by being written in factor form first. For example, $5^3 \times 5^2 = (5 \times 5 \times 5) \times (5 \times 5) = 5^5$.
- The simpler and faster way to multiply numbers or variables in index form with the same base is to use the First Index Law. The First Index Law states: $a^m \times a^n = a^{m+n}$. This means that when numbers in index form with the same base are multiplied by each other, the powers (indices) are added together. For example, $5^3 \times 5^2 = 5^{3+2} = 5^5$ (as above).
- If the variables in index form that are being multiplied have coefficients, the coefficients are multiplied together and the variables in index form are multiplied, and simplified using the First Index Law. For example, $2a^4 \times 3a^5 = (2 \times 3) \times (a^4 \times a^5) = 6a^9$.

↑ Coefficients multiplied

↑ Variables multiplied, which means indices are added

WORKED EXAMPLE 5

Simplify $2^3 \times 2^6$ after first writing in factor form, leaving the answer in index form.

THINK	WRITE
1 Write the problem.	$2^3 \times 2^6$
2 Write in factor form.	$= (2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2 \times 2)$
3 Simplify by writing in index form.	$= 2^9$

WORKED EXAMPLE 6

Simplify $7^4 \times 7 \times 7^3$, giving your answer in index form.

THINK	WRITE
1 Write the problem.	$7^4 \times 7 \times 7^3$
2 Show that the 7 in the middle has an index of 1.	$= 7^4 \times 7^1 \times 7^3$
3 Check to see if the bases are the same. They are all 7.	
4 Simplify by using the First Index Law (add indices).	$= 7^{4+1+3}$ $= 7^8$

WORKED EXAMPLE 7

Simplify $5e^{10} \times 2e^3$.

THINK	WRITE
1 Write the problem.	$5e^{10} \times 2e^3$
2 The order is not important when multiplying, so place the numbers first.	$= 5 \times 2 \times e^{10} \times e^3$
3 Multiply the numbers.	$= 10 \times e^{10} \times e^3$
4 Check to see if the bases are the same. They are both e.	
5 Simplify by using the First Index Law (add indices).	$= 10e^{10+3}$ $= 10e^{13}$

Multiplying expressions containing numbers in index form with different bases

- When there is more than one variable involved in the multiplication question, the First Index Law is applied to each variable separately.

WORKED EXAMPLE 8

Simplify $7m^3 \times 3n^5 \times 2m^8 \times n^4$.

THINK	WRITE
1 Write the problem.	$7m^3 \times 3n^5 \times 2m^8 \times n^4$
2 The order is not important when multiplying, so place numbers first and group the same variables together.	$= 7 \times 3 \times 2 \times m^3 \times m^8 \times n^5 \times n^4$
3 Simplify by multiplying the numbers and using the First Index Law for bases that are the same (add indices).	$= 42 \times m^{3+8} \times n^{5+4}$ $= 42m^{11}n^9$

Exercise 3.3 First Index Law (multiplying numbers in index form with the same base)

assessON

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3a–f, 4, 5, 6a–f, 11	Questions: 1–7, 10, 11, 13	Questions: 2g–l, 3g–l, 4, 5, 6f–j, 7–13

Individual pathway interactivity int-4402 eBookplus

REFLECTION
The First Index Law can only be applied if the bases are the same. Why is that so?

eBookplus
Digital doc
Spreadsheet
Multiplying with indices
doc-2160

FLUENCY

1 WE5 Simplify the following after first writing in factor form.

a $2^2 \times 2^4 = (2 \times 2) \times (\square \times \square \times \square \times \square)$
$= 2^{\square}$

b $5^3 \times 5^5 = (5 \times 5 \times 5) \times (\square \times \square \times \square \times \square \times \square)$
$= 5^{\square}$

c $f^6 \times f \times f^2 = (\square \times \square \times \square \times \square \times \square \times \square) \times \square \times (\square \times \square)$
$= f^{\square}$

2 Simplify each of the following.

a $3^7 \times 3^2$ **b** $6^{14} \times 6^3$ **c** $10^6 \times 10^4$
d $11^3 \times 11^3$ **e** $7^8 \times 7$ **f** $2^{11} \times 2^3$
g $5^2 \times 5^2$ **h** $8^9 \times 8^2$ **i** $13^7 \times 13^8$
j $q^{23} \times q^{24}$ **k** $x^7 \times x^7$ **l** $e \times e^3$

3 WE6 Simplify each of the following, giving your answer in index form.

a $3^4 \times 3^6 \times 3^2$ **b** $2^{10} \times 2^3 \times 2^5$ **c** $5^4 \times 5^4 \times 5^9$
d $6^8 \times 6 \times 6^2$ **e** $10 \times 10 \times 10^4$ **f** $17^2 \times 17^4 \times 17^6$
g $p^7 \times p^8 \times p^7$ **h** $e^{11} \times e^{10} \times e^2$ **i** $g^{15} \times g \times g^{12}$
j $e^{20} \times e^{12} \times e^6$ **k** $3 \times b^2 \times b^{10} \times b$ **l** $5 \times d^4 \times d^5 \times d^7$

4 a MC What does $6 \times e^3 \times b^2 \times b^4 \times e$ equal?

A $6b^6e^4$ **B** $6b^6e^3$ **C** $6b^9e$
D $6b^{10}e$ **E** $6b^8e^3$

b What does $3 \times f^2 \times f^{10} \times 2 \times e^3 \times e^8$ equal?

A $32e^{11}f^{12}$ **B** $6e^{11}f^{12}$ **C** $6e^{23}f$
D $6e^{24}f^{20}$ **E** $3e^{24}f^{12}$

5 WE7 Simplify each of the following.

a $4p^7 \times 5p^4$ **b** $2x^2 \times 3x^6$ **c** $8y^6 \times 7y^4$
d $3p \times 7p^7$ **e** $12t^3 \times t^2 \times 7t$ **f** $6q^2 \times q^5 \times 5q^8$

6 WE8 Simplify each of the following.

a $2a^2 \times 3a^4 \times e^3 \times e^4$ **b** $4p^3 \times 2h^7 \times h^5 \times p^3$
c $2m^3 \times 5m^2 \times 8m^4$ **d** $2gh \times 3g^2h^5$
e $5p^4q^2 \times 6p^2q^7$ **f** $8u^3w \times 3uw^2 \times 2u^5w^4$
g $9dy^8 \times d^3y^5 \times 3d^7y^4$ **h** $7b^3c^2 \times 2b^6c^4 \times 3b^5c^3$
i $4r^2s^2 \times 3r^6s^{12} \times 2r^8s^4$ **j** $10h^{10}v^2 \times 2h^8v^6 \times 3h^{20}v^{12}$

UNDERSTANDING

7 Simplify each of the following.

a $3^x \times 3^4$ **b** $3^y \times 3^{y+2}$

c $3^{2y+1} \times 3^{4y-6}$ **d** $3^{\frac{1}{2}} \times 3^{\frac{2}{3}} \times 3^{\frac{3}{4}}$

8 **a** Express the following basic numerals in index form: 9, 27 and 81.

b Use your answers to part **a** to help you simplify each of the following expressions. (Give each answer in index form.)

i $3^4 \times 81 \times 9$

ii $27 \times 3^n \times 3^{n-1}$

REASONING

9 Explain why $2^x \times 3^y$ does not equal $6^{(x+y)}$.

10 Step 1: The prime number 5 is multiplied by itself n times.

Step 2: The prime number 5 is multiplied by itself m times.

Step 3: The answers from steps 1 and step 2 are multiplied together.

Explain how you arrive at your final answer. What is your answer?

PROBLEM SOLVING

11 One dollar is placed on a square of a chess board, two dollars on the next square, four dollars on the next square, eight dollars on the next square and so on.

a Write the number of dollars on the 10th square in index form.

b Write the number of dollars on the rth square in index form.

c How much money is on the 6th and 7th squares in total?

d How much money is on the 14th and 15th squares in total? Write your answer in index form.

e Simplify your answer to part **d** by first taking out a common factor.

f Compare the numerical value of the number of dollars on the 64th square of the chess board to the numerical value of the distance in kilometres to the nearest star, Proxima Centauri.

12 **a** If $x^2 = x \times x$, what does $(x^3)^2$ equal?

b If the sides of a cube are 2^4 cm long, what is the volume of the cube in index form? (*Hint:* The volume of a cube of side length s cm is s^3 cm^3.)

c What is the side length of a cube of volume 5^6 mm^3?

d What is the side length of a cube of volume $(a^n)^{3p}$ mm^3?

13 If I square a certain number, then multiply the result by three times the cube of the certain number before adding one, the result is 97. What is the certain number?

CHALLENGE 3.1

A cabinetmaker uses a power saw to cut a piece of timber into 5 pieces in 2 minutes. How long would it take to cut a similar piece of timber into 8 pieces?

3.4 Second Index Law (dividing numbers in index form with the same base)

- The numbers in index form with the same base can be divided by first being written in factor form. For example:

$$2^6 \div 2^4 = \frac{2^6}{2^4} = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2}$$
$$= \frac{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times 2 \times 2}{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2}} = 2 \times 2$$
$$= 2^2$$

- The simpler and faster way to divide the numbers in index form is to apply the Second Index Law. The Second Index Law states: $a^m \div a^n = a^{m-n}$. This means that when the numbers or variables in index form with the same base are divided, the powers are subtracted. For example, $2^6 \div 2^4 = 2^{6-4} = 2^2$ (as above).

WORKED EXAMPLE 9

Simplify $\frac{5^{10}}{5^3}$ after first writing in factor form, leaving your answer in index form.

THINK	WRITE
1 Write the problem.	$\frac{5^{10}}{5^3}$
2 Write in factor form.	$= \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5}$
3 Cancel 5s.	$= 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$
4 Write in index form.	$= 5^7$

WORKED EXAMPLE 10

Simplify $d^{12} \div d^4$ using an index law.

THINK	WRITE
1 Write the problem and express it as a fraction.	$d^{12} \div d^4$
2 Check to see if the bases are the same. They are both d.	$= \frac{d^{12}}{d^4}$
3 Simplify by using the Second Index Law (subtract indices).	$= d^{12-4}$ $= d^8$

Dividing with coefficients

- When the coefficients are present, we divide them as we would divide any other numbers and then apply the Second Index Law to the variables.
- In examples where the cofficients do not divide evenly, we simplify the fraction that is formed by them.
- When there is more than one variable involved in the division question, the Second Index Law is applied to each variable separately.

WORKED EXAMPLE 11

Simplify $36d^7 \div 12d^3$ giving your answer in index form.

THINK	WRITE
1 Write the problem and express it as a fraction.	$36d^7 \div 12d^3 = \frac{36d^7}{12d^3}$
2 Divide the numbers (or coefficients) and apply the Second Index Law to the variables.	$= \frac{3d^7}{d^3}$
3 Simplify by using the Second Index Law (subtract indices).	$= 3d^{7-3}$ $= 3d^4$

WORKED EXAMPLE 12

Simplify $\frac{7t^3 \times 4t^8}{12t^4}$.

THINK	WRITE
1 Write the problem.	$\frac{7t^3 \times 4t^8}{12t^4}$
2 Multiply the numbers in the numerator and apply the First Index Law (add indices) in the numerator.	$= \frac{28t^{11}}{12t^4}$
3 Simplify the fraction formed and apply the Second Index Law for the variable (subtract indices).	$= \frac{7t^7}{3}$

assessON

Exercise 3.4 Second Index Law (dividing numbers in index form with the same base)

REFLECTION
How will you remember that when numbers in index form are divided, powers are subtracted but coefficients are divided?

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–6, 7a, b, 14	Questions: 1–8, 11, 12, 14	Questions: 1, 2g–l, 3g–l, 4, 5, 6e–i, 7–14

Individual pathway interactivity int-4403 eBookplus

FLUENCY

1 WE9 Simplify each of the following after first writing in factor form, leaving your answer in index form.

a $\frac{2^5}{2^2}$ b $\frac{7^7}{7^3}$ c $\frac{10^8}{10^5}$

2 WE10 Simplify each of the following using the index law, leaving your answer in index form.

a $3^3 \div 3^2$ b $11^9 \div 11^2$ c $5^8 \div 5^4$ d $12^6 \div 12$

e $3^{45} \div 3^{42}$ f $13^{75} \div 13^{74}$ g $6^{23} \div 6^{19}$ h $\frac{10^{13}}{10^9}$

i $\frac{15^{456}}{15^{423}}$ j $\frac{h^{78}}{h}$ k $\frac{b^{77}}{b^7}$ l $\frac{f^{1000}}{f^{100}}$

eBookplus
Digital docs
SkillSHEET Equivalent fractions doc-6836
SkillSHEET Simplifying algebraic expressions doc-6837
SkillSHEET Simplifying algebraic fractions doc-6838

3 WE11 Simplify each of the following, giving your answer in index form.

a $3x^5 \div x^3$ **b** $6y^7 \div y^5$ **c** $8w^{12} \div w^5$

d $12q^{34} \div 4q^{30}$ **e** $16f^{12} \div 2f^3$ **f** $100h^{100} \div 10h^{10}$

g $80j^{15} \div 20j^5$ **h** $\frac{45p^{14}}{9p^4}$ **i** $\frac{48g^8}{6g^5}$

j $\frac{12b^7}{8b}$ **k** $\frac{81m^6}{18m^2}$ **l** $\frac{100n^{95}}{40n^5}$

4 a MC What does $21r^{20} \div 14r^{10}$ equal?

A $7r^{10}$ **B** $\frac{3r^2}{2}$ **C** $7r^2$ **D** $\frac{3r^{10}}{2}$ **E** $\frac{2}{3}r^{10}$

b What does $\frac{2m^{33}}{16m^{11}}$ equal?

A $\frac{m^{22}}{8}$ **B** $\frac{8}{m^{22}}$ **C** $8m^{22}$ **D** $\frac{m^3}{8}$ **E** None of the above

5 WE12 Simplify each of the following.

a $\frac{15p^{12}}{5p^8}$ **b** $\frac{18r^6}{3r^2}$ **c** $\frac{45a^5}{5a^2}$

d $\frac{60b^7}{20b}$ **e** $\frac{100r^{10}}{5r^6}$ **f** $\frac{9q^2}{q}$

6 WE12 Simplify each of the following.

a $\frac{8p^6 \times 3p^4}{16p^5}$ **b** $\frac{12b^5 \times 4b^2}{18b^2}$ **c** $\frac{25m^{12} \times 4n^7}{15m^2 \times 8n}$

d $\frac{27x^9y^3}{12xy^2}$ **e** $\frac{16h^7k^4}{12h^6k}$ **f** $\frac{12j^8 \times 6f^5}{8j^3 \times 3f^2}$

g $\frac{8p^3 \times 7r^2 \times 2s}{6p \times 14r}$ **h** $\frac{27a^9 \times 18b^5 \times 4c^2}{18a^4 \times 12b^2 \times 2c}$ **i** $\frac{81f^{15} \times 25g^{12} \times 16h^{34}}{27f^9 \times 15g^{10} \times 12h^{30}}$

UNDERSTANDING

7 Simplify each of the following.

a $2^{10} \div 2^p$ **b** $2^{7e} \div 2^{3e-4}$

c $\frac{5^{4x} \times 5^{3y}}{5^{2y} \times 5^x}$ **d** $\frac{3^{2-3m} \times 3^{7m}}{3^{5m} \times 3}$

8 Consider the fraction $\frac{8 \times 16 \times 4}{2 \times 32}$.

a Rewrite the fraction, expressing each basic numeral as power of 2.

b Simplify by giving your answer

i in index form **ii** as a basic numeral.

c Now check your answer by cancelling and evaluating the fraction in the ordinary way.

9 Consider the fraction $\frac{6 \times 27 \times 36}{12 \times 81}$.

a Rewrite the fraction, expressing each basic numeral as the product of its prime factors.

b Simplify, giving the answer:

i in index form **ii** as a basic numeral.

REASONING

10 Explain why $\frac{12^x}{3^y}$ does not equal $4^{(x-y)}$.

11 Step 1: The prime number 3 is multiplied by itself p times.
Step 2: The prime number 3 is multiplied by itself q times.
Step 3: The answer from step 1 is divided by the answer from step 2.
Explain how you arrive at your final answer. What is your answer?

PROBLEM SOLVING

eBookplus
Digital doc
WorkSHEET 3.1
doc-6851

12 By considering $a^p \div a^p$, show that any base raised to the power of zero equals 1.

13 By considering $m^6 \div m^8$, show that $m^{-2} = \frac{1}{m^2}$.

14 I cube a certain number, then multiply the result by six. I now divide the result by the certain number to the power of five. The result is 216. What is the certain number?

CHALLENGE 3.2

Heather uses 27 similar cubes to make a large solid cube. To do this, she used one dab of glue to join every pair of touching faces. How many dabs of glue did Heather need to use to complete the task?

3.5 Third Index Law (the power of zero)

- Consider the following two different methods of simplifying $2^3 \div 2^3$.

Method 1

$$2^3 \div 2^3 = \frac{2 \times 2 \times 2}{2 \times 2 \times 2}$$
$$= \frac{8}{8}$$
$$= 1$$

Method 2

$$2^3 \div 2^3 = \frac{2^3}{2^3}$$
$$= 2^{3-3} \text{ (using the Second Index Law)}$$
$$= 2^0$$

As the two results should be the same, 2^0 must equal 1.

- Any base that has an index power of 0 is equal to 1.
- The Third Index Law states: $a^0 = 1$. This means that any base that is raised to the power of zero is equal to 1.
- If it is in brackets, any numeric or algebraic expression that is raised to the power of zero is equal to 1. For example, $(2 \times 3)^0 = 1$, $(2abc^2)^0 = 1$.

WORKED EXAMPLE 13

Find the value of 15^0.

THINK	WRITE
1 Write the problem.	15^0
2 Any base with an index of zero is equal to one.	$= 1$

WORKED EXAMPLE 14

Find the value of $(25 \times 36)^0$.

THINK	WRITE
1 Write the problem.	$(25 \times 36)^0$
2 Everything within the brackets has an index of zero, so the answer is 1.	$= 1$

WORKED EXAMPLE 15

Find the value of $19e^5a^0$.

THINK	WRITE
1 Write the problem.	$19e^5a^0$
2 Only a has a power of zero, so replace it with a 1 and simplify.	$= 19e^5 \times 1$ $= 19e^5$

WORKED EXAMPLE 16

Simplify $\dfrac{6m^3 \times 11m^{14}}{3m^{10} \times 2m^7}$.

THINK	WRITE
1 Write the problem.	$\dfrac{6m^3 \times 11m^{14}}{3m^{10} \times 2m^7}$
2 Multiply the numbers and apply the First Index Law in both the numerator and denominator.	$= \dfrac{66m^{17}}{6m^{17}}$
3 Divide the numbers and simplify using the Second Index Law.	$= 11m^{17-17}$ $= 11m^0$
4 Simplify using the Third Index Law.	$= 11 \times 1$ $= 11$

Exercise 3.5 Third Index Law (the power of zero)

assessON

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8, 12	Questions: 1–8, 10, 12	Questions: 1–7, 8f–j, 9–12

Individual pathway interactivity int-4404 eBookplus

REFLECTION
Explain how the Second and Third Index Laws are connected.

FLUENCY

1 WE13 Find the value for each of the following.

a 16^0 **b** 44^0 **c** f^0 **d** h^0

2 WE14 Find the value of each of the following.

a $(23 \times 8)^0$ **b** $7^0 \times 6^0$ **c** $(35z^4)^0$ **d** $(12w^7)^0$

3 Find the value of each of the following.

a 4×3^0 **b** $9^0 + 11$ **c** $c^0 - 10$ **d** $3p^0 + 19$

4 WE15 Find the value of each of the following.

a $12m^3k^0$ **b** $7c^0 + 14m^0$ **c** $32g^0 + 40h^0$ **d** $\dfrac{8k^0}{7j^0}$

e $\dfrac{16t^0}{8y^0}$ **f** $\dfrac{6b^2 \times 5c^0}{3s^0}$ **g** $\dfrac{4d^0 \times 9p^2}{12q^0}$ **h** $\dfrac{3p \times 4d^0}{2z^0 \times 6p}$

5 Find each of the following.

a $e^{10} \div e^{10}$ **b** $a^{12} \div a^{12}$ **c** $(4b^3)^0 \div (4b^3)^0$

d $84f^{11} \div 12f^{11}$ **e** $30z^9 \div 10z^9$ **f** $99t^{13} \div 33t^{13}$

UNDERSTANDING

6 Simplify each of the following.

a $\dfrac{21p^4}{21p^4}$ **b** $\dfrac{40f^{33}}{10f^{33}}$ **c** $\dfrac{54p^6q^8}{27p^6q^8}$ **d** $\dfrac{16p^{11}q^{10}}{8p^2q^{10}}$

e $\dfrac{24a^9e^{10}}{16a^9e^6}$ **f** $\dfrac{x^4y^2z^{11}}{x^4yz^{11}}$ **g** $\dfrac{7i^7m^6r^4}{21i^7m^3r^4}$ **h** $\dfrac{3c^5d^3l^9}{12c^2d^3l^9}$

7 a MC You are told that there is an error in the statement $3p^7q^3r^5s^6 = 3p^7s^6$. To make the statement correct, what should the left-hand side be?

A $(3p^7q^3r^5s^6)^0$ **B** $(3p^7)^0q^3r^5s^6$ **C** $3p^7(q^3r^5s^6)^0$ **D** $3p^7(q^3r^5)^0s^6$ **E** $3(p^7q^3r^5)^0s^6$

b You are told that there is an error in the statement $\dfrac{8f^6g^7h^3}{6f^4g^2h} = \dfrac{8f^2}{g^2}$. To make the statement correct, what should the left-hand side be?

A $\dfrac{8f^6(g^7h^3)^0}{(6)^0f^4g^2(h)^0}$ **B** $\dfrac{8(f^6g^7h^3)^0}{(6f^4g^2h)^0}$ **C** $\dfrac{8(f^6g^7)^0h^3}{(6f^4)^0g^2h}$ **D** $\dfrac{8f^6g^7h^3}{(6f^4g^2h)^0}$ **E** None of the above

c What does $\dfrac{6k^7m^2n^8}{4k^7(m^6n)^0}$ equal?

A $\dfrac{3}{2}$ **B** $\dfrac{3n^8}{2}$ **C** $\dfrac{3m^2}{2}$ **D** $\dfrac{3m^2n^8}{2}$ **E** None of the above

8 WE16 Simplify each of the following.

a $\dfrac{2a^3 \times 6a^2}{12a^5}$ **b** $\dfrac{3c^6 \times 6c^3}{9c^9}$ **c** $\dfrac{5b^7 \times 10b^5}{25b^{12}}$ **d** $\dfrac{8f^3 \times 3f^7}{4f^5 \times 3f^5}$

e $\dfrac{9k^{12} \times 4k^{10}}{18k^4 \times k^{18}}$ **f** $\dfrac{2h^4 \times 5k^2}{20h^2 \times k^2}$ **g** $\dfrac{p^3 \times q^4}{5p^3}$ **h** $\dfrac{m^7 \times n^3}{5m^3 \times m^4}$

i $\dfrac{8u^9 \times v^2}{2u^5 \times 4u^4}$ **j** $\dfrac{9x^6 \times 2y^{12}}{3y^{10} \times 3y^2}$

REASONING

9 Explain why $x^0 = 1$.

10 Simplify $\dfrac{2^0\, x^2}{2^2\, x^0}$, explaining each step of your method.

PROBLEM SOLVING

11 I raise a certain number to the power of three, then multiply the answer by three to the power of zero. I then multiply the result by the certain number to the power of four and divide the answer by three times the certain number to the power of seven. If the final answer is three multiplied by the certain number squared, find the certain number. Show each line of your method.

12 A Mathematics class is asked to simplify $\dfrac{64x^6y^6z^3}{16x^2y^6z^3}$. Peter's answer is $4x^3$. Explain why this is incorrect, pointing out Peter's error. What is the correct answer? Can you identify another source of possible error involving indices?

3.6 Fourth Index Law (raising a power to another power)

- The Fourth Index Law states that when raising a power to another power, the indices are multiplied; that is, $(a^m)^n = a^{m \times n}$. For example, $(5^3)^2 = 5^{3 \times 2} = 5^6$.
- Every number and variable inside the brackets should have its index multiplied by the power outside the brackets. That is,

$$(a \times b)^m = a^m \times b^m$$
$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

(These are sometimes called the Fifth and Sixth Index Laws.)
- Any number or variable that does not appear to have an index really has an index of one; that is, $2 = 2^1$, $a = a^1$.
- Every number or variable inside the brackets must be raised to the power outside the brackets. For example, $(3 \times 2)^4 = 3^4 \times 2^4$ and $(2a^4)^3 = 2^3 \times a^{4 \times 3} = 8a^{12}$.

WORKED EXAMPLE 17

Simplify the following, leaving answers in index form.

a $(7^4)^8$ **b** $\left(\dfrac{3^2}{5^3}\right)^3$

THINK	WRITE
a 1 Write the problem.	**a** $(7^4)^8$
2 Simplify using the Fourth Index Law (multiply the indices).	$= 7^{4 \times 8}$ $= 7^{32}$
b 1 Write the problem.	**b** $\left(\dfrac{3^2}{5^3}\right)^3$
2 Multiply the indices.	$= \dfrac{3^{2 \times 3}}{5^{3 \times 3}}$
3 Simplify.	$= \dfrac{3^6}{5^9}$

WORKED EXAMPLE 18

Simplify $(2b^5)^2 \times (5b^8)^3$.

THINK	WRITE
1 Write the problem.	$(2b^5)^2 \times (5b^8)^3$
2 Simplify using the Fourth Index Law.	$= 2^{1\times2}b^{5\times2} \times 5^{1\times3}b^{8\times3}$ $= 2^2b^{10} \times 5^3b^{24}$
3 Calculate the coefficient.	$= 4b^{10} \times 125b^{24}$ $= 500b^{10} \times b^{24}$
4 Simplify using the First Index Law.	$= 500b^{34}$

WORKED EXAMPLE 19

Simplify $\left(\frac{2a^5}{d^2}\right)^3$.

THINK	WRITE
1 Write the problem.	$\left(\frac{2a^5}{d^2}\right)^3$
2 Simplify using the Fourth Index Law for each term inside the grouping symbols.	$= \frac{2^{1\times3}a^{5\times3}}{d^{2\times3}}$ $= \frac{2^3a^{15}}{d^6}$
3 Calculate the coefficient.	$= \frac{8a^{15}}{d^6}$

assess on

Exercise 3.6 Fourth Index Law (raising a power to another power)

REFLECTION
How will you remember to raise all coefficients to the power outside the brackets?

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–4, 5a–i, 6a–c, 11, 12	Questions: 1–7, 9, 11, 12	Questions: 1d–i, 2d–i, 3d–i, 4, 5g–i, 6, 7i–o, 8g–o, 9–11, 13

Individual pathway interactivity int-4405 eBook plus

FLUENCY

1 WE17 Simplify each of the following, leaving your answers in index form.

a $(3^2)^3$ b $(6^8)^{10}$ c $(11^{25})^4$

d $(5^{12})^{12}$ e $(3^2 \times 10^3)^4$ f $(13 \times 17^3)^5$

g $\left(\frac{3^3}{2^2}\right)^{10}$ h $(3w^9q^2)^4$ i $\left(\frac{7e^5}{r^2q^4}\right)^2$

2 WE18 Simplify each of the following.

a $(p^4)^2 \times (q^3)^2$ **b** $(r^5)^3 \times (w^3)^3$ **c** $(b^5)^2 \times (n^3)^6$

d $(j^6)^3 \times (g^4)^3$ **e** $(q^2)^2 \times (r^4)^5$ **f** $(h^3)^8 \times (j^2)^8$

g $(f^4)^4 \times (a^7)^3$ **h** $(t^5)^2 \times (u^4)^2$ **i** $(i^3)^5 \times (j^2)^6$

3 Simplify each of the following.

a $(2^3)^4 \times (2^4)^2$ **b** $(t^7)^3 \times (t^3)^4$ **c** $(a^4)^0 \times (a^3)^7$

d $(b^6)^2 \times (b^4)^3$ **e** $(e^7)^8 \times (e^5)^2$ **f** $(g^7)^3 \times (g^9)^2$

g $(3a^2)^4 \times (2a^6)^2$ **h** $(2d^7)^3 \times (3d^2)^3$ **i** $(10r^{12})^4 \times (2r^3)^2$

4 a MC What does $(p^7)^2 \div p^2$ equal?

A p^7 **B** p^{12} **C** p^{16} **D** $p^{4.5}$ **E** p^{11}

b What does $\dfrac{(w^5)^2 \times (p^7)^3}{(w^2)^2 \times (p^3)^5}$ equal?

A w^2p^6 **B** $(wp)^6$ **C** $w^{14}p^{36}$ **D** w^2p^2 **E** w^6p^{19}

c What does $(r^6)^3 \div (r^4)^2$ equal?

A r^3 **B** r^4 **C** r^8 **D** r^{10} **E** r^{12}

5 Simplify each of the following.

a $(a^3)^4 \div (a^2)^3$ **b** $(m^8)^2 \div (m^3)^4$

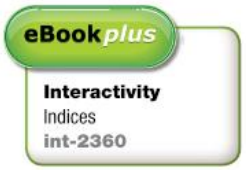
c $(n^5)^3 \div (n^6)^2$ **d** $(b^4)^5 \div (b^6)^2$

e $(f^7)^3 \div (f^2)^2$ **f** $(g^8)^2 \div (g^5)^2$

g $(p^9)^3 \div (p^6)^3$ **h** $(y^4)^4 \div (y^7)^2$

i $\dfrac{(c^6)^5}{(c^5)^2}$ **j** $\dfrac{(f^5)^3}{(f^2)^4}$

k $\dfrac{(k^3)^{10}}{(k^2)^8}$ **l** $\dfrac{(p^{12})^3}{(p^{10})^2}$



6 WE19 Simplify each of the following.

a $\left(\dfrac{3b^4}{d^3}\right)^2$ **b** $\left(\dfrac{5h^{10}}{2j^2}\right)^2$

c $\left(\dfrac{2k^5}{3t^8}\right)^3$ **d** $\left(\dfrac{7p^9}{8q^{22}}\right)^2$

e $\left(\dfrac{5y^7}{3z^{13}}\right)^3$ **f** $\left(\dfrac{4a^3}{7c^5}\right)^4$

UNDERSTANDING

7 Simplify each of the following using the index laws.

a $g^3 \times 2g^5$ **b** $2p^6 \times 4p^2$ **c** $(w^3)^6$

d $12x^6 \div 2x$ **e** $(2d^3)^2$ **f** $5a^6 \times 3a^2 \times a^2$

g $15s^8 \div 5s^2$ **h** $4bc^6 \times 3b^3 \times 5c^2$ **i** $\dfrac{14x^8}{7x^4}$

j $(f^4g^3)^2$ **k** $\dfrac{16u^6v^5}{6u^3v}$ **l** $x^2y^4 \times xy^3$

m $5a^6b^2 \times a^2 \times 3ab^3$ **n** $x^2y^4 \div xy^3$ **o** $(4p^2q^5)^3$

REASONING

8 Simplify each of the following, giving your answer in index form. Justify your answer in each case.

a $(w^3)^4 \div w^2$

b $\dfrac{4x^5 \times 3x}{2x^4}$

c $(2a^3)^2 \times 3a^5$

d $12x^6 \times 2x \div 3x^5$

e $2d^3 + d^2 + 5d^3$

f $\dfrac{(2k^3)^2}{4k^4}$

g $\dfrac{4p^5}{p^4 \times 6p}$

h $15s^8t^3 \div 5s^2t^2 \times 2st^4$

i $12b^4c^6 \div 3b^3 \div 4c^2$

j $(f^4g^3)^2 - fg^3 \times f^7g^3$

k $\dfrac{(3p^3)^2 \times 4p^7}{2(p^4)^3}$

l $2(x^2y)^4 \times 8xy^3$

m $5a^6b^2 + a^2 \times 3a^4b^2$

n $24x^2y^4 \div 12xy^3 - xy$

o $\dfrac{4p^2q^7 \times (3p^3q)^2}{6(pq)^3 \times p^5q^4}$

9 Explain why $((a^b)^c)^0 = 1$.

10 A Mathematics class is asked to simplify $(r^4)^3 \div (r^3)^2$. Karla's answer is r. Explain why this is incorrect, pointing out Karla's error. What is the correct answer?

PROBLEM SOLVING

11 Show that $2^{(2^{(2^2)})}$ is not equal to $(((2)^2)^2)^2$.

12 Arrange these numbers in ascending order.

3^{2^2}, $2^{2^{2^2}}$, 2^{3^3}, 5^{2^2}, 2^{5^2}, 2^{2^5}

13 **a** Identify as many different expressions as possible that when raised to a power will result in $16x^8y^{12}$.

b Identify as many different expressions as possible that when raised to a power will result in $3^{12n}a^{6n}b^{12n}$.

eBookplus
Digital doc
WorkSHEET 3.2
doc-6852

3.7 Review

www.jacplus.com.au

The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:

- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

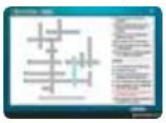
Review questions

Download the Review questions document from the links found in your eBookPLUS.

Interactivities

Word search
int-2619

Crossword
int-2620



Sudoku
int-3183

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in wr ting or using a concept map, a poster or technology.

base	expanded form	index form
basic numeral	factor form	index laws
coefficient	index	variable

Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

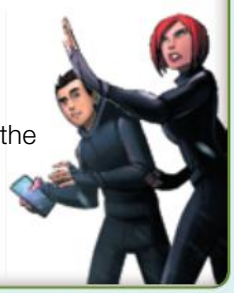
assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

www.assesson.com.au

Link to SpyClass, an exciting online game combining a comic book–style story with problem-based learning in an immersive environment.

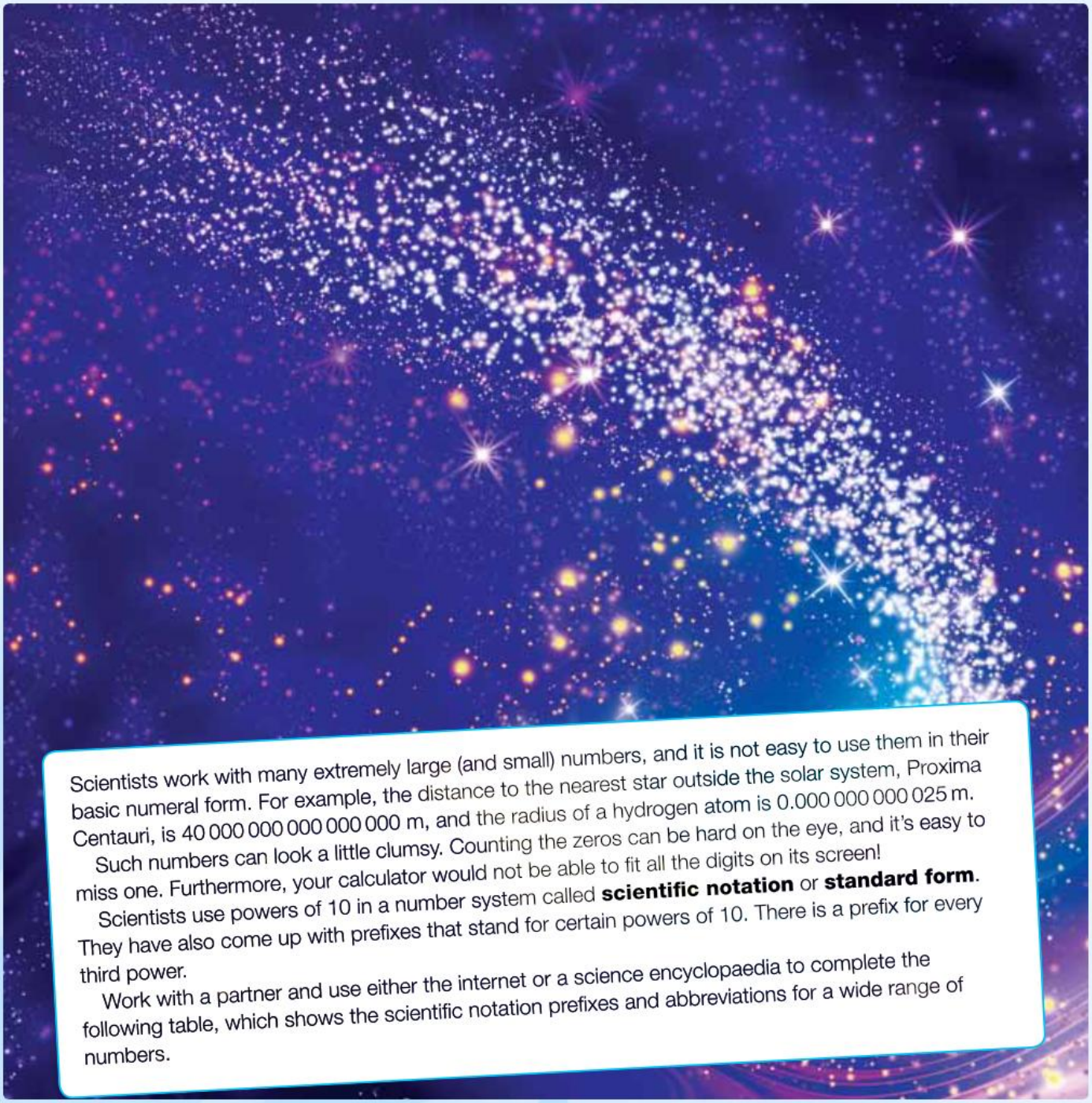
Join Jesse, Toby and Dan and help them to tackle some of the world's most dangerous criminals by using the knowledge you've gained through your study of mathematics.

www.spyclass.com.au



RICH TASK

Scientific notation and standard form



Scientists work with many extremely large (and small) numbers, and it is not easy to use them in their basic numeral form. For example, the distance to the nearest star outside the solar system, Proxima Centauri, is 40 000 000 000 000 000 m, and the radius of a hydrogen atom is 0.000 000 000 025 m.

Such numbers can look a little clumsy. Counting the zeros can be hard on the eye, and it's easy to miss one. Furthermore, your calculator would not be able to fit all the digits on its screen!

Scientists use powers of 10 in a number system called **scientific notation** or **standard form**. They have also come up with prefixes that stand for certain powers of 10. There is a prefix for every third power.

Work with a partner and use either the internet or a science encyclopaedia to complete the following table, which shows the scientific notation prefixes and abbreviations for a wide range of numbers.

Standard form	Basic numeral	Name	SI prefix	SI symbol
1.0×10^{12}	1 000 000 000 000		tera	
1.0×10^{9}		Billion		
1.0×10^{6}	1 000 000		mega	M
1.0×10^{3}				
	100	Hundred	hecto	
1.0×10^{1}			deca	da
1.0×10^{-1}	0.1			
1.0×10^{-2}	0			
1.0×10^{-3}	0.001	Thousandth		
1.0×10^{-6}		Millionth	micro	μ
1.0×10^{-9}			nano	
1.0×10^{-12}	0.000 000 000 001	Trillionth		p

Note: SI is the abbreviation for International System of Units.

Your calculator will accept very large or very small numbers when they are entered because it uses scientific notation. Use the following steps to write the number 825 460 in scientific notation.

Step 1: Place a decimal point so that the number appears to be between 1 and 10.

8.254 60

Step 2: Count how many decimal places the decimal point is from its old position. (For whole numbers, this is at the right-hand end of the number.) In this case, it is five places away.

8.254 60

Step 3: Multiply the number in step 1 by the power of 10 equal to the number of places in step 2.

$8.254\,60 \times 10^{5}$

Note: **If your number was made smaller in step 1, multiply it by a positive power to increase it to its true value. If your number was made larger in step 1, multiply it by a negative power to reduce it to its true value.**

Proxima Centauri, near the Southern Cross, is the closest star to Earth and is 4.2 light-years away. A light-year is the distance that light travels in 1 year. Light travels at 300 000 kilometres per second.

1 Write 300 000 km/s in scientific notation.

2 Find the distance travelled by light in 1 minute.

3 Find the distance travelled by light in 1 hour.

4 Find the distance travelled by light in 1 day.

5 Multiply your answer in question **4** by 365.25 to find the length of a light-year in kilometres. (Why do we multiply by 365.25?) Write this distance in scientific notation.

6 Calculate the distance from Earth to Proxima Centauri in kilometres.

7 Calculate the distance from Earth to some other stars in both light-years and kilometres.

CODE PUZZLE

The largest gland in the body

Simplify the expressions to find the puzzle's answer code.

$\frac{15f^7}{5f^4}$ = **C**	$\frac{24e^{11}c^{15}}{8e^9c^{15}}$ = **V**	$18a^{21} \div 3a^{16}$ = **P**	$e^{10} \div e^7$ = **I**
$50t^{12} \div 25t^{11}$ = **F**	$a^6 \div a^2$ = **A**	$\frac{18n^7u^{12}}{3n^3u}$ = **T**	$(6n^7u^7) \div (2n^5u^6)$ = **N**
$(12c^{20}) \div (2c^{13})$ = **K**	$(16c^9) \div (8c^5)$ = **D**	$\frac{a^{60}}{a^{20}}$ = **W**	$\frac{10e^3k^{12}}{2k^5e^3}$ = **R**
$\frac{9c^2t^7e}{3ct^4ec}$ = **O**	$\frac{8k^6e^2}{4e^2k^4}$ = **H**	$6e^{10} \div 3e^2$ = **B**	$35f^{21} \div 7f^{12}$ = **U**
$7c^8 \div 7c^7$ = **S**	$\frac{28f^{10}}{4f^8}$ = **L**	$\frac{8n^6u^{11}}{2nu^8}$ = **E**	$(20k^8t^6) \div (5k^6t^2k^2)$ = **Y**

$6n^4u^{11}$	$2k^2$	$4n^5u^3$	$7f^2$	e^3	$3e^2$	$4n^5u^3$	$5k^7$	$6a^5$	$5k^7$	$3t^3$	$2c^4$	$5f^9$	$3f^3$	$4n^5u^3$	c	
a^4	$7f^2$	e^3	$6n^4u^{11}$	$5k^7$	$4n^5u^3$	$3t^3$	$2t$	$2e^8$	e^3	$7f^2$	$4n^5u^3$	a^4	$2c^4$	a^4	$4t^4$	
e^3	$6n^4u^{11}$	$2k^2$	$4n^5u^3$	$7f^2$	$6a^5$	c	$2e^8$	$5k^7$	$4n^5u^3$	a^4	$6c^7$	$2c^4$	$3t^3$	a^{40}	$3n^2u$	$2t$	a^4	$6n^4u^{11}$

eBookplus Activities

3.2 Review of index form

Digital docs
- SkillSHEET (doc-6834) Factor trees
- SkillSHEET (doc-6835) Squaring numbers

Interactivity
- IP interactivity 3.2 (int-4401) Review of index form

3.3 First Index Law

Digital doc
- Spreadsheet (doc-2160) Multiplying with indices

Interactivity
- IP interactivity 3.3 (int-4402) First Index Law

3.4 Second Index Law

Digital docs
- SkillSHEET (doc-6836) Equivalent fractions
- SkillSHEET (doc-6837) Simplifying algebraic expressions
- SkillSHEET (doc-6838) Simplifying algebraic fractions
- Spreadsheet (doc-2161) Dividing with indices
- WorkSHEET 3.1 (doc-6851)

Interactivity
- IP interactivity 3.4 (int-4403) Second Index Law

3.5 Third Index Law

Interactivity
- IP interactivity 3.5 (int-4404) Third Index Law

3.6 Fourth Index Law

Digital docs
- WorkSHEET 3.2 (doc-6852)
- Spreadsheet (doc-2162) Raising a power to another power

Interactivities
- Indices (int-2360)
- IP interactivity 3.6 (int-4405) Fourth Index Law

3.7 Review

Interactivities
- Word search (int-2619)
- Crossword (int-2620)
- Sudoku (int-3183)

Digital docs
- Topic summary (doc-10755)
- Concept map (doc-10768)
- Topic review (Word doc-14928, PDF doc-14929)

To access eBookPLUS activities, log on to www.jacplus.com.au

ANSWERS

TOPIC 3 Index laws

3.2 Review of index form

1 **a** Base = 8, power = 4
b Base = 7, power = 10
c Base = 20, power = 11
d Base = 19, power = 0
e Base = 78, power = 12
f Base = 3, power = 100
g Base = m, power = 5
h Base = c, power = 24
i Base = n, power = 36
j Base = d, power = 42

2 **a** 2^6 **b** 4^4 **c** x^5
d 9^3 **e** $11l^7$ **f** $44m^5$

3 **a** 4×4 **b** $5 \times 5 \times 5 \times 5$
c $7 \times 7 \times 7 \times 7 \times 7$ **d** $6 \times 6 \times 6$
e $3 \times 3 \times 3 \times 3 \times 3 \times 3$
f $n \times n \times n \times n \times n \times n \times n$
g $a \times a \times a \times a$
h $k \times k \times k \times k \times k \times k \times k \times k \times k \times k \times k$

4 **a** 243 **b** 256 **c** 256 **d** 1331
e 2401 **f** 216 **g** 1 **h** 625

5 **a** B **b** D

6 **a** $2^2 \times 4^4 \times 6$ **b** $3^4 \times 7^4$
c $2^3 \times 19^5$ **d** $4^4 \times 13^2$
e $66m^5p^2s^2$ **f** $378i^3n^2r^3$
g $192e^3kp^2$ **h** $99j^5lp^2$

7 **a** $15 \times f \times f \times f \times j \times j \times j \times j$
b $7 \times k \times k \times k \times k \times k \times k \times s \times s$
c $4 \times b \times b \times b \times c \times c \times c \times c \times c$
d $19 \times a \times a \times a \times a \times m \times n \times n \times n$
e $8 \times l \times l \times l \times l \times r \times r \times t \times t$

8 **a** $64 = 2^6$ **b** $40 = 2^3 \times 5$
c $36 = 2^2 \times 3^2$ **d** $400 = 2^4 \times 5^2$
e $225 = 3^2 \times 5^2$ **f** $2000 = 2^4 \times 5^3$

9 **a** 120 **b** 100
c 216 **d** 308
e 1575 **f** 760 000

10 **a** **i** 10^1 **ii** 10^2 **iii** 10^3 **iv** 10^6
b **ii** 5×10^2
iii $4 \times 10^2 + 7 \times 10^1$
iv $2 \times 10^3 + 3 \times 10^2 + 6 \times 10^1$
v $1 \times 10^3 + 9 \times 10^2 + 8 \times 10^1$
vi $5 \times 10^3 + 4 \times 10^2 + 3 \times 10^1$
c **i** 75 000
ii 30 600
iii 5 200 480

11 Factors multiplied together in 'shorthand' form:
$a \times a \times a \times b \times b \times b \times b$

12 **a** $2^3a^2b^3c^4$ **b** 2 074 464

13 **a** $1 \times 10^2 + 4 \times 10^1$, $6 \times 10^2 + 8 \times 10^1$
b $7 \times 10^2 + 12 \times 10^1 = 8 \times 10^2 + 2 \times 10^1$
c 820
d 820
e Check with your teacher.

14 **a** $1 \times 2^3 + 1 \times 2^1$, $1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1$
b $2 \times 2^3 + 1 \times 2^2 + 2 \times 2^1 = 1 \times 2^4 + 1 \times 2^2 + 1 \times 2^2$
$= 1 \times 2^4 + 2 \times 2^2 = 1 \times 2^4 + 1 \times 2^3$
c 24
d 24
e Check with your teacher.

3.3 First Index Law (multiplying numbers in index form with the same base)

1 **a** $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$
b $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^8$
c $f \times f \times f \times f \times f \times f \times f \times f \times f = f^9$

2 **a** 3^9 **b** 6^{17} **c** 10^{10} **d** 11^6
e 7^9 **f** 2^{14} **g** 5^4 **h** 8^{11}
i 13^{15} **j** q^{47} **k** x^{14} **l** e^4

3 **a** 3^{12} **b** 2^{18} **c** 5^{17} **d** 6^{11}
e 10^6 **f** 17^{12} **g** p^{22} **h** e^{23}
i g^{28} **j** e^{38} **k** $3b^{13}$ **l** $5d^{16}$

4 **a** A **b** B

5 **a** $20p^{11}$ **b** $6x^8$
c $56y^{10}$ **d** $21p^8$
e $84t^6$ **f** $30q^{15}$

6 **a** $6a^6e^7$ **b** $8h^{12}p^6$ **c** $80m^9$
d $6g^3h^6$ **e** $30p^6q^9$ **f** $48u^9w^7$
g $27d^{11}y^{17}$ **h** $42b^{14}c^9$ **i** $24r^{16}s^{18}$
j $60h^{38}v^{20}$

7 **a** 3^{x+4} **b** 3^{2y+2} **c** 3^{6y-5} **d** $3^{\frac{23}{12}}$

8 **a** $9 = 3^2$; $27 = 3^3$; $81 = 3^4$
b **i** 3^{10} **ii** 3^{2n+2}

9 The bases, 2 and 3, are different. The index laws do not apply.

10 Step 1: 5^n
Step 2: 5^m
Step 3: 5^{n+m}

11 **a** 2^9 **b** 2^{r-1} **c** $2^5 + 2^6 = \$96$
d $2^{13} + 2^{14}$ **e** $2^{13}(1 + 2) = 3(2^{13})$
f Check with your teacher.

12 **a** x^6 **b** 2^{12} **c** 5^2 **d** $(a^n)^p = a^{np}$

13 2

Challenge 3.1

$3\frac{1}{2}$ minutes

3.4 Second Index Law (dividing numbers in index form with the same base)

1 **a** 2^3 **b** 7^4 **c** 10^3

2 **a** 3 **b** 11^7 **c** 5^4 **d** 12^5
e 3^3 **f** 13 **g** 6^4 **h** 10^4
i 15^{33} **j** h^{77} **k** b^{70} **l** f^{900}

3 **a** $3x^2$ **b** $6y^2$ **c** $8w^7$ **d** $3q^4$
e $8f^9$ **f** $10h^{90}$ **g** $4j^{10}$ **h** $5p^{10}$
i $8g^3$ **j** $\frac{3b^6}{2}$ **k** $\frac{9m^4}{2}$ **l** $\frac{5n^{90}}{2}$

4 **a** D **b** A

5 **a** $3p^4$ **b** $6r^4$ **c** $9a^3$ **d** $3b^6$
e $20r^4$ **f** $9q$

6 **a** $\frac{3p^5}{2}$ **b** $\frac{8b^5}{3}$ **c** $\frac{5m^{10}n^6}{6}$ **d** $\frac{9x^8y}{4}$ **e** $\frac{4hk^3}{3}$
f $3f^3j^5$ **g** $\frac{4p^2rs}{3}$ **h** $\frac{9a^5b^3c}{2}$ **i** $\frac{20f^6g^2h^4}{3}$

7 **a** 2^{10-p} **b** 2^{4e+4} **c** 5^{3x+y} **d** 3^{1-m}

8 **a** $\frac{2^3 \times 2^4 \times 2^2}{2^1 \times 2^5}$
b **i** 2^3 **ii** 8
c 8

9 **a** $\frac{2 \times 3 \times 3^3 \times 2^2 \times 3^2}{2^2 \times 3 \times 3^4}$
b **i** $2^1 \times 3^1$ **ii** 6

10 The bases, 12 and 3, are different. The laws of indices do not apply.

11 3^{p-q}

12 $a^p \div a^p = 1$ and $a^p \div a^p = a^{p-p} = a^0 \Rightarrow a^0 = 1$

13 $m^6 \div m^8 = m^{6-8} = m^{-2}$ but $m^6 \div m^8 = \frac{m^6}{m^8} = \frac{1}{m^2} \Rightarrow m^{-2} = \frac{1}{m^2}$

14 $\frac{1}{6}$

Challenge 3.2

54 dabs

3.5 Third Index Law (the power of zero)

1 **a** 1 **b** 1 **c** 1 **d** 1

2 **a** 1 **b** 1 **c** 1 **d** 1

3 **a** 4 **b** 12 **c** −9 **d** 22

4 **a** $12m^3$ **b** 21 **c** 72 **d** $\frac{8}{7}$
e 2 **f** $10b^2$ **g** $3p^2$ **h** 1

5 **a** 1 **b** 1 **c** 1 **d** 7
e 3 **f** 3

6 **a** 1 **b** 4 **c** 2 **d** $2p^9$
e $\frac{3}{2}e^4$ **f** y **g** $\frac{m^3}{3}$ **h** $\frac{c^3}{4}$

7 **a** D **b** A **c** D

8 **a** 1 **b** 2 **c** 2 **d** 2
e 2 **f** $\frac{h^2}{2}$ **g** $\frac{q^4}{5}$ **h** $\frac{n^3}{5}$
i v^2 **j** $2x^6$

9 Any base raised to the power of one equals zero.

10 $2^0 = 1, x^0 = 1 \Rightarrow \frac{2^0x^2}{2^2x^0} = \frac{x^2}{2^2} = \frac{x^2}{4}$

11 $\frac{1}{3}$

12 Peter has treated the x's incorrectly. He has calculated $x^{6 \div 2}$ instead of x^{6-2}.
The answer is $4x^4$.
If 64 and 16 are converted to base 2 or base 4, and the indices divided not subtracted, an error will occur.

3.6 Fourth Index Law (raising a power to another power)

1 **a** 3^6 **b** 6^{80} **c** 11^{100} **d** 5^{144} **e** $3^8 \times 10^{12}$
f $13^5 \times 17^{15}$ **g** $\frac{3^{30}}{2^{20}}$ **h** $3^4w^{36}q^8$ **i** $\frac{7^2e^{10}}{r^4q^8}$

2 **a** p^8q^6 **b** $r^{15}w^9$ **c** $b^{10}n^{18}$
d $j^{18}g^{12}$ **e** q^4r^{20} **f** $h^{24}j^{16}$
g $f^{16}a^{21}$ **h** $t^{10}u^8$ **i** $i^{15}j^{12}$

3 **a** 2^{20} **b** t^{33} **c** a^{21} **d** b^{24} **e** e^{66}
f g^{39} **g** $324a^{20}$ **h** $216d^{27}$ **i** $40\,000r^{54}$

4 **a** B **b** B **c** D

5 **a** a^6 **b** m^4 **c** n^3 **d** b^8
e f^{17} **f** g^6 **g** p^9 **h** y^2
i c^{20} **j** f^7 **k** k^{14} **l** p^{16}

6 **a** $\frac{9b^8}{d^6}$ **b** $\frac{25h^{20}}{4j^4}$ **c** $\frac{8k^{15}}{27t^{24}}$ **d** $\frac{49p^{18}}{64q^{44}}$
e $\frac{125y^{21}}{27z^{39}}$ **f** $\frac{256a^{12}}{2401c^{20}}$

7 **a** $2g^8$ **b** $8p^8$ **c** w^{18} **d** $6x^5$ **e** $4d^6$
f $15a^{10}$ **g** $3s^6$ **h** $60b^4c^8$ **i** $2x^4$ **j** f^8g^6
k $\frac{8u^3v^4}{3}$ **l** x^3y^7 **m** $15a^9b^5$ **n** xy **o** $64p^6q^{15}$

8 **a** w^{10} **b** $6x^2$ **c** $12a^{11}$ **d** $8x^2$ **e** $7d^3 + d^2$
f k^2 **g** $\frac{2}{3}$ **h** $6s^7t^5$ **i** bc^4 **j** 0
k $18p$ **l** $16x^9y^7$ **m** $8a^6b^2$ **n** xy **o** $6q^2$

9 Any base, even a complex one like this, raised to the power of one equals zero.

10 Karla has added the powers instead of multiplying.
$r^7 \div r^6 = r^1 = r$
The correct answer is r^6.

11 $2^{(2^{(2^2)})} = 2^{2^4} = 2^{16}$; $(((2)^2)^2)^2 = 2^8$

12 2^{2^5}; 2^{3^3}; 2^{5^2}; $2^{2^{2^2}}$; 3^{2^2}; 5^2

13 **a** $(16x^8y^{12})^1$, $(4x^4y^6)^2$, $(2x^2y^3)^4$

b $(3^{12n}a^{6n}b^{12n})^1$, $(3^{12}a^6b^{12})^n$, $(3^6a^3b^6)^{2n}$, $(3^2a^1b^2)^{6n}$, $(3^{6n}a^{3n}b^{6n})^2$, $(3^{2n}a^nb^{2n})^6$

Investigation — Rich task

Standard form	Basic numeral	Name	SI prefix	SI symbol
1.0×10^{12}	1 000 000 000 000	Trillion	tera	T
1.0×10^9	1 000 000 000	Billion	giga	G
1.0×10^6	1 000 000	Million	mega	M
1.0×10^3	1000	Thousand	kilo	k
1.0×10^2	100	Hundred	hecto	h
1.0×10^1	10	Ten	deca	da
1.0×10^{-1}	0.1	Tenth	deci	d
1.0×10^{-2}	0.01	Hundredth	centi	c
1.0×10^{-3}	0.001	Thousandth	milli	m
1.0×10^{-6}	0.000 001	Millionth	micro	m
1.0×10^{-9}	0.000 000 001	Billionth	nano	n
1.0×10^{-12}	0.000 000 000 001	Trillionth	pico	p

1 3.0×10^5 km/s

2 1.8×10^7 km/min

3 1.08×10^9 km/h

4 2.592×10^{10} km/day

5 9.47×10^{12} km. On average, there are 365.25 days in one year.

6 4.07×10^{13} km

7 Check with your teacher.

Code puzzle

The liver produces a litre of bile a day. It helps break down fat.

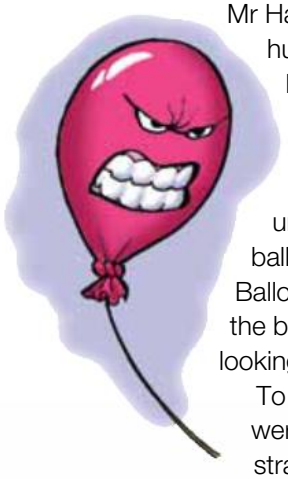
projectsplus

Attack of the killer balloons

Searchlight ID: pro-0095

Scenario

Ms Lovely is a Math teacher at Scholar High School and during last week she had her 30th birthday.



Mr Handsome, her adoring husband, secretly placed a balloon and flowers in her classroom before the beginning of her first class. Unfortunately, he had unknowingly purchased a killer balloon from Mr Loon, of Angry Balloons! During Ms Lovely's class, the balloon developed a strange-looking face and burst open.

To everyone's amazement, there were now five balloons, all with strange faces!

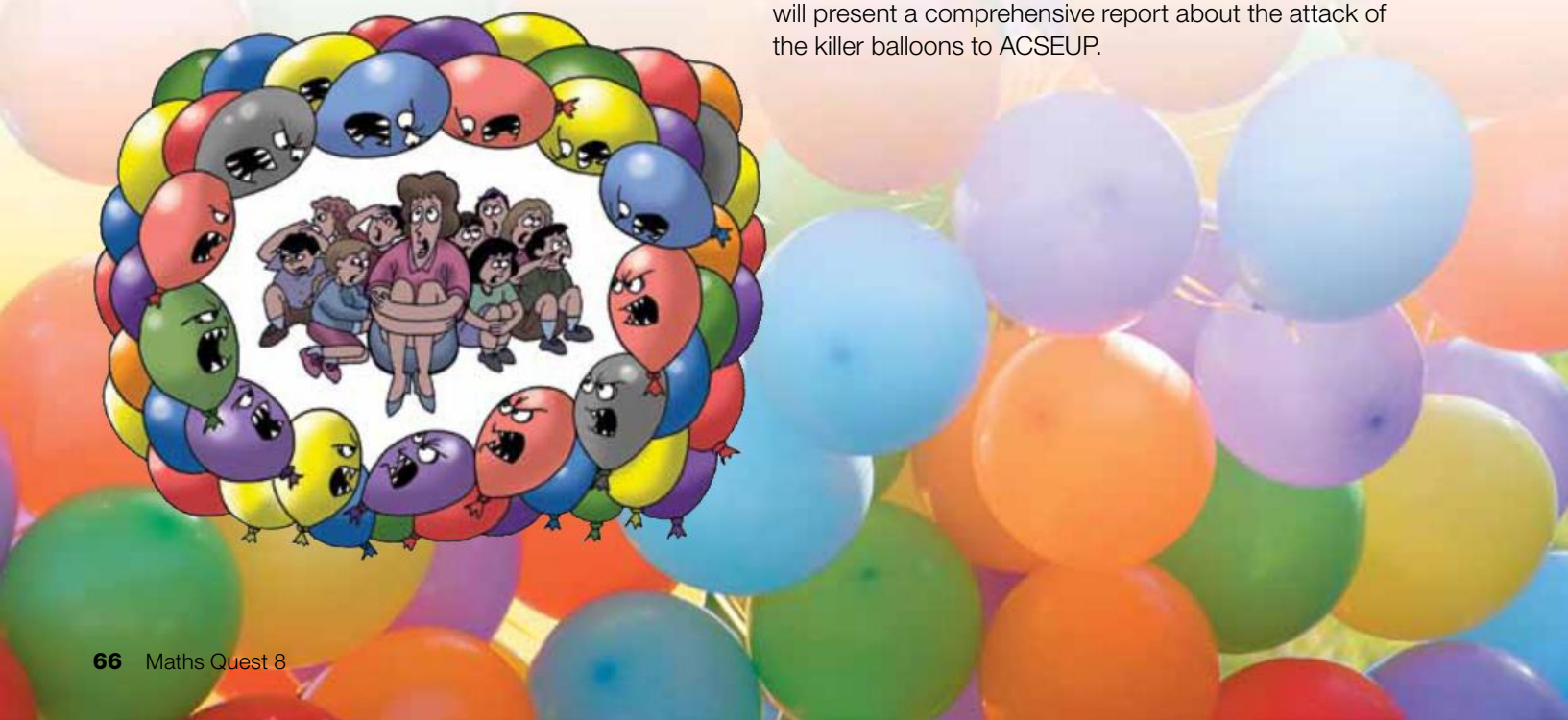
During the following period when Ms Lovely was taking a new class, the five balloons also burst, and during the day they continued to multiply in this way. Eventually it became apparent that the balloons were hostile and were going to attack.

Terrified, Ms Lovely and the threatened children escaped, but the killer balloons had taken control of her classroom and were threatening to spread. The students and staff were evacuated from the school, and the Australian Centre for the Study of Extraordinary and Unexplained Phenomena (ACSEUP) was called to investigate. ACSEUP re-created the events to produce a simulation video so they could study the balloons and work out how to defeat them. They realised that if they could work out the mathematical pattern of the increasing numbers of balloons perhaps they could find a plan for defeating them. However, after extensive analysis, they were stumped. The ACSEUP researchers had heard that your school had some clever maths students, and have asked your mathematics class to help Ms Lovely's students solve the mystery of the killer balloons.

Task

Watch the case study video that ACSEUP has provided for your class, and simultaneously track the growth of the killer balloons on a chart. Use the data that you have collected to create a graph showing the growth rate of the killer balloons. From these, create a mathematical equation that simulates the attack.

When you have finished this research and understand the problem, you will answer some other questions that ACSEUP has asked about similar balloon scenarios. These will include how to go about defeating the killer balloons. Using your solutions to these questions, combined with your charts, graphs and equations, you will present a comprehensive report about the attack of the killer balloons to ACSEUP.



Summary of tasks

- Record and graph the growth of the balloons.
- Create a mathematical formula that simulates the growth of the balloons.
- Answer questions about other balloon growth scenarios.
- Record and graph the popping of the balloons.
- Create a mathematical formula that simulates the destruction of the balloons.
- Answer questions about other balloon destruction scenarios.
- Prepare a report on the attack of the killer balloons for ACSEUP.

Process

- Open the ProjectsPLUS application for this topic in your eBookPLUS. Watch the Attack of the Killer Balloons case study, navigate to your Media Centre, and print Template 1 provided in the Template section. Fill in the required data in Template 1. Your Media Centre includes the re-enactment of the killer balloon take-over.
- Next, press the 'Start Project' button and then set up your project group. You will need to create a group of two or three of your classmates before you begin this project. Save your group settings and the project will be launched.
- Navigate to the Media Centre, then to the Documents section and answer the questions in the six ACSEUP worksheets.
- Navigate to the Media Centre, then to the Template section and print Presentation guidelines.
- Use the Presentation guidelines to choose a topic for your presentation, then read about the guidelines for the project.
- Navigate to the Media Centre to access GeoGebra and the corresponding user manual.
- If possible, use a SMARTBoard or overhead projector and computer to display your GeoGebra graph and tables during the presentation.

SUGGESTED SOFTWARE

- ProjectsPLUS
- Microsoft Word
- GeoGebra

Your ProjectsPLUS application is available in this topic's Student Resources tab inside your eBookPLUS. Visit www.jacplus.com.au to locate your digital resources.

Interactivity

KILLER BALLOONS

SEARCHLIGHT ID: int-2447

If a quantity doubles or triples over a consistent time period, it is said to be increasing exponentially. Use this interactivity to change the exponential rate of growth and then predict the number of balloons that will appear. Watch the graph of the number of balloons against time plotted on the wall.

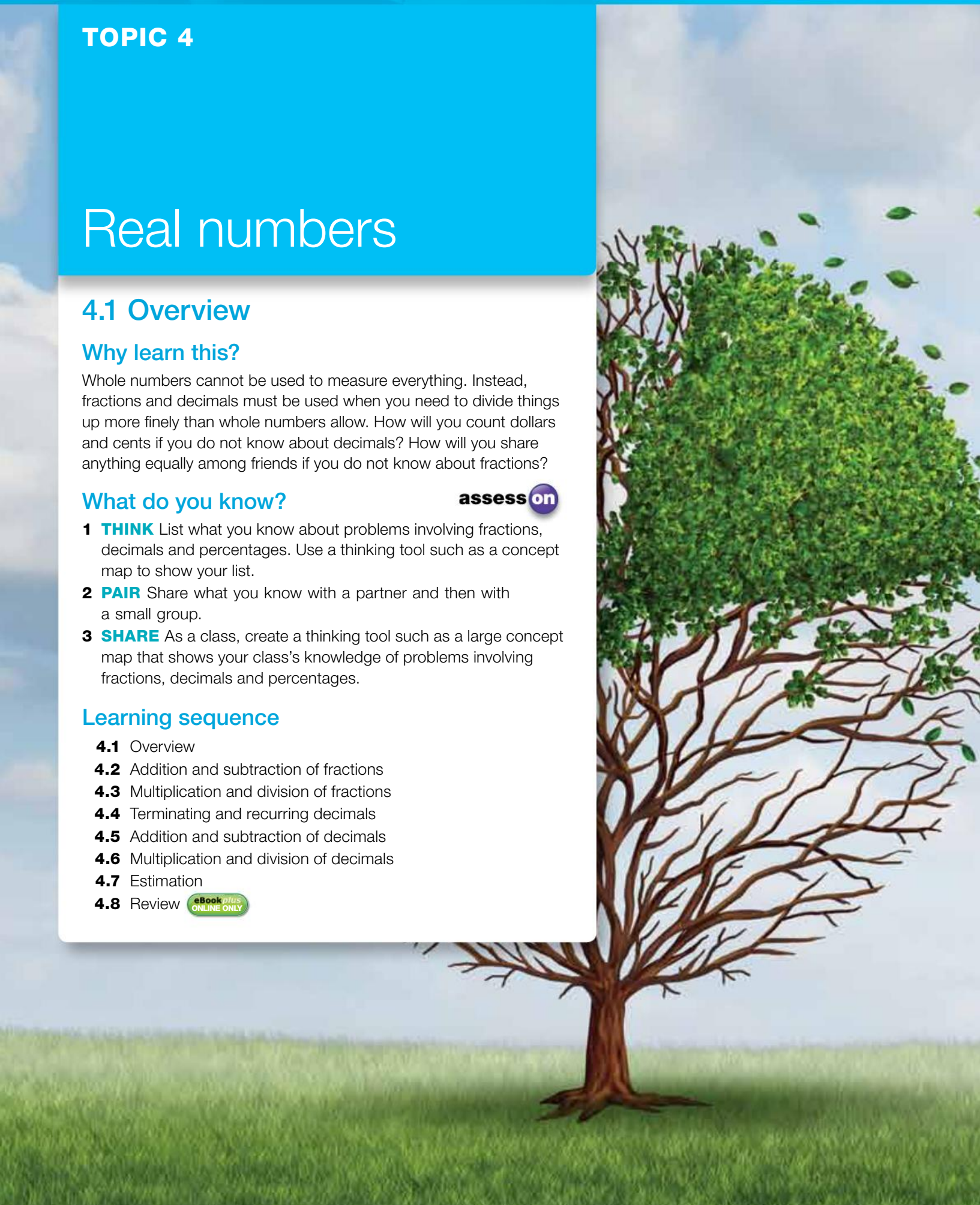
MEDIA CENTRE

Your Media Centre contains:

- a Document section with ACSEUP worksheets
- a Template section with presentation guidelines
- GeoGebra software and its user manual
- an assessment rubric.

TOPIC 4

Real numbers



4.1 Overview

Why learn this?

Whole numbers cannot be used to measure everything. Instead, fractions and decimals must be used when you need to divide things up more finely than whole numbers allow. How will you count dollars and cents if you do not know about decimals? How will you share anything equally among friends if you do not know about fractions?

What do you know?

assessON

1 **THINK** List what you know about problems involving fractions, decimals and percentages. Use a thinking tool such as a concept map to show your list.
2 **PAIR** Share what you know with a partner and then with a small group.
3 **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of problems involving fractions, decimals and percentages.

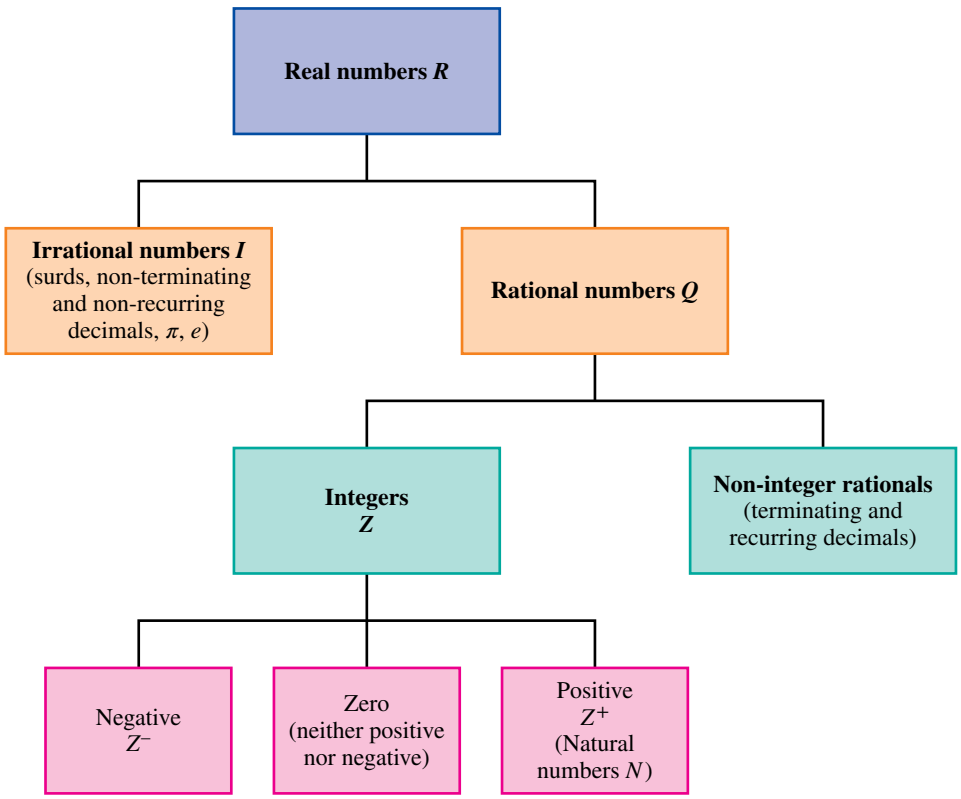
Learning sequence

4.1 Overview
4.2 Addition and subtraction of fractions
4.3 Multiplication and division of fractions
4.4 Terminating and recurring decimals
4.5 Addition and subtraction of decimals
4.6 Multiplication and division of decimals
4.7 Estimation
4.8 Review eBookplus ONLINE ONLY

4.2 Addition and subtraction of fractions

Introduction to real numbers

- Number systems have become more sophisticated as societies have developed.
- Different types of numbers can be classified into specific groups.
- The **real number** system contains the set of **rational** and **irrational** numbers. It can be denoted by the symbol R.
- The set of real numbers contains a number of groups or subsets that can be classified as shown in the chart below.



Rational numbers

- A rational number is any number that can be expressed as a ratio of two whole numbers in the form $\frac{a}{b}$, where a and b are whole numbers and $b \neq 0$.
- The set of rational numbers is given the symbol Q. Examples include $\frac{1}{4}$, -3, $\frac{4}{7}$, 100, -63, 1.5, -0.8 and $1.\overline{45}$.
- Rational numbers include all integers (symbol Z), as integers can be expressed in the form $\frac{a}{b}$. For example, $100 = \frac{100}{1}$, $-3 = \frac{-3}{1}$.
- Rational numbers include all fractions, both those that can be expressed as terminating decimals and those that can be expressed as recurring decimals. For example, $\frac{2}{5} = 0.4$ is a terminating decimal and $\frac{1}{3} = 0.33..... = 0.\dot{3}$ is a recurring decimal. These are rational numbers as they can be expressed in the form $\frac{a}{b}$, where a and b are whole numbers and $b \neq 0$.

Irrational numbers

- Numbers that cannot be expressed in the form $\frac{a}{b}$, where a and b are whole numbers and $b \neq 0$, are called irrational numbers. Examples of these numbers are $\sqrt{5}$, $\sqrt{7}$ and $2\sqrt{11}$. These numbers can be expressed as decimals, but their decimal values do not terminate or repeat in any pattern.
- A special irrational number is π (pi). This is the distance around the circumference of a circle with a diameter of 1. It can be approximated as a decimal that is non-terminating and non-recurring.
  In decimal form, $\pi = 3.141\,592\ldots$
  The value of π has been calculated to many millions of decimal places with the aid of a computer, but it is still non-terminating and non-recurring.

Review of addition and subtraction of proper fractions

- As can be seen from the diagram on page 70, fractions are rational numbers.
- A fraction has two parts. The top part is called the **numerator** and the bottom part is called the **denominator**.

  $\frac{2}{3}$ ⟵ Numerator
  ⟵ Denominator

  This line means divide.
- A **proper fraction** has a numerator that is less than the denominator; for example, $\frac{3}{5}$.
- **Equivalent fractions** are fractions that are equal in value; for example, $\frac{1}{2} = \frac{2}{4}$.
- When we add and subtract fractions, the denominators *must* be the same.

WORKED EXAMPLE 1

Simplify $\frac{4}{5} - \frac{3}{4}$.

THINK	WRITE
1 Write the question.	$\frac{4}{5} - \frac{3}{4}$
2 Find the lowest common denominator, that is, the lowest multiple common to both.	$= \frac{4 \times 4}{5 \times 4} - \frac{3 \times 5}{4 \times 5}$
3 Write both fractions with the same denominator, that is, the lowest common denominator.	$= \frac{16}{20} - \frac{15}{20}$
4 Subtract the fractions.	$= \frac{1}{20}$
5 Write the answer.	

Addition and subtraction of mixed numbers

- An **improper fraction** has a numerator greater than the denominator; for example, $\frac{7}{3}$.
- A **mixed number** contains a whole number part and a proper fraction part; for example, $7\frac{5}{8}$.
- When we are adding and subtracting mixed numbers, we must first change them to improper fractions, and then solve them using the method shown in Worked example 1. However, it is often better to add or subtract the whole number parts first and then the fraction parts.

WORKED EXAMPLE 2

Calculate $2\frac{2}{3} + 3\frac{1}{2}$.

THINK	WRITE
1 Write the question.	$2\frac{2}{3} + 3\frac{1}{2}$
2 Change each mixed number to an improper fraction.	$= \frac{8}{3} + \frac{7}{2}$
3 Write both fractions with the same denominator using equivalent fractions.	$= \frac{8 \times 2}{3 \times 2} + \frac{7 \times 3}{2 \times 3}$ $= \frac{16}{6} + \frac{21}{6}$
4 Add the fractions.	$= \frac{37}{6}$
5 Write the answer as a mixed number if appropriate.	$= 6\frac{1}{6}$

- A rough estimate can be found by adding or subtracting the whole number parts. For example, $2 + 3 = 5$, so $2\frac{2}{3} + 3\frac{1}{2} > 5$.

WORKED EXAMPLE 3

Calculate $4\frac{1}{5} - 1\frac{1}{2}$.

THINK	WRITE
1 Write the question.	$4\frac{1}{5} - 1\frac{1}{2}$
2 Change each mixed number to an improper fraction.	$= \frac{21}{5} - \frac{3}{2}$
3 Write both fractions with the same denominator using equivalent fractions.	$= \frac{21 \times 2}{5 \times 2} - \frac{3 \times 5}{2 \times 5}$ $= \frac{42}{10} - \frac{15}{10}$
4 Subtract the second fraction from the first.	$= \frac{27}{10}$
5 Write the answer as a mixed number if appropriate.	$= 2\frac{7}{10}$

Negative fractions

- Positive and negative numbers have both size and direction, so they are often called **directed numbers**.
- Directed numbers include zero, integers and fractions.

WORKED EXAMPLE 4

Graph each of the following sets of directed numbers on a number line.

a $x > -2\frac{1}{2}$ **b** $x \leq 1\frac{3}{4}$ **c** $-2\frac{1}{3} < x \leq 3.4$

THINK

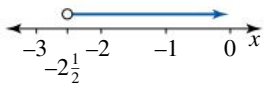
a $>$ means greater than, so x takes on all values to the right of $-2\frac{1}{2}$. Use an *open* or *unshaded* circle to indicate that the number itself is not included.

b $\leq$ means less than or equal to, so x takes on all values to the left of and including $1\frac{3}{4}$. Show all points fitting the description. Use a *closed* or *shaded* circle to show that the number itself is included.

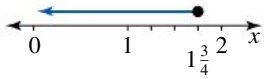
c $-2\frac{1}{3} < x \leq 3.4$ means all numbers lying between the given boundaries, including 3.4 itself. Use an open dot to show that $-2\frac{1}{3}$ is not included and a closed dot to show that 3.4 is included. Use estimation to locate the boundary points.

DRAW

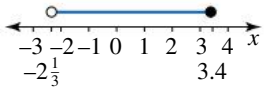
a



b



c



Addition and subtraction of positive and negative fractions

- The rules for adding and subtracting integers apply to the addition and subtraction of positive and negative fractions.
  1. When adding same signs, add and keep the sign.
  2. When adding different signs, find the difference between the 'sign-less' numbers (that is ignoring the sign) then use the sign of the number furthest from zero.
  3. When subtracting, add the opposite.
- The rules for addition and subtraction of fractions apply:
  Write all fractions with the same denominator, then add or subtract the numerators:

$$\frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6} = \frac{7}{6} = 1\frac{1}{6}$$

WORKED EXAMPLE 5

Calculate $-\frac{1}{3} + \frac{1}{2}$.

THINK

1. Write the expression.
2. Write both fractions with the same denominator.
3. Add the numerators.
   Note: They are different signs, so find the difference and use the sign of the number furthest from zero.
4. Write the answer.

WRITE

$-\frac{1}{3} + \frac{1}{2}$

$= -\frac{2}{6} + \frac{3}{6}$

$= \frac{1}{6}$

WORKED EXAMPLE 6

Calculate $-\frac{3}{4} - \frac{1}{6}$.

THINK	WRITE
1 Write the expression.	$-\frac{3}{4} - \frac{1}{6}$
2 To subtract, add the opposite.	$= -\frac{3}{4} + -\frac{1}{6}$
3 Write both fractions with the same denominator.	$= -\frac{9}{12} + -\frac{2}{12}$
4 Add the numerators. *Note:* They are the same sign, so add and keep the sign.	$= -\frac{11}{12}$
5 Write the answer.	

assess on

Exercise 4.2 Addition and subtraction of fractions

REFLECTION
Why do we normally leave the answer as a mixed number instead of an improper fraction?

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–10, 12, 18	Questions: 1–3, 4d–j, 5–8, 9d–l, 10–13, 16–18	Questions: 1–3, 4g–j, 5–8, 9g–l, 10–17, 19

Individual pathway interactivity int-4406 eBookplus

eBookplus
Digital docs
SkillSHEET Order of operations I doc-6854
SkillSHEET Factors doc-6855
SkillSHEET Multiples doc-6856
SkillSHEET Adding and subtracting fractions I doc-6857

FLUENCY

1 Simplify the following fractions, working from left to right.
a $\frac{3}{17} + \frac{6}{17}$ b $\frac{21}{27} - \frac{16}{27}$ c $\frac{6}{17} + \frac{2}{17} + \frac{4}{17}$ d $\frac{3}{15} + \frac{11}{15} - \frac{2}{15}$

2 Simplify the following fractions, writing the answer as a mixed number if appropriate.
a $\frac{3}{5} + \frac{4}{5}$ b $\frac{7}{8} + \frac{3}{8}$ c $\frac{7}{8} + \frac{3}{8} + \frac{6}{8}$ d $\frac{41}{50} - \frac{24}{50} + \frac{6}{50}$

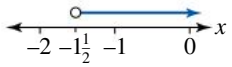
3 WE1 Simplify the following.
a $\frac{2}{5} + \frac{1}{4}$ b $\frac{3}{4} + \frac{5}{8}$ c $\frac{6}{10} - \frac{2}{5}$ d $\frac{8}{25} + \frac{34}{50} - \frac{7}{25}$
e $\frac{3}{4} + \frac{5}{6}$ f $\frac{9}{10} - \frac{2}{7}$ g $\frac{8}{12} + \frac{5}{7} - \frac{1}{2}$ h $\frac{21}{30} + \frac{5}{6} + \frac{9}{10}$

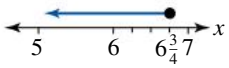
4 WE2 Calculate the following and check answers with the aid of a calculator.
a $4\frac{1}{5} - 2\frac{3}{5}$ b $6\frac{7}{9} - 3\frac{5}{9}$ c $8\frac{4}{5} - 4\frac{1}{5}$ d $8\frac{5}{6} - 4\frac{1}{6}$
e $6\frac{7}{8} - 4\frac{3}{8}$ f $1\frac{4}{9} + 5\frac{5}{9}$ g $6\frac{1}{4} + 3\frac{2}{8}$ h $12\frac{2}{5} + 8\frac{7}{9}$
i $4\frac{3}{4} - 5\frac{1}{6} + 3\frac{2}{12}$ j $1\frac{2}{5} + 3\frac{1}{3} - 2\frac{4}{15}$

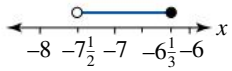
5 WE3 Calculate the following and check answers with the aid of a calculator.
a $5\frac{3}{5} - 2\frac{3}{10}$ b $6\frac{1}{2} - 3\frac{5}{6}$ c $10\frac{1}{4} - 5\frac{2}{5}$ d $4\frac{1}{8} - 2\frac{3}{5}$

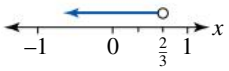
6 WE4 Graph each of the following sets of directed numbers on a number line.
a $x > -3\frac{1}{2}$ b $x < -2\frac{1}{3}$ c $x \geq -4.5$ d $x \leq 6.25$
e $-2 < x < 1$ f $-1\frac{2}{3} < x \leq \frac{1}{2}$ g $-6\frac{3}{4} \leq x < -4\frac{1}{2}$ h $x \geq 2.75$
i $x < -1\frac{4}{5}$ j $2\frac{3}{5} \leq x \leq 3.8$

7 Describe the directed numbers graphed on each number line. Use number sentences such as: $x > -2\frac{1}{2}$, $x \leq 4.8$ and so on.

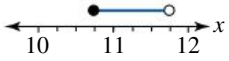
a


b


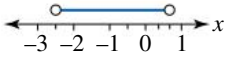
c


d


e

f


g

h


8 Arrange the following in ascending order.

a $6, -3, 0, 5.8, -4\frac{1}{2}$ **b** $3\frac{2}{3}, -1, -4.2, 1\frac{1}{2}, -\frac{3}{4}$ **c** $0, -\frac{2}{3}, \frac{1}{4}, -6.8, -8.6$

9 WE5 Calculate the following.

a $-\frac{3}{5} + \frac{1}{5}$ **b** $-\frac{3}{8} + -\frac{5}{8}$ **c** $\frac{1}{4} + -\frac{1}{2}$

d $-\frac{1}{6} + -\frac{2}{3}$ **e** $\frac{1}{2} + -\frac{3}{8}$ **f** $-\frac{1}{2} + -\frac{1}{3}$

g $-\frac{3}{5} + \frac{1}{2}$ **h** $\frac{3}{4} + -\frac{1}{3}$ **i** $-2\frac{1}{2} + -1\frac{3}{4}$

j $1\frac{2}{3} + -2\frac{3}{5}$ **k** $-2\frac{1}{2} - 3\frac{1}{3}$ **l** $3\frac{1}{4} + 1\frac{3}{5}$

10 WE6 Calculate the following.

a $\frac{1}{2} - \frac{3}{4}$ **b** $-\frac{1}{2} - \frac{1}{3}$ **c** $-\frac{1}{3} - \frac{2}{5}$

d $-\frac{3}{4} - -\frac{2}{5}$ **e** $2\frac{1}{2} - 3\frac{1}{4}$ **f** $-3\frac{3}{5} - -1\frac{1}{3}$

eBookplus
Interactivity
Adding and subtracting fractions
int-2357

UNDERSTANDING

11 If Mary eats $\frac{5}{8}$ of a block of chocolate for afternoon tea and $\frac{3}{8}$ of the block after dinner, how much of the block has she eaten altogether?

12 Seven bottles of soft drink were put out onto the table at a birthday party. How much soft drink was left over after $5\frac{2}{9}$ bottles were consumed?

REASONING

13 Frank has a part-time job at the local newsagency. If he spends $\frac{1}{3}$ of his pay on comic books and $\frac{2}{5}$ on lollies, what fraction of his pay does he have left over? Show your working.

14 In my class, $\frac{1}{3}$ of the students ride their bikes to school, $\frac{1}{4}$ catch the bus and the rest get a lift. What fraction of my class get a lift to school? Show your working.

15 A Year 8 class organised a cake stall to raise some money. If they had 10 whole cakes to start with, sold $2\frac{3}{4}$ cakes at recess and then $5\frac{7}{8}$ at lunch time, how much cake was left over? Show your working.

PROBLEM SOLVING

16 Polly and Neda had divided up some coins. Neda was upset as Polly had more coins than Neda. Polly said 'Here's one-third of my coins'. Neda was moved by Polly's generosity and gave back one-half of her total. Polly gave her one-quarter of her new total and an extra coin.

a Assuming that Polly started with x coins and Neda started with y coins, show that after their final exchange of coins Polly had $\frac{15x + 9y - 24}{24}$ coins and Neda had $\frac{9x + 15y + 24}{24}$ coins.

b Show that the total number of coins Polly and Neda had between them after their final exchange was the same total with which they started.

17 You need to fill a container with exactly $2\frac{5}{12}$ cups of water. If you have cups that measure $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$ and 1 cup of water, what is the quickest way to measure this amount?

1 cup $\frac{1}{2}$ cup $\frac{1}{3}$ cup $\frac{1}{4}$ cup

18 A tray used to cook a muesli bar slice holds 3 cups of ingredients. For the recipe shown, determine whether or not the mixture will fit in the tray.

Muesli bar slice

Ingredients

- $\frac{1}{2}$ cup of dried fruit
- $\frac{3}{4}$ cup of grated apple
- $1\frac{1}{4}$ cups of muesli
- $\frac{2}{3}$ cup of apple juice
- 1 tablespoon ($\frac{2}{25}$ cup) of canola margarine

19 Some fractions, when added, have the following property:
The numerator of the sum of the fractions is equal to the sum of the numerators, and the denominator of the sum of the fractions is equal to the sum of the denominators.
Find two pairs of fractions that have this property.

CHALLENGE 4.1

Use the following clues to find the values of A, B, C and D.

- A is the third multiple of D.
- Both C and D are factors of A.
- B is the only number that is neither composite nor prime.
- All numbers are different and add up to 20.

	A	
B		C
	D	

4.3 Multiplication and division of fractions

Multiplication of fractions

- Change mixed numbers to improper fractions before multiplying.
- Cancel if possible
- Multiply the numerators and multiply the denominators.

WORKED EXAMPLE 7

Simplify $2\frac{1}{4} \times 1\frac{5}{7}$.

THINK	WRITE
1 Write the question.	$2\frac{1}{4} \times 1\frac{5}{7}$
2 Change the mixed numbers to improper fractions and cancel if possible.	$= \frac{9}{\cancel{4}^{1}} \times \frac{\cancel{12}^{3}}{7}$
3 Multiply the numerators and then multiply the denominators.	$= \frac{27}{7}$
4 Change to a mixed number and simplify if appropriate.	$= 3\frac{6}{7}$

- To divide fractions, multiply the first fraction by the reciprocal of the second fraction. Cancel where possible, then multiply the numerators and multiply the denominators.

WORKED EXAMPLE 8

Find $2\frac{1}{4} \div \frac{3}{8}$.

THINK	WRITE
1 Write the question.	$2\frac{1}{4} \div \frac{3}{8}$
2 Change mixed numbers to improper fractions.	$= \frac{9}{4} \div \frac{3}{8}$
3 Change ÷ to × and invert the second fraction, (× and invert), and cancel if appropriate.	$= \frac{\cancel{9}^{3}}{\cancel{4}^{1}} \times \frac{\cancel{8}^{2}}{\cancel{3}^{1}}$
4 Multiply the numerators and then multiply the denominators.	$= \frac{3}{1} \times \frac{2}{1}$
5 Simplify if appropriate.	$= 6$

Multiplication and division of positive and negative fractions

- To multiply fractions, express any mixed numbers as improper fractions. Cancel where possible. Multiply the numerators and denominators, applying the rules for multiplying positive and negative numbers.

WORKED EXAMPLE 9

Simplify $\frac{2}{5} \times -\frac{5}{8}$.

THINK	WRITE
1 Write the expression and cancel the common factors in numerators and denominators.	$\frac{\cancel{2}^{1}}{\cancel{5}^{1}} \times \frac{-\cancel{5}^{1}}{\cancel{8}^{4}}$
2 Multiply the numerators and then multiply the denominators. *Note:* positive × negative = negative	$= \frac{1}{1} \times -\frac{1}{4}$
3 Write the answer.	$= -\frac{1}{4}$

- When dividing by a fraction, express any mixed numbers as improper fractions and multiply by the reciprocal.

WORKED EXAMPLE 10

Evaluate $-\frac{3}{4} \div -1\frac{1}{2}$.

THINK	WRITE
1 Write the expression using a division sign.	$-\frac{3}{4} \div -1\frac{1}{2}$
2 Change the divisor to an improper fraction.	$= -\frac{3}{4} \div -\frac{3}{2}$
3 Change ÷ to ×, tip the divisor (multiply and tip) and cancel common factors in numerators and denominators.	$= -\frac{\cancel{3}^{1}}{\cancel{4}^{2}} \times -\frac{\cancel{2}^{1}}{\cancel{3}^{1}}$
4 Multiply the numerators and then multiply the denominators. *Note:* negative × negative = positive	$= -\frac{1}{2} \times -\frac{1}{1}$
5 Write the answer.	$= \frac{1}{2}$

assess on

Exercise 4.3 Multiplication and division of fractions

REFLECTION
Do the rules of operation order apply to positive and negative fractions?

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7, 9, 14	Questions: 1d–l, 2d–i, 3d–i, 4d–i, 5d–i, 6d–i, 7–9, 10, 13, 14	Questions: 1g–l, 2f–i, 3f–i, 4f–i, 5d–i, 6f–i, 7–12, 14–16

Individual pathway interactivity int-4407 eBookplus

eBookplus
Digital doc
SkillSHEET
Multiplying and dividing fractions
doc-6858

FLUENCY

1 Simplify the following.

a $\frac{3}{4} \times \frac{1}{2}$ **b** $\frac{1}{8} \times \frac{1}{7}$ **c** $\frac{2}{5} \times \frac{3}{5}$

d $\frac{5}{7} \times \frac{1}{3}$ **e** $\frac{1}{2} \times \frac{5}{6}$ **f** $\frac{3}{7} \times \frac{7}{9}$

g $\frac{11}{20} \times \frac{2}{3}$ **h** $\frac{1}{3} \times \frac{3}{5}$ **i** $\frac{5}{8} \times \frac{11}{20}$

j $\frac{2}{3} \times \frac{9}{10}$ **k** $\frac{6}{7} \times \frac{14}{15}$ **l** $\frac{5}{6} \times \frac{3}{10}$

2 WE7 Simplify the following and check your answers with the aid of a calculator.

a $3\frac{1}{2} \times 1\frac{3}{5}$ **b** $1\frac{2}{10} \times 1\frac{1}{5}$ **c** $2\frac{2}{3} \times 1\frac{1}{2}$

d $3\frac{2}{4} \times 2\frac{1}{2}$ **e** $8\frac{9}{10} \times \frac{7}{10}$ **f** $5\frac{3}{4} \times 2\frac{2}{5}$

g $6 \times 2\frac{1}{6}$ **h** $1\frac{3}{5} \times \frac{5}{8}$ **i** $4\frac{3}{4} \times 2\frac{1}{2}$

3 Simplify the following.

a $\frac{1}{3} \div \frac{1}{2}$ **b** $\frac{7}{8} \div \frac{3}{2}$ **c** $\frac{4}{14} \div \frac{1}{3}$

d $\frac{2}{5} \div \frac{1}{4}$ **e** $\frac{3}{4} \div \frac{7}{8}$ **f** $\frac{5}{6} \div \frac{8}{9}$

g $\frac{12}{15} \div \frac{4}{3}$ **h** $\frac{1}{5} \div \frac{10}{12}$ **i** $\frac{3}{4} \div \frac{3}{8}$

4 WE8 Find the following.

a $1\frac{6}{10} \div 1\frac{3}{5}$ **b** $3\frac{5}{7} \div 2\frac{1}{6}$ **c** $1\frac{1}{6} \div \frac{2}{1}$

d $1\frac{5}{7} \div \frac{1}{3}$ **e** $1\frac{1}{3} \div \frac{5}{6}$ **f** $\frac{7}{9} \div 1\frac{7}{18}$

g $3\frac{1}{2} \div 1\frac{3}{5}$ **h** $10\frac{4}{5} \div 2\frac{1}{2}$ **i** $7\frac{8}{9} \div 7\frac{1}{2}$

eBookplus
Interactivity
Dividing fractions
int-2358

5 WE9 Simplify the following.

a $-\frac{1}{2} \times \frac{1}{3}$ **b** $-\frac{3}{4} \times -\frac{1}{5}$ **c** $-\frac{2}{3} \times 7$

d $\frac{1}{3} \times -\frac{3}{4}$ **e** $-\frac{3}{4} \times \frac{5}{6}$ **f** $-\frac{5}{6} \times \frac{3}{10}$

g $-\frac{8}{9} \times 1\frac{3}{4}$ **h** $-3\frac{1}{7} \times -\frac{7}{8}$ **i** $-2\frac{1}{2} \times \frac{4}{5}$

6 WE10 Evaluate the following.

a $-\frac{1}{5} \div \frac{1}{2}$ **b** $\frac{2}{3} \div -\frac{3}{4}$ **c** $-\frac{7}{4} \div -\frac{2}{1}$

d $\frac{3}{2} \div -4$ **e** $-\frac{1}{8} \div \frac{3}{4}$ **f** $-1\frac{4}{5} \div 6$

g $-2\frac{1}{4} \div -\frac{1}{2}$ **h** $2\frac{2}{3} \div -1\frac{1}{9}$ **i** $-\frac{3}{5} \div 2\frac{5}{8}$

eBookplus
Digital doc
Spreadsheet
Four operations with fractions
doc-2130

UNDERSTANDING

7 Find $\frac{3}{4}$ of 16. ('Of' has the same meaning as multiplying.)

8 An assortment of 75 lollies is to be divided evenly among 5 children.

a What fraction of the total number of lollies will each child receive?

b How many lollies will each child receive?

9 Evaluate the following.

a $-\frac{2}{3} + \frac{1}{6} \times -\frac{2}{5}$ **b** $1\frac{1}{2} \times -\frac{5}{6} \div \frac{4}{7}$

c $-\frac{7}{8} \div -1\frac{3}{4} - \frac{1}{2}$ **d** $\left(\frac{2}{5} - \frac{6}{7}\right) \times -3\frac{1}{3}$

e $\left(-1\frac{1}{2} - 3\frac{4}{5}\right) \div \frac{3}{5}$ **f** $\frac{9}{10} \times -\frac{5}{3} \div \left(1\frac{2}{7} - 2\frac{1}{2}\right)$

REASONING

10 Sam has been collecting caps from all around the world. If he has a total of 160 caps and $\frac{1}{5}$ of them are from the USA, how many non-USA caps does he have? Show your working.

11 In the staff room there is $\frac{7}{8}$ of a cake left over from a meeting. If 14 members of staff would all like a piece, what fraction of the original cake will they each receive? Show how you reached your answer.

12 Year 8's cake stall raised $120. If they plan to give $\frac{1}{4}$ to a children's charity, and $\frac{2}{3}$ to a charity for the prevention of cruelty to animals, how much will each group receive and how much is left over? Show your working.

PROBLEM SOLVING

13 The distance between Melbourne and Adelaide is 728 km. The Overland train takes $10\frac{1}{2}$ hours to travel from Melbourne to Adelaide. The driving time by car is $9\frac{3}{4}$ hours, and it takes $\frac{5}{6}$ hours to fly. If the average speed is found by dividing the total distance by the travelling time, calculate the average speed when travelling from Melbourne to Adelaide by:

a train **b** car **c** plane.

14 You come across a fantastic recipe for chocolate pudding that you want to try, but it says that the recipe feeds five people. Explain what you would do to modify the recipe so that it feeds only you.

Chocolate pudding (serves 5)

Ingredients

- $1\frac{1}{2}$ cups plain flour
- 1 teaspoon baking soda
- $\frac{1}{2}$ cup sugar
- 1 teaspoon vanilla essence
- $1\frac{1}{2}$ cups melted chocolate
- $\frac{1}{3}$ cup cocoa powder
- $\frac{1}{2}$ cup canola oil
- $\frac{1}{2}$ cup water
- 1 cup milk

15 Fractions of the type $\cfrac{1}{a + \cfrac{1}{b + \cfrac{1}{c}}}$ are called continued fractions.

a Investigate continued fractions and write an explanation.

b Write $\frac{7}{30}$ as a continued fraction. What are the values of a, b and c?

16 A wealthy merchant died and left 17 camels to be shared among his three children. The eldest was to have half of the camels, the second child one-third, and the third child one-ninth. 'It's not possible!', protested the eldest. A wise man lent his camel to the children, raising the total to 18. The eldest child then took half (nine camels); the second child took one-third (six camels); and the youngest child took one-ninth (two camels). The wise man then departed on his own camel and everyone was happy.

a Explain what the eldest child meant when they said 'It's not possible!'.

b Calculate the value of $\frac{1}{2} + \frac{1}{3} + \frac{1}{9}$ and comment on the relationship between the numerator, the denominator and the number of camels.

eBookplus
Digital doc
WorkSHEET 4.1
doc-6870

CHALLENGE 4.2

When the numerator and denominator of a certain fraction are placed together, the resulting number is a two-digit prime number. The sum of these two digits is 11 and the digit in the tens place is the higher. If the numerator is less than the denominator, find the fraction and the prime number.

4.4 Terminating and recurring decimals

Converting fractions to decimals

- Numbers that can be written as fractions are called rational numbers.
- To write a fraction as a decimal, divide the numerator by the denominator.
- Fractions expressed as decimals will be **terminating decimals**, which have a fixed number of decimal places; or **recurring decimals**, which have an infinite number of decimal places.

- Recurring decimals that have one recurring digit are written with a dot above the recurring digit. Recurring decimals with more than one digit are written with a line above the recurring digits:

$$0.333333\ldots\ldots = 0.\dot{3}$$
$$0.484848\ldots\ldots = 0.\overline{48}$$

(Sometimes two dots are used instead of a line, with the dots shown above the first and last digits of the repeating pattern; for example, $0.484848\ldots\ldots = 0.\dot{4}\dot{8}$).
- If the only factors of the denominator are powers of 2 or 5, you will be able to find an exact decimal equivalent.
- Unless you are told otherwise, round to 2 decimal places.
- Negative fractions are converted using the same method (remember that a negative fraction will result in a negative decimal).
- A calculator can also be used to convert fractions to decimals.

WORKED EXAMPLE 11

Convert the following fractions to decimals. State if the decimal is a recurring decimal or a terminating decimal.

a $\frac{1}{5}$ **b** $4\frac{5}{12}$ **c** $-\frac{5}{8}$ **d** $\frac{2}{3}$

THINK	WRITE
a 1 Write the question.	**a** $\frac{1}{5}$
2 Rewrite the question using division.	$= 1 \div 5$
3 Divide, adding zeros as required.	$5\overline{)1.0}$ (quotient: 0.2)
4 Write the answer.	$\frac{1}{5} = 0.2$ This is a terminating decimal.
b 1 Write the question.	**b** $4\frac{5}{12}$
2 Convert mixed numbers to improper fractions.	$= \frac{53}{12}$
3 Rewrite the question using division.	$= 53 \div 12$
4 Since 12 is not a power of 2 or 5, it will be a recurring decimal.	
5 Divide, adding zeros so until a pattern occurs.	$12\overline{)53.^{5}0^{2}0^{8}0^{8}0^{8}0}$ (quotient: $4.\ 4\ 1\ 6\ 6\ 6$)
6 Write the answer with a dot above the recurring number.	$4\frac{5}{12} = 4.41\dot{6}$ This is a recurring decimal.
c 1 The size of the decimal will be the same as for $\frac{5}{8}$.	**c** $\frac{5}{8}$
2 Rewrite the question using division.	$= 5 \div 8$
3 $8 = 2^3$, so an exact answer will be found. Divide, adding zeros as required.	$8\overline{)5.^{5}0^{2}0^{4}0}$ (quotient: $0.\ 6\ 2\ 5$)

4	Write the answer, remembering that the original fraction was negative.	$-\frac{5}{8} = -0.625$ This is a terminating decimal.
d 1	Rewrite the question using division.	**d** $\frac{2}{3} = 2 \div 3$
2	Since 3 is not a power of 2 or 5, it will be a recurring decimal.	$3\overline{)2.000}$ (.666)
3	Write the answer, remembering to place a dot above the recurring number.	$\frac{2}{3} = 0.\dot{6}$ This is a recurring decimal.

- When changing a decimal to a fraction, rewrite the decimal as a fraction with the same number of zeros in the denominator as there are decimal places in the question. Simplify the fraction by cancelling.
- If the decimal has a whole part, it is easier to write it in expanded form; for example, $2.365 = 2 + 0.365$.
- Negative decimals are converted using the same method (remember that a negative decimal will result in a negative fraction).

WORKED EXAMPLE 12

Convert the following decimals to fractions in simplest form.
a 0.25 **b 1.342** **c −0.8**

THINK		WRITE
a 1	Write the question.	**a** 0.25
2	Rewrite as a fraction with the same number of zeros in the denominator as there are decimal places in the question. Simplify the fraction by cancelling.	$= \frac{25^{1}}{100^{4}}$
3	Write the answer.	$= \frac{1}{4}$
b 1	Write the question.	**b** 1.342
2	Rewrite the decimal in expanded form.	$= 1 + 0.342$
3	Write as a mixed number with the same number of zeros in the denominator as there are decimal places in the question and cancel.	$= 1 + \frac{342^{171}}{1000^{500}}$
4	Write the answer.	$= 1\frac{171}{500}$
c 1	Write the question.	**c** -0.8
2	Rewrite as a fraction with the same number of zeros in the denominator as there are decimal places in the question. Simplify the fraction by cancelling.	$= -\frac{8^{4}}{10^{5}}$
3	Write the answer.	$= -\frac{4}{5}$

Exercise 4.4 Terminating and recurring decimals

assessON

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–4, 6, 10, 14	Questions: 1–6, 8, 10, 11, 13, 14	Questions: 1f–j, 2f–j, 3f–j, 4k–r, 5–13, 15

Individual pathway interactivity int-4408 eBookplus

REFLECTION
How would you decide whether it is better to write a number as a fraction or a decimal?

eBookplus
Digital docs
Spreadsheet
Converting fractions to decimals
doc-2131
Spreadsheet
Converting decimals to fractions
doc-2132

FLUENCY

1 WE11a Convert the following fractions to decimals, giving exact answers or using the correct notation for recurring decimals where appropriate.

a $\frac{4}{5}$ **b** $\frac{1}{4}$ **c** $\frac{3}{4}$ **d** $\frac{5}{12}$ **e** $\frac{9}{11}$
f $\frac{21}{25}$ **g** $\frac{7}{4}$ **h** $\frac{13}{6}$ **i** $\frac{7}{15}$ **j** $\frac{2}{3}$

2 WE11b Convert the following mixed numbers to decimal numbers, giving exact answers, or using the correct notation for recurring decimals where appropriate. Check your answers using a calculator.

a $1\frac{5}{6}$ **b** $1\frac{3}{4}$ **c** $3\frac{2}{5}$ **d** $8\frac{4}{5}$ **e** $12\frac{9}{10}$
f $6\frac{3}{4}$ **g** $5\frac{2}{3}$ **h** $11\frac{11}{15}$ **i** $6\frac{1}{2}$ **j** $4\frac{1}{3}$

3 WE11c Convert the following fractions to decimal numbers, giving exact answers, or correct to 2 decimal places where appropriate.

a $-\frac{4}{5}$ **b** $-\frac{7}{9}$ **c** $-\frac{8}{5}$ **d** $-\frac{4}{15}$ **e** $-\frac{3}{8}$
f $-2\frac{1}{4}$ **g** $-6\frac{3}{5}$ **h** $-3\frac{1}{7}$ **i** $-1\frac{5}{6}$ **j** $-5\frac{8}{9}$

4 WE12 Convert the following decimal numbers to fractions in simplest form.

a 0.4 **b** 0.8 **c** 1.2 **d** 3.2 **e** 5.6
f 0.75 **g** 1.30 **h** 7.14 **i** 4.21 **j** 10.04
k 1.333 **l** 8.05 **m** 7.312 **n** 9.940 **o** 12.045
p 84.126 **q** 73.90 **r** 0.0042

5 Of the people at a school social $\frac{3}{4}$ were boys. Write this fraction as a decimal number.

UNDERSTANDING

6 On a recent science test Katarina worked the bonus question correctly as well as everything else, and her score was $\frac{110}{100}$. What is this as a decimal value?

7 Alison sold the greatest number of chocolates in her Scouting group. She sold $\frac{5}{9}$ of all chocolates sold by the group. Write this as a decimal number, correct to 2 decimal places.

8 Alfonzo ordered a pizza to share with three friends, but he ate 0.6 of it. What fraction was left for his friends?

9 Using examples, explain the difference between rational and irrational numbers.

REASONING

10 By converting to decimal fractions, arrange the following in order from lowest to highest, showing all of your working.
$\frac{6}{10}, \frac{3}{4}, \frac{2}{3}, \frac{5}{8}, \frac{6}{7}$

11 To change $0.\overline{14}$ from a recurring decimal to a vulgar fraction, put the digits after the decimal place (14) on the numerator. The denominator has the same number of digits as the numerator (2), with all of these digits being 9's. Thus $0.\overline{14} = \frac{14}{99}$.

- **a** Write $0.\dot{7}$ as a vulgar fraction, showing all of your working.
- **b** Write $0.\overline{306}$ as a vulgar fraction, showing all of your working.
- **c** Write $2.\dot{2}$ as a proper fraction, showing all of your working.

12 **a** When you add two recurring decimals, do you get a recurring decimal? Explain.

- **b** When you add a recurring decimal and a terminating decimal, do you get a recurring decimal? Explain.
- **c** When you subtract two recurring decimals, do you get a recurring decimal? Explain.
- **d** When you subtract a recurring decimal from a terminating decimal, do you get a recurring decimal? Explain.
- **e** When you subtract a terminating decimal from a recurring decimal, do you get a recurring decimal? Explain.

PROBLEM SOLVING

13 Robin Hoot, the captain of a motley pirate crew known throughout the Seven Seas as 'The Ferry Men', kept $0.\dot{3}$ of all stolen loot for himself. The rest was split between his crew.

- **a** If there were 20 Ferry Men, what fraction of the loot did each Ferry Man receive?
- **b** Little George, one of the Ferry Men, received 15 gold doubloons. How much loot was there altogether?
- **c** If there were p pirates not including Robin Hoot, and each of them received d doubloons, how much loot was there altogether?

14 The pig ate $\frac{2}{5}$ of the chocolate cake the animals had stolen from Mrs Brown's window sill where it had been cooling. The goat and the cow ate $0.\dot{2}$ and 0.35 of the cake respectively.

- **a** If the duck and the turkey finished the rest, having equal shares, what fraction did the duck eat?
- **b** If the duck and the turkey had r grams of the cake each, how much of the cake did the pig have?

15 James had 18 litres of water shared unequally between three buckets. Then he:

- **a** Poured three quarters of the water in bucket 1 into bucket 2.
- **b** Poured half the water that was now in bucket 2 into bucket 3.
- **c** Poured a third of the water that was now in bucket 3 into bucket 1. After the pouring, all the buckets contained equal amounts of water. How much water did each bucket start with?

4.5 Addition and subtraction of decimals

Addition and subtraction of positive decimals

- When decimals are being added or subtracted, the decimal points are lined up one underneath the other.

WORKED EXAMPLE 13

Find 4.622 + 38 + 210.07 + 21.309.

THINK

Write the numbers one underneath the other with the decimal points lined up and fill the spaces with zeros. Then add as for whole numbers putting the decimal point in the answer directly under the decimal points in the question.

WRITE

$$\begin{array}{r} 4.622 \\ 38.000 \\ 210.070 \\ +\ 21_{1}1_{1}.3_{1}0_{1}9 \\ \hline 274.001 \end{array}$$

WORKED EXAMPLE 14

Find 37.6 − 12.043.

THINK

Write the numbers one under the other with the larger number on top and the decimal points lined up. Add in the required zeros and subtract using the method shown.

WRITE

$$\begin{array}{r} 37.600 \\ -\ 12.043 \\ \hline 25.557 \end{array}$$

Addition and subtraction of positive and negative decimals

- The rules for addition and subtraction of negative numbers apply.
  - To add two numbers with the same sign, add and keep the sign.
  - To add two numbers with different signs, find the difference between the signed numbers, then use the sign of the number further from 0.
  - To subtract a number, add the opposite.

WORKED EXAMPLE 15

Calculate:
a **−3.64 + −2.9** **b** **−5.7 + 2.4**
and check each answer by using estimation.

THINK

a 1 Write the question in columns with the decimal points directly beneath each other. Include the zeros.

2 They have the same sign, so add and keep the sign.

3 Check the answer by using estimation.

WRITE

a
$$\begin{array}{r} -3.64 \\ +\ -2.90 \\ \hline -6.54 \end{array}$$

$-4 + -3 = -7$

b **1** Repeat step 1 of part **a**.

b
$$\begin{array}{r} 5.7 \\ -2.4 \\ \hline 3.3 \end{array}$$

2 They have different signs, so subtract the smaller number from the larger number and use the sign of the number furthest from zero.

$-5.7 + 2.4 = -3.3$

3 Check the answer by using estimation.

$-6 + 2 = -4$

WORKED EXAMPLE 16

Calculate −5.307 − 0.62 and check the answer by using estimation.

THINK	WRITE
1 Write the question.	$-5.307 - 0.62$
2 Change to addition of the opposite.	$= -5.307 + -0.62$
3 Rewrite the question in columns with the decimal points directly beneath each other. Include the zeros.	$\begin{array}{r} -5.307 \\ +-0.620 \\ \hline \end{array}$
4 Evaluate.	-5.927
5 Check the answer by using estimation.	$-5 - 1 = -6$

assess on

Exercise 4.5 Addition and subtraction of decimals

INDIVIDUAL PATHWAYS

REFLECTION
When decimals are being added, why are the decimal points lined up one underneath the other?

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–6, 8, 11, 14	Questions: 1–8, 11, 12, 14, 15	Questions: 1g–n, 2g–l, 3, 4e–j, 5–15

Individual pathway interactivity int-4409 eBookplus

eBookplus
Digital doc
SkillSHEET
Adding and subtracting decimals
doc-6859

FLUENCY

1 WE13 Find the following.

a $8.3 + 4.6$
b $7.2 + 5.8$
c $16.45 + 3.23$
d $7.9 + 12.4$
e $13.06 + 4.2$
f $5.34 + 2.80$
g $128.09 + 4.35$
h $5.308 + 33.671 + 3.74$
i $0.93 + 4.009 + 1.3$
j $5.67 + 3 + 12.002$
k $56.830 + 2.504 + 0.1$
l $306 + 5.2 + 6.032 + 76.9$
m $25.3 + 89 + 4.087 + 7.77$
n $34.2 + 7076 + 2.056 + 1.3$

2 WE14 Find the following.

a $4.56 - 2.32$
b $19.97 - 12.65$
c $124.99 - 3.33$
d $63.872 - 9.051$
e $43.58 - 1.25$
f $1709.53 - 34.6$
g $87.25 - 34.09$
h $125.006 - 0.04$
i $24.86 - 1.963$
j $35 - 8.97$
k $42.1 - 9.072$
l $482 - 7.896$

3 MC **a** The difference between 47.09 and 21.962 is:

A 17.253 **B** 26.93 **C** 25.932 **D** 26.128 **E** 25.128

b The sum of 31.5 and 129.62 is:

A 98.12 **B** 161.12 **C** 150.12 **D** 444.62 **E** 132.77

4 Calculate the following.

a $56.3 + 52.09 + 6.7$ **b** $7.9 + 3 + 21.053$

c $908.52 - 87.04$ **d** $53.091 + 6 + 1895.2$

e $1495.945 - 2.07$ **f** $439.98 - 6$

g $7.286 + 5.4 + 2.083 + 1538.82$ **h** $12.784 - 3.9$

i $603.9 - 5.882$ **j** $3965.09 + 3.2 + 256 + 0.006$

5 WE15 Calculate the following and check each answer by using estimation.

a $-0.4 + -0.5$ **b** $0.2 + -0.9$ **c** $-0.8 + 0.23$

d $-0.021 + -0.97$ **e** $13.69 + -6.084$ **f** $-0.0037 + 0.638$

6 WE16 Calculate the following and check each answer by using estimation.

a $0.8 - 1.5$ **b** $-0.6 - 0.72$ **c** $-3 - -6.4$

d $-2.6 - 1.7$ **e** $-3.2 - -0.65$ **f** $0.084 - 0.902$

eBookplus

Digital docs
Spreadsheet
Adding decimals
doc-2133
Spreadsheet
Subtracting decimals
doc-2134

UNDERSTANDING

7 Round to the nearest whole number to find an approximate answer to the following.

a $33.2 + 4.8 - 10.5$ **b** $59.62 - 17.71 + 3.6$

c $29.5 - 15.3 + 5.7$ **d** $99.9 + 35.3 - 5.5$

8 a On a recent shopping trip, Salmah spent the following amounts: $45.23, $102.78, $0.56 and $8.65. How much did he spend altogether?

b If Salmah started with $200.00, how much did he have left after the trip?

9 Dagmar is in training for the school athletic carnival. The first time she ran the 800 m it took her 187.04 seconds. After a week of intensive training she had reduced her time to 145.67 seconds. By how much had she cut her time?

10 Kathie runs each morning before school. On Monday she ran 1.23 km, on Tuesday she ran 3.09 km, she rested on Wednesday and on both Thursday and Friday she ran 2.78 km. How many kilometres has she run for the week?

REASONING

11 Students in your school are planning to raise money for a needy cause by holding running days for different Year levels. They hope to run a total of 1946 km.

The Year 7s ran 278.2 km, the Year 8s ran 378.4 km and the Year 9s ran 526 km. How many more kilometres must the students run to reach their goal?

12 A group of bushwalkers — Alan, Brett, Chris and Dan — were staying at Port Stephens. From their camp at Hawks Nest, they wanted to walk 8.2 km to Barry Creek. At the 5-km mark, Dan developed blisters because he had incorrect footwear and was accompanied back 3.8 km by Alan. There they met another walking group who had band-aids with them. After fixing Dan's feet, Dan and Alan decided to walk back past

their Hawks Nest camp and continue on another 2.1 km until they came to Myall Lakes National Park.

a How far did they have to walk to get back to Hawks Nest after Dan fixed his blisters?

b How far apart are Barry Creek and Myall Lakes National Park?

c If Alan and Dan had decided to meet Chris and Brett at Barry Creek, instead of going to Myall Lakes National Park, how far would they have walked in total to get to Barry Creek? Justify your answer.

13 During 2011, international visitors to Australia spent $58.3 billion. During 2012, international visitors spent $47.8 billion. By how much did spending by international visitors increase or decrease from 2011 to 2012? Show your working.

PROBLEM SOLVING

14 a A friend incorrectly rounded 23.925 38 to 2 decimal places as 23.92. Where did your friend make a mistake? Explain how to correctly round to 2 decimal places.

b Your friend started work at a milk bar and was trying to round numbers in his head. The total came to $12.97 and the person gave your friend $15.00. Your friend realised that the change would come to $2.03. Because 3 is less than 5, your friend rounded it down and gave the person $2.00 back. Explain why the customer might discuss their change with your friend.

Rounding guidelines for cash transactions

Final cash amount	Round to:
1 and 2 cents	nearest 10
3 and 4 cents	nearest 5
6 and 7 cents	nearest 5
8 and 9 cents	nearest 10

Source: www.accc.gov.au

15 By writing each of these recurring decimals correct to 4 decimal places and showing your working, find:

a $0.\dot{3} + 0.\dot{5}$

b $0.\dot{3} + 0.\dot{8}$

c $0.\dot{3} + 0.\dot{7}$

d $0.\dot{4} + 0.\overline{53}$

e $0.\dot{7} - 0.\dot{6}$

f $1.1\dot{2} - 0.0\dot{3}$

g $2.\overline{534} - 0.\overline{12}$.

4.6 Multiplication and division of decimals

Multiplication of positive decimals

- To multiply decimals, ignore the decimal point and multiply as for whole numbers. Count the number of digits after the decimal point in each of the multiplying numbers, then add these together to find the number of decimal places in the answer.
- It is a good idea to estimate the answer to make sure that your answer makes sense.

WORKED EXAMPLE 17

Calculate, giving an exact answer, 125.678 × 0.23.

THINK	WRITE
1 Write the numbers with the larger one on top. Multiply, starting with the last digit and ignoring the decimal point.	$\begin{array}{r} {\scriptstyle 1\ \ 1\ 1\ 1} \\ {\scriptstyle 1\ \ 2\ 2\ 2} \\ 12\,5678 \\ \times\ 023 \\ \hline 37\ \ 7034 \\ 251_{1}3560 \\ \hline 289\ \ 0594 \end{array}$
2 Count the number of digits after the decimal point in each of the multiplying numbers and use this total as the number of decimal places in the answer. There are 3 decimal places in 125.678 and 2 in 0.23, so there will be 5 decimal places in the answer.	$125.678 \times 0.23 = 28.905\,94$

Division of positive decimals

- When dividing decimals, make sure that the divisor (the number you are dividing by) is a whole number.
- If the divisor is not a whole number, make it a whole number by either:
  - writing the question as a fraction and multiplying the numerator and denominator by an appropriate power of 10, or
  - multiplying the dividend and divisor by an appropriate power of 10.
- Once the divisor is a whole number, divide the numbers as usual, making sure the decimal point in the answer is directly above the decimal point in the question.
- Extra zeros can be placed after the decimal point in the dividend if needed.

WORKED EXAMPLE 18

Calculate:
a 54.6 ÷ 8 **b 89.356 ÷ 0.06.**
Give answers correct to 2 decimal places.

THINK	WRITE
a 1 Write the question as shown, adding zeros to one more decimal place than is required. Write the decimal point in the answer directly above the decimal point in the question and divide as for short division.	a $\begin{array}{r} 6.\ 8\ 2\ 5 \\ 8\overline{)54.^{6}6^{2}0^{4}0} \end{array}$
2 Write the question and answer, rounded to the required number of decimal places.	$54.6 \div 8 \approx 6.83$ (2 decimal places)

b

1	Write the question.	**b** $89.356 \div 0.06$
2	Multiply both parts by an appropriate multiple of 10 so that the divisor is a whole number (in this case, 100).	$= (89.356 \times 100) \div (0.06 \times 100)$ $= 8935.6 \div 6$
3	Divide, adding zeros to one more decimal place than required. Write the decimal point in the answer directly above the decimal point in the question and divide as for short division.	$1\ 4\ 8\ 9.\ 2\ 6\ 6$ $6\overline{)8^{2}9^{5}3^{5}5.^{1}6^{4}0^{4}0}$
4	Write the question and answer, rounded to the required number of decimal places.	$89.356 \div 0.06 \approx 1489.27$ (2 decimal places)

Multiplication and division of positive and negative decimals

- When positive and negative decimals are being multiplied and divided, the rules for multiplying and dividing positive and numbers apply.
  - If both numbers are positive or negative, the result is positive.
  - If one number is positive and the other is negative, the result is negative.

WORKED EXAMPLE 19

Simplify -3.8×0.05.

THINK	WRITE
1 Multiply as for whole numbers.	$^{4}3\ 8$ $\times\ 5$ 190
2 Count the number of decimal places in the question and insert the decimal point. The signs are different, so insert a negative sign in the answer.	$-3.8 \times 0.05 = -0.190$ $= -0.19$

WORKED EXAMPLE 20

Find the quotient of $-0.015 \div -0.4$, giving an exact answer.

THINK	WRITE
1 Write the question.	$-0.015 \div -0.4$ $= -(0.015 \times 10) \div -(0.4 \times 10)$
2 Multiply both parts by 10 to produce a whole number divisor.	$= -0.15 \div -4$
3 Divide until an exact answer is achieved or until a recurring pattern is evident.	$0.03\ 7\ 5$ $4\overline{)0.15^{3}0^{2}0}$
4 The signs are the same, so the answer is positive.	$-0.015 \div -0.4 = 0.0375$

Exercise 4.6 Multiplication and division of decimals

assessON

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–5, 6, 8, 13, 16	Questions: 1–7, 11, 13, 15, 16	Questions: 1g–o, 2, 3g–l, 4g–l, 5g–l, 6–16

Individual pathway interactivity int-4410 eBookplus

REFLECTION
Why is it a good idea to make an estimate of the answer when you are multiplying decimals?

FLUENCY

eBookplus
Digital doc
SkillSHEET
Multiplying and dividing decimals
doc-6860

1 WE17 Calculate the following, giving an exact answer.

a 6.2×0.8	**b** 7.9×1.2	**c** 65.7×3.2
d 109.5×5.6	**e** 5.09×0.4	**f** 32.76×2.4
g 123.97×4.7	**h** 576.98×2	**i** 3.4×642.1
j 0.6×67.9	**k** 23.4×6.7	**l** 0.006×43.6
m 52.003×12	**n** 22.97×0.015	**o** 13.42×0.011

2 WE18a Calculate the following. Give answers correct to 2 decimal places.

a $43.2 \div 7$	**b** $523.9 \div 4$	**c** $6321.09 \div 8$
d $286.634 \div 3$	**e** $76.96 \div 12$	**f** $27.8403 \div 11$

3 WE18b Calculate the following. Give answers correct to 2 decimal places, where appropriate.

a $53.3 \div 0.6$	**b** $960.43 \div 0.5$	**c** $21.42 \div 0.004$
d $3219.09 \div 0.006$	**e** $478.94 \div 0.016$	**f** $76.327 \div 0.000\,08$
g $25.865 \div 0.004$	**h** $26.976 \div 0.0003$	**i** $0.0673 \div 0.0005$
j $12.000\,53 \div 0.007$	**k** $35.064 \div 0.005$	**l** $0.059 \div 0.009$

4 WE19 Simplify each of the following.

a 0.3×-0.2	**b** -0.8×0.9	**c** -0.4×-0.06
d $(-0.6)^2$	**e** $(-0.3)^2$	**f** -0.004×40
g 4000×-0.5	**h** -0.02×-0.4	**i** $(-0.05)^2$
j -4.9×0.06	**k** $(0.2)^2 \times -40$	**l** $(1.2)^2 \times (0.3)^2$

eBookplus
Digital docs
Spreadsheet
Multiplying decimals
doc-2135
Spreadsheet
Dividing decimals
doc-2136

5 WE20 Find the quotient of each of the following, giving an exact answer.

a $-8.4 \div 0.2$	**b** $0.15 \div -0.5$	**c** $-0.0405 \div -0.3$
d $-15 \div 0.5$	**e** $0.049 \div -0.07$	**f** $-3.2 \div -0.008$
g $-0.0036 \div 0.06$	**h** $270 \div -0.03$	**i** $-0.04 \div -800$
j $0.8 \div -0.16$	**k** $(1.2)^2 \div 0.04$	**l** $(1.5)^2 \div 0.05$

UNDERSTANDING

6 Evaluate the following, giving answers correct to 1 decimal place.

a $4.6 \times 2.1 + 1.2 \times 3.5$	**b** $5.9 \times 1.8 - 2.4 \times 3.8$
c $6.2 + 4.5 \div 0.5 - 7.6$	**d** $11.4 - 7.6 \times 1.5 + 2$

7 MC **a** $\frac{1}{2}(3.6 + 1.4 \times 7.5)$ is equal to:

A 18.75 **B** 14.1 **C** 9.375 **D** 7.05 **E** 28.2

b Rounded to 2 decimal places, $\frac{3}{4}(10.5 - 5.8 \div 4 \times 1.2)$ is equal to:

A 1.06 **B** 6.57 **C** 0.73 **D** 11.68 **E** 2.19

8 Round each of the following to the nearest whole number to find an estimate.

a $3.5 \times 24.9 + 33.2$ **b** $4.8 \times 19.6 - 10.4$

c $15.6 + 50.1 \times 9.5 - 15.4$ **d** $49.8 - 20.3 \div 4.7$

9 A group of 21 Year 8 students were going on an excursion to the planetarium. If the total cost is $111.30, how much would each student have to pay?

10 Simplify $\frac{0.16}{0.28} \times \frac{56}{3.2}$.

11 Your car's tank holds 55 litres of fuel. The car consumes 7.6 L of fuel for each 100 km travelled. You make a journey from Melbourne to Traralgon, a distance of 145 km. How much fuel is left in the tank after the journey?

12 Find the decimal exactly halfway between 2.01 and 2.02.

REASONING

13 A shoebox with a mass of 150 g measures 30.1 cm by 16.5 cm along the bottom with a height of 10.3 cm.

a The volume of the shoebox is found by multiplying its three linear dimensions together. What is its volume?

b Density is defined as mass divided by volume. What is the density of the shoebox in g/cm^3?

14 A square of grass has a side length of 3.4 metres. There is a path 1.2 metres wide around the outside of the square of grass.

a Find the area of the grassed area.

b Find the area of the path, explaining your method.

PROBLEM SOLVING

15 The prestigious Collins Street Lawyers, Messrs Waffle, Pockett and Runne, have been retained to secure the patent on the Global Coffee Constant, or GCC as it is better known.

If t represents all the tea in China, c represents all the coffee in Brazil and g represents the total number of tea and coffee drinkers globally, find a simple expression for the GCC in terms of t, c and g, given the following information: the GCC equals 0.4 of all the tea in China multiplied by $\frac{5}{9}$ of all the coffee in Brazil divided by $0.\dot{5}$ of the number of tea and coffee drinkers globally.

16 Speed equals distance travelled divided by the time taken.

a If Septimus Squirrel travels 27.9 m in 8.4 seconds and Fortescue Fieldmouse travels 18.7 m in 6.4 seconds, which of them travels at the greater speed?

b Travelling at 5 metres per second, how long will it take Genevieve Goose to travel round the equator, assuming the circumference of the Earth at the equator is approximately 40 075 kilometres?

4.7 Estimation

- Sometimes an **estimate** of the answer is all that is required (an estimate is answer close to the actual answer, but found using easier numbers).
- Some estimation methods are detailed in this section.

Clustering around a common value

- If the calculation involves similar values, the values can be rounded to the same number.

WORKED EXAMPLE 21

Marilyn and Kim disagree about the answer to the following calculation: 7.3 + 7.1 + 6.9 + 6.8 + 7.2 + 7.3 + 7.4 + 6.6. Marilyn says the answer is 56.6, but Kim thinks it is 46.6. Obtain an estimate for the calculation and determine who is correct.

THINK	WRITE
1 Carefully analyse the values and devise a method to estimate the total.	Each of the values can be approximated to 7 and there are 8 values.
2 Perform the calculation using the rounded numbers.	$7 \times 8 = 56$
3 Answer the question.	Marilyn is correct, because the approximate value is very close to 56.6.

Rounding, rounding up, rounding down

- When we are **rounding** to a given place value:
  - if the next lower place value digit is less than 5, leave the place value digit as it is, and add zeros to all lower place values, if necessary
  - if the next lower place value digit is 5 or greater, increase the given place value digit by 1 and add zeros to all lower place values, if necessary.
- When rounding 25 354:
  - to the nearest thousand, the result is 25 000
  - to the nearest hundred, the result is 25 400.
- When **rounding up**, the digit in the desired place value is increased by 1 regardless of the digits in the lower place positions (as long as they are not all zeros). Zeros are added to the lower place positions to retain the place value.
- When rounding up 3100:
  - to the nearest thousand, the result is 4000
  - to the nearest hundred, the result is 3100.
- When **rounding down**, all digits following the desired place value are replaced by zeros, leaving the given place value unchanged.
- When rounding down 635 to the nearest ten, the result is 630.

WORKED EXAMPLE 22

Consider the number 39 461 and perform the following.
a Round to the nearest thousand.
b Round up to the nearest hundred.
c Round down to the nearest ten.

THINK	WRITE
a 1 Consider the digit in the thousands place position and the digit in the next lower place position.	a The digit 9 lies in the thousands position. The digit 4, which is less than 5, lies in the hundreds position.

	2 Write the answer, adding the required number of zeros.	The number 39 461 rounded to the nearest thousand is 39 000.
b	1 Consider the digit in the hundreds place position and the digit in the next lower place position.	b The digit 4 lies in the hundreds position. The digit 6 lies in the tens position. When rounding up to the nearest hundred, the 4 will increase to 5.
	2 Write the answer, adding the required number of zeros.	The number 39 461 rounded up to the nearest hundred is 39 500.
c	1 Consider the digit in the tens place position and the digit in the next lower place position.	c The digit 6 lies in the tens position. The digit 1 lies in the units position. When rounding down to the nearest ten, the 1 will be converted to 0.
	2 Write the answer, adding the required number of zeros.	The number 39 461 rounded down to the nearest ten is 39 460.

Rounding to the first digit

- When estimating answers to calculations, sometimes it is simplest to round all numbers in the calculation to the first digit and then perform the operation.

WORKED EXAMPLE 23

Provide an estimate to the following calculations by first rounding each number to its first digit. Check your estimate with a calculator. Comment on the accuracy of your estimate.

a $394 + 76 - 121$ b $\dfrac{692 \times 32}{19 \times 87}$

THINK	WRITE
a 1 Round each of the numbers to the first digit.	a Rounded to the first digit, 394 becomes 400, 76 becomes 80 and 121 becomes 100.
2 Perform the calculation using the rounded numbers.	$394 + 76 - 121 \approx 400 + 80 - 100$ ≈ 380
3 Check using a calculator. Comment on how the rounded result compares with the actual answer.	Using a calculator, the result is 349. The estimate compares well to the actual (calculator) value.
b 1 Round each of the numbers to the first digit.	b Rounded to the first digit, 692 becomes 700, 32 becomes 30, 19 becomes 20 and 87 becomes 90.
2 Perform the calculation using the rounded numbers.	$\dfrac{692 \times 32}{19 \times 87} \approx \dfrac{700^{35} \times 30^{1}}{20_{1} \times 90_{3}}$ $\approx \frac{35}{3}$ ≈ 12
3 Check using a calculator. Comment on how the rounded result compares with the actual answer.	Using a calculator, the result is 13.4 (rounded to 1 decimal place). The estimate is very close to the actual (calculator) value.

Rounding the dividend to a multiple of the divisor

- To make division easier, the dividend can be rounded to a multiple of the divisor. For example, in 20 532 ÷ 7, the 20 532 (the dividend) could be rounded to 21 000. Because we know that 21 is a multiple of 7, we could see, through mental approximation, that the answer is close to 3000 (the exact answer is 2933).

WORKED EXAMPLE 24

Provide estimates for the calculation $\frac{537}{40}$ by:

a rounding the dividend up to the nearest hundred
b rounding the dividend to the nearest ten
c rounding the dividend to a multiple of the divisor.

THINK

a 1 Round the dividend up to the nearest hundred.

2 Perform the division. Write the estimation.

b 1 Round the dividend to the nearest ten.

2 Perform the division. Write the estimation.

c 1 Round the dividend to a multiple of the divisor.

2 Perform the division. Write the estimation.

WRITE

a 537 rounded up to the nearest hundred is 600.

$$\frac{537}{40} \approx \frac{\cancel{600}^{15}}{\cancel{40}_{1}}$$
$$\approx 15$$

b 537 rounded up to the nearest ten is 540.

$$\frac{537}{40} \approx \frac{\cancel{540}^{27}}{\cancel{40}^{2}}$$
$$\approx 13.5$$

c 52 is a multiple of 4.

$$\frac{537}{40} \approx \frac{\cancel{520}^{13}}{\cancel{40}_{1}}$$
$$\approx 13$$

- Different methods will give slightly different estimates.

WORKED EXAMPLE 25

The exact answer to $\frac{132 \times 77}{55}$ has the digits 1848. Use any estimation technique to locate the position of the decimal point.

THINK

1 Round each of the numbers to the first digit.

2 Perform the calculation using the rounded numbers and write the estimate, ignoring the decimal.

3 Use the estimate obtained to locate the position of the decimal point. Write the correct answer.

WRITE

Rounded to the first digit, 132 becomes 100, 77 becomes 80 and 55 becomes 60.

$$\frac{132 \times 77}{55} \approx \frac{100 \times \cancel{80}^{4}}{\cancel{60}_{3}}$$
$$\approx \frac{400}{3}$$
$$\approx 133$$

The estimate gives an answer between 100 and 200. This indicates that the decimal point should be between the last two digits. The correct answer is 184.8.

Exercise 4.7 Estimation

INDIVIDUAL PATHWAYS

REFLECTION
Can you think of some examples of when an estimate rather than the exact answer is required?

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7, 10, 12	Questions: 1–14	Questions: 1, 2, 4, 5i–p, 6–15

Individual pathway interactivity int-4411

eBookplus

Digital docs
SkillSHEET Rounding to the first (leading) digit doc-6861
SkillSHEET Operations with fractions doc-6862
SkillSHEET Operations with decimals doc-6863

FLUENCY

1 WE21 Marilyn and Kim disagree about the answer to the following calculation: 8.6 + 9.2 + 8.7 + 8.8 + 8.9 + 9.3 + 9.4 + 8.6. Marilyn says the answer is 81.5, but Kim thinks it is 71.5. Obtain an estimate for the calculation and determine who is correct.

2 WE22 For each of the following numbers:

i round to the first digit
ii round up to the first digit
iii round down to the first digit.

a 239 b 4522 c 21
d 53 624 e 592 f 1044

3 Round each of the numbers in question **2** down to the nearest ten.

4 Round each of the numbers in question **2** up to the nearest hundred.

5 WE23 Find an estimate for each of the following by rounding each number to the first digit.

a $78 \div 21$ b $297 + 36$ c $587 - 78$
d $235 + 67 + 903$ e $1256 - 678$ f 789×34
g 56×891 h $1108 \div 53$ i $345 + 8906 - 23 + 427$
j $907 \div 88$ k $326 \times 89 \times 4$ l $2378 \div 109$
m $(426 + 1076) \times 21$ n $7 \times 211 - 832$ o $977 \div 10 \times 37$
p $(12\,384 - 6910) \times (214 + 67)$

6 WE24 Provide estimates for each of the following by first rounding the dividend to a multiple of the divisor.

a $35\,249 \div 9$ b $2396 \div 5$ c $526\,352 \div 7$
d $145\,923 \div 12$ e $92\,487 \div 11$ f $5249 \div 13$

7 WE25 Use any of the estimation techniques to locate the position of the decimal point in each of the following calculations. The correct digits for each one are shown in brackets.

a $\dfrac{369 \times 16}{288}$ (205)
b $\dfrac{42\,049}{14 \times 20}$ (150 175)
c $\dfrac{99 \times 270}{1320}$ (2025)
d $\dfrac{285 \times 36}{16 \times 125}$ (513)
e $\dfrac{256 \times 680}{32 \times 100}$ (544)
f $\dfrac{7290 \times 84}{27 \times 350}$ (648)

8 If 127 people came to a school social and each paid $5 admission, find an estimate for the amount of money collected.

UNDERSTANDING

9 Estimate the whole numbers between which each of the following will lie.

a $\sqrt{20}$ **b** $\sqrt{120}$ **c** $\sqrt{180}$ **d** $\sqrt{240}$

10 Complete the table below with the rounded question, the estimated answer and the exact answer. Use rounding to the first digit. The first one has been completed.

	Question	Rounded question	Estimated answer	Exact answer
a	789×56	800×60	48 000	44 184
b	$124 \div 5$			
c	$678 + 98 + 46$			
d	235×209			
e	$7863 - 908$			
f	63×726			
g	$39\,654 \div 227$			
h	$1809 - 786 + 467$			
i	$21 \times 78 \times 234$			
j	$942 \div 89$			
k	$\begin{array}{r} 492 \times 94 \\ \hline 38 \times 49 \end{array}$			
l	$\begin{array}{r} 54\,296 \\ \hline 97 \times 184 \end{array}$			

REASONING

11 Find an approximate answer to each of the worded problems below by rounding to the first digit. Remember to write each answer in a sentence.

a A company predicted that it would sell 13 cars in a month at $28 999 each. About how much money would they take in sales?

b A tap was leaking 8 mL of water each hour. Approximately how many millilitres of water would be lost if the tap was allowed to leak for 78 hours?

c The Year 8 cake stall sold 176 pieces of cake for 95 cents each. How much money did they make?

d Steven swam 124 laps of a 50 m pool and, on average, each lap took him 47 seconds. If he swam non-stop, for approximately how many seconds was he swimming?

e An audience of 11 784 people attended a recent Kylie concert at Rod Laver Arena and paid $89 each for their tickets. How much money was taken at the door?

f A shop sold 4289 articles at $4.20 each. How much money was paid altogether?

g On Clean Up Australia Day, 19 863 people volunteered to help. If they each picked up 196 pieces of rubbish, how many pieces of litter were collected altogether?

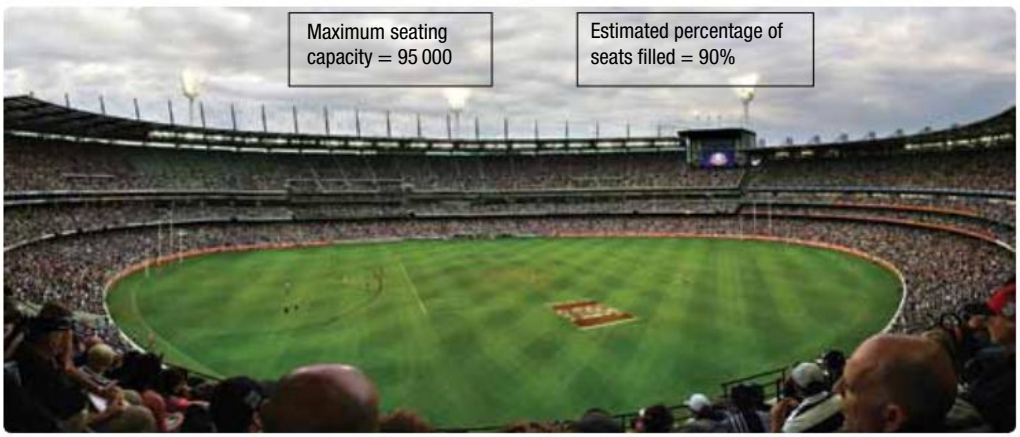
PROBLEM SOLVING

12 Sports commentators often estimate the crowd size at sports events. They estimate the percentage of the seats that are occupied, and then use the venue's seating capacity to estimate the crowd size.

a The photograph below shows the crowd at an AFL match between Collingwood and Essendon at the Melbourne Cricket Ground. Use the information given with the photograph to calculate the estimated number of people in attendance.

b Choose two well-known sports venues and research the maximum seating capacity for each.

c If you estimate that three out of every four seats are occupied at each of the venues chosen in part **b** for particular sports events, how many people are in attendance at each venue?



13 For each of the following multiplications, calculate the product when only:

i the first number is rounded to the nearest 10

ii the second number is rounded to the nearest 10.

a 18×45 **b** 18×46 **c** 18×47 **d** 18×48 **e** 18×49

14 Describe any patterns that you notice in your answers to question **13**.

15 Explore the rounding error patterns that occur in calculating the following multiplications when only:

i the first number is rounded to the nearest 10

ii the second number is rounded to the nearest 10.

a 15×44 **b** 16×44 **c** 17×44

eBookplus
Digital doc
WorkSHEET 4.2
doc-11438

eBookplus ONLINE ONLY

4.8 Review

www.jacplus.com.au

The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:

- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

Review questions

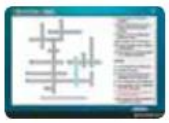
Download the Review questions document from the links found in your eBookPLUS.

eBookplus

Interactivities

Word search
int-2752

Crossword
int-2753



Sudoku
int-3184

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

common value	integer	real number
denominator	irrational number	reciprocal
directed numbers	mixed numbers	recurring decimals
dividend	non-terminating decimal	rounding
divisor	numerator	rounding down
equivalent fraction	proper fraction	rounding up
estimate	quotient	surd
improper fraction	rational number	terminating decimals

Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

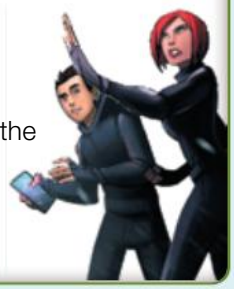
assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

www.assesson.com.au

Link to SpyClass, an exciting online game combining a comic book–style story with problem-based learning in an immersive environment.

Join Jesse, Toby and Dan and help them to tackle some of the world's most dangerous criminals by using the knowledge you've gained through your study of mathematics.

www.spyclass.com.au



INVESTIGATION

RICH TASK

A growing nation

Details about a population are collected in a census. Australia's next census will take place in August 2016, on a day known as 'census day'. The information provided by the population is collected and analysed by the Australian Bureau of Statistics over a period of 2 years. After this, the information is released to the public.

The following table displays some selected characteristics for Australia based on the information collected in the 2011 census.

Characteristics	Total persons
Males in Australia	10 634 013
Females in Australia	10 873 704
People aged 15 years and over	17 363 694
People aged 65 years and over	3 012 289
People born in Australia	15 017 847
People who speak English only	16 509 291

1. What was the total population of Australia on census day 2011?
2. According to the values in the table, how many people were not born in Australia?
3. Express the male population as a proportion of the entire population. Give your answer as a decimal. (Divide the male population by the total population.) Repeat this for the female population.
4. How many people were aged 14 years and younger?

The table below shows the same characteristics from the first table, but it relates to Victoria only.

Characteristics	Total persons
Males in Victoria	2 632 619
Females in Victoria	2 721 423
People aged 15 years and over	4 355 249
People aged 65 years and over	761 580
People born in Australia	3 670 933
People who speak English only	3 874 861

States of Australia

5 What was the population of Victoria on census day 2011?

6 Express both the male and female population of Victoria as a proportion of the total population of the state.

7 How does the proportion of males and females in Victoria compare with the proportion of males and females in the entire Australian population?

8 On a separate page, compare the other characteristics in the table with those for the entire population.

Australia's population has increased since Federation in 1901. The table below shows some of the information collected in the 1901 census.

Characteristics	Total persons
Males in Australia	1 977 928
Females in Australia	1 795 873
People living in Victoria	1 201 070
People born in Australia	2 908 303

9 How much did Australia's population grow in the 105 years since 1901?

10 Compare the proportion of people born in Australia at the 1901 census with the 2011 census.

The Australian Bureau of Statistics (ABS) website (www.abs.gov.au) can be used to investigate other characteristics from the 2011 census.

11 Use the ABS website to investigate another characteristic from the 2011 census. State your findings and include all working to justify your conclusions.

CODE PUZZLE

Funny definitions

Answer the division questions to solve the puzzle.

Y = 1.2 ÷ 6 =	L = 0.87 ÷ 0.6 =
W = 17.6 ÷ 8 =	J = 1.16 ÷ 0.2 =
V = 6.8 ÷ 4 =	I = 0.648 ÷ 0.8 =
U = 15.9 ÷ 3 =	H = 0.92 ÷ 0.4 =
T = 32.4 ÷ 9 =	G = 4.2 ÷ 1.2 =
S = 7.49 ÷ 7 =	F = 0.462 ÷ 0.11 =
R = 9.8 ÷ 2 =	E = 26.1 ÷ 10 =
P = 13.75 ÷ 5 =	D = 56.1 ÷ 30 =
O = 0.35 ÷ 0.7 =	C = 0.304 ÷ 0.08 =
N = 2.79 ÷ 0.9 =	B = 1.4 ÷ 0.25 =
M = 1.32 ÷ 0.3 =	A = 0.564 ÷ 0.12 =

Boycott	3.8	4.9	4.7	1.87	1.45	2.61	4.2	0.5	4.9	4.4	4.7	1.45	2.61	5.6	4.7	5.6	0.81	2.61	1.07
Each	4.4	0.81	3.1	0.5	4.9	0.81	4.9	4.9	0.81	3.6	4.7	3.6	0.81	0.5	3.1				
Flattery	1.45	0.81	1.7	0.81	3.1	3.5	0.81	3.1	4.7	3.1	4.7	2.75	4.7	4.9	3.6	4.4	2.61	3.1	3.6
Inkling	4.7	5.6	4.7	5.6	0.2	2.75	2.61	3.1											
Kidney	5.8	0.5	0.81	3.1	3.6	0.81	3.1	4.7	3.8	2.3	0.81	1.45	1.87	1.07	1.45	2.61	3.5		
Tortoise	2.2	2.3	4.7	3.6	0.5	5.3	4.9	3.6	2.61	4.7	3.8	2.3	2.61	4.9	1.87	0.81	1.87		

eBookplus Activities

4.2 Addition and subtraction of fractions

Digital docs
- SkillSHEET (doc-6854) Order of operations 1
- SkillSHEET (doc-6855) Factors
- SkillSHEET (doc-6856) Multiples
- SkillSHEET (doc-6857) Adding and subtracting fractions 1

Interactivities
- Adding and subtracting fractions (int-2357)
- IP interactivity 4.2 (int-4406) Addition and subtraction of fractions

4.3 Multiplication and division of fractions

Digital docs
- SkillSHEET (doc-6858) Multiplying and dividing fractions
- Spreadsheet (doc-2130) Four operations with fractions
- WorkSHEET 4.1 (doc-6870)

Interactivities
- Dividing fractions (int-2358)
- IP interactivity 4.3 (int-4407) Multiplication and division of fractions

4.4 Terminating and recurring decimals

Digital docs
- Spreadsheet (doc-2131) Converting fractions to decimals
- Spreadsheet (doc-2132) Converting decimals to fractions

Interactivity
- IP interactivity 4.4 (int-4408) Terminating and recurring decimals

4.5 Addition and subtraction of decimals

Digital docs
- SkillSHEET (doc-6859) Adding and subtracting decimals
- Spreadsheet (doc-2133) Adding decimals
- Spreadsheet (doc-2134) Subtracting decimals

Interactivity
- IP interactivity 4.5 (int-4409) Addition and subtraction of decimals

4.6 Multiplication and division of decimals

Digital docs
- SkillSHEET (doc-6860) Multiplying and dividing decimals
- Spreadsheet (doc-2135) Multiplying decimals
- Spreadsheet (doc-2136) Dividing decimals

Interactivity
- IP interactivity 4.6 (int-4410) Multiplication and division of decimals

4.7 Estimation

Digital docs
- SkillSHEET (doc-6861) Rounding to the first (leading) digit
- SkillSHEET (doc-6862) Operations with fractions
- SkillSHEET (doc-6863) Operations with decimals
- WorkSHEET 4.2 (doc-11438)

Interactivity
- IP interactivity 4.7 (int-4411) Estimation

4.8 Review

Interactivities
- Word search (int-2752)
- Crossword (int-2753)
- Sudoku (int-3184)

Digital docs
- Topic summary (doc-10756)
- Concept map (doc-10769)
- Topic review (Word doc-14930, PDF doc-14931)

To access eBookPLUS activities, log on to www.jacplus.com.au

Answers

TOPIC 4 Real numbers

4.2 Addition and subtraction of fractions

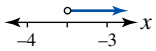
1 **a** $\frac{9}{17}$ **b** $\frac{5}{27}$ **c** $\frac{12}{17}$ **d** $\frac{4}{5}$

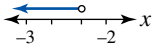
2 **a** $1\frac{2}{5}$ **b** $1\frac{1}{4}$ **c** 2 **d** $\frac{23}{50}$

3 **a** $\frac{13}{20}$ **b** $1\frac{3}{8}$ **c** $\frac{1}{5}$ **d** $\frac{18}{25}$
e $1\frac{7}{12}$ **f** $\frac{43}{70}$ **g** $\frac{37}{42}$ **h** $2\frac{13}{30}$

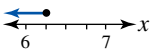
4 **a** $1\frac{3}{5}$ **b** $3\frac{2}{9}$ **c** $4\frac{3}{5}$ **d** $4\frac{2}{3}$ **e** $2\frac{1}{2}$
f 7 **g** $9\frac{1}{2}$ **h** $21\frac{8}{45}$ **i** $2\frac{3}{4}$ **j** $2\frac{7}{15}$

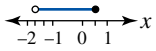
5 **a** $3\frac{3}{10}$ **b** $2\frac{2}{3}$ **c** $4\frac{17}{20}$ **d** $1\frac{21}{40}$

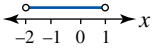
6 **a** 

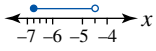
b 

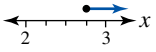
c

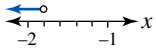
d 

e 

f 

g 

h 

i 

j

7 **a** $x > -1\frac{1}{2}$ **b** $x \leq 6\frac{3}{4}$
c $-7\frac{1}{2} < x \leq -6\frac{1}{3}$ **d** $x < \frac{2}{3}$
e $x \geq -4\frac{2}{5}$ **f** $10\frac{3}{4} \leq x < 11\frac{3}{4}$
g $x > -8.7$ **h** $-2\frac{1}{2} < x < \frac{2}{3}$

8 **a** $-4\frac{1}{2}, -3, 0, 5.8, 6$ **b** $-4.2, -1, -\frac{3}{4}, 1\frac{1}{2}, 3\frac{2}{3}$
c $-8.6, -6.8, -\frac{2}{3}, 0, \frac{1}{4}$

9 **a** $-\frac{2}{5}$ **b** $-\frac{8}{8}$ or -1 **c** $-\frac{1}{4}$ **d** $-\frac{5}{6}$
e $\frac{1}{8}$ **f** $-\frac{5}{6}$ **g** $-\frac{1}{10}$ **h** $\frac{5}{12}$
i $-4\frac{1}{4}$ **j** $-\frac{14}{15}$ **k** $-5\frac{5}{6}$ **l** $4\frac{17}{20}$

10 **a** $-\frac{1}{4}$ **b** $-\frac{5}{6}$ **c** $-\frac{11}{15}$ **d** $-\frac{7}{20}$
e $-\frac{3}{4}$ **f** $-2\frac{4}{15}$

11 All of it

12 $1\frac{7}{9}$ bottles were left over.

13 He has $\frac{4}{15}$ of his pay left.

14 $\frac{5}{12}$ of the class get a lift.

15 $\frac{11}{8}$ or $1\frac{3}{8}$ of the cake was left.

16 Check with your teacher.

17 1 cup + $\frac{1}{2}$ cup + $\frac{1}{4}$ cup + two $\frac{1}{3}$ cups

18 $\frac{1}{2} + \frac{3}{4} + 1\frac{1}{4} + \frac{2}{3} + \frac{2}{25} = 3\frac{37}{150}$
The tray holds 3 cups of ingredients, which means the mixture will not fit.

19 Answers will vary.

Challenge 4.1

A = 12, B = 1, C = 3 and D = 4

4.3 Multiplication and division of fractions

1 **a** $\frac{3}{8}$ **b** $\frac{1}{56}$ **c** $\frac{6}{25}$ **d** $\frac{5}{21}$
e $\frac{5}{12}$ **f** $\frac{1}{3}$ **g** $\frac{11}{30}$ **h** $\frac{1}{5}$
i $\frac{11}{32}$ **j** $\frac{3}{5}$ **k** $\frac{4}{5}$ **l** $\frac{1}{4}$

2 **a** $5\frac{3}{5}$ **b** $1\frac{11}{25}$ **c** 4 **d** $8\frac{3}{4}$ **e** $6\frac{23}{100}$
f $13\frac{4}{5}$ **g** 13 **h** 1 **i** $11\frac{7}{8}$

3 **a** $\frac{2}{3}$ **b** $\frac{7}{12}$ **c** $\frac{6}{7}$ **d** $1\frac{3}{5}$ **e** $\frac{6}{7}$
f $\frac{15}{16}$ **g** $\frac{3}{5}$ **h** $\frac{6}{25}$ **i** 2

4 **a** 1 **b** $1\frac{5}{7}$ **c** $\frac{7}{12}$ **d** $5\frac{1}{7}$ **e** $1\frac{3}{5}$
f $\frac{14}{25}$ **g** $2\frac{3}{16}$ **h** $4\frac{8}{25}$ **i** $1\frac{7}{135}$

5 **a** $-\frac{1}{6}$ **b** $\frac{3}{20}$ **c** $-4\frac{2}{3}$ **d** $-\frac{1}{4}$ **e** $-\frac{5}{8}$
f $-\frac{1}{4}$ **g** $-1\frac{5}{9}$ **h** $2\frac{3}{4}$ **i** -2

6 **a** $-\frac{2}{5}$ **b** $-\frac{8}{9}$ **c** $\frac{7}{8}$ **d** $-\frac{3}{8}$ **e** $-\frac{1}{6}$
f $-\frac{3}{10}$ **g** $4\frac{1}{2}$ **h** $-2\frac{2}{5}$ **i** $-\frac{8}{35}$

7 12

8 **a** $\frac{1}{5}$ **b** 15 lollies each

9 **a** $-\frac{11}{15}$ **b** $-2\frac{3}{16}$ **c** 0 **d** $1\frac{11}{21}$
e $-8\frac{5}{6}$ **f** $1\frac{4}{17}$

10 128 non-American caps

11 $\frac{1}{16}$ of the cake each

12 $30 to the children's charity, $80 to the charity for the prevention of cruelty to animals, $10 left over

13 **a** $69\frac{1}{3}$ km/h
b $74\frac{2}{3}$ km/h
c $873\frac{3}{5}$ km/h

14 To modify the recipe so that it feeds only you, divide all the ingredient amounts by 5. This will create a portion that is $\frac{1}{5}$ of the original size.

15 **a** Answers will vary.
b $a = 4, b = 3, c = 2$

16 **a** It is not possible to evenly divide 17 by $\frac{1}{2}$, $\frac{1}{3}$ or $\frac{1}{9}$. The eldest child meant that it was not possible to have a portion of a camel.
b $\frac{17}{18}$. Half of the 18 camels = 9; one-third of the 18 camels = 6; one-ninth of the 18 camels = 2; 9 + 6 + 2 = 17. The numerator, 17, is the number of camels that the merchant had; the denominator, 18, is the total number of camels after the wise man lent one camel.

Challenge 4.2

Fraction is $\frac{3}{8}$; prime number is 83.

4.4 Terminating and recurring decimals

1 **a** 0.8 **b** 0.25 **c** 0.75 **d** $0.41\dot{6}$ **e** $0.\overline{81}$
f 0.84 **g** 1.75 **h** $2.1\dot{6}$ **i** $0.4\dot{6}$ **j** $0.\dot{6}$

2 **a** $1.8\dot{3}$ **b** 1.75 **c** 3.4 **d** 8.8 **e** 12.9
f 6.75 **g** 5.6 **h** $11.7\dot{3}$ **i** 6.5 **j** $4.\dot{3}$

3 **a** −0.8 **b** −0.78 **c** −1.6 **d** −0.27 **e** −0.375
f −2.25 **g** −6.6 **h** −3.14 **i** $-1.8\dot{3}$ **j** $-5.8\dot{9}$

4 **a** $\frac{2}{5}$ **b** $\frac{4}{5}$ **c** $1\frac{1}{5}$ **d** $3\frac{1}{5}$ **e** $5\frac{3}{5}$
f $\frac{3}{4}$ **g** $1\frac{3}{10}$ **h** $7\frac{7}{50}$ **i** $4\frac{21}{100}$ **j** $10\frac{1}{25}$
k $1\frac{333}{1000}$ **l** $8\frac{1}{20}$ **m** $7\frac{39}{125}$ **n** $9\frac{47}{50}$ **o** $12\frac{9}{200}$
p $84\frac{63}{500}$ **q** $73\frac{9}{10}$ **r** $\frac{21}{5000}$

5 0.75
6 1.1
7 0.56
8 $\frac{2}{5}$
9 Check with your teacher.
10 $\frac{6}{10}, \frac{5}{8}, \frac{2}{3}, \frac{3}{4}, \frac{6}{7}$
11 **a** $\frac{7}{9}$
b $\frac{34}{111}$
c $2\frac{2}{9}$
12 **a** No, the sum of two recurring decimals may be an exact number; for example, $\frac{1}{3} + \frac{2}{3} = \frac{3}{3} = 1$.
b Yes, the recurring part is maintained; for example, $\frac{1}{3} + \frac{1}{2} = \frac{5}{6} = 0.8\dot{3}$.
c No, it could result in zero; for example, $\frac{1}{3} - \frac{1}{3} = 0$.
d Yes, the recurring part is maintained.
e Yes, the recurring part is maintained.
13 **a** $\frac{1}{30}$
b 450 gold doubloons
c $1.5pd$
14 **a** $\frac{1}{72}$
b $\frac{144r}{5}$g
15 Bucket 1: 12 L, Bucket 2: 3 L, Bucket 3: 3 L

4.5 Addition and subtraction of decimals

1 **a** 12.9 **b** 13 **c** 19.68
d 20.3 **e** 17.26 **f** 8.14
g 132.44 **h** 42.719 **i** 6.239
j 20.672 **k** 59.434 **l** 394.132
m 126.157 **n** 7113.556
2 **a** 2.24 **b** 7.32 **c** 121.66
d 54.821 **e** 42.33 **f** 1674.93
g 53.16 **h** 124.966 **i** 22.897
j 26.03 **k** 33.028 **l** 474.104
3 **a** E **b** B
4 **a** 115.09 **b** 31.953 **c** 821.48
d 1954.291 **e** 1493.875 **f** 433.98
g 1553.589 **h** 8.884 **i** 598.018
j 4224.296
5 **a** −0.9 **b** −0.7 **c** −0.57
d −0.991 **e** 7.606 **f** 0.6343
6 **a** −0.7 **b** −1.32 **c** 3.4
d −4.3 **e** −2.55 **f** −0.818
7 **a** 27 **b** 46 **c** 21 **d** 129
8 **a** Salmah spent $157.22.
b He would have $42.78 left.
9 41.37 seconds
10 9.88 km
11 763.4 km
12 **a** 1.2 km **b** 10.3 km **c** 15.8 km in total
13 Spending decreased by $10.5 billion.
14 **a** The third decimal place is 5, and so should be rounded up. The friend made the mistake of rounding down. 23.925 38 rounded to 2 decimal places is 23.93.
b $2.03 should be rounded to the nearest 5 cents, but the friend rounded to the nearest 10 cents. $2.03 rounded to the nearest 5 cents is $2.05.
15 **a** 0.8889 **b** 1.2222 **c** 1.1111
d 0.9798 **e** 0.1111 **f** 1.0889
g 2.4133

4.6 Multiplication and division of decimals

1 **a** 4.96 **b** 9.48 **c** 210.24
d 613.2 **e** 2.036 **f** 78.624
g 582.659 **h** 1153.96 **i** 2183.14
j 40.74 **k** 156.78 **l** 0.2616
m 624.036 **n** 0.344 55 **o** 0.147 62
2 **a** 6.17 **b** 130.98 **c** 790.14
d 95.54 **e** 6.41 **f** 2.53
3 **a** 88.83 **b** 1920.86
c 5355 **d** 536 515
e 29 933.75 **f** 954 087.50
g 6466.25 **h** 89 920
i 134.60 **j** 1714.36
k 7012.80 **l** 6.56
4 **a** −0.06 **b** −0.72 **c** 0.024
d 0.36 **e** 0.09 **f** −0.16
g −2000 **h** 0.008 **i** 0.0025
j −0.294 **k** −1.6 **l** 0.1296
5 **a** −42 **b** −0.3 **c** 0.135 **d** −30
e −0.7 **f** 400 **g** −0.06 **h** −9000
i 0.000 05 **j** −5 **k** 36 **l** 45
6 **a** 13.9 **b** 1.5 **c** 7.6 **d** 2.0
7 **a** D **b** B
8 **a** 133 **b** 90 **c** 501 **d** 46
9 $5.30
10 10
11 43.98 L
12 2.015
13 **a** 5115.495 cm^3
b 0.0293 g/cm^3
14 **a** 11.56 m^2
b $5.8^2 - 11.56 = 22.08$ m^2
15 $\frac{2ct}{5g}$
16 **a** Septimus is faster, with a speed of 3.32 m/s; Fortescue's speed is 2.92 m/s.
b 2226.39 hours

4.7 Estimation

1 Estimate: 72. Kim is correct.
2 **a** **i** 200 **ii** 300 **iii** 200
b **i** 5000 **ii** 5000 **iii** 4000
c **i** 20 **ii** 30 **iii** 20
d **i** 50 000 **ii** 60 000 **iii** 50 000
e **i** 600 **ii** 600 **iii** 500
f **i** 1000 **ii** 2000 **iii** 1000
3 **a** 230 **b** 4520 **c** 20
d 53 620 **e** 590 **f** 1040
4 **a** 300 **b** 4600 **c** 100
d 53 700 **e** 600 **f** 1100
5 **a** 4 **b** 340 **c** 520 **d** 1170
e 300 **f** 24 000 **g** 54 000 **h** 20
i 9680 **j** 10 **k** 108 000 **l** 20
m 28 000 **n** 600 **o** 4000 **p** 810 000
6 **a** 4000 **b** 500
c 70 000 **d** 12 000
e 8000 **f** 400
7 **a** 20.5 **b** 150.175 **c** 20.25
d 5.13 **e** 54.4 **f** 64.8

8 $500

9 **a** 4 and 5 **b** 10 and 11
c 13 and 14 **d** 15 and 16

10

	Question	Simplified question	Estimated answer	Exact answer
a	789×56	800×60	48 000	44 184
b	$124 \div 5$	$100 \div 5$	20	24.8
c	$678 + 98 + 46$	$700 + 100 + 50$	850	822
d	235×209	200×200	40 000	49 115
e	$7863 - 908$	$8000 - 900$	7100	6955
f	63×726	60×700	42 000	45 738
g	$39\,654 \div 227$	$40\,000 \div 200$	200	174.69
h	$1809 - 786 + 467$	$2000 - 800 + 500$	1700	1490
i	$21 \times 78 \times 234$	$20 \times 80 \times 200$	320 000	383 292
j	$942 \div 89$	$900 \div 90$	10	10.58
k	$\frac{492 \times 94}{38 \times 49}$	$\frac{500 \times 100}{40 \times 50}$	25	24.84
l	$\frac{54\,296}{97 \times 184}$	$\frac{50\,000}{100 \times 200}$	2.5	3.04

11 **a** $300 000 **b** 640 mL
c $200 **d** 5000 seconds
e $1 080 000 **f** $16 000
g 4 000 000 pieces of litter

12 **a** 85 500
b, c Answers will vary.

13 **a** **i** 900
ii 900
b **i** 920
ii 900
c **i** 940
ii 900
d **i** 960
ii 900
e **i** 980
ii 900

14 When the first number is rounded, the answers increase. When the second number is rounded, the answers remain constant.

15 **a** **i** 880
ii 600
b **i** 880
ii 640
c **i** 880
ii 680
The error increases if the smaller number is rounded. The error decreases when the rounded number is closer to the actual number.

Investigation — Rich task

1 21 507 717
2 6 489 870
3 Male: 0.494; female: 0.506
4 4 144 023
5 5 354 042
6 Male: 0.492; female: 0.508
7 The proportions of males and females in Victoria are similar to those in the whole of Australia.
8 Answers will vary; check with your teacher.
9 The population of Australia grew by 17 733 916 in the years between 1901 and 2011.
10 In 1901, the proportion of people born in Australia was 0.771. In 2011, the proportion of people born in Australia was 0.698. The proportion of people born in Australia in 1901 is greater than the proportion born in Australia in 2011.
11 Answers will vary; check with your teacher.

Code puzzle

Boycott: Cradle for male babies
Each: Minor irritation
Flattery: Living in an apartment
Inkling: A baby pen
Kidney: Joint in a child's leg
Tortoise: What our teacher did

TOPIC 5

Ratios and rates

5.1 Overview

Why learn this?

Ratios are used to compare different quantities. For example, they can be used to describe money exchanges, ingredients used in cooking or perspective in drawings. Some ratios have been found to occur in nature, and in ancient times some philosophers believed that numbers could explain the nature of the universe. Rates enable us to compare quantities expressed in different units. For example, how far will the family car travel on 30 litres of unleaded petrol when travelling in traffic at an average speed of 55 km/h?

What do you know?

assessON

1 **THINK** List what you know about ratios and rates. Use a thinking tool such as a concept map to show your list.
2 **PAIR** Share what you know with a partner and then with a small group.
3 **SHARE** As a class, create a 'thinking tool' such as a large concept map that shows your class's knowledge of ratios and rates.

Learning sequence

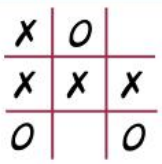
5.1 Overview
5.2 Introduction to ratios
5.3 Simplifying ratios
5.4 Proportion
5.5 Comparing ratios
5.6 Dividing in a given ratio
5.7 Rates
5.8 Interpreting graphs
5.9 Review eBook plus ONLINE ONLY

5.2 Introduction to ratios

- **Ratios** are used to compare quantities of the same kind.
- Ratios do not have a name or unit of measurement.
- The order of the numbers in a ratio is important. If the ratio of flour to water is 1 : 4, the 1 corresponds to the quantity of flour and the 4 corresponds to the quantity of water.
- Ratios can also be written in fractional form: $1 : 4 \Leftrightarrow \frac{1}{4}$.
- Ratios can also be written as percentages; for example, if 20% of a class of Year 8 students walked to school, then 80% did not walk to school — a ratio of 1 : 4.
- Before ratios are written, the numbers must be expressed in the *same* units of measurement. Once the units are the same, they can be omitted.
- Simplified ratios contain only whole numbers.

WORKED EXAMPLE 1

Look at the completed game of noughts and crosses at right and write the ratios of:
a crosses to noughts
b noughts to unmarked spaces.



THINK	WRITE
a Count the number of crosses and the number of noughts. Write the 2 numbers as a ratio (the number of crosses must be written first).	a 4 : 3
b Count the number of noughts and the number of unmarked spaces. Write the 2 numbers as a ratio, putting them in the order required (the number of noughts must be written first).	b 3 : 2

WORKED EXAMPLE 2

Rewrite the following statement as a ratio: 7 mm to 1 cm.

THINK	WRITE
1 Express both quantities in the same units. To obtain whole numbers, convert 1 cm to mm (rather than 7 mm to cm).	7 mm to 1 cm 7 mm to 10 mm
2 Omit the units and write the 2 numbers as a ratio.	7 : 10

assess on

Exercise 5.2 Introduction to ratios

REFLECTION
Why doesn't a simplified ratio have units?

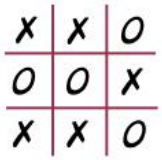
INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7, 9, 12, 13	Questions: 1–4, 5a–p, 6–8, 10, 12–14	Questions: 1–4, 5k–t, 7–14

Individual pathway interactivity int-4412 eBookplus

FLUENCY

1 **WE1** Look at the completed game of noughts and crosses and write the ratios of:



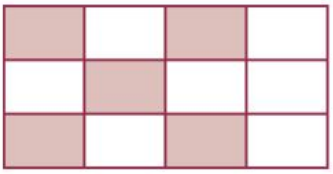
- **a** noughts to crosses
- **b** crosses to noughts
- **c** crosses to total number of spaces
- **d** total number of spaces to noughts
- **e** noughts in the top row to crosses in the bottom row.

eBookplus
Digital docs
SkillSHEET
Converting units of length, capacity and time
doc-6888
SkillSHEET
Highest common factor
doc-6889

2 Look at the coloured circles on the right and then write the following ratios.

- **a** Black : red
- **b** Red : black
- **c** Aqua : black
- **d** Black : aqua
- **e** Aqua : red
- **f** Black : (red and aqua)
- **g** Aqua : (black and red)
- **h** Black : total circles
- **i** Aqua : total circles
- **j** Red : total circles

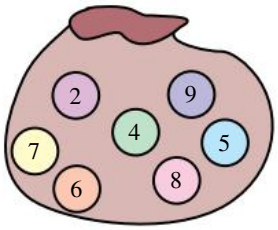
3 For the diagram shown, write the following ratios.



- **a** Shaded parts : unshaded parts
- **b** Unshaded parts : shaded parts
- **c** Shaded parts : total parts

eBookplus
Interactivity
Fruit salad ratio
int-0076

4 In the bag of numbers shown to the right, write the ratios of:



- **a** even numbers to odd numbers
- **b** prime numbers to composite numbers
- **c** numbers greater than 3 to numbers less than 3
- **d** multiples of 2 to multiples of 5
- **e** numbers divisible by 3 to numbers not divisible by 3.

5 **WE2** Rewrite each of the following statements as a ratio.

- **a** 3 mm to 5 mm
- **b** 6 s to 19 s
- **c** $4 to $11
- **d** 7 teams to 9 teams
- **e** 1 goal to 5 goals
- **f** 9 boys to 4 boys
- **g** 3 weeks to 1 month
- **h** 3 mm to 1 cm
- **i** 17 seconds to 1 minute
- **j** 53 cents to $1
- **k** 11 cm to 1 m
- **l** 1 g to 1 kg
- **m** 1 L to 2 kL
- **n** 7 hours to 1 day
- **o** 5 months to 1 year
- **p** 1 km to 27 m
- **q** 7 apples to 1 dozen apples
- **r** 13 pears to 2 dozen pears
- **s** 3 females to 5 males
- **t** 1 teacher to 22 students

6 Out of 100 people selected for a school survey, 59 were junior students, 3 were teachers and the rest were senior students. Write the following ratios.

- **a** Teachers : juniors
- **b** Juniors : seniors
- **c** Seniors : teachers
- **d** Teachers : students
- **e** Juniors : other members of the survey

7 Write each of the following as a mathematical ratio.

- **a** In their chess battles, Lynda has won 24 games and Karen has won 17.
- **b** There are 21 first-division teams and 17 second-division teams.
- **c** Nathan could long jump twice as far as Rachel.
- **d** On the camp there were 4 teachers and 39 students.
- **e** In the mixture there were 4 cups of flour and 1 cup of milk.

- **f** Elena and Alex ran the 400 m in the same time.
- **g** The radius and diameter of a circle were measured.
- **h** The length of a rectangle is three times its width.
- **i** On Friday night 9 out of every 10 people enjoyed the movie.
- **j** The length of one side of an equilateral triangle compared to its perimeter.
- **k** The length of a regular hexagon compared to its perimeter.
- **l** The number of correct options compared to incorrect options in a multiple-choice question containing five options.

UNDERSTANDING

8 A pair of jeans originally priced at $215 was purchased for $179. What is the ratio of:

- **a** the original price compared to the selling price?
- **b** the original price compared to the discount?

9 Matthew received a score of 97% for his Maths test. What is the ratio of:

- **a** the marks received compared to marks lost?
- **b** the marks lost compared to total marks possible?

10 For each comparison that follows, state whether a ratio could be written and give a reason for your answer. (*Remember:* Before ratios are written, numbers must be expressed in the same unit of measurement.)

- **a** Anna's mass is 55 kg. Her cat has a mass of 7 kg.
- **b** Brian can throw a cricket ball 40 metres, and John can throw the same ball 35 metres.
- **c** The cost of painting the wall was $55; its area is 10 m^2.
- **d** For a trip, the car's average speed was 85 km/h; the trip took 4 h.

- **e** Brett's height is 2.1 m. Matt's height is 150 cm.
- **f** Jonathon apples cost $2.40 per dozen; Delicious apples cost $3.20 per dozen.
- **g** Mary is paid $108; she works 3 days a week.
- **h** David kicked 5 goals and 3 behinds (33 points); his team scored 189 points.

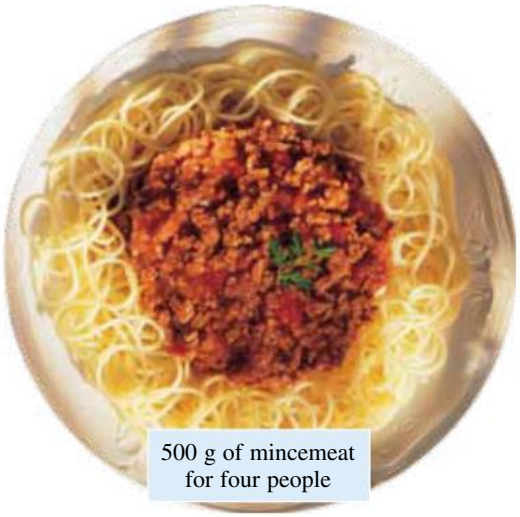
11 **a** If 17% of students in a class have sports training once a week, write as a ratio the number of students who have sports training once a week compared to the students who do not.

b A survey found that only 3% of workers take their lunch to work on a regular basis. Write as a ratio the number of workers who do not take their lunch to work compared to those who do.

REASONING

12 A recipe to make enough spaghetti bolognaise to feed four people needs 500 grams of mincemeat. According to this recipe, how much mincemeat would be needed to make enough spaghetti bolognaise to feed a group of nine people?

Show all working.



500 g of mincemeat for four people

PROBLEM SOLVING

13 Out of 100 people selected for a school survey, 68 were students, 5 were teachers and the rest were support staff.

a Write the ratio of teachers to students.

b Write the ratio of support staff to teachers.

c Write the ratio of students to people surveyed.

d Write the number of students as both a fraction and a percentage of the number of people surveyed.

e Write the number of students as both a fraction and a percentage of the number of teachers surveyed.

14 In class 8A, there are 4 boys and 5 girls. In class 8B, there are 12 boys and 15 girls. In class 8C, there are 6 girls and 8 boys.

a For each class, write the ratio of boys to girls.

b In which two classes are the proportions of boys and girls the same?

c Class 8A joined class 8B to watch a program for English. What was the resulting boy-to-girl ratio?

d Class 8C joins class 8B for PE lessons. What is the boy-to-girl ratio in the combined PE class?

e Is the ratio of boys to girls watching the English program equivalent to the ratio of boys to girls in the combined PE class?

CHALLENGE 5.1

The ratio of gold coins to silver coins in a purse is 2 : 5. If there are 10 gold coins in the purse, what is the smallest number of coins that needs to be added to the purse so that the ratio of gold coins to silver coins changes from 3 : 4?

5.3 Simplifying ratios

- When the numbers in a ratio are multiplied or divided by the same number to obtain another ratio, these two ratios are said to be **equivalent**. (This is a similar process to obtaining equivalent fractions).
- Like fractions, ratios are usually written in simplest form; that is, reduced to lowest terms.

WORKED EXAMPLE 3

Express the ratio 16 : 24 in simplest form.

THINK

1. Write the ratio.
2. Determine the largest number by which both 16 and 24 can be divided (that is, what is the highest common factor?). It is 8.
3. Divide both 16 and 24 by 8 to obtain an equivalent ratio in simplest form.

WRITE

16 : 24
÷ 8 ÷ 8
2 : 3

WORKED EXAMPLE 4

Write the ratio of 45 cm to 1.5 m in simplest form.

THINK

1. Write the question.
2. Express both quantities in the same units by changing 1.5 m into cm. (1 m = 100 cm)
3. Omit the units and write the 2 numbers as a ratio.
4. Simplify the ratio by dividing both 45 and 150 by 15 — the HCF.

WRITE

45 cm to 1.5 m

45 cm to 150 cm

45 : 150
÷ 15 ÷ 15
3 : 10

WORKED EXAMPLE 5

Simplify the following ratios.

a $\frac{2}{5} : \frac{7}{10}$ b $\frac{5}{6} : \frac{5}{8}$

THINK

a 1. Write the fractions in ratio form.
2. Write equivalent fractions using the lowest common denominator (in this case, 10).
3. Multiply both fractions by 10.
4. Check if the remaining whole numbers that form the ratio can be simplified. In this case they cannot.

WRITE

a $\frac{2}{5} : \frac{7}{10}$

$\frac{4}{10} : \frac{7}{10}$
× 10 × 10
4 : 7

b 1 Write the fractions in ratio form.

2 Write equivalent fractions using the lowest common denominator (in this case, 24).

3 Multiply both fractions by 24.

4 Check if the remaining whole numbers that form the ratio can be simplified. In this case divide each by the HCF of 5.

b $\frac{5}{6} : \frac{5}{8}$

$\times 24$ ($\frac{20}{24} : \frac{15}{24}$) $\times 24$

$\div 5$ ($20 : 15$) $\div 5$

$4 : 3$

- If the ratio uses decimals, we multiply by the smallest power of 10 that will produce a whole number for both parts of the ratio.

WORKED EXAMPLE 6

Write the following ratios in simplest form.
a 2.1 to 3.5 **b 1.4 : 0.75**

THINK

a 1 Write the decimals in ratio form.

2 Both decimals have one decimal place, so multiplying each by 10 will produce whole numbers.

3 Simplify by dividing both numbers by the HCF of 7.

b 1 Write the decimals in ratio form.

2 Because 0.75 has two decimal places, we multiply each decimal by 100 to produce whole numbers.

3 Simplify by dividing both numbers by the HCF of 5.

WRITE

a $2.1 : 3.5$

$\times 10$ $\times 10$

$21 : 35$

$\div 7$ $\div 7$

$3 : 5$

b $1.4 : 0.75$

$\times 100$ $\times 100$

$140 : 75$

$\div 5$ $\div 5$

$28 : 15$

- If the ratio contains algebraic terms, divide both parts of the ratio by the highest common factor (HCF) including common algebraic terms.

WORKED EXAMPLE 7

Simplify the ratios:
a $10a^2b : 15ab^2$ **b $3mn : 6mn$.**

THINK

a 1 Write the ratios.

2 Simplify the ratio by dividing both sides by $5ab$ (the HCF).

WRITE

a $10a^2b : 15ab^2$

$\frac{10a^2b}{5ab} : \frac{15ab^2}{5ab}$

3 Cancel common factors to obtain the ratio in simplest form. $2a : 3b$

b 1 Write the ratios. **b** $3mn : 6mn$

2 Simplify the ratio by dividing both sides by $3mn$ (the HCF). $\frac{3mn}{3mn} : \frac{6mn}{3mn}$

3 Cancel common factors to obtain the ratio in simplest form. $1 : 2$

assess on

Exercise 5.3 Simplifying ratios

REFLECTION
In what ways is simplifying ratios similar to simplifying fractions?

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7, 9, 12, 14, 15	Questions: 1, 2, 3a–l, 4a–f, 5, 6–11, 14, 15	Questions: 1m–t, 2, 3m–t, 4f–j, 5–10, 13–16

Individual pathway interactivity int-4413 eBookplus

eBookplus
Digital docs
SkillSHEET
Simplifying fractions
doc-6890
SkillSHEET
Finding and converting to lowest common denominator
doc-6891

FLUENCY

1 WE3 Express each ratio in simplest form.

a 2 : 4	**b** 3 : 9	**c** 5 : 10	**d** 6 : 18
e 12 : 16	**f** 15 : 18	**g** 24 : 16	**h** 21 : 14
i 25 : 15	**j** 13 : 26	**k** 15 : 35	**l** 27 : 36
m 36 : 45	**n** 42 : 28	**o** 45 : 54	**p** 50 : 15
q 56 : 64	**r** 75 : 100	**s** 84 : 144	**t** 88 : 132

2 Complete the patterns of equivalent ratios.

a	**b**	**c**	**d**	**e**
1 : 3	2 : 1	2 : 3	64 : 32	48 : 64
2 : 6	4 : 2	4 : 6	__ : 16	24 : __
__ : 9	__ : 4	6 : __	__ : 8	12 : __
__ : 12	__ : 8	__ : 12	8 : __	__ : 8
5 : __	20 : __	__ : 24	__ : 1	__ : __

eBookplus
Interactivity
Simplifying ratios
int-3383

3 WE4 Write the following ratios in simplest form.

a 8 cm to 12 cm	**b** $6 to $18	**c** 50 s to 30 s
d 80 cm to 2 m	**e** 75 cents to $3	**f** 2 h to 45 min
g 300 mL to 4 L	**h** 500 g to 2.5 kg	**i** 45 mm to 2 cm
j $4 to $6.50	**k** 2500 L to 2 kL	**l** 2500 m to 2 km
m 30 cents to $1.50	**n** 2 h 45 min to 30 min	**o** 200 m to 0.5 km
p 0.8 km to 450 m	**q** $1\frac{1}{2}$ min to 300 s	**r** 1.8 cm to 12 mm
s 3500 mg to 1.5 g	**t** $1.75 to $10.50	

4 Compare each of the following, using a mathematical ratio (in simplest form).

a The Hawks won 8 games and the Lions won 10 games.
b This jar of coffee costs $4 but that one costs $6.
c While Joanne made 12 hits, Holly made 8 hits.
d In the first innings, Ian scored 48 runs and Adam scored 12 runs.

e During the race, Rebecca's average speed was 200 km/h and Donna's average speed was 150 km/h.

f In the basketball match, the Tigers beat the Magic by 105 points to 84 points.

g The capacity of the plastic bottle is 250 mL and the capacity of the glass container is 2 L.

h Joseph ran the 600 m in 2 minutes while Maya ran the same distance in 96 seconds.

i In the movie audience, there were 280 children and 35 adults.

j On a page in the novel *Moby Dick* there are 360 words. Of these, 80 begin with a vowel.

5 A serving of a particular breakfast cereal contains:

3.6 g of protein	0.4 g of fat
20 g of carbohydrate	1 g of sugar
3.3 g of dietary fibre	84 mg of sodium.

Find the following ratios in simplest form:

a sugar to carbohydrate **b** fat to protein

c protein to fibre **d** sodium to protein.

6 WE5 Write the following ratios in simplest form.

a $\frac{1}{3}$ to $\frac{2}{3}$ **b** $\frac{5}{7}$ to $\frac{6}{7}$ **c** $\frac{1}{4} : \frac{1}{2}$

d $\frac{2}{3} : \frac{5}{6}$ **e** $\frac{4}{5}$ to $\frac{9}{20}$ **f** $\frac{2}{3} : \frac{3}{5}$

g $\frac{3}{10} : 1$ **h** $1\frac{2}{3}$ to $\frac{1}{3}$ **i** $1\frac{1}{4} : 1\frac{1}{2}$

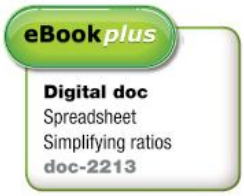
j $3\frac{1}{3}$ to $2\frac{1}{2}$ **k** $1 : 1\frac{3}{5}$ **l** $3\frac{1}{4}$ to $2\frac{4}{5}$

7 WE6 Write the following ratios in simplest form.

a 0.7 to 0.9 **b** 0.3 : 2.1 **c** 0.05 to 0.15 **d** 0.8 : 1

e 0.25 : 1.5 **f** 0.375 to 0.8 **g** 0.95 : 0.095 **h** 1 to 1.25

i 0.01 : 0.1 **j** 1.2 : 0.875 **k** 0.004 to 0.08 **l** 0.5 : 0.92



8 WE7 Simplify these ratios.

a $2a : 10b$ **b** $6p : 3p$ **c** $2x^2 : 3x$ **d** $7xy^2 : 14xy$

e $36m^3n^2 : 48m^2n^2$ **f** $ab : 4ab^2$ **g** $10^3x : 10x^3$ **h** $50(cd)^2 : 25c$

UNDERSTANDING

9 MC Toowoomba's population is 80 000 and Brisbane's population is 1.8 million. The ratio of Toowoomba's population to that of Brisbane is:

A 2 : 45 **B** 4 : 9 **C** 1 : 1.8 **D** 9 : 4 **E** none of these

10 MC When he was born, Samuel was 30 cm long. Now, on his 20th birthday, he is 2.1 m tall. The ratio of his birth height to his present height is:

A 3 : 7 **B** 1 : 21 **C** 7 : 10 **D** 1 : 7 **E** none of these

11 MC The cost of tickets to two different concerts is in the ratio 3 : 5. If the more expensive ticket is $110, the cheaper ticket is:

A $180 **B** $80 **C** $50 **D** $45 **E** $66

12 MC A coin was tossed 100 times and Tails appeared 60 times. The ratio of Heads to Tails was:

A 2 : 3 **B** 3 : 5 **C** 3 : 2 **D** 5 : 3 **E** 2 : 5

13 MC Out of a 1.25 L bottle of soft drink, I have drunk 500 mL. The ratio of soft drink remaining to the original amount is:

A 2 : 3 **B** 3 : 5 **C** 3 : 2 **D** 5 : 3 **E** 2 : 5

REASONING

14 The total attendance at a large outdoor concert was 32 200 people. Of this total, 13 800 people were female.

- **a** How many males attended the concert?
- **b** Compare the number of females to males by writing the ratio in simplest form.
- **c** Write the number of females as a fraction of the number of males.
- **d** What percentage of the crowd was female? How does this compare with your answer to part **c**?

PROBLEM SOLVING

15 In a primary school that has 910 students, 350 students are in the senior school and the remainder are in the junior school. Of the senior school students, 140 are females. There are the same number of junior males and junior females. Write the following ratios in simplest form:

- **a** senior students to junior students
- **b** senior females to senior males
- **c** senior males to total senior students
- **d** junior males to senior males
- **e** junior females to whole school population.

16 Compare the following, using a mathematical ratio (in simplest form):

- **a** Of the 90 000 people who attended the test match, 23 112 were females. Compare the number of males to females.
- **b** A Concorde jet cruises at 2170 km/h but a Cessna cruises at 220 km/h. Compare their speeds.
- **c** A house and land package is sold for $250 000. If the land was valued at $90 000, compare the land and house values.
- **d** In a kilogram of fertiliser, there are 550 g of phosphorus. Compare the amount of phosphorus to other components of the fertiliser.
- **e** Sasha saves $120 out of his take-home pay of $700 each fortnight. Compare his savings with his expenses.

17 The table at right represents the selling price of a house over a period of time.

Date of sale	Selling price
March 2014	$1.275 million
December 2010	$1.207 million
April 2010	$1.03 million
December 2006	$850 000

- **a** Compare the price of the house purchased in December 2006 with the purchase price in April 2010 as a ratio in simplest form.
- **b** Compare the price of the house purchased in December 2006 with the purchase price in December 2010 as a ratio in simplest form.
- **c** Compare the price of the house purchased in December 2006 with the purchase price in March 2014 as a ratio in simplest form.
- **d** **i** How much has the value of the house increased from December 2006 to March 2014?
  - **ii** Compare the increase obtained in part **d i** to the price of the house purchased in December 2006 as a ratio in simplest form.
- **e** Comment on the results obtained in part **d**.

5.4 Proportion

- A **proportion** is a statement of equality of two ratios.
- In general, if $\frac{a}{b} = \frac{c}{d}$, then, using cross-multiplication, $a \times d = c \times b$.

WORKED EXAMPLE 8

Use the cross-multiplication method to determine whether the following pair of ratios is in proportion:
6 : 9; 24 : 36.

THINK	WRITE
1 Write the ratios in fraction form.	$\frac{6}{9}$; $\frac{24}{36}$
2 Perform a cross-multiplication.	$6 \times 36 = 216$; $24 \times 9 = 216$
3 Check whether the products are equal.	$216 = 216$ Therefore, the ratios are in proportion.

WORKED EXAMPLE 9

Find the value of a in the following proportion: $\frac{a}{3} = \frac{6}{9}$.

THINK	WRITE
1 Write the proportion statement.	$\frac{a}{3} = \frac{6}{9}$
2 Cross-multiply and equate the products.	$a \times 9 = 6 \times 3$
3 Solve for a by dividing both sides of the equation by 9.	$9a = 18$ $\frac{9a}{9} = \frac{18}{9}$ $a = 2$

WORKED EXAMPLE 10

The ratio of girls to boys on the school bus was 4 : 3. If there were 28 girls, how many boys were there?

THINK	WRITE
1 Let the number of boys be b and write a proportion statement. (Since the first number in the ratio represents girls, place 28 (the number of girls) as the numerator.)	$\frac{4}{3} = \frac{28}{b}$
2 Cross-multiply and equate the products.	$4 \times b = 28 \times 3$

3 Solve for b by dividing both sides by 4.

$$4b = 84$$
$$\frac{4b}{4} = \frac{84}{4}$$
$$b = 21$$

4 Write the answer. There are 21 boys.

assessON

Exercise 5.4 Proportion

REFLECTION
Can you think of an example of how a proportion statement might be used?

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–4, 7, 8, 11, 13	Questions: 1a–h, 2a–h, 3–6, 8, 9, 11, 13, 14	Questions: 1g–l, 2g–l, 3f–j, 4f–j, 5–10, 12–14

Individual pathway interactivity int-4414 eBookplus

FLUENCY

1 WE8 Use the cross-multiplication method to determine whether the following pairs of ratio are in proportion.

a 2 : 3; 8 : 12 **b** 4 : 7; 8 : 14 **c** 5 : 7; 10 : 14

d 5 : 8; 10 : 16 **e** $\frac{7}{9}; \frac{21}{25}$ **f** $\frac{3}{8}; \frac{12}{32}$

g $\frac{14}{16}; \frac{5}{9}$ **h** $\frac{11}{12}; \frac{7}{8}$ **i** $\frac{13}{15}; \frac{6}{7}$

j $\frac{8}{9}; \frac{24}{27}$ **k** $\frac{3}{5}; \frac{6}{8}$ **l** $\frac{21}{18}; \frac{49}{42}$

2 WE9 Find the value of a in each of the following proportions.

a $\frac{a}{2} = \frac{4}{8}$ **b** $\frac{a}{6} = \frac{8}{12}$ **c** $\frac{a}{9} = \frac{2}{3}$

d $\frac{3}{a} = \frac{9}{12}$ **e** $\frac{7}{a} = \frac{14}{48}$ **f** $\frac{10}{a} = \frac{3}{15}$

g $\frac{3}{7} = \frac{a}{28}$ **h** $\frac{12}{10} = \frac{a}{5}$ **i** $\frac{8}{12} = \frac{a}{9}$

j $\frac{35}{7} = \frac{5}{a}$ **k** $\frac{24}{16} = \frac{6}{a}$ **l** $\frac{30}{45} = \frac{2}{a}$

3 WE10 Solve each of the following, using a proportion statement and the cross-multiplication method.

a The ratio of boys to girls in a class is 3 : 4. If there are 12 girls, how many boys are in the class?

b In a room the ratio of length to width is 5 : 4. If the width is 8 m, what is the length?

c The team's win–loss ratio is 7 : 5. How many wins has it had if it has had 15 losses?

d A canteen made ham and chicken sandwiches in the ratio 5 : 6. If 20 ham sandwiches were made, how many chicken sandwiches were made?

e The ratio of concentrated cordial to water in a mixture is 1 : 5. How much concentrated cordial is needed for 25 litres of water?

f The ratio of chairs to tables is 6 : 1. If there are 42 chairs, how many tables are there?

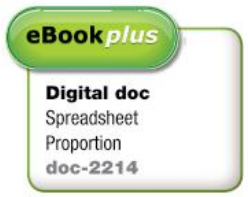
g The ratio of flour to milk in a mixture is 7 : 2. If 14 cups of flour are used, how much milk is required?

h The ratio of protein to fibre in a cereal is 12 : 11. If there are 36 grams of protein, what is the mass of fibre?

i In a supermarket, the ratio of 600 mL cartons of milk to litre cartons is 4 : 5. If there are sixty 600 mL cartons, how many litre cartons are there?

j In a crowd of mobile-phone users, the ratio of men to women is 7 : 8. How many women are there if there are 2870 men?

4 Although we know that only whole numbers are used in ratios, sometimes in a proportion statement the answer can be a fraction or a mixed number. Consider the following proportion:



$$\frac{a}{6} = \frac{7}{4}$$
$$a \times 4 = 7 \times 6$$
$$4a = 42$$
$$a = 10.5 \text{ (or } 10\tfrac{1}{2}\text{)}$$

Calculate the value of a in each of the following proportion statements. Write your answer correct to 1 decimal place.

a $\frac{a}{7} = \frac{8}{5}$ **b** $\frac{a}{6} = \frac{4}{5}$ **c** $\frac{a}{3} = \frac{7}{10}$ **d** $\frac{a}{9} = \frac{9}{10}$

e $\frac{5}{a} = \frac{7}{10}$ **f** $\frac{8}{a} = \frac{6}{7}$ **g** $\frac{9}{7} = \frac{a}{6}$ **h** $\frac{13}{6} = \frac{a}{5}$

i $\frac{9}{15} = \frac{7}{a}$ **j** $\frac{7}{8} = \frac{9}{a}$

UNDERSTANDING

5 Write a proportion statement for each situation and then solve the problem. If necessary, write your answer correct to 1 decimal place.

a A rice recipe uses the ratio of 1 cup of rice to 3 cups of water. How many cups of rice can be cooked in 5 cups of water?

b Another recipe states that 2 cups of rice are required to serve 6 people. If you have invited 11 people, how many cups of rice will you need?

c In a chemical compound there should be 15 g of chemical A to every 4 g of chemical B. If my compound contains 50 g of chemical A, how many grams of chemical B should it contain?

d A saline solution contains 2 parts of salt to 17 parts of water. How much water should be added to 5 parts of salt?

e To mix concrete, 2 buckets of sand are needed for every 3 buckets of blue metal. For a big job, how much blue metal will be needed for 15 buckets of sand?

6 Decide whether a proportion statement could be made for each of the following ratios.

a Height : age **b** Mass : age

c Intelligence : age **d** Distance : time

e Cost : number **f** Age : shoe size

g Sausages cooked : number of people **h** Eggs : milk (in a recipe)

i Number of words : pages typed **j** Length : area (of a square)

7 MC **a** If $\frac{p}{q} = \frac{l}{m}$, then:

A $p \times q = l \times m$ **B** $p \times l = q \times m$ **C** $p \times m = l \times q$

D $\frac{p}{m} = \frac{l}{q}$ **E** none of these is true.

b If $\frac{x}{3} = \frac{y}{6}$, then:

A $x = 2$ and $y = 4$ **B** $x = 1$ and $y = 2$ **C** $x = 3$ and $y = 6$

D $x = 6$ and $y = 12$ **E** all of these are true.

c If $\frac{23}{34} = \frac{x}{19}$ then, correct to the nearest whole number, x equals:

A 13 **B** 12 **C** 34 **D** 28 **E** 17

d The directions on a cordial bottle suggest mixing 25 mL of cordial with 250 mL of water. How much cordial should be mixed with 5.5 L of water?

A 0.55 mL **B** 5.5 mL **C** 55 mL **D** 550 mL **E** 5500 mL

REASONING

8 In a family, 3 children receive their allowances in the ratio of their ages, which are 16 years, 14 years and 10 years. If the oldest child receives $32, how much do the other two children receive? Show your working.

9 In jewellery, gold is often combined with other metals. 'Pink gold' is a mixture of pure gold, copper and silver in the ratio of 15 : 4 : 1. 'White gold' is a mixture of pure gold and platinum in the ratio of 3 : 1.

a What fraction of both pink and white gold jewellery is pure gold?

Pure gold is 24 carats and is not mixed with other metals. For most jewellery, however, 18-carat gold is used.

b Using your answer to part **a**, show why jewellery gold is labelled 18 carats.

c If the copper and silver in an 18-carat bracelet weigh a combined 2 grams, what is the weight of gold in the bracelet?

d If the price of gold is $35 per gram, what is the cost of the gold in the bracelet?

10 Two classes each contain 8 boys. In one class, the ratio of boys to girls is 1 : 2; in the other it is 2 : 1. If the two classes combine, what will the new ratio be? Justify your answer.

PROBLEM SOLVING

11 The nutritional panel for a certain cereal is shown. Sugar is one form of carbohydrate. Calculate the ratio of sugar to total carbohydrates in the cereal.

	Quantity per 30-g serving	Percentage daily intake per 30-g serving	Quantity per 30-g serving with $\frac{1}{2}$ cup skim milk
Energy	480 kJ	5.5%	670 kJ
Protein	6.6 g	13.1%	11.2 g
Fat — total — saturated	 0.2 g 0.1 g	 0.3% 0.1%	 0.3 g 0.2 g
Carbohydrate — total — sugars	 20.8 g 9.6 g	 6.7% 10.7%	 27.3 g 16.1 g

12 The solution to this question contains some gaps. Rewrite the solution, replacing the empty boxes with the appropriate numbers.

On a map for which the scale is 1 : 20 000, what distance in cm represents 2.4 km on the ground?

$1 : 20\,000 = x \text{ cm} : 2.4 \text{ km}$

$1 : 20\,000 = x \text{ cm} : 2.4 \times 1000 \times 100 \text{ cm}$

$1 : 20\,000 = x : 240\,000$

$$\frac{1}{20\,000} = \frac{x}{\square}$$

$$\frac{1}{20\,000} \times \square = \frac{x}{240\,000} \times \square$$

$x = \square$

So, $\square$ cm on the map represents 2.4 km on the ground.

13 Simplify the following three-part ratios.

a 50 : 20 : 15 **b** 0.4 : 1.8 : 2.2

14 A concreter needs to make 1.0 m^3 of concrete for the base of a garden shed. How many cubic metres of each of the components should be used?

To make concrete mix: 1 part cement, 2 parts sand and 4 parts gravel.

eBookplus
Digital doc
WorkSHEET 5.1
doc-2216

5.5 Comparing ratios

- In some cases it is necessary to know which of two given ratios is the larger.
- In some cases it is necessary to know if two given ratios are equal.
- To compare ratios, write them as fractions with a common denominator.

WORKED EXAMPLE 11

Which is the larger ratio in the following pair?
3 : 5 2 : 3

THINK	WRITE
1 Write each ratio in fraction form.	$\frac{3}{5}$; $\frac{2}{3}$
2 Change each fraction to the lowest common denominator (which is 15).	$\frac{9}{15}$; $\frac{10}{15}$
3 Compare the fractions: since both fractions have a denominator of 15, the larger the numerator, the larger the fraction.	$\frac{9}{15} < \frac{10}{15}$
4 The second fraction is larger and it corresponds to the second ratio in the pair. State your conclusion.	Therefore, 2 : 3 is the larger ratio.

- When measuring the steepness of various slopes or hills, we need to compare the **gradient**.
- Gradient is calculated by finding the ratio

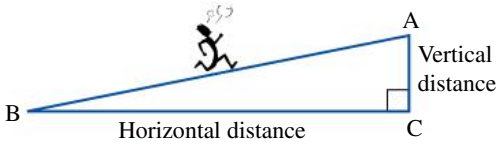
$$\frac{\text{vertical distance}}{\text{horizontal distance}}$$

between any 2 points on a hill. This is also known as calculating $\frac{\text{rise}}{\text{run}}$.

- The larger the gradient, the steeper the hill.

WORKED EXAMPLE 12

Find the gradient of the hill (AB) if AC = 2 m and BC = 10 m.



THINK	WRITE
1 Write the rule for finding the gradient.	Gradient $= \frac{\text{vertical distance}}{\text{horizontal distance}}$
2 The vertical distance is 2 m and the horizontal distance is 10 m. Substitute these values into the formula.	$= \frac{2}{10}$
3 Simplify by dividing both numerator and denominator by 2.	$= \frac{1}{5}$

assess on

Exercise 5.5 Comparing ratios

REFLECTION
What does it mean if the gradient is 0?

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–5, 8	Questions: 1–6, 8, 9	Questions: 1–9

Individual pathway interactivity int-4415 eBookplus

FLUENCY

1 WE11 Which is the greater ratio in each of the following pairs?

a 1 : 4, 3 : 4
b 5 : 9, 7 : 9
c 6 : 5, 2 : 5
d 3 : 5, 7 : 10
e 7 : 9, 2 : 3
f 2 : 5, 1 : 3
g 2 : 3, 3 : 4
h 5 : 6, 7 : 8
i 5 : 9, 7 : 12
j 9 : 8, 6 : 5

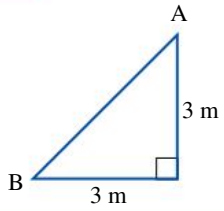
2 In each of the following cases, decide which netball team has the better record.

a Yarra Valley won 2 matches out of 5. Peninsula won 5 out of 10.
b Boroondara won 13 out of 18. Geelong won 7 out of 12.
c City West won 12 out of 20. Monash won 14 out of 25.
d VU Western won 8 out of 13. DC North won 13 out of 20.

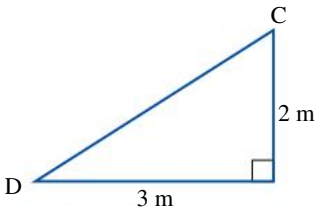
3 In a cricket match, Jenny bowled 5 wides in her 7 overs and Lisa bowled 4 wides in her 6 overs. Which bowler had the higher wides per over ratio?

4 a WE12 Find the gradient of each of the hills represented by the following triangles.

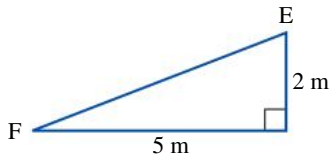
i



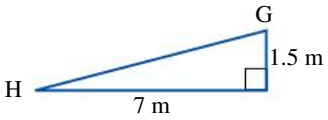
ii



iii



iv



b Which slope has the largest gradient?

c Which slope has the smallest gradient?

d List the hills in order of increasing steepness.

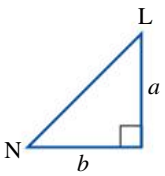
5 Draw triangles that demonstrate a gradient of:

a $\frac{2}{1}$ **b** $\frac{3}{1}$ **c** $\frac{4}{3}$ **d** $\frac{3}{2}$ **e** $\frac{2}{5}$

eBookplus
Digital doc
Spreadsheet
Comparing ratios
doc-2215

UNDERSTANDING

6 MC **a** If the gradient of LN in the triangle at right is 1, then:



A $a > b$ **B** $a < b$ **C** $a = b$

D $a = 1$ **E** $b = 1$

b If $\frac{5}{6} > \frac{a}{5}$, then a could be:

A 4 **B** 5 **C** 6

D 7 **E** all of these numbers.

REASONING

7 Draw a right-angled triangle on a piece of graph paper so that the two sides at right angles to each other are 6 cm and 8 cm. Measure the third side length of the triangle, which should be 10 cm.

a What is the ratio of the three sides of this triangle?

b If you change the size of your triangle but keep the shape the same, what happens to the ratio of the three sides of the triangle?

c A piece of string is used to mark out a triangle with its sides in the same ratio as the one you have drawn. The smallest side of the triangle is 75 cm long. How long are the other two sides?

PROBLEM SOLVING

8 An internet search for a homemade lemonade recipe yielded the following results. All of the recipes had water added, but the sweetness of the lemonade is determined by the ratio of lemon juice to sugar. For each of the recipe ratios shown below, determine the mixtures that have the same taste. If they do not have the same taste, determine which site has the sweeter lemonade recipe.

a Website 1: 3 tablespoons of sugar for every 15 tablespoons of lemon juice
Website 2: 4 tablespoons of sugar for every 20 tablespoons of lemon juice
b Website 3: 3 tablespoons of sugar for every 9 tablespoons of lemon juice
Website 4: 5 tablespoons of sugar for every 16 tablespoons of lemon juice
c Website 5: 2 cups of sugar for every 3 cups of lemon juice
Website 6: 5 tablespoons of sugar for every 8 tablespoons of lemon juice
d Website 7: 3 tablespoons of sugar for every 8 tablespoons of lemon juice
Website 8: 7 tablespoons of sugar for every 12 tablespoons of lemon juice

9 The steepness of a hill can be written as a ratio of the vertical distance (height) to the horizontal distance. Hill A has a ratio of 1 : 5 and hill B has a ratio of 7 : 36.
a Use diagrams to determine which hill has the steeper slope. Is this an accurate method of determining the slope?
b Without using diagrams, investigate another method to determine which ratio produces the steeper slope.
c Will your method from part **b** always work when you want to compare ratios?

5.6 Dividing in a given ratio

eBookplus
eLesson
Dividing in ratios
eles-0041

When something is shared, we often use ratios to ensure that the sharing is fair.

Consider this situation. Two people buy a lottery ticket for $3. They win a prize of $60. How is the prize divided fairly?

Each person contributes $1.50	One person contributes $1 and the other $2
• The contribution for the ticket is in the ratio 1 : 1. • The prize is divided in the ratio 1 : 1. • Each person receives $30.	• The contribution for the ticket is in the ratio 1 : 2. • The prize is divided in the ratio 1 : 2. • The person who contributed $1 receives $20 and the other receives $40.

In the situation above:

- the 1 : 1 ratio has 1 + 1 = 2 total parts. Each person receives $\frac{1}{2}$ of the prize money ($30).
- the 1 : 2 ratio has 1 + 2 = 3 total parts. Person 1 paid for 1 part of the ticket so receives $\frac{1}{3}$ of the prize money ($20); person 2 paid for 2 parts of the ticket so receives $\frac{2}{3}$ of the prize money ($40).

WORKED EXAMPLE 13

Share the amount of $500 000 in the ratio 3 : 7.

THINK	WRITE
1 Determine how many shares (parts) are in the ratio.	Total number of parts = 3 + 7 = 10

2	The first share represents 3 parts out of a total of 10, so find $\frac{3}{10}$ of the total amount.	First share $= \frac{3}{10} \times \$500\,000$ $= \$150\,000$
3	The second share is the remainder, so subtract the first share amount from the total amount.	Second share $= \$500\,000 - \$150\,000$ $= \$350\,000$

- In Worked example 12, the second share represents 7 parts out of the total of 10. Another way to calculate the size of the second share would be to find $\frac{7}{10}$ of the total amount.

$$\text{Second share} = \frac{7}{10} \times \$500\,000$$
$$= \$350\,000$$

WORKED EXAMPLE 14

Concrete mixture for a footpath was made up of 1 part of cement, 2 parts of sand and 4 parts of blue metal. How much sand was used to make 4.2 m^3 of concrete?

THINK	WRITE
1 Find the total number of parts.	Total number of parts $= 1 + 2 + 4$ $= 7$
2 There are 2 parts of sand to be used in the mixture, so find $\frac{2}{7}$ of the total amount of concrete made.	Amount of sand $= \frac{2}{7} \times 4.2\ \text{m}^3$ $= 1.2\ \text{m}^3$

Exercise 5.6 Dividing in a given ratio

assessON

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7, 9, 13	Questions: 1–9, 11, 13, 14	Questions: 1–13, 15

Individual pathway interactivity int-4416 eBook plus

REFLECTION
Think of some examples of instances where you need to divide in a ratio other than 1 : 1.

FLUENCY

1 Write the total number of parts for each of the following ratios.

a 1 : 2	**b** 2 : 3	**c** 3 : 1	**d** 3 : 5	**e** 4 : 9
f 5 : 8	**g** 6 : 7	**h** 9 : 10	**i** 1 : 2 : 3	**j** 3 : 4 : 5

2 WE13 Share the amount of $1000 in the following ratios.

a 2 : 3	**b** 3 : 1	**c** 1 : 4	**d** 1 : 1	**e** 3 : 5
f 5 : 3	**g** 3 : 7	**h** 9 : 1	**i** 7 : 13	**j** 9 : 11

3 If Nat and Sam decided to share their lottery winnings of $10 000 in the following ratios, how much would each receive?

a 1 : 1	**b** 2 : 3	**c** 3 : 2	**d** 3 : 7	**e** 7 : 3
f 1 : 4	**g** 9 : 1	**h** 3 : 5	**i** 12 : 13	**j** 23 : 27

4 Rosa and Mila bought a lottery ticket costing $10. How should they share the first prize of $50 000 if their respective contributions were:

a $2 and $8? **b** $3 and $7? **c** $4 and $6?

d $5 and $5? **e** $2.50 and $7.50?

5 WE14 Concrete mixture is made up of 1 part cement, 2 parts sand and 4 parts blue metal.

a How much sand is needed for 7 m^3 of concrete?

b How much cement is needed for 3.5 m^3 of concrete?

c How much blue metal is required for 2.8 m^3 of concrete?

d How much sand is used for 5.6 m^3 of concrete?

e How much cement is needed to make 8.4 m^3 of concrete?

6 Three of your teachers buy a Lotto ticket costing $20. How should they share the first prize of $600 000 if they each contribute:

a $3, $7 and $10?

b $6, $6 and $8?

c $1, $8 and $11?

d $5, $6 and $9?

e $5, $7.50 and $7.50?

eBookplus
Digital doc
Spreadsheet
Dividing in a given ratio
doc-2489

7 In a family, 3 children receive their allowances in the ratio of their ages, which are 15 years, 12 years and 9 years. If the total of the allowances is $60, how much does each child receive?

UNDERSTANDING

8 In a school, the ratio of girls in Years 8, 9 and 10 is 6 : 7 : 11. If there is a total of 360 girls in the three year levels:

a how many Year 8 girls are there?

b how many more Year 10 girls are there than Year 8 girls?

9 In a moneybox, there are 5-cent, 10-cent and 20-cent coins in the ratio 8 : 5 : 2. If there are 225 coins altogether:

a how many 5-cent coins are there?

b how many more 10-cent coins than 20-cent coins are there?

c what is the total value of the 5-cent coins?

d what is the total value of the coins in the moneybox?

10 MC **a** A square of side length 4 cm has its area divided into two sections in the ratio 3 : 5. The area of the larger section is:

A 3 cm^2 **B** 5 cm^2 **C** 8 cm^2 **D** 10 cm^2 **E** 16 cm^2

b A block of cheese is cut in the ratio 2 : 3. If the smaller piece is 150 g, the mass of the original block was:

A 75 g **B** 200 g **C** 300 g **D** 375 g **E** 450 g

c Contributions to the cost of a lottery ticket were $1.75 and $1.25. What fraction of the prize should the larger share be?

A $\frac{7}{12}$ **B** $\frac{5}{7}$ **C** $\frac{7}{5}$ **D** $\frac{3}{5}$ **E** $\frac{5}{12}$

d A television channel that telecasts only news, movies and sport does so in the ratio 2 : 3 : 4 respectively. How many movies, averaging a length of $1\frac{1}{2}$ hours, would be shown during a 24-hour period?

A 2 **B** 3 **C** 4 **D** 5 **E** 6

REASONING

11 Three angles of a triangle are in the ratio 1 : 2 : 3. What is the magnitude of each angle? Justify your answer.

12 The angles of a quadrilateral are in the ratio 2 : 3 : 4 : 6. What is the difference in magnitude between the smallest and largest angles? Show your working.

PROBLEM SOLVING

13 The ratio of boys to girls in Year 8 at a school is 3 : 2.

- **a** If there are 75 boys in Year 8, how many girls are there? Each class in Year 8 contains 25 students. One class contains boys only, but the remaining classes are a mixture of boys and girls.
- **b** How many classes in Year 8 contain a mixture of boys and girls?
- **c** What is the ratio of mixed classes to boys only classes?

14 The wing of a model aeroplane is 4 cm long, and the scale of the model is 1 : 300.

- **a** What is the ratio of the length of the model's wing to the length of the wing of the actual aeroplane?
- **b** How long is the wing of the actual aeroplane?

15 **a** There are apples, oranges and bananas in a fruit bowl. The ratio of apples to oranges is 3 : 4, and the number of apples is $\frac{3}{8}$ of the total number of pieces of fruit. How many bananas are there?

- **b** Next to the fruit bowl is a bowl of nuts. The ratio of cashews to peanuts is 15 : 4, the ratio of peanuts to walnuts is 4 : 7 and the ratio of hazelnuts to peanuts is 11 : 4. Calculate the minimum number of nuts there can be in the bowl.
- **c** Somebody added some macadamias to the nut bowl in part **b**. The ratio of peanuts to macadamias is now 1 : 3. Calculate:
  - **i** the number of macadamia nuts added
  - **ii** the total number of nuts now in the bowl.

5.7 Rates

- A **rate** is used to compare how quantities change.
- Unlike ratios, rates have units.
- An example of a rate is speed (measured in km/h or m/s).
- A rate is in its simplest form if it is *per one unit*.

WORKED EXAMPLE 15

Express the following statement using a rate in simplest form:
The 30-litre container was filled in 3 minutes.

THINK	WRITE
1 A suitable rate would be litres per minute (L/min). Put the capacity of the container in the numerator and the time in which it was filled in the denominator of the fraction.	$\text{Rate} = \frac{30\text{ L}}{3\text{ min}}$ $= \frac{10\text{ L}}{1\text{ min}}$
2 Simplify the fraction.	$= 10\text{ L/min}$

WORKED EXAMPLE 16

Joseph is paid $8.50 per hour as a casual worker. At this rate, how much does he receive for 6 hours of work?

THINK	WRITE
1 The rate is given in $ per hour. So it actually tells us the amount of money earned in each hour; that is, the hourly payment.	Payment per 1 hour = $8.50
2 State the number of hours worked.	Hours worked = 6
3 To find the total payment, multiply the hourly payment by the total number of hours worked.	Total payment = $8.50 × 6 = $51

assessON

Exercise 5.7 Rates

REFLECTION
Why do we need units with rates?

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7, 9, 12, 14, 15	Questions: 1, 2, 3a–l, 4a–f, 5, 6–11, 14, 15	Questions: 1m–t, 2, 3m–t, 4f–j, 5–10, 13–16

Individual pathway interactivity int-4417 eBookplus

eBookplus

Digital docs
SkillSHEET
Converting a mixed number to an improper fraction
doc-6892
SkillSHEET
Multiplying decimals by 10, 100 and 1000
doc-6893
SkillSHEET
Multiplying a whole number by a fraction
doc-6894
SkillSHEET
Converting minutes to a fraction of an hour
doc-6895

FLUENCY

1 WE15 Express each of the following statements using a rate in simplest form.

- **a** A lawn of 600 m^2 was mown in 60 min.
- **b** A tank of capacity 350 kL is filled in 70 min.
- **c** A balloon of volume 4500 cm^3 was inflated in 15 s.
- **d** The cost of 10 L of fuel was $13.80.
- **e** A car used 16 litres of petrol in travelling 200 km.
- **f** A 12 m length of material cost $30.
- **g** There were 20 cows grazing in a paddock that was 5000 m^2 in area.
- **h** The gate receipts for a crowd of 20 000 people were $250 000.
- **i** The cost of painting a 50 m^2 area was $160.
- **j** The cost of a 12 minute phone call was $3.00.
- **k** The team scored 384 points in 24 games.
- **l** Last year 75 kg of fertiliser cost $405.
- **m** The winner ran the 100 m in 12 s.
- **n** To win, Australia needs to make 260 runs in 50 overs.
- **o** For 6 hours work, Bill received $159.
- **p** The 5.5 kg parcel cost $19.25 to post.
- **q** Surprisingly, 780 words were typed in 15 minutes.

r From 6 am to noon, the temperature changed from 10 °C to 22 °C.

s When Naoum was 10 years old he was 120 cm tall. When he was 18 years old he was 172 cm tall.

t A cyclist left home at 8.30 am and at 11.00 am had travelled 40 km.

2 WE16 Sima is paid $15.50 per hour. At this rate, how much does she earn in a day on which she works 7 hours?

3 A basketball player scores, on average, 22 points per match. How many points will he score in a season in which he plays 18 matches?

4 A car's fuel consumption is 11 L/100 km. How much fuel would it use in travelling 550 km?

5 To make a solution of fertiliser, the directions recommend mixing 3 capfuls of fertiliser with 5 L of water. How many capfuls of fertiliser should be used to make 35 L of solution?

6 Anne can type 60 words per minute. How long will she take to type 4200 words?

7 Marie is paid $42 per day. For how long will she have to work to earn $504?

8 The rate of 1 teacher per 16 students is used to staff a school. How many teachers will be required for a school with 784 students?

9 Land is valued at $42 per m^2. How much land could be bought for $63 000?

10 On average, a test bowler took 1 wicket every 4.5 overs. How many wickets did he take in a season in which he bowled 189 overs?

UNDERSTANDING

11 What quantities (such as distance, time, volume) are changing if the units of rate are:

a km/h? **b** cm^3/sec? **c** L/km? **d** $ per h?

e $ per cm? **f** kL/min? **g** cents/litre? **h** $ per dozen?

i kg/year? **j** cattle/hectare?

12 What units would you use to measure the changes taking place in each of the following situations?

a A rainwater tank being filled **b** A girl running a sprint race

c A boy getting taller **d** A snail moving across a path

e An ink blot getting larger **f** A car consuming fuel

g A batsman scoring runs **h** A typist typing a letter

13 Water flows from a hose at a rate of 3 L/min. How much water will flow in 2 h?

14 Tea bags in a supermarket can be bought for $1.45 per pack (pack of 10) or for $3.85 per pack (pack of 25). Which is the cheaper way of buying the tea bags?

15 Car A uses 41 L of petrol in travelling 500 km. Car B uses 34 L of petrol in travelling 400 km. Which car is the more economical?

16 Coffee can be bought in 250 g jars for $9.50 or in 100 g jars for $4.10. Which is the cheaper way of buying the coffee and how large is the saving?

17 MC **a** A case containing 720 apples was bought for $180. The cost could be written as:

A 30 cents each **B** 20 cents each **C** $3.00 per dozen
D $2.00 per dozen **E** $2.80 for 10

b Mark, a test cricketer, has a batting strike rate of 68, which means he has made 68 runs for every 100 balls faced. What is Steve's strike rate if he has faced 65 overs and has made 280 runs? (*Note:* Each over contains 6 balls.)

A 65 **B** 68.2 **C** 71.8 **D** 73.2 **E** 74.1

c A carport measuring 8 m × 4 m is to be paved. The paving tiles cost $36 per m^2 and the tradesperson charges $12 per m^2 to lay the tiles. How much will it cost to pave the carport (to the nearest $50)?

A $1400 **B** $1450 **C** $1500 **D** $1550 **E** $1600

d A tank of capacity 50 kL is to be filled by a hose whose flow rate is 150 L/min. If the tap is turned on at 8 am, when will the tank be filled?

A Between 1.00 pm and 1.30 pm **B** Between 1.30 pm and 2.00 pm
C Between 2.00 pm and 2.30 pm **D** Between 2.30 pm and 3.00 pm
E Between 3.00 pm and 3.30 pm

REASONING

18 A chiropractor sees 160 patients every week.

a What is his rate of seeing patients each hour if he works a 40-hour week?

b How long, on average, does he spend with each patient?

c Using these rates, if the chiropractor wants to make at least $10 000 every week, what is the minimum he must charge each patient? Justify your answer.

19 If 4 monkeys eat 4 bananas in 4 minutes, how long does it take 12 monkeys to eat 12 bananas? Show your working.

20 If Bill takes 3 hours to paint a room and James takes 5 hours to paint a room, how long will it take to paint a room if they work together? Show your working.

PROBLEM SOLVING

21 Beaches are sometimes unfit for swimming if heavy rain has washed pollution into the water. A beach is declared unsafe for swimming if the concentration of bacteria is more than 5000 organisms per litre. A sample of 20 millilitres was tested and found to contain 55 organisms. Calculate the concentration in the sample (in organisms/litre) and state whether or not the beach should be closed.

22 **a** The Earth is a sphere with a mass of 6.0×10^{24} kg and radius 6.4×10^{6} m.

i Use the formula $V = \frac{4}{3}\pi r^3$ to calculate the volume of the Earth.

ii Hence calculate the density of the Earth.

b The planet Jupiter has a mass of 1.9×10^{27} kg and a radius of 7.2×10^{7} m.

i Calculate the volume of Jupiter.

ii Calculate the density of Jupiter.

c Different substances have their own individual density. Does it seem that the Earth and Jupiter are made of the same substance? Explain.

23 The Stawell Gift is a 120 m handicap footrace. Runners who start from scratch run the full 120 m; for other runners, their handicap is how far in front of scratch that they start. Joshua Ross has won the race twice. In 2003, with a handicap of 7 m, his time was 11.92 seconds. In 2005, from scratch, he won in 12.36 seconds. In which race did he run faster?

eBookplus
Digital doc
WorkSHEET 5.2
doc-2217

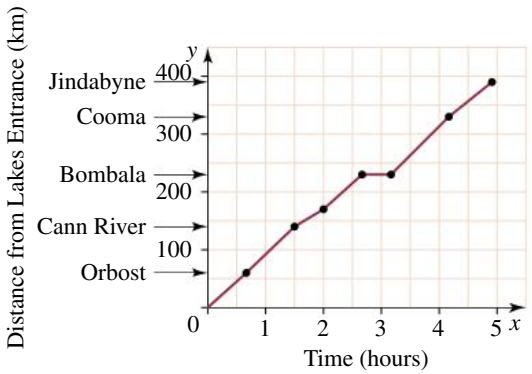
CHALLENGE 5.2

Jennifer's work is 40 km from her home. On the way to work one morning, her average speed was 80 km/h. Due to bad weather and roadworks, her average speed for the trip home was only 30 km/h. How much time did Jennifer spend travelling to and from work on this day?

5.8 Interpreting graphs

- All graphs tell a story or describe a situation. They compare two related quantities against each other. As one quantity changes, it affects the other.
- Graphs are used to represent all sorts of data in everyday life and are frequently used in fields outside mathematics, including science, geography and economics.
- The gradient (also called slope) of a straight line is a measure of how steep a straight line is.
- In a distance–time graph, the gradient of the graph represents the speed.
- The units used for distance and time are also used for speed.
- The greater the gradient (slope), the greater the speed; and vice-versa.
- On a distance–time graph, time is represented on the horizontal axis (x-axis).

- On a distance–time graph, distance is represented on the vertical axis (y-axis).
- The graph below shows the story of a car trip from Lakes Entrance to Jindabyne (distances are approximate). Describe the trip.



  - The trip could be described like this: '*We left Lakes Entrance and passed through Orbost 40 minutes into our trip. We reached Cann River approximately 1.5 hours after we set off. The trip out of Cann River was slow at first until we reached the top of the mountain. We then travelled faster on our way to Bombala where we stopped for a 30-minute break. We reached Cooma an hour after leaving Bombala, then the trip from Cooma to Jindabyne was slow due to snow traffic. The whole trip took us just under 5 hours.*'
  - Estimated travel times are determined by reading points along the graph.
  - A line's gradient can indicate whether distance is being travelled quickly (speed is fast), distance is being travelled slowly (speed is slow), or no distance is being travelled (speed is non-existent).
  - The line's gradient in this case is directly linked to the speed of the car.

The use of units

- The gradient of a line describes the change in one variable with respect to the other variable.
- When calculations of the gradient are done using the units from the graph, interpretation of the gradient becomes easier.
- The gradient of the graph between Orbost and Cann River is calculated below.

$$m = \frac{\text{rise}}{\text{run}}$$

$$= \frac{\text{distance}_{\text{Orbost} \rightarrow \text{Cann River}}}{\text{time}_{\text{Orbost} \rightarrow \text{Cann River}}}$$

$$= \frac{140 \text{ km} - 60 \text{ km}}{1.5 \text{ h} - 0.6667 \text{ h}}$$

$$= \frac{80 \text{ km}}{0.8333 \text{ h}}$$

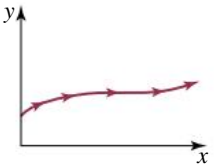
$$= 96 \text{ km/h}$$

- The gradient of the line is the average speed of the car over this section.
- Each straight line section of graph indicates a constant speed.

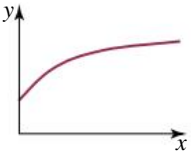
WORKED EXAMPLE 17

Three graphs (I, II and III) and three descriptions (A, B and C) are shown below. Match each graph to the description that best suits it and explain your choice.

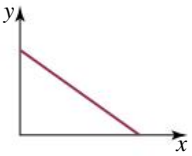
i



ii



iii



a The circumference of a tree trunk over a calendar year
b The distance run versus the distance from the finishing line for a runner in a 400 m race
c The weight of a newborn baby in the first year of life

THINK

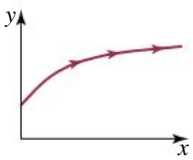
1 How does a tree grow in a normal year? You could expect that the growth rate of a tree will change with the seasons; the faster it grows, the steeper the slope. Over a year:

Summer	Autumn	Winter	Spring	Summer
Quickest growth	Growth → slowing	Growth very slow	Growth increasing	Quickest growth
→	→	→	→	↗

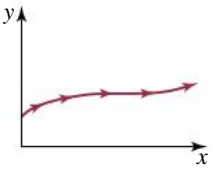
2 As the runner runs further, the distance to the finish line becomes less. The gradient of the curve should be decreasing. In fact:

Distance run (m)	0	100	200	300	400
Distance to finish (m)	400	300	200	100	0

3 Babies grow rapidly at first, then this growth rate begins to slow.



WRITE



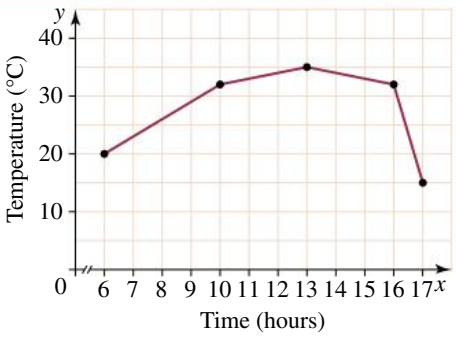
The circumference of a tree, A, is best represented by Graph I.

As the runner runs, (x) the distance (y) to the finish line becomes less. The graph should be nearing $y = 0$. Graph III is the only graph which does this. The distance run versus the distance to run is best represented by graph III.

The growth rate of babies is quick at first then slows but continues to increase. Graph II shows an increasing gradient that becomes less steep; therefore, graph II best represents the growth of a baby in the first year.

WORKED EXAMPLE 18

Use the following graph to answer the following questions.
a What units are shown on the graph?
b What does a change in the y-value represent?
c What does a change in the x-value represent?
d Calculate the gradient of the graph over the section in which the temperature was rising the fastest.
e What does the gradient represent?



THINK	WRITE
a Units are displayed on axis labels.	The units shown on the graph are °C (degrees Celsius) for temperature and hours for time.
b The y-axis shows the temperature.	A change in the y-value indicates a change in temperature.
c The x-axis shows time.	A change in the x-value indicates a change in time.
d The temperature is rising the fastest when the graph is increasing most steeply. This occurs between 6 and 10 am, when the temperature increases from 20 °C to ~32 °C.	$m = \frac{\text{rise}}{\text{run}}$ $= \frac{12\ °C}{4h}$ $= 3\ °C/h$
e The gradient has units of °C/h, which are read as degrees Celsius per hour.	The gradient represents the rate at which the temperature is changing per hour.

assessON

Exercise 5.8 Interpreting graphs

REFLECTION
How can a graph be used to tell a story?

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–9, 13	Questions: 1–10, 13, 14	Questions: 1–14

Individual pathway interactivity int-4418 eBookplus

FLUENCY

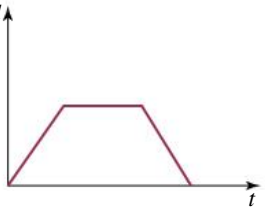
1 Which axis represents distance on a distance–time graph?

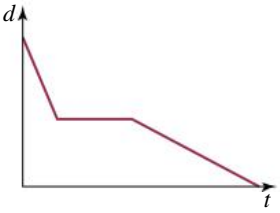
2 If distance is measured in metres (vertical axis) and time is measured in seconds (horizontal axis), how is speed measured?

3 The express train through country Victoria is delayed for one hour because of a herd of cattle on the line. How is this event graphed on a distance–time graph?

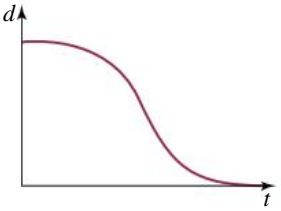
4 How can a graph tell a story?

5 MC Which of the following graphs best describes a student walking to school who dawdles at first, meets up with some friends and drops in at the corner shop before stepping up the pace to get to school on time?

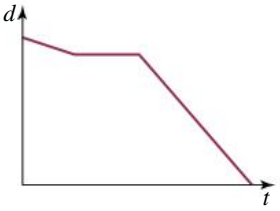
A 

B 

C



D

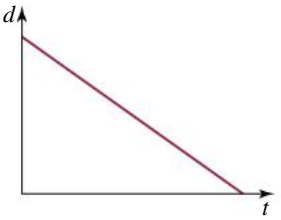


6 Explain your reasons for choosing your answer in question **5**.

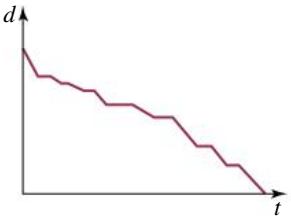
UNDERSTANDING

7 **MC** Which of the following graphs best describes a student travelling to school on a bus?

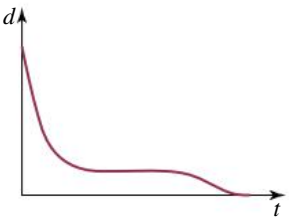
A



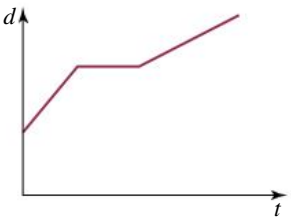
B



C



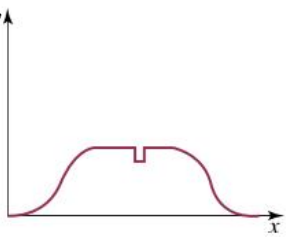
D



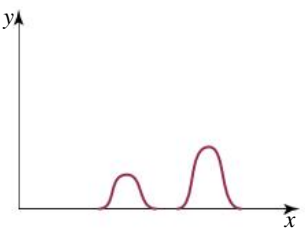
8 Explain your reasons for choosing your answer in question **7**.

9 **WE17** Three descriptions (a, b and c) and three graphs (I, II and III) are shown below. Match each graph to the description that best suits it and explain your choice.

i



ii



iii

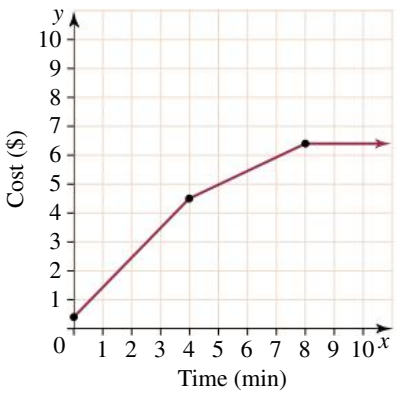
a The number of students in the canteen line over a school day

b The number of cans in a school's drink vending machine over a school day

c The number of students on school grounds over a school day

REASONING

10 **WE18** The graph shows the cost of a mobile telephone call. Use the graph to answer the following questions.



a What units are shown on the graph?

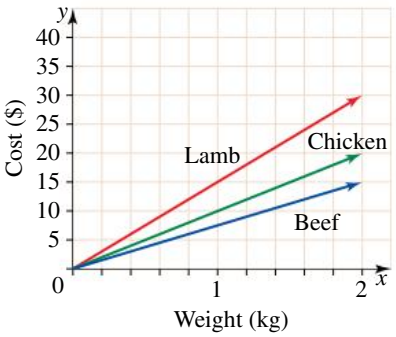
b What does a change in the y-value represent?

c What does a change in the x-value represent?

d Calculate the gradient for each straight line section of the graph using units.

e What does the gradient represent?

11 The price per kilogram for 3 different types of meat is illustrated in the graph.

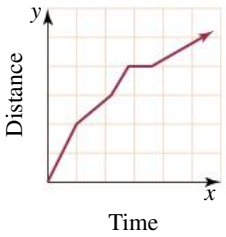


- **a** Calculate the gradient (using units) for each graph.
- **b** What is the cost of 1 kg of each type of meat?
- **c** What is the cost of purchasing:
  - **i** 1 kg of lamb?
  - **ii** 0.5 kg of chicken?
  - **iii** 2 kg of beef?
- **d** What is the total cost of the order in part **c**?
- **e** Copy and complete the table below to confirm your answer from part **d**.

Meat type	Cost per kilogram ($/kg)	Weight required (kg)	Cost = $/kg × kg
Lamb		1	
Chicken		0.5	
Beef		2	
		Total cost	

PROBLEM SOLVING

12 The graph represents the distance travelled by a vehicle versus time. Copy the graph onto some grid paper and, using red pen, show what the graph would look like if the vehicle at any point in time was travelling twice as fast.



13 Write a story for each of the graphs in question **9**.

14 Draw sketches to represent each of the stories below and explain your sketches.

- **a** The mass of a pig over its lifetime
- **b** The altitude of a plane during a flight
- **c** The height of water in a bath tub as it is emptied

eBookplus ONLINE ONLY

5.9 Review

www.jacplus.com.au

The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:

- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

Review questions

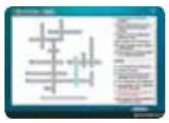
Download the Review questions document from the links found in your eBookPLUS.

eBookplus

Interactivities

Word search
int-2623

Crossword
int-2624



Sudoku
int-3185

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

common denominator
cross-multiplication
equivalent
gradient
horizontal distance
interpreting
proportion
rates
ratio
rise
run
share
simplify
simplest form
vertical distance

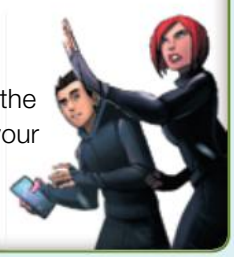
Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

www.assesson.com.au

Link to SpyClass, an exciting online game combining a comic book–style story with problem-based learning in an immersive environment.

Join Jesse, Toby and Dan and help them to tackle some of the world's most dangerous criminals by using the knowledge you've gained through your study of mathematics.



www.spyclass.com.au

INVESTIGATION

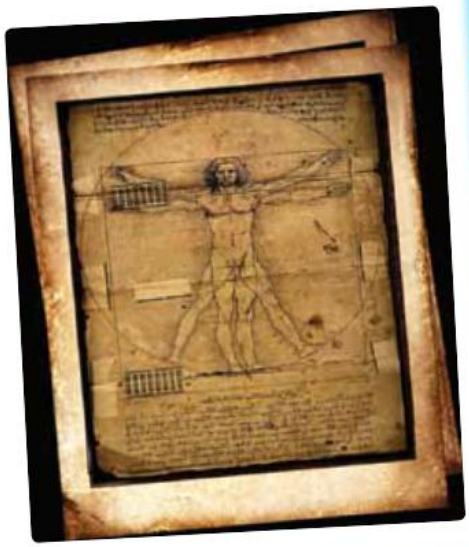
RICH TASK

The golden ratio

The Greeks believed that using a special ratio of numbers in building designs, paintings, sculpture etc. would automatically make them beautiful. This number is known as the golden ratio or golden number. The human body has many examples of the golden ratio.

$$\text{The golden number} = \frac{\sqrt{5}+1}{2}.$$

Work out this number as a decimal to 3 decimal places.



Part A

The golden ratio is often represented by phi (φ). One of the interesting relationships of this ratio to the design of the human body is that there are:

- five appendages on the torso: arms, legs and head
- five appendages on each of these: fingers, toes and five openings on the face
- five senses: sight, hearing, touch, taste and smell.

The golden number is also based on 5 because the number phi can be written as:

$$5^{0.5} \times 0.5 + 0.5.$$

Leonardo da Vinci's drawings of the human body emphasised its proportions. The ratios of the following distances equal the golden ratio:

- foot to navel : navel to head
- length of forearm : length of hand
- length of upper arm : length of hand and forearm.

Your task is to explore the golden ratio as it applies to your body.

Work in pairs to measure parts of your body. On the next page are some of the measurements you could take. Choose other parts of your body to measure and find as many golden ratios as possible.

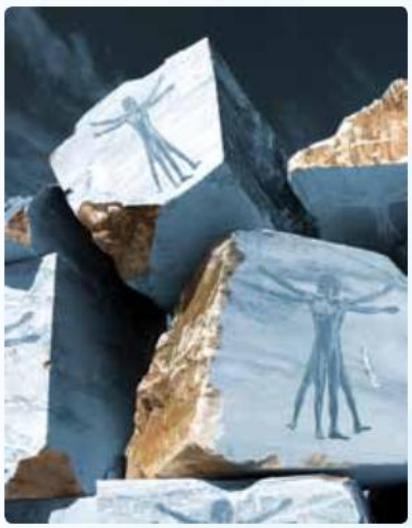
Body measurement (cm)	Ratio	Decimal value
Foot to navel : navel to head		
Length of forearm : length of hand		
Length of upper arm : length of hand and forearm		

Part B

Another way to find the golden ratio is using the Fibonacci sequence:

1, 1, 2, 3, 5, 8, 13, …

1. Write the first 20 terms of the sequence in the table.
2. Take the terms two at a time and divide the larger number by the smaller (e.g. divide 2 by 1, 3 by 2, 5 by 3 etc.), working as accurately as possible. Record your answers in the table.
3. Use the internet to discover other ways of finding the golden ratio.

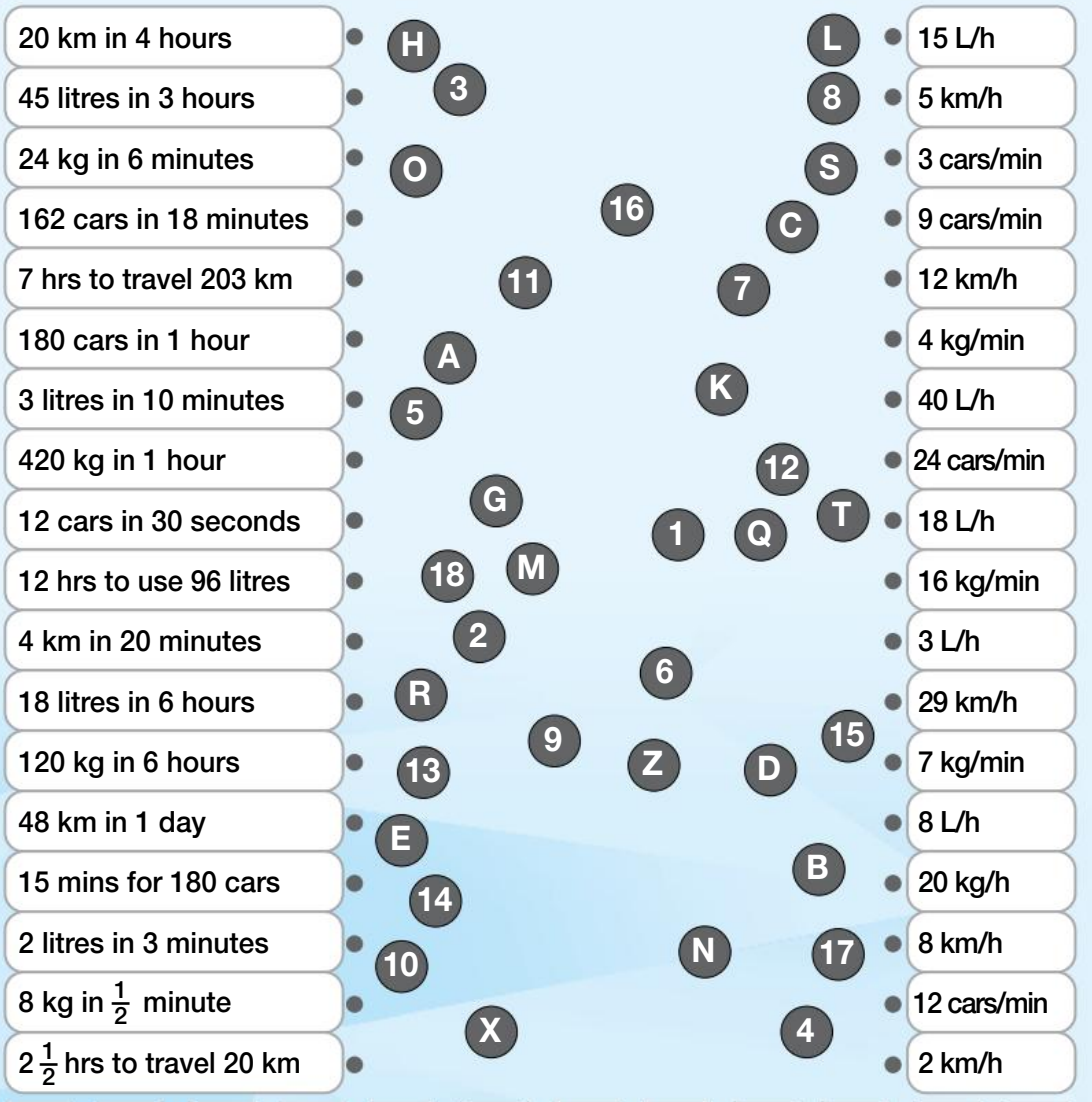


Sequence	Ratio	Decimal value
1	–	–
1	$\frac{1}{1}$	1
2		2
3	$\frac{3}{2}$	
5	$\frac{5}{3}$	$1.\dot{6}$
8		
13	$\frac{13}{8}$	1.625

CODE PUZZLE

Who created Sherlock Holmes?

Convert the information on the left of the page to rates. Join the dots next to the data and the corresponding rates with straight lines. Each line will pass through a number and letter to give the solution code.



eBookplus Activities

5.2 Introduction to ratios

Digital docs
- SkillSHEET (doc-6888) Converting units of length, capacity and time
- SkillSHEET (doc-6889) Highest common factor

Interactivities
- Fruit salad ratio (int-0076)
- IP interactivity 5.2 (int-4412) Introduction to ratios

5.3 Simplifying ratios

Digital docs
- SkillSHEET (doc-6890) Simplifying fractions
- SkillSHEET (doc-6891) Finding and converting to lowest common denominator
- Spreadsheet (doc-2213) Simplifying ratios

Interactivities
- IP interactivity 5.3 (int-4413) Simplifying ratios
- Simplifying ratios (int-3383)

5.4 Proportion

Digital docs
- Spreadsheet (doc-2214) Proportion
- WorkSHEET 5.1 (doc-2216)

Interactivity
- IP interactivity 5.4 (int-4414) Proportion

5.5 Comparing ratios

Digital doc
- Spreadsheet (doc-2215) Comparing ratios

Interactivity
- IP interactivity 5.5 (int-4415) Comparing ratios

5.6 Dividing in a given ratio

Digital doc
- Spreadsheet (doc-2489) Dividing in a given ratio

eLesson
- Dividing in ratios (eles-0041)

Interactivity
- IP interactivity 5.6 (int-4416) Dividing in a given ratio

5.7 Rates

Digital docs
- SkillSHEET (doc-6892) Converting a mixed number to an improper fraction
- SkillSHEET (doc-6893) Multiplying decimals by 10, 100 and 1000
- SkillSHEET (doc-6894) Multiplying a whole number by a fraction
- SkillSHEET (doc-6895) Converting minutes to a fraction of an hour
- WorkSHEET 5.2 (doc-2217)

Interactivity
- IP interactivity 5.7 (int-4417) Rates

5.8 Interpreting graphs

Interactivity
- IP interactivity 5.8 (int-4418) Interpreting graphs

5.9 Review

Interactivities
- Word search (int-2623)
- Crossword (int-2624)
- Sudoku (int-3185)

Digital docs
- Topic summary (doc-10757)
- Concept map (doc-10770)
- Topic review (Word doc-14932, PDF doc-14933)

To access eBookPLUS activities, log on to www.jacplus.com.au

Answers

TOPIC 5 Ratios and rates

5.2 Introduction to ratios

1 **a** 4 : 5 **b** 5 : 4 **c** 5 : 9 **d** 9 : 4 **e** 1 : 2
2 **a** 5 : 3 **b** 3 : 5 **c** 1 : 5 **d** 5 : 1 **e** 1 : 3
f 5 : 4 **g** 1 : 8 **h** 5 : 9 **i** 1 : 9 **j** 1 : 3
3 **a** 5 : 7 **b** 7 : 5 **c** 5 : 12
4 **a** 4 : 3 **b** 3 : 4 **c** 6 : 1 **d** 4 : 1 **e** 2 : 5
5 **a** 3 : 5 **b** 6 : 19 **c** 4 : 11
d 7 : 9 **e** 1 : 5 **f** 9 : 4
g 3 : 4 **h** 3 : 10 **i** 17 : 60
j 53 : 100 **k** 11 : 100 **l** 1 : 1000
m 1 : 2000 **n** 7 : 24 **o** 5 : 12
p 1000 : 27 **q** 7 : 12 **r** 13 : 24
s 3 : 5 **t** 1 : 22
6 **a** 3 : 59 **b** 59 : 38 **c** 38 : 3 **d** 3 : 97 **e** 59 : 41
7 **a** 24 : 17 **b** 21 : 17 **c** 2 : 1 **d** 4 : 39 **e** 4 : 1
f 1 : 1 **g** 1 : 2 **h** 3 : 1 **i** 9 : 1 **j** 1 : 3
k 1 : 6 **l** 1 : 4
8 **a** 215 : 179 **b** 215 : 36
9 **a** 97 : 3 **b** 3 : 100
10 **a** Yes (same units) **b** Yes (same units)
c No (different units) **d** No (different units)
e Yes (same units) **f** Yes (same units)
g No (different units) **h** Yes (same units)
11 **a** 17 : 83 **b** 97 : 3
12 1125 g
13 **a** 5 : 68 **b** 27 : 5 **c** 68 : 100
d $\frac{17}{25}$, 68% **e** $\frac{68}{5}$, 1360%
14 **a** Class 8A — 4 : 5; class 8B — 4 : 5; class 8C — 4 : 3
b Classes 8A and 8B
c 4 : 5 **d** 20 : 21 **e** No

Challenge 5.1

Fourteen coins need to be added to the purse. Three silver coins and 11 gold coins will change the ratio of gold to silver to 3 : 4.

5.3 Simplifying ratios

1 **a** 1 : 2 **b** 1 : 3 **c** 1 : 2 **d** 1 : 3 **e** 3 : 4
f 5 : 6 **g** 3 : 2 **h** 3 : 2 **i** 5 : 3 **j** 1 : 2
k 3 : 7 **l** 3 : 4 **m** 4 : 5 **n** 3 : 2 **o** 5 : 6
p 10 : 3 **q** 7 : 8 **r** 3 : 4 **s** 7 : 12 **t** 2 : 3
2 **a** 1 : 3
2 : 6
3 : 9
4 : 12
5 : 15
b 2 : 1
4 : 2
8 : 4
16 : 8
20 : 10
c 2 : 3
4 : 6
6 : 9
8 : 12
16 : 24
d 64 : 32
32 : 16
16 : 8
8 : 4
2 : 1
e 48 : 64
24 : 32
12 : 16
6 : 8
3 : 4
3 **a** 2 : 3 **b** 1 : 3 **c** 5 : 3 **d** 2 : 5 **e** 1 : 4
f 8 : 3 **g** 3 : 40 **h** 1 : 5 **i** 9 : 4 **j** 8 : 13
k 5 : 4 **l** 5 : 4 **m** 1 : 5 **n** 11 : 2 **o** 2 : 5
p 16 : 9 **q** 3 : 10 **r** 3 : 2 **s** 7 : 3 **t** 1 : 6
4 **a** 4 : 5 **b** 2 : 3 **c** 3 : 2 **d** 4 : 1 **e** 4 : 3
f 5 : 4 **g** 1 : 8 **h** 5 : 4 **i** 8 : 1 **j** 9 : 2
5 **a** 1 : 20 **b** 1 : 9 **c** 12 : 11 **d** 7 : 300
6 **a** 1 : 2 **b** 5 : 6 **c** 1 : 2 **d** 4 : 5 **e** 16 : 9
f 10 : 9 **g** 3 : 10 **h** 5 : 1 **i** 5 : 6 **j** 4 : 3
k 5 : 8 **l** 65 : 56
7 **a** 7 : 9 **b** 1 : 7 **c** 1 : 3 **d** 4 : 5 **e** 1 : 6
f 15 : 32 **g** 10 : 1 **h** 4 : 5 **i** 1 : 10 **j** 48 : 35
k 1 : 20 **l** 25 : 46
8 **a** $a : 5b$ **b** 2 : 1 **c** $2x : 3$ **d** $y : 2$ **e** $3m : 4$
f $1 : 4b$ **g** $10^2 : x^2$ **h** $2cd^2 : 1$
9 A **10** D **11** E **12** A **13** B
14 **a** 18 400 **b** 3 : 4 **c** $\frac{3}{4}$
d 42.86%. In part **c** the number of females was compared with the number of males (part-to-part comparison), whereas in part **d** the number of females was compared with the total number in the crowd (part-to-whole comparison).
15 **a** 5 : 8 **b** 2 : 3 **c** 3 : 5 **d** 4 : 3 **e** 4 : 13
16 **a** 929 : 321 **b** 217 : 22 **c** 9 : 16 **d** 11 : 9 **e** 6 : 29
17 **a** 85 : 103 **b** 50 : 71 **c** 2 : 3
d **i** $425 000 **ii** 1 : 2
e In a period of just over 6 years, the price of the house has increased by half its purchase price (in 2006).

5.4 Proportion

1 **a** Yes **b** Yes **c** Yes **d** Yes **e** No
f Yes **g** No **h** No **i** No **j** Yes
k No **l** Yes
2 **a** $a = 1$ **b** $a = 4$ **c** $a = 6$ **d** $a = 4$ **e** $a = 24$
f $a = 50$ **g** $a = 12$ **h** $a = 6$ **i** $a = 6$ **j** $a = 1$
k $a = 4$ **l** $a = 3$
3 **a** 9 boys **b** 10 m **c** 21 wins
d 24 chicken sandwiches **e** 5 litres **f** 7 tables
g 4 cups **h** 33 g **i** 75 cartons
j 3280 women
4 **a** 11.2 **b** 4.8 **c** 2.1 **d** 8.1 **e** 7.1
f 9.3 **g** 7.7 **h** 10.8 **i** 11.7 **j** 10.3
5 **a** $\frac{1}{3} = \frac{n}{5}$ $n = 1.7$ **b** $\frac{2}{6} = \frac{n}{11}$ $n = 3.7$ **c** $\frac{15}{4} = \frac{50}{n}$ $n = 13.3$
d $\frac{2}{17} = \frac{5}{n}$ $n = 42.5$ **e** $\frac{2}{3} = \frac{15}{n}$ $n = 22.5$
6 **a** No **b** No **c** No **d** Yes **e** Yes
f No **g** Yes **h** Yes **i** Yes **j** No
7 **a** C **b** E **c** A **d** D
8 The 14-year-old receives $28 and the 10-year-old receives $20.
9 **a** White gold $\frac{3}{4}$, pink gold $\frac{3}{4}$
b Because $\frac{18}{24} = \frac{3}{4}$. **c** 6 g **d** $210
10 4 : 5
11 9.6 : 20.8 = 6 : 13
12 1 : 20 000 = x cm : 2.4 km
1 : 20 000 = x cm : 2.4 × 1000 × 100 cm
1 : 20 000 = x : 240 000
$\frac{1}{20\,000} = \frac{x}{240\,000}$
$\frac{1}{20\,000} \times 240\,000 = \frac{x}{240\,000} \times 240\,000$
$x = 12$
So, 12 cm on the map represents 2.4 km on the ground.
13 **a** 10 : 4 : 3 **b** 2 : 9 : 11
14 0.14 m^3 cement, 0.29 m^3 sand, 0.57 m^3 gravel

5.5 Comparing ratios

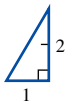
1 **a** 3 : 4 **b** 7 : 9 **c** 6 : 5 **d** 7 : 10 **e** 7 : 9
f 2 : 5 **g** 3 : 4 **h** 7 : 8 **i** 7 : 12 **j** 6 : 5

2 **a** Peninsula **b** Boroondara
c City West **d** DC North

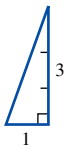
3 Jenny

4 **a** **i** 1 **ii** $\frac{2}{3}$ **iii** $\frac{2}{5}$ **iv** $\frac{3}{14}$
b AB **c** GH **d** iv, iii, ii, i

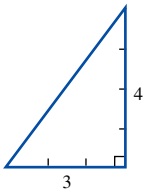
5 **a**



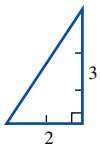
b



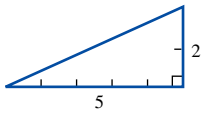
c



d



e

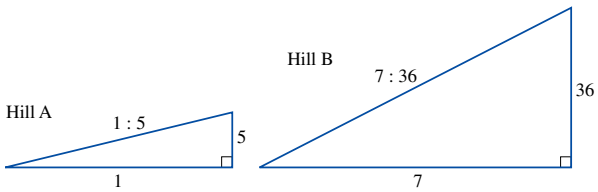


6 **a** C **b** A

7 **a** 3 : 4 : 5
b The ratio of the 3 sides stays the same.
c 1 m, 1.25 m

8 **a** $\frac{3}{15} = \frac{4}{20}$; same taste
b $\frac{3}{9} \neq \frac{5}{16}$; website 3 has the sweeter recipe.
c $\frac{2}{3} \neq \frac{5}{8}$; website 5 has the sweeter recipe.
d $\frac{3}{8} \neq \frac{7}{12}$; website 8 has the sweeter recipe.

9 **a**



Hill B is steeper. Diagrams are accurate only if a scale drawing is used and there is a significant difference in slope.
b Answers will vary. **c** Answers will vary.

5.6 Dividing in a given ratio

1 **a** 3 **b** 5 **c** 4 **d** 8 **e** 13
f 13 **g** 13 **h** 19 **i** 6 **j** 12

2 **a** \$400, \$600 **b** \$750, \$250
c \$200, \$800 **d** \$500, \$500
e \$375, \$625 **f** \$625, \$375
g \$300, \$700 **h** \$900, \$100
i \$350, \$650 **j** \$450, \$550

3 **a** \$5000, \$5000 **b** \$4000, \$6000
c \$6000, \$4000 **d** \$3000, \$7000
e \$7000, \$3000 **f** \$2000, \$8000
g \$9000, \$1000 **h** \$3750, \$6250
i \$4800, \$5200 **j** \$4600, \$5400

4 **a** \$10 000, \$40 000 **b** \$15 000, \$35 000
c \$20 000, \$30 000 **d** \$25 000, \$25 000
e \$12 500, \$37 500

5 **a** 2 m^3 **b** 0.5 m^3 **c** 1.6 m^3
d 1.6 m^3 **e** 1.2 m^3

6 **a** \$90 000, \$210 000, \$300 000
b \$180 000, \$180 000, \$240 000
c \$30 000, \$240 000, \$330 000
d \$150 000, \$180 000, \$270 000
e \$150 000, \$225 000, \$225 000

7 \$25, \$20, \$15

8 **a** 90 **b** 75

9 **a** 120 **b** 45 **c** \$6 **d** \$19.50

10 **a** D **b** D **c** A **d** D

11 30°, 60°, 90°

12 96°

13 **a** 50 **b** 4 **c** 4 : 1

14 **a** 1 : 300 **b** 1200 cm

15 **a** 1 **b** 37
c **i** 12 **ii** 49

5.7 Rates

1 **a** 10 m^2/min **b** 5 kL/min
c 300 cm^3/s **d** \$1.38/L
e 8 L/100 km or 12.5 km/L **f** \$2.50/m
g 40 cows/hectare or (250 m^2/cow) **h** \$12.50/person
i \$3.20/m^2 **j** 25 c/min
k 16 points/game **l** \$5.40/kg
m $8\frac{1}{3}$ m/s **n** 5.2 runs/over
o \$26.50/h **p** \$3.50/kg
q 52 words/min **r** 2 °C/h
s 6.5 cm/year **t** 16 km/h

2 \$108.50

3 396 points

4 60.5 L

5 21

6 70 min

7 12 days

8 49

9 1500 m^2

10 42

11 **a** $\frac{\text{distance}}{\text{time}}$ **b** $\frac{\text{volume}}{\text{time}}$ **c** $\frac{\text{capacity}}{\text{distance}}$
d $\frac{\text{money}}{\text{time}}$ **e** $\frac{\text{money}}{\text{length}}$ **f** $\frac{\text{capacity}}{\text{time}}$
g $\frac{\text{money}}{\text{capacity}}$ **h** $\frac{\text{money}}{\text{number}}$ **i** $\frac{\text{mass}}{\text{time}}$
j $\frac{\text{number}}{\text{area}}$

12 **a** mL/min **b** m/s **c** cm/year
d cm/h **e** mm^2/sec **f** L/km
g Runs/over **h** Words/min

13 360 L

14 Packs of 10

15 Car A

16 250 g jar; 75c

17 **a** C **b** C **c** D **d** B

18 **a** 4 patients per hour **b** 15 min
c \$62.50 per patient

19 4 min

20 $1\frac{7}{8}$ hours or 1 hour 52 minutes 30 seconds

21 2750 organisms/litre. The beach should not be closed.

22 **a** **i** $V = 1.098\,07 \times 10^{21}$ cm^3 **ii** 5464.13 kg/m^3
b **i** $V = 1.563\,46 \times 10^{24}$ cm^3 **ii** 1215.25 kg/m^3
c Earth and Jupiter do not seem to be made of the same substance, because Jupiter's density is much less than that of Earth even though Jupiter is much larger.

23 The 2005 race

Challenge 5.2

To work: 30 mins.
From work: $1\frac{1}{3}$ hours (1 hour and 20 minutes)
Total travel time: $1\frac{5}{6}$ hours (1 hour and 50 minutes)

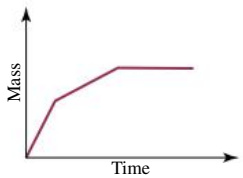
5.8 Interpreting graphs

1 The vertical axis is distance.
2 m/s
3 Horizontal line stretching for 60 minutes (with time measured horizontally)
4 Graphs can tell a story by showing how quantities of one variable differ in relation to another variable.
5 D
6 The graph shows how the student dawdles at first and the rate of movement is slow. Then there is no change at all as the student goes into the shop. Finally the speed increases as the student hurries to school.
7 B
8 The graph shows how the bus makes its way to the school. The graph is not a completely straight line because there would be unevenness in relation to traffic conditions and pick-up and drop-off stops.
9 **a** II **b** III **c** I
10 **a** Time (min) and cost ($)
b Change in cost
c Change in time
d $m = \$1$ per min, $m = 50$c per min, $m =$ free calls, i.e. 0c
e The gradient represents the increase in cost for each increase in time.
11 **a** Lamb: $m = \$15$/kg
Chicken: $m = \$10$/kg
Beef: $m = \$7.50$/kg
b Lamb: $15
Chicken: $10
Beef: $7.50
c **i** $15 **ii** $5 **iii** $15
d $35
e

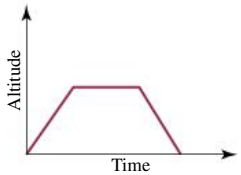
Meat type	Cost per kilogram ($/kg)	Weight required (kg)	Cost = $/kg × kg
Lamb	15	1	15
Chicken	10	0.5	5
Beef	7.50	2	15
		Total cost	35

12 Answers will vary.
Students should understand that the distance travelled at the end of each leg of the journey should not change but that the time taken to reach the end of each leg should be half the original time taken.
The amount of time taken for a rest is not affected and, as such, the overall amount of time taken for the whole journey will not be exactly half the original time.
13 Answers will vary.
14 Answers will vary. Example:

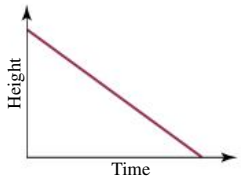
a



b



c



Investigation — Rich task

$\frac{\sqrt{5}+1}{2} = 1.618$

Part A: Teacher to check
Part B:
1 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765
2

Sequence	Ratio	Decimal value
1	—	—
1	$\frac{1}{1}$	1
2	$\frac{2}{1}$	2
3	$\frac{3}{2}$	1.5
5	$\frac{5}{3}$	$1.\dot{6}$
8	$\frac{8}{5}$	1.6
13	$\frac{13}{8}$	1.625
21	$\frac{21}{13}$	1.615 38
34	$\frac{34}{21}$	1.619 05
55	$\frac{55}{34}$	1.617 65
89	$\frac{89}{55}$	1.618 18
144	$\frac{144}{89}$	1.617 98
233	$\frac{233}{144}$	1.618 06
377	$\frac{377}{233}$	1.618 03
610	$\frac{610}{377}$	1.618 04
987	$\frac{987}{610}$	1.618 03
1597	$\frac{1597}{987}$	1.618 03
2584	$\frac{2584}{1597}$	1.618 03
4181	$\frac{4181}{2584}$	1.618 03
6765	$\frac{6765}{4181}$	1.618 03

Code puzzle

Sir Arthur Conan Doyle

TOPIC 6

Application of percentages

%
%
%

10%
0%
50%
10*
0%
%
0*
%
30%
50%
10%
0%
%
0%
10%

6.2 Percentages, fractions and decimals

- The term **per cent** means 'per hundred'.
- The symbol for percentage is %. For example, 60% means 60 parts out of 100 parts.
- Percentages, fractions and decimals are different ways of expressing the same quantity.
- Percentage is another way of writing a fraction with a denominator of 100, or of writing the number of hundredths in a decimal.

$$60\% = \frac{60}{100} = 0.60$$

- There are a number of common percentages, and their fraction and decimal equivalents, with which you should be familiar.

Percentage	Fraction	Decimal
50%	$\frac{1}{2}$	0.5
25%	$\frac{1}{4}$	0.25
75%	$\frac{3}{4}$	0.75
$33\frac{1}{3}\%$	$\frac{1}{3}$	$0.\dot{3}$
100%	1	1

WORKED EXAMPLE 1

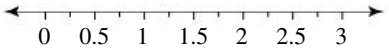
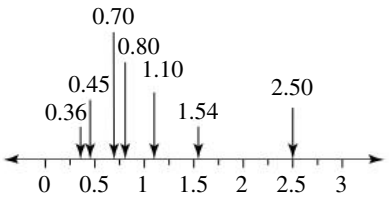
Convert the following percentages to fractions and then decimals.
a 67% **b 55%**

THINK	WRITE
a 1 To convert to a fraction, write the percentage, then change it to a fraction with a denominator of 100.	a $67\% = \frac{67}{100}$
2 To convert 67% to a decimal, think of it as 67.0%, then divide it by 100 by moving the decimal point two places to the left.	$67\% = 0.67$
b 1 To convert 55% to a fraction, write the percentage, then change it to a fraction by adding a denominator of 100.	b $55\% = \frac{55}{100}$
2 This is not in simplest form, so cancel by dividing the numerator and the denominator by 5.	$55\% = \frac{\cancel{55}}{\cancel{100}} = \frac{11}{20}$
3 To convert 55% to a decimal, think of it as 55.0%, then divide it by 100 by moving the decimal point two places to the left.	$55\% = 0.55$

- The easiest method of comparing percentages, fractions and decimals is to convert all of them to their decimal form and use place values to compare them.

WORKED EXAMPLE 2

Place the following quantities in ascending order, and then place them on a number line.
45%, $\frac{7}{10}$, 0.36, 80%, $2\frac{1}{2}$, 110%, 1.54

THINK	WRITE/DRAW
1 Convert all of the quantities into their decimal equivalents.	0.45, 0.7, 0.36, 0.80, 2.5, 1.10, 1.54
2 Place them in ascending order.	0.36, 0.45, 0.7, 0.80, 1.10, 1.54, 2.5
3 Place them in ascending order in their original form.	0.36, 45%, $\frac{7}{10}$, 80%, 110%, 1.54, $2\frac{1}{2}$
4 Draw a number line from 0 to 3, with increments of 0.25.	
5 Place the numbers on the number line.	

Percentage increases and decreases

- Percentage increases and decreases can be used to calculate and compare prices, mark ups, discounts, population changes, company profits and many other quantities.

WORKED EXAMPLE 3

Calculate the percentage increase when 52 increases to 72.

THINK	WRITE
1 The difference between 52 and 72 is 20.	$72 - 52 = 20$
2 The percentage increase can be calculated by creating the fraction 20 out of 52 and then multiplying by 100.	$\frac{20}{52} \times 100 = 38.46$
3 Write the answer.	The percentage increase is 38.46%.

WORKED EXAMPLE 4

Calculate the percentage decrease when the population of a town falls from 62 000 people to 48 000 people.

THINK	WRITE
1 The difference between 62 000 and 48 000 is 14 000.	$62\,000 - 48\,000 = 14\,000$
2 The percentage decrease can be calculated by creating the fraction 14 000 out of 62 000 and then multiplying by 100.	$\frac{14\,000}{62\,000} \times 100 = 22.58$
3 Write the answer.	The percentage decrease is 22.58%.

Percentage error

- Percentage error is used to compare the difference between an estimate of a quantity and the actual value. For example, manufacturers and scientists use percentage error to determine the reliability of their equipment and processes, and the validity of their experiments. The closer the percentage error is to zero, the better the estimate.

Calculating percentage error

- If the approximate value is greater than the exact value:

$$\text{Percentage error} = \frac{\text{approximate value} - \text{exact value}}{\text{exact value}} \times 100$$

- If the approximate value is less than the exact value:

$$\text{Percentage error} = \frac{\text{exact value} - \text{approximate value}}{\text{exact value}} \times 100$$

WORKED EXAMPLE 5

a The estimated weight of a newborn baby was 3500 grams, but the baby's actual weight was 4860 grams. Calculate the percentage error.
b The estimated distance between two towns was 70 km, but the actual distance was 65.4 km. Calculate the percentage error.

THINK	WRITE
a 1 The estimated weight was less than the actual weight.	**a** Percentage error $= \frac{\text{exact value} - \text{approximate value}}{\text{exact value}} \times 100$
2 Calculate the percentage error.	Percentage error $= \frac{4860 - 3500}{4860} \times 100$ $= 27.98\%$
3 Write the answer.	The percentage error is 27.98%.
b 1 The estimated distance was greater than the actual distance.	**b** Percentage error $= \frac{\text{approximate value} - \text{exact value}}{\text{exact value}} \times 100$
2 Calculate the percentage error.	Percentage error $= \frac{70 - 65.4}{65.4} \times 100$ $= 7.03\%$
3 Write the answer.	The percentage error is 7.03%.

assess on

Exercise 6.2 Percentages, fractions and decimals

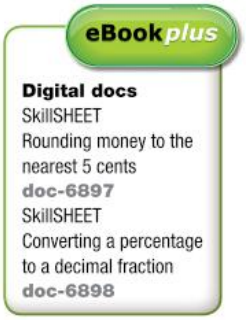
INDIVIDUAL PATHWAYS

REFLECTION
Where might it be necessary in daily life to convert between percentages and fractions or decimals?

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–11, 14, 16	Questions: 1–16	Questions: 1–17

Individual pathway interactivity int-4419 eBookplus

FLUENCY



1 Express each of the following fractions as a percentage.

a $\frac{7}{8}$ **b** $\frac{3}{5}$ **c** $\frac{5}{6}$ **d** $2\frac{1}{3}$

2 Express each of the following decimals as a percentage.

a 0.15 **b** 0.85 **c** 3.10 **d** 0.024

3 WE1 Express the following percentages as fractions in simplest form.

a 20% **b** 35% **c** 61% **d** 105%

4 WE1 Express the following percentages as decimals.

a 24% **b** 13% **c** 1.5% **d** 250%

UNDERSTANDING

5 WE2 For the following sets of numbers, write them in ascending order and then place them on a number line.

a 1.6, 25%, $\frac{7}{8}$, 75%, 10%, $3\frac{1}{2}$, 2.4 **b** $3\frac{4}{5}$, 330%, 4.5, 150%, 3, $2\frac{1}{3}$, 2.8

6 WE3 Calculate the percentage increase when 250 increases to 325.

7 WE4 Calculate the percentage decrease, correct to 2 decimal places, when the population of fish in a pond decreases from 1500 to 650.

8 Express \$120 as a percentage of \$400.

9 In a library, there are 24 children, 36 women and 42 men. What is the percentage of women visiting the library? Give your answer to 2 decimal places.

10 During a sale, a jacket originally priced at \$79.99 is decreased in price to \$55.99. What is the percentage decrease?

11 WE5 **a** The estimated grocery bill budgeted for the week was \$250, but the actual bill was \$262.20. Calculate the percentage error.

b A long-distance runner estimated her run took 120 minutes, but the official time recorded was 118.3 minutes. Calculate the percentage error.

REASONING

12 A group of students was practising their basketball free throws. Each student had four shots and the results are displayed in the table.

Free throw results	Number of students	Percentage of students
No shots in	3	
One shot in, three misses	11	
Two shots in, two misses	10	
Three shots in, one miss	4	
All shots in	2	

a How many students participated in the game?

b Complete the table to show the percentage of students for each result.

c How many students made exactly 25% of their shots?

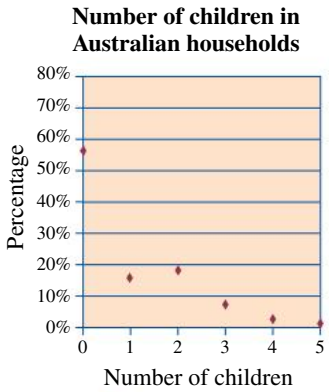
d What percentage of students made less than 50% of their shots?

13 WE5 In supermarkets, potatoes are frequently sold in 2 kg bags. As potatoes are discrete objects, the bags rarely weigh exactly 2 kg. For reasons relating to both customer satisfaction and profit, the warehouse supervisor knows that a percentage error of more than 10% is unacceptable.

Two bags of potatoes are chosen at random and weighed. Bag A weighs 2.21 kg and Bag B weighs 1.88 kg. Calculate the percentage error for each of these bags and determine if either or both will pass the inspection.

PROBLEM SOLVING

14 The graph at right shows the percentage of households with 0 to 5 children.



- **a** What percentage of these households have six or more children?
- **b** What percentage of these households have fewer than two children?
- **c** What fraction of these households have no children?
- **d** What fraction of these households have between one and three children?

15 Use the bunch of flowers shown to answer these questions.

- **a** What percentage of the flowers are yellow?
- **b** What fraction of the flowers are pink?
- **c** Write two of your own questions and swap with a classmate.

16 Survey your classmates on the brand of mobile phone that they have. Present your results in a table showing the percentage, fraction and decimal amount of each brand.

17 The Australian government capped the level of ethanol in petrol at 10%, because petrol with 20% or more ethanol may cause engine problems in some older vehicles.

- **a** Explain what is meant by the expression 'capped at 10%'.
- **b** What is the highest fraction of ethanol allowed in Australian petrol?
- **c** Above what fraction of ethanol can car engines experience problems?

CHALLENGE 6.1

The price of entry into a theme park has increased by 10% every year since the theme park opened. If the latest price rise increased the tickets to $8.80, what was the price of a ticket 2 years ago?

6.3 Finding percentages of an amount

- Sally wants to buy a book but is not sure whether she has enough money. The original price was $35 but the sale notice says that all books have been reduced by 20%.
- If you were the sales assistant, could you help her solve the problem?

To find a percentage of an amount (for example 20% of $35):

1. write the percentage as a fraction with a denominator of 100
2. change the 'of' to a ×
3. write the amount over one if it is not already a fraction
4. cancel
5. multiply the numerators and multiply the denominators
6. divide the numerator by the denominator.

WORKED EXAMPLE 6

Find 20% of 35.

THINK	WRITE
1 Write the question.	20% of 35
2 Write the percentage as a fraction with a denominator of 100, change the 'of' to a '×', write the amount as a faction over 1 and cancel.	$= \frac{20}{\cancel{100}^{20}} \times \frac{\cancel{35}^{7}}{1}$
3 Cancel again.	$= \frac{\cancel{20}^{1}}{\cancel{20}^{1}} \times \frac{7}{1}$
4 Multiply numerators and multiply denominators.	$= \frac{7}{1}$
5 Simplify by dividing the numerator by the denominator.	$= 7$

- If Sally has $30, does she have enough money to buy the book? The price has been reduced by $7. The sale price is $28 so she will have enough.

WORKED EXAMPLE 7

Find 46% of 75 and write the answer as a mixed numeral.

THINK	WRITE
1 Write the question.	46% of 75
2 Write the percentage as a fraction, change the 'of' to a '×', write the number as a faction over 1. Cancel.	$= \frac{46}{\cancel{100}^{4}} \times \frac{\cancel{75}^{3}}{1}$
3 Cancel again.	$= \frac{\cancel{46}^{23}}{\cancel{4}^{2}} \times \frac{3}{1}$
4 Multiply numerators and multiply denominators.	$= \frac{69}{2}$
5 Write the answer as a mixed numeral by dividing the denominator into the numerator.	$= 34\frac{1}{2}$

- To find a percentage of an amount using decimals:
  1. write the percentage as a decimal
  2. change the 'of' to × (multiplication)
  3. multiply.

WORKED EXAMPLE 8

Of the 250 students selected at random to complete a survey, 16% were in Year 11. How many students were in Year 11?

THINK	WRITE
1 Decide what percentage of the total is required and write an expression to find the percentage of the total.	16% of 250
2 Write the percentage as a decimal. Change the 'of' to a '×.'	$= 0.16 \times 250$
3 Multiply.	$= 40$
4 Answer the question by writing a sentence.	40 of the 250 students were in Year 11.

assess on

Exercise 6.3 Finding percentages of an amount

REFLECTION
In what situations would it be useful to use the shortcut methods for common percentages?

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–10, 14, 18, 24	Questions: 1a–e, 2a–l, 3a–i, 4, 5a–g, 6a–l, 7–18, 24–26	Questions: 1e–j, 2j–u, 3g–o, 4, 5i–l, 6i–r, 7–26

Individual pathway interactivity int-4420 eBookplus

FLUENCY

1 Copy each of the following problems, then find the answers by completing the working.

a $90\% \text{ of } 200 = \frac{90}{100} \times \frac{200}{1} =$

b $8\% \text{ of } 50 = \frac{8}{100} \times \frac{50}{1} =$

c $50\% \text{ of } 120 = \frac{50}{100} \times \frac{120}{1} =$

d $20\% \text{ of } 90 = \frac{20}{100} \times \frac{90}{1} =$

e $30\% \text{ of } 150 = \frac{30}{100} \times \frac{150}{1} =$

f $75\% \text{ of } 16 = \frac{75}{100} \times \frac{16}{1} =$

g $5\% \text{ of } 30 = \frac{5}{100} \times \frac{30}{1} =$

h $80\% \text{ of } 55 = \frac{80}{100} \times \frac{55}{1} =$

i $15\% \text{ of } 70 = \frac{15}{100} \times \frac{70}{1} =$

j $65\% \text{ of } 120 = \frac{65}{100} \times \frac{120}{1} =$

2 WE6 Find the following.

a 50% of 20
b 20% of 80
c 5% of 60
d 10% of 30
e 9% of 200
f 31% of 300
g 40% of 15
h 12% of 50
i 35% of 80
j 70% of 110
k 52% of 75
l 90% of 70
m 80% of 5000
n 44% of 150
o 68% of 25
p 24% of 175
q 38% of 250
r 95% of 200
s 110% of 50
t 150% of 8
u 125% of 20

3 WE7 Find the following and write the answer as a mixed numeral.

a 18% of 20	**b** 16% of 30	**c** 11% of 70	**d** 8% of 120
e 74% of 25	**f** 66% of 20	**g** 2% of 95	**h** 55% of 45
i 15% of 74	**j** 32% of 220	**k** 95% of 62	**l** 32% of 65
m 18% of 80	**n** 82% of 120	**o** 27% of 60	

4 MC **a** 45% written as a fraction is:

A $\frac{45}{100}$ **B** $\frac{45}{1}$ **C** $\frac{450}{1}$ **D** $\frac{1}{45}$

b When finding 17% of 22, the 'of' will be changed to:

A ÷ **B** from **C** + **D** ×

c Which of the following would find 15% of 33?

A 15 of 33 **B** $\frac{15}{1} \times 33$ **C** $\frac{15}{100} \times \frac{33}{100}$ **D** $\frac{15}{100} \times \frac{33}{1}$

d 60% of 30 is:

A $19\frac{4}{5}$ **B** $\frac{31}{5}$ **C** 186 **D** 18

5 For each of the following, express the percentage as a decimal first and then solve, remembering to round your answer to the nearest 5c.

a 15% of $12.00	**b** 15% of $8.00	**c** 15% of $20.00
d 15% of $60.00	**e** 25% of $30.00	**f** 25% of $45.00
g 25% of $90.00	**h** 25% of $220.00	**i** 30% of $15.00
j 30% of $25.00	**k** 30% of $47.50	**l** 30% of $102.20

6 Find 1% of the following. Round the answers to the nearest 5c.

a $268	**b** $713	**c** $573	**d** $604
e $5.60	**f** $12	**g** $13	**h** $14.80
i $21.70	**j** $81.75	**k** $19.89	**l** $429.50
m $4.25	**n** $6.49	**o** $9.99	**p** $0.24
q $0.77	**r** $1264.37		

7 Find the following. Round the answers to the nearest 5c.

a 12% of $11	**b** 21% of $50	**c** 11% of $30
d 3% of $22	**e** 6% of $40	**f** 22% of $10
g 13% of $14	**h** 35% of $210	**i** 12% of $150
j 9% of $17	**k** 2% of $53	**l** 7% of $29
m 45% of $71.50	**n** 33% of $14.50	**o** 42% of $3.80
p 31% of $1.45	**q** 64% of $22.50	**r** 41% of $1200

8 MC **a** 10% of $7.25 equals:

A $725 **B** $7.30 **C** $72.50
D $0.73 **E** $7250

b 1% of $31.48 equals:

A $3.14 **B** $0.31 **C** $0.32
D $31.50 **E** $0.03

c 15% of $124 equals:

A $18.60 **B** $1.24 **C** $6.20
D $13.64 **E** $15.24

d 22% of $5050 equals:

A $60.60 **B** $50.50 **C** $1111
D $43.56 **E** $1010

UNDERSTANDING

9 Maria is buying a new set of golf clubs. The clubs are marked at $950, but if Maria pays cash, the shop will take 10% off the marked price. How much will the clubs cost if Maria pays cash?

10 WE8 Thirty per cent of residents in the shire of Booroondara are over the age of 65. If there are 180 000 residents, how many are over the age of 65?

11 Jay is buying a new lounge suite worth $2150. Jay has to leave a 15% deposit and then pay the balance in monthly instalments. How much deposit does Jay have to pay?

12 Ninety per cent of students at a school were present for school photographs. If the school has 1100 students, how many were absent on the day the photographs were taken?

13 Jim can swim 50 m in 31 seconds. If he improves his time by 10%, what will Jim's time for 50 m be?

14 In a survey, 40 people were asked if they liked or disliked Vegemite. Of the people surveyed, 5% said they disliked Vegemite. How many people:

a disliked Vegemite? **b** liked Vegemite?

15 Thirty-two thousand four hundred people went to the Gabba to watch a Brisbane versus Collingwood football match. Of the crowd, 42% went to the game by car and 55% caught public transport. How many people:

a arrived by car? **b** caught public transport?

16 Explain how to use the shortcut method (that is, without using a calculator) to leave a $17\frac{1}{2}$% tip for a bill of $76.

17 If a train fare of $12 was increased by 10%, how much will you pay for the return trip? (Assume that the fare is the same each way.)

REASONING

18 When I am 5% older than I am now, I will be 21 years old. How old am I now?

19 The price of bread has increased to 250% of its price 20 years ago. If a loaf of bread costs $2.00 now, how much would it have cost 20 years ago? Show your working.

20 I am six months old. If I gain 10% of my current mass I will be three times my birth mass. If my birth mass was 3 kg, what is my mass now? Round your answer to one decimal place. Show your working.

21 I am 33 years old. I have lived in England for 8 years. If I stay in England, how old will I be when the number of years I have lived there is 50% of my age? Show your working.

22 My mother is four times older than I am. My sister is 75% of my age, and 10% of my grandfather's age. My father is 50, 2 years older than my mother. How old are my sister and grandfather? Justify your answer.

PROBLEM SOLVING

23 Broadcasting regulations specify that 55% of television programs shown between 6 pm and midnight must be Australian and that, between 6 pm and midnight, there should be no more than 13 minutes per hour of advertising.

- **a** How many minutes of advertising are allowed between 6 pm and midnight?
- **b** For how many minutes are programs screened between 6 pm and midnight?
- **c** What is the maximum percentage time spent screening advertising?
- **d** How many minutes of Australian content must be screened between 6 pm and midnight?

24 In a Maths competition, the top 8% of students across the state achieve a score of 40 or more out of a possible 50.

- **a** In a school where 175 students are entered in the Maths competition, how many scores of more than 40 would you expect?
- **b** In one school, there were 17 scores of 40 or more, and 204 scores that were less than 40. Did the students perform better than state average?

25 My mother is four times older than I am. My sister is 75% of my age and 10% of my grandmother's age. My father is 50, which is two years older than my mother. How old are my sister and grandmother?

26 I am 27 years old and have lived in Australia for 12 years. If I continue to live in Australia, how old will I be when the number of years I have lived here is 75% of my age?

eBookplus
Digital doc
WorkSHEET 6.1
doc-6912

6.4 Discount

- To get rid of old stock (for example out-of-date fashions at the end of a season), store managers often reduce prices by giving discounts.
- A discount is a reduction in price.
- A discount can be shown as an amount in dollars.

- A discount can be shown as a percentage of the marked price (that is, the price marked on the article).

- If the discount is expressed as a percentage, to find the actual amount of a discount, we calculate the percentage of the marked price by multiplying the marked price by the percentage. For example, a 10% discount on an item marked at $120 gives a discount amount of $12.

Calculating selling price of a discounted item

- Method 1
  - Use the percentage remaining after the percentage discounted has been subtracted from 100%; that is, if an item for sale has a 10% discount then the price must be 90% of the marked price.

WORKED EXAMPLE 9

Find the sale price on a hat marked $72 if a 10% discount is given.

THINK	WRITE
1 Find the percentage of the marked price that is paid, by subtracting the percentage discount.	100% − 10% = 90%
2 Find the sale price of the hat.	90% of $72 = 0.9 × $72 = $64.80
3 Write the answer in a sentence.	The sale price of the hat is $64.80.

- Method 2
  - The new sale price of the item can be solved by calculating the amount of the discount, then subtracting the discount from the marked price.
  - Alternative solution:

Discount = 10% of $72.00
= $7.20

Sale price = marked price − discount
= \$72.00 − \$7.20
= \$64.80

WORKED EXAMPLE 10

Peddles is a bicycle store that has offered a discount of 15% on all goods.
Find:
a the cash discount allowed on a bicycle costing \$260
b the sale price of the bicycle.

THINK	WRITE
a Find the discount, which is 15% of the marked price.	a Discount = 15% of \$260 = 0.15 × \$260 = \$39 The cash discount allowed is \$39.
b 1 To find the sale price, subtract the discount from the marked price.	b Sale price = marked price − discount = \$260 − \$39 = \$221
2 Answer the question in a sentence.	The sale price of the bicycle is \$221.

- To calculate the percentage discount, write the monetary amount as a percentage of the original price.

$$\text{Percentage discount} = \frac{\text{cash discount}}{\text{original price}} \times \frac{100}{1}\%$$

WORKED EXAMPLE 11

At Peddles, the price of a bicycle is reduced from \$260 to \$200. Calculate the percentage discount.

THINK	WRITE
1 Calculate the amount of the discount.	Discount = \$260 − \$200 = \$60
2 Write the discount as a percentage of the original price.	Percentage discount = $\frac{60}{260} \times 100\%$ = 23%
3 Answer the question in a sentence.	The percentage discount is about 23%.

Exercise 6.4 Discount

assess on

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7, 12, 13, 20, 27	Questions: 1–20, 25, 27	Questions: 1–27

Individual pathway interactivity int-4421 eBook plus

REFLECTION
How are discounts used to encourage people to purchase?

eBookplus

Digital docs
SkillSHEET
Decreasing a quantity by a percentage
doc-6899
SkillSHEET
Finding a percentage of a quantity (money)
doc-6900

FLUENCY

1 Calculate the discount on each of the items in the table, using the percentage shown.

	Item	Marked price	Discount
a	MP3 player	$210	20%
b	Skateboard	$185	25%
c	Rollerblades	$330	15%
d	Mobile phone	$190	40%

2 Without the use of a calculator, calculate the percentage discount for each of the following.

	Marked price	Discount
a	$100	$10
b	$250	$125
c	$90	$30
d	$80	$20

3 WE9 Find the sale price of each article when the marked price and discount are shown as in this table.

	Marked price (RRP)	Discount
a	$1000	15%
b	$250	20%
c	$95	12%
d	$156	$33\frac{1}{3}\%$
e	$69.95	$7\frac{1}{2}\%$

4 Decrease the amount by the percentages.

a $50 by 10% **b** $90 by 50% **c** $45 by 20%

5 Find the percentage discount given on the items shown in the table. Round to the nearest per cent.

	Original price	Selling price
a	$25	$15
b	$100	$72
c	$69	$50
d	$89.95	$70

UNDERSTANDING

6 A tablet computer was advertised with a saving of $148. Estimate the percentage discount being offered.

A$599
SAVE $148.00

7 The following items are all discounted.

$380
25% discount

$450
20% discount

$260
$33\frac{1}{3}$% discount

$600
15% discount

a Which has the largest dollar discount?

b Which have the same dollar discount?

c What is the difference between the largest and the smallest dollar discount?

d If the surfboard has a discount of 20%, would $470 be enough to buy it?

8 **WE10** A sale discount of 20% was offered by the music store Solid Sound. Find:

a the cash discount allowed on a $350 sound system

b the sale price of the system.

9 A wristwatch is advertised at $69.95, less 10% discount. Find the sale price.

10 A store-wide clearance sale advertised 15% off everything.

a What would be the selling price of a pair of jeans marked at $49?

b If a camera marked at $189 was sold for $160.65, was the correct percentage deducted?

11 T-shirts are advertised at $15.95 less 5% discount. How much would Jim pay for five T-shirts?

12 **WE11** Calculators were advertised at $20, discounted from $25. What percentage discount was given?

13 A tennis racquet marked at $79.95 sells for $60. What percentage discount is this, to the nearest whole number?

14 CDs normally selling for $28.95 were cleared for $23.95. What percentage discount was given (correct to 1 decimal place)?

15 A shirt was reduced from $90 to $63. Express the reduction as a percentage of the original price.

16 At a sale, Ann bought a $120 jacket for $48. What percentage of the original price did she save?

17 You bought a mobile phone priced $199.95 and signed up for a 1-year plan. You received a 10% discount on the telephone and 15% discount on the $75 connection fee. How much did you pay altogether (correct to the nearest 5 cents)?

18 Aanh bought two hairdryers for $128 each. She sold one at a loss of 5% and the other for a profit of 10%.

a Find the selling price of each.

b Will she have made a profit or a loss?

19 **MC** Kristen's car insurance was $670, but she had a 'No claim bonus' discount of 12%. Which of the following will not give the amount she must pay?

A First find 12% of $670 and add your answer to $670.

B Calculate $88 \div 100 \times 670$.

C Find 88% of $670.

D First find 12% of $670, and subtract your answer from $670.
E Calculate $0.12 \times \$670$ and subtract your answer from $670.

REASONING

20 Movie tickets sell for $12.00 each, but if you buy 4 or more you get $1.00 off each ticket. What percentage discount is this (correct to 2 decimal places)? Show your working. *Hint*: Find $1 as a percentage of $12.

21 MC I am allowed a discount of 10% off the total price of 6 articles that cost $\$x$ each. The price finally paid is:

A $\$60x$ **B** $\$5.4x$ **C** $\$0.06x$ **D** $\$0.6x$ **E** $\$6x$

Justify your answer.

22 You are in a surf shop and you hear 'For today only: take fifty per cent off the original price and then a further forty per cent off that.' You hear a customer say 'This is fantastic! You get ninety per cent off the original price!' Is this statement correct? Explain why.

23 Is there a difference between 75% off $200 and 75% of $200? Explain.

24 Henry buys a computer priced at $1060, but with a 10% discount. Sancha finds the same computer selling at $840 plus a tax of 18%. Who has the better buy? Explain.

PROBLEM SOLVING

25 A classmate was completing a discount problem where she needed to calculate a 25% discount on $79. She misread the question and calculated a 20% discount to get $63.20. She then realised her mistake and took a further 5% from $63.20. Is this the same as taking 25% on $79? Use calculations to support your answer.

26 A store had to increase its prices by 10% to cover increasing expenses. A particular DVD player was originally priced at $220. Use the questions below to help you calculate the new price of the DVD player using two different methods.

a i Calculate the cost increase and add it to the original price.
ii Add the percentage amount to 100% and multiply your answer by the original price.
b What do you notice about the answers to part **a**?
c Describe this alternative method in your own words.

27 What would you multiply the original prices of items by to get their new prices with:

a a 20% discount? **b** a 15% discount?
c a 25% increase? **d** a 5% increase?
e a 35% discount? **f** an 11% increase?
g a 6% discount? **h** a 100% increase?

eBookplus
Digital doc
Investigation
Successive discounts
doc-2228

6.5 Profit and loss

- When a manufacturer produces a product, it is usually sold to a wholesaler who subsequently sells the product on to retail outlets. At each stage the product is marked up by a certain percentage.

- When a retailer calculates the price to be marked on an article (the *selling price*, SP), many overhead costs must be taken into account (staff wages, rent, store improvements, electricity, advertising and so on).
- The total price the retail shop owner pays for the product including overhead costs is the *cost price*, CP.
- The *profit* is the difference between the total of the retailer's costs (cost price) and the price for which the goods actually sell (selling price).
  – If SP > CP, then a profit is made.

$$\text{Profit} = \text{selling price} - \text{cost price}$$

  – If SP < CP, then a loss is made.

$$\text{Loss} = \text{cost price} - \text{selling price}$$

Selling price

- To calculate the selling price of an item given the cost price and the percentage profit, increase the cost price by the given percentage.

$$\text{Selling price} = (100\% + \text{percentage profit}) \text{ of cost price}$$

- To calculate the selling price of an item given the cost price and the percentage loss, decrease the cost price by the given percentage.

$$\text{Selling price} = (100\% - \text{percentage loss}) \text{ of cost price}$$

WORKED EXAMPLE 12

Ronan operates a sports store at a fixed profit margin of 65%. For how much would he sell a pair of running shoes that cost him $40?

THINK	WRITE
1 Find the selling price by first adding the percentage profit to 100% then finding this percentage of the cost price.	Selling price = 165% of $40 = 1.65 × $40 = $66
2 Write the answer in a sentence.	The running shoes would sell for $66.

WORKED EXAMPLE 13

David bought a surfboard for $300 and sold it at a 20% loss a year later. What was the selling price?

THINK	WRITE
1 Find the selling price by first subtracting the percentage loss from 100% then finding this percentage of the cost price.	Selling price = 80% of $300 = 0.80 × $300 = $240
2 Write the answer in a sentence.	David sold the surfboard for $240.

- Profit or loss is usually calculated as a percentage of the cost price.

$$\text{Percentage profit} = \frac{\text{profit}}{\text{cost}} \times 100\%$$

$$\text{Percentage loss} = \frac{\text{loss}}{\text{cost}} \times 100\%$$

WORKED EXAMPLE 14

A music store buys CDs at $15 each and sells them for $28.95 each. What is the percentage profit made on the sale of a CD?

THINK	WRITE
1 Calculate the profit on each CD: selling price − cost.	Profit = $28.95 − $15 = $13.95
2 Calculate the percentage profit: $\frac{\text{profit}}{\text{cost}} \times 100\%$.	Percentage profit = $\frac{13.95}{15} \times 100\%$ = 93%
3 Write the answer in a sentence, rounding to the nearest per cent if applicable.	The profit is 93% of the cost price.

- Modern accounting practice favours calculating profit or loss as a percentage of the selling price. This is because commissions, discounts, taxes and other items of expense are commonly based on the selling price.

$$\text{Percentage profit} = \frac{\text{profit}}{\text{selling price}} \times 100\%$$

$$\text{Percentage loss} = \frac{\text{loss}}{\text{selling price}} \times 100\%$$

assessON

Exercise 6.5 Profit and loss

REFLECTION
How can you tell if an item is being sold for a profit or a loss?

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8, 11, 19	Questions: 1–15, 18, 19	Questions: 1–20

Individual pathway interactivity int-4422 eBookplus

eBookplus

Digital docs
SkillSHEET
Expressing one quantity as a percentage of another
doc-6901
SkillSHEET
Increasing a quantity by a percentage
doc-6902

Assume percentage profit or loss is calculated on the cost price unless otherwise stated.

FLUENCY

1 WE12,13 Find the selling price for each of the following:

	Cost price	%	Profit/loss
a	$18	40%	profit
b	$116	25%	loss

	Cost price	%	Profit/loss
c	$1300	30%	profit
d	$213	75%	loss
e	$699	$33\frac{1}{3}\%$	profit

2 **WE14** For each of the following items, find the percentage profit or loss.

	Cost price	Selling price
a	$15	$20
b	$40	$50
c	$40	$30
d	$75	$85
e	$38.50	$29.95

UNDERSTANDING

3 A supermarket buys frozen chickens for $3.50 each and sells them for $5.60. What is the percentage profit made on the sale of each chicken?

4 A restored motorbike was bought for $350 and later sold for $895.

a How much profit was made?

b What percentage was profit? Give your answer correct to the nearest whole number.

5 James' Second Hand Bookshop buys second hand books for $4.80 and sells them for $6.00.

a What is the ratio of the profit to the cost price?

b What is the percentage profit on the cost price?

c What is ratio of the profit to the selling price?

d What is the percentage profit on the selling price?

e Discuss how **a** and **b** are related.

6 A retailer bought a laptop for $1200 and advertised it for $1525.

a How much profit was made?

b What is the percentage profit (to the nearest whole number) on the cost price?

c What is the percentage profit (to the nearest whole number) on the selling price?

d Compare the differences between the answers to **b** and **c**.

7 Rollerblades bought for $139.95 were sold after six months for $60.

a How much was the loss?

b What was the percentage loss? Give your answer to the nearest whole number.

8 A sports card collection costing $80 was sold for $65. What was the percentage loss?

9 Running shoes bought by a sports store for $30 per pair were sold at $79.95. What percentage profit was made?

10 Kyle runs a jewellery business that uses a fixed profit margin of 98%. For how much would he sell a necklace that cost him $830?

11 Find the selling price for each item.

- **a** Jeans costing $20 are sold with a profit margin of 95%.
- **b** A soccer ball costing $15 is sold with a profit margin of 80%.
- **c** A sound system costing $499 is sold at a loss of 45%.
- **d** A skateboard costing $30 is sold with a profit margin of 120%.

12 A fruit-and-veg shop bought 500 kg of tomatoes for $900 and sold them for $2.80 per kg.

- **a** What is the profit per kilogram?
- **b** Calculate the profit as a percentage of the cost price (round to 1 decimal place).
- **c** Calculate the profit as a percentage of the selling price (round to 1 decimal place).
- **d** Compare the answers to parts **b** and **c**.

13 Sonja bought an old bike for $20. She spent $47 on parts and paint and renovated it. She then sold it for $115 through her local newspaper. The advertisement cost $10.

- **a** What were her total costs?
- **b** What percentage profit (to the nearest whole number) did she make on costs?
- **c** What percentage profit (to the nearest whole number) did she make on the selling price?

14 MC A clothing store operates on a profit margin of 150%. The selling price of an article bought for $\$p$ is:

A $\$151p$ **B** $\$150p$ **C** $\$2.5p$ **D** $\$1.5p$ **E** $\$0.15p$

REASONING

15 A fruit and vegetable retailer buys potatoes by the tonne for $180 and sells them in 5-kg bags for $2.45. What percentage profit does he make (to the nearest whole number)? Show your working.

16 Two business partners bought a business for $158 000 and sold it for $213 000. The profit was to be shared between the two business partners in the ratio of 3 : 2. What percentage share (to the nearest whole number) does each person receive? How much does each receive?

17 What is the maximum discount a retailer can offer on her marked price of $100 so that she ends up selling at no profit and no loss, if she had initially marked her goods up by $50? Justify your answer.

PROBLEM SOLVING

18 To produce a set of crockery consisting of a dinner plate, soup bowl, bread plate and coffee mug, the costs per item are $0.98, $0.89, $0.72 and $0.69 respectively. These items are packaged in boxes of 4 sets and sell for $39. If the company sells 4000 boxes in a month, what is their total profit?

19 Copy and complete the table below.

Cost per item	Items sold	Sale price	Total profit
$4.55	504	$7.99	
$20.00		$40.00	$8040.00
$6.06	64 321		$225 123.50
	672	$89.95	$28 425.60

20 The method used to calculate profits can make a difference when comparing different profits.

Cost = $20.00
Price = $120.00

Cost = $26 500.00
Price = $32 000.00

Cost: $1.00 (Homemade)
Price: $3.50

a i Describe the profits on each of the items above as a raw amount.
ii List the items from largest profit to smallest profit.
iii Discuss whether this is a fair method of comparing the profits.
b i Express the profit on each of the items as a percentage of its cost.
ii List the items from largest profit to smallest profit.
iii Discuss whether this is a fair method of comparing the profits.
c i Express the profit on each of the items as a percentage of its price.
ii List the items from largest profit to smallest profit.
iii Discuss whether this is a fair method of comparing the profits.

eBookplus
Digital doc
WorkSHEET 6.2
doc-6913

CHALLENGE 6.2

Max bought a car for $6000.00. He sold it to Janine for 80% of the price he paid for it. Janine sold it to Jennifer at a 10% loss. Jennifer then sold it to James for 75% of the price she paid. What did James pay for the car? What was the total percentage loss on the car from Max to James?

6.6 Goods and Services Tax (GST)

- **GST**: This is a tax levied by the Australian federal government on goods and services. (As with all taxes, there are exemptions, but these will not be considered here.)

- **Goods**: A tax of 10% is added to new items that are purchased, such as some foods, petrol and lollies.
- **Services**: A tax of 10% is added to work (services) that is paid for, such as work performed by plumbers, painters and accountants.

WORKED EXAMPLE 15

A packet of potato chips costs $1.84 before GST. Find:
a the GST charged on the packet of chips
b the total price the customer has to pay.

THINK	WRITE
a GST is 10%.	10% of $1.84 = $0.18 cents (rounded)
b Total equals GST plus pre-GST price.	$1.84 + $0.18 = $2.02 (rounded up by the seller to $2.05)

- To find the pre-GST amount when the total you are given includes GST, divide the GST-inclusive amount by 110 and multiply by 100.

WORKED EXAMPLE 16

A plumber's hourly charge includes GST. If she worked for 5 hours and the total bill including GST was $580, what was her hourly price before GST?

THINK	WRITE
1 Find the hourly price including GST.	$\frac{\$580}{5} = \$116$
2 Find the hourly price excluding GST.	110% of pre-GST hourly rate $= \$116$ 100% of pre-GST hourly rate $= \frac{\$116}{110} \times \frac{100}{1}$ The plumber's hourly rate is $105.45 before GST.

assess on

Exercise 6.6 Goods and Services Tax (GST)

REFLECTION
What do the terms *inclusive* and *exclusive of GST* mean?

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–13, 15	Questions: 1–15	Questions: 1–16

Individual pathway interactivity int-4423 eBookplus

FLUENCY

1 Explain the GST in your own words.
2 Does GST apply below? Answer yes or no for each example.

a Petrol
b A lawyer's fee
c Hotel accommodation
d Lounge room carpet
e Floor tiling
f Wages at a fast-food restaurant

3 **WE15** With or without a calculator:
i calculate the GST payable on each of the following pre-GST prices
ii calculate the total price including GST.
a 500 g laundry powder at $4.50
b 400-g tin of canned peaches at $2.12

4 **WE16** The prices below are inclusive of GST. What is the pre-GST price of each?
a 1 kg apples at $3.85
b 500 g laundry powder at $4.50
c 400-g tin of canned peaches at $2.20
d 5 kg potatoes at $6.50

5 The telephone company Ringtel charges home customers $42.50 per month plus $0.24 per local call. Find the monthly phone bill, including GST, if a customer makes 51 local calls in a month.

6 All car rental agencies use similar charging plans. Drivo charges $44 per day plus $0.47 per kilometre travelled. A customer wishes to rent a car for four days and travels 1600 km. What is her total bill, including GST?

7 Expresso is a company that operates in the 'we-visit-you' car repair business. It charges $85 per hour plus a flat $40 visiting fee.
a Set up an expression, which includes GST, for the cost of a repair that takes t hours.
b If the repair takes 3 hours and 30 minutes, what is the final cost?

8 A company that installs floor tiles charges $35 per square metre for the actual tiles, and a fee of $100 plus $10 per square metre to install the tiles in a home. Let the area of the floor to be tiled be x m^2.
a Find an expression, including GST, that represents the total cost of tiling in terms of x.
b What would be the total cost for a 20 m^2 floor?

9 A new bicycle costs $450, including GST. How much is the GST?

10 To buy my new super-dooper mobile phone outright I must pay $30 per month, including GST, for 3 years. How much GST will I pay?

11 I buy a pack of batteries and pay 25 cents GST. How much did I pay in total for the batteries?

UNDERSTANDING

12 In the United Kingdom a similar tax, called the Value Added Tax or VAT, is levied at 20%. If I paid A$67 for a jumper purchased in a shop in Bond Street, London:
a how much VAT did I pay?
b what was the pre-VAT price of the jumper?

13 In New Zealand GST is levied at 15% of the purchase price of goods. If I buy a pair of jeans and pay NZ$12 in GST, what total price did I pay for the jeans in NZ dollars?

REASONING

14 The Goods and Services Tax or GST rate is 10%. This means that when a business sells something or provides a service it must charge an extra $\frac{1}{10}$ of the price/cost. That extra money then must be sent to the tax office. For example, an item that would

otherwise be worth $100 now has GST of $10 added. So the price tag will show $110. The business will then send to the tax office that $10 with all the other GST it has collected on behalf of the government.

a Suppose a shopkeeper made sales totalling $15 400. How much GST must he put aside?

b Is there a number he can quickly divide by to figure out the GST?

PROBLEM SOLVING

15 In the country Snowdonia, GST is 12.5%. Igor's girlfriend Karla has purchased a new hair drier that cost her in total, including GST, 111 Kopeks. There are 100 Plens in 1 Kopek.

a How much GST did Karla pay?

b If 1 Australian dollar = 2 Kopeks, how much GST would Karla have paid if she had purchased the hair drier in Melbourne, where GST is currently 10%?

16 Taking GST to be 10%:

a what is the GST payable on an item whose pre-GST price is $\$P$, and what is the price payable?

b What is the pre-GST price of an item for which I paid $\$A$, and how much GST did I pay?

eBookplus ONLINE ONLY

6.7 Review

www.jacplus.com.au

The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:

- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

Review questions

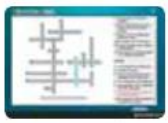
Download the Review questions document from the links found in your eBookPLUS.

eBookplus

Interactivities

Word search
int-2625

Crossword
int-2626



Sudoku
int-3186

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

cost price	marked price	percentage profit
discount	overhead costs	profit
GST	percentage discount	sale price
loss	percentage loss	selling price

Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

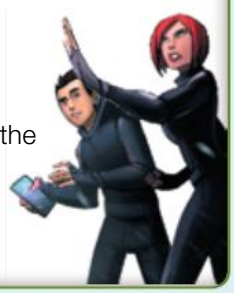
assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

www.assesson.com.au

Link to SpyClass, an exciting online game combining a comic book–style story with problem-based learning in an immersive environment.

Join Jesse, Toby and Dan and help them to tackle some of the world's most dangerous criminals by using the knowledge you've gained through your study of mathematics.

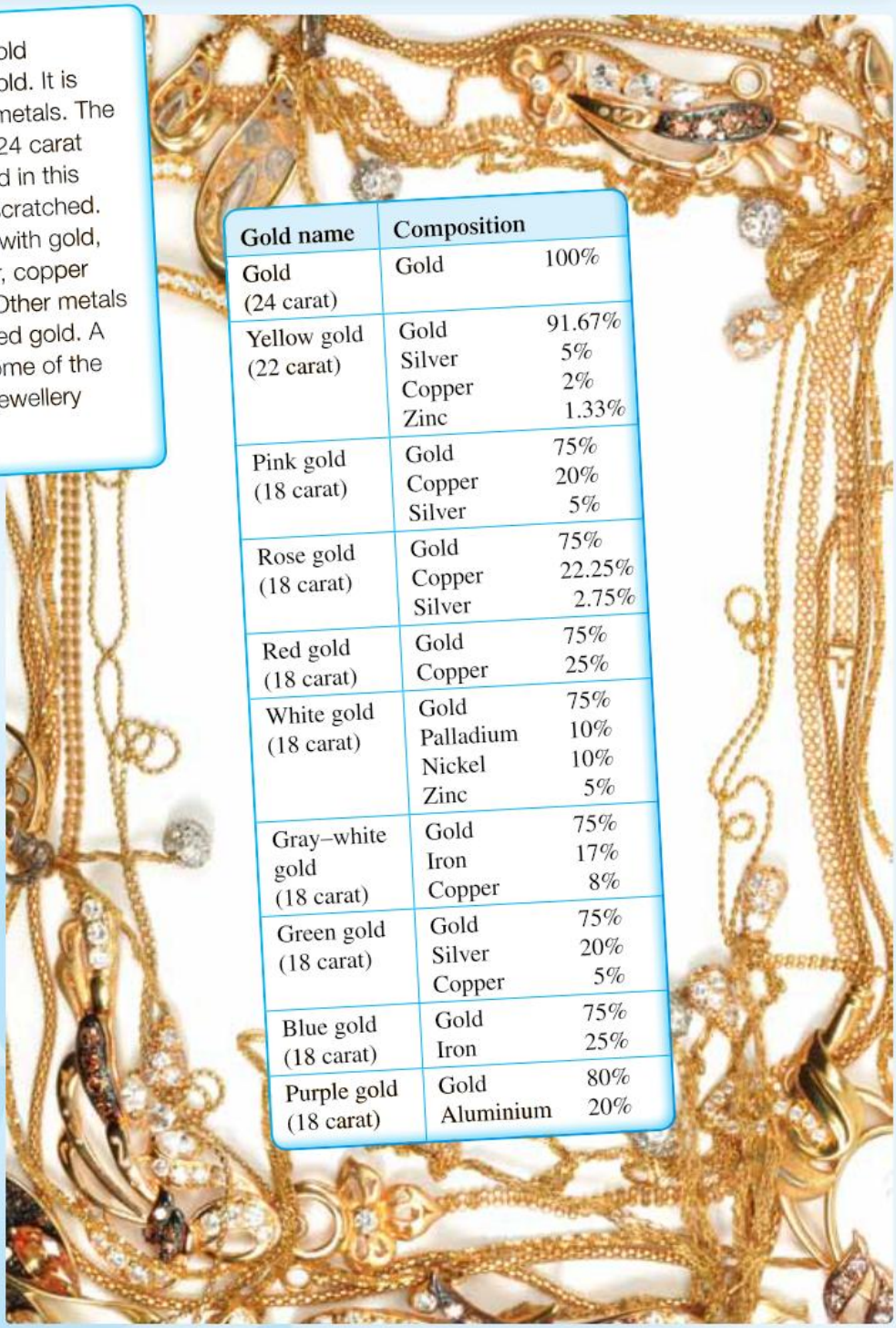
www.spyclass.com.au



RICH TASK

The composition of gold in jewellery

You may be aware that most gold jewellery is not made of pure gold. It is actually an alloy, or mixture of metals. The finest gold used in jewellery is 24 carat and is known as fine gold. Gold in this form is very soft and is easily scratched. Most metals will form an alloy with gold, the most common being silver, copper and zinc in jewellery making. Other metals may be used to create coloured gold. A table of the composition of some of the common gold alloys used in jewellery pieces is shown at right.



Gold name	Composition	
Gold (24 carat)	Gold	100%
Yellow gold (22 carat)	Gold Silver Copper Zinc	91.67% 5% 2% 1.33%
Pink gold (18 carat)	Gold Copper Silver	75% 20% 5%
Rose gold (18 carat)	Gold Copper Silver	75% 22.25% 2.75%
Red gold (18 carat)	Gold Copper	75% 25%
White gold (18 carat)	Gold Palladium Nickel Zinc	75% 10% 10% 5%
Gray–white gold (18 carat)	Gold Iron Copper	75% 17% 8%
Green gold (18 carat)	Gold Silver Copper	75% 20% 5%
Blue gold (18 carat)	Gold Iron	75% 25%
Purple gold (18 carat)	Gold Aluminium	80% 20%

Use the table to answer the following questions.

1 Study the table and list the metals used to create the alloys of gold mentioned.

2 A particular rose-gold bracelet weighs 36 grams. Calculate the masses of the various components in the bracelet.

3 How much more gold would be in a yellow-gold bracelet of the same mass? What fraction is this of the mass of the bracelet?

4 Pink, rose and red gold all contain 75% gold. In addition, they each contain copper, and pink and rose gold also contain silver. Describe the effect you feel the composition of the alloy has on the colour of the gold.

5 Why does white gold not contain any copper?

6 Compare the composition of the alloys in red gold and blue gold.

7 24-carat gold is classed as 100% gold. On this basis, an alloy of gold containing 75% gold has a carat value of 18 carat. Note this fact in the table above. The purple gold is 80% gold. What would its carat value be?

8 Just as there are various qualities of gold used in jewellery making, the same is true of silver jewellery. Sterling silver, which is commonly used, is actually not pure silver. Find out about the composition of silver used in jewellery making. Write a short report on your findings on a separate sheet of paper.

CODE PUZZLE

What did the grass say to the dirt?

The answer to each percentage question below, and the letter beside it, give the puzzle's solution code.

A 35% of $200	**B** 80% of 70kg	**C** $33\frac{1}{3}$% of 2h	**D** 62% of 1.5m
E 75% of 60min	**F** 20% of $250	**H** 150% of 28cm	**L** 5% of 380kg
M 90% of 50kg	**N** $12\frac{1}{2}$% of 96cm	**O** 8% of 10h	**Q** $6\frac{1}{2}$% of $1000	**R** 15% of 2m
S $66\frac{2}{3}$% of 45min	**T** 25% of $360	**U** 7% of 600kg	**X** 18% of $350

$70	93cm	93cm	$70	19kg	45min	$90	$90	45min	30cm	$90	48min

40min	12cm	45kg	,	30min	19kg	12cm	42kg	93cm	.	42cm	,	42kg	93cm

$50	12cm	30min	$63	12cm	$90	56kg	12cm	42kg	93cm	$65	93cm	40min .

eBookplus Activities

6.2 Percentages, fractions and decimals

Digital docs

- SkillSHEET (doc-6897) Rounding money to the nearest 5 cents
- SkillSHEET (doc-6898) Converting a percentage to a decimal fraction

Interactivity

- IP interactivity 6.2 (int-4419) Percentages, fractions and decimals

6.3 Finding percentages of an amount

Digital doc

- WorkSHEET 6.1 (doc-6912)

Interactivity

- IP interactivity 6.3 (int-4420) Finding percentages of an amount

6.4 Discount

Digital docs

- SkillSHEET (doc-6899) Decreasing a quantity by a percentage
- SkillSHEET (doc-6900) Finding a percentage of a quantity (money)
- Investigation (doc-2228) Successive discounts

Interactivity

- IP interactivity 6.4 (int-4421) Discount

6.5 Profit and loss

Digital docs

- SkillSHEET (doc-6901) Expressing one quantity as a percentage of another
- SkillSHEET (doc-6902) Increasing a quantity by a percentage
- WorkSHEET 6.2 (doc-6913)

Interactivity

- IP interactivity 6.5 (int-4422) Profit and loss

6.6 Goods and Services Tax (GST)

Interactivity

- IP interactivity 6.6 (int-4423) Goods and Services Tax (GST)

6.7 Review

Interactivities

- Word search (int-2625)
- Crossword (int-2626)
- Sudoku (int-3186)

Digital docs

- Topic summary (doc-10758)
- Concept map (doc-10771)
- Topic review (Word doc-14934, PDF doc-14935)

To access eBookPLUS activities, log on to www.jacplus.com.au

Answers

TOPIC 6 Application of percentages

6.2 Percentages, fractions and decimals

1 **a** 87.5%
b 60%
c 83.33% (correct to 2 decimal places)
d 233.33% (correct to 2 decimal places)

2 **a** 15% **b** 85% **c** 310% **d** 2.4%

3 **a** $\frac{1}{5}$ **b** $\frac{7}{20}$ **c** $\frac{61}{100}$ **d** $\frac{21}{20}$

4 **a** 0.24 **b** 0.13 **c** 0.015 **d** 2.50

5 **a** 10%, 25%, 75%, $\frac{7}{8}$, 1.6, 2.4, $3\frac{1}{2}$
b 150%, $2\frac{1}{3}$, 2.8, 3, 330%, $3\frac{4}{5}$, 4.5

6 30%

7 56.67%

8 30%

9 35.29%

10 30%

11 **a** 4.65% **b** 1.44%

12 **a** 30
b

Free throw results	Number of students	Percentage of students
No shots in	3	10%
One shot in, three misses	11	$36.\dot{6}\%$
Two shots in, two misses	10	$33.\dot{3}\%$
Three shots in, one miss	4	$13.\dot{3}\%$
All shots in	2	$6.\dot{6}\%$

c 11 **d** $46.\dot{6}\%$

13 Bag B will pass but Bag A will not pass.

14 **a** 0% **b** 72% **c** $\frac{14}{25}$ **d** $\frac{41}{100}$

15 **a** 38% **b** $\frac{8}{21}$ **c** Answers will vary.

16 Answers will vary.

17 **a** The expression 'capped at 10%' means that petrol can contain a maximum of 10% ethanol.
b $\frac{1}{10}$ **c** $\frac{1}{5}$

Challenge 6.1

$7.27

6.3 Finding percentages of an amount

1 **a** 180 **b** 4 **c** 60 **d** 18 **e** 45
f 12 **g** $1\frac{1}{2}$ **h** 44 **i** $10\frac{1}{2}$ **j** 78

2 **a** 10 **b** 16 **c** 3 **d** 3 **e** 18
f 93 **g** 6 **h** 6 **i** 28 **j** 77
k 39 **l** 63 **m** 4000 **n** 66 **o** 17
p 42 **q** 95 **r** 190 **s** 55 **t** 12
u 25

3 **a** $3\frac{3}{5}$ **b** $4\frac{4}{5}$ **c** $7\frac{7}{10}$ **d** $9\frac{3}{5}$ **e** $18\frac{1}{2}$
f $13\frac{1}{5}$ **g** $1\frac{9}{10}$ **h** $24\frac{3}{4}$ **i** $11\frac{1}{10}$ **j** $70\frac{2}{5}$
k $58\frac{9}{10}$ **l** $20\frac{4}{5}$ **m** $14\frac{2}{5}$ **n** $98\frac{2}{5}$ **o** $16\frac{1}{5}$

4 **a** A **b** D **c** D **d** D

5 **a** \$1.80 **b** \$1.20 **c** \$3.00 **d** \$9.00 **e** \$7.50
f \$11.25 **g** \$22.50 **h** \$55.00 **i** \$4.50 **j** \$7.50
k \$14.25 **l** \$30.65

6 **a** \$2.70 **b** \$7.15 **c** \$5.75 **d** \$6.05 **e** \$0.05
f \$0.10 **g** \$0.15 **h** \$0.15 **i** \$0.20 **j** \$0.80
k \$0.20 **l** \$4.30 **m** \$0.05 **n** \$0.05 **o** \$0.10
p \$0.00 **q** \$0.00 **r** \$12.65

7 **a** \$1.30 **b** \$10.50 **c** \$3.30 **d** \$0.65 **e** \$2.40
f \$2.20 **g** \$1.80 **h** \$73.50 **i** \$18.00 **j** \$1.55
k \$1.05 **l** \$2.05 **m** \$32.20 **n** \$4.80 **o** \$1.60
p \$0.45 **q** \$14.40 **r** \$492.00

8 **a** D **b** B **c** A **d** C

9 \$855

10 54 000 residents

11 \$322.50

12 110 students

13 27.9 seconds

14 **a** 2 people **b** 38 people

15 **a** 13 608 people **b** 17 820 people

16 10% + 5% + $2\frac{1}{2}$% = \$7.60 + \$3.80 + \$1.90 = \$13.30

17 \$26.40

18 20 years old

19 \$0.80

20 8.2 kg

21 50 years old

22 9 years old, 90 years old

23 **a** 78 minutes
b 282 minutes or 4 hours 42 minutes
c $21\frac{2}{3}$%
d 155.1 minutes or 155 minutes 6 seconds

24 **a** 14
b 7.69% of students achieved a score of 40 or more, which is just below the state average.

25 Sister: 9; grandmother: 90

26 60

6.4 Discount

1 **a** \$42 **b** \$46.25 **c** \$49.50 **d** \$76

2 **a** 10% **b** 50% **c** $33\frac{1}{3}$% **d** 25%

3 **a** \$850 **b** \$200 **c** \$83.60 **d** \$104 **e** \$64.70

4 **a** \$45 **b** \$45 **c** \$36

5 **a** 40% **b** 28% **c** 28% **d** 22%

6 Estimate 25%

7 **a** Mobile phone \$95
b Surfboard and bike
c \$8.35 **d** No

8 **a** \$70 **b** \$280

9 \$62.96

10 **a** \$41.65 **b** Yes

11 \$75.76

12 20%

13 25%

14 17.3%

15 30%

16 60%

17 \$243.70

18 **a** \$121.60 \$140.80 **b** Gain

19 A

20 \$1.00/\$12.00 × 100% = 8.33%, so this is a 8.33% discount.

21 B

22 No, the statement is not correct. For example, if you have a cost of \$100, a 50% discount = \$50 and a 40% discount = \$20. Total discount = \$70; this represents a 70% discount, not 90%.

23 Yes (difference in the meanings) 75% off \$200 = \$150 off the price so would pay only \$50.
75% of \$200 = \$150, i.e. $\frac{3}{4}$ of \$200.

24 Henry pays \$954; Sancha pays \$991.20. Henry has the best buy.

25 95% of \$63.20 = \$60.05; 75% of \$79 = \$59.25. The two methods calculate percentages of different amounts so result in different answers.

26 **a** **i** 10% of \$220 = \$22
\$220 + \$22 = \$242
ii 110% of \$220 = \$242
b The answers are the same.
c The price increase is 10%. Add this to 100% to get 110% and then multiply 110% by the original price to give the new price.

27 **a** 80% **b** 85% **c** 125% **d** 105%
e 65% **f** 111% **g** 94% **h** 200%

6.5 Profit and loss

1 **a** \$25.20 **b** \$87 **c** \$1690 **d** \$53.25 **e** \$932

2 **a** $33\frac{1}{3}$% profit **b** 25% profit
c 25% loss **d** $13\frac{1}{3}$% profit
e 22.2% loss

3 60%

4 **a** \$545 **b** 156%

5 **a** 1 : 4 **b** 25% **c** 1 : 5 **d** 20%
e The ratio of the profit to the cost price as a fraction is the same as the percentage profit on the cost price.

6 **a** \$325 **b** 27% **c** 21%
d The percentage profit is greater on the cost price.

7 **a** \$79.95 **b** 57%

8 18.75%

9 166.5%

10 \$1643.40

11 **a** \$39 **b** \$27 **c** \$274.45 **d** \$66

12 **a** \$1.00 profit per kg
b 55.6%
c 35.7%
d The percentage profit is greater on the cost price.

13 **a** \$77 **b** 49% **c** 33%

14 C

15 172%

16 60%, 40%; \$33 000, \$22 000

17 50%

18 \$103 520

19

Cost per item	Items sold	Sale price	Total profit
\$4.55	504	\$7.99	\$1 733.76
\$20.00	402	\$40.00	\$8 040.00
\$6.06	64 321	\$9.56	\$225 123.50
\$47.65	672	\$89.95	\$28 425.60

20 **a** **i** \$100, \$5500, \$2.50
ii Car, shoes, cake
iii Not fair; profit should be compared as a proportion of cost.
b **i** 500% 20.75%, 250%
ii Shoes, cake, car
iii Fairer than in questions in 1 and 3
c **i** $83.\dot{3}$%, 17%, 71.4%
ii Shoes, cake, car
iii Not fair; the profit should be calculated on the cost.

Challenge 6.2

James paid \$3240. The total percentage loss was 46%.

6.6 Goods and Services Tax (GST)

1 GST is a tax levied by the Australian federal government on goods and services.

2 **a**–**e** Yes **f** No

3 **a** **i** 45 cents **ii** \$4.94
b **i** 21 cents **ii** \$2.33

4 **a** \$3.50 **b** \$4.09 **c** \$2.00 **d** \$5.91

5 \$60.21

6 \$1020.80

7 **a** $1.1(85t + 40)$ **b** \$371.25

8 **a** $1.1(45x + 100)$ **b** \$1100

9 \$40.91

10 \$98.18

11 \$2.75

12 **a** \$11.17 **b** \$55.83

13 NZ\$92

14 **a** \$1400 **b** 11

15 **a** 12 Kopeks, 33 Plens
b \$4.94 GST, total price ≈ \$54.27

16 **a** $\$\frac{P}{10}, \$\frac{11P}{10}$ **b** $\frac{10A}{11}, \frac{A}{11}$

Investigation — Rich task

1 Metals used as alloying elements with gold are silver, copper, zinc, palladium, nickel, iron and aluminium.

2 27 g gold, 8.01 g copper, 0.99 g silver

3 6 g, $\frac{1}{6}$

4 From pink to rose to red gold the percentage of silver decreases, causing the gold alloy to darken in colour. At the same time, the percentage of copper increases, also contributing to the darker colour.

5 The copper would colour the gold with its familiar reddish colour so that it would not be white.

6 Red gold and blue gold each have 75% gold and 25% of another metal. In the case of red gold the contributing metal is copper; blue gold contains iron.

7 19 K

8 Answers will vary but should include that sterling silver is 92.5% silver and 7.5% copper. Silver from Mexico is of a lower quality. Teacher to check student answers.

Code puzzle

Don't move. I've got you covered.

TOPIC 7

Congruence

7.1 Overview

Why learn this?

From backyard sheds and home extensions to architecture itself, the design of buildings is founded on congruence. Many structures, from the modern to the very old, were created by people who had the chance to read a chapter similar to this one. Grand structures such as Flinders St Station, and even the very house in which you are living, are all based on the principles of congruence.

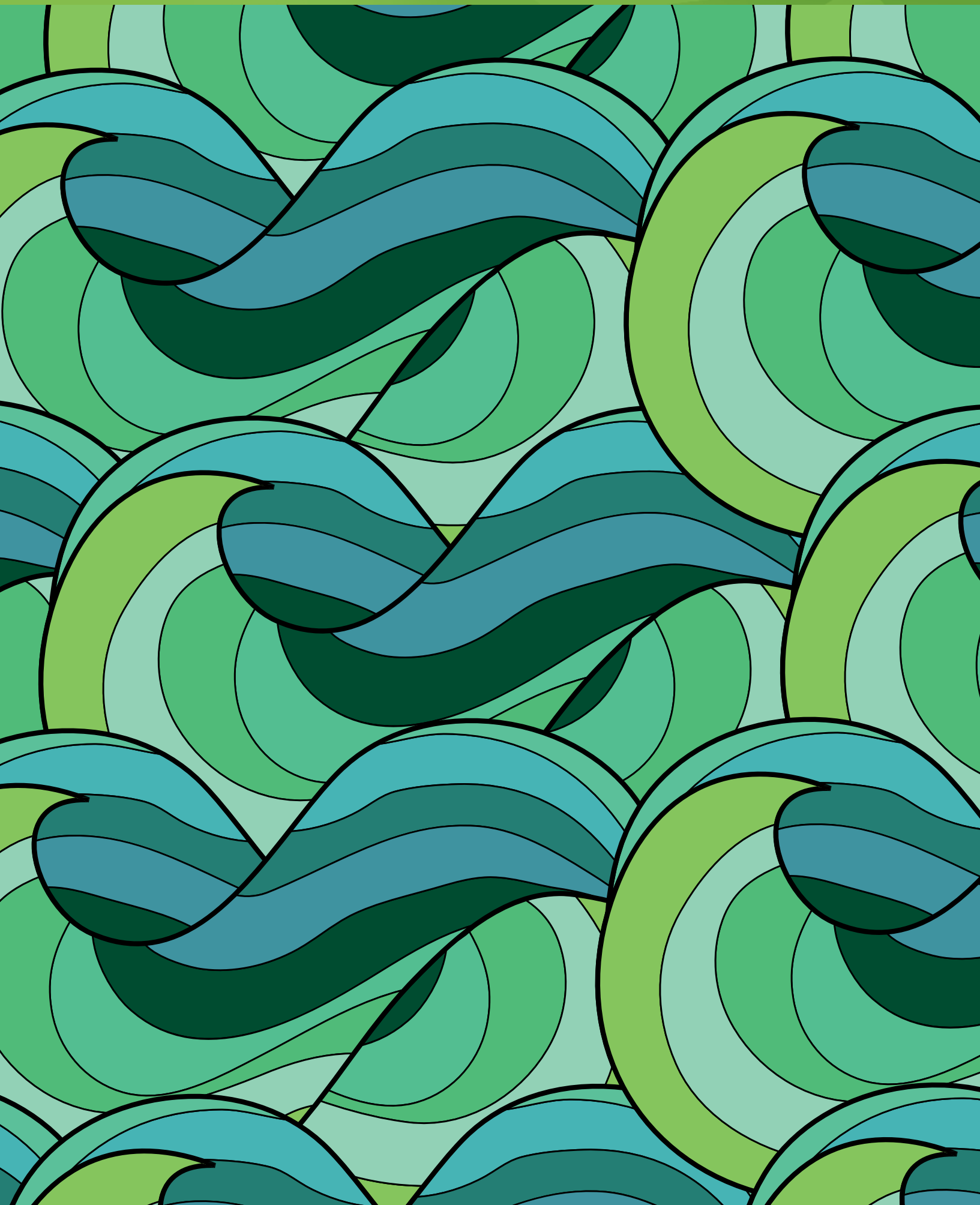
What do you know?

assessON

1 **THINK** List what you know about congruence and transformations. Use a thinking tool such as a concept map to show your list.
2 **PAIR** Share what you know with a partner and then with a small group.
3 **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of congruence and transformations.

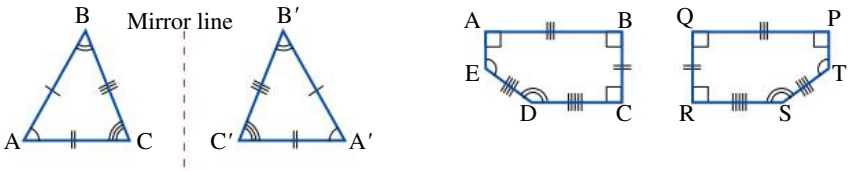
Learning sequence

7.1 Overview
7.2 Congruent figures
7.3 Triangle constructions
7.4 Congruent triangles
7.5 Quadrilaterals
7.6 Nets, polyhedra and Euler's rule
7.7 Review eBook plus ONLINE ONLY



7.2 Congruent figures

- **Congruent figures** are identical in size and shape.
- The transformations of reflection, rotation and translation do not change the shape and size of a figure. The original and transformed figure are said to be congruent.
- Repeated transformations of a congruent shape or a group of congruent shapes so that an entire surface is covered with a regular pattern are called tessellations.
- The symbol used for congruence is ≡.
- When writing congruence statements, we name the vertices of the figures in corresponding (or matching) order.
- Sometimes you will need to rotate, reflect or translate the figures for them to be orientated the same way.
- If, through one or more translations, rotations and reflections, one shape can lie exactly on top of the other, then these two figures are congruent.
- The figures below are congruent. For the triangle, we can write ABC ≡ A′B′C′; for the pentagon, we can write ABCDE ≡ PQRST.

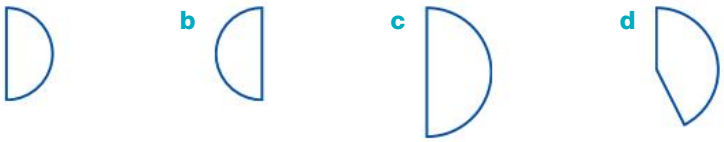


Note that equivalent sides and angles are labelled with the same symbol.

WORKED EXAMPLE 1

Select a pair of congruent shapes from the following set.

a **b** **c** **d**



THINK

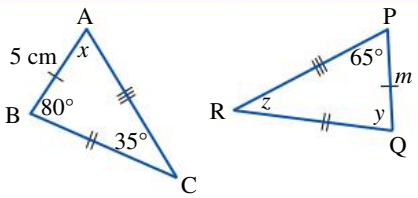
Figures **a**, **b** and **c** have the same shape (that is, a semicircle). Figure **d** is not a semicircle and thus is not congruent to any other figures. Figure **c** is larger than figures **a** and **b** and so is not congruent to any one of them. Figures **a** and **b** are identical in shape and size (**b** is a reflection of **a**) and therefore, are congruent to each other.

WRITE

Shape **a** ≡ Shape **b**

WORKED EXAMPLE 2

Find the value of the variables in the pair of congruent triangles at right.



THINK

1. Since ΔABC ≡ ΔPQR, the corresponding angles are equal in size. Corresponding angles are included between the sides of equal length. So, by looking at the markings on the sides of the triangles, we can conclude that:
   ∠BAC corresponds to ∠RPQ
   ∠ABC corresponds to ∠PQR
   ∠ACB corresponds to ∠PRQ.
   So, match the variables with the corresponding angles whose sizes are given.

WRITE

$x = 65°$
$y = 80°$
$z = 35°$

2 In congruent triangles, the corresponding sides are equal in length. Using the markings on the sides of the triangles, observe that the unknown side PQ corresponds to side AB. State the value of the variable (which represents the length of PQ).

$m = 5$ cm

The principle of congruence is applied in the construction of many everyday objects. For example the frame of the bicycle shown is partly made up of two congruent triangles; the wheels are congruent circles.

Exercise 7.2 Congruent figures

assessON

INDIVIDUAL PATHWAYS

■ PRACTISE	■ CONSOLIDATE	■ MASTER
Questions: 1–5, 7	Questions: 1–5, 7	Questions: 1–8

■■■ Individual pathway interactivity int-4424 eBookplus

REFLECTION
When we write a congruence statement, why are the vertices listed in corresponding order?

FLUENCY

1 Plot the following coordinates on a Cartesian plane: (1, −1), (1, 0), (1, 1), (2, 1), (1, 2), (1, 3), (2, 3), (3, 3). Join the points with straight lines to make the shape of the letter F.

- **a** Reflect this figure in the *y*-axis and describe the new location using the coordinate points.
- **b** Translate the original figure by three units to the right and two units down, and describe the new location using the coordinate points.
- **c** Rotate the original figure through 90° about the origin and describe the new location using the coordinate points.
- **d** Look at the transformations in parts **a**, **b** and **c**. Are the shapes congruent? Why?

2 WE1 Select a pair of congruent shapes from each of the following sets.

a **i**

ii

iii

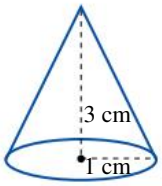
iv

eBookplus

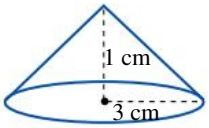
Digital docs
SkillSHEET
Naming angles and shapes
doc-6915
SkillSHEET
Calculating angles in triangles
doc-6916

eBookplus
Interactivity
Transformations and the Cartesian plane
int-2368

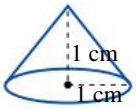
b **i**



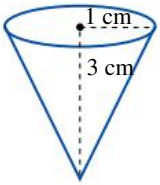
ii



iii



iv



c **i**

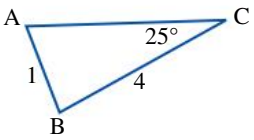
ii

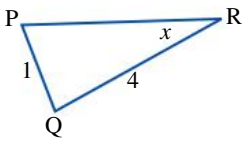
iii

iv

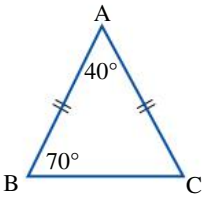
3 **WE2** All of these pairs of triangles are congruent. Name the congruent triangles and find the value of the variables in each case. (*Remember:* The vertices must be listed in corresponding order.)

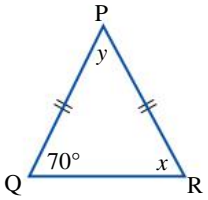
a



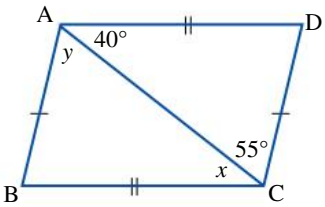


b

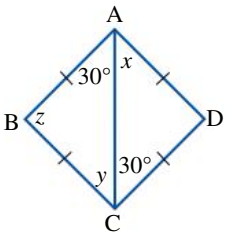




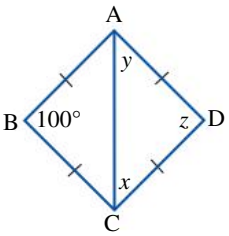
c



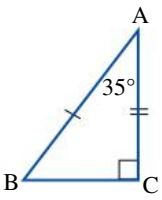
d

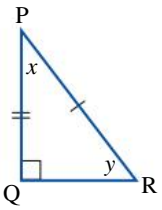


e

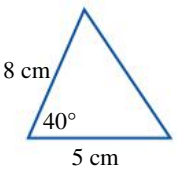


f

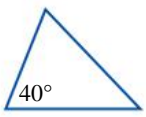




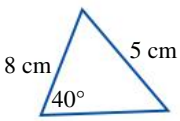
4 **MC** Which of the following is congruent to the triangle shown?



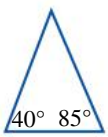
a



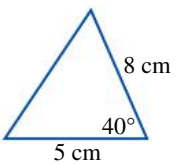
b



c



d



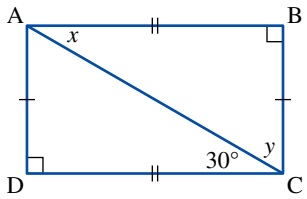
A **a** only **B** **a** and **b** **C** **d** only **D** **b** and **d**

E None of the above

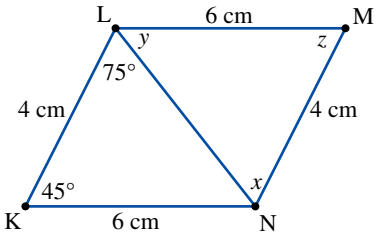
UNDERSTANDING

5 Name the congruent triangles in each question and find the value of the variable(s).

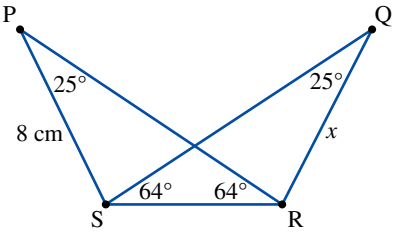
a



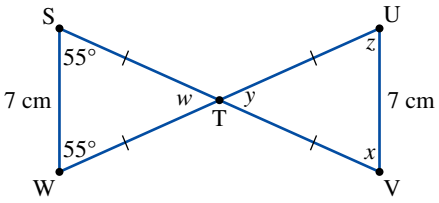
b



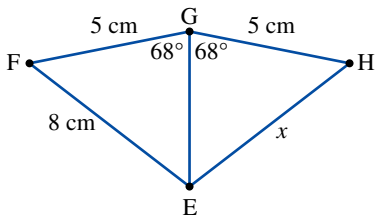
c



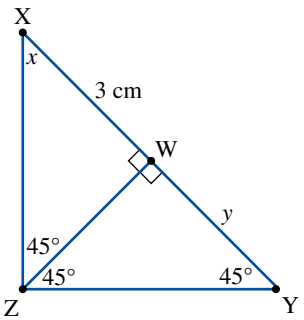
d



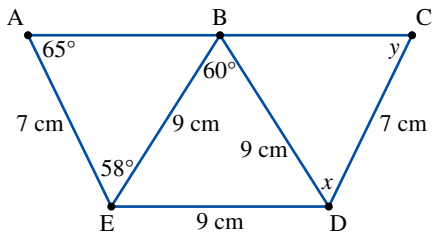
e



f



g

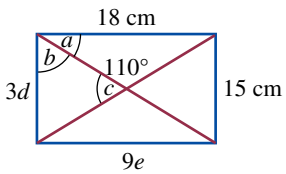


REASONING

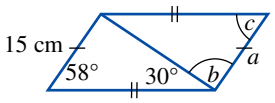
6 Give an example to show that triangles with two angles of equal size and a pair of non-corresponding sides of equal length may not be congruent.

PROBLEM SOLVING

7 Given that the four right-angled triangles in the figure below are congruent, find the values of the pronumerals.



8 Given that the right-hand triangle is congruent to the left-hand triangle in the figure below, find the values of the pronumerals.



CHALLENGE 7.1
Determine which regular polygons will tessellate on their own without any spaces or overlaps.

7.3 Triangle constructions

- Using a ruler, protractor and a pair of compasses, you can construct any triangle from three pieces of information.
- Sometimes, only one triangle can be drawn from the information. Sometimes more than one triangle can be drawn.

Constructing a triangle given three side lengths

- If the lengths of the three sides of a triangle are known, it can be constructed with the help of a ruler and a pair of compasses.

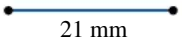
WORKED EXAMPLE 3

Using a ruler and a pair of compasses, construct a triangle with side lengths 15 mm, 20 mm and 21 mm.

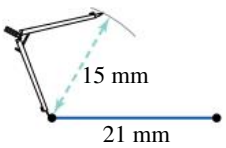
THINK

DRAW

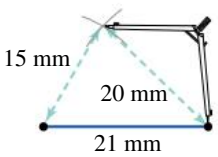
1. Rule out the longest side (21 mm).



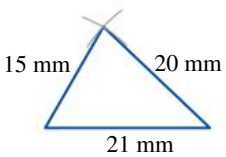
2. Open the compasses to the shortest side length (15 mm).
3. Draw an arc from one end of the 21 mm side.



4. Open the compasses to the length of the third side (20 mm) and draw an arc from the other end of the 21 mm side.



5. Join the point of intersection of the two arcs and the end points of the 21 mm side with lines.

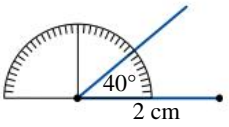
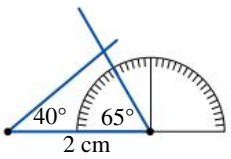
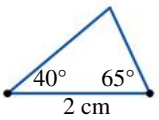


Constructing a triangle given two angles and the side between them

- If the size of any two angles of a triangle and the length of the side between these two angles are known, the triangle can be constructed with the aid of a ruler and a protractor.

WORKED EXAMPLE 4

Use a ruler and protractor to construct a triangle with angles 40° and 65°, and the side between them of length 2 cm.

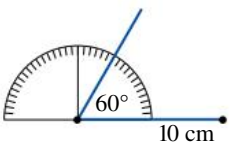
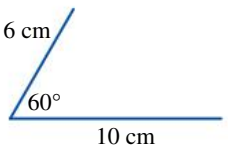
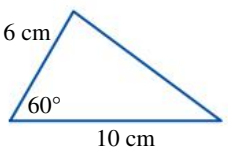
THINK	DRAW
1 Rule a line of length 2 cm.	
2 Place the centre of your protractor on one end point of the line and measure out a 40° angle. Draw a line so that it makes an angle of 40° with the 2 cm line.	
3 Place the centre of your protractor on the other end point of the 2 cm line and measure an angle of 65°. Draw a line so that it makes a 65° angle with the 2 cm line.	
4 If necessary, continue the lines until they intersect each other to form a triangle.	

Constructing a triangle given two sides and the angle between them

- The angle between two sides is called the **included angle**.
- If the length of two sides and the size of the included angle are known, the triangle can be constructed with a protractor and a ruler.

WORKED EXAMPLE 5

Use a ruler and protractor to construct a triangle with sides 6 cm and 10 cm long, and an angle between them of 60°.

THINK	DRAW
1 Rule a line 10 cm long.	
2 Place the centre of your protractor on one end point of the line and mark an angle of 60°. *Note:* These figures have been reduced.	
3 Join the 60° mark and the end point of the 10 cm side with the straight line. Extend the line until it is 6 cm long.	
4 Join the end points of the two lines to complete the triangle.	

assessON

Exercise 7.3 Triangle constructions

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–4, 8	Questions: 1, 2a–d, 3a–d, 4–6, 8, 9	Questions: 1, 2e–h, 3e–h, 4–7, 10, 11

Individual pathway interactivity int-4425 eBookplus

REFLECTION
When you are constructing a triangle and you know two angles and a side, do you need to know the side between the angles?

FLUENCY

1 WE3 Using a ruler and a pair of compasses, construct 2 congruent triangles with the following side lengths:

- **a** 7 cm, 6 cm, 4 cm
- **b** 5 cm, 4 cm, 5 cm
- **c** 6 cm, 5 cm, 3 cm
- **d** 6 cm, 6 cm, 6 cm
- **e** 7.5 cm, 4.5 cm, 6 cm
- **f** 2 cm, 6.5 cm, 5 cm
- **g** an equilateral triangle of side 3 cm
- **h** an equilateral triangle of side 4.5 cm.

2 WE4 Use a ruler and protractor to construct these triangles:

- **a** angles 60° and 60° with the side between them 5 cm long
- **b** angles 50° and 50° with the side between them 6 cm long
- **c** angles 30° and 40° with the side between them 4 cm long
- **d** angles 60° and 45° with the side between them 3 cm long
- **e** angles 30° and 60° with the side between them 4 cm long
- **f** angles 65° and 60° with the side between them 3.5 cm long
- **g** angles 60° and 90° with the side between them 5 cm long
- **h** angles 60° and 36° with the side between them 4.5 cm long.

eBookplus
Interactivity
Random triangles
int-2369

3 WE5 Use a ruler and protractor to construct the following triangles:

- **a** two sides 10 cm and 5 cm long, angle of 30° between them
- **b** two sides 8 cm and 3 cm long, angle of 45° between them
- **c** two sides 6 cm and 6 cm long, angle of 60° between them
- **d** two sides 4 cm and 5 cm long, angle of 90° between them
- **e** two sides 7 cm and 6 cm long, angle of 80° between them
- **f** two sides 9 cm and 3 cm long, angle of 110° between them
- **g** two sides 6 cm and 6 cm long, angle of 50° between them
- **h** two sides 5 cm and 4 cm long, angle of 120° between them.

UNDERSTANDING

4 a Use your ruler and compass to draw an isosceles triangle with two sides 5 cm long and one side 7 cm.

- **b** Use your protractor to measure the size of the largest angle.
- **c** Complete this sentence using one of the words below: This triangle is an _____ angled triangle.
  - **i** acute
  - **ii** right
  - **iii** obtuse

REASONING

5 **a** Construct a triangle with angles of 45°, 55° and 80°.

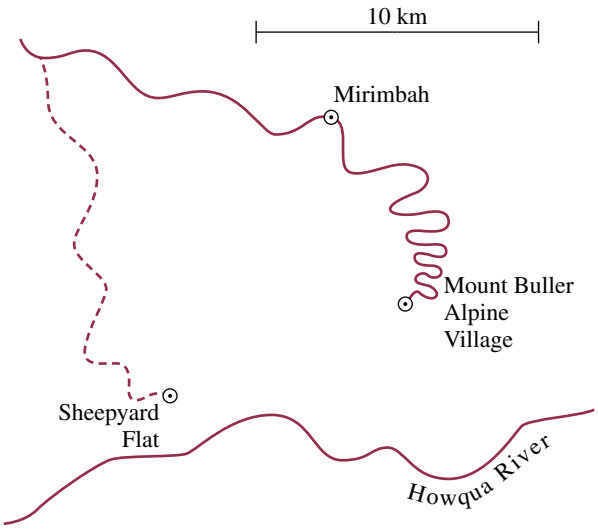
b How many different triangles can be constructed from this information? Justify your answer.

6 Construct a right-angled triangle with the longest side 13 cm and one of the other sides 5 cm.

7 Mike and Susan were constructing a triangle ABC where angle ABC = 30°, BC = 12 cm and AC = 8 cm. Mike's triangle was an acute-angled triangle and Susan's was an obtuse-angled triangle. Draw both triangles.

PROBLEM SOLVING

8 A bushwalker is injured and cannot walk. Before the battery of his mobile phone died, he reported that he was equidistant from the peak of Mount Buller and Sheepyard Flat where he parked his car. He estimated that he had walked 6 km and had crossed Howqua River before his fall. At rescue headquarters, the SES captain looked at a map. Sheepyard Flat is approximately 10 km from Mount Buller. Trace the map at right into your workbook and draw the point where the SES should send the rescue helicopter.



9 Use a ruler and protractor to construct triangles with the following features.

a Angles 30° and 90° with a 3-cm edge between them

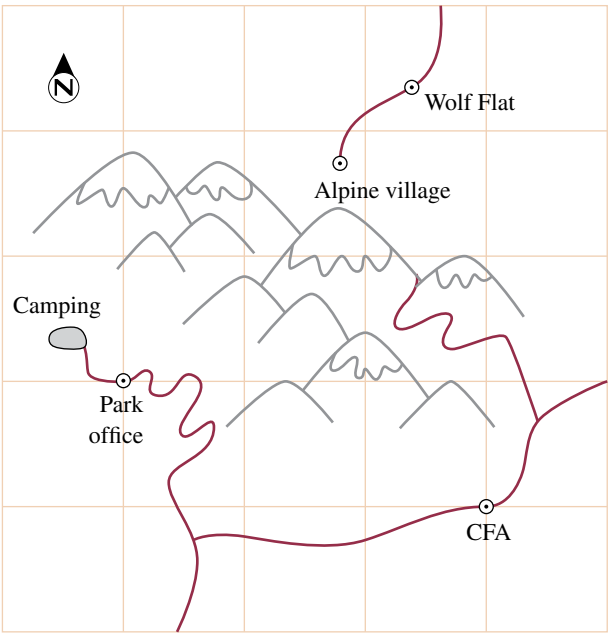
b Angles 45° and 45° with a 2.5-cm edge between them

c Angles 60° and 100° with a 40-mm edge between them

d Angles 22° and 33° with a 33-mm edge between them

e Isosceles triangle with adjacent angles of 57° and a 4-cm baseline

10 A forestry ranger informs the CFA that he sees smoke rising from behind a mountain range at 40° east of due north from his park office. The CFA chief sees the same smoke at an angle of 30° west of due north from her station. Trace the map at right into your workbook and mark the location of the fire.



11 Use a ruler and protractor to construct triangles with the following features.

a Edge lengths of 5.2 cm and 3 cm and an angle between them of 45°

b Edge lengths of 2.5 cm and 2.5 cm and an angle between them of 60°

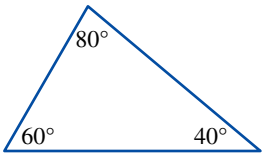
c Edge lengths of 28 mm and 40 mm and an angle between them of 120°

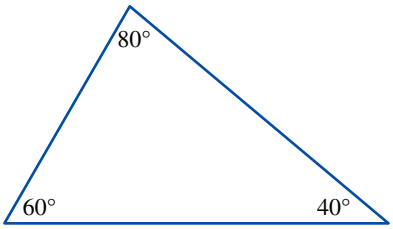
d Edge lengths of 63 mm and 33 mm and an angle between them of 135°

e A right-angled triangle with edge lengths of 4.5 cm and 2.5 cm creating the right angle

7.4 Congruent triangles

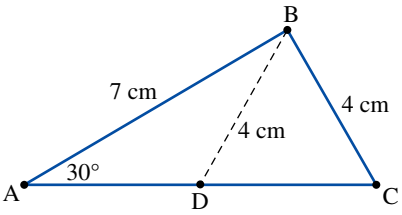
- From section 7.3 it can be seen that a triangle can be described with three pieces of information about side length and/or angle size.
- If three angles are known, an infinite number of triangles can be drawn. For example the triangles below have angle sizes of 60°, 40° and 80°.





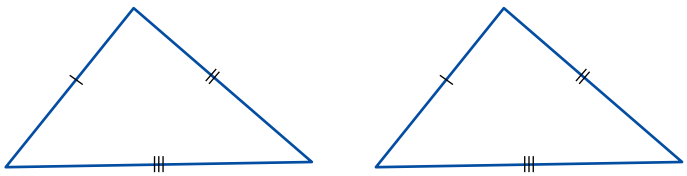
∴ If only three angles are known, this is not a test for congruency.

- There can be two possible triangles drawn from two sides and the non-included angle. In the diagram at right, angles ABC and ABD both have sides of 7 cm and 4 cm, and an angle of 30° between the 7-cm side and the unknown side.
  ∴ If two sides and the non-included angle are given, this is not a test for congruence.

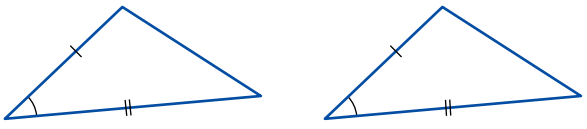


Congruency tests

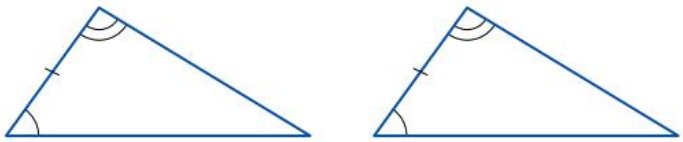
- There are four tests that can be used to demonstrate that two triangles are congruent.
- It is not enough for the diagrams to look the same; you need evidence!
- Two triangles are congruent if:
  – three sides of one triangle are equal to the three sides of the other (SSS)



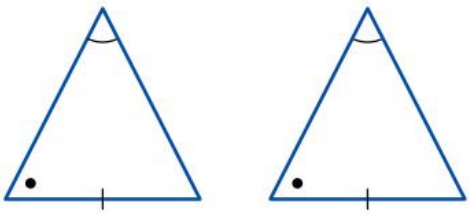
  – two sides and the included angle of one triangle are equal to two sides and the included angle of the other (SAS)



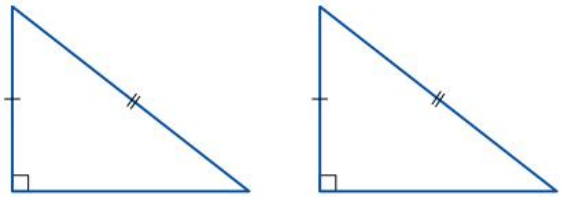
– two angles and a side of one triangle are equal to the two angles and the corresponding side of the other (AAS)



– a pair of corresponding angles and a non-included side are equal (ASA)

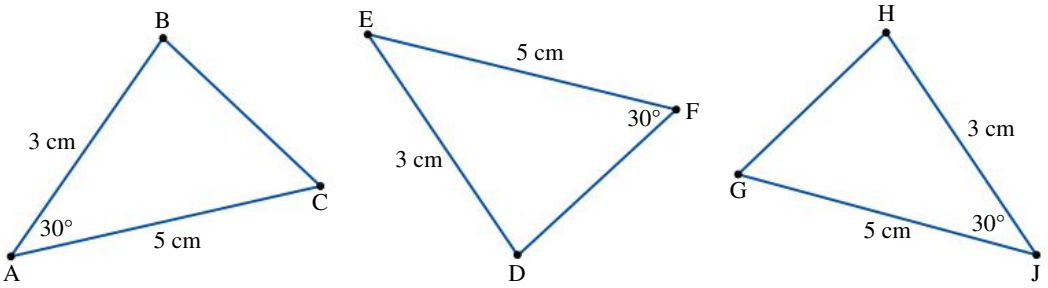


– the triangles are right-angled and the hypotenuse and a side of one are equal to the hypotenuse and a side of the other (RHS).



WORKED EXAMPLE 6

Which of the following triangles are definitely congruent? Give a reason for your answer.



THINK

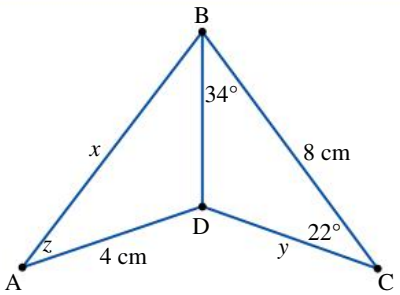
In all three triangles, two given sides are of equal length (3 cm and 5 cm). In triangles ABC and JHG, the included angle is 30°. In triangle AFD, 30° is not the included angle.

WRITE

$\Delta ABC \equiv \Delta JHG$ (SAS)

WORKED EXAMPLE 7

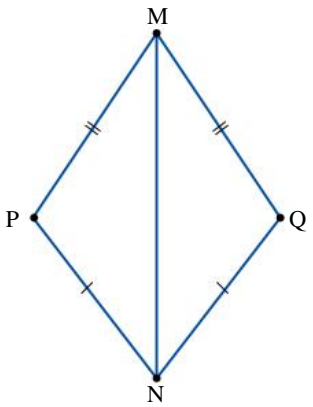
Given that ΔABD ≡ ΔCBD, find the value of the variables.



THINK	WRITE
1 The corresponding sides are equal in length. ΔABD ≡ ΔCBD. AB and CD are unknown. AB corresponds to CB. CD corresponds to AD.	ΔABD ≡ ΔCBD AB = CB $x = 8$ cm CD = AD $y = 4$ cm
2 The corresponding angles are equal. ∠BAD is unknown. ∠BAD corresponds to ∠BCD.	∠BAD = ∠BCD $z = 22°$

WORKED EXAMPLE 8

Prove that ΔMNP is congruent to ΔMNQ.



THINK	WRITE
1 Study the diagram and state which sides and/or angles are equal.	NP = NQ (given) PM = QM (given) MN is common
2 Select the appropriate congruency test (in this case SSS, as the triangles have all corresponding sides congruent in length).	ΔNPM ≡ ΔNQM (SSS)

Exercise 7.4 Congruent triangles

assessON

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–3	Questions: 1–6, 9	Questions: 1–10

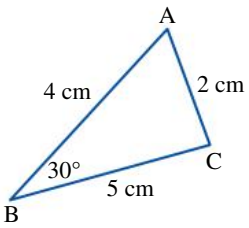
Individual pathway interactivity int-4426 eBookplus

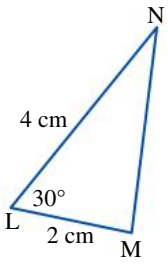
REFLECTION
When using the congruency rule ASA, why is it not necessary for the side to be between the two angles?

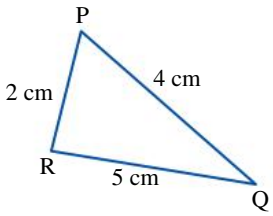
FLUENCY

1 WE6 In each part of the question, which of the triangles are congruent? Give a reason for your answer.

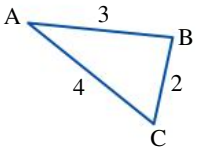
a

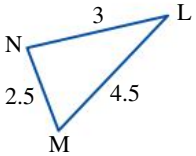


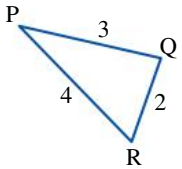




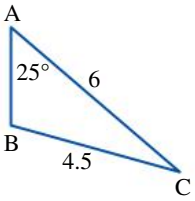
b

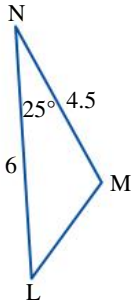


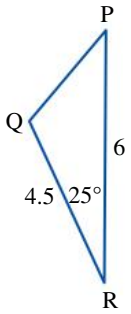




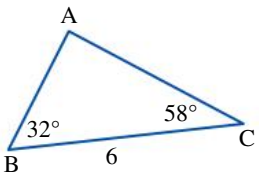
c

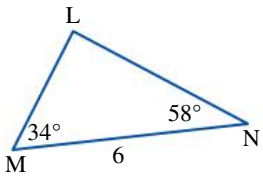


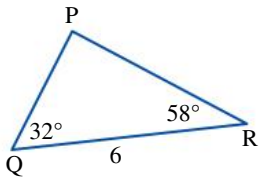




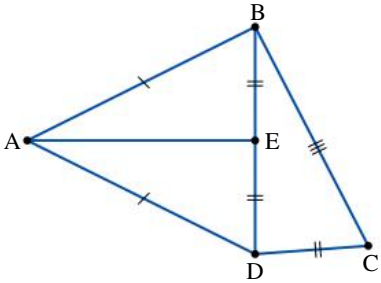
d



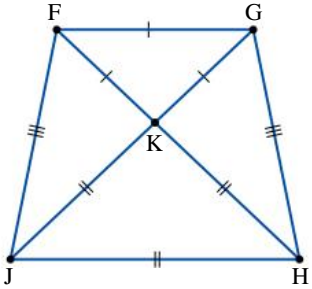




e

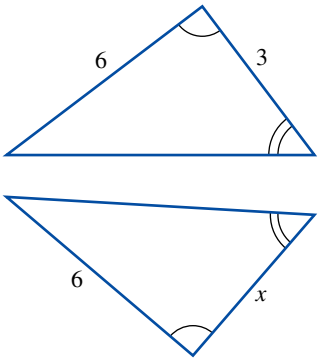


f

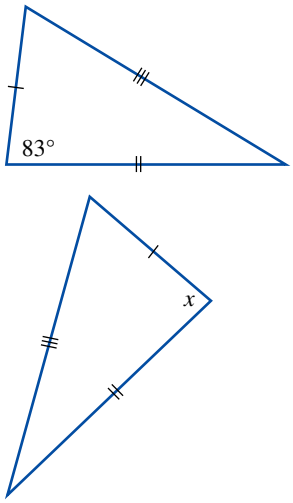


2 WE7 Find the values of the variables in each of the following pairs of congruent triangles. All side lengths are in centimetres.

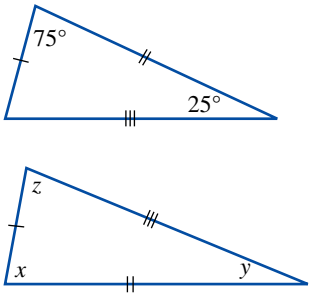
a



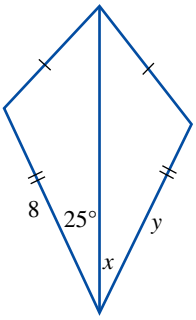
b



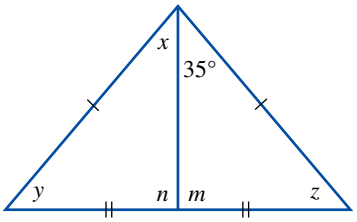
c



d

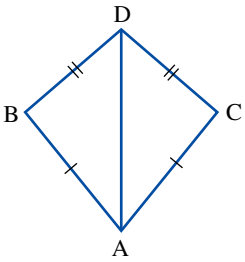


e



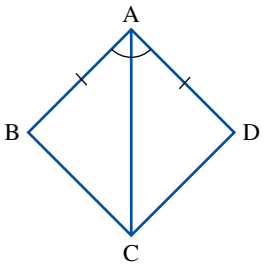
3 WE8 For each of the following, prove that:

a



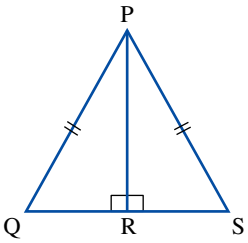
ΔDBA ≡ ΔDCA

b



ΔABC ≡ ΔADC

c



ΔPQR ≡ ΔPSR

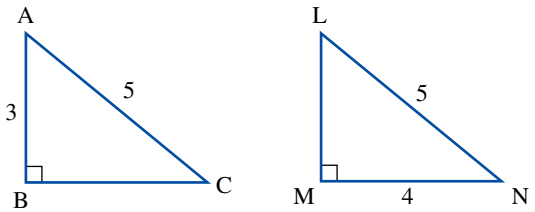
UNDERSTANDING

4 Draw an example of two different triangles that have two corresponding sides equal and one pair of equal angles.

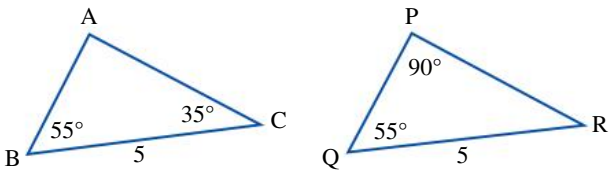
5 Draw an example of two different triangles that have three pairs of equal angles.

6 Are the following triangles congruent? Give a reason for your answer. *Hint:* Pythagoras' theorem (Topic 15) is needed for this question.

a



b



REASONING

7 In an equilateral triangle ABC, the midpoints of the sides are labelled D, E and F. Prove that triangle DEF is an equilateral triangle.

8 All triangles have three sides and three angles. Using mathematical reasoning, including examples, give the minimum information required to construct a unique triangle.

PROBLEM SOLVING

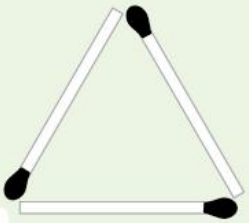
9 Prove that the construction of a diagonal in a parallelogram creates two congruent triangles.

10 From the right angle of a right-angled isosceles triangle, a straight line is that which bisects the hypotenuse. Prove that two congruent triangles are created.

eBookplus
Digital doc
WorkSHEET 7.1
doc-2250

CHALLENGE 7.2

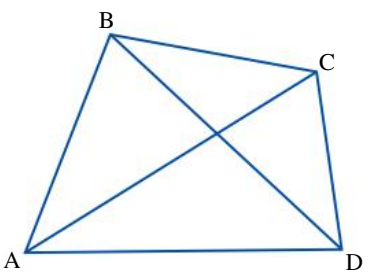
The figure shows how you can construct an equilateral triangle from three matchsticks. Show how you can construct five equilateral triangles using only nine matchsticks.



7.5 Quadrilaterals

Review of terms and rules

- The figure ABCD is a quadrilateral.
- A **quadrilateral** is a four-sided figure.
- The sum of the interior angles at the four vertices of a quadrilateral is 360°.
- **Opposite angles** do not have rays in common. ∠ABC and ∠ADC are opposite angles. ∠BCD and ∠BAD are also opposite angles.

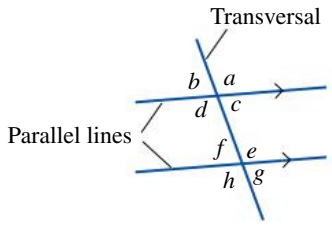


- **Opposite sides** do not intersect. BC and AD are opposite sides. AB and CD are opposite sides.
- **Diagonals** connect opposite angles. AC is one diagonal and BD is the other.
- In some quadrilaterals, the *diagonals bisect each other*. In the diagram above, BD would bisect AC only if it divided AC into two equal sections. AC would bisect BD only if it divided it into two equal sections. In the previous diagram, the diagonals do not bisect each other.
- In some quadrilaterals, the *diagonals bisect the angles*. Diagonal BD bisects ∠ABC if ∠ABD and ∠DBC are congruent. In the diagram, above, BD does not bisect ∠ABC.
- There are several special quadrilaterals.
  - A **parallelogram** is a quadrilateral where the opposite pairs of sides are parallel.
  - A **rhombus** is a parallelogram in which all sides are equal. (Because it is a parallelogram, the properties of a parallelogram are also properties of a rhombus.)
  - A **rectangle** is a parallelogram in which all angles are 90°. (Because it is a parallelogram, the properties of a parallelogram are also properties of a rectangle.)
  - A **square** is a rectangle with all sides equal. (Because rectangles are parallelograms and a rhombus is a parallelogram with all sides congruent, a square is also a right-angled rhombus.)
  - A **kite** is a quadrilateral which has two pairs of adjacent sides that are equal.
  - A **trapezium** is a quadrilateral with one pair of parallel unequal sides.

Properties of quadrilaterals

- By considering triangles in quadrilaterals, the properties of quadrilaterals can be determined.

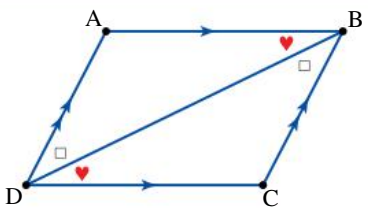
Angles and parallel lines



- Parallel lines are denoted by identical arrowheads.
- A straight line cutting a parallel line is called a **transversal**.
- **Corresponding angles** are equal in magnitude. For example, in the diagram, $a = e$, $c = g$, $b = f$ and $d = h$.
- **Vertically opposite angles** are equal in magnitude. For example, in the diagram, $a = d$, $b = c$, $f = g$ and $e = h$.
- **Alternate angles** are equal in magnitude. For example, in the diagram, $c = f$ and $d = e$.
- **Co-interior angles** (also known as allied angles) sum to 180°. For example, in the diagram, $c + e = 180°$ and $d + f = 180°$.
- Angles and parallel lines can be used to establish congruency of triangles in parallelograms.

WORKED EXAMPLE 9

When a diagonal is drawn in a parallelogram, two pairs of equal angles are formed, as shown (alternate angles).



a State the congruency relationship between the two triangles formed, giving a reason.

b Hence write a relationship between:

i AB and DC **ii DA and CB** **iii ∠BAD and ∠BCD.**

c What conclusion can be drawn about:

i opposite sides of a parallelogram? **ii opposite angles of a parallelogram?**

THINK

a There are two equal angles, and BD is in both triangles.

b **i** AB and DC are corresponding sides in the triangles.

ii DA and CB are corresponding sides in the triangles.

iii ∠BAD and ∠BCD are corresponding angles.

c **i** AB and DC are opposite sides. DA and CB are opposite sides.

ii ∠BAD and ∠BCD are opposite angles.
∠ABC and ∠ADC are opposite angles.

WRITE

a ∠ABD = ∠CDB (given)
∠ADB = ∠CBD (given)
BD is common to both
∴ ΔABD ≡ ΔCDB (ASA)

b **i** AB = DC

ii DA = CB

iii ∠BAD = ∠BCD

c **i** AB = DC and DA = CB; therefore, opposite sides are equal.

ii ∠BAD = ∠BCD
∠ABD = ∠BDC (given)
∠ADB = ∠CBD (given)
∴ ∠ABC = ∠ADC
Opposite angles are equal.

Exercise 7.5 Quadrilaterals

assessON

INDIVIDUAL PATHWAYS

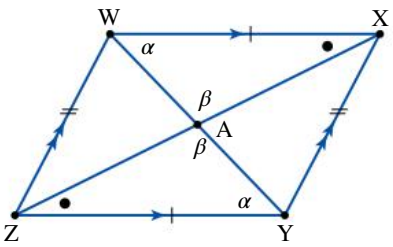
PRACTISE	CONSOLIDATE	MASTER
Questions: 1–5, 9, 10	Questions: 1–6, 9, 10	Questions: 1–11

Individual pathway interactivity int-4427 eBookplus

REFLECTION
Why is it important to be able to show congruency using mathematical reasoning?

FLUENCY

1 WE9a,b Consider the parallelogram WXYZ. From the worked example, we know that opposite sides are equal. Both diagonals are drawn giving 3 pairs of angles as shown (alternate angles and opposite angles).



eBookplus
Digital doc
SkillSHEET
Angles and parallel lines
doc-6917

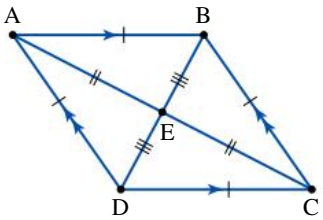
a State the congruency relationship between ΔWXA and ΔYZA, giving a reason.

b Hence state the relationship between:

i WA and YA **ii** XA and ZA.

c Do the diagonals bisect each other?

2 Consider the rhombus ABCD. Because the rhombus is also a parallelogram, we know that the diagonals bisect each other.



a State the congruency relationship between ΔABE and ΔBEC, giving a reason.

b Hence state the relationship between:

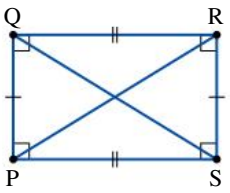
i ∠AEB and ∠CEB **ii** ∠ABE and ∠CBE.

c What is the magnitude of ∠AEB?

d Name all the triangles congruent to ΔABE.

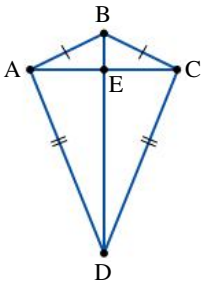
e Do the diagonals bisect the angles?

3 Consider the rectangle PQRS.



- **a** Show that ΔPQR and ΔSRP are congruent.
- **b** What can you say about the diagonals of a rectangle?

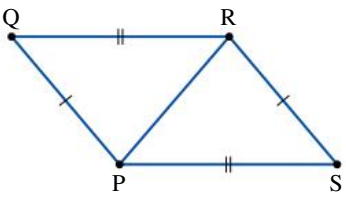
4 Consider the kite ABCD.



- **a** Show that ΔABD and ΔCBD are congruent.
- **b** What can you say about:
  - **i** ∠ABD and ∠CBD?
  - **ii** ∠BAD and ∠BCD?
  - **iii** ∠BDA and ∠BDC?
- **c** Use part **b** to show that ΔABD ≡ ΔCBD.
- **d** What can you say about:
  - **i** AE and CE?
  - **ii** ∠AEB and ∠CEB?
- **e** Are ΔBEC and ΔDCE congruent?
- **f** Use the above results to answer the following questions about kites.
  - **i** Are the opposite sides equal?
  - **ii** Are the opposite angles equal?
  - **iii** Do the diagonals bisect each other?
  - **iv** Do the diagonals intersect at 90°?
  - **v** Do the diagonals bisect the angles?

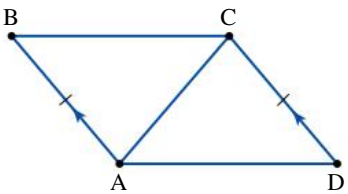
UNDERSTANDING

5 The quadrilateral PQRS has opposite sides that are equal.



- **a** Show that the triangles are congruent.
- **b** Complete the following.
  - **i** ∠QPR = _____
  - **ii** ∠QRP = _____
- **c** Complete the following.
  - **i** QP is parallel to _____.
  - **ii** QR is parallel to _____.
- **d** What type of quadrilateral is PQRS?

6 In quadrilateral ABCD, AB and DC are both parallel and equal in length.

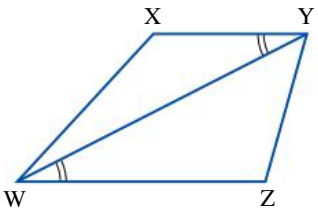


- **a** Identify one pair of alternate angles.
- **b** Show that the two triangles are congruent.
- **c** Hence demonstrate that BC and AD are parallel.
- **d** What type of quadrilateral is ABCD?

7 In a quadrilateral MNOP, use 2 pairs of adjacent triangles to show that the sum of the interior angles of a quadrilateral is 360°.

REASONING

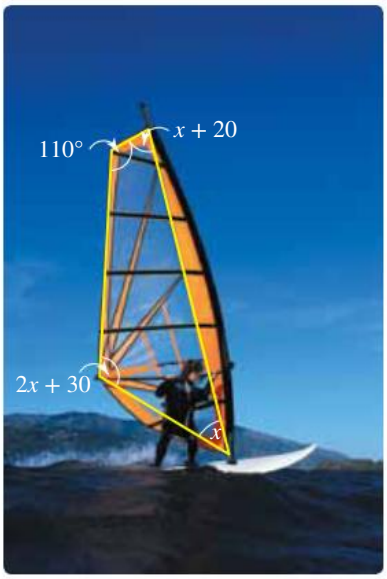
8 WXYZ is a quadrilateral, where ∠XYW = ∠YWZ. Use mathematical reasoning to determine the type of quadrilateral.



PROBLEM SOLVING

9 The frame of the swing shown in the photograph is a trapezium. The diagonal sides make an angle of 73° with the ground. Evaluate the angle, x, that the horizontal crossbar makes with the sides.

10 The pattern for a storm sail for a sailboard is shown in the photograph. Calculate the size of each interior angle.



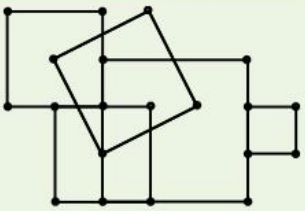
11 **a** Prove that a quadrilateral can have no more than one interior angle that is greater than 180°.

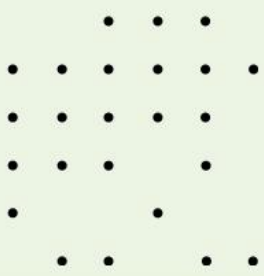
b If all interior angles are integers, what is the largest possible interior angle that a quadrilateral can have? Use your protractor to draw it.

eBookplus
Digital doc
WorkSHEET 7.2
doc-2251

CHALLENGE 7.3

The diagram shows five squares drawn among a network of dots. The dots represent a corner of a square and no two squares share a corner, although some squares can share part of a side. Use the network of dots below to draw squares that follow the same rules — that is, with corners represented by the dots and no two squares sharing a corner. How many squares can be drawn?

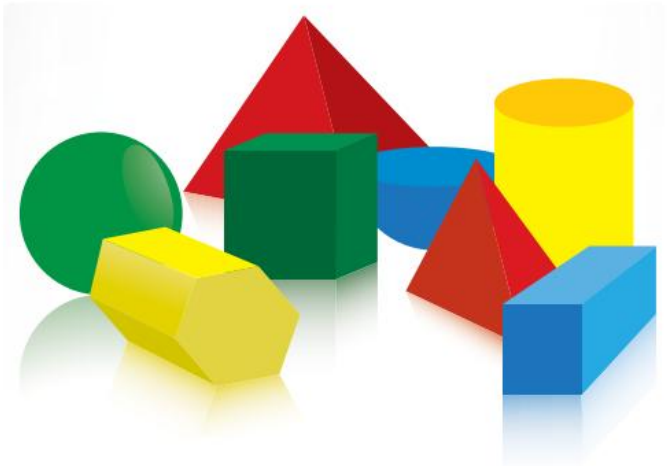




7.6 Nets, polyhedra and Euler's rule

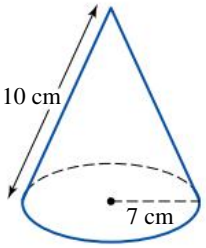
Nets

- The faces forming a solid can be drawn as plane shapes, which are joined across the edges to form the solid. The complete set of faces forming a solid is called its net. Note that for some figures, different nets can be drawn.



WORKED EXAMPLE 10

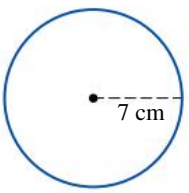
Draw a possible net for a cone, which has a radius of 7 cm and a slanting height of 10 cm.



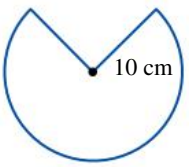
THINK

DRAW

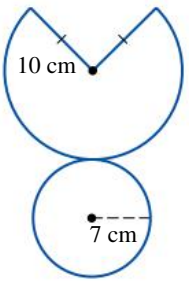
1. The base of a cone is a circle of radius 7 cm.



2. The other (slanted) part of the cone when split open will form a sector of a circle of radius 10 cm.



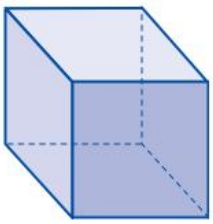
3. If the two parts (from steps 1 and 2) are put together, the complete net of a cone is obtained.



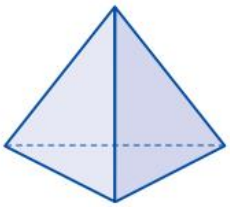
Polyhedron construction

- A 3-dimensional solid where each of the faces is a polygon is called a **polyhedron**. If all the faces are congruent, the solid is called a regular polyhedron or a Platonic solid.
- Platonic solids are shown below.

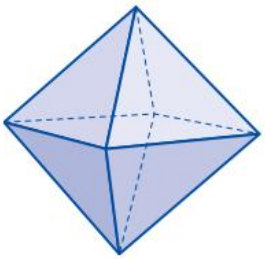
1. A **cube**, with 6 faces, each of which is a square.



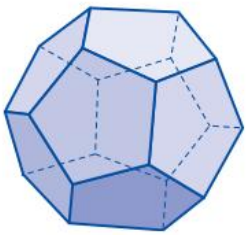
2. A **tetrahedron**, with 4 faces, each of which is an equilateral triangle.



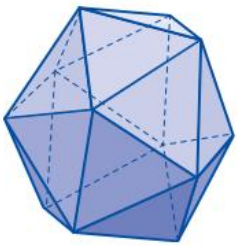
3. An **octahedron**, with 8 faces, each of which is an equilateral triangle.



4. A **dodecahedron**, with 12 faces, each of which is a regular pentagon.



5. An **icosahedron**, with 20 faces, each of which is an equilateral triangle.



- Construction of polyhedra is conveniently done from nets.

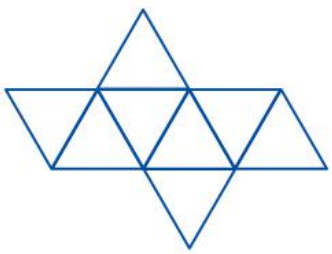
WORKED EXAMPLE 11

Draw a net of an octahedron.

THINK

An octahedron is a polyhedron with 8 faces, each of which is an equilateral triangle. So its net will consist of 8 equilateral triangles. Draw a possible net of an octahedron.

DRAW



Euler's rule

- **Euler's rule** shows the relationship between the number of **edges**, the number of **faces** and the number of vertices in any polyhedron. Note that a **vertex** (the singular of vertices) is a point or a corner of a shape where the straight edges meet.

Euler's rule states that for any polyhedron:
Number of faces (F) + number of vertices (V) − 2 = number of edges (E),
or $F + V - 2 = E$.

assessON

Exercise 7.6 Nets, polyhedra and Euler's rule

INDIVIDUAL PATHWAYS

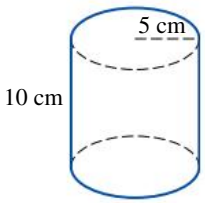
REFLECTION
When would you find it a useful skill to be able to construct 3D shapes?

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7, 13	Questions: 1–13	Questions: 1–14

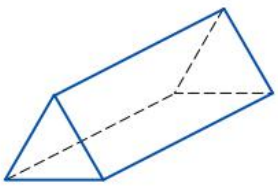
Individual pathway interactivity int-4428 eBook plus

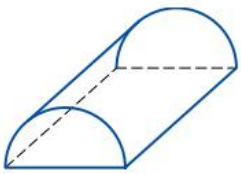
FLUENCY

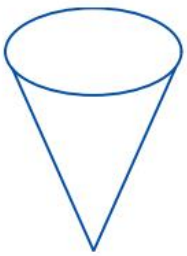
1 WE10 Draw a possible net for the cylinder shown below.



2 Draw a possible net for each of the following solids.

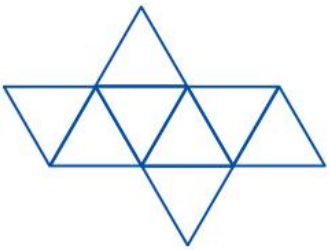
a


b


c


3 WE11 Draw 2 different nets to form a cube.

4 Draw a net for the octahedron that is different from the one shown below.

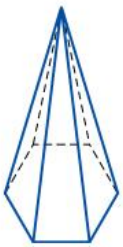


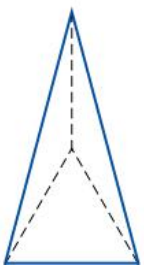
Cut out your net and fold it to form the octahedron.

5 Using nets generated from geometry software or elsewhere, construct:

a a cube **b** a tetrahedron **c** a dodecahedron.

6 Construct the pyramids shown in the figures below.

a


b


UNDERSTANDING

7 Draw some 3-dimensional shapes of your choice. How many of these shapes are polyhedra? Can you name them?

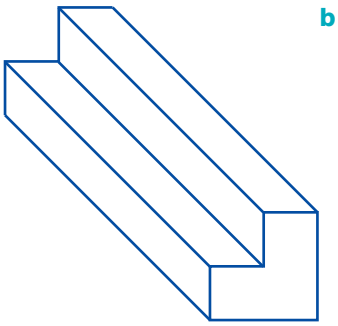
8 Verify Euler's rule for the following Platonic solids.

a A cube

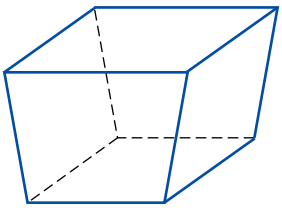
b An octahedron

9 Show that Euler's rule holds true for these solids.

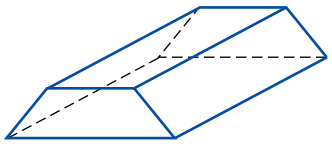
a



b

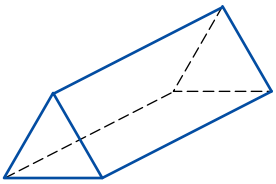


c

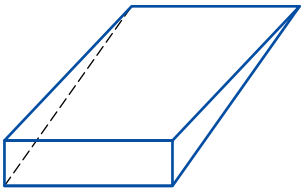


10 Make the prisms shown below. Verify Euler's rule for each shape.

a



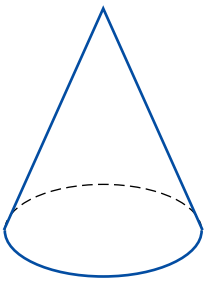
b



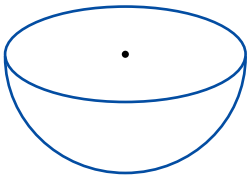
REASONING

11 Can you verify Euler's rule for the following shapes? Give reasons for your answer.

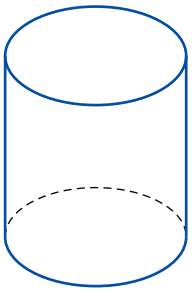
a



b



c

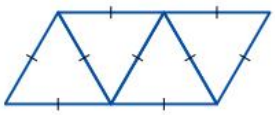


12 A spherical scoop of ice-cream sits on a cone. Draw this shape. Can you verify Euler's rule for this shape? Explain your reasoning.

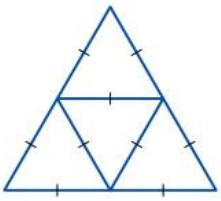
PROBLEM SOLVING

13 Which of the following compound shapes are nets that can be folded into a 3-D solid? You may wish to enlarge them onto paper, cut them out and fold them.

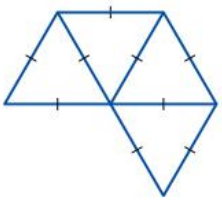
a



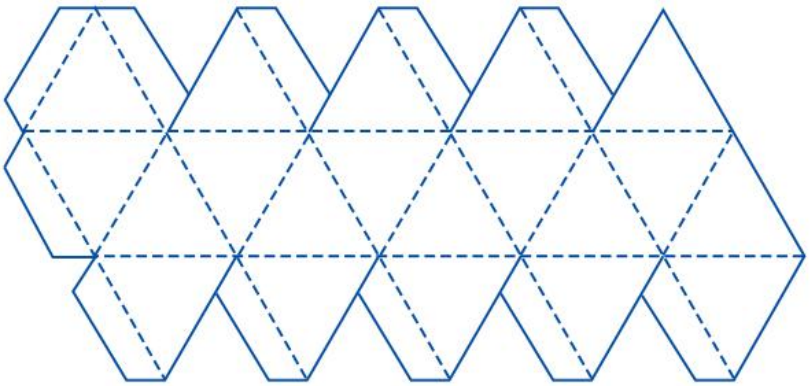
b



c



14 a The net below forms an icosahedron. Copy the net onto grid paper and fold it to make the icosahedron. You may wish to use the computer program called Poly to help you with the construction.



b Verify Euler's rule for this shape, showing your working.

7.7 Review

eBookplus ONLINE ONLY

www.jacplus.com.au

The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:

- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

Review questions

Download the Review questions document from the links found in your eBookPLUS.

eBookplus

Interactivities

Word search:
int-2627

Crossword:
int-2628

Sudoku:
int-3187

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

alternate angles	kite	rhombus
co-interior angles	opposite angles	square
congruent	opposite sides	transversal
corresponding angles	parallelogram	trapezium
corresponding sides	quadrilateral	triangle
diagonals	rectangle	vertically opposite angles

Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

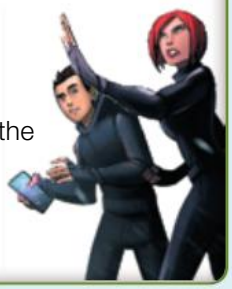
assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

www.assesson.com.au

Link to SpyClass, an exciting online game combining a comic book–style story with problem-based learning in an immersive environment.

Join Jesse, Toby and Dan and help them to tackle some of the world's most dangerous criminals by using the knowledge you've gained through your study of mathematics.

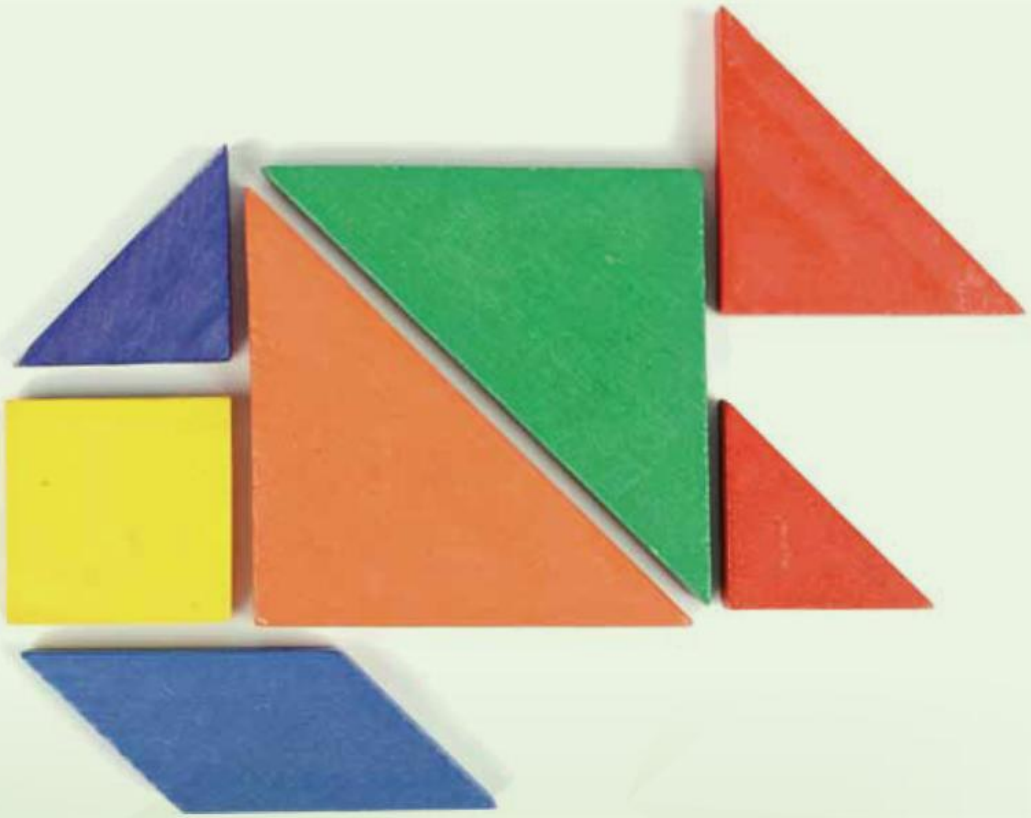
www.spyclass.com.au



INVESTIGATION

RICH TASK

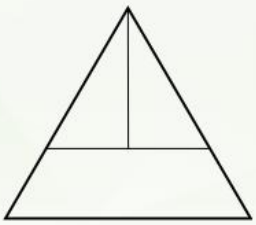
Puzzling constructions



How well do you know the properties of the different triangles and quadrilaterals? This activity will challenge you to create as many triangles and quadrilaterals as you can from three different types of puzzles.

Follow the instructions in this activity to create your three puzzles. Draw each shape onto a firm sheet of A4 paper. Carefully space the puzzles, so they will all fit onto the one sheet.

Three-piece triangle

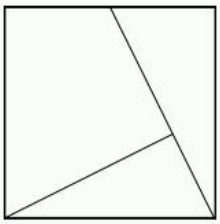


- Construct an equilateral triangle with side lengths of 10 cm. Mark the centre of each side.
- Lightly rule a line from each corner to the midpoint on the opposite side. These lines will intersect at one point.
- From this point, rule a line to the top corner. Rule another line parallel to the base side that goes through the centre point. Cut out the three shapes.

1 Name the three shapes created in this puzzle. List the main properties of each shape.

2 Use any number of pieces to construct as many triangles and quadrilaterals as you can. Draw each solution and list the main properties of each shape.

Three-piece square

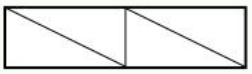


- Draw a square with side lengths of 10 cm. Mark the centre of the top and right sides.
- Rule a line from the centre of the top side to the bottom right corner.
- Rule another line from the bottom left corner towards the centre on the right side. Stop when you meet the previous line. Cut out the three shapes.

3 Name the three shapes created in this puzzle. List the main properties of each shape.

4 Repeat question **2** using the pieces from this puzzle.

Four-piece rectangle

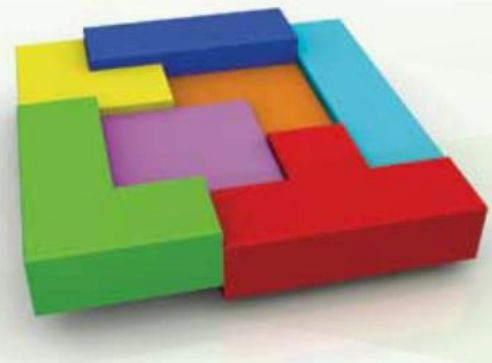


- Draw a rectangle measuring 20 cm by 5 cm.
- Mark the centre of the long side and divide the rectangle into two.
- Rule a diagonal line in each smaller rectangle. Cut out the four shapes.

5 What do you notice about the shapes you created? List their main properties.

6 Repeat question **2** using the pieces from this puzzle.

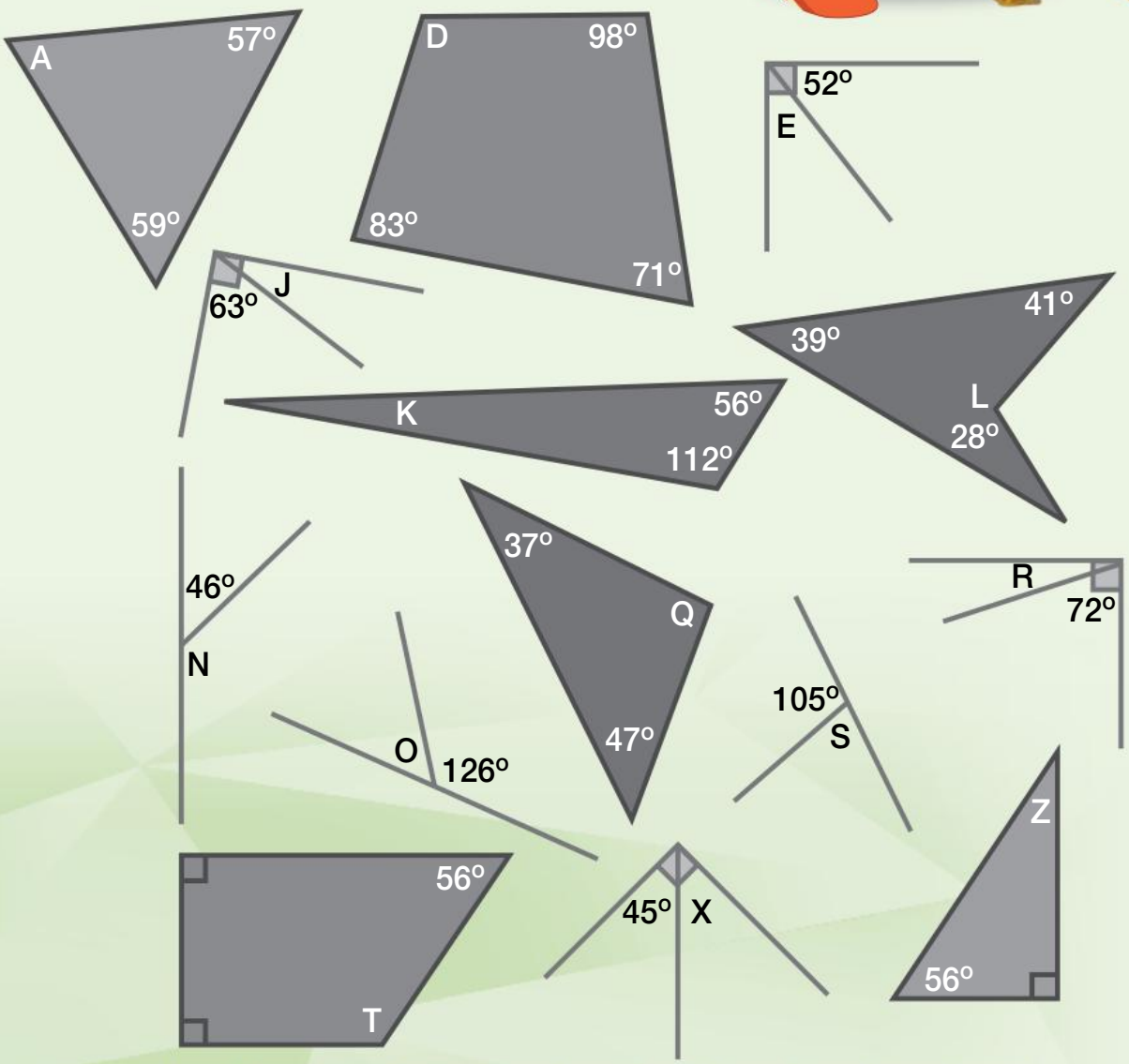
7 What other types of polygons can you construct from each individual puzzle?



CODE PUZZLE

What did the pencil say to the eraser?

The size of the lettered angles gives the puzzle answer code.



64°	108°	108°	64°	252°	38°	124°	124°	38°	18°	124°	54°
75°	34°	27°	108°	252°	108°	75°	134°				
45°	134°	124°	96°	96°	124°	12°	108°	96°			

eBookplus Activities

7.2 Congruent figures

Digital docs
- SkillSHEET (doc-6915) Naming angles and shapes
- SkillSHEET (doc-6916) Calculating angles in triangles

Interactivities
- Transformations and the Cartesian plane (int-2368)
- IP interactivity 7.2 (int-4424) Congruent figures

7.3 Triangle constructions

Interactivities
- Random triangles (int-2369)
- IP interactivity 7.3 (int-4425) Triangle constructions

7.4 Congruent triangles

Digital doc
- WorkSHEET 7.1 (doc-2250)

Interactivity
- IP interactivity 7.4 (int-4426) Congruent triangles

7.5 Quadrilaterals

Digital docs
- SkillSHEET (doc-6917) Angles and parallel lines
- WorkSHEET 7.2 (doc-2251)

Interactivity
- IP interactivity 7.5 (int-4427) Quadrilaterals

7.6 Nets, polyhedra and Euler's rule

Interactivity
- IP interactivity 7.6 (int-4428) Nets, polyhedra and Euler's rule

7.7 Review

Interactivities
- Word search (int-2627)
- Crossword (int-2628)
- Sudoku (int-3187)

Digital docs
- Topic summary (doc-10759)
- Concept map (doc-10772)
- Topic review (Word doc-14936, PDF doc-14937)

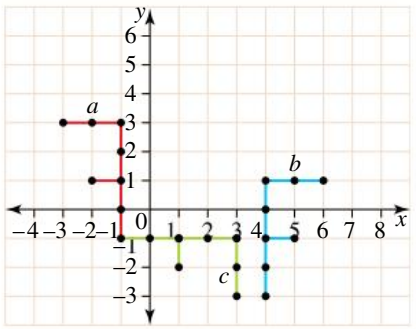
To access eBookPLUS activities, log on to www.jacplus.com.au

Answers

TOPIC 7 Congruence

7.2 Congruent figures

1 a–c



d Yes

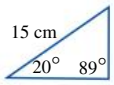
2 **a** **i** and **iii** **b** **i** and **iv** **c** **ii** and **iv**

3 **a** $\Delta ABC \equiv \Delta PQR$, $x = 25°$
b $\Delta ABC \equiv \Delta PQR$, $x = 70°$, $y = 40°$
c $\Delta ABC \equiv \Delta CDA$, $x = 40°$, $y = 55°$
d $\Delta ABC \equiv \Delta ADC$, $x = 30°$, $y = 30°$, $z = 120°$
e $\Delta ABC \equiv \Delta ADC$, $x = 40°$, $y = 40°$, $z = 100°$
f $\Delta ABC \equiv \Delta PRQ$, $x = 35°$, $y = 55°$

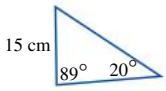
4 C

5 **a** $\Delta ABC \equiv \Delta ADC$ $x = 30°$ $y = 60°$
b $\Delta KLN \equiv \Delta MNL$ $x = 75°$ $y = 60°$ $z = 45°$
c $\Delta PSR \equiv \Delta QRS$ $x = 8$ cm
d $\Delta SWT \equiv \Delta UVT$ $w = y = 70°$ $x = z = 55°$
e $\Delta EFG \equiv \Delta EHG$ $x = 8$ cm
f $\Delta WXZ \equiv \Delta WYZ$ $x = 45°$ $y = 3$ cm
g $\Delta ABE \equiv \Delta CDB$ $x = 58°$ $y = 65°$

6



and



7 $a = 35°$, $b = 55°$, $c = 70°$, $d = 5$ cm, $e = 2$ cm

8 $a = 15$ cm, $b = 92°$, $c = 58°$

Challenge 7.1

Equilateral triangles, squares and regular hexagons

7.3 Triangle constructions

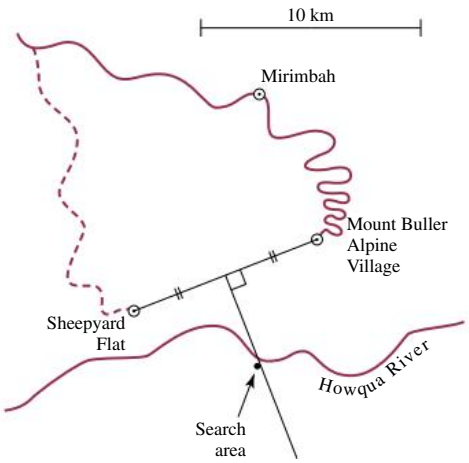
1 to **3** Check with your teacher.

4 **a** Check with your teacher.
b 89°
c **i**

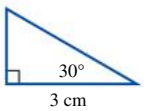
5 **a** Check with your teacher.
b 1 triangle

6, **7** Check with your teacher.

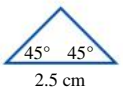
8



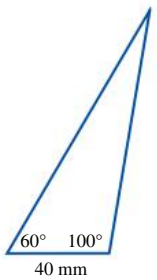
9 **a**



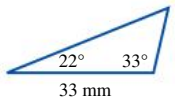
b



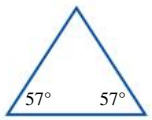
c



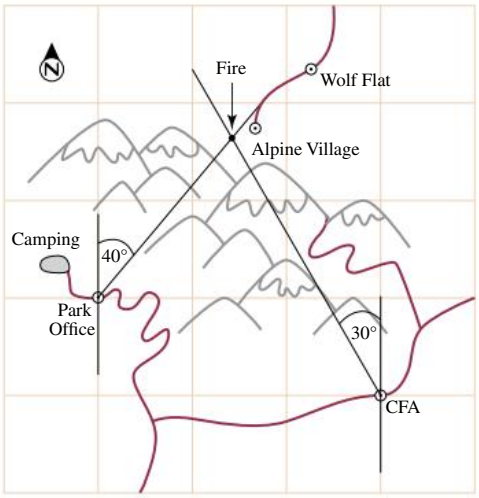
d



e

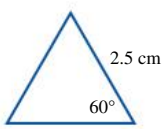


10

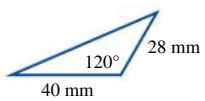


11 **a**

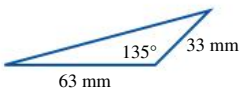
b



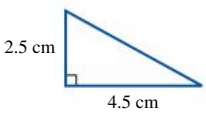
c



d



e

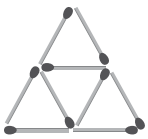


7.4 Congruent triangles

1 **a** $\Delta ABC \equiv \Delta PQR$ (SSS)
b $\Delta ABC \equiv \Delta PQR$ (SSS)
c $\Delta LMN \equiv \Delta PQR$ (SAS)
d $\Delta ABC \equiv \Delta PQR$ (ASA)
e $\Delta ABE \equiv \Delta ADE$ (SSS)
f $\Delta FKJ \equiv \Delta GKH$ (SSS)

2 **a** $x = 3$ cm
b $x = 83°$
c $x = 75°$ $y = 25°$ $z = 80°$
d $x = 25°$ $y = 8$ cm
e $x = 35°$ $y = z = 55°$ $n = m = 90°$
3 **a** AB = AC
BD = CD
AD is common.
ΔDBA ≡ ΔDCA (SSS)
b AB = AD
AC is common.
∠BAC = ∠CAD
ΔABC ≡ ΔADC (SAS)
c PQ = PS
PR is common.
∠PRQ = ∠PRS = 90°
ΔPQR = ΔPSR (RHS)
4, **5** Check with your teacher.
6 **a** Congruent (SSS) **b** Congruent (ASA)
7 to **10** Check with your teacher.

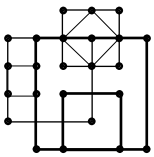
Challenge 7.2



7.5 Quadrilaterals

1 **a** ΔWXA = ΔYZA (ASA)
b **i** WA = YA
ii XA = ZA
c Diagonals bisect each other.
2 **a** ΔABE = ΔCBE (SSS)
b **i** ∠AEB = ∠CEB
ii ∠ABE = ∠CBE
c 90°
d ΔCBE, ΔADE, ΔCDE
e Diagonals bisect the angles.
3 **a** ΔPQR ≡ ΔSRP (SAS)
b Diagonals of a rectangle are equal.
4 **a** ΔABD ≡ ΔCBD (SSS)
b **i** ∠ABD = ∠CBD
ii ∠BAD = ∠BCD
iii ∠BDA = ∠BDC
c ΔABD ≡ ΔCBD (SAS)
d **i** AE = CE
ii ∠AEB = ∠CEB
e No
f **i** No
ii No (one pair of opposite angles are equal)
iii No (one diagonal bisects the other)
iv Yes
v No (one diagonal bisects the unequal angles)
5 **a** ΔQPR ≡ ΔSRP (SSS)
b **i** ∠SRP
ii ∠SPR
c **i** RS
ii PS
d Parallelogram
6 **a** ∠BAC = ∠DCA
b ΔABC ≡ ΔCDA (SAS)
c ∠BCA = ∠DAC (alternate angles)
d Parallelogram
7 All quadrilaterals can be divided into two adjacent triangles. The interior angle sum of a triangle is 180°, so the interior angle sum of a quadrilateral is equal to 360° (180° × 2).
8 Trapezium
9 $x = 107°$
10 50°, 70°, 110°, 130°
11 **a** Check with your teacher. **b** 357°

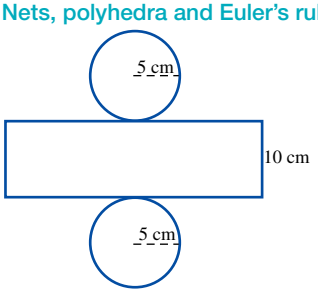
Challenge 7.3



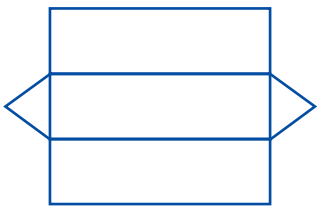
Six squares can be drawn.

7.6 Nets, polyhedra and Euler's rule

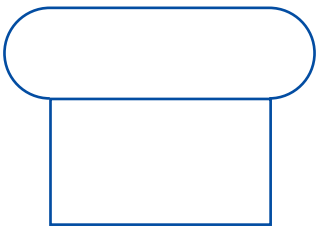
1



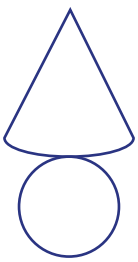
2 **a**



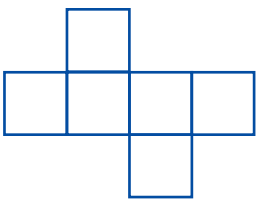
b



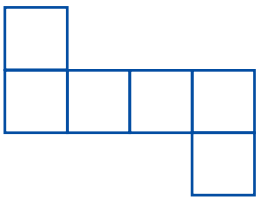
c



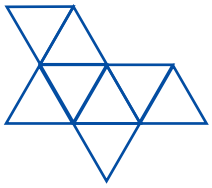
3 Two possible nets are:



or



4 A possible net is:



5 to **7** Check with your teacher.

8 **a** $E = 12$, $V = 8$, $F = 6$, $8 + 6 - 2 = 12$

b $E = 12$, $V = 6$, $F = 8$, $6 + 8 - 2 = 12$

9 **a** $E = 18$, $V = 12$, $F = 8$, $8 + 12 - 2 = 18$

b and **c** $E = 12$, $V = 8$, $F = 6$, $6 + 8 - 2 = 12$

10 For each shape, $E = 9$, $V = 6$ and $F = 5$ where $5 + 6 - 2 = 9$.

11 No. Edges, vertices and faces can not be identified on a solid with curves.

12 No

13 **a** Yes

b Yes

c No

14 **a** Check with your teacher.

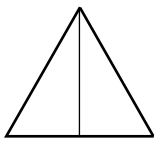
b 20 triangular faces, 30 edges and 12 vertices

$20 + 12 - 2 = 30$

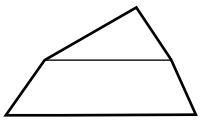
Investigation — Rich task

1 Two right-angled triangles and one trapezium with two parallel sides

2 Some possible solutions are shown; teacher to check for others.



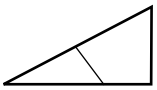
Equilateral triangle with two pieces; all angles = 60°



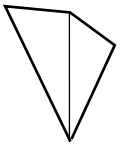
Irregular quadrilateral with two pieces; all sides have different lengths

3 One large and one smaller right-angled triangle and an irregular quadrilateral

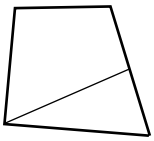
4 Some possible solutions are shown; teacher to check for others.



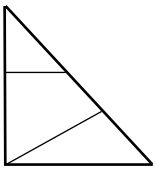
Right-angled triangle with two pieces



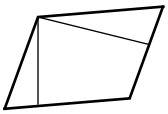
Irregular quadrilateral with two pieces



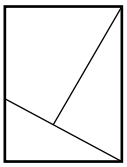
Trapezium with two pieces



Right-angled triangle with three pieces



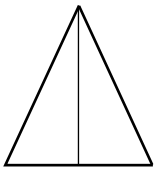
Parallelogram with three pieces



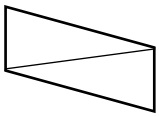
Rectangle with three pieces

5 Four identical right-angled triangles

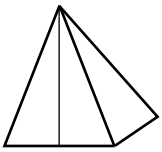
6 Some possible solutions are shown; teacher to check for others.



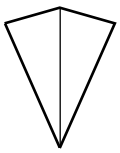
Isosceles triangle with two pieces



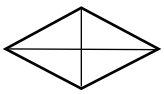
Parallelogram with two pieces



Irregular quadrilateral with three pieces



Kite with two pieces



Rhombus with all four pieces

7 Teacher to check.

Code puzzle

Take me to your ruler.

TOPIC 8

Algebra

8.1 Overview

Why learn this?

Humankind could not have travelled to the moon without algebra. Without algebra there would be no television, no iPod, no iPad — nothing electrical at all. A knowledge of algebra also makes possible complex geometry, which is reflected in structures ranging from the Colosseum to the simple suburban house. Algebra is the fundamental building block of mathematics.

You need a knowledge of algebra to succeed in mathematics at school. The further you advance in your studies, the more useful you will find algebra.

What do you know?

assessON

1 **THINK** List what you know about algebra. Use a thinking tool such as a concept map to show your list.
2 **PAIR** Share what you know with a partner and then with a small group.
3 **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of algebra.

Learning sequence

8.1 Overview
8.2 Using variables
8.3 Substitution
8.4 Working with brackets
8.5 Substituting positive and negative numbers
8.6 Number laws and variables
8.7 Simplifying expressions
8.8 Multiplying and dividing expressions with variables
8.9 Expanding brackets
8.10 Factorising
8.11 Review eBookplus ONLINE ONLY

8.2 Using variables

eBookplus
eLesson
Using variables
eles-0042

- A **variable** (or **pronumeral**) is a letter or symbol that represents a value in an algebraic expression or equation.
- In algebraic expressions such as $a + b$, the variables represent any number.
- In algebraic equations such as $x + y = 9$, variables are referred to as **unknowns** because the variable represents a specific value that is not yet known.
- When we write expressions with variables, the multiplication sign is omitted. For example, $8n$ means '$8 \times n$' and $12ab$ means '$12 \times a \times b$'.
- The division sign is rarely used. For example, $y \div 6$ is usually written as $\frac{y}{6}$.

WORKED EXAMPLE 1

Suppose we use b to represent the number of ants in a nest.

a Write an expression for the number of ants in the nest if 25 ants died.

b Write an expression for the number of ants in the nest if the original ant population doubled.

c Write an expression for the number of ants in the nest if the original population increased by 50.

d What would it mean if we said that a nearby nest contained $b + 100$ ants?

e What would it mean if we said that another nest contained $b - 1000$ ants?

f Another nest in very poor soil contains $\frac{b}{2}$ ants. How much smaller than the original is this nest?

THINK	WRITE
a The original number of ants (b) must be reduced by 25.	a $b - 25$
b The original number of ants (b) must be multiplied by 2. It is not necessary to show the $\times$ sign.	b $2b$
c 50 must be added to the original number of ants (b).	c $b + 50$
d This expression tells us that the nearby nest has 100 more ants.	d The nearby nest has 100 more ants.
e This expression tells us that the nest has 1000 fewer ants.	e This nest has 1000 fewer ants.
f The expression $\frac{b}{2}$ means $b \div 2$, so this nest is half the size of the original nest.	f This nest is half the size of the original nest.

Exercise 8.2 Using variables

assessON

INDIVIDUAL PATHWAYS

■ PRACTISE	■ CONSOLIDATE	■ MASTER
Questions: 1–8, 12	Questions: 1–9, 12, 13	Questions: 1–14

■■■ Individual pathway interactivity int-4429 eBookplus

REFLECTION
List some reasons for using variables instead of numbers.

FLUENCY

eBookplus
Digital doc
SkillSHEET
Alternative expressions used to describe the four operations
doc-6922

1 WE1 Suppose we use x to represent the number of ants in a nest.

- **a** Write an expression for the number of ants in the nest if 420 ants were born.
- **b** Write an expression for the number of ants in the nest if the original ant population tripled.
- **c** Write an expression for the number of ants in the nest if the original ant population decreased by 130.
- **d** What would it mean if we said that a nearby nest contained $x + 60$ ants?
- **e** What would it mean if we said that a nearby nest contained $x - 90$ ants?
- **f** Another nest in very poor soil contains $\frac{x}{4}$ ants. How much smaller than the original is this nest?

2 Suppose x people are in attendance at the start of an Aussie Rules football match.

- **a** If a further y people arrive during the first quarter, write an expression for the number of people at the ground.
- **b** Write an expression for the number of people at the ground if a further 260 people arrive prior to the second quarter commencing.

c At half-time 170 people leave. Write an expression for the number of people at the ground after they have left.

d In the final quarter a further 350 people leave. Write an expression for the number of people at the ground after they have left.

3 The canteen manager at Browning Industries orders m Danish pastries each day. Write a paragraph that could explain the table below.

Time	Number of Danish pastries
9.00 am	m
9.15 am	$m - 1$
10.45 am	$m - 12$
12.30 pm	$m - 12$
1.00 pm	$m - 30$
5.30 pm	$m - 30$

4 Imagine that your cutlery drawer contains a knives, b forks and c spoons.

a Write an expression for the total number of knives and forks you have.

b Write an expression for the total number of items in the drawer.

c You put 4 more forks in the drawer. Write an expression for the number of forks now.

d Write an expression for the number of knives in the drawer after 6 knives are removed.

5 If y represents a certain number, write expressions for the following numbers.

a A number 7 more than y.

b A number 8 less than y.

c A number that is equal to five times y.

d The number formed when y is subtracted from 14.

e The number formed when y is divided by 3.

f The number formed when y is multiplied by 8 and 3 is added to the result.

6 Using a and b to represent numbers, write expressions for:

a the sum of a and b

b the difference between a and b

c three times a subtracted from two times b

d the product of a and b

e twice the product of a and b

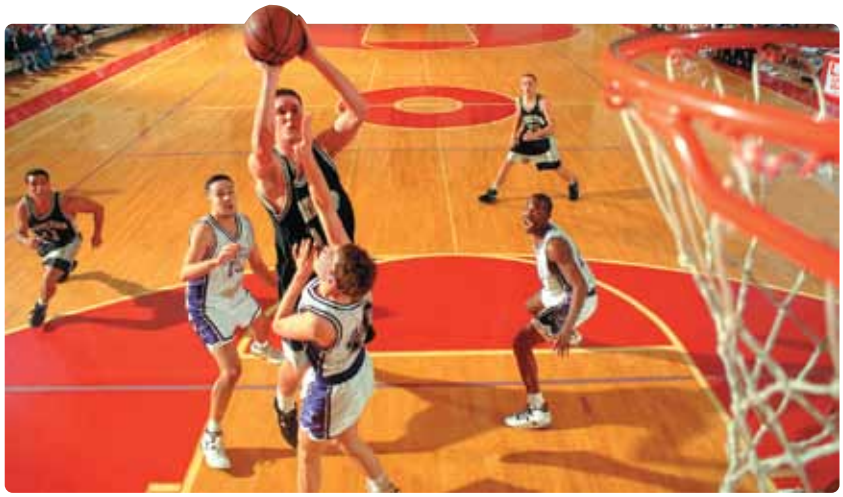
f the sum of $3a$ and $7b$

g a multiplied by itself

h a multiplied by itself and the result divided by 5.

7 If tickets to a basketball match cost $27 for adults and $14 for children, write an expression for the cost of:

a y adult tickets **b** d child tickets **c** r adult and h child tickets.



UNDERSTANDING

8 Naomi is now t years old.

a Write an expression for her age in 2 years' time.

b Write an expression for Steve's age if he is g years older than Naomi.

c How old was Naomi 5 years ago?

d Naomi's father is twice her age. How old is he?

9 James is travelling by train into town one particular evening and observes that there are t passengers in his carriage. He continues to take note of the number of people in his carriage each time the train departs from a station, which occurs every 3 minutes. The table below shows the number of passengers.

Time	Number of passengers
7.10 pm	t
7.13 pm	$2t$
7.16 pm	$2t + 12$
7.19 pm	$4t + 12$
7.22 pm	$4t + 7$
7.25 pm	t
7.28 pm	$t + 1$
7.31 pm	$t - 8$
7.34 pm	$t - 12$

a Write a paragraph explaining what happened.

b When did passengers first begin to alight the train?

c At what time did the carriage have the most number of passengers?

d At what time did the carriage have the least number of passengers?

REASONING

10 A microbiologist places m bacteria onto an agar plate. She counts the number of bacteria at approximately 3-hour intervals. The results are shown in the table below.

Time	Number of bacteria
9.00 am	m
Noon	$2m$
3.18 pm	$4m$
6.20 pm	$8m$
9.05 pm	$16m$
Midnight	$32m - 1240$

a Explain what happens to the number of bacteria in the first 5 intervals.
b What might be causing the number of bacteria to increase in this way?
c What is different about the last bacteria count?
d What may have happened to cause this?

11 n represents an even number.
a Is the number $n + 1$ odd or even?
b Is $3n$ odd or even?
c Write expressions for:
i the next three even numbers that are greater than n
ii the even number that is 2 less than n.

PROBLEM SOLVING

12 If the side of a square tile box is x cm long and the height is h cm, write expressions for the total surface area and the volume of the tile box.

13 If a rectangular tile box has the same width and height as the square tile box in question **12** but is one and a half times as long, write expressions for the total surface area and the volume of the tile box.

14 If the square tile box in question **12** has a side length of 20 cm and both boxes in questions **12** and **13** have a height of 15 cm, calculate the surface area and volume of the square tile box and the surface area and volume of the rectangular tile box using your expressions.

CHALLENGE 8.1

Bill was describing his age to a group of people using two different algebraic expressions. Four times a certain number minus one and three times a certain number plus three, both reveal Bill's age. What is the number used and what is Bill's age?

8.3 Substitution

- If the value of a variable (or variables) is known, it is possible to **evaluate** (work out the value of) an expression by using **substitution**. The variable is replaced with the number.
- Substitution can also be used with a formula or rule.

WORKED EXAMPLE 2

Find the value of the following expressions if $a = 3$ and $b = 15$.

a $6a$

b $7a - \frac{2b}{3}$

THINK	WRITE
a 1 Substitute the variable (a) with its correct value and replace the multiplication sign.	**a** $6a = 6 \times 3$
2 Evaluate and write the answer.	$= 18$
b 1 Substitute each variable with its correct value and replace the multiplication signs.	**b** $7a - \frac{2b}{3} = 7 \times 3 - \frac{2 \times 15}{3}$
2 Perform the first multiplication.	$= 21 - \frac{2 \times 15}{3}$
3 Perform the next multiplication.	$= 21 - \frac{30}{3}$
4 Perform the division.	$= 21 - 10$
5 Perform the subtraction and write the answer.	$= 11$

WORKED EXAMPLE 3

The formula for finding the area (A) of a rectangle of length l and width w is $A = l \times w$. Use this formula to find the area of the rectangle at right.

THINK	WRITE
1 Write the formula.	$A = l \times w$
2 Substitute each variable with its value.	$= 270 \times 32$
3 Perform the multiplication and state the correct units.	$= 8640 \text{ m}^2$

assessON

Exercise 8.3 Substitution

REFLECTION
Can any value be substituted for a variable in every expression?

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–4, 6, 8, 13	Questions: 1a–l, 2a–l, 3a–l, 4–8, 11, 13	Questions: 1m–t, 2i–p, 3i–p, 4–13

Individual pathway interactivity int-4430 eBook plus

FLUENCY

1 WE2 Find the value of the following expressions, if $a = 2$ and $b = 5$.

a $3a$	**b** $7a$	**c** $6b$	**d** $\frac{a}{2}$
e $a + 7$	**f** $b - 4$	**g** $a + b$	**h** $b - a$
i $5 + \frac{b}{5}$	**j** $3a + 9$	**k** $2a + 3b$	**l** $\frac{8}{a}$
m $\frac{25}{b}$	**n** ab	**o** $2ab$	**p** $7b - 30$
q $6b - 4a$	**r** $\frac{ab}{5}$	**s** $\frac{15}{a} + \frac{7}{b}$	**t** $\frac{9}{a} - \frac{3}{b}$

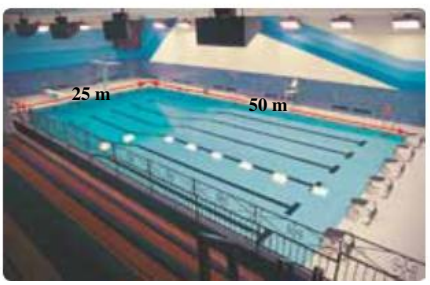
2 Substitute $x = 6$ and $y = 3$ into the following expressions and evaluate.

a $6x + 2y$	**b** $\frac{x}{3} + \frac{y}{3}$	**c** $3xy$	**d** $\frac{24}{x} - \frac{9}{y}$
e $\frac{12}{x} + 4 + y$	**f** $3x - y$	**g** $2.5x$	**h** $\frac{7x}{2}$
i $3.2x + 1.7y$	**j** $11y - 2x$	**k** $\frac{13y}{3} - 2x$	**l** $\frac{4xy}{15}$
m $4.8x - 3.5y$	**n** $8.7y - 3x$	**o** $12.3x - 9.6x$	**p** $\frac{3x}{9} - \frac{y}{12}$

3 Evaluate the following expressions, if $d = 5$ and $m = 2$.

a $d + m$	**b** $m + d$	**c** $m - d$	**d** $d - m$
e $2m$	**f** md	**g** $5dm$	**h** $\frac{md}{10}$
i $-3d$	**j** $-2m$	**k** $6m + 5d$	**l** $\frac{3md}{2}$
m $25m - 2d$	**n** $\frac{7d}{15}$	**o** $4dm - 21$	**p** $\frac{15}{d} - m$

4 WE3 The formula for finding the perimeter (P) of a rectangle of length l and width w is $P = 2l + 2w$. Use this formula to find the perimeter of the rectangular swimming pool at right.



5 The formula for the perimeter (P) of a square of side length l is $P = 4l$. Use this formula to find the perimeter of a square of side length 2.5 cm.

6 The formula $c = 0.1a + 42$ is used to calculate the cost in dollars (c) of renting a car for one day from Poole's Car Hire Ltd, where a is the number of kilometres travelled

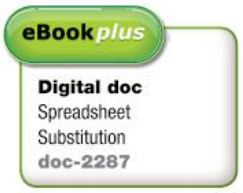
on that day. Find the cost of renting a car for one day if the distance travelled is 220 kilometres.

7 The area (A) of a rectangle of length l and width w can be found using the formula $A = lw$. Find the area of the rectangles below:

a length 12 cm, width 4 cm

b length 200 m, width 42 m

c length 4.3 m, width 104 cm.



UNDERSTANDING

8 The formula $F = \frac{9}{5}C + 32$ is used to convert temperatures measured in degrees Celsius to an approximate Fahrenheit value. F represents the temperature in degrees Fahrenheit (°F) and C the temperature in degrees Celsius (°C).

a Find F when $C = 100$ °C.

b Convert 28 °C to Fahrenheit.

c Water freezes at 0 °C. What is the freezing temperature of water in Fahrenheit?

9 The formula $D = 0.6T$ can be used to convert distances in kilometres (T) to the approximate equivalent in miles (D). Use this rule to convert the following distances to miles:

a 100 kilometres **b** 248 kilometres **c** 12.5 kilometres.

REASONING

10 Ben says that $\frac{4x^2}{2x} = 2x$. Emma says that is not correct if $x = 0$. Explain Emma's reasoning.

PROBLEM SOLVING

11 If $s = ut + \frac{1}{2}at^2$, evaluate s if $u = 5$, $t = 10$ and $a = 9.81$.

12 The width of a cuboid is x cm.

a If the length is 5 cm more than the width and the height is 2 cm less than the width, find the volume, V cm^3, of the cuboid in terms of x.

b Evaluate V if x equals 10.

c Explain why x cannot equal 1.5.

13 On the space battleship RAN *Fantasie*, there are p Pletons, each with 2 legs, $(p - 50)$ Argors, each with 3 legs, and $(2p + 35)$ Kleptors, each with 4 legs.

a Find the total number of legs, L, on board the *Fantasie*, in terms of p, in simplified form.

b If $p = 200$, find L.

8.4 Working with brackets

- Brackets are grouping symbols. The expression $3(a + 5)$ can be thought of as 'three groups of $(a + 5)$', or $(a + 5) + (a + 5) + (a + 5)$.
- When substituting into an expression with brackets, remember to place a multiplication sign ($\times$) next to the brackets. For example, $3(a + 5)$ is thought of as $3 \times (a + 5)$.
- Following operation order, evaluate the brackets first and then multiply by the value outside of the brackets.

WORKED EXAMPLE 4

a **Substitute $r = 4$ and $s = 5$ into the expression $5(s + r)$ and evaluate.**
b **Substitute $t = 4$, $x = 3$ and $y = 5$ into the expression $2x(3t - y)$ and evaluate.**

THINK	WRITE
a 1 Place the multiplication sign back into the expression.	**a** $5(s + r) = 5 \times (s + r)$
2 Substitute the variables with their correct values.	$= 5 \times (5 + 4)$
3 Evaluate the expression in the pair of brackets first.	$= 5 \times 9$
4 Perform the multiplication and write the answer.	$= 45$
b 1 Place the multiplication signs back into the expression.	**b** $2x(3t - y) = 2 \times x \times (3 \times t - y)$
2 Substitute the variables with their correct values.	$= 2 \times 3 \times (3 \times 4 - 5)$
3 Perform the multiplication inside the pair of brackets.	$= 2 \times 3 \times (12 - 5)$
4 Perform the subtraction inside the pair of brackets.	$= 2 \times 3 \times 7$
5 Perform the multiplication and write the answer.	$= 42$

assess on

Exercise 8.4 Working with brackets

REFLECTION
Is operation order followed when substituting values for variables?

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–4, 7	Questions: 1a–l, 2a–l, 3–5, 7	Questions: 1j–p, 2j–p, 3–8

Individual pathway interactivity int-4431 eBookplus

FLUENCY

1 WE4 Substitute $r = 5$ and $s = 7$ into the following expressions and evaluate.

a $3(r + s)$	**b** $2(s - r)$	**c** $7(r + s)$
d $9(s - r)$	**e** $s(r + 3)$	**f** $s(2r - 5)$
g $3r(r + 1)$	**h** $rs(3 + s)$	**i** $11r(s - 6)$
j $2r(s - r)$	**k** $s(4 + 3r)$	**l** $7s(r - 2)$
m $s(3rs + 7)$	**n** $5r(24 - 2s)$	**o** $5sr(sr + 3s)$
p $8r(12 - s)$		

2 Evaluate each of the expressions below, if $x = 3$, $y = 5$ and $z = 9$.

a $xy(z - 3)$ **b** $\frac{12}{x}(z - y)$ **c** $\frac{z}{3}\left(\frac{2y}{10} + x - 2\right)$

d $(x + y)(z - y)$ **e** $(z - 3)4x$ **f** $zy(17 - xy)$

g $\frac{y}{5}(7 - x + 3)$ **h** $(8 - y)(z + x)$ **i** $\left(7 - \frac{12}{x}\right)4y$

j $\frac{6}{x}(xz + y - 3)$ **k** $(y + 2)\frac{z}{x}$ **l** $2x(xyz - 105)$

m $12(y - 1)(z + 3)$ **n** $(3x - 7)\left(\frac{27}{x} + 7\right)$ **o** $-2(4x + 1)\left(\frac{36}{z} - 3\right)$

p $-3(2y - 11)\left(\frac{z}{x} + 8\right)$

3 The formula for the perimeter (P) of a rectangle of length l and width w is $P = 2l + 2w$. This rule can also be written as $P = 2(l + w)$. Use the rule to find the perimeter of rectangular comic covers with the following measurements.

a $l = 20$ cm, $w = 11$ cm

b $l = 27.5$ cm, $w = 21.4$ cm

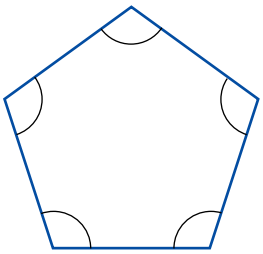
UNDERSTANDING

4 MC When $a = 8$ and $b = 12$ are substituted into the expression $\frac{a}{6}(15 - b + 9)$, the expression is equal to

A 32 **B** 16 **C** $21\frac{1}{3}$

D 24 **E** 27

5 A rule for finding the sum of the interior angles in a many-sided figure such as a pentagon is $S = 180(n - 2)^\circ$, where S represents the sum of the angles inside the figure and n represents the number of sides. The diagram at right shows the interior angles in a pentagon.



Use the rule to find the sum of the interior angles for the following figures:

a a hexagon (6 sides) **b** a pentagon

c a triangle **d** a quadrilateral (4 sides)

e a 20-sided figure.

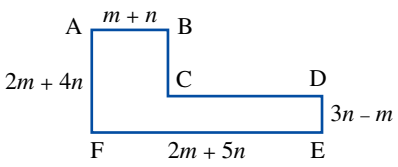
REASONING

6 The dimensions of the figure shown are given in terms of m and n. Write, in terms of m and n, an expression for:

a the length of CD **b** the length of BC

c the perimeter of the figure.

Show all of your working.



PROBLEM SOLVING

7 **a** Find an expression for the area of a triangle whose base length is $(m + n)$ cm and whose height is $(m - n)$ cm.

b If $m = 15$ and $n = 6$, find the area of the triangle.

c Show that $m > n$.

d Explain what happens to the triangle as m and n move closer in value.

8 It can be shown that $(x - a)(x - a) = x^2 - 2ax + a^2$. By substitution, show that this is true if:

a $x = 4, a = 1$

b $x = 3p, a = 2p$

c $x = 0$

8.5 Substituting positive and negative numbers

- If the variable you are substituting for has a negative value, simply remember the following rules for directed numbers:
  1. For addition and subtraction, signs that occur together can be combined.

Same signs positive	for example,	$7 + +3 = 7 + 3$
	and	$7 - -3 = 7 + 3$
Different signs negative	for example,	$7 - +3 = 7 - 3$
	and	$7 + -3 = 7 - 3$

  2. For multiplication and division.

Same signs positive	for example,	$+7 \times +3 = +21$
	and	$-7 \times -3 = +21$
Different signs negative	for example,	$+7 \times -3 = -21$
	and	$-7 \times +3 = -21$

WORKED EXAMPLE 5

a Substitute $m = 5$ and $n = -3$ into the expression $m - n$ and evaluate.
b Substitute $m = -2$ and $n = -1$ into the expression $2n - m$ and evaluate.
c Substitute $a = 4$ and $b = -3$ into the expression $5ab - \frac{12}{b}$ and evaluate.

THINK	WRITE
a 1 Substitute the variables with their correct value.	**a** $m - n = 5 - -3$
2 Combine the two negative signs and add.	$= 5 + 3$
3 Write the answer.	$= 8$
b 1 Replace the multiplication sign.	**b** $2n - m = 2 \times n - m$
2 Substitute the variables with their correct values.	$= 2 \times -1 - -2$
3 Perform the multiplication.	$= -2 - -2$
4 Combine the two negative signs and add.	$= -2 + 2$
5 Write the answer.	$= 0$

c 1 Replace the multiplication signs.
2 Substitute the variables with their correct values.
3 Perform the multiplications.
4 Perform the division.
5 Combine the two negative signs and add.
6 Write the answer.

c $5ab - \frac{12}{b} = 5 \times a \times b - \frac{12}{b}$
$= 5 \times 4 \times -3 - \frac{12}{-3}$
$= -60 - \frac{12}{-3}$
$= -60 - -4$
$= -60 + 4$
$= -56$

Exercise 8.5 Substituting positive and negative numbers

assessON

INDIVIDUAL PATHWAYS

■ PRACTISE	■ CONSOLIDATE	■ MASTER
Questions: 1–4	Questions: 1a–l, 2, 3a–l, 4, 6, 7	Questions: 1m–t, 2g–l, 3m–t, 4–8

■■■ Individual pathway interactivity int-4432 eBookplus

REFLECTION
What can you say about the sign of x^2?

FLUENCY

1 WE5a Substitute $m = 6$ and $n = -3$ into the following expressions and evaluate.

a $m + n$ **b** $m - n$ **c** $n - m$
d $n + m$ **e** $3n$ **f** $-2m$
g $2n - m$ **h** $n + 5$ **i** $2m + n - 4$
j $11n + 20$ **k** $-5n - m$ **l** $\frac{m}{2}$
m $\frac{mn}{9}$ **n** $\frac{4m}{n-5}$ **o** $\frac{4m}{n}$
p $\frac{12}{2n}$ **q** $\frac{9}{n} + \frac{m}{2}$ **r** $6mn - 1$
s $-\frac{3n}{2} + 1.5$ **t** $14 - \frac{mn}{9}$

2 WE5b Substitute $x = 8$ and $y = -3$ into the following expressions and evaluate.

a $3(x - 2)$ **b** $x(7 + y)$ **c** $5y(x - 7)$
d $2(3 - y)$ **e** $(y + 5)x$ **f** $xy(7 - x)$
g $(3 + x)(5 + y)$ **h** $5(7 - xy)$ **i** $\frac{x}{2}(5 - y)$
j $\left(\frac{x}{4} - 1\right)\left(\frac{2y}{6} + 4\right)$ **k** $\frac{9}{y}(6 - x)$ **l** $3(x - 1)\left(\frac{y}{3} + 2\right)$

eBookplus
Digital docs
SkillSHEET
Order of operations II
doc-6923
SkillSHEET
Order of operations with brackets
doc-6924
SkillSHEET
Operations with directed numbers
doc-6925

3 WE5c Substitute $a = -4$ and $b = -5$ into the following expressions and evaluate.

a $a + b$	**b** $a - b$	**c** $b - 2a$	**d** $2ab$
e $12 - ab$	**f** $-2(b - a)$	**g** $a - b - 4$	**h** $3a(b + 4)$
i $\frac{4}{b}$	**j** $\frac{8}{a}$	**k** $\frac{16}{4a}$	**l** $\frac{6b}{5}$
m $45 + 4ab$	**n** $8ab - 3b$	**o** $\frac{a}{2} + \frac{3b}{5}$	**p** $2.5b$
q $11a + 6b$	**r** $(a - 5)(8 - b)$	**s** $(9 - a)(b - 3)$	**t** $1.5b + 2a$

UNDERSTANDING

4 If $p = -2$ and $q = -3$, evaluate $\frac{3(-pq - p^2)}{q + 2p}$.

REASONING

5 Consider the expression $1 - 5x$. If x is a negative integer, explain why the expression will have a positive value.

PROBLEM SOLVING

6 Consider the equation $(a - b)(a + b) = a^2 - b^2$.

a By substituting $a = -3$ and $b = -2$, show that this is true.

b By substituting $a = -q$ and $b = -2q$, show that this is true.

7 If $x = -2r$ is substituted into $\frac{(r - x)(x + r)}{(r - 2x)}$, will the answer be positive or negative if:

a $r > 0$?

b $r < 0$?

8 A circle is cut out of a square.

a If the side length of the square is x, the radius of the circle is $0.25x$ and the area of the circle is approximately $0.196\ x^2$, find an expression for the remaining area.

b Calculate the area when $x = 2$ by substituting into your expression. Give your answer to 3 decimal places.

c What is the largest radius the circle can have?

eBookplus
Digital doc
WorkSHEET 8.1
doc-2290

8.6 Number laws and variables

- When dealing with any type of number, we must obey particular rules.

Commutative Law

- The **Commutative Law** refers to the order in which two numbers may be added, subtracted, multiplied or divided.
- The Commutative Law holds true for addition and multiplication because the order in which two numbers are added or multiplied does not affect the result.

$$3 + 2 = 2 + 3$$
$$3 \times 2 = 2 \times 3$$

- Since variables take the place of numbers, the Commutative Law holds true for the addition and multiplication of variables.

$$x + y = y + x$$
$$x \times y = y \times x$$

- The Commutative Law does not hold true for subtraction or division because the results obtained are different.

$$3 - 2 \neq 2 - 3$$
$$3 \div 2 \neq 2 \div 3$$

- Since variables take the place of numbers, the Commutative Law does not hold true for the subtraction and division of variables.

$$x - y \neq y - x$$
$$x \div y \neq y \div x$$

WORKED EXAMPLE 6

Find the value of the following expressions if $x = 4$ and $y = 7$. Comment on the results obtained.

a i $x + y$ **ii** $y + x$ **b i** $x - y$ **ii** $y - x$
c i $x \times y$ **ii** $y \times x$ **d i** $x \div y$ **ii** $y \div x$

THINK	WRITE
a i 1 Substitute each variable with its correct value.	**a i** $x + y = 4 + 7$
2 Evaluate and write the answer.	$= 11$
ii 1 Substitute each variable with its correct value.	**ii** $y + x = 7 + 4$
2 Evaluate and write the answer.	$= 11$
3 Compare the result with the answer obtained in part **a i**.	The same result is obtained; therefore, order is not important when adding two terms.
b i 1 Substitute each variable with its correct value.	**b i** $x - y = 4 - 7$
2 Evaluate and write the answer.	$= -3$
ii 1 Substitute each variable with its correct value.	**ii** $y - x = 7 - 4$
2 Evaluate and write the answer.	$= 3$
3 Compare the result with the answer obtained in part **b i**.	Two different results are obtained; therefore, order is important when subtracting two terms.
c i 1 Substitute each variable with its correct value.	**c i** $x \times y = 4 \times 7$
2 Evaluate and write the answer.	$= 28$

ii **1** Substitute each variable with its correct value.

$y \times x = 7 \times 4$

2 Evaluate and write the answer.

$= 28$

3 Compare the result with the answer obtained in part **c i**.

The same result is obtained; therefore, order is not important when multiplying two terms.

d i **1** Substitute each variable with its correct value.

$x \div y = 4 \div 7$

2 Evaluate and write the answer.

$= \frac{4}{7} (\approx 0.57)$

ii **1** Substitute each variable with its correct value.

$y \div x = 7 \div 4$

2 Evaluate and write the answer.

$= \frac{7}{4} (1.75)$

3 Compare the result with the answer obtained in part **d i**.

Two different results are obtained; therefore, order is important when dividing two terms.

eBookplus
Interactivity
The Associative Law
int-2370

Associative Law

- The **Associative Law** refers to the order in which three numbers may be added, subtracted, multiplied or divided, taking two at a time.
  Note: The Associative Law refers to the order in which the addition (or other operation) is performed, and this order is indicated by the use of brackets. The order in which the variables are written does not change.
- Like the Commutative Law, the Associative Law holds true for addition and multiplication of numbers.

$$5 + (10 + 3) = (5 + 10) + 3$$
$$5 \times (10 \times 3) = (5 \times 10) \times 3$$

- Since variables take the place of numbers, the Associative Law holds true for the addition and multiplication of variables.

$$x + (y + z) = (x + y) + z$$
$$x \times (y \times z) = (x \times y) \times z$$

- Like the Commutative Law, the Associative Law does not hold for subtraction and division of numbers.

$$5 - (10 - 3) \neq (5 - 10) - 3$$
$$5 \div (10 \div 3) \neq (5 \div 10) \div 3$$

- Since variables take the place of numbers, the Associative law does not hold true for the subtraction and division of variables.

$$x - (y - z) \neq (x - y) - z$$
$$x \div (y \div z) \neq (x \div y) \div z$$

WORKED EXAMPLE 7

Find the value of the following expressions if $x = 12$, $y = 6$ and $z = 2$. Comment on the results obtained.

a i $x + (y + z)$ **ii** $(x + y) + z$ **b i** $x - (y - z)$ **ii** $(x - y) - z$
c i $x \times (y \times z)$ **ii** $(x \times y) \times z$ **d i** $x \div (y \div z)$ **ii** $(x \div y) \div z$

THINK	WRITE
a i 1 Substitute each variable with its correct value.	**a i** $x + (y + z) = 12 + (6 + 2)$
2 Evaluate the expression in the pair of brackets.	$= 12 + 8$
3 Perform the addition and write the answer.	$= 20$
ii 1 Substitute each variable with its correct value.	**ii** $(x + y) + z = (12 + 6) + 2$
2 Evaluate the expression in the pair of brackets.	$= 18 + 2$
3 Perform the addition and write the answer.	$= 20$
4 Compare the result with the answer obtained in part **a i**.	The same result is obtained; therefore, order is not important when adding 3 terms.
b i 1 Substitute each variable with its correct value.	**b i** $x - (y - z) = 12 - (6 - 2)$
2 Evaluate the expression in the pair of brackets.	$= 12 - 4$
3 Perform the subtraction and write the answer.	$= 8$
ii 1 Substitute each variable with its correct value.	**ii** $(x - y) - z = (12 - 6) - 2$
2 Evaluate the expression in the pair of brackets.	$= 6 - 2$
3 Perform the subtraction and write the answer.	$= 4$
4 Compare the result with the answer obtained in part **b i**.	Two different results are obtained; therefore, order is important when subtracting 3 terms.
c i 1 Substitute each variable with its correct value.	**c i** $x \times (y \times z) = 12 \times (6 \times 2)$
2 Evaluate the expression in the pair of brackets.	$= 12 \times 12$
3 Perform the multiplication and write the answer.	$= 144$
ii 1 Substitute each variable with its correct value.	**ii** $(x \times y) \times z = (12 \times 6) \times 2$

	2 Evaluate the expression in the pair of brackets.		$= 72 \times 2$
	3 Perform the multiplication and write the answer.		$= 144$
	4 Compare the result with the answer obtained in part **c i**.		The same result is obtained; therefore, order is not important when multiplying 3 terms.
d i	1 Substitute each variable with its correct value.	**d i**	$x \div (y \div z) = 12 \div (6 \div 2)$
	2 Evaluate the expression in the pair of brackets.		$= 12 \div 3$
	3 Perform the division and write the answer.		$= 4$
ii	1 Substitute each variable with its correct value.	**ii**	$(x \div y) \div z = (12 \div 6) \div 2$
	2 Evaluate the expression in the pair of brackets.		$= 2 \div 2$
	3 Perform the division and write the answer.		$= 1$
	4 Compare the result with the answer obtained in part **d i**.		Two different results are obtained; therefore, order is important when dividing 3 terms.

Identity Law

- The **Identity Law** for addition states that when zero is added to any number, the original number remains unchanged. For example, $5 + 0 = 0 + 5 = 5$.
- The Identity Law for multiplication states that when any number is multiplied by one, the original number remains unchanged. For example, $3 \times 1 = 1 \times 3 = 3$.
- Since variables take the place of numbers:

$$x + 0 = 0 + x = x$$

$$x \times 1 = 1 \times x = x$$

Inverse Law

- The **Inverse Law** for addition states that when a number is added to its additive inverse (opposite sign), the result is zero. For example, $5 + -5 = 0$.
- The Inverse Law for multiplication states that when a number is multiplied by its multiplicative inverse (reciprocal), the result is one. For example, $3 \times \frac{1}{3} = 1$.
- Since variables take the place of numbers:

$$x + -x = -x + x = 0$$

$$x \times \frac{1}{x} = \frac{1}{x} \times x = 1$$

Exercise 8.6 Number laws and variables

assessON

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–6, 10	Questions: 1a–e, 2a–e, 3a–h, 4a–d, 5a–d, 6, 7, 9, 10	Questions: 1e–h, 2e–h, 3g–l, 4c–f, 5c–f, 6–11

Individual pathway interactivity int-4433 eBook plus

REFLECTION
The Commutative Law does not hold for subtraction. What can you say about the results of $x - a$ and $a - x$?

FLUENCY

1 WE6a,b Find the value of the following expressions if $x = 3$ and $y = 8$. Comment on the results obtained.

a i $x + y$ ii $y + x$
b i $3x + 2y$ ii $2y + 3x$
c i $5x + 2y$ ii $2y + 5x$
d i $8x + y$ ii $y + 8x$
e i $x - y$ ii $y - x$
f i $2x - 3y$ ii $3y - 2x$
g i $4x - 5y$ ii $5y - 4x$
h i $3x - y$ ii $y - 3x$

2 WE6c,d Find the value of the following expressions if $x = -2$ and $y = 5$. Comment on the results obtained.

a i $x \times y$ ii $y \times x$
b i $6x \times 3y$ ii $3y \times 6x$
c i $4x \times y$ ii $y \times 4x$
d i $7x \times 5y$ ii $5y \times 7x$
e i $x \div y$ ii $y \div x$
f i $10x \div 4y$ ii $4y \div 10x$
g i $6x \div 3y$ ii $3y \div 6x$
h i $7x \div 9y$ ii $9y \div 7x$

3 Indicate whether each of the following is true or false for all values of the variables.

a $a + 5b = 5b + a$ **b** $6x - 2y = 2y - 6x$ **c** $7c + 3d = -3d + 7c$
d $5 \times 2x \times x = 10x^2$ **e** $4x \times -y = -y \times 4x$ **f** $4 \times 3x \times x = 12x \times x$
g $\frac{5p}{3r} = \frac{3r}{5p}$ **h** $-7i - 2j = 2j + 7i$ **i** $-3y \div 4x = 4x \div -3y$
j $-2c + 3d = 3d - 2c$ **k** $\frac{0}{3s} = \frac{3s}{0}$ **l** $15 \times -\frac{2x}{3} = \frac{2x}{3} \times -15$

4 WE7a,b Find the value of the following expressions if $x = 3$, $y = 8$ and $z = 2$. Comment on the results obtained.

a i $x + (y + z)$ ii $(x + y) + z$
b i $2x + (y + 5z)$ ii $(2x + y) + 5z$
c i $6x + (2y + 3z)$ ii $(6x + 2y) + 3z$
d i $x - (y - z)$ ii $(x - y) - z$

e **i** $x - (7y - 9z)$ **ii** $(x - 7y) - 9z$

f **i** $3x - (8y - 6z)$ **ii** $(3x - 8y) - 6z$

5 **WE7c,d** Find the value of the following expressions if $x = 8$, $y = 4$ and $z = -2$. Comment on the results obtained.

a **i** $x \times (y \times z)$ **ii** $(x \times y) \times z$

b **i** $x \times (-3y \times 4z)$ **ii** $(x \times -3y) \times 4z$

c **i** $2x \times (3y \times 4z)$ **ii** $(2x \times 3y) \times 4z$

d **i** $x \div (y \div z)$ **ii** $(x \div y) \div z$

e **i** $x \div (2y \div 3z)$ **ii** $(x \div 2y) \div 3z$

f **i** $-x \div (5y \div 2z)$ **ii** $(-x \div 5y) \div 2z$

UNDERSTANDING

6 Indicate whether each of the following is true or false for all values of the variables.

a $a - 0 = 0$ **b** $a \times 1\,000\,000 = 0$ **c** $15t \times -\frac{1}{15t} = 1$

d $3d \times \frac{1}{3d} = 1$ **e** $\frac{8x}{9y} \div \frac{8x}{9y} = 1$ **f** $\frac{11t}{0} = 0$

7 **MC** The value of the expression $x \times (-3y \times 4z)$ when $x = 4$, $y = 3$ and $z = -3$ is:

A 108 **B** −432 **C** 432

D 112 **E** −108

8 **MC** The value of the expression $(x - 8y) - 10z$ when $x = 6$, $y = 5$ and $z = -4$ is:

A −74 **B** 74 **C** −6

D 6 **E** −36

REASONING

9 **a** If $x = -1$, $y = -2$ and $z = -3$, find the value of:

i $-x - y - z$

ii $-x - (y + z)$

b Comment on the answers with special reference to the Associative Law.

PROBLEM SOLVING

10 Evaluate each of the following expressions for $x = -3$, $y = 2$ and $z = -1$.

a $2x - (3y + 2z)$

b $x \times (y - 2z)$

11 **a** State the additive inverse of $(3p - 4q)$.

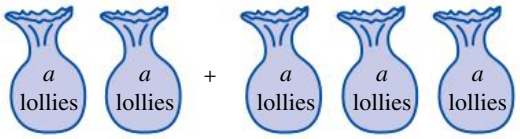
b State the multiplicative inverse of $(3p - 4q)$.

c Evaluate **a** and **b** when $p = -1$ and $q = 3$.

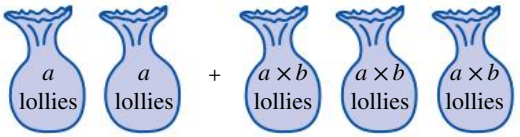
d What happens if you multiply a term or pronumeral by its multiplicative inverse?

8.7 Simplifying expressions

- Expressions can often be written in a more simple form by collecting (adding or subtracting) like terms.
- **Like terms** are terms that contain exactly the same variables, raised to the same power. To understand why $2a + 3a$ *can* be added but $2a + 3ab$ can *not* be added, consider the following identical bags of lollies, each containing a lollies.



So, $2a + 3a = 5a$. Therefore we have 5 bags each containing a lollies.
Then consider the following 2 bags containing a lollies and 3 bags containing $a \times b$ lollies.



So $2a + 3ab$ cannot be added as they are not identical and we do not have any further information.
So, $2a + 3ab = 2a + 3ab$. Therefore all we can say is that we have 2 bags containing a lollies and 3 bags containing ab lollies.
For example:

$3x$ and $4x$ are like terms.
$3ab$ and $7ab$ are like terms.
$2bc$ and $4cb$ are like terms.
$3g^2$ and $45g^2$ are like terms.

$3x$ and $3y$ are not like terms.
$7ab$ and $8a$ are not like terms.
$8a$ and $3a^2$ are not like terms.

WORKED EXAMPLE 8

Simplify the following expressions.
a $3a + 5a$ **b** $7ab - 3a - 4ab$ **c** $2c - 6 + 4c + 15$

THINK		WRITE
a 1	Write the expression and check that the two terms are like terms, that is, they contain the same variables.	a $3a + 5a$
2	Add the like terms and write the answer.	$= 8a$
b 1	Write the expression and check for like terms.	b $7ab - 3a - 4ab$
2	Rearrange the terms so that the like terms are together. Remember to keep the correct sign in front of each term.	$= 7ab - 4ab - 3a$
3	Subtract the like terms and write the answer.	$= 3ab - 3a$
c 1	Write the expression and check for like terms.	c $2c - 6 + 4c + 15$
2	Rearrange the terms so that the like terms are together. Remember to keep the correct sign in front of each term.	$= 2c + 4c - 6 + 15$
3	Simplify by collecting like terms and write the answer.	$= 6c + 9$

assessON

Exercise 8.7 Simplifying expressions

REFLECTION
What do you need to remember when checking for like terms?

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3a–i, 4	Questions: 1, 2a–o, 3a–l, 4, 7	Questions: 1m–x, 2m–v, 3m–u, 4–7

Individual pathway interactivity int-4434 eBookplus

eBookplus
Digital docs
SkillSHEET Combining like terms doc-6926
SkillSHEET Simplifying fractions doc-6927

FLUENCY

1 WE8a Simplify the following expressions.

a $4c + 2c$
b $2c - 5c$
c $3a + 5a - 4a$
d $6q - 5q$
e $-h - 2h$
f $7x - 5x$
g $3a - 7a - 2a$
h $-3f + 7f$
i $4p - 7p$
j $-3h + 4h$
k $11b + 2b + 5b$
l $7t - 8t + 4t$
m $9m + 5m - m$
n $x - 2x$
o $7z + 13z$
p $5p + 3p + 2p$
q $9g + 12g - 4g$
r $18b - 4b - 11b$
s $13t - 4t + 5t$
t $-11j + 4j$
u $-12l + 2l - 5l$
v $13m - 2m - 4m + m$
w $m + 3m - 4m$
x $t + 2t - t + 8t$

2 WE8b,c Simplify the following expressions.

a $3x + 7x - 2y$
b $3x + 4x - 12$
c $11 + 5f - 7f$
d $3u - 4u + 6$
e $2m + 3p + 5m$
f $-3h + 4r - 2h$
g $11a - 5b + 6a$
h $9t - 7 + 5$
i $12 - 3g + 5$
j $6m + 4m - 3n + n$
k $5k - 5 + 2k - 7$
l $3n - 4 + n - 5$
m $2b - 6 - 4b + 18$
n $11 - 12h + 9$
o $12y - 3y - 7g + 5g - 6$
p $8h - 6 + 3h - 2$
q $11s - 6t + 4t - 7s$
r $2m + 13l - 7m + l$
s $3h + 4k - 16h - k + 7$
t $13 + 5t - 9t - 8$
u $2g + 5 + 5g - 7$
v $17f - 3k + 2f - 7k$

UNDERSTANDING

3 Simplify the following expressions.

a $x^2 + 2x^2$
b $3y^2 + 2y^2$
c $a^3 + 3a^3$
d $d^2 + 6d^2$
e $7g^2 - 8g^2$
f $3y^3 + 7y^3$
g $2b^2 + 5b^2$
h $4a^2 - 3a^2$

i $g^2 - 2g^2$
j $a^2 + 4 + 3a^2 + 5$
k $11x^2 - 6 + 12x^2 + 6$
l $12s^2 - 3 + 7 - s^2$
m $3a^2 + 2a + 5a^2 + 3a$
n $11b - 3b^2 + 4b^2 + 12b$
o $6t^2 - 6g - 5t^2 + 2g - 7$
p $11g^3 + 17 - 3g^3 + 5 - g^2$
q $12ab + 3 + 6ab$
r $14xy + 3xy - xy - 5xy$
s $4fg + 2s - fg + s$
t $11ab + ab - 5$
u $18ab^2 - 4ac + 2ab^2 - 10ac$

REASONING

4 Rose owns an art gallery and sells items supplied to her by various artists. She receives a commission for all items sold. She uses the following method to keep track of the money she owes the artists when their items are sold.

- Ask the artist how much they want for the item.
- Add 50% to that price, then mark the item for sale at this new price.
- When the item sells, take one-third of the sale price as commission, then return the balance to the artist.

Use algebra to show that this method does return the correct amount to the artist.

5 Explain, using mathematical reasoning and with diagrams if necessary, why the expression $2x + 2x^2$ cannot be simplified.

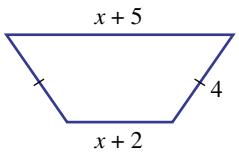
PROBLEM SOLVING

6 Three members of a fundraising committee are making books of tickets to sell for a raffle. Each book of tickets contains t tickets. If the first person has 14 books of tickets, the second person has 12 books of tickets and the third person has 13 books of tickets, write an expression for:

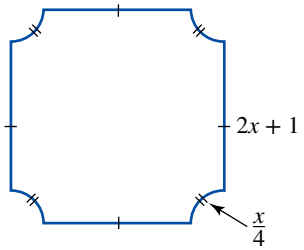
a the number of tickets that the first person has
b the number of tickets that the second person has
c the number of tickets that the third person has
d the total number of tickets for the raffle.

7 Write an expression for the total perimeter of the following shapes.

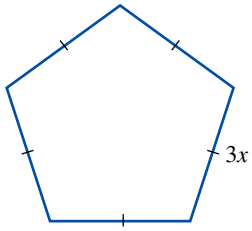
a



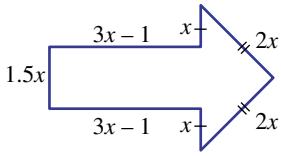
b



c

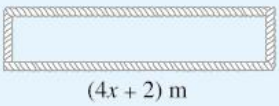


d



CHALLENGE 8.2

If 32 metres of rope are required to make the rectangular shape shown in the figure, what is the value of x?



8.8 Multiplying and dividing expressions with variables

Multiplying variables

- When we multiply variables (as already stated) the Commutative Law holds, so order is not important. For example:

$$3 \times 6 = 6 \times 3$$
$$6 \times w = w \times 6$$
$$a \times b = b \times a$$

- The multiplication sign ($\times$) is usually omitted for reasons of convention.

$$3 \times g \times h = 3gh$$
$$2 \times x^2 \times y = 2x^2y$$

- Although order is not important, conventionally the variables in each term are written in alphabetical order. For example,

$$2 \times b^2 \times a \times c = 2ab^2c$$

WORKED EXAMPLE 9

Simplify the following.
a $5 \times 4g$ **b** $-3d \times 6ab \times 7$

THINK	WRITE
a 1 Write the expression and replace the hidden multiplication signs.	**a** $5 \times 4g$ $= 5 \times 4 \times g$
2 Multiply the numbers.	$= 20 \times g$
3 Remove the multiplication sign.	$= 20g$
b 1 Write the expression and replace the hidden multiplication signs.	**b** $-3d \times 6ab \times 7$ $= -3 \times d \times 6 \times a \times b \times 7$
2 Place the numbers at the front.	$= -3 \times 6 \times 7 \times d \times a \times b$
3 Multiply the numbers.	$= -126 \times d \times a \times b$
4 Remove the multiplication signs and place the variables in alphabetical order.	$= -126abd$

Dividing expressions with variables

- When dividing expressions with variables, rewrite the expression as a fraction and simplify by cancelling.
- Remember that when the same variable appears as a factor on both the numerator and denominator, it may be cancelled.

WORKED EXAMPLE 10

a **Simplify** $\frac{16f}{4}$.

b **Simplify** $15n \div 3n$.

THINK	WRITE
a 1 Write the expression.	**a** $\frac{16f}{4} = \frac{{}^{4}\cancel{16}f}{\cancel{4}_{1}}$
2 Simplify the fraction by cancelling 16 with 4 (divide both by 4).	$= \frac{4f}{1}$
3 No need to write the denominator since we are dividing by 1.	$= 4f$
b 1 Write the expression and then rewrite it as a fraction.	**b** $15n \div 3n = \frac{15n}{3n}$ $= \frac{{}^{5}\cancel{15}n}{{}_{1}\cancel{3}n}$

2 Simplify the fraction by cancelling 15 with 3 and n with n. $= \frac{5}{1}$

3 No need to write the denominator since we are dividing by 1. $= 5$

WORKED EXAMPLE 11

Simplify $-12xy \div 27y$.

THINK

1 Write the expression and then rewrite it as a fraction.

2 Simplify the fraction by cancelling 12 with 27 (divide both by 3) and y with y.

WRITE

$$-12xy \div 27y = -\frac{12xy}{27y}$$
$$= -\frac{{}^{4}\cancel{12}xy}{{}_{9}\cancel{27}\cancel{y}}$$
$$= -\frac{4x}{9}$$

WORKED EXAMPLE 12

Simplify the following.

a $3m^3 \times 2m$ **b** $5p^{10} \times 3p^3$

c $36x^7 \div 12x^4$ **d** $\dfrac{6y^3 \times 4y^8}{12y^4}$

THINK

a 1 Write the problem.

2 The order is not important when multiplying, so place the numbers first.

3 Multiply the numbers.

4 Check to see if the bases are the same. They are both m.

5 Simplify by adding the indices.

b 1 Write the problem.

2 The order is not important when multiplying, so place the numbers first.

3 Multiply the numbers.

4 Check to see if the bases are the same. They are both p.

5 Simplify by adding the indices.

WRITE

a $3m^3 \times 2m$
$= 3 \times 2 \times m^3 \times m$
$= 6 \times m^3 \times m$
$= 6 \times m^{3+1}$
$= 6m^4$

b $5p^{10} \times 3p^3$
$= 5 \times 3 \times p^{10} \times p^3$
$= 15 \times p^{10} \times p^3$
$= 15 \times p^{10+3}$
$= 15p^{13}$

c **1** Write the problem and express it as a fraction.

c $36x^7 \div 12x^4$

$= \frac{36x^7}{12x^4}$

2 Divide the numbers.

$= \frac{3x^7}{x^4}$

3 Check to see if the bases are the same. They are both x.

4 Subtract the powers.

$= 3x^{7-4}$

$= 3x^3$

d **1** Write the problem.

d $\frac{6y^3 \times 4y^8}{12y^4}$

2 Multiply the numbers in the numerator. Simplify the numbers in index form in the numerator.

$= \frac{24y^{11}}{12y^4}$

3 Divide the numbers and subtract the powers.

$= 2y^{11-4}$

$= 2y^7$

Exercise 8.8 Multiplying and dividing expressions with variables

assessON

INDIVIDUAL PATHWAYS

■ PRACTISE	■ CONSOLIDATE	■ MASTER
Questions: 1–5, 9, 10	Questions: 1a–p, 2a–l, 3a–l, 4a–l, 5a–l, 6a–l, 7a–e, 8–10	Questions: 1m–z, 2m–t, 3m–t, 4m–u, 5m–t, 6m–r, 7e–j, 8–11

■ ■ ■ **Individual pathway interactivity** int-4435 eBook plus

REFLECTION
How are multiplication and division of expressions with variables similar to multiplication and division of numbers?

FLUENCY

1 WE9 Simplify the following.

a $4 \times 3g$ **b** $7 \times 3h$ **c** $4d \times 6$ **d** $3z \times 5$

e $6 \times 5r$ **f** $5t \times 7$ **g** $4 \times 3u$ **h** $7 \times 6p$

i $7gy \times 3$ **j** $2 \times 11ht$ **k** $4x \times 6g$ **l** $10a \times 7h$

m $9m \times 4d$ **n** $3c \times 5h$ **o** $9g \times 2x$ **p** $2.5t \times 5b$

q $13m \times 12n$ **r** $6a \times 12d$ **s** $2ab \times 3c$ **t** $4f \times 3gh$

u $2 \times 8w \times 3x$ **v** $11ab \times 3d \times 7$ **w** $16xy \times 1.5$ **x** $3.5x \times 3y$

y $11q \times 4s \times 3$ **z** $4a \times 3b \times 2c$

2 WE10 Simplify the following.

a $\frac{8f}{2}$ **b** $\frac{6h}{3}$ **c** $\frac{15x}{3}$ **d** $9g \div 3$

e $10r \div 5$ **f** $4x \div 2x$ **g** $8r \div 4r$ **h** $\frac{16m}{8m}$

i $14q \div 21q$ **j** $\frac{3x}{6x}$ **k** $\frac{12h}{14h}$ **l** $50g \div 75g$

m $\frac{8f}{24f}$ **n** $35x \div 70x$ **o** $24m \div 36m$ **p** $y \div 34y$

q $27h \div 3h$ **r** $\frac{20d}{48d}$ **s** $\frac{64q}{44q}$ **t** $81l \div 27l$

3 Simplify the following.

a $\frac{15fg}{3}$ **b** $12cd \div 4$ **c** $\frac{8xy}{12}$ **d** $24cg \div 24$

e $\frac{11xy}{11x}$ **f** $\frac{9pq}{18q}$ **g** $\frac{21ab}{28b}$ **h** $\frac{9dg}{12g}$

i $\frac{5jk}{kj}$ **j** $55rt \div 77t$ **k** $\frac{10mxy}{35mx}$ **l** $36bc \div 27c$

m $13xy \div x$ **n** $\frac{16cd}{40cd}$ **o** $14abc \div 7bc$ **p** $3gh \div 6h$

q $\frac{132mnp}{60np}$ **r** $\frac{11ad}{66ad}$ **s** $18adg \div 45ag$ **t** $\frac{bh}{7h}$

4 Simplify the following.

a $3 \times -5f$ **b** $-6 \times -2d$ **c** $11a \times -3g$

d $-9t \times -3g$ **e** $-5t \times -4dh$ **f** $6 \times -3st$

g $-3 \times -2w \times 7d$ **h** $-4a \times -3b \times 2c \times e$ **i** $11ab \times -3f$

j $3as \times -3b \times -2x$ **k** $-5h \times -5t \times -3q$ **l** $4 \times -3w \times -2 \times 6p$

m $-7a \times 3b \times g$ **n** $17ab \times -3gh$ **o** $-3.5g \times 2h \times 7$

p $5h \times 8j \times -k$ **q** $75x \times 1.5y$ **r** $12rt \times -3z \times 4p$

s $2ab \times 3c \times 5$ **t** $-4w \times 34x \times 3$ **u** $-3ab \times -5cd \times -6ef$

5 WE11 Simplify the following.

a $\frac{-4a}{8}$ **b** $\frac{-11ab}{33b}$ **c** $60jk \div -5k$

d $-3h \div -6dh$ **e** $\frac{-32g}{40gl}$ **f** $-12xy \div 48y$

g $\frac{12ab}{-14ab}$ **h** $\frac{6fgh}{30ghj}$ **i** $-4xyz \div 6yz$

j $\frac{-rt}{6rt}$ **k** $-5mn \div 20n$ **l** $-14st \div -28$

m $34ab \div -17ab$ **n** $\frac{-ab}{-3a}$ **o** $\frac{-7dg}{35gh}$

p $-60mn \div 55mnp$ **q** $\frac{28def}{18d}$ **r** $-72xyz \div 28yz$

s $\frac{54pq}{36pqr}$ **t** $-\frac{121oc}{132oct}$

UNDERSTANDING

6 WE12 Simplify the following.

a $2a \times a$ **b** $-5p \times -5p$ **c** $-5 \times 3x \times 2x$

d $ab \times 7a$ **e** $3b^2 \times 2cd$ **f** $-5xy \times 4 \times 8x$

g $7pq \times 3p \times 2q$ **h** $5m \times n \times 6nt \times -t$ **i** $-3 \times xyz \times -3z \times -2y$

j $-7a \times -3b \times -2c^2$ **k** $2mn \times -3 \times 2n \times 0$ **l** $w^2x \times -9z^2 \times 2xy^2$

m $2a^4 \times 3a^7$ **n** $2x^2y^3 \times x^3$ **o** $20m^{12} \div 2m^3$

p $\dfrac{25p^{12} \times 4q^7}{15p^2 \times 8q^2}$ **q** $\dfrac{8x^3 \times 7y^2 \times 2z^2}{6x \times 14y}$ **r** $\dfrac{a \times ab \times 3b^2}{5a^2b^2}$

7 Simplify the following.

a $\dfrac{3}{a} \times \dfrac{2}{a}$ **b** $\dfrac{5b}{2} \times \dfrac{4b}{3}$ **c** $w \times \dfrac{5}{w^2}$

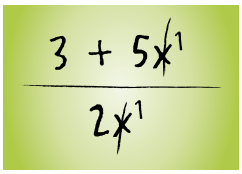
d $\dfrac{3rk}{2s} \times \dfrac{6st}{5rt}$ **e** $-\dfrac{15gt}{10ag} \times \dfrac{2g}{5t}$ **f** $-\dfrac{4ht}{3dk} \times \dfrac{-12hk}{9dt}$

g $\dfrac{5t}{gn} \div \dfrac{1}{g}$ **h** $\dfrac{-9th}{4g} \div \dfrac{tg}{6h}$ **i** $\dfrac{4xy}{7wz} \div \dfrac{x}{14z}$

j $\dfrac{-10f}{3w} \div \dfrac{5}{-9wz}$

REASONING

8 The student's working below shows incorrect cancelling. Explain why the working is not correct.

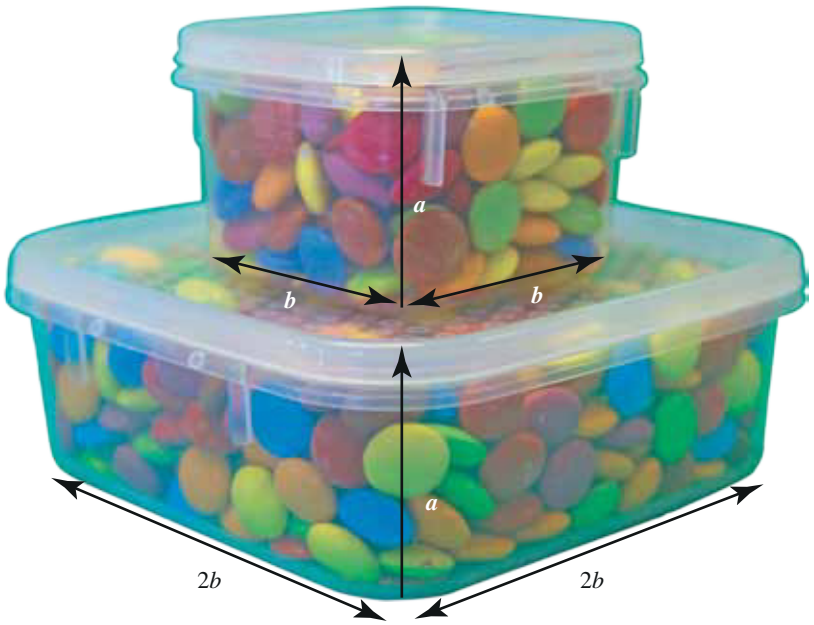


PROBLEM SOLVING

9 Evaluate $\dfrac{2x - (3y + 2z)}{-(-x - y) + z}$ if $x = 2$, $y = -1$ and $z = -4$.

10 Evaluate $\dfrac{x \times (y - 2z)}{3x - (y - z)}$ if $x = -2$, $y = 5$ and $z = -1$.

11 a Write an expression for the volume of each of the containers shown.



(*Hint:* The volume of a container is found by multiplying the length by the width by the height.)

b How many times will the contents of the smaller container fit inside the larger container? (*Hint:* Divide the volumes.)

8.9 Expanding brackets

The Distributive Law

- The **Distributive Law** is the name given to the following process.

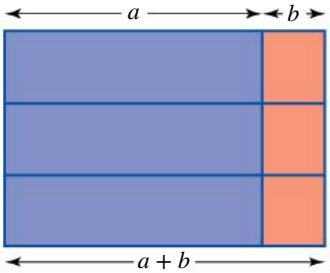
$$3(5 + 8) = 3 \times 5 + 3 \times 8$$

This is because the number out in front is *distributed* to each of the terms in the bracket.

- Since variables take the place of numbers, the Distributive Law also holds true for algebraic expressions.

$$a(b + c) = ab + ac$$

- The Distributive Law can be demonstrated using the concept of area. As can be seen in the diagram at right, $3(a + b) = 3a + 3b$



- We can think of $3(a + b) = (a + b) + (a + b) + (a + b)$
  Collecting like terms, $a + a + a + b + b + b = 3a + 3b$
- An expression containing a bracket multiplied by a number can be written in expanded or factorised form.

Factorised form = Expanded form

$$\overbrace{3(a + b)} = \overbrace{3a + 3b}$$

Expanding and factorising are the inverse of each other.

- The Distributive Law can be used when the terms inside the brackets are either added or subtracted.

$$a(b - c) = ab - ac$$

- The Distributive Law is not used when the terms inside the brackets are multiplied or divided. You can see this with numbers $2(4 \times 5) = 2 \times 4 \times 5$; *not* $(2 \times 4) \times (2 \times 5)$.
- When simplifying expressions, we can leave the result in either factorised form or expanded form, but not a combination of both.

WORKED EXAMPLE 13

Use the Distributive Law to expand the following expressions.
a $3(a + 2)$ **b** $x(x - 5)$

THINK		WRITE
a 1	Write the expression.	**a** $3(a + 2) = 3(a + 2)$
2	Use the Distributive Law to expand the brackets.	$= 3 \times a + 3 \times 2$
3	Simplify by multiplying.	$= 3a + 6$
b 1	Write the expression.	**b** $x(x - 5) = x(x - 5)$
2	Use the Distributive Law to expand the brackets.	$= x \times x + x \times -5$
3	Simplify by multiplying.	$= x^2 - 5x$

- Some expressions can be simplified further by collecting like terms after any brackets have been expanded.

WORKED EXAMPLE 14

Expand the expressions below and then simplify by collecting any like terms.

a $3(x - 5) + 4$ **b** $4(3x + 4) + 7x + 12$

c $2x(3y + 3) + 3x(y + 1)$ **d** $4x(2x - 1) - 3(2x - 1)$

THINK	WRITE
a 1 Write the expression.	**a** $3(x - 5) + 4$
2 Expand the brackets.	$= 3 \times x + 3 \times -5 + 4$ $= 3x - 15 + 4$
3 Collect the like terms (−15 and 4).	$= 3x - 11$
b 1 Write the expression.	**b** $4(3x + 4) + 7x + 12$
2 Expand the brackets.	$= 4 \times 3x + 4 \times 4 + 7x + 12$ $= 12x + 16 + 7x + 12$
3 Rearrange so that the like terms are together. (Optional)	$= 12x + 7x + 16 + 12$
4 Collect the like terms.	$= 19x + 28$
c 1 Write the expression.	**c** $2x(3y + 3) + 3x(y + 1)$
2 Expand the brackets.	$= 2x \times 3y + 2x \times 3 + 3x \times y + 3x \times 1$ $= 6xy + 6x + 3xy + 3x$
3 Rearrange so that the like terms are together. (Optional)	$= 6xy + 3xy + 6x + 3x$
4 Simplify by collecting the like terms.	$= 9xy + 9x$
d 1 Write the expression.	**d** $4x(2x - 1) - 3(2x - 1)$
2 Expand the brackets. Take care with negative terms.	$= 4x \times 2x + 4x \times -1 - 3 \times 2x - 3 \times -1$ $= 8x^2 - 4x - 6x + 3$
3 Simplify by collecting the like terms.	$= 8x^2 - 10x + 3$

Exercise 8.9 Expanding brackets

assessON

INDIVIDUAL PATHWAYS

■ PRACTISE	■ CONSOLIDATE	■ MASTER
Questions: 1–4, 8	Questions: 1a–l, 2a–l, 3a–l, 4a–f, 5a–f, 8	Questions: 1m–t, 2m–r, 3j–p, 4f–j, 5f–j, 6–9

■■■ Individual pathway interactivity int-4436 eBookplus

REFLECTION
Why doesn't the Distributive Law apply when there is a multiplication sign inside the brackets, that is for $a(b \times c)$?

FLUENCY

1 WE13 Use the Distributive Law to expand the following expressions.

a $3(d + 4)$ **b** $2(a + 5)$ **c** $4(x + 2)$ **d** $5(r + 7)$

e $6(g + 6)$ **f** $2(t - 3)$ **g** $7(d + 8)$ **h** $9(2x - 6)$

i $12(4 + c)$ **j** $7(6 + 3x)$ **k** $45(2g + 3)$ **l** $1.5(t + 6)$
m $11(t - 2)$ **n** $3(2t - 6)$ **o** $t(t + 3)$ **p** $x(x + 4)$
q $g(g + 7)$ **r** $2g(g + 5)$ **s** $3f(g + 3)$ **t** $6m(n - 2m)$

2 Expand the following.

a $3(3x - 2)$ **b** $3x(x - 6y)$ **c** $5y(3x - 9y)$ **d** $50(2y - 5)$
e $-3(c + 3)$ **f** $-5(3x + 4)$ **g** $-5x(x + 6)$ **h** $-2y(6 + y)$
i $-6(t - 3)$ **j** $-4f(5 - 2f)$ **k** $9x(3y - 2)$ **l** $-3h(2b - 6h)$
m $4a(5b + 3c)$ **n** $-3a(2g - 7a)$ **o** $5a(3b + 6c)$ **p** $-2w(9w - 5z)$
q $12m(4m + 10)$ **r** $-3k(-2k + 5)$

3 WE14 Expand the expressions below and then simplify by collecting any like terms.

a $7(5x + 4) + 21$ **b** $3(c - 2) + 2$
c $2c(5 - c) + 12c$ **d** $6(v + 4) + 6$
e $3d(d - 4) + 2d^2$ **f** $3y + 4(2y + 3)$
g $24r + r(2 + r)$ **h** $5 - 3g + 6(2g - 7)$
i $4(2f - 3g) + 3f - 7$ **j** $3(3x - 4) + 12$
k $-2(k + 5) - 3k$ **l** $3x(3 + 4r) + 9x - 6xr$
m $12 + 5(r - 5) + 3r$ **n** $12gh + 3g(2h - 9) + 3g$
o $3(2t + 8) + 5t - 23$ **p** $24 + 3r(2 - 3r) - 2r^2 + 5r$

4 Expand the following and then simplify by collecting like terms.

a $3(x + 2) + 2(x + 1)$ **b** $5(x + 3) + 4(x + 2)$
c $2(y + 1) + 4(y + 6)$ **d** $4(d + 7) - 3(d + 2)$
e $6(2h + 1) + 2(h - 3)$ **f** $3(3m + 2) + 2(6m - 5)$
g $9(4f + 3) - 4(2f + 7)$ **h** $2a(a + 2) - 5(a^2 + 7)$
i $3(2 - t^2) + 2t(t + 1)$ **j** $m(n + 4) - mn + 3m$

UNDERSTANDING

5 Simplify the following expressions by removing the brackets and then collecting like terms.

a $3h(2k + 7) + 4k(h + 5)$ **b** $6n(3y + 7) - 3n(8y + 9)$
c $4g(5m + 6) - 6(2gm + 3)$ **d** $11b(3a + 5) + 3b(4 - 5a)$
e $5a(2a - 7) - 5(a^2 + 7)$ **f** $7c(2f - 3) + 3c(8 - f)$
g $7x(4 - y) + 2xy - 29$ **h** $11v(2w + 5) - 3(8 - 5vw)$
i $3x(3 - 2y) + 6x(2y - 9)$ **j** $8m(7n - 2) + 3n(4 + 7m)$

REASONING

6 a Using the concept of area as shown above, explain with diagrams and mathematical reasoning why $5(6 - 2) = 5 \times 6 - 5 \times 2$.

b Using the concept of area as shown above, explain with diagrams and mathematical reasoning why $4(x - y) = 4 \times x - 4 \times y$.

7 Expressions of the form $(a + b)(c + d)$ can be expanded by using the Distributive Law twice. Distribute one of the factors over the other; for example,

$(a + b)(c + d)$. The expression can then be fully expanded following Worked example 13.

a $(x + 1)(x + 2)$ **b** $(a + 3)(a + 4)$
c $(c + 2)(c - 3)$ **d** $(y + 4)(y - 4)$
e $(u - 2)(u - 3)$ **f** $(k - 5)(k - 2)$

PROBLEM SOLVING

8 The price of a pair of jeans is \$50. During a sale, the price of the jeans is discounted by \$$d$.

a Write an expression to represent the sale price of the jeans.

b If you buy three pairs of jeans during the sale, write an expression to represent the total purchase price:

i containing brackets **ii** in expanded form (without brackets).

c Write an expression to represent the total change you would receive from \$200 for the three pairs of jeans purchased during the sale.

9 A triptych is a piece of art that is divided into three sections or panels. The middle panel is usually the largest panel and is flanked by two related panels.

a Write a simplified expression for the area of the three paintings (excluding the frame).

b Write a simplified expression for the combined area of the triptych.

c The value of f is $m + 102.5$. Substitute $(m + 102.5)$ into your combined area formula and simplify the expression.

d The actual value of m is 122.5 cm. Sketch the shape of the three paintings in your workbook and show the actual measurements of each, including length, width and area.

8.10 Factorising

- **Factorising** is the opposite process to expanding.
- Factorising involves identifying the highest common factors of the algebraic terms.
- To find the highest common factor of the algebraic terms:
  1. Find the highest common factor of the number parts.
  2. Find the highest common factor of the variable parts.
  3. Multiply these together.

WORKED EXAMPLE 15

Find the highest common factor of $6x$ and 10.

THINK	WRITE
1 Find the highest common factor of the number parts.	
Break 6 down into factors.	$6 = 3 \times 2$
Break 10 down into factors.	$10 = 5 \times 2$
The highest common factor is 2.	HCF = 2
2 Find the highest common factor of the variable parts.	
There isn't one, because only the first term has a variable part.	The HCF of $6x$ and 10 is 2.

WORKED EXAMPLE 16

Find the highest common factor of $14fg$ and $21gh$.

THINK	WRITE
1 Find the highest common factor of the number parts.	
Break 14 down into factors.	$14 = 7 \times 2$
Break 21 down into factors.	$21 = 7 \times 3$
The highest common factor is 7.	HCF = 7
2 Find the highest common factor of the variable parts.	
Break fg down into factors.	$fg = f \times g$
Break gh down into factors.	$gh = g \times h$
Both contain a factor of g.	HCF = g
3 Multiply these together.	The HCF of $14fg$ and $21gh$ is $7g$.

- To factorise an expression, place the highest common factor of the terms outside the brackets and the remaining factors for each term inside the brackets.

WORKED EXAMPLE 17

Factorise the expression $2x + 6$.

THINK	WRITE
1 Break down each term into its factors.	$2x + 6$
	$= 2 \times x + 2 \times 3$
2 Write the highest common factor outside the brackets.	$= 2 \times (x + 3)$
Write the other factors inside the brackets.	
3 Remove the multiplication sign.	$= 2(x + 3)$

WORKED EXAMPLE 18

Factorise $12gh - 8g$.

THINK	WRITE
1 Break down each term into its factors.	$12gh - 8g$ $= 4 \times 3 \times g \times h - 4 \times 2 \times g$
2 Write the highest common factor outside the brackets. Write the other factors inside the brackets.	$= 4 \times g \times (3 \times h - 2)$
3 Remove the multiplication signs.	$= 4g(3h - 2)$

Exercise 8.10 Factorising

assess on

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–4, 12	Questions: 1, 2, 3a–l, 4a–l, 5, 6, 12, 13	Questions: 1, 2, 3j–r, 4m–v, 5–13

Individual pathway interactivity int-4437 eBookplus

REFLECTION
What strategies will you use to find the highest common factor?

eBookplus
Digital doc
SkillSHEET
Highest common factor
doc-6928

FLUENCY

1 WE15 Find the highest common factor of the following.

a 4 and 6 b 6 and 9 c 12 and 18 d 13 and 26
e 14 and 21 f $2x$ and 4 g $3x$ and 9 h $12a$ and 16

2 WE16 Find the highest common factor of the following.

a $2gh$ and $6g$ b $3mn$ and $6mp$ c $11a$ and $22b$
d $4ma$ and $6m$ e $12ab$ and $14ac$ f $24fg$ and $36gh$
g $20dg$ and $18ghq$ h $11gl$ and $33lp$ i $16mnp$ and $20mn$
j $28bc$ and $12c$ k $4c$ and $12cd$ l x and $3xz$

3 WE17 Factorise the following expressions.

a $3x + 6$ b $2y + 4$ c $5g + 10$
d $8x + 12$ e $6f + 9$ f $12c + 20$
g $2d + 8$ h $2x - 4$ i $12g - 18$
j $11h + 121$ k $4s - 16$ l $8x - 20$
m $12g - 24$ n $14 - 4b$ o $16a + 64$
p $48 - 12q$ q $16 + 8f$ r $12 - 12d$

4 WE18 Factorise the following.

a $3gh + 12$ b $2xy + 6y$ c $12pq + 4p$
d $14g - 7gh$ e $16jk - 2k$ f $12eg + 2g$
g $12k + 16$ h $7mn + 6m$ i $14ab + 7b$
j $5a - 15abc$ k $8r + 14rt$ l $24mab + 12ab$
m $4b - 6ab$ n $12fg - 16gh$ o $ab - 2bc$
p $14x - 21xy$ q $11jk + 3k$ r $3p + 27pq$
s $12ac - 4c + 3dc$ t $4g + 8gh - 16$ u $28s + 14st$
v $15uv + 27vw$

eBookplus
Digital doc
Spreadsheet
Finding the HCF
doc-2289

UNDERSTANDING

5 Find the highest common factor of $4ab$, $6a^2b^3$ and $12a^3b$.

6 Find the lowest common multiple of $3ab$, $4a^3bc$ and $6a^2b^2$.

7 Simplify: $\frac{4x-4}{10x-20} \times \frac{15x+15}{3x-3} \times \frac{6x-12}{20x+20}$.

8 Simplify: $\frac{3x+6}{x-1} \div \frac{12x+24}{6(x+1)}$.

9 Factorise and hence simplify: $\frac{2x^2y-6xy^2}{a+2b} \times \frac{4a+8b}{3xy} \div \frac{2x-6y}{7}$.

REASONING

10 Simplify $(5ax^2y - 6bxy + 2ax^2y - bxy) \div (ax^2 - bx)$. Show your working.

PROBLEM SOLVING

11 A cuboid measures $(3x - 6)$ cm by $(2x + 8)$ cm by $(ax - 5a)$ cm.

a Write an expression for its volume.

b If the cuboid weighs $(8x - 16)$ g, find a factorised expression for its density in g/cm^3.

12 A farmer's paddock is a rectangle of length $(2x - 6)$ m and width $(3x + 6)$ m.

a Find the area of the paddock in factorised form.

b State the smallest possible value of x. Explain your reasoning.

13 Factorise and hence simplify:

a $\frac{3x^2y}{z^2} \times \frac{z^2}{12xy} \times \frac{6y^3}{z}$

b $\frac{4x^2(y-1)}{(z-1)^2} \times \frac{z-1}{24x(y-1)} \times \frac{3(y-1)^2}{9z}$

c $\frac{5xz^2}{y^2} \times \frac{y^2}{15xz} \div \frac{5y^3}{xz^2}$

d $\frac{24z(x+1)}{(y-1)^2} \div \frac{6y^2(x+1)}{5x} \div \frac{15(y-1)}{z^2(x+1)^2}$

eBookplus
Digital doc
WorkSHEET 8.2
doc-2291

eBookplus ONLINE ONLY

8.11 Review

www.jacplus.com.au

The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:

- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

Review questions

Download the Review questions document from the links found in your eBookPLUS.

eBookplus

Interactivities

Word search
int-2629

Crossword
int-2630

Sudoku
int-3188

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

associative law
brackets
commutative law
distributive law
evaluate
expanded form
expanding
expression
factorised form
factorising
highest common factor
identity law
inverse law
like terms
pronumeral
substitution
unknowns
variable

Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

www.assesson.com.au

Link to SpyClass, an exciting online game combining a comic book–style story with problem-based learning in an immersive environment.

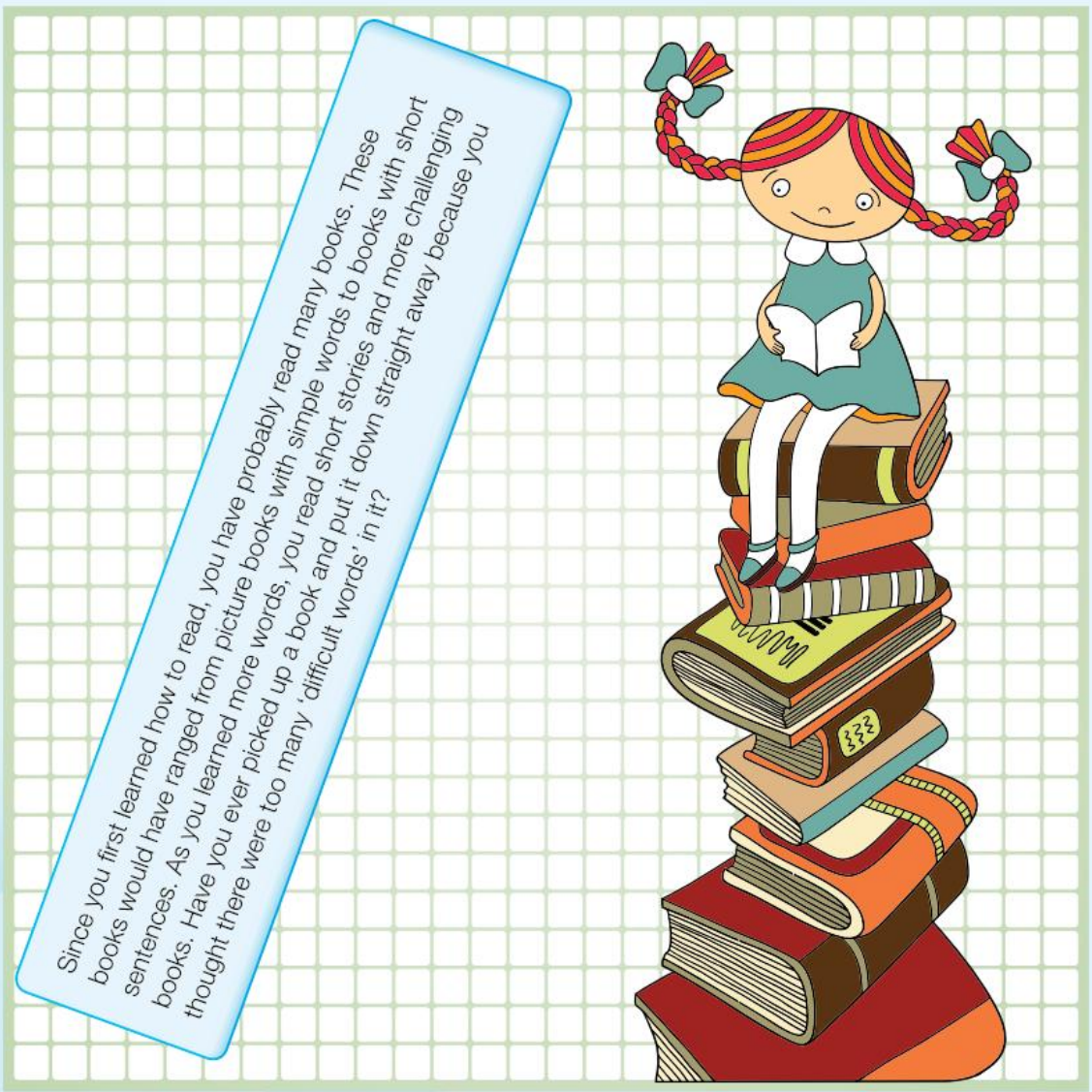
Join Jesse, Toby and Dan and help them to tackle some of the world's most dangerous criminals by using the knowledge you've gained through your study of mathematics.

www.spyclass.com.au

INVESTIGATION

RICH TASK

Readability index



The reading difficulty of a text can be described by a readability index. There are several different methods used to calculate reading difficulty, and one of these methods is known as the Rix index. The Rix index is obtained by dividing the number of long words by the number of sentences.

1 Use a variable to represent the number of long words and another to represent the number of sentences. Write a formula that can be used to calculate the Rix index.

When using the formula to determine the readability index, follow these guidelines:

- A long word is a word that contains seven or more letters.
- A sentence is a group of words that ends with a full stop, question mark, exclamation mark, colon or semi-colon.
- Headings and numbers are not included and hyphenated words count as one word.

Consider this passage from a Science textbook.

2 How many sentences and long words appear in this passage of text?

3 Use your formula to calculate the Rix index for this passage. Round your answer to 2 decimal places.

A fatal fall . . . or was it murder?

In 1991, some German hikers found a body preserved in ice near the Italy–Austria border. Scientists used radiometric dating and found that the body was about 5300 years old! They thought that the person, known now as the Iceman, had died of hypothermia (extreme cold). Ten years later, another group of scientists using high-tech X-rays found the remains of an arrowhead lodged near his left lung. Specialists have not yet confirmed whether the Iceman fell back onto his arrow, or if he was murdered. And without any witnesses to question, the truth may never be known!

Once you have calculated the Rix index, the table below can be used to work out the equivalent year level of the passage of text.

Rix index	Equivalent year level
Below 0.2	1
0.2–0.49	2
0.5–0.79	3
0.8–1.29	4
1.3–1.79	5
1.8–2.39	6
2.4–2.99	7

Rix index	Equivalent year level
3.0–3.69	8
3.7–4.49	9
4.5–5.29	10
5.3–6.19	11
6.2–7.19	12
Above 7.2	University level

4 What year level is the passage of text equivalent to?

When testing the reading difficulty of a book, it is not necessary to consider the entire book. Choose a section of text with at least 10 sentences and collect the required information for the formula.

5 Choose a passage of text from one of your school books, a magazine or newspaper. Calculate the Rix index and use the table above to determine the equivalent year level.

6 Repeat question **5** using another section of the book, magazine or newspaper. Did the readability level change?

7 On a separate page, rewrite the passage from question **5** with minimum changes, so that it is now suitable for a higher or lower year level. Explain the method you used to achieve this. Provide a Rix index calculation to prove that you changed the level of reading difficulty.

CODE PUZZLE

Doctor, I've swallowed the film out of my camera!

The factorised form of the expressions and the letter beside each of them gives the puzzle code.

$8y(x-9)$	$2(4-x)$	$2y(3x-5)$	$-3(4x+7)$				
$-(2x-1)$	$-7(7+4x)$	$2y(x-4)$	$2(3x-7)$				
$6(3-7x)$	$-y(x-1)$	$x(y-2)$	$-5(4x+5)$	$4(3x+5)$	$-(x-2)$	$5(3x+2)$	
$3(2x-1)$	$-8(x+3)$	$7(8x-5)$	$3x(2y+1)$	$y(2x-3)$	$-2(x+1)$	$3(x-4)$	$y(3x-2)$

D = $6x - 3$ =

E = $8 - 2x$ =

G = $15x + 10$ =

H = $-2x + 1$ =

I = $12x + 20$ =

L = $2xy - 3y$ =

N = $18 - 42x$ =

O = $-2x - 2$ =

P = $2xy - 8y$ =

S = $-12x - 21$ =

T = $xy - 2x$ =

V = $56x - 35$ =

E = $6xy + 3x$ =

H = $-20x - 25$ =

L = $8xy - 72y$ =

N = $-x + 2$ =

E = $6x - 14$ =

O = $-xy + y$ =

P = $3x - 12$ =

E = $-8x - 24$ =

S = $3xy - 2y$ =

T = $6xy - 10y$ =

O = $-49 - 28x$ =

eBookplus ACTIVITIES

8.2 Using variables

Digital doc
- SkillSHEET (doc-6922) Alternative expressions used to describe the four operations

eLesson
- Using variables (eles-0042)

Interactivity
- IP interactivity 8.2 (int-4429) Using variables

8.3 Substitution

Digital doc
- Spreadsheet (doc-2287) Substitution

Interactivity
- IP interactivity 8.3 (int-4430) Substitution

8.4 Working with brackets

Interactivity
- IP interactivity 8.4 (int-4431) Working with brackets

8.5 Substituting positive and negative numbers

Digital docs
- SkillSHEET (doc-6923) Order of operations II
- SkillSHEET (doc-6924) Order of operations with brackets
- SkillSHEET (doc-6925) Operations with directed numbers
- WorkSHEET 8.1 (doc-2290)

Interactivity
- IP interactivity 8.5 (int-4432) Substituting positive and negative numbers

8.6 Number laws and variables

Interactivities
- The Associative Law (int-2370)
- IP interactivity 8.6 (int-4433) Number laws and variables

8.7 Simplifying expressions

Digital docs
- SkillSHEET (doc-6926) Combining like terms
- SkillSHEET (doc-6927) Simplifying fractions

Interactivity
- IP interactivity 8.7 (int-4434) Simplifying expressions

8.8 Multiplying and dividing expressions with variables

Interactivity
- IP interactivity 8.8 (int-4435) Multiplying and dividing expressions with variables

8.9 Expanding brackets

Digital doc
- Spreadsheet (doc-2288) Expanding brackets

Interactivity
- IP interactivity 8.9 (int-4436) Expanding brackets

8.10 Factorising

Digital docs
- SkillSHEET (doc-6928) Highest common factor
- Spreadsheet (doc-2289) Finding the HCF
- WorkSHEET 8.2 (doc-2291)

Interactivity
- IP interactivity 8.10 (int-4437) Factorising

8.11 Review

Interactivities
- Word search (int-2629)
- Crossword (int-2630)
- Sudoku (int-3188)

Digital docs
- Topic summary (doc-10760)
- Concept map (doc-10773)
- Topic review (Word doc-14938, PDF doc-14939)

To access eBookPLUS activities, log on to www.jacplus.com.au

Answers

TOPIC 8 Algebra

8.2 Using variables

1 **a** $x + 420$
b $3x$
c $x - 130$
d The nearby nest has 60 more ants.
e The nearby nest has 90 fewer ants.
f The nest is one quarter of the size of the original nest.

2 **a** $x + y$ **b** $x + y + 260$
c $x + y + 90$ **d** $x + y - 260$

3 Between 9.00 am and 9.15 am one Danish pastry was sold. In the next hour-and-a-half, a further 11 Danish pastries were sold. No more Danish pastries had been sold at 12.30 pm, but in the next half-hour 18 more were sold. No Danish pastries were sold after 1.00 pm.

4 **a** $a + b$ **b** $a + b + c$
c $b + 4$ **d** $a - 6$

5 **a** $y + 7$ **b** $y - 8$
c $5y$ **d** $14 - y$
e $\frac{y}{3}$ **f** $8y + 3$

6 **a** $a + b$ **b** $a - b$
c $2b - 3a$ **d** ab
e $2ab$ **f** $3a + 7b$
g a^2 **h** $\frac{a^2}{5}$

7 **a** $\$27y$ **b** $\$14d$ **c** $\$(27r + 14h)$

8 **a** $t + 2$ **b** $t + g$
c $t - 5$ **d** $2t$

9 **a** Various answers are possible; an example is shown. The number of passengers doubled at the next stop and continued to increase, more than quadrupling in the first nine minutes. At 7.22 pm, 5 people alighted the train, and by 7.25 pm the same number of passengers were on the train as there were at the beginning. By 7.34 pm there were 12 fewer passengers than there were at the beginning.
b 7.22 pm **c** 7.19 pm **d** 7.34 pm

10 **a** The number of bacteria in each of these intervals is double the number of bacteria in the previous interval.
b The bacteria could be dividing in two.
c It is lower than expected, based on the previous pattern of growth.
d Some of the bacteria may have died, or failed to divide and reproduce.

11 **a** Odd **b** Even
c **i** $n + 2$, $n + 4$ and $n + 6$
ii $n - 2$

12 TSA $= 2x^2 + 4xh$; $V = x^2h$

13 TSA $= 5xh + 3x^2$; $V = \frac{3}{2}x^2h$

14 $\text{TSA}_{\text{square box}} = 2000\ \text{cm}^2$; $V_{\text{square box}} = 6000\ \text{cm}^3$
$\text{TSA}_{\text{rectangular box}} = 2700\ \text{cm}^2$; $V_{\text{rectangular box}} = 9000\ \text{cm}^3$

Challenge 8.1

Number is 4; Bill's is age 15.

8.3 Substitution

1 **a** 6 **b** 14 **c** 30 **d** 1
e 9 **f** 1 **g** 7 **h** 3
i 6 **j** 15 **k** 19 **l** 4
m 5 **n** 10 **o** 20 **p** 5
q 22 **r** 2 **s** $\frac{89}{10}(8\frac{9}{10})$ **t** $\frac{39}{10}(3\frac{9}{10})$

2 **a** 42 **b** 3 **c** 54 **d** 1
e 9 **f** 15 **g** 15 **h** 21
i 24.3 **j** 21 **k** 1 **l** 4.8
m 18.3 **n** 8.1 **o** 16.2 **p** $\frac{7}{4}(1\frac{3}{4})$

3 **a** 7 **b** 7 **c** −3 **d** 3
e 4 **f** 10 **g** 50 **h** 1
i −15 **j** −4 **k** 37 **l** 15
m 40 **n** $2\frac{1}{3}$ **o** 19 **p** 1

4 150 m

5 10 cm

6 $C = \$64$

7 **a** 48 cm^2 **b** 8400 m^2 **c** 4.472 m^2 or 44 720 cm^2

8 **a** $F = 212$ °F **b** 28 °C = 82.4 °F **c** 32 °F

9 **a** 100 km ≈ 60 miles **b** 248 km ≈ 148.8 miles
c 12.5 km ≈ 7.5 miles

10 If $x = 0$, then the expression becomes $\frac{0}{0}$, which is indeterminate.

11 540.5

12 **a** $V = x(x + 5)(x - 2)$
b 1200
c Because $1.5 - 2 < 0$

13 **a** $13p - 10$
b 2590

8.4 Working with brackets

1 **a** 36 **b** 4 **c** 84 **d** 18
e 56 **f** 35 **g** 90 **h** 350
i 55 **j** 20 **k** 133 **l** 147
m 784 **n** 250 **o** 9800 **p** 200

2 **a** 90 **b** 16 **c** 6 **d** 32
e 72 **f** 90 **g** 7 **h** 36
i 60 **j** 58 **k** 21 **l** 180
m 576 **n** 32 **o** −26 **p** 33

3 **a** 62 cm **b** 97.8 cm

4 B

5 **a** 720° **b** 540° **c** 180°
d 360° **e** 3240°

6 **a** CD $= m + 4n$ **b** BC $= 3m + n$
c Perimeter $= 8m + 18n$

7 **a** $A = \frac{1}{2}(m + n)(m - n)$
b $A = \frac{1}{2}(21 \times 9) = 94.5\ \text{cm}^2$
c If $m < n$, then $(m - n) < 0$. Negative height is impossible.
d If m and n move closer in value, then the length of the base of the triangle gets closer to $2m$ (or $2n$) and the height gets closer to zero.

8 Check with your teacher.

8.5 Substituting positive and negative numbers

1 **a** 3 **b** 9 **c** −9 **d** 3
e −9 **f** −12 **g** −12 **h** 2
i 5 **j** −13 **k** 9 **l** 3
m −2 **n** −3 **o** −8 **p** −2
q 0 **r** −109 **s** 6 **t** 16

2 **a** 18 **b** 32 **c** −15 **d** 12
e 16 **f** 24 **g** 22 **h** 155
i 32 **j** 3 **k** 6 **l** 21

3 **a** −9 **b** 1 **c** 3 **d** 40
e −8 **f** 2 **g** −3 **h** 12
i $-\frac{4}{5}$ **j** −2 **k** −1 **l** −6
m 125 **n** 175 **o** −5 **p** −12.5
q −74 **r** −117 **s** −104 **t** −15.5

4 $\frac{30}{7}$ or $4\frac{2}{7}$

5 If x is negative then $5x$ will also be a negative integer (less than or equal to -5). Subtracting this number is equivalent to adding a positive integer. The result will be positive.
6 Check with your teacher.
7 **a** Negative
b Positive
8 **a** $\approx 0.804x^d$
b ≈ 3.216
c $\frac{x}{2}$

8.6 Number laws and variables

1 **a** **i** 11 **ii** 11, same
b **i** 25 **ii** 25, same
c **i** 31 **ii** 31, same
d **i** 32 **ii** 32, same
e **i** -5 **ii** 5, different
f **i** -18 **ii** 18, different
g **i** -28 **ii** 28, different
h **i** 1 **ii** -1, different
2 **a** **i** -10 **ii** -10, same
b **i** -180 **ii** -180, same
c **i** -40 **ii** -40, same
d **i** -350 **ii** -350, same
e **i** $-\frac{2}{5}$ **ii** $-\frac{5}{2}$, different
f **i** -1 **ii** -1, same
g **i** $-\frac{4}{5}$ **ii** $-\frac{5}{4}$, different
h **i** $-\frac{14}{45}$ **ii** $-\frac{45}{14}$, different
3 **a** True **b** False **c** False
d True **e** True **f** True
g False **h** False **i** False
j True **k** False **l** True
4 **a** **i** 13 **ii** 13, same
b **i** 24 **ii** 24, same
c **i** 40 **ii** 40, same
d **i** -3 **ii** -7, different
e **i** -35 **ii** -71, different
f **i** -43 **ii** -67, different
5 **a** **i** -64 **ii** -64, same
b **i** 768 **ii** 768, same
c **i** -1536 **ii** -1536, same
d **i** -4 **ii** -1, different
e **i** -6 **ii** $-\frac{1}{6}$, different
f **i** $\frac{8}{5}$ **ii** $\frac{1}{10}$, different
6 **a** False **b** False **c** False
d True **e** True **f** False
7 C
8 D
9 **a** **i** 6 **ii** 6
b The answers are equal because of the use of the Associative Law.
10 **a** -10 **b** -12
11 **a** $(-3p + 4q)$ **b** $\frac{1}{(3p - 4q)}$
c $15, -\frac{1}{15}$ **d** The result is the identity, which is 1.

8.7 Simplifying expressions

1 **a** $6c$ **b** $-3c$ **c** $4a$
d q **e** $-3h$ **f** $2x$
g $-6a$ **h** $4f$ **i** $-3p$
j h **k** $18b$ **l** $3t$
m $13m$ **n** $-x$ **o** $20z$
p $10p$ **q** $17g$ **r** $3b$
s $14t$ **t** $-7j$ **u** $-15l$
v $8m$ **w** 0 **x** $10t$
2 **a** $10x - 2y$ **b** $7x - 12$ **c** $11 - 2f$
d $6 - u$ **e** $7m + 3p$ **f** $4r - 5h$
g $17a - 5b$ **h** $9t - 2$ **i** $17 - 3g$
j $10m - 2n$ **k** $7k - 12$ **l** $4n - 9$
m $12 - 2b$ **n** $20 - 12h$ **o** $9y - 2g - 6$
p $11h - 8$ **q** $4s - 2t$ **r** $14l - 5m$
s $3k - 13h + 7$ **t** $5 - 4t$ **u** $7g - 2$
v $19f - 10k$
3 **a** $3x^2$ **b** $5y^2$ **c** $4a^3$
d $7d^2$ **e** $-g^2$ **f** $10y^3$
g $7b^2$ **h** a^2 **i** $-g^2$
j $4a^2 + 9$ **k** $23x^2$ **l** $11s^2 + 4$
m $8a^2 + 5a$ **n** $b^2 + 23b$ **o** $t^2 - 4g - 7$
p $8g^3 - g^2 + 22$ **q** $18ab + 3$ **r** $11xy$
s $3fg + 3s$ **t** $12ab - 5$ **u** $20ab^2 - 14ac$
4 Check with your teacher.
5 Check with your teacher.
6 **a** $14t$ **b** $12t$ **c** $13t$ **d** $39t$
7 **a** $2x + 15$ **b** $9x + 4$ **c** $15x$ **d** $13.5x - 2$

Challenge 8.2

$x = 3$

8.8 Multiplying and dividing expressions with variables

1 **a** $12g$ **b** $21h$ **c** $24d$ **d** $15z$
e $30r$ **f** $35t$ **g** $12u$ **h** $42p$
i $21gy$ **j** $22ht$ **k** $24gx$ **l** $70ah$
m $36dm$ **n** $15ch$ **o** $18gx$ **p** $12.5bt$
q $156mn$ **r** $72ad$ **s** $6abc$ **t** $12fgh$
u $48wx$ **v** $231abd$ **w** $24xy$ **x** $10.5xy$
y $132qs$ **z** $24abc$
2 **a** $4f$ **b** $2h$ **c** $5x$ **d** $3g$
e $2r$ **f** 2 **g** 2 **h** 2
i $\frac{2}{3}$ **j** $\frac{1}{2}$ **k** $\frac{6}{7}$ **l** $\frac{2}{3}$
m $\frac{1}{3}$ **n** $\frac{1}{2}$ **o** $\frac{2}{3}$ **p** $\frac{1}{34}$
q 9 **r** $\frac{5}{12}$ **s** $\frac{16}{11}(1\frac{5}{11})$ **t** 3
3 **a** $5fg$ **b** $3cd$ **c** $\frac{2xy}{3}$ **d** cg
e y **f** $\frac{p}{2}$ **g** $\frac{3a}{4}$ **h** $\frac{3d}{4}$
i 5 **j** $\frac{5r}{7}$ **k** $\frac{2y}{7}$ **l** $\frac{4b}{3}$
m $13y$ **n** $\frac{2}{5}$ **o** $2a$ **p** $\frac{g}{2}$
q $\frac{11m}{5}$ **r** $\frac{1}{6}$ **s** $\frac{2d}{5}$ **t** $\frac{b}{7}$
4 **a** $-15f$ **b** $12d$ **c** $-33ag$
d $27gt$ **e** $20dht$ **f** $-18st$
g $42dw$ **h** $24abce$ **i** $-33abf$
j $18absx$ **k** $-75hqt$ **l** $144pw$
m $-21abg$ **n** $-51abgh$ **o** $-49gh$
p $-40hjk$ **q** $112.5xy$ **r** $-144prtz$
s $30abc$ **t** $-408wx$ **u** $-90abcdef$
5 **a** $-\frac{a}{2}$ **b** $-\frac{a}{3}$ **c** $-12j$ **d** $\frac{1}{2d}$
e $-\frac{4}{5l}$ **f** $-\frac{x}{4}$ **g** $-\frac{6}{7}$ **h** $\frac{f}{5j}$
i $-\frac{2x}{3}$ **j** $-\frac{1}{6}$ **k** $-\frac{m}{4}$ **l** $\frac{st}{2}$
m -2 **n** $\frac{b}{3}$ **o** $-\frac{d}{5h}$ **p** $-\frac{12}{11p}$
q $\frac{14ef}{9}$ **r** $-\frac{18x}{7}$ **s** $\frac{3}{2r}$ **t** $-\frac{11}{12t}$

6 **a** $2a^2$ **b** $25p^2$ **c** $-30x^2$
d $7a^2b$ **e** $6b^2cd$ **f** $-160x^2y$
g $42p^2q^2$ **h** $-30mn^2t^2$ **i** $-18xy^2z^2$
j $-42abc^2$ **k** 0 **l** $-18w^2x^2y^2z^2$
m $6a^{11}$ **n** $2x^5y^3$ **o** $10m^9$
p $\frac{5p^{10}q^5}{6}$ **q** $\frac{4x^2yz^2}{3}$ **r** $\frac{3b}{5}$

7 **a** $\frac{6}{a^2}$ **b** $\frac{10b^2}{3}$ **c** $\frac{5}{w}$ **d** $\frac{9k}{5}$ **e** $-\frac{3g}{5a}$
f $\frac{16h^2}{9d^2}$ **g** $\frac{5t}{n}$ **h** $-\frac{27h^2}{2g^2}$ **i** $\frac{8y}{w}$ **j** $6fz$

8 The pronumeral must be a common factor of every term of the numerator and every term of the denominator before it can be cancelled.

9 -5

10 $\frac{7}{6}$

11 **a** $V_{\text{small container}} = ab^2$, $V_{\text{large container}} = 4ab^2$ **b** 4 times

8.9 Expanding brackets

1 **a** $3d + 12$ **b** $2a + 10$
c $4x + 8$ **d** $5r + 35$
e $6g + 36$ **f** $2t - 6$
g $7d + 56$ **h** $18x - 54$
i $48 + 12c$ **j** $42 + 21x$
k $90g + 135$ **l** $1.5t + 9$
m $11t - 22$ **n** $6t - 18$
o $t^2 + 3t$ **p** $x^2 + 4x$
q $g^2 + 7g$ **r** $2g^2 + 10g$
s $3fg + 9f$ **t** $6mn - 12m^2$

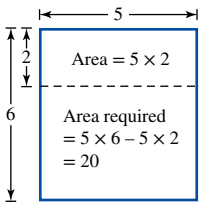
2 **a** $9x - 6$ **b** $3x^2 - 18xy$
c $15xy - 45y^2$ **d** $100y - 250$
e $-3c - 9$ **f** $-15x - 20$
g $-5x^2 - 30x$ **h** $-12y - 2y^2$
i $-6t + 18$ **j** $-20f + 8f^2$
k $27xy - 18x$ **l** $-6bh + 18h^2$
m $20ab + 12ac$ **n** $-6ag + 21a^2$
o $15ab + 30ac$ **p** $-18w^2 + 10wz$
q $48m^2 + 120m$ **r** $6k^2 - 15k$

3 **a** $35x + 49$ **b** $3c - 4$
c $22c - 2c^2$ **d** $6v + 30$
e $5d^2 - 12d$ **f** $11y + 12$
g $26r + r^2$ **h** $9g - 37$
i $11f - 12g - 7$ **j** $9x$
k $-5k - 10$ **l** $18x + 6rx$
m $8r - 13$ **n** $18gh - 24g$
o $11t + 1$ **p** $24 + 11r - 11r^2$

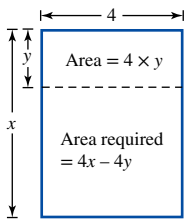
4 **a** $5x + 8$ **b** $9x + 23$
c $6y + 26$ **d** $d + 22$
e $14h$ **f** $21m - 4$
g $28f - 1$ **h** $4a - 3a^2 - 35$
i $6 - t^2 + 2t$ **j** $7m$

5 **a** $10hk + 21h + 20k$ **b** $15n - 6ny$
c $8gm + 24g - 18$ **d** $18ab + 67b$
e $5a^2 - 35a - 35$ **f** $11cf + 3c$
g $28x - 5xy - 29$ **h** $37vw + 55v - 24$
i $6xy - 45x$ **j** $77mn - 16m + 12n$

6 **a**



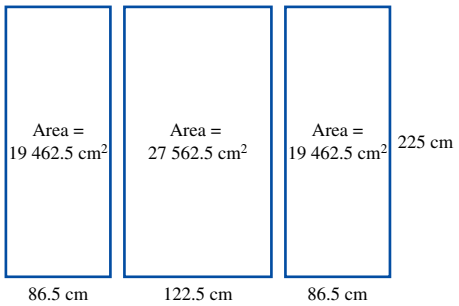
b



7 **a** $x^2 + 3x + 2$ **b** $a^2 + 7a + 12$
c $c^2 - c - 6$ **d** $y^2 - 16$
e $u^2 - 5u + 6$ **f** $k^2 - 7k + 10$

8 **a** $\$50 - d$
b **i** Sale price $(P) = 3(50 - d)$ **ii** $P = 150 - 3d$
c Amount of change $(C) = 200 - (150 - 3d) = 50 + 3d$

9 **a** $A_{\text{left}} = fm - 36f$; $A_{\text{centre}} = fm$; $A_{\text{right}} = fm - 36f$
b $A = 3fm - 72f$ **c** $A = 3m^2 + 235.5m - 7380$
d



8.10 Factorising

1 **a** 2 **b** 3 **c** 6 **d** 13
e 7 **f** 2 **g** 3 **h** 4

2 **a** $2g$ **b** $3m$ **c** 11 **d** $2m$
e $2a$ **f** $12g$ **g** $2g$ **h** $11l$
i $4mn$ **j** $4c$ **k** $4c$ **l** x

3 **a** $3(x + 2)$ **b** $2(y + 2)$ **c** $5(g + 2)$
d $4(2x + 3)$ **e** $3(2f + 3)$ **f** $4(3c + 5)$
g $2(d + 4)$ **h** $2(x - 2)$ **i** $6(2g - 3)$
j $11(h + 11)$ **k** $4(s - 4)$ **l** $4(2x - 5)$
m $12(g - 2)$ **n** $2(7 - 2b)$ **o** $16(a + 4)$
p $12(4 - q)$ **q** $8(2 + f)$ **r** $12(1 - d)$

4 **a** $3(gh + 4)$ **b** $2y(x + 3)$
c $4p(3q + 1)$ **d** $7g(2 - h)$
e $2k(8j - 1)$ **f** $2g(6e + 1)$
g $4(3k + 4)$ **h** $m(7n + 6)$
i $7b(2a + 1)$ **j** $5a(1 - 3bc)$
k $2r(4 + 7t)$ **l** $12ab(2m + 1)$
m $2b(2 - 3a)$ **n** $4g(3f - 4h)$
o $b(a - 2c)$ **p** $7x(2 - 3y)$
q $k(11j + 3)$ **r** $3p(1 + 9q)$
s $c(12a - 4 + 3d)$ **t** $4(g + 2h - 4)$
u $14s(2 + t)$ **v** $3v(5u + 9w)$

5 $2ab$

6 $12a^3b^2c$

7 $\frac{3}{5}$

8 $\frac{3(x + 1)}{2(x - 1)}$

9 $\frac{28}{3}$ or $9\frac{1}{3}$

10 $7y$

11 **a** $(3x - 6)(2x + 8)(ax - 5a)$ **b** $\frac{4}{3a(x + 4)(x - 5)}$

12 **a** $6(x - 3)(x + 2)$ **b** $x > 3$

13 **a** $\frac{3xy^3}{2z}$ **b** $\frac{x(y - 1)^2}{18z(z - 1)}$ **c** $\frac{xz^3}{15y^3}$ **d** $\frac{4xz^3(x + 1)^2}{3y^2(y - 1)^3}$

Investigation — Rich task

1 Let l represent the number of long words and s represent the number of sentences. The Rix index $= \frac{l}{s}$.

2 Six sentences and 19 long words

3 3.17

4 Grade 8

5 to **7** Check with your teacher.

Code puzzle

Let's hope nothing develops.

TOPIC 9

Numeracy 2

9.1 Overview

Why learn this?

Why should you learn to count numbers? Imagine going into a shop and asking for a dozen eggs. How could you tell how many eggs you were getting? On trust? Was your change correct? How could you be sure?

You cannot go through life without learning to accurately add, subtract, multiply and divide.

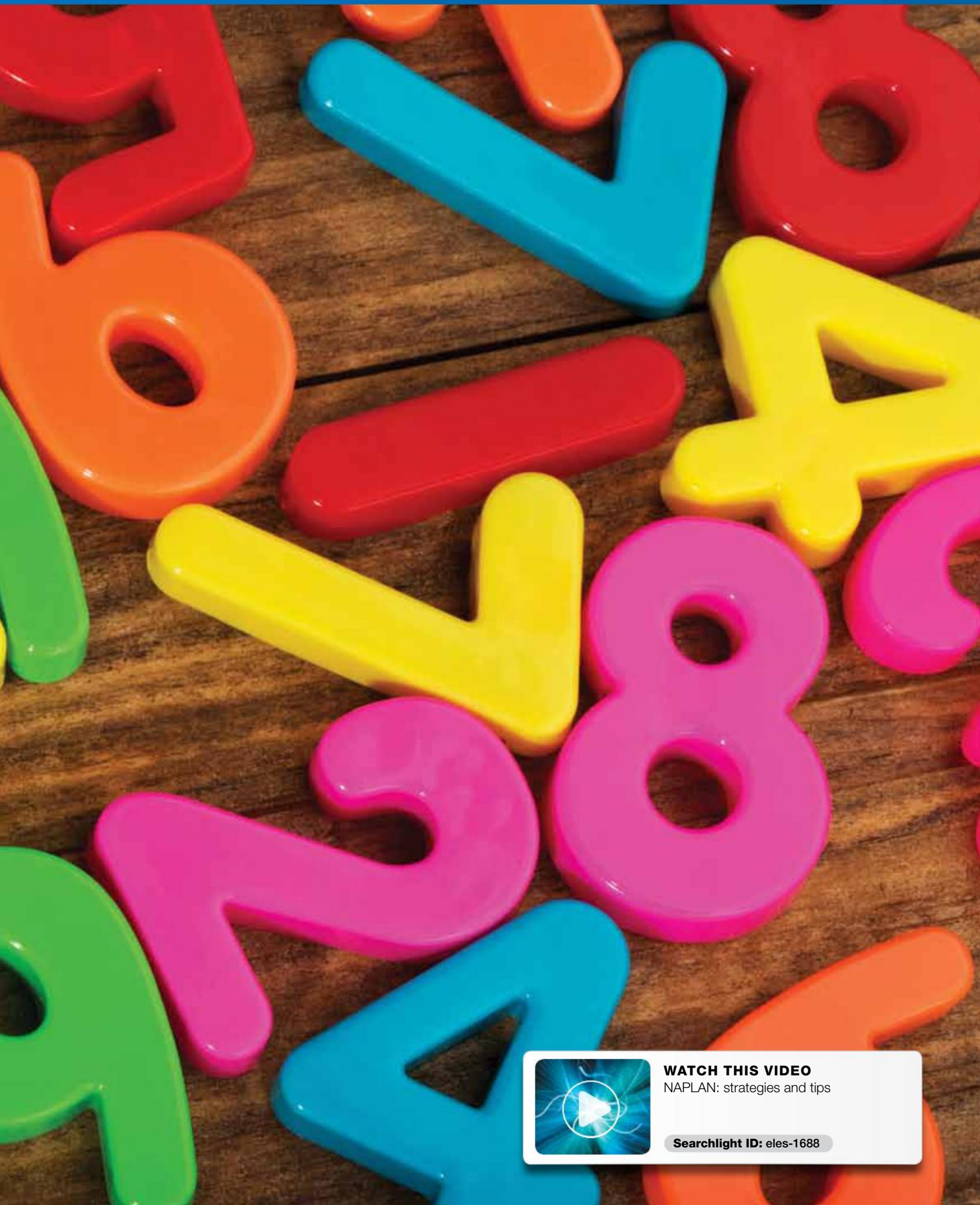
What do you know?

assessON

1 **THINK** List what you know about numeracy. Use a thinking tool such as a concept map to show your list.
2 **PAIR** Share what you know with a partner and then with a small group.
3 **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of numeracy.

Learning sequence

9.1 Overview
9.2 Set C
9.3 Set D



WATCH THIS VIDEO
NAPLAN: strategies and tips

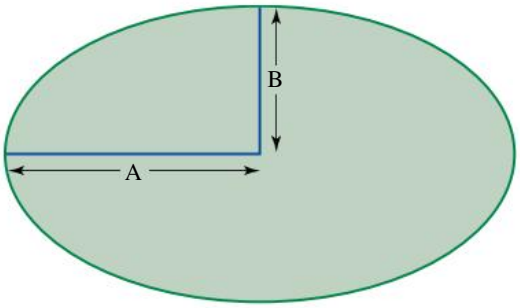
Searchlight ID: eles-1688

assessON

9.2 Set C

Calculator allowed

1 Each year, the area of tree logging in the Otway Ranges is approximately equivalent to clearing 200 football ovals. The area of a football oval is given by Area = $3.142 \times A \times B$ (see the diagram). If the width of the football oval is 110 m and the length is 160 m, approximately how many square metres of trees are felled each year in the Otways?



A 1.9×10^4 **B** 2.76×10^4 **C** 1.9×10^6 **D** 2.76×10^6

2 A designer wants to enlarge a cylinder to 150% of its original size. If the diameter is 37 mm, what was the radius of the copy of the cylinder?

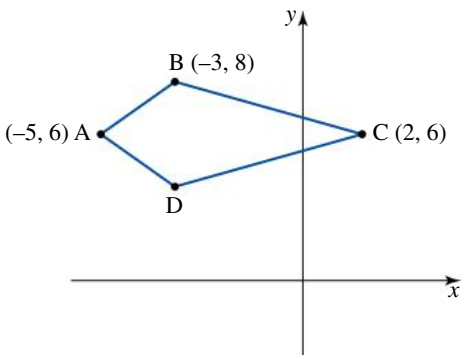
A 27.75 mm **B** 46.25 mm
C 55.5 mm **D** 92.5 mm

3 A teenager is to receive 750 mL of saline solution. The drip rate is adjusted to 60 mL per hour. When will it be necessary to change the saline solution if the drip rate begins at 10.30 am?

4 The area of a square is 73 square metres. Which is the closest to the length of each side?

A 8.4 m **B** 8.5 m **C** 8.6 m **D** 8.7 m

5 A kite is drawn with the coordinates of D being omitted. What are the coordinates of D?



A (−3, −3) **B** (−3, −8) **C** (−3, 4) **D** (−3, 6)

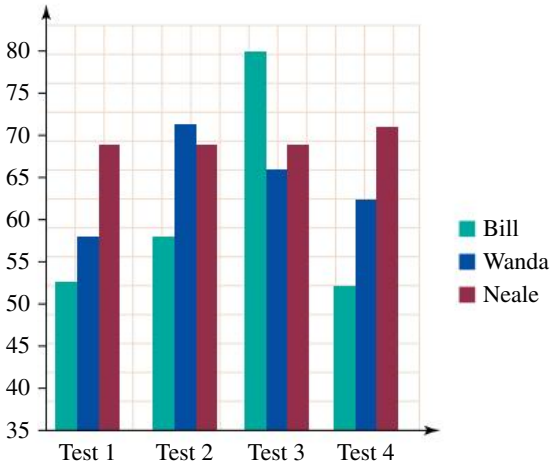
6 Your friend is planning a trip of 2320 km. The plan is to drive between 400 km and 480 km each day. At this rate, which of the following would be a reasonable number of days to complete the trip?

A Fewer than 4 days **B** Between 4 and 6 days
C Between 6 and 8 days **D** More than 8 days

7 Last year there were 225 students at Top End High School. This year there are 20 per cent fewer students than there were last year. Approximately how many students are at Top End High School this year?

A 205 **B** 245 **C** 270 **D** 180

8 This column graph shows four test results for Bill, Wanda and Neale. Which of the following options represents the test results?



A Bill has a higher average than Neale but a lower average than Wanda.

B Bill has a lower average than Wanda but a higher average than Neale.

C Wanda has a higher average than Bill but a lower average than Neale.

D Wanda has a higher average than Neale and Bill.

9 A prepared workout on a treadmill consists of intervals of walking at various rates and angles of incline. A 3% incline means 3 units of vertical rise for every 100 units of horizontal run. My treadmill, when set at a 3% incline, has a horizontal run of 1.6 m. What will be the vertical rise?

A 4.8 m

B 48 cm

C 48 mm

D 4.8 mm

10 You are about to play your final game in a computer tournament. Your previous scores have been 134, 99, 109, 117 and 101. To win the tournament your average must be at least 114. What is the minimum score you must achieve in this game to win?

11 Suppose your heart rate is 72 beats per minute. How many days will it take your heart to beat 1 000 000 times? Round your answer to the nearest number of days.

12 A gardener wants to put 12 cm of mulch on his garden, whose dimensions are 20 m by 13 m. How many trailer loads will he require if his trailer holds 1.5 m^3.

A 20

B 21

C 46

D 47

13 A machine packs grain at a rate of $1\frac{1}{5}$ tonnes of grain per hour. How long will the machine take to pack 18 000 kg of grain?

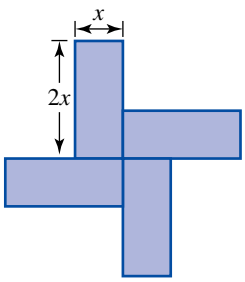
A 15 hours

B 21 hours 6 minutes

C 21 hours 24 minutes

D 216 hours

14 In the following diagram, the shape is made from four identical rectangles.



Each rectangle has a width of x and a length of $2x$. If the perimeter of the shape is 48 cm, what is the area of the shape?

A 112.5 cm^2 **B** 72 cm^2 **C** 60 cm^2 **D** 180 cm^2

15 A student recorded the times for 25 people running a 100-metre race. The stem-and-leaf diagram shows the results.
Key: 13 | 7 represents 13.7 seconds

Stem	*Leaf*
13	7
14	2 3 4 4
14	5 5 7 7 8 9
15	0 1 2 2 3 4
15	5 5 6 7 9
16	0 1 2

What percentage of students ran for 14.8 seconds or less?

A 36% **B** 40% **C** 45% **D** 48%

16 A student recorded the times for 25 people running a 200-metre race. The stem-and-leaf diagram shows the results.
Key: 13 | 7 represents 13.7 seconds

Stem	*Leaf*
13	7
14	2 3 4 4
14	5 5 7 7 8 9
15	0 1 2 2 3 4
15	5 5 6 7 9
16	0 1 2

What percentage of students ran for more than 14.8 seconds but less than 15.5 seconds?

A 28% **B** 32% **C** 36% **D** 40%

17 Tsing works in a bakery. One of her chores is to take cardboard sheets and to fold them into small and large trays. It takes 2 minutes to fold a small tray and 3 minutes to fold a large tray. Can she complete 80 small and 45 large trays in the allocated time of 3.5 hours?

A Yes, Tsing will finish in 2.6 hours.
B No, Tsing will take 4 hours and 25 minutes.
C Yes, Tsing will finish exactly in 3.5 hours.
D No, Tsing will take 4 hours and 55 minutes.

18 The maximum quantity of air that can fill your lungs is called Force Vital Capacity (L). This can be modelled by the formula $L = 4.43 \times H - 0.026 \times A - 2.89$ where H = your height (metres) and A = your age (years). If you are 165 cm tall and 15 years old, what is your capacity (to the nearest litre)?

A 4 **B** 5
C 70 **D** 106

19 What is the surface area of a rectangular prism that has a length of 11 cm, a width of 6 cm and a height of 50 millimetres?

A 170 cm^2 **B** 302 cm^2
C 330 cm^2 **D** 1832 cm^2

20 You have a piece of string 3.7 metres long. You cut it into 6 pieces of equal length to tie to 6 balloons, and there is 22 cm of string left. How long were each of the 6 pieces?

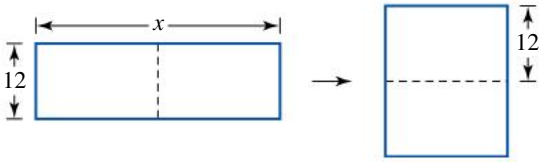
21 My teacher and I boarded a tram together. At the second stop, three people got on. At the third stop, three people got on and one got off. At the fourth stop, three got off. At the fifth stop, six people got off. At the sixth stop, one-half of the passengers got off and I was the only passenger left on the tram. How many passengers were on the tram when my teacher and I got on?

22 Examine the expression $3k^2 + 6k - 5 + 6k^2 + 2$. When it is simplified, which of the following is the equivalent expression?

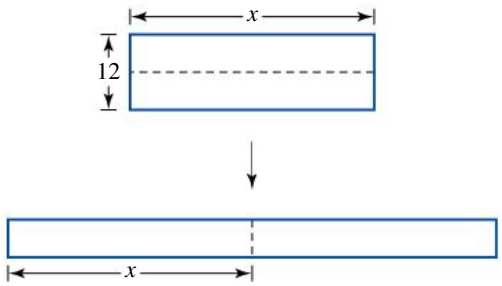
A $3k^2 + 12k - 3$ **B** $9k^2 + 6k - 3$
C $9k^2 + 6k + 7$ **D** $15k^2 + 6k - 3$

23 Jane and Lance each have a rectangular piece of paper of the same dimensions. The length is labelled x cm and the width is 12 cm. Jane and Lance each cut their paper in half in two different ways as illustrated.

Jane's cutting



Lance's cutting



Which of the following represents the sum of the perimeters of the two new designs?

A $P = 5x + 48$ **B** $P = 5(x + 12)$
C $P = 6x + 60$ **D** $P = 6(x + 12)$

24 The process for making a Chinese dish called 'Dragon's beard noodles' by hand is as follows.

Take a 100-cm strand of dough and fold it in half. Stretch the dough back to its original length so that the two thinner strands are formed. Repeat this process over and over increasing the number of noodles as they get progressively thinner.

After the 6th fold how many noodles strands will you have?

A 12

B 32

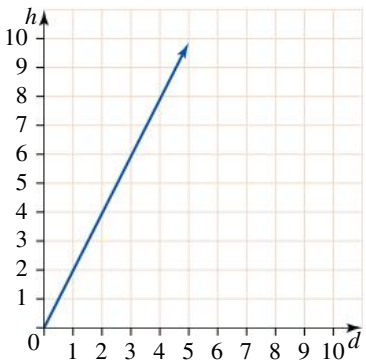
C 64

D 128

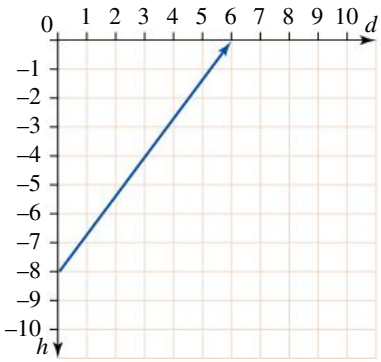
25 The steps on the foreshore have a horizontal width of 24 cm and a height of 18 cm, as shown.

For someone climbing the steps, which of the following graphs models the height above the ground (h) against the distance from the first step (d)?

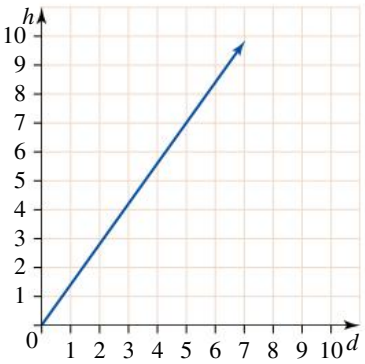
A



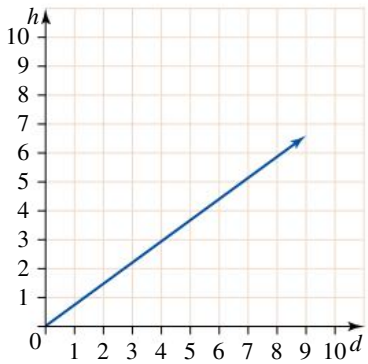
B



C



D



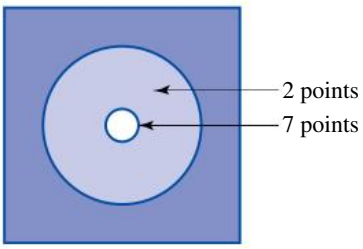
26 A recipe for pizza dough requires a 20 cm × 40 cm rectangular pan. However, you have only a square pan, with dimensions of 25 cm × 25 cm. What will be the approximate difference in the size of your pizza due to using the square pan?

A Smaller by 22%
B Larger by 22%
C Smaller by 28%
D Larger by 28%

27 Alice ripped a piece of paper into three parts, and tore each of those parts into three more parts. If she repeated this action 3 times, how many pieces of paper would she have?

A 15
B 81
C 243
D 729

28 Ning was throwing darts at a target as shown at right. When his dart landed on or inside the circle (bullseye) of the target board, he earned 7 points. However, when his dart landed outside the circle he earned 2 points. After 50 throws his friend reported his score to be 140 points. Ning wanted to know how many bullseyes he had hit, but his friend did not know.



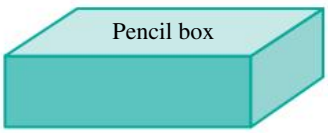
Let x = number of hits on the bullseye. Which of the following equations could be used to solve for x?

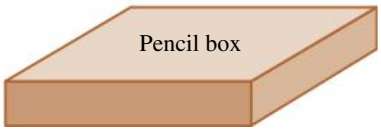
A $7x + 50 - x = 140$
B $7x + 2(50 - x) = 140$
C $7x + 2x = 140$
D $2x + 2(50 - 7x) = 140$

29 John bushwalked 36 km through the Grampians in 3 days. On the first day he hiked 50% of the total distance. On the second day he hiked 25% of the distance that remained. How many kilometres did he hike on the third day?

A $11\frac{1}{2}$
B $13\frac{1}{2}$
C $22\frac{1}{2}$
D 27

30 Two pencil boxes as shown have the following information. The volume of one pencil box is 24 cm^2 more than the volume of the other pencil box. One box has two of its sides measuring 3 cm and 4 cm, while the other box has sides that measure 1.5 cm and 6 cm. The third sides of the boxes are the same length. What is that length?





A 2 cm
B 4 cm
C 6 cm
D 8 cm

9.3 Set D

Non-calculator

1 A 130 cm long strand of wire is cut into three pieces.
The longest piece is three times as long as the second shortest piece.
The second longest piece is three times as long as the shortest piece.
How long is the shortest piece?

A 18 cm **B** 10 cm **C** 17.5 cm **D** 15 cm

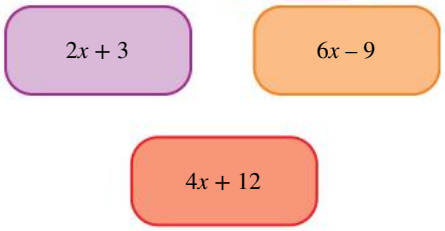
2 One litre of paint covers an area of 20 square metres.
How much paint will cover one square metre?

A 0.005 litre
B 0.002 litre
C 0.05 litre
D 0.02 litre

3 A pancake recipe requires $4\frac{1}{2}$ cups of milk. If you wish to make one-fifth of the recipe, how many cups of milk will you need?

A 0.2 **B** 0.5
C 0.8 **D** 0.9

4 Each card pictured is labelled with a value.
What is the mean value of these cards?

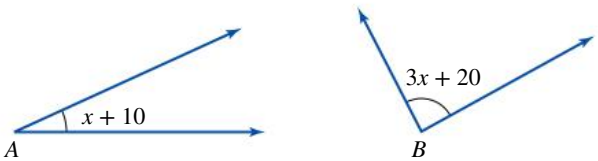


A $12x + 6$
B $12x + 24$
C $4x + 6$
D $4x + 2$

5 In order to purchase a new iPod you must save at least $260. What inequality represents the amount of money, m, that you must save?

A $m \leq 260$ **B** $m < 260$ **C** $m \geq 260$ **D** $m > 260$

6 In the diagram below, $\angle A$ and $\angle B$ are complementary.



What is the measure of $\angle B$?

A 65° **B** 45° **C** 30° **D** 15°

7 A swimming pool is being filled with water. The pool already contained 5000 litres of water. The table shows the number of litres of water in the pool after t hours.

Litres of water in pool (L)	Number of hours (t)
5 000	0
7 500	1
10 000	2
12 500	3
15 000	4

Which rule can be used to determine the number of litres, L, of water in the pool after t hours?

A $L = 2500t$ **B** $L = 5000t$

C $L = 5000t + 2500$ **D** $L = 5000 + 2500t$

8 Anne wants to solve the equation shown.

$$2x - 3 = 13.$$

Which steps could she use to find the solution?

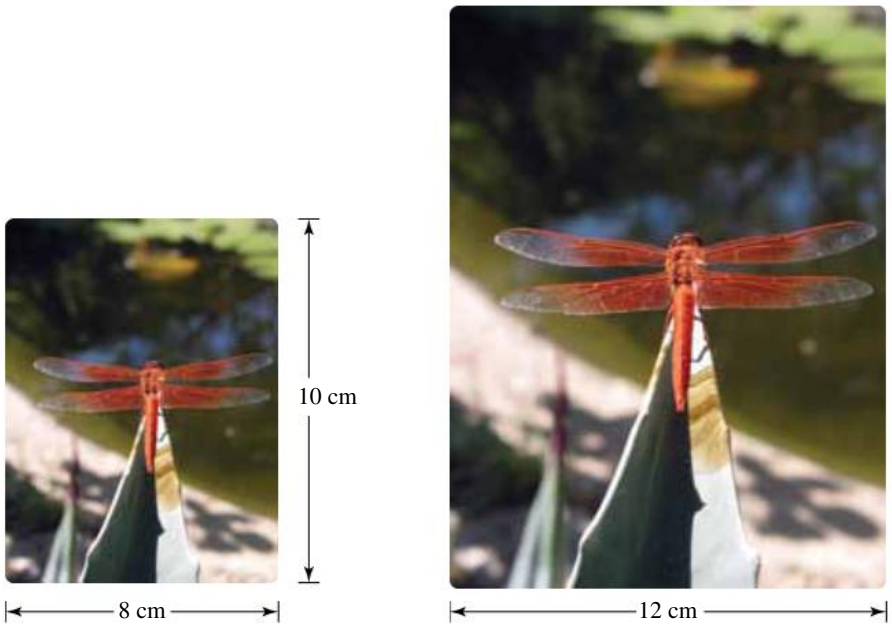
A Add 3 to both sides, then divide both sides by 2.

B Subtract 3 from both sides, then divide both sides by 2.

C Divide both sides by 2, then add 3 to both sides.

D Multiply both sides by 2, then subtract 3 from both sides.

9 The photograph of a dragonfly is shown with its dimensions.



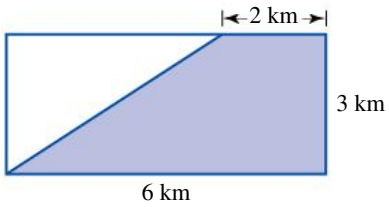
You decide to enlarge the photograph, and the new width is 12 cm. What is the new length?

A 14 cm **B** 15 cm **C** 16 cm **D** 17 cm

10 Madeline noticed that in one minute she blinked 50 times. At this rate, approximately how many days will it take her to blink 1 000 000 times?

A 8 **B** 10 **C** 12 **D** 14

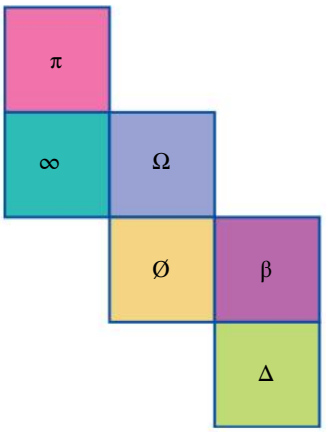
11 A parcel of land is to be subdivided as shown below. The shaded area is to be sold.



What percentage of the total area does this represent?

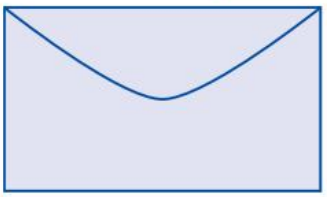
A $33\frac{1}{3}$ **B** 50 **C** $66\frac{2}{3}$ **D** 75

12 When the diagram shown is folded to make a cube, what symbol is on the face opposite the face marked Δ?



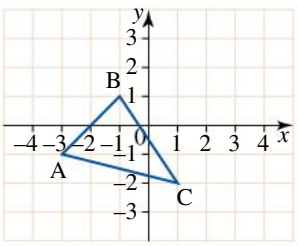
A β
B Ω
C ø
D ∞

13 You have an envelope that has a perimeter of 35 cm. If the ratio of the length to the width is 4 : 3, what are the dimensions of the envelope?



A $L = 20$ cm, $W = 15$ cm
B $L = 10$ cm, $W = 7.5$ cm
C $L = 15$ cm, $W = 20$ cm
D $L = 7.5$ cm, $W = 10$ cm

14 A triangle ABC was drawn on coordinate axes as shown. What would be the coordinates of the triangle reflected in the y axis?



A (1, 3) (1, 1) (−1, −3)
B (3, −1) (1, 1) (−1, −2)
C (−3, 1) (−1, −1) (1, 3)
D (3, 1) (1, 1) (1, −3)

15 Evaluate the following expression.

$$2\frac{2}{5} - 1\frac{2}{3}$$

16 An electronic device counted 4500 vehicles passing through an intersection during an 8-hour period. If the number of vehicles passing through the intersection per hour remains the same, what proportion can be used to find x, the number of vehicles that would be counted during a 10-hour period?

A $\frac{4500}{8} = \frac{x}{10}$ **B** $\frac{8}{4500} = \frac{x}{10}$

C $\frac{8}{x} = \frac{10}{4500}$ **D** $\frac{8}{4500} = \frac{10}{x}$

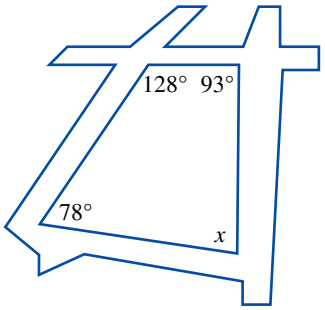
17 John has five fewer marbles than Liam, and Tang has three times as many as John. If Liam has n marbles, which of the following represents the number of marbles that Tang has?

A $3n - 5$ **B** $3n$ **C** $5 - 3n$ **D** $3(n - 5)$

18 The directions for using a concentrated cleaning product say to add 3 capfuls of the product to every 4 litres of water. Which of the following equations can be used to calculate c, the number of capfuls of the product needed for 7 litres of water?

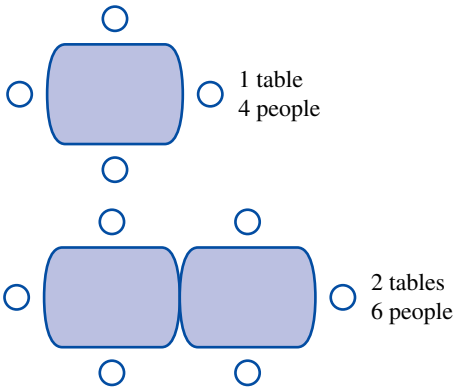
A $\frac{3}{4} = \frac{c}{7}$ **B** $\frac{3}{4} = \frac{c}{11}$ **C** $\frac{4}{3} = \frac{c}{7}$ **D** $\frac{4}{3} = \frac{c}{11}$

19 Intersecting paths have been constructed to surround a decorative garden bed. What is the angle measurement for x?



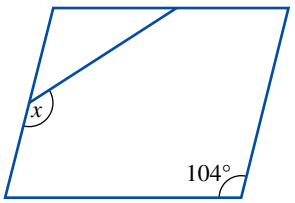
A 128°
B 102°
C 87°
D 61°

20 The diagrams show the seating arrangements that will be used if tables are placed end to end. Which formula represents the relationship between the number of people (P) that can be seated and the number of tables (t) placed end to end?



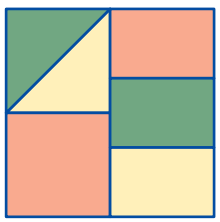
A $P = 4t$
B $P = 3t$
C $P = 4t - 2$
D $P = 2t + 2$

21 The diagram shows a rhombus. The midpoints of two of its sides are joined with a straight line. What is the measure of angle x?



A 76°
B 104°
C 142°
D 152°

22 A square dart board is shown below.



Suppose a dart, thrown randomly, hits the board. Determine the probability of the dart's landing on a green segment.

A $\frac{7}{12}$ **B** $\frac{1}{8}$ **C** $\frac{7}{24}$ **D** $\frac{1}{14}$

23 In how many different ways could these plastic bottles be arranged in a line?

24 What is the solution for the following expression?

$5 + \frac{70}{10} \times (1 + 2)^2 - 1$

A 95 **B** 71 **C** 67 **D** 46

25 A helicopter has a rotor that moves at a rate of 720 revolutions per minute. Through how many degrees does the rotor turn per second?

A 0.03 **B** 12
C 4320 **D** 259 200

26 You counted a total of 40 goldfish in a pond in the Botanical Gardens. The gardener told you that the ratio of female fish to male fish was 3 : 5. What was the total of the number of male goldfish?

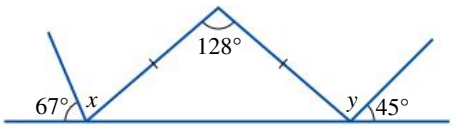
A 24 **B** 25 **C** 26 **D** 30

27 Based on the table, which statement is true?

Polygons	
Number of sides (n)	**Sum of interior angle measures (S)**
3	180°
4	360°
5	540°
6	720°
7	900°

A The sum of the interior angle measures decreases by $\frac{1}{2}$ for each side increase of 1.
B The sum of the interior angle measures increases by 180° for each side increase of 1.
C The sum of the interior angle measures doubles for each side increase of 1.
D The sum of the interior angle measures is a whole number multiple of 360°.

28 The sum of x and y is:



A 196° **B** 180°
C 135° **D** 113°

29 A box is displayed on the top of a shop counter as shown in the photograph. Its dimensions are length $= 1\frac{1}{4}$ m, width $= 1\frac{2}{5}$ m and height $= 0.8$ m. What is the volume of the box?

A $1.2\,m^3$ **B** $1.25\,m^3$
C $1.4\,m^3$ **D** $1.65\,m^3$

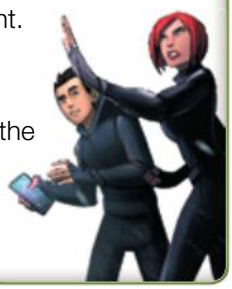
30 If two sides of a triangle are 12 cm and 20 cm, the third side must be:

A between and including 8 cm and 32 cm.
B between but not including 8 cm and 32 cm.
C greater than 8 cm.
D less than 32 cm.

Link to SpyClass, an exciting online game combining a comic book–style story with problem-based learning in an immersive environment.

Join Jesse, Toby and Dan and help them to tackle some of the world's most dangerous criminals by using the knowledge you've gained through your study of mathematics.

www.spyclass.com.au



Answers

TOPIC 9 Numeracy 2

9.2 Set C Calculator allowed

1 D	**2** A	**3** 11 pm	**4** B
5 C	**6** B	**7** D	**8** C
9 C	**10** 124	**11** 10 days	**12** B
13 A	**14** B	**15** B	**16** A
17 D	**18** A	**19** B	**20** 58 cm
21 4	**22** B	**23** B	**24** C
25 D	**26** A	**27** C	**28** B
29 B	**30** D		

9.3 Set D Non-calculator

1 B	**2** C	**3** D	**4** D
5 C	**6** A	**7** D	**8** A
9 B	**10** D	**11** C	**12** B
13 A	**14** B	**15** $\frac{11}{15}$	**16** A
17 D	**18** A	**19** D	**20** D
21 C	**22** C	**23** 24	**24** C
25 C	**26** B	**27** B	**28** A
29 C	**30** B		

TOPIC 10

Measurement

10.1 Overview

Why learn this?

Measurement is important in many aspects of everyday life. If you do not have an understanding of metric measurement — length, area and volume — many professions and jobs will be closed to you. Imagine a designer, dressmaker, architect or builder who did not understand measurement. Their job would be impossible!

What do you know?

assessON

1 **THINK** List what you know about measurement. Use a thinking tool such as a concept map to show your list.
2 **PAIR** Share what you know with a partner and then with a small group.
3 **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of measurement.

Learning sequence

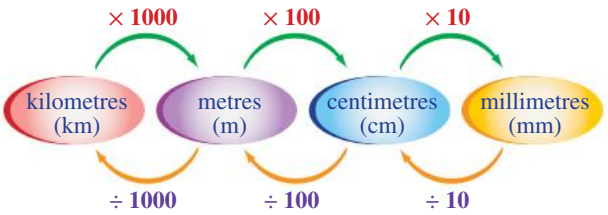
10.1 Overview
10.2 Perimeter
10.3 Circumference
10.4 Area of rectangles, triangles, parallelograms, rhombuses and kites
10.5 Area of a circle
10.6 Area of trapeziums
10.7 Volume of prisms and other solids
10.8 Time
10.9 24-hour clock and time zones
10.10 Review eBook plus ONLINE ONLY

83
84
85
86
71
74
73
72
71
70
52
53

10.2 Perimeter

Units of length

- Metric units of length include millimetres (mm), centimetres (cm), metres (m) and kilometres (km).
- To convert between the units of length, we use the following conversion chart:



- When converting from a larger unit to a smaller unit, multiply by the conversion factor; when converting from a smaller unit to a larger unit, divide by the conversion factor.

WORKED EXAMPLE 1

Complete the following metric length conversions.

a **1.027 m = ____ cm**
b **0.0034 km = ____ m**
c **76 500 m = ____ km**
d **3.069 m = ____ mm**

THINK

a Look at the conversion table. To convert metres to centimetres, we need to multiply by 100. So, move the decimal point two places to the right.

b To convert kilometres to metres, we need to multiply by 1000. So, move the decimal point three places to the right.

c To convert metres to kilometres, divide by 1000. This can be done by moving the decimal point three places to the left.

d Look at the conversion table. To convert metres to millimetres, we need to multiply by 100 and then by 10. This is the same as multiplying by 1000, so move the decimal point three places to the right.

WRITE

a $1.027 \times 100 = 102.7$ cm

b $0.0034 \times 1000 = 3.4$ m

c $76\,500 \div 1000 = 76.5$ km

d $3.069 \times 100 \times 10 = 3069$ mm

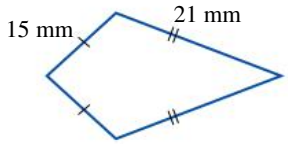
The perimeter

- The **perimeter** of a shape is the total distance around the shape.
- To find the perimeter of a shape:
  - identify the length of each side
  - ensure that all measurements are in the same units
  - add all side lengths together and include units with your answer.

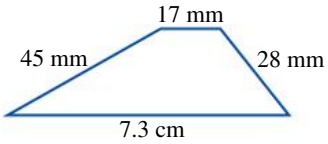
WORKED EXAMPLE 2

Find the perimeter of each of the shapes below.

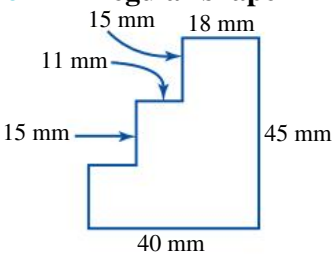
a A kite



b A trapezium



c An irregular shape



THINK

a 1 Make sure that all the measurements are in the same units and add them together.

2 Write the answer in words, including the units.

b 1 Notice that the measurements are not all in the same metric units. Convert to the smaller unit (in this case convert 7.3 cm to mm).

2 Add the measurements.

3 Write the answer in words, including the units.

c 1 Make sure that given measurements are in the same units.

2 Determine the lengths of the unknown sides and label them on the diagram.

3 Add the measurements.

4 Write the answer in words, including the units.

WRITE/DRAW

a $P = 21 \times 2 + 15 \times 2$
$= 72$

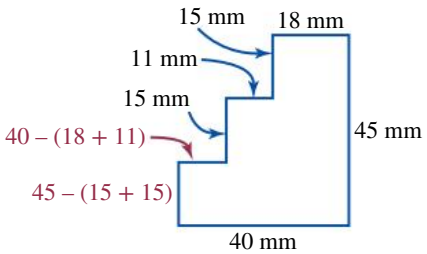
The perimeter of the kite shown is 72 mm.

b 7.3 cm = 73 mm

$P = 45 + 17 + 28 + 73$
$= 163$

The perimeter of the trapezium shown is 163 mm.

c

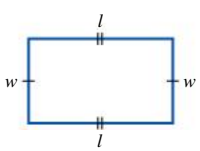


$P = 45 + 18 + 15 + 11 + 15 + [40 - (18 + 11)] + [45 - (15 + 15)] + 40$
$= 45 + 18 + 15 + 11 + 15 + 11 + 15 + 40$
$= 170$ mm

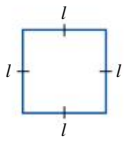
The perimeter of the irregular shape shown is 170 mm.

Finding the perimeter of a square and a rectangle

- The perimeter (P) of a rectangle is given by the formula $P = 2(l + w)$, where l is the length and w is the width of the rectangle.

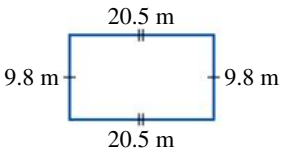


- The perimeter (P) of a square is given by the formula $P = 4l$, where l is the side length of the square.



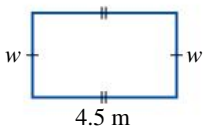
WORKED EXAMPLE 3

Find the perimeter of a rectangular block of land that is 20.5 m long and 9.8 m wide.

THINK	WRITE/DRAW
1 Draw a diagram of the block of land and write in the measurements.	
2 Write the formula for the perimeter of a rectangle.	$P = 2(l + w)$
3 Substitute the values of l and w into the formula, and calculate.	$P = 2 \times (20.5 + 9.8)$ $= 2 \times 30.3$ $= 60.6$ m
4 Write the worded answer with the correct units.	The perimeter of the block of land is 60.6 m.

WORKED EXAMPLE 4

A rectangular billboard advertising country Victoria has a perimeter of 16 m. Calculate its width if the length is 4.5 m.

THINK	WRITE/DRAW
1 Draw a diagram of the rectangular billboard and write in the measurements.	 $P = 16$ m
2 Write the formula for the perimeter of a rectangle.	$P = 2(l + w)$ $= 2l + 2w$
3 Substitute the values of P and l into the formula, and solve the equation: (a) subtract 9 from both sides (b) divide both sides by 2 (c) simplify if appropriate.	$16 = 2 \times 4.5 + 2w$ $16 = 9 + 2w$ $16 - 9 = 9 - 9 + 2w$ $7 = 2w$ $\frac{7}{2} = \frac{2w}{2}$ $3.5 = w$ $w = 3.5$
4 Write the worded answer with the correct units.	The width of the rectangular billboard is 3.5 m.

Exercise 10.2 Perimeter

assessON

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 7, 8, 14, 17	Questions: 1a, c, e, g, i, k, m, 2a, c, d, f, g, h, i, j, 3, 5, 8, 14, 16–18	Questions: 1b, d, f, h, j, l, n, 2b, d, f, h, i, k, l, 4, 6, 7, 9–19

REFLECTION
What is the best way to remember the units conversion chart?

Individual pathway interactivity int-4438 eBookplus

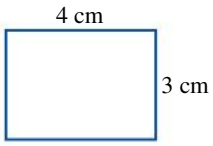
FLUENCY

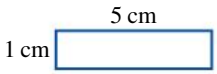
eBookplus
Digital docs
SkillSHEET
Multiplying and dividing by powers of 10
doc-6956
SkillSHEET
Converting units of length
doc-6957

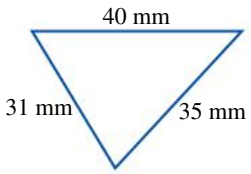
1 WE1 Fill in the gaps for each of the following.

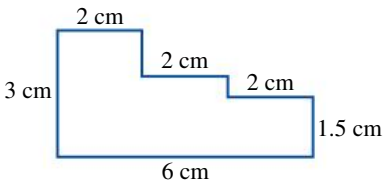
a 20 mm = _____ cm
b 13 mm = _____ cm
c 130 mm = _____ cm
d 1.5 cm = _____ mm
e 0.03 cm = _____ mm
f 2.8 km = _____ m
g 0.034 m = _____ cm
h 2400 mm = _____ cm = _____ m
i 1375 mm = _____ cm = _____ m
j 2.7 m = _____ cm = _____ mm
k 0.08 m = _____ mm
l 6.071 km = _____ m
m 670 cm = _____ m
n 0.0051 km = _____ m

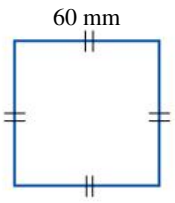
2 WE2 Find the perimeter of the shapes below.

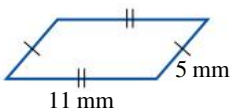
a


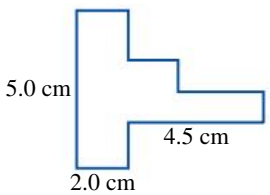
b


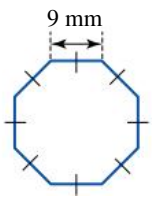
c


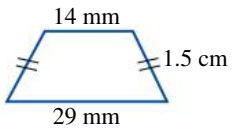
d


e


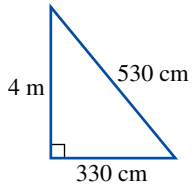
f


g


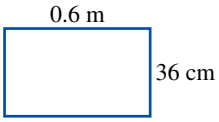
h


i


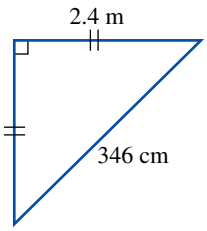
j



k



l



UNDERSTANDING

3 Chipboard sheets are sold in three sizes. Convert each of the measurements below into centimetres and then into metres:

a 1800 mm × 900 mm **b** 2400 mm × 900 mm **c** 2700 mm × 1200 mm.

4 A particular type of chain is sold for $2.25 per metre. What is the cost of 2.4 m of this chain?

5 Fabric is sold for $7.95 per metre. How much will 4.8 m of this fabric cost?

6 The standard marathon distance is 42.2 km. If a marathon race starts and finishes with one lap of a stadium which is 400 m in length, what distance is run on the road outside the stadium?

7 Maria needs 3 pieces of timber of lengths 2100 mm, 65 cm and 4250 mm to construct a clothes rack.

a What is the total length of timber required, in metres?

b How much will the timber cost at $3.80 per metre?

8 WE3 Find the perimeter of a basketball court, which is 28 m long and 15 m wide.

9 A woven rectangular rug is 175 cm wide and 315 cm long. Find the perimeter of the rug.

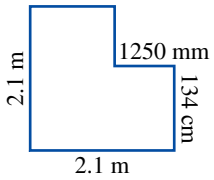
10 A line is drawn to form a border 2 cm from each edge of a piece of A4 paper. If the paper is 30 cm long and 21 cm wide, what is the length of the border line?

11 A rectangular paddock 144 m long and 111 m wide requires a new single-strand wire fence.

a What length of fencing wire is required to complete the fence?

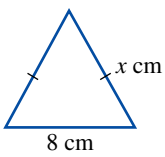
b How much will it cost to rewire the fence if the wire cost $1.47 per metre.

12 A computer desk needs to have table edging. If the edging cost $1.89 per metre, find the cost of the table edging required for the desk.



13 Calculate the unknown side lengths in each of the given shapes.

a

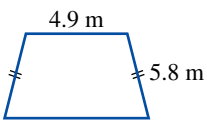


Perimeter = 30 cm

b

Perimeter = 176 cm

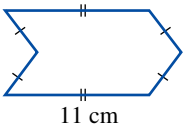
c



Perimeter = 23.4 m

14 WE4 The rectangular billboard has a perimeter of 25 m. Calculate its width if the length is 7 m.

15 The ticket at right has a perimeter of 42 cm.



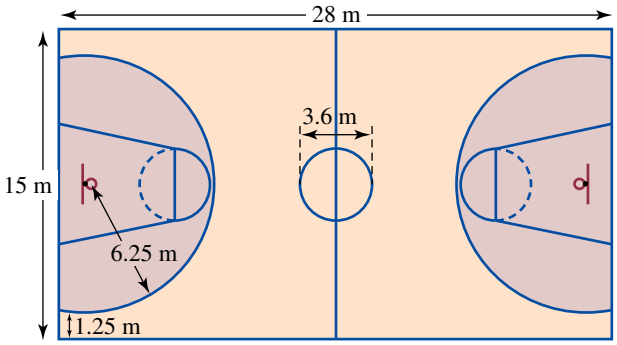
- **a** Calculate the unknown side length.
- **b** Olivia wishes to decorate the ticket by placing a gold line along the slanted sides. How long is the line on each ticket?
- **c** A bottle of gold ink will supply enough ink to draw 20 m of line. How many bottles of ink should be purchased if 200 tickets are to be decorated?

REASONING

16 A square and an equilateral triangle have the same perimeter. The side of the triangle is 3 cm longer than the side of the square. How long is the side of the square? Show your working.

PROBLEM SOLVING

17 As a warm-up activity for PE class, you are required to run laps of the basketball court. The dimensions of a full-size basketball court are shown.



- **a** How far will you run if you run three full laps of the court?
- **b** How far will you run if you run three laps of half the court?

18 **a** A pool fence is to be placed around a rectangular pool that is 6 m by 8 m. If the fence is to be 1 m away from the edge of the pool and also rectangular in shape, what length of fencing is required? (Include the width of the gate in the length of the fence.)

- **b** Write a rule that relates the length of the fence to the length and width of any rectangular pool.

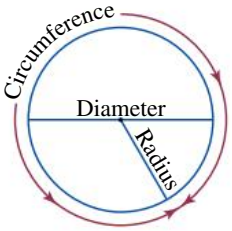
19 A present is to be wrapped using a box, as shown at right.

- **a** If the dimensions of the box are 30 cm long by 25 cm wide by 18 cm high, what is the total length of ribbon that would be needed if the bow used 35 cm of ribbon? (Assume no overlap at the start or finish of the ribbon.)
- **b** Write a rule that will allow you to calculate the amount of ribbon required for any rectangular box.

10.3 Circumference

- The **circumference (*C*)** is another term for the perimeter of a circle.
- The **diameter (*D*)** of a circle is the name given to the straight-line distance across a circle though its centre.
- The straight-line distance from the centre of the circle to the circumferences is called the **radius**.
- There is a relationship between the diameter and the circumference of a circle.

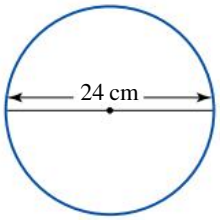
- The ratio $\frac{C}{D}$ is approximately 3. When this ratio is calculated exactly, it is called *pi* or π.
- The symbol π represents the ratio of the circumference of a circle to its diameter. It is an infinite, non-recurring and non-terminating decimal that begins as 3.141 592 653 5 . . .
- The circumference of a circle is given by the formula $C = \pi D$.
- A diameter of a circle is twice as long as its radius, that is, $D = 2r$. Therefore, the other way to write the formula for the circumference is $C = 2\pi r$, where r is the radius of a circle.



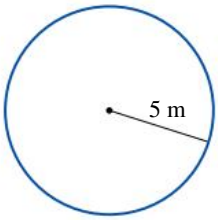
WORKED EXAMPLE 5

Find the circumference of each of the following circles, giving answers i in terms of π and ii correct to 2 decimal places.

a



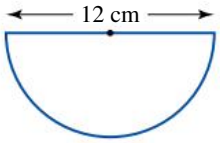
b



THINK	WRITE
a i 1 Write the formula for the circumference of a circle. *Note:* Since the diameter of the circle is given, use the formula that relates the circumference to the diameter.	**a i** $C = \pi D$
2 Substitute the value $D = 24$ into the formula.	$= \pi \times 24$
3 Write the answer and include the correct units.	$= 24\pi$ cm
ii 1 Write the formula for the circumference of a circle.	**ii** $C = \pi D$
2 Substitute the values $D = 24$ and $\pi = 3.14\ldots$ into the formula.	$= 3.14\ldots \times 24$
3 Evaluate and include the correct units.	$= 75.398\ldots$ $= 75.40$ cm
b i 1 Write the formula for the circumference of a circle. *Note:* Since the radius of the circle is given, use the formula that relates the circumference to the radius.	**b i** $C = 2\pi r$
2 Substitute the value $r = 5$ into the formula.	$= 2 \times \pi \times 5$
3 Write the answer and include the correct units.	$= 10\pi$ m
ii 1 Write the formula for the circumference of a circle.	**ii** $C = 2\pi r$
2 Substitute the values $r = 5$ and $\pi = 3.14\ldots$ into the formula.	$= 2 \times 3.14\ldots \times 5$
3 Evaluate and include the correct units.	$= 31.415\ldots$ $= 31.42$ m

WORKED EXAMPLE 6

Find the perimeter of the following shape, correct to 2 decimal places.



THINK	WRITE
1 Identify the parts that constitute the perimeter of the given shape.	$P = \frac{1}{2}$ circumference + straight-line section
2 Write the formula for the circumference of a circle. *Note:* If the circle were complete, the straight-line segment shown would be its diameter. So the formula that relates the circumference to the diameter is used.	$P = \frac{1}{2}\pi D$ + straight-line section
3 Substitute the values $D = 12$ and $\pi = 3.14\ldots$ into the formula.	$= \frac{1}{2} \times 3.14\ldots \times 12 + 12$
4 To find the perimeter of the given shape, halve the value of the circumference and add the length of the straight section.	$= 18.849\ldots + 12$
5 Evaluate and include the correct units.	$= 30.85$ cm

Exercise 10.3 Circumference

assessON

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–6, 8, 11, 12, 16, 18	Questions: 1a, c, e, 2a, c, e, 3a, c, e, 4b, d, 5, 6, 8, 11, 12, 15, 16, 18–20	Questions: 1a, d, 2a, b, f, 3c, e, f, 4b, c, d, f, i, 5–7, 9, 11c, 12c, 13–17, 19–21

Individual pathway interactivity int-4439 eBookplus

REFLECTION
If you needed to estimate the value of π using any round object, how would you do it?

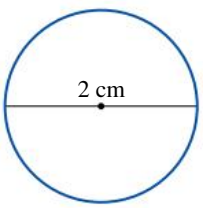
FLUENCY

1 WE5a Find the circumference of each of these circles, giving answers

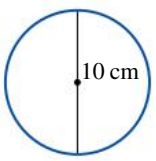
i in terms of π

ii correct to 2 decimal places.

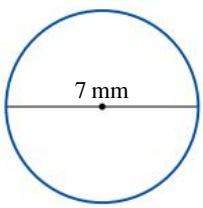
a



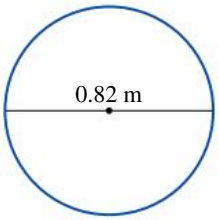
b



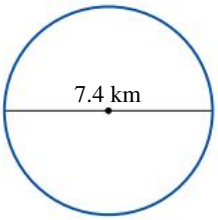
c



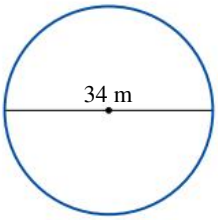
d



e



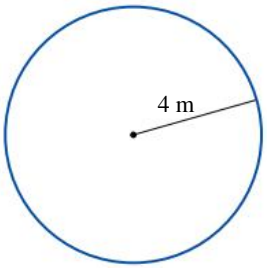
f



2 **WE5b** Find the circumference of each of the following circles, giving answers

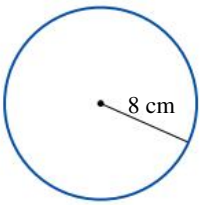
i in terms of π **ii** correct to 2 decimal places.

a



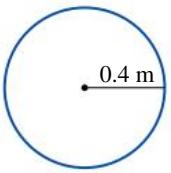
b

c

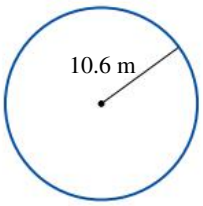


d

e



f

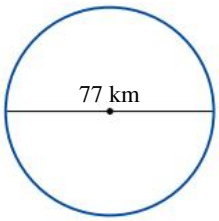


eBookplus

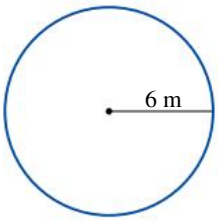
Digital doc
Investigation
The diameter of a circle and its circumference — any connection?
doc-2318

3 Choose the appropriate formula and find the circumference of these circles.

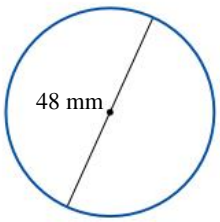
a



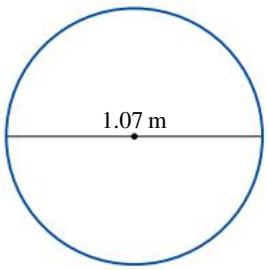
b



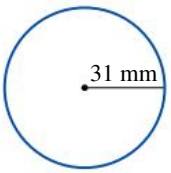
c



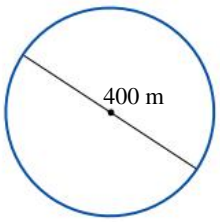
d



e

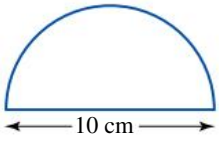


f

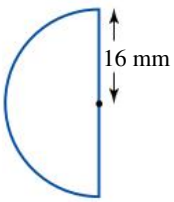


4 **WE6** Find the perimeter of each of the shapes below. (Remember to add the lengths of the straight sections.) Give your answers to 2 decimal places.

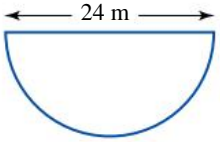
a



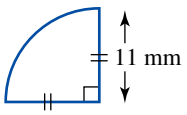
b



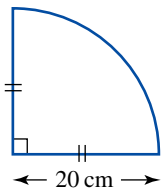
c



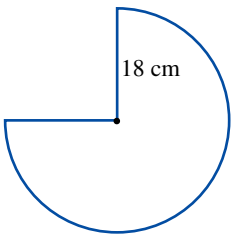
d



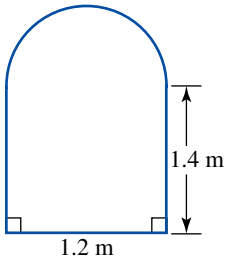
e



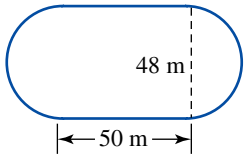
f



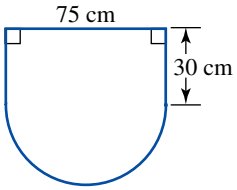
g



h



i



5 **MC** The circumference of a circle with a radius of 12 cm is:

A $\pi \times 12$ cm **B** $2 \times \pi \times 12$ cm **C** $2 \times \pi \times 24$ cm

D $\pi \times 6$ cm **E** $\pi \times 18$ cm

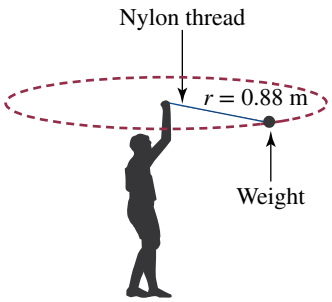
6 **MC** The circumference of a circle with a diameter of 55 m is:

A $2 \times \pi \times 55$ m **B** $\pi \times \frac{55}{2}$ m **C** $\pi \times 55$ m

D $\pi \times 110 \times 2$ m **E** $2 \times \pi \times 110$ m

UNDERSTANDING

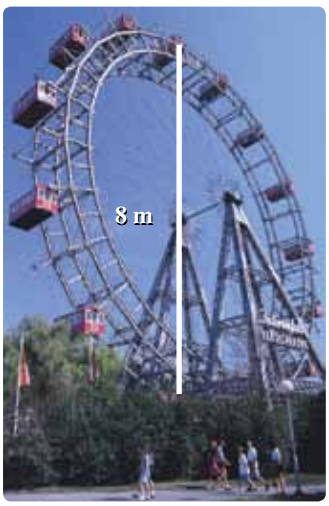
7 In a Physics experiment, students spin a metal weight around on the end of a nylon thread. How far does the metal weight travel if it completes 10 revolutions on the end of a 0.88 m thread? Give your answer to 2 decimal places.



8 A scooter tyre has a diameter of 32 cm. What is the circumference of the tyre? Give your answer to 2 decimal places.

9 Find the circumference of the seaweed around the outside of this sushi roll, correct to 2 decimal places.

10 Find the circumference of the Ferris wheel shown below, correct to 2 decimal places.



11 Calculate the diameter of a circle (correct to 2 decimal places where appropriate) with a circumference of:

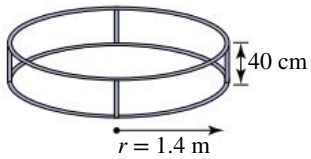
- **a** 18.84 m
- **b** 64.81 cm
- **c** 74.62 mm.

12 Calculate the radius of a circle (correct to 2 decimal places where appropriate) with a circumference of:

- **a** 12.62 cm
- **b** 47.35 m
- **c** 157 mm.

13 Calculate the radius of a tyre with a circumference of 135.56 cm. Give your answer to 2 decimal places.

14 Calculate the total length of metal pipe needed to assemble the wading pool frame shown at right. Give your answer in metres to 2 decimal places.



15 Nathan runs around the inside lane of a circular track that has a radius of 29 m. Rachel runs in the outer lane, which is 2.5 m further from the centre of the track. How much longer is the distance Rachel runs each lap? Give your answer to 2 decimal places.

REASONING

16 In *Around the world in eighty days* by Jules Verne, Phileas Fogg boasts that he can travel around the world in 80 days or fewer. This was in the 1800s, so he couldn't take a plane. What average speed is needed to go around the Earth at the equator in 80 days? Assume you travel for 12 hours each day and that the radius of the Earth is approximately 6390 km. Give your answer in km/h to 2 decimal places.

17 Liesel's bicycle covers 19 m in 10 revolutions of her bicycle wheel while Jared's bicycle covers 20 m in 8 revolutions of his bicycle wheel. What is the difference between the radii of the two bicycle wheels? Give your answer in cm to 2 decimal places.

PROBLEM SOLVING

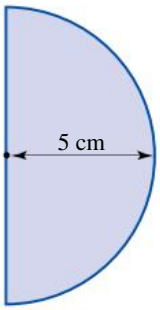
18 A shop sells circular trampolines of four different sizes. Safety nets that go around the trampoline are optional and can be purchased separately. The entrance to the trampoline is via a zip in the net.

The following table shows the diameters of all available trampolines and their net lengths. Which safety net matches which trampoline?

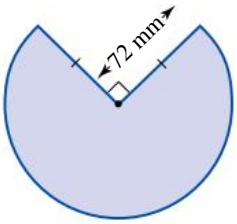
	Diameter of trampoline	Length of safety net
a	1.75 m	**i** 6.03 m
b	1.92 m	**ii** 9.86 m
c	2.46 m	**iii** 5.50 m
d	3.14 m	**iv** 7.73 m

19 Calculate the perimeter of each of the following shapes. Give your answers correct to 2 decimal places.

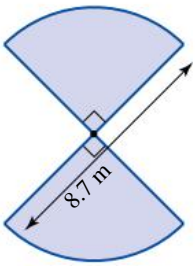
a



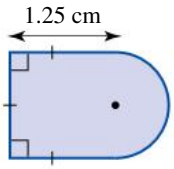
b



c

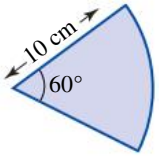


d

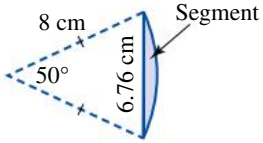


20 To cover a total distance of 1.5 km, a student needs to run around a circular track three times. Calculate the radius of the track correct to the nearest metre.

21 **a** A sector is a section of a circle contained between two radii (like a pizza slice). Investigate how to find the perimeter of this sector.

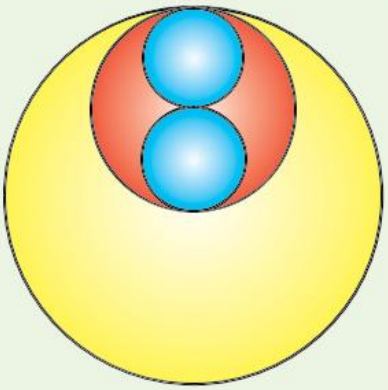


b A segment is a section of a circle contained between a chord and an arc, as shown in this diagram. Investigate how to get the perimeter of the segment shown.



CHALLENGE 10.1

The two small circles in the diagram have a diameter that is equal to the radius of the medium-sized circle. The diameter of the medium-sized circle is equal to the radius of the large circle. If the large circle has a radius of 8 cm, determine the area of the shaded section.

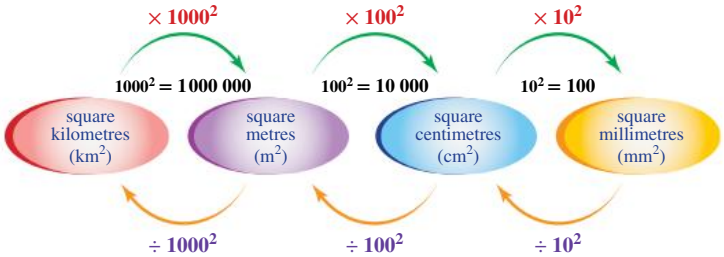


10.4 Area of rectangles, triangles, parallelograms, rhombuses and kites

Area

- The **area** of a shape is the amount of flat surface enclosed by the shape.
- Area is measured in square units, such as square millimetres (mm^2), square centimetres (cm^2), square metres (m^2) and square kilometres (km^2)

- Area units can be converted using the chart below.



- Area units are the squares of those for the corresponding linear units.
- Large areas of land can be measured in hectares (ha). 1 ha = 10 000 m^2.

WORKED EXAMPLE 7

Complete the following metric conversions.
a **0.081 km^2 = _____ m^2**
b **19 645 mm^2 = _____ m^2**

THINK

a Look at the metric conversion chart. To convert square kilometres to square metres, multiply by 1 000 000; that is, move the decimal point 6 places to the right.

b Look at the metric conversion chart. To convert square millimetres to square metres, divide by 1 000 000; that is, move the decimal point 6 places to the left.

WRITE

a $0.081\ km^2 = 0.081 \times 1\,000\,000\ m^2$
$= 81\,000\ m^2$

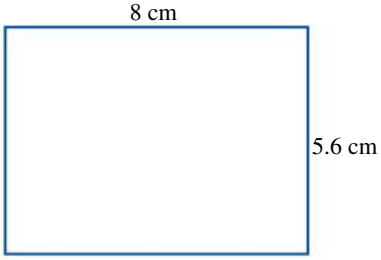
b $19\,645\ mm^2 = 19\,645 \div 1\,000\,000\ m^2$
$= 0.019\,645\ m^2$

Area of a rectangle

- The area of a rectangle can be found using the formula $A_R = l \times w$, where l is the length and w is the width of the rectangle.
- The area of a square can be found using the formula $A_S = l^2$, where l is the side length of the square.

WORKED EXAMPLE 8

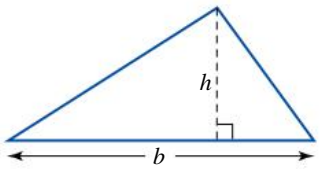
Find the area of a rectangle with dimensions shown below.



THINK	WRITE
1 Write the formula for the area of a rectangle.	$A = l \times w$
2 Identify the values of l and w.	$l = 8$ and $w = 5.6$
3 Substitute the values of l and w into the formula and evaluate. Include the appropriate units.	$A = 8 \times 5.6$ $= 44.8 \text{ cm}^2$

Area of a triangle

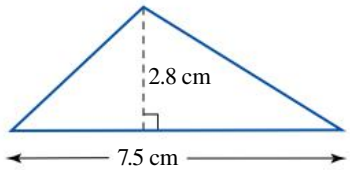
- The area of a triangle, A_T, is given by the formula $A_T = \frac{1}{2} \times b \times h$, where b is the base and h is the height of the triangle.
- The base and the height of a triangle are perpendicular (at right angles) to each other.



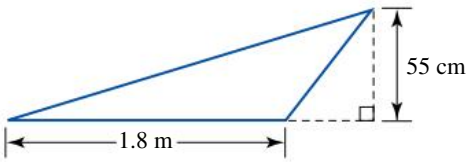
WORKED EXAMPLE 9

Find the area of each of these triangles in the smaller unit of measurement.

a



b

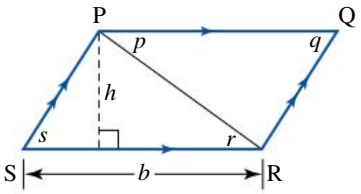


THINK	WRITE
a 1 Write the formula for the area of a triangle.	**a** $A = \frac{1}{2}bh$
2 Identify the values of b and h.	$b = 7.5, h = 2.8$
3 Substitute the values of b and h into the formula.	$A = \frac{1}{2} \times 7.5 \times 2.8$
4 Evaluate. Remember to include the correct units (cm^2).	$= 3.75 \times 2.8$ $= 10.5 \text{ cm}^2$
b 1 Write the formula for the area of a triangle.	**b** $A = \frac{1}{2}bh$
2 Convert measurements to cm.	$1.8 \text{ m} = 1.8 \times 100 \text{ cm}$ $= 180 \text{ cm}$
3 Identify the values of b and h.	$b = 180, h = 55$
4 Substitute the values of b and h into the formula.	$A = \frac{1}{2} \times 180 \times 55$
5 Evaluate. Remember to include the correct units (cm^2).	$= 90 \times 55$ $= 4950 \text{ cm}^2$

Area of a parallelogram

eBookplus
Digital doc
Investigation
Area of a parallelogram
doc-2319

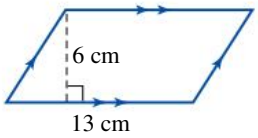
- A **parallelogram** is a quadrilateral with two pairs of parallel sides. Each parallel pair is of equal length.
- A parallelogram can be divided into two triangles by drawing a diagonal.
  In the parallelogram PQRS below, are ΔPQR and ΔRSP congruent?
  As their corresponding sides are of equal length, then ΔPQR ≡ ΔRSP (SSS congruency condition).



- As the area of a triangle is given as $A_T = \frac{1}{2} \times b \times h$, then the area of a parallelogram must be twice the area of each triangle, so A_P, area of a parallelogram is given by the formula $A_P = b \times h$, where b is the base and h is the height of a parallelogram.
- The base and the height of a parallelogram are perpendicular to each other.

WORKED EXAMPLE 10

Find the area of the parallelogram shown.



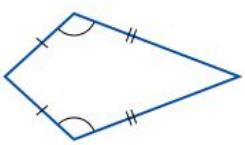
THINK	WRITE
1 Write the formula for the area of a parallelogram.	$A = bh$
2 Identify the values of b and h.	$b = 13, h = 6$
3 Substitute 6 for h and 13 for b.	$A = 13 \times 6$
4 Multiply the numbers together and include the correct units.	$= 78 \text{ cm}^2$

Area of a rhombus

- A **rhombus** is a parallelogram with all four sides of equal length and each pair of opposite sides parallel.
- The area of a rhombus can be determined by multiplying the length of the diagonals and dividing by 2: $A_{RH} = \frac{d_1 \times d_2}{2}$.

Area of a kite

- A **kite** is a quadrilateral with two pairs of equal, adjacent sides and one pair of equal angles.
- The area of a kite can be determined by dividing the kite into two equal triangles and using the formula for the area of a triangle: $A_K = \frac{d_1 \times d_2}{2}$.



Exercise 10.4 Area of rectangles, triangles, parallelograms, rhombuses and kites

assessON

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–6, 7a, c, e, g, i, 8, 10, 12, 16, 17, 24	Questions: 1a, c, e, g, i, 2a, e, f, 3b, 4, 5, 6a, d, f, 7a, e, g, i, 8, 9, 11, 13, 14, 17, 19, 22, 24, 25	Questions: 1b, d, f, h, j, 2c, e, f, 3c, 4, 5, 6c, e, f, 8–10, 12, 15–26

Individual pathway interactivity int-4440 eBookplus

REFLECTION
Why is it important to know that the height and the base of any triangle and any parallelogram are perpendicular?

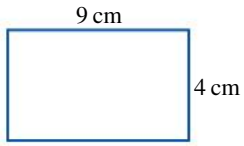
FLUENCY

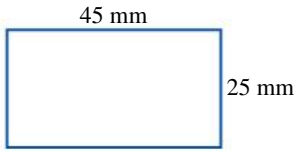
1 WE7 Complete the following metric conversions.

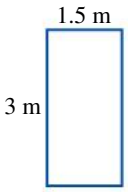
a 0.53 km^2 = _____ m^2
b 235 mm^2 = _____ cm^2
c 2540 cm^2 = _____ mm^2
d 542 000 cm^2 = _____ m^2
e 74 000 mm^2 = _____ m^2
f 3 000 000 m^2 = _____ km^2
g 98 563 m^2 = _____ ha
h 1.78 ha = _____ m^2
i 0.987 m^2 = _____ mm^2
j 0.000 127 5 km^2 = _____ cm^2

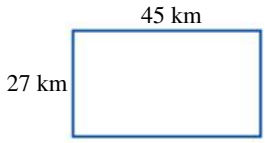
eBookplus
Digital doc
SkillSHEET
Areas of squares, rectangles and triangles
doc-6958

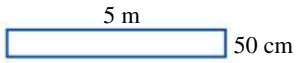
2 WE8 Find the area of each of the rectangles below.

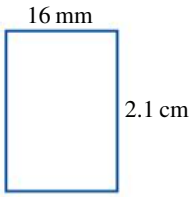
a


b


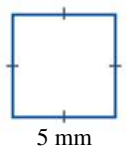
c


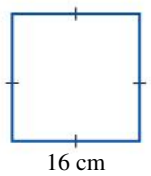
d


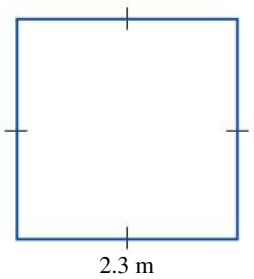
e


f


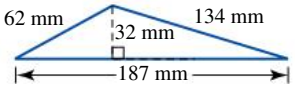
3 Find the area of each of the squares below.

a


b


c


Questions **4** and **5** relate to the diagram at right.



4 MC The height and base respectively of the triangle are:

A 32 mm and 62 mm
B 32 mm and 134 mm
C 32 mm and 187 mm
D 62 mm and 187 mm
E 134 mm and 187 mm

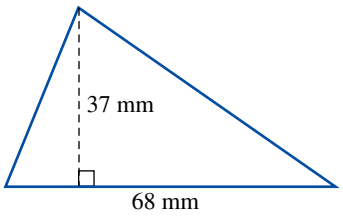
5 **MC** The area of the triangle is:

A 2992 mm **B** 2992 mm^2 **C** 5984 mm

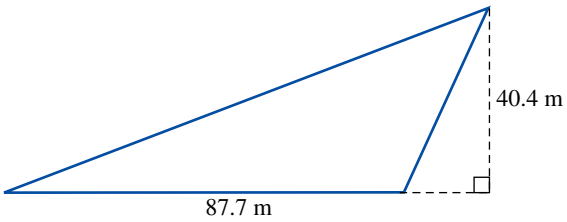
D 5984 mm^2 **E** 6128 mm^2

6 **WE9** Find the areas of the following triangles. Where there are two units given, answer using the smaller units.

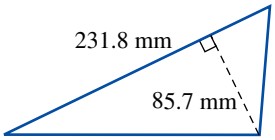
a



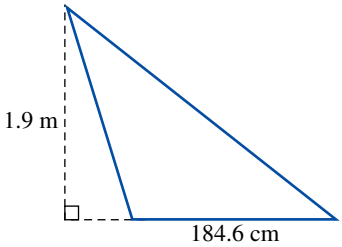
b



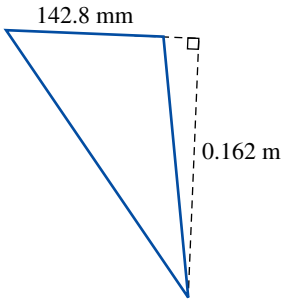
c



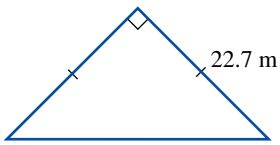
d



e

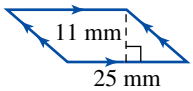


f

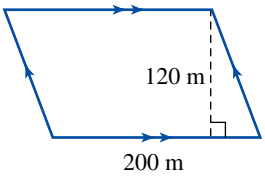


7 **WE10** Find the areas of the following parallelograms.

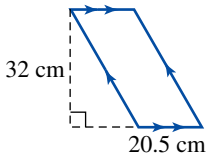
a



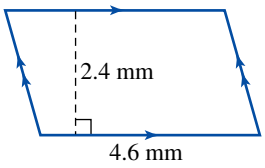
b



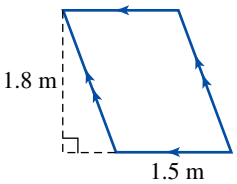
c



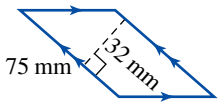
d



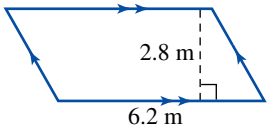
e



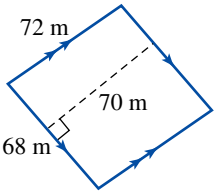
f



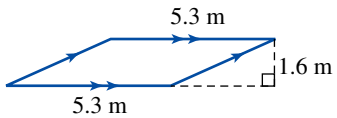
g



h



i



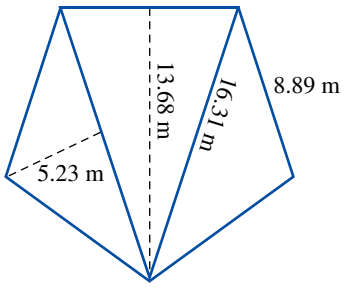
8 **a** Find the area of a rhombus whose diagonals are 10 cm and 6 cm.

b Find the area of a rhombus whose diagonals are 8 cm and 6 cm.

c Find the area of a kite whose diagonals are 20 cm and 9 cm.

UNDERSTANDING

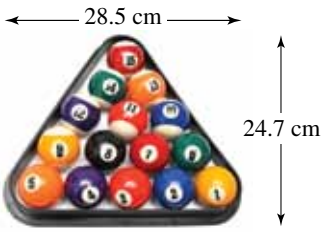
9 Zorko has divided his vegetable patch, which is in the shape of a regular (all sides equal) pentagon, into 3 sections as shown in the diagram at right.



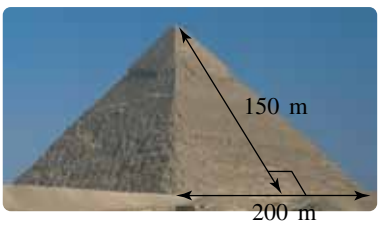
a Calculate the area of each individual section, correct to 2 decimal places.

b Calculate the area of the vegetable patch, correct to 2 decimal places.

10 Find the area of the triangle used to rack up the pool balls at right.



11 The pyramid at right has 4 identical triangular faces with the dimensions shown. Calculate:



a the area of one of the triangular faces

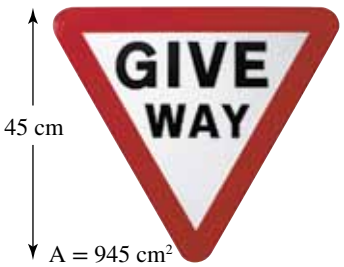
b the total area of the 4 faces.

12 Georgia is planning to create a feature wall in her lounge room by painting it a different colour. The wall is 4.6 m wide and 3.4 m high.

a Calculate the area of the wall to be painted.

b Georgia knows that a 4 litre can of paint is sufficient to cover 12 square metres of wall. How many cans must she purchase if she needs to apply two coats of paint?

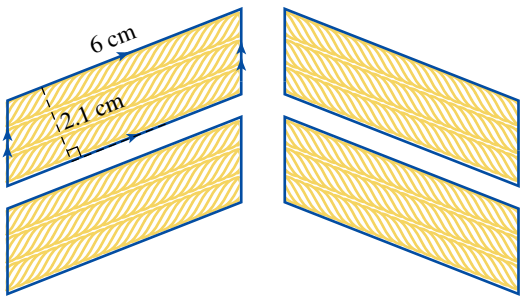
13 Calculate the base length of the give-way sign at right.



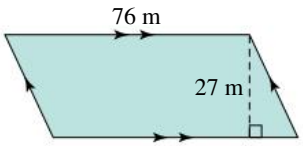
14 **a** Calculate the width of a rectangular sportsground if it has an area of 30 ha and a length of 750 m.

b The watering system at the sports-ground covers 8000 square metres in 10 minutes. How long does it take to water the sportsground?

15 Find the area of gold braid needed to make the four military stripes shown.



16 What is the area of the block of land in the figure at right?



17 MC Which statement about a parallelogram is false?

- **A** The opposite sides of a parallelogram are parallel.
- **B** The height of the parallelogram is perpendicular to its base.
- **C** The area of a parallelogram is equal to the area of the rectangle whose length is the same as the base and whose width is the same as the height of the parallelogram.
- **D** The perimeter of the parallelogram is given by the formula $P = 2(b + h)$.
- **E** The area of a parallelogram is given by the formula $A = bh$.

18 The base of a parallelogram is 3 times as long as its height. Find the area of the parallelogram, given that its height is 2.4 cm long.

19 A designer vase has a square base of side length 12 cm and four identical sides, each of which is a parallelogram. If the vertical height of the vase is 30 cm, find the total area of the glass used to make this vase. (Assume no waste and do not forget to include the base.)

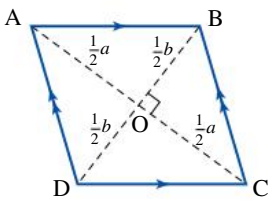
20 **a** Find the length of the base of a parallelogram whose height is 5.2 cm and whose area is 18.72 cm^2.

b Find the height of a parallelogram whose base is 7.5 cm long and whose area is 69 cm^2.

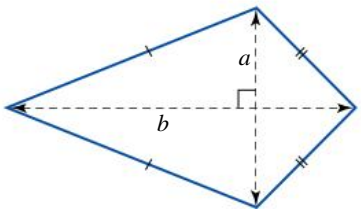
21 The length of the base of a parallelogram is equal to its height. If the area of the parallelogram is 90.25 cm^2, find its dimensions.

REASONING

22 If the diagonals of a rhombus bisect each other and intersect at right angles, use mathematical reasoning to show and explain that $A_{RH} = \frac{1}{2} \times a \times b$, where a and b are the lengths of the diagonals of the rhombus.



23 Using mathematical reasoning, determine the area of the kite below in terms of a and b.



PROBLEM SOLVING

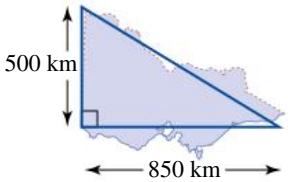
24 Show possible dimensions for each of the following shapes so that they each have an area of 36 cm^2.

a Rectangle **b** Parallelogram **c** Rhombus

25 A rectangular flower bed measures 20 m by 16 m. A gravel path 2 m wide surrounds it.

- **a** Draw a diagram representing the flower bed and path.
- **b** Calculate the area of the flower bed.
- **c** Calculate the area of the gravel path.
- **d** If gravel costs $5 per square metre, how much will it cost to cover the path?

26 The area of Victoria can be approximated using a right-angled triangle with the measurements shown at right.

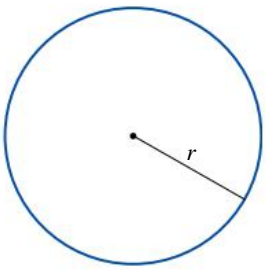


- **a** Estimate the area of Victoria by calculating the area of the triangle.
- **b** Use your atlas or the internet to compare your estimate with the actual area of Victoria.
- **c** Explain why the answer you obtained in part **a** can be regarded only as an estimate.
- **d** How could a more accurate estimate of the area of Victoria be made?

10.5 Area of a circle

- The area of a circle, A, can be found using the formula $A = \pi r^2$, where r is the radius of the circle.

eBookplus
Digital doc
Investigation
Area of a circle
doc-2321

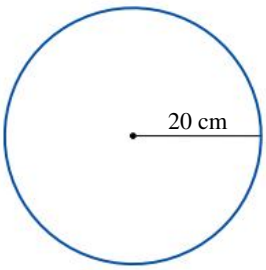


- The radius of a circle, r, is equal to a half of its diameter, D: $r = \frac{D}{2}$.

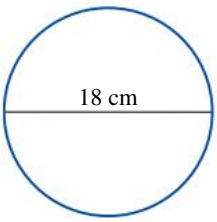
WORKED EXAMPLE 11

Find the area of each of the following circles correct to 2 decimal places.

a



b



THINK	WRITE
a 1 Write the formula for the area of a circle.	**a** $A = \pi r^2$
2 Substitute 20 for r and 3.14... for π.	$A = 3.14... \times 20^2$
3 Evaluate (square the radius first) and include the correct units.	$= 3.14... \times 400$ $= 1256.637...$ $= 1256.64 \text{ cm}^2$
b 1 Write the formula for the area of a circle.	**b** $A = \pi r^2$
2 We need radius, but are given the diameter. State the relation between the radius and the diameter.	$D = 18; r = D \div 2$
3 Halve the value of the diameter to get the radius.	$r = 18 \div 2$ $= 9$

4 Substitute 9 for r and 3.14... for π. $A = 3.14... \times 9^2$
$= 3.14... \times 81$

5 Evaluate (square the radius first) and include the correct units. $= 254.469...$
$= 254.47 \text{ cm}^2$

assess on

Exercise 10.5 Area of a circle

INDIVIDUAL PATHWAYS

REFLECTION
Think of a way to remember the difference between the formula for the circumference and the formula for the area of a circle.

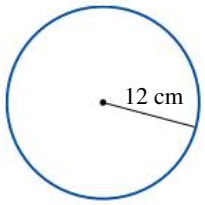
PRACTISE	CONSOLIDATE	MASTER
Questions: 1–3, 4a–d, 6, 9, 10	Questions: 1–3, 4a–f, 5, 6, 9, 10, 12	Questions: 1a, c, f, 2b, d, 3a, c, 4–12

Individual pathway interactivity int-4441 eBookplus

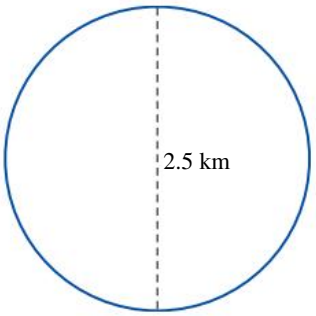
FLUENCY

1 WE11 Find the area of each of the following circles correct to 2 decimal places.

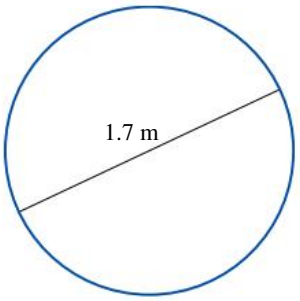
a



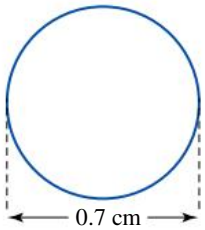
b



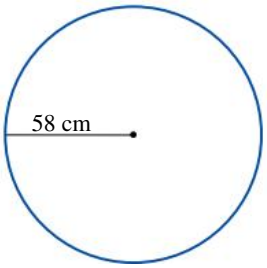
c



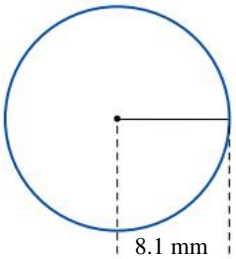
d



e



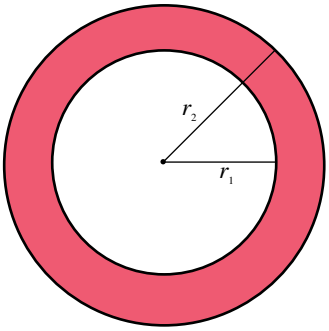
f



2 Find the area, correct to 2 decimal places, of:
- **a** a circle of radius 5 cm
- **b** a circle of radius 12.4 mm
- **c** a circle of diameter 28 m
- **d** a circle of diameter 18 cm.

UNDERSTANDING

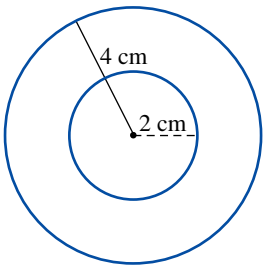
3 The word **annulus** is the Latin word for *ring*. An annulus is the shape formed between two circles with a common centre (called concentric circles). To find the area of an annulus, calculate the area of the smaller circle and subtract it from the area of the larger circle. Find the area of the annulus, correct to 2 decimal places, for each of the following sets of concentric circles.



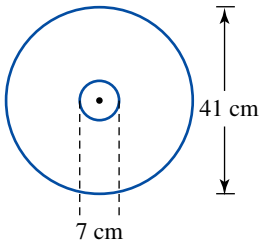
An annulus is the shaded area between the concentric circles.

r_1 = radius of smaller circle
r_2 = radius of larger circle
$\text{Area}_{\text{annulus}} = \pi r_2^2 - \pi r_1^2$

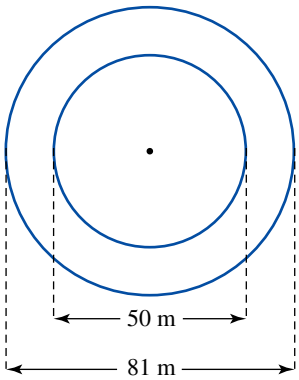
a



b

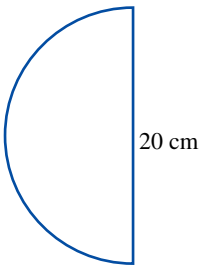


c

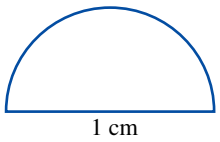


4 Find the area of each of the following shapes. Give your answers to 2 decimal places.

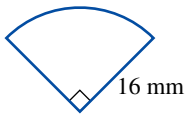
a



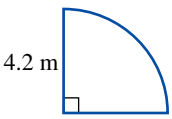
b



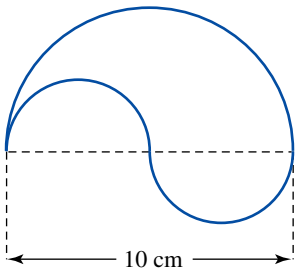
c



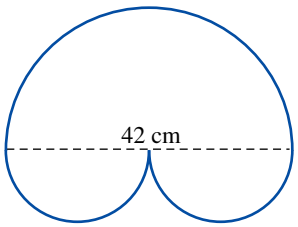
d



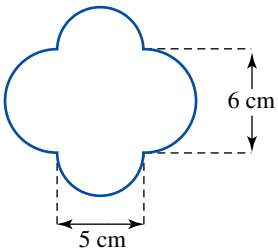
e



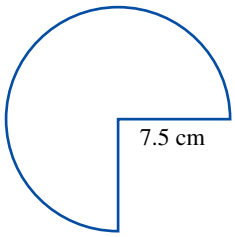
f



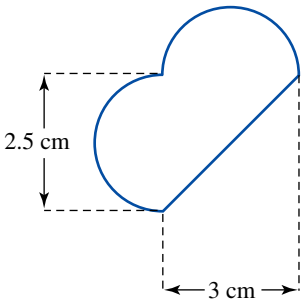
g



h



i



5 Find the minimum area of aluminium foil, correct to 2 decimal places, that could be used to cover the top of the circular tray with diameter 38 cm.

6 What is the area of material in a circular mat of diameter 2.4 m? Give your answer correct to 2 decimal places.

7 How many packets of lawn seed should Joanne buy to sow a circular bed of diameter 27 m, if each packet of seed covers 23 m^2?

8 A landscape gardener wishes to spread fertiliser on a semicircular garden bed that has a diameter of 4.7 m. How much fertiliser is required if the fertiliser is applied at the rate of 20 g per square metre? Give your answer to the nearest gram.

REASONING

9 Investigate what happens to the diameter of a circle if its area is:

- **a** doubled
- **b** quadrupled
- **c** halved
- **d** changed by factor of n.

10 Investigate what happens to the circumference of a circle if its area is:

- **a** doubled
- **b** quadrupled
- **c** halved
- **d** changed by factor of n.

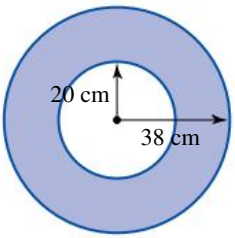
PROBLEM SOLVING

11 The total area of wood used to make a set of six identical round coasters is 425 cm^2.

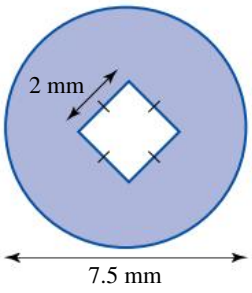
- **a** Calculate the area of the wood used in each coaster.
- **b** Calculate the radius of each coaster correct to 2 decimal places.
- **c** An 8.5 cm wide cylindrical coffee mug is placed on one of the coasters. Use your answer to part **b** to decide whether it will fit entirely within the coaster's surface.

12 Calculate the shaded area in each of the following shapes.

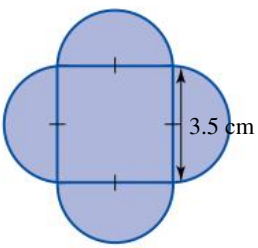
a



b

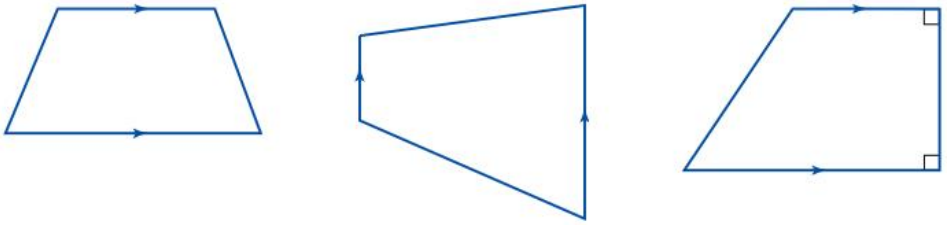


c

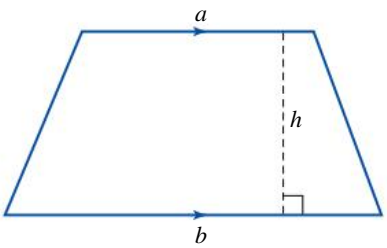


10.6 Area of trapeziums

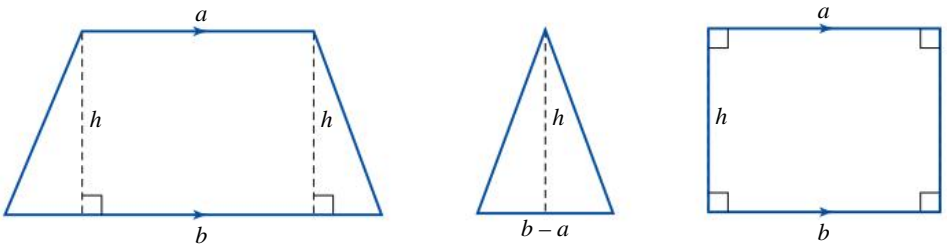
- A **trapezium** is a quadrilateral with one pair of parallel unequal sides.
- The following figures are all trapeziums.



- The height of the trapezium is perpendicular to each of its parallel bases.



To determine the area of a trapezium, draw two lines to create two triangles and one rectangle as shown.



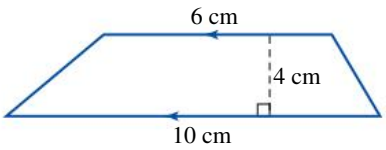
Placing the two triangles together creates one triangle and one rectangle.

The total area of the two shapes, A, will give $A = a \times h + \frac{1}{2} \times h \times \times (b - a)$.
Simplifying this gives $A = \frac{1}{2}(a + b) \times h$.

- The area of a trapezium, A, is given by the formula $A = \frac{1}{2}(a + b) \times h$, where a and b are the lengths of parallel sides and h is the height of the trapezium.

WORKED EXAMPLE 12

Find the area of the trapezium at right.



THINK	WRITE
1 Write the formula for the area of the trapezium.	$A = \frac{1}{2}(a + b) \times h$
2 Identify the values of a, b and h. *Note:* It does not matter which of the parallel sides is a and which one is b, since we will need to add them together.	$a = 10$, $b = 6$ and $h = 4$

3 Substitute the values of a, b and h into the formula. $A = \frac{1}{2} \times (10 + 6) \times 4$

4 Evaluate (work out the brackets first) and include the correct units. $A = \frac{1}{2} \times 16 \times 4$

$= 32 \text{ cm}^2$

Exercise 10.6 Area of trapeziums

INDIVIDUAL PATHWAYS

REFLECTION
Can you think of a strategy to help you remember the formula for finding the area of a trapezium?

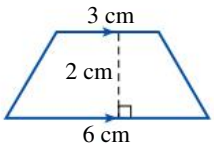
PRACTISE	CONSOLIDATE	MASTER
Questions: 1–4, 9	Questions: 1b, d, e, f, 2, 3, 5, 7, 9, 10	Questions: 1b, e, f, 2, 3–10

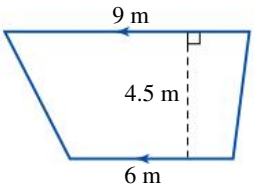
Individual pathway interactivity int-4442 eBookplus

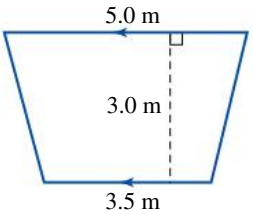
eBookplus Digital doc Investigation Area of a trapezium doc-2320

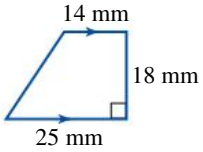
FLUENCY

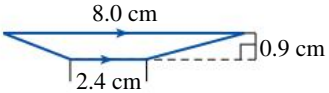
1 WE12 Find the area of each of the following trapeziums.

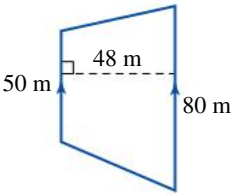
a 

b 

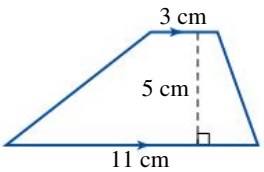
c 

d 

e 

f 

2 MC Which of the following is the correct way to calculate the area of the trapezium shown?



A $\frac{1}{2} \times (3 + 5) \times 11$

B $\frac{1}{2} \times (3 + 5 + 11)$

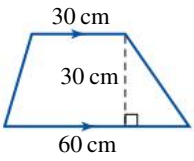
C $\frac{1}{2} \times (11 - 3) \times 5$

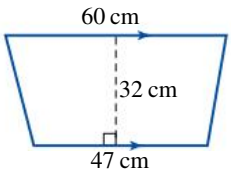
D $\frac{1}{2} \times (11 + 5) \times 3$

E $\frac{1}{2} \times (3 + 11) \times 5$

UNDERSTANDING

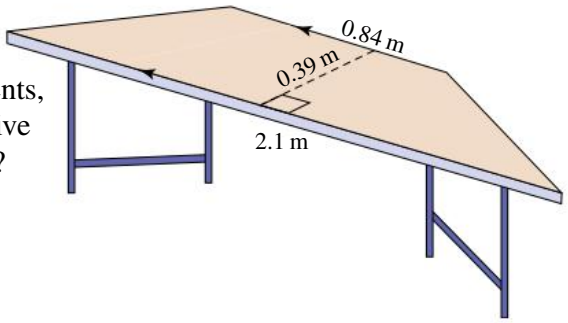
3 A dress pattern contains these two pieces.



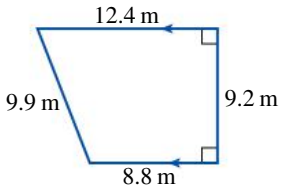


Find the total area of material needed to make both pieces.

4 A science laboratory has four benches with the dimensions shown at right. What would be the cost, to the nearest 5 cents, of covering all four benches with a protective coating that costs $38.50 per square metre?



5 Stavros has accepted a contract to concrete and edge the yard, the dimensions of which are shown in the figure at right.



a What will be the cost, to the nearest 5 cents, of concreting the yard if concrete costs $28.00 per square metre?

b The yard must be surrounded by edging strips, which cost $8.25 per metre. Find:

i the cost of the edging strips, to the nearest 5 cents

ii the total cost of materials for the job.

6 The side wall of this shed is in the shape of a trapezium and has an area of 4.6 m^2. Find the perpendicular distance between the parallel sides if one side of the wall is 2.6 m high and the other 2 m high.

7 MC Two trapeziums have corresponding parallel sides of equal length. The height of the first trapezium is twice as large as the height of the second. The area of the second trapezium is:

A twice the area of the first trapezium

B half the area of the first trapezium

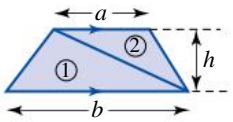
C quarter of the area of the first trapezium

D four times the area of the first trapezium

E impossible to say

REASONING

8 The formula for the area of a trapezium can be proved by dividing it into two triangles as shown.



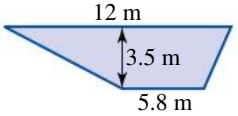
a Find the area of triangle 1 in terms of the pronumerals shown in the diagram.

b Find the area of triangle 2 in terms of the pronumerals shown in the diagram.

c Find the area of the trapezium by adding the areas of the two triangles together.

PROBLEM SOLVING

9 A section of a garden is in the shape of a trapezium as shown.



a Calculate the area of this section of the garden.

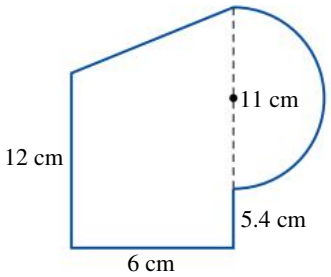
b This section of the garden is to be covered with mulch. According to the information on the pack, each bag contains enough mulch to cover 5 m^2 of surface. How many bags of mulch are needed to complete the job?

c If each bag costs $8.95, calculate the total cost of the mulch.

10 Calculate the area of the shape in the figure as:

a an exact value

b rounded to 5 significant digits.

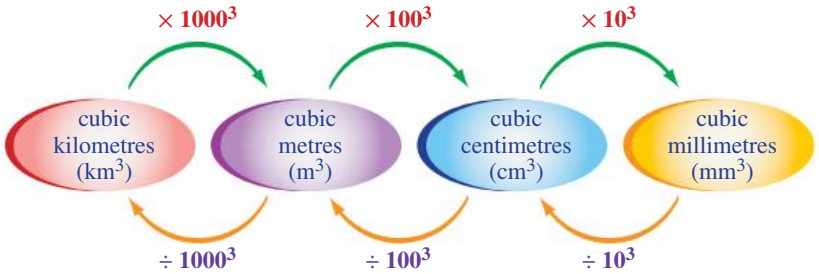


eBookplus
Digital doc
WorkSHEET 10.1
doc-2322

10.7 Volume of prisms and other solids

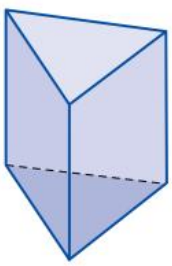
Volume

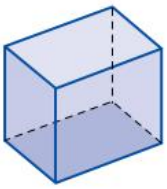
- **Volume** is the amount of space inside a three-dimensional object.
- Volume is measured in cubic units such as mm^3, cm^3 or m^3.
- Volume units can be converted using the chart below.

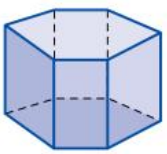


Prisms

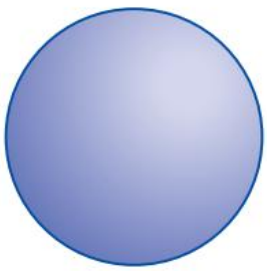
- **Prisms** are solid shapes with identical opposite polygonal ends which are joined by straight edges. They are three-dimensional objects that can be cut into identical slices, called cross-sections.
- Prisms are named according to the shape of their cross-section. The objects below are all prisms.

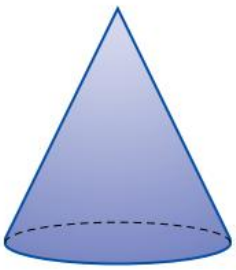

Triangular prism

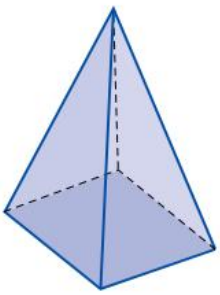

Rectangular prism


Hexagonal prism

- Objects that do not have a uniform cross-section can not be classified as prisms. For example, the objects below are not prisms.

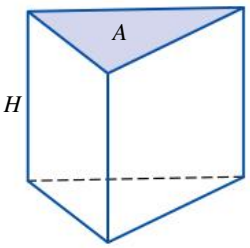

Sphere


Cone

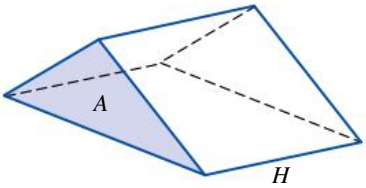

Square pyramid

- The volume of any prism is given by the formula $V = A \times H$, where A is the cross-sectional area of a prism and H is the height of a prism.
- The volume of a rectangular prism is given by the formula $V = lwh$, where l is the length, w is the width and h is the height of the prism.
- A rectangular prism can be cut to form two equal triangular prisms.
- The volume of a cube is given by the formula $V = l^3$, where l is the side length of a cube.
- The height of the prism is not necessarily the height of the object in a true sense of the word. It is just the dimension perpendicular to the cross-section.

eBookplus
Interactivity
Volumes of prisms
int-2754



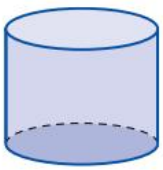
$V = A \times H$
The base of this prism, A, *is* its cross-section; H *is* the height of the prism.

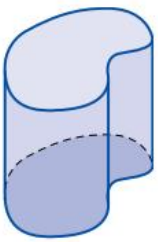


The base of this prism *is not* its cross-section. H represents the depth (or length) of the prism; A is its cross-section.

Volume of solids with uniform cross-section that are not prisms

- The objects with a uniform cross-section whose ends are not polygons (that is, those whose ends are not joined by straight edges) can not be classified as prisms. For example, the shapes below are not prisms, even though they have uniform cross-sections.



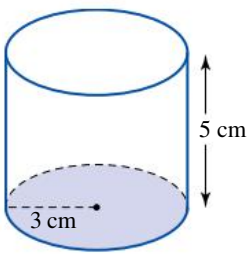


- The formula $V = AH$, where A is the cross-sectional area and H is the dimension perpendicular to it, will give the volume of any solid with uniform cross-section, even if it is not a prism.

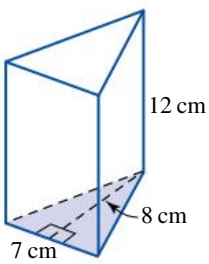
WORKED EXAMPLE 13

Find the volume of each of the following. Give answers correct to 2 decimal places where appropriate.

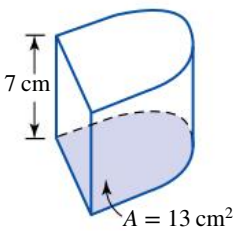
a



b



c



THINK		WRITE	
a	1 Write the formula for the volume of the given shape.	a	$V = A \times H$
	2 Identify the shape of the cross-section and, hence, write the formula to find its area		$A_{\text{circle}} = \pi r^2$
	3 State the value of r.		$r = 3$
	4 Substitute the value of r into the formula and evaluate.		$A = 3.14... \times 3^2$ $= 28.27...\ \text{cm}^2$
	5 State the value of H.		$H = 5$
	6 To find the volume, multiply the cross-sectional area by the height and include the correct units.		$V = 28.27... \times 5$ $= 141.371...$ $= 141.37\ \text{cm}^3$
b	1 Write the formula for the volume of a prism.	b	$V = A \times H$
	2 Identify the shape of the cross-section and, hence, write the formula to find its area.		$A_{\text{triangle}} = \frac{1}{2}bh$
	3 State the values of the variables. (*Note:* h is the height of the triangle, *not* of the prism.)		$b = 7, h = 8$
	4 Substitute the values of b and h into the formula and evaluate.		$A = \frac{1}{2} \times 7 \times 8$ $= 28\ \text{cm}^2$
	5 State the value of H, the height of the prism.		$H = 12$
	6 To find the volume of the prism, multiply the cross-sectional area by the height and include the correct units.		$V = 28 \times 12$ $= 336\ \text{cm}^3$
c	1 Write the formula for the volume of the given shape.	c	$V = A \times H$
	2 State the values of the cross-sectional area and the height of the shape.		$A = 13, H = 7$
	3 Multiply the cross-sectional area by the height and include the correct units.		$V = 13 \times 7$ $= 91\ \text{cm}^3$

Exercise 10.7 Volume of prisms and other solids

assessON

INDIVIDUAL PATHWAYS

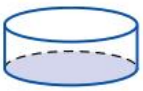
PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2a, d, g, j, 3, 4, 7, 10	Questions: 1, 2b, e, h, k, 3, 5, 6, 8, 10, 11	Questions: 1, 2b, c, i, l, 4–12

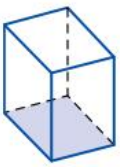
REFLECTION
How can you say if a solid has a uniform cross-section?

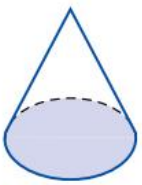
Individual pathway interactivity int-4443 eBookplus

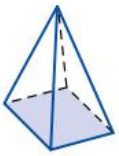
FLUENCY

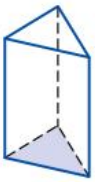
1 Which of the three-dimensional shapes below are prisms?

a 

b 

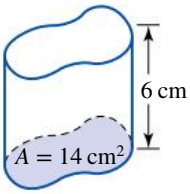
c 

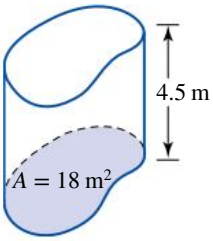
d 

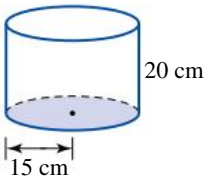
e 

eBookplus
Digital doc
SkillSHEET
Volume of cubes and rectangular prisms
doc-6959

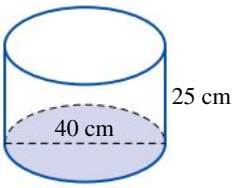
2 WE13 Find the volume of each of the following. Give your answers to 2 decimal places where appropriate.

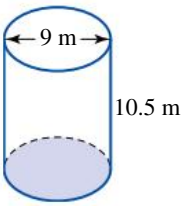
a 

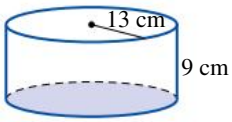
b 

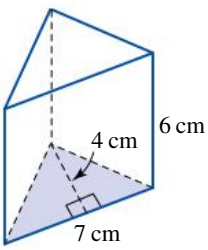
c 

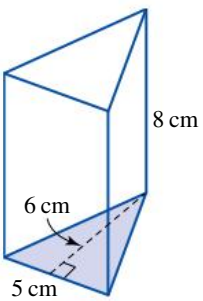
eBookplus
Digital doc
Spreadsheet
Volume of a prism
doc-2324

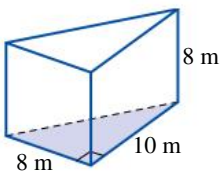
d 

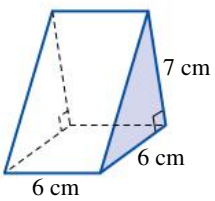
e 

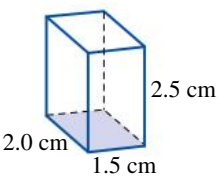
f 

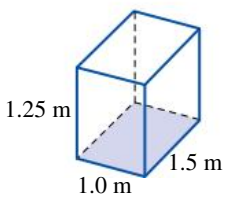
g 

h 

i 

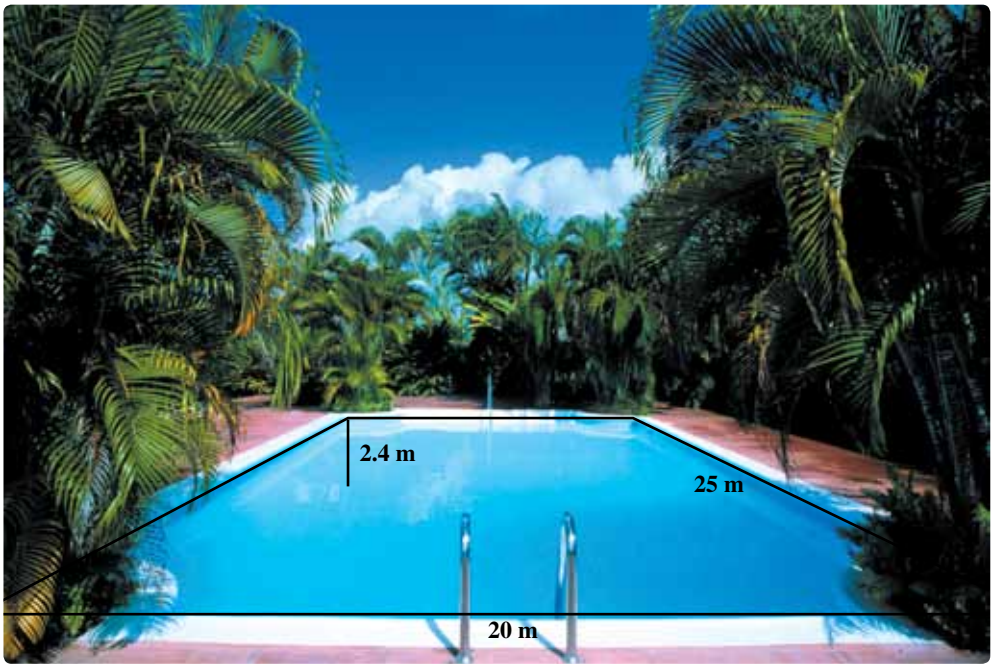
j 

k 

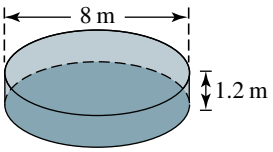
l 

UNDERSTANDING

3 What volume of water will a rectangular swimming pool with dimensions shown in the following photograph hold if it is completely filled? The pool has no shallow or deep end. It is all the same depth.

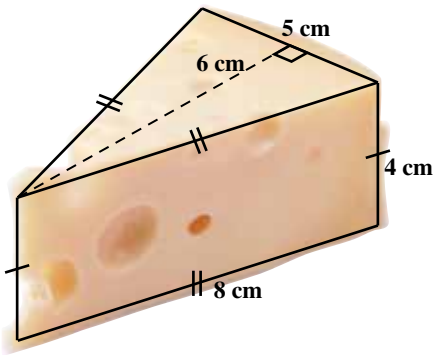


4 How many cubic metres of cement will be needed to make the cylindrical foundation shown in the figure at right? Give your answer to 2 decimal places.

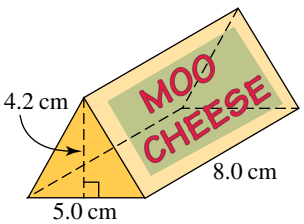


5 What are the volumes of these pieces of cheese?

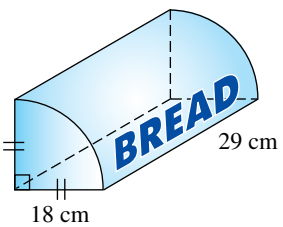
a



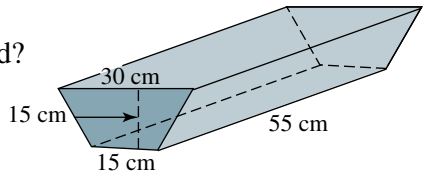
b



6 What is the volume of the bread bin shown at right? Give your answer to the nearest whole number.

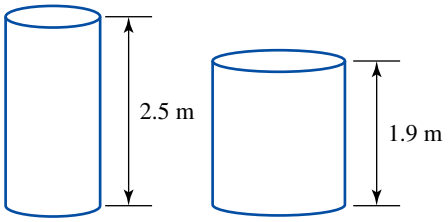


7 How much water will this pig trough, with dimensions shown in the figure at right, hold if it is completely filled? Give your answer in litres correct to 1 decimal place. (*Hint:* 1 litre = 1000 cm^3.)



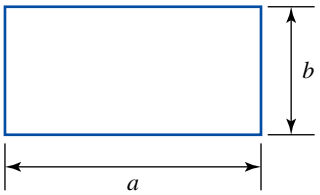
REASONING

8 A rectangular metal sheet with length 2.5 m and width 1.9 m is used to make a cylinder. The metal sheet can be rolled on its length or on its width. The two cylinders that can be formed are shown in the diagram.



- **a** Calculate the radius to one decimal place for:
  - **i** the taller cylinder
  - **ii** the shorter cylinder.
- **b** Calculate the two volumes.
- **c** Which volume is larger? Explain why this is the case.

9 Consider the same situation as in the previous question but this time the metal sheet has length a and width b ($a > b$).



- **a** Write a formula for the radius of base of each cylinder.
- **b** Write a formula for each of the volumes.
- **c** Explain which volume is always going to be larger.
- **d** Does this explain your answer in the previous question?

PROBLEM SOLVING

10 A new (unsharpened) pencil is in the shape of a hexagonal prism. If the area of the hexagonal end is 80 mm^2 and the volume of the whole pencil is 14.4 cm^3, calculate the length of the pencil.

11 Consider a set of three food containers of different sizes, each of which is in the shape of a rectangular prism.

The smallest container is 6 cm long, 4 cm wide and 3 cm high. All the dimensions of the medium container are double those of the smallest one; all the dimensions of the largest container are triple those of the smallest one.

- **a** Calculate the volume of the smallest container.
- **b** State the dimensions and hence calculate the volume of:
  - **i** the medium container
  - **ii** the largest container.
- **c** Calculate the ratio of the volumes of:
  - **i** the smallest container to the medium container
  - **ii** the smallest container to the largest container.
- **d** Consider your answers to part **c** and use them to copy and complete the following: If all dimensions of a rectangular prism are increased by a factor of n, the volume of the prism is increased by a factor of _____.

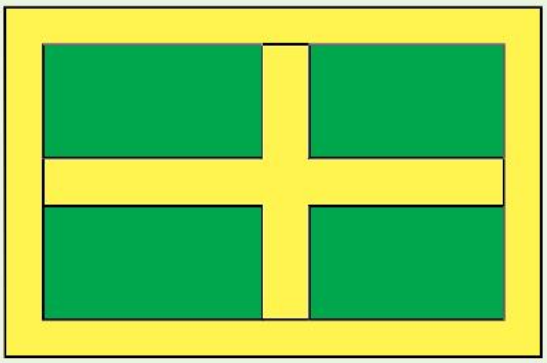
12 A set of cabin tents is being made for a historical documentary. The dimensions of each tent are shown.

a Calculate the area of material required to make each tent, including the floor.

b Calculate the amount of space inside the tent.

CHALLENGE 10.2

Below is a sketch of a city park. It is 24 m long and 18 m wide. The footpaths are 3 m wide. What is the surface area of the footpaths?



10.8 Time

Time — an introduction

- Time is one of our most useful systems of measurement. We can use it to work out how long we have been doing something, or how long we have until we must do something. We can use it to make future arrangements.
- Time is something we all use, every day.
- Time is divided into units. There are:

60 seconds	in	1 minute
60 minutes	in	1 hour
24 hours	in	1 day
7 days	in	1 week
2 weeks	in	1 fortnight
about 4 weeks	in	1 month
12 months	in	1 year
about 365 days	in	1 year
10 years	in	1 decade
100 years	in	1 century
1000 years	in	1 millennium.

- We also use the word time to refer to an instant (for example, 3 o'clock) rather than a period.

- A clock or watch can display the time in one of two ways.
  - Some clocks display the time in analogue form using hour, minute and second hands that move continuously as they point to numbers on the clock face.
  - Other clocks and watches display a set of digits that change in steps as time passes.

Time calculations

WORKED EXAMPLE 14

How many minutes are there in $4\frac{1}{4}$ hours?

THINK	WRITE
1 Convert the mixed number to an improper fraction.	$4\frac{1}{4}$ hours $= \frac{4 \times 4 + 1}{4}$ $= \frac{17}{4}$ hours
2 Multiply the improper fraction by the number of minutes in 1 hour.	$\frac{17}{4}$ hours $= \frac{17}{4} \times 60$ minutes $= \frac{1020}{4}$ minutes
3 Evaluate.	$= 255$ minutes

WORKED EXAMPLE 15

How many minutes are there in 5 days?

THINK	WRITE
1 Convert the number of days to hours, that is, multiply 5 by 24 hours.	5 days $= 5 \times 24$ hours
2 Evaluate.	$= 120$ hours
3 Convert the number of hours to minutes, that is, multiply 120 by 60 minutes.	120 hours $= 120 \times 60$
4 Evaluate.	$= 7200$ minutes

WORKED EXAMPLE 16

Change the following into hours and minutes.
a 300 minutes **b 425 minutes**

THINK	WRITE
a 1 Convert minutes to hours, that is, divide 300 by 60.	a 300 minutes $= 300 \div 60$
2 Evaluate.	$= 5$ hours
b 1 Convert minutes to hours, that is, divide 425 by 60.	b 425 minutes $= 425 \div 60$ $= \frac{425}{60}$
2 Evaluate.	$= 7$ remainder 5
3 Write the answer in hours and minutes.	425 minutes $= 7$ hours 5 minutes

- The suffixes am and pm are used to indicate morning and afternoon, and are derived from *ante meridiem* (meaning before midday) and *post meridiem* (after midday).
  Note: Noon (or midday) may be written as 12.00 pm, and midnight may be written as 12.00 am.
- One way to find time differences is to mentally calculate how long it is to the next hour, then to the next noon or midnight. The following worked example illustrates this technique.

WORKED EXAMPLE 17

Find the time difference between:
a 7.15 pm and 10.25 pm
b 2.50 am and 8.20 pm
c 3.40 am on Tuesday and 5.10 pm on Wednesday (next day).

THINK

a
1. Construct a vertical column starting at 7.15 pm and ending at 10.25 pm. Set up two columns with the headings 'hours' and 'minutes' next to the first time.
2. Write key times between the start and finish times.
3. Calculate the times added to get each key time, and write them under the hours and minutes headings as shown.
4. Add the time differences to find the total time difference.
5. Convert any minute values over 60 to hours and minutes and adjust the answer.
6. Answer the question.

b Repeat steps 1 to 6 in part **a**.

c Repeat steps 1 to 6 in part **a**.

WRITE

a

	Hours	Minutes
7.15 pm		
8.00 pm		45
10.25 pm	2	25
Total time	2	70
	= 3	10

The time difference is 3 hours 10 minutes.

b

	Hours	Minutes
2.50 am		
3.00 am		10
12.00 pm	9	00
8.20 pm	8	20
Total time	17	30

The time difference is 17 hours 30 minutes.

c

	Hours	Minutes
3.40 am Tuesday		
4.00 am Tuesday		20
4.00 am Wednesday	24	00
12.00 pm Wednesday	8	00
5.10 pm Wednesday	5	10
Total time	37	30

The time difference is 37 hours 30 minutes.

Exercise 10.8 Time

assessON

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7, 8a, d, g, 9a, d, g, 10, 11, 12a, c, 13–16, 24, 26, 27	Questions: 1–7, 8b, e, h, 9b, e, h, 10a, d, g, i, l, 11, 12b, d, 13–18, 22, 24–28	Questions: 1–5, 6b, e, h, 7c, f, i, 8c, f, i, 9c, g, h, i, 10c, h, i, k, 11a, g, k, 12b, d, 13–29

REFLECTION
What strategies will you use to help you remember how to convert between different units of time?

Individual pathway interactivity int-4444 eBookplus

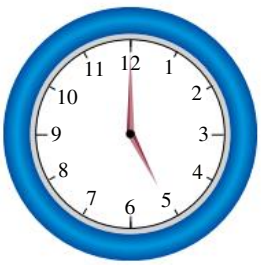
FLUENCY

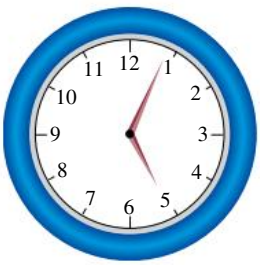
1 Match the following activities with the most likely amount of time.

a	a cricket test match	1 year
b	writing your name	10 seconds
c	eating breakfast	15 minutes
d	building a house	5 days
e	flying time from Melbourne to Hong Kong	7 hours
f	being in Year 7	6 months

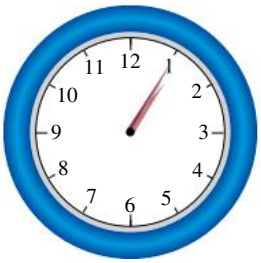
eBookplus
Digital doc
SkillSHEET
Reading the time (from analogue clocks)
doc-6960

2 What are the times shown on each of the following analogue clocks?

a


b


c

d


e

f

3 For each of the following, draw a 12-hour clock face and show the time.

a 8.20 am	**b** 8.45 pm	**c** 10.50 am
d 12.00 am	**e** 11.05 pm	**f** 5.11 pm
g 7.32 pm	**h** 9.24 am	**i** 11.16 am

4 Today we use clocks and watches to tell the time. Name two other devices that have been used in the past to measure time.

5 Explain the difference between 2 hours 25 minutes and 2.25 hours.

6 WE14,15 How many minutes are there in each of the following periods?

a 2 hours **b** $2\frac{1}{2}$ hours **c** $3\frac{1}{4}$ hours **d** $\frac{1}{2}$ hour

e $\frac{3}{4}$ hour **f** $7\frac{3}{4}$ hours **g** 1 day **h** 9 days

7 WE16 Change the following to hours and minutes.

a 200 minutes **b** 185 minutes **c** 160 minutes

d 230 minutes **e** 405 minutes **f** 95 minutes

g 610 minutes **h** 72 minutes **i** 305 minutes

8 Change the following to minutes.

a 1 hour 15 minutes **b** 2 hours 10 minutes **c** 1 hour 50 minutes

d 4 hours 25 minutes **e** 7 hours 35 minutes **f** 3 hours 12 minutes

g $16\frac{3}{4}$ hours **h** $5\frac{1}{2}$ hours **i** $6\frac{1}{4}$ hours

9 How many:

a days in a week? **b** weeks in a year?

c seconds in a minute? **d** seconds in an hour?

e seconds in a day? **f** hours in a day?

g hours in a year? **h** minutes in a day?

i minutes in a year?

Note: Assume 1 year = 365 days.

10 What is the time:

a 1 hour after 4.00 pm?

b 1 hour before 4.00 pm?

c 1 hour after 5.30 am?

d 1 hour 20 minutes after half past 10 am?

e 1 hour 20 minutes before 10.00 am?

f 3 hours 18 minutes after 2.00 pm?

g 2 hours 30 minutes before 4.30 am?

h 4 hours 35 minutes after quarter to 10 am?

i 4 hours 35 minutes before 10.05 pm?

j 5 hours 27 minutes after 1.08 am?

k 5 hours 27 minutes before 1.08 am?

l 1 hour 10 minutes before 11.30 am?

11 WE17a,b Find the time difference between:

a 8.20 pm and 8.35 pm **b** 4.15 pm and 5.20 pm

c 7.15 am and 8.28 am **d** 5.17 am and 6.32 am

e 9.15 pm and 10.08 pm **f** 3.16 pm and 5.09 pm

g 11.28 pm and midnight **h** 9.21 am and 1.06 pm

i 11.10 am and 4.25 pm **j** 6.05 am and 6.05 pm

k half past 6 pm and quarter to 3 am.

12 WE17c Find the time difference between:

a 7.20 am on Monday and 6.30 pm the next day, Tuesday

b 4.38 am on Saturday and 1.25 pm the next day, Sunday.

c 8.45 pm on Wednesday and 10.16 am the next day, Thursday

d 1.20 pm on Wednesday and 9.09 am the first Friday.

eBookplus
Digital doc
Spreadsheet
Time differences
doc-3445

13 **MC** 225 minutes is the same as:

A 2 hours 25 minutes **B** 2 hours 15 minutes **C** 3 hours 45 minutes
D 3 hours 25 minutes **E** 2 hours 45 minutes

14 **MC** If the time was 8.45 pm, what would be the time 5 hours 20 minutes later?

A 1.05 am **B** 1.05 pm **C** 2.05 pm **D** 2.05 am **E** 1.10 pm

15 **MC** If a train takes 2 hours and 18 minutes to arrive at its destination and it arrives at 6.03 pm, at what time did it leave?

A 3.45 pm **B** 8.21 pm **C** 4.15 pm **D** 3.21 pm **E** 3.45 am

UNDERSTANDING

16 If it takes Joanne 16 minutes to write one page of a letter, how long will it take her to write a letter of three pages?

17 Mathew spends half an hour doing homework each week night. How many hours of homework has he done after 3 weeks?

18 If the time is now 7.55 am in Melbourne, what is the time in Adelaide if South Australian time is half an hour behind Victorian time?

19 If the time is now 8.30 am in Melbourne, what is the time in Perth if Western Australia is two hours behind Victoria?

20 If the time is 9.45 pm in Perth, what is the time in Melbourne? (See question **19**.)

21 If the time is 10.05 pm in Adelaide, what is the time in Melbourne? (See question **18**.)

22 If Susan takes 25 minutes to walk to the station, waits for the train for 7 minutes, travels on the train for 12 minutes and then takes 8 minutes to walk to her friend's house, how long has she taken altogether?

23 If James spends 35 minutes on Friday, 2 hours and 12 minutes on Saturday and $1\frac{1}{4}$ hours on Sunday to complete his assignment, how much time did he take altogether?

REASONING

24 In a mirror reflection, the time on an analogue clock face appears to be 25 minutes to 2. What time is it?

25 When would an analogue clock and its mirror image appear exactly the same? Explain your answer.

PROBLEM SOLVING

26 You have a training schedule for tennis. If you follow the schedule, how long will you train each week?

27 You are allowed to stay up until 10:30 pm on Saturday night. What time will you need to start a DVD if you watch:

a *Finding Nemo* (97 minutes)?

b *Aladdin* (86 minutes)?

c *Star Wars* (124 minutes)?

28 A classmate attempted to calculate some time differences. Explain where he may have gone wrong in his calculations and then find the correct answers.

a
$$\begin{array}{r} 2240 \\ -2005 \\ \hline 235 \text{ min} \end{array}$$

b
$$\begin{array}{r} 9{:}22 \\ -7{:}38 \\ \hline 1{:}84 \end{array}$$

29 The battery on your new MP3 player is supposed to last for 10 hours. If you listen to music on your MP3 player for 40 minutes per day, how many days will the battery last until you have to charge it again?

10.9 24-hour clock and time zones

eBookplus
Interactivity
Time zones
int-0006

- Sometimes the time is given using a 24-hour system or 24-hour clock.
- The hours are numbered from 1 to 24, beginning at midnight.
- The times from midnight to 12.59 pm look very similar to the 12-hour time. At 1.00 pm you need to add 12 hours, making the time 1300 hours.
- Times in the 24-hour system are written as 'fourteen hundred hours' (1400) or 'twenty-three thirty hours' (2330).
- Notice that in 24-hour time, we always use 4 digits. Some examples are given in the table below.

12-hour time	24-hour time
6.00 am	0600 hours
10.00 am	1000 hours
12.30 pm	1230 hours
2.00 pm	1400 hours (add 12 hours to 2.00)
8.00 pm	2000 hours (add 12 hours to 8.00)
11.30 pm	2330 hours (add 12 hours to 11.30)

WORKED EXAMPLE 18

Find the difference in hours and minutes between the following 24-hour times:
a 0635 and 2150 **b 1055 and 1543.**

THINK

a 1 Set up two columns with the headings 'hours' and 'minutes'.

2 Write the values, putting the highest hour value first.

3 Calculate by subtraction.

4 Write as hours and minutes.

WRITE

a

Hours	Minutes
21	50
06	35
15	15

15 hours 15 minutes

b 1 Set up two columns with the headings 'hours' and 'minutes'.

2 Write the values, putting the highest hour value first.

3 You cannot take 55 minutes from 43 minutes. Convert 1 hour into 60 minutes and add it to the minutes column. Then rewrite 15 43 as 14 103.
(Using 14 103 helps us to do the subtraction, but it is not a real time.)

4 Calculate by subtraction.

5 Write as hours and minutes.

b

Hours	Minutes
15	43
10	55
14	103
10	55
4	48

4 hours 48 minutes

Time zones

- Time changes as we travel around the world. In Europe it is night time when in Australia it is day and it is day time in Europe when we have night.
- To help everyone know what time it is in different countries, the world is divided into 24 time zones. (See the map showing world time zones on page 320.)

Time in Perth	Time in Melbourne	Time in Jakarta
3.00 pm	5.00 pm	2.00 pm
6.00 pm	8.00 pm	5.00 pm
10.00 am	Midday	9.00 am
10.00 pm	Midnight	9.00 pm

- If we travel east, we need to set our watches ahead.
- If we travel west, we need to set our watches back.
- Australia is divided into three time zones.
  - Queensland, New South Wales, the ACT, Victoria and Tasmania observe Eastern Standard Time (EST).
  - The Northern Territory and South Australia observe Central Standard Time (CST), which is $\frac{1}{2}$ hour behind EST.
  - Western Australia observes Western Standard Time (WST), which is 2 hours behind EST.
- The time in England is called Greenwich Mean Time (GMT) and is 10 hours behind EST.

eBookplus
Digital doc
Spreadsheet
Time zones
doc-3449

Time in London (GMT)	Time in Jakarta	Time in Perth	Time in Melbourne (EST)
7.00 am	2.00 pm	3.00 pm	5.00 pm
4.00 pm	11.00 pm	Midnight	2.00 am (next day)
Midday	7.00 pm	8.00 pm	10.00 pm
Midnight	7.00 am	8.00 am	10.00 am
3.00 am	10.00 am	11.00 am	1.00 pm

WORKED EXAMPLE 19

a What is the time in Melbourne if the time in London (GMT) is 8.00 am?
b What is the time in London (GMT) if the time in Melbourne is 8.00 am?

THINK	WRITE
a 1 Melbourne is 10 hours ahead of London. Add 10 hours to the London time.	a London time: 8.00 am Melbourne time: 8.00 am + 10 hours
2 Calculate.	It is 6.00 pm.
b 1 London is 10 hours behind Melbourne. Subtract 10 hours from the Melbourne time.	b Melbourne time: 8.00 am London time: 8.00 am − 10 hours
2 Calculate.	It is 10.00 pm the previous day.

Daylight-saving time

- Many countries around the world have daylight-saving time during summer so that people can make the most of the warm weather.
- When daylight-saving time begins, clocks are turned forward by 1 hour at 2.00 am.
- At the end of daylight-saving time, the clocks are then turned back by 1 hour at 3.00 am.
- In Australia, the states Queensland and Western Australia, and the Northern Territory do not observe daylight-saving time, while the ACT and other states do.
- During daylight-saving, the time in Tasmania, Victoria, New South Wales and ACT is known as Eastern Daylight-Saving Time (EDT). The time in South Australia is known as Central Daylight-Saving Time (CDT).
- During daylight-saving time:
  - Queensland (EST) is 1 hour behind EDT.
  - Northern Territory (CST) is $1\frac{1}{2}$ hours behind EDT.
  - South Australia (CDT) is $\frac{1}{2}$ hour behind EDT.
  - Western Australia (WST) is 3 hours behind EDT.

WORKED EXAMPLE 20

You call your friend in Darwin on Christmas Day from your home in Sydney. It is 7 pm in Sydney. What time is it in Darwin?

THINK	WRITE
1 Because it is Christmas Day (the middle of summer), Sydney time will be at EDT, which is $1\frac{1}{2}$ hours ahead of Darwin time (CST).	Sydney time is EDT and Darwin time is CST. The time in Sydney is $1\frac{1}{2}$ hours ahead of the time in Darwin.
2 Subtract $1\frac{1}{2}$ hours from the Sydney time to give the time in Darwin.	Time in Darwin = 7 pm − 1 h 30 min = 5.30 pm

Exercise 10.9 24-hour clock and time zones

assessON

INDIVIDUAL PATHWAYS

PRACTISE
Questions:
1, 2a, d, g, j, 3a, d, g, j, 4a, c, e, g, i, 5–11, 12a, d, g, j, 13a, d, g, j, 14, 16, 17, 21, 24

CONSOLIDATE
Questions:
1, 2b, e, h, k, 3b, e, h, k, 4b, d, f, h, 5–11, 12b, e, h, k, 13b, e, h, k, 14, 15, 17, 18, 20, 22, 24, 25

MASTER
Questions:
1, 2c, f, i, l, 3c, f, i, l, 4a, e, h, 5–11, 12c, f, i, l, 13c, f, i, l, 14, 15, 19–25

Individual pathway interactivity int-4445 eBookplus

REFLECTION
Why is it important that arrival and departure times at airports are in 24-hour time?

FLUENCY

1 Match the 12-hour times with the appropriate 24-hour times.

10.12 am	1212	Noon	1120
12.12 pm	1200	11.20 pm	0000
Midnight	1012	11.20 am	2320

eBookplus
Digital doc
SkillSHEET
24-hour clock
doc-6961

2 Write each of the following times using the 24-hour clock.

a 10.20 am **b** 11.30 am **c** 5.10 am
d 4.15 am **e** 5.15 pm **f** 6.30 pm
g 8.30 am **h** 8.40 pm **i** Midday
j 11.30 pm **k** 4.35 am **l** 2.30 pm

3 Convert each of the following 24-hour times to 12-hour time.

a 2315 **b** 1310 **c** 0815
d 0115 **e** 1818 **f** 1220
g 0005 **h** 1005 **i** 2005
j 1135 **k** 1520 **l** 1414

4 WE18 Find the difference between each of the following times. (The first time is the earlier time in each case.)

a 1005 and 2315 **b** 1000 and 1215
c 1430 and 1615 **d** 1530 and 1615
e 1023 and 2312 **f** 1135 and 1440
g 0820 and 1550 **h** 0712 and 2008
i 1455 and 0015

eBookplus
Digital docs
Spreadsheet
12-h to 24-h time
doc-3447
Spreadsheet
24-h to 12-h time
doc-3448

5 Mary-Jane always arrived at school at 0855 and left at 1526. How long was she at school?

6 Peter wished to record a movie on video. He had a 180-minute tape and a 240-minute tape. The film he wished to record started at 2030 and finished at 2345.

a How long was the movie?
b Which tape should Peter use?

7 An aircraft left the airport at 0920 and arrived at its destination at 1305. How long was the flight?

8 MC A clock shows the time as 1543. What is the correct time if it is known that the clock is 33 minutes slow?

A 1576 **B** 1616 **C** 1510
D 1516 **E** 1606

9 MC A clock shows the time as 2345. What is the correct time if it is known that the clock is 27 minutes slow?

A 2318 **B** 2372 **C** 2412
D 0042 **E** 0012

10 MC A clock shows the time 0857. What is the correct time if it is known that the clock is 48 minutes fast?

A 0809 **B** 0811 **C** 0945
D 0805 **E** 0905

11 MC A clock shows the time 1004. What is the correct time if it is known that the clock is 31 minutes fast?

A 1035 **B** 0935 **C** 0973
D 1013 **E** 0933

12 WE19a What is the time in Melbourne if the time in London (GMT) is as given below?

a 11.00 am **b** 12.30 pm
c 10.20 am **d** Midnight
e Noon **f** 2.15 am
g 3.30 pm **h** 5.45 pm
i 8.10 am **j** 9.35 pm
k 1.12 pm **l** 1.12 am?

13 WE19b What is the time in London (GMT) if the time in Melbourne is as given below?

a 3.30 am **b** Noon **c** Midnight
d 5.50 pm **e** 6.40 am **f** 7.20 pm
g 11.10 am **h** 1325 **i** 1550
j 0615 **k** 1855 **l** 1935?

14 WE20 You call your friend in Adelaide on Christmas Day from your home in Cairns, Queensland. It is 5 pm in Cairns. What time is it in Adelaide?

UNDERSTANDING

15 Jamie has made a schedule for football training. He will start exercising at 0615 and stop at 0705. He will also train from 1600 until 1810. For how long will Jamie train each day?

16 MC If Suzie wanted to telephone her friend in England before he went to work for the day, what would be the best time for her to ring from Melbourne, Australia?

A 0700 **B** 1700
C 1900 **D** 2100
E 2300

17 If the time was 5.00 am in London, what would the time be in:

a Perth? **b** Melbourne? **c** Jakarta?

18 If the time was 8.00 pm in Perth, what would the time be in:

a London? **b** Melbourne? **c** Jakarta?

19 If the time was 3.30 pm in Jakarta, what would the time be in:

a London? **b** Perth? **c** Melbourne?

20 Joe lives in Perth. He wants to ring his brother who lives in Hobart, and his parents who live in Darwin at 1 minute to midnight on New Year's Eve. What time (Perth time) should he make each call?

21 Using the internet and the preceding map of time zones of the world, investigate other countries in the world that are in the same time zones as Australia. List three cities in other countries that are in the same time zone as Australian capital cities.

REASONING

22 Jessica and Connor have to catch the airport train at 8.20 am so they will be on time to catch their plane at the airport. Jessica's watch is 10 minutes fast, but she thinks it is 5 minutes slow. Connor's watch is 10 minutes slow, but he thinks it is 10 minutes fast. Each leaves home expecting to arrive at the station just in time to catch the train. Do they both catch the train? Show your working.

PROBLEM SOLVING

23 A flight from Melbourne to Perth can take 4 hours. If your flight departs Melbourne airport at 2:10 pm (EST), at what time will you arrive in Perth (WST)?

A flight from Melbourne to Perth takes about 4 hours.

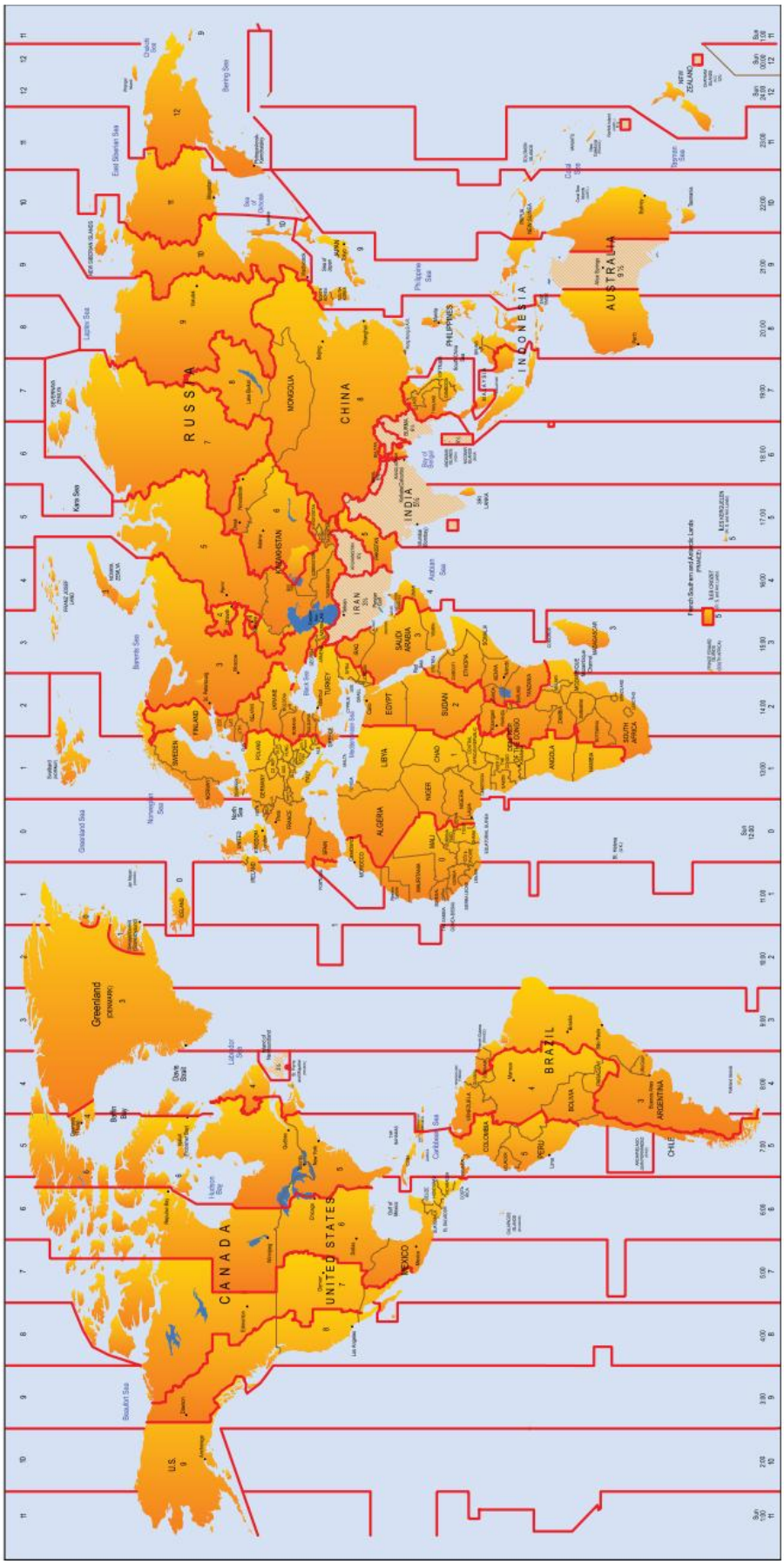
24 A plane flew a direct route from Melbourne to London. Some of the details of the trip are listed below.

- Departure date: Friday 5 August
- Departure time: 8:00 am
- Flight time: 25 hours

a What was the time and date in Melbourne when the plane arrived in London?

b What was the local time (UTC) when the plane arrived?

eBookplus
Digital doc
WorkSHEET 10.2
doc-6932


RUSSIA
CHINA
INDONESIA
AUSTRALIA
PHILIPPINES
MONGOLIA
KAZAKHSTAN
IRAN
INDIA
SAUDI ARABIA
EGYPT
SUDAN
LIBYA
ALGERIA
NIGER
CHAD
TURKEY
FINLAND
SWEDEN
POLAND
GERMANY
FRANCE
NORWAY
ANGOLA
SOUTH AFRICA
NEW ZEALAND
Greenland
CANADA
UNITED STATES
MEXICO
BRAZIL
ARGENTINA
CHILE
COLOMBIA
PERU
BOLIVIA
U.S.

10.10 Review

www.jacplus.com.au

The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:

- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

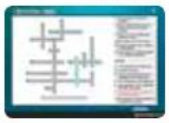
Review questions

Download the Review questions document from the links found in your eBookPLUS.

Interactivities

Word search: int-2756

Crossword: int-2757



Sudoku: int-3189

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

24-hour clock	diameter	radius
annulus	EST	sphere
area	GMT	square pyramid
circumference	metric units	time
cone	perimeter	time zones
CST	pi (π)	volume
daylight saving	prism	WST

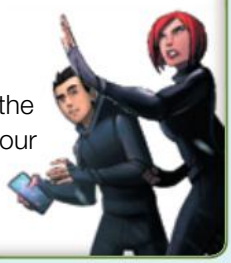
Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

www.assesson.com.au

Link to SpyClass, an exciting online game combining a comic book–style story with problem-based learning in an immersive environment.

Join Jesse, Toby and Dan and help them to tackle some of the world's most dangerous criminals by using the knowledge you've gained through your study of mathematics.



www.spyclass.com.au

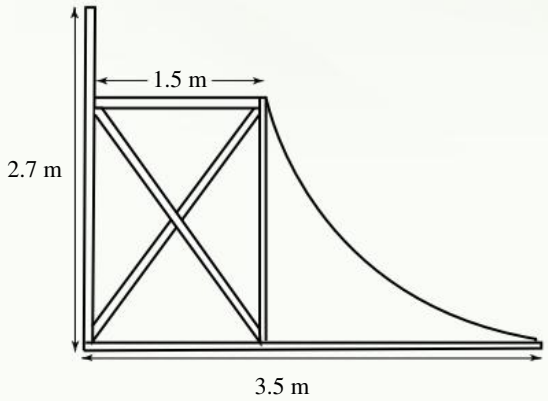
INVESTIGATION

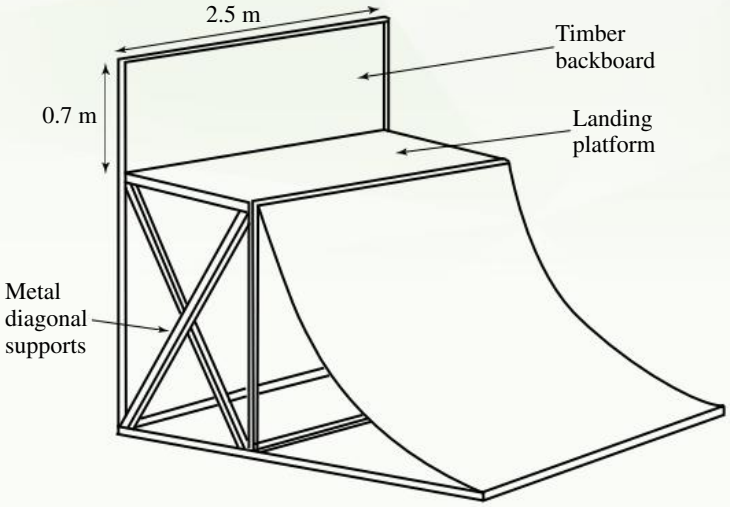
RICH TASK

Designing a skate park

Skateboarding is an extremely popular activity. Skateboard parks are constructed in many areas for enthusiasts to demonstrate their skills and practise new moves. Two popular pieces of equipment in skate parks are ramps known as the half-pipe and the quarter-pipe. Your local council has decided to build a skate park in your area. The park will have, among the collection of permanent equipment, a half-pipe ramp and a quarter-pipe ramp. You have been asked to provide precise diagrams and measurements to assist in the building of these two structures.

Two different views of your quarter-pipe design are shown below with measurements. The ramp section is made from metal and its length is equivalent to the arc length of a quarter circle with a radius of 2 m.





1. What is the length of the curved part of the ramp?
2. Use a scale diagram to estimate the length of the diagonal metal supports within the frame.
3. Calculate the combined area of the timber backboard, landing platform and the curved section of the ramp.
4. The frame is made from strong square metal piping. If there are 10 horizontal supports in the frame, what is the total length of metal piping used in the construction?
5. How many times will a wheel of a skateboard turn on the curved section of the ramp, given that the diameter of the skateboard wheel is 55 mm?

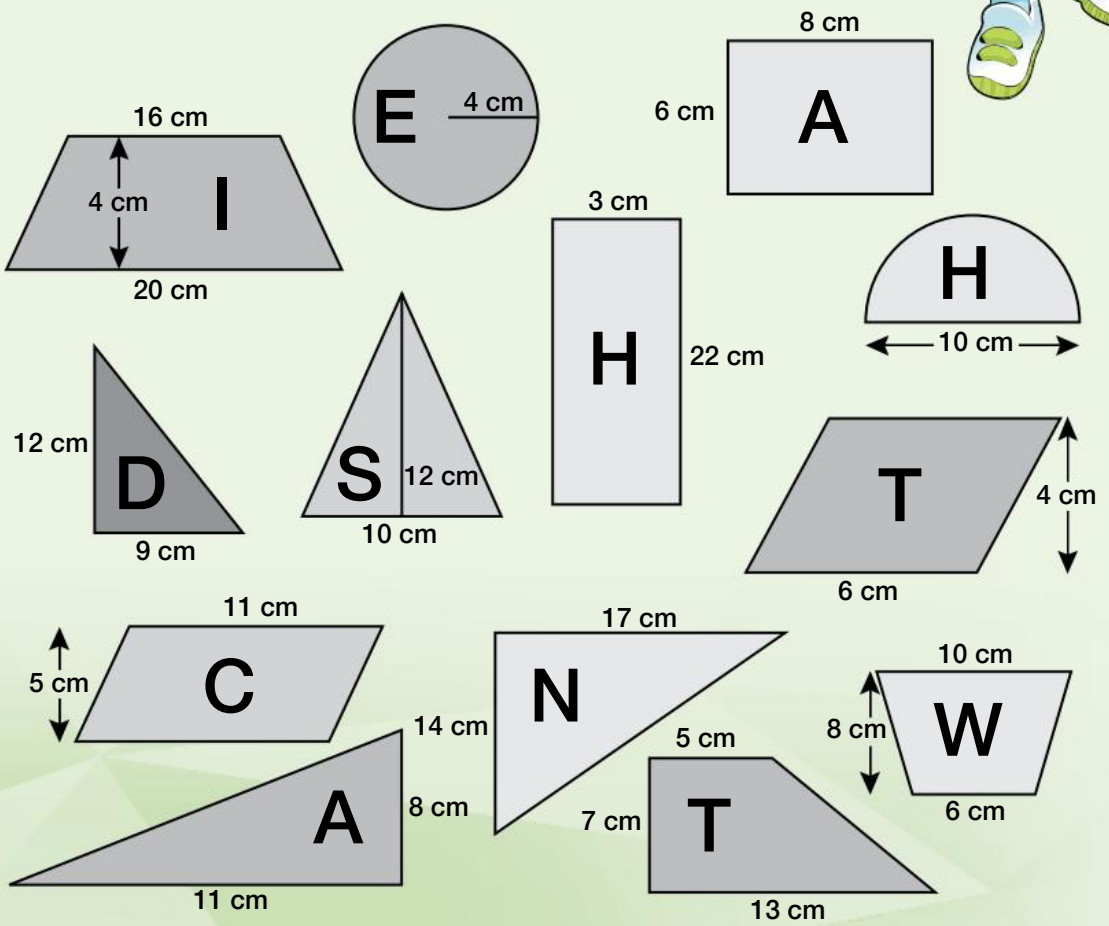
Your design for the half-pipe contains two quarter-pipes separated by a 2-metre flat section. The quarter-pipes contain the same measurements shown earlier.

6. Draw a side-on view of your half-pipe showing all relevant measurements.
7. What is the total area of metal required to construct the skating section of the half-pipe and landing platforms?
8. Construct a scale model of your design. Your teacher will provide you with materials to use.

CODE PUZZLE

Two Australians who have won at Wimbledon

Calculate the areas of the figures below to give the puzzle's answer code.



55	44	60	66		48	119	54	
39.25	50.24	64	72	24	63			

eBookplus Activities

10.2 Perimeter

Digital docs
- SkillSHEET (doc-6956) Multiplying and dividing by powers of 10
- SkillSHEET (doc-6957) Converting units of length

Interactivity
- IP interactivity 10.2 (int-4438) Perimeter

10.3 Circumference

Digital doc
- Investigation (doc-2318) The diameter of a circle and its circumference — any connection?

Interactivity
- IP interactivity 10.3 (int-4439) Circumference

10.4 Area of rectangles, triangles, parallelograms, rhombuses and kites

Digital docs
- SkillSHEET (doc-6958) Areas of squares, rectangles and triangles
- Investigation (doc-2319) Area of a parallelogram

Interactivity
- IP interactivity 10.4 (int-4440) Area of rectangles, triangles, parallelograms, rhombuses and kites

10.5 Area of a circle

Digital doc
- Investigation (doc-2321) Area of a circle

Interactivity
- IP interactivity 10.5 (int-4441) Area of a circle

10.6 Area of trapeziums

Digital docs
- Investigation (doc-2320) Area of a trapezium
- WorkSHEET 10.1 (doc-2322)

Interactivity
- IP interactivity 10.6 (int-4442) Area of trapeziums

10.7 Volume of prisms and other solids

Digital docs
- SkillSHEET (doc-6959) Volumes of cubes and rectangular prisms
- Spreadsheet (doc-2324) Volume of a prism

Interactivities
- Volume of prisms (int-2754)
- IP interactivity 10.7 (int-4443) Volume of prisms and other solids

10.8 Time

Digital docs
- SkillSHEET (doc-6960) Reading the time (from analogue clocks)
- Spreadsheet (doc-3445) Time differences

Interactivity
- IP interactivity 10.8 (int-4444) Time

10.9 24-hour clock and time zones

Digital docs
- SkillSHEET (doc-6961) 24-hour clock
- WorkSHEET 10.2 (doc-6932)
- Spreadsheet (doc-3447) 12-h to 24-h time
- Spreadsheet (doc-3448) 24-h to 12-h time
- Spreadsheet (doc-3449) Time zones

Interactivities
- Time zones (int-0006)
- IP interactivity 10.9 (int-4445) 24-hour clock and time zones

10.10 Review

Interactivities
- Word search (int-2756)
- Crossword (int-2757)
- Sudoku (int-3189)

Digital docs
- Topic summary (doc-10761)
- Concept map (doc-10774)
- Topic review (Word doc-14940, PDF doc-14941)

To access eBookPLUS activities, log on to www.jacplus.com.au

Answers

TOPIC 10 Measurement

10.2 Perimeter

1 **a** 20 mm = 2 cm **b** 13 mm = 1.3 cm
c 130 mm = 13 cm **d** 1.5 cm = 15 mm
e 0.03 cm = 0.3 mm **f** 2.8 km = 2800 m
g 0.034 m = 3.4 cm
h 2400 mm = 240 cm = 2.4 m
i 1375 mm = 137.5 cm = 1.375 m
j 2.7 m = 270 cm = 2700 mm
k 0.08 m = 80 mm
l 6.071 km = 6071 m
m 670 cm = 6.7 m
n 0.0051 km = 5.1 m

2 **a** 14 cm **b** 12 cm
c 106 mm **d** 18 cm
e 240 mm **f** 32 mm
g 23 cm **h** 72 mm
i 73 mm **j** 1260 cm (12.6 m)
k 192 cm (1.92 m) **l** 826 cm

3 **a** 1800 mm × 900 mm = 180 cm × 90 cm
= 1.8 m × 0.9 m
b 2400 mm × 900 mm = 240 cm × 90 cm
= 2.4 m × 0.9 m
c 2700 mm × 1200 mm = 270 cm × 120 cm
= 2.7 m × 1.2 m

4 $5.40
5 $38.16
6 41 400 m or 41.4 km
7 **a** 7 m **b** $26.60
8 86 m
9 980 cm
10 86 cm
11 **a** 510 m **b** $749.70
12 $15.88
13 **a** 11 cm **b** 22 cm **c** 6.9 m
14 5.5 m
15 **a** 5 cm **b** 20 cm **c** 2 bottles
16 9 cm
17 **a** 258 m
b 174 m
18 **a** Fence length (l) = 10 m and width (w) = 8 m, so perimeter = 36 m.
b Fence length = $2(l + w + 4)$
19 **a** 217 cm
b Length = $2w + 2l + 4h$ + bow

10.3 Circumference

1 **a** **i** 2π cm **ii** 6.28 cm
b **i** 10π cm **ii** 31.42 cm
c **i** 7π mm **ii** 21.99 mm
d **i** 0.82π m **ii** 2.58 m
e **i** 7.4π km **ii** 23.25 km
f **i** 34π m **ii** 106.81 m

2 **a** **i** 8π m **ii** 25.13 m
b **i** 34π mm **ii** 106.81 mm
c **i** 16π cm **ii** 50.27 cm
d **i** 2.86π km **ii** 8.98 km
e **i** 0.8π m **ii** 2.51 m
f **i** 21.2π m **ii** 66.60 m

3 **a** 241.90 km **b** 37.70 m
c 150.80 mm **d** 3.36 m
e 194.78 mm **f** 1256.64 m

4 **a** 25.71 cm **b** 82.27 mm
c 61.70 m **d** 39.28 mm
e 71.42 cm **f** 120.82 cm
g 5.88 m **h** 250.80 m
i 252.81 cm

5 B
6 C
7 55.29 m
8 100.53 cm
9 119.38 mm
10 25.13 m
11 **a** 6.00 m
b 20.63 cm
c 23.75 mm
12 **a** 2.01 cm
b 7.54 m
c 24.99 mm
13 21.58 cm
14 19.19 m
15 15.71 m
16 41.82 km/h
17 9.55 cm
18 **a** iii **b** i **c** iv **d** ii
19 **a** 25.71 cm **b** 483.29 mm
c 31.07 m **d** 5.71 cm
20 80 m
21 **a** 60° = $\frac{1}{6}$ of circle. Therefore, length of curved line = $\frac{1}{6} \times 2\pi r$ and P = 30.47 cm.
b Length of arc = 6.98 cm; perimeter = 13.74 cm.

Challenge 10.1

25.13 cm^2

10.4 Area of rectangles, triangles, parallelograms, rhombuses and kites

1 **a** 530 000 m^2 **b** 2.35 cm^2
c 254 000 mm^2 **d** 54.2 m^2
e 0.074 m^2 **f** 3 km^2
g 9.8563 ha **h** 17 800 m^2
i 987 000 mm^2 **j** 1 275 000 cm^2

2 **a** 36 cm^2 **b** 1125 mm^2
c 4.5 m^2 **d** 1215 km^2
e 2.5 m^2 **f** 336 mm^2

3 **a** 25 mm^2
b 256 cm^2
c 5.29 m^2

4 C
5 B

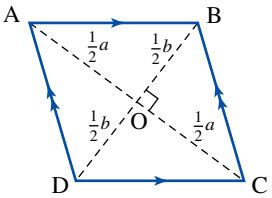
6 **a** 1258 mm^2 **b** 1771.54 m^2
c 9932.63 mm^2 **d** 17 537 cm^2
e 11 566.8 mm^2 **f** 257.645 m^2

7 **a** 275 mm^2 **b** 24 000 m^2
c 656 cm^2 **d** 11.04 mm^2
e 2.7 m^2 **f** 2400 mm^2
g 17.36 m^2 **h** 4760 m^2
i 8.48 m^2

8 **a** 30 cm^2 **b** 24 cm^2
c 90 cm^2

9 **a** 42.65 m^2, 60.81 m^2, 42.65 m^2
b 146.11 m^2

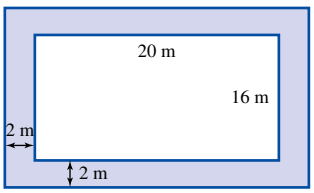
10 351.98 cm^2
11 **a** 15 000 m^2
b 60 000 m^2
12 **a** 15.64 m^2 **b** 3
13 42 cm
14 **a** 400 m **b** 375 min or $6\frac{1}{4}$ h
15 50.4 cm^2
16 2052 m^2
17 D
18 17.28 cm^2
19 1584 cm^2
20 **a** 3.6 cm **b** 9.2 cm
21 $b = h = 9.5$ cm
22



Area of $\Delta ABC = \frac{1}{2} \times$ base $\times$ height
$= \frac{1}{2} \times a \times \frac{1}{2}b$
$= \frac{1}{4} \times a \times b$

Area of rhombus ABCD $= 2 \times$ area of ΔABC
$= 2 \times \frac{1}{4} \times a \times b$
$= \frac{1}{2} \times a \times b$

23 $A_K = \frac{1}{2} \times a \times b$
24 Answers will vary. Examples are given.
a Length = 12 cm, width = 3 cm
b Base = 4 cm, height = 9 cm
c Diagonals 12 cm and 6 cm
25 **a**



b 320 m^2
c 160 m^2
d \$800
26 **a** 212 500 km^2
b 227 600 km^2
c Because Victoria is not a triangle
d Divide Victoria into smaller regular shapes.

10.5 Area of a circle

1 **a** 452.39 cm^2 **b** 4.91 km^2
c 2.27 m^2 **d** 0.38 cm^2
e 10 568.32 cm^2 **f** 206.12 mm^2
2 **a** 78.54 cm^2 **b** 483.05 mm^2
c 615.75 m^2 **d** 254.47 cm^2
3 **a** 37.70 cm^2
b 1281.77 cm^2
c 3189.50 m^2
4 **a** 157.08 cm^2 **b** 0.39 cm^2
c 201.06 mm^2 **d** 13.85 m^2
e 39.27 cm^2 **f** 1039.08 cm^2
g 77.91 cm^2 **h** 132.54 cm^2
i 9.74 cm^2
5 1134.11 cm^2
6 4.52 m^2
7 25 packets
8 173 g
9 **a** Diameter changes by a factor of 1.414 ($= \sqrt{2}$).
b Diameter changes by a factor of 2.
c Diameter changes by a factor of 0.707 ($= \sqrt{\frac{1}{2}}$).
d Diameter changes by a factor of $\sqrt{n}$.
10 **a** Circumference changes by a factor of 1.414 ($= \sqrt{2}$).
b Circumference changes by a factor of 2.
c Circumference changes by a factor of 0.707 ($= \sqrt{\frac{1}{2}}$).
d Circumference changes by a factor of $\sqrt{n}$.
11 **a** $A = 70.83$ cm^2
b $r = 4.75$ cm
c The diameter of the coaster is larger than that of the mug, so the coffee mug will fit within the coaster's surface.
12 **a** 3279.82 cm^2 **b** 40.18 mm^2
c 31.49 cm^2

10.6 Area of trapeziums

1 **a** 9 cm^2 **b** 33.75 m^2
c 12.75 m^2 **d** 351 mm^2 (3.51 cm^2)
e 4.68 cm^2 **f** 3120 m^2
2 E
3 3062 cm^2
4 \$88.30
5 **a** \$2730.55
b **i** \$332.50
ii \$3063.05
6 2 m
7 B
8 **a** $A_1 = \frac{1}{2}bh$ **b** $A_2 = \frac{1}{2}ah$
c $A = A_1 + A_2 + = \frac{1}{2}ah + \frac{1}{2}bh$
9 **a** 31.15 m^2 **b** 7 **c** \$62.65
10 **a** $85.2 + 15.125\pi$ cm^2 **b** 132.72 cm^2

10.7 Volume of prisms and other solids

1 **b** and **e**
2 **a** 84 cm^3 **b** 81 m^3
c 14 137.17 cm^3 **d** 31 415.93 cm^3
e 667.98 m^3 **f** 4778.36 cm^3
g 84 cm^3 **h** 120 cm^3
i 320 m^3 **j** 126 cm^3
k 7.5 cm^3 **l** 1.875 m^3
3 1200 m^3
4 60.32 m^3
5 **a** 60 cm^3 **b** 84 cm^3
6 7380 cm^3
7 18.6 L
8 **a** **i** 0.3 m **ii** 0.4 m
b 0.72 m^3, 0.94 m^3
c The cylinder with the bigger radius is larger. The area of the base is πr^2. The radius has a bigger influence on the volume than the height because the radius is squared.
9 **a** $R = \frac{\text{circumference}}{2\pi}$ and $r = \frac{\text{circumference}}{2\pi}$

$= \frac{a}{2\pi}$ $= \frac{b}{2\pi}$

b $V_a = \pi\left(\frac{a}{2\pi}\right)^2 b$ and $V_b = \pi\left(\frac{b}{2\pi}\right)^2 a$

$= \frac{a^2 b}{4\pi}$ $= \frac{ab^2}{4\pi}$

c The cylinder with the bigger radius has a larger volume.

$V_a = \frac{a^2b}{4\pi} = \frac{ab}{4\pi}a$ and $V_b = \frac{ab^2}{4\pi} = \frac{ab}{4\pi}b$.

Both volumes have a common fraction, $\frac{ab}{4\pi}$, multiplied by the radius, but $a > b$.

d Yes

10 18 cm

11 **a** 72 cm^3

b **i** 12 cm long, 8 cm wide, 6 cm high; $V = 576$ cm^3

ii 18 cm long, 12 cm wide, 9 cm high; $V = 1944$ cm^3

c **i** 1 : 8 **ii** 1 : 27

d If all dimensions of a rectangular prism are increased by a factor of n, the volume of the prism is increased by a factor of n^3.

12 **a** 29.38 m^2

b 10.5 m^3

Challenge 10.2

297 m^2

10.8 Time

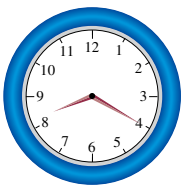
1 **a–f** A cricket test match — 5 days, writing your name — 10 seconds, eating breakfast — 15 minutes, building a house — 6 months, flying time from Melbourne to Hong Kong — 7 hours, being in Year 7 — 1 year.

2 **a** 5 o'clock **b** 4 minutes past 5

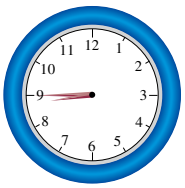
c 23 minutes past 7 **d** 5 minutes past 1

e 14 minutes past 9 **f** 13 minutes to 7

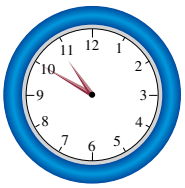
3 **a**



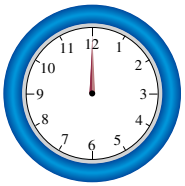
b



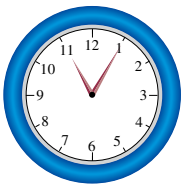
c



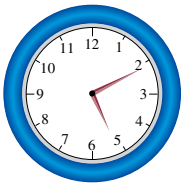
d



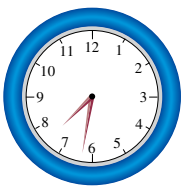
e



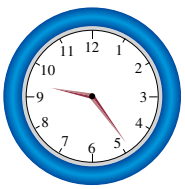
f



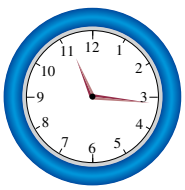
g



h



i



4 Sundial and hourglass

5 2.25 hours = $2\frac{1}{4}$ hours which is 2 hours and 15 minutes

6 **a** 120 **b** 150 **c** 195

d 30 **e** 45 **f** 465

g 1440 **h** 12 960

7 **a** 3 h 20 min **b** 3 h 5 min

c 2 h 40 min **d** 3 h 50 min

e 6 h 45 min **f** 1 h 35 min

g 10 h 10 min **h** 1 h 12 min

i 5 h 5 min

8 **a** 75 min **b** 130 min

c 110 min **d** 265 min

e 455 min **f** 192 min

g 1005 min **h** 330 min

i 375 min

9 **a** 7 **b** 52

c 60 **d** 3600

e 86 400 **f** 24

g 8760 **h** 1440

i 525 600

10 **a** 5 pm **b** 3 pm

c 6.30 am **d** 11.50 am

e 8.40 am **f** 5.18 pm

g 2.00 am **h** 2.20 pm

i 5.30 pm **j** 6.35 am

k 7.41 pm **l** 10.20 am

11 **a** 15 min **b** 1 h 5 min

c 1 h 13 min **d** 1 h 15 min

e 53 min **f** 1 h 53 min

g 32 min **h** 3 h 45 min

i 5 h 15 min **j** 12 h

k 8 h 15 min

12 **a** 35 h 10 min **b** 32 h 47 min

c 13 h 31 min **d** 43 h 49 min

13 C

14 D

15 A

16 48 min

17 $7\frac{1}{2}$ h

18 7.25 am

19 6.30 am

20 11.45 pm

21 10.35 pm

22 52 min

23 4 h 2 min

24 25 minutes past 10

25 12 o'clock and 6 o'clock

26 2 hours 50 minutes

27 **a** 8.53 pm **b** 9.04 pm **c** 8.26 pm

28 **a** The calculation is correct but the interpretation of the answer is incorrect. It should be 2 hours and 35 minutes rather than 235 minutes.

b The calculation is incorrect; the times have been subtracted as if they were decimals. The correct answer is 1 hour 44 minutes.

29 15 days

10.9 24-hour clock and time zones

1 10.12 am = 1012, 12.12 pm = 1212, Midnight = 0000, Noon = 1200, 11.20 pm = 2320, 11.20 am = 1120

2 **a** 1020 **b** 1130

c 0510 **d** 0415

e 1715 **f** 1830

g 0830 **h** 2040
i 1200 **j** 2330
k 0435 **l** 1430

3 **a** 11.15 pm **b** 1.10 pm
c 8.15 am **d** 1.15 am
e 6.18 pm **f** 12.20 pm
g 12.05 am **h** 10.05 am
i 8.05 pm **j** 11.35 am
k 3.20 pm **l** 2.14 pm

4 **a** 13 h 10 min **b** 2 h 15 min
c 1 h 45 min **d** 45 min
e 12 h 49 min **f** 3 h 5 min
g 7 h 30 min **h** 12 h 56 min
i 9 h 20 min

5 6 h 31 min

6 **a** 3 h 15 min **b** 240-min tape

7 3 h 45 min

8 B

9 E

10 A

11 E

12 **a** 9.00 pm **b** 10.30 pm
c 8.20 pm **d** 10 am (next day)
e 10 pm **f** 12.15 pm
g 1.30 am (next day) **h** 3.45 am (next day)
i 6.10 pm **j** 7.35 am (next day)
k 11.12 pm **l** 11.12 am

13 **a** 5.30 pm (previous day)
b 2.00 am
c 2.00 pm
d 7.50 am
e 8.40 pm (previous day)
f 9.20 am
g 1.10 am
h 0325
i 0550
j 2015 (previous day)
k 0855
l 0935

14 5.30 pm

15 3 hours

16 B

17 **a** 1 pm **b** 3 pm **c** 12 pm

18 **a** Midday **b** 10 pm **c** 7 pm

19 **a** 8.30 am **b** 4.30 pm **c** 6.30 pm

20 Brother 8.59 pm, parents 10.29 pm

21 Answers could include:
Perth — Singapore, Kuala Lumpur, Cebu City in the Philippines, Denpasar in Bali Brisbane — Port Moresby in Papua New Guinea.

22 Connor misses the train by 20 minutes.

23 4.10 pm

24 **a** 9.00 am on 6 August
b 11.00 pm on 5 August

Investigation — Rich task

1 3.14 m

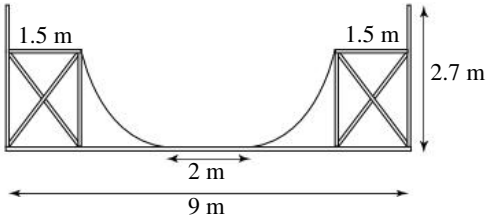
2 2.5 m

3 13.35 m^2

4 44.4 m

5 18.18 times

6



7 28.2 m^2

8 Teacher to check.

Code puzzle

Cash and Hewitt

TOPIC 11

Linear equations

11.1 Overview

Why learn this?

Linear equations are a form of algebra, and they are often used to describe everyday situations using mathematics. One of the most useful skills that you learn in algebra is how to solve equations. Solving equations means finding numbers whose values you do not know. Once you learn the techniques of solving equations, you will find that many problems can be expressed and solved using linear equations.

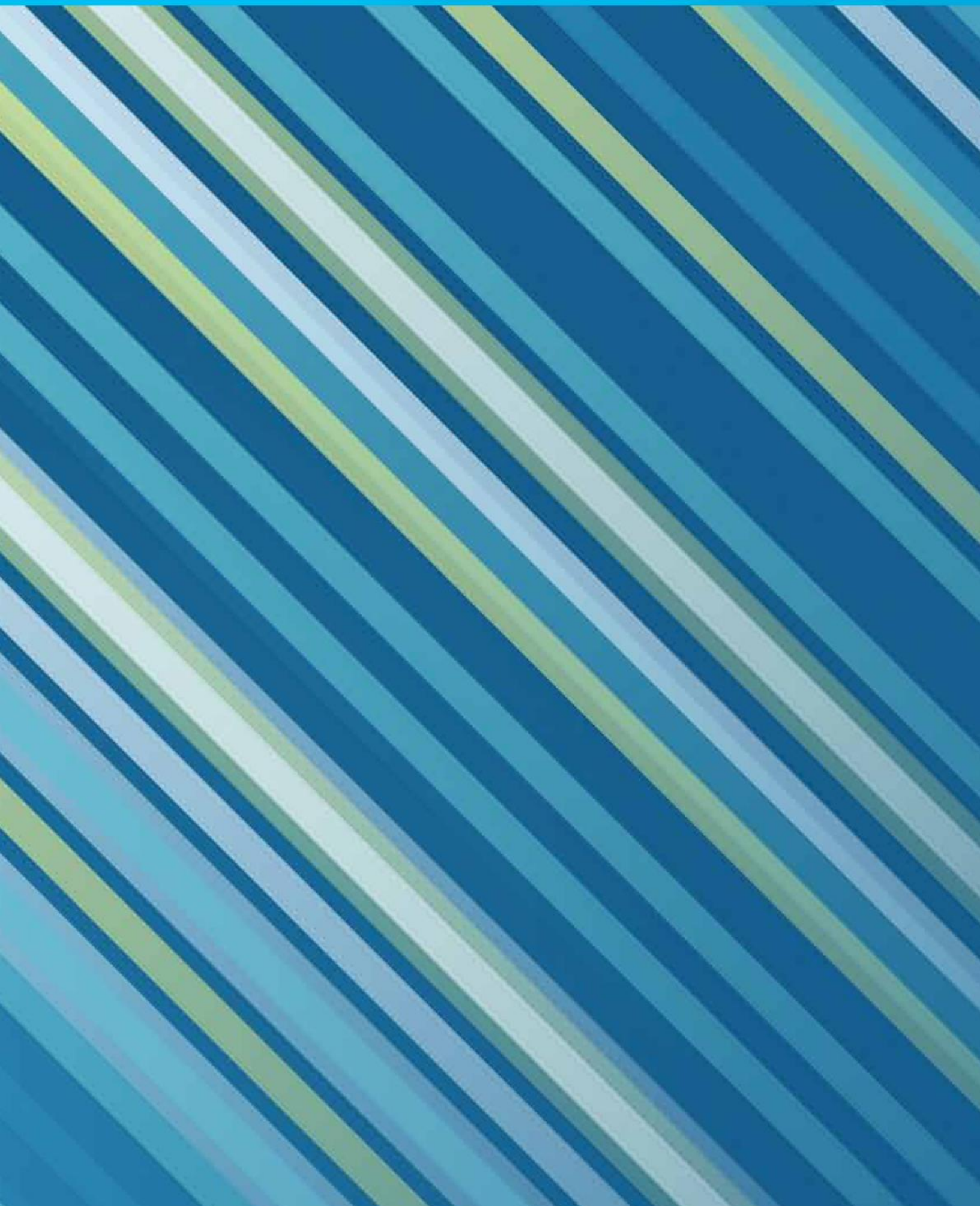
What do you know?

assessON

1 **THINK** List what you know about equations. Use a thinking tool such as a concept map to show your list.
2 **PAIR** Share what you know with a partner and then with a small group.
3 **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge equations.

Learning sequence

11.1 Overview
11.2 Identifying patterns
11.3 Backtracking and inverse operations
11.4 Keeping equations balanced
11.5 Using algebra to solve problems
11.6 Equations with the unknown on both sides
11.7 Review eBookplus ONLINE ONLY



11.2 Identifying patterns

- Mathematics is used to describe *relationships* in the world around us.
- Mathematicians study *patterns* in numbers and shapes found in nature to discover rules.
- These rules can then be applied to other, more general situations.
- Looking at the number pattern 1, 4, 7, 10, . . . we can see that by adding 3 to any of these numbers, we obtain the next number.
- This number pattern is called a **sequence**.
- Each number in the sequence is called a **term**.
- Each sequence has a *rule* that describes the pattern. For the sequence above, the rule is 'add 3'.

WORKED EXAMPLE 1

Describe the following number patterns in words then write down the next three numbers in the pattern.

a 4, 8, 12, . . . **b 4, 8, 16, . . .**

THINK	WRITE
a 1 The next number is found by adding 4 to the previous number.	a Next number = previous number + 4
2 Add 4 each time to get the next three numbers.	
3 Write down the next three numbers.	Next three numbers are 16, 20, 24.
b 1 The next number is found by multiplying the previous number by 2.	b Next number = previous number × 2
2 Multiply by 2 each time to get the next three numbers.	
3 Write the next three numbers.	Next three numbers are 32, 64, 128.

WORKED EXAMPLE 2

Using the following rules, write down the first five terms of the number pattern.

a Start with 32 and divide by 2 each time.

b Start with 2, multiply by 4 and subtract 3 each time.

THINK	WRITE
a 1 Start with 32 and divide by 2.	a $32 \div 2 = 16$
2 Keep dividing the previous answer by 2 until five numbers have been calculated.	$16 \div 2 = 8$ $8 \div 2 = 4$ $4 \div 2 = 2$ $2 \div 2 = 1$
3 Write the answer.	The first five numbers are 16, 8, 4, 2 and 1.

b	1 Start with 2 then multiply by 4 and subtract 3.	b $2 \times 4 - 3 = 5$ $5 \times 4 - 3 = 17$
	2 Continue to apply this rule to the answer until five numbers have been calculated.	$17 \times 4 - 3 = 65$ $65 \times 4 - 3 = 257$ $257 \times 4 - 3 = 1025$
	3 Write the five numbers.	The first five numbers are 5, 17, 65, 257, 1025.

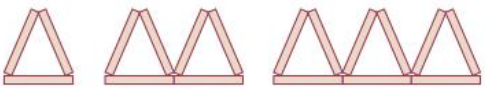
WORKED EXAMPLE 3

Describe the pattern that occurs in the final digit of the number set represented by: $7^1, 7^2, 7^3, \ldots$

THINK	WRITE
1 Write the powers as basic numerals.	$7^1 = 7$
2 Continue until a pattern in the last digit is noticed. When the number becomes large, be concerned with only the last digit.	$7^2 = 49$ $7^3 = 49 \times 7 = 343$ $7^4 = 343 \times 7 = 2401$ $7^5 = 2401 \times 7 = 16\,807$ $7^6 = \ldots 7 \times 7 = \ldots 9$
3 Write the pattern.	The pattern in the last digit is 7, 9, 3, 1 repeated.

Geometric patterns

- Patterns can be found in geometric shapes.
- If we examine the three shapes below, we can see patterns by investigating the changes from one shape to the next. For example, look at the number of matchsticks in each set of triangles.



By using a table of values we can see a number pattern developing:

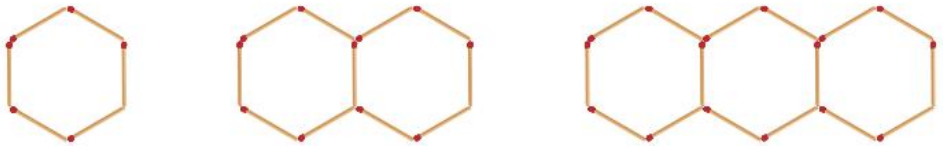
Number of triangles	Number of matchsticks
1	3
2	6
3	9
4	12
5	15
6	18

The pattern in the final row is 3, 6, 9, . . .; we can see that the rule here is 'add 3'.

We can also look for a relationship between the number of triangles and the number of matchsticks in each shape. If you examine the table, you will see that a relationship can be found. In words, the relationship is 'the number of matchsticks equals 3 times the number of triangles'.

WORKED EXAMPLE 4

Consider a set of hexagons constructed according to the pattern shown below.



a Using matches, pencils or similar objects, construct the above figures. Draw the next two figures in the series.
b Draw up a table showing the relationship between the number of hexagons in the figure and the number of matches used to construct the figure.
c Devise a rule to describe the number of matches required for each figure in terms of the number of hexagons in the figure.
d Use your rule to determine the number of matches required to make a figure consisting of 20 hexagons.

THINK

a 1 Construct the given figures with matches. Note the number of additional matches it takes to progress from one figure to the next — 5 in this case.

2 Draw the next two figures, adding another 5 matches each time.

b Draw up a table showing the number of matches needed for each figure in terms of the number of hexagons. Fill it in by looking at the figures.

c 1 Look at the pattern in the number of matches going from one figure to the next. It is increasing by 5 each time.

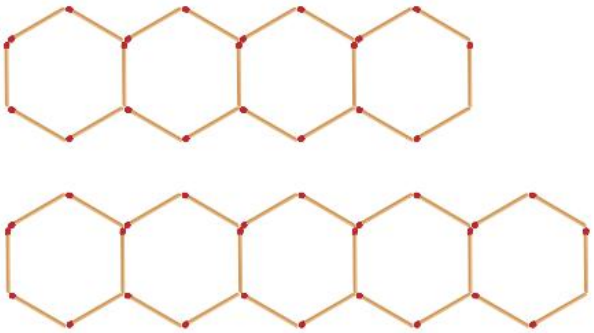
2 If we take the number of hexagons and multiply this number by 5, it does not give us the number of matches. However, if we add 1 to this number, it does give us the number of matches in each shape.

d 1 Use the rule to find the number of matches to make a figure with 20 hexagons.

2 Work out the answer and write it down.

WRITE

a The next two figures are:



b

Number of hexagons	1	2	3	4	5
Number of matches	6	11	16	21	26

c The number of matches increased by 5 in going from one figure to the next.

Number of matches = number of hexagons × 5 + 1

d Number of matches for 20 hexagons
= 20 × 5 + 1
= 101

So 101 matches would be required to construct a figure consisting of 20 hexagons.

Exercise 11.2 Identifying patterns

assessON

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7	Questions: 1–8	Questions: 1–9

Individual pathway interactivity int-4446 eBookplus

REFLECTION
What should you look for when trying to determine number patterns?

FLUENCY

1 WE1 Copy the patterns below, describe the pattern in words and then write down the next three numbers in the pattern.

a 2, 4, 6, 8, . . .
b 3, 8, 13, 18, . . .
c 27, 24, 21, 18, . . .
d 1, 3, 9, 27, . . .
e 128, 64, 32, 16, . . .
f 1, 4, 9, 16, . . .

2 Fill in the missing numbers in the following number patterns.

a 3, . . ., 9, 12, . . ., . . .
b 8, . . ., . . ., 14, . . .
c 4, 8, . . ., 32, . . .
d . . ., . . ., 13, 15, . . .
e 66, 77, . . ., 99, . . . 121
f 100, . . ., . . ., 85, 80, . . .

3 WE2 Using the following rules, write down the first five terms of the number patterns.

a Start with 1 and add 4 each time.
b Start with 5 and multiply by 3 each time.
c Start with 50 and take away 8 each time.
d Start with 64 and divide by 2 each time.
e Start with 1, multiply by 2 and add 2 each time.
f Start with 1, add 2 then multiply by 2 each time.

eBookplus
Digital docs
SkillSHEET
Number patterns
doc-6978
SkillSHEET
Using tables to show number patterns
doc-6979
SkillSHEET
Describing a number pattern from a table
doc-6980

UNDERSTANDING

4 Each of the following represents a special number set. What is common to the numbers in the set?

a 2, 4, 6, 8, 10
b 1, 8, 27, 64
c 2, 3, 5, 7, 11
d 1, 1, 2, 3, 5, 8
e 1, 2, 3, 4, 6, 12
f 3, 9, 27, 81

5 WE3 Investigate the pattern that occurs in the final digit of the following sets. Describe the pattern in each case.

a $2^1, 2^2, 2^3, \ldots$
b $3^1, 3^2, 3^3, \ldots$
c $4^1, 4^2, 4^3, \ldots$
d $5^1, 5^2, 5^3, \ldots$
e $8^1, 8^2, 8^3, \ldots$

6 WE4 For each of the sets of shapes below, follow these instructions to investigate the pattern.

i Construct the shapes using matches. Draw the next two shapes in the series.
ii Construct a table to show the relationship between the number of shapes in each figure and the number of matchsticks used to construct it.
iii Devise a rule in words that describes the pattern relating the number of shapes in each figure and the number of matchsticks used to construct it.

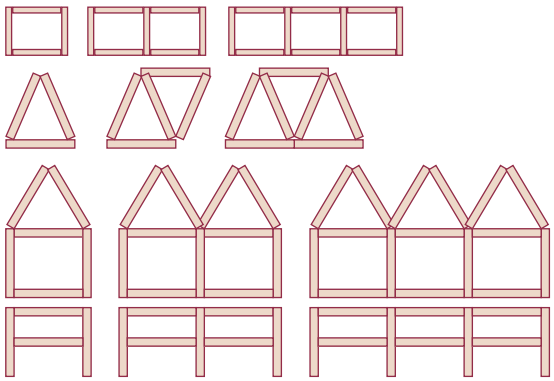
iv Use your rule to work out the number of matchsticks required to construct a figure made up of 20 such shapes.

a

b

c

d



REASONING

7 Consider the triangular pattern of even numbers shown below.

2

4 6

8 10 12

14 16 18 20

a Complete the next two lines of the triangle using this pattern.

b Complete the triangle as far as necessary to find the position of the number 60.

c Explain how, without completing any more of the triangle, you could find the position of the number 100.

d Create a similar triangle using odd numbers. Look for the patterns in this triangle. Are they the same as or different from those for the triangle of even numbers? Justify your answers by illustrations from the triangle.

PROBLEM SOLVING

8 A grain of rice is placed on the first square of a chess board, three grains of rice on the next square, nine grains of rice on the next square, and so on.

a Write a rule in index form to give the number of grains of rice on the nth square.

b Which square has 2187 pieces of rice on it?

9 0, 1, 1; 1, 1, 2; 1, 2, 3; 2, 3, 5; 3, 5, 8; 5, 8, 13…

a What is the next term in this sequence of numbers?

b Which term would follow the term a, b, c?

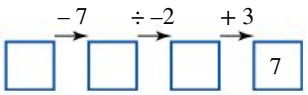
11.3 Backtracking and inverse operations

- An equation links two expressions with an equals sign.
- Adding and subtracting are inverse operations.
- Multiplying and dividing are inverse operations.
- A flowchart can be used to represent a series of operations.

- In a flowchart, the starting number is called the input number and the final number is called the output number.
- By using inverse operations, it is possible to reverse the flowchart and work from the output number to the input number.

WORKED EXAMPLE 5

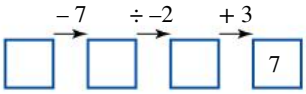
Find the input number for this flowchart.

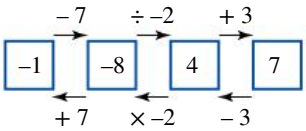


THINK

1. Copy the flowchart.
2. Backtrack to find the input number.
   The inverse operation of +3 is −3 (7 − 3 = 4).
   The inverse operation of + −2 is × −2 (4 × −2 = −8).
   The inverse operation of −7 is +7 (−8 + 7 = −1).
   Fill in the missing numbers.
3. State the input number.

WRITE/DRAW

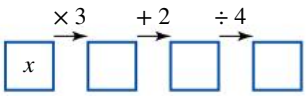




The input number is −1.

WORKED EXAMPLE 6

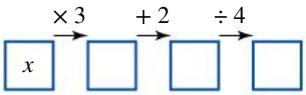
Find the output expression for this flowchart.

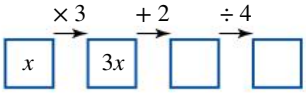


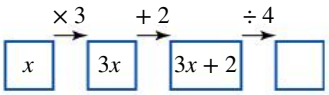
THINK

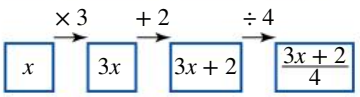
1. Copy the flowchart and look at the operations that have been performed.
2. Multiplying x by 3 gives $3x$.
3. Adding 2 gives $3x + 2$.
4. Now place a line beneath all of $3x + 2$ and divide by 4.
5. State the output expression.

WRITE/DRAW









The output expression is $\frac{3x + 2}{4}$.

WORKED EXAMPLE 7

Starting with x, draw the flowchart whose output number is given by the expressions:

a $6 - 2x$ **b** $-2(x + 6)$.

THINK

a 1 Rearrange the expression.
Note: $6 - 2x$ is the same as $-2x + 6$.

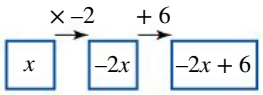
2 Multiply x by -2, and then add 6.

b 1 The expression $x + 6$ is grouped in a pair of brackets, so we must obtain this part first. Therefore, add 6 to x.

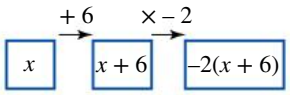
2 Multiply the whole expression by -2.

WRITE/DRAW

a



b



assess on

Exercise 11.3 Backtracking and inverse operations

INDIVIDUAL PATHWAYS

REFLECTION
What do you need to be careful of when you are backtracking equations?

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3a, d, e, m, 4, 5	Questions: 1a, c, e, g, i, k, 2a, c, e, g, i, k 3b, e, h, k, n, 4, 5	Questions: 1b, d, f, h, j, l, 2b, f, j, l, p, 3a, b, e, i, k–p, 4–6

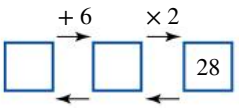
Individual pathway interactivity int-4447 eBookplus

eBookplus
Digital docs
SkillSHEET Flowcharts doc-6981
SkillSHEET Inverse operations doc-6982
SkillSHEET Solving equations by backtracking doc-6983

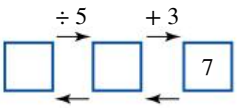
FLUENCY

1 WE5 Find the input number for each of the following flowcharts.

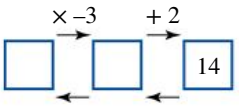
a



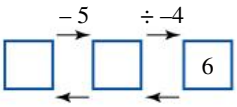
b



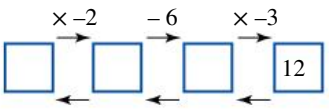
c



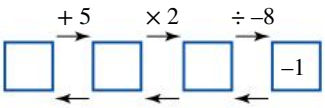
d



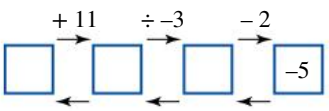
e



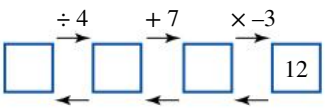
f



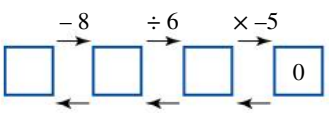
g



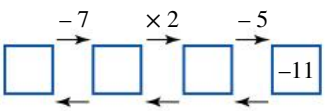
h



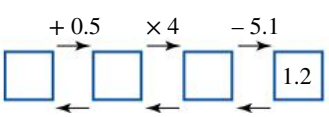
i



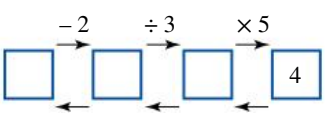
j



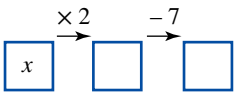
k

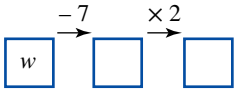


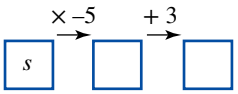
l

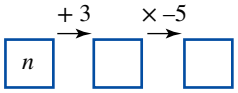


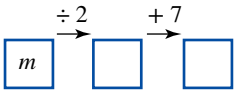
2 **WE6** Find the output expression for each of the following flowcharts.

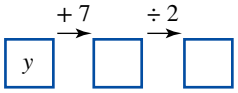
a 

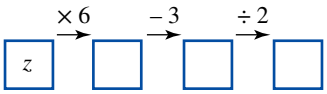
b 

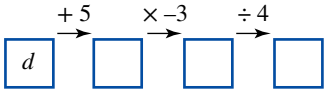
c 

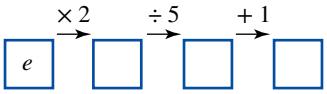
d 

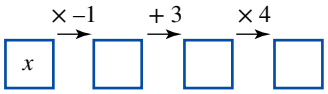
e 

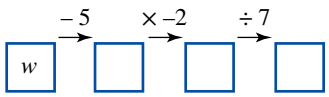
f 

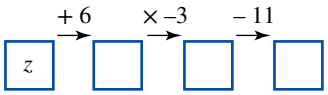
g 

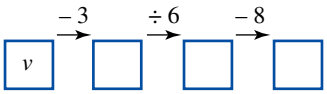
h 

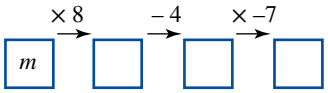
i 

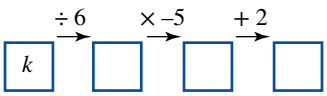
j 

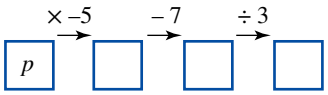
k 

l 

m 

n 

o 

p 

UNDERSTANDING

3 **WE7** Starting with x, draw the flowchart whose output expression is:

a $2(x + 7)$ **b** $-2(x - 8)$ **c** $3m - 6$

d $-3m - 6$ **e** $\frac{x - 5}{8}$ **f** $\frac{x}{8} - 5$

g $-5x + 11$ **h** $-x + 11$ **i** $-x - 13$

j $5 - 2x$ **k** $\frac{3x - 7}{4}$ **l** $\frac{-3(x - 2)}{4}$

m $\frac{x + 5}{8} - 3$ **n** $-7\left(\frac{x}{5} - 2\right)$ **o** $3\left(\frac{2x}{7} + 4\right)$

p $\frac{1}{4}\left(\frac{6x}{11} - 3\right)$

REASONING

4 **a** Draw a flowchart to convert from degrees Fahrenheit (F) to degrees Celsius (C) using the formula $C = \frac{5}{9}(F - 32)$.

b Use the flowchart to convert 50 °F to degrees Celsius by substituting into the flowchart.

c Backtrack through the flowchart to find what 35 °C is in degrees Fahrenheit.

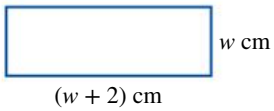
PROBLEM SOLVING

5 Consider the following puzzle.

> *Think of a number.*
> *Double it.*
> *Add 10.*
> *Divide by 2.*
> *Subtract the number you first thought of.*

- **a** Represent this puzzle as a flowchart, with n representing the unknown number.
- **b** What do you notice about the final number?
- **c** Repeat this with different starting numbers.

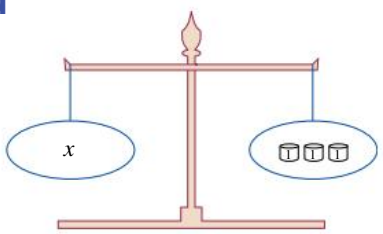
6 The rectangle shown has an area of 255 cm^2.



- **a** Discuss what strategies you might use to find the value of w. Can you use backtracking?
- **b** Find the value of w and explain the method you use.

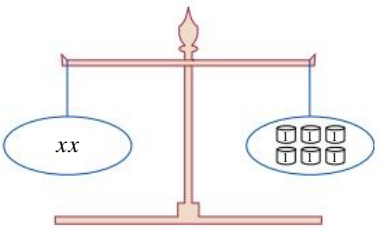
11.4 Keeping equations balanced

- As an equation can be thought of as two expressions with an equals sign between them, an equation can be thought of as a balanced scale. The diagram at right represents the simple equation $x = 3$.



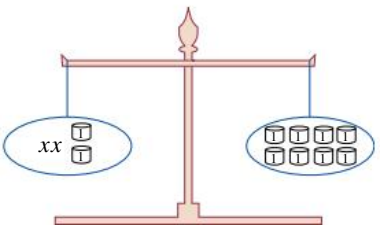
$x = 3$

- If the amount of the left-hand side (LHS) is doubled, the scale will stay balanced provided that the amount on the right-hand side (RHS) is doubled.



$2x = 6$

- Similarly, the scale will stay balanced if we add a quantity to both sides.
- The scales will remain balanced as long as we do the same to both sides.



$2x + 2 = 8$

WORKED EXAMPLE 8

Starting with the equation $x = 4$, write the new equation when we:
a multiply both sides by 4
b take 6 from both sides
c divide both sides by $\frac{2}{5}$.

THINK	WRITE
a 1 Write the equation.	**a** $x = 4$
2 Multiply both sides by 4.	$x \times 4 = 4 \times 4$
3 Simplify by removing the multiplication signs. Write numbers before variables.	$4x = 16$
b 1 Write the equation.	**b** $x = 4$
2 Subtract 6 from both sides.	$x - 6 = 4 - 6$
3 Simplify.	$x - 6 = -2$
c 1 Write the equation.	**c** $x = 4$
2 Dividing by a fraction is the same as multiplying by its reciprocal. Multiply both sides by $\frac{5}{2}$.	$x \div \frac{2}{5} = 4 \div \frac{2}{5}$ $x \times \frac{5}{2} = 4 \times \frac{5}{2}$
3 Simplify.	$\frac{5x}{2} = \frac{20}{2}$ $\frac{5x}{2} = 10$

Exercise 11.4 Keeping equations balanced

assess on

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7, 10–13	Questions: 1–13	Questions: 1–14

Individual pathway interactivity int-4448 eBookplus

REFLECTION
How does using inverse operations keep the equations balanced?

FLUENCY

1 WE8 Starting with the equation $x = 6$, write the new equation when we:

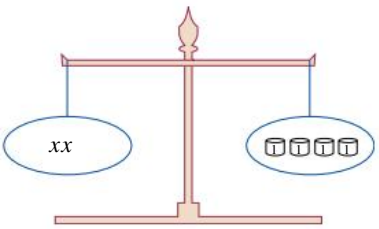
a add 5 to both sides
b multiply both sides by 7
c take 4 from both sides
d divide both sides by 3
e multiply both sides by -4
f multiply both sides by -1
g divide both sides by -1
h take 9 from both sides
i multiply both sides by $\frac{2}{3}$
j divide both sides by $\frac{2}{3}$
k take $\frac{2}{3}$ from both sides.

eBookplus
Interactivity
Balancing equations
int-0077

UNDERSTANDING

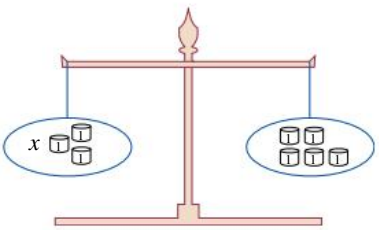
2 **a** Write the equation that is represented by the diagram at right.

b Show what happens when you halve the amount on both sides. Write the new equation.



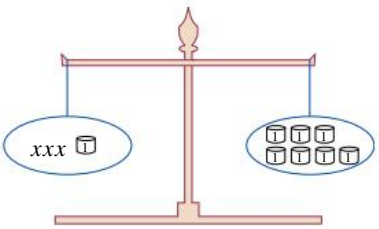
3 **a** Write the equation that is represented by the diagram at right.

b Show what happens when you take three from both sides. Write the new equation.



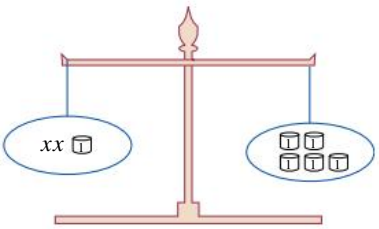
4 **a** Write the equation that is represented by the diagram at right.

b Show what happens when you add three to both sides. Write the new equation.



5 **a** Write the equation that is represented by the diagram at right.

b Show what happens when you double the amount on each side. Write the new equation.



6 MC If we start with $x = 5$, which of these equations is not true?

A $x + 2 = 7$ **B** $3x = 8$ **C** $-2x = -10$

D $\frac{x}{5} = 1$ **E** $x - 2 = 3$

7 MC If we start with $x = 3$, which of these equations is not true?

A $\frac{2x}{3} = 2$ **B** $-2x = -6$ **C** $2x - 6 = 0$

D $\frac{x}{5} = \frac{3}{5}$ **E** $x - 5 = 2$

8 MC If we start with $x = -6$, which of these equations is not true?

A $-x = 6$ **B** $2x = -12$ **C** $x - 6 = 0$

D $x + 4 = -2$ **E** $x - 2 = -8$

9 MC If we start with $2x = 12$, which of these equations is not true?

A $\frac{2x}{3} = 4$ **B** $-2x = -12$ **C** $2x - 6 = 2$

D $4x = 24$ **E** $2x + 5 = 17$

REASONING

10 Make an equivalent equation for each of the equations listed below by performing the operation given in brackets to both sides.

a $m + 8 = 9 \quad (+ 8)$
b $n - 3 = 5 \quad (- 2)$
c $2p + 3 = 9 \quad (+ 3)$
d $3m = 12 \quad (\times 4)$
e $12n = 36 \quad (\div 6)$
f $\frac{p}{3} = 5 \quad (\times 6)$
g $3p + 7 = 2 \quad (\times 4)$
h $5m - 3 = -1 \quad (\div 3)$
i $x + 5 = 7 \quad (\times a)$

PROBLEM SOLVING

11 Some simple equations are hard to represent on a balance scale. What would you do to balance the following equations?

a $x - 4 = 10$
b $x - 6 = 3$
c $\frac{w}{3} = 7$
d $\frac{e}{2} = 9$
e $\frac{q}{4} - 2 = 10$
f $\frac{q - 2}{4} = 10$

12 **a** Why is it difficult to represent the problems in question **11** on a balance scale?
b What is the difference between parts **e** and **f** in question **11**?

13 Solve the equations in question **11**.

14 Represent the following problems on balance scales and solve for the unknown quantity.

a Five bags of sugar weigh three kilos. What does one bag of sugar weigh?
b Four chocolate bars and an ice-cream cost $9.90. If the ice-cream costs $2.70, what does a chocolate bar cost?
c A shopkeeper weighed three apples at 920 g. He thought this was heavy, and realised somebody had left a 500-g weight on his balance scale. What was the average weight of one apple?

CHALLENGE 11.1

You have eight $1 coins, one of which is heavier than the rest. Using a set of balance scales, describe how you can find the heavy coin in the least number of weighings.

eBookplus
Digital doc
WorkSHEET 11.1
doc-2351

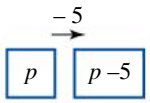
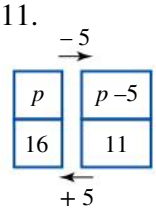
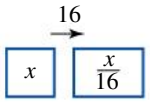
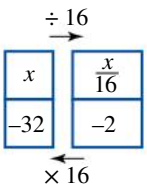
11.5 Using algebra to solve problems

- A **linear equation** is an equation where the variable has an index (power) of 1. This means that it never contains terms like x^2 or $\sqrt{x}$.
- To solve a linear equation, perform the same operations on both sides until the variable or unknown is left by itself.
- Sometimes the variable or unknown is called a pronumeral.
- A flowchart is useful to show you the order of operations applied to x, so that the reverse order and inverse operation can be used to solve the equation. As you become confident with solving equations algebraically, you can leave out the flowchart steps.

WORKED EXAMPLE 9

Solve these one-step equations by doing the same to both sides.

a $p - 5 = 11$ **b** $\frac{x}{16} = -2$

THINK	WRITE
a 1 Write the equation.	**a** $p - 5 = 11$
2 Draw a flowchart and fill in the arrow to show what has been done to p. 	
3 Backtrack from 11. 	
4 Add 5 to both sides.	$p - 5 + 5 = 11 + 5$
5 Give the solution.	$p = 16$
b 1 Write the equation.	**b** $\frac{x}{16} = -2$
2 Draw a flowchart and fill in the arrow to show what has been done to x. 	
3 Backtrack from −2. 	
4 Multiply both sides by 16.	$\frac{x}{16} \times 16 = -2 \times 16$
5 Give the solution.	$x = -32$

- The equations in Worked example 9 are called one-step equations because only one operation needs to be undone to obtain the value of the unknown.

WORKED EXAMPLE 10

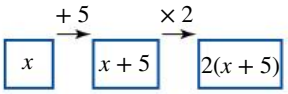
Solve these two-step equations by doing the same to both sides.

a $2(x + 5) = 18$ **b** $\frac{x}{3} + 1 = 7$

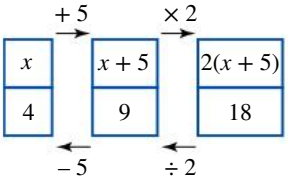
THINK

a 1 Write the equation.

2 Draw a flowchart and fill in the arrow to show what has been done to x.



3 Backtrack from 18.



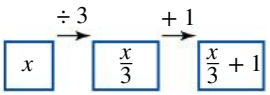
4 Divide both sides by 2.

5 Subtract 5 from both sides.

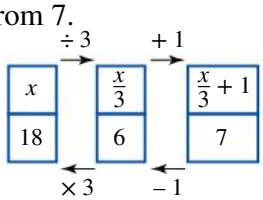
6 Give the solution.

b 1 Write the equation.

2 Draw a flowchart and fill in the arrows to show what has been done to x.



3 Backtrack from 7.



4 Subtract 1 from both sides.

5 Multiply both sides by 3.

6 Give the solution.

WRITE

a $2(x + 5) = 18$

$$\frac{2(x + 5)}{2} = \frac{18}{2}$$
$$x + 5 = 9$$
$$x + 5 - 5 = 9 - 5$$
$$x = 4$$

b $\frac{x}{3} + 1 = 7$

$$\frac{x}{3} + 1 - 1 = 7 - 1$$
$$\frac{x}{3} = 6$$
$$\frac{x}{3} \times 3 = 6 \times 3$$
$$x = 18$$

- The equations in Worked example 10 are called two-step equations because two operations need to be undone to obtain the value of the unknown.

WORKED EXAMPLE 11

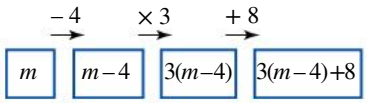
Solve the following equations by doing the same to both sides.

a $3(m-4)+8=5$ **b** $6\left(\frac{x}{2}+5\right)=-18$

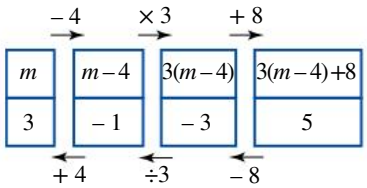
THINK

a 1 Write the equation.

2 Draw a flowchart and fill in the arrows to show what has been done to m.



3 Backtrack from 5.



4 Subtract 8 from both sides.

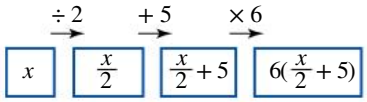
5 Divide both sides by 3.

6 Add 4 to both sides.

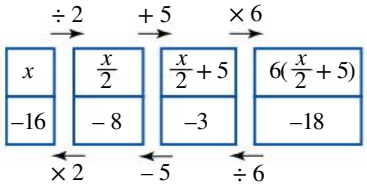
7 Give the solution.

b 1 Write the equation.

2 Draw a flowchart and fill in the arrows to show what has been done to x.



3 Backtrack from −18.



WRITE

a $3(m-4)+8=5$

$3(m-4)+8-8=5-8$

$3(m-4)=-3$

$\frac{3(m-4)}{3}=\frac{-3}{3}$

$m-4=-1$

$m-4+4=-1+4$

$m=3$

b $6\left(\frac{x}{2}+5\right)=-18$

4 Divide both sides by 6.

$$\frac{6\left(\frac{x}{2}+5\right)}{6}=\frac{-18}{6}$$
$$\frac{x}{2}+5=-3$$

5 Subtract 5 from both sides.

$$\frac{x}{2}+5-5=-3-5$$
$$\frac{x}{2}=-8$$

6 Multiply both sides by 2.

$$\frac{x}{2}\times 2=-8\times 2$$

7 Give the solution.

$$x=-16$$

Exercise 11.5 Using algebra to solve problems

assessON

INDIVIDUAL PATHWAYS

PRACTISE
Questions:
1, 2a, b, e, f, i, j, 3a, e, i, m, q, r, 4: columns 1 and 2;
5: columns 1 and 2;
6, 7: column 1; 8, 11, 12

CONSOLIDATE
Questions:
1, 2, 3a–l, 4, 5a–l, 6–12

MASTER
Questions:
1i–l, 2i–l, 3i–l, 4i–l, 6–14

REFLECTION
How do you decide on the order to undo the operations?

Individual pathway interactivity int-4449 eBookplus

FLUENCY

1 WE9a Solve these one-step equations by doing the same to both sides.

a $x+8=7$ **b** $12+r=7$
c $31=t+7$ **d** $w+4.2=6.9$
e $\frac{5}{8}=m+\frac{1}{8}$ **f** $\frac{2}{7}=j+3$
g $q-8=11$ **h** $-16+r=-7$
i $21=t-11$ **j** $y-5.7=8.8$
k $-\frac{11}{7}=z-\frac{2}{3}$ **l** $-\frac{9}{13}=f-1$

2 WE9b Solve these one-step equations by doing the same to both sides.

a $11d=88$ **b** $7p=-98$
c $5u=4$ **d** $2.5g=12.5$
e $8m=\frac{1}{4}$ **f** $-\frac{3}{5}=9j$
g $\frac{t}{8}=3$ **h** $\frac{k}{5}=-12$
i $-5.3=\frac{l}{4}$ **j** $\frac{v}{6}=\frac{2}{3}$
k $\frac{c}{9}=-\frac{5}{27}$ **l** $-\frac{7}{12}=\frac{h}{5}$

3 WE10a Solve these two-step equations by doing the same to both sides.

a $3m+5=14$ **b** $-2w+6=16$
c $-5k-12=8$ **d** $4t-3=-15$

eBookplus
Digital doc
Spreadsheet
2-step equations
doc-2353

e $2(m-4)=-6$ **f** $-3(n+12)=18$
g $5(k+6)=-15$ **h** $-6(s+11)=-24$
i $2m+3=10$ **j** $40=-5(p+6)$
k $5-3g=14$ **l** $11-4f=-9$
m $2q-4.9=13.2$ **n** $7.6+5r=-8.4$
o $13.6=4t-0.8$ **p** $-6k+7.3=8.5$
q $-4g-\frac{1}{5}=\frac{4}{5}$ **r** $-\frac{3}{8}=2f-\frac{18}{8}$

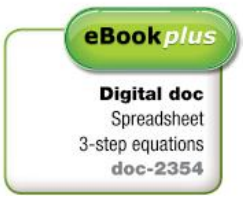
4 **WE10b** Solve these two-step equations by doing the same to both sides.

a $\frac{x}{3}+2=9$ **b** $\frac{x-5}{4}=1$
c $\frac{m+3}{2}=-7$ **d** $\frac{h}{-3}+1=5$
e $\frac{-m}{5}-3=1$ **f** $\frac{2w}{5}=-4$
g $\frac{-3m}{7}=-1$ **h** $\frac{c-7}{3}=-2$
i $\frac{-5m}{4}=10$ **j** $\frac{t+2}{7}=-5$
k $\frac{c-21}{9}=-4.5$ **l** $\frac{x}{8}-3.2=-5.8$

5 **WE11** Solve these equations by doing the same to both sides. They will need more than two steps.

a $2(m+3)+7=3$ **b** $\frac{-2(x+5)}{5}=6$ **c** $\frac{5m+6}{3}=4$
d $\frac{3x-2}{7}=1$ **e** $\frac{4-2x}{3}=6$ **f** $\frac{-x+3}{2}=-4$
g $\frac{3x}{7}-2=1$ **h** $\frac{4b}{5}-3=5$ **i** $\frac{7f}{9}+2=-5$
j $6-\frac{4z}{3}=-2$ **k** $8-\frac{6m}{5}=2$ **l** $-9-\frac{5u}{11}=-4$
m $\frac{3m-5}{-2}=7$ **n** $-7(5w+3)=35$ **o** $5\left(\frac{x}{2}-6\right)=-10$
p $\frac{d-7}{2}+10=8$ **q** $\frac{3n+1}{4}-5=2$ **r** $\frac{3(t-5)}{7}+9=6$

UNDERSTANDING



6 Below is Alex's working to solve the equation $2x+3=14$.

$$2x+3=14$$
$$\frac{2x}{2}+3=\frac{14}{2}$$
$$x+3=7$$
$$x+3-3=7-3$$
$$x=4$$

a Is the solution correct?
b If not, can you find where the error is and correct it?

7 Simplify the left-hand side of the following equations by collecting like terms, and then solve.

a $3x + 5 + 2x + 4 = 19$
b $13v - 4v + 2v = -22$
c $-3m + 6 - 5m + 1 = 15$
d $-3y + 7 + 4y - 2 = 9$
e $-3y - 7y + 4 = 64$
f $5t + 4 - 8t = 19$
g $5w + 3w - 7 + w = 13$
h $w + 7 + w - 15 + w + 1 = -5$
i $7 - 3u + 4 + 2u = 15$
j $7c - 4 - 11 + 3c - 7c + 5 = 8$

REASONING

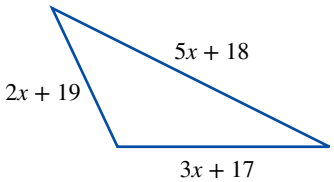
8 A repair person calculates his service fee using the equation $F = 40t + 55$, where F is the service fee in dollars and t is the number of hours spent on the job.

a How long did a particular job take if the service fee was $155?

b Explain what the numbers 40 and 55 could represent as costs in the service fee equation.

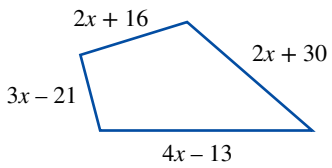
9 Lyn and Peta together raised $517 from their cake stalls at the school fete. If Lyn raised l dollars and Peta raised $286, write an equation that represents the situation and determine the amount Lyn raised.

10 a Write an equation that represents the perimeter of the figure below and then solve for x.



Perimeter = 184 cm

b Write an equation that represents the perimeter of the figure below and then solve for x.



Perimeter = 287 cm

PROBLEM SOLVING

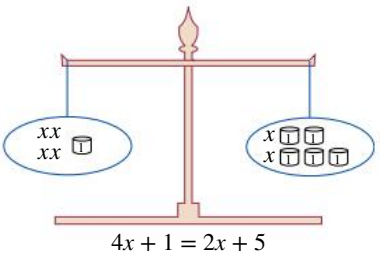
11 If four times a certain number equals nine minus a half of the number, find the number.

12 Tom is 5 years old and his dad is 10 times his age, being 50 years old. Is it possible, at any stage, for Tom's dad to be twice the age of his son? Explain your answer.

13 Lauren earns the same amount for mowing four of the neighbours' lawns every month. Each month she saves all her pay except $30, which she spends on her mobile phone. If she has $600 at the end of the year, how much did she earn each month? (Write an equation to solve for this situation first.)

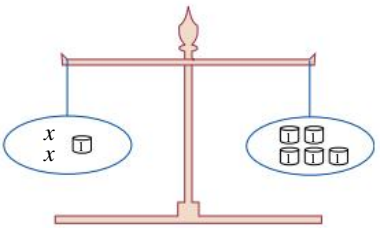
11.6 Equations with the unknown on both sides

- Some equations have unknowns on both sides of the equation.
- If an equation has unknowns on both sides, eliminate the unknowns from one side and then solve as usual.
- Consider the equation $4x + 1 = 2x + 5$.
  - Drawing the equation on a pair of scales looks like this:



$4x + 1 = 2x + 5$

  - The scales remain balanced if $2x$ is eliminated from both sides:



$2x + 1 = 5$

  - Writing this algebraically, we have:

$$\begin{aligned} 4x + 1 &= 2x + 5 \\ 4x + 1 - 2x &= 2x + 5 - 2x \\ 2x + 1 &= 5 \\ 2x + 1 - 1 &= 5 - 1 \\ 2x &= 4 \\ \frac{2x}{2} &= \frac{4}{2} \\ x &= 2 \end{aligned}$$

WORKED EXAMPLE 12

Solve the equation $5t - 8 = 3t + 12$ and check your solution.

THINK	WRITE
1 Write the equation.	$5t - 8 = 3t + 12$
2 Subtract the smaller unknown (that is, $3t$) from both sides and simplify.	$5t - 8 - 3t = 3t + 12 - 3t$ $2t - 8 = 12$
3 Add 8 to both sides and simplify.	$2t - 8 + 8 = 12 + 8$ $2t = 20$
4 Divide both sides by 2 and simplify.	$\frac{2t}{2} = \frac{20}{2}$ $t = 10$

5 Check the solution by substituting $t = 10$ into the left-hand side and then the right-hand side of the equation.	If $t = 10$, LHS $= 5t - 8$ $= 50 - 8$ $= 42$ If $t = 10$, RHS $= 3t + 12$ $= 30 + 12$ $= 42$
6 Comment on the answers obtained.	Since the LHS and RHS are equal, the equation is true when $t = 10$.

WORKED EXAMPLE 13

Solve the equation $3n + 11 = 6 - 2n$ and check your solution.

THINK	WRITE
1 Write the equation.	$3n + 11 = 6 - 2n$
2 The inverse of $-2n$ is $+2n$. Therefore, add $2n$ to both sides and simplify.	$3n + 11 + 2n = 6 - 2n + 2n$ $5n + 11 = 6$
3 Subtract 11 from both sides and simplify.	$5n + 11 - 11 = 6 - 11$ $5n = -5$
4 Divide both sides by 5 and simplify.	$\frac{5n}{5} = \frac{-5}{5}$ $n = -1$
5 Check the solution by substituting $n = -1$ into the left-hand side and then the right-hand side of the equation.	If $n = -1$, LHS $= 3n + 11$ $= -3 + 11$ $= 8$ If $n = -1$, RHS $= 6 - 2n$ $= 6 - -2$ $= 6 + 2$ $= 8$
6 Comment on the answers obtained.	Since the LHS and RHS are equal, the equation is true when $n = -1$.

WORKED EXAMPLE 14

Expand the brackets and then solve the following equation, checking your solution.
a $3(s + 2) = 2(s + 7) + 4$ **b** $4(d + 3) - 2(d + 7) + 4 = 5(d + 2) + 7$

THINK	WRITE
a 1 Write the equation.	a $3(s + 2) = 2(s + 7) + 4$
2 Expand the brackets on each side of the equation first and then simplify.	$3s + 6 = 2s + 14 + 4$ $3s + 6 = 2s + 18$

3 Subtract the smaller unknown term (that is, $2s$) from both sides and simplify.

$$3s + 6 - 2s = 2s + 18 - 2s$$
$$s + 6 = 18$$

4 Subtract 6 from both sides and simplify.

$$s + 6 - 6 = 18 - 6$$
$$s = 12$$

5 Check the solution by substituting $s = 12$ into the left-hand side and then the right-hand side of the equation.

If $s = 12$,

$$\begin{aligned} \text{LHS} &= 3(s + 2) \\ &= 3(12 + 2) \\ &= 3(14) \\ &= 42 \end{aligned}$$

If $s = 12$,

$$\begin{aligned} \text{RHS} &= 2(s + 7) + 4 \\ &= 2(12 + 7) + 4 \\ &= 2(19) + 4 \\ &= 38 + 4 \\ &= 42 \end{aligned}$$

6 Comment on the answers obtained.

Since the LHS and RHS are equal, the equation is true when $s = 12$.

b **1** Write the equation.

b $4(d + 3) - 2(d + 7) + 4 = 5(d + 2) + 7$

2 Expand the brackets on each side of the equation first, and then simplify.

$$4d + 12 - 2d - 14 + 4 = 5d + 10 + 7$$
$$2d + 2 = 5d + 17$$

3 Subtract the smaller unknown term (that is, $2d$) from both sides and simplify.

$$2d - 2d + 2 = 5d - 2d + 17$$
$$2 = 3d + 17$$

4 Rearrange the equation so that the unknown is on the left-hand side of the equation.

$$3d + 17 = 2$$

5 Subtract 17 from both sides and simplify.

$$3d + 17 - 17 = 2 - 17$$
$$3d = -15$$

6 Divide both sides by 3 and simplify.

$$\frac{3d}{3} = -\frac{15}{3}$$
$$d = -5$$

7 Check the solution by substituting $d = -5$ into the left-hand side and then the right-hand side of the equation.

If $d = -5$,

$$\begin{aligned} \text{LHS} &= 4(d + 3) - 2(d + 7) + 4 \\ &= 4(-5 + 3) - 2(-5 + 7) + 4 \\ &= 4(-2) - 2(2) + 4 \\ &= -8 - 4 + 4 \\ &= -8 \end{aligned}$$

If $d = -5$,

$$\begin{aligned} \text{RHS} &= 5(-5 + 2) + 7 \\ &= 5(-3) + 7 \\ &= -15 + 7 \\ &= -8 \end{aligned}$$

8 Comment on the answers obtained.

Since the LHS and RHS are equal, the equation is true when $d = -5$.

- *Note:* When solving equations with the unknown on both sides, it is good practice to remove the unknown with the smaller coefficient from the relevant side.

Exercise 11.6 Equations with the unknown on both sides

assessON

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–4, 6, 8, 9, 13, 15	Questions: 1, 2, 3a–l, 4–10, 13–15	Questions: 1g–i, 2g–i, 3k–p, 4–16

Individual pathway interactivity int-4450 eBookplus

REFLECTION
Does it matter to which side of the equation the unknowns are moved?

FLUENCY

eBookplus
Digital docs
SkillSHEET
Combining like terms
doc-6984
SkillSHEET
Expanding expressions containing brackets
doc-6985
SkillSHEET
Checking solutions by substitution
doc-6986
SkillSHEET
Writing equations from worded statements
doc-6987

1 WE12 Solve the following equations and check your solutions.

a $8x + 5 = 6x + 11$
b $5y - 5 = 2y + 7$
c $11n - 1 = 6n + 19$
d $6t + 5 = 3t + 17$
e $2w + 6 = w + 11$
f $4y - 2 = y + 9$
g $3z - 15 = 2z - 11$
h $5a + 2 = 2a - 10$
i $2s + 9 = 5s + 3$
j $k + 5 = 7k - 19$
k $4w + 9 = 2w + 3$
l $7v + 5 = 3v - 11$

2 WE13 Solve the following equations and check your solutions.

a $3w + 1 = 11 - 2w$
b $2b + 7 = 13 - b$
c $4n - 3 = 17 - 6n$
d $3s + 1 = 16 - 2s$
e $5a + 12 = -6 - a$
f $7m + 2 = -3m + 22$
g $p + 7 = -p + 15$
h $3 + 2d = 15 - 2d$
i $5 + m = 5 - m$
j $7s + 3 = 15 - 5s$
k $3t - 7 = -17 - 2t$
l $16 - 2x = x + 4$

eBookplus
Digital doc
Spreadsheet
Unknowns on both sides
doc-2355

3 **WE14** Expand the brackets and then solve the following equations, checking your solutions.

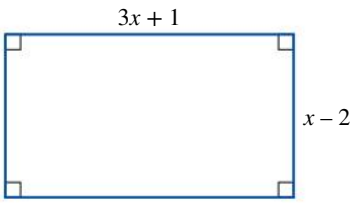
a $3(2x + 1) + 3x = 30$
b $2(4m - 7) + m = 76$
c $3(2n - 1) = 4(n + 5) + 1$
d $t + 4 = 3(2t - 7)$
e $3d - 5 = 3(4 - d)$
f $4(3 - w) = 5w + 1$
g $2(k + 5) - 3(k - 1) = k - 7$
h $4(2 - s) = -2(3s - 1)$
i $-3(z + 3) = 2(4 - z)$
j $5(v + 2) = 7(v + 1)$
k $2m + 3(2m - 7) = 4 + 5(m + 2)$
l $3d + 2(d + 1) = 5(3d - 7)$
m $4(d + 3) - 2(d + 7) + 5 = 5(d + 12)$
n $5(k + 11) + 2(k - 3) - 7 = 2(k - 4)$
o $7(v - 3) - 2(5 - v) + 25 = 4(v + 3) - 8$
p $3(l - 7) + 4(8 - 2l) - 7 = -4(l + 2) - 6$

UNDERSTANDING

4 Solve the equation $\frac{-3(x + 4)}{7} - \frac{5(1 - 3x)}{3} = 3x - 4$.

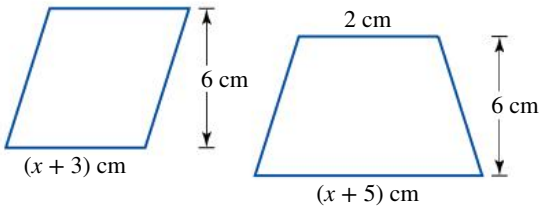
5 Solve the equation $\frac{(x - 2)}{3} + 5 = 2x$.

6 Find the value of x given the perimeter of the rectangle is 48 cm.



7 The two shapes shown have the same area.

a Write an equation to show that the parallelogram and the trapezium have the same area.
b Solve the equation for x.
c State the dimensions of the shapes.



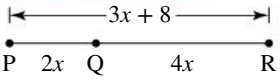
REASONING

8 A maths class has equal numbers of boys and girls. Eight of the girls left early to play in a netball match. This left 3 times as many boys in the class as girls. How many students are in the class? Show your working.

9 Judy is thinking of a number. First she doubles it and adds 2. She realizes that if she multiplies it by 3 and subtracts 1, she gets the same result. Find the number.

10 Mick's father was 28 years old when Mick was born. If his father is now 3 times as old as Mick is, how old are they both now?

11 Given the following number line for a line segment PR, determine the length of PR?



12 In 8 years' time, Tess will be 5 times as old as her age 8 years ago. How old is Tess now?

eBookplus
Interactivity
Solving equations
int-2373

PROBLEM SOLVING

13 You have 12 more than three times the number of marker pens in your pencil case than your friend has in his pencil case. The teacher has 5 more than four times the number of marker pens in your friend's pencil case.

a Write an expression for the number of marker pens in:

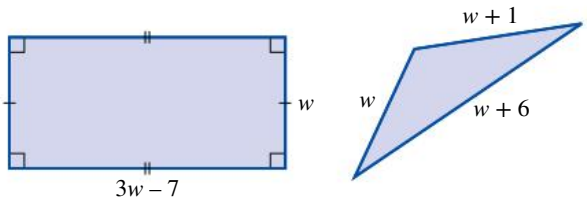
i your pencil case

ii your teacher's pencil case.

b You have the same number of marker pens as the teacher. Write an equation to show this.

c Calculate the number of marker pens in your friend's pencil case by solving the equation from part **b**.

14 The length of the rectangle below is 7 centimetres less than three times its width. If the perimeter of the rectangle is the same as the perimeter of the triangle, calculate the side lengths of the rectangle and triangle.



15 **a** How many rides do you need so that option 1 and option 2 cost the same?

b If you were planning on having only two rides, explain which option you would choose and why.

eBookplus
Digital doc
WorkSHEET 11.2
doc-2352

16 On her birthday today, a mother is three times as old as her daughter will be in eight years time. It just so happens that the mother's age today is the same as their house number, which is four times the value of thirteen minus the daughter's age.

a Write each sentence above as an expression. (*Hint*: Make the daughter's age the unknown variable.)

b Use these two expressions to make an equation and solve it.

c How old are the mother and the daughter?

d What is the number of their house?

CHALLENGE 11.2

A balanced scale containing boxes of Smarties and loose Smarties in the pans. There are three full boxes of Smarties and another box with four Smarties missing in one pan. The other pan contains four empty boxes and 48 loose Smarties. How many Smarties are in a full box?

eBookplus ONLINE ONLY

11.7 Review

www.jacplus.com.au

The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:

- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

Review questions

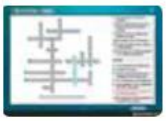
Download the Review questions document from the links found in your eBookPLUS.

eBookplus

Interactivities

Word search
int-2633

Crossword
int-2634



Sudoku
int-3190

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

backtracking
equation
flowchart
geometric pattern
input number
inverse operations
linear equation
output number
patterns
relationships
rule
sequence
term

Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

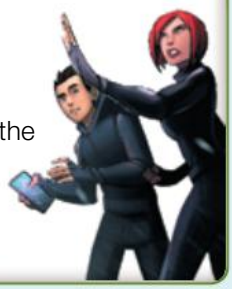
assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

www.assesson.com.au

Link to SpyClass, an exciting online game combining a comic book–style story with problem-based learning in an immersive environment.

Join Jesse, Toby and Dan and help them to tackle some of the world's most dangerous criminals by using the knowledge you've gained through your study of mathematics.

www.spyclass.com.au



RICH TASK

The Olympic freestyle final

Swimming is one of the most popular sports of the Olympic Games. Over the years the Olympics have been held, competitors have been swimming more quickly and their times have been correspondingly reduced.

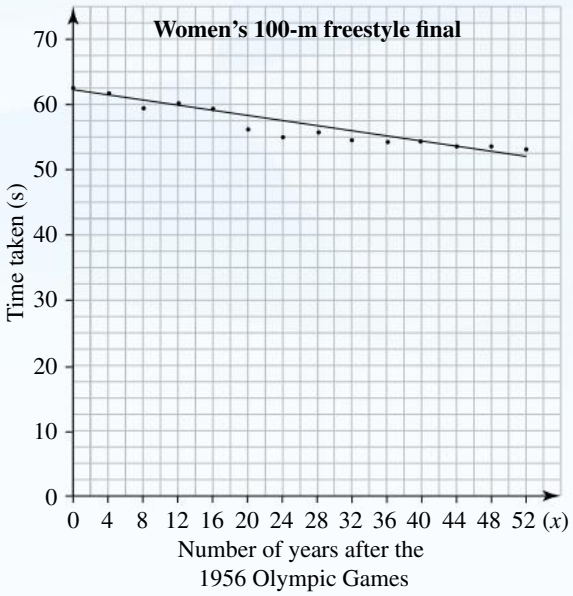
The table below displays the winning times for the men's and women's 100-metre freestyle final for the Olympic Games from 1956 to 2008.

Men's and women's 100-metre freestyle results — Olympic Games 1956–2008

Year	Men's time (seconds)	Women's time (seconds)
1956	55.4	62.0
1960	55.2	61.2
1964	53.4	59.5
1968	52.2	60.0
1972	51.2	58.6
1976	50.0	55.7
1980	50.4	54.8
1984	49.8	55.9
1988	48.6	54.9
1992	49.0	54.7
1996	48.7	54.5
2000	48.3	53.8
2004	48.2	53.8
2008	47.2	53.1

Note: All times have been rounded to 1 decimal place.

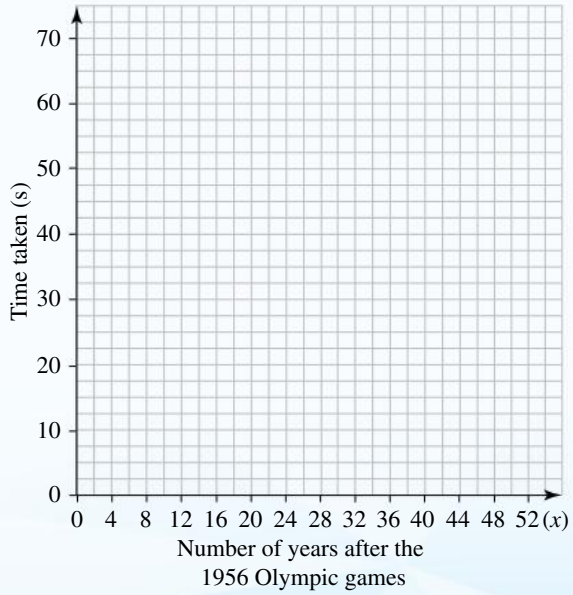
The women's times have been graphed on the set of axes at right. There is no straight line that passes through all the points, so a line of best fit has been selected to approximate the swimming times. The equation of this line is given by $t = 62 - 0.1864x$, where t represents the time taken and x represents the number of years after 1956.



1. What year is represented by $x = 16$?
2. Substitute $x = 16$ into the equation to find an approximation for the time taken. How close is this approximation to the actual time given in the table?
3. What x-value would you use for the 2012 Olympics? Use the equation to predict the women's 100-metre freestyle final time for the 2012 Olympics.
4. The women's time in 2012 was 53.0 seconds. How does your prediction compare with the actual time?
5. Use backtracking to solve the equation to the nearest whole number when $t = 56$ seconds. What year does your solution represent?

An equation to approximate the men's times is given by $t = 54 - 0.1364x$. Use this equation to answer the following questions.

6. The men's time in 1984 was 49.8 seconds. How does this compare with the time obtained from the equation?
7. Use the equation to predict the men's time for the 2012 Olympic Games.
8. Plot the men's times on the set of axes provided. Between the points, draw in a line of best fit.
9. Extend your line of best fit so that it passes the x-value that represents the year 2012. How does this value compare with the prediction you obtained in question **7** above?



10. The men's time in 2012 was 47.52 seconds. How does your prediction compare with the actual time?
11. Use your graphs to compare both the men's and women's times with the actual results obtained during the 2004 Athens Olympics.

How long will it be before the men and women are swimming identical times?

12. Investigate this by plotting the times given on the same set of axes and drawing a line of best fit. Extend both lines until they intersect. The point of intersection represents the time when the swimming times are identical. Present your findings on graph paper.

CODE PUZZLE

Kidneys . . .

Solve the equations to discover the puzzle's answer code.

		4	9	–8	–6	–3	3	2	13	13	
–8	1	–6	3	–3	15	13	4	4	–8	–16	9	2
2	5	1	–8	–1	5	–5	2	6	5	–7	–3	2
–8	9	–6	3	–3	15	13	4	–16	3	1	–5	–3

$2A - 5 = A$	$3D = 10 - 2D$	$7 + 5E = 2E - 2$	$15 - F = F + 7$
$6I + 11 = 74 - I$	$3K + 8 = K - 6$	$L = 4L + 24$	$M + 6 = 3M - 6$
$6(N + 3) = 2N - 2$	$2O + 4 = 3O - 9$	$12 - 4I = 8I$	$3(2R - 1) = 5R$
$3S - 6 - S = S + 9$	$2(T - 6) = 3(2T + 4)$	$U = 48 + 4U$	$9 - 2Y = 5Y + 16$

eBookplus Activities

11.2 Identifying patterns

Digital docs
- SkillSHEET (doc-6978) Number patterns
- SkillSHEET (doc-6979) Using tables to show number patterns
- SkillSHEET (doc-6980) Describing a number pattern from a table

Interactivity
- IP interactivity 11.2 (int-4446) Identifying patterns

11.3 Backtracking and inverse operations

Digital docs
- SkillSHEET (doc-6981) Flowcharts
- SkillSHEET (doc-6982) Inverse operations
- SkillSHEET (doc-6983) Solving equations by backtracking

Interactivity
- IP interactivity 11.3 (int-4447) Backtracking and inverse operations

11.4 Keeping equations balanced

Digital doc
- WorkSHEET 11.1 (doc-2351)

Interactivities
- Balancing equations (int-0077)
- IP interactivity 11.4 (int-4448) Keeping equations balanced

11.5 Using algebra to solve problems

Digital docs
- Spreadsheet (doc-2353) 2-step equations
- Spreadsheet (doc-2354) 3-step equations

Interactivity
- IP interactivity 11.5 (int-4449) Using algebra to solve problems

11.6 Equations with the unknown on both sides

Digital docs
- SkillSHEET (doc-6984) Combining like terms
- SkillSHEET (doc-6985) Expanding expressions containing brackets
- SkillSHEET (doc-6986) Checking solutions by substitution
- SkillSHEET (doc-6987) Writing equations from worded statements
- Spreadsheet (doc-2355) Unknowns on both sides
- WorkSHEET 11.2 (doc-2352)

Interactivities
- Solving equations (int-2373)
- IP interactivity 11.6 (int-4450) Equations with the unknown on both sides

11.7 Review

Interactivities
- Word search (int-2633)
- Crossword (int-2634)
- Sudoku (int-3190)

Digital docs
- Topic summary (doc-10762)
- Concept map (doc-10775)
- Topic review (Word doc-14942, PDF doc-14943)

To access eBookPLUS activities, log on to www.jacplus.com.au

Answers

TOPIC 11 Linear equations

11.2 Identifying patterns

1 **a** Add 2; 10, 12, 14
b Add 5; 23, 28, 33
c Subtract 3; 15, 12, 9
d Multiply by 3; 81, 243, 729
e Divide by 2; 8, 4, 2
f Squares of numbers 1; 25, 36, 49

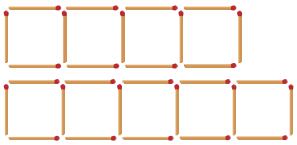
2 **a** 6, 15, 18
b 10, 12, 16
c 16, 64
d 9, 11, 17
e 88, 110
f 95, 90, 75

3 **a** 1, 5, 9, 13, 17
b 5, 15, 45, 135, 405
c 50, 42, 34, 26, 18
d 64, 32, 16, 8, 4
e 1, 4, 10, 22, 46
f 1, 6, 16, 36, 76

4 **a** Even numbers/multiples of 2
b Cubes
c Prime numbers
d Fibonacci sequence
e Factors of 12
f Multiples/powers of 3

5 **a** 2, 4, 8, 6
b 3, 9, 7, 1
c 4, 6
d 5
e 8, 4, 2, 6

6 **a** **i**



ii

Number of squares	1	2	3	4	5
Number of matches	4	7	10	13	16

iii Number of matches = number of squares × 3 + 1
iv 61

b **i**

ii

Number of triangles	1	2	3	4	5
Number of matches	3	5	7	9	11

iii Number of matches = number of triangles × 2 + 1
iv 41

c **i**

ii

Number of houses	1	2	3	4	5
Number of matches	6	11	16	21	26

iii Number of matches = number of houses × 5 + 1
iv 101

d **i**

ii

Number of fence units	1	2	3	4	5
Number of matches	4	7	10	13	16

iii Number of matches = number of fence units × 3 + 1
iv 61

7 **a, b**

2
4 6
8 10 12
14 16 18 20
22 24 26 28 30
32 34 36 38 40 42
44 46 48 50 52 54 56
58 60

c–d Check with your teacher.

8 **a** Grains of rice = 3^{n-1}
b 8th square

9 **a** 8, 13, 21
b b, c, $b + c$

11.3 Backtracking and inverse operations

1 **a** 8 **b** 20 **c** −4
d −19 **e** −1 **f** −1
g −2 **h** −44 **i** 8
j 4 **k** 1.075 **l** 4.4

2 **a** $2x - 7$ **b** $2(w - 7)$ **c** $-5s + 3$
d $-5(n + 3)$ **e** $\frac{m}{2} + 7$ **f** $\frac{y + 7}{2}$
g $\frac{6z - 3}{2}$ **h** $\frac{-3(d + 5)}{4}$ **i** $\frac{2e}{5} + 1$
j $4(3 - x)$ **k** $\frac{-2(w - 5)}{7}$ **l** $-3(z + 6) - 11$
m $\frac{v - 3}{6} - 8$ **n** $-7(8m - 4)$
o $\frac{-5k}{6} + 2$ **p** $\frac{-5p - 7}{3}$

3 **a** $x \xrightarrow{+7} x + 7 \xrightarrow{\times 2} 2(x + 7)$
b $x \xrightarrow{-8} x - 8 \xrightarrow{\times -2} -2(x - 8)$
c $m \xrightarrow{\times 3} 3m \xrightarrow{-6} 3m - 6$
d $m \xrightarrow{\times -3} -3m \xrightarrow{-6} -3m - 6$
e $x \xrightarrow{-5} x - 5 \xrightarrow{\div 8} \frac{x - 5}{8}$
f $x \xrightarrow{\div 8} \frac{x}{8} \xrightarrow{-5} \frac{x}{8} - 5$
g $x \xrightarrow{\times -5} -5x \xrightarrow{+11} -5x + 11$
h $x \xrightarrow{\times -1} -x \xrightarrow{+11} -x + 11$
i $x \xrightarrow{\times -1} -x \xrightarrow{-13} -x - 13$
j $x \xrightarrow{\times -2} -2x \xrightarrow{+5} -2x + 5$
k $x \xrightarrow{\times 3} 3x \xrightarrow{-7} 3x - 7 \xrightarrow{\div 4} \frac{3x - 7}{4}$
l $x \xrightarrow{-2} x - 2 \xrightarrow{\times -3} -3(x - 2) \xrightarrow{\div 4} \frac{-3(x - 2)}{4}$
m $x \xrightarrow{+5} x + 5 \xrightarrow{\div 8} \frac{x + 5}{8} \xrightarrow{-3} \frac{x + 5}{8} - 3$
n $x \xrightarrow{\div 5} \frac{x}{5} \xrightarrow{-2} \frac{x}{5} - 2 \xrightarrow{\times -7} -7(\frac{x}{5} - 2)$

o $\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{\div 7} \boxed{\frac{2x}{7}} \xrightarrow{+4} \boxed{\frac{2x}{7}+4} \xrightarrow{\times 3} \boxed{3\left(\frac{2x}{7}+4\right)}$

p $\boxed{x} \xrightarrow{\times 6} \boxed{6x} \xrightarrow{\div 11} \boxed{\frac{6x}{11}} \xrightarrow{-3} \boxed{\frac{6x}{11}-3} \xrightarrow{\times \frac{1}{4}} \boxed{\frac{1}{4}\left(\frac{6x}{11}-3\right)}$

4 a $\boxed{F} \xrightarrow{-32} \boxed{F-32} \xrightarrow{\times 5} \boxed{5(F-32)} \xrightarrow{\div 9} \boxed{\frac{5(F-32)}{9}}$

b 10

c 95

5 a $\boxed{n} \xrightarrow{\times 2} \boxed{2n} \xrightarrow{+10} \boxed{2n+10} \xrightarrow{\div 2} \boxed{\frac{2n+10}{2}}$

b It is 5. **c** It is always 5.

6 a Strategies include 'guess and check'. Backtracking will not work as the pronumeral w appears more than once in the equation and cannot be simplified.

b $w = 15$ cm

11.4 Keeping equations balanced

1 a $x + 5 = 11$ **b** $7x = 42$

c $x - 4 = 2$ **d** $\frac{x}{3} = 2$

e $-4x = -24$ **f** $-x = -6$

g $-x = -6$ **h** $x - 9 = -3$

i $\frac{2x}{3} = 4$ **j** $\frac{3x}{2} = 9$

k $x - \frac{2}{3} = 5\frac{1}{3}$

2 a $2x = 4$ **b** $x = 2$

3 a $x + 3 = 5$ **b** $x = 2$

4 a $3x + 1 = 7$ **b** $3x + 4 = 10$

5 a $2x + 1 = 5$ **b** $4x + 2 = 10$

6 B **7** E

8 C **9** C

10 a $m + 16 = 17$ **b** $n - 5 = 3$

c $2p + 6 = 12$ **d** $12m = 48$

e $2n = 6$ **f** $2p = 30$

g $12p + 28 = 8$ **h** $\frac{5m + 3}{3} = \frac{-1}{3}$

i $ax + 5a = 7a$

11 a Add 4 to both sides.

b Add 6 to both sides.

c Multiply both sides by 3.

d Multiply both sides by 2.

e Add 2 to both sides, then multiply both sides by 4.

f Multiply both sides by 4, then add 2 to both sides.

12 a They have subtraction and division in the expressions and these cannot be represented on a pan balance scale.

b In part **e**, the pronumeral is divided by 4 and then 2 is subtracted from the result. In part **f**, 4 is subtracted from the pronumeral and then the result is divided by 2.

13 a $x = 14$ **b** $x = 9$ **c** $w = 21$

d $e = 18$ **e** $q = 48$ **f** $q = 42$

14 a $5x = 3$

$x = \frac{3}{5}$. One bag of sugar weighs $\frac{3}{5}$ kg.

b $4x + 2.70 = 9.90$

$x = 1.8$. One chocolate bar costs \$1.80.

c $3x + 500 = 920$

$x = 140$. One apple weighs 140 g.

Challenge 11.1

Weigh 1:

The least number of weighings is two. Take 6 coins and place 3 coins on each side of the balance. There is a remainder of 2 unweighed coins. If the 6 coins balances, the heavy coin must be one of the 2 unweighed coins.

Weigh 2:

If the 6 coins do not balance, the heavy coin must be one of the 3 coins on the heavy side of the balance (the side that drops lower than the other). Take the 3 coins, place one coin on each side of the empty balance, so that there are 2 coins on the scale at a time. If the balance balances, the heavy coin is the coin not on the balance. If the balance does not balance, the heavy coin is on the heavy side of the balance.

11.5 Using algebra to solve problems

1 a $x = -1$ **b** $r = -5$

c $t = 24$ **d** $w = 2.7$

e $m = \frac{1}{2}$ **f** $j = -\frac{19}{7}$ or $-2\frac{5}{7}$

g $q = 19$ **h** $r = 9$

i $t = 32$ **j** $y = 14.5$

k $z = -\frac{19}{21}$ **l** $f = \frac{4}{13}$

2 a $d = 8$ **b** $p = -14$

c $u = \frac{4}{5}$ or 0.8 **d** $g = 5$

e $m = \frac{1}{32}$ **f** $j = -\frac{1}{15}$

g $t = 24$ **h** $k = -60$

i $l = -21.2$ **j** $v = 4$

k $c = -\frac{5}{3}$ or $-1\frac{2}{3}$ **l** $h = -\frac{35}{12}$ or $-2\frac{11}{12}$

3 a $m = 3$ **b** $w = -5$

c $k = -4$ **d** $t = -3$

e $m = 1$ **f** $n = -18$

g $k = -9$ **h** $s = -7$

i $m = 3.5$ **j** $p = -14$

k $g = -3$ **l** $f = 5$

m $q = 9.05$ **n** $r = -3.2$

o $t = 3.6$ **p** $k = -0.2$

q $g = -\frac{1}{4}$ or -0.25 **r** $f = \frac{15}{16}$

4 a $x = 21$ **b** $x = 9$

c $m = -17$ **d** $h = -12$

e $m = -20$ **f** $w = -10$

g $m = 2\frac{1}{3}$ **h** $c = 1$

i $m = -8$ **j** $t = -37$

k $c = -19.5$ **l** $x = -20.8$

5 a $m = -5$ **b** $x = -20$

c $m = 1\frac{1}{5}$ **d** $x = 3$

e $x = -7$ **f** $x = 11$

g $x = 7$ **h** $b = 10$

i $f = -9$ **j** $z = 6$

k $m = 5$ **l** $u = -11$

m $m = -3$ **n** $w = -1\frac{3}{5}$

o $x = 8$ **p** $d = 3$

q $n = 9$ **r** $t = -2$

6 a The solution is not correct.

b Alex should have subtracted 3 from both sides first before dividing both sides by 2. The solution should be $x = 5\frac{1}{2}$. The error made was that he didn't divide all parts of the equation by 2.

7 a $x = 2$ **b** $v = -2$

c $m = -1$ **d** $y = 4$

e $y = -6$ **f** $t = -5$

g $w = 2\frac{2}{9}$ **h** $w = \frac{2}{3}$

i $u = -4$ **j** $c = 6$

8 a $2\frac{1}{2}$ hours

b 40 could be the charge per hour; 55 could be the flat fee covering travel and other expenses.

9 $l + 286 = 517$, $l = \$231$

10 a $10x + 54 = 184$, $x = 13$ cm **b** $11x + 12 = 287$, $x = 25$ cm

11 2
12 Yes, when Tom is 45 and his dad is 90.
13 $80 per month

11.6 Equations with the unknown on both sides

1 **a** $x = 3$ **b** $y = 4$ **c** $n = 4$
d $t = 4$ **e** $w = 5$ **f** $y = 3\frac{2}{3}$
g $z = 4$ **h** $a = -4$ **i** $s = 2$
j $k = 4$ **k** $w = -3$ **l** $v = -4$

2 **a** $w = 2$ **b** $b = 2$ **c** $n = 2$
d $s = 3$ **e** $a = -3$ **f** $m = 2$
g $p = 4$ **h** $d = 3$ **i** $m = 0$
j $s = 1$ **k** $t = -2$ **l** $x = 4$

3 **a** $x = 3$ **b** $m = 10$ **c** $n = 12$
d $t = 5$ **e** $d = 2\frac{5}{6}$ **f** $w = 1\frac{2}{9}$
g $k = 10$ **h** $s = -3$ **i** $z = -17$
j $v = 1\frac{1}{2}$ **k** $m = 11\frac{2}{3}$ **l** $d = 3\frac{7}{10}$
m $d = -19$ **n** $k = -10$ **o** $v = 2$
p $l = 18$

4 $x = -\frac{13}{33}$

5 $x = \frac{13}{5}$ or $2\frac{3}{5}$

6 6.25 cm

7 **a** $6(x + 3) = \frac{1}{2} \times 6 \times (x + 5 + 2)$
b $x = 1$ cm
c Parallelogram: base 4 cm, height 6 cm, Trapezium: base 6 cm, top 2 cm, height 6 cm.

8 24
9 3
10 Mick is 14, his father is 42.
11 16
12 12
13 **a** **i** $3x + 12$ **ii** $4x + 5$
b $3x + 12 = 4x + 5$
c $x = 7$

14 $2(3w - 7) + 2w = 3w + 7$
$6w - 14 + 2w = 3w + 7$
$5w = 21$
$w = 4.2$
Rectangle lengths are 4.2 cm and 5.6 cm; triangle lengths are 4.2 cm, 5.2 cm and 10.2 cm.

15 **a** $2.5x + 10 = 3.5x + 5$
$x = 5$
Both options cost the same for 5 rides.
b Option 1: $2.5 \times 2 + 10 = \$15$; option 2: $3.5 \times 2 + 5 = \$12$.
Option 2 is cheaper for 2 rides.

16 **a** Let daughter's age = x. Mother's age = $3(x + 8)$ and $= 4(13 - x)$.
b $3(x + 8) = 4(13 - x)$
$3x + 24 = 52 - 4x$
$x = 4$
c The daughter is 4 and the mother is 36.
d The number of their house is 36.

Challenge 11.2

13 Smarties

Investigation — Rich task

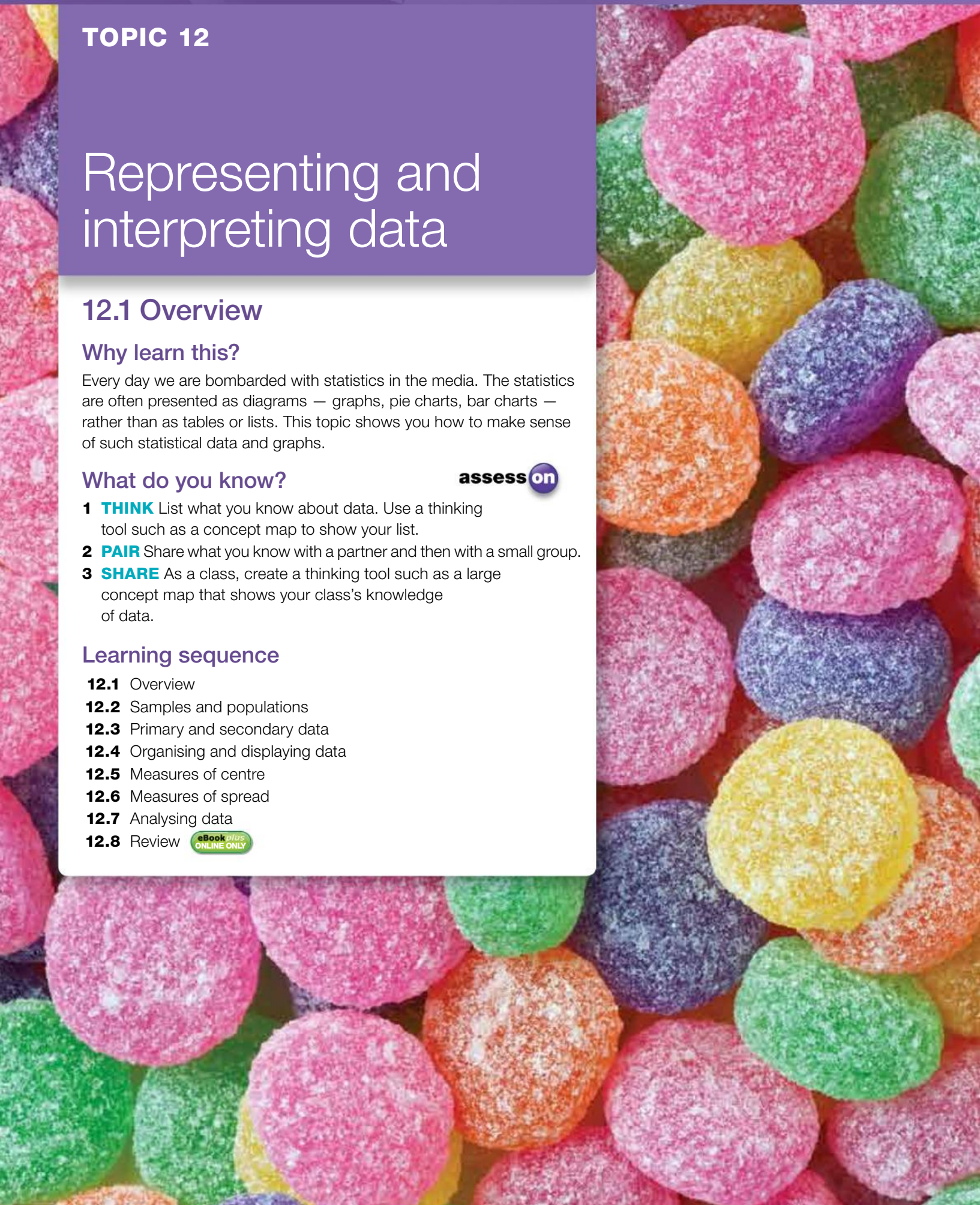
1 1972
2 $t = 59.0$ s; difference of 0.4 s
3 $x = 56$, $t = 51.6$ s
4 Teacher to check.
5 $x = 32$; year is 1988
6 $t = 50.2$ s; difference of 0.4 s
7 $t = 46.4$ s
8–12 Teacher to check.

Code puzzle

Filter 200 litres of fluid daily and make 2 litres of urine.

TOPIC 12

Representing and interpreting data



12.1 Overview

Why learn this?

Every day we are bombarded with statistics in the media. The statistics are often presented as diagrams — graphs, pie charts, bar charts — rather than as tables or lists. This topic shows you how to make sense of such statistical data and graphs.

What do you know?

assessON

1 **THINK** List what you know about data. Use a thinking tool such as a concept map to show your list.
2 **PAIR** Share what you know with a partner and then with a small group.
3 **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of data.

Learning sequence

12.1 Overview
12.2 Samples and populations
12.3 Primary and secondary data
12.4 Organising and displaying data
12.5 Measures of centre
12.6 Measures of spread
12.7 Analysing data
12.8 Review eBook plus ONLINE ONLY

12.2 Samples and populations

Collecting data

- Information or **data** is constantly being collected.
- Different organisations collect different types of data. For example, at a cricket match, some of the **statistics** gathered for a batsman are: time spent batting, the number of balls faced, the runs off a particular delivery, where the ball was hit, the number of 4s or 6s hit, and so on.
- Once the data are collected, they can be organised, analysed and interpreted.
- A *survey* is the process of collecting data. If every member of a target population is surveyed, the process is called a **census**.
- A census is conducted in Australia every 5 years to obtain an accurate profile of Australians. On census night each person in Australia is required to complete a detailed booklet containing a series of questions relating to age, marital status, employment, income, housing, education, modes of transport and so on. This allows the government to analyse the population and make decisions on how to improve services.
- Due to limitations in time, cost and practicality, in many cases a **sample** of the population is selected at **random** (not in any particular order or pattern) to prevent biased (leaning in a favoured direction) results.

eBookplus
Digital docs
Investigation
Data and the Olympics
doc-7013
Spreadsheet
Data and the Olympics
doc-7106

WORKED EXAMPLE 1

In each of the following, state if the information was obtained by census or survey.

a A school uses a roll to count the number of students absent each day.
b The television ratings, in which 2000 families complete a questionnaire on the programs they watch over a one-week period.
c A battery manufacturer tests every hundredth battery off the production line.
d A teacher records the examination marks of her class.

THINK	WRITE
a Every student is counted at roll call each morning.	a Census
b Not every family is asked to complete a ratings questionnaire.	b Survey
c Not every battery is tested.	c Survey
d The marks of every student are recorded.	d Census

- Data can be collected using a variety of techniques. Three common methods are:
  - observation
  - survey
  - experiment.

Surveys

- Collecting data by survey is the form most frequently used. The survey is administered with the aid of a questionnaire. The success of obtaining meaningful and relevant data from a questionnaire depends largely on the care taken in designing the questions. Methods used to collect data include:
  - personal interviews, where the interviewer usually asks prepared questions, then records the replies from the respondent
  - telephone interviews, where the interview is conducted over the telephone
  - self-administered questionnaires, which are usually mailed to individuals who complete the questionnaire, then return it in a pre-paid envelope, or hold it for collection.

Questionnaires

- When a **questionnaire** is designed, we must keep in mind that the collected data should be in a form that is easy to analyse. Questions can be **open** or **closed**.
- Open questions are those where the respondent has no guided boundaries within which to answer. Questions that belong to this class include:
  'Who is your favourite singer?'
  'What is your favourite food?'
  The main problem with open questions is that their answers are often difficult to classify and analyse.
- Closed questions are of the type where the respondent must answer within a category. The question about food above could be rewritten as:
  'Which one of the following foods do you prefer most?'
  - Meat
  - Seafood
  - Poultry
  - Vegetables
  - Fruit

  These types of answers are easier to analyse than answers to open questions. It must be noted that options such as:
  - None of the above or
  - Don't know

  should be avoided, if possible, as they provide the respondent with a 'way out'.

WORKED EXAMPLE 2

Ten students were asked the open question:
'What do you dislike about your school uniform?'
These were their responses.

1. **It's too old-fashioned.**
2. **It costs too much.**
3. **It's just yucky.**
4. **It's a dreadful colour.**
5. **The girls aren't allowed to wear slacks.**
6. **The colour is drab.**
7. **The skirt is too long.**
8. **It would be better to have it all one colour rather than three different colours.**
9. **The shoes are too uncomfortable.**
10. **We should be allowed to wear leggings in winter.**

Classify the responses into categories to identify the two main reasons given.

THINK	WRITE
1 Since two main reasons have to be identified, look for, say, 3 categories which could be used to classify the data.	Two main categories are apparent: 1 Style 2 Colour A third category is not apparent.
2 Classify the responses.	**Style** Responses 1, 5, 7, 9, 10 **Colour** Responses 4, 6, 8 Responses 2 and 3 can be in a miscellaneous category.
3 Identify the main reasons.	The main reasons students dislike the uniform are its style and colour.

It can be seen from this worked example that it is probably worth taking the time to write closed questions for a questionnaire, as this reduces the incidence of answers that are difficult to classify.

Planning a questionnaire

- To prepare good questions you should keep the following points in mind.
  1. The questions should flow smoothly from one to the next.
  2. Introductory remarks should be included, outlining the aim and purpose of the questionnaire, along with any necessary instructions.
  3. Jargon, slang and abbreviations should be avoided.
  4. Do not ask questions that are vague or ambiguous.
  5. Avoid bias and emotional language.
  6. Avoid double-barrel questions.
  7. Do not ask leading questions (those that lead to an expected response).
  8. Make sure your questions are capable of being answered by your respondents.
  9. Avoid questions with double negatives.
  10. At the conclusion, thank the respondent for answering.

WORKED EXAMPLE 3

Identify the areas of concern with the following questions, then rewrite each so that the meaning is clear, unbiased and understandable.

a Don't you agree that the school uniform should be changed?

b Didn't you know that students are not allowed to go to the school canteen at morning tea?

c Those poor, distraught animals in the zoo should be running wild and not restricted in cages. Don't you agree?

d How old are you?

THINK	WRITE
a This is a leading question — leading the respondent to agree.	**a** This is a leading question. Are you in favour of a change in the school uniform?
b This question has two negative parts causing confusion for the respondent.	**b** This is a double negative question. Are you aware of the rule banning students from the school canteen at morning tea?
c This question plays on the emotions of the respondent.	**c** This question raises an emotional issue. What's your opinion about keeping animals in cages at zoos?
d People (particularly older people) often don't like disclosing personal details.	**d** This is a personal question. To which age group do you belong? • Less than 10 years • 10–20 years • 21–50 years • Over 50 years of age

Selecting samples

- When a sample is selected from a population, care must be taken to ensure that the composition of the sample is as close as possible to that of the population.
- One of the simplest ways to select a reliable sample is by using a random sampling technique.
- A way in which this can be achieved is by numbering each individual item (element) of the population, then randomly selecting items for the sample by using random digit tables, the random function on a calculator or (without looking) drawing numbers from a container.

WORKED EXAMPLE 4

As part of their quality control program, staff members at Perplexing Puzzles select 5 puzzles from a batch of 50 coming off a production line, for testing. Use the following table of random numbers to select the 5 puzzles to be tested.

94, 01, 54, 68, 74	**32, 44, 44, 82, 77**	**59, 82, 09, 61, 63**	**64, 65, 42, 58, 43**
71, 10, 88, 82, 22	**88, 57, 07, 40, 15**	**25, 70, 49, 10, 35**	**01, 75, 51, 47, 50**
62, 88, 08, 78, 73	**95, 16, 05, 92, 21**	**22, 30, 49, 03, 14**	**75, 87, 71, 73, 34**
11, 74, 81, 21, 02	**80, 58, 01, 18, 67**	**18, 71, 05, 96, 21**	**06, 55, 40, 78, 50**
17, 94, 40, 56, 00	**60, 47, 80, 33, 43**	**25, 85, 02, 89, 05**	**57, 21, 63, 96, 18**

THINK	WRITE
1 Assign each puzzle on the production line a number, according to its order, ranging from 1 to 50. *Note:* 01 will represent the first puzzle on the production line.	01 — First puzzle on the production line. 02 — Second puzzle on the production line. 03 — Third puzzle on the production line. . . . . . . 50 — Fiftieth puzzle on the production line.

2 Randomly select a starting point and move in a systematic order, jotting down 5 pairs of digits that range from 01 to 50. *Note:* Reject any values that equal 00 or are greater than 50. Reject any repeated values.	Starting position: fourth row, first block, fourth pair of digits; that is, 21. 21 accept 02 accept 80 reject since value is greater than 50 58 reject since value is greater than 50 01 accept 18 accept 67 reject since value is greater than 50 18 reject since value is repeated 71 reject since value is greater than 50 05 accept
3 List the puzzles whose numbers correspond to those selected in step 2 and interpret the results.	The puzzles numbered 21, 02, 01, 18 and 05 will be selected for testing.

eBookplus
Digital doc
Investigation
Bias
doc-2473

Biased samples

- If a sample is not representative of the population, it is said to be **biased**.
- To avoid bias, the size of the sample should be about $\sqrt{\text{population size}}$.

WORKED EXAMPLE 5

Comment on the selection of the sample in the following cases.

a Members of the men's basketball team were interviewed regarding the height of the mirrors in the gym changing room.

b Spectators at a music concert were interviewed regarding the sporting facilities in the town.

c Of the 100 people in the bush walking group, 5 were interviewed regarding increasing the club's fees.

THINK	WRITE
a Are those in the men's basketball team of typical height compared with those who use the gym?	a Those in the men's basketball team would be taller than most of the people who use the gym change room. They would prefer the mirrors to be higher. This is a biased sample.
b Those at a music concert probably would not use the sporting facilities very frequently.	b This is a biased sample as it does not include those who would make greatest use of the sporting facilities.
c This sample is too small.	c This sample of 5 from the group of 100 is too small to represent the views of the whole group.

assessON

Exercise 12.2 Samples and populations

INDIVIDUAL PATHWAYS

REFLECTION
Are the random numbers generated by a computer or calculator truly random?

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–10, 14, 16	Questions: 1–9, 11, 12, 15	Questions: 1–17

Individual pathway interactivity int-4451 eBookplus

FLUENCY

eBookplus
Digital docs
Investigation
Collecting data for surveys and questionnaires
doc-2167

eBookplus
Interactivity
Random numbers
int-0089

1 Copy and complete the following.
When we obtain data from the whole population, we conduct a _____; however, a survey obtains data from a _____ of the population.

2 A school conducts an election for a new school captain. Every teacher and student in the school casts a vote. Is this an example of a census or a survey? Explain your answer.

3 A questionnaire is conducted by the school council to see what library facilities the community needs. If 500 people who live in the community participate, is this an example of a census or a survey?

4 WE1 For each of the following, state whether a census or a survey has been used.

- **a** Fifty people at a shopping centre are asked to nominate the supermarket where they do most of their grocery shopping.
- **b** To find the most popular new car on the road, 300 new-car buyers are asked what make and model they purchased.
- **c** To find the most popular new car on the road, the make and model of every new car registered are recorded.
- **d** To test the life of a light bulb, every 100th bulb is tested.

5 For each of the following, recommend whether you would use a census or a survey to find:

- **a** the most popular TV program on Sunday night at 8.30 pm
- **b** the number of 4-wheel-drive cars sold in a year
- **c** the number of cars travelling on a toll road each day
- **d** the percentage of defective SIM cards produced by a mobile phone manufacturing company.

6 Explain what you understand by the terms ‘open’ and ‘closed’ questions. Give an example of an open question, then rewrite your question in a closed format.

7 WE2 Ten students were asked their opinions about the cause of congestion at the school’s front gate. Analyse their responses below, suggest categories into which they could be classified and identify the most commonly stated reasons for the congestion.

1. The cars shouldn’t come up the front driveway.
2. The front entrance is too small.
3. There should be another entrance.
4. The buses are the problem.
5. Bike riders should have a separate entrance.
6. Parents don’t care where they park.
7. Kids just sit around there talking.
8. The buses all arrive at the same time.
9. The road is not wide enough.
10. There should be a traffic control officer there to direct the traffic.

8 Fifteen students were asked to identify the one thing in their Maths class that they particularly didn’t like. Classify the responses below into appropriate categories, then identify the main reasons.

1. It’s too hard.
2. There’s too much homework.
3. I can’t understand the teacher.
4. It’s boring.

5. The boys are too distracting.
6. The teacher favours the boys.
7. It's too much work.
8. I can't concentrate for that length of time.
9. We do something new every lesson.
10. The teacher expects too much.
11. I don't like working in groups.
12. Our teacher is too strict.
13. Our class is too big.
14. The work is not interesting.
15. I don't like doing presentations to the class.

9 WE3 Identify the areas of concern with the following questions, then rewrite each so that the meaning is clear, unbiased and understandable.

- **a** How much money do you earn?
- **b** Do you exercise regularly?
- **c** Which venue do you prefer — the SCG, the MCG, the Gabba or the WACA?
- **d** Do you support the Premier's policy on wildlife preservation?
- **e** How tall are you in feet and inches?
- **f** Did you buy your sneakers for comfort and quality?

UNDERSTANDING

10 An opinion poll is conducted to try to predict the outcome of an election. People are asked to ring one number if they would vote for the current political party, and a different number if they favour the opposition. Comment on this method of collecting data.

11 Rewrite the following open questions in closed format.

- **a** How much pocket money do you get each week?
- **b** How do you travel to school?
- **c** What type of destination do you prefer for a holiday?

REASONING

12 WE5 Comment on the selection of the sample in the following cases.

- **a** The choir members were interviewed regarding the opening hours of the school swimming pool.
- **b** Spectators at a football match were interviewed after the match to get their views on the Prime Minister's new health package.
- **c** Visitors arriving at the Sydney international airport were interviewed regarding the transport system from there into the city.
- **d** Twenty spectators at the Australian Tennis Open championships were interviewed regarding an appropriate time of the year to hold the event.

13 Use technology to select at random the following samples:

- **a** 6 shoppers from 75 in a store
- **b** 4 babies from 30 born on a particular day
- **c** 6 products from 100 coming off a production line.

PROBLEM SOLVING

14 WE4 As part of their quality control program, staff members at Perplexing Puzzles select 5 puzzles from a batch of 50 coming off a production line, for testing. Use the following table of random numbers to select the 5 puzzles to be tested.

94, 01, 54, 68, 74	32, 44, 44, 82, 77	59, 82, 09, 61, 63	64, 65, 42, 58, 43
71, 10, 88, 82, 22	88, 57, 07, 40, 15	25, 70, 49, 10, 35	01, 75, 51, 47, 50
62, 88, 08, 78, 73	95, 16, 05, 92, 21	22, 30, 49, 03, 14	75, 87, 71, 73, 34
11, 74, 81, 21, 02	80, 58, 01, 18, 67	18, 71, 05, 96, 21	06, 55, 40, 78, 50
17, 94, 40, 56, 00	60, 47, 80, 33, 43	25, 85, 02, 89, 05	57, 21, 63, 96, 18

15 Use the table of random numbers given in question **14** to select at random the following samples:

- **a** 3 people from a population of 99 (work horizontally from top left)
- **b** 4 students from a class of 60 (work horizontally from the bottom row of the second block)
- **c** 3 raffle tickets from a book of 50 (diagonally in the third block, starting at bottom left).

16 MC Using the table of random numbers from question **14**, the sample '5 students from a group of 50' (working horizontally and beginning from the first pair of digits in row 2, block 3) would correspond to:

A 25, 70, 49, 10, 35 **B** 95, 16, 05, 92, 21 **C** 16, 05, 21, 22, 30
D 25, 49, 10, 35, 01 **E** 09, 42, 43, 10, 22

17 Describe how you could use a standard pack of cards to select a random sample from a small population.

CHALLENGE 12.1

A survey was conducted on students to see whether they preferred watching sports or playing sports. Eighteen people preferred watching sports and 16 preferred playing sports. If 30 people completed the survey, and 2 people don't like to watch or play sports, how many like to do both?

12.3 Primary and secondary data

Primary data

- **Primary data** are data that you have collected yourself.
- This means that you have ownership of the data, and no one else has access to the data until it is released or published.
- A variety of methods of collecting primary data is available. These include observation, measurement, survey, experiment or simulation.

Observation

- This involves recording the behavioural patterns of people, objects and events in a systematic manner.
- The data can be collected as a disguised observation (respondents are unaware they are being observed) or undisguised observation (the respondent is aware). CCT cameras are an example of people knowing that their movements are being recorded, but are not always aware of where the recording takes place.
- Observations can be in a natural environment (for example, in a food hall), or a contrived environment (a food-tasting session for a food company).
- Mechanical devices (video cameras, closed circuit television, counting devices across a road) can also be used.

Measurement

- Measurement involves using some measuring device to collect data.
- This generally involves conducting an experiment of some type.
  - The height of everyone in your class can be measured.
  - The mass of all newborn babies can be collected.
  - A pedometer measures the number of paces the wearer takes.

Surveys

- Surveys involve designing a questionnaire to interview people. Often the questionnaire requires many rewrites to obtain one which is clear and unbiased.
- The interview can be in person — face to face or by telephone. The advantage of this method is that you are able to see the reactions of those you are interviewing, and explain particular questions, if necessary.
- Most frequently these days, email is used; however, there are advantages and disadvantages to using this type of survey.
- Advantages
  - Can cover a large number of people or organisations
  - A wide geographic coverage is possible
  - It avoids embarrassment on the part of the respondent
  - There is no interviewer bias
  - The respondent has time to consider responses
  - It's relatively cheap
- Disadvantages
  - The questions have to be relatively simple
  - The response rate is often quite low (inducements are often given as an incentive to return)
  - The reliability of the answers is questionable
  - No control over who actually completes the questionnaire
  - Problems with incomplete questionnaires

Experiment

- Generally, when conducting an experiment the data collected are quantitative.
- Particular care should be taken to ensure that the experiment is conducted in a manner that would produce similar results if repeated.
- Care must be taken with the recording of results.
- The results must be in a form that can readily be analysed.
- All results need to be recorded, including the weird or unexpected outcomes.

Simulation

- Experiments such as rolling a die, tossing a coin or drawing a card from a deck may be conducted to model some real-life situation.
- Simulations occur in areas such as business, engineering, medical and scientific research.
- They are often used to imitate real-life situations, which may be dangerous, impractical or too expensive to explore by other means.

WORKED EXAMPLE 6

It is widely believed that there is equal chance of having a boy or girl with each birth. Genetics and the history of births in a family sometimes have a great influence on the sex of the child as well. Ignore those factors in this question.

a Design an experiment to simulate the chance of giving birth to a boy or a girl.

b Describe how your experiment could be conducted to determine the number of children a couple should have, on average, to ensure they have offspring of both sexes.

THINK

a Use a device that can simulate two outcomes which are equally likely. This could be a random number generator to generate two integers, say a 0 (representing a boy) and 1 (representing a girl).
A fair coin could be tossed, such that, a Head represents a boy, and a Tail represents a girl.

b 1 Describe how the experiment will be conducted.

2 Display the table of results.

WRITE

a A fair coin will be tossed with a Head representing a boy (B), and a Tail representing a girl (G).

b The experiment will be conducted 50 times, and a record kept of each experiment.
For each experiment, the coin will be tossed until both sexes result. This may mean that there could be 7 trials in an experiment (GGGGGGB) before both sexes are represented.

The table below shows the results of the 50 experiments.

Exp. no.	Results	No. of trials	Exp. no.	Results	No. of trials
1	BG	2	26	GGGB	4
2	GGB	3	27	GGGGB	5
3	BG	2	28	GGGB	4
4	GGGGB	5	29	BG	2
5	BBBBBBG	7	30	BBBG	4

(*Continued*)

Exp. no.	Results	No. of trials	Exp. no.	Results	No. of trials
6	GGGB	4	31	BG	2
7	BBG	3	32	GB	2
8	BBG	3	33	GGGB	4
9	BBBBG	5	34	BG	2
10	GB	2	35	GGGGGGB	7
11	BG	2	36	BBBBBBG	7
12	GGGB	4	37	GB	2
13	BBG	3	38	BG	2
14	BBG	3	39	GGB	3
15	GB	2	40	GGGGB	5
16	BG	2	41	BBG	3
17	GGB	3	42	BBBBBG	6
18	GB	2	43	GGB	3
19	GGB	3	44	GGB	3
20	BBBG	4	45	BBBG	4
21	BG	2	46	BBG	3
22	GB	2	47	GGGGGGB	7
23	GGGGB	5	48	BG	2
24	BG	2	49	BBG	3
25	GGGGB	5	50	GGGGB	6
				Total	175

This table shows that 175 trials were undertaken in 50 experiments where each experiment resulted in both sexes.

3 Determine the average number of children required to produce offspring of both sexes.

Average number of children $= \frac{175}{50} = 3.5$

4 Write a conclusion.

The average number of children a couple should have to reach the goal of having both sexes is 4.

- Before collecting any primary data, it must be clear what data are to be collected.
- A decision must be made as to the method of collection.
- The advantages and disadvantages of the collection method must be acknowledged.
- The reason for the data collection should be clear from the outset.

WORKED EXAMPLE 7

You have been asked to obtain primary data to determine the methods of transport used to travel to school by the students at your school. The data collected are to provide support for the Student Council's proposal for a school bus.

a What data should be collected?

b Outline possible methods which could be used to collect this data.

c Decide which method you consider to be the best option, and discuss its advantages and disadvantages.

THINK	WRITE
a Outline the various forms of transport available to the students.	**a** The modes of transport available to students at the school are: car, bus, train, bicycle and walking
b Consider all the alternatives for collecting the data.	**b** Several methods could be used to collect the data. • Could stand at the school gate one morning and ask students as they arrive. • A questionnaire could be designed. • Students could be asked to write their mode of transport on a piece of paper and place in a collection tin.
c 1 Decide on the best option.	**c** The first option of standing at the school gate is very time-consuming, and students could arrive at the back gate. The third option does not seem reliable, as some students may not comply, and other students may place multiple pieces of paper in the collection tin. The second option seems the best of the three.
2 Discuss the advantages and disadvantages.	The advantages of a questionnaire include: • There is a permanent record on paper. • It is not time-consuming to distribute or collect. • Students can complete it at their leisure. Disadvantages include: • Students may not return it. • Expense involved in producing copies.

Note: This example does not represent the views of all those collecting such data. It merely serves to challenge students to explore and discuss available options.

- Sometimes the primary data required are not obvious at the outset of the investigation.
- For example, you are asked to investigate the claim:
  Most students do not eat a proper breakfast before school.
  What questions would you ask to prove or refute this claim?

Secondary data

- **Secondary data** are data that have already been collected by someone else.
- The data can come from a variety of sources:
  – Paper — books, journals, magazines, company reports
  – Electronic — online databases, internet, broadcasts, DVDs
  – Government sources — ABS provides a wealth of statistical data
  – General business sources — academic institutions, stockbroking firms, sporting clubs
  – Media — newspapers, TV reports.
- Secondary data sources often provide data which would not be possible for an individual to collect.
- The data can be qualitative or quantitative.
- The accuracy and reliability of the data sometimes needs to be questioned, depending on its source.
- The age of the data should always be considered.
- Often the data which surrounds us passes by unnoticed.
- It is important to learn the skills to be able to critically analyse secondary data.

WORKED EXAMPLE 8

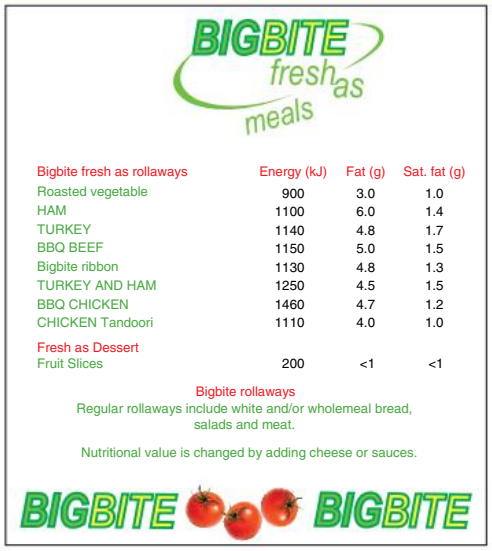
Bigbite advertise the energy and fat content of some of their rollaways on their placemats.

a What information can you gain from this data?

b Bigbite advertise that they have a range of rollaways with less than 6 grams of fat. Comment on this claim.

c This could be the starting point of a statistical investigation. How could you proceed from here?

d Investigations are not conducted simply for the sake of investigating. Suggest some aims for investigating further.



THINK

a Look at the data on the placemat to gain as much information as possible.

b Examine the data to discover if there is evidence to support the claim. Make further comment.

c What would be the next step in the investigation?

d What are some interesting facts which could be revealed through a deeper investigation?

WRITE

a The placemat reveals the following information:
- A higher energy content of a rollaway does not necessarily mean that its fat content is higher.
- As the fat content of a rollaway increases, generally the saturated fat content also increases.
- The addition of some types of protein (ham, turkey, beef, chicken) increases the energy content of the rollaway.
- These data are only for those rollaways on white or wholemeal bread with salads and meat.
- The addition of condiments (sauces) or cheese will alter these figures.
- A fruit slice has much less energy and fat than a rollaway.

b All the rollaways displayed have less than 6 grams of fat, so Bigbite's claim is true.
It must be remembered that the addition of cheese and sauce to these rollaways would increase their fat content. Also, if the rollaway was on any bread other than white or wholemeal, the fat content could go beyond 6 grams.

c The placemat displays a toll-free phone number for further information.
Their web site also contains additional detailed information.

d Suggested aims for investigating further could be:
- How much extra fat is added to a rollaway by the addition of cheese and/or sauce?
- What difference does a different type of bread make to the fat content of the rollaway?
- Which rollaway contains the highest fat content?
- What is the sugar content of the rollaways?

Exercise 12.3 Primary and secondary data

assess on

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–4, 6, 8, 13	Questions: 1–5, 7, 9, 10, 12, 13	Questions: 1a, d, g, 2, 3, 4–14

Individual pathway interactivity int-4452 eBook plus

REFLECTION
When using secondary data from other countries, what different unit classifications could you encounter?

FLUENCY

1 WE6 Devise an experiment to simulate each of the following situations and specify the device used to represent the outcomes.

- **a** A true/false test in which answers are randomly distributed.
- **b** A casino game with outcomes grouped in colours of either red or black.
- **c** Breakfast cereal boxes containing 4 different types of plastic toys.
- **d** In a group of six people, one person is to be chosen as the leader.
- **e** A choice of three main meals on a restaurant's menu, all of which are equally popular.
- **f** Five possible holiday destinations offered by a travel agent; such that all destinations are equally available and equally priced.
- **g** Five types of takeaway fast foods available in one area, where one pizza is twice as popular as each of the others types of takeaway food (the other 4 are equally popular).

2 WE7 You have been asked to obtain primary data from students at your school to determine internet access students have at home. The data collected are to provide support for opening the computer room for student use at night.

- **a** What data should be collected?
- **b** Outline possible methods which could be used to collect this data.
- **c** Decide which method you consider to be the best option, and discuss its advantages and disadvantages.

3 WE8 This label shows the nutritional information of Brand X rolled oats.

Nutrition Information			
Servings Per Package: 25			Serving Size 30g
	Per Serving 30g	%DI* Per Serving	Per 100g
Energy	486kJ	6%	1620kJ
Protein	4.3g	9%	14.3g
Fat - Total	2.8g	4%	9.3g
- Saturated	0.5g	2%	1.7g
- Trans	Less than 0.1g	-	Less than 0.1g
- Polyunsaturated	1.0g	-	3.2g
- Monounsaturated	1.3g	-	4.4g
Carbohydrate	16.8g	5%	56.0
- Sugars	0.9g	1%	3.0g
Dietary Fibre	3.1g	10%	10.4g
Sodium	0.7mg	0.1%	2mg

* % DI = Percentage daily intake

- **a** What information can you gain from this data?
- **b** This could be the starting point of a statistical investigation. How could you proceed from here?
- **c** Suggest some aims for investigating further.

4 **a** Provide a list of methods you could use to collect primary data.

b Describe which method you would use to collect the following primary data.

i Heights of trees along the footpaths of a tree-lined street

ii Number of buses that transport students to your school in the morning

iii Sunrise times during summer

iv Student opinion regarding length of lessons

UNDERSTANDING

For questions **5** and **6**, design an experiment to simulate the situation, carry out the experiment and give the results of the experiment.

5 A mouse in a maze can make left or right turns at each junction. Assuming each turn is equally likely, how many junctions on average must the mouse go through before each type of turn will have been made?

6 A restaurant menu features 4 desserts which are assumed to be equally popular. How many dessert orders must be filled (on average) before the owner can be sure all types will have been ordered?

7 This label shows the nutritional information of Brand Y rolled oats.

Nutrition information

Servings per package: 30
Serving size: 30g

	Avg. quantity per serving 30g	Avg. quantity per 100g
Energy	480kJ (115Cal)	1600kJ (383Cal)
Protein	3.2g	10.5g
Fat, total	2.4g	8.0g
- saturated	LESS THAN 1g	1.5g
Carbohydrate	18.3g	61.0g
- sugars	0.0g	0.0g
Dietary Fibre, total	3.3g	10.0g
Sodium	LESS THAN 5mg	LESS THAN 5mg

Ingredients

Oats (100%)

Attention

THIS PRODUCT CONTAINS GLUTEN.

Storage

Store in a cool, dry place.

Compare the nutritional information with that on the Brand X label in question **3**.

8 Comment on this claim.

> We did a survey on 100 people regarding eating chocolate.
> 60 of these people said they regularly ate chocolate.
> We then measured the heights of all 100 people.
> **** The result ****
> Eating chocolate makes you taller!!

9 Russel operates a computer software sales outlet. He keeps a log of all complaints from customers. Suggest how he could organise his log.

10 The following claim has been made regarding secondary data.
There's a lot more secondary data than primary data, it's a lot cheaper and it's easier to acquire.
Comment on this statement.

REASONING

11 The local Bed Barn was having a sale on selected beds by Sealy and Sleepmaker. Four of the beds on sale were:

Sealy Posturepremier	on sale for $1499	a saving of $1000
Sealy Posturepedic	on sale for $2299	a saving of $1600
Sleepmaker Casablanca	on sale for $1199	a saving of $800
Sleepmaker Umbria	on sale for $2499	a saving of $1800

The store claimed that all these beds had been discounted by at least 40%. Comment on whether this statement is true, supporting your comments with sound mathematical reasoning.

12 Hamish is planning on running a stall as a fund-raiser selling ice-cream. There are 1000 students in his school ranging from Year 7 to Year 12. There are 5 Year 8 classes each of 25 students, boys and girls.

Hamish intends to ask a group of 10 students chosen at random from each of these 5 classes to select their favourite three ice-cream flavours. Hamish is confident that this random sampling method encompassing a total of 50 students should give him an accurate picture of the ice-cream preferences for the school. Is Hamish correct or is he facing a financial disaster?

PROBLEM SOLVING

13 Addison, a prospective home buyer, wishes to get up to date with the cost of a mortgage from financial institutions. She realises that there are a lot of lenders in the market place. Explain how she would collect the necessary information in the form of:

a primary data

b secondary data.

14 Kirsty, chief marketing manager of FARMCO Cheeses, has decided to run a major TV advertising campaign.

a How should she choose a TV channel and time slot to run her advertisements on?

b How should she decide which demographic/age groups to target?

c Does the answer to **b** have any bearing on the answer to **a**?

12.4 Organising and displaying data

Examining data

- Once collected, the data must be organised.
- When this has been done, any anomalies in the data will be highlighted.
- Before we rush into doing calculations, we must consider how to treat any anomalies. These could have occurred because of:
  – recording errors
  – unusual responses.
- Sometimes a decision is made just to disregard these anomalies, which are regarded as outliers.
- Outliers can greatly affect the results of calculations, as we shall see later.

Frequency tables

- Organising the raw data into a frequency table is the first step in allowing us to see trends in the data.

WORKED EXAMPLE 9

In a suburb of 350 houses, a sample of 20 households was surveyed to find the number of children in them. The data were collected and recorded as follows:
0, 2, 3, 2, 1, 3, 5, 2, 0, 1, 2, 0, 2, 1, 2, 1, 2, 3, 1, 0.
a Organise the data into a frequency table.
b Comment on the distribution of the data.
c What could you say about the number of children per household in the suburb?

THINK

a Draw up a frequency table and complete the entries.

b Look at how the data are distributed.

c Does this sample seem to reflect the population characteristics?

WRITE

a

Children per household	Frequency
0	4
1	5
2	7
3	3
4	0
5	1

b The data value of 5 appears to be an outlier. This is probably not a recording error, but it is not typical of the number of children per household. Most households seem to have 1 or 2 children.

c The sample is an appropriate size, and would probably reflect the characteristics of the population. It would be reasonably safe to say that most houses in the suburb contained 1 or 2 children.

- Sometimes there is too much data to treat as single entries, and it is necessary to group the data into **class intervals**.
- The choice for the size of class intervals should lead to between 5 and 10 groups being formed.

- Class intervals are set so that each score belongs to one group only.
- This means that it is not possible to have intervals such as 5–10, 10–20, . . ., because an entry of 10 would then be in two class intervals.

WORKED EXAMPLE 10

A sample of 40 people was surveyed about the number of hours per week they spent watching TV. The results, rounded to the nearest hour, are listed below.
12, 18, 9, 17, 20, 7, 24, 16, 9, 27, 7, 16, 26, 15, 7, 28, 11, 20, 9, 11, 23, 19, 29, 12, 19, 12, 16, 21, 8, 4, 16, 20, 17, 10, 24, 21, 5, 13, 29, 26

a Organise the data into a frequency table using class intervals of 5–<10, 10–<15 etc. Show the midpoint of each class interval.

b Comment on the distribution of the data.

THINK

a 1 Draw up a frequency table with three columns: class interval (hours of TV), midpoint and frequency.

2 The midpoint is calculated by adding the two extremes of the class interval and dividing by 2. For example, the midpoint of the first class interval is $\frac{5 + 10}{2} = 7.5$.

3 Systematically go through the list, determine how many times each score occurs and enter the information into the frequency column.

b Look at how the data are distributed.

WRITE

a

Hours of TV	Midpoint	Frequency
5–<10	7.5	9
10–<15	12.5	7
15–<20	17.5	10
20–<25	22.5	8
25–<30	27.5	6

b The TV viewing times are fairly evenly distributed, but the most-frequent number of hours per week viewing was about 15 to 20 hours.

Column graphs

- Once a frequency table has been constructed from the data, it can be displayed in graphical form. The most important statistical displays are column graphs. A special type of column graph is called a **histogram**. It has the following characteristics.
  1. All columns are of equal width.
  2. No gaps are left between columns.
  3. Each column 'straddles' an x-axis score; that is, the column starts and finishes halfway between scores.
  4. Usually a half-interval is left at the beginning and end of the graph. That is, the first score is one unit in from the frequency (y)-axis.

WORKED EXAMPLE 11

Consider the frequency table created in Worked example 9.

Children per household	Frequency
0	4
1	5
2	7
3	3
4	0
5	1

a Display the data as a histogram.
b Comment on the shape of the graph.
c Are these comments consistent with those in Worked example 9?

THINK

a 1 Rule a set of axes on graph paper. Give the graph a title. Label the horizontal axis 'Number of children per household' and the vertical axis 'Frequency'.

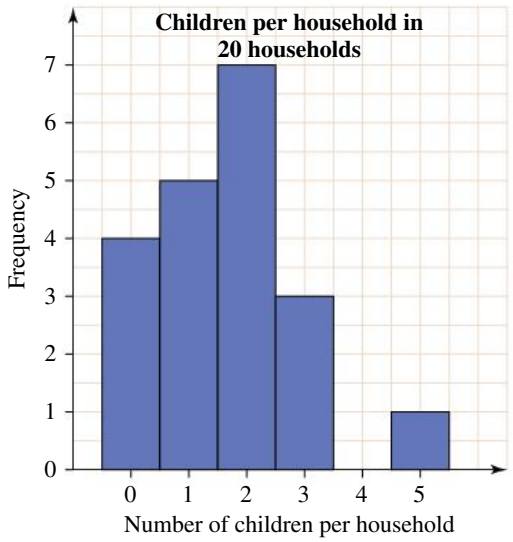
2 Leaving a $\frac{1}{2}$ unit space at the beginning, draw the first column so that it starts and finishes halfway between each household, and reaches a vertical height of 4 units. Repeat this technique for the other scores.

b Look at how the data are distributed.

c Look for any similarities or differences between the observations in Worked example 9 and those made here.

WRITE/DRAW

a



b The data show that the score of '5 children per household' seems to be an outlier, and that the majority of households seem to have 1 or 2 children.

c These comments are consistent with those made in Worked example 9.

- When presenting grouped data graphically, we label the horizontal axis (score) with either:
  - the class interval, or
  - the midpoint of each class interval.

WORKED EXAMPLE 12

Consider the grouped frequency table created in Worked example 10.
a Display the data as a histogram.
b Comment on the shape of the graph.

Hours of TV	Midpoint	Frequency
5–<10	7.5	9
10–<15	12.5	7
15–<20	17.5	10
20–<25	22.5	8
25–<30	27.5	6

THINK

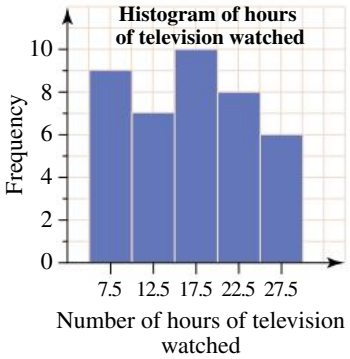
a 1 Rule a set of axes on graph paper. Give the graph a title. Label the horizontal axis 'Number of hours of television watched' and the vertical axis 'Frequency'.

2 Leaving a $\frac{1}{2}$ unit space at the beginning, draw the first column so that it starts and finishes halfway between class intervals, and reaches a vertical height of 9 units. Repeat this technique for the other scores.

b Look at how the data are distributed.

WRITE/DRAW

a

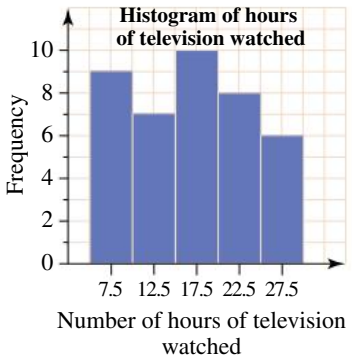


b The hours of TV watching is fairly consistent throughout the week. A maximum number of people watch about 15 to 20 hours per week.

- If we join the midpoints of the tops of the columns of a histogram, then extend the ends to the *x*-axis, we form what is called a **frequency polygon**.

WORKED EXAMPLE 13

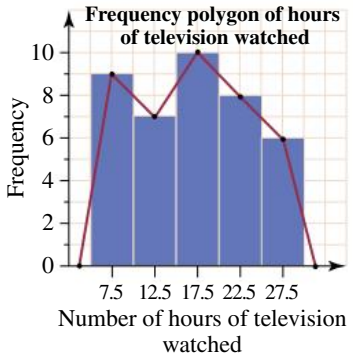
Use the histogram created in Worked example 12 to draw a frequency polygon of the data.



THINK

1. Mark the midpoints of the tops of the columns of the histogram.
2. Join the midpoints with straight lines.
3. Close the polygon by drawing lines at each end down to the *x*-axis.

WRITE/DRAW



- The procedure for dealing with a large amount of data, organising it into class intervals, then producing a histogram and a frequency polygon can be seen in this next example.

WORKED EXAMPLE 14

The following data are the results of testing the lives (in hours) of 100 torch batteries.
20, 31, 42, 49, 46, 36, 42, 25, 28, 37, 48, 49, 45, 35, 25, 42, 30, 23, 25, 26,
29, 31, 46, 25, 40, 30, 31, 49, 38, 41, 23, 46, 29, 38, 22, 26, 31, 33, 34, 32,
41, 23, 29, 30, 29, 28, 48, 49, 31, 49, 48, 37, 38, 47, 25, 43, 38, 48, 37, 20,
38, 22, 21, 33, 35, 27, 38, 31, 22, 28, 20, 30, 41, 49, 41, 32, 43, 28, 21, 27,
20, 39, 40, 27, 26, 36, 36, 41, 46, 28, 32, 33, 25, 31, 33, 25, 36, 41, 28, 33

a Choose a suitable class interval for the given data and present the results in a frequency distribution table.
b Draw a histogram of the data.
c Add a polygon to the histogram.

THINK

a
1. Choose a suitable size for the class interval.
2. Obtain the range for the given data; that is, subtract the smallest value from the largest.
3. Divide the results obtained for the range by 5 and round to the nearest whole number.
   Note: A class interval of 5 hours will result in 6 groups.
4. Draw a frequency table and list the class intervals in the first column, beginning with the smallest value.
   Note: The class interval 20–<25 includes hours ranging from and including 20 to less than 25.
5. Systematically go through the data and determine the frequency of each class interval.
6. Calculate the total of the frequency column.

WRITE

a

Range = largest value − smallest value
= 49 − 20
= 29

Number of class intervals: $\frac{29}{5} = 5.8$

$= 6$

Life time (hours)	Tally	Frequency (f)
20–<25	𝍸 𝍸 II	12
25–<30	𝍸 𝍸 𝍸 𝍸 III	23
30–<35	𝍸 𝍸 𝍸 𝍸	20
35–<40	𝍸 𝍸 𝍸 I	16
40–<45	𝍸 𝍸 III	13
45–<50	𝍸 𝍸 𝍸 I	16
	Total	100

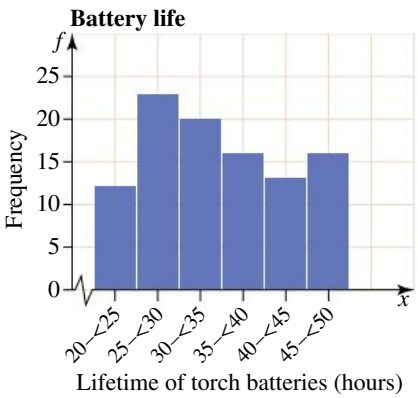
b 1 Rule and label a set of axes on graph paper. Give the graph a title.

2 Add scales to the horizontal and vertical axes. *Note:* Leave a half interval at the beginning and end of the vertical axis.

3 Draw in the first column so that it starts and finishes halfway between class intervals and reaches a vertical height of 12 units.

4 Repeat step 3 for each of the other scores.

b

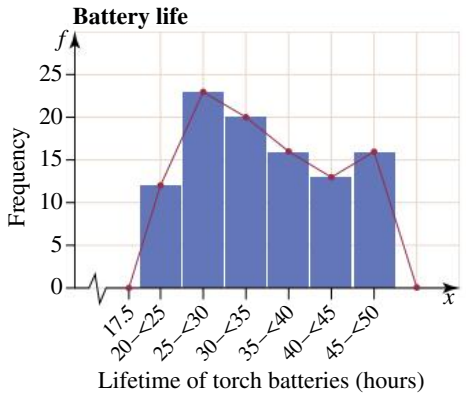


c 1 Mark the midpoints of the tops of the columns obtained in the histogram from part **b**.

2 Join the midpoints by straight line intervals.

3 Close the polygon by drawing lines at each end down to the class interval (x) axis.

c



Exercise 12.4 Organising and displaying data

assess on

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7, 9, 12	Questions: 1, 2, 4, 6, 8, 10–12	Questions: 1, 2c, 3, 5, 7, 10–13

Individual pathway interactivity int-4453 eBookplus

REFLECTION
How does a column graph for categorical data differ from a histogram for numerical data?

FLUENCY

1 **WE9, 11, 13** In a suburb of roughly 1500 houses, a random sample of 40 households were chosen, and surveyed to find the number of children in each of them. The data were collected and recorded as follows.

0, 3, 2, 4, 1, 2, 3, 2, 2, 2, 2, 1, 3, 4, 5, 2, 3, 1, 1, 1,
0, 0, 2, 3, 4, 1, 3, 4, 2, 2, 0, 1, 2, 3, 2, 0, 2, 4, 5, 1.

a Comment on the sample.
b Organise the data into a frequency table.
c Display the data as a combined histogram and frequency polygon.
d Comment on the distribution of the data and the shape of the graph.
e What could you say about the number of children per household in the suburb?

eBookplus
Digital doc
SkillSHEET
Presenting data in a frequency table
doc-6989

2 Consider the following frequency tables.

a

x	f
1	4
2	3
3	5
4	0
5	2
6	6
Total	

b

x	f
150	2
151	6
152	9
153	18
154	11
155	3
Total	

c

x	f
50	3
52	3
54	2
56	2
58	1
60	5
Total	

For each table:

i draw a combined histogram and frequency polygon to suit the data

ii comment on the shape of the graph.

3 A quality control officer selected 25 boxes of CD players at random from a production line. She tested every CD player in each box and displayed the number of defective CD players in each box as follows:
1, 3, 2, 5, 2, 2, 1, 5, 2, 1, 2, 4, 3, 0, 5, 3, 2, 1, 3, 2, 1, 3, 4, 2, 1.

a Comment on the sample.

b Organise the data into a frequency table.

c Comment on the distribution of the data.

d What could you say about the population of CD players?

4 This table shows the number of hours of sport played per week by a group of Year 8 students.

Score (hours of sport played)	Frequency (f)
1	3
2	8
3	10
4	12
5	16
6	8
7	7
Total	64

a Draw a histogram and frequency polygon to display the data.

b Comment on the shape of the graph.

c Discuss whether you feel this sample reflects the sporting habits of Year 8 students generally.

5 **WE10, 12, 13** A block of houses in a suburb was surveyed to find the size of each house (in m^2). The results are shown in the following table.

Size of house (m^2)	Frequency
100–<150	13
150–<200	18
200–<250	19
250–<300	17
300–<350	14
350–<400	11
Total	92

- **a** Draw a histogram and frequency polygon to display the data.
- **b** Comment on the shape of the graph.
- **c** Discuss whether you feel this sample reflects the size of the houses in the suburb.

6 Forty people joined a weight-loss program. Their mass (in kg) was recorded at the beginning of the program and is shown in this frequency table.

Class interval	Frequency
60–<70	2
70–<80	5
80–<90	9
90–<100	12
100–<110	7
110–<120	3
120–<130	2
Total	40

- **a** Draw a histogram and frequency polygon to display the data.
- **b** Comment on the shape of the graph.
- **c** Discuss whether you feel this sample reflects the mass of people in the community.

7 **WE14** Forty people in a shopping centre were asked about the number of hours per week they spent watching TV. The result of the survey is shown below.

10, 13, 7, 12, 16, 11, 6, 14, 6, 11, 5, 14, 12, 8, 27, 17, 13, 8, 14, 10,
13, 7, 15, 10, 16, 8, 18, 14, 21, 28, 9, 12, 11, 13, 9, 13, 29, 5, 24, 11

- **a** Organise the data into class intervals of 5–<10 hours etc., and draw up a frequency table.
- **b** Draw a histogram and frequency polygon to display the data.
- **c** Comment on the shape of the graph.
- **d** Discuss whether you feel this sample reflects the TV viewing habits of the community.

UNDERSTANDING

8 A Year 8 class sat for a 10-question multiple-choice test. These are their results.

2, 6, 5, 9, 8, 7, 3, 6, 9, 4, 8, 8, 6, 7, 6, 4, 7, 8, 7, 8, 6, 7, 8, 5, 3, 9, 2, 6, 5, 8

- **a** Organise the data into a frequency table.
- **b** Display the data as a combined histogram and frequency polygon.
- **c** Comment on the performance of the class in the test.

9 The number of hours of sleep during school week nights for a Year 8 class are recorded below.

6, 9, 7, 8, 7, $8\frac{1}{2}$, $6\frac{1}{2}$, 8, $7\frac{1}{2}$, $7\frac{1}{2}$, 8, $8\frac{1}{2}$, $6\frac{1}{2}$, 8, 8, 7, $7\frac{1}{2}$, 8, 9, 8

- **a** Organise the data into a frequency table.
- **b** Display the data as a histogram.

c Comment on the sleeping habits of the Year 8 students.

d Discuss whether you feel these sample results reflect those of Year 8 students generally.

REASONING

10 The amount of pocket money (in dollars) available to a random sample of 13-year-olds each week was found to be as shown below.

10, 15, 5, 4, 8, 10, 4, 15, 5, 6, 10, 6, 5, 10, 8, 10, 5, 10, 10, 6

a Organise the data into a frequency table.

b Display the data as a histogram.

c Comment on the shape of the histogram.

d Discuss whether you feel these sample results reflect those of 13-year-olds generally.

11 Data were collected on the number of times people go to the cinema in a month.

4, 5, 7, 9, 1, 2, 5, 2, 4, 8, 3, 6,
2, 3, 8, 1, 1, 4, 5, 3, 3, 6, 1, 2,
7, 1, 3, 2, 2, 4, 10, 0, 1, 3, 4, 6

a Put the data in a frequency table.

b Draw a histogram to represent the data.

c How many people go to the cinema less than three times a month?

d How many people go to the cinema at least three times a month?

e Is it reasonable to draw conclusions about the whole population based on this sample? Give reasons for your answer.

eBook*plus*
Digital doc
Spreadsheet
Histograms and frequency polygons
doc-2183

PROBLEM SOLVING

12 The following data give the results of testing the lives (in hours) of 100 torch batteries.
25, 36, 30, 34, 21, 40, 36, 46, 29, 38, 20, 41, 34, 45, 25, 40, 31, 39, 24, 45, 27, 44, 23, 35, 47, 49, 20, 37, 43, 26, 35, 28, 48, 30, 20, 36, 41, 26, 32, 42, 21, 31, 45, 42, 26, 37, 33, 24, 45, 38, 36, 43, 21, 34, 38, 35, 28, 41, 30, 22,

29, 32, 39, 25, 44, 21, 35, 38, 41, 35, 30, 23, 37, 43, 33,
34, 28, 39, 22, 31, 35, 42, 38, 27, 36, 46, 28, 34, 37, 29,
24, 30, 39, 44, 31, 24, 36, 28, 47, 21

a Choose a suitable class interval for the given data, and present the results in a frequency distribution table.

b Draw a histogram and frequency polygon of the data.

c Comment on the trends shown by the histogram.

d Discuss whether you feel these results reflect those of the battery population.

13 A random sample of 30 students in Year 9 undertook a survey to investigate the height of Year 9 students. These were their measured heights (in cm).

146, 163, 156, 168, 159, 170, 152, 174, 156, 163, 157, 161, 178, 151, 148,
167, 162, 157, 166, 154, 150, 166, 160, 155, 164, 157, 171, 168, 158, 162

a Organise the data into class intervals 145–<150 cm etc., and draw up a frequency distribution table.

b Draw a histogram displaying the data.

c Reorganise the class intervals to be 145–<148 cm etc., and construct a new frequency distribution table.

d Draw a new histogram displaying the data in part **c**.

e Comment on the similarities and differences between the two histograms.

eBookplus
Digital doc
WorkSHEET 12.1
doc-7005

12.5 Measures of centre

- Simple calculations based on collected data can help give us typical values, or values that show how the data cluster.
- These typical values are commonly referred to as averages.
- We will look at 3 different types of averages used in interpreting data: **mean**, **median** and **mode**.

Mean

- The mean or average of a set of scores is the sum of all the scores divided by the number of scores.
- It is denoted by the symbol $\bar{x}$ (pronounced x bar).

WORKED EXAMPLE 15

Jan's basketball scores were 18, 24, 20, 22, 14 and 12. What was his mean score? Calculate your answer, correct to 1 decimal place.

THINK	WRITE
1 Calculate the total of the basketball scores.	Total score $= 18 + 24 + 20 + 22 + 14 + 12$ $= 110$
2 Count the number of basketball scores.	Number of scores $= 6$
3 Define the rule for the mean.	Mean $= \dfrac{\text{total score}}{\text{number of scores}}$
4 Substitute the known values into the rule.	$\bar{x} = \dfrac{110}{6}$
5 Evaluate, rounding to 1 decimal place. *Note:* The mean is often not one of the given scores.	$= 18.333\,33\ldots$ $= 18.3$

- Sometimes calculations need to be performed from a frequency distribution table.

WORKED EXAMPLE 16

Calculate the mean of the frequency distribution data given below.

Score (x)	1	2	3	4	5	6
Frequency (f)	3	2	4	0	1	5

THINK

1 Rearrange the rows as columns and include an extra column headed:
'Score × frequency ($x \times f$)'.

2 Enter the information into the third column; that is, the score of 1 occurred 3 times. Therefore, $x \times f = 1 \times 3 = 3$.
The score of 2 occurred 2 times.
Therefore, $x \times f = 2 \times 2 = 4$.
Continue this process for each pair of data.

WRITE

Score (x)	Frequency (f)	Score × frequency ($x \times f$)
1	3	$1 \times 3 = 3$
2	2	$2 \times 2 = 4$
3	4	$3 \times 4 = 12$
4	0	$4 \times 0 = 0$
5	1	$5 \times 1 = 5$
6	5	$6 \times 5 = 30$
Total	15	54

3 Determine the total of the 'Frequency' column. This shows how many scores there are altogether.

4 Determine the total of the 'Score × frequency' column. This shows the overall value of all the scores.

5 Define the rule for the mean.

$$\text{Mean} = \frac{\text{total of score} \times \text{frequency values}}{\text{total frequency values}}$$

$$\bar{x} = \frac{54}{15}$$
$$= 3.6$$

6 Substitute the known values into the rule.

7 Evaluate the answer to 1 decimal place.
Note: The mean is often not one of the given scores.

Median

- The median is the middle score for an odd number of scores and the average of the two middle scores for an even number of scores.
- Alternatively, if a set of data contains n scores, the median is given by the $\frac{n+1}{2}$ th score.
- To obtain the median, the scores must be arranged in numerical order.

WORKED EXAMPLE 17

Find the median of each of the following sets of scores.
a 10, 8, 11, 5, 17 **b** 9, 3, 2, 6, 3, 5, 9, 8

THINK	WRITE
a 1 Arrange the values in ascending order.	**a** 5, 8, 10, 11, 17
2 Select the middle value. *Note:* There are an odd number of scores, that is, 5. Hence, the third value is the middle number or median. Alternatively, use the rule $\frac{n+1}{2}$, where $n = 5$ gives the position of the median. The location of the median is $\left(\frac{5+1}{2} = 3\right)$; that is, the 3rd score.	
3 Answer the question.	The median of the scores is 10.
b 1 Arrange the values in ascending order.	**b** 2, 3, 3, 5, 6, 8, 9, 9
2 Select the middle values. *Note:* There are an even number of scores that is, 8. Hence, the fourth and fifth values are the middle numbers, or median. Again the rule $\frac{n+1}{2}$ could be used to locate the position of the median.	
3 Obtain the average of the two middle values.	Median $= \frac{5+6}{2}$ $= \frac{11}{2}$ $= 5\frac{1}{2}$ (or 5.5)
4 Answer the question. *Note:* The median in this case is not one of the actual scores.	The median of the score is $5\frac{1}{2}$ or 5.5.

Note: For sets of data containing an odd number of scores, the median will be one of the actual scores, while for the sets with an even number of scores, the median will be positioned halfway between the two scores.

Mode

- The mode is the most common score in a set of data.
- It is the score with the highest frequency. It measures clustering of scores.
- Some sets of scores have more than one mode or no mode at all. There is no mode when all values occur an equal number of times.

WORKED EXAMPLE 18

Find the mode of each of the following sets of scores.
a 5, 7, 9, 8, 5, 8, 5, 6 **b 10, 8, 11, 5, 17** **c 9, 3, 2, 6, 3, 5, 9, 8**

THINK	WRITE
a 1 Look at the set of data and circle any values that have been repeated.	**a** ⑤, 7, 9, ⑧, ⑤, ⑧, ⑤, 6
2 Choose the values that have been repeated the most.	The number 5 occurs 3 times.
3 Answer the question.	The mode for the given set of values is 5.
b 1 Look at the set of data and circle any values that have been repeated.	**b** 10, 8, 11, 5, 17 No values have been repeated.
2 Answer the question. *Note:* No mode is not the same as having a mode which equals 0.	The following set of data has no mode, since none of the scores correspond to a highest frequency. Each of the numbers occur only once.
c 1 Look at the set of data and circle any values that have been repeated.	**c** ⑨, ③, 2, 6, ③, 5, ⑨, 8
2 Choose the values that have been repeated the most.	The number 3 occurs twice. The number 9 occurs twice.
3 Answer the question.	The modes for the given set of values are 3 and 9.

WORKED EXAMPLE 19

The data from a survey asking people how often per week they purchased take-away coffee from a café are shown below.
2, 9, 11, 8, 5, 5, 5, 8, 7, 4, 5, 3.
Use the data to calculate each of the following.
a The mean number of coffees purchased per week
b The median number of coffees purchased per week
c The modal number of coffees purchased per week

THINK	WRITE
a 1 To calculate the mean, add all the values in the data set and divide by how many data values there are. There are 12 values in the data set.	$\bar{x} = \frac{\text{sum of all the values}}{\text{number of values}}$ $= \frac{2 + 9 + 11 + 8 + 5 + 5 + 5 + 8 + 7 + 4 + 5 + 3}{12}$ $= \frac{72}{12}$ $= 6$
2 Answer the question.	The mean number of coffees purchased per week is 6.

b **1** The median is the middle value by position. There are 12 values in the data set, so the middle position is between the 6th and 7th values.

$$\text{Median position} = \frac{n+1}{2} = \frac{12+1}{2} = \frac{13}{2} = 6.5$$

2 Arrange the data set in order from lowest to highest.
The 6th value is 5.
The 7th value is 5.

$$2, 3, 4, 5, 5, 5, 5, 7, 8, 8, 9, 11$$

$$\text{Median} = \frac{5+5}{2} = \frac{10}{2} = 5$$

3 Answer the question.

The median number of coffees purchased per week is 5.

c The mode is the most common value in the data set. The most common value is 5.

The modal number of coffees purchased per week is 5.

The effect of outliers on measures of centre

- An **outlier** is a data value that is considerably different from the rest of the values in a set of data.
- The presence of one or more outliers may have a considerable effect on the measures of centre of a particular set of data.
  Consider a netball team where the team scored the following in 10 games:
  17, 23, 31, 19, 50, 29, 16, 23, 30, 32.
  Rearrange the scores in order:
  16, 17, 19, 23, 23, 29, 30, 31, 32, 50

$$\text{Mean} = \frac{16+17+19+23+23+29+30+31+32+50}{10} = 27$$

$$\text{Median} = \frac{23+29}{2} = 26$$

Mode = 23

The following Saturday, the regular goal shooter was ill, and Lauren, who plays in a higher division, was asked to play. The team's score for that game was 200.
When the measures of centre were recalculated for 11 games, the following results were obtained:
16, 17, 19, 23, 23, 29, 30, 31, 32, 50, 200

$$\text{Mean} = \frac{16+17+19+23+23+29+30+31+32+50+200}{11} = 43$$

Median = 29
Mode = 23
The inclusion of an extreme value or outlier has dramatically increased the mean of the data, marginally increased the median and left the mode unchanged.
The important point to learn from this illustration is that when a set of data includes extreme values, the mean may not be truly representative of the data.

Which measure of centre is most useful?

- Now that we have discussed the three measures of central tendency, it is important to know which measure will be most useful in a given situation.
  - The mean is appropriate when no extreme values or outliers distort the picture.
  - The median is appropriate when outliers are present.
  - The mode is appropriate when the most common result is significant.

Summary of measures of centre		
Mode	**Median**	**Mean**
The most common value or category. It tells us nothing about the rest of the data. Data may have no mode, one mode or more than one mode.	The score in the exact middle of the values placed in numerical order. It tells us nothing about other values. It is unaffected by exceptionally large or small values.	Uses all the scores as a total, divided by the number of scores. It is affected by exceptionally large or small scores.

assessON

Exercise 12.5 Measures of centre

REFLECTION
Why is the mean affected more by an outlier than the median or the mode?

INDIVIDUAL PATHWAYS

■ PRACTISE	■ CONSOLIDATE	■ MASTER
Questions: 1–15, 17, 20, 23, 27	Questions: 1–15, 16, 18, 22, 25, 27, 28, 30	Questions: 1–15, 19, 21, 24–30

■■■ Individual pathway interactivity int-4454 eBookplus

FLUENCY

eBookplus
Digital docs
SkillSHEET
Finding the mean of ungrouped data
doc-6991
SkillSHEET
Arranging a set of data in ascending order
doc-6992
SkillSHEET
Finding the median
doc-6993
SkillSHEET
Finding the score in a data set that occurs most frequently
doc-6995

1 WE15 Caroline's basketball scores were 28, 25, 29, 30, 27 and 22. What was her mean score? Give the answer correct to 1 decimal place.

2 Find the mean (average) of each set of the following scores. Give the answers correct to 2 decimal places.
   a 1, 2, 3, 4, 7, 9
   b 2, 7, 8, 10, 6, 9, 11, 4, 9
   c 3, 27, 14, 0, 2, 104, 36, 19, 77, 81
   d 4, 8.4, 6.6, 7.0, 7.5, 8.0, 6.9

3 Francesca's soccer team has the following goals record this season: 2, 0, 1, 3, 1, 2, 4, 0, 2, 3.
   a What total number of goals have they scored?
   b How many games have they played?
   c Find the team's average score.

eBookplus
Interactivity
Measures of centre
int-2362

4 MC Frisco's athletics coach timed 5 consecutive 200 m training runs. He recorded times of 25.1, 23.9, 24.8, 24.5 and 27.3 seconds. His mean 200 m time (in seconds) is:
   A 24.60 B 25.20 C 25.12 D 25.42 E 26.12

5 An Olympic figure skater was given these scores by the panel of judges:
4.8, 4.6, 4.5, 4.7, 4.8, 4.9, 4.2, 4.0, 4.8.
Find the average score correct to 1 decimal place.

6 Two Year 8 groups did the same Mathematics test. Their results out of 10 were:
Group A: 5, 8, 7, 9, 6, 7, 8, 5, 4, 2
Group B: 5, 6, 4, 5, 9, 7, 8, 8, 9, 7.
Which group had the highest mean?

7 WE16 Calculate the mean of this frequency distribution, correct to 2 decimal places.

Score (x)	1	2	3	4	5
Frequency (f)	4	3	6	1	0

8 Calculate the mean of this frequency distribution, correct to 2 decimal places.

Score (x)	6	7	8	9	10
Frequency (f)	2	8	3	6	2

9 These scores show the number of people in each apartment in a block of flats. Use a frequency table to calculate the mean number of people per unit, correct to 1 decimal place.
1, 3, 2, 4, 2, 1, 3, 5, 3, 2, 4, 1, 3, 2, 1, 2

10 The mean of 10 scores is 8. What is the total of all the scores?

11 WE17a Find the median of the following scores.

a 5, 5, 7, 12, 13 **b** 28, 13, 17, 21, 18, 17, 14

12 WE17b Find the median of each of the following sets of scores.

a 2, 52, 46, 52, 48, 52, 48

b 4, 1.5, 1.7, 2.0, 1.8, 1.5, 1.7, 1.8, 1.9

13 WE18 For each set of scores in questions **11** and **12**, find the mode.

Questions **14** and **15** refer to the following set of scores.
1, 1, 1, 4, 4, 5, 5, 6, 3, 3, 7, 6, 5, 4, 6, 2, 1, 8

14 MC The median of the given scores is:

A 1 **B** 4.5 **C** 4
D 5 **E** 8

15 MC The mode of the given scores is:

A 5 **B** 6 **C** 4
D 3 **E** 1

UNDERSTANDING

16 A third Year 8 group had the following results in the same test as in question **6**:
5, 7, 8, 4, 6, 8, 5, 9, 8.

a What is the average score of this group, correct to 1 decimal place?

b What must a tenth student (who was originally absent) score to bring this group's average to 7?

eBookplus

Digital docs
Investigation
How many red M&Ms?
doc-2166
Spreadsheet
Mean
doc-2184
Spreadsheet
Median
doc-2185
Spreadsheet
Mode
doc-2187

17 A survey of the number of occupants in each house in a street gave the following data:
2, 5, 1, 6, 2, 3, 2, 4, 1, 2, 0, 2, 3, 2, 4, 5, 4, 2, 3, 4.
Prepare a frequency distribution table with an $x \times f$ column and use it to find the average number of people per household.

18 The mean of 5 scores is 7.2.
- **a** What is the sum of the scores?
- **b** If four of the scores are 9, 8, 7 and 5, what is the fifth?

19 WE19 Over 10 matches, a soccer team scored the following number of goals:
2, 3, 1, 0, 4, 5, 2, 3, 3, 4.
- **a** What was the most common number of goals scored?
- **b** What was the median number of goals scored?
- **c** In this case, does the mode or the median give a score that shows a typical performance?

20 Here are Tiger Woods's scores (numbers of strokes) hole by hole for the first 9 holes of a major golf tournament.

Hole number	1	2	3	4	5	6	7	8	9
Score	4	4	3	2	4	3	3	2	4

- **a** How many strokes were most commonly hit?
- **b** What was his median score?
- **c** As Woods prepares to tee off towards the next hole, how many strokes could the crowd expect him to take to complete the hole? Discuss factors which could influence the outcome.

21 The following scores represent the number of museli bars sold in a school canteen each day over two weeks:
54, 64, 51, 58, 56, 59, 10, 34, 48, 56.
- **a** Calculate the mean.
- **b** Calculate the median.
- **c** Calculate the mode.
- **d** Which of the mean, median or mode best represents a typical day's sales in the school canteen? Explain.

22 A small business pays the following annual wages (in thousands of dollars) to its employees:
18, 18, 18, 18, 26, 26, 26, 40, 80.
- **a** What is the mode of the distribution?
- **b** What is the median wage?
- **c** What is the mean wage?
- **d** Which measure would you expect the employee's union to use in wage negotiations?
- **e** Which might the boss use in such negotiations?

REASONING

23 Find five numbers that have a mean of 10 and a median of 12.

24 The mean of 5 different test scores is 15. What are the largest and smallest possible test scores, given that the median is 12? All test scores are whole numbers. Justify your answer.

25 The mean of 5 different test scores is 10. What are the largest and smallest possible values for the median? All test scores are whole numbers. Justify your answer.

26 The mean of 9 different test scores that are whole numbers percentage is 85. The median is 80. What is the greatest possible range between the highest and lowest possible test scores? Justify your answer.

PROBLEM SOLVING

27 The data at right show the number of hours Year 8 students used a computer in a particular week.

a Calculate the mean, median and mode.

b What do you notice about the value of the mean compared with the median?

c What do you notice about the value of the mean compared with the mode?

d Which is the best indicator of the centre of the data set?

5, 3, 6, 7, 3, 5, 2, 5, 2, 3, 6, 7

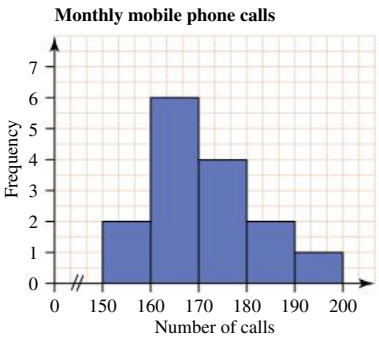
28 Identify the mean, median and mode in the following paragraph:

It was an amazing game of cricket today. The winning team hit more sixes than any other number of runs. This meant that, even though the middle value of runs per over was 3, the sixes brought the average up to about five runs per over. What an incredible game!

29 Calculate the mean number of DVDs presented in this frequency table, using the median of each interval as the x-value for the interval.

Number of DVDs (x)	Frequency (f)
1–15	3
16–30	9
31–45	8
46–60	11
61–75	10
76–90	14
91–105	15
106–120	18

30 Calculate the mean number of calls made on mobile phones in the month shown in this graph.



CHALLENGE 12.2

If the mean of six items of data is 24 and five of the items are 19, 26, 22, 30 and 27, find the sixth item.

12.6 Measures of spread

- In analysing a set of scores, it is helpful to see not only how the scores tend to cluster, or how the middle of the set looks, but also how they spread or scatter.
- Two classes may have the same average mark, but the spread of scores may differ considerably.

Range

- The **range** of a set of scores is the difference between the highest and lowest scores.

WORKED EXAMPLE 20

Find the range of the following sets of data.

a 7, 3, 5, 2, 1, 6, 9, 8

b

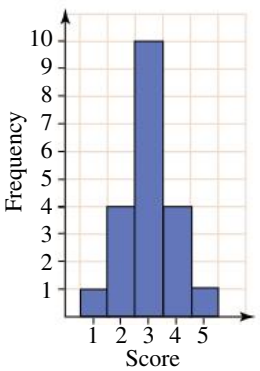
x	7	8	9	10
f	1	3	5	2

THINK	WRITE
a 1 Obtain the highest and lowest values.	**a** Highest value = 9 Lowest value = 1
2 Define the range.	Range = highest value − lowest value
3 Substitute the known values into the rule.	= 9 − 1
4 Evaluate.	= 8
5 Answer the question.	The set of values has a range of 8.
b 1 Obtain the highest and lowest values. *Note:* Consider the values only, not the frequencies.	**b** Highest value = 10 Lowest value = 7
2 Define the range.	Range = highest value − lowest value
3 Substitute the known values into the rule.	= 10 − 7
4 Evaluate.	= 3
5 Answer the question.	The frequency distribution table data have a range of 3.

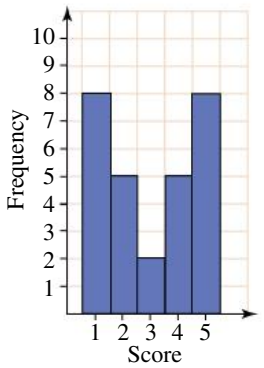
- Although the range identifies both the lowest and highest scores, the spread of the scores within the range is frequently not uniform.

Skewness

- The mean and median are measures of centre.
- They represent typical values around which the data are centred.
- In a symmetrical distribution, values of the mean, median and mode are very close, and any of these measures could be used to describe a 'central' value of the data set.
- In reality, not many data sets have a symmetrical distribution; they are said to be **skewed**.
- Consider the graph that has been drawn at right. This graph is symmetrical, and we can see that the mean, median and mode are all equal to 3.



- The majority of scores are clustered around the mean. This is an example of a **normal distribution**.
- The second graph shows a data set in which the scores are not clustered and there are two modes at either end of the distribution. In this example, although it is still symmetrical there are two modes, 1 and 5, while the mean and median are still 3.



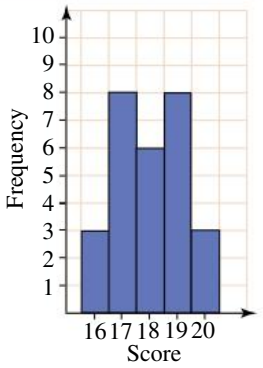
WORKED EXAMPLE 21

The figure at right shows the distribution of a set of scores on an exam.

a Is the graph symmetrical?

b What is the mode?

c Can the mean and median be seen from the graph and, if so, what are their values?



THINK

a Consider the columns either side of the middle. If they are equal, then the graph is symmetrical.

b Look for the score or scores that occur the most often.

c Since the graph is symmetrical, the middle score will be the mean and the median.

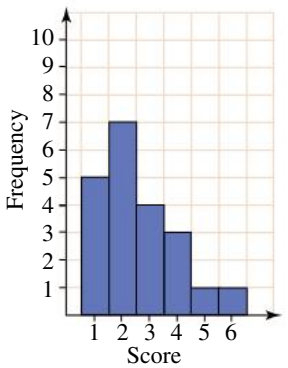
WRITE

a The graph is symmetrical.

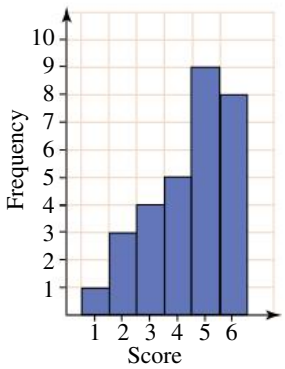
b Mode = 17 and 19

c Mean = 18
Median = 18

- When a graph is not symmetrical, the mean and median cannot be easily seen from the graph.
- Consider the graph at right where the scores are gathered to the lower end of the distribution.
- The way in which the data are gathered to one end of the distribution is called the skewness.
- When a greater number of scores are distributed at the lower end of the distribution, the data are said to be positively skewed.

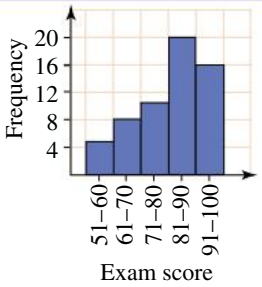


- Similarly, when most of the scores are distributed at the upper end, the data are said to be negatively skewed, as shown at right.



WORKED EXAMPLE 22

The distribution at right shows the results of the Mathematics exam at a certain school.
a What is the modal class?
b Describe the skewness of the data set.



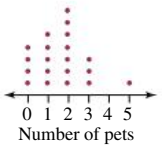
THINK	WRITE
a Locate the modal class by considering which class occurs most often. Which column is the largest?	a Modal class = 81 – 90
b Observe where the majority of data are located. In this case it is at the upper end of the distribution.	b The data are negatively skewed.

- There are many reasons why a data set may be skewed.
- In the case of an exam, an easier exam may lead to negatively skewed data with more students obtaining a higher mark, while a more difficult exam may lead to more students at the lower end of the distribution and hence the data will be positively skewed.
- The skewness of a distribution is also evident in other graphical forms.

Key: 1 | 6 = 16 people

Stem	*Leaf*
0	2 4
0	7 7 9
1	0 1 4 4 4 4
1	5 6 6 7 8 8 9
2	1 2 2 3 3 3
2	7

- This stem plot shows the data concentrated towards the upper end of the distribution, making the data set negatively skewed.
- The distribution of this dot plot is concentrated towards the lower end, making it positively skewed.
- It is also possible to see the skewness of a distribution from a frequency table.



Children per household	Frequency
0	4
1	5
2	7
3	3
4	0
5	1

- This frequency table produced in Worked example 9 shows that the distribution of children per household is positively skewed, with the scores being concentrated at the lower end of the distribution.
- The histogram displaying the data also confirms this.

Exercise 12.6 Measures of spread

assessON

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2a, c, 3–8, 10	Questions: 1, 2b, d, 3–10, 12, 14, 15	Questions: 1, 2d, 3–5, 7–9, 10–16

Individual pathway interactivity int-4455 eBookplus

REFLECTION
Could a distribution be symmetrical and have three modes?

FLUENCY

eBookplus
Digital doc
SkillSHEET
Finding the middle score for data arranged in a dot plot
doc-6994

1 WE20a Find the range of the following scores.

a 5, 5, 7, 12, 13
b 28, 13, 17, 21, 18, 17, 14
c 2, 52, 46, 52, 48, 52, 48
d 4, 1.5, 1.7, 2.0, 1.8, 1.5, 1.7, 1.8, 1.9

2 WE20b Find the range of the following sets of data.

a

x	6	7	8	9	10
f	1	5	10	7	3

b

x	1	2	3	4	5	6
f	7	9	6	8	10	10

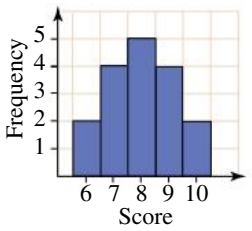
c

x	5	10	15	20
f	1	5	10	7

d

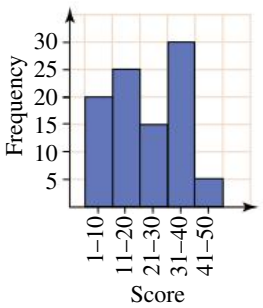
x	110	111	112	113	114
f	2	2	2	3	3

3 WE21 The following figure shows the distribution of a set of scores.



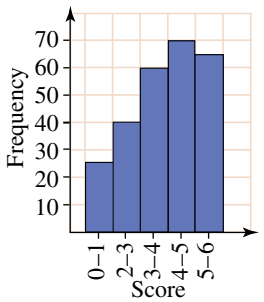
a Is the graph symmetrical?
b What is the mode?
c Can the mean and median be seen from the graph and, if so, what are their values?

4 Consider the following distribution.



a Are the data symmetrical?
b What is the modal class?
c Can the mean and median be seen from the graph and, if so, what are their values?

5 WE22 Consider the distribution shown at right.



a What is the modal class?

b Describe the skewness of the distribution.

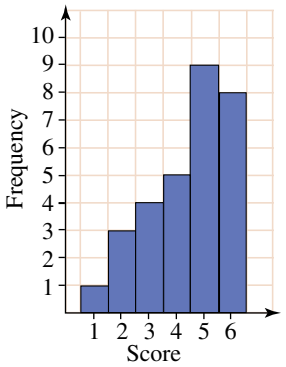
6 The table below shows the number of goals scored by a netball team throughout a season.

Number of goals	Frequency
1–10	13
11–20	16
21–30	27
31–40	33
41–50	31

a Draw a frequency histogram of the data.

b Describe the data set in terms of its skewness.

7 MC The distribution represented by the graph at right is:



A positively skewed

B negatively skewed

C symmetrical

D normally distributed

E none of the above.

UNDERSTANDING

8 Consider the stem plots below.

a Key: 1 | 5 = 15

Stem	*Leaf*
1	1 2 7 8 9
2	2 8
3	1 3 7 9
4	0 1 2 6

b Key: 24 | 7 = 247

Stem	*Leaf*
24	2 7
25	2 4 6 6 8
26	0 1 3 5 9
28	5 6 6 8

c Key: 17 | 4 = 174

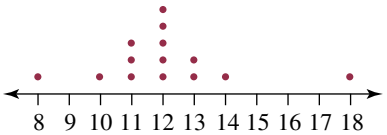
Stem	*Leaf*
15	6 2 4
16	8 6 1 3 9
17	0 2 1 8 6 7 3 4
18	4 1 5 2 7 1

For each stem plot:

i give the range

ii describe the skewness of the distribution.

9 Describe the skewness of the distribution in this dot plot.



10 The table below shows the number of goals scored by a soccer team throughout a season.

Number of goals	Frequency
0	5
1	7
2	7
3	7
4	7
5	5

a Show this information in a frequency histogram.
b Are the data symmetrical?
c What is the mode?
d Can the mean and median be seen for this distribution and, if so, what are their values?

11 A movie is shown at 30 cinemas across Melbourne on its opening day. The number of people attending at each cinema is shown in the table below.

Number of people	Frequency
1–50	3
51–100	2
101–150	6
151–200	9
201–250	10

a Present the data in a frequency histogram.
b Are the data symmetrical?
c What is the modal class?
d Describe the skewness of the distribution and explain possible reasons for the skewness.

REASONING

12 Year 10 students at Merrigong High School sit exams in Science and Maths. The results are shown in the table below.

Mark	Number of students (Science)	Number of students (Maths)
51–60	7	6
61–70	10	7
71–80	8	12
81–90	8	9
91–100	2	6

a Is either distribution symmetrical?
b If either distribution is not symmetrical, state whether it is positively or negatively skewed.
c Discuss the possible reasons for any skewness.
d State the modal class of each distribution.

13 Draw an example of a graph that is:
- **a** symmetrical
- **b** positively skewed with one mode
- **c** negatively skewed with two modes.

PROBLEM SOLVING

14 The mean of the following values is 4.1.
x, $(x + 1)$, $(x - 4)$, $(x + 2)$, $(x - 2)$, $(x - 1)$, x, $(x - 5)$, $(x + 3)$, $(x - 3)$
- **a** Find x.
- **b** What is the median?
- **c** What is the mode?
- **d** Discuss the skewness or otherwise of the distribution.

15 Consider a set of values with a mean of 15 and a median of 15.5.
- **a** If another number is added that has the effect of lowering the mean to 14.9, what happens to the median? Give reasons for your answer.
- **b** If another number is added that has the effect of raising the mean to 15.1, what happens to the median? Give reasons for your answer.

16 A data set containing 9 pieces of data has a highest value of b and a range of a. The mean is m.
- **a** If another value, $(m + 1)$, is added to the data set, by how much is the value of the mean increased?
- **b** What is the new range if $(m + 1) > b$?

12.7 Analysing data

- You are now armed with the skills to calculate measures of centre and spread for a data set, and to construct and analyse various forms of graphs.
- It must be remembered that most of the data we analyse are the results of surveys conducted on samples of the population.
- It is appropriate, then, that we consider data collected from a survey in light of the population from which the sample originated.
- To understand what information the data give, and perhaps to draw conclusions from it, we must appreciate what each statistical measure does.

Statistical measures	Definition and purpose
Mode	The most common score or category. It tells us nothing about the rest of the data. Data may have no mode, one mode or more than one mode.
Median	The score in the exact middle of the values placed in numerical order. It tells us nothing about the rest of the data. It is unaffected by exceptionally large or small scores.
Mean	The sum of all the scores divided by the number of scores. It is affected by exceptionally large or small scores.
Range	The difference between the highest score and the lowest score. It shows how far the scores are spread apart. It is particularly useful when combined with the mean or the median.

WORKED EXAMPLE 23

Explain which statistical measure is referred to in these statements.
a The majority of people surveyed prefer Activ-8 sports drink.
b The ages of fans at the Rolling Stones concert varied from 8 to 80.
c The average Australian family has 2.5 children.

	THINK	WRITE
a	1 Write the statement and highlight the key word(s).	a The majority of people surveyed prefer Activ-8 sports drink.
	2 Relate the highlighted word to one of the statistical measures.	Majority implies most, which refers to the mode.
	3 Answer the question.	This statement refers to the mode.
b	1 Write the statement and highlight the key word(s).	b The ages of fans at the Rolling Stones concert varied from 8 to 80.
	2 Relate the highlighted word to one of the statistical measures.	The statement refers to the range of fans' ages at the concert.
	3 Answer the question.	This statement refers to the range.
c	1 Write the statement and highlight the key word(s).	c The average Australian family has 2.5 children.
	2 Relate the highlighted word to one of the statistical measures.	The statement deals with surveying the population (census) and finding out how many children are in each family.
	3 Answer the question.	This statement refers to the mean.

Using sample properties to predict those of the population

- Once survey data has been collated and analysed, we can use the data set to predict the characteristics of the population from which it was taken. Consider this example.

WORKED EXAMPLE 24

The 153 students in Year 8 all sat for a 10-question multiple-choice test as a practice test for an upcoming exam. A random sample of the results of 42 of the students gave this distribution.

Score (x)	1	2	3	4	5	6	7	8	9	10
Frequency (f)	2	3	6	7	11	8	4	0	0	1

a Calculate the mean mark.
b Determine the median mark.
c Give the modal mark.
d Which measure of centre best represents the data?
e Describe the distribution.
f Comment on any prediction about the properties of the population from this sample.

THINK

a Add a third column to the table to calculate the total of all the marks. Use the total to then calculate the mean.

WRITE

a

Mark(x)	Frequency(f)	$f \times x$
1	2	2
2	3	6
3	6	18
4	7	28
5	11	55
6	8	48
7	4	28
8	0	0
9	0	0
10	1	10
Total	42	195

$$\text{Mean} = \frac{\text{Total of all scores}}{\text{Number of scores}}$$
$$= \frac{195}{42}$$
$$= 4.6$$

b The median is the score in the middle, i.e. the $\frac{42+1}{2}$th score — the average of the 21st and 22nd score. Add a cumulative frequency column to the original frequency distribution. (Find the total frequency at that point for each mark.) Look in the cumulative frequency column to see where the 21st and 22nd scores lie.

b

Mark (x)	Frequency (f)	Cumulative frequency
1	2	2
2	3	$2 + 3 = 5$
3	6	$5 + 6 = 11$
4	7	$11 + 7 = 18$
5	11	$18 + 11 = 29$
6	8	$29 + 8 = 37$
7	4	$37 + 4 = 41$
8	0	$41 + 0 = 41$
9	0	$41 + 0 = 41$
10	1	$41 + 1 = 42$
Total	42	42

The 21st and 22nd scores are both 5.
The median is 5.

c The modal mark is the one that occurs most frequently. Look for the one with the highest frequency.

c The mode is 5.

d Compare the results for the mean, median and mode. Look for similarities and differences.	**d** The mean is 4.6, the median is 5 and the mode is 5. It seems that any of these measures of centre would be appropriate to use as a measure of centre of the data. However, check the outlier of 10. When the outlier is disregarded, the mean is calculated to be $185 \div 41 = 4.5$. It seems that the outlier in this case does not have a great effect on the mean. So, the mean, median or mode could be used as a measure of the centre of the data.
e Look for skewness of the data.	**e** The score of 10 seems to be an outlier. It is most likely not due to a recording error. The remaining scores form a cluster towards the lower end of the data set, indicating that the distribution is positively skewed.
f Consider whether these results from the sample would reflect those of the population.	**f** It seems likely that these results would reflect those of the whole population. The sample is random, and of sufficient size. The outlier of 10 indicates that there would be a few students with full marks, and at least half the students passed the test.

Exercise 12.7 Analysing data

assessON

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–6, 10, 12, 14	Questions: 1–6, 12–14	Questions: 1–16

Individual pathway interactivity int-4456 eBookplus

REFLECTION
Why are measures of centre important statistical measures for businesses?

FLUENCY

1 WE23 Explain which statistical measure is referred to in these statements.

a There was a 15 °C temperature variation during the day.

b Children at this school are absent 3.4 days per semester, on average.

c Most often you have to pay $79.95 for those sports shoes.

d The average Australian worker earns about $470 per week.

e A middle-income family earns about $35 000 per annum.

2 **WE24** This frequency table shows the results of a random sample of 15 students (from a class of 30) who sat for a 10-question multiple-choice test.

Score (x)	Frequency (f)
4	1
5	2
6	5
7	4
8	3
Total	15

a Calculate the mean mark.
b Determine the median mark.
c Give the modal mark.
d Which measure of centre best represents the data?
e Describe the distribution.
f Comment on any prediction of properties of the population from this sample.

3 Consider the following frequency distribution tables.

a

x	1	2	3	4	5
f	4	3	2	1	0

b

x	6	7	8	9	10
f	2	8	3	4	2

For each one:
i calculate the mean score to 1 decimal place
ii determine the median score
iii give the modal score
iv indicate which measure of centre best describes the distribution
v describe the skewness of the distribution.

4 Consider the following frequency stem plots.

a Key: 1 | 0 = 10

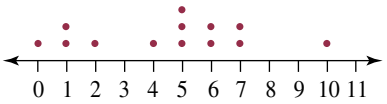
Stem	*Leaf*
1	0 2
2	1 3 3 5
3	
4	4

b Key: 10 | 0 = 100

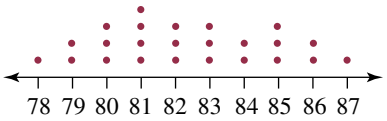
Stem	*Leaf*
10	0
11	0 2 2 2
12	0 4 6 6
13	3

For each one:
i calculate the mean score to 1 decimal place
ii determine the median score
iii give the modal score
iv indicate which measure of centre best describes the distribution
v describe the skewness of the distribution.

5 **a**



b



For each one:

- **i** calculate the mean score to 1 decimal place
- **ii** determine the median score
- **iii** give the modal score
- **iv** indicate which measure of centre best describes the distribution
- **v** describe the skewness of the distribution.

UNDERSTANDING

6 A survey of the number of people in each house in a street produced these data:
2, 5, 1, 6, 2, 3, 2, 1, 4, 3, 4, 3, 1, 2, 2, 0, 2, 4.

- **a** Prepare a frequency distribution table with an $f \times x$ column and use it to find the average (mean) number of people per household, correct to 1 decimal place.
- **b** Draw a dot plot of the data and use it to find the median number per household.
- **c** Find the modal number per household.
- **d** Which of the measures would be most useful to:
  - **i** a real estate agent renting out houses?
  - **ii** a government population survey?
  - **iii** an ice-cream mobile vendor?

7 The contents of 20 packets of matches were counted after random selection and the following numbers obtained:
138, 139, 139, 141, 137, 140, 137, 141, 139, 142, 140, 141, 141, 139, 141, 138, 139, 140, 141, 138.

- **a** Compile a frequency distribution table, including an $f \times x$ column.
- **b** Find the mode, median and mean of the distribution. Give answers correct to 1 decimal place where necessary.
- **c** Which of the 3 measures best supports the manufacturer's claim that there are 140 matches per box?
- **d** Display the data as a combined histogram and frequency polygon.

8 A class of 26 students had a median mark of 54 in Mathematics; however, no-one actually obtained this result.

- **a** Explain how this is possible.
- **b** Explain how many must have scored below 54.

9 A soccer team had averaged 2.6 goals per match after 5 matches. After their sixth match, the average had dropped to 2.5. How many goals did they score in that latest match?

10 A tyre manufacturer selects 48 tyres at random from the production line for testing. The total distance travelled during the safe life of each tyre is shown in the table.

Distance in km ('000)	82	78	56	52	50	46
Number of tyres	2	4	10	16	12	4

- **a** Calculate the mean, median and mode.
- **b** Recalculate the mean with the 6 longest-lasting tyres removed. By how much is it lowered?
- **c** If you selected a tyre at random, what tyre life would it most likely have?
- **d** In a production run of 10 000 tyres, how many could be expected to last for a maximum of 50 000 km?
- **e** As the manufacturer, for what distance would you be prepared to guarantee your tyres? Why?

REASONING

11 At a preview cinema session, the ages of the viewers were recorded and displayed in a stem-and-leaf plot (at right). For these data, find the:

- **a** range
- **b** mean
- **c** median
- **d** mode.

Key: 1 | 6 = 16 years

Stem	*Leaf*
1	5 6 7 7 8 9 9
2	1 2 4 8 8
3	0 1 1 1 5
4	2 3
5	3

12 Create a data set that fits each of the following descriptions.

- **a** Five data values with a mean of 3 and a mode of 3
- **b** Five data values with a mean of 3 and a mode of 4
- **c** Five data values with a mean of 3 and a median of 2

PROBLEM SOLVING

13 Read the following paragraph and explain what statistics are represented and what they mean.

It's been an exciting day at the races today. There was a record fast time of 38 seconds, and also a record slow time of 4 minutes and 52 seconds. We had an unbelievable number of people who ran the race in exactly 1 minute. Despite this, the average time was well over 2 minutes, due to the injury of a few runners.

14 Write a story to represent the following statistics: mean = 45.6, median = 34, mode = 24 and range = 58.

15 The number of goals a netballer scored in the 12 games of a season was as follows: 1, 1, 1, 1, 2, 2, 2, 3, 3, 3, 8, 12. A local newspaper reporter asked the netballer what his average was for the season.

- **a** Which measure of centre (mean, median or mode) should the netballer give the reporter as his 'average' so that the value of the average is as high as possible?
- **b** Which measure of centre would you choose to best describe the 'average' number of goals the netballer scored each game? Why?

eBook plus
Digital doc
WorkSHEET 12.2
doc-7012

12.8 Review

eBookplus ONLINE ONLY

The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:

- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

www.jacplus.com.au

Review questions

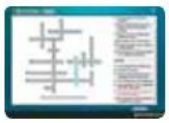
Download the Review questions document from the links found in your eBookPLUS.

eBookplus

Interactivities

Word search:
int-2759

Crossword:
int-2760



Sudoku:
int-3191

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

bias
census
class intervals
closed question
column graph
data
displaying data
frequency tables
mean
measure of centre
measure of spread
median
mode
normal distribution
open question
organising data
outlier
population
primary source of data
questionnaire
random
range
representative data
sample
secondary source of data
skewed

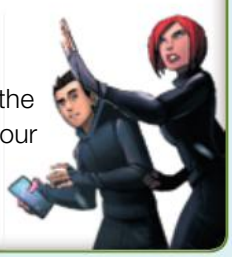
Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

www.assesson.com.au

Link to SpyClass, an exciting online game combining a comic book–style story with problem-based learning in an immersive environment.

Join Jesse, Toby and Dan and help them to tackle some of the world's most dangerous criminals by using the knowledge you've gained through your study of mathematics.



www.spyclass.com.au

RICH TASK

Analysing the English language

In the English language, our alphabet contains 26 letters that combine to form words. Have you ever wondered why some letters appear more often than others in many of the words we read? Scrabble® is a game that allows players to form interlocking words in a crossword style. The words are formed using lettered tiles that carry a numerical value. Players compete against each other to form words of the highest score until all the tiles have been used.

The following table displays the letter distribution and the value of each letter tile in the game of Scrabble.

Letter	Number of tiles	Letter score	Letter	Number of tiles	Letter score	Letter	Number of tiles	Letter score	Letter	Number of tiles	Letter score
A	9	1	B	2	3	C	2	3	D	4	2
E	12	1	F	2	4	G	3	2	H	2	4
I	9	1	J	1	8	K	1	5	L	4	1
M	2	3	N	6	1	O	8	1	P	2	3
Q	1	10	R	6	1	S	4	1	T	6	1
U	4	1	V	2	4	W	2	4	X	1	8
Y	2	4	Z	1	10	Blank	2	0			

1 Which letters are most common in Scrabble and which are least common?
2 What do you notice about the relationship between the number of tiles used for each letter and their letter score?

The following passage about newspapers and magazines is taken from an English textbook.

3 Complete a frequency table for the distribution of letters in this passage.
4 How do the results from this frequency table compare with the results from the Scrabble game's frequency table?

Who reads news?

Newspapers and magazines are produced for different categories of readers. These are known as the 'target audience'. Particular groups of readers buy certain types of newspapers and magazines because these publications have content which interests them as a reader. Magazines and newspapers cover different types of news; for example, a local paper contains information on local issues whereas a magazine like *Movie* has all the latest news on films and film stars. General newspapers like *The Age* and *The Australian* try to appeal to a broader audience by including sections on various topics such as sport, business and property.

Knowing which letters in the English language are most common can be very helpful when we are trying to solve coded messages. The following paragraph is written in code.

Tnlmktebt bl ehvtmxw bg max lhnmaxkg axfbliaxkx tgw bl ftwx ni hy lbq lmtmxl tgw mph mxkkbmhkbxl. Max vtibmte hy Tnlmktebt bl Vtguxkkt.

5 Study the coded message carefully. Use an appropriate method to decode the message.
6 Explain the strategies you used to decode the message.
7 Using different codes, write your own messages on a separate page and test them on your classmates.

CODE PUZZLE

An important discovery in 1921

Complete the frequency table for the number of hours of television watched per week by a group of people. The completed table will give the puzzle's answer code.

Hours of TV watched per week

Number of hours	Tally	Frequency	
0–4			= A
5–9			= B
10–14			= D
15–19			= E
20–24			= F
25–29			= H
30–34			= I
35–39			= L
40–44			= M
45–49			= N
50–54			= O
55–59			= R
60–64			= S
65–69			= T
Total			= U

12, 41, 20, 34, 6,
50, 0, 28, 15, 30,
25, 20, 39, 23, 16,
11, 34, 5, 35, 23,
22, 44, 20, 28, 12,
40, 7, 38, 16, 33,
14, 51, 29, 8, 24,
24, 10, 27, 35, 20,
25, 16, 30, 0, 47, 15, 49, 25, 11,
57, 33, 8, 27, 38, 15, 23, 51, 21, 37, 7, 52, 21, 15,
42, 13, 37, 6, 54, 17, 13, 43, 26, 17, 36, 3, 24, 61, 18,
60, 14, 34, 21, 25, 43, 2, 32, 41, 21, 32, 26, 12, 53, 9,
45, 17, 47, 6, 24, 26, 45, 15, 63, 10, 34, 65, 19, 58, 21

10	5	3	108	8	10	5	108	3	13	11	10	5	1	12	13
1	2	13	4	1	7	13	5	1	6	17	11	10	4	9	13	1	13	3

eBookplus Activities

12.2 Samples and populations

Digital docs
- Investigation (doc-2473) Bias
- Investigation (doc-2167) Collecting data for surveys and questionnaires
- Investigation (doc-7013) Data and the Olympics
- Spreadsheet (doc-7106) Data and the Olympics

Interactivities
- Random numbers (int-0089)
- IP interactivity 12.2 (int-4451) Samples and populations

12.3 Primary and secondary data

Interactivity
- IP interactivity 12.3 (int-4452) Primary and secondary data

12.4 Organising and displaying data

Digital docs
- SkillSHEET (doc-6989) Presenting data in a frequency table
- Spreadsheet (doc-2183) Histograms and frequency polygons
- WorkSHEET 12.1 (doc-7005)

Interactivity
- IP interactivity 12.4 (int-4453) Organising and displaying data

12.5 Measures of centre

Digital docs
- SkillSHEET (doc-6991) Finding the mean of ungrouped data
- SkillSHEET (doc-6992) Arranging a set of data in ascending order
- SkillSHEET (doc-6993) Finding the median
- SkillSHEET (doc-6995) Finding the score in a data set that occurs most frequently
- Investigation (doc-2166) How many red M&Ms?
- Spreadsheet (doc-2184) Mean
- Spreadsheet (doc-2185) Median
- Spreadsheet (doc-2187) Mode

Interactivities
- Measures of centre (int-2362)
- IP interactivity 12.5 (int-4454) Measures of centre

12.6 Measures of spread

Digital doc
- SkillSHEET (doc-6994) Finding the middle score for data arranged in a dot plot

Interactivity
- IP interactivity 12.6 (int-4455) Measures of spread

12.7 Analysing data

Digital doc
- WorkSHEET 12.2 (doc-7012)

Interactivity
- IP interactivity 12.7 (int-4456) Analysing data

12.8 Review

Interactivities
- Word search (int-2759)
- Crossword (int-2760)
- Sudoku (int-3191)

Digital docs
- Topic summary (doc-10763)
- Concept map (doc-10776)
- Topic review (Word doc-14944, PDF doc-14945)

To access eBookPLUS activities, log on to www.jacplus.com.au

Answers

TOPIC 12 Representing and interpreting data

12.2 Samples and populations

1 Census, sample
2 Census — every member of the population participates.
3 Survey
4 **a** Survey
b Survey
c Census
d Survey
5 **a** Survey
b Census
c Census
d Survey
6 Open questions have no boundaries for response. Closed questions require answers to fall within a category. A variety of examples is possible.
7 A variety of categorical responses is suitable; check with your teacher.
8 A variety of categorical responses is suitable; check with your teacher.
9 **a** This is a personal question.
b This question is vague. Define what is meant by 'exercise' and 'regularly'.
c What do the abbreviations stand for?
d What is the Premier's policy on wildlife preservation?
e Is this question capable of being answered?
What is a 'foot'?
f This is a double-barrelled question.
Check with your teacher for alternative wording of questions.
10 Generally only those people with strong points of view participate in telephone surveys. This type of sample is often not representative of the whole population. The high telephone cost for the response is also a deterrent to responding.
11 Check with your teacher.
12 A variety of answers is available here.
a Typically, the choir members would not be the ones using the pool. The sample is biased.
b These people may or may not know about the PM's health-care package. In any case, the sample is biased as it is not truly representative of the population.
c If the visitors had just arrived at the airport, they would not have had time to experience the transport system into the city. The sample is biased.
d This sample size is far too small.
13 A variety of answers is possible.
14 22, 30, 49, 3, 14 is one possible solution.
15 **a** 94, 1, 54
b 60, 47, 33, 43
c 25, 49, 10
16 D
17 Discuss this question as a class.

Challenge 12.1

6

12.3 Primary and secondary data

1 These are simply examples of simulations that could be conducted.
a A coin could be flipped (Heads representing 'True' and Tails representing 'False').
b A coin could be flipped (Heads representing 'red' and Tails representing 'black').
c Spinner with 4 equal sectors (each sector representing a different toy)
d Roll a die (each face represents a particular person).
e Spinner with 3 equal sectors (each one representing a particular meal)
f Spinner with 5 equal sectors (each one representing a particular destination)
g Spinner with 5 sectors, one with an angle size of 120° and four with an angle size of 60° (each one representing a particular fast food)
2 Answers will vary, but some possible suggestions include:
Which students have internet access at home?
Do the students need access at night?
What hours would be suitable?
How many would make use of this facility?
3 Check with your teacher.
4 Answers will vary, but some possible suggestions include:
a Census, survey, questionnaire, interview, observation, experiment, on-line response
b **i** Measurement
ii Observation
iii Newspaper recordings
iv Survey
5–7 Check with your teacher.
8 The claim is false. It is not a logical deduction.
9 Check with your teacher.
10 Check with your teacher.
11 Sealy Posturepremier: 40% off $(\frac{1000}{2499} \times 100\%)$
Sealy Posturepedic: 41% off $(\frac{1600}{3899} \times 100\%)$
Sleepmaker Casablanca: 40% off $(\frac{800}{1999} \times 100\%)$
Sleepmaker Umbria: 42% off $(\frac{1800}{4299} \times 100\%)$
True; the discount is at least 40% off these beds.
12 A sample from one Year group only will not give an indication of the preferences of the whole school. He should ask students from all year levels.
13 Answers will vary, but some possible suggestions include:
Primary data: She could visit each of the institutions in turn either in person or on-line.
Secondary data: She could seek advice from friends and colleagues. She could enlist the help of a mortgage broker or similar professional.
14 Answers will vary, but some possible suggestions include:
a The most watched channel and the time slot with highest viewing ratings
b Conduct a survey in a supermarket to establish the typical customer for her product
c Yes. If the typical customer was a five-year-old child, there would be little point in advertising during the 6 o'clock news, even though that time slot may have the highest rating on the most watched channel.

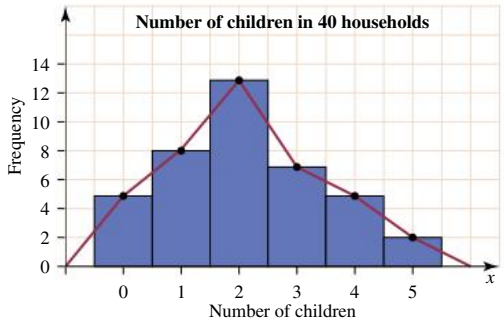
12.4 Organising and displaying data

1 a The sample is random and is of sufficient size, so calculations from the data could be considered reliable.

b

x	Frequency
0	5
1	8
2	13
3	7
4	5
5	2
Total	40

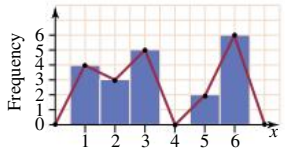
c



d The data are distributed fairly evenly around 2 children per household, and there appear to be no outliers. The graph clearly shows a maximum at 2 children per household.

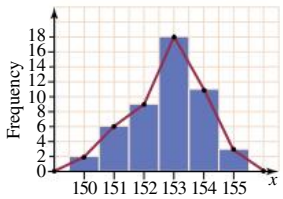
e The sample is a random one, and of sufficient size; we can be confident that the suburb also exhibits these same properties.

2 a i



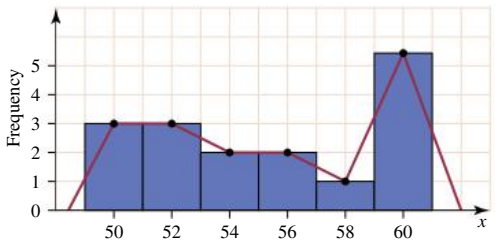
ii This graph is quite inconsistent, showing no particular trend.

b i



ii This graph is almost symmetrical, rising to a maximum at 153, then decreasing.

c i



ii This graph is quite unusual. It shows a group of smaller values, then a maximum at the upper limit of the data.

3 a The sample is a random one, so it seems to be a reliable reflection of the population of the portable CD players.

b

x	Frequency
0	1
1	6
2	8
3	5
4	2
5	3
Total	25

c There was only 1 box with no defective CD players. Most boxes had only 1, 2 or 3 defective CD players, while 3 boxes were found to have 5 defective CD players.

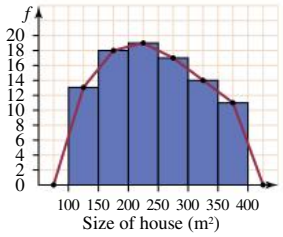
d Since the sample was randomly selected, it seems to be a reliable reflection of the characteristics of the population. It would be reasonably safe to say that most boxes would have only 1, 2 or 3 defective CD players.

4 a

b The graph rises steadily to a maximum, then falls away sharply at the upper end of the data.

c This is likely to be a true reflection of the sporting habits of Year 8 students. Some do a minimum of only 1 hour per week, quite a few do 2, 3 or 4 hours per week, with the maximum number doing 5 hours per week. The committed sports players would put in 6 or 7 hours per week.

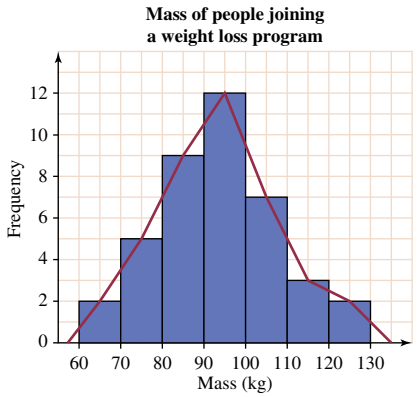
5 a



b The graph is roughly symmetrical, rising to a maximum at the 200-m^2 to 250-m^2 size, then decreasing slowly.

c Since this is one block of houses in the suburb, it is not a random sample. We often find all the houses in a block of a suburb are of similar style. For this reason, we could not say it reflects the size of houses in the whole suburb.

6 a



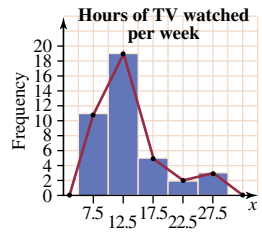
b The graph is quite symmetrical, rising to a maximum at the 90-kg to 100-kg mass, then decreasing more rapidly to the 130-kg mass.

c This would most likely not reflect the masses of people in the community because these are people who have enrolled in a program to lose weight.

7 a

Class interval	Midpoint (class centre)	Tally	Frequency
5–<10	7.5	𝍸 𝍸 |	11
10–<15	12.5	𝍸 𝍸 𝍸 𝍸	19
15–<20	17.5	𝍸	5
20–<25	22.5	||	2
25–<30	27.5	|||	3
		Total	40

b



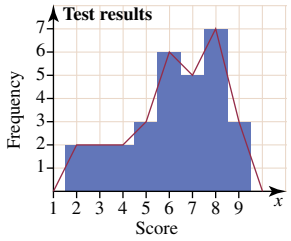
c The graph is heavily weighted towards the lower end of the scale, with most people watching fewer than 15 hours of TV per week. There were 3 people who watched almost 30 hours of TV per week.

d Since these people were interviewed in a shopping centre, the sample is not a random one. It could not therefore be taken to reflect the viewing habits of the community.

8 a

Test results	Frequency
2	2
3	2
4	2
5	3
6	6
7	5
8	7
9	3
Total	30

b

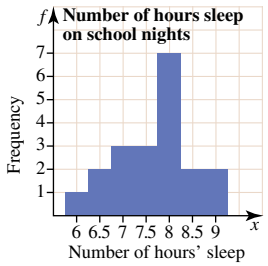


c Most Year 8 students achieved a score of about 5 to 9. There were 6 students who received less than 5. This is probably a typical spread of marks for a multiple-choice test.

9 a

Hours of sleep	Frequency
6	1
$6\frac{1}{2}$	2
7	3
$7\frac{1}{2}$	3
8	7
$8\frac{1}{2}$	2
9	2
Total	20

b



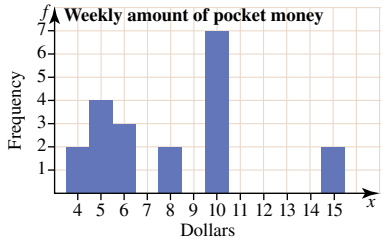
c The histogram peaks sharply at the 8-hours sleep mark, indicating that generally, Year 8 students get 8 hours of sleep per night during the week. Some get less, and a few get more.

d It seems likely that these sample results would reflect the sleeping habits of Year 8 students generally.

10 a

Pocket money ($)	Frequency
4	2
5	4
6	3
7	0
8	2
9	0
10	7
11	0
12	0
13	0
14	0
15	2
Total	20

b



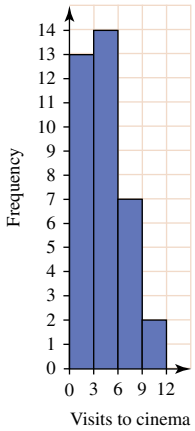
c The histogram shows no general trend. The maximum is at $10, indicating that a popular amount of pocket money is $10 per week. Quite a few receive less than this, with only 2 receiving more.

d Since this is a random sample, it is quite likely that these results reflect the general population of 13-year-olds when it comes to pocket money.

11 a Answers will depend on class intervals chosen. An example is given.

Number of visits to the cinema	Tally	Frequency (f)
0–2	𝍸 𝍸 III	13
3–5	𝍸 𝍸 IIII	14
6–8	𝍸 II	7
9–11	II	2
	Total	36

b



c 13

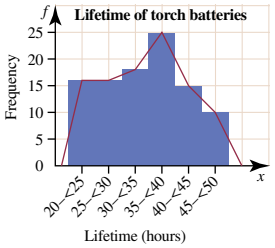
d 23

e No. Check with your teacher.

12 a

Life time (hours)	Frequency
20–<25	16
25–<30	16
30–<35	18
35–<40	25
40–<45	15
45–<50	10
Total	100

b



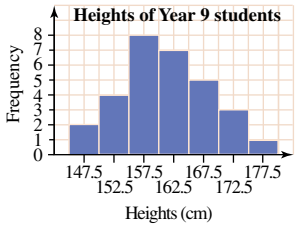
c The histogram shows that the majority of torch batteries last for about 40 hours. A few last longer than this.

d It seems reasonable that the torch battery population would display a similar trend.

13 a

Class interval	Midpoint (class centre)	Tally	Frequency
145–<150	147.5	II	2
150–<155	152.5	IIII	4
155–<160	157.5	𝍸 II	8
160–<165	162.5	𝍸 II	7
165–<170	167.5	𝍸	5
170–<175	172.5	III	3
175–<180	177.5	I	1
		Total	30

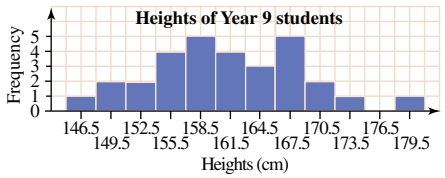
b



c

Class interval	Midpoint (class centre)	Tally	Frequency
145–<148	146.5	I	1
148–<151	149.5	II	2
151–<154	152.5	II	2
154–<157	155.5	IIII	4
157–<160	158.5	𝍸	5
160–<163	161.5	IIII	4
163–<166	164.5	III	3
166–<169	167.5	𝍸	5
169–<172	170.5	II	2
172–<175	173.5	I	1
175–<178	176.5	—	0
178–<181	179.5	I	1
		Total	30

d



e The two histograms represent the same data set, but appear to be quite different. The first histogram appears to be roughly symmetrical, with a maximum number of students having a height about 155 cm to 160 cm. The second histogram has two modes, with the most common height for students being about 157 cm to 160 cm, or 166 cm to 169 cm. It illustrates the fact that the interpretation of a histogram displaying grouped data is dependent on the class interval used.

12.5 Measures of centre

1 26.8
2 **a** 4.33 **b** 7.33 **c** 36.30 **d** 6.91
3 **a** 18 **b** 10 **c** 1.8 goals
4 C
5 4.6
6 Group A: 6.1; Group B: 6.8. Group B has a larger mean.
7 2.29
8 7.90
9

x	f	xf
1	4	4
2	5	10
3	4	12
4	2	8
5	1	5
Total	16	39

Mean = 2.4
10 80
11 **a** 7 **b** 17
12 **a** 48 **b** 1.8
13 Question 11:
a 5
b 17
Question 12:
a 52
b 1.5, 1.7, 1.8
14 C
15 E
16 **a** 6.7 **b** 10
17

x	f	xf
0	1	0
1	2	2
2	7	14
3	3	9
4	4	16
5	2	10
6	1	6
Total	20	57

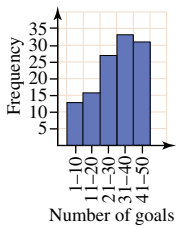
Mean = 2.85
18 **a** 36 **b** 7
19 **a** 3 **b** 3 **c** Both
20 **a** 4 **b** 3
c Three strokes. Factors include wind, difficulty of hole, performance on the day, and so on.
21 **a** 49 **b** 55 **c** 56 **d** Median
22 **a** $18 000 **b** $26 000 **c** $30 000 **d** Mean **e** Mode
23 One possible answer is 5, 6, 12, 13, 14. If the five numbers are in ascending order, the third number must be 12 and the other 4 numbers must total 38.
24 Highest score is 49 and lowest score is 0.
The scores would be 0, 1, 12, 13, 49.
25 Largest value for median is 15. Scores would be 0, 2, 15, 16, 17. Smallest value for median is 2. Scores would be 0, 1, 2, a, b, where $a + b = 47$.
26 43; highest score: 100; lowest score: 57. The scores would be 57, 77, 78, 79, 80, 97, 98, 99, 100.
27 **a** Mean = 4.5; median = 5; modes = 3 and 5
b The median is higher than the mean.
c The mean is between the two modal values.
d As there are no outliers, the mean is probably the best indicator of the centre of the data set.
28 Mean = 5; median = 3; mode = 6
29 73.45
30 171

Challenge 12.2

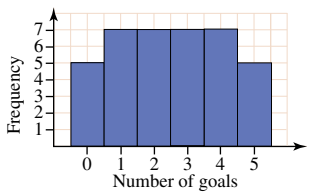
20

12.6 Measures of spread

1 **a** 8 **b** 15 **c** 50 **d** 2.5
2 **a** 4 **b** 5 **c** 15 **d** 4
3 **a** Yes **b** 8 **c** Both equal 8.
4 **a** No **b** 31–40 **c** No
5 **a** 4–5 **b** Negatively skewed
6 **a**

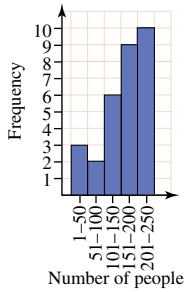


b Negatively skewed
7 B
8 **a** **i** 35
ii The data are irregular, with no obvious skewness.
b **i** 46
ii The data are negatively skewed.
c **i** 35
ii The data are negatively skewed.
9 This distribution seems to have two outliers — one at 8 and the other at 18. If these two scores are disregarded, the remaining part of the distribution is almost symmetrical.
10 **a**



b Yes
c 1, 2, 3 and 4
d Yes, both equal 2.5.

11 a



b No
c 201–250
d The distribution is negatively skewed. Reasons could include the size of cinemas or the target audience of the movie.

12 a No
b Science: positively skewed, Maths: negatively skewed
c The Science test may have been more difficult.
d Science: 61–70, Maths: 71–80

13 Answers will vary. Check with your teacher.

14 a 5
b 4.5
c 5
d The data are relatively symmetrical and there is no obvious skewness.

15 a Lowered
b Raised

16 a 0.1
b $(m + 1) - (b - a)$

12.7 Analysing data

1 a Range **b** Mean **c** Mode **d** Mean **e** Median

2 a 6.4 **b** 6 **c** 6
d They are all quite close, so any would do.
e The distribution is negatively skewed, with more scores towards the upper end.
f Since these were the results of half the class, and the sample was random, it seems likely that the population results would be similar.

3 a i 2 **ii** 2 **iii** 1 **iv** Mean or median
v Positively skewed
b i 7.8 **ii** 7 **iii** 7 **iv** Any of the three
v Slightly positively skewed

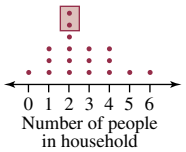
4 a i 22.6 **ii** 23 **iii** 23 **iv** Any of the three
v Possibly positively skewed with an outlier at 44
b i 117.5 **ii** 116 **iii** 112 **iv** Mean
v Symmetrical

5 a i 4.5 **ii** 5 **iii** 5 **iv** Any of the three
v The distribution is very scattered, with an outlier at 10.
b i 82.4 **ii** 82 **iii** 81 **iv** Mean or median
v Slightly positively skewed

6 a

Score (x)	Frequency (f)	Freq. × score ($f \times x$)
0	1	0
1	3	3
2	6	12
3	3	9
4	3	12
5	1	5
6	1	6
	$n = 18$	$\Sigma fx = 47$

b $\bar{x} = 2.6$



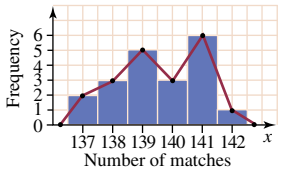
c 2
d i Median **ii** Mean **iii** Mode

7 a

Score (x)	Frequency (f)	Freq. × score ($f \times x$)
137	2	274
138	3	414
139	5	695
140	3	420
141	6	846
142	1	142
	$n = 20$	$\Sigma fx = 2791$

b 141, 139.5, 139.6 **c** Mean

d



8 a The median was calculated by taking the average of the 2 middle scores.
b 13

9 2

10 a 55 250 km, 52 000 km, 52 000 km
b 51 810 km; it is reduced by 3440 km.
c 52 000 km
d 2500
e 50 000 km; 92% last that distance or more.

11 a 38 years
b 27 years
c 26 years
d 31 years

12 a Answers will vary. An example is 0, 1, 3, 3, 8.
b Answers will vary. An example is 0, 1, 4, 4, 6.
c Answers will vary. An example is 1, 2, 2, 4, 6.

13 The range was 4 minutes 14 seconds (fastest time of 38 seconds, slowest time of 4 minutes 52 seconds). The most common time (mode) was 1 minute. The average (mean) time was more than 2 minutes.

14 Answers will vary.

15 a The mean (3.25). The median is 2 and the mode is 1.
b Answers will vary.

Investigation — Rich task

1 The letters E, A, I and O are the most common; the letters J, K, Q, X and Z are least common.

2 The more tiles for a letter results in a lower tile score. The less tiles for a letter results in a higher tile score.

3 The frequency distribution table includes the letters in the passage heading.

Letter	Frequency	Letter	Frequency
A	61	**O**	31
B	6	**P**	23
C	18	**Q**	0
D	18	**R**	39
E	73	**S**	50
F	13	**T**	34
G	10	**U**	14
H	14	**V**	4
I	31	**W**	11
J	0	**X**	1
K	3	**Y**	6
L	18	**Z**	4
M	10		
N	41		

4 Results from both frequency tables show letters J, K, Q, X and Z to be the least common. However, while results from the scrabble frequency table show letters A, E, I and O to be the most common; results from the textbook frequency table show letters A, E, I, N, O, R, S and T to be most common.
5 Australia is located in the southern hemisphere and is made up of six states and two territories. The capital of Australia is Canberra.
6 Teacher to check. Common strategies involve looking for common letters and frequently used two- and three-lettered words.
7 Teacher to check.

Code puzzle

Insulin used in the treatment of diabetes

projectsplus

How to burglar-proof your bedroom

Searchlight ID: pro-0097

Scenario

As new employees of *Kidnapper Nappers*, you and your team have been asked to design a burglar alarm for Alvin's bedroom. Alvin is a little tired of his younger brother, Michael, going into his bedroom and using Alvin's electronic equipment whenever he chooses. Alvin wants a simple alarm installed to deter anyone entering his bedroom without permission.

Alvin has asked that both his door and window be alarmed as added security.

Your brief is to design the circuitry that will then be built by the development team. This will involve:

1. learning the three basic building blocks of logic — NOT, AND and OR — using truth tables
2. learning the concept of a logic circuit, which consists of INPUTS (0's or 1's), OUTPUTS (0's or 1's) and GATES that control the logic
3. learning how to design a simple burglar alarm to meet Alvin's needs
4. learning how to design a logic circuit for the alarm consisting of a series of INPUTS, GATES and OUTPUTS.

This is your team's first big project and it is very important that your presentation is first class.

Task

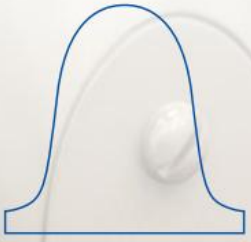
You will produce a PowerPoint presentation to be shown to Alvin and your manager, clearly explaining the features of the circuit you have designed, and the problems it will solve. It is important that you use appropriate circuit diagrams and terminology.

NOT gate

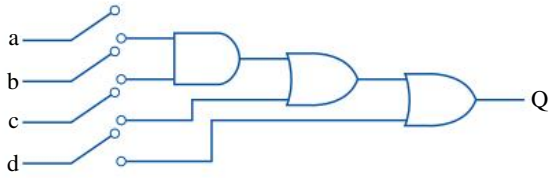
AND gate

OR gate

Input switch


Output alarm

As your manager understands your inexperience with this type of project, he has provided an Interactivity to help you understand and also a series of Learning tasks to be completed to start your team on its way.



Logic circuit

Process

- Open the ProjectsPLUS application for this topic in your eBookPLUS. Watch the introductory video lesson, click the Start project button and then set up your project group. You can complete this project individually or invite other members of your class to form a group. Save your settings, and the project will be launched.
- Go to your Media centre and find the interactivity **Logic gates** (int-2448) to determine how Logic gates can be used.
- Navigate to your Media Centre. Here you will find a series of learning tasks (Worksheets) that will help you to complete the project. These must be completed and submitted to your teacher before you commence the next step.
- Research: Navigate again to your Research forum and make notes of important facts and ideas that you discover during your research. Enter your findings as articles under your topics in the Research forum. Each of your team members should find at least three different sources. You can view and comment on other group members' research, and rate the information they have entered. When your research is complete, print your Research report to hand in to your teacher.
- Visit the Media Centre and download the PowerPoint template (alarm template.ppt), to help you prepare your presentation. Your Media centre also includes images that can help to liven up your presentations.
- Use the Powerpoint template (alarm template.ppt) to develop your presentation. Remember that you are trying to convince Alvin and your manager that your design and proposal will be simple and easy to construct. Make sure that you include all the information they requested, and that your presentation will grab their attention.

SUGGESTED SOFTWARE

- ProjectsPLUS
- Microsoft PowerPoint
- Microsoft Excel

Your ProjectsPLUS application is available in this topic's Student Resources tab inside your eBookPLUS. Visit www.jacplus.com.au to locate your digital resources.

Interactivity

LOGIC GATES

Searchlight ID: int-2448

Boolean logic is at the heart of the digital age. It was developed by George Boole in the 1800s. Logic gates are the basic building blocks of a digital circuit. Using different combinations of these gates, it is possible to implement any digital component. Use this interactivity to investigate the input and output functions of logic gates.

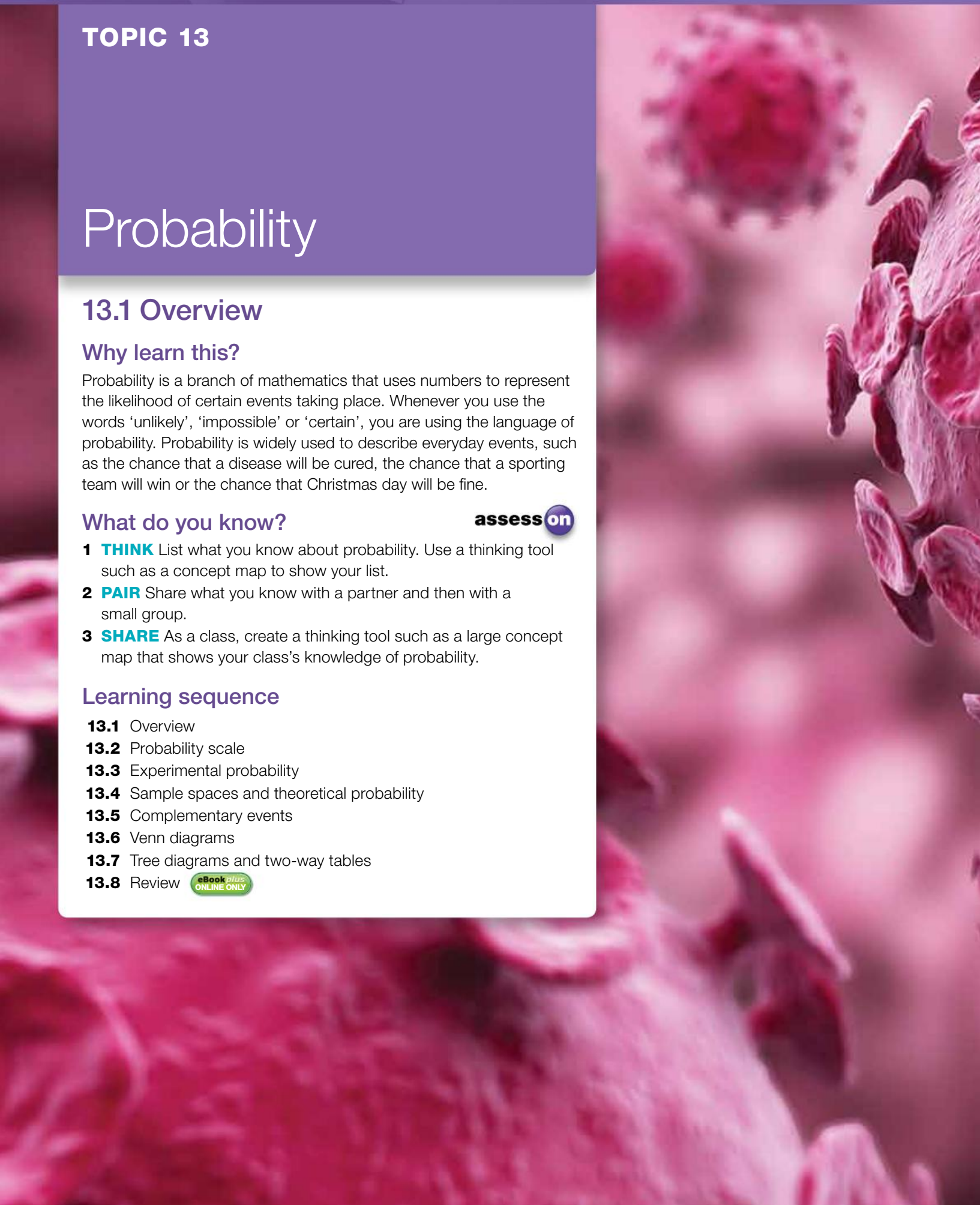
MEDIA CENTRE

Your Media Centre contains:

- a Document section with worksheets
- a Template section with presentation guidelines and circuit templates
- some images you can use in your presentation
- an assessment rubric.

TOPIC 13

Probability



13.1 Overview

Why learn this?

Probability is a branch of mathematics that uses numbers to represent the likelihood of certain events taking place. Whenever you use the words 'unlikely', 'impossible' or 'certain', you are using the language of probability. Probability is widely used to describe everyday events, such as the chance that a disease will be cured, the chance that a sporting team will win or the chance that Christmas day will be fine.

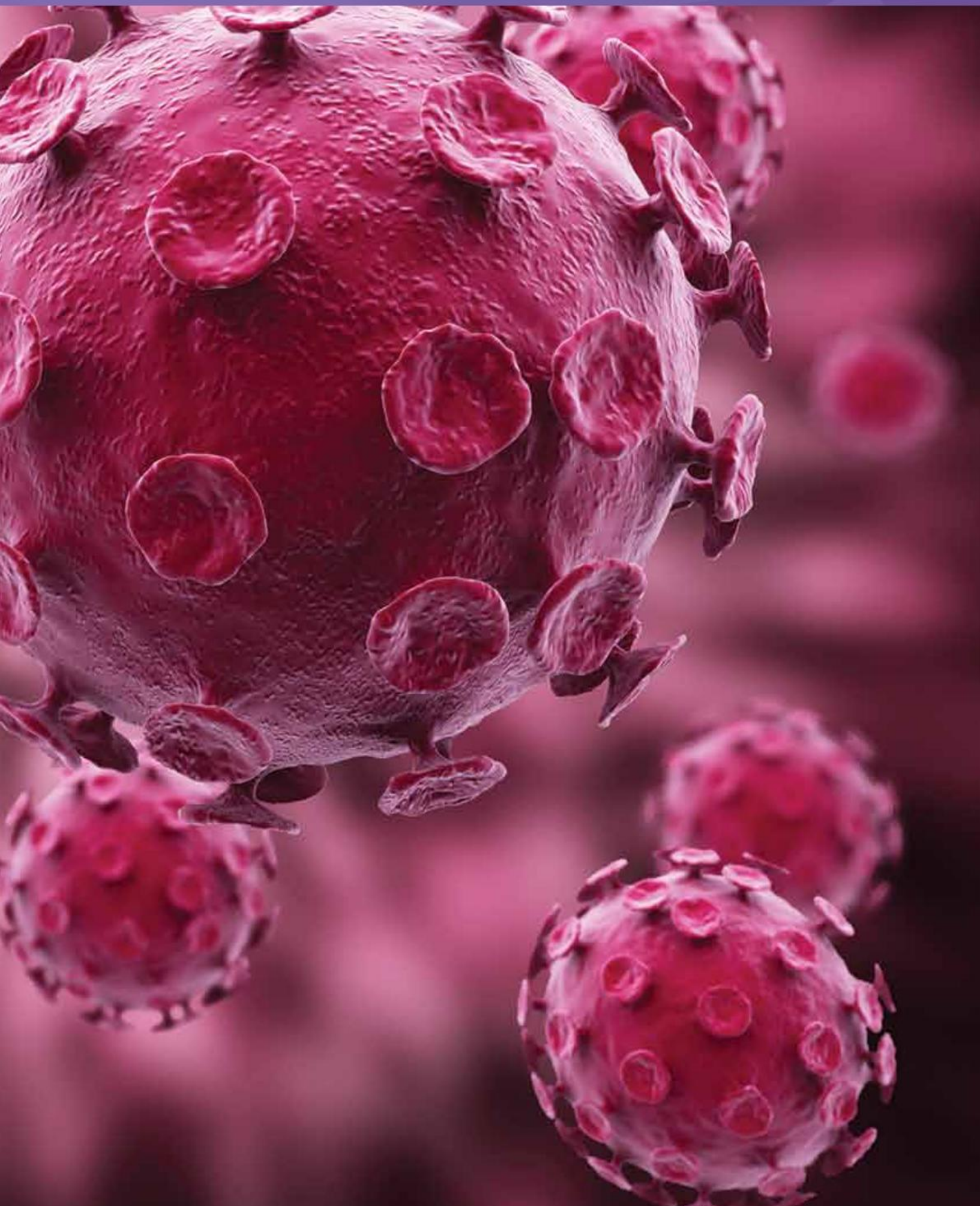
What do you know?

assessON

1 **THINK** List what you know about probability. Use a thinking tool such as a concept map to show your list.
2 **PAIR** Share what you know with a partner and then with a small group.
3 **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of probability.

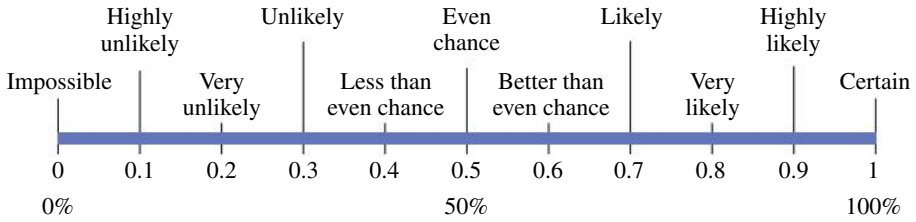
Learning sequence

13.1 Overview
13.2 Probability scale
13.3 Experimental probability
13.4 Sample spaces and theoretical probability
13.5 Complementary events
13.6 Venn diagrams
13.7 Tree diagrams and two-way tables
13.8 Review eBook plus ONLINE ONLY



13.2 Probability scale

- **Probability** is defined as the chance of an event occurring.
- A scale from 0 to 1 inclusive is used to allocate the probability of an event as follows:



- A probability of 0 implies that the chance of an event happening is **impossible**.
- A probability of 1 implies that the chance of an event happening is **certain**.
- Probabilities may be written as fractions, decimals or percentages.

WORKED EXAMPLE 1

Describe the probability of each of the following events occurring, using a term from this list.

impossible **highly unlikely** **very unlikely**
less than even chance **even chance**
better than even chance **very likely**
highly likely **certain**

a February follows January.
b You draw the queen of diamonds from a standard deck of playing cards.
c You will compete in gymnastics at the Olympics.
d You roll a standard die and obtain an even number.
e Every Mathematics student will obtain a score of 99.95% as shown in an examination.

THINK	WRITE
a 1 Read the statement and associate the likelihood of the event occurring with one of the given words from the list. *Note:* Provide reasoning.	**a** This is a true statement. February always follows January.
2 Answer the question.	It is *certain* this event will occur.
b Repeat steps 1 and 2 of part **a**.	**b** In a standard deck of 52 playing cards there is only one queen of diamonds. Thus, you have an extremely slim chance of drawing this particular card. It is *highly unlikely* this event will occur.
c Repeat steps 1 and 2 of part **a**.	**c** The chance of a person competing in the Olympics is very small. However, it could happen. It is *very unlikely* this event will occur.
d Repeat steps 1 and 2 of part **a**.	**d** There are six possible outcomes when rolling a die, each of which are equally likely. Three of the outcomes are even while three are odd. There is an *even chance* this event will occur.

e Repeat steps 1 and 2 of part **a**.	**e** Due to each student having different capabilities, this situation could never occur. It is *impossible* that this event will occur.

- It is important to note that the responses for particular situations, such as part **c** in Worked example 1, are not always straightforward and may differ for each individual. A careful analysis of each event is required before making any predictions about their future occurrences.

WORKED EXAMPLE 2

Assign a fraction to represent the estimated probability of each of the following events occurring:

a a high tide will be followed by a low tide
b everyone in the class will agree on every matter this year
c a tossed coin lands Heads
d a standard die is rolled and the number 5 appears uppermost
e one of your 15 tickets in a 20-ticket raffle will win.

THINK	WRITE
a 1 Determine the likelihood of an event occurring, with reasoning. 2 Answer the question.	**a** The tide pattern occurs daily; this event seems *certain*. The probability of this event occurring is equal to 1.
b Repeat steps 1 and 2 of part **a**.	**b** Total agreement among many people on every subject over a long time is virtually *impossible*. The probability of this event occurring is equal to 0.
c Repeat steps 1 and 2 of part **a**.	**c** When tossing a coin there are two equally likely outcomes, a head or a tail. The probability of this event occurring is equal to $\frac{1}{2}$.
d Repeat steps 1 and 2 of part **a**.	**d** When rolling a die there are six equally likely outcomes: 1, 2, 3, 4, 5, 6. The probability of this event occurring is equal to $\frac{1}{6}$.
e Repeat steps 1 and 2 of part **a**.	**e** There are 15 chances out of 20 of winning. The probability of this event occurring is equal to $\frac{15}{20}$, which when simplified is equal to $\frac{3}{4}$.

Exercise 13.2 Probability scale

REFLECTION
Can you think of some events that would have a probability of 0, 1 or $\frac{1}{2}$ of occurring?

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–6, 8	Questions: 1–8	Questions: 1–9

Individual pathway interactivity int-4457 eBookplus

UNDERSTANDING

eBookplus
Digital doc
SkillSHEET
Understanding chance words
doc-6969

1 WE1 Describe the probability of each of the following events occurring using a term from the list below.

Impossible	Highly unlikely	Very unlikely
Less than even chance	Even chance	Better than even chance
Very likely	Highly likely	Certain

a The sun will set today.
b Every student in this class will score 100% in the next Mathematics exam.
c It will rain tomorrow.
d Your shoelace will break next time you tie your shoes.
e Commercial TV stations will reduce time devoted to ads.
f A comet will collide with Earth this year.
g The year 2020 will be a leap year.
h You roll a standard die and an 8 appears uppermost.
i A tossed coin lands on its edge.
j World records will be broken at the next Olympics.
k You roll a standard die and an odd number appears uppermost.
l You draw the queen of hearts from a standard deck of playing cards.
m You draw a heart or diamond card from a standard deck of playing cards.
n One of your 11 tickets in a 20-ticket raffle will win.
o A red marble will be drawn from a bag containing 1 white marble and 9 red marbles.
p A red marble will be drawn from a bag containing 1 red and 9 white marbles.

2 Write two examples of events that are:
a impossible **b** certain **c** highly likely
d highly unlikely **e** equally likely (even chance).

3 WE2 Assign a fraction or decimal to represent the estimated probability of each of the following events occurring.
a A Head appears uppermost when a coin is tossed.
b You draw a red marble from a bag containing 1 white and 9 red marbles.
c A standard die shows a 7 when rolled.
d You draw a yellow disk from a bag containing 8 yellow disks.
e The next baby in a family will be a boy.
f A standard die will show a 1 or a 2 when rolled.

g You draw the queen of hearts from a standard deck of playing cards.
h One of your 11 tickets in a 20-ticket raffle will win.
i A standard die will show a number less than or equal to 5 when rolled.
j You draw an ace from a standard deck of playing cards.
k A class captain will be elected from five candidates.
l You draw a king or queen card from a standard deck of playing cards.
m You spin a seven-sided spinner and obtain an odd number.
n Heads or Tails will show uppermost when a coin is tossed.

UNDERSTANDING

4 MC The word that best describes the probability for a standard die to show a prime number is:

A impossible **B** very unlikely **C** even chance
D very likely **E** certain

5 MC The probability of Darwin experiencing a white Christmas this year is closest to:

A 1 **B** 0.75 **C** 0.5
D 0.25 **E** 0

REASONING

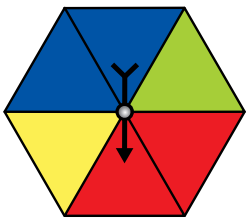
6 The letters of the word *MATHEMATICS* are each written on a small piece of card and placed in a bag. If one card is selected from the bag, what is the probability that it is:

a a vowel **b** a consonant
c the letter M **d** the letter C?

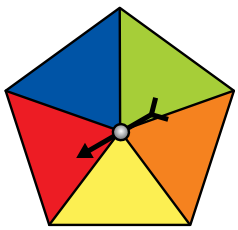
7 Answer the following for each of the spinners shown.

i Is there an equal chance of landing on each colour? Explain.
ii List all the possible outcomes.
iii Find the probability of each outcome.

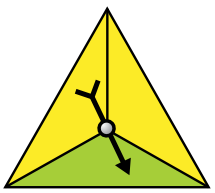
a



b



c



PROBLEM SOLVING

8 All the jelly beans in the photograph are placed in a bag for a simple probability experiment.

a Which colour jelly bean is most likely to be randomly selected from the bag? Explain.
b Which colour jelly bean is least likely to be randomly selected from the bag? Explain.
c Find the probability of randomly selecting each coloured jelly bean from the bag.

9 Draw a spinner with the following probabilities.

a $\Pr(\text{blue}) = \frac{1}{3}$ and $\Pr(\text{white}) = \frac{2}{3}$
b $\Pr(\text{blue}) = \frac{1}{2}$, $\Pr(\text{white}) = \frac{1}{4}$, $\Pr(\text{green}) = \frac{1}{8}$ and $\Pr(\text{red}) = \frac{1}{8}$
c $\Pr(\text{blue}) = 0.75$ and $\Pr(\text{white}) = 0.25$

CHALLENGE 13.1

Four different types of coloured card are enclosed in a bag: red, blue, green and yellow. There is a 40% chance of drawing a blue card and the probability of a red card is 0.25. There are 4 yellow cards. If there is a total of 20 cards in the bag, what is the probability of drawing a green card? How many green cards are in the bag?

13.3 Experimental probability

- **Experiments** are performed to provide data, which can then be used to forecast the outcome of future similar events.
- An experiment that is performed in the same way each time is called a **trial**.
- An **outcome** is a particular result of a trial.
- A **favourable outcome** is one that we are looking for.
- An **event** is the set of favourable outcomes in each trial.
- The **relative frequency** of an event occurring is the experimental probability of it occurring.
- Relative frequency of an event $= \dfrac{\text{frequency of an event}}{\text{total number of trials}}$

WORKED EXAMPLE 3

The table at right shows the results of a fair coin that was tossed 20 times. What are the relative frequencies of:
a Heads?
b Tails?

Event	Frequency
Heads	8
Tails	12
Total	20

THINK

a 1 Write the frequency of the number of Heads and the total number of trials; that is, tosses.

2 Write the rule for the relative frequency.

3 Substitute the known values into the rule.

4 Evaluate and simplify if possible.

5 Answer the question.

b 1 Write the frequency of the number of Tails and the total number of trials; that is, tosses.

2 Write the rule for the relative frequency.

WRITE

a Frequency of Heads = 8
Total number of tosses = 20

$$\text{Relative frequency} = \frac{\text{frequency of a Head}}{\text{total number of tosses}}$$

$$\text{Relative frequency of Heads} = \frac{8}{20}$$

$$= \frac{2}{5} \text{ (or } 0.4\text{)}$$

The relative frequency of obtaining Heads is $\frac{2}{5}$.

b Frequency of Tails = 12
Total number of tosses = 20

$$\text{Relative frequency} = \frac{\text{frequency of Tails}}{\text{total number of tosses}}$$

3 Substitute the known values into the rule.

Relative frequency of Tails $= \frac{12}{20}$

4 Evaluate and simplify if possible.

$= \frac{3}{5}$ (or 0.6)

5 Answer the question.

The relative frequency of obtaining Tails is $\frac{3}{5}$.

WORKED EXAMPLE 4

Forty people picked at random were asked where they were born. The results were coded as follows:
Place of birth
1. Melbourne **2. Elsewhere in Victoria**
3. Interstate **4. Overseas.**
Responses
1, 3, 2, 1, 1, 4, 3, 1, 2, 1, 2, 1, 3, 4, 1, 2, 3, 1, 3, 4, 4, 3, 2, 1, 2, 3, 1, 4, 1, 2, 3, 4, 1, 2, 3, 1, 1, 4, 2, 3

a Organise the data into a frequency table.
b Find the relative frequency of each category as a fraction and a decimal.
c What is the total of the relative frequencies?
d Where is a person selected at random most likely to have been born?
e How many people out of 100 would you expect to be born overseas?

THINK

a 1 Draw a table with 3 columns. The column headings are in order, Score, Tally and Frequency.

2 Enter the codes 1, 2, 3 and 4 into the score column.

3 Place a stroke into the tally column each time a code is recorded.
Note: 𝍸 represents a score of five.

4 Count the number of strokes corresponding to each code and record in the frequency column.

5 Add the total of the frequency column.

b 1 Write the rule for the relative frequency.

2 Substitute the known values into the rule for each category.

3 Evaluate and simplify where possible.

WRITE

a

Score	Tally	Frequency
Code 1	𝍸 𝍸 ||||	14
Code 2	𝍸 ||||	9
Code 3	𝍸 𝍸	10
Code 4	𝍸 ||	7
	Total	40

b Relative frequency $= \frac{\text{frequency of category}}{\text{total number of people}}$

Category 1: People born in Melbourne

Relative frequency $= \frac{14}{40}$

$= \frac{7}{20}$ or 0.35

Category 2: People born elsewhere in Victoria

Relative frequency $= \frac{9}{40}$ or 0.255

Category 3: People born interstate

Relative frequency $= \frac{10}{40}$

$= \frac{1}{4}$ or 0.25

Category 4: People born overseas

Relative frequency $= \frac{7}{40}$ or 0.175

c 1 Add each of the relative frequency values.

c Total $= \frac{7}{20} + \frac{9}{40} + \frac{1}{4} + \frac{7}{40}$
$= 0.35 + 0.225 + 0.25 + 0.175$
$= 1$

2 Answer the question.

The relative frequencies sum to a total of 1.

d 1 Using the results from part **b**, obtain the code that corresponds to the largest frequency.
Note: A person selected at random is *most* likely to have been born in the place with the largest frequency.

d Melbourne (Code 1) corresponds to the largest frequency.

2 Answer the question.

A person selected at random is most likely to have been born in Melbourne.

e 1 Write the relative frequency of people born overseas and the number of people in the sample.

e Relative frequency (overseas) $= \frac{7}{40}$
Number of people in the sample $= 100$

2 Write the rule for the expected number of people.
Note: Of the 100 people, $\frac{7}{40}$ or 0.175 would be expected to be born overseas.

Expected number = relative frequency × number of people

3 Substitute the known values into the rule.

Expected number $= \frac{7}{40} \times 100$

4 Evaluate.

$= \frac{700}{40}$
$= 17.5$

5 Round the value to the nearest whole number.
Note: We are dealing with people. Therefore, the answer must be represented by a whole number.

≈ 18

6 Answer the question.

We would expect 18 of the 100 people to be born overseas.

assessON

Exercise 13.3 Experimental probability

REFLECTION
Explain why the experimental probability of an event can never be greater than 1. What does it mean if the experimental probability is equal to 1?

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8, 14, 16	Questions: 1–10, 14, 16, 17	Questions: 1, 4, 5, 6, 8, 9–18

Individual pathway interactivity int-4458 eBookplus

FLUENCY

1 WE3 The table at right shows the results of tossing a fair coin 150 times. What are the relative frequencies of:
- **a** Heads?
- **b** Tails?

Event	Frequency
Heads	84
Tails	66
Total	150

eBookplus
Digital doc
SkillSHEET
Understanding a deck of playing cards
doc-6970

2 A fair coin was tossed 300 times. A Head came up 156 times.
- **a** Find the relative frequency of the Head outcome as a fraction.
- **b** Calculate the relative frequency of Tails as a decimal.

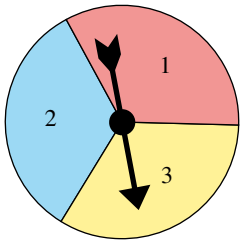
3 A die is thrown 50 times, with 6 as the favourable outcome. The 6 came up 7 times. Find the relative frequency of:

a a 6 occurring

b a number that is not a 6 (that is, any number other than a 6) occurring.

4 A spinner with 3 equal sectors, as shown at right, was spun 80 times, with results as shown in the table:

Score	1	2	3
Frequency	29	26	25



a What fraction of the spins resulted in a 3?

b What fraction of the spins resulted in a 2?

c Express the relative frequency of the spins that resulted in a 1 as a decimal.

5 A die was rolled 200 times and the results recorded in the table below.

Score	1	2	3	4	5	6
Frequency	30	34	35	32	35	34

a Name the outcomes that make up each of the following events:

i an even number

ii a number less than 3

iii a number ≥ 3

iv a prime number

v a number > 6

vi the number 5 or more

vii a non-prime number

viii the number 4 or less

ix a multiple of three

x a number that is divisible by 5.

b Express the relative frequency of each of the face numbers as a percentage.

c What percentage of outcomes turned out to be even?

d What was the relative frequency of non-prime numbers, as a percentage?

e What was the relative frequency of numbers divisible by 5, as a percentage?

f What was the relative frequency of numbers greater than or equal to ($\geq$) 3, as a percentage?

g What was the relative frequency of odd numbers, as a percentage?

h What was the relative frequency of numbers that are multiples of 3, as a percentage?

i What was the relative frequency of numbers that are 5 or greater, as a percentage?

j What was the relative frequency of numbers that are 2 or less, as a percentage?

6 **WE4** 100 people picked at random were asked which Olympic event they would most like to see. The results were coded as follows:

1. Swimming 2. Athletics 3. Gymnastics 4. Rowing.

The recorded scores were:

1, 1, 4, 3, 2, 2, 2, 4, 4, 3, 1, 1, 4, 2, 1, 1, 1, 4, 2, 2, 1, 3, 3, 3, 4, 1, 1, 3, 2, 2, 1, 2, 1, 1, 1, 1, 2, 3, 3, 3, 3, 2, 2, 4, 1, 1, 1, 3, 2, 2, 4, 1, 1, 1, 3, 3, 3, 3, 2, 1, 2, 2, 2, 2, 3, 4, 4, 1, 1, 1, 2, 3, 3, 2, 1, 4, 3, 2, 3, 1, 1, 2, 4, 1, 1, 3, 2, 2, 3, 3, 4, 4, 2, 1, 1, 3, 1, 2, 4, 1.

a Organise the data into a frequency table.

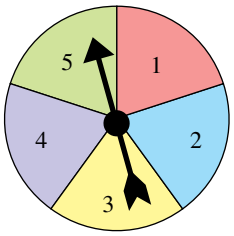
b Find the relative frequency of each category as a fraction and a decimal.

c What is the total of the relative frequencies?

d Which Olympic event selected at random is most likely to be seen?

e How many people out of 850 would you expect to see the gymnastics?

7 The following are results of 20 trials conducted for an experiment involving the 5-sector spinner at right.
1, 4, 2, 5, 3, 4, 5, 3, 2, 5, 1, 3, 2, 4, 2, 1, 4, 3, 3, 2



- **a** Organise the data into a frequency table.
- **b** Find the relative frequency of each outcome.
- **c** How many times would you have expected each outcome to have appeared? How did you come to this conclusion?
- **d** Which was the most common outcome?
- **e** What is the total of all the relative frequencies?

8 A card is *randomly* (with no predictable pattern) drawn 60 times from a hand of 5 cards, it is recorded, then returned and the five cards are reshuffled. The results are shown in the frequency distribution table at right. For each of the following, give:

Card	Frequency
3 ♥	13
Q ♦	15
3 ♦	12
3 ♣	9
3 ♠	11

- **i** the favourable outcomes that make up the event
- **ii** the relative frequency of these events.
  - **a** A heart
  - **b** A diamond
  - **c** A red card
  - **d** A 3
  - **e** A spade or a heart
  - **f** A 3 or a queen
  - **g** The king of spades
  - **h** A 3 or a diamond

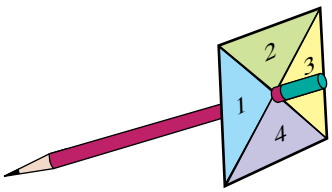
UNDERSTANDING

9 The following table shows the progressive results of a coin-tossing experiment.

Number of coin tosses	Outcome		Relative frequency	
	Heads	Tails	Heads (%)	Tails (%)
10	6	4	60	40
100	54	46	54	46
1000	496	504	49.6	50.4

- **a** What do you notice about the relative frequencies for each trial?
- **b** If we were to repeat the same experiment in the same way, would the results necessarily be identical to those in the table? Explain your answer.

10 The square spinner at right was trialled 40 times and the results of how it landed were recorded as shown below.
2, 4, 3, 1, 3, 2, 1, 4, 4, 3, 3, 1, 4, 2, 1, 2, 3, 1, 4, 2,
4, 2, 1, 2, 1, 3, 1, 4, 3, 1, 3, 1, 4, 2, 3, 1, 3, 2, 4, 4



- **a** What would you expect the relative frequency of each outcome to be?
- **b** Organise the data into a frequency table and calculate the actual experimental relative frequency of each number.
- **c** Find the relative frequency of the event, *odd number*, from the table obtained in part **b**.
- **d** What outcomes make up the event, *prime number*? *Hint:* Remember a prime number has exactly 2 factors: itself and 1.
- **e** Calculate the relative frequency of the event, *prime number*, from the table obtained in part **b**.

11 When 60 light bulbs were tested, 3 were found to be faulty.

a What was the relative frequency of faulty bulbs?

b What fraction of the bulbs were not faulty?

c In a carton of 600 such bulbs, how many would you expect to be faulty?

12 **MC** Olga observed that, in 100 games of roulette, red came up 45 times. Out of 20 games on the same wheel, how many would she expect to come up red?

A 4.5 **B** $\frac{4}{9}$ **C** $\frac{9}{4}$

D 9 **E** None of these

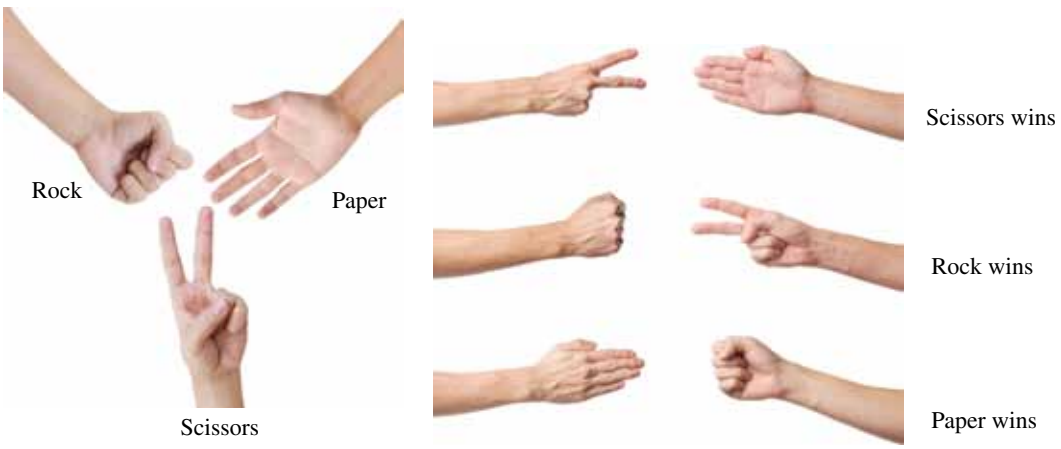
13 **MC** A fair coin was tossed 40 times and it came up Tails 18 times. The relative frequency of Heads was:

A $\frac{9}{11}$ **B** $\frac{11}{20}$ **C** $\frac{9}{20}$

D $\frac{20}{11}$ **E** unable to be calculated

REASONING

14 The game 'rock, paper, scissors' is played all over the world, not just for fun but also for settling disagreements.
The game uses the three different hand signs.
Simultaneously, two players 'pound' the fist of one hand into the air three times.
On the third time each player displays one of the hand signs. Possible results are shown below.



a Play 20 rounds of 'rock, paper, scissors' with a partner. After each round, record each player's choice and the result in a table like the following one. (Use R for rock, P for paper and S for scissors.)

Round number	Player 1	Player 2	Result
1	P	R	Player 1 wins
2	S	R	Player 2 wins
3	S	S	Tie

b Based on the results of your 20 rounds, what is the experimental probability of
- **i** you winning?
- **ii** your partner winning?
- **iii** a tie?

c Do you think playing 'rock, paper, scissors' is a fair way to settle a disagreement? Explain.

15 There is a total of 200 green and red marbles in a box. A marble is chosen, its colour is noted, and it is replaced in the box. This experiment is conducted 65 times. Only 13 green marbles are chosen.

a What is a reasonable estimate of the number of red marbles in the box? Show all of your working.

b If this experiment is conducted n times and g green marbles are chosen, what is a reasonable estimate of the number of red marbles in the box? Show all of your working.

PROBLEM SOLVING

16 The gender of babies in a set of triplets is simulated by flipping 3 coins. If a coin lands tails up, the baby is a boy. If a coin lands heads up, the baby is a girl. In the simulation, the trial is repeated 40 times and the following results show the number of heads obtained in each trial:

0, 3, 2, 1, 1, 0, 1, 2, 1, 0, 1, 0, 2, 0, 1, 0, 1, 2, 3, 2, 1, 3, 0, 2, 1, 2, 0, 3, 1, 3, 0, 1, 0, 1, 3, 2, 2, 1, 2, 1.

a Calculate the probability that exactly one of the babies in a set of triplets is female.

b Calculate the probability that more than one of the babies in the set of triplets is female.

17 A survey of the favourite foods of Year 8 students was conducted, with the following results.

a Estimate the probability that macaroni and cheese is the favourite food among Year 8 students.

b Estimate the probability that a vegetarian dish is the favourite food.

c Estimate the probability that a beef dish is the favourite food.

Meal	Tally
Hamburger	45
Fish and chips	31
Macaroni and cheese	30
Lamb souvlaki	25
BBQ pork ribs	21
Cornflakes	17
T-bone steak	14
Banana split	12
Corn-on-the-cob	9
Hot dogs	8
Garden salad	8
Veggie burger	7
Smoked salmon	6
Muesli	5
Fruit salad	3

18 A standard deck of 52 playing cards consists of four suits, clubs, diamonds, hearts and spades, as shown in the table (sample space) at right.

a Copy and complete the sample space for the deck of cards.

b One card is chosen at random. What is the probability that the card is:

- **i** a red card
- **ii** a picture card (jack, queen or king)
- **iii** an ace
- **iv** an ace or a heart
- **v** an ace and a heart
- **vi** not a diamond
- **vii** a club or a 7
- **viii** neither a heart nor a queen
- **ix** a card worth 10 (all picture cards are worth 10)
- **x** a red card or a picture card?

Clubs	Diamonds	Hearts	Spades
A♣	A♦	A♥	A♠
2♣		2♥	
3♣		3♥	
4♣		4♥	
5♣		5♥	
6♣		6♥	
7♣		7♥	
8♣		8♥	
9♣		9♥	
10♣		10♥	
J♣		J♥	
Q♣		Q♥	
K♣		K♥	

CHALLENGE 13.2

There are 7 blue balls, 3 yellow balls and 2 green balls in a bag. How many yellow balls need to be added to the bag to make the probability of drawing a yellow ball $\frac{3}{4}$?

eBookplus
Digital doc
WorkSHEET 13.1
doc-7107

13.4 Sample spaces and theoretical probability

- The **theoretical probability** (or empirical probability) of a particular event occurring is denoted by the symbol P(event).
- The **sample space**, S, is the set of all the possible outcomes.
- $\text{P(event)} = \dfrac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$
- If a very large number of trials is conducted, the relative frequency of an event will match the theoretical probability of the event.

WORKED EXAMPLE 5

A standard 6-sided die is rolled.

a List the sample space for this experiment.

b Determine the probability of obtaining the following appearing uppermost:

- **i 4**
- **ii an odd number**
- **iii 5 or less.**

THINK	WRITE
a Write all the possible outcomes for the given experiment.	**a** $S = \{1, 2, 3, 4, 5, 6\}$
b i 1 Write the number of possible outcomes.	**b i** Number of possible outcomes = 6
2 Write the number of favourable outcomes. *Note:* The favourable outcome is 4.	Number of favourable outcomes = 1
3 Write the rule for probability.	$\text{P(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$
4 Substitute the known values into the rule and evaluate.	$\text{P}(4) = \frac{1}{6}$
5 Answer the question.	The probability of 4 appearing uppermost is $\frac{1}{6}$.
ii 1 Write the number of possible outcomes.	**ii** Number of possible outcomes = 6
2 Write the number of favourable outcomes. The favourable outcomes are 1, 3, 5.	Number of favourable outcomes = 3
3 Write the rule for probability.	$\text{P(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$
4 Substitute the known values into the rule.	$\text{P(odd number)} = \frac{3}{6}$
5 Evaluate and simplify.	$= \frac{1}{2}$
6 Answer the question.	The probability of an odd number appearing uppermost is $\frac{1}{2}$.
iii Repeat steps 1 to 5 of part **b i**. *Note:* 5 or less means the favourable outcomes are 1, 2, 3, 4, 5. Therefore, the number of favourable outcomes is 5.	**iii** Number of possible outcomes = 6 Number of favourable outcomes = 5 $\text{P(5 or less)} = \frac{5}{6}$ The probability of obtaining 5 or less is $\frac{5}{6}$.

WORKED EXAMPLE 6

A card is drawn at random from a standard well-shuffled pack. Find the probability of drawing:
a a club
b a king or an ace
c not a spade.
Express each answer as a fraction and as a percentage.

THINK	WRITE
a 1 Write the number of outcomes in the sample space. There are 52 cards in a pack.	**a** Number of possible outcomes = 52
2 Write the number of favourable outcomes. There are 13 cards in each suit.	Number of favourable outcomes = 13
3 Write the rule for probability.	$\text{P(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$

4 Substitute the known values into the rule and simplify.

5 Convert the fraction to a percentage; that is, multiply by 100%.

6 Answer the question.

$$\text{P(a club)} = \frac{13}{52}$$
$$= \frac{1}{4}$$
$$\text{Percentage} = \frac{1}{4} \times 100\%$$
$$= \frac{100}{4}\%$$
$$= 25\%$$

The probability of drawing a club is $\frac{1}{4}$ or 25%.

b 1 Write the number of outcomes in the sample space.

2 Write the number of favourable outcomes. There are 4 kings and 4 aces.

3 Write the rule for probability.

4 Substitute the known values into the rule and simplify.

5 Convert the fraction to a percentage, rounded to one decimal place.

6 Answer the question.

b Number of possible outcomes = 52

Number of favourable outcomes = 8

$$\text{P(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

$$\text{P(a king or an ace)} = \frac{8}{52}$$
$$= \frac{2}{13}$$
$$\text{Percentage} = \frac{2}{13} \times 100\%$$
$$= \frac{200}{13}\%$$
$$\approx 15.4\%$$

The probability of drawing a king or an ace is $\frac{2}{13}$ or approximately 15.4%.

c Repeat steps 1 to 6 of part **a**.
Note: Not a spade means clubs, hearts or diamonds. Therefore, the number of favourable outcomes is 39.

c Number of possible outcomes = 52
Number of favourable outcomes = 39
$\text{P(not a spade)} = \frac{39}{52} = \frac{3}{4}$
$\text{Percentage} = \frac{3}{4} \times 100\% = \frac{300}{4} = 75\%$
The probability of drawing a card that is not a spade is $\frac{3}{4}$ or 75%.

WORKED EXAMPLE 7

A shopping centre car park has spaces for 10 buses, 300 cars and 20 motorbikes. If all vehicles have an equal chance of leaving at any time, find the probability that the next vehicle to leave will be:
a a motorbike
b a bus or a car
c not a car.

THINK	WRITE
a 1 Write the number of outcomes in the sample space. There are 330 vehicles.	**a** Number of possible outcomes = 330
2 Write the number of favourable outcomes. There are 20 motorbikes. Therefore, the number of favourable outcomes is 20.	Number of favourable outcomes = 20
3 Write the rule for probability.	$\text{P(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$
4 Substitute the known values into the rule and simplify.	$\text{P(a motorbike)} = \frac{20}{330} = \frac{2}{33}$
5 Answer the question.	The probability of a motorbike next leaving the car park is $\frac{2}{33}$.
b 1 Write the number of outcomes in the sample space.	**b** Number of possible outcomes = 330
2 Write the number of favourable outcomes. There are 10 buses and 300 cars. Therefore, the number of favourable outcomes is 310.	Number of favourable outcomes = 310
3 Write the rule for probability.	$\text{P(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$
4 Substitute the known values into the rule and simplify.	$\text{P(a bus or a car)} = \frac{310}{330} = \frac{31}{33}$
5 Answer the question.	The probability of a bus or car next leaving the car park is $\frac{31}{33}$.
c Repeat steps 1 to 5 of part **a**. *Note:* There are 10 buses and 20 motorbikes. Therefore, the number of favourable outcomes is 30.	**c** Number of possible outcomes = 330 Number of favourable outcomes = 30 $\text{P(not a car)} = \frac{30}{330} = \frac{1}{11}$ The probability of a vehicle that is not a bus next leaving the car park is $\frac{1}{11}$.

assessON

Exercise 13.4 Sample spaces and theoretical probability

REFLECTION
How is theoretical probability similar to experimental probability?

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–6, 8, 9, 14, 17, 22, 23	Questions: 1–5, 7, 9–17, 22–24	Questions: 1a, e, h, k, 2–4, 5a, b, d, h, 6–8, 12–24

Individual pathway interactivity int-4459 eBook plus

FLUENCY

eBookplus

Digital docs
SkillSHEET
Simplifying fractions
doc-6971
SkillSHEET
Converting a fraction into a decimal
doc-6972
SkillSHEET
Converting a fraction into a percentage
doc-6973
SkillSHEET
Multiplying a fraction by a whole number
doc-6974

1 List the sample spaces for these experiments:

- **a** tossing a coin
- **b** selecting a vowel from the word ASTRONAUT
- **c** selecting a day of the week to go to the movies
- **d** drawing a marble from a bag containing 3 reds, 2 whites and 1 black
- **e** rolling a standard 6-sided die
- **f** drawing a picture card from a standard pack of playing cards
- **g** spinning an 8-sector circular spinner numbered from 1 to 8
- **h** selecting even numbers from the first 20 counting numbers
- **i** selecting a piece of fruit from a bowl containing 2 apples, 4 pears, 4 oranges and 4 bananas
- **j** selecting a magazine from a rack containing 3 *Dolly*, 2 *Girlfriend*, 1 *Smash Hits* and 2 *Mathsmag magazines*
- **k** selecting the correct answer from the options A, B, C, D, E on a multiple-choice test
- **l** winning a medal at the Olympic games.

2 WE5 A standard 6-sided die is rolled.

- **a** List the sample space for this experiment.
- **b** Determine the probability of obtaining the following appearing uppermost:
  - **i** a 6
  - **ii** an even number
  - **iii** at most, 4
  - **iv** a 1 or a 2
  - **v** a prime number
  - **vi** a number greater than 4
  - **vii** a 7
  - **viii** a number that is a factor of 60.

3 WE6 A card is drawn at random from a standard well-shuffled pack. Find the probability of drawing:

- **a** the king of spades
- **b** a 10
- **c** a jack or a queen
- **d** a club
- **e** a red card
- **f** an 8 or a diamond
- **g** an ace.

Express each answer as a fraction and as a percentage, correct to 1 decimal place.

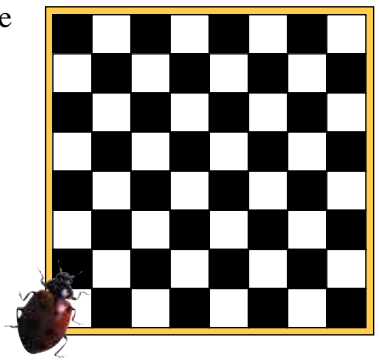
4 WE7 A shopping centre car park has spaces for 8 buses, 160 cars and 12 motorbikes. If all vehicles have an equal chance of leaving at any time, find the probability that the next vehicle to leave will be:

- **a** a bus
- **b** a car
- **c** a motorbike or a bus
- **d** not a car.

5 A bag contains 3 red, 2 black, 1 pink, 2 yellow, 3 green and 3 blue marbles. If a marble is drawn at random, calculate the chance that it is:

- **a** red
- **b** black
- **c** yellow
- **d** red or black
- **e** not blue
- **f** red or black or green
- **g** white
- **h** not pink.

6 A beetle drops onto *one* square of a chessboard. What are its chances of landing on a square that is:



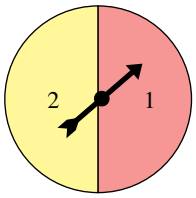
- **a** black?
- **b** white?
- **c** neither black nor white?
- **d** either black or white?

7 What chance is there that the next person you meet has his/her birthday:

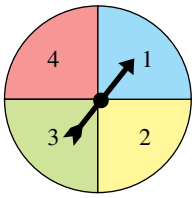
- **a** next Monday?
- **b** sometime next week?
- **c** in September?
- **d** one day next year?

8 For each of the following spinners:

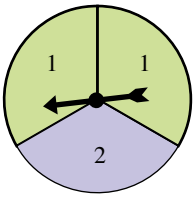
a



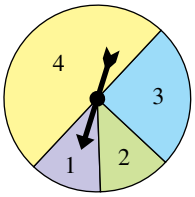
b



c



d



- **i** state whether each of the outcomes are equally likely. Explain your answer.
- **ii** find the probability of the pointer stopping on 1.

UNDERSTANDING

9 Hanna flipped a coin 5 times and each time a Tail showed. What are the chances of Tails showing on the *sixth* toss?

10 **a** Design a circular spinner coloured red, white, black, yellow and green so that each colour is equally likely to result from any trial.

- **b** What will be the angle between each sector in the spinner?

11 **a** Design a circular spinner with the numerals 1, 2 and 3 so that 3 is twice as likely to occur as 2 or 1 in any trial.

- **b** What will be the size of the angles in each sector at the centre of the spinner?

12 **a** Design a circular spinner labelled A, B, C and D so that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{3}$, $P(C) = \frac{1}{6}$, $P(D) = \frac{1}{4}$.

- **b** What will be the size of the angles between each sector in the spinner?

13 **a** What is the total of all the probabilities in question **12**?

- **b** What is the angle sum of the sectors in question **12**?

14 **a** List all the outcomes for tossing a coin once, together with their individual probabilities.

- **b** Find the total.

15 **a** List all the outcomes for tossing a coin twice, together with their individual probabilities.

- **b** Find the total.

16 **a** List the probabilities for the elements of the sample space for rolling a 6-faced die.

- **b** Find the total of the probabilities.
- **c** Do the totals for questions **14b**, **15b** and **16b** agree with that in **13a**?
- **d** What conclusion can you draw?

17 **MC** If a circular spinner has 3 sectors, A, B and C, such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$, then P(C) must be:

A $\frac{1}{4}$ **B** $\frac{2}{5}$ **C** $\frac{1}{6}$

D $\frac{5}{6}$ **E** none of these

18 **MC** For an octagonal spinner with equal sectors, numbered from 1 to 8, the chance of getting a number between (but not including) 5 and 8 is:

A $\frac{1}{4}$ **B** $\frac{1}{2}$ **C** $\frac{3}{8}$

D $\frac{5}{8}$ **E** none of these

19 **a** What is the sample space for rolling a standard 6-sided die?

b How many elements are in the sample space for rolling 2 standard 6-sided dice (think of the dice having different colours)? *Hint:* The answer is not 12.

c Complete the following sample space for rolling two dice.

Note: Each colour in the table corresponds to the colour of the die.

		Die 2					
		1	2	3	4	5	6
Die 1	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2						
	3						
	4						
	5						
	6						

d Using the sample space, complete the following table.

Sum	2	3	4	5	6	7	8	9	10	11	12
Probability											

e Do you notice a pattern involving the probabilities in the table? Explain your answer.

The pattern observed in part **e** relates to symmetry. Investigate the symmetry property of tossing three coins.

f List the sample space for tossing three coins.

g Using the sample space obtained in part **f**, complete the following table.

Number of Heads	0	1	2	3
Probability				

h From the table above, which event is the probability of tossing three Tails the same as?

i By symmetry, which event has the same probability as tossing Heads twice and Tails once?

20 Toss a coin 60 times and record whether it shows Heads (H) or Tails (T) uppermost in a table like the following one.

Outcome	Tally	Frequency	% Relative frequency $\left(\frac{\text{frequency}}{60} \times 100\%\right)$
Heads (H)			
Tails (T)			

Once your results have been recorded, obtain the frequency results from each member of the class and add them together to obtain a total value for the class frequency. This is called *pooling* your results. Determine the percentage relative class frequency by using the rule given in the following table.

Outcome	Class frequency	% Relative class frequency $\left(\frac{\text{class frequency}}{\text{number of students} \times 60} \times 100\%\right)$
Heads (H)		
Tails (T)		

Now that both tables have been completed, answer the following questions.

a Does a larger sample group alter the percentage results?

b Are the class results closer to those predicted by theory?

c Compare your own result with the class result. Which do you think would be more reliable? Explain your answer.

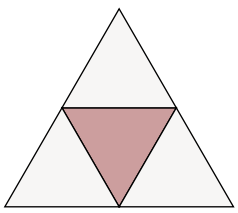
d Complete the following sentence:

In the long run the *relative frequency* of an event will _____.

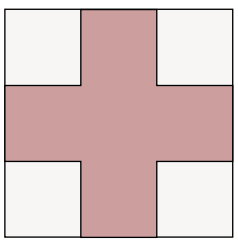
REASONING

21 The targets shown are an equilateral triangle, a square and a circle with coloured regions that are also formed from equilateral triangles, squares and circles. If a randomly thrown dart hits each target, find the probability that the dart hits each target's coloured region.

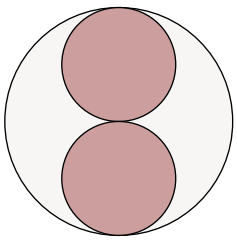
a



b



c



22 A fair coin is flipped 3 times. Calculate the probability of obtaining:
- **a** at least two Heads or at least two Tails
- **b** exactly two Tails.

23 Two dice are rolled and the product of the two numbers is found. Calculate the probability that the product of the two numbers is:
- **a** an odd number
- **b** a prime number
- **c** more than 1
- **d** at most 36.

24 A fair die is rolled and a fair coin flipped. Calculate the probability of obtaining:
- **a** an even number and a Head
- **b** a Tail from the coin
- **c** a prime number from the die
- **d** a number less than 5 and a Head.

13.5 Complementary events

- In some situations, there are only two possible outcomes.
- When there is nothing in common between these events and together they form the sample, they are called **complementary events**.
- For example, when tossing a coin, there are only two possible outcomes, a Head or a Tail. They are complementary events.
- If an event is denoted by the letter A, its complement is denoted by the letter A', and the sum of their probabilities is equal to one.
- If we know the probability of one event, subtracting this probability from 1 will give us the probability of the complementary event.
- If events A and A' are complementary, then:
  $P(A) + P(A') = 1$
  and $P(A') = 1 - P(A)$ or $P(A) = 1 - P(A')$.

WORKED EXAMPLE 8

Find the complement of each of the following events:
- **a** **selecting a red card from a standard deck**
- **b** **rolling two dice and getting a total greater than 9**
- **c** **selecting a red marble from a bag containing 50 marbles.**

THINK	WRITE
a Selecting a black card will complete the sample space for this experiment.	**a** The complement of selecting a red card is selecting a black card.
b When rolling two dice, rolling a total less than 10 will complete the sample space.	**b** The complement of rolling a total greater than 9 is rolling a total less than 10.
c Selecting a marble that is not red is the only way to define the rest of the sample space for this experiment.	**c** The complement of selecting a red marble in this experiment is not selecting a red marble.

WORKED EXAMPLE 9

If a card is drawn from a pack of 52 cards, what is the probability that the card is not a diamond?

THINK	WRITE
1 Determine the probability of drawing a diamond.	Number of diamonds, $n(E) = 13$ Number of cards, $n(S) = 52$ $P(E) = \frac{n(E)}{n(S)}$ $P(\text{diamond}) = \frac{13}{52}$ $= \frac{1}{4}$
2 Write down the rule for obtaining the complement of drawing a diamond; that is, not drawing a diamond.	$P(A') = 1 - P(A)$ $P(\text{not a diamond}) = 1 - P(\text{diamond})$
3 Substitute the known values into the given rule and simplify.	$= 1 - \frac{1}{4}$ $= \frac{3}{4}$
4 Answer the question.	The probability of drawing a card that is not a diamond is $\frac{3}{4}$.

assessON

Exercise 13.5 Complementary events

REFLECTION
Explain why the probability of an event and the probability of the complement of the event always sum to 1.

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–11	Questions: 1–13	Questions: 1–14

Individual pathway interactivity int-4460 eBook plus

FLUENCY

1 WE8 For each of the following, state the complementary event.

- **a** From a bag of numbered marbles selecting an even number
- **b** From the letters of the alphabet selecting a vowel
- **c** Tossing a coin and it landing Heads
- **d** Rolling a die and getting a number less than 3
- **e** Rolling two dice and getting a total less than 12
- **f** Selecting a diamond from a deck of cards
- **g** Selecting an E from the letters of the alphabet
- **h** Selecting a blue marble from a bag of marbles

2 MC *Note:* There may be more than one correct answer.
A student is to be chosen from a class of 30 students. Each student in the class is either 14 or 15 years old. Which of the following represent complementary events?

- **A** Selecting a 14-year-old boy and selecting a 14-year-old girl
- **B** Selecting a boy and selecting a girl
- **C** Selecting a 14-year-old and selecting a 15-year-old
- **D** Selecting a 14-year-old boy and selecting a 15-year-old girl
- **E** All of the above

UNDERSTANDING

3 For each of the following, state whether the pair of events are complementary or not. Explain your answer.

- **a** Having Weet Bix or Corn Flakes for breakfast.
- **b** Walking to your friend's house or riding your bike to your friend's house.
- **c** Watching TV at night or listening to the radio.
- **d** Passing your Mathematics test or failing your Mathematics test.
- **e** Rolling a number less than 4 on a die or rolling a total greater than 4.

4 WE9 If a card is drawn from a pack of 52 cards, what is the probability that the card is not a queen?

5 MC The statement that does *not* involve complementary events is:

- **A** travelling to school by bus and travelling to school by car
- **B** drawing a red card from a pack of 52 playing cards and drawing a black card from 52 playing cards
- **C** drawing a vowel from cards representing the 26 letters of the alphabet or drawing a consonant
- **D** obtaining an even number on a six-sided die or obtaining an odd number on a die
- **E** All of the above

6 When a six-sided die is rolled 3 times, the probability of getting 3 sixes is $\frac{1}{216}$. What is the probability of not getting 3 sixes?

7 Eight athletes compete in a 100 m race. The probability that the athlete in lane 1 will win is $\frac{1}{5}$. What is the probability that one of the other athletes wins? (Assume that there are no dead heats.)

8 A pencil case has 4 red pens, 3 blue pens and 5 black pens. If a pen is drawn randomly from the pencil case, find:

- **a** P(drawing a blue pen)
- **b** P(not drawing a blue pen)
- **c** P(drawing a red or a black pen)
- **d** P(drawing neither a red nor a black pen).

9 Holty is tossing two coins. He claims that getting two Heads and getting zero Heads are complementary events. Is he right? Explain your answer.

10 Seventy Year 9 students were surveyed. Their ages ranged from 13 years to 15 years, as shown in the following table.

Age	13	14	15	Total
Boys	10	20	9	39
Girls	7	15	9	31
Total	17	35	18	70

A student from the group is selected at random. Find:

- **a** P(selecting a student of the age of 13 years)
- **b** P(not selecting a student of the age of 13 years)
- **c** P(selecting a 15-year-old boy)
- **d** P(not selecting a 15-year-old boy).

REASONING

11 In a bag there are 4 red cubes and 7 green cubes. If Clementine picks a cube at random, what is the probability that it is not:
a red or green? **b** red? **c** green?

12 In a bag there are 4 red cubes and 7 green cubes. Clementine picks a cube at random, looks at it and notes that it is red. Without putting it back, she picks a second cube from the bag. What is the probability that it is not green? Show your working.

PROBLEM SOLVING

13 In a hand of n cards there are r red cards. I choose a card at random. What is the probability that it is:
a black? **b** not black?

14 There are three cyclists in a road race. Cyclist A is twice as likely to win as cyclist B and three times as likely to win as cyclist C. Find the probability that:
a cyclist B wins **b** cyclist A does not win.

13.6 Venn diagrams

- A **set** is a collection of things or numbers that belong to a well defined category. For example, a bird is a member of the set of two-legged creatures; odd numbers belong to the set of integers.
- The elements of a set are enclosed in curly brackets, or braces. { }
- A **Venn diagram** is made up of a rectangle and one or more circles. It is used to show the relationships between different groups or sets of objects.
- The rectangle contains all the objects under consideration and is called the **universal set**.
- Each group or set of objects within the universal set is enclosed in its own circle inside the rectangle.
- The symbol for the universal set is ξ.

WORKED EXAMPLE 10

Draw a Venn diagram representing the relationship between the following sets below. Show the position of all the elements in the Venn diagram.
ξ = {counting numbers up to 10}
A = {first 3 prime numbers}
B = {odd numbers less than 9}

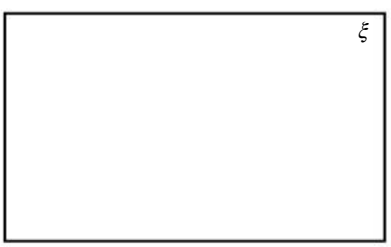
THINK

1. List the elements in each of the sets.

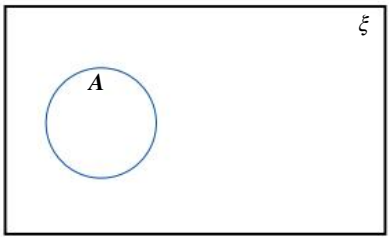
2. Draw the universal set as a rectangle.
Note: This contains the elements 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10.

WRITE/DRAW

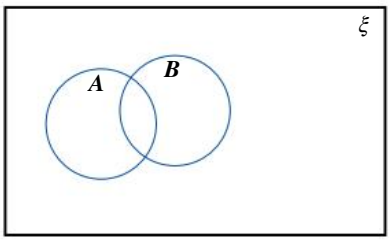
$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{2, 3, 5\}$
$B = \{1, 3, 5, 7\}$



3 Draw and label a circle within the rectangle to represent set A. This circle contains the elements 2, 3 and 5.

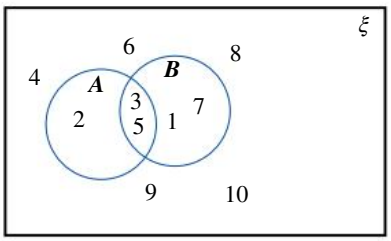


4 Draw and label a circle within the rectangle to represent set B. This circle contains the elements 1, 3, 5 and 7.
Note: Circles A and B will overlap as they have common elements, that is, 3 and 5.



5 Enter the elements into the appropriate section of the Venn diagram.

- **a** First label the overlapping section with the elements common to both A and B.
- **b** Next label sets A and B with the elements not already included in the overlapping section.
- **c** Lastly, label the rectangle with those elements in the universal set not already listed in sets A or B.



6 Check that circle A contains all the elements of set A. Similarly, check circle B and the universal set.

Intersection of sets (∩) and union of sets (∪)

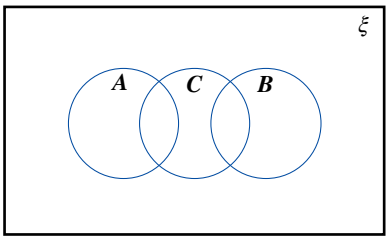
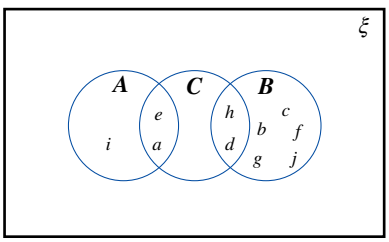
eBookplus
eLesson
Union and intersection of sets
eles-0049

- When sets A and B overlap as in Worked example 10, this overlap is defined as the intersection of the two sets.
- This region is represented using notation $A \cap B$ (read as 'A intersection B'). In Worked example 10, $A \cap B = \{3, 5\}$.
- The union of two sets, A and B, is the set of all elements in A or in B (or in both).
- This region is represented using the notation $A \cup B$ (read as 'A union B'). In Worked example 10, $A \cup B = \{1, 2, 3, 5, 7\}$.

WORKED EXAMPLE 11

a Draw a Venn diagram representing the relationship between the following sets. Show the position of all the elements in the Venn diagram.
ξ = {first 10 letters of the English alphabet}
A = {vowels}
B = {consonants}
C = {letters of the word *head*}

b Use the Venn diagram to list the elements in the following sets.
i B' **ii** $B \cap C$
iii $A \cup C$ **iv** $(A \cap C) \cap (B \cap C)$

THINK	WRITE/DRAW
a **1** List the elements in each of the sets.	**a** $\xi = \{a, b, c, d, e, f, g, h, i, j\}$ $A = \{a, e, i\}$ $B = \{b, c, d, f, g, h, j\}$ $C = \{a, e, d, h\}$
2 Draw the universal set as a rectangle.	
3 Draw and label two separate circles within the rectangle to represent the disjoint sets A and B.	
4 Draw and label a third circle within the rectangle that overlaps set A and set B. *Note:* Circle C is positioned between circles A and B as it has elements common to both sets.	
5 Enter the elements into the appropriate section of the Venn diagram; that is, fill in the letters in the overlapping areas first, and then work outwards to the universal set.	Sets A and C have a and e in common. Sets B and C have h and d in common. There are no remaining elements in C. The remaining element in A is i. The remaining elements in B are b, c, f, g, j. 
b **i** Carefully analyse the Venn diagram and identify the set required. *Note:* B' is the complement of set B and includes all the elements that are part of the universal set and not in set B.	**b** **i** $B' = \{a, e, i\}$
ii Carefully analyse the Venn diagram and identify the set required. *Note:* The intersection of B and C is the overlapping area of these two circles.	**ii** $B \cap C = \{d, h\}$
iii Carefully analyse the Venn diagram and identify the set required. *Note:* The union of A and C contain all the elements in circles A and C.	**iii** $A \cup C = \{a, d, e, h, i\}$
iv Carefully analyse the Venn diagram and consider the intersection of A with C. Next consider the intersection of B with C. Compare the sets obtained and answer the question.	**iv** $(A \cap C) \cap (B \cap C) = \{a, e\} \cap \{d, h\}$ $= \{\}$

WORKED EXAMPLE 12

An ice-creamery conducted a survey of 60 customers on a Monday and obtained the following results on two new ice-cream flavours. The results showed that 35 customers liked Product A, 40 liked Product B, and 24 liked both equally.

a Draw a Venn diagram to illustrate the above information.

b Use the Venn diagram to answer the following questions.

- **i How many customers liked Product A only?**
- **ii How many customers liked Product B only?**
- **iii How many customers liked neither product?**

c If a customer was selected at random on this Monday morning, what is the probability they would have liked neither new flavour?

d Calculate the probability that a customer liked Product A given that they liked Product B.

THINK

a 1 Draw the universal set as a rectangle.

2 Draw and label two overlapping circles within the rectangle to represent Product A and Product B. *Note:* Circles for products A and B overlap because 24 customers liked both products equally.

3 Working from the overlapping area outwards, determine the number of customers in each region. *Note:* The total must equal the number of customers surveyed, that is, 60.

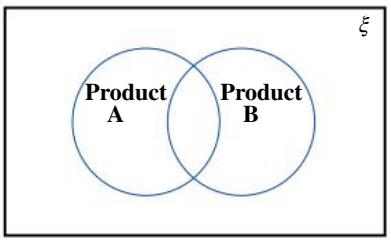
b i Refer to the Venn diagram and answer the question. *Note:* The non-overlapping part of Product A's circle refers to the customers that like Product A only.

ii Refer to the Venn diagram and answer the question. *Note:* The non-overlapping part of Product B's circle refers to the customers that like Product B only.

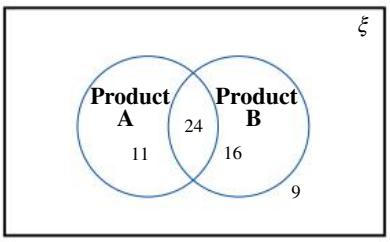
iii Refer to the Venn diagram and answer the question.

WRITE/DRAW

a



There are 24 customers in both sets.
Product A's set contains 11 customers (that is, 35 − 24) who like Product A but not Product B.
Product B's set contains 16 customers (that is, 40 − 24) who like Product B but not Product A.
The remaining 9 customers (that is, [60 − (11 + 24 + 16)]) like neither product.



b i 11 customers liked Product A only.

ii 16 customers liked Product B only.

iii 9 customers liked neither product.

c There was a total of 60 customers and 9 customers liked neither new flavour.

c P(liking neither new flavour) $= \frac{9}{60}$
$= \frac{3}{20}$

d Forty customers liked Product B and 24 customers liked Products A and B.

d P(Liking Product A/Liked product B) $= \frac{24}{40} = \frac{3}{5}$

assessON

Exercise 13.6 Venn diagrams

REFLECTION
When is it best to use a Venn diagram to represent data?

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–10, 13	Questions: 1–11, 13	Questions: 1–14

Individual pathway interactivity int-4461 eBookplus

eBookplus
Interactivity
Venn diagrams
int-2377

FLUENCY

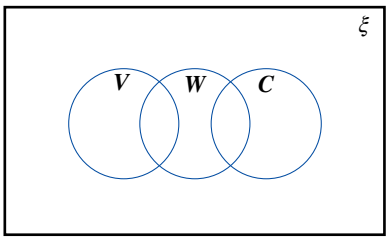
1 WE10 Draw a Venn diagram representing the relationship between the following sets. Show the position of all the elements in the Venn diagram.
ξ = {integers ranging from 10 to 20}
B = {odd numbers greater than 12 and less than 18}
A = {composite numbers ranging from 10 to 20}

2 Draw a Venn diagram representing the relationship between the following sets. Show the position of all the elements in the Venn diagram.
ξ = {alphabet letters a to j}
V = {vowels}
H = {letters of the word *high*}

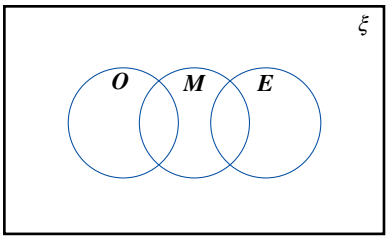
3 Draw a Venn diagram representing the relationship between the following sets. Show the position of all the elements in the Venn diagram.
ξ = {counting numbers up to 10}
P = {prime numbers}
E = {even numbers}

4 Draw a Venn diagram representing the relationship between the following sets. Show the position of all the elements in the Venn diagram.
ξ = {months of the year}
J = {months of the year beginning with j}
W = {winter months}
S = {summer months}

5 The Venn diagram at right shows the following sets.
ξ = {first 16 letters of the alphabet}
V = {vowels}
C = {consonants}
W = {letters of the word *padlock*}
Show the position of all the elements in the Venn diagram.



6 The Venn diagram at right shows the following sets.
ξ = {positive integers less than 15}
O = {odd numbers}
E = {even numbers}
M = {multiples of 3}
Show the position of all the elements in the Venn diagram.



7 **WE11** **a** Draw a Venn diagram representing the relationship between the following sets. Show the position of all the elements in the Venn diagram.
ξ = {$a, c, e, g, i, k, m, o, q, s, u, w, y$}
A = {vowels}
B = {consonants}
C = {letters of the word *cages*}

b Use the Venn diagram to list the elements in the following sets.
i B' **ii** $B \cap C$ **iii** $A \cup C$ **iv** $(A \cap C) \cup (B \cap C)$

UNDERSTANDING

8 **WE12** A tyre manufacturer conducting a survey of 2200 customers obtained the following results on two tyres: 1390 customers preferred Tyre A, 1084 preferred Tyre B, and 496 preferred both equally.

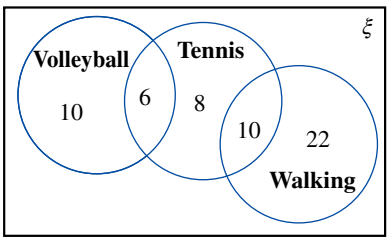
a Draw a Venn diagram to illustrate the above information.

b Use the Venn diagram to answer the following questions.
i How many customers preferred Tyre A only?
ii How many customers preferred Tyre B only?
iii How many customers preferred neither tyre?

c If a customer was selected at random, calculate the probability that they would have preferred neither type of tyre.

d Calculate the probability that a customer preferred Tyre A, given that they preferred Tyre B.

9 A sporting club has its members playing different sports, as shown in the Venn diagram.



a Copy the given Venn diagram and shade the areas that represent:
i members playing tennis only
ii members walking only
iii members both playing tennis and walking.

b How many members:
  i play volleyball? **ii** are involved in all three activities?
c How many members belong to the sporting club?
d How many members do not:
  i play tennis? **ii** walk?
e Calculate the probability that a member likes playing volleyball or tennis but does not like walking.
f Calculate the probability that a member likes playing volleyball and tennis but does not like walking.
g Calculate the probability that a member likes playing tennis, given that they like walking.

REASONING

10 Margaret is in charge of distributing team uniforms for students representing the school in music, athletics and debating. Margaret knows that representing the school are 43 students, of whom some are involved in more than one activity. They must purchase a uniform for each activity in which they participate.

Margaret has the following information: 36 students are in the concert band, 31 students are in the athletics team, 12 students are in the debating team, 6 students are involved in all three. Nine students are involved in music and debating, 7 in athletics and debating and 26 students are in music and athletics.

a Show this information on a Venn diagram.
b Calculate the probability that a student will be required to purchase only the music uniform.
c Calculate the probability that a student will be required to purchase only the athletics uniform.
d Calculate the probability that a student will be required to purchase only the debating uniform.
e Calculate the probability that a student will be required to purchase music and debating uniforms but not an athletics uniform.
f Calculate the probability that a student will be required to purchase a music or athletics uniform but not a debating uniform.
g Calculate the probability that a student will be required to purchase a debating uniform given that they purchased a music uniform.

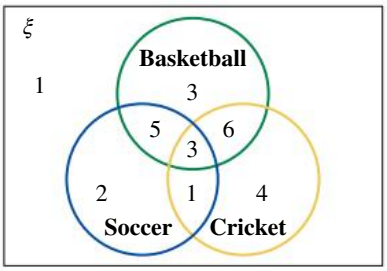
11 Draw a Venn diagram that displays the following information: $n(A) = 10$, $n(B) = 5$, $n(A \cap B) = 4$.

PROBLEM SOLVING

12 A survey of a Year 8 class found the numbers of class members who play basketball, cricket and soccer. Use the following Venn diagram to calculate the number of students who:

a were in the class
b play basketball

c play cricket and basketball
d play cricket and basketball but not soccer
e play soccer but not cricket
f play all three sports
g do not play cricket, basketball or soccer
h do not play cricket
i play cricket or basketball
j play basketball or cricket or soccer.



13 A group of 40 university lecturers were asked which free-to-air TV stations they watched on a particular evening. Twelve watched SBS, twenty-five watched ABC1 and ten watched neither SBS nor ABC1.

a Show this information on a fully labelled Venn diagram.
b How many watched both SBS and ABC1?

14 A survey of 140 fifteen-year-old girls investigated how many read magazines (M), crime novels (C) and science fiction (S). It found:

- 23 read both magazines and science fiction
- 21 read both magazines and crime novels
- 25 read both crime novels and science fiction
- 15 read all three
- 40 read magazines only
- 38 read crime novels only
- 10 read science fiction only.

a Show this information on a fully labelled Venn diagram.
b How many girls read magazines?
c How many girls read only crime?
d How many girls read science fiction?
e How many girls read none of these three?

13.7 Tree diagrams and two-way tables

- A **tree diagram** is a branching diagram that lists all the possible outcomes (the sample space).
- A **two-way table** can also be used to represent the sample space.

WORKED EXAMPLE 13

a Show the sample space for tossing a coin twice (or 2 coins together) by using:
i a tree diagram **ii a two-way table.**
b What is the probability of obtaining:
i Heads twice? **ii Heads and Tails?**

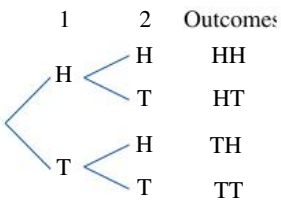
THINK

a i **1** Use branches to show the individual outcomes for the first toss. Place a **1** above the first toss outcomes.

2 Link each outcome from the first toss with the outcomes of the second toss. Place a **2** above the second toss outcomes.

WRITE/DRAW

a i



3 List each of the possible outcome pairs in the order they occur; that is, the first toss result followed by the second toss result.

ii 1 Draw a table consisting of three rows and columns. Leave the first cell blank.

2 Label the second and third cells of the first row as H and T respectively. Place a **1** above the first row.

3 Label the second and third cells of the first column as H and T respectively. Place a **2** beside the first column.

4 Combine the outcome pairs in the order in which they occur in each of the remaining cells; that is, the first toss result followed by the second toss result.

ii

Coin		1	
		H	T
2	H	H H	T H
	T	H T	T T

b i 1 Using either the tree diagram or the two-way table, write the number of favourable outcomes and the total number of possible outcomes.
Note: The outcome of two Heads occurs once.

b i Number of favourable outcomes = 1
Total number of possible outcomes = 4

2 Write the rule for probability.

$$\text{P(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

3 Substitute the known values into the rule and evaluate.

$$\text{P(2 Heads)} = \frac{1}{4}$$

4 Answer the question.

The probability of obtaining 2 Heads when a coin is tossed twice is $\frac{1}{4}$.

ii 1 Using either the tree diagram or the two-way table, write the number of favourable outcomes and the total number of possible outcomes.
Note: The outcome of 1 Head and 1 Tail occurs twice.

ii Number of favourable outcomes = 2
Total number of possible outcomes = 4

2 Write the rule for probability.

$$\text{P(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

3 Substitute the known values into the rule and simplify.

$$\text{P(1 Head and 1 Tail)} = \frac{2}{4} = \frac{1}{2}$$

4 Answer the question.

The probability of obtaining 1 Head and 1 Tail when a coin is tossed twice is $\frac{1}{2}$.

WORKED EXAMPLE 14

a A coin is tossed and then a die is rolled. Use i a tree diagram ii a two-way table to show all the possible outcomes.
b What is the probability of obtaining i Heads and an even number? ii an odd number?

THINK

a i

1. Use branches to show the individual outcomes for the first event; that is, the toss of the coin. Place a **1** above the first event outcome.
2. Link each outcome from the first event with each of the outcomes from the second event; that is, the roll of the die. Place a **2** above the second event outcomes.
3. List each of the possible outcome pairs in the order they occur; that is, the first event result followed by the second event result.

ii

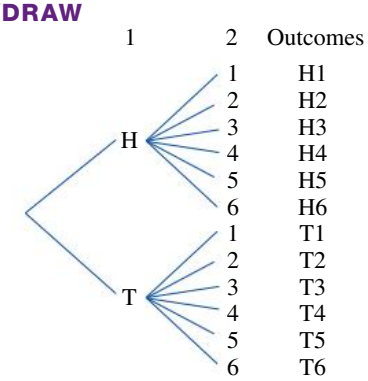
1. Draw a table consisting of seven rows and three columns. Leave the first cell blank.
2. Label the second and third cells of the first row as H and T respectively. Place a **1** above the first row.
3. Label cells two to seven of the first column as 1, 2, 3, 4, 5, 6 respectively. Place a **2** beside the first column.
4. Combine the outcome pairs in the order they occur in each of the remaining cells; that is, the first event result followed by the second event result.

b i

1. Using either the tree diagram or the two-way table, write the number of favourable outcomes and the total number of possible outcomes.
   Note: The outcome of Heads and an even number occurs 3 times.
2. Write the rule for probability.
3. Substitute the known values into the rule and simplify.
4. Answer the question.

WRITE/DRAW

a i



ii

2 \ 1	H	T
1	H 1	T 1
2	H 2	T 2
3	H 3	T 3
4	H 4	T 4
5	H 5	T 5
6	H 6	T 6

b i

Number of favourable outcomes = 3
Total number of possible outcomes = 12

$$\text{P(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

$$\text{P(Heads and an even number)} = \frac{3}{12} = \frac{1}{4}$$

The probability of obtaining Heads and an even number when a coin is tossed and a die is rolled is $\frac{1}{4}$.

ii

1. Using either the tree diagram or the two-way table, write the number of favourable outcomes and the total number of possible outcomes.
   Note: The outcome of an odd number occurs 6 times.

   ii Number of favourable outcomes = 6
   Total number of possible outcomes = 12

2. Write the rule for probability.

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

3. Substitute the known values into the rule and simplify.

$$P(\text{an odd number}) = \frac{6}{12}$$
$$= \frac{1}{2}$$

4. Answer the question.

The probability of obtaining an odd number when a coin is tossed and a die is rolled is $\frac{1}{2}$.

WORKED EXAMPLE 15

a Draw a tree diagram to show the combined experiment, tossing a coin and spinning a circular spinner with four equal sectors labelled 1, 2, 3, 4.
b Determine the probability of each possible result.
c What do you notice about the sum of the probabilities?
d Calculate the probability of obtaining:
i Heads and an even number **ii a prime number** **iii Tails.**

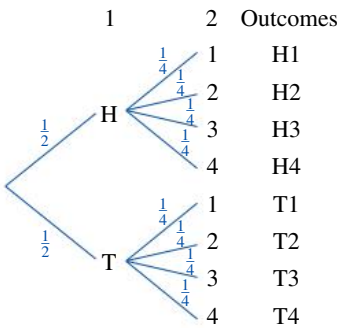
THINK

a

1. Use branches to show the individual outcomes for the first part of the experiment (that is, tossing a coin). Place a **1** above the coin toss outcomes. Label the ends of the branches H and T and place the probability of each along their respective branch.
2. Link each outcome from the coin toss with the outcomes of the second part of the experiment (that is, spinning a circular spinner). Place a **2** above the spinning outcomes. Label the ends of the branches 1, 2, 3 and 4 and place the probability of each along its respective branch.
3. List each of the possible outcome pairs in the order they occur; that is, the tossing of the coin result followed by the spinning result.
   Note: It doesn't matter which part of the experiment goes first, because the parts do not affect each other. They are called independent events. The coin toss will have two branches, and the spinning of the circular spinner will have four branches.

WRITE/DRAW

a



b Determine the probability of each possible result by multiplying the first result probability by the second result probability of the ordered pair.

b $P(H, 1) = \frac{1}{2} \times \frac{1}{4}$ $P(T, 1) = \frac{1}{2} \times \frac{1}{4}$
$= \frac{1}{8}$ $= \frac{1}{8}$
$P(H, 2) = \frac{1}{2} \times \frac{1}{4}$ $P(T, 2) = \frac{1}{2} \times \frac{1}{4}$
$= \frac{1}{8}$ $= \frac{1}{8}$
$P(H, 3) = \frac{1}{2} \times \frac{1}{4}$ $P(T, 3) = \frac{1}{2} \times \frac{1}{4}$
$= \frac{1}{8}$ $= \frac{1}{8}$
$P(H, 4) = \frac{1}{2} \times \frac{1}{4}$ $P(T, 4) = \frac{1}{2} \times \frac{1}{4}$
$= \frac{1}{8}$ $= \frac{1}{8}$

c Add the probabilities together and answer the question.

c $\text{Total} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$
$= 1$
The probabilities of each combined result in the tree diagram add up to 1.

d i 1 Add the probability of each of the outcome pairs that comprise Heads and an even number.

2 Evaluate and simplify.

d i $P(H, \text{even}) = P(H, 2) + P(H, 4))$
$P(H, \text{even}) = \frac{1}{8} + \frac{1}{8}$
$= \frac{2}{8}$
$= \frac{1}{4}$

ii 1 Add the probability of each of the outcome pairs that comprise a prime number; that is, 2 or 3.
Note: Heads or Tails are not specified; therefore, pairs consisting of either coin outcome are acceptable.

2 Evaluate and simplify.

ii $P(\text{prime}) = P(H, 2) + P(H, 3) + P(T, 2) + P(T, 3)$
$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$
$= \frac{4}{8}$
$= \frac{1}{2}$

iii 1 Add the probability of each of the outcome pairs that comprise Tails.
Note: Numbers are not specified; therefore, pairs consisting of any spinner outcome are acceptable.

2 Evaluate and simplify.

iii $P(\text{tails}) = P(T, 1) + P(T, 2) + P(T, 3) + P(T, 4)$
$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$
$= \frac{4}{8}$
$= \frac{1}{2}$

Exercise 13.7 Tree diagrams and two-way tables

assess on

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–14, 18	Questions: 1–15, 17–19	Questions: 1–20

Individual pathway interactivity int-4462 eBookplus

REFLECTION
What are some things you need to be careful of when you are calculating the number of possible outcomes?

eBookplus

Digital docs
SkillSHEET
Listing the sample space
doc-6975
SkillSHEET
Multiplying proper fractions
doc-6976

FLUENCY

1 a WE13 Show the sample space for tossing a coin twice (or 2 coins together) by using:
- **i** a tree diagram
- **ii** a two-way table.

b What is the probability of obtaining:
- **i** 2 Tails?
- **ii** Heads and then Tails?
- **iii** Tails and then Heads?
- **iv** one of each?
- **v** both the same?

2 a Use a tree diagram to show the sample space for 2 children that are born into a family.

b What are the chances that they are:
- **i** 2 girls?
- **ii** 2 boys?
- **iii** both the same sex?
- **iv** a boy, then a girl?
- **v** a girl, then a boy?
- **vi** one of each sex?

3 a Use a tree diagram to show the sample space for an electrical circuit that contains two switches, each of which can be on or off.

b What chance is there that the switches are:
- **i** both on?
- **ii** both off?
- **iii** both in the same position?
- **iv** one off, one on?

4 a Use a tree diagram to show the sample space for a true/false test that has 2 questions.

b What is the probability that the answers are:
- **i** true, then false?
- **ii** false, then true?
- **iii** both false?
- **iv** both true?
- **v** one true, one false?

5 a Use a tree diagram and two-way table to show the sample space for the following. A light may be on or off and a door open or closed.

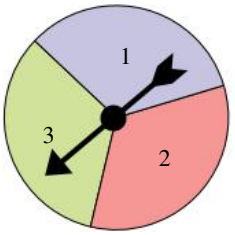
b What are the chances of the following situations?
- **i** door open, light on
- **ii** door closed, light off
- **iii** door closed, light on
- **iv** door open, light off

6 a WE14 A coin is tossed and then a die is rolled. Use **i** a tree diagram **ii** a two-way table to show all the possible outcomes.

b What is the probability of obtaining:
- **i** Tails and the number 5?
- **ii** an even number?
- **iii** Heads and a prime number?
- **iv** the number 3?

7 a Use a two-way table to show the sample space for the following. A coin is tossed and the spinner at right is twirled.

b What are the chances of obtaining:
- **i** Heads and a 1?
- **ii** Tails and a 2?
- **iii** Tails and a 3?
- **iv** Heads and a 2?
- **v** Tails and an odd number?



vi Heads and an even number?

vii Tails and a prime number?

8 a Use a two-way table to show the sample space for the following. Zipper sports cars come in 3 colours (red, white and yellow) with manual or automatic transmissions available.

b What are the chances of a car selected at random being:

i red?

ii automatic?

iii yellow and automatic?

iv white and manual?

v red and automatic?

9 a Use a two-way table to show the sample space for the following. A die is rolled and a coin is tossed.

b What is the probability of obtaining the following?

i H, 6 ii T, 1 iii H, even

iv T, odd v H, ≥ 4 vi T, 3 or 5

vii H, not 6

10 A bag contains two red balls and one black. A ball is drawn, its colour noted, and then it is replaced. A second draw is then made.

a Use a tree diagram to list all possible outcomes. (*Hint:* Place each red ball on a separate branch.)

b Find the probability of drawing:

i black, black ii red, red iii red, then black

iv black, then red v different colours vi the same colour each time

vii no reds viii no blacks ix at least one red

x neither red nor black xi at least one black.

11 a WE15 Draw a tree diagram to show the combined experiment, tossing a coin and spinning a circular spinner with six equal sectors labelled 1, 2, 3, 4, 5, 6.

b Determine the probability of each possible result.

c What do you notice about the sum of the probabilities?

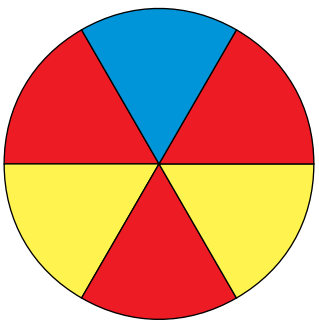
d Calculate the probability of obtaining:

i a head and an even number

ii a prime number

iii a tail.

12 a Draw a tree diagram to show the combined experiment, rolling a die and spinning the circular spinner shown at right.



b Determine the probability of each possible result.

c What do you notice about the sum of the probabilities?

d Calculate the probability of obtaining:

i the number 2

ii the colour yellow

iii the colour red

iv an even number and the colour blue.

eBookplus
Digital doc
Spreadsheet
Coin tossing
doc-2408

UNDERSTANDING

13 MC Two sets of traffic lights each show red, amber or green for equal amounts of time. The chance of encountering 2 red lights in succession are:

A $\frac{1}{3}$ B $\frac{1}{6}$ C $\frac{2}{3}$ D $\frac{2}{9}$ E $\frac{1}{9}$

14 MC In the situation described in question **13**, the probability of experiencing amber and green in any order is:

A $\frac{2}{3}$ B $\frac{1}{9}$ C $\frac{1}{2}$ D $\frac{1}{3}$ E none of these

15 To get to school each morning, you are driven along Smith Street and pass through an intersection controlled by traffic lights. The traffic light for Smith Street drivers has a cycle of green and amber for a total of 40 seconds and then red for 20 seconds.

a What is the probability that the traffic light will be red as you approach the intersection?

b Over 3 school weeks, how many days would you expect the traffic light to be red as you approach the intersection?

c Design an experiment in which you simulate the operation of the traffic light to record whether the light is red as you approach the intersection. Explain what device you will use to represent the outcomes.

d Complete this experiment 10 times and calculate the experimental probability that the traffic light will be red as you approach the intersection.

e Repeat the experiment a further 40 times so that you have 50 results. Again calculate the probability that the traffic light will be red as you approach the intersection.

f Use the probabilities calculated in parts **d** and **e** to estimate the number of days that the traffic light would be red over 3 school weeks.

g Compare your results.

h What is the chance that the traffic light will be red every morning for a school week?

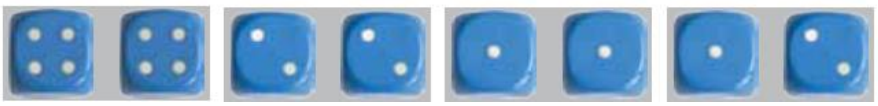
REASONING

16 People around the world have played games with dice for thousands of years. Dice were first mentioned in print in the *Mahabharata*, a sacred epic poem written in India more than 2000 years ago. The six-sided dice used today are almost identical to those used in China about 600 BC and in Egypt about 2000 BC.

Barbooth is a popular game in Greece and Mexico. Two players take turns rolling 2 dice until one of the following winning or losing rolls is obtained.

Winning rolls:

Losing rolls:



a Calculate the probability of getting a winning roll.

b Calculate the probability of getting a losing roll.

c Calculate the probability of getting neither a winning nor a losing roll.

d Play the game a number of times with a partner. Set up an experiment to investigate the experimental probabilities of getting a winning roll and getting a losing roll. Compare your results.

17 In your drawers at home, there are two white T-shirts, a green T-shirt and a red T-shirt. There are also a pair of black pants and a pair of khaki pants.

a Draw a tree diagram to show all the possible combinations of T-shirts and pants that you could wear.

b If you get dressed in the dark and put on one T-shirt and one pair of pants, calculate the probability that you put on the red T-shirt and khaki pants. Show your working.

PROBLEM SOLVING

18 In the last Science test, your friend guessed the answers to three true/false questions.

a Use a tree diagram to show all the different answer combinations for the three questions.

b Calculate the probability that your friend:

i got all three answers correct

ii got two correct answers

iii got no correct answers.

19 Assuming that the chance of a baby being a boy or a girl is the same, answer the following questions.

a Calculate the probability that a family with three children has:

i all boys

ii two girls and one boy

iii three children of the same gender

iv at least one girl

v two children of the same gender.

b If the family was expecting another baby, what is the probability that the new baby will be a boy?

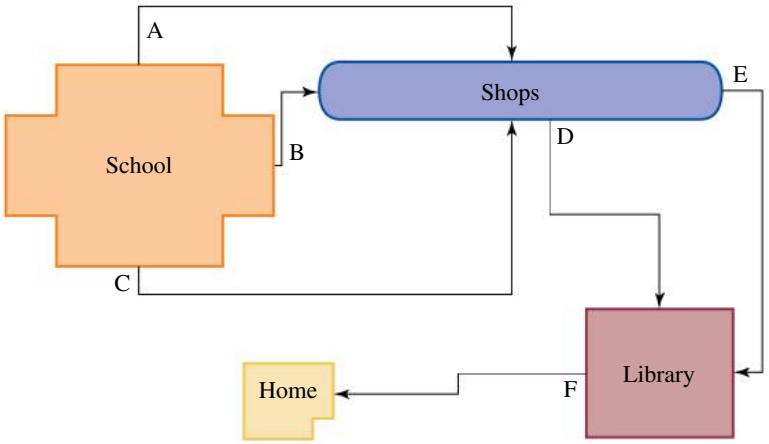
c If the family already has three boys, what is the probability that the new baby will be a boy?

d If the family has three girls, what is the probability that the new baby will be a boy?

e How likely is the combination of children shown in the photo above?

20 There are three different ways to go from school to the shops. There are two different ways to go from the shops to the library. There is only one way to go from the library to home.

This afternoon you need to travel from school to home via the shops and the library.



eBookplus
Digital doc
WorkSHEET 13.2
doc-7108

a Use a tree diagram to calculate the number of different routes you could use on your journey. (*Hint:* Use the letters A, B and C to represent the different routes from school to the shops and D and E to represent the different routes from the shops to the library. Use F to represent the route from the library to home.)

b Would the number of outcomes be different if we omitted the last leg of the journey from the library to home? Why?

eBookplus ONLINE ONLY

13.8 Review

www.jacplus.com.au

The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:

- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

Review questions

Download the Review questions document from the links found in your eBookPLUS.

eBookplus

Interactivities

Word search:
int-2637

Crossword:
int-2638

Sudoko:
int-3192

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

certain
complementary events
event
experiment
favourable outcome
impossible
intersection
outcome
probability
random
relative frequency
sample space
set
theoretical probability
tree diagram
trial
two-way table
union
universal set
Venn diagram

Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

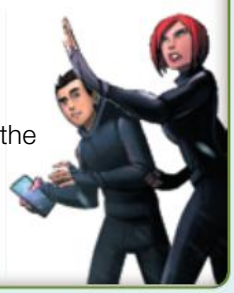
assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

www.assesson.com.au

Link to SpyClass, an exciting online game combining a comic book–style story with problem-based learning in an immersive environment.

Join Jesse, Toby and Dan and help them to tackle some of the world's most dangerous criminals by using the knowledge you've gained through your study of mathematics.

www.spyclass.com.au

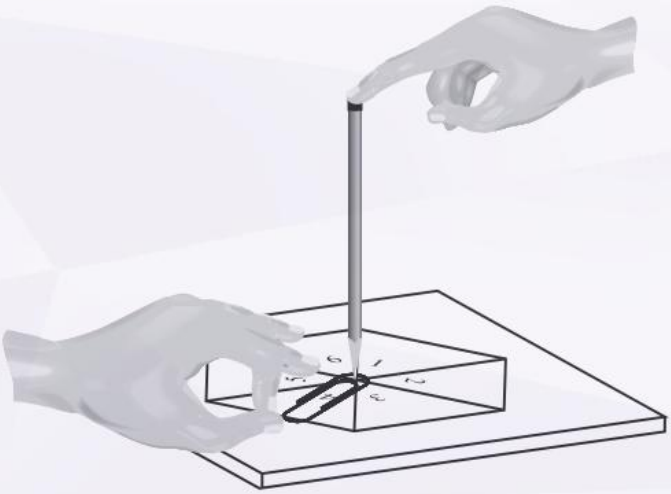


RICH TASK

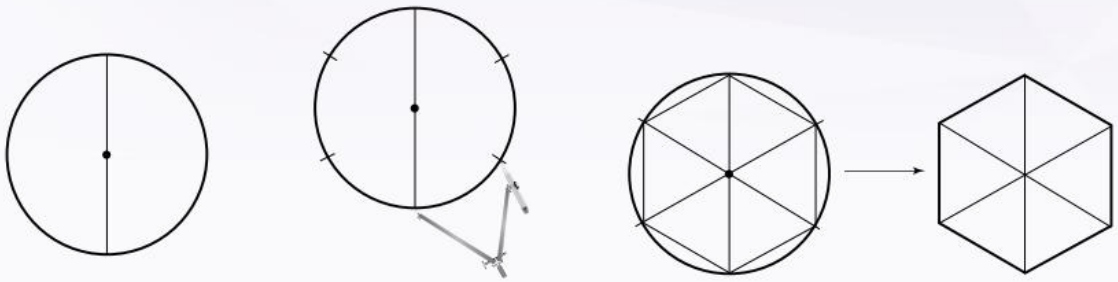
In a spin

When dealing with events involving chance, we try to predict what the outcome will be. Some events have an even chance of occurring, whereas others have little or no chance of occurring. If the outcomes for events are equally likely, we can predict how often each outcome will appear. But, will our predictions always be exact?

Spinners are often used to help calculate the chance of an event occurring. There are many different types of spinners, and one with 6 equal sections is shown below. A paperclip is flicked around the pencil placed at the centre. The instructions below will enable you to construct a spinner to use in a probability exercise.



- Draw a circle with a radius of 8 cm onto a piece of cardboard using a pair of compasses. Using a ruler, draw a line to indicate the diameter of the circle.
- Keep the compasses open at the same width. Place marks on the circumference on both sides of the points where the diameter meets the circle.
- Join the marks around the circumference to produce your hexagon. Draw a line with a ruler to join the opposite corners and cut out the hexagon.
- Number the sections 1 to 6 or colour each section in a different colour.



1 Flick the paperclip around the pencil 120 times. Record each outcome in a copy of the table below.

Score	Tally	Frequency
1		
2		
3		
4		
5		
6		
	Total	

Note: If the paperclip lands on a line, re-spin the spinner.

2 Are the outcomes of each number in your spinner equally likely? Explain why.

3 What would be the theoretical probability of spinning a 6?

4 Based on the results you obtained from your spinner, list the relative frequencies of each outcome on the spinner.

5 How many times would you expect the paperclip to land on each number when you perform 120 spins?

6 How close were your results to the expected results?

Combine your results with the results obtained from your classmates.

7 Design a new frequency table for the class results.

8 How do the relative frequencies of the pooled class results compare with your results? Are they closer to the results you expected?

9 If time permits, continue spinning the spinner and pooling your results with the class. Investigate the results obtained as you increase the number of trials for the experiment.

CODE PUZZLE

What did the cannibal have for breakfast?

Draw a Venn diagram to help solve each of these problems. The letters give the puzzle's answer code.

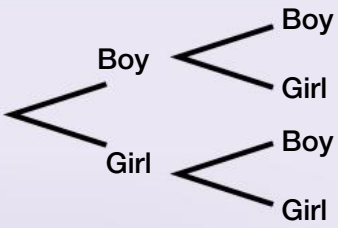
G In a club of 35 members, everyone plays table tennis or badminton. Twenty-five play table tennis and 15 play badminton. What is the probability of selecting a player who plays both sports?

K Twelve boys have both a football and a basketball. Eighteen boys have a football and 15 have a basketball. What is the probability of selecting a boy who has only a football?

I Of 50 people, 38 went to the movies and 42 went shopping. If every person went either to the movies or shopping, find the probability of selecting a person who went both to the movies and shopping.

B In a class election, 12 people voted for Karin and 18 voted for Lynette. If there were 25 in the class, find the probability of selecting a student who voted for both.

The following is a tree diagram showing the possible outcomes of a family having two children. Extend it for three children and answer the following questions.



S The probability that the family has 3 girls

E The probability that the family has 2 boys and a girl

D The probability that the family has 3 of the same sex

N The probability that the family has at least 2 girls

A The probability that a third child will be a girl if the family already has 2 girls

$\frac{1}{5}$	$\frac{1}{3}$	$\frac{2}{7}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{3}{8}$	$\frac{3}{5}$	$\frac{1}{2}$	$\frac{1}{7}$	$\frac{1}{8}$

eBookplus Activities

13.2 Probability scale

Digital doc

- SkillSHEET (doc-6969) Understanding chance words

Interactivity

- IP interactivity 13.2 (int-4457) Probability scale

13.3 Experimental probability

Digital docs

- SkillSHEET (doc-6970) Understanding a deck of playing cards
- WorkSHEET 13.1 (doc-7107)

Interactivity

- IP interactivity 13.3 (int-4458) Experimental probability

13.4 Sample spaces and theoretical probability

Digital docs

- SkillSHEET (doc-6971) Simplifying fractions
- SkillSHEET (doc-6972) Converting a fraction into a decimal
- SkillSHEET (doc-6973) Converting a fraction into a percentage
- SkillSHEET (doc-6974) Multiplying a fraction by a whole number

Interactivity

- IP interactivity 13.4 (int-4459) Sample spaces and theoretical probability

13.5 Complementary events

Interactivity

- IP interactivity 13.5 (int-4460) Complementary events

13.6 Venn diagrams

Interactivities

- Venn diagrams (int-2377)
- IP interactivity 13.6 (int-4461) Venn diagrams

eLesson

- Union and intersection of sets (eles-0049)

13.7 Tree diagrams and two-way tables

Digital docs

- SkillSHEET (doc-6975) Listing the sample space
- SkillSHEET (doc-6976) Multiplying proper fractions
- Spreadsheet (doc-2408) Coin tossing
- WorkSHEET 13.2 (doc-7108)

Interactivity

- IP interactivity 13.7 (int-4462) Tree diagrams and two-way tables

13.8 Review

Interactivities

- Word search (int-2637)
- Crossword (int-2638)
- Sudoku (int-3192)

Digital docs

- Topic summary (doc-10764)
- Concept map (doc-10777)
- Topic review (Word doc-14946, PDF doc-14947)

To access eBookPLUS activities, log on to **www.jacplus.com.au**

Answers

TOPIC 13 Probability

13.2 Probability scale

1 **a** Certain **b** Highly unlikely
c Discuss **d** Discuss
e Highly unlikely **f** Highly unlikely
g Certain **h** Impossible
i Highly unlikely **j** Highly likely
k Even chance **l** Highly unlikely
m Even chance **n** Better than even chance
o Highly likely **p** Highly unlikely

2 **a–e** Should be discussed in class

3 **a** $\frac{1}{2}$ **b** $\frac{9}{10}$ **c** 0 **d** 1
e $\frac{1}{2}$ **f** $\frac{1}{3}$ **g** $\frac{1}{52}$ **h** $\frac{11}{20}$
i $\frac{5}{6}$ **j** $\frac{1}{13}$ **k** $\frac{1}{5}$ **l** $\frac{2}{13}$
m $\frac{4}{7}$ **n** 1

4 C

5 E

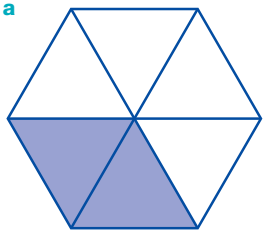
6 **a** $\frac{4}{11}$ **b** $\frac{7}{11}$ **c** $\frac{2}{11}$ **d** $\frac{1}{11}$

7 **a** **i** No
ii {blue, green, red, yellow}
iii Blue: $\frac{1}{3}$; green: $\frac{1}{6}$; red: $\frac{1}{3}$; yellow: $\frac{1}{6}$
b **i** Yes
ii {blue, green, orange, yellow, red}
iii Blue: $\frac{1}{6}$; green: $\frac{1}{6}$; orange: $\frac{1}{6}$; yellow: $\frac{1}{6}$; red: $\frac{1}{6}$
c **i** No
ii {yellow, green}
iii Yellow:$\frac{2}{3}$; green:$\frac{1}{3}$

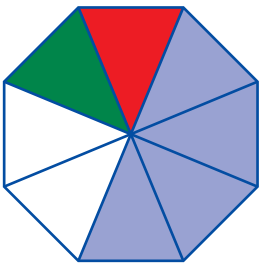
8 **a** Blue. There are more blue jelly beans than any other colour.
b Yellow. There are fewer yellow jelly beans than any other colour.
c Blue: $\frac{6}{17}$; green: $\frac{5}{17}$; red: $\frac{4}{17}$; yellow: $\frac{2}{17}$

9 Answers will vary. Some examples are shown.

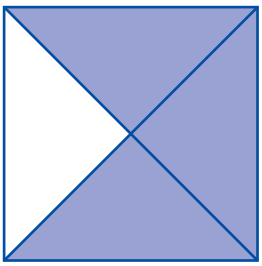
a



b



c



Challenge 13.1

The probability of a green card is 0.15. There are 3 green cards in the bag.

13.3 Experimental probability

1 **a** $\frac{14}{25}$ (0.56) **b** $\frac{11}{25}$ (0.44)

2 **a** $\frac{13}{25}$ **b** 0.48

3 **a** $\frac{7}{50}$ (0.14) **b** $\frac{43}{50}$ (0.86)

4 **a** $\frac{5}{16}$ **b** $\frac{13}{40}$ **c** 0.3625

5 **a** **i** 2, 4, 6
ii 1, 2
iii 3, 4, 5, 6
iv 2, 3, 5
v Impossible
vi 5, 6
vii 1, 4, 6
viii 1, 2, 3, 4
ix 3, 6
x 5
b 1 → 15%
2 → 17%
3 → 17.5%
4 → 16%
5 → 17.5%
6 → 17%
c 50% **d** 48% **e** 17.5%
f 68% **g** 50% **h** 34.5%
i 34.5% **j** 32%

6 **a**

Score	Frequency
1	34
2	27
3	24
4	15
Total	100

b Swimming = $\frac{34}{100}$ (0.34)
Athletics = $\frac{27}{100}$ (0.27)
Gymnastics = $\frac{24}{100}$ (0.24)
Rowing = $\frac{15}{100}$ (0.15)
c 1 **d** Swimming **e** 204

7 **a**

Score	Frequency
1	3
2	5
3	5
4	4
5	3
Total	20

b 1 → $\frac{3}{20}$ (0.15)
2 → $\frac{1}{4}$ (0.25)
3 → $\frac{1}{4}$ (0.25)
4 → $\frac{1}{5}$ (0.2)
5 → $\frac{3}{20}$ (0.15)
c 4
There are five possible outcomes, and each has an equal chance of occurring. Therefore, in 20 trials each outcome would be expected to occur 4 times.
d 2 and 3 **e** 1

8 **a** **i** 3 of hearts **ii** $\frac{13}{60}$
b **i** Queen of diamonds and 3 of diamonds
ii $\frac{9}{20}$
c **i** 3 of hearts, queen of diamonds and 3 of diamonds
ii $\frac{2}{3}$

d **i** 3 of each suit
ii $\frac{3}{4}$
e **i** 3 of both spades and hearts
ii $\frac{2}{5}$
f **i** All cards drawn
ii 1
g **i** None of the cards drawn
ii 0
h **i** All cards drawn
ii 1

9 **a** The greater the number of trials, the closer the results come to what we would expect; that is, a relative frequency of 50% for each event.
b No, the results would not be identical because this is an experiment and values will differ for each trial.

10 **a** $\frac{1}{4}$
b

Score	Frequency
1	11
2	9
3	10
4	10
Total	40

c $\frac{21}{40}$ **d** 2, 3 **e** $\frac{19}{40}$

11 **a** $\frac{1}{20}$ (0.05) **b** $\frac{19}{20}$ (0.95) **c** 30

12 D **13** B

14 Check with your teacher.

15 **a** 160 **b** $200\left(1-\frac{g}{n}\right)$

16 **a** $\frac{7}{20}$ **b** $\frac{2}{5}$

17 **a** $\frac{30}{241}$ **b** $\frac{91}{241}$ **c** $\frac{59}{241}$

18 **a**

Clubs	Diamonds	Hearts	Spades
A♣	A♦	A♥	A♠
2♣	2♦	2♥	2♠
3♣	3♦	3♥	3♠
4♣	4♦	4♥	4♠
5♣	5♦	5♥	5♠
6♣	6♦	6♥	6♠
7♣	7♦	7♥	7♠
8♣	8♦	8♥	8♠
9♣	9♦	9♥	9♠
10♣	10♦	10♥	10♠
J♣	J♦	J♥	J♠
Q♣	Q♦	Q♥	Q♠
K♣	K♦	K♥	K♠

b **i** $\frac{1}{2}$ **ii** $\frac{3}{13}$ **iii** $\frac{1}{13}$ **iv** $\frac{4}{13}$ **v** $\frac{1}{52}$
vi $\frac{3}{4}$ **vii** $\frac{4}{13}$ **viii** $\frac{9}{13}$ **ix** $\frac{4}{13}$ **x** $\frac{8}{13}$

Challenge 13.2
24

13.4 Sample spaces and theoretical probability

1 **a** {Heads, Tails} **b** {a, a, o, u}
c {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}
d {R, R, R, W, W, B} **e** {1, 2, 3, 4, 5, 6}
f {king of hearts, diamonds, clubs, spades; queen of hearts, diamonds, clubs, spades; jack of hearts, diamonds, clubs, spades}
g {1, 2, 3, 4, 5, 6, 7, 8}
h {2, 4, 6, 8, 10, 12, 14, 16, 18, 20}
i {apple, apple, pear, pear, pear, pear, orange, orange, orange, orange, banana, banana, banana, banana}
j {*Dolly*, *Dolly*, *Dolly*, *Girlfriend*, *Girlfriend*, *Smash Hits*, *Mathsmag*, *Mathsmag*}
k {A, B, C, D, E} **l** {gold, silver, bronze}

2 **a** {1, 2, 3, 4, 5, 6}
b **i** $\frac{1}{6}$ **ii** $\frac{1}{2}$ **iii** $\frac{2}{3}$ **iv** $\frac{1}{3}$
v $\frac{1}{2}$ **vi** $\frac{1}{3}$ **vii** 0 **viii** 1

3 **a** $\frac{1}{52}$, 1.9% **b** $\frac{1}{13}$, 7.7%
c $\frac{2}{13}$, 15.4% **d** $\frac{1}{4}$, 25%
e $\frac{1}{2}$, 50% **f** $\frac{4}{13}$, 30.8% **g** $\frac{1}{13}$, 7.7%

4 **a** $\frac{2}{45}$ **b** $\frac{8}{9}$ **c** $\frac{1}{9}$ **d** $\frac{1}{9}$

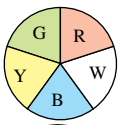
5 **a** $\frac{3}{14}$ **b** $\frac{1}{7}$ **c** $\frac{1}{7}$ **d** $\frac{5}{14}$
e $\frac{11}{14}$ **f** $\frac{4}{7}$ **g** 0 **h** $\frac{13}{14}$

6 **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** 0 **d** 1

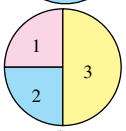
7 **a** $\frac{1}{365}$ (or $\frac{1}{366}$) **b** $\frac{7}{365}$ (or $\frac{7}{366}$)
c $\frac{30}{365}=\frac{6}{73}$ (or $\frac{30}{366}=\frac{5}{61}$) **d** $\frac{1}{365}$ (or $\frac{1}{366}$)
Note: In a leap year there are 366 days.

8 **a** **i** Yes, equal sectors **ii** $\frac{1}{2}$
b **i** Yes, equal sectors **ii** $\frac{1}{4}$
c **i** No, sector 1 occupies a larger area. **ii** $\frac{2}{3}$
d **i** No, sector 1 occupies the smallest area. **ii** $\frac{1}{8}$

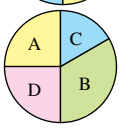
9 $\frac{1}{2}$

10 **a**


b Each sector has an angle of 72° at the centre of the spinner.

11 **a**


b Sectors 1 and 2 have angles of 90° and sector 3 has an angle of 180° at the centre of the spinner.

12 **a**


b Sectors A and D: 90°, sector B: 120° and sector C: 60°.

13 **a** 1 **b** 360°

14 **a** Heads, $\frac{1}{2}$; Tails, $\frac{1}{2}$ **b** 1

15 **a** H H, $\frac{1}{4}$ H T, $\frac{1}{4}$ T H, $\frac{1}{4}$ T T, $\frac{1}{4}$
b 1

16 **a** $\frac{1}{6}$ **b** 1 **c** Yes
d The total of probabilities for all elements in a sample space is always 1.

17 C

18 A

19 **a** {1, 2, 3, 4, 5, 6} **b** 36

c

		Die 2					
		1	2	3	4	5	6
Die 1	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

***d** See table at foot of page.

e The probabilities are symmetric.

f {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

g

Number of Heads	0	1	2	3
Probability	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

h 3 Heads **i** Heads once and Tails twice

20 Ask your teacher.

21 **a** $\frac{1}{4}$ **b** $\frac{5}{9}$ **c** $\frac{1}{2}$

22 **a** 1 **b** $\frac{3}{8}$

23 **a** $\frac{1}{4}$ **b** $\frac{1}{6}$ **c** $\frac{35}{36}$ **d** 1

24 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$ **d** $\frac{1}{3}$

13.5 Complementary events

1 **a** Selecting an odd number
b Selecting a consonant
c The coin landing Tails
d Getting a number greater than 2
e Getting a total of 12
f Not selecting a diamond
g Not selecting an E
h Not selecting a blue marble

2 B and C

3 **a** Not complementary, as there are other things that you could have for breakfast.
b Not complementary, as there are other ways of travelling to your friend's house.
c Not complementary, as there are other things that you could be doing.
d Complementary, as this covers all possible outcomes.
e Not complementary, as neither case covers the possibility of rolling a 4.

4 $\frac{12}{13}$ **5** A

6 $\frac{215}{216}$ **7** $\frac{4}{5}$

8 **a** $\frac{1}{4}$ **b** $\frac{3}{4}$ **c** $\frac{3}{4}$ **d** $\frac{1}{4}$

9 No, the two events are not complementary, as the sum of their probabilities does not equal one. Getting one Head is also an outcome.

10 **a** $\frac{17}{70}$ **b** $\frac{53}{70}$ **c** $\frac{9}{70}$ **d** $\frac{61}{70}$

11 **a** 0 **b** $\frac{7}{11}$ **c** $\frac{4}{11}$

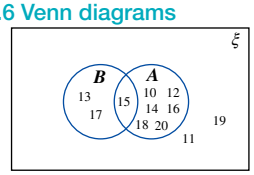
12 $\frac{3}{10}$

13 **a** $\frac{n-r}{n}$ **b** $\frac{r}{n}$

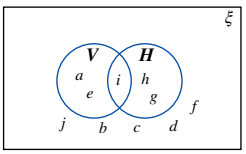
14 **a** $\frac{3}{11}$ **b** $\frac{5}{11}$

13.6 Venn diagrams

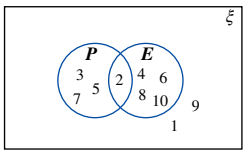
1



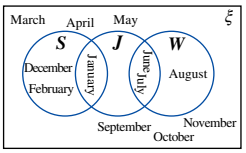
2



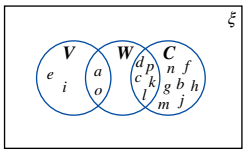
3



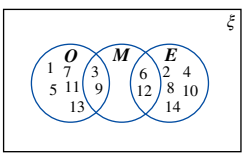
4



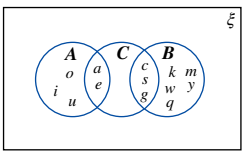
5



6

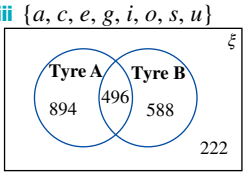


7 **a**



b **i** {*a*, *e*, *i*, *o*, *u*} **ii** {*c*, *g*, *s*}
iii {*a*, *c*, *e*, *g*, *i*, *o*, *s*, *u*} **iv** {*c*, *a*, *g*, *e*, *s*}

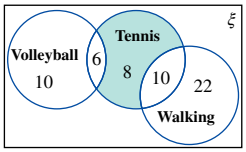
8 **a**



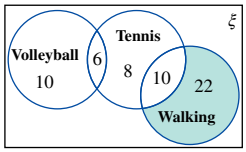
b **i** 894 **ii** 588 **iii** 222

c $\frac{222}{2200} = \frac{111}{1100}$ **d** $\frac{124}{271}$

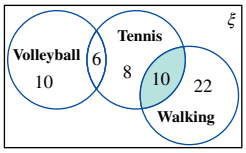
9 **a** **i**



ii



iii



b **i** 16 **ii** 0

c 56

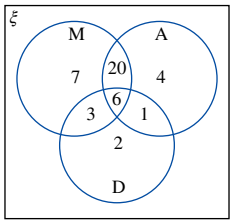
d **i** 32 **ii** 24 **e** $\frac{24}{56} = \frac{3}{7}$

f $\frac{6}{56} = \frac{3}{28}$ **g** $\frac{10}{32} = \frac{5}{16}$

***

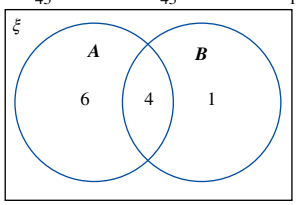
Sum	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36} = \frac{1}{18}$	$\frac{3}{36} = \frac{1}{12}$	$\frac{4}{36} = \frac{1}{9}$	$\frac{5}{36}$	$\frac{6}{36} = \frac{1}{6}$	$\frac{5}{36}$	$\frac{4}{36} = \frac{1}{9}$	$\frac{3}{36} = \frac{1}{12}$	$\frac{2}{36} = \frac{1}{18}$	$\frac{1}{36}$

10 a



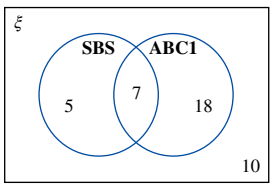
b $\frac{7}{43}$ **c** $\frac{4}{43}$ **d** $\frac{2}{43}$
e $\frac{3}{43}$ **f** $\frac{31}{43}$ **g** $\frac{1}{12}$

11



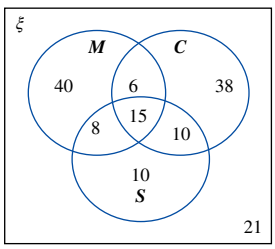
12 a 25 **b** 17 **c** 9 **d** 6 **e** 7
f 3 **g** 1 **h** 11 **i** 22 **j** 24

13 a



b 7

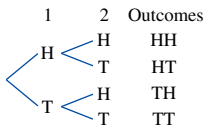
14 a



b 69 **c** 38 **d** 43 **e** 21

13.7 Tree diagrams and two-way tables

1 a i

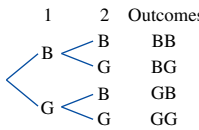


ii

2 \ 1	H	T
H	H H	T H
T	H T	T T

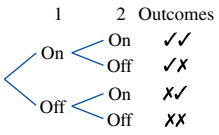
b i $\frac{1}{4}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{4}$ **iv** $\frac{1}{2}$ **v** $\frac{1}{2}$

2 a



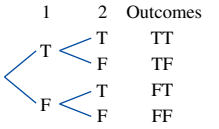
b i $\frac{1}{4}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{2}$ **iv** $\frac{1}{4}$ **v** $\frac{1}{4}$ **vi** $\frac{1}{2}$

3 a



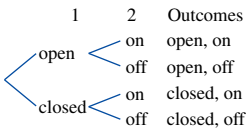
b i $\frac{1}{4}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{2}$ **iv** $\frac{1}{2}$

4 a



b i $\frac{1}{4}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{4}$ **iv** $\frac{1}{4}$ **v** $\frac{1}{2}$

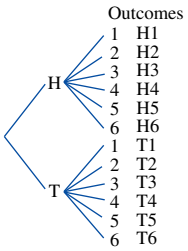
5 a



Door \ Light	On	Off
Open	open, on	open, off
Closed	closed, on	closed, off

b i $\frac{1}{4}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{4}$ **iv** $\frac{1}{4}$

6 a i



ii

2 \ 1	H	T
1	H 1	T 1
2	H 2	T 2
3	H 3	T 3
4	H 4	T 4
5	H 5	T 5
6	H 6	T 6

b i $\frac{1}{12}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{4}$ **iv** $\frac{1}{6}$

7 a

Coin \ Spinner	1	2	3
H	H 1	H 2	H 3
T	T 1	T 2	T 3

b i $\frac{1}{6}$ **ii** $\frac{1}{6}$ **iii** $\frac{1}{6}$ **iv** $\frac{1}{6}$
v $\frac{1}{3}$ **vi** $\frac{1}{6}$ **vii** $\frac{1}{3}$

8 a

Transmission \ Colour	Red	White	Yellow
Manual	MR	MW	MY
Auto	AR	AW	AY

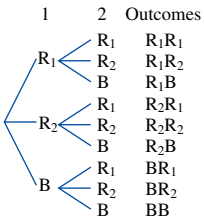
b i $\frac{1}{3}$ ii $\frac{1}{2}$ iii $\frac{1}{6}$ iv $\frac{1}{6}$ v $\frac{1}{6}$

9 a

Coin \ Die	1	2	3	4	5	6
H	H 1	H 2	H 3	H 4	H 5	H 6
T	T 1	T 2	T 3	T 4	T 5	T 6

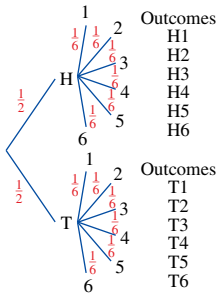
b i $\frac{1}{12}$ ii $\frac{1}{12}$ iii $\frac{1}{4}$ iv $\frac{1}{4}$
v $\frac{1}{4}$ vi $\frac{1}{6}$ vii $\frac{5}{12}$

10 a



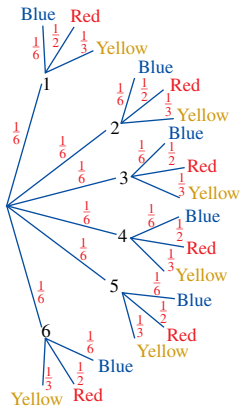
b i $\frac{1}{9}$ ii $\frac{4}{9}$ iii $\frac{2}{9}$ iv $\frac{2}{9}$
v $\frac{4}{9}$ vi $\frac{5}{9}$ vii $\frac{1}{9}$ viii $\frac{4}{9}$
ix $\frac{8}{9}$ x 0 xi $\frac{5}{9}$

11 a



b P(H, 1) → P(T, 6) = $\frac{1}{12}$
c 1
d i $\frac{1}{4}$ ii $\frac{1}{2}$ iii $\frac{1}{2}$

12 a



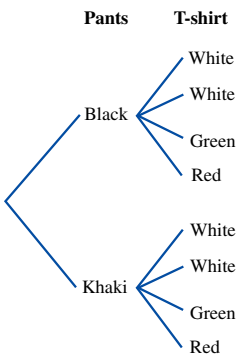
b P(1, Blue) → P(6, Blue) = $\frac{1}{36}$
P(1, Red) → P(6, Red) = $\frac{1}{12}$
P(1, Yellow) → P(6, Yellow) = $\frac{1}{18}$
c 1
d i $\frac{1}{6}$ ii $\frac{1}{3}$ iii $\frac{1}{2}$ iv $\frac{1}{12}$

13 E **14** E

15 a $\frac{1}{3}$ **b** 5
c–g Check with your teacher.
h For the lights described, P(red every morning of a school week) = $\frac{1}{243}$.

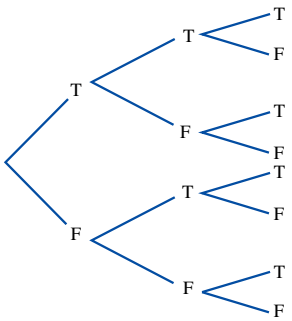
16 a P(winning roll) = $\frac{5}{36}$ **b** P(losing roll) = $\frac{5}{36}$
c P(neither win or lose) = $\frac{26}{36}$ **d** Check with your teacher.

17 a



b $\frac{1}{8}$

18 a



b i $\frac{1}{8}$ ii $\frac{1}{8}$ iii $\frac{1}{8}$

19 a i $\frac{1}{8}$ ii $\frac{3}{8}$ iii $\frac{1}{4}$
iv $\frac{7}{8}$ v $\frac{3}{4}$
b $\frac{1}{2}$
c $\frac{1}{2}$
d $\frac{1}{2}$
e Pr(2 girls, 1 boy) = $\frac{1}{8}$

20 a 6 (ADF, AEF, BDF, BEF, CDF, CEF)
b No; there is only one way from the library to home.

Investigation — Rich task

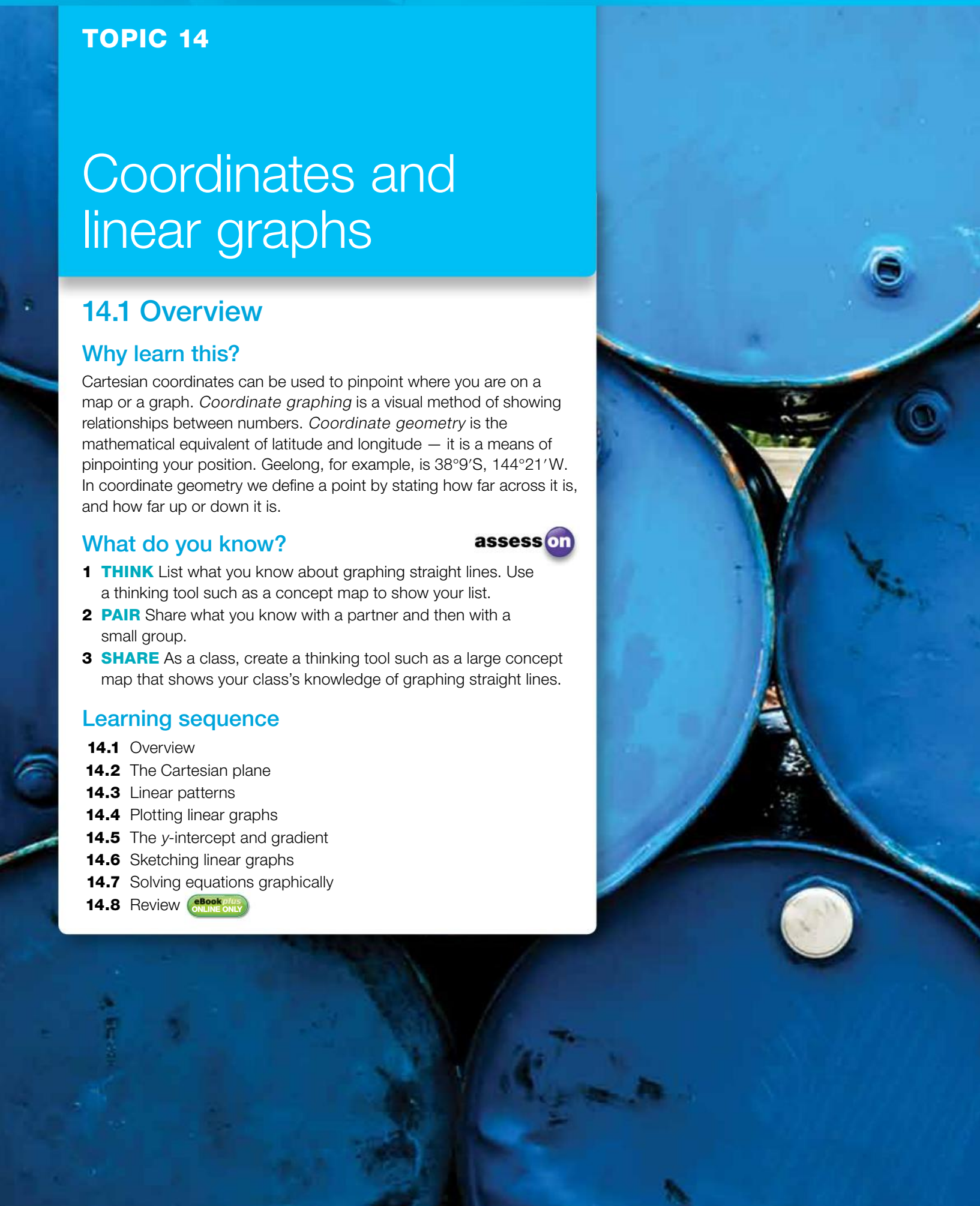
1 Teacher to check
2 They are equally likely because each section is the same size.
3 $\frac{1}{6}$
4 Teacher to check
5 20
6 to **9** Teacher to check

Code puzzle

Baked beings

TOPIC 14

Coordinates and linear graphs



14.1 Overview

Why learn this?

Cartesian coordinates can be used to pinpoint where you are on a map or a graph. *Coordinate graphing* is a visual method of showing relationships between numbers. *Coordinate geometry* is the mathematical equivalent of latitude and longitude — it is a means of pinpointing your position. Geelong, for example, is 38°9′S, 144°21′W. In coordinate geometry we define a point by stating how far across it is, and how far up or down it is.

What do you know?

assess on

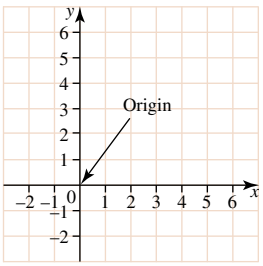
1 **THINK** List what you know about graphing straight lines. Use a thinking tool such as a concept map to show your list.
2 **PAIR** Share what you know with a partner and then with a small group.
3 **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of graphing straight lines.

Learning sequence

14.1 Overview
14.2 The Cartesian plane
14.3 Linear patterns
14.4 Plotting linear graphs
14.5 The *y*-intercept and gradient
14.6 Sketching linear graphs
14.7 Solving equations graphically
14.8 Review eBook plus ONLINE ONLY

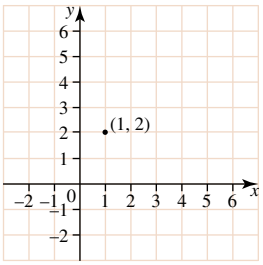
14.2 The Cartesian plane

The Cartesian plane



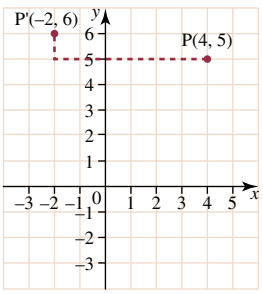
- A **Cartesian plane** (named after its inventor René Descartes) consists of two axes. The horizontal axis is called the *x*-axis; the vertical axis is called the *y*-axis.
- The centre of a Cartesian plane (where *x*- and *y*-axis intersect) is called the **origin**.
- Both axes are evenly scaled and numbered, with 0 (zero) placed at the origin. On the *x*-axis the numbers increase from left to right, while on the *y*-axis the numbers increase from bottom to top.
- Arrows are placed on the end of each axis to show that they continue infinitely.

Coordinates



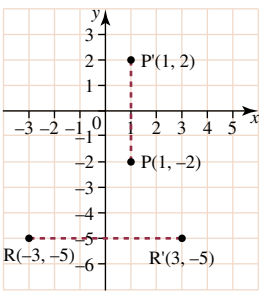
- To describe a position on a Cartesian plane, coordinates are used.
- A Cartesian coordinate is written as (x, y), where x and y are any numbers. The first number refers to the horizontal position of the point and is called 'the *x*-coordinate' of a point. The second number refers to the vertical position of the point and is called 'the *y*-coordinate' of a point. The coordinates of the origin are (0, 0).
- To locate a point on the Cartesian plane, move along the *x*-axis to the number indicated by the *x*-coordinate and then along the *y*-axis to the number indicated by the *y*-coordinate. For example, to locate the point with coordinates (1, 2), move 1 unit to the right of the origin and then 2 units up.

Translation



- Any point may be **translated**. A translation is simply a movement, usually sideways or up or down. A translation to the right or a translation upwards is regarded as positive.
- For example, if the point P(4, 5) is translated 6 units to the left (negative) and 1 unit up (positive), the resulting point, usually named P′, is P′(−2, 6).

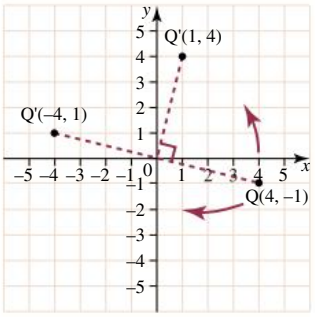
Reflection



- Any point P may be **reflected** in either axis. For example, if P(1, −2) is reflected in the *x*-axis, the result is P(1, 2). On the other hand, if R(−3, −5) is reflected in the *y*-axis, the result is R′(3, −5).

Rotation

- Any point may be **rotated** about the origin in either direction, clockwise or anticlockwise. For example, if Q(4, −1) is rotated through 90° in an anticlockwise direction about the origin, the result is Q′(1, 4). On the other hand if Q(4, −1)) is rotated through 180° in a clockwise direction about the origin, the result is Q′(−4, 1).



WORKED EXAMPLE 1

a Draw a Cartesian plane that goes from −3 to 5 on the *x*-axis and −3 to 5 on the *y*-axis and plot the following points.
A(1, 2) B(3, −2) C(−2, $-2\frac{1}{2}$) D(0, −1) E(−2, 0)
b State the coordinates of each of the original five points if they are translated 2 units to the right and 3 units down. Label the new points A′, B′ and so on.
c State the coordinates of each of the original five points if they are reflected in the *x*-axis, labelling the new points A″, B″ and so on.

THINK

a 1 Draw a Cartesian plane that extends from −3 to 5 on both the *x*- and *y*-axes.

2 Find and plot the position of each point, and then label.

A Across to 1 on the *x*-axis and then up to 2 on the *y*-axis.

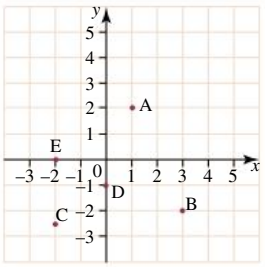
B Across to 3 on the *x*-axis and then down to −2 on the *y*-axis.

C Across to −2 on the *x*-axis and then down to $-2\frac{1}{2}$ on the *y*-axis.

D Stay at the origin (0 on the *x*-axis) and then down to −1 on the *y*-axis.

E Across to −2 on the *x*-axis and then stay at 0 on the *y*-axis.

WRITE/DRAW



b To translate 2 units to the right, add 2 to *x*.
To translate 3 units down, subtract 3 from *y*.

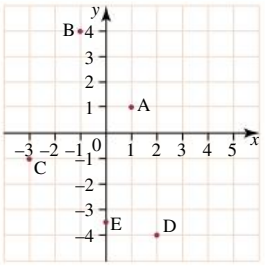
A(1, 2) ⇒ A′(3, −1)
B(3, −2) ⇒ B′(5, −5)
C(−2, $-2\frac{1}{2}$) ⇒ C′(0, $-5\frac{1}{2}$)
D(0, −1) ⇒ D′(2, −4)
E(−2, 0) ⇒ E′(0, −3)

c For reflection in the *x*-axis, change the sign of *y*.

A(1, 2) ⇒ A″(1, −2)
B(3, −2) ⇒ B″(3, 2)
C(−2, $-2\frac{1}{2}$) ⇒ C″(−2, $2\frac{1}{2}$)
D(0, −1) ⇒ D″(0, 1)
E(−2, 0) ⇒ E″(−2, 0)

WORKED EXAMPLE 2

a Write the Cartesian coordinates of the points A to E marked on the Cartesian plane at right.
b The points A to E are rotated through 90° in an anticlockwise direction about the origin. Find their new coordinates.



THINK **WRITE**

a 1 Trace along the x-axis to find the first number, and then along the y-axis to find the second number.
Point A is at 1 on the x-axis and 1 on the y-axis.
Point B is at -1 on the x-axis and 4 on the y-axis.
Point C is at -3 on the x-axis and -1 on the y-axis.
Point D is at 2 on the x-axis and -4 on the y-axis.
Point E is at 0 on the x-axis and $-3\frac{1}{2}$ on the y-axis.

2 Write each point as a pair of coordinates.

A(1, 1) B(−1, 4) C(−3, −1) D(2, −4) E(0, $-3\frac{1}{2}$)

b For anticlockwise rotation, interchange x and y and change the sign of the new x.

A(1, 1) ⇒ A(−1, 1)
B(−1, 4) ⇒ B(−4, −1)
C(−3, −1) ⇒ C(1, −3)
D(2, −4) ⇒ D(4, 2)
E(0, $-3\frac{1}{2}$) ⇒ E($3\frac{1}{2}$, 0)

assess on

Exercise 14.2 The Cartesian plane

REFLECTION
Think of an alternative way (that is, other than Cartesian coordinates) to specify the position of a point on a plane.

INDIVIDUAL PATHWAYS

■ PRACTISE	■ CONSOLIDATE	■ MASTER
Questions: 1–10	Questions: 1–11, 13	Questions: 1–14

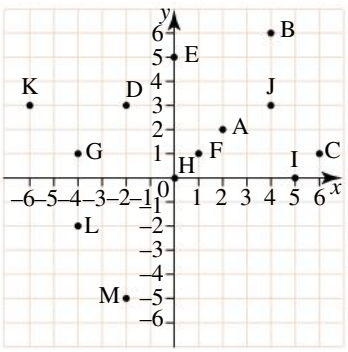
■■■ Individual pathway interactivity int-4463 eBookplus

eBookplus
Digital docs
SkillSHEET Grid coordinates I doc-6934
SkillSHEET Grid coordinates II doc-6935
SkillSHEET Plotting coordinate points doc-6936

FLUENCY

1 WE1a Draw a Cartesian plane that extends from −6 to 6 on the x-axis and −6 to 6 on the y-axis, and plot and label the following points.

a A(3, 3) **b** B(2, 5) **c** C(5, 1)
d D(−1, 4) **e** E(−4, 2) **f** F(−2, 0)
g G(−2, −3) **h** H(−4, −5) **i** I(0, −3)
j J(1, −2) **k** K(3, $-1\frac{1}{2}$) **l** L($4\frac{1}{2}$, 0)



2 WE2a Write the Cartesian coordinates of the points A to M marked on the Cartesian plane at right.

UNDERSTANDING

3 MC *Note:* There may be more than one correct answer.

a The point (3, 4) gives a position on the Cartesian plane of:

- **A** 3 on the *y*-axis, 4 on the *x*-axis
- **B** 3 left, 4 up
- **C** 4 right, 3 up
- **D** 3 on the *x*-axis, 4 on the *y*-axis
- **E** 3 right, 4 up

b The point (−2, 0) gives a position on the Cartesian plane of:

- **A** left 2, up 0
- **B** left 0, down 2
- **C** −2 on the *x*-axis, 0 on the *y*-axis
- **D** −2 on the *y*-axis, 0 on the *x*-axis.
- **E** 2 on the *x*-axis, 0 on the *y*-axis

4 On 1-cm graph paper, draw a Cartesian plane with an *x*-axis from −5 to 5 and a *y*-axis from −3 to 5. Connect these three groups of points.
START (0, 5) (−3, −3) (5, 2) (−5, 2) (3, −3) (0, 5) STOP
START (5, 2) (0, 0) (−5, 2) STOP
START (3, −3) (0, 0) (−3, −3) STOP

5 On 1-cm graph paper, draw a Cartesian plane with an *x*-axis from −6 to 6 and a *y*-axis from −6 to 6. Connect these groups of points.
START (4, 6) (−4, 6) (−6, 0) (−4, −6) (4, −6) (6, 0) (4, 6) (−6, 0) (4, −6) (4, 6) STOP
START (−4, 6) (−4, −6) (6, 0) (−4, 6) STOP
START (4, 0) (2, 2.5) (−2, 2.5) (−4, 0) (−2, −2.5) (2, −2.5) (4, 0) STOP
Colour the 6 triangles between the star and the hexagon. For example the triangle (6, 0) (4, 6) (4, 1) could be coloured red. Colour the 6 triangles inside the star. For example (4, 0) (4, 1) (2, 2.5) could be coloured green.

6 Draw a Cartesian plane using the following points. First check the coordinates to find the lowest and highest *x*- and *y*-values needed on each axis. Connect these groups of points.
START (4, 1) (3, 1) (4, 3) (5, 3) (5.5, 4) (5.5, 5) (6, 6) (6, 7) (5, 7.5) (4, 7.5) (3, 7.5) (2, 8) (1.5, 9) (1.5,10) (2, 11) (3, 12) (4, 12.5) (5, 13) (6, 14) (7, 15) (8, 15) (10, 14) (11, 11) (11, 10) (10.5, 8.5) (10, 7.5) (9, 10.5) (8.5, 8) (7.5, 7) (8, 6.5) (8, 5) (9, 4) (10, 3) (10, 1) (4, 1) STOP
START (2, 8) (1.5, 7.5) (1, 8) (1.5, 9) STOP
START (4, 1) (4, 2) (5, 2.5) (6.5, 2.5) (5.5, 4) STOP
START (6, 6.5) (8, 6.5) (8, 6) (6, 6) STOP
EYE AT (5.5, 11)

7 Draw a Cartesian plane. Check the following coordinates to find the lowest and highest *x*- and *y*-value needed on the axes. Then draw a cartoon character.
START (6, 7) (7.5, 9) (5, 9) (4.5, 12) (2, 11) (0, 13) (−1.5, 10) (−5, 11) (−5, 8) (−8, 6) (−6, 4) (−8, 2)
(−6, 1) (−7, −2) (−4, −1.5) (−4, −3.5) (−1.5, −3) (−2, −4) (−4, −7)
(−5, −8) STOP
START (−2, −9) (−1, −7) (1, −8) (3, −8) (4, −7.5) (5, −10) STOP
START (4, −7.5) (3.5, −6) (3.5, −4) (4, −3) (5, −2.5) (5, −2) (4, −1.5) (4, −1) (5, 0.5) (7, 1) (8, 2) (8, 2.5) (6.5, 3) STOP

START (4, −2.5) (2, −3) (0.5, −3) (0, −2) (1, −1) (2, −0.5) (3, 0) (7, 1) STOP
START (6, 2.5) (6.5, 3) (6.5, 4) (6, 4) (4, 3) STOP
START (6, 7) (5, 7.5) (4, 7) (3, 6) (1, 6) (0, 5) (−1, 4) (0, 2) (1.5, 1.5) (3, 2) (4, 4) (6.5, 4) (7, 5) (7, 6) (6, 7) STOP
START (4, 4) (4, 5) (3, 6) STOP
START (1, −1) (5, 0) STOP
EYES AT (1, 3) AND (5, 5)
EYE LASHES (−1, 4) TO (−2, 4.5), (0, 5) TO (−0.5, 6), (1, 6) TO (0.5, 7), (2, 6) TO (2, 7), (4, 7) TO
(3.5, 8), (5, 7.5) TO (5, 8.5), (6, 7) TO (6.5, 8), (6.5, 6.5) TO (7, 7)

8 On a piece of graph paper, draw a Cartesian plane that covers the entire page. Draw a simple picture on your graph paper. List the coordinates and instructions showing when to start and stop joining the points. The coordinates should be chosen carefully so that someone else can draw your picture, following your instructions.

9 **WE1b,c** A triangle is formed by the points A(3, −2), B(−5, 1) and C(4, 7).

- **a** If the points are reflected in the x–axis, find the coordinates of the vertices of the new triangle.
- **b** The new triangle is now translated 5 units to the left and 1 unit down. Find the coordinates of the vertices of this new triangle.

10 A quadrilateral whose vertices are given by the points P(1, 2), Q(5, −1), R(6, 5) and S(−1, −3) is rotated clockwise through 270° about the origin. Find the coordinates of the vertices of the rotated quadrilateral.

REASONING

11 The point P(a, $-b$) is reflected in the y-axis and then rotated through 180° about the origin. Find the coordinates of the new point. Show your working.

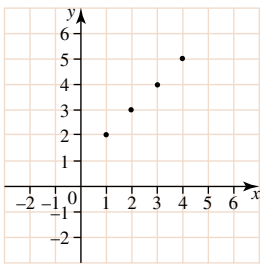
12 Using the point (x, $-y$), prove that a rotation of 180° gives an identical result to a rotation of −180°.

PROBLEM SOLVING

13 The point A(a, b) is rotated anticlockwise through −90°, about the origin, then reflected in the x-axis, translated by 4 units in the negative x direction and finally reflected in the y-axis. What are the final coordinates of A?

14 Using the point R($-r$, $-q$) and showing all working, prove that a rotation of 180° followed by a reflection in the y-axis is equivalent to a reflection in the x-axis.

14.3 Linear patterns



- A set of points may be such that the x- and y-coordinates of each point are connected by the same rule. For example, each of the points (1, 2) (2, 3) (3, 4) (4, 5) have the same relationship (rule) between their x- and y-values; namely, the y-coordinate of every point is one more than its x-coordinate.
- If the set of points where x- and y-coordinates are connected by a certain rule is plotted on the Cartesian plane, the points will form a pattern.
- If the pattern formed by the set of points is a straight line, we refer to it as a linear pattern. For example, the points (1, 2) (2, 3) (3, 4) (4, 5), when plotted, will form a linear pattern as shown.

- A diagram formed by plotting a set of points on the Cartesian plane is referred to as a graph. If the points form a straight line, then the graph is linear (a straight-line graph).
- A set of points with their coordinates can be presented in the form of a table. For example, the points (−2, 4) (−1, 2) (0, 0) (1, −2) (2, −4) can be presented as shown.

x	−2	−1	0	1	2
y	4	2	0	−2	−4

WORKED EXAMPLE 3

Plot the following points on a Cartesian plane and comment on any pattern formed. Check the lowest and highest values to help you decide the numbers to mark on the axes.
(−2, 1) (−1, 2) (0, 3) (1, 4) (2, 5)

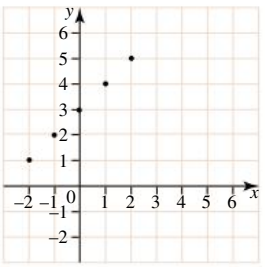
THINK

1 Look at the x- and y-values of the points and draw a Cartesian plane.
The lowest value for the x-axis is −2, the highest is 2.
The lowest value for the y-axis is 1, the highest is 5.
Extend each axis slightly beyond these values.

2 Plot each point.

3 Comment on any pattern formed.

WRITE/DRAW



A linear pattern is obtained as the points form a straight line.

WORKED EXAMPLE 4

a Plot the points in the following table on a Cartesian plane.

x	−4	−3	−2	−1	0	1
y	−2	−1	0	1	2	3

b Do the points form a linear graph? if so, what would the next point in the pattern be?

THINK

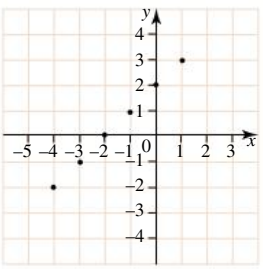
a 1 Look at the x- and y-values of the points and draw a Cartesian plane.

2 Plot each point.

b 1 Look at the position of the points and answer the question.
Note: The points form a straight line, so we have a linear graph.

WRITE/DRAW

a



b Yes, the points do form a linear graph.

2 Study the pattern and answer the question.
Note: The pattern shows that the x-values increase by 1, and the y-values increase by 1. The next x-value will be 2 and the next y-value will be 4.

The next point in the pattern is (2, 4).

assessON

Exercise 14.3 Linear patterns

REFLECTION
Is it possible to say whether or not a set of points will form a linear graph without actually plotting them?

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7	Questions: 1–8	Questions: 1–10

Individual pathway interactivity int-4464 eBook plus

FLUENCY

1 WE3 Plot the points on a Cartesian plane and comment on any pattern formed. (Check the lowest and highest values to help you decide which ***scale*** to mark on the axes.)

a (−2, −3) (−1, −2) (0, −1) (1, 0) (2, 1)
b (−2, 0) (−1, 1) (0, 2) (1, 3) (2, 4)
c (−2, −4) (−1, −2) (0, 0) (1, 2) (2, 4)
d (−2, −5) (−1, −2) (0, 1) (1, 4) (2, 7)
e (−2, 2) (−1, 1) (0, 0) (1, −1) (2, −2)
f (−2, 0) (−1, −1) (0, −2) (1, −3) (2, −4)

2 WE4a Plot the following points on a Cartesian plane.

a

x	−2	−1	0	1	2
y	1	2	3	4	5

b

x	−2	−1	0	1	2
y	−2	−1	0	1	2

c

x	−2	−1	0	1	2
y	−7	−4	−1	2	5

d

x	−2	−1	0	1	2
y	3	2	1	0	−1

e

x	−2	−1	0	1	2
y	−1	−0.5	0	0.5	1

f

x	−2	−1	0	1	2
y	4	2	0	−2	−4

3 WE4b Plot the following points on a Cartesian plane. Do the points form a linear graph? If so, state the next point in the pattern.

a (−3, −3) (−2, −1) (−1, 1) (0, 3) (1, 5) (2, 7) (3, 9)
b (−3, −5) (−2, −3) (−1, 0) (0, 1) (1, 4) (2, 5) (3, 7)

c

x	−2	−1	0	1	2
y	3	−1	−2	−3	−4

d

x	−2	−1	0	1	2
y	−6	−3	0	3	6

UNDERSTANDING

4 MC (*Note:* There may be more than one correct answer.)

a The next point in the linear pattern made by (−2, 0) (−1, 1) (0, 2) (1, 3) (2, 4) is:
A (5, 3) **B** (−3, −5) **C** (3, −5) **D** (3, 5) **E** (4, 6)

b The next point in the linear pattern made by (−2, 9) (−1, 8) (0, 7) (1, 6) (2, 5) is:
A (−3, 8) **B** (3, 4) **C** (3, 6) **D** (4, 3) **E** (6, 3)

c The next point in the linear pattern made by (−2, −18) (−1, −14) (0, −10) (1, −6) (2, −2) is:
A (−3, 3) **B** (−3, −20) **C** (3, 2) **D** (4, 6) **E** (6, 16)

d Which of the following set of points would make a linear pattern?
A (−2, −1) (−1, −2) (0, −3) (1, −4) (2, −5)
B (−2, 12) (−1, 10) (0, 8) (1, 6) (2, 4)
C (−2, −1) (−1, 0) (0, 1) (1, −1) (2, 0)
D (−2, −5) (−1, 0) (0, 4) (1, 5) (2, 8)
E (−2, 0) (−1, 3) (0, 6) (1, 9) (2, 12)

eBookplus
Digital doc
Spreadsheet
Plotting points
doc-2439

REASONING

5 Plot the following points on a Cartesian plane and join the points with a straight line.
a (−3, −2) (0, 1) (1, 2) (4, 5)
b (−2, 4) (0, 2) (1, 1) (3, −1)
c $(-1, -1)\ (0, -3)\ (1\frac{1}{2}, -6)\ (2, -7)$
d $(-\frac{3}{2}, -2)\ (-\frac{1}{2}, 0)\ (1, 3)\ (3, 7)$

6 By extending each of the lines in question **5**, give three examples of points that also fit the same pattern and lie on the line.

PROBLEM SOLVING

7 A student starts the 'Get fit' plan below.

Day (d)	Distance (D)
Monday (1)	1 km
Tuesday (2)	1.25 km
Wednesday (3)	1.5 km
Thursday (4)	
Friday (5)	

a Copy and complete the pattern for the 'Get fit' plan to show the distances run for 7 days.
b Determine whether the increasing distance represents a linear function.
c What is the constant amount by which the student increases her run each day?
d Calculate the total distance run on the 10th day.

8 The time taken for your teacher to write a maths test is 12 minutes per question plus a 10 minute rest after each lot of 5 questions.
a How much time does it take your teacher to write:
i one question? **ii** two questions? **iii** six questions?
b Generate a table of values for the number of questions written, q, versus the total time taken, t, for up to 20 questions.
c Determine whether this relationship represents a linear function.

9 The latest version of a new computer operating system is offered for sale on 1 January. An expert software engineer claims there are 400 bugs in this release; however, the manufacturer claims that it can fix 29 bugs each month. When will the software be bug-free?

10 Consider the relationship $\frac{1}{y} = \frac{1}{x} + 1$. It appears that both variables are being raised to the power of 1. The relationship can be transposed as shown below.

$$\frac{1}{y} = \frac{1}{x} + 1$$
$$\frac{1}{y} = \frac{1}{x} + \frac{x}{x}$$
$$\frac{1}{y} = \frac{1+x}{x}$$
$$y = \frac{x}{1+x}$$

eBook*plus*
Digital doc
WorkSHEET 14.1
doc-2416

a Generate a table of values for x versus y.
b Plot a graph to show the points contained in the table of values.
c Use your plot to confirm whether this relationship is linear.

14.4 Plotting linear graphs

- To plot the linear graphs whose equation is given, follow these steps.
  1. Create a table of values first.
     – Draw a table.
     – Select some x-values.
     – Substitute the selected x-values into the rule to find corresponding y-values.
  2. Draw a Cartesian plane.
  3. Plot the point from the table and join them with the straight line.
  4. Label the graph.

WORKED EXAMPLE 5

Draw a table of values and plot the graph of $y = 2x + 1$ and label the line.

THINK

1 Write the rule.

2 Draw a table and choose simple x-values.

3 Use the rule to find each y-value and enter them in the table.
When $x = -2$, $y = 2 \times -2 + 1 = -3$.
When $x = -1$, $y = 2 \times -1 + 1 = -1$.
When $x = 0$, $y = 2 \times 0 + 1 = 1$.
When $x = 1$, $y = 2 \times 1 + 1 = 3$.
When $x = 2$, $y = 2 \times 2 + 1 = 5$.

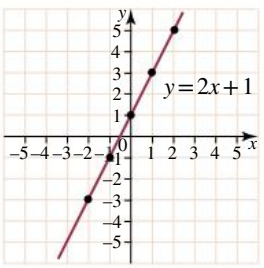
4 Draw a Cartesian plane and plot the points.

5 Join the points to form a straight line and label the graph.

WRITE/DRAW

$y = 2x + 1$

x	-2	-1	0	1	2
y	-3	-1	1	3	5



Function notation

- In mathematics, linear rules are also called **linear functions**.
- A linear function can be thought of as a machine with an 'input' of x and an 'output' of y.
- To write a rule in function notation, $f(x)$ is used in place of y.
- When using functional notation, assume x can take any value unless otherwise stated.

WORKED EXAMPLE 6

Draw a table of values and plot the graph of $f(x) = -x + 2$.

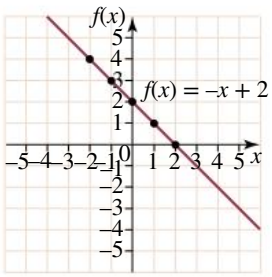
THINK

1. Write the rule.
2. Draw a table and choose simple x-values.
3. Use the rule to find each function value and enter them in the table.
   Remember that $-x$ means $-1x$.
   When $x = -2, f(-2) = -1 \times -2 + 2 = 4$.
   When $x = -1, f(-1) = -1 \times -1 + 2 = 3$.
   When $x = 0, f(0) = -1 \times 0 + 2 = 2$.
   When $x = 1, f(1) = -1 \times 1 + 2 = 1$.
   When $x = 2, f(2) = -1 \times 2 + 2 = 0$.
4. Draw a Cartesian plane and plot the points.
5. Join the points to form a straight line and label the graph.

WRITE/DRAW

$f(x) = -x + 2$

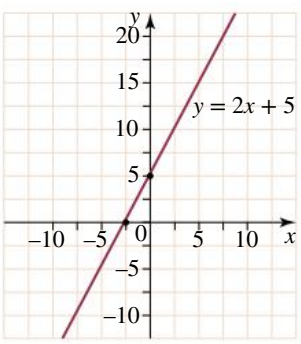
x	-2	-1	0	1	2
$f(x)$	4	3	2	1	0

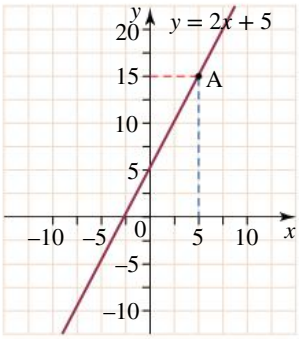


- Linear graphs can also be used to solve linear equations.

WORKED EXAMPLE 7

Use the following graph to solve the linear equation $2x + 5 = 15$.



THINK	WRITE/DRAW
1 Rule a horizontal line (red) at $y = 15$. This is the right-hand side of the original equation. This line meets the graph at point A. Rule a vertical line (blue) from point A to the x-axis. The line meets the x-axis at 5.	
2 The solution to the linear equation $2x + 5 = 15$ is $x = 5$.	The solution to the linear equation $2x + 5 = 15$ is $x = 5$.

assess on

Exercise 14.4 Plotting linear graphs

REFLECTION
How many points are needed to plot a linear graph?

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–11, 13	Questions: 1–11, 13–15	Questions: 1–16

Individual pathway interactivity int-4465 eBookplus

FLUENCY

eBookplus
Digital doc
SkillSHEET
Substitution into rules
doc-6937
SkillSHEET
Completing a table of values for a given rule
doc-6938
SkillSHEET
Plotting a line using a table of values
doc-6939

1 WE5 Complete the following tables of values, plot the points on a Cartesian plane, and join them to make a linear graph. Label the graphs with the rules.

a Rule: $y = x + 3$

x	−2	−1	0	1	2
y	1	2	3		

b Rule: $f(x) = x - 5$

x	−2	−1	0	1	2
$f(x)$		−6			−3

c Rule: $y = 5x$

x	−2	−1	0	1	2
y	−10		0		

d Rule: $f(x) = 2x + 4$

x	−2	−1	0	1	2
$f(x)$	0			6	

e Rule: $y = 3x + 2$

x	−2	−1	0	1	2
y					

f Rule: $f(x) = 2x - 2$

x	−2	−1	0	1	2
$f(x)$		−4			

g Rule: $y = 4x - 3$

x	−2	−1	0	1	2
y					

h Rule: $f(x) = -3x + 2$

x	−2	−1	0	1	2
$f(x)$		5			−4

2 a MC $y = 3x + 4$ means:

- **A** The y-value equals the x-value, add 3 and then times 4.
- **B** The y-value equals the x-value times 3 and then add 4.
- **C** The x-value equals the y-value times 3 and then add 4.
- **D** The y-value equals 4 times the x-value divided by 3.
- **E** The x-value equals 4 times the y-value and then add 3.

b A table of values shows:

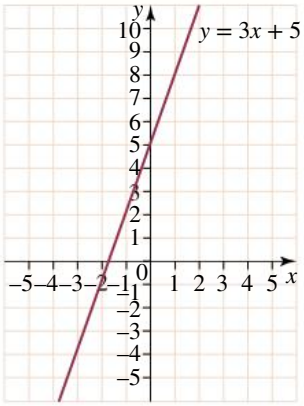
A a rule **B** coordinates **C** a linear graph
D an axis **E** that a rule continues forever

3 **WE6** Draw a table of values and plot the graph for each of the following rules. Label each graph.

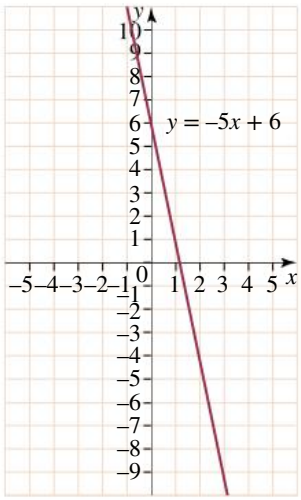
a $y = x + 2$ **b** $f(x) = x - 4$ **c** $y = x - 1$ **d** $f(x) = x + 5$
e $y = 3x$ **f** $f(x) = 7x$ **g** $y = 4x + 1$ **h** $f(x) = 2x - 3$
i $y = 3x - 5$ **j** $f(x) = -2x$ **k** $y = -6x + 2$ **l** $f(x) = -5x + 4$

4 **WE7** For each of the following, use the graph shown to solve the linear equation given.

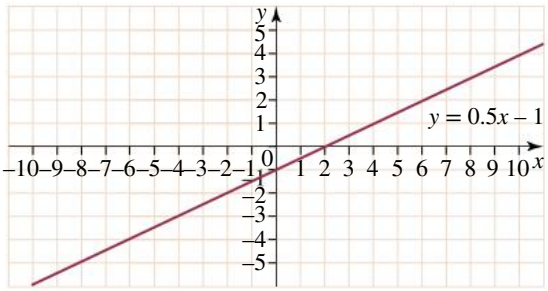
a $3x + 5 = 8$



b $-5x + 6 = -9$



c $0.5x - 1 = -4$



UNDERSTANDING

5 Plot a graph of the following rules from the tables of values provided. Label the graphs. Then copy and complete the sentences.

a $y = 4$

x	−2	−1	0	1	2
y	4	4	4	4	4

For the rule $y = 4$, the y-value of all coordinates is _____.

b $y = 1$

x	−2	−1	0	1	2
y	1	1	1	1	1

For the rule $y = 1$, the y-value of all coordinates is _____.

c $y = -2$

v	−2	−1	0	1	2
y	−2	−2	−2	−2	−2

For the rule $y = -2$, the y-value of all coordinates is _____.

d $y = -5$

x	−2	−1	0	1	2
y	−5	−5	−5	−5	−5

For the rule $y = -5$, the y-value of all coordinates is _____.

eBookplus
Digital doc
Spreadsheet
Plotting graphs with tables
doc-2441

6 Make a table of values and plot the graph for each of the following rules.

a $y = 3$ **b** $y = 2$ **c** $y = -2$ **d** $y = -4$

7 Plot the graph of each of the following rules from the table of values provided. Label the graph. Then copy and complete the sentence.

a $x = 1$

x	1	1	1	1	1
y	−2	−1	0	1	2

For the rule $x = 1$, the x-value of all coordinates is _____.

b $x = 3$

x	3	3	3	3	3
y	−2	−1	0	1	2

For the rule $x = 3$, the x-value of all coordinates is _____.

c $x = -2$

x	−2	−2	−2	−2	−2
y	−2	−1	0	1	2

For the rule $x = -2$, the x-value of all coordinates is _____.

d $x = -7$

x	−7	−7	−7	−7	−7
y	−2	−1	0	1	2

For the rule $x = -7$, the x-value of all coordinates is _____.

8 Make a table of values; then plot and label the graph for each of the following.

a $x = 2$ **b** $x = 5$ **c** $x = -5$ **d** $x = 0$

REASONING

9 Draw a table of values and then graph each of these rules on the same Cartesian plane.

a $y = 2x$ **b** $y = 2x - 1$ **c** $y = 2x + 1$

Describe the relationship between these lines.

10 Draw a table of values and graph each of these rules on the same Cartesian plane.

a $f(x) = 3x + 1$ **b** $f(x) = -2x + 1$

What do you notice?

11 Draw a table of values and graph each of these rules on the same Cartesian plane.

a $y = -x$ **b** $y = x + 2$

What do you notice?

12 The monthly cost in dollars, C, of renting a mobile phone is given by the equation $C = 15 + 0.5x$, where x is the call time in minutes.

a Plot the graph of the equation.

b If July's bill was $100, what was your call time?

c If August's bill was only $50, what was your call time?

PROBLEM SOLVING

13 **a** Create a table of values that shows the cost of lengths of fabric in the photo at right from 0 to 8 metres.

b Plot a graph showing the cost of 0 to 8 metres of fabric.

c Is this a linear relationship? Explain.

14 When you make a call from a mobile phone, you usually pay a flagfall (charge for just making the call) and then you are charged for how long you talk.

One phone company offers 25 cents flagfall and 35 cents per 30 seconds.

a Create a table of values that shows the cost of a phone call between 0 and 5 minutes (in 30-second blocks).

b Plot a graph showing the cost of a phone call between 0 and 5 minutes.

15 Find the equation for each of the following tables of values. Complete the tables and plot the linear graphs on separate Cartesian planes.

a

x	−2	−1	0	1	2	3	4
y	−3		−2		−1	−0.5	0

b

x	0	10	20	30	40	50
y	5	7	9		13	

16 a Use the equation $y = 2x - 5$ to complete the table below and plot the linear graph.

b What did you do differently to find the values for this table?

x				2			
y	−13	−9	−5		3	7	11

CHALLENGE 14.1

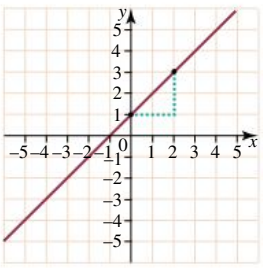
Kane's Taxi Service charges $2.50 for the first kilometre and $1.10 for each additional kilometre. If the fare was $47.60, how far did the taxi go?

14.5 The *y*-intercept and gradient

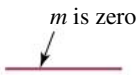
The gradient

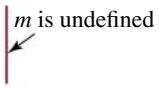
- **Gradient** gives a measure of steepness. For example, the gradient of a road going up the hill would inform us of the steepness of the slope of the hill.
- The gradient of a straight line shows the change in vertical height as the horizontal distance increases by one unit.
- The gradient of a straight line is denoted by the letter m and is given by the formula: $m = \frac{\text{rise}}{\text{run}}$, where *rise* is the vertical distance between any two points on the line and *run* is the horizontal distance between the same two points.
- To find the gradient of a straight line on a Cartesian plane, draw a right-angled triangle anywhere along the line as shown below and use it to measure *rise* and *run*.

  The value of the gradient is given by the ratio $\frac{\text{rise}}{\text{run}}$. For example, in the following diagram, rise = 2 and run = 2, so the gradient $m = \frac{2}{2} = 1$.

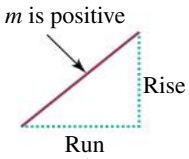


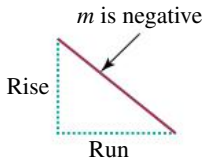
- The gradient of a straight line is constant; that is, it will be the same when measured anywhere along the line.
- Since the gradient is the measure of steepness, the greater the value of the gradient, the steeper the line.
- If the line is horizontal, there is no slope; hence the value of the gradient is zero. If the line is vertical, we say that its gradient is **infinite** or **undefined**.





- If the line slopes upwards from left to right (that is, it *rises*), the gradient is positive. If it slopes downwards from left to right (that is, it *falls*), the gradient is negative.





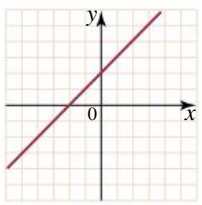
The *y*-intercept

- The *y*-intercept is the *y*-coordinate of the point where the graph crosses the *y*-axis.
- The symbol used to denote the *y*-intercept is *c*.
- The graph crosses the *y*-axis at the point (0, *c*).

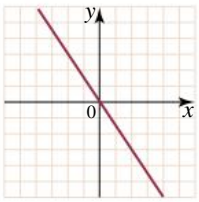
WORKED EXAMPLE 8

State whether these lines have a positive, negative, zero or undefined gradient.

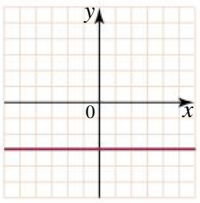
a



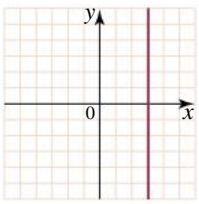
b



c



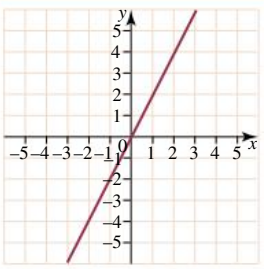
d

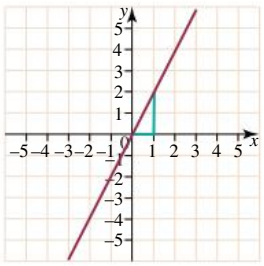


THINK	WRITE
a A line that rises from left to right, /, has a positive gradient.	**a** Positive gradient
b A line that drops from left to right, \, has a negative gradient.	**b** Negative gradient
c A line that is horizontal has a zero gradient.	**c** Zero gradient
d A line that is vertical has an undefined gradient.	**d** Undefined gradient

WORKED EXAMPLE 9

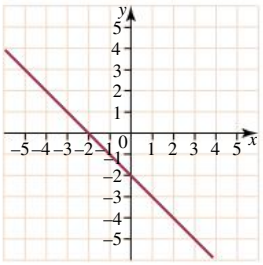
Find the gradient of the linear graph at right.

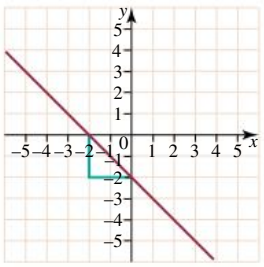


THINK	WRITE/DRAW
1 Choose two convenient points on the line and draw a triangle to find the rise and the run. *Note:* It does not matter which two points are chosen because the gradient of a straight line is constant.	
2 Read the rise and the run from the graph.	rise = 2, run = 1
3 Calculate the gradient.	Gradient: $m = \frac{\text{rise}}{\text{run}}$ $= \frac{2}{1}$ $= 2$

WORKED EXAMPLE 10

Find:
a the gradient, *m*
b the *y*-intercept, *c*, of the linear graph at right.



THINK	WRITE/DRAW
a 1 Choose two convenient points on the line and draw a triangle to find the rise and the run.	a 
	rise = −2, run = 2
2 Calculate the gradient. In this case the rise is negative since the line slopes down from left to right.	Gradient: $m = \frac{\text{rise}}{\text{run}} = \frac{-2}{2}$ $= -1$
b The *y*-intercept is where the graph crosses the *y*-axis, and *c* is the *y*-value at this point.	b $c = -2$

Finding the rule for the linear graph

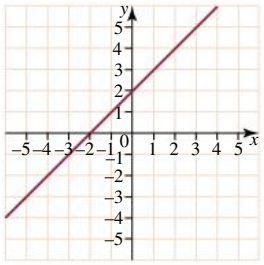
- The general rule for all linear graphs is given by the equation $y = mx + c$, where x and y are variables, m is the gradient and c is the y-intercept.
- To find the equation of the line from its graph:
  – select any two points on the line, measure run and rise between these points and find the gradient; this gives the value of m

– locate the y-intercept (the point where the line crosses the y-axis) and read its y-coordinate; this gives the value of c
– substitute the values of m and c into $y = mx + c$ to obtain the equation of the given line.

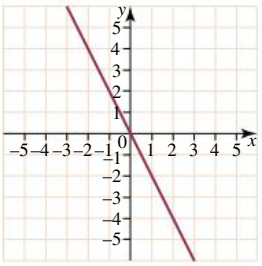
WORKED EXAMPLE 11

For each of the linear graphs shown, find the gradient, m, the y-intercept, c, and, using m and c, work out the rule.

a



b



THINK

a 1 Choose two convenient points on the line and draw a triangle to find the rise and run. Use the triangle formed by the axes.

2 Calculate the gradient.

3 State the value of c. The y-intercept is where the graph crosses the y-axis.

4 Substitute the values of m and c into the general rule.

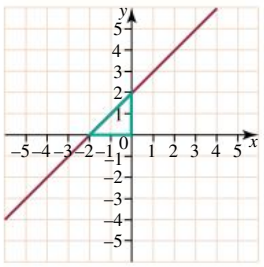
5 State the rule.

b 1 Choose two convenient points on the line and draw a triangle to find the rise and run. The rise is negative as the line slopes downwards from left to right.

2 Calculate the gradient.

WRITE/DRAW

a



Rise $= 2$, run $= 2$

Gradient: $m = \frac{\text{rise}}{\text{run}} = \frac{2}{2}$

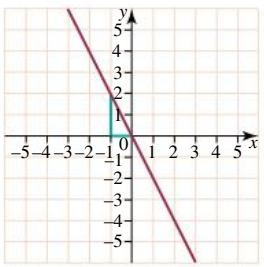
$= 1$

$c = 2$

$y = mx + c$
$= 1 \times x + 2$
$= x + 2$

The rule is $y = x + 2$.

b



Rise $= -2$, run $= 1$

Gradient: $m = \frac{\text{rise}}{\text{run}} = \frac{-2}{1}$

$= -2$

3	State the value of c. The y-intercept is where the graph crosses the y-axis. *Note:* The graph passes through the origin.	$c = 0$
4	Substitute the values of m and c into the general rule.	$y = mx + c$ $= -2 \times x + 0$ $= -2x$
5	State the rule.	The rule is $y = -2x$.

WORKED EXAMPLE 12

State the gradient and y-intercept for each of the following linear rules.
a $y = 2x - 3$ **b** $y = -x + 1$

	THINK	WRITE
a 1	Write the rule.	a $y = 2x - 3$
2	Compare with the general rule.	$y = mx + c$
3	The gradient is given by m.	Gradient: $m = 2$
4	The y-intercept is given by c.	y-intercept: $c = -3$
b 1	Write the rule.	b $y = -x + 1$
2	Compare with the general rule.	$y = mx + c$
3	The gradient is given by m.	Gradient: $m = -1$
4	The y-intercept is given by c.	y-intercept: $c = 1$

Exercise 14.5 The y-intercept and gradient

assessON

INDIVIDUAL PATHWAYS

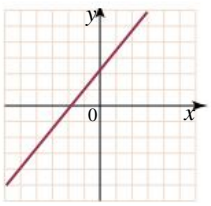
PRACTISE	CONSOLIDATE	MASTER
Questions: 1–11	Questions: 1–11, 13	Questions: 1, 2f–i, 3c–f, 4–5, 6c–f, 7g–l, 8–14

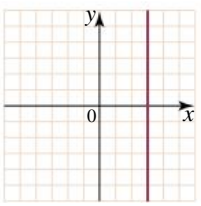
REFLECTION
How might positive and negative gradients be reflected in real-life situations?

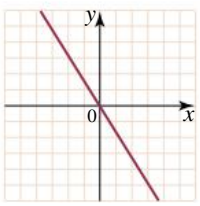
Individual pathway interactivity int-4466 eBookplus

FLUENCY

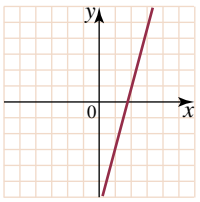
1 WE8 State whether each of the following lines has a positive, negative, zero or undefined gradient.

a 

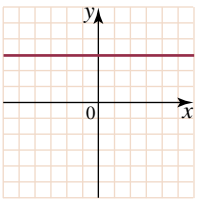
b 

c 

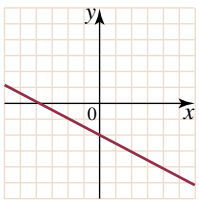
d



e

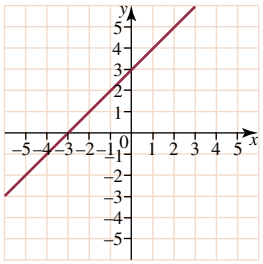


f

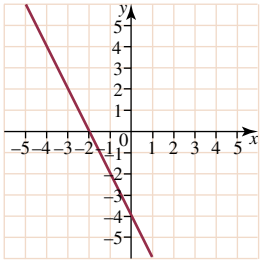


2 **WE9** Find the gradient of the following linear graphs.

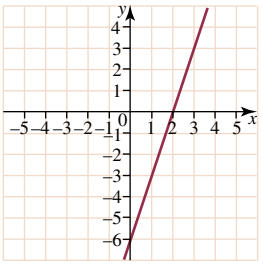
a



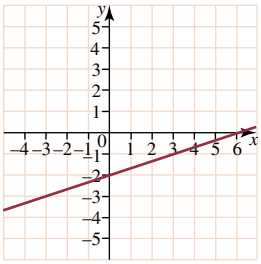
b



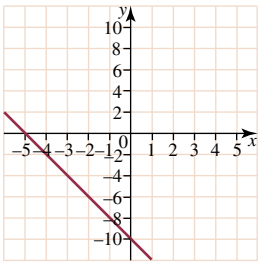
c



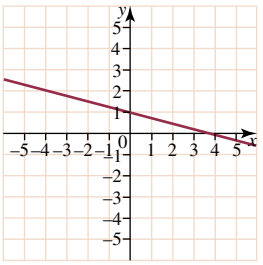
d



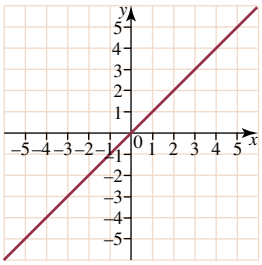
e



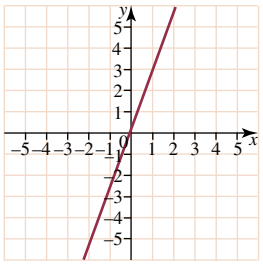
f



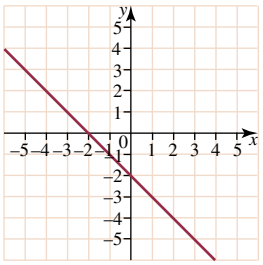
g



h



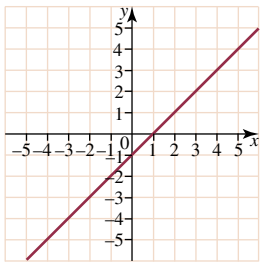
i



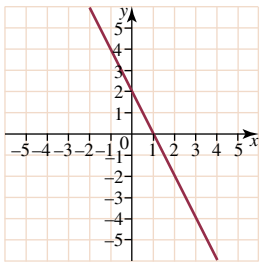
3 **WE10** For each of the following linear graphs, find:

i the gradient, m **ii** the y-intercept, c.

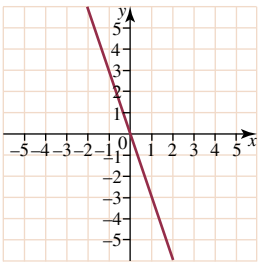
a



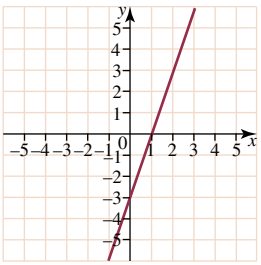
b



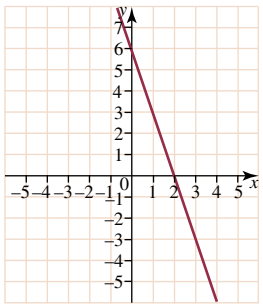
c



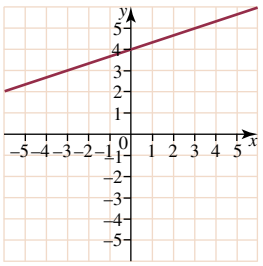
d



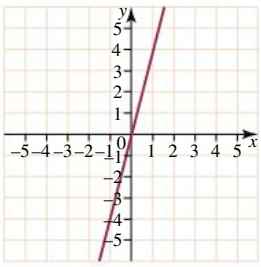
e



f

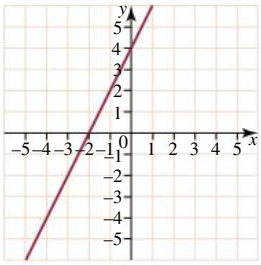


4 **MC** For each of these graphs, choose the correct alternative.



a The gradient of this graph is:

A $\frac{1}{4}$
B -4
C 4
D $-\frac{1}{4}$
E 0



b The gradient and y-intercept of this graph are, respectively:

A $2, 4$
B $-2, 2$
C $-2, 4$
D $2, 2$
E $-4, -2$

eBookplus
Digital doc
Spreadsheet
Plotting graphs with rules
doc-2443

5 **MC** Hugo has developed greater skills with his skateboard. Earlier he mastered a ramp that has a run of 2 m and a rise of 1.5 m. The ramp has a gradient of:

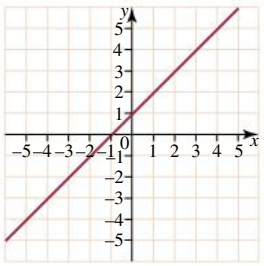
A 0.75
B 3
C 1.5
D $-\frac{3}{2}$
E 2

eBookplus
Digital doc
Spreadsheet
Gradient and intercepts
doc-2444

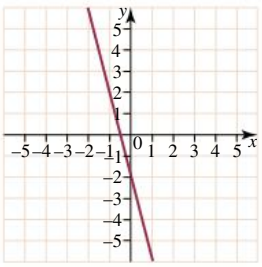
6 **WE11** For each of the following linear graphs:

i find the gradient, m
ii find the y-intercept, c
iii using m and c, work out the rule.

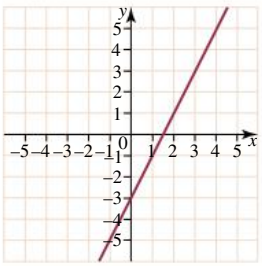
a



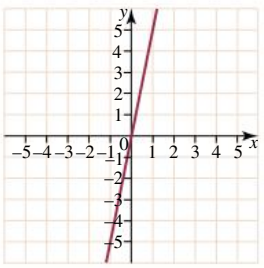
b



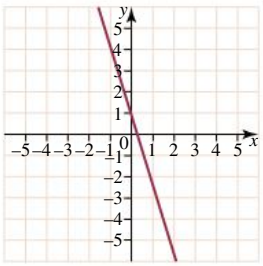
c



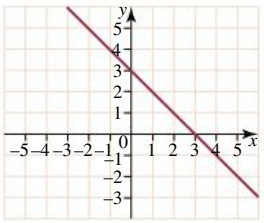
d



e



f

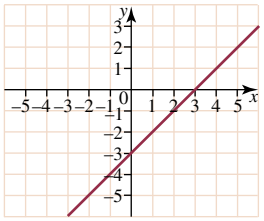


7 **WE12** State the gradient and y-intercept for each of the following linear rules.

a $y = x + 3$ **b** $y = x - 4$ **c** $y = 3x + 1$ **d** $y = 5x - 2$
e $y = 6x + 10$ **f** $y = 8x - 7$ **g** $y = 5x + 3$ **h** $y = 9x - 4$
i $y = -3x + 4$ **j** $y = -6x + 2$ **k** $y = -4x$ **l** $y = x$

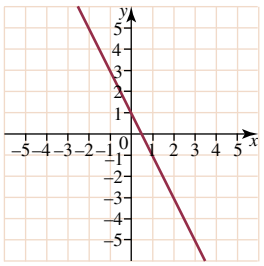
8 **MC** For each of the following graphs, choose the correct response.

a The rule for this linear graph is:



A $y = -3x$ **B** $y = x + 3$ **C** $y = 3x - 3$
D $y = x - 3$ **E** $y = x$

b The rule for this linear graph is:



A $y = -2x + 1$ **B** $y = -x + 1$ **C** $y = x + 1$
D $y = 2x + 1$ **E** $y = -\frac{1}{2}x + 1$

9 **a** **MC** A linear graph with the rule $y = 2x + 6$ would have:

A $m = 6, c = 2$ **B** $m = 2, c = 6$ **C** $m = -2, c = 6$
D $m = 1, c = 6$ **E** $m = 2, c = 2$

b A linear graph with the rule $y = 4x - 7$ would have:

A $m = -7, c = 4$ **B** $m = 7, c = 4$ **C** $m = 4, c = 4$
D $m = 4, c = 7$ **E** $m = 4, c = -7$

UNDERSTANDING

10 Chris's fridge is not working. He called a repair company, and they are sending someone to repair the fridge.
Company charges are: \$55 to come to your home, plus
\$45 for every $\frac{1}{2}$ hour the repairer is there.

a Copy and complete the table to show how much it could cost Chris to have his fridge repaired.

Time (hours)	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
Cost (\$)	55	100	145				

b Draw a graph of this information. Place time on the x-axis and cost on the y-axis.

c State the independent and dependent variables.

d Is there a linear relationship between cost and time?

e Find the gradient of the graph.

f Find the y-intercept.

g Find the rule for this graph using $y = mx + c$.

h In your equation replace y with cost and replace x with time:
Cost = ___ × time + ___.

i Use your equation to find the cost if a repair person takes 4 hours to fix Chris's fridge.

REASONING

11 Kyle was very bored on his holidays and decided to measure how much the grass grew every day for one week. His results are shown in the table.

Day number	0	1	2	3	4	5	6	7
Height of grass (mm)	10	12	14	16	18	20	22	24

a Kyle knew his dad would want the grass cut as soon as it was 2.5 cm (25 mm) long. On which day would this occur?

b Plot the points from the graph on a Cartesian plane putting day on the x-axis and height on the y-axis.

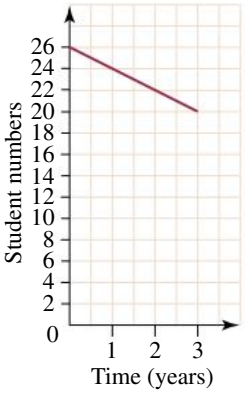
c Do the points form a linear graph?

d Find the gradient and y-intercept of the graph.

e Find an equation for the height of the grass by filling in the blanks. Height = ___ × no. of days + _____ or $h =$ ___ $d +$ _____

f How long would the grass be after 14 days if it had not been cut?

12 Samantha has noticed that there are fewer students in her home group now than when she started school 3 years ago. The graph at right shows how the number of students has changed.



a If the pattern continues, how many students will there be in Samantha's home group next year?

b Find the gradient and y-intercept of the graph.

c The equation for this graph would be:
Number of students = _____ × time + _____.
This could be shortened to $s =$ ____ $t +$ ____.
Fill in the blanks in the equation above with the gradient and y-intercept.

d Using the equation from part **c**, find the number of students in Samantha's home group after 6 years.

13 **a** If t represents the time in hours and C represents cost ($), construct a table of values for 0–3 hours for the cost of playing ten-pin bowling at the new alley.

> Save $$$ with Supa-Bowl!!!
> NEW Ten-Pin Bowling Alley
> *Shoe rental just $2 (fixed fee)*
> Rent a lane for ONLY $6/hour!

b Use your table of values to plot a graph of time versus cost. (*Hint:* Ensure your time axis (horizontal axis) extends to 6 hours and your cost axis (vertical axis) extends to $40.)

c **i** What is the y-intercept?

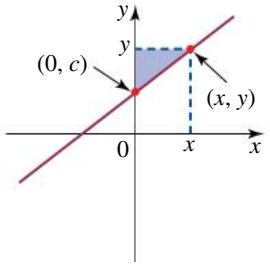
ii What does the y-intercept represent in terms of the cost?

d Calculate the gradient.

e Write a linear equation to describe the relationship between cost and time.

f Use your linear equation from part **e** to calculate the cost of a 5 hour tournament.

g Use your graph to check your answer to part **f**.



14 **a** Using the graph at right, write a general formula for the gradient m in terms of x, y and c.

b Transpose your formula to make y the subject. What do you notice?

CHALLENGE 14.2

A certain linear pattern has its Cartesian coordinates in fraction form as:
$(x, y)\ (\frac{1}{8}, \frac{7}{12})(\frac{1}{4}, \frac{5}{6})(\frac{3}{8}, \frac{13}{12})(\frac{1}{2}, \frac{4}{3})$.

Find the rule that satisfies these Cartesian coordinates.

14.6 Sketching linear graphs

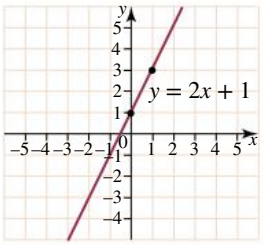
- To sketch a straight-line graph only two points are needed.
- The two points can be obtained by substituting any two x-values into the rule to get the corresponding y-values. (The same process was used to obtain the table of values to plot the line.)
- There are two other convenient techniques of sketching straight lines: (a) gradient and y-intercept method and (b) intercept method. These are discussed below.

Sketching linear graphs using gradient and y-intercept method

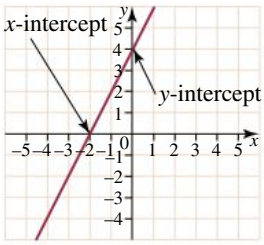
- To sketch the line using the gradient-intercept method, follow these steps.
  - Identify the values of the gradient and the y-intercept from the equation; write the gradient in the form $m = \frac{\text{rise}}{\text{run}}$.
  - Plot the y-intercept; this is your first point.
  - To locate the second point, begin with the y-intercept and move up (or down) the number of units suggested by the rise, and then move forward the number of units suggested by the run.
  - Join the two points with the straight line; label your graph.

WORKED EXAMPLE 13

Sketch the graph of $y = 2x + 1$ using the gradient and y-intercept method.

THINK	WRITE/DRAW
1 Write the equation and compare with the general equation $y = mx + c$.	$y = 2x + 1 \quad m = 2, c = 1$
2 Plot a point at the y-intercept.	
3 Write the gradient as a fraction to identify the rise and the run.	$m = \frac{2}{1}$ So, rise = 2 and run = 1.
4 From the y-intercept at 1, move up 2 units and across 1 to plot the second point.	
5 Join the 2 points to form a straight line. Extend and label the line.	

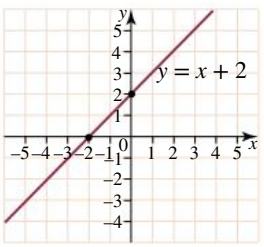
Intercept method



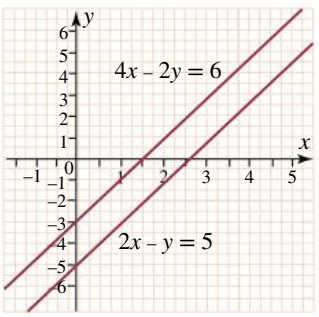
- The intercept method involves finding the coordinates of both the x- and the y-intercepts, and plotting them.
- At the x-intercept, $y = 0$. Therefore, to obtain the value of the x-intercept, substitute $y = 0$ into the equation and solve the equation for x.
- The y-intercept can be obtained directly from the equation (it is the value of c). Alternatively, at the y-intercept, $x = 0$. Therefore, to obtain the value of the y-intercept, substitute $x = 0$ into the equation and evaluate.
- To sketch the graph, plot both intercepts on the Cartesian axes and join them with the straight line. Label the line.

WORKED EXAMPLE 14

Sketch the graph of $y = x + 2$ using the intercept method.

THINK	WRITE/DRAW
1 Write the equation.	$y = x + 2$
2 At the y-intercept, $x = 0$. To find it, substitute $x = 0$ into the equation.	At y-intercept, $x = 0$ $y = 0 + 2$ $= 2$ y-intercept $(0, 2)$
3 At the x-intercept, $y = 0$. To find it, substitute $y = 0$ into the equation.	At x-intercept, $y = 0$ $0 = x + 2$
4 Rearrange the equation so that x is on the left-hand side.	$x + 2 = 0$
5 Subtract 2 from both sides of the equation.	$x + 2 - 2 = 0 - 2$ $x = -2$ x-intercept $(-2, 0)$
6 Plot the 2 intercepts and join them to form a straight line. Label the line.	

Parallel lines



- If two or more lines have the same slope — that is, their gradients are equal — then the lines are parallel. For example, the lines $y = 3x - 2$; $y = 3x + 6$; $y = 3x - 80$ are all parallel, each having a gradient of 3.
- Now consider the following lines.
  $2x - y = 5$ [1]
  $4x - 2y = 6$ [2]
- By using the intercept method we obtain the graph shown.

- We can confirm that the two lines are in fact parallel by rearranging each equation into the form $y = mx + c$ and checking the gradient of each.

$$\begin{aligned} 2x - y &= 5 \quad [1] & 4x - 2y &= 6 \quad [2] \\ -y &= 5 - 2x & -2y &= 6 - 4x \\ -y &= -2x + 5 & -2y &= -4x + 6 \\ y &= 2x - 5 & y &= 2x - 3 \end{aligned}$$

Gradient $m = 2$ Gradient $m = 2$

assess on

Exercise 14.6 Sketching linear graphs

REFLECTION
Is one method of sketching linear graphs easier than the other methods?

INDIVIDUAL PATHWAY INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a, d, g, j, 2a, d, g, j, 3–6	Questions: 1b, e, h, k, 2b, e, h, k, 3–6, 8, 9	Questions: 1c, f, i, l, 2c, f, i, l, 3–10

Individual pathway interactivity int-4467 eBookplus

FLUENCY

1 WE13 Sketch and label the following graphs using the gradient and y-intercept method.

a $y = x + 1$ **b** $y = x + 3$ **c** $y = x - 3$
d $y = x - 2$ **e** $y = 2x + 2$ **f** $y = 2x - 1$
g $y = 4x - 2$ **h** $y = 6x - 4$ **i** $y = -x + 4$
j $y = -x + 2$ **k** $y = -x - 5$ **l** $y = -2x + 3$

2 WE14 Sketch and label the following graphs using the intercept method.

a $y = x + 3$ **b** $y = x + 6$ **c** $y = x - 4$ **d** $y = x - 5$ **e** $y = x + 10$
f $y = x - 7$ **g** $y = x + 1$ **h** $y = x - 8$ **i** $y = x + 8$ **j** $y = x + 9$
k $y = -x + 1$ **l** $y = -x - 4$

eBookplus
Interactivity
Graphs of linear functions
int-2379

3 a MC The gradient and y-intercept of the graph given by $y = 3x - 5$ is:

A $m = 3, c = 5$ **B** $m = 5, c = 3$ **C** $m = -3, c = 5$
D $m = 3, c = -5$ **E** $m = -5, c = 3$

b If $m = -6$ and $c = 4$, the rule for the linear graph would be:

A $y = 4x - 6$ **B** $y = -6x + 4$ **C** $y = 6x - 4$
D $y = -4x + 6$ **E** $y = -6x - 4$

4 a MC The x- and y-intercepts for the linear graph whose rule is $y = x + 9$ are:

A 9 and 9 **B** 9 and −9 **C** −9 and 9
D 0 and 9 **E** 0 and −9

b The x- and y-intercepts for the linear graph whose rule is $y = x - 15$ are:

A 15 and −15 **B** −15 and −15 **C** 15 and 0
D 15 and 15 **E** 0 and 15

UNDERSTANDING

5 For the graph of the linear equation $y = 3x - 6$, show that:

a the y-intercept is −6
b the x-intercept is 2
c the gradient is 3 (using your answers to parts **a** and **b**).

REASONING

6 A linear graph has a y-intercept of 6 and a gradient of -2.

- **a** What is its equation?
- **b** What is the value of the x-intercept?

7 A linear graph has an x-intercept of p and a y-intercept of q.

- **a** What is its equation?
- **b** What is the value of the gradient?

PROBLEM SOLVING

8 The cost, C, in dollars, of a visit to an amusement park is given by the equation

$$C = 2R + 15$$

where R is the number of rides that you go on.

- **a** Sketch a graph representing the cost (y-axis) against the number of rides (x-axis).
- **b** What is the entry fee for this amusement park?
- **c** What is the cost for each ride you take?
- **d** What does it cost, in total, if you go on five rides?

Stay all day: $C(\$) = 2R + 15$

9 The total time, T, in minutes, that it takes a Year 8 student to complete a Maths homework sheet consisting of 20 questions is given by the equation

$$T = 4.5Q + 1.5$$

where Q is the number of questions that a student answers.

- **a** Sketch a graph, with Q on the x-axis and T on the y-axis.
- **b** How long does it take to complete all 20 questions?
- **c** What is the possible significance of the number 1.5 in the equation given?

10 **a** Sketch the linear equation $y = -\frac{5}{7}x - \frac{3}{4}$:

- **i** using the y-intercept and the gradient
- **ii** using the x- and y-intercepts
- **iii** using two other points.

- **b** Compare and contrast the methods and generate a list of advantages and disadvantages for each method. Which method do you think is best? Why?

eBookplus
Digital doc
WorkSHEET 14.2
doc-2417

14.7 Solving equations graphically

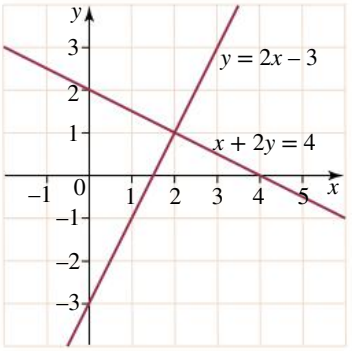
- Any two linear graphs will meet at a point unless they are parallel. (That is, they have the same gradient.) It is at the point where the two graphs intersect that the two equations share the same x- and y-coordinates. This point is the solution to the two equations.
- Pairs of equations may be solved graphically.

- This method involves drawing the graph of each equation on the same set of axes, and then finding the point of intersection of these lines.
- The point of intersection is the point that is common to both equations and is therefore a solution to both the equations.
- When solving equations graphically, obtaining an accurate solution depends on drawing accurate graphs. For this reason, it is best to use graph paper or graphing software.

WORKED EXAMPLE 15

Use the graphs of the given straight lines at right to determine the point of intersection and, hence, the solution of the equations.

$x + 2y = 4$
$y = 2x - 3$



THINK	WRITE
1 Write the equations and number them.	$x + 2y = 4$ [1] $y = 2x - 3$ [2]
2 Locate the point of intersection of the two lines. This gives the solution.	Point of intersection (2, 1) Solution: $x = 2$ and $y = 1$
3 Check the solution by substituting $x = 2$ and $y = 1$ into the given equations. Comment on the results obtained.	Check equation [1]: LHS $= x + 2y$ RHS $= 4$ $= 2 + 2(1)$ $= 4$ LHS = RHS Check equation [2]: LHS $= y$ RHS $= 2x - 3$ $= 1$ $= 2(2) - 3$ $= 4 - 3$ $= 1$ LHS = RHS In both cases LHS = RHS, therefore the solution set (2, 1) is correct.

WORKED EXAMPLE 16

Solve the following pair of linear equations using a graphical method to find their point of intersection.

$x + y = 6$
$2x + 4y = 20$

THINK	WRITE/DRAW
1 Write the equations, one under the other and number them.	$x + y = 6$ [1] $2x + 4y = 20$ [2]

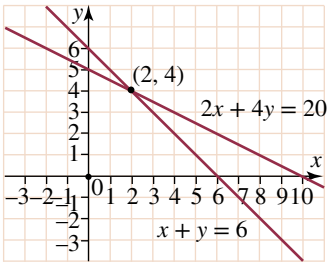
2 Calculate the x- and y-intercepts for equation [1].
For the x-intercept, substitute $y = 0$ into equation [1].
For the y-intercept, substitute $x = 0$ into equation [1].

Equation [1]
x-intercept: when $y = 0$,
$x + 0 = 6$
$x = 6$
The x-intercept is at (6, 0).
y-intercept: when $x = 0$,
$0 + y = 6$
$y = 6$
The y-intercept is at (0, 6).

3 Calculate the x- and y-intercepts for equation [2].
For the x-intercept, substitute $y = 0$ into equation [2].
Divide both sides by 2.
For the y-intercept, substitute $x = 0$ into equation [2].
Divide both sides by 4.

Equation [2]
x-intercept: when $y = 0$,
$2x + 0 = 20$
$2x = 20$
$x = 10$
The x-intercept is at (10, 0).
y-intercept: when $x = 0$,
$0 + 4y = 20$
$4y = 20$
$y = 5$
The y-intercept is at (0, 5).

4 Use graph paper to rule up a set of axes and label the x-axis from -3 to 10 and the y-axis from -3 to 6.



5 Plot the x- and y-intercepts for each equation.

6 Produce a graph of each equation by ruling a straight line through its intercepts.

7 Label each graph.

8 Locate the point of intersection of the lines.

The point of intersection is (2, 4).

9 Check the solution by substituting $x = 2$ and $y = 4$ into each equation.

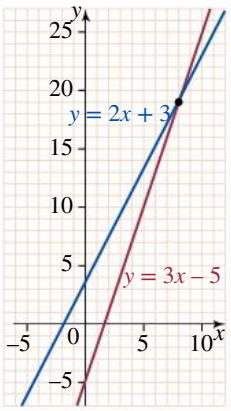
Check [1]: LHS $= x + y$ RHS $= 6$
$= 2 + 4$
$= 6$
LHS $=$ RHS
Check [2]: LHS $= 2x + 4y$ RHS $= 20$
$= 2(2) + 4(4)$
$= 4 + 16$
$= 20$
LHS $=$ RHS

10 State the solution.

In both cases, LHS = RHS. Therefore, the solution set (2, 4) is correct.
The solution is $x = 2$, $y = 4$.

WORKED EXAMPLE 17

Solve the equation $2x + 3 = 3x - 5$ graphically.

THINK	DRAW/WRITE
1 Sketch the graphs of $y = 2x + 3$ and $y = 3x - 5$ on the same set of axes. (See Worked example 16.)	
2 Write the equations and number them.	$y = 3x - 5$ [1] $y = 2x + 3$ [2]
3 Locate the point of intersection of the two lines. This gives the solution.	Point of intersection (8, 19) Solution: $x = 8$ and $y = 19$
4 Check the solution by substituting $x = 8$ and $y = 19$ into each equation.	Check [1]: LHS = 19 RHS $= 3x - 5$ $= 3(8) - 5$ $= 24 - 5$ $= 19$ LHS = RHS Check [2]: LHS =19 RHS $= 2x + 3$ $= 2(8) + 3$ $= 16 + 3$ $= 19$ LHS = RHS
5 State the solution.	In both cases LHS = RHS, therefore. Therefore, the solution set (8, 19) is correct. The solution is $x = 8$, $y = 19$.

assess on

Exercise 14.7 Solving equations graphically

REFLECTION
Why is it that only two points are necessary to sketch a straight line?

INDIVIDUAL PATHWAYS

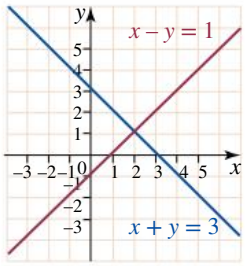
PRACTISE	CONSOLIDATE	MASTER
Questions: 1–5, 7, 9, 12, 13	Questions: 1–8, 11–13	Questions: 1c–f, 2c–e, 3c–f, 4–14

Individual pathway interactivity int-4468 eBookplus

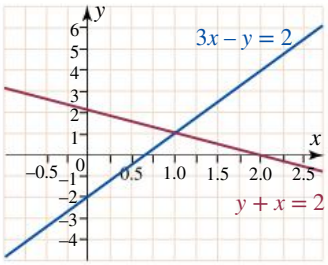
FLUENCY

1 **WE15** For each of the following graphs, determine the coordinates of the point of intersection.

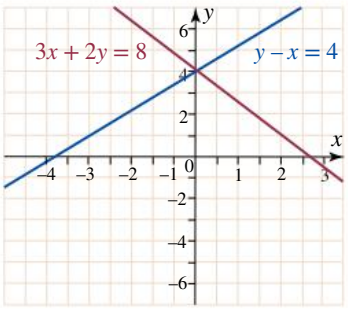
a



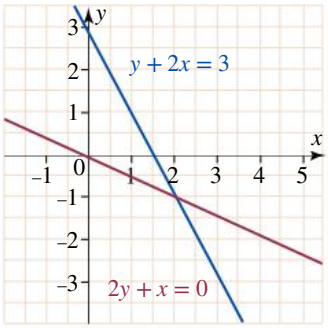
b



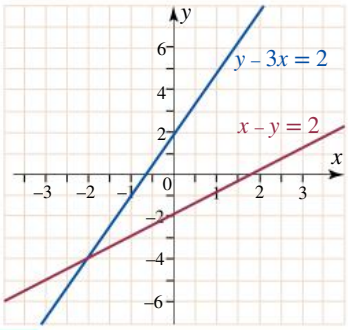
c



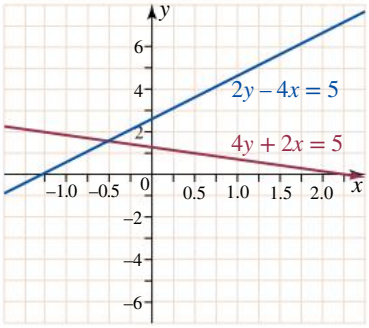
d



e



f



2 **WE16** Solve each of the following pairs of equations by graphing the equations and identifying the point of intersection.

a $y = 2x + 3$
$y = 8 - 3x$

b $y + 2x = -8$
$y = 2x + 4$

c $2y = 12x + 16$
$3y = -6x - 24$

d $2x - y = -1$
$3x + y = 11$

e $y = \frac{2x}{3} + 2$
$y = 2x - 2$

3 For each pair of equations below, calculate the gradients and indicate whether the lines are parallel.

a $y = 2x - 4$
$6y - 12x = 20$

b $2y = 5 - 6x$
$3y = -9x + 18$

c $3y + 2x = 9$
$6y + 4x = 18$

d $y = -2x + 3$
$y = \frac{1}{2}x - 5$

e $y = 4x + 5$
$2y - 15x = 10$

f $4y = 6 - x$
$y = 4x + 6$

4 Draw the graphs of the equations in question **3e** and use the graphs to then solve the equations.

5 **a** Identify the gradient and y-intercept of the line $2x - y = 4$.

b Write another equation for a line that is:

i not parallel to the line in part **a**.

ii parallel to the line in part **a**.

UNDERSTANDING

6 Solve the linear equations $y + x = 10$ and $-2y + x = -5$ graphically.

7 WE17 Solve the equation $3x - 7 = -2x + 3$ graphically.

8 Use the graph of the line $y = -2x - 3$ to solve the equation $-2x + 3 = 0$.

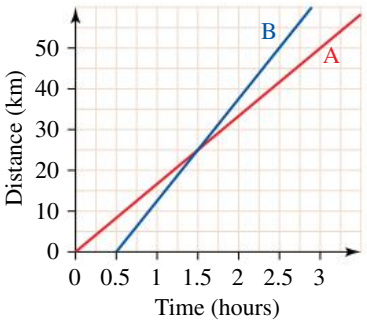
9 Use the graph of the line $y = 6x$, to solve the equation $6x + 1 = 0$.

10 Solve the equation $\frac{x}{3} + 1 = 2 - \frac{2x}{5}$ graphically.

REASONING

11 A triangle is formed by the graphs of $x + y = 13$, $x + 2y = 9$ and $2x + y = 9$. Determine the vertices of the triangle by graphing the equations to identify the coordinates of the points at which each pair of lines cross.

12 The photo below shows two friends during a race. The graph below right shows the distance in kilometres covered by both friends in a given time in hours. At what time in the race was the photo taken?



PROBLEM SOLVING

13 A fire truck and an ambulance have been called to an accident scene. The ambulance starts at the hospital; the equation of the distance it travels with respect to time is $d = 1.8t$. The fire truck leaves the garage and travels following the equation $d = 1.5t + 3.6$. In both equations, d is the distance travelled in kilometres, and t represents the time in minutes.

- **a** Plot the two graphs on the same set of axes with d on the vertical axis and t on the horizontal axis.
- **b** How far from the hospital was the ambulance after 5 minutes?
- **c** If the two vehicles meet at the scene of the accident, how long did it take the two vehicles to arrive?
- **d** What distance did the two vehicles travel to the scene of the accident?

14 You are given some money for your birthday and decide to save a certain amount per week to buy a bike. This information is represented in the equation $A = 17.5w + 110$, where A is the amount in dollars and w is the number of weeks.

- **a** Sketch the graph of the equation.
- **b** Use the graph to determine how much you will have saved after 10 weeks.
- **c** How much money did you get for your birthday?
- **d** How many weeks would it take for you to save a total of $670?
- **e** If the bike costs $800, how long will it take you to save for it at this rate?

eBookplus ONLINE ONLY

14.8 Review

www.jacplus.com.au

The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:

- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

Review questions

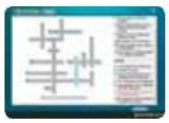
Download the Review questions document from the links found in your eBookPLUS.

eBookplus

Interactivities

Word search:
int-2639

Crossword:
int-2640



Sudoku:
int-3193

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

Cartesian plane
coordinates
function notation
general rule
gradient
infinite gradient
intercept method
linear functions
negative gradient
plotting linear graphs
positive gradient
rise
run
table of values
undefined gradient
y-intercept
zero gradient

Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

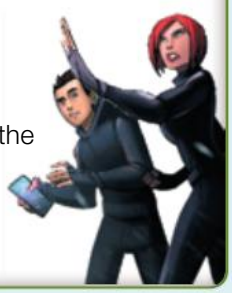
assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

www.assesson.com.au

Link to SpyClass, an exciting online game combining a comic book–style story with problem-based learning in an immersive environment.

Join Jesse, Toby and Dan and help them to tackle some of the world's most dangerous criminals by using the knowledge you've gained through your study of mathematics.

www.spyclass.com.au



RICH TASK

Choosing the right hire car

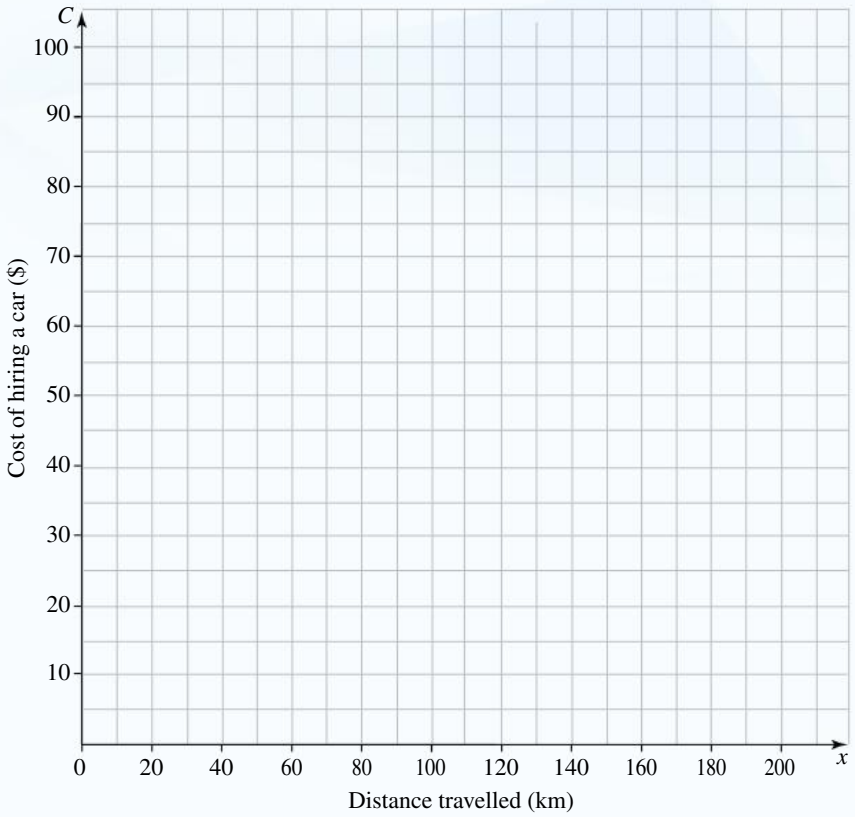
A group of tourists have just arrived at Sydney airport and are investigating the best hire car deals. They decide to study the different options offered by Bonza Car Rentals.

Option 1	$60 per day unlimited kilometres
Option 2	$30 per day and 25 cents/km
Option 3	$40 per day and 35 cents for every kilometre over 100 each day

The group know that on their first day they will be visiting the local attractions close to Sydney, so they will not be travelling many kilometres.

1. How much would each option cost if the total kilometres travelled in a day was 90 km?
2. Write an equation to show the cost of hiring a car for a day for options 1 and 2. Use C to represent the total cost of hiring a car for a day and x to represent the number of kilometres travelled in a day.

3 Plot the graphs of the three options on the set of axes provided to show the cost of hiring a car for a day to travel 200 km. Use different colours to represent the three options.



4 Examine the graphs of the three options carefully. Write a brief statement to explain the costs associated with each option over 200 km.

5 How much does it cost to travel 200 km in one day with each option?

The group decide to drive to Melbourne and plan to spend 2 days travelling there, stopping overnight in Canberra. On arrival in Melbourne, they plan to stay for 4 days and return to Sydney on the seventh day, driving the whole distance in one day and returning the car to Bonza Car Rentals.

6 In their 4 days in Melbourne, the group travelled a total of 180 km. What was the total distance travelled for this trip?

7 Work out how much it would cost to hire a car for each option to cover this trip. (Option 3 gives 100 km 'free travel' per day. For a 7-day hire, you get 700 km 'free'.)

8 The group took option 1. Was this the best deal? Comment.

9 Investigate some different companies' rates for car hire. On a separate page, draw graphs to represent their rates and present your findings to the class.

CODE PUZZLE

The first African country to win an Olympic gold medal!

Join the dots between each table of values and its rule with a straight line. Each line will pass through a letter and a number giving the puzzle's code.

x	−6	−4	−2	0	2
y	−2	−1	0	1	2

x	−3	−2	−1	0	1	2	3
y	0	1	2	3	4	5	6

$y = -x$

$y = 3x + 2$

$y = 2x + 1$

B

D

4

T

x	1	2	3	4	5	6
y	6	7	8	9	10	11

x	−3	−1	1	3	5
y	3	1	−1	−3	−5

$y = 6x$

$y = 2x$

x	−2	−1	0	1	2
y	−5	−3	−1	1	3

5

Q

7

S

13

x	−3	−2	−1	0
y	1	2	3	4

N

$y = \frac{1}{2}x + 1$

$y = x - 2$

14

x	2	3	4
y	0	1	2

G

12

10

x	0	1	2
y	0	6	12

1

3

E

$y = \frac{1}{2}x$

$y = x - 3$

2

x	−2	−1	0	1	2
y	−6	−3	0	3	6

9

x	−2	−1	0	1	2
y	−3	−1	1	3	5

O

Z

6

$y = 2x - 1$

$y = x + 4$

8

x	−2	−1	0	1	2
y	−4	−1	2	5	8

11

H

L

$y = x + 5$

x	−2	−1	0	1	2
y	−4	−2	0	2	4

A

R

x	−2	−1	0	1	2
y	−5	−4	−3	−2	−1

x	2	4	6
y	1	2	3

$y = x + 3$

$y = 3x$

2	5	5	2	9	7	3	3	7	8	3	12

8	10	3	6	14	1	7	13	11	4	1

eBookplus Activities

14.2 The Cartesian plane

Digital docs
- SkillSHEET (doc-6934) Grid coordinates I
- SkillSHEET (doc-6935) Grid coordinates II
- SkillSHEET (doc-6936) Plotting coordinate points

Interactivity
- IP interactivity 14.2 (int-4463) The Cartesian plane

14.3 Linear patterns

Digital docs
- Spreadsheet (doc-2439) Plotting points
- WorkSHEET 14.1 (doc-2416)

Interactivity
- IP interactivity 14.3 (int-4464) Linear patterns

14.4 Plotting linear graphs

Digital docs
- SkillSHEET (doc-6937) Substitution into rules
- SkillSHEET (doc-6938) Completing a table of values for a given rule
- SkillSHEET (doc-6939) Plotting a line using a table of values
- Spreadsheet (doc-2441) Plotting graphs with tables

Interactivity
- IP interactivity 14.4 (int-4465) Plotting linear graphs

14.5 The *y*-intercept and gradient

Digital docs
- Spreadsheet (doc-2443) Plotting graphs with rules
- Spreadsheet (doc-2444) Gradient and intercepts

Interactivity
- IP interactivity 14.5 (int-4466) The *y*-intercept and gradient

14.6 Sketching linear graphs

Digital doc
- WorkSHEET 14.2 (doc-2417)

Interactivities
- Graphs of linear functions (int-2379)
- IP interactivity 14.6 (int-4467) Sketching linear graphs

14.7 Solving equations graphically

Interactivity
- IP interactivity 14.7 (int-4468) Solving equations graphically

14.8 Review

Interactivities
- Word search (int-2639)
- Crossword (int-2640)
- Sudoku (int-3193)

Digital docs
- Topic summary (doc-10765)
- Concept map (doc-10778)
- Topic review (Word doc-14948, PDF doc-14949)

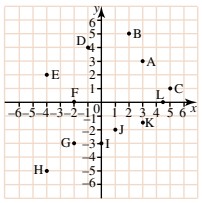
To access eBookPLUS activities, log on to www.jacplus.com.au

Answers

TOPIC 14 Coordinates and linear graphs

14.2 The Cartesian plane

1 a–l



2 A(2, 2) B(4, 6) C(6, 1) D(−2, 3) E(0, 5) F(1, 1) G(−4, 1) H(0, 0) I(5, 0) J(4, 3) K(−6, 3) L(−4, −2) M(−2, −5)

3 a E **b** A, C

4–8 Check with your teacher.

9 a A (3, 2), B (−5, −1), C (4, −7)

b A (−2, 1), B (−10, −2), C (−1, −8)

10 P(−2, 1), Q(1, 5), R(−5, 6), S(3, −1)

11 P$(a, -b)$ ⇒ P$(-a, -b)$ ⇒ P(a, b)

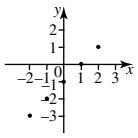
12 Check with your teacher.

13 $(4 - b, a)$

14 $(-r, -q)$ ⇒ (r, q) ⇒ $(-r, q)$; $(-r, -q)$ ⇒ $(-r, q)$

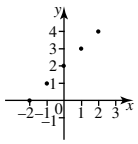
14.3 Linear patterns

1 a



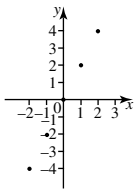
Linear

b



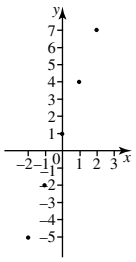
Linear

c



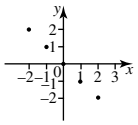
Linear

d



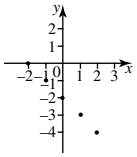
Linear

e



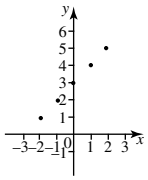
Linear

f

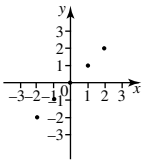


Linear

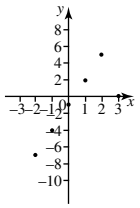
2 a



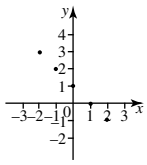
b



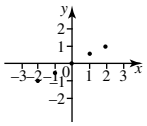
c



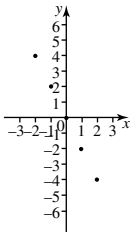
d



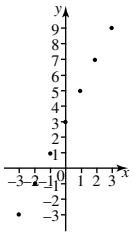
e



f

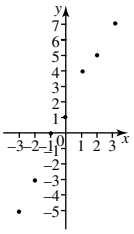


3 a



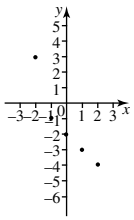
Yes (4, 11)

b



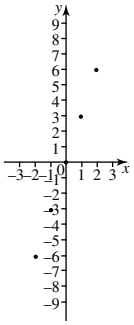
No

c



No

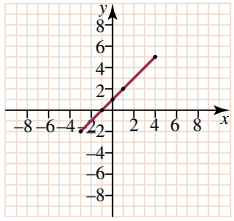
d



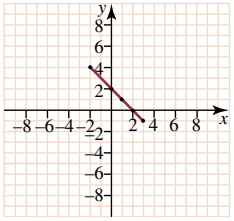
Yes(3, 9)

4 a D **b** B **c** C **d** A, B, E

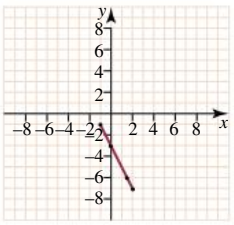
5 a



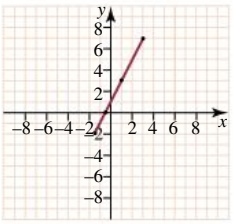
b



c



d



6 Check with your teacher.

7 a

Day (d)	Distance (D)
Monday (1)	1 km
Tuesday (2)	1.25 km
Wednesday (3)	1.5 km
Thursday (4)	1.75 km
Friday (5)	2 km
Saturday (6)	2.25 km
Sunday (7)	2.5 km

b Linear
c 0.25 km per day
d 3.25 km

8 a **i** 12 minutes **ii** 24 minutes **iii** 82 minutes

b

q	0	1	2	3	4	5	6	7	8	9	10
t	0	12	14	36	48	60	82	94	106	118	130
q	11	12	13	14	15	16	17	18	19	20	
t	152	164	176	188	200	222	234	246	258	270	

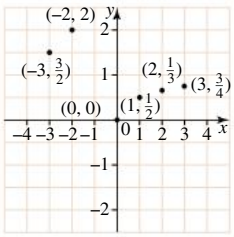
c Not linear

9 22 February of the following year

10 a

x	−3	−2	−1	0	1	2	3
y	$\frac{3}{2}$	2	Undefined	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$

b

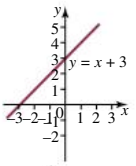


c Not linear

14.4 Plotting linear graphs

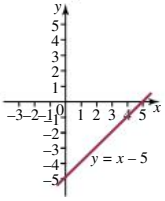
1 a

x	−2	−1	0	1	2
y	1	2	3	4	5



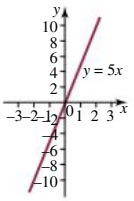
b

x	−2	−1	0	1	2
$f(x)$	−7	−6	−5	−4	−3



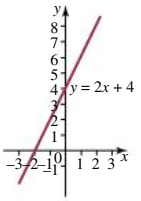
c

x	−2	−1	0	1	2
y	−10	−5	0	5	10



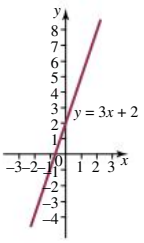
d

x	−2	−1	0	1	2
$f(x)$	0	2	4	6	8



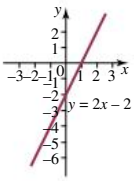
e

x	−2	−1	0	1	2
y	−4	−1	2	5	8



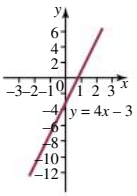
f

x	−2	−1	0	1	2
$f(x)$	−6	−4	−2	0	2



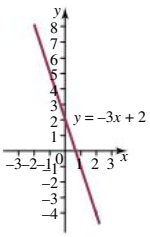
g

x	−2	−1	0	1	2
y	−11	−7	−3	1	5



h

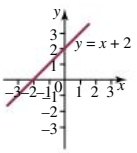
x	−2	−1	0	1	2
$f(x)$	8	5	2	−1	−4



2 a B **b** B

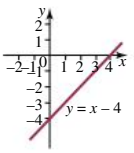
3 a

x	−2	−1	0	1	2
$f(x)$	0	1	2	3	4



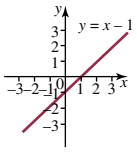
b

x	−2	−1	0	1	2
$f(x)$	−6	−5	−4	−3	−2



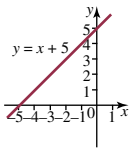
c

x	−2	−1	0	1	2
y	−3	−2	−1	0	1



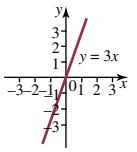
d

x	−2	−1	0	1	2
y	3	4	5	6	7



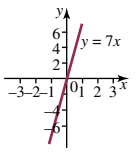
e

x	−2	−1	0	1	2
y	−6	−3	0	3	6



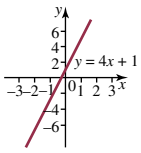
f

x	−2	−1	0	1	2
$f(x)$	−14	−7	0	7	14



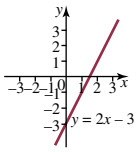
g

x	−2	−1	0	1	2
y	−7	−3	1	5	9



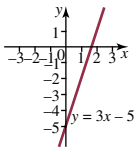
h

x	−2	−1	0	1	2
$f(x)$	−27	−5	−3	−1	1



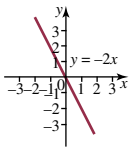
i

x	−2	−1	0	1	2
y	−11	−8	−5	−2	1



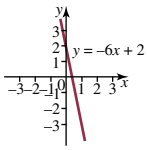
j

x	−2	−1	0	1	2
$f(x)$	4	2	0	−2	−4



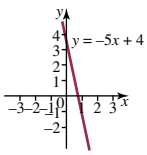
k

x	−2	−1	0	1	2
$f(x)$	14	8	2	−4	−10



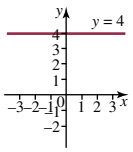
l

x	−2	−1	0	1	2
$f(x)$	14	9	4	−1	−6

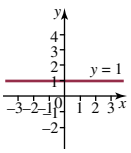


4 **a** $x = 1$ **b** $x = 3$ **c** $x = -6$

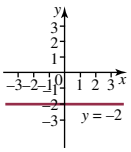
5 **a** 4



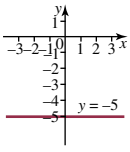
b 1



c −2

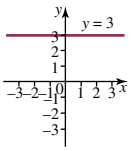


d −5



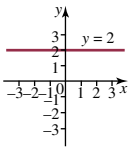
6 **a**

x	−2	−1	0	1	2
y	3	3	3	3	3



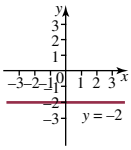
b

x	−2	−1	0	1	2
y	2	2	2	2	2



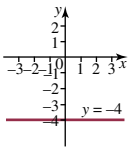
c

x	−2	−1	0	1	2
y	−2	−2	−2	−2	−2

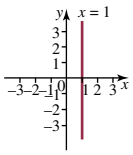


d

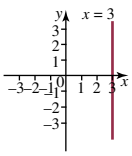
x	−2	−1	0	1	2
y	−4	−4	−4	−4	−4



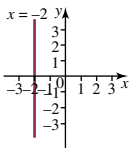
7 **a** 1



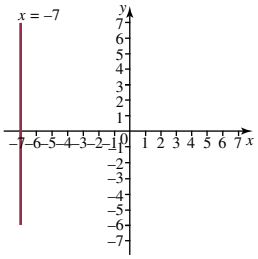
b 3



c −2

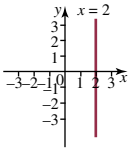


d −7



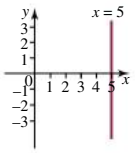
8 **a**

x	2	2	2	2	2
y	−2	−1	0	1	2



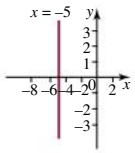
b

x	5	5	5	5	5
y	−2	−1	0	1	2



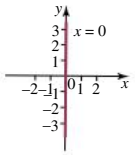
c

x	−5	−5	−5	−5	−5
y	−2	−1	0	1	2



d

x	0	0	0	0	0
y	−2	−1	0	1	2

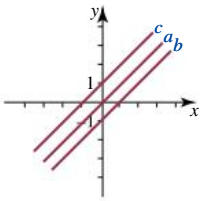


9 a

x	−2	−1	0	1	2
y	−4	−2	0	2	4

b

x	−2	−1	0	1	2
y	−5	−2	−1	1	3



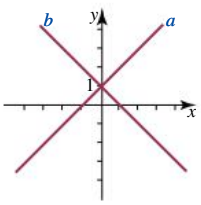
c

x	−2	−1	0	1	2
y	−3	−1	1	3	5

Lines are parallel.

10 a

x	−2	−1	0	1	2
y	−5	−2	1	4	7



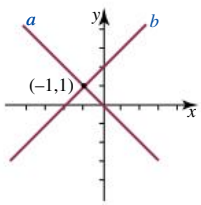
b

x	−2	−1	0	1	2
y	5	3	1	−1	−3

Lines meet at (0, 1).

11 a

x	−2	−1	0	1	2
y	2	1	0	−1	−2

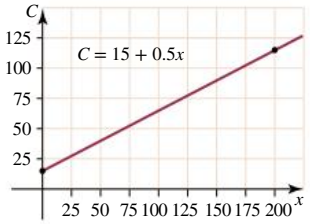


b

x	−2	−1	0	1	2
y	0	1	2	3	4

Lines meet at (−1, 1).

12 a

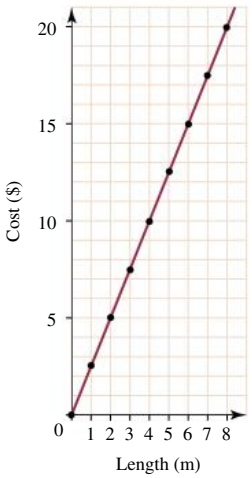


b 170 minutes **c** 70 minutes

13 a

Length (m)	0	1	2	3	4	5	6	7	8
Cost ($)	0	2.50	5.00	7.50	10.00	12.50	15.00	17.50	20.00

b

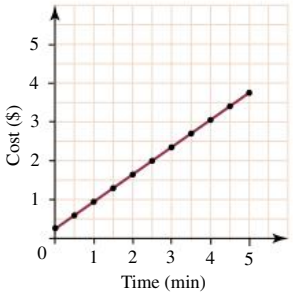


c This is a linear relationship as the graph is a straight line.

14 a

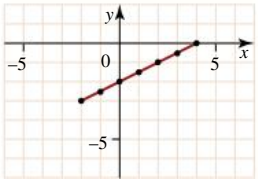
Time (min)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
Cost ($)	0.25	0.60	0.95	1.30	1.65	2.00	2.35	2.70	3.05	3.40	3.75

b



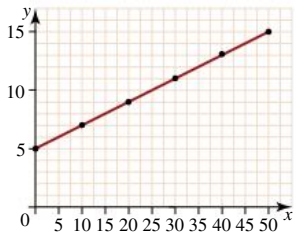
15 a

x	−2	−1	0	1	2	3	4
y	−3	−2.5	−2	−1.5	−1	−0.5	0



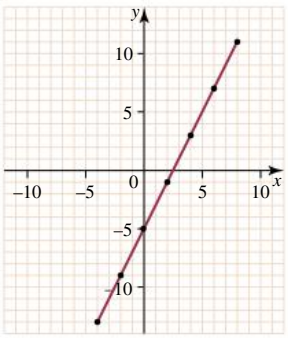
b

x	0	10	20	30	40	50
y	5	7	9	11	13	15



16 a

x	−4	−2	0	2	4	6	8
y	−13	−9	−5	−1	3	7	11



b Look at the pattern of y-values to find the missing y-value. Use the y-values to calculate the x-values.

Challenge 14.1

41 km

14.5 The *y*–intercept and gradient

1 a Positive **b** Undefined **c** Negative
d Positive **e** Zero **f** Negative

2 a 1 **b** −2 **c** 3 **d** $\frac{1}{3}$ **e** −2
f $-\frac{1}{4}$ **g** 1 **h** 3 **i** −1

3 a i $m = 1$ **ii** $c = -1$
b i $m = -2$ **ii** $c = 2$
c i $m = -3$ **ii** $c = 0$
d i $m = 3$ **ii** $c = -3$
e i $m = -3$ **ii** $c = 6$
f i $m = \frac{1}{3}$ **ii** $c = 4$

4 a C **b** A

5 A

6 a i $m = 1$ **ii** $c = 1$ **iii** $y = x + 1$
b i $m = -4$ **ii** $c = -2$ **iii** $y = -4x - 2$
c i $m = 2$ **ii** $c = -3$ **iii** $y = 2x - 3$
d i $m = 5$ **ii** $c = 0$ **iii** $y = 5x$
e i $m = -3$ **ii** $c = 1$ **iii** $y = -3x + 1$
f i $m = -1$ **ii** $c = 3$ **iii** $y = -x + 3$

7 a Gradient = 1, y-intercept = 3
b Gradient = 1, y-intercept = −4
c Gradient = 3, y-intercept = 1
d Gradient = 5, y-intercept = −2
e Gradient = 6, y-intercept = 10
f Gradient = 8, y-intercept = −7
g Gradient = 5, y-intercept = 3
h Gradient = 9, y-intercept = −4
i Gradient = −3, y-intercept = 4
j Gradient = −6, y-intercept = 2
k Gradient = −4, y-intercept = 0
l Gradient = 1, y-intercept = 0

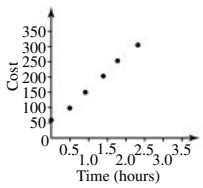
8 a D **b** A

9 a B **b** E

10 a

Times (hours)	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
Cost ($)	55	100	145	190	235	280	325

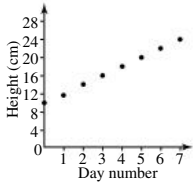
b



c Independent: time Dependent: cost
d Yes
e $m = 90$
f $c = 55$
g $y = 90x + 55$
h Cost = 90 × time + 55
i $C = \$415$

11 a Day 8

b



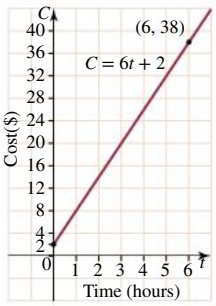
c Yes
d $m = 2$, $c = 10$
e $h = 2d + 10$
f $h = 38$ mm

12 a 18 **b** $m = -2$, $c = 26$
c $s = -2t + 26$ **d** $s = 14$

13 a

t	0	1	2	3
C	2	8	14	20

b



c i (0, 2)
ii The y-intercept represents the initial cost of bowling at the alley, which is the shoe rental.

d $m = 6$
e $C = 6t + 2$
f $32
g Answers will vary.

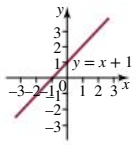
14 a $m = \dfrac{y - c}{x}$
b $y = mx + c$

Challenge 14.2

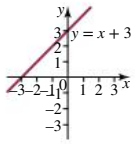
$y = 2x + \frac{1}{3}$

14.6 Sketching linear graphs

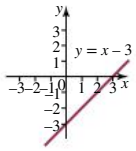
1 a



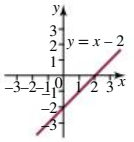
b



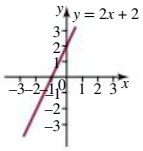
c



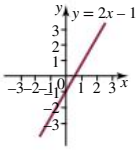
d



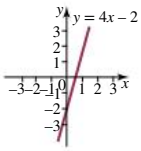
e



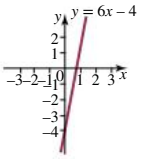
f



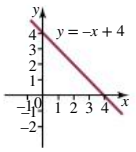
g



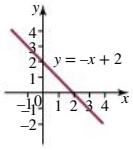
h



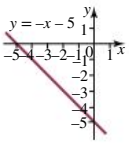
i



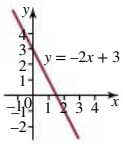
j



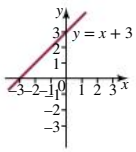
k



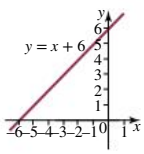
l



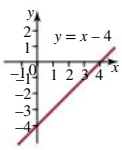
2 a



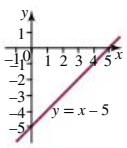
b



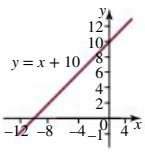
c



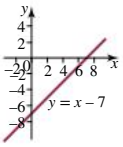
d



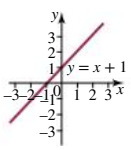
e



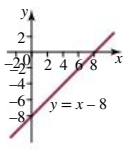
f



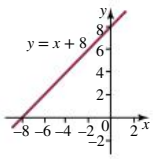
g



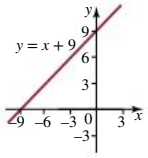
h



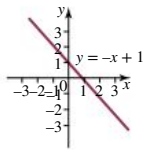
i



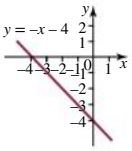
j



k



l



3 a D **b** B

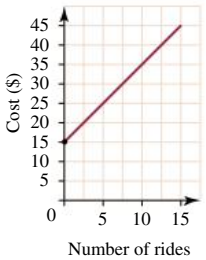
4 a C **b** A

5 Check with your teacher.

6 a $y = -2x + 6$ **b** 3

7 a $y = -\frac{p}{q}x + q$ **b** $-\frac{q}{p}$

8 a

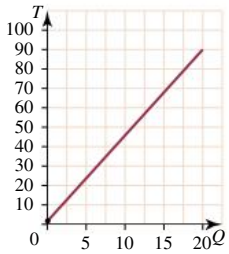


b $15.00

c $2.00

d $25.00

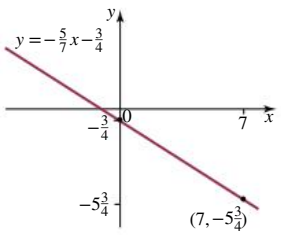
9 a



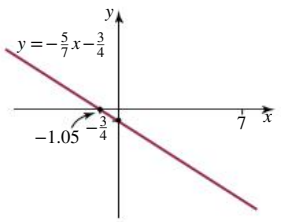
b 91.5 minutes

c Answers will vary. For example, 1.5 minutes might be the amount of time that it takes a student to set up their work and get ready.

10 a i



ii



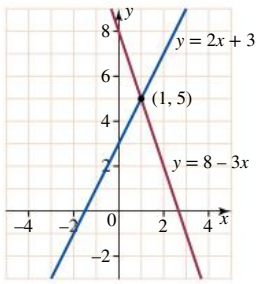
iii Answers will vary.

b Answers will vary.

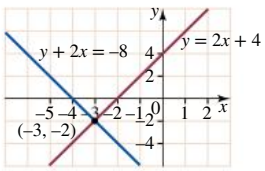
14.7 Solving equations graphically

1 **a** (2, 1)
b (1, 1)
c (0, 4)
d (2, −1)
e (−2, −4)
f (−0.5, 1.5)

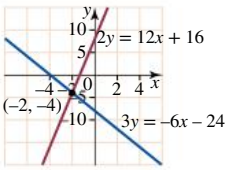
2 **a** Intersection point (1, 5)



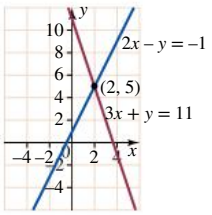
b Intersection point (−3, −2)



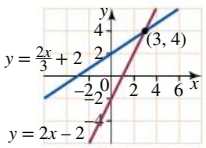
c Intersection point (−2, −4)



d Intersection point (2, 5)



e Intersection point (3, 4)



3 **a** Gradient = 2 and 2; parallel
b Gradient = −3 and −3; parallel
c Gradient = $-\frac{2}{3}$ and $-\frac{2}{3}$; parallel
d Gradient = −2 and $\frac{1}{2}$; not parallel
e Gradient = 4 and $\frac{15}{2}$; not parallel
f Gradient = $-\frac{1}{4}$ and 4; not parallel

4 The lines intersect on the y-axis. The point of intersection is (0, 5).

5 **a** Rearrange: $y = 2x - 4$. The gradient is 2 and the y-intercept is −4.
b **i** $y = 3x - 4$ **ii** $y = 2x + 5$

6 (5, 5)

7 $x = 2$

8 $x = \frac{3}{2}$

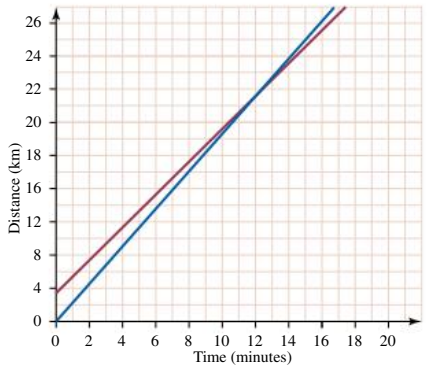
9 $x = -\frac{1}{6}$

10 $x = \frac{15}{11}$

11 (3, 3), (−4, 17), (17, −4)

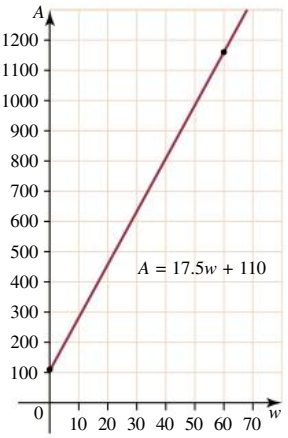
12 1.5 hours

13 **a**



b 9 km
c 12 min
d 21.6 km

14 **a**



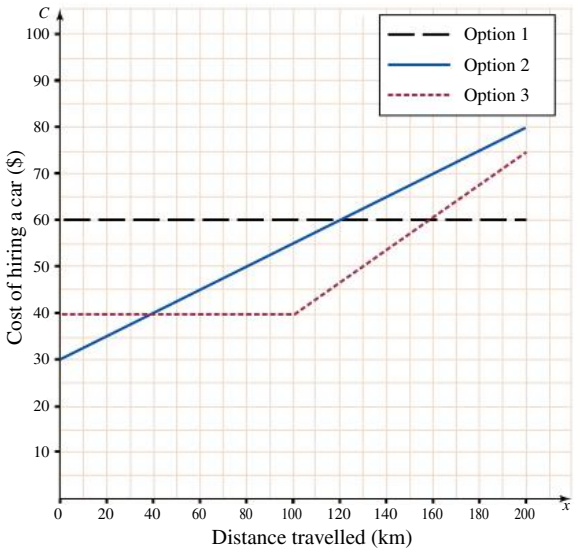
b $285 **c** $110 **d** 32 weeks **e** 40 weeks

Investigation — Rich task

1 Option 1: $60; Option 2: $52.50; Option 3: $40

2 Option 1: $C = \$60$
Option 2: $C = \$(30 + 0.25x)$

3



4 Option 1 starts off the most expensive but by the end of the 200 km, it is the least expensive, remaining at $60 for the day. Option 2 looks the least expensive at the start, but after travelling 200 km, its cost is the most expensive at $80. Option 3's cost is $40 up to 100 km travelled, but it increases to $75 by the end of 200 km of travel.
5 Option 1: $60; Option 2: $80; Option 3: $75
6 2052 km
7 Option 1: $420; Option 2: $723; Option 3: $753.20
8 Yes. Teacher to check comments.
9 Teacher to check.

Code puzzle

South Africa

TOPIC 15

Pythagoras' theorem

15.1 Overview

Why learn this?

Pythagoras was a famous mathematician who lived about 2500 years ago. He is credited with being the first person to prove that in any right-angled triangle there is a special relationship between the squares of the three sides. The theorem has an important role to play in everyday life because right-angled triangles occur in construction, navigation, planning, design and packaging. Pythagoras' theorem is one of the great geometrical theorems.

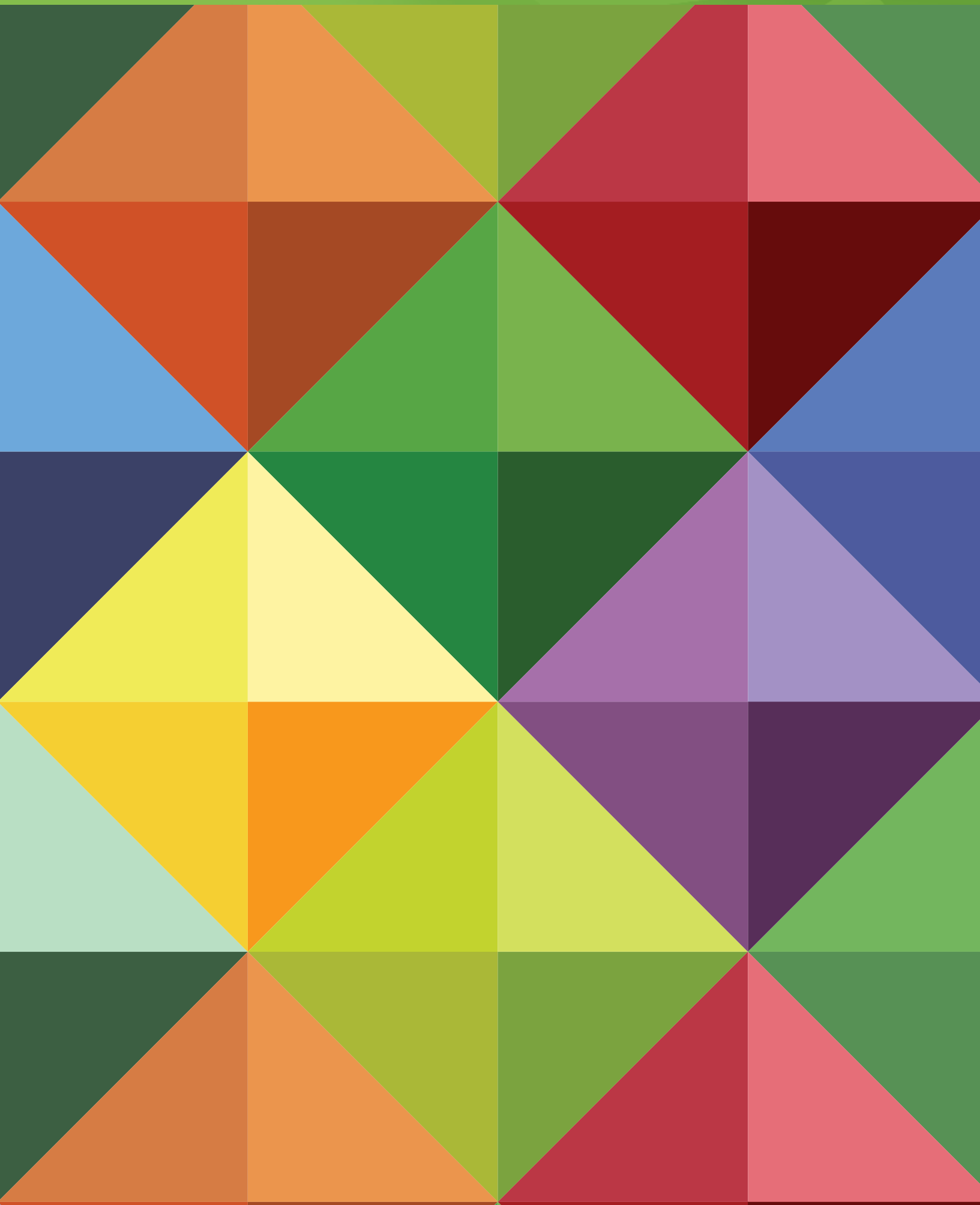
What do you know?

assessON

1 **THINK** List what you know about Pythagoras' theorem. Use a thinking tool such as a concept map to show your list.
2 **PAIR** Share what you know with a partner and then with a small group.
3 **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of Pythagoras' theorem.

Learning sequence

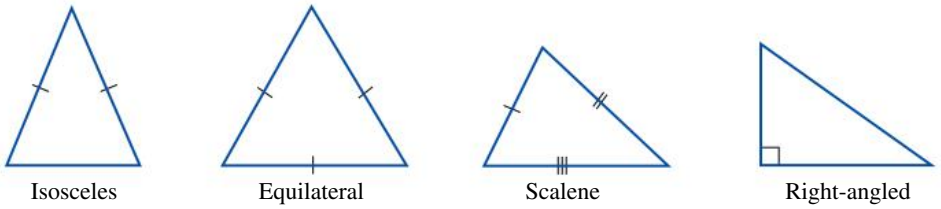
15.1 Overview
15.2 Right-angled triangles
15.3 Finding the hypotenuse
15.4 Finding a shorter side
15.5 Working with different units
15.6 Composite shapes
15.7 Pythagorean triads
15.8 Pythagoras in 3-D
15.9 Review eBookplus ONLINE ONLY



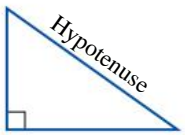
15.2 Right-angled triangles

- This topic investigates one of the most important ideas in geometry related to **right-angled triangles**. Triangles can be classified according to the length of their sides. They can be classified as either equilateral, isosceles or scalene. Triangles can be classified by their angles as well. A triangle with one right angle is said to be a right-angled triangle.

Types of triangles



- In any triangle, the longest side is opposite the largest angle; so in a right-angled triangle, the longest side is opposite the right angle. This side has a special name. It is called the **hypotenuse**.



- The theorem, or rule, that you will learn more about was named after an ancient Greek mathematician, called Pythagoras (580–501 BC). It will enable you to solve all kinds of practical problems related to right-angled triangles.

assessON

Exercise 15.2 Right-angled triangles

REFLECTION
How will you remember the meaning of the pronumerals *a*, *b* and *c* in Pythagoras' theorem?

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–6, 8, 10	Questions: 1–5, 7, 8, 10, 11	Questions: 1–5, 7–11

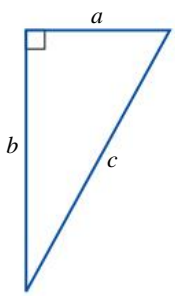
Individual pathway interactivity int-4469 eBookplus

eBookplus
Digital docs
SkillSHEET
Measuring lengths and angles
doc-11425
SkillSHEET
Finding the square of a number
doc-11426

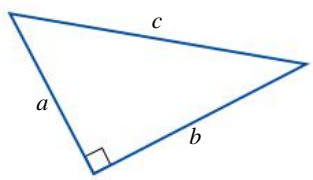
FLUENCY

1 For each of the following triangles, the lengths of the sides are recorded in the table that follows. Note that the hypotenuse is always marked as *c*. Complete the table.

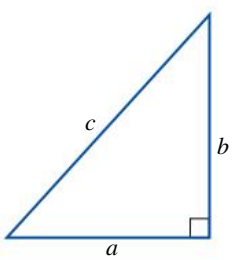
a



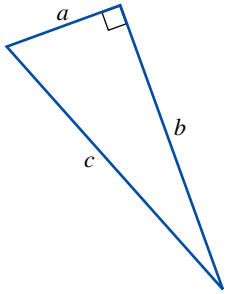
b



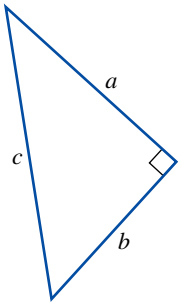
c



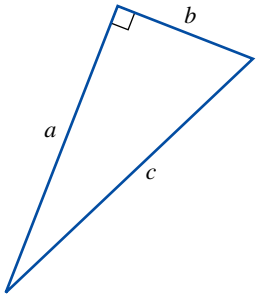
d



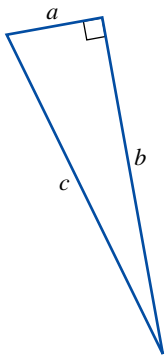
e



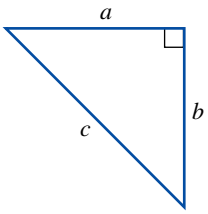
f



g



h



Lengths (mm)	a	b	c	d	e	f	g	h
a	19	24	27	16	31	41	13	24
b	35	32	30	40	25	19	45	24
c	40	40	40	43	40	45	47	34
a^2								
b^2								
$a^2 + b^2$								
c^2								

2 What do you notice about the results in the table in question **1**?

3 On a sheet of paper, carefully draw 6 right-angled triangles of different sizes. You will need to use a protractor, set square or template to make sure the triangles are right-angled. Carefully measure the sides of each triangle, and complete a table like the one in question **1**. What do you notice about these results?

4 Now draw some triangles that are not right-angled, measure their sides and complete the same kind of table. Remember to label the longest side as c. What do you notice this time?

5 Explain why it is not possible to have a triangle with two right angles in it.

6 For this activity you will need graph paper, coloured pencils, glue and scissors.

a On a sheet of graph paper, draw a right-angled triangle with a base of 4 cm and a height of 3 cm.

b Carefully draw a square on each of the three sides of the triangle and mark a grid on each so that the square on the base is divided into 16 small squares, while the square on the height is divided into 9 small squares.

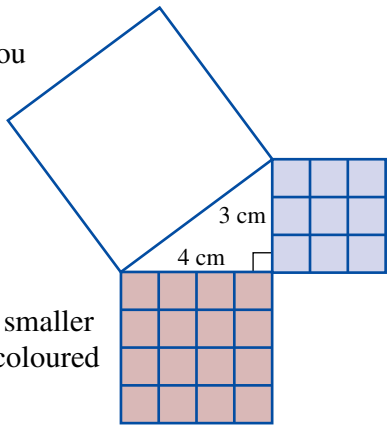
c Colour the square on the base, and the square on the height in different colours as shown at right so that you can still see the grid lines.

d Carefully cut out the two coloured squares from the triangle.

e Now stick the larger of the coloured squares on the uncoloured square of the triangle (the square on the hypotenuse).

f Using the grid lines as a guide, see if you can cut the smaller square up and fit it on the remaining space. The two coloured squares should have exactly covered the third square.

g What do you notice about the hypotenuse and the other two sides of a right-angled triangle?



UNDERSTANDING

7 A right-angled triangle is a special triangle, but it has some properties that are common to all triangles. One of these common properties is that the sum of the lengths of any two sides must be greater than the length of the third side.

Using a ruler and compass, attempt to construct a triangle whose side lengths are 7 cm, 4 cm and 12 cm. What do you find?

REASONING

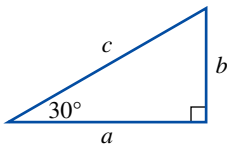
8 What is the sum of the two smaller angles in a right-angled triangle? Justify your answer.

9 If the two smaller angles in a right-angled triangle are equal in size, make a statement about the triangle. What size is each of the smaller angles?

PROBLEM SOLVING

10 Can you draw an equilateral triangle that has a right angle? Explain.

11 a Draw four triangles with a 30° angle like the one shown at right, ensuring that the length of a is different for each.



b For each triangle you have drawn in part **a**, find the value of:

i $\frac{b}{a}$ **ii** $\frac{b}{c}$ **iii** $\frac{a}{c}$

c What do you notice about your answers to part **b**?

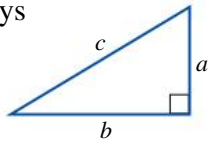
d Repeat questions **a** to **c** for another four right-angled triangles using an angle other than 30°.

15.3 Finding the hypotenuse

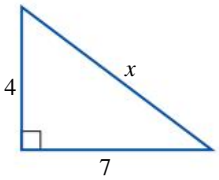
- Pythagoras was perhaps the first mathematician to recognise and investigate a very important property of right-angled triangles. The theorem, or rule, named after him states that:

In any right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. The rule is written as $c^2 = a^2 + b^2$ where a and b are the two shorter sides and c is the hypotenuse.

- The hypotenuse is the longest side of a right-angled triangle and is always the side that is opposite the right angle.

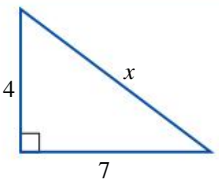


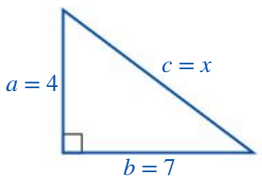
- Pythagoras' theorem gives us a way of finding the length of the third side in a triangle, if we know the lengths of the two other sides.
- We are able to find the length of the hypotenuse when we are given the length of the two shorter sides by substituting into the formula $c^2 = a^2 + b^2$ to calculate the hypotenuse c.
- Calculations with Pythagoras' theorem often result in a number under the root sign $(\sqrt{\ })$ that is not a square number. In such case answers may be left unsimplified, and this is called exact (surd) form. A calculator may be used to find an approximate answer, usually given to a specified number of decimal places.



WORKED EXAMPLE 1

For the triangle at right, find the length of the hypotenuse, x, correct to 1 decimal place.

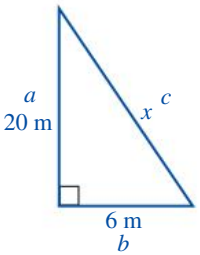


THINK	WRITE/DRAW
1 Copy the diagram and label the sides a, b and c. Remember to label the hypotenuse as c.	
2 Write Pythagoras' theorem.	$c^2 = a^2 + b^2$
3 Substitute the values of a, b and c into this rule and simplify.	$x^2 = 4^2 + 7^2$ $= 16 + 49$ $= 65$
4 Calculate x by taking the square root of 65. As 65 is not a square number, a calculator can be used to find an approximate value for x. Round your answer correct to 1 decimal place.	$x = \sqrt{65}$ This is the exact (surd) form. $x \approx 8.1$

- The $\approx$ symbol means 'is approximately equal to'. An alternative symbol, $\fallingdotseq$, can also be used.
- In many cases we are able to use Pythagoras' theorem to solve practical problems. We can model the problem by drawing a diagram, and use Pythagoras' theorem to solve the right-angled triangle. We then use the result to give a worded answer.

WORKED EXAMPLE 2

A fire is on the twelfth floor of a building. A child needs to be rescued from a window that is 20 metres above ground level. If the rescue ladder can be placed no closer than 6 m from the foot of the building, what is the minimum length ladder needed to make the rescue? Leave your answer in exact form.

THINK	WRITE/DRAW
1 Draw a diagram and label the sides a, b and c. Remember to label the hypotenuse as c.	
2 Write Pythagoras' theorem.	$c^2 = a^2 + b^2$
3 Substitute the values of a, b and c into this rule and simplify.	$x^2 = 20^2 + 6^2$ $= 400 + 36$ $= 436$
4 Calculate x by taking the square root of 436.	$x = \sqrt{436}$
5 Give a worded answer.	The ladder needs to be $\sqrt{436}$ metres long.

assess on

Exercise 15.3 Finding the hypotenuse

REFLECTION
How do you know which side is the hypotenuse?

INDIVIDUAL PATHWAYS

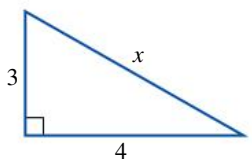
PRACTISE	CONSOLIDATE	MASTER
Questions: 1–5, 9	Questions: 1–6, 9, 10	Questions: 1–11

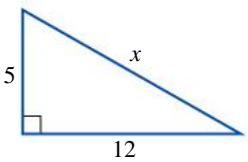
Individual pathway interactivity int-4470 eBookplus

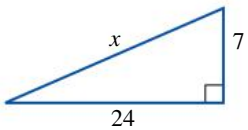
eBookplus
Digital docs
SkillSHEET Finding the square root of a number doc-11427
SkillSHEET Rounding to a given number of decimal places doc-11428

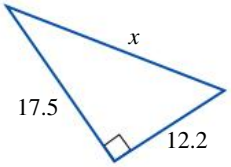
FLUENCY

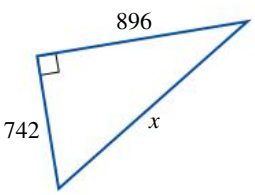
1 WE1 For the following triangles, find the length of the hypotenuse, x, correct to 1 decimal place where necessary.

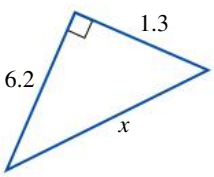
a


b


c


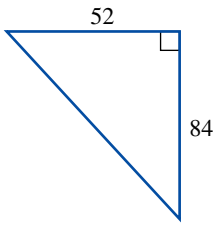
d


e


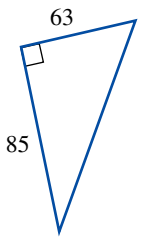
f


2 For each of the following triangles, find the length of the hypotenuse. Leave your answers in exact form.

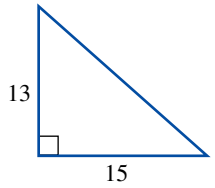
a



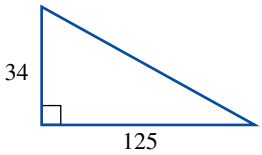
b



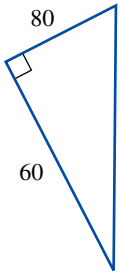
c



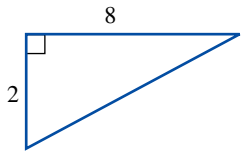
d



e



f



UNDERSTANDING

3 A right-angled triangle has a base of 5 cm and a perpendicular height of 11 cm. Find the length of the hypotenuse. Leave your answers in exact form.

4 Find the lengths, correct to 2 decimal places, of the diagonals of squares that have side lengths of:

a 12 cm **b** 20 mm **c** 4.9 cm

5 What is the length, correct to 2 decimal places, of the diagonal of a rectangle whose sides are:

a 10 cm and 8 cm? **b** 620 cm and 400 cm? **c** 17 cm and 3 cm?

6 An isosceles triangle has a base of 30 cm and a perpendicular height of 10 cm. Find the length of the two equal sides of the isosceles triangle. Give your answer correct to 2 decimal places.

REASONING

7 A right-angled triangle has a perpendicular height of 17.2 cm, and a base that is half the height. Find the length of the hypotenuse, correct to 2 decimal places.

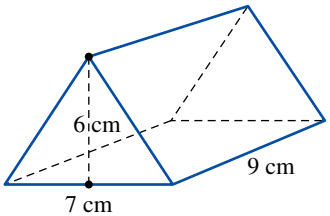
8 WE2 A ladder leans against a vertical wall. The foot of the ladder is 1.2 m from the wall, and the top of the ladder reaches 4.5 m up the wall. How long is the ladder? Give your answer correct to 2 decimal places.

9 Wally is installing a watering system in his garden. The pipe is to go all around the edge of the rectangular garden, and have a branch diagonally across the garden. The garden measures 5 m by 7.2 m. If the pipe costs $2.40 per metre (or part thereof), what will be the total cost of the pipe? Show your working.

PROBLEM SOLVING

10 **a** Calculate the volume of the prism.

b Find the length of the sloping edge of the cross-section.



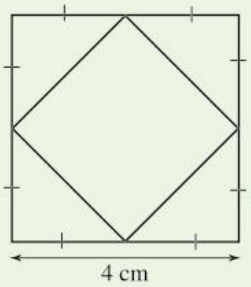
11 A ladder rests against a vertical wall with the top of the ladder being 7 m above the ground. If the bottom of the ladder was moved 1 m further away from the foot of the wall, the top of the ladder would rest against the foot of the wall. What is the length of the ladder?

12 A right-angled isosceles triangle has an area of 200 cm^2. Find the area of the semi-circle which sits on the hypotenuse.

CHALLENGE 15.1

A smaller square is drawn inside a larger square as shown in the diagram.

Use Pythagoras' theorem to determine the side length of the smaller square and hence the area of the smaller square.

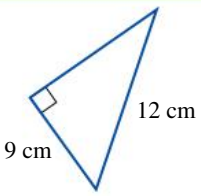


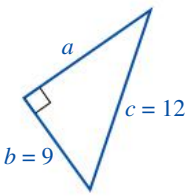
15.4 Finding a shorter side

- Sometimes a question will give you the length of the hypotenuse and ask you to find one of the shorter sides. In such examples, we need to rearrange Pythagoras' formula.
- Given that $c^2 = a^2 + b^2$, we can rewrite this as:
  $a^2 = c^2 - b^2$
  or $b^2 = c^2 - a^2$.

WORKED EXAMPLE 3

Find the length of the unmarked side of the triangle at right. Leave your answer in exact form.

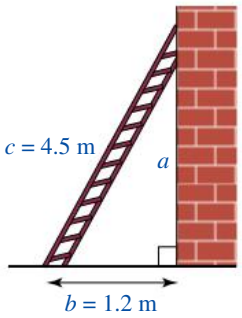


THINK	WRITE/DRAW
1 Copy the diagram and label the sides a, b and c. Remember to label the hypotenuse as c.	
2 Write Pythagoras' theorem for a shorter side.	$a^2 = c^2 - b^2$
3 Substitute the values of a, b and c into this rule and simplify.	$a^2 = 12^2 - 9^2$ $= 144 - 81$ $= 63$
4 Find a by taking the square root of 63.	$a = \sqrt{63}$ cm

- Practical problems may also involve you being required to find the shorter side of a right-angled triangle.

WORKED EXAMPLE 4

A ladder that is 4.5 m long leans up against a vertical wall. The foot of the ladder is 1.2 m from the wall. How far up the wall does the ladder reach? Give your answer correct to 2 decimal places.

THINK	WRITE/DRAW
1 Draw a diagram and label the sides a, b and c. Remember to label the hypotenuse as c.	
2 Write Pythagoras' theorem for a shorter side.	$a^2 = c^2 - b^2$
3 Substitute the values of a, b and c into this rule and simplify.	$a^2 = 4.5^2 - 1.2^2$ $= 20.25 - 1.44$ $= 18.81$
4 Find a by taking the square root of 18.81. Round to 2 decimal places.	$a = \sqrt{18.81}$ ≈ 4.34 m
5 Give a written answer.	The ladder will reach a height of 4.34 m up the wall.

- Some questions will require you to decide which method is needed to solve the problem. A diagram will help you decide whether you are finding the hypotenuse or one of the shorter sides.

Exercise 15.4 Finding a shorter side

assessON

INDIVIDUAL PATHWAYS

■ PRACTISE	■ CONSOLIDATE	■ MASTER
Questions: 1–6, 8, 10, 14	Questions: 1–10, 13, 14	Questions: 1–15

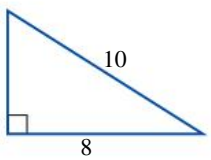
■■■ Individual pathway interactivity int-4471 eBookplus

REFLECTION
What should you look out for when using Pythagoras' theorem to calculate the value of missing sides?

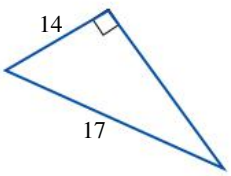
FLUENCY

1 WE3 Find the length of the unmarked side in each of the following triangles. Leave your answer in exact form.

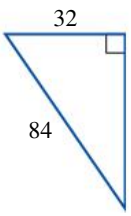
a



b

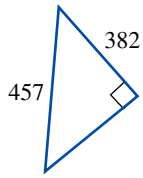


c

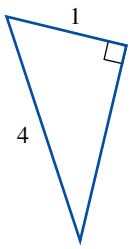


eBookplus
Digital doc
SkillSHEET
Rearranging formulas
doc-11429

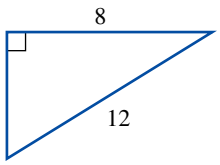
d



e

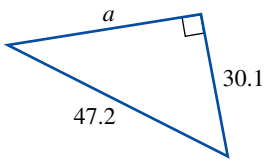


f

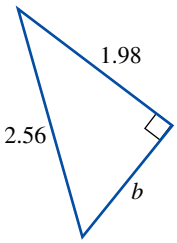


2 Find the value of the pronumeral, correct to 2 decimal places.

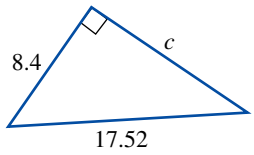
a



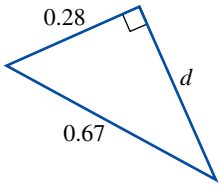
b



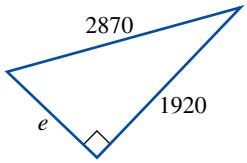
c



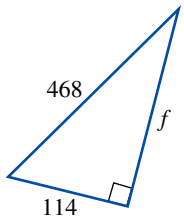
d



e



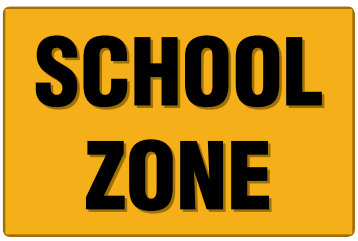
f



UNDERSTANDING

For questions **3** to **13**, give your answers correct to 2 decimal places.

3 The diagonal of the rectangular sign at right is 34 cm. If the height of this sign is 25 cm, find the width.

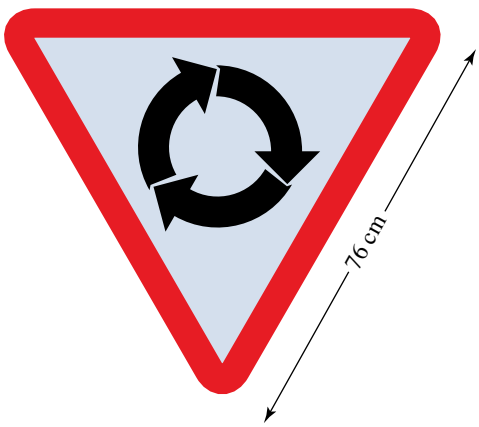


4 The diagonal of a rectangle is 120 cm. One side has a length of 70 cm. Find:

- **a** the length of the other side
- **b** the perimeter of the rectangle
- **c** the area of the rectangle.

5 An equilateral triangle has sides of length 20 cm. Find the height of the triangle.

6 The roundabout sign shown at right is in the form of an equilateral triangle. Find the height of the sign and, hence, find its area.



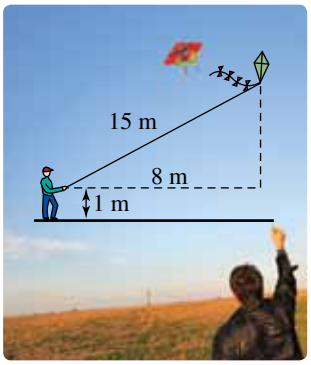
7 WE4 A ladder that is 7 metres long leans up against a vertical wall. The top of the ladder reaches 6.5 m up the wall. How far from the wall is the foot of the ladder?

8 A tent pole that is 1.5 m high is to be supported by ropes attached to the top. Each rope is 2 m long. How far from the base of the pole can each rope be pegged?

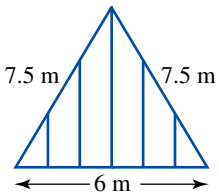
9 Ben's dog 'Macca' has wandered onto a frozen pond, and is too frightened to walk back. Ben estimates that the dog is 3.5 m from the edge of the pond. He finds a plank, 4 m long, and thinks he can use it to rescue Macca. The pond is surrounded by a bank that is 1 m high. Ben uses the plank to make a ramp for Macca to walk up. Will he be able to rescue his dog?

REASONING

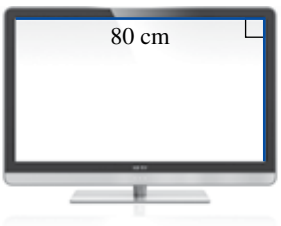
10 A kite is attached to a string 15 m long. Sam holds the end of the string 1 m above the ground, and the horizontal distance of the kite from Sam is 8 m as shown at right. How far above the ground is the kite? Show your working.



11 Penny, the carpenter, is building a roof for a new house. The roof has a gable end in the form of an isosceles triangle, with a base of 6 m and sloping sides of 7.5 m. She decides to put 5 evenly spaced vertical strips of wood as decoration on the gable as shown below. How many metres of this decorative wood does she need? Show your working.



12 The size of a rectangular television screen is given by the length of its diagonal. The television below has a 92-cm screen. What is the length of the shorter side? Show your working.



PROBLEM SOLVING

13 An art student is trying to hang her newest painting on an existing hook in a wall. She leans a 1.2-m ladder against the wall so that the distance between the foot of the ladder and the wall is 80 cm.

- **a** Draw a sketch showing the ladder leaning on the wall.
- **b** How far up the wall does the ladder reach?
- **c** The student climbs the ladder to check whether she can reach the hook from the step at the very top of the ladder. Once she extends her arm, the distance from her feet to her fingertips is 1.7 m. If the hook is 2.5 m above the floor, will the student reach it from the top step?

eBookplus
Digital doc
Worksheet 15.1
doc-11431

14 a A rectangle has two sides of length 3 cm and diagonals of length 5 cm. Calculate the length of the other two sides.

b It is known that the diagonals of this rectangle bisect each other at the point of intersection. Using this fact, check (using appropriate calculations) whether the diagonals are perpendicular to each other.

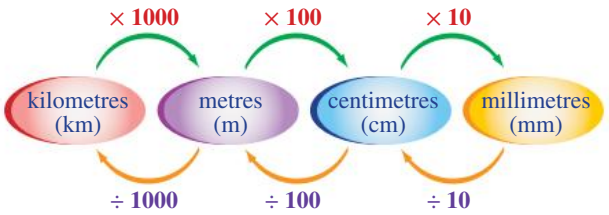
15.5 Working with different units

- When we use Pythagoras' theorem, we are usually working with a practical situation where measurements have been given. In any calculation, it is essential that all of the measurements are in the same units (for example, cm).
- Do you remember the relationship between the units of length? Here is a reminder:

10 mm = 1 cm
100 cm = 1 m
1000 m = 1 km

To change to a larger unit, divide by the conversion factor.
To change to a smaller unit, multiply by the conversion factor.

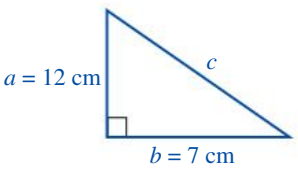
For example, to change kilometres to metres, multiply by 1000.
To change centimetres to metres, divide by 100.
The following chart shows all the conversions of length that you are likely to need.



- When using Pythagoras' theorem, always check the units given for each measurement. If necessary, convert all measurements to the same units before using the rule.

WORKED EXAMPLE 5

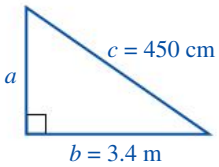
Find the length, in mm, of the hypotenuse of a right-angled triangle if the 2 shorter sides are 7 cm and 12 cm. Give your answer correct to 1 decimal place.

THINK	WRITE/DRAW
1 Draw a diagram and label the sides a, b and c. Remember to label the hypotenuse as c.	
2 Check that all measurements are in the same units. They are.	
3 Write Pythagoras' theorem for the hypotenuse.	$c^2 = a^2 + b^2$
4 Substitute the values of a and b into this rule and simplify.	$c^2 = 12^2 + 7^2$ $= 144 + 49$ $= 193$

5	Find c by taking the square root. Give the units in the answer.	$c = \sqrt{193}$ ≈ 13.89 cm
6	Check the units required in the answer and convert if necessary.	$= 13.89 \times 10$ mm $= 138.9$ mm

WORKED EXAMPLE 6

The hypotenuse and one other side of a right-angled triangle are 450 cm and 3.4 m respectively. Find the length of the third side, in cm, correct to the nearest whole number.

	THINK	WRITE/DRAW
1	Draw a diagram and label the sides a, b and c. Remember to label the hypotenuse as c.	
2	Check that all measurements are in the same units. They are different, so convert 3.4 m into cm.	$3.4 \text{ m} = 3.4 \times 100$ cm $= 340$ cm
3	Write Pythagoras' theorem for a shorter side.	$a^2 = c^2 - b^2$
4	Substitute the values of b and c into this rule and simplify.	$a^2 = 450^2 - 340^2$ $= 202\,500 - 115\,600$ $= 86\,900$
5	Find a by taking the square root of 86 900. Round to the nearest whole number.	$a = \sqrt{86\,900}$ ≈ 295
6	Give a worded answer.	The third side will be approximately 295 cm long.

Exercise 15.5 Working with different units

assessON

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8, 10, 11, 14, 17	Questions: 1, 2a–e, 3a–e, 4a–d, 5–12, 14, 17	Questions: 1, 2e–h, 3e–h, 4c–f, 5–18

Individual pathway interactivity int-4472 eBookplus

REFLECTION
How do you remember whether to multiply or divide when converting units?

FLUENCY

Where appropriate, give answers correct to 2 decimal places.

1 WE5 Find the length, in mm, of the hypotenuse of a right-angled triangle, if the two shorter sides are 5 cm and 12 cm.

2 Find the length of the hypotenuse of the following right-angled triangles, giving the answer in the units specified.

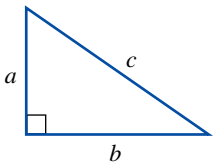
- **a** Sides 456 mm and 320 mm, hypotenuse in cm.
- **b** Sides 12.4 mm and 2.7 cm, hypotenuse in mm.

eBookplus
Digital doc
SkillSHEET
Converting units of length
doc-11430

c Sides 32 m and 4750 cm, hypotenuse in m.
d Sides 2590 mm and 1.7 m, hypotenuse in mm.
e Sides 604 cm and 249 cm, hypotenuse in m.
f Sides 4.06 km and 4060 m, hypotenuse in km.
g Sides 8364 mm and 577 cm, hypotenuse in m.
h Sides 1.5 km and 2780 m, hypotenuse in km.

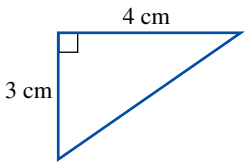
3 WE6 The hypotenuse and one other side of a right-angled triangle are given for each case below. Find the length of the third side in the units specified.
a Sides 46 cm and 25 cm, third side in mm.
b Sides 843 mm and 1047 mm, third side in cm.
c Sides 4500 m and 3850 m, third side in km.
d Sides 20.3 cm and 123 mm, third side in cm.
e Sides 6420 mm and 8.4 m, third side in cm.
f Sides 0.358 km and 2640 m, third side in m.
g Sides 491 mm and 10.8 cm, third side in mm.
h Sides 379 000 m and 82 700 m, third side in km.

4 Two sides of a right-angled triangle are given. Find the third side in the units specified. The diagram shows how each triangle is to be labelled. *Remember:* c is always the hypotenuse.



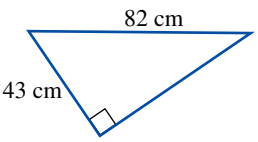
a $a = 37$ cm, $c = 180$ cm, find b in cm.
b $a = 856$ mm, $b = 1200$ mm, find c in cm.
c $b = 4950$ m, $c = 5.6$ km, find a in km.
d $a = 125\,600$ mm, $c = 450$ m, find b in m.
e $a = 0.0641$ km, $b = 0.153$ km, find c in m.
f $a = 639\,700$ cm, $b = 2.34$ km, find c in m.

5 MC *Note*: There may be more than one correct answer.
a What is the length of the hypotenuse in this triangle?



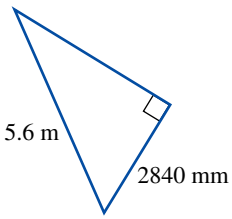
A 25 cm **B** 50 cm
C 50 mm **D** 500 mm

b What is the length of the third side in this triangle?



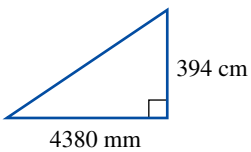
A 0.698 m **B** 69.8 cm
C 6.98 cm **D** 69.8 mm

c The most accurate measure for the length of the third side in the triangle at right is:



A 4.83 m **B** 23.3 cm
C 2330 mm **D** 4826 mm

d What is the length of the third side in this triangle?



A 34.71 m **B** 5.89 m
C 1722 cm **D** 4.4 m

6 A rectangle measures 35 mm by 4.2 cm. Find the length of the diagonal in mm.

7 A sheet of A4 paper measures 210 mm by 297 mm. Find the length of the diagonal in centimetres.

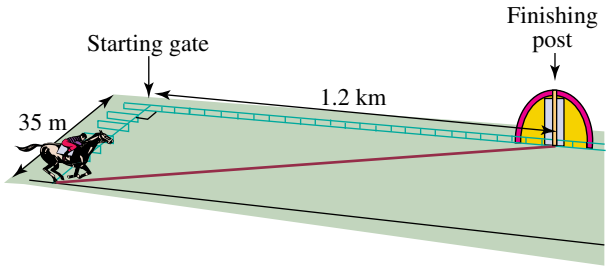
8 A rectangular envelope has a length of 21 cm and a diagonal measuring 350 mm. Find:

a the width of the envelope **b** the area of the envelope.

UNDERSTANDING

9 A right-angled triangle has a hypotenuse of 47.3 cm and one other side of 0.308 m. Find the area of the triangle.

10 A horse race is 1.2 km. The track is straight, and 35 m wide. How much further than 1.2 km will a horse run if it starts on the outside and finishes on the inside as shown?



11 A ramp is 9 metres long, and rises to a height of 250 cm. What is the horizontal distance, in metres, between the bottom and the top of the ramp?

12 Sarah is making a gate, which has to be 1200 mm wide. It must be braced with a diagonal strut made of a different type of timber. She has only 2 m of this kind of timber available. What is the maximum height of the gate that she can make?

REASONING

13 A rectangular park is 260 m by 480 m. Danny usually trains by running around the edge of the park. After heavy rain, two adjacent sides are too muddy to run along, so he runs a triangular path along the other two sides and the diagonal. Danny does 5 circuits of this path for training. How far does he run? Give your answer in km.

14 A swimming pool is 50 m by 25 m. Peter is bored by his usual training routine, and decides to swim the diagonal of the pool. How many diagonals must he swim to complete his normal distance of 1.2 km?

15 A hiker walks 4.5 km west, then 3.8 km south. How far in metres is she from her starting point?

16 A square has a diagonal of 10 cm. What is the length of each side in mm?

PROBLEM SOLVING

17 The triangular sides of a wheat field measure 2.4 km, 2500 m and 700 000 mm. Is the triangle right-angled? Show your working.

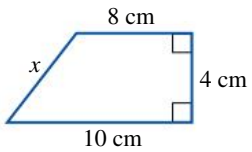
18 The sides of a rectangle measure a cm by $5a$ mm. The diagonal measures $\frac{b}{10}$ m. Find the relationship between a and b.

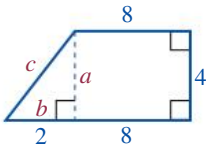
15.6 Composite shapes

- In all of the exercises so far, we have been working with only one right-angled triangle. Many situations involve more complex diagrams, where the right-angled triangle is not as obvious. A neat diagram is essential for these questions.

WORKED EXAMPLE 7

Find the length of the side, x. Give your answer correct to 2 decimal places.

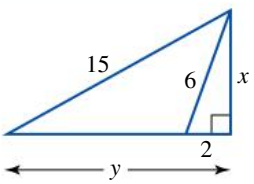


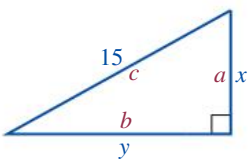
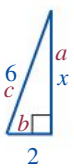
THINK	WRITE/DRAW
1 Copy the diagram. On the diagram, create a right-angled triangle and use the given measurements to work out the lengths of 2 sides.	
2 Label the sides of your right-angled triangle as a, b and c. Remember to label the hypotenuse as c.	
3 Check that all measurements are in the same units. They are the same.	
4 Write Pythagoras' theorem for the hypotenuse.	$c^2 = a^2 + b^2$
5 Substitute the values of a, b and c into this rule and simplify.	$x^2 = 4^2 + 2^2$ $= 16 + 4$ $= 20$
6 Find x by taking the square root of 20. Round your answer correct to 2 decimal places.	$x = \sqrt{20}$ ≈ 4.47 cm

- Some situations will involve shapes that contain more than one triangle. In this case it is a good idea to split the diagram into separate right-angled triangles first.

WORKED EXAMPLE 8

For the diagram at right, find the length of the sides marked x and y to 2 decimal places.



THINK	WRITE/DRAW
1 Copy the diagram.	 
2 Find and draw any right-angled triangles contained in the diagram. Label their sides.	
3 To find an unknown side in a right-angled triangle, we need to know 2 sides, so find x first.	

4 For the triangle containing x, write down Pythagoras' theorem for a shorter side. Use it to find x.

$$a^2 = c^2 - b^2$$
$$x^2 = 6^2 - 2^2$$
$$= 36 - 4$$
$$= 32$$
$$x = \sqrt{32}$$
$$\approx 5.66$$

5 We now know 2 sides for the other triangle because we can substitute $x = 5.66$.

6 For the triangle containing y, write down Pythagoras' theorem for a shorter side and use it to find y.

$$b^2 = c^2 - a^2$$
$$y^2 = 15^2 - 5.66^2$$
$$= 225 - 32$$
$$= 193$$
$$y = \sqrt{193}$$
$$\approx 13.89$$

Exercise 15.6 Composite shapes

assess on

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8, 13	Questions: 1–10, 13, 14	Questions: 1–15

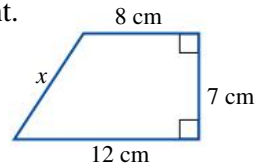
Individual pathway interactivity int-4473 eBook plus

REFLECTION
What do you look for when dividing composite shapes?

Where appropriate, give answers correct to 2 decimal places.

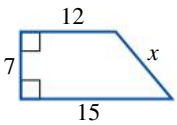
FLUENCY

1 WE7 Find the length of the side x in the figure at right.

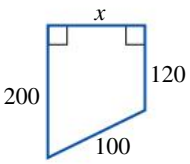


2 For the following diagrams, find the length of the sides marked x.

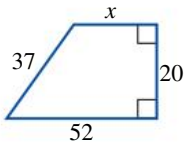
a



b

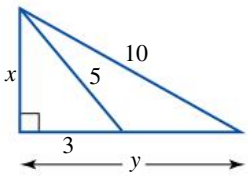


c

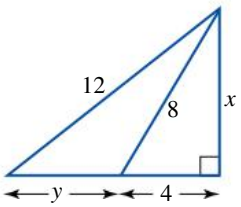


3 WE8 For each of the following diagrams, find the length of the sides marked x and y.

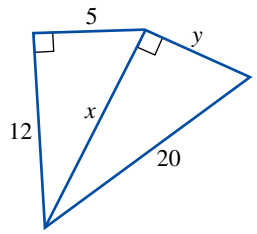
a



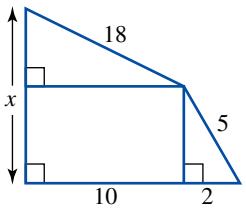
b



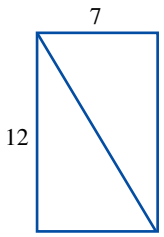
c



d

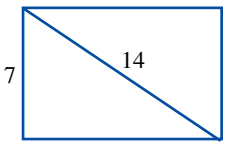


4 MC **a** The length of the diagonal of the rectangle at right is:



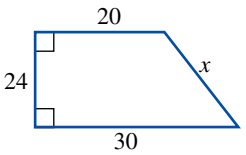
A 9.7 B 19
C 13.9 D 12.2

b The area of the rectangle at right is:



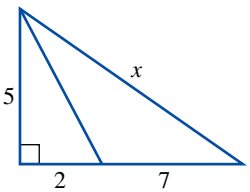
A 12.1 B 84.9
C 98 D 169.7

c The value of x in this shape is:



A 24 B 26
C 38.4 D 10

d What is the value of x in this figure?



A 5.4 B 8.6
C 10.1 D 10.3

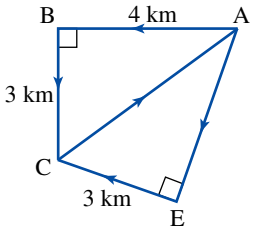
UNDERSTANDING

5 A garden bed is in the shape of a right-angled trapezium with the sloping edge 2.0 m, and parallel sides of 3.2 m and 4.8 m. Find the width of the garden and, hence, the area.

6 Two buildings, 10 and 18 m high, are directly opposite each other on either side of a European street that is 6 m wide (as shown in the following figure). What is the distance between the top of the two buildings?

7 Jess paddles a canoe 1700 m to the west, then 450 m south, and then 900 m to the east. She then stops for a rest. How far is she from her starting point?

8 A yacht race starts and finishes at A and consists of 6 legs; AB, BC, CA, AE, EC, CA, in that order as shown in the figure at right. If AB = 4 km, BC = 3 km and CE = 3 km, find:

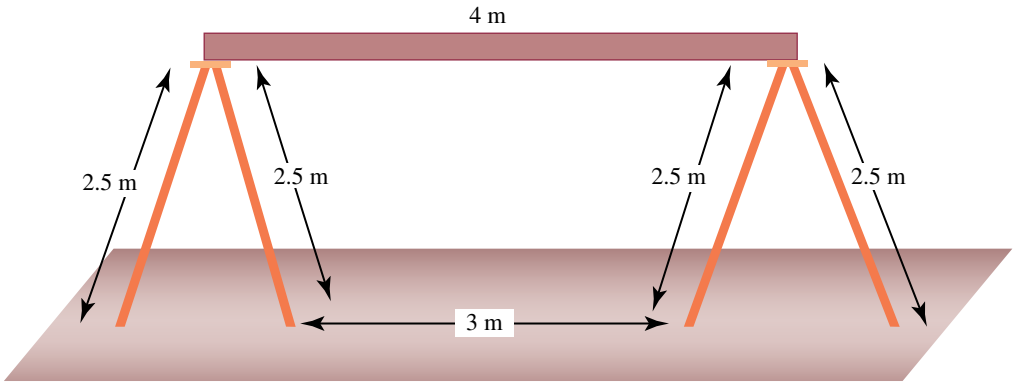


a AC

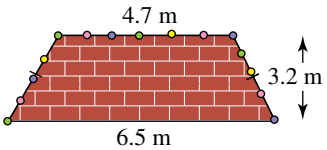
b AE

c the total length of the race.

9 A painter uses a trestle to stand on in order to paint a ceiling. It consists of 2 stepladders connected by a 4 m long plank. The inner feet of the 2 stepladders are 3 m apart, and each ladder has sloping sides of 2.5 m. How high off the ground is the plank?

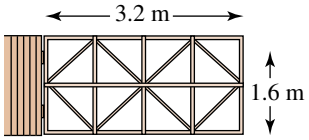


10 A feature wall in a garden is in the shape of a trapezium, with parallel sides of 6.5 m and 4.7 m. The wall is 3.2 m high. It is to have fairy lights around the perimeter (except for the base). How many metres of lighting are required?

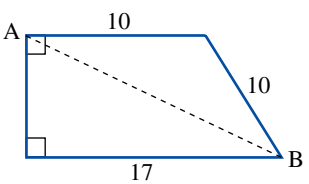


REASONING

11 A rectangular gate is 3.2 m long and 1.6 m high, and consists of three horizontal beams, and five vertical beams as shown in the diagram at right. Each section is braced with diagonals. How much timber is needed for the gate?

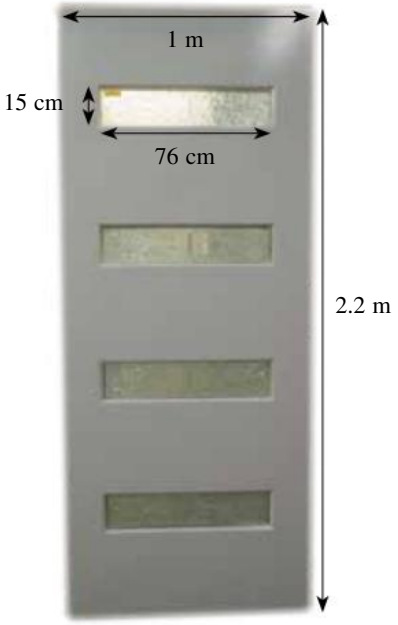


12 Find the distance AB in the following plan of the paddock. Distances are in metres.



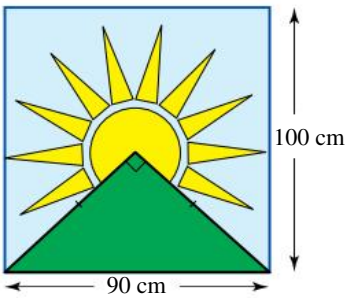
PROBLEM SOLVING

13 The front door shown is 1 m wide and 2.2 m high and has four identical glass panels, each 76 cm long and 15 cm wide.



- **a** Calculate the total area of the glass panels.
- **b** The door is to be painted inside and outside. Calculate the total area to be painted.
- **c** Two coats of paint are needed on each side of the door. If the paint is sold in 1-L tins at \$24.95 per litre and each litre covers 8 m^2 of the surface, calculate the total cost of painting the door.

14 The leadlight panel shown depicts a sunrise over the mountains. The mountain is represented by a green triangle 45 cm high. The yellow sun is represented by a section of a circle with an 18-cm radius. There are 10 yellow sunrays in the shape of isosceles triangles with a base of 3 cm and a height of 12 cm. The sky is blue. Calculate the area of the leadlight panel made of:



- **a** green glass
- **b** yellow glass
- **c** blue glass

15 On each of the four sides of a square an equilateral triangle is drawn, the final figure resembling a four pointed star. If the sides of the square are $\frac{a}{b}$ cm long, what is the area of the complete figure?

15.7 Pythagorean triads

- Pythagorean triads, or triads for short, are 3 integers (whole numbers) that satisfy Pythagoras' theorem. This means that the 3 numbers could be the sides of a right-angled triangle.
- We test Pythagorean triads by first calculating c^2, then calculating $a^2 + b^2$. If both calculations yield the same result, the original 3 numbers form a Pythagorean triad.
- The numbers 3, 4 and 5 form a Pythagorean triad because $3^2 + 4^2 = 25$ and $5^2 = 25$. If we multiply each of these 3 numbers by another number, the resulting numbers will also form a Pythagorean triad. For example, if we multiply 3, 4 and 5 by 5, we get 15, 20 and 25. These 3 numbers satisfy Pythagoras' theorem.
  Check: $c^2 = 25^2 = 625$
  $a^2 + b^2 = 15^2 + 20^2 = 625$
  so $c^2 = a^2 + b^2$
- Algebra can be used to find sets of numbers that are Pythagorean triads. There are two ways in which this can be done.

Method 1

- Start with any odd number, and make this the shortest length of the triangle.

- Use the formula $M = \frac{s^2 - 1}{2}$ where S = shortest side and M = middle side, to calculate the middle side of the triangle.
- You now have two sides of the triangle and can use $c^2 = a^2 + b^2$ to calculate the third side (hypotenuse, c).

WORKED EXAMPLE 9

If the smallest number of a Pythagorean triad is 7, find the middle number and, hence, find the third number.

THINK	WRITE
1 Write down the given information, and the formula to find the middle side, M.	$S = 7$ $M = \frac{S^2 - 1}{2}$
2 Substitute $S = 7$ into the formula and evaluate.	$= \frac{7^2 - 1}{2}$ $= \frac{49 - 1}{2}$ $= \frac{48}{2}$ $= 24$ The middle number is 24.
3 Use Pythagoras' theorem to find the third number.	$c^2 = a^2 + b^2$ $= 7^2 + 24^2$ $= 49 + 576$ $= 625$ $c = \sqrt{625}$ $= 25$ The third number is 25.
4 State the solution.	The Pythagorean triad required is 7, 24, 25.

- Why do you think this rule works only when the smallest side is an odd number?

Method 2

- This method takes any 2 numbers, and applies a set of rules to them, in order to generate Pythagorean triads. Select 2 numbers, x and y, with the following rules:
  Rule 1 The number chosen for x must be larger than y; that is $x > y$.
  Rule 2 One number must be odd and the other even, to find the first triad.
  Rule 3 The numbers chosen for x and y must have no common factors. For example, we could not choose 6 and 9 since they have a common factor of 3.
- The Pythagorean triad is given by $2xy$, $x^2 - y^2$, $x^2 + y^2$. Additional triads can be found by multiplying each of these side lengths by a common multiple.

WORKED EXAMPLE 10

Find a Pythagorean triad using the values $x = 7$ and $y = 2$.

THINK	WRITE
1 Write down the values given in the question.	$x = 7, y = 2$
2 Substitute these values into the expression for each term.	$2xy = 2 \times 7 \times 2$ $= 28$ $x^2 - y^2 = 7^2 - 2^2$ $= 49 - 4$ $= 45$ $x^2 + y^2 = 7^2 + 2^2$ $= 49 + 4$ $= 53$
3 Check that the numbers obtained satisfy Pythagoras' theorem. Does $c^2 = a^2 + b^2$?	$c^2 = 53^2$ $= 2809$ $a^2 + b^2 = 28^2 + 45^2$ $= 784 + 2025$ $= 2809$ Therefore, $c^2 = a^2 + b^2$.
4 State the solution.	The Pythagorean triad required is 28, 45, 53.

assessON

Exercise 15.7 Pythagorean triads

REFLECTION
What strategies could you use to memorise as many common triads as possible?

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–9	Questions: 1–10, 12	Questions: 1–13

Individual pathway interactivity int-4474 eBookplus

eBookplus
Interactivity
Pythagorean triples
int-2765

FLUENCY

1 Use Pythagoras' theorem to decide which of these triangles are right-angled.

a 6, 8, 10 **b** 5, 12, 13 **c** 4, 5, 6
d 24, 7, 25 **e** 16, 20, 12 **f** 14, 16, 30

2 Use the triads below to create three other triads, and check that they satisfy Pythagoras' theorem.

a 3, 4, 5 **b** 5, 12, 13

3 WE9 The smallest numbers of four Pythagorean triads are given below. Find the middle number and, hence, find the third number.

a 9 **b** 11 **c** 13 **d** 29

4 What do you notice about the triads in question **3**?

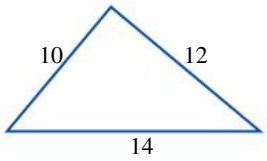
5 WE10 For each of the following, find a Pythagorean triad using the given values of x and y. (In each case check that the 3 numbers found satisfy Pythagoras' theorem.)

a $x = 6$ and $y = 1$ **b** $x = 7$ and $y = 4$ **c** $x = 8$ and $y = 3$
d $x = 11$ and $y = 6$ **e** $x = 14$ and $y = 9$ **f** $x = 15$ and $y = 2$

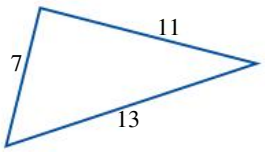
UNDERSTANDING

6 Are the following triangles right-angled? Use Pythagoras' theorem to check.

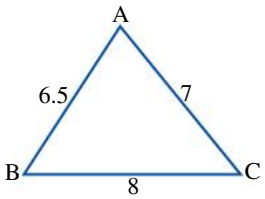
a



b



7 Use Pythagoras' theorem to prove that the following triangle is not right-angled.



8 **a** Does the method used in question **5** work if both x and y are even?

b Does it work if both x and y are odd?

Try some examples of your own.

REASONING

9 If 32, x, 68 is a Pythagorean triad, what is the value of x? Show your working.

10 Find a Pythagorean triad in which the smallest number is 33. Explain how you reached the solution.

PROBLEM SOLVING

11 Prove that $(a^2 - b^2)$, $2ab$, $(a^2 + b^2)$ form a Pythagorean triad.

12 The lengths of the sides of a particular triangle are a cm, $2a$ cm and $3a$ cm. Prove that the triad formed is not a Pythagorean triad.

13 **a** If $(p - q)$, p, $(p + q)$ form a Pythagorean triad, find a relationship between p and q.

b If $p = 8$, find the Pythagorean triad.

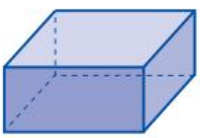
eBookplus
Digital doc
WorkSHEET 15.2
doc-11433

CHALLENGE 15.2

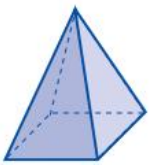
In a right-angled triangle the smallest angle is two-thirds the size of the other angle, which is not a right angle. What is the size of the smallest angle?

15.8 Pythagoras in 3-D

- Many real-life situations involve 3-dimensional (3-D) shapes: shapes with length, width and height. Some common 3-D shapes used in this section include boxes and pyramids.
- The important thing about 3-D shapes is that in a diagram, right angles may not look like right angles, so it is important to redraw sections of the diagram in 2 dimensions, where the right angles can be seen accurately.



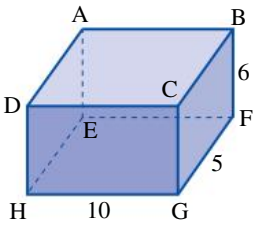
Box

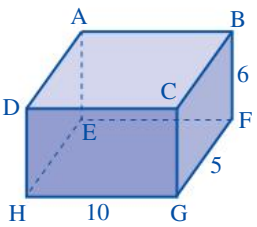
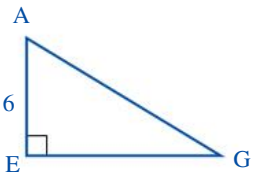
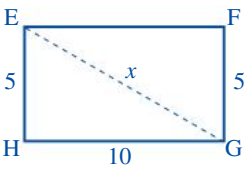
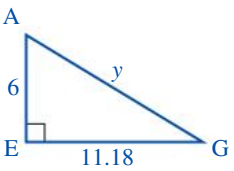


Pyramid

WORKED EXAMPLE 11

Find the length AG in this box.



THINK	WRITE/DRAW
1 Draw the diagram in 3-D.	
2 Draw, in 2-D, a right-angled triangle that contains AG and label the sides. Only 1 side is known, so we need to find another right-angled triangle to use.	
3 Draw EFGH in 2-D and show the diagonal EG. Label the side EG as x.	
4 Use Pythagoras' theorem to find EG.	$c^2 = a^2 + b^2$ $x^2 = 5^2 + 10^2$ $= 25 + 100$ $= 125$ $x = \sqrt{125}$ ≈ 11.18
5 Place this information on triangle AEG. Label the side AG as y.	
6 Solve this triangle for AG.	$c^2 = a^2 + b^2$ $y^2 = 6^2 + 11.18^2$ $= 36 + 125$ $= 161$ $y = \sqrt{161}$ ≈ 12.69
7 Write your answer.	The length of AG is 12.69.

Exercise 15.8 Pythagoras in 3-D

assess on

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–6, 10	Questions: 1–8, 10	Questions: 1–11

Individual pathway interactivity int-4475 eBookplus

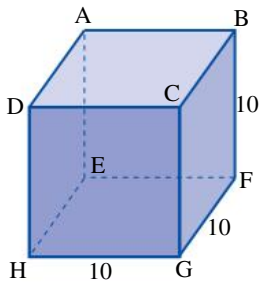
REFLECTION
What are the key features to look for when solving a 3-D problem involving triangles?

Where appropriate, give answers correct to 2 decimal places.

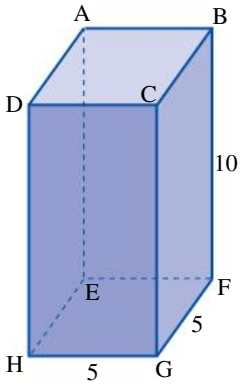
FLUENCY

1 WE11 Find **i** length EG and hence **ii** length AG.

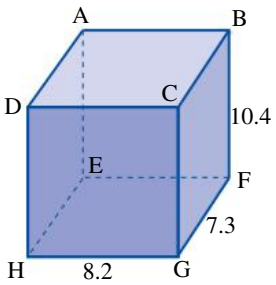
a



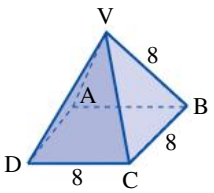
b



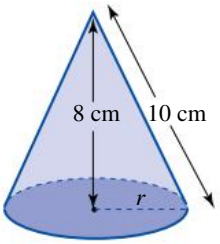
c



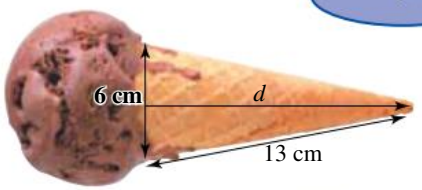
2 Find BD and, hence, the height of the pyramid at right.



3 The sloping side of a cone is 10 cm and the height is 8 cm. Find the radius, r, of the base.



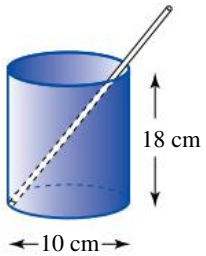
4 An ice-cream cone has a diameter across the top of 6 cm, and sloping side of 13 cm. How deep is the cone?



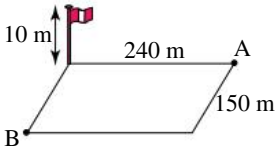
5 Jodie travels to Bolivia, taking with her a suitcase as shown in the photograph. She buys a carved walking stick 1.2 m long. Will she be able to fit it in her suitcase for the flight home?

UNDERSTANDING

6 A desk tidy is shaped like a cylinder, height 18 cm and diameter 10 cm as shown. A pencil that is 24 cm long rests inside. What length of the pencil is above the top of the cylinder?



7 A 10 m high flagpole is in the corner of a rectangular park that measures 240 m by 150 m.



a Find:

i the length of the diagonal of the park

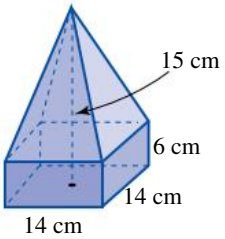
ii the distance from A to the top of the pole

iii the distance from B to the top of the pole.

b A bird flies from the top of the pole to the centre of the park. How far does it fly?

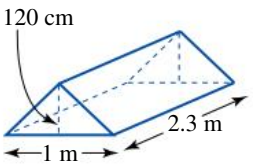
REASONING

8 The total height of the shape at right is 15 cm. Find the length of the sloping side of the pyramid.



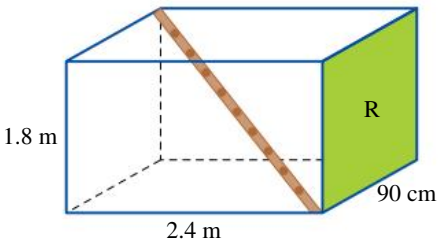
9 A tent is in the shape of a triangular prism, with a height of 120 cm as shown at right. The width across the base of the door is 1 m, and the tent is 2.3 m long.

Find the length of each sloping side, in metres. Then find the area of fabric used in the construction of the sloping rectangles which form the sides.



PROBLEM SOLVING

10 A wooden plank of the greatest possible length is placed inside a garden shed. Use the diagram to calculate:



a the length of the diagonal of the floor of the shed

b the length of the plank of wood to 1 decimal place.

11 A square-based pyramid has a height of h m and a base side length of b m.

a Find the length of the slant height in terms of h and b.

b If the volume of a pyramid is $\frac{1}{3}$ of the volume of the prism into which it snugly fits, find the volume of the pyramid in terms of h and b.

eBookplus ONLINE ONLY

15.9 Review

www.jacplus.com.au

The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:

- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

Review questions

Download the Review questions document from the links found in your eBookPLUS.

eBookplus

Interactivities

Word search
int-3385

Crossword
int-3386

Sudoku
int-3194

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

3 dimensions	Pythagoras' theorem	right-angled triangle
composite shape	Pythagorean triads	solve
hypotenuse	right angle	units

Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

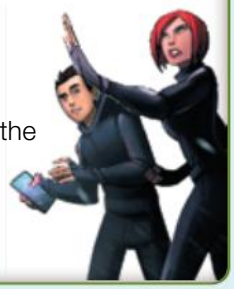
assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

www.assesson.com.au

Link to SpyClass, an exciting online game combining a comic book–style story with problem-based learning in an immersive environment.

Join Jesse, Toby and Dan and help them to tackle some of the world's most dangerous criminals by using the knowledge you've gained through your study of mathematics.

www.spyclass.com.au

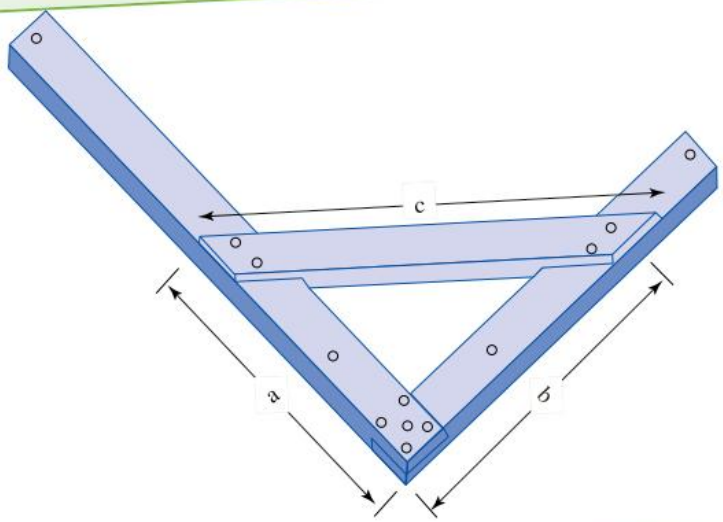


INVESTIGATION

RICH TASK

Are these walls at right angles?

Builders often use what is called a 'builder's square' when pegging out the foundations of a building to ensure that adjacent walls are at right angles. Even an error of just 1 or 2 degrees could mean that walls at the opposite end of a building may not end up in line. A builder's square uses the properties of Pythagoras' theorem. The diagram below shows how a builder's square is constructed. The hypotenuse side, *c*, acts as a brace to keep the two adjacent sides, *a* and *b*, in the correct position.



We will investigate the use of the builder's square. Cut two thin strips of paper to represent the arms a and b as shown in the diagram above. Join these two strips at one point with a pin to make the shape of the builder's square. The length of c can be obtained by measuring the distance from the end of a to the end of b. Complete questions 1 to 4, and record your results in the table on the following page.

1 Open the arms so that they make an angle of 90°. Use a protractor to measure the angle. Carefully measure the strips to obtain the values for *a*, *b* and *c* in millimetres. Record your results in the first row of the table.

2 Open the arms so that they make an angle less than 90°. Use a protractor to measure the angle. Carefully measure the strips to obtain the values for *a*, *b* and *c* in millimetres. Complete the second row of the table.

3 Repeat question **2** with the arms opened up to an angle greater than 90°. Complete the third row of the table.

4 Change the lengths of *a* and *b* by constructing a new builder's square. Repeat questions **1** to **3** and complete the last three rows of the table.

Length of a	Length of b	Length of c	$a^2 + b^2$	c^2	Angle

5 Consider the last three columns of the table. Using your results, what conclusions can you draw?

6 Construct a new builder's square and open it to an angle that you estimate to be 90°. Take measurements of *a*, *b* and *c* and record them below. What conclusions would you draw regarding your angle estimate? Check by measuring your angle with a protractor.

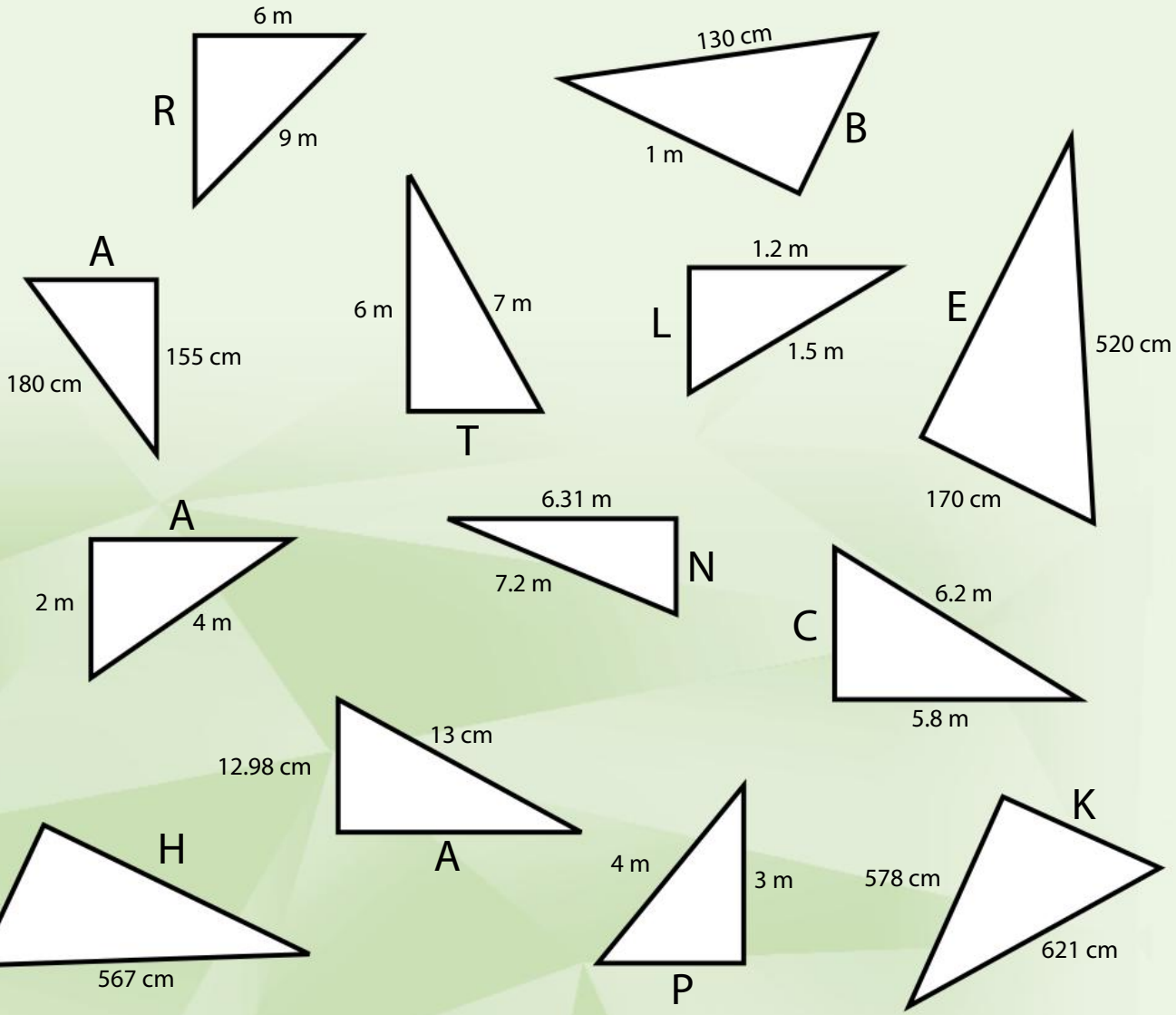
7 From your investigations, write a paragraph outlining how the measurements of the lengths of *a*, *b* and *c* on a builder's square can be used by the builder to determine whether the angle between adjacent walls would be equal to, greater than or less than a right angle.

CODE PUZZLE

What is Bagheera in *The Jungle Book*?

Calculate the lengths of the lettered sides in the triangles. Place the lengths with their letters above them, in ascending order in the boxes below. The letters will spell out the puzzle's answer.

Give lengths accurate to 2 decimal places.



eBookplus Activities

15.2 Right-angled triangles

Digital docs
- SkillSHEET (doc-11425) Measuring lengths and angles
- SkillSHEET (doc-11426) Finding the square of a number

Interactivity
- IP interactivity 15.2 (int-4469) Right-angled triangles

15.3 Finding the hypotenuse

Digital docs
- SkillSHEET (doc-11427) Finding the square root of a number
- SkillSHEET (doc-11428) Rounding to a given number of decimal places

Interactivity
- IP interactivity 15.3 (int-4470) Finding the hypotenuse

15.4 Finding a shorter side

Digital docs
- SkillSHEET (doc-11429) Rearranging formulas
- WorkSHEET 15.1 (doc-11431)

Interactivity
- IP interactivity 15.4 (int-4471) Finding a shorter side

15.5 Working with different units

Digital doc
- SkillSHEET (doc-11430) Converting units of length

Interactivity
- IP interactivity 15.5 (int-4472) Working with different units

15.6 Composite shapes

Interactivity
- IP interactivity 15.6 (int-4473) Composite shapes

15.7 Pythagorean triads

Interactivities
- Pythagorean triples (int-2765)
- IP interactivity 15.6 (int-4474) Sketching linear graphs

15.8 Pythagoras in 3-D

Interactivity
- IP interactivity 15.8 (int-4475) Pythagoras in 3-D

Digital doc
- WorkSHEET 15.2 (doc-11433)

15.9 Review

Interactivities
- Word search (int-3385)
- Crossword (int-3386)
- Sudoku (int-3194)

Digital docs
- Topic summary (doc-10766)
- Concept map (doc-10779)
- Topic review (Word doc-14950, PDF doc-14951)

To access eBookPLUS activities, log on to www.jacplus.com.au

Answers

TOPIC 15 Pythagoras' theorem

15.2 Right-angled triangles

1

	a	b	c	d	e	f	g	h
a	19	24	27	16	31	41	13	24
b	35	32	30	40	25	19	45	24
c	40	40	40	43	40	45	47	34
a^2	361	576	729	256	961	1681	169	576
b^2	1225	1024	900	1600	625	361	2025	576
$a^2 + b^2$	1586	1600	1629	1856	1586	2042	2194	1152
c^2	1600	1600	1600	1849	1600	2025	2209	1156

2 $a^2 + b^2 \approx c^2$
3 $a^2 + b^2 = c^2$
4 $a^2 + b^2 \neq c^2$
5 The angle sum of a triangle is 180°. If a triangle contained two right angles, the third angle would be 0°, which would mean it was a straight line rather than a triangle.
6 g The square on the hypotenuse is equal to the sum of the squares on the two other sides.
7 It is not a possible triangle because $7 + 4 < 12$.
8 90°. The right angle is 90° and the angle sum of a triangle is 180°. Therefore the sum of the other two angles must equal 90°.
9 The triangle is isosceles; the smaller angles are 45°.
10 It is not possible as all angles are 60° in an equilateral triangle.
11 a Answers will vary.
b i 0.577
ii 0.5
iii 0.866
c The values are the same for all the triangles.

15.3 Finding the hypotenuse

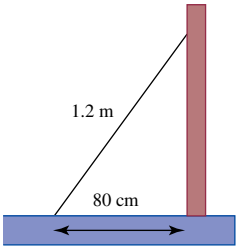
1 a $x = 5$ **b** $x = 13$ **c** $x = 25$
d $x = 21.3$ **e** $x = 1163.3$ **f** $x = 6.3$
2 a $\sqrt{9760}$ **b** $\sqrt{11\,194}$ **c** $\sqrt{394}$
d $\sqrt{16\,781}$ **e** 100 **f** $\sqrt{68}$
3 $\sqrt{146}$ cm
4 a 16.97 cm **b** 28.28 mm **c** 6.93 cm
5 a 12.81 cm **b** 737.83 cm **c** 17.26 cm
6 18.03 cm
7 19.23 cm
8 4.66 m
9 $81.60
10 a 189 cm^3 **b** 6.95 cm
11 25 m
12 314.16 cm^2

Challenge 15.1

Side length ≈ 2.83 cm; area = 8 cm^2

15.4 Finding a shorter side

1 a 6 **b** $\sqrt{93}$ **c** $\sqrt{6032}$
d $\sqrt{62\,925}$ **e** $\sqrt{15}$ **f** $\sqrt{80}$
2 a 36.36 **b** 1.62 **c** 15.37
d 0.61 **e** 2133.19 **f** 453.90
3 23.04 cm
4 a 97.47 cm **b** 334.94 cm **c** 6822.90 cm^2
5 17.32 cm **6** 65.82 cm; 2501.16 cm^2
7 2.60 m **8** 1.32 m
9 Yes **10** 13.69 m
11 20.61 m **12** 45.43 cm
13 a



b 89.44 cm
c Yes, she will reach the hook from the top step.
14 a 4 cm
b The diagonals are perpendicular to each other because they create right-angled triangles.

15.5 Working with different units

1 130 mm
2 a 55.71 cm **b** 29.71 mm **c** 57.27 m
d 3098.08 mm **e** 6.53 m **f** 5.74 km
g 10.16 m **h** 3.16 km
3 a 386.13 mm **b** 62.09 cm **c** 2.33 km
d 16.15 cm **e** 541.70 cm **f** 2615.61 m
g 478.97 mm **h** 369.87 km
4 a 176.16 cm **b** 147.40 cm **c** 2.62 km
d 432.12 m **e** 165.88 m **f** 6811.55 m
5 a C **b** A, B **c** D **d** B
6 54.67 mm **7** 36.37 cm
8 a 28 cm **b** 588 cm^2
9 552.86 cm^2
10 0.51 m **11** 8.65 m
12 1600 mm **13** 6.43 km
14 21.46 diagonals, so would need to complete 22
15 5889.82 m **16** 70.71 mm
17 Yes **18** $a^2 = 80b^2$

15.6 Composite shapes

1 8.06 cm
2 a $x = 7.62$ **b** $x = 60$ **c** $x = 20.87$
3 a $x = 4$, $y = 9.17$ **b** $x = 6.93$, $y = 5.80$
c $x = 13$, $y = 15.20$ **d** $x = 19.55$
4 a C **b** B **c** B **d** D
5 1.2 m; 4.8 m^2
6 10 m **7** 917.88 m
8 a 5 km **b** 4 km **c** 24 m
9 2.45 m **10** 11.35 m
11 26.65 m **12** 18.44 m
13 a 4560 cm^2 or 0.456 m^2
b 34 880 cm^2 or 3.488 m^2
c The paint job would need 1 tin of paint, costing $24.95. The cost of the paint used would be $21.76.
14 a 2025 cm^2 **b** 943.41 cm^2 **c** 6031.59 cm^2
15 $(1 + \sqrt{3})\frac{a^2}{b^2}$

15.7 Pythagorean triads

1 Yes: **a**, **b**, **d**, **e** No: **c**, **f**
2 Answers will vary. Possible answers:
a 6, 8, 10; 9, 12, 15; 12, 16, 20
b 10, 24, 26; 15, 36, 39; 20, 48, 52
3 a 40, 41 **b** 60, 61 **c** 84, 85 **d** 420, 421

4 The two larger numbers are consecutive numbers.
5 **a** 12, 35, 37 **b** 33, 56, 65 **c** 48, 55, 73
d 85, 132, 157 **e** 115, 252, 277 **f** 60, 221, 229
6 **a** $10^2 + 12^2 \neq 14^2$ No **b** $7^2 + 11^2 \neq 13^2$ No
7 $6.5^2 + 7^2 = 91.25$
$8^2 = 64$
Therefore, the triangle is not right-angled.
8 **a** Yes **b** Yes
9 60
10 The following Pythagorean triads can be found: 33, 44, 55; 33, 56, 65; 33, 180, 183; 33, 544, 545. Explanations will vary.
11, **12** Check with your teacher.
13 **a** $p = 4q$ **b** 6, 8, 10

Challenge 15.2
36°

15.8 Pythagoras in 3-D
1 **a** **i** 14.14 **ii** 17.32
b **i** 7.07 **ii** 12.25
c **i** 10.98 **ii** 15.12
2 11.31, 5.66
3 6 cm
4 12.65 cm
5 No: maximum stick can only be 115 cm long.
6 3.41 cm
7 **a** **i** 283.02 m **ii** 240.21 m **iii** 150.33 m
b 141.86 m
8 13.38 cm
9 1.3 m, 5.98 m^2
10 **a** 2.563 m **b** 3.1 m
11 **a** $\sqrt{h^2 + \frac{b^2}{4}}$
b $\frac{1}{3}b^2h$

Investigation — Rich task
1 to **4** Answers will vary. Teacher to check.
5 When the angle is equal to 90°, $c^2 = a^2 + b^2$.
For an angle greater than 90°, $c^2 > a^2 + b^2$.
For an angle smaller than 90°, $c^2 < a^2 + b^2$.
6 If $c^2 = a^2 + b^2$, then the angle is equal to 90°. Check by measuring the angle.
7 Paragraph explanation of the results in question **5**; teacher to check.

Code puzzle
A black panther

TOPIC 16

Numeracy 3

16.1 Overview

Why learn this?

Why should you learn to count numbers? Imagine going into a shop and asking for a dozen eggs. How could you tell how many eggs you were getting? On trust? Was your change correct? How could you be sure?

You cannot go through life without learning to accurately add, subtract, multiply and divide.

What do you know?

assessON

1 **THINK** List what you know about numeracy. Use a thinking tool such as a concept map to show your list.
2 **PAIR** Share what you know with a partner and then with a small group.
3 **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of numeracy.

Learning sequence

16.1 Overview
16.2 Set E
16.3 Set F

WATCH THIS VIDEO
NAPLAN: strategies and tips

Searchlight ID: eles-1688

16.2 Set E

Calculator allowed

1 The students in a class measure their heights in cm. The following stem-and-leaf plot shows their heights.
Key: 13 | 5 means 135 cm.

Stem	*Leaf*
13	5 8
14	0 4 9
15	3 5 5
16	3

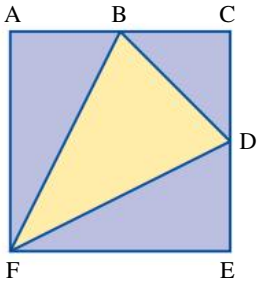
What is the mean height?

A 148 cm **B** 149 cm **C** 155 cm **D** 163 cm

2 You have a rectangular box with a lid. The top of the lid has an area of 392 square centimetres. The ratio of the width to the length of the lid is 1 : 8. What are the dimensions of the lid?

A 4 cm by 98 cm **B** 7 cm by 56 cm
C 8 cm by 49 cm **D** 8 cm by 64 cm

3 The following is a design of a quilt square (ACEF) whose sides are 5 cm long. B and D are the midpoints of sides AC and CE respectively. What is the area of the triangle BDF?



A 6.25
B 9.375
C 12.5
D 15.625

4 A photograph is placed on a card measuring 8.5 cm by 10 cm. A 1.5-cm edging is left all around. What area (in square cm) does the photograph cover?

A 59.5 cm^2
B 49 cm^2
C 46.75 cm^2
D 38.5 cm^2

5 A student recorded the temperature outside her classroom every hour for one school day. She drew up the following table.

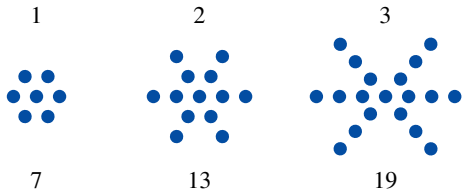
Time	9 am	10 am	11 am	12 pm	1 pm	2 pm	3 pm
Temperature °C	11	13		34	36	33	29

If the temperature increased from 11 am to midday by $41\frac{2}{3}\%$, what was the temperature at 11 am?

A 22° **B** 23°
C 24° **D** 25°

6 The school bus took $1\frac{1}{4}$ hours to travel 80 km. How far did it travel in $2\frac{1}{2}$ hours?

7 Consider the following pattern.



Which one of the following equations could model the design, where C represents the total number of dots (the numbers shown below) and n represents the series number (the numbers shown above)?

A $C = 7n - 1$ **B** $C = 6n$
C $C = 6n + 1$ **D** $C = 6n - 1$

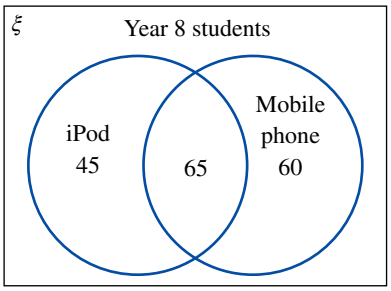
8 A bag of fertiliser is made by mixing nitrates, potash and phosphates in the ratio of 3 : 2 : 5. How much fertiliser will be produced if 15 kg of nitrates are used?

A 22 kg **B** 25 kg **C** 45 kg **D** 50 kg

9 Fuel for a two-stroke engine is made by mixing petrol and oil in the ratio 30 : 1. If you have 250 mL of oil, how much petrol needs to be added to make a two-stroke mixture?

A 0.12 litre **B** 1.2 litres **C** 7.5 litres **D** 8.3 litres

10 The Venn diagram shows how many of the 200 Year 8 students have an iPod only, a mobile phone only, or both an iPod and a mobile phone. Use the information in the diagram to find the probability that a student chosen at random has neither an iPod nor a mobile phone.



A $\frac{1}{30}$ **B** $\frac{3}{20}$
C $\frac{6}{10}$ **D** $\frac{1}{20}$

11 The Orion Nebula is approximately 1.5×10^3 light-years away from Earth. It is visible with the naked eye. One light-year is approximately 9.5×10^{12} km. What is the approximate distance in kilometres between Earth and the Orion Nebula?

A 1.425×10^{15}
B 1.425×10^{16}
C 1.1×10^{11}
D 6.33×10^9

12 An express train leaves Geelong at 8.55 am and arrives in Altona station at 9.37 am. If the train travelled 52 km, what was the train's average speed in km/h?

A 98 km/h **B** 84 km/h **C** 74 km/h **D** 71 km/h

13 A packet of M&Ms contains red, orange, blue, green, yellow and brown chocolates. The probability of choosing a colour from the packet is shown in the table below.

Colour	Red	Orange	Blue	Green	Yellow	Brown
Probability	0.3	0.13	0.12		0.25	0.09

What is the probability of choosing a green M&M?

A 0.9 **B** 0.38

C 0.11 **D** 0.1

14 The following table shows the number of M&Ms of different colours in a particular bag.

Colour	**Number**	**Expression**
Red		x
Orange		$x - 2$
Blue		$x - 1$
Green		$x + 1$
Yellow		$x - 3$
Brown		$x + 2$
Total	33	

How many yellow M&Ms are in the bag?

A 30 **B** 20 **C** 10 **D** 3

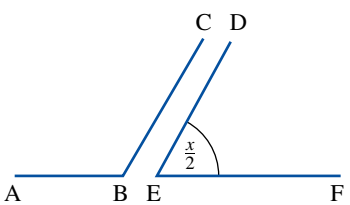
15 In a bag of 200 jelly beans, the following distribution of colours was found.
Brown 13%, yellow 14%, red 13%, blue 24%, orange 20%, green 16%
How many blue jelly beans are in the bag?

A 20 **B** 24 **C** 48 **D** 40

16 'Dragon beards' is a special Chinese dish. The noodles are hand-pulled until they are extraordinarily fine. It has been calculated that a piece of paste prepared with 1.5 kilograms of wheat flour can make 144 000 hair-thin noodles, each 20 centimetres long. If you joined all the strands together, what would be their total length?

A 28.8 km **B** 288 km **C** 28 800 km **D** 288 000 km

17 The angles below are supplementary. The measure of ∠DEF is $\frac{x}{2}$.



What expression represents the measure of ∠ABC?

A $90 - \frac{x}{2}$ **B** $90 + \frac{x}{2}$ **C** $180 - \frac{x}{2}$ **D** $180 + \frac{x}{2}$

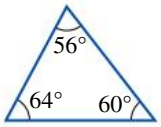
18 A driver drives 8 km south, then 6 km east followed by 2 km south. He then travels 3 km west. Next, in order to avoid a traffic jam, he turns and travels 6 km north. How many kilometres is the driver south of his starting point?

19 If the ratios of the sides of a quadrilateral are given in order as 3 : 3 : 4 : 4, what type of quadrilateral must it be?

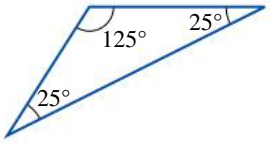
A Rectangle **B** Square
C Kite **D** Parallelogram

20 The angles of a triangle are in the ratio 3 : 5 : 7. Which diagram has angles in this ratio?

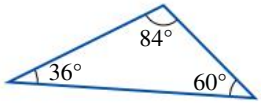
A



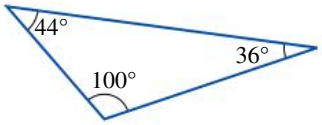
B



C

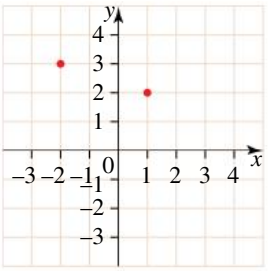


D



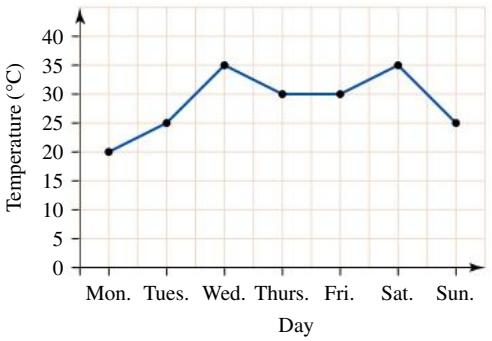
21 A trapezium was created on a Cartesian plane using the coordinates A (2, 0), B (−2, 0), C (−3, 4) and D (x, 4). What is the value for x if the area of the trapezium is 24 units?

22 Ann is drawing a right-angled isosceles triangle on a coordinate grid. She has plotted two of the corners of the triangle at (−2, 3) and (1, 2). Which of the coordinates below would make the third corner a right-angled isosceles triangle?



A (−1, 4)
B (4, 3)
C (0, −1)
D (3, 5)

23 Ann recorded the highest temperature each day over a 7-day period in February. Which statement describes the data?



A Median > mean
B Mean = mode
C Median = mean
D Median < mean

24 Ethan wants to solve for x in this equation.

$1\frac{2}{5} - x \times \frac{2}{3} = 1\frac{2}{3}$

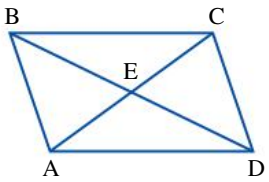
Which step should he perform first?

A Divide both sides by $\frac{2}{3}$.
B Multiply both sides by $\frac{2}{3}$.
C Subtract $1\frac{2}{5}$ from both sides.
D Add $1\frac{2}{5}$ to both sides.

25 Al-Samaw'al was simplifying the expression $6x - 3(x - 5)$. Which is the equivalent expression?

A $3x - 5$ **B** $3x + 5$ **C** $3x - 15$ **D** $3x + 15$

26 In the parallelogram ABCD below, AC and DB intersect at E.
AE = $3x - 3$ and EC = $x + 13$.



What is the value of x?

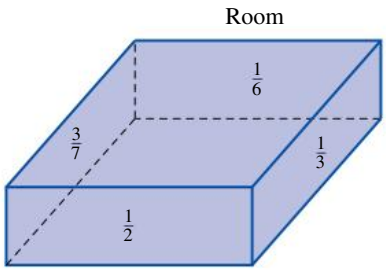
A 8 **B** 16 **C** 20 **D** 40

assessON

16.3 Set F

Non-calculator

1 A room has four identical walls that are being painted. For decorative purposes, each of the four walls is being painted in a variety of colours. One of the colours is blue, and the four walls will each have different areas that are painted this colour. The fraction of each wall that is painted blue is shown on the diagram.
What fraction of all 4 walls will be painted with this colour?



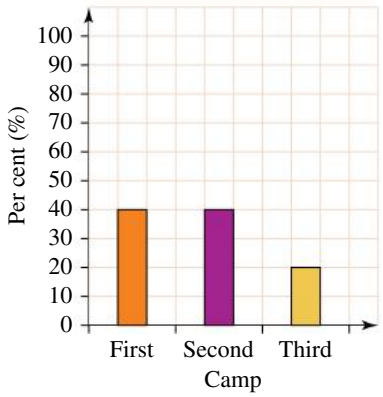
A $\frac{5}{7}$ **B** $\frac{6}{7}$
C $\frac{5}{14}$ **D** $1\frac{5}{7}$

2 Five friends earned money by helping to build a wall in a garden. After they divided the money equally, they each received $280. Which of the following equations could be used to determine x, the total amount (in dollars) that the five friends earned?

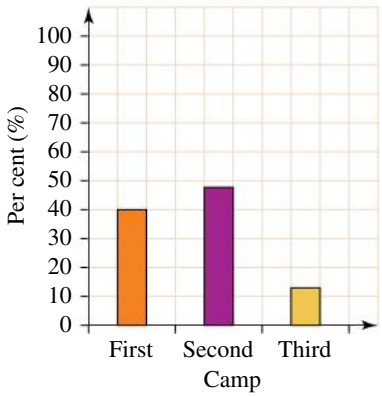
A $5x = 280$ **B** $x - 5 = 280$
C $x + 5 = 280$ **D** $\frac{x}{5} = 280$

3 There are 150 Scouts at a camp. For 40% of them, this is their first camp. One-fifth of the remainder have been to one other camp. The others have been to three camps. Which of the following represents the percentages of students on camp for the first, second and third times?

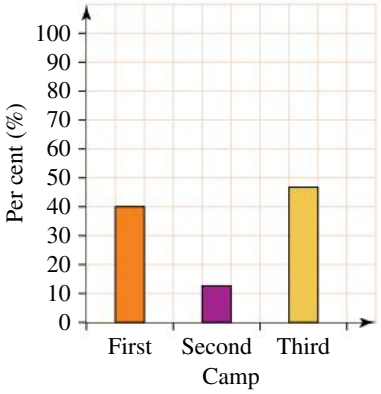
A



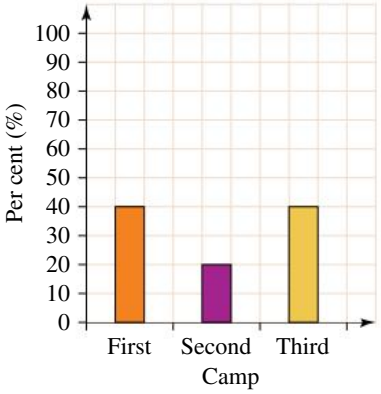
B



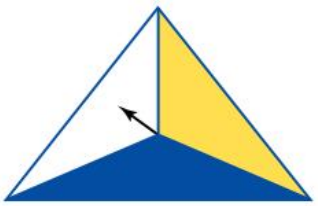
C



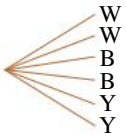
D



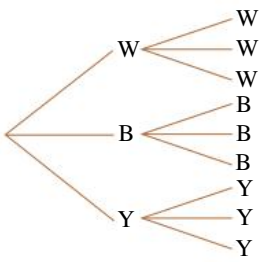
4 Ash spins the arrow twice on the spinner. Which tree diagram shows all the possible outcomes?



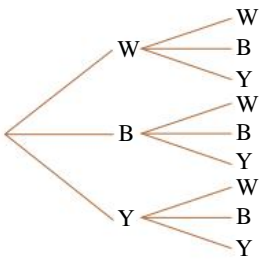
A



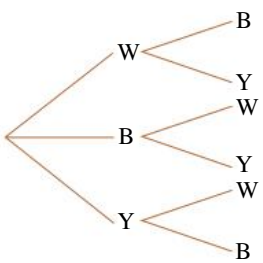
B



C



D



5 The human heart beats an average of 37 800 000 times in one year. Which of the following is an equivalent value?

A 3.78×10^{-7} **B** 3.78×10^{-6} **C** 3.78×10^{6} **D** 3.78×10^{7}

6 Which unit of measurement would be most appropriate for measuring the area of a page of newspaper?

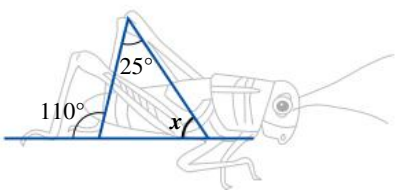
A mm^2 **B** cm^2 **C** m^2 **D** km^2

7 A hockey team played n games, losing five of them and winning the rest. The ratio of games won to games lost is:

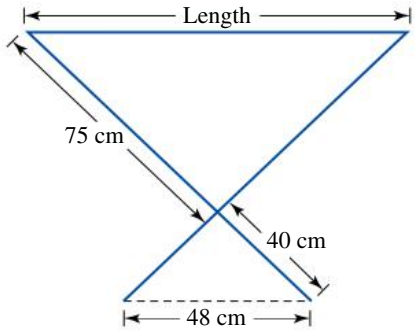
A $\frac{n}{5}$ **B** $\frac{n-5}{5}$ **C** $\frac{5}{n}$ **D** $\frac{5}{n-5}$

8 What is the value of the angle x?

A 85°
B 70°
C 110°
D 130°



9 An ironing board with the measurements shown is advertised online.



When the ironing board is set up, two similar triangles are formed. How long is the top?

A 90 cm **B** 62.5 cm **C** 51.2 cm **D** 48 cm

10 The stem-and-leaf plot below shows the age of each member of a bicycle club.

Bike club members' ages

Key: 1 | 8 represents 18

Stem	*Leaf*
1	8 8 9
2	3 4 6 6 6 7 9
3	1 2 5 7 8
4	0 2 5
5	2 7

What is the range of the ages of the bicycle club members?

A 57 **B** 39

C 36 **D** 30

11 The stem-and-leaf plot below shows the age of each member of a running club.

Running club members' ages

Key: 1 | 3 represents 13

Stem	*Leaf*
1	3 5 6 7 8 8 9
2	0 1 2 3 4 4 4 5
3	0 2 3 4 5

What is the median age of the club members?

A 22 **B** 22.5

C 24 **D** 24.5

12 The *Chef and Cook* television show produces two square-based cakes. The small cake has a side length of 8 cm while the large cake has a side length of 16 cm. Which of the following statements is true?

A The area of the base of the large cake is 2 times the area of the small cake.

B The area of the base of the large cake is 4 times the area of the small cake.

C The area of the base of the large cake is 8 times the area of the small cake.

D The area of the base of the large cake is 16 times the area of the small cake.

13 Calculate the value of the following.

$25 \times (2768 + 2768 + 2768 + 2768)$

14 The stage in the hall at League School has the shape of a quadrilateral, as shown.
Which of these are the most likely values of x and y?

	x	y
A	30	160
B	45	145
C	63	117
D	72	118

15 The numbers of passengers on a train over a 20-day period were recorded as follows:
59, 65, 73, 83, 90, 83, 71, 92, 60, 58, 96, 66, 75, 76, 85, 77, 86, 79, 87, 79
The data are displayed below on a stem-and-leaf plot.
Key: 5 | 9 represents 59

Stem	*Leaf*
5	8 9
6	0 5 ?
7	1 3 5 6 7 9 9
8	3 3 5 6 7
9	0 2 6

What is the missing number?

16 The table shows the results of a survey, which asked drivers how many accidents they had over the previous 5 years. What is the median number of accidents per year?

Number of accidents	0	1	2	3	4	5	6
Number of drivers	16	14	21	4	3	1	1

A 0.5 **B** 1 **C** 1.5 **D** 2

17 Alex wants to know the answer to the following expression.
$(5 \times 10^4) + (2 \times 10^2) + (4 \times 10) = \square$
Which of the following is correct?

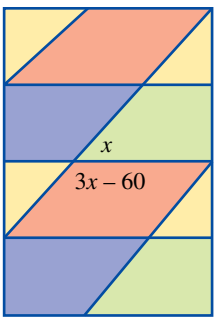
A 502 400 **B** 52 400 **C** 50 240 **D** 5240

18 What is the missing number in the following equation?

$$\frac{4}{3} \div \frac{?}{3} = \frac{1}{8}$$

19 My mother is 4 times as old as I am. My sister is 75% of my age and 10% of my grandfather's age. My father is 50, which is 2 years older than my mother. How old are my sister and grandfather?

20 In the quilt design below, what is the measure of x?

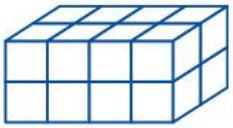


A 10° **B** 30° **C** 40° **D** 60°

21 Is the following equation true or false?

$12 + (5 \times 9) - (108 \div 2) = 3$

22 The solid brick shown is made of small cubic bricks of sides 1 unit. When the large brick is disassembled into its component small bricks, the total surface area of all the small bricks is how much greater than the surface area of the large brick?

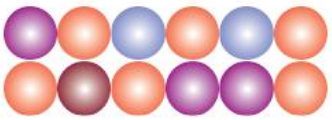


A 32 **B** 40 **C** 56 **D** 96

23 Jane ate 0.2 parts of pizza and her friend ate 0.02 parts of the same pizza. Find the ratio between the parts they ate.

A 10 : 1 **B** 5 : 2 **C** 1 : 1 **D** 1 : 10

24 A marble is selected at random. What is the probability that the marble will not be blue?



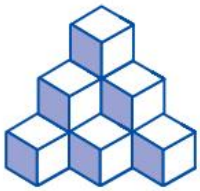
A $\frac{5}{6}$ **B** $\frac{1}{6}$ **C** $\frac{3}{14}$ **D** $\frac{3}{7}$

25 Write 70 as a percentage of 200.

26 If $4x + 3 = 22$, what is the value of $4x - 3$?

A -3 **B** 0 **C** 16 **D** 19

27 If this pattern continues, how many cubes will it take to make 10 layers?



28 The first number in a pattern is 48. To go from one number to the next the rule is to divide by 4. What is the fourth number in the pattern?

A $\frac{4}{3}$ **B** $\frac{3}{4}$ **C** $\frac{1}{3}$ **D** $\frac{1}{4}$

29 Which of the following is equivalent to the expression $2^6 \times 2^4$?

A 2^{24} **B** 2^{10} **C** 4^{24} **D** 4^{10}

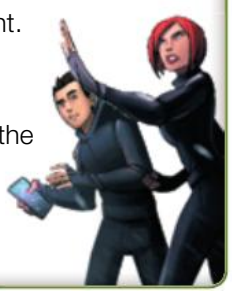
30 Andrew wants to find the value for the expression $\frac{6x + 5}{8x + 5}$ when $x = -10$. What is the value?

A $\frac{2}{3}$ **B** $\frac{-2}{3}$ **C** $\frac{11}{15}$ **D** $\frac{-11}{15}$

Link to SpyClass, an exciting online game combining a comic book–style story with problem-based learning in an immersive environment.

Join Jesse, Toby and Dan and help them to tackle some of the world's most dangerous criminals by using the knowledge you've gained through your study of mathematics.

www.spyclass.com.au



Answers

TOPIC 16 Numeracy 3

16.2 Set E Calculator allowed

1 A	**2** B	**3** B	**4** D
5 C	**6** 160 km	**7** C	**8** D
9 C	**10** B	**11** B	**12** C
13 C	**14** D	**15** C	**16** A
17 C	**18** 4 km	**19** C	**20** C
21 5	**22** C	**23** A	**24** C
25 D	**26** A		

16.3 Set F Non-calculator

1 C	**2** D	**3** C	**4** C
5 D	**6** B	**7** B	**8** A
9 A	**10** B	**11** B	**12** B
13 276 800	**14** C	**15** 6	**16** C
17 C	**18** 32	**19** 9, 90	**20** D
21 True	**22** C	**23** A	**24** A
25 35%	**26** C	**27** 220	**28** B
29 B	**30** C		

TOPIC 17

Programming

17.1 Overview

Why learn this?

A computer program creates instructions for a computer to follow to perform tasks; it makes thousands of decisions every second. Computer programs are written in precise codes that consist of words, letters and symbols specific to the particular computer language being used.

In this topic the computer language JavaScript is used in most of the examples and questions. JavaScript has been chosen for its popularity and availability on nearly all web browsers.

The end of the topic will focus on the importance of testing code to identify errors and problems. Once the errors are found, the program can be fixed, ensuring confidence in the program that has been written.

What do you know?

assess on

1 **THINK** Write what you know about computer programming languages. Use a thinking tool such as a concept map to display your ideas.
2 **PAIR** Share what you know with a partner and then with a small group.
3 **SHARE** As a class, combine all the ideas into one large concept map or poster.

Learning sequence

eBook plus ONLINE ONLY

17.1 Overview
17.2 Expressions
17.3 Decisions
17.4 Functions
17.5 Testing
17.6 Debugging
17.7 Review

eBookplus
Digital doc
How to write your own program
doc-18769

17.2 Expressions

Values

- Three of the most common types of values used in computer programming are:
  - **numbers** (numerical values): for example `12`, `3`, `-13`, `1.5`, `3.14`
  - **strings** (any text surrounded in quotation marks `""`): for example `"A string"`, `"9876"`, `"Some information"`, `"A name"`, `""`
  - **Booleans** (used for logic; this type only has two possible values): either `false` or `true`.
- In JavaScript the expression `typeof value` returns either `"number"`, `"string"` or `"boolean"`.

WORKED EXAMPLE 1

Answer these questions for each of the following values.
i Is the value a number, a string, or boolean?
ii Apply the expression `typeof value` to each value
iii What is the result of the expression `typeof value`?

a `"Benjamin"` **b** `-12.3` **c** `"A string"`
d `false` **e** `"12345"`

THINK	WRITE
a i `"Benjamin"` is in quotation marks, so this value is a string.	**a i** String
ii 1 Apply the expression `typeof value` to the value `"Benjamin"`.	**ii** `typeof "Benjamin"`
iii `typeof "Benjamin"` is `"string"`.	`"string"`
b i `-12.3` is a numerical value not in quotation marks, so this value is a number.	**b i** Number
ii 1 Apply the expression `typeof value` to the value `-12.3`.	**ii** `typeof -12.3`
iii `typeof -12.3` is `"number"`.	`"number"`
c i `"A string"` is in quotation marks, so this value is a string.	**c i** String
ii 1 Apply the expression `typeof value` to the value `"A string"`.	**ii** `typeof "A string"`
iii `typeof "A string"` is `"string"`.	`"string"`
d i `false` is a boolean value.	**d i** Boolean
ii 1 Apply the expression `typeof value` to the value `false`.	**ii** `typeof false`
iii `typeof false` is `"boolean"`.	`"boolean"`

e **i** `"12345"` is in quotation marks, so it is a string.		**e** **i** String
ii **1** Apply the expression `typeof value` to the value `"12345"`.		**ii** `typeof "12345"`
iii `typeof "12345"` is `"string"`.		`"string"`

Variables

- Computer languages use memory locations to store values. These named containers are called **variables**. There are complex rules as to what is a valid variable name. For simplicity, this topic will restrict the variable names to three simple rules.
- Variable names:
  - must not start with a number
  - can only contain upper case and lower case letters, numbers, and the underscore character (_), and cannot contain spaces
  - cannot be JavaScript keywords. The following are JavaScript keywords that should not be used as variable names.

```
abstract, arguments, Array, boolean, break, byte,
case, catch, char, class, const, continue, Date,
debugger, default, delete, do, double, else, enum,
eval, export, extends, final, finally, float, for,
function, goto, hasOwnProperty, if, implements,
import, in, Infinity, instanceof, int, interface,
isFinite, isNaN, isPrototypeOf, length, let, long,
Math, name, NaN, native, new, null, Number, Object,
package, private, protected, prototype, public,
return, short, static, String, super, switch,
synchronized, this, throw, throws, toString,
transient, true, try, typeof, undefined, valueOf,
var, void, volatile, while, with, yield, false
```

WORKED EXAMPLE 2

Each of the following is a valid variable name. True or false?

a `aVariable` **b** `2_second` **c** `a|or`
d `endsInNumber_1` **e** `case`

THINK	WRITE
a `aVariable` is valid because it only uses letters.	**a** True
b `2_second` is invalid because it uses a number as the first character.	**b** False
c `a|or` is invalid because it uses a `|` character.	**c** False
d `endsInNumber_1` is valid as it only uses letters, numbers and the _ character. It also does not start with a number.	**d** True
e `case` is invalid because `"case"` is a JavaScript keyword.	**e** False

Numerical expressions

eBookplus
Interactivity
CodeBlocks Activity 1
int-6573

- In JavaScript, numerical expressions involving numbers, brackets, and plus and minus signs should be evaluated as in any mathematical expression.
  - Multiplication uses the character `*`.
  - Division uses the character `/`.
  - Fractions can be evaluated using the division character, but the numerator and denominator expressions must be put in brackets.

$$\frac{9+8+7}{1+3+4} = (9+8+7)/(1+3+4)$$

WORKED EXAMPLE 3

Write the following mathematical expressions as JavaScript expressions.

a $1.21 + 3.3$ **b** $3 + 9 - (6 - 4)$ **c** 18×1.6

d $24 \div (3 \times 4)$ **e** $\frac{15-3}{7-1}$

THINK	WRITE
a This is a simple numerical expression with no special characters required.	**a** `1.21+3.3`
b This is a simple numerical expression with no special characters required.	**b** `3+9-(6-4)`
c Replace × with `*`.	**c** `18*1.6`
d Replace × with `*` and ÷ with `/`.	**d** `24/(3*4)`
e The fraction is a division. The numerator and denominator require brackets.	**e** `(15-3)/(7-1)`

- The `%` symbol is used to find the remainder after an integer division. For example, `17%5` evaluates to `2`, as 5 divides into 17 three times with a 2 remainder.

WORKED EXAMPLE 4

Evaluate the following JavaScript expressions manually.

a `21%2` **b** `18%7` **c** `123%25`

THINK	WRITE
a Divide 2 into 21: 2 goes into 21 ten times. There is a remainder of 1.	**a** `1`
b Divide 7 into 18: 7 goes into 18 two times. There is a remainder of 4.	**b** `4`
c Divide 25 into 123: 25 goes into 123 four times. There is a remainder of 23.	**c** `23`

String expressions

- Expressions are not restricted to numbers. Strings can also belong to expressions.
- Two or more strings can be 'concatenated' (joined together) using the `+` symbol. For example, `"We"+" are "+"joined"` evaluates to `"We are joined"`.

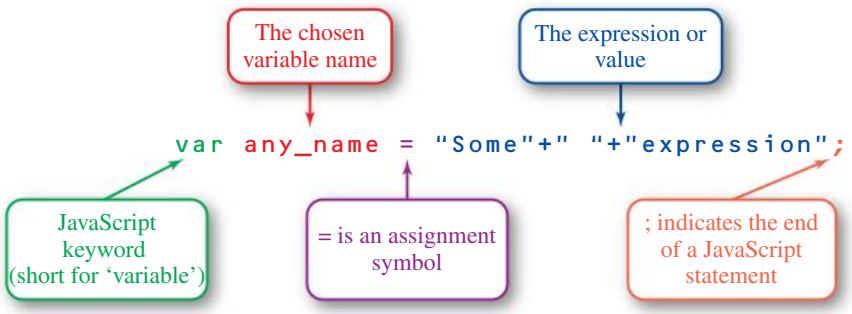
WORKED EXAMPLE 5

a Write a JavaScript expression to concatenate the strings `"Join ","to","get","her"`.
b Write the string `"I can separate strings"` as an expression of single words and spaces.

THINK	WRITE
a Replace the commas with `+`.	a `"Join "+"to"+"get"+"her"`
b Replace the spaces with `..."+" "+"...`	b `"I"+" "+"can" +" "+"separate" +" "+"strings"`

Assigning values or expressions

- When a value is assigned to a variable for the first time, the statement should begin with the JavaScript keyword `var`. JavaScript uses the following structure to assign a value or expression to a variable.



WORKED EXAMPLE 6

Write JavaScript statements to make the following variable assignments.
a Assign the value `100` to the variable `hundred`.
b Assign `false` to the Boolean variable `inUniversity`.
c Assign `"English"` to the variable `language`.
d Assign the expression `365/7` to the variable `weeks`.
e Assign the expression `42*1000` to the variable `metres`.

THINK	WRITE
a 1 Identify the variable name, which in this case is `hundred`.	a
2 Identify the value, which in this case is `100`.	
3 Apply the JavaScript structure to write the statement.	`var hundred = 100;`

b 1 Identify the variable name, which in this case is `inUniversity`.

2 Identify the value which in this case is `false`.

3 Apply the JavaScript structure to write the statement.

b

```
var inUniversity = false;
```

c 1 Identify the variable name, which in this case is `language`.

2 Identify the value, which in this case is `"English"`.

3 Apply the JavaScript structure to write the statement.

c

```
var language = "English";
```

d 1 Identify the variable name, which in this case is `weeks`.

2 Identify the expression, which in this case is `365/7`.

3 Apply the JavaScript structure to write the statement.

d

```
var weeks = 365/7;
```

e 1 Identify the variable name, which in this case is `metres`.

2 Identify the value, which in this case is `42*1000`.

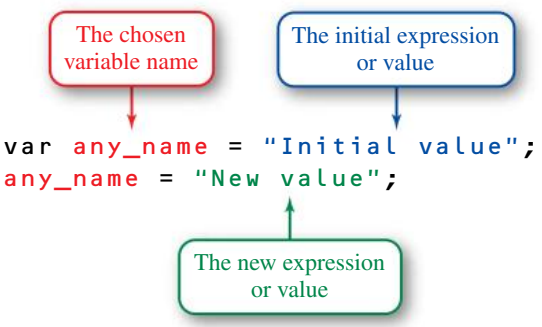
3 Apply the JavaScript structure to write the statement.

e

```
var metres = 42*1000;
```

Reassigning values or expressions

- A variable can change value as the program runs through a sequence of statements in order. When a variable is reassigned (i.e. changes value), the statement does not require the JavaScript keyword `var`. JavaScript uses the following sequence to assign and then reassign a value or expression to a variable. The value of the variable changes with each assignment.



WORKED EXAMPLE 7

Manually simulate running the following programs. What is the final value stored in the variable x for each program?

a
```
var x = 20;
x = 10;
```

b
```
var x = 2+12/4;
x = 2+4-3;
```

c
```
var x = 1;
var y = 12;
x = 12;
x = 15;
y = 10;
```

d
```
var x = 6;
x = x + 7;
```

	THINK	WRITE
a	1 Write the initial assignment.	a `x = 20`
	2 Write the reassignment.	`x = 10`
b	1 Write the initial assignment.	b `x = 2+12/4`
	2 Evaluate the expression.	`x = 5`
	3 Write the reassignment.	`x = 2+4-3`
	4 Evaluate the expression. (Note that the value of x can change.)	`x = 3`
c	1 Write the initial assignment to x.	c `x = 1`
	2 Write the initial assignment to y.	`y = 12`
	3 Write the reassignment to x.	`x = 12`
	4 Write the reassignment to x.	`x = 15`
	5 Write the reassignment to y.	`y = 10`
	Rewrite the last assignment to x.	`x = 15`
d	1 Write the initial assignment to x.	d `x = 6`
	2 Write the reassignment to x. Note that the statement `x = x + 7` is a reassignment, not an algebraic expression.	`x = x + 7`
	3 Substitute 6 for x only on the right-hand side of the assignment. Note `x + 7` is first calculated, then reassigned to x again.	`x = 6 + 7`
	4 Evaluate the expression (note that the value of x can change).	`x = 13`

Exercise 17.2 Expressions

assessON

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8, 14	Questions: 1–12, 14	Questions: 1–15

REFLECTION
What is the difference between a value and a variable in computer programming?

FLUENCY

1 **WE1** Answer these questions for each of the following values.

i Is the value a number, string or Boolean?

ii Apply the expression `typeof value` to each value. What is the result of the expression `typeof value`?

a `1.243`
b `"Is this a string?"`
c `false`
d `""`
e `"True"`
f `"Laura"`
g `-13.2`
h `"Classify this"`
i `true`
j `"One"`
k `123456`
l `"true"`
m `"Time"`

2 Classify the results of the following expressions as number, string or Boolean.

a `1+2+12`
b `"f"+"als"+"e"`
c `19*3`
d `"This"+" is"+" a stri"+"ng"`
e `""+""+""+""`
f `"t"+"r"+"u"+"e"`
g `15/4`
h `13.2-12`
i `typeof 10`

3 **WE2** Each of the following is a valid variable name. True or false?

a `for`
b `while`
c `an&and`
d `ten_10`
e `10tenTen10`
f `storeHere`
g `12_twelve`
h `A1234`
i `modulus%this`
j `variable`
k `1234A`

4 **WE4** Evaluate the following JavaScript expressions manually.

a `21%4`
b `212%5`
c `400%25`
d `82%17`
e `63%15`
f `31%1.2`

5 What is the value stored in the variable `y` in each of the following expressions?

a
```
var y = 20+23;
```
b
```
var y = 20/(7+3);
```
c
```
var y = (1+2+3+4)/(1+1+1+1);
```
d
```
var y = ((12-(6+5))*100+99+1)/2;
```
e
```
var y = "One"+" "+"hundred"+" "+"and"+" "+"ten";
```
f
```
var y = typeof 182;
```
g
```
var y = typeof true;
```
h
```
var y = typeof "false";
```
i
```
var y = typeof false;
```

6 **WE7** Manually simulate running the following programs. What is the final value stored in the variable `z` for each program?

a
```
var z = "First";
    z = "Second";
```
b
```
var z = "Init"+"i"+"al"+" "+"value";
    z = "Fin"+"al"+" val"+"ue";
    z = typeof z;
```
c
```
var z = "Started here";
var y = "Why is this needed?";
    z = "Time to change";
    z = "Change again";
    y = "Final value";
```

d
```
var z = 12;
    z = z * 2;
```
e
```
var z = "value";
    z = "New "+z;
```
f
```
var z = 12;
var y = 15;
var x = z*y/z;
    z = x;
    z = x*x
    y = 10;
```

UNDERSTANDING

7 WE3 Write the following mathematical expressions as JavaScript expressions.

a $2.234 + 1.2$ **b** $12(3 + 4) \div (3 - 1)$

c 82×0.5 **d** $(23 + 25) \div (6 \times 4)$

e $\dfrac{(15 - 318 - 2)}{(5 - 1)}$

8 Write the following JavaScript expressions as mathematical expressions.

a `(15-3*4-1)/(4-3)` **b** `12*3*4`

c `((12-1)/(3-1))/2` **d** `(1+2)+(3*3)/(13-4)`

e `2+1/3+(13+1)/3`

9 WE5 **a** Write a JavaScript expression to concatenate the strings `"It "`, `"takes "`, `"under"` and `"standing"`.

b Write the string `"Separate as seven strings"` as an expression of single words and spaces.

10 WE6 Write a JavaScript statement for each of the following variable assignments.

a Assign the value `1000` to the variable `thousand`.

b Assign `true` to the Boolean variable `inSecondarySchool`.

c Assign `"JavaScript"` to the variable `computerLanguage`.

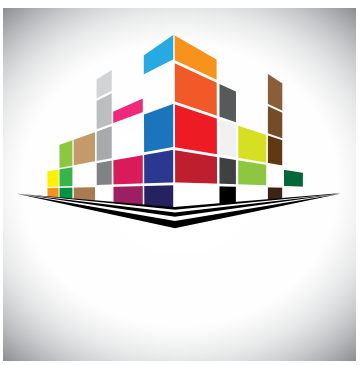
d Assign the expression `365*24` to the variable `hoursPerYear`.

e Assign the expression `15*100` to the variable `fifteenMetresInCentimetres`.

11 For each of the following JavaScript statements, identify:

i the variable name **ii** the expression **iii** the stored value.

a
```
var variable = "val"+"ue";
```
b
```
var x = "a"+"b"+"c";
```
c
```
var two = (8*3*7*6*4*5*2)/(5*6*8*3*7*4);
```
d
```
var thirty = "3"+"0";
```
e
```
var number10 = 5*2;
```
f
```
var valueType = typeof 23;
```



REASONING

12 Write a program to:

- store the value `32` in the variable `inKilometres`
- convert `inKilometres` to metres and store into the variable `inMetres`.

13 Write a program to:

- store the value `10` in the variable `width`
- store the value `14` in the variable `height`

- calculate the perimeter of a rectangle given the `width` and `height` and store the result in the variable `rectanglePerimeter`
- calculate the area of a rectangle given the `width` and `height` and store the result in the variable `rectangleArea`.

PROBLEM SOLVING

14 Consider the following 3 statements.

Reference	Statement
A	`var t=5;`
B	`t=3*(t+2);`
C	`t=t/t+t;`

a Find all of the different ways to rearrange the 3 statements.
b The first time a variable is declared, it requires the `var` keyword. Find all of the different valid JavaScript programs that were created in part **a**.
c Find the final value of `t` for each of the valid JavaScript programs from part **b**.

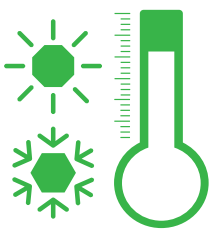
15 The first time a variable is declared, it requires the `var` keyword. The current order of the following statements is not valid because it divides by zero.

Reference	Statement
A	`var x=5;`
B	`var y=x-1;`
C	`x = 2 - y/2;`
D	`y = 2/x;`

a Find the only valid JavaScript program that can be created by rearranging the following statements.
b What is the final value of `x`?
c What is the final value of `y`?

17.3 Decisions

- Programs are often required to make decisions. For example, "Does this password match?", "Is this temperature greater than 20 degrees?" and "Does this data exist?".
- Programs make decisions based on Boolean values and expressions.



Boolean compare expressions

- Two numerical values can be compared with each other by using specific combinations of symbols, as shown in the table below. The result of these expressions is a Boolean value (i.e. either true or false).

Expression	Description
`x1===x2`	`true` if `x1` equals `x2`, otherwise `false`
`x1!==x2`	`true` if `x1` does not equal `x2`, otherwise `false`

Expression	Description
`x1>x2`	`true` if `x1` is greater than `x2`, otherwise `false`
`x1>=x2`	`true` if `x1` is greater than or equal to `x2`, otherwise `false`
`x1<x2`	`true` if `x1` is less than `x2`, otherwise `false`
`x1<=x2`	`true` if `x1` is less than or equal to `x2`, otherwise `false`

WORKED EXAMPLE 8

Evaluate the following JavaScript expressions manually.

a `2===1` b `3!==4` c `6>-3`
d `9>=9` e `3<3.01` f `2<=1`

THINK	WRITE
a The statement "2 equals 1" is `false`.	a `false`
b The statement "3 does not equal 4" is `true`.	b `true`
c The statement "6 is greater than −3" is `true`.	c `true`
d The statement "9 is greater than or equal to 9" is `true`.	d `true`
e The statement "3 is less than 3.01" is `true`.	e `true`
f The statement "2 is less than or equal to 1" is `false`.	f `false`

- Two string values can be compared with each other by using `===` (compare if the two strings are equal) or `!==` (compare if the two strings are not equal). The result of these expressions is a Boolean value (i.e. either true or false).

WORKED EXAMPLE 9

Evaluate the following JavaScript expressions manually.

a `"2"==="2"` b `"A"!=="B"`
c `"Longer"==="longer"` d `"longer"!=="shorter"`
e `"Expression"===("Express"+"i"+"on")`

THINK	WRITE
a The statement `"2"==="2"` is `true`, since the two strings are identical.	a `true`
b The statement "`A"!=="B"` is `true`, since the two strings are not identical.	b `true`
c 1 `"Longer"` and `"longer"` do not match exactly, as there is a case difference "L" and "l".	
2 The statement `"Longer"==="longer"` is `false`.	c `false`
d The statement "`longer"!=="shorter"` is `true`, since the two strings are not identical.	d `true`
e 1 `("Express"+"i"+"on")` evaluates to "`Expression`".	
2 The statement `"Expression"===("Express"+"i"+"on")` is `true`, since the two strings are identical.	e `true`

Boolean AND expressions

- The statement `boolean1&&boolean2` returns `true` if `boolean1` and `boolean2` are both `true`. Otherwise, the statement returns `false`.
- The statement can be extended indefinitely. The statement `boolean1&&boolean2&&boolean3` returns `true` if `boolean1`, `boolean2` and `boolean3` are all `true`. Otherwise, the statement returns `false`.

WORKED EXAMPLE 10

Evaluate the following JavaScript expressions manually.

a `false&&false`
b `false&&true`
c `true&&false`
d `true&&true`
e `false&&false&&false`
f `true&&false&&true&&true`
g `true&&true&&true&&true`

THINK	WRITE
a At least one Boolean is `false`.	a `false`
b At least one Boolean is `false`.	b `false`
c At least one Boolean is `false`.	c `false`
d All the Booleans are `true`.	d `true`
e At least one Boolean is `false`.	e `false`
f At least one Boolean is `false`.	f `false`
g All the Booleans are `true`.	g `true`

Boolean OR expressions

- The statement `boolean1||boolean2` returns `true` if either `boolean1` or `boolean2` are `true`. Otherwise, the statement returns `false`.
- The statement can be extended indefinitely. The statement `boolean1||boolean2||boolean3` returns `true` if either `boolean1`, `boolean2` or `boolean3` are `true`. Otherwise, the statement returns `false`.

WORKED EXAMPLE 11

Evaluate the following JavaScript expressions manually.

a `false||false`
b `false||true`
c `true||false`
d `true||true`
e `false||false||false`
f `true||true||true||true`
g `true||false||true||true`

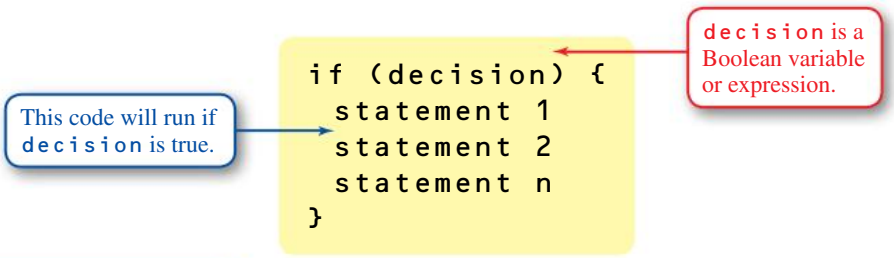
THINK	WRITE
a All the Booleans are `false`.	a `false`
b At least one Boolean is `true`.	b `true`
c At least one Boolean is `true`.	c `true`
d At least one Boolean is `true`.	d `true`

e	All the Booleans are `false`.	`false`
f	At least one Boolean is `true`.	`true`
g	At least one Boolean is `true`.	`true`

If structure

- Decisions are based on Boolean values. In JavaScript, the `if` structure is used to make a decision to run a section of code if the decision value is true. For example, the following program will run `{statement 1, statement 2, ..., statement n}` if `decision` is `true`.

eBookplus
Interactivity
CodeBlocks Activity 2
int-6574



WORKED EXAMPLE 12

Manually simulate running the following programs. What is the final value stored in the variable x for each program?

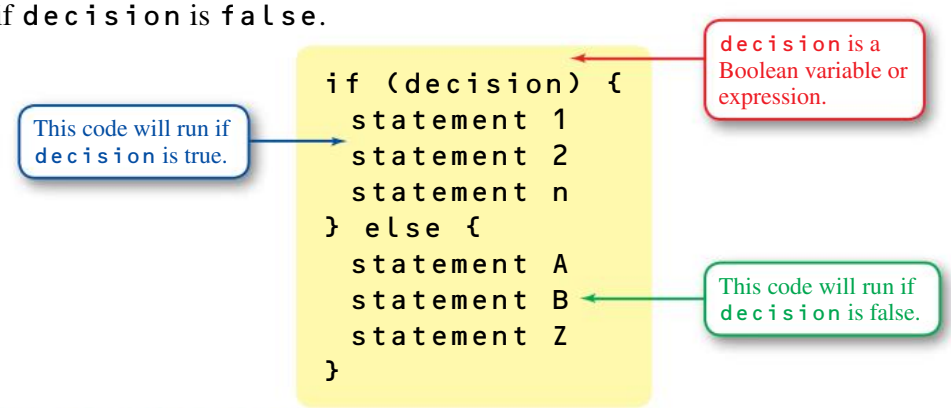
a
```
var x=10;
if (true) {
  x=20;
}
```
b
```
var x=3.14;
if (false) {
  x=2*x;
}
```
c
```
var x=1.2;
var isDayTime=true;
if (isDayTime) {
  x=2*x+0.6;
  x=x*x;
}
```

	THINK	WRITE
a 1	Write the first assignment.	a `x=10`
2	`decision` is `true`, so run the statement inside the `{}` block.	
3	Write the assignment.	`x=20`
b 1	Write the first assignment.	b `x=3.14`
2	`decision` is `false`, so ignore the statement inside the `{}` block.	
c 1	Write the first assignment.	c `x=1.2`
2	Write the second assignment.	`isDayTime=true`

3	The Boolean decision value `isDayTime` is `true`, so run the statements inside the second `{}` block.	
4	Write the equation.	`x=2*x+0.6`
5	Substitute `1.2` for `x` only on the right-hand side of the reassignment.	`x=2*1.2+0.6`
6	Evaluate the expression.	`x=3`
7	Write the equation.	`x=x*x`
8	Substitute `3` for `x` only on the right-hand side of the reassignment.	`x=3*3`
9	Evaluate the expression.	`x=9`

If else structure

- In JavaScript the `if else` structure is used execute one of two different sections of code.
  1. Execute `{statement 1, statement 2, ..., statement n}` if `decision` is `true`.
  2. Execute `{statement A, statement B, ..., statement Z}` if `decision` is `false`.



WORKED EXAMPLE 13

Manually simulate running the following programs. What is the final value stored in the variable `y` for each program?

a
```
var y=1;
if (true) {
  y=2;
} else {
  y=3;
}
```

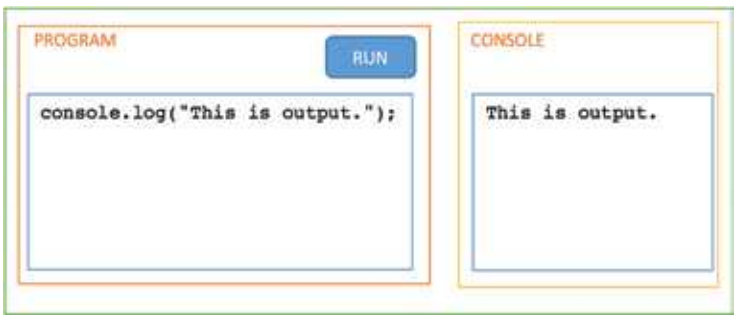
b
```
var y=6;
if (false) {
  y=2*y;
} else {
  y=y*y;
}
```

c
```
var y=9;
var isWet=false;
if (isWet) {
  y=3*y;
} else {
  y=(y*y-1)/8;
}
```

	THINK	WRITE
a	1 Write the first assignment.	a `y=1`
	2 `decision` is `true`, so run the statement inside the first `{}` block.	
	3 Write the assignment. Note the second block is ignored.	`y=2`
b	1 Write the first assignment.	b `y=6`
	2 `decision` is `false`, so run the statement inside the second `{}` block. Note the first block is ignored.	
	3 Write the equation.	`y=y*y`
	4 Substitute `6` for `y` only on the right-hand side of the reassignment.	`y=6*6`
	5 Evaluate the expression.	`y=36`
c	1 Write the first assignment.	c `y=9`
	2 Write the second assignment.	`isWet=false`
	3 `decision isWet` is `false`, so run the statement inside the second `{}` block.	
	4 Write the equation.	`y=(y*y-1)/8`
	5 Substitute `9` for `y` only on the right-hand side of the reassignment.	`y=(9*9-1)/8`
	6 Evaluate the expression.	`y=10`

Output

- The **console** is a special region in a web browser for monitoring the running of JavaScript programs. Most web browsers, including Chrome, Firefox, Safari, Internet Explorer, Microsoft Edge and Opera, allow you to activate the console through the menu options.
- In order to see the result of an expression or value, the JavaScript `console.log` function can be used. This function outputs results to the console.



Note that the quotation marks at the beginning and end of strings are not shown in the output.

WORKED EXAMPLE 14

What is the output to the console after each of the following statements runs?

a `console.log(2*(1+5)/(4-1)<3);`
b `console.log(true&&(false||true));`
c `console.log("Conso"+"le output");`

THINK		WRITE
a 1	Show the original statement.	a `console.log(2*(1+5)/(4-1)<3);`
2	`2*(1+5)/(4-1)` simplifies to 4.	`console.log(4<3);`
3	`4<3` simplifies to `false` as 4 is not less than 3.	`console.log(false);`
4	Write the output to the console.	`false`
b 1	Show the original statement.	b `console.log(true&&(false||true));`
2	`(false||true)` simplifies to `true` as at least one is true.	`console.log(true&&true);`
3	`true&&true` simplifies to `true` as both are true.	`console.log(true);`
4	Write the output to the console.	`true`
c 1	Show the original statement.	c `console.log("Conso"+"le output");`
2	`"Conso"+"le output"` simplifies to `"Console output"`.	`console.log("Console output");`
3	Write the output to the console.	`Console output`

Exercise 17.3 Decisions

REFLECTION
Can you explain the term Boolean?

INDIVIDUAL PATHWAYS

■ PRACTISE	■ CONSOLIDATE	■ MASTER
Questions: 1–7, 11, 13	Questions: 1–9, 12, 14	Questions: 1–15

FLUENCY

1 WE8 Evaluate the following JavaScript expressions manually.

a `2.2===2.2`
b `32231!==32321`
c `75>-31`
d `37631>=3763`
e `3.03<3.04`
f `22<=1321`
g `1.24424===1.24244`
h `4984!==3333`
i `-6>-9`
j `91>=91`
k `13.0026>13.0027`
l `243<=134`
m `3762>=3763`

2 WE9 Evaluate the following JavaScript expressions manually.

a `"3"==="3"`
b `"C"!=="D"`
c `"Case matters"==="Case Matters"`
d `"ten"!=="10"`
e `typeof true==="boolean"`

3 WE10 Evaluate the following JavaScript expressions manually.

a `true&&false`
b `false&&true`
c `false&&false`
d `true&&true`
e `false&&false&&false&&false&&false`
f `true&&true&&true&&true&&true&&true`
g `true&&false&&true&&false&&true`
h `true&&true&&false&&true&&true&&true&&true`
i `true&&false&&true&&false&&true&&true`
j `true&&false&&true&&true&&false&&true&&true`

4 WE11 Evaluate the following JavaScript expressions manually.

a `true||false`
b `true||true`
c `false||false`
d `false||true`
e `false||false||false||false||false`
f `true||true||true||true||true||true||true`
g `true||false||true||true||false||true||true`
h `true||false||true||false||true||true`
i `false||false||false||true||false||false||false||true`

5 WE12 Manually simulate running the following programs. What is the final value stored in the variable `z` for each program?

a
```
var z=911;
if (false) {
  z=0*3;
}
```
b
```
var z=3.141;
if (true) {
  z=z/3;
}
```
c
```
var z="5:00";
var isPM=true;
if (isPM) {
  z="17:00";
}
```
d
```
var z=6;
if (true) {
  z=z/3;
  z=z*111;
}
```
e
```
var z=3.141;
if (typeof z==="number") {
  z=z/2;
}
```
f
```
var z="3:15";
var isPM=true;
var is24Clock=true
if (is24Clock&&isPM) {
  z="15:15";
}
```

6 WE14 What is the output to the console after each of the following statements runs?

a
```
console.log((11+5)*2/(14-10)<3);
```
b
```
console.
 log((false&&true)||true);
```
c
```
console.log("console.l"+"og
 i"+"s a use"+"ful function");
```
d
```
console.log(19%2>0);
```
e
```
console.log(false||(3>12)||
 ((false||true)&&true));
```
f
```
console.log((typeof
 "a string")!=="string");
```

UNDERSTANDING

7 Evaluate the following JavaScript expressions manually.

a `(5*4*3*2*1)<=(20*3*2*1)`
b `(21 + 0.2)===21.2`
c `123!==(1*100+2*10+3)`
d `“Zero"!=="0"`
e `(“String "+"expression")===(“String"+" ex"+"pression")`
f `(100*3/4)>(70+5)`
g `“Join words"===("Join"+" "+"words")`
h `"3"===("2"+"1")`
i `(25*3)>=(150/2)`
j `“21"===("2"+"1")`
k `"Empty"===""`
l `(typeof "Empty")===(typeof "")`

8 WE13 Manually simulate running the following programs. What is the final value stored in the variable `finalValue` for each program?

a
```
var finalValue = 18;
if (false) {
  finalValue = finalValue / 2;
} else {
  finalValue = finalValue / 3;
}
```
b
```
var finalValue = 18;
if (finalValue%2 === 0) {
  finalValue = finalValue / 2;
} else {
  finalValue = finalValue + 1;
  finalValue = finalValue / 2;
}
```
c
```
var finalValue = "";
var hour = 13;
if (hour >= 12) {
  finalValue = "Afternoon";
} else {
  finalValue = "Morning";
}
```

d
```
var finalValue = 1.25;
if (true) {
  finalValue = 4*finalValue;
} else {
  finalValue = 4*(finalValue-0.25);
}
if (finalValue > 1) {
  finalValue = finalValue - 1;
} else {
  finalValue = finalValue + 1;
}
```

9 Evaluate the following JavaScript expressions manually.

a `(-1<3)&&(3>=100)`
b `(-11<=13)||(36>101)`
c `("3"===("1"+"2"))&&(3===(1+2))`
d `(-12<-13)||(-1>-2)`
e `(34123>=34123)||(2<(14-1-10))`
f `(12>13)&&(1<2)`
g `(34>=34)&&(32<(34-1))`
h `("21"!==("1"+"2"))||(4!==(101+2))`
i `("98"===("9"+"8"))||(17===(9+8))`
j `("12"===("1"+"2"))&&(3===(1+2))`

10 Evaluate the following JavaScript expressions manually.

a `(false&&false&&false)||true`
b `(true||true||false)&&(true||false)`

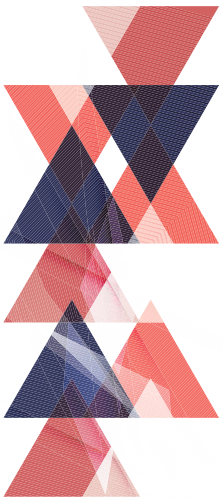
11 Consider the following program.

```
var hour=17;
var isAfternoon = hour>=12;
var before="";
var after="";
if (isAfternoon) {
  after="PM";
  hour = hour - 12;
} else {
  after = "AM";
}
if (hour === 3) {
  before="3:00";
}
if (hour === 4) {
  before="4:00";
}
if (hour === 5) {
  before="5:00";
}
var time = before + after;
console.log(time);
```

a What is the output to the console after the program is run?
b Change the first statement of the program from `var hour=17;` to `var hour=5;`. What is the output to the console after the updated program is run?
c Change the first statement of the program to `var hour=16;`. What is the output to the console after the updated program is run?

REASONING

12 The program below is designed to output if a triangle has an `"Odd number of area units."` or `"Even number of area units."`.



```
var base = __;
var height = __;
var area = base * _______ / 2;
var hasEvenAreaUnits = (____%2)===0;
var prefix = "";
var oddString = "Odd";
var evenString = _______;
if (hasEvenAreaUnits) {
  prefix = _________;
} else {
  prefix = _________;
}
console.log(prefix + " number of area units.");
```

a Given a triangle that has a `base` of 13 units and `height` of 10 units, fill in the gaps for the program.
b What is the output to the console of this program?

13 **a** Given a `number`, write a JavaScript program to:
- output "`Fizz`" if the `number` is divisible by 3
- output "`Buzz`" if the `number` is divisible by 5
- output "`Fizz Buzz`" if the `number` is divisible by 3 and 5
- otherwise output the `number`.

The program has been started below.

```
var number=1;
var isFizz=(number%3)===0;
var isBuzz=
```

b Record the output of the program to the console.
c For each value in the following table, change the assignment to `number`, run your program, and record the output in the table below.

number	First statement	Output to console
1	`var number = 1;`	
2	`var number = 2;`	
3		
4		
5		
14		
15		

number	First statement	Output to console
16		
135		

PROBLEM SOLVING

14 Consider the following repeated code block.

Block	`var x=53;`
1	`if ((x%2)===0) { x=x/2; } else { x=3*x+1; }`
2	`if ((x%2)===0) { x=x/2; } else { x=3*x+1; }`
3	`if ((x%2)===0) { x=x/2; } else { x=3*x+1; }`
:	
n	`if ((x%2)===0) { x=x/2; } else { x=3*x+1; }`

a Calculate the final value of `x` after running the above sequence with $n = 4$.
b What the smallest value of n required for `x` to finish with a value of `1`?
c Which two other values of n are required for `x` to finish with a value of `1`?

15 Consider the following code blocks.

Block	`var x=1;`
1	`if ((x%2)===0) { x=x/2; } else { x=3*x+1; }`
2	`if ((x%3)===0) { x=x/3; } else { x=4*x+1; }`
3	`if ((x%5)===0) { x=x/5; } else { x=6*x+1; }`
4	`if ((x%7)===0) { x=x/7; } else { x=8*x+1; }`

a Calculate the final value of `x` after running the above sequence.
b Rearrange the 4 blocks so `x` starts with `1` and finishes with `33` when the new sequence is run.
c Rearrange the 4 blocks so `x` starts with `1` and finishes with `2` when the new sequence is run.

17.4 Functions

Defining a function

- In complex programs, it is often useful to wrap a section of code in a function. This gives the section of code a descriptive name.
- In JavaScript a function is used to give a `name` to a section of code: `{statement 1, statement 2, ..., statement n}`.

```
function name() {
  statement 1
  statement 2
  statement n
}
```

Calling a function

- A defined function can be called with the statement `name();` and the section of code is run: `{statement 1, statement 2, ..., statement n}`.

```
function name() {
  statement 1
  statement 2
  statement n
}
name();
name();
name();
```

The defined function can now be called by its name many times without having to rewrite the code each time.

WORKED EXAMPLE 15

What is the output to the console of each of the following programs?

a
```
function outputTime() {
  console.log("7:05AM");
}
outputTime();
```

b
```
function trianglePerimeter() {
  var a=10;
  var b=12;
  var c=8;
  console.log(a+b+c);
}
trianglePerimeter();
```

THINK		WRITE
a 1	Identify the function being called.	a `outputTime();`
2	Write the output statement.	`console.log("7:05AM")`
3	Write the output to the console.	`7:05AM`
b 1	Identify the function being called.	b `trianglePerimeter();`
2	The variable `a` equals `10`.	`a=10`
3	The variable `b` equals `12`.	`b=12`
4	The variable `c` equals `8`.	`c=8`
5	Write the output statement.	`console.log(a+b+c)`
6	Substitute `10` for `a`, `12` for `b` and `8` for `c`.	`console.log(10+12+8)`
7	Evaluate the expression.	`console.log(30)`
8	Write the output to the console.	`30`

Input

- In JavaScript a function can be given inputs to vary the function's behaviour. A function with a `name` can be given zero, one or more inputs `x1,x2,...`

```
function name(x1,x2,...) {
  statement 1
  statement 2
  statement n
}
name(v1,v2,...);
```

- Once the function is defined, it can be called with the statement `name(v1,v2,...);`. The value `v1` is assigned to the input `x1`, the value `v2` is assigned to the input `x2`, and so on.

WORKED EXAMPLE 16

What is the output to the console of each of the following programs?

a
```
function outputTriple(y) {
  console.log(y+y+y);
}
outputTriple(3.2);
```

b
```
function rectanglePerimeter(w,h) {
  console.log(w+h+w+h);
}
rectanglePerimeter(7,11);
```

c
```
function nothingIn() {
  console.log("Same output");
}
nothingIn();
```

THINK

a
1. The input `y` equals `3.2`
2. Write the output statement.
3. Substitute `3.2` for `y`.
4. Evaluate the expression.
5. Write the output to the console.

b
1. The input `w` equals `7`.
2. The input `h` equals `11`.
3. Write the output statement.
4. Substitute `7` for `w` and `11` for `h`.

WRITE

a
```
y=3.2
console.log(y+y+y)
console.log(3.2+3.2+3.2)
console.log(9.6)
9.6
```

b
```
w=7
h=11
console.log(w+h+w+h)
console.log(7+11+7+11)
```

	5	Evaluate the expression.	`console.log(36)`
	6	Write the output to the console.	`36`
c	1	The function has no inputs.	c
	2	Write the output statement.	`console.log("Same output");`
	3	Write the output to the console.	`Same output`

Returning a value

- Most of the time it is not useful to just return the output to the console. It is important for most functions to return the results to be used elsewhere.
- In JavaScript a function `name` can `return` an internal value `output` to be `stored`.

```
function name(x1,x2,...) {
  statement 1
  statement 2
  statement n
  return output;
}
var stored = name(v1,v2,...);
```

WORKED EXAMPLE 17

Manually simulate running the following programs. What is the final value stored in the variable `p` for each program?

a
```
function volumeBox(h,w,d) {
  return h*w*d;
}
var p=volumeBox(3,6,9);
```

b
```
function perimeter(a,b,c,d) {
  total=a+b+c+d;
  return total;
}
var p=perimeter(11,5,13,19);
```

		THINK	WRITE
a	1	Write the assignment.	a `p=volumeBox(3,6,9)`
	2	The input `h` equals 3.	`h=3`
	3	The input `w` equals 6.	`w=6`
	4	The input `d` equals 9.	`d=9`
	5	Write the return expression.	`volumeBox(3,6,9)=h*w*d`

6	Substitute 3 for h, 6 for w and 9 for d.		volumeBox(3,6,9)=3*6*9
7	Evaluate the return expression.		volumeBox(3,6,9)=162
8	Write the assignment.		p=volumeBox(3,6,9)
9	Substitute 162 for volumeBox(3,6,9).		p=162
b 1	Write the assignment.	b	p=perimeter(11,5,13,19)
2	The input a equals 11.		a=11
3	The input b equals 5.		b=5
4	The input c equals 13.		c=13
5	The input d equals 19.		d=19
6	Write the first assignment inside the function.		total=a+b+c+d
7	Substitute 11 for a, 5 for b, 13 for c and 19 for d.		total=11+5+13+19
8	Evaluate the expression.		total=48
9	Write the return expression.		perimeter(11,5,13,19)=total
10	Substitute 48 for total.		perimeter(11,5,13,19)=48
11	Write the assignment.		p=perimeter(11,5,13,19)
12	Substitute 48 for perimeter (11,5,13,19).		p=48

Return at any point

- A function can have many return points within itself. Once the return point is reached, the function does not execute any further. This is useful when the result is known before running through the rest of the function.

```
function name() {
...
  if (decision) {
    return early;
  }
...
  return output;
}
```

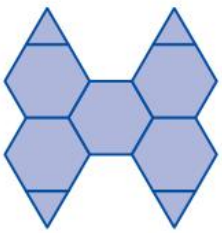
WORKED EXAMPLE 18

Manually simulate running the following program. What is the final value stored in the variable `k`?

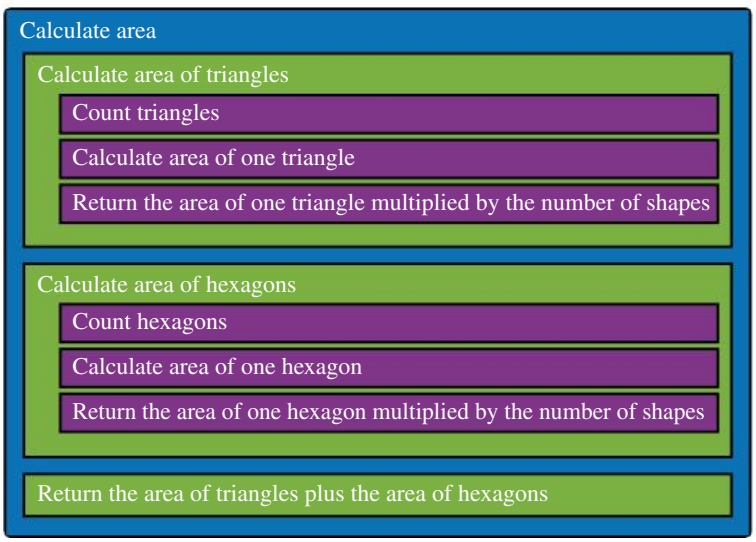
```
function oddEven(a) {
  var decision = a%2!==0;
  if (decision) {
    return "Odd";
}
  return "Even";
}
var k=oddEven(19);
```

THINK	WRITE
1 Write the assignment.	`k=oddEven(19)`
2 The input `a` equals `19`.	`a=19`
3 Write the first assignment inside the function `oddOrEven`.	`decision = a%2!==0`
4 Substitute `19` for `a`.	`decision = 19%2!==0`
5 Substitute `1` for `19%2`.	`decision = 1!==0`
6 Substitute `true` for `1!==0`.	`decision = true`
7 `decision` is `true`, so run the statement inside the `{}` block. The function is now finished.	`return "Odd"`
8 Write the return expression.	`oddEven(19)="Odd"`
9 Rewrite the assignment outside the function.	`k=oddEven(19)`
10 Substitute `"Odd"` for `oddEven(19)`.	`k="Odd"`

Functions calling functions

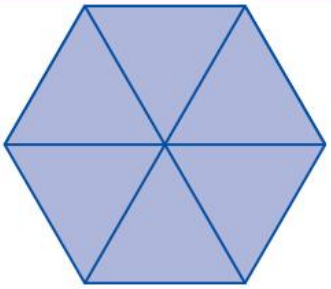


- The design of complex functions requires starting with the higher-level tasks and breaking them down into smaller and smaller tasks.
  For example, the task is to calculate the area of the shape shown.
  The top-level function is to calculate the area. Each task can be broken down into smaller functions until the lowest-level function becomes too simple to break down any further.



WORKED EXAMPLE 19

Manually simulate running the following program. What is the final value stored in the variable `area`?



```
function equilateral(x) {
  var output=0.43301*x*x;
  return output;
}
function areaHexagon(side) {
  var triangle=equilateral(side)
  var output=6*triangle;
  return output;
}
var area=areaHexagon(10);
```

THINK	WRITE
1 Write the assignment outside the functions.	`area=areaHexagon(10)`
2 Identify the called function `areaHexagon`.	
3 The input `side` equals `10`.	`side=10`
4 Write the first assignment inside the function `areaHexagon`.	`triangle=equilateral(side)`
5 Substitute `10` for `side`.	`triangle=equilateral(10)`
6 Identify the called function `equilateral`.	
7 The input `x` equals `10`.	`x=10`
8 Write the first assignment inside the function `equilateral`.	`output=0.43301*x*x`
9 Substitute `10` for `x`.	`output=0.43301*10*10`
10 Evaluate the expression.	`output=43.301`
11 Write the return expression inside the function `equilateral`.	`equilateral(10)=output`
12 Substitute `43.301` for `output`.	`equilateral(10)=43.301`
13 Rewrite the `triangle` simplified assignment.	`triangle=equilateral(10)`
14 Substitute `43.301` for `equilateral(10)`.	`triangle=43.301`
15 Write the second assignment inside the function `areaHexagon`.	`output=6*triangle`
16 Substitute `43.301` for `triangle`.	`output=6*43.301`
17 Evaluate the expression.	`output=259.806`

18	Write the return expression inside the function `areaHexagon`.	`areaHexagon(10)=output`
19	Substitute `259.806` for `output`.	`areaHexagon(10)=259.806`
20	Rewrite the assignment outside the functions.	`area=areaHexagon(10)`
21	Substitute `259.806` for `areaHexagon(10)`.	`area=59.806`

Testing values

- Boolean expressions can be used to test conditions. For example, to test if `value1` is greater than `value2`, use the expression `value1>value2`.
- For more complex expressions, it is often useful to define the expression as a function with a meaningful name.

WORKED EXAMPLE 20

Define functions with the following names and returning the Boolean expressions shown.

a `isEqual, x===y` b `isEven, (z%2)===0`

c `aRemainder, (a%b)!==0`

		THINK	WRITE
a	1	Identify the variables in the Boolean expression and convert them into the inputs `(x,y)`.	a
	2	Define the function name with the inputs.	`function isEqual(x,y) {`
	3	Assign the expression to the `output`.	`  var output = x===y;`
	4	Return the `output` variable.	`  return output;`
	5	Finish the function definition.	`}`
b	1	Identify the variable in the Boolean expression and convert it into the input `(z)`.	b
	2	Define the function name with the inputs.	`function isEven(z) {`
	3	Assign the expression to the `output`.	`  var output = (z%2)===0;`
	4	Return the `output` variable.	`  return output;`
	5	Finish the function definition.	`}`
c	1	Identify the variables in the Boolean expression and convert them into the inputs `(a,b)`.	c

2. Define the function name with the inputs.
3. Assign the expression to the `output`.
4. Return the `output` variable.
5. Finish the function definition.

```
function aRemainder(a,b) {
  var output = (a%b)!==0;
  return output;
}
```

Exercise 17.4 Functions

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7, 13	Questions: 1–12, 13	Questions: 1–14

REFLECTION
Why is it useful to define a function in computer programming?

FLUENCY

1 WE15 What is the output to the console of each of the following programs?

a
```
function outputYearLevel() {
  console.log("Year 8");
}
outputYearLevel();
```

b
```
function sumTriangleAngles() {
  var a=130;
  var b=20;
  var c=30;
  console.log(a+b+c);
}
sumTriangleAngles();
```

2 WE16 What is the output to the console of each of the following programs?

a
```
function outputDoubleThenTriple(z) {
  var y = z*2;
  console.log(y+y+y);
}
outputDoubleThenTriple(3.5);
```

b
```
function rectangleArea(small,large) {
  var square = small*small;
  var extra = (large-small)*small;
  console.log(square+extra);
}
rectangleArea(3,6);
```

c
```
function whyInputs(f) {
  console.log("No inputs here");
}
whyInputs();
```

3 WE17 Manually simulate running the following programs. What is the final value stored in the variable `v` for each program?

a
```
function totalValues(a,b,c,d,e) {
  return e+d+c+b+a;
}
var v=totalValues(1,10,100,1000,10000);
```
b
```
function max(x,y) {
  var output=x;
  if (x<y) {
    output=y;
  }
  return output;
}
var v=max(11,12);
```

4 **WE18** Manually simulate running the following program. What is the final value stored in the variable `v`?
```
function max(x,y) {
  if (x<y) {
    return y;
  }
  return x;
}
var v=max(111,122);
```

5 **WE19** Manually simulate running the following program. What is the final value stored in the variable `area`?
```
function squareArea(x) {
  var output=x*x;
  return output;
}
function allSquaresArea(number,side) {
  var square=squareArea(side)
  var output=number*square;
  return output;
}
var area= allSquaresArea(12,4);
```

UNDERSTANDING

6 **WE20** Define functions with the following names and returning the Boolean expressions shown.
- **a** `isNotEqual, x!==y`
- **b** `isOdd, (z%2)!==0`
- **c** `noRemainders, (a%b)===0`

7 What is the output to the console of the following program?
```
function outputYearLevel(year) {
  if (year===8) {
    console.log("Year eight");
  } else {
    console.log("Not year eight");
  }
}
outputYearLevel(9);
```

8 Consider the following function definitions.

```
function isValidTriangle(a1,a2,a3) {
  var allPositive = a1>0 && a2>0 && a3>0;
  var sumTo180 = (a1+a2+a3)===180;
  var output = allPositive && sumTo180;
  return output;
}
function outputValidTriangle(angle1,angle2,angle3) {
  if (isValidTriangle(angle1,angle2,angle3)) {
    console.log("Valid");
  } else {
    console.log("Invalid");
  }
}
```

What are the outputs to the console for the following function calls?

a `outputValidTriangle(12,120,48);`
b `outputValidTriangle(-12,140,52);`
c `outputValidTriangle(90,90,0);`
d `outputValidTriangle(1,2,3);`
e `console.log(isValidTriangle(90,45,45));`

9 Consider the following function definitions.

```
function min(a,b) {
  var output=0;
  if (a<b) {
    output=a;
  } else {
    output=b;
  }
  return output;
}
function probOneDie(x) {
  return 1/x;
}
function probTwoDice(total,sides) {
  var combinations = min(total-1, 2*sides+1-total);
  var probTwo = probOneDie(sides)*probOneDie(sides);
  return combinations*probTwo;
}
```

What is the returned value to 4 decimal places for each of the following function calls?

a `probOneDie(6);`
b `probOneDie(20);`
c `probTwoDice(7,6);`
d `probTwoDice(2,6);`
e `probTwoDice(12,6);`
f `probTwoDice(1,6);`
g `probTwoDice(13,6);`
h `probTwoDice(21,20);`
i `min(1,6);`
j `min(13,6);`

10 Describe the aim of the following function.

```
function unknown(percentage,decimal) {
  var decimalToPercentage = decimal*100;
  var output = percentage===decimalToPercentage;
  return output;
}
```

REASONING

11 Define a function to take an input `a` and return the probability that a fair 6-sided die will show a number greater than `a`. Assume `a` is `1`, `2`, `3`, `4`, `5` or `6`.

12 Define a function to take inputs `a`, `x` and return the probability that a fair `x`-sided die will return a number less than `a`. Assume `x` is a positive integer and `a` is `1`, `2`, `3`, . . ., or `x`.

PROBLEM SOLVING

13 Manually simulate running the program below. What is the final value stored in the variable `w`?

```
function f1() { return 1; }
function f2() { return 1; }
function f3() { return f1()+f2(); }
function f4() { return f2()+f3(); }
function f5() { return f3()+f4(); }
function f6() { return f4()+f5(); }
function f7() { return f5()+f6(); }
function f8() { return f6()+f7(); }
function f9() { return f7()+f8(); }
var w = f9();
```

14 Explain what the problem is with the program below.

```
function again(a) {
  return a+again(a/2);
}
var w = again(1);
```

17.5 Testing

Test cases

- Once a function is written, it is important that it returns expected results for the inputs. Most of the time it is impossible to test all the combinations of inputs, so it is important

to test typical uses of the function. It is also important to test the inputs at the limit of what the function expects.

- A test case is a table of different inputs with expected outputs for a function.

WORKED EXAMPLE 21

The required behaviour of the function `min` is to return the minimum value of the two inputs `a` and `b`. Complete the test case table.

Case	Inputs		min(a,b)
	a	b	Expected
a	1	2	
b	2	2	
c	99	8	
d	9	9	
e	12	12	

THINK

Start the test case table.

a 1 is the minimum of 1 and 2.

b 2 is the minimum of 2 and 2.

c 8 is the minimum of 99 and 8.

d 9 is the minimum of 9 and 9.

e 12 is the minimum of 12 and 12.

WRITE

Case	Inputs		min(a,b)
	a	b	Expected
a	1	2	1
b	2	2	2
c	99	8	8
d	9	9	9
e	12	12	12

Outputting actual results

- Once the function `name` is defined, the actual result from a test case can be sent to the console with the statement `console.log(name(a1,b1,...));`. The values `a1,b1,...` are the inputs for the first test case, the values `a2,b2,...` are the inputs for the second test case, and so on.

```
function name(a,b,...) {
  statement 1
  statement 2
  statement n
}
console.log(name(a1,b1,...));
console.log(name(a2,b2,...));
```

WORKED EXAMPLE 22

For each test case, write the statement to output to the console the result of calling the function `area` with the inputs `h` and `w`.

Case	h	w	area(h,w)
a	1	2	2
b	6	7	54
c	5	9	45

THINK

a 1 Think about the general output statement: `console.log(area(h,w));`.

2 Substitute 1 for `h` and 2 for `w`.

b 1 Think about the general output statement: `console.log(area(h,w));`.

2 Substitute 6 for `h` and 7 for `w`.

c 1 Think about the general output statement: `console.log(area(h,w));`.

2 Substitute 5 for `h` and 9 for `w`.

WRITE

a

```
console.log(area(1,2));
```

b

```
console.log(area(6,7));
```

c

```
console.log(area(5,9));
```

Executing test cases

- Executing a test case involves recording the actual results by calling the function with the test case inputs. The actual function's result is compared with the expected test case result to determine if the test case passed or failed.

WORKED EXAMPLE 23

The following function is faulty and is to be tested with the inputs from the table below.

```
function min(a,b) {
  if (a<b) { return a; }
  if (b<a) { return b; }
  return 0;
}
```

Execute each test by filling in the table with the actual result and if it passed or failed. Note that not all the expected results are correct.

Case	Inputs		min(a,b)		Pass or fail
	a	b	Expected	Actual	
a	1	2	1		
b	2	2	0		
c	99	8	9		
d	9	9	8		
e	12	12	12		

THINK

Write the table header.

a The actual result equals 1 using `console.log(min(1,2));`.
The test passed as 1 equals 1.

b The actual result equals 0 using `console.log(min(2,2));`.
The test passed as 0 equals 0.

c The actual result equals 8 using `console.log(min(99,8));`.
The test failed as 9 does not equal 8.

d The actual result equals 0 using `console.log(min(9,9));`.
The test failed as 7 does not equal 0.

e The actual result equals 0 using `console.log(min(12,12));`.
The test failed as 12 does not equal 0.

WRITE

Case	Inputs		min(a,b)		Pass or fail
	a	b	Expected	Actual	
a	1	2	1	1	Pass
b	2	2	0	0	Pass
c	99	8	9	8	Fail
d	9	9	7	0	Fail
e	12	12	12	0	Fail

Fixing test cases

- Once a test case has been executed, the test case will either pass or fail.
  - If a test case passes, this usually indicates there is no problem, but there could be a problem with both the function and the test case.
  - If a test case fails, this indicates a problem with either the test case, the function or both.

Expected result	Actual result	Pass or fail	Situation description
Correct	Incorrect	Fail	A problem with the function
Incorrect	Correct	Fail	A problem with the test case
Incorrect	Incorrect	Fail	A problem with both the function and the test case
Correct	Correct	Pass	Everything is working as expected.
Incorrect	Incorrect	Pass	There is a problem with both the function and the test case, but everything looks like it is working as expected.

WORKED EXAMPLE 24

The required behaviour of the function `min` is to return the minimum value of the two inputs `a` and `b`. Below are the results of the test cases.

Case	Inputs		min(a,b)		Pass or fail
	a	b	Expected	Actual	
a	1	2	1	1	
b	2	2	0	0	
c	99	8	9	8	
d	9	9	7	0	
e	12	12	12	0	

For each test case:

i **are the expected results correct or incorrect?**
ii **are the actual function's results correct or incorrect?**
iii **did the test case pass or fail?**
iv **how would you describe the situation?**

THINK

a 1 is the minimum of 1 and 2.

i 1 equals the test case's expected value, 1.

ii 1 equals the function's actual value, 1.

iii The expected value, 1, equals the actual value, 1.

iv Select the description describing the situation.

b 2 is the minimum of 2 and 2.

i 2 does not equal the test case's expected value, 0.

ii 2 does not equal the function's actual value, 0.

iii The expected value, 0, equals the actual value, 0.

iv Select the description describing the situation.

c 8 is the minimum of 99 and 8.

i 8 does not equal the test case's expected value, 9.

ii 8 equals the function's actual value, 8.

iii The expected value, 9, does not equal the actual value, 8.

WRITE

a

i Correct

ii Correct

iii Pass

iv Everything is working as expected.

b

i Incorrect

ii Incorrect

iii Pass

iv There is a problem with both the function and the test case, but everything looks like it is working as expected.

c

i Incorrect

ii Correct

iii Fail

iv Select the description describing the situation.	iv A problem with the test case
d 9 is the minimum of 9 and 9.	d
i 9 does not equal the test case's expected value, 7.	i Incorrect
ii 9 does not equal the function's actual value, 0.	ii Incorrect
iii The expected value, 7, does not equal the actual value, 0.	iii Fail
iv Select the description describing the situation.	iv A problem with both the function and the test case
e 12 is the minimum of 12 and 12.	e
i 12 equals the test case's expected value, 12.	i Correct
ii 12 does not equal the function's actual value, 0.	ii Incorrect
iii The expected value, 12, does not equal the actual value, 0.	iii Fail
iv Select the description describing the situation.	iv A problem with the function

- When programmers write test cases or functions, they can make mistakes. The first place to start looking for these mistakes is when a test case fails. If a test case fails, double check that the test case has the correct expected result.
- If there is no problem with the test case but the function still produces an error, the function will require fixing. Fixing a function is called debugging and is the focus of the next section.

Exercise 17.5 Testing

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7, 13	Questions: 1–9, 11, 13	Questions: 1–14

REFLECTION
What can you assume if:
a a test case passes?
b a test case fails?

FLUENCY

1 WE21 The required behaviour of the function `max` is to return the maximum value of the two inputs `x` and `y`. Complete the test case table.

Case	Inputs		max(x,y)
	x	y	Expected
a	11	21	
b	24	24	

Case	Inputs		max(x,y)
	x	y	Expected
c	18	9	
d	1.2	1.4	
e	-1	-1	

2 WE22 For each of the following test cases, write the statement to output to the console the result of calling the function `volume` with the inputs `h`, `w` and `d`.

Case	h	w	d	volume(h,w,d)	Output statement
a	4	5	6	120	
b	6	7	0	0	
c	1.2	9.5	1.2	13.68	

3 WE23 The following function is faulty and is to be tested with the inputs from the table below.

```
function perimeter(width,height) {
  return width+height+height+height;
}
```

Execute each test by filling in the table with the actual result and if it passed or failed.

Test	Inputs		perimeter(width,height)		Pass or fail
	width	height	Expected	Actual	
a	4	4	16		
b	6	7	26		
c	1.3	1.3	5.2		
d	2.2	1.8	8		

4 The following function is faulty and is to be tested with the inputs from the table below.

```
function max(x,y) {
  if (x<y) { return x; }
  return y;
}
```

Execute each test by filling in the table with the actual result and if it passed or failed. Note that not all the expected results are correct.

Test	Inputs		max(x,y)		Pass or fail
	x	y	Expected	Actual	
a	11	21	21		
b	24	24	24		
c	18	9	9		
d	1.2	1.2	2.4		
e	-1	-1	-2		

5 WE24 The required behaviour of the function `max` is to return the maximum value of the two inputs `x` and `y`. Below are the results of the test cases.

Test	Inputs		max(x,y)		i Expected correct	ii Actual correct	iii Pass or fail
	x	y	Expected	Actual			
a	11	21	21	11			
b	24	24	24	24			
c	18	9	9	9			
d	1.2	1.2	2.4	1.2			
e	-1	-1	-2	-1			

For each test case:

i are the expected results correct or incorrect?

ii are the actual function's results correct or incorrect?

iii did the test case pass or fail?

UNDERSTANDING

6 a Write a function `equalFractions` with the inputs `a`, `b`, `c` and `d` that returns `true` if $\frac{a}{b} = \frac{c}{d}$ and otherwise returns `false`.

b Complete the test case table:

i with the expected results

ii with the actual function results

iii with the pass or fail outcomes.

Test	Inputs				equalFractions(a,b,c,d)		iii Pass or fail
	a	b	c	d	i Expected	ii Actual	
A	1	2	3	6			
B	3	4	9	16			
C	99	3	66	2			

7 a Write a function `unknown` with the inputs `x1`, `x2`, `x3` and `x4` that will pass all the test cases in the table below.

b Complete the test case table:

i with the actual function results

ii with the pass or fail outcomes.

Test	Inputs				unknown(x1,x2,x3,x4)		Pass or fail
	x1	x2	x3	x4	Expected	Actual	
A	1	2	3	4	10		
B	3.1	4.2	0.1	1.7	9.1		
C	2	2	2	2	8		
D	-1	1	-1	1	0		
E	1	-2	3	-4	-2		

8 The following function is faulty and is to be tested with the inputs from the table below.

```
function f(x) {
  if (x <= 2) {
    return 1;
  }
  return f(x-1)+f(x-2);
}
```

Execute each test by filling in the table with the actual result and if it passed or failed.

Test	Inputs	f(x)		Pass or fail
	x	Expected	Actual	
a	1	1		
b	2	2		
c	4	3		
d	10	55		

9 a The following function is faulty and is to be tested with the inputs from the table below.

```
function average(x,y) {
  if (x !== y) { return x; }
  return (x+y)/2;
}
```

Execute each test by filling in the table with the actual result and if it passed or failed.

Test	Inputs		average(x,y)		Pass or fail
	x	y	Expected	Actual	
i	3	5	4		
ii	6	6	6		
iii	10	20	15		

b The following is a partially fixed version of the function from part **a**.

```
function average(x,y) {
  if (x === y) { return x; }
  return (x+y)/2;
}
```

Execute each test again by filling in the table with the new actual result and if it passed or failed.

Test	Inputs		average(x,y)		Pass or fail
	x	y	Expected	Actual	
i	3	5	4		
ii	6	6	6		
iii	10	20	15		

c The first line of the function in part **b** is redundant. Removing that line gives the following function.

```
function average(x,y) {
  return (x+y)/2;
}
```

Execute each test again by filling in the table with the new actual result and if it passed or failed.

Test	Inputs		average(x,y)		Pass or fail
	x	y	Expected	Actual	
i	3	5	4		
ii	6	6	6		
iii	10	20	15		

10 The required behaviour of the function `median` is to return the middle value of the three inputs `x`, `y` and `z`. The function is defined below.

```
function median(x,y,z) {
  if ((x<=y) && (y<=z)) { return y; }
  if ((x<=z) && (z<=y)) { return z; }
  if ((y<=x) && (x<=z)) { return x; }
  if ((y<=z) && (z<=x)) { return z; }
  return y;
}
```

Complete the test case table.

Test	Inputs			anyEqual(x,y,y)		Pass or fail
	x	y	z	Expected	Actual	
a	1	2	3			
b	4	4	5			
c	18	29	9			
d	6	3	118			

REASONING

11 The required behaviour of the function `max` is to return the largest value of the three inputs `x`, `y` and `z`.

a Write the function `max` with the three inputs `x`, `y` and `z`.

b Write the expected results in the test case table below.

c Execute the 5 test cases.

Test	Inputs			max(x,y,z)		c Pass or fail
	x	y	z	a Expected	b Actual	
i	1	2	4			
ii	1	6	2			
iii	61	61	61			
iv	6.2	2	1			
v	-1	-2	-5			

12 The required behaviour of the function `volume` is to return the volume of a box given the inputs `width`, `height` and `depth`.

a Write the function `volume` with the three inputs `width`, `height` and `depth`.

b Write the expected results in the test case table below.

c Execute the 5 test cases.

Test	Inputs			volume(width,height, depth)		c Pass or fail
	width	height	depth	b Expected	c Actual	
i	1	2	3			
ii	1.2	1.3	1.1			
iii	10	10	10			
iv	3	2	1			
v	0	3	6			

PROBLEM SOLVING

13 The required behaviour of the function `anyEqual` is to return `true` if any of the three inputs `x`, `y` and `z` are equal; otherwise, it must return `false`. The function is defined below.

```
function anyEqual(x,y,z) {
  return (x===y)||(x===z);
}
```

Complete the test case table below.

Test	Inputs			anyEqual(x,y,y)		Pass or fail
	x	y	z	Expected	Actual	
a	2	2	3			
b	−2	24		true		Pass
c	18	9		true	false	
d	6	7	8			

14 The required behaviour of the function `median` is to return the middle value of the three inputs `x`, `y` and `z`. The function is defined below, but the version shown is incorrect.

```
function median(x,y,z) {
  if ((x<=y) && (y<=z)) { return y; }
  if ((x<=z) && (z<=y)) { return z; }
  if ((y<=x) && (x<=z)) { return x; }
  if ((y<=z) && (z<=x)) { return z; }
  return x;
}
```

a Write and execute a test case that passes.

b Write and execute a test case that fails from incorrect actual results.

Test	Inputs			median(x,y,y)		Pass or fail
	x	y	z	Expected	Actual	
a						Pass
b						Fail

c Rewrite the function to return the correct results for all inputs.

17.6 Debugging

Testing functions

- If any errors are found after the test cases are created and executed, the function or test case is fixed and the testing is repeated. Manually testing functions in this way can be a time-consuming and error-prone process. To reduce the effort involved, the testing process can be automated in various ways. One method to automate the testing is to call a `testing` function.
- Before defining the `testing` function, a helper function `addQuotes` is created to add single quotation marks to any string `value`. The single quotation marks will make it clear which values are strings, numbers or Booleans.

```
function addQuotes(value) {
  if (typeof value === "string") {
    return "'"+value+"'";
  }
  return value;
}
```

WORKED EXAMPLE 25

Given the function `addQuotes` as shown above, what is the output to the console of the following statements?

```
a console.log(addQuotes("A string"));
b console.log(addQuotes(100));
c console.log(addQuotes(true));
d console.log(addQuotes("true"));
e console.log(addQuotes(""));
```

THINK	WRITE
a typeof `"A string"` equals `"string"`, so add quotation marks.	a `'A string'`
b typeof `100` equals `"number"`, so do not add quotation marks.	b `100`
c typeof `true` equals `"boolean"`, so do not add quotation marks.	c `true`
d typeof `"true"` equals `"string"`, so add quotation marks.	d `'true'`
e typeof `""` equals `"string"`, so add quotation marks.	e `''`

- The `testing` function compares the `actual` result with the `expected` result and outputs to the console a "Pass" or "Fail" statement, a reason and a description of the `test` case.
- This `testing` function will be referred to throughout this section.

```
function testing(actual, expected, test) {
  actual = addQuotes(actual);
  expected = addQuotes(expected);
  if (actual===expected) {
    console.log("Pass, "+actual+"="+expected+",
     "+test);
    } else {
    console.log("Fail, "+actual+"≠"+expected+",
     "+test);
    }
}
```

WORKED EXAMPLE 26

Using the `testing` function, what is the output to the console of the following testing statements?

a `testing(10,10,"Two equal values");`
b `testing(98.0,98,"Decimal and integer");`
c `testing(true,false,"Two Booleans");`
d `testing("AB","CD","Two strings");`

THINK	WRITE
a The test passes as `10` equals `10`.	a `Pass, 10=10, Two equal values`
b The test passes as `98.0` equals `98`.	b `Pass, 98=98, Decimal and integer`
c The test fails as `true` does not equal `false`.	c `Fail, true≠false, Two Booleans`
d The test fails as `"AB"` does not equal `"CD"`.	d `Fail, "AB"≠"CD", Two strings`

Automatic testing

- A function `name` to be tested has the inputs `a,b,...`

```
function name(a,b,...) {
  statement 1
  statement 2
  statement n
  return output;
}
```

- Once the functions `testing` and `name` are defined, they can be used together to create a set of automated tests.

```
testing(name(a1,b1,...), E1, "Test 1");
testing(name(a2,b2,...), E2, "Test 2");
testing(name(aM,bM,...), EM, "Test M");
```

– For `"Test 1"`, `E1` is the expected result of the expression `name(a1,b1,...)`.
– For `"Test 2"`, `E2` is the expected result of the expression `name(a2,b2,...)`.
– For `"Test M"`, `EM` is the expected result of the expression `name(aM,bM,...)`.

WORKED EXAMPLE 27

The function `pRed` returns the probability of picking a red marble from a bag that contains `r` red and `g` green marbles.

```
function pRed(r,g) {
  var marbles = r+g;
  if (marbles===0) {
    return 0.5;
  }
  return r/marbles;
}
```

a Write a program with the functions `addQuotes`, `testing` and `pRed`.

b Convert the test case table into a set of automated testing statements and add them to the program.

Test	Inputs		pRed(r,g)	
	r	g	Expected	Actual
i	1	1	0.5	pRed(1,1)
ii	10	0	1	pRed(10,0)
iii	0	7	0	pRed(0,7)
iv	3	5	0.375	pRed(3,5)
v	0	0	0	pRed(0,0)

c What is the output from each testing statement?

THINK

a 1 Before adding the `testing` function, the helper function `addQuotes` is required. Note the definition was cut down in the interest of space, see the full definition of `addQuotes` in a previous section.

2 Add the `testing` function. Note the definition was cut down in the interest of space; see the full definition of `testing` in the previous section.

WRITE

a
```
function addQuotes( ......}
```

```
function testing( ......}
```

3 Add the function to be tested, `pRed`.

```
function pRed(r,g) {
  var marbles = r+g;
  if (marbles===0) {
    return 0.5;
  }
  return r/marbles;
}
```

b Think about the general testing statement.
`testing(pRed(r,g),expected, test);`

- **i** Substitute `r`, `g`, `expected` and `test`.
- **ii** Substitute `r`, `g`, `expected` and `test`.
- **iii** Substitute `r`, `g`, `expected` and `test`.
- **iv** Substitute `r`, `g`, `expected` and `test`.
- **v** Substitute `r`, `g`, `expected` and `test`.

b

```
testing(pRed(1,1), 0.5, "a");
testing(pRed(10,0), 1, "b");
testing(pRed(0,7), 0, "c");
testing(pRed(3,5), 0.375, "d");
testing(pRed(0,0), 0, "e");
```

c Run the entire program from parts **a** and **b**.

- **i** Note the output from running `testing(pRed(1,1), 0.5, "a");`
- **ii** Note the output from running `testing(pRed(10,0), 1, "b");`
- **iii** Note the output from running `testing(pRed(0,7), 0, "c");`
- **iv** Note the output from running `testing(pRed(3,5),0.375,"d");`
- **v** Note the output from running `testing(pRed(0,0), 0, "e");`

c

```
i Pass, 0.5=0.5, a
ii Pass, 1=1, b
iii Pass, 0=0, c
iv Pass, 0.375=0.375, d
v Fail, 0.5≠0, e
```

Fixing test cases

- The failed test case is the first stage of finding an error. Before you try to fix the code, double check that the expected value of your test case is correct. If any of the test cases were updated, run the tests again.

WORKED EXAMPLE 28

The function `pBlue` returns the probability of picking a blue marble from a bag that contains `r` red, `g` green and `b` blue marbles.

```
function pBlue(r,g,b) {
  var marbles = g+b+g;
  var output = b/marbles;
  return output;
}
```

Answer the following for each of the test cases.

- **i Did the test case pass or fail?**
- **ii If required, correct the expected value for the second test run.**
- **iii Did the test case pass or fail for the second test run?**

Test	Inputs			Actual	First test run		Second test run	
	r	g	b		Expected	i Pass or fail	ii Expected	iii Pass or fail
a	1	1	2	0.5	0.25			
b	3	3	4	0.4	0.4			
c	0	3	2	0.25	0.4			

THINK

a i The actual value, 0.5, does not equal the expected value, 0.25.

ii The expected value, $\frac{2}{1+1+2} = 0.5$, requires correction.

iii The actual value, 0.5, equals the expected value, 0.5.

b i The actual value, 0.4, equals the expected value, 0.4.

ii The expected value, $\frac{4}{3+3+4} = 0.4$, requires no correction.

iii The actual value, 0.4, equals the expected value, 0.4.

c i The actual value, 0.25, does not equal the expected value, 0.4.

ii The expected value, $\frac{2}{0+3+2} = 0.4$, requires no correction.

iii The actual value, 0.25, does not equal the expected value, 0.4.

WRITE

a i Fail

ii 0.5

iii Pass

b i Pass

ii 0.5

iii Pass

c i Fail

ii 0.5

iii Fail

Bugs

- If you have confidence in the test cases and they still fail, the second stage is to find what part of the code is causing the problem. These problems are referred to as **bugs**. The term 'bug' dates back to 1947, when a problem in an early computer was traced back to a moth trapped in an electronic switch. The bug was removed and taped to a log book.

The first report of a computer bug, recorded in 1947

Debugging

- **Debugging** refers to the process of finding and removing bugs in the computer code. One method is to use the failed test case and step through each statement in the code. At each step, check the variables and flow are what is expected.

WORKED EXAMPLE 29

The function `pBlue` returns the probability of picking a blue marble from a bag that contains `r` red, `g` green and `b` blue marbles.

```
function pBlue(r,g,b) {
  var marbles = g+b+g;
  var output = b/marbles;
  return output;
}
```

Inputs			pBlue(r,g,b)		
r	g	b	Actual	Expected	Pass or fail
0	3	2	0.25	0.4	Fail

a Use the failed test case `r=0`, `g=3` and `b=2` to call the function with the test inputs.
b Step through each statement in the code.
c For each statement in the code, check the variables and flow are what is expected. Find and correct the error in the function.
d Retest the corrected function using the test case.
i Find the new actual value.
ii Show if the corrected function passed or failed the test case.

Inputs			Fixed pBlue(r,g,b)		
r	g	b	i Actual	Expected	ii Pass or fail
0	3	2		0.4	

THINK

a Call the function with the test inputs.

b 1 The input `r` equals `0`.

2 The input `g` equals `3`.

3 The input `b` equals `2`.

4 Write the assignment expression.

5 Substitute `0` for `r`, `3` for `g` and `2` for `b`.

6 Substitute `8` for `3+2+3`.

c 1 The expected value for `marbles` is 0 + 3 + 2 = 5, which does not equal the computed value, 8. Therefore the formula to calculate the value for `marbles` is incorrect.

WRITE

a `pBlue(0,3,2)=0.25`

b `r=0`

`g=3`

`b=2`

`marbles = g+b+g`

`marbles = 3+2+3`

`marbles = 8`

c

2 Write the start of the function again.

```
function pBlue(r,g,b) {
```

3 Correct the assignment expression.

```
  var marbles = r+g+b;
```

4 Write out the rest of the function.

```
  var output = b/marbles;
  return output;
}
```

d i Use `console.log(pBlue(0,3,2))` to find the new actual value of `0.4`.

d i `pBlue(0,3,2))=0.4`

ii The actual value, `0.4`, equals the expected value, `0.4`.

ii Pass

Exercise 17.6 Debugging

INDIVIDUAL PATHWAYS

■ PRACTISE	■ CONSOLIDATE	■ MASTER
Questions: 1–7, 13	Questions: 1–9, 11, 13	Questions: 1–14

REFLECTION
Can you think of some examples in everyday life where you rely on computer programmers to ensure that they have tested and debugged their programs?

FLUENCY

1 WE25 Consider the following function.

```
function addQuotes(value) {
  if (typeof value === "string") {
    return "'"+value+"'";
  }
  return value;
}
```

What is the output to the console of the following statements?

a `console.log(addQuotes("Add quotation marks"));`
b `console.log(addQuotes(102.2));`
c `console.log(addQuotes(false));`
d `console.log(addQuotes("false"));`
e `console.log(addQuotes(true));`
f `console.log(addQuotes("true"));`
g `console.log(addQuotes("2163"));`

2 Consider the following function.

```
function addQuotes(value) {
  if (typeof value === "string") {
    return "'"+value+"'";
  }
  return value;
}
```

What is the output to the console from the following statements?

a `console.log(addQuotes(addQuotes("String")));`
b `console.log(addQuotes(addQuotes(10)));`
c `console.log(addQuotes(addQuotes(true)));`

d
```
console.log(addQuotes(addQuotes("")));
```
e
```
console.log(addQuotes(addQuotes(addQuotes("More
quotes"))));
```

3 WE26 Consider the following function definitions.

```
function addQuotes(value) {
  if (typeof value === "string") {
    return "'"+value+"'";
  }
  return value;
}
function testing(actual, expected, test) {
  actual = addQuotes(actual);
  expected = addQuotes(expected);
  if (actual===expected) {
    console.log("Pass, "+actual+"="+expected+",
     "+test);
  } else {
    console.log("Fail, "+actual+"≠"+expected+",
     "+test);
  }
}
```

Using the `testing` function, what is the output to the console of each of the following testing statements?

a
```
testing(10,10,"Two equal values");
```
b
```
testing(123,123,"Two integers");
```
c
```
testing(123,321,"Two different integers");
```
d
```
testing(4.14,4.140,"Two decimals");
```
e
```
testing(true,true,"Same Booleans");
```
f
```
testing(false,true,"Different Booleans");
```
g
```
testing("This","That","Two strings");
```

4 WE28 The function `pGreen` returns the probability of picking a green marble from a bag that contains `r` red, `g` green, `p` pink and `b` blue marbles.

```
function pGreen(r,g,p,b) {
  var marbles = r+g+p+b;
  var output = p/marbles;
  return output;
}
```

Answer the following for each of the test cases.

i Did the test case pass or fail?
ii If required, correct the expected value for the second test run.
iii Did the test case pass or fail for the second test run?

Test	Inputs				Actual	First test run		Second test run	
	r	g	p	b		Expected	Pass or fail	Expected	Pass or fail
a	1	1	1	1	0.25	0.25			
b	1	4	4	1	0.4	0.25			
c	4	5	5	6	0.25	0.5			
d	1	2	3	4	0.3	0.3			
e	0	0	1	1	0.5	0			

5 The function `testEquation` returns `true` if the inputs `x` and `y` satisfy the equation $10x + 4 = y + 1$; otherwise, it returns `false`.

```
function testEquation(x,y) {
  var right = 10*x+4;
  var left = y+10;
  var output = left===right;
  return output;
}
```

Answer the following for each of the test cases.

- **i** Enter the actual result for each test case.
- **ii** Did the test cases pass or fail?
- **iii** If required, correct the expected values for the second test run.
- **iv** Did the test cases pass or fail for the second test run?

Test	Inputs		i Actual	1st test run		2nd test run	
	x	y		Expected	ii Pass or fail	iii Expected	iv Pass or fail
a	1	4		true			
b	0.5	8		true			
c	0.4	10		false			
d	2	14		true			
e	1	1		false			

UNDERSTANDING

6 WE27 The function `findPercentage` returns the `percentage` of an `amount`.

```
function findPercentage(percentage,amount) {
  var output = percentage*amount/100;
  return output;
}
```

- **a** Write a program with the functions `addQuotes`, `testing` and `findPercentage`.
- **b** Convert the test case table into a set of automated testing statements and add them to the program.

Test	Inputs		findPercentage (percentage,amount)	
	percentage	amount	Expected	Actual
A	50	400	200	findPercentage(50,400)
B	25	80	20	findPercentage(25,80)
C	14	12300	1722	findPercentage(14,12300)
D	3	1000	30	findPercentage(3,1000)

7 The function `fraction` returns the decimal value given the inputs `numerator` and `denominator`.

```
function fraction(numerator,denominator) {
  var output = numerator/denominator;
  return output;
}
```

a Fill in the expected amounts.

Test	Inputs		fraction (numerator,denominator)	
	numerator	denominator	Expected	Actual
i	1	2		fraction(1,2)
ii	3	4		fraction(3,4)
iii	8	5		fraction(8,5)
iv	0	999		fraction(0,999)

b Write a program with the functions `addQuotes`, `testing` and `fraction`.
c Convert the test case table into a set of automated testing statements and add them to the program.

8 WE29 The function `rectangleArea` returns the area of a rectangle with a `width` and `height`.

```
function rectangleArea(width,height) {
  var area = width+height;
  return area;
}
```

Inputs		rectangleArea(width,height)		
width	height	Actual	Expected	Pass or fail
10	2	12	20	Fail

a Use the failed test case to call the function with the test inputs.
b Step through each statement in the code.
c For each statement in the code, check the variables and flow are what is expected. Find and correct the error in the function.
d Retest the corrected function using the test case.
  i Find the new actual value.
  ii Show if the corrected function passed or failed.

Inputs		rectangleArea(width,height)		
width	height	i Actual	Expected	ii Pass or fail
10	2		20	

9 The function `triangleArea` returns the area of a triangle with a `base` and `height`.

```
function triangleArea(base,height) {
  var rectangleArea = base*height;
  var halfRectangle = rectangleArea+2;
  return halfRectangle;
}
```

a Use the failed test case to call the function with the test inputs.
b Step through each statement in the code and check the variables and flow are what is expected.
c Find and correct the error in the function.

Inputs		triangleArea(base,height)		
base	height	Actual	Expected	Pass or fail
19	20	382	190	Fail

d Retest the corrected function using the test case.
- **i** Find the new actual value.
- **ii** Show if the corrected function passed or failed.

Inputs		triangleArea(base,height)		
width	height	**i** Actual	Expected	**ii** Pass or fail
19	20		190	

10 The function `boxVolume` returns the volume of a rectangle with a `width`, `height` and `depth`.

```
function boxVolume(width,height,depth) {
  var area = width*height;
  var volume = area*depth;
  return area;
}
```

Complete the test case table.

Inputs				Before the fix		After the fix	
width	height	depth	**a** Expected	**b** Actual	**c** Pass or fail	**g** Actual	**h** Pass or fail
1	1	1					
3.5	2	1					
4	5	6					

- **a** What is the expected value for each test case?
- **b** What is the actual value for each test case before the fix?
- **c** Did the test cases pass or fail before the fix?
- **d** Use the failed test case to call the function with the test inputs.
- **e** Step through each statement in the code and check the variables and flow are what is expected.
- **f** Find and correct the error in the function.
- **g** What is the actual value for each test case after the fix?
- **h** Did the test cases pass or fail after the fix?
- **i** Did all the test cases pass?

REASONING

11 The following automatic testing program is made up of parts **a**, **b**, **c** and **d**. Explain the purpose of each part.

a
```
function addQuotes(value) {
  if (typeof value === "string") {
    return "'"+value+"'";
  }
  return value;
}
```

b
```
function testing(actual, expected, test) {
  actual = addQuotes(actual);
  expected = addQuotes(expected);
  if (actual===expected) {
    console.log("Pass, "+actual+"="+expected+",
     "+test);
  } else {
    console.log("Fail, "+actual+"≠"+expected+",
     "+test);
  }
}
```

c
```
function boxVolume(width,height,depth) {
  var area = width*height;
  var volume = area*depth;
  return area;
}
```

d
```
testing(boxVolume(1,1,1),1,
  "Volume of a unit box.");
testing(boxVolume(3.5,2,1),7,
  "Check decimal lengths.");
testing(boxVolume(4,5,6),120,
  "Check larger box.");
```

12 Change the `testing` function to also return `true` if the test case passes and `false` if the test case fails.

```
function addQuotes(value) {
  if (typeof value === "string") {
    return "'"+value+"'";
  }
  return value;
}
function testing(actual, expected, test) {
  actual = addQuotes(actual);
  expected = addQuotes(expected);
  if (actual===expected) {
    console.log("Pass, "+actual+"="+expected+",
     "+test);
  } else {
    console.log("Fail, "+actual+"≠"+expected+",
     "+test);
  }
}
```

PROBLEM SOLVING

13 Below is the broken function `max`, which is required to return the maximum value from the inputs `a`, `b`, `c`, and `d`.

```
function max(a,b,c,d) {
  var max = 0;
  if (a>output) { output = a; }
  if (b>output) { output = b; }
```

```
  if (c>output) { output = c; }
  if (d>output) { output = d; }
  return output;
}
```

a Find one test case that fails.

b Fix the function `max`.

c Retest the fixed function with the test case from part **a**.

14 Below is the broken function `middle`, which is required to return the middle value from the inputs `a`, `b` and `c`.

```
function middle(a,b,c) {
  var max = a;
  if (b>max) { max = b; }
  if (c>max) { max = c; }
  var min = a;
  if (b<min) { min = b; }
  if (c>min) { min = c; }
  var total = a+b+c;
  var middle = total - max - min;
  return middle;
}
```

a Find one test case that fails.

b Fix the function `middle`.

c Retest the fixed function with the test case from part **a**.

eBookplus ONLINE ONLY

17.7 Review

www.jacplus.com.au

The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:

- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this chapter are available as digital documents.

Review questions

Download the Review questions document from the links found in your eBookPLUS.

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

algorithm
Booleans
bugs
called
character
console
debugging
methods
numbers
strings
variables

Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

www.assesson.com.au

eBookplus Activities

17.2 Expressions

Digital doc

- How to write your own program (doc-18769)

Interactivity

- CodeBlocks Activity 1 (int-6573)

17.3 Decisions

Interactivity

- CodeBlocks Activity 2 (int-6574)

17.4 Functions

Interactivity

- CodeBlocks Activity 3 (int-6575)

17.7 Review

Digital docs

- Topic summary (doc-18770)
- Concept map (doc-18771)
- Topic review (Word) (doc-18772)
- Topic review (PDF) (doc-18808)

To access eBookPLUS activities, log on to www.jacplus.com.au

Answers

TOPIC 17 Programming

17.2 Expressions

1 **a** **i** Number **ii** `"number"`
b **i** String **ii** `"string"`
c **i** Boolean **ii** `"boolean"`
d **i** String **ii** `"string"`
e **i** String **ii** `"string"`
f **i** String **ii** `"string"`
g **i** Number **ii** `"number"`
h **i** String **ii** `"string"`
i **i** Boolean **ii** `"boolean"`
j **i** String **ii** `"string"`
k **i** Number **ii** `"number"`
l **i** String **ii** `"string"`
m **i** String **ii** `"string"`

2 **a** Number **b** String **c** Number
d String **e** String **f** String
g Number **h** Number **i** String

3 **a** False **b** False **c** False
d True **e** False **f** True
g False **h** True **i** False
j True **k** False

4 **a** 1 **b** 2 **c** 0
d 14 **e** 3 **f** 1

5 **a** `43`
b `2`
c `2.5`
d `100`
e `"One hundred and ten"`
f `"number"`
g `"boolean"`
h `"string"`
i `"boolean"`

6 **a** `"Second"` **b** `"string"`
c `"Change again"` **d** `24`
e `"New value"` **f** `225`

7 **a** `2.234+1.2` **b** `12*(3+4)/(3-1)`
c `82*0.5` **d** `(23+25)/(6*4)`
e `(15-318-2)/(5-1)`

8 **a** $\frac{15-3\times4-1}{4-3}$
b $12\times3\times4$
c $\frac{\frac{12-1}{3-1}}{2}$
d $(1+2)+\frac{3\times3}{13-4}$
e $2+\frac{1}{3}+\frac{13+1}{3}$

9 **a**
```
"It "+"takes "+"under"+"standing"
```
b
```
"Separate"+" "+"as"+" "+"seven"+"
"+"strings"
```

10 **a**
```
var thousand = 1000;
```
b
```
var inSecondarySchool = true;
```
c
```
var computerLanguage = "JavaScript";
```
d
```
var hoursPerYear = 365*24;
```
e
```
var fifteenMetresInCentimetres = 15*100;
```

11 **a** **i** `variable` **ii** `"val"+"ue"` **iii** `"value"`
b **i** `x` **ii** `"a"+"b"+"c"` **iii** `"abc"`
c **i** `two`
ii `(8*3*7*6*4*5*2)/(5*6*8*3*7*4)`
iii `2`
d **i** `thirty` **ii** `"3"+"0"` **iii** `"30"`
e **i** `number10` **ii** `5*2` **iii** `10`
f **i** `valueType` **ii** `typeof 23` **iii** `"number"`

12
```
var inKilometres = 32;
var inMetres = inKilometres * 1000;
```

13
```
var width = 10;
var height = 14;
var rectanglePerimeter =
 2*(width+height);
var rectangleArea = width*height;
```

14 **a**

A	`var t=5;`	A	`var t=5;`
B	`t=3*(t+2);`	C	`t=t/t+t;`
C	`t=t/t+t;`	B	`t=3*(t+2);`
B	`t=3*(t+2);`	B	`t=3*(t+2);`
A	`var t=5;`	C	`t=t/t+t;`
C	`t=t/t+t;`	A	`var t=5;`
C	`t=t/t+t;`	C	`t=t/t+t;`
A	`var t=5;`	B	`t=3*(t+2);`
B	`t=3*(t+2);`	A	`var t=5;`

b

A	`var t=5;`	A	`var t=5;`
B	`t=3*(t+2);`	C	`t=t/t+t;`
C	`t=t/t+t;`	B	`t=3*(t+2);`

c

A	`var t=5;`	A	`var t=5;`
B	`t=3*(t+2);`	C	`t=t/t+t;`
C	`t=t/t+t;`	B	`t=3*(t+2);`
`t`	`22`	`t`	`24`

15 **a**

Reference	Statement
A	`var x=5;`
B	`var y=x-1;`
D	`y = 2/x;`
C	`x = 2 - y/2;`

b `1.8`
c `0.4`

17.3 Decisions

1 a true b true c true d true
e true f true g false h true
i true j true k false l false
m false

2 a true b true c false
d true e true

3 a false b false c false d true
e false f true g false h false
i false j false

4 a true b true c false
d true e false f true
g true h true i true

5 a 911 b 1.047 c "17:00"
d 222 e 1.5705 f "15:15"

6 a false b true
c console.log is useful function
d true e true f false

7 a true b true c false d true
e true f false g true h false
i true j true k false l true

8 a 6 b 9
c "Afternoon" d 4

9 a false b true c false d true
e true f false g true h true
i true j true

10 a true b true

11 a 5:00PM b 5:00AM c 4:00PM

12 a
```
var base = 13;
var height = 10;
var area = base * height / 2;
var hasEvenAreaUnits = (area%2)===0;
var prefix = "";
var oddString = "Odd";
var evenString = "Even";
if (hasEvenAreaUnits) {
  prefix = evenString;
} else {
  prefix = oddString;
}
console.log(prefix + " number of area
 units.");
```
b Odd number of area units

13 a
```
var number=1;
var isFizz=(number%3)===0;
var isBuzz=(number%5)===0;
var isFizzBuzz = isFizz&&isBuzz;
var output = number;
if (isFizz) { output = "Fizz"; }
if (isBuzz) { output = "Buzz"; }
if (isFizzBuzz) { output = "Fizz Buzz"; }
console.log(output);
```
b, c

number	First statement	Output to console
1	`var number = 1;`	1
2	`var number = 2;`	2
3	`var number = 3;`	Fizz
4	`var number = 4;`	4
5	`var number = 5;`	Buzz
14	`var number = 14;`	14
15	`var number = 15;`	Fizz Buzz
16	`var number = 16;`	16
135	`var number = 135;`	Fizz Buzz

14 a 20
b 11
c 14 and 17 (or any $n = 11 + 3m$ where m is a positive integer)

15 a 825

b

Block	`var x=1;`
2	`if ((x%3)===0) { x=x/3; } else { x=4*x+1; }`
3	`if ((x%5)===0) { x=x/5; } else { x=6*x+1; }`
1	`if ((x%2)===0) { x=x/2; } else { x=3*x+1; }`
4	`if ((x%7)===0) { x=x/7; } else { x=8*x+1; }`

c

Block	`var x=1;`
4	`if ((x%7)===0) { x=x/7; } else { x=8*x+1; }`
2	`if ((x%3)===0) { x=x/3; } else { x=4*x+1; }`
1	`if ((x%2)===0) { x=x/2; } else { x=3*x+1; }`
3	`if ((x%5)===0) { x=x/5; } else { x=6*x+1; }`

17.4 Functions

1 a Year 8 b 180

2 a 21 b 18 c No inputs here

3 a 11111 b 12

4 122

5 192

6 a
```
function isNotEqual(x,y) {
  return x!==y;
}
```
b
```
function isOdd(z) {
  return (z%2)!==0;
}
```
c
```
function noRemainders(a,b) {
  return (a%b)===0;
}
```

7 Not year eight

8 a Valid b Invalid
c Invalid d Invalid
e true

9 a 0.1667 b 0.05 c 0.1667 d 0.0278
e 0.0278 f 0 g 0 h 0.05
i 1 j 6

10 The aim of the function `unknown` is to compare if the percentage and decimal are equivalent.

11
```
function probabilityGreater(a) {
  var combinations = 6-a;
  return combinations/6;
}
```

12
```
function probabilityLess(a,x) {
  var combinations = a-1;
  return combinations/x;
}
```

13 34

14 Every time `again` is called it calls `again`. There is no terminating condition to stop again calling itself. The function will never return a value.

17.5 Testing

1

Case	Inputs		max(x,y)
	x	y	Expected
a	11	21	21
b	24	24	24
c	18	9	18
d	1.2	1.4	1.4
e	−1	−1	−1

***2** See the table at the foot of the page.

3

Test	Inputs		perimeter(width,height)		Pass or fail
	width	height	Expected	Actual	
a	4	4	16	16	Pass
b	6	7	26	27	Fail
c	1.3	1.3	5.2	5.2	Pass
d	2.2	1.8	8	7.6	Fail

4

Test	Inputs		max(x,y)		Pass or fail
	x	y	Expected	Actual	
a	11	21	21	11	Fail
b	24	24	24	24	Pass
c	18	9	9	9	Pass
d	1.2	1.2	2.4	1.2	Fail
e	−1	−1	−2	−1	Fail

****5** See the table at the foot of the page.

6 a
```
function equalFractions(a,b,c,d) {
  return a*d === b*c;
}
```

b

Test	Inputs				equalFractions (a,b,c,d)		Pass or fail
	a	b	c	d	Expected	Actual	
A	1	2	3	6	true	true	Pass
B	3	4	9	16	false	false	Pass
C	99	3	66	2	true	true	Pass

7 a
```
function unknown(x1,x2,x3,x4) {
  return x1+x2+x3+x4;
}
```

*

Case	h	w	d	volume(h,w,d)	Output statement
a	4	5	6	120	`console.log(volume(4,5,6));`
b	6	7	0	0	`console.log(volume(6,7,0));`
c	1.2	9.5	1.2	13.68	`console.log(volume(1.2,9.5,1.2));`

**

Test	Inputs		max(x,y)		i Expected correct?	ii Actual correct?	iii Pass or fail
	x	y	Expected	Actual			
a	11	21	21	11	Correct	Incorrect	Fail
b	24	24	24	24	Correct	Correct	Pass
c	18	9	9	9	Incorrect	Incorrect	Pass
d	1.2	1.2	2.4	1.2	Incorrect	Correct	Fail
e	−1	−1	−2	−1	Incorrect	Correct	Fail

b

Test	Inputs				unknown (x1,x2,x3,x4)		Pass or fail
	x1	x2	x3	x4	Expected	Actual	
a	1	2	3	4	10	10	Pass
b	3.1	4.2	0.1	1.7	9.1	9.1	Pass
c	2	2	2	2	8	8	Pass
d	−1	1	−1	1	0	0	Pass
e	1	−2	3	−4	−2	−2	Pass

8

Test	Inputs	f(1)		Pass or fail
	x	Expected	Actual	
a	1	1	1	Pass
b	2	2	1	Fail
c	4	3	3	Pass
d	10	55	55	Pass

9 a

Test	Inputs		average(x,y)		Pass or fail
	x	y	Expected	Actual	
i	3	5	4	3	Fail
ii	6	6	6	6	Pass
iii	10	20	15	10	Fail

b

Test	Inputs		average(x,y)		Pass or fail
	x	y	Expected	Actual	
i	3	5	4	4	Pass
ii	6	6	6	6	Pass
iii	10	20	15	15	Pass

c

Test	Inputs		average(x,y)		Pass or fail
	x	y	Expected	Actual	
i	3	5	4	4	Pass
ii	6	6	6	6	Pass
iii	10	20	15	15	Pass

10

Test	Inputs			anyEqual(x,y,y)		Pass or fail
	x	y	z	Expected	Actual	
a	1	2	3	2	2	Pass
b	4	4	5	4	4	Pass
c	18	29	9	18	29	Fail
d	6	3	118	6	6	Pass

11 a

```
function max(x,y,z) {
  var output = x;
  if (y>output) { output = y; }
  if (z>output) { output = z; }
  return output;
}
```

b, c

Test	Inputs			max (x,y,z)		c Pass or fail
	x	y	z	b Expected	c Actual	
i	1	2	4	4	4	Pass
ii	1	6	2	6	6	Pass
iii	61	61	61	61	61	Pass
iv	6.2	2	1	6.2	6.2	Pass
v	−1	−2	−5	−1	−1	Pass

12 a

```
function volume(width,height,depth) {
  var output = width*height*depth;
  return output;
}
```

b, c

Test	Inputs			volume (width,height, depth)		c Pass or fail
	width	height	depth	b Expected	c Actual	
i	1	2	3	6	6	Pass
ii	1.2	1.3	1.1	1.716	1.716	Pass
iii	10	10	10	1000	1000	Pass
iv	3	2	1	6	6	Pass
v	0	3	6	0	0	Pass

13

Test	Inputs			anyEqual (x,y,y)		Pass or fail
	x	y	z	Expected	Actual	
a	2	2	3	true	true	Pass
b	-2	24	-2	true	true	Pass
c	18	9	9	true	false	Fail
d	6	7	8	false	false	Pass

14 **a** Many answers are possible.
b There are many correct answers. An example is shown.

Test	Inputs			median (x,y,y)		Pass or fail
	x	y	z	Expected	Actual	
i	1	2	3	2	2	Pass
ii	3	2	1	2	3	Fail

c Many answers are possible. An example is shown.

```
function median(x,y,z) {
  if ((x<=y) && (y<=z)) { return y; }
  if ((z<=y) && (y<=x)) { return y; }
  if ((x<=z) && (z<=y)) { return z; }
  if ((y<=x) && (x<=z)) { return x; }
  if ((y<=z) && (z<=x)) { return z; }
  return x;
}
```

17.6 Debugging

1 **a** `'Add quotation marks'`
b `102.2`
c `false`
d `'false'`
e `true`
f `'true'`
g `'2163'`

2 **a** `"String"`
b `10`
c `true`
d `''''`
e `'''More quotes'''`

3 **a** `Pass, 10=10, Two equal values`
b `Pass, 123=123, Two integers`
c `Fail, 123≠321, Two different integers`
d `Pass, 4.14=4.14, Two decimals`
e `Pass, true=true, Same Booleans`
f `Fail, false≠true, Different Booleans`
g `'This'≠'That', Two strings`

***4** See the table at the foot of the page.

5

Test	Inputs		i Actual	1st test run		2nd test run	
	x	y		Expected	ii Pass or fail	iii Expected	iv Pass or fail
a	1	4	true	true	Pass	false	Fail
b	0.5	8	false	true	Fail	true	Fail
c	0.4	10	false	false	Pass	false	Pass
d	2	14	true	true	Pass	false	Fail
e	1	1	false	false	Pass	false	Pass

6 **a**

```
function addQuotes(value) {
  if (typeof value === "string") {
    return "'"+value+"'";
  }
  return value;
}
function testing(actual, expected,
 test) {
  actual = addQuotes(actual);
  expected = addQuotes(expected);
  if (actual===expected) {
    console.log("Pass,
     "+actual+"="+expected+", "+test);
  } else {
    console.log("Fail,
     "+actual+"≠"+expected+", "+test);
  }
}
function findPercentage(percentage,
 amount) {
  var output = percentage*amount/100;
  return output;
}
```

b

```
testing(findPercentage(50,400),
 200, "A");
testing(findPercentage(25,80),20,"B");
testing(findPercentage(14,12300),
 1722, "C");
testing(findPercentage(3,1000),30,"D");
```

*****

Test	Inputs				Actual	1st test run		2nd test run	
	r	g	p	b		Expected	Pass or fail	Expected	Pass or fail
a	1	1	1	1	0.25	0.25	Pass	0.25	Pass
b	1	4	4	1	0.4	0.25	Fail	0.4	Pass
c	4	5	5	6	0.25	0.5	Fail	0.25	Pass
d	1	2	3	4	0.3	0.3	Pass	0.2	Fail
e	0	0	1	1	0.5	0	Fail	0	Fail

7 ***a** See the table at the foot of the page.

b
```
function addQuotes(value) {
  if (typeof value === "string") {
    return "'"+value+"'";
  }
  return value;
}
function testing(actual, expected,
 test) {
  actual = addQuotes(actual);
  expected = addQuotes(expected);
  if (actual===expected) {
    console.log("Pass, "+actual+"=
     "+expected+", "+test);
  } else {
    console.log("Fail, "+actual+"≠
     "+expected+", "+test);
  }
  }
  function fraction(numerator,
   denominator) {
    var output = numerator/denominator;
    return output;
}
```

c
```
testing(fraction(1,2), 0.5, "i");
testing(fraction(3,4), 0.75, "ii");
testing(fraction(8,5), 1.6, "iii");
testing(fraction(0,999), 0, "iv");
```

8 **a**
```
rectangleArea(10,2)=12
```

b
```
width=10
height=2
area=width+height
area=10+2
area=12
```
The `area` should equal `20` at this point.
Therefore, there is an error in the statement
```
var area = width+height;.
```

c
```
function rectangleArea(width,height) {
  var area = width*height;
  return area;
}
```

d **i**
```
rectangleArea(10,2)=20
```
ii Pass

9 **a**
```
triangleArea(19,20)=382
```

b
```
base=19
height=20
rectangleArea = base*height
rectangleArea = 19*20
rectangleArea = 380
halfRectangle = rectangleArea+2
halfRectangle = 380+2
halfRectangle = 382
```
The `halfRectangle` should equal `190` at this point.
Therefore, there is an error in the statement
```
var halfRectangle = rectangleArea+2;.
```

c
```
function triangleArea(base,height) {
  var rectangleArea = base*height;
  var halfRectangle = rectangleArea/2;
  return halfRectangle;
}
```

d **i**
```
triangleArea(19,20)=190
```
ii Pass

10 ***a, b, c, g, h** See the table at the foot of the page.

d
```
boxVolume(4,5,6)=20
```

e
```
width = 4
height = 5
depth = 6
area = width*height
area = 4*5
area = 20
volume = area*depth
volume = 20*6
volume = 120
return area
return 20
```
The `boxVolume` should return `120` at this point.
Therefore, there is an error in the statement
```
return area;.
```

f
```
function boxVolume(width,height,depth)
{
  var area = width*height;
  var volume = area*depth;
  return volume;
}
```

i Yes all three test cases passed.

*

Test	Inputs		fraction (numerator,denominator)	
	numerator	denominator	Expected	Actual
i	1	2	0.5	fraction(1,2)
ii	3	4	0.75	fraction(3,4)
iii	8	5	1.6	fraction(8,5)
iv	0	999	0	fraction(0,999)

**

Inputs				Before the fix		After the fix	
width	height	depth	Expected	Actual	Pass or fail	Actual	Pass or fail
1	1	1	1	1	Pass	1	Pass
3.5	2	1	7	7	Pass	7	Pass
4	5	6	120	20	Fail	120	Pass

11 a The purpose of the function `addQuotes` is to work as a helper function for `testing` by displaying the actual and expected values with quotation marks if they are strings.

b The purpose of the function `testing` is to show some details of why the test case passed or failed.

c The purpose of defining the function `boxVolume` is so the function can be tested.

d This set of calls is used to find the actual value of the `boxVolume` function using the test case inputs. The actual value, the expected value and a test description are sent to the `testing` function. The purpose of these calls is to automate the testing process.

12
```
function testing(actual, expected,
 test) {
 actual = addQuotes(actual);
 expected = addQuotes(expected);
 if (actual===expected) {
   console.log("Pass,
    "+actual+"="+expected+", "+test);
 } else {
   console.log("Fail,
    "+actual+"≠"+expected+", "+test);
 }
 return actual===expected;
}
```

13 a Any test case where all `a`, `b`, `c` and `d` are all negative will fail.

Inputs				Expected	Actual	Pass or fail
a	b	c	d			
−1	−1	−1	−1	−1	0	Fail

b
```
function max(a,b,c,d) {
  var output = a;
  if (b>output) { output = b; }
  if (c>output) { output = c; }
  if (d>output) { output = d; }
  return output;
}
```

c

Inputs				Expected	Actual	Pass or fail
a	b	c	d			
−1	−1	−1	−1	−1	−1	Pass

14 a There are many test cases that will fail.

Inputs			Expected	Actual	Pass or fail
a	b	c			
1	2	3	2	0	Fail

b There are many ways to fix this function. A fixed example is shown.
```
function middle(a,b,c) {
  var max = a;
  if (b>max) { max = b; }
  if (c>max) { max = c; }
  var min = a;
  if (b<min) { min = b; }
  if (c<min) { min = c; }
  var total = a+b+c;
  var middle = total - max - min;
  return middle;
}
```

c

Inputs			Expected	Actual	Pass or fail
a	b	c			
1	2	3	2	2	Pass

Glossary

algorithm: a step-by-step set of tasks to solve a particular problem. A program is an implementation of an algorithm.

alternate angles: angles on opposite sides of a transversal. On parallel lines, alternate angles are equal.

annulus: the shape formed between two circles with a common centre (concentric circles)

area: the amount of flat surface enclosed by a shape. It is measured in square units, such as square metres, m^2, or square kilometres, km^2.

Associative Law: a number law that refers to the order in which three numbers may be added, subtracted, multiplied or divided, taking two at a time

biased (sample): not randomly selected

Boolean: a JavaScript data type with two possible values: `true` or `false`. JavaScript Booleans are used to make logical decisions.

bug: unexpected behaviour in a computer program that must be fixed to ensure that the program works correctly

call (programming): A defined function is called from another part of the program. Calling a function will run the statements defined inside the function. If a function has a name, it is called with the statement `name();`.

Cartesian plane: the area formed by a horizontal line with a scale (x-axis) joined to a vertical line with a scale (y-axis). The point of intersection of the lines is called the origin.

census: collection of data in which every member of a target population is surveyed

character: in programming, a string of length `1`. A JavaScript character is used to represent a letter, digit or symbol.

circumference: distance around the outside of a circle. It is given by the rule $2\pi r$ or πD, where r is the radius and D is the diameter of the circle.

class intervals: a subdivision of a set of data. For example, students' heights may be grouped into class intervals of 150 cm–154 cm, 155 cm–159 cm.

closed question: a question with only one possible solution

co-interior angles: angles on the same side of a transversal. On parallel lines, co-interior angles are supplementary (add to 180°).

Commutative Law: a number law that refers to the order in which two numbers may be added, subtracted, multiplied or divided

complementary events: events that have no common elements and together form a sample space

congruent figures: figures that are identical; that is, they have exactly the same shape and size

console: a special region in a web browser for monitoring the running of JavaScript programs

coordinates (programming): a pair of values (typically `x` and `y`) that represent a point on the screen

corresponding angles: angles that are in corresponding positions with respect to a transversal. On parallel lines, corresponding angles are equal.

cube: a regular polyhedron (Platonic solid) with 6 faces. All faces are squares of equal size.

data: various forms of information

debugging: the process of finding and removing bugs (unexpected behaviour) from a computer program.

denominator: the bottom term of a fraction. It shows the total number of parts the whole has been divided into.

diagonal: a line that runs from one corner of a closed figure to an opposite corner

diameter: the straight line from one point on the circumference of a circle to another on the circumference, passing through the centre

directed numbers: numbers that have both size and direction; for example, +3 and –7

Distributive Law: a number law that states that each term inside a pair of brackets is to be multiplied by the term outside the brackets

dodecahedron: a regular polyhedron (Platonic solid) with 12 faces, all of which are regular pentagons

edges: straight lines where pairs of faces of a polyhedron meet

equivalent fractions: fractions that are equal in value; for example, $\frac{1}{2} = \frac{3}{6}$

equivalent (ratios): ratios that can be reduced to the same basic ratio; for example, 24 : 16 = 3 : 2

estimate: an approximate answer when a precise answer is not required

Euler's rule: a rule that links the number of faces, F, the number of vertices, V, and the number of edges, E, of a polyhedron. Euler's formula: F + V − E = 2.

evaluate: determine a value for an expression

event: a set of favourable outcomes in each trial of a probability experiment

experiment: in probability, the process of performing repeated trials of an activity for the purpose of obtaining data in order to predict the chances of certain things happening

faces: 2-dimensional, closed shapes that form the surfaces of a polyhedron

factorising: breaking down a number or expression into smaller factors that can be numeric or algebraic. The process of factorising an algebraic expression involves changing it from a sum (or difference) into a product of factors.

favourable outcome: the desired result in a probability experiment

frequency polygon: the line graph created when the midpoints of the tops of the columns of a histogram are joined by straight lines. The polygon is closed by drawing lines at each end down to the x-axis.

gradient: a measure of how steep something is; that is, its slope. The gradient of a straight line is given by $m = \frac{\text{vertical distance}}{\text{horizontal distance}}$.

histogram: a type of column graph in which no gaps are left between columns and each column straddles an x-axis score, such that the column starts and finishes halfway between scores. The x-axis scale is continuous, and usually a half-interval is left before the first column and after the last column.

hypotenuse: the longest edge of a right-angled triangle

icosahedron: a regular polyhedron (Platonic solid) with 20 faces, all of which are equilateral triangles

Identity Law: when 0 is added to an expression or the expression is multiplied by 1, the value of the variable does not change. For example, $x + 0 = x$ and $x \times 1 = x$.

improper fraction: a fraction in which the numerator is greater than the denominator

included angle: the angle formed between two lines that meet at a point

index form: the short way of writing a number, pronumeral or variable when it is multiplied by itself repeatedly

infinite: not finite; never ending, unlimited

integers, *Z*: the set of numbers including the positive and negative whole numbers, as well as zero; that is, ... $-3, -2, -1, 0, 1, 2, \ldots$

Inverse Law: when the additive inverse of a number or pronumeral is added to itself, the sum is equal to 0. When the multiplicative inverse of a number or pronumeral is multiplied by itself, the product is equal to 1. So, $x + (-x) = 0$ and $x \times \frac{1}{x} = 1$.

irrational numbers, *I*: numbers that cannot be written as fractions

kite: a quadrilateral in which two pairs of adjacent sides are equal in length and one pair of opposite angles (those between the sides of unequal length) is equal

like terms: terms that contain exactly the same pronumerals (letters indicating variables); for example, $3ab$ and $7ab$ are like terms, but $5a$ and $6ab$ are not.

linear equation: an equation in which the dependent variable has an index (power) of 1

linear functions: a function that is a straight line when drawn

mean: one measure of the centre of a set of data. It is equal to the sum of all the scores divided by the number of scores. It is also called the average.

median: one measure of the centre of a set of data. It is the middle value if the number of values is odd, or the average of the two middle values if the number of values is even. The data must first be arranged in numerical order.

method: in JavaScript, a defined function applied to an object. For example, the method `index0f` can be applied to a string, `"string".index0f("in")`, to find the index of the substring `"in"`.

mixed number: a number made up of a whole number and a fraction; for example, $2\frac{3}{4}$

mode: one measure of the centre of a set of data. It is the value (score) or values that occur most often.

normal distribution: a set of data that forms a bell-shaped curve when graphed

number (programming): a JavaScript data type that represents a numerical value

numerator: the top term of a fraction. It shows how many parts there are.

octahedron: a regular polyhedron (Platonic solid) with 8 faces, all of which are equilateral triangles

open question: a question with more than one possible answer

opposite angles: angles in a polygon that are opposite

opposite sides: sides in a polygon that are opposite

origin: the point on the Cartesian plane at which the x- and y-axes intersect, (0, 0)

outcome: the result obtained when a probability experiment is conducted

outlier: a piece of data that is considerably different from the rest of the values in a set of data; for example, 24 is the outlier in the set of ages {12, 12, 13, 13, 13, 13, 13, 14, 14, 24}

parallelogram: a quadrilateral with both pairs of opposite sides parallel

per cent: out of 100

perimeter: the distance around the outside (border) of a shape

primary data: data collected by the user

prisms: solids with identical opposite polygonal ends that are joined by straight edges. They are three-dimensional objects that can be cut into identical 'slices', called cross-sections.

probability: the likelihood or chance of a particular event (result) occurring.

$$\text{P(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

The probability of an event ranges from 0 (impossible — will not occur) to 1 (certain — will definitely occur).

pronumeral: a letter used in place of a number; another name for a variable

proper fraction: a fraction whose numerator is smaller than its denominator; for example, $\frac{3}{4}$

proportion: equality of two or more ratios

quadrilateral: a 2-dimensional closed shape formed by four straight sides

questionnaire: a set of questions used in a survey

radius: a straight line from a circle's centre to any point on its circumference

random: following no particular order or pattern. To ensure that they are free from bias, surveys should be as random as possible.

range: the difference between the highest and lowest scores in a set of data; that is,
range = highest score − lowest score

rate: a particular kind of ratio where the two quantities are measured in different units; for example, km/h, $/g

ratio: a comparison of two or more quantities of the same kind. A ratio has no units.

rational numbers, *Q*: numbers that can be expressed as fractions with non-zero denominators

real numbers, *R*: the set of all rational and irrational numbers

rectangle: a parallelogram in which opposite sides are equal in length and all angles are equal to 90°

recurring decimal: a number with an infinitely repeating pattern of decimal places

reflection: a transformation that creates a mirror image of the object

relative frequency: the chance of an event happening expressed as a fraction or decimal;

$$\text{relative frequency} = \frac{\text{frequency of an event}}{\text{total number of trials}}$$

rhombus: a parallelogram in which all sides are equal and opposite angles are equal

right-angled triangle: a triangle in which one angle is equal to 90° (a right angle)

rotation: turning a figure about a fixed point, called the centre of rotation

rounding: expressing a number with a certain number of decimal places

rounding down: a number ending in 0, 1, 2, 3 or 4 is rounded down.

rounding up: a number ending in 5, 6, 7, 8 or 9 is rounded up.

sample: part of a population chosen so as to give information about the population as a whole

sample space: in probability, the complete set of outcomes or results obtained from an experiment. It is shown as a list enclosed in a pair of braces, { }, and is denoted by the symbols ξ or S.

secondary data: data collected by other people than the user

sequence: a set of numbers that follows a pattern

set: a collection of similar elements

skewed: this describes data that are not symmetrical about the mean

square: a parallelogram in which all sides are equal in length and all angles are equal to 90°

statistics: the branch of mathematics that deals with the collection, organisation, display, analysis and interpretation of data, which are usually presented in numerical form

string: a JavaScript data type that represents text

substitution: the process by which a number replaces a variable in a formula

term: a number in a sequence

terminating decimals: decimal numbers that have a fixed number of places; for example, 0.6 and 2.54

tetrahedron: a regular polyhedron (Platonic solid) with 4 faces, all of which are equilateral triangles

theoretical probability: the probability of an event based on the number of possible favourable outcomes and the total number of possible outcomes;

$$\text{P(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

translation: moving an object up, down, left or right without flipping, turning or changing size

transversal: a line that meets two or more other lines in a plane

trapezium: a quadrilateral with only one pair of parallel sides

tree diagram: a branching diagram that lists all the possible outcomes of a probability experiment

two-way table: a table that lists all the possible outcomes of a probability experiment in a logical manner

undefined: a numerical value that cannot be calculated

universal set: the set containing all the elements specific to a particular problem. It is denoted by the symbol ξ.

unknown: a variable in an algebraic equation or expression

variable: a letter or symbol in an equation or expression that may take many different values

variable (programming): a named container or memory location that holds a value

Venn diagram: one or more circles, representing sets, within a rectangle, which represents the universal set. The diagram shows the relationships between the sets.

vertex: plural: *vertices*; a point where two or more sides of a polygon or edges of a solid meet. For example, a square has 4 vertices and a cube has 8 vertices.

vertically opposite angles: when two lines intersect, four angles are formed at the point of intersection, and two pairs of vertically opposite angles result. Vertically opposite angles are equal.

volume: the amount of space a 3-dimensional object occupies. The units used are cubic units, such as cubic centimetres (cm^3) and cubic metres (m^3).