# Essential Insight Exam Guide

## Mathematics Applications

Year 12 WACE

Western Australian Curriculum

2025 Edition

Jeremy Chen

# Essential Insight Exam Guide
# Mathematics Applications
# Year 12 WACE

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|---|---|
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# Unit 3

## Unit 3.1 – Bivariate data analysis

### Section 1

**2023 Section 1 Question 1**

**Bivariate data analysis**

A survey was conducted by a film studio executive of filmgoers' favourite genres. The categories chosen were action, drama and comedy. The information is displayed in the table below.

| | | Genre | | | |
|---|---|---|---|---|---|
| | | Action | Drama | Comedy | Total |
| Age (years) | 1–10 | 40 | 45 | | 125 |
| | 11–20 | 17 | | 43 | 86 |
| | 21–30 | | 25 | | 125 |
| | Total | | 96 | 113 | 336 |

(a) Complete the two-way table above. (3 marks)

(b) Identify the response variable for these data. (1 mark)

(c) The incomplete two-way percentaged table is shown below.

| | | Genre | | | |
|---|---|---|---|---|---|
| | | Action | Drama | Comedy | Total |
| Age (years) | 1–10 | | 36 | | |
| | 11–20 | | 30 | | |
| | 21–30 | 56 | | 24 | |
| | Total | | | | |

(i) Complete the table above by using either row percentages or column percentages, as appropriate. (3 marks)

(ii) State an association that can be observed from the two-way percentaged table. (1 mark)

**2022 Section 1 Question 3**

**Bivariate data analysis**

An analysis was done on hospital patients based on their age and injuries received from various outdoor activities. The information is displayed below.

| | | Activity | | | |
|---|---|---|---|---|---|
| | | Trampoline | Bike | Skateboard | Total |
| Age (years) | 0–8 | 150 | 120 | 30 | 300 |
| | 9–16 | 180 | | 90 | 450 |
| | 17–24 | 120 | 80 | | 400 |
| | 25–32 | 20 | | | 250 |
| | Total | 470 | | 400 | 1400 |

(a) Complete the two-way table above. (2 marks)

(b) Identify the explanatory variable for these data. (1 mark)

(c) The incomplete two-way percentaged table is shown below.

| | | Activity | | | |
|---|---|---|---|---|---|
| | | Trampoline | Bike | Skateboard | Total |
| Age (years) | 0–8 | 50 | | | |
| | 9–16 | | | 20 | |
| | 17–24 | 30 | | | |
| | 25–32 | | 60 | | |
| | Total | | | | |

(i) Show how the value of 20% was calculated. (1 mark)

(ii) Complete the table by using either row percentages **or** column percentages, as appropriate. (3 marks)

(d) State an association that can be observed from the two-way percentaged table. (1 mark)

**2021 Section 1 Question 4**

**Bivariate data analysis**

A public opinion survey was conducted on the statement 'our overwhelming dependence on computers is a good thing', with partial results being shown in the table below.

| | | Opinion | | | |
|---|---|---|---|---|---|
| | | **Agree** | **Disagree** | **Undecided** | **Total** |
| **Age** | **20–39 years** | 40 | 28 | | 80 |
| | **40–59 years** | 38 | | 20 | 100 |
| | **60–79 years** | 20 | | 18 | |
| | **Total** | | | | 230 |

(a) Complete the table above. (3 marks)

(b) Identify the response variable. (1 mark)

(c) Use the template below to construct a percentaged two-way frequency table showing either column or row percentages as appropriate, to investigate if there is an association between age and opinion. (4 marks)

| | | Opinion | | | |
|---|---|---|---|---|---|
| | | **Agree** | **Disagree** | **Undecided** | |
| **Age** | **20–39 years** | | | | |
| | **40–59 years** | | | | |
| | **60–79 years** | | | | |
| | | | | | |

(d) State an association that can be observed from the percentaged two-way frequency table. (1 mark)

**2021 Section 1 Question 7**

**Bivariate data analysis**

The ages in years, and salaries in thousands of dollars ($'000), of eight employees at a company are shown below. The equation of the least-squares line for these data is $y = 0.2x + 38$.

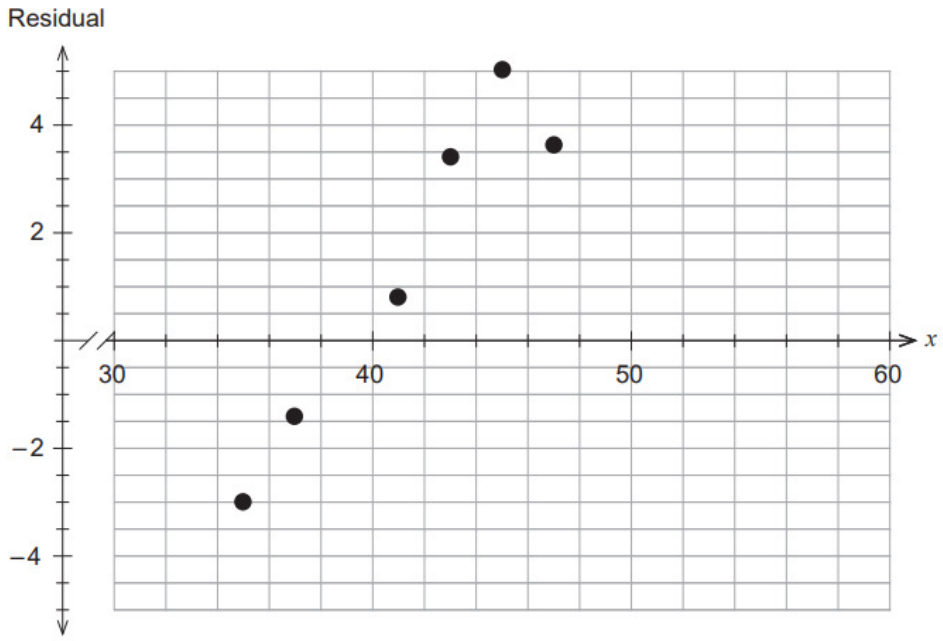
| **Age** $(x)$ | 35 | 37 | 41 | 43 | 45 | 47 | 53 | 55 |
|---|---|---|---|---|---|---|---|---|
| **Salary** $(y)$ | 42 | 44 | 47 | 50 | 52 | 51 | 49 | 45 |

The table below shows the predicted y-values, obtained from the equation of the least-squares line, and the corresponding residuals.

| $x$ | $y$ | Predicted $y$-value | Residual |
|---|---|---|---|
| 35 | 42 | 45.0 | −3.0 |
| 37 | 44 | 45.4 | −1.4 |
| 41 | 47 | 46.2 | 0.8 |
| 43 | 50 | 46.6 | 3.4 |
| 45 | 52 | 47.0 | 5.0 |
| 47 | 51 | 47.4 | 3.6 |
| 53 | 49 | 48.6 | 0.4 |
| 55 | 45 | **A** | **B** |

(a) Determine the value of **A** and **B**. (2 marks)

(b) Plot the last two residuals on the graph below. (2 marks)




(c) Justify, using the residual plot in part (b), whether the least-squares line is a good model for these data. (2 marks)

The calculated correlation coefficient for these data is 0.42.

(d) Describe how this supports your response in part (c). (1 mark)

**2020 Section 1 Question 1**

**Bivariate data analysis**

The owner of a bicycle shop recorded the type of repairs he made to bicycles with different purchase prices.

| | | Purchase price | | | |
|---|---|---|---|---|---|
| | | Less than $500 | From $500 to $1000 | Greater than $1000 | Total |
| Type of repair | Wheels and tyres | 36 | 6 | 18 | 60 |
| | Gears and brakes | 20 | 12 | 8 | 40 |
| | Frame and suspension | 15 | 2 | 3 | 20 |

(a) Identify the explanatory variable for the table above. (1 mark)

The percentages in each row of the following table show the proportion of bicycles with different purchase prices requiring that type of repair.

(b) Complete the table. (2 marks)

| | | Purchase price | | | |
|---|---|---|---|---|---|
| | | Less than $500 | From $500 to $1000 | Greater than $1000 | Total |
| Type of repair | Wheels and tyres | | 10 | | 100 |
| | Gears and brakes | | 30 | 20 | 100 |
| | Frame and suspension | 75 | | | 100 |

(c) Using the information from the table in part (b), describe **one** association between these variables. (2 marks)

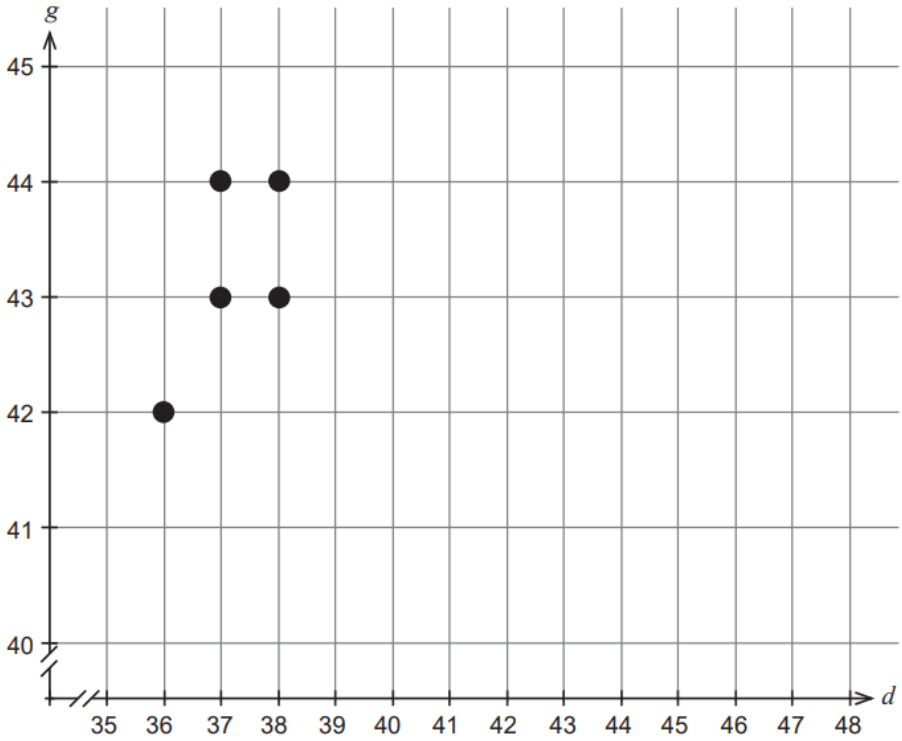
**2020 Section 1 Question 4**

**Bivariate data analysis**

The table shows data comparing the gestation period (in days) with the birth weight (in grams) for ten Tasmanian possums.

| **Gestation period in days ($d$)** | 36 | 37 | 37 | 38 | 38 | 42 | 43 | 44 | 44 | 45 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Birth weight in grams ($g$)** | 42 | 43 | 44 | 43 | 44 | 41 | 42 | 43 | 41 | 42 |

(a) Plot the last five data points on the axes below. (2 marks)




The correlation coefficient for these observations is approximately –0.6 and the least-squares line is $g = -0.17d + 49$.

(b) Describe what this correlation suggests about the general pattern of association between gestation period and birth weight. (2 marks)

(c) Determine the coefficient of determination for these data. (1 mark)

(d) State the meaning of the coefficient of determination in the context of the question. (1 mark)

(e) Use the least-squares line to predict the birth weight of a possum after 40 days gestation. (1 mark)

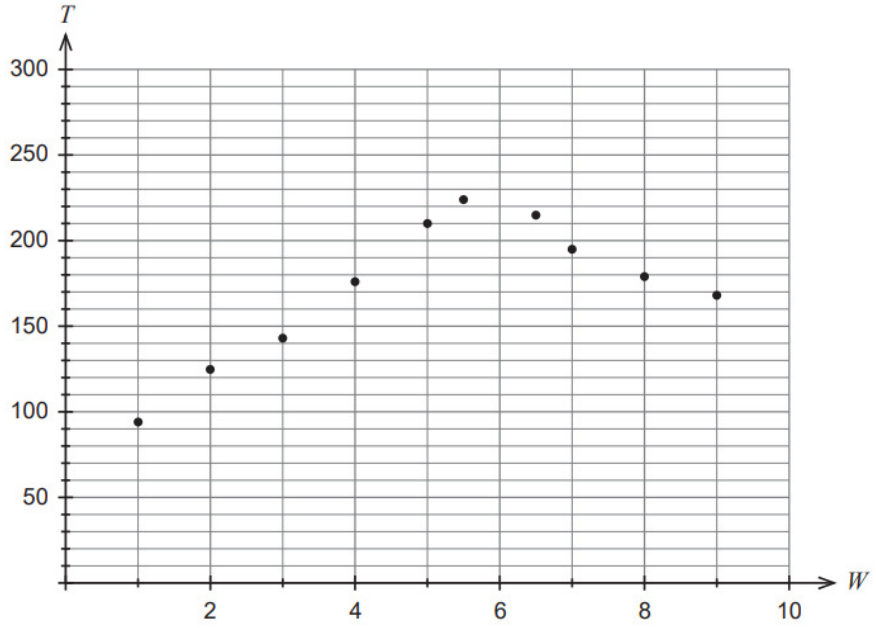
(f) Comment on the validity of this prediction. (2 marks)

(g) Is there any statistical evidence to support the research view that a higher birth weight will cause a shorter gestation period? Justify your answer. (2 marks)

**2019 Section 1 Question 2**

**Bivariate data analysis**

Katie is a hobby farmer who has been experimenting with a species of tomato plant growing under the same soil and climatic conditions. She varied the amount of water ($W$), in millimetres, used during each week and recorded the total number of tomatoes ($T$) produced by each plant. The scatterplot showing her results is drawn below.




Katie determined the following summary information:

- $r_{WT} = 0.66$
- the equation of the least-squares line is $T = 10.55W + 119.11$

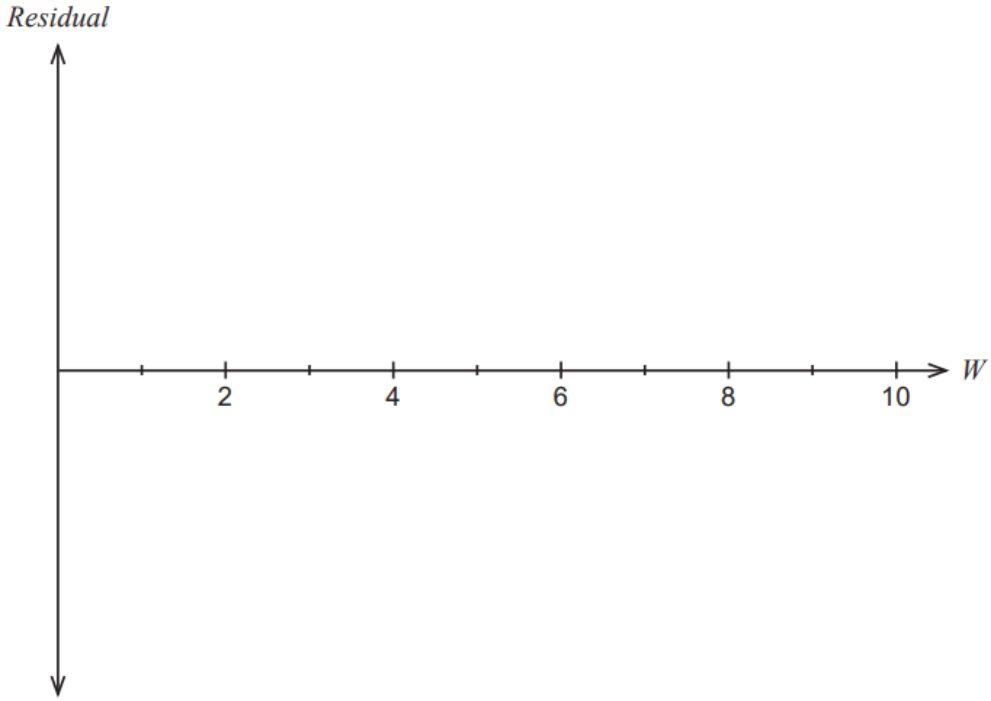
(a) Identify the response variable. (1 mark)

(b) Use the equation of the least-squares line to predict the total number of tomatoes produced when 10 millimetres of water are given to a plant during each week. (2 marks)

(c) Fit the least-squares line to the scatterplot. (2 marks)

Katie decided to draw a residual plot to gather more information about her results.

(d) (i) Sketch a residual plot she would have likely drawn for the given data. Note: you do not have to calculate actual values. (2 marks)




(ii) Use your residual plot to discuss the appropriateness of fitting a linear model to the data. (2 marks)

## Section 2

**2023 Section 2 Question 7**

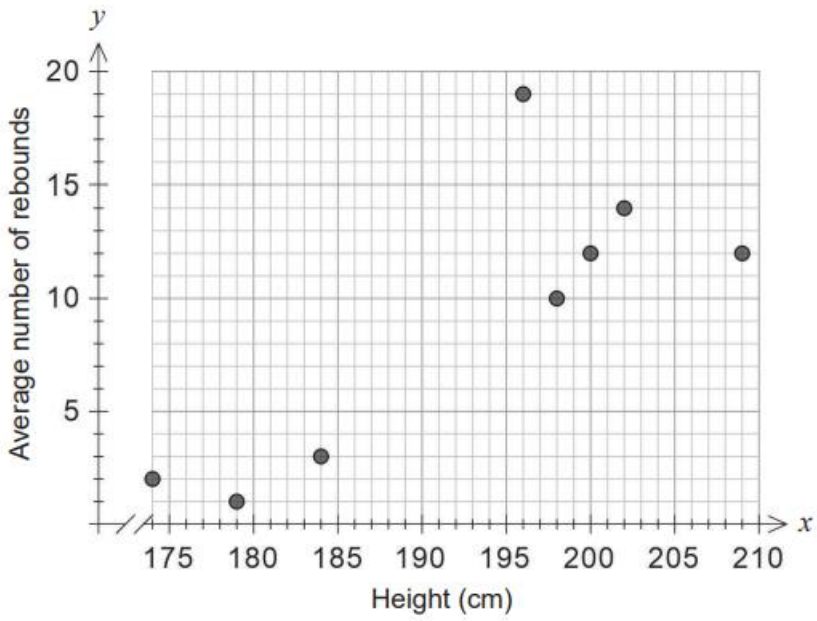
**Bivariate data analysis**

The heights of players and the average number of rebounds per game were recorded for a basketball team over the course of a 30 game season. The data collected is shown in the table below.

| Player | Height ($x$) (cm) | Average number of rebounds ($y$) |
|---|---|---|
| **1** | 198 | 10 |
| **2** | 174 | 2 |
| **3** | 200 | 12 |
| **4** | 182 | 2 |
| **5** | 184 | 3 |
| **6** | 209 | 12 |
| **7** | 196 | 19 |
| **8** | 185 | 8 |
| **9** | 202 | 14 |
| **10** | 179 | 1 |

(a) Identify the explanatory variable. (1 mark)

(b) Complete the scatter graph below by plotting the missing data from the table. (2 marks)




The data has a correlation coefficient of 0.814, and the equation of the least-squares line is y = 0.43x – 74.23.

(c) Draw the least-squares line on the graph above. (2 marks)

(d) Describe the association between players' heights and average number of rebounds in terms of direction and strength. (2 marks)

(e) Determine the coefficient of determination and state its meaning in the context of the question. (2 marks)

**2023 Section 2 Question 10**

**Bivariate data analysis**

Data concerning rental properties have been collected from 10 suburbs of a city. The data is for median property value ($p$) ($'000), median weekly rent ($\$w$) and percentage vacancy rate ($v$%) within each suburb.

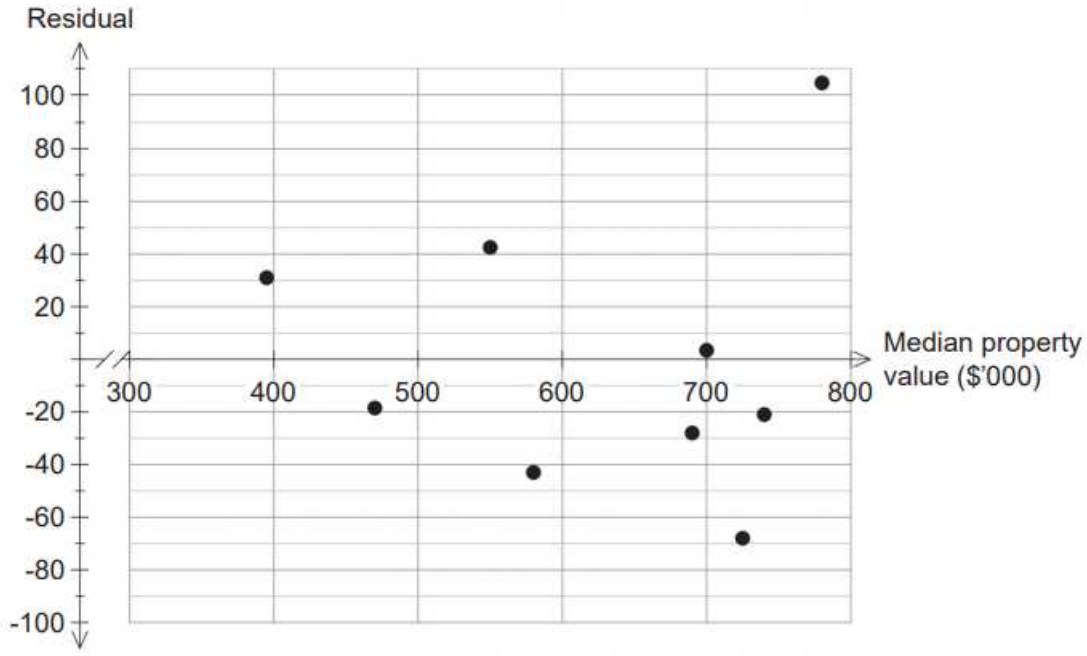
The data in the table below show the median property value and the median weekly rent for the 10 suburbs.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Median property value ($p$) ($'000)** | 395 | 470 | 550 | 725 | 580 | 780 | 700 | 740 | 690 | 585 |
| **Median weekly rent ($w$) ($)** | 445 | 460 | 590 | 630 | 530 | 850 | 680 | 690 | 640 | 575 |

(a) Calculate the correlation coefficient and the equation of the least-squares line for these data. (3 marks)

(b) In the context of this question, interpret the gradient of the least-squares line determined in part (a). (2 marks)

(c) The graph below shows the residual plot for the first nine suburbs as given in the table. Determine the residual for the 10th suburb and plot this value on the graph. (2 marks)




(d) State a conclusion that can be drawn from the residual plot. (1 mark)

(e) The predicted weekly rent of a property was calculated to be $612. What property value was this based on? (2 marks)

(f) If the data point (780, 850) was removed from all calculations, would the gradient of the least-squares line determined in part (a) increase, decrease or stay the same? (1 mark)

Bivariate data analysis between percentage vacancy rate and median weekly rent produced the following: $r^2_{vw} = 0.85$ and $w = -82.64v + 940.64$.

(g) Explain why $r_{vw} = -0.92$. (2 marks)

(h) A property has a vacancy rate of 4.1% and a median property value of $605 000. Predict the median weekly rent using the most reliable predictor. Justify which predictor is used. (2 marks)

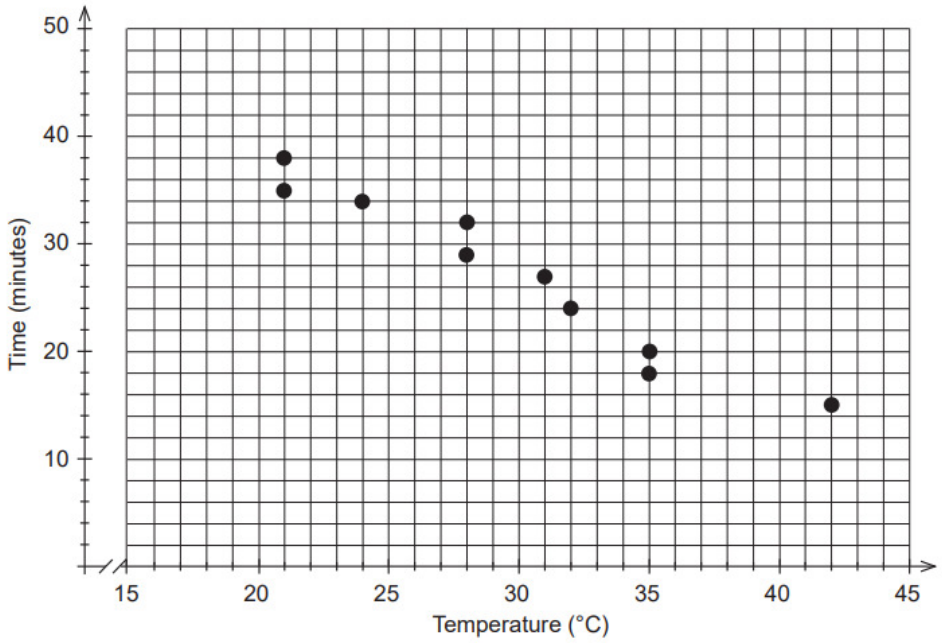
(i) Calculate the expected change in the weekly rent if the percentage vacancy rate increases by 0.4%. (1 mark)

(j) Comment on the statement 'it is clear both property price and vacancy rate will cause changes to the median weekly rent'. Justify your answer. (2 marks)

**2022 Section 2 Question 11**

**Bivariate data analysis**

Nullah wanted to see if there was a relationship between outside temperature and the time taken to dry his laundry. The following data was collected over a 10 day period.




| **Temperature (°C)** | 28 | 35 | 42 | 31 | 24 | 21 | 21 | 35 | 32 | 28 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Time (minutes)** | 29 | 20 | 15 | 27 | 34 | 38 | 35 | 18 | | |

(a) Complete the table by locating the data in the graph. (2 marks)

(b) Determine the equation of the least-squares line and state the correlation coefficient. (2 marks)

(c) Draw the least-squares line onto the graph above. (2 marks)

(d) Describe the association between the two variables in terms of direction and strength. (2 marks)

(e) What percentage of the variation in drying time can be explained by the variation in outside temperature? (1 mark)

(f) Identify at least one other factor that could explain the variation in drying time. (1 mark)

(g) The temperature on Day 11 is predicted to be 17 °C.

(i) Use the equation for the least-squares line from part (b) to predict the time Nullah should expect his laundry to dry on this day. (1 mark)

(ii) Is this prediction reliable? Justify your answer. (2 marks)

**2022 Section 2 Question 13**

**Bivariate data analysis**

Data have been collected for nine suburbs within a city about the number of mobile phone towers and the number of births in the last 12 months for each suburb.

| | Suburb | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| **Number of mobile phone towers ($n$)** | 4 | 6 | 7 | 8 | 6 | 10 | 5 | 8 | 7 |
| **Number of births in the last 12 months ($b$)** | 25 | 29 | 35 | 45 | 38 | 54 | 22 | 38 | 39 |

The data has a correlation coefficient of 0.92, and the equation of the least-squares line is $b = 5.13n + 1.31$.

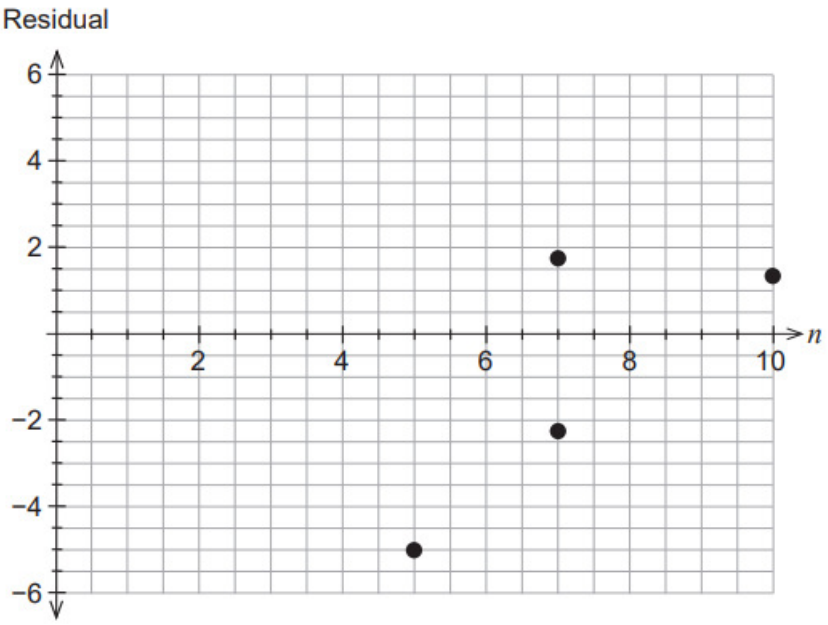
(a) Interpret the gradient of the least-squares line in the context of the question. (2 marks)

(b) Explain the significance of the correlation coefficient in the context of the question. (2 marks)

(c) (i) Predict the number of births for a suburb in this city that has nine mobile phone towers. (1 mark)

(ii) Comment on the validity of the prediction in part (c)(i). Justify your response. (2 marks)

(d) Complete the residual plot below.




(e) Based on the residual plot, comment on whether the least-squares line is a suitable model for these data. (2 marks)

(f) A 10th suburb has a data point (5,12) which has been verified as correct. State a practical explanation of how this could be a correct data point. (1 mark)

(g) A journalist has followed the mathematics involved in working with bivariate data and is writing a report for a newspaper. What is a valid statement that could be made about the observed association between the number of mobile phone towers and the number of births in the last 12 months for suburbs within the city? (2 marks)

**2021 Section 2 Question 9**

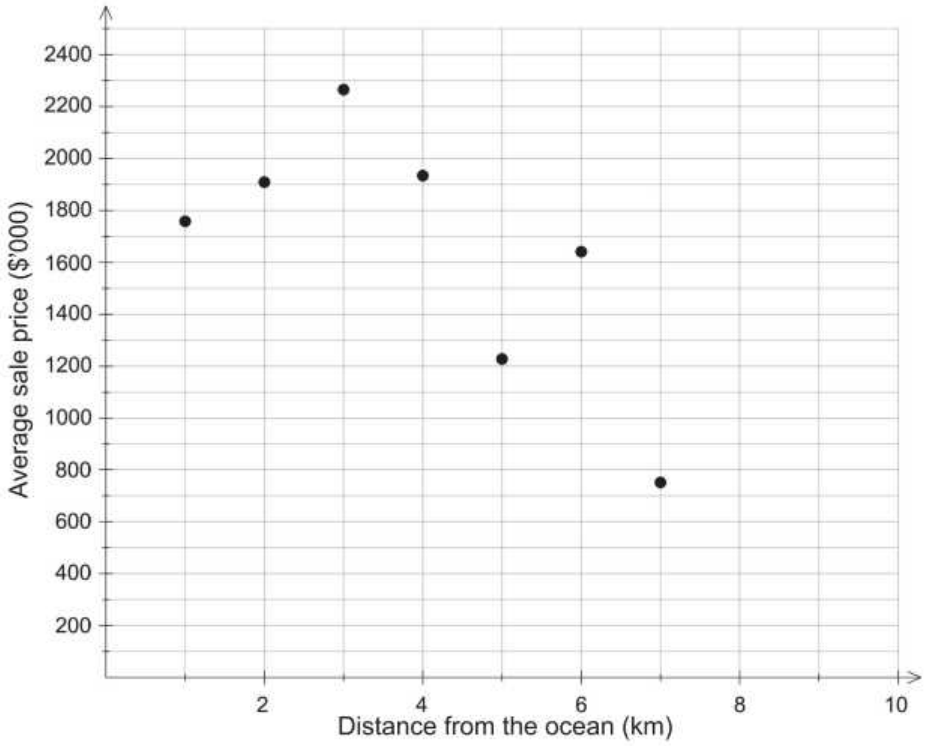
**Bivariate data analysis**

A real estate agent is analysing data on the sale of houses over the last six months. The table shows the average sale price of houses, in thousands of dollars ($'000), and their distance from the ocean, to the nearest kilometre.

| Distance from the ocean (km) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Average sale price ($'000) | 1758 | 1909 | 2265 | 1934 | 1228 | 1641 | 751 | 967 | 676 |

(a) State the explanatory variable. (1 mark)

(b) On the scatterplot below, plot the last two data points from the table. (1 mark)




(c) Determine the equation of the least-squares line for these data. (1 mark)

(d) Interpret the slope of the least-squares line from part (c) in the context of this question. (2 marks)

(e) (i) State the value of the correlation coefficient for these data. (1 mark)

(ii) What does the correlation coefficient measure? (1 mark)

(iii) Describe the association between the variables in terms of direction and strength. (2 marks)

(f) What percentage of the variation in average sale price can be explained by the variation in the distance from the ocean? (1 mark)

(g) In six months time, a homebuyer will have saved enough money for a deposit on a house. He would like to live about four kilometres from the ocean.

(i) Use the equation of the least-squares line from part (c) to predict the average sale price of houses four kilometres from the ocean. (1 mark)

(ii) Explain why your prediction is different from the average sale price given in the table. (1 mark)

(h) Give a reason why extrapolation in the context of this question would not make sense. (1 mark)

(i) The real estate agent was talking to some potential buyers and was heard to make the statement, “Having property closer to the ocean causes higher selling prices”. Comment on this statement. (2 marks)

**2020 Section 2 Question 10**

**Bivariate data analysis**

A football club records body measurements for all of their players. Shown below are the waistline measurements (cm) and percentage body fat for eleven players.

| Player | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Waistline measurement ($w$) | 89 | 100 | 87 | 96 | 94 | 83 | 81 | 83 | 84 | 97 | 98 |
| Percentage body fat ($p$) | 14 | 17 | 11 | 19 | 17 | 12 | 9 | 10 | 8 | 14 | 19 |

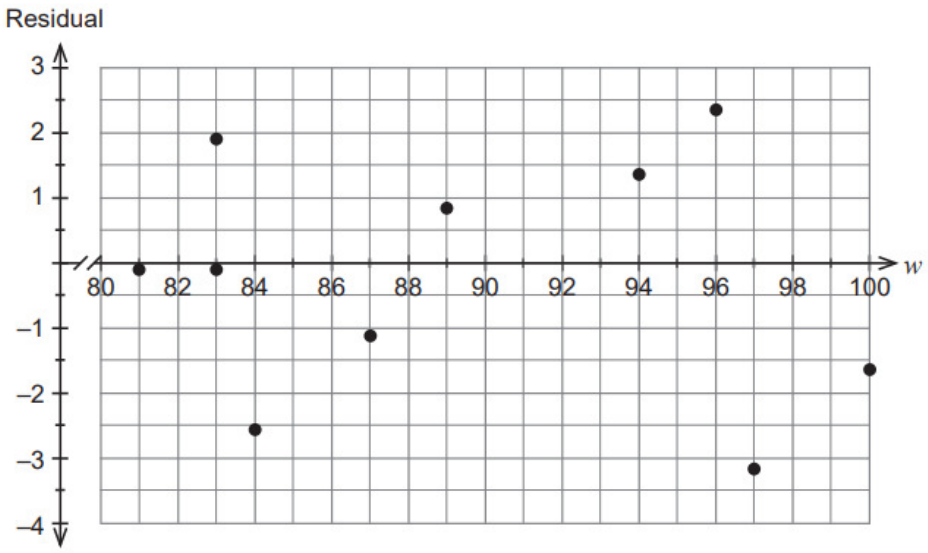
Research has shown that estimates for percentage body fat can be determined by using waistline measurements.

(a) Calculate the correlation coefficient $r_{wp}$ for these data. (1 mark)

(b) Determine the equation of the least-squares line for these data. (1 mark)

(c) In the context of this question, interpret the slope of the line found in part (b). (2 marks)

(d) The residual plot shown below is for the first 10 players' data. Calculate the residual for player number 11 and plot this point on the graph. (2 marks)



(e) Comment on the appropriateness of fitting a linear model to the data. Justify your answer. (2 marks)

(f) What percentage of the variation in the percentage body fat measurements is **unexplained** by the variation in the waistline measurements? (2 marks)

(g) Wayne is player number 12 and has a waistline measurement of 105 cm.

(i) Determine his predicted percentage of body fat. (1 mark)

(ii) Comment on the validity of the prediction and give a justification for your answer. (2 marks)

(h) Player number 13 has a residual of –2.6. What information does this provide about the percentage body fat for this player? (2 marks)

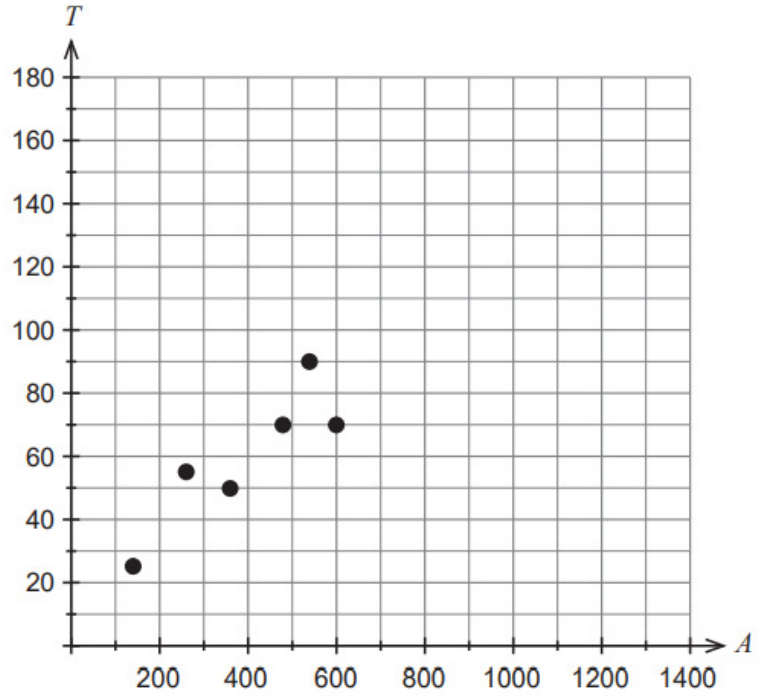
**2019 Section 2 Question 8**

**Bivariate data analysis**

Abdul has a lawnmowing business and is investigating if there is a relationship between the size of a lawn and the length of time it takes to cut the lawn. He takes a random sample of eight customers and measures the areas of their lawns and notes the times, in minutes, it takes to mow their lawns. The results are in the table below, where $A$ is the area of the lawn in square metres and $T$ is the time in minutes. (Note: some values are missing.)

| Customer | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| $A$ (m$^2$) | | 260 | | 480 | 540 | 600 | 860 | 1180 |
| $T$ (min) | 25 | 55 | 50 | 70 | 90 | 70 | 135 | 140 |

(a) Complete the scatterplot below. (1 mark)




(b) From the information below, determine the equation of the least-squares line in terms of $A$ and $T$ and state the coefficient of determination for these data. (2 marks)

Linear Reg
$y = ax+b$
a = 0.114691
b = 16.008241
r = 0.9510026
$r^2$ = 0.9044059

(c) Interpret the value of the gradient of the least-squares line in the context of the question. (2 marks)

(d) Given that Abdul charges $30 per hour, estimate the charge for mowing a customer's lawn with an area of 500 m2 . (2 marks)

(e) Explain whether the estimate determined in part (d) would be valid. (2 marks)

(f) Using the least-squares line correct to three decimal places

(i) calculate the residuals for Customers B and D. (2 marks)

(ii) explain the significance of the sign and the size of these residuals in reference to the least-squares line. (2 marks)

**2019 Section 2 Question 14**

**Bivariate data analysis**

The table below contains data provided by the Australian Bureau of Statistics. It shows the number of households with and without internet access from 2014–2017. All values are in thousands of households.

**Internet access**

| State/territory | 2014–15 | | | 2016–17 | | |
|---|---|---|---|---|---|---|
| | Households with internet access '000 | Households without internet access '000 | Total '000 | Households with internet access '000 | Households without internet access '000 | Total '000 |
| New South Wales | 2407.9 | 414.5 | 2822.4 | 2439.9 | 421.8 | 2861.7 |
| Victoria | 1934.2 | 305.1 | **A** | 2008.2 | 305.8 | 2314.0 |
| Queensland | 1552.4 | 248.5 | 1800.9 | 1591.9 | 249.8 | 1841.7 |
| South Australia | 565.1 | 121.4 | 686.5 | 575.5 | **B** | 696.6 |
| Western Australia | 843.6 | 113.0 | 956.6 | 859.7 | 112.6 | 972.3 |
| Tasmania | 172.0 | 38.7 | 210.7 | 177.7 | 36.2 | 213.9 |
| Northern Territory | 58.1 | 6.3 | 64.4 | 57.6 | 7.3 | 64.9 |
| Australian Capital Territory | 137.2 | 9.0 | 146.2 | 140.1 | 9.7 | 149.8 |
| Total | 7670.5 | 1256.5 | 8927.0 | 7850.6 | 1264.3 | 9114.9 |

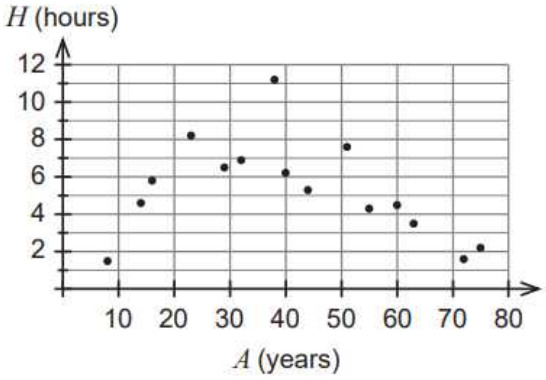
(a) (i) Determine the value of **A** and **B** in the table above. (2 marks)

(ii) Compare the percentages, correct to two decimal places, of households with internet access in New South Wales between 2014–15 and 2016–17. Comment on your results. (3 marks)

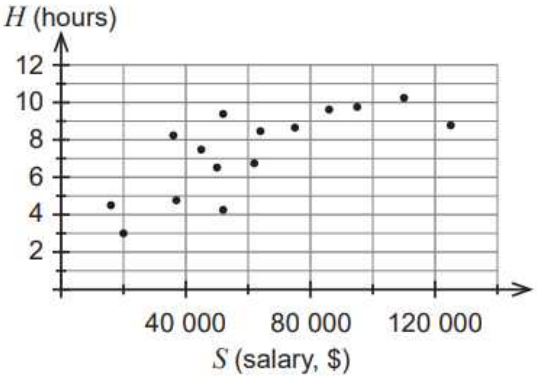
(iii) What is the difference in the data for households with internet access for the Northern Territory over the time period shown, compared to other States and Territories? (1 mark)

An internet service provider from Tasmania wanted to determine whether a person's age, *A*, or salary, *S*, affected the number of hours, *H*, of internet usage per day. The graphs below each show the recorded data for people surveyed.

**Daily Internet Usage According to Age**




**Daily Internet Usage According to Salary**




(b) (i) Describe the association between a person's salary and the number of hours of internet usage per day, in terms of direction and form. (2 marks)

(ii) The internet service provider calculated the correlation coefficient for the data contained in each graph. The values they calculated are contained in the following list.

–1.25, –0.95, –0.75, –0.3, 0.1, 0.3, 0.75, 0.95, 1.25

Choose the best estimate from the list for each of the graphs shown above. (2 marks)

| Graph | Correlation coefficient |
|---|---|
| Daily internet usage according to age | |
| Daily internet usage according to salary | |

**2019 Section 2 Question 16**

**Bivariate data analysis**

The table below records the altitude (metres above sea level), latitude (° S) and mean maximum temperature (°C) during January for eight cities in the southern hemisphere.

| Altitude ($A$) | Latitude ($L$) | Mean maximum temperature ($T$) |
|---|---|---|
| 15 | 31.95 | 25 |
| 20 | 43.53 | 20 |
| 24 | 42.88 | 18 |
| 314 | 45.03 | 16 |
| 8 | 6.18 | 28 |
| 154 | 12.05 | 26 |
| 37 | 12.46 | 29 |
| 8 | 34.60 | 25 |

Comparing altitude and the mean maximum temperature, it was determined that the least-squares line for these data was $T = -0.022A + 24.97$ and $r_{AT} = -0.50$.

(a) Determine the coefficient of determination for altitude and the mean maximum temperature and interpret this value. (2 marks)

(b) Determine the equation of the least-squares line for comparing latitude and the mean maximum temperature and state the correlation coefficient. (2 marks)

Rio de Janeiro has a latitude of 22.93° S and an altitude of 9 metres.

(c) Use the two least-squares lines above to predict the mean maximum temperature in January for Rio de Janeiro. Which prediction is more valid? Justify your choice. (3 marks)

**2023 Section 1 Question 1**

**Bivariate data analysis**

A survey was conducted by a film studio executive of filmgoers' favourite genres. The categories chosen were action, drama and comedy. The information is displayed in the table below.

| | | Genre | | | |
|---|---|---|---|---|---|
| | | Action | Drama | Comedy | Total |
| Age (years) | 1–10 | 40 | 45 | | 125 |
| | 11–20 | 17 | | 43 | 86 |
| | 21–30 | | 25 | | 125 |
| | Total | | 96 | 113 | 336 |

(a) Complete the two-way table above. (3 marks)

| | | Genre | | | |
|---|---|---|---|---|---|
| | | Action | Drama | Comedy | Total |
| Age (years) | 1–10 | 40 | 45 | **40** | 125 |
| | 11–20 | 17 | **26** | 43 | 86 |
| | 21–30 | **70** | 25 | **30** | 125 |
| | Total | **127** | 96 | 113 | 336 |

| Solution |
|---|
| see table above |
| **Specific behaviours** |
| ✓ determines three correct values<br>✓ determines four correct values<br>✓ determines all correct values |

(b) Identify the response variable for these data. (1 mark)

| Solution |
|---|
| genre |
| **Specific behaviours** |
| ✓ correct response variable |

(c) The incomplete two-way percentaged table is shown below.

| | | Genre | | | |
|---|---|---|---|---|---|
| | | **Action** | **Drama** | **Comedy** | **Total** |
| **Age (years)** | **1–10** | | 36 | | |
| | **11–20** | | 30 | | |
| | **21–30** | 56 | | 24 | |
| | **Total** | | | | |

(i) Complete the table above by using either row percentages or column percentages, as appropriate. (3 marks)

| | | Genre | | | |
|---|---|---|---|---|---|
| | | **Action** | **Drama** | **Comedy** | **Total** |
| **Age (years)** | **1–10** | **32** | 36 | **32** | **100** |
| | **11–20** | **20** | 30 | **50** | **100** |
| | **21–30** | 56 | **20** | 24 | **100** |
| | **Total** | | | | |

| **Solution** |
|---|
| see table above |
| **Specific behaviours** |
| ✓ determines row percentages are required<br>✓ determines three correct values<br>✓ determines all correct values |

(ii) State an association that can be observed from the two-way percentaged table. (1 mark)

| **Solution** |
|---|
| as age increases, the percentage of the drama genre decreases |
| **Specific behaviours** |
| ✓ correctly states association |

**2022 Section 1 Question 3**

**Bivariate data analysis**

An analysis was done on hospital patients based on their age and injuries received from various outdoor activities. The information is displayed below.

| | | Activity | | | |
|---|---|---|---|---|---|
| | | **Trampoline** | **Bike** | **Skateboard** | **Total** |
| **Age (years)** | **0–8** | 150 | 120 | 30 | 300 |
| | **9–16** | 180 | **180** | 90 | 450 |
| | **17–24** | 120 | 80 | **200** | 400 |
| | **25–32** | 20 | **150** | **80** | 250 |
| | **Total** | 470 | **530** | 400 | 1400 |

(a) Complete the two-way table above. (2 marks)

| **Solution** |
|---|
| See table above |
| **Specific behaviours** |
| ✓ correctly completes three or more entries<br>✓ correctly completes all entries |

(b) Identify the explanatory variable for these data. (1 mark)

| **Solution** |
|---|
| Age |
| **Specific behaviours** |
| ✓ correctly identifies age |

(c) The incomplete two-way percentaged table is shown below.

| | | Activity | | | |
|---|---|---|---|---|---|
| | | **Trampoline** | **Bike** | **Skateboard** | **Total** |
| **Age (years)** | **0–8** | 50 | **40** | **10** | **100** |
| | **9–16** | **40** | **40** | 20 | **100** |
| | **17–24** | 30 | **20** | **50** | **100** |
| | **25–32** | **8** | 60 | **32** | **100** |
| | **Total** | | | | |

(i) Show how the value of 20% was calculated. (1 mark)

| **Solution** |
|---|
| $\frac{90}{450} = \frac{1}{5} = 20\%$ |
| **Specific behaviours** |
| ✓ shows correct calculation |

(ii) Complete the table by using either row percentages **or** column percentages, as appropriate. (3 marks)

| Solution |
|---|
| See above table |
| **Specific behaviours** |
| ✓ correctly identifies row percentages required and row totals equal 100<br>✓ correctly calculates values for age 25–32<br>✓ correctly calculates remaining values |

(d) State an association that can be observed from the two-way percentaged table. (1 mark)

| Solution |
|---|
| As a patient's age increases, injuries received due to trampoline decreases. |
| **Specific behaviours** |
| ✓ correctly states association |
| Accept other relevant answers. |

**2021 Section 1 Question 4**

**Bivariate data analysis**

A public opinion survey was conducted on the statement 'our overwhelming dependence on computers is a good thing', with partial results being shown in the table below.

| | | Opinion | | | |
|---|---|---|---|---|---|
| | | **Agree** | **Disagree** | **Undecided** | **Total** |
| **Age** | **20–39 years** | 40 | 28 | | 80 |
| | **40–59 years** | 38 | | 20 | 100 |
| | **60–79 years** | 20 | | 18 | |
| | **Total** | | | | 230 |

(a) Complete the table above. (3 marks)

| Solution |
|---|
| See table above |
| **Specific behaviours** |
| ✓ determines at least 3 correct entries<br>✓ determines at least 5 correct entries<br>✓ determines correctly all entries |

(b) Identify the response variable. (1 mark)

| Solution |
|---|
| Opinion |
| **Specific behaviours** |
| ✓ correct answer |

(c) Use the template below to construct a percentaged two-way frequency table showing either column or row percentages as appropriate, to investigate if there is an association between age and opinion. (4 marks)

| | | Opinion | | | |
|---|---|---|---|---|---|
| | | Agree | Disagree | Undecided | Total |
| Age | 20–39 years | 50 | 35 | 15 | 100 |
| | 40–59 years | 38 | 42 | 20 | 100 |
| | 60–79 years | 40 | 24 | 36 | 100 |
| | | | | | |

| Solution |
|---|
| See table above |
| **Specific behaviours** |
| ✓ determines row percentages are required<br>✓ determines at least 3 correct entries<br>✓ determines at least 6 correct entries<br>✓ determines correctly all entries |

(d) State an association that can be observed from the percentaged two-way frequency table. (1 mark)

| Solution |
|---|
| As age increases, the percentage undecided increases. |
| **Specific behaviours** |
| ✓ gives correct association |

**2021 Section 1 Question 7**

**Bivariate data analysis**

The ages in years, and salaries in thousands of dollars ($'000), of eight employees at a company are shown below. The equation of the least-squares line for these data is $y = 0.2x + 38$.

| **Age** $(x)$ | 35 | 37 | 41 | 43 | 45 | 47 | 53 | 55 |
|---|---|---|---|---|---|---|---|---|
| **Salary** $(y)$ | 42 | 44 | 47 | 50 | 52 | 51 | 49 | 45 |

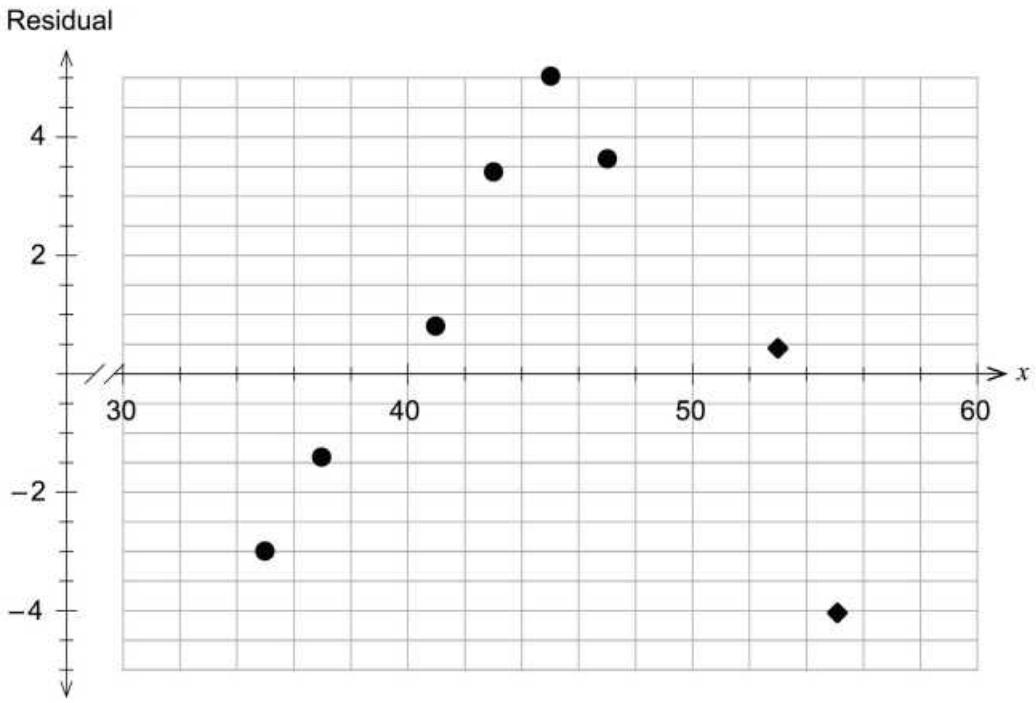
The table below shows the predicted y-values, obtained from the equation of the least-squares line, and the corresponding residuals.

| $x$ | $y$ | Predicted $y$-value | Residual |
|---|---|---|---|
| 35 | 42 | 45.0 | −3.0 |
| 37 | 44 | 45.4 | −1.4 |
| 41 | 47 | 46.2 | 0.8 |
| 43 | 50 | 46.6 | 3.4 |
| 45 | 52 | 47.0 | 5.0 |
| 47 | 51 | 47.4 | 3.6 |
| 53 | 49 | 48.6 | 0.4 |
| 55 | 45 | **A** | **B** |

(a) Determine the value of **A** and **B**. (2 marks)

| **Solution** |
|---|
| $y = 0.2(55) + 38$<br>$= 11 + 38$<br>$= 49$<br>Therefore, **A** = 49<br>**B** = 45 − 49 = −4.0 |
| **Specific behaviours** |
| ✓ correctly calculates the value of **A**<br>✓ correctly calculates the value of **B** |

(b) Plot the last two residuals on the graph below. (2 marks)

| Solution |
|---|
|   |
| **Specific behaviours** |
| ✓ correctly plots residuals at (53, 0.4)<br>✓ correctly plots residual at (55, **B**) i.e. (55,−4.0) |

(c) Justify, using the residual plot in part (b), whether the least-squares line is a good model for these data. (2 marks)

| Solution |
|---|
| The residual plot shows a pattern, therefore, this suggests that the least-squares line is unsuitable |
| **Specific behaviours** |
| ✓ correctly states the model is unsuitable<br>✓ correctly states the residual plot shows a pattern |

The calculated correlation coefficient for these data is 0.42.

(d) Describe how this supports your response in part (c). (1 mark)

| Solution |
|---|
| As the correlation coefficient is low, this supports the answer to part (c), that the least squares lines is unsuitable |
| **Specific behaviours** |
| ✓ states a low correlation coefficient supports unsuitability |

**2020 Section 1 Question 1**

**Bivariate data analysis**

The owner of a bicycle shop recorded the type of repairs he made to bicycles with different purchase prices.

| | | Purchase price | | | |
|---|---|---|---|---|---|
| | | Less than $500 | From $500 to $1000 | Greater than $1000 | Total |
| Type of repair | Wheels and tyres | 36 | 6 | 18 | 60 |
| | Gears and brakes | 20 | 12 | 8 | 40 |
| | Frame and suspension | 15 | 2 | 3 | 20 |

(a) Identify the explanatory variable for the table above. (1 mark)

| Solution |
|---|
| Purchase price |
| **Specific behaviours** |
| ✓ identifies correct explanatory variable |

The percentages in each row of the following table show the proportion of bicycles with different purchase prices requiring that type of repair.

(b) Complete the table. (2 marks)

| | | Purchase price | | | |
|---|---|---|---|---|---|
| | | Less than $500 | From $500 to $1000 | Greater than $1000 | Total |
| Type of repair | Wheels and tyres | **60** | 10 | **30** | 100 |
| | Gears and brakes | **50** | 30 | 20 | 100 |
| | Frame and suspension | 75 | **10** | **15** | 100 |

| Solution |
|---|
| See table above |
| **Specific behaviours** |
| ✓ calculates 3 correct values<br>✓ calculates all correct values |

(c) Using the information from the table in part (b), describe **one** association between these variables. (2 marks)

| Solution |
|---|
| The higher the purchase price, the lower the percentage of gears and brakes repairs that are needed. |
| **Specific behaviours** |
| ✓ identifies an association between gears and brakes repairs and purchase price<br>✓ describes association in terms of differences in the percentages from the information in part (b) |

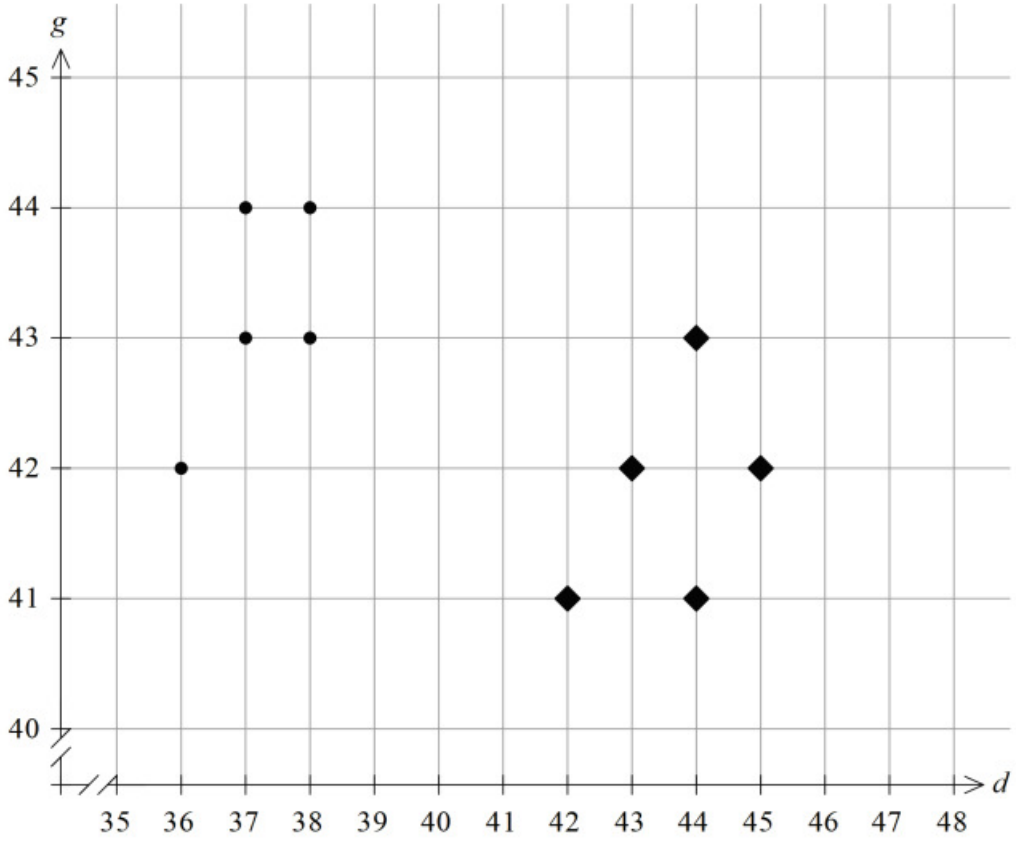
**2020 Section 1 Question 4**

**Bivariate data analysis**

The table shows data comparing the gestation period (in days) with the birth weight (in grams) for ten Tasmanian possums.

| **Gestation period in days ($d$)** | 36 | 37 | 37 | 38 | 38 | 42 | 43 | 44 | 44 | 45 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Birth weight in grams ($g$)** | 42 | 43 | 44 | 43 | 44 | 41 | 42 | 43 | 41 | 42 |

(a) Plot the last five data points on the axes below. (2 marks)




| **Solution** |
|---|
| See graph above |
| **Specific behaviours** |
| ✓ correctly plots at least three points |
| ✓ correctly plots all points |

The correlation coefficient for these observations is approximately –0.6 and the least-squares line is $g = -0.17d + 49$.

(b) Describe what this correlation suggests about the general pattern of association between gestation period and birth weight. (2 marks)

| **Solution** |
|---|
| Moderate negative linear relationship |
| **Specific behaviours** |
| ✓ correctly mentions negative linear relationship |
| ✓ correctly mentions moderate relationship |

(c) Determine the coefficient of determination for these data. (1 mark)

| Solution |
|---|
| 36% (0.36) |
| **Specific behaviours** |
| ✓ correctly calculates the coefficient of determination |

(d) State the meaning of the coefficient of determination in the context of the question. (1 mark)

| Solution |
|---|
| 36% of the variation in their birth weight can be explained by the variation in the gestation period |
| **Specific behaviours** |
| ✓ correctly states the meaning in the context of the question |

(e) Use the least-squares line to predict the birth weight of a possum after 40 days gestation. (1 mark)

| Solution |
|---|
| $g = -0.17 \times 40 + 49$<br>$g = 42.2$ grams |
| **Specific behaviours** |
| ✓ correctly calculates birth weight |

(f) Comment on the validity of this prediction. (2 marks)

| Solution |
|---|
| It may not be valid (even though it is interpolation) as the correlation is moderate. |
| **Specific behaviours** |
| ✓ correctly states prediction may not be valid<br>✓ correctly states correlation is moderate |

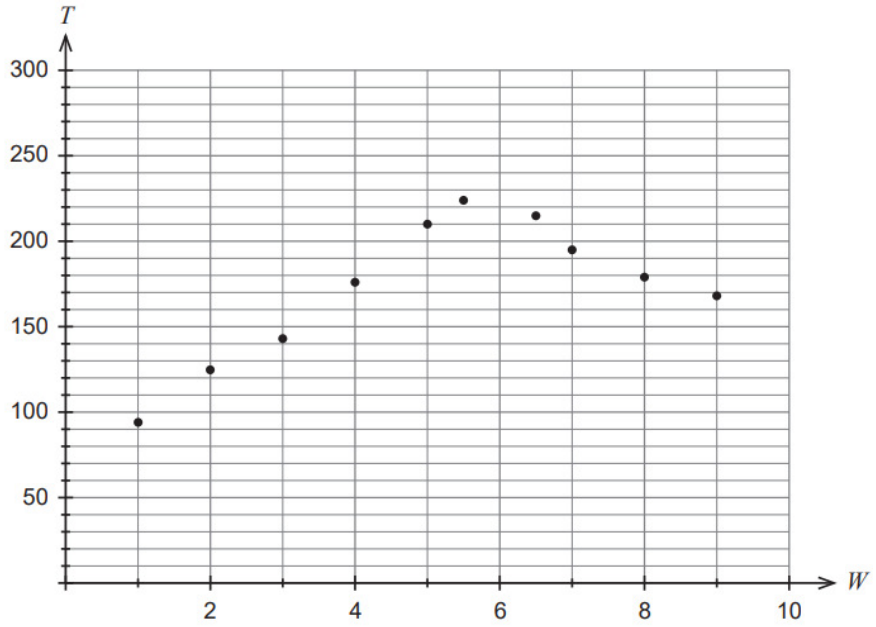
(g) Is there any statistical evidence to support the research view that a higher birth weight will cause a shorter gestation period? Justify your answer. (2 marks)

| Solution |
|---|
| There is not enough evidence to say there is a causal relationship between birth weight and a shorter gestation period, there may be other factors involved. |
| **Specific behaviours** |
| ✓ correctly states there is not enough supporting evidence<br>✓ states there may be other factors involved |

**2019 Section 1 Question 2**

**Bivariate data analysis**

Katie is a hobby farmer who has been experimenting with a species of tomato plant growing under the same soil and climatic conditions. She varied the amount of water ($W$), in millimetres, used during each week and recorded the total number of tomatoes ($T$) produced by each plant. The scatterplot showing her results is drawn below.




Katie determined the following summary information:

- $r_{WT} = 0.66$
- the equation of the least-squares line is $T = 10.55W + 119.11$

(a) Identify the response variable. (1 mark)

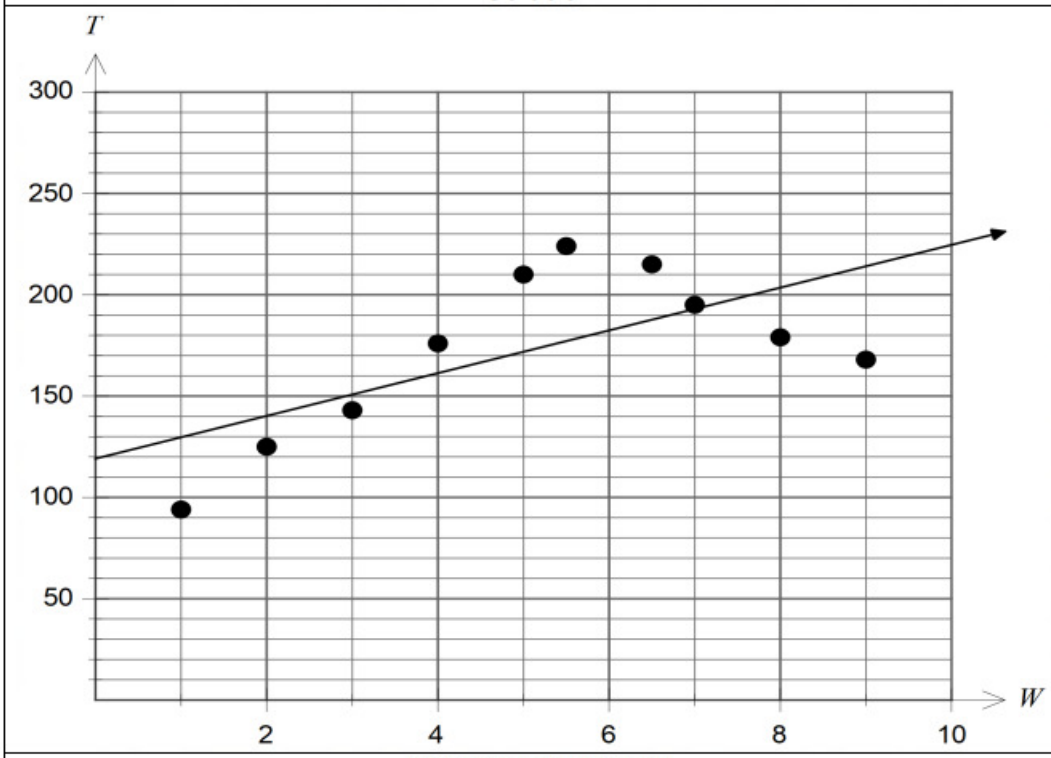
| Solution |
|---|
| Number of tomatoes |
| **Specific behaviours** |
| ✓ identifies correct variable |

(b) Use the equation of the least-squares line to predict the total number of tomatoes produced when 10 millimetres of water are given to a plant during each week. (2 marks)

| Solution |
|---|
| $T = 10.55 \times 10 + 119.11$<br>$T = 105.5 + 119.11$<br>$T = 224.61 \approx 224 / 225$ |
| **Specific behaviours** |
| ✓ correctly substitutes 10 into least-squares line<br>✓ rounds correctly to a whole number of tomatoes |

(c) Fit the least-squares line to the scatterplot. (2 marks)

**Solution**

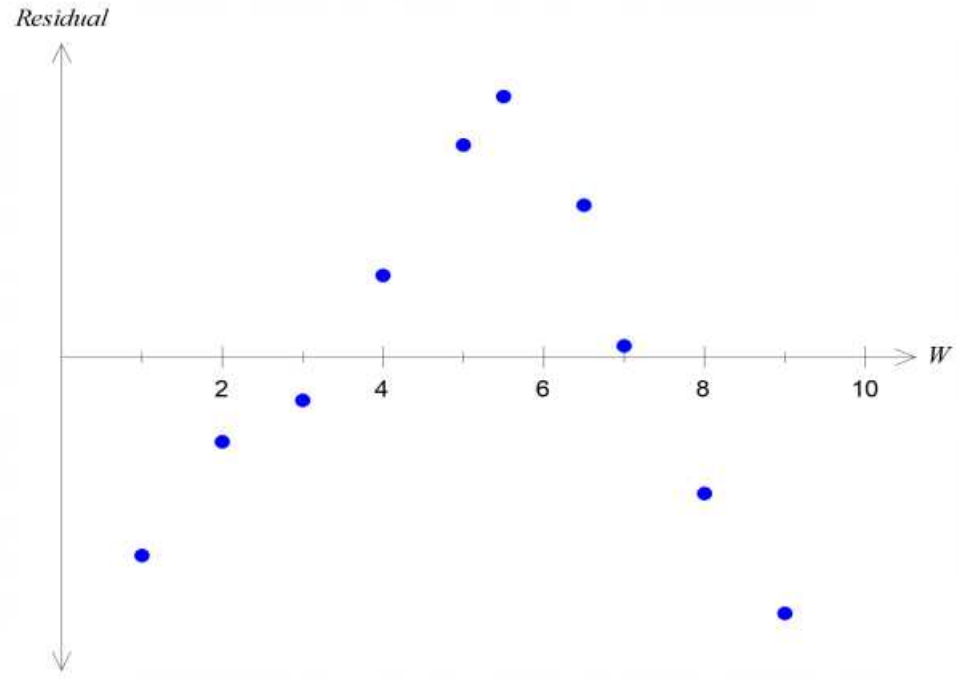



**Specific behaviours**

✓ plots one correct point
✓ plots a second correct point and draws least-squares line

Katie decided to draw a residual plot to gather more information about her results.

(d) (i) Sketch a residual plot she would have likely drawn for the given data. Note: you do not have to calculate actual values. (2 marks)

| Solution |
| --- |
|   |
| **Specific behaviours** |
| ✓ correctly shows shape of residual plot<br>✓ plots all points on correct side of horizontal axis |

(ii) Use your residual plot to discuss the appropriateness of fitting a linear model to the data. (2 marks)

| Solution |
| --- |
| A linear model is not appropriate as a pattern is evident in the residual plot. |
| **Specific behaviours** |
| ✓ states linear model is not appropriate<br>✓ states a valid reason |

**2023 Section 2 Question 7**

**Bivariate data analysis**

The heights of players and the average number of rebounds per game were recorded for a basketball team over the course of a 30 game season. The data collected is shown in the table below.

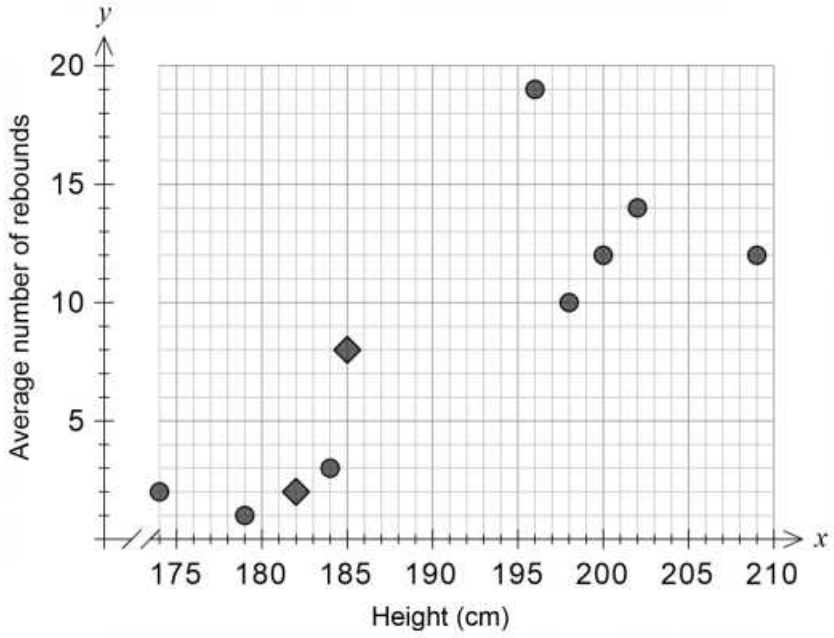
| Player | Height ($x$) (cm) | Average number of rebounds ($y$) |
|---|---|---|
| **1** | 198 | 10 |
| **2** | 174 | 2 |
| **3** | 200 | 12 |
| **4** | 182 | 2 |
| **5** | 184 | 3 |
| **6** | 209 | 12 |
| **7** | 196 | 19 |
| **8** | 185 | 8 |
| **9** | 202 | 14 |
| **10** | 179 | 1 |

(a) Identify the explanatory variable. (1 mark)

| **Solution** |
|---|
| height |
| **Specific behaviours** |
| ✓ identifies correct variable |

(b) Complete the scatter graph below by plotting the missing data from the table. (2 marks)

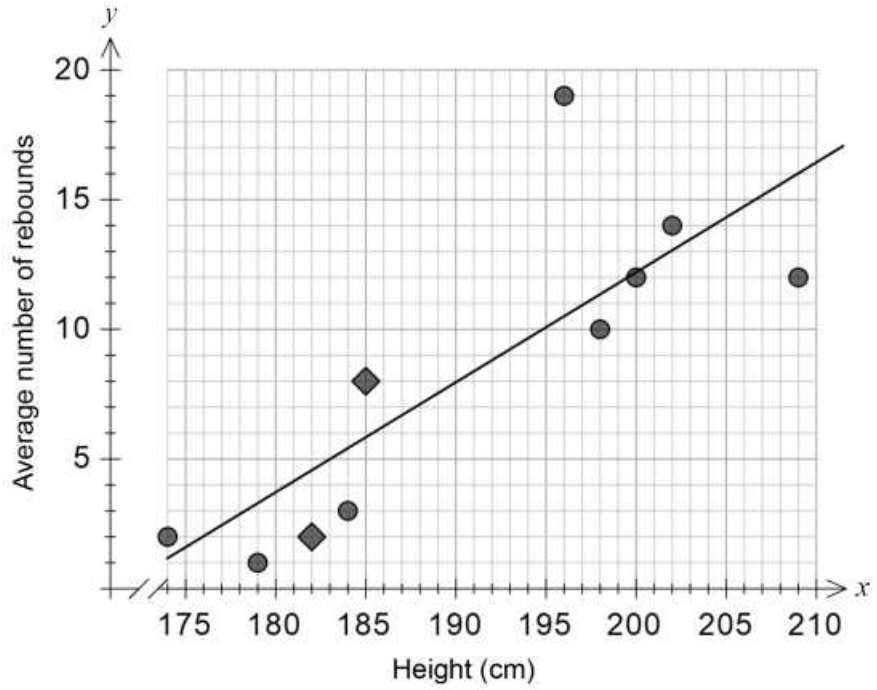
**Solution**




| **Specific behaviours** |
|---|
| ✓ plots one correct coordinate |
| ✓ plots both correct coordinates |

The data has a correlation coefficient of 0.814, and the equation of the least-squares line is y = 0.43x – 74.23.

(c) Draw the least-squares line on the graph above. (2 marks)

| Solution |
|---|
|  |
| **Specific behaviours** |
| ✓ shows correct slope<br>✓ shows that line passes approximately through (200, 12) |

(d) Describe the association between players' heights and average number of rebounds in terms of direction and strength. (2 marks)

| Solution |
|---|
| strong positive association |
| **Specific behaviours** |
| ✓ correctly states strong<br>✓ correctly states positive association |

(e) Determine the coefficient of determination and state its meaning in the context of the question. (2 marks)

| Solution |
|---|
| $r^2 = 0.663$<br>Therefore 66.3% of the variation in average number of rebounds can be explained by the variation of the height of the basketball players. |
| **Specific behaviours** |
| ✓ determines correct coefficient of determination<br>✓ determines correct meaning |

**2023 Section 2 Question 10**

**Bivariate data analysis**

Data concerning rental properties have been collected from 10 suburbs of a city. The data is for median property value ($p$) ($'000), median weekly rent ($\$w$) and percentage vacancy rate ($v$%) within each suburb.

The data in the table below show the median property value and the median weekly rent for the 10 suburbs.

| **Median property value ($p$) ($'000)** | 395 | 470 | 550 | 725 | 580 | 780 | 700 | 740 | 690 | 585 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Median weekly rent ($w$) ($)** | 445 | 460 | 590 | 630 | 530 | 850 | 680 | 690 | 640 | 575 |

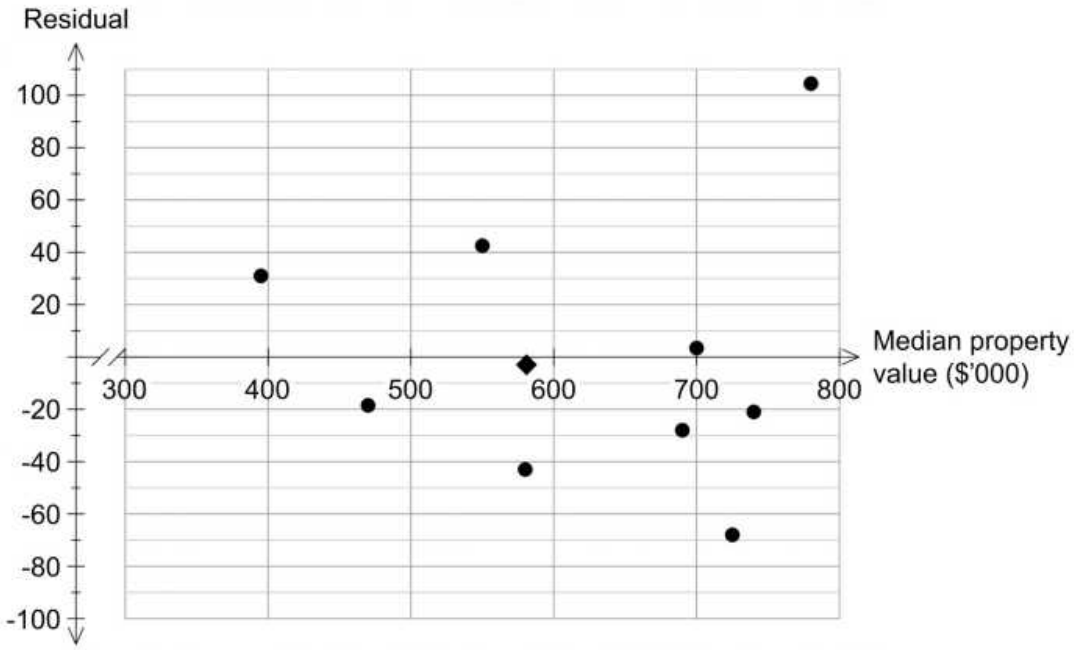
(a) Calculate the correlation coefficient and the equation of the least-squares line for these data. (3 marks)

| **Solution** |
|---|
| $r = 0.91$ $\quad w = 0.861p + 73.84$ |
| **Specific behaviours** |
| ✓ determines correct correlation coefficient<br>✓ determines correct least squares line<br>✓ uses correct variables |

(b) In the context of this question, interpret the gradient of the least-squares line determined in part (a). (2 marks)

| **Solution** |
|---|
| For each $1000 increase in median property value, median weekly rent will increase by $0.86 |
| **Specific behaviours** |
| ✓ uses median property value and median weekly rent<br>✓ uses correct values |

(c) The graph below shows the residual plot for the first nine suburbs as given in the table. Determine the residual for the 10th suburb and plot this value on the graph. (2 marks)




| Solution |
|---|
| Residual: (585, –2.6)<br><br>See graph above for plot |
| **Specific behaviours** |
| ✓ determines correct point<br>✓ plots point correctly |

(d) State a conclusion that can be drawn from the residual plot. (1 mark)

| Solution |
|---|
| It is appropriate to use a linear model for these data as there is no clear pattern in the residual plot |
| **Specific behaviours** |
| ✓ states correct conclusion |

(e) The predicted weekly rent of a property was calculated to be $612. What property value was this based on? (2 marks)

| Solution |
|---|
| $612 = 0.861p + 73.84 \quad \Rightarrow p = 625.04$<br>Property value $625 000 |
| **Specific behaviours** |
| ✓ solves for $p$<br>✓ expresses answer in thousands of dollars |

(f) If the data point (780, 850) was removed from all calculations, would the gradient of the least-squares line determined in part (a) increase, decrease or stay the same? (1 mark)

| Solution |
|---|
| decrease |
| **Specific behaviours** |
| ✓ gives correct answer |

Bivariate data analysis between percentage vacancy rate and median weekly rent produced the following: $r^2_{vw} = 0.85$ and $w = -82.64v + 940.64$.

(g) Explain why $r_{vw} = -0.92$. (2 marks)

| Solution |
|---|
| $\sqrt{0.85} = 0.92$, but the gradient of least squares line is negative therefore $r$ is negative. |
| **Specific behaviours** |
| ✓ determines square root<br>✓ explains why negative |

(h) A property has a vacancy rate of 4.1% and a median property value of $605 000. Predict the median weekly rent using the most reliable predictor. Justify which predictor is used. (2 marks)

| Solution |
|---|
| the strength of $r_{vw} > r_{pw}$, therefore use vacancy rate<br>predicted value $\approx \$602$ |
| **Specific behaviours** |
| ✓ explains why vacancy rate is the best predictor<br>✓ determines correct predicted value |

(i) Calculate the expected change in the weekly rent if the percentage vacancy rate increases by 0.4%. (1 mark)

| Solution |
|---|
| $-82.64 \times 0.4 = -\$33.06$ (A decrease of $33.06) |
| **Specific behaviours** |
| ✓ determines correct change |

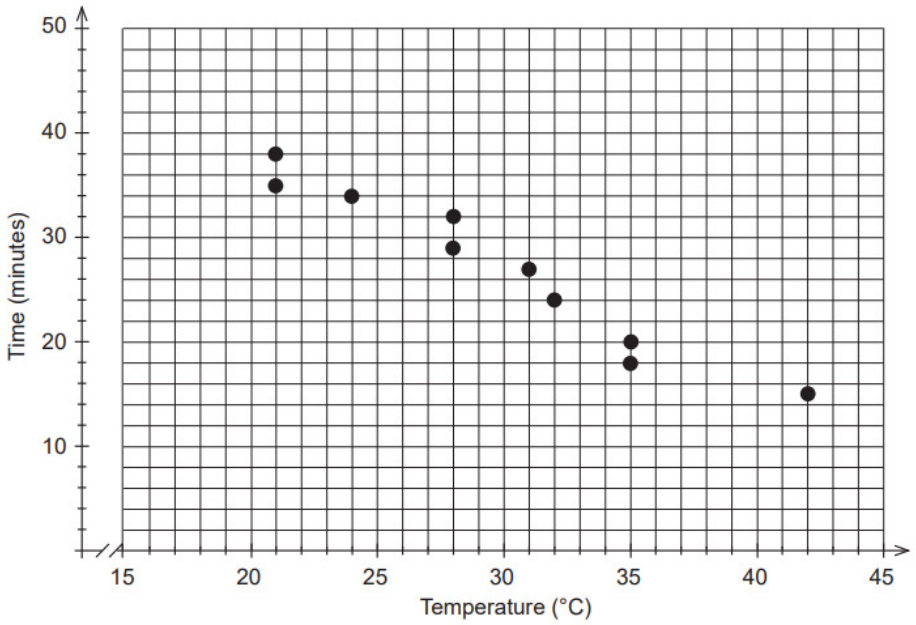
(j) Comment on the statement 'it is clear both property price and vacancy rate will cause changes to the median weekly rent'. Justify your answer. (2 marks)

| Solution |
|---|
| statement incorrect, cause not established |
| **Specific behaviours** |
| ✓ comments statement incorrect<br>✓ gives correct justification |

**2022 Section 2 Question 11**

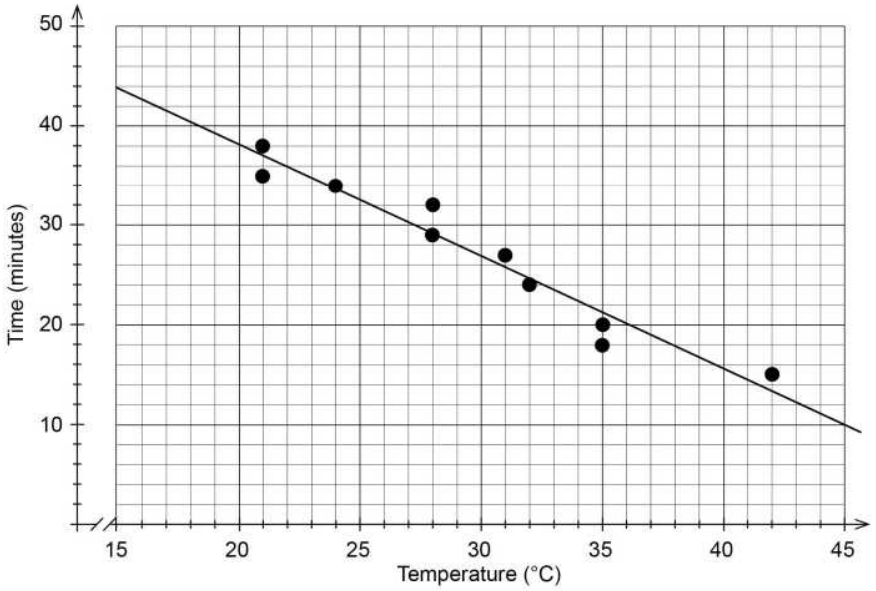
**Bivariate data analysis**

Nullah wanted to see if there was a relationship between outside temperature and the time taken to dry his laundry. The following data was collected over a 10 day period.




| **Temperature (°C)** | 28 | 35 | 42 | 31 | 24 | 21 | 21 | 35 | 32 | 28 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Time (minutes)** | 29 | 20 | 15 | 27 | 34 | 38 | 35 | 18 | | |

(a) Complete the table by locating the data in the graph. (2 marks)




| **Temperature (°C)** | 28 | 35 | 42 | 31 | 24 | 21 | 21 | 35 | 32 | 28 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Time (minutes)** | 29 | 20 | 15 | 27 | 34 | 38 | 35 | 18 | **24** | **32** |

| Solution |
|---|
| see table above |
| **Specific behaviours** |
| ✓ one correct entry<br>✓ both correct entries |

(b) Determine the equation of the least-squares line and state the correlation coefficient. (2 marks)

| Solution |
|---|
| $y = -1.13x + 60.74, r = -0.97$ |
| **Specific behaviours** |
| ✓ states correct equation<br>✓ states correct correlation coefficient |

(c) Draw the least-squares line onto the graph above. (2 marks)

| Solution |
|---|
| see graph above |
| **Specific behaviours** |
| ✓ line goes through approximately (28, 29)<br>✓ line has correct slope |

(d) Describe the association between the two variables in terms of direction and strength. (2 marks)

| Solution |
|---|
| The two variables have a strong, negative association |
| **Specific behaviours** |
| ✓ correctly states strong association<br>✓ correctly states negative association |

(e) What percentage of the variation in drying time can be explained by the variation in outside temperature? (1 mark)

| Solution |
|---|
| $r^2 = 0.945 = 94.5\%$<br>∴ 94.5% of the variation in the drying time can be explained by the variation in outside temperature |
| **Specific behaviours** |
| ✓ states correct percentage |

(f) Identify at least one other factor that could explain the variation in drying time. (1 mark)

| Solution |
|---|
| Answers could include:<br>• wind<br>• type of laundry material<br>• shade. |
| **Specific behaviours** |
| ✓ identifies at least one valid factor |
| Accept other relevant answers. |

(g) The temperature on Day 11 is predicted to be 17 °C.

(i) Use the equation for the least-squares line from part (b) to predict the time Nullah should expect his laundry to dry on this day. (1 mark)

| Solution |
|---|
| $y = -1.13(17) + 60.74 = 41.53$<br>time = 41.53 minutes |
| **Specific behaviours** |
| ✓ calculates correct time |

(ii) Is this prediction reliable? Justify your answer. (2 marks)

| Solution |
|---|
| No, it is unreliable since it is extrapolation |
| **Specific behaviours** |
| ✓ correctly states it is unreliable<br>✓ correctly states extrapolation |

**2022 Section 2 Question 13**

**Bivariate data analysis**

Data have been collected for nine suburbs within a city about the number of mobile phone towers and the number of births in the last 12 months for each suburb.

| | Suburb | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| **Number of mobile phone towers ($n$)** | 4 | 6 | 7 | 8 | 6 | 10 | 5 | 8 | 7 |
| **Number of births in the last 12 months ($b$)** | 25 | 29 | 35 | 45 | 38 | 54 | 22 | 38 | 39 |

The data has a correlation coefficient of 0.92, and the equation of the least-squares line is $b = 5.13n + 1.31$.

(a) Interpret the gradient of the least-squares line in the context of the question. (2 marks)

| Solution |
|---|
| There is an average increase of approximately 5 births per year for each extra mobile phone tower |
| **Specific behaviours** |
| ✓ correctly states increase<br>✓ correctly states 5 births per year |

(b) Explain the significance of the correlation coefficient in the context of the question. (2 marks)

| Solution |
|---|
| There is a strong, positive relationship between the number of mobile phone towers and the number of births in the last 12 months |
| **Specific behaviours** |
| ✓ correctly mentions both variables<br>✓ correctly identifies strength and/or direction |

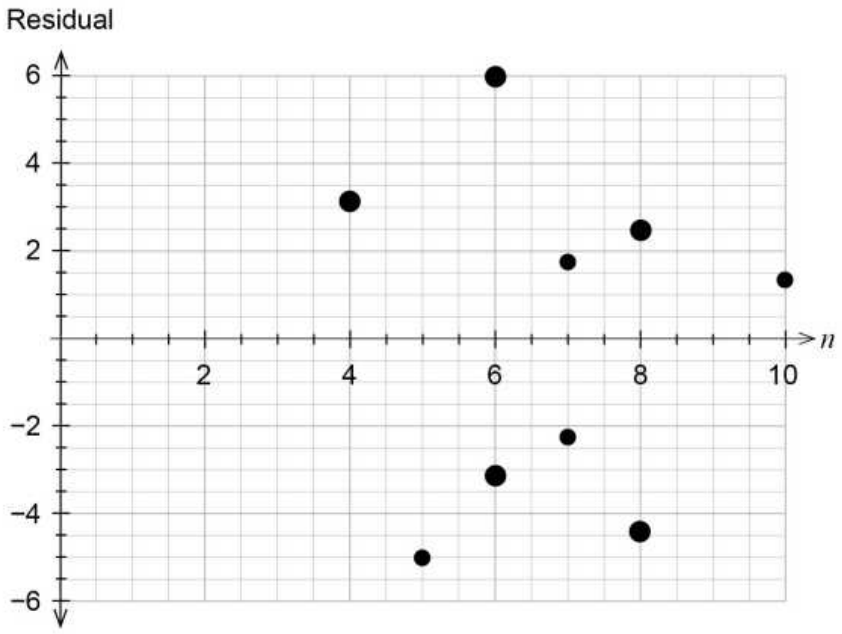
(c) (i) Predict the number of births for a suburb in this city that has nine mobile phone towers. (1 mark)

| Solution |
|---|
| $b = 5.13(9) + 1.31 = 47.48 \approx 48$ |
| **Specific behaviours** |
| ✓ correct prediction |

(ii) Comment on the validity of the prediction in part (c)(i). Justify your response. (2 marks)

| Solution |
|---|
| Prediction is valid since $r$ is strong and it is interpolation |
| **Specific behaviours** |
| ✓ states prediction is valid<br>✓ states $r$ is strong and it is interpolation |

(d) Complete the residual plot below.




| Solution |
|---|
| (4, 3.15), (6, –3.12), (6, 5.88), (8, 2.61), (8, –4.39) |
| **Specific behaviours** |
| ✓ correctly plots one point<br>✓ correctly plots at least three points<br>✓ correctly plots all points |

(e) Based on the residual plot, comment on whether the least-squares line is a suitable model for these data. (2 marks)

| Solution |
|---|
| Least-squares line is suitable because the residual plot shows no pattern |
| **Specific behaviours** |
| ✓ states line is suitable<br>✓ gives valid reason |

(f) A 10th suburb has a data point (5,12) which has been verified as correct. State a practical explanation of how this could be a correct data point. (1 mark)

| Solution |
|---|
| New suburb with mobile towers installed and not yet fully populated |
| **Specific behaviours** |
| ✓ correct comment |

(g) A journalist has followed the mathematics involved in working with bivariate data and is writing a report for a newspaper. What is a valid statement that could be made about the observed association between the number of mobile phone towers and the number of births in the last 12 months for suburbs within the city? (2 marks)

| Solution |
|---|
| While the mathematics shows a strong association there is no evidence of a causal relationship. Other factors would need to be considered to establish a causal relationship. |
| **Specific behaviours** |
| ✓ states strong mathematical relationship<br>✓ states causal relationship not evident |

**2021 Section 2 Question 9**

**Bivariate data analysis**

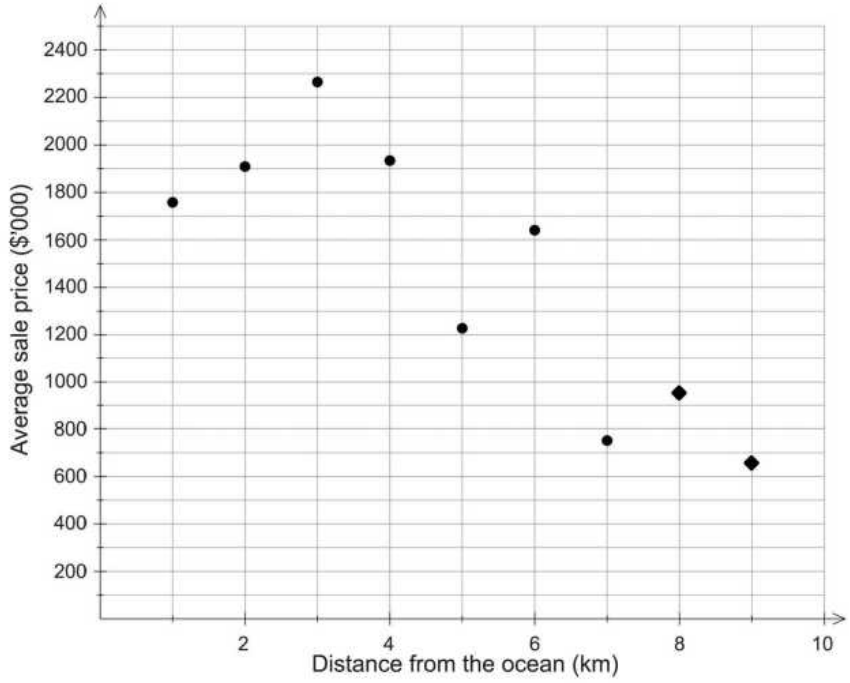
A real estate agent is analysing data on the sale of houses over the last six months. The table shows the average sale price of houses, in thousands of dollars ($'000), and their distance from the ocean, to the nearest kilometre.

| Distance from the ocean (km) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Average sale price ($'000) | 1758 | 1909 | 2265 | 1934 | 1228 | 1641 | 751 | 967 | 676 |

(a) State the explanatory variable. (1 mark)

| Solution |
|---|
| Distance from the ocean |
| **Specific behaviours** |
| ✓ states correct variable |

(b) On the scatterplot below, plot the last two data points from the table. (1 mark)




| Solution |
|---|
| see graph above |
| **Specific behaviours** |
| ✓ both points are plotted correctly |

(c) Determine the equation of the least-squares line for these data. (1 mark)

| Solution |
|---|
| $p = -174.58d + 2331.7 \qquad (y = -174.58x + 2331.7)$ |
| **Specific behaviours** |
| ✓ correctly determines equation |

(d) Interpret the slope of the least-squares line from part (c) in the context of this question. (2 marks)

| Solution |
|---|
| For each kilometre extra from the ocean, the selling price drops by an average of $174 580 |
| **Specific behaviours** |
| ✓ refers to correct variables<br>✓ correctly states average price drop of $174 580 |

(e) (i) State the value of the correlation coefficient for these data. (1 mark)

| Solution |
|---|
| $r = -0.84$ |
| **Specific behaviours** |
| ✓ correct answer |

(ii) What does the correlation coefficient measure? (1 mark)

| Solution |
|---|
| Strength of the linear relationship |
| **Specific behaviours** |
| ✓ correct answer |

(iii) Describe the association between the variables in terms of direction and strength. (2 marks)

| Solution |
|---|
| Strong, negative association |
| **Specific behaviours** |
| ✓ correctly states direction<br>✓ correctly states strength |

(f) What percentage of the variation in average sale price can be explained by the variation in the distance from the ocean? (1 mark)

| Solution |
|---|
| $r^2 = 0.7007 \approx 70.1\%$ |
| **Specific behaviours** |
| ✓ calculates correct percentage |

(g) In six months time, a homebuyer will have saved enough money for a deposit on a house. He would like to live about four kilometres from the ocean.

(i) Use the equation of the least-squares line from part (c) to predict the average sale price of houses four kilometres from the ocean. (1 mark)

| Solution |
|---|
| $d = 4 \Rightarrow p = 1633.38 \quad \Rightarrow \text{price} = \$1\ 633\ 380$ |
| **Specific behaviours** |
| ✓ determines correct price estimate |

(ii) Explain why your prediction is different from the average sale price given in the table. (1 mark)

| Solution |
|---|
| The table value is the true average value, whereas the prediction comes from the least-squares line. |
| **Specific behaviours** |
| ✓ gives a valid reason |

(h) Give a reason why extrapolation in the context of this question would not make sense. (1 mark)

| Solution |
|---|
| Following this model, the selling price would eventually become less than zero. |
| **Specific behaviours** |
| ✓ gives a valid reason |

(i) The real estate agent was talking to some potential buyers and was heard to make the statement, "Having property closer to the ocean causes higher selling prices". Comment on this statement. (2 marks)

| Solution |
|---|
| Not a valid statement. Only **average** price increases the closer to the ocean.<br>**or**<br>Not a valid statement. One variable does not cause the other to occur (not causally related). |
| **Specific behaviours** |
| ✓ states comment not valid<br>✓ gives an appropriate reason |

**2020 Section 2 Question 10**

**Bivariate data analysis**

A football club records body measurements for all of their players. Shown below are the waistline measurements (cm) and percentage body fat for eleven players.

| Player | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Waistline measurement ($w$) | 89 | 100 | 87 | 96 | 94 | 83 | 81 | 83 | 84 | 97 | 98 |
| Percentage body fat ($p$) | 14 | 17 | 11 | 19 | 17 | 12 | 9 | 10 | 8 | 14 | 19 |

Research has shown that estimates for percentage body fat can be determined by using waistline measurements.

(a) Calculate the correlation coefficient $r_{wp}$ for these data. (1 mark)

| Solution |
|---|
| $r = 0.88$ |
| **Specific behaviours** |
| ✓ correct answer |

(b) Determine the equation of the least-squares line for these data. (1 mark)

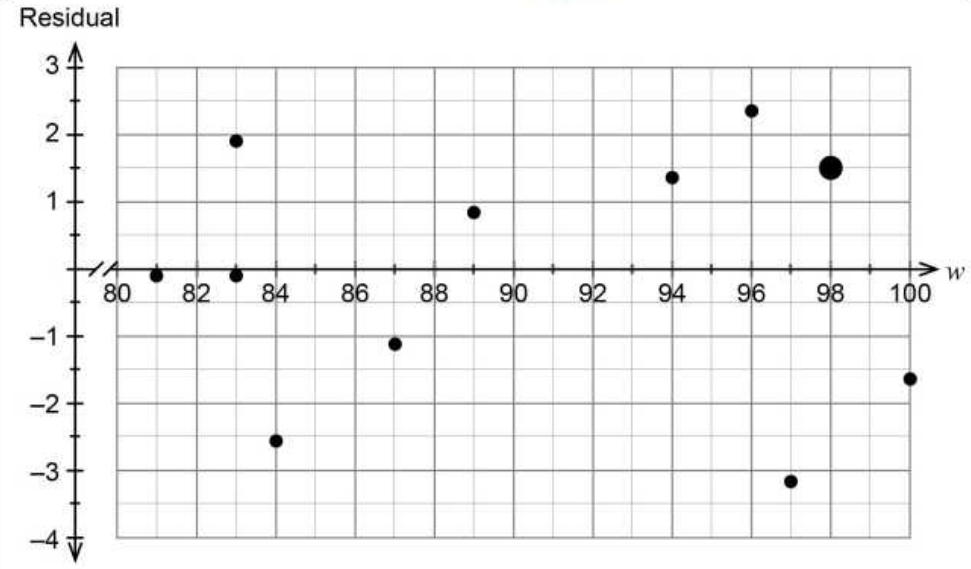
| Solution |
|---|
| $p = 0.5w - 31.4$ |
| **Specific behaviours** |
| ✓ correct answer |

(c) In the context of this question, interpret the slope of the line found in part (b). (2 marks)

| Solution |
|---|
| For each 1 cm increase in waistline measurement, the percentage body fat increases by 0.5. |
| **Specific behaviours** |
| ✓ matches waistline measurement to percentage body fat<br>✓ average 0.5 increase in percentage body fat for each 1 cm increase in waistline measurement |

(d) The residual plot shown below is for the first 10 players' data. Calculate the residual for player number 11 and plot this point on the graph. (2 marks)

**Solution**




Residual = 1.5

**Specific behaviours**

✓ correct residual
✓ correctly plotted

(e) Comment on the appropriateness of fitting a linear model to the data. Justify your answer. (2 marks)

| **Solution** |
|---|
| Appropriate as there is no clear pattern in the residuals |
| **Specific behaviours** |
| ✓ correctly states that fitting a linear model is appropriate<br>✓ correctly states there is no clear pattern in residuals |

(f) What percentage of the variation in the percentage body fat measurements is **unexplained** by the variation in the waistline measurements? (2 marks)

| **Solution** |
|---|
| $r^2 = d = 0.77$<br>$\therefore$ 23% unexplained |
| **Specific behaviours** |
| ✓ correctly determines coefficient of determination<br>✓ correctly gives unexplained variation percentage |

(g) Wayne is player number 12 and has a waistline measurement of 105 cm.

(i) Determine his predicted percentage of body fat. (1 mark)

| **Solution** |
|---|
| $p = 0.5 \times 105 - 31.4 = 21$ |
| **Specific behaviours** |
| ✓ correct answer |

(ii) Comment on the validity of the prediction and give a justification for your answer. (2 marks)

| Solution |
| --- |
| Not valid, extrapolation |
| **Specific behaviours** |
| ✓ correctly states prediction is not valid<br>✓ correctly states it is due to extrapolation |

(h) Player number 13 has a residual of –2.6. What information does this provide about the percentage body fat for this player? (2 marks)

| Solution |
| --- |
| The percentage body fat for player 13 is 2.6 below their predicted percentage body fat. |
| **Specific behaviours** |
| ✓correctly states that percentage body fat is below/under …<br>✓correctly relates actual percentage body fat measurement to the predicted percentage body fat (from the least-squares line) |

**2019 Section 2 Question 8**

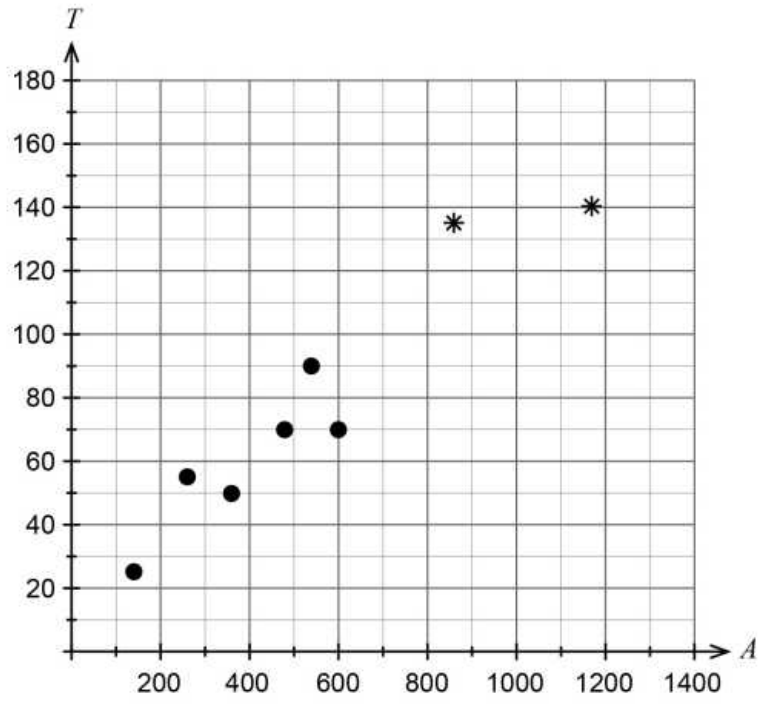
**Bivariate data analysis**

Abdul has a lawnmowing business and is investigating if there is a relationship between the size of a lawn and the length of time it takes to cut the lawn. He takes a random sample of eight customers and measures the areas of their lawns and notes the times, in minutes, it takes to mow their lawns. The results are in the table below, where $A$ is the area of the lawn in square metres and $T$ is the time in minutes. (Note: some values are missing.)

| Customer | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| $A$ (m$^2$) | | 260 | | 480 | 540 | 600 | 860 | 1180 |
| $T$ (min) | 25 | 55 | 50 | 70 | 90 | 70 | 135 | 140 |

(a) Complete the scatterplot below. (1 mark)

**Solution**




**Specific behaviours**

✓ plots both points correctly

(b) From the information below, determine the equation of the least-squares line in terms of $A$ and $T$ and state the coefficient of determination for these data. (2 marks)

Linear Reg
$y = ax+b$
a = 0.114691
b = 16.008241
r = 0.9510026
$r^2$ = 0.9044059

| Solution |
|---|
| Equation of least-squares line is: $T = 0.115A + 16.008$<br>The coefficient of determination is 0.904 |
| **Specific behaviours** |
| ✓ states correct equation<br>✓ states correct coefficient of determination |

(c) Interpret the value of the gradient of the least-squares line in the context of the question. (2 marks)

| Solution |
|---|
| the time taken to mow the lawn increases by 0.115 minutes per square metre |
| **Specific behaviours** |
| ✓ correctly defines an increase<br>✓ gives rate with correct units |

(d) Given that Abdul charges $30 per hour, estimate the charge for mowing a customer's lawn with an area of 500 m2 . (2 marks)

| Solution |
|---|
| 16.008 + 0.115 × 500 = 73.508<br>Estimated charge = $\frac{73.508}{60} \times 30 = \$36.75$ |
| **Specific behaviours** |
| ✓ correctly calculates an estimate for $T$<br>✓ correctly calculates the charge |

(e) Explain whether the estimate determined in part (d) would be valid. (2 marks)

| Solution |
|---|
| Estimate would be valid since it is interpolation and the correlation coefficient is strong |
| **Specific behaviours** |
| ✓ correctly explains validity<br>✓ explains validity with reference to either interpolation or correlation coefficient |

(f) Using the least-squares line correct to three decimal places

(i) calculate the residuals for Customers B and D. (2 marks)

| Solution |
| --- |
| Residual for customer B is 9.092<br>Residual for customer D is –1.208 |
| **Specific behaviours** |
| ✓ correctly calculates residual for customer B<br>✓ correctly calculates residual for customer D |

(ii) explain the significance of the sign and the size of these residuals in reference to the least-squares line. (2 marks)

| Solution |
| --- |
| The change in sign indicates the residuals are above and below the least-squares line<br>The size indicates that the residual for D is closer to the line than the residual for customer B (or vice versa) |
| **Specific behaviours** |
| ✓ states correct meaning of residual sign<br>✓ states correct meaning of residual size |

**2019 Section 2 Question 14**

**Bivariate data analysis**

The table below contains data provided by the Australian Bureau of Statistics. It shows the number of households with and without internet access from 2014–2017. All values are in thousands of households.

**Internet access**

| State/territory | 2014–15 | | | 2016–17 | | |
|---|---|---|---|---|---|---|
| | Households with internet access '000 | Households without internet access '000 | Total '000 | Households with internet access '000 | Households without internet access '000 | Total '000 |
| New South Wales | 2407.9 | 414.5 | 2822.4 | 2439.9 | 421.8 | 2861.7 |
| Victoria | 1934.2 | 305.1 | **A** | 2008.2 | 305.8 | 2314.0 |
| Queensland | 1552.4 | 248.5 | 1800.9 | 1591.9 | 249.8 | 1841.7 |
| South Australia | 565.1 | 121.4 | 686.5 | 575.5 | **B** | 696.6 |
| Western Australia | 843.6 | 113.0 | 956.6 | 859.7 | 112.6 | 972.3 |
| Tasmania | 172.0 | 38.7 | 210.7 | 177.7 | 36.2 | 213.9 |
| Northern Territory | 58.1 | 6.3 | 64.4 | 57.6 | 7.3 | 64.9 |
| Australian Capital Territory | 137.2 | 9.0 | 146.2 | 140.1 | 9.7 | 149.8 |
| Total | 7670.5 | 1256.5 | 8927.0 | 7850.6 | 1264.3 | 9114.9 |

(a) (i) Determine the value of **A** and **B** in the table above. (2 marks)

| Solution |
|---|
| **A** = 1934.2 + 305.1 = 2239.3<br>**B** = 696.6 – 575.5 = 121.1 |
| **Specific behaviours** |
| ✓ correctly determines **A**<br>✓ correctly determines **B** |

(ii) Compare the percentages, correct to two decimal places, of households with internet access in New South Wales between 2014–15 and 2016–17. Comment on your results. (3 marks)

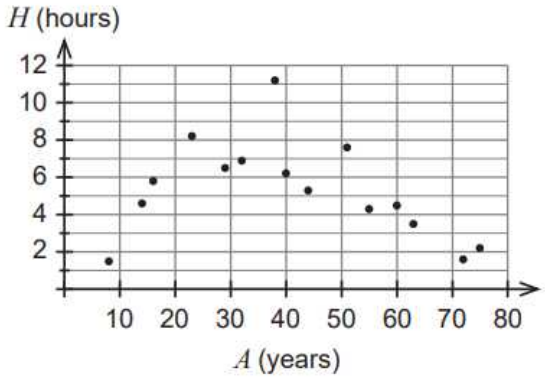
| Solution |
|---|
| 2014–15: $\frac{2407.9}{2822.4} = 85.31\%$, 2016–17: $\frac{2439.9}{2861.7} = 85.26\%$<br><br>There is a slight decrease in the percentage of households with internet access from 2014–15 to 2016–17. |
| **Specific behaviours** |
| ✓ calculates the correct percentages<br>✓ correctly rounds percentages to two decimal places<br>✓ states there is a small drop in the percentage of households with internet access from 2014–15 to 2016–17 |

(iii) What is the difference in the data for households with internet access for the Northern Territory over the time period shown, compared to other States and Territories? (1 mark)

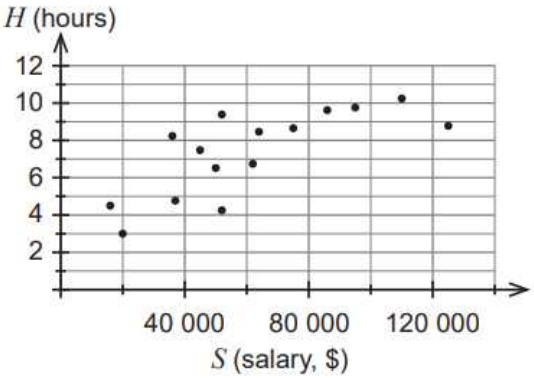
| Solution |
|---|
| The number of households with internet access decreases from 2014–15 to 2016–17. |
| **Specific behaviours** |
| ✓ correctly states how they are different |

An internet service provider from Tasmania wanted to determine whether a person's age, *A*, or salary, *S*, affected the number of hours, *H*, of internet usage per day. The graphs below each show the recorded data for people surveyed.

**Daily Internet Usage According to Age**




**Daily Internet Usage According to Salary**




(b) (i) Describe the association between a person's salary and the number of hours of internet usage per day, in terms of direction and form. (2 marks)

| Solution |
|---|
| Positive and linear |
| **Specific behaviours** |
| ✓ states correct direction<br>✓ correctly identifies a linear form |

(ii) The internet service provider calculated the correlation coefficient for the data contained in each graph. The values they calculated are contained in the following list.

–1.25, –0.95, –0.75, –0.3, 0.1, 0.3, 0.75, 0.95, 1.25

Choose the best estimate from the list for each of the graphs shown above. (2 marks)

| Solution | |
|---|---|
| **Graph** | **Correlation coefficient** |
| Daily internet usage according to age | –0.3 |
| Daily internet usage according to salary | 0.75 |
| **Specific behaviours** | |
| ✓ chooses correct correlation coefficient for internet usage according to age<br>✓ chooses correct correlation coefficient for internet usage according to salary | |

**2019 Section 2 Question 16**

**Bivariate data analysis**

The table below records the altitude (metres above sea level), latitude (° S) and mean maximum temperature (°C) during January for eight cities in the southern hemisphere.

| Altitude ($A$) | Latitude ($L$) | Mean maximum temperature ($T$) |
|---|---|---|
| 15 | 31.95 | 25 |
| 20 | 43.53 | 20 |
| 24 | 42.88 | 18 |
| 314 | 45.03 | 16 |
| 8 | 6.18 | 28 |
| 154 | 12.05 | 26 |
| 37 | 12.46 | 29 |
| 8 | 34.60 | 25 |

Comparing altitude and the mean maximum temperature, it was determined that the least-squares line for these data was $T = -0.022A + 24.97$ and $r_{AT} = -0.50$.

(a) Determine the coefficient of determination for altitude and the mean maximum temperature and interpret this value. (2 marks)

| Solution |
|---|
| $r^2 = 0.25$<br>Approximately 25% of the variation in temperature can be explained by the variation in altitude. |
| **Specific behaviours** |
| ✓ correctly determines the coefficient of determination<br>✓ gives correct description of its meaning |

(b) Determine the equation of the least-squares line for comparing latitude and the mean maximum temperature and state the correlation coefficient. (2 marks)

| Solution |
|---|
| $T = -0.264L + 30.94$<br>$r = -0.88$ |
| **Specific behaviours** |
| ✓ correctly states equation of least-squares line<br>✓ correctly states correlation coefficient |

Rio de Janeiro has a latitude of 22.93° S and an altitude of 9 metres.

(c) Use the two least-squares lines above to predict the mean maximum temperature in January for Rio de Janeiro. Which prediction is more valid? Justify your choice. (3 marks)

| Solution |
| --- |
| $T = -0.264 \times 22.93 + 30.94 = 24.89$<br>$T = -0.022 \times 9 + 24.97 = 24.77$<br>The prediction using latitude is more valid as the correlation coefficient is much stronger. |
| **Specific behaviours** |
| ✓ correctly determines a prediction using latitude and altitude<br>✓ correctly states that the prediction using latitude is more valid<br>✓ correctly explains that latitude has a stronger correlation coefficient |

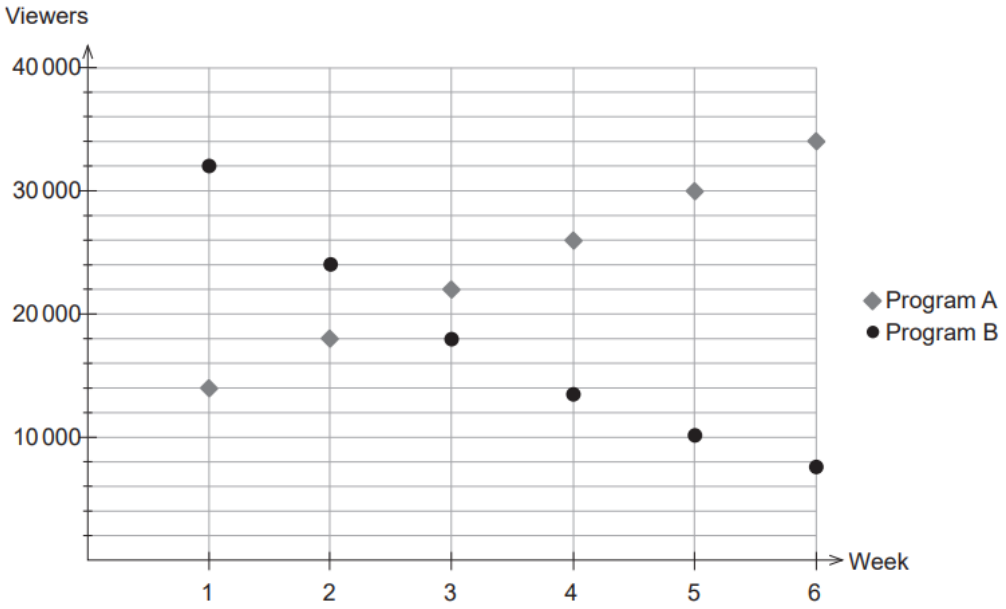
# Unit 3.2 – Growth and decay in sequences

## Section 1

| | |
|---|---|
| **2023 Section 1 Question 3**<br><br>**Growth and decay in sequences** | From January 1, 2020, a company offered its employees an income package with a starting wage of $4000 per month, paid at the end of each month. Also, as an incentive to stay with the company, there was a monthly increase of $50 each month.<br><br>(a) Determine a recursive rule for the monthly wage. (2 marks)<br><br>(b) Deduce a simplified rule for the nth term of the monthly wage. (2 marks)<br><br>(c) Determine the monthly wage for December 2020. (2 marks)<br><br>The company has decided to make the monthly increase $60 from the end of December 2023.<br><br>(d) Calculate the monthly wage for March 2024. (3 marks) |

| | |
|---|---|
| **2021 Section 1 Question 1**<br><br>**Growth and decay in sequences** | Hanai is a successful college basketball player. His coach has warned him that he will lose his scholarship if he scores 54% or below on a weekly assessment. On his first three weekly assessments he scored 84%, 81% and 78% respectively.<br><br>Assume Hanai's weekly assessments continue to follow this pattern.<br><br>(a) Deduce a rule for the $n^{th}$ term of this sequence. (2 marks)<br><br>(b) Determine Hanai's score on his sixth weekly assessment. (1 mark) |

| | |
|---|---|
| | (c) Predict when Hanai will lose his scholarship. (2 marks) |

| | |
|---|---|
| **2021 Section 1 Question 6**<br><br>**Growth and decay in sequences** | A television network programmer was analysing the number of viewers for two children's programs over a period of several weeks, to decide which program should be given the better time slot. The viewing numbers, displayed on the graph below, formed an arithmetic sequence and a geometric sequence.<br><br>  <br><br><br>(a) Write a recursive rule for the arithmetic sequence. (2 marks)<br><br>(b) Using the first two data points, deduce a rule for the $n^{th}$ term of the geometric sequence. (2 marks) |

| | |
|---|---|
| | (c) Explain which program should be given the better time slot. (2 marks)<br><br>(d) Determine the number of viewers for the more successful program in Week 8. (1 mark) |

| | |
|---|---|
| **2019 Section 1 Question 6**<br><br>**Growth and decay in sequences** | The population of turtles in an artificial lake at a wildlife sanctuary is initially 32 and research has shown a natural decrease in population of 50% each year. Twenty extra turtles are introduced to the lake at the end of each year.<br><br>(a) Determine a recursive rule for the turtle population. (2 marks)<br><br>(b) Determine the long-term steady state of the turtle population. (2 marks) |

| | |
|---|---|
| | (c) If the wildlife sanctuary preferred a long-term steady state of 80 turtles, what yearly addition of turtles would be required to produce this steady state? Assume all other conditions remain the same. (2 marks) |

## Section 2

| | |
|---|---|
| **2022 Section 2 Question 9**<br><br>**Growth and decay in sequences** | A study of a penguin colony on an island was conducted and it found the initial population size of 1200 was dropping by 14% each year due to the introduction of non-native predators.<br><br>(a) Explain why the population after $n$ weeks is $1200 \times (0.86)^n$ penguins. (2 marks)<br><br>After eight weeks, the Parks and Wildlife Service set traps to reduce the predator numbers. This saw the penguin population increase weekly by 6%.<br><br>(b) State the recursive formula that models the new population growth. (2 marks)<br><br>(c) How many weeks will it take to get the population back up to the initial size? (1 mark) |

Once the population returns to the initial size, it is further helped by the introduction of penguins from a breeding program at the zoo.

The new population growth model can be represented by

$P_{n+1} = -0.25P_n + 3000$, $P_0 = 1200$ .

(d) Discuss the long-term behaviour of the penguin population, now that it is being supported by the breeding program. (2 marks)

| | |
|---|---|
| **2022 Section 2 Question 17**<br><br>**Growth and decay in sequences** | Indie was in a line with 24 other people for a slide at a water park. She noticed that the approximate number of people ($P$) in the line for the slide increased by 1.5% every minute ($m$).<br><br>(a) Write an exponential equation in the form $P = ar^m$ to represent this situation. (2 marks)<br><br>(b) Determine the approximate number of people in the line after 2 hours. (2 marks)<br><br>After 3 hours, the line started to decrease by 1% per minute.<br><br>(c) Using this new information, calculate the approximate number of people in line, 5 hours after Indie initially lined up. (4 marks) |

**2021 Section 2 Question 11**

**Growth and decay in sequences**

Judith monitors the water quality in her garden pond at the same time every day. She likes to maintain the concentration of algae at between 200 and 250 units per 100 litres (L). Her measurements show that the concentration increases daily according to the recursive rule $C_{n+1} = 1.025C_n$ , where $C_1 = 200$ units per 100 L (the minimum concentration).

When the concentration gets above the 250 units per 100 L limit, she treats the water to bring the concentration back to the minimum 200 units per 100 L.

(a) If Judith treated the water on Sunday, 6 December 2020, determine

(i) the concentration on Wednesday, 9 December 2020. (2 marks)

(ii) the day and date when she next treated the water. (2 marks)

(b) During the first week of January 2021, Judith monitored the water and recorded the following readings.

| Day | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Concentration ($C$) | 200 | 206 | 212.18 | 218.55 | 225.10 | 231.85 | 238.81 |

(i) Determine the revised recursive rule. (2 marks)

(ii) If she treated the water on 10 January and went on holiday until 20 January, when she next treated the water, calculate the concentration of the water on her return, assuming the recursive rule from part (b)(i) is used. (2 marks)

| | |
|---|---|
| **2020**<br>**Section 2**<br>**Question 7**<br><br>**Growth and decay in sequences** | The world's tallest man was recorded as 60 cm long at birth. He grew 28 cm in his first year, 26 cm in his second year and so on, always 2 cm less than in the previous year until he stopped growing.<br><br>(a) Calculate his annual growth (in cm) in his fourth and fifth years. (1 mark)<br><br>(b) Deduce the rule for his annual growth in the $n^{th}$ year, until he stopped growing. (2 marks)<br><br>(c) In which year did he first not grow any taller? (1 mark)<br><br>(d) Calculate his maximum height. (2 marks) |

| | |
|---|---|
| **2020**<br>**Section 2**<br>**Question 8**<br><br>**Growth and decay in sequences** | A farmer has a large lake on his farm and has started stocking it with fish of a variety that will flourish in the conditions in this lake. Monitoring has shown that the number of adult fish is increasing at a consistent rate of 9% per month and at the beginning of 2020 the lake holds 660 of the adult fish.<br><br>(a) Write a recursive rule to give the number of adult fish in the lake at the end of each month from the beginning of 2020. (2 marks)<br><br>(b) Deduce a rule for the nth term of this sequence. (2 marks) |

| | |
|---|---|
| | The farmer plans to allow the general public to pay to fish in the lake. This will commence at the beginning of the next month after the adult fish population first reaches 4000.<br><br>(c) Determine how many months after the beginning of 2020 fishing will commence. (2 marks)<br><br>(d) The farmer wishes to maintain a steady state in the adult fish population once fishing commences. Calculate how many adult fish can be taken from the lake each month. (3 marks) |

| | |
|---|---|
| **2019 Section 2 Question 7**<br><br>**Growth and decay in sequences** | A water tank is full. When a tap at the bottom of the tank is opened, 84 litres run out in the first minute, 78 litres in the second minute and 72 litres in the third minute. This pattern continues until the tank is empty.<br><br>(a) Write a rule for the $n^{th}$ term of a sequence in the form $T_n = A + Bn$, which will model this situation where $T_n$ is the amount of water that runs out in the $n^{th}$ minute. (2 marks)<br><br>(b) How many litres run out in the seventh minute? (1 mark)<br><br>(c) How many litres have run out after eight minutes? (1 mark)<br><br>(d) What is the capacity of the tank? (2 marks) |

| | |
|---|---|
| **2019 Section 2 Question 10**<br><br>**Growth and decay in sequences** | Ruby Ducks Coffee shops commenced operations in 1992 and had 15 stores open by the end of the year. They have been so successful over the years that the number of stores worldwide has continued to grow exponentially since then. The number of shops operating, $T$, at the end of 2017 was 22 579 and at the end of 2018 was 30 256.<br><br>The number of shops operating at the end of n years can be represented by the recursive rule $T$n = $1.34T_n – 1$, $T_1 = 15$.<br><br>(a) Show mathematically that the common ratio is approximately 1.34. (1 mark)<br><br>(b) Write the rule for the $n^{th}$ term of this sequence. (1 mark)<br><br>(c) Determine the first year in which there is likely to be over 200 000 Ruby Ducks Coffee shops. (2 marks)<br><br>Typically, each store has twelve employees working during the day across different shifts. Each employee earns, on average, $114.80 per day.<br><br>(d) Calculate the total daily wages for all stores at the beginning of 2012. (3 marks) |

**2023 Section 1 Question 3**

**Growth and decay in sequences**

From January 1, 2020, a company offered its employees an income package with a starting wage of $4000 per month, paid at the end of each month. Also, as an incentive to stay with the company, there was a monthly increase of $50 each month.

(a) Determine a recursive rule for the monthly wage. (2 marks)

| Solution |
|---|
| $T_{n+1} = T_n + 50, T_1 = 4000$ |
| **Specific behaviours** |
| ✓ states correct recursive rule<br>✓ states correct first term |

(b) Deduce a simplified rule for the nth term of the monthly wage. (2 marks)

| Solution |
|---|
| $T_n = 4000 + (n-1)(50)$<br>$T_n = 3950 + 50n$ |
| **Specific behaviours** |
| ✓ uses correct arithmetic formula<br>✓ gives correct simplified rule for the $n^{th}$ term |

(c) Determine the monthly wage for December 2020. (2 marks)

| Solution |
|---|
| $T_{12} = 3950 + 50(12)$<br>$= 4550$<br>Therefore, the monthly wage for December 2020 is $4550 |
| **Specific behaviours** |
| ✓ correctly identifies term 12<br>✓ correctly calculates the $4550 |

The company has decided to make the monthly increase $60 from the end of December 2023.

(d) Calculate the monthly wage for March 2024. (3 marks)

| Solution |
|---|
| $12 \times 4 = 48$<br>$T_{48} = 3950 + 50(48) = 6350$<br>Therefore, the monthly wage for March 2024 is<br>6350 + 60 + 60 + 60 = $6530 |
| **Specific behaviours** |
| ✓ correctly calculates $T_{48}$<br>✓ calculates correct term for March 2024<br>✓ states correct solution for wage |

**2021 Section 1 Question 1**

**Growth and decay in sequences**

Hanai is a successful college basketball player. His coach has warned him that he will lose his scholarship if he scores 54% or below on a weekly assessment. On his first three weekly assessments he scored 84%, 81% and 78% respectively.

Assume Hanai's weekly assessments continue to follow this pattern.

(a) Deduce a rule for the $n^{th}$ term of this sequence. (2 marks)

| Solution |
|---|
| $T_n = 84 + (n-1)(-3)$<br>$= 87 - 3n$ |
| **Specific behaviours** |
| ✓ correctly identifies an arithmetic sequence<br>✓ correctly states the rule for the $n^{th}$ term |

(b) Determine Hanai's score on his sixth weekly assessment. (1 mark)

| Solution |
|---|
| $T_6 = 87 - 3(6)$<br>$= 69$<br>Therefore, he gets 69% on his sixth assessment |
| **Specific behaviours** |
| ✓ calculates the correct value |

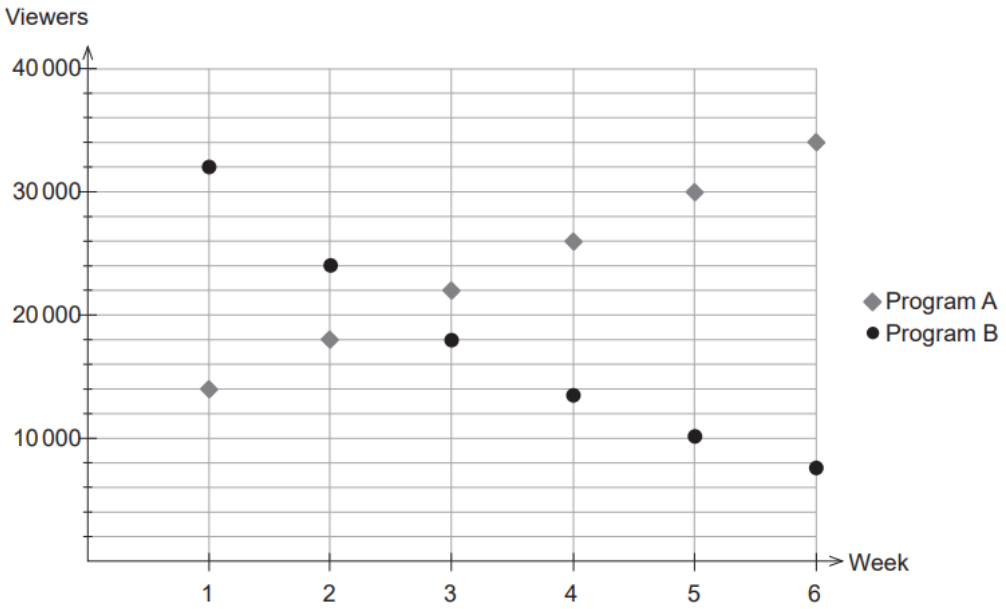
(c) Predict when Hanai will lose his scholarship. (2 marks)

| Solution |
|---|
| $54 = 87 - 3n$<br>$3n = 33$<br>$n = 11$<br>Therefore, Hanai will lose his scholarship after the 11$^{th}$ weekly assessment |
| **Specific behaviours** |
| ✓ substitutes 54 correctly<br>✓ identifies correct weekly assessment |

**2021 Section 1 Question 6**

**Growth and decay in sequences**

A television network programmer was analysing the number of viewers for two children's programs over a period of several weeks, to decide which program should be given the better time slot. The viewing numbers, displayed on the graph below, formed an arithmetic sequence and a geometric sequence.




(a) Write a recursive rule for the arithmetic sequence. (2 marks)

| Solution |
|---|
| $T_{n+1} = T_n + 4000, T_1 = 14\,000$ |
| **Specific behaviours** |
| ✓ states a correct recursive rule<br>✓ states correct first term |

(b) Using the first two data points, deduce a rule for the $n^{th}$ term of the geometric sequence. (2 marks)

| Solution |
|---|
| Using 32 000 and 24 000<br>Ratio = $\frac{24\,000}{32\,000} = \frac{3}{4} = 0.75$<br>Therefore $T_n = 32\,000\left(\frac{3}{4}\right)^{n-1} = 32\,000(0.75)^{n-1}$ |
| **Specific behaviours** |
| ✓ determines correct ratio<br>✓ states correct $n^{th}$ term |

(c) Explain which program should be given the better time slot. (2 marks)

| Solution |
|---|
| Program A because viewer numbers are increasing |
| **Specific behaviours** |
| ✓ states correct program<br>✓ explains why Program A should be given the better time slot |

(d) Determine the number of viewers for the more successful program in Week 8. (1 mark)

| Solution |
|---|
| $T_8 = 34\,000 + 8\,000 = 42\,000$<br>Therefore, the number of viewers is 42 000 |
| **Specific behaviours** |
| ✓ correctly states number of viewers |

**2019 Section 1 Question 6**

**Growth and decay in sequences**

The population of turtles in an artificial lake at a wildlife sanctuary is initially 32 and research has shown a natural decrease in population of 50% each year. Twenty extra turtles are introduced to the lake at the end of each year.

(a) Determine a recursive rule for the turtle population. (2 marks)

| Solution |
|---|
| $T_{n+1} = 0.5T_n + 20, T_1 = 32$ |
| **Specific behaviours** |
| ✓ states correct rule<br>✓ states correct first term |

(b) Determine the long-term steady state of the turtle population. (2 marks)

| Solution 1 |
|---|
| $T_2 = 0.5(32) + 20$<br>$= 36$<br>$T_3 = 0.5(36) + 20$<br>$= 38$<br>$T_4 = 0.5(38) + 20$<br>$= 39$<br>$T_5 = 0.5(39) + 20$<br>$39.5$<br>Approaching a steady state of 40 turtles |
| **Specific behaviours** |
| ✓ correctly calculates a further 4 terms<br>✓ correctly determines the long-term steady state |

**OR**

| Solution 2 |
|---|
| $x = 0.5x + 20$<br>$0.5x = 20$<br>$x = 40$ |
| **Specific behaviours** |
| ✓ correctly writes steady state equation<br>✓ correctly determines the long-term steady state |

(c) If the wildlife sanctuary preferred a long-term steady state of 80 turtles, what yearly addition of turtles would be required to produce this steady state? Assume all other conditions remain the same. (2 marks)

| **Solution 1** |
| --- |
| By trial and error.<br>Example: if $k$ = 30, $T_2 = 46, T_3 = 53, T_4 = 56.5$ , not approaching 80<br>Example: if $k$ = 40, $T_2 = 56, T_3 = 68, T_4 = 74, T_5 = 77$ , approaching 80 |
| **Specific behaviours** |
| ✓ uses trial and error to correctly generate a sequence of at least 4 terms<br>✓ correctly determines the yearly addition of turtles |

**OR**

| **Solution 2** |
| --- |
| $80 = 0.5 \times 80 + k$<br>$80 = 40 + k$<br>$k = 40$ |
| **Specific behaviours** |
| ✓ correctly writes steady state equation<br>✓ correctly determines the yearly addition of turtles |

**2022 Section 2 Question 9**

**Growth and decay in sequences**

A study of a penguin colony on an island was conducted and it found the initial population size of 1200 was dropping by 14% each year due to the introduction of non-native predators.

(a) Explain why the population after $n$ weeks is $1200 \times (0.86)^n$ penguins. (2 marks)

| Solution |
|---|
| Assuming population size dropping by 14% per week:<br><br>The population of penguins in each week is 86% of that of the previous week, so $r = 0.86$. The initial population ($P_0$) is 1200. |
| **Specific behaviours** |
| ✓ identifies that 14% decrease is equivalent to 86% of previous value<br>✓ identifies 1200 is initial population size |

| Alternate solution |
|---|
| Assuming population size dropping by 14% per year:<br><br>The population of penguins is not $1200\times(0.86)^n$ after $n$ weeks. The given expression would be the population of penguins after $n$ years, not weeks. For $n$ years the expression would be $1200\times\left(1-\frac{0.14}{52}\right)^n = 1200\times 0.99731^n$. |
| **Specific behaviours** |
| ✓ states given expression is not the population after $n$ weeks<br>✓ states valid reason |

After eight weeks, the Parks and Wildlife Service set traps to reduce the predator numbers. This saw the penguin population increase weekly by 6%.

(b) State the recursive formula that models the new population growth. (2 marks)

| Solution |
|---|
| If used given expression from part (a):<br>$P_8 = 1200\times 0.86^8 \approx 359$ penguins<br>Therefore, the recursive formula is<br>$P_{n+1} = 1.06P_n,\ P_0 = 359$<br><br>or<br><br>If used alternate solution from part (a):<br>$P_8 = 1200\times 0.99731^8 \approx 1174$ penguins<br>Therefore, the recursive formula is<br>$P_{n+1} = 1.06P_n,\ P_0 = 1174$ |
| **Specific behaviours** |
| ✓ states recursive formula<br>✓ includes correct $P_0$ value |

(c) How many weeks will it take to get the population back up to the initial size? (1 mark)

| Solution |
|---|
| If used given expression from part (a):<br><br>It will take 21 weeks for population to return to 1200.<br><br>or<br><br>If used alternate solution from part (a):<br><br>The population will return to 1200 within the first week (0.37 weeks) |
| **Specific behaviours** |
| ✓ determines correct number of weeks |

Once the population returns to the initial size, it is further helped by the introduction of penguins from a breeding program at the zoo.

The new population growth model can be represented by

$P_{n+1} = -0.25P_n + 3000$, $P_0 = 1200$ .

(d) Discuss the long-term behaviour of the penguin population, now that it is being supported by the breeding program. (2 marks)

| Solution |
|---|
| The long-term trend for the penguin population is to reach a steady state of 2400 penguins. |
| **Specific behaviours** |
| ✓ identifies steady state solution<br>✓ states 2400 penguins |

**2022 Section 2 Question 17**

**Growth and decay in sequences**

Indie was in a line with 24 other people for a slide at a water park. She noticed that the approximate number of people ($P$) in the line for the slide increased by 1.5% every minute ($m$).

(a) Write an exponential equation in the form $P = ar^m$ to represent this situation. (2 marks)

| Solution |
|---|
| $P = 25 \times 1.015^m$ |
| **Specific behaviours** |
| ✓ states correct value of $a$<br>✓ states correct value of $r$ |

(b) Determine the approximate number of people in the line after 2 hours. (2 marks)

| Solution |
|---|
| $P = 25 \times 1.015^{120}$<br>$P = 149.23$<br>$\sim 149$ people |
| **Specific behaviours** |
| ✓ states correct value<br>✓ recognises integer value required |

After 3 hours, the line started to decrease by 1% per minute.

(c) Using this new information, calculate the approximate number of people in line, 5 hours after Indie initially lined up. (4 marks)

| Solution |
|---|
| $P = 25 \times 1.015^{180}$<br>$P = 364.61$<br>$\sim 365$<br><br>$P = 365 \times 0.99^{m}$<br>$P = 365 \times 0.99^{120}$<br>$P = 109.3$<br>$\sim 109$ people |
| **Specific behaviours** |
| ✓ calculates $P$ for $m = 180$<br>✓ states new ratio of 0.99<br>✓ identifies $m = 120$<br>✓ uses equation to calculate $P \sim 109$ |

**2021 Section 2 Question 11**

**Growth and decay in sequences**

Judith monitors the water quality in her garden pond at the same time every day. She likes to maintain the concentration of algae at between 200 and 250 units per 100 litres (L). Her measurements show that the concentration increases daily according to the recursive rule $C_{n+1} = 1.025C_n$ , where $C_1 = 200$ units per 100 L (the minimum concentration).

When the concentration gets above the 250 units per 100 L limit, she treats the water to bring the concentration back to the minimum 200 units per 100 L.

(a) If Judith treated the water on Sunday, 6 December 2020, determine

(i) the concentration on Wednesday, 9 December 2020. (2 marks)

| Solution |
|---|
| $C_4 = 215.38$ units per 100 L |
| **Specific behaviours** |
| ✓ correctly identifies Wednesday is $C_4$<br>✓ correctly calculates the concentration |

(ii) the day and date when she next treated the water. (2 marks)

| Solution |
|---|
| $C_{10} = 249.77$ , $C_{11} = 256.02$ i.e. Wednesday 16 December ($11^{th}$ term) |
| **Specific behaviours** |
| ✓ calculates correct day<br>✓ calculates correct date |

(b) During the first week of January 2021, Judith monitored the water and recorded the following readings.

| Day | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Concentration ($C$) | 200 | 206 | 212.18 | 218.55 | 225.10 | 231.85 | 238.81 |

(i) Determine the revised recursive rule. (2 marks)

| Solution |
|---|
| $\frac{206}{200} = 1.03 \ \ \therefore C_{n+1} = 1.03C_n, C_1 = 200$ |
| **Specific behaviours** |
| ✓ calculates correct ratio<br>✓ states correct initial value |

(ii) If she treated the water on 10 January and went on holiday until 20 January, when she next treated the water, calculate the concentration of the water on her return, assuming the recursive rule from part (b)(i) is used. (2 marks)

| Solution |
|---|
| 20 January is the eleventh term. $C_{11} = 268.78$ units per 100 L |
| **Specific behaviours** |
| ✓ determines correct term<br>✓ calculates correct value |

**2020 Section 2 Question 7**

**Growth and decay in sequences**

The world's tallest man was recorded as 60 cm long at birth. He grew 28 cm in his first year, 26 cm in his second year and so on, always 2 cm less than in the previous year until he stopped growing.

(a) Calculate his annual growth (in cm) in his fourth and fifth years. (1 mark)

| Solution |
|---|
| Fourth year: 22 cm<br>Fifth year: 20 cm |
| **Specific behaviours** |
| ✓ calculates correct values |

(b) Deduce the rule for his annual growth in the $n^{th}$ year, until he stopped growing. (2 marks)

| Solution |
|---|
| $T_n = 30 - 2n$ |
| **Specific behaviours** |
| ✓ determines the correct constant term<br>✓ determines the correct coefficient of $n$ |

(c) In which year did he first not grow any taller? (1 mark)

| Solution |
|---|
| In the fifteenth year |
| **Specific behaviours** |
| ✓ calculates correct value |

(d) Calculate his maximum height. (2 marks)

| Solution |
|---|
| $60 + (28 + 26 + 24 + \ldots + 2) = 270$ cm |
| **Specific behaviours** |
| ✓ correctly sums terms from 28 to 2<br>✓ correctly adds 60 to the sum |

**2020 Section 2 Question 8**

**Growth and decay in sequences**

A farmer has a large lake on his farm and has started stocking it with fish of a variety that will flourish in the conditions in this lake. Monitoring has shown that the number of adult fish is increasing at a consistent rate of 9% per month and at the beginning of 2020 the lake holds 660 of the adult fish.

(a) Write a recursive rule to give the number of adult fish in the lake at the end of each month from the beginning of 2020. (2 marks)

| Solution |
|---|
| $T_{n+1} = 1.09T_n \qquad T_0 = 660$ |
| **Specific behaviours** |
| ✓ correctly states recursive rule<br>✓ correctly states $T_0$ |

(b) Deduce a rule for the nth term of this sequence. (2 marks)

| Solution |
|---|
| $A_n = 660 \times 1.09^n$ |
| **Specific behaviours** |
| ✓ gives formula in exponential form<br>✓ states correct rule |

The farmer plans to allow the general public to pay to fish in the lake. This will commence at the beginning of the next month after the adult fish population first reaches 4000.

(c) Determine how many months after the beginning of 2020 fishing will commence. (2 marks)

| Solution |
|---|
| $4000 = 660 \times 1.09^n \quad n = 20.9$<br>Therefore fishing will commence 21 months after the beginning of 2020<br>**or**<br>$T_{21} = 4031.8$<br>Therefore fishing will commence 21 months after the beginning of 2020 |
| **Specific behaviours** |
| ✓ correctly solves for $n$<br>✓ correctly states correct number of months after the beginning of 2020 |

(d) The farmer wishes to maintain a steady state in the adult fish population once fishing commences. Calculate how many adult fish can be taken from the lake each month. (3 marks)

| Solution |
|---|
| $T_{21} = 4031$<br>$T_{n+1} = 1.09T_n - x$<br>$4031 = 1.09 \times 4031 - x \qquad x = 362.79$<br><br>362 fish per month |
| **Specific behaviours** |
| ✓ correctly determines $T_{21} = 4031$<br>✓ correctly determines monthly increase<br>✓ correctly rounds down for number of fish |

**2019 Section 2 Question 7**

**Growth and decay in sequences**

A water tank is full. When a tap at the bottom of the tank is opened, 84 litres run out in the first minute, 78 litres in the second minute and 72 litres in the third minute. This pattern continues until the tank is empty.

(a) Write a rule for the $n^{th}$ term of a sequence in the form $T_n = A + Bn$, which will model this situation where $T_n$ is the amount of water that runs out in the $n^{th}$ minute. (2 marks)

| Solution |
|---|
| $T_n = 84 + (n-1)(-6)$<br>$= 90 - 6n$ |
| **Specific behaviours** |
| ✓ states correct value for $A$<br>✓ states correct value for $B$ |

(b) How many litres run out in the seventh minute? (1 mark)

| Solution |
|---|
| 48 L |
| **Specific behaviours** |
| ✓ states correct value |

(c) How many litres have run out after eight minutes? (1 mark)

| Solution |
|---|
| 48 L |
| **Specific behaviours** |
| ✓ states correct value |

(d) What is the capacity of the tank? (2 marks)

| Solution |
|---|
| $T_{15} = 0$, Sum of first 15 terms is 630 L |
| **Specific behaviours** |
| ✓ states that the 15th term is zero<br>✓ states correct capacity |

**2019 Section 2 Question 10**

**Growth and decay in sequences**

Ruby Ducks Coffee shops commenced operations in 1992 and had 15 stores open by the end of the year. They have been so successful over the years that the number of stores worldwide has continued to grow exponentially since then. The number of shops operating, $T$, at the end of 2017 was 22 579 and at the end of 2018 was 30 256.

The number of shops operating at the end of n years can be represented by the recursive rule $T$n = $1.34T_n - 1$, $T_1 = 15$.

(a) Show mathematically that the common ratio is approximately 1.34. (1 mark)

| Solution |
|---|
| $r = \frac{30\,256}{22\,579} \approx 1.34$ |
| **Specific behaviours** |
| ✓ shows correct calculation of the ratio |

(b) Write the rule for the $n^{th}$ term of this sequence. (1 mark)

| Solution |
|---|
| $T_n = 15 \times 1.34^{n-1}$ or $T_n = 11.19 \times 1.34^n$ |
| **Specific behaviours** |
| ✓ states correct rule |

(c) Determine the first year in which there is likely to be over 200 000 Ruby Ducks Coffee shops. (2 marks)

| Solution |
|---|
| After 33 years there are approximately 234 719 shops<br>Therefore, the first year is 2025 |
| **Specific behaviours** |
| ✓ determines correct number of years<br>✓ determines correct year |

Typically, each store has twelve employees working during the day across different shifts. Each employee earns, on average, $114.80 per day.

(d) Calculate the total daily wages for all stores at the beginning of 2012. (3 marks)

| Solution |
|---|
| Stores $\approx 3900$<br>Employees $= 3900 \times 12 = 46\,800$<br>Daily wages $= 46\,800 \times 114.80 = \$5\,372\,640$ |
| **Specific behaviours** |
| ✓ correctly determines the number of stores<br>✓ correctly determines the number of employees<br>✓ correctly determines the total wages |

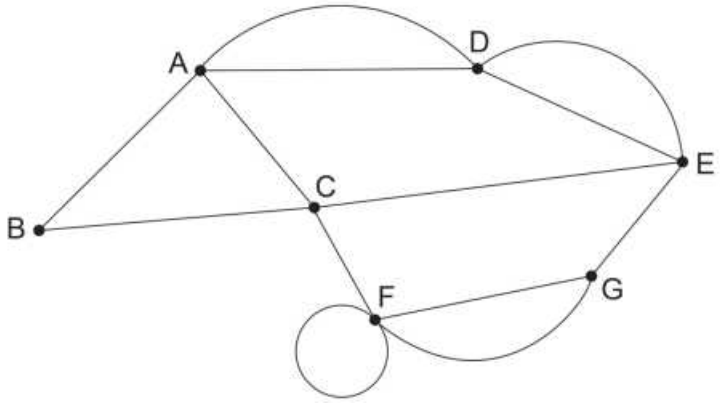
# Unit 3.3 – Graphs and networks

## Section 1

**2023 Section 1 Question 2**

**Graphs and networks**

The network shown below is for walking tracks around a wildlife sanctuary. Located at each vertex is an undercover area with picnic tables and viewing stations for visitors.




(a) Complete the adjacency matrix for the network shown. (3 marks)

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| A | 0 | 1 | 1 | 2 | 0 | 0 | 0 |
| B | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| C | 1 | 1 | 0 | ___ | ___ | ___ | ___ |
| D | 2 | 0 | 0 | ___ | ___ | ___ | ___ |
| E | 0 | 0 | 1 | ___ | ___ | ___ | ___ |
| F | 0 | 0 | 1 | ___ | ___ | ___ | ___ |
| G | 0 | 0 | 0 | ___ | ___ | ___ | ___ |

(b) Identify the feature of the adjacency matrix that indicates it is an undirected graph. (1 mark)

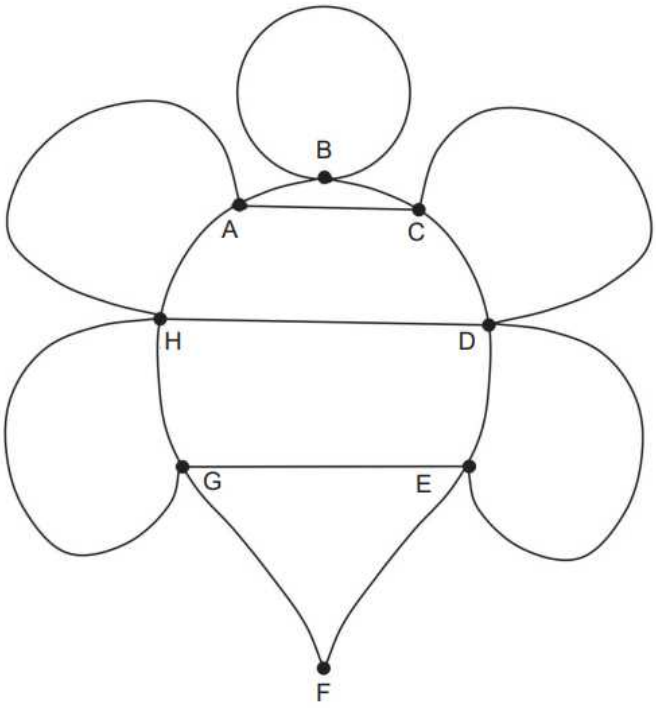
(c) List all possible open paths of length two starting from G. (3 marks)

(d) Each morning before visitors are admitted, all fences along walking tracks between vertices must be checked for safety. Is it possible to carry out the safety check with an Eulerian trail, a semi-Eulerian trail or neither? Justify your answer. (2 marks)

**2022 Section 1 Question 1**

**Graphs and networks**

The network below shows the different paths around a botanical exhibition.

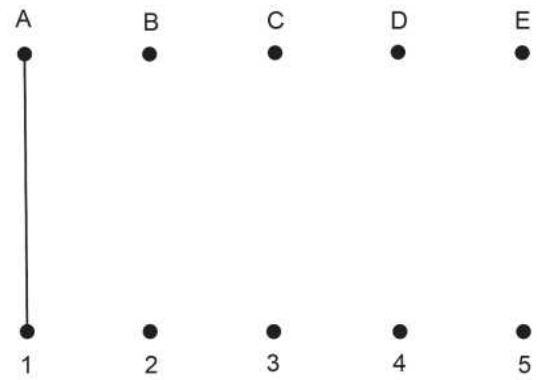



(a) Explain why this is not a simple graph. (1 mark)

(b) Is this network Eulerian or semi-Eulerian? Justify your response. (2 marks)

(c) Use Euler's formula to prove that this network is planar. (2 marks)

| | |
|---|---|
| | (d) A visitor to the exhibition has taken her child in a pram and wishes to walk along every path exactly once to see all the floral displays, excluding the walk back to the car park. If the only car parks are located next to D and F, where would you suggest they park? Use mathematical reasoning to support your answer. (2 marks) |

| | |
|---|---|
| **2021 Section 1 Question 2**<br><br>**Graphs and networks** | A construction company uses five different machines and has five employees who operate those machines.<br><br>The adjacency matrix below shows each of the five employees (A, B, C, D, E) and the five machines they are trained to operate. These are the only machines they may use.<br><br>Employee<br>A B C D E<br>Machine<br>1 [1 0 0 1 0]<br>2 [0 0 0 0 1]<br>3 [1 1 0 0 0]<br>4 [0 0 1 0 1]<br>5 [0 0 1 1 0]<br><br>(a) Draw the adjacency matrix as a bipartite graph. The A1 connection has already been drawn on the graph. (2 marks)<br><br>  |

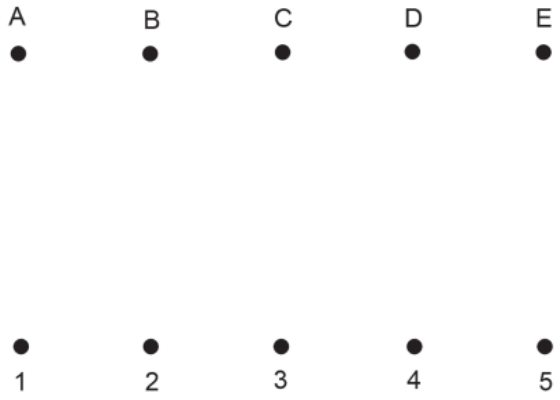
(b) Does the bipartite graph in part (a) represent

(i) a planar graph? (1 mark)

(ii) a connected graph? (1 mark)

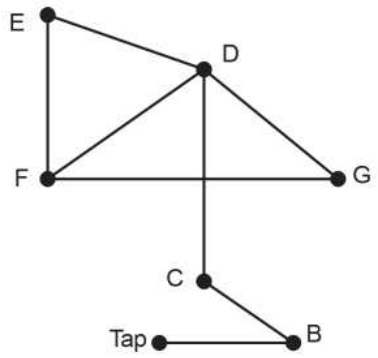
(c) Explain why the bipartite graph in part (a) is a simple graph. (2 marks)

(d) Complete the table below to show how an allocation of all employees to a machine could be achieved. (2 marks)




| Employee | A | B | C | D | E |
|---|---|---|---|---|---|
| Machine | | | | | |

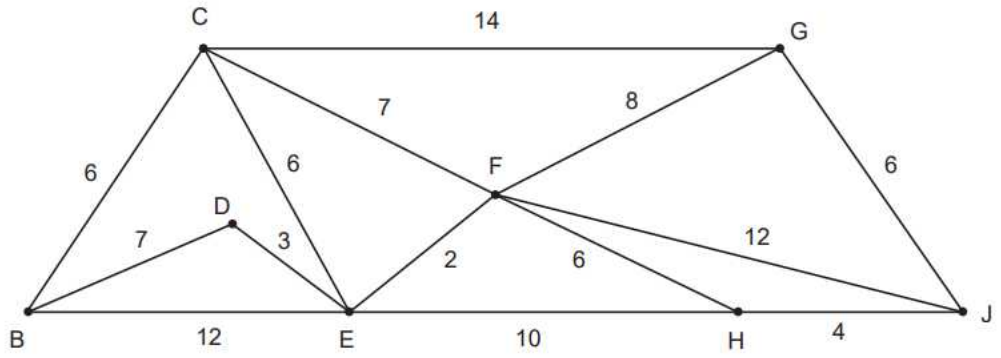
(e) The company foreman decides to allocate Employee E to Machine 4. Explain what effect this will have to the allocation of the remaining employees to the machines. (2 marks)

| | |
|---|---|
| **2021**<br>**Section 1**<br>**Question 3**<br><br>**Graphs and networks** | The graph below shows the current network of reticulation pipes in Tarik's garden.<br><br>  <br><br><br>(a) Using Euler's formula, stating the number of vertices, edges and faces, show that the graph is planar. (2 marks)<br><br>The water needs to travel from the tap and through all pipes.<br><br>(b) List a possible route for the water. (1 mark)<br><br>(c) What is the mathematical term for the route listed in part (b)? (1 mark)<br><br>(d) Tarik would like to increase the water pressure by removing one edge (pipe).<br><br>(i) Identify any edge that **cannot** be removed. (1 mark)<br><br>(ii) What is the name given to the type of edge identified in part (d)(i)? (1 mark) |

**2021 Section 1 Question 5**

**Graphs and networks**

The network below shows the relative distances, in hundreds of metres, between wi-fi hotspots around a university campus.




The control room for this system of hotspots is located at B. A problem has been reported with the hotspot at J.

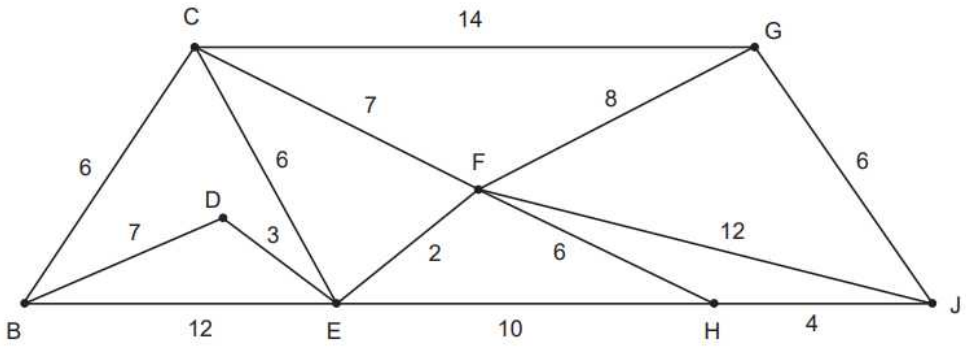
(a) A technician is sent from the control room to solve the problem at J. To get there as quickly as possible, she wants to use the shortest path, travelling from hotspot to hotspot. Determine the required path and its length. (3 marks)

(b) A second technician is sent from the control room to J to help resolve the problem. The technician uses an open path travelling from hotspot to hotspot. If the technician travels 2700 metres, determine the path he used. (1 mark)

(c) After repairs have been made, all hotspots need to be checked. A technician is sent from the control room, travelling to all hotspots once only, and finishing back at the control room.

(i) State the name given to this type of path. (2 marks)

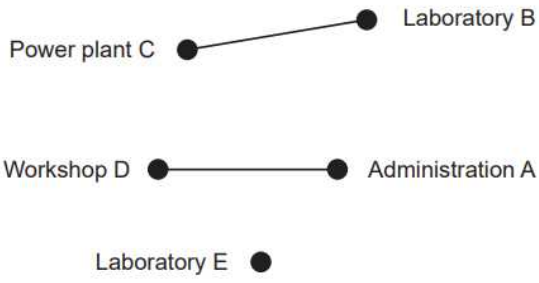
(ii) Determine the length of the shortest path possible and state the path used. (2 marks)




**2020 Section 1 Question 2**

**Graphs and networks**

A small research facility consists of five buildings with walkways represented by the edges in this network:




(a) Determine the smallest number of edges (walkways) to be added to ensure that the network is

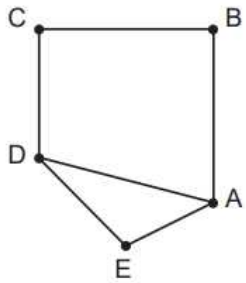
(i) connected (1 mark)

(ii) complete (1 mark)

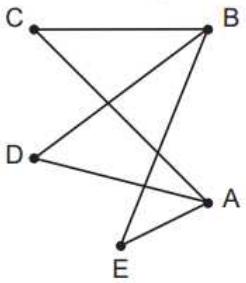
(iii) planar with 4 regions. (2 marks)

Three different plans for completing the network with the addition of four walkways are shown:

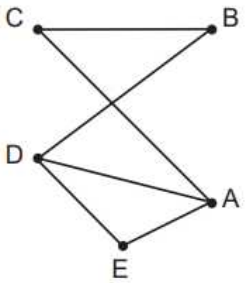
Plan *P*




Plan *Q*




Plan *R*




(b) State which, if any, of these plans has a graph that is

(i) bipartite (1 mark)

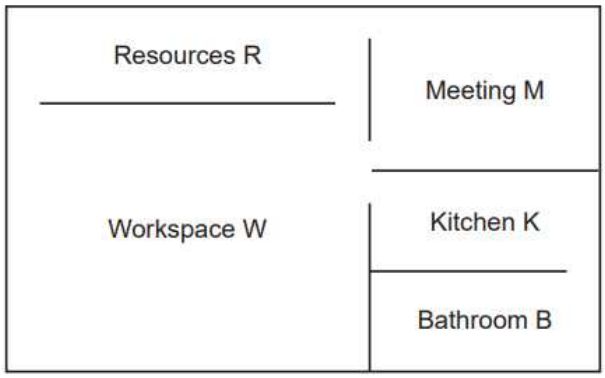
(ii) Eulerian (1 mark)

(iii) a Hamiltonian cycle. (2 marks)

**2020 Section 1 Question 6**

**Graphs and networks**

A small business office has five separate areas connected by doorways shown as gaps in this diagram:




This adjacency matrix below represents the number of doorways directly between each area:

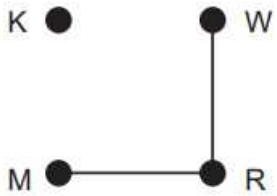
| | B | K | M | R | W |
|---|---|---|---|---|---|
| B | 0 | 1 | 0 | 0 | 0 |
| K | 1 | 0 | 0 | 0 | 1 |
| M | 0 | 0 | 0 | 1 | 1 |
| R | 0 | 0 | 1 | 0 | $Y$ |
| W | 0 | $X$ | 1 | 2 | 0 |

(a) State the meaning of the zero entries in the matrix. (1 mark)

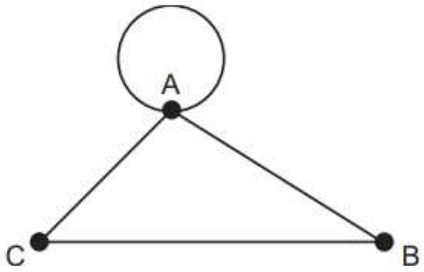
(b) Determine the value of X and Y. (2 marks)

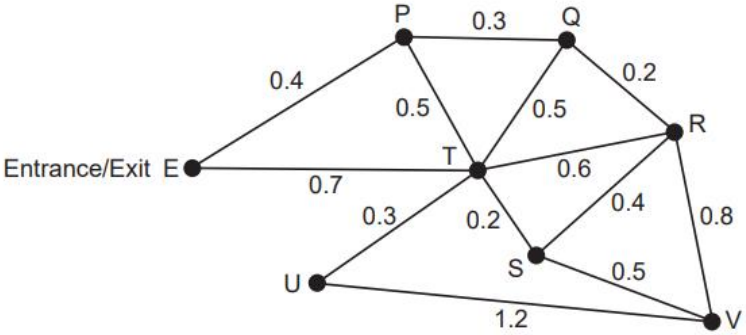
(c) Describe how the total number of doorways for each area can be found from the adjacency matrix. (1 mark)

(d) Complete this network with vertices corresponding to the office areas and the edges representing the doorways. (3 marks)




(e) Determine how many different routes there are between the meeting room and the workspace that pass through exactly two doorways. (1 mark)

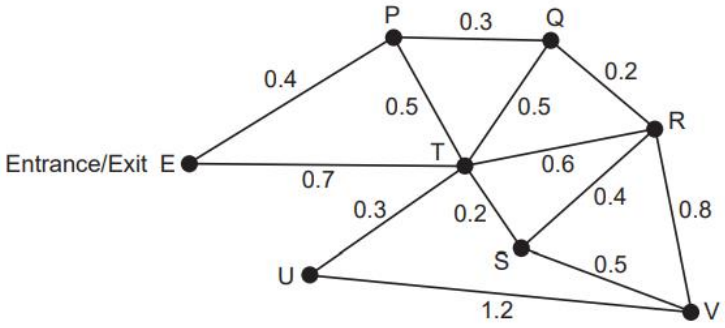
| | |
|---|---|
| **2019**<br>**Section 1**<br>**Question 1**<br><br>**Graphs and networks** | The graph shown represents three buildings A, B and C, with connecting walkways, at a local school.<br><br>  <br><br><br>(a) Why is the graph planar? (1 mark)<br><br>(b) Show that the graph satisfies Euler's formula. (2 marks)<br><br>(c) Construct the adjacency matrix for the graph. (3 marks)<br><br>A student wishes to carry out closed walks of length two from Building A.<br><br>(d) List all his possible walks. (2 marks) |

| | |
|---|---|
| **2019**<br>**Section 1**<br>**Question 4**<br><br>**Graphs and networks** | A marine park has attractions with paths connecting them. The vertices on the graph represent the attractions and the numbers on the edges represent the path distances (km) between the attractions. Visitors can either walk around the park or take one of the many shuttle buses that run between attractions.<br><br>  <br><br><br>The manager of the marine park leaves his office, which is located at the entrance/exit (E) and walks to attraction V.<br><br>(a) (i) Determine the shortest distance from E to V. (1 mark)<br><br>(ii) If the manager needs to pick up some tools left at U on the way, determine the route he should take and the corresponding distance, given he wants to take the shortest route from E to V. (2 marks)<br><br>Rachel arrives at the entrance. She wants to complete a Hamiltonian cycle.<br><br>(b) State the route she should take. (2 marks) |

Drinking water is already being supplied at E. The manager has recently received funding to establish drinking fountains at each attraction. For this to happen, pipelines will need to be laid along the paths to each attraction. He has drawn up a table to show the distances between attractions.

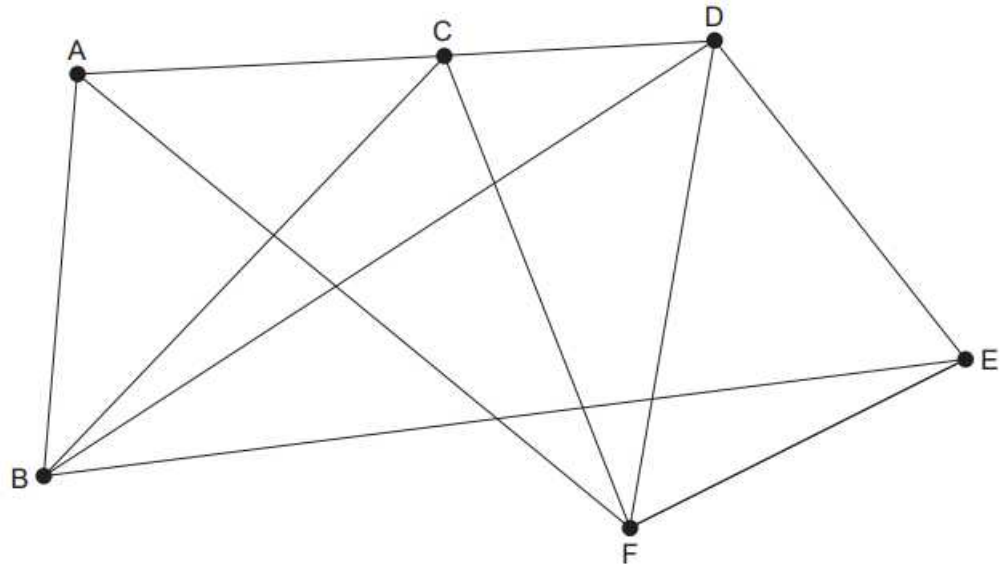
| | E | P | Q | R | S | T | U | V |
|---|---|---|---|---|---|---|---|---|
| E | – | 0.4 | – | – | – | 0.7 | – | – |
| P | 0.4 | – | 0.3 | – | – | 0.5 | – | – |
| Q | – | 0.3 | – | 0.2 | – | 0.5 | – | – |
| R | – | – | 0.2 | – | 0.4 | 0.6 | – | 0.8 |
| S | – | – | – | 0.4 | – | 0.2 | – | 0.5 |
| T | 0.7 | 0.5 | 0.5 | 0.6 | 0.2 | – | 0.3 | – |
| U | – | – | – | – | – | 0.3 | – | 1.2 |
| V | – | – | – | 0.8 | 0.5 | – | 1.2 | – |

(c) (i) Use Prim's algorithm, or otherwise, to determine the minimum total length of pipelines. Highlight the required pipelines on the diagram below. (4 marks)




(ii) The manager has been told that a pipeline of length 0.2 km could be laid from S to U. How, if at all, will this affect the total length of pipelines that should be laid in order to maintain a minimum length? (2 marks)

## Section 2

| | |
|---|---|
| **2023 Section 2 Question 12**<br><br>**Graphs and networks** | An event organiser is setting up a music festival with multiple stages. The graph below shows the stages as vertices, and the edges are the walkways that connect each stage.<br><br>  <br><br><br>(a) To avoid congestion, the organisers want to move the stages and re-position some connecting walkways so no walkways intersect. Redraw the graph to meet these conditions. (2 marks)<br><br>(b) Use Euler's formula, stating the number of vertices, edges and faces, to verify that this graph is planar. (2 marks) |

(c) State a reason why the graph is not Eulerian. (1 mark)

One attendee wishes to visit every stage to make the most of the festival.

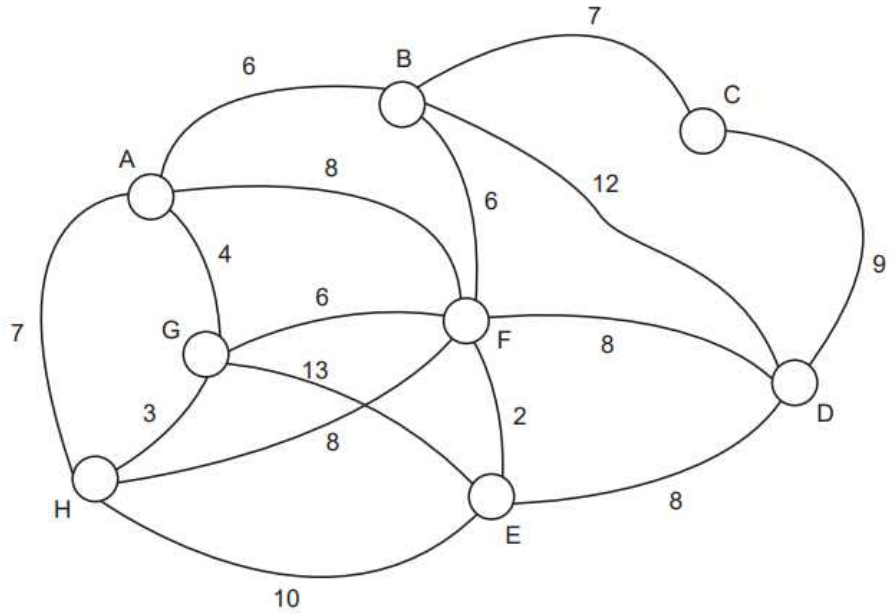
(d) (i) Write down a pathway the attendee can take to visit every stage exactly once if they start at Stage F. (1 mark)

(ii) What is the name given to this path? (1 mark)

**2022 Section 2 Question 10**

**Graphs and networks**

The network shown below represents the cycle tracks connecting the various points of interest on an island. The numbers indicate the distance between points of interest to the nearest 100 metres.




(a) Each morning all tracks must have a safety check. Is it possible to check all individual tracks by an Eulerian trail? Justify your answer. (2 marks)

(b) Rubbish bins at each point of interest are emptied daily. The rubbish truck leaves from position C and must return to C after emptying all rubbish bins.

(i) What name is given to the type of path taken by the rubbish truck? (2 marks)

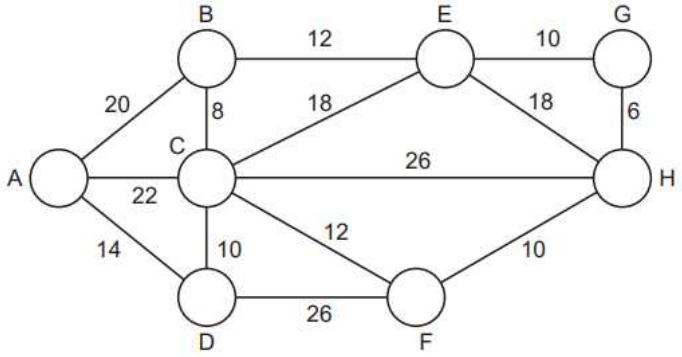
(ii) The rubbish truck wants to follow the path of least distance. State the path and its length. (3 marks)

| | |
|---|---|
| | (c) A tourist at G wishes to travel to C but path AG is closed for maintenance. Determine the shortest path possible and its length. (2 marks) |

**2020 Section 2 Question 13**

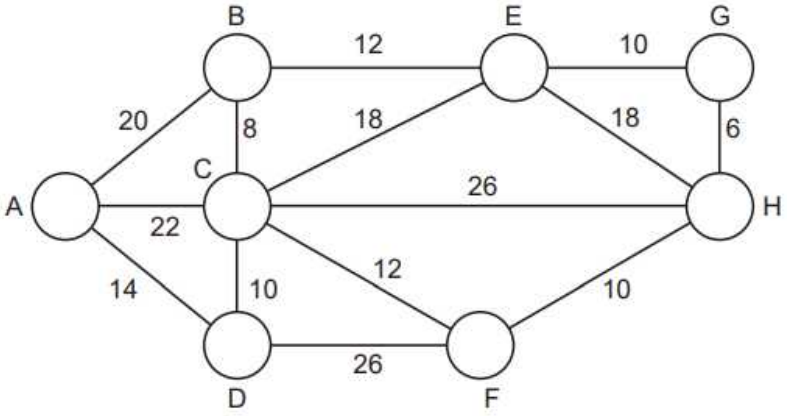
**Graphs and networks**

The graph below represents a road transport network from a warehouse at A to seven retail outlets B, C, D, E, F, G and H. The number on each edge represents the distance, in kilometres, along each road.



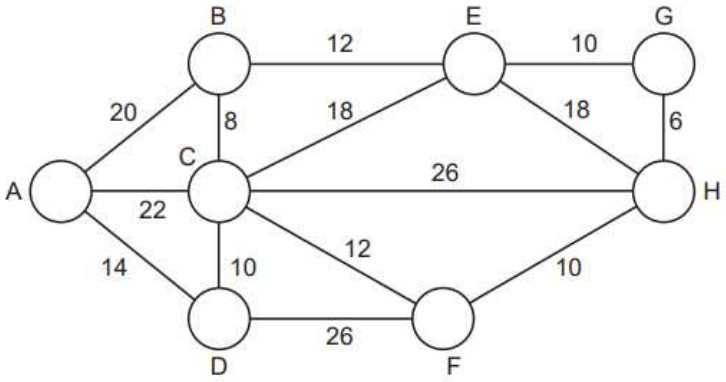

(a) Identify the shortest Hamiltonian path from the warehouse and state its length. (3 marks)

A special delivery must be made from the warehouse to retail outlet H.

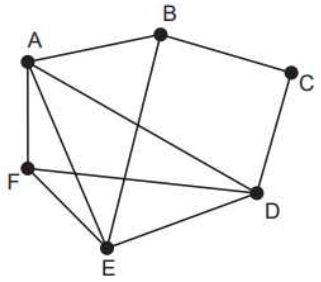
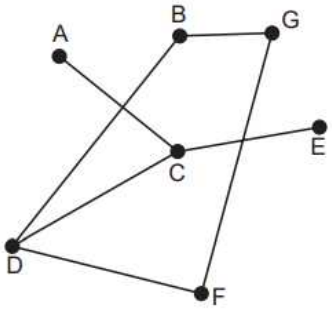



(b) Determine the shortest path and the distance travelled for this delivery. Working **must** appear on the network to show an appropriate method has been used. (3 marks)

Road CH presently goes around what is now a dry salt lake. It is proposed that a direct road be constructed that will reduce the distance between retail outlets C and H.



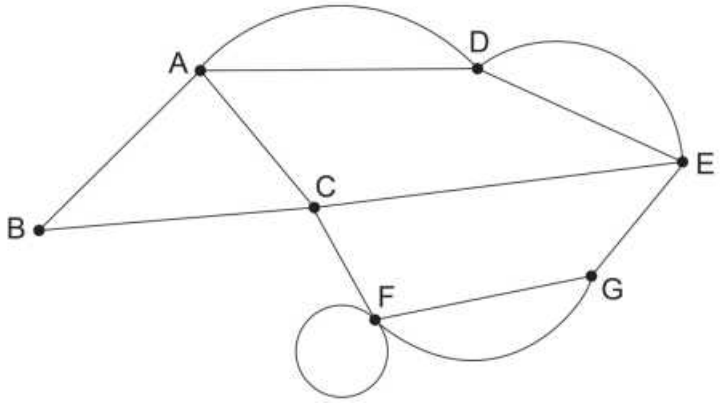

(c) By how much can the direct road between C and H be reduced, so that the shortest path from the warehouse to H includes the direct road CH? (3 marks)

| | |
|---|---|
| **2019 Section 2 Question 12**<br><br>**Graphs and networks** | Jake, a park ranger, is giving a presentation at a National Park and Wildlife Conference on possible designs for a new park. Unfortunately, Jake made mathematical errors in his presentation about the paths (represented by edges) and shelter huts (represented by vertices) in the park.<br><br>(a) For each of the following statements, the graph drawn by Jake was incorrect. Redraw the graph to match the statement correctly.<br><br>(i) This park plan has been drawn as a connected planar graph containing six vertices. (2 marks)<br><br>  <br><br><br>(ii) This park plan has been drawn as a bipartite graph. (3 marks)<br><br>  <br><br><br>Jake also makes the following incorrect statement in his presentation. ‘A park plan can be a complete graph with 21 paths and six shelter huts’.<br><br>(b) If the plan must be a complete graph with 21 paths, how many shelter huts should Jake have quoted? (1 mark) |

**2023 Section 1 Question 2**

**Graphs and networks**

The network shown below is for walking tracks around a wildlife sanctuary. Located at each vertex is an undercover area with picnic tables and viewing stations for visitors.




(a) Complete the adjacency matrix for the network shown. (3 marks)

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| A | 0 | 1 | 1 | 2 | 0 | 0 | 0 |
| B | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| C | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| D | 2 | 0 | 0 | 0 | 2 | 0 | 0 |
| E | 0 | 0 | 1 | 2 | 0 | 0 | 1 |
| F | 0 | 0 | 1 | 0 | 0 | 1 | 2 |
| G | 0 | 0 | 0 | 0 | 1 | 2 | 0 |

| Solution |
|---|
| see diagram above |
| **Specific behaviours** |
| ✓ determines at least 12 correct entries<br>✓ determines at least 16 correct entries<br>✓ determines all correct entries |

(b) Identify the feature of the adjacency matrix that indicates it is an undirected graph. (1 mark)

| Solution |
|---|
| matrix is symmetric across leading diagonal |
| **Specific behaviours** |
| ✓ correctly identifies feature |

(c) List all possible open paths of length two starting from G. (3 marks)

| Solution |
|---|
| GEC, GED (twice), GFC (twice) |
| **Specific behaviours** |
| ✓ determines at least three correct paths<br>✓ does not list paths that include the loop<br>✓ determines all five correct paths |

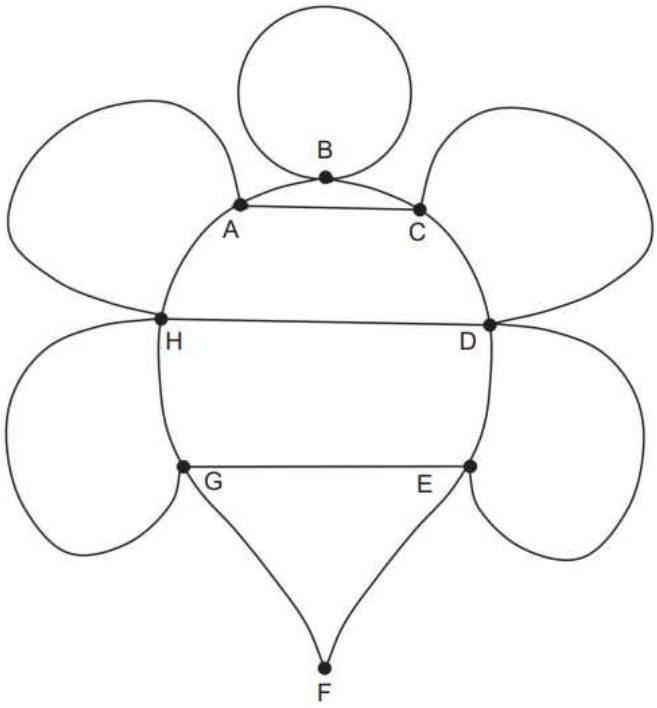
(d) Each morning before visitors are admitted, all fences along walking tracks between vertices must be checked for safety. Is it possible to carry out the safety check with an Eulerian trail, a semi-Eulerian trail or neither? Justify your answer. (2 marks)

| Solution |
|---|
| semi-Eulerian, F and G are odd vertices |
| **Specific behaviours** |
| ✓ states semi-Eulerian<br>✓ justifies semi-Eulerian |

**2022 Section 1 Question 1**

**Graphs and networks**

The network below shows the different paths around a botanical exhibition.




(a) Explain why this is not a simple graph. (1 mark)

| Solution |
|---|
| This network is not a simple graph as it contains loops or multiple edges |
| **Specific behaviours** |
| ✓ states a correct reason |

(b) Is this network Eulerian or semi-Eulerian? Justify your response. (2 marks)

| Solution |
| --- |
| This network is semi-Eulerian because there are exactly two odd vertices (D and H have degree of 5). |
| **Specific behaviours** |
| ✓ states network is semi-Eulerian<br>✓ Identifies two odd vertices |

(c) Use Euler's formula to prove that this network is planar. (2 marks)

| Solution |
| --- |
| $f + v - e = 2$<br>$10 + 8 - 16 = 2$<br>$LHS = RHS$ $\therefore$ planar |
| **Specific behaviours** |
| ✓ states correct number of faces, vertices and edges<br>✓ correctly shows LHS = RHS |

(d) A visitor to the exhibition has taken her child in a pram and wishes to walk along every path exactly once to see all the floral displays, excluding the walk back to the car park. If the only car parks are located next to D and F, where would you suggest they park? Use mathematical reasoning to support your answer. (2 marks)

| Solution |
| --- |
| Network is semi-Eulerian. Suggestion would be to park near start vertex D<br>$\therefore$ car park near D. |
| **Specific behaviours** |
| ✓ states car park D<br>✓ provides valid reason |

**2021 Section 1 Question 2**

**Graphs and networks**

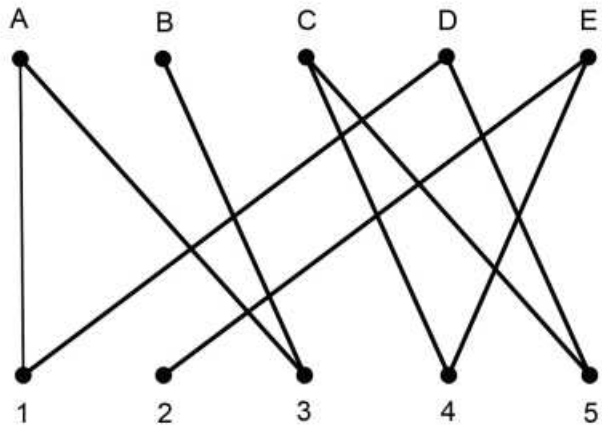
A construction company uses five different machines and has five employees who operate those machines.

The adjacency matrix below shows each of the five employees (A, B, C, D, E) and the five machines they are trained to operate. These are the only machines they may use.

| Machine | Employee A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 1 | 0 |
| 2 | 0 | 0 | 0 | 0 | 1 |
| 3 | 1 | 1 | 0 | 0 | 0 |
| 4 | 0 | 0 | 1 | 0 | 1 |
| 5 | 0 | 0 | 1 | 1 | 0 |

(a) Draw the adjacency matrix as a bipartite graph. The A1 connection has already been drawn on the graph. (2 marks)

**Solution**




**Specific behaviours**

✓ correctly allocates at least 6 connections
✓ correctly allocates all connections

(b) Does the bipartite graph in part (a) represent

(i) a planar graph? (1 mark)

**Solution**

Yes

**Specific behaviours**

✓ correct answer

(ii) a connected graph? (1 mark)

**Solution**

Yes

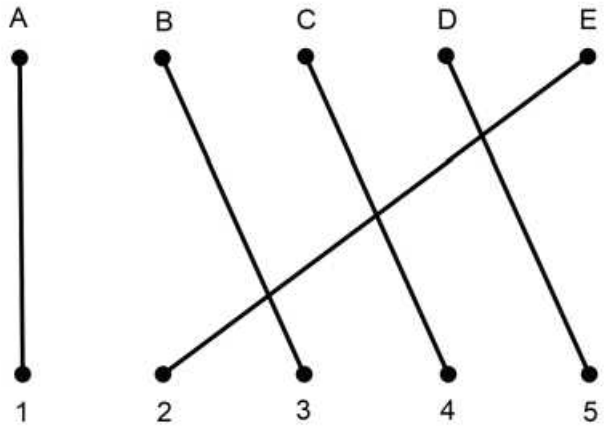
**Specific behaviours**

✓ correct answer

(c) Explain why the bipartite graph in part (a) is a simple graph. (2 marks)

| Solution |
|---|
| A simple graph has no loops or multiple edges. |
| **Specific behaviours** |
| ✓ correctly states there are no loops<br>✓ correctly states there are no multiple edges |

(d) Complete the table below to show how an allocation of all employees to a machine could be achieved. (2 marks)




| Employee | A | B | C | D | E |
|---|---|---|---|---|---|
| Machine | 1 | 3 | 4 | 5 | 2 |

| Solution |
|---|
| See table above (or graph) |
| **Specific behaviours** |
| ✓ states at least 3 correct allocations<br>✓ all correct allocations |

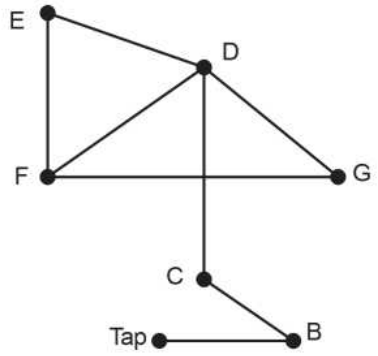
(e) The company foreman decides to allocate Employee E to Machine 4. Explain what effect this will have to the allocation of the remaining employees to the machines. (2 marks)

| Solution |
|---|
| $E4 \Rightarrow C5 \Rightarrow D1 \Rightarrow A3 \quad \Rightarrow B$ cannot connect to Machine 2<br>**or**<br>$E4 \Rightarrow C5 \Rightarrow D1 \Rightarrow B3 \quad \Rightarrow A$ cannot connect to Machine 2<br>**or**<br>$E4 \Rightarrow C5 \Rightarrow A1 \Rightarrow B3 \quad \Rightarrow D$ cannot connect to Machine 2<br>**or**<br>$E4 \Rightarrow D5 \Rightarrow A1 \Rightarrow B3 \quad \Rightarrow C$ cannot connect to Machine 2 |
| **Specific behaviours** |
| ✓ correctly states that one employee will always not be allocated<br>✓ correctly states it is not possible to allocate Machine 2 to an employee |

**2021 Section 1 Question 3**

**Graphs and networks**

The graph below shows the current network of reticulation pipes in Tarik's garden.




(a) Using Euler's formula, stating the number of vertices, edges and faces, show that the graph is planar. (2 marks)

| Solution |
|---|
| $f = 3$, $v = 7$ and $e = 8$<br>Using Euler's formula $v + f - e = 2$<br>L.H.S = $7 + 3 - 8 = 2$<br>Therefore, the graph is planar. |
| **Specific behaviours** |
| ✓ correctly states the number of vertices and faces<br>✓ correctly states the number of edges and verifies Euler's formula |

The water needs to travel from the tap and through all pipes.

(b) List a possible route for the water. (1 mark)

| Solution |
|---|
| Possibilities:<br>Tap-B-C-D-G-F-D-E-F<br>Tap-B-C-D-G-F-E-D-F<br>Tap-B-C-D-F-E-D-G-F<br>Tap-B-C-D-E-F-D-G-F<br>Tap-B-C-D-F-G-D-E-F<br>Tap-B-C-D-E-F-G-D-F |
| **Specific behaviours** |
| ✓ correctly states a possible route |

(c) What is the mathematical term for the route listed in part (b)? (1 mark)

| Solution |
|---|
| semi-Eulerian trail |
| **Specific behaviours** |
| ✓ states correct term |

(d) Tarik would like to increase the water pressure by removing one edge (pipe).

(i) Identify any edge that **cannot** be removed. (1 mark)

| Solution |
|---|
| He cannot remove:<br>Tap-B **or**<br>B-C **or**<br>C-D |
| **Specific behaviours** |
| ✓ states a correct edge |

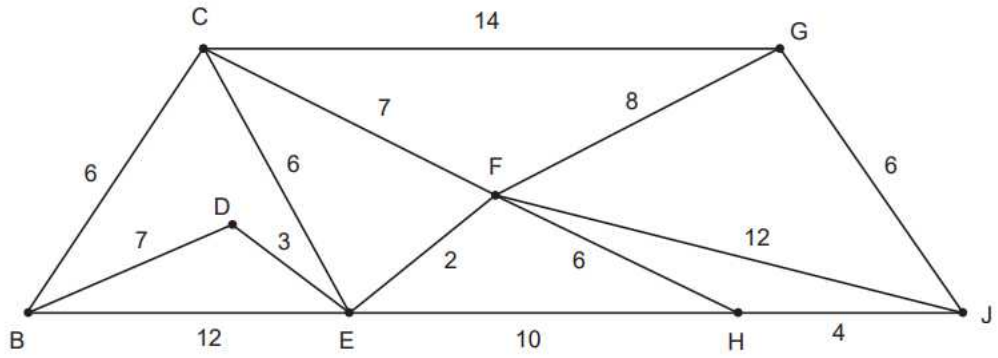
(ii) What is the name given to the type of edge identified in part (d)(i)? (1 mark)

| Solution |
|---|
| A bridge |
| **Specific behaviours** |
| ✓ states correct name |

**2021**
**Section 1**
**Question 5**

**Graphs and networks**

The network below shows the relative distances, in hundreds of metres, between wi-fi hotspots around a university campus.




The control room for this system of hotspots is located at B. A problem has been reported with the hotspot at J.

(a) A technician is sent from the control room to solve the problem at J. To get there as quickly as possible, she wants to use the shortest path, travelling from hotspot to hotspot. Determine the required path and its length. (3 marks)

| Solution |
|---|
| BDEFHJ $\Rightarrow$ 22<br>$\therefore$ 2200 metres |
| **Specific behaviours** |
| ✓ identifies correct path<br>✓ path = 22<br>✓ path length 2200 metres |

(b) A second technician is sent from the control room to J to help resolve the problem. The technician uses an open path travelling from hotspot to hotspot. If the technician travels 2700 metres, determine the path he used. (1 mark)

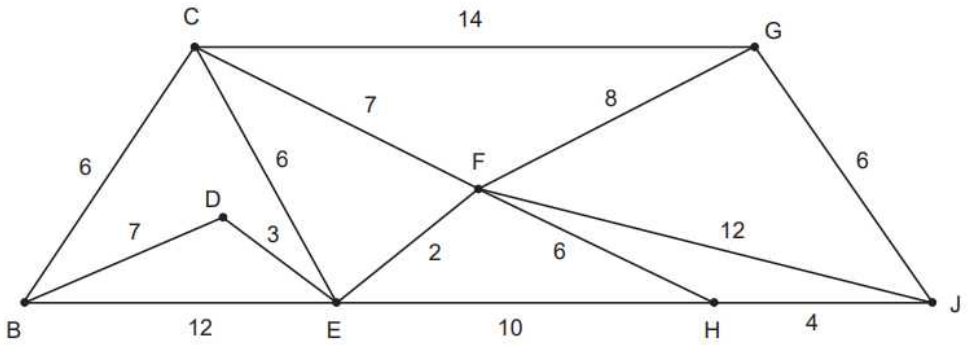
| Solution |
|---|
| BCFGJ |
| **Specific behaviours** |
| ✓ identifies correct path |

(c) After repairs have been made, all hotspots need to be checked. A technician is sent from the control room, travelling to all hotspots once only, and finishing back at the control room.

(i) State the name given to this type of path. (2 marks)

| Solution |
|---|
| Hamiltonian cycle |
| **Specific behaviours** |
| ✓ correctly states it is a cycle<br>✓ correctly states it is Hamiltonian |

(ii) Determine the length of the shortest path possible and state the path used. (2 marks)

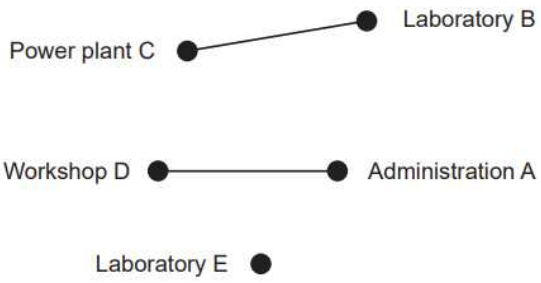



| Solution |
|---|
| BCGJHFEDB $\Rightarrow$ 4800 metres |
| **Specific behaviours** |
| ✓ correctly gives the shortest closed path<br>✓ states correct length |

**2020 Section 1 Question 2**

**Graphs and networks**

A small research facility consists of five buildings with walkways represented by the edges in this network:




(a) Determine the smallest number of edges (walkways) to be added to ensure that the network is

(i) connected (1 mark)

| Solution |
|---|
| 2 edges |
| **Specific behaviours** |
| ✓ states correct number |

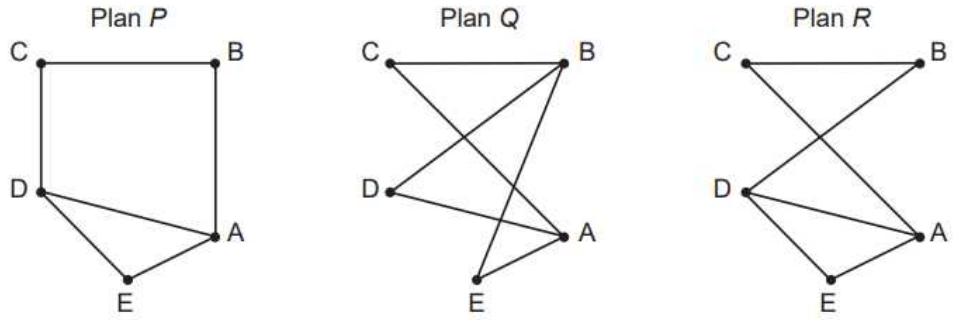
(ii) complete (1 mark)

| Solution |
|---|
| 8 edges |
| **Specific behaviours** |
| ✓ states correct number |

(iii) planar with 4 regions. (2 marks)

| Solution | | |
|---|---|---|
| If connected,<br>$v+f-e=2$<br>$5+4-e=2$<br>$e=7$<br>Therefore, 5 edges are required. | **or** | If not connected,<br>Euler's formula does not hold.<br>Therefore, 3 edges are required. |
| **Specific behaviours** | | |
| ✓ determines total number of edges required<br>✓ states correct number of additional edges required | | |

Three different plans for completing the network with the addition of four walkways are shown:




(b) State which, if any, of these plans has a graph that is

(i) bipartite (1 mark)

| Solution |
|---|
| Plan *Q* |
| **Specific behaviours** |
| ✓ states correct plan |

(ii) Eulerian (1 mark)

| Solution |
|---|
| None |
| **Specific behaviours** |
| ✓ states correct answer |

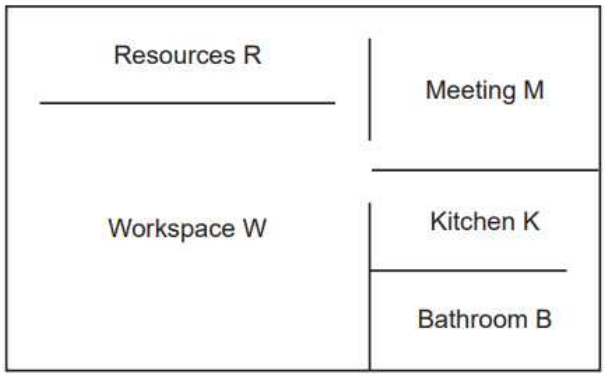
(iii) a Hamiltonian cycle. (2 marks)

| Solution |
|---|
| Plan *P* and Plan *R* |
| **Specific behaviours** |
| ✓ states Plan *P*<br>✓ states Plan *R* |

**2020 Section 1 Question 6**

**Graphs and networks**

A small business office has five separate areas connected by doorways shown as gaps in this diagram:




This adjacency matrix below represents the number of doorways directly between each area:

| | B | K | M | R | W |
|---|---|---|---|---|---|
| B | 0 | 1 | 0 | 0 | 0 |
| K | 1 | 0 | 0 | 0 | 1 |
| M | 0 | 0 | 0 | 1 | 1 |
| R | 0 | 0 | 1 | 0 | $Y$ |
| W | 0 | $X$ | 1 | 2 | 0 |

(a) State the meaning of the zero entries in the matrix. (1 mark)

| **Solution** |
|---|
| There is no doorway between the area(s) |
| **Specific behaviours** |
| ✓ correctly states that there is no doorway |

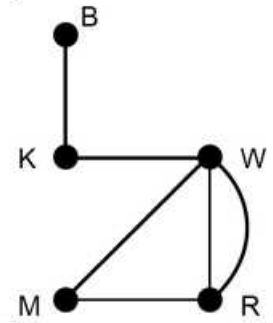
(b) Determine the value of X and Y. (2 marks)

| **Solution** |
|---|
| $X = 1$<br>$Y = 2$ |
| **Specific behaviours** |
| ✓ states correct value of $X$<br>✓ states correct value of $Y$ |

(c) Describe how the total number of doorways for each area can be found from the adjacency matrix. (1 mark)

| **Solution** |
|---|
| Sum of the row or column corresponding to the specific area |
| **Specific behaviours** |
| ✓ correctly identifies a row or column sum |

(d) Complete this network with vertices corresponding to the office areas and the edges representing the doorways. (3 marks)

| Solution |
|---|
|   |
| **Specific behaviours** |
| ✓ correctly adds **B** connected to **K**<br>✓ correctly connects **W** to both **K** and **M**<br>✓ correctly connects **W** to **R** with a second edge |

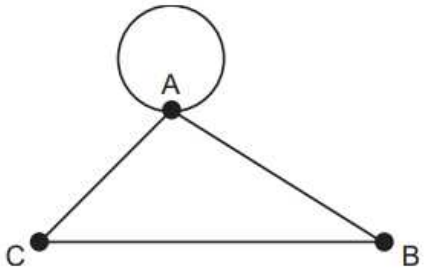
(e) Determine how many different routes there are between the meeting room and the workspace that pass through exactly two doorways. (1 mark)

| Solution |
|---|
| Two (two via **R**) |
| **Specific behaviours** |
| ✓ states correct answer |

**2019 Section 1 Question 1**

**Graphs and networks**

The graph shown represents three buildings A, B and C, with connecting walkways, at a local school.




(a) Why is the graph planar? (1 mark)

| Solution |
|---|
| No two edges cross |
| **Specific behaviours** |
| ✓ states correct reason |

(b) Show that the graph satisfies Euler's formula. (2 marks)

| Solution |
|---|
| $v = 3$, $e = 4$, $f = 3$. $3 + 3 - 4 = 2$, verified. |
| **Specific behaviours** |
| ✓ gives correct values for the number of vertices, edges and faces<br>✓ correctly verifies Euler's formula |

(c) Construct the adjacency matrix for the graph. (3 marks)

| Solution |
|---|
| $\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$ |
| **Specific behaviours** |
| ✓ gives correct entry for $A$ to $A$<br>✓ gives the remainder of correct entries<br>✓ correctly labels rows and columns |

A student wishes to carry out closed walks of length two from Building A.

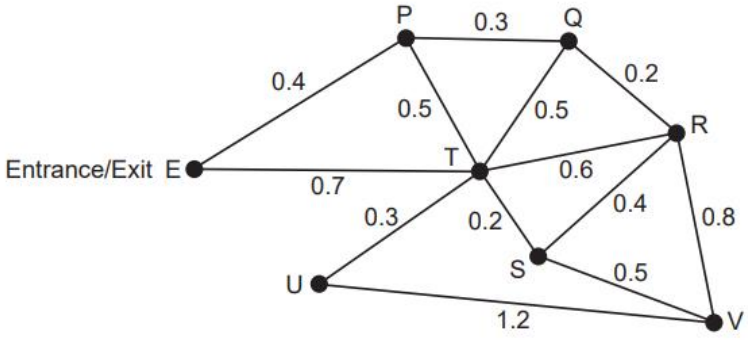
(d) List all his possible walks. (2 marks)

| Solution |
|---|
| A-B-A<br>A-C-A<br>A-A-A<br><br>A total of 3 walks |
| **Specific behaviours** |
| ✓ lists at least two walks<br>✓ lists all 3 walks |

**2019 Section 1 Question 4**

**Graphs and networks**

A marine park has attractions with paths connecting them. The vertices on the graph represent the attractions and the numbers on the edges represent the path distances (km) between the attractions. Visitors can either walk around the park or take one of the many shuttle buses that run between attractions.




The manager of the marine park leaves his office, which is located at the entrance/exit (E) and walks to attraction V.

(a) (i) Determine the shortest distance from E to V. (1 mark)

| Solution |
|---|
| 1.4 km |
| **Specific behaviours** |
| ✓ correctly determines shortest distance |

(ii) If the manager needs to pick up some tools left at U on the way, determine the route he should take and the corresponding distance, given he wants to take the shortest route from E to V. (2 marks)

| Solution |
| --- |
| ETUTSV<br>Total = 2 km |
| **Specific behaviours** |
| ✓ correctly states shortest route<br>✓ correctly determines shortest distance |

Rachel arrives at the entrance. She wants to complete a Hamiltonian cycle.

(b) State the route she should take. (2 marks)

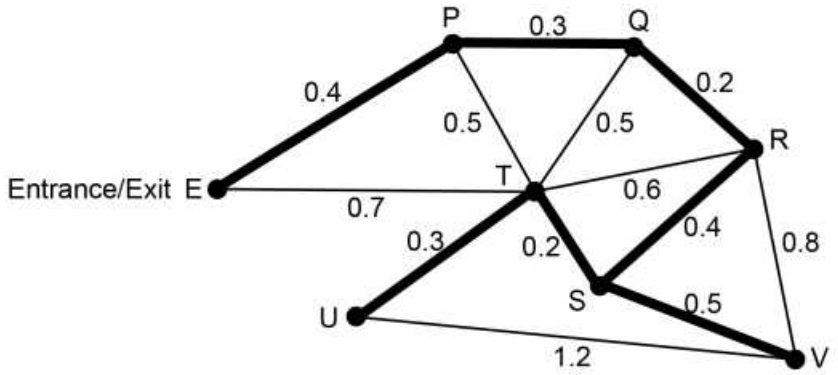
| Solution |
| --- |
| EPQRSVUTE |
| **Specific behaviours** |
| ✓ states a path containing all vertices<br>✓ states the correct Hamiltonian cycle (starting and finishing at the entrance/exit) |

Drinking water is already being supplied at E. The manager has recently received funding to establish drinking fountains at each attraction. For this to happen, pipelines will need to be laid along the paths to each attraction. He has drawn up a table to show the distances between attractions.

| | E | P | Q | R | S | T | U | V |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| E | – | 0.4 | – | – | – | 0.7 | – | – |
| P | 0.4 | – | 0.3 | – | – | 0.5 | – | – |
| Q | – | 0.3 | – | 0.2 | – | 0.5 | – | – |
| R | – | – | 0.2 | – | 0.4 | 0.6 | – | 0.8 |
| S | – | – | – | 0.4 | – | 0.2 | – | 0.5 |
| T | 0.7 | 0.5 | 0.5 | 0.6 | 0.2 | – | 0.3 | – |
| U | – | – | – | – | – | 0.3 | – | 1.2 |
| V | – | – | – | 0.8 | 0.5 | – | 1.2 | – |

(c) (i) Use Prim's algorithm, or otherwise, to determine the minimum total length of pipelines. Highlight the required pipelines on the diagram below. (4 marks)

| Solution |
|---|
| EP(0.4), PQ(0.3), QR(0.2), RS(0.4), VS(0.5), ST(0.2), UT(0.3)<br><br>The minimum length is 2.3 km. |
| **Specific behaviours** |
| ✓ gives at least 4 correct connections<br>✓ gives all correct connections<br>✓ correctly states minimum length |

| Solution |
|---|
|  |
| **Specific behaviours** |
| ✓ highlights correct spanning tree |

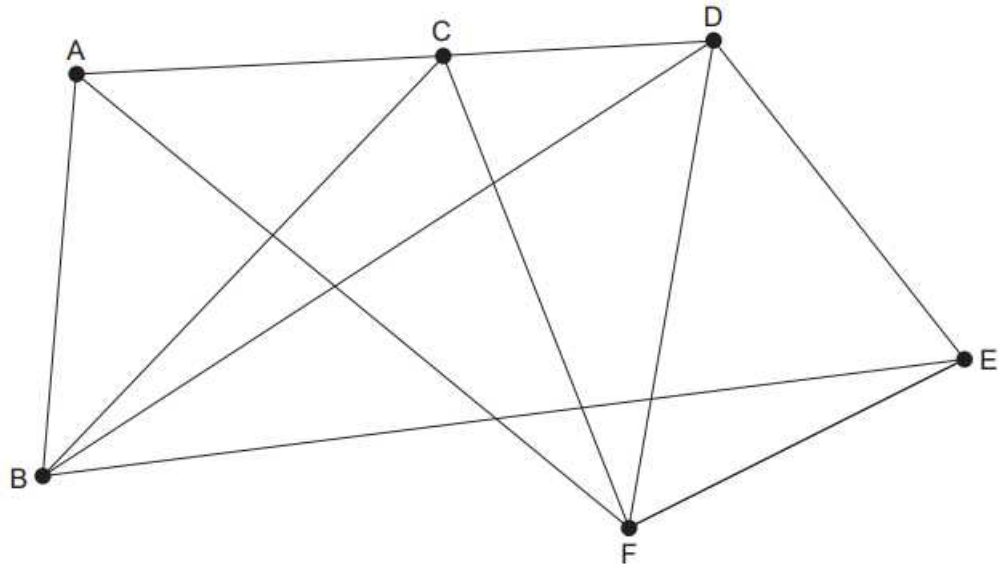
(ii) The manager has been told that a pipeline of length 0.2 km could be laid from S to U. How, if at all, will this affect the total length of pipelines that should be laid in order to maintain a minimum length? (2 marks)

| Solution |
|---|
| The minimum length will decrease by 0.1 km (as SU would be used instead of TU). |
| **Specific behaviours** |
| ✓ states it is a decrease<br>✓ gives the decrease as 0.1 km |

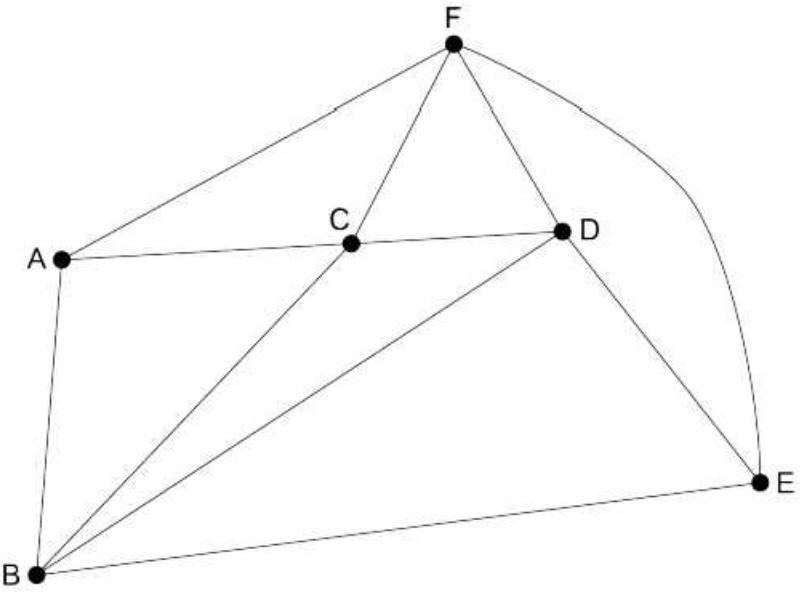
**2023 Section 2 Question 12**

**Graphs and networks**

An event organiser is setting up a music festival with multiple stages. The graph below shows the stages as vertices, and the edges are the walkways that connect each stage.




(a) To avoid congestion, the organisers want to move the stages and re-position some connecting walkways so no walkways intersect. Redraw the graph to meet these conditions. (2 marks)

| Solution |
|---|
|  |
| **Specific behaviours** |
| ✓ draws the graph with no edges crossing<br>✓ draws the graph with all edges maintained |
| Accept other relevant answers. |

(b) Use Euler's formula, stating the number of vertices, edges and faces, to verify that this graph is planar. (2 marks)

| Solution |
| --- |
| 7 faces, 6 vertices and 11 edges<br>$v+f-e=6+7-11=2$ which verifies Euler's formula. |
| **Specific behaviours** |
| ✓ correctly states the number of vertices, edges and faces<br>✓ verifies Euler's formula |

(c) State a reason why the graph is not Eulerian. (1 mark)

| Solution |
| --- |
| it contains odd vertices |
| **Specific behaviours** |
| ✓ states that the graph contains odd vertices |

One attendee wishes to visit every stage to make the most of the festival.

(d) (i) Write down a pathway the attendee can take to visit every stage exactly once if they start at Stage F. (1 mark)

| Solution |
| --- |
| FACDEB or FEBACD or FEDCAB |
| **Specific behaviours** |
| ✓ states a correct pathway |
| Accept other relevant answers. |

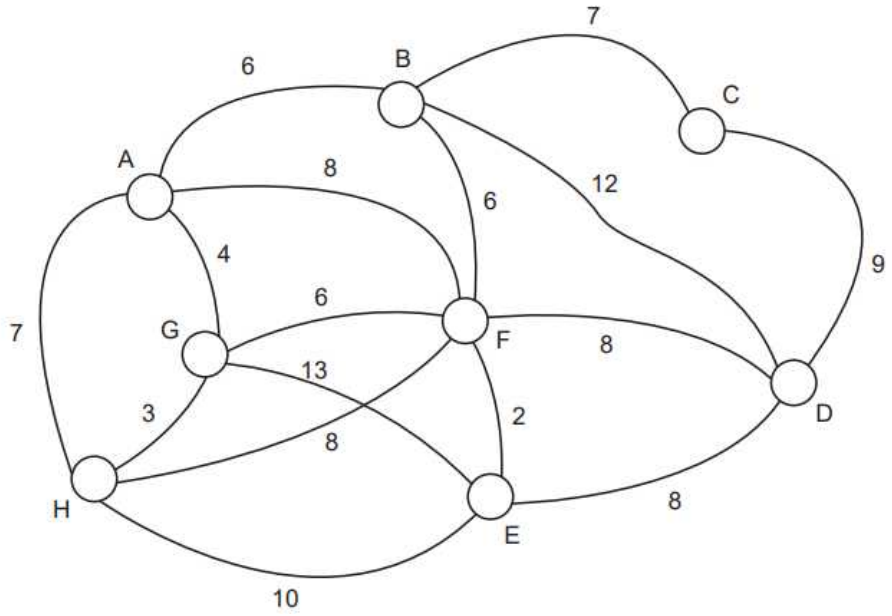
(ii) What is the name given to this path? (1 mark)

| Solution |
| --- |
| Hamiltonian |
| **Specific behaviours** |
| ✓ states correct name |

**2022 Section 2 Question 10**

**Graphs and networks**

The network shown below represents the cycle tracks connecting the various points of interest on an island. The numbers indicate the distance between points of interest to the nearest 100 metres.




(a) Each morning all tracks must have a safety check. Is it possible to check all individual tracks by an Eulerian trail? Justify your answer. (2 marks)

| Solution |
|---|
| Yes, it is possible to check all individual tracks by an Eulerian trail as the network has 0 odd vertices |
| **Specific behaviours** |
| ✓ states yes it is possible<br>✓ identifies network has 0 odd vertices |

(b) Rubbish bins at each point of interest are emptied daily. The rubbish truck leaves from position C and must return to C after emptying all rubbish bins.

(i) What name is given to the type of path taken by the rubbish truck? (2 marks)

| Solution |
|---|
| Hamiltonian cycle |
| **Specific behaviours** |
| ✓ correctly states Hamiltonian<br>✓ correctly states cycle |

(ii) The rubbish truck wants to follow the path of least distance. State the path and its length. (3 marks)

| Solution |
|---|
| CBAGHFEDC<br>7 + 6 + 4 + 3 + 8 + 2 + 8 + 9 = 47<br>4700 metres |
| **Specific behaviours** |
| ✓ path starts and ends at C and visits all vertices<br>✓ correct length<br>✓ correct distance |

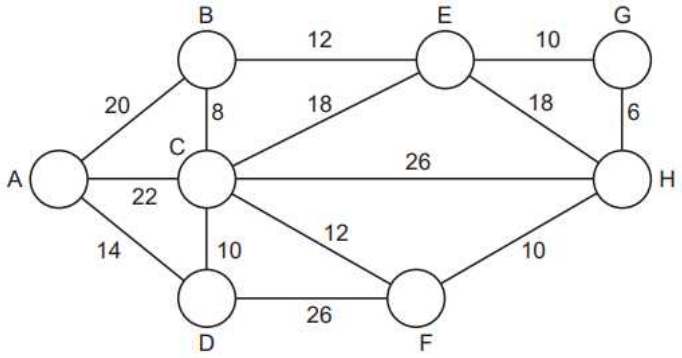
(c) A tourist at G wishes to travel to C but path AG is closed for maintenance. Determine the shortest path possible and its length. (2 marks)

| Solution |
|---|
| GFBC 1900 metres |
| **Specific behaviours** |
| ✓ correct path<br>✓ correct length |

**2020 Section 2 Question 13**

**Graphs and networks**

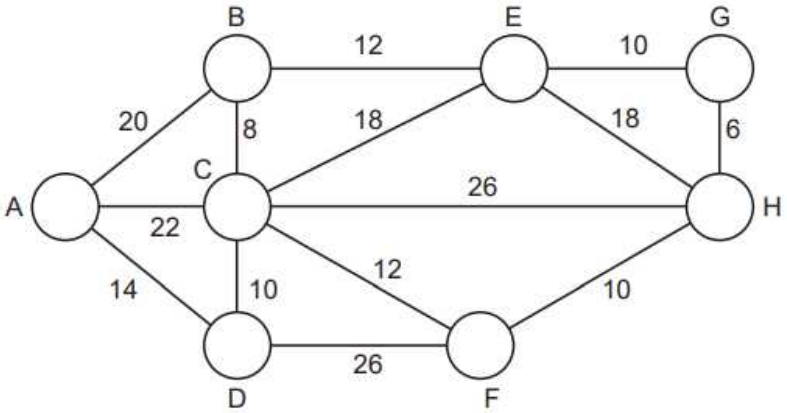
The graph below represents a road transport network from a warehouse at A to seven retail outlets B, C, D, E, F, G and H. The number on each edge represents the distance, in kilometres, along each road.




(a) Identify the shortest Hamiltonian path from the warehouse and state its length. (3 marks)

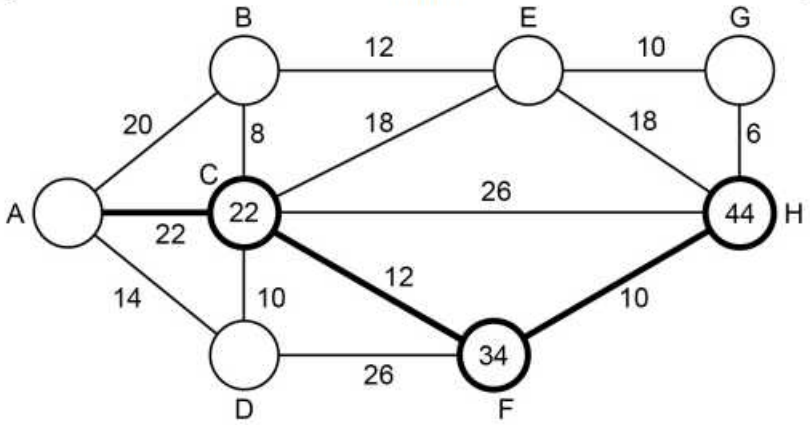
| Solution |
|---|
| ADCBEGHF<br><br>14 + 10 + 8 + 12 + 10 + 6 + 10 = 70 km |
| **Specific behaviours** |
| ✓ gives a Hamiltonian path to all vertices<br>✓ determines correct Hamiltonian path<br>✓ states correct length of path |

A special delivery must be made from the warehouse to retail outlet H.



(b) Determine the shortest path and the distance travelled for this delivery. Working **must** appear on the network to show an appropriate method has been used. (3 marks)

**Solution**

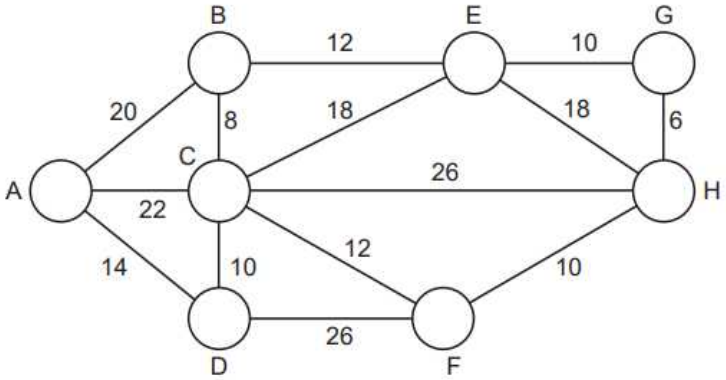



Shortest path is ACFH
Distance = 44 km

**Specific behaviours**

- ✓ shows correct working on the network
- ✓ states correct path
- ✓ states correct distance

Road CH presently goes around what is now a dry salt lake. It is proposed that a direct road be constructed that will reduce the distance between retail outlets C and H.




(c) By how much can the direct road between C and H be reduced, so that the shortest path from the warehouse to H includes the direct road CH? (3 marks)

**Solution**

$22+(26-x)<44 \quad \Rightarrow x>4$

*also* $\quad x<26$

$4<x<26$

**Specific behaviours**

- ✓ determines that the path ACH is less than 44
- ✓ correctly states that direct road needs to be reduced by more than 4
- ✓ correctly states that direct road needs to be reduced by less than 26

**2019 Section 2 Question 12**

**Graphs and networks**

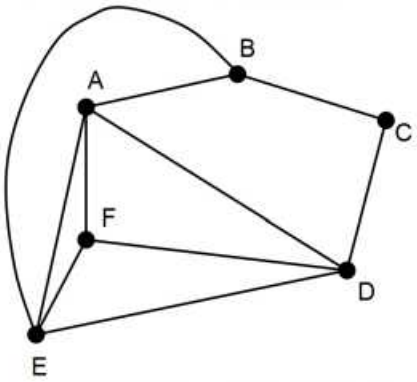
Jake, a park ranger, is giving a presentation at a National Park and Wildlife Conference on possible designs for a new park. Unfortunately, Jake made mathematical errors in his presentation about the paths (represented by edges) and shelter huts (represented by vertices) in the park.

(a) For each of the following statements, the graph drawn by Jake was incorrect. Redraw the graph to match the statement correctly.

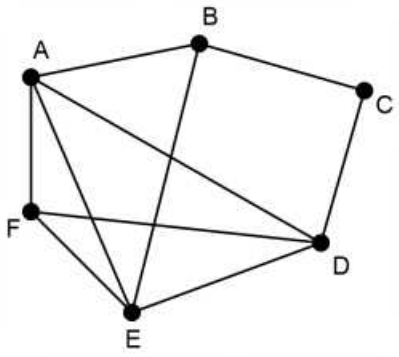
(i) This park plan has been drawn as a connected planar graph containing six vertices. (2 marks)

**Solution**

For example,




OR

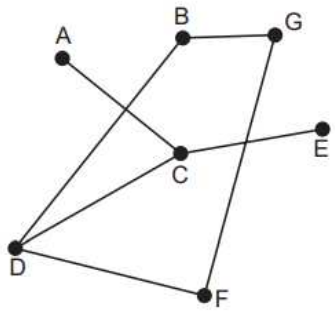


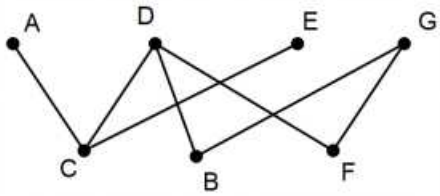

(the original shape is a connected planar graph)

**Specific behaviours**

✓ the graph is drawn with no two edges crossing
✓ the graph is drawn with all connections
OR
✓✓correctly draws or states that the original is a connected planar graph

(ii) This park plan has been drawn as a bipartite graph. (3 marks)




| Solution |
| --- |
|  |
| **Specific behaviours** |
| ✓ correctly arranges the groups<br>✓ correctly joins at least 4 vertices<br>✓ correctly joins all vertices |



Jake also makes the following incorrect statement in his presentation. 'A park plan can be a complete graph with 21 paths and six shelter huts'.

(b) If the plan must be a complete graph with 21 paths, how many shelter huts should Jake have quoted? (1 mark)

| Solution |
| --- |
| Correct number of huts is seven |
| **Specific behaviours** |
| ✓ states correct number of huts |

# Unit 4

## Unit 4.1 – Time series analysis

### Section 1

**2022 Section 1 Question 5**

**Time series analysis**

A stall at a local market sells leather handbags. Sales over the past three weeks are tabulated, along with other calculations.

| Week | Day | $t$ | Sales | 3 point moving average | Daily mean | Percentage of daily mean | Deseasonalised sales |
|---|---|---|---|---|---|---|---|
| 1 | Friday | 1 | 2 | – | 5 | 40 | 4 |
| | Saturday | 2 | 9 | 5 | | 180 | 6 |
| | Sunday | 3 | 4 | **5.33** | | 80 | 4 |
| 2 | Friday | 4 | 3 | 4.33 | 5 | 60 | 6 |
| | Saturday | 5 | 6 | 5 | | **120** | 4 |
| | Sunday | 6 | 6 | 5 | | 120 | 6 |
| 3 | Friday | 7 | 3 | 6 | **6** | 50 | |
| | Saturday | 8 | 9 | 6 | | 150 | 6 |
| | Sunday | 9 | 6 | – | | 100 | 6 |

(a) Show clearly how each of the numbers in bold type was calculated. (3 marks)

**5.33**

**120**

**6**

(b) Determine the seasonal index for Friday and explain its meaning. (2 marks)

(c) (i) Give a reason why time series data are deseasonalised. (1 mark)

(ii) Calculate the deseasonalised value for Friday of Week 3. (2 marks)

The equation of the least-squares line for the deseasonalised sales, based on time t, is $y = 0.2t + 4.3$.

(d) How does this equation support the observation that sales are increasing? (1 mark)

(e) Predict the sales for Friday of Week 4. (3 marks)

## Section 2

**2023 Section 2 Question 8**

**Time series analysis**

The table below shows the average price per kilogram of watermelon in a supermarket for each of the four seasons of the year over a four-year period.

| Year | Quarter | Time ($t$) | Average price ($) | 4-point centred moving average | Quarterly mean | Average price as a percentage of the quarterly mean | Deseasonalised data |
|---|---|---|---|---|---|---|---|
| 2019 | Summer | 1 | 1.06 | - | 1.85 | 57.30 | 1.80 |
| | Autumn | 2 | 1.36 | - | | 73.51 | 1.79 |
| | Winter | 3 | 2.94 | 1.87 | | 158.92 | 1.89 |
| | Spring | 4 | 2.03 | 1.92 | | 109.73 | 1.86 |
| 2020 | Summer | 5 | 1.24 | 1.97 | **P** | 59.05 | 2.11 |
| | Autumn | 6 | 1.58 | 2.04 | | 75.24 | 2.07 |
| | Winter | 7 | 3.12 | 2.12 | | 148.57 | 2.01 |
| | Spring | 8 | 2.44 | 2.20 | | 116.19 | 2.23 |
| 2021 | Summer | 9 | 1.47 | 2.29 | 2.39 | 61.51 | 2.50 |
| | Autumn | 10 | 1.93 | 2.36 | | 80.75 | 2.53 |
| | Winter | 11 | 3.48 | 2.41 | | 145.61 | 2.23 |
| | Spring | 12 | 2.69 | 2.44 | | 112.55 | 2.46 |
| 2022 | Summer | 13 | **Q** | **R** | 2.75 | 57.09 | 2.69 |
| | Autumn | 14 | 2.07 | 2.75 | | **S** | 2.72 |
| | Winter | 15 | 4.65 | - | | 169.09 | 2.99 |
| | Spring | 16 | 2.71 | - | | 98.55 | 2.48 |

(a) Determine the values **P**, **Q**, **R** and **S** in the table above. (4 marks)

(b) Complete the following table. (1 mark)

| Season | Summer | Autumn | Winter | Spring |
|---|---|---|---|---|
| **Seasonal index** | 58.83% | 76.19% | | 109.26% |

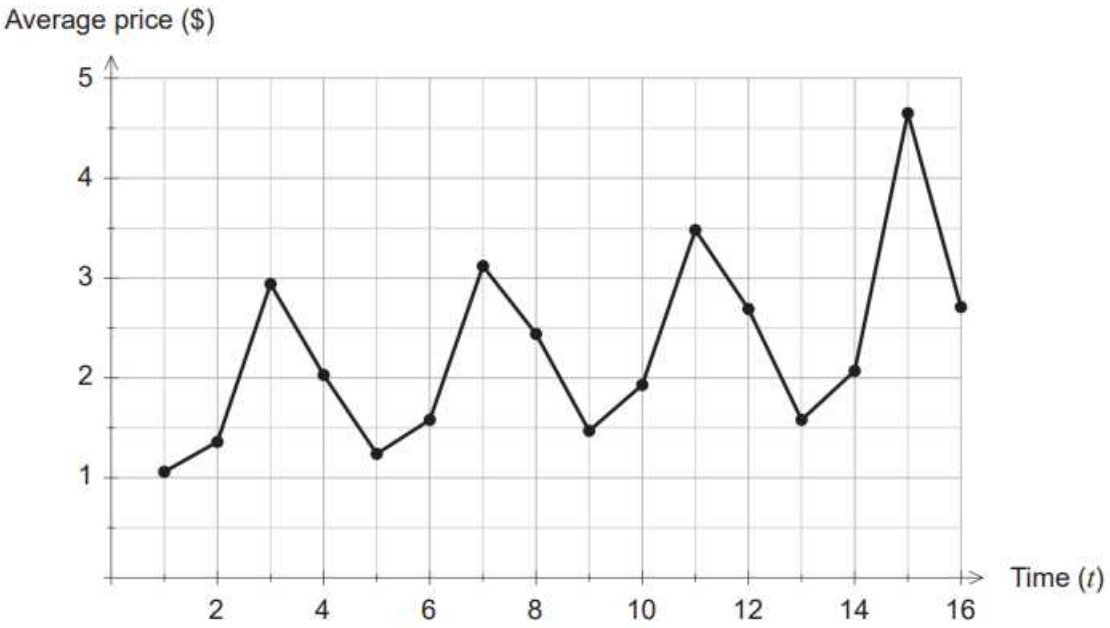
(c) Interpret the seasonal index for Summer in the context of this question. (1 mark)

(d) Define 'deseasonalised data'. (1 mark)

(e) Using information from the tables on page 6, show how the deseasonalised data value for Autumn 2021 was calculated. (1 mark)

(f) Determine the equation of the least-squares line using the deseasonalised data. (1 mark)

(g) The graph below is the time series plot for these data. On this graph draw the least-squares line determined in part (f). (2 marks)

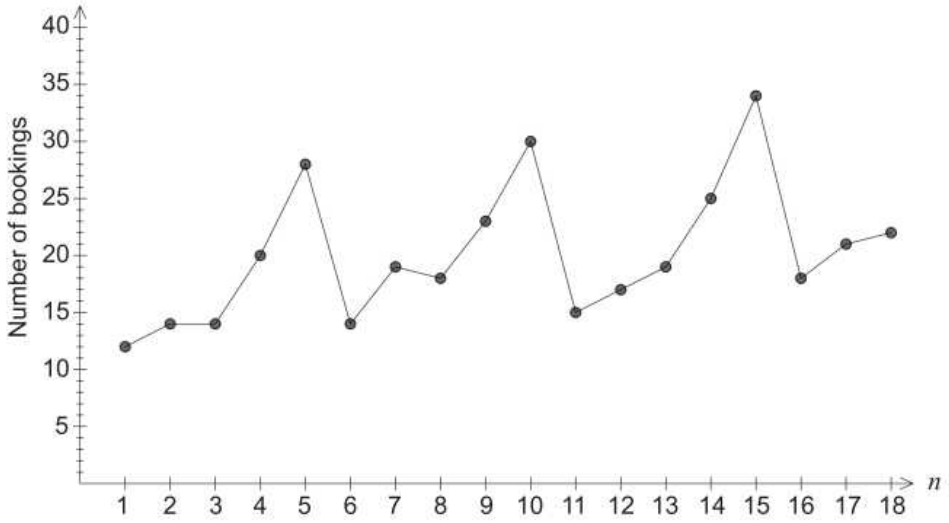



(h) Predict the average price per kilogram of watermelon from the supermarket in Spring 2024. (3 marks)

(i) Discuss the reliability of the prediction in part (h). (2 marks)

| | |
|---|---|
| **2022**<br>**Section 2**<br>**Question 8**<br><br>**Time series analysis** | Yash is saving for a deposit on an apartment, which is about to begin construction.<br><br>(a) Two saving plans are found in advertisements in a newspaper.<br>Option 1: 3.05% per annum compounded at the end of each month.<br>Option 2: 3.01% per annum compounded at the end of each day.<br><br>Which of these two options would give the better return? Justify your answer mathematically. (3 marks) |

| | |
|---|---|
| | Yash locates a better investment option online. This offers 3.5% per annum compounded at the end of each day. Yash will begin with an initial deposit of $12 300 and a weekly deposit of $300. The deposit required to secure the apartment in 12 months is $30 000.<br><br>(b) (i) Show that this option will not be sufficient to reach the total needed to secure the apartment. State how far short of the required amount this option will be. (4 marks)<br><br>(ii) Determine what the weekly deposit needs to be for a total of $30 000 to be achieved by the end of 12 months. (2 marks) |

| | |
|---|---|
| **2021**<br>**Section 2**<br>**Question 8**<br><br>**Time series analysis** | The graph below shows the number of bookings at a dog grooming salon over its first few weeks of business.<br><br>  <br><br><br>(a) Which simple moving average would be the **most** suitable for the data displayed in this graph? (1 mark) |

A more detailed view of the same data is given in the table below.

| Week | Day | $n$ | Number of bookings | Seasonal mean | Number of bookings as a percentage of the seasonal mean | Seasonally adjusted figures |
|---|---|---|---|---|---|---|
| 1 | Tuesday | 1 | 12 | 17.6 | **A** | 17.7 |
| | Wednesday | 2 | 14 | | 79.55 | 16.9 |
| | Thursday | 3 | 14 | | 79.55 | 16.6 |
| | Friday | 4 | 20 | | 113.64 | 17.8 |
| | Saturday | 5 | 28 | | 159.09 | 18.3 |
| 2 | Tuesday | 6 | 14 | **B** | 67.31 | 20.6 |
| | Wednesday | 7 | 19 | | 91.35 | 23.0 |
| | Thursday | 8 | 18 | | 86.54 | 21.4 |
| | Friday | 9 | 23 | | 110.58 | 20.4 |
| | Saturday | 10 | 30 | | 144.23 | 19.7 |
| 3 | Tuesday | 11 | **C** | 22 | 68.18 | 22.1 |
| | Wednesday | 12 | 17 | | 77.27 | 20.6 |
| | Thursday | 13 | 19 | | 86.36 | 22.6 |
| | Friday | 14 | 25 | | 113.64 | 22.2 |
| | Saturday | 15 | 34 | | 154.55 | 22.3 |
| 4 | Tuesday | 16 | 18 | – | – | – |
| | Wednesday | 17 | 21 | | – | – |
| | Thursday | 18 | 22 | | – | – |

(b) Calculate the value of **A**, **B** and **C** in the table. (3 marks)

(c) Calculate the seasonal index for Saturday. (1 mark)

(d) The equation of the least-squares line using the seasonally adjusted figures is $y = 0.40n + 16.94$. Draw this line on the graph. (2 marks)

(e) (i) Use the equation of the least-squares line given in part (d) to predict the number of bookings that will be made for the Saturday of Week 5. (2 marks)

(ii) Comment on this prediction. (2 marks)

**2020 Section 2 Question 14**

**Time series analysis**

The table below shows the number of sprinkler systems installed by a local reticulation business over the past four years.

| Year | Season | $n$ | Number of systems | Seasonal mean | Number of systems as a percentage of the seasonal mean | Seasonally adjusted figures |
|---|---|---|---|---|---|---|
| 2017 | Summer | 1 | **A** | 14 | 71.4 | 10.4 |
| | Autumn | 2 | 18 | | **B** | 15.7 |
| | Winter | 3 | 11 | | 78.6 | 14.7 |
| | Spring | 4 | 17 | | 121.4 | 14.7 |
| 2018 | Summer | 5 | 15 | **C** | 105.3 | 15.7 |
| | Autumn | 6 | 16 | | 112.3 | 14.0 |
| | Winter | 7 | 11 | | 77.2 | 14.7 |
| | Spring | 8 | 15 | | 105.3 | 13.0 |
| 2019 | Summer | 9 | 13 | 11.75 | 110.6 | 13.6 |
| | Autumn | 10 | 12 | | 102.1 | 10.5 |
| | Winter | 11 | 8 | | 68.1 | 10.7 |
| | Spring | 12 | 14 | | 119.1 | 12.1 |
| 2020 | Summer | 13 | 16 | — | — | — |
| | Autumn | 14 | 15 | | — | — |

(a) Calculate the value of **A**, **B** and **C**. (3 marks)

(b) Complete the table showing the seasonal index for each season. (2 marks)

| **Season** | Summer | Autumn | Winter | Spring |
|---|---|---|---|---|
| **Seasonal index** | 95.8 | 114.3 | | |

(c) Show how the seasonally adjusted figure of 13.6 for Summer 2019 was calculated. (2 marks)

(d) During which season could more employees be given annual holidays with least disruption to sprinkler installations? Use mathematical evidence to support your answer. (2 marks)

(e) Determine the least-squares line using the seasonally adjusted figures. (1 mark)

(f) Using your line from part (e), estimate the number of sprinkler systems that will be installed in Summer 2021. (2 marks)

(g) Comment on the long-term prospects of the business. (2 marks)

**2019 Section 2 Question 11**

**Time series analysis**

Data for the total occupancy of rooms for each season of the year at a Perth hotel is shown below.

| $n$ | Year | Season | Total rooms occupied | Seasonal mean | 4-point centred moving average | Total rooms occupied as a percent of seasonal mean |
|---|---|---|---|---|---|---|
| 1 | 2015/16 | Spring | 1770 | 1660.5 | | 106.59 |
| 2 | | Summer | 1904 | | | **B** |
| 3 | | Autumn | 1591 | | 1644.375 | 95.81 |
| 4 | | Winter | 1377 | | 1622.5 | 82.93 |
| 5 | 2016/17 | Spring | 1641 | 1610.25 | 1618 | 101.91 |
| 6 | | Summer | 1858 | | 1614.75 | 115.39 |
| 7 | | Autumn | 1601 | | 1602.25 | 99.43 |
| 8 | | Winter | 1341 | | 1584.75 | 83.28 |
| 9 | 2017/18 | Spring | 1577 | 1524.0 | 1558 | 103.48 |
| 10 | | Summer | **A** | | 1532.375 | 116.93 |
| 11 | | Autumn | 1463 | | 1526.875 | 96.00 |
| 12 | | Winter | 1274 | | 1525.125 | 83.60 |
| 13 | 2018/19 | Spring | 1600 | 1519.75 | **C** | 105.28 |
| 14 | | Summer | 1745 | | 1525.25 | 114.82 |
| 15 | | Autumn | 1504 | | | 98.96 |
| 16 | | Winter | 1230 | | | 80.93 |

(a) Calculate the value of **A**, **B** and **C**. (3 marks)

(b) Complete the table showing the seasonal index for each season. (1 mark)

| Summer | Autumn | Winter | Spring |
|---|---|---|---|
| 1.1545 | | 0.8268 | 1.0432 |

(c) Calculate the deseasonalised value for Winter 2017/18. (2 marks)

(d) Comment on the effect the seasonal index had on the value found in part (c). (1 mark)

(e) The least-squares line using deseasonalised data is $R = -12.071_n + 1681.25$. Use this line to predict the total number of rooms occupied during Spring 2020/21. (2 marks)

When a prediction was made for Spring 2020/21, using the least-squares line based on the 4-point centred moving averages, the answer was 1481.

(f) Explain why this is different from the answer obtained in part (e). (1 mark)

The manager of the hotel attended a meeting with the owners of the hotel. She explained to the owners that the reduction in occupancy was due to the downturn in the Western Australian economy in recent years.

(g) Comment on the statement made by the hotel manager. (2 marks)

(h) What practical advice, in the context of the question, would you give to the manager of the hotel? (1 mark)

**2022 Section 1 Question 5**

**Time series analysis**

A stall at a local market sells leather handbags. Sales over the past three weeks are tabulated, along with other calculations.

| Week | Day | $t$ | Sales | 3 point moving average | Daily mean | Percentage of daily mean | Deseasonalised sales |
|---|---|---|---|---|---|---|---|
| 1 | Friday | 1 | 2 | – | 5 | 40 | 4 |
| | Saturday | 2 | 9 | 5 | | 180 | 6 |
| | Sunday | 3 | 4 | **5.33** | | 80 | 4 |
| 2 | Friday | 4 | 3 | 4.33 | 5 | 60 | 6 |
| | Saturday | 5 | 6 | 5 | | **120** | 4 |
| | Sunday | 6 | 6 | 5 | | 120 | 6 |
| 3 | Friday | 7 | 3 | 6 | **6** | 50 | |
| | Saturday | 8 | 9 | 6 | | 150 | 6 |
| | Sunday | 9 | 6 | – | | 100 | 6 |

(a) Show clearly how each of the numbers in bold type was calculated. (3 marks)

| Solution |
|---|
| **5.33:** $\frac{9+4+3}{3}=\frac{16}{3}=5.33$<br>**120:** $\frac{6}{5}\times 100=120$<br>**6:** $\frac{3+9+6}{3}=\frac{18}{3}=6$ |
| **Specific behaviours** |
| ✓ correctly shows addition $9+4+3$ and division by 3<br>✓ correctly shows $6\div 5$ and conversion to a percentage<br>✓ correctly shows addition $3+9+6$ and division by 3 |

(b) Determine the seasonal index for Friday and explain its meaning. (2 marks)

| Solution |
|---|
| $\frac{40+60+50}{3}=\frac{150}{3}=50$<br>On average, Friday values are 50% of the daily mean |
| **Specific behaviours** |
| ✓ correctly calculates 50<br>✓ states the link to daily mean |

(c) (i) Give a reason why time series data are deseasonalised. (1 mark)

| Solution |
|---|
| To remove the seasonal effect |
| **Specific behaviours** |
| ✓ states a correct reason |

(ii) Calculate the deseasonalised value for Friday of Week 3. (2 marks)

| Solution |
| --- |
| $3 \div 50\% = 3 \div 0.5 = 6$ |
| **Specific behaviours** |
| ✓ correctly shows division by 50% or 0.5<br>✓ correctly calculates the value of 6 |

The equation of the least-squares line for the deseasonalised sales, based on time t, is $y = 0.2t + 4.3$.

(d) How does this equation support the observation that sales are increasing? (1 mark)

| Solution |
| --- |
| Positive gradient on a straight line means the values are increasing |
| **Specific behaviours** |
| ✓ correctly identifies the positive gradient |

(e) Predict the sales for Friday of Week 4. (3 marks)

| Solution |
| --- |
| $y(t = 10) = 0.2 \times 10 + 4.3 = 6.3$<br>$6.3 \times 50\% = 3.15$<br>Rounded to 3 handbags |
| **Specific behaviours** |
| ✓ correctly substitutes $t = 10$<br>✓ correctly complete calculation<br>✓ gives the correct rounded value of 3 |

**2023 Section 2 Question 8**

**Time series analysis**

The table below shows the average price per kilogram of watermelon in a supermarket for each of the four seasons of the year over a four-year period.

| Year | Quarter | Time ($t$) | Average price ($) | 4-point centred moving average | Quarterly mean | Average price as a percentage of the quarterly mean | Deseasonalised data |
|---|---|---|---|---|---|---|---|
| 2019 | Summer | 1 | 1.06 | - | 1.85 | 57.30 | 1.80 |
| | Autumn | 2 | 1.36 | - | | 73.51 | 1.79 |
| | Winter | 3 | 2.94 | 1.87 | | 158.92 | 1.89 |
| | Spring | 4 | 2.03 | 1.92 | | 109.73 | 1.86 |
| 2020 | Summer | 5 | 1.24 | 1.97 | **P** | 59.05 | 2.11 |
| | Autumn | 6 | 1.58 | 2.04 | | 75.24 | 2.07 |
| | Winter | 7 | 3.12 | 2.12 | | 148.57 | 2.01 |
| | Spring | 8 | 2.44 | 2.20 | | 116.19 | 2.23 |
| 2021 | Summer | 9 | 1.47 | 2.29 | 2.39 | 61.51 | 2.50 |
| | Autumn | 10 | 1.93 | 2.36 | | 80.75 | 2.53 |
| | Winter | 11 | 3.48 | 2.41 | | 145.61 | 2.23 |
| | Spring | 12 | 2.69 | 2.44 | | 112.55 | 2.46 |
| 2022 | Summer | 13 | **Q** | **R** | 2.75 | 57.09 | 2.69 |
| | Autumn | 14 | 2.07 | 2.75 | | **S** | 2.72 |
| | Winter | 15 | 4.65 | - | | 169.09 | 2.99 |
| | Spring | 16 | 2.71 | - | | 98.55 | 2.48 |

(a) Determine the values **P**, **Q**, **R** and **S** in the table above. (4 marks)

**Solution**

$$P = \frac{1.24 + 1.58 + 3.12 + 2.44}{4} = 2.10$$

$$Q = 2.75 \times 4 - (2.07 + 4.65 + 2.71) = 1.57 \text{ or } Q = 2.75 \times 57.09\% = 1.57$$

$$R = \frac{\frac{3.48}{2} + 2.69 + 1.57 + 2.07 + \frac{4.65}{2}}{4} = 2.60$$

$$S = \frac{2.07}{2.75} \times 100 = 75.27$$

**Specific behaviours**

✓ calculates correct P value
✓ calculates correct Q value
✓ calculates correct R value
✓ calculates correct S value

(b) Complete the following table. (1 mark)

| Season | Summer | Autumn | Winter | Spring |
|---|---|---|---|---|
| **Seasonal index** | 58.83% | 76.19% | **155.72%** | 109.26% |

| Solution |
|---|
| see table above |
| **Specific behaviours** |
| ✓ calculates correct value |
| Note: the value in the table is the sum to 400<br>155.55% is the value when the winter components are averaged. |

(c) Interpret the seasonal index for Summer in the context of this question. (1 mark)

| Solution |
|---|
| on average Summer season is 41.17% below trend value or 58.83% of the trend value |
| **Specific behaviours** |
| ✓ interprets the context correctly |

(d) Define 'deseasonalised data'. (1 mark)

| Solution |
|---|
| Data where periodic variations due to seasonal effects have been removed. |
| **Specific behaviours** |
| ✓ determines correct definition |

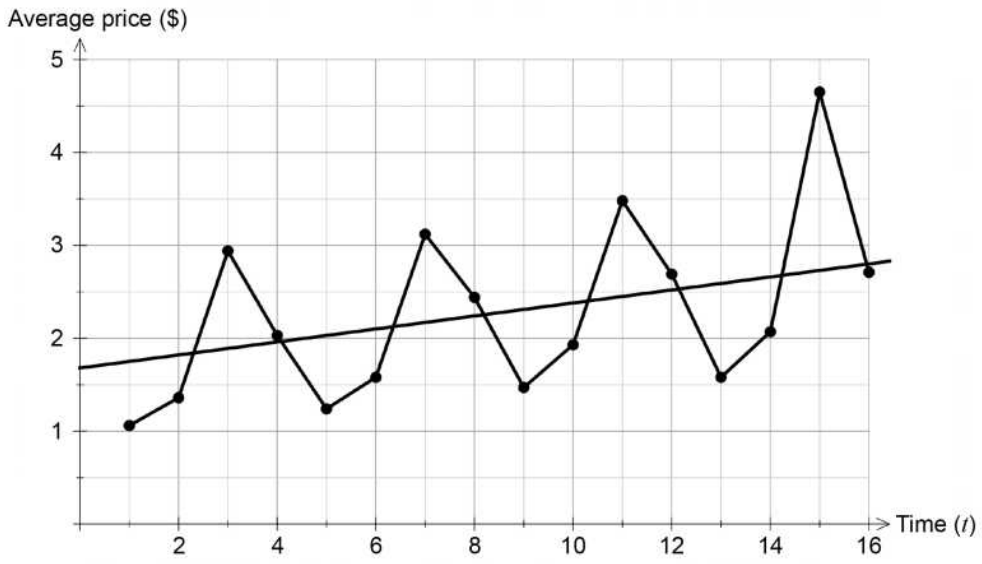
(e) Using information from the tables on page 6, show how the deseasonalised data value for Autumn 2021 was calculated. (1 mark)

| Solution |
|---|
| $\frac{1.93}{0.7619} = 2.53$ |
| **Specific behaviours** |
| ✓ determines correct calculation |

(f) Determine the equation of the least-squares line using the deseasonalised data. (1 mark)

| Solution |
|---|
| $y = 0.07t + 1.68$ |
| **Specific behaviours** |
| ✓ determines correct equation |

(g) The graph below is the time series plot for these data. On this graph draw the least-squares line determined in part (f). (2 marks)




| Solution |
|---|
| see graph above |
| **Specific behaviours** |
| ✓ shows correct $y$-intercept<br>✓ draws line with correct slope |

(h) Predict the average price per kilogram of watermelon from the supermarket in Spring 2024. (3 marks)

| Solution |
|---|
| $t = 24$<br>$y = 0.07(24) + 1.68 = 3.36$<br>$3.36 \times 1.0926 = 3.67$ |
| **Specific behaviours** |
| ✓ uses $t = 24$<br>✓ predicts trend value<br>✓ seasonally adjusts |

(i) Discuss the reliability of the prediction in part (h). (2 marks)

| Solution |
|---|
| prediction not reliable as it is extrapolation |
| **Specific behaviours** |
| ✓ states prediction not reliable<br>✓ states it is extrapolation |

**2022 Section 2 Question 8**

**Time series analysis**

Yash is saving for a deposit on an apartment, which is about to begin construction.

(a) Two saving plans are found in advertisements in a newspaper.
Option 1: 3.05% per annum compounded at the end of each month.
Option 2: 3.01% per annum compounded at the end of each day.

Which of these two options would give the better return? Justify your answer mathematically. (3 marks)

| Solution |
|---|
| Option 1 $i = 0.0305$, $n = 12$ effective interest = 3.09%<br>Option 2 $i = 0.0301$, $n = 365$ effective interest = 3.06%<br>∴ Option 1 is better |
| **Specific behaviours** |
| ✓ correctly calculates option 1 effective interest<br>✓ correctly calculates option 2 effective interest<br>✓ correct conclusion |

Yash locates a better investment option online. This offers 3.5% per annum compounded at the end of each day. Yash will begin with an initial deposit of $12 300 and a weekly deposit of $300. The deposit required to secure the apartment in 12 months is $30 000.

(b) (i) Show that this option will not be sufficient to reach the total needed to secure the apartment. State how far short of the required amount this option will be. (4 marks)

| Solution |
|---|
| N = 52, I = 3.5, PV = –12300, PMT = –300, P/Y = 52, C/Y = 365<br>FV = 28608.96<br><br>Short by $1391.04 |
| **Specific behaviours** |
| ✓ uses at least 4 correct values for N, I, PV, PMT, P/Y, C/Y<br>✓ uses all correct values for N, I, PV, PMT, P/Y, C/Y<br>✓ determines correct FV<br>✓ determines correct shortfall |

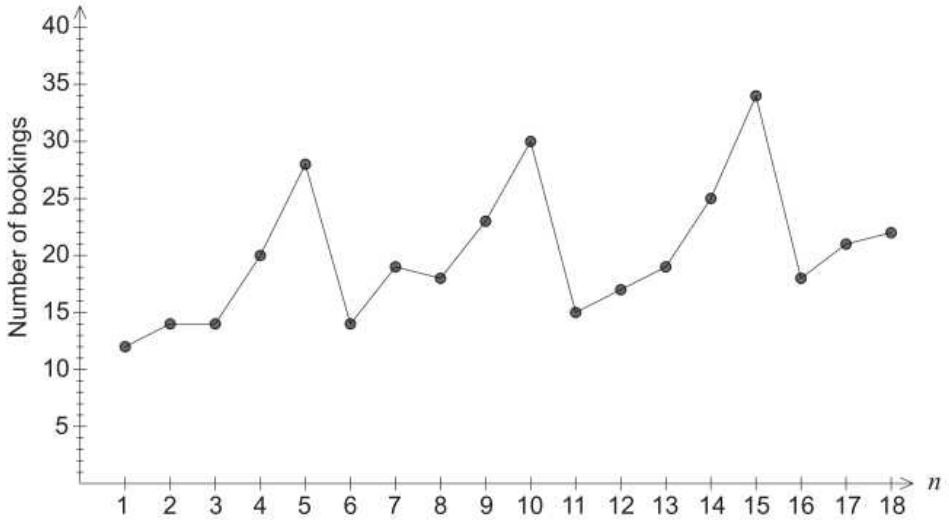
(ii) Determine what the weekly deposit needs to be for a total of $30 000 to be achieved by the end of 12 months. (2 marks)

| Solution |
|---|
| N = 52, I = 3.5, PV = –12300, FV = 30000, P/Y = 52, C/Y = 365<br>PMT = –326.29<br>The weekly deposit is $326.29 (to the nearest cent) |
| **Specific behaviours** |
| ✓ uses all correct values for N, I, PV, FV, P/Y, C/Y<br>✓ determines correct payment |

**2021 Section 2 Question 8**

**Time series analysis**

The graph below shows the number of bookings at a dog grooming salon over its first few weeks of business.




(a) Which simple moving average would be the **most** suitable for the data displayed in this graph? (1 mark)

| Solution |
|---|
| 5 point moving average |
| **Specific behaviours** |
| ✓ states correct moving average |

A more detailed view of the same data is given in the table below.

| Week | Day | $n$ | Number of bookings | Seasonal mean | Number of bookings as a percentage of the seasonal mean | Seasonally adjusted figures |
|---|---|---|---|---|---|---|
| 1 | Tuesday | 1 | 12 | 17.6 | **A** | 17.7 |
| | Wednesday | 2 | 14 | | 79.55 | 16.9 |
| | Thursday | 3 | 14 | | 79.55 | 16.6 |
| | Friday | 4 | 20 | | 113.64 | 17.8 |
| | Saturday | 5 | 28 | | 159.09 | 18.3 |
| 2 | Tuesday | 6 | 14 | **B** | 67.31 | 20.6 |
| | Wednesday | 7 | 19 | | 91.35 | 23.0 |
| | Thursday | 8 | 18 | | 86.54 | 21.4 |
| | Friday | 9 | 23 | | 110.58 | 20.4 |
| | Saturday | 10 | 30 | | 144.23 | 19.7 |
| 3 | Tuesday | 11 | **C** | 22 | 68.18 | 22.1 |
| | Wednesday | 12 | 17 | | 77.27 | 20.6 |
| | Thursday | 13 | 19 | | 86.36 | 22.6 |
| | Friday | 14 | 25 | | 113.64 | 22.2 |
| | Saturday | 15 | 34 | | 154.55 | 22.3 |
| 4 | Tuesday | 16 | 18 | – | – | – |
| | Wednesday | 17 | 21 | | – | – |
| | Thursday | 18 | 22 | | – | – |

(b) Calculate the value of **A**, **B** and **C** in the table. (3 marks)

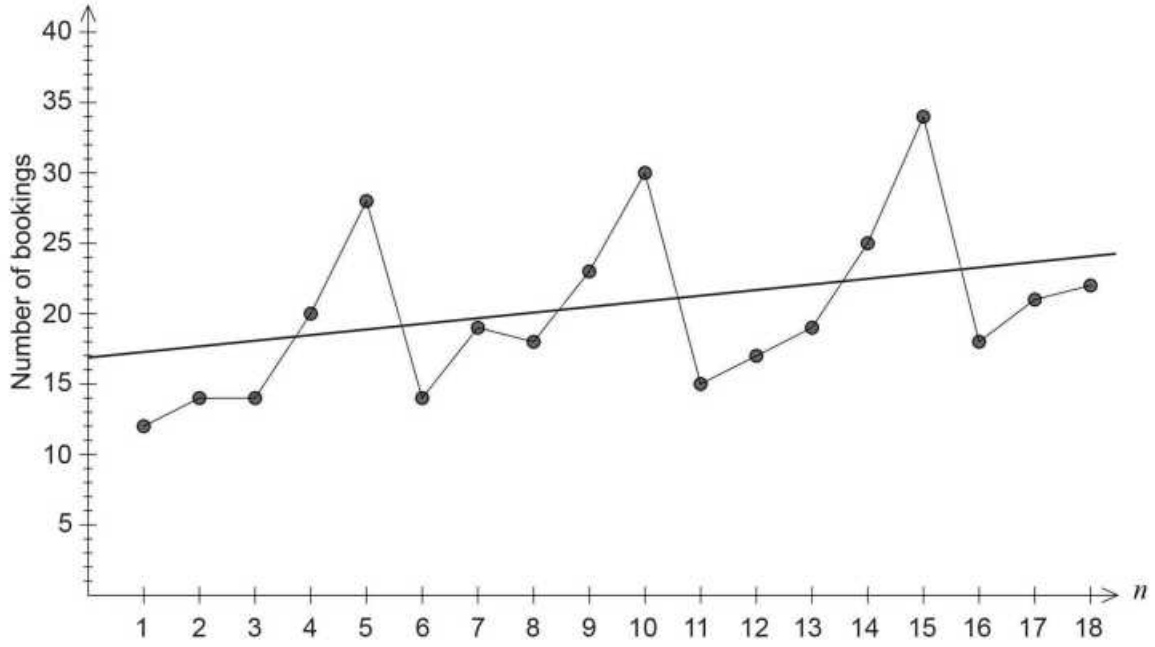
| Solution |
|---|
| **A**: $12 \div 17.6 \times 100 = 68.18$<br>**B**: $(14+19+18+23+30) \div 5 = 20.8$<br>**C**: $(22 \times 5) - (17+19+25+34) = 15$ |
| **Specific behaviours** |
| ✓ correctly calculates the value of **A**<br>✓ correctly calculates the value of **B**<br>✓ correctly calculates the value of **C** |

(c) Calculate the seasonal index for Saturday. (1 mark)

| Solution |
|---|
| SI = 1.5262 (152.62%) |
| **Specific behaviours** |
| ✓ correctly calculates the seasonal index |

(d) The equation of the least-squares line using the seasonally adjusted figures is $y = 0.40n + 16.94$. Draw this line on the graph. (2 marks)

| Solution |
|---|
| See graph below |
| **Specific behaviours** |
| ✓ plots one correct point<br>✓ plots a second correct point and draws the least-squares line across the given data |



(e) (i) Use the equation of the least-squares line given in part (d) to predict the number of bookings that will be made for the Saturday of Week 5. (2 marks)

| Solution |
| --- |
| Saturday of Week 5: $n = 25$<br>$y = 0.40(25) + 16.94 = 26.94$<br>Therefore, predicted number of bookings is 26.94 × 1.5262 = 41.12 (approx. 41) |
| **Specific behaviours** |
| ✓ correctly uses $n = 25$<br>✓ correctly multiplies by the seasonal index to predict the number of bookings |

(ii) Comment on this prediction. (2 marks)

| Solution |
| --- |
| Not valid, since it is extrapolation |
| **Specific behaviours** |
| ✓ correctly states it is not valid<br>✓ correctly states it is outside the range of the data (extrapolation) |

**2020 Section 2 Question 14**

**Time series analysis**

The table below shows the number of sprinkler systems installed by a local reticulation business over the past four years.

| Year | Season | $n$ | Number of systems | Seasonal mean | Number of systems as a percentage of the seasonal mean | Seasonally adjusted figures |
|---|---|---|---|---|---|---|
| 2017 | Summer | 1 | **A** | 14 | 71.4 | 10.4 |
| | Autumn | 2 | 18 | | **B** | 15.7 |
| | Winter | 3 | 11 | | 78.6 | 14.7 |
| | Spring | 4 | 17 | | 121.4 | 14.7 |
| 2018 | Summer | 5 | 15 | **C** | 105.3 | 15.7 |
| | Autumn | 6 | 16 | | 112.3 | 14.0 |
| | Winter | 7 | 11 | | 77.2 | 14.7 |
| | Spring | 8 | 15 | | 105.3 | 13.0 |
| 2019 | Summer | 9 | 13 | 11.75 | 110.6 | 13.6 |
| | Autumn | 10 | 12 | | 102.1 | 10.5 |
| | Winter | 11 | 8 | | 68.1 | 10.7 |
| | Spring | 12 | 14 | | 119.1 | 12.1 |
| 2020 | Summer | 13 | 16 | — | — | — |
| | Autumn | 14 | 15 | | — | — |

(a) Calculate the value of **A**, **B** and **C**. (3 marks)

**Solution**

$$\frac{A+18+11+17}{4}=14 \Rightarrow A=10$$

$$B=\frac{18}{14}\times 100=128.6$$

$$C=\frac{15+16+11+15}{4}=14.25$$

**Specific behaviours**

✓ correctly calculates the value of $A$
✓ correctly calculates the value of $B$
✓ correctly calculates the value of $C$

(b) Complete the table showing the seasonal index for each season. (2 marks)

| **Season** | Summer | Autumn | Winter | Spring |
|---|---|---|---|---|
| **Seasonal index** | 95.8 | 114.3 | **74.6** | **115.3** |

**Solution**

See table above

**Specific behaviours**

✓ correctly calculates value for winter
✓ correctly calculates value for spring

(c) Show how the seasonally adjusted figure of 13.6 for Summer 2019 was calculated. (2 marks)

| Solution |
| --- |
| $\frac{13}{0.958} = 13.57 \approx 13.6$ |
| **Specific behaviours** |
| ✓ correctly uses division<br>✓ uses correct values |

(d) During which season could more employees be given annual holidays with least disruption to sprinkler installations? Use mathematical evidence to support your answer. (2 marks)

| Solution |
| --- |
| Winter, as the seasonal index (74.6) is the lowest |
| **Specific behaviours** |
| ✓ chooses correct season<br>✓ gives valid reason |

(e) Determine the least-squares line using the seasonally adjusted figures. (1 mark)

| Solution |
| --- |
| $y = -0.24n + 14.88$ |
| **Specific behaviours** |
| ✓ states correct equation |

(f) Using your line from part (e), estimate the number of sprinkler systems that will be installed in Summer 2021. (2 marks)

| Solution |
| --- |
| $y(17) = 10.8$<br>Therefore, predicted value is $10.8 \times 0.958 = 10.35$<br>i.e. 10 sprinkler systems. |
| **Specific behaviours** |
| ✓ calculates the value of $y$ when $n = 17$<br>✓ correctly multiplies by seasonal index and rounds appropriately |

(g) Comment on the long-term prospects of the business. (2 marks)

| Solution |
| --- |
| The number of systems installed is declining since the gradient of the least-squares line is negative. |
| **Specific behaviours** |
| ✓ correctly mentions declining number of sprinkler systems installed<br>✓ correctly refers to gradient of least-squares line |

**2019 Section 2 Question 11**

**Time series analysis**

Data for the total occupancy of rooms for each season of the year at a Perth hotel is shown below.

| $n$ | Year | Season | Total rooms occupied | Seasonal mean | 4-point centred moving average | Total rooms occupied as a percent of seasonal mean |
|---|---|---|---|---|---|---|
| 1 | 2015/16 | Spring | 1770 | 1660.5 | | 106.59 |
| 2 | | Summer | 1904 | | | **B** |
| 3 | | Autumn | 1591 | | 1644.375 | 95.81 |
| 4 | | Winter | 1377 | | 1622.5 | 82.93 |
| 5 | 2016/17 | Spring | 1641 | 1610.25 | 1618 | 101.91 |
| 6 | | Summer | 1858 | | 1614.75 | 115.39 |
| 7 | | Autumn | 1601 | | 1602.25 | 99.43 |
| 8 | | Winter | 1341 | | 1584.75 | 83.28 |
| 9 | 2017/18 | Spring | 1577 | 1524.0 | 1558 | 103.48 |
| 10 | | Summer | **A** | | 1532.375 | 116.93 |
| 11 | | Autumn | 1463 | | 1526.875 | 96.00 |
| 12 | | Winter | 1274 | | 1525.125 | 83.60 |
| 13 | 2018/19 | Spring | 1600 | 1519.75 | **C** | 105.28 |
| 14 | | Summer | 1745 | | 1525.25 | 114.82 |
| 15 | | Autumn | 1504 | | | 98.96 |
| 16 | | Winter | 1230 | | | 80.93 |

(a) Calculate the value of **A**, **B** and **C**. (3 marks)

**Solution**

$$\frac{1577+A+1463+1274}{4}=1524 \Rightarrow A=1782$$

$$\frac{1904}{1660.5}\times 100 = B \Rightarrow B = 114.66$$

$$\frac{\frac{1463}{2}+1274+1600+1745+\frac{1504}{2}}{4}=1525.625=C$$

**Specific behaviours**

✓ correctly calculates the value of $A$
✓ correctly calculates the value of $B$
✓ correctly calculates the value of $C$

(b) Complete the table showing the seasonal index for each season. (1 mark)

| Summer | Autumn | Winter | Spring |
|---|---|---|---|
| 1.1545 | **0.9755** | 0.8268 | 1.0432 |

**Solution**

See table above

**Specific behaviours**

✓ correctly calculates seasonal index for Autumn

(c) Calculate the deseasonalised value for Winter 2017/18. (2 marks)

| Solution |
| --- |
| $\frac{1274}{0.8268} = 1540.88$ (1541 to nearest whole number) |
| **Specific behaviours** |
| ✓ correctly shows division by seasonal index<br>✓ correctly calculates deseasonalised value |

(d) Comment on the effect the seasonal index had on the value found in part (c). (1 mark)

| Solution |
| --- |
| Increased value to underlying trend |
| **Specific behaviours** |
| ✓ states correct reason |

(e) The least-squares line using deseasonalised data is $R = -12.071_n + 1681.25$. Use this line to predict the total number of rooms occupied during Spring 2020/21. (2 marks)

| Solution |
| --- |
| $n = 21, R = 1427.8$<br>Prediction is 1427.8 × 1.0432 = 1489.4 (1489 to nearest whole number) |
| **Specific behaviours** |
| ✓ uses correct value of $n$<br>✓ calculates the predicted total number of rooms occupied |

When a prediction was made for Spring 2020/21, using the least-squares line based on the 4-point centred moving averages, the answer was 1481.

(f) Explain why this is different from the answer obtained in part (e). (1 mark)

| Solution |
| --- |
| Different method used for smoothing |
| **Specific behaviours** |
| ✓ states correct reason |

The manager of the hotel attended a meeting with the owners of the hotel. She explained to the owners that the reduction in occupancy was due to the downturn in the Western Australian economy in recent years.

(g) Comment on the statement made by the hotel manager. (2 marks)

| Solution |
| --- |
| Cause not established<br>Comment not appropriate |
| **Specific behaviours** |
| ✓ states the cause in not established<br>✓ states the comment is not appropriate |

(h) What practical advice, in the context of the question, would you give to the manager of the hotel? (1 mark)

| Solution |
| --- |
| Drop room rate<br>Advertise<br>etc… |
| **Specific behaviours** |
| ✓ gives a valid reason |

# Unit 4.2 – Loans, investments and annuities

## Section 1

| | |
|---|---|
| **2022 Section 1 Question 2**<br><br>**Loans, investments and annuities** | A gardener purchases a new lawnmower valued at $4800. The lawnmower depreciates at a constant rate of $250 per year.<br><br>(a) (i) Determine a recursive rule for the value of the lawnmower after $n$ years. (2 marks)<br><br>(ii) Deduce a rule for the $n^{th}$ term of this sequence. (1 mark)<br><br>(b) Determine the value of the lawnmower after 4 years. (1 mark)<br><br>As part of his business plan, the gardener will sell his lawnmower when its value drops below $1300.<br><br>(c) Calculate when the lawnmower will be sold. (3 marks) |

## Section 2

| | |
|---|---|
| **2023 Section 2 Question 9**<br><br>**Loans, investments and annuities** | Sonia secures a bank loan to buy a professional gaming computer. The loan has reducible interest. Information about the loan is shown below.<br><br>Loan issued: Start of October 2023.<br>Starting balance: $9200.<br>Interest: Compounded monthly.<br>Repayments: $290 per month.<br><br>After the first monthly payment at the end of October 2023, Sonia's balance is $8992.80.<br><br>(a) Use the information above to show that the annual interest rate is 10.8%. (2 marks)<br><br>(b) Determine a recursive rule to model the balance of the loan at the end of each month. (2 marks)<br><br>(c) Determine<br><br>(i) the balance of the loan at the end of November 2023. (1 mark)<br><br>(ii) the total amount of interest incurred in the first three months. (2 marks)<br><br>(iii) the balance of the loan at the end of May 2024. (1 mark)<br><br>(d) Determine how many months it takes to repay the loan. (1 mark) |

(e) Determine the final repayment and the total amount repaid. (2 marks)

(f) Calculate the total interest paid on the loan. (1 mark)

(g) Sonia is paid every fortnight in her employment. Instead of monthly repayments of $290, she is now considering making fortnightly repayments of $145, with the interest calculated fortnightly. Use mathematical evidence to show what difference this would make and advise Sonia what her savings might be. (4 marks)

**2023 Section 2 Question 13**

**Loans, investments and annuities**

Chau purchased a brand-new motorcycle valued at $17 000. Once Chau drove the motorcycle out of the dealer's yard, it immediately depreciated in value by 15%. By the end of the first year, it depreciated a further 5%. After the first year the motorcycle depreciated at a constant rate of 7% per year.

(a)

| $n$ (years) | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Value of the motorcycle ($) | 13 727.50 | | | |

(i) Show how the value of the motorcycle at the end of the first year was calculated. (2 marks)

(ii) Complete the remainder of the table. (2 marks)

(b) Determine the percentage by which the motorcycle depreciated in the first year. (2 marks)

(c) Write a recursive rule that represents the amount the motorcycle will be worth at the end of each year, after the first year. (2 marks)

| | |
|---|---|
| | (d) Determine how much the motorcycle will be worth at the end of eight years. (1 mark)<br><br>(e) When will the motorcycle be first worth less than $5000? (1 mark) |

| | |
|---|---|
| **2023 Section 2 Question 14**<br><br>**Loans, investments and annuities** | Ravi retired at the beginning of the month, with a superannuation balance of $945 864. He has a written guarantee of a 7.5% per annum return on his superannuation with interest added at the end of each month. Ravi will receive an annuity of $3200 paid at the end of each fortnight.<br><br>Ravi has also committed to purchasing a new car for a price of $37 000. There is a 12-month waiting period for the delivery of the car and he plans to pay for the car by withdrawing the cost from his superannuation account at time of delivery, when payment is due.<br><br>(a) Calculate the balance in the superannuation account at the end of two years. (6 marks) |

| | |
|---|---|
| | (b) After 15 years of retirement Ravi expects to have a less active lifestyle and his living expenses will not be as high. He also understands that the balance of his superannuation needs to last for his remaining years. Ravi decides the most suitable plan for him is to place the balance of his superannuation into a perpetuity with a fortnightly payment.<br><br>Assuming the 7.5% interest rate is maintained, what fortnightly amount can Ravi expect from the perpetuity? (5 marks) |

| | |
|---|---|
| **2022 Section 2 Question 15**<br><br>**Loans, investments and annuities** | Paris, a retired hairdresser, has a superannuation balance of $775 320.<br><br>(a) She intends to set up an annual scholarship of $5000 to be awarded to a worthy apprentice. Paris is able to negotiate a fixed annual interest rate of 7.2% through a share trading company. Show that $69 444.44 (to the nearest cent) of Paris's superannuation is needed to be invested in this company to meet the requirements of the annual scholarship. (1 mark) |

After the scholarship is in place, Paris considers her future investment plans with the remaining balance. Assume a financial advisor has indicated an interest rate of 6.1% per annum, added monthly, can be locked in.

(b) Paris decides on receiving a monthly annuity of $7000, with the interest added before the annuity is paid, at the end of each month.

(i) The annuity can be modelled by the recurrence relation
$T_n+1 = (1 + 0.00508)T_n - 7000$ , $T_0 = a$.
Using the information given in the question, show how to obtain the value 0.00508 and state the value of $a$. (2 marks)

(ii) For how many months will Paris be able to receive the annuity of $7000? (1 mark)

(iii) Determine the amount of the final payment. (2 marks)

(iv) Determine the total interest received on the annuity. (3 marks)

(c) Paris also considers receiving a perpetuity after the scholarship is in place. Determine the monthly amount she would receive. (3 marks)

**2022 Section 2 Question 16**

**Loans, investments and annuities**

After paying a deposit for his new apartment, Declan obtains a bank loan for the remaining amount of $112 000 at 3.26% per annum compounded monthly. He can currently afford to repay $970 per month at the end of every month.

(a) Calculate how much he would owe after the 40th repayment. (3 marks)

(b) Declan decided to deposit a one-off extra amount of $1600, after the 16th repayment. Calculate the new amount he would owe after the 40th repayment. (4 marks)

**2021 Section 2 Question 10**

**Loans, investments and annuities**

Wendy moved into an apartment and organised a loan of $16 000 to purchase new furniture. To pay off the loan Wendy makes repayments of $600 at the end of each month. The spreadsheet below shows the progress of her loan.

| Month | Opening balance | Interest | Repayment | Closing balance |
|---|---|---|---|---|
| 1 | 16 000.00 | 98.67 | 600.00 | 15 498.67 |
| 2 | 15 498.67 | 95.58 | 600.00 | 14 994.24 |
| 3 | 14 994.24 | 92.46 | 600.00 | 14 486.71 |
| 4 | | | | |

(a) Write a calculation to show that the yearly interest rate is approximately 7.4%. (2 marks)

(b) Complete the fourth row of the spreadsheet. (3 marks)

(c) Write a recursive rule to determine the closing balance of the loan at the end of each month. (2 marks)

(d) Determine how many months it will take Wendy to pay off the loan. (1 mark)

(e) Calculate how much interest is paid over the duration of the loan. (3 marks)

On reflection, Wendy realised she could have repaid $800 each month.

(f) Determine the maximum amount Wendy would have been able to borrow, if all other details of the loan and repayment time remained the same. (3 marks)

| | |
|---|---|
| | |

| | |
|---|---|
| **2021 Section 2 Question 12**<br><br>**Loans, investments and annuities** | Virat purchases a new motor vehicle for $24 500. For the first two years the vehicle depreciates at a rate of 13% per year and for the third year it depreciates at a lower rate of 9.5% per year.<br><br>(a) Calculate the value of the vehicle after one year. (1 mark)<br><br>(b) Calculate the value of the vehicle after the first three years. (2 marks)<br><br>For the next three years the rate of depreciation is constant at $r$% per year. The average rate of depreciation for the first six years is 11% per year.<br><br>(c) Calculate the value of r as a percentage. (3 marks) |

| | |
|---|---|
| **2021 Section 2 Question 14**<br><br>**Loans, investments and annuities** | Patrick has retired and invested his lump sum superannuation payout of $717 850 at a rate of 5.7% per annum compounded monthly. He begins the investment strategy from 1 January.<br><br>(a) Patrick will receive $4500 at the end of each month for general living expenses and will also receive a further $4000 at the end of each year for an annual holiday.<br><br>(i) Identify this type of investment account. (1 mark)<br><br>(ii) Determine the balance in the account at the end of the first year. (4 marks) |

(iii) Determine the balance in the account at the end of the second year. (3 marks)

(b) When Patrick retired, he also considered the option of setting up a perpetuity with his superannuation payout still at 5.7% per annum compounded monthly. Calculate the quarterly payments Patrick would have received with this perpetuity in place. (3 marks)

**2020 Section 2 Question 9**

**Loans, investments and annuities**

Giuseppe wishes to set up an annuity. He is told that an annuity with quarterly investment returns and quarterly payments is modelled by the recursive rule:

$A_{n+1} = A_n \times 1.019 - P$, $A_0 = Q$ with the values of $P$ and $Q$ consistent with the spreadsheet below.

| Quarter | Opening balance | Investment gain | Payment | Closing balance |
|---|---|---|---|---|
| 1 | \$648 000 | \$12 312 | \$15 000 | $X$ |
| 2 | | $Y$ | \$15 000 | |
| 3 | | | | |

(a) Determine the values of $P$, $Q$, $X$ and $Y$ and write them in the table below. (4 marks)

| $P$ | $Q$ | $X$ | $Y$ |
|---|---|---|---|
| | | | |

(b) What is the annual compound interest rate for this investment? (1 mark)

When the balance in the annuity first falls below \$300 000, Giuseppe converts the payment to a perpetuity so that his children are left with some inherited benefits. The interest rate remains the same as that calculated in part (b).

(c) Determine the number of years the annuity operates before the perpetuity starts. (2 marks)

(d) What are the quarterly payments under this perpetuity? (2 marks)

(e) Giuseppe believes that his investment returns are at an effective interest rate of 7.93% p.a. Use a clear calculation to comment on the accuracy of this belief. (2 marks)

| | |
|---|---|
| **2020 Section 2 Question 11**<br><br>**Loans, investments and annuities** | Shari requires a loan of $325 000 for the purchase of a new house. She wishes to make two equally-spaced repayments of $700 each month.<br><br>Shari is offered a choice of two loan options for the first three years, both of which have interest calculated daily.<br>• Option 1 An introductory compound interest rate of 2.55% per annum for the first year which changes to 2.99% per annum for the next two years.<br>• Option 2 A compound interest rate of 2.85% per annum fixed for the first three years.<br><br>(a) Describe briefly the benefit of making two repayments of $700 each month instead of one repayment of $1400 at the end of each month. (1 mark)<br><br>(b) For Option 1, calculate<br><br>(i) the loan balance at the end of the first year. (3 marks)<br><br>(ii) the loan balance at the end of the third year. (2 marks)<br><br>(c) Determine which option gives the best result for Shari after three years and by how much. (3 marks) |

**2020 Section 2 Question 12**

**Loans, investments and annuities**

Jessica wants to borrow $15 000 from her parents to purchase a car. They will be charging her compound interest at the rate of 4% per annum, with interest added yearly.

(a) Jessica is currently studying so she will not want to be making any regular repayments.

(i) Complete the table below to show the amount she will owe her parents at the end of each year. (2 marks)

| **Number of years ($n$)** | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| **Amount owing ($)** | 15 000 | | | |

(ii) Write a recursive rule to calculate the amount owing at the end of each year. (2 marks)

Jessica's parents are encouraging her to get a part-time job so that she can make repayments along the way. Jessica estimates that she will be able to earn enough money to pay off $2400 each year.

(b) If interest is charged yearly and she repays the $2400 at the end of each year, write a recursive rule to calculate the amount owing at the end of each year. (1 mark)

(c) If interest is charged monthly and she makes equal monthly repayments,

(i) write a recursive rule to calculate the amount owing at the end of each month.(1 mark)

(ii) calculate how many months it will take to repay the loan. (1 mark)

(iii) calculate the total amount Jessica would pay over the duration of the loan. (3 marks)

| | |
|---|---|
| **2019 Section 2 Question 9**<br><br>**Loans, investments and annuities** | Corbie and Grant are investigating the possibility of purchasing their own apartment. They will need to open a bank account to save for a deposit. They make an initial contribution of $7000 into the account and add a further $800 at the end of each month. The account has an interest rate of 2.6% per annum, compounded monthly.<br><br>(a) How much would be in the account at the end of two years? (3 marks)<br><br>Corbie and Grant are willing to pay $280 000 for the apartment and will require an additional $22 000 in fees associated with the purchase. A deposit of 20% of the total cost will be needed in order to qualify for a bank loan.<br><br>(b) (i) Show that the required deposit is $60 400. (1 mark)<br><br>(ii) How long would it take to save enough for the deposit? (2 marks)<br><br>(c) If, at the end of two years, their parents agree to give them a total of $10 000 as a reward for their dedicated saving effort, determine the minimum monthly contribution Corbie and Grant will need to make if they are to have enough for the full deposit after four years. (3 marks) |

| | |
|---|---|
| **2019 Section 2 Question 13**<br><br>**Loans, investments and annuities** | Mehmet has saved $3600 from wages received at a part-time job. He is keen to invest this money in an account which earns 3.65% per annum, compounded monthly.<br><br>Over the next three years, Mehmet plans to continue working part-time and is aiming to make deposits of $250 at the end of each month.<br><br>(a) Write a recursive relation to give the value of the investment at the end of each month. (2 marks)<br><br>(b) Mehmet hopes that this investment will double his initial savings in one year. Justify whether this is possible. (2 marks) |

| | |
|---|---|
| | (c) Determine the total amount of interest Mehmet would receive after three years. (3 marks)<br><br>Unfortunately, after two years, Mehmet's working hours are reduced and he is only able to deposit $120 at the end of each month.<br><br>(d) By how much would this reduce the value of his investment by the end of the three years? (3 marks) |

| | |
|---|---|
| **2019 Section 2 Question 17**<br><br>**Loans, investments and annuities** | Joel has set up a special investment fund that has a current balance of $350 000. He contributes 7.5% of his monthly income to the investment and has an overseas pension which contributes a further $355 per month. The investment fund has an interest rate of 6.5% per annum, compounded monthly. Joel's annual salary is $101 000 and he has just turned 60 years of age.<br><br>(a) Calculate Joel's total monthly contribution to the fund. (2 marks)<br><br>(b) Calculate the lump sum that he could receive if he retires on his 67th birthday. (2 marks)<br><br>Joel retires at 67 and wants to use his lump sum payment to set up a regular income. He decides to look at two options that offer monthly payments.<br><br>Option 1: A reducing balance annuity at 7% per annum, compounded monthly.<br>Option 2: A perpetuity at 7.5% per annum, compounded monthly.<br><br>(c) Calculate his maximum monthly income for the next 20 years using Option 1. (2 marks)<br><br>(d) Calculate his monthly income using Option 2. (2 marks) |

**2022 Section 1 Question 2**

**Loans, investments and annuities**

A gardener purchases a new lawnmower valued at $4800. The lawnmower depreciates at a constant rate of $250 per year.

(a) (i) Determine a recursive rule for the value of the lawnmower after $n$ years. (2 marks)

| Solution |
|---|
| $T_{n+1} = T_n - 250, T_0 = 4800$ |
| **Specific behaviours** |
| ✓ states correct rule<br>✓ states $T_0 = 4800$ |

(ii) Deduce a rule for the $n^{th}$ term of this sequence. (1 mark)

| Solution |
|---|
| $T_n = 4800 - 250n$ |
| **Specific behaviours** |
| ✓ states correct rule |

(b) Determine the value of the lawnmower after 4 years. (1 mark)

| Solution |
|---|
| $T_4 = 4800 - 250(4)$<br>$= \$3800$ |
| **Specific behaviours** |
| ✓ determines correct value |

As part of his business plan, the gardener will sell his lawnmower when its value drops below $1300.

(c) Calculate when the lawnmower will be sold. (3 marks)

| Solution |
|---|
| $1300 = 4800 - 250n$<br>$250n = 3500$<br>$n = 14 \quad \therefore$ after 14 years |
| **Specific behaviours** |
| ✓ identifies difference of 3500<br>✓ correctly calculates the value of $n$<br>✓ correctly states after 14 years |

**2023 Section 2 Question 9**

**Loans, investments and annuities**

Sonia secures a bank loan to buy a professional gaming computer. The loan has reducible interest. Information about the loan is shown below.

Loan issued: Start of October 2023.
Starting balance: $9200.
Interest: Compounded monthly.
Repayments: $290 per month.

After the first monthly payment at the end of October 2023, Sonia's balance is $8992.80.

(a) Use the information above to show that the annual interest rate is 10.8%. (2 marks)

| Solution |
|---|
| Interest for the first month = $(8992.80+290)-9200=82.80$<br>Annual interest rate = $\frac{82.80}{9200}\times 100\times 12=10.8\%$ |
| **Specific behaviours** |
| ✓ correctly calculates the interest for the first month<br>✓ correctly calculates annual interest rate |

(b) Determine a recursive rule to model the balance of the loan at the end of each month. (2 marks)

| Solution |
|---|
| $T_{n+1}=1.009T_n-290, T_0=9200$ |
| **Specific behaviours** |
| ✓ states correct rule<br>✓ states correct initial value |

(c) Determine

(i) the balance of the loan at the end of November 2023. (1 mark)

| Solution |
|---|
| $8783.74 (Term 2 in the sequence) |
| **Specific behaviours** |
| ✓ determines correct balance |

(ii) the total amount of interest incurred in the first three months. (2 marks)

| Solution |
|---|
| 82.80 + (0.009 × 8992.80) + (0.009 × 8783.74) = $242.79 |
| **Specific behaviours** |
| ✓ determines correct interest for each month<br>✓ sums each interest |

(iii) the balance of the loan at the end of May 2024. (1 mark)

| Solution |
|---|
| $7489.24 (Term 8 in the sequence) |
| **Specific behaviours** |
| ✓ determines correct balance |

(d) Determine how many months it takes to repay the loan. (1 mark)

| Solution |
|---|
| 38 months |
| **Specific behaviours** |
| ✓ determines correct value |

(e) Determine the final repayment and the total amount repaid. (2 marks)

| Solution |
|---|
| $T_{37} = 150.64$<br>Therefore, the final repayment is \$150.64 × 1.009 = \$152<br><br>Total amount repaid = $37 \times 290 + 152 = \$10\,882$<br>or<br>38 × 290 – 138.00 = \$10 882 |
| **Specific behaviours** |
| ✓ determines correct final repayment<br>✓ determines correct total amount repaid |

(f) Calculate the total interest paid on the loan. (1 mark)

| Solution |
|---|
| 10 882 – 9200 = \$1682 |
| **Specific behaviours** |
| ✓ determines correct value |

(g) Sonia is paid every fortnight in her employment. Instead of monthly repayments of \$290, she is now considering making fortnightly repayments of \$145, with the interest calculated fortnightly. Use mathematical evidence to show what difference this would make and advise Sonia what her savings might be. (4 marks)

| Solution |
|---|
| Interest each fortnight $\frac{10.8}{26} = 0.41538\%$ (5 d.p.)<br>$A_{n+1} = \left(1 + \frac{0.108}{26}\right) A_n - 145, A_0 = 9200$<br><br>74 repayments and total repaid = 74 × 145 – 28.92 = \$10 701.08<br>Sonia should use fortnightly repayments to save 10 882 – 10 701.08 = \$180.92<br>She would also pay off the loan quicker. |
| **Specific behaviours** |
| ✓ determines correct number of repayments<br>✓ calculates the total repaid<br>✓ determines correct final savings<br>✓ gives correct advice |

**2023 Section 2 Question 13**

**Loans, investments and annuities**

Chau purchased a brand-new motorcycle valued at $17 000. Once Chau drove the motorcycle out of the dealer's yard, it immediately depreciated in value by 15%. By the end of the first year, it depreciated a further 5%. After the first year the motorcycle depreciated at a constant rate of 7% per year.

(a)

| $n$ (years) | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Value of the motorcycle ($) | 13 727.50 | | | |

(i) Show how the value of the motorcycle at the end of the first year was calculated. (2 marks)

| Solution |
|---|
| $17\,000 \times 0.85 \times 0.95 = 13\,727.50$ |
| **Specific behaviours** |
| ✓ multiplies by 0.85<br>✓ multiplies answer by 0.95 |

(ii) Complete the remainder of the table. (2 marks)

| Solution | | | | |
|---|---|---|---|---|
| $n$ (years) | 1 | 2 | 3 | 4 |
| Value of the motorcycle ($) | 13 727.50 | **12 766.58** | **11 872.91** | **11 041.81** |
| **Specific behaviours** | | | | |
| ✓ determines one correct value<br>✓ correctly completes table | | | | |

(b) Determine the percentage by which the motorcycle depreciated in the first year. (2 marks)

| Solution |
|---|
| $\frac{3272.50}{17\,000} \times 100 = 19.25\%$ |
| **Specific behaviours** |
| ✓ calculates the actual decrease of 3272.50<br>✓ calculates correct percentage |

(c) Write a recursive rule that represents the amount the motorcycle will be worth at the end of each year, after the first year. (2 marks)

| Solution |
|---|
| $T_{n+1} = 0.93T_n,\ T_0 = 13\,727.50$ |
| **Specific behaviours** |
| ✓ states correct value for $T_0$<br>✓ states correct recursive rule |

(d) Determine how much the motorcycle will be worth at the end of eight years. (1 mark)

| Solution |
|---|
| $T_7 = 8259.848$<br>therefore the motorcycle will be worth \$8259.85 to the nearest cent |
| **Specific behaviours** |
| ✓ determines correct value |

(e) When will the motorcycle be first worth less than \$5000? (1 mark)

| Solution |
|---|
| $T_{13} = 5344.04$<br>$T_{14} = 4969.96$<br><br>therefore during the 15th year |
| **Specific behaviours** |
| ✓ determines correct time |

**2023 Section 2 Question 14**

**Loans, investments and annuities**

Ravi retired at the beginning of the month, with a superannuation balance of \$945 864. He has a written guarantee of a 7.5% per annum return on his superannuation with interest added at the end of each month. Ravi will receive an annuity of \$3200 paid at the end of each fortnight.

Ravi has also committed to purchasing a new car for a price of \$37 000. There is a 12-month waiting period for the delivery of the car and he plans to pay for the car by withdrawing the cost from his superannuation account at time of delivery, when payment is due.

(a) Calculate the balance in the superannuation account at the end of two years. (6 marks)

| Solution |
|---|
| N = 26, I = 7.5, PV = –945 864, PMT = 3200, P/Y = 26, C/Y = 12 ⇒ FV = 933 028.76<br><br>Balance after 1 year = 933 028.76 – 37 000 = \$896 028.76<br><br>N = 26, I = 7.5, PV = –896 028.76, PMT = 3200, P/Y = 26, C/Y = 12 ⇒<br>FV = 879 324.68<br><br>Balance after 2 years = \$879 324.68 |
| **Specific behaviours** |
| ✓ determines at least four correct parameters for first year calculation<br>✓ determines all correct parameters for first year calculation<br>✓ determines correct balance after one year<br>✓ determines correct starting value for second year<br>✓ determines all correct parameters for second year<br>✓ determines correct balance after two years |

(b) After 15 years of retirement Ravi expects to have a less active lifestyle and his living expenses will not be as high. He also understands that the balance of his superannuation needs to last for his remaining years. Ravi decides the most suitable plan for him is to place the balance of his superannuation into a perpetuity with a fortnightly payment.

Assuming the 7.5% interest rate is maintained, what fortnightly amount can Ravi expect from the perpetuity? (5 marks)

| Solution |
| --- |
| After 13 more years<br><br>N = 338, I = 7.5, PV = –879 324.68, PMT = 3200, P/Y = 26, C/Y = 12 ⇒ FV = 498 326.85<br><br>Perpetuity:<br><br>N = 26, I = 7.5, PV = –498 326.85, FV = 498 326.85, P/Y = 26, C/Y = 12<br>⇒ PMT = 1435.07<br>Fortnightly payment of $1435.07 |
| **Specific behaviours** |
| ✓ determines thirteen extra years needed<br>✓ determines all correct parameters<br>✓ determines correct value after fifteen years<br>✓ determines all correct parameters for perpetuity<br>✓ determines correct fortnightly payment |

**2022 Section 2 Question 15**

**Loans, investments and annuities**

Paris, a retired hairdresser, has a superannuation balance of $775 320.

(a) She intends to set up an annual scholarship of $5000 to be awarded to a worthy apprentice. Paris is able to negotiate a fixed annual interest rate of 7.2% through a share trading company. Show that $69 444.44 (to the nearest cent) of Paris's superannuation is needed to be invested in this company to meet the requirements of the annual scholarship. (1 mark)

| Solution |
| --- |
| $\frac{5000}{0.072} = 69\,444.44$ |
| **Specific behaviours** |
| ✓ calculates correct value |

After the scholarship is in place, Paris considers her future investment plans with the remaining balance. Assume a financial advisor has indicated an interest rate of 6.1% per annum, added monthly, can be locked in.

(b) Paris decides on receiving a monthly annuity of $7000, with the interest added before the annuity is paid, at the end of each month.

(i) The annuity can be modelled by the recurrence relation
$T_n+1 = (1 + 0.00508)T_n - 7000$ , $T_0 = a$.
Using the information given in the question, show how to obtain the value 0.00508 and state the value of $a$. (2 marks)

| **Solution** |
| --- |
| $\frac{6.1\%}{12} = 0.00508$ and $a = 775\,320 - 69\,444.44 = 705\,875.56$ |
| **Specific behaviours** |
| ✓ correctly shows how 0.00508 was obtained<br>✓ correctly states the value of $a$ |

(ii) For how many months will Paris be able to receive the annuity of $7000? (1 mark)

| **Solution** |
| --- |
| 141 months |
| **Specific behaviours** |
| ✓ calculates correct number of months |

(iii) Determine the amount of the final payment. (2 marks)

| **Solution** |
| --- |
| 7000 – 2141.58 = 4858.42<br>Therefore, the final payment is $4858.42 |
| **Specific behaviours** |
| ✓ determines amount of over payment<br>✓ determines correct final payment |
| Note: If use $\frac{6.1}{1200}$, the final payment is $5161.09 |

(iv) Determine the total interest received on the annuity. (3 marks)

| **Solution** |
| --- |
| $141 \times 7000 + 4858.42 - 705\,875.56 = 285\,982.86$<br>Therefore, the total interest received is $285 982.86 |
| **Specific behaviours** |
| ✓ uses correct number of months of full payment<br>✓ forms correct calculation<br>✓ correct value |
| Note: If final payment is $5161.09, total interest is $286 285.53 |

(c) Paris also considers receiving a perpetuity after the scholarship is in place. Determine the monthly amount she would receive. (3 marks)

| Solution |
| --- |
| 705 875.56 × 1.00508 − 705 875.56<br>= $3585.85 |
| **Specific behaviours** |
| ✓ correctly calculates monthly interest rate<br>✓ forms correct calculation<br>✓ correct payment |

Alternate solution:

| Solution |
| --- |
| N = 1, I = 6.1, PV = –705 875.56, FV = 705 875.56, P/Y = 12, C/Y = 12<br>PMT = −3588.20<br>Therefore, the monthly amount received is $3588.20 |
| **Specific behaviours** |
| ✓ uses at least 4 correct values for N, I, PV, FV, P/Y, C/Y<br>✓ uses all correct values for N, I, PV, FV, P/Y, C/Y<br>✓ correct PMT |

**2022 Section 2 Question 16**

**Loans, investments and annuities**

After paying a deposit for his new apartment, Declan obtains a bank loan for the remaining amount of $112 000 at 3.26% per annum compounded monthly. He can currently afford to repay $970 per month at the end of every month.

(a) Calculate how much he would owe after the 40th repayment. (3 marks)

| Solution |
| --- |
| Using the financial app with<br>N = 40, I = 3.26, PV = −112 000, PMT = 970, P/Y = C/Y = 12<br>FV = 83 910.19<br>He would owe $83 910.19 after the 40th repayment |
| **Specific behaviours** |
| ✓ states at least 4 correct entries<br>✓ states all correct entries<br>✓ states correct answer |

(b) Declan decided to deposit a one-off extra amount of $1600, after the 16th repayment. Calculate the new amount he would owe after the 40th repayment. (4 marks)

| Solution |
| --- |
| Step 1. Using the financial app<br>N = 16, I = 3.26, PV = −112 000, PMT = 970, P/Y = C/Y = 12<br>FV = $101 128.46<br>New PV = $101 128.46 − 1600 = $99 528.46<br><br>Step 2. Using financial app<br>N = 24(40 − 16), I = 3.26, PV = −99 528.46, PMT = 970, P/Y = C/Y = 12<br>FV = $82 202.54<br><br>New amount owing after the 40th repayment is $82 202.54 |
| **Specific behaviours** |
| ✓ states all correct entries in step 1<br>✓ correctly subtracts 1600 from FV in step 1 to give new PV<br>✓ states all correct entries in step 2<br>✓ correctly determines new FV |

**2021 Section 2 Question 10**

**Loans, investments and annuities**

Wendy moved into an apartment and organised a loan of $16 000 to purchase new furniture. To pay off the loan Wendy makes repayments of $600 at the end of each month. The spreadsheet below shows the progress of her loan.

| Month | Opening balance | Interest | Repayment | Closing balance |
|---|---|---|---|---|
| 1 | 16 000.00 | 98.67 | 600.00 | 15 498.67 |
| 2 | 15 498.67 | 95.58 | 600.00 | 14 994.24 |
| 3 | 14 994.24 | 92.46 | 600.00 | 14 486.71 |
| 4 | | | | |

(a) Write a calculation to show that the yearly interest rate is approximately 7.4%. (2 marks)

**Solution**

$$\frac{98.67}{16\,000}\times 100 = 0.6166875\%$$

$$0.6166875\times 12 = 7.40025\% \approx 7.4\%$$

**Specific behaviours**

✓ correctly calculates monthly interest
✓ correctly calculates yearly interest rate

(b) Complete the fourth row of the spreadsheet. (3 marks)

**Solution**

| Month | Opening balance | Interest | Repayment | Closing balance |
|---|---|---|---|---|
| 1 | 16 000.00 | 98.67 | 600.00 | 15 498.67 |
| 2 | 15 498.67 | 95.58 | 600.00 | 14 994.24 |
| 3 | 14 994.24 | 92.46 | 600.00 | 14 486.71 |
| 4 | **14 486.71** | **89.33** | **600.00** | **13 976.04** |

**Specific behaviours**

✓ correct Opening balance and Repayment
✓ correct Interest
✓ correct Closing balance

(c) Write a recursive rule to determine the closing balance of the loan at the end of each month. (2 marks)

**Solution**

$$T_{n+1} = \left(1+\frac{0.074}{12}\right)T_n - 600,\ T_0 = 16\,000$$

**or**

$$T_{n+1} = 1.00617\,T_n - 600,\ T_0 = 16\,000$$

**Specific behaviours**

✓ states correct rule
✓ states correct term zero

(d) Determine how many months it will take Wendy to pay off the loan. (1 mark)

**Solution**

30 months

**Specific behaviours**

✓ gives correct number of months

(e) Calculate how much interest is paid over the duration of the loan. (3 marks)

| Solution |
|---|
| $30 \times 600 - 465.56 = \$17\,534.44$<br>Interest = $\$17\,534.44 - 16\,000 = 1534.44$<br>**or**<br>$29 \times 600 = 17\,400$<br>$\left(1+\frac{0.074}{12}\right)\times 133.62 = 134.44$<br>$17\,400 + 134.44 = 17\,534.44$<br>Interest = $\$17\,534.44 - 16\,000 = 1534.44$ |
| **Specific behaviours** |
| ✓ 30 months of \$600 less overpayment (or second method)<br>✓ calculates correct total<br>✓ calculates correct interest |

On reflection, Wendy realised she could have repaid \$800 each month.

(f) Determine the maximum amount Wendy would have been able to borrow, if all other details of the loan and repayment time remained the same. (3 marks)

| Solution |
|---|
| N = 30, I = 7.4, PMT = 800, FV = 0, P/Y = 12, C/Y = 12<br>PV = −21849.52<br>∴ Wendy could borrow \$21 849.52 |
| **Specific behaviours** |
| ✓ uses at least 4 correct values for N, I, PMT, FV, P/Y, C/Y<br>✓ uses all correct values for N, I, PMT, FV, P/Y, C/Y<br>✓ determines correct value for PV |

**2021 Section 2 Question 12**

**Loans, investments and annuities**

Virat purchases a new motor vehicle for \$24 500. For the first two years the vehicle depreciates at a rate of 13% per year and for the third year it depreciates at a lower rate of 9.5% per year.

(a) Calculate the value of the vehicle after one year. (1 mark)

| Solution |
|---|
| $24\,500(1-0.13)^1 = \$21\,315$ |
| **Specific behaviours** |
| ✓ correctly calculates the value of the vehicle after 1 year |

(b) Calculate the value of the vehicle after the first three years. (2 marks)

| Solution |
|---|
| $24\,500(1-0.13)^2(1-0.095)^1 = \$16\,782.37$ |
| **Specific behaviours** |
| ✓ correctly calculates the value of the vehicle after 2 years<br>✓ calculates the value of the vehicle after 3 years using the correct rate of depreciation for the third year |

For the next three years the rate of depreciation is constant at $r$% per year. The average rate of depreciation for the first six years is 11% per year.

(c) Calculate the value of r as a percentage. (3 marks)

| **Solution** |
| --- |
| Value of vehicle at the end of 6 years using average rate of depreciation:<br>$24\,500(1-0.11)^6 = \$12\,176.04$<br><br>Therefore $\$12176.04 = 16782.37\left(1-\frac{r}{100}\right)^3 \Rightarrow r = 10.14\%$ |
| **Specific behaviours** |
| ✓ correctly calculates the value of the vehicle after 6 years using average rate of depreciation<br>✓ correctly sets up an equation to solve for $r$<br>✓ correctly determines the value of $r$ |
| |

**2021 Section 2 Question 14**

**Loans, investments and annuities**

Patrick has retired and invested his lump sum superannuation payout of $717 850 at a rate of 5.7% per annum compounded monthly. He begins the investment strategy from 1 January.

(a) Patrick will receive $4500 at the end of each month for general living expenses and will also receive a further $4000 at the end of each year for an annual holiday.

(i) Identify this type of investment account. (1 mark)

| Solution |
|---|
| Annuity |
| **Specific behaviours** |
| ✓ states correct answer |

(ii) Determine the balance in the account at the end of the first year. (4 marks)

| Solution |
|---|
| N = 12, I = 5.7, PV = –717 850, PMT = 4500, P/Y = 12, C/Y = 12<br>FV = 704 420.20<br>Balance at end of year 1 = 704 420.20 – 4000 = $700 420.20 |
| **Specific behaviours** |
| ✓ uses at least 4 correct values for N, I, PV, PMT, P/Y, C/Y<br>✓ uses all correct values for N, I, PV, PMT, P/Y, C/Y<br>✓ determines correct value for FV<br>✓ determines correct end of year balance |

(iii) Determine the balance in the account at the end of the second year. (3 marks)

| Solution |
|---|
| N = 12, I = 5.7, PV = –700 420.20, PMT = 4500, P/Y = 12, C/Y = 12<br>FV = 685 970.53<br>Balance at end of year 2 = $685 970.53 – 4000 = $681 970.53 |
| **Specific behaviours** |
| ✓ uses correct value for PV<br>✓ determines correct FV<br>✓ determines correct end of year 2 balance |

(b) When Patrick retired, he also considered the option of setting up a perpetuity with his superannuation payout still at 5.7% per annum compounded monthly. Calculate the quarterly payments Patrick would have received with this perpetuity in place. (3 marks)

| Solution |
|---|
| N = 2 (can be any value), I = 5.7, PV = –717 850, FV = 717 850, P/Y = 4, C/Y = 12<br>Quarterly payments = $10 278.03 |
| **Specific behaviours** |
| ✓ uses at least 4 correct values for N, I, PV, FV, P/Y, C/Y<br>✓ uses all correct values for N, I, PV, FV, P/Y, C/Y<br>✓ states correct quarterly payments |

**2020 Section 2 Question 9**

**Loans, investments and annuities**

Giuseppe wishes to set up an annuity. He is told that an annuity with quarterly investment returns and quarterly payments is modelled by the recursive rule:

$A_{n+1} = A_n \times 1.019 - P$, $A_0 = Q$ with the values of $P$ and $Q$ consistent with the spreadsheet below.

| Quarter | Opening balance | Investment gain | Payment | Closing balance |
|---|---|---|---|---|
| 1 | \$648 000 | \$12 312 | \$15 000 | $X$ |
| 2 | | $Y$ | \$15 000 | |
| 3 | | | | |

(a) Determine the values of $P$, $Q$, $X$ and $Y$ and write them in the table below. (4 marks)

| $P$ | $Q$ | $X$ | $Y$ |
|---|---|---|---|
| \$15 000 | \$648 000 | \$645 312 | \$12 260.93 |

| Solution |
|---|
| See table above |
| **Specific behaviours** |
| ✓ correctly determines the value of $P$<br>✓ correctly determines the value of $Q$<br>✓ correctly determines the value of $X$<br>✓ correctly determines the value of $Y$ |

(b) What is the annual compound interest rate for this investment? (1 mark)

| Solution |
|---|
| $1.9 \times 4 = 7.6\%$ |
| **Specific behaviours** |
| ✓ correctly calculates the annual interest rate |

When the balance in the annuity first falls below \$300 000, Giuseppe converts the payment to a perpetuity so that his children are left with some inherited benefits. The interest rate remains the same as that calculated in part (b).

(c) Determine the number of years the annuity operates before the perpetuity starts. (2 marks)

| Solution |
|---|
| Using the recursive rule, n = 66 and the balance is \$299 501.07, so it would last 66 quarters, i.e. 16.5 years. |
| **Specific behaviours** |
| ✓ correctly determines the value for n = 66<br>✓ correctly states the number of years |

(d) What are the quarterly payments under this perpetuity? (2 marks)

| Solution |
|---|
| I = 7.6, PV = –299 501.07, FV = 299 501.07, P/Y = C/Y = 4<br>which gives PMT = 5690.52<br>**or**<br>0.019 × 299 501.07<br>= 5690.52<br>Quarterly payments will be $5690.52 |
| **Specific behaviours** |
| ✓ uses correct values for I, PV, PMT, P/Y, C/Y<br>✓ correctly calculates the quarterly payments |

(e) Giuseppe believes that his investment returns are at an effective interest rate of 7.93% p.a. Use a clear calculation to comment on the accuracy of this belief. (2 marks)

| Solution |
|---|
| $i_{effective} = \left(1 + \frac{0.076}{4}\right)^4 = 1.0782$<br>i.e. effective interest rate is 7.82%, therefore his belief is not true |
| **Specific behaviours** |
| ✓ correctly calculates the effective interest rate<br>✓ correctly concludes that the belief is not true |

**2020 Section 2 Question 11**

**Loans, investments and annuities**

Shari requires a loan of $325 000 for the purchase of a new house. She wishes to make two equally-spaced repayments of $700 each month.

Shari is offered a choice of two loan options for the first three years, both of which have interest calculated daily.

- Option 1 An introductory compound interest rate of 2.55% per annum for the first year which changes to 2.99% per annum for the next two years.
- Option 2 A compound interest rate of 2.85% per annum fixed for the first three years.

(a) Describe briefly the benefit of making two repayments of $700 each month instead of one repayment of $1400 at the end of each month. (1 mark)

| Solution |
|---|
| Loan amount reduces at a quicker rate or less interest is paid |
| **Specific behaviours** |
| ✓ correct answer |

(b) For Option 1, calculate

(i) the loan balance at the end of the first year. (3 marks)

| Solution |
|---|
| N = 24, I = 2.55, PV = 325 000, PMT = –700, P/Y = 24, C/Y = 365<br><br>Loan balance = $316 386.79 |
| **Specific behaviours** |
| ✓ uses at least 4 correct values for N, I, PV, PMT, P/Y, C/Y<br>✓ uses all correct values for N, I, PV, PMT, P/Y, C/Y<br>✓ correctly determines balance |

(ii) the loan balance at the end of the third year. (2 marks)

| Solution |
|---|
| N = 48, I = 2.99, PV = 316 386.79, PMT = –700, P/Y = 24, C/Y = 365<br><br>Loan balance = $301 279.69 |
| **Specific behaviours** |
| ✓ uses correct values for N, I, PV<br>✓ correctly determines balance |

(c) Determine which option gives the best result for Shari after three years and by how much. (3 marks)

| Solution |
|---|
| N = 72, I = 2.85, PV = 325 000, PMT = –700, P/Y = 24, C/Y = 365<br><br>Loan balance = $301 422.81<br><br>Option 1 best by 301 422.81 – 301 279.69 = $143.12 |
| **Specific behaviours** |
| ✓ uses all correct values for N, I, PV, PMT, P/Y, C/Y<br>✓ correctly determines balance for option 2<br>✓ correctly determines Option 1 is best by $143.12 |

**2020 Section 2 Question 12**

**Loans, investments and annuities**

Jessica wants to borrow $15 000 from her parents to purchase a car. They will be charging her compound interest at the rate of 4% per annum, with interest added yearly.

(a) Jessica is currently studying so she will not want to be making any regular repayments.

(i) Complete the table below to show the amount she will owe her parents at the end of each year. (2 marks)

| **Number of years ($n$)** | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| **Amount owing ($)** | 15 000 | | | |

(ii) Write a recursive rule to calculate the amount owing at the end of each year. (2 marks)

| Solution |
|---|
| $A_{n+1} = 1.04A_n, \quad A_0 = 15\,000$ |
| **Specific behaviours** |
| ✓ states recursive part of rule<br>✓ states starting value |

Jessica's parents are encouraging her to get a part-time job so that she can make repayments along the way. Jessica estimates that she will be able to earn enough money to pay off $2400 each year.

(b) If interest is charged yearly and she repays the $2400 at the end of each year, write a recursive rule to calculate the amount owing at the end of each year. (1 mark)

| Solution |
| --- |
| $T_{n+1} = 1.04T_n - 2400, T_0 = 15000$ |
| **Specific behaviours** |
| ✓ states the correct recursive rule |

(c) If interest is charged monthly and she makes equal monthly repayments,

(i) write a recursive rule to calculate the amount owing at the end of each month.(1 mark)

| Solution |
| --- |
| $B_{n+1} = 1.0033B_n - 200, B_0 = 15000$ |
| **Specific behaviours** |
| ✓ states the correct recursive rule |

(ii) calculate how many months it will take to repay the loan. (1 mark)

| Solution |
| --- |
| 87 months |
| **Specific behaviours** |
| ✓ states the correct number of months |

(iii) calculate the total amount Jessica would pay over the duration of the loan. (3 marks)

| Solution |
| --- |
| Amount owing at end of 86th month = $89.46<br>Interest for 87th month = $89.46 \times \frac{0.04}{12} = 0.30$<br>Total paid = (86 × 200) + (89.46 + 0.30) = $17 289.76 |
| **Specific behaviours** |
| ✓ correctly determines amount owing at end of 86th month<br>✓ correctly determines interest for 87th month<br>✓ correctly states total amount paid |

**2019 Section 2 Question 9**

**Loans, investments and annuities**

Corbie and Grant are investigating the possibility of purchasing their own apartment. They will need to open a bank account to save for a deposit. They make an initial contribution of $7000 into the account and add a further $800 at the end of each month. The account has an interest rate of 2.6% per annum, compounded monthly.

(a) How much would be in the account at the end of two years? (3 marks)

| Solution |
|---|
| N = 24<br>I = 2.6<br>PV = –7000<br>PMT = –800 FV = $27 059.30<br>P/Y = 12<br>C/Y = 12<br>**or** $T_{n+1} = T_n\left(1+\frac{2.6}{1200}\right)+800, T_0 = 7000$<br>$T_{24} = 27059.30$ |
| **Specific behaviours** |
| ✓ states correct value for N<br>✓ states correct values for PV and PMT (both negative or both positive)<br>✓ determines correct future value |

Corbie and Grant are willing to pay $280 000 for the apartment and will require an additional $22 000 in fees associated with the purchase. A deposit of 20% of the total cost will be needed in order to qualify for a bank loan.

(b) (i) Show that the required deposit is $60 400. (1 mark)

| Solution |
|---|
| 20% of (280 000 + 22 000) = $60 400 |
| **Specific behaviours** |
| ✓ shows how to calculate the deposit of $60400 |

(ii) How long would it take to save enough for the deposit? (2 marks)

| Solution |
|---|
| I = 2.6<br>PV = –7000<br>PMT = –800<br>FV = 60 400 N = 61.3 months Therefore it would take 62 months.<br>P/Y = 12<br>C/Y = 12 |
| **Specific behaviours** |
| ✓ Uses correct FV in calculations<br>✓ determines correct time |

(c) If, at the end of two years, their parents agree to give them a total of $10 000 as a reward for their dedicated saving effort, determine the minimum monthly contribution Corbie and Grant will need to make if they are to have enough for the full deposit after four years. (3 marks)

| Solution |
|---|
| N = 24<br>I = 2.6<br>PV = –37 059.30<br>FV = 60 400 PMT = $868.22<br>P/Y = 12<br>C/Y = 12 |
| **Specific behaviours** |
| ✓ states correct value for PV<br>✓ states correct value for N<br>✓ determines correct payment |

**2019 Section 2 Question 13**

**Loans, investments and annuities**

Mehmet has saved $3600 from wages received at a part-time job. He is keen to invest this money in an account which earns 3.65% per annum, compounded monthly.

Over the next three years, Mehmet plans to continue working part-time and is aiming to make deposits of $250 at the end of each month.

(a) Write a recursive relation to give the value of the investment at the end of each month. (2 marks)

| Solution |
|---|
| $T_{n+1} = T_n + \frac{0.0365}{12} \times T_n + 250, \quad T_0 = 3600$ |
| **Specific behaviours** |
| ✓ states recursive part of rule<br>✓ states correct starting value |

(b) Mehmet hopes that this investment will double his initial savings in one year. Justify whether this is possible. (2 marks)

| Solution |
|---|
| At the end of the 12th month Mehmet has only saved $6784.32, therefore he does not double his money in one year. |
| **Specific behaviours** |
| ✓ correctly states that Mehmet does not save $7200 in one year<br>✓ correctly justifies this decision |

(c) Determine the total amount of interest Mehmet would receive after three years. (3 marks)

| Solution |
|---|
| Value = 13 511.92<br>Deposits = $3600 + 36 \times 250 = 12\,600$<br>Interest = 13 511.92 – 12600 = $911.92 |
| **Specific behaviours** |
| ✓ correctly determines final value<br>✓ correctly determines total of all deposits<br>✓ correctly determines interest received |

Unfortunately, after two years, Mehmet's working hours are reduced and he is only able to deposit $120 at the end of each month.

(d) By how much would this reduce the value of his investment by the end of the three years? (3 marks)

| Solution |
| --- |
| Value after 24 months = $10 086.83<br>Value after next 12 months = $11 925.56<br>Difference = 13 511.92 − 11 925.56 = $1586.36<br>i.e. $1586.36 less |
| **Specific behaviours** |
| ✓ correctly determines value after two years<br>✓ correctly determines value after next 12 months<br>✓ correctly determines the difference between future values |

**2019 Section 2 Question 17**

**Loans, investments and annuities**

Joel has set up a special investment fund that has a current balance of $350 000. He contributes 7.5% of his monthly income to the investment and has an overseas pension which contributes a further $355 per month. The investment fund has an interest rate of 6.5% per annum, compounded monthly. Joel's annual salary is $101 000 and he has just turned 60 years of age.

(a) Calculate Joel's total monthly contribution to the fund. (2 marks)

| Solution |
| --- |
| Monthly amount = $0.075 \times \frac{101000}{12} + 355 = \$986.25$ |
| **Specific behaviours** |
| ✓ calculates correct monthly income contribution<br>✓ calculates correct total monthly contribution |

(b) Calculate the lump sum that he could receive if he retires on his 67th birthday. (2 marks)

| Solution |
| --- |
| N = 12 × 7, I = 6.5, PV = 350 000, PMT = 986.25, P/Y = C/Y = 12<br><br>FV = $655 539.45 |
| **Specific behaviours** |
| ✓ correctly uses positive (or negative) values for both PV and PMT<br>✓ calculates correct lump sum |

Joel retires at 67 and wants to use his lump sum payment to set up a regular income. He decides to look at two options that offer monthly payments.

Option 1: A reducing balance annuity at 7% per annum, compounded monthly.
Option 2: A perpetuity at 7.5% per annum, compounded monthly.

(c) Calculate his maximum monthly income for the next 20 years using Option 1. (2 marks)

| **Solution** |
| --- |
| N = 12 × 20, I = 7, PV = 655 539.45, FV = 0, P/Y = C/Y = 12<br><br>PMT = $5082.39 |
| **Specific behaviours** |
| ✓ uses correct values for N and FV<br>✓ calculates correct monthly income |

(d) Calculate his monthly income using Option 2. (2 marks)

| **Solution** |
| --- |
| N = (any positive integer), I = 7.5, PV = 655 539.45, FV = –655 539.45, P/Y = C/Y = 12<br><br>PMT = $4097.12 |
| **Specific behaviours** |
| ✓ uses correct values for PV and FV<br>✓ calculates correct monthly income |

# Unit 4.3 – Networks and decision mathematics

## Section 1

**2023 Section 1 Question 4**

**Networks and decision math-ematics**

The following tables show activities and their completion requirements for a project to build a shearing shed.

| Activity | Activity description |
|---|---|
| A | Design the shearing shed |
| B | Prepare the site |
| C | Purchase materials |
| D | Construct the frame and walls |
| E | Install roof and windows |
| F | Add ventilation and insulation |
| G | Build the shearing stand |
| H | Install electrics |
| J | Paint and finish the shed |
| K | Install shearing equipment |

| Requirements |
|---|
| A, B and C start the project together |
| D starts when A is completed |
| E starts when B is completed |
| F starts when C is completed |
| G starts when D and E are completed |
| J starts when F is completed |
| H starts when D, E and F are completed |
| K starts when G, H and J are completed |

Draw the project network for the information given in the tables. (4 marks)

**2023 Section 1 Question 5**

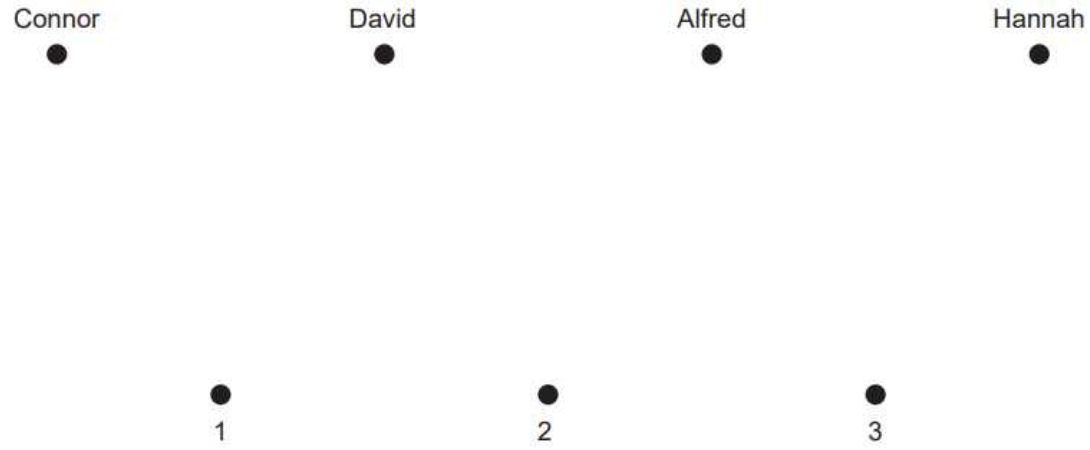
**Networks and decision math-ematics**

Four camp leaders, Connor, David, Alfred and Hannah, are responsible for setting up the camp site for the upcoming school camp. Today there are three tasks available. Each task will only have one camp leader assigned to it.

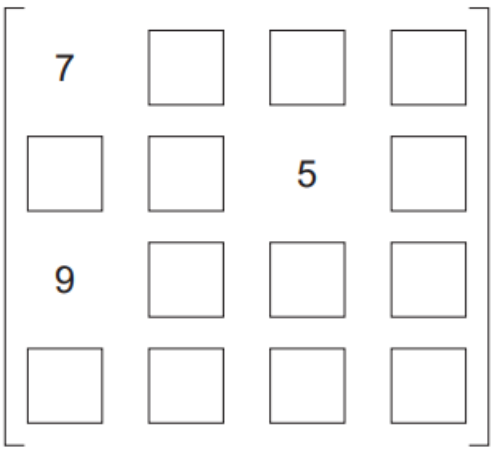
The number of hours each camp leader takes to complete each task is shown in the table below.

| Camp leader | Task 1 | Task 2 | Task 3 |
|---|---|---|---|
| Connor | 7 | 5 | 8 |
| David | 3 | 8 | 5 |
| Alfred | 9 | 6 | 7 |
| Hannah | 8 | 8 | 6 |

(a) Draw a weighted bipartite graph showing all the possible allocations for each of the camp leaders. (2 marks)




(b) Complete the 4 x 4 matrix below to represent the allocation of tasks to camp leaders. (2 marks)




(c) Use the Hungarian algorithm to determine the allocation of camp leader to the task that will minimise the time taken. (4 marks)

(d) Show the task allocated to each camp leader and calculate the total time taken to complete all tasks. (2 marks)

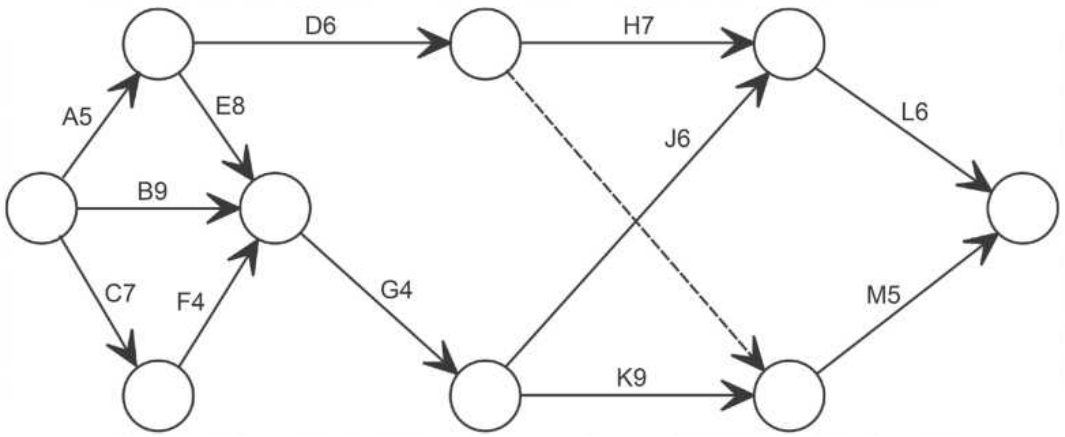
| Camp leader | Connor | David | Alfred | Hannah |
|---|---|---|---|---|
| Task | | | | |

Total time: _____

**2023 Section 1 Question 6**

**Networks and decision math-ematics**

A landscape architect has produced the following project network for the development of a community market garden. The digraph shows the order of completion of the various tasks and their expected completion time in hours.




(a) Complete the immediate predecessor/s column in the table below. (3 marks)

| Task | Time (hours) | Immediate predecessor/s |
|---|---|---|
| A | 5 | |
| B | 9 | |
| C | 7 | |
| D | 6 | |
| E | 8 | |
| F | 4 | |
| G | 4 | |
| H | 7 | |
| J | 6 | |
| K | 9 | |
| L | 6 | |
| M | 5 | |

(b) Determine the critical path and the minimum completion time for the project. Workings must be shown to verify your answer. (3 marks)

(c) Determine which task/s have a float time of exactly 2 hours. (2 marks)

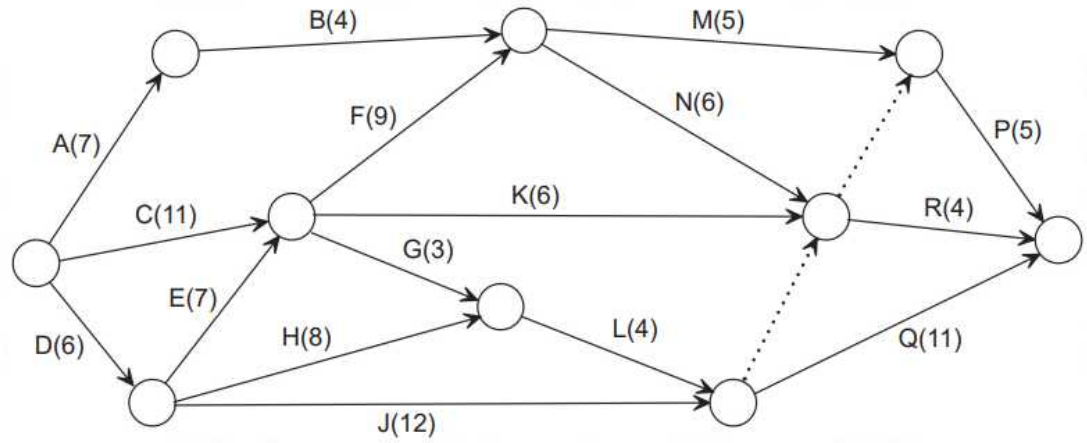
(d) Describe why Task D can be delayed by 6 hours and not affect the minimum completion time. (2 marks)

(e) Due to the release of a new piece of technology for reticulation control, Task G is no longer required. Redraw the network showing how the removal of Task G will change the configuration of the network. Task times are **not** required to be shown. (2 marks)

**2022 Section 1 Question 4**

**Networks and decision math-ematics**

The project network shown below is to be followed when installing a new conveyor belt system for a mining company. The tasks, with their completion time in days, are shown on the network. The project will commence on 20 November, with work continuing for seven days each week.

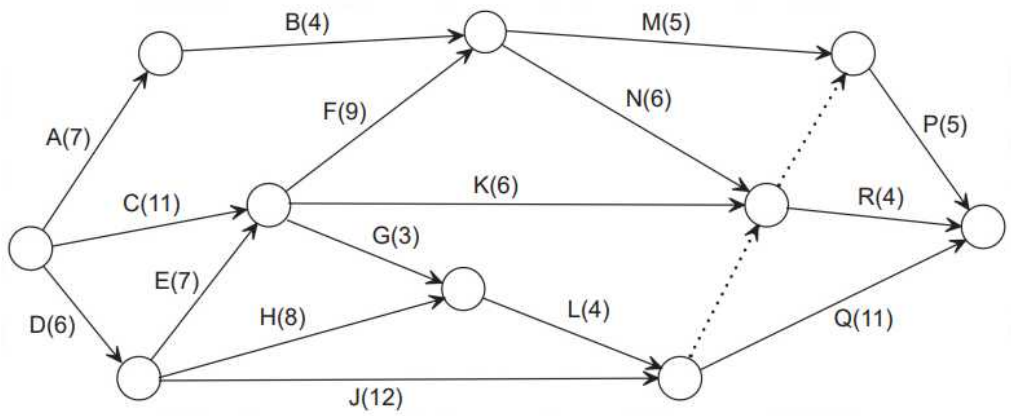



(a) Determine the critical path and the minimum completion time. (2 marks)

(b) Determine the immediate predecessors for Task P. (2 marks)

(c) What is the latest date in November that Task H can commence? (2 marks)

(d) It has been confirmed that there is a delay in obtaining some parts from overseas. This means Task G will take 5 days longer to complete. Discuss all implications for the completion of the project. A copy of the **original** network is given below. (3 marks)




**2022 Section 1 Question 6**

**Networks and decision math-ematics**

A company has four different models of robotic welders that are used to assemble components that require welding. Each of these welders must be set up to assemble just one type of component. The number of components that each welder can assemble per hour for the three different types of components required is shown below.

| | | Welders | | | |
|---|---|---|---|---|---|
| | | **A** | **B** | **C** | **D** |
| **Components** | **1** | 15 | 12 | 13 | 11 |
| | **2** | 13 | 16 | 13 | 9 |
| | **3** | 15 | 16 | 12 | 14 |

(a) Use the Hungarian Algorithm to determine the allocation of welder to component that will maximise the number of components assembled. (6 marks)

(b) State how many of each component will be produced each hour with the allocation determined in part (a). (2 marks)

**2022 Section 1 Question 7**

**Networks and decision math-ematics**

The following table shows activities and their immediate predecessors for a project to develop, produce and sell a new interactive game for children.

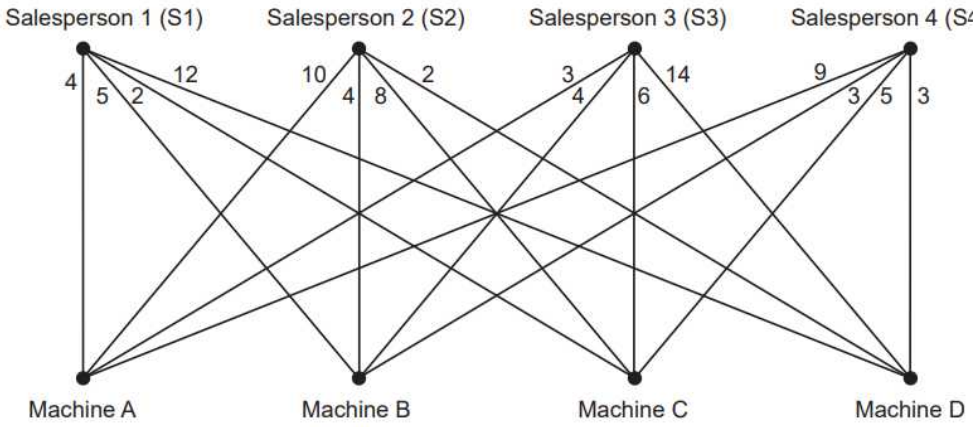
| Activity | Activity description | Immediate Predecessor |
|---|---|---|
| A | Market analysis | - |
| B | Product design | A |
| C | Manufacture product | B |
| D | Develop software | B |
| E | Software testing | C, D |
| F | Design and produce packaging | C |
| G | Assemble product | E, F |
| H | Sales promotion | G |

Draw the project network for the information given in the table.

**2020 Section 1 Question 3**

**Networks and decision math-ematics**

The bipartite graph below shows the average number of sales per day of four specialist pieces of machinery by four salespersons. The salespersons went through a rotation process of spending a number of full days selling only one type of machine before moving to the next. The company that makes these machines wants to maximise the sales by allocating the sales of each machine to only one person.




(a) Complete the matrix below showing the information from the bipartite graph. (1 mark)

| | A | B | C | D |
|---|---|---|---|---|
| S1 | 4 | 5 | 2 | 12 |
| S2 | 10 | 4 | 8 | 2 |
| S3 | 3 | 4 | | |
| S4 | | | | |

(b) Show use of the Hungarian algorithm to allocate each salesperson to a specialist machine in order to maximise sales. The allocation is to be written in the table below. (5 marks)

| Salesperson | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Machine | | | | |

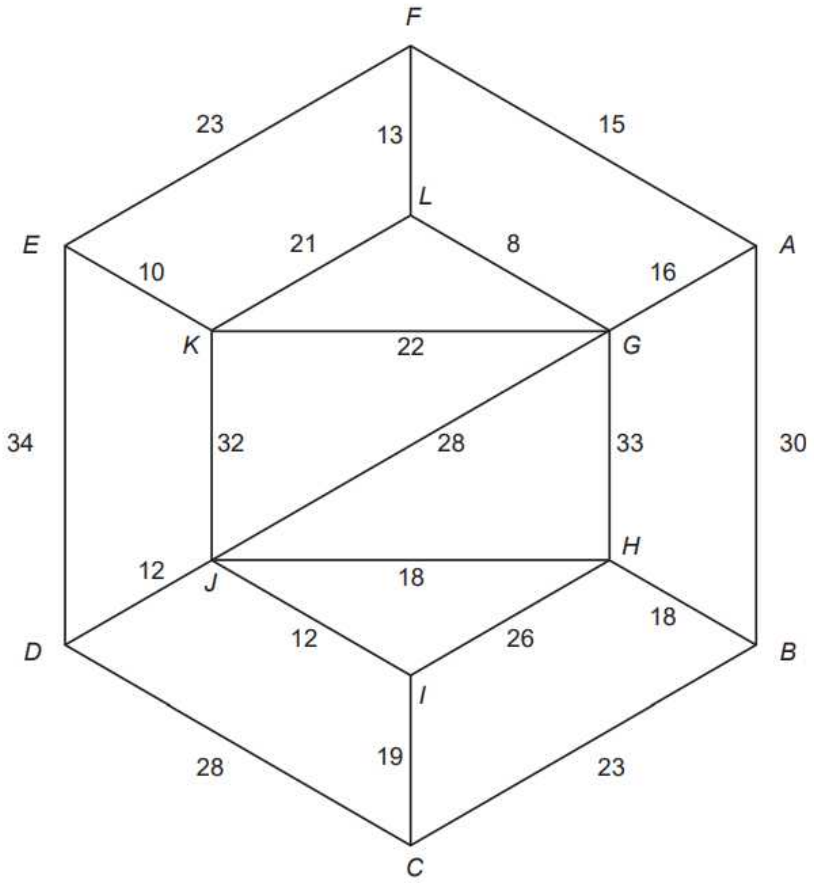
(c) How many more sales are made by using the allocation in part (b) compared to the allocation: S1 – A, S2 – B, S3 – C and S4 – D? (2 marks)

**2020 Section 1 Question 5**

**Networks and decision math-ematics**

A communication wi-fi network is to be installed to service a shopping centre connecting 12 shops located at vertices *A, B, C, D, E, F, G, H, I, J, K* and *L*. The only practical connections between vertices are shown on the following network.

The number on each edge is the quoted price, in hundreds of dollars, for the direct link between the vertices.

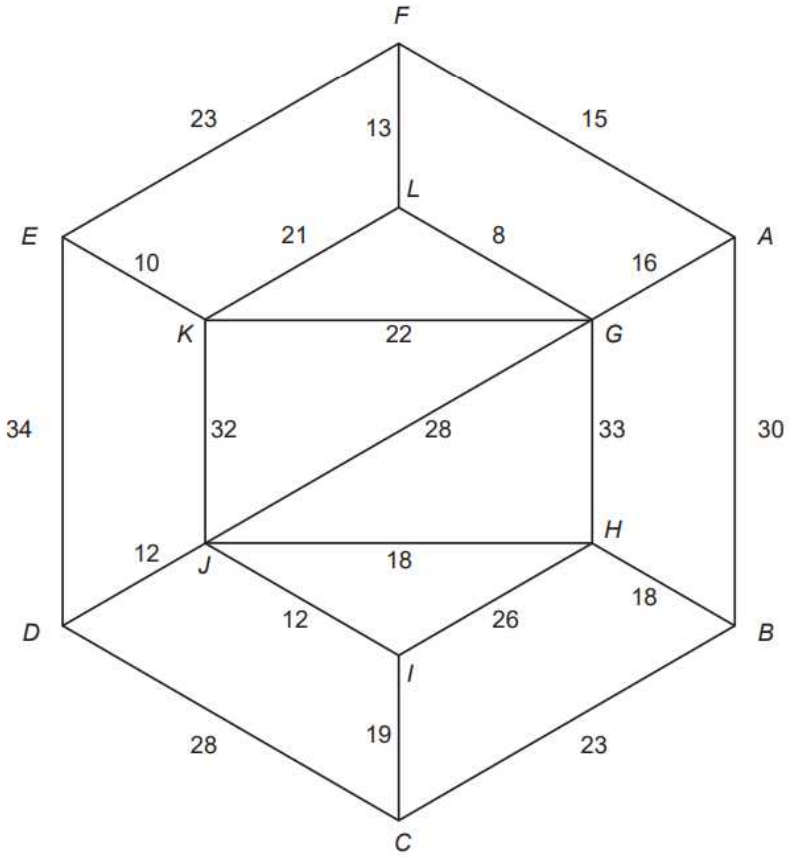



(a) A minimal spanning tree is to be used to determine the minimum cost of this installation.

(i) Show clearly on the network the minimum spanning tree solution. (3 marks)

(ii) Determine the minimum cost. (2 marks)

(b) Due to further construction at the shopping centre, edge GJ is now not feasible. Explain how this will change the solution for part (a). (2 marks)



**2019 Section 1 Question 3**

**Networks and decision math-ematics**

A company has four small workshops that each produce four different types of outdoor furniture. The annual cost of production of the furniture at each workshop is shown in the table below, with all values in thousands of dollars.

| | Type 1 $'000 | Type 2 $'000 | Type 3 $'000 | Type 4 $'000 |
|---|---|---|---|---|
| **Workshop A** | 25 | 43 | 50 | 39 |
| **Workshop B** | 33 | 31 | 56 | 39 |
| **Workshop C** | 28 | 47 | 59 | 38 |
| **Workshop D** | 36 | 32 | 56 | 41 |

The cost matrix is given by

$$\begin{bmatrix} 25 & 43 & 50 & 39 \\ 33 & 31 & 56 & 39 \\ 28 & 47 & 59 & 38 \\ 36 & 32 & 56 & 41 \end{bmatrix}$$

The company is interested in knowing what the minimum annual cost would be if each furniture type was allocated to its own individual workshop. The Hungarian Algorithm is to be used to determine the allocation and the minimum annual cost. The first step of the Hungarian Algorithm, where the smallest number in each row is subtracted from all other numbers in that row, is shown below.

(a) Continue the steps of the Hungarian Algorithm to determine the appropriate allocation of workshops to furniture type and state the **minimum** annual cost. (5 marks)

$$\begin{bmatrix} 0 & 18 & 25 & 14 \\ 2 & 0 & 25 & 8 \\ 0 & 19 & 31 & 10 \\ 4 & 0 & 24 & 9 \end{bmatrix}$$

| **Type** | Type 1 | Type 2 | Type 3 | Type 4 |
|---|---|---|---|---|
| **Workshop** | | | | |

Total minimum annual cost _____

The revenue matrix, in thousands of dollars, for the sale of the furniture produced annually at each workshop is given by

$$\begin{bmatrix} 37 & 61 & 60 & 53 \\ 45 & 52 & 73 & 50 \\ 38 & 65 & 75 & 55 \\ 44 & 54 & 76 & 45 \end{bmatrix}$$

(b) Given that *Profit* = *Revenue* – *Cost*, complete the Profit matrix below. (1 mark)

$$\text{Profit matrix} = \begin{bmatrix} 12 & 18 & 10 & 14 \\ 12 & 21 & 17 & 11 \\ 10 & 18 & & \end{bmatrix}$$

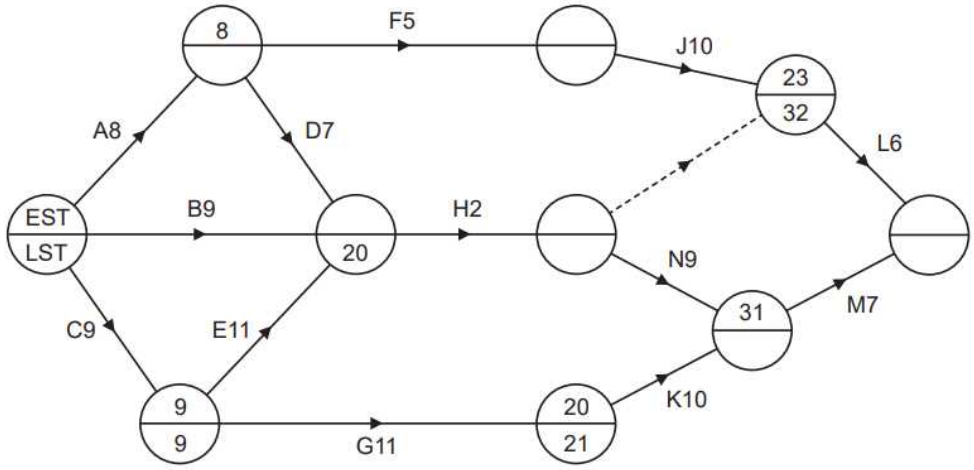
(c) Use the Hungarian Algorithm to determine the appropriate allocation of workshops to furniture type that will produce the **maximum** annual profit. (4 marks)

| **Type** | Type 1 | Type 2 | Type 3 | Type 4 |
|---|---|---|---|---|
| **Workshop** | | | | |

**2019 Section 1 Question 5**

**Networks and decision math-ematics**

The network below represents a construction project. The number on each edge gives the time, in hours, to complete the activity. Each activity requires one worker.




(a) Complete the precedence table below. (2 marks)

| Activity | A | B | C | D | E | F | G | H | J | K | L | M | N |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Time (hours) | 8 | 9 | 9 | 7 | 11 | 5 | 11 | 2 | 10 | 10 | 6 | 7 | 9 |
| Immediate predecessor | - | - | - | A | C | A | C | B,D,E | | | | | |

(b) Complete the network showing the earliest starting time (EST) and latest starting time (LST) for each node. (Note: the first node indicates which is the EST and the LST.) (2 marks)

(c) Determine the critical path and the minimum completion time for the project. (2 marks)

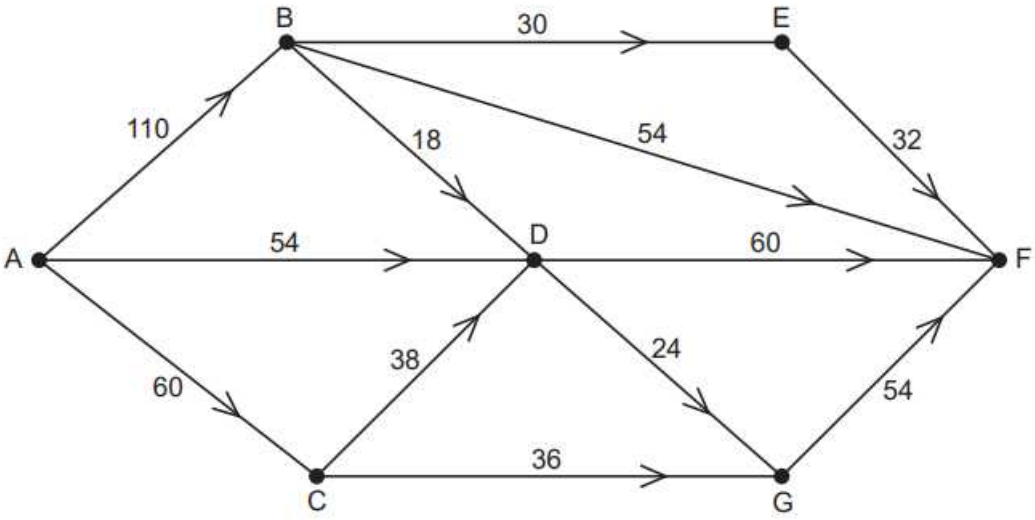
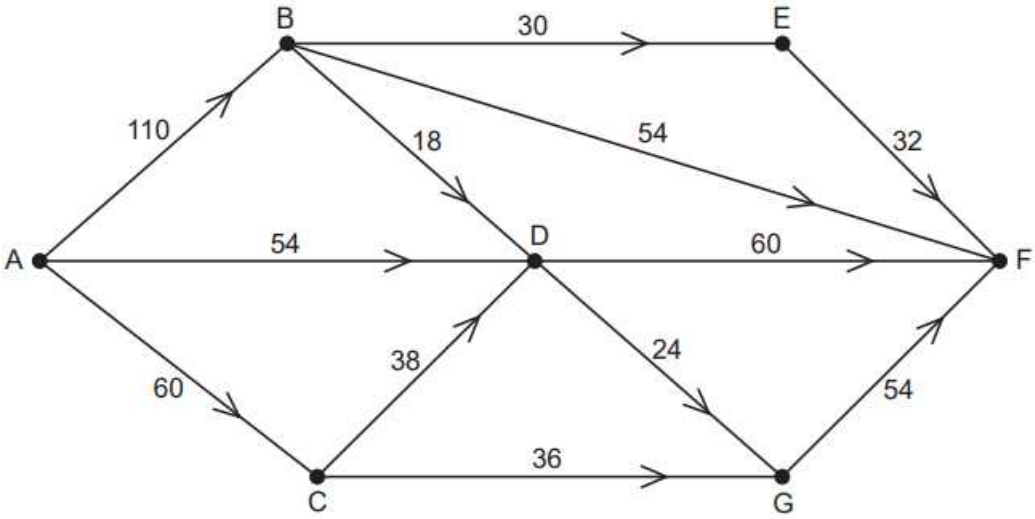
(d) Calculate the float times for Activities D and F. (2 marks)

(e) Given that the sum of all the times of the activities is 104 hours, calculate the minimum number of workers required to complete the project in the minimum completion time. (1 mark)

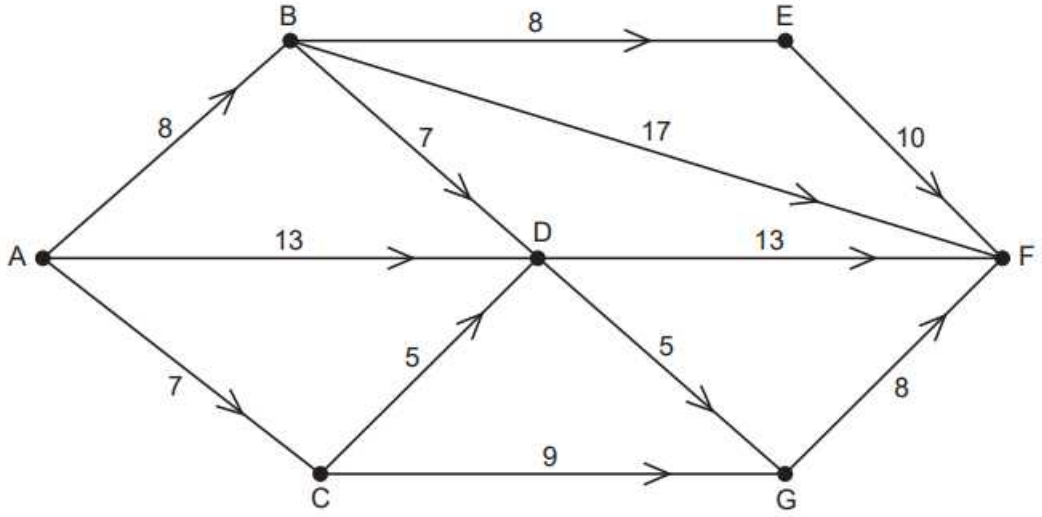
(f) What is the latest time into the project that Activity F could start without affecting the minimum completion time? (1 mark)

(g) Explain the purpose of the dotted line on the network. (1 mark)

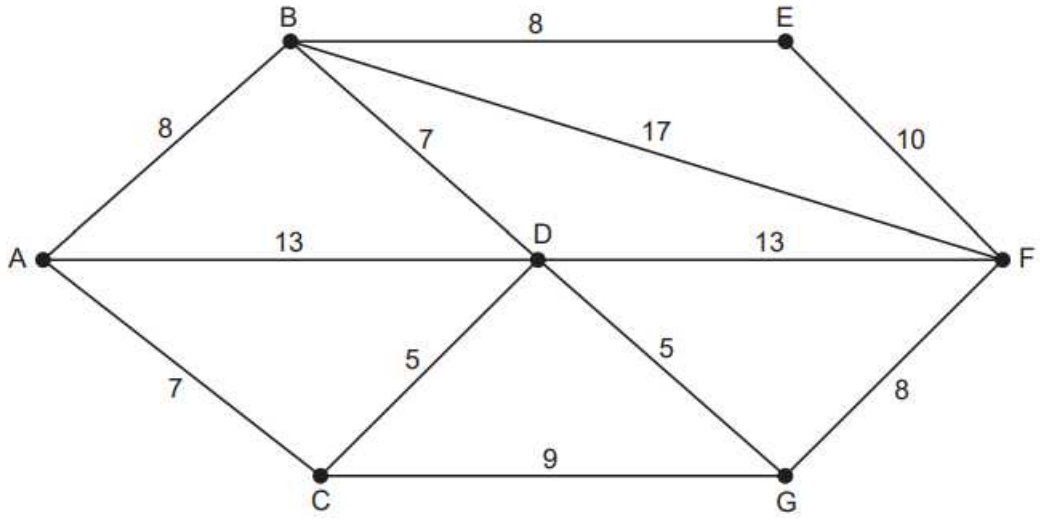
## Section 2

| | |
|---|---|
| **2023 Section 2 Question 11**<br><br>**Networks and decision math-ematics** | A ferry service connects a group of islands in the Pacific Ocean.<br><br>(a) The network below is for the hourly departures from A and shows the number of passengers that can be carried between each island, A, B, C, D, E, F and G. Determine the maximum number of passengers that can be carried from A to F each hour. Show systematic workings. (4 marks)<br><br>  <br><br><br>(b) On the network below, draw the minimum cut that corresponds to the maximum flow determined in part (a). (1 mark)<br><br>  <br> |

(c) The network below shows the distance in kilometres between the islands. Ferry passengers are charged a $10 booking fee plus $1.50 for each kilometre they plan to travel. Calculate the minimum cost of travelling from A to F. (3 marks)



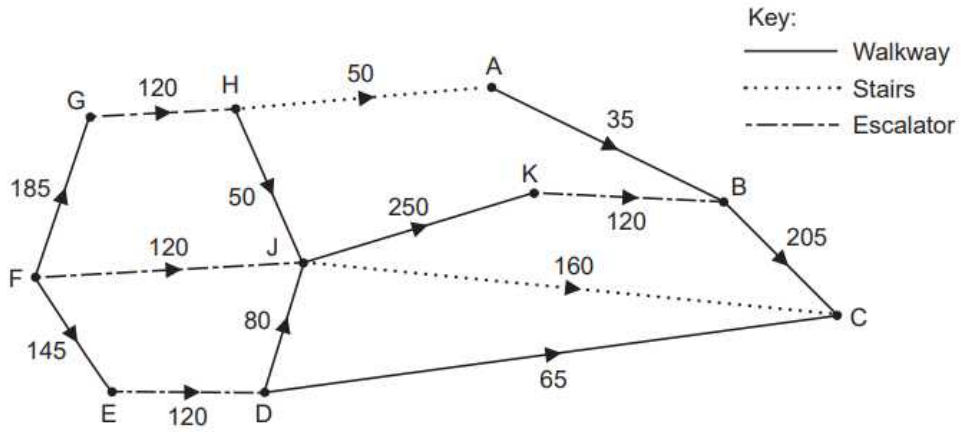

(d) During the off-season when there are fewer passengers, the ferry company will only maintain the services that form the minimum spanning tree. On the network below, highlight clearly the services that will be maintained. (2 marks)



**2022 Section 2 Question 12**

**Networks and decision math-ematics**

The network below shows the different routes commuters can take, travelling from a bus stop to a designated train platform. The edges are a combination of stairs, escalators and covered walkways, with the numbers indicating the maximum number of people per minute that each route can manage.



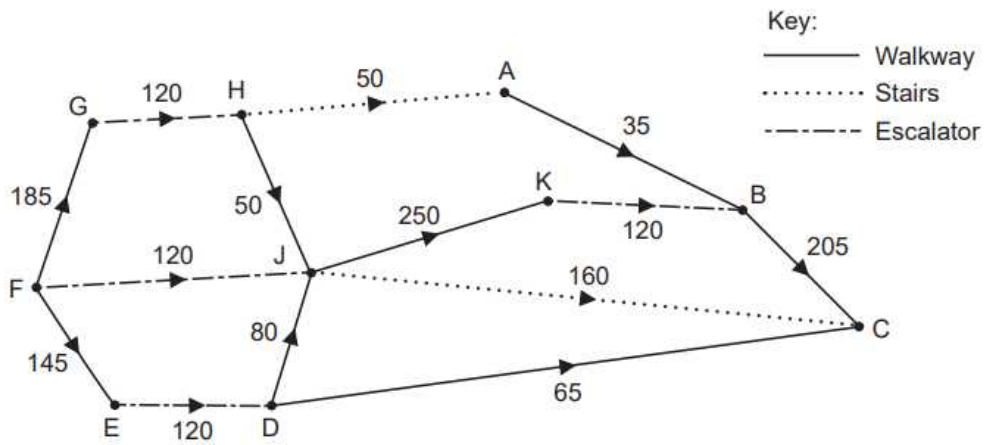

(a) Which vertex represents

(i) the bus stop? (1 mark)

(ii) the train platform? (1 mark)

(b) Determine the maximum number of people per minute that can travel from the bus stop to the train platform, listing each path and the corresponding flow. (3 marks)

(c) Draw the minimum cut that corresponds to the maximum flow on the network diagram. (1 mark)

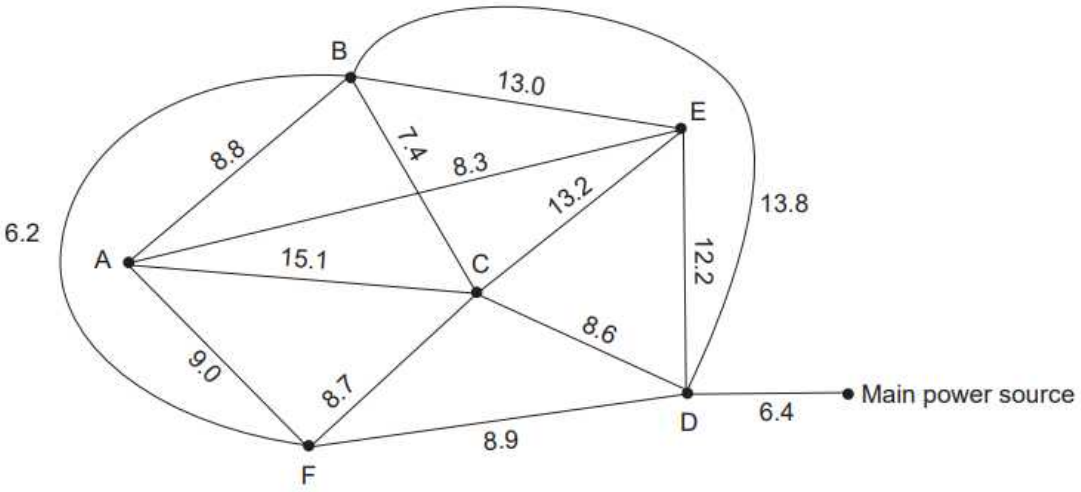



(d) The Transport Authority has approved upgrades for one **escalator** connection to increase the capacity from 120 to 150 people per minute. Which connection should be upgraded? Justify your response. (2 marks)

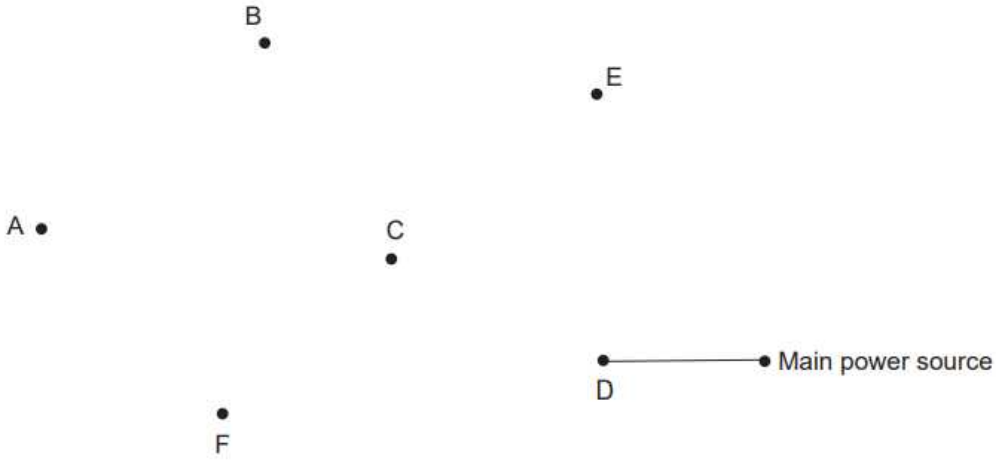
**2022 Section 2 Question 14**

**Networks and decision math-ematics**

Electric wiring is being installed to six powered sites (A–F) at a new caravan park from the main power source. The distance, in metres, between each site is shown below.




(a) Complete the minimum spanning tree below. (3 marks)



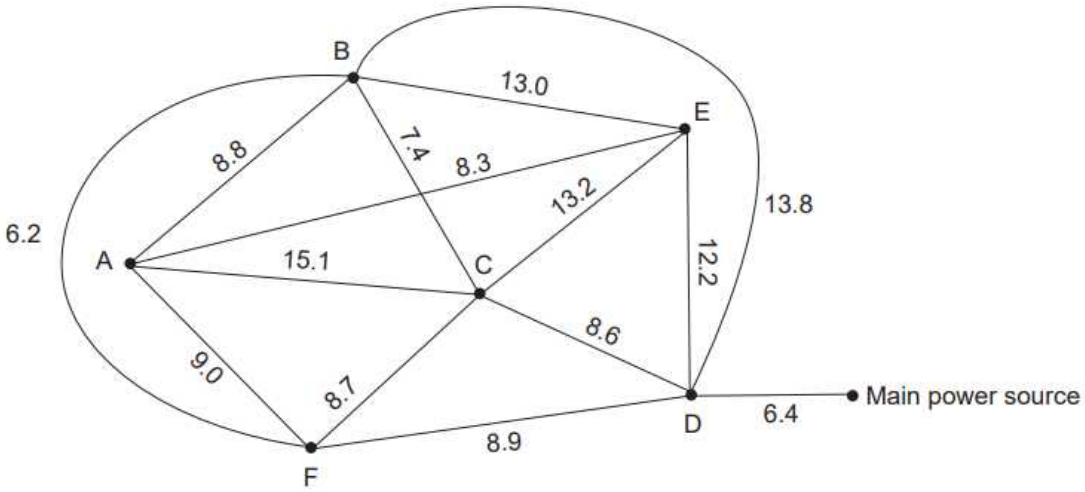

(b) The cost of installing the electric wiring is $14.20 per metre. If the wiring must include the connection between D and the main power source

(i) calculate the minimum length of wiring required. (2 marks)

(ii) calculate the total cost of installing the wiring. (1 mark)

It is discovered that between Sites B and F there is a large section of rock. The owner can still install the wiring directly between these two sites but the cost for that section will double in price.

(c) What effect, if any, will this have on the minimum spanning tree and total cost of the installation? A copy of the original graph has been provided below. (3 marks)



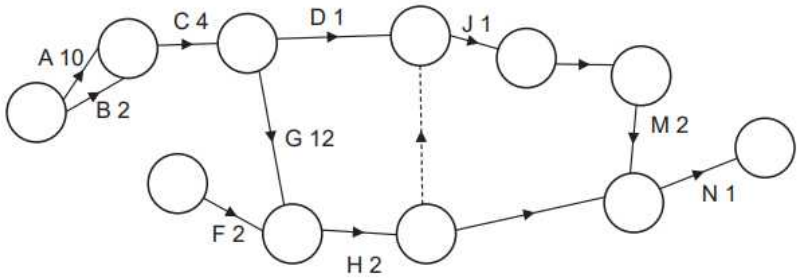
**2021 Section 2 Question 13**

**Networks and decision math-ematics**

A kitchen renovation project consists of a number of tasks of different durations and completed in different orders. One such renovation has this information summarised in the table below.

| Task | Task description | Duration (days) | Immediate predecessor(s) |
|---|---|---|---|
| A | Prepare plans | 10 | None |
| B | Select contractor | 2 | None |
| C | Review plans | 4 | A, B |
| D | Purchase appliances | 1 | C |
| E | Remove old appliances and benches | 2 | C |
| F | Prepare electrics and plumbing | 2 | E |
| G | Construct new cupboards and benches | 12 | C |
| H | Install cupboards and benches | 2 | F, G |
| J | Install appliances | 1 | see part (b) |
| K | Tiling and splashbacks | 5 | J |
| L | Flooring | 3 | H |
| M | Complete electrical and plumbing | 2 | K |
| N | Test and handover | 1 | L, M |

A partially-completed project network is shown below for this table.




(a) Complete the network diagram. (2 marks)

(b) Identify the immediate predecessor(s) of Task J. (1 mark)

(c) What does the dotted line on the network indicate? (1 mark)

(d) Determine the critical path and the minimum completion time for the project. (2 marks)

(e) Determine the float time for Task D and explain its meaning in terms of the renovation. (2 marks)

| | |
|---|---|
| | (f) Once Task E (removal of old appliances and benches) begins, the kitchen cannot be used. What is the least amount of time the occupants of the house will be without a working kitchen? (1 mark)<br><br>(g) If Task G was actually completed in nine days, how would this affect the critical path and minimum completion time? (2 marks)<br><br>Tasks E and F are both delayed.<br><br>(h) What is the maximum possible delay that does not affect the original minimum completion time? (1 mark) |

**2021 Section 2 Question 15**

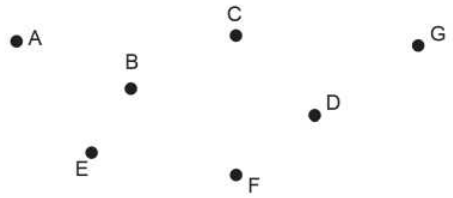
**Networks and decision math-ematics**

A number of streets connecting locations A to G inclusive have been identified as needing lighting upgrades with energy-efficient LED lamps. The council has decided that each location must have at least one connecting street that receives the lighting upgrade. The table below shows the cost, in dollars, of completing the upgrade in each street.

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| **A** | – | 7900 | 10 300 | – | 7100 | – | – |
| **B** | 7900 | – | 7600 | 6200 | 6500 | – | 15 400 |
| **C** | 10 300 | 7600 | – | 8500 | – | 12 200 | 9200 |
| **D** | – | 6200 | 8500 | – | 8000 | 4700 | 9800 |
| **E** | 7100 | 6500 | – | 8000 | – | 4800 | – |
| **F** | – | – | 12 200 | 4700 | 4800 | – | 10 100 |
| **G** | – | 15 400 | 9200 | 9800 | – | 10 100 | – |

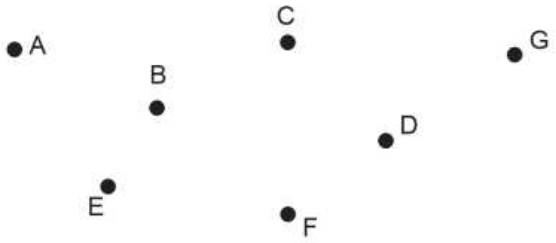
The council has a limited budget, so it needs to complete the upgrades at minimum cost.

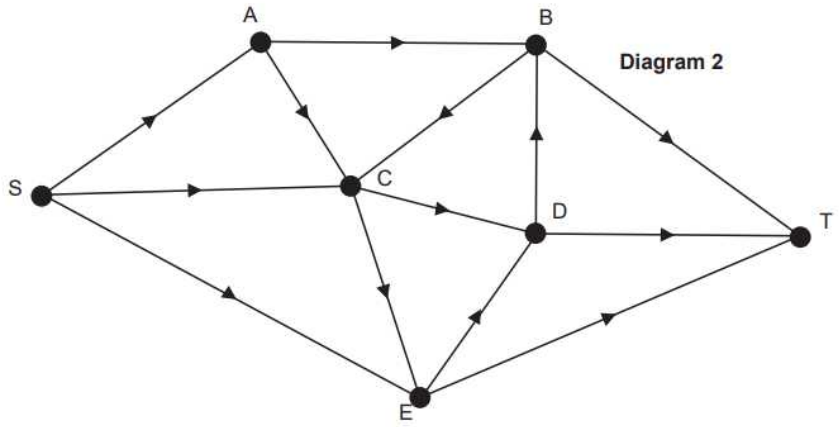
(a) Demonstrate the use of Prim's algorithm on the table above to determine the minimum spanning tree and draw it on the diagram below. (3 marks)



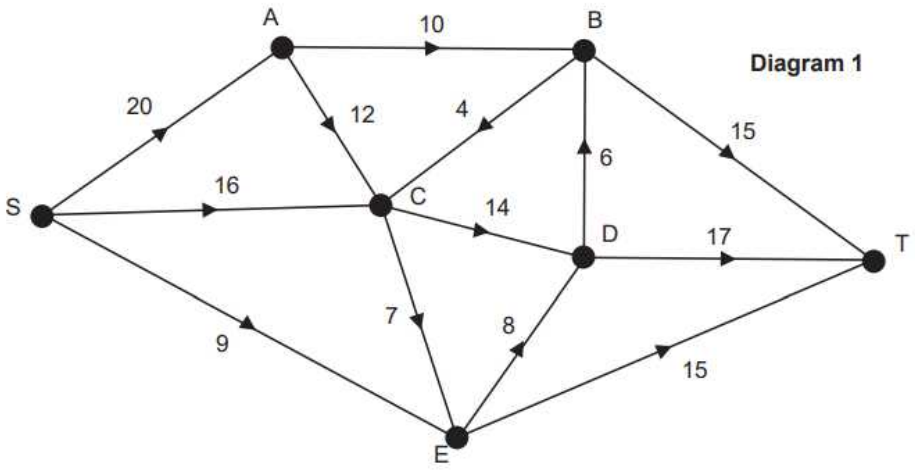

(b) The council has set aside $42 000 to complete the lighting upgrades. Does it have enough in its budget to make the necessary upgrades? Justify your answer. (2 marks)

(c) Due to the location of the police station, the upgrade from D to C must be included. What effect will this have on the minimum cost and the spanning tree? (2 marks)



| | |
|---|---|
| **2021 Section 2 Question 16**<br><br>**Networks and decision math-ematics** | The graph below shows a network of water pipes. The water source and main pumping station are located at S. The distribution centre is at T and the other vertices are intermediate pumping stations. The weights on the edges show the capacities in kilolitres per hour that can flow through each pipe.<br><br>(a) (i) Determine the value of the cut shown in Diagram 1. (1 mark)<br><br>(ii) Using your answer to part (a)(i), what can be said about the maximum flow of water through the network? (1 mark)<br><br>(b) State the maximum possible flows along the paths SABT and SCDT. (2 marks)<br><br>(c) Determine the maximum flow from S to T, listing each path and the corresponding flow. (3 marks)<br><br>(d) Using **Diagram 2** below, indicate a possible flow along each pipe corresponding to the maximum flow calculated in part (c). (2 marks)<br><br>  <br><br>**Diagram 2** |

(e) Determine the minimum cut that corresponds to the maximum flow and illustrate this on the copy of Diagram 1 shown below. (1 mark)




**2020 Section 2 Question 15**

**Networks and decision math-ematics**

The graph below shows a network of sewage pipes. The numbers on the edges indicate the number of litres per minute that can flow along each pipe.

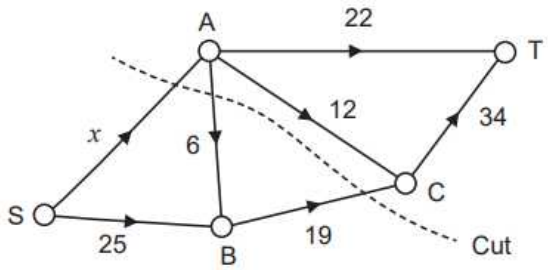



Diagram 1

(a) Show that the value of x is 38, given that the value of the cut is 57. (2 marks)

(b) Calculate the value of the maximum flow through the network. (2 marks)

(c) Indicate on Diagram 2 below a possible flow along each pipe corresponding to the maximum flow calculated in part (b). (2 marks)

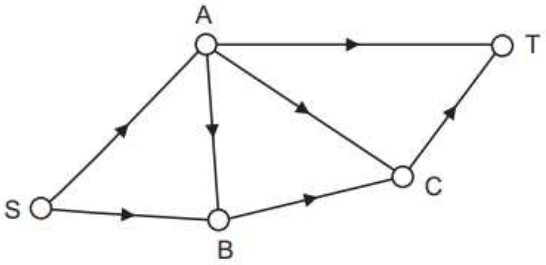
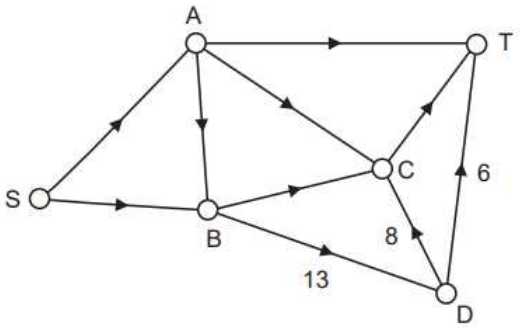



**Diagram 2**

Extra pipes, BD, DC and DT, are added to form a new system shown below. The capacities of the new pipes are indicated on the diagram. The original pipes have the same capacity as shown in **Diagram 1**.




(d) How will the addition of these pipes affect the new maximum flow through the system? (2 marks)

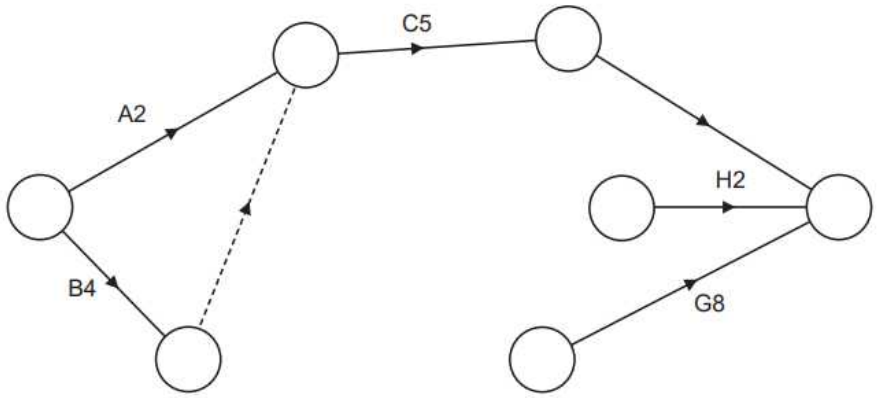
**2020 Section 2 Question 16**

**Networks and decision math-ematics**

Valeska and her sister Katrin are planning a small building project. The table below shows the required activities, together with the times taken (in days) and the immediate predecessors for each activity.

| Activity | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| Time (days) | 2 | 4 | 5 | 3 | 6 | 3 | 8 | 2 |
| Immediate predecessors | – | – | A, B | B | C | C | D | D, F |

(a) Complete the project network below, showing all activities and durations. (3 marks)




(b) Determine all critical activities and the minimum completion time for the project. (3 marks)

(c) Calculate the float times for each of the non-critical activities. (3 marks)

(d) If Activity H is delayed by three days, what effect will this have on the minimum completion time and the critical activities? (2 marks)

| | |
|---|---|
| | (e) Extra resources become available that can be used to shorten the duration of **one** of Activities B, E or F (on the original network) by one day. Which of these activities should be shortened and why? (3 marks) |

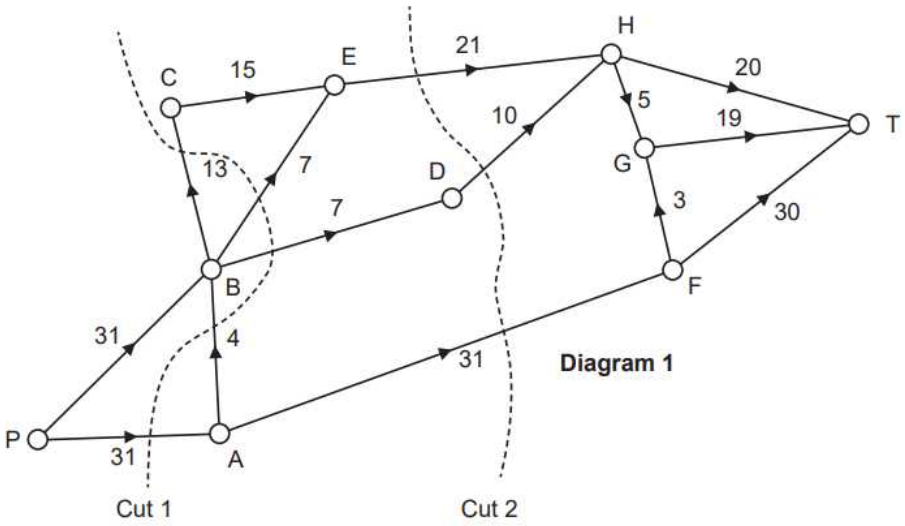
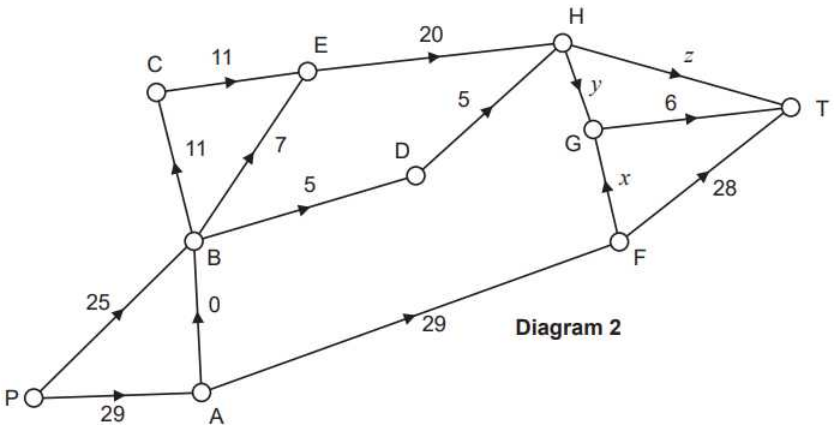
| | |
|---|---|
| **2019 Section 2 Question 15**<br><br>**Networks and decision math-ematics** | The directed network below shows the maximum available capacity for transferring power between different sub-stations on a small island. The number on each edge gives the capacity in kilovolts (kV).<br><br>  <br><br>**Diagram 1**<br><br>(a) State the capacity of each cut in Diagram 1. (2 marks) |

Diagram 2 shows a possible flow through the same network.




(b) Determine the initial flow in Diagram 2. (1 mark)

(c) Calculate the value of *x, y* and *z* in Diagram 2. (3 marks)

(d) Determine the maximum flow for the original network (Diagram 1). (2 marks)

Engineers wish to increase the maximum capacity to sub-station T. They propose to add a new transmission line from E to T of capacity 3 kV **or** a new transmission line from D to G of capacity 3 kV

(e) Determine which of these proposals will increase the maximum capacity to sub-station T. Justify your answer. (3 marks)

**2023 Section 1 Question 4**

**Networks and decision math-ematics**

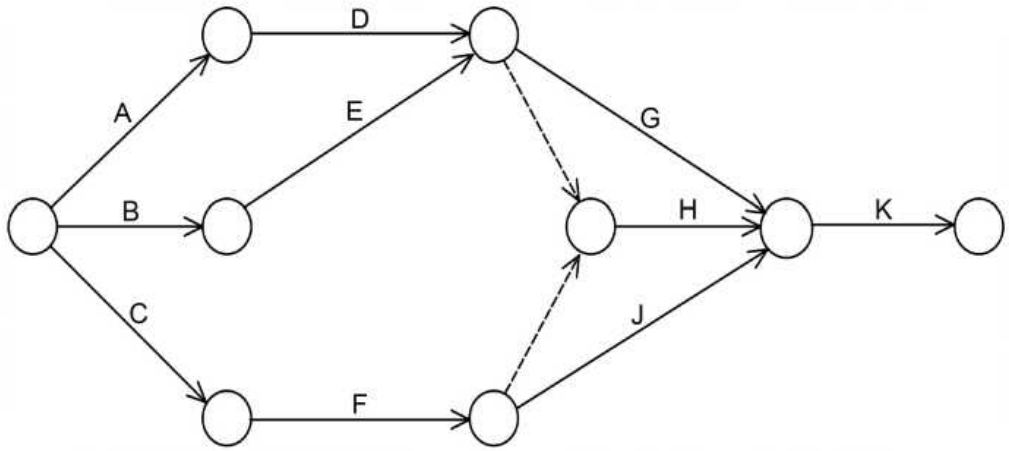
The following tables show activities and their completion requirements for a project to build a shearing shed.

| Activity | Activity description |
|---|---|
| A | Design the shearing shed |
| B | Prepare the site |
| C | Purchase materials |
| D | Construct the frame and walls |
| E | Install roof and windows |
| F | Add ventilation and insulation |
| G | Build the shearing stand |
| H | Install electrics |
| J | Paint and finish the shed |
| K | Install shearing equipment |

| Requirements |
|---|
| A, B and C start the project together |
| D starts when A is completed |
| E starts when B is completed |
| F starts when C is completed |
| G starts when D and E are completed |
| J starts when F is completed |
| H starts when D, E and F are completed |
| K starts when G, H and J are completed |

Draw the project network for the information given in the tables. (4 marks)

**Solution**




**Specific behaviours**

- ✓ all edges show task and direction
- ✓ shows at least six correct edges
- ✓ shows at least one dummy link
- ✓ shows all other correct edges

**2023 Section 1 Question 5**

**Networks and decision math-ematics**

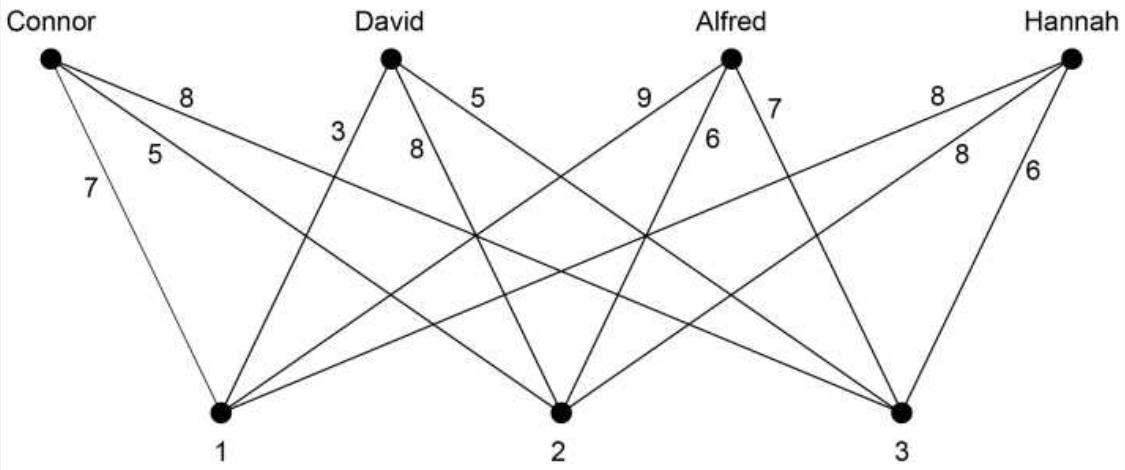
Four camp leaders, Connor, David, Alfred and Hannah, are responsible for setting up the camp site for the upcoming school camp. Today there are three tasks available. Each task will only have one camp leader assigned to it.

The number of hours each camp leader takes to complete each task is shown in the table below.

| Camp leader | Task 1 | Task 2 | Task 3 |
|---|---|---|---|
| Connor | 7 | 5 | 8 |
| David | 3 | 8 | 5 |
| Alfred | 9 | 6 | 7 |
| Hannah | 8 | 8 | 6 |

(a) Draw a weighted bipartite graph showing all the possible allocations for each of the camp leaders. (2 marks)

**Solution**




**Specific behaviours**

✓ correctly allocates edges
✓ correctly allocates weights on edges

(b) Complete the 4 x 4 matrix below to represent the allocation of tasks to camp leaders. (2 marks)

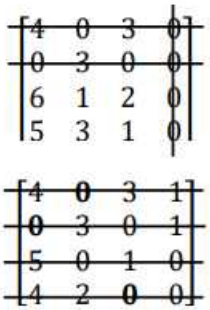
**Solution**

$$\begin{bmatrix} 7 & 5 & 8 & 0 \\ 3 & 8 & 5 & 0 \\ 9 & 6 & 7 & 0 \\ 8 & 8 & 6 & 0 \end{bmatrix}$$

**Specific behaviours**

✓ correctly enters a column of zeros
✓ correct values in matrix

(c) Use the Hungarian algorithm to determine the allocation of camp leader to the task that will minimise the time taken. (4 marks)

| Solution |
|---|
| $\begin{bmatrix} 4 & 0 & 3 & 0 \\ 0 & 3 & 0 & 0 \\ 6 & 1 & 2 & 0 \\ 5 & 3 & 1 & 0 \end{bmatrix}$   |
| **Specific behaviours** |
| ✓ subtracts smallest values from every column<br>✓ shows that every zero can be covered by 3 horizontal/vertical lines<br>✓ determines that smallest uncovered value is 1 and subtracts 1 from every uncovered value and adds 1 to every value covered by 2 lines<br>✓ shows that every zero can be covered by 4 horizontal/vertical lines |

(d) Show the task allocated to each camp leader and calculate the total time taken to complete all tasks. (2 marks)

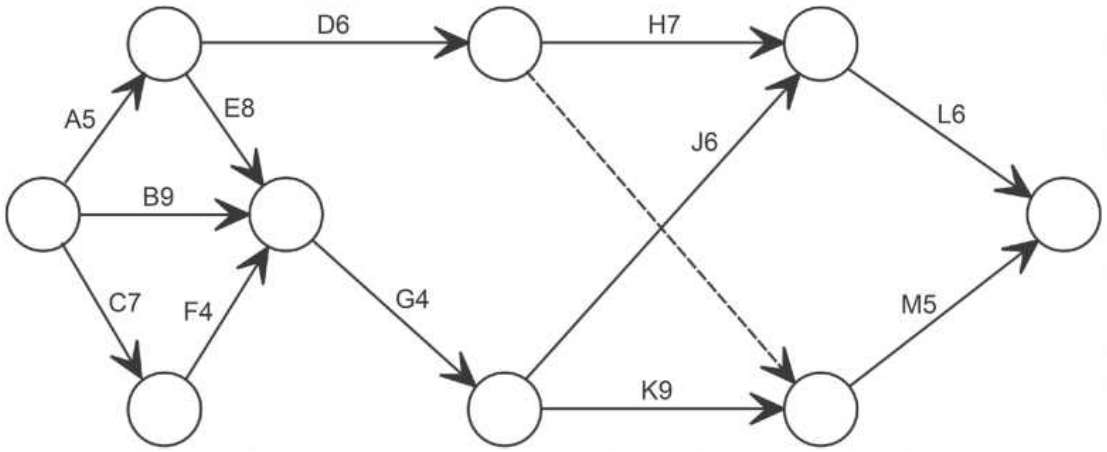
| Camp leader | Connor | David | Alfred | Hannah |
|---|---|---|---|---|
| Task | 2 | 1 | - | 3 |

| Solution |
|---|
| see table above for task allocation |
| total time: 14 hours |
| **Specific behaviours** |
| ✓ states correct allocation<br>✓ calculates correct time |

**2023 Section 1 Question 6**

**Networks and decision math-ematics**

A landscape architect has produced the following project network for the development of a community market garden. The digraph shows the order of completion of the various tasks and their expected completion time in hours.



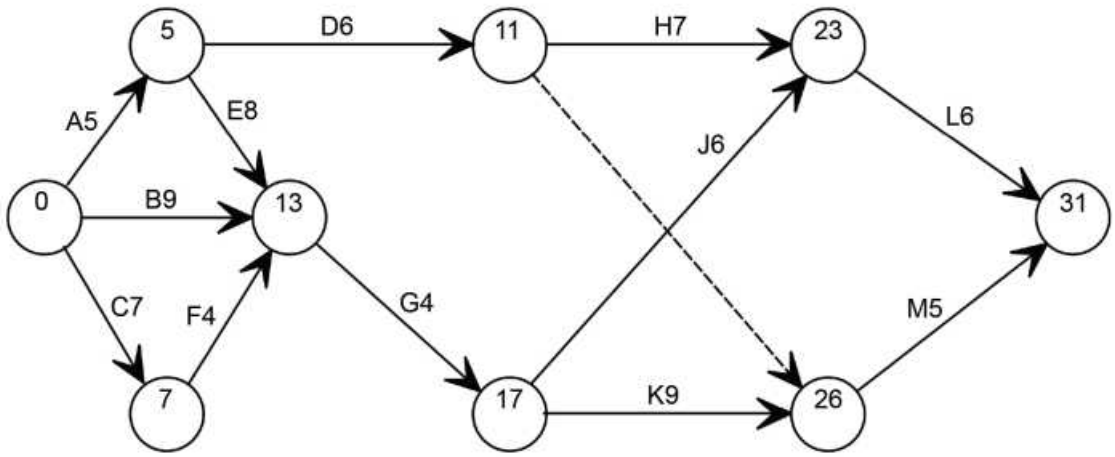

(a) Complete the immediate predecessor/s column in the table below. (3 marks)

| Task | Time (hours) | Immediate Predecessor/s |
|---|---|---|
| A | 5 | - |
| B | 9 | - |
| C | 7 | - |
| D | 6 | **A** |
| E | 8 | **A** |
| F | 4 | **C** |
| G | 4 | **B, E, F** |
| H | 7 | **D** |
| J | 6 | **G** |
| K | 9 | **G** |
| L | 6 | **H, J** |
| M | 5 | **D, K** |

| **Solution** |
|---|
| see table above |
| **Specific behaviours** |
| ✓ states correct predecessors for 10 tasks<br>✓ states correct predecessors for M<br>✓ states all correct predecessors |

(b) Determine the critical path and the minimum completion time for the project. Workings must be shown to verify your answer. (3 marks)

**Solution**



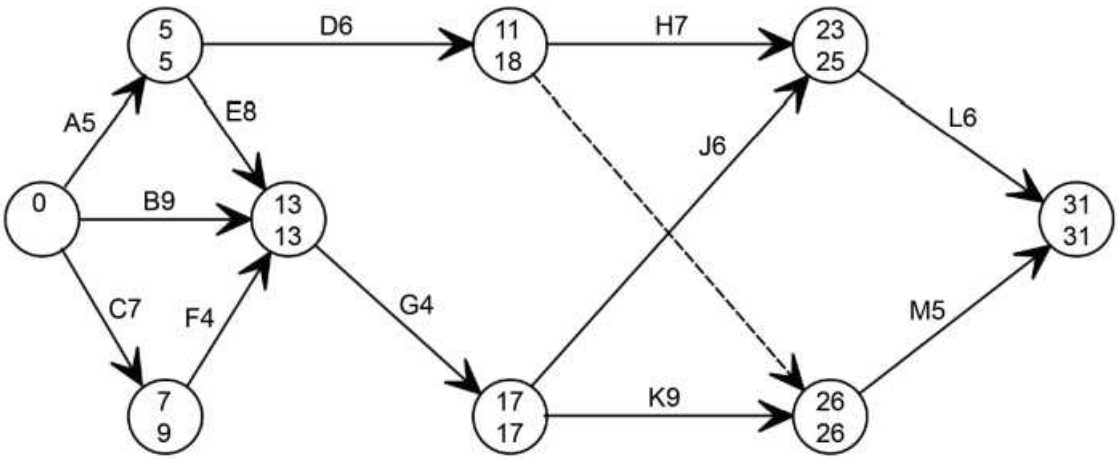

AEGKM, 31 hours

**Specific behaviours**

✓ shows systematic working
✓ gives correct path
✓ gives correct length

(c) Determine which task/s have a float time of exactly 2 hours. (2 marks)

**Solution**




Task F, C, L and J have a float time of exactly 2 hours.

**Specific behaviours**

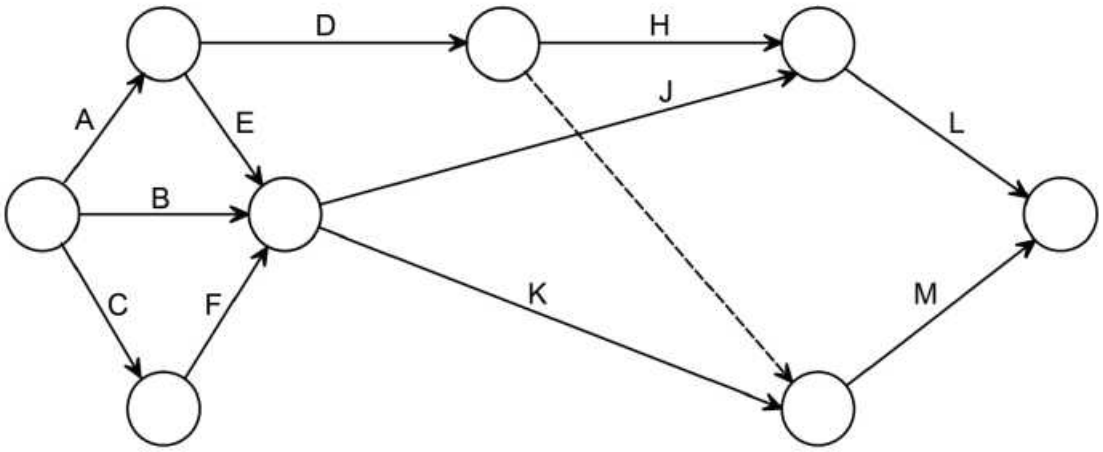
✓ identifies two correct tasks
✓ identifies all correct tasks

(d) Describe why Task D can be delayed by 6 hours and not affect the minimum completion time. (2 marks)

| Solution |
| --- |
| float time for H is 7 hours, hence, D can be delayed by 6 hours |
| **Specific behaviours** |
| ✓ identifies the float time for H is 7 hours<br>✓ gives correct explanation why D can be delayed by 6 hours |

(e) Due to the release of a new piece of technology for reticulation control, Task G is no longer required. Redraw the network showing how the removal of Task G will change the configuration of the network. Task times are **not** required to be shown. (2 marks)

**Solution**

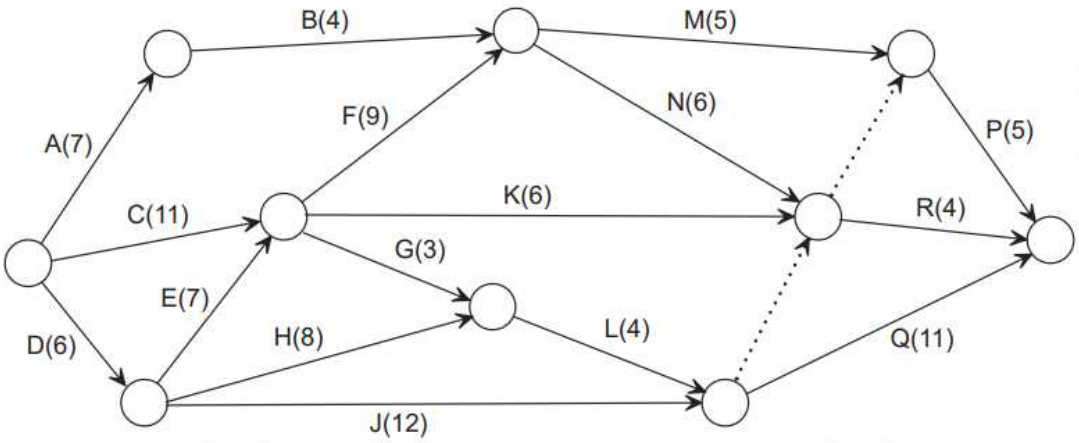



| **Specific behaviours** |
| --- |
| ✓ shows task G removed<br>✓ shows tasks J and K with immediate predecessors E, B, and F |

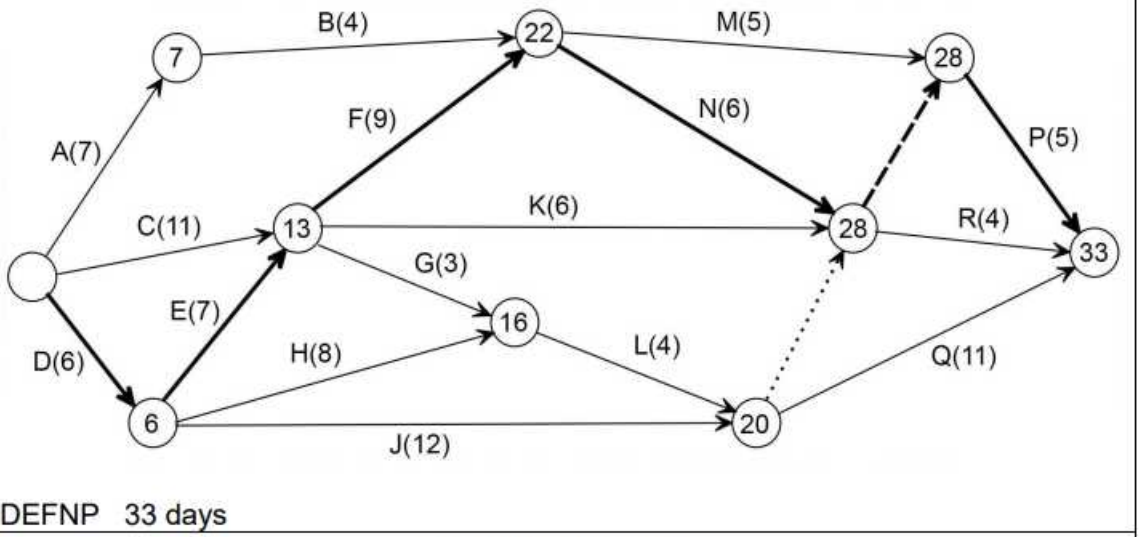
**2022 Section 1 Question 4**

**Networks and decision math-ematics**

The project network shown below is to be followed when installing a new conveyor belt system for a mining company. The tasks, with their completion time in days, are shown on the network. The project will commence on 20 November, with work continuing for seven days each week.




(a) Determine the critical path and the minimum completion time. (2 marks)

| Solution |
|---|
|  |
| DEFNP 33 days |
| **Specific behaviours** |
| ✓ correct path<br>✓ correct length |



(b) Determine the immediate predecessors for Task P. (2 marks)

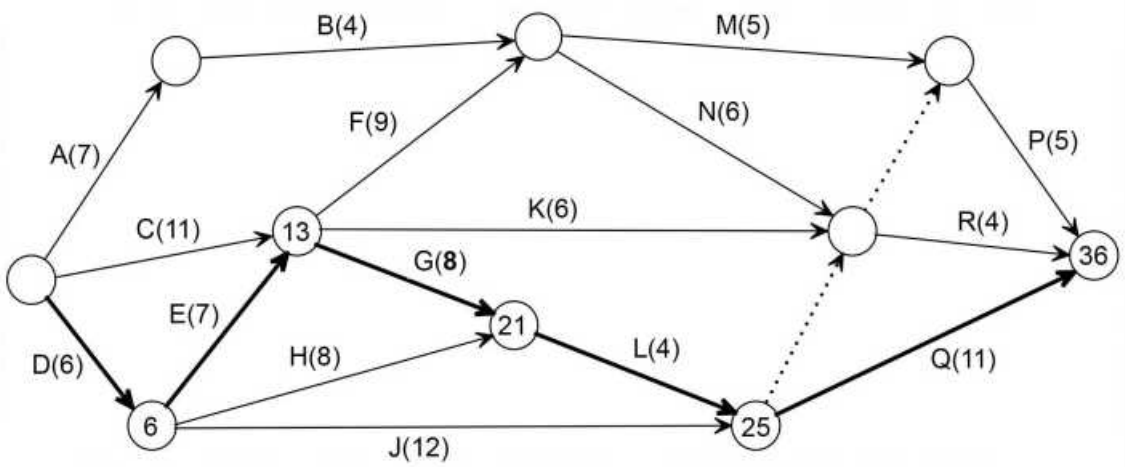
| Solution |
|---|
| M, N, K, L and J |
| **Specific behaviours** |
| ✓ at least 3 correct predecessors<br>✓ all correct predecessors |

(c) What is the latest date in November that Task H can commence? (2 marks)

| Solution |
| --- |
| 33 – (8 + 4 + 11) = 10 days<br>Latest start date is 30 November |
| **Specific behaviours** |
| ✓ correctly determines number from start of the project<br>✓ correctly determines correct date |

(d) It has been confirmed that there is a delay in obtaining some parts from overseas. This means Task G will take 5 days longer to complete. Discuss all implications for the completion of the project. A copy of the **original** network is given below. (3 marks)

**Solution**




New critical path is: DEGLQ
New completion time is: 36 days

| Specific behaviours |
| --- |
| ✓ correctly indicates change to task G<br>✓ correctly states the new critical path<br>✓ correctly states the new completion time |

**2022 Section 1 Question 6**

**Networks and decision math-ematics**

A company has four different models of robotic welders that are used to assemble components that require welding. Each of these welders must be set up to assemble just one type of component. The number of components that each welder can assemble per hour for the three different types of components required is shown below.

| | | Welders | | | |
|---|---|---|---|---|---|
| | | A | B | C | D |
| Components | 1 | 15 | 12 | 13 | 11 |
| | 2 | 13 | 16 | 13 | 9 |
| | 3 | 15 | 16 | 12 | 14 |

(a) Use the Hungarian Algorithm to determine the allocation of welder to component that will maximise the number of components assembled. (6 marks)

**Solution**

$$\begin{bmatrix} 15 & 12 & 13 & 11 \\ 13 & 16 & 13 & 9 \\ 15 & 16 & 12 & 14 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 3 & 5 \\ 3 & 0 & 3 & 7 \\ 1 & 0 & 4 & 2 \\ 16 & 16 & 16 & 16 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 3 & 2 & 4 \\ 3 & 0 & 3 & 7 \\ 1 & 0 & 4 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \cancel{0} & \cancel{3} & 2 & 4 \\ \cancel{3} & \cancel{0} & 3 & 7 \\ \cancel{1} & \cancel{0} & 4 & 2 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{bmatrix}$$

$$\begin{bmatrix} \cancel{0} & \cancel{3} & \cancel{0} & \cancel{2} \\ \cancel{3} & \cancel{0} & \cancel{1} & \cancel{5} \\ \cancel{1} & \cancel{0} & \cancel{2} & \cancel{0} \\ \cancel{2} & \cancel{2} & \cancel{0} & \cancel{0} \end{bmatrix}$$

Allocation: A – 1, B – 2, D – 3

**Specific behaviours**

- ✓ shows square matrix with row of zeros
- ✓ shows subtraction of each score from 16
- ✓ shows subtraction of lowest score in each row from each score in that row
- ✓ shows all zeros can be covered by just 3 lines
- ✓ shows subtraction of 2 from each uncovered score and 2 added to each score covered by 2 lines
- ✓ determines correct allocation

(b) State how many of each component will be produced each hour with the allocation determined in part (a). (2 marks)

| Solution |
|---|
| Component 1 – 15<br>Component 2 – 16<br>Component 3 – 14 |
| **Specific behaviours** |
| ✓ three correct values<br>✓ all values linked to correct components |

**2022 Section 1 Question 7**

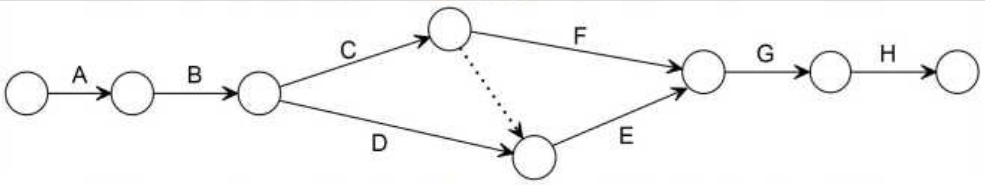
**Networks and decision math-ematics**

The following table shows activities and their immediate predecessors for a project to develop, produce and sell a new interactive game for children.

| Activity | Activity description | Immediate Predecessor |
|---|---|---|
| A | Market analysis | - |
| B | Product design | A |
| C | Manufacture product | B |
| D | Develop software | B |
| E | Software testing | C, D |
| F | Design and produce packaging | C |
| G | Assemble product | E, F |
| H | Sales promotion | G |

Draw the project network for the information given in the table.

**Solution**



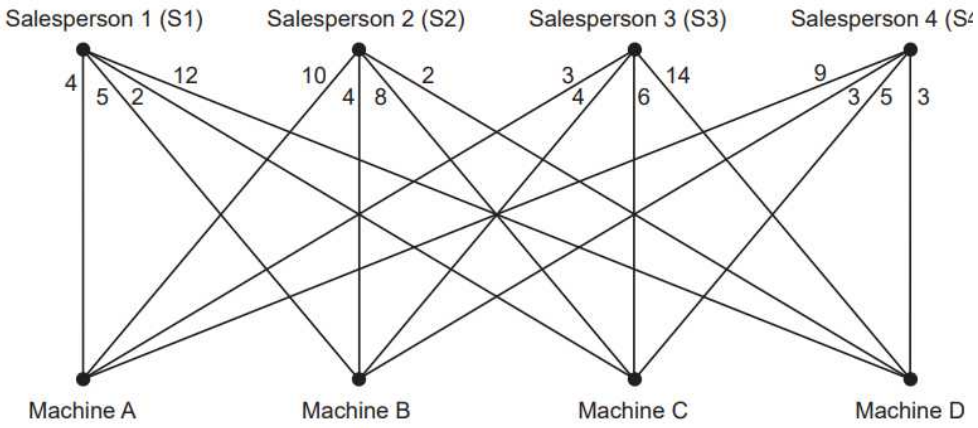

**Specific behaviours**

✓ correctly shows direction of flow throughout the network
✓ correctly shows at least 6 correct activity positions
✓ correctly shows all activities

**2020 Section 1 Question 3**

**Networks and decision math-ematics**

The bipartite graph below shows the average number of sales per day of four specialist pieces of machinery by four salespersons. The salespersons went through a rotation process of spending a number of full days selling only one type of machine before moving to the next. The company that makes these machines wants to maximise the sales by allocating the sales of each machine to only one person.




(a) Complete the matrix below showing the information from the bipartite graph. (1 mark)

**Solution**

| | A | B | C | D |
|---|---|---|---|---|
| S1 | 4 | 5 | 2 | 12 |
| S2 | 10 | 4 | 8 | 2 |
| S3 | 3 | 4 | **6** | **14** |
| S4 | **9** | **3** | **5** | **3** |

**Specific behaviours**

✓ completes matrix correctly

(b) Show use of the Hungarian algorithm to allocate each salesperson to a specialist machine in order to maximise sales. The allocation is to be written in the table below. (5 marks)

**Solution**

$$\begin{bmatrix} 10 & 9 & 12 & 2 \\ 4 & 10 & 6 & 12 \\ 11 & 10 & 8 & 0 \\ 5 & 11 & 9 & 11 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 7 & 10 & 0 \\ 0 & 6 & 2 & 8 \\ 11 & 10 & 8 & 0 \\ 0 & 6 & 4 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 1 & 8 & 0 \\ 0 & 0 & 0 & 8 \\ 11 & 4 & 6 & 0 \\ 0 & 0 & 2 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 7 & \boxed{0} & 7 & 0 \\ 0 & 0 & \boxed{0} & 9 \\ 10 & 3 & 5 & \boxed{0} \\ \boxed{0} & 0 & 2 & 7 \end{bmatrix}$$

| **Salesperson** | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| **Machine** | **B** | **C** | **D** | **A** |

**Specific behaviours**

- ✓ correctly subtracts each entry from largest value of 14
- ✓ correctly subtracts the smallest value in that row from each value in that row and then correctly subtracts the smallest value in that column from each value in that column
- ✓ determines that all zeros in the matrix can be covered by just 3 horizontal/vertical lines
- ✓ correctly determines the smallest uncovered value = 1 and subtracts 1 from each uncovered value and adds 1 to each value covered by 2 intersecting lines
- ✓ determines the correct allocation of machines

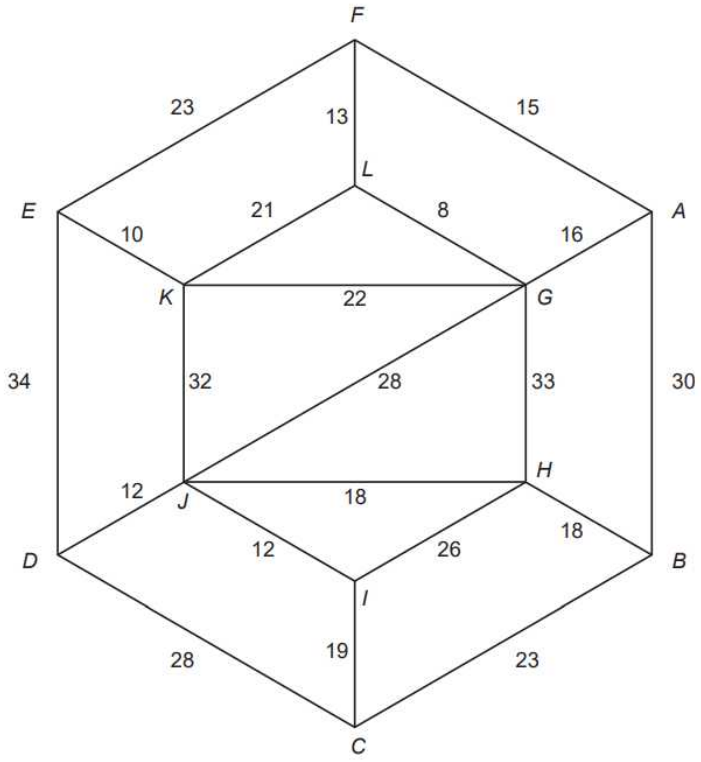
(c) How many more sales are made by using the allocation in part (b) compared to the allocation: S1 – A, S2 – B, S3 – C and S4 – D? (2 marks)

**Solution**

S1 – A, S2 – B, S3 – C and S4 – D gives a total of 17 sales
Solution to part (b) gives a total of 36 sales.
Greater by 19.

**Specific behaviours**

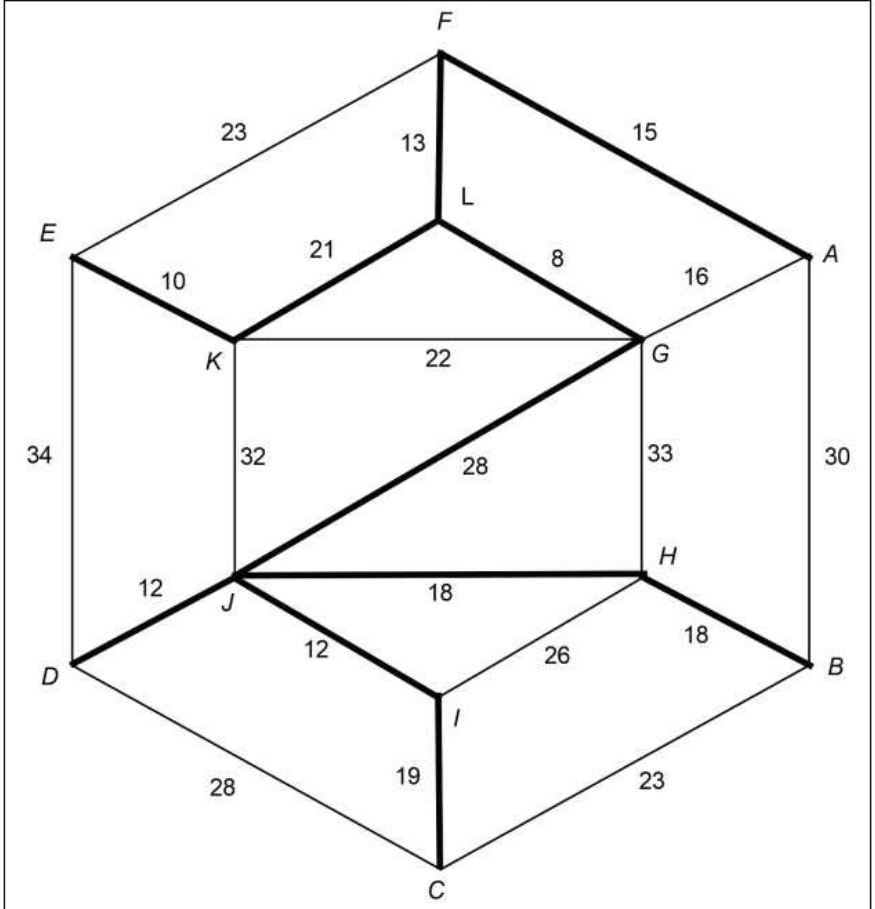
- ✓ determines number of sales for each allocation
- ✓ determines the correct difference

| | |
|---|---|
| **2020 Section 1 Question 5**<br><br>**Networks and decision math-ematics** | A communication wi-fi network is to be installed to service a shopping centre connecting 12 shops located at vertices *A, B, C, D, E, F, G, H, I, J, K* and *L*. The only practical connections between vertices are shown on the following network.<br><br>The number on each edge is the quoted price, in hundreds of dollars, for the direct link between the vertices.<br><br>  <br> |

(a) A minimal spanning tree is to be used to determine the minimum cost of this installation.

(i) Show clearly on the network the minimum spanning tree solution. (3 marks)

**Solution**




**Specific behaviours**

✓ shows at least 5 correct edges
✓ shows at least 8 correct edges
✓ shows correct minimum spanning tree

(ii) Determine the minimum cost. (2 marks)

**Solution**

10 + 21 + 13 + 15 + 8 + 28 + 18 + 18 + 12 + 12 + 19 = 174

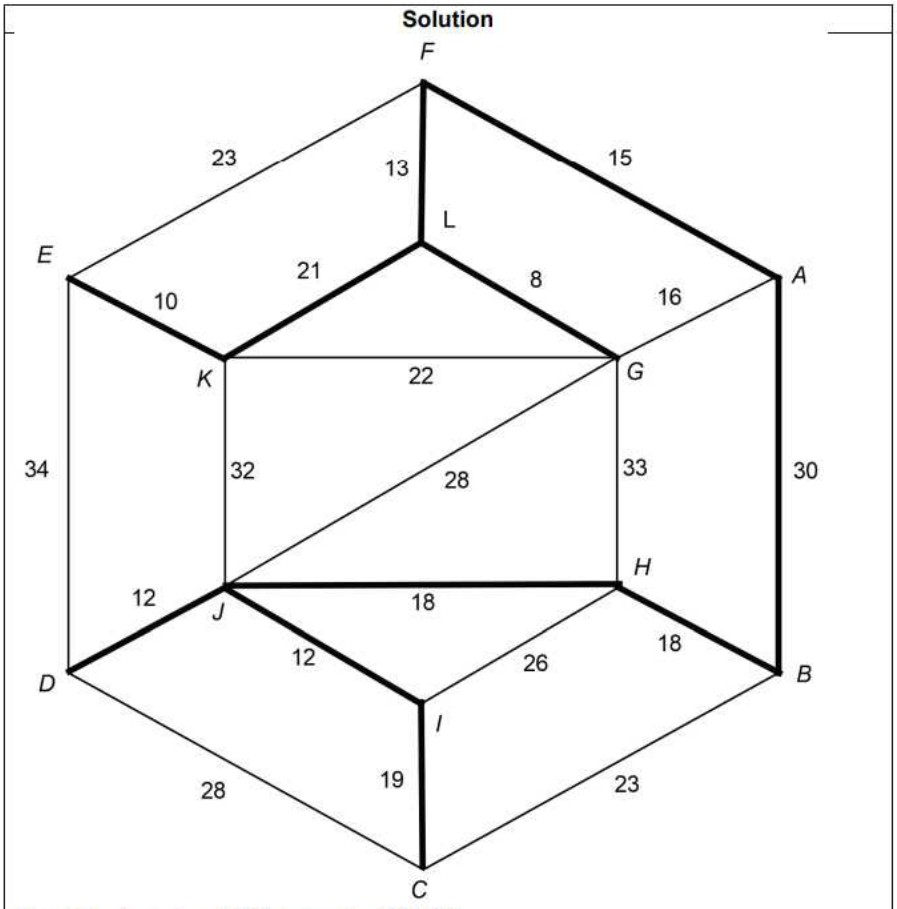
Cost = $17 400

**Specific behaviours**

✓ determines sum of edges is 174
✓ determines cost is $17 400

(b) Due to further construction at the shopping centre, edge GJ is now not feasible. Explain how this will change the solution for part (a). (2 marks)

**Solution**




Now includes edge *AB*. Total cost = $17 600

| Specific behaviours |
|---|
| ✓ includes *AB* instead of *GJ* |
| ✓ determines cost is $17 600 (increases by $200) |

**2019 Section 1 Question 3**

**Networks and decision math-ematics**

A company has four small workshops that each produce four different types of outdoor furniture. The annual cost of production of the furniture at each workshop is shown in the table below, with all values in thousands of dollars.

| | Type 1 $'000 | Type 2 $'000 | Type 3 $'000 | Type 4 $'000 |
|---|---|---|---|---|
| **Workshop A** | 25 | 43 | 50 | 39 |
| **Workshop B** | 33 | 31 | 56 | 39 |
| **Workshop C** | 28 | 47 | 59 | 38 |
| **Workshop D** | 36 | 32 | 56 | 41 |

The cost matrix is given by

$$\begin{bmatrix} 25 & 43 & 50 & 39 \\ 33 & 31 & 56 & 39 \\ 28 & 47 & 59 & 38 \\ 36 & 32 & 56 & 41 \end{bmatrix}$$

The company is interested in knowing what the minimum annual cost would be if each furniture type was allocated to its own individual workshop. The Hungarian Algorithm is to be used to determine the allocation and the minimum annual cost. The first step of the Hungarian Algorithm, where the smallest number in each row is subtracted from all other numbers in that row, is shown below.

(a) Continue the steps of the Hungarian Algorithm to determine the appropriate allocation of workshops to furniture type and state the **minimum** annual cost. (5 marks)

$$\begin{bmatrix} 0 & 18 & 25 & 14 \\ 2 & 0 & 25 & 8 \\ 0 & 19 & 31 & 10 \\ 4 & 0 & 24 & 9 \end{bmatrix}$$

**Solution**

$$\begin{bmatrix} \cancel{0} & 18 & 1 & 6 \\ \cancel{2} & \cancel{0} & \cancel{1} & \cancel{0} \\ \cancel{0} & 19 & 7 & 2 \\ \cancel{4} & \cancel{0} & \cancel{0} & \cancel{1} \end{bmatrix}$$

Subtracts smallest number in each column from each element

Least amount of strike-throughs is 3, therefore need to add smallest element not crossed out (i.e. one) to the intersections and subtract the smallest element from the uncrossed elements to give:

$$\begin{bmatrix} \cancel{0} & \cancel{17} & \cancel{0} & \cancel{5} \\ \cancel{3} & \cancel{0} & \cancel{1} & \cancel{0} \\ \cancel{0} & \cancel{18} & \cancel{6} & \cancel{1} \\ \cancel{5} & \cancel{0} & \cancel{0} & \cancel{1} \end{bmatrix}$$

Least amount of strike-throughs is now four

Therefore, the allocation matrix is

$$\begin{bmatrix} 0 & 17 & \boxed{0} & 5 \\ 3 & 0 & 1 & \boxed{0} \\ \boxed{0} & 18 & 6 & 1 \\ 5 & \boxed{0} & 0 & 1 \end{bmatrix}$$

| **Type** | Type 1 | Type 2 | Type 3 | Type 4 |
|---|---|---|---|---|
| **Workshop** | **C** | **D** | **A** | **B** |

Total minimum annual cost is $149 000

**Specific behaviours**

✓ subtracts smallest number in each column from other numbers in that column
✓ correctly makes adjustments to matrix
✓ correctly allocates workshops
✓ states the sum of the allocations correctly
✓ states minimum annual cost in thousands of dollars

The revenue matrix, in thousands of dollars, for the sale of the furniture produced annually at each workshop is given by

$$\begin{bmatrix} 37 & 61 & 60 & 53 \\ 45 & 52 & 73 & 50 \\ 38 & 65 & 75 & 55 \\ 44 & 54 & 76 & 45 \end{bmatrix}$$

(b) Given that *Profit* = *Revenue* – *Cost*, complete the Profit matrix below. (1 mark)

| **Solution** |
|---|
| $\begin{bmatrix} 12 & 18 & 10 & 14 \\ 12 & 21 & 17 & 11 \\ 10 & 18 & \boxed{16} & \boxed{17} \\ \boxed{8} & \boxed{22} & \boxed{20} & 4 \end{bmatrix}$ |
| **Specific behaviours** |
| ✓ correctly completes all entries |

(c) Use the Hungarian Algorithm to determine the appropriate allocation of workshops to furniture type that will produce the **maximum** annual profit. (4 marks)

**Solution**

$\begin{bmatrix} 10 & 4 & 12 & 8 \\ 10 & 1 & 5 & 11 \\ 12 & 4 & 6 & 5 \\ 14 & 0 & 2 & 18 \end{bmatrix}$ Subtracts each element in the profit matrix from 22

$\begin{bmatrix} 6 & 0 & 8 & 4 \\ 9 & 0 & 4 & 10 \\ 8 & 0 & 2 & 1 \\ 14 & 0 & 2 & 18 \end{bmatrix}$ Subtracts smallest number in each row from each element

$\begin{bmatrix} \boxed{0} & 0 & 6 & 3 \\ 3 & \boxed{0} & 2 & 9 \\ 2 & 0 & 0 & \boxed{0} \\ 8 & 0 & \boxed{0} & 17 \end{bmatrix}$ Subtracts smallest number in each column from each element

| **Type** | Type 1 | Type 2 | Type 3 | Type 4 |
|---|---|---|---|---|
| **Workshop** | **A** | **B** | **D** | **C** |

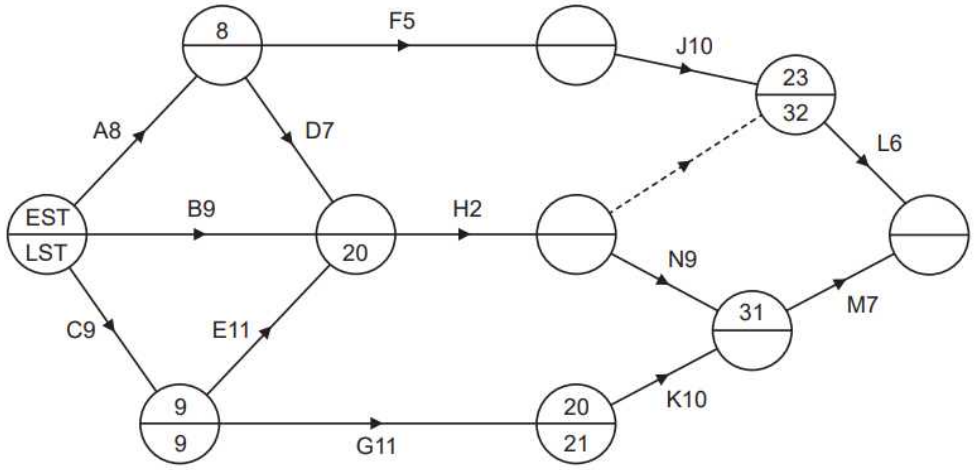
**Specific behaviours**

✓ correctly obtains first matrix
✓ correctly obtains second matrix
✓ correctly obtains third matrix
✓ correctly allocates workshops

**2019 Section 1 Question 5**

**Networks and decision math-ematics**

The network below represents a construction project. The number on each edge gives the time, in hours, to complete the activity. Each activity requires one worker.



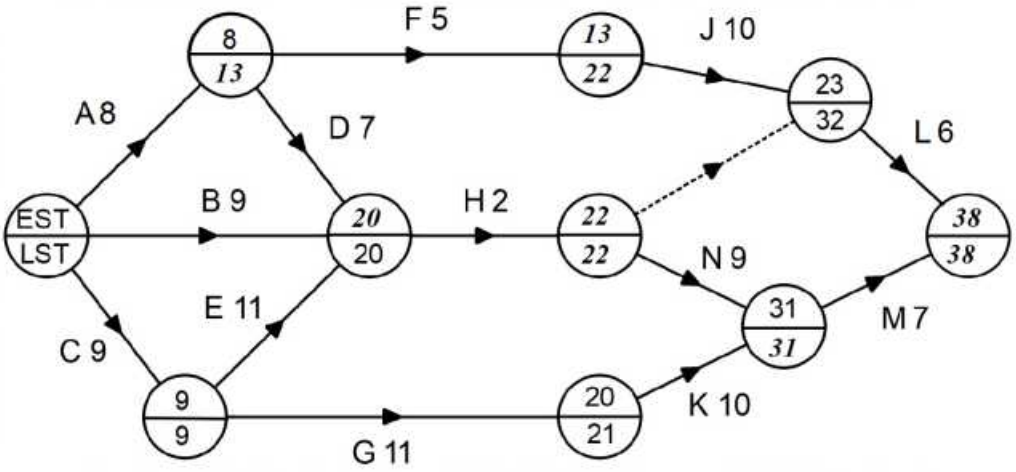

(a) Complete the precedence table below. (2 marks)

| **Activity** | A | B | C | D | E | F | G | H | J | K | L | M | N |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Time (hours)** | 8 | 9 | 9 | 7 | 11 | 5 | 11 | 2 | 10 | 10 | 6 | 7 | 9 |
| **Immediate predecessor** | - | - | - | A | C | A | C | B,D,E | **F** | **G** | **H,J** | **N,K** | **H** |

| **Solution** |
|---|
| See table above |
| **Specific behaviours** |
| ✓ correctly allocates predecessors for activity L<br>✓ correctly allocates all predecessors |

(b) Complete the network showing the earliest starting time (EST) and latest starting time (LST) for each node. (Note: the first node indicates which is the EST and the LST.) (2 marks)

**Solution**




| **Specific behaviours** |
|---|
| ✓ completes at least 4 nodes correctly<br>✓ completes all nodes correctly |

(c) Determine the critical path and the minimum completion time for the project. (2 marks)

| Solution |
|---|
| Critical path is CEHNM. Minimum completion time is 38 hours |
| **Specific behaviours** |
| ✓ states correct path<br>✓ states correct time |

(d) Calculate the float times for Activities D and F. (2 marks)

| Solution |
|---|
| Float time for D is 5 hours<br>Float time for F is 9 hours |
| **Specific behaviours** |
| ✓ gives correct float for D<br>✓ gives correct float for F |

(e) Given that the sum of all the times of the activities is 104 hours, calculate the minimum number of workers required to complete the project in the minimum completion time. (1 mark)

| Solution |
|---|
| three |
| **Specific behaviours** |
| ✓ states correct number of workers required |

(f) What is the latest time into the project that Activity F could start without affecting the minimum completion time? (1 mark)

| Solution |
|---|
| Seventeen hours |
| **Specific behaviours** |
| ✓ states correct latest time |

(g) Explain the purpose of the dotted line on the network. (1 mark)

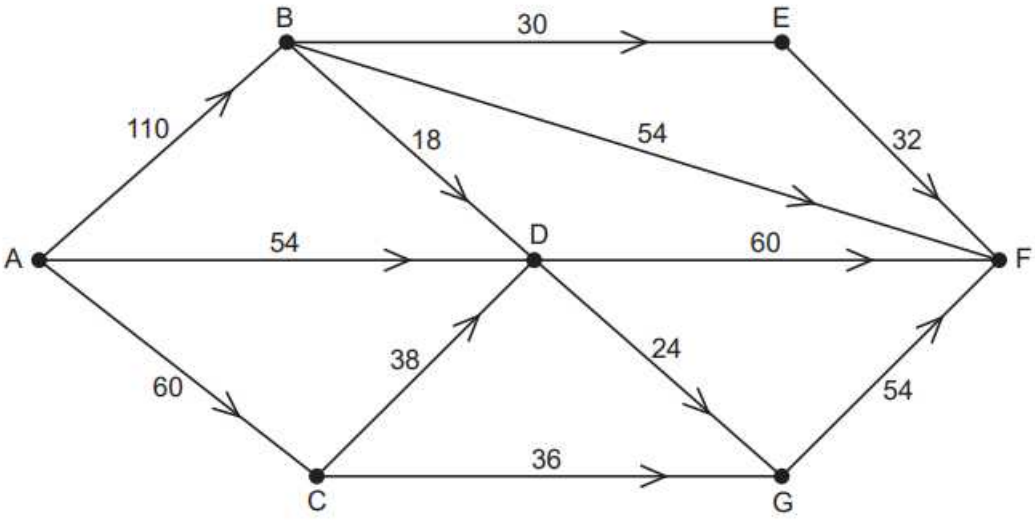
| Solution |
|---|
| Activity L depends on activities H and J |
| **Specific behaviours** |
| ✓ states correct purpose |

**2023 Section 2 Question 11**

**Networks and decision math-ematics**

A ferry service connects a group of islands in the Pacific Ocean.

(a) The network below is for the hourly departures from A and shows the number of passengers that can be carried between each island, A, B, C, D, E, F and G. Determine the maximum number of passengers that can be carried from A to F each hour. Show systematic workings. (4 marks)




**Solution**

Possible solutions:

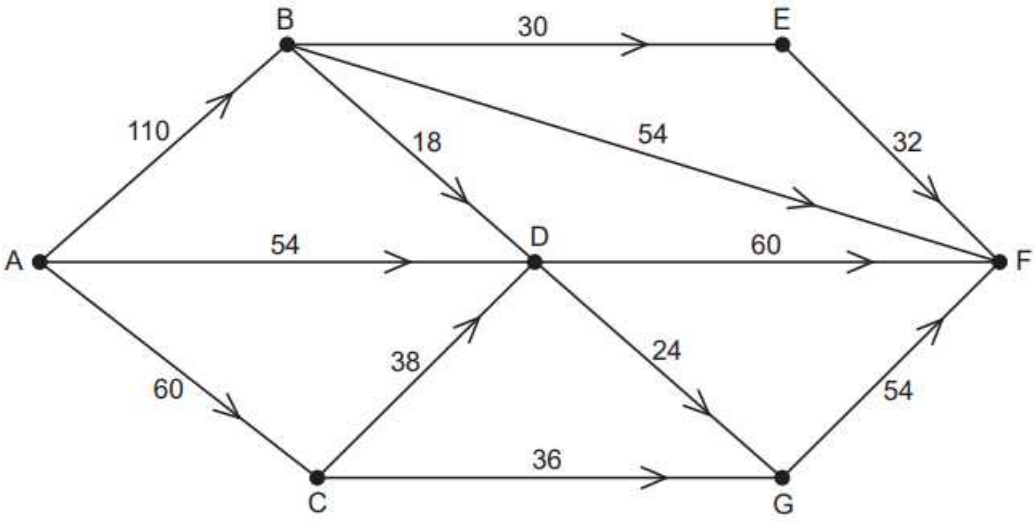
| | | | |
|---|---|---|---|
| ABEF | 30 | ABEF | 30 |
| ABF | 54 | ACGF | 36 |
| ABDF | 18 | ADF | 54 |
| ADF | 42 | ABF | 54 |
| ADGF | 12 | ABDF | 6 |
| ACDGF | 12 | ACDGF | 18 |
| ACGF | 30 | | |

Total passengers = 198

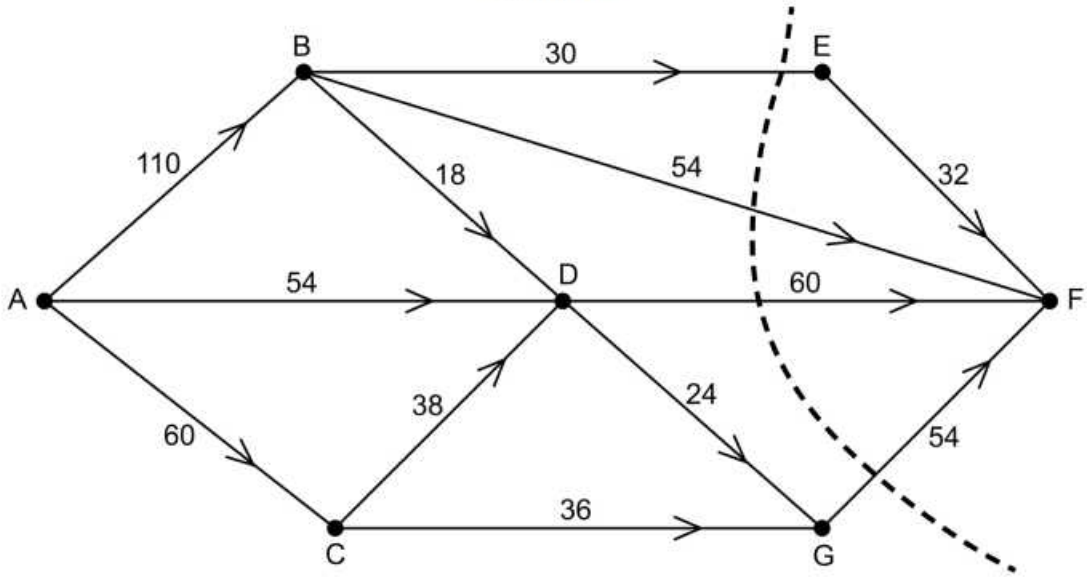
**Specific behaviours**

- ✓ shows systematic working
- ✓ lists at least four correct paths with passenger numbers
- ✓ lists complete set of paths with passenger numbers
- ✓ states correct total

(b) On the network below, draw the minimum cut that corresponds to the maximum flow determined in part (a). (1 mark)



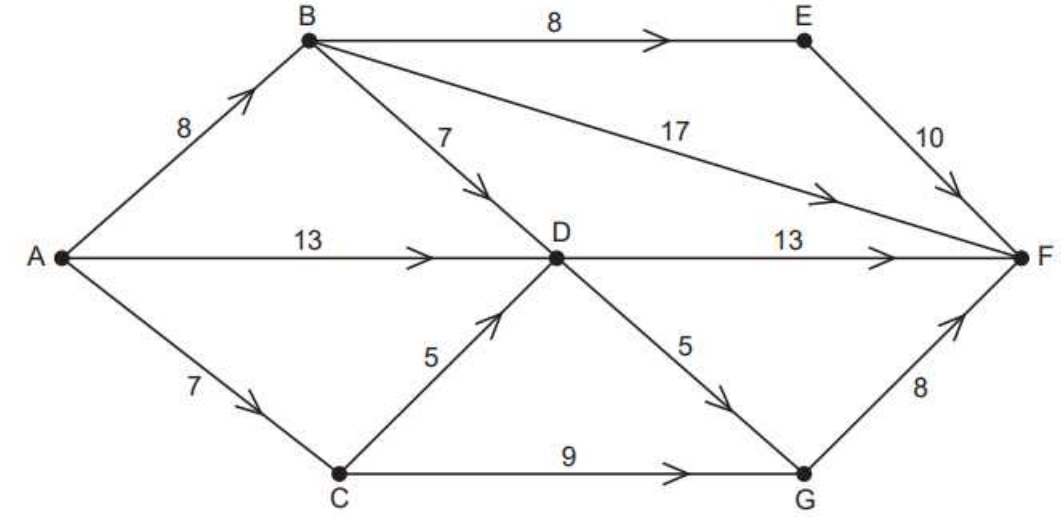

**Solution**




**Specific behaviours**

✓ draws correct cut

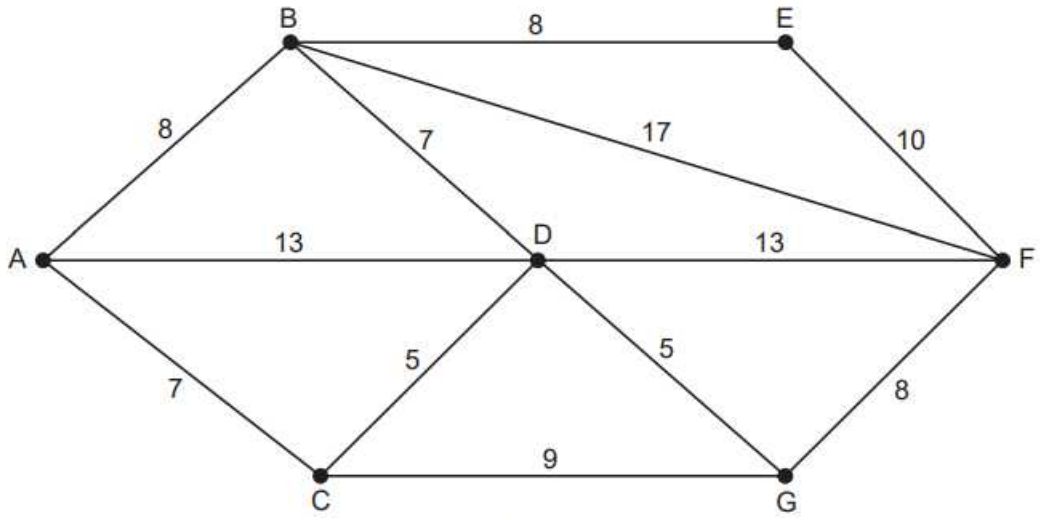
(c) The network below shows the distance in kilometres between the islands. Ferry passengers are charged a $10 booking fee plus $1.50 for each kilometre they plan to travel. Calculate the minimum cost of travelling from A to F. (3 marks)



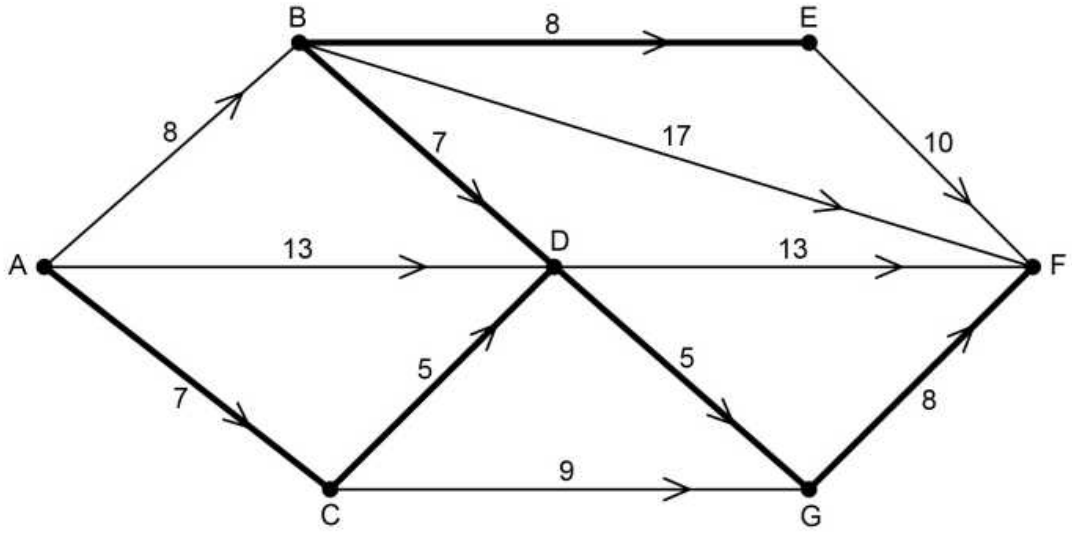

B
8
E
8
7
17
10
A
13
D
13
F
5
5
7
8
9
C
G

| Solution |
| --- |
| Shortest path is ACGF, distance = 24 km<br>Cost = 10 + 1.50 × 24 = $46 |
| **Specific behaviours** |
| ✓ determines shortest path<br>✓ determines shortest distance<br>✓ determines minimum cost |

(d) During the off-season when there are fewer passengers, the ferry company will only maintain the services that form the minimum spanning tree. On the network below, highlight clearly the services that will be maintained. (2 marks)




**Solution**



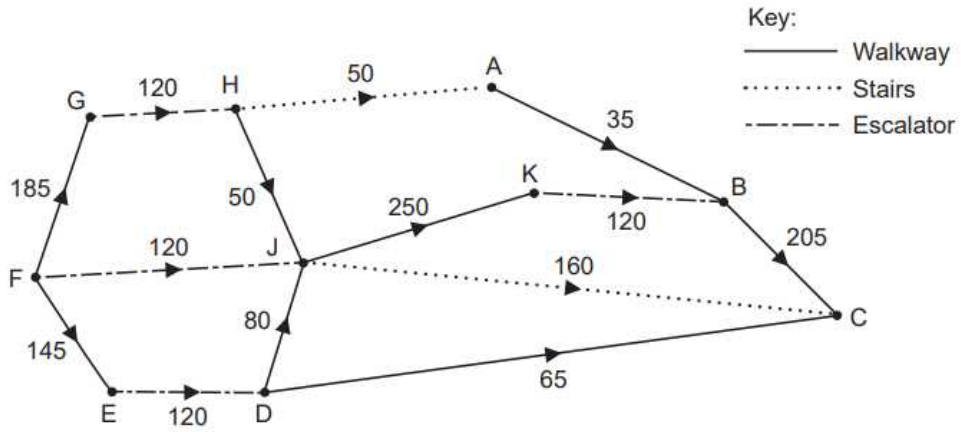

**Specific behaviours**

✓ determines a spanning tree with four correct edges
✓ determines all correct edges

**2022 Section 2 Question 12**

**Networks and decision math-ematics**

The network below shows the different routes commuters can take, travelling from a bus stop to a designated train platform. The edges are a combination of stairs, escalators and covered walkways, with the numbers indicating the maximum number of people per minute that each route can manage.




(a) Which vertex represents

(i) the bus stop? (1 mark)

| Solution |
|---|
| vertex F |
| **Specific behaviours** |
| ✓ states correct vertex |

(ii) the train platform? (1 mark)

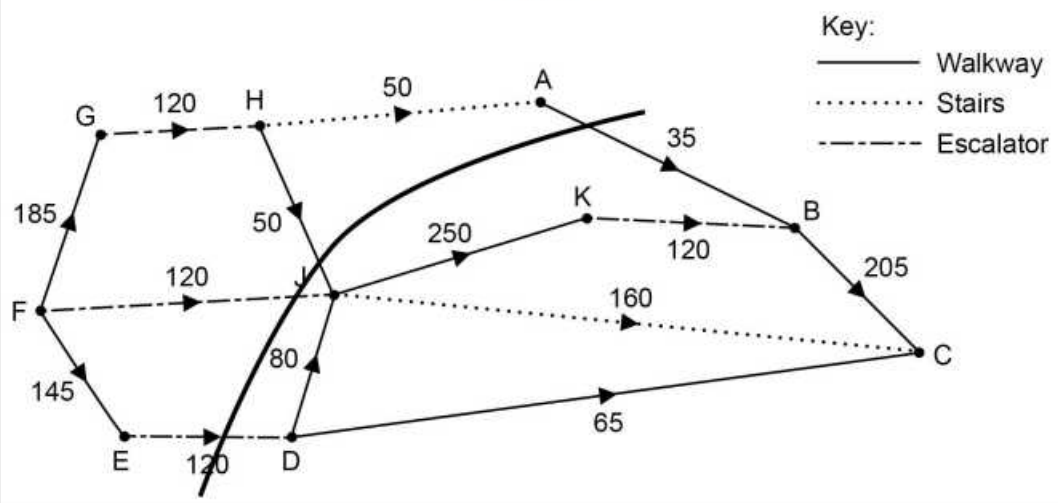
| Solution |
|---|
| vertex C |
| **Specific behaviours** |
| ✓ states correct vertex |

(b) Determine the maximum number of people per minute that can travel from the bus stop to the train platform, listing each path and the corresponding flow. (3 marks)

| Solution | | |
|---|---|---|
| F-G-H-A-B-C = 35<br>F-G-H-J-K-B-C = 50<br>F-J-K-B-C = 70<br>F-E-D-J-C = 80<br>F-J-C = 50<br>F-E-D-C = 40 | OR | F-E-D-C = 65<br>F-E-D-J-K-B-C = 55<br>F-J-C = 120<br>F-G-H-A-B-C = 35<br>F-G-H-J-K-B-C = 50 |
| Maximum flow = 325 people per minute | | |
| **Specific behaviours** | | |
| ✓ use of a systematic approach is evident<br>✓ has all possible flow paths<br>✓ correctly calculates maximum flow | | |

(c) Draw the minimum cut that corresponds to the maximum flow on the network diagram. (1 mark)

**Solution**




**Specific behaviours**

✓ draws correct cut

(d) The Transport Authority has approved upgrades for one **escalator** connection to increase the capacity from 120 to 150 people per minute. Which connection should be upgraded? Justify your response. (2 marks)

**Solution**

Four possible escalator connections:

- GH: has not used maximum flow
- FJ: has more than 30 units flowing in and out
- ED: has 25 units flowing in and out
- KB: has no spare flow entering from HJ or FJ

∴ FJ would be the best connection to increase by 30 people per minute.

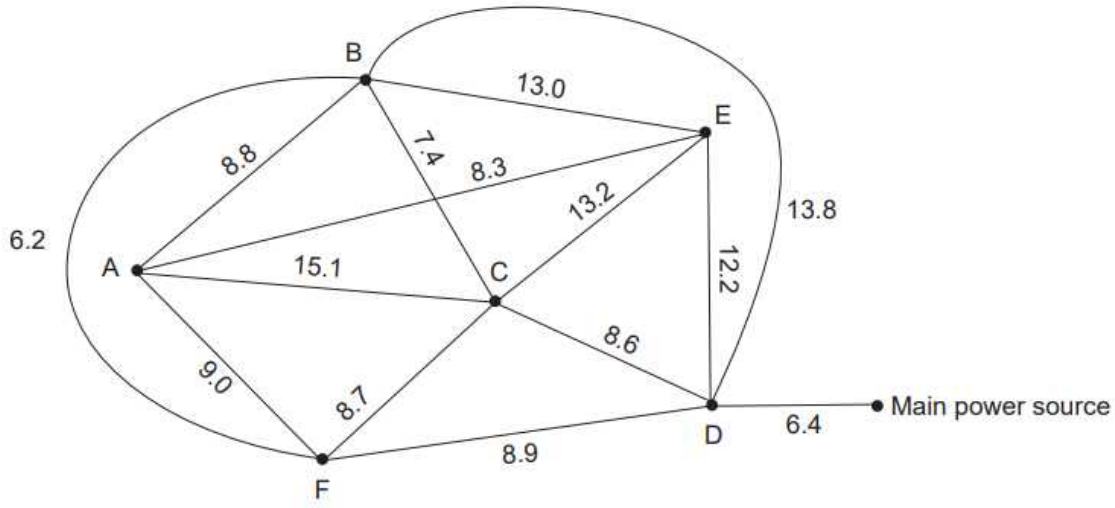
**Specific behaviours**

✓ correctly identifies FJ
✓ provides valid justification

**2022 Section 2 Question 14**

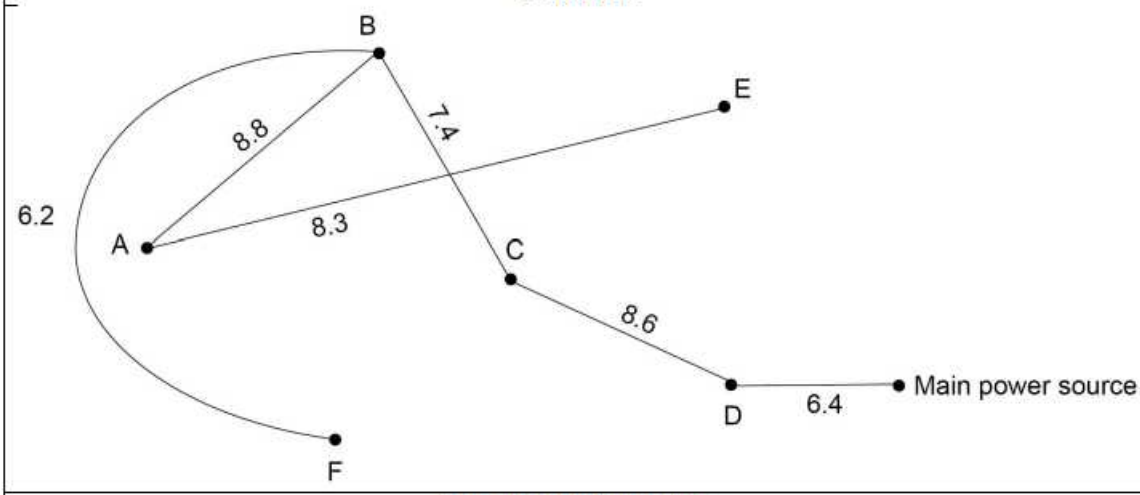
**Networks and decision math-ematics**

Electric wiring is being installed to six powered sites (A–F) at a new caravan park from the main power source. The distance, in metres, between each site is shown below.




(a) Complete the minimum spanning tree below. (3 marks)

**Solution**




| Specific behaviours |
|---|
| ✓ shows appropriate method for determining minimum spanning tree |
| ✓ draws minimum spanning tree with at least 3 correct connections |
| ✓ draws minimum spanning tree with all correct connections |

(b) The cost of installing the electric wiring is $14.20 per metre. If the wiring must include the connection between D and the main power source

(i) calculate the minimum length of wiring required. (2 marks)

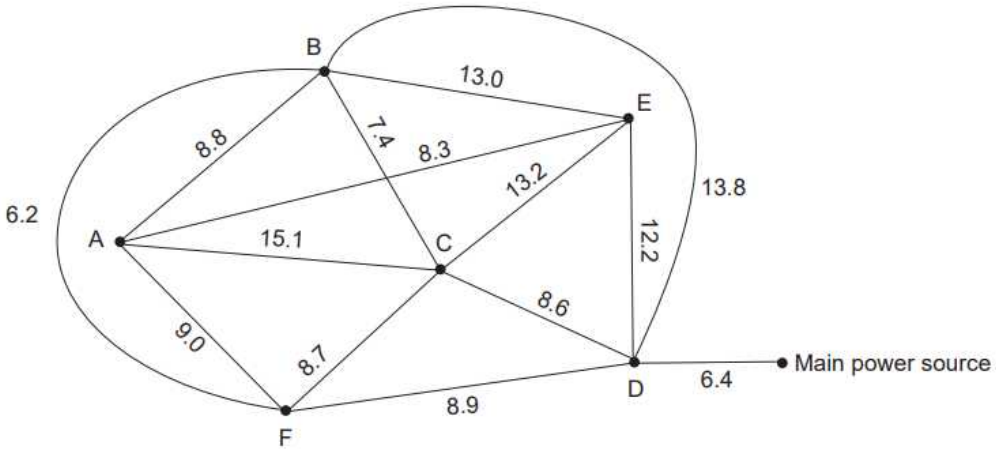
| Solution |
|---|
| Total = $(8.3 + 8.8 + 6.2 + 7.4 + 8.6) + 6.4$<br>$= 39.3 + 6.4$<br>$= 45.7$ metres of wiring required |
| **Specific behaviours** |
| ✓ calculates total on tree (39.3m)<br>✓ calculates total including main connection |

(ii) calculate the total cost of installing the wiring. (1 mark)

| Solution |
| --- |
| Cost = $45.7 \times 14.20$<br>= $648.94 |
| **Specific behaviours** |
| ✓ calculates product of wiring required and cost per metre |

It is discovered that between Sites B and F there is a large section of rock. The owner can still install the wiring directly between these two sites but the cost for that section will double in price.

(c) What effect, if any, will this have on the minimum spanning tree and total cost of the installation? A copy of the original graph has been provided below. (3 marks)




| Solution |
| --- |
| The doubling of the price would effectively make connection B–F 12.4 instead of 6.2. The minimum spanning tree would no longer use the connection between B and F and would instead use F–C. The required wiring would increase to 48.2 metres and the total cost would increase from $648.94 to $684.44. |
| **Specific behaviours** |
| ✓ states B–F connection would not be used<br>✓ identifies replacement connection C–F<br>✓ calculates new total cost |

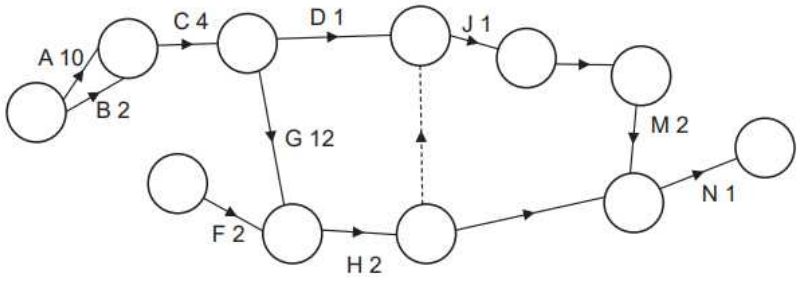
**2021 Section 2 Question 13**

**Networks and decision math-ematics**

A kitchen renovation project consists of a number of tasks of different durations and completed in different orders. One such renovation has this information summarised in the table below.

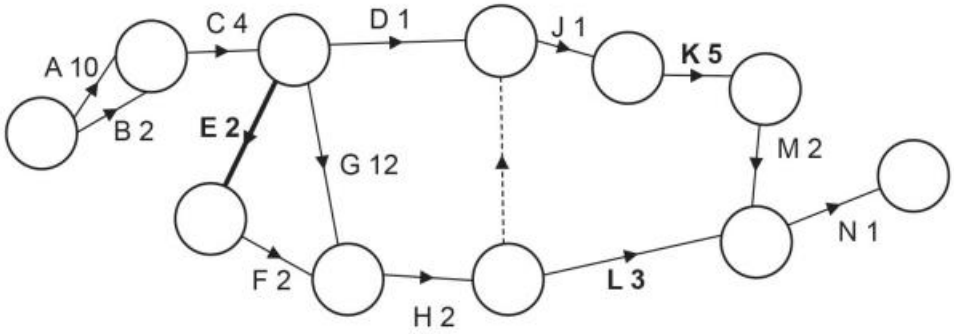
| Task | Task description | Duration (days) | Immediate predecessor(s) |
|---|---|---|---|
| A | Prepare plans | 10 | None |
| B | Select contractor | 2 | None |
| C | Review plans | 4 | A, B |
| D | Purchase appliances | 1 | C |
| E | Remove old appliances and benches | 2 | C |
| F | Prepare electrics and plumbing | 2 | E |
| G | Construct new cupboards and benches | 12 | C |
| H | Install cupboards and benches | 2 | F, G |
| J | Install appliances | 1 | see part (b) |
| K | Tiling and splashbacks | 5 | J |
| L | Flooring | 3 | H |
| M | Complete electrical and plumbing | 2 | K |
| N | Test and handover | 1 | L, M |

A partially-completed project network is shown below for this table.




(a) Complete the network diagram. (2 marks)

**Solution**




**Specific behaviours**

✓ correctly draws edge E
✓ writes correct weight on edges E, K, L

(b) Identify the immediate predecessor(s) of Task J. (1 mark)

**Solution**

Immediate predecessors of Task J are D and H.

**Specific behaviours**

✓ identifies correct immediate predecessors for task J

(c) What does the dotted line on the network indicate? (1 mark)

| Solution |
| --- |
| H is the predecessor for J. |
| **Specific behaviours** |
| ✓ states correct reason |

(d) Determine the critical path and the minimum completion time for the project. (2 marks)

| Solution |
| --- |
| ACGHJKMN. Minimum completion time is 37 days. |
| **Specific behaviours** |
| ✓ states correct critical path<br>✓ states correct minimum completion time |

(e) Determine the float time for Task D and explain its meaning in terms of the renovation. (2 marks)

| Solution |
| --- |
| Float time = 13 days. This means Task D can be delayed by up to 13 days. |
| **Specific behaviours** |
| ✓ states correct float time<br>✓ gives correct explanation in terms of the renovation |

(f) Once Task E (removal of old appliances and benches) begins, the kitchen cannot be used. What is the least amount of time the occupants of the house will be without a working kitchen? (1 mark)

| Solution |
| --- |
| 15 days |
| **Specific behaviours** |
| ✓ states correct time |

(g) If Task G was actually completed in nine days, how would this affect the critical path and minimum completion time? (2 marks)

| Solution |
| --- |
| Critical path stays the same. Minimum completion time is now 34 days. |
| **Specific behaviours** |
| ✓ states the critical path stays the same<br>✓ states correct new minimum completion time |

Tasks E and F are both delayed.

(h) What is the maximum possible delay that does not affect the original minimum completion time? (1 mark)

| Solution |
| --- |
| 8 days |
| **Specific behaviours** |
| ✓ states correct time |

**2021 Section 2 Question 15**

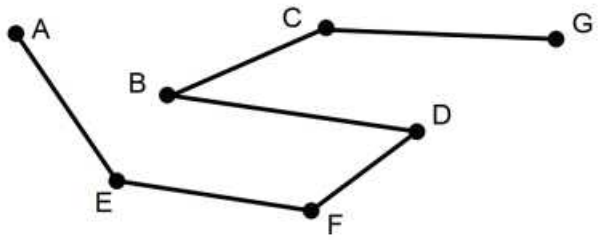
**Networks and decision math-ematics**

A number of streets connecting locations A to G inclusive have been identified as needing lighting upgrades with energy-efficient LED lamps. The council has decided that each location must have at least one connecting street that receives the lighting upgrade. The table below shows the cost, in dollars, of completing the upgrade in each street.

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| **A** | – | 7900 | 10300 | – | 7100 | – | – |
| **B** | 7900 | – | 7600 | 6200 | 6500 | – | 15400 |
| **C** | 10300 | 7600 | – | 8500 | – | 12200 | 9200 |
| **D** | – | 6200 | 8500 | – | 8000 | 4700 | 9800 |
| **E** | 7100 | 6500 | – | 8000 | – | 4800 | – |
| **F** | – | – | 12200 | 4700 | 4800 | – | 10100 |
| **G** | – | 15400 | 9200 | 9800 | – | 10100 | – |

The council has a limited budget, so it needs to complete the upgrades at minimum cost.

(a) Demonstrate the use of Prim's algorithm on the table above to determine the minimum spanning tree and draw it on the diagram below. (3 marks)




**Solution**

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| ~~A~~ | – | ~~7900~~ | ~~10300~~ | – | ~~7100~~ | – | – |
| ~~B~~ | ~~7900~~ | – | ~~7600~~ | (6200) | ~~6500~~ | – | ~~15400~~ |
| ~~C~~ | ~~10300~~ | (7600) | – | ~~8500~~ | – | ~~12200~~ | ~~9200~~ |
| ~~D~~ | | ~~6200~~ | ~~8500~~ | | ~~8000~~ | (4700) | ~~9800~~ |
| ~~E~~ | (7100) | ~~6500~~ | | ~~8000~~ | | ~~4800~~ | |
| ~~F~~ | | | ~~12200~~ | ~~4700~~ | (4800) | | ~~10100~~ |
| ~~G~~ | – | ~~15400~~ | (9200) | ~~9800~~ | – | ~~10100~~ | – |

See diagram above for graph of spanning tree.

**Specific behaviours**

✓ correctly demonstrates use of Prim's algorithm on the table
✓ obtains correct solution using Prim's algorithm
✓ correctly draws minimum spanning tree

(b) The council has set aside $42 000 to complete the lighting upgrades. Does it have enough in its budget to make the necessary upgrades? Justify your answer. (2 marks)

| Solution |
|---|
| Cost = 7100 + 4800 + 4700 + 6200 + 7600 + 9200<br>= $39 600<br>∴ sufficient funds |
| **Specific behaviours** |
| ✓ calculates minimum cost in dollars<br>✓ states there are sufficient funds |

(c) Due to the location of the police station, the upgrade from D to C must be included. What effect will this have on the minimum cost and the spanning tree? (2 marks)

| Solution |
|---|
| 39 600 – 7600 (BC) = 32 000. 32 000 + 8500 (DC) = 40 500.<br>This means the cost now increases by $900 to $40 500.<br>The spanning tree will not include BC. |
| **Specific behaviours** |
| ✓ states that cost increases by $900<br>✓ correctly states that connection BC no longer exists |

**2021 Section 2 Question 16**

**Networks and decision math-ematics**

The graph below shows a network of water pipes. The water source and main pumping station are located at S. The distribution centre is at T and the other vertices are intermediate pumping stations. The weights on the edges show the capacities in kilolitres per hour that can flow through each pipe.

(a) (i) Determine the value of the cut shown in Diagram 1. (1 mark)

| Solution |
|---|
| Cut = 9 + 7 +14 + 0 + 15 = 45 |
| **Specific behaviours** |
| ✓correctly calculates the value of cut |

(ii) Using your answer to part (a)(i), what can be said about the maximum flow of water through the network? (1 mark)

| Solution |
|---|
| The maximum flow through the network is ≤ 45. |
| **Specific behaviours** |
| ✓correct answer |

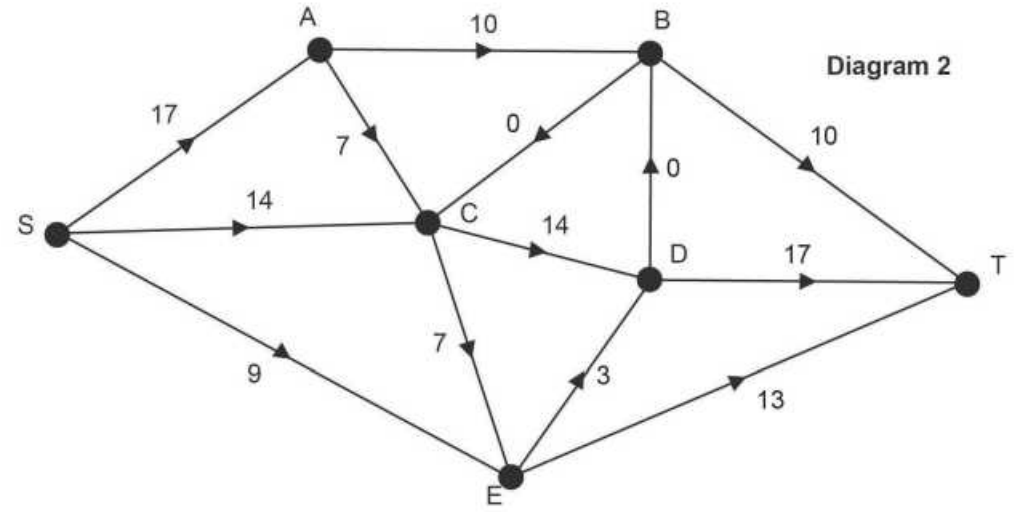
(b) State the maximum possible flows along the paths SABT and SCDT. (2 marks)

| Solution |
|---|
| SABT = 10<br>SCDT = 14 |
| **Specific behaviours** |
| ✓correctly states maximum flow for SABT<br>✓correctly states maximum flow for SCDT |

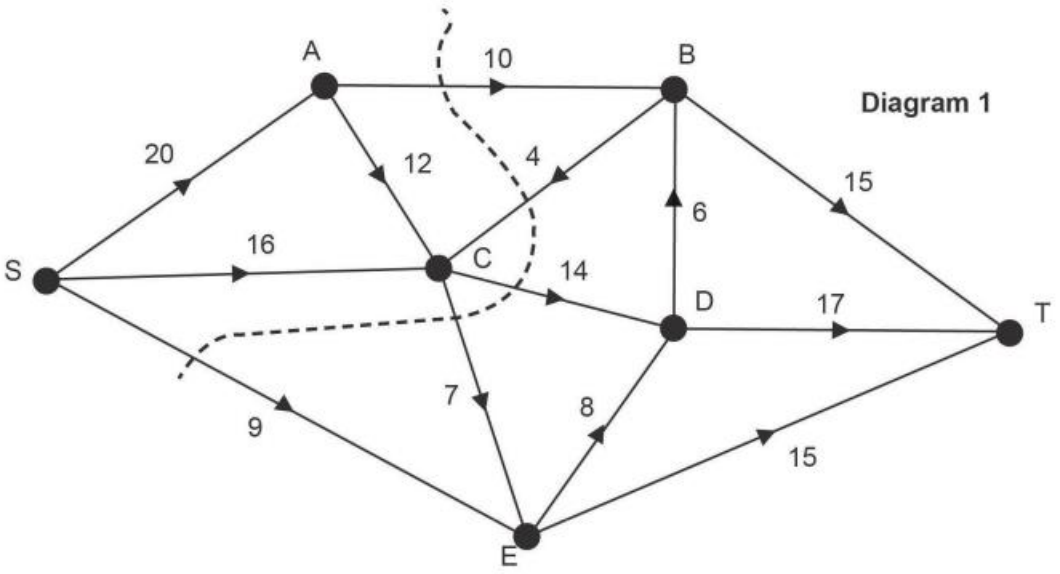
(c) Determine the maximum flow from S to T, listing each path and the corresponding flow. (3 marks)

| Solution |
|---|
| One possibility,<br>Using answers from part (b):<br>SABT = 10, SCDT = 14, SACEDT = 3, SAET = 4, SET = 9<br>∴ maximum flow = 40 kilolitres per hour<br>**or**<br>Another possibility: SABT = 10, SACDBT = 5, SACDT = 5, SCDT = 4, SCEDT = 7, SEDT = 1, SET = 8<br>∴ maximum flow = 40 kilolitres per hour |
| **Specific behaviours** |
| ✓correctly gives at least one other path<br>✓correctly gives 2 correct paths<br>✓states correct maximum flow |

(d) Using **Diagram 2** below, indicate a possible flow along each pipe corresponding to the maximum flow calculated in part (c). (2 marks)

| Solution |
|---|
| Using the first possibility:<br>  <br> |
| **Specific behaviours** |
| ✓correctly shows flow from S = flow into T<br>✓correctly labels diagram |

(e) Determine the minimum cut that corresponds to the maximum flow and illustrate this on the copy of Diagram 1 shown below. (1 mark)




| Solution |
|---|
| See above diagram for cut |
| **Specific behaviours** |
| ✓correctly draws the minimum cut |

**2020 Section 2 Question 15**

**Networks and decision math-ematics**

The graph below shows a network of sewage pipes. The numbers on the edges indicate the number of litres per minute that can flow along each pipe.

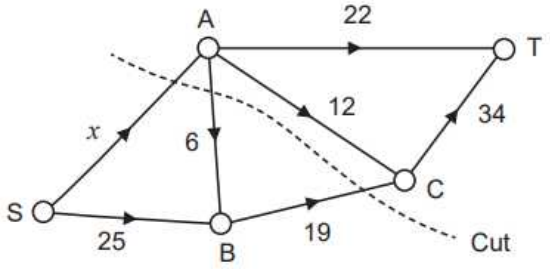



Diagram 1

(a) Show that the value of x is 38, given that the value of the cut is 57. (2 marks)

| Solution |
|---|
| $x + 19 = 57$<br>$x = 38$ |
| **Specific behaviours** |
| ✓ interprets the cut correctly to establish an equation<br>✓ shows that $x = 38$ |

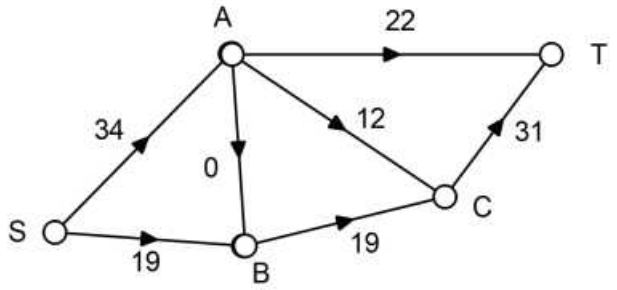
(b) Calculate the value of the maximum flow through the network. (2 marks)

| Solution |
|---|
| Listing:<br>**1.** SBCT-19, SAT- 22, SACT-12 gives a total of 53 L per minute<br>**2.** SACT-12, SAT- 22, SABCT- 4, SBCT-15 gives a total of 53 L per minute<br>**or**<br>minimum cut through AT, AC and BC = 53 |
| **Specific behaviours** |
| ✓ lists at least 2 correct paths **or** shows the minimum cut<br>✓ states correct maximum flow |

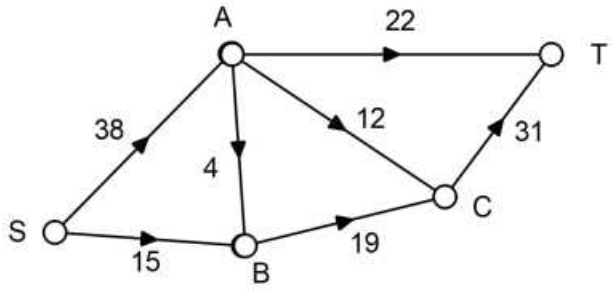
(c) Indicate on Diagram 2 below a possible flow along each pipe corresponding to the maximum flow calculated in part (b). (2 marks)

**Solution**

Using Listing **1**:



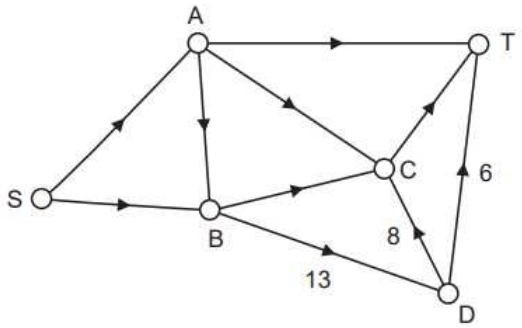

Using Listing **2:**




**Specific behaviours**

✓ indicates at least 4 correct capacities
✓ indicates all correct capacities

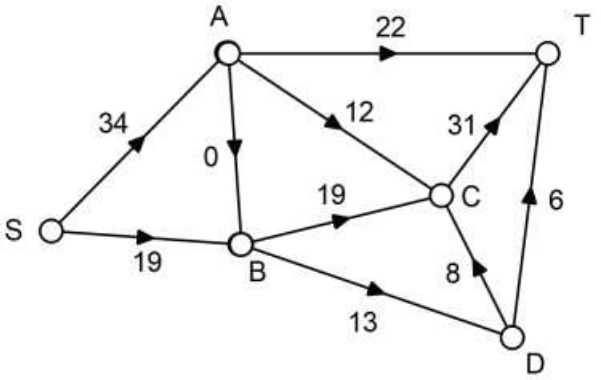
Extra pipes, BD, DC and DT, are added to form a new system shown below. The capacities of the new pipes are indicated on the diagram. The original pipes have the same capacity as shown in **Diagram 1**.



(d) How will the addition of these pipes affect the new maximum flow through the system? (2 marks)

**Solution**

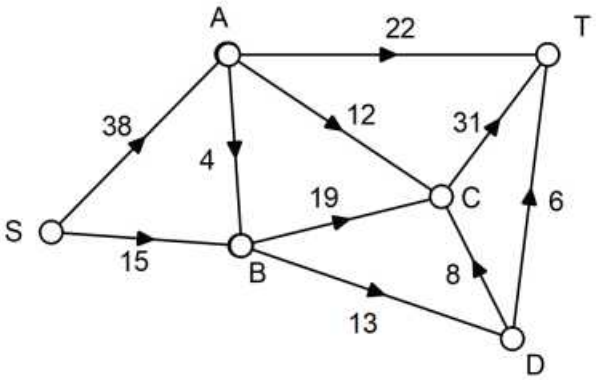
Using Listing **1**:
SBDT = 6
SABDCT = 3
Therefore, max flow = 53 + 9 = 62
New max flow = 62 L per min
This is an increase of 9 L per min




Using Listing **2**:
SBDT = 6
SBDCT = 3
Therefore, max flow = 53 + 9 = 62
New max flow = 62 L per min
This is an increase of 9 L per min

**or**

Min cut through AT, CT and DT
gives 62 as before
This is an increase of 9 L per min




**Specific behaviours**

✓ correctly states the flow will increase
✓ correctly calculates increase of 9 L per min (from 53 to 62)

**2020 Section 2 Question 16**

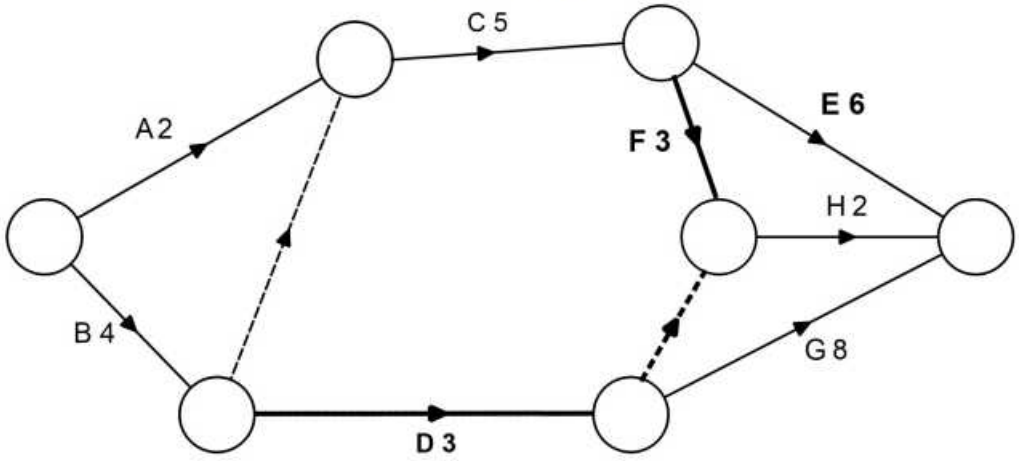
**Networks and decision math-ematics**

Valeska and her sister Katrin are planning a small building project. The table below shows the required activities, together with the times taken (in days) and the immediate predecessors for each activity.

| Activity | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| Time (days) | 2 | 4 | 5 | 3 | 6 | 3 | 8 | 2 |
| Immediate predecessors | – | – | A, B | B | C | C | D | D, F |

(a) Complete the project network below, showing all activities and durations. (3 marks)

**Solution**




**Specific behaviours**

✓ draws diagram with D following B as well as F following C
✓ contains a dotted line linking D with H
✓ network is labelled with tasks, durations and arrows

(b) Determine all critical activities and the minimum completion time for the project. (3 marks)

**Solution**

There are two critical paths: BCE and BDG, so the critical activities are B, C, E, D and G.
Minimum completion time = 15 days

**Specific behaviours**

✓ states at least 3 critical activities
✓ states all critical activities
✓ states correct minimum completion time

(c) Calculate the float times for each of the non-critical activities. (3 marks)

**Solution**

A has a float time of 2 days
F has a float time of 1 day
H has a float time of 1 day

**Specific behaviours**

✓ correctly states float time for A
✓ correctly states float time for F
✓ correctly states float time for H

(d) If Activity H is delayed by three days, what effect will this have on the minimum completion time and the critical activities? (2 marks)

| Solution |
|---|
| H has a float time of 1 day so a delay of 3 days would increase the minimum completion time to 17 days. The critical activities would now be B, C, F and H (i.e. the new critical path is BCFH) |
| **Specific behaviours** |
| ✓ states correct increase of 2 days<br>✓ states new critical activities |

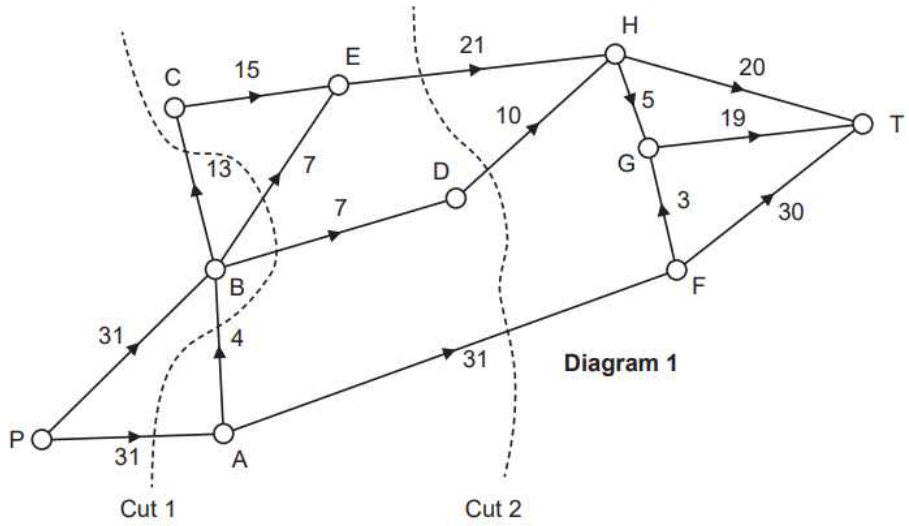
(e) Extra resources become available that can be used to shorten the duration of **one** of Activities B, E or F (on the original network) by one day. Which of these activities should be shortened and why? (3 marks)

| Solution |
|---|
| Shorten B, which will reduce the minimum completion time by 1 day, as it is on both critical paths.<br><br>Shortening F will have no effect since it is not a critical activity.<br>Shortening E would only be effective if D or G were reduced as well, as E is only on one critical path. |
| **Specific behaviours** |
| ✓ states correct activity to shorten<br>✓ states that B is on both critical paths<br>✓ provides a valid reason for not shortening E and F |

**2019 Section 2 Question 15**

**Networks and decision math-ematics**

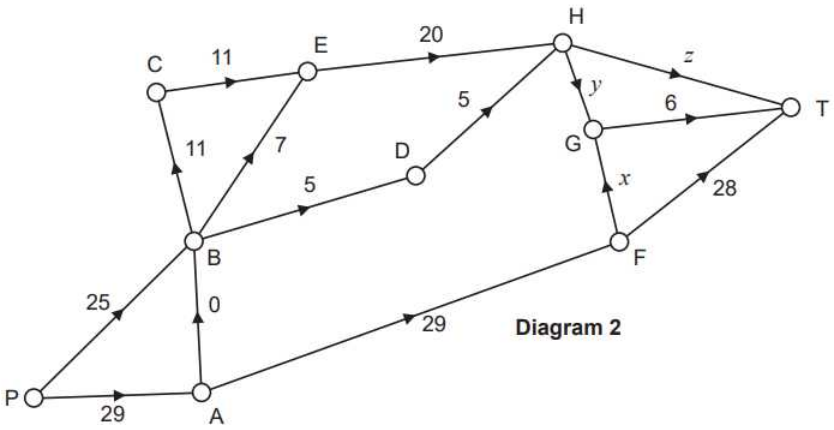
The directed network below shows the maximum available capacity for transferring power between different sub-stations on a small island. The number on each edge gives the capacity in kilovolts (kV).




(a) State the capacity of each cut in Diagram 1. (2 marks)

| Solution |
|---|
| Cut 1 = 58, Cut 2 = 62 |
| **Specific behaviours** |
| ✓ states correct value of cut 1<br>✓ states correct value of cut 2 |

Diagram 2 shows a possible flow through the same network.




(b) Determine the initial flow in Diagram 2. (1 mark)

| Solution |
|---|
| 25 + 29 = 54 which is the flow out of the source 7 |
| **Specific behaviours** |
| ✓ correctly shows that the initial flow is 54 kV |

(c) Calculate the value of *x, y* and *z* in Diagram 2. (3 marks)

| Solution |
|---|
| $x = 1, y = 5, z = 20$ |
| **Specific behaviours** |
| ✓ states correct value of $x$<br>✓ states correct value of $y$<br>✓ states correct value of $z$ |

(d) Determine the maximum flow for the original network (Diagram 1). (2 marks)

| Solution |
|---|
| Minimum cut through HT, HG and AF on Diagram 1<br>**or** PAFT 30, PBCEHT 13, PBDHGT 5, PBEHT 7, PAFGT 1<br>**or** PAFT on Diagram 2 can increase by two<br>Maximum flow = 56 |
| **Specific behaviours** |
| ✓ correctly chooses a suitable method to determine the maximum flow<br>✓ correctly determines maximum flow |

Engineers wish to increase the maximum capacity to sub-station T. They propose to add a new transmission line from E to T of capacity 3 kV **or** a new transmission line from D to G of capacity 3 kV

(e) Determine which of these proposals will increase the maximum capacity to sub-station T. Justify your answer. (3 marks)

| **Solution** |
| --- |
| ET will increase the flow by 2 (PBCET)<br>DG will increase the flow by 2 (PBDGT)<br>**or** cut 1 = minimum cut with edge ET or DG, i.e. an increase of 2<br>Therefore, either proposal will increase the flow by 2 |
| **Specific behaviours** |
| ✓ correctly shows that ET will increase the flow<br>✓ correctly shows that DG will increase the flow<br>✓ correctly justifies that either will increase the maximum capacity |