NewQMaths

Ross **Brodie**
Stephen **Swift**

11A

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Contents

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Introduction

New QMaths 11A textbooks are designed to assist students to build competence in the computational, estimation and measurement skills they need for informed citizenship and lifelong learning, to increase their confidence in using mathematics to solve problems, to encourage them to develop positive attitudes towards mathematics, and to provide a basis for them to make sound judgements in real life. Students are encouraged to further develop key competencies, including the opportunity to work with others and in teams.

New QMaths 11A provides ample opportunity for students to develop skills and demonstrate performance in the dimensions of communication and justification, knowledge and procedures, use of technology, use of mathematical instruments, and modelling and problem solving. It is a gender-inclusive and culturally inclusive text that accommodates a variety of student learning styles through a modern approach that will engage students and provide impetus for the affective objectives of the Mathematics B syllabus. The Mathematics B texts are sequenced to allow for the systematic development of mathematical concepts through a spiralling and integrated organisation that provides a seamless transition from Junior mathematics.

All elective topics are included in the *New QMaths A* textbooks, and there is considerable support of the use of appropriate technologies and application of concepts in a variety of situations from real-world to simplified life-related tasks. Students are provided with opportunities to demonstrate initiative, and there is variation from simple, single-step tasks to tasks that are complex in nature.

What you will find in *New QMaths 11A* and the accompanying NelsonNet website

The textbook includes:

- modern full-colour layout to help you to distinguish between examples, exercises, definitions and investigations
- clear definitions and explanations of important terms and methods
- artwork, photographs and maps in colour for maximum clarity and appeal
- new content and exercises to improve development of concepts and skills
- extensive explanations to help you to get the most from graphics calculators
- detailed instructions in the text for the Casio and Texas Instruments graphics calculators and on the NelsonNet website for the Sharp graphics calculator; the authors have assumed that calculators are in default settings—different results may be obtained if this is not the case
- explanations of the use of spreadsheets and other technology
- modelling and problem-solving practice within exercises to develop concepts beyond basic knowledge and procedures
- a chapter summary and chapter review at the at the end of each chapter to assist in revision for end-of-term and end-of-semester assessment; the review questions are labelled to indicate the exercises where concepts were introduced
- a clear linkage from the text to the syllabus at the beginning of each chapter.

The accompanying NelsonNet website includes:

- Additional exercises (with answers) for every exercise to give opportunities for extra practice of concepts
- Graphics calculator programs in downloadable form
- Graphics calculator instructions for the Sharp graphics calculator
- Extra material as background or extension for interested students and their teachers
- Spreadsheets for investigations and exercises, including interactive spreadsheets
- A complete syllabus guide for Years 11 and 12 with appropriate sections linked to chapter openings
- Teacher notes and copy masters
- An extensive glossary explaining important terms with examples

To access these resources, please visit **www.nelsonnet.com.au**.

Please note: All resources listed throughout the book as available on the CD-ROM can now be found the NelsonNet website.

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Symbols and abbreviations

While every effort has been made to ensure that calculator instructions are accurate, results may vary when graphics calculators are not set in default mode or as a result of manufacturers' changes.

$=$	is equal to	1 : 650 000	at a scale of 1 to 650 000
$\neq$	is not equal to	π	pi (approximately 3.141 592 7)
$\approx$	is approximately equal to	52°15′42″	52 degrees 15 minutes 42 seconds
$\sqrt{}$	square root	$\angle$	angle
%	percentage	|||	is similar to
$<$	is less than	$\triangle$	triangle
$>$	is greater than	$\perp$	perpendicular
$\leqslant$	is less than or equal to	$\bar{x}$	mean of x values
$\geqslant$	is greater than or equal to	σ	standard deviation
$\pm$	plus or minus	5 : 8	ratio of 5 to 8
Σ	summation	/	per
Q_3	3rd quartile	060°G	grid bearing of 60°
P_{87}	87th percentile	157°M	magnetic bearing of 157°
D_4	4th decile	123°T	true bearing of 123°

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Solving triangles

1

Contents

Syllabus Guide
Chapter 1

Syllabus subject matter

Elements of applied geometry

- Applications of trigonometry using sine, cosine and tangent ratios
- Applications of Pythagoras's Theorem
- Simple algebraic manipulation of relevant formulas for this topic

Quantitative concepts and skills

- Metric measurement including measurement of mass, length, area and volume in practical contexts
- Calculation and estimation with and without instruments
- Basic algebraic manipulations

Trigonometry is the study of the sides and the angles of triangles. Long before trigonometry developed as a formal branch of mathematics, the ancient Babylonians used trigonometric functions in their study of astronomy and the ancient Egyptians used trigonometry to assist in building the pyramids and surveying land. Since that time, trigonometry has found many practical applications, including navigation, building, surveying and computer graphics.

1.1 Right-angled triangles

The Greek mathematician Pythagoras (582–500 BC) is generally given credit for first proving the relationship between the sides of a right-angled triangle, although it is known that the ancient Egyptians, Babylonian and Chinese were familiar with some of the properties.

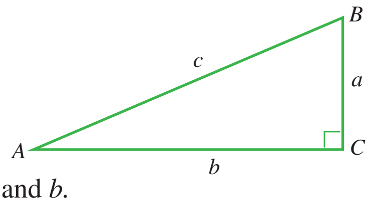
!

In a right-angled triangle the longest side is opposite the right angle and is called the **hypotenuse**.

The square of the hypotenuse is equal to the sum of the squares of the other two sides.

$$c^2 = a^2 + b^2$$

where c is the hypotenuse and the other two sides are a and b.

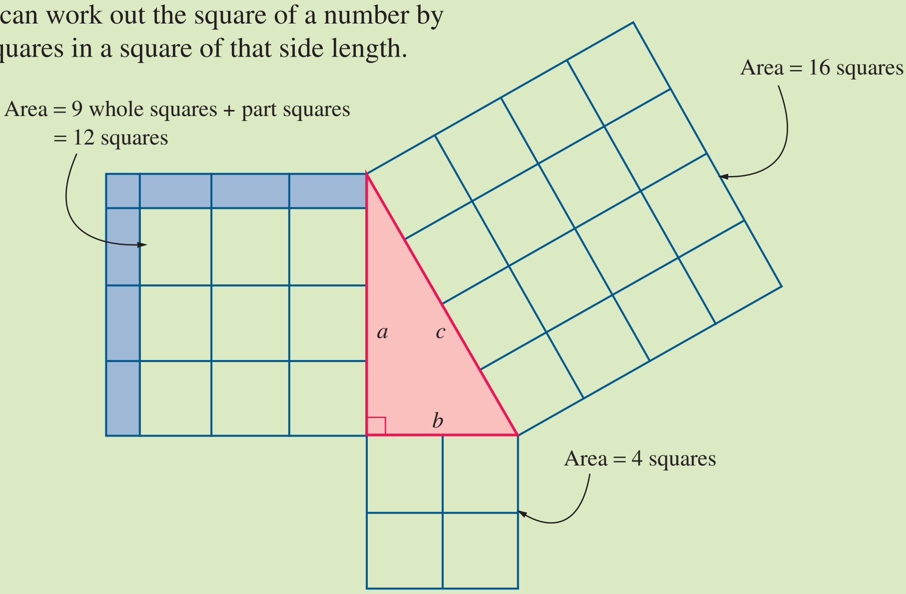


The **naming convention** for triangles is that the vertices (corners) have capital letters and the opposite sides have the same lower-case letter. In the triangle above, $AC = b$, $AB = c$ and $BC = a$. The right angle is shown by a small square in the corner.

Teacher Notes

Investigation Phythagoras's Theorem

Geometrically, the area of a square is equal to x^2, where x is equal to the length of the side. This means we can work out the square of a number by counting unit squares in a square of that side length.

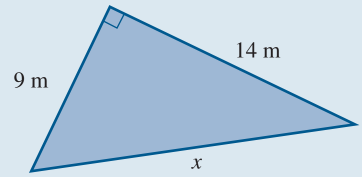


We can show that Pythagoras's Theorem is true by counting the area in squares drawn on the sides of the triangle. The area is counted by dividing the squares into unit squares as shown at the bottom of page 2.

Work in groups of two or three to show Pythagoras's Theorem using this method. Each group should have different-sized triangles and compare the areas counted in the squares. You may have to count some parts of squares by adding them together.

Example 1

Find the unknown length in this triangle.

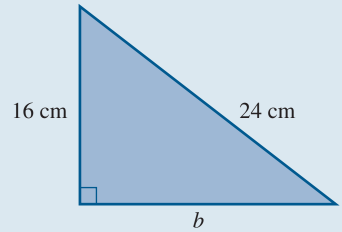


Solution

Write Pythagoras's Theorem.	$c^2 = a^2 + b^2$
x is the hypotenuse.	$x^2 = 9^2 + 14^2$
Work out the squares.	$= 81 + 196$
Calculate the sum.	$= 277$
Find the square root.	$x = \sqrt{277}$ m
Use $\sqrt{\ }$ on your calculator.	$= 16.6433\ldots$ m
Round off.	≈ 16.6 m
Write the answer in a sentence.	The unknown length is about 16.6 m.

Example 2

Calculate the third side of the following triangle.



Solution

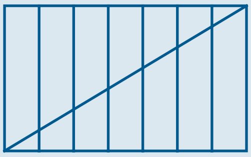
Write Pythagoras's Theorem.	$c^2 = a^2 + b^2$
18 is the hypotenuse.	$24^2 = 16^2 + b^2$
Work out the squares.	$576 = 256 + b^2$
Now use subtraction.	$b^2 = 576 - 256$
	$= 320$
Find the square root.	$b = \sqrt{320}$ cm
Use $\sqrt{\ }$ on your calculator.	$= 17.8885\ldots$ cm
Round off.	≈ 17.9 cm
Write the answer.	The third side is about 17.9 cm.

Example 3

The frame for a wall is 2400 high and 3100 long. A diagonal brace must be added. How long will it be? (Building lengths in Australia are always in millimetres.)

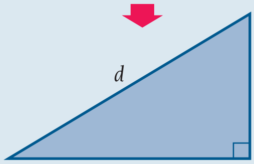
Solution

Sketch the frame (2.4 m by 3.1 m).



Show the triangle.

Write a letter (d) for the brace length.



Write Pythagoras's Theorem. $c^2 = a^2 + b^2$

d is the hypotenuse. $d^2 = 3.1^2 + 2.4^2$

Use your calculator. $= 9.61 + 5.76$

$= 15.37$

Find the square root. $d = \sqrt{15.37}$ m

Use $\sqrt{\ }$ on your calculator. $= 3.9204 \ldots$ m

$= 3920.4 \ldots$ mm

≈ 3920 (to nearest mm)

Write the answer. The brace will be 3920 mm long.

Teacher Notes

Investigation Pythagorean triples

Pythagoras's Theorem can be used to find some standard right-angled triangles where the values of the sides are whole numbers. These are called **Pythagorean triples**. The 3, 4, 5 and 5, 12, 13 triples and their multiples are the most common, but there are many others.

1 Work in groups of two or three to find which of the following are Pythagorean triples. You should also check whether the triples have a common factor to find whether they are multiples of simpler ones.

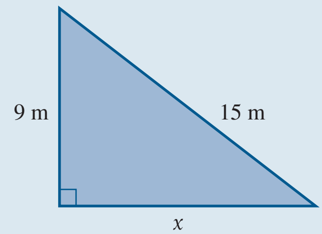
5, 6, 7	6, 8, 10	9, 12, 15	12, 16, 20
10, 24, 26	8, 15, 17	10, 14, 18	7, 24, 25
9, 40, 41	12, 35, 37	16, 63, 65	20, 24, 32

2 Can you find others?

Pythagorean triples and their multiples can sometimes be used to calculate missing sides without squaring. To use them, you must be able to recognise the most common ones: 3, 4, 5 and 5, 12, 13. Others such as 8, 15, 17 and 7, 24, 25 are encountered more rarely.

Example 4

Find the unknown side in this triangle.



Solution

The hypotenuse is 3×5 and the shortest side is 3×3. — There is a 3, 4, 5 triple.

The sides are 3 times bigger than the 3, 4, 5 triple. — Unknown side $x = 3 \times 4 = 12$ cm.

Write the answer. — The unknown side is 12 cm long.

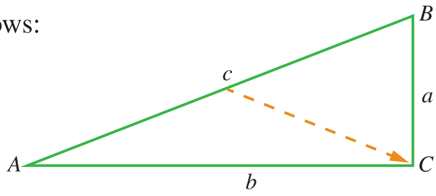
Pythagoras's Theorem is commonly used in reverse to find whether or not an angle is 90°. This is a common use in building and surveying applications.

!

The **reverse of Pythagoras's Theorem** says that, if the sides of a triangle satisfy $c^2 = a^2 + b^2$, then the angle opposite side c is a right angle.

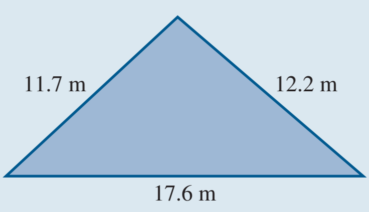
In fact, the *size* of angle C can be checked as follows:

- If $c^2 < a^2 + b^2$, then $C < 90°$ (C is acute).
- If $c^2 = a^2 + b^2$, then $C = 90°$ (C is right).
- If $c^2 > a^2 + b^2$, then $C > 90°$ (C is obtuse).



Example 5

Determine whether this triangle is acute, obtuse or right-angled.



Solution

Find the sum of squares of the shorter sides. — $11.7^2 + 12.2^2 = 136.89 + 148.84 = 285.73$

Find the square of the longest side. — $17.6^2 = 309.76$

Compare the results. — $17.6^2 > 11.7^2 + 12.2^2$

Compare with Pythagorean result. — The longer side is longer than a hypotenuse.

Write the answer. — The triangle is obtuse-angled.

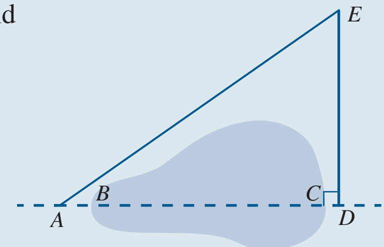
Pythagoras's Theorem is sometimes used in surveying to work out a distance that cannot be directly measured because there is an obstruction. This is done by offsetting at right angles from one point until a distance can be measured from the second point.

Example 6

A surveyor wants to know the length of a small lake and makes the following measurements between the points shown in the diagram.

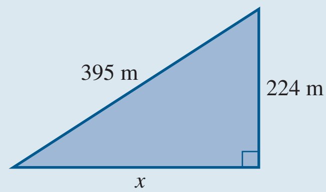
$AB = 35$ m
$CD = 15$ m
$AE = 395$ m
$DE = 224$ m

Find the length of the lake: BC.



Solution

Draw the triangle.
Write the known sides.
Label the unknown side.



Use Pythagoras's Theorem.

$$c^2 = a^2 + b^2$$
$$395^2 = x^2 + 224^2$$
$$156\,025 = x^2 + 50\,176$$

Now use subtraction.

$$x^2 = 156\,025 - 50\,176$$
$$= 105\,849$$

Find the square root.

$$x = \sqrt{105\,849} \text{ m}$$
$$= 325.3444\ldots \text{ m}$$

Round off to measurement accuracy.

$$\approx 325 \text{ m}$$

Now use the original diagram.

$$BC \approx 325 \text{ m} - 35 \text{ m} - 15 \text{ m}$$
$$= 275 \text{ m}$$

Write the answer.

The lake is about 275 m long.

Additional Exercise 1.1

Exercise 1.1 Right-angled triangles

1 Find the unknown side in each of the following triangles.

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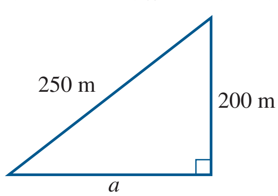

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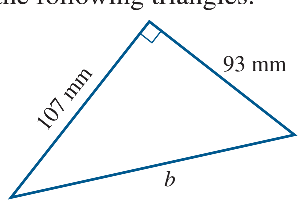

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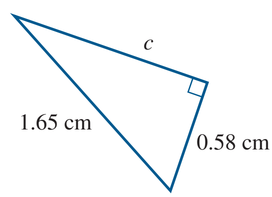

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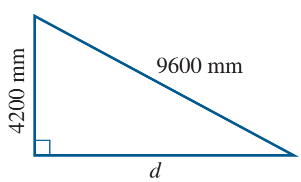

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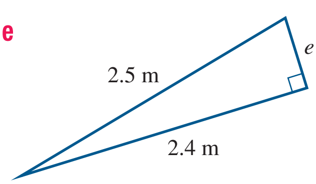

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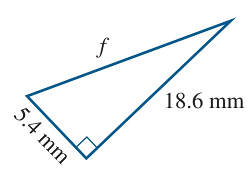

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2 Find the length of the diagonal in each of the following rectangles.

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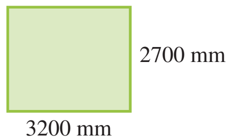

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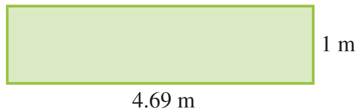

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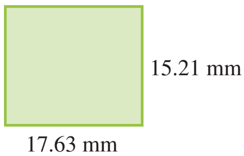

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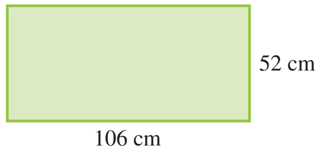

3 Find the unknown side in each of the following triangles.

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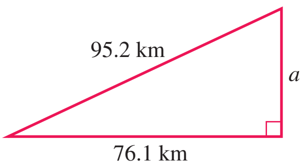

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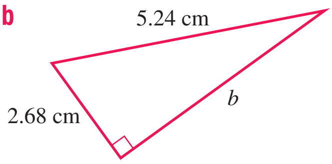

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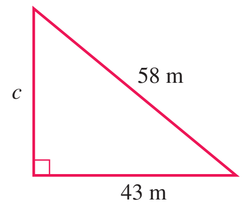

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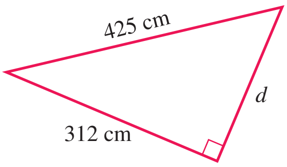

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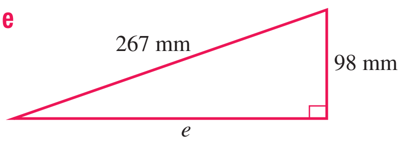

4 Use Pythagoras's Theorem to find the unknown side in each of the following triangles.

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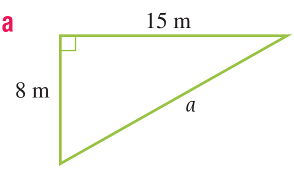

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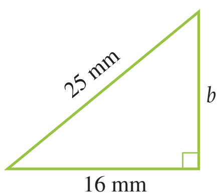

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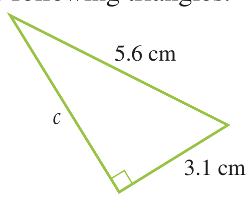

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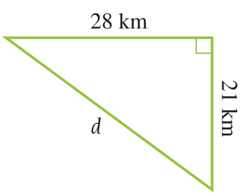

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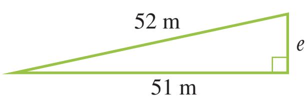

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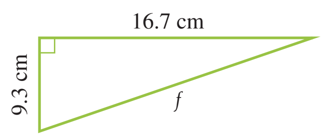
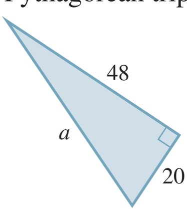


5 Use Pythagorean triples to find the missing side in each of the following triangles.

a


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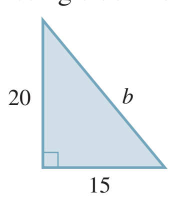
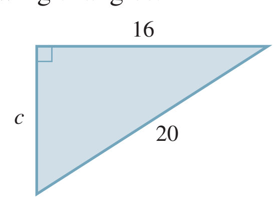


c


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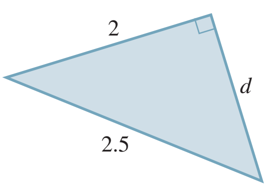
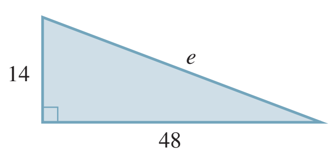
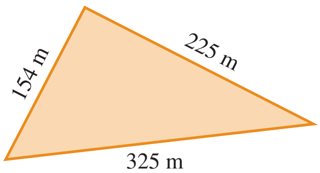


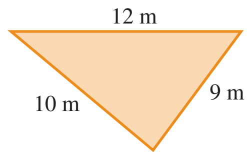
e


6 Determine whether each of the following triangles is acute, obtuse or right-angled.

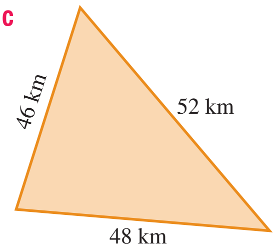
a



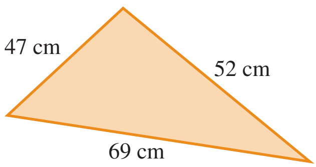
b



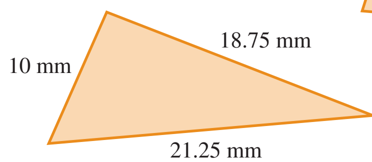
c



d



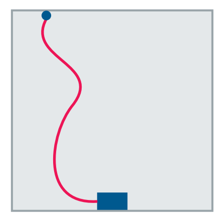
e



Modelling and problem solving

7 A wall frame 3500 long and 2100 high is to have a diagonal brace fitted. How long must the brace be?

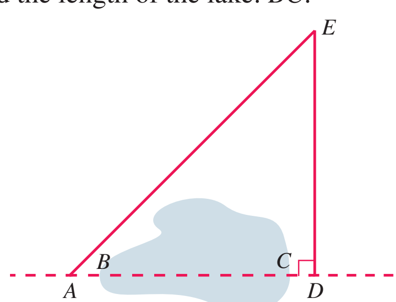
8 A power point is at floor level 1 m from the corner of a square room. A projector is to be placed in the centre of the opposite wall. If the room is 6 m wide, what is the minimum length of the power cord required?



9 A surveyor wants to know the length of a small lake, and makes these measurements between the points shown in the diagram:

$AB = 28$ m $CD = 25$ m
$AE = 627$ m $DE = 452$ m

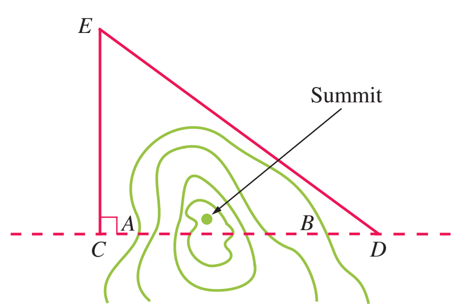
Find the length of the lake: BC.



10 A survey cannot measure directly over a small hill, so the following measurements are made between the points shown in the diagram, where A and B are on the same contour line:

$AC = 45$ m $BD = 55$ m
$CE = 248$ m $DE = 427$ m

Find the horizontal distance AB.

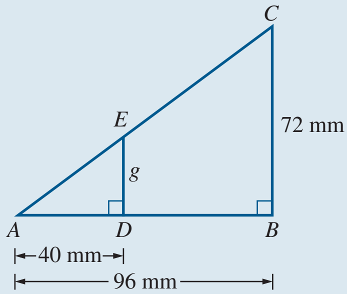


1.2 Using similar triangles

Triangles that are multiples of the 3, 4, 5 triple are exactly the same shape. The only difference between them is their size. We say that the triangles are **similar** and the ratio of their sides is the **scale factor**. The same applies to any other triples or even non-right-angled triangles that are the same shape. We can use the scale factor to find unknown sides in one of the triangles.

Example 7

Find the scale factor and unknown side in the diagram of similar triangles shown at right.



Solution

Compare corresponding known sides. Scale factor from $\triangle ADE$ to $\triangle ABC$ is:

$$\frac{AD}{AB} = \frac{40}{96}$$

Keep exact number on your calculator. $= 0.4166 \ldots$

Now multiply to get the unknown side. $g = 72 \text{ mm} \times \text{scale factor}$

Use the exact number. $= 72 \text{ mm} \times 0.4166 \ldots$

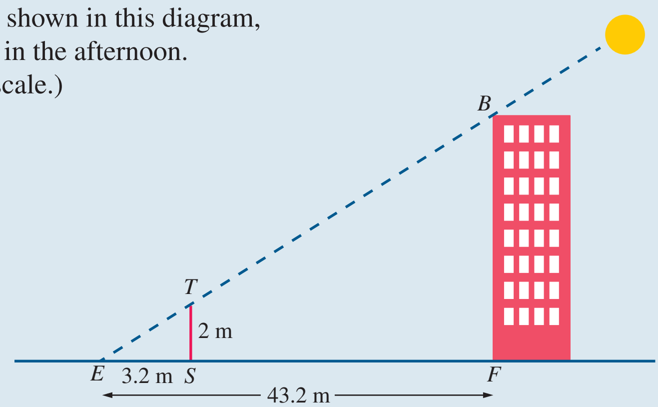
$= 30 \text{ mm}$

Write the answer. The unknown side is of length 30 mm.

The similarity principle is applied to height measurement with the aid of a **shadow stick**. To measure the height of an object, a stick of known height is placed in the shadow of the object. The stick is put on the ground so that its top is just on the edge of the shadow.

Example 8

Find the height of the building shown in this diagram, showing a shadow stick set up in the afternoon. (The diagram is not drawn to scale.)



Solution

The sunlight forms the hypotenuse of the triangles $\triangle EST$ and $\triangle EFB$. The shadow forms the sides ES and EF, and the stick and the building make the third sides. The triangles are similar.

Compare corresponding known sides.

Scale factor from $\triangle EFB$ to $\triangle EST$ is:

$$\frac{EF}{ES} = \frac{43.2}{3.2}$$

Keep the number on your calculator.

$$= 13.5$$

Now multiply to get the unknown side.

$$BF = 2 \text{ m} \times \text{scale factor}$$
$$= 2 \text{ m} \times 13.5 = 27 \text{ m}$$

Write the answer in terms of the question.

The building is 27 m high.

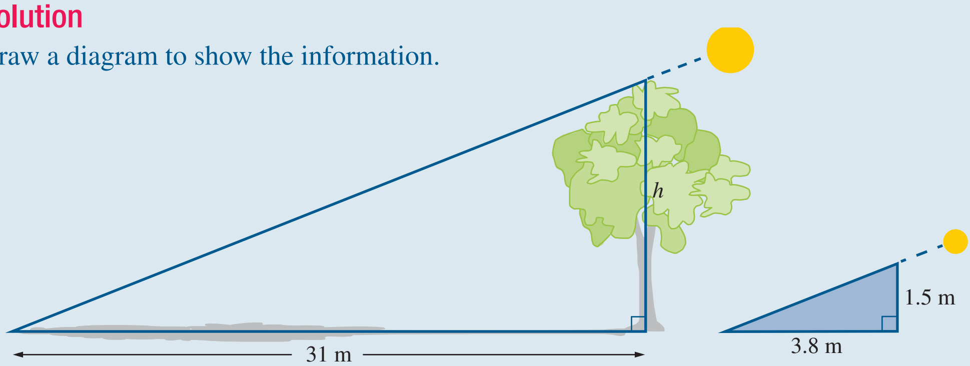
The shadow stick can be used even when the stick is not in the shadow of the building. However, the shadows must be measured *at the same time* so that the triangles remain similar. After all, the length of the shadow cast by the stick will be the same wherever it is placed at a given time.

Example 9

A stick of height 1.5 m casts a shadow of length 3.8 m. At the same time, a tree casts a shadow 31 m long. What is the height of the tree?

Solution

Draw a diagram to show the information.



Find the scale factor.

Scale factor (tree to stick) is:

$$\frac{\text{length of tree shadow}}{\text{length of stick shadow}} = \frac{31}{3.8}$$

Keep exact number on your calculator.

$$= 8.1578\ldots$$

Now multiply to get the unknown side.

$$\text{Tree height} = 1.5 \text{ m} \times \text{scale factor}$$

Use the exact number on your calculator.

$$= 1.5 \text{ m} \times 8.1578\ldots$$
$$= 12.2368\ldots \text{ m}$$

Round and write the answer.

The tree is about 12.2 m high.

9780170226912

Similar triangles can also be used in surveying to find distances that cannot be measured exactly. Using triangles in this way to measure aspects of the Earth is called **triangulation**.

Example 10

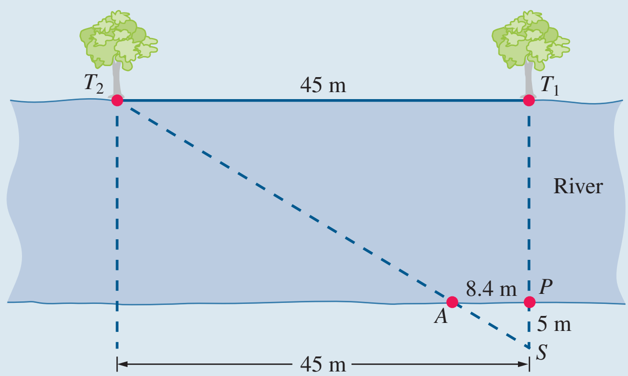
A surveyor needs to measure the distance across a river. There are two trees on the opposite bank. After pacing along the river bank, the surveyor finds that they are 45 m apart. She then moves back 5 m from the bank, directly opposite the first tree. Her assistant has to move 8.4 m along the bank to place a stick directly in her line of sight to the second tree. Find the width of the river.

Solution

Draw a diagram to show the information.

Label points.

Write lengths.



Find the scale factor.

Scale factor is $\dfrac{T_1T_2}{AP} = \dfrac{45}{8.4}$

Keep exact number on your calculator.

$= 5.3571\ \ldots$

$ST_1 = SP \times \text{scale factor}$

Use exact number on your calculator.

$= 5\text{ m} \times 5.3571\ \ldots$

Keep exact number on your calculator.

$= 26.7857\ \ldots\text{ m}$

Work out desired distance.

$PT_1 = ST_1 - SP$

$= 26.7857\ \ldots - 5\text{ m}$

$= 21.7857\ \ldots\text{ m}$

Round and write the answer.

The river is about 22 m wide.

Investigation Shadow stick use

A metre ruler makes an excellent shadow stick because the calculations are simplified by having the stick exactly 1 m high. Use a metre ruler and a long tape in groups of two or three to measure the heights of buildings and trees around the school.

You will need to make sure the metre ruler is vertical when you are measuring its shadow. This can be done using a spirit level. If you don't have a spirit level, you can check that the stick is vertical by line of sight against something known to be vertical, such as the corner of a building or a telephone or light pole.

Check your measurements by comparing the results of your group with the results of other groups.

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Calculator Program

Technology

The program SHADOW can be used to find the height of an object from the height and length of the shadow of a stick and the length of the shadow of the object. The lengths of the shadows must be taken at the same time. The program is given in full on the CD-ROM. Enter the program (or load it from the CD-ROM) and try it with various measurements. Below are parts of the program as it might appear on the Casio, TI–84 and Sharp calculators respectively..

```
1
STICK SHADOW
?
2.3
OBJECT SHADOW
?
5.8
```

```
STICK SHADOW 3.5
OBJECT SHADOW 10
OBJECT HEIGHT I…
        7.142857143
               Done
```

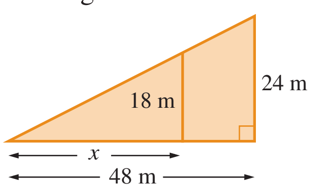
```
SHADOW STICK
STICK HEIGHT
H=
3
STICK SHADOW
S=
1.5
```

Additional Exercise 1.2

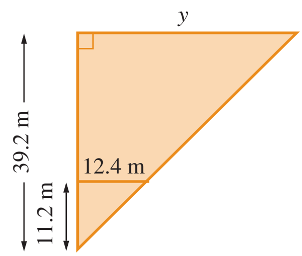
Exercise 1.2 Using similar triangles

1 These diagrams show similar triangles. Use scale factors to find the unknown sides.

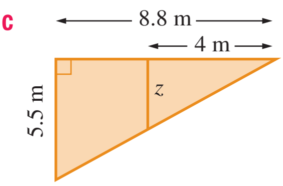
a



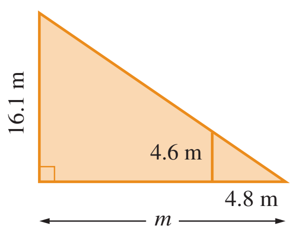
b



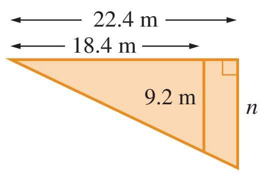
c



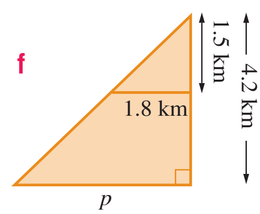
d



e



f



Modelling and problem solving

2 The shadow from a 3 m high post is measured to be 2.4 m long. At the same time, the shadow of a building is measured and found to be 18 m. How high is the building?

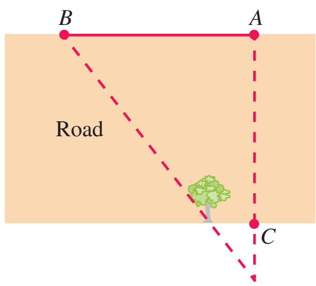
3 A group of students used a straight stick 1.4 m long as a shadow stick to help calculate the height of a tree on the edge of the school oval. The shadow of the tree was 44.1 m long at the same time as the shadow of the stick was 1.8 m long. How high is the tree?

4 A student used a metre ruler as a shadow stick to measure the heights of some school buildings. At 2 pm the length of the shadow cast by the ruler was 60 cm and the length of the shadow of A-block was 4.86 m. The student then moved to the oval and measured the length of the shadow of the gym, which was 3.5 m long. After that the student moved to C-block and measured its shadow as 7.2 m. It was then 2:45 pm and the student made a last check of the stick's shadow length before going inside. It was now 90 cm long. The students were to calculate the heights of the buildings for homework.

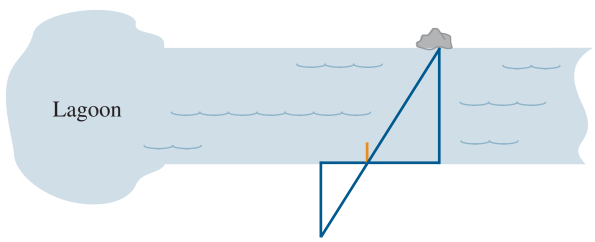
- **a** What did this student do incorrectly?
- **b** What was the height of A-block?
- **c** What was the height of C-block?
- **d** What do you estimate the height of the gym to be?

5 The facade of a historic building was preserved when the building was demolished for a new shopping centre. In the morning the sun shone through the facade, casting a shadow across the street. There were bright patches in the shadow from the windows of the facade. If the whole shadow was 25 m long and the bright patch from a 1.8 m-high window was 2.4 m long, how high was the facade?

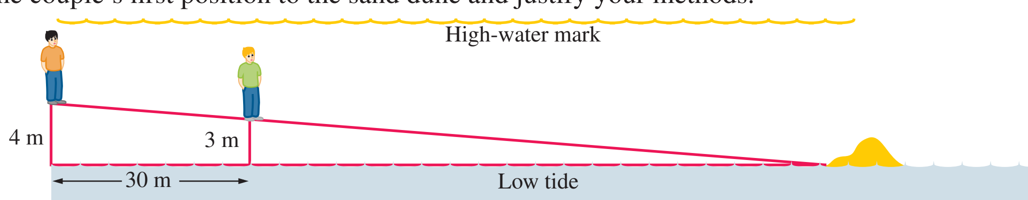
6 The distance across a road cutting is to be calculated using triangulation. Sightings are taken of two points A and B on the opposite side of the road that are known to be 120 m apart. The sides of the road cutting are parallel, and point C on the surveyor's side is directly opposite point A. From a point 8 m away from the road edge at C, the surveyor notices that a small tree on the edge of the road just obscures point B. The tree is found to be 6.7 m from C on the surveyor's side of the road. What is the width of the road cutting?



7 Water is rushing out the long, narrow entrance to a lagoon at low tide. There is a large rock on the opposite bank of the entrance. A fisherman is directly opposite the rock. He walks 4 m along the bank of the entrance and puts a stick in the sand. After walking a further 2 m along the bank, he then has to move 4.5 m back from the bank in order to line up the stick and rock. How wide is the lagoon entrance?

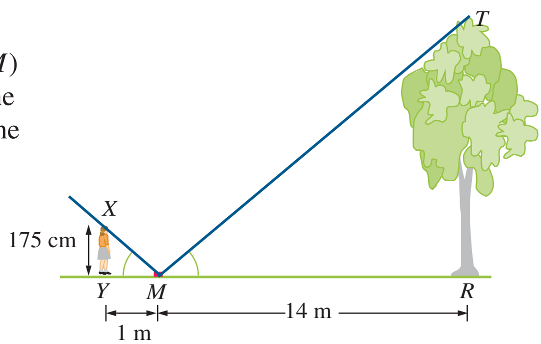


8 From a point at the water's edge at low tide, two friends can see a sand dune directly down the beach. One walks 30 m along the water-line towards the dune and 3 m towards the high-water mark. The other walks 4 m towards the high-water mark to make a straight line with the first person and the sand dune. Use triangulation methods to find the distance from the couple's first position to the sand dune and justify your methods.



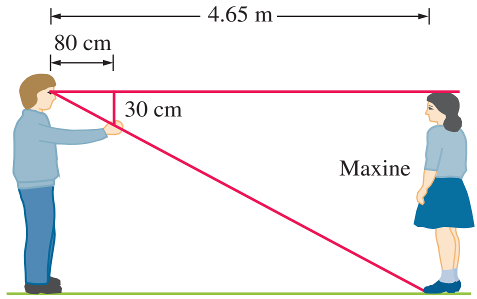
9 You can estimate the height of a tree (TR), as shown in the diagram, by placing a mirror (M) on level ground and moving backwards in line with the tree to a point Y where you can see the image of the top of the tree in the mirror.

Sally is standing at Y and her eyes are 175 cm above the ground. When the mirror is placed 1 m from her and 14 m away from the tree, she can see the top of the tree. How high is the tree?



10 If you hold a ruler in front of yourself, you can line it up so that the head and feet of a friend standing some distance from you coincide with the top and bottom of the ruler, as shown in the diagram.

Steve is holding a 30 cm ruler at a distance of 80 cm from his eyes as shown. He finds that when Maxine is standing 4.65 m away from him, the top of her head lines up with the top of the ruler and her feet line up with the bottom. How tall is Maxine?



1.3 Using the tangent ratio

The trigonometric ratios relate the *sides* to the *angles* of right-angled triangles. Before looking at the ratios we will examine the naming of the sides. The three sides are named in relation to the angle in which you are interested.

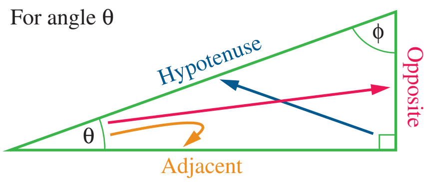
!

The longest side is opposite the right angle and is called the **hypotenuse**.

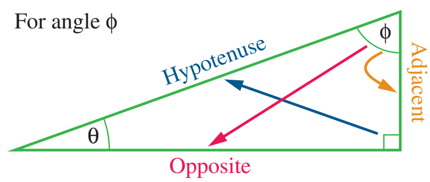
The **opposite** side is furthest away from the angle. It is diagonally opposite the angle.

The **adjacent** side is next to the angle and makes one 'arm' of the angle. (The hypotenuse is the other 'arm'.)

For angle θ



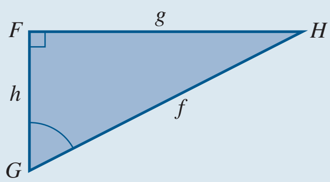
For angle ϕ



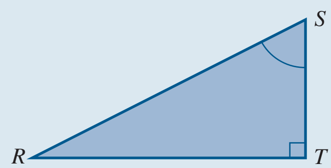
Example 11

Name the opposite, adjacent and hypotenuse for the angles marked in these triangles.

a



b



Solution

a The hypotenuse is opposite the right angle. — The hypotenuse is side *f*.

The side opposite *G* is *g* (naming convention). — The opposite to angle *G* is side *g*.

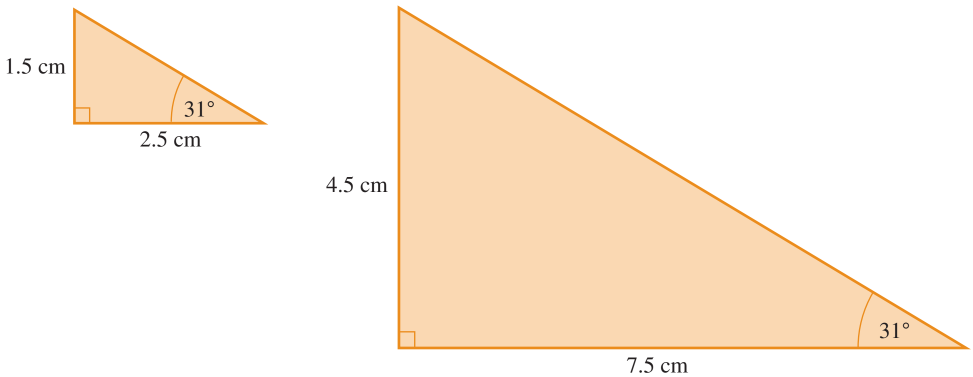
The side next to the angle is side *h*. — The adjacent to angle *G* is side *h*.

b The hypotenuse is opposite the right angle. — The hypotenuse is side *SR*.

The side opposite *S* is *RT*. — The opposite to angle *S* is side *RT*.

The side next to the angle is side *ST*. — The adjacent to angle *S* is side *ST*.

Two right-angled triangles with the same corner angle will always be the same shape. The triangles will be **similar** and so will be related by a **scale factor**. The triangles below have a corner angle of 31°, and the scale factor is 3.



The opposite and adjacent sides in each triangle will divide to give the same amount.

$$\frac{1.5}{2.5} = 0.6 \quad \text{and} \quad \frac{4.5}{7.5} = 0.6$$

The ratios of the opposite and adjacent sides will actually be the same for any similar right-angled triangle. We can say that every right-angled triangle with a 31° corner angle will have the same ratio of opposite to adjacent (0.6). In the past these ratios were calculated for different angles and printed in a look-up table called the **tangent table**. Nowadays we use a calculator to find the ratios.

!

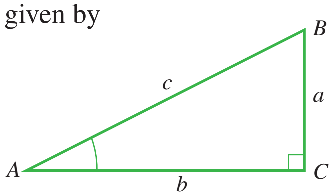
Tangent ratio

In any right-angled triangle, the tangent ratio of an angle is given by

$$\text{Tangent of the angle} = \frac{\text{opposite side}}{\text{adjacent side}}$$

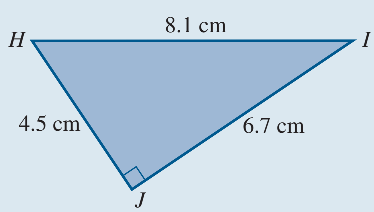
We usually abbreviate the tangent ratio as **tan** and write

$$\tan A = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b}$$



Example 12

Find the value of tan H for this triangle.



Solution

Tangent uses opposite and adjacent.	$\tan H = \dfrac{\text{opposite}}{\text{adjacent}}$
The opposite side is 6.7 and the adjacent side is 4.5.	$= \dfrac{6.7}{4.5}$ ≈ 1.49
Write the answer.	tan H is approximately 1.49.

When using a calculator for this work, make sure it is set to **degree mode**. Most scientific calculators will show a small DEG on the display if they are set in this mode. If the display shows RAD or GRAD, it will not give the correct answers. The angle mode is set in graphics calculators using the SETUP or MODE menu.

Example 13

Use a calculator to find tan 34.8° correct to 4 decimal places.

Solution

Use the tan key.	tan 34.8 = → 0.6950181
Round and write the answer.	tan 34.8 is about 0.6950.

Example 14

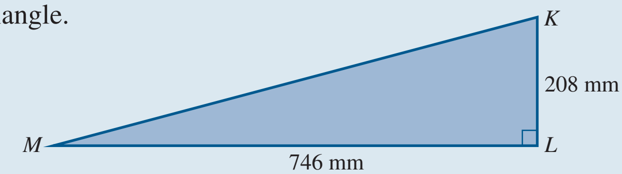
Using a calculator, find M such that tan M = 1.89.

Solution

Use $\tan^{-1}$.	2ndF tan 1.89 = → 62.116659
Round and write the answer.	M is about 62.1°.

Example 15

Find the angles in the following triangle.



Solution

Consider sides in relation to angle M. $\quad \tan M = \dfrac{\text{opposite}}{\text{adjacent}}$

Opposite is 208 and adjacent is 746. $\quad = \dfrac{208}{746}$

Keep the exact number on your calculator. $\quad = 0.2788\ \ldots$

Use $\tan^{-1}$ to find M, then round. $\quad$ So $\quad M \approx 15.6°$

Use sum of angles in a triangle. $\quad$ Thus $\quad K \approx 180° - 90° - 15.6°$

$\quad = 74.4°$

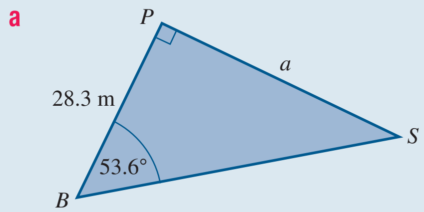
Write the answers. $\quad$ The angles are about 15.6° and 74.4°.

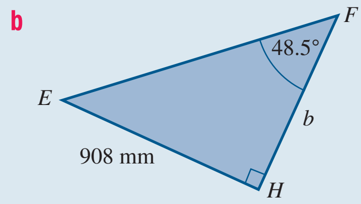
Note: We could find K using tangent again, but it is easier to use the sum of angles in a triangle.

The tangent ratio can be used to find one of the shorter sides when the other side is known. We use the angle opposite the missing side to make the calculation easier. Pythagoras's Theorem can then be used to find the hypotenuse, if desired.

Example 16

Find the marked sides in the following triangles.





Solution

a The missing side is opposite the known angle B.

$$\tan B = \frac{\text{opposite}}{\text{adjacent}}$$

The adjacent is known. The opposite is required.

$$\tan 53.6° = \frac{a}{28.3}$$

Multiply to get a.

$$a = 28.3 \times \tan 53.6°$$

Use your calculator, keeping exact number.

$$= 28.3 \times 1.3563 \ldots$$
$$= 38.3851 \ldots \text{ m}$$

Round and write the answer.

a is about 38.4 m.

b The missing side is not opposite the known angle. The angle opposite the required side can be worked out using the sum of angles in a triangle.

$$E = 180° - 90° - 48.5°$$
$$= 41.5°$$

Use the tangent ratio.

$$\tan E = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 41.5° = \frac{b}{908}$$

Multiply to get b.

$$b = 908 \times \tan 41.5°$$

Use your calculator, keeping exact number.

$$= 908 \times 0.8847 \ldots$$
$$= 803.33 \ldots \text{ mm}$$

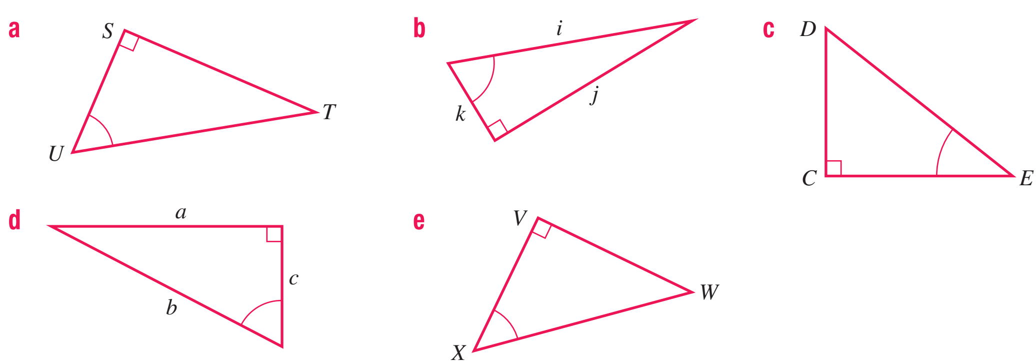
Round and write the answer.

b is about 803 mm.

Exercise 1.3 Using the tangent ratio

Additional Exercise 1.3

1 Name the opposite, adjacent and hypotenuse for each of the indicated angles in the diagrams shown below.



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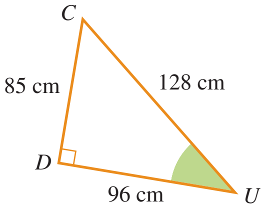
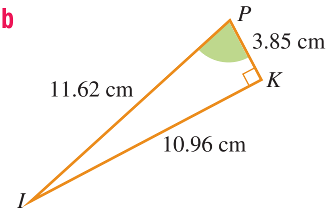
2 Find the tangent of each indicated angle in the following triangles.

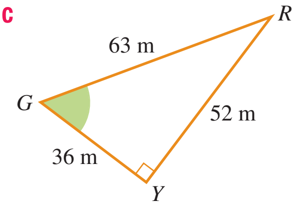
a



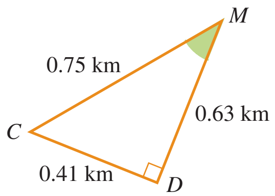
b



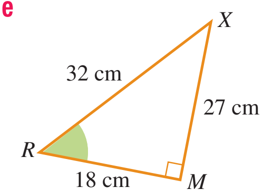
c



d



e



3 Find the following values correct to 4 decimal places.

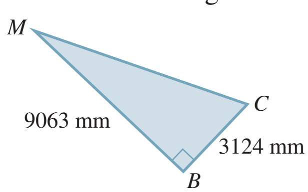
a $\tan 76.8°$ **b** $\tan 5°$ **c** $\tan 32°$
d $\tan 84.6°$ **e** $\tan 17.2°$

4 Find the angle, correct to 1 decimal place, for which:

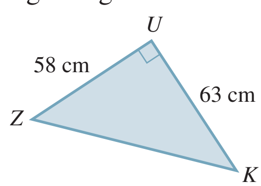
a $\tan A = 0.8$ **b** $\tan L = 0.976$ **c** $\tan C = 1.2$
d $\tan F = 3.4$ **e** $\tan H = 0.15$

5 Find the unknown angles in the following triangles correct to 1 decimal place.

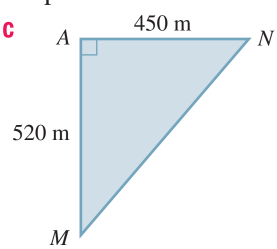
a



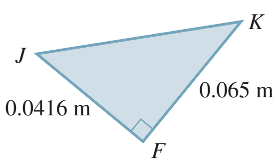
b



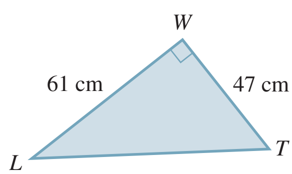
c



d

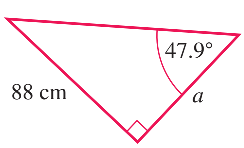


e

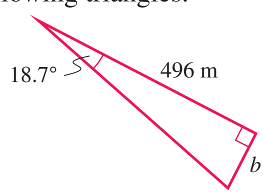


6 Find the marked side in each of the following triangles.

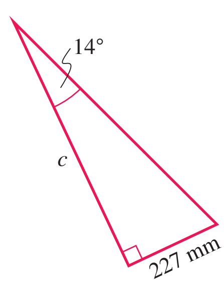
a



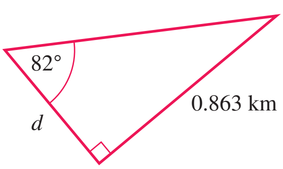
b



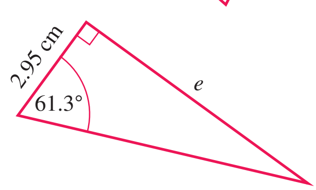
c



d



e



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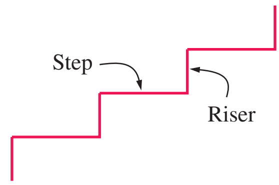
Modelling and problem solving

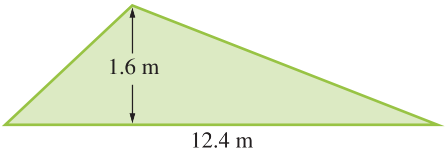
7 A staircase has 16 steps that are 265 mm wide and risers that are 165 mm high. The maximum angle of a wheelchair ramp is 7°.



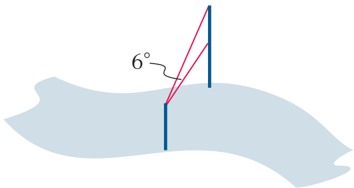
a What will be the angle of a ramp placed directly on the stairs?

b How can a wheelchair ramp of the correct slope be fitted? Fully explain and justify your answer.

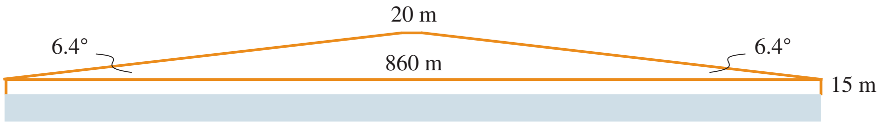
8 A roof truss has a height of 1.6 m and an overall width of 12.4 m. The highest point of the truss is 1 m from the centre. What is the slope of each side?



9 A surveyor's assistant has a 3.5 m-high stick on the other side of a stream. The surveyor finds that he has to turn his theodolite up an angle of 6° to see the top of the stick. The theodolite is 1 m off the ground. How wide is the stream?



10 A bridge slopes up from each end at an angle of 6.4° and has a flat section 20 m long in the middle. If the river it crosses is 860 m wide and the ends are 15 m above the water, how high is the middle section above the water? Carefully explain your reasoning.



11 When it is standing, the legs of an ironing board make an angle of 42.5° with the floor. The crossed legs just fit under the board, which is 1.2 m long. How high is the ironing board? Carefully explain your working.

1.4 Using the sine ratio

The ratio of the opposite and adjacent is the same for similar right-angled triangles. This is also true for the ratio of the opposite and hypotenuse in similar right-angled triangles.

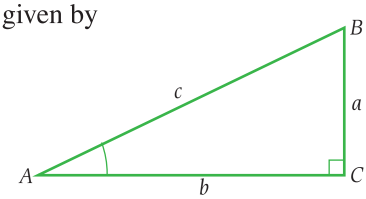
Sine ratio

In any right-angled triangle, the sine ratio of an angle is given by

$$\text{Sine of the angle} = \frac{\text{opposite side}}{\text{hypotenuse}}$$

We usually abbreviate the sine ratio as **sin** and write

$$\sin A = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{c}$$



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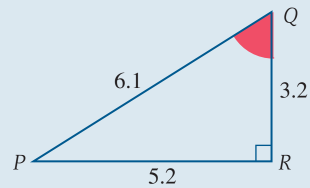
Example 17

Find the value of sin Q for this triangle.



Solution

Sine uses opposite and hypotenuse.

$$\sin Q = \frac{\text{opposite}}{\text{hypotenuse}}$$

The opposite side is 5.2 and the hypotenuse is 6.1.

$$= \frac{5.2}{6.1}$$

$$= 0.8524 \ldots$$

Round and write the answer.

sin Q is approximately 0.85.

As with the tangent ratio, we can use a calculator to find the sine ratio of an angle. Again, make sure the calculator is set in degree mode.

Example 18

Use a calculator to find sin 54° correct to 4 decimal places.

Solution

Use the sin key.

sin 54 = → 0.8090169

Round and write the answer.

sin 54° is about 0.8090.

Example 19

Use a calculator to find A correct to 1 decimal place, if sin A = 0.66.

Solution

Use $\sin^{-1}$.

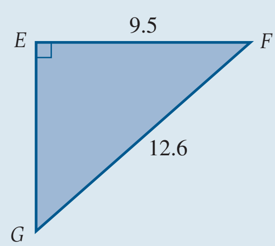
2ndF sin 0.66 = → 41.299872

Round and write the answer.

A is about 41.3°.

Example 20

Find the unknown angles in this triangle.



Solution

Consider angle G so that the sides are opposite and hypotenuse.

$$\sin G = \frac{\text{opposite}}{\text{hypotenuse}}$$

Opposite is 9.5 and hypotenuse is 12.6.

$$= \frac{9.5}{12.6}$$

Keep the exact number on your calculator.

$$= 0.7539\ \ldots$$

Use $\sin^{-1}$ to find G, then round.

So $G \approx 48.9°$

Use sum of angles in a triangle.

Thus $F \approx 180° - 90° - 48.9°$

$= 41.1°$

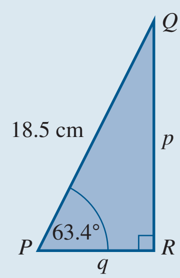
Write the answers.

The angles are about 48.9° and 41.1°.

The measurement of many engineering, surveying and construction projects is often incomplete. The sine ratio can be used to find the missing measurements. In a right-angled triangle, one side and one angle must be known.

Example 21

Find the unknown angle and sides in the following triangle.



Solution

Use the angle sum of a triangle.

$$63.4° + Q + 90° = 180°$$
$$Q = 26.6°$$

To find p, use the angle *opposite* the side.

$$\sin P = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 63.4° = \frac{p}{18.5}$$

Multiply to get p.

$$p = 18.5 \times \sin 63.4°$$

Use your calculator, keeping accuracy.

$$= 18.5 \times 0.8941\ \ldots$$
$$= 16.5418\ \ldots$$

Round to accuracy of question.

$$\approx 16.5 \text{ cm}$$

To find q, use the angle *opposite* the side.

$$\sin Q = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 26.6° = \frac{q}{18.5}$$

Multiply to get q.

$$q = 18.5 \times \sin 26.6°$$

Use your calculator, keeping accuracy.

$$= 18.5 \times 0.4477\ \ldots$$
$$= 8.2835\ \ldots$$

Round to 3 figures, the question accuracy.

$$\approx 8.28 \text{ cm}$$

Write the answers.

Unknown angle Q is 26.6°. Unknown sides are about 16.5 cm and 8.28 cm.

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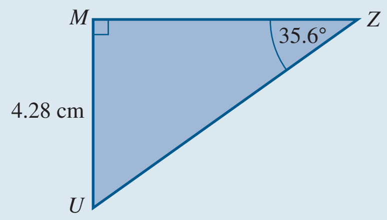
The sine ratio can be used to calculate the length of the hypotenuse from an angle and the opposite side.

Example 22

Find the unknown sides in the following triangle.



Solution

MU is the opposite and *UZ* the hypotenuse, so use sine.

$$\sin Z = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 35.6° = \frac{4.28}{UZ}$$

Multiply by *UZ*.

$$UZ \times \sin 35.6° = 4.28$$

Now divide by sin 35.6° to get *UZ*.

$$UZ = \frac{4.28}{\sin 35.6°}$$

Use your calculator, keeping accuracy.

$$= \frac{4.28}{0.5821\ \ldots}$$

You may want to use the memory.

$$= 7.3523\ \ldots$$

Round, but keep accuracy on calculator.

$$\approx 7.35 \text{ cm}$$

Find the other angle.

$$U + 35.6° + 90° = 180°$$

$$U = 54.4°$$

Use sine again.

$$\sin U = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 54.4° = \frac{MZ}{UZ}$$

Multiply to get *MZ*.

$$MZ = UZ \times \sin 54.4°$$

Use the accurate number kept on your calculator.

$$= 7.3523\ \ldots \times 0.8131\ \ldots$$

$$= 5.9782\ \ldots$$

Round to 3 figures, the question accuracy.

$$\approx 5.98 \text{ cm}$$

Write the answers.

Unknown sides are $UZ \approx 7.35$ cm and $MZ \approx 5.98$ cm.

Note: Pythagoras's Theorem or tangent could be used to work out the third side instead of using the sine ratio again.

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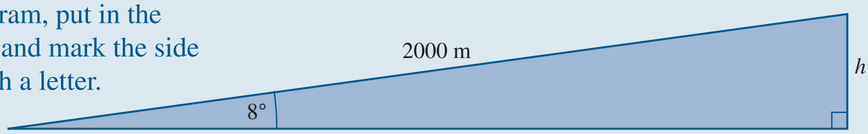
Example 23

A train travels 2 km along a sloping track. The track is at an angle of 8°. Through what height does the train rise?

Solution

Draw a diagram, put in the information and mark the side we want with a letter.



Opposite and hypotenuse are involved, so use sine.

$$\sin 8° = \frac{h}{2000}$$

Multiply to get h.

$$h = 2000 \times \sin 8°$$
$$= 2000 \times 0.1391\ \ldots$$
$$= 278.3462\ \ldots$$

Round to accuracy of question.

$$\approx 300 \text{ m}$$

Write the answer in a sentence.

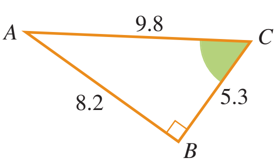
The train rises approximately 300 m.

Exercise 1.4 Using the sine ratio

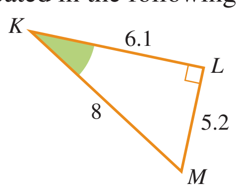
Additional Exercise 1.4

1 Find the sine ratios of the angles indicated in the following triangles.

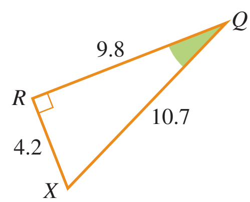
a



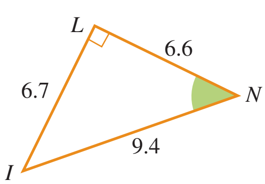
b



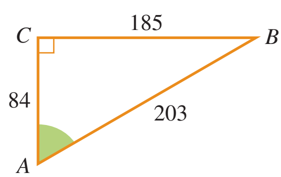
c



d

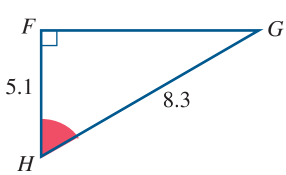


e

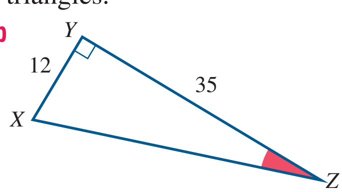


2 Use Pythagoras's Theorem to find the third side and then find the sine ratio of the angle indicated in each of the following triangles.

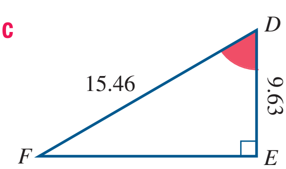
a



b



c



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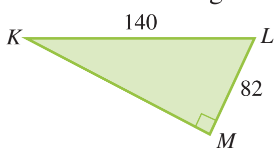
3 Use a calculator to find the following correct to 4 decimal places.

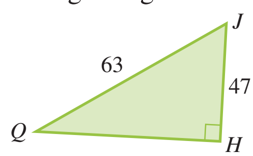
a sin 63° **b** sin 15°
c sin 87° **d** sin 17°
e sin 45° **f** sin 31.8°
g sin 22.1° **h** sin 30.7°
i sin 59.5° **j** sin 74.6°

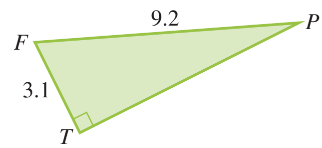
4 Find the angle (correct to 1 decimal place) for which:

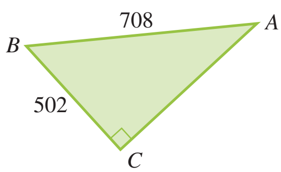
a $\sin Q = 0.58$ **b** $\sin H = 0.95$
c $\sin J = 0.06$ **d** $\sin Y = 0.36$
e $\sin D = 0.83$

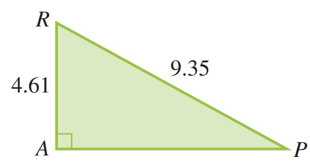
5 Find the unknown angles in the following triangles.

a


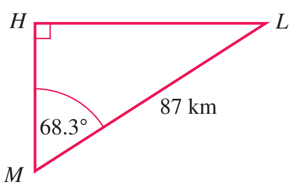
b


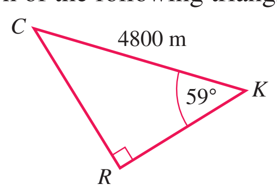
c


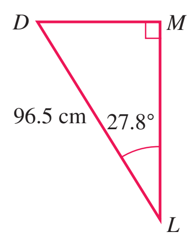
d


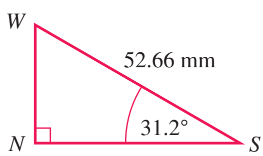
e


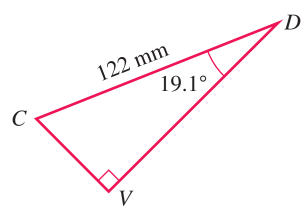
6 Find the unknown angle and sides in each of the following triangles.

a


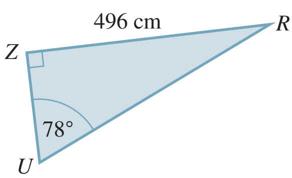
b


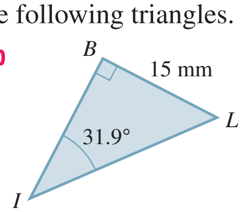
c


d


e


7 Find the unknown sides in each of the following triangles.

a


b


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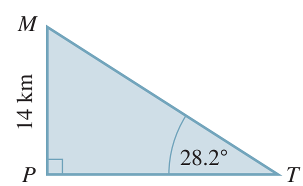

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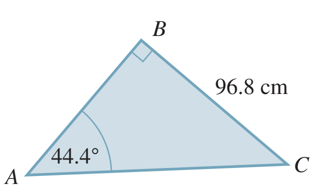

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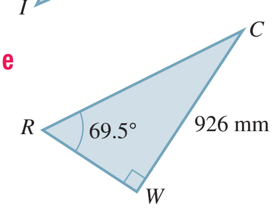
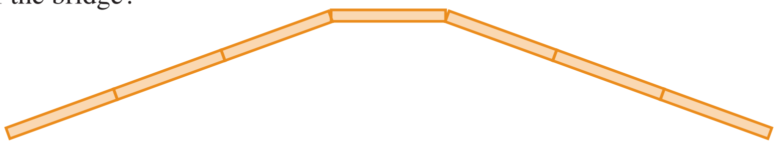


Modelling and problem solving

8 A slot-car enthusiast has 7 straight sections of track to make a bridge with. She plans to use 3 to slope up and 3 to slope back down, with one flat in the middle. The bridge can have a maximum slope of 15°, and the track sections are each 30 cm long. What is the maximum height of the bridge?



9 Aluminium planks 6 m long are used on a building site. The maximum safe slope to run wheelbarrows of concrete along from one floor to another is 12°. Each floor is 2.7 m higher than the previous one.

a What is the maximum height rise for one plank?

b What is the horizontal distance covered when the slope is 12°?

c How many planks must be used to go up three floors? Explain and justify your answer.

10 A wire 30 m long is used to hold a radio mast in place. The wire is at an angle of 72° to the ground and is tied 2 m from the top of the mast. How high is the radio mast?

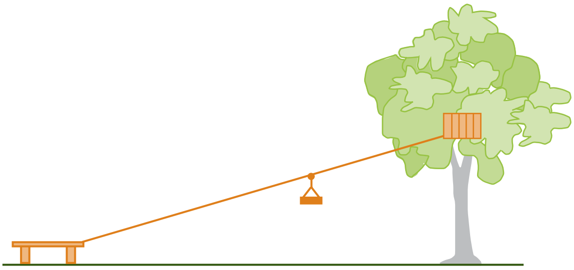
11 A jet climbs at an angle of 26° to the ground. The cruising height for the jet is 8000 m, the climbing speed is 300 km/h and the cruising speed is 450 km/h.

a How far does the jet travel horizontally to reach its cruising height?

b How far is it from the airport 5 minutes after taking off? Carefully justify your reasoning.

12 The mast of a yacht is 5.4 m from the bow. A taut line from the top of the mast to the bow makes an angle of 74.8° with the deck. How long is the line and how high is the mast?

13 At a scout rally a flying fox is set up to travel between a treehouse and a raised platform. The platform is 2 m off the ground, and the treehouse is 6.5 m from the ground. To work properly the rope must be at an angle of at least 8°. What are the maximum length of the rope and the maximum distance of the platform from the treehouse?



1.5 Using the cosine ratio

The cosine ratio is an alternative to the sine ratio. It is similar, except that the *adjacent* side is used instead of the opposite.

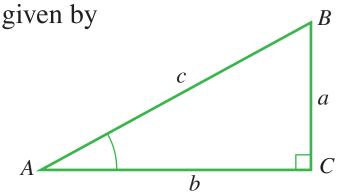
Cosine ratio

In any right-angled triangle, the cosine ratio of an angle is given by

$$\text{Cosine of the angle} = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

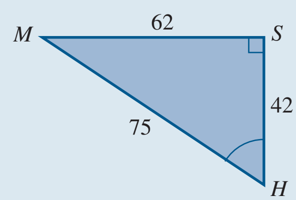
We usually abbreviate the cosine ratio as **cos** and write

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$



Example 24

Find the value of cos H from the triangle shown.



Solution

Cosine uses adjacent and hypotenuse.

$$\cos H = \frac{\text{adjacent}}{\text{hypotenuse}}$$

The adjacent side is 42 and the hypotenuse is 75.

$$= \frac{42}{75} = 0.56$$

Write the answer.

cos H is 0.56.

Example 25

Use a calculator to find cos 46.9°, correct to 4 decimal places.

Solution

Make sure the calculator is in degree mode.

Use the cos key.

cos 46.9 = → 0.6832737

Round and write the answer.

cos 46.9° is about 0.6833.

Example 26

If cos $G = 0.5782$, what is G?

Solution

Use $\cos^{-1}$.

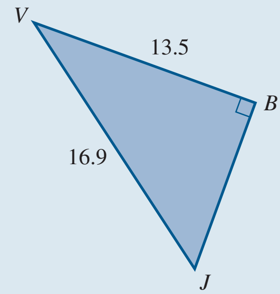
2ndF cos 0.5782 = → 54.675960

Round and write the answer.

G is about 54.7°.

Example 27

Use cosine to find the angles in this triangle.



Solution

Consider $\angle V$ so that the sides are the adjacent and hypotenuse: adjacent = 13.5, hypotenuse = 16.9.

$$\cos V = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$= \frac{13.5}{16.9}$$

Keep the exact value on your calculator.

$$= 0.7988 \ldots$$

Use $\cos^{-1}$ to find V and round.

So $V \approx 37.0°$

Use sum of angles in a triangle to find J.

Thus $J \approx 180° - 90° - 37.0°$

$$= 53.0°$$

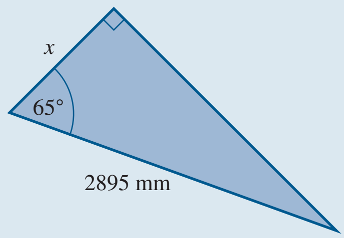
Write the answers, showing correct accuracy.

The angles are about 37.0° and 53.0°.

Cosine can be used to find missing sides, in a similar way to sine. Cosine is most useful when the adjacent side is to be worked out from the hypotenuse or vice versa.

Example 28

Find the marked side in the following triangle.



Solution

Adjacent and hypotenuse are involved, so use cosine.

$$\cos 65° = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$= \frac{x}{2895}$$

Multiply to get x.

$$x = 2895 \times \cos 65°$$

Use your calculator, keeping accuracy.

$$= 2895 \times 0.4226 \ldots$$

$$= 1223.47 \ldots$$

Round to accuracy of question.

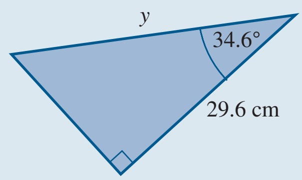
$$\approx 1223 \text{ mm}$$

Write the answer.

The marked side is about 1223 mm long.

Example 29

Find the hypotenuse in the following triangle.



Solution

Adjacent is known and hypotenuse is required, so use cosine.	$\cos 34.6° = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
	$= \dfrac{29.6}{y}$
Multiply by y.	$y \times \cos 34.6° = 29.6$
Now divide by cos 34.6° to get y.	$y = \dfrac{29.6}{\cos 34.6°}$
Use your calculator, keeping accuracy.	$= \dfrac{29.6}{0.8231 \ldots}$
You may want to use the memory.	$= 35.9600 \ldots$
Round to accuracy of question.	≈ 36.0 cm
Write the answer.	The hypotenuse is about 36.0 cm long.

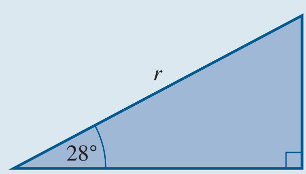
Note: We could use sine instead of cosine by finding the third angle first.

Example 30

The slope of a suburban road up a steep hill is 28°. The survey map shows that the road covers a horizontal distance of 807 m to reach the top of the hill. What is the actual length of road surface that must be used for calculations of guttering and sealing requirements?

Solution

Draw a diagram, put in the information and mark the wanted side with a letter.

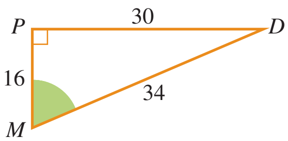


Adjacent and hypotenuse are involved, so use cosine.	$\cos 28° = \dfrac{807}{r}$
Multiply by r.	$r \times \cos 28° = 807$
Now divide by cos 28° to get r.	$r = \dfrac{807}{\cos 28°}$
Use your calculator, keeping accuracy.	$= \dfrac{807}{0.8829 \ldots}$
You may want to use the memory.	$= 913.9840 \ldots$
Round to accuracy of question.	≈ 914 m
Write the answer.	The actual length of road surface is about 914 m.

Exercise 1.5 Using the cosine ratio

Additional Exercise 1.5

1 Find the cosines of the angles indicated in the triangles below.

a


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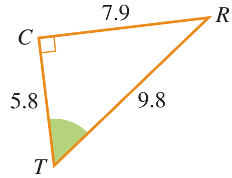
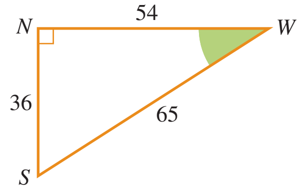
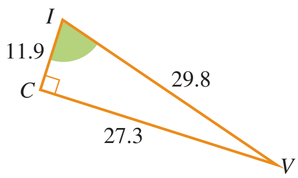
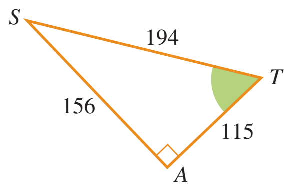


c


d


e


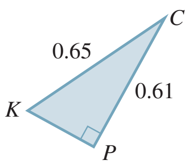
2 Use your calculator to find the following correct to 4 decimal places.

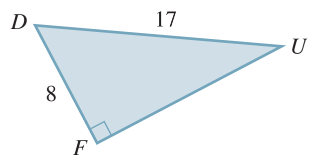
a $\cos 58°$ b $\cos 27.5°$ c $\cos 72.9°$
d $\cos 47.2°$ e $\cos 22.1°$

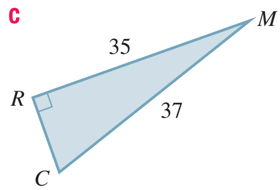
3 Find the angle (correct to 1 decimal place) for which:

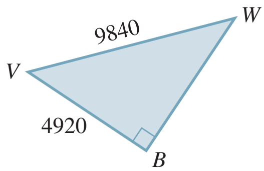
a $\cos R = 0.458$ b $\cos Y = 0.9879$ c $\cos P = 0.4$
d $\cos T = 0.0035$ e $\cos A = 0.1976$

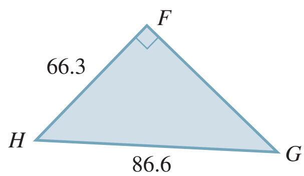
4 Use cosine to find the unknown angles in the following triangles.

a


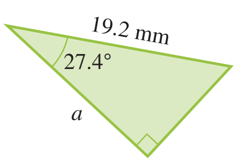
b


c


d


e


5 Find the marked sides in the following triangles.

a


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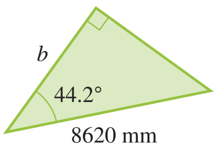

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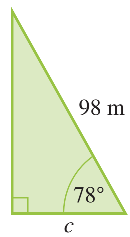

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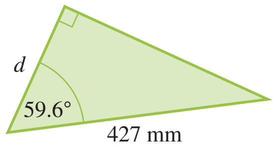
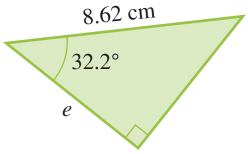


e


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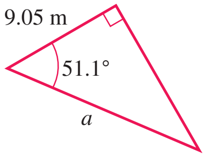
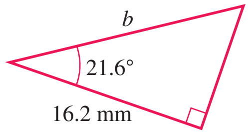
6 Find the hypotenuse in each of the following.

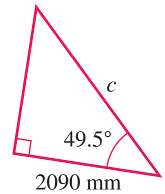
a



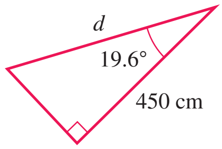
b



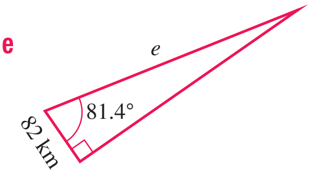
c



d

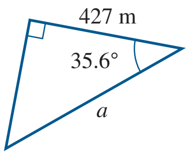


e

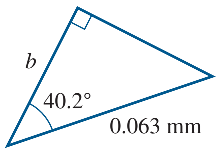


7 Find the marked side in each of the following triangles.

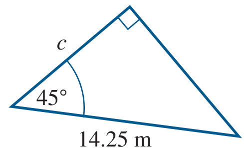
a



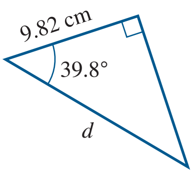
b



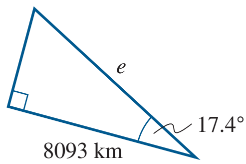
c



d

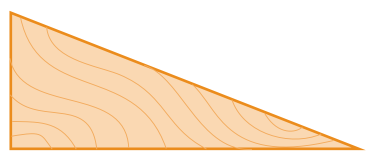


e



Modelling and problem solving

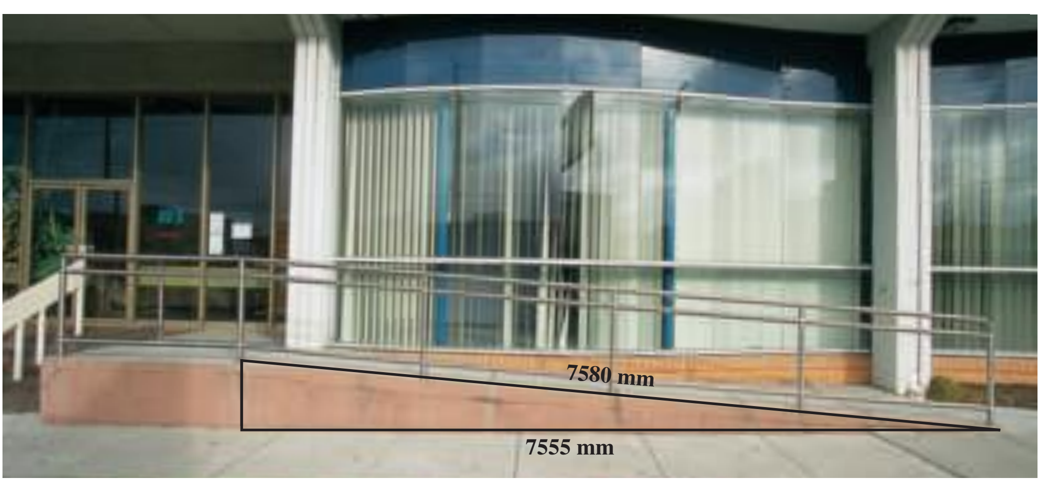
8 A carpenter needs to cut a right-angled wedge out of timber. The wedge has to be 25 cm long, and the sloping edge must be 28 cm. Find the angle the carpenter should use to cut the timber, and explain your reasoning.



9 A wheelchair ramp up to an entrance has a horizontal length of 7555 mm and a sloping length of 7580 mm. The maximum angle allowed for such ramps by Australian Standards is 4.76°.

a What is the angle of the ramp?

b Does the ramp meet the standard? Justify your answer.



10 A long slope for a train line is inclined at an angle of 8.4°. To lay the track for the slope, 248 sections of 20 m track were used. What horizontal distance is travelled by the train in going up the slope?

11 A tent rope is looped around a peg that is 4825 mm away from the tent pole. When stretched tight from the peg to the top of the pole it makes an angle of 27.3° with the ground. The rope is 6 m long, and after looping around the peg the rope goes to a cleat partway up the tight rope. Calculate how far up the rope the cleat is from the peg and justify your reasoning.

12 A fire hose is tied to the top of a pole to dry. It is pulled 12.5 m away from the pole so that none of it lies on the ground, and it makes an angle of 65.4° with the ground. How long is the fire hose?

1.6 Applying trig ratios

In solving trigonometric problems, you must remember the three ratios. One of the mnemonics that people use to help to remember the ratios is shown below.

!

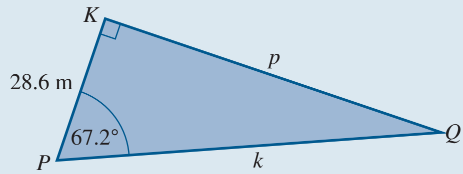
'Signals of help	$\rightarrow$	$\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}}$
cause all hands	$\rightarrow$	$\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
to offer assistance'	$\rightarrow$	$\tan A = \dfrac{\text{opposite}}{\text{adjacent}}$

To **solve a triangle**, we find the unknown sides and angles.

Solving a right-angled triangle depends on choosing the correct trigonometric ratio. The correct ratio is chosen by considering the sides involved as hypotenuse, opposite and adjacent to the known or required angle. In cases where two sides are known, Pythagoras's Theorem may be used to find the third side.

Example 31

Solve this triangle.



Solution

For angle P, p is opposite and q is adjacent, so use tangent.
$$\tan P = \frac{\text{opposite}}{\text{adjacent}}$$

Put in values.
$$\tan 67.2° = \frac{p}{28.6}$$

Multiply by 28.6 to find p.
$$\begin{aligned} p &= 28.6 \times \tan 67.2° \\ &= 28.6 \times 2.3789\ldots \\ &= 68.0367\ldots \end{aligned}$$

Round to accuracy of question.
$$\approx 68.0 \text{ m}$$

For angle P, k is hypotenuse and q is adjacent, so use cosine.
$$\cos P = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Put in values.
$$\cos 67.2° = \frac{28.6}{k}$$

Multiply by k.
$$k \times \cos 67.2° = 28.6$$

Divide by cos 67.2° to find k.
$$\begin{aligned} k &= \frac{28.6}{\cos 67.2°} \\ &= \frac{28.6}{0.3875\ldots} \\ &= 73.8034\ldots \end{aligned}$$

Round to accuracy of question.
$$\approx 73.8 \text{ m}$$

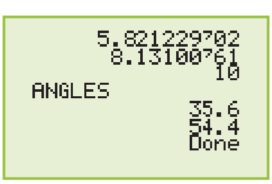
Use sum of angles in a triangle to find Q.
$$\begin{aligned} Q &= 180° - 90° - 67.2° \\ &= 22.8° \end{aligned}$$

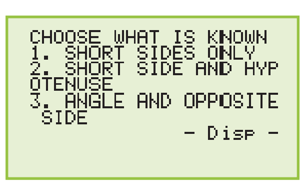
Write the answers.
$p \approx 68.0$ m, $k \approx 73.8$ m and $Q = 22.8°$.

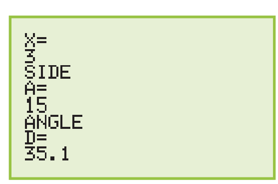
Technology

Calculator Program

You can use a small program on a graphics calculator to solve triangles. The program TRISOL can be used to find the unknown sides or angles in any right-angled triangle. The program is given in full on the CD-ROM. Enter the program (or load it from the CD-ROM) and try it with various triangles. The calculator must be set in degree mode. If you are unsure how to enter and run the program, refer to your calculator manual or check with your teacher.

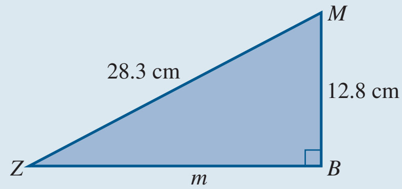






Example 32

Find the unknown side and angles in this triangle.



Solution

For angle M, z is adjacent and b is hypotenuse, so use cosine.

$$\cos M = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{12.8}{28.3}$$

Keep exact value on your calculator. $= 0.4522\ldots$

Use $\cos^{-1}$ to find M. $M = 63.1088\ldots°$

Round, but keep accuracy on calculator. $\approx 63.1°$

Use sum of angles to find Z. $Z = 180° - 90° - M \approx 26.9°$

For angle M, m is opposite and b is hypotenuse, so use sine.

$$\sin M = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{m}{28.3}$$

Multiply by 28.3 to find m. $m = 28.3 \times \sin M$

Use exact value for M from calculator.

$$= 28.3 \times \sin 63.1088\ldots° = 28.3 \times 0.8918\ldots = 25.2398\ldots$$

Round. ≈ 25.2 cm

Write the answers to the accuracy of the question. $M \approx 63.1°$, $Z \approx 26.9°$ and $m \approx 25.2$ cm.

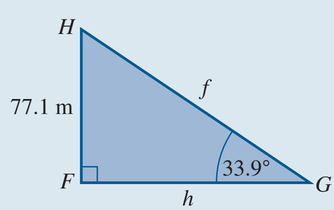
Note: You could find m using Pythagoras's Theorem instead of sine.

Example 33

In $\triangle FGH$, $F = 90°$, $G = 33.9°$ and $g = 77.1$ m. Solve $\triangle FGH$.

Solution

Draw a diagram.



Use sum of angles to find H.

$$H = 180° - 90° - 33.9° = 56.1°$$

Use angle H to find h, so the unknown is opposite. Then h is opposite and g is adjacent, so use tangent.

$$\tan H = \frac{\text{opposite}}{\text{adjacent}}$$

Put in values.	$\tan 56.1^\circ = \dfrac{h}{77.1}$
Multiply by 77.1 to find h.	$h = 77.1 \times \tan 56.1^\circ$
	$= 77.1 \times 1.4881\ \ldots$
	$= 114.7369\ \ldots$
Round, but keep accuracy on calculator.	≈ 115 m
Use Pythagoras to find f.	$f^2 = h^2 + g^2$
Substitute values, using exact value of h on calculator.	$= (114.7369\ \ldots)^2 + 77.1^2$
	$= 13\,164.5572\ \ldots + 5944.41$
	$= 19\,108.9672\ \ldots$
Use the √ key.	$f = \sqrt{19\,108.9672\ \ldots}$
	$= 138.2351\ \ldots$
Round to 3-figure accuracy.	≈ 138 m
Write answers to the accuracy of the question.	$H = 56.1^\circ$, $h \approx 115$ m and $f \approx 138$ m.

In surveying and mapping, angles may need to be specified very exactly. This has traditionally been done using the smaller units of **minutes and seconds of arc**.

!

Angle units

Each degree is divided into 60 **minutes of arc**. Each minute of arc is divided into 60 **seconds of arc**. The symbols used for degrees, minutes and seconds are °, ′ and ″ respectively.

$1^\circ = 60'$ (1 degree = 60 minutes)

$1' = 60''$ (1 minute = 60 seconds)

Modern calculators work out the trigonometric ratios in decimal degrees. Before we can use a calculator for sine, cosine or tangent, we must change degrees, minutes and seconds to degrees only.

Example 34

Change 10° 18′24″ to degrees, correct to 4 decimal places.

Solution

Change 24″ to minutes by dividing by 60.	$24'' = \dfrac{24'}{60} = 0.4'$
Write the angle.	$10^\circ 18' 24'' = 10^\circ 18.4'$
Change 18.4′ to degrees by dividing by 60.	$18.4' = \dfrac{18.4^\circ}{60}$
	$= 0.3066\ \ldots^\circ$
Round and write the angle.	$10^\circ 18' 24'' \approx 10.3067^\circ$
On most scientific and graphics calculators, the DMS or ° ′ ″ key can be used.	10 DMS 18 DMS 24 DMS → 10.306667

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Example 35

Change 14.8674° to degrees, minutes and seconds, correct to the nearest second.

Solution

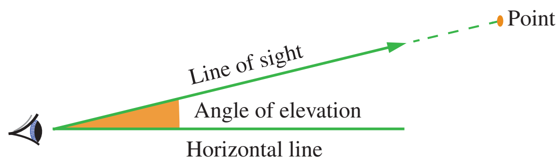
Multiply the decimal part by 60 to change to minutes.	$0.8674° = 0.8674 \times 60'$ $= 52.044'$
Write the angle.	$14.8674° = 14°52.044'$
Multiply the decimal part by 60 to change to seconds.	$0.044' = 0.044 \times 60''$ $= 2.64''$
Round and write the answer.	$14.8674° \approx 14°52'3''$
On most scientific and graphics calculators, the **DMS** or **° ′ ″** key can be used in reverse.	14.8674 **INV** **DMS** → 14°52°2.64

Angles are described in a variety of ways. In order to apply trigonometry effectively, you must be familiar with the common methods.

!

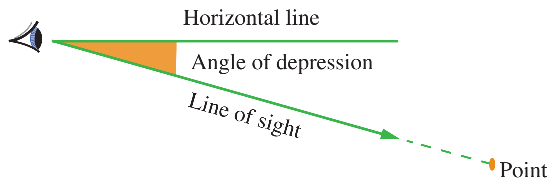
An **angle of elevation** is formed when you have to look up from a horizontal line to see a point.

The angle you have to look up is the angle of elevation.



An **angle of depression** is formed when you have to look down from a horizontal line to see a point.

The angle you have to look down is the angle of depression.

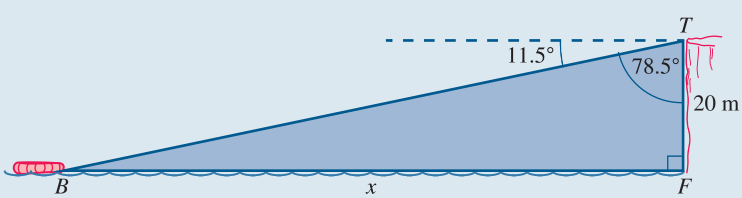


Example 36

From the top of a cliff 20 m high, the angle of depression of a surf-boat is 11.5°. How far is the boat from the foot of the cliff?

Solution

Draw a diagram, label points and show the information.



Find an angle in the triangle.	$\angle FTB = 90° - 11.5°$ $= 78.5°$
Use tangent as opposite and adjacent are involved.	$\tan 78.5° = \frac{\text{opposite}}{\text{adjacent}}$
Put in values.	$\tan 78.5° = \frac{x}{20}$
Multiply by 20 to find x.	$x = 20 \times \tan 78.5°$ $= 20 \times 4.9151 \ldots$ $= 98.301 \ldots$
Round to the accuracy of the question.	≈ 98 m
Write the answer in terms of the question.	The boat is about 98 m from the cliff.

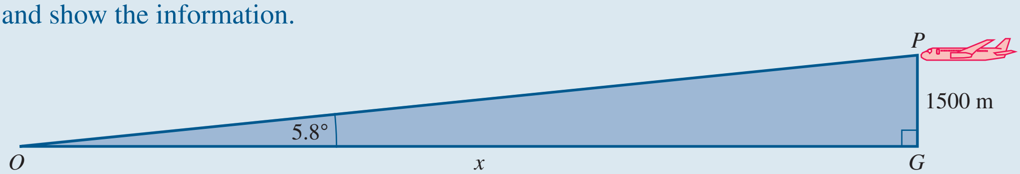
Example 37

A plane flying at a height of 1500 m is spotted by an observer. She notes that the plane has an angle of elevation of 5°48′. How far away (on the ground) is the plane?

Solution

Change the angle to degrees. $5°48' = 5.8°$

Draw a diagram, label points and show the information.



Use tangent as opposite and adjacent are involved.	$\tan 5.8° = \frac{\text{opposite}}{\text{adjacent}}$
Put in values.	$\tan 5.8° = \frac{1500}{x}$
Multiply by x.	$x \times \tan 5.8° = 1500$
Divide by tan 5.8° to find x.	$x = \frac{1500}{\tan 5.8°}$ $= \frac{1500}{0.1015 \ldots}$ $= 14\,767.2248 \ldots$
Round to accuracy of question.	$\approx 15\,000$ m or 15 km
Write the answer.	The plane is about 15 km away.

Directions on the Earth are given in terms of the basic compass points **north**, **south**, **east** and **west**.

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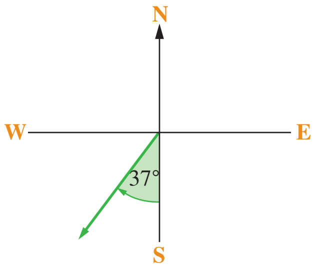
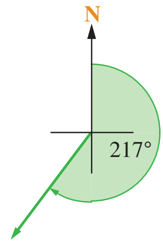
!

The **bearing** of an object is given as the 3-digit *clockwise* angle the observer turns *from due north* in order to face the object. An object due west has a bearing of 270°.

A bearing can also be given as the angle from north or south towards either east or west.

The direction shown in the diagram can be given as either S37°W or 217°.



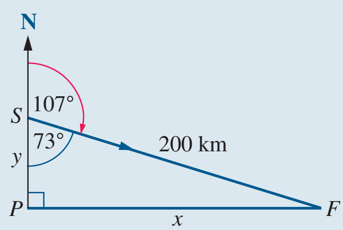


Example 38

A boat sails at a bearing of 107° for 200 km. How far south and east does it travel (to the nearest kilometre)?

Solution

Draw a diagram, label points and show the information.



Find an angle in the triangle. $\angle PSF = 180° - 107° = 73°$

For x, opposite and hypotenuse are involved.

$$\sin 73° = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$= \frac{x}{200}$$

Multiply by 200 to find x.

$$x = 200 \times \sin 73°$$
$$= 200 \times 0.9563 \ldots$$
$$= 191.2609 \ldots$$

Round.

$$\approx 191 \text{ km}$$

For y, adjacent and hypotenuse are involved.

$$\cos 73° = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$= \frac{y}{200}$$

Multiply by 200 to find y.

$$y = 200 \times \cos 73°$$
$$= 200 \times 0.2923 \ldots$$
$$= 58.4743 \ldots$$

Round.

$$\approx 58 \text{ km}$$

Write the answers in a sentence. The boat travels about 58 km south and 191 km east.

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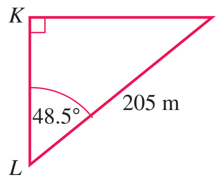
Additional Exercise 1.6

Exercise 1.6 Applying trig ratios

1 Solve these triangles.

a


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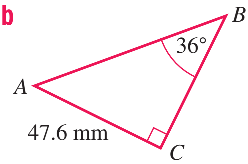
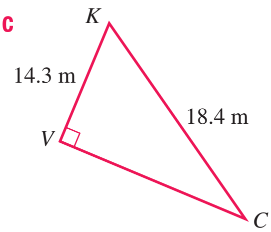
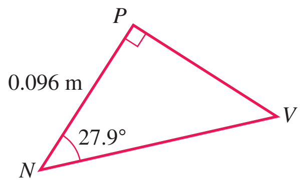
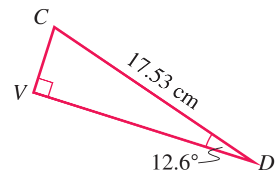
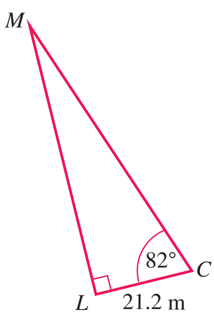


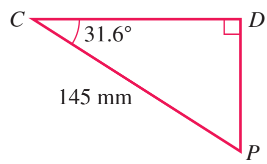
c


d


e


2 Find the unknown sides and angles in these triangles.

a


b


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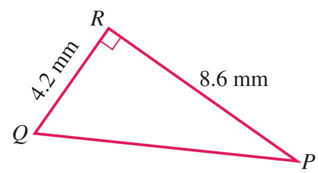

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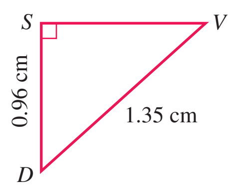
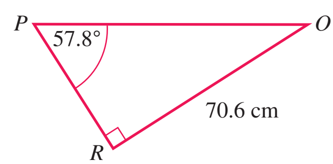


e


3 Solve these triangles.

a $\triangle ABC$, $A = 90°$, $C = 28°$, $b = 8$ cm

b $\triangle PDG$, $D = 90°$, $p = 5$ m, $g = 7$ m

c $\triangle KLM$, $M = 90°$, $L = 34°$, $l = 6$ mm

d $\triangle RMS$, $S = 90°$, $R = 57.2°$, $m = 5.3$ km

e $\triangle EFG$, $E = 90°$, $e = 76$ m, $f = 57$ m

4 Change the following to decimal degrees, correct to 4 decimal places.

a $58°24'27''$ **b** $82°54'18''$ **c** $32°12'45''$

d $17°16'$ **e** $4°28'17''$

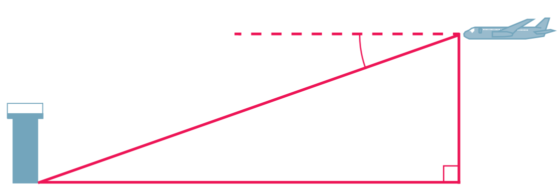
5 Change the following to degrees, minutes and seconds, correct to the nearest second.

a 14.665° **b** 48.55° **c** 78.2575°

d 68.9° **e** 26.7244°

Modelling and problem solving

6 The pilot of a plane observes that the angle of depression of the control tower at the airport is 3.7°. If the altimeter shows the plane is at a height of 2400 m, find the horizontal distance to the control tower.



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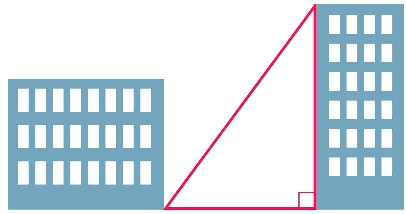
7 From the other side of the street, the angle of elevation of the top of a building is 72°. If the street is 35 m wide, find the height of the building.



8 From the top of a lighthouse 47 m high, the angle of depression of a boat at sea is 4.25°. At sea-level, how far from the lighthouse is the boat?

9 Sharon knows that she takes 12 paces every 10 m. She paces out a distance of 60 paces from the foot of a tree and looks up at the top. Its angle of elevation is then 38°. Sharon's eyes are 1.5 m from the ground. What is the height of the tree?

10 The height of a mountain is shown on the map as 1700 m. From the top of a hill 370 m high, the angle of elevation to the top of the mountain is 6.7°. Find the horizontal distance between the hill and the mountain, in kilometres.

11 A ship travels 150 km on a bearing of 134°. How far south and east does it travel?

12 After travelling for 2 hours in a direction S36°E an aeroplane is 600 km south of its starting point. Find the actual distance travelled and its speed.

13 A train is travelling at 60 km/h on a track running from west to east. An observer south of the track notes that the train is directly north of him. Half an hour later the train is at a bearing of 020°. What is the distance of the observer south of the track?

14 A cyclist travelling on a road running north–south notices that a hill is directly west of him. After cycling at 20 km/h for a further 2 hours, he notices that the direction of the hill is now N75°W. How far is the hill from the road, and how far is it from the new position of the cyclist?

15 One town is directly north of another. From a point 20 km to the west of the road joining them, the towns are at bearings of 050° and 120°. How far apart are the towns?

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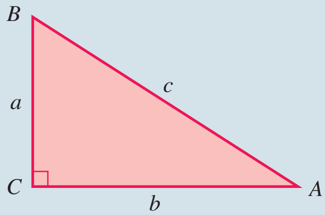
Chapter summary

- The **naming convention** for triangles is that the vertices (corners) are named using capital letters and the opposite sides have the same lower-case letter.
- **Pythagoras's Theorem** states that in a right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides. The hypotenuse is the side opposite the right angle. $c^2 = a^2 + b^2$, where c is the hypotenuse and a and b are the other two sides.
- **Pythagorean triples** are whole numbers that satisfy Pythagoras's Theorem, such as 3, 4, 5 and 5, 12, 13.
- **Similar triangles** have the same shape. The ratio of their sides is the **scale factor**. Similar triangles can be used with a shadow stick of known height to measure the height of an object using their shadows.
- **Triangulation** is the use of triangles to measure aspects of the Earth.
- The sides of a right-angled triangle are related to one angle as the **hypotenuse**, which is opposite the right angle, the **opposite**, which is diagonally opposite the angle, and the **adjacent**, which is the third side next to the angle.
- In any right-angled triangle, the **tangent ratio** of an angle is given by

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$



- In any right-angled triangle, the **sine ratio** of an angle is given by

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

- In any right-angled triangle, the **cosine ratio** of an angle is given by

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

- Each degree is divided into 60 **minutes of arc**. Each minute of arc is divided into 60 **seconds of arc**. The symbols used for degrees, minutes and seconds are °, ′ and ″ respectively.

  1° = 60′ (1 degree = 60 minutes) and 1′ = 60″ (1 minute = 60 seconds).
- An **angle of elevation** is formed by looking up from a horizontal line to see a point.
- An **angle of depression** is formed by looking down from a horizontal line to see a point.
- The **bearing** of the object is given as the 3-digit clockwise angle the observer turns from due north in order to face the object. A bearing can also be given as the angle from north or south towards either east or west.

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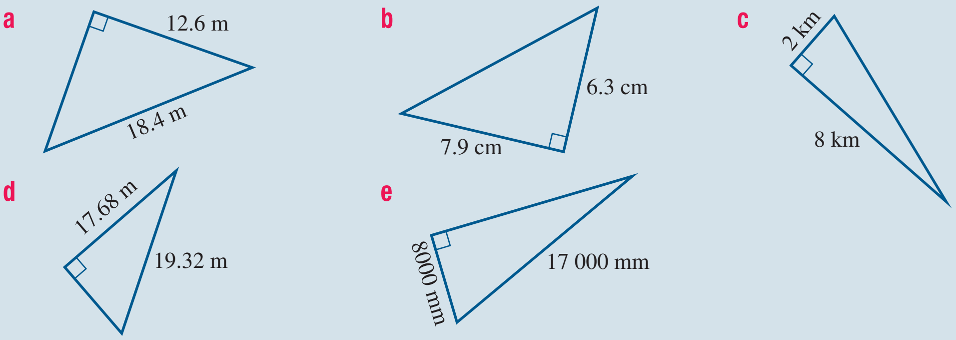
Chapter review

Knowledge and procedures

1 Use Pythagoras's Theorem to find the missing sides in the following triangles. Ex 1.1

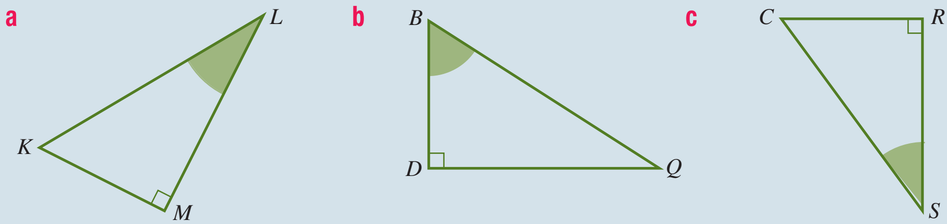


2 Use Pythagoras's Theorem to work out whether triangles with the following dimensions are acute, right-angled or obtuse. Ex 1.1

a 3, 5, 7 **b** 8, 10, 12 **c** 8, 15, 17 **d** 13, 15, 17

3 A shadow stick 2.5 m high casts a shadow of length 4 m. At the same time a building casts a shadow of length 30 m. How high is the building? Ex 1.2

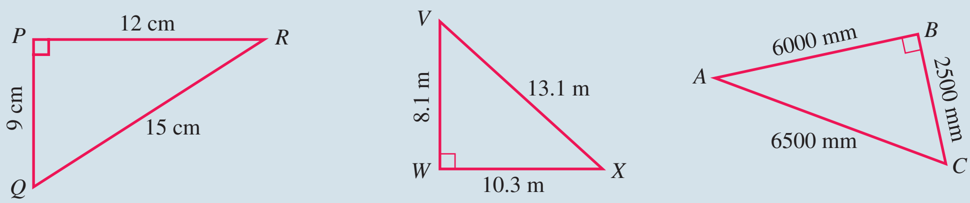
4 Name the opposite, adjacent and hypotenuse for the angle marked in each of the following triangles. Ex 1.3



5 Use the diagrams below to find: Ex 1.3–5

a $\sin Q$ **b** $\sin V$ **c** $\sin A$ **d** $\sin C$ **e** $\cos Q$

f $\cos R$ **g** $\cos X$ **h** $\cos C$ **i** $\tan Q$ **j** $\tan A$



6 Use your calculator to find, correct to 4 decimal places: Ex 1.3–5

a $\sin 47°$ **b** $\cos 3.8°$ **c** $\tan 85°$ **d** $\tan 45°$

e $\cos 72°$ **f** $\sin 54.9°$ **g** $\cos 19.6°$ **h** $\sin 45°$

i $\cos 12°18'$ **j** $\tan 28°19'8''$

7 Find the value (correct to 1 decimal place) of the angle for which: Ex 1.3–5

a $\sin T = 0.55$ **b** $\cos R = 0.247$ **c** $\tan E = 3.5$

d $\tan K = 0.34$ **e** $\cos P = 0.8023$ **f** $\sin W = 0.2$

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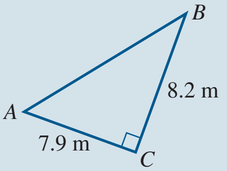
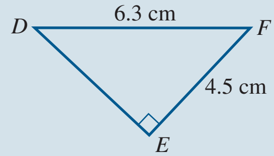
Chapter review

Ex 1.3–5 **8** Find the unknown angles in the following triangles.

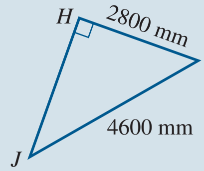
a



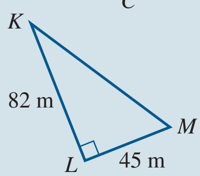
b



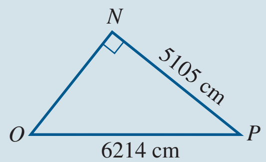
c



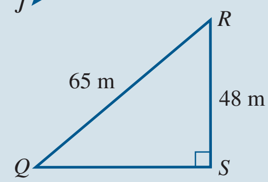
d



e

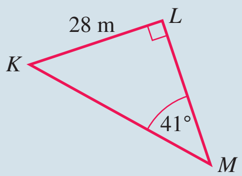


f

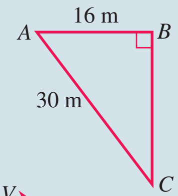


Ex 1.3–5 **9** Solve the following triangles.

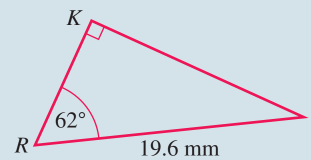
a



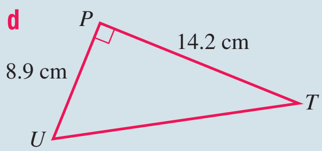
b



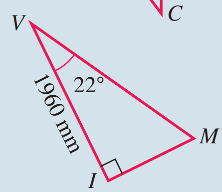
c



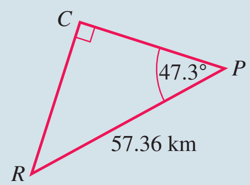
d



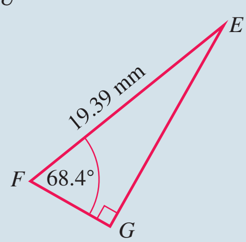
e



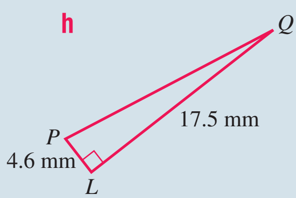
f



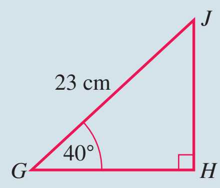
g



h



i



Modelling and problem solving

Ex 1.1 **10** Find the length of steel bracing needed to fit a diagonal brace to a wall frame 6.2 m long and 2.4 m high.

Ex 1.1 **11** A triangular course for a yacht race has a north and an east leg, and the final leg returns to the start. The final leg is 7 km long and the first leg is 4.5 km long. How long is the second leg?

Ex 1.6 **12** From a point 100 m from the base of a cliff the angle of elevation of the top of the cliff is 16°. The angle of elevation of the top of a lighthouse built on the cliff is 30°. Find the height of the cliff and the height of the lighthouse.

Ex 1.6 **13** A cabinet-maker has splayed the legs of a table out at an angle of 15° to the vertical. If the height of the table surface is 850 mm and the table-top is 45 mm thick, find the length of the legs.

Ex 1.6 **14** A kite line is let out 70 m while the kite rises 45 m. If the line is tight, find the angle of elevation of the kite from the boy holding the string.

Ex 1.6 **15** An aircraft is 370 km north of its original position. Its bearing is 342° from the original position. Find the distance actually travelled.

Gathering information

2

Contents

Syllabus Guide Chapter 2

Syllabus subject matter

Data collection and presentation

- Types of data and variables (continuous and discrete)
- Practical aspects of collecting and handling data for observation, experimentation or survey, including possible data problems

Quantitative concepts and skills

- Rates, percentages, ratio and proportion

How do chains like McDonald's, Red Rooster, Bunnings or Autobarn decide where to put a new store? How do publicity agencies design effective ads? How do governments know whether people will support regional plans? They all use information gathered from surveys and other sources. The work in this chapter will help you understand how surveys are designed.

2.1 Data types

Peter has fair hair, is 16 years old, is 175 cm tall, has three brothers and prefers chocolate ice-cream to strawberry, but likes vanilla least. Each of these pieces of information about Peter is called an item of **data**. However, they are not all the same kind of data.

!

Types of data

Categorical data has no particular order because the data values are just names. This is sometimes called **nominal** data.

Ordinal data has an order, but the differences between steps are not meaningful.

Discrete data can have only certain values. It is sometimes called **count** data because it often arises from counting.

Continuous data can have any value, although there may be a maximum and/or minimum. Continuous data is sometimes collected as discrete data.

Data is collected about the same **attributes** for each member of a group. Each attribute for which data is collected is called a statistical **variable**. We classify the variables according to the type of data collected. Each value of a variable that is collected is called a **score**.

Example 1

Classify each of the data items above about Peter.

Solution

Fair hair is a name.	Hair colour is a categorical variable.
Age can be any value from 0 up.	Age is a continuous variable.
His height can be any value.	Height is a continuous variable.
He has a whole number of brothers.	Number of brothers is discrete.
Ice-cream flavour preference has an order.	Ice-cream flavour preference is ordinal.

In Example 1, both age and height are usually stated discretely, but the underlying variable is continuous. Similarly, ordinal or categorical variables are sometimes stated discretely. It doesn't make sense to find the average or to find the difference between categorical or ordinal scores. The next example shows how each variable type could be stated discretely.

Example 2

Classify each of the variables from the following part of a questionnaire.

Circle the number that best shows your answer.

1 I barrack for the Cowboys in the NRL.

Strongly agree	*Agree*	*Don't know*	*Disagree*	*Strongly disagree*
1	*2*	*3*	*4*	*5*

2 My favourite colour is:

Green	*Blue*	*Red*	*Yellow*	*Orange*
1	*2*	*3*	*4*	*5*

3 The number of living grandparents I have is:

0	*1*	*2*	*3*	*4*

4 My weekly income is:

$0–$500	*$501–$1000*	*$1001–$1500*	*$1501–$2000*	*$2001 +*
1	*2*	*3*	*4*	*5*

Solution

Football team preference has an order.	Football team preference is ordinal.
Favourite colours are names, even when coded as numbers.	Favourite colour is categorical.
People are counted as whole numbers.	Number of people is discrete.
Weekly income can be any value (to 1 cent).	Weekly income is considered continuous.

When we collect data or use data collected by other people, it should be examined to determine whether it seems reasonable. One way to check this is to work out a reasonable **range** for the data that is collected—that is, to determine the highest and lowest values you would reasonably expect. This does not mean that values outside the range are automatically discarded, but they may be regarded with some suspicion.

Example 3

What is the expected data range for:

a an adult's height (in centimetres)? **b** a person's annual income?

Solution

a People with dwarfism are usually at least 120 cm and even basketballers are rarely over 240 cm.	Expected height range is 120–240 cm.
b Very few individuals have an income over $200 000 per year.	Expected annual income range is $0–$200 000.

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Additional Exercise 2.1

Exercise 2.1 Data types

1 For each of the following, classify the data as categorical, ordinal, discrete or continuous.
   a memory sizes of computers at your school
   b overall performances of different tyres
   c distances to shops from your school
   d costs of restaurant meals
   e types of trees native to your area
   f placings in the final of the 100 m race
   g eye colours of members of your family
   h pulse rates of athletes after an event

2 Identify the variable type for each question on the following form.

> *1 What is your age?* ________________
> *2 What is your height?* ________________
> *3 What is your weight?* ________________
> *4 Do you use a gym for exercise?* ________________
> *5 How many times do you exercise each week?* ________________
> *6 How do you travel to school/work?* ________________

3 Identify the variable type for each question on the following questionnaire.

> *1 What is your name?* ________________
> *2 What is your age?* ________________
> *3 For how long have you been out of work (months)?* ________________
> *4 How many job advertisements did you reply to last week?* ________________
> *5 How many interviews did you attend last week?* ________________
> *6 How far (km) from home are you prepared to travel for work?* ________________

4 Give the expected data range for each of the following variables.
   a number of students in a Queensland secondary school
   b mass of a passenger car
   c price of a new car
   d value of a current Australian coin
   e value of an old Australian coin

5 Some schools in Queensland allow adult students to enrol in Years 11 and 12 and attend normal classes.
   a Does your school seem to allow this?
   b What is the expected range for student age in Year 11 in *your* school?
   c What is the expected range for student age in Year 11 in *Queensland*?

Modelling and problem solving

6 The travel destinations of people leaving Brisbane International Airport were coded as follows.

New Zealand = 1, Asia = 2, Pacific Islands = 3, America = 4, Africa = 5, Europe = 6

The average for 50 people was worked out to be 2.2, and this was given as evidence that the most common destination was Asia. What is wrong with this conclusion? Explain your reasons.

7 A **Likert scale** is a number scale that indicates preference. It is usually set out as:

Strongly agree	*Agree*	*Don't know*	*Disagree*	*Strongly disagree*
1	*2*	*3*	*4*	*5*

In a survey, people were asked to respond to the following questions on Likert scales.

1 *Violent criminals should not be allowed any privileges such as library access in gaol.*

2 *Capital punishment should be brought back for murders involving torture.*

The responses were then averaged, giving results of 3.4 and 3.6 respectively. Why is it incorrect to say that this means that people are more in favour of capital punishment than of withdrawal of privileges for violent criminals? Give reasons for your answer.

8 Jan said that the distance for a javelin throw is a discrete variable because it is always measured to the nearest 2 cm. Deirdre disagreed with Jan, saying that the distance for a throw is continuous. Explain who you think is correct, carefully justifying your answer.

9 The table below shows the ages of students in a Year 11 class.

Age	15	16	17	18
Number of students	4	15	3	1

a What kind of variable is student age? Give reasons for your answer.

b What kind of variable is the number of students? Justify your answer.

10 The prices of fruit and vegetables vary during the year according to availability. Oranges sold in Australia are often imported from California in the off-season for Australian oranges. Give an estimate of the price range of oranges (per kilogram) during the year, justifying your answer.

2.2 Designing surveys

In a **survey**, the same information is collected from many people. Surveys are the most common method of data collection. Most surveys are conducted by means of a **questionnaire**. The questionnaire can be given by an interviewer, by telephone, by post or by delivery. It can also be completed by an observer making notes. No matter how the data is collected, the questionnaire needs to be designed carefully to collect the desired information.

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Questionnaire design

1 The questionnaire should start with a statement regarding the purpose of the survey.
2 All questions should be numbered.
3 Questions should be worded in a clear, unbiased and unambiguous fashion.
4 Questions should require only short answers.
5 Questions may be open-ended or may have answers provided that can be ticked, circled or numbered.
6 Very few open-ended questions should be incorporated.
7 Categories should be used for sensitive information such as income or age.
8 The questionnaire should not be too long.
9 The investigator should be clear how the responses will be compiled and used.

Example 4

Rewrite the following question so that it is more likely to be answered correctly.

6 *What is your weekly income?* ____________________

Solution

Many people have variable income or are reluctant to give an exact answer. The question is best written using categories. The categories should not overlap. The question should also be more precise about the income meant.

6 *Please circle the range that best shows your normal gross weekly income (before tax and deductions):*

\$0–\$199	*\$200–\$399*	*\$400–\$599*
\$600–\$799	*\$800–\$999*	*\$1000–\$1199*
\$1200–\$1399	*\$1400–\$1599*	*\$1600 or more*

Example 5

Rewrite the following question in an improved manner.

4 *How much pocket-money does your father give you?* ____________________

Solution

The question is biased as their father may not be the person who gives them pocket-money.

The question is imprecise because it does not give a time period for the pocket-money.

4 *How much money do you usually get each week from your parent or guardian to spend on yourself (not including bus fares, clothes money, etc.)?* ____________________

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Even as it now stands, the question in Example 5 does not clearly allow for the fact that some students get regular pocket-money and others get pocket-money for doing jobs around the house. However, the question does need to be short. If a question needs more explanation, that should be given before the question is asked on a written survey. For an interview, the interviewer can answer questions when the person interviewed is unsure of the meaning of a question.

Example 6

Rewrite the question from Example 5 so that it explains what is meant and has category answers.

Solution

Explanation in the question would make it too long, so give the explanation before the question. Clearly separate the question from the explanation.

> ***4*** *Most students get some money from their parent or guardian to spend on themselves. Some students get the money for doing jobs such as washing dishes, washing the car or cleaning the house. This money, which is not for bus fares, clothes, books or other necessities, is usually called pocket-money. It is not money you get from an outside job like McDonald's.*
> *Circle the response that best shows the pocket-money you get in a week.*
> *\$0–\$5 \$6–\$10 \$11–\$15 \$16–\$20 \$21–\$25 More than \$25*

When you are compiling the results of a survey, it is often useful to have responses **coded** as numbers, because it is quicker to count them, put them in a table or on computer if they are done this way. This coding can be done in the question or can be done later from the response form.

Example 7

Design a suitable question to find the types of letters people send, with coded responses.

Solution

People send personal letters, bill payments, letters to officials, letters to order goods and services, packets and parcels. These can be coded with numbers. There may also be other kinds of letters. A time frame should also be established.

> *You may send a number of kinds of letters. For each category below, please write the number of letters you have sent in that category in the last 2 weeks.*
> ***1*** *Personal letters to friends and relatives* ☐
> ***2*** *Bill payments* ☐
> ***3*** *Letters to officials* ☐
> ***4*** *Letters to order things* ☐
> ***5*** *Packets or parcels* ☐
> ***6*** *Others* ☐

Example 8

Design a short questionnaire to find whether some students would prefer to start and finish school earlier.

Solution

Start with purpose.	*This survey asks about school start and finish times.*
Give clear instructions.	*Please circle the response that you most agree with.*
Check respondent.	***1*** *What year are you in?*
	8 9 10 11 12
	2 *Are you male or female?*
	M F
Key question.	***3*** *Would you like to start and finish school earlier?*
	Y N
Clear instruction.	*If you answered No to question **3**, stop now.*
Follow-up question.	***4*** *How much earlier would you prefer?*
	$\frac{1}{2}$ *1* $1\frac{1}{2}$ *2 hours*

Example 9

The following questionnaire was designed for an interviewer to ask householders about their insurance needs. Critically examine the questionnaire.

1 *Greet householder. If a child comes to the door, ask for Daddy.*
2 *Say 'I'm conducting a market research survey about insurance.'*
3 *Ask 'Have you got house and contents insurance?'*
4 *Ask 'What risks are you covered for?'*
5 *Ask 'What insurance do you have on your car?'*
6 *Ask 'Do you have suitable life insurance?'*
7 *Say 'Thank-you for your time.'*

Response sheet — Please complete with householder's responses.

1 *Street and house number* ______________________________
2 *House and contents insurance   Y   N*
3 *Risks—List* ______________________________
4 *Car insurance* ______________________________
5 *Life insurance* ______________________________

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Solution

This questionnaire has the advantage that the interviewer is given specific direction about questions, so the way the questions are asked should be the same for all the respondents.

The instruction to 'ask for Daddy' indicates bias.

The questionnaire guide numbers do not correspond to response sheet numbers.

The question about risks is vague and difficult to write responses for. It is also difficult to see how this could be compiled to a result.

The questions about car insurance and life insurance are also vague. The car insurance question does not allow for cases where the person has no car or more than one car.

There is no provision to show the day and time on the response sheet. Responses obtained at 11 am on Monday could be very different from those obtained at 4 pm on Saturday.

Investigation Newspaper questionnaires

Teacher Notes

Questionnaires are often published in newspapers and magazines. In some cases, they are designed to help readers find out something about themselves, and in other cases readers are invited to send their responses to the newspaper or magazine as an opinion poll. Your school library probably keeps a range of newspapers and magazines.

1. Collect as many questionnaires from newspapers and magazines as you can.
2. What is the purpose of each questionnaire?
3. Critically examine the questionnaires.

Exercise 2.2 Designing surveys

Additional Exercise 2.2

1. Rewrite the following questions so that they have category answers.
   - a What is your height?
   - b How many hours did you spend watching TV last week?
   - c How long did you spend on homework last night?
   - d How many DVDs are there at home?
   - e How long does it take you to get to school?
2. Rewrite the following questions so that they are not biased.
   - a Do you prefer Coke or other soft-drinks?
   - b Is your calculator a Casio or another brand?
   - c Is your house one-storey or two-storey?
   - d Do you mostly watch Channel 10?
   - e Is Cooking your favourite subject?
3. Rewrite the following questions with coded answers.
   - a How many brothers and sisters do you have?
   - b How many of your grandparents are still alive?
   - c Which radio station do you listen to most?
   - d What is your main food at breakfast?
   - e What do you have to drink with your evening meal?

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4 Rewrite the following questions with clear explanations of what is meant.
   a What kind of sport do you prefer?
   b Are you a good runner?
   c What kind of family do you have?
   d Do you have any spare money?
   e Do you do a lot of homework?

Modelling and problem solving

5 Design suitable survey questions to determine:
   a the most popular soaps on TV
   b the career aspirations of students in your year level at school
   c the ages of people entering a movie theatre
   d the occupations of people who eat breakfast at a certain fast-food chain
   e the highest standards of education completed by the workers on a construction project at an inner-city development site.

6 Design a short questionnaire to determine whether students who spend more time studying are more successful in their subjects. Explain how you worked out each question.

7 Design a short questionnaire to determine the types of holidays people prefer. Explain how you worked out each question.

8 The following questionnaire was designed to obtain information about leisure activities. Critically examine the questionnaire, then rewrite it.

> *1 What is your age?* ________________
>
> *2 What is your sex?* ________________
>
> *3 What do you do on the weekends?* ________________
>
> *4 What do you do after work?* ____________________________________
>
> *5 Do you watch a lot of TV?* ________________________
>
> *6 Do you like a lot of sport?* ________________________

9 The following questionnaire was designed to find what kinds of films people like to watch. Critically examine the questionnaire.

> *1 What is your age?* ________________
>
> *2 What is your sex?* ________________
>
> *3 Do you go to the cinema a lot?* ________________
>
> *4 What kind of films do you watch at the cinema?* ________________
>
> *5 What kind of films do you watch on video or DVD?* ________________
>
> *6 Do you buy or hire films for your video or DVD?* ________________

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10 The survey form below was intended to find whether doing more homework improved students' results. Critically examine the survey and rewrite it, explaining your reasons for changing questions.

> *1 What class are you in?* ________________
> *2 What is your sex?* ________________
> *3 How much time do you spend doing homework?* ________________
> *4 How well do you do at school?* ________________
> *5 How much more homework would you need to do to do better?* ________________

2.3 Survey methods and data quality

The survey method used to collect information will influence the usefulness of the information collected. **Interviews** are expensive to conduct, but with trained interviewers they can give the most reliable information.

Written questionnaires are relatively cheap to administer, but their distribution and collection can be costly if it is desirable to obtain responses from a large majority of those targeted. The Australian Bureau of Statistics (ABS) has a system of distribution and collection of the 5-yearly Australian Census to ensure responses from virtually every household in Australia, but this system costs an enormous amount of money. Some commercial surveys are distributed in pads to shops so that customers can fill them out and place their responses in a box. To ensure that at least some people fill out the questionnaire, this type of distribution usually incorporates a lottery to give prizes to some of the respondents.

Telephone polls are fairly cheap to conduct, but are limited in the information they can gather. Since the advent of telemarketing, people are less willing to answer questions on the phone than they were 20 years ago.

Some 'surveys' use SMS to get responses. These are often very biased and the respondent may even have to pay to take part.

The quality of data collected will vary according to the survey method. Even if every respondent was absolutely honest, the data could be unreliable because some groups of people were not surveyed. Telephone polls cannot be used for people who do not have a phone. The time at which a telephone poll is conducted will also influence who can be interviewed. There are advantages and disadvantages to each kind of survey method.

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Investigation Conducting interviews

1 Work in groups of two or three to write a question to find out what kind of car (for example, sedan, ute, 4WD, station wagon) students would like to get after they have been working for a few years.

2 If your teacher can arrange suitable times, interview students in other classes to obtain answers to your question from at least 15 students. Each group should ask students from a different class if possible. If this cannot be arranged, then each group should obtain answers at lunchtime or when travelling to or from school.

3 Each group should calculate the percentages of students who want each type of car.

4 Discuss your results and try to find reasons for variations in results between groups.

Example 10

A telephone poll is conducted between 3 pm and 5 pm on a Wednesday afternoon to determine the voting intentions of people in the next election. The telephone numbers are selected from residential entries on pages 235, 864 and 1024 of the *White Pages* of the current Brisbane telephone directory. What are the advantages and disadvantages of this method of survey?

Solution

Advantages

- The survey is quick, relatively cheap and easy to conduct.

Disadvantages

- The survey may be biased to a particular group—for example, if page 864 contains mainly people whose surname is Singh.
- The time and day of the survey mean that most people with full-time jobs will not be surveyed.
- There is no guarantee that the person who answers is old enough to vote.
- The survey is biased towards people who live in the city.
- The survey may be biased because many people will not answer questions on the phone.
- The survey may be biased because it excludes people who do not have a landline phone.

Additional Exercise 2.3

Exercise 2.3 Survey methods and data quality

Modelling and problem solving

1 A form addressed to 'The householder' is posted to every house in one suburb of Townsville, asking people about the kinds of groceries they buy. The form is to be sent back to the survey company within a fortnight. Explain the advantages and limitations of this kind of survey.

2 People with telephone numbers on a randomly selected page from the Gold Coast *White Pages* are phoned between 6 pm and 8 pm on a weeknight. They are asked whether they would support a new high-rise shopping centre and residential development on Burleigh Head. Explain why some people would not answer. Explain also the advantages and disadvantages of this kind of survey.

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3 An SMS survey is sent to all people who have subscribed to Optus in the last month. The survey asks how much they would pay for ring tone downloads of the latest hits. Explain the problems of this survey.

4 People walking through the middle of the Queen Street Mall in Brisbane at lunchtime on Tuesday are handed a survey form. It asks them to write down the kind of lunch they prefer to have and to put the survey in a box at the same location the next day. One person who answers the survey will win a free dinner at a top-class Brisbane restaurant. The survey is being conducted for a chain of sandwich bars. Is the survey likely to give good results for the sandwich bar company?

5 The editor of a health and fitness magazine thinks that the weight-and-height tables for Australians are out of date. The editor asks staff to put a survey form in the next issue asking subscribers to send in their current weight, height and age so that new tables can be compiled. Ten people who send in replies will win a year's free subscription to the magazine. Give reasons why this would be a biased survey.

6 A form is printed in a local newspaper to find the attitudes of people to a proposal to ban the planting of certain trees in Cairns. The form is below an article about the problems caused by invasive tree roots in the sewerage system. People are asked to send the form back to the newspaper office. This survey is designed to get a particular response. How has this been done?

7 A 'reality' TV show asks viewers to SMS the name of the contestant they want removed from the show, by 10 am in 3 days time. It costs 55 cents to take part. Give some advantages and disadvantages of this type of survey, explaining your reasons carefully.

8 A magazine asks readers to SMS their favourite celebrity by the end of the week in which the magazine came out. They will publish the results in the following week. It costs only the standard SMS charge to take part. Give reasons why this might help increase the magazine's circulation. Explain why the results are not likely to be representative.

9 A current affairs TV show has interviewed the family of two sisters who were killed by an 18-year-old who lost control of a high-powered sports car and veered onto the footpath. The TV show asks viewers to ring one of three numbers to register their votes about young drivers using high-powered cars. The votes are:

1. They should not be allowed.
2. They should be allowed.
3. It wouldn't make any difference.

What is wrong with this survey?

10 An entertainment website urges people to 'Vote now' for their all-time favourite movie. Do you think this is likely to give an accurate result? Justify your answer.

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2.4 Observing and experimenting

Some information has to be collected by observation or experiment. For example, behavioural information such as how much people talk to each other at work is often collected by a **timed observation**. In cases like this it is important to devise an observation schedule so that the information is collected in an objective manner.

Example 11

Devise an observation schedule to collect information about behaviour in a bus queue.

Solution

Types of behaviour must be listed.
They should be directly observable and not need interpretation.
There should not be too many.

1. Standing quietly
2. Talking to next person in queue
3. Listening to next person
4. Looking in wallet or bag
5. Gazing at surroundings
6. Signalling to approaching bus

Write down the timing.
Draw up an easily used schedule.

Each person in the queue should be observed in turn each minute.

1 Standing	2 Talking	3 Listening	4 Bag	5 Gazing	6 Signalling

Place a tally mark each minute for each person in the queue.
Number of people in queue: ________________
Total time of observation: ________________

Example 12

The observation schedule from Example 11 was completed in the 5 minutes before a bus came. Some people joined the queue during that time. The results are shown below, but some drink was spilt on part of the form.

1 Standing	2 Talking	3 Listening	4 Bag	5 Gazing	6 Signalling
卌 卌 卌 |	卌 ||	卌 卌 |	卌 |	卌 卌 |||	|||

Place a tally mark each minute for each person in the queue.
Number of people in ________________
Total time of observation: 5 minutes

a How many people were seen signalling to the approaching bus?
b What was the most common behaviour?
c What was the smallest possible number of people in the queue when the bus arrived?

Solution

a The signalling column has 3 marks	Three people were signalling.
b The standing column has the most marks, 16.	The most common behaviour was standing quietly.
c There are 56 tally marks altogether. The most anyone could get was 5 marks if they were there the whole time. Round up to allow for someone coming at the last minute.	$56 \div 5 = 11.2$ There were at least 12 people when the bus arrived.

Experiments are also performed to find information. These may be as simple as measuring people's bodily dimensions or as complex as making judgments at a wine tasting. In the case of measurements, they must be made in the same way for each person. In the case of subjective judgments, such as tasting, there should be an attempt to exclude other factors that may influence judgment. In a wine tasting, judges are given the wines in unmarked carafes or masked bottles that are identified only by lot number, so that they are not influenced by a winemaker's previous reputation.

A **double-blind experiment** is sometimes used in medical experiments. In this type of test of the usefulness of a drug, neither the doctor nor the patient knows whether the test drug or a placebo (a 'pretend' drug) is being administered until after the trial.

Example 13

Devise an experiment to find the best-tasting thickened cream from four different brands.

Solution

Hide identity of creams.	Repackage the creams in similar containers, labelling them as 1, 2, 3 and 4.
Make conditions the same.	Place equal dollops of cream on fresh scones.
Equalise conditions that vary.	Change the order of creams for each taster.
Obtain information.	Ask each taster to rate the creams in order.

In Example 13, we could also try different uses of the cream to obtain more information. There should be sufficient tasters to obtain a reasonable answer.

Investigation Using an observation schedule

Work in small groups for this investigation.

1 Devise an observation schedule to find what students do at lunchtime when they are not playing sport. You may want to observe some students before devising the schedule to find what needs to be placed on the schedule. This is called a **pilot study**.

2 Now observe some students at lunchtime to fill in the observation schedule.

3 As a class group, discuss the problems encountered in completing the schedules.

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Additional Exercise 2.4

Exercise 2.4 Observing and experimenting

1 The following observation schedule was completed by watching the behaviour of people waiting in a queue at a theme park. Each person was observed once.

Standing quietly	Talking to neighbour	Listening to neighbour	Watching people on ride	Eating while waiting	Looking in bag
𝍸 𝍸 𝍸 ǀǀǀ	𝍸 𝍸 𝍸 ǀ	𝍸 𝍸 𝍸 𝍸 ǀǀ	𝍸 𝍸ǀ ǀǀǀ	𝍸 𝍸	𝍸 ǀǀ

a How many people were watching others on a ride?

b What was the most common behaviour?

2 Five different sweetening substances were tested to find the one people liked most. The following schedule shows the ratings of different people for each substance. The most liked was rated 1, the next 2, and so on up to 5.

Aspartame	Saccharin	Sucralose	Sugar	Xylitol
4, 5, 2, 3, 3, 5, 4, 4, 5, 2	5, 4, 5, 4, 5, 4, 5, 5, 4, 5	2, 1, 3, 1, 2, 3, 1, 2, 1, 3	1, 2, 1, 2, 1, 1, 2, 1, 2, 1	3, 3, 4, 5, 4, 2, 3, 3, 3, 4

a How many people tasted the sweeteners?

b Which sweetener had the most 1s?

c Add up the numbers and use the totals to find the overall order of preference from most to least liked.

Modelling and problem solving

For each of the following, explain the decisions you made in devising the schedule or experiment.

3 Devise an observation schedule to find the number and types of vehicles travelling on a road next to the school.

4 Devise an observation schedule to find the behaviour of a teacher during a lesson.

5 Devise an experiment to measure the masses of students' books in your grade.

6 Devise an experiment to measure the masses of students in the school.

7 Devise an observation schedule to find how students use the school library.

8 Devise an observation schedule to find how people cross the road. Explain the decisions you make.

9 Devise an experiment to find the best-tasting kind of chocolate from a number of brands and flavours.

10 Devise an experiment to find the best kind of facial cream.

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2.5 Organising data

The results of a survey, experiment or observation are often organised into **frequency tables** or **spreadsheets** to make them useful. During this process, data that appears to be recorded incorrectly or has other defects may be discarded. However, it is important to discard data only after carefully considering whether it may be useful.

!

Defective data

It may be necessary to discard data that is:

- incomplete to the extent that it cannot be properly counted
- duplicated, such as when people put in multiple returns in order to maximise their chances of getting a prize
- incorrectly recorded, such as a person giving a height of 360 cm
- contradictory, such as when people answer the same question in different ways
- carelessly completed, such as when people just put answer A to all questions.

A **computer spreadsheet** or **database** can show all the data, and yet still be used to find frequencies and other statistical measures. It is usual to set up the table in a spreadsheet or database so that each row represents the response of one person, and each column shows the response to a particular question.

Teacher Notes

Example 14

Set up a spreadsheet to store the responses to the survey form shown below.

1 What year are you in at school? 8 9 10 11 12

2 Are you male or female? *M* *F*

3 Would you like to start and finish school earlier? *Y* *N*

*If you answered No to question **3**, stop now.*

4 How much earlier would you prefer? $\frac{1}{2}$ 1 $1\frac{1}{2}$ 2 *hours*

Solution

Put a heading at the top of the spreadsheet. Leave the first column for respondent number. Put a heading at the top of each column for the answer to each question. Widen the first column and centre the headings and data. Enter the data.

	A	B	C	D	E	F	G
1	New Qmaths 11A			SURVEY COMPILATION			
2	Survey of school starting times						
3	Respondent	Q1:Year	Q2: Sex	Q3: Y/N	Q4: Time		
4	1	9	M	Y	1		
5	2	11	F	N	2		

Spreadsheet

The spreadsheet can also be loaded from the CD-ROM.

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Example 15

Use the following table of responses for Example 14 to complete frequency tables for year and time.

Respondent	Q1: Year	Q2: Sex	Q3: Y/N	Q4: Time
1	11	M	Y	1
2	10	F	N	
3	12	M	N	
4	11	F	N	
5	10	F	N	
6		M	N	
7	7	M	Y	0.5
8	9	F	N	
9	9	F	Y	0.5
10	11	M	Y	1.5
11	11	M		
12	11	M	N	
13	11	M	Y	1
14	9	F	N	
15	12	F	Y	2
16	8	F	Y	1.5
17	9	M	Y	0.5
18	11	M	N	
19	10	F	Y	1.5
20	9	F	Y	0.5

Solution

Response 6 is incomplete: we cannot count for the year. Response 7 has the wrong year. Response 11 looks as if the survey was not completed.

Discarding these three responses, we get the following frequency tables.

Year	Tally	Frequency
8	|	1
9	𝍸	5
10	|||	3
11	𝍸 |	6
12	||	2

Time	Tally	Frequency
0.5	|||	3
1	||	2
1.5	|||	3
2	|	1

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Exercise 2.5 Organising data

Additional Exercise 2.5

Spreadsheet

1 Set up a spreadsheet to store the responses to the survey form shown below.

1 What is your age?
10–20 21–30 31–40 41–50 51–60 61–70 71–80 81+ years
2 What is your sex? M F
3 Do you have an MP3 player? Y N
If you do not have an MP3 player, ignore the rest of this survey.
4 How often do you use your player?
A Every day B Most days C Some days D Sometimes
5 How many songs do you have on your player?
A Less than 20 B 20–99 C 100–199 D 200–499 E 500 or more

2 Set up a spreadsheet to store the responses to the survey form shown below.

1 What grade are you in? ______________
2 What is your sex? M F
3 What kind of music do you like most?
____
4 How many CDs/DVDs do you have of this kind of music? ______________
5 How many songs of this kind have you downloaded onto an MP3?
A Less than 20 B 20–99 C 100–199
D 200–499 E 500 or more
6 Who is your favourite singer/band?
____
7 How many CDs/DVDs do you have of this singer/band? ______________

3 Set up a spreadsheet to store the responses to the survey form shown below.

1 What grade are you in? ______________
2 Are you male or female? M F
3 For how many hours a week do you usually watch TV? ______________
4 What is your height? ______________ cm
5 What is your weight? ______________ kg
6 For how many hours a week do you normally exercise or play sport? ______________

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Modelling and problem solving

4 The following table shows the results for a survey of the grades of some students.

1 What grade are you in? ______________
2 How many As did you get in your last report for achievement? ____________
3 How many Bs did you get in your last report for achievement? ____________
4 How many Cs did you get in your last report for achievement? ____________
5 How many Ds did you get in your last report for achievement? ____________
6 How many Es did you get in your last report for achievement? ____________

Respondent	1 Grade	2 As	3 Bs	4 Cs	5 Ds	6 Es
1	12	1	0	4	1	0
2	10	0	0	1	1	4
3	8	0	1	3	1	1
4	8	0	0	1	4	1
5	12	2	2	1	1	0
6	9	1	1	3	1	0
7	12	3	3	0	0	0
8	10	3	3	0	0	0
9	11	0	0	3	1	2
10	8	1	0	4	0	1
11	10	0	1	3	2	0
12	8	0	0	1	1	4
13	11	1	2	3	0	0
14	12	3	1	2	0	0
15	10	0	1	5	0	0
16	10	0	1	2	3	0
17	9	1	1	2	2	0
18	9	0	0	6	0	0
19	11	4	0	1	1	0
20	11	1	0	2	2	1
21	9	1	1	3	1	0
22	10	4	2	0	0	0
23	11	1	1	2	2	0
24	9	2	0	4	0	0
25	10	2	4	0	0	0
26	10	1	0	5	0	0
27	9	1	0	5	0	0
28	10	1	3	2	0	0
29	9	1	1	4	0	0
30	9	0	3	2	0	1

a Compile a frequency table for grade.
b Compile a frequency table for As.
c Only grades A to C are passes. Compile a frequency table for passes.

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5 The following observation schedule shows the traffic that passed through an intersection during a 5-hour period. Compile a frequency table for the results.

Cars	Motorbikes	Trucks	Taxis	Other
卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 III	卌 IIII	卌 卌 卌 卌 卌 卌	卌 卌	卌 卌 卌 卌 卌 卌 卌 卌 II

6 Four different kinds of cream were tasted by a class of Year 11 students. The creams were re-packaged and labelled as 1, 2, 3 and 4. The students' orders of preference for the creams were as follows, with the best cream given first in each case:

2-1-3-4 1-4-2-3 2-1-4-3 4-3-1-2 4-2-3-1 1-4-2-3 2-4-3-1 2-4-3-1 4-3-1-2
2-3-4-1 3-4-1-2 3-2-1-4 2-4-3-1 1-2-4-3 4-3-1-2 2-1-3-4 3-2-1-4 4-2-1-3
2-3-4-1 1-4-3-2 4-1-2-3 4-1-2-3 4-2-3-1 3-4-2-1 3-2-1-4

a Draw up a frequency table to find the overall order of preference of the creams according to the students' first preferences only.

b Devise a method to use the students' preference orders to find the overall order of preference. You must explain your method carefully.

7 The data below shows the masses (in kg) of some people exercising in a gym. Make a frequency table using appropriate categories to show the masses. You need to explain the choice of categories you make.

109	104	80	72	106	70	92	108	71	87	114	54	74	105	69
56	111	100	101	92	56	89	97	102	71	91	69	80	79	64
64	69	121	108	105	51	71	97	53	61	65	120	93	97	

8 The table below shows the responses of people to a survey of exercise habits. They were asked for their height and weight and the number of hours of exercise they did each week. The body mass index for each person was calculated using the formula:

$$\text{BMI} = \frac{\text{mass in kg}}{(\text{height in m})^2}$$

Note that a BMI between 19 and 25 is considered to be in the normal range.

BMI	27	27	24	29	24	31	22	22	13	29	22	20	25	21
Exercise (hours)	8	4	7	3	9	4	15	15	20	2	11	15	10	13

BMI	25	36	23	23	22	27	27	24	25	23	28	24	28	29
Exercise (hours)	6	2	8	13	15	4	4	8	8	10	9	11	7	7

a Make a frequency table of the BMIs of this group.

b Make a frequency table of the hours of exercise of this group.

c Decide whether any of the group seem particularly unhealthy and give reasons.

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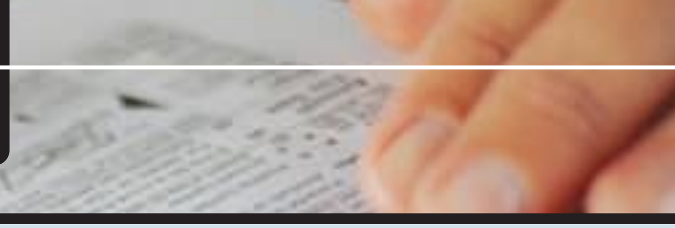

Chapter summary

- **Categorical** data is names. It is sometimes called **nominal** data.
- **Ordinal** data has an order, but the differences between steps are not meaningful.
- **Discrete** data can have only certain values. It is sometimes called **count** data because it often arises from counting.
- **Continuous** data can have any value, although there may be a maximum and/or minimum.
- An **attribute** is a property of an object or person. Each attribute for which data is collected is called a statistical **variable**. Variables are classified according to the data type.
- Each value of a variable that is collected is called a **score**.
- A **questionnaire** is a group of questions designed to find information.
- Data collected by seeking the same information from many people is called a **survey**.
- **Interviews** generally give more reliable data than telephone polls or written questionnaires.
- **Observation** is a data collection technique where a real situation is counted. It may be done using an observation schedule over time.
- **Experimentation** is a deliberate attempt to gather data by measuring or testing.
- **Spreadsheets** or **databases** may be used to store information in a response table.
- **Frequency tables** are tables listing how many times (frequency) each score appears.

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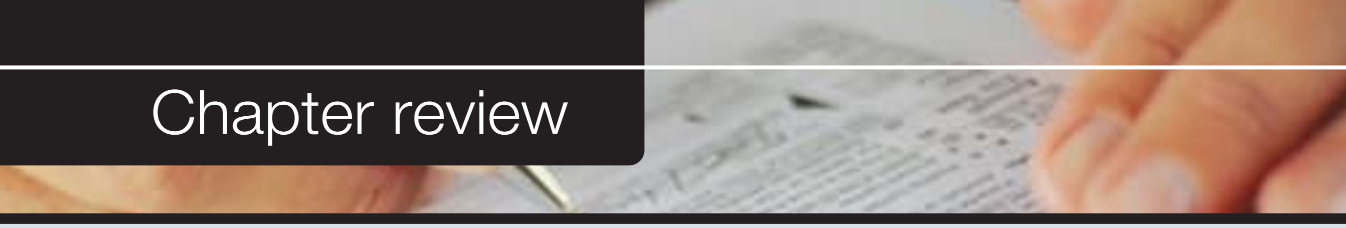

Chapter review

Knowledge and procedures

1 Classify each of the following data types as categorical, ordinal, discrete or continuous. **Ex 2.1**
- **a** prices of chocolates
- **b** diameters of trees
- **c** makes of cars driven by teachers

2 What is the expected data range for each of the following? **Ex 2.1**
- **a** diameter of a tree
- **b** price of a mattress
- **c** age of a primary school child

3 Rewrite the following questions in an improved fashion. **Ex 2.2**
- **a** Do you watch *Neighbours* or other programs?
- **b** How much junk food do you eat?
- **c** For how long have your parents been married?

4 Set up a spreadsheet to store the responses to the survey form shown below. **Ex 2.5**

1 What is your age? ______________

2 Are you male or female? *M* *F*

3 What football team do you support? ________________

4 How long do you spend watching football each week? _______ *hours*

Modelling and problem solving

5 The ages of people entering a cinema were averaged to find the general age of people attending a particular picture. Is there any problem with this approach? **Ex 2.1**

6 Design a short questionnaire to find the radio programs listened to by people in different age groups. **Ex 2.2**

7 Critically examine the following questionnaire. **Ex 2.2**

1 How long have you had your licence for? _________ *years*

2 Do you do long trips? *Y* *N*

3 How long do you drive for? __________ *hours*

4 How many accidents have you had? ________________

5 Are you generally a safe driver? *Y* *N*

8 A TV station conducted a poll asking people to phone two different numbers to register Yes or No votes for capital punishment after a TV program about murders of children. Would there be any problems with the results of this poll? **Ex 2.3**

9 Devise an experiment to find the best instant coffee. **Ex 2.4**

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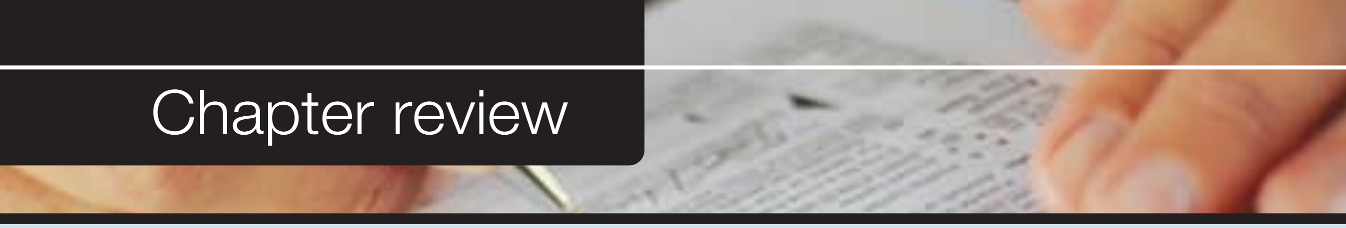

Chapter review

Ex 2.5

10 Responses to this survey on longer prison terms are shown below. Compile frequency tables for sentence lengths for assault with a weapon and for actual bodily harm.

1 *What is your age group?*
10–19 20–29 30–39 40–49 50–59 60–69 70–79 80–89 90+

2 *What is your sex? M F*

3 *Do you believe prison terms should be longer for crimes of violence? Y N*

4 *How long should the sentence be for assault? ________ years*

5 *How long should the sentence be for assault with a weapon? ________ years*

6 *How long should the sentence be for actual bodily harm? ________ years*

7 *How long should the sentence be for grievous bodily harm? ________ years*

8 *How long should the sentence be for manslaughter? ________ years*

9 *How long should the sentence be for rape? ________ years*

10 *How long should the sentence be for murder? ________ years*

Respondent	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
1	60–69	F	N	6	11	11	11	13	13	14
2	30–39	M	Y	3	6	11	12	17	21	25
3	30–39	F	Y	6	11	14	15	18	21	22
4	20–29	F	Y	3	3	7	11	12	16	20
5	80–89	F	Y	2	7	7	7	8	13	14
6	50–59	M	Y	1	2	4	9	13	17	18
7	20–29	M	Y	2	2	3	6	10	10	13
8	40–49	M	Y	1	6	6	10	13	18	20
9	40–49	M	Y	4	8	8	9	12	16	21
10	60–69	F	Y	4	6	10	12	15	15	16
11	60–69	M	Y	6	11	15	19	21	26	29
12	50–59	M	N	5	7	9	9	10	12	15
13	30–39	M	N	3	8	11	15	15	15	17
14	80–89	F	Y	5	6	7	8	9	12	15
15	80–89	M	N	1	4	9	10	11	15	16
16	40–59	M	Y	5	10	15	17	18	21	24
17	40–59	M	Y	6	9	11	11	13	17	17
18	10–19	M	N	6	10	11	15	17	20	24
19	30–39	F	Y	3	5	9	13	14	15	15
20	30–39	F	Y	1	4	6	8	8	10	10

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Measuring shapes and spaces

3

Contents

Syllabus Guide
Chapter 3

Syllabus subject matter

Elements of applied geometry

- Area, volume and capacity in life-related situations
- Simple algebraic manipulation of relevant formulas for this topic

Quantitative concepts and skills

- Metric measurement including measurement of mass, length, area and volume in practical contexts
- Calculation and estimation with and without instruments
- Basic algebraic manipulations

The metric system is a decimalised system of measurement used in most parts of the world. While length can be measured directly, the area of a region or the volume of a space is usually calculated from length measurements using some indirect method. Most people can estimate distances reasonably well, but the estimation of areas and volumes is less easy. The ability to calculate the volume of concrete needed for a building slab, or the area of tiles to cover a floor, is an important practical skill. The work in this chapter will help you to develop these skills.

3.1 Measuring perimeters

Measurements of length in the metric system are based on the standard unit: the metre (m). Other length units are derived from the metre using **prefixes**. A list of metric system prefixes is shown in Appendix 1. The common metric system prefixes for measuring length are:

$$\textbf{kilo (k)} = 10^3 = 1000$$
$$\textbf{centi (c)} = 10^{-2} = 0.01$$
$$\textbf{milli (m)} = 10^{-3} = 0.001$$

So, 1 *kilo*metre means 1 × 1000 metres or 1000 m, and so on.

Example 1

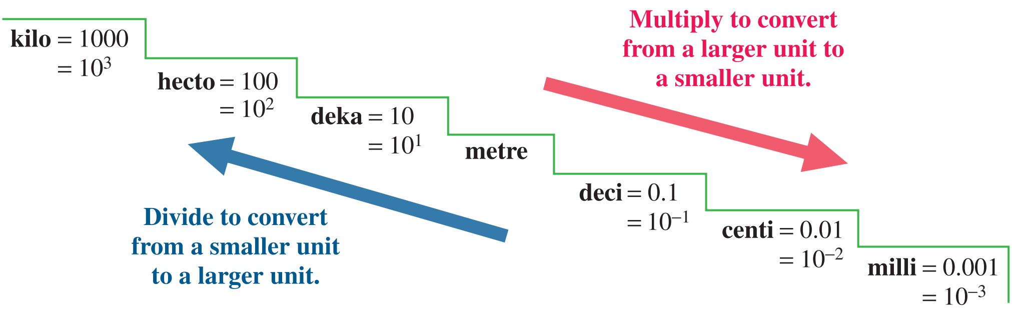
Convert 150 cm to m.

Solution

Centi = 10^{-2}.	1 cm = 0.01 m
Now apply to 150 cm.	150 cm = 150 × 0.01 m
Move the decimal point 2 places to the left.	= 1.5 m
Write the answer.	150 cm = 1.5 m

The following diagram helps to show how to convert from one unit to another in the metric system. Each 'step' on the diagram represents multiplication or division by 10. When converting metric measures, remember that when you convert from a larger unit to a smaller unit (stepping down), you need to multiply. When converting from a smaller unit to a larger unit (stepping up), you need to divide.

! **Metric length conversions**



Example 2

Convert 3.5 km to mm.

Solution

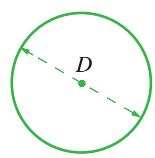
First, to convert from km to m = 3 'steps' = 10^3.	
Larger unit → smaller unit, so multiply.	$3.5 \text{ km} = 3.5 \times 10^3 \text{ m}$
Move the decimal point 3 places to the right.	$= 3500 \text{ m}$
Now, to convert from m to mm = 3 'steps' = 10^3.	
Larger unit → smaller unit, so multiply.	$3500 \text{ m} = 3500 \times 10^3 \text{ mm}$
Move the decimal point 3 places to the right.	$= 3\ 500\ 000 \text{ mm}$
Write the answer.	$3.5 \text{ km} = 3\ 500\ 000 \text{ mm}$

The **perimeter** of a flat shape is the length around the outside. When finding perimeters, we sometimes need to use the formula for the circumference of a circle.

! **Circumference of a circle**

Circumference $C = \pi D$

where $\pi \approx 3.14$ and D is the diameter of the circle.



Example 3

Find the circumference of a circle with radius 3.4 m.

Solution

Diameter twice the radius.	$D = 2 \times 3.4 \text{ m} = 6.8 \text{ m}$
Write the formula.	$C = \pi D$
Substitute for D.	$= \pi \times 6.8 \text{ m}$
Use the π key on your calculator.	$= 21.3628\ldots \text{ m}$
Round and write the answer, indicating rounding.	The circumference is about 21.4 m.

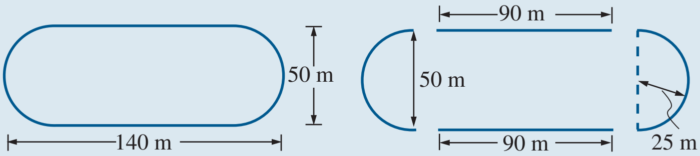
Perimeter is used in the calculation of fencing costs.

Example 4

A sports-ground 140 m long and 50 m wide has semicircular ends. A fence is to be erected all the way round, with four gates, each 2 m wide. The gates cost \$150 each and the fencing costs \$45/m. Find the cost of fencing the ground.

Solution

Sketch the sports-ground.
Show the measurements.
Divide the perimeter into sections.



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Calculate the length round each end.	End $= \frac{1}{2} \times \pi D$
	$= 0.5 \times \pi \times 50$ m $= 78.5398\ldots$ m
Find the whole length.	Perimeter $= 90 + 78.5398\ldots + 90 + 78.5398\ldots$ m
	$= 337.0796\ldots$ m
Take off gate lengths (4×2 m).	Fence length $= 337.0796\ldots - 4 \times 2$ m
	$= 329.0796\ldots$ m
Find the total cost.	Total cost $= 329.0796\ldots \times \$45 + 4 \times \$150$
	$\approx \$14\,808.58 + \$600$
	$= \$15\,408.58$
Write the answer.	The cost of fencing the ground is about \$15 408.58.

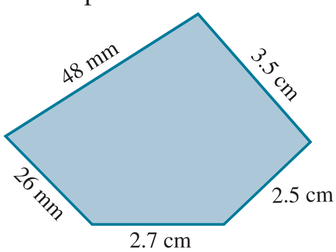
Additional Exercise 3.1

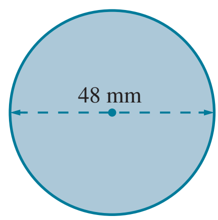
Exercise 3.1 Measuring perimeters

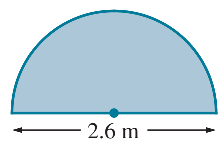
1 Change each of the following measurements to the unit indicated.

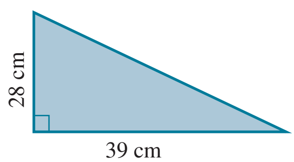
a 320 cm → m **b** 5.23 km → m **c** 1460 mm → cm
d 58 cm → m **e** 537 m → km **f** 32.7 m → cm
g 0.054 m → mm **h** 47 350 cm → km **i** 0.000 64 km → mm

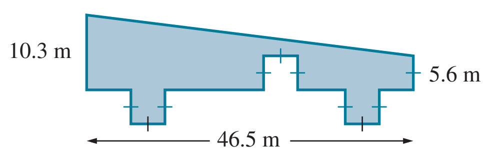
2 Find the perimeters of the following shapes.

a


b


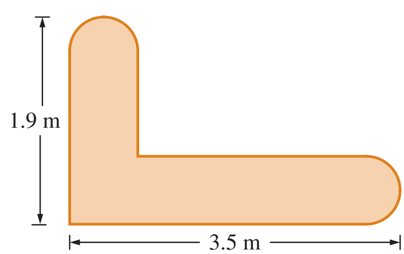
c


d


e


Modelling and problem solving

3 The L-shaped kitchen bench with semicircular ends shown here has been made from chipboard veneer. The bench is 600 mm wide. It must have an edging strip of veneer attached all the way along the edge to finish the job. What length of edging veneer is required?



4 A fence is to be erected around a block of land that has a slanting front boundary and parallel side boundaries of 27 m and 42 m. The rear boundary is 20 m long and is at right angles to the side boundaries. The posts are to be a maximum of 2 m apart, there are two railings, and 100 mm palings are to be spaced 20 mm apart. The posts cost \$12.50 each, the railings cost \$5.40/m and the palings cost \$1.40 each. A 2.5 m gate costing \$130 is to be placed at the front. Find the cost of materials to complete the job.

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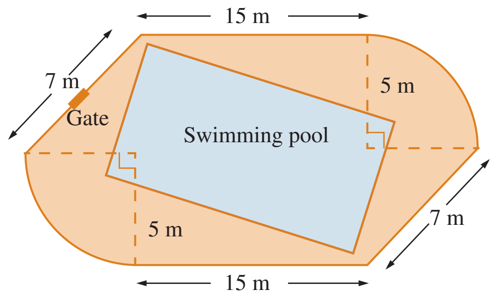
5 A swimming pool is set in a wooden deck as shown. A pool fence is to be erected around the border, with a 1200 mm wide gate. The cost of materials for a 1200 mm high fence is \$42/m, and the gate costs \$120. Installation costs are \$10/m for the fence and gate.

a Calculate the cost of materials to complete the job.

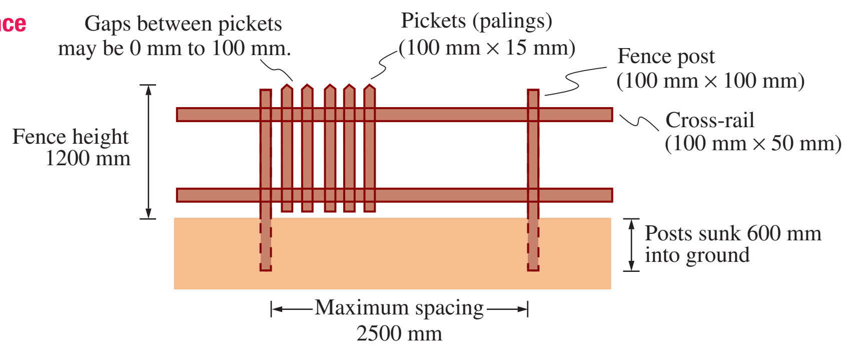
b Find the cost of installing the fence and gate.



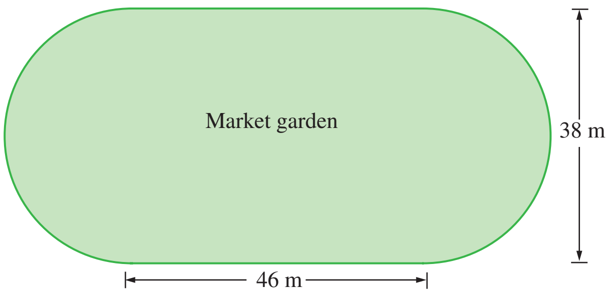
6 A circular horse-racing track is 700 m across the inside and is 15 m wide. The track has to be re-fenced on both sides with post-and-rail fencing at a cost of \$34/m for materials and labour. Find the cost of erecting the fences.

7 The market garden plot shown below is to be enclosed with the wooden post-and-rail fence as illustrated.

Post-and-rail fence



Market garden plot



a Determine the number of posts and the length of cross-rails needed to complete the task.

b Calculate the minimum number of 100 mm × 15 mm palings required if the maximum spacing desired is 75 mm.

c Work out the cost of the materials if posts cost \$14.50 each, rails cost \$3.40/m and palings cost \$2.10 each.

3.2 Calculating areas of shapes

When measuring the areas of surfaces, square units are used. The units are derived from units of length. The area unit used will depend on the size of the surface being measured. The most convenient units and their conversions are shown below.

!

Area units

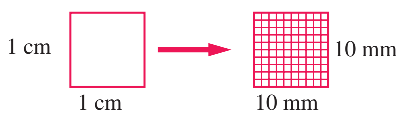
Unit	*Abbreviation*	*Examples*
Square millimetre	mm^2	Precious metals for jewellery
Square centimetre	cm^2	Paper, pavers, posters
Square metre	m^2	Walls, house floorspace, blocks of land
Hectare	ha	Larger land areas such as farms and parks
Square kilometre	km^2	Countries, lakes, national parks

Conversions

$1 \text{ cm}^2 = 100 \text{ mm}^2$ $\quad$ $1 \text{ ha} = 100 \text{ m} \times 100 \text{ m} = 10\,000 \text{ m}^2$

$1 \text{ m}^2 = 10\,000 \text{ cm}^2$ $\quad$ $1 \text{ km}^2 = 100 \text{ ha} = 1\,000\,000 \text{ m}^2$

A square centimetre is 1 cm long and 1 cm wide. If this is divided into square millimetres, then it is 10 mm long and 10 mm wide. This gives the conversion shown above. The other conversions are obtained in a similar fashion.



$$\mathbf{1 \text{ cm}^2 = 10 \text{ mm} \times 10 \text{ mm} = 100 \text{ mm}^2}$$

Example 5

Convert:

a $120\,000 \text{ cm}^2$ to m^2 $\quad$ **b** 2.34 km^2 to ha.

Solution

a Write the conversion.

$$1 \text{ m}^2 = 100 \text{ cm} \times 100 \text{ cm} = 10\,000 \text{ cm}^2$$

Smaller unit → larger unit, so divide by 10 000 (move decimal point 4 places to left).

$$120\,000 \text{ cm}^2 = 120\,000 \div 10\,000 \text{ m}^2 = 12 \text{ m}^2$$

b Write the conversion.

$$1 \text{ km}^2 = 100 \text{ ha}$$

Larger unit → smaller unit, so multiply by 100 (move decimal point 2 places to right).

$$2.34 \text{ km}^2 = 2.34 \times 100 \text{ ha} = 234 \text{ ha}$$

In most situations, an area can be determined using the areas of basic shapes. The formulas for the areas of the common shapes are given on the next page.

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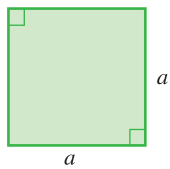
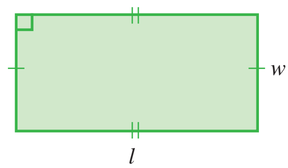
Areas of common shapes

Square



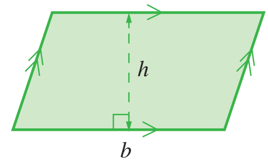
Area = $a \times a = a^2$

Rectangle



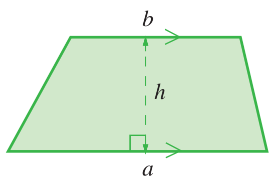
Area = $l \times w$

Parallelogram



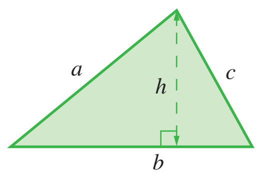
Area = $b \times h$

Trapezium



Area = $\frac{1}{2}(a + b) \times h$

Triangle



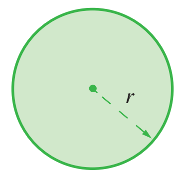
Area = $\frac{1}{2} \times b \times h$

or by Heron's Formula:

Area = $\sqrt{s(s-a)(s-b)(s-c)}$

where $s = \frac{1}{2}(a + b + c)$ is the **semiperimeter**.

Circle

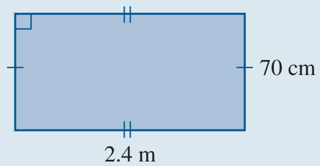


Area = πr^2

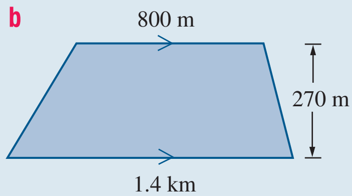
Example 6

Calculate the areas of the following, writing answers in the larger unit.

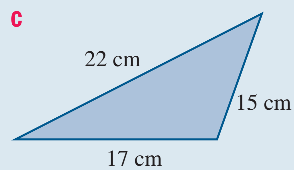
a



b



c



Solution

a Put measurements in the same units. $70 \text{ cm} = 0.7 \text{ m}$

Write the formula. $A = l \times w$

Put in values. $= 2.4 \times 0.7 \text{ m}^2$

Calculate answer. $= 1.68 \text{ m}^2$

b Put measurements in the same units. $800 \text{ m} = 0.8 \text{ km}$

$270 \text{ m} = 0.27 \text{ km}$

Write the formula. $A = \frac{1}{2}(a + b) \times h$

Write $\frac{1}{2}$ as $\div 2$. $= (1.4 + 0.8) \times 0.27 \div 2 \text{ km}^2$

Do brackets first. $= 2.2 \times 0.27 \div 2 \text{ km}^2$

Calculate answer. $= 0.297 \text{ km}^2$

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c We use Heron's Formula because we know all three sides, but not the height.

Calculate the semiperimeter.	$s = \frac{1}{2}(a + b + c)$
Write $\frac{1}{2}$ as $\div 2$.	$= (22 + 17 + 15) \div 2$ cm
	$= 54 \div 2$ cm $= 27$ cm
Write the formula.	$A = \sqrt{s(s-a)(s-b)(s-c)}$
Put in values.	$= \sqrt{27(27-22)(27-17)(27-15)}$ cm^2
	$= \sqrt{27 \times 5 \times 10 \times 12}$ cm^2
	$= \sqrt{16\,200}$ cm^2
	$= 127.2792 \ldots$ cm^2
Round the answer.	≈ 130 cm^2

Technology

Calculator Program

The program CAREA can be used to find the areas of common shapes. The program is given in full on the CD-ROM. Enter the program (or load it from the CD-ROM) and try it with different shapes.

```
CHOOSE ONE OF
1. SQUARE
2. RECTANGLE
3. PARALLELOGRAM
4. TRIANGLE
5. TRAPEZIUM
6. CIRCLE
?
```

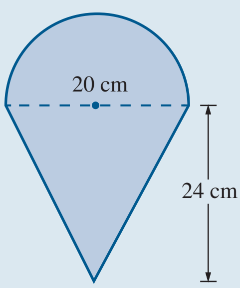
```
6. CIRCLE
?
4

2.BASE HEIGHT
3.ALL SIDES
?
2
```

```
3.ALL SIDES
X=
3
SIDES
A=
5
B=
8
```

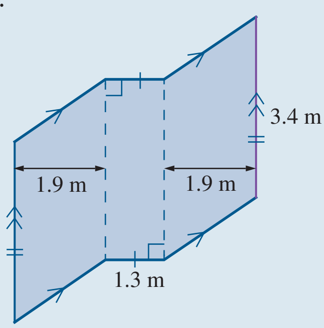
Example 7

Find the areas of the shapes shown here.

a



b



Solution

a The shape is a semicircle joined to a triangle.

Find the area of a full circle.	Circle $= \pi r^2$
The radius is 10 cm.	$= \pi \times 10^2$ cm^2
	$= 314.1592 \ldots$ cm^2
Semicircle is half.	Semicircle $= 314.1592 \ldots \div 2$ cm^2
Put into memory.	$= 157.0796 \ldots$ cm^2

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Now do the triangle.	Triangle $= \frac{1}{2} b \times h$
	$= 0.5 \times 20 \times 24$ (or $20 \times 24 \div 2$) cm^2
	$= 240$ cm^2
Put together.	Area of shape $= 157.0796 \ldots + 240$ cm^2
	$= 397.0796 \ldots$ cm^2
Round the answer.	≈ 397 cm^2

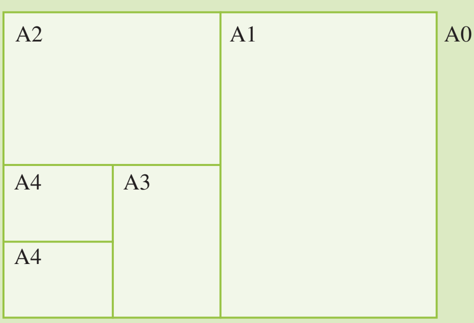
b This shape has a rectangle and two equal parallelograms.

Area of rectangle.	$A = l \times w$
	$= 1.3 \times 3.4$ m^2
	$= 4.42$ m^2
Area of parallelogram.	$A = b \times h$
	$= 3.4 \times 1.9$ m^2
	$= 6.46$ m^2
Put together.	Total area $= 4.42 + 2 \times 6.46$ m^2
	$= 17.34$ m^2
Round to fit question accuracy.	≈ 17 m^2

Investigation Paper sizes

Paper comes in sizes A0, A1, A2, A3, A4 and so on. All sizes are generated from the A0 size, which is a rectangular shape with an area of 1 m^2.

The sides of the rectangle are in the ratio $\sqrt{2} : 1$. The sizes A1, A2 and so on are formed by halving the longer side.



1 Determine the dimensions of a sheet of A0 paper.

2 Now use the dimensions that you have just worked out to calculate the dimensions of a sheet of A4 paper. Measure a sheet of A4 and compare its dimensions with those you have calculated.

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Additional Exercise 3.2

Exercise 3.2 Calculating areas of shapes

1 Change each of the following measurements to the unit of square measure indicated.

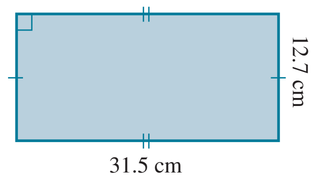
a 250 000 $cm^2 \rightarrow m^2$ **b** 0.000 24 $m^2 \rightarrow mm^2$ **c** 5800 $m^2 \rightarrow km^2$

d 258 500 $m^2 \rightarrow$ ha **e** 0.036 $km^2 \rightarrow m^2$ **f** 0.0021 ha $\rightarrow m^2$

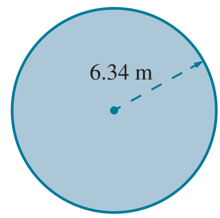
g 6 720 000 $mm^2 \rightarrow m^2$ **h** 56.8 $mm^2 \rightarrow cm^2$

2 Find the area of each of the following.

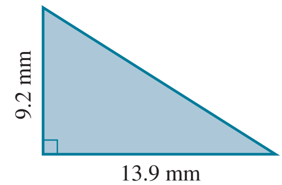
a



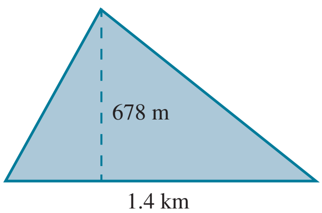
b



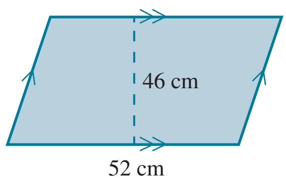
d



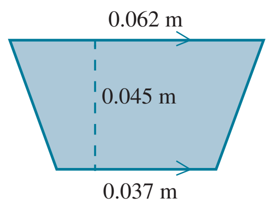
d



e

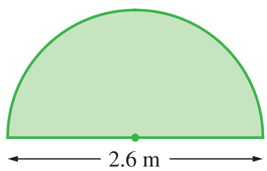


f

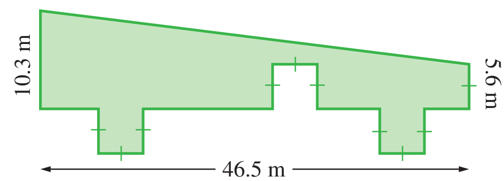


3 Find the area of each of the following.

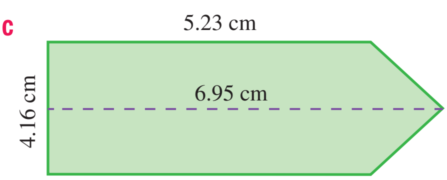
a



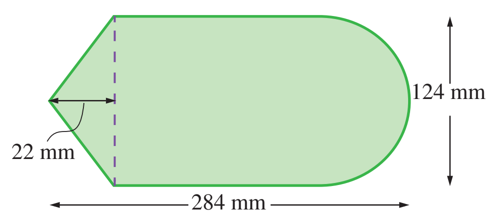
b



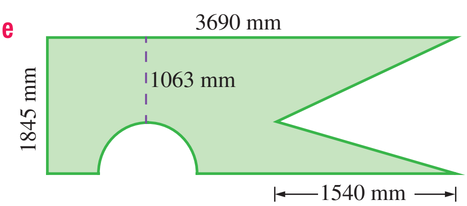
c



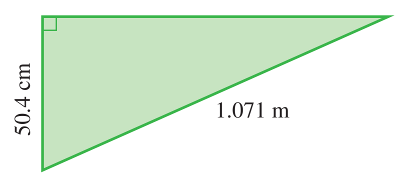
d



e



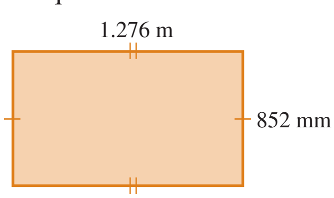
f



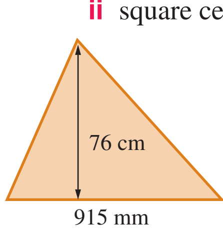
4 Find the areas of the following in:

i square metres **ii** square centimetres.

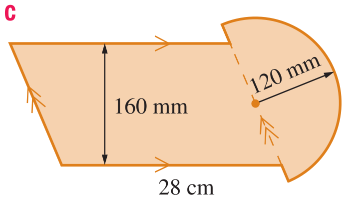
a



b

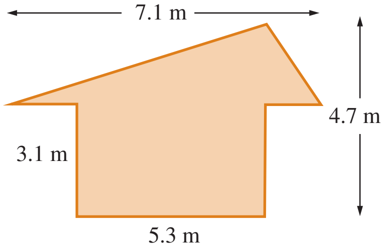


c

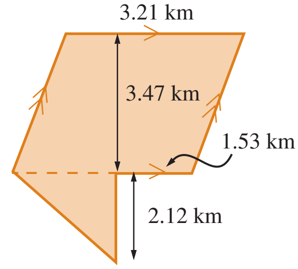


5 Find the areas of the following shapes.

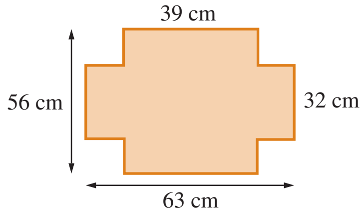
a



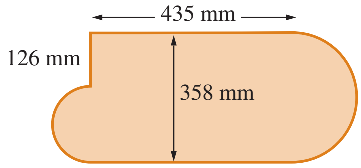
b



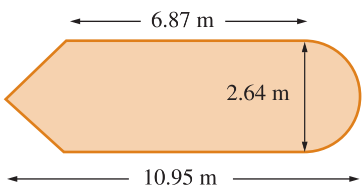
c



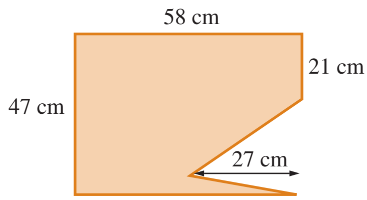
d



e



f



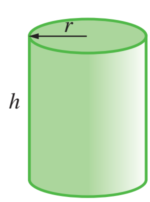
3.3 Areas of 3D shapes

The **surface area** of a 3D shape is the area of the faces of the shape. For shapes with flat faces, the surface area can be calculated simply by adding the areas of the faces. Some shapes have special formulas for their surface areas because they have curved faces.

!

Curved surface areas (CSA)

Cylinder



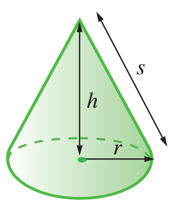
$\mathbf{CSA} = \mathbf{2\pi} \boldsymbol{rh}$

Top $= \pi r^2$

Bottom $= \pi r^2$

Total SA $= 2\pi r^2 + 2\pi rh$

Cone



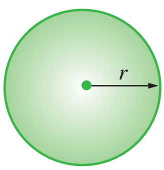
$\mathbf{CSA} = \boldsymbol{\pi rs}$

where $s = \sqrt{r^2 + h^2}$

Bottom $= \pi r^2$

Total SA $= \pi r^2 + \pi rs$

Sphere



$\mathbf{CSA} = \mathbf{4\pi} \boldsymbol{r^2}$

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Example 8

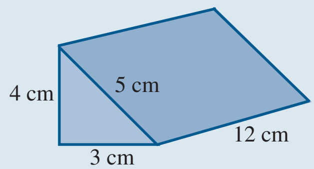
Find the surface areas of:

a a prism with a triangular end 5 cm by 4 cm by 3 cm and length 12 cm

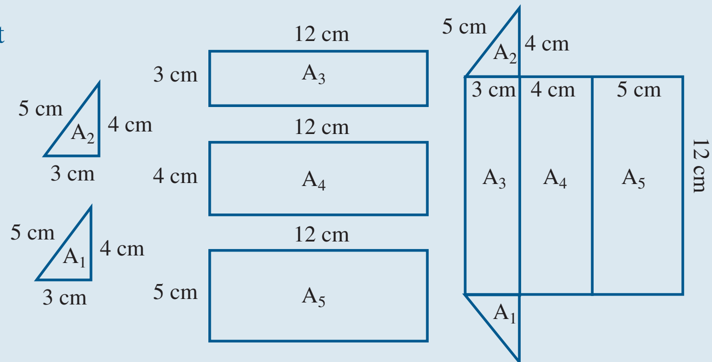
b a cylinder with an open top of diameter 16 cm and height 12 cm.

Solution

a Draw a sketch.
Label the sides.



Then draw each face flat (or draw flat as a net).
In this case, the triangles are right-angled (3–4–5).
Label the areas.



Write formulas.
Calculate areas.

$$A_1 = \tfrac{1}{2} b \times h = 0.5 \times 3 \times 4 \text{ cm}^2 = 6 \text{ cm}^2$$

$$A_2 = \tfrac{1}{2} b \times h = 0.5 \times 3 \times 4 \text{ cm}^2 = 6 \text{ cm}^2$$

$$A_3 = l \times w = 12 \times 3 \text{ cm}^2 = 36 \text{ cm}^2$$

$$A_4 = l \times w = 12 \times 4 \text{ cm}^2 = 48 \text{ cm}^2$$

$$A_5 = l \times w = 12 \times 5 = 60 \text{ cm}^2$$

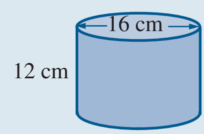
Work out the total.

$$\text{Total area} = A_1 + A_2 + A_3 + A_4 + A_5 = 6 + 6 + 36 + 48 + 60 = 156 \text{ cm}^2$$

Write the answer.

The surface area is 156 cm^2.

b Draw a sketch.



Work out curved surface area.

$$\text{CSA} = 2\pi rh = 2 \times \pi \times 8 \times 12 \text{ cm}^2 = 603.1857\ldots \text{ cm}^2$$

Keep on calculator.
Work out area of base.

$$\text{Area of bottom} = \pi r^2 = \pi \times 8^2 \text{ cm}^2 = 201.0619\ldots \text{ cm}^2$$

Work out the total.
Use figures on calculator.

$$\text{Total area} = 603.1857\ldots + 201.0619\ldots \text{ cm}^2 = 804.2477\ldots \text{ cm}^2$$

Round and write the answer.

The surface area is about 800 cm^2.

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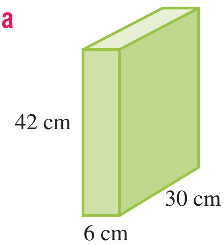
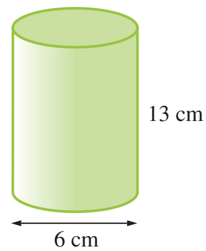
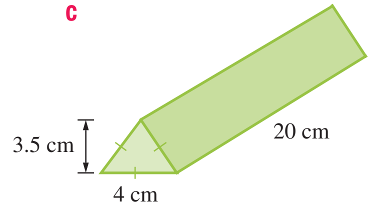
Exercise 3.3 Areas of 3D shapes

Additional Exercise 3.3

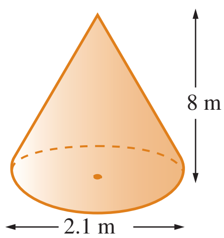
1 Calculate the surface areas of the following shapes.

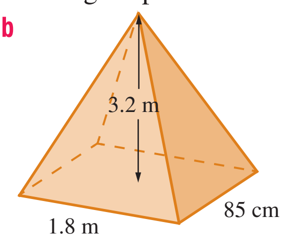
a


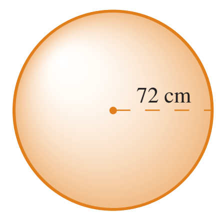
b


c


2 Calculate the surface areas of the following shapes.

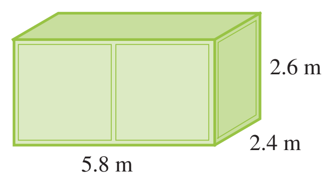
a


b


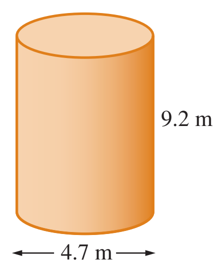
c


Modelling and problem solving

3 This shipping container needs to be painted all over.
   a Find the total area to be painted.
   b Find the cost of painting the container if the paint comes only in 4 L tins that cost \$52, each litre of paint covers 8 m^2, and labour costs are \$7.50/m^2.

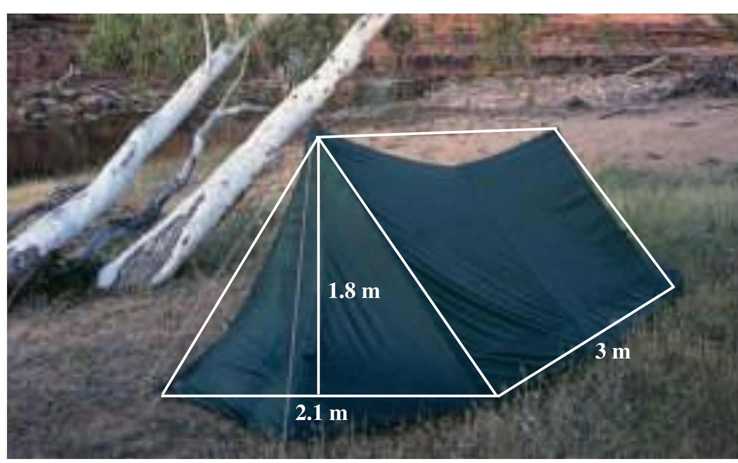


4 This silo is constructed from an aluminium alloy. It has a top and a base. The material costs \$8.50/m^2 and 5% needs to be allowed for wastage. Labour costs for the construction of the silo are \$45.50/h.
   a How much metal is needed?
   b What is the total material and labour cost of constructing the silo if it takes 20 hours?



5 A cereal box is 8 cm wide, 25 cm long and 20 cm high. What area of cardboard is needed to make 5000 boxes, allowing 15% extra for wastage and joins?

6 A tent with a floor is in the shape of a triangular prism 3 m long, 1.8 m high and 2.1 m wide. What area of cloth is needed to make the tent if no allowance is made for window and doorflaps and joins?



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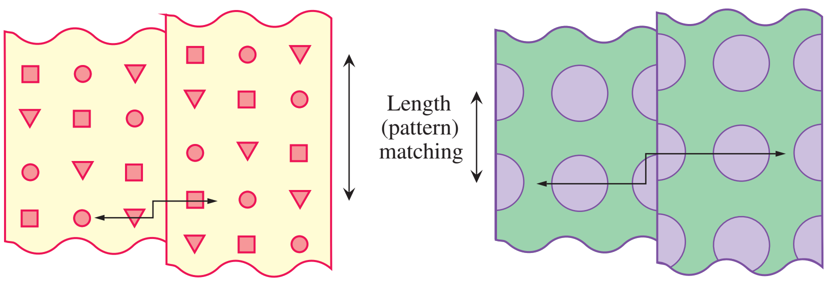
3.4 Applying area calculations

Areas are important in many jobs that need to be done in or around the house. Area is also important in some sports.

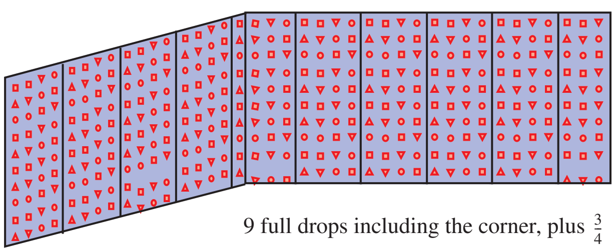
Curtains, wallpaper and some carpets have patterns that must be matched, and they also come only in particular widths. The extra amount that must be allowed for pattern-matching can be as much as the space taken by the pattern of the material. Fabrics are usually sold in widths of 90 cm, 115 cm or 150 cm. Carpet is usually 3.66 m wide and wallpaper rolls are usually 50 cm wide and 10 m long.

Fabric sales are usually rounded *up* to the next 10 cm, carpet *up* to the next 0.1 m on the roll (linear metre) for each 'run', and wallpaper of course *up* to the next full roll.

With patterned materials, allowance must be made for **pattern match** (also called **pattern length**).



When wallpaper is hung, the number of **drops** is worked out first. Each drop is the width of the paper, and the length of the drop is the height of the wall. The drops are cut at the corners and *slightly* overlapped at these corners to keep the paper plumb, so there is almost no wastage at the corners. The total length of the walls being papered is used to work out the number of drops. It is usual to omit doors from the calculation but to include windows to balance the quantities. If two walls were papered, the result could be as shown below.



9 full drops including the corner, plus $\frac{3}{4}$ at the end of one wall.

In the case of carpet, it is usual to keep joins to a minimum.

When laying tiles on a floor or wall, it is usual to work out the number of tiles needed to stretch along the sides and multiply to calculate the number needed. The lengths are always *rounded up* to the nearest half-tile. For ceramic tiles, an allowance must be made for grout. Small tiles (less than 100 mm) normally have a 3 mm joint, but larger tiles have a 5 mm joint.

When estimating the area of walls for painting, openings such as windows and doors are usually ignored. House paint is normally available in 1 L and 4 L tins.

In most practical applications, some allowance should be made for waste caused by accidents or mistakes. This is typically 5%.

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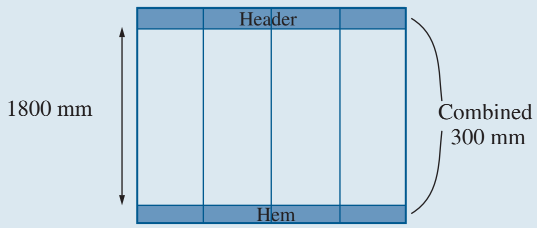
Example 9

A window 1.8 m high is to be curtained with four lengths of fabric. The pattern repeats every 480 mm. There must be a 30 cm allowance for the header and the hem. What length of fabric is needed, ignoring wastage?

Solution

Calculate the drop length.

$$\begin{aligned}\text{Length of drop} &= 1800 + 300 \text{ mm}\\ &= 2100 \text{ mm} = 2.1 \text{ m}\end{aligned}$$



Find the number of patterns in a drop.

$$\begin{aligned}\text{Patterns per drop} &= 2100 \text{ mm} \div 480 \text{ mm}\\ &= 4.375 \text{ patterns}\end{aligned}$$

You will need to allow for 5 patterns in a drop.

$$\begin{aligned}\text{Fabric length per drop} &= 5 \times 480 \text{ mm}\\ &= 2400 \text{ mm}\\ &= 2.4 \text{ m}\end{aligned}$$

There are 4 drops.

$$\begin{aligned}\text{Total fabric length} &= 4 \times 2.4 \text{ m}\\ &= 9.6 \text{ m}\end{aligned}$$

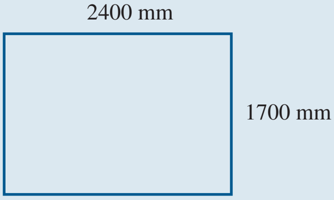
It is not necessary to allow for wastage when using this method to calculate the length of fabric for curtains.

Example 10

Calculate the number of 200 mm × 200 mm tiles needed to tile the floor of a laundry measuring 2400 mm by 1700 mm.

Solution

Draw a floor plan.



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Work out the size, with 5 mm of grout.	Tile width $= 200 + 5$ mm $= 205$ mm
Work out number for the length.	Number for 2400 mm wall $= 2400 \div 205$ $= 11.7073\ldots$
More than a half, so round up.	≈ 12 tiles
Work out number for the width.	Number for 1700 mm wall $= 1700 \div 205$ $= 8.2926\ldots$
Less than a half, so make $\frac{1}{2}$.	$\approx 8\frac{1}{2}$ tiles
Multiply to get total.	Number of tiles $= 12 \times 8.5$ $= 102$ tiles
Add 5% wastage.	Total with wastage $= 105\%$ of 102 tiles $= 1.05 \times 102$ ≈ 107 tiles
Write the answer.	107 tiles are needed for the laundry.

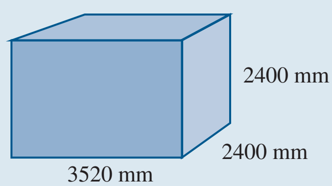
Example 11

a How many litres of paint are needed to give the walls of a bedroom 2400 mm by 3520 mm two coats of paint covering 14 m^2/L, assuming the ceiling height is 2400 mm?

b How much ceiling white is needed at the same coverage?

Solution

Draw a sketch, ignoring doors and windows.



a	Calculate total wall length in metres.	Wall length $= 2 \times 2.4 + 2 \times 3.52 = 11.84$ m
	Calculate wall area.	Wall area $= 11.84 \times 2.4 = 28.416\ m^2$
	Allow for 2 coats.	Area to paint $= 2 \times 28.416 = 56.832\ m^2$
	Calculate quantity of paint.	Paint needed $= 56.832 \div 14$ L $= 4.0594 \ldots$ L
	Add 5% wastage.	Wastage $= 5\%$ of $4.0594\ldots$ L $= 0.2029\ldots$ L Total needed $= 4.2624$ L
	Write the answer.	A 4 L tin and a 1 L tin are needed.
b	Calculate ceiling area.	Ceiling area $= 2.4 \times 3.52\ m^2$ $= 8.448\ m^2$
	Allow for 2 coats.	Area to paint $= 2 \times 8.448 = 16.896\ m^2$
	Calculate quantity of paint.	Paint needed $= 16.896 \div 14$ L $= 1.2068\ldots$ L
	Write the answer, and explain wastage.	Two 1 L tins of ceiling white are needed, leaving about 0.8 L for wastage.

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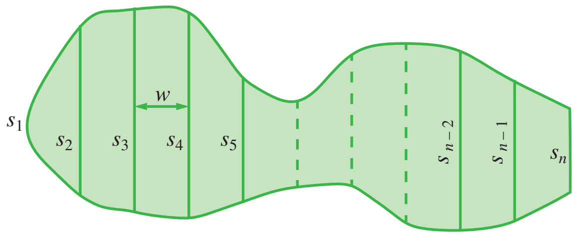
The area of an irregular shape is sometimes needed in practical applications. In this case, a method of approximation can be used.

Trapezoidal rule

The area of an irregular shape can be *approximated* by dividing the shape into strips of equal width, as shown below.



The following rule comes from the rule for the area of a trapezium, $A = \frac{1}{2}(a + b) \times h$.

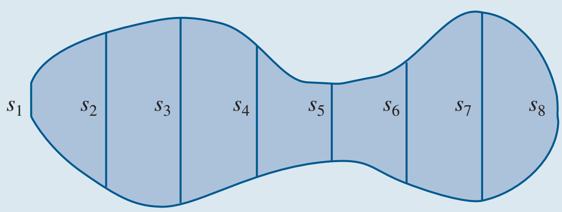
$$\text{Area} \approx \frac{w}{2}(s_1 + 2s_2 + 2s_3 + \ldots + 2s_{n-1} + s_n)$$

$$= \frac{\text{width}}{2} \times (\text{total of end lengths} + 2 \times \text{total of middle lengths})$$

$$= \text{width} \times (\text{total of end lengths} + 2 \times \text{total of middle lengths}) \div 2$$

Example 12

Find the area of the garden bed shown below, which is divided into strips 5 m wide, with the lengths given in the table.



Strip	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8
Length (m)	0.2	10.1	12.2	8.6	5.0	7.9	12.4	2.1

Solution

Find total of end lengths.

$$\text{End lengths} = 0.2 + 2.1 \text{ m}$$
$$= 2.3 \text{ m}$$

Find total of middle lengths.

$$\text{Middle lengths} = 10.1 + 12.2 + 8.6 + 5.0 + 7.9 + 12.4 \text{ m}$$
$$= 56.2 \text{ m}$$

Write the formula.

$$\text{Area} = \text{width} \times (\text{end lengths} + 2 \times \text{middle lengths}) \div 2$$

Work out area.

$$= 5 \times (2.3 + 2 \times 56.2) \div 2 \text{ m}^2$$
$$= 5 \times 114.7 \div 2 \text{ m}^2$$
$$= 286.75 \text{ m}^2$$

Round and write the answer.

The area of the garden bed is about 287 m^2.

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Example 13

For the 'Sandalwood' house design on the opposite page, find the cost of carpeting the Family and Meals areas and the 1.8 m-wide passageway between the Alfresco area and the Kitchen. The carpet costs $215/m, does not have a pattern and costs $10.40/m^2 to lay.

Solution

Carpet is 3.66 m wide. The most economical way to lay the carpet is across the house, with one join across the Family and Meals areas and another across the passageway near the entrance to the Kitchen.

Calculate the length of carpet.	Length of carpet = 6.3 + 6.3 m = 12.6 m
Find the cost of carpet.	Cost of carpet = 12.6 × \$215 = \$2709
Find the area of carpet.	Area of carpet = 6.3 × 5.1 + 1.8 × 4 m^2 = 39.33 m^2
Find the laying cost.	Laying cost = \$10.4 × 39.33 = \$409.03
Find the total cost.	Total cost = \$2709 + \$409.03 = \$3118.03

Additional Exercise 3.4

Exercise 3.4 Applying area calculations

Modelling and problem solving

Questions **1** to **5** refer to the 'Sandalwood' house design on the opposite page.

1 The ensuite (ENS) floor is to be tiled with ceramic tiles 230 mm square. Allow for 5 mm grout and wastage of 5%. The tiles cost $4.80 each, the glue costs $40/L and the grout costs $44.50 a bag. A litre of glue will cover 2 m^2, and a bag of grout is enough for 5 m^2 of tiles.

a How many tiles are needed?
b How much glue is needed?
c How much grout is needed?
d What is the total cost?

2 The four bedrooms are to be carpeted with carpet costing $85/m^2, including the cost of laying. The floors of the wardrobes (WIR and R) are not to be carpeted. What will the carpeting cost?

3 Assuming the walls are 2400 mm high, how much paint is needed to do two coats on the bedroom walls, if the paint covers 12 m^2/L?

4 Estimate how much ceiling white would be needed for two coats of the whole house (not including the garage or the Alfresco area), if coverage is 10 m^2/L.

5 Assuming that the bath (850 mm wide) is not tiled, how many 100 mm square tiles with 3 mm grout are needed for the bathroom and toilet floors?

Questions **6** to **10** refer to the 'Cypress' house design on page 86.

6 Four drops of curtain material are to be used on the sliding windows (SW) and two drops on the fixed windows (FG). The windows are 1200 mm high and a 30 cm allowance is needed for headers and hems. The pattern spacing for the bedroom curtains is 600 mm. How much material is needed for the bedroom windows?

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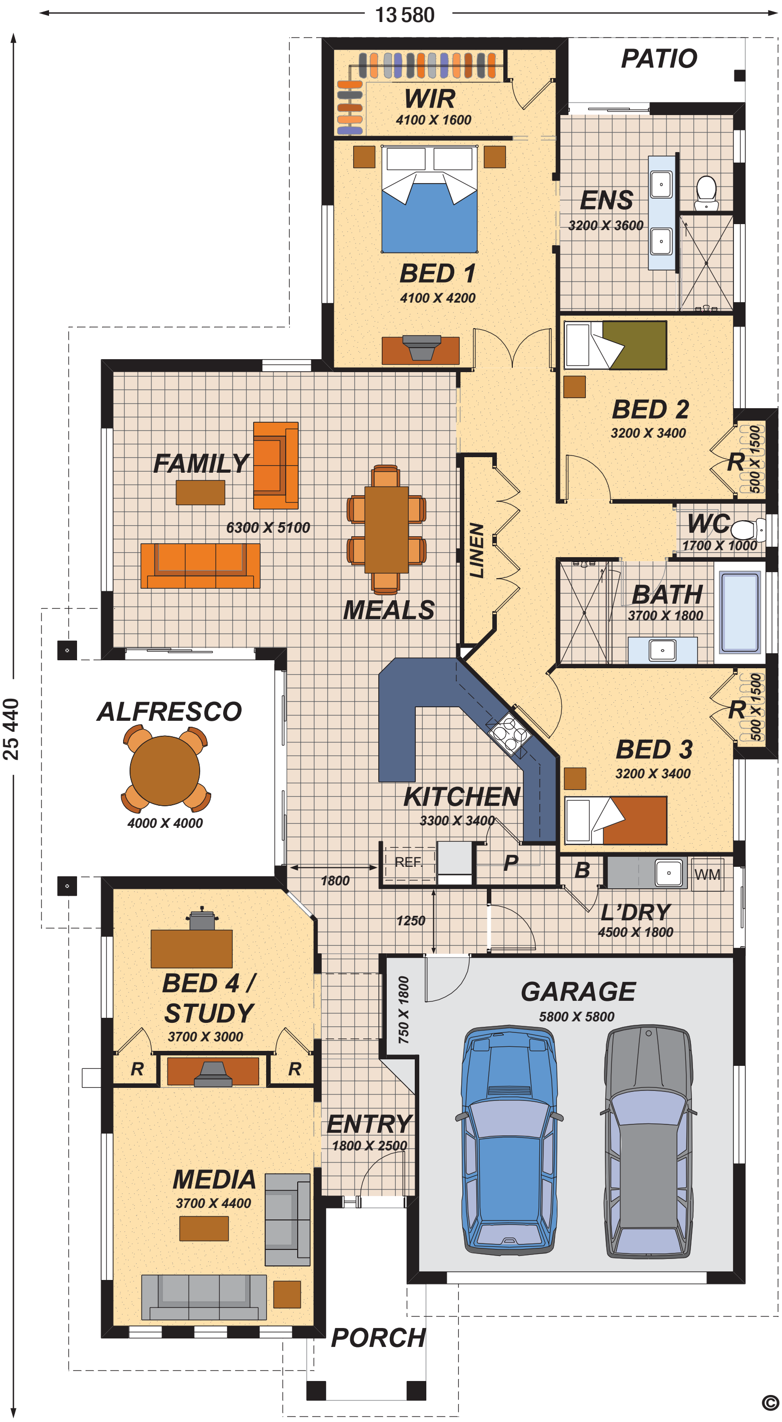
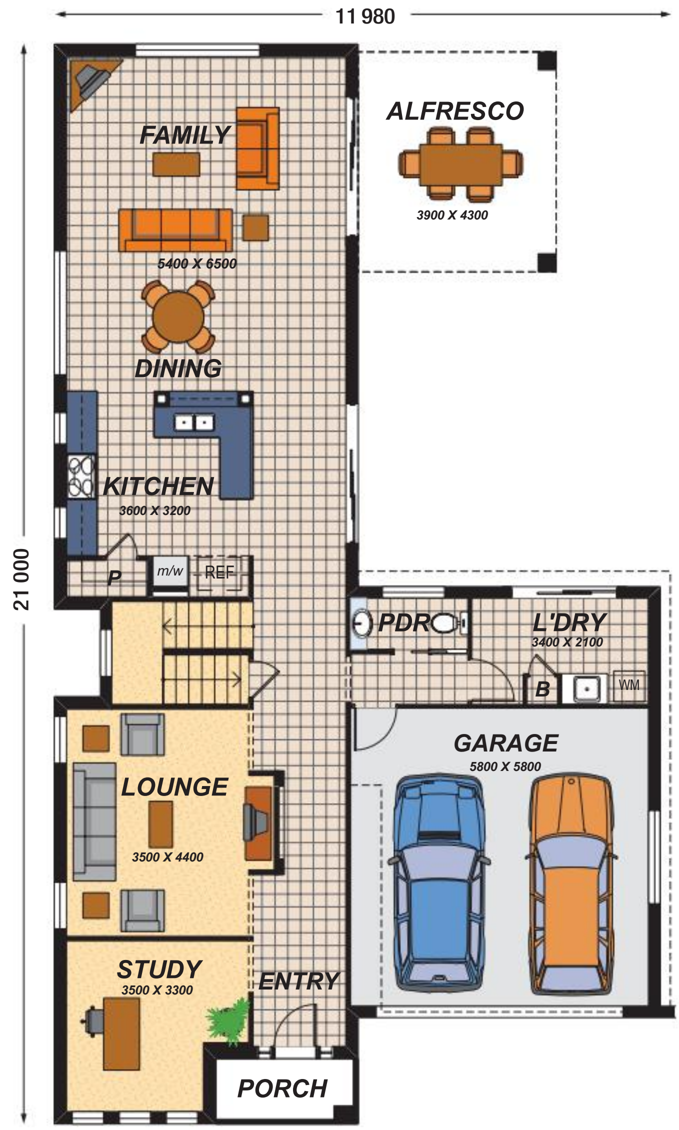
Sandalwood


13 580
25 440
PATIO
WIR
4100 X 1600
ENS
3200 X 3600
BED 1
4100 X 4200
BED 2
3200 X 3400
R
500 X 1500
WC
1700 X 1000
FAMILY
6300 X 5100
MEALS
LINEN
BATH
3700 X 1800
ALFRESCO
4000 X 4000
KITCHEN
3300 X 3400
BED 3
3200 X 3400
R
500 X 1500
REF.
P
B
WM
1800
1250
L'DRY
4500 X 1800
BED 4 /
STUDY
3700 X 3000
R
R
GARAGE
5800 X 5800
750 X 1800
ENTRY
1800 X 2500
MEDIA
3700 X 4400
PORCH

Upper floor

Cypress



Ground floor

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7 The laundry is to be tiled with 200 mm tiles with 5 mm grout.

a How many tiles are needed?

b Estimate the cost if the tiles cost $440 for 100 tiles.

8 The Family, Dining, Lounge and Study are to be carpeted with carpet costing $325/m. The laying cost is $10/m². Use some drawings with strips marked on them to find how much carpet is needed and what the cost is if the carpet is laid:

a front to back

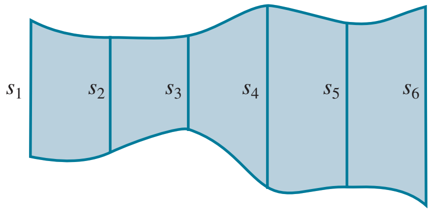
b side to side.

9 Estimate the cost of painting the floor of the garage with anti-slip paving paint (three coats) if it costs $118 for a 10 L bucket and the coverage is 9 m²/L.

10 What is the cost of two coats of paint for the walls (2400 mm high) of bedrooms 2, 3 and 4 if the coverage is 12 m²/L and the paint comes in 1 L tins costing $28.40 and 4 L tins costing $68.90?

11 A large garden bed is to be covered with weed mat to stop weeds growing. The gardener took the following measurements of 6 m-wide strips. What area of weed mat is needed?

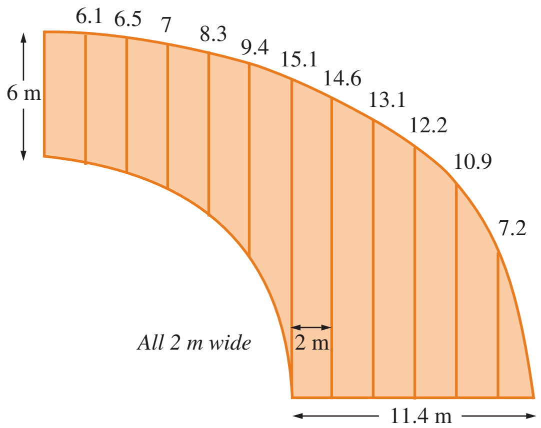
Strip	s_1	s_2	s_3	s_4	s_5	s_6
Length (m)	10.4	8.7	7.1	13.8	12.4	15.2



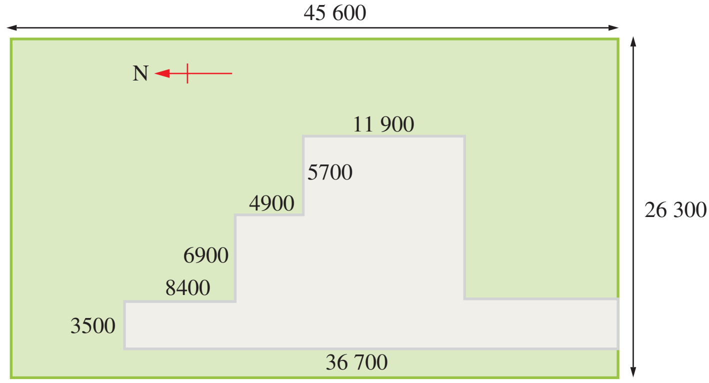
12 The plan of a sweeping driveway is shown.

a Use the trapezoidal rule to find the area of the driveway.

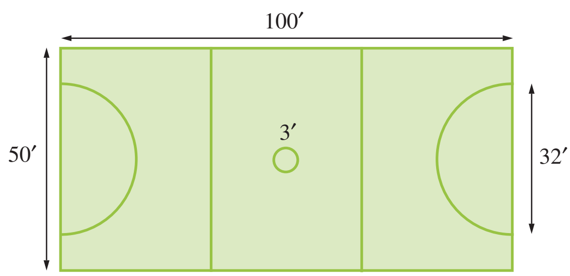
b If concreting costs $30/m², find the cost of having the driveway concreted.



13 If A-grade turf costs $6.00/m^2, find the cost of turfing the yard shown below, assuming that the green-shaded area is to be turfed.

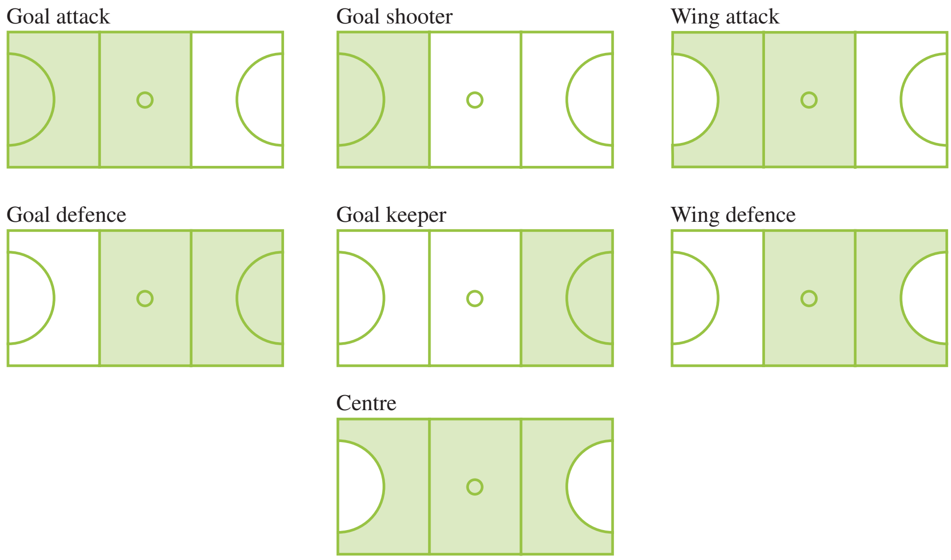


14 A netball court is shown here, with measurements in feet. The court is divided into three congruent sections, which are referred to as 'thirds'.



a Convert the dimensions to metric measurements using 1 foot (1′) ≈ 0.3048 m.

b Each player may enter only certain areas, as shown below. Calculate the area that each player may enter and express it as a percentage of the whole court area.



15 A netball is 230 mm in diameter and the hoop is 380 mm in diameter. What percentage of the hoop is occupied when a goal is scored?

3.5 Calculating volumes

Volume measures 3-dimensional space. It is calculated as the number of cubes that will fit into the space, so is given in cubic units. **Capacity** is another term for volume, but is usually used for volumes of containers for liquids or gases. The common units and conversions are shown below.

! **Volume and capacity units**

Unit	*Abbreviation*	*Examples*
Cubic centimetre	cm^3	Volumes of small objects
Cubic metre	m^3	Volumes of sand, gravel, soil
Millilitre	mL	Capacities of spoons, glasses, small cans
Litre	L	Capacities of large containers, drums, tanks
Kilolitre	kL	Capacities of swimming pools, farm dams
Megalitre	ML	Capacities of reservoirs

Conversions

$1\ m^3 = 1\,000\,000\ cm^3$ $\quad$ $1\ L = 1000\ mL$ $\quad$ $1\ ML = 1\,000\,000\ L = 1000\ m^3$

$1\ m^3 = 1\ kL = 1000\ L$ $\quad$ $1\ mL = 1\ cm^3$

Example 14

Convert:

a $120\,000\ cm^3$ to m^3 $\quad$ **b** $2.34\ m^3$ to L.

Solution

a Smaller unit → larger unit, so divide.

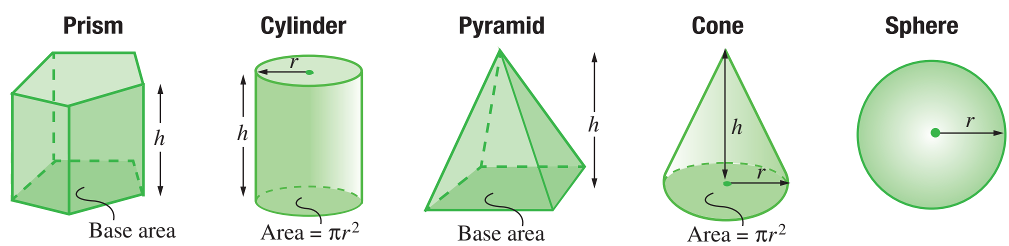
$$120\,000\ cm^3 = 120\,000 \div 1\,000\,000\ m^3$$
$$= 0.12\ m^3$$

b Larger unit → smaller unit, so multiply.

$$2.34\ m^3 = 2.34 \times 1000\ L$$
$$= 2340\ L$$

Many objects encountered in everyday life can be considered as prisms or pyramids, so their volumes may be calculated using the basic formulas for these objects. A true **prism** has a constant polygonal cross-sectional area known as the **base**, while a true **pyramid** comes to a point from the polygonal base. The most common formulas are shown below.

! **Volumes of common 3D shapes**



$V = \text{base area} \times h$ $\quad$ $V = \pi r^2 h$ $\quad$ $V = \frac{1}{3} \times \text{base area} \times h$ $\quad$ $V = \frac{1}{3}\pi r^2 h$ $\quad$ $V = \frac{4}{3}\pi r^3$

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Example 15

Find the volume of the can of pineapple shown at right.

Solution

A cylinder is like a prism. $V = A \times h$

The base is a circle. $= \pi r^2 \times h$

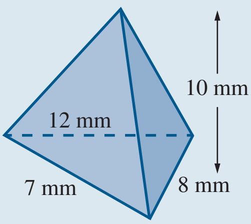
Radius is 4.2 cm. $= \pi \times 4.2^2 \times 8.6\ \text{cm}^3$

$= 476.5921\ \ldots\ \text{cm}^3$

Round and write the answer. The volume is about 477 cm^3.

Example 16

Find the volume of this triangular pyramid.



Solution

Find the area of the base using Heron's Formula.

Find s. $s = \frac{1}{2}(a + b + c)$

$= \frac{1}{2}(12 + 8 + 7)$ mm

$= 13.5$ mm

Find the area. $A = \sqrt{s \times (s - a) \times (s - b) \times (s - c)}$

$= \sqrt{13.5 \times 1.5 \times 5.5 \times 6.5}\ \text{mm}^2$

$= \sqrt{723.9375}\ \text{mm}^2$

Keep on calculator. $= 26.9060\ \ldots\ \text{mm}^2$

Now find volume of pyramid. $V = \frac{1}{3} A \times h$

Write $\frac{1}{3}$ as $\div 3$. $= 26.9060\ \ldots \times 10 \div 3\ \text{mm}^3$

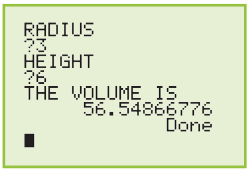
$= 89.6869\ \ldots\ \text{mm}^3$

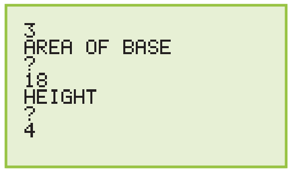
Round and write the answer. The volume is about 90 mm^3.

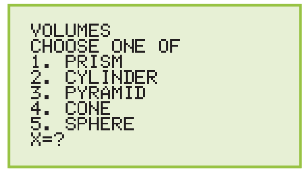
Technology

Calculator Program

The program CVOL can be used to find the volumes of common 3D shapes. The program is given in full on the CD-ROM. Enter the program (or load it from the CD-ROM) and try it with different shapes.

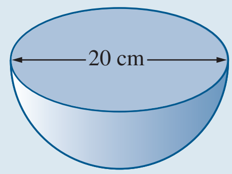






Example 17

Find the capacity of this hemispherical bowl.



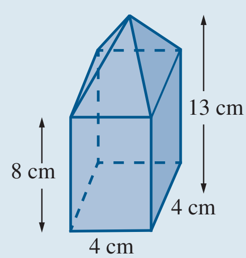
Solution

Find volume of sphere.	$V = \frac{4}{3}\pi r^3$
Radius is 10 cm.	$= \frac{4}{3} \times \pi \times 10^3\ \text{cm}^3$
Keep on calculator.	$= 4188.7902 \ldots\ \text{cm}^3$
Divide by 2 for hemisphere.	Volume of bowl $= 4188.7902 \ldots \div 2\ \text{cm}^3$
	$= 2094.3951 \ldots\ \text{cm}^3$
Convert to appropriate units.	$= 2.0943 \ldots$ L
Round and write the answer.	The bowl has a capacity of about 2.1 L.

Volumes of some 3D shapes may be worked out as combinations of prisms, pyramids or spheres.

Example 18

Calculate the volume of this 3D shape.



Solution

The shape can be considered as a prism with a pyramid on top.

Calculate prism volume.	Area of base $= 4 \times 4\ \text{cm}^2$
	$= 16\ \text{cm}^2$
	Volume of prism $= A \times h$
	$= 16 \times 8\ \text{cm}^3 = 128\ \text{cm}^3$

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Calculate pyramid volume.

$$\text{Area of base} = 16\text{ cm}^2$$
$$\text{Height of pyramid} = 13 - 8\text{ cm}$$
$$= 5\text{ cm}$$
$$\text{Volume of pyramid} = \frac{1}{3} \times A \times h$$
$$= 16 \times 5 \div 3\text{ cm}^3$$
$$= 26.6666\ldots\text{ cm}^3$$

Add to find combined volume.

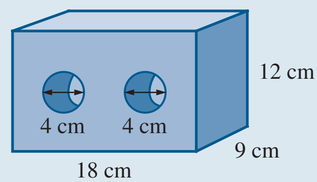
$$\text{Volume of shape} = 128 + 26.6666\ldots\text{ cm}^3$$
$$= 154.6666\ldots\text{ cm}^3$$

Round and write the answer.

The volume of the 3D shape is about 155 cm^3.

Example 19

Find the volume of clay needed to make this brick.



Solution

The shape is a rectangular prism with two cylinders taken out.

Find volume of the prism.

$$V = A \times h$$
$$= 18 \times 9 \times 12\text{ cm}^3$$
$$= 1944\text{ cm}^3$$

Find volume of a cylinder.
Radius is 2 cm.

$$V = \pi r^2 h$$
$$= \pi \times 2^2 \times 9\text{ cm}^3$$
$$= 113.0973\ldots\text{ cm}^3$$

Subtract to find combined volume.

$$\text{Volume of clay} = 1944 - 2 \times 113.0973\ldots\text{ cm}^3$$
$$= 1717.8053\ldots\text{ cm}^3$$

Round and write the answer.

The volume of clay needed is about 1720 cm^3.

Additional Exercise 3.5

Exercise 3.5 Calculating volumes

1 What would be the most appropriate unit for the volume of each of these objects?
- a a can of softdrink
- b a tablespoon
- c a shipping container
- d Wivenhoe dam
- e an engine's cylinders
- f a laundry tub
- g a dessertspoon
- h a bath
- i a house
- j a car tyre

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2 Convert to the units indicated.

a $26 \text{ m}^3 \rightarrow \text{cm}^3$ **b** $320\,000 \text{ cm}^3 \rightarrow \text{m}^3$

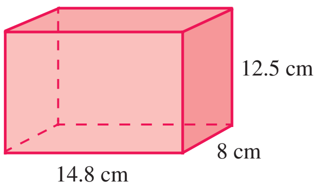
c $2.8 \times 10^{11} \text{ cm}^3 \rightarrow \text{ML}$ **d** $800 \text{ mL} \rightarrow \text{L}$

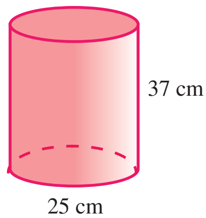
e $56 \text{ kL} \rightarrow \text{m}^3$

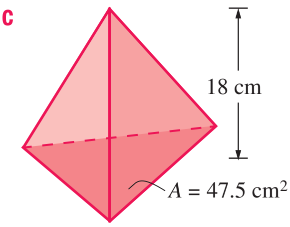
3 What is the volume of a 2 L bottle of milk in:

a cubic centimetres? **b** millilitres? **c** cubic metres?

4 Find the volumes of the shapes below.

a


b


c


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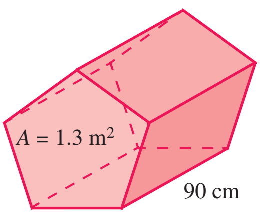

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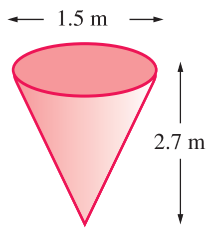

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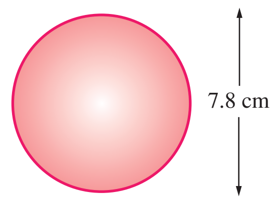

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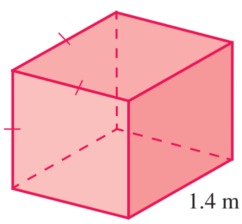

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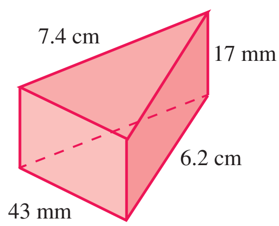

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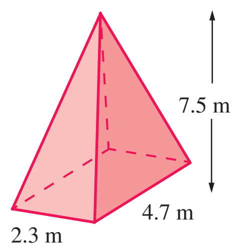
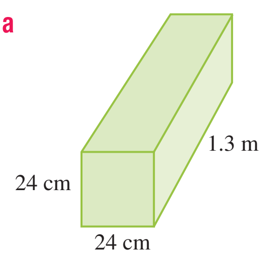


5 Find the volumes of the shapes below.

a


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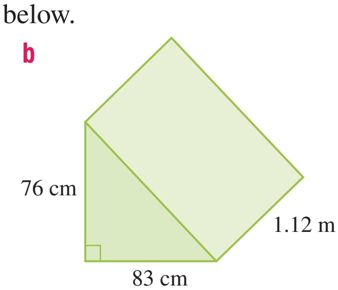

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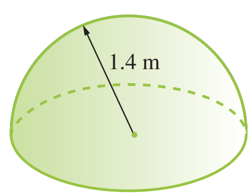

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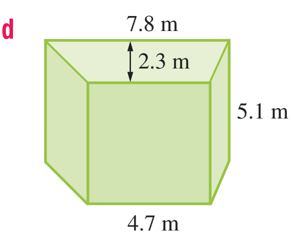

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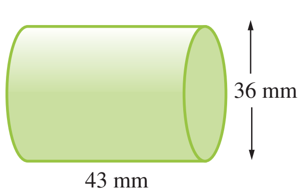

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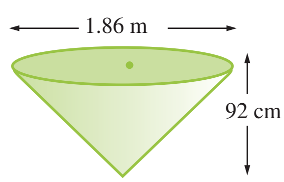
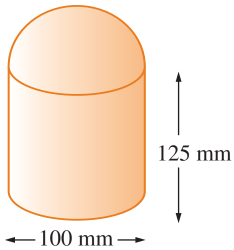
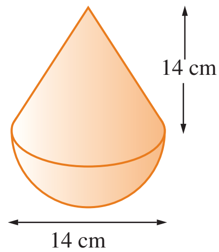


6 Find the volumes of the following shapes.

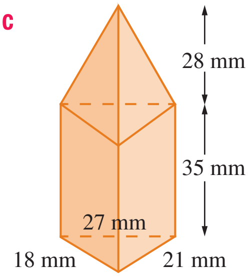
a



b

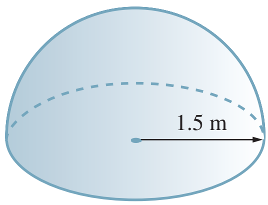


c

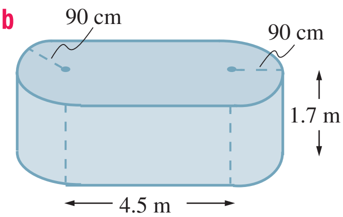


7 Find the volumes of the following shapes.

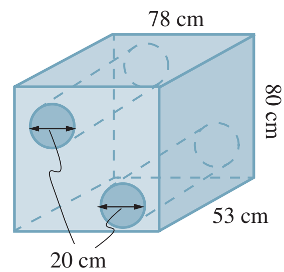
a



b



c



Modelling and problem solving

8 A house brick measures 230 mm × 110 mm × 76 mm. What is its volume in:

a cubic millimetres?

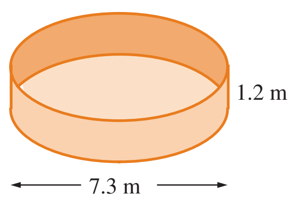
b cubic centimetres?

9 Some swimming pools are drawn below. For each pool, find:

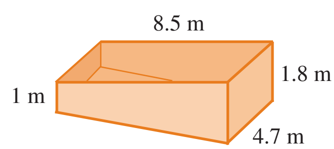
i the capacity in litres

ii the time taken to fill the pool at a rate of 8 L/minute.

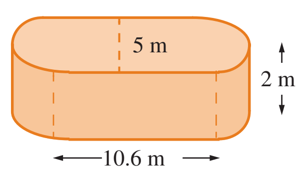
a



b



c



10 An engineering company wants to build a truck body that will hold 18 m^3 of material. The truck body will be in the shape of a rectangular prism. If the dimensions of the base are 1.8 m by 4.8 m, how high will the sides need to be made?

3.6 Applying volume calculations

Many households in Australia rely on the water that they collect themselves from rainwater. To calculate the volume of water that may be collected in a storage tank, we need to know the rainfall and the area covered by the roof for collection.

Example 20

A farmer is considering boosting his water supply by collecting water from the roof of the machinery shed. The shed is 8 m long and 12 m deep, and the annual rainfall in the area varies from 200 mm to 700 mm, with about 80% of the rain falling in the period from December to February. How big would the storage tank need to be to hold all the water collected?

Solution

Calculate the area covered by the roof.

$$\begin{aligned}\text{Area} &= 8 \times 12 \text{ m}^2 \\ &= 96 \text{ m}^2\end{aligned}$$

The maximum amount of rain that will need to be held in the storage tank is the amount that falls in the December–February period of a good year. For calculation purposes, we can consider this as all being on the roof at once and then flowing into the tank.

Calculate the maximum depth, if all the water was on the roof.

$$\begin{aligned}\text{Depth} &= 80\% \text{ of } 700 \text{ mm} \\ &= 0.80 \times 700 \text{ mm} \\ &= 560 \text{ mm} = 0.56 \text{ m}\end{aligned}$$

Calculate the volume to be held.

$$\begin{aligned}\text{Volume} &= 96 \times 0.56 \text{ m}^3 \\ &= 53.76 \text{ m}^3\end{aligned}$$

Convert to appropriate units.

$$= 53\,760 \text{ L}$$

Round and write the answer.

The farmer would need a 50 000 L tank to store all the water from December to February in a good year.

Investigation Storage tanks

Families who rely on rainwater for their daily water needs must be able to determine the size of storage tank that will be adequate to service their needs. Three main questions need to be answered:

- How much water will the family use?
- How much water can be collected from the roof of the house?
- What will the dimensions of the storage tank be?

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Investigation continued

The data in the following table outlines the amounts of water used in a typical household.

Purpose	Water used
Toilet	13 L (6.5 L for a half-flush)
Bath	50 to 120 L (half-full)
Shower	40 to 250 L for an average 8-minute shower
Dishwashing by hand	18 L per wash
Dishwasher	23 to 60 L per wash
Clothes washing	73 to 265 L per load
Garbage disposal unit	30 L per day
Handbasin (washing hands)	5 L
Drinking, cooking and household cleaning	8 L per day per person (average)
Tap running while cleaning teeth	5 L
Garden sprinkler	1500 L per hour
Car washing with a hose	100 to 300 L

1 Discuss the water usage figures in the table above. Do you think that they are reasonable?

2 As a result of your discussions, decide on the amount of water that a family of two adults and two children would use in a week, a month and a year.

The average monthly rainfall for Brisbane is as follows.

Month	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Rainfall (mm)	164	174	143	94	87	76	67	44	32	95	96	127

The rainwater that falls on the roof is collected and run into a storage tank. To simplify calculations, the floor area of the house may be assumed to be the catchment area — the area on which water falls.

3 Use the data here to calculate the average volume of water that would be collected from a 100 m^2 home in Brisbane each month. (Remember, 1 m^3 = 1000 L.)

4 Compare the monthly rainwater collection figures with the water consumption figures that you calculated previously.

5 Most rainwater tanks are cylindrical in shape. Use this fact and your previous calculations to determine the dimensions of a rainwater tank that would be sufficient for the needs of a family of four living in a 100 m^2 house in the Brisbane area.

In some cases, a method of approximation is needed to calculate volumes. For example, the removal of earth in a road cutting requires an estimation of the amount of material to be removed. Estimation is also needed when calculating the volume of a ventilation duct that is circular at one end and rectangular at the other. There are two methods for this situation, and both are like the trapezoidal rule for the calculation of areas.

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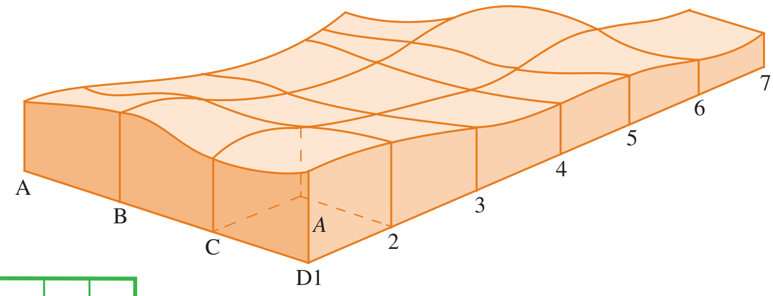
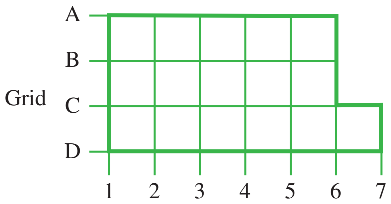
Approximation methods for volume calculation

Grid heights method

The heights (or depths) at the intersections of a grid such as that shown below are used to calculate the volume.





Some intersections are at the corners of 4 blocks, some are at 3 blocks, some are at 2 blocks and some are at only 1 block. In the above, these are:

Corners of 1 block: A1, A6, C7, D1, D7

Corners of 2 blocks: A2, A3, A4, A5, B1, B6, C1, D2, D3, D4, D5, D6

Corners of 3 blocks: C6

Corners of 4 blocks: B2, B3, B4, B5, C2, C3, C4, C5

The approximate volume is given by

$$\text{Volume} \approx \frac{1}{4} \times A \times (H_1 + 2H_2 + 3H_3 + 4H_4)$$

where H_1 = total of heights at corners of 1 block

H_2 = total of heights at corners of 2 blocks

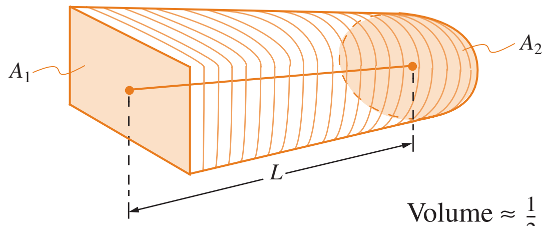
H_3 = total of heights at corners of 3 blocks

H_4 = total of heights at corners of 4 blocks

A = area of grid square.

Average end areas method

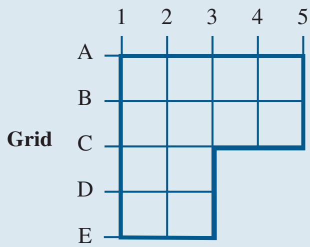
A section that gradually changes from one shape to another can have its volume estimated from the average of the end areas, provided the difference is not too large.



$$\text{Volume} \approx \frac{1}{2} \times (A_1 + A_2) \times L$$

Example 21

A grid of 20 m × 20 m squares is placed over a site for a new factory. The depth of soil and rock to be removed below each grid point is shown in the following table.



Depth of soil (m)

	1	2	3	4	5
A	4.575	4.002	3.917	3.571	3.000
B	5.001	4.597	3.718	3.200	3.199
C	5.213	4.777	2.517	2.818	3.222
D	4.876	4.213	3.000		
E	4.213	3.917	3.517		

Work out:

a the volume of material to be removed

b the volume of material to be trucked away if, when it is loosened with earth-moving equipment, its volume increases by 15%

c the number of truckloads required to remove the material if each semitrailer tipper can carry 18 m^3.

Solution

a Add depths at corners of 1 block (A1, A5, C5, E1, E3).

$H_1 = 4.575 + 3.000 + 3.222 + 4.213 + 3.517$ m
$= 18.527$ m

Add depths at corners of 2 blocks (A2, A3, A4, B1, B5, C1, C4, D1, D3, E2).

$H_2 = 4.002 + 3.917 + 3.571 + 5.001 + 3.199 + 5.213 + 2.818 + 4.876 + 3.000 + 3.917$ m
$= 39.514$ m

Depth at corner of 3 blocks (C3).

$H_3 = 2.517$ m

Add depths at corners of 4 blocks (B2, B3, B4, C2, D2).

$H_4 = 4.597 + 3.718 + 3.200 + 4.777 + 4.213$ m
$= 20.505$ m

Work out area of grid square.

$A = 20 \times 20$ m^2
$= 400$ m^2

Use formula for volume.

$\text{Volume} \approx \frac{1}{4} \times A \times (H_1 + 2H_2 + 3H_3 + 4H_4)$
$\approx 400 \times (18.527 + 2 \times 39.514 + 3 \times 2.517 + 4 \times 20.505) \div 4$ m^3
$\approx 18\,712.6$ m^3

Round and write the answer.

The volume to be removed is about 18 710 m^3.

b 15% extra makes 115%.

Volume after loosening
= 115% of volume to be removed

Use calculator accuracy.

$= 1.15 \times 18\,712.6$ m^3
$= 21\,519.49$ m^3

Round and write the answer.

Volume to be trucked away is about 21 520 m^3.

c Divide by truck volume.

Number of truckloads $= 21\,519.49 \div 18$
$= 1195.5272 \ldots$ loads

Round up and write the answer.

About 1196 truckloads are required.

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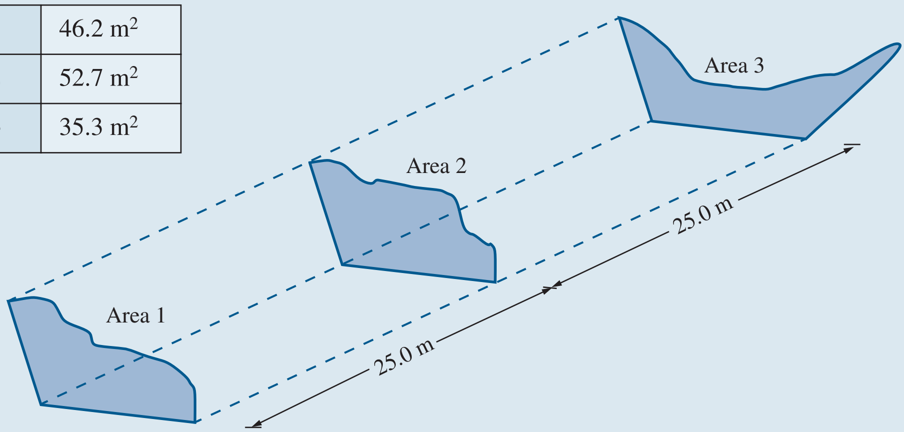
Example 22

A segment of road cutting is shown below. The separation between each area segment is 25.0 m. How much material must be removed to make the road?

Area 1	46.2 m^2
Area 2	52.7 m^2
Area 3	35.3 m^2



Solution

Find volume for the first section.

$$V_1 \approx \frac{1}{2} \times (A_1 + A_2) \times L$$
$$\approx (46.2 + 52.7) \times 25 \div 2 \text{ m}^3$$
$$\approx 1236.25 \text{ m}^3$$

Find volume for the second section.

$$V_2 \approx \frac{1}{2} \times (A_2 + A_3) \times L$$
$$\approx (52.7 + 35.3) \times 25 \div 2 \text{ m}^3$$
$$\approx 1100 \text{ m}^3$$

Add for total volume.

$$\text{Total} \approx 1236.25 + 1100 \text{ m}^3$$
$$\approx 2336.25 \text{ m}^3$$

Round up and write the answer.

The volume to be removed is about 2340 m^3.

Exercise 3.6 Applying volume calculations

Additional Exercise 3.6

Modelling and problem solving

1 Use the information listed on page 96 to estimate the weekly water consumption of a household with:

a two adults and an infant

b two adults and three children who attend high school

c five adults—two of whom work from home.

2 If 20 mm of rain falls on a catchment area of 80 m^2, how high would the level rise in a cylindrical rainwater tank with diameter 3 m?

3 A house has a roof with an effective catchment area of 140 m^2 and it is built in an area where the rainfall never exceeds 100 mm per month. A cylindrical rainwater tank is to be made with a diameter of 3.5 m. How high should it be made so that it will hold rainfall for any month, assuming that none is consumed?

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4 Below is data concerning a family living in an area where rain is the only source of water.

Month	Rainwater collected (L)	Water consumed (L)	Monthly surplus (+) or deficit (–) (L)	Cumulative surplus or deficit (L)
Jan.	12 120	10 700	+1420	+1420
Feb.	9 460	11 400	–1940	–520
Mar.	7 080	10 300	–3220	
Apr.	8 350	9 800		
May	7 130	10 600		
Jun.	10 240	8 700		
Jul.	11 350	8 900		
Aug.	9 900	10 800		
Sep.	8 960	10 400		
Oct.	10 730	10 200		
Nov.	13 450	10 800		
Dec.	14 760	12 200		
Totals				

a Complete the table.

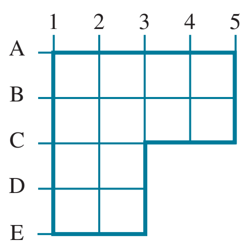
b How much water would be needed in the rainwater tank at the start of the year to avoid running out during the year?

5 An underground concrete tank is a rectangular prism 20 m long, 5 m wide and 3 m deep. It takes the run-off from the farm buildings of a large dairy. The roofs have a total area of 450 m^2. At the end of a long dry spell the tank is only 12% full. How much rain is needed to fill the tank?

6 The following grid and table show the depths of material to be removed for a pool at an aquatic theme park. The grid has 15 m squares.

a Find the volume of material to be removed.

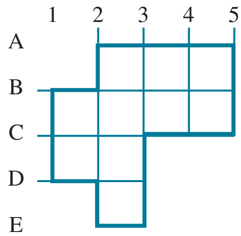
b How many 12 m^3 truckloads will be needed to take the soil away if it expands by 15% when loosened?



Depth of soil (m)

	1	2	3	4	5
A	10.175	10.213	10.317	10.795	10.112
B	9.975	8.500	7.713	7.517	8.946
C	9.102	7.236	4.203	5.130	7.957
D	8.766	6.715	4.789		
E	7.924	6.987	5.918		

7 This grid and depth table concern the overburden above a seam of high-grade coal in central Queensland. The grid has 20 m squares. Calculate the volume of overburden that must be removed to expose the seam for open-cut mining using draglines.

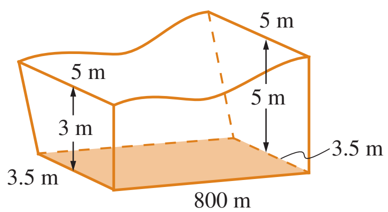


Depth of overburden (m)

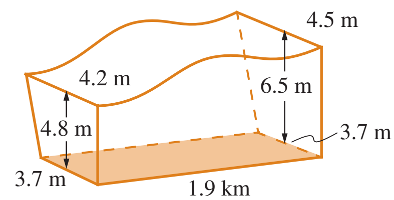
	1	2	3	4	5
A		5.180	7.203	4.182	3.980
B	6.732	6.108	5.819	8.234	7.410
C	4.287	7.154	6.104	6.205	5.657
D	5.345	5.254	8.290		
E		5.452	7.235		

8 Find the volumes of earth that must be excavated for the road cuttings in the sketches below.

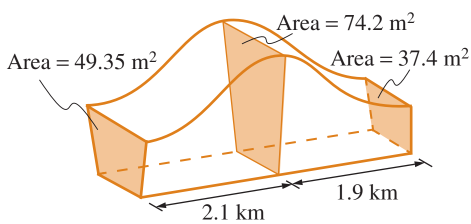
a



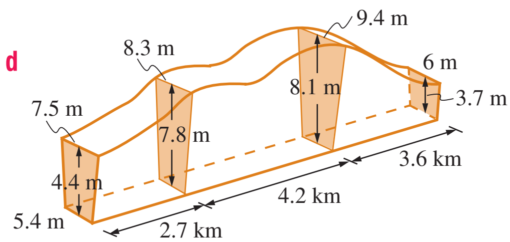
b



c

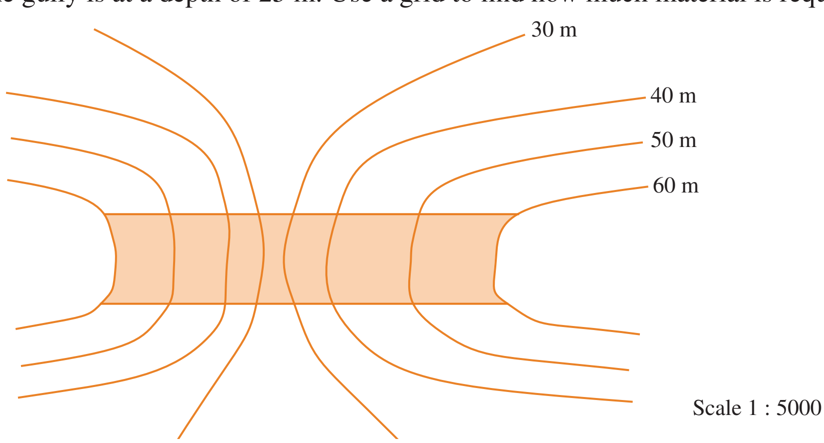


d



9 Where it exits the roof, a ventilation duct has a circular end with a diameter of 1200 mm. Inside the ventilation shaft, the section is rectangular, with dimensions 1200 mm by 900 mm. The section of duct that changes shape is 3 m long. What is the volume of air in this section?

10 This contour map shows a gully to be filled with earth taken from a cutting in roadworks. The straight lines and shading show the area to be filled, which is to be taken up to the contour height of 60 m. The width of the area to be filled (shaded) is effectively 60 m. The bottom of the gully is at a depth of 25 m. Use a grid to find how much material is required.



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Chapter summary

- The **perimeter** of a shape is the distance around the outside.
- The **circumference** of a circle is given by $C = \pi D$, where $\pi \approx 3.14$.
- Common metric measures of length are the millimetre, centimetre, metre (m) and kilometre.
  1 millimetre (mm) $= 10^{-3} = 0.001$ metres   1 centimetre (cm) $= 10^{-2} = 0.01$ metres
  1 kilometre (km) $= 10^{3} = 1000$ metres
- In Australia, building measurements are normally given in millimetres.
- **Area** is a measure of two-dimensional space, obtained by working out how many squares will fit in the space.
- The common metric units of area are the square centimetre, square metre, hectare and square kilometre.
  1 square kilometre (km^2) = 100 hectares = 1 000 000 square metres
  1 hectare (ha) = 10 000 square metres
  1 square metre (m^2) = 10 000 square centimetres (cm^2)
- The formulas for the areas of common shapes are:
  Rectangle: $A = l \times w$ Parallelogram: $A = b \times h$
  Trapezium: $A = \frac{1}{2} \times (a + b) \times h$ Triangle: $A = \frac{1}{2} \times b \times h$
  Triangle (Heron's Formula): $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a + b + c)$
  Circle: $A = \pi r^2$
- The **surface area** of a 3D shape is the total area of the faces of the shape.
- The formulas for the **curved surface areas** of some common shapes are:
  Cylinder: CSA $= 2\pi rh$
  Cone: CSA $= \pi rs$ where $s = \sqrt{r^2 + h^2}$
  Sphere: CSA $= 4\pi r^2$
- The **trapezoidal rule** for the area of an irregular shape is:
  $$\text{Area} \approx \frac{1}{2}\ \text{width} \times (\text{total of end lengths} + 2 \times \text{total of middle lengths})$$
- **Volume** measures 3D space by finding the number of unit cubes that will fit in the space.
- **Capacity** is used to indicate the volume of a container for liquids.
- The common units of volume and capacity are the cubic centimetre (cm^3), cubic metre (m^3), litre (L), millilitre (mL), kilolitre (kL) and megalitre (ML).
  1 cubic metre = 1 000 000 cubic centimetres
  1 cm^3 = 1 mL and 1 m^3 = 1 kL = 1000 L
  1 L = 1000 mL, 1 kL = 1000 L and 1 ML = 1 000 000 L
- A **prism** is a 3D shape with a constant polygonal cross-section, usually called the **base**.
- A **pyramid** is a 3D shape that tapers from the polygonal base to a point.
- The formulas for the volumes of some common 3D shapes are:
  Prism: $V = A \times h$ Cylinder: $V = \pi r^2 h$
  Pyramid: $V = \frac{1}{3} \times A \times h$ Cone: $V = \frac{1}{3}\pi r^2 h$ Sphere: $V = \frac{4}{3}\pi r^3$
- The formula for the approximate volume of material obtained from **grid heights** is:
  $$V \approx \frac{1}{4} \times A \times (H_1 + 2H_2 + 3H_3 + 4H_4)$$
- The formula for the approximate volume of an object calculated by **average end areas** is:
  $$V \approx \frac{1}{2} \times (A_1 + A_2) \times L$$

Chapter review

Knowledge and procedures

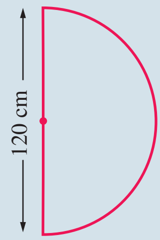
1 The circumference of a circle is given by: Ex 3.1

A $C = \pi r$ **B** $C = 2\pi D$ **C** $C = \pi r^2$

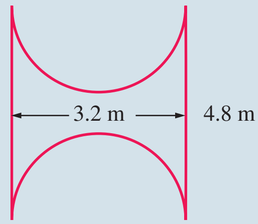
D $C = \pi D$ **E** $C = \pi D^2$

2 Calculate the perimeters of the following shapes. Ex 3.1

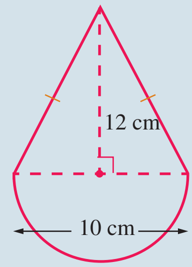
a



b



c

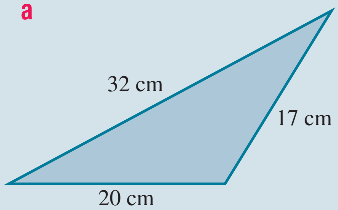


3 State the formula for the area of: Ex 3.2

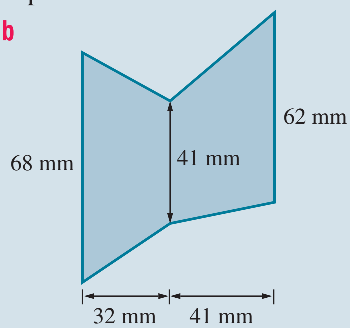
a a circle **b** a triangle (2 formulas) **c** a trapezium.

4 Find the areas of the following shapes. Ex 3.2

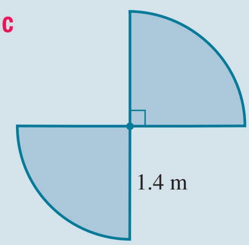
a



b

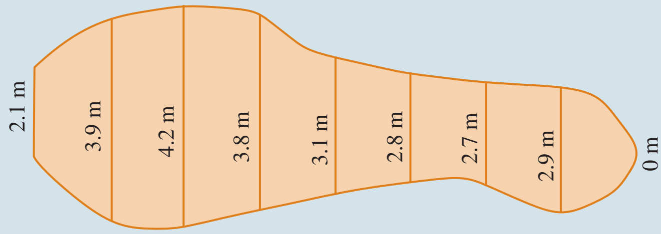


c



5 Use a diagram to explain the trapezoidal rule for the area of an irregular shape. Ex 3.4

6 Calculate the area of the following shape, where the lines are 1.8 m apart. Ex 3.4



7 Convert the following. Ex 3.5

a $15\ m^3$ to L **b** 280 mL to L

c $570\ cm^3$ to mL **d** $570\ cm^3$ to m^3

8 State the formula for the volume of: Ex 3.5

a a prism **b** a pyramid

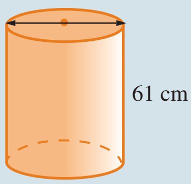
c a cylinder **d** a sphere.

Chapter review

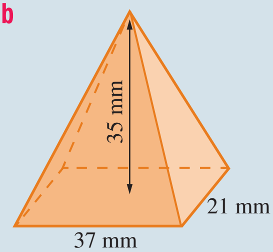
Ex 3.5

9 Find the volumes of shapes **a**, **b** and **c**. Find the surface areas of shapes **a** and **b**.

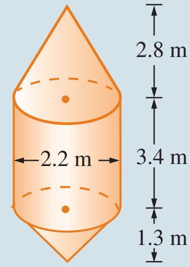
a



b



c



Ex 3.6

10 The grid heights formula for the calculation of volume is:

A $V \approx \frac{1}{3} \times (H_1 + H_2 + H_3 + H_4)$

B $V \approx \frac{1}{4} \times A \times (H_1 + H_2 + H_3 + H_4)$

C $V \approx \frac{1}{4} \times A \times (H_1 + 2H_2 + 3H_3 + 4H_4)$

D $V \approx \frac{1}{2} \times A \times (H_1 + H_2 + H_3 + H_4)$

E $V \approx \frac{1}{2} \times A \times (H_1 + 2H_2 + 3H_3 + 4H_4)$

Modelling and problem solving

Use the site plan below to answer questions **11** to **15**.

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Chapter review

11 Calculate the area of: Ex 3.4

a the patio
b the office
c the house
d the path and the driveway.

12 The back yard from the property line to the two fences is to be turfed. It turf costs \$9.50/m^2, find the cost of buying the turf needed. Ex 3.4

13 The roof of the garage is flat. It is to be covered with metal sheets measuring 2400 mm × 750 mm with a 50 mm overlap. Find the number of sheets required. Ex 3.4

14 The ceiling of the office is 3000 mm high. The office is to be air-conditioned. Each cubic metre of office space will require approximately 60 W of air-conditioning power to cope with the site conditions and daytime use. Determine the size in watts (W) of the air-conditioning units required for the office area. Ex 3.6

15 Calculate the cost of purchasing Weldmesh fencing material to run along the side and back property lines, given the following data: Ex 3.1

- Weldmesh fence panels come in 2400 mm lengths but may be cut to any length.
- 2400 mm fence panels (1200 mm high) cost \$71 each.
- 50 mm × 50 mm fence posts cost \$28 each.

16 Find the following using the 'Hawthorn' house design on the next page, assuming the ceilings are 2400 mm high. Ex 3.4

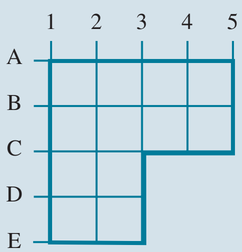
a The least amount of carpet (in metres) needed for the Lounge, bedrooms and Home Theatre.

b The estimated cost of giving the ceilings (including the garage) two coats of paint at a cost of \$78 for a 4 L tin if the coverage is 9 m^2/L.

c The number of 150 mm square tiles needed for the bathroom floor, allowing for 5 mm grout and 5% wastage. There is no need to tile under the bath, which is 900 mm wide. The shower is 900 mm square.

d The number of rolls of the same wallpaper needed for bedrooms 2, 3 and 4 if there is a 30 cm pattern repeat. Assume the wallpaper is 50 cm wide and comes in 10 m rolls. An allowance of 15 cm per drop is required for trimming the top and bottom of each drop of wallpaper.

17 Find the volume of material shown by the following grid and table, assuming that the grid has 6 m squares. Ex 3.6



Depth of material (m)

	1	2	3	4	5
A	1.27	1.21	1.32	1.79	1.83
B	1.12	1.10	1.23	1.57	1.65
C	0.98	1.21	1.24	1.48	1.55
D	0.34	0.56	0.87		
E	0.25	0.35	0.55		

18 Estimate the volume of water that could be collected from the roof of the 'Hawthorn' house if there is 28 mm of rain. Ex 3.6

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Hawthorn

Ex 3.6

19 A duct is 20 m long. It has a 10 m-long central rectangular section which is 600 mm by 800 mm, a 5 m-long section changing to an 800 mm round section at one end, and a 5 m-long section changing to a 700 mm square section at the other end. Find the volume of gas in the duct.

Using scale drawings

4

Contents

Syllabus Guide
Chapter 4

Syllabus subject matter

Linking two and three dimensions

- Interpretation of scale drawings and plans
- Drawing simple scale drawings and plans

Quantitative concepts and skills

- Metric measurement including measurement of mass, length, area and volume in practical contexts
- Calculation and estimation with and without instruments
- Basic algebraic manipulations

A scale drawing is one that represents a real object. The scale of the drawing is the ratio of the size of the drawing to the actual or real size of the object. Building plans and maps are examples of commonly used scale drawings. Scale drawings are helpful in allowing people to visualise and gain a better perspective of very large or very small objects.

4.1 Using scales

The **scale** of a drawing relates the drawing size to the size of the real object that it represents.

!

A **scale** is normally written with the size of the drawing first, followed by the size of the real object. It may be written in any of three ways:

- unit equivalence, such as 1 cm : 50 m
- ratio, such as 1 : 5000
- fraction, such as 1/5000.

The ratio and fraction forms are written without units.

Example 1

Change a scale of 1 cm : 200 m to a ratio.

Solution

Make the units the same using 1 m = 100 cm.

$$\begin{aligned} 1\text{ cm} : 200\text{ m} &= 1\text{ cm} : 200 \times 100\text{ cm} \\ &= 1\text{ cm} : 20\,000\text{ cm} \end{aligned}$$

Remove the units.

$$= 1 : 20\,000$$

Example 2

Complete the following: 1 : 500 is the same as 1 cm : … m.

Solution

The real object is 500 times bigger.

Real size for 1 cm $= 1 \times 500$ cm $= 500$ cm

Change to metres.

$= 5$ m

Write the answer.

1 : 500 is the same as 1 cm : 5 m.

Example 3

The scale of a drawing is 1 : 50. What is the real length of a 24 mm line on the drawing?

Solution

The real object is bigger, so multiply by the scale factor, 50. Change to a sensible unit.

Real length $= 24 \times 50$ mm $= 1200$ mm
$= 1.2$ m

Write the answer.

The real length is 1.2 m.

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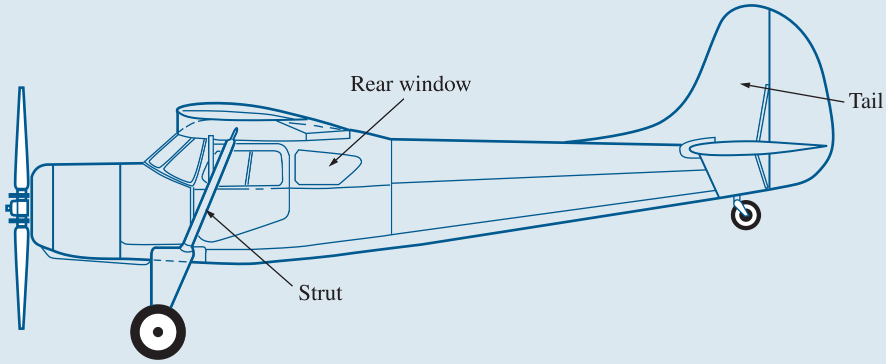
Example 4

The plan below shows a fixed wing aircraft drawn to a scale of 1 : 125. What is the length of the propeller?



Solution

Measure the propeller length on the drawing. Drawing length = 32 mm

Multiply by the scale factor, 125. Real length = 32×125 mm

= 4000 mm

Write the answer in metres. The length of the propeller is 4 m.

Exercise 4.1 Using scales

Additional Exercise 4.1

1 Write each of the following scales in ratio form.

a 1 cm : 1 m
b 1 cm = 1 km
c 1 mm = 1 m
d 1 mm : 10 m
e 2 cm : 1 m
f 5 mm = 1 km

2 Complete each of the following.

a 1 : 100 is the same as 1 cm: … m
b 1 : 1000 is the same as 1 mm = … m
c 1 : 500 is the same as 1 cm = … m
d 1 : 10 000 is the same as 1 cm: … m
e 1 : 650 000 is the same as 1 cm = … km
f 1 : 50 000 is the same as 1 mm: … m

3 The following measurements were made on drawings at the given scales. Find the real measurements.

a 28 mm at 1 : 200
b 3.2 cm at 1 : 5000
c 13 cm at 1 : 100
d 15 mm at 1 : 10 000
e 5.8 cm at 1 : 50 000
f 46 mm at 1 : 20 000

4 Use the aircraft plan in Example 4 to find the real measurements of:

a the overall length of the aircraft
b the length of the strut joining the wing and the forward wheel housing
c the diameter of the forward wheel
d the distance from the top of the tail to the bottom of the rear wheel
e the width and height of the rear window.

5 The distance between two points on a map is 275 mm. If the scale is 1 : 25 000, what is the true distance between the points?

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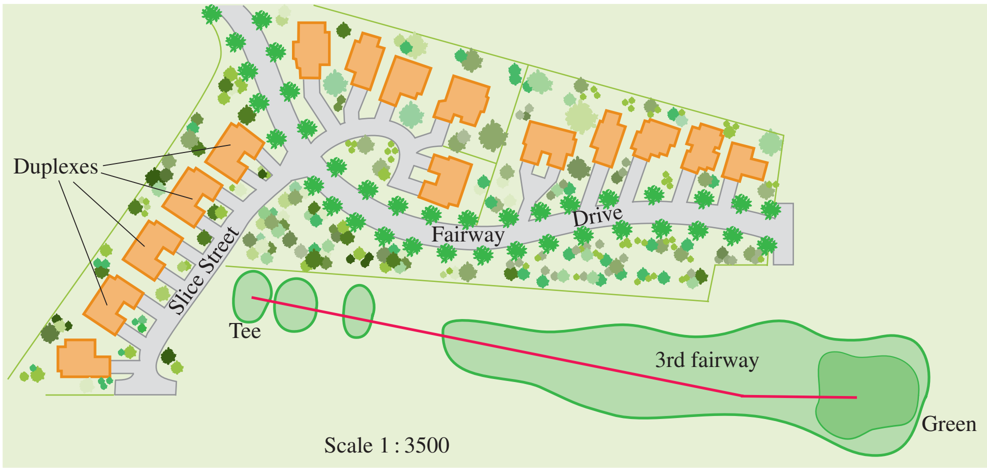
Modelling and problem solving

6 Use this scale drawing of part of a golf course housing development to answer the questions.

a How wide is Fairway Drive?

b The path of a golf ball hit from the tee to the green of the third hole is shown in red. How far did each of the two shots travel?

c What is the overall length of each duplex on Slice Street (measured parallel to the street)?



7 Use the bathroom plan on the right to find the real measurement (in millimetres) for:

a the length of the bath

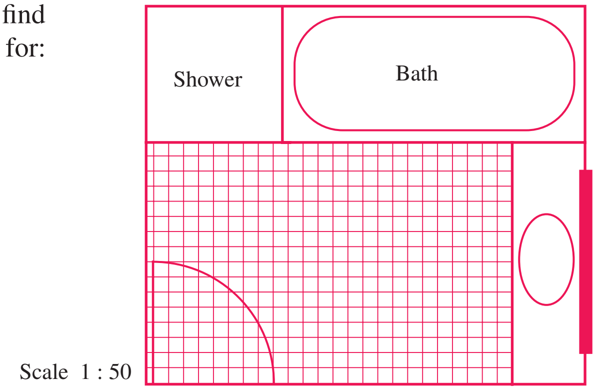
b the width of the shower

c the width of the door

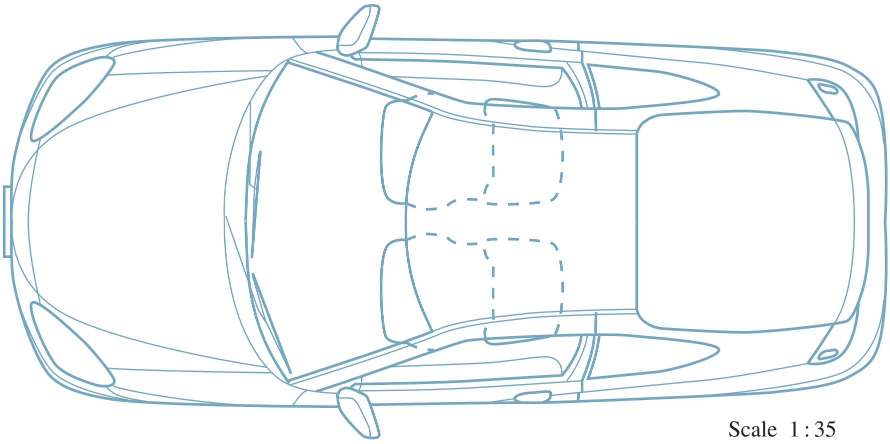
d the width of the window

e the length of the vanity unit

f the width of the vanity unit.



8 Use the scale drawing of the car below to answer the following questions.



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a How long is the car?

b How wide is the car body?

c How wide is the front car seat?

d What is the smallest distance between the seats?

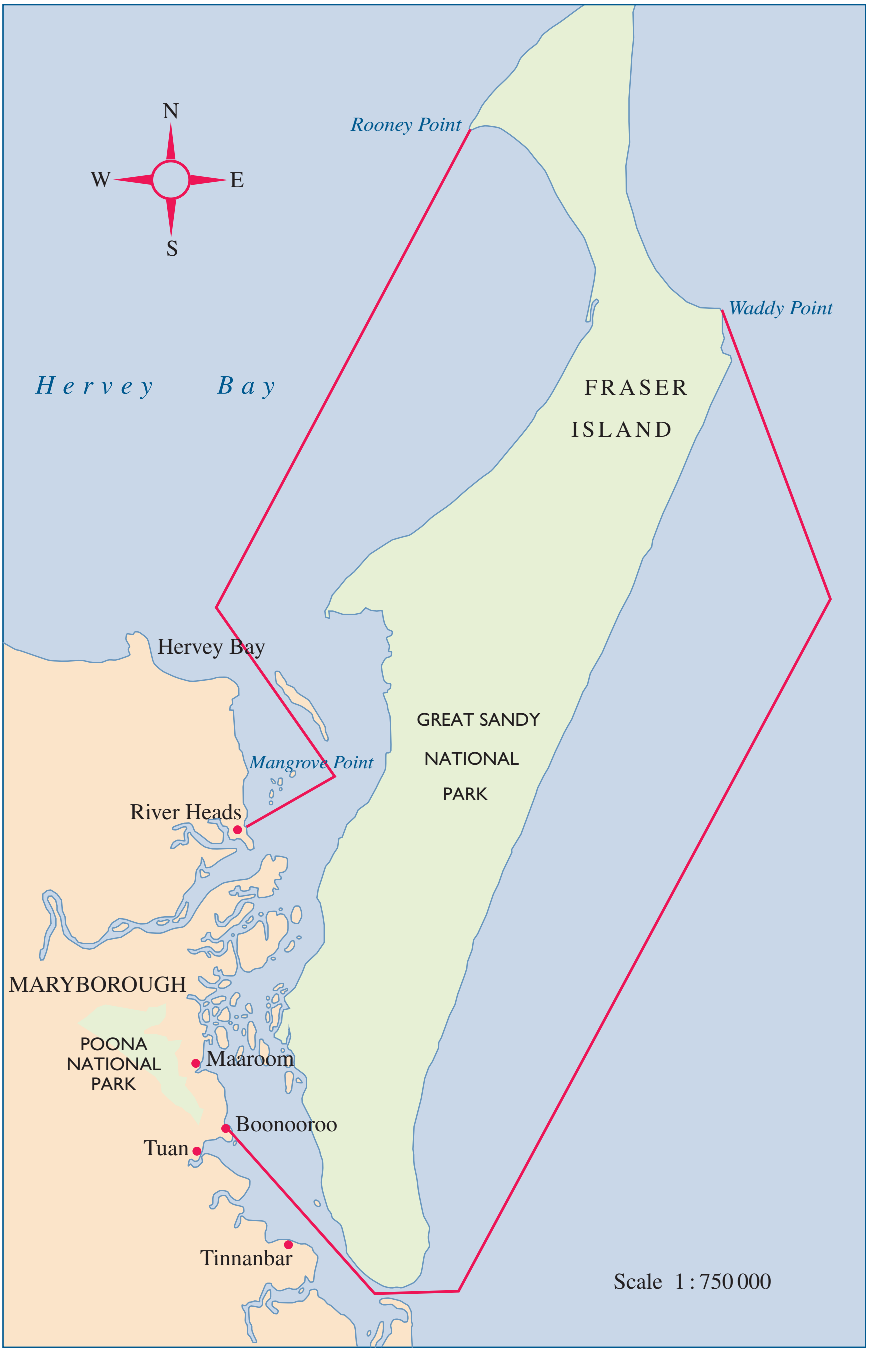
9 Use the map of the Hervey Bay area below to find:

a the actual distance of the route shown between River Heads and Rooney Point

b the actual distance of the route shown between Boonooroo and Waddy Point

c the actual distance between Boonooroo and River Heads

d the greatest actual width of Fraser Island, measured east–west.



4.2 Constructing scale drawings

A **scale drawing** is made on a piece of paper (or computer screen), so the drawing has to fit on the paper. Scales such as 1 : 36, 1 : 5280 and 1 : 63 360 were once used for scale drawings because they were useful in the imperial system of measurement. These days, we try to make the scales fit the metric system, so we use scales such as 1 : 10, 1 : 20, 1 : 25, 1 : 40, 1 : 50, 1 : 100, 1 : 500, 1 : 1000, 1 : 5000, 1 : 10 000, 1 : 50 000, 1 : 100 000, 1 : 500 000 wherever possible.

!

To work out the scale for a drawing:

1. Change the measurements of the real object to the same units as the drawing paper.
2. Work out the size of the space available. Allow for margins on the paper.
3. Divide the size of the real object by the size of the space for the drawing in *both* directions to obtain a **scale factor** for each direction.
4. Choose the larger scale factor (smaller scale) from the two directions.
5. Round *up* to a simple number.

Example 5

A house 16 m long and 9 m wide is to be drawn on A4 paper. What scale should be used?

Solution

Change units of the house.	16 m = 16 000 mm 9 m = 9000 mm
Measure the paper.	A4 paper size = 297 mm × 210 mm
Allow for margins.	Margins should be 25 mm.
Find size of drawing space.	Drawing space = 247 mm × 160 mm
Divide longer dimensions.	Length scale factor = 16 000 mm ÷ 247 mm ≈ 64.8
Divide shorter dimensions.	Width scale factor = 9000 mm ÷ 160 mm ≈ 56.3
Choose the larger scale factor.	Use 64.8.
Round the larger scale factor up.	Make the scale 1 : 100.
Write the answer.	A scale of 1 : 100 should be used.

Example 6

A desk 120 cm by 50 cm is shown on a scale drawing at a scale of 1 : 50. What are the dimensions of the drawing?

Solution

Divide the width in mm by the scale factor.	Width = 1200 mm ÷ 50 = 24 mm
Divide the depth in mm by the scale factor.	Depth = 500 mm ÷ 50 = 10 mm
Write the answer.	The dimensions of the drawing are 24 mm by 10 mm.

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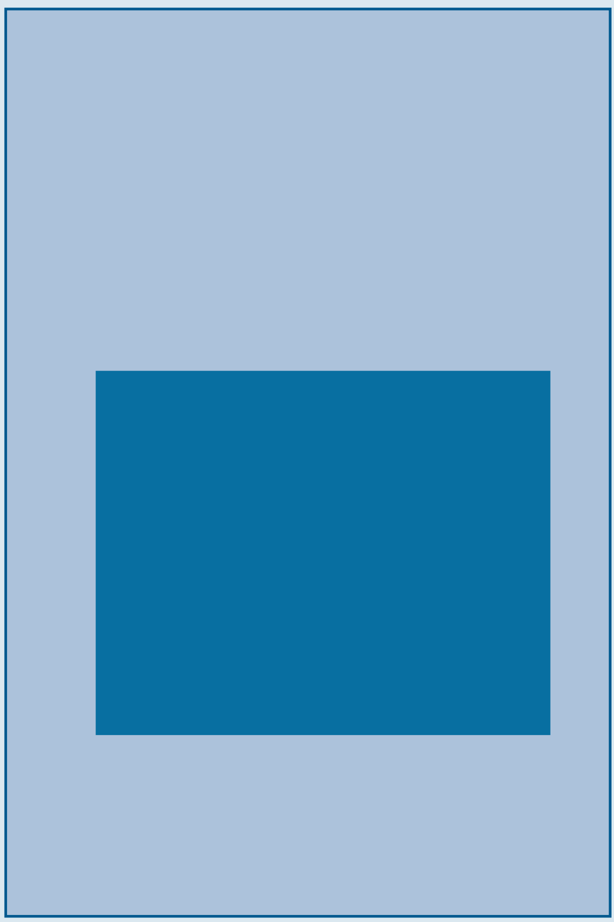
Example 7

A scale drawing of a house on a block of land is to be made. The block is 20 m wide and 30 m deep, and the house is 15 m wide and 12 m deep. The house is 3 m from the left boundary and 6 m from the front boundary. The drawing must fit into a space 10 cm wide by 12 cm deep. Choose a scale and make the drawing.

Solution

Change the units.	20 m = 2000 cm, 30 m = 3000 cm
Divide the dimensions.	2000 cm ÷ 10 cm = 200 3000 cm ÷ 12 cm = 250
Choose the scale.	Use a scale of 1 : 250.
Calculate the drawing dimensions in mm.	Block width = 20 000 mm ÷ 250 = 80 mm Block depth = 30 000 mm ÷ 250 = 120 mm House width = 15 000 mm ÷ 250 = 60 mm House depth = 12 000 mm ÷ 250 = 48 mm
Calculate the house position in mm.	Distance from left side = 3000 mm ÷ 250 = 12 mm Distance from front = 6000 mm ÷ 250 = 24 mm
Make the drawing. Write the scale on the drawing	



Scale 1 : 250

Front boundary

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Additional Exercise 4.2

Exercise 4.2 Constructing scale drawings

1 What scale should be used for a drawing on A4 paper (allowing for 25 mm margins) of:
- a a house block 70 m by 28 m?
- b a park 2 km by 1.8 km?
- c a room 6 m by 3.5 m?
- d a house 18 m by 10 m?
- e a subdivision 3 km by 5 km?
- f a cattle station 20 km by 16 km?

2 Objects of the following sizes are drawn to scale. Find the dimensions of each drawing.
- a 30 m by 40 m at 1 : 100
- b 16 m by 11 m at 1 : 200
- c 3.0 m by 4.5 m at 1 : 50
- d 900 mm by 2100 mm at 1 : 50
- e 28 km by 60 km at 1 : 50 000
- f 450 m by 800 m at 1 : 500

Modelling and problem solving

3 A house 16 m wide by 19 m deep is on a block of land 20 m wide by 35 m deep. The house is set back 7 m from the front boundary and 2 m from either side. Make a scale drawing of the block and house on A4 paper.

4 A 16-year-old girl wants to rearrange the furniture in her bedroom. She has measured the room and finds that it is 3.00 m by 2.40 m, with the 820 mm door in one corner on the 2.40 m wall and a 1200 mm-wide window in the centre of the opposite wall. Her bed is 1950 mm by 900 mm, her wardrobe is 1200 mm by 500 mm, her desk is 900 mm by 500 mm and her dressing table is 900 mm by 450 mm. Make a scale drawing on A4 paper of her bedroom, and use cutouts of furniture to work out an arrangement of her furniture. Justify any decisions you make.

5 An office has to accommodate 6 staff in a room 10 m long and 6 m wide with two 900 mm wide doors. Each staff member needs a desk 1200 mm by 600 mm, a filing cabinet 450 mm wide and 600 mm deep, and space for a chair. In addition, space is needed for a photocopier, shared fax and shared printer in the office. These are 1200 mm by 600 mm, 600 mm by 450 mm and 600 mm by 600 mm respectively. Dividers may be placed in the room to ensure privacy. Choose a scale and design a layout to fit:
- a A4 paper
- b A3 paper (297 × 420 mm).

Investigation Crowded classrooms

1 Work out the scale required to plot your classroom so that it fits on an A4 sheet with some room left on the edges as margins.

2 Now work in groups to make a scale drawing of the classroom showing door(s), windows and any furniture.

3 Calculate the floor space left free when the furniture is taken into account. Find the ratio of the free floor space to total floor space. Find the amount of floor space per person, both in terms of total space and free space.

4 Discuss the amount of floor space needed to avoid feeling 'crowded' in a classroom or work environment. Does this depend on the person, or is there general agreement?

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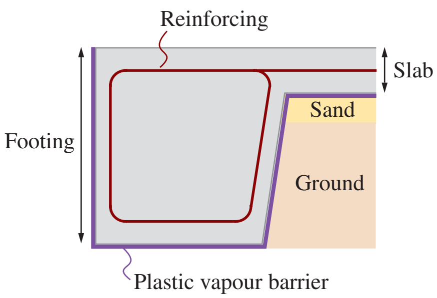
4.3 Using scale drawings

Scale drawings are used extensively in the building industry to show the detail of constructions. Many houses and other buildings are now built on reinforced concrete slabs. These normally have deeper **footings** underneath the load-bearing walls of the building. In houses and other small buildings, the load-bearing walls will be the external walls. The depth of footings will depend on the soil and the type of construction.



When footings are poured, the concrete is vibrated to ensure that there are no air pockets, so it sets solid.

Required quantities and costs of wall cladding, roofing, wallpaper, carpet and painting can also be calculated from scale drawings.

Example 8

The floor plan of a double garage is shown. It has a reinforced concrete slab 100 mm thick with footings 450 mm deep by 300 mm wide at the edges. The concrete costs $\$180/m^3$ delivered. Find the cost of the concrete for the slab and footings.

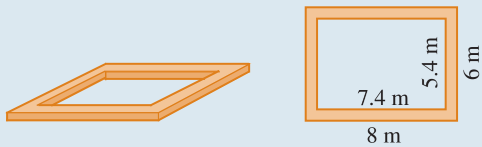
Solution

Measure the plan. Plan dimensions are 6 cm by 8 cm.

Work out real size. Real slab is 6 m by 8 m.

Find volume of the slab. Volume of slab $= 6 \times 8 \times 0.1 \text{ m}^3 = 4.8 \text{ m}^3$

Sketch footings (below ground). (Ignore the slight taper and treat as a rectangular prism.)



Find area of footings. Area $= 8 \times 6 - 7.4 \times 5.4 \text{ m}^2 = 8.04 \text{ m}^2$

Find depth of footings. Allow for slab. Depth $= 450 \text{ mm} - 100 \text{ mm} = 0.35 \text{ m}$

Find volume of footings. Volume of footings $=$ area $\times$ depth $= 8.04 \times 0.35 \text{ m}^3 = 2.814 \text{ m}^3$

Find total volume. Volume of slab and footings $= 4.8 + 2.814 \text{ m}^3 = 7.614 \text{ m}^3$

Round up to allow for waste. $\approx 8 \text{ m}^3$

Find cost. Cost of concrete $= 8 \times \$180 = \$1440$

Write answer. Concrete for the slab and footings costs \$1440.

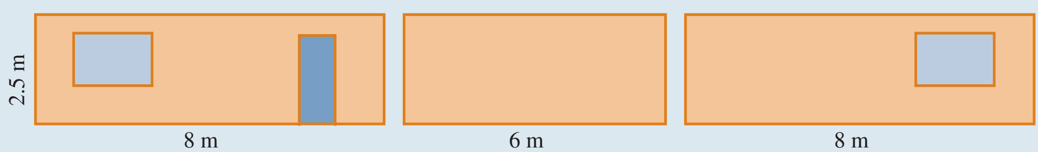
The website www.humes.com.au/Toolbox/Calculators/calcConcreteSlabOnGrade.asp can be used to work out the volume of concrete footings.

Example 9

With allowance for waste and mortar, it is usual to assume that it requires 53 bricks to complete a square metre of a single wall. The side and back walls of the garage in Example 8 are to be brick and will be 2.5 m high. How many bricks are needed, allowing for a side entry door 820 mm by 2030 mm and two windows 1800 mm by 1200 mm?

Solution

Sketch the walls.



Calculate area of walls. Wall area $= 2.5 \times 6 + 2 \times 2.5 \times 8 \text{ m}^2 = 55 \text{ m}^2$

Allow for windows and door. Area of door and windows $= 0.82 \times 2.03 + 2 \times 1.8 \times 1.2 \text{ m}^2 \approx 6 \text{ m}^2$

Calculate area of bricks. Area of bricks $= 55 - 6 \text{ m}^2 = 49 \text{ m}^2$

Calculate quantity. Number of bricks $= 49 \times 53 = 2597$ bricks

Write the answer. The walls will need about 2600 bricks.

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Example 10

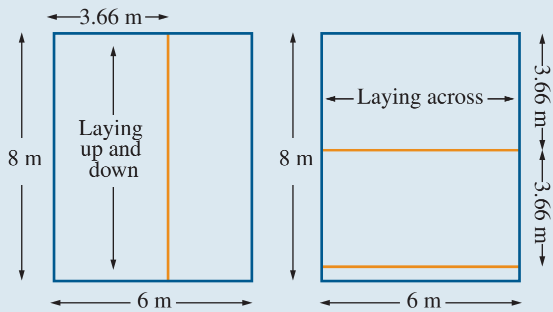
The garage shown in Example 8 on page 115 is to be converted into a games room. What length of 3.66 m-wide carpet will be needed to carpet the floor?

Solution

Measure the dimensions of the floor.
In the drawing, the garage floor is 6 cm by 8 cm.

Change to real dimensions.
The scale is 1 cm : 1 m.
Drawing width = 6 cm, so real width = 6 m.
Drawing length = 8 cm, so real length = 8 m.

Sketch the floor and show how the carpet could be laid each way.

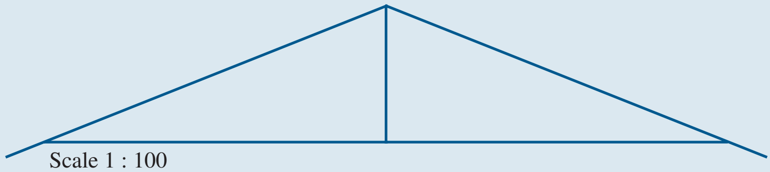


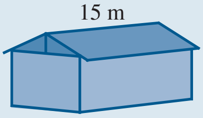
Decide which way to lay the carpet.
Laying the carpet 'up and down' will require two 8 m lengths or 16 m of carpet.
Laying the carpet 'across' will require three 6 m lengths or 18 m of carpet.

Write the answer.
16 m of carpet will be needed, but a 1.32 m by 8 m strip will be wasted.

Example 11

The scale drawing below shows the end of a gable roof. The roof is 15 m long. It is to be covered with corrugated iron costing $48.70/sheet. Each sheet is 2 m long and 900 mm wide and must be overlapped both lengthwise and endwise with the next sheet by 8 cm. How many sheets are needed for the job, and how much will they cost?





Solution

Measure slope of the roof.	Length on drawing $= 5.4$ cm
Work out real length.	Real length of slope $= 5.4$ m
Allow for overlaps.	Effective width of sheeting $= 0.9\text{ m} - 0.08\text{ m}$ $= 0.82$ m
	Effective length of sheeting $= 2\text{ m} - 0.08\text{ m}$ $= 1.92$ m
Calculate number of sheets down roof slope.	Sheets down slope $= 5.4 \div 1.92$ ≈ 2.81 sheets
Round up.	≈ 3 sheets
Calculate number of sheets along roof.	Sheets along roof $= 15 \div 0.82$ ≈ 18.29 sheets
Can use $\frac{1}{2}$ sheet on other slope.	$\approx 18\frac{1}{2}$ sheets
Calculate number of sheets required.	Sheets per slope $= 3 \times 18.5$ $= 55.5$ sheets
	Total sheets $= 2 \times 55.5$ $= 111$ sheets
Work out cost.	Cost $= 111 \times \$48.70$ $= \$5405.70$
Write the answer.	111 sheets of corrugated iron costing \$5405.70 are needed.

Investigation Landscaping

1 Select an area in your school grounds that you believe could benefit from some landscaping work. Take some measurements and make a scale drawing of the area. Include all existing features such as garden beds, trees, shrubs, large rocks and seating.

2 Work with other members of your group to draw a landscaping plan that shows the extent of work that you propose should be carried out. Make sure you include the scale and a legend. Clearly show all existing features. You could use colour coding to make the plan easier to read.

3 Once your plan has been completed, make up costing tables for all aspects of the proposed work. Next, go to a local landscape supplies outlet and a hardware store to collect the pricing information needed to complete the costing tables.

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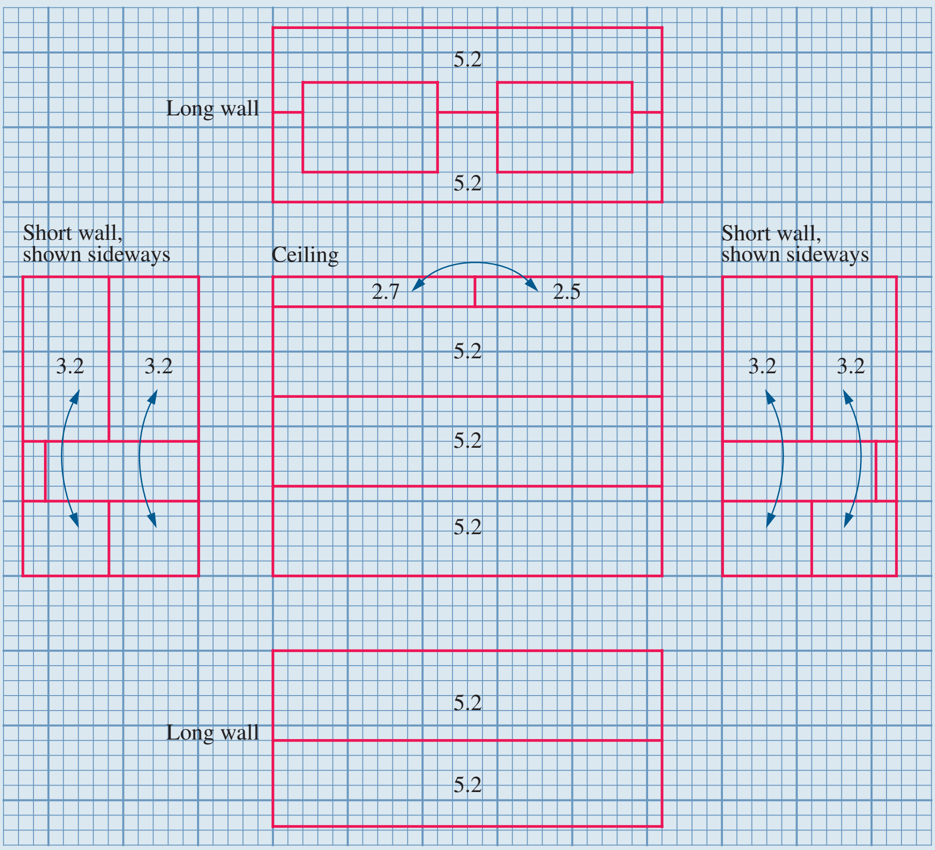
Plasterboard has a standard width of 1200 mm and is available in lengths of 2.4, 2.7, 3, 3.6, 4.2, 4.8, 5.4 and 6 m. A smaller range of lengths is available in widths of 900 mm and 1350 mm. On walls, sheets are fastened lengthwise across the walls of the room. Scale drawings help to work out the number and lengths of sheets needed for a job. It may be useful to use graph paper for this type of calculation.

Example 12

A room 4 m by 5.2 m has a ceiling 2350 mm high, two windows 1800 mm long and 1200 mm high in one of the long walls, and a door 820 mm by 2030 mm in each of the short walls. Use scale drawings to find the plasterboard that should be ordered to cover the ceiling and walls.

Solution

Make small drawings of the ceiling and walls at a scale of (say) 1 : 100.
Draw plasterboard sheets onto the drawings.



Note that it is usual to use full sheets around windows, and to use the scrap above doorways. A shorter sheet can be cut in two for the last part of the ceiling. If this is just above the windows, a join will be less noticeable. The measurements are shown on the drawings.

Considering the sizes available, the plasterboard needed is 7×5.4 m, 4×3.6 m and 1×2.7 m.

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Tradespeople commonly use graph paper in the way shown in Example 12 to estimate quantities for other materials such as floor coverings, external cladding, insulation and roof covering.

Additional Exercise 4.3

Exercise 4.3 Using scale drawings

1 The map of the Goodna area below has small grid squares that show an actual size of 250 m square. Use the scale drawing to find:

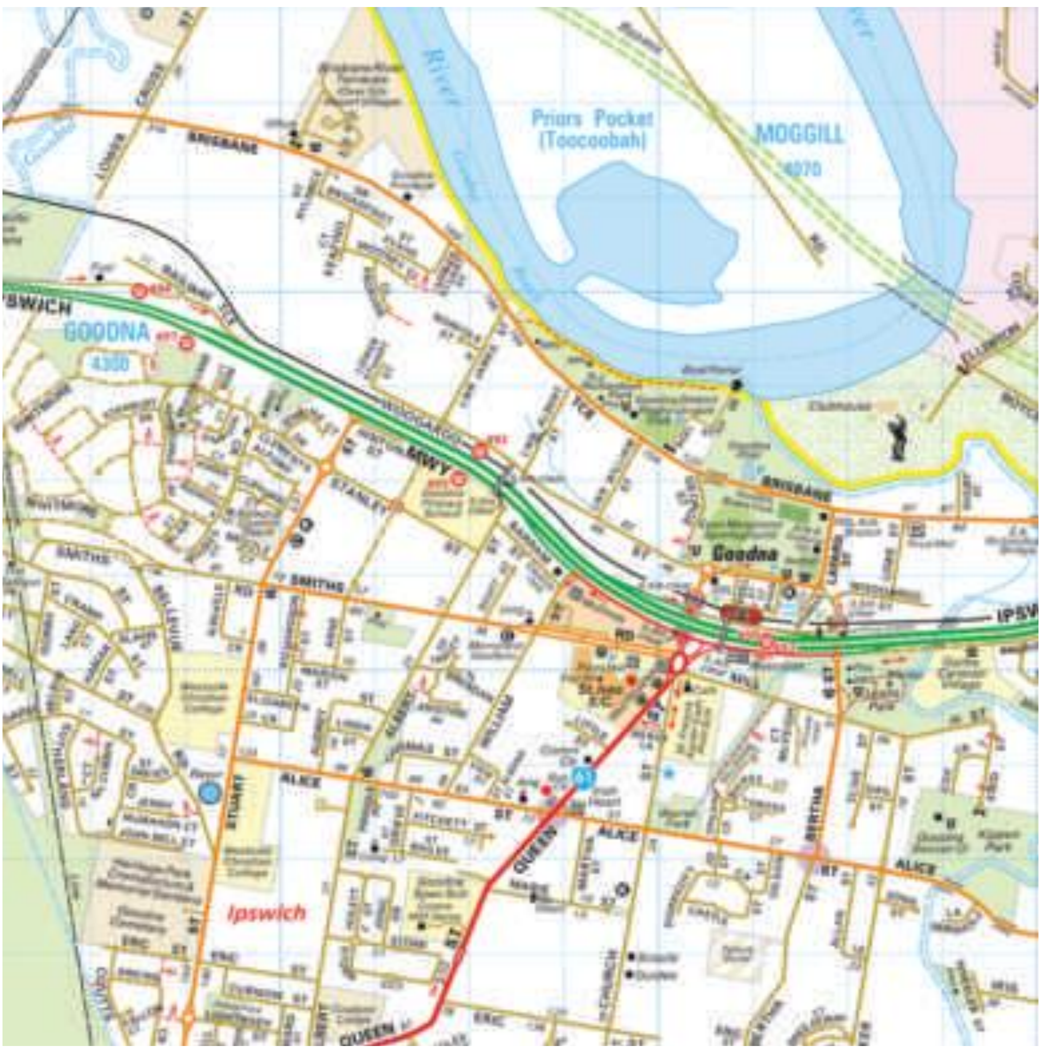
a the distance between the Richardson Park boat ramp ➡ and the Goodna train station

b the distance between the Business Centre ● in Queen Street and the public telephone ☎ in Smiths Road

c the distance between the churches on Alice Street and Smiths Road

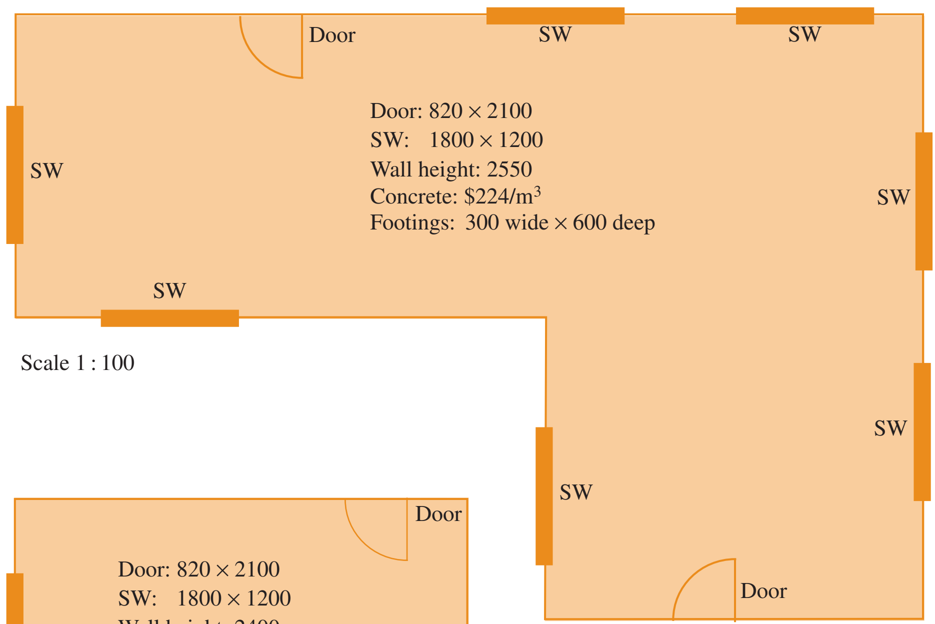
d the distance between the police station ★ on Church Street and the reservoir on Stuart Street

e the area of the Heritage Park Crematorium and Memorial Gardens and Goodna Cemetery.

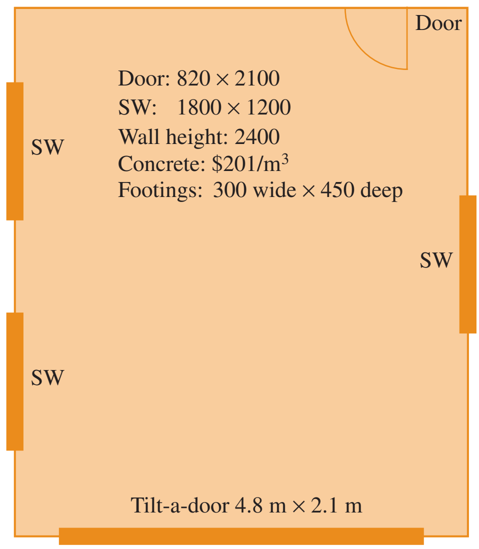


Questions **2** to **5** refer to the building floor plans shown below.

a



b



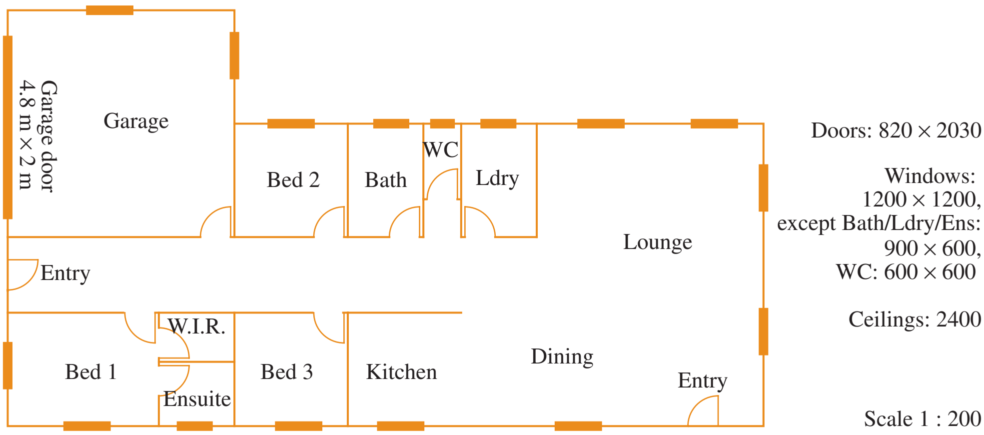
2 Find the cost of the concrete for the slab and footings of each building. The cost of the concrete per cubic metre and the size of footings are given with each plan. Assume that 100 mm slabs are used.

3 Assume that both buildings are to have single-brick walls with 53 bricks/m^2. If no allowance is made for windows and doors, find the number of bricks needed for each building.

4 Each sliding window (SW) in the buildings is 1800 mm × 1200 mm, and the doors are 820 mm × 2100 mm. Recalculate the quantities of bricks needed for the walls, allowing for windows and doors. Hence find the cost of bricks for each building at $840/1000, assuming that you can buy to the nearest 100 bricks.

5 Assume that the walls are to be weatherboard, not brick. Weatherboards are 225 mm wide and need an overlap of 25 mm on the long side only. Find the total length of weatherboards needed for each building, with allowance for windows and doors. Hence find the cost of weatherboards for each building at $6.30/m. Show all steps in logical order.

Questions **6** to **9** refer to the house plan shown below.



6 Find the length of 3.66 m-wide carpet needed to carpet the entire house, excluding the bathroom, kitchen, laundry, ensuite, WC and garage.

7 Find the number of rolls of wallpaper needed for the bedroom walls, given that the chosen wallpaper comes in 50 cm-wide rolls of length 10 m, with no pattern-matching required.

8 Find the cost of painting the ceilings of the house and garage with two coats, given that the paint covers 11 m^2/L and only comes in 4 L tins costing \$78.60 each. Fully explain your reasoning.

9 Draw scale plans to find the plasterboard needed for the following rooms, including walls and ceilings.

a Bedroom 2 **b** Bedroom 1, including walk-in-robe and ensuite **c** Garage

10 A flat roof is 6.9 m wide and 10 m long.

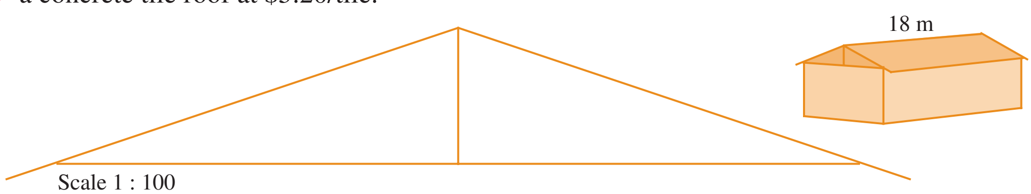
a Find the number of sheets of corrugated iron needed to cover the roof if the sheets are 2 m by 900 mm and must be overlapped by 8 cm at their sides and ends.

b Find the number of concrete tiles needed for the same roof if they are 30 cm wide and 50 cm long and must be overlapped by 6 cm in each direction.

11 The gable roof shown below is 18 m long. Assume that corrugated roofing iron is 2 m long and 900 mm wide (with overlap of 8 cm each way) and that concrete roof tiles are 30 cm wide and 50 cm long (with overlap of 6 cm each way). Find the quantity and cost for:

a a corrugated iron roof at \$39.02/sheet

b a concrete tile roof at \$5.20/tile.



12 A carpark is to be marked out on a vacant block of land that is 40 m deep and has a 50 m frontage. Spaces of 2.6 m by 5 m and turning circles of 8 m are to be allowed. Two 3 m-wide entrances are needed also. Use a scale drawing to work out a plan for the carpark, and hence find the number of cars that can be accommodated when it is full. Explain your reasoning and justify your decisions.

Chapter summary

- The **scale** of a drawing relates the size of the drawing to the size of the real object that it represents.
- A scale is normally written with the size of the drawing first, followed by the size of the real object. It may be written as a ratio, unit equivalence or fraction.
- To work out the scale for a drawing:
  - Change the measurements of the real object to the same units as the drawing paper.
  - Divide the size of the real object by the size of the space for the drawing in *both* directions to obtain a **scale factor** for each direction.
  - Choose the larger scale factor (smaller scale) from the two directions.
  - Round *up* to a simple number, such as 10, 20, 25, 40, 50 or a multiple of these.
- **Scale drawings** may be used to calculate quantities and costs for footings, slabs, brickwork, wall cladding, roofing, wallpaper, carpet and painting.
- It is usual to allow 53 bricks/m^2. Plasterboard is normally available in a width of 1200 mm, carpet in a width of 3.66 m and wallpaper in 50 cm-wide rolls that are 10 m long.
- It is often useful to use graph paper to assist in making scale drawings.

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Chapter review

Knowledge and procedures

Ex 4.1 **1** Write each of the following scales in ratio form.

a 1 cm : 2 m **b** 1 mm = 50 cm **c** 2 cm : 10 m

Ex 4.1 **2** Complete each of the following.

a 1 : 500 is the same as 1 cm : … m. **b** 1 : 2000 is the same as 1 mm : … m.
c 1 : 5000 is the same as 1 cm : … m. **d** 1 : 800 000 is the same as 1 cm : … km.

Ex 4.1 **3** The following measurements were made on drawings at the given scales. Find the real measurements.

a 17 mm at 1 : 100 **b** 8.6 cm at 1 : 2000
c 25 mm at 1 : 50 000 **d** 7.2 cm at 1 : 1000

Ex 4.1 **4** The distance between two points on a map is 53 mm. If the scale is 1 : 20 000, what is the true distance between the points?

Ex 4.2 **5** What scale should be used for a drawing (with margins) on A4 paper of:

a a house block 60 m by 25 m? **b** a reserve 3.4 km by 2.1 km?
c a room 4 m by 3.5 m? **d** a house 17 m by 12 m?

Ex 4.2 **6** Objects of the following sizes are drawn at the given scales. What are the dimensions of each drawing?

a 15 m by 20 m at 1 : 100 **b** 35 m by 21 m at 1 : 200
c 2.4 m by 3.5 m at 1 : 50 **d** 1200 mm by 1800 mm at 1 : 25.

Modelling and problem solving

Ex 4.1–4.3 **7** The map of the Brisbane CBD area shown on the opposite page has grid squares that show an actual size of 250 m square. Use the scale drawing to find:

a the distance along Adelaide Street from King George Square to Samuel Griffith Place
b the distance between Strike Bowling ■ at the Wintergarden and the RACQ outlet ■ near the GPO
c the distance between the police station ★ in Charlotte Street and the public telephone ☎ near the corner of Mary Street and Edward Street
d the total area bound by Mary Street, Adelaide Street, Edward Street and Albert Street.

Ex 4.1–4.3 **8** A house 18 m wide by 24 m deep is to be shown on a piece of A4 paper. The house is set back 4 m from the front boundary and 2 m from the left boundary of the block, which is 23 m wide by 35 m deep. Make a scale drawing of the house and block.

Ex 4.1–4.3 **9** Peter wants to rearrange his bedroom. The bedroom is 3.0 m by 2.8 m and has an 820 mm-wide door in the corner of one shorter wall. In the opposite corner there are two 1200 mm-wide windows, one in each wall. His bed is 2000 mm by 900 mm and his desk is 1200 m by 450 mm. He has a wardrobe 900 mm by 500 mm and a chest of drawers 900 mm by 450 mm. Make a scale drawing of his room, together with cutouts of the furniture.

Chapter review

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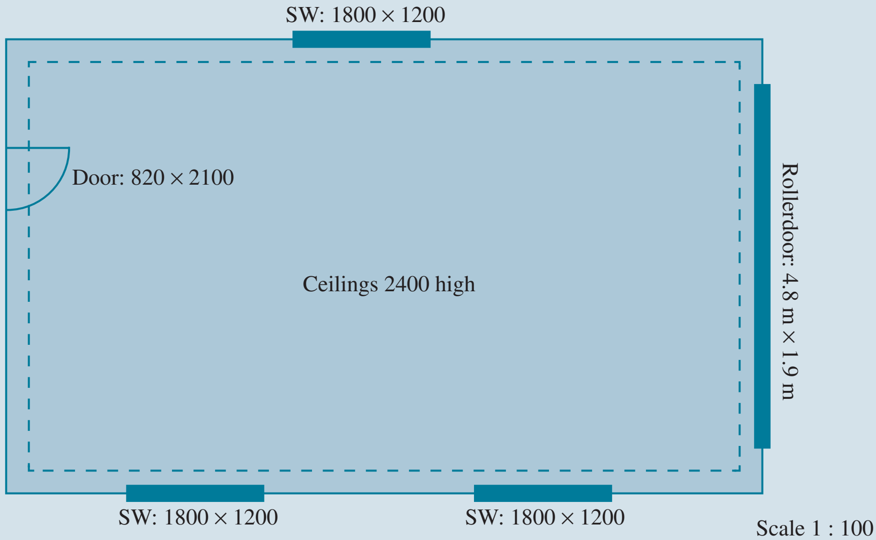
Chapter review

Questions **10** to **13** refer to the shed plan shown below.



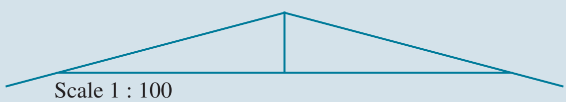
Scale 1 : 100

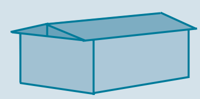
Ex 4.3 **10** Find the amount of concrete needed for the slab of the shed if footings 300 mm wide by 600 mm deep are required at the edges. If the concrete costs $185/m^3, find the cost of the concrete. Assume that a 100 mm slab is to be laid.

Ex 4.3 **11** Find the number of bricks needed for the walls of the shed. Fully justify your answer.

Ex 4.3 **12** Use a scale drawing on graph paper to find what plasterboard would be needed to line the walls and ceiling of the shed.

Ex 4.3 **13** The scale diagram below shows the end of a gable roof for the shed. The roof is to project 600 mm from the front and back of the shed. Find the number of sheets of aluminium needed for the roof if the sheets can be supplied cut to length, have a width of 1.2 m and must be overlapped by 10 cm. Explain your reasoning.





Income and taxes

5

Contents

Syllabus Guide Chapter 5

Syllabus subject matter

Managing money I

- Earnings, including salary, wages, overtime, commission, piece rate, and means-tested income; an industrial award should be used where appropriate
- Taxation, including taxable income, gross income, net income, goods and services tax (GST), deductions, rebates, and levies

Quantitative concepts and skills

- Calculation and estimation with and without instruments
- Rates, percentages, ratio and proportion

Most people engage in paid work at some time in their life, so it is important for you to have an understanding of the various ways in which you can earn money. The conditions of employment for particular trades or industries are set out in industrial awards. These conditions include things like pay rates, job classifications, hours of work, penalty rates, leave entitlements and additional payments called allowances. People who belong to an industrial union rely on their union to negotiate the terms and conditions of their employment via a process called collective or enterprise bargaining. Some people, however, choose to negotiate their terms and conditions of employment directly with their employer using an individual agreement or contract.

5.1 Earning a wage

There are many ways in which payment for work is calculated and workers are paid. Incomes are usually compared on an annual basis.

!

Gross income is the income earned before any deductions are made. Income tax deductions must be made (compulsory). You may also choose to have medical insurance, savings, union dues, personal superannuation contributions, and other voluntary deductions made. This means your gross income is not what you actually receive.

A **wage** is income that is paid by the week for the hours that are worked. People earning wages can normally quit or be fired on 1 or 2 weeks' notice.

A **salary** is income that is calculated by the year. It is usually paid each fortnight or month. The hours of work are more flexible than for wages, and notice of dismissal or resignation is usually at least a month.

Example 1

An assistant childcare worker is paid $545.30 per week. What is her annual gross income?

Solution

There are 52 weeks in a year.
Multiply weekly wage by 52.

$$\text{Annual gross income} = \$545.30 \times 52$$
$$= \$28\,355.60$$

Example 2

A new graduate starts on a salary of $42 500. What is her fortnightly gross pay?

Solution

There are 26 fortnights in a year.
Divide salary by 26 and round off.

$$\text{Fortnightly gross pay} = \$42\,500 \div 26$$
$$= \$1634.62$$

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There are many ways in which people are employed. **Permanent** employees may work on a full-time or part-time basis. A **full-time** employee usually works a 38-hour week and **part-time** employees work for fewer hours. Hours worked in excess of ordinary working hours, or outside usual starting and ceasing times, are paid at a higher rate called **overtime**. Overtime rates are normally stated using terms such as 'time-and-a-half' and 'double-time', and the rate depends on how many overtime hours are worked. Permanent employees receive benefits such as sick leave, holiday pay and long-service leave.

Casual employees are engaged to work by the hour. There is usually a minimum of 2 or 3 hours per engagement, but casual workers may also work full days, and the hours worked may be irregular or regular. Casual workers are not entitled to leave benefits and are paid a **loading** to compensate for this. In most cases, the loading means that casual workers receive an hourly rate that is about 20% more than that paid to permanent workers.

Example 3

A casual 18-year-old fast-food worker usually works 25 hours a week, and his rate of pay is \$12.44 per hour. He has 3 weeks off a year. Calculate his gross weekly wage and gross annual income.

Solution

Multiply rate by hours worked.	Weekly wage = \$12.44 × 25
Casual employees are not paid for holidays.	= \$311.00
Multiply by 49 to find the annual income.	Annual income = \$311.00 × 49
Evaluate.	= \$15 239.00

Example 4

A metal worker is paid \$658.20 per 38-hour week. The loading for a metal worker employed as a casual is 23%.

a What is the hourly rate for a metal worker employed as a casual?

b What would a casual metal worker be paid for working 30 hours in a week?

Solution

a Calculate the hourly rate.	Hourly rate = \$658.20 ÷ 38
	= \$17.32
Multiply by the loading.	Casual hourly rate = \$17.32 × 123%
Evaluate and round off.	= \$21.30
b Multiply rate by hours.	Casual wage = \$21.30 × 30
Evaluate.	= \$639.00

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Example 5

Jack, a permanent hospitality worker, is paid $658.20 for a normal 38-hour week. Overtime is time-and-a-half for the first 3 hours, then double-time. Jack receives paid meal and rest breaks. Calculate Jack's gross wage for the week shown on the roster below.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Start		9:30 am	8:30 am	11:30 am	7:30 am	7:30 am	7:30 am
Finish		2:30 pm	4:30 pm	8:30 pm	5:30 pm	3:30 pm	12:30 pm

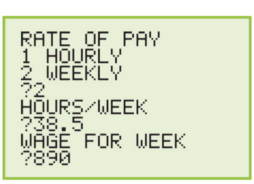
Solution

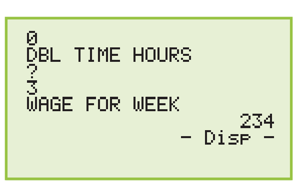
Calculate the hours Jack worked.	Tuesday = 9:30 am to noon + noon to 2:30 pm
	= 2 h 30 min + 2 h 30 min
Evaluate.	= 5 h
Repeat for other days.	Wednesday = 8 h
	Thursday = 9 h
	Friday = 10 h
	Saturday = 8 h
	Sunday = 5 h
Calculate total hours.	Total = 5 + 8 + 9 + 10 + 8 + 5 h
Evaluate.	= 45 h
Calculate hours of overtime.	Overtime = 45 – 38 h
Evaluate.	= 7 h
Calculate hourly rate.	Hourly rate = $658.20 ÷ 38
Evaluate and round off.	= $17.32
Calculate first 3 h of overtime.	First 3 hours = $\$17.32 \times 3 \times 1\frac{1}{2}$
Evaluate.	= $77.94
Calculate remaining overtime.	Remainder = $\$17.32 \times 4 \times 2$
Evaluate.	= $138.56
Calculate gross for the week.	Gross wage = $658.20 + $77.94 + $138.56
Evaluate.	= $874.70
Write the answer.	Jack's gross wage for the week is $874.70.

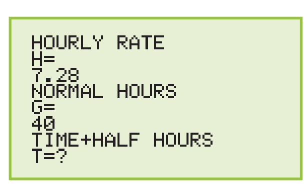
Calculator Program

Technology

The program WAGES can be used to perform wage calculations. The program is given in full on the CD-ROM. Enter the program (or load it from the CD-ROM) and try it with different scenarios. Try entering just normal hours, overtime at time-and-a-half or at double-time. You can enter the hourly rate or the normal weekly wage for a 38-hour week to perform the calculations.







Investigation Payment systems

- Work in small groups for this activity. You will probably find that people in the group already know something about different systems of payment.
- We know that casual employees work for a higher hourly rate than permanent employees doing the same job. Many employers prefer to have casual workers instead of permanent employees, and some employees prefer to be casual workers.
  - Find out why employers prefer to employ a casual worker, even though they have to pay a higher hourly rate.
  - Talk to friends, family and others to find out whether they prefer to work as a casual or as a permanent employee.
  - Document your findings.
- Some transport companies employ **owner-drivers** under contract, and others prefer to have permanent employees driving company trucks. Investigate some local companies to find why they prefer one or the other.
- Many people in the information technology (IT) industry work on **contracts**. Find out why this is the case. (Both the *Australian* and the *Courier-Mail* have regular IT features where employment opportunities and other industry issues are discussed.)
- **Piecework** is very common in the clothing industry. Many people in this industry are employed as **outworkers**. Find out what an outworker is. What does the term 'sweat-shop' mean? What is the system of payment for outworkers doing piecework, and how is it different for someone doing piecework in a factory?

Exercise 5.1 Earning a wage

Additional Exercise 5.1

For all questions in this exercise, assume that:

- a normal working week is 38 hours for permanent full-time workers
- overtime is paid to permanent employees at the rate of time-and-a-half for the first 3 hours and double-time thereafter
- casual employees receive a 23% loading on the hourly rate paid to permanent employees.

1 Change these to gross annual income.

a $248.60 a week **b** $746.80 a week

c $510.40 a week **d** $463.70 a week

2 Change these to gross annual income.

a $895.40 a fortnight **b** $972.50 a fortnight

c $1125.60 a fortnight **d** $1386.20 a fortnight

3 A taxi-driver averages $168 for a 12-hour shift. He works 5 shifts a week for 48 weeks of the year. What is his gross annual income?

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4 An experienced childcare director gets a salary of $49 913. What is her fortnightly gross pay?

5 A systems analyst starts work on a salary of $55 712. How much is his weekly gross pay?

6 An administrative officer with a shire council gets a salary of $36 441. What is her gross fortnightly pay?

7 A casual cleaner works 25 hours a week and is paid $15.72 per hour. Calculate his annual gross income if he takes 3 weeks holiday unpaid during the year.

8 A grader driver gets $687.00 a week. What is his annual gross income?

9 A cost accountant working for a manufacturer gets a salary of $68 500. What is her gross monthly pay?

10 A clerical worker employed permanent full-time is paid $659.10 a week.
- a Calculate the hourly rates for permanent and casual clerical workers employed at the same level.
- b How much will a clerk employed at the same level on a casual basis earn for working 25 hours in a week?

11 Sally, an assistant childcare worker who is employed on a casual basis, is paid $19.10 an hour. Her friend Max works permanent part-time as a childcare assistant at the same childcare centre.
- a Calculate Max's hourly rate.
- b What do Sally and Max earn in a week in which they both work for 28 hours?

12 A firm employs signwriters on a casual and a permanent basis. If a permanent signwriter earns $627.53 for a full-time week, what hourly rate should be paid to casual signwriters?

13 Cara is employed as a permanent full-time beverage attendant at Sloppy Joe's restaurant and is paid $618.50 per week. She receives paid meal and rest breaks. Her roster for one week is shown here.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Start		10:30 am	11:30 am	10:00 am	11:30 am	12:15 pm	5:15 pm
Finish		8:30 pm	5:30 pm	7:45 pm	10:00 pm	11:00 pm	9:30 pm

- a How many hours did Cara work?
- b How many hours overtime did she work?
- c What was her gross wage for the week?

14 Kyle works at Brodie's Chicken Haven over the December–January school holiday period. As a 17-year-old, his hourly casual rate is $10.77 and he isn't paid any overtime. His roster for a week in mid-December is shown. Paid meal and rest breaks are included.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Start	10:00 am	10:30 am		9:30 am	10:00 am	noon	noon
Finish	4:30 pm	5:45 pm		6:15 pm	7:30 pm	9:15 pm	7:30 pm

What was Kyle's gross wage for this week?

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Modelling and problem solving

15 Ralph is a permanent full-time plasterer who works Monday to Friday and currently earns $668.42 a week. He doesn't work any overtime. Ralph wants to spend one day a week looking after his grandchildren. If his employer agrees to let Ralph work as a casual employee, how many hours a week will he need to work to earn the same gross weekly wage?

16 Shelly is employed 5 days per week as a cook in a BYO restaurant. During the year, she takes 4 weeks off for a vacation and has 6 days off due to illness. According to the relevant award, the weekly wage for a permanent employee in Shelly's job is $639.30.

a Calculate Shelly's annual gross income if she is employed as a permanent and works 38 hours a week.

b Calculate Shelly's annual gross income for this year if she is employed as a casual and usually works 38 hours a week.

c Should Shelly prefer casual or permanent employment? Justify your decision.

5.2 Commission and piecework

Some workers earn money on **commission**, which means that they get a percentage of the value of sales or of a service provided. Most people working on a commission or part-commission basis are involved in direct sales. For example, insurance agents, travelling salespeople and real-estate salespeople usually work on a commission basis.

Sometimes a **retainer** is paid as well as a commission. This is a small payment made regardless of sales to act as a 'safety net' for times when the commission payment is small.

Investigation Commission payments

- Collect information on various types of commission payments.
- Find out percentages paid as commissions and relate these to the value of the goods sold.
- For the situations you investigate, find out whether or not a retainer is paid. If a retainer is paid, relate this to the level of commission paid.
- You could try to contact people employed as:
  - car salespeople
  - real-estate agents
  - insurance salespeople
  - taxi-drivers
  - party-plan sales organisers
  - door-to-door salespeople.
- Document your findings and write down some conclusions about the variety of commission and retainer payments you found.

Example 6

A party-plan hostess gets 25% commission on the value of cosmetics sold at the 'party'. What does she earn when sales of $740 are made?

Solution

This is a 'straight' commission.	Amount earned = commission
Calculate the percentage of total sales.	= 25% of $740
Convert % to a decimal.	$= 0.25 \times \$740$
Evaluate.	= $185

Example 7

A confectionery saleswoman who sells to corner stores gets a retainer of $180 a week, plus a commission of 12%. What does she get in a week when she sells $2500 worth of lollies?

Solution

Calculate the value of the commission.	Commission = 12% of $2500
	$= 0.12 \times \$2500$
Evaluate.	= $300
Calculate income for the week.	Income = retainer + commission
	= $180 + $300
Evaluate.	= $480

Some people are paid according to the number of items that they make or handle. This is called **piecework** and is common among people who work from home. People working from home are called **outworkers**. In the clothing industry, outworkers are paid according to the number of garments they finish. Some accountants who specialise in income-tax returns are paid according to the number of returns they process. The delivery of advertising material may be paid on the basis of how much is delivered. Piecework in a factory is sometimes paid for by means of an **incentive** or **bonus** in addition to a normal wage.

Example 8

A seamstress is paid $8.60 an hour plus 85c for each shirt collar she sews on in a factory assembly line. How much does she earn for an 8-hour day if she attaches 90 shirt collars?

Solution

Calculate the time payment.	Time payment $= \$8.60 \times 8$
Evaluate.	= $68.80
Calculate the piecework payment.	Piecework payment $= \$0.85 \times 90$
Evaluate.	= $76.50
Calculate total earnings.	Total earnings = $68.80 + $76.50
Evaluate.	= $145.30

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Additional Exercise 5.2

Exercise 5.2 Commission and piecework

1 A taxi-driver works on 45% commission. After working a normal 12-hour shift he has taken $260 in fares. What does he earn for the shift?

2 Another taxi-driver works on 52.5% commission. During one week she takes $985 in fares. What does she earn for the week?

3 If a driver working on 45% commission makes $427.50, what is the total taken in fares?

4 A life-insurance saleswoman works on a retainer of $150 a week plus a commission of 0.2% of the value of policies sold. One week she sells policies worth $120 000, $85 000 and $240 000. What is her gross income for that week?

5 A real-estate saleswoman works on a straight commission basis, but the commission varies with the value of the property sold. For each property, the commission is separately calculated as 1.5% of the value of the property between $200 000 and $300 000 and 0.5% for the remainder. One month she sells properties for $280 000, $475 000 and $245 000. What is her gross income for that month?

6 Grape pickers are paid $2.90 a tin.
  a How much would a picker earn if she picked 80 tins of grapes in a 12-hour day?
  b What would her hourly rate be?

7 An advertising material delivery firm pays its delivery people 6.5 cents per leaflet.
  a How much would you earn if you delivered to 850 houses?
  b How many houses would you have to deliver material to in a day to make $180?
  c If you are delivering a second leaflet as well, you get an extra 2.45 cents a leaflet for the second one. How many deliveries would you need to make in this case to get $180?

8 An outworker for a clothing manufacturer makes $3.60 for every pair of shorts she sews. One day she sews 50 pairs of shorts. What does she earn?

9 A computer manufacturer pays $18.50 for each motherboard that is assembled and tested. How much would you earn for doing 23 motherboards?

Modelling and problem solving

10 A pizza parlour pays $5.20 per delivery. One night, Peter delivered 24 pizzas between 5:30 pm and 11:30 pm, but also covered 210 km in his car.
  a How much did he get from the pizza parlour?
  b If his car uses 11 L/100 km and petrol costs him $1.23/L, how much did he pay for petrol?
  c What were Peter's real earnings and hourly rate. Explain your reasoning.

11 Aziz sells music CDs that retail for $28 each. He has the option of working for either a straight commission of 15% or a commission of 8% and a retainer of $150 a week. How many CDs would he need to sell in a week to be better off on commission? Justify your decision.

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5.3 Industrial awards

Many people work under an **industrial award** that sets out the exact conditions of work and pay. It is usually negotiated between employers and unions representing workers in an industrial court or arbitration commission. Awards provide employment condition details such as:

- pay rates, overtime rates and special payments
- leave (sick, parental, annual, etc.)
- breaks for meals
- termination of employment
- superannuation
- hours of work.

Awards are public, legal documents that are available for everyone to see, so no secret conditions apply. Some employees and employers will not have an industrial award but may work according to an individual contract or agreement. Awards are registered to ensure that the interests of all parties are protected.

Investigation Industrial awards

- Use the summary sheet for the Fast Food Industry Award in Appendix 2 on pages 388–94 and work in small groups to find the wage for one week for a fast-food industry worker who:
  - is employed full-time;
  - has a standard 38-hour week; and
  - worked 46 hours including 5 hours between 11:00 pm and 4:00 am.
- Obtain the full text of some industrial awards and work out what your starting wages would be if you obtained a job at the end of Year 12. This will make more sense if you choose to investigate a job you would like to have.
- What could you expect to earn by the time you were 20? How about at 25?
- You can obtain awards from Wageline at *www.wageline.qld.gov.au* or by looking in the *Industrial Gazette*. Other information about employment and industrial relations can be obtained by visiting the website of the Department of Employment and Industrial Relations at *www.deir.qld.gov.au*.

Summary sheets from other industrial awards may be found on the CD-ROM.

Extra Material
Wageline award summary sheets

Additional Exercise 5.3

Exercise 5.3 Industrial awards

Use the award summary sheets in Appendix 2 on pages 388–94 to answer the following questions. Note that the words ‘weekly’ and ‘part-time’ refer to permanent employees.

1 What is the minimum period of work per day that a part-time hairdresser can be asked to do?

2 What is the penalty rate for a registered nurse on an afternoon shift?

3 What is the weekend penalty rate for a fast-food worker?

4 Work out the gross wage for a week for each of the following.
 a Mandy, an 18-year-old Level 2 full-time fast-food worker
 b Max, a Level 5 part-time hairdresser, who works 23 hours a week including 5 hours on Saturday
 c Michelle, a full-time Level 3 registered nurse in her first year who works 5 days a week and wears a uniform but does not assist with X-ray treatments

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5 What is the hourly rate for a 17-year-old Level 3 casual fast-food worker for hours worked between 1:00 am and 4:00 am on a Friday morning?

6 What is the hourly rate for the first 3 hours of overtime worked by a 25-year-old part-time Level 2 hairdresser?

7 Work out the week's gross wage for each of the following.
   a a Level 2 part-time fast-food worker who works 25 hours over 5 days
   b a 16-year-old full-time Level 1 hairdresser who works 38 hours
   c a casual first-year Level 2 registered nurse who wears a uniform and works 30 hours over a 7-day period including 4 hours on each of Saturday and Sunday
   d a 19-year-old Level 2 casual fast-food worker who works 43 hours, including 12 hours over the weekend
   e a full-time apprentice hairdresser employed on Level 3 for 38 hours
   f a part-time registered specialist medical centre nurse in his fifth year who works 30 hours including 4 hours on Saturday and 3 hours on Sunday
   g a full-time manager of a hairdressing business who works a normal 38-hour week and in addition works 5 hours overtime and 4 hours on Sunday
   h a Level 3 casual fast-food worker who works 28 ordinary hours and in addition works 4 hours on Sunday and 3 hours between 1:00 am and 4:00 am on Thursday

Investigation Government payments

The Federal Government uses the taxes collected to pay for things such as building major infrastructure (for example, national highways), running the defence forces of Australia and providing a system of **social security**. The government social security system is designed to give financial help to people who need assistance. The aged, sick, unemployed, disabled and others who are in genuine need of assistance are able to go to the Federal Government agency called Centrelink, which is responsible for providing social security assistance.

This investigation aims to help you to understand Australia's social security system. You could visit the Centrelink website at *www.centrelink.gov.au,* go to a Centrelink office or see the Guidance Officer at your school to find out the information needed to answer the following questions.

People who apply for assistance under the system of social security are assessed by Centrelink staff to determine their eligibility for certain forms of payment or other assistance. The term 'income support' is often used to refer to all forms of social security payments.

- Find out about various types of social security payments, such as the Age Pension, Family Tax Benefit, Disability Support Pension, Sickness Allowance, Carer Payment and Carer Allowance, Newstart Allowance and Youth Allowance, also Austudy and Abstudy.
- For each type of payment you investigate, find out what the payment is designed to do, who is eligible to claim the payment and how much recipients of the payment receive.

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Investigation continued

Most forms of income support are subject to a **means test**. A means test determines whether or not an individual or family is eligible to receive certain types of benefits from the government and, if they are eligible, their level of payment. The funds available for income support are limited, so means tests make sure that the people most in need receive the greatest level of assistance. The means test may apply to the income and assets of both the applicant and their spouse.

- Find out whether Youth Allowance, Austudy and Abstudy are means tested. If so, describe the means test(s) and explain how any means test could affect payments to recipients.

Australia's social security system has evolved over time to its current form. Other countries have similar systems of social security.

- Select a number of major countries from South-East Asia, Oceania, Europe and South America and add these to a list with Canada and the USA.
- Use the Internet or any other method to find out which of these countries have social security systems.
- How many of these countries have a payment similar to the Youth Allowance?
- For those countries with a payment like the Youth Allowance, compare the methods of calculating entitlements for individuals.
- Using the Australian Youth Allowance as your basis for comparison, comment on the complexity, fairness and level of payments made for any of the youth payment systems that you find.

5.4 Income tax and take-home pay

When you earn a wage or salary, your employer must deduct **income tax** from your weekly or fortnightly pay and send it to the Australian Taxation Office (ATO). The tax paid is called **Pay As You Go (PAYG)** withholding tax. It is called a **withholding tax** because it must be withheld from your pay. PAYG is paid by your employer to the Federal Government through the ATO. Nobody likes paying tax, but the money the government spends comes from taxes. Up-to-date information about PAYG and other aspects of taxation may be obtained from the ATO website at *www.ato.gov.au*.

The amount of PAYG withholding (or income) tax depends on:

- how much you earn
- whether you claim the tax-free threshold
- whether you claim the Family Tax Benefit or various rebates
- whether you have a tax file number.

The **tax-free threshold** (or **general exemption**) is the amount of income that can be earned in a year before you start paying tax. You claim it by completing a **Tax File Number Declaration** form for your employer. The tax-free threshold of $6000 can only be claimed from one employer at a time, so if you have more than one job at the same time it can only be claimed for one. The ATO gives employers a schedule of the tax instalments (**tax tables**) so that they can withhold the correct amounts. It is important that they do so, or you could be faced with a big bill when you complete your yearly income tax return.

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The tax tables for weekly earnings up to $2500 are given in Appendix 3 on pages 395–402. The tables show two columns where the tax-free threshold has been claimed (columns 2 and 3). Employees who are entitled to leave loading are paid an additional amount as a percentage of their annual leave payment. The column for the tax-free threshold *with* leave loading (column 2) has slightly higher tax instalments to take account of the tax to be paid on the leave loading.

If a **tax file number** has not been supplied to your employer, 46.5% of your income will be withheld as tax.

Some people are entitled to a **Family Tax Benefit** or other tax **rebate**. These are amounts of tax paid that the ATO returns to the employee because he or she has a dependent spouse, has dependent children or lives in an isolated area, or for other special reasons. Employees can choose to anticipate their yearly tax rebate or Family Tax Benefit and reduce their PAYG withholding tax, rather than wait to collect the rebate at the end of the financial year.

The following ready reckoner is used to calculate the reduction in PAYG withholding tax for various rebates (or **tax offsets**). The ready reckoner is repeated on the CD-ROM.

Extra Material

Ready reckoner tax table

READY RECKONER FOR FAMILY TAX BENEFIT AND TAX OFFSETS

Amount claimed ($)	Weekly value ($)	Amount claimed ($)	Weekly value ($)	Amount claimed ($)	Weekly value ($)	Amount claimed ($)	Weekly value ($)	Amount claimed ($)	Weekly value ($)
1	–	20	–	200	4.00	1000	19.00	1800	34.00
2	–	30	1.00	300	6.00	1100	21.00	1900	36.00
3	–	40	1.00	338	6.00	1173	22.00	2000	38.00
4	–	50	1.00	400	8.00	1200	23.00	2051	39.00
5	–	57	1.00	500	10.00	1300	25.00	2100	40.00
6	–	60	1.00	600	11.00	1400	27.00	2500	48.00
7	–	70	1.00	700	13.00	1500	29.00	3000	57.00
8	–	80	2.00	770	15.00	1540	29.00		
9	–	90	2.00	800	15.00	1600	30.00		
10	–	100	2.00	900	17.00	1700	32.00		

If the exact rebate claimed is not shown in the ready reckoner, add the values for an appropriate combination of rebates. For example, if an annual rebate of $452 is claimed, add the values for rebates of $400, $50 and $2.

$$\begin{aligned}\text{Reduction} &= \$8.00 + \$1.00 + \$0.00\\ &= \$9.00\end{aligned}$$

Reduce the tax amount shown in column 2 ('With tax-free threshhold with leave loading') of the tax tables (Appendix 3) by $9.

Example 9

Janice works as a cleaner and receives a gross wage of $630.70 a week. She claims the tax-free threshold and is entitled to leave loading. Use the tax tables in Appendix 3 to find the tax that will be deducted.

Solution

Whole dollars are used and cents ignored.	$630.70 ≈ $630
Look up $630 in the 'With tax-free threshold with leave loading' column.	PAYG tax = $97.00

Example 10

Peta is an IT worker and her husband stays home to look after the children. She claims a Family Tax Benefit of $1450 and earns a salary of $52 800. Peta is paid a leave loading. What are her fortnightly gross pay and the tax that will be deducted from it?

Solution

We need to work in weekly amounts.	
Calculate gross weekly pay.	Gross pay = $52 800 ÷ 52
Evaluate.	= $1015.38
Locate relevant PAYG amount for $1015.	Tax = $219.00
Use the ready-reckoner to calculate the tax reduction due to $1450 Family Tax Benefit.	Reduction = $27.00 + $1.00 = $28.00
Calculate reduced weekly tax.	Tax (reduced) = $219.00 – $28.00
Evaluate.	= $191.00
Calculate fortnightly amounts and state the answer.	Her fortnightly gross pay is $2030.76 and the tax deducted is $382.00.

The Australian Tax Office has an online calculator that can be used to work out tax.

Example 11

Konrad earns $845.70 a week and claims a rebate to the value of $630. He is not paid leave loading. What is his tax?

Solution

Use the ‘no leave loading’ column in Appendix 3.	Tax = $163.00
Use the ready-reckoner to calculate the reduction for the rebate.	Reduction = $11.00 + $1.00 = $12.00
Calculate the reduced weekly tax.	Tax (reduced) = $163.00 – $12.00
Evaluate.	= $151.00

Income tax is only one deduction that may be made from your pay. Some people have superannuation deducted from their pay, and many choose to have such things as medical insurance, union dues, house payments and car payments deducted. They find it easier to have these deducted each week instead of saving for them. **Gross pay** is the amount you earn before any deductions are made. **Net pay** is the amount left after all deductions are made—what you actually get in your ‘pay packet’. Some people are paid by **Electronic Funds Transfer (EFT)**, which pays the net pay straight into their bank, credit union or building society account.

Example 12

Sonia earns \$880.60 a week. Her income tax is \$176 and she pays 5% of her gross pay for superannuation. She has medical insurance (\$46.60 a week), car payments (\$112.18 a week) and car insurance (\$28.65 a week) deducted. What are her total deductions and net pay?

Solution

First calculate the deduction for 'super'.

$$\begin{aligned}\text{Superannuation} &= 5\% \text{ of } \$880.60\\ &= 0.05 \times \$880.60\\ &= \$44.03\end{aligned}$$

Next total all deductions.

Deduction	*Amount*
Income tax	\$176.00
Superannuation	\$44.03
Medical insurance	\$46.60
Car payment	\$112.18
Car insurance	\$28.65
Total	\$407.46

Calculate net pay.

$$\begin{aligned}\text{Net pay} &= \text{gross pay} - \text{deductions}\\ &= \$880.60 - \$407.46\\ &= \$473.14\end{aligned}$$

State the result.

Total deductions are \$407.46, net pay is \$473.14.

Exercise 5.4 Income tax and take-home pay

Additional Exercise 5.4

For the following questions, assume that the tax-free threshold is claimed and leave loading is paid, unless otherwise stated. Tax instalments are calculated in whole dollars.

1 Andrea is a nurse earning \$721.40 a week. What weekly tax instalments are deducted?

2 Roger doesn't bother to give his tax file number to his employer, as he only plans to work for a few weeks as a fruit picker. He earns \$694.76 for the first week's picking.
  a What tax is deducted?
  b How much tax would he pay if he submitted a tax file number and claimed the tax-free threshold?

3 Catherine earns \$476.02 a week as a cake finisher. She doesn't know how to fill in the Tax File Number Declaration form properly, so she doesn't claim the tax-free threshold. She does give her tax file number though.
  a How much tax is deducted from her pay?
  b How much tax would she pay if she claimed the tax-free threshold?

4 Rae earns \$1196.64 a fortnight as an administrative assistant. Because she is employed as a casual, she is not paid leave loading. Rae claims a Family Tax Benefit of \$2431 and claims the tax-free threshold. What tax is deducted from her pay each week?

5 Paul has a dependent wife and two children. He earns \$1859.70 a fortnight. He claims Family Tax Benefit of \$1865. What is the tax on a fortnight's pay?

6 Robyn usually earns \$654.80 a week, and she does claim the tax-free threshold. One week she works a couple of hours overtime, so her gross pay goes up by \$42.38. How much of this does she lose in tax?

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7 Darrin is a psychiatric nurse and earns $2113.63 a fortnight. He lives by himself and doesn't claim the tax-free threshold as a way of forcing himself to save, because he will receive the tax he has overpaid as a refund when he completes his tax return.
   a How much extra is deducted in tax per week by his not claiming the tax-free threshold?
   b How much extra tax does he pay in a full year?

8 John works on a mining site. He is able to claim a rebate of $865. What tax does he pay on his $987.55 weekly wage?

9 Tran has two jobs. He earns $477.80 a week as a sales assistant at a Northfield Shoppingtown and $369.70 a week as a delivery driver at Pizzaworld. He claims the tax-free threshold for his major source of income and isn't paid leave loading.
   a How much tax does Tran pay a week?
   b How much would he pay if he earned the same gross income in a single job?

10 Charlotte earns $1236.90 a fortnight. She has union dues of $5.80 a week and medical insurance of $38.80 a week deducted from her pay. Find the tax she pays, her total deductions and net pay per fortnight.

11 Adam is paid a gross fortnightly wage of $1714.70. He has his medical insurance of $86.40 a fortnight deducted from his pay and pays a superannuation contribution of 9% of his gross wage. Work out the tax Adam pays, his total deductions and net pay each fortnight.

Modelling and problem solving

12 Dianna works for 42 hours one week. She is paid at ordinary time for 38 hours, at $21.72 an hour. Overtime is paid at time-and-a-half for the first 2 hours and double-time thereafter. She has $125.45 a week deducted for a car payment and pays 6% of her gross basic pay (not including overtime) for superannuation. She is not paid leave loading but does claim the tax-free threshold. She does not have medical insurance. Work out her gross pay, tax, total deductions and net pay for the week.

Extra Material

Income tax returns

13 Michelle works at a resort as a cleaner. Her base rate is $18.60 an hour, but because she is a casual employee she gets a 23% loading but does not receive a leave loading. She works for 32 hours in one week. She has union dues of $5.30 a week deducted from her pay. Find her gross pay, tax, total deductions and net pay for the week.

5.5 Other government taxes

About half of all Australian Federal Government revenue comes from personal income tax, but other taxes are also important. Tax on company profits contributes about 15% of revenue, and the **goods and services tax (GST)** is the next most important tax.

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The GST is a broad-based tax of 10% on the supply of most goods, services and anything else consumed in Australia. GST-free items include most food, most health and educational services, local government rates and charges, exports, religious services and non-commercial activities of charitable institutions.

The amount of GST payable on taxable goods and services is always 10% of their value, and the amount of GST is always included in the price of a supplied item or service. To work out how much GST is included in the price, divide the price by 11.

! **Goods and services tax**

$$\text{GST} = \text{price} \div 11$$

Example 13

A retailer calculates the pre-GST price of a watch to be \$420. Find the GST that must be added and the price paid for the watch by the customer.

Solution

Find 10% of the price.

$$\begin{aligned}\text{GST} &= 10\% \text{ of } \$420 \\ &= \$42\end{aligned}$$

Add GST to price.

$$\begin{aligned}\text{Price paid} &= \$420 + \$42 \\ &= \$462\end{aligned}$$

Example 14

Pam pays \$3300 for a new computer at a major retail store. How much GST is included in the price?

Solution

Divide price by 11.

$$\begin{aligned}\text{GST} &= \text{price} \div 11 \\ &= \$3300 \div 11 \\ &= \$300\end{aligned}$$

GST is paid to the ATO at each step of the supply chain, with businesses charging GST in the prices of goods, services and anything else they supply. Businesses claim any GST they have paid as **tax credits**, so the GST is really only paid once, by the end consumer.

Example 15

Mike is a timber merchant and he sells Guilia some timber for \$110. Guilia makes furniture and she uses the timber to make a coffee table, which she sells to Ali, a furniture retailer, for \$330. Ali sells the coffee table to a customer for \$660.

a Calculate the GST involved in each of the transactions.

b Calculate the GST amount paid to the ATO by each person in the supply chain after tax credits are claimed.

c Calculate the total GST paid to the ATO.

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Solution

Draw a diagram to show the supply chain.

Mike (timber merchant)
Sells timber (GST-inclusive price = \$110)
Guilia (furniture manufacturer)
Sells coffee table (GST-inclusive price = \$330)
Ali (furniture retailer)
Sells coffee table (GST-inclusive price = \$660)
Customer

a Calculate GST paid by Guilia to Mike.

$$\text{GST} = \$110 \div 11 = \$10$$

Calculate GST paid by Ali to Guilia.

$$\text{GST} = \$330 \div 11 = \$30$$

Calculate GST paid by customer to Ali.

$$\text{GST} = \$660 \div 11 = \$60$$

b Calculate GST received by Mike. GST received by Mike = \$10

Calculate GST paid to ATO by Mike. GST paid to ATO = \$10

Calculate GST received by Guilia. GST received by Guilia = \$30

Calculate GST paid by Guilia to ATO. GST paid to ATO = \$30 − \$10 = \$20

Calculate GST received by Ali. GST received by Ali = \$60

Calculate GST paid by Ali to ATO. GST paid to ATO = \$60 − \$30 = \$30

c Calculate total paid to ATO. GST paid to ATO = \$10 + \$20 + \$30 = \$60

You can see from Example 15 that, even though GST is paid at each step of the supply chain, the total amount of GST collected by the ATO is the same as the GST paid by the customer in the retail price of the item purchased.

Example 16

A furniture shop buys lounge suites from a distributor for \$1420. The shop works on a mark-up of 75%. Calculate the retail price of the lounge suites including GST.

Calculate the mark-up. Mark-up $= 75\%$ of \$1420

Evaluate. $= 0.75 \times \$1420$

$= \$1065$

Calculate the marked-up price. Marked-up price $= \$1420 + \$1065$

Evaluate. $= \$2485$

Calculate the GST. GST $= 10\%$ of \$2485

Evaluate. $= \$248.50$

Calculate the GST-inclusive retail price. Retail price = marked-up price + GST

$= \$2485 + \$248.50$

$= \$2733.50$

Duties are an important source of tax revenue for state governments. Duties are taxes on certain transactions and documents. For example, duty is paid on insurance contracts, transfers of motor vehicle registration, property purchases and mortgage agreements. The rates of duty vary, but some examples are given below.

Situation	Duty rate
Registration or transfer of registration of a motor vehicle	\$2 for each \$100, or part of \$100, of the full purchase price of the vehicle
Purchase of a residence* that is intended to be the home of the purchaser:	
Up to \$320 000	\$1.00 per \$100, or part of \$100
\$320 001 to \$500 000	\$3200 + \$3.50 per \$100, or part of \$100, over \$320 000
\$500 001 to \$700 000	\$9500 + \$4.00 per \$100, or part of \$100, over \$500 000
More than \$700 000	\$17 500 + \$4.50 per \$100, or part of \$100, over \$700 000
Purchase of a property that is not intended to be the home of the purchaser:	
\$100 000 to \$250 000	\$2350 + \$3.25 per \$100, or part of \$100, over \$100 000
\$250 001 to \$500 000	\$7225 + \$3.50 per \$100, or part of \$100, over \$250 000
\$500 001 to \$700 000	\$15 975 + \$4.00 per \$100, or part of \$100, over \$500 000
More than \$700 000	\$23 975 + \$4.50 per \$100, or part of \$100, over \$700 000

* Concessions are available if the residence will be the first home of the purchaser.

Example 17

Dwayne buys a second-hand car for \$12 000 on hire-purchase, paying \$5000 deposit. How much duty will he pay for the transfer of registration?

Solution

To find the duty on the transfer of registration, we need to work out how many \$100s there are in the purchase price of the car.

Divide purchase price by \$100. Number of \$100s $= \$12\,000 \div \$100 = 120$

Calculate the duty. Duty $= \$2 \times 120$

Evaluate. $= \$240$

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Example 18

Julia bought a home unit for \$325 000 to rent out to a tenant. How much duty must she pay on the purchase of the property?

Solution

The property will not be Julia's home.	Duty = \$7225 + \$3.50 per \$100 over \$250 000
\$3.50 per \$100 is 3.5%.	= \$7225 + 3.5% of \$75 000
	= \$7225 + 0.035 × \$75 000
	= \$7225 + \$2625
Evaluate.	= \$9850

Additional Exercise 5.5

Exercise 5.5 Other government taxes

1 Calculate the amount of GST that needs to be added to the pre-GST price of each of the following items.

a new car: \$15 000
b camera: \$320
c novel: \$45
d chainsaw: \$350
e dinner set: \$220
f suitcase: \$180

2 How much GST is paid by the consumer in the purchase price of each of the following?

a boat: \$8800
b computer: \$3300
c chair: \$154
d bottled water: \$2.75
e Lego set: \$29.15
f tennis racquet: \$200

3 The Maxwell family is reviewing some of the bills paid for various services in the last 6 months. How much GST did they pay in each case?

a new stormwater drain: \$1364
b tax return preparation: \$598
c car service: \$352
d telephone account: \$462
e electricity account: \$444.80
f driving lessons: \$120

4 Calculate the duty that would be paid on each of the following.

a registration transfer for a car worth \$25 900
b registration transfer for a prime mover worth \$85 000
c transfer of registration for a motorbike worth \$7200
d sale of an investment unit for \$423 000
e sale of a house to an owner-occupier for \$390 000
f sale of an investment property for \$620 000

Modelling and problem solving

5 Wendy makes craft items for a gift shop. She sells a teddy bear to Julie, who runs All Occasions Gifts, for \$33 (including GST). Julie sells the teddy bear to a customer for \$71.50.

a Calculate the GST paid by Julie and the customer.
b How much GST is paid to the ATO by each person in the supply chain after tax credits are claimed?

6 Tran sells 20 m of fabric to John, who is an upholsterer. John pays a GST-inclusive price of \$924 for the fabric. John uses the fabric to re-cover a lounge suite for his customer, Mandy. Mandy pays John \$2310, which includes GST.

   a Calculate the GST paid by John and Mandy.

   b How much GST is paid to the ATO by each person in the supply chain after tax credits are claimed?

7 A computer retailer has a mark-up of 85% on all items. She pays \$123 to a supplier for an inkjet printer. Calculate the retail price of the printer and the amount of GST paid by the customer.

8 A sports store marks up brand-name soccer boots by 150%. If a customer pays \$140.25 for a pair of boots, how much did the store pay for them?

Investigation Barter

A system of **barter** is one in which goods and services are exchanged for other goods and services without money changing hands. Historically, barter was superseded because money is more efficient. In recent years there have been attempts to establish registers of people willing to barter.

A **swap column** in a local newspaper is an example of a simple barter system. If you barter goods or services, you must declare this on your income tax return. The ATO will assess what income tax is payable on bartered goods or services.

A **barter register** is like an organised swap column. In this investigation you will draw up a barter register for the class.

■ **1** Work in groups of 4 or 5 to draw up a list under the headings shown below.

2 In the 'Have' column, put things you no longer want or are willing to swap.

3 In the 'Want' column, put things that you want to get by swapping what you have.

Name	Have	Want

4 Now pool the results of the groups to create a class register. Perhaps some people can actually find a swap in the class.

■ Discuss the register you have created. What problems does your register indicate with barter systems? Are the swaps that people envisage of equal value?

■ Discuss problems that you think could be encountered in a larger barter register.

■ What do you think are the potential problems with bartering services?

■ Bartercard and other organised barter systems actually used a form of currency to assist in the exchange of goods and services. Visit the websites of Bartercard, BigVine, BarterTrust, Tradeway, BarterNet and others to see how they work.

■ Do these organised barter systems overcome the potential difficulties associated with barter that you discussed previously?

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Chapter summary

- **Gross income** is the income earned before any deductions are made.
- A **wage** is income that is paid by the week for the hours that are worked, while a **salary** is income that is calculated by the year and paid each fortnight or month.
- A person employed **full-time** usually works a 38-hour week, and **part-time** employees work for fewer hours.
- **Permanent** employees may be full-time or part-time and are entitled to benefits such as paid sick leave and annual leave. **Casual** workers are engaged to work by the hour and do not receive these benefits. They are paid a **loading** to compensate for this.
- Hours worked in excess of ordinary working hours are paid at a higher rate called **overtime**.
- A **commission** is a percentage of the value of sales or of the service provided. Some people are paid a commission instead of a wage or salary. A **retainer** is a small payment that is made regardless of sales and may be paid as well as a commission.
- Being paid according to the number of items produced is called **piecework**. People working from home doing piecework are called **outworkers**.
- Many people work under an **industrial award**, which sets out the exact conditions of work and pay. Some people's work conditions are set according to an individual contract or agreement.
- The Federal Government uses some of the taxes it collects to fund the **social security** system. The social security system provides payments to people who need help.
- Social security payments are subject to a **means test**, which may cover both an income test and an assets test.
- Employers must deduct **income tax** from wages and salaries. Income tax is withheld by the employer and sent to the Australian Taxation Office (ATO).
- The income tax required to be paid can be reduced if a worker is entitled to a **Family Tax Benefit** or other tax **rebate (tax offset)**.
- **Gross pay** is the amount you earn before any deductions are made. **Net pay** is the amount left after all deductions have been made.
- The **goods and services tax** (GST) is a broad-based tax of 10% on the supply of most goods, services and anything else consumed in Australia.

  $$\text{GST} = \text{price} \div 11$$
- **Duty** is generally paid on signed documents such as the transfer of vehicle registration, hire-purchase agreements and property purchases.
- A system of **barter** is one in which goods and services are exchanged for other goods and services without money changing hands.

Chapter review

Knowledge and procedures

1 A delivery-van driver earns $625.40 a week. What is his annual gross income? Ex 5.1

2 A pharmacist working at a large chemist shop making up prescriptions is on a salary of $57 890. What is her fortnightly pay? Ex 5.1

3 The hourly rate for bar attendants at the Happy Vale Country Club is $17.46. Permanent full-time staff work a 38-hour week and overtime is paid at time-and-a-half for the first 3 hours overtime and double-time for any hours thereafter. Casual employees are paid a 23% loading on the normal hourly rate. All Happy Vale employees receive paid meal and rest breaks. Calculate the gross weekly wages for each of the following Happy Vale employees. Ex 5.1

a Rachael works full-time as a permanent employee. Her start and finish times for the week are as follows.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Start		11:30 am	10:30 am	11:00 am	11:30 am	12:15 pm	4:15 pm
Finish		8:30 pm	5:30 pm	7:45 pm	10:00 pm	10:00 pm	9:30 pm

b Rowan is a casual whose start and finish times for the week are as follows.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Start	11:30 am			10:00 am	noon	2:00 pm	2:30 pm
Finish	6:30 pm			6:30 pm	10:00 pm	midnight	7:30 pm

4 Suzanne is a manufacturer's representative who works on a 15% commission with a retainer of $250 a week. One week she persuades 40 shops to buy a complete display of hand tools at $145 each. How much does she earn? Ex 5.2

5 Lee does quality control for an importer of radio-controlled toy cars. He gets paid $3.90 for every car he inspects. One day he checks 43 cars. How much does he get? Ex 5.2

6 Use the industrial awards in Appendix 2 on pages 388–94 to find the gross weekly wages for: Ex 5.3

a Anne, a Level 2 fast-food worker, who worked 5 hours overtime in addition to her normal 38 hours for the week

b Maxine, a Level 5 casual hairdresser, who worked 7 hours overtime in addition to her normal 24-hour week

c Sam, a second-year Level 2 registered nurse, who completed his 38-hour week from Monday to Friday and then worked 6 hours overtime on Sunday.

7 Alina earns $680.75 a week. She is paid a leave loading and claims the tax-free threshold. What tax is taken out of her pay? Ex 5.4

8 Andrew is supporting his mum and earns $1220 a fortnight. He is paid a leave loading, claims the tax-free threshold and also claims rebates totalling $2460. How much tax does he pay each fortnight? Ex 5.4

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Chapter review

Ex 5.4 **9** Sondra's ordinary gross pay is $693.50 for a 38-hour week. She claims the tax-free threshold, is paid a leave loading and has deductions of $6.80 a week for union dues and $59.60 a week for medical insurance. What is her net pay in a week in which she does 2 hours overtime at time-and-a-half?

Ex 5.5 **10** A hardware store buys a handbasin from an importer for $215.60, including GST. The store works on a mark-up of 75%. Work out:

a the GST paid to the importer

b the retail price of the handbasin

c the GST paid by the customer.

Ex 5.5 **11** Carly bought herself a second-hand car for $19 500. What duty did she pay on the transfer of registration?

Ex 5.5 **12** Andrea and Matt bought a home unit for $352 700, to live in. What was the duty on the purchase of the unit?

Ex 5.5 **13** What is the duty payable by an investor on the purchase of a house for $552 000?

Modelling and problem solving

Ex 5.2 **14** Nina is a representative for a gourmet food company selling food to delicatessens all over the city. She works for a commission of 19% on all sales but also has the option to be paid a retainer. If she opted for the retainer of $250, her commission rate would drop to 13%. What value of sales does she need make in a week in order to be better off on a straight commission basis?

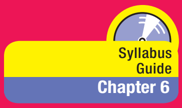
Ex 5.4 **15** Dillon works for 46 hours one week. He is paid at ordinary time for 38 hours, at $23.50 an hour. Overtime is paid at time-and-a-half for the first 2 hours and double-time thereafter. He has $135.50 a week deducted for car payments and pays 5% of his gross basic pay (not including overtime) for superannuation. He has $88.70 deducted weekly for private health insurance. Work out his gross pay, tax, total deductions and net pay.

Ex 5.5 **16** An electrical goods retailer has a mark-up of 115% on all items. He pays $875 to a manufacturer for a refrigerator. Calculate the retail price of the refrigerator and the amount of GST paid by the customer.

Location, distance and time on Earth

6

Contents



Syllabus subject matter

Elements of applied geometry

- Latitude, longitude and measurement of time and distance
- Simple algebraic manipulation of relevant formulas for this topic

Quantitative concepts and skills

- Metric measurement including measurement of mass, length, area and volume in practical contexts
- Calculation and estimation with and without instruments

For many years, it wasn't possible to know the exact positions of ships and aircraft at any instant. The worldwide **Global Positioning System (GPS)** now makes it possible to answer the simple question 'Where am I?' almost instantaneously and with a high degree of precision. GPS technology does this by utilising atomic clocks that keep time to within a billionth of a second, in conjunction with a network of navigational satellites.

The system made its public debut in the 1991 Gulf War, but has since found many applications in the civilian sector. GPS is now used for a vast array of military and civilian applications, including locating vessels lost at sea, assisting emergency vehicles to find their destinations, and helping freight and transport companies to keep track of their fleets. Car navigation systems use GPs to show your location on a street map.

6.1 Location and position

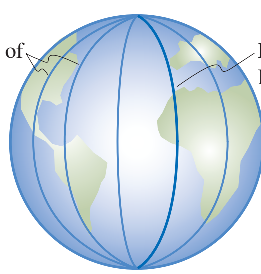
Latitude and longitude

Great circles are drawn on the surface so that their centres are at the centre of the Earth. **Small circles** do not have their centres at the centre of the Earth.

Great circles

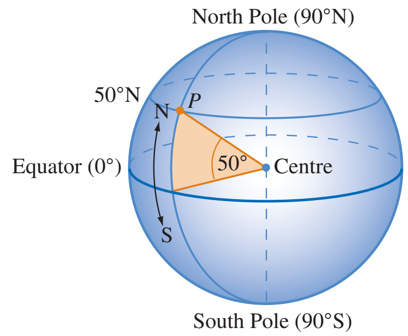
Small circles

A position on the Earth is described by its **latitude** and **longitude**. Latitude and longitude are both stated as angles, because they are measured by angles at the centre of the Earth between circles drawn on the Earth's surface. **Meridians** are great semicircles drawn between the North and South Poles. The **Prime Meridian of longitude** is the meridian passing through Greenwich, England.

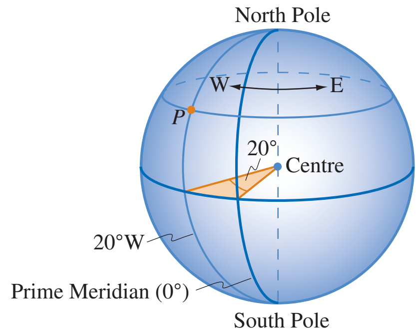


The **Equator** also is a great circle. **Parallels of latitude** are small circles drawn parallel to the Equator with their centres at the axis through the North and South Poles.

The **latitude** of a point is the angle between the Equator and the parallel of latitude passing through the point. It is measured north or south from the Equator.



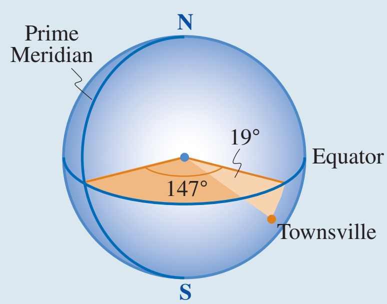
The **longitude** of a point on the Earth is the angle between the Prime Meridian and a meridian passing through the point, measured east or west to make the angle less than 180º.



When the position of a point is stated, the latitude is stated first and then the longitude—without a comma between them. So the position of Mexico City, for example, is 19ºN 99ºW.

Example 1

Use the following diagram to find the latitude and longitude of Townsville. The Prime Meridian and Equator are both shown on the diagram.



Solution

Townsville is shown as about 147º to the east of the Prime Meridian and about 19º south of the Equator.

Townsville is at 19ºS 147ºE.

Most people cannot measure angles of latitude and longitude for themselves. To find the latitude and longitude of a place, we actually look up the position in an **atlas**. More precise information can also be obtained from government mapping authorities or from the internet.

Example 2

Use an atlas to find the latitude and longitude of Mumbai (Bombay).

Solution

Large atlases may have the precise positions of major cities shown in the index. Otherwise you can find the position on a map showing India.

Mumbai is at 19ºN 73ºE.

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Investigation Using Google Earth to find locations

You can use Google Earth to find locations on the Earth.

1 Open Google Earth on your computer.
Type 'London' into the 'Fly to' input and press Enter. Wait for the location to be shown.
Now Type 'Brisbane' into the 'Fly to' input and press Enter. Wait for the location to be shown.
Now type 'Townsville' into the 'Fly to' input and press Enter. Wait for the location to be shown.

2 The latitude and longitude of the location are shown in the bottom left-hand corner. This is the location of the pointer. Move the pointer around and notice that the position shown changes. The position shown is accurate to within a metre.

3 Try zooming in and out to change the detail that is shown.

4 See whether you can find your own house and its precise location.

Additional Exercise 6.1

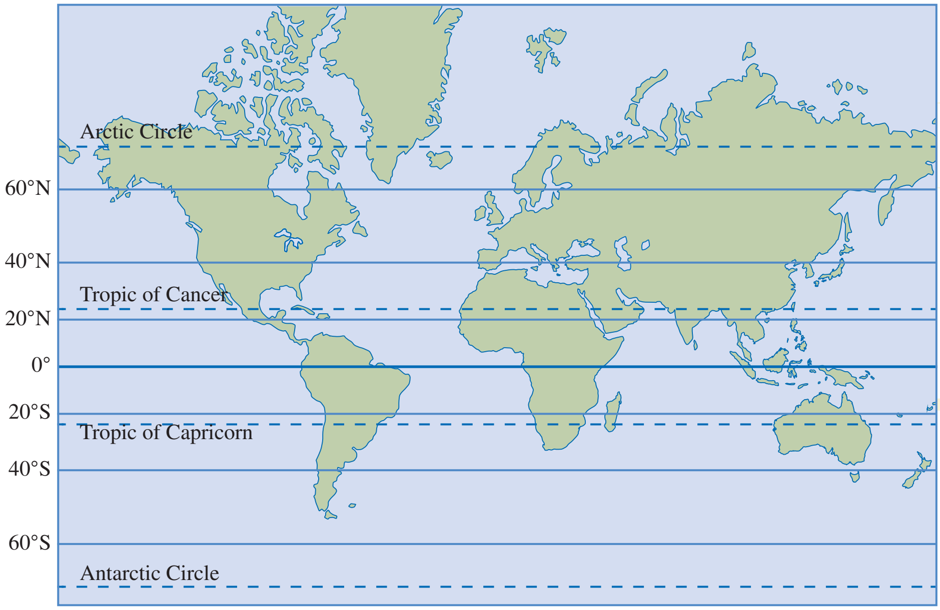
Exercise 6.1 Location and position

1 a Are all small circles the same length?
  b Which parallel of latitude is the greatest length?
  c What is at latitude 90ºS?
  d Are all meridians of longitude the same length?

2 The 80º meridian of longitude passes through the Indian Ocean. Would it be east or west?

3 Use the map below to match each of the countries with its corresponding latitude from the following list. (Consult an atlas if you are unsure of the positions of countries.)

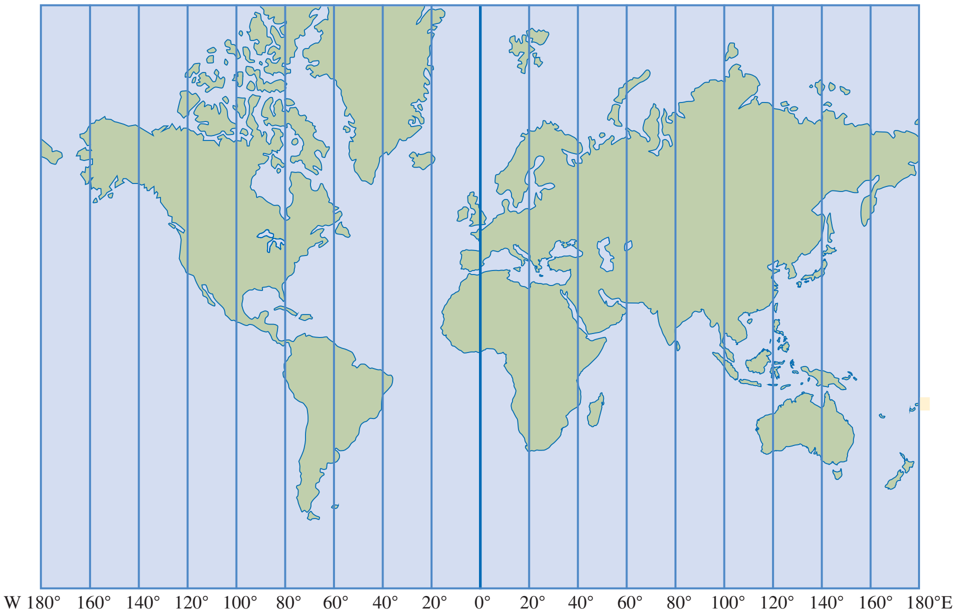
40ºS 5ºS 50ºN 20ºN 65ºN 0º 20ºS 45ºN 50ºN 25ºS



a Zimbabwe
b Germany
c Burma
d Singapore
e Mongolia
f Tanzania
g Paraguay
h New Zealand
i Finland
j Ukraine

4 Use the map below to match each of the countries named with its corresponding longitude from the following list. (Consult an atlas if you are unsure of the positions of countries.)

40ºE 100ºE 60ºE 70ºW 20ºW 140ºE 90ºW 15ºE 170ºE 80ºE



a Sri Lanka
b Iceland
c Japan
d Syria
e Honduras
f Thailand
g New Zealand
h Turkmenistan
i Chile
j Austria

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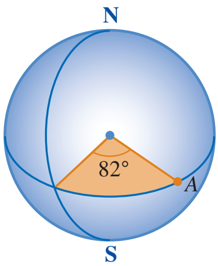
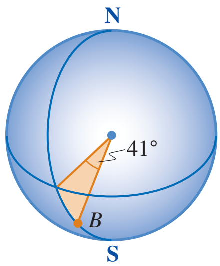
5 State the latitude and longitude of each point shown on the diagrams below. The Prime Meridian and Equator are shown on each diagram.

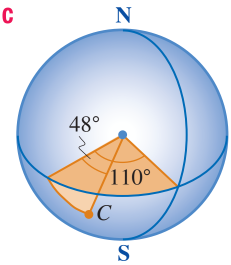
a



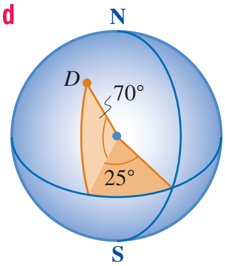
b



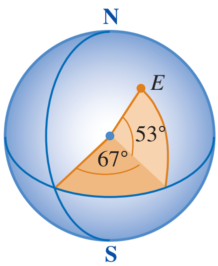
c



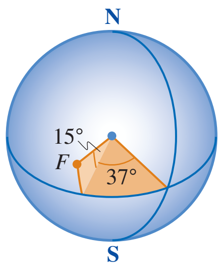
d



e



f



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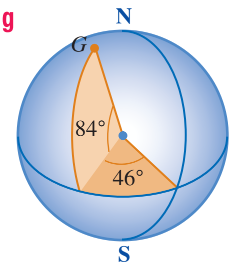
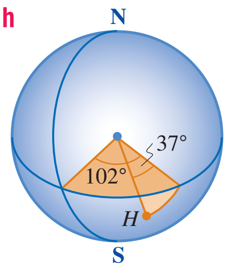


h



6 Use an atlas or Google Earth to name the place situated at:

a 24ºS 134ºE　**b** 51ºN 4ºE　**c** 23ºS 43ºW　**d** 17ºN 96ºE
e 11ºN 105ºE　**f** 33ºS 71ºW　**g** 31ºN 121ºE　**h** 23ºN 82ºW
i 50ºN 14ºE　**j** 56ºN 13ºE.

7 Use an atlas, Google Earth or Whereis to state the position of:

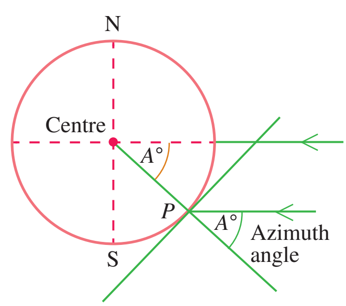
a Cairo　**b** Helsinki　**c** Brisbane　**d** Gibraltar
e Dublin　**f** Hong Kong　**g** Darwin　**h** San Juan, Puerto Rico
i Mexico City　**j** Montevideo.

Modelling and problem solving

8 The angle between the vertical and the direction of the Sun is called the **azimuth**. At point *P* on the diagram, the azimuth is *A*°. At midday on 21 March and 21 September the Sun is directly overhead at the Equator. At this time in Brisbane, the azimuth is $27\frac{1}{2}$°.

a What is the latitude of Brisbane?

b What is the azimuth for New York (41ºN 74ºW) at the same time?



9 Longreach Airport is exactly on the Tropic of Capricorn (23°26′22″ south of the Equator). On 22 December, the Sun is directly overhead at noon, so its azimuth is 0°. Brisbane is at $27\frac{1}{2}$°S 153°E and Cairns is further north at 17°S 146°E. To the nearest $\frac{1}{2}$°, what is the azimuth of the Sun on 22 December:

a in Brisbane?　**b** in Cairns?

10 During winter, the Sun moves north of the Equator, so that on 21 June it is over the Tropic of Cancer (23°26′22″ north of the Equator). To the nearest $\frac{1}{2}$°, what is the azimuth of the Sun on 21 June:

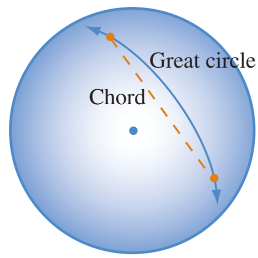
a in Brisbane?　**b** in Cairns?

6.2 Distance and speed on the Earth

The Earth rotates once a day, from west to east. You are carried with the Earth, taking 24 hours for one complete circle. The circle is a parallel of latitude, so unless you are at the Equator, it is a small circle. The gravity of the Earth makes it round, but the spinning makes it fattest at the Equator. The radius at the Equator is 6378.14 km, but the radius at the poles is only 6356.75 km. The average radius is good enough for nearly all calculations of distances on the Earth.

> **!** The average radius of the Earth is about 6371 km.

The *shortest* distance between two points is a straight line. However, we cannot tunnel through the Earth to make a straight-line path between positions on the surface of the Earth. The shortest distance on the *surface* of the Earth will be directly above the line going through the Earth. It will be a curve following a great circle route, because the straight line is a chord of the great circle directly above it.



> **!** The shortest distance between any two points on Earth is part of a great circle.

Investigation Great circle routes

1 Visit the websites of major international airlines to see whether their air routes are displayed. Other websites, such as the Great Circle Mapper, *http://gc.kls2.com/*, allow you to find great circle air routes by identifying the departure and destination locations. The Qantas website was used to find international routes from Sydney, as shown below.

Investigation continued

2 Try to find the routes from Hawaii to Sydney, San Francisco and Tokyo. What shape are the routes as they appear on the map?

3 Find the route from London to San Francisco. Why does it seem to go off the top of the map?

Now work with a partner, using a small globe.

4 Use a piece of string to find the shortest surface routes between different places on the Earth. What do you find about the routes?

5 Compare with your findings from examination of maps of air routes. What do you conclude?

Points that lie on the same meridian are on a great circle. We can calculate the distance between them using the difference in their latitudes. Great circles have a radius of about 6371 km, so their circumference is

$$2 \times \pi \times 6371 \approx 40\,030 \text{ km}$$

Travel through an angle of 1 degree on a great circle will involve a distance of 1/360th of the circumference. Dividing 40 030 km by 360, we find that:

! On a great circle $1^\circ \approx 111.2$ km

Example 3

Find the distance between Auckland Island (51ºS 166ºE) and Noumea (22ºS 166ºE).
Find the time it would take to travel in a boat averaging 15 km/h.

Solution

Since the two positions have the same longitude they are on the same meridian, which is a great circle. The angle travelled is given by the difference in latitudes.

Calculate the difference in latitudes.	Latitude difference $= (51 - 22)^\circ$ $= 29^\circ$
Calculate the distance.	Distance $\approx 111.2 \times 29$ km
Evaluate and round.	≈ 3225 km
Write down rule for speed.	Speed $= \dfrac{\text{distance}}{\text{time}}$
Substitute known values.	$15 \text{ km/h} \approx \dfrac{3225 \text{ km}}{\text{time}}$
Rearrange.	Time $\approx \dfrac{3225 \text{ km}}{15 \text{ km/h}}$
Evaluate.	$= 215$ h
State the result.	The distance between Auckland Island and Noumea is about 3225 km and the trip would about take about 215 hours (almost 9 days).

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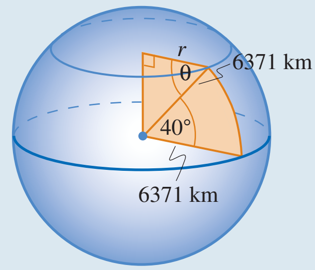
Example 4

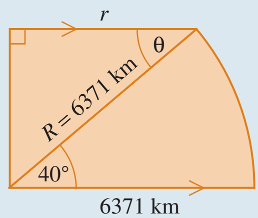
What is the radius of the parallel of latitude at 40ºN?

Solution

Start by drawing a diagram showing the Equator, the latitude angle and the radius of the Earth.



Next draw a sketch using the triangle to work out the radius of the circle, r.



Calculate θ using alternate angles.	$\theta = 40°$
r is adjacent to θ and R is the hypotenuse. Use cos θ to calculate r.	$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
Rewrite using information from the sketch.	$\cos 40° = \dfrac{r}{R}$
Rearrange and substitute for R.	$r = 6371 \times \cos 40°$
Find cos 40º.	$= 6371 \times 0.7660 \ldots$
Evaluate and round.	≈ 4880 km
State the result.	The radius of the parallel of latitude at 40ºN is about 4880 km.

Once you know the radius of a parallel of latitude, you can work out the distance around the Earth at that latitude and the speed of rotation of a point on the Earth at that latitude.

Example 5

Calculate the circumference of the parallel of latitude at 40ºN and the speed of rotation of points at 40ºN.

Solution

From Example 4, the radius of the parallel of latitude at 40°N is about 4880 km.

Write down the rule for circumference (C).	$C = 2\pi r$
Substitute for r.	$C = 2 \times \pi \times 4880$ km
Evaluate and round.	$\approx 30\,662$ km
Any point on the Earth travels around it in 24 hours.	
Write down the rule for speed.	$\text{Speed} = \dfrac{\text{distance}}{\text{time}}$
Substitute known values.	$\approx 30\,662 \text{ km} \div 24 \text{ h}$
Evaluate and round.	≈ 1278 km/h
State the result.	The parallel of latitude at 40°N has a circumference of 30 662 km and points at 40°N rotate at about 1278 km/h.

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The circumference of a parallel of latitude is $2\pi \times$ (radius of the parallel of latitude). The radius of the parallel of latitude at latitude θ is $R \times \cos\theta$, where R is the average radius of the Earth. Compared to a great circle, distances along a parallel of latitude are reduced by the factor $\cos\theta$.

> **!** On a parallel of latitude $1° \approx 111.2 \cos\theta$ km where θ is the angle of latitude.

Example 6

Find the distance along the parallel of latitude between Bowen (20°S 148°E) and Port Hedland (20°S 119°E).

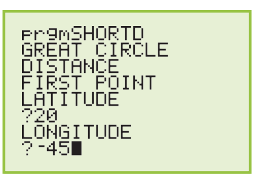
Solution

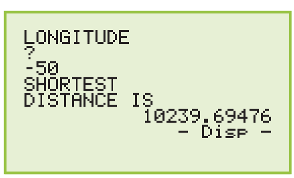
Write the formula.	Distance for $1° \approx 111.2 \cos\theta$ km
Substitute the latitude.	$= 111.2 \cos 20°$ km
Calculate the difference in longitude.	Longitude diff. $= (148 - 119)° = 29°$
Calculate the distance for a difference of 29° of longitude at a latitude of 20°.	Distance $\approx 29 \times 111.2 \times \cos 20°$ km
	$\approx 29 \times 111.2 \times 0.9397$ km
Evaluate and round.	≈ 3030 km
State the result.	It is about 3030 km along the parallel of latitude from Bowen to Port Hedland.

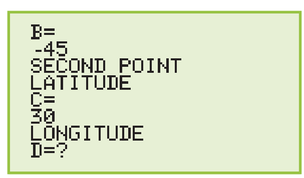
Calculator Program

Technology

Finding the shortest distance between two points on the Earth involves finding the great circle that passes through the points. This is so difficult that sailors often did not try to sail the shortest distance, but followed a rhumb line of constant bearing instead. The shortest distance can be calculated using the program SHORTD, given in full on the CD-ROM. Enter the program (or load it from the CD-ROM) and calculate the shortest distance between different points on the Earth. You need to enter north and east as positive and south and west as negative.







Additional Exercise 6.2

Exercise 6.2 Distance and speed on the Earth

1 Find the distance between Melbourne (38°S 145°E) and Wewak, Papua New Guinea (4°S 145°E).

2 What is the distance from Boston (42°N 71°W) to Santiago, Chile (33°S 71°W)?

3 Find the distance between Stockholm (59°N 18°E) and Cape Town (34°S 18°E). How long would it take to fly between these cities at 600 km/h?

4 What is the distance from Brisbane (27°S 153°W) to Corrientes, Argentina (27°S 59°W) along the parallel of latitude?

5 What is the distance between the Indian cities Aurangabad (20ºN 75ºE) and Puri (20ºN 86ºE) along the parallel of latitude?

6 What is the distance between Montevideo (35ºS 56ºW) and Canberra (35ºS 149ºE) along the parallel of latitude?

Modelling and problem solving

7 How long would it take a light aircraft to fly at 210 km/h from Barcelona (41ºN 2ºE) to Tirana, Albania (41ºN 20ºE) along the parallel of latitude?

8 A plane flying at 400 km/h travelled directly north for 6 hours and 40 minutes before making an emergency landing. If the plane took off from Hobart (43ºS 147ºE), where did it land?

9 Find the time taken for a migrating bird to fly from Townsville (19ºS 147ºE) to Chimoio, Mozambique (19ºS 33ºE) if it flies along the parallel of latitude at 19 km/h.

10 Calculate the radius and circumference of the parallel of latitude and the speed of rotation for each of these places.

a Brisbane (27ºS 153ºE)

b Townsville (19ºS 147ºE)

c Cape Canaveral (28ºN 81ºW)

d Bamaga on Cape York (11ºS 142ºE)

Investigation Great circle distances

You have seen how to calculate distances along meridians and along parallels of latitude. The distance along a parallel of latitude is not actually the shortest distance between points. That is always along a great circle route. The formula below can be used to calculate the great circle distance d between any two points A ($\text{Lat}_1 \text{Lon}_1$) and B ($\text{Lat}_2 \text{Lon}_2$).

$$d \approx 111.2 \times \cos^{-1} [\cos (\Delta\text{Lon}) \cos \text{Lat}_1 \cos \text{Lat}_2 + \sin \text{Lat}_1 \sin \text{Lat}_2]$$

ΔLon is the difference between longitudes. ΔLon must be less than 180º.

For example, the shortest distance between Flinders Island (40ºS 148ºE) and Perth (32ºS 116ºE) is given by:

$$\begin{aligned} d &\approx 111.2 \times \cos^{-1} [\cos (148^\circ - 116^\circ) \times \cos 40^\circ \times \cos 32^\circ + \sin 40^\circ \times \sin 32^\circ] \\ &\approx 111.2 \times \cos^{-1} [\cos 32^\circ \times \cos 40^\circ \times \cos 32^\circ + \sin 40^\circ \times \sin 32^\circ] \\ &\approx 111.2 \times \cos^{-1} [0.550\,928 + 0.340\,626] \\ &\approx 111.2 \times 26.930\,868 \\ &\approx 2995 \end{aligned}$$

So the distance from Flinders Island to Perth is about 2995 km.

1 Work with a partner to calculate the shortest distances between different positions on the Earth. Compare the distance along the parallel of latitude with the shortest distance between:

- Bowen (20ºS 148ºE) and Port Hedland (20ºS 119ºE)
- Shanghai (31ºN 121ºE) and Jerusalem (32ºN 35ºE)
- Canberra (35.5ºS 149ºE) and Montevideo (35ºS 56ºW)
- Los Angeles (34ºN 118ºW) and Tokyo (36ºN 140ºE)

2 Discuss the differences in distance with your partner.

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6.3 Days and their lengths

Extra Material

Seasons and calendars

The week was originally part of the Jewish calendar but is now used throughout the world as a convenient unit of time. In different cultures the days have had different names. The present English names are derived from the names of the Sun, Moon, planets and Norse gods. The days of the week gradually cycle through the dates of the year, so that New Year's Day can fall on any day of the week, depending on the year. A **perpetual calendar** shows the days of the week for any date in any year. There are many perpetual calendar websites, such as *www.calendarhome.com/tyc/*, which has a 10 000-year calendar.

Investigation Days of the week

1 Work in groups of three or four to find the day of the week:
- for your birthday this year
- for your birthday next year
- on which you were actually born
- on which Christmas (25 December) will fall this year
- on which Australia Day (26 January) fell this year
- on which the summer solstice falls this year.

2 Use a perpetual calendar to check your answers to question **1**.

There are 365 days in a year. Because $365 = 52 \times 7 + 1$ and $366 = 52 \times 7 + 2$, the day of the week on which New Year's Day falls moves through the week as the years advance. If the previous year was a leap year, it advances by 2 days; otherwise it advances by 1 day.

Example 7

In 2007, New Year's Day was a Monday. On what day of the week will it occur in 2014?

Solution

It is easiest to work it out year by year.

2008—It moves on 1 day to Tuesday.

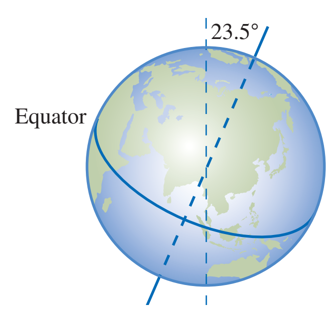
2009—It moves on 2 days because the previous year is a leap year, so it will be Thursday.

2010—Friday 2011—Saturday

2012—Sunday 2013—It moves on 2 days to Tuesday.

So New Year's Day in the year 2014 will be on a Wednesday.

The Earth rotates on an imaginary axis passing through the North and South Poles. This axis is tilted at about 23.5° to the vertical. The tilt in the axis causes the seasons because it permits the Sun's rays to shine more directly and for longer periods on certain locations on the Earth's surface at different times of the year. In this way, when it is summer in the Northern Hemisphere the Sun's rays are more directly overhead and more intense than in the Southern Hemisphere, where it is winter at this time.



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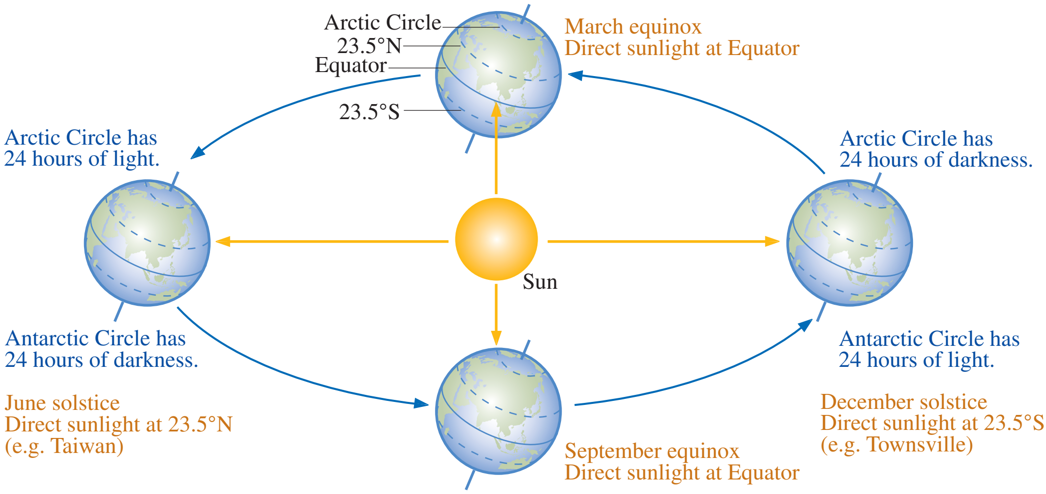
The length of the day changes during the year. The lengths of day and night are the same at an **equinox**. For the Southern Hemisphere, the autumn equinox occurs on 21 March and the spring (vernal) equinox occurs on 22 September. The longest and shortest days of the year occur at the summer and winter **solstices**—22 December and 21 June respectively in the Southern Hemisphere.



The following table gives the approximate day length for various latitudes at different times of the year.

Approximate day length

Southern Hemisphere: read down				
Latitude	**22 September**	**22 December**	**21 March**	**21 June**
0°	12 h	12.0 h	12 h	12.0 h
10°	12 h	12.6 h	12 h	11.4 h
20°	12 h	13.2 h	12 h	10.8 h
30°	12 h	13.9 h	12 h	10.1 h
40°	12 h	14.9 h	12 h	9.1 h
50°	12 h	16.3 h	12 h	7.7 h
60°	12 h	18.4 h	12 h	5.6 h
70°	12 h	24 h	12 h	0 h
80°	12 h	24 h	12 h	0 h
90°	12 h	24 h	12 h	0 h
Latitude	**21 March**	**21 June**	**22 September**	**22 December**
Northern Hemisphere: read up				

Example 8

What is the approximate day length in Camooweal (20°S 138°E) in the middle of winter?

Solution

Camooweal is in the Southern Hemisphere, so the middle of winter is at 21 June.

Look up the table of day lengths and state how you got the result.

From the table, the day length = 10.8 h

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The exact lengths of the days change in a very complicated way between these extremes because the speed of the Earth in its orbit is not constant. The lengths of the longest and shortest days depend on the latitude.

!

For a latitude $\theta < 66.5°$:

Daylight minutes of shortest day $\approx 8 \times \cos^{-1}(0.4338 \times \tan\theta)$

Daylight minutes of longest day $\approx 1440 - 8 \times \cos^{-1}(0.4338 \times \tan\theta)$

These formulas do not take the width of the Sun's disc or the refraction of the atmosphere into account. A correction of about 10 minutes (longer) will account for these factors. At latitudes greater than 66.5º, the longest day is actually 24 hours and the shortest day is 0 hours. Hence, polar regions are sometimes called the Land of the Midnight Sun.

This photograph shows the summer solstice at Mawson, Antartica.

Example 9

What are the lengths of the shortest and longest days in Brisbane, at latitude 27ºS?

Solution

State the rule for shortest day.	Shortest day $\approx 8 \times \cos^{-1}(0.4338 \times \tan\theta)$ min
Replace θ with 27º, evaluate and round.	≈ 618 min
Add 10-minute correction.	Corrected time $\approx 618 + 10 = 628$ min
Convert to hours and minutes.	$= 10$ h 28 min
State the rule for longest day.	Longest day $\approx 1440 - 8 \times \cos^{-1}(0.4338 \times \tan\theta)$ min
Replace θ with 27º, evaluate and round.	$\approx 1440 - 618$ min $= 822$ min
Add 10-minute correction.	Corrected time $\approx 822 + 10$ min $= 832$ min
Convert to hours and minutes.	$= 13$ h 52 min

Solar noon is the time when the Sun is at its highest point in the sky. It is not always at 12 o'clock because of variation in the Earth's orbit and differences in longitude between places in the same time zone. However, solar noon is always halfway between sunrise and sunset. If you know the latitude and time when the Sun rises, from the length of the day you can work out the time when the Sun will set and the time when it will be highest in the sky.

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Example 10

On 3 April the Sun rises at 5:59 am in Brisbane and the day is 11 h 46 min in length. At what times are solar noon and sunset?

Solution

Calculate the half-day length.

$$\text{Length of half-day} = \tfrac{1}{2} \text{ of } 11 \text{ h } 46 \text{ min}$$
$$= 5 \text{ h } 53 \text{ min}$$

Since solar noon is halfway through the day, it will occur 5 h 53 min after sunrise.

Calculate when solar noon occurs.

$$\text{Solar noon} = 5{:}59 \text{ am} + 5 \text{ h } 53 \text{ min}$$
$$= 11{:}52 \text{ am}$$

Calculate when sunset occurs.

$$\text{Sunset} = 5{:}59 \text{ am} + 11 \text{ h } 46 \text{ min}$$
$$= 5{:}45 \text{ pm}$$

Exercise 6.3 Days and their lengths

Additional Exercise 6.3

1 Use the table on page 163 to find the approximate day length in:
- a Wanganui (40ºS) on 21 June
- b Oslo (60ºN) on 21 June
- c Charters Towers (20ºS) in late December
- d Flinders Island (40ºS) in the middle of the southern winter
- e Bowen (20ºS) in the middle of the southern summer
- f Cairo (30ºN) in the middle of the northern winter
- g Oslo (60ºN) in the middle of the northern summer.

2 a What are the lengths of the shortest and longest days in Alice Springs (24ºS)?
b If sunrise is at 7:02 am in Alice Springs on the shortest day, when are solar noon and sunset?

3 a What are the lengths of the shortest and longest days in Melbourne (38ºS)?
b If sunrise is at 5:48 am in Melbourne on the longest day, when is sunset?

4 What are the lengths of the shortest and longest days at the Tropic of Capricorn (22.5ºS)?

5 What is the difference between the shortest and longest days in Columbus, Ohio (40ºN)?

6 What is the difference between the shortest and longest days in Anchorage, Alaska (61ºN)?

Modelling and problem solving

7 Which city receives more intense sunlight in June: Sydney (34ºS 151ºE) or Quebec (47ºN 71ºW)? Why?

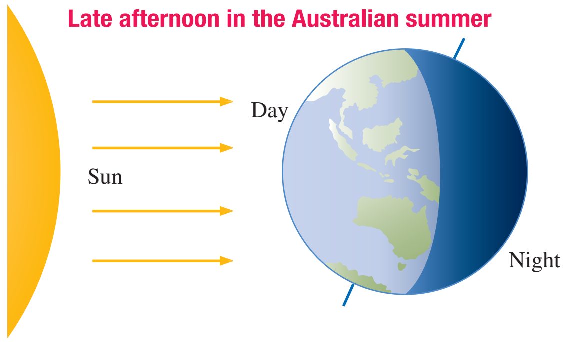
8 What would happen to the seasons if the Earth were tilted 40º instead of its current 23.5º?

9 What would happen to the seasons if the Earth were tilted at 23.5º in the opposite direction?

10 Work out the day of the week for Christmas Day in:
a 2009 b 1989 c 1974 d 2020 e 2014

6.4 Time zones on the Earth

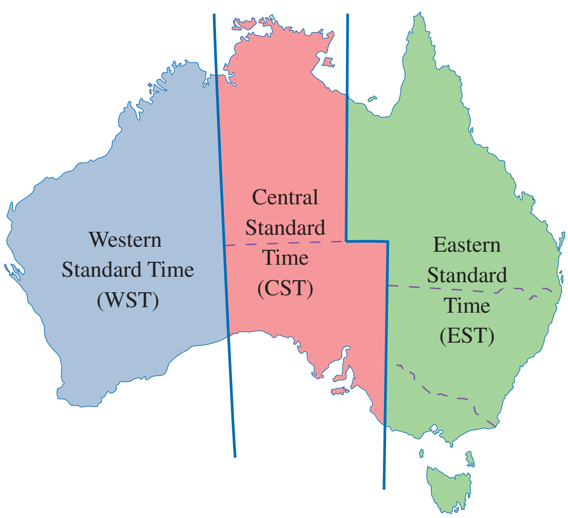
The side of the Earth facing the Sun is illuminated, so it is daylight on that side. As the Earth spins, different parts turn to face the Sun. The **circle of illumination** sweeps around the Earth once a day, bringing dawn at one edge and sunset at the other.



We find it convenient to have our clocks set so that dawn, solar noon and sunset are at about the same time each day. However, when it is dawn in Brisbane it is still dark in Toowoomba. It is dawn in Toowoomba about 4 minutes later. It is another 20 minutes before it is dawn in Townsville. It would be silly to have clocks set to different times in every town down the eastern coast of Australia.

Australia is divided into three **time zones**. In addition, some States change the clocks in summer to include **daylight saving**, but this is not a standard time zone. The eastern and western Australian standard time zones are part of an international system of time zones.

The **Eastern Standard Time** zone covers the whole of Queensland, New South Wales, Victoria and Tasmania. It is 10 hours ahead of Greenwich Mean Time. Internationally, Australian Eastern Standard Time is abbreviated to **AEST**. The **Central Standard Time** zone covers South Australia and the Northern Territory but it is not an international standard time zone. It is only half an hour behind Eastern Standard Time. The **Western Standard Time** zone is 2 hours behind Eastern Standard Time and covers Western Australia. It is 8 hours ahead of Greenwich and is an international standard time zone. Australian time zones are shown below.



Since $360° \div 24 = 15°$, each hour corresponds to a difference in longitude of 15°; and since $24 \times 60 \div 360° = 4$, each degree of longitude corresponds to a time difference of 4 minutes.

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1 h ≡ 15° of longitude 1° of longitude ≡ 4 minutes

The world is divided into **standard time zones** with the clocks set 1 hour apart in neighbouring time zones. This screen dump is taken from website *http://www.qlock.com/time/gmaps*.

The clocks in a time zone are set so that solar noon is at 12 o'clock in the middle of the zone. The date is changed at the **International Date Line**, which passes through the middle of the Pacific Ocean. All time zones are referred to Greenwich (at 0º longitude). Areas with eastern longitudes are 'ahead' of Greenwich in time, and areas with western longitudes are 'behind' Greenwich.

Throughout the world, time zone boundaries are modified to state and country boundaries. There are a few countries, such as Saudi Arabia, which do not use the appropriate standard time zone. A map of the time zones of different areas of the world is often given in an atlas. Time differences for various countries are also listed in the back of the *White Pages* telephone directory to help you avoid ringing people overseas at inappropriate times. It lists the time difference for each country in the form AEST − x hours. For example, Denmark is AEST − 9 hours and the USA is AEST − 15–21 hours. The diagram on page 169 shows world time zones referred to Greenwich.

Investigation Time zones and the telephone

The theoretical time difference between two places on the Earth can be worked out from the difference in longitude. Every degree of longitude makes a difference of 4 minutes. The difference in longitude between Townsville (19ºS 146ºE) and Wellington, New Zealand (41ºS 174ºE) is 174º − 146º = 28º. Thus the theoretical time difference between Townsville and Wellington is 28 × 4 minutes = 112 minutes. The time difference shown in the *White Pages* is 2 hours.

1 Work in groups of three or four with an atlas and a copy of the *White Pages* to find the theoretical and actual time differences between your area and New York, London, Paris, Moscow, Tokyo, Jakarta, San Francisco and Johannesburg.

There are many websites that give the time and date of places relative to Greenwich Mean Time, such as *www.timeanddate.com/worldclock/* and *www.worldtimezone.com/*.

2 Use a website of your choice to investigate the time differences between places of interest to you.

The following example demonstrates that differences in time zones need to be taken into consideration when travelling long distances.

Example 11

It takes $6\frac{1}{2}$ hours to fly from Brisbane to Perth.

a If you take off at 8 am in Brisbane, at what time will you arrive in Perth?

b If you take off at 8 am in Perth, at what time will you arrive in Brisbane?

a Calculate the arrival time in Perth. This is Brisbane time.

$$\text{Arrival time} = 8 \text{ am} + 6\tfrac{1}{2} \text{ h} = 2{:}30 \text{ pm}$$

Perth is 2 hours behind Brisbane time. Adjust for the time difference.

$$\text{Perth time} = 2{:}30 \text{ pm} - 2 \text{ h} = 12{:}30 \text{ pm}$$

State the result.

The flight will arrive in Perth at 2:30 pm Brisbane time or 12:30 pm local time.

b Calculate the arrival time in Brisbane. This is Perth time.

$$\text{Arrival time} = 8 \text{ am} + 6\tfrac{1}{2} \text{ h} = 2{:}30 \text{ pm}$$

Brisbane is 2 hours ahead of Perth time. Adjust for the time difference.

$$\text{Brisbane time} = 2{:}30 \text{ pm} + 2 \text{ h} = 4{:}30 \text{ pm}$$

State the result.

The flight will arrive in Brisbane at 2:30 pm Perth time or 4:30 pm local time.

International air flights are sufficiently fast to cause problems with time zones. It can be confusing for air travellers when their flights cross the International Date Line. It is possible to land in Los Angeles 'before' taking off in Brisbane—provided you refer to local time. If you visit the website of Qantas (*www.qantas.com.au/*) or other international airlines you will be able to compare departure and arrival dates and times for overseas flights.

Example 12

A flight from Brisbane to Los Angeles takes 17 hours including a stopover at Honolulu. If the flight leaves Brisbane at 1:30 pm on Sunday, when will it arrive in Los Angeles:

a in Brisbane time? **b** in Los Angeles time?

Solution

a Calculate the arrival time in Los Angeles. This is Brisbane time.

$$\text{Arrival time} = 1{:}30 \text{ pm Sun} + 17 \text{ h} = 6{:}30 \text{ am Mon}$$

State the result.

The flight will arrive in Los Angeles at 6:30 am on Monday—Brisbane time.

b Convert Brisbane time to Los Angeles (local) time. Los Angeles is on the western coast of the USA. Look up the time zone diagram on page 169.

Calculate the time difference between Brisbane and Los Angeles.

$$\text{Time difference} = (-8) - (+10) = -18 \text{ hours}$$

This means that Los Angeles is 18 hours behind Brisbane time.

Convert arrival time to local time.

$$\text{Arrival time} = 6{:}30 \text{ am Mon} - 18 \text{ hours} = 12{:}30 \text{ pm Sun}$$

State the result.

The flight will arrive in Los Angeles at 12:30 pm on Sunday—Los Angeles time.

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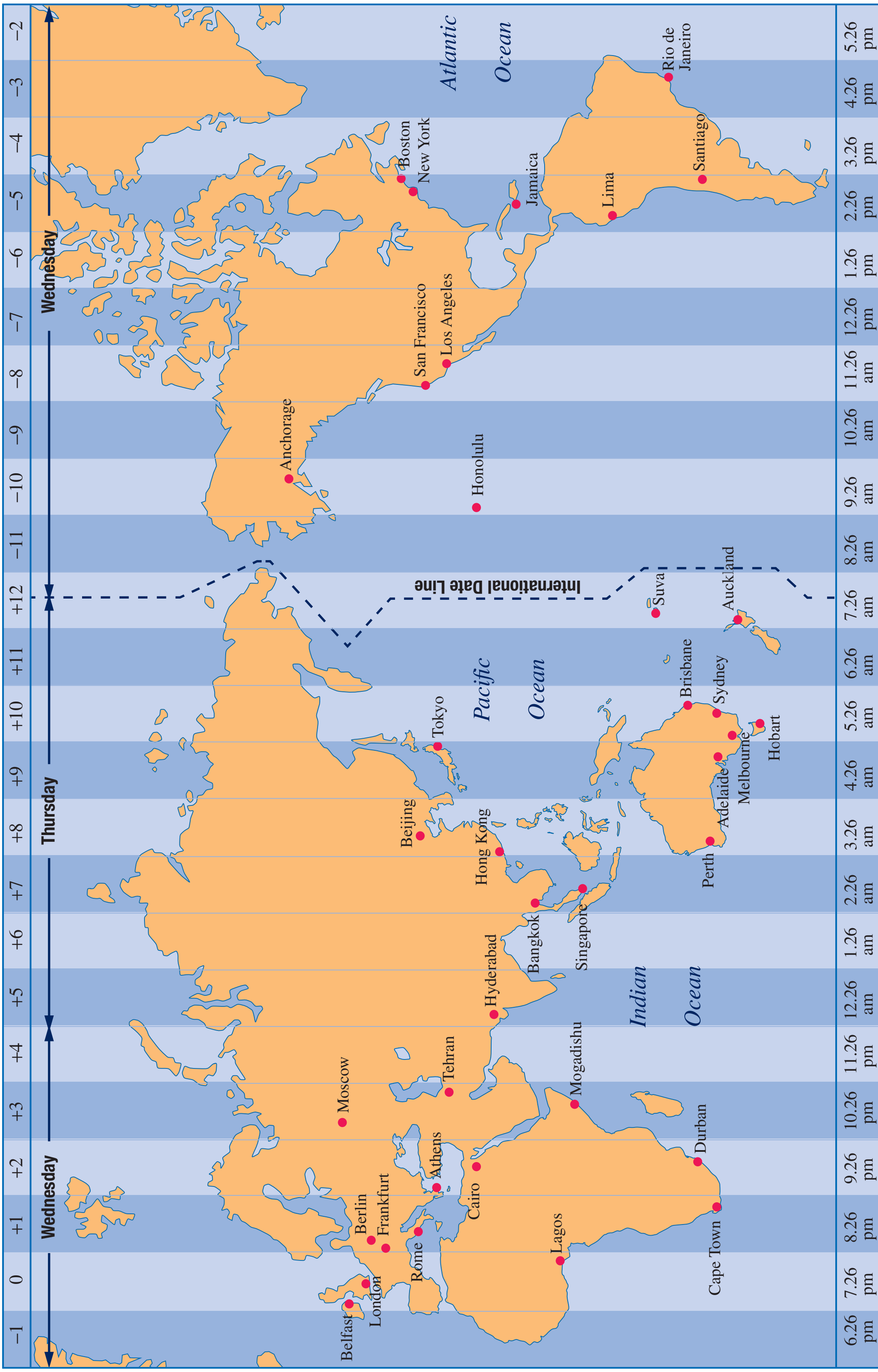

Greenwich Mean Time

World times when it is 5:26 am on Thursday in Queensland

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Example 12 shows that travellers on the flight between Brisbane and Los Angeles arrive in Los Angeles an hour before leaving Brisbane (in local time)! Travelling across time zones may cause people to suffer from 'jet lag', which can make it difficult for them to adjust their sleep patterns.

Additional Exercise 6.4

Exercise 6.4 Time zones on the Earth

Use the time zone map shown on page 169 for this exercise.

1 Calculate the time in Brisbane when it is:
- **a** 3:30 am in Adelaide
- **b** 3:20 pm in Suva
- **c** 2:45 pm in Belfast
- **d** 10:15 am in Rome
- **e** 10:15 am in Hong Kong.

2 A fax is sent at 11:00 am on Wednesday from Sydney. Assuming that there are no transmission delays, work out the time and day when it will arrive in:
- **a** Suva
- **b** Bangkok
- **c** Santiago
- **d** Athens
- **e** Cape Town
- **f** Lagos
- **g** Lima
- **h** Beijing.

Modelling and problem solving

3 A plane leaves Brisbane at 7:00 am on Saturday for Narita airport, Tokyo. The direct flight takes 9 hours 30 minutes. When it arrives, what will be the day and time:
- **a** in Brisbane?
- **b** in Tokyo?

4 A plane leaves Sydney at 1:20 pm on Tuesday for Heathrow airport, London. The flight takes 25 hours 45 minutes, including a stopover at Changi airport, Singapore. When it arrives, what will be the day and time:
- **a** in Sydney?
- **b** in London?

Answer the following questions in local time (i.e. the time at the destination).

5 It takes $13\frac{1}{2}$ hours to fly from Sydney to Taipei, Taiwan. If a plane leaves Sydney at 2:30 pm on Monday, when will it arrive at Chiang Kai Chek airport in Taipei?

6 It takes 26 hours and 45 minutes to fly from Brisbane to Vancouver, Canada. If a plane leaves Brisbane at 6:30 am Wednesday, when will it get to Vancouver?

7 It takes 26 hours 40 minutes to fly from Charles De Gaulle airport in Paris to Sydney. If a flight leaves Paris at 7:30 pm on Sunday, when will it arrive in Sydney?

8 A direct flight from Los Angeles to Brisbane takes 12 hours and 25 minutes. At what time will a flight leaving Los Angeles at 10 am arrive in Brisbane?

9 It takes $24\frac{3}{4}$ hours, including stopovers, to fly from Frankfurt in Germany to Brisbane. When would a flight leaving Frankfurt at 7 am on Wednesday arrive in Brisbane?

10 A flight leaves Brisbane at 4:30 pm on Tuesday and flies to Rome. When does it arrive if the flight takes 23 hours 45 minutes (including stopovers).

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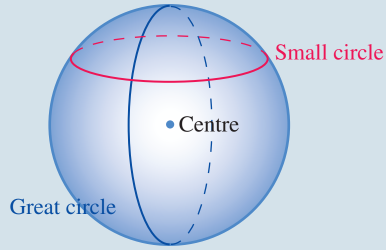
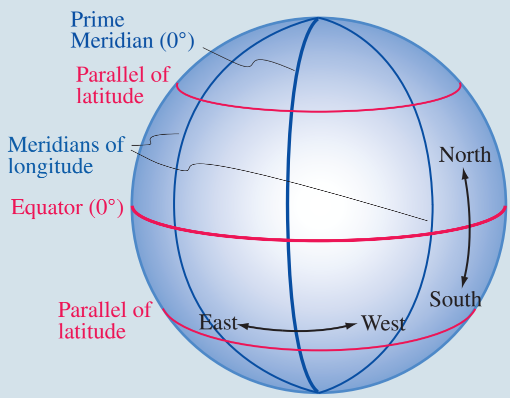
Chapter summary

- **Great circles** are drawn on the surface so that their centre is at the centre of the Earth. **Small circles** do not have their centre at the centre of the Earth.



- **Parallels of latitude** are small circles drawn parallel (north and south) to the **Equator** (0º). **Meridians of longitude** are great semicircles drawn between the North and South Poles. The **Prime Meridian** (0º) passes through Greenwich, England. The position of a point on the Earth is stated in terms of degrees north or south of the Equator and east or west of the Prime Meridian.



- The Earth rotates on its axis once a day, from west to east. The **speed of rotation** of points on the surface of the Earth decreases the further they are away from the Equator.
- The **radius of the Earth** varies but averages 6371 km.
- The **shortest distance** on the surface of the Earth is part of a great circle. On a great circle, $1° \approx 111.2$ km. On a parallel of latitude, $1° \approx 111.2 \cos\theta$ km where θ is the angle of latitude.
- The length of the day changes during the year. The lengths of day and night are the same at an **equinox** (21 March and 22 September in the Southern Hemisphere), while the shortest and longest days occur at the summer and winter **solstices** (22 December and 21 June respectively in the Southern Hemisphere).
- The exact lengths of the days change in a very complicated way between these extremes because the speed of the Earth in its orbit is not constant. The lengths of the longest and shortest days vary depending on the latitude (θ). For a latitude $\theta < 66.5º$:

  Daylight minutes of shortest day $\approx 8 \times \cos^{-1}(0.4338 \times \tan\theta)$

  Daylight minutes of longest day $\approx 1440 - 8 \times \cos^{-1}(0.4338 \times \tan\theta)$

- The world is divided into **standard time zones** with the clocks set 1 hour apart in neighbouring time zones. Each hour corresponds to a difference in longitude of 15º, and each 1° of longitude corresponds to a time difference of 4 minutes. The date changes at the **International Date Line**, which passes through the middle of the Pacific Ocean. All time zones are referred to Greenwich (at 0º longitude), with eastern longitudes ‘ahead’ of Greenwich in time and western longitudes ‘behind’ Greenwich.
- Australia is divided into three time zones: **Eastern Standard**, **Central Standard** and **Western Standard**. Some states change the clocks in summer to include **daylight saving**, but this is not a standard time zone.

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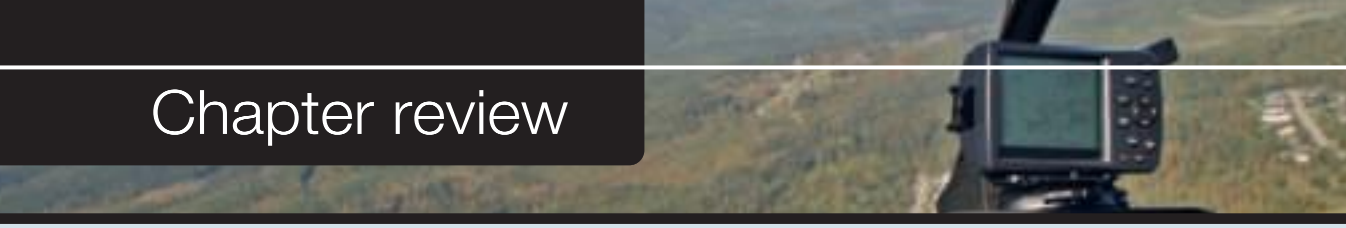
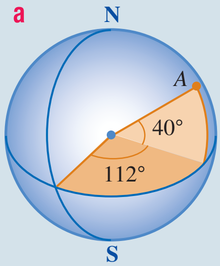


Chapter review

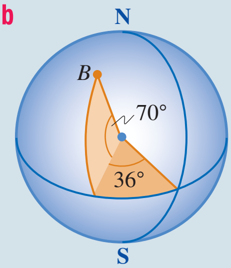
Knowledge and procedures

Ex 6.1 **1** State the latitude and longitude of each of the points shown on the diagrams below. The Prime Meridian and Equator are shown on each diagram.

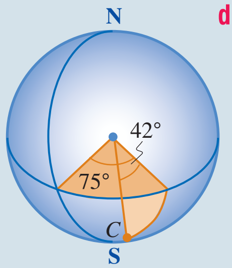
a



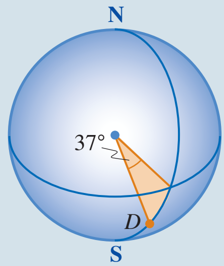
b



c



d



Ex 6.1 **2** Use an atlas to find the positions of the following cities.

a Sydney **b** Rome **c** Kuala Lumpur **d** Kyoto

Ex 6.2 **3** Calculate the radius and circumference of the parallel of latitude and speed of rotation for each of these places.

a Mt Isa (22ºS 140ºE) **b** Buenos Aires (34ºS 58ºW) **c** Madras (13ºN 80ºE)

Ex 6.2 **4** Find the distance between:

a Budapest (48ºN 19ºE) and Cape Town (34ºS 19ºE) along the meridian.

b Perth (32ºS 116ºE) and Newcastle (32ºS 152ºE) along the parallel of latitude

Ex 6.3 **5** The Australian Federation began on a Tuesday. Use this to work out the day of the week of New Year's Day in 1893 and 1907.

Ex 6.3 **6** Use the table on page 163 to find the day length in Port of Spain, Trinidad (10ºN 62ºW) on 21 June.

Ex 6.3 **7** Use the formulas given on page 164 to work out the lengths of the longest and shortest days in Mt Isa (22ºS 140ºE).

Ex 6.4 **8** What time is it in Brisbane when it is 5:30 pm in Alice Springs?

Ex 6.4 **9** What is the time in Port Hedland, WA when it is 11 am in Brisbane?

Ex 6.3 **10** What countries first celebrate New Year's Day? (*Hint:* They are in the Pacific Ocean.)

Modelling and problem solving

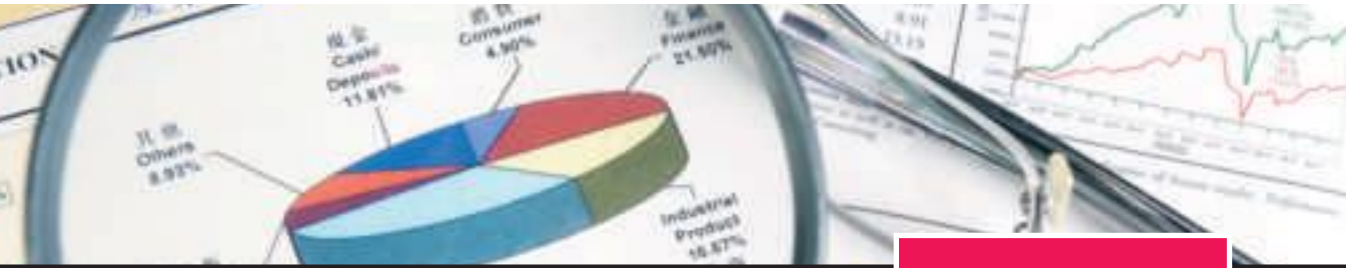
Ex 6.2 **11** A fairy penguin swims along a line of latitude from Albany, WA (35°S 118°E) to Glenelg, SA (35°S $138\frac{1}{2}$°E), except that it has to detour around Cape Spencer. The detour adds 35 minutes to its trip, but otherwise it swims at an average of 10 km/h. If it leaves Albany at 9 am on Wednesday, what are the time and day in Albany when it arrives at Glenelg?

Ex 6.4 **12** A plane takes 9 hours and 15 minutes to fly from Perth, WA to Durban, South Africa. If it leaves Perth at 8:50 am, at what time does it arrive in Durban?

Ex 6.4 **13** A plane leaves Brisbane bound for Boston on the eastern coast of the USA at 2:40 pm on Monday. The trip takes $26\frac{1}{2}$ hours, including stopovers. What are the local time and day when it arrives?

Ex 6.4 **14** A flight to Belfast leaves Brisbane at 7:50 am on Wednesday. The plane lands $29\frac{1}{2}$ hours later in Belfast. What are the time and day in Belfast on arrival?

Graphs and charts



7

Contents

Syllabus Guide Chapter 7

Syllabus subject matter

Data collection and presentation

- Types of data and variables (continuous and discrete)
- Descriptions of key features of data with reference to suitable elections of graphical and tabular displays
- Data displays including scatterplots, simple and compound stem-and-leaf plots, and box-and-whisker plots

Quantitative concepts and skills

- Rates, percentages, ratio and proportion
- Plotting points using Cartesian coordinates

Information presented in tables or as sets of numbers may be accurate, but is not easily understood. Visual presentation of information as graphs allows us to see a trend, identify outstanding features and understand the overall meaning of information at a glance.

7.1 Using graphs

There are a number of basic graph types that display information as lines, rectangles, pictures or parts of a whole.

!

A **picture graph** (**pictograph** or **pictogram**) is made by using pictures or symbols to represent the data. The number of pictures, or the size of pictures, shows the numbers. When constructing a picture graph it is usual to use a maximum of about ten symbols for any one piece of data.

A **sector graph** (**pie chart** or **circle graph**) has a circle divided into sectors. The whole circle represents the total of the data, and the angle (area) of each sector shows the individual pieces of data.

A **divided bar graph** (or **segmented bar chart**) has a rectangle divided into smaller sections. The whole rectangle represents the total of the data, and the length (area) of each section shows the individual pieces of data.

A **bar graph** (**bar chart**) is constructed by using rectangles (bars or columns) of equal width for each item of data. The bars may be separated by small equal spaces for ease of reading and comparison of data and may be arranged horizontally or vertically. A vertical bar graph is often called a **column graph**.

Line graphs are used when variations or trends in data are of major importance. Both the vertical and the horizontal axes are numeric. When constructing a line graph, the data is plotted as a set of points, then the points are joined. The line joining the points is often used to *estimate* in-between values, and this is called **interpolation**.

When drawing graphs, we try to choose a scale that makes it easy to draw the graph, in a similar way to choosing scales for scale drawings.

Example 1

Draw a picture graph to show the sales of novels from a bookshop, shown in the table.

Type	Horror	Romance	Historical	Mystery	Crime	Other
Sales	4097	14 875	6172	8942	7241	12 320

Solution

You should have at most 10 pictures for the biggest number.

$$\text{Books/picture} = 14\,875 \div 10 \approx 1500$$

Round *up* to an easy number.

Use a scale of 2000 books/picture.

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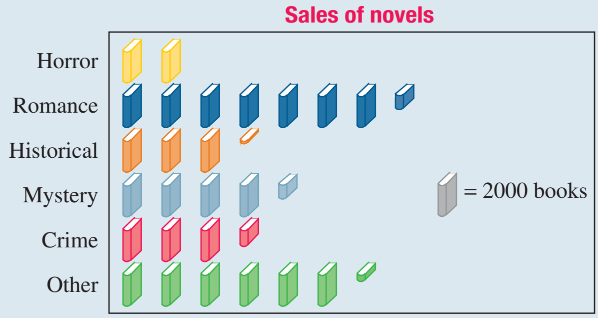
Work out the number of pictures for each type of novel.

$4097 \div 2000 \approx 2.0$
$14\ 875 \div 2000 \approx 7.4$
and so on

Type	Horror	Romance	Historical	Mystery	Crime	Other
Pictures	2.0	7.4	3.1	4.5	3.6	6.2

Draw a graph, including the title and scale.

Use part-drawings to show decimals.



Example 2

When some students were asked for their favourite flavour of ice-blocks, 8 said raspberry, 6 said lemon, 9 said chocolate, 4 said lime and 5 said orange. Show this as a pie chart.

Solution

Find the proportion of each flavour.

Multiply each fraction by 360° to find the angle.

Alternatively, you can work out 360° ÷ 32, then multiply each number by this value.

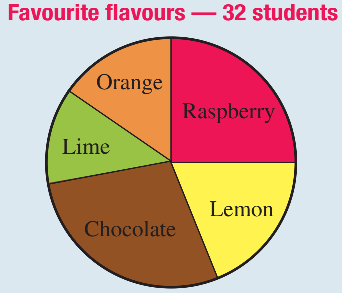
Results are best shown in a table.

Sometimes the total will be 359° or 361° because of rounding errors.

Flavour	Number	Fraction	Angle
Raspberry	8	$\frac{8}{32} = \frac{1}{4}$	$\frac{1}{4} \times 360° = 90°$
Lemon	6	$\frac{6}{32} = \frac{3}{16}$	$\frac{3}{16} \times 360° \approx 68°$
Chocolate	9	$\frac{9}{32}$	$\frac{9}{32} \times 360° \approx 101°$
Lime	4	$\frac{4}{32} = \frac{1}{8}$	$\frac{1}{8} \times 360° = 45°$
Orange	5	$\frac{5}{32}$	$\frac{5}{32} \times 360° \approx 56°$
Total	**32**	$\frac{32}{32} = 1$	**360°**

Draw the graph, labelling each sector.

Give the total for the graph.



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Example 3

The numbers of thickshakes of different flavours sold in an hour were as follows.

Flavour	Strawberry	Chocolate	Lime	Banana	Pineapple	Mango
Number	20	30	15	18	9	6

Draw a divided bar graph to show this information.

Solution

The length of the bar for a page should be about 12 cm. The total number to show is 98, so divide to work out the scale.

$$\text{Thickshakes/cm} = 98 \div 12 = 8.166\ldots$$

Round up to an easy number.

$$\approx 10$$

Work out the total length.

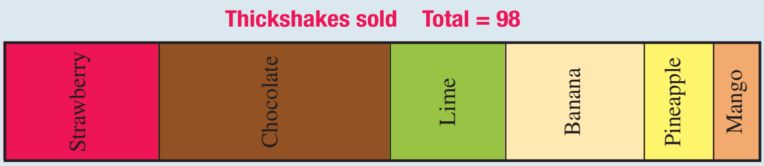
$$\text{Total length} = 98 \div 10 \text{ cm} = 9.8 \text{ cm}$$

Divide by the scale to work out the length of each section.

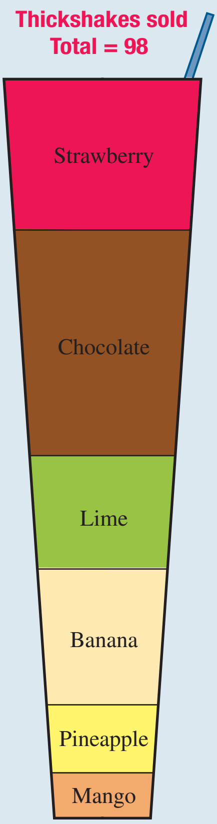
$$20 \div 10 = 2 \text{ cm}$$
$$30 \div 10 = 3 \text{ cm} \quad \text{and so on}$$

Flavour	Strawberry	Chocolate	Lime	Banana	Pineapple	Mango
Length (cm)	2	3	1.5	1.8	0.9	0.6

Draw and label the bar, giving the total at the top.



This divided bar graph could be shown as a thickshake container to make it more interesting, but this introduces another problem which will be dealt with later in the chapter.



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When line graphs are drawn, one value often depends on the other. The dependent value is put on the vertical axis. Where time is involved, it is most often on the horizontal axis.

Example 4

The amount of acid that must be added to a pool for different pH values is given below. Draw a line graph of information to fit in a space about 10 cm high.

pH	7.8	8.0	8.2	8.4	8.6	8.8	9.0
Acid (mL)	200	400	700	1000	1400	2000	2700

Solution

The pH does not start from 0. There is about 12 cm across the page for the pH, which covers a range of 2.2. There is about 10 cm vertically for the acid, which goes up to 2700 mL.

Work out the horizontal scale.

$$\text{pH scale} = 2.2 \div 12/\text{cm}$$
$$= 0.1833\ldots/\text{cm}$$

Round up.

$$\approx 0.2/\text{cm}$$

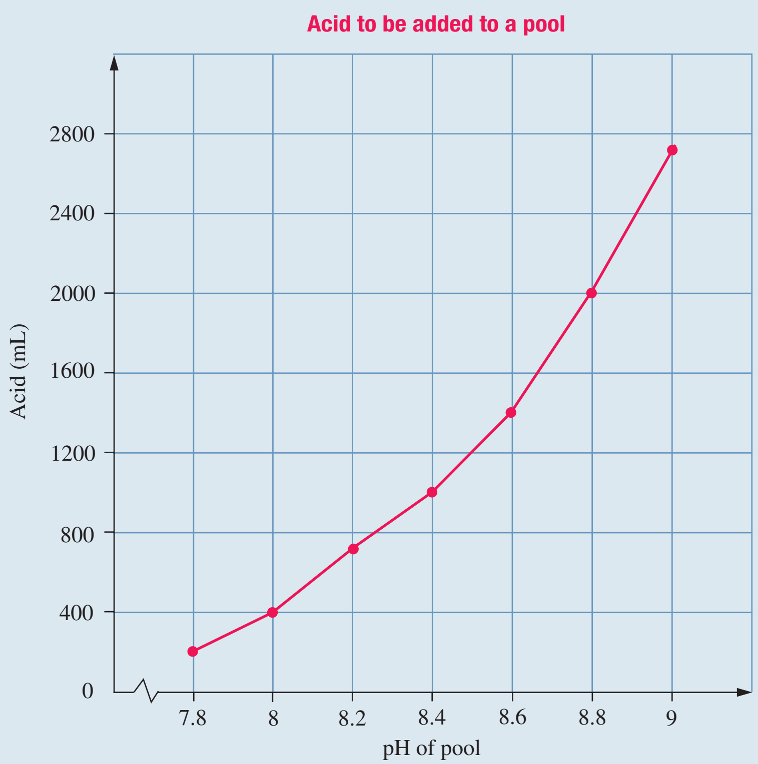
Work out the vertical scale.

$$\text{Acid scale} = 2700 \div 10\text{ mL/cm}$$
$$= 270\text{ mL/cm}$$

Round up.

$$\approx 400\text{ mL/cm}$$

Draw the axes and the graph, showing that the horizontal axis does not start from 0 by a zigzag on the axis.

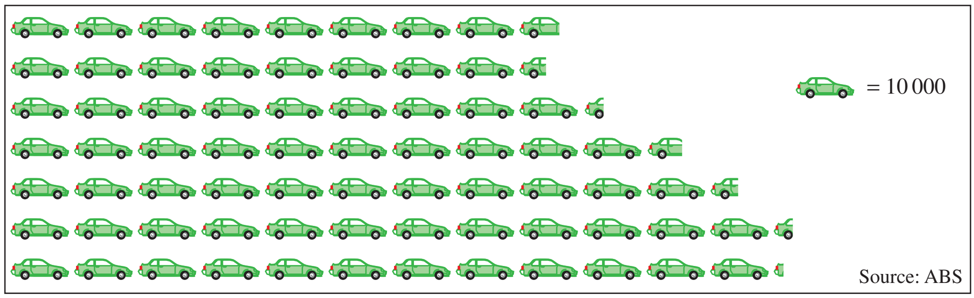


Additional Exercise 7.1

Exercise 7.1 Using graphs

1 The graph below shows the number of new passenger vehicles sold in Queensland each year from 2000 to 2006.

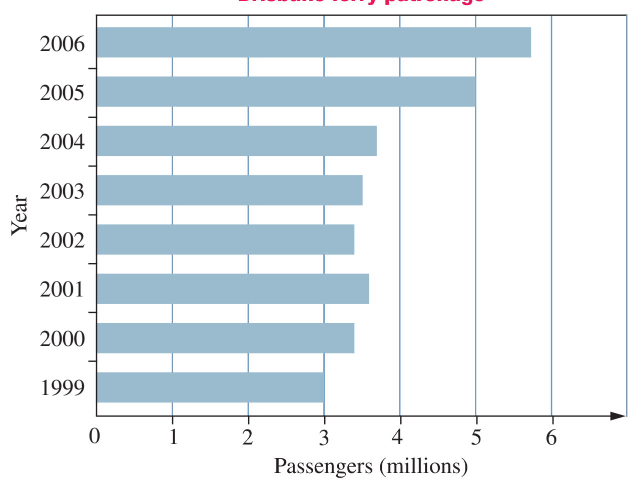
New passenger vehicles, Queensland



- **a** What information is represented by this graph?
- **b** What is the value of each car symbol?
- **c** In which years did the number of new vehicles drop?
- **d** How many new passenger vehicles were sold in 2006?
- **e** Give an example of new vehicles not included in this data.

2 The bar graph on the right shows the numbers of passengers carried by Brisbane ferries, including CityCats.

Brisbane ferry patronage



- **a** What is the scale?
- **b** How many passengers were carried in 2002?
- **c** In which year was the biggest change in patronage?
- **d** Suggest a reason why patronage jumped in that year.

3 In 1986, 18% of Queensland's resident population was born overseas. The graph at the top of the next page shows the numbers of overseas-born residents of Queensland in 2006.
- **a** In what part of the world were most Queenslanders born?
- **b** In what part of the world were the greatest number of overseas-born Queenslanders born?
- **c** The angle for Oceania is 94°. What percentage of overseas-born Queenslanders were from there?

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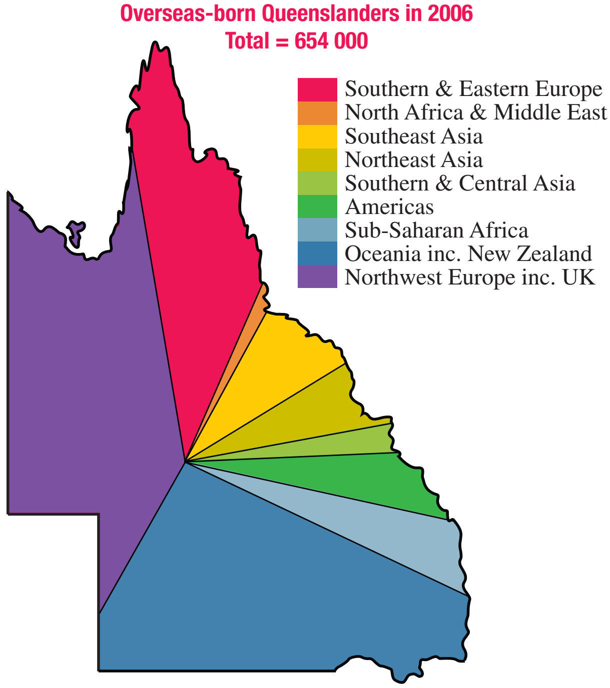
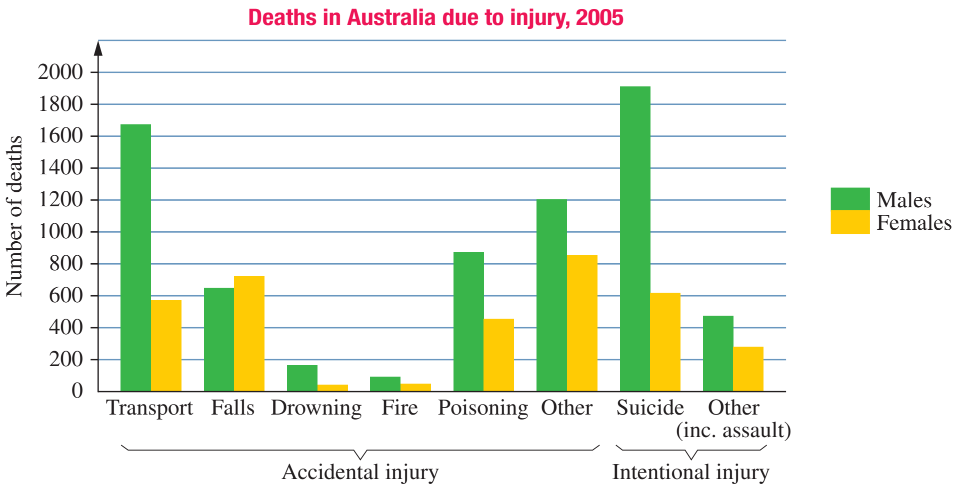
d The angle for the Americas is 15°. How many overseas-born Queenslanders were from there?

e The angle for Southeast Asia is 30°. How many overseas-born Queenslanders were from there?

f Measure the angle for Asia and calculate the percentage of overseas-born Queenslanders from Asia.



4 The graph below shows deaths in Australia from injury in 2005.



a From which kind of injury did more females than males die?

b What was the leading cause of death in males?

c What was the leading cause of death in females (apart from 'Other')?

d How many men died in transport accidents?

e How many women committed suicide?

5 The causes of death for young people (15–24 age group) in Australia are shown below.

Cause of death	Motor vehicles	Other accidents	Suicide	Cancer	Non-infectious diseases	Other
Per cent	48	12	10	8	15	7

Represent this information using:

a a pictograph b a pie chart c a segmented bar chart.

6 The table on the right shows the results of a survey of the number of countries in which each of the languages given is an official language. Use the information to draw a sector graph.

Language	Number of countries
Arabic	26
Bantu	8
Chinese	6
Creole	14
English	78
French	41
German	7
Hindi	5
Italian	5
Portuguese	6
Spanish	24

7 The data below relates to births to unmarried mothers in Queensland over a 10-year period.

Year	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005
Ex-nuptial % of births	32.7	33.4	33.7	33.8	34.1	35.6	36.8	37.3	38.0	38.4

a Construct a line graph of ex-nuptial births as a percentage of all births.
b Explain the trend shown by the graph.
c Explain why this graph could not be used to predict the percentage in 2015.

8 The average humidity at 9 am in different months on Thursday Island is shown below. Use the information to draw a line graph.

Month	J	F	M	A	M	J	J	A	S	O	N	D
% humidity	84	86	85	82	82	81	80	78	75	73	73	78

7.2 Frequency graphs

!

A **stem-and-leaf plot** (or **stemplot**) is a simple way to display a frequency distribution. For two-digit scores, the first digit is placed in order from lowest to highest in a column to make the stem. The second digits are written in a row across from the first digit to make the leaves. The lengths of the rows form a graph showing the spread of the data.

Example 5

Below are the ages of mothers of Year 11 students. Make a stem-and-leaf plot of the data.
38 43 35 52 55 57 47 49 39 44 46 43 48 44 40 51 53 36 42 49 52 39 44

Solution

The first digits are 3, 4 and 5.
Put these in the stem on the left.
Then put the second digits of the first few scores.

Stem	Leaf
3	8 5
4	3
5	2

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Continue with the rest of the scores, leaving spaces between the leaves.

Stem	Leaf
3	8 5 9 6 9
4	3 7 9 4 6 3 8 4 0 2 9 4
5	2 5 7 1 3 2

When you have put in all the data, arrange the leaves in ascending order, and add a title and key.

Ages of mothers of Year 11 students

Stem	Leaf
3	5 6 8 9 9
4	0 2 3 3 4 4 4 6 7 8 9 9
5	1 2 2 3 5 7

Key: 4 | 3 = 43

Data with more than two digits is arranged with the extra digits in the most sensible place. It may be necessary to add a note giving information about the data.

Example 6

The times (in seconds) for 16 runners to complete 400 m are recorded as:

48.8 54.6 49.3 49.1 50.4 52.8 49.9 50.7 48.6 50.3 49.4 51.1 53.7 52.6 50.8 50.3

Draw a stem-and-leaf plot for this data.

Solution

All values are about 50 s.

Put two digits in the stem.

Add the title and key.

400 m times (s)

Stem	Leaf
48	6 8
49	1 3 4 9
50	3 3 4 7 8
51	1
52	6 8
53	7
54	6

Key: 50 | 7 = 50.7

Example 7

The gross turnovers for 12 consecutive trading days of a local small business were:

\$956 \$754 \$578 \$1098 \$502 \$413 \$738 \$299 \$886 \$592 \$789 \$801

Show this information using a stemplot.

Solution

In this case the leaves have two digits, or there would be too many stems.

Separate the leaves by spaces and arrange in ascending order.

Gross daily turnover (\$)

Stem	Leaf
2	99
3	
4	13
5	02 78 92
6	
7	38 54 89
8	01 86
9	56
10	98

Key: 5 | 78 = 578

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Frequency histograms and **polygons** are the most common ways to display the information in a frequency table.

!

For both **frequency histograms** and **frequency polygons**:

- The horizontal axis is continuous and shows the variable.
- Frequency is always shown on the vertical axis.

For **histograms**:

- The columns are shown with no spaces between them, with the scores in the centres of the columns. It is usual to number the lines and draw the sides of the columns halfway between the lines.

For **polygons**:

- The points are shown on the graph paper lines directly above the scores.
- The polygon is closed at either end, so extra scores with zero frequency need to be added.
- A frequency polygon may also be drawn by joining the midpoints of the tops of the columns of a histogram.

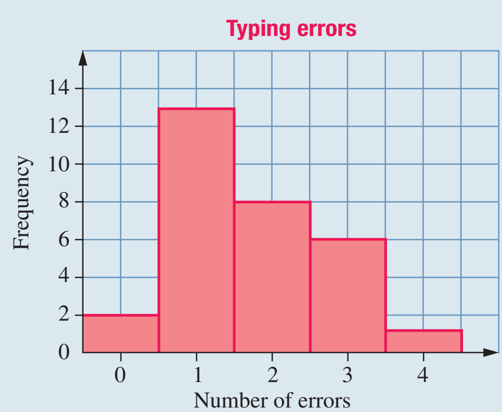
Example 8

Draw a histogram of the numbers of typing errors made by a class of students.

Number of errors	0	1	2	3	4
Number of students	2	13	8	6	1

Solution

Five columns are needed, so make each 2 grid squares wide.
The highest frequency is 13, so go up in 2s.
Add a title to the graph.



Example 9

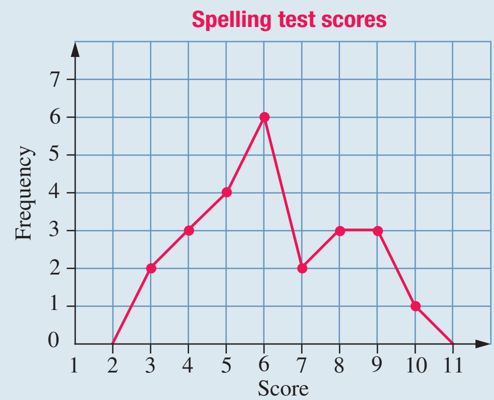
The following were the scores of some students on a spelling test. Make a frequency table and draw a frequency polygon of the scores.

3 5 6 7 8 3 4 5 6 4 5 8 9 6 8 9 6 6 7 4 5 9 10 6

Solution

The scores range from 3 to 10. The polygon must go from 2 to 11.

Score	Tally	Frequency
3	||	2
4	|||	3
5	||||	4
6	~~||||~~ |	6
7	||	2
8	|||	3
9	|||	3
10	|	1
	Total	**24**



Technology

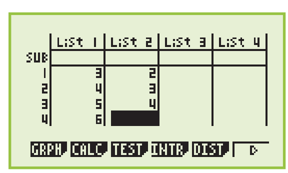
A graphics calculator can be used to draw statistical graphs. The following shows how to enter the data of spelling scores from Example 9.

Score	3	4	5	6	7	8	9	10
Frequency	2	3	4	6	2	3	3	1

Casio fx-9860G AU

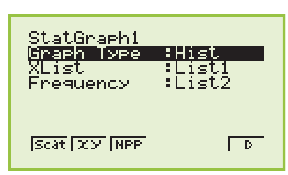
To enter the data, choose the STAT menu. If there is already data in List 1, delete it using **F6** **F4** **F1** .

Enter the scores in List 1, and enter the frequencies in List 2, pressing **EXE** after each item and using the cursor arrows to move between the lists.



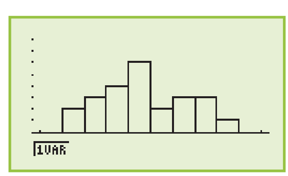
When all the data is entered, enter the GRPH submenu. From the screen above, press **F1** .

Then enter the SET submenu, pressing **F6** . Set the Graph Type to Hist using **F6** **F1** , the XList to List 1 using **F1** 1 **EXE** and the Frequency to List 2 using **F2** 2 **EXE** .

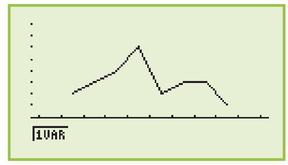


EXIT from the SET submenu and use GPH1, **F1** .

Press **F6** to draw the graph, setting the start to 3 and width to 1 if necessary.



To draw a polygon, change the graph type to Broken by using F6 F5 from the SET submenu and redraw the graph.

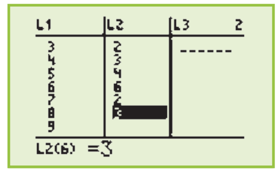


Texas Instruments TI-84

The TI-83 works in a similar way to the Casio.

Press the STAT key and Choose the Edit menu. If there is already data in a list, clear it by moving up to the heading and pressing CLEAR followed by ENTER.

Enter the scores in L1, and enter the frequencies in L2.
To draw a histogram, enter the STATPLOT menu using 2nd Y= . Press ENTER to set Plot1.



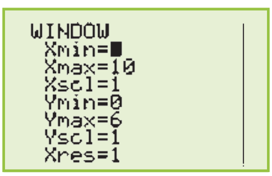
Set it to On, and choose the histogram from the icons shown.

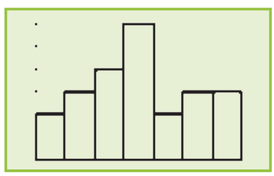
Set the Xlist to L1 using 2nd 1.

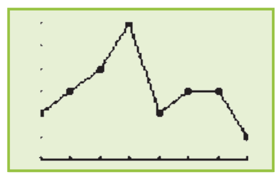
Set the Freq to L2 using 2nd 2.

Press the WINDOW key to set the parameters for the graph, setting Xmin to 3, Xmax to 10, Xscl to 1, Ymin to 0 and Ymax to 6.

Press the GRAPH key to see the graph.







To change to a polygon, select the line graph icon in the STATPLOT menu and redraw the graph.

Calculator Instructions

Sharp EL-9900

Instructions for the Sharp EL-9900 are on the CD-ROM.

Additional Exercise 7.2

Exercise 7.2 Frequency graphs

1 The gross turnovers for 16 consecutive trading days of a suburban video shop were:

\$956	\$754	\$578	\$1098
\$502	\$413	\$738	\$299
\$886	\$592	\$789	\$801
\$1203	\$880	\$923	\$344

Show this information using a stemplot.

2 The times (in seconds) for 20 runners to complete 400 m were recorded as:

48.8	54.6	49.3	49.1	50.4	52.8	49.9	50.7	49.7	50.4
48.6	50.3	49.4	51.1	53.7	52.6	50.8	50.3	54.7	51.2

Draw a stem-and-leaf plot for this data.

3 Use the information in the table below to construct:

a a histogram

b a polygon.

Score	8	9	10	11	12	13	14
Frequency	4	1	3	8	5	7	2

4 The following information gives the ages of the parents of some Year 11 students:

46	42	45	46	42	51	44	38	45	43	39	40	44	41
37	51	44	44	43	48	45	41	47	50	43	41	41	40
38	36	47	43	49	39	47	41	48	43	43	42		

a Construct a frequency table.

b Draw a histogram.

c Draw a polygon.

Modelling and problem solving

5 The following information was obtained from the *ourbrisbane.realestate.com.au* website. It shows the houses available for rent on one particular day in Tarragindi. Not all details have been shown.

a Make a stem-and-leaf plot of the weekly rents.

b Comment on the range of rents and justify your answer.

TARRAGINDI

$400 p.w. Move into this beautiful home straight away and you'll be able to enjoy all the conveniences …

$290 p.w. This three bedroom large timber highset home has a sunroom, a combined lounge/dining …

$550 p.w *Situated high on the hill this highset brick home has everything *3 bedrooms upstairs all …

$300 p.w. Brick 4 Bedroom, lounge, dining, Great Location, NO PETS …

$450 p.w. This double storey 4 bedroom brick home is ideal for the large family. The open plan living …

$330 p.w. This family home is located in a quiet street in Tarragindi close to local shops, schools and …

$320 p.w. Newly renovated post war 2 bedroom home (built-ins & air con), situated in one of …

$325 p.w. THIS TWO BEDROOM & SLEEP OUT HOME HAS JUST BECOME AVAILABLE FOR …

$340 p.w. SOLID 3 BEDROOM [ALL WITH BUILT INS] HOME. POLISHED TIMBER FLOORS …

$320 p.w. 3 Bedroom low set brick house. New paint, carpet & curtains. Good size kitchen, lots of …

$550 p.w. An inspection of this property will not disappoint. This beautifully presented 4B/R home has …

$270 p.w. * 2 bedroom + sleepout * Reverse cycle air conditioning in lounge room * Timber highset …

$300 p.w. FEATURES - 4 BEDROOM LOW SET BRICK HOME + STORAGE ROOM, DOUBLE …

$310 p.w. Newly renovated two bedroom house with newly polished floors and fresh paint! Brand new …

$400 p.w. Five bedrooms, two bathrooms, timber floors, large internal laundry, new bathrooms and …

$350 p.w. This refurbished post war home in a convenient location is an ideal family home. The …

$340 p.w. Neat and tidy 3 bedroom highset home – Freshly painted – New carpets –Built in wardrobes …

$240 p.w. Ground level older style stucco two bedroom home, no wardrobes, separate lounge, roomy …

$350.00 weekly This very well maintained four bedroom home in Tarragindi features modern kitchen …

$500 p.w. Two storey modern home, on small block, with u/cover entertaining at rear, & patio off …

$300.00 weekly Surprisingly large three bedroom home with beautiful polished floors throughout, …

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6 The following ads for motorcycles appeared in a weekend newspaper.

a Make a stemplot of the prices.

b Comment on the range of prices and justify your answer.

BOLWEL Blue Devil, 2006 3,200 km. 9 mths reg. 9 mths wrnty, helmet. $2,400

Rare Offering

HARLEY DAVIDSON 1983, Sportster, completely rebuilt bike, new leather saddle bags and spares. $8,000

HONDA CBR 250RR 1991, unregistered. 16,000 km, 2K black paint. VFR400 brakes. $3500 ono

HONDA CR85R As new, hardly ridden, never raced, 6 spd, serviced with manual, spares, $4,000

HONDA VFR400 V4 engine Haynes workshop manual, red/black, single sided swingarm, vgc $5000

STYLISH

HONDA VTX 1300, 2003 model, 7000 km’s, first to see will buy $13,000 ono.

HONDA VT750 Shadow ’07 – NEW only 154 km, 2 yr warranty, safety certificate + rego, white pearl. Cost $11,000, sell $9000 – MUST SELL!

HONDA VT750 Shadow 2005, 4,900 km, still under warranty, always garaged, $8,400

HONDA XR 600R, ’90, 600cc, 5sp, no money to spend, trial weapon. Perfect condition, $3500

HONDA XR650R ’04 – Excellent condition. Commuter use only by current owner. 10 months rego. 7500 kms, $7200

GREAT BUY

HONDA CBR1000, 1992. Red and white, good front tyre, batteries, chain + sprockets. Need back tyre for rego, gear sack rack, 4 into 1 exhaust. $3800

HONDA CR250 Motor Cross 2001 Model, $3400. Runs well

HONDA CR80 2003. Ex Cond, new front tyre, full spares, $2,900

A REAL GOER

HONDA Quad TRX 450R with HRC kit/pro taper bars, extremely quick, EC, bargain $7750

KAWASAKI GPZ900 Ninja, 1986, 20,000 Original miles. Very good condition. $2,000

KAWASAKI KLR 250 ’91 dual purpose, great learners bike, good on fuel, reliable, good tyres, 2 mth reg, no rwc. $2,140

ONE OWNER

KAWASAKI ZX10 ’04 PC, low kms, pol, rims, r/stand, completely orig, rarely used, reluctant sale, $13,990

KAWASAKI ZZR 250 roadbike, 1996, 48,000 kms. Good condition, 9 months rego, $3000

KTM 250 SXF 06 model, DEP full exhaust, immaculate cond, $7500

KTM 400 SX ’02. Protaper bars, bark busters x2. Good cond, $4200

Immaculate

SUZUKI GSXR 100 2006, black/silver, 2000 klms, perfect condition, price $14,500

SUZUKI GS 1100GK ’84 exceptional bike, 39,000 ks, side/rear panniers, cruise control, black, silver, chrome $4800

SUZUKI GS450S Dohc, 1981, unregistered, rebuilt, new battery, tyres and exhaust. $1400

SUZUKI 650 Katana, shaft drive, sporty, econ, reliable, roadworthy, must sell, $2400, consider swap veh.

TRAIL BIKE Near new, 110 pit bike, EC, yard ridden only, runs perfectly, $1500

TRIUMPH Bonniville, ’78. T-model 140V 750, forward controls, plus extras, VGC, 4 months rego, 590 AF, $8500 ono

TRIUMPH 790 CC American Cruiser, ’06, 4 months rego, low km, 8T-tune pipes, taco 50 bar + more, $15,000

VIRAGO 250 ’02 As New. Only 2600 k, $4250 ono

YAMAHA SRV 250 V-twin sports bike, 39,000 klms, green, well maintained, $2700

YAMAHA XTZ 660 1999 Blue/Orange stripes. Dual purpose. Excellent condition. Low 34,000 kms. $6000 ono

YAMAHA ZEAL 250cc For Sale: $3500, 1998, 3 months rego. Great bike

BRAND NEW

YAMAHA Virago, XV, 250w, brand new, 07 model, 100 kms, overseas transfer, must sell, $5,500

YAMAHA WR 250F 2003, hardly used, excellent condition, unregistered, $5500 neg.

BARGAIN

YAMAHA WR250F, 02, 700 km from new, YZ + stain tune mufflers, spare kit, tyres and gear. $6800.

YAMAHA XT500G-80. Totally original, near new cond, $1900

YAMAHA YZ125, 2001 model, brand new, in mint condition, hardly ridden, $3000 ono.

YAMAHA 250 Virago, 2000 model, custom paint, 12 months rego, 6600 kms, $3500 ono

7.3 Histograms and ogives

It may be useful to know the number of items that lie below (or above) a particular value. The **cumulative frequency** is used for this. Graphs of cumulative frequency are also useful.

!

- **Cumulative frequency** is the sum of the frequencies up to and including an item.
- A **cumulative frequency histogram** is a histogram drawn using cumulative frequencies.
- A **cumulative frequency polygon** is drawn using cumulative frequencies.
- An **ogive** results when the points of a cumulative frequency polygon are connected by a smooth curve.
- A **percentage ogive** has the cumulative frequencies expressed as percentages.

Example 10

The following table shows the hours of sleep of a group of 16-year-olds.

Hours	5	6	7	8	9	10	11
Frequency	2	4	12	15	16	8	4

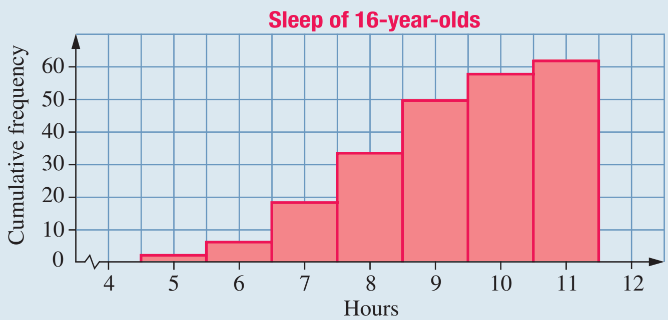
a Redraw the table with a cumulative frequency column.
b Draw a cumulative frequency histogram.
c Draw a cumulative frequency polygon.
d Find the number of people who slept for less than 8 hours.
e Find the number who slept for more than 8 hours.

Solution

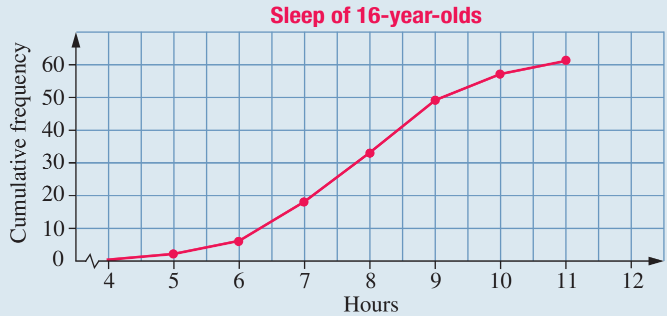
a Redraw the table vertically. Add the frequencies as you go down the table.

Hours of sleep	Frequency	Cumulative frequency
5	2	2
6	4 → 4 + 2 =	6
7	12	18
8	15	33
9	16 → 16 + 33 =	49
10	8	57
11	4	61
Total	**61**	

b The graph has to go up to 61, the highest cumulative frequency, so go up in 10s.



c The graph starts at zero for 4 hours.
The polygon has the points joined with straight lines.



d Less than 8 is the same as 7 or less.
18 people slept for less than 8 hours.

e 33 of the 61 people slept for 8 hours or less.
$61 - 33 = 28$
Write the result.
28 people slept for more than 8 hours.

Example 11

The table below shows the amounts that some people at the beach said they would be willing to pay to gain access to Kings Beach if they had to pay a levy to maintain the beach.

Amount ($)	2	3	4	5	6	7	8	9	10
Frequency	12	4	8	16	14	7	5	3	2

a Calculate cumulative percentages.
b Draw a cumulative frequency ogive.
c Find the percentage who would be willing to pay at least $5.
d If there were 6000 people at the beach, how many would be willing to pay $5 or more?

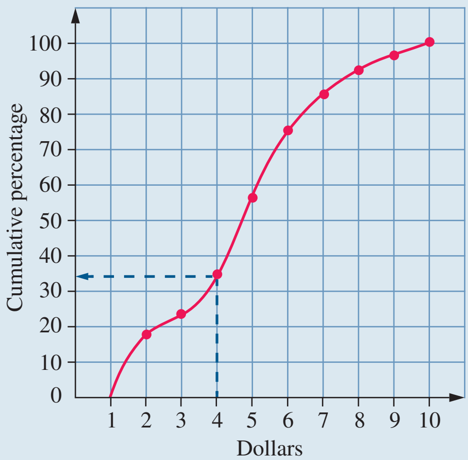
Solution

a Add cumulative frequency and cumulative percentage columns to the table.

Amount ($)	Frequency	Cumulative frequency	Cumulative percentage
2	12	12	$\frac{12}{71} \times 100\% = 17$
3	4	16	$\frac{16}{71} \times 100\% = 23$
4	8	24	$\frac{24}{71} \times 100\% = 34$
5	16	40	$\frac{40}{71} \times 100\% = 56$
6	14	54	$\frac{54}{71} \times 100\% = 76$
7	7	61	$\frac{61}{71} \times 100\% = 86$
8	5	66	$\frac{66}{71} \times 100\% = 93$
9	3	69	$\frac{69}{71} \times 100\% = 97$
10	2	71	$\frac{71}{71} \times 100\% = 100$
Total	**71**		

b The ogive has the points joined with a smooth curve.

Amount to access King's Beach



c Less than $5 is 34%.

This can be read from the graph.

At least $5 = 100% − 34%
= 66%

Write the result.

66% would be willing to pay at least $5.

d 66% is $\frac{66}{100} = 0.66$

$5 or more = 66% of 6000
= 0.66 × 6000
= 3960

Write the result in a sentence.

About 4000 people would pay $5 or more.

Exercise 7.3 Histograms and ogives

Additional Exercise 7.3

1 Calculate cumulative frequencies and draw a cumulative frequency histogram for each table.

a

Score	3	4	5	6	7	8	9	10
Frequency	2	5	9	11	7	4	3	1

b

Score	25	26	27	28	29	30	31	32
Frequency	7	12	16	30	31	14	6	3

c

Score	5	6	7	8	9	10	11	12
Frequency	9	7	6	6	5	4	3	1

2 Calculate cumulative frequencies and draw a cumulative frequency polygon for each table.

a

Score	12	13	14	15	16	17	18	19
Frequency	3	8	12	16	9	7	4	2

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b

Score	7	8	9	10	11	12	13	14
Frequency	3	9	19	43	28	17	9	5

c

Score	5	6	7	8	9	10	11	12
Frequency	3	4	8	10	12	14	8	2

Modelling and problem solving

3 The shoe sizes of some 16-year-old girls were as follows.

Size	5	$5\frac{1}{2}$	6	$6\frac{1}{2}$	7	$7\frac{1}{2}$	8	$8\frac{1}{2}$	9	$9\frac{1}{2}$
Frequency	3	8	9	10	12	11	7	5	3	2

a Work out the cumulative percentages and draw a percentage ogive.
b What percentage of the girls have a shoe size less than 7?
c From a group of 240 16-year-old girls, how many would you expect to have a shoe size bigger than 8?
d If you surveyed a larger group, would you expect other sizes to be given? Justify your answer.

4 The numbers of weeks taken by a construction company to build some houses are shown below:

6	5	7	9	10	8
6	5	7	8	9	10
12	6	5	7	8	9
10	6	6	6	8	7
7	7	8	6	5	11

a Make a frequency table including a cumulative percentage column.
b Construct an ogive.
c What percentage of the houses were built in 6 weeks or less?
d Is the company justified in saying that it normally takes about 6 weeks to build a house?

5 The numbers of accidents at a busy intersection in Brisbane were recorded each day as follows.

Accidents	0	1	2	3	4	5
Frequency	10	12	15	14	8	2

a Work out cumulative frequencies and construct an ogive.
b On how many days were there more than 2 accidents?
c What percentage of days had fewer than 2 accidents?
d Could you use this data to calculate accidents at intersections throughout Brisbane? Explain your answer.

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7.4 Graphing grouped data

For continuous variables, or discrete variables with many values, a frequency table with individual values may be too big to be useful. In this case, a frequency table is simplified by using **classes** or **class intervals**. It is usually best to have between 5 and 15 classes.

!

The upper and lower boundaries of a class are called the **upper class limit** and **lower class limit** respectively.

The **stated class limits** may not be the same as the **true (real) class limits**, particularly for continuous variables.

- For continuous variables, the true class limits generally include the lower limit and exclude the upper limit, and are usually at 'half' values since most measurements are rounded.
- For discrete variables, the class limits are generally inclusive. The true class limits are considered to be at the 'half' values for consistency.

The **class width** (sometimes called the **class length** or **class size**) is the difference between consecutive upper (or lower) class limits.

The **class midpoint** (**class centre, class score** or **class mark**) is the average of the upper and lower class limits.

$$\text{Class midpoint} = \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

Example 12

The numbers of spark plugs replaced in cars each day by a mobile mechanic were:

17 16 11 14 9 10 15 14 16 12 20 18 7 13 15 22 18 19
20 9 16 17 23 18 17 15 24 18 16 18 21 17 18 20 17 24

Present this data as a grouped frequency distribution table.

Solution

The lowest value is 7 and the highest is 24.
The range is 24 − 7 = 17.
A class width of 2 will give a suitable number of classes.
The number of spark plugs used is discrete, so the class limits will be: 7–8, 9–10, 11–12, etc.
These are both the true and the stated class limits.

Number of spark plugs used each day

Class	Tally	Frequency
7–8	|	1
9–10	|||	3
11–12	||	2
13–14	|||	3
15–16	~~||||~~ ||	7
17–18	~~||||~~ ~~||||~~ |	11
19–20	||||	4
21–22	||	2
23–24	|||	3
	Total	**36**

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When a histogram or polygon is constructed from grouped data, the class score is approximated by the class midpoint. For cumulative frequency, we can only be certain that the cumulative frequency value has been achieved at the end of the class interval.

!

Histograms and polygons of grouped data

The edges of the columns for **frequency histograms** and **cumulative frequency histograms** are at the *true class limits*.

The points for **frequency polygons** are placed at the *class midpoints*. A frequency polygon can be drawn over a histogram by joining the centres of the columns.

The points for **cumulative frequency polygons** are placed at the *true upper class limits*. A cumulative frequency polygon can be drawn over a histogram by joining the right-hand ends of the columns.

Example 13

Draw a histogram of the spark plug data from Example 12.

Solution

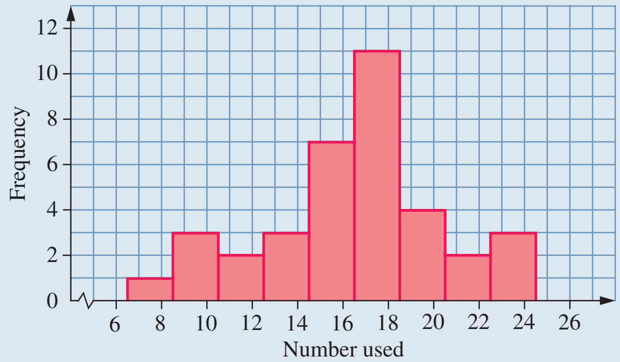
Add a true class limits column to the table. For discrete data, this is on the 'halves'.

Number of spark plugs used each day

Class	True class limits	Frequency
7–8	6.5–8.5	1
9–10	8.5–10.5	3
11–12	10.5–12.5	2
13–14	12.5–14.5	3
15–16	14.5–16.5	7
17–18	16.5–18.5	11
19–20	18.5–20.5	4
21–22	20.5–22.5	2
23–24	22.5–24.5	3
	Total	**36**

Draw the histogram, making sure you place the edges of the columns at the true class limits.

Spark plugs used each day



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Example 14

The speeds of motorists recorded by police are given in the table on the right. Draw a frequency polygon and a cumulative frequency polygon for the data.

Speed (km/h)	Frequency
45–49	2
50–54	6
55–59	12
60–64	13
65–69	6
70–74	6
75–79	3
80–84	2
Total	**50**

Solution

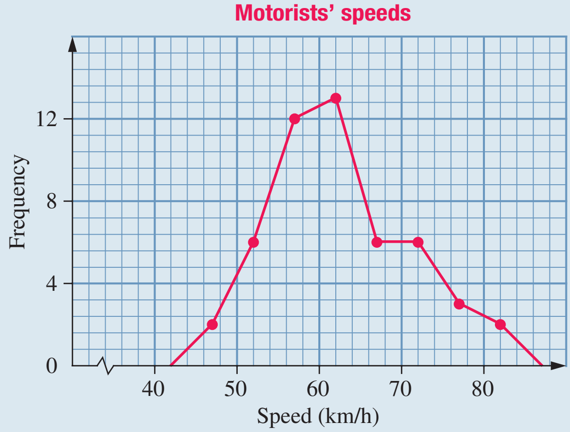
Put the true class limits.
Put the class centres.
Calculate the cumulative frequencies.

Motorists' speeds (km/h)

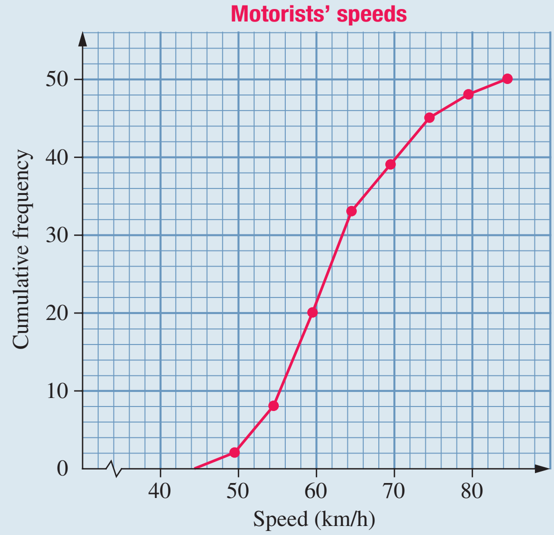
True class limits	Class centre	Frequency	Cumulative frequency
44.5–49.5	47	2	2
49.5–54.5	52	6	8
54.5–59.5	57	12	20
59.5–64.5	62	13	33
64.5–69.5	67	6	39
69.5–74.5	72	6	45
74.5–79.5	77	3	48
79.5–84.5	82	2	50
	Total	**50**	

Draw the frequency polygon using the class centres.
Start and finish at 42 and 87, the next centres.

Motorists' speeds



Draw the cumulative frequency polygon using the true upper class limits.



Additional Exercise 7.4

Exercise 7.4 Graphing grouped data

1 Draw frequency histograms of the following grouped data.

a

Score	5–9	10–14	15–19	20–24	25–29	30–34
Frequency	3	7	13	10	7	5

b

Score	15–24	25–34	35–44	45–54	55–64	65–74
Frequency	2	8	12	17	9	4

2 Draw frequency polygons of the following grouped data.

a

Score	15–19	20–24	25–29	30–34	35–39	40–44	45–49	50–54
Frequency	1	8	12	16	10	6	3	1

b

Score	240–249	250–259	260–269	270–279	280–289	290–299
Frequency	3	9	15	16	9	4

3 Draw cumulative frequency histograms of the following grouped data.

a

Score	30–33	34–37	38–41	42–45	46–49	50–53	54–57
Frequency	3	8	12	18	11	7	3

b

Score	40–54	55–69	70–84	85–99	100–114	115–129
Frequency	2	9	13	16	8	3

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4 Draw cumulative frequency polygons of the following grouped data.

a

Score	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89
Frequency	3	8	26	35	28	15	8	5

b

Score	0–99	100–199	200–299	300–399	400–499	500–599
Frequency	5	15	22	21	14	4

Modelling and problem solving

5 a Bunjee jumpers must be weighed before they 'take the plunge'. The weights (in kg) of 40 jumpers were recorded as follows:

41	58	63	37	49	58	71	33	85	58
60	73	81	46	55	38	80	48	50	62
61	59	63	44	77	62	58	73	62	75
52	60	69	61	55	47	76	42	66	70

Use the following class limits to construct a frequency distribution table and draw a frequency polygon: 30–39, 40–49, … 80–89.

b From 200 bunjee jumpers, how many would you expect to weigh 70 kg or more? Justify your answer.

6 a The numbers of litres of oil used in servicing cars each day by a mobile mechanic were:

172	164	115	142	98	104	157	149	163	124	201	187	79	133
152	221	184	226	205	98	162	171	213	188	173	156	224	184
166	152	210	172	181	207	178	224	167	185	110	108		

Present this data as a grouped frequency distribution table and draw an ogive.

b How much oil would you expect to be used in a fortnight? Justify your answer.

7 Fifty schools in south-east Queensland were surveyed regarding the percentage of non-English-speaking background (NESB) students in attendance.

a Draw a histogram for this data.

b Construct a frequency polygon for the data.

c What percentage of NESB students would you expect in Queensland schools? Explain your answer.

% NESB students	**Frequency**
0–9	5
10–19	9
20–29	18
30–39	12
40–49	5
50–59	1

8 A couple who make specialty soaps for sale at a local craft market want to check the masses of the cakes of soap. Over a period of a few weeks, the masses of 50 cakes of soap were recorded before the soap was wrapped for sale.

a Construct a cumulative frequency histogram.

b What percentage of the soaps had a mass less than 270 g?

c From 800 cakes of soap, how many would you expect to have a mass less than 265 g? Justify your answer.

Mass (g)	**Frequency**
255 to 259	2
260 to 264	12
265 to 269	15
270 to 274	15
275 to 279	4
280 to 284	2

7.5 Back-to-back graphs and scatterplots

To compare data, it is common to set out the information as a back-to-back graph. This is particularly useful with stem-and-leaf plots because it can be done quickly and easily.

Example 15

The parents of some students in Year 11 have the following ages. Make a back-to-back stemplot showing the ages of both parents and comment on your results.

Mothers' ages:

38 43 35 52 55 57 47 49 39 44 46 43 48 44 40 51 53 36 42 49 52 39 44

Fathers' ages:

41 46 38 55 58 57 48 48 40 49 45 49 52 48 43 57 61 38 42 51 54 41 44

Solution

Put the fathers' ages on one side of the stem and the mothers' ages on the other side.

Put the leaves on the left in descending order.

Ages of parents of Year 11 students (years)

Father Leaf	Stem	Mother Leaf
8 8	3	5 6 8 9 9
9 9 8 8 8 6 5 4 3 2 1 1 0	4	0 2 3 3 4 4 4 6 7 8 9 9
8 7 7 5 4 2 1	5	1 2 2 3 5 7
1	6	

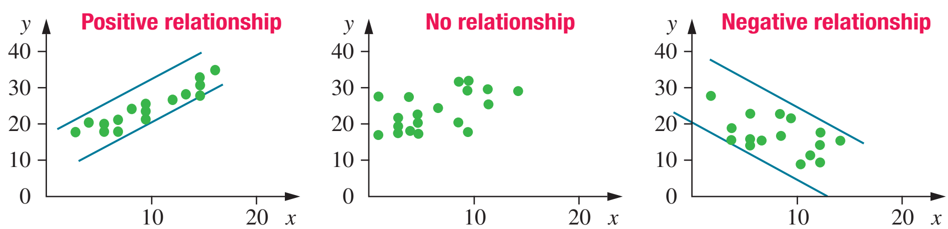
Key: 8 | 3 = 38 = 3 | 8

It is clear from the pattern that the fathers are in general a little older than the mothers.

While back-to-back graphs are useful, they do not show us the information in a precise way, because it is not clear from the graph whether the students with older mothers also have older fathers, or whether the parents' ages are mixed up. To find whether there is a general relationship between data such as this, we use a **scatterplot**.

!

A **scatterplot** (**scatter graph** or **scattergram**) is constructed from data that occurs in pairs by plotting points on a coordinate system. One variable is plotted on the x-axis, and the other on the y-axis. **Positive** and **negative relationships** may be found by observing the general pattern of the points.



- In a positive relationship, as one variable increases, the other generally increases.
- In a negative relationship, as one variable increases, the other generally decreases.

These trends can be shown by sloping lines that mostly enclose the points on the graph.

Example 16

Use a scatterplot to find whether there is any relationship between the ages of the mothers and fathers of Year 11 students given in Example 15.

Mother's age:

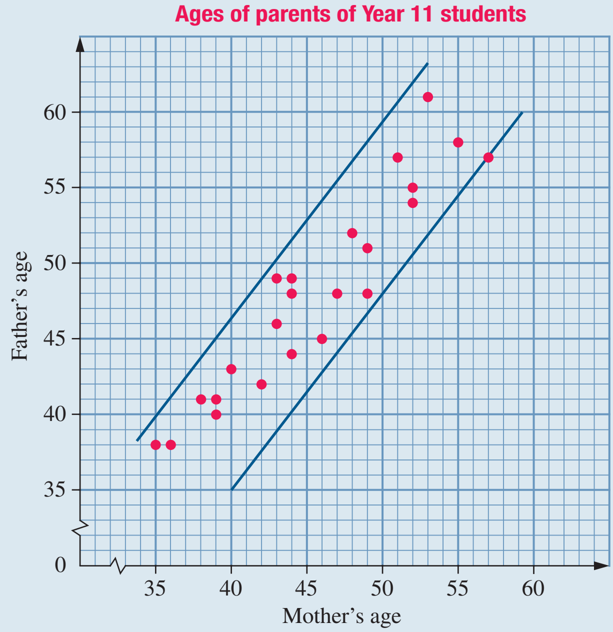
38 43 35 52 55 57 47 49 39 44 46 43 48 44 40 51 53 36 42 49 52 39 44

Corresponding father's age:

41 46 38 55 58 57 48 48 40 49 45 49 52 48 43 57 61 38 42 51 54 41 44

Solution

Plot the mothers' ages on the x-axis and the fathers' ages on the y-axis.



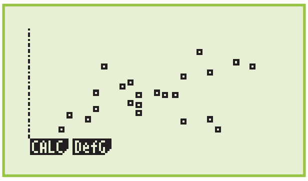
The scatterplot shows a clear positive relationship between the ages of students' mothers and fathers.

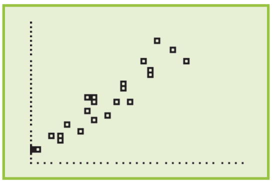
Technology

Scatterplots

You can use your graphics calculator to make scatterplots. Enter the data in lists as you did for histograms and polygons, but enter corresponding values in List 1 and List 2. Instead of choosing the histogram or polygon, set the scatterplot as the graph type.

Remember to change the **WINDOW** for the TI-84 and Sharp calculators. You also need to select the XY type on the Sharp calculator. The data from Example 16 gives the following when done on a graphics calculator.





Spreadsheet

You can also use a spreadsheet to make scatterplots. You can use the prepared spreadsheet on the CD-ROM. Enter the data from Example 16 into the spreadsheet and look at the scatterplot. The first pair of values has been entered for you.

Once the data is all entered, block the data. Click on the Chart Wizard, choose XY(Scatter) and follow the instructions to draw the scatterplot.

Investigation — Demographic relationships in countries

The table below shows some statistics for fertility, maternal death and use of modern contraception for selected countries. Work in groups to find whether there are relationships between the different statistics given.

Country	Fertility	Maternal death ratio/100 000	Modern contraception %
Afghanistan	7.48	1900	4
Algeria	2.53	140	50
Australia	1.75	8	72
Bangladesh	3.25	380	44
Brazil	2.35	260	70
Cambodia	4.14	450	19
Canada	1.51	6	73
Chad	6.65	1100	2
China	1.70	56	83
Egypt	3.29	84	54
France	1.87	17	69
Germany	1.32	8	72
Indonesia	2.37	230	57
Italy	1.28	5	39
Japan	1.33	10	53
Malaysia	2.93	41	30
Mexico	2.40	83	60
New Zealand	1.96	7	72
Norway	1.79	16	69
Papua New Guinea	4.10	300	20
Philippines	3.22	200	33
Russian Federation	1.33	67	53
Saudi Arabia	4.09	23	29
Singapore	1.35	30	53
South Africa	2.80	230	55
Sweden	1.64	2	71
Turkey	2.46	70	38
United Kingdom	1.66	13	81
United States of America	2.04	17	71

Investigation Weight for age

Work as a class group to collect information about class members' weights (mass in kilograms), ages (in months), heights and scores on the last Maths test. You may want to add some other variables to the list. When you have collected the data, work in groups to determine whether there are any relationships between the variables you have information for.

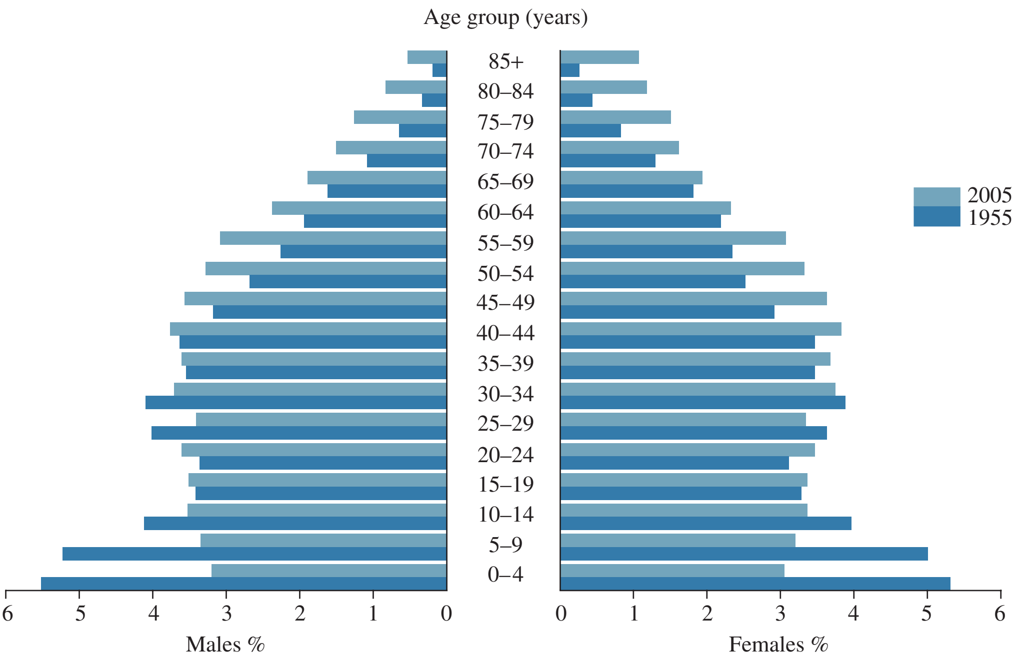
Exercise 7.5 Back-to-back graphs and scatterplots

Additional Exercise 7.5

Modelling and problem solving

1 The back-to-back graph below compares the age distribution of the Australian population in 1955 to that in 2005.

Age distribution of population, 1955 and 2005



a Which age groups were a larger percentage in 1955 than in 2005?

b Were there more babies in 1955 than in 2005? Justify your answer.

c What has happened to the age distribution of the Australian population in the 50 years from 1955 to 2005? Justify your answer.

d How are the male and female population distributions different? Explain your answer.

2 The numbers of words per sentence in a computer magazine article were:

10	28	31	17	23	27	18	15	26	24	20	19
36	27	14	25	15	22	11	21	24	27	17	29

The numbers of words per sentence in a newspaper article were:

27	39	33	24	28	19	32	41	33	27	35	12
38	41	27	13	22	23	18	46	32	22	18	32

a Construct a back-to-back stemplot showing both articles.

b What conclusion can you draw about the articles?

3 The relative humidities at 3 pm in two towns over a period of 2 weeks were:

Town A:	33	35	67	45	48	67	84	56	58	57	45	48	68	56
Town B:	45	48	67	78	79	84	65	58	43	59	69	89	78	69

a Draw a back-to-back stemplot of these humidities.

b Compare the results and suggest which town got more storms.

4 The following figures are the mean minimum monthly temperatures (in °C) for Hobart and Brisbane.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Brisbane	20.9	20.8	19.6	17.1	13.7	11.1	9.56	10.1	12.6	15.7	18.1	19.9
Hobart	11.8	11.7	10.6	8.72	6.4	4.5	3.9	4.5	5.8	7.3	9.0	10.6

a Compare the two sets of data by drawing a back-to-back stem-and-leaf plot.

b Compare the two sets of data by drawing a back-to-back histogram.

c Explain the variation of temperature between summer and winter.

5 The following information shows the cost of a single room in, and the star rating of, some Queensland motels. Use a scatterplot to find if there is a relationship.

Cost ($)	160	104	110	92	68	72	270	150	280	150
Stars	4	4	4	2	2	2	5	$3\frac{1}{2}$	4	4

Cost ($)	110	120	120	104	120	84	74	126	210	70
Stars	$2\frac{1}{2}$	$3\frac{1}{2}$	$3\frac{1}{2}$	$2\frac{1}{2}$	3	3	$2\frac{1}{2}$	$3\frac{1}{2}$	$3\frac{1}{2}$	$2\frac{1}{2}$

6 The following figures were collected by the Australian Bureau of Statistics. They concern where men and women work. The figures show the percentage of each sex working in each industry shown in the table.

Industry	**Male**	**Female**
Administrative, managerial	9.2	2.5
Professional, technical	13.5	19.2
Service, sport, recreation	5.9	16.1
Trades, process and production workers	40.5	8.9
Farmers, fishers, timber workers	9.1	4.0
Transport and communication	7.3	2.1
Clerical	8.0	34.6
Retailing	6.5	12.6

a Use a scatterplot to find whether there is a relationship between male and female participation in different industries.

b What do the results suggest about the employment of men and women? *Hint:* If there was equal participation, what would be the relationship?

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7 Use a scatterplot to find whether there is a relationship between how long 10 obese people were on a weight-loss program and the number of kilograms that each lost in that period.

Months on program	7	4	12	9	22	10	1	14	17	4
Weight lost (kg)	24	12	42	38	71	46	1	53	64	18

8 A heavy machinery firm keeps records of weekly usage and annual maintenance expenses for its equipment, as shown in the table.

Weekly usage (hours)	Annual maintenance expenses ($100s)
13	17.3
11	21.0
21	31.0
28	36.0
32	47.5
18	30.5
23	32.2
31	40.0
40	51.4
39	40.5

a Draw a scatterplot for this data.

b Does there appear to be a relationship between the variables?

7.6 Misleading graphs

A graph is always drawn by a person. Even if you use a computer or calculator to draw a graph, you decide what the scales and points will be. Accordingly, you will choose to emphasise the aspects of the information you think are most important. This may also cause someone to misinterpret the graph and thus gain a misleading impression. In some cases, a person may choose to draw a graph in a way that is deliberately intended to mislead.

!

Graphs can be drawn to change the emphasis (or even mislead) by:

Non-zero origin – exaggerating changes by starting above zero

Uneven scale(s) – changing appearance by going up in uneven steps on one or both axes

Using 2D or 3D – exaggerating differences by using 2D or 3D representation

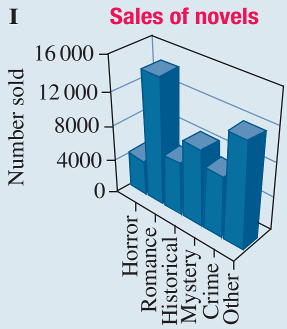
Uneven axes – exaggerating differences by making one axis much longer than the other

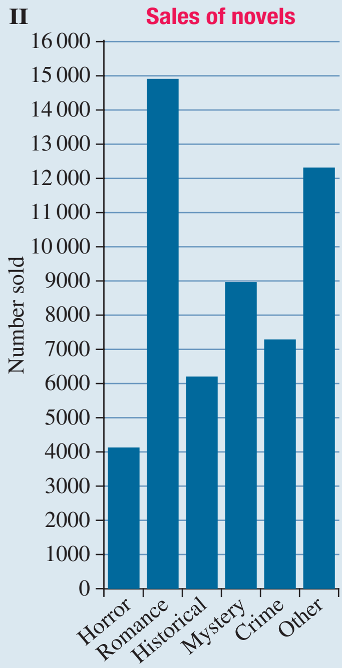
Missing data – misleading by leaving out points or data.

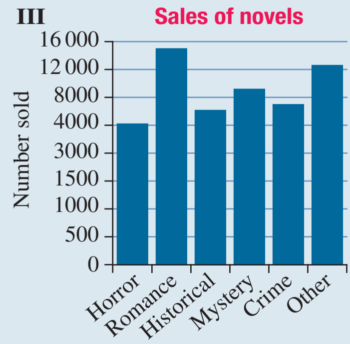
Example 17

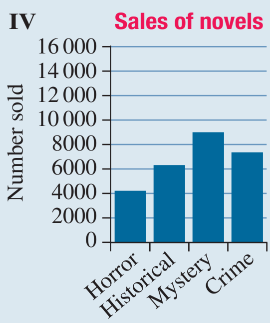
a For each of the following graphs, state the method that has been used to change the emphasis on the data from Example 1 on page 174.

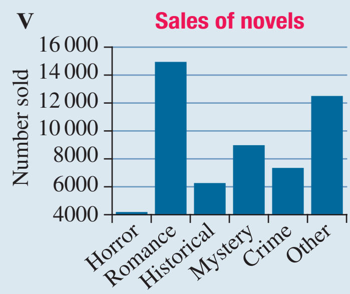
b Redraw the graph correctly.











Solution

a Graph I: The sales of Romance and Other look nearly the same. The rest appear small. The graph uses the effect of 3D and perspective to do this.

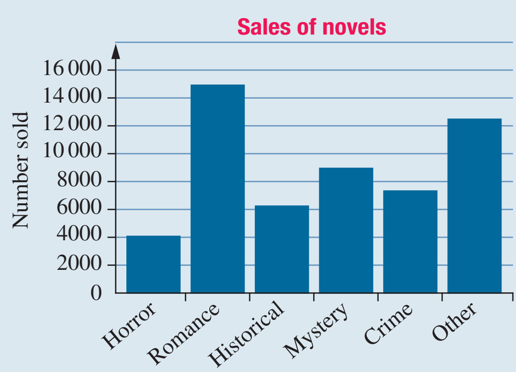
Graph II: The differences between sales appear very large. The graph uses uneven axes—the vertical axis is much larger than the horizontal axis.

Graph III: The differences in sales appear much smaller in this graph. It uses an uneven scale on the vertical axis. The first step is 500 and the last steps go up by 4000.

Graph IV: On this graph, the sales of Mystery novels appear greatest. It is missing data—the Romance and Other sales are not included.

Graph V: The sales of Horror novels seem very small because the graph starts from 4000, a non-zero origin.

b A correctly drawn column graph for the data from Example 1 is shown on the right.



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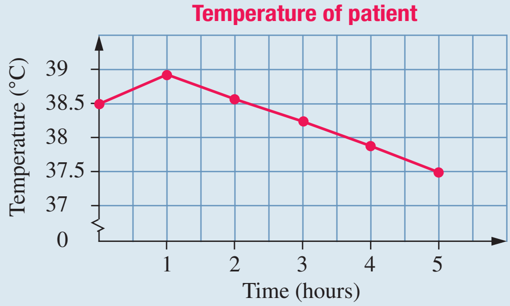
Example 18

The following data shows the temperature of a patient during the first few hours of hospitalisation. Draw a graph that emphasises the changes of temperature of the patient.

Time (hours)	0	1	2	3	4	5
Temperature (°C)	38.5	38.9	38.6	38.2	37.8	37.5

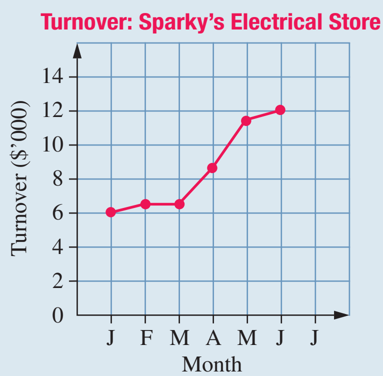
Solution

If the graph was drawn from 0°C, it would appear flat. To emphasise the changes, the vertical axis should be drawn only from 37° to 39°. This is shown below.



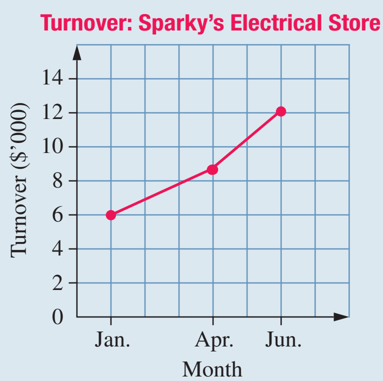
Example 19

Choose from the points shown on the graph at right to make it appear that there has been a steady increase in sales.



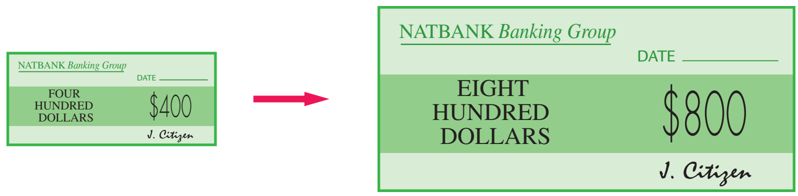
Solution

Choosing only the points that lie almost straight from January to June will have the required effect.



2D picture graphs can easily mislead the reader. The following pictogram was supposed to show how the average weekly earnings of Australian workers doubled between 1985 and 2000.

How average weekly earnings have grown



1985 2000

However, although the length and width of the second cheque are double those of the first, the area of the second cheque is *quadruple* the area of the first cheque. This creates the impression that average weekly earnings quadrupled, not doubled, over this period.

Similarly, the second divided bar graph in Example 3 on page 176 is misleading. The sales of strawberry flavour look much bigger than the sales of banana flavour because the strawberry section is drawn wider at the top of the thickshake container.

Additional Exercise 7.6

Exercise 7.6 Misleading graphs

Modelling and problem solving

1 Draw a line graph of the following average monthly temperatures in Brisbane to:

a emphasise the changes **b** minimise the changes.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Temperature (°C)	29.1	28.9	28.1	26.4	23.5	21.2	20.6	21.7	23.9	25.7	27.3	28.8

2 Select points from the table to draw a line graph of the monthly sales of a real estate office to:

a emphasise variation **b** emphasise steady sales.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Sales ($'00 000)	34	28	36	31	13	21	32	17	42	35	27	18

3 Draw a column graph of the following information about the turnover of a shop to:

a show an increase **b** show steady earnings.

Year	2003	2004	2005	2006	2007
Turnover ($'000)	256	640	495	587	610

4 Draw a horizontal bar graph of the following information about primary causes of fatal car accidents to make it appear that:

a one cause is more frequent than the others

b the causes are not very different in frequency.

Cause	Excessive speed	Alcohol or drug use	Road conditions	Driver inexperience	Driver inattention	Other
Number of accidents	39	65	24	52	47	37

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5 The consumption of different types of meat by Australians in a recent year was the following.

Type of meat	Beef and veal	Mutton and lamb	Pig meat	Poultry
Consumption ('000 tonnes)	713	311	330	569

Draw a picture graph to make it appear that:

a about the same amount of each type of meat was eaten

b more poultry than other types of meat was eaten.

6 The amounts of time each week that a student spent on each subject outside class were as follows.

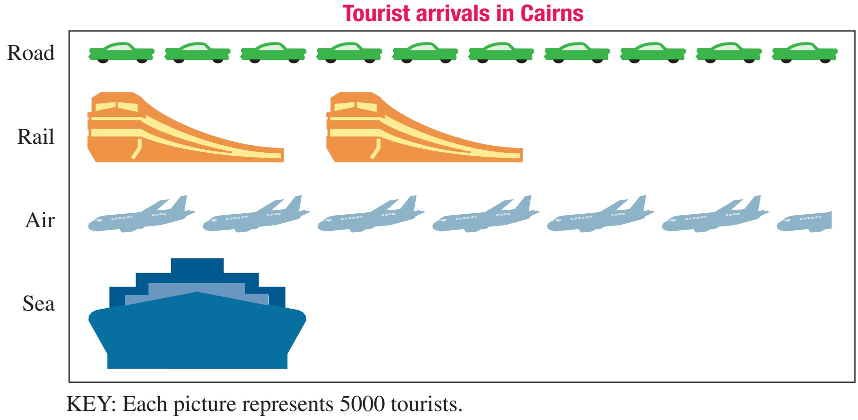
Subject	Maths	Biology	HPE	BCT	Art	English
Time (min)	100	90	55	32	140	80

Draw a column graph to make it appear that:

a the subjects got about the same amount of time

b a few subjects got much more time than the others.

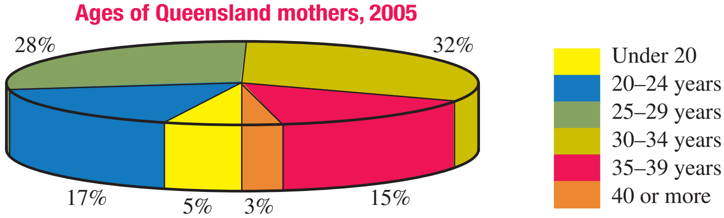
7 The graph below shows the numbers of tourists arriving in Cairns by each form of transport.



a Give reasons why this graph is misleading.

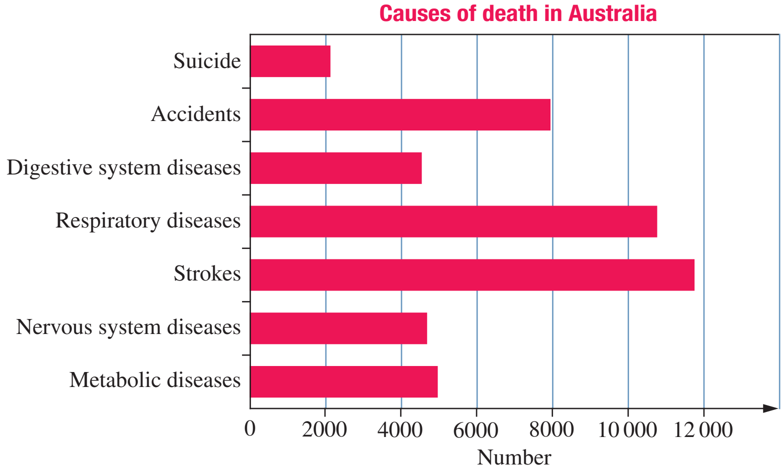
b How could it be drawn to be more correct?

8 The graph below shows the ages of Queensland mothers giving birth in 2005.



a Give reasons how the shape of the graph is misleading.
b Give reasons how the colours of the graph are misleading.
c How could the graph be redrawn to be more correct?

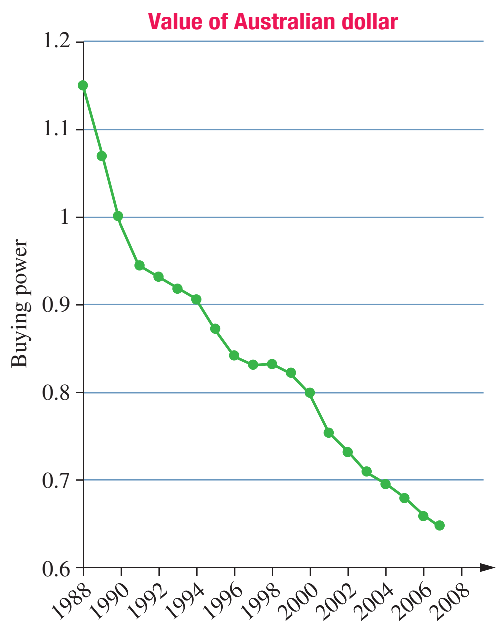
9 The graph below shows causes of death among Australians.



a Explain how this graph is seriously misleading.
b Explain what must be done to make it more correct.

10 The graph on the right shows the value of the Australian dollar over a period of 20 years (indexed to 1990).

a Explain how this graph is misleading.
b Redraw the graph so that it is more correct and justify the changes you make.



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Chapter summary

- A **picture graph** (**pictograph** or **pictogram**) is made by using pictures or symbols to represent the data.
- A **sector graph** (**pie chart** or **circle graph**) has a circle divided into sectors.
- A **divided bar graph** (or **segmented bar chart**) has a rectangle divided into smaller pieces.
- A **bar graph** (**bar chart**) is constructed by using rectangles (bars or columns) of equal width for each item of data. A vertical bar graph is often called a **column graph**.
- **Line graphs** are constructed by plotting data as a set of points and joining the points.
- A **stem-and-leaf plot** (or **stemplot**) is created by placing the first digit(s) in order from lowest to highest in a column to make the stem. The other digits are written in a row across from the first digit to make the leaves.
- A **frequency histogram** is a column graph of a frequency distribution. A **frequency polygon** is a line graph of a frequency distribution. For both histograms and polygons, the horizontal axis is continuous and shows the variable, while the frequency is always shown on the vertical axis.
- **Cumulative frequency** is the sum of the frequencies up to and including an item. An **ogive** is a smoothed cumulative frequency polygon.
- Continuous and large data distributions are tabulated in **classes**. The **class midpoint** (**class centre**, **class score** or **class mark**) is the average of the **upper class limits** and **lower class limits**.
- Class midpoint = $\dfrac{\text{upper class limit + lower class limit}}{2}$
- The edges of the columns for **frequency histograms** and **cumulative frequency histograms** of grouped data are at the true class limits.
- The points for **frequency polygons** of grouped data are placed at the class midpoints.
- The points for **cumulative frequency polygons** and **ogives** of grouped data are placed at the true upper class limits.
- A **scatterplot** (**scatter graph** or **scattergram**) is constructed by plotting one variable as the x-coordinate and the other as the y-coordinate. **Positive** and **negative relationships** may be found by observing the general pattern of the points.
- The appearance of a graph may be altered for emphasis or to mislead using a **non-zero origin**, **uneven scale(s)**, **2D or 3D**, **uneven axes** or by **missing data**.

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Chapter review

Knowledge and procedures

Ex 7.1

1 The following pictograph shows the numbers of offences in certain categories of crime that were reported to police one year in Australia.

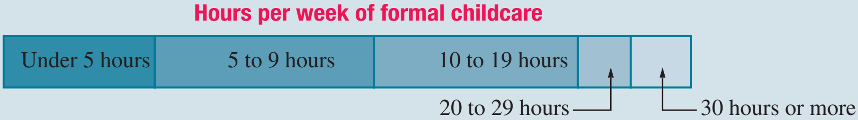
a Approximately how many offences in each category were reported?

b What percentage of the total number were sexual assaults?

c It is estimated that only 55% of assaults are actually reported to police. If this is the case, approximately how many assaults occurred during this period?

Ex 7.1

2 In Australia one year, about 455 000 children under 12 years old received formal childcare. The segmented bar chart below shows the numbers of hours per week of formal childcare that they received.



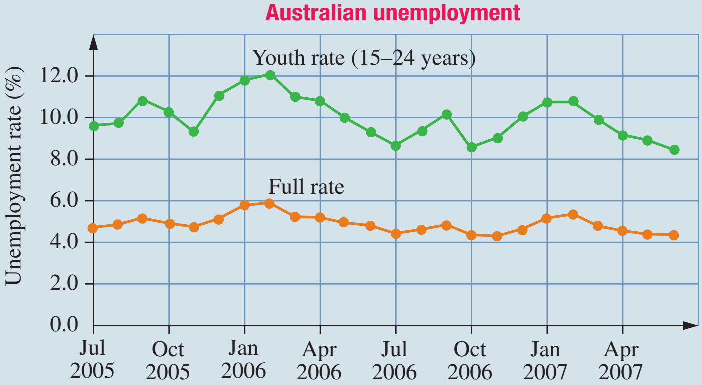
a What period of formal childcare was most common for these children?

b How many of these children received less than 5 hours a week formal childcare?

c Represent this data as a pie chart.

Ex 7.1

3 The following graph shows the unemployment rate for Australia over 2 years.



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Chapter review

a For what period is actual data shown?

b At what intervals is the unemployment rate shown?

c How do changes in the full and youth unemployment rates compare?

d What was the highest youth unemployment rate in this period?

4 The information below shows the temperature of a cup of coffee after it is poured. Use the information to draw a line graph of the temperature. Ex 7.1

Time (minutes)	0	5	10	15	20	25	30	35
Temperature (°C)	77	64	55	48	42	37	33	30

5 Make a stem-and-leaf plot of the response times (in seconds) shown below for consecutive emergency calls by 000 telephone operators: Ex 7.2

12 75 45 52 61 49 53 45 64 22 54 15 23 45 68 51 40
39 60 61 55 58 41 69 66 52 38 79 18 54 22 31 47

6 The data below shows the numbers of new golden staph infections recorded at a large hospital over 60 days.

Number of infections	0	1	2	3	4	5	6
Number of days	9	7	9	11	6	12	6

a Draw a histogram to show the data. Ex 7.2

b Draw a cumulative frequency polygon. Ex 7.3

7 The results of some students in Maths A and English are given below: Ex 7.5

Maths A: 35 55 64 84 41 51 55 82 54 87 35 42 51 77 62 74 67 52 66 51

English: 66 50 55 59 71 45 63 55 78 72 44 59 62 57 77 50 44 63 52 71

Draw a back-to-back stemplot of the information and comment on the results.

8 How is a class midpoint worked out? Ex 7.4

9 Sketch scatterplots that show positive, negative and no relationship. Ex 7.5

Modelling and problem solving

10 The weights in kilograms lost by new members of a well-known weight-loss program after completing a special 3-month introductory offer are given here: Ex 7.4

12.2 2.1 33.9 0.9 4.2 8.7 1.8 28.4 7.6 19.4 3.8 2.7 23.5 9.4 8.2 1.3
7.1 1.6 2.1 6.9 4.6 0.3 5.6 2.5 4.8 4.4 0 7.8 5.8 4.2 9.3 1.3
2.6 14.1 2.3 12.6 10.9 3.8 0.3 12.5 7.5 4.6 8.9 2.1 12.5 9.8 11.1 7.1

a Use suitable class groups to make a frequency distribution table.

b Draw an ogive.

c What percentage of the participants lost more than 4 kg?

d Comment on the effectiveness of the program and explain your answer.

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Chapter review

Ex 7.5

11 This data gives the proportions of employees who were members of a trade union.

Age group	15–19	20–24	25–34	35–44	45–54	55–59	60–64	65+
Males	27%	38%	47%	52%	53%	56%	51%	13%
Females	27%	32%	35%	35%	41%	43%	26%	3%

a Use a back-to-back histogram to represent this data.
b Use the resulting histogram to make any relevant comparisons.

Ex 7.5

12 The number of police and number of prisoners held in custody in a particular region of Queensland are shown below.

Year	1961	1966	1971	1976	1981	1986	1991	1996	2001
Police	40	41	48	53	73	97	134	221	264
Prisoners	325	221	338	243	375	546	1178	1354	1643

a Draw a scatterplot for the information.
b Does the scatterplot suggest any relationship between the number of police and the number of prisoners?
c Does this suggest that increasing police numbers increases crime? Justify your answer.

Ex 7.6

13 The numbers of students who completed different subjects in Year 12 in 2006 in Queensland are shown in the table below.

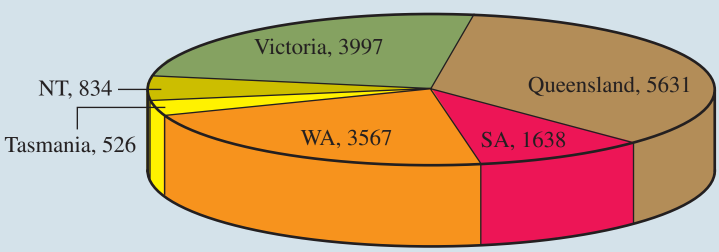
Subject	English	Maths A	Maths B	Biology	Chemistry	Visual Art	Physical Education	Legal Studies	BCT
Number	33 033	22 015	15 988	11 831	8765	6917	10 055	7046	7121

Draw a column graph to make it appear that:
a Biology had the largest number of students.
b English had the second-largest number of students.
c The numbers of students completing Chemistry, Visual Art, Legal Studies and BCT were very small.

Ex 7.6

14 The following graph shows prisoners in Australian gaols in 2006.

Prisoners in full-time custody, 2006



a Explain at least two ways in which the graph is misleading.
b What should be done to make the graph more correct?

Introduction to navigation

8

Contents

Syllabus Guide Chapter 8

Syllabus subject matter

Maps and compasses—navigation

- Compass bearings and reverse bearings
- Magnetic variation
- Plot and determine compass bearings and reverse bearings
- Use magnetic variation to explain the link between True bearings and Magnetic bearings

Quantitative concepts and skills

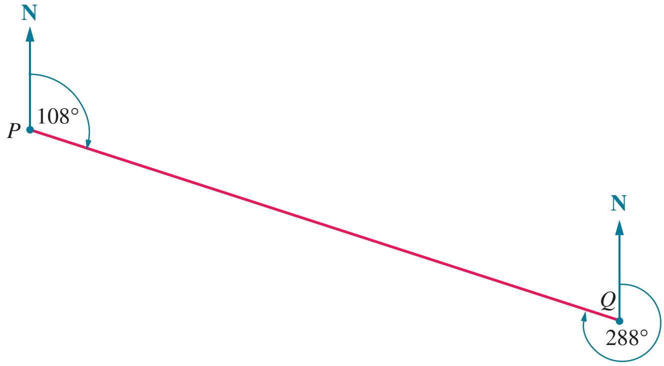
- Metric measurement including measurement of mass, length, area and volume in practical contexts
- Calculation and estimation with and without instruments

The Earth is a sphere, but small parts of it can be considered to be flat. This means that for most purposes we can use a flat map on paper to find our way around. It is only when we want to travel large distances that the curvature of the Earth becomes important.

To find our way around, we need a sense of direction. We use bearings to state directions. In ancient times, compasses were made from magnetic lodestones. Magnetic compasses are still used today to find directions.

8.1 Finding bearings and distances

A direction on the Earth is given as a **bearing**. A bearing is the horizontal direction of an object from an observer expressed as the **clockwise angle from north to the direction of the object**. The bearing is measured in degrees and given as three figures from 000° to 360°. Maps are normally drawn with north running vertically up the map. In the diagram below, the bearing of *PQ* (from *P* to *Q*) is 108°.



A **back bearing** (sometimes called a **reverse bearing**) is the bearing of the opposite direction. A back bearing differs from the bearing by 180°. In the diagram on page 212, the back bearing of *PQ* (from *Q* to *P*) is 288°.

Back bearings

- If a bearing is less than 180°, add 180° to find the back bearing.
- If the bearing is greater than 180°, subtract 180°.

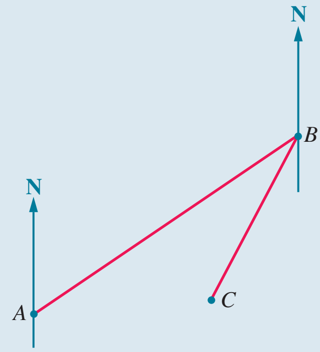
Example 1

For the diagram shown, use a protractor to find:

a the bearing of B from A

b the bearing of BC

c the back bearing of BC.



Solution

a Position the protractor over A as shown below. Read off the clockwise angle from north to B.

Bearing of B from A is 056°.

b Position the protractor over B as shown. Read off the clockwise angle to C.

Bearing of BC is $180° + 27° = 207°$

c Calculate the back bearing of BC from the bearing of BC.

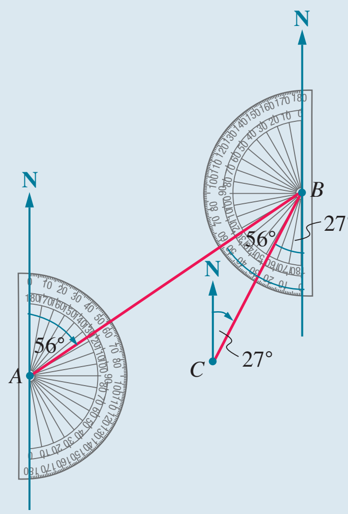
Back bearing of BC

$= \text{bearing of } BC - 180°$

$= 207° - 180°$

$= 027°$

Note: The back bearing of BC is the same as the bearing of CB.



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Example 2

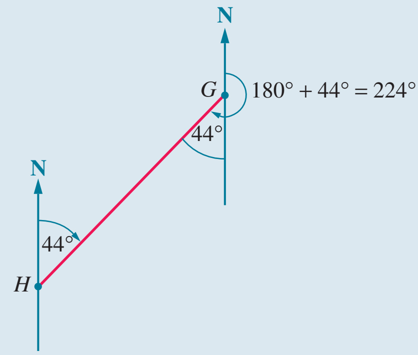
The bearing of *GH* is 224°. What is the back bearing?

Solution

The bearing is greater than 180°, so subtract 180°.

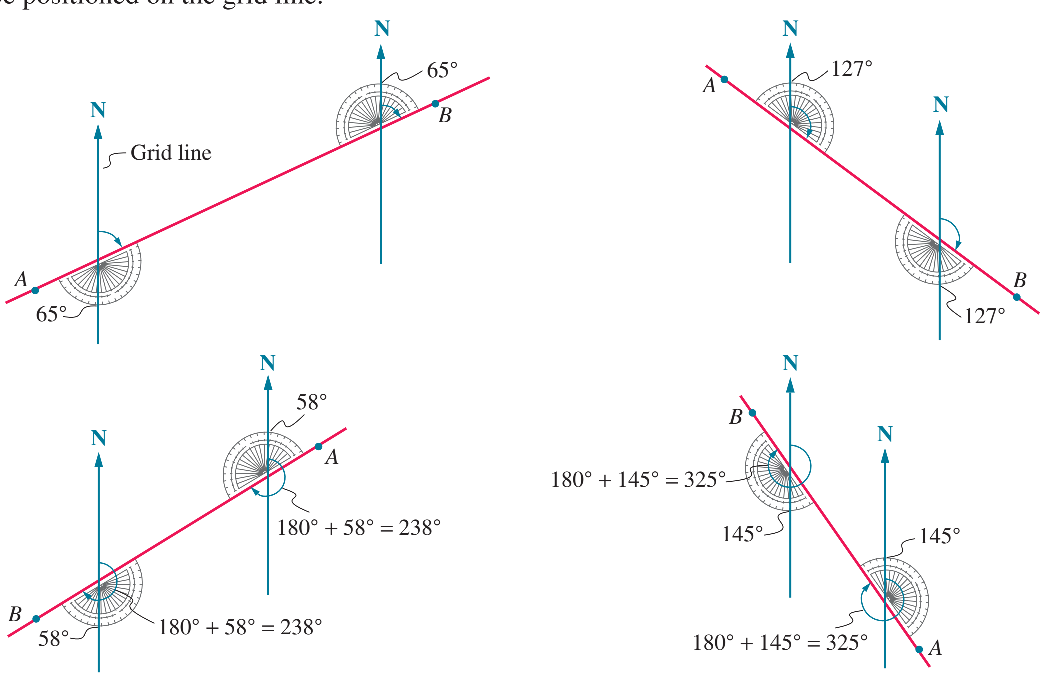
Back bearing = 224° – 180°
= 044°

You can draw a sketch to confirm the result.



Maps are produced with lines in a criss-cross pattern called **grid lines**. Bearings taken from the vertical grid lines are referred to as **grid north** because on most maps the grid lines are not exactly north–south.

You can use a clear plastic ruler and 180° protractor to measure bearings directly from the vertical grid lines on a map. This technique may be used for any bearing, but in some cases you must add 180° to the protractor reading to obtain the correct bearing. The protractor may be placed above or below the ruler, but care must be taken to read the correct scale. Some possible set-ups to measure the various bearings of *AB* are shown below. The centre of the protractor must be positioned on the grid line.



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Example 3

Measure the grid bearing from Monkey Point on Great Keppel Island to Double Head on the Yeppoon/Keppel Isles map shown below.

Solution

Line up a plastic ruler on the map from Monkey Point to Double Head.

Put your protractor against the ruler.

Slide the protractor along until the centre cross is on a vertical grid line and read the angle.

This gives the bearing.

$$\text{Grid bearing} = 180° + 103°$$
$$= 283°$$

Yeppoon/Keppel Isles area **Scale 1 : 250 000**

Most survey maps are produced to an international standard based on the **Universal Transverse Mercator Projection (UTM)**. This system establishes grids that cover the entire Earth with grid systems that overlap each other, for latitudes from 80°S to 80°N. Another system is used for areas near the poles.

Areas and distances are shown very accurately on maps produced with the UTM projections. The grids have lines 10 km (10 000 m) apart or 1 km (1000 m) apart, depending on the scale of the map. The map in Example 3 has a 10 km grid and a scale of 1 : 250 000. Distances can be calculated using the map scale or measured against a distance scale on the map.

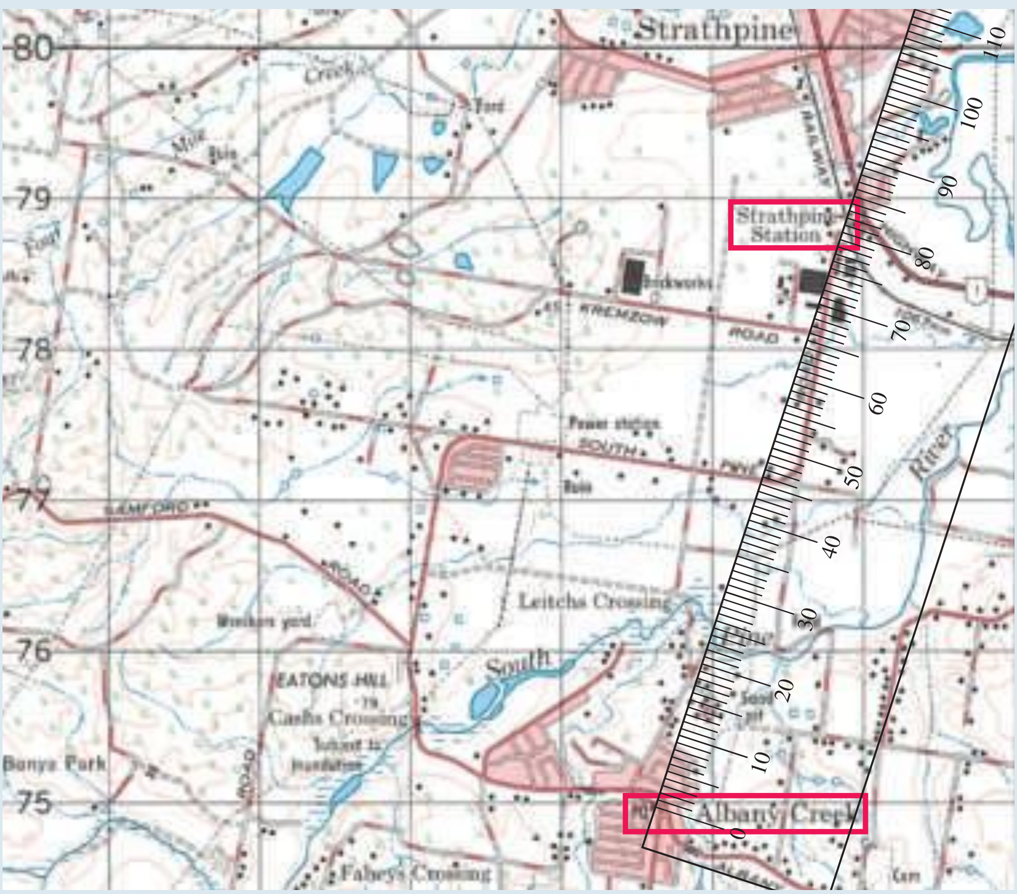
Example 4

Find the distance from the Strathpine Station to the Albany Creek post office (PO) on the Albany Creek map shown below.

Solution

The distance on the map is measured using a clear plastic ruler. The map is at a scale of 1 : 50 000 with 1 km grid squares.

Measure the distance on the map from the railway station to the post office (in mm).	Map distance = 82 mm
Calculate the true distance.	True distance = 82 mm × 50 000
Multiply by 50 000 as the scale is 1 : 50 000.	= 4 100 000 mm
Convert mm to m (÷1000).	= 4100 m
Convert m to km (÷1000).	= 4.1 km



Albany Creek area **Scale 1 : 50 000**

We can use the distance and bearing to find a point on a map from a given point. A protractor and plastic ruler can be used to avoid marking the map. The distance is converted to the map scale and the protractor is lined up against the ruler and north lines.

Example 5

From the Yeppoon/Keppel Isles map shown below, find what geographical feature is near a distance of 26.75 km and on a grid bearing of 262° from Monkey Beach on Great Keppel Island.

Solution

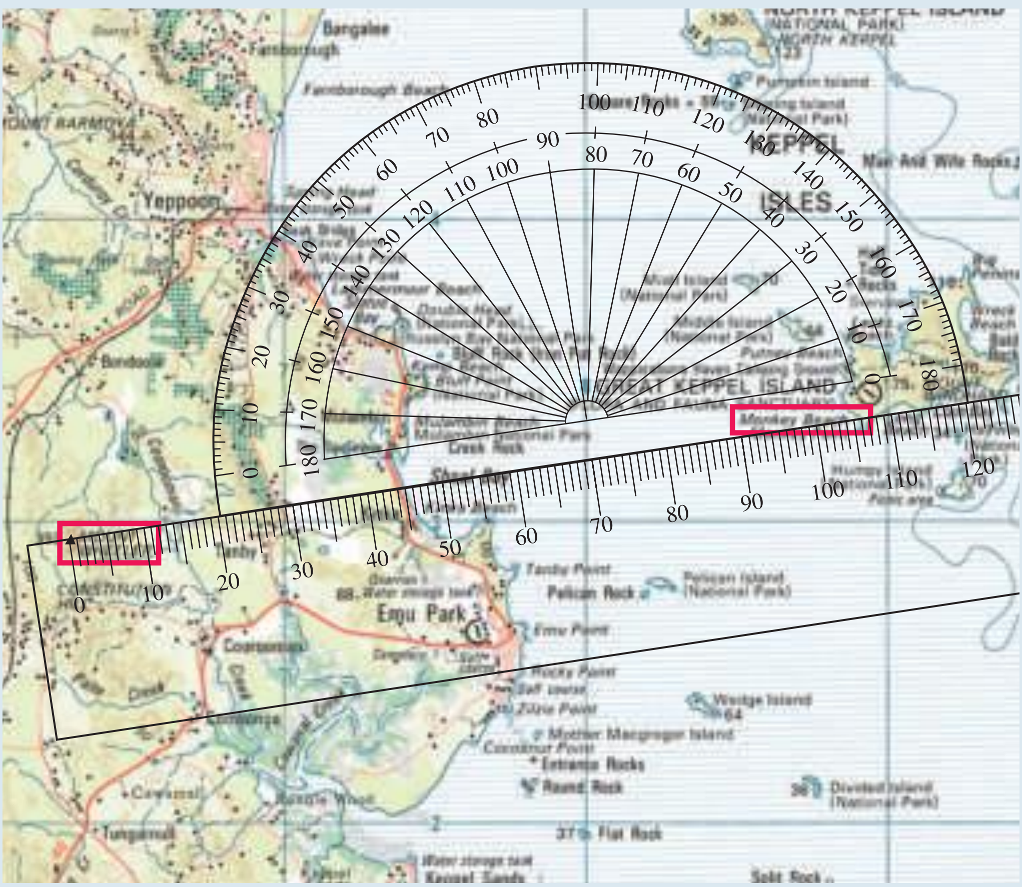
Find the distance on the map by dividing by the map scale and converting to mm.	Map distance = 26.75 km ÷ 250 000 = 107 mm

Since 262° = 180° + 82°, we want the protractor to measure a further 82° past the south direction (180°).

Place the protractor so that the base passes through Monkey Beach and it measures the correct angle on a north–south line.

Slide the ruler along the protractor to measure the distance of 107 mm on the map.

Write the answer.	The geographical feature is Mt Wheeler.



Yeppoon/Keppel Isles area **Scale 1 : 250 000**

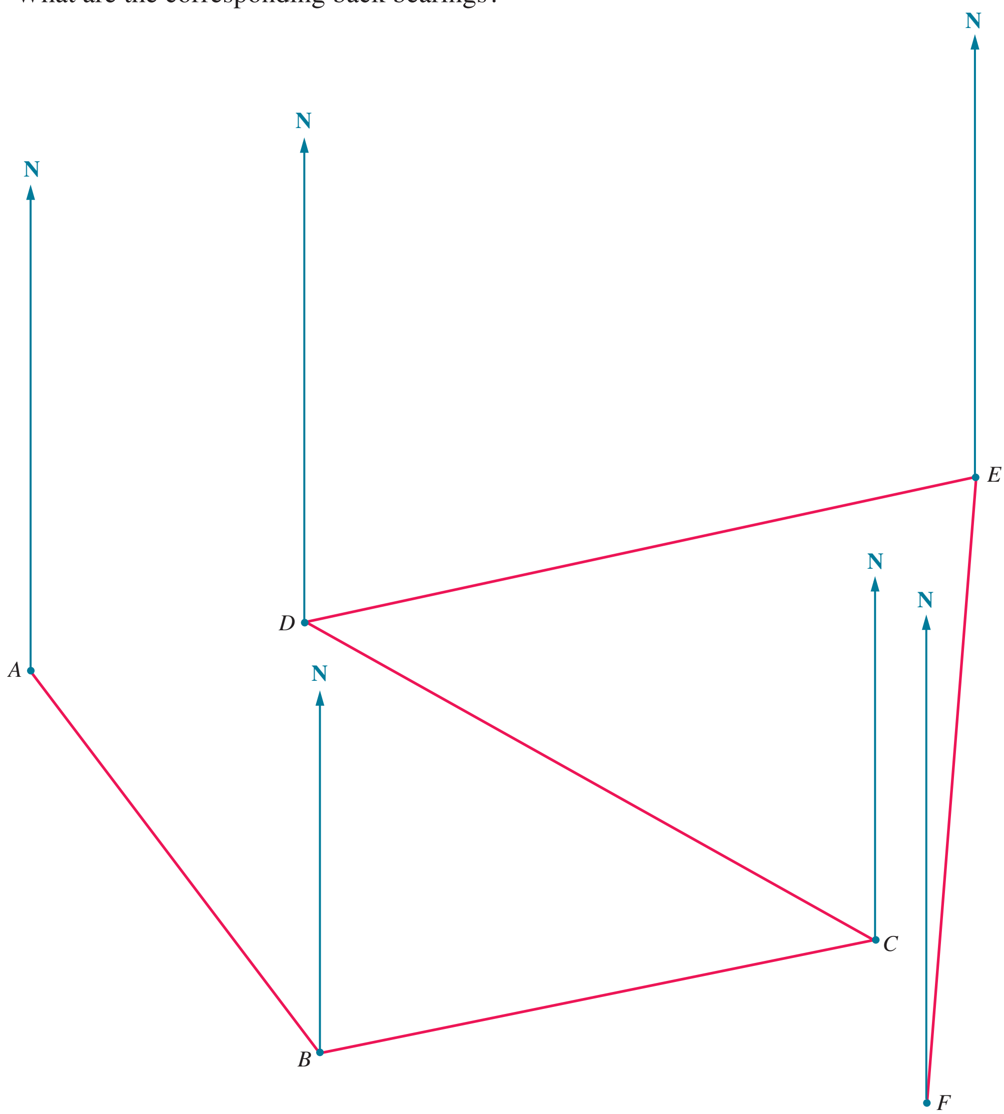
Note: When using a map to find distances or bearings of features that are not given by a precise point, it is usual practice to use a central point on the feature. In Example 5, a point at the centre of Monkey Beach was used. The same practice applies for buildings and other features that have been constructed. Mt Wheeler also covers a large area.

Additional Exercise 8.1

Exercise 8.1 Finding bearings and distances

1 a Use a protractor to find the bearings of *AB*, *BC*, *CD*, *DE* and *EF* in the following diagram.

b What are the corresponding back bearings?



2 What are the back bearings for these grid bearings?

a 305° **b** 174° **c** 090°

d 243° **e** 032°

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3 Use a clear ruler and protractor on the map of the Gladstone area shown below to find the grid bearing:

a from Datum Point to Friend Point

b from Worthington Island to Farmer Point

c from Mt Sugarloaf to Hamilton Point

d from the quarry on Boyne Island to Mt Larcom

e from the highway road bridge over the Calliope River to the railway bridge downstream.

Gladstone area **Scale 1 : 250 000**

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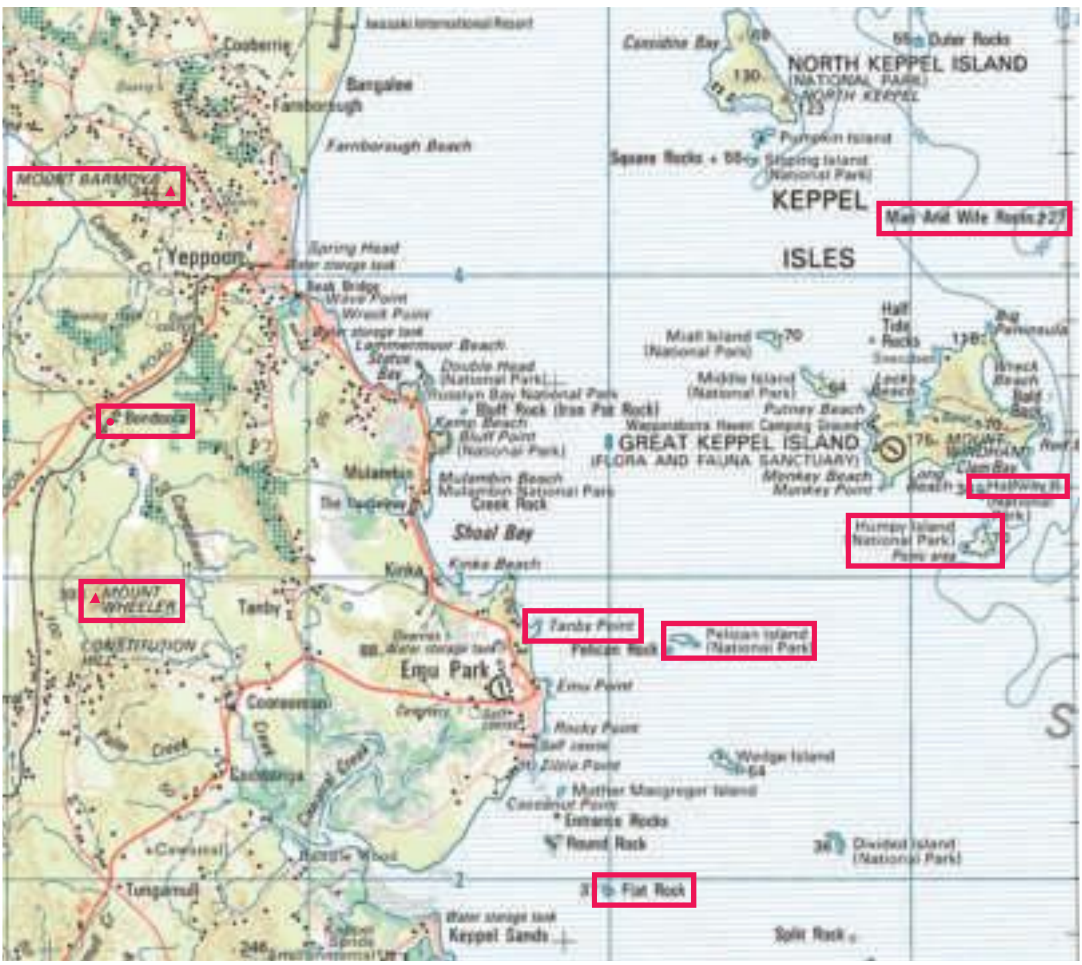
4 Use a clear plastic ruler and protractor on the map of the Yeppoon/Keppel Isles area shown below to find the grid bearing of:

- **a** Bondoola railway station from Mt Barmoya
- **b** Halfway Island from Tanby Point
- **c** Man and Wife Rocks from Mt Wheeler
- **d** Mt Barmoya from Pelican Island
- **e** Flat Rock from the closest point on Humpy Island.



Yeppoon/Keppel Isles area **Scale 1 : 250 000**

5 Use the Albany Creek map shown below to find the distance:

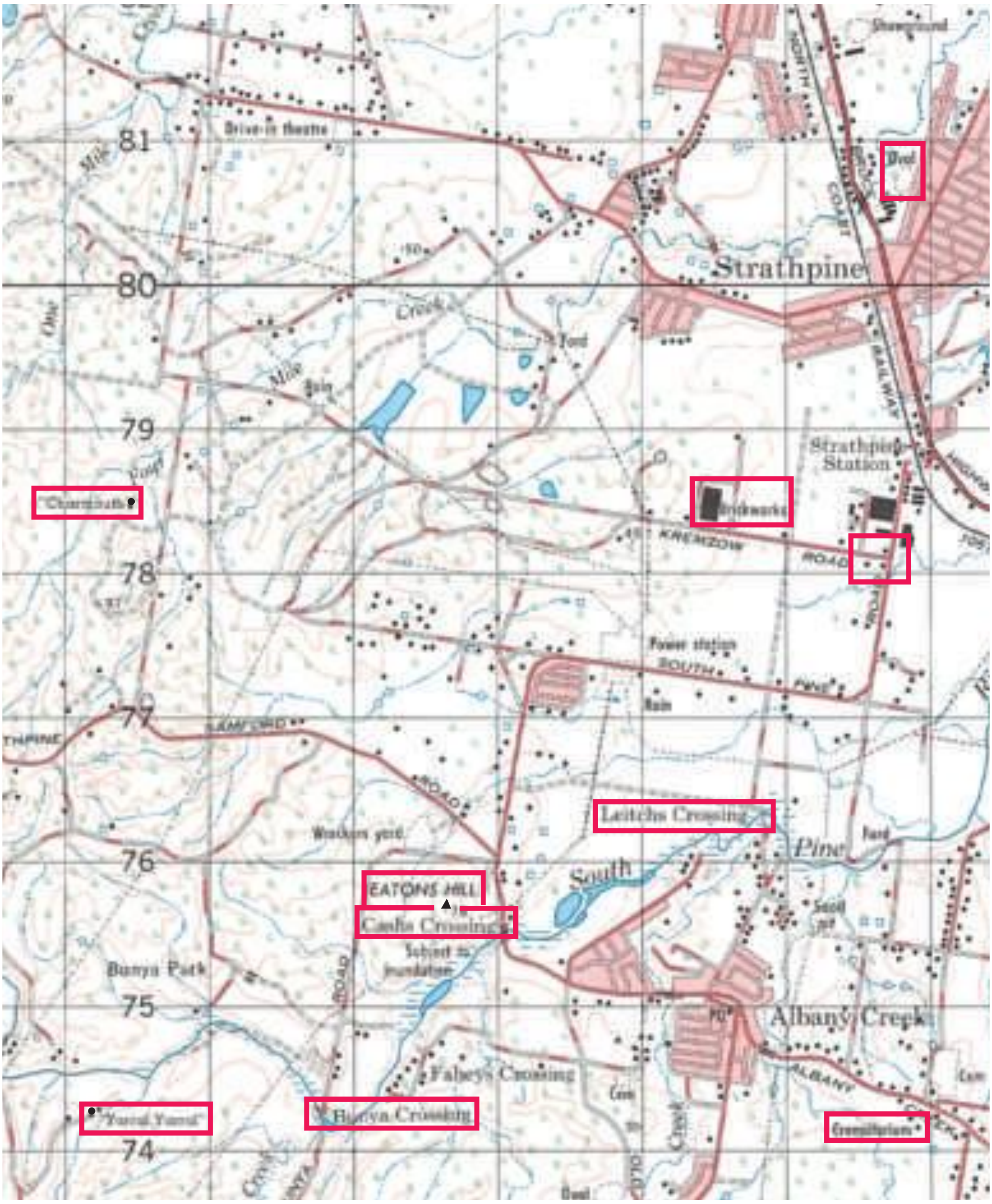
a from Cashs Crossing to Leitchs Crossing of the South Pine River

b from ‘Charmouth’ to ‘Yarral Yarral’

c from the intersection of South Pine Road and Kremzow Road to Eatons Hill

d from the crematorium near Albany Creek Road to the oval at Strathpine

e from the brickworks near Kremzow Road to Bunya Crossing.



Albany Creek area **Scale 1 : 50 000**

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6 Use the map of Redcliffe shown below to find the distance:

- **a** from the Redcliffe showground to the tower at Deception Bay
- **b** from Clontarf Point to Scarborough Point
- **c** from Dohles Rocks to the gravel plant near North Pine River
- **d** from the outfall of the sewage treatment plant's pipeline to Scotts Point
- **e** along the straight section of the Houghton Highway.

Map courtesy Australian Surveying and Land Information Group, Canberra, Australia. Crown Copyright ©. All rights reserved. *www.auslig.gov.au*

Redcliffe area **Scale 1 : 100 000**

7 From the railway station at Northgate, find the features located near the following distances and grid bearings on the map of the lower Brisbane River shown on the opposite page.

- **a** 3.9 km, 183°
- **b** 7.8 km, 071°
- **c** 3.5 km, 321°
- **d** 11 km, 123°
- **e** 7.9 km, 078°
- **f** 1.2 km, 027°
- **g** 3.2 km, 119°
- **h** 9.2 km, 078°

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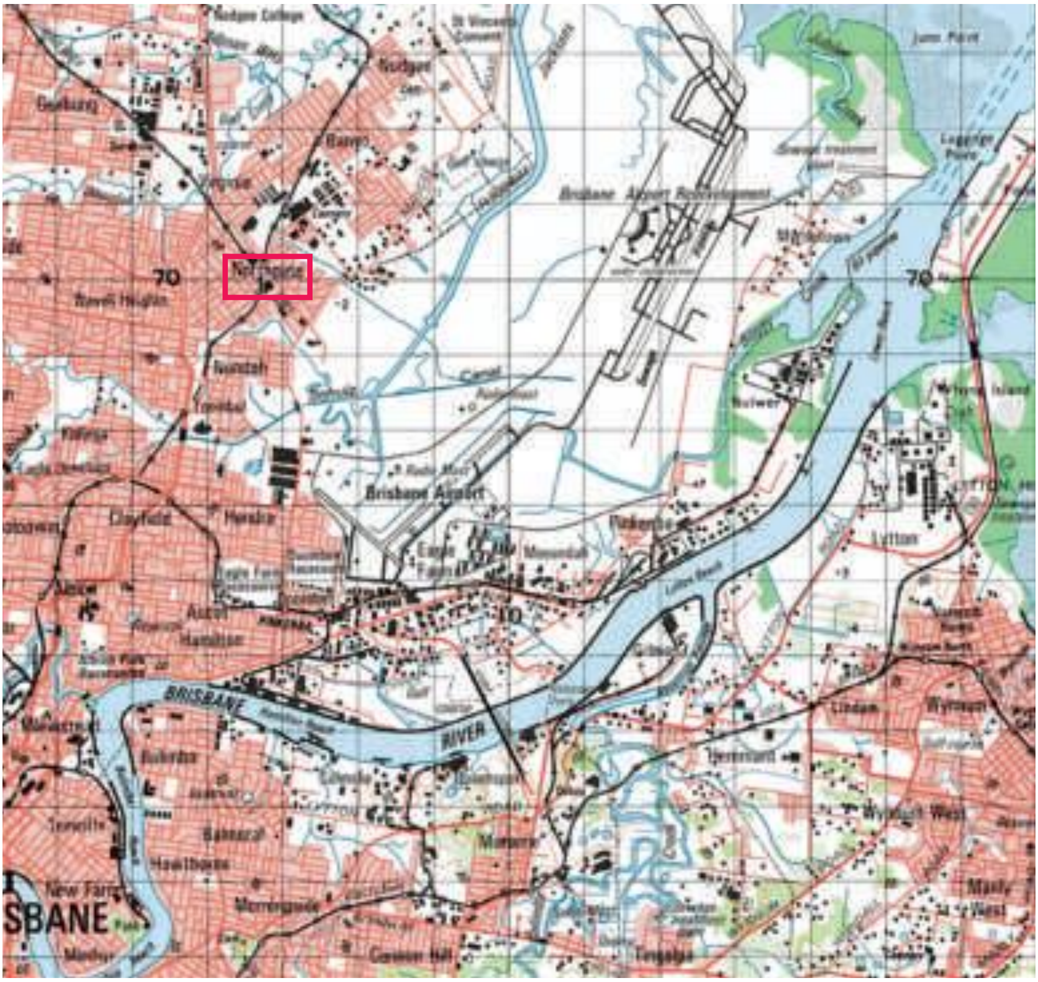

Lower Brisbane River area **Scale 1 : 100 000**

8.2 Using scales and directions

A protractor and ruler can be used to draw a route specified by pairs of bearings and distances. Each **leg** of the route is drawn from the last point reached. The final position can then be measured from the starting position.

Example 6

a Draw the route that would be followed from A to D if the legs of the trip have the following grid bearings and distances.

- AB is 48 km at a bearing of 120°.
- BC is 60 km at a bearing of 200°.
- CD is 35 km at a bearing of 065°.

b Use the drawing to find the distance and grid bearing of AD.

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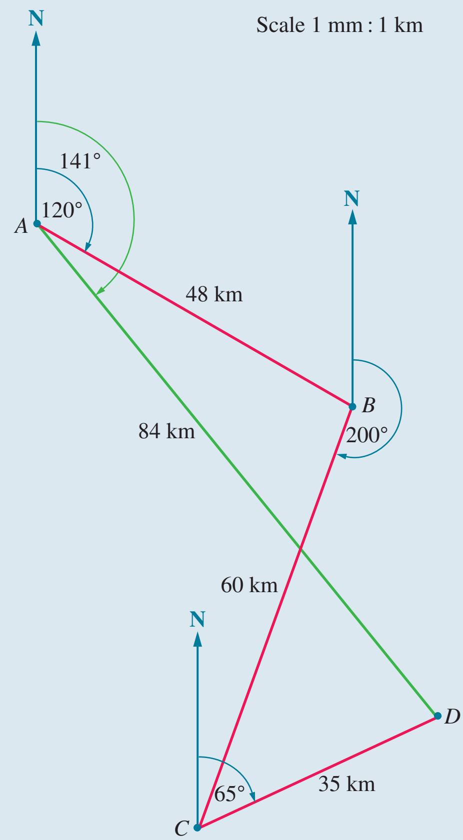
Solution

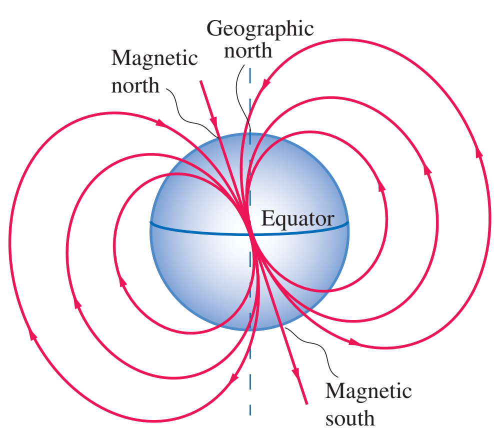
a Use a scale of 1 mm to 1 km.
Mark the starting point, A.
Draw a north–south line through A.
Use the protractor to mark the direction 120°.
Draw a line from A along this direction.
Measure 48 mm and mark in B.
Draw a north–south line through B.
Use a protractor to keep the north–south lines parallel.
Repeat the steps as described above for BC.
Repeat for CD.



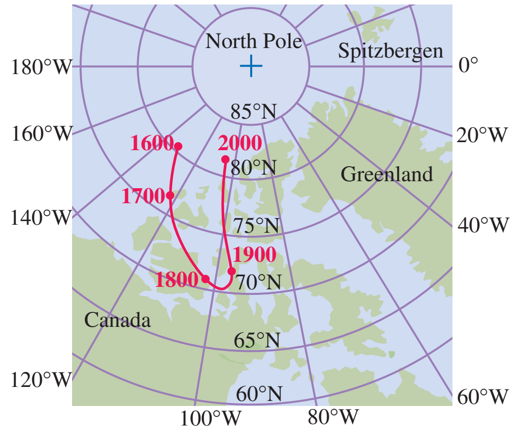
b Draw in AD

Measure the distance AD.	$AD = 84$ mm
Convert to km (×1 000 000).	$= 84$ km
Measure the bearing AD.	Bearing of $AD = 141°$
State the result.	AD is 84 km at a bearing of 141°.

The Earth behaves like a huge magnet. This has been known for so long that the north and south directions are used to describe the poles of a magnet. On the Earth, a suspended magnet will turn so that it is in line with the **magnetic field**. In most places on the Earth the magnetic field runs in a roughly north–south direction. The pole that turns towards the north is called the **north-seeking pole** of the magnet, but this is usually abbreviated to **north**. The **south-seeking pole** is also abbreviated, to **south**.



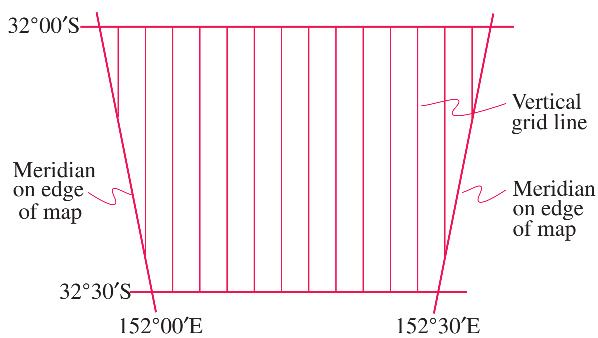
The Earth's magnetic field varies in direction and strength from place to place. It also changes direction over the years. The poles of the Earth's magnetic field are not at the North and South Poles, and also gradually change their positions. The diagram at right shows how the magnetic north pole moved from 1600 to 2000.



A magnetic compass is actually a very small light magnet made in the shape of a needle. The needle turns to point in the direction of the magnetic field. If there are no other magnetic materials nearby, this will be the direction of the Earth's magnetic field. In most places on the Earth, this will be nearly north–south.

The difference between true north and the direction of the Earth's magnetic field is called the **variation** (or **declination**) **of the magnetic field**. The variation must be taken into account when using a map and compass.

As previously mentioned, the **grid lines** on a map do not usually point exactly north. The grid lines are drawn parallel to one another on the Universal Transverse Mercator (UTM) Projection, but the meridians that point true north converge. On detailed maps derived from the UTM, the meridians are very nearly parallel, but in a slightly different direction from the grid lines. The difference in direction is called the **grid convergence**. Maps derived from the UTM are usually drawn between meridians, and these are shown on the edges of the map. The width of one of these maps varies a little between the top and bottom because the meridians converge. The meridians at the edges can be used to determine the direction of true north. If the meridian divergence of a UTM-derived map was exaggerated, the map edges would look like this:



So there are three 'north' directions that are important when using a map:

- **Grid north (GN)** is the direction shown by the grid lines on a map.
- **True north (TN)** is the direction shown by meridians. It is the direction of the North Pole of the Earth.
- **Magnetic north (MN)** is the direction of the Earth's magnetic field.

True north does not change. Grid north depends on the map-maker, but is now determined by the international use of the UTM projection. Magnetic north changes from place to place and from time to time. A **true bearing (T)** is the angle to true north. A **magnetic bearing (M)** is the angle to magnetic north. A **grid bearing (G)** is the angle to grid north.

The relationship between the three directions is normally shown on the map. The rate of change of magnetic north is also given. On marine charts, the directions are shown by **compass roses**.

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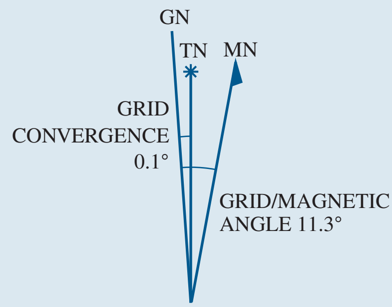
Example 7

The map of the lower Brisbane River area shown on page 223 has the relationship shown here between GN, TN and MN.

Use the information to find the true bearing and magnetic bearing for a grid bearing of 207° in 2013.



TRUE NORTH, GRID NORTH AND MAGNETIC NORTH ARE SHOWN DIAGRAMMATICALLY FOR THE CENTRE OF THE MAP. MAGNETIC NORTH IS CORRECT FOR 1983 AND MOVES EASTERLY BY 0.1° IN ABOUT TWO YEARS.

Solution

Magnetic north moves east by 0.1° in 2 years.

Calculate years to 2013. $\quad 2013 - 1983 = 30$ years

Calculate movement of magnetic north in 30 years.

$$\text{MN movement} = (30 \div 2) \times 0.1° = 1.5°$$

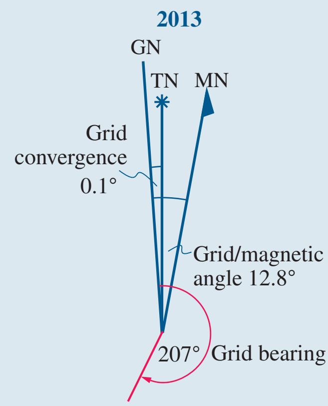
Calculate grid/magnetic angle in 30 years.
We *add* because magnetic north is moving *east*.

$$\text{GN/MN angle} = 11.3° + 1.5° = 12.8°$$

The angle between grid north and true north is 0.1°.
From the diagram:

$$\begin{aligned}\text{True bearing} &= \text{grid bearing} - \text{grid convergence}\\ &= 207° - 0.1°\\ &= 206.9°\text{T}\end{aligned}$$

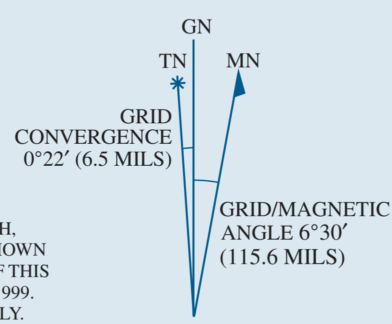
$$\begin{aligned}\text{Magnetic bearing} &= \text{grid bearing} - \text{grid/magnetic angle}\\ &= 207° - 12.8°\\ &= 194.2°\text{M}\end{aligned}$$



Example 8

Using a compass, a magnetic bearing of 038° is obtained in 2014 in an area with the relationship between grid, true and magnetic north as shown here. What are the true bearing and grid bearing?

THE RELATIONSHIP BETWEEN TRUE NORTH, GRID NORTH AND MAGNETIC NORTH IS SHOWN DIAGRAMMATICALLY FOR THE CENTRE OF THIS MAP. MAGNETIC VALUE IS CORRECT FOR 1999. ANNUAL CHANGE IS 01′ (0.3 MILS) EASTERLY.



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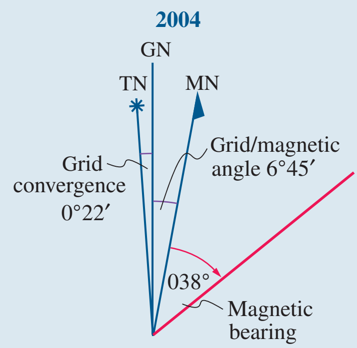
Solution

The change in magnetic north is 01′ every year after 1999. (Ignore MILS.)
The year 2014 is 15 years after 1999, so MN has moved 15 × 01′ = 15′ east.
Grid/magnetic angle is 6°30′ + 15′ = 6°45′.

From the diagram:

Grid bearing = magnetic bearing + grid/magnetic angle
= 038° + 6°45′
= 044°45′G

True bearing = grid bearing + grid convergence
= 044°45′ + 0°22′ = 044°67′
= 045°07′T (1° = 60′)



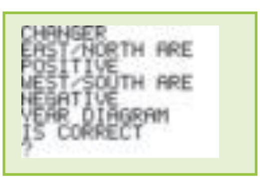
Technology

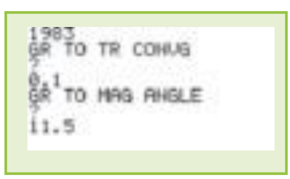
Calculator Program

Changing between magnetic bearings and grid or true bearings is a little tricky. You can use the program MAGTGR to check your calculations. The program is given in full on the CD-ROM. You need to enter the following in the program:

- The year for which the three norths diagram is correct.
- The grid convergence. If grid north is east of true north, it is negative.
- The change in magnetic north per year. An easterly change is positive and a westerly change is negative.
- The year the bearing is taken.
- The three-figure bearing you want to change.
- The kind of bearing that it is—magnetic, true or grid.

Enter the program (or load it from the CD-ROM) and try it with different changes of bearing.





Exercise 8.2 Using scales and directions

Additional Exercise 8.2

1 Make an accurate scale drawing of each of the following routes.
   a *AB* 65 km at 060°, *BC* 57 km at 153°, *CD* 57 km at 243°
   b *AB* 2700 m at 139°, *BC* 2300 m at 244°, *CD* 4 km at 039°
   c *AB* 4.4 km at 233°, *BC* 4.2 km at 160°, *CD* 8.4 km at 37°
   d *AB* 270 km at 117°, *BC* 460 km at 252°, *CD* 600 km at 16°
   e *AB* 82 km at 327°, *BC* 46 km at 115°, *CD* 42 km at 214°

2 For each of the routes given in question **1**, find the distance and the bearing from the start to the finish.

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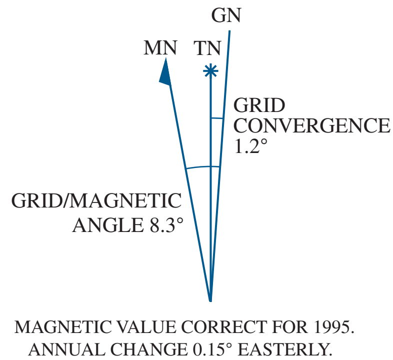
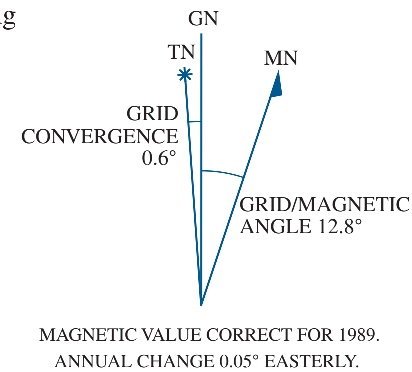
3 Use this diagram and information to find the true bearing and magnetic bearing in 2008 for the following grid bearings.

a 063°G **b** 125°G
c 246°G **d** 196°G
e 355°G



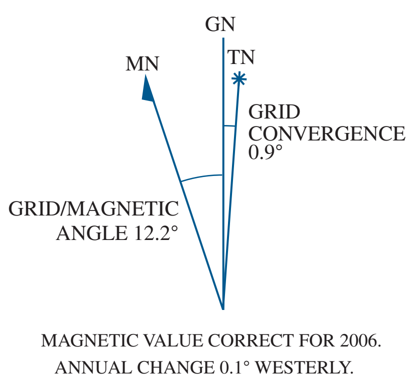
4 Use this diagram and information to find the true bearing and magnetic bearing in 2014 for the following grid bearings.

a 207°G **b** 092°G
c 164°G **d** 009°G
e 327°G



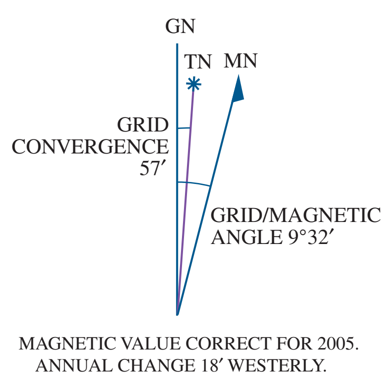
5 Use this diagram and information to find the true bearing and grid bearing for the following magnetic bearings in 2015.

a 284°M **b** 074°M
c 129°M **d** 010°M
e 303°M



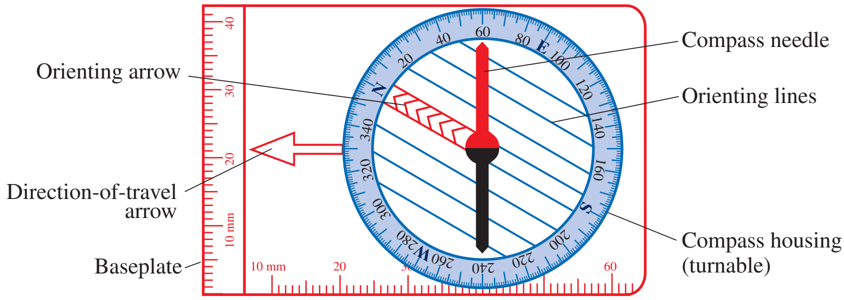
6 Use this diagram and information to find the true bearing and grid bearing for the following magnetic bearings in 2022.

a 006°M **b** 038°M
c 127°M **d** 214°M
e 356°M

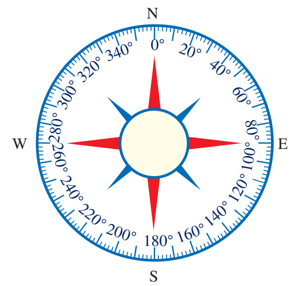


8.3 Using a map and compass

There are many kinds of compasses. The one we will use is called an **adjustable dial compass**. This type of compass is also called a **baseplate compass** or **orienteering compass**. An adjustable dial compass looks like this.



It has a built-in movable protractor or compass housing and is transparent so that maps can be read through the compass. Other features include orienting lines, a direction-of-travel arrow, and a red and black compass needle held in a liquid-damped enclosure. The red part of the compass needle always points towards the Earth's magnetic north pole.

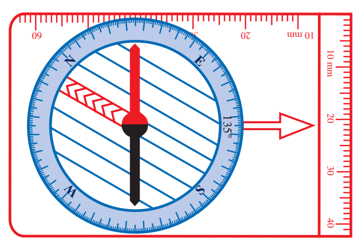


You can see that as well as having N, S, E and W indicated, the compass housing shows numbers of degrees to indicate direction.

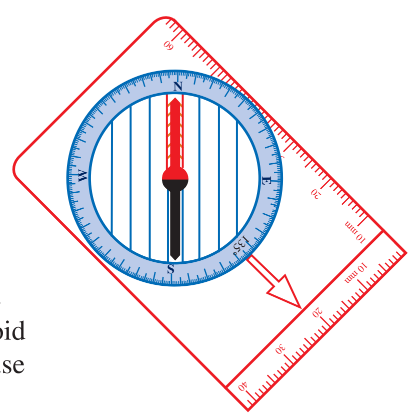
This means that north is 0° (or 360°), east is 90°, south is 180° and west is 270°.

If you want to travel north, just make sure that the arrow moves freely by holding the compass horizontally and follow the direction of the red arrow. If you want to travel in a different direction, you need to use the turnable compass housing.

1 Let's say you want to travel in a direction halfway between south and east (i.e. south-east). First find south-east on the compass housing (i.e. 135°). Then turn the compass housing so that south-east on the housing comes exactly where the large direction-of-travel arrow meets the housing, as shown in the diagram on the right.



2 Hold the compass horizontally in your hand so that the compass needle can turn. While watching the compass, turn your body until the compass needle is aligned with the orienting arrow inside the compass housing, as shown in the diagram on the right. Make sure that the red part of the compass needle points at north in the compass housing—otherwise you could walk off in the opposite direction to what you want.



Now simply walk off in the direction that the travel arrow is pointing in. You need to check the compass frequently to avoid getting off course. Later in this chapter we will see how to use an adjustable dial compass and map to navigate a course.

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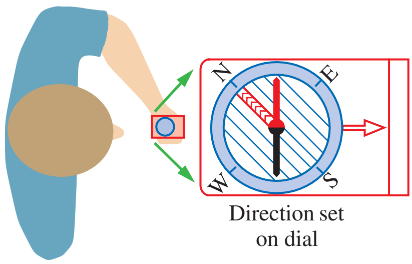
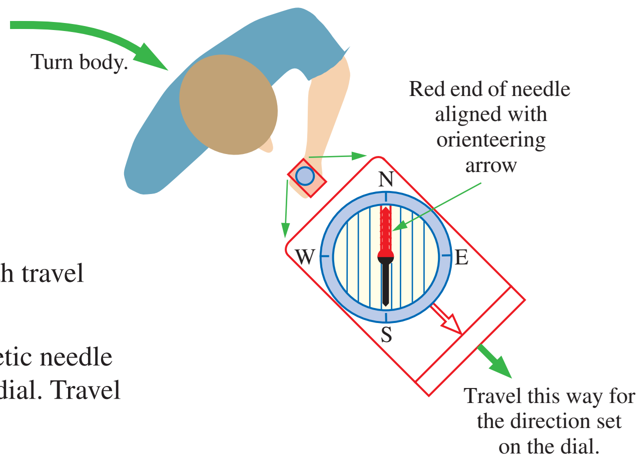
Magnetic materials will affect the working of the compass. Don't expect it to work properly if you are wearing steel belt buckles or if you hold the compass near an iron railing or car bonnet.

The following diagram summarises this method of finding a direction using a compass.

Finding a direction





1 Set direction on compass dial with travel arrow pointing directly ahead.

2 Turn body until red end of magnetic needle lines up with orienting arrow on dial. Travel arrow now shows direction.

Investigation Using a compass

For this investigation you will need the following:

- a large, open area (for example, your school oval)
- a coin or any other small flat object
- an adjustable dial compass.

Although you can do this on your own, it is more fun to work in a small group and compare your efforts with those of other group members.

1 Select a starting point and place the coin on the ground. Stand on the coin and set the adjustable dial to a direction of 150°, as described on page 229.

2 Face the direction of 150° with the compass held horizontally in your hand in front of you. Take 20 paces, making sure that your paces are the same size and that you stay on the selected direction. Remember to stop when you reach 20 paces.

3 Now use the compass to set a new direction of 270°. Take 20 paces in that direction and stop.

4 Next take 20 paces in the direction of 30°.

5 Look down on the ground and see whether the coin is there.

6 Now it's time for others to try their compass skills. See how close other group members are to the coin when they finish the course. The things to remember are:
   - keep your paces regular
   - stay on course by checking the compass
   - carefully set the direction on the compass housing
   - turn your whole body to face the direction that you should be travelling in.

Extra Material
Using a compass

7 Draw a scale diagram of the course that you should have travelled. One example of a scale diagram is shown on the CD-ROM. Compare your drawing with the one provided on the CD-ROM.

8 What is the name of the figure formed by the course?

Magnetic compasses must be used with magnetic bearings, so true bearings or grid bearings must be converted to magnetic bearings by correcting for the magnetic variation. A compass is oriented to magnetic north. When using an orienteering compass with a map, it is convenient to mark in some magnetic north lines (**magnetic meridians**). These are normally shown with arrow-heads to distinguish them from grid lines.

An adjustable dial compass is easier to use with a map than a conventional compass. The baseplate is placed in line with the desired direction, then the dial (compass housing) is turned until the orienting lines are in the north direction. The bearing is read directly from the dial. The magnetic needle is ignored.

Example 9

Find the magnetic bearing of Mount Quincan from Tula on the Malanda map shown below using an adjustable dial compass.

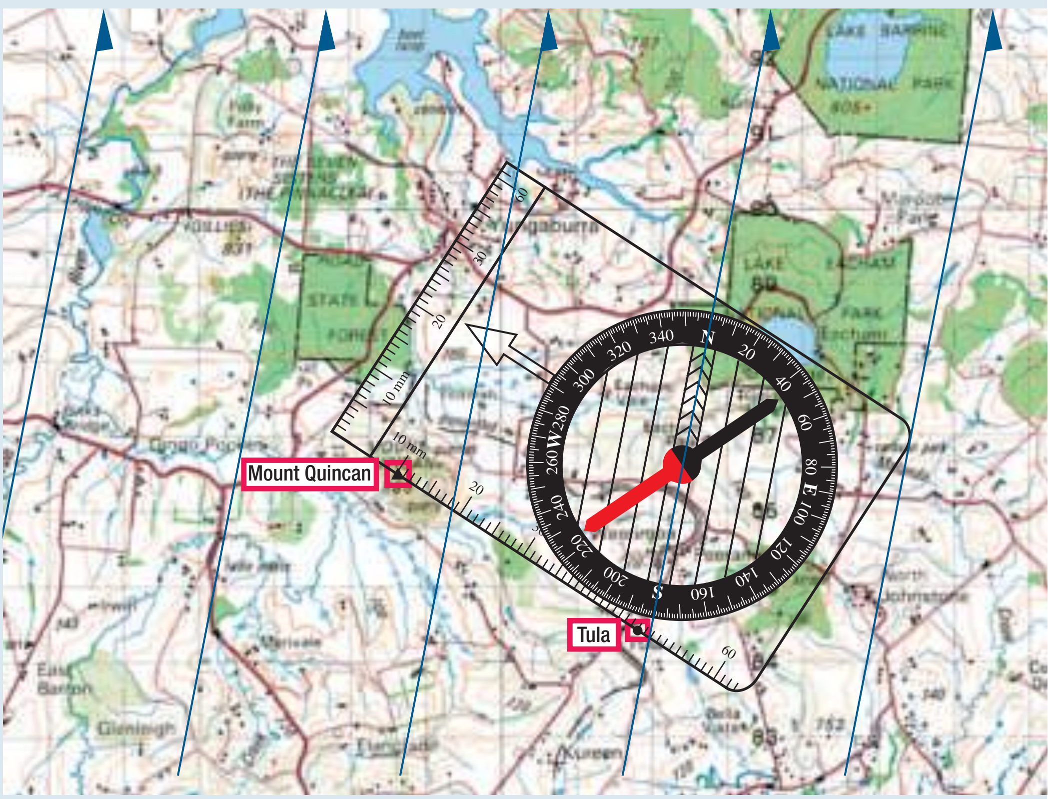
Solution

The map has magnetic north lines drawn in.

The distance is short enough to use the baseplate of the compass without a ruler.

Line up the baseplate between the two points and turn the dial so that the orienting lines are in the direction of the Earth's magnetic field (i.e. aligned with the magnetic north lines).

The dial reading is 292°, so the magnetic bearing is 292°M.



Malanda area **Scale 1 : 100 000**

You can also use an orienteering compass to find a map location using a bearing and a distance from a starting point. To use the compass in this way:

1. Use the map scale to change the distance into a distance on the map.
2. Set the bearing on the dial of the compass.
3. Place the edge of the compass baseplate on the starting point.
4. Turn the baseplate until the orienting lines are in line with the magnetic north lines (meridians), keeping the edge on the starting point.
5. Now use a plastic ruler against the baseplate to measure the distance.

Example 10

Use the Rathdowney area map shown below to find the feature that is near a distance of 3150 m and a bearing of 129°M from the windmill west of the township of Rathdowney.

Solution

Calculate the distance on the map.

$$\text{Distance} = 3150 \div 25\,000 \text{ m} = 0.126 \text{ m} = 12.6 \text{ cm}$$

Set the compass dial at 129°.
Place the compass and ruler on the map.

The feature is the top of the hill with an elevation of 160 m.

Rathdowney area **Scale 1 : 25 000**

Investigation Small orienteering courses

Teacher Notes

Extra Material

Pace length

Before undertaking this investigation, make sure that you discuss safety issues relating to orienteering with your teacher. You will need to know your pace length. Instructions for calculating your pace length are given on the CD-ROM.

1 Work in groups of three for this activity. Each group must set up a mini-course in the school grounds in the following way:
   - Choose an identifiable starting point.
   - Choose a visible feature as your first control point.
   - Use the compass to find the magnetic bearing of the control point. Walk to the control point and use your pace length to work out the distance. Average the distances obtained by the three group members.
   - Write down the control point, distance and bearing.
   - Choose the next control point and repeat the procedure.
   - Make a course with three or four legs.
   - Write down the starting point, and the distance and bearing of each of your control points in order.
   - Put a piece of paper with the name of each member of the group at each control point. Write 'Finished' on the last one.

2 Swap your instructions with the instructions from another group. Follow the course you have been given.

3 When the activity is finished, collect any unfound pieces of paper. Discuss any difficulties you encountered with other groups.

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Additional Exercise 8.3

Exercise 8.3 Using a map and compass

1 Use the Malanda map shown below to obtain the magnetic bearing:

a to Merivale from Elanglade

b to the centre of Lake Eacham from the centre of the quarry at Mt Quincan

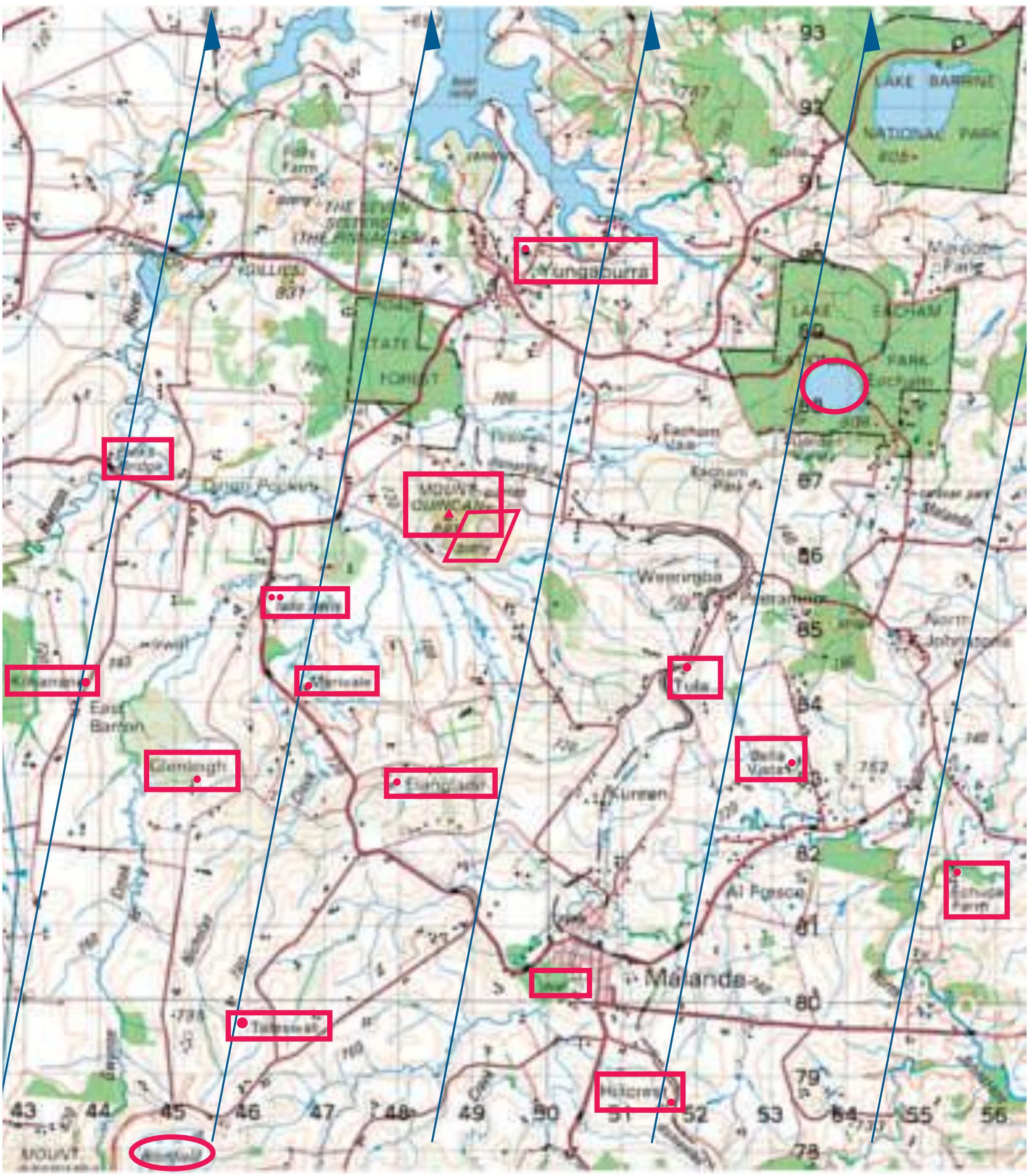
c to the centre of Bromfield swamp from Tula

d to Pinks Bridge from Mount Quincan

e to Yungaburra from Echuca Farm

f to Hillcrest from Glenleigh

g to the southern oval at Malanda from the radio masts north of Merivale.



Malanda area **Scale 1 : 100 000**

2 Use the map of the central part of Boonah shown below to obtain the magnetic bearing:

a along Treatment Works Road from its intersection with Cemetery Road

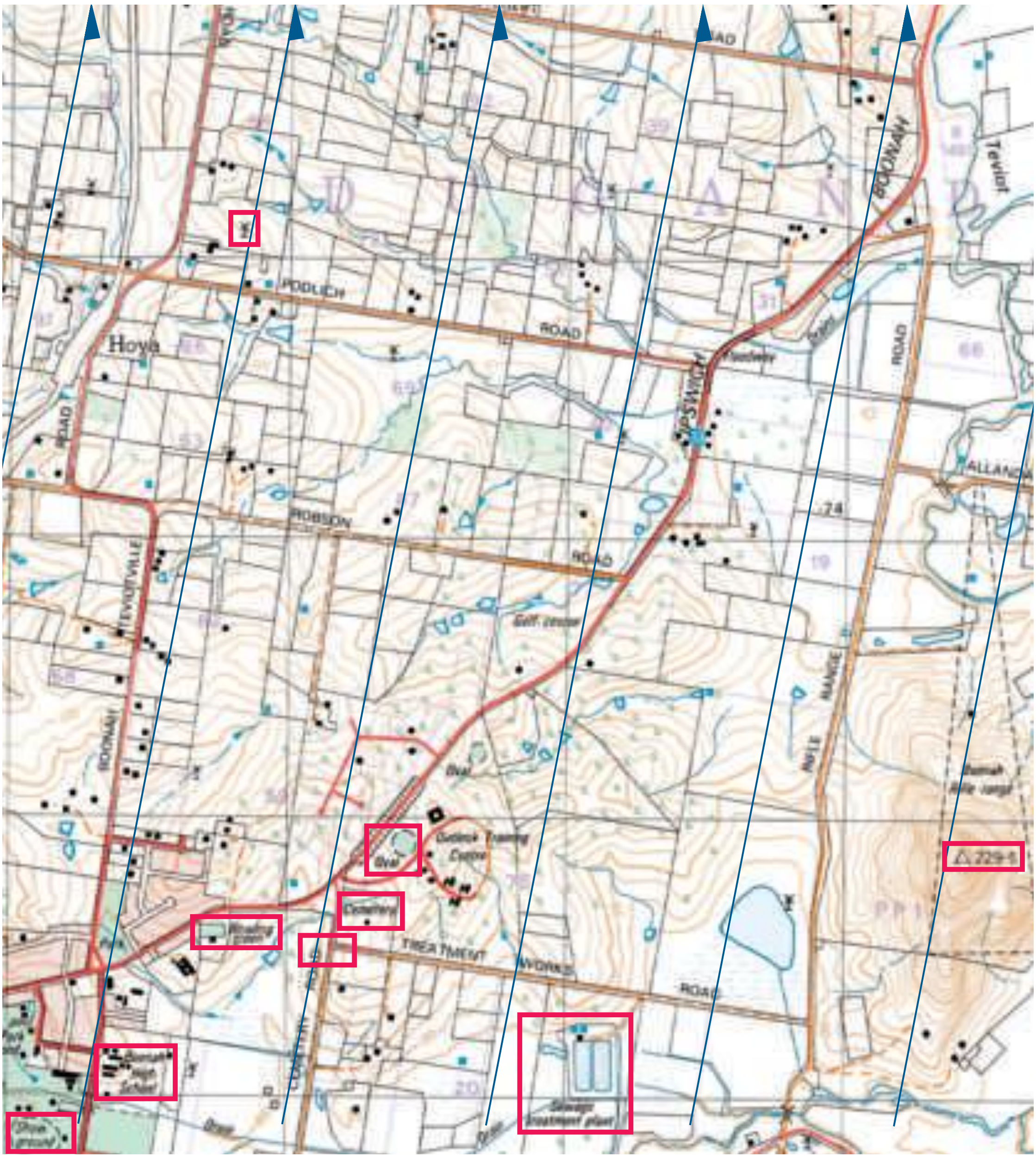
b to the windmill that is north-east of Hoya from the building at the cemetery

c along Cemetery Road southwards from its intersection with Treatment Works Road

d to the centre of the oval west of the Outlook Training Centre from the centre of Boonah High School

e to the building at the bowling green from the building at the sewage treatment plant

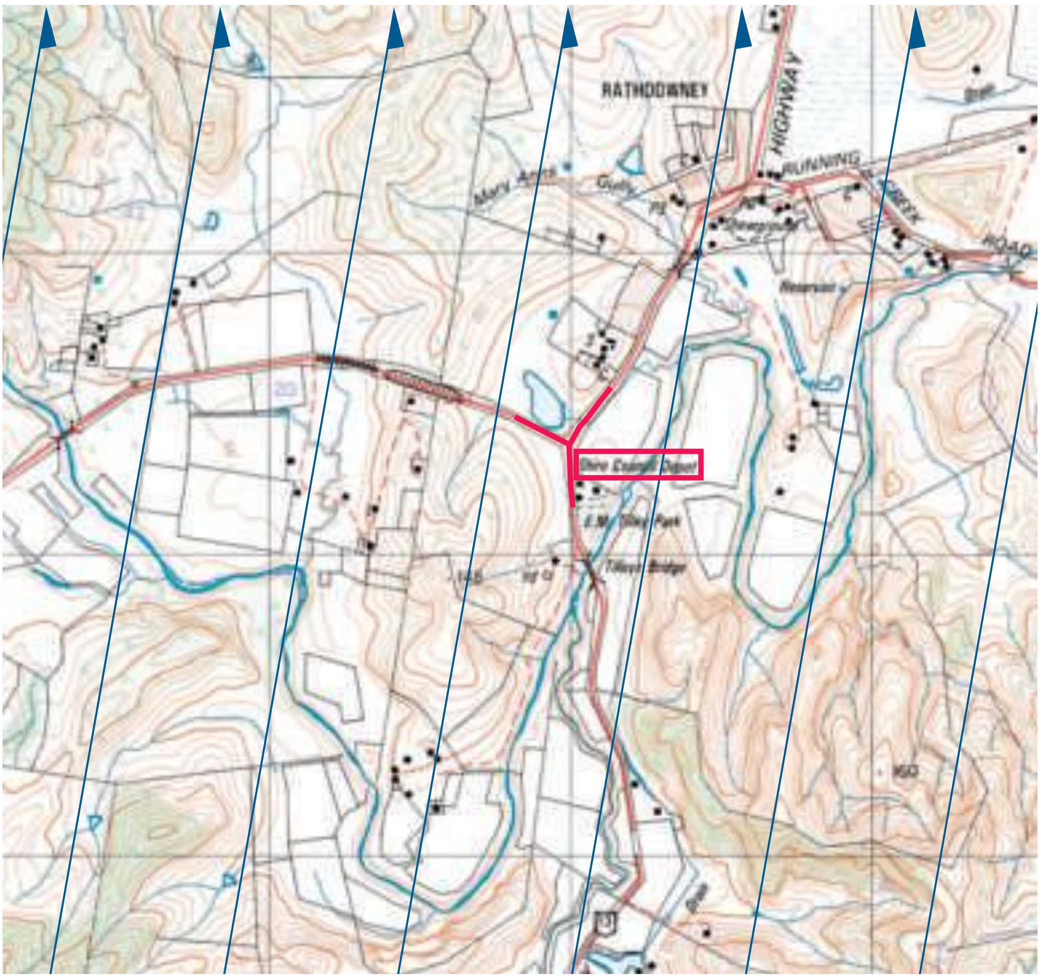
f to the hill 229.5 m high on the rifle range from the centre of the showground.



Boonah area **Scale 1 : 25 000**

3 Use the map of Rathdowney shown below to locate features at the following distances and bearings from the intersection of the roads just north of the Shire Council depot.

a 425 m at 179°M
b 1650 m at 310°M
c 1475 m at 126°M
d 1050 m at 050°M
e 1675 m at 261°M



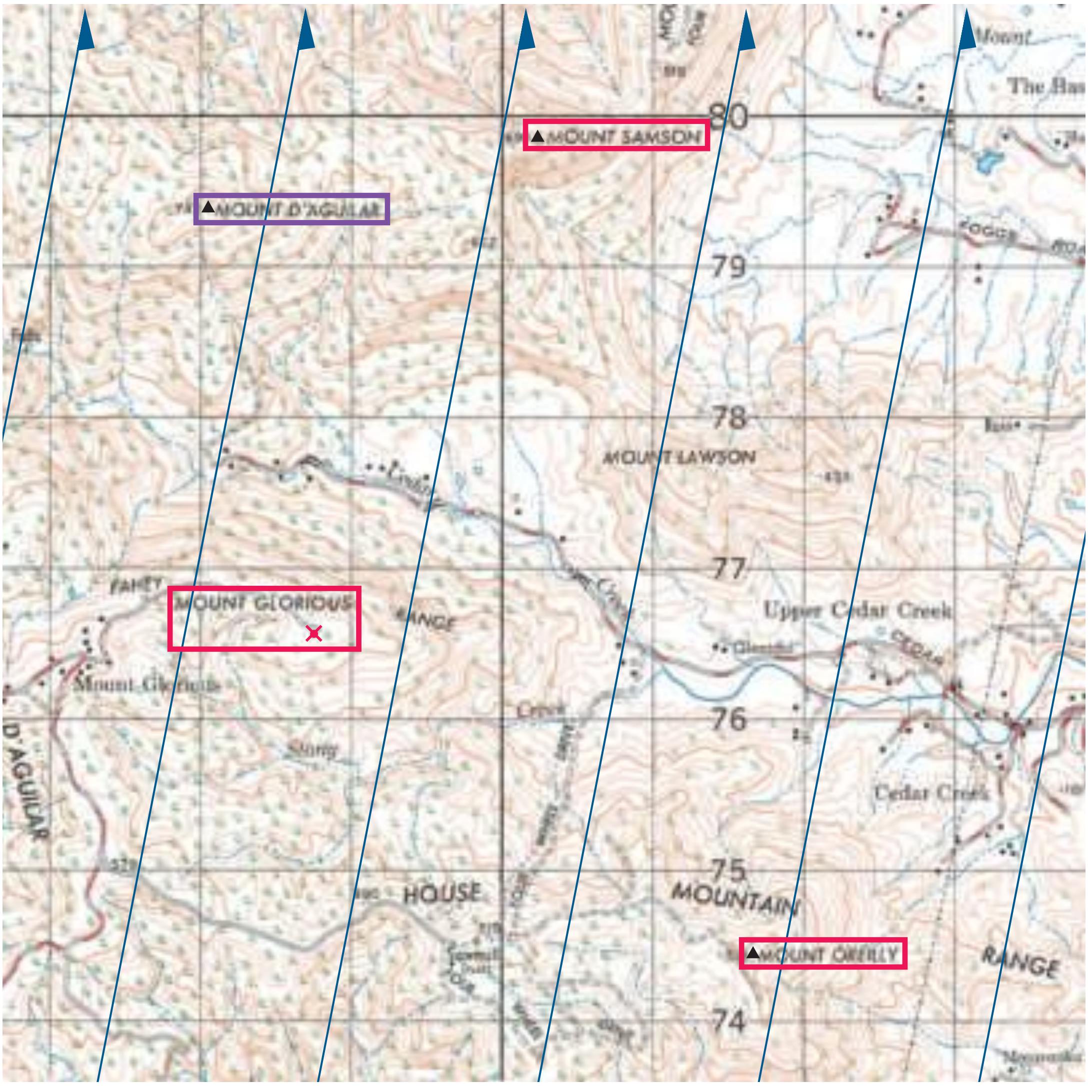
Rathdowney area **Scale 1 : 25 000**

4 Use the Malanda map shown on page 234 to find features:

a 5.5 km at 060°M from Mount Quincan
b 7.5 km at 107°M from Kilkarran
c 5.9 km at 325°M from Tula
d 11.5 km at 041°M from Tideswall
e 4.1 km at 215°M from Bella Vista.

5 Use the map of Mt Glorious shown on the opposite page to find features at the following distances and bearings from the hut marked by the cross on top of Mt Glorious.

a 2900 m at 335°M
b 3600 m at 014°M
c 2725 m at 081°M
d 2350 m at 146°M
e 5500 m at 111°M



Mt Glorious area **Scale 1 : 50 000**

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6 An orienteering course is marked on the map of the northern part of Moreton Island shown below. Work out the distance and magnetic bearing of each control point from the previous one, in the order Start, 1, 2, 3, 4, 5 and Finish.

Moreton Island area **Scale 1 : 100 000**

7 The map of the northern part of North Stradbroke Island shown on the opposite page has control points marked for an orienteering course. Work out the distance and magnetic bearing for each leg of the course.

Modelling and problem solving

8 A bushwalker is in the Mt Glorious area but is uncertain of her exact position. Using her compass, she can see Mt D'Aguilar at a bearing of 300°M, Mt Samson at a bearing of 330°M and Mt O'Reilly at a bearing of 164°M. Use the map of the Mt Glorious area shown on page 237 to calculate her approximate position.

North Stradbroke Island area **Scale 1 : 100 000**

9 A hiker uses a map to find the bearing from his current position to a lagoon, which is 10 km away. Unfortunately, when taking the bearing, he forgets to account for the magnetic variation of 10°E. Assuming that he walks 10 km along the bearing he calculated from the map, how far will he be from the lagoon?

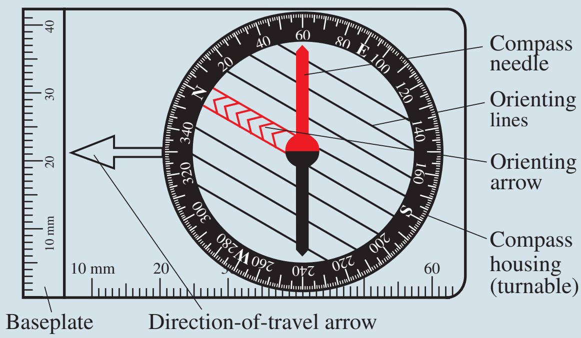
10 Two orienteering friends set out from the start of a course in opposite directions. Max heads due west for 400 m and then goes due south for 2.4 km to arrive at the first control point. Clarissa starts off heading due east, but after a while she changes direction and heads straight for the first control point. If they both travelled the same distance:

a how far east did Clarissa travel before changing direction?

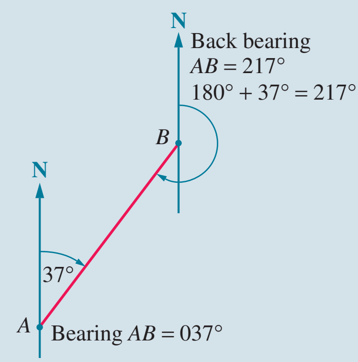
b what bearing did Clarissa follow to reach the first control point?

Chapter summary

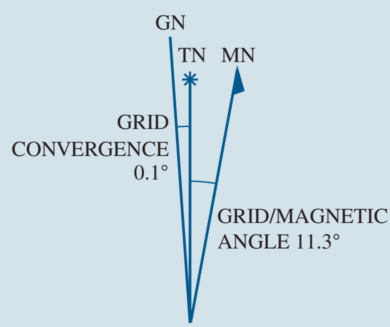
- A compass is used to find directions. An **adjustable dial** or **orienteering compass** is useful for map work as it has a built-in movable protractor and is transparent so that maps can be read through the compass.
- To use an orienteering compass to find a direction, set the direction on the dial, then turn your body until the compass needle is aligned with the orienting lines. You then follow the direction of the travel arrow.



- **Orienteering** is a sport in which runners navigate a course between **control points** marked on a map, using a map and compass
- A direction on the Earth is given as a **bearing**: the horizontal direction (in degrees) of an object from an observer, expressed as the clockwise angle from north to the object direction. A bearing is given as three figures from 000° to 360°. A **back** or **reverse bearing** is the bearing of the opposite direction. A back bearing differs from the bearing by 180°.



- Maps are normally drawn with north running vertically up the map. Maps are produced with lines in a criss-cross pattern called **grid lines**. Bearings taken from these grid lines are referred to as **grid north**.
- You can use a clear plastic ruler and 180° protractor to measure bearings directly from the grid lines on a map.
- Survey maps are produced to an international standard based on the **Universal Transverse Mercator (UTM) Projection**, which establishes grids that cover the entire Earth.
- Areas and distances are shown very accurately on maps produced with the UTM projections. Distances can be calculated using the map scale or measured against the map distance scale.
- We can use the distance and bearing to find a point on a map from a given point.
- The Earth is like a large magnet with the Earth's magnetic field running in a roughly north–south direction. A suspended magnet will turn so that it is in line with the Earth's magnetic field.
- There are three 'north' directions that are important when using a map: **grid north** (the direction shown by the grid lines on a map), **true north** (the direction shown by meridians) and **magnetic north** (the direction of the Earth's magnetic field). True north does not change, and magnetic north changes with time and place.
- Magnetic compasses must be used with **magnetic bearings**, so **true bearings** or **grid bearings** must be converted to magnetic bearings by correcting for the **magnetic variation**.



TRUE NORTH, GRID NORTH AND MAGNETIC NORTH ARE SHOWN DIAGRAMMATICALLY FOR THE CENTRE OF THE MAP. MAGNETIC NORTH IS CORRECT FOR 2003 AND MOVES EASTERLY BY 0.1° IN ABOUT TWO YEARS.

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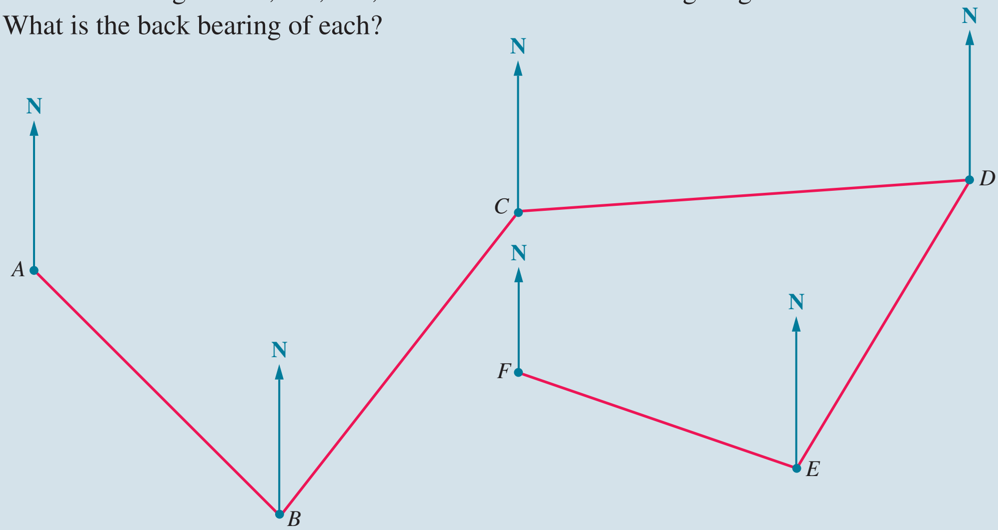
Chapter review

Knowledge and procedures

1 **a** Find the bearings of *AB*, *BC*, *CD*, *DE* and *EF* in the following diagram. Ex 8.1

b What is the back bearing of each?



2 Use a protractor and clear ruler on the Gordonvale map shown on page 242 to find the grid bearing of: Ex 8.1

a Aloomba railway station from Walshs Pyramid

b Kamma railway station from Charringa railway station

c the intersection of the road south of Kamma and the Bruce Highway from Green Hill

d the oval west of Gordonvale from Araluen

e Grey Peaks from Banna.

3 Use a clear ruler to find the distances between the points in question **2**. Ex 8.1

4 Find features on the Gordonvale map shown on the next page located at the following distances and grid bearings. Ex 8.1

a 3.5 km at 041° from Meringa

b 5.2 km at 041° from Meringa

c 10.1 km at 205° from Green Hill

d 7.2 km at 327° from Banna

e 9.3 km at 159° from Kamma

5 Draw the following routes, and find the distance and bearing from the start to the finish. Ex 8.2

a *AB* 6 km at 191°, *BC* 6 km at 106°, *CD* 4.5 km at 337°

b *AB* 1500 m at 133°, *BC* 900 m at 235°, *CD* 1200 m at 285°, *DE* 1800 m at 005°

c *AB* 100 km at 245°, *BC* 80 km at 315°, *CD* 140 km at 162°, *DE* 120 km at 078°

6 Use the diagram at right and the information it contains to find the true bearing and magnetic bearing in 2015 for each of the following grid bearings. Ex 8.2

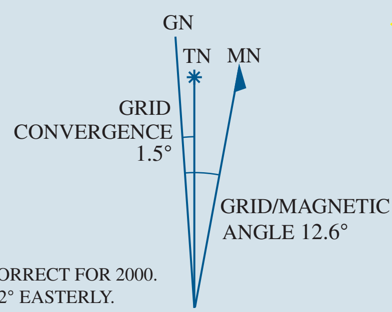
a 063°G

b 008°G

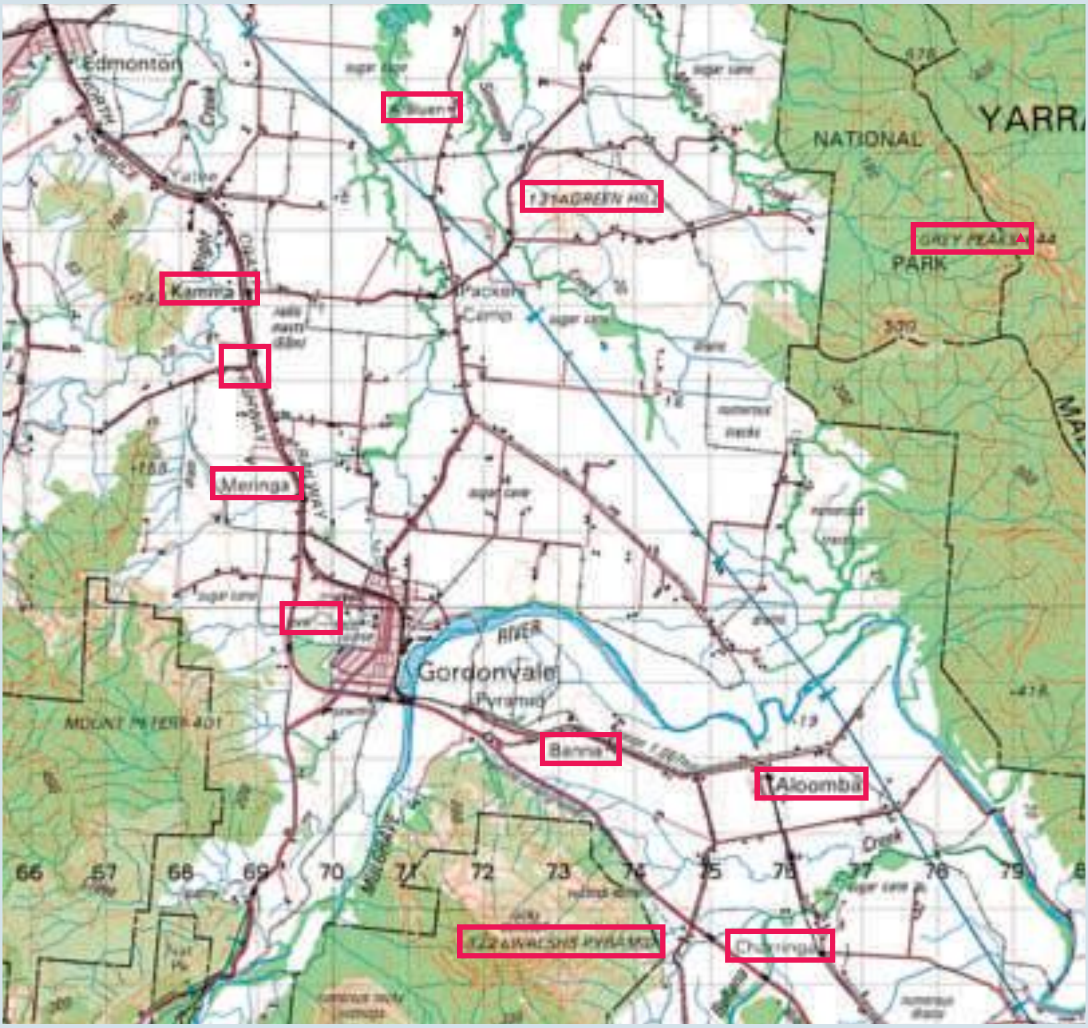
c 354°G

d 198°G

e 241°G



Chapter review

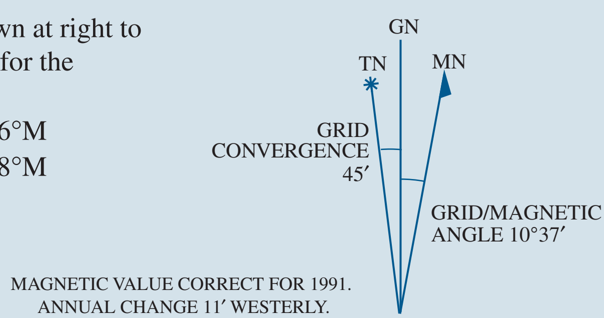


Gordonvale area **Scale 1 : 100 000**

Ex 8.2

7 Use the diagram and information shown at right to find the true bearing and grid bearing for the following magnetic bearings in 2016.

a 209°M **b** 006°M
c 156°M **d** 358°M
e 088°M



Ex 8.3

8 Use the Rockhampton map shown on the opposite page to find the magnetic bearing of:

a Rockhampton Airport from Pine Mountain **b** Mt Lion from Mt Archer
c the radio mast near Lion Lagoons from Neerkol **d** Boomerang Lagoon from Mt Lion
e Nerimbera railway station from Mt Etna.

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Chapter review

9 Use the Rockhampton map shown below to find the feature:

a 11 km at 057°M from Pandoin railway station

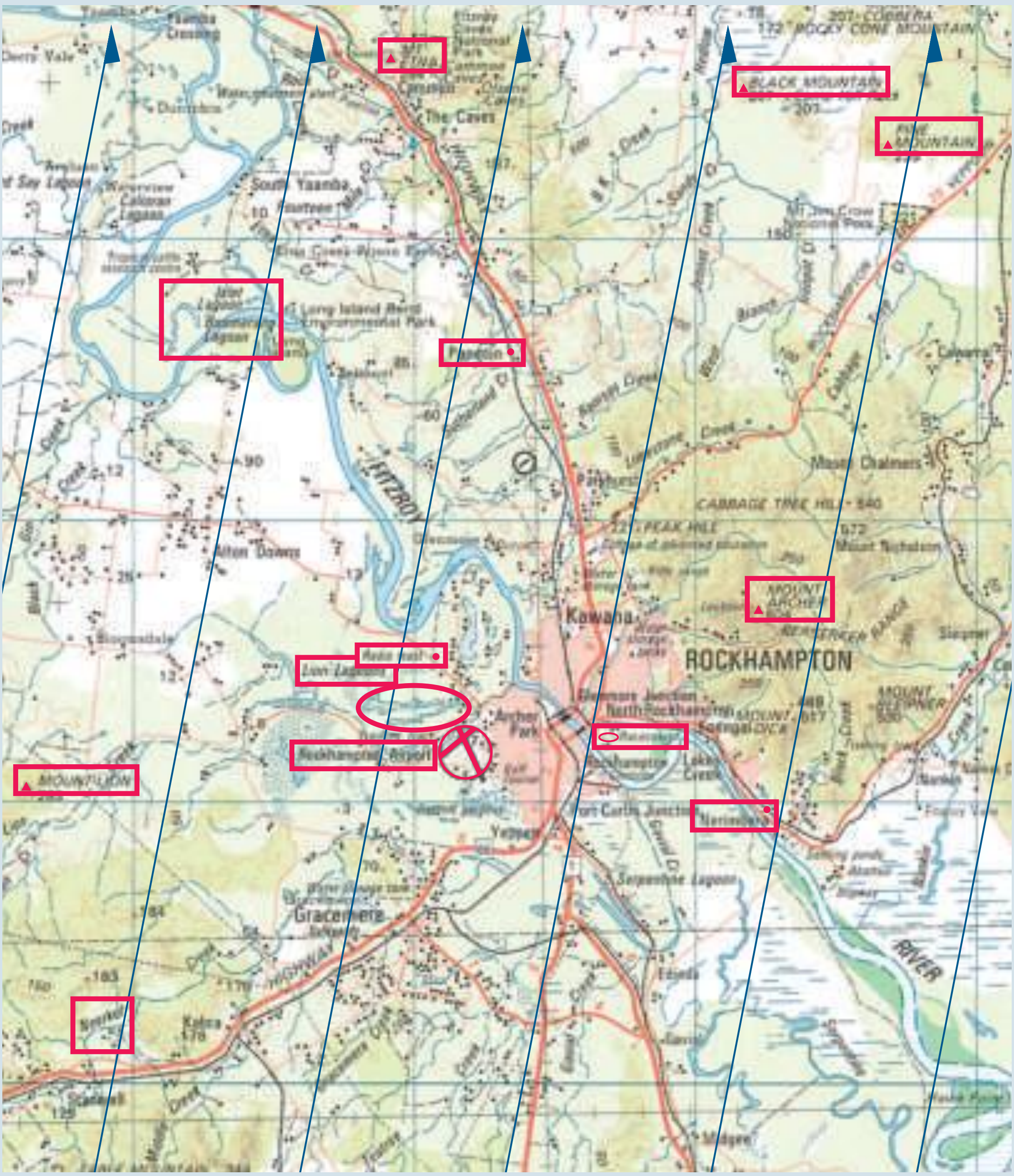
b 23.25 km at 329°M from Lion Lagoons

c 25 km at 218°M from Black Mountain

d 17.5 km at 143°M from Islet Lagoon

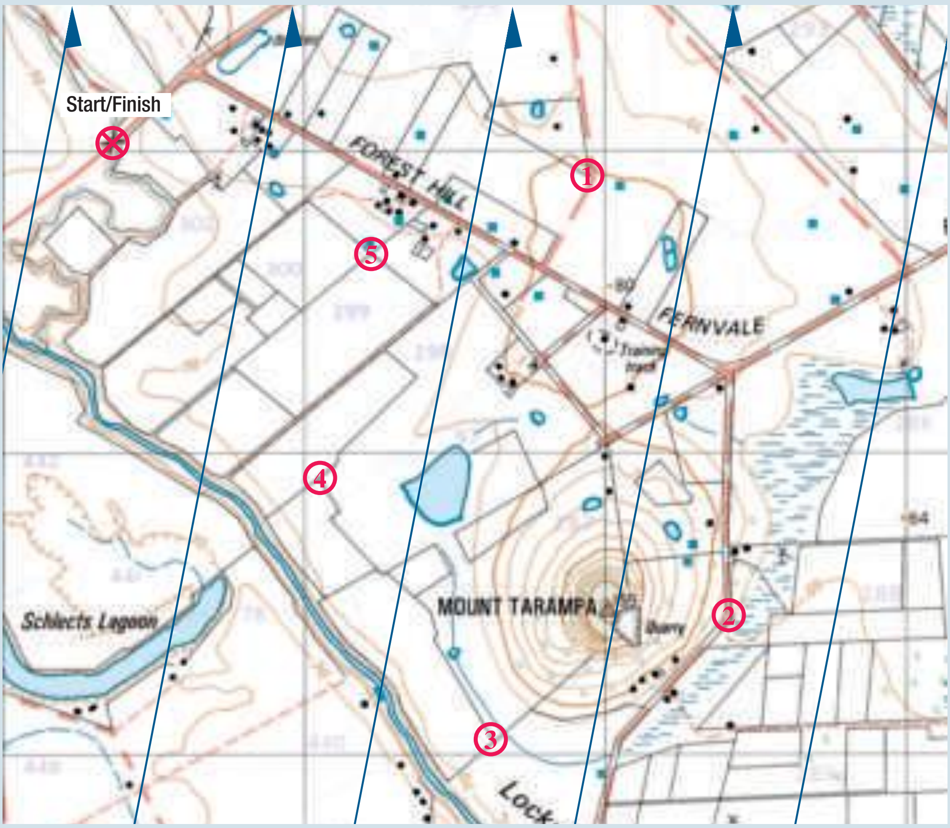
e 18 km at 331°M from Rockhampton racecourse.

Ex 8.3



Rockhampton area Scale 1 : 250 000

Ex 8.3 **10** The Mt Tarampa map shown below has five numbered control points and a Start/Finish marked on it for an orienteering course. Find the distance and bearing of each control point from the previous one, in order.



Mt Tarampa area **Scale 1 : 25 000**

Modelling and problem solving

Ex 8.3 **11** A hiker is in the Gordonvale area but is uncertain of his exact position. He knows that the magnetic variation is 11° east. Using his compass, he can see Walshs Pyramid at a bearing of 232°M, Green Hill at a bearing of 352°M and Grey Peaks at a bearing of 033°M. Use the map of the Gordonvale area (page 242) to calculate his approximate position.

Ex 8.3 **12** A bushwalker finds the bearing from her current position to a homestead which she calculates to be 5 km away. When taking the bearing, instead of adding the magnetic variation of 9° she subtracts it. After following this bearing for 5 km, how far will she be from the homestead?

Ex 8.3 **13** Zena and Conan leave from the first control point of an orienteering course. Zena goes north for 300 m, then changes direction and heads due west for 1.2 km to arrive at the second control point. Conan goes south but soon finds he is heading in the wrong direction, so he changes course and heads for the second control point.

a If they both travelled the same distance, how far south did Conan travel?

b What bearing did Conan follow to reach the second control point?

Introduction to land measurement

9

Contents

Syllabus Guide Chapter 9

Syllabus subject matter

Maps and compasses—land measurement

- Compass bearings and reverse bearings
- Position fixing using directions, and vertical and horizontal measurements in relation to a datum
- Calculation of perimeters and areas

Quantitative concepts and skills

- Metric measurement including measurement of mass, length, area and volume in practical contexts
- Calculation and estimation with and without instruments

The great Egyptian civilisation established over 5000 years ago was based on the fertility of the Nile valley and delta. Each year, the Nile flooded and brought down rich alluvial deposits that allowed two crops to be grown. After the floods, it was necessary to re-establish field boundaries before people planted their crops. The ancient Egyptians needed to survey the land so that everyone got their correct entitlement.

Some of the methods they used, such as 3-4-5 triangles for right angles, are still in use today. Even today, a block of land must be surveyed before you build a house. If a house is built in the wrong place, it can be extremely expensive to correct the problem, even if the house does not need to be demolished.

The Nile delta (from space)

9.1 Using surveys of distances

Surveys of large areas of land are called **geodetic surveys**. Surveys of smaller land areas are known as **plane surveys**. Surveys can be classified according to either the purposes for which they are conducted or the method used to carry them out.

!

The main categories of surveys according to their purposes are:

- **topographical surveys**—concerned with the location of natural and constructed features such as hills, lakes, vegetation, buildings and fences;
- **cadastral surveys**—concerned with the establishment of property boundaries and other administrative information such as land ownership and place names;
- **engineering surveys**—concerned with the design of engineering works such as roads, railways and dams.

The Queensland division of the Institution of Surveyors (Australia) (*www.isaqld.org.au*) has a website that provides information about surveying and links to a large number of related sites. The Queensland Department of Natural Resources and Water website (*www.nrw.qld.gov.au*) has a wide variety of information about all forms of geospatial (mapping) data and surveying.

Other types of surveys, such as hydrographic, geographical, geological and military surveys, are aimed at the specific purposes suggested by their names.

One of the earliest forms of measuring length is **pacing**. It is not highly accurate, but is useful because it does not rely on any equipment except your body. The accuracy of surveying using pacing can be improved if you have a good idea of the length of your pace.

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Investigation Pace length

You will need to calculate your pace length if you want to be able to do some elementary surveying. It is best to work in groups for this activity.

1 Set out a length of at least 20 m in a straight line on level ground.

2 Pace the distance by starting the count with your first step after the starting point and finishing the count with your first step past the endpoint of the length that you have marked out. It is important that you remember to take *natural paces*.

3 Calculate the length of your pace as follows:

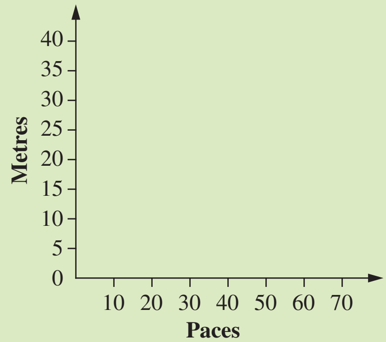
$$\text{Pace length} = \frac{\text{number of metres}}{\text{number of paces}}$$

4 Repeat this procedure at least three times and average the results to work out your pace length.

5 Now, if you multiply your pace length by the number of paces taken over a given length, you will have the given length in metres.

6 Check the accuracy of your pace length calculation by pacing the length and width of a large structure such as a school building or a sporting field. Use your pace length to calculate the dimensions of the structure. Check them with a tape measure.

7 Using your data, construct a conversion graph showing paces (horizontal axis) and metres (vertical axis). Keep it for future use.



Once the length of your pace is known, you can estimate various lengths with moderate accuracy. In this way the areas of various shapes also can be calculated.

Example 1

A person who takes 25 paces to cover 20 m finds that a square field has a side that is 370 paces long. Calculate the length of the side and the area of the field.

Solution

Calculate pace length.	$\text{Pace length} = \frac{\text{number of metres}}{\text{number of paces}}$
Substitute known values and evaluate.	$= \frac{20 \text{ m}}{25 \text{ paces}} = 0.8 \text{ m/pace}$
Calculate side length.	Length of field = 370 paces × 0.8 m/pace
	= 296 m
Calculate area of field.	Area of field = side × side
Substitute known values.	= 296 m × 296 m
Evaluate.	= 87 616 m^2
Write the answers.	The field has a side length of 296 m and an area of 87 616 m^2.

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Introduction to land measurement

The area rules for a number of different plane shapes are outlined in Chapter 3 on page 73. You may need to review these before you complete some of the problems in this chapter.

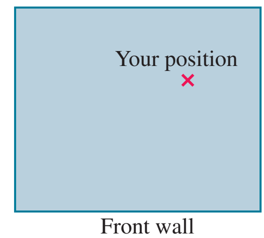
Extra Material

Land measurement units

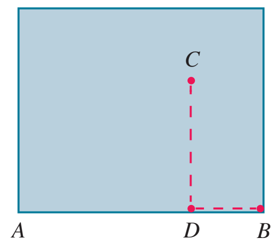
These days, the metric system of measurement is used in Australia and most other parts of the world. However, in some countries other units of measurement continue to be used for length and area measures. Some examples of these measures and their metric equivalents are provided on the CD-ROM for your information.

There are many ways of locating and describing the position of an object.

Imagine that you are standing somewhere in your classroom and you want to explain to a classmate where you are in relation to the front wall.



You could measure CD (the perpendicular distance from your position to the front wall) and BD (the perpendicular distance from the end of the wall to the line CD). The distance CD is called an **offset** from the wall.

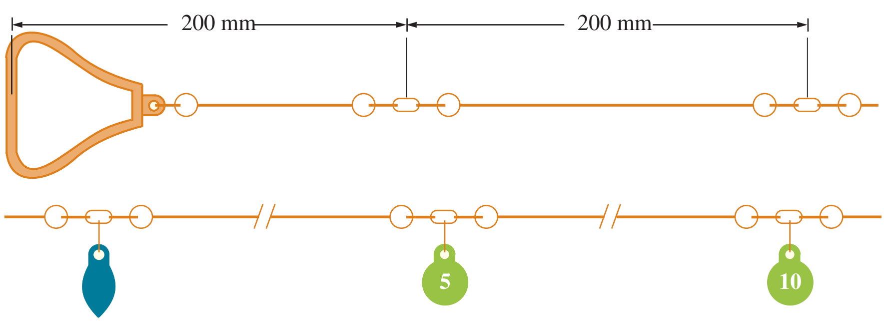


Chain surveying or **chaining** involves producing a plan of a limited area using simple distance-measuring instruments. It is most appropriate for areas that are relatively free of obstructions and not too rough. In this form of surveying, distances can be measured in many ways, including:

- pacing
- tapes or chains
- trundle wheels and other odometer devices
- graduated measuring rods (stadia)
- electronic devices.

Chaining takes its name from the fact that a very early measuring device was the **chain**. A typical surveyor's chain is 20 m long, consisting of 100 lengths of metal called **links**, which are joined by oval rings. Each link is 200 mm. Unnumbered yellow markers are fixed every metre. At the 5th, 10th and 15th metre marks, the yellow tag is replaced by a red tag that is numbered. So that the tape may be read in either direction, the 5th and 15th metre marks are simply numbered with a 5, while the 10th metre is marked with a 10.

Web sites such as *www.survey.history.org, www.usyd.edu.au/museums/whatson/exhibitions/clandsea.shtm*, *www.gemmary.com/instcat/index.html* and *www.theodolite.com* have more information about the tools used by surveyors of today and yesterday.



The term ‘**chaining**’ applies to any linear survey, no matter what measuring equipment is used. It involves the recording of a number of linear measurements taken in relation to a main survey line. You will need a number of field skills to be able to conduct a chain survey. These include:

- setting out straight lines (**ranging**)
- setting perpendicular **offsets**
- measuring horizontal distances on sloping ground (**step chaining**)
- setting **ties** and forming **well-conditioned triangles**.

Investigation Ranging

Distances measured in surveys need to be straight. So before you can start chain surveying you need to be able to set out straight lines, called **survey lines**. The process of setting out straight lines is called ranging because it uses a series of **ranging poles**.

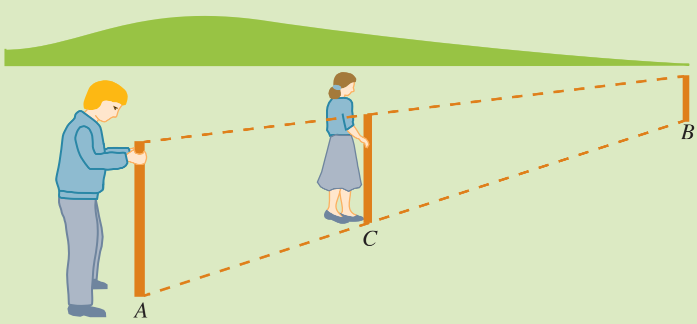
For this activity you will need:

- four or five pieces of straight timber at least 1200 mm long (ranging poles)
- a spirit level
- a 20 m or 30 m tape
- some markers
- a hammer.

1 Go onto the school oval or another suitable area of reasonably flat land. Select two points that are separated by more than 50 m. (You can pace out the length to check it is actually longer than 50 m.) Now you are going to measure the distance between the two points, using the tape.

2 Set up ranging poles at the two points (A and B) that you have selected. Use the spirit level to make sure that the poles are vertical and use the hammer to fix them in place.

3 You need to establish intermediate points in a straight line between A and B so that you can use the tape to measure the distance. One person should stand at a ranging pole (e.g. A) while someone else holds another ranging pole at C—approximately in the line AB. Distance AC should be within the range of the measuring tape that you are going to use.



4 When the poles at A, B and C are in a straight line, the pole at C can be hammered in place. Further ranging poles can then be fixed in the same way. You can minimise errors when the poles are being fixed by making sure of the following:
   - The assistant stands to the side of the survey line to avoid obstructing the view of the line.
   - Ranging poles are erected vertically and sighted as near to their bases as possible.

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Investigation continued

5 Now measure *AB* using the measuring tape. A marker should be used each time the limit of the tape is reached.

6 Once your group has measured the distance between the two points you selected, remove the ranging poles between *A* and *B* and swap with another group.

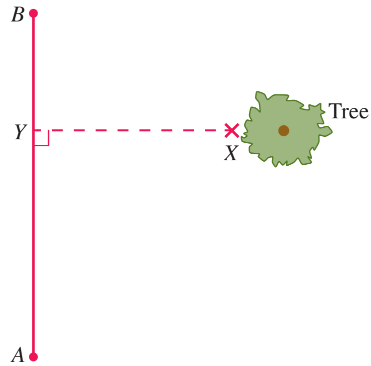
7 Measure the distance between the two points selected by the group with whom you swapped, and compare your measurement of the distance with theirs.

8 Continue swapping with other groups if time permits.

The previous investigation describes how you can establish a main survey line. This acts as a reference for locating the positions of objects of interest within the survey area. One way of locating the position of an object is to measure its perpendicular distance or **offset** from a point on the survey.

In the diagram at right, the survey line, *AB*, has been set out, and there is a tree (at *X*) to the side of the survey line that needs to be located.

One of the easiest methods of locating the position of *X* is to find the length *AY* and the length *XY*, perpendicular to *AB*.



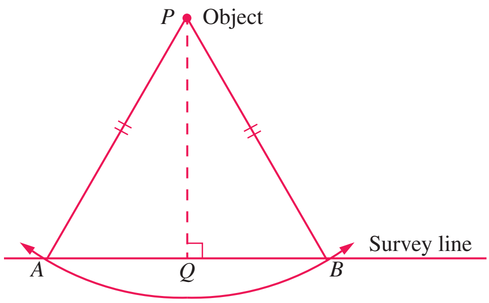
Extra Material

Perpendicular offsets

Commercially manufactured instruments called **optical squares** can be used, but it is unlikely that one will be available at your school for you to use. The CD-ROM describes how to make and use two inexpensive alternatives to these optical instruments.

You can also use some elementary geometry to set a perpendicular offset. Once a survey line (*AB*) has been set out, select any convenient point *A* and measure the distance to the object using a string line or measuring tape.

Next, use the string line or tape to measure the same distance from the object to the survey line on the other side of the perpendicular offset, *PQ*, to locate *B*. This is known as '**swinging the arc**'.



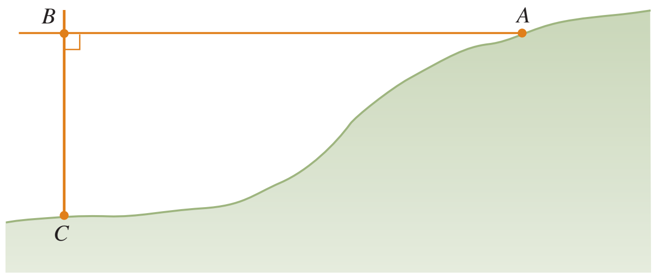
Once *B* has been located, *AB* can be bisected to locate *Q*—the point on the survey line perpendicular to *P*. The required distance, *PQ*, can then be measured.

Investigation Perpendicular offsets

For this investigation, you will need the same equipment as for the ranging investigation on pages 249–50. In addition, you need a roll of string or measuring tape about 50–100 m long.

1. Go to the school oval or some other convenient area.
2. Set out a survey line so that some object is located to one side of line. Make sure that the object is within the range of the roll of string or measuring tape.
3. Measure the offset distance by 'swinging the arc' as described on the opposite page.
4. Once you have finished, swap with another group and check each other's measurements.

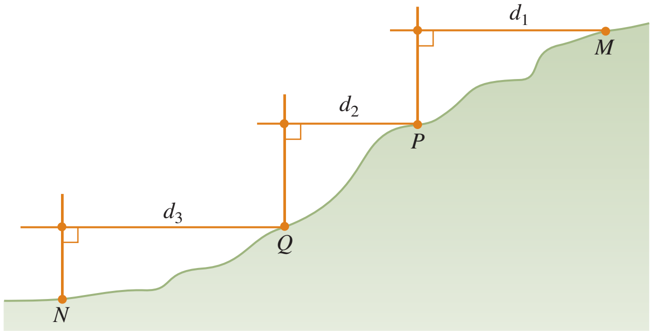
For sloping ground, a procedure known as **step chaining** is used to find horizontal measurements. Imagine that the horizontal distance between two features at *A* and *C* has to be found.



The horizontal distance (AB) can be found in the following way:

1. Hold one end of a tape at *A*.
2. Hold the tape in a horizontal position, using a spirit level.
3. With the tape taut, use a plumb-bob to locate *B* directly above *C*.
4. Read the measurement *AB* on the tape.

If the slope of the ground is steep, a number of steps may need to be taken. Consider the horizontal distance between two features at *M* and *N* as shown below. In these cases, horizontal distances are transferred to the ground using a plumb-bob at *P*, *Q* and then finally at *N*. The horizontal distance between *M* and *N* is found by adding d_1, d_2 and d_3.



When step chaining, you should remember the following points:

- Keep the tape as taut as possible.
- Make sure that there are no twists in the tape.
- Do not use steps longer than about 8 m.
- Record the measurements as you go.

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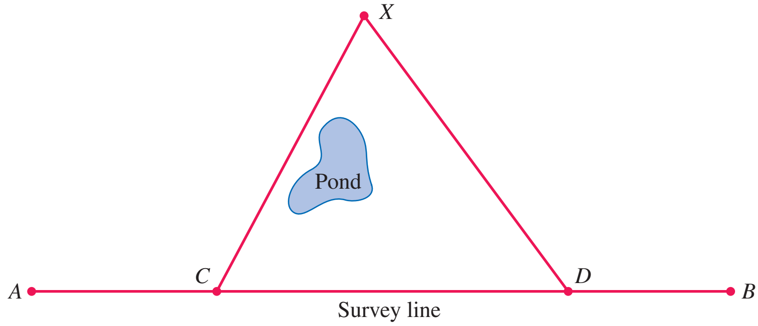
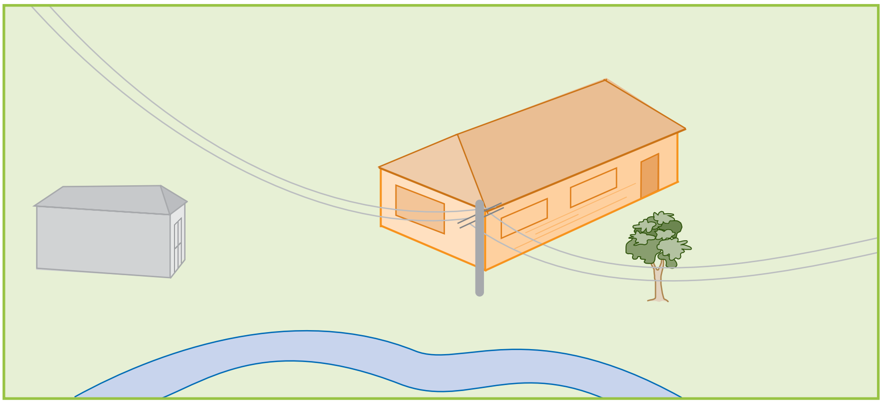
Sometimes an obstacle between the feature and the survey line means that the position of a feature cannot be measured as an offset. In such a case, **ties** are measured. Imagine that the position of the feature at X (shown below) is to be fixed.



The pond between X and the survey line makes it difficult to range a line and make the measurement of the offset. Instead, the position of X is 'tied' to the survey line by CX and DX. Lines are ranged from C to X and from D to X as described previously. Once the distances AC, CX, DX and CD have been measured, the position of X is known.

When using ties, care must be taken to form **well-conditioned triangles** (e.g. $\triangle CXD$ above). Well-conditioned triangles have all angles between 45° and 75°. Triangles should be drawn as close to equilateral as possible. The use of such triangles will result in a more accurate plot. Examples of badly conditioned triangles are shown below.

Suppose you want to carry out a survey of the area of land below. How should you go about it?



1 When you arrive at the area to be surveyed, begin with a **reconnaissance survey** of the site. This involves walking over the area, looking for the best position for the survey line and identifying any features to be included in the survey.

2 Draw a rough sketch of the area. Surveyors use a **field book** to do this and to record other details such as measurements from the survey.

3 Next determine the position of the survey line. All features have to be connected to the survey line by ties or offsets. Ideally, the survey line should:
   - be close to all features to be surveyed
   - be clear of obstacles that would prevent ranging the line, and
   - be approximately the same length as the area to be surveyed.

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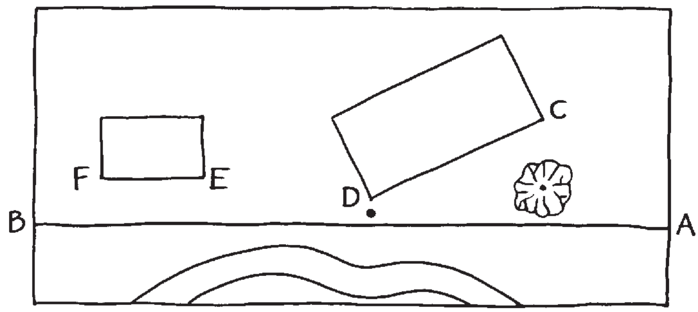
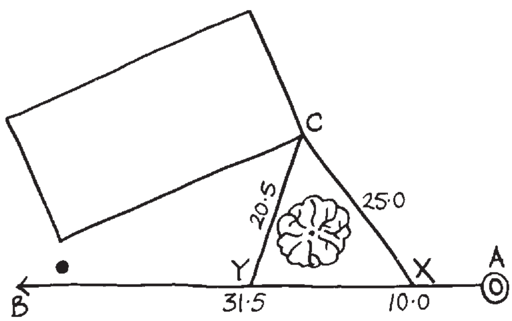
Following these rules, the survey line *AB* would be a good choice. Drive pegs into the ground at *A* and *B*, and draw the line on the rough sketch made previously, as shown here.



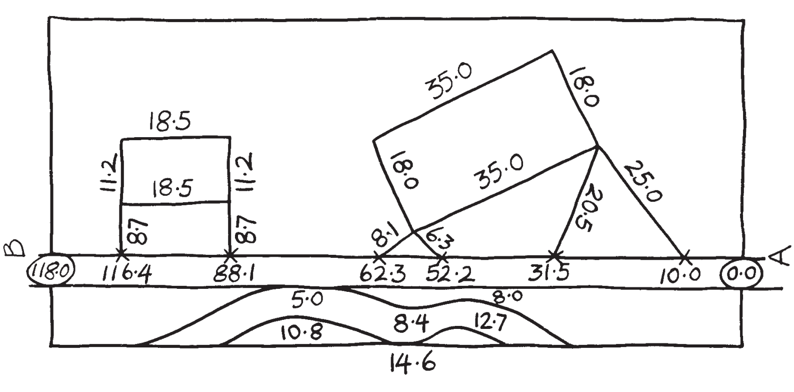
4 Now you can start to record the survey data, remembering that the sketch will not be a scale drawing. When the survey details are recorded, we say they are **booked**—hence 'booking a survey'.

Start by ranging the line between *A* and *B* as described on pages 249–50. Because you are starting at *A*, book it as 0.0 in the field notes.

Next, locate the corner of the house (*C*) in relation to the survey line using ties. Do this by selecting a point (*X*) on the survey line *AB* so that *CX* makes an angle of about 60° with the survey line, as shown here. (Remember that the angle must be between 45° and 75° for the resulting points to form a well-conditioned triangle.) Then locate another point (*Y*) on the survey line so that $\triangle CYX$ is well-conditioned. Book all the measurements *CX*, *CY*, *AX* and *AY* in the field notes.



Now continue booking the survey details, such as buildings, roads and streams, using offsets and ties as appropriate. Below are the booking details for the area under consideration.



Additional Exercise 9.1

Exercise 9.1 Using surveys of distances

1 Calculate the pace length for each of the following people.

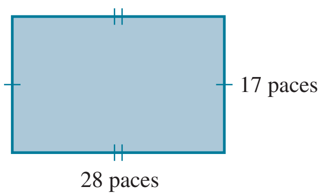
a Max takes 50 paces to cover 40 m.
b Jacqui takes 44 paces to cover 40 m.
c Hung covers 35 m using 44 paces.
d Santina covers 50 m using 72 paces.

2 Mandy has a pace length of 0.9 m. Calculate the following distances paced out by Mandy.

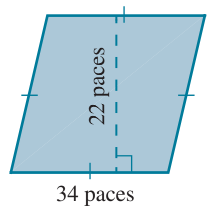
a 28 paces across her front yard
b 12 paces along the side of her house
c 73 paces from her form room to the science lab
d 458 paces from the bus stop to her front door

3 Jason knows his pace length is 0.8 m. Calculate the area of each of the following figures paced out by Jason.

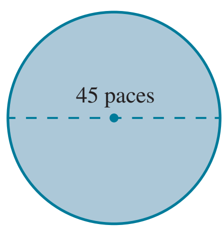
a



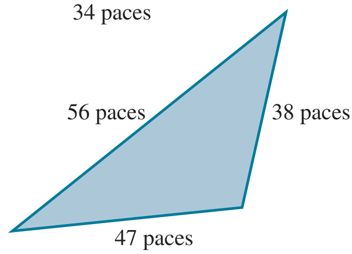
b



c

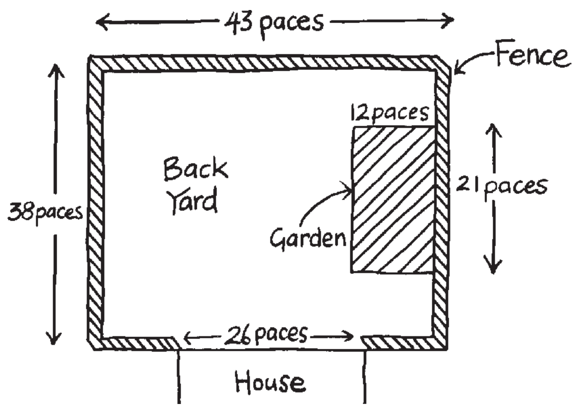


d



Modelling and problem solving

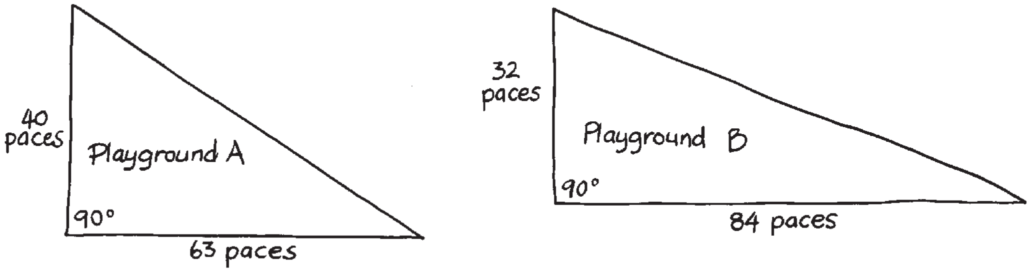
4 Jacqui calculates that she takes, on average, 28 paces to cover 20 m. She decides to help her parents measure their backyard and garden bed. She makes some notes as shown here.



Use her sketch to calculate:

a the dimensions of the backyard
b the dimensions of the garden
c the area of the backyard
d the area of the garden
e the length of the fence.

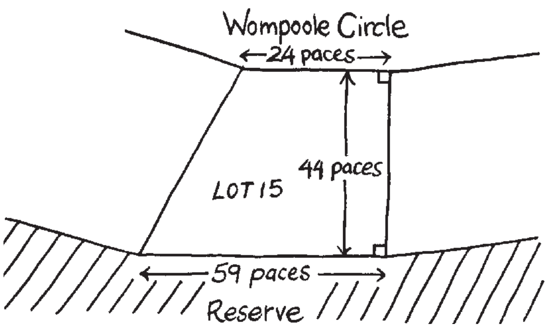
5 Nick knows that he covers 26 m with every 30 paces. He wants to compare the areas of two playgrounds that are in the shape of right-angled triangles. He makes the following sketches.



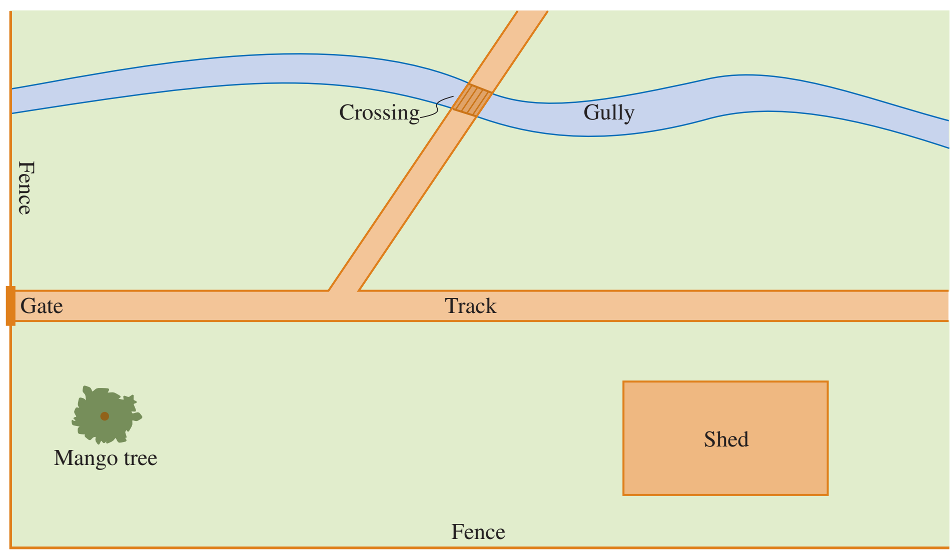
- **a** What is Nick's pace length?
- **b** What are the dimensions of each playground?
- **c** What are the areas of the two playgrounds?

6 Santo is looking at some building blocks at a suburban subdivision. He paces out one block as shown below. Santo takes 29 paces to cover 20 m.

- **a** What is his pace length?
- **b** What are the actual lengths that he paced?
- **c** What is the area of the building block?



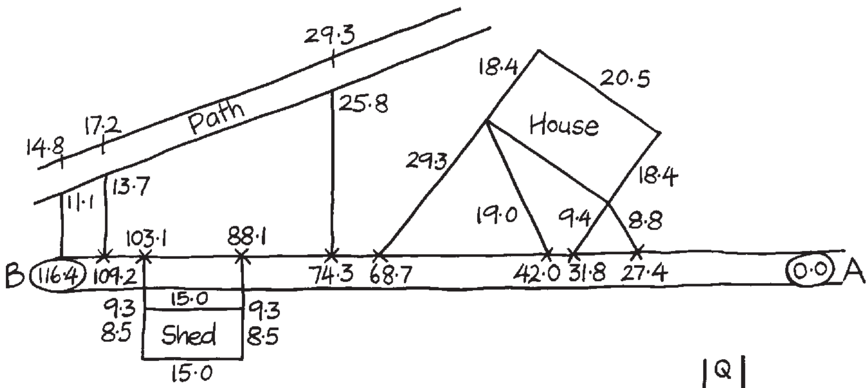
7 Here is the plan of the bottom corner of a property, drawn to a scale of 1 : 1000.



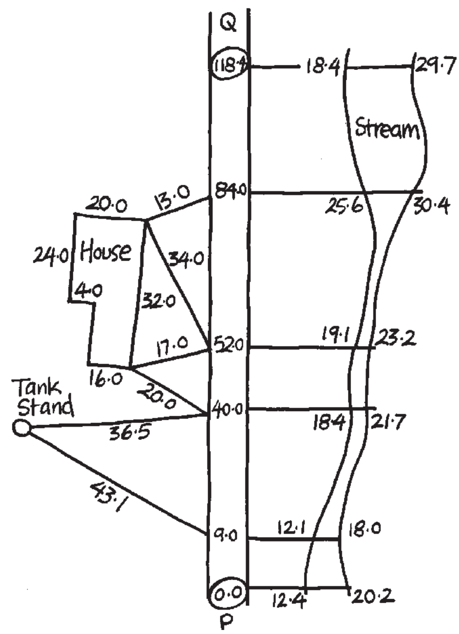
Scale: 1 cm = 10 m

- **a** The property owner wants the area shown above surveyed. Make a copy of the plan in your book.
- **b** Mark in the position that you would choose to range the survey line, AB, so that A is somewhere near the gate on the track.
- **c** Book the details of the survey, so that all relevant features are located. Assume that the drawing is to the scale shown.

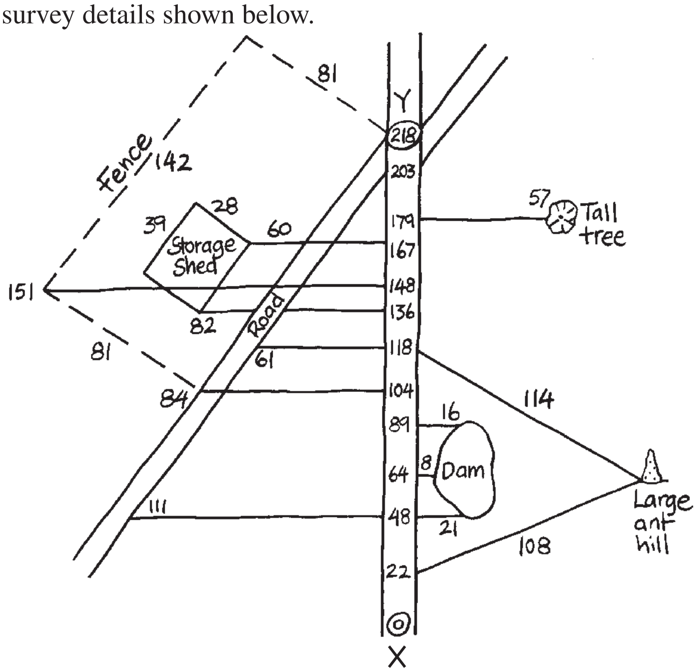
8 The details of a survey are booked as follows. Use the sketch and field measurements to make a scale drawing of the surveyed region, using the scale 1 : 1000.



9 The results of a survey are booked as shown on the right. Use the survey details to make a scale drawing of the area. Select a suitable scale.



10 Use a suitable scale to make a drawing of the survey details shown below.



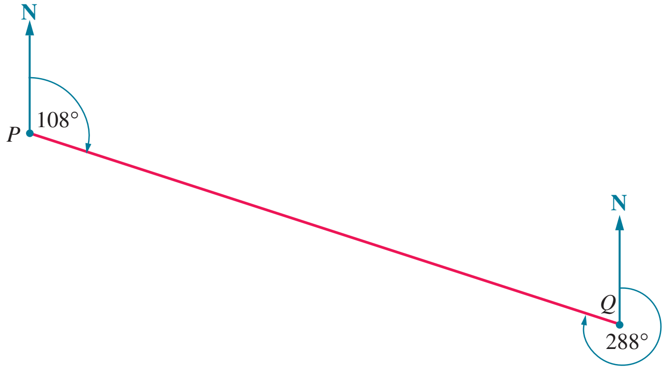
9.2 Using bearings and traverses

Surveyors use a range of sophisticated equipment in their work. The '**total station**' is an instrument that has the combined features of an electronic measuring meter to find distance and a theodolite to measure direction. It can also be linked to a hand-held computer to store thousands of field observations.

A modern total station can typically measure distances to within ±5 mm over a maximum range of 2800 m and angles to within ±5″ (about 0.083°). It can even calculate its own location from three known positions in the field, store this information and then automatically calculate the coordinates of any point that can be sighted within its range. Web sites such as *www.surveyhistory.org* and *www.theodolite.com* have more information about surveyors' instruments.

You may be able to obtain some experience with a theodolite if your teacher can arrange to make one available for the class to use. You may even be able to visit a local surveyor to see how a theodolite is used in the field. While surveyors use a theodolite to accurately measure direction, an orienteering or adjustable dial compass will be sufficient for our purposes.

Directions in surveying are given as **bearings**. A bearing is an angle measured clockwise from north (000°) and is written as a three-digit number between 000° and 360°. A **back** (or **reverse**) **bearing** is the bearing of the opposite direction. In this figure, the bearing of PQ (from P to Q) is 108°. The back bearing of PQ (from Q to P) is 288°.



Back bearings

- If a bearing is less than 180°, add 180° to find the back bearing.
- If the bearing is greater than 180°, subtract 180°.

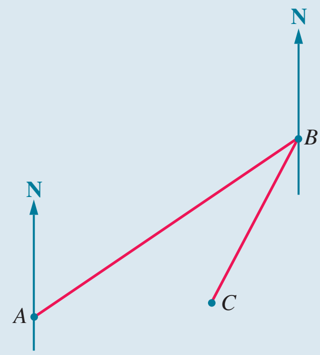
Example 2

For the diagram shown, use a protractor to find:

a the bearing of *B* from *A*

b the bearing of *BC*

c the back bearing of *BC*.



Solution

a Position the protractor over *A* as shown below. Read off the clockwise angle from north to *B*.

Bearing of *B* from *A* is 056°.

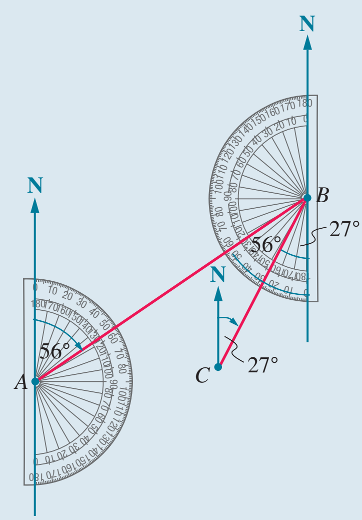
b Position the protractor over *B* as shown. Read off the clockwise angle to *C*.

Bearing of *BC* is 180° + 27° = 207°

c Calculate the back bearing of *BC* from the bearing of *BC*.

Back bearing of *BC*
= bearing of *BC* − 180°
= 207° − 180°
= 027°

Note: The back bearing of *BC* is the same as the bearing of *CB*.



Example 3

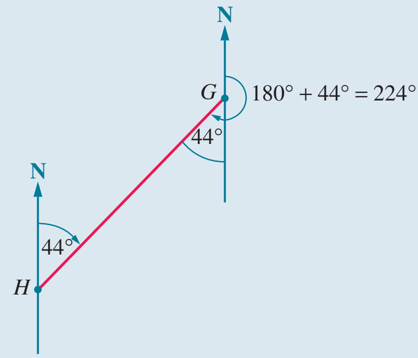
The bearing of *GH* is 224°. What is the back bearing?

Solution

The bearing is greater than 180°, so subtract 180°.

Back bearing = 224° − 180°
= 044°

You can draw a sketch to confirm the result.



Example 4

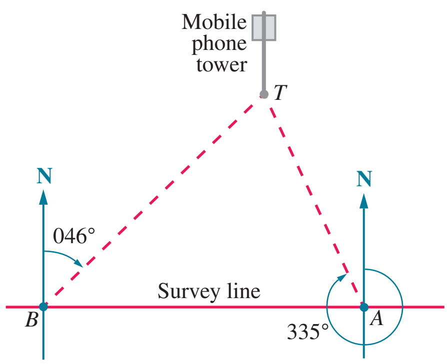
Find the back bearing of the bearing XY 121°.

Solution

The bearing is less than 180°, so add 180°.	Back bearing = 121° + 180° = 301°
State the result.	Back bearing of XY is 301°.

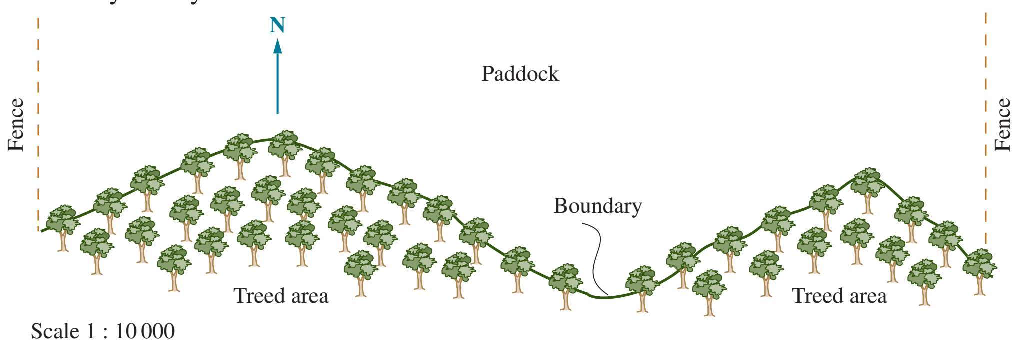
In section 9.1 we saw how perpendicular offsets or ties can be used to locate the position of an object relative to a survey line. Bearings can be used for the same purpose. The mobile phone tower T shown here can be located by finding bearings from any two suitable points (say A and B) on a survey line.

This method works well so long as the resulting triangle ($\triangle BAT$) is well-conditioned (all angles between 45° and 75°).

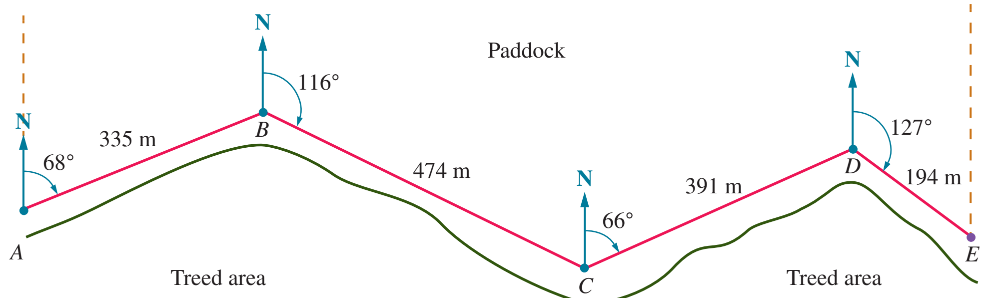


A **traverse** is a consecutive series of lines or **legs**. The legs are chained and their directions are fixed using a compass or theodolite.

The diagram below shows the plan of a treed area adjacent to a paddock. A farmer wishes to have the boundary surveyed in order to sell off some land.

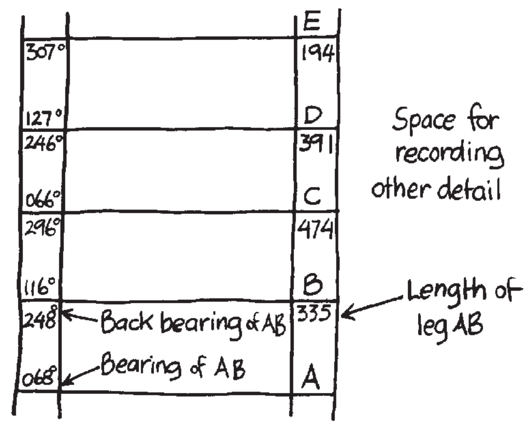


The stations *A*, *B*, *C*, *D* and *E* must be chosen carefully because they will determine the shape of the treed area as it appears on the plan. To complete the traverse we take bearings *AB*, *BC*, *CD* and *DE* and chain the distances as indicated below. This is known as an **open traverse** because the last line does not return to the starting point.



The details of the traverse and the way it would be booked are as follows.

Leg	Distance	Bearing
AB	335	068°
BC	474	116°
CD	391	066°
DE	194	127°



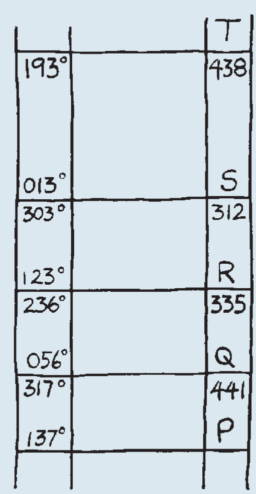
The field notes are recorded so that there is room on either side of the survey line data for the recording of additional detail as required. The column on the left is used to record the bearing and back bearing of each leg.

Example 5

Here are the details of an open traverse survey. (Measurements are in metres.)

Leg	Forward bearing	Back bearing
PQ	137°	317°
QR	056	236°
RS	123°	303°
ST	013°	193°

Make a scale drawing of the traverse.



Solution

You can see the forward and back bearings for each leg displayed in the table. Start by marking point P on your page.

Select a suitable scale. The legs are between 312 m and 441 m in length, so a scale of 1 mm : 10 m (1 : 10 000) will be appropriate. This means that 1 mm on the drawing represents 10 m on the traverse.

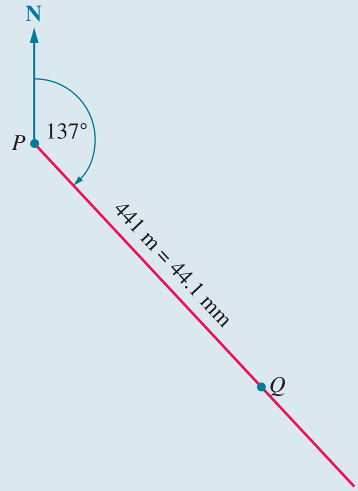
For the leg PQ, the forward bearing is 137°. Mark in a north–south line through P.

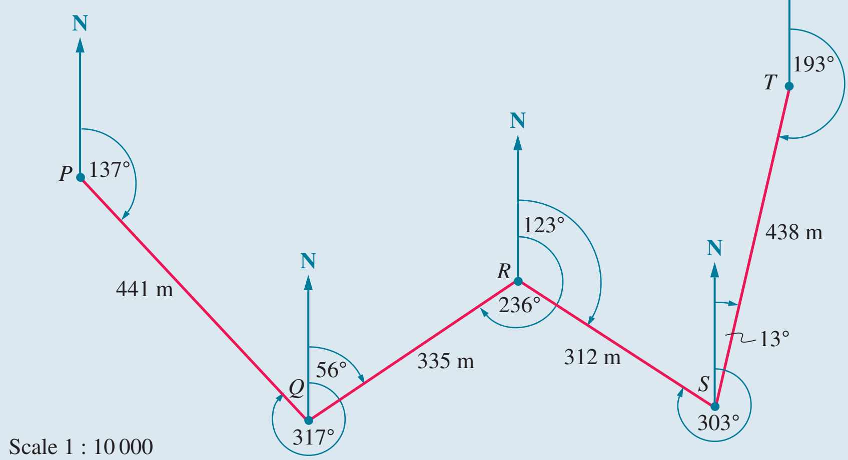
Measure a bearing of 137° from P and draw in a light pencil line along the bearing.

PQ is 441 m, so measure 44.1 mm along the 137° bearing. Mark in point Q. Draw in PQ with a heavy rule.

Continue with the same procedure for the remaining legs of the traverse.

Record the scale on the diagram.





Technology

A route or traverse can be checked using the Route/Traverse spreadsheet on the CD-ROM. Note that the vertical and horizontal scales may be a little different in this spreadsheet.

Spreadsheet

	A	B	C	D	E	F
1	New Qmaths 11A			ROUTE/OPEN TRAVERSE		
2		Forward		Start/Finish	Distance	946.53244
3	Leg	bearing	Distance	Shown	Bearing	82.615862
4	PQ	137	441			
5	QR	56	335			
6	RS	123	312			
7	ST	13	438			

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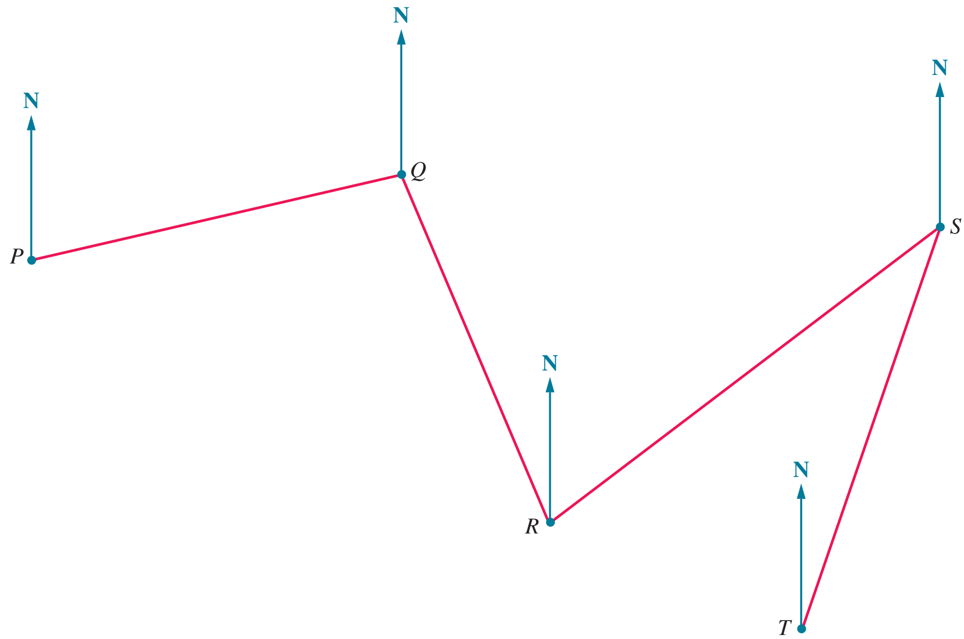
Additional Exercise 9.2

Exercise 9.2 Using bearings and traverses

1 a Find the bearings of *PQ*, *QR*, *RS* and *ST*.

b What is the back bearing of each?



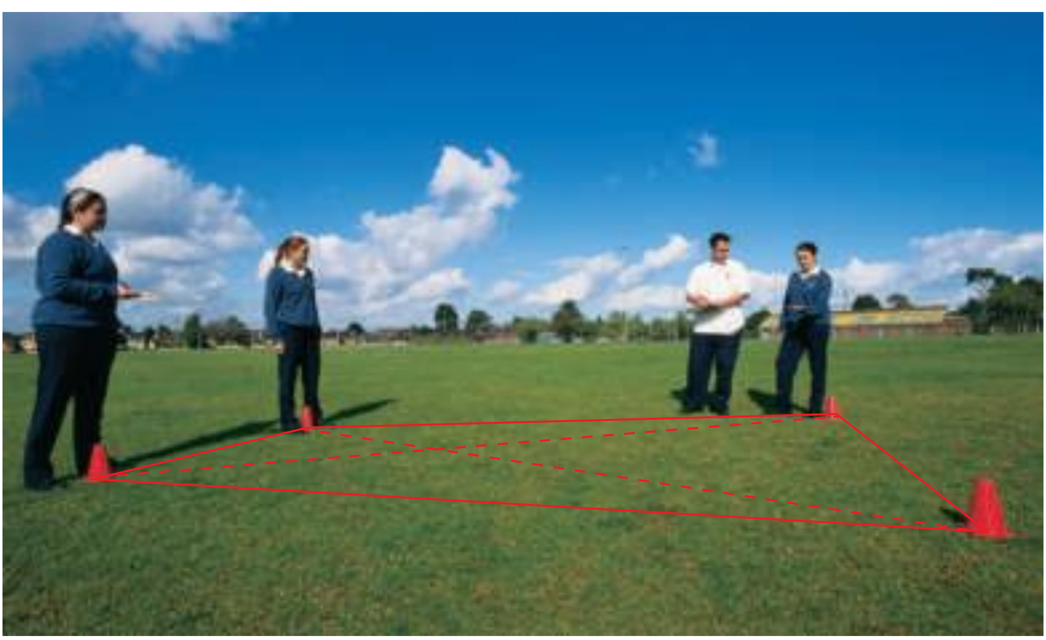
2 What is the back bearing for each of the following?

a 305° **b** 174° **c** 090°

d 243° **e** 032°

Modelling and problem solving

3 Go onto your school oval and mark out four positions forming a four-sided figure.



a From each marker, find the bearings of the other markers. Record the information on a sketch showing the positions of the markers.

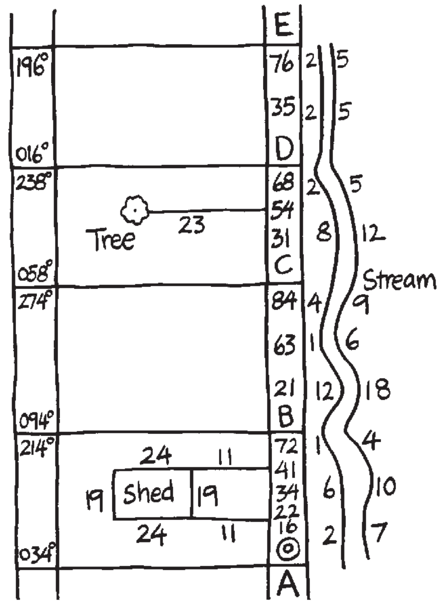
b Calculate the back bearing of each bearing recorded.

4 Make scale drawings of the following routes, and find the distance and bearing from the start to the finish.

a *AB* 650 m at 075°, *BC* 825 m at 138°, *CD* 1120 m at 268°

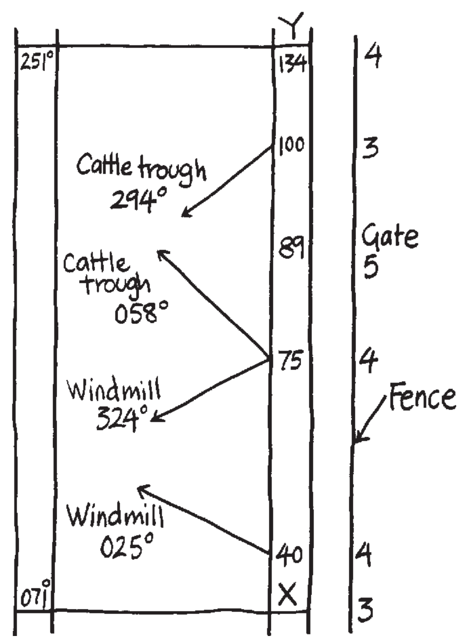
b *VX* 78 km at 116°, *XW* 62 km at 343°, *WY* 81 km at 114°, *YZ* 122 km at 258°

5 These field notes show the details of part of a property boundary survey. Use the notes and a scale of 1 : 2000 to make a sketch of this portion of the property boundary. All measurements are in metres.



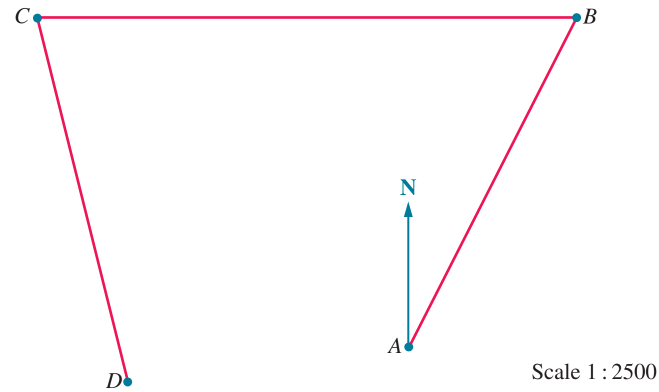
6 a Choose a suitable scale to make a map of the traverse from the following field notes.

b How far from the survey line are the cattle trough and the windmill?

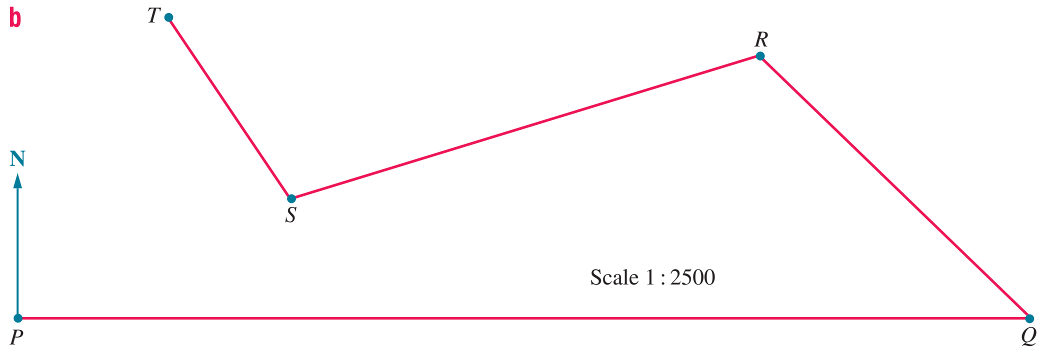


7 The diagrams below show the routes of two open-traverse surveys, *ABCD* and *PQRST*. Both have been drawn using the scale 1 : 2500. In each case, book the details of the survey.

a



b



8 The following bearings were noted in the course of an open traverse.

Leg	Forward bearing	Back bearing	Distance (km)
AB	052°	232°	31
BC	127°	307°	43
CD	086°	266°	47
DE	220°	040°	62
EF	116°	296°	28
FG	041°	221°	53

a Use the information in the table to make a scale drawing of the traverse. (A scale of 1 : 1 000 000 would be suitable.)

b Find the distance and bearing of the finish point from the starting point.

9.3 Compasses and orienteering

Magnetic compasses must be used with magnetic bearings, so true bearings or grid bearings must be converted to magnetic bearings by correcting for the magnetic variation. A compass is oriented to magnetic north. When using an orienteering compass with a map, it is convenient to mark in some magnetic north lines (**magnetic meridians**). These are normally shown with arrow-heads to distinguish them from grid lines.

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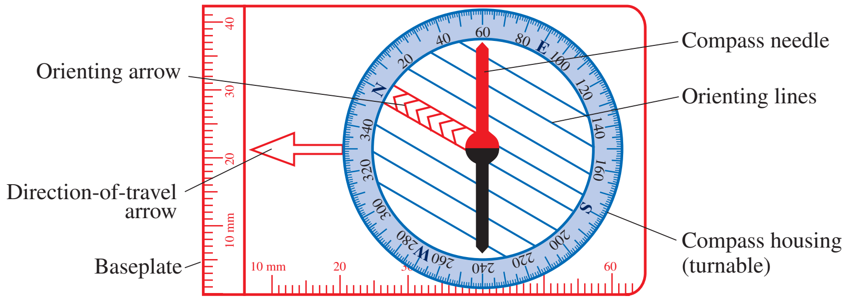
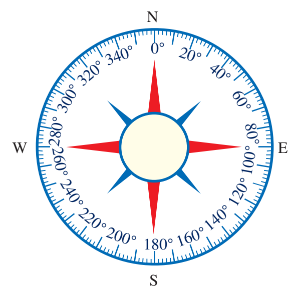
The bearings found with a compass are known as **magnetic bearings**; a capital M is used after the bearing to signify this. For example, magnetic north is 000° M and magnetic west is 270° M.

There are many kinds of compasses. The one we will use is called an **adjustable dial compass**. This type of compass is also called a **baseplate compass** or **orienteering compass**. An adjustable dial compass looks like this.

Teacher Notes



It has a built-in movable protractor or compass housing and is transparent so that maps can be read through the compass. Other features include orienting lines, a direction-of-travel arrow, and a red and black compass needle held in a liquid-damped enclosure. The red part of the compass needle always points towards the Earth's magnetic north pole.

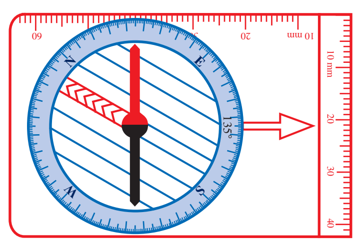


You can see that as well as having N, S, E and W indicated, the compass housing shows numbers of degrees to indicate direction.

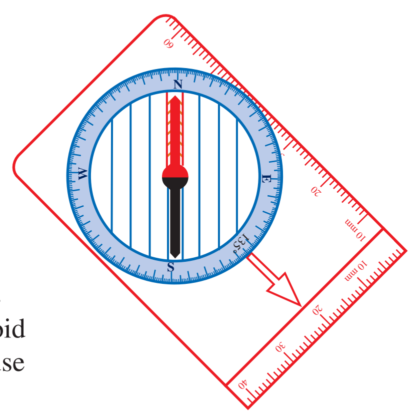
This means that north is 0° (or 360°), east is 90°, south is 180° and west is 270°.

If you want to travel north, just make sure that the arrow moves freely by holding the compass horizontally and follow the direction of the red arrow. If you want to travel in a different direction, you need to use the turnable compass housing.

1 Let's say you want to travel in a direction halfway between south and east (i.e. south-east). First find south-east on the compass housing (i.e. 135°). Then turn the compass housing so that south-east on the housing comes exactly where the large direction-of-travel arrow meets the housing, as shown in the diagram on the right.



2 Hold the compass horizontally in your hand so that the compass needle can turn. While watching the compass, turn your body until the compass needle is aligned with the orienting arrow inside the compass housing, as shown in the diagram on the right. Make sure that the red part of the compass needle points at north in the compass housing—otherwise you could walk off in the opposite direction to what you want.



Now simply walk off in the direction that the travel arrow is pointing in. You need to check the compass frequently to avoid getting off course. Later in this chapter we will see how to use an adjustable dial compass and map to navigate a course.

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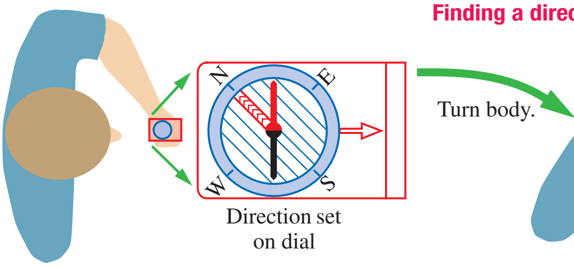
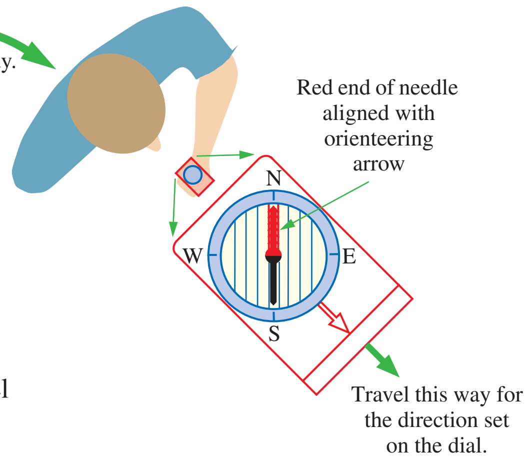
Magnetic materials will affect the working of the compass. Don't expect it to work properly if you are wearing steel belt buckles or if you hold the compass near an iron railing or car bonnet.

The following diagram summarises this method of finding a direction using a compass.

Finding a direction





1 Set direction on compass dial with travel arrow pointing directly ahead.

2 Turn body until red end of magnetic needle lines up with orienting arrow on dial. Travel arrow now shows direction.

Investigation Using a compass

For this investigation you will need the following:

- a large, open area (for example your school oval)
- a coin or any other small flat object
- an adjustable dial compass.

Although you can do this on your own, it is more fun to work in a small group and compare your efforts with those of other group members.

1 Select a starting point and place the coin on the ground. Stand on the coin and set the adjustable dial to a direction of 150°, as described on page 265.

2 Face the direction of 150° with the compass held horizontally in your hand in front of you. Take 20 paces, making sure that your paces are the same size and that you stay on the selected direction. Remember to stop when you reach 20 paces.

3 Now use the compass to set a new direction of 270°. Take 20 paces in that direction and stop.

4 Next take 20 paces in the direction of 30°.

5 Look down on the ground and see whether the coin is there.

6 Now it's time for others to try their compass skills. See how close other group members are to the coin when they finish the course. The things to remember are:
   - keep your paces regular
   - stay on course by checking the compass
   - carefully set the direction on the compass housing
   - turn your whole body to face the direction that you should be travelling in.

Extra Material
Using a compass

7 Draw a scale diagram of the course that you should have travelled. One example of a scale diagram is shown on the CD-ROM. Compare your drawing with the one provided on the CD-ROM.

8 What is the name of the figure formed by the course?

Magnetic compasses must be used with magnetic bearings, so true bearings or grid bearings must be converted to magnetic bearings by correcting for the magnetic variation. A compass is oriented to magnetic north. When using an orienteering compass with a map, it is convenient to mark in some magnetic north lines (**magnetic meridians**). These are normally shown with arrow-heads to distinguish them from grid lines.

An adjustable dial compass is easier to use with a map than a conventional compass. The baseplate is placed in line with the desired direction, then the dial (compass housing) is turned until the orienting lines are in the north direction. The bearing is read directly from the dial. The magnetic needle is ignored.

Example 6

Find the magnetic bearing of Mount Quincan from Tula on the Malanda map shown below using an adjustable dial compass.

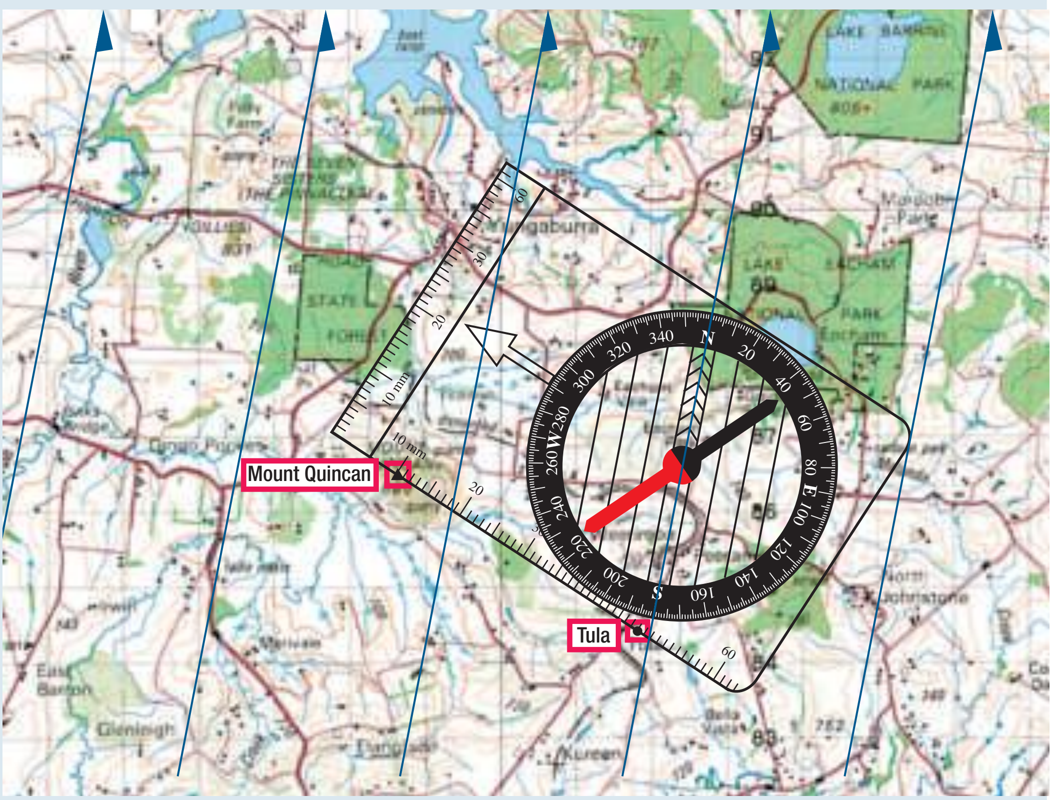
Solution

The map has magnetic north lines drawn in.

The distance is short enough to use the baseplate of the compass without a ruler.

Line up the baseplate between the two points and turn the dial so that the orienting lines are in the direction of the Earth's magnetic field (i.e. aligned with the magnetic north lines).

The dial reading is 292°, so the magnetic bearing is 292°M.



Malanda area **Scale 1 : 100 000**

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You can also use an orienteering compass to find a map location using a bearing and a distance from a starting point. To use the compass in this way:

1. Use the map scale to change the distance into a distance on the map.
2. Set the bearing on the dial of the compass.
3. Place the edge of the compass baseplate on the starting point.
4. Turn the baseplate until the orienting lines are in line with the magnetic north lines (meridians), keeping the edge on the starting point.
5. Now use a plastic ruler against the baseplate to measure the distance.

Example 7

Use the Rathdowney area map shown below to find the feature that is near a distance of 3150 m and a bearing of 129°M from the windmill west of the township of Rathdowney.

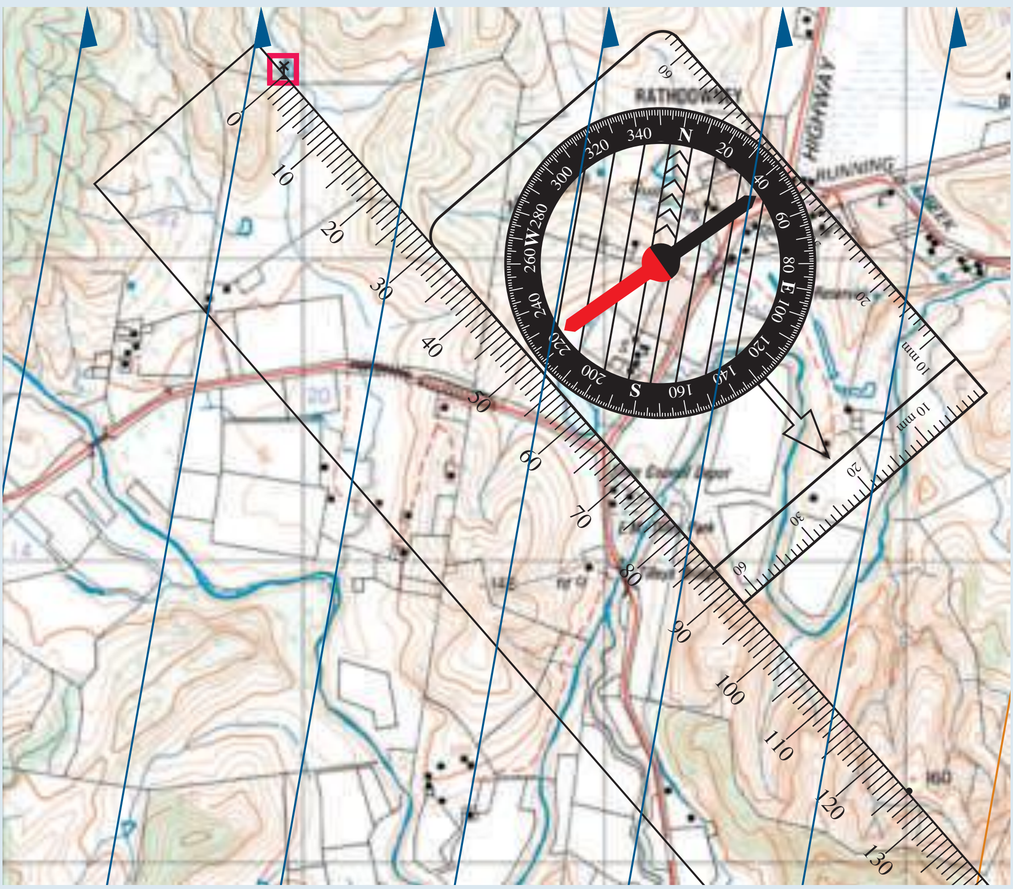
Solution

Calculate the distance on the map.

$$\text{Distance} = 3150 \div 25\,000 \text{ m} = 0.126 \text{ m} = 12.6 \text{ cm}$$

Set the compass dial at 129°.
Place the compass and ruler on the map.

The feature is the top of the hill with an elevation of 160 m.



Rathdowney area **Scale 1 : 25 000**

Investigation Small orienteering courses

Teacher Notes

Before undertaking this investigation, make sure that you discuss safety issues relating to orienteering with your teacher. You will need to remember your pace length, which you calculated earlier in this chapter (page 247).

1 Work in groups of three for this activity. Each group must set up a mini-course in the school grounds in the following way:
   - Choose an identifiable starting point.
   - Choose a visible feature as your first control point.
   - Use the compass to find the magnetic bearing of the control point. Walk to the control point and use your pace length to work out the distance. Average the distances obtained by the three group members.
   - Write down the control point, distance and bearing.
   - Choose the next control point and repeat the procedure.
   - Make a course with three or four legs.
   - Write down the starting point, and the distance and bearing of each of your control points in order.
   - Put a piece of paper with the name of each member of the group at each control point. Write 'Finished' on the last one.

2 Swap your instructions with the instructions from another group. Follow the course you have been given.

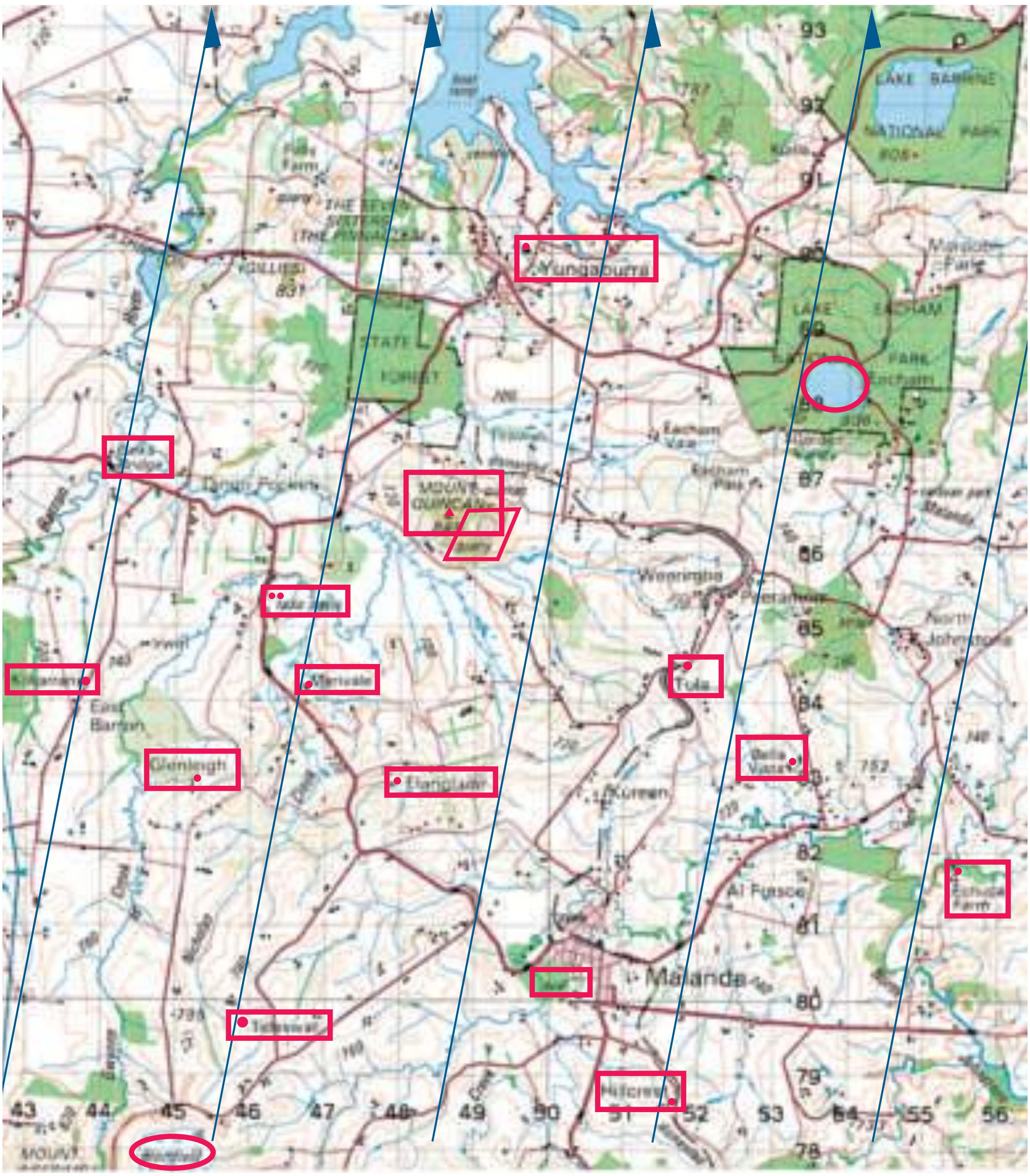
3 When the activity is finished, collect any unfound pieces of paper. Discuss any difficulties you encountered with other groups.

Additional Exercise 9.3

Exercise 9.3 Compasses and orienteering

1 Use the Malanda map shown below to obtain the magnetic bearing:

  a to Merivale from Elanglade
  b to the centre of Lake Eacham from the centre of the quarry at Mt Quincan
  c to the centre of Bromfield swamp from Tula
  d to Pinks Bridge from Mount Quincan
  e to Yungaburra from Echuca Farm
  f to Hillcrest from Glenleigh
  g to the southern oval at Malanda from the radio masts north of Merivale.



Malanda area **Scale 1 : 100 000**

2 Use the map of the central part of Boonah shown below to obtain the magnetic bearing:

a along Treatment Works Road from its intersection with Cemetery Road

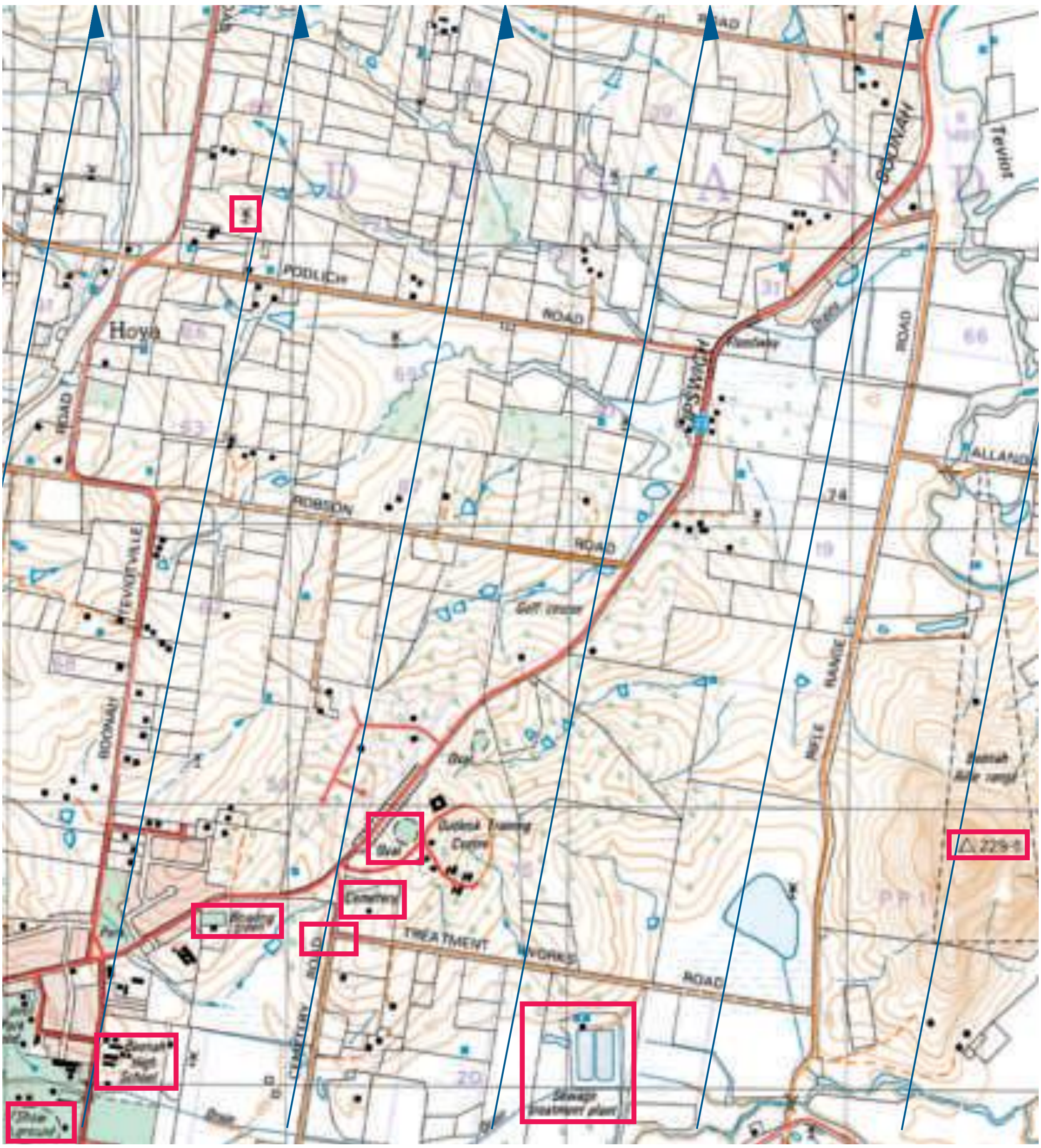
b to the windmill that is north-east of Hoya from the building at the cemetery

c along Cemetery Road southwards from its intersection with Treatment Works Road

d to the centre of the oval west of the Outlook Training Centre from the centre of Boonah High School

e to the building at the bowling green from the building at the sewage treatment plant

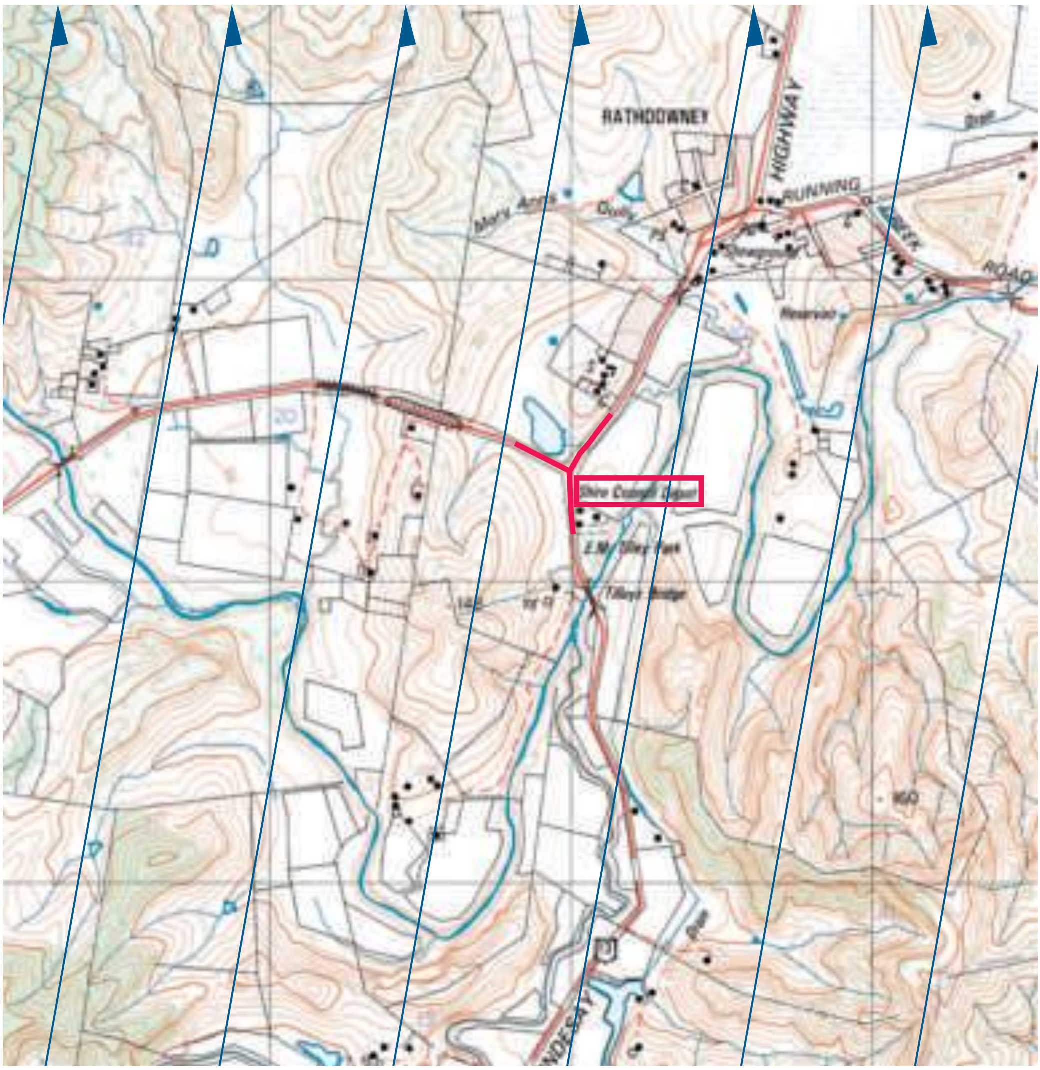
f to the hill 229.5 m high on the rifle range from the centre of the showground.



Boonah area **Scale 1 : 25 000**

3 Use the map of Rathdowney shown below to locate features at the following distances and bearings from the intersection of the roads just north of the Shire Council depot.

a 425 m at 179°M
b 1650 m at 310°M
c 1475 m at 126°M
d 1050 m at 050°M
e 1675 m at 261°M



Rathdowney area **Scale 1 : 25 000**

4 Use the Malanda map shown on page 270 to find features:

a 5.5 km at 060°M from Mount Quincan
b 7.5 km at 107°M from Kilkarran
c 5.9 km at 325°M from Tula
d 11.5 km at 041°M from Tideswall
e 4.1 km at 215°M from Bella Vista.

5 Use the map of Mt Glorious shown below to find features at the following distances and bearings from the hut marked by the cross on top of Mt Glorious.

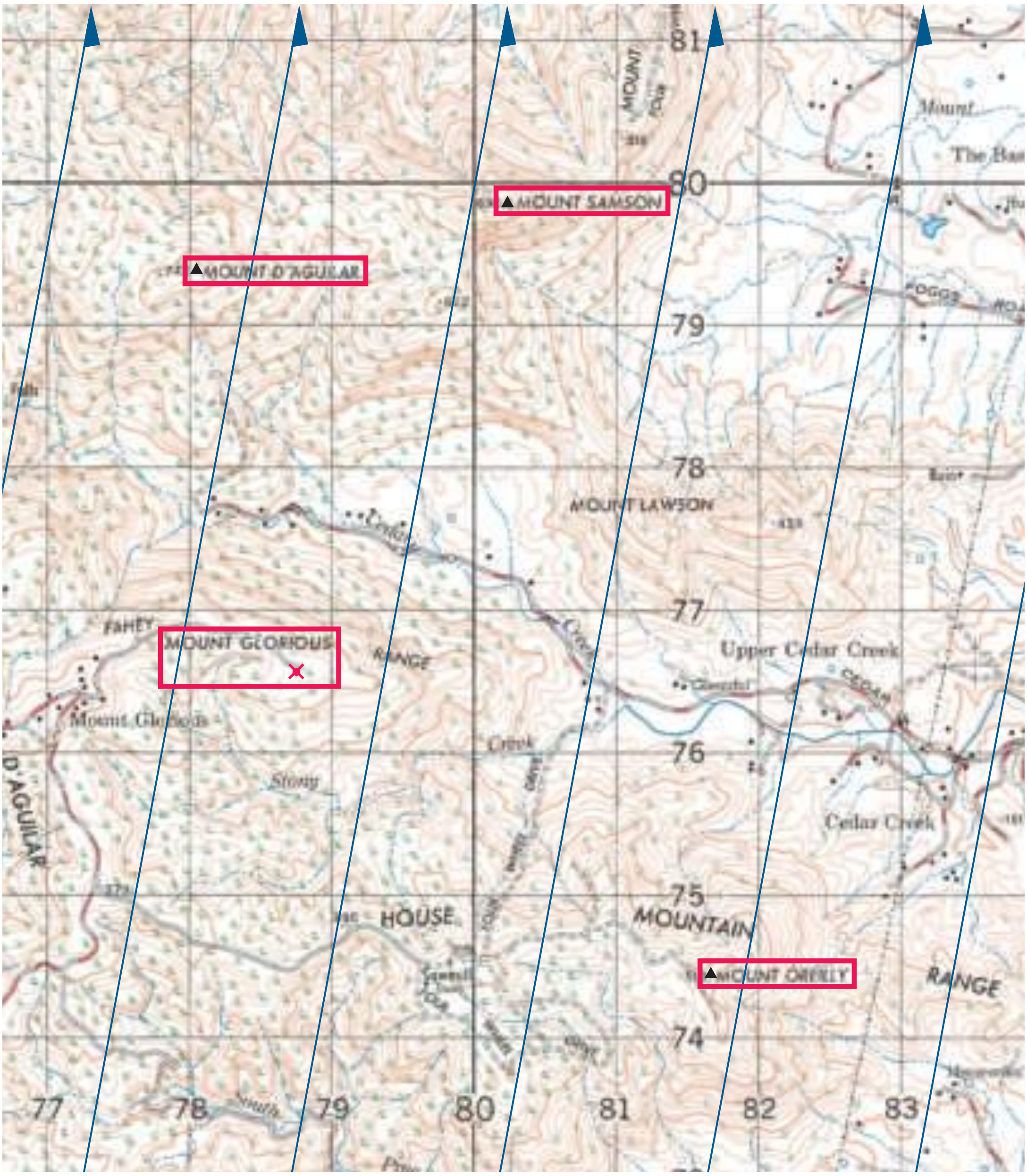
a 2900 m at 335°M

b 3600 m at 014°M

c 2725 m at 081°M

d 2350 m at 146°M

e 5500 m at 111°M



Mt Glorious area **Scale 1 : 50 000**

6 An orienteering course is marked on the map of the northern part of Moreton Island shown below. Work out the distance and magnetic bearing of each control point from the previous one, in the order Start, 1, 2, 3, 4, 5 and Finish.

Moreton Island area **Scale 1 : 100 000**

7 The map of the northern part of North Stradbroke Island shown on the opposite page has control points marked for an orienteering course. Work out the distance and magnetic bearing for each leg of the course.

Modelling and problem solving

8 A bushwalker is in the Mt Glorious area but is uncertain of her exact position. Using her compass, she can see Mt D'Aguilar at a bearing of 300°M, Mt Samson at a bearing of 330°M and Mt O'Reilly at a bearing of 164°M. Use the map of the Mt Glorious area shown on page 273 to calculate her approximate position.

North Stradbroke Island area **Scale 1 : 100 000**

9 Two orienteering friends set out from the start of a course in opposite directions. Max heads due west for 400 m and then goes due south for 2.4 km to arrive at the first control point. Clarissa starts off heading due east, but after a while she changes direction and heads straight for the first control point. If they both travelled the same distance:

a how far east did Clarissa travel before changing direction?

b what bearing did Clarissa follow to reach the first control point?

9.4 Calculating land areas

One of the reasons for conducting a survey of a property is to determine the area of land included within the boundaries. Land areas are calculated using one of the following metric units:

1 square metre	$= 1\text{ m} \times 1\text{ m}$	$= 1\text{ m}^2$
1 hectare (ha)	$= 100\text{ m} \times 100\text{ m}$	$= 10\,000\text{ m}^2$
1 square kilometre	$= 1000\text{ m} \times 1000\text{ m}$	$= 1\,000\,000\text{ m}^2$

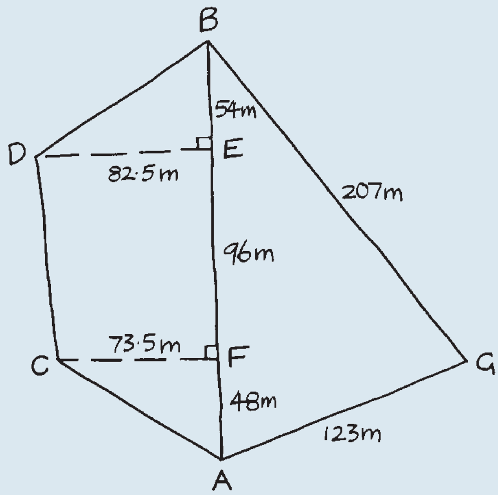
The unit that is used depends on the size of the property being measured. Suburban building blocks of land, for instance, are measured in square metres.

There are a number of ways of finding area, depending on whether it is calculated from a map or from measurements obtained in a survey (**field measurements**). We will now look at the sorts of calculations that are commonly necessary in elementary surveying work.

When a large straight-edged (polygonal) area must be calculated, it is normal to divide it into a number of smaller regions. The areas of the smaller regions are then calculated and added to give the area of the entire region.

Example 8

A sketch of a paddock is made and some field measurements are taken. Calculate the area of the paddock.



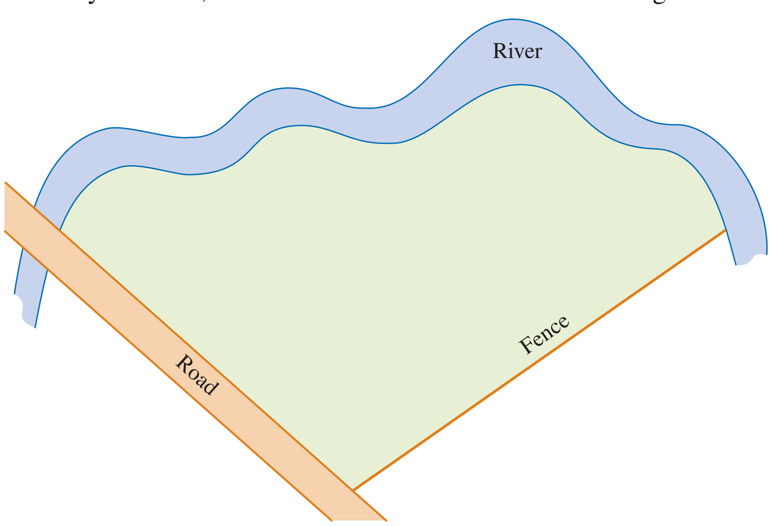
Solution

This paddock is a polygon and can be conveniently divided into three triangles ($\triangle AFC$, $\triangle DEB$ and $\triangle AGB$) and a trapezoid ($CFED$). The area of the paddock is the sum of these areas.

Write rule for area of $\triangle AFC$.	Area of $\triangle AFC = \frac{1}{2} AF \times CF$
Substitute relevant lengths.	$= \frac{1}{2} \times 48 \times 73.5$
Evaluate.	$= 1764\text{ m}^2$

Write rule for area of $\triangle DEB$.	Area of $\triangle DEB = \frac{1}{2} EB \times DE$
Substitute relevant lengths.	$= \frac{1}{2} \times 54 \times 82.5$
Evaluate.	$= 2227.5\ \text{m}^2$
To find the area of $\triangle AGB$ we use Heron's Formula.	Area of $\triangle AGB = \sqrt{s(s-a)(s-b)(s-c)}$
	where $s = \frac{1}{2}(a+b+c)$
Calculate s for $\triangle AGB$.	$s = \frac{1}{2}(207 + 198 + 123)$
	$= 264$ m
Write Heron's Formula for $\triangle AGB$.	Area of $\triangle AGB = \sqrt{264(264-207)(264-198)(264-123)}$
Evaluate and round.	$= \sqrt{264 \times 57 \times 66 \times 141}$
	$\approx 11\ 833.7\ \text{m}^2$
Write rule for area of trapezoid $CFED$.	Area of $CFED = \frac{1}{2}(DE + CF) \times EF$
Substitute.	$= \frac{1}{2}(82.5 + 73.5) \times 96$
Evaluate.	$= 7488\ \text{m}^2$
Calculate area of paddock.	Area of $AGBDC = \triangle AFC + \triangle DEB + \triangle AGB + CFED$
Substitute.	$\approx 1764 + 2227.5 + 11\ 833.7 + 7488$
Evaluate and round.	$\approx 23\ 310\ \text{m}^2$
State the result using appropriate unit.	Area of the paddock is about 2.331 ha.

In practice, many regions that need to be measured have irregular or curved boundaries, such as the region bounded by the river, the fence and the road shown in the diagram below.



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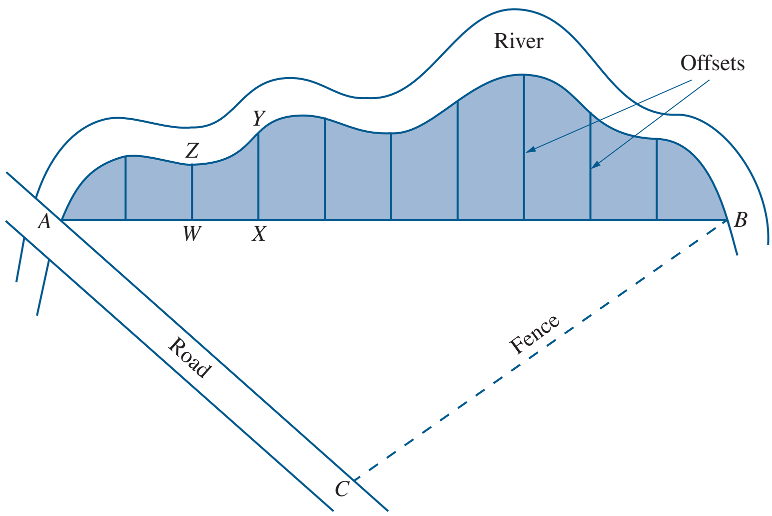
In the field, such boundaries are sketched by using a survey line (AB) and a number of equally spaced offsets from AB to the irregular boundary. In this case, the area of $\triangle ABC$ can be readily found. The irregular region between AB and the river can be found by examining each region formed by the offsets, the river and the survey line AB.



The area of this irregular portion could be calculated by treating each region formed by the offsets individually. For example, $WXYZ$ could be treated like a trapezoid with the offsets WZ and XY the parallel sides, and WX the perpendicular distance between them. In this way,

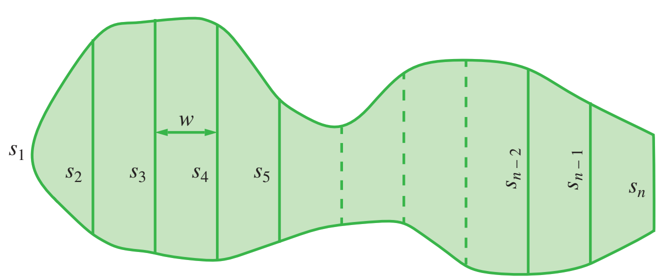
$$\text{Area of } WXYZ \approx \tfrac{1}{2}(WZ + XY) \times WX$$

This could be done for each region and then the sum of all areas found —but this would be time consuming. In Chapter 3 we saw how the **trapezoidal rule** can be used to find irregular areas.

!

Trapezoidal rule

The area of an irregular shape can be *approximated* by dividing the shape into strips of equal width, as shown below.

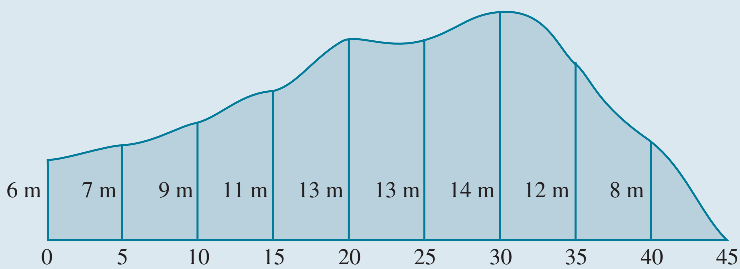


The following rule comes from the rule for the area of a trapezium, $A = \tfrac{1}{2}(a + b) \times h$.

$$\begin{aligned}\text{Area} &\approx \frac{w}{2}(s_1 + 2s_2 + 2s_3 + \ldots + 2s_{n-1} + s_n)\\ &= \frac{\text{width}}{2} \times (\text{total of end lengths} + 2 \times \text{total of middle lengths})\\ &= \text{width} \times (\text{total of end lengths} + 2 \times \text{total of middle lengths}) \div 2\end{aligned}$$

Example 9

Find the area of this region.



Solution

The last length in this case, after the 8 m, is 0 m. The width of each strip is 5 m.

Calculate total of end lengths. Sum of end lengths $= 6 + 0$

$= 6$ m

Calculate total of middle lengths. Total of middle lengths $= 7 + 9 + 11 + 13 + 13 + 14 + 12 + 8$

Evaluate. $= 87$ m

Write the trapezoidal rule. Area $\approx \frac{5}{2}(6 + 2 \times 87)$

Evaluate. $= \dfrac{5 \times 180}{2}$

$= 450 \text{ m}^2$

State the result. The area of the region is approximately 450 m^2.

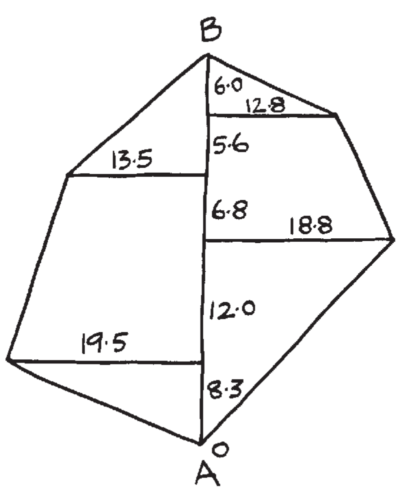
Exercise 9.4 Calculating land areas

Additional Exercise 9.4

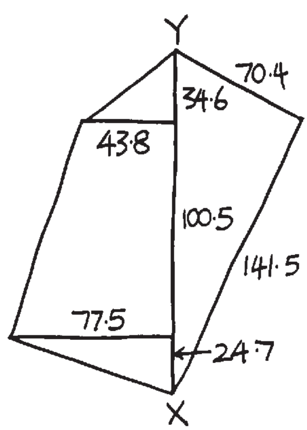
Modelling and problem solving

1 The following field notes relate to separate regions. Calculate the area of each. (Measurements are in metres.)

a



b



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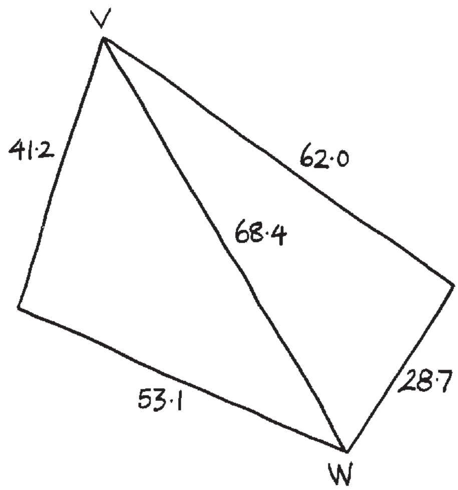
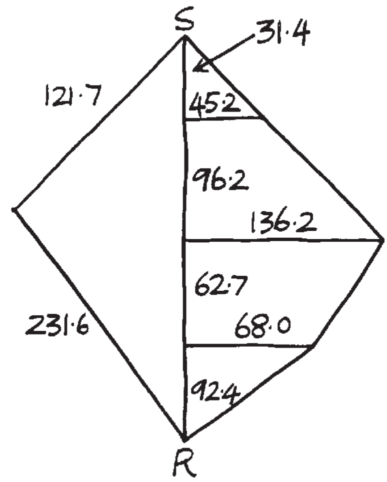
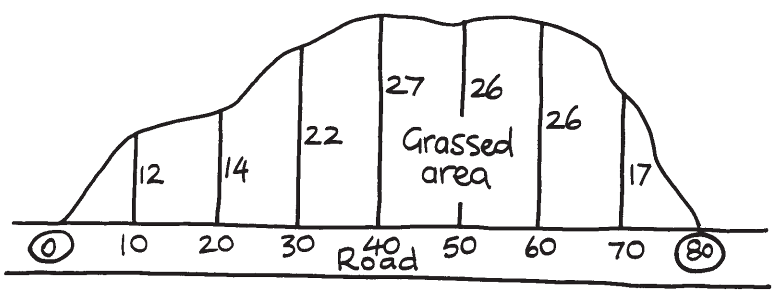
c



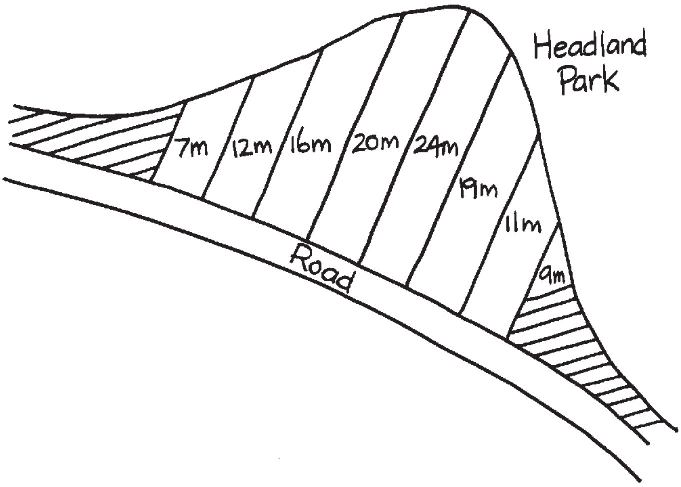
d



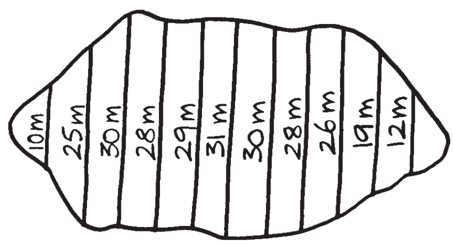
2 A grassed area to the side of a road has been surveyed and the following perpendicular offsets have been taken. Calculate the area of the grassed region using the trapezoidal rule.



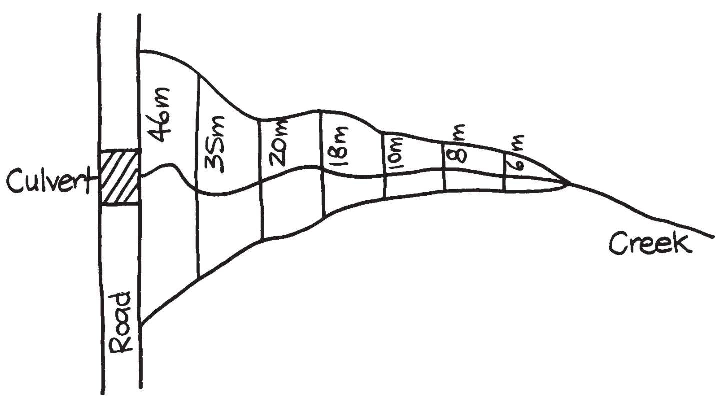
3 Calculate the area of the proposed park shown in the following field notes. The strips are 8 m in width.



4 An artificial lake was surveyed using 5 m-wide strips as shown in the field notes below. What is the area of the lake?



5 This recreational reserve was surveyed using 8 m-wide strips. What is the area of the reserve?



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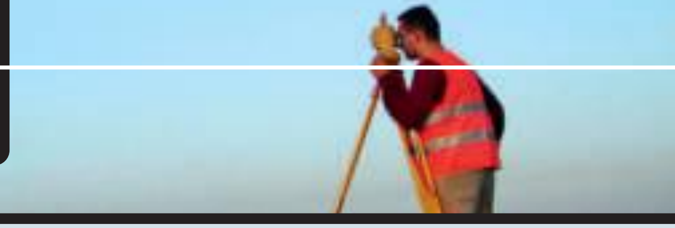
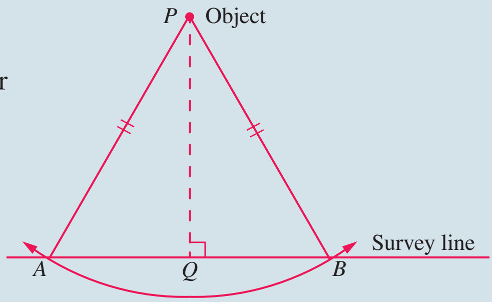
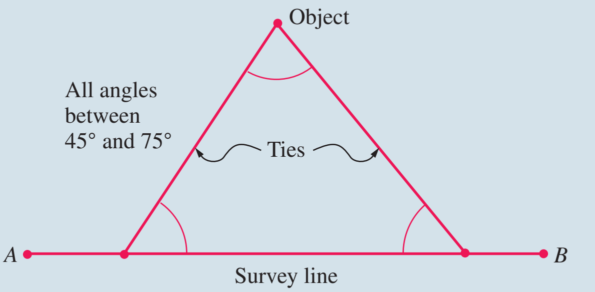


Chapter summary

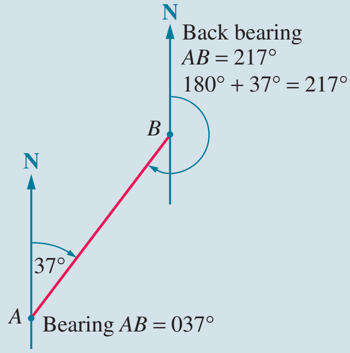
- Surveys of large areas of land are called **geodetic surveys**, while surveys of smaller land areas are known as **plane surveys**. Plane surveys include:
  topographical surveys—concerned with the location of natural and artificial features;
  cadastral surveys—concerned with the establishment of property boundaries;
  engineering surveys—concerned with the design of engineering works.
- It is possible to estimate distances using **pacing**, but this is not accurate. Surveyors use a variety of instruments such as tapes, chains, trundle wheels and electronic devices to measure length.
- **Chain surveying** or **chaining** involves producing a plan of an area of land using distance-measuring instruments. This is done by recording a number of linear measurements relative to a main **survey line**.
- The position of an object relative to a survey line can be described using **perpendicular offsets**. Objects can be located in a direction perpendicular to a survey line by '**swinging an arc**'.



- The position of an object can also be **tied** to the main survey line. When using this method, the triangle formed by the ties and the survey line must be **well-conditioned**; that is, all angles must be between 45° and 75°.



- To conduct a **chain survey**, first determine the position of the survey line. The survey line needs to be positioned so that all features can be connected to it using ties or offsets.
- The survey data are recorded (or **booked**) as **field notes** with the starting point on the survey line booked as 0.0. All features of interest are then booked relative to points on the survey line.
- A direction on the Earth is given as a **bearing**: the horizontal direction (in degrees) of an object from an observer, expressed as the clockwise angle from north to the object direction. A bearing is given as three figures from 000° to 360°. A **back** or **reverse bearing** is the bearing of the opposite direction. A back bearing differs from the bearing by 180°.



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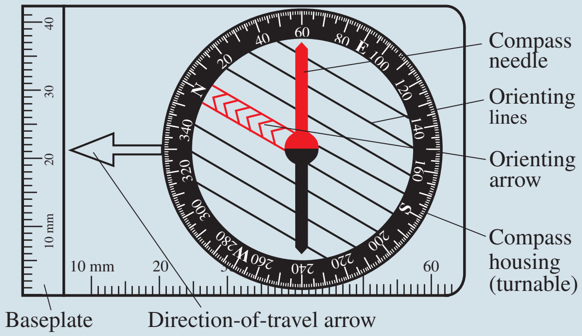
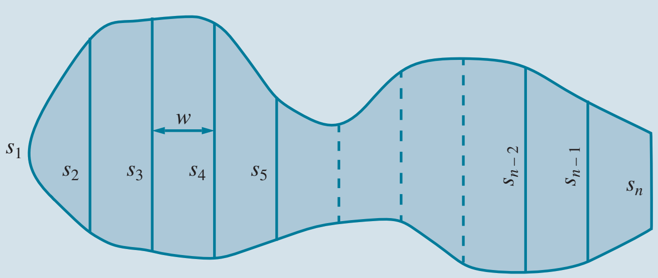
Chapter summary

- An **open traverse** maps out a path between two points using a consecutive series of lines or **legs**.
- Surveyors use a **theodolite** to measure direction. An **adjustable dial** (or **orienteering**) **compass** also can be used to measure direction.



- To use an orienteering compass to find a direction, set the direction on the dial, then turn your body until the compass needle is aligned with the orienting lines. You then follow the direction-of-travel arrow.
- A bearing obtained using a compass is called a **magnetic bearing** and is distinguished by the letter **M** after the bearing, such as 057°M.
- **Orienteering** is a sport in which runners navigate a course between **control points** marked on a map, using a map and compass.
- **Land area** is measured in square metres, hectares or square kilometres, depending on the size of the area being measured.
- To determine the area of a polygonal area of land, divide it into smaller regions such as triangles and rectangles.
- An area with irregular or curved boundaries can be calculated using the **trapezoidal rule**.
- **Trapezoidal rule**

  The area of an irregular shape can be *approximated* by dividing the shape into strips of equal width, as shown below.



The following rule comes from the rule for the area of a trapezium, $A = \frac{1}{2}(a + b) \times h$.

$$\text{Area} \approx \frac{w}{2}(s_1 + 2s_2 + 2s_3 + \ldots + 2s_{n-1} + s_n)$$

$$= \frac{\text{width}}{2} \times (\text{total of end lengths} + 2 \times \text{total of middle lengths})$$

$$= \text{width} \times (\text{total of end lengths} + 2 \times \text{total of middle lengths}) \div 2$$

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Chapter review

Knowledge and procedures

Ex 9.1

1 Calculate the following pace lengths.

a 55 paces cover 48 m. **b** 39 m is covered using 43 paces.

Ex 9.1

2 Riad has a pace length of 0.8 m. Calculate the following distances paced out by Riad.

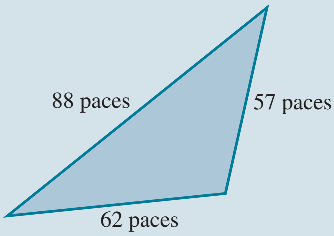
a 68 paces around his house

b 1671 paces along from his house to the park

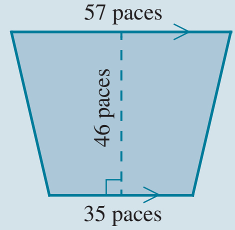
Ex 9.1

3 Jac knows that his pace length is 0.9 m. Calculate the areas of these figures paced out by Jac.

a



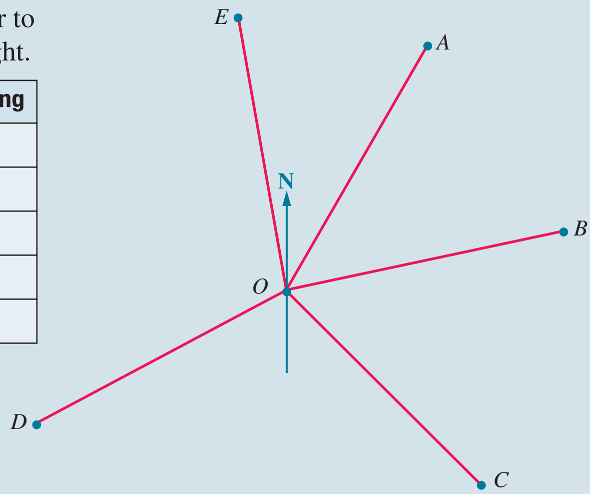
b



Ex 9.2

4 Complete the table by using a protractor to measure the angles in the diagram at right.

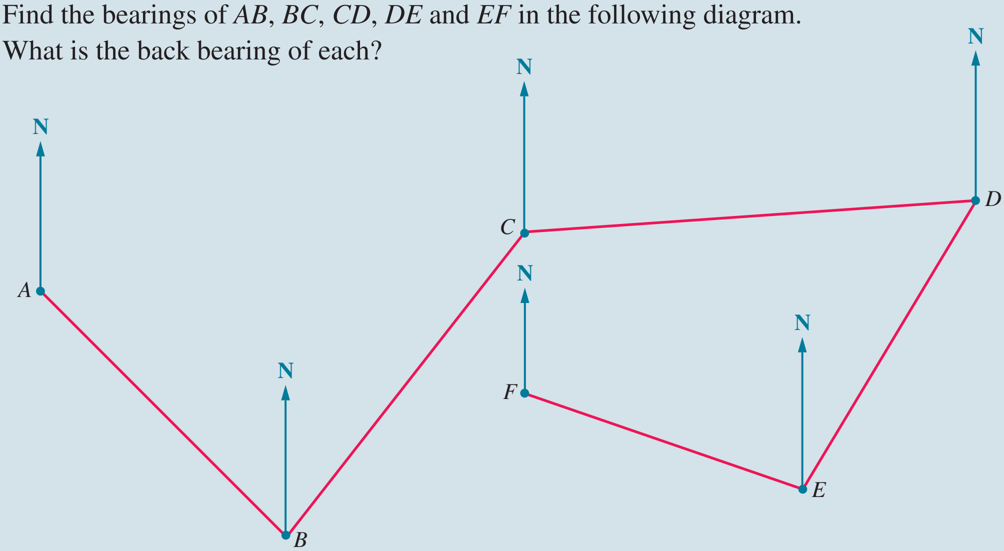
Leg	Forward bearing	Back bearing
OA		
OB		
OC		
OD		
OE		



Ex 9.2

5 **a** Find the bearings of *AB*, *BC*, *CD*, *DE* and *EF* in the following diagram.

b What is the back bearing of each?



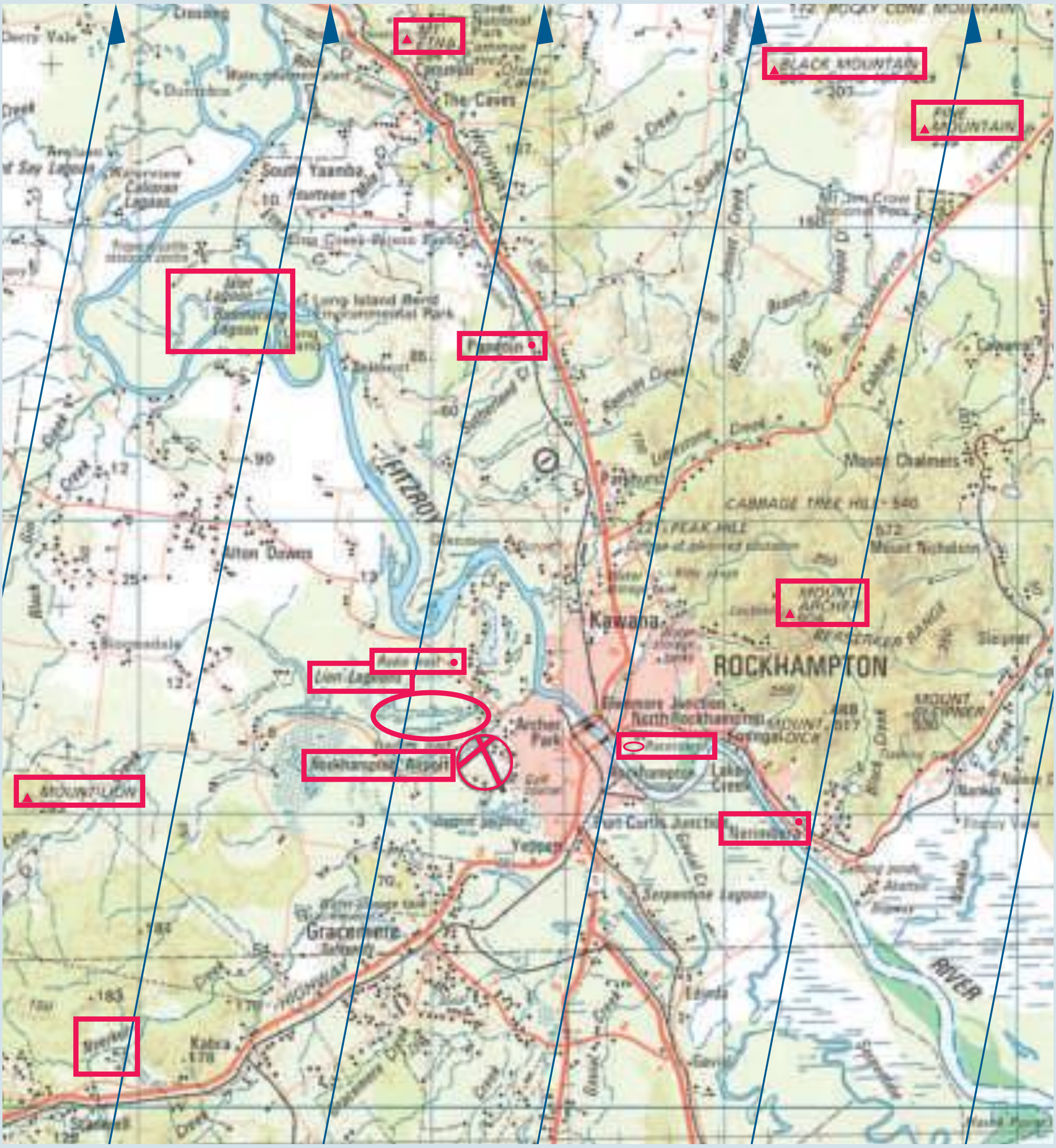
Chapter review

6 What is the back bearing for each of the following bearings? Ex 9.2

a 220° b 187° c 047° d 321° e 122°

7 Use the Rockhampton map shown below to find the magnetic bearing of: Ex 9.3

a Rockhampton Airport from Pine Mountain
b Mt Lion from Mt Archer
c the radio mast near Lion Lagoons from Neerkol
d Boomerang Lagoon from Mt Lion
e Nerimbera railway station from Mt Etna.



Rockhampton area **Scale 1 : 250 000**

Chapter review

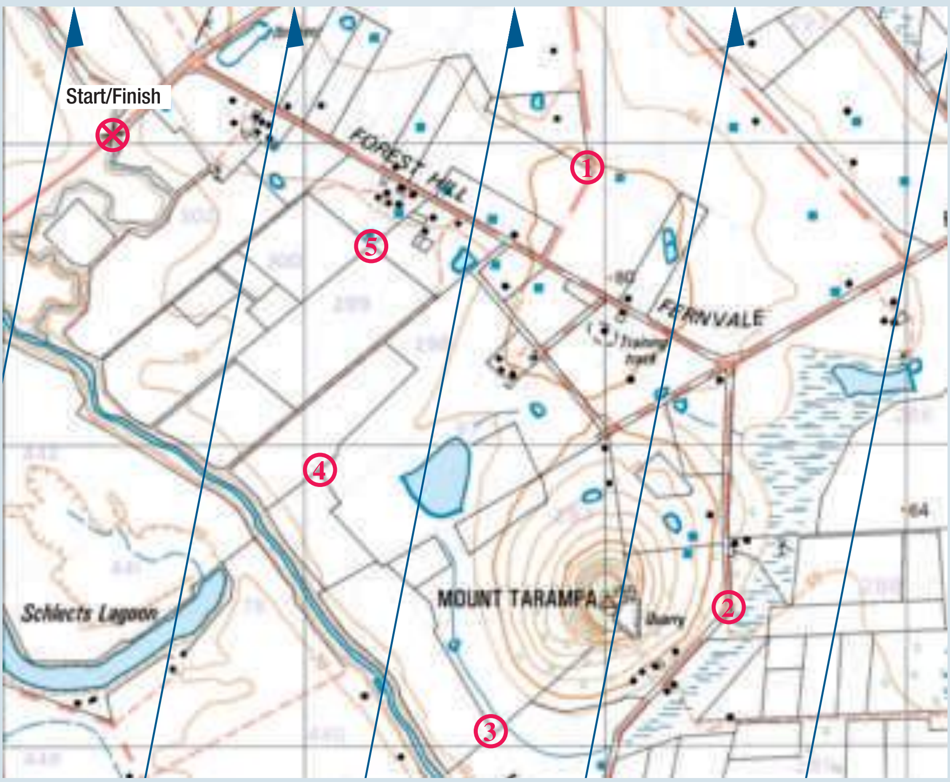
Ex 9.3

8 Use the Rockhampton map shown on the previous page to find the feature:

a 11 km at 057°M from Pandoin railway station
b 23.25 km at 329°M from Lion Lagoons
c 25 km at 218°M from Black Mountain
d 17.5 km at 143°M from Islet Lagoon
e 18 km at 331°M from Rockhampton racecourse.

Ex 9.3

9 The Mt Tarampa map shown below has five numbered control points and a Start/Finish marked on it for orienteering course. Find the distance and bearing of each control point from the previous one, in order.

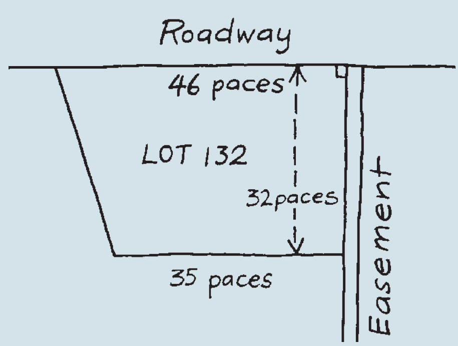


Mt Tarampa area Scale 1 : 25 000

Modelling and problem solving

Ex 9.1

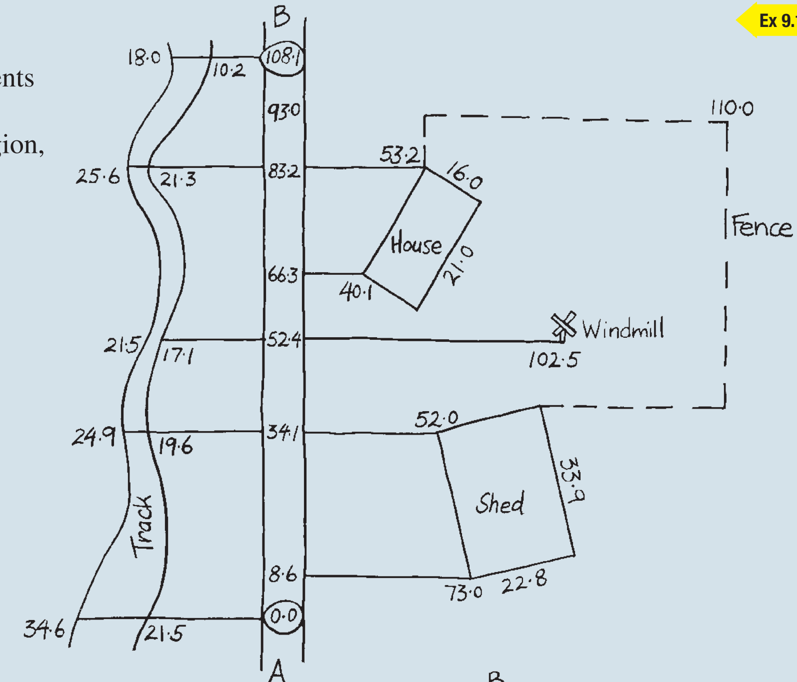
10 Jake has made a rough drawing of a block of ground he is thinking about buying, as shown. If Jake takes 32 paces to cover 28 m, what is the area of the building block?



Chapter review

11 The details of a survey are booked as follows. Use the sketch and field measurements (in metres) to make a scale drawing of the surveyed region, using the scale 1 : 1000.

Ex 9.1

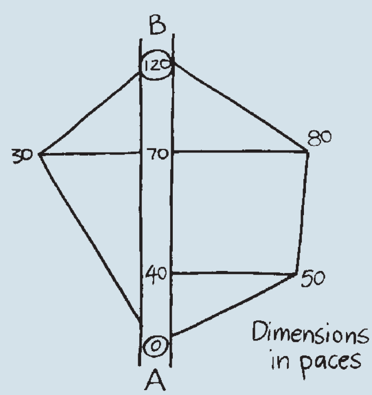


12 The survey notes to the right relate to a play area at a school. The area was paced out by Roberto, who covers 20 m every 27 paces.

a Make a scale drawing of the area using a scale of 1 : 1000.

b What is the area of this region?

Ex 9.1, 9.4



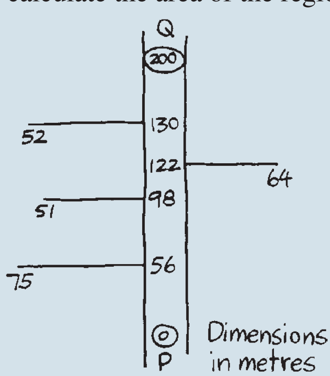
13 The field notes of two separate surveys are shown. In each case:

i select a suitable scale and make a scale drawing of the area

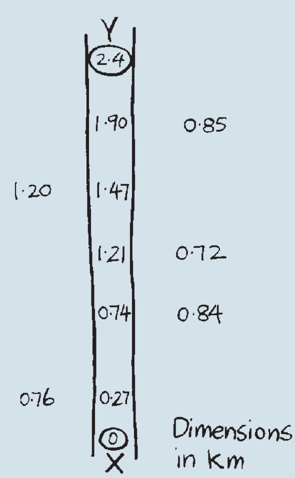
ii calculate the area of the region surveyed.

Ex 9.1, 9.4

a



b



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Chapter review

Ex 9.2

14 **a** Make a scale drawing of the following route:

AB 630 m at 120°, *BC* 800 m at 035°, *CD* 870 m at 167°, *DE* 660 m at 241°

b Find the distance and bearing from the start to the finish.

Ex 9.3

15 A hiker is in the Rockhampton area but is uncertain of his exact position. Using his compass, he can see Mount Archer at a bearing of 178°M, Pine Mountain at a bearing of 010°M and the radio mast at a bearing of 227°M. Use the map of the Rockhampton area shown on page 285 to calculate his approximate position.

Ex 9.3

16 Zena and Conan leave from the first control point of an orienteering course. Zena goes north for 300 m, then changes direction and heads due west for 1.2 km to arrive at the second control point. Conan goes south but soon finds he is heading in the wrong direction, so he changes course and heads for the second control point.

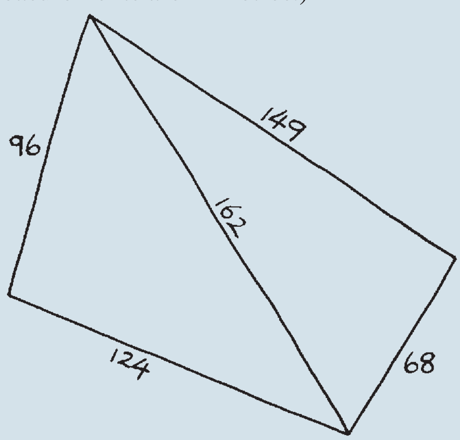
a If they both travelled the same distance, how far south did Conan travel?

b What bearing did Conan follow to reach the second control point?

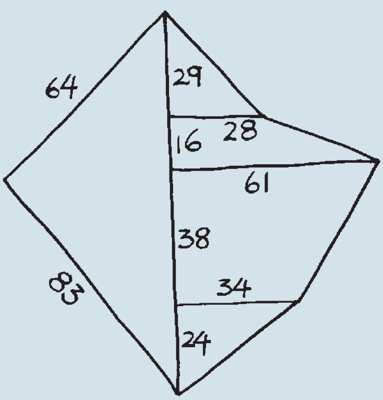
Ex 9.4

17 The following field notes relate to separate regions. Calculate the area of each. (Measurements are in metres.)

a

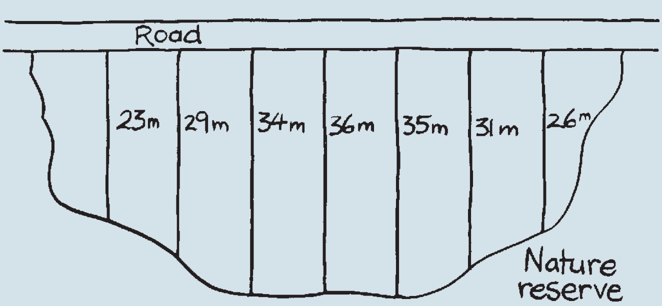


b



Ex 9.4

18 Calculate the area of the proposed nature reserve shown in these field notes. The strips are 12 m in width.



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Managing your money

10

Contents

Syllabus Guide Chapter 10

Syllabus subject matter

Managing money I

- Budgeting, including the preparation of a personal budget plan
- Spending, including discount and foreign exchange
- Business applications, including profit, loss and markup

Quantitative concepts and skills

- Calculation and estimation with and without instruments
- Rates, percentages, ratio and proportion

A budget is a fundamental and effective financial management tool that can be used by almost anyone. No matter how much you earn, it is important to know how much money you have to spend and where you spend it. Personal budgeting allows you to know how much money you have at your disposal. It can be a self-education tool that shows you how your funds are allocated and how far you are from reaching your goals. Once you know these things, you are in a better position to make well-informed financial decisions.

10.1 Marking prices up and down

When a retailer (shop owner) buys goods from a wholesaler or manufacturer at **cost price**, a **markup** is usually added to the price to cover costs and make a profit. The amount added depends on many factors, including the turnover, competition, market share, manufacturer's recommendation, freshness and storage life. The price shown on an item offered for sale is called the **marked price** or **retail price**. A **markdown** is the amount that is taken off the price of goods to get rid of them when they are selling slowly, damaged or shop-soiled.

!

The **markup** on an item is the difference between its marked price and its original cost.

$$\text{Markup} = \text{marked price} - \text{cost price}$$

The **markdown** on an item is the difference between the original marked price and the final marked price.

$$\text{Markdown} = \text{original marked price} - \text{final marked price}$$

Example 1

A shop had CD players for sale for \$449. The players cost the shop \$320 each. What was the markup?

Solution

Write the formula.	Markup = marked price − cost price
Substitute known information.	= \$449 − \$320
Evaluate.	= \$129

Example 2

Six months after its release, the Electronics Superstore sold a Dreamware computer game at a markdown of \$55 from the original marked price. The marked price of the game was \$49.50. What was the original marked price?

Solution

Write the formula.	Markdown = original price − marked-down price
Substitute known information.	\$55 = original price − \$49.50
Rearrange.	Original price = \$49.50 + \$55
Evaluate.	= \$104.50

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When markup is shown as a percentage, the cost price is usually used as the base. In mathematics, indexes are usually expressed in terms of *original amounts.* In the case of markup, the cost price is the original amount.

Example 3

An electrical store uses a 40% markup on large items. What is the retail price for a fridge that cost the store \$840?

Solution

Write the formula for markup.	Markup = 40% of \$840
Evaluate.	$= 0.40 \times \$840 = \$336$
Write the formula for retail price.	Retail price = cost price + markup
Substitute known values and evaluate.	$= \$840 + \$336 = \$1176$

Alternative method

Write the formula for markup.	Markup = 40% of cost price
Write the formula for retail price.	Retail price = cost price + markup
Substitute known values.	= cost price + 40% of cost price
	= 140% of cost price
Change % to decimal and evaluate.	$= 1.4 \times \$840 = \$1176$

Example 4

The retail price of a tyre is \$125 and the cost price was \$75. What was the percentage markup?

Solution

Calculate markup.	Markup $= \$125 - \$75 = \$50$
% markup is found using the cost price.	% markup $= \dfrac{\$50}{\$75} \times 100\%$
Evaluate.	$= 0.6666 \ldots$
Express as a percentage.	$= 66\frac{2}{3}\%$

Example 5

A shop uses a 70% markup. What was the cost price of a pair of jeans sold for \$157.25?

Solution

Write a rule for marked price.	Marked price = cost price + 70% of cost price
	= 170% of cost price
Substitute known information.	\$157.25 = 170% of cost price
Write % as a decimal.	$= 1.70 \times$ cost price
Rearrange.	Cost price $= \$157.25 \div 1.70$
Evaluate.	= \$92.50
State the result.	The cost price of the jeans was \$92.50.

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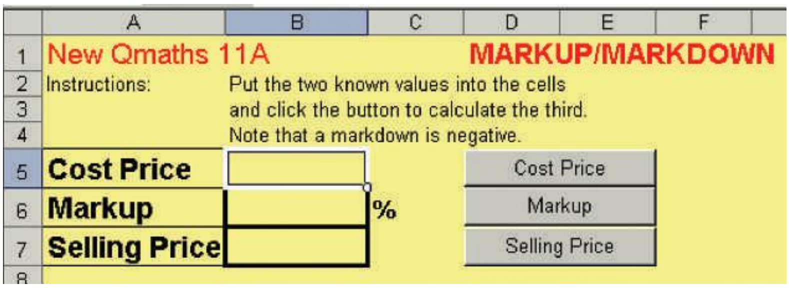
Spreadsheet

Technology

You can use the Markup/Markdown spreadsheet on the CD-ROM to check your calculations for markups or markdowns. A markdown must be entered as a negative.



Additional Exercise 10.1

Exercise 10.1 Marking prices up and down

1 Copy and complete the following table of cost prices, marked prices, markups and markdowns.

	Item	Original (cost) price	Marked price	Up or down?	Amount of markup/ markdown
a	DVD player	\$240	\$310		
b	Table		\$785	Markup	\$225
c	Lamp	\$154	\$140		
d	Software bundle	\$235		Markup	\$110
e	Bottle of soft drink	\$0.90	\$2.10		
f	Magazine	\$4.05	\$7.50		
g	Travel bag	\$28	\$62		
h	Digital TV	\$1700		Markup	\$1200

2 A hardware shop works on a 65% markup. What would the shop charge for:

a spades with a cost price of \$12?
b drills that cost the shop \$140?
c packs of ceiling insulation that cost \$60 each?
d tubes of silicon costing \$3 each?
e a chainsaw with a cost price of \$280?

3 A furniture store will sell a three-piece lounge suite for \$5000, or the chairs and sofa separately at \$1400 for each chair and \$2800 for the sofa. If the cost price of the chairs was \$760 and the cost price of the sofa was \$1240, work out the percentage markup on:

a the lounge suite as a set
b the sofa sold separately
c a chair sold separately.

4 A takeaway works on a markup of 120%. What was the cost price of each of the following?

a Spring roll sold for \$2.20
b Cod sold for \$4.20 a piece
c Mini dim-sims sold for \$1.10 each
d Seafood snacks sold for \$2.85 each

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5 After a fire, smoke-damaged goods are sold at a markdown of 15% to clear stock.
  a What is the marked price of a tracksuit with an original price of $175?
  b What would a dress that originally cost $240 sell for?
  c A jacket is marked down to $238. What was the original price?
  d What was the original price of a top sold for $68?

6 A pool shop sells pumps for $540. If they cost the shop $400, what is the percentage markup?

7 An aluminium dinghy that cost a boatyard $1200 is sold for $2160. What is the percentage markup?

8 One day before the use-by date, packets of biscuits are marked down to $1.50 each. If the original price was $2.10, what was the percentage markdown?

9 Mirrors costing $140 are sold at a markup of 60%.
  a What is the markup? **b** What is the retail price?

10 An electronics shop buys small resistors for $30 for each 100 and sells them for 75 cents each. What is the percentage markup?

10.2 Discounting prices

Some manufacturers and distributors suggest the price at which goods should be sold. This is called the **recommended retail price** (**RRP**) or **list price**. Retail shops are not bound to sell at this price, but many do. For example, many books even have a recommended retail price printed on the back. Suppliers who set a recommended retail price usually offer the goods to retailers at a **trade discount**. This means that they sell the goods to the shop below the recommended retail price. The trade discount is often expressed as a percentage of the list price (RRP).

Example 6

Calculators are sold to shops at a trade discount of 40%. What do the shops pay for a calculator with a list price of $45?

Solution

Write a formula for the discount.	Trade discount = 40% of list price
Substitute list price.	= 40% of $45
	= 0.40 × $45
Evaluate.	= $18
Write a rule for cost price.	Cost price = list price − discount
Evaluate.	= $45 − $18 = $27

Alternative method

Write a formula for cost price.	Cost price = RRP − discount
Discount is 40% of RRP.	= RRP − 40% of RRP
	= 60% of $45
Evaluate.	= 0.60 × $45 = $27
State the result in a sentence.	The price paid by shops is $27.

Wholesalers and other suppliers often offer a discount for early payment. When goods are supplied, the **invoice** (bill) is normally payable within 7, 14, 30, 60 or 90 days. A discount offered for earlier payment shows the percentage and time as a fraction—for example:

$$\frac{5}{10}, \frac{2}{20}, \frac{n}{30} \quad \text{or} \quad 5/10, 2/20, n/30$$

This means:

- Payment made within 10 *calendar days* of the invoice date gives a 5% discount.
- Payment within 20 calendar days gives a 2% discount.
- The net amount is due within 30 calendar days of the invoice date. (The **net amount** is the total amount of the invoice, less any credit for return of damaged or unsatisfactory goods.)

Retailers who fail to pay invoices by the due date may have to pay additional charges, or find that their credit is reduced or discontinued.

Example 7

All-Pro sports goods shop orders 20 tennis racquets, which are invoiced to the shop as shown.

SMASH HIT RACQUETS			
DATE:	25 August 2008		
INVOICE TO:	All-Pro Sporting Goods		
Goods Supplied	*Unit Price*	*Number*	*Total*
Tennis Racquets	$65	20	$1300
TERMS: 3/10, n/30			

a How much is due if the invoice is paid on 3 September?
b How much is due if the invoice is paid on 14 September?
c What is the last date for payment?

Solution

a 3 September is the 9th day after the invoice date, so the 3% discount applies.

Calculate discount. Discount = 3% of $1300

Evaluate. = 0.03 × $1300 = $39

Calculate amount due. Amount due = $1300 − $39

= $1261

Alternative method

3 September is the 9th day after the invoice date, so the 3% discount applies.

Calculate discount. Discount = 3% of invoice price

Write a rule for amount due. Amount due = invoice price − discount

Substitute known values. = 97% of $1300

Evaluate. = 0.97 × $1300 = $1261

b Discount applies for 10 days after the invoice date of 25 August, i.e. until 4 September. 14 September is after 4 September.

The full amount of $1300 must be paid.

c The last date for payment is 30 days after the invoice date, 25 August.

Calculate August days left. 31 − 25 = 6 days

Calculate September days. 30 − 6 = 24 days

Write the answer. The last date for payment is 24 September.

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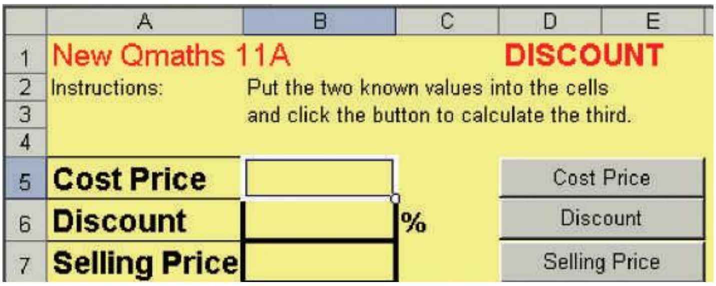
Technology

Spreadsheet

You can use the Discount spreadsheet provided on the CD-ROM to check your calculations for discounts.



In cases where several discounts are given, the order of discounting can make a difference to the final amount. This depends on whether the discounts are given as percentages or cash amounts. When manufacturers want to promote a particular product, they sometimes offer a discount in the form of a cash refund to customers for purchasing the product. This is usually called a **'cash-back'** offer.

Investigation Multiple discounts

Work in groups of two or three for this investigation. Conclusions can be shared with the whole class at the end of the investigation.

An oven is priced at \$1600. A customer can use two discounts:

- a cash discount of \$100 using a voucher
- a percentage discount of 15%.

1 Work out whether it is better to get the cash discount and *then* the percentage discount, or the percentage and *then* the cash.

2 Now make up some multiple discounts of your own for your group to work on. The object is to find what the best order for discounts is when:
- multiple cash discounts are available
- multiple percentage discounts are available
- mixed cash and percentage discounts are available.

3 Write down the conclusions of your group and try to explain them to the rest of the class.

Example 8

A store bought 5 mini stereo systems with a RRP of $850 each. The store first gets a trade discount of 40% and then an early payment discount of 5%.

a What is the total amount paid (with both discounts)?

b What is the overall percentage discount?

Solution

a Calculate the total RRP.	Total RRP $= 5 \times \$850 = \$4250$
Write the given information.	Trade discount $= 40\%$
Find % paid after trade discount.	Cost price $= 60\%$ of $4250
Write the given information.	Early payment discount is 5%
Find % paid after early discount.	After-discount price $= 95\%$ of cost price
Find final amount paid.	Amount paid $= 95\%$ of 60% of $4250
	$= 0.95 \times 0.6 \times \$4250$
Evaluate.	$= \$2422.50$
b Calculate discount.	Discount = RRP − price paid
Substitute known values.	$= \$4250 - \$2422.50$
	$= \$1827.50$
Calculate % discount.	% discount $= \dfrac{\text{discount}}{\text{RRP}} \times 100\%$
Substitute known values.	$= \dfrac{\$1827.50}{\$4250} \times 100\%$
Evaluate.	Overall % discount $= 43\%$
Alternative method	
Use percentage values.	Final % paid $= 95\%$ of 60% of 100%
Evaluate.	$= 0.95 \times 0.6 \times 100\% = 57\%$
Find overall % discount.	Overall % discount $= 43\%$

You can see from Example 8 that the overall discount is not just the sum of the discounts. *Multiple discounts must be worked out in steps as shown. We cannot just add them.*

Example 9

A car dealership gets a trade discount of 25% of the RRP of a new car (without on-road costs). The dealership adds $2000 for preparing the car for sale. The RRP of the car, without on-road costs and dealer charges, is $25 900. Registration and insurance cost a total of $1470.

a What does the dealership make on selling the car?

b What is the final price of the car 'on the road' when someone buys it?

Solution

a Write a formula for the dealer's take. Dealer's take = trade discount + charges

Substitute known values. = 25% of RRP + \$2000

= 0.25 × \$25 900 + \$2000

Evaluate. = \$6475 + \$2000

= \$8475

Write the result. The dealership makes \$8475 on selling the car.

b The final price paid by the customer is the sum of the RRP, dealer charges and the on-road costs of registration and insurance.

Calculate price paid by customer. Final price = \$25 900 + \$2000 + \$1470

Evaluate. = \$29 370

Write the result. The customer pays \$29 370 for the car 'on the road'.

Additional Exercise 10.2

Exercise 10.2 Discounting prices

1 A bookseller buys stock at a trade discount of 35%. What does she pay for books with these recommended retail prices?

a \$25.90 **b** \$36.40 **c** \$45 **d** \$9.95 **e** \$14.95

2 A new car distributor gets a trade discount of 20% of the RRP of a new car (without on-road costs). The distributor also adds \$1500 'delivery costs' for detailing the car. The RRP of the car, without delivery and on-road costs, is \$39 800, and registration and insurance cost \$1750.

a What does the dealer make on selling the car?

b What is the final price of the car when someone buys it?

3 A sweet manufacturer offers packs of sweets at a RRP of \$2 to organisations for use in fund-raising. The trade discount offered is 60%. One school buys 5000 packets and sells all of them.

a How much does the manufacturer get?

b How much does the school make?

Another school buys 7000 packets but can only sell 5000 at the full price. It sells off the remainder at \$1 a packet.

c How much does the manufacturer get from this school?

d How much does the school make?

4 A furniture store orders 5 dining tables from a manufacturer at \$840 each. Terms are 5/10, 2/30, n/60 and the invoice is dated 16 May.

a What is the total due if the invoice is paid on 20 May?

b What is the latest payment date to obtain a 5% discount?

c What is the latest payment date to obtain a 2% discount?

d How much would be due if the invoice was paid on 8 June?

e What is the latest possible payment date?

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5 A mattress manufacturer is invoiced on 17 December for 20 000 pocket springs at \$1.40 each. However, 400 are immediately returned because they have sharp edges. The terms on the invoice are 2/10, n/30.

a How much is due on the invoice?
b How much is due if payment is made immediately after the mattresses are checked?
c What is the last possible payment date?
d What is the net amount due on the last possible payment date?

6 A joiner runs an account with a local timber merchant. The joinery receives the following invoice for timber supplied.

WOOD WORKS	
DATE:	24 October 2008
INVOICE TO:	Joe's Joinery
Goods supplied	200 m of 19 × 45 hoop pine @ \$2.60/m 75 m of 31 × 63 Tasmanian oak @ \$5.60/m
TERMS: 4/20, n/60	

a What is the total amount payable?
b What discount is available?
c What is the latest date of payment to obtain the discount?
d What is the last possible payment date?

7 A beach hire operator ordered 5 new surfboards at a cost of \$850 each. She is invoiced on 28 November at terms of 5/10, 2/20, n/60.

a What is the net amount due?
b What is the smallest total she can pay (with the larger discount)?
c When must she pay to get the larger discount?
d By what date must she pay to get the smaller discount?
e What amount does she pay with the smaller discount?
f What is the last date for payment?

8 A large chain store has a '$12\frac{1}{2}$% off' day where everything in the shop is discounted by $12\frac{1}{2}$%. The garden section is discounting wheelbarrows by \$10 on top of the $12\frac{1}{2}$%.

a What are the final price and discount on a wheelbarrow marked at \$72?
b What is the overall percentage discount?

9 The RRP of an MP3 player is \$350. A trade discount of 45% is given to retailers, and on a particular delivery the terms of the invoice are 5/10, 2/20, n/30. What is the price paid by the retailer if the retailer pays:

a on delivery? **b** 15 days later? **c** on the last possible day?

10 A whitegoods wholesaler buys a freezer for \$800 and applies a markup of 30%, but allows discounts of 5% for regular customers and 5% for early payment.

a How much does a retailer who is not a regular customer pay for the freezer?
b How much does a retailer who is a regular customer pay if the payment is early?

Retailers of whitegoods usually use a markup of 40%.

c What price would the first retailer charge for the freezer?
d What price would the retailer who is a regular customer of the wholesaler charge?
e If the second retailer offers a discount of 10% at a sale, what is the sale price?

Investigation From manufacturer to customer

The markups applied by a manufacturer, distributor, wholesaler and retailer add to the final price paid by a customer, and so do taxes. A discount lowers the price. The change in the price from manufacturer to retailer can be worked out as a series of small problems. Work in groups to answer the following problem.

> A bed costs \$300 to make and the manufacturer applies a markup of 30% (inclusive of GST). The wholesale markup is 25% (inclusive of GST) and the retail markup is 50%, but a discount of 15% is given. Finally, 10% GST is added after the discount. What is the final price paid by the customer?

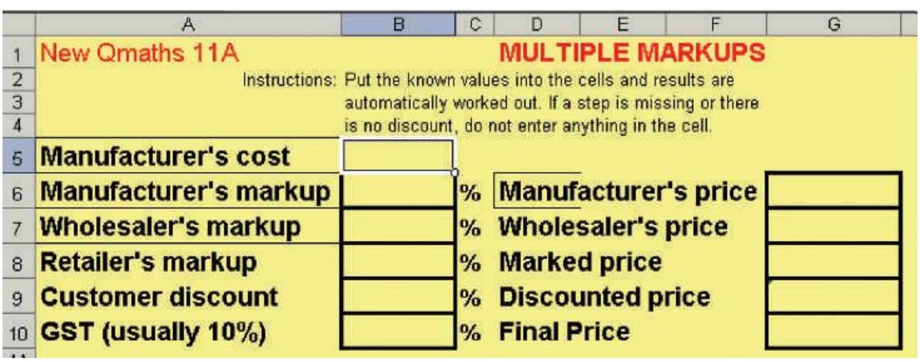
The calculations involved are time-consuming and repetitive and can be done by a computer. It is possible to set up a spreadsheet to perform these calculations for you.

- Set up a spreadsheet that can be used to calculate the final price paid by the customer for an item that has multiple markups, discount and GST applied, as in the case of the bed described above.
- Try it out for the above case and then try other cases.

Technology

Spreadsheet

You can use the Multiple Markups spreadsheet on the CD-ROM to check your calculations for multiple marksups.



10.3 Profit and loss

Businesses always try to make a profit. Losses must be minimised. All businesses need money to pay for their running costs. This money is called the **capital** of the business. It is invested by the owners, shareholders or partners in the business and used to rent or buy premises, pay wages, obtain materials and so on.

The amount of money made by different businesses is compared by people with money to invest. Obviously, they are keen to get the best return on their investment. Businesses are often compared by working out the profit as a percentage of the capital. Banks use percentage profit to help decide whether to lend money to a company.

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The idea of percentage profit or loss is also used with individual items. However, in a full calculation, the costs of selling the item must also be taken into account. For example, it costs money just to have an item on the shelf in a shop.

It is usual to use **percentage profit** or **percentage loss** to compare returns for different items.

!

Profit = selling price – cost price (when selling price > cost price)

Loss = cost price – selling price (when selling price < cost price)

Unless otherwise stated, percentage profit and percentage loss are calculated using the cost price as the denominator.

$$\textbf{Percentage profit} = \frac{\text{profit}}{\text{cost price}} \times 100\%$$

$$\textbf{Percentage loss} = \frac{\text{loss}}{\text{cost price}} \times 100\%$$

A loss is sometimes written as a negative profit.

Example 10

An electrical store sells fridges for \$850. It buys them for \$600. What percentage profit does the store make?

Solution

Write a rule for profit.	Profit = selling price – cost price
Substitute known values.	= \$850 – \$600
Evaluate.	= \$250
Change to a percentage of cost price.	$\% \text{ profit} = \frac{\$250}{\$600} \times 100\%$
Evaluate.	$\approx 41.7\%$

Example 11

A car dealer values a trade-in at \$8000 to help clinch the sale of a new car, but can only get \$6900 for it from a wholesale dealer. What percentage loss does the car dealer make on the trade-in?

Solution

Write a rule for loss.	Loss = cost price – selling price
Substitute known values.	= \$8000 – \$6900
Evaluate.	= \$1100
Change to a percentage of cost price.	$\% \text{ loss} = \frac{\$1100}{\$8000} \times 100\%$
Evaluate.	$= 13.75\%$

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The total of all sales in a period (usually a year) is called the **turnover**. **Net profit** (gross profit less 'overheads' such as wages and lease payments) is sometimes expressed as a percentage of turnover.

Example 12

A local convenience store has a turnover of \$315 000. The owner's gross profit is \$87 000, but overheads, including wages, power, accounting fees, advertising, rent and other charges, come to \$54 600. What is the net profit as a percentage of turnover?

Solution

Write a rule for net profit.	Net profit = gross profit − overheads
Substitute known values.	= \$87 000 − \$54 600
Evaluate.	= \$32 400
Express as a % of turnover.	% profit = $\frac{\$32\,400}{\$315\,000} \times 100\%$
Evaluate.	≈ 10.3%
Write the result in a sentence.	Net profit (as a % of turnover) is about 10.3%.

Example 13

A used car dealer, working on a profit margin of 35%, bought a car for \$10 300. What price did the dealer sell the car for?

Solution

Write the rule for selling price.	Selling price = cost price + profit
Use the given information.	= cost price + 35% of cost price
Simplify.	= 135% of cost price
Use the given information.	= 1.35 × \$10 300
Evaluate.	= \$13 905

Example 14

An agricultural supplier works on a profit margin of 28%. Drums of pesticide are sold for \$544 each. What was the cost price?

Solution

Write the rule for selling price.	Selling price = cost price + profit
Use the given information.	= cost price + 28% of cost price
Simplify.	= 128% of cost price
Use the given information.	\$544 = 1.28 × cost price
Rearrange and evaluate.	Cost price = \$544 ÷ 1.28
	= \$425
Write the result.	The cost price was \$425.

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Example 15

Mead Murphy, an antique dealer, made a profit of $68 when he sold an antique bisque doll. His percentage profit was 17%. What did he pay for the doll? What was the sale price?

Solution

Write the profit as a % of cost price.	Profit = 17% of cost price
Use the given information.	$\$68 = 0.17 \times$ cost price
Rearrange and evaluate.	Cost price $= \$68 \div 0.17 = \$400$
Write the rule for selling price.	Selling price = cost price + profit
Substitute known values and evaluate.	$= \$400 + \$68 = \$468$
Write the result.	Mead paid $400 for the doll and sold it for $468.

Additional Exercise 10.3

Exercise 10.3 Profit and loss

1 Calculate the percentage profit for each of the following.

	Item	Cost price	Sale price
a	Kitchen blender	$48	$69
b	Bath scales	$54	$86
c	51 cm TV	$265	$524
d	Pair of jeans	$145	$230
e	3-burner barbecue	$270	$415

2 A second-hand dealer bought the following items: fishing reel ($15), video camera ($220), jewellery box ($35), filing cabinet ($80) and printer ($340). The fishing reel sold the next day for $20, the video camera sold for $290 and the filing cabinet sold for $130. The jewellery box and printer were in the window for several weeks, and eventually sold for $25 and $255 respectively. Work out the percentage profit or loss on each item.

3 A snack bar in a shopping centre paid $15 000 a month rent over the previous year. The snack-bar's payroll, insurance and other wages costs were $95 000 a year. Its power, light and cleaning costs were $28 000 a year. Its turnover for the year was $910 000, and its suppliers charged a total of $540 000 for the year.

a What was the percentage profit on the snacks, without taking costs into account?

b What was the net profit?

c What was the net profit as a percentage of turnover?

4 A sales representative bought a new car for $32 500 and after using it for 2 years sold it for $18 600. What was the percentage loss?

5 A newsagent makes a profit of 30% on stationery. What was the cost price of each of these?

a Graph pad sold for $1.95 **b** A4 lined paper sold for $2.60

c Exercise book sold for $3.25

6 A motor spares firm uses a profit margin of 120%. For what price would it sell the following?

a Carburettor kit that cost $85 **b** Petrol pump that cost $220

c Universal joints that cost $58 each **d** Head gasket that cost $145

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7 A coin dealer bought a rare penny and sold it at a profit of 20%, making $64 profit. What were the buying price and the selling price?

8 Hugh bought a used car for $4800 and spent several weekends detailing the car and tuning it up. He spent about $250 on materials. Eventually he sold the car and made a profit of 14%. How much did he sell it for?

Modelling and problem solving

9 Peter made 15% profit when he sold some pine bookcases he had made, taking all costs into account. If he considered only the cost of materials, it would have been a 55% profit. The sale price was $372 each. How much did the materials for a bookcase cost?

10 Maxine bought an old washstand at a deceased estate auction. She cleaned it up and sold it to an antiques dealer for a profit of 15%. The dealer sold the washstand to a customer for $2159, including a $20 delivery charge. If the dealer made 55% profit, how much did Maxine pay for the washstand at auction?

10.4 Constructing a budget

It costs you and your parents money for you to come to school. The cost of books, pens, paper, transport, excursions and other school expenses must be met. It also costs money for your food, accommodation and leisure activities. Some expenses are paid once a year, but some are paid every day. You may get money from Youth Allowance, from part-time work or from your parents.

When you start work and live on your own, you will probably have more money but also more expenses to deal with. You will have to decide how you are going to use your money.

A **budget** is a financial plan for a period of time or a particular purpose. Many people work out a yearly budget for their personal finances. You may prepare a budget for a holiday or for running a car. In accounting it is usual to show a budget as a **balance sheet**, listing **income** in a column on the left and **expenses** on the right. A **balanced budget** has equal totals of income and expenses. A business normally prepares budgets for different parts of the operation.

Example 16

The costs for a child's birthday party are as follows:

Birthday cake:	$35.00
Lollies and chips:	$4.20 per child
Balloons and streamers:	$45.00
Soft drink:	$3.30 per child
Pizzas:	5 @ $8.90
Hot snacks:	$2.75 per child
Ice-cream:	$1.60 per child
Computer game hire:	$38.00

a Prepare a budget for the party if 9 children are invited and there are 2 other children in the family besides the guest of honour.

b Calculate the cost per child for the party.

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Solution

a Since this is only an expenditure budget, a single column is used. There are 12 children at the party.

Item	Cost
Birthday cake	$35.00
Lollies and chips	$12 \times 4.20 = \$50.40$
Balloons and streamers	$45.00
Soft drink	$12 \times 3.30 = \$39.60$
Pizzas	$5 \times 8.90 = \$44.50$
Hot snacks	$12 \times 2.75 = \$33.00$
Ice-cream	$12 \times 1.60 = \$19.20$
Computer game hire	$38.00
Total	**$304.70**

State the result. The party will cost $304.70.

b Divide to find cost per child. Cost per child $= \$304.70 \div 12 \approx \$25.39$

Round and state the result. The party will cost about $25 per child.

As seen in Example 16, the amount spent or obtained for many types of goods depends on the number and the **unit price**. This is the cost or amount received for one item. In many cases it is useful to include columns of unit cost and number of items in a budget.

Example 17

A cake stall run by a P. & C. Association is stocked with cakes donated by parents at the school. The coordinator of the stall decides to price the cakes as follows:

large sponges $8.50	large fruit cakes $10.50
medium sponges $5.50	medium fruit cakes $6.00
fairy cakes $1.00	slices 80c

The stall has 20 large sponges, 8 large fruit cakes, 35 medium sponges, 15 medium fruit cakes, 80 fairy cakes and 120 slices. Work out a budget to predict the total takings if all items sell.

Solution

Item	Unit price	Number	Amount
Large sponge	$8.50	20	$170.00
Large fruit cake	$10.50	8	$84.00
Medium sponge	$5.50	35	$192.50
Medium fruit cake	$6.00	15	$90.00
Fairy cake	$1.00	80	$80.00
Slice	$0.80	120	$96.00
Total			**$712.50**

State the result. The cake stall will take $712.50 if all items are sold.

Most budgets include both income and expenses. A **personal budget** of living expenses usually has only one or two items on the income side and many items on the expense side.

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When you first start working out a personal budget, try to do it on a weekly or fortnightly basis, depending on how often you are paid.

A budget can be used to help work out how much money must be put aside for expenses that come less frequently, such as power and phone bills, clothes and insurance. Known expenses should be converted to a weekly or fortnightly basis.

When preparing a budget, it is usual to count 4 weeks in a month and 12 weeks in a quarter. When converting a yearly figure to weekly, divide by 50. This will be accurate enough for budgeting and provide a buffer in case prices rise during the year.

Example 18

Karen earns \$610 a week after tax. She rents a furnished unit with a friend and her share of the rent is for \$350 a fortnight. She spends about \$210 a week on food, household items and toiletries. She catches the train to work each weekday and pays \$4.90 for a return ticket. The power (electricity and gas) bills total about \$280 a quarter and she pays half of this. Karen estimates that she spends \$310 a month on clothes.

Draw up a weekly budget and work out how much she can save in a year if she sticks with this budget.

Solution

Convert each item to a weekly basis.

Weekly income	
Wages	\$610
Total	**\$610**

Weekly expenses	
Rent (\$350 ÷ 2)	\$175.00
Food, etc.	\$210.00
Transport (\$4.90 × 5)	\$24.50
Power (\$280 ÷ 2 ÷ 12)	\$11.67
Clothes (\$310 ÷ 4)	\$77.50
Total	**\$498.67**

Calculate approximate weekly savings.

$$\begin{aligned}\text{Weekly savings} &\approx \$610 - \$499\\ &= \$111\end{aligned}$$

Calculate yearly savings.

$$\begin{aligned}\text{Yearly savings} &\approx \$111 \times 50\\ &= \$5550\end{aligned}$$

Karen in Example 18 won't save as much as her budget predicts because she has missed out some expenses in working out her budget. She has allowed nothing for entertainment and leisure activities. Very few people would stay home all weekend and every night of the week, as she seems to be planning to do.

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Investigation Personal budget

Can you afford to move away from home when you start work? This investigation will help you to use a budget to help answer this question. You should work in groups of three or four for this investigation.

- Use the newspaper to find how much it would cost to rent a furnished flat, home unit or house in your area.
- Work together to write down lists of the items on which you or other people in your group spend money. Don’t forget things like haircuts, soap, deodorant, cosmetics, train and bus fares, Mars Bars, shoes, sports fees, DVD hire and movies.
- Use your lists to estimate what you spend on clothing, entertainment, personal hygiene and grooming, transport and miscellaneous expenses.
- Estimate the cost of the food you eat in a week. You need to include breakfast cereal, fruit, vegetables, butter and cooking ingredients as well as main meals. Don’t forget takeaway food.
- Now try to estimate the cost of floor-cleaning materials, washing-up liquid, clothes detergent, toilet paper and other such household expenses.
- Estimate the cost of power bills.
- You should take out insurance to cover the value of your belongings. Estimate the total value of your possessions and find out how much it will cost to insure them.
- Find out how much it costs to have a telephone (line rental and equipment charges) and estimate your call costs. Compare this with mobile phone plans that suit your needs.

When you don’t know the cost of a particular item, work as a group to estimate the cost. If you are working overnight on the investigation, divide up the items that must be checked so that everyone in the group does a few.

Spreadsheet

A spreadsheet named Personal Budget Planner is included on the CD-ROM. Open the spreadsheet and enter your income and expenses in the relevant orange sections. Before entering any data, note the following points:

- Decide whether you are going to be living by yourself or with friends. If you will be sharing, adjust expenses that can be shared, such as rent, so that you only enter the amount you will be required to pay.
- The spreadsheet has been designed to cover a wide range of circumstances, so it’s alright to leave rows blank if they don’t apply to you.
- Convert your income and expenses to monthly amounts before entering them. For many expenses it’s easiest to estimate a yearly amount and then divide by 12 to come up with the monthly expense.

When you have entered your data, click on ‘Update results’.

Investigate the types of jobs and the number of hours a week that you would need to work in order to cover your expenses.

	A	B	C	D	E	F
1	New Qmaths 11A		PERSONAL BUDGET PLANNER			
2		Update results				
3			Jan	Feb	Mar	Apr
4	A	INCOME				
5		Salary, wages, etc				
6		Investment income (interest, dividends)				
7		Other income				
8		TOTAL MONTHLY INCOME	$0.00	$0.00	$0.00	$0.00

Exercise 10.4 Constructing a budget

Additional Exercise 10.4

Modelling and problem solving

Questions **1** to **4** are parts of the same story.

1 A club is having a food stall at a local market to raise funds. The treasurer gets the following prices from a wholesaler:

Wieners:	\$60/100
Bread rolls:	\$55/100
Tomato sauce:	\$10.80 for a 4 L bottle

The wieners will be served in a roll with tomato sauce. The treasurer thinks that most people will have sauce and that 20 mL will be used for each hot dog. Work out the cost of providing 1000 hot dogs.

2 The president of the club also wants to sell hamburgers with onion, tomato and lettuce. The treasurer gets the costs for the hamburger ingredients listed below:

Hamburger patties:	\$95/100
Bread rolls:	\$55/100
Tomato sauce:	\$10.80 for a 4 L bottle
Tomatoes:	\$6.80/kg
Onions:	\$4.60/kg
Lettuce:	\$4.20 each

She tells the president that each hamburger will need 20 mL of tomato sauce. One lettuce and a kilogram of tomatoes will be enough for 20 hamburgers, but a kilogram of onions is enough for only 10 hamburgers.

Work out the cost of providing 1000 hamburgers with 'the lot'.

3 The committee decide to sell hot dogs, hamburgers and cans of soft drink, which are bought for 80 cents each. Hot dogs will be sold for \$3.50 each, hamburgers for \$4.50 each and cans of soft drink for \$2.00 each. At the market, 460 hot dogs, 320 hamburgers and 510 cans of soft drink are sold. What income does the club get from the food stall?

4 When the committee ordered the food, they got enough for 400 hamburgers and 600 hot dogs. At the end of the day they sell off the leftover bread rolls for 20 cents each, the wieners for 20 cents and the hamburger patties for 40 cents, but can't sell the other perishable leftovers. Cans of drink can be returned for a full refund. Work out a budget statement for the food stall and thus how much the club makes from the day.

5 A plant nursery supplier grows vegetable seedlings for retail nurseries. For punnets of tomato seedlings the supplier plants 20 seeds in the punnet and then, if too many come up, pinches some out. Each plastic punnet costs 12 cents and the seeds are about \$1.20 for 100. A mixture of sand and loam is used for the punnets, and this costs \$25 for enough to fill 1000 punnets. The supplier estimates that one person takes about an hour to prepare 150 punnets of seedlings, and that watering and weeding take another 20 minutes before the punnets are ready for sale.

a Work out the cost of the punnets, sand/loam mixture and seeds for 3000 punnets of tomato seedlings

b Work out the work time taken to prepare 3000 punnets.

c Work out the cost of the labour for 3000 punnets at \$30 an hour.

d What is the total cost of 3000 punnets?

e What is the cost of preparing one punnet?

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6 A retail nursery attached to a supermarket sells tomato seedlings for $3 a punnet, lettuce seedlings for $2.50 a punnet and sweet corn seedlings for $3.50 a punnet. In one week the nursery sells 508 tomato, 356 lettuce and 187 sweet corn punnets.

a How much does the nursery get for the punnets of seedlings?

The punnets cost the nursery $1.20 each for tomato, $0.80 for lettuce and $1.35 for sweet corn. It bought 600 tomato, 400 lettuce and 200 sweet corn punnets.

b How much did the seedlings cost the nursery?

c At the end of the week the nursery had to throw out the unsold seedlings. How much profit did it make?

7 Macka earns $1072 a fortnight after tax. He shares a house with three other people. The house costs $350 a week and the shared power bill is usually about $420 for a quarter. Macka buys lots of takeaways and spends about $220 a week on food and other household items. He is paying off his car at $160 a month and spends about $75 a fortnight to run it. Macka spends about $1000 a year on clothes. He finds it impossible to save any money, but does go to the football on Saturday and usually goes out on Saturday night. Use the assumptions on page 305 to work out Macka's budget and what he could save in a year if he gave up his Saturday entertainment.

8 Aya is sharing a small furnished flat in a big house with her best friend. She earns $510 a week after tax and the flat costs them $410 a fortnight, but that includes power. Aya hardly ever eats out, so she spends only $95 a week on food and household necessities. She walks to the train station and pays $24 for a weekly train ticket to get to and from work. Aya spends about $1900 a year on clothes and is trying to save up for an overseas trip. Work out her budget and how much she could save in a year.

10.5 Budgeting for a car or holiday

It costs money to own a car even before you take it out of the garage. The **fixed costs** of running a car include registration, insurance and minimum maintenance. If you borrow money to buy a car, the payments also have to be made. When using a car you must pay for **running costs** such as petrol, servicing, replacement of tyres and other parts, and general maintenance.

Registration and third-party insurance are compulsory. In Queensland, third-party insurance is included in registration. Insurance on the car itself is optional. Insurance rates can be very high for under 25-year-olds.

A car must be serviced even if it is not driven. Older cars should be serviced at least every 3 months, but newer cars may go for 12 months between services. The **service interval** of a car is normally stated in terms of the maximum distance and maximum time between services. A service interval of 5000 km/3 months means that the car must be serviced every 5000 km or 3 months, whichever comes sooner.

Example 19

Suzy owns a car that she is paying off. The loan costs \$234.50 a month. Registration costs \$680 a year and car insurance is \$840 a year. She drives about 20 000 km a year and the service interval is 10 000 km/6 months. Services cost about \$475 each. The car uses petrol at the rate of 12 L/100 km and petrol costs \$1.40 a litre. Suzy needs to replace the tyres every 2 years at a cost of \$650 a set. From experience, Suzy knows that she needs to allow an extra \$600 a year for repairs. Draw up a budget for Suzy's car and work out the weekly cost of owning and running (using) the car.

Solution

It is easiest to work out the budget for the whole year and divide by 50 to estimate the weekly cost. Construct a table of yearly costs associated with owning and running the car.

Item	Associated cost	Yearly cost
Loan repayments	\$234.50/month	$\$234.50 \times 12 = \$2814$
Registration	\$680/year	\$680
Insurance	\$840/year	\$840
Services	\$475/10 000 km	$\$475 \times 2 = \$950$
Petrol	\$1.40/L	$\$1.40 \times 12 \times 20\,000 \div 100 = \$3360$
Tyres	\$650/2 years	$\$650 \div 2 = \$325$
Repairs	\$600/year	\$600
Total		**\$9569**

Calculate the weekly cost.

Weekly cost $= \$9569 \div 50$
$= \$191.38$

Round and state the result.

The car costs Suzy about \$191 a week.

The fixed and running costs associated with owning a car are significant, but there is another cost that is not as apparent: **depreciation**. For most cars, the value decreases as a result of the car being used. The amount that it depreciates depends on many things, including the initial age of the vehicle, the amount of 'wear and tear', and the make and model. It is not unusual for a car purchased new for \$30 000 to be worth about \$12 000 after 5 years. This means that the car effectively costs about \$3600 a year to own in addition to the fixed and running costs.

Motoring organisations like the Royal Automobile Club of Queensland (RACQ) can be contacted by members to obtain information about the various costs associated with owning and running a car. The RACQ has a website at *www.racq.com.au*.

Investigation The full cost of using a car

Work in groups to investigate the real cost of owning and running a second-hand car. You will need to allocate different tasks to the people in the group.

- First, select the make and model of a car that you would like to own. (Be reasonable!)
  You can obtain car prices from a website such as:
  – *http://www.redbookasiapacific.com/au/*
  – *http://carsguide.news.com.au/*
  – *http://www.carprices.com/*
  You can also obtain car prices from a newspaper (Saturday is usually best).

Investigation continued

- Find out how much the car is likely to depreciate each year.
- Contact an insurer to find out how much car insurance would cost for an 18-year-old.
- Check with a local service station to find the cost of services.
- Ask a service station operator how much you are likely to spend on repairs.
- Find out the typical fuel consumption for the car you have chosen and estimate the number of kilometres you are likely to travel in a year.
- Contact Queensland Transport to find how much registration (including third-party insurance) will cost a year.
- Find out what interest rate may be charged for a personal loan to buy the car.
- Ask the RACQ about the costs and benefits of being a member. Consider whether you would join.
- Collate the information you have gathered and use it to find the real cost of owning and running a second-hand car for an 18-year-old.
- Compare this with the likely earnings of someone of this age.
- What proportion of earnings would be spent on the car you selected?

When you go on a holiday or undertake some other kind of trip, extra expenses are involved. It is a good idea to work these out in advance and to get advice on costs. The budget for a group excursion must be worked out carefully so that everyone knows what costs are involved. When planning a trip, the expenses can usually be classified as **transport**, **accommodation**, **admissions**, **meals** and **pocket money** for souvenirs and snacks.

Example 20

Three friends plan a cycling tour of the Brisbane Valley. They plan to cycle from Brisbane through Ipswich, Fernvale, Esk, Toogoolawah, Harlin, Kilcoy, Woodford, Caboolture and back to Brisbane. They have worked out that they will stay overnight at Fernvale, Esk, Kilcoy and Caboolture and that bed and breakfast at hotels or motels will cost about $80 each for each night. They plan to carry only clothing, drinks and spare parts for their bikes. They will buy takeaways for lunch and have a good meal at night. They estimate that breakfast will cost about $8 each, takeaway snacks and drinks at lunchtime about $10 each and evening meals about $35 each. They will be away for 5 full days but will eat at home for breakfast on the first day and dinner on the last day. They buy maps and repair materials for their bikes, totalling $96. Work out a budget for the trip.

Solution

Construct a table for the costs for each person.

Item	Cost
Overnight accommodation	$\$80 \times 4 = \$320$
Breakfast	$\$8 \times 4 = \$32$
Lunch	$\$10 \times 5 = \$50$
Evening meal	$\$35 \times 4 = \$140$
Maps and repair materials	$\$96 \div 4 = \$24$
Total	**$566**

State the result.

It will cost each person about $566 for the trip.

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Additional Exercise 10.5

Exercise 10.5 Budgeting for a car or holiday

Modelling and problem solving

1 Mandy drives about 400 km a fortnight. The service interval is 5000 km/3 months and each service costs $320. She needs new tyres every 4 years at $620 a set and repairs cost about $900 a year. Registration costs $580 a year and car insurance $640 a year. The car uses 10.5 L of petrol/100 km, and petrol costs on average $1.38/L. Work out a budget for Mandy's car expenses and so find the weekly cost of running the car.

2 Trevor is a travelling salesman who drives about 120 000 km a year. He has a company car but has to pay for petrol, service and repairs. The service interval is 10 000 km/6 months and each service costs $335. He needs two sets of tyres a year at $675 a set and repairs cost about $1500 a year. The car uses 10.5 L of petrol/100 km, and petrol costs on average $1.36/L. Work out a budget for Trevor's car expenses and so find the weekly cost of running the car.

3 Corey has a $2000 deposit to buy a car. Payments will be $260 a month and she expects to drive about 600 km a week. The car uses 14 L of petrol/100 km and services cost $285 each. Registration costs $600 and car insurance $2010 (because she had a major accident in the previous year). She gets 40 000 km from a set of tyres, which cost $550, and the average cost of petrol is $1.28/L. The service interval is 5000 km or 3 months. Major repairs will be about $1125 a year. Work out a yearly budget and thus weekly cost of owning and running the car.

4 Allan bought a second-hand car with no deposit and easy terms of $105 a week over 4 years. He immediately bought a set of tyres for $556, and replaced them every 35 000 km. He drove 65 000 km in the first 2 years, and the same average amount in the third year. Registration cost $660 a year and car insurance $1950. He had no major repairs. Services cost about $68 each for oil and parts (he did them himself). The service interval was 5000 km/3 months. Petrol cost on average $1.33/L, and the car used 15 L/100 km. After 3 years he sold the car for $4500 and paid out the loan. He got a discount for early repayment and had to pay only $3500 for the final year's payments.

a What was the annual cost of owning and running the car for 3 years?

b What was the weekly cost?

c What did the car cost Allan altogether?

5 A couple plan to fly to Townsville from Brisbane for a 7-day holiday on Magnetic Island. The return airfare is $449 per person, and airport transfers to and from Townsville and Magnetic Island are $165. The tariff for their room is $185 a night, which includes breakfast. They think that lunch and dinner will be a total of about $65 a day each. They plan to spend about $25 each a day on other purchases. Work out a budget for the couple's trip.

6 A family of two adults and three children (aged 6, 9 and 13) are planning a day trip to a Gold Coast theme park. They will travel by car and it costs about $64 to fill up with petrol. They will buy snacks and lunch at the theme park. Admission is $65 for adults, $45 for 10- to 15-year-olds and $25 for children under 10. Ice-creams cost $4.50 each, drinks $4, and lunch $28 each for adults and $17 each for children. If they each have two ice-creams and three drinks, work out a budget for the family outing.

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7 A 5-day Year 8 camp is planned. It is anticipated that 50 girls and 43 boys will attend. The camp is to be held at a campsite that costs \$100 per day for each cabin. The five teachers will be accommodated in one cabin, and the students will use separate cabins for boys and girls. There are six bunks in each cabin. The catering cost for the week is \$82 per person. Buses seating 55 people will be hired for the trip, at a cost of \$482 each, to get there and back.

- **a** Draw up a budget to work out the total cost of the camp.
- **b** If the total cost is to be paid by the students attending the camp, what is the exact cost per student?
- **c** When working out costs for trips of this nature, it is usual to allow 5% extra for absentees and other costs. The amount for each student is then rounded up to the next multiple of \$5. Under these conditions, what will students be asked to pay?

8 The theatre students in a school are going on an excursion to see a play. The entry cost is \$10.50 for each student, and buses cost \$275 for a 25-seater and \$482 for a 45-seater. There are 110 students going on the excursion. Draw up budgets showing the options for buses and work out the cost for each student for the excursion (to the nearest 5 cents).

Investigation An overseas trip

Work in groups of three or four for this investigation.

- Decide on an overseas destination for a group of four people. Let your mind wander—Paris, Rome, New York, Istanbul, Kathmandu …
- Visit the Australian government travel advisory service at *www.smartraveller.gov.au* for travel tips and other travel assistance.
- Once you have selected your destination, work out how long you wish to be away from home, and how you are going to travel—both to your destination and when you arrive there. It's probably best to visit a travel agent or to do some surfing on the web.
- Draw up an itinerary for the entire time you will be away. It doesn't need to be too detailed and you can look in some travel brochures for ideas.
- One person should investigate the costs of passports, visas and travel insurance, and other costs to do with leaving Australia and entering the country of your choice, such as vaccinations and departure tax.
- Someone else should gather information about the cost of travelling to your destination and with travelling within the country you intend visiting.
- Another person should investigate the likely amounts the group will spend on:
  - Accommodation
  - Meals
  - Visiting tourist attractions
  - Entertainment
  - Incidental purchases
  - Buying souvenirs
  - Additional clothing
  - Laundry expenses.
- Once you have estimated all costs, prepare a budget and work out the cost for each person on the trip.
- Compare your findings with those of other groups.

See whether you can find a similar package prepared by a travel agent and compare the costs of the trip you have devised with those of the travel agent.

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10.6 Buying and selling foreign money

If you travel overseas you must change your Australian money to the money of other countries. The money of a country is called its **currency**. The price of a currency is called its **exchange rate**. Before 1983, the government fixed the exchange rate of the Australian dollar, but now it 'floats' according to supply and demand.

Banks and other financial institutions do not change your money free of charge. They charge a commission, which is usually included in the exchange rate. They give different rates, depending on whether you are *buying* or *selling* Australian dollars. Different banks charge different commissions, so the buying and selling rates will be different even though the exchange rate is the same. The actual exchange rate is sometimes called the **midrate** to distinguish it from the buying and selling rates. It is close to the average of the buying and selling rates.

The exchange rate of the Australian dollar with important currencies such as the Japanese yen, American dollar and the European euro are published almost every day by newspapers.

The International Organisation for Standardisation (ISO) has assigned each currency a three-letter code. The table below shows the currency unit and the ISO code that represents the currency for various countries and regions. Also shown are examples of the buying and selling rates in Australian dollars (AUD) on a certain day.

Country/Region	Currency unit	Code	Buying	Selling
Canada	Dollar ($)	CAD	0.9401	0.8688
China	Yuan (¥)	CNY	6.9285	5.9098
Denmark	Krone	DKK	5.1440	4.4568
European Union	Euro (€)	EUR	0.6594	0.5992
Fiji	Dollar ($)	FJD	1.4373	1.2642
Hong Kong	Dollar ($)	HKD	7.0154	6.3785
India	Rupee	INR	35.566	32.104
Japan	Yen (¥)	JPY	110.46	99.99
Malaysia	Ringgit	MYR	3.0929	2.5918
New Zealand	Dollar ($)	NZD	1.1798	1.0818
Norway	Kroner	NOK	5.5059	4.7135
Papua New Guinea	Kina	GPK	2.7899	2.2439
Philippines	Peso	PHP	41.751	35.837
Singapore	Dollar ($)	SGD	1.3872	1.2455
South Africa	Rand	ZAR	6.5485	5.7154
Sri Lanka	Rupee	LKR	97.95	85.45
Sweden	Krona	SEK	6.3466	5.5643
Switzerland	Franc	CHF	1.1024	0.9877
Thailand	Baht	THB	31.79	25.33
Great Britain (UK)	Pound (£)	GBP	0.4404	0.4112
USA	Dollar ($)	USD	0.8649	0.8261

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Example 21

On a particular day, the exchange rate (midrate) for the Australian dollar (AUD) and American dollar (USD) is 1 AUD = 0.8455 USD. How much is:

a 5000 AUD worth in USD?

b 5000 USD worth in AUD?

Solution

a The exchange rate is stated in AUD, so we can just multiply by the exchange rate.

Write the AUD to USD exchange rate.	$1\text{ AUD} = 0.8455\text{ USD}$
Multiply by 5000.	$5000\text{ AUD} = 0.8455\text{ USD} \times 5000$
Evaluate.	$= 4227.50\text{ USD}$

b To change from USD to AUD, we can reverse the method (divide) or work out the reverse exchange.

Write the exchange rate.	$1\text{ AUD} = 0.8455\text{ USD}$
Multiply by 5000.	$5000\text{ AUD} = 0.8455\text{ USD} \times 5000$
Divide by 0.8455.	$\dfrac{5000\text{ AUD}}{0.8455} = \dfrac{0.8455\text{ USD} \times 5000}{0.8455}$
Evaluate and round off.	$5914\text{ AUD} \approx 5000\text{ USD}$
State the result.	5000 USD is worth about 5914 AUD.

International currency dealers buy and sell currencies on a day-to-day basis, and this determines the price of currencies in the short term. Over longer periods, international competitiveness, interest rates, inflation and a range of other economic factors determine the exchange rate.

When you change from Australian currency to another currency, you are **selling** Australian dollars, so you use the **selling rate** to work out how much of the other currency you will get.

When you change another currency into Australian dollars, you are **buying** Australian dollars, so you use the **buying rate** to work out how many Australian dollars you will get.

Example 22

Use the exchange rates table on page 313 to calculate how much you would get for:

a 5000 AUD in USD

b 5000 USD in AUD

c 2400 AUD in JPY

d 630GBP in AUD.

Solution

a You are selling Australian dollars.

State the selling rate.	$1\text{ AUD} = 0.8261\text{ USD}$
Multiply by 5000.	$5000\text{ AUD} = 0.8261\text{ USD} \times 5000$
Evaluate.	$= 4130.50\text{ USD}$

b You are buying Australian dollars.

State the exchange rate.	$1\text{ AUD} = 0.8649\text{ USD}$
Multiply by 5000.	$5000\text{ AUD} = 0.8649\text{ USD} \times 5000$
Divide by 0.9401.	$\dfrac{5000\text{ AUD}}{0.8649} = \dfrac{0.8649\text{ USD} \times 5000}{0.8649}$
Evaluate and round off.	$5781.02\text{ AUD} \approx 5000\text{ USD}$
Reverse the equation.	$5000\text{ USD} \approx 5781.02\text{ AUD}$

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Alternative method

State the buying rate.	1 AUD = 0.8649 USD
Divide by 0.8649 to find the rate for 1 USD.	1 AUD ÷ 0.8649 = 1 USD
Evaluate.	1.1562... AUD = 1 USD
Calculate for 5000 USD.	1.1562… AUD × 5000 = 5000 USD
Evaluate and round off.	5781.02 AUD ≈ 5000 USD
Reverse the equation.	5000 USD ≈ 5781.02 AUD

c You are selling Australian dollars.

State the selling rate.	1 AUD = 99.99 JPY
Multiply by 2400.	5000 AUD = 99.99 JPY × 2400
Evaluate.	= 239 976 JPY

d You are buying Australian dollars.

State the buying rate.	0.4404 GBP = 1 AUD
Multiply by 630.	0.4404 GBP × 630 = 630 AUD
Divide by 0.4404.	630 GBP = 630 AUD ÷ 0.4404
Evaluate and round off.	≈ 1430.52 AUD

Exercise 10.6 Buying and selling foreign money

Additional Exercise 10.6

Refer to the exchange rates in the table on page 313 for questions in this exercise.

1 Assuming that the midrate is the average of the buying and selling rates, calculate the midrate (exchange rate) for each currency in the table on page 313.

2 Using the relevant midrate calculated in question **1**, calculate how much:

a 3000 AUD is worth in NZD
b 1670 AUD is worth in GBP
c 580 AUD is worth in JPY
d 4900 AUD is worth in HKD
e 4000 EUR is worth in AUD
f 2300 USD is worth in AUD
g 610 NZD is worth in AUD
h 50 000 JPY is worth in AUD.

3 Use the relevant buying or selling rate shown in the table to calculate how much you would actually receive if you exchanged each of the amounts shown in question **2**.

4 How much would 500 USD be worth in Japanese yen?

5 If Angelique was changing 400 AUD to Fiji dollars, how much would she get?

6 When Yoshi came back from Japan he had 400 000 JPY left. How much would he get for it in AUD?

7 When Hannah was travelling overseas, she wanted to get some travellers' cheques in US dollars, some in euros and some in Thai baht. If she split 6000 AUD evenly between the currencies, how much of each did she get?

8 Alin was on exchange to Australia, but was paid in USD through electronic funds transfer. His salary was 1200 USD a fortnight after tax. How much in AUD did he get when he drew this out at an automatic teller machine in Australia?

9 Cara thought that a sewing machine advertised in a British magazine was especially cheap at £800. What would be the equivalent price in Australian currency?

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Chapter summary

- **Markup** is usually added to the **cost price** to cover costs and make a profit. The price shown on an item offered for sale is called the **marked price** or **retail price**. The markup on an item is the difference between its marked price and its original cost.

$$\text{Markup} = \text{marked price} - \text{cost price}$$

- **Markdown** is the amount that is taken off the price of goods to make them sell faster. The markdown on an item is the difference between the original marked price and the final marked price.

$$\text{Markdown} = \text{original marked price} - \text{final marked price}$$

- **% markup** usually uses the cost price as the base.
- The **recommended retail price (RRP)** or **list price** is the price at which manufacturers and distributors suggest goods should be sold.
- A **trade discount** is a reduction of the RRP offered to retailers. The trade discount is often expressed as a percentage of the list price (RRP). Other discounts are also offered for reasons such as early payment of an **invoice**. The **net amount** paid for goods by a retailer is the total amount of the invoice, less any credit.
- '**Cash-back**' discounts are sometimes offered instead of percentage discounts, to promote a product.
- **Multiple discounts** must be worked out in steps, not just added.
- The **capital** of a business is the money needed to run it. Businesses always try to make a profit and minimise any losses.

$$\textbf{Profit} = \text{selling price} - \text{cost price} \quad (\text{when selling price} > \text{cost price})$$
$$\textbf{Loss} = \text{cost price} - \text{selling price} \quad (\text{when selling price} < \text{cost price})$$

- Unless otherwise stated, **percentage profit** and **percentage loss** are calculated using the cost price as the denominator.

$$\text{Percentage profit} = \frac{\text{profit}}{\text{cost price}} \times 100\%$$
$$\text{Percentage loss} = \frac{\text{loss}}{\text{cost price}} \times 100\%$$

- A **budget** is a financial plan for a period of time or a particular purpose. In accounting it is usual to show a budget as a **balance sheet**, listing **income** in a column on the left and expenses on the right. A **balanced budget** has equal totals of income and expenses. A **personal budget** may be used to determine how much may be saved or to plan for a significant purchase.
- Owning a car involves certain **fixed costs** such as registration and insurance and **running costs** such as petrol and maintenance. Cars also **depreciate** as a result of being used.
- The money of a country is called its **currency**. The International Organisation for Standardisation (ISO) has assigned each currency a three-letter code.
- When you change from Australian currency to another currency you are **selling** Australian dollars and **buying** the other currency. The price of a currency is called its **exchange rate**. The **midrate** is the actual exchange rate and it is close to the average of the buying and selling rates.

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Chapter review

Knowledge and procedures

1 A bait and tackle shop uses a markup of 45%. Ex 10.1
  a For what price would the shop sell reels that cost it $60?
  b What did a rod marked at $116 cost the shop?

2 A chocolate sold for $1.20 cost a shop 75c. What was the percentage markup? Ex 10.1

3 A coffee table costing $320 is sold at a markdown of 20%. What is the sale price? Ex 10.1

4 After reducing the price by 15%, an anklet sold for $323. What was the original price? Ex 10.1

5 The recommended retail price of liquid chlorine is $1.40/L. A hardware shop bought 2000 L at a trade discount of 20%. What did the shop pay for the chlorine? Ex 10.2

6 After a trade discount of 25%, a furniture retailer paid $360 for a chair. What is the retail price? Ex 10.2

7 A clothes shop ordered 250 T-shirts at $35 each. On the invoice terms are 5/10 and n/30. The jeans were delivered on 25 October, but 10 faulty T-shirts were returned to the manufacturer. Ex 10.2
  a What is the net total?
  b What is the last date to obtain a discount?
  c What is the discount price?
  d What is the last date for payment?

8 A shop obtains a trade discount of 35% on scuba-diving gear. The terms of an invoice are 5/15, n/60. If the shop gets 25 wetsuits with a RRP of $360 each, what is: Ex 10.2
  a the least the shop can pay for them?
  b the total percentage discount?

9 The usual price of a small fibreglass boat is $13 000. The boat dealer is having a '20% off everything' sale. In addition, the manufacturer is offering a cash-back of $500. What is the real price of the boat if the dealer's discount is worked out first? Ex 10.2

10 A pet food supplier wants to reduce stock and offers a discount of 20% in addition to a regular customer discount of 10%. What total percentage discount will regular customers receive? Ex 10.2

11 A pet shop bought five puppies for $300 and, after paying $30 each for vaccinations, sold them for $150 each. Ex 10.3
  a What was the shop's total profit?
  b What was the profit as a percentage of cost price?
  c What was the profit as a percentage of the shop's costs, including vaccination?
  d What was the profit as a percentage of sale price?

12 A DVD holder costing $28 was sold at a profit of 25%. What was the selling price? Ex 10.3

13 A car was sold for $18 800 at a profit of 60%. What were the cost price and dollar value of the profit? Ex 10.3

14 A second-hand truck was sold for a profit of $7200, which was 45% of the cost price. What were the cost price and selling price? Ex 10.3

15 On a certain day the buying rate for the Indian rupee is stated as 1 AUD = 36.988 INR and the selling rate as 1 AUD = 33.388 INR. Ex 10.6
  a What is the midrate?
  b What is 1350 AUD worth in rupees?
  c What would you get if you changed 1350 AUD to rupees?
  d How much would you get if you changed 2500 INR to AUD?

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Chapter review

Modelling and problem solving

Ex 10.1 **16** The manufacturing cost of a motor is \$260. The manufacturer applies a markup of 30%. The wholesaler has to pay \$20 transport to get the motor from the manufacturer and then adds a markup of 20%. It costs a retailer another \$15 to take the motor to his shop. The retailer adds a markup of 35%. Finally, GST of 10% is added. What will the customer pay for the motor, and what is the total percentage increase from the manufactured cost?

Ex 10.4 **17** A Year 5 class decide to sell fruit salad at their school fete. They buy 20 kg of bananas at \$3.90/kg, 10 pineapples at \$8 each, 25 kg of watermelon at \$1.70/kg, 12 rockmelons at \$10 each and 20 kg of apples at \$5.60/kg. By the time the fruit is prepared, 25% of the weight has been lost in skins, seeds, cores, etc. The fruit salad is put into 250 g containers that cost 10 cents each. The fruit salad sells out at \$4 per container for a total of \$1020.

- **a** How many containers are sold?
- **b** What weight of fruit salad is sold?
- **c** What weight of fruit is bought?
- **d** What is the cost of the fruit?
- **e** Draw up a budget to work out the total cost of the fruit salad and the profit made by the class.

Ex 10.4 **18** Phan has just moved into her own flat. She earns \$880 a week after tax and pays \$380 a fortnight for her flat. Her food and other day-to-day expenses come to \$320 a week, and transport costs her \$70 a week. She is paying off a CD player at \$112 a month and also spends about \$380 a month on clothes. Her aerobics class costs \$18 a week. Draw up a budget and work out how much she could save in a year.

Ex 10.5 **19** Josif is paying off his car at \$245 a month and registration costs \$690 a year. His car insurance is \$1675 a year and he drives about 30 000 km a year. The service interval for the car is 5000 km/3 months and services cost about \$280 each. Petrol costs Josif \$1.30/L and the car uses 13 L/100 km. The tyres must be replaced every 2 years at a cost of \$560 a set and Josif allows \$1600 a year for major repairs to the car. Draw up a budget and work out the weekly cost of owning and running the car.

Ex 10.4 **20** Four friends plan a trip to a rock concert in Sydney. They are driving down in a car that uses 15 L/100 km and the trip will be about 1500 km altogether. Petrol costs \$1.32 a litre. Tickets for the concert are \$150 each and they are staying overnight in an on-site cabin that costs \$225 for up to six people. They think that food will cost them about \$110 each for the trip. Draw up a budget and work out the cost for each person for the trip.

Ex 10.6 **21** Esrah has an Alfa Romeo but loses a side mirror taking his car into the garage. To get the part quickly he rings an Italian spare-parts dealer who quotes a price of 180 EUR. The cost of posting the part by air is given as 65 EUR. Government charges are 15% of the price quoted for the part and GST of 10% is also charged on this price. The buying and selling rates for 1 AUD are 0.6428 EUR and 0.6246 EUR respectively.

- **a** the quoted price of the part in AUD
- **b** the quoted price landed in Australia
- **c** the import duty
- **d** the GST
- **e** the total cost for the mirror.

Measures of data

11

Contents

Syllabus Guide
Chapter 11

Syllabus subject matter

Data collection and presentation

- What a sample represents, how it relates to populations and whether it is appropriate
- Descriptions of key features of data with reference to suitable selections of graphical and tabular displays
- Data displays including scatterplots, simple and compound stem-and-leaf plots, and box-and-whisker plots
- Sample means and medians as measures of central tendency
- Sample standard deviations and interquartile range as descriptors of spread

Quantitative concepts and skills

- Calculation and estimation with and without instruments
- Basic algebraic manipulations
- Plotting points using Cartesian coordinates

Statistical data is usually collected to help make decisions. In order to do this effectively, it is often useful to summarise the extensive information from a survey or statistical experiment. You may hear statements like 'Most Australians believe that …', 'The average Queensland family …' and so on. These summaries are obtained using data measures. In this chapter, you will learn about the most frequently used measures and their purposes.

11.1 Finding the centre of data

The 'typical' value of data is called the **central tendency** and there are three ways to measure the central tendency. These are the **mode**, the **mean** and the **median**. You have probably used these already in Junior mathematics.

!

Measures of central tendency

The **mode** of a set of data is the most common score. It is the score with the highest frequency. If there are two scores with equal highest frequency, we say that the distribution is **bimodal** and give both values.

The **mean** (symbol $\bar{x}$) is the arithmetic average of the scores. It is calculated by either:

- adding the individual scores and dividing by the number of scores, or
- adding the products of the scores and frequencies and dividing by the total frequency.

This is written as $\bar{x} = \dfrac{\Sigma x}{n}$ or $\bar{x} = \dfrac{\Sigma fx}{\Sigma f}$ where the symbol Σ means sum (the total of).

The **median** is the middle score, when all scores are written in order from smallest to largest. If there is an even number of scores, then it is the average of the middle two scores. In general, the median is the $\dfrac{n+1}{2}$ th score, provided the scores are in order.

Example 1

For the following scores, 9 6 7 14 3 5 14 5 8 5 12, find the:

a mode **b** mean **c** median.

Solution

a There are more 5s than any other. The mode is 5.

b Find the total. $\Sigma x = 9 + 6 + 7 + 14 + 3 + 5 + 14 + 5 + 8 + 5 + 12$
$= 88$

There are 11 scores. $n = 11$

Find the mean. $\bar{x} = \dfrac{\Sigma x}{n} = \dfrac{88}{11} = 8$

Write the answer. The mean is 8.

c Find which is the median score. $\dfrac{n+1}{2} = \dfrac{11+1}{2} = \dfrac{12}{2} = 6\text{th score}$

Arrange in order to find the median. 3, 5, 5, 5, 6, 7, 8, 9, 12, 14, 14

Choose the middle score. The median is 7.

It is unusual for all three measures to be very different, and the mean is rarely a whole number. In most cases, the mode, mean and median will all be about the same. However, when they are different the choice of which to use will depend on the data and the intended use.

> **!**
>
> ### Choosing measures of central tendency
>
> The *mode* is the score *most likely* to occur. It is the only measure of central tendency that can be used for non-numeric data.
>
> The *mean* is most useful when the data is continuous (such as measurements like heights), *for performing calculations*, or in cases where the data is evenly spread.
>
> The *median* is most useful *when data is unevenly spread* on one side or for discrete data (such as numbers of children) where a decimal or fraction would sound silly.

Example 2

The ages at which a group of mothers had their first child are shown below:

28 25 37 24 33 28 24 28 35 25 21 27

a Find the mode, mean and median of the ages.

b What was the typical age at which this group had their first child?

Solution

a There are more 28s than any other. The mode is 28 years.

Find the total. $\Sigma x = 335$

There are 12 scores. $n = 12$

Find the mean.

$$\bar{x} = \frac{\Sigma x}{n} = \frac{335}{12}$$
$$= 27.9166\ldots$$
$$\approx 27.9 \text{ years}$$

Find which is the median score.

$$\frac{n+1}{2} = \frac{12+1}{2}$$
$$= \frac{13}{2}$$
$$= 6.5\text{th score}$$

Arrange in order to find the median. 21, 24, 24, 25, 25, 27, 28, 28, 28, 33, 35, 37

There are two 'middle' scores. The median is the average of 27 and 28.

The median is 27.5 years.

Write the answers. The mode is 28 years, mean about 27.9 years and median 27.5 years.

b Even though we normally state ages as whole numbers, they are really continuous because they measure the time from birth to the present.

Use the mean for continuous data. The typical age was 27.9 years.

For tabulated data, the mean is most easily found using an fx (frequency × score) column and the median by using a cumulative frequency column.

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Example 3

a Find the mean, median and mode of the following test scores.

Score	0	1	2	3	4	5	6	7	8	9	10
Frequency	1	4	6	8	9	10	15	12	7	5	4

b What is the typical test score for this group?

Solution

a Redraw the table vertically and put in fx (frequency × score) and cumulative frequency (c.f.) columns.

Test score, x	**Frequency, f**	**fx**	**c.f.**
0	1	0	1
1	4	4	5
2	6	12	11
3	8	24	19
4	9	36	28
5	10	50	38
6	15	90	53
7	12	84	65
8	7	56	72
9	5	45	77
10	4	40	81
Totals	$\Sigma f = 81$	$\Sigma fx = 441$	

Find the totals of the frequency and fx columns.

The score 6 has the highest frequency. The mode is 6.

Find which is the median score.

$$\frac{n+1}{2} = \frac{81+1}{2} = \frac{82}{2}$$
$$= 41\text{st score}$$

From the cumulative frequency column, the first 38 scores go up to 5, and the next 15 scores, from the 39th to 53rd scores, are all 6.

The 41st score is a 6. The median is 6.

Find the mean.

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{441}{81} = 5.4444\ldots$$
$$\approx 5.4$$

Write the answer. The mean is about 5.4.

b As the scores appear to be spread unevenly, the median should be used as the typical score. The typical test score for this group is 6.

Example 4

Find the median of the following ages of a sample of people clubbing in Brisbane on Saturday night.

Age	18	19	20	21	22	23	24	25	26
Number	15	18	19	15	10	10	8	7	2

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Solution

Redraw the table vertically and put in the cumulative frequency column.

Age	Frequency	c.f.
18	15	15
19	18	33
20	19	52
21	15	67
22	10	77
23	10	87
24	8	95
25	7	102
26	2	104
Total	**104**	

Find which is the median score.

$$\frac{n+1}{2} = \frac{104+1}{2} = \frac{105}{2}$$

$= 52.5$th score

From the cumulative frequency column, the first 52 scores go up to 20, and the next 15 scores, from the 53rd to the 67th, are all 21.

The 52nd and 53rd scores are 20 and 21 respectively.

$$\text{Median} = \frac{20+21}{2} = \frac{41}{2} = 20.5$$

Write the answer.

The median is 20.5.

You can use your scientific calculator to work out the mean of ungrouped data. Some calculators can also be used for grouped data.

Example 5

Use a scientific calculator to work out the mean of the following data:

28 25 37 24 33 28 24 28 35 25 21 27

Solution

Change the calculator to the STAT mode. You usually have to press the **MODE** key to do this. For some calculators there are several STAT modes. Choose the SD, lowest-numbered or 1-var mode.

A small STAT or SD should appear on the screen.

Put in the numbers, pressing **M+** after each.

28 **M+** 25 **M+** 37 **M+** …

To get the mean, you need to access the $\bar{x}$ function.

You usually press the **SHIFT**, **2ndF** or **ALPHA** key and a number to get it.

SHIFT 1 → 27.91666667 SD

If you don't get the result shown, check with your teacher for your calculator.

Round and write the answer and say how you got it.

From the calculator, the mean is about 27.9.

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Example 6

Use a graphics calculator to find the mean, median and mode (if possible) of each of the following data sets.

a 3, 6, 8, 10, 5, 4, 6, 8, 8, 3, 4, 6

b

x	3	4	5	6	7	8	9
f	5	7	6	3	2	0	1

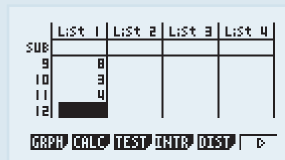
Solution

a For any of the calculators, enter the data in List 1 as shown on pages 183–84.

Casio fx-9860G AU

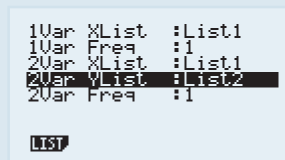
Choose the CALC submenu by pressing **F2**.

Choose the SET submenu by pressing **F6**.



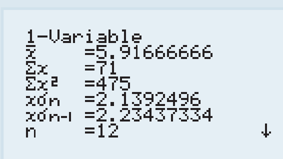
Set the 1Var XList to List 1.

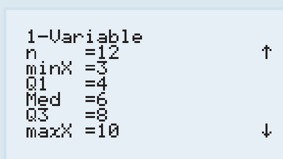
Set the 1Var Freq to 1.

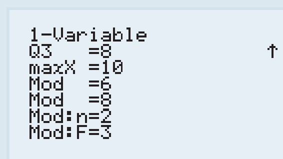


EXIT and choose 1-Var by pressing **F1**.

Use the cursor keys to move down the list.





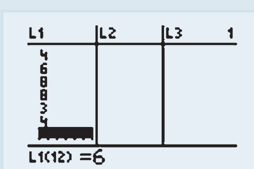


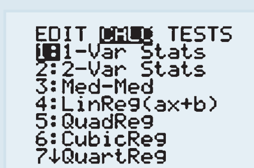
The mean is about 5.92, the median is 6 and there are two modes, 6 and 8, both with frequency of 3.

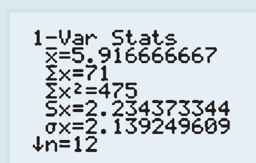
Texas Instruments TI-84

After the data is entered in L1, press the **STAT** key and choose the CALC menu.

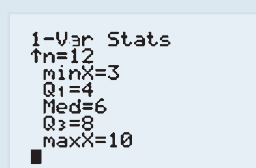
Choose 1-Var Stats, press **ENTER** and put in L1 by pressing **2nd** 1.







Use the cursor keys to move down the list.



The mean is about 5.92 and the median is 6.

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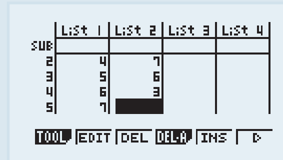
b For any of the calculators, enter the scores in List 1 and the frequencies in List 2 as shown on pages 183–84.

Casio fx-9860G AU

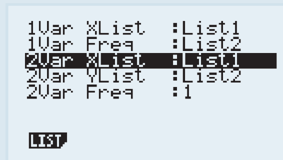
Choose the CALC submenu by pressing **F2**.

Choose the SET submenu by pressing **F6**.

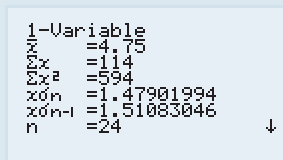


Set the 1Var XList to List 1.

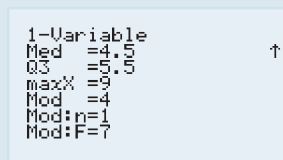
Set the 1Var Freq to List 2.



EXIT and choose 1-Var by pressing **F1**.



Use the cursor keys to move down the list.

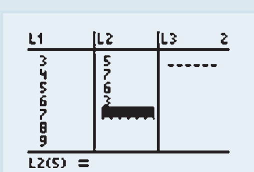


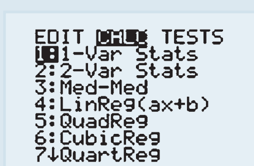
The mean is 4.75, the median is 4.5 and the mode is 4.

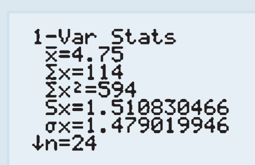
Texas Instruments TI-84

After the data is entered in L1 and L2, press **STAT** and choose the CALC menu.

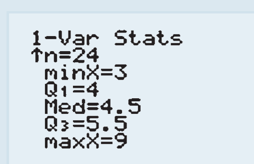
Choose 1-Var Stats, press **ENTER** and put in L1, L2 by pressing **2nd** 1 **,** **2nd** 2.







Use the cursor keys to move down the list.



The mean is 4.75 and the median is 4.5.

Sharp EL-9900

Instructions for the Sharp calculator are on the CD-ROM.

Calculator Instructions

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Investigation Hand spans

Work in groups of three or four.

1 How big is your hand span? Measure it.

2 Compare your hand span with the hand spans of other members of your group.

3 Now estimate the average and median hand spans of the whole class. (If you have only a small class, you could estimate the average and median hand spans of all Year 11 Maths A students at your school.)

4 Collect the measurements that you need to calculate the mean and median hand spans, then make the calculations.

5 Compare your estimates with the actual figures and comment on the results.

6 If a similar number of Year 8 students were added to the group under consideration, how do you think the values of mean and median might alter?

Additional Exercise 11.1

Exercise 11.1 Finding the centre of data

1 Find the mean, median and mode of each of the following sets of data.

a Strokes on a nine-hole golf course: 6 4 7 5 9 6 8 6 7

b Shoe sizes of the Brisbane Bullets: 11 15 14 12 14 13 14

c Average speed (km/h) for the lead car in the first 10 laps of the Formula One Grand Prix in Albert Park in Melbourne:

205 212 208 212 217 220 215 225 210 217

2 Find the mean, median and mode of each of the following sets of scores.

a

Score	5	6	7	8	9	10	11	12	13	14
Frequency	2	4	5	7	8	6	3	2	2	1

b

Score	0	1	2	3	4	5	6	7	8	9	10
Frequency	4	6	5	10	12	13	6	5	3	0	2

c

Score	45	46	47	48	49	50	51	52	53
Frequency	5	8	12	18	24	22	19	8	6

3 Use a graphics calculator to find the mean, median and mode of each of these sets of scores.

a 12, 8, 16, 14, 12, 13, 9, 10, 11, 12, 13, 9, 14, 12, 11, 9

b 7, 3, 4, 2, 6, 5, 4, 8, 4, 3, 2, 8, 4, 0, 1

c

Score	3	4	5	6	7	8	9	10	11	12	13
Frequency	2	4	6	10	12	8	6	5	2	1	3

d

Score	17	18	19	20	21	22	23	24	25
Frequency	2	0	5	8	12	9	6	4	1

Modelling and problem solving

4 The shoe sizes of a group of Year 11 students were recorded as follows:

8 9 5 6 8 7 6 8 12 9 8 7 6
10 8 6 7 5 8 11 6 8 9 10 8

a Find the mean, median and modal shoe sizes.
b What is the typical shoe size of this group?

5 Thirty employees of a local engineering firm were surveyed as to the number of days they were absent last year, with the following results.

Days absent	0	1	2	3	4	5	6	7	8	9	10
Frequency	4	3	6	6	3	1	3	2	1	0	1

a Draw a histogram of the results.
b Find the mean, mode and median for this data.
c If the firm employed 125 people in total, how many days of absence would you expect altogether?
d How many days' absence did the 'average' employee have last year?

6 The number of cars parked in a side street to a major shopping centre was recorded every hour for 8 trading hours over 5 weekdays. The results were as follows:

15 20 18 19 20 19 18 15 18 20 12 18 21 17
19 14 16 20 17 18 17 19 15 20 16 19 18 20
19 17 20 18 20 15 18 12 19 18 17 19

a Find the mode, median and mean of this data.
b What was the typical number of cars parked in the side street over this period?

11.2 Finding the spread of data

Sets of data may have the same centres but be quite different, as shown by these three sets of data:

Set 1:	8	8	9	9	9	9	10	10	10	10	10	11	11	11	11	12	12
Set 2:	2	5	6	7	8	9	9	9	10	11	11	11	12	13	14	15	18
Set 3:	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

The mean and median of all three sets are 10.

It is clear that the second set of data is spread more than the first, and that the third set is even more spread (though it does start and finish at the same numbers as set 2). Three different **measures of dispersion** (or **spread**) are used to calculate the spread of data. They are the **range**, **interquartile range** and **standard deviation**.

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Measures of dispersion

The **range** measures the overall spread of the scores.

Range = highest score − lowest score

The **interquartile range** measures the spread of the middle half of the data.

Interquartile range = third quartile − first quartile

The **first quartile** (**Q_1, lower quartile**) is the value that lies a quarter of the way through the data when the data is arranged in ascending order. The **third quartile** (**Q_3, upper quartile**) is the value that lies three-quarters of the way through the data. The first quartile is the median of the lower half of the data, while the third quartile is the median of the upper half. In general:

- If n is odd: First quartile $= \frac{n+1}{4}$ th data item.
- If n is even: First quartile $= \frac{n+2}{4}$ th data item.

In both cases, the third quartile is the corresponding item back from the last data item.

Teacher Notes

The **standard deviation** measures how far every data item is from the mean. It is abbreviated as **SD**, has the symbol σ and is calculated using the formulas

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} \text{ for individual scores}$$

or

$$\sigma = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2} \text{ for tables} \quad \text{where } \Sigma \text{ means the sum.}$$

Example 7

For the following scores: 12 17 6 9 3 3 7 8 10 19 10 12 11

a find the range

b find the interquartile range

c find the standard deviation.

Solution

a Lowest score = 3 and highest = 19.

Range = 19 − 3 = 16.

b Arrange in order to find Q_1 and Q_3.

$$\underbrace{3\ \ 3\ \ 6\ \ 7\ \ 8\ \ 9}_{\text{Lower half of data}}\ \ 10\ \ \underbrace{10\ \ 11\ \ 12\ \ 12\ \ 17\ \ 19}_{\text{Upper half of data}}$$

$Q_1 = 6.5$ Median $Q_3 = 12$

Find the interquartile range.

$$\text{Interquartile range} = Q_3 - Q_1 = 12 - 6.5 = 5.5$$

Write the answer.

The interquartile range is 5.5.

c Work out the mean by adding the scores.

$\Sigma x = 127$ and $n = 13$

Keep the exact value on your calculator.

$$\bar{x} = \frac{\Sigma x}{n} = \frac{127}{13} = 9.7692\ldots$$

Find the sum of the squares.

$$\Sigma x^2 = 3^2 + 3^2 + 6^2 + \ldots + 17^2 + 19^2 = 9 + 9 + 36 + \ldots + 289 + 361 = 1507$$

Use the mean and the sum of the squares.	$\sigma = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$
Use the exact value of $\bar{x}$ on your calculator.	$= \sqrt{1507 \div 13 - 9.7692\ldots^2}$
	$= \sqrt{115.9230\ldots - 95.4378\ldots}$
Keep the exact number.	$= \sqrt{20.4852\ldots}$
Use the square root.	$= 4.5260\ldots$
Round.	≈ 4.53
Write the answer.	The standard deviation is about 4.53.

When finding the quartiles of a large number of scores, we use the rule to find which score should be used to find the first quartile. When finding which score should be used to find the third quartile, the last item must be counted. In order to do this, we subtract *one less* than the number of scores for the first quartile from the total number of scores.

If there were 34 scores, the first quartile would be the $\dfrac{34 + 2}{4} = 9$th score, so the third quartile would be the $34 - 8 = 26$th score.

Example 8

Find for the data below:

a the range

b the interquartile range.

Score	7	8	9	10	11	12	13	14	15	16
Frequency	3	4	6	7	8	9	6	8	4	2

Solution

Redraw the table with cumulative frequency, fx (frequency × score), x^2 and fx^2 (frequency × score^2) columns. Add up the f, fx and fx^2 columns.

x	f	c.f.	fx	x^2	fx^2
7	3	3	21	49	147
8	4	7	32	64	256
9	6	13	54	81	486
10	7	20	70	100	700
11	8	28	88	121	968
12	9	37	108	144	1296
13	6	43	78	169	1014
14	8	51	112	196	1568
15	4	55	60	225	900
16	2	57	32	256	512
Totals	$\Sigma f = 57$		$\Sigma fx = 655$		$\Sigma fx^2 = 7847$

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a The scores go from 7 to 16.

Range = highest − lowest
= 16 − 7 = 9

b 57 is an odd number of scores. The first 13 scores go to 9 and the next 7 scores are all 10, so the 14th and 15th scores are both 10.

$Q_1 = \dfrac{57+1}{4}$ th score
$= 14\tfrac{1}{2}$ th score = 10

To count back $14\tfrac{1}{2}$ scores from the final score, take $13\tfrac{1}{2}$ from 57.

$Q_3 = 57 - 13\tfrac{1}{2}$ th score
$= 43\tfrac{1}{2}$ th score

The 43rd score is 13 and the 44th score is 14.

$= \dfrac{13+14}{2} = 13.5$

Find the interquartile range.

Interquartile range = $Q_3 - Q_1$
= 13.5 − 10 = 3.5

Write the answer.

The interquartile range is 3.5.

Students are generally expected to use calculators or computers to work out data measures.

Example 9

Use a scientific calculator to work out the standard deviation of the following data:
28 25 37 24 33 28 24 28 35 25 21 27

Solution

Change to the STAT mode and enter the data as shown in Example 5 on page 323.

28 M+ 25 M+ 37 M+ …

To get the standard deviation, you need to access the σ_x, σ_{xn} or SD function. You usually press the SHIFT, 2ndF or ALPHA key and a number to get it.

SHIFT 2 → 4.609018936 SD

If you don't get the result shown, check with your teacher for your calculator.

Round and write the answer and say how you got it.

From the calculator, the standard deviation is about 4.61.

Strictly speaking, the formula for the standard deviation of a sample is different from that for a population. You will nearly always deal with samples, so you should use the σ_{xn-1} or s_x function, which would give the slightly different result 4.81 in Example 9. Ask your teacher which you should use.

Example 10

Use your graphics calculator to find the range, interquartile range and standard deviation of each of the following sets of data.

a 5, 8, 12, 4, 3, 6, 7, 9, 10, 4, 6, 6, 13, 15, 18, 5, 4, 11, 9

b

x	22	23	24	25	26	27	28	29	30	31
f	3	4	5	7	9	8	5	3	3	2

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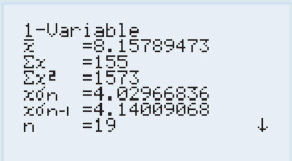
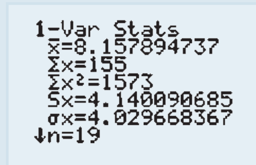
Solution

a Enter the data and find the 1-Var statistics as shown in Example 6a on page 324.

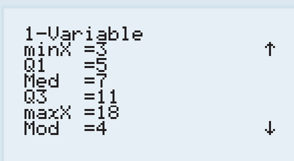
Casio fx-9860G AU | **Texas Instruments TI-84**

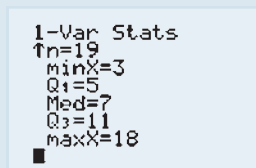




Use the cursor keys to move down the list to find the other values.

Casio fx-9860G AU | **Texas Instruments TI-84**





Casio: $x\sigma n = 4.0296\ldots$ $\quad$ minX $= 3$
$Q_1 = 5$ $\quad Q_3 = 11$ $\quad$ maxX $= 18$

TI-84: $\sigma x = 4.0296\ldots$ $\quad$ minX $= 3$
$Q_1 = 5$ $\quad Q_3 = 11$ $\quad$ maxX $= 18$

Work out answers from the values.

Range $= 18 - 3$
$= 15$
Interquartile range $= 11 - 5$
$= 6$
Standard deviation ≈ 4.03

b Enter the data and find the 1-Var statistics as in Example 6b on page 325. Use the cursor keys to move down the list to find the values. Work out the answers from the values.

$\sigma = 2.3154\ldots$ $\quad$ minX $= 22$,
$Q_1 = 24.5$ $\quad Q_3 = 28$ $\quad$ maxX $= 31$

Range $= 31 - 22$
$= 9$
Interquartile range $= 28 - 24.5$
$= 3.5$
Standard deviation ≈ 2.32

Sharp EL-9900

Instructions for the Sharp calculator are provided on the CD-ROM.

Calculator Instructions

The intended use of a measure of spread will help determine whether you should use the range, interquartile range or standard deviation.

The range measures the gross spread, while the interquartile range measures the spread of the middle half of the scores.

The standard deviation should be used when making comparisons between data sets as it uses all the data to measure the spread of scores from the mean. For normal distributions, 68.3% of all scores lie within 1 standard deviation from the mean, 95.4% within 2 standard deviations and 99.7% within 3 standard deviations.

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Example 11

The shoe sizes of men's shoes sold one Friday in two different stores are shown below:

City store: 8 9 10 11 8 9 10 13 10 12 10 9 8 6 8 9 10 10

Suburban store: 6 8 10 12 10 8 9 9 9 6 8 10 12 11

Which store has the greater concentration of sizes sold?

Solution

Use a graphics calculator to find the statistics for each store. Results are shown in the table.

	Mean	Median	Range	IQ range	SD
City	9.44	9.5	7	2	1.57
Suburban	9.14	9	6	2	1.81

While the range in the city store is more than that in the suburban store, this is the result of only one person buying a size 13 pair of shoes.
The standard deviation in the city store is much smaller than that in the suburban store, so the concentration of sizes sold is greater in the city store.

Additional Exercise 11.2

Exercise 11.2 Finding the spread of data

1 Find the range, interquartile range and standard deviation of each of the following sets of data. You should use your calculator to find the standard deviation.

a 4, 2, 5, 2, 7, 5, 8, 8, 3, 5, 8, 3, 5, 9

b 18, 15, 14, 12, 13, 17, 18, 16, 14, 16, 17, 19, 12, 16, 16, 19

c 24, 26, 29, 33, 39, 42, 41, 27, 36, 34, 28, 27, 24, 27, 44

2 Find the range and interquartile range of each of the following sets of scores.

a

Score	0	1	2	3	4	5	6	7	8	9
Frequency	2	5	5	7	7	6	6	3	2	1

b

Score	8	9	10	11	12	13	14	15	16	17	18
Frequency	3	7	6	10	12	13	6	5	3	0	2

c

Score	25	26	27	28	29	30	31	32	33
Frequency	3	6	10	16	22	20	17	6	4

3 Use a graphics calculator to find the range, interquartile range and standard deviation of each of the following sets of scores.

a 10, 11, 12, 13, 9, 14, 12, 11, 9, 12, 8, 16, 14, 12, 13, 9

b 4, 3, 2, 8, 4, 0, 1, 7, 3, 4, 2, 6, 5, 4, 8

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c

Score	5	6	7	8	9	10	11	12	13	14	15
Frequency	2	4	5	7	11	8	6	5	2	1	2

d

Score	23	24	25	26	27	28	29	30	31	32	33
Frequency	1	3	5	8	12	9	6	4	0	2	3

Modelling and problem solving

4 Tanya and Juan are employed as quality controllers at a factory where computer mother boards are assembled. They inspect the completed mother boards as they come off the assembly line. The numbers of mother boards that they each inspect each hour over a 9-hour period are shown below:

Tanya: 30 31 29 30 29 23 33 35 30

Juan: 28 30 26 12 29 31 35 48 31

Who is the more consistent worker?

5 The times (seconds) taken for 25 job applicants to complete a manual dexterity test are shown below:

14 11 10 12 19 14 5 17 12
10 11 19 15 19 10 21 14 27
16 23 15 14 18 20 17

Within how many seconds would you expect half of the applicants to complete the test? (*Hint:* What does the median mean?)

6 The numbers of births per week were counted at Royal Brisbane Hospital. The data appears in the table below.

Number of births per week	10	11	12	13	14	15	16	17	18
Frequency	3	3	4	7	8	12	7	5	3

a Over what period was the data taken?

b What is the typical number of births?

c What conclusions can you reach about the spread of the numbers of births?

d Can you find any information about the time of year at which most births occur?

11.3 Using grouped data

The methods already seen can be modified to use with grouped data, such as heights in categories such as 140–149 cm, 150–159 cm, etc. We have to take account of the fact that a class covers a range of possible scores. In the case of calculations involving the values of scores, we use the class midpoint as an approximation of all the scores in a class. For the positions of scores, we use the true class limits. These were discussed on pages 191–94.

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Grouped data tables

For calculation of the mean and standard deviation, the **class midpoints** are used.

For calculation of the median, range and interquartile range, **true class limits** are used.

Interpolation is used to find the median, Q_1 and Q_3.

The formula for the value of the ***m*th term** in a class interval is $L + \frac{m}{f} \times w$

where L is the lower class limit, f is the frequency of the class and w is the class width.

A **cumulative frequency polygon** or **ogive** can be used in these calculations. The median and quartiles are found using 50%, 25% and 75% of the total frequency.

We cannot find a mode, but only a **modal class**.

Example 12

The values of orders for a party-plan sales campaign are tabulated below. Find the mean, median, modal class, range and interquartile range from the table.

Value ($)	50–59	60–69	70–79	80–89	90–99	100–109
Number of orders	8	19	28	46	72	85

Value ($)	110–119	120–129	130–139	140–149	150–159
Number of orders	62	41	26	18	12

Solution

Redraw the table with true class limits, class midpoint, cumulative frequency and fx (frequency × score) columns.

Use the class midpoint in calculations for the score. Add up the f and fx columns.

True class limits ($)	Class midpoint, x	f	c.f.	fx
49.5–59.5	54.5	8	8	436.0
59.5–69.5	64.5	19	27	1225.5
69.5–79.5	74.5	28	55	2086.0
79.5–89.5	84.5	46	101	3887.0
89.5–99.5	94.5	72	173	6804.0
99.5–109.5	104.5	85	258	8882.5
109.5–119.5	114.5	62	320	7099.0
119.5–129.5	124.5	41	361	5104.5
129.5–139.5	134.5	26	387	3497.0
139.5–149.5	144.5	18	405	2601.0
149.5–159.5	154.5	12	417	1854.0
Totals		$\Sigma f = 417$		$\Sigma fx = 43\,476.5$

Mean

Write the formula. $\bar{x} = \frac{\Sigma fx}{\Sigma f}$

Substitute values from the table. $= \frac{43\,476.5}{417}$

$= 104.2601\ \ldots$

Round. $\approx \$104.26$

Median

Find which is the median score. $\frac{n+1}{2} = \frac{417+1}{2} = \frac{418}{2} = 209\text{th score}$

From the table, there are 173 scores up to 99.5, and 85 scores from 99.5 to 109.5.

Write the value of the lower class limit. $L = 99.5$

Find which term in the class we want. $m = 209 - 173$

$= 36$

Write the class width. $w = 10$

Write the frequency of the class. $f = 85$

Find the median. $\text{Median} = L + \frac{m}{f} \times w$

The median is $\frac{36}{85}$ths of the way from 99.5 to 109.5. $= 99.5 + \frac{36}{85} \times 10$

$= 103.7352\ \ldots$

Round. $\approx \$103.74$

Modal class

The highest frequency is 85. The modal class is \$100–\$109.

Range

The true lower limit of the first class is 49.5 and the true upper limit of the last class is 159.5, so these are the lowest and highest possible values. $\text{Range} = 159.5 - 49.5$

$= \$110$

Interquartile range

Find which is the first quartile score. There is an odd number of scores. $\frac{n+1}{4} = \frac{417+1}{4} = \frac{418}{4} = 104\frac{1}{2}\text{th score}$

From the table, there are 101 scores up to 89.5, and 72 scores from 89.5 to 99.5.

Write the value of the lower class limit. $L = 89.5$

Find which term in the class we want. $m = 104.5 - 101$

$= 3.5$

Write the class width. $w = 10$

Write the frequency of the class. $f = 72$

Find Q_1. $Q_1 = L + \frac{m}{f} \times w$

Q_1 is $\frac{3.5}{72}$ths of the way from 89.5 to 99.5. $= 89.5 + \frac{3.5}{72} \times 10$

Keep the exact value on your calculator. $= \$89.9861\ \ldots$

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Find which is the third quartile score, by taking *one less* than the first quartile score number from the total number.	$417 - 103\frac{1}{2} = 313\frac{1}{2}$ th score

From the table, there are 258 scores up to 109.5, and 62 scores from 109.5 to 119.5.

Write the value of the lower class limit.	$L = 109.5$
Find which term in the class we want.	$m = 313.5 - 258$ $= 55.5$
Write the class width.	$w = 10$
Write the frequency of the class.	$f = 62$
Find Q_3.	$Q_3 = L + \frac{m}{f} \times w$
Q_3 is $\frac{55.5}{62}$ ths of the way from 109.5 to 119.5.	$= 109.5 + \frac{55.5}{62} \times 10$ $= \$118.4516\ldots$
Find the interquartile range.	IQ range $= Q_3 - Q_1$
Use the stored value of Q_1.	$= 118.4516\ldots - 89.9861\ldots$ $= 28.4655\ldots$
Round.	$\approx \$28.47$
Write the answers.	The mean value is about \$104.26, median about \$103.74, modal class \$100–\$109, range \$110 and interquartile range about \$28.47.

Although it is tedious to draw the graph, it is often easier to find the median and interquartile range from a cumulative frequency polygon than by using interpolation.

Example 13

Use a graph to find the median and interquartile range of the following hourly air pollution data in Brisbane over a short period.

Pollution level	Frequency
0–19	30
20–39	21
40–59	15
60–79	6
80–99	6
100–119	5
120–139	1
140–159	4

Solution

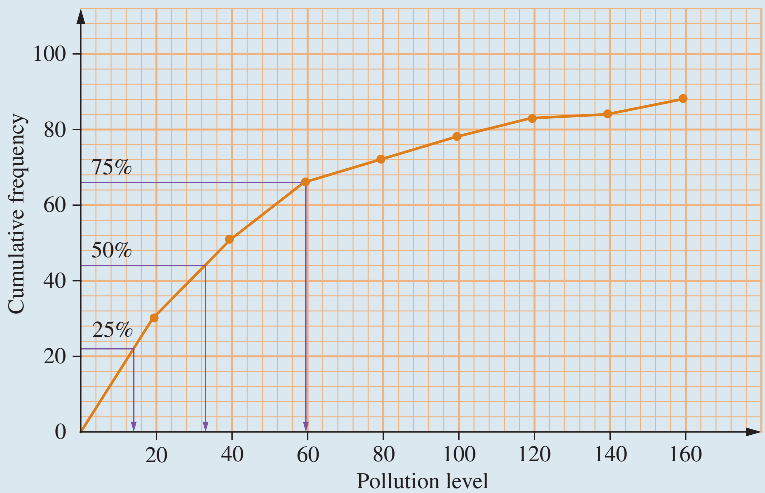
The true class limits are 0–19.5, 19.5–39.5, etc.

Redraw the table with the true class limits and add a cumulative frequency column.

Draw a cumulative frequency polygon, remembering to put the values at the true upper class limits.

True class	Frequency	c.f.
0–19.5	30	30
19.5–39.5	21	51
39.5–59.5	15	66
59.5–79.5	6	72
79.5–99.5	6	78
99.5–119.5	5	83
119.5–139.5	1	84
139.5–159.5	4	88

Air pollution in Brisbane



Median is at 50% of the total. 50% of $88 = 0.50 \times 88 = 44$

Find the score for c.f. = 44 on the graph. Median ≈ 33

Q_1 is at 25% of the total. 25% of $88 = 22$

Find the score for c.f. = 22 on the graph. $Q_1 \approx 14$

Q_3 is at 75% of the total. 75% of $88 = 66$

Find the score for c.f. = 66 on the graph. $Q_3 \approx 59.5$

Find the interquartile range.

$$\begin{aligned} \text{IQ range} &= Q_3 - Q_1 \\ &\approx 59.5 - 14 = 45.5 \end{aligned}$$

Write the answers. The median is about level 33 and the interquartile range about 45.5 levels.

A graphics calculator can be used with grouped frequencies to find the mean and standard deviation similarly to the method for tabulated scores shown in Example 10b on page 331, by putting the class centres as the first list. However, the median, range, quartiles and interquartile range obtained by this method will be incorrect.

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Example 14

Use a graphics calculator to find the mean and standard deviation of these pollution levels.

Pollution level	0–19	20–39	40–59	60–79	80–99	100–119	120–139	140–159
Frequency	30	21	15	6	6	5	1	4

Solution

Enter the class centres 9.5, 29.5, …, 129.5, 149.5 into List 1 and the frequencies into List 2 of the graphics calculator. Find the 1-Var statistics as in Example 6a on page 325.

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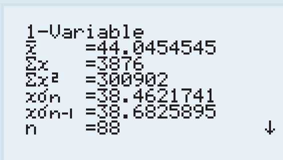
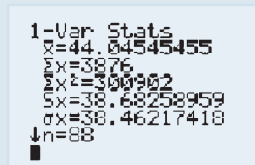


Texas Instruments TI-84



Write the answers.

The mean is about level 44.0 and the standard deviation about 38.5 levels.

Note: The values of minX = 9.5, Q_1 = 9.5, median = 29.5, Q_3 = 59.5 and maxX = 149.5 obtained from the calculator for this example are incorrect (compared with Example 13).

Calculator Instructions

Sharp EL-9900

Instructions for the Sharp calculator are provided on the CD-ROM.

Additional Exercise 11.3

Exercise 11.3 Using grouped data

1 Find the mean, median, modal class, range and interquartile range for each table below.

a

Class	Frequency
0–4	1
5–9	4
10–14	9
15–19	14
20–24	2

b

Class	Frequency
40–49	4
50–59	8
60–69	12
70–79	23
80–89	9
90–99	5

c

Class	Frequency
105–109	28
110–114	43
115–119	17
120–124	5
125–129	2

d

Class	Frequency
70–109	4
110–149	7
150–189	11
190–229	18
230–269	37
270–309	22

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2 Use graphs to find the median and interquartile range for each of the following data sets.

a **Heights of some Year 11 students**

Height (cm)	145–149	150–154	155–159	160–164	165–169	170–174	175–179	180–184
Number	2	3	7	16	28	32	10	2

b **Production for a central Queensland coal mine**

Production ('000 t)	50–54.9	55–59.9	60–64.9	65–69.9	70–74.9	75–79.9	80–84.9	85–89.9
Years	1	4	3	4	5	5	2	1

3 Use a graphics calculator to find the mean and standard deviation of each set of data.

a

Score	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90–99
f	2	5	7	11	10	8	4	1

b

Score	35–39	40–44	45–49	50–54	55–59	60–64	65–69	70–74	75–79	80–84	85–89
f	2	8	12	15	13	12	10	7	5	3	2

Modelling and problem solving

4 The monthly incomes of the employees of a Queensland manufacturer are shown in the table on the right.

a Find the mean, median and modal class for the employees' incomes.

b What is the typical employee's income? Explain your answer.

c Find the range, interquartile range and standard deviation of the employees' incomes.

d What income range would cover the middle half of the employees' incomes?

Income ($)	Number
2500–2999	12
3000–3499	29
3500–3999	44
4000–4499	39
4500–4999	38
5000–5499	25
5500–5999	17
6000–6499	9
6500–6999	5
7000–7499	2
7500–7999	1

5 The masses (kilograms) of chickens were recorded by a quality controller as follows:

1.6 1.7 1.2 1.4 1.5 1.4 2.0 1.1 1.8 1.9 1.7 1.6 2.2 1.6 1.4 1.3
1.5 1.5 1.7 1.5 1.3 1.6 1.8 1.1 1.7 1.4 1.2 1.9 2.1 1.4 1.6 2.2
1.5 1.6 1.9 1.5 1.4 2.2 1.5 1.9 1.6 1.5 1.8 1.5 1.6 2.4 1.7 1.5
1.8 2.3 1.7 1.6 2.2 1.7 1.6 1.7 1.4 1.6 1.7 1.8 1.6 1.0 1.4 1.6
1.5 2.2 1.7 1.8 1.6 1.9 1.7 1.6 1.5 2.3 1.8 2.0 1.9 1.5 1.2 2.5

a Choose appropriate class intervals and arrange this data into a frequency distribution table.

b Draw an ogive for the data by joining the points with a smooth curve.

c Find the mean, modal class and median of the data.

d Find the range, interquartile range and standard deviation of the data.

11.4 Finding quantiles

You have worked out the median and quartiles. The median is halfway through the data, while the quartiles divide the data into 4 quarters. **Quantiles** divide the data into other fractions.

!

Quantiles

A **quantile** is a score that divides the data in a frequency distribution into particular quantities. Quantiles may be calculated using interpolation or graphs.

Percentiles divide the data into percentage groups. The 35th percentile (shown as P_{35}) is the score below which 35% of all scores lie.

Deciles divide the data into tenths. The 7th decile (shown as D_7) is the score below which $\frac{7}{10}$ ths of the data lies.

Example 15

Find the 45th percentile for the data shown below.

x	0–4	5–9	10–14	15–19	20–24	25–29	30–34	35–39	40–44	45–49
f	3	5	8	9	11	12	8	7	4	2

Solution

Redraw the table with true class limits.

Complete a cumulative frequency column.

True classes	*f*	c.f.
0–4.5	3	3
4.5–9.5	5	8
9.5–14.5	8	16
14.5–19.5	9	25
19.5–24.5	11	36
24.5–29.5	12	48
29.5–34.5	8	56
34.5–39.5	7	63
39.5–44.5	4	67
44.5–49.5	2	69

Find 45% of the total frequency. 45% of 69 = $0.45 \times 69 = 31.05$

From the table, there are 25 scores up to 19.5, and 11 scores from 19.5 to 24.5.

Write the value of the lower class limit. $L = 19.5$

Find which term in the class we want. $m = 31.05 - 25 = 6.05$

Write the class width. $w = 5$

Write the frequency of the class. $f = 11$

Find P_{45}. $P_{45} = L + \frac{m}{f} \times w$

P_{45} is $\frac{6.05}{11}$ ths of the way from 19.5 to 24.5. $= 19.5 + \frac{6.05}{11} \times 5 = 22.25$

Write the answer. The 45th percentile is 22.25.

Example 16

Use a cumulative frequency polygon to find the 8th decile of the distribution below.

x	30–39	40–49	50–59	60–69	70–79	80–89	90–99	100–109	110–119
f	12	20	45	93	87	52	38	18	7

Solution

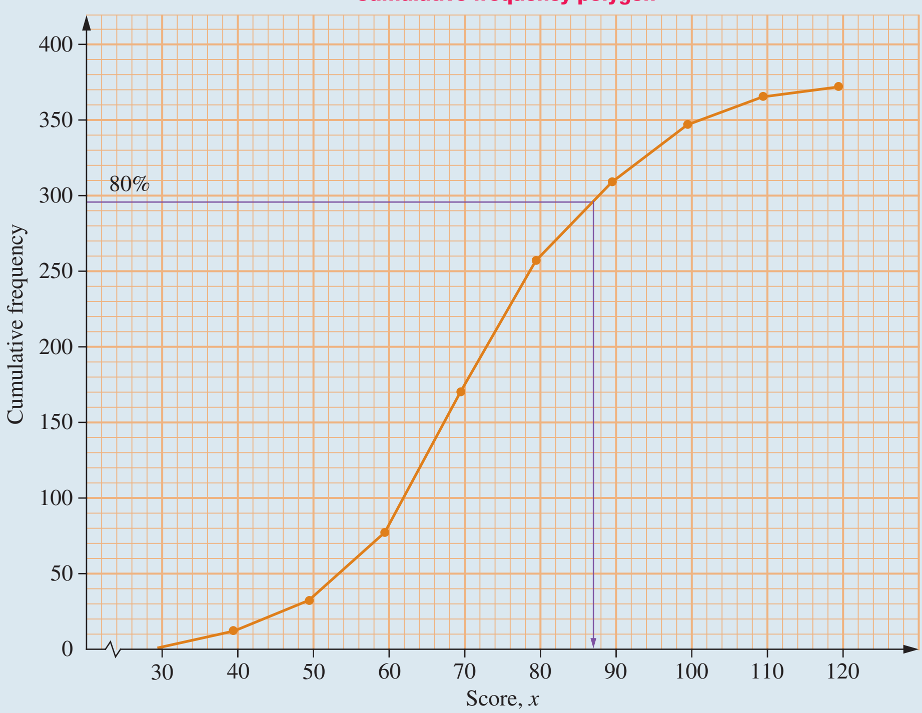
Redraw the table with true class limits.

Complete a cumulative frequency column.

True classes	*f*	c.f.
29.5–39.5	12	12
39.5–49.5	20	32
49.5–59.5	45	77
59.5–69.5	93	170
69.5–79.5	87	257
79.5–89.5	52	309
89.5–99.5	38	347
99.5–109.5	18	365
109.5–119.5	7	372

Draw a cumulative frequency polygon, remembering to use the true upper class limits.

Cumulative frequency polygon



Find 0.8 of the total frequency. $0.8 \times 372 = 297.6$

Use the graph to find the corresponding score. The 8th decile is about 87.

Some people prefer to use a cumulative percentage polygon or ogive in this work so that the results can be read directly from the graph.

Example 17

Use a cumulative percentage ogive to find the 57th percentile and 3rd decile for the distribution below.

x	12–15	16–19	20–23	24–27	28–31	32–35	36–39	40–43	44–47
f	8	12	24	35	32	25	18	8	5

Solution

Redraw the table with true class limits, and complete a cumulative frequency column.

Use the total frequency to calculate cumulative percentages.

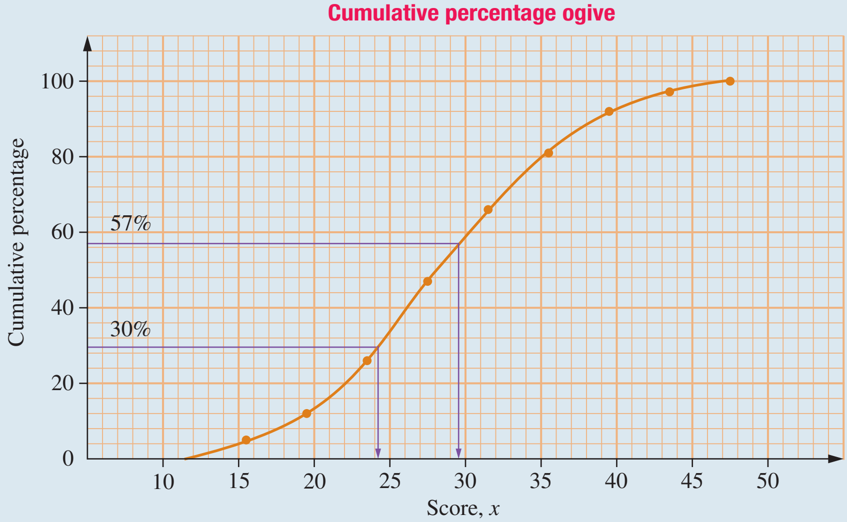
$\frac{8}{167} \times 100\% \approx 4.8\%$

$\frac{20}{167} \times 100\% \approx 12.0\%$, etc.

True classes	*f*	c.f.	Cumulative percentage (%)
11.5–15.5	8	8	4.8
15.5–19.5	12	20	12.0
19.5–23.5	24	44	26.3
23.5–27.5	35	79	47.3
27.5–31.5	32	111	66.5
31.5–35.5	25	136	81.4
35.5–39.5	18	154	92.2
39.5–43.5	8	162	97.0
43.5–47.5	5	167	100.0

Draw a cumulative percentage ogive using a smooth curve with the upper class limits.

Cumulative percentage ogive



Read the 57th and 30th percentiles from the graph.

The 57th percentile is about 29.4.

The 3rd decile is about 24.3.

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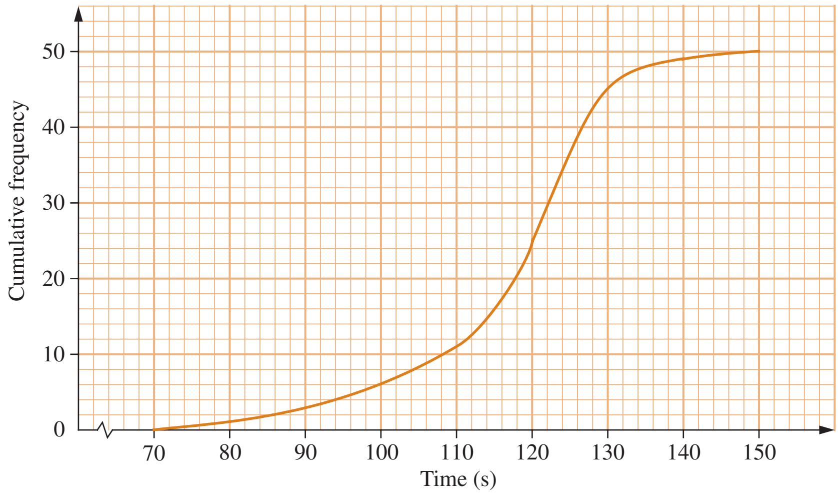
Exercise 11.4 Finding quantiles

Additional Exercise 11.4

1 The ogive below shows the times in seconds taken by a group of young drivers to complete an error-free lap of a defensive-driving circuit.

Defensive-driving times



a Find the median time.

b Find P_{82} and P_{35}.

c Find the quartiles.

d Find D_6 and D_9.

e If the drivers had to complete an error-free circuit in under 125 s to pass the defensive-driving course, how many passed?

2 The percentage ogive below shows the heights of some candidates who applied to join the Queensland Police Service.

Police candidates' heights

a Find the median and quartiles.

b Find the 90th percentile.

c Until 1990, officers had to be at least 178 cm tall. If this regulation still applied, what percentage of these candidates would have been ineligible?

d Find P_{15}, P_{42} and P_{85}.

e Find D_2, D_6 and D_4.

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3 The table below shows the times at which some students said they left for school.

Time (am)	7:30–7:45	7:45–8:00	8:00–8:15	8:15–8:30	8:30–8:45	8:45–9:00
Number	5	12	18	16	8	3

Use interpolation to find:

a P_{35}, P_{45} and P_{78}

b D_1, D_4 and D_9.

4 The table below shows the masses of a group of students.

Mass (kg)	40–49	50–59	60–69	70–79	80–89	90–99	100–109	110–119
Number	6	13	17	19	12	8	5	2

Use interpolation to find:

a P_{32}, P_{65} and P_{18}

b D_2, D_4 and D_8.

Modelling and problem solving

5 A market research firm interviewed shoppers leaving the Myer Centre in the Brisbane in the week before Christmas. They asked shoppers what was the greatest amount that they had spent on a single Christmas present. The results are summarised in the table on the right.

a Use the table to draw an ogive.

b Use the ogive to find how many people spent less than \$30.00.

c What percentage of shoppers spent at least \$85.00?

d What is the value of the median purchase?

e Up to what value present was purchased by the 35% of customers who spent least?

f What was the least value present purchased by the 15% of customers who spent most?

Amount spent (\$)	Number of customers
0.00–19.99	20
20.00–39.99	62
40.00–59.99	38
60.00–79.99	19
80.00–99.99	10
100.00–119.99	4
120.00–139.99	7
140.00–159.99	8
160.00–179.99	2
180.00–199.99	1

6 The numbers of hours of work experience gained by students in a school–industry link program were recorded as shown on the right.

a A period of 70 hours or more in work experience is required for students to be awarded a merit certificate. How many students spent too few hours in work experience to gain this award?

b What percentage of students were eligible for an advanced certificate (greater than 85 hours)?

c What was the least number of hours worked by the top 35% of students in the program?

d What was the maximum number of hours worked by the bottom 15% of students?

Hours	Number of students
10–19	3
20–29	9
30–39	15
40–49	39
50–59	31
60–69	13
70–79	6
80–89	4
90–99	2

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7 An assembly worker fitted the following numbers of electronic components to a motherboard in 65 consecutive quarter-hour periods:

42	36	26	15	13	43	28	33	38	24	32
44	20	38	31	18	38	47	11	16	40	26
43	26	23	32	27	38	24	33	38	41	46
47	36	43	24	26	34	45	30	30	32	24
24	44	27	35	48	18	22	46	40	30	26
42	30	28	33	23	27	32	39	27	17	

a Using the classes 10–14, 15–19, 20–24, etc., present the data in a cumulative frequency table.

b Draw a percentage ogive for this data.

c For what percentage of the time could the assembly worker be expected to fit at least 30 components in 15 minutes?

d A supervisor decides to give a bonus for 'high productivity'. What number of components would you suggest as the minimum to be fitted in a quarter of an hour to gain a bonus?

8 The table shows the Australian population on the Census night of 8 August 2006.

Age	0–4	5–9	10–14	15–19	20–24	25–29	30–34
Number	1 260 403	1 308 863	1 367 940	1 356 910	1 347 362	1 276 929	1 399 459

Age	35–39	40–44	45–49	50–54	55–59	60–64	65–69
Number	1 466 184	1 471 658	1 446 725	1 315 787	1 234 602	958 077	757 386

Age	70–74	75–79	80–84	85–89	90–94	95–99	100+
Number	616 051	543 611	404 484	214 319	85 732	19 649	3157

a Find the median age of the Australian population.

b Find the 35th and 65th percentiles and hence the age range that contains the middle 30% of the Australian population.

c Find the age range that contains the middle 90% of the Australian population.

11.5 Using summary statistics

A convenient way to show the dispersion of a set of data in a visual form is the **box-and-whisker plot**. Box-and-whisker plots can also be used to compare different sets of data.

!

A **box-and-whisker plot** (**boxplot**) is constructed on a scale as follows:

1. Place 5 points on the scale to show the lowest score (Q_0), Q_1, median (Q_2), Q_3 and highest score (Q_4).
2. Draw in a box from Q_1 to Q_3 with a division at the median.
3. Draw lines (whiskers) from the box to the extreme values.

The boxplot may be drawn vertically or horizontally.

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The five values (Q_0, Q_1, Q_2, Q_3, Q_4) used in a box-and-whisker plot are sometimes called a **five-number summary**.

Example 18

a Construct a box-and-whisker plot to show these dress sizes of a group of women:

20 12 16 18 10 16 16 14 20 18 16 12 14 12 16 16 18 10 22 10 12 18

b Use a graphics calculator to show the boxplot.

Solution

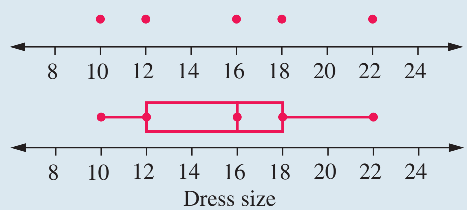
a A graphics calculator can be used to find the 5 points, entering the data in List 1, as shown in Examples 6 and 10.

Lowest score = 10, Q_1 = 12, median = 16, Q_3 = 18, highest score = 22

Draw a scale from 8 to 24.

Put each point on the scale.

Draw the box and whiskers.



b In each case, the data should be entered into List 1. The graph type must be set to a boxplot instead of a histogram or line graph, and the X variable set to L1 with the frequency set to 1. Refer to pages 183–184 to check the method. The results are below.

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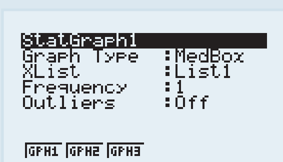

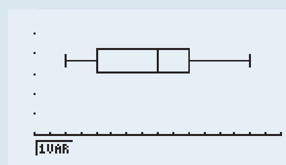

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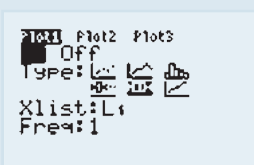

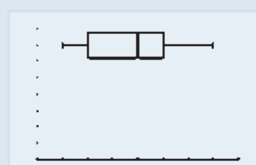

Sharp EL-9900

Calculator Instructions

Instructions for the Sharp calculator are provided on the CD-ROM.

Example 19

The data below shows the times (seconds) taken by some 14-year-old boys and girls to run 50 m. Draw side-by-side boxplots and hence compare the times of boys and girls.

Boys: 8.3 8.1 8.1 7.8 8.8 8.1 7.4 8.0 7.8 9.1 8.2 8.6 8.1 8.2 7.6
8.0 7.6 7.9 8.8 8.5 8.3 8.3 8.3 8.5 8.2 8.5 8.0 8.0 8.6 7.9

Girls: 8.4 8.3 8.8 8.2 8.9 9.7 8.5 8.4 8.8 9.0 8.7 8.3 8.1 8.4 9.0
8.6 8.9 8.4 9.3 9.1 8.5 8.4 9.5 8.5 8.5 8.5 9.7 8.9 8.6 8.4

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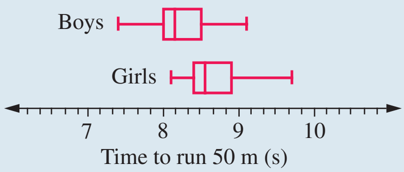
Solution

Use a graphics calculator for the boys.
minX = 7.4, Q_1 = 8.0, median = 8.15, Q_3 = 8.5, maxX = 9.1

Use a graphics calculator for the girls.
minX = 8.1, Q_1 = 8.4, median = 8.55, Q_3 = 8.9, maxX = 9.7

Draw both boxplots on the same scale.



In general, the boys run faster than the girls at this age, but some girls do run faster than many of the boys.

Note: Side-by-side boxplots can be drawn on a graphics calculator by entering the boys and girls into separate lists.

The choice of which measures of central tendency and dispersion should be used depends on two main factors: the nature of the data and the proposed use of the measures.

!

Properties of the mean, median and mode

- For large data sets of a symmetrical distribution, they are about the same.
- The mean and median cannot be calculated for nominal data.
- The mean may not be sensible for some sets of discrete data.
- All the data is used to calculate the mean.
- The mean is affected by extreme values, but the median and mode are not affected.
- The mode is the most probable value, if an item is chosen at random.
- The median is the most central value.
- The mean is the most commonly used measure.

Properties of the range, interquartile range and standard deviation

- The range measures the overall spread of data.
- The interquartile range measures the spread of the middle half of the scores.
- The standard deviation is based on all the scores.
- The standard deviation should be used when making comparisons between data sets as it uses all the data.
- The standard deviation is the most commonly used measure of spread.

Example 20

For each of the following situations, which measure of central tendency should be used?

a The maximum temperatures in Toowoomba every day in January are recorded over a 20-year period. What is the most likely maximum temperature on 15 January?

b The house prices in Aspley are monitored over a 12-month period. What is the typical house price?

c The competitors in an iron-man competition on the Gold Coast are weighed before competing. What is the typical mass of an iron-man competitor?

Solution

a The most likely temperature is the mode. It occurs the greatest number of times.

b There could be a very expensive house sold, which would greatly affect the mean, so the median should be used.

c There is no particular reason to use the mode or median, and the data is continuous, so the mean should be used.

Example 21

For each of the following situations, which measure of dispersion should be used?

a To determine how unusual a particular person's weight is

b To determine the spread of the majority of some data

c To determine the spread of students' test scores on the QCS multiple-choice test

Solution

a The SD should be used to find how many SDs from the mean the person's weight is: the more SDs away, the more unusual the weight.

b The interquartile range should be used to find the spread of the middle half of the data.

c The range should be used because the entire spread is needed.

Investigation Media use of statistics

Use the collection of newspapers and magazines in your school library to investigate the uses of statistics. Try to determine which measures are used in each article. You can also use Internet sites to find uses and misuses of statistics. For each article, list the measures that seem to be used or misused and classify the use as appropriate or inappropriate.

Discuss your findings as a class to determine whether the media generally use statistics appropriately or not. You may find the site *http://ink.news.com.au/mercury/mathguys/mercindx.htm* useful in this discussion.

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Example 22

Fran scored 25/45 in her Biology test and 30/55 in her Physics test. In Biology, the class mean was 20 and the SD was 12, while in Physics the class mean was 24 and the SD was 14. Use the SD to find in which subject Fran did better, compared with the rest of the class.

Solution

The SD measures the spread of scores from the mean.

Find how many SDs Fran is above the mean in each case.

In Biology, she was 5 above the mean.	SDs above mean = $5 \div 12 \approx 0.42$
In Physics, she was 6 above the mean.	SDs above mean = $6 \div 14 \approx 0.43$
Write the answer.	Fran did slightly better in Physics.

Exercise 11.5 Using summary statistics

Additional Exercise 11.5

1 An archaeologist discovered 25 coins that had been produced in the Middle Ages. The masses (in grams) of the coins were:

8.12 8.21 8.33 8.12 8.26 8.14 8.21 8.56 7.78 8.24 8.34 8.62 8.45
7.89 8.25 8.65 8.19 8.77 7.91 7.80 8.32 8.56 8.64 8.73 9.84

Draw a box-and-whisker diagram to show these results.

2 The numbers of typing mistakes in a page of typing were recorded as follows for 24 business college students:

4	1	0	12	9	4	5	7
2	0	1	9	5	9	10	21
4	7	6	3	5	4	8	0

Draw a boxplot for this data.

Modelling and problem solving

3 The relative humidities (%) at 3 pm in two towns over a period of 2 weeks were:

Town A:	33	35	67	45	48	67	84	56	58	57	45	48	68	56
Town B:	45	48	67	78	79	84	65	58	43	59	69	89	78	69

a Draw a side-by-side boxplot of these humidities.

b Compare the results and suggest which town may have received more storms.

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4 The maximum temperatures (°C) in two Queensland towns over a fortnight were:

Town A:	22	24	28	32	35	27	24	29	28	29	27	31	23	30
Town B:	25	26	26	24	26	28	28	26	29	30	27	24	29	31

a Draw boxplots to compare the temperatures.

b Which town is more likely to be near the sea? Give reasons for your answer.

5 Two golf ball manufacturers produce balls with a nominal diameter of 4.3 cm. A sample of 20 balls was taken from those produced by each company and measured. The results (in centimetres) were:

Beaut Ball Company:

4.29 4.30 4.29 4.30 4.34 4.30 4.32
4.30 4.29 4.29 4.28 4.31 4.31 4.27
4.28 4.30 4.29 4.30 4.28 4.28

E-Zee Ball Company:

4.27 4.36 4.31 4.30 4.30 4.29 4.28
4.30 4.37 4.31 4.30 4.37 4.28 4.30
4.26 4.34 4.31 4.35 4.34 4.39

Use a side-by-side box-and-whisker plot to compare the outputs of the companies and decide which company has the better production processes.

6 A new program to help people stop smoking was trialled by the Queensland Department of Health. Twenty-five smokers who entered the program were randomly selected from all participants, and the number of cigarettes that each smoked per day was recorded. Two months after the completion of the program, the same people were again contacted in relation to their cigarette consumption. The results were as follows:

Cigarettes per day on entry into the program:

9 30 8 32 10 36 18 20 10 27 17 24 18
11 12 24 17 21 15 23 15 22 18 13 22

Cigarettes per day 2 months after completing the program:

8 34 9 21 11 23 8 23 12 9 24 19 25
17 23 16 22 17 27 16 24 12 10 21 9

Use side-by-side boxplots to determine the success of the program. What conclusion about the trial would you give to the Department of Health?

7 The percentage efficiencies of some pieces of machinery were found to be:

90 91 89 91 89 90 87 88 90 87 90 91

a Find the values of the mean, median and mode.

b What is the general efficiency level?

8 The weekly numbers of accidents on a dangerous stretch of road were recorded over 26 consecutive weeks of a study.

2 1 6 3 2 4 0 5 2 3 4 7 3 1 4 2 0 5 6 3 2 4 5 2 5 4

a What is the modal number of accidents per week?

b What is the mean number of accidents per week?

c Calculate the median.

d Which measure should be used as the typical number of accidents?

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9 The minimum daily temperatures (°C) in Stanthorpe were recorded over a 50-day period:

3 4 7 8 2 5 6 3 1 7 1 5 2 3 7 3 5 1 4 2 4 3 6 4 0

3 6 3 1 6 4 3 1 7 8 3 4 9 6 2 3 5 8 3 9 1 7 2 3 5

a What is the most likely minimum?

b What is the typical minimum?

10 At the start of a weight-watchers' course, all participants take part in a weigh-in. Their masses are summarised in the table below.

a Find the mean, median and mode.

b Which result would best typify the 'weight' of the participants?

Mass (kg)	Number of weight-watchers
70–74	4
75–79	8
80–84	10
85–89	14
90–94	11
95–99	9
100–104	4

11 The 12 runners in a horse race are given the following weights by the handicapper.

No. 1	58 kg	No. 7	53 kg
No. 2	56 kg	No. 8	52 kg
No. 3	55 kg	No. 9	52 kg
No. 4	54.5 kg	No. 10	52 kg
No. 5	54.5 kg	No. 11	50 kg
No. 6	54 kg	No. 12	48 kg

a Find the mean, mode and median of the handicapper's weights.

b What is the typical weight?

c If the top weight (58 kg) is given a 2.5 kg allowance because it is ridden by an apprentice jockey, which (if any) of the three measures calculated in part **a** will remain unchanged?

12 Hasim and David were comparing their results. Hasim got 56 in Maths A and 45 in Chemistry, while David got 48 in Maths A and 56 in Physical Recreation. Hasim claimed that since he did better in Maths A than David, Physical Recreation must be easier than Chemistry. The mean and SD in Chemistry were 55 and 8 respectively, while in Physical Recreation they were 60 and 5. Use these figures to determine whether Hasim was correct.

13 Peter's weight is 85 kg and his height is 183 cm. The average weight and height are 78 kg and 175 cm respectively, with SDs of 15 kg and 12 cm. Would you expect Peter to appear thin, fat or ordinary?

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Chapter summary

- The **central tendency** of data is measured by the mode, the mean and the median.
- The **mode** of a set of data is the most common score. If there are two scores with equal highest frequency, we say the distribution is **bimodal** and give both values.
- The **mean** is the arithmetic average of the scores with the symbol $\bar{x}$. It is calculated by either:
  - adding the individual scores and dividing by the number of scores, or
  - adding the products of the scores and frequencies and dividing by the total frequency.

  This is written as: $\bar{x} = \frac{\Sigma x}{n}$ or $\bar{x} = \frac{\Sigma fx}{\Sigma f}$ where Σ means sum (the total of).
- The **median** is the middle score, when all scores are written in order from smallest to largest. If there is an even number of scores, then it is the average of the middle two scores.

  In general, the median is the $\frac{n+1}{2}$ th score, provided the scores are in order.
- Three different **measures of dispersion** are used to calculate the **spread** of data: the range, the interquartile range and the standard deviation.
- The **range** measures the overall spread of the scores.

  Range = highest score − lowest score
- The **interquartile range** measures the spread of the middle half of the data.

  Interquartile range = third quartile − first quartile
- The **first quartile** (**Q_1, lower quartile**) is the value that lies a quarter of the way through the data. The **third quartile** (**Q_3, upper quartile**) is the value that lies three-quarters of the way.

  If n is odd: $Q_1 = \frac{n+1}{4}$ th data item. If n is even: $Q_1 = \frac{n+2}{4}$ th data item.

  The third quartile is the corresponding item back from the last data item.
- The **standard deviation** (**SD** or σ) (where Σ means the sum) is calculated using the formula:

  $\sigma = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$ for individual scores or $\sigma = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$ for tables.
- For grouped data tables:
  - The mean and standard deviation are calculated using **class midpoints**.
  - The median, range and interquartile range are calculated using **true class limits**.
  - **Interpolation** is use to find the median, Q_1 and Q_3.
  - The formula for the value of the ***m*****th term** in a class interval is $L + \frac{m}{f} \times w$

    where L is the lower class limit, f is the frequency of the class and w is the class width.
  - A **cumulative frequency polygon** or **ogive** can also be used to find the median and quartiles as the scores at 50%, 25% and 75% of the total frequency.
  - A **modal class** is stated for grouped frequency tables.
- **Quantiles** divide the data into fractions, and may be calculated using interpolation or graphs.
  - **Percentiles** divide the data into percentage groups. The 35th percentile (shown as P_{35}) is the score below which 35% of all scores lie.
  - **Deciles** divide the data into tenths. The 7th decile (D_7) is the score below which $\frac{7}{10}$ ths of the data lies.
- The dispersion of a data set can be shown in a **box-and-whisker plot** (**boxplot**), which shows the positions of the lowest score (Q_0), Q_1, median (Q_2), Q_3 and highest score (Q_4) on a scale. These five values are sometimes called a **five-number summary**.

Chapter review

Knowledge and procedures

1 What is the formula for calculating the mean of data? Ex 11.1

2 How is the interquartile range calculated? Ex 11.2

3 Find the mean, median, mode, range, interquartile range and standard deviation of this data. Ex 11.1, 2, 3

a 23, 28, 29, 25, 26, 25, 29, 28, 22, 24, 21, 31, 32, 24, 27, 24, 26

b

x	f
12	3
13	4
14	7
15	9
16	6
17	5
18	4
19	3
20	1

c

Score	f
5–9	3
10–14	6
15–19	12
20–24	18
25–29	19
30–34	23
35–39	21
40–44	17
45–49	14
50–54	8
55–59	4

4 Use a graph to find the median and interquartile range of the following data. Ex 11.3

Score	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90–99
f	4	6	8	9	11	8	6	5

5 Find P_{23}, P_{86}, D_1 and D_6 for the data in the table below. Ex 11.4

Score	85–89	90–94	95–99	100–104	105–109	110–114	115–119	120–124	125–129
f	3	7	8	10	11	9	7	5	4

6 Construct a boxplot for the following data: Ex 11.5

16 15 12 18 19 21 22 14 10 15 16 17 14 18 17 19 15

Modelling and problem solving

7 The ages of spectators at a hockey match are given in the table on the right. Ex 11.1, 3, 5

a Calculate the mean, median and modal age of the people at the match.

b What was the typical age of the spectators?

Age	Number
15–24	57
25–34	188
35–44	221
45–54	75
55–64	117
65–74	51

8 During a chickenpox epidemic, the numbers of cases seen each day by general practitioners in Logan City were recorded as follows. Use a graphics calculator to calculate the mean and standard deviation of this data. Ex 11.3

Number of cases	100–109	110–119	120–129	130–139	140–149
Frequency	3	7	14	9	7

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Chapter review

Ex 11.3, 4

9 Police in a 100 km/h zone registered the following speeds on a radar gun:

98 100 92 102 99 110 114 108 101 94 95 98 108 102 103 107 111
114 78 96 103 101 98 109 99 118 79 91 108 93 119 102 95 97
100 112 94 103 96 120 91 104 113 101 100 99 115 109 82 97

a Make a cumulative frequency distribution table for this data and draw a percentage ogive.

b The police gave a ticket to every motorist who was travelling at more than 104 km/h. How many tickets were issued?

c Find the speed below which:
i 22% of the values lie **ii** 75% of the values lie **iii** 50% of the values lie.

Ex 11.3, 4

10 The working life of a particular electronic component is tested by a consumers' association. The results are shown in the table.

a Use a percentage ogive to find the 10th, 21st and 83rd percentiles.

b The manufacturer wants to guarantee that the components will last for a certain minimum life. Which of the values that you have calculated would you recommend for this purpose? Explain why.

Life of component (hours)	Number
0–299	8
300–599	15
600–899	31
900–1199	127
1200–1499	382
1500–1799	142
1800–2099	78

Ex 11.3, 5

11 A particularly dangerous stretch of road was studied and, among other things, the ages of drivers involved in serious accidents were collected. The ages collected were:

20 18 21 38 19 20 68 22 51 43 25 28 17 23 28
57 20 18 49 69 60 23 18 21 67 72 18 24 32 66
21 19 18 37 69 22 23 54 73 19 20 59 68 18 17
45 73 67 17 25 21 32 41 49 64 71 22 24 21 67

a Make a cumulative frequency distribution table with suitable classes.

b Find the range, interquartile range, standard deviation, mean and median.

c Construct a boxplot for this data.

Ex 11.1

12 The mean weight of the Brisbane Broncos football squad is 115 kg. If two new recruits weighing 125 kg and 130 kg join the existing squad of 18 players, what will be the new mean weight of the squad?

Ex 11.5

13 A golfer had the following scores for her last 20 rounds of golf:

86 102 81 90 105 78 89 86 80 91 82 79 81 88 90 101 94 82 86 88

A new member of the club has the following scores for her first 10 rounds of golf:

132 118 128 141 120 92 138 90 147 144

Use a side-by-side box-and-whisker plot to compare the performances of the two golfers.

Ex 11.5

14 Two food-packaging machines are used to pack 2 kg containers of flour. A sample of 20 packs from each machine was taken and weighed. The results (kilograms) were:

Machine 1: 2.088 2.076 2.099 2.070 2.095 2.068 2.008 2.099 2.016 2.087
2.012 2.021 2.091 2.037 2.074 2.085 2.002 2.070 2.068 2.006

Machine 2: 2.070 2.068 2.061 2.076 2.077 2.070 2.081 2.071 2.062 2.069
2.066 2.079 2.064 2.072 2.068 2.072 2.077 2.071 2.068 2.066

Use the mean and standard deviation to compare the outputs of the two machines, and comment on the effectiveness of each machine.

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Building and construction calculations

12

Contents

Syllabus Guide
Chapter 12

Syllabus subject matter

Elements of applied geometry

- Applications of Pythagoras's Theorem
- Area, volume and capacity in life-related situations

Linking two and three dimensions

- Interpretation of scale drawings and plans
- The geometry of bracing for rigidity
- Practical tests for squareness, plumbness and levels
- Estimation of quantities and costs in a variety of construction areas

Quantitative concepts and skills

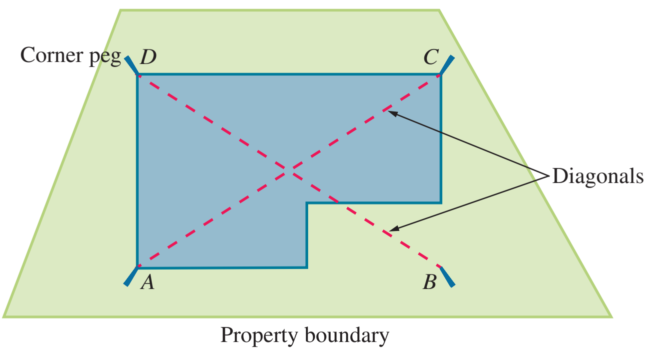
- Metric measurement including measurement of mass, length, area and volume in practical contexts
- Calculation and estimation with and without instruments

Even though the ancient Egyptians didn't know the theory behind Pythagoras's Theorem, they knew from experience that they could form right angles by using a Pythagorean triple. They used this principle to determine the right angles for the square base of the pyramid of Khufu with amazing accuracy.

It is highly likely that Egyptian Pharaohs employed people to monitor the cost of building the pyramids. In modern times, the role of controlling construction costs by accurate measurement of the work required and by knowledge of the costs and prices of the work, labour, materials and plant required is undertaken by a quantity surveyor.

12.1 Squaring up

When a house is set out on a site, the first step is to position four corner pegs to show the outside corners of the house. Even if the house is not rectangular, four pegs are still used. Pythagoras's Theorem may be used to square up the pegs when the property boundaries are not parallel. The diagonals should be equal when the corner pegs have been positioned.

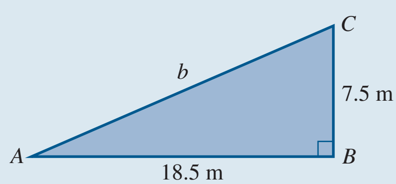


Example 1

The front pegs, *A* and *B*, of a house (as shown in the diagram above) are 3 m from the front boundary and 18.5 m apart. Peg *C* is to be set back 7.5 m from peg *B*. How long should the diagonal be for the corner of the building to be square (that is, for $\angle ABC$ to be a right angle)?

Solution

Draw a diagram showing *A*, *B* and *C*.



Write Pythagoras's Theorem.
Calculate the value of *b*.

$$b^2 = 7.5^2 + 18.5^2$$
$$= 56.25 + 342.25$$
$$= 398.5$$
$$b = \sqrt{398.5}$$
$$= 19.9624\ldots \text{ m}$$

Round to nearest mm and write the answer. The diagonal should be 19.962 m long.

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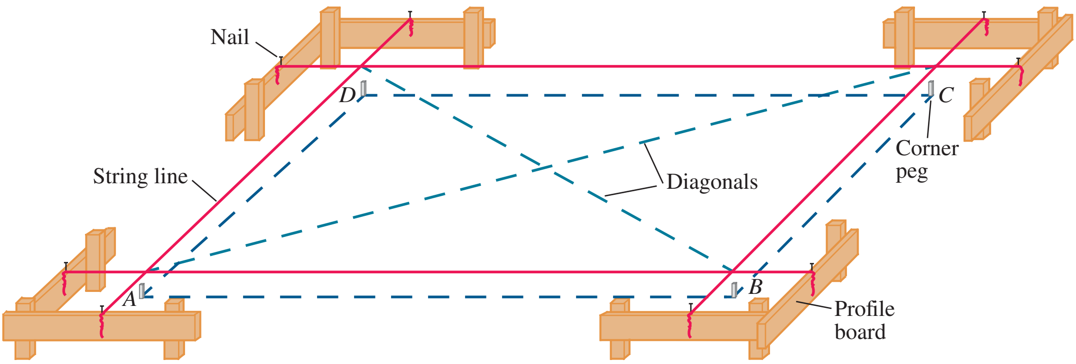
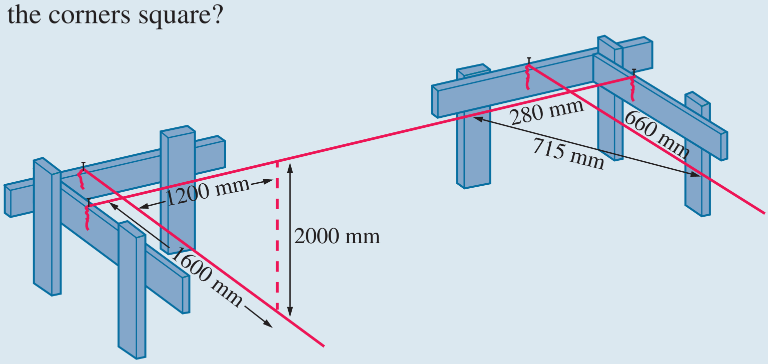
In Example 1, once *C* was positioned by two workers with tapes stretched out to the lengths *AC* and *BC*, peg *D* would be positioned in the same way. After the pegs of a house are in place, **profiles** are set up. These are set back about 1200 mm from the corners of the house to mark the edges. The **profile boards** must be set level and have saw cuts (**kerfs**) or nails to mark the external wall edges, the foundations and other lines needed to construct the building. The corner profiles are squared up using Pythagoras's Theorem (usually a 3–4–5 triangle) and checked using string lines that should intersect over the corner pegs of the house. Once the profiles are in place, the corner pegs are no longer needed; they are discarded when the foundations are dug out.



Example 2

A builder takes measurements from the string lines that are attached to profiles as shown below. Are the corners square?



Solution

Use Pythagoras's Theorem to check.

Change measurements to metres.

$$\begin{aligned} 1.2^2 &= 1.44 \\ +\,1.6^2 &= \underline{2.56} \\ &\ 4.00 \end{aligned} \qquad 2.0^2 = 4.00$$

State the result. Since $1.2^2 + 1.6^2 = 2.0^2$, the corner with the 1200, 1600 and 2000 mm measurements is square.

Change measurements to metres.

$$\begin{aligned} 0.28^2 &= 0.0784 \\ +\,0.66^2 &= \underline{0.4356} \\ &\ 0.5140 \end{aligned} \qquad 0.715^2 = 0.511\,225$$

State and explain the result. Since $0.28^2 + 0.66^2 \neq 0.715^2$, the corner with the 280, 660 and 715 mm measurements is not square. The corner must be opened out to increase the hypotenuse.

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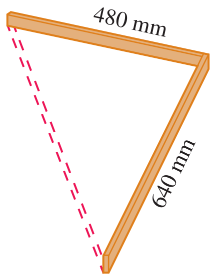
Additional Exercise 12.1

Exercise 12.1 Squaring up

1 The front pegs, A and B, of a house are 12.8 m apart and peg C is 8.4 m behind peg B. How long should the diagonal from A to C be to ensure that this corner of the house is square?

2 Two pieces of timber 480 mm and 640 mm in length are to be used to make a builder's square. How long should the other piece of timber be cut?

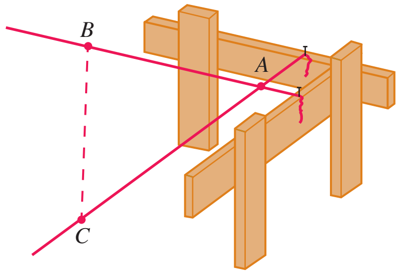


3 A builder is about to square up string lines between profiles as shown here. In each case, calculate the distance that BC will need to be for the profile hurdles to be square.

a $AB = 1740$ mm and $AC = 2320$ mm

b $AB = 600$ mm and $AC = 1440$ mm

c $AB = 3420$ mm and $AC = 1860$ mm



4 For a building with a rectangular base measuring 10 m by 5 m, how long will the diagonal need to be if the base is square?

5 A builder sets out pegs and finds that one pair of opposite sides measures 15 700 mm and the other pair is 6400 mm. Can the builder assume that the profile is square? Explain your answer.

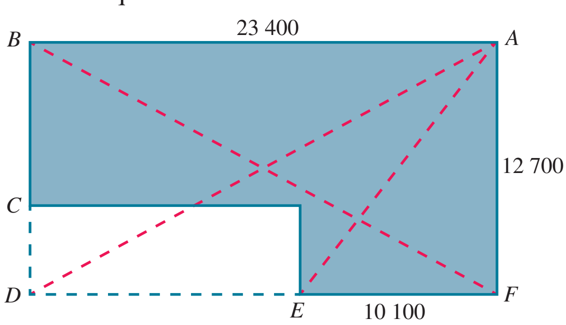
6 After the pegs for a house are set up, each diagonal is found to be 16 200 mm. Will it be necessary to measure the lengths of the sides to check that they form a rectangle? Justify your result.

Modelling and problem solving

7 The front pegs of a house are 16.2 m apart and so are the rear pegs. The diagonal from A to C is 0.4 m shorter than the diagonal from B to D. The side measurements are both 9.3 m. How should the back pegs be moved to square the house? Explain your answer.

8 **a** This is the plan of an L-shaped house. Calculate the length that diagonals AD and FB will need to measure in order for the base to be square.

b What length will AE need to measure if this portion of the base is to be square? Show your steps in logical order.



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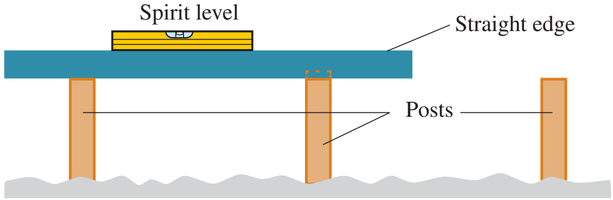
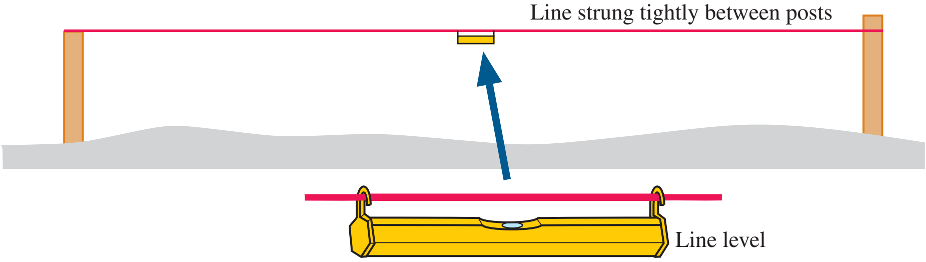
12.2 Finding levels

In building, the tops of vertical posts such as supports for a pergola or fence are often required to be at the same height. Unfortunately, they are rarely situated on level ground. This means that a method of **levelling** the tops other than measuring the distance from the ground must be used. The methods used to establish levels depend on the distance over which the level is required.

For short distances a **spirit level** may be useful. Distances longer than the length of the spirit level can be managed by using a straight edge and transferring levels via a series of intermediate posts, as shown below. However, this method is quite inaccurate.

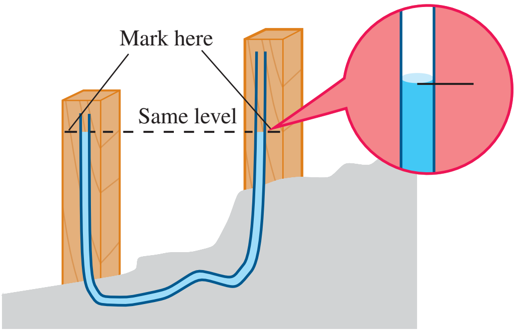


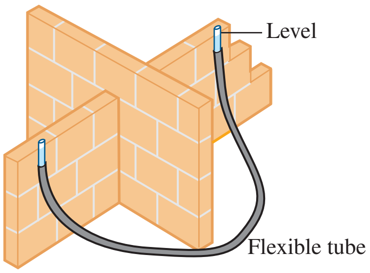
Another method of establishing levels in this situation is by use of a **line level**, thus eliminating the need for intermediate posts. However, a line level is a small instrument, and this means it is also inaccurate. In addition, the sag in the line introduces further error.



A **water level** can also be used to find levels. A water level is made by filling a length of clear plastic tubing with water. Provided the ends are open to the atmosphere and the tube is free from air bubbles and obstructions, the level of the water at one end of the tube is always the same as the level of the water at the other end.

To use a water level, the tube is stretched between two points and then adjusted until the water level at one end matches a mark at the required height. The water level at the other end is then marked to show the same level. This is a quick and accurate method of obtaining level points. It also has the advantage that the flexible tube can be moved around obstacles.





Optical levels are used by builders when a higher degree of accuracy is required. The major optical levels are the dumpy level, automatic level and laser level.

Dumpy level

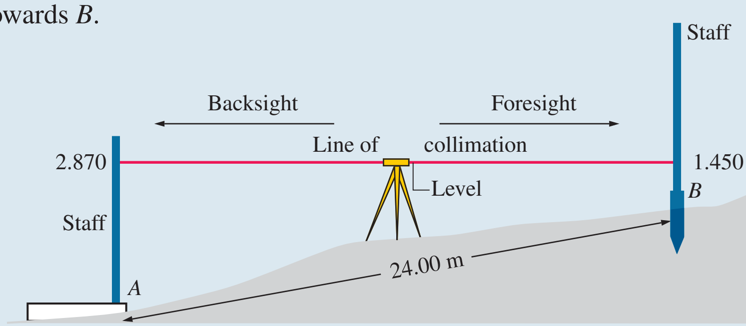
Automatic level

Laser level

Once an optical level is set up on its tripod, the required level is established as a reference. The level can be swivelled to sight other objects that require levelling. The positions of these points are then measured in relation to the reference level. These levels can be used with a **levelling staff** to quickly determine the rise or fall of a building site. The line of sight is called a **line of collimation**. The levels are normally referenced to some particular point on the site called the **datum point**. This point is often the lowest point on the site, but it may also be a survey point.

Example 3

A builder's level is set up between points A and B as shown. The levelling staff readings are shown. Calculate the rise between the two staffs and the angle at which the land slopes from A towards B.

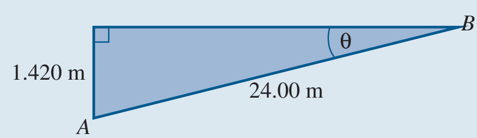


Solution

Find the rise by subtraction.

Rise $= 2.870 - 1.450$ m $= 1.420$ m

Draw a triangle to find the angle.



Mark the information on the triangle.
1.420 m is opposite and 24.00 m is the hypotenuse, so use the sine ratio.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1.420}{24.00}$$

Keep the exact number on your calculator.

$$= 0.0591\ldots$$

Use $\sin^{-1}$ on your calculator.

$$\theta = 3.3919\ldots°$$

Round and write the complete answer.

The angle is about 3.4°, so the ground slopes upwards from A to B at an angle of 3.4°.

Investigation Levelling

Work in groups of three or four. For this investigation your group will require:

- a number of stakes of equal length, suitable for hammering into the ground
- a length of 'brickies' string
- a spirit level
- a line level
- a length of clear plastic tubing.

1 Select a site in the school grounds where two stakes can be hammered into ground that is not level. Make sure that the rise or fall isn't more than the length of a stake.

2 Mark two stakes about 10 cm up from the sharp end, then hammer in the stakes at least 10 m apart so that the same length of stake is left above the ground.

3 Mark a point near the top of one of the stakes, then transfer this level to the other stake, using:

  a a spirit level and intermediate stakes **b** a line level.

4 Compare the results using a water level.

5 Discuss your findings with those obtained by other groups and comment on the accuracy of each method used.

6 By how much does the ground rise or fall over the distance between the two stakes?

7 Use this information and the distance that the two stakes are apart to calculate the rise or fall for the ground in degrees.

In many cases, a level must be established in order to make sure that there is sufficient fall for drainage. The requirements for drainage are often specified as a ratio, giving the fall as a ratio of distance. Thus a required fall of 1 : 50 would mean that the fall must be 1 m for every 50 m along the line of drainage.

Example 4

For the rainwater gutter to drain properly, the fall of the guttering must be at least 1 : 500. How much fall is required for a gutter that runs along the front of a 16.4 m-wide house to a downpipe at one end?

Solution

Write the ratio as a fraction. $1:500 = \frac{1}{500}$

Multiply to find the fall. $\text{Fall} = 16.4 \times \frac{1}{500}\text{ m}$

It can be done as division. $= 16.4 \div 500\text{ m} = 0.0328\text{ m}$

Round *up* to get the fall in appropriate units. The fall needed is about 3.3 cm.

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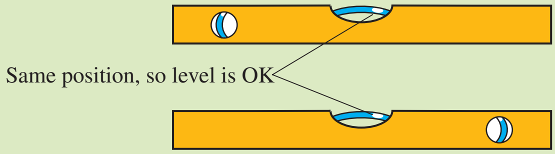
Investigation Vertical walls

In this investigation you will use a spirit level or plumb-bob to test whether walls around your school are vertical. Work in groups of three or four. Each group will require the following equipment:

- a spirit level
- a straight-edge (straight piece of timber about 2 m long)
- small wooden blocks of equal thickness (about 2 cm thick)
- a plumb-bob (which could simply be a fishing sinker tied to a piece of string).

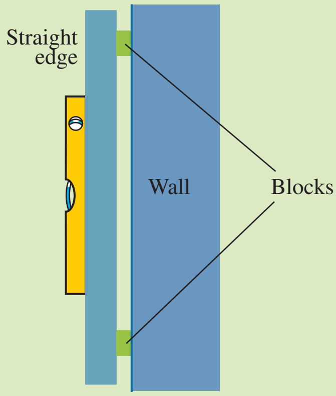
Some groups could use a plumb-bob while others are using a spirit level, to reduce the amount of equipment required.

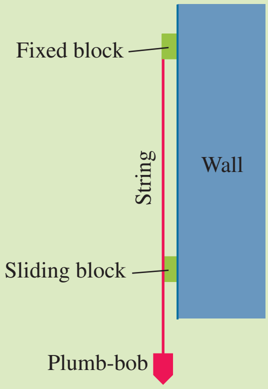
Before beginning the work, check that the spirit level is accurate. This is most easily accomplished by reversing the level in the same position on a flat surface. If the level shows the same reading either way, it is accurate. If the level is not accurate, it may have an adjustment screw that can be used (with your teacher's permission) to make it more accurate.



Attach the small blocks to each end of the straight-edge to make it stand out to avoid surface irregularities. Then you can use it with the level to check for plumbness, as below on the left.

Attach the end of the plumb-bob string to one block and hold it against the top of the wall. Slide the other block between the string and the wall to check for plumbness. If the wall is plumb, the block will just fit between the string and the wall, as shown below on the right.





1. Check the plumbness of:
   - the walls in your classroom
   - the external walls of the building
   - flagpoles and goalposts
   - retaining walls used in the landscaping.
2. When you have finished, decide which method is easier to use when checking for plumbness, which method is cheaper and which method is more accurate.
3. Also discuss whether some walls that you measured should *not* be plumb for any reason.

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Exercise 12.2 Finding levels

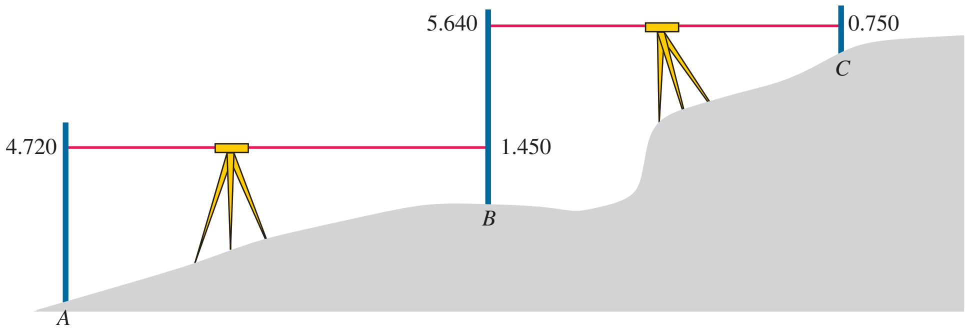
1 A fall of 1 : 200 is needed for a concrete gutter. What fall is needed for a drain that is 48.6 m long?

2 What fall is needed for guttering that runs 13.6 down one side of a house if a fall of 1 : 300 is desired?

3 Using a dumpy level, measurements of 4.43 m and 2.95 m are taken at two corners of a block of land. What is the fall between the corners?

Modelling and problem solving

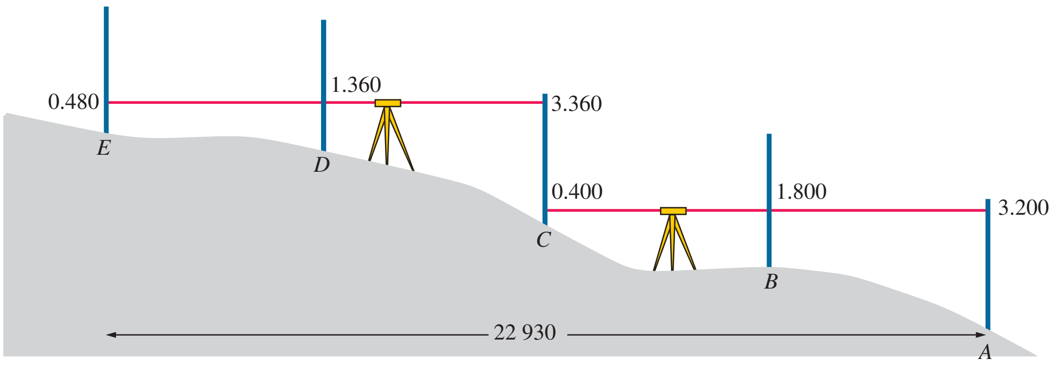
4 A dumpy level and measuring staff are used to find the change in level from point *A* to point *C* as shown in the diagram below. Measurements are in metres. Calculate the difference in the ground level between *A* and *C*, showing all necessary steps.



5 The readings (in metres) in the following diagram were taken at a proposed building site.

a How much higher than point *A* is point *E*?

b Calculate the average angle that the slope makes with the horizontal.



6 The site plan at the top of the next page shows the position of a proposed house site. The contour lines on the plan show points of equal level, with each line 500 mm higher than the previous one. Prior to building, a dumpy level is set up between points *A* and *B* and readings of 2.240 m and 3.450 m respectively are taken.

a Calculate the fall across the proposed house site.

b For the house site, find the average angle that the ground makes with the horizontal plane.

c Estimate the fall across the whole block of land, from point *X* to point *Y*, clearly justifying your estimate.

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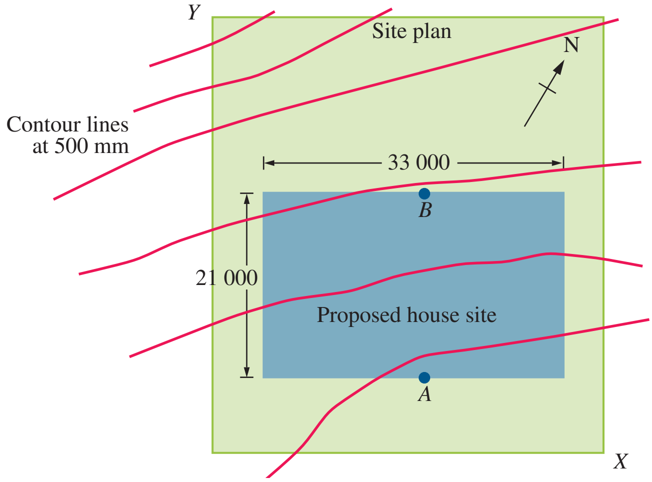
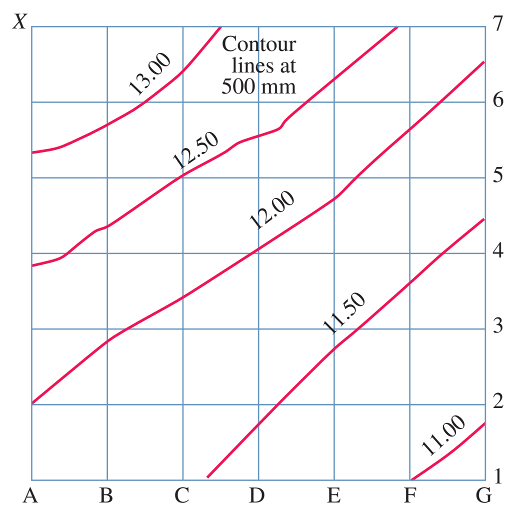


7 The diagram at right shows a large building block prior to its preparation for building. The measurements shown are in metres, and each grid square is 5 m square.

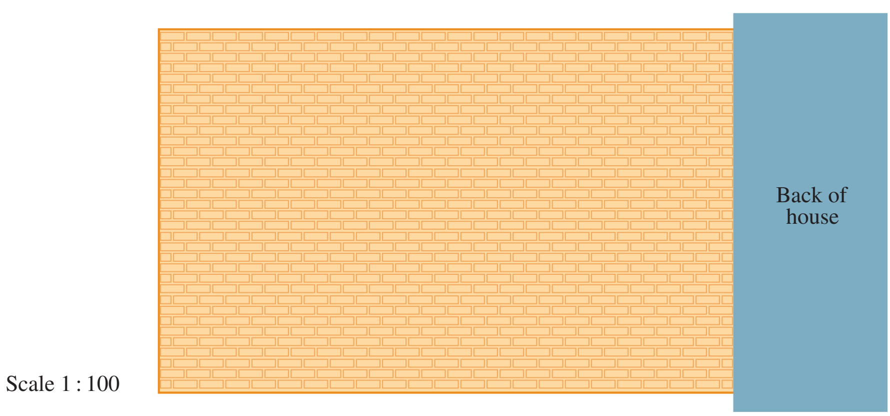


a Estimate the level at each of the following grid points:
   i F1   ii D3   iii B4

b Estimate the total fall across the building site (from point *X* to G1).

c If the site was levelled by moving material from one side of a diagonal line from G7 to A1, how much material would need to be moved? (*Hint:* Use the grid heights method and fully explain your logic.)

8 Paving needs a fall of between 1 : 200 and 1 : 100 to drain properly. The diagram below shows an area to be paved. The area has to drain away from the house.

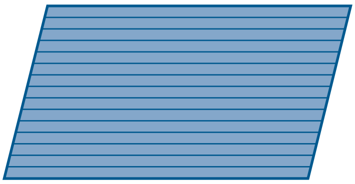


a What actual fall is needed?

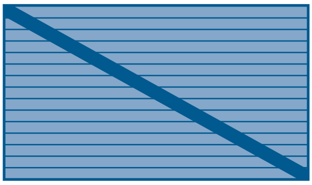
b What angle will the pavers make with the horizontal?

12.3 Bracing for strength

Bracing is used to ensure that the framework of a building remains square. A wall frame that distorts in strong winds or some other force will cause cracking of plaster and paint and could even lead to the collapse of the building. A diagonal timber or metal **brace** is the most common method of bracing, but structural plywood or fibre–cement sheets may also be used.



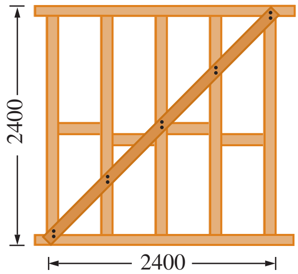
Unbraced wall may distort.



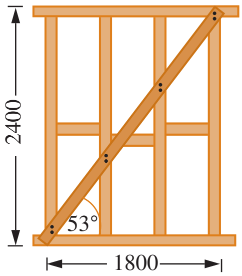
Braced wall is more rigid.

The direction of potential movement of a frame may tend to shorten the diagonal bracing (as shown above) or lengthen the bracing. Shortening of the diagonal is known as **compression** and stretching the diagonal as **tension**. Timber bracing, metal angle bracing and sheeting all provide bracing for both compression and tension. However, flat metal bracing (speed brace) must be used in both directions as it provides only tension bracing.

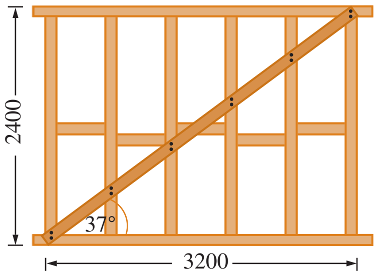
Wall bracing is normally put into position before the frame is stood up, but final nailing is delayed until all the wall frames are erected to ensure that the whole building can be plumbed and squared. More than one brace may need to be used in long walls. Diagonal bracing is strongest when it is at an angle of 45°, but is still sufficiently strong between angles 37° and 53°.



Diagonal bracing at 45 degrees



Diagonal bracing should not be any steeper than the above.



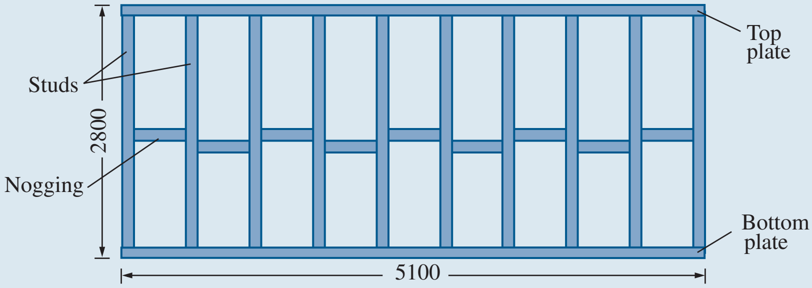
Diagonal bracing should not be any less steep than the above.

Example 5

a Find the longest brace that could be used on the wall frame shown.

b How far would it reach along the bottom plate?

c Could two braces be fitted?



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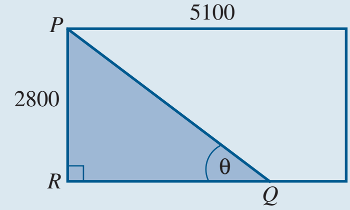
Solution

a Draw a sketch of the triangle formed by the brace.
Name the points and angle.
The longest brace PQ will have the smallest angle θ.



Write the smallest allowable angle. — Smallest value of $\theta = 37°$

PR is the opposite and PQ is the hypotenuse, so use the sine ratio.

$$\sin 37° = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{2800}{PQ}$$

Multiply by PQ. — $PQ \times \sin 37° = 2800$

Now divide by sin 37°. — $PQ = \dfrac{2800}{\sin 37°}$

Use your calculator and keep the number. — $= 4652.5923 \ldots$ mm

Round to the nearest mm. — The longest brace would be 4653 mm long.

b The bottom is the adjacent, so use cos.

$$\cos 37° = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{RQ}{PQ}$$

Use the exact value in your calculator.

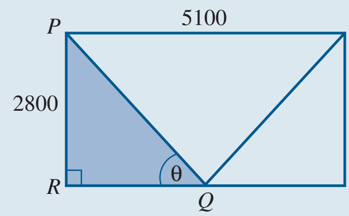
$$= \frac{RQ}{4652.5923 \ldots}$$

Multiply by 4652.5923 … — $RQ = 4652.5923 \ldots \times \cos 37°$

Evaluate. — $= 3715.7255 \ldots$ mm

Round to the nearest mm. — The brace would reach 3716 mm along the bottom plate.

c Two braces should be placed equally, so each would take half the space along the bottom.
Draw a diagram with shorter braces.



RQ would be half the length. — $RQ = 5100 \div 2 = 2550$ mm

Find θ, using tan.

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{2800}{2550} = 1.0980 \ldots$$

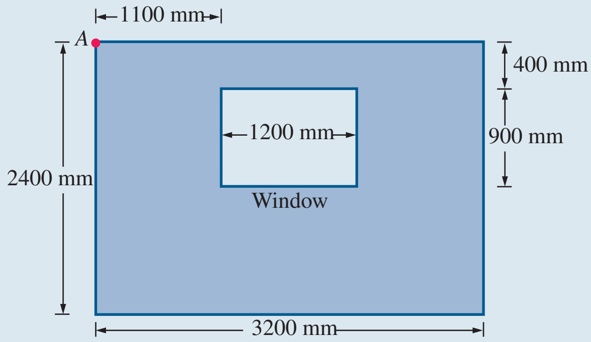
Use $\tan^{-1}$. — $\theta \approx 47.7°$

The angle is between 37° and 53°. — Since θ is acceptable, two braces could be used.

It is usual to put braces on each side of doorways and very large windows to make sure that both sides are rigid. Small windows may influence the position of a diagonal brace. If a diagonal brace cannot be fitted, sheets of structural plywood may need to be used.

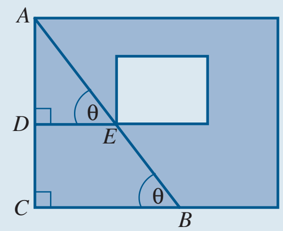
Example 6

For the wall shown on the right, calculate the greatest possible length of bracing from corner A, and the distance along the bottom plate that the brace will reach.



Solution

Sketch the frame and mark the brace so that it touches the corner of the window.
A 45° brace would pass through the window.
The angle can be calculated using the triangle from A to the window corner E.



Calculate AD.

$$AD = 400 + 900 = 1300 \text{ mm}$$

In $\triangle AED$, AD is opposite θ and DE is adjacent, so use tan to find θ.

$$\tan\theta = \frac{AD}{DE}$$

$$= \frac{1300}{1100} = 1.1818\ldots$$

Use $\tan^{-1}$ and keep the exact value.

$$\theta = 49.7636\ldots°$$

θ is acceptable, so find AB using sin in $\triangle ABC$.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{AC}{AB}$$

Put in values.

$$\sin 49.7636\ldots° = \frac{2400}{AB}$$

Multiply by AB.

$$AB \times \sin 49.7636\ldots° = 2400$$

Divide by sin 49.7636 …°.

$$AB = \frac{2400}{\sin 49.7636\ldots°}$$

$$= \frac{2400}{0.7633\ldots}$$

Keep the exact value on your calculator.

$$= 3143.8867\ldots \text{ mm}$$

Now use cos in $\triangle ABC$ to find BC.

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{BC}{AB}$$

Put in values.

$$\cos 49.7636\ldots° = \frac{BC}{3143.8867\ldots}$$

Multiply by 3143.8867 … Use the exact values on your calculator.

$$BC = 3143.8867\ldots \times \cos 49.7636\ldots°$$

$$= 2030.7692\ldots \text{ mm}$$

Round and write the answers.

The brace can be 3144 mm long and will reach 2031 mm along the bottom plate.

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Investigation Simple frames

You have no doubt seen many structures in which frameworks are prominent. They include roof trusses, bridges, towers, high-tension power pylons and crane arms.

The triangle is common to all these structures. All the frames are polygons that have had diagonals placed inside to provide strength and rigidity. The diagonals and other cross-members form triangles, which allow the load to be shared, with each member being placed under compression or tension by the load. The cantilever construction invented by Frank Lloyd Wright and used in many sports stadium roofs is an excellent example of this.

In this activity, you use the principles you have learnt to construct a structure to support an object. Work in groups of three or four. Each group should obtain the following materials:

- 40 plastic straws
- scissors
- adhesive tape
- a marble.

The objective of the investigation is to build structures attached to a desk by adhesive tape to support the marble at:

- the greatest possible *horizontal* distance from the desk
- the greatest possible *vertical* distance from the desk.

Sketch all final structures. Record the greatest horizontal and vertical distances from the desk at which the marble was supported.

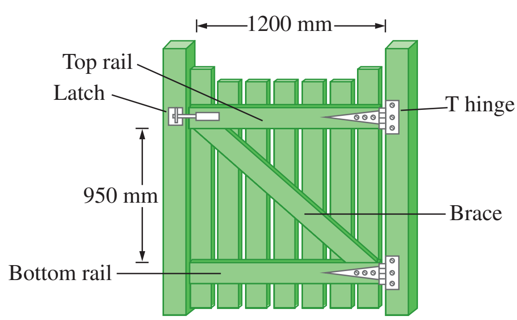
Additional Exercise 12.3

Exercise 12.3 Bracing for strength

Modelling and problem solving

1 a Calculate the length of the brace required for the wooden gate illustrated at right.

b Has the correct diagonal been used for the brace shown, or should the brace run from the top hinge to the bottom rail? Clearly justify your answer.

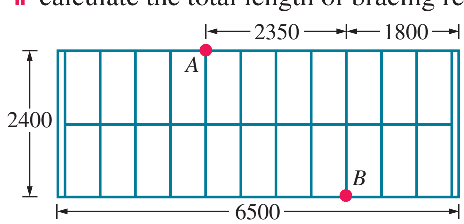


2 For each of the wall frames shown below:

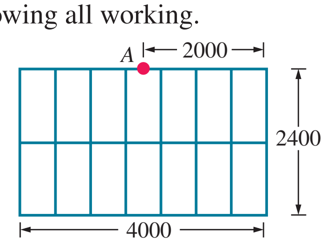
i redraw the diagram and mark in the approximate positions of permanent braces from the point(s) indicated.

ii calculate the total length of bracing required, showing all working.

a



b



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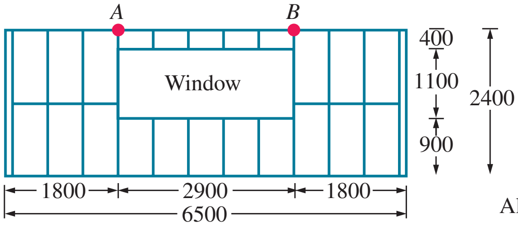
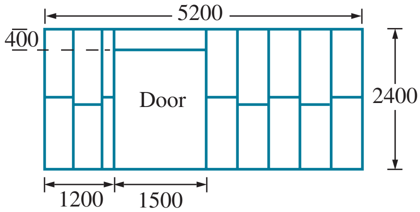
c

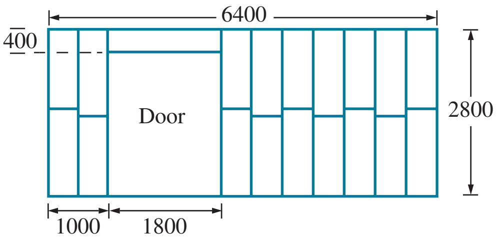


3 For each of the following wall frames, calculate the longest brace that could safely be used.

a

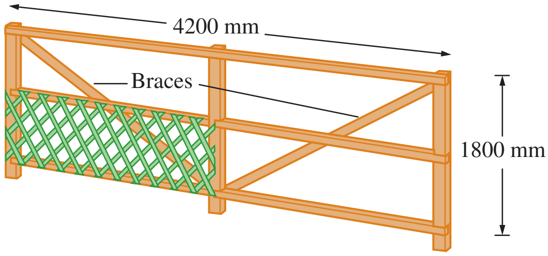


b



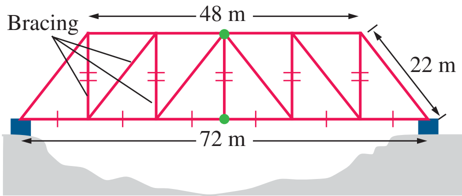
All measurements are in mm.

4 The screen shown below is to be constructed. Calculate the length of the bracing required.

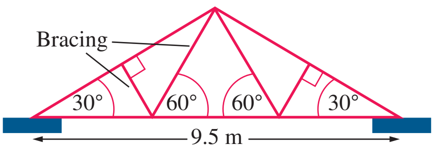


5 Calculate the total length of internal bracing material indicated in each of these structures, showing logical steps.

a



b



12.4 Estimating building costs

The basic cost that a builder quotes for a building can be estimated from its area, assuming that a particular type of construction is used. On level sites, a single-storey building, with a concrete slab and footings, will cost about $600 per square metre. The quotation would include allowances for council inspections, basic light, power, plumbing and hardware.

In some cases, costs are still quoted per **square**, which is an old measure of area. In this case, costs would be about $5500 per square.

> **!** A **square** is an area 10 feet by 10 feet. 1 square ≈ 9.29 m^2.

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Example 7

Two different houses have floor areas of 250 m^2 and 28 squares. Which is bigger, and by how much?

Solution

Change the second to square metres. $28 \text{ squares} \approx 28 \times 9.29 \text{ m}^2$
$\approx 260 \text{ m}^2$

Write the answer. The 28-square house is about 10 m^2 bigger.

Example 8

A builder works on a basis of \$620/m^2 when estimating basic house-building costs. What would be the builder's quotation for a house with an area of:

a 180.7 m^2? **b** 16 squares?

Solution

a Multiply cost by area.

$$\begin{aligned}\text{Estimated cost} &= \$620 \times 180.7\\ &= \$112\,034\end{aligned}$$

b Change area to m^2.

$$\text{Area} \approx 16 \times 9.29 \text{ m}^2$$

Multiply cost by area.

$$\begin{aligned}\text{Estimated cost} &= \$620 \times 16 \times 9.29\\ &\approx \$92\,157\end{aligned}$$

Example 9

Tim and Grace wish to build a new 24-square house on their block. The block slopes 700 mm from one side to the other, and the builder estimates that the extra siteworks will cost \$12 500. The couple also decide they would like a clay tile roof instead of Colorbond steel. This will cost an additional \$5800. The builder works on a basis of \$645/m^2. What will be the builder's quotation for their house?

Solution

Change area to m^2.

$$\text{Area} \approx 24 \times 9.29 \text{ m}^2$$

Multiply cost by area.

$$\begin{aligned}\text{Estimated basic cost} &= \$645 \times 24 \times 9.29\\ &\approx \$143\,809\end{aligned}$$

Add extras.

$$\begin{aligned}\text{Estimated total cost} &= \$143\,809 + \$12\,500 + \$5800\\ &= \$162\,109\end{aligned}$$

Basic building costs may include very limited choices of door furniture such as knobs and handles, painting colours, bathroom fixtures and electrical fittings. Floor coverings, curtains, wallpapers and other fittings are not included in normal building quotations. The costs of these items, or particular requirements such as selected doors and additional power points, must be considered separately. Fences, paths, driveways and landscaping are also separate from basic building costs.

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Example 10

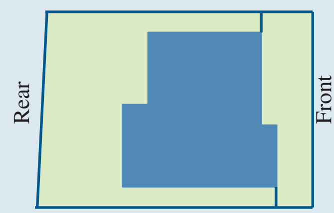
The quoted cost of building a house is \$140 500, including 12 single power points. Upgrading a single power point to a double power point costs an additional \$30 per power point. Extra power points or lights cost \$50 each, and the customer decides to put in another 8 double power points. There are 10 light fittings in the house, and the customer decides to put in another 3 lights. The lightshades chosen cost about \$85 each.

Solution

Find number of extra fittings.	Extra fittings = 8 + 3 = 11
Find cost of extra electrical fittings.	Extra fittings cost = \$50 × 11 = \$550
Find number of upgrades.	Double points = 12 + 8 = 20
Find cost of double points	Double points cost = \$30 × 20 = \$600
Find lightshades cost.	Lightshades cost = \$85 × 13 = \$1105
Find total cost.	Total cost = \$550 + \$600 + \$1105 = \$2255
Write the answer.	The extra electrical fittings will cost \$2255.

Example 11

Lily Tsang has small children and wants fences for her new house. The rear and side fences are to be 19 m, 26 m and 27 m long respectively, and small fences, each 2 m long, are required from the side boundaries to the house. What will the fences cost, at \$106/m?



Solution

Find total length of fences.	Length = 19 + 26 + 27 + 2 + 2 = 76 m
Find cost.	Cost = \$106 × 76 = \$8056
Write the answer.	The fences will cost \$8056.

Example 12

The cost of additional concrete paths for a new house is \$81/m for a 1 m-wide path, while a driveway 2.5 m wide costs \$207/m. A customer wants a path 6 m long to the clothesline and another path, which would be 10.5 m long, down one side of the house to get to the back door. The driveway is to be 7.8 m long. What will the paths and driveway cost?

Solution

Find length of paths.	Total path length = 10.5 + 6 = 16.5 m
Find cost of paths.	Cost of paths = 16.5 × \$81 = \$1336.50
Find driveway cost.	Cost of driveway = 7.8 × \$207 = \$1614.60
Add to find total cost.	Total cost = \$1336.50 + \$1614.60 = \$2951.10
Write the answer.	The paths and driveway will cost \$2951.10.

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Investigation Kitchen fittings

The kitchen fittings are very important in a house. Many modern appliances are standard sizes, so that the design of kitchen spaces can be considered without knowing what brand of appliances will be used. There are also some important principles of kitchen design that should be borne in mind.

- Stoves are normally 900 mm high, 600 mm deep, and 450 mm or 600 mm wide.
- Dishwashers are normally designed to fit in a 600 mm-wide and 600 mm-deep space, under a 900 mm-high benchtop.
- Fridges and freezers usually need a 700 mm-deep space, with at least 200 mm of space above them to allow for air circulation. They vary considerably in width.
- Benchtops are normally 900 mm high so that most people can comfortably work at them while standing. They should be at least 400 mm wide.
- Drawers should not be placed above bench height.
- Cupboards placed above workbenches should be set back at least 300 mm from the edge of the bench and leave at least 600 mm of space above the bench for headroom.
- Benchtops should protrude over cupboards by at least 20 mm.
- Kickboards underneath cupboards should be set back about 50 mm from the front of cupboards and be about 100 mm high to allow ‘toe room’ while accessing the cupboard.
- If possible, range hoods and extractor fans should be vented to the outside of the house.
- Sinks, drains and taps should be placed so that plumbing is on or in an outside wall, for ease of access and to reduce costs.
- The placement of the stove, sink and refrigerator should allow easy access during food preparation. It is often said that they should form a small triangle.
- If possible, the kitchen should not be an access route for other parts of the house.
- The dining area should be adjacent to the kitchen.

There are many kitchen design and installation companies that will custom-design kitchens, install prefabricated kitchen systems or perform a complete kitchen refurbishment. Many have display rooms and websites that show what they can do.

Work in small groups to design ‘ideal’ kitchens for:

- a family with three small children
- a family with adult children
- a working couple
- a ‘share’ house with four bedrooms.

You should obtain brochures from some local kitchen designers and explain how you made your decisions.

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Exercise 12.4 Estimating building costs

Additional Exercise 12.4

1 Change the following to square metres.

a 15 squares b 19 squares c 24 squares d 32 squares e 17 squares

2 A builder estimates basic building costs using a figure of $615/m^2. What would be the quotations for buildings with the following areas?

a 210 m^2 b 334 m^2 c 275 m^2 d 23 squares e 18 squares

3 A quantity surveyor uses a costing of $647/m^2 to estimate house-building costs. What would be the estimated costs of buildings with the following areas?

a 36 squares b 340 m^2 c 260 m^2 d 29 squares e 21 squares

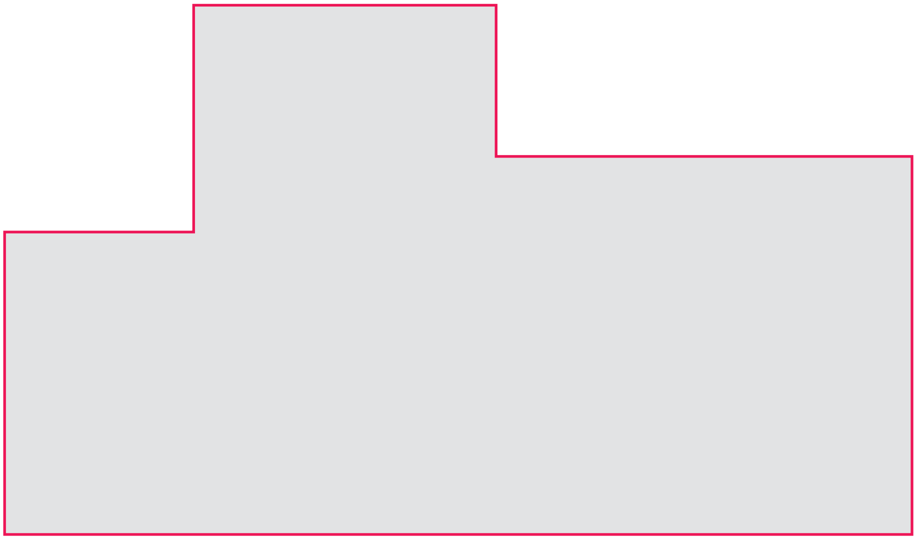
4 The standard plan for a three-bedroom house includes a single power point in each bedroom, the bathroom, the lounge, the dining room and family room, and two power points in the kitchen. The customer wants a double power point in each bedroom, three doubles in the lounge room and the kitchen, two doubles in the family room and one double in the garage. It costs $20 a point to upgrade to doubles and $64 for each additional power point. How much extra will the power points cost?

5 A customer requires timber panel internal doors and solid timber external doors for the house. There are seven internal doors and two external doors. Timber panel doors are an extra $345 each, and solid timber external doors are an extra $405 each. How much extra will the specified doors cost?

6 Instead of standard chrome taps, a customer wants flickmixers in the bath, handbasin and kitchen sink. These cost an extra $270 each, and the different handbasin and sink are an extra $114 each. Matching taps costing $90/set extra are wanted for the laundry tub and the washing machine outlets. How much extra will the change of tapware cost?

Modelling and problem solving

7 A builder uses a basic costing of $635/m^2. A tile roof costs about $18/m^2 more than the Colorbond steel roof, and the sloping block for the house outline plan shown below entails additional siteworks estimated to cost $8700. What would be the estimated cost for the house with a tile roof?



Scale 1 : 200

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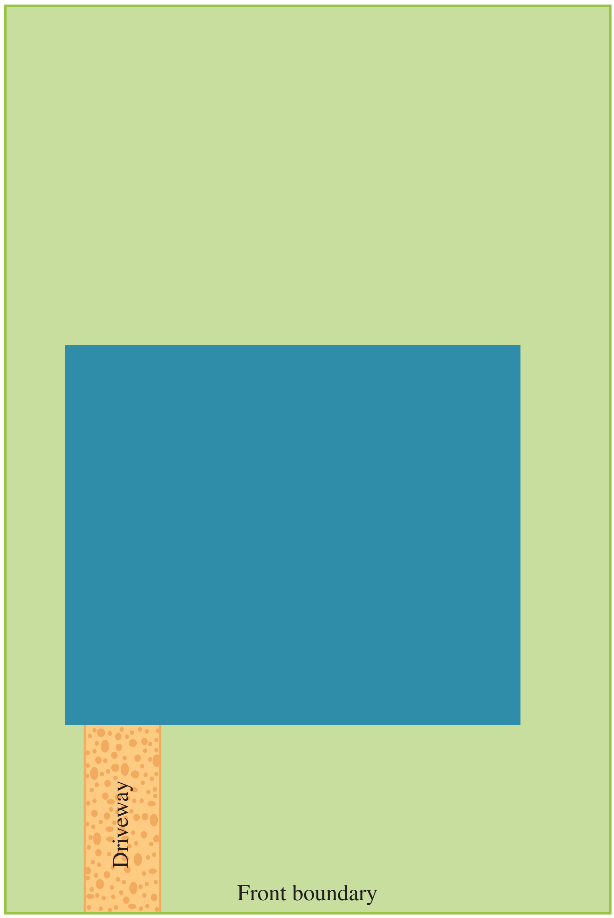
8 A builder is asked to quote on a house on a sloping block with a fall of 1200 mm. In order to cope with the fall, a retaining wall will be needed. The additional siteworks will cost \$11 600, and the retaining wall and drainage will cost \$8400. The house is to be 28 squares, and the basic building cost is \$640/m^2. Estimate the cost of building the house, showing all steps in logical order.

9 The house shown below on its block is to have fences built along the back and full side boundaries, with short fences in from the side boundaries to the front of the house. If the fencing costs \$152/m, how much will this cost? Fully justify your working.



Scale 1 : 250

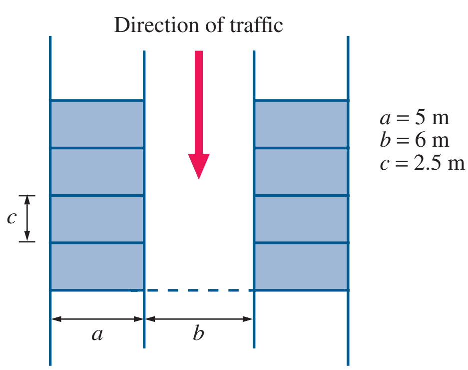
10 Instead of standard ‘Gainsborough’ door furniture, a customer prefers to use ‘selected’ door furniture that will cost \$120 extra for each door. There are nine doors in the house. How much extra will the ‘selected’ door furniture cost?

11 For the house in question **9**, the customer also wants a 2.5 m-wide driveway from the front boundary to the front of the house, and 1 m-wide paths right around the outside of the house and from the rear of the house to the back fence. The driveway will cost \$184.50/m, and the paths will be \$70.50/m. The rear path is to start from the edge of the path along the back of the house, and the path along the front of the house is not to cover the driveway. What will the paths and driveway cost in total?

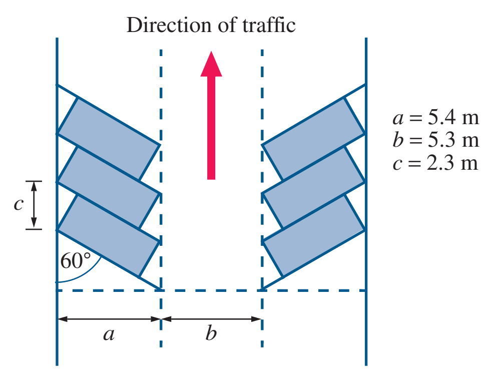
12 A kitchen is to be arranged in a space 3 m long and 2.4 m wide, with entry from one end. There is a window in the centre of the long side, and plumbing must be against the outside wall. A fridge space 700 mm deep and 1200 mm wide is to be left, and the stove will occupy a space 600 mm square. The dishwasher will take a space 600 mm square, and the freezer is 600 m wide and 700 mm deep. The dishwasher can be placed underneath a bench. Work out an arrangement of the kitchen, allowing for a sink and drainboard at least 1200 mm long. Show the arrangement as a plan (top view). Justify all decisions you make.

12.5 Car parks and office areas

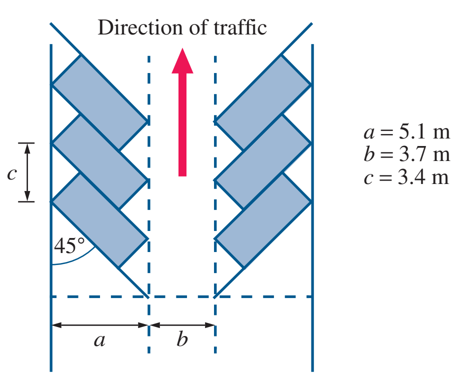
Car parks are needed everywhere—at shopping centres, sporting venues, schools and on the street. Because land for parking is usually limited and expensive to provide, it is usual to try to limit the size of car parks. Other considerations such as safety and the ease of movement of vehicles must also be considered when designing car parks. Here are some standard designs used for car parks.



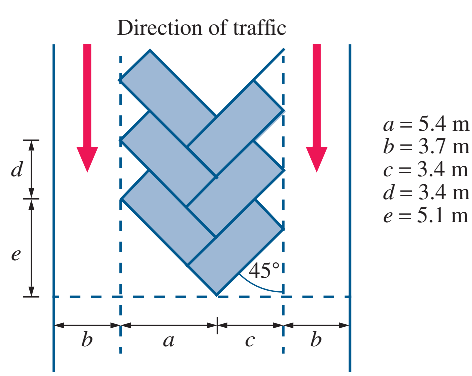
90° approach angle



60° approach angle



45° approach angle (centre access)



45° approach angle (outer access)

Example 13

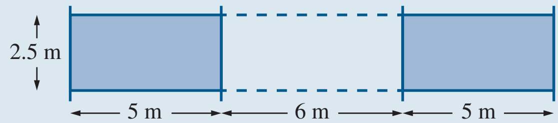
Calculate the overall area required for each car for the 90º approach angle car park design shown on page 375.

Solution

The overall area includes the roadway. To calculate the overall area required for a single car, any number of bays can be used as long as the roadway is taken into account.

Draw a sketch for two car bays, including the roadway.

Put the dimensions on the sketch.



The overall area for two cars is a rectangle.

Calculate the area of the rectangle.

$$\begin{aligned}\text{Rectangle area} &= \text{length} \times \text{breadth}\\ &= 16\text{ m} \times 2.5\text{ m}\\ &= 40\text{ m}^2\end{aligned}$$

Calculate the overall area for a single car.

$$\begin{aligned}\text{Area for 1 car} &= 40\text{ m}^2 \div 2\\ &= 20\text{ m}^2\end{aligned}$$

Write the answer.

The area needed for each car is 20 m^2.

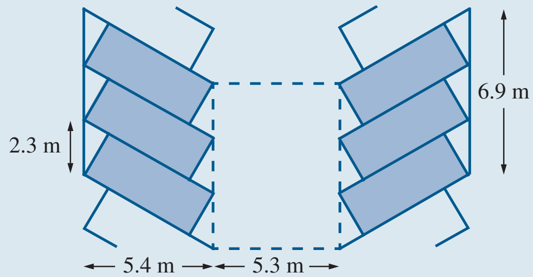
Example 14

Calculate the overall area required for each car for the 60º approach angle car park design shown on page 375.

Solution

Draw a sketch for six car bays, including the roadway.

Put the dimensions on the sketch.



The overall area for six cars consists of two parallelograms and a rectangle.

Calculate the area of the rectangle.

$$\begin{aligned}\text{Rectangle area} &= \text{length} \times \text{breadth}\\ &= 6.9\text{ m} \times 5.3\text{ m}\\ &= 36.57\text{ m}^2\end{aligned}$$

Calculate the area of each parallelogram.

$$\begin{aligned}\text{Parallelogram area} &= \text{base} \times \text{height}\\ &= 6.9\text{ m} \times 5.4\text{ m}\\ &= 37.26\text{ m}^2\end{aligned}$$

Calculate the overall area for six cars.

$$\begin{aligned}\text{Area for 6 cars} &= 36.57 + 2 \times 37.26\text{ m}^2\\ &= 111.09\text{ m}^2\end{aligned}$$

Calculate the overall area for a single car.

$$\begin{aligned}\text{Area for 1 car} &= 111.09\text{ m}^2 \div 6\\ &= 18.515\text{ m}^2\end{aligned}$$

Round and write the answer.

The area needed for each car is about 18.5 m^2.

Note that this design wastes some space at the ends of each car bay.

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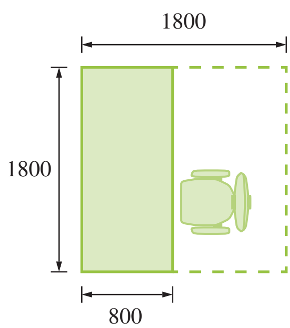
Investigation Car park design

A rectangular piece of land measuring 80 m by 50 m is to be converted into a car park. The owners of the land want the car park to have the following features.

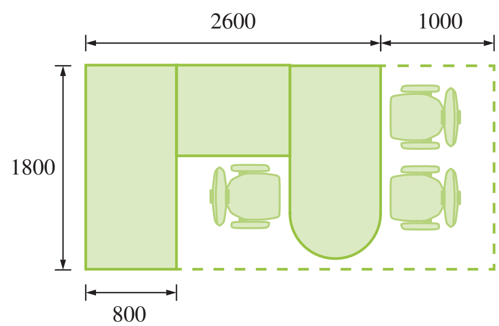
- The car park needs to have one entrance and one exit, each measuring 5.5 m wide.
- The flow of traffic is to be one-way, so that only cars entering the car park may use the entrance and only those leaving may use the exit.
- The entrance and exit may be located anywhere along the 80 m-long sides of the land.
- The car park needs to cater for vehicles with a turning circle of 11.5 m. (The **turning circle** is the distance a car travels to complete a circle with the steering wheel at full lock.)

Work in a small group and use a scale drawing or any other model to come up with a design that accommodates the maximum number of vehicles. You can use any design or combination of designs for the car park. Some typical car park designs are shown on page 375.

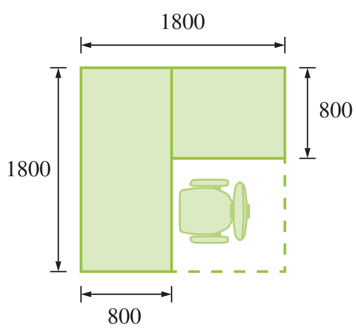
The use of floor area is an important issue for office buildings. Office workers need to have adequate room to work, but floor space can be expensive. For this reason, **open plan** offices are now very common. In an open plan office, workstations are separated by low partitions rather than walls. Some typical workstation designs are shown below.



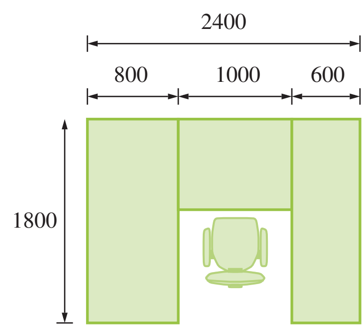
Type 1 workstation



Type 2 workstation



Type 3 workstation



Type 4 workstation

The floor area required for a workstation is known as its **footprint**. The footprint of a workstation includes the floor area taken up by the desk and the floor space required for the chair(s). In addition to the footprint of the workstation, room is also required around the workstation to allow for movement around the office. As a rule of thumb, each workstation should occupy a minimum floor space of 2.1 m by 2.1 m.

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Example 15

Calculate the percentage of the minimum workstation floor area that is occupied by the footprint of a type 1 workstation as shown on the previous page.

Solution

Calculate the area of the footprint. Area of footprint $= 1.8 \text{ m} \times 1.8 \text{ m} = 3.24 \text{ m}^2$

Calculate the minimum workstation area. Minimum area $= 2.1 \text{ m} \times 2.1 \text{ m} = 4.41 \text{ m}^2$

Calculate the footprint percentage. Footprint % $= \dfrac{3.24}{4.41} \times 100\%$

Evaluate and round off. $\approx 73\%$

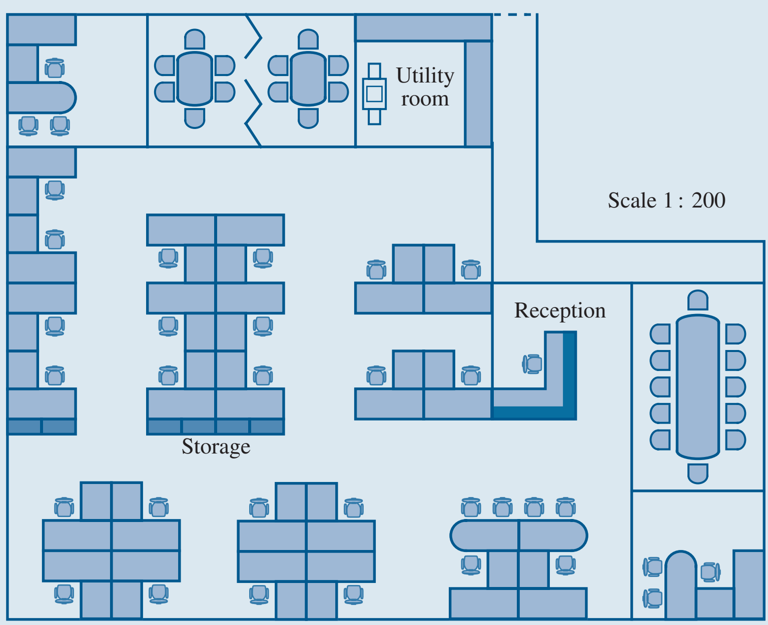
State the result. The footprint of a type 1 workstation occupies about 73% of the minimum area required for a workstation.

In addition to the space required for workstations, lighting and air-conditioning requirements also need to be considered. The **lux (lx)** is the SI unit of illuminance—the measure of the intensity of light. The illuminance of a family living room is about 50 lx. The illuminance of general office areas is about 320 lx. Approximately 13.5 W/m^2 is required in order to achieve this level of illumination. Offices usually use 36 W fluorescent tubes, singly or in 72 W pairs.

The air-conditioning requirement for rooms is based on the amount of heat that needs to be removed (cooling) or added (heating). Air-conditioner outputs are measured in **kilowatts (kW)**. While every situation is slightly different, a rule of thumb for the air-conditioning requirement for normal office areas is 125 W/m^2 (0.125 kW/m^2).

Example 16

Use the office floor plan shown below to answer the questions that follow.



a How many pairs of 36 W lights are required to illuminate the entire office?
b What air-conditioning capacity (kW) is required for this office?

Solution

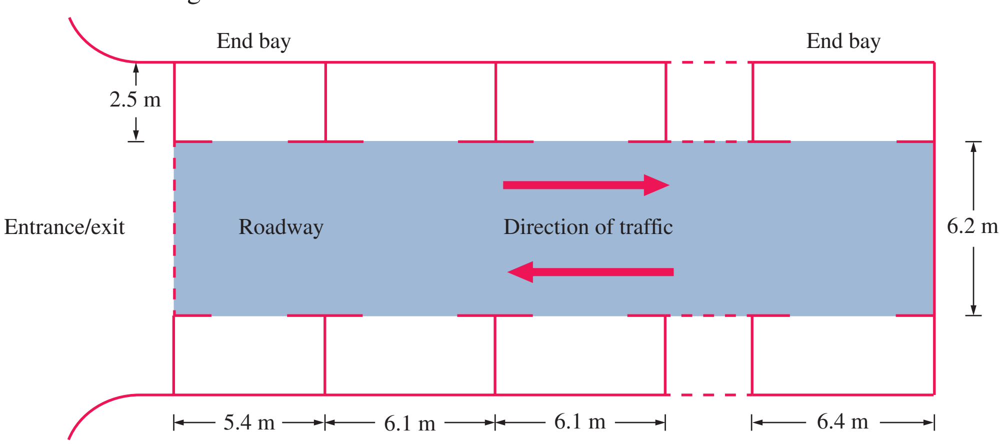
Calculate the dimensions of the office.	Scale of drawing = 1 : 200 1 cm on drawing = 1×200 cm (actual) = 2 m Dimensions of office = $20\text{ m} \times 16\text{ m}$
Calculate the area of the office.	Area of office = $20\text{ m} \times 16\text{ m} - 6\text{ m} \times 6\text{ m}$ = 284 m^2
a Calculate power required for lighting.	Lighting power = $13.5\text{ W/m}^2 \times 284\text{ m}^2$ = 3834 W
Find the number of pairs of lights required.	Number of lights = $3834 \div 72$ = 53.25
Round up and state the answer.	Fifty-four pairs of 36 W lights are required to illuminate the office.
b Calculate power required for air-conditioning.	Air-conditioning power = $0.125\text{ kW/m}^2 \times 284\text{ m}^2$ = 35.5 kW
Round if necessary and state the answer.	Air-conditioning capacity of 35.5 kW is required for the office.

Exercise 12.5 Car parks and office areas

Additional Exercise 12.5

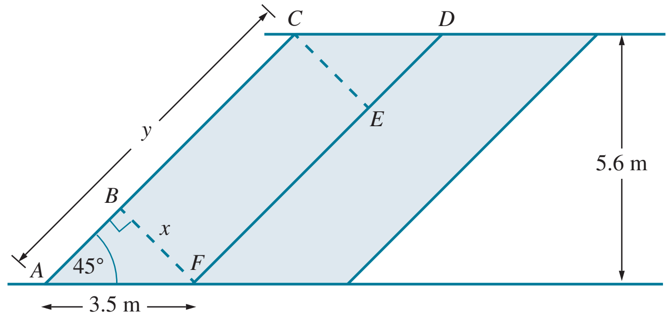
1 Calculate the overall area required for each car for the 45º approach angle (centre access) car park design shown on page 375.

2 The diagram below shows a design for parallel parking. The end bays near the entrance/exit are 5.4 m long, while the other end bays are 6.4 m long. Parking bays between the end bays are all 6.1 m long.



a Calculate the area of each of the different types of car bays.
b A car park for 20 cars is constructed using this design. What area is taken up by the car bays?
c For the car park mentioned in part **b**, what percentage is taken up by the roadway?

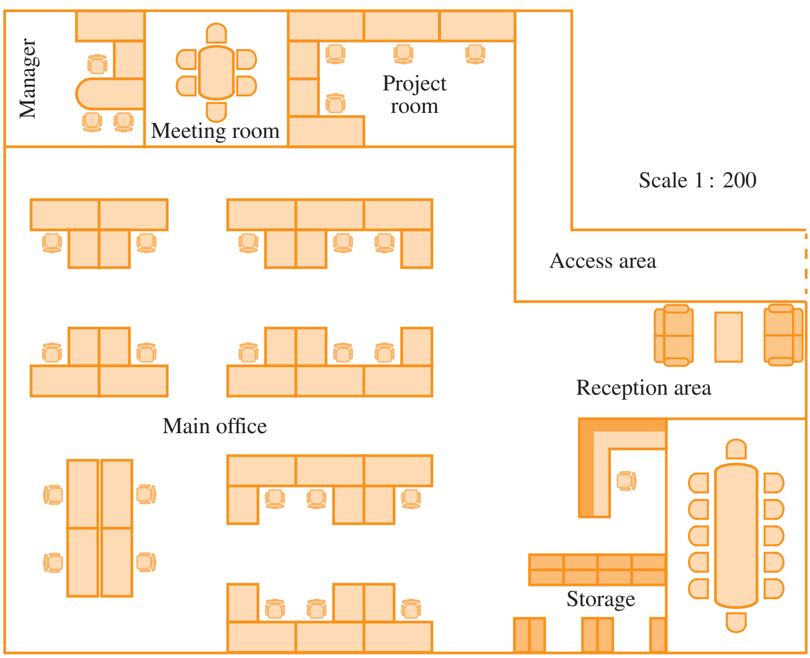
3 The diagram below shows two bays in a 45° approach angle car park.



a Use trigonometry or Pythagoras's Theorem to calculate the value of x.
b Similarly, calculate the value of y.
c Calculate the area of the parallelogram $ACDF$—the total car bay.
d Calculate the area of $FBCE$—the area available for parking.

4 Calculate the percentage of the minimum workstation floor area that is occupied by the footprint of a type 2 workstation as shown on page 377.

5 Use the office floor plan shown below to answer the questions that follow.



a The manager's office has a type 2 workstation as shown on page 377. Calculate the percentage of the floor area of the manager's office that is occupied by the footprint of the workstation.
b The main office area contains type 1 and type 3 workstations as shown on page 377. Calculate the total footprint for these workstations.
c Calculate the area of the entire office, including the access area.
d How many pairs of 36 W lights are required to illuminate the entire office?
e What air-conditioning capacity (kW) is required for this office?

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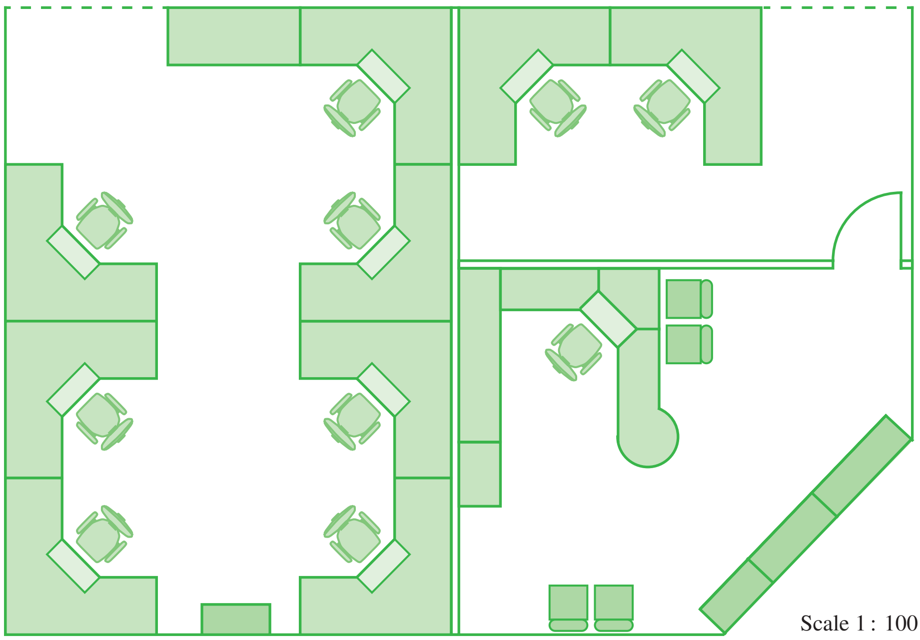
6 Use the office floor plan shown below to answer the questions that follow.



a Calculate the floor area of the office.

b How many 36 W lights are required to illuminate the entire office?

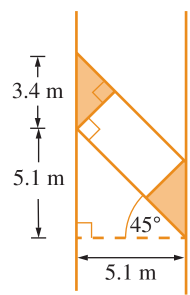
c What air-conditioning capacity (kW) is required for this office?

Modelling and problem solving

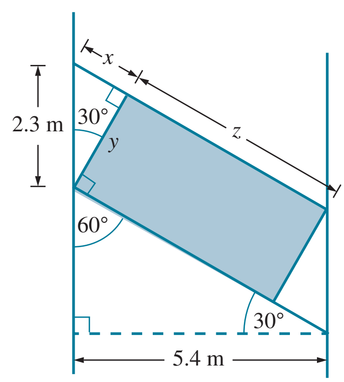
7 The diagram on the right shows a detail of the 45° approach angle (centre access) car park design shown on page 375.

a Calculate the width of the car bay measured perpendicular to its oblique sides.

b The shaded areas on the car bay are not available for parking. Calculate the area available for a car to park in. Show all steps logically.



8 The diagram on the right shows a detail of the 60° approach angle car park design shown on page 375. The shaded area in the diagram is the area available for a car to park in. Calculate this area. Explain any assumptions you make.



Chapter summary

- Buildings are set out **square** using Pythagoras's Theorem.
- The **3–4–5 triangle** is commonly used for squaring corners.
- Spirit levels, line levels, water levels, dumpy levels, automatic levels and laser levels are used to establish **levels** on a building site, by reference to a **datum point**.
- The angle of a slope can be calculated from the fall and length or horizontal distance of the slope using trigonometry.
- **Bracing** is used in building frames to ensure rigidity. The angle for diagonal bracing should be between 37° and 53°, with 45° bracing being strongest. **Sheeting** is used in situations where diagonal bracing is not possible.
- A spirit level or plumb-bob can be used to check that walls are vertical.
- A **square** is an old area measure equal to about 9.29 m^2.
- The basic cost of building can be estimated by multiplying the floor area by the building cost per unit area.
- Building contracts normally specify what fittings or choices are included. Additional work or fittings must be paid for as extras.
- Car parks are usually designed to try to limit the area required. Other considerations include safety and the ease of movement of vehicles. Calculations of the overall area require for each car need to include the roadway.
- Many office buildings use an **open plan** in which workstations are separated by low partitions rather than walls. The floor area required for a workstation is known as its **footprint**—the floor area taken up by the desk and the chair(s). Workstations should occupy a minimum floor space of 2.1 m by 2.1 m.
- The **lux (lx)** is the SI unit of illuminance and is the measure of the intensity of light. The illuminance of a family living room is about 50 lx. The illuminance of general office areas is approximately 320 lx, which requires approximately 13.5 W/m^2 (fluorescent tubes) to achieve.
- Air-conditioner outputs are measured in **kilowatts (kW)**. As a rule of thumb, the air-conditioning requirement for normal office areas is 125 W/m^2 (0.125 kW/m^2).

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Chapter review

Knowledge and procedures

1 What triangle dimensions do builders commonly use to check for squareness? Ex 12.1

2 The front pegs of a house are 15.6 m apart and the rear pegs are placed 8.4 m behind. How long should the diagonals be to ensure that the house is square? Ex 12.1

3 The minimum fall on corrugated iron sheeting should be 1 : 12. What fall is needed for a roof that measures 6.4 m horizontally from the ridge to the gutter? Ex 12.2

4 Using a dumpy level, the front corners of a block are found to have levels of 1.24 m and 3.20 m. What is the fall across the front of the block? Ex 12.2

5 Can braces be placed from the top corner at one end to the bottom corner at the other end of walls with the following dimensions? Ex 12.3

a 2.4 m high, 3.0 m long
b 2.4 m high, 1.6 m long
c 2.1 m high, 2.6 m long

6 Change these areas to m^2. Ex 12.4

a 8 squares
b 27 squares

7 A builder estimates basic construction costs at $425/$m^2$. How much would it cost to build: Ex 12.4

a a house of area 270 m^2?
b a house of area 26 squares?

8 A house has a laundry, garage, kitchen, lounge, family room, bathroom, separate toilet and four bedrooms. There is a single power point in every room except the toilet. A customer wants double power points instead of singles, two extra double points in the kitchen, family room and lounge, and an extra double point in the master bedroom. It costs $20 extra per power point to upgrade to double and $54 extra for each additional power point. How much will the extra electrical work cost? Ex 12.4

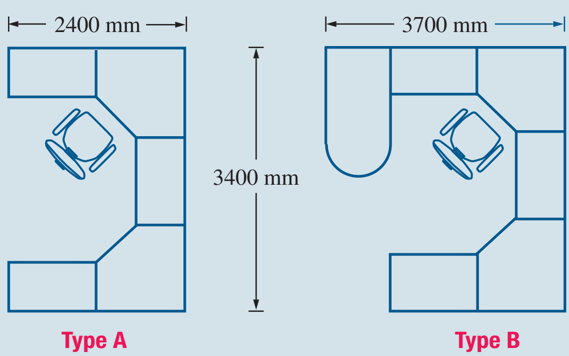
9 A builder quotes $188 000 to build a house on a level block. The block actually slopes by 5° from one side to the other. The house is 18 m wide and 22 m long, so 14 m^3 of soil has to be moved to level the site. A retaining wall 800 mm high is needed along one side. It will cost $112/$m^3$ to move and compact the soil, and $240/m to build and drain the retaining wall. Ex 12.4

a What is the additional cost of the required sitework?
b What is the percentage increase in the cost of the house?

10 A new house is built on a rectangular block 19 m wide and 26 m deep. A fence is erected right around the block, including a front fence. The fencing contractor quotes $116/m, including gates. What will the fence cost? Ex 12.4

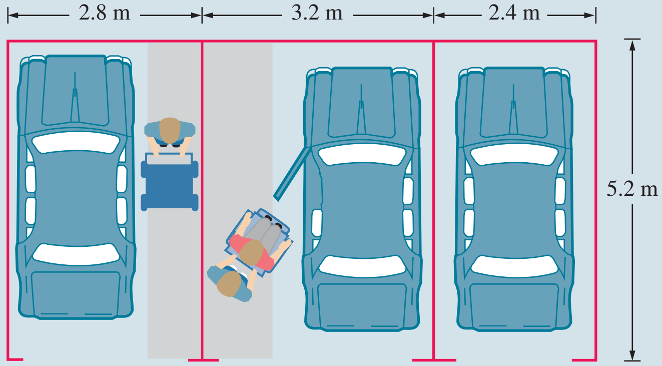
11 Two different styles of office workstation are shown on the right. Ex 12.5

a Calculate the area of the footprint of each type of workstation.
b How does the footprint of each workstation compare with the minimum floor space required for a workstation?



Ex 12.5

12 The diagram below shows three different types of car bays in a car park. The family car bay is designed to make it easier to load prams and the like, the disabled car bay is designed to make it easier for people in wheelchairs or with movement restrictions to get in and out of a car, while the standard car bay is for the use of the general public.



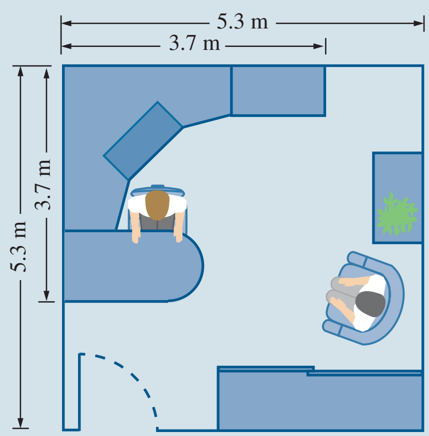
a Calculate the area of each type of car bay.

b How much bigger is the family car bay than the standard one?

c How much bigger is the disabled car bay than the standard one?

Ex 12.5

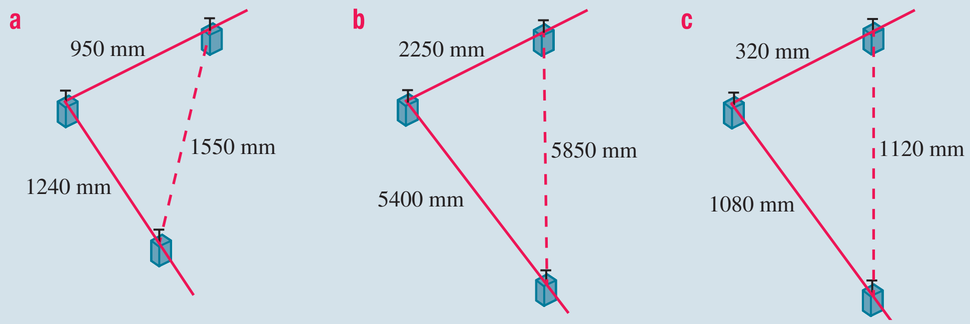
13 The diagram on the right shows an office. What percentage of the office floor are is taken up by the footprint of the workstation?



Modelling and problem solving

Ex 12.1

14 Which of the string-line arrangements below will result in corners that are square?



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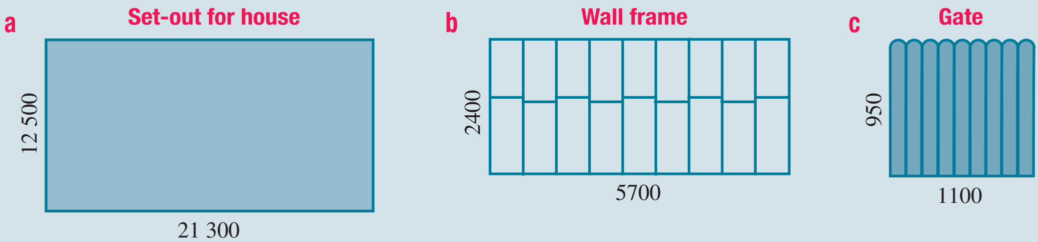
Chapter review

15 Calculate the length of the diagonals of each of the following structures if the structure is square.

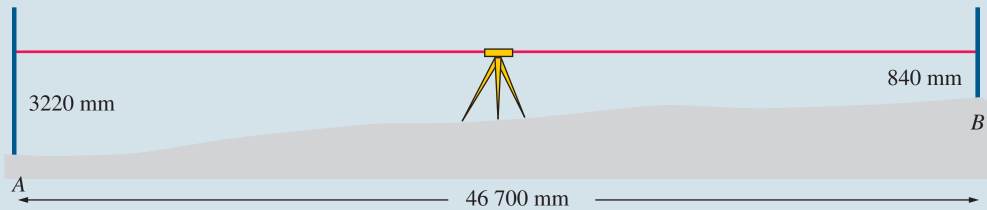


All measurements are in mm.

Ex 12.1

16 Here is a cross-sectional view of a building site where levels were taken.

Ex 12.2

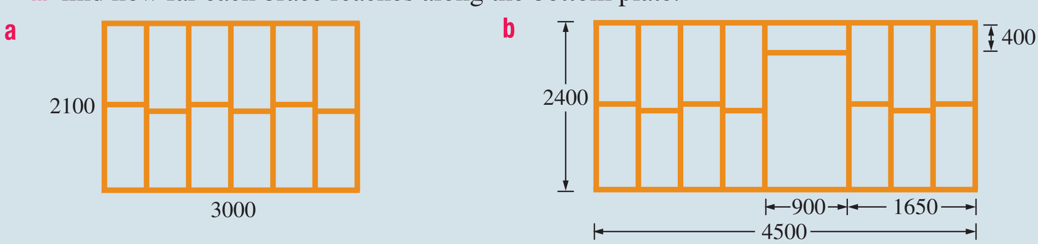


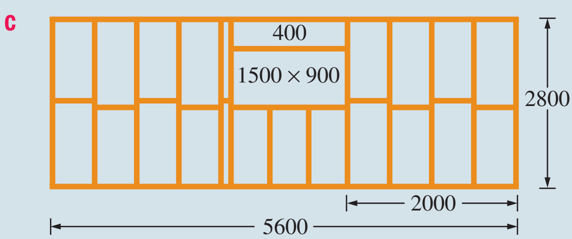
a Calculate the rise from point A to point B.
b Calculate the average angle at which the ground rises. Justify your result.

17 For each of the wall frames shown below:

Ex 12.3

i redraw the diagram and mark the approximate position(s) of the brace(s)
ii find the length of each brace
iii find how far each brace reaches along the bottom plate.





All measurements are in mm.

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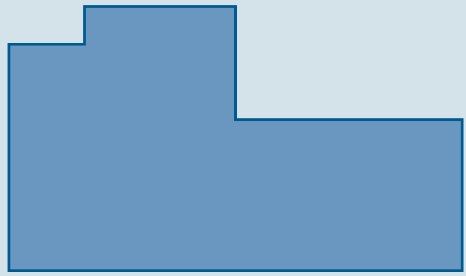
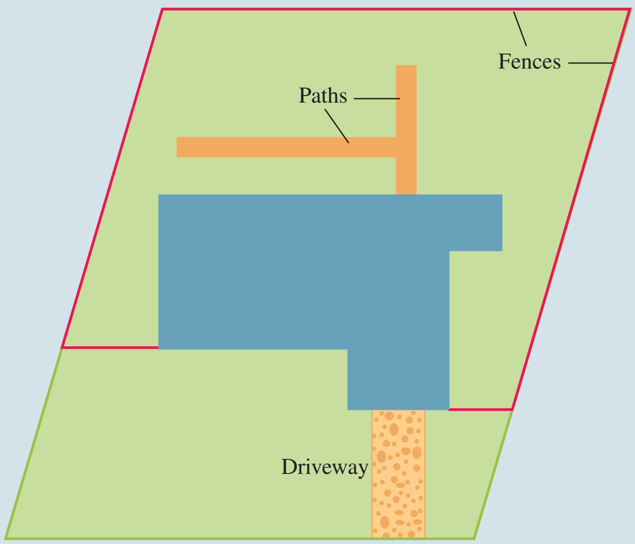
Ex 12.4

18 Find the cost of building the house with the floor plan shown below, at a cost of $6375/square. Fully explain your steps.



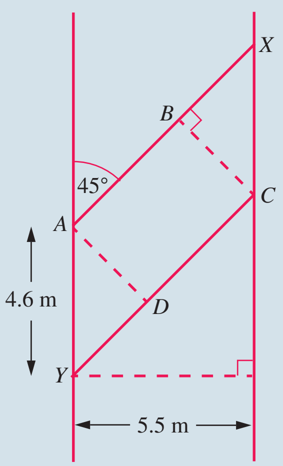
Ex 12.4

19 The house shown on the block plan below is to have fences, paths and a driveway installed as shown on the plan. If the fence costs $158/m, the driveway costs $268/m and the paths cost $96/m, find the total cost.



Ex 12.5

20 The diagram on the right shows a detail of a 45° approach angle car park. Calculate the area available for a car to park in ($ABCD$). Explain any assumptions you make.



Appendix 1

Metric system prefixes

Prefix (number)	Abbreviation	Meaning
tera- (trillion)	T	$\times$ 1 000 000 000 000 = 10^{12}
giga- (billion)	G	$\times$ 1 000 000 000 = 10^9 Computer memory: $\times$ 1 073 741 824 = 2^{30}
mega- (million)	M	$\times$ 1 000 000 = 10^6 Computer memory: $\times$ 1 048 576 = 2^{20}
kilo- (thousand)	k	$\times$ 1000 = 10^3 Computer memory: $\times$ 1024 = 2^{10}
hecto- (hundred)	h	$\times$ 100 = 10^2
deka- (ten)	da	$\times$ 10 = $\times 10^1$
		1 unit
deci- (tenth)	d	$\div$ 10 = $\times$ 10^{-1}
centi- (hundredth)	c	$\div$ 100 = $\times$ 10^{-2}
milli- (thousandth)	m	$\div$ 1000 = $\times$ 10^{-3}
micro- (millionth)	μ	$\div$ 1 000 000 = $\times$ 10^{-6}
nano- (billionth)	n	$\div$ 1 000 000 000 = $\times$ 10^{-9}
pico- (trillionth)	p	$\div$ 1 000 000 000 000 = $\times$ 10^{-12}

A megabyte refers to the size of computer memory, but is 2^{20} bytes, not the standard metric multiple.

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Appendix 2

Wageline Award Summary Sheets

The following Award Summary Sheets contains a summary of the major provisions of the relevant awards as provided by the Queensland Industrial Relations Commission pursuant to the Industrial Relations Act 1999 ('the Act'). The information contained in each summary sheet is not an award as defined by the Act and must not be taken to be a definitive statement of what the award prescribes. Whilst every care has been exercised in the preparation of the information contained herein, a user should not rely upon the information and should seek recourse to the award. The Department of Employment and Industrial Relations hereby expressly excludes any liability to a user for damages incurred as a result of reliance upon the information contained herein.
Employers are required by the Act to display a full copy of the award at their place of business. The content of these documents is managed by Industrial Relations Services on behalf of Wageline. For further information regarding these summary sheets or to purchase a copy of the award please contact Wageline on the numbers below.
Telephone: 1300 369 945 Facsimile: (07) 3872 0519

Wageline Award Summary Sheet – Fast Food Industry Award – South-Eastern Division 2003

Application This Award applies to all employees as defined in clause 1.6, engaged in, or in connection with, Fast Food Operations (as defined) throughout the South-Eastern Division of the State of Queensland, employed by; Toocom Pty Ltd trading as Hungry Jacks Qld, and franchises thereto; Amalgamated Food & Poultry Pty Ltd (Inc.) WA trading as Red Rooster and Big Rooster, franchises thereto; Chicken World; Dominos Pizza Australia Pty Ltd and franchises thereto; Eagle Boys Dial-a-Pizza Australia Pty Ltd, and franchises thereto; Uncle Tony's Kebabs Pty Ltd trading as Uncle Tony's Kebabs and franchises thereto; Brodies Enterprises Pty Ltd and Brodies Franchises Pty Ltd trading as Brodies Meal Makers and franchises thereto:

Provided that this Award shall not apply to employees covered by any other Award or Industrial Agreement, nor to any establishment which has a licence to sell alcohol.

This Award shall also apply to all employees as defined herein, engaged in, or in connection with, Fast Food Operations (as defined) throughout the South-Eastern Division of the State of Queensland, employed by Subway Systems Australia Pty Ltd and franchises thereto, and to their employers, provided that to accommodate the transition to this Award, employees engaged prior to 28 July 2004 will be exempt from clauses 6.5.1, 6.5.2 and 6.5.3 of this Award and will instead be subject to clauses 6.5.1 and 6.5.2 of the Retail Take-Away Food Award – South Eastern Division 2003. No other terms and conditions of the Retail Take-Away Food Award – South Eastern Division 2003 shall apply.'

Effective from September 1, 2006

Classification	Weekly	Part-time	Casual
Level 1	$503.80	13.2579	16.3072
Level 2	$519.20	13.6632	16.8057
Level 3	$539.00	14.1842	17.4466

Juniors	% of adult rate	Level	Weekly	Part-time	Casual
Under 17 years	55%	1	$277.10	7.2921	8.9693
		2	$285.60	7.5158	9.2444
		3	$296.50	7.8026	9.5972
17 years	65%	1	$327.50	8.6184	10.6007
		2	$337.50	8.8816	10.9243
		3	$350.40	9.2211	11.3419
18 years	75%	1	$377.90	9.9447	12.2320
		2	$389.40	10.2474	12.6043
		3	$404.30	10.6395	13.0866
19 years	85%	1	$428.20	11.2684	13.8602
		2	$441.30	11.6132	14.2842
		3	$458.20	12.0579	14.8312

Trainees	Refer to the Order *Apprentices' and Trainees' Wages and Conditions (Excluding Certain Queensland Government Entities)* 2003, Queensland Government Industrial Gazette, 11 July 2003, Vol 173, No.11, pages 878–927. Refer also clause 4.9.
Ordinary hours	The ordinary hours of work shall be an average of 38 hours per week, to be worked within a cycle not exceeding either 7, 14 or 28 consecutive days (or a combination of any of those). Minimum of 4 hours and maximum of 10 hours per day. Provision for banking of rostered days off. No employee under the age of 18 years shall work, or be permitted to work, later than 8.00pm without the consent of his/her parents or legal guardians. See clause 6.1.
Penalty rates	*Weekend Penalty* – All ordinary time worked by employees (other than part-time and casual employees) on a Saturday or Sunday shall be paid at the rate of time and a quarter. See clause 6.5.1. *Late Work Penalty* – All ordinary time worked by employees (other than casuals) Monday to Friday between 11.00 pm and 12.30 am, employees shall be paid an additional $1.327 (as from 1/9/06) per hour. See clause 6.5.2. All ordinary time worked by an employee between 12.30 am and 5.00 am Monday to Friday shall attract an additional payment at the rate of $1.995 (as from 1/9/06) per hour or part thereof. See clause 6.5.3.
Overtime	Time worked outside or in excess of the ordinary hours or outside the usual commencing and ceasing times shall be paid for at the rate of time and a half for the first 3 hours and double time thereafter in any one day. All time worked on an employee's rostered day off shall be paid for at overtime rates with a minimum payment of 2 hours. See clause 6.6.
Part-time	Not less than 10 hours and not more than 32 hours per week, to be worked on not more than 5 days of the week. Minimum of 2 hours and a maximum of 10 hours per day. Part-time employees shall be entitled to paid leave on a pro rata basis. See clause 4.7.
Casual	Employee engaged by the hour. Minimum of 2 hours for each engagement. Casual loadings are payable as such; **23%** for all ordinary hours worked **73%** where the rate of pay is prescribed as time and a half **123%** where the rate of pay is prescribed as double time **173%** where the rate of pay is prescribed as double time and a half The loadings are payable separately and are not to be compounded.
Superannuation	An employer is required to meet the minimum requirements set out in both the Federal Superannuation Guarantee legislation and this award. Employers and employees should telephone 13 10 20 to determine an employer's possible obligation under the federal legislation and should read the superannuation clause (cl. 5.4) contained within the award to determine award entitlements/obligations. The approved funds named in the award are: Rest, Sunsuper and Westpac Master Plan

Notice by Employer And Employee (other than casual)

Period of Continuous Service	*Period of Notice*
Not more than 1 year	1 week
More than 1 year up to 3 years	2 weeks
More than 3 years up to 5 years	3 weeks
More than 5 years	4 weeks

Where the employee is over 45 years of age and has had more than two years service, an additional week's notice is due when the employment is terminated by the employer.

Redundancy Refer to clause 4.4. New provisions operative as from 1/12/03.

Classifications and Definitions

Fast Foods means and include specialty take-away foods and proprietary items packaged, sold and served in such a manner as to facilitate their being taken from the point of distribution to be consumed elsewhere; provided that this definition shall not be construed so as to include the re-heating of pre-cooked foods which are sold in sandwich bars, milk bars, grocery shops and shops trading as delicatessens or roadhouses attached to service stations.

Fast Food Operations means the preparation and serving of Fast Foods in Fast Foods Outlets.

Fast Foods Outlet means an establishment or section thereof which is exclusively engaged in the preparation and/or serving of Fast Foods as defined in clause 1.6.3 and shall include any company premises, whether within such establishment or otherwise, where Fast Foods are prepared or partially prepared, which by custom and practice is open for 7 days of the week.

Fast Food Worker Level 1 means an employee undergoing training in a fast food establishment who is in the first 15 weeks of service and who performs basic tasks under supervision. For the purpose of this definition, 'service' shall mean any work performed for any employer involving the preparation and/or sale of prepared food.

Fast Food Worker Level 2 means an employee with at least 15 weeks service who can competently perform designated operations functions.

Fast Food Worker Level 3 means a senior employee who is proficient in all operations functions and who is appointed by the employer to assist and supervise employees at Levels 1 and 2. Such level does not apply to employees engaged in one on one training.

Award Summary Sheet – Hairdressers' Industry Award – State 2003 (Southern Division Western District)

Application Employees in the Queensland Hairdressing Industry performing and/or carrying out, duties in any hairdressing salon.

Partial exemption As an alternative to being subject to all Award provisions an employee remunerated 15% in excess of the rate prescribed for Level 6 – Manager ($736.41), will mutually agree in writing with the employer not to be bound by the conditions of the Award, except for: clauses 3.1, 4.6, 4.7, 4.8, 5.3, 7.1, 7.2, 7.3, 7.4, 7.5, 7.6, 11.3. (these clauses may still be applicable)

Effective from September 1, 2006

Classification	Weekly	Part-time	Casual
Level 1	$ 521.55	13.7250	16.8818
Level 2	$ 546.55	14.3829	17.6910
Level 3	$ 598.65	15.7539	19.3774
Level 4	$ 609.05	16.0276	19.7140
Level 5	$ 619.45	16.3013	20.0506
Level 6	$ 640.35	16.8513	20.7271

Juniors		Level			Level 2		
Age	%	Weekly	Part-time	Casual	Weekly	Part-time	Casual
Under 16	45%	$234.73	6.1771	7.5978	$246.03	6.4745	7.9636
16 years	50%	$260.83	6.8639	8.4427	$273.33	7.1929	8.8473
17 years	55%	$286.83	7.5482	9.2842	$300.53	7.9087	9.7277

Apprentices/Trainees

Level	% of trade	Weekly	Part-time
Wage Level 1	40%	$239.50	6.3026
Wage Level 2	55%	$329.30	8.6658
Wage Level 3	75%	$449.00	11.8158
Wage Level 4	90%	$538.80	14.1789

Ordinary hours The ordinary hours of work shall be an average of 38 per week to be worked on either a 7, 14, 21 or 28 consecutive day work cycle on any five days in the week, Monday to Sunday inclusive between the hours of 8.00 am and 7.00 pm (9.00 pm on the day designated as the late night trading day).
Scope to allow up to 10 ordinary hours per day to be worked (11 hours on the day of late night trading).
See clause 6.1.

Penalty rates Ordinary hours worked by full time and part-time employees on a Saturday shall be paid at the rate of time and one-quarter.
Ordinary hours worked by all employees on a Sunday shall be paid at the rate of double time.
See clause 6.1.2.

Overtime All time worked in excess of the ordinary hours is paid at the rate of time and a-half for the first three hours and double time thereafter.
All work performed on a Sunday will be paid for at the rate of double time.
See clause 6.2.

Part-time An employee who has reasonably predictable hours and who is employed for not less than 15 hours per week and less than 38 ordinary hours per week, to be worked on not more than 5 days of the week with a minimum of 3 hours per day. At the time of engagement the employer and employee will agree in writing on the pattern of work required including specifying the number of ordinary hours per week, the days on which the work is to be performed and the usual commencing and ceasing times. A part-time employee is entitled to paid leave on a pro rata basis. See clause 4.3.

Casual An employee whose employment may be terminated by either party at a moment's notice. A casual shall be engaged for a minimum period of 3 hours on each occasion and a maximum of 32 ordinary hours per week. Casuals shall be paid 1/38th of the appropriate weekly rate plus 23% loading for ordinary hours worked Monday to Saturday. Casual employees will be paid at double the casual hourly rate of pay on Sundays. See clause 4.4.

Superannuation An employer is required to meet the minimum requirements set out in both the Federal Superannuation Guarantee legislation and this award. Employers and employees should telephone 13 10 20 to determine an employer's possible obligation under the federal legislation and should read the superannuation clause (cl. 5.3) contained within the award to determine award entitlements/obligations.

The approved funds named in the award are: Sunsuper, Hairdressers' Association Superannuation Fund (H.A.S.F) and The Stefan Group Superannuation Scheme.

Notice by Employer And Employee (other than casual)

Period of Continuous Service	*Period of Notice*
Not more than 1 year	1 week
More than 1 year up to 3 years	2 weeks
More than 3 years up to 5 years	3 weeks
More than 5 years	4 weeks

Where the employee is over 45 years of age and has had more than two years service, an additional week's notice is due.

Notice shall not be counted as annual leave.

Notice by Employee (other than casuals) One week

Notice shall not be counted as annual leave.

Redundancy Refer to clause 4.8. New provisions operative as from 1/12/03.

Classifications

Level 1 – Hairdressing Industry Employee

Points of Entry – An employee at this level has very little or no previous experience in the industry. Employees will remain at this level for a maximum of 6 months.

Skills/Duties:

(a) Responsible for the quality of their work subject to detailed direction.
(b Works in a team environment and/or under routine supervision.
(c) Undertakes duties in a safe and responsible manner.
(d) Exercises discretion within their level of skills and training.
(e) Possesses basic interpersonal and communication skills.
(f) Indicative of the tasks which an employee at this level may perform are the following:
- Basic reception duties;
- General cleaning duties;
- Responsible for periodic stock checks.

Level 2 – Tea & tidies, Receptionists

Points of Entry – An employee at this level possesses the skills of the Hairdressing Industry Employee – Level 1, and is not currently undertaking an apprenticeship, or equivalent and appropriate training.

Skills/Duties:

(a) Responsible for the quality of their work subject to detailed direction.
(b) Works in a team environment and/or under routine supervision.
(c) Undertakes duties in a safe and responsible manner.
(d) Exercises discretion within their level of skills and training.
(e) Possesses basic interpersonal and communication skills.
(f) Indicative of the tasks which an employee at this level may perform include maintaining a beauty salon in a clean condition and making and serving tea to clientele.

Level 3 – Hairdresser

(a) Men

Points of Entry – A 'Hairdresser' applies the skills acquired through successful completion of an apprenticeship in men's hairdressing, or equivalent and appropriate training.

Skills/Duties:

(i) Understands and is responsible for quality control standards.
(ii) Possesses a high degree of interpersonal and communication skills.
(iii) May perform work requiring no supervision either individually or in a team environment.
(iv) Ability to supervise and provide direction and guidance to apprentices and trainees.
(v) Men's hairdressing includes arranging, dressing, cleansing, cutting, trimming, shaving the hair or beard of any person, whether by hand, or by mechanical or electrical apparatus or appliances; and massaging, cleansing or stimulating the scalp, face or neck of any person, whether with the use of cosmetic, antiseptic or similar preparations, or of tonics, lotions or cream or otherwise. It also includes the sharpening or setting of razors.

(b) Ladies

Points of Entry – A Hairdresser applies the skills acquired through successful completion of an apprenticeship in Ladies Hairdressing or equivalent and appropriate training.

Skills/Duties:

(i) Understands and is responsible for quality control standards.
(ii) Possesses a high degree of interpersonal and communication skills.
(iii) May perform work requiring no supervision either individually or in a team environment.
(iv) Ability to supervise and provide direction and guidance to apprentices and trainees.
(v) Ladies hairdressing includes cutting ladies' hair and arranging, dressing, curling, waving, cleansing, trimming, shaving, bleaching, tinting, colouring or otherwise treating the hair of the head of any person, whether by hand, or by any mechanical or electrical apparatus or appliances; and massaging, cleansing or stimulating the scalp, face or neck of any person, whether with the use of cosmetic, antiseptic or similar preparations or of tonics, lotions or cream or otherwise. In clause 5.4.3 (b) (v), 'trimming' means such trimming as is required in connection with curling, waving, bleaching, tinting and colouring.

Level 4 – Hairdresser training for dual competencies

Points of Entry – A Hairdresser who applies the skills acquired through the successful completion of an apprenticeship or equivalent and appropriate training, in either Ladies or Men's Hairdressing and is undertaking further study to attain the qualifications as per Level 5.

Skills/Duties:

(a) Understands and is responsible for quality control standards.
(b) Possesses a high degree of interpersonal and communication skills.
(c) May perform work requiring no supervision either individually or in a team environment.
(d) A Hairdresser at this level performs duties as qualified in either Men's or Ladies Hairdressing in accordance with Level 3; as well as performing duties with appropriate supervision in accordance with their training towards a second qualification.

Level 5 – Hairdresser – Ladies and men's hairdressing

Points of Entry – A Hairdresser applies the skills acquired through successful completion of apprenticeships or equivalent and appropriate training, in Ladies and Men's Hairdressing or recognised equivalent qualifications.

Skills/Duties:

(a) Understands and is responsible for quality control standards.
(b) Possesses a high degree of interpersonal and communication skills.
(c) Performs work requiring no supervision either individually or in a team environment.
(d) Men's and ladies hairdressing includes arranging, dressing, curling, waving, cleansing, cutting, trimming, shaving, bleaching, tinting, colouring or otherwise treating the hair or beard of any person, whether by hand or by any mechanical or electrical apparatus or appliances and massaging, cleansing or stimulating the scalp, face or neck of any person whether with the use of cosmetic antiseptic or similar preparations or of tonics lotions, or cream or otherwise. It also includes the sharpening or setting of razors.

Level 6 – Manager

Points of Entry – A person recognised as and employed by the employer as a Manager.

Skills/Duties:

(a) Implements quality control techniques and procedures.
(b) Understands and is responsible for the salon.
(c) Highly developed level of interpersonal and communication skills.
(d) Ability to supervise and provide direction and guidance to other employees.

(e) Exercises discretion within the scope of this level.
(f) A Manager's functions may include:
- Liaising with upper management and/or owners, suppliers and customers with respect to the salon's operations;
- Maintaining control registers including inventory control and being responsible for the preparation and reconciliation of regular reports, accounts, and the day to day handling of money;
- Has sound knowledge of the employer's operations;
- The ability to assist in the provision of on-the-job training and induction.

Award Summary Sheet – Nurses' Award – State 2005 (Southern Division Western District)

Application This Award shall apply to all nursing staff employed in non-institutional health settings including: Industrial, Commercial and Retail Establishments, Local Government Authorities, Doctors Surgeries, Specialist Medical Centres, Creches and Kindergartens, Independent Schools, and Pathology Laboratories. *(This Award shall not apply to employees of the Queensland Government or the Mater Misericordiae Public Hospital who are covered by any other Award, Certified Agreement or Industrial Agreement, or who are members of a religious Order.)*

Effective from September 1, 2006

Doctor's rooms	Weekly	Part-time	Casual
Registered Nurse Level 1			
1st Year	$650.95	17.1303	21.0702
2nd Year	$674.55	17.7513	21.8341
3rd Year	$696.15	18.3197	22.5333
4th Year	$721.65	18.9908	23.3587
Registered Nurse Level 2			
1st Year	$837.55	22.0408	27.1102
2nd Year	$851.25	22.4013	27.5536
Registered Nurse Level 3			
1st Year	$912.15	24.0039	29.5249
2nd Year	$929.85	24.4697	30.0978
Specialist medical centres	**Weekly**	**Part-time**	**Casual**
Registered Nurse Level 1			
1st Year	$650.95	17.1303	21.0702
2nd Year	$674.55	17.7513	21.8341
3rd Year	$696.15	18.3197	22.5333
4th Year	$721.65	18.9908	23.3587
5th Year	$745.25	19.6118	24.1226
6th Year	$768.85	20.2329	24.8865

Refer to clause 5.1 for rates in other establishments.

Ordinary hours Average of 38 ordinary hours per week to be worked on the following basis:
152 hours within a work cycle not exceeding 28 consecutive days. Scope to allow the method of implementation of the 38 hour week to be varied for individual employees, groups or sections of employees. Scope to allow up to 10 ordinary hours per day to be worked. See clause 6.1.

Penalty rates *Weekend work – extra payment (clause 6.5)* – Ordinary time worked between midnight Friday and midnight Sunday shall be paid for at the rate of ordinary time plus the following additional percentage of the employee's ordinary time rate:

Saturday – 50%
Sunday – 75%

Afternoon and night duty – extra payment (clause 6.6) – **Afternoon shift** is a shift where the majority of hours are worked after 12 midday and finished at or before 6.00 pm. Afternoon shift workers shall be paid an allowance of **12.5%** for each shift of ordinary hours. **Night shift** is a shift commencing at or after 6.00 pm or before 7.30 am the following day, the major portion of which is worked between 6.00 pm and 7.30 am. Night shift workers shall be paid an allowance of **15%** for each shift of ordinary hours. See clause 6.6.

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Overtime All time worked in excess of the ordinary working hours shall be paid for at the following rates:

- in the case of shift workers at the rate of double time;
- in the case of all other employees at the rate of time and a half for the first 3 hours and double time thereafter on any one day.
- All overtime worked on a Sunday shall be paid at the rate of double time.

Payment shall be made for all overtime, no provision for time off in lieu of overtime. See clause 6.7.

Part-time Engaged to work regular hours each week. Ordinary daily working hours shall be a minimum of 4 hours and maximum of 8 hours such hours shall be fewer than 32 ordinary hours per week. Paid at the rate of 1/38th of the weekly rate. Minimum payment of 4 hours on any day when work is performed. Receives pro rata entitlement to paid leave. See clauses 1.4.9 and 5.1.4.

Casual Employee engaged on a daily basis for not more than 32 hours in any one week. Loading of 23%. Entitled to allowances on a pro rata basis. Minimum of 2 hours shall be paid for each engagement. See clauses 1.4.4 and 5.1.3.

Allowances *X-Ray and Radium Allowance* (cl. 5.2.4) – $8.00 per week
Uniform and Laundry Allowance (cl. 5.2.5) – $159 per annum paid on a pro rata basis each day. (Uniform)
$1.85 per week (Laundry)
In charge allowance – Independent Schools (cl. 5.2.6) – $3.10 per week
For other allowances refer to clause 5.2.

Previous Service *Total Experience to Count (Recognition of Previous Service)*
An employee shall be given credit for all previous continuous nursing service which shall include time spent obtaining additional nursing certificates other than the General Nursing Certificate.
A part-time or casual employee shall be required to complete the equivalent of a full working year (1976 hours) from the time of their first appointment, enrolment or registration or of their last increment before being eligible for the next increment. A person who has completed 1976 hours of duty, or has received payment for 1976 hours, including annual, sick, bereavement and other paid leave, shall be deemed to have completed a full year. See clause 5.1.7.

Superannuation An employer is required to meet the minimum requirements set out in both the Federal Superannuation Guarantee legislation and this award. Employers and employees should telephone 13 10 20 to determine an employer's possible obligation under the federal legislation and should read the superannuation clause (cl. 5.3) contained within the award to determine award entitlements/obligations.
The approved funds named in the award are: HESTA and Sunsuper.

Notice by Employer And Employee (other than casual)

Period of Continuous Service	*Period of Notice*
More than 1 year up to 3 years	2 weeks
More than 3 years up to 5 years	3 weeks
More than 5 years	4 weeks

Where the employee is over 45 years of age and has had more than two years service, an additional week's notice is due.

Notice by Employee (other than casuals) 2 week

Redundancy Refer to clause 4.4.

Definitions

Registered Nurse means an employee registered under the Nursing Act 1992 as a Registered Nurse; and

- who is subject to the regulations and/or bylaws of the Queensland Nursing Council and who holds a current Annual Licensing Certificate; and
- who is employed on the basis of that qualification.

Enrolled Nurse means an employee who is enrolled under the Nursing Act 1992 as an Enrolled Nurse; and

- who is subject to the regulations and/or bylaws of the Queensland Nursing Council and who holds a current Annual Licensing Certificate as such.

Assistant-In-Nursing is an employee who is assisting in nursing duties but who is not a Registered Nurse or an Enrolled Nurse.

Specialist Medical Centre is a centre in which services are provided by one or more registered specialist medical practitioners.

Appendix 3

Weekly tax table

Weekly earnings	Amount to be withheld: With tax-free threshold with leave loading	With tax-free threshold no leave loading	No tax-free threshold	Weekly earnings	Amount to be withheld: With tax-free threshold with leave loading	With tax-free threshold no leave loading	No tax-free threshold	Weekly earnings	Amount to be withheld: With tax-free threshold with leave loading	With tax-free threshold no leave loading	No tax-free threshold	Weekly earnings	Amount to be withheld: With tax-free threshold with leave loading	With tax-free threshold no leave loading	No tax-free threshold
1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
1	–	–	–	71	–	–	12.00	141	5.00	5.00	23.00	211	15.00	15.00	35.00
2	–	–	–	72	–	–	12.00	142	5.00	5.00	23.00	212	16.00	15.00	35.00
3	–	–	–	73	–	–	12.00	143	5.00	5.00	24.00	213	16.00	15.00	35.00
4	–	–	1.00	74	–	–	12.00	144	5.00	5.00	24.00	214	16.00	16.00	35.00
5	–	–	1.00	75	–	–	12.00	145	5.00	5.00	24.00	215	16.00	16.00	35.00
6	–	–	1.00	76	–	–	13.00	146	6.00	5.00	24.00	216	16.00	16.00	36.00
7	–	–	1.00	77	–	–	13.00	147	6.00	5.00	24.00	217	16.00	16.00	36.00
8	–	–	1.00	78	–	–	13.00	148	6.00	6.00	24.00	218	16.00	16.00	36.00
9	–	–	1.00	79	–	–	13.00	149	6.00	6.00	25.00	219	17.00	16.00	36.00
10	–	–	2.00	80	–	–	13.00	150	6.00	6.00	25.00	220	17.00	16.00	36.00
11	–	–	2.00	81	–	–	13.00	151	6.00	6.00	25.00	221	17.00	17.00	36.00
12	–	–	2.00	82	–	–	14.00	152	6.00	6.00	25.00	222	17.00	17.00	37.00
13	–	–	2.00	83	–	–	14.00	153	7.00	6.00	25.00	223	17.00	17.00	37.00
14	–	–	2.00	84	–	–	14.00	154	7.00	7.00	25.00	224	17.00	17.00	37.00
15	–	–	2.00	85	–	–	14.00	155	7.00	7.00	26.00	225	18.00	17.00	37.00
16	–	–	3.00	86	–	–	14.00	156	7.00	7.00	26.00	226	18.00	17.00	37.00
17	–	–	3.00	87	–	–	14.00	157	7.00	7.00	26.00	227	18.00	17.00	37.00
18	–	–	3.00	88	–	–	15.00	158	7.00	7.00	26.00	228	18.00	18.00	38.00
19	–	–	3.00	89	–	–	15.00	159	8.00	7.00	26.00	229	18.00	18.00	38.00
20	–	–	3.00	90	–	–	15.00	160	8.00	7.00	26.00	230	18.00	18.00	38.00
21	–	–	3.00	91	–	–	15.00	161	8.00	8.00	27.00	231	18.00	18.00	38.00
22	–	–	4.00	92	–	–	15.00	162	8.00	8.00	27.00	232	19.00	18.00	38.00
23	–	–	4.00	93	–	–	15.00	163	8.00	8.00	27.00	233	19.00	18.00	38.00
24	–	–	4.00	94	–	–	16.00	164	8.00	8.00	27.00	234	19.00	19.00	39.00
25	–	–	4.00	95	–	–	16.00	165	8.00	8.00	27.00	235	19.00	19.00	39.00
26	–	–	4.00	96	–	–	16.00	166	9.00	8.00	27.00	236	19.00	19.00	39.00
27	–	–	4.00	97	–	–	16.00	167	9.00	8.00	28.00	237	19.00	19.00	39.00
28	–	–	5.00	98	–	–	16.00	168	9.00	9.00	28.00	238	19.00	19.00	39.00
29	–	–	5.00	99	–	–	16.00	169	9.00	9.00	28.00	239	20.00	19.00	39.00
30	–	–	5.00	100	–	–	16.00	170	9.00	9.00	28.00	240	20.00	19.00	40.00
31	–	–	5.00	101	–	–	17.00	171	9.00	9.00	28.00	241	20.00	20.00	40.00
32	–	–	5.00	102	–	–	17.00	172	9.00	9.00	28.00	242	20.00	20.00	40.00
33	–	–	5.00	103	–	–	17.00	173	10.00	9.00	29.00	243	20.00	20.00	40.00
34	–	–	6.00	104	–	–	17.00	174	10.00	10.00	29.00	244	20.00	20.00	40.00
35	–	–	6.00	105	–	–	17.00	175	10.00	10.00	29.00	245	21.00	20.00	40.00
36	–	–	6.00	106	–	–	17.00	176	10.00	10.00	29.00	246	21.00	20.00	41.00
37	–	–	6.00	107	–	–	18.00	177	10.00	10.00	29.00	247	21.00	20.00	41.00
38	–	–	6.00	108	–	–	18.00	178	10.00	10.00	29.00	248	21.00	21.00	41.00
39	–	–	6.00	109	–	–	18.00	179	11.00	10.00	30.00	249	21.00	21.00	41.00
40	–	–	7.00	110	–	–	18.00	180	11.00	10.00	30.00	250	21.00	21.00	41.00
41	–	–	7.00	111	–	–	18.00	181	11.00	11.00	30.00	251	21.00	21.00	41.00
42	–	–	7.00	112	–	–	18.00	182	11.00	11.00	30.00	252	22.00	21.00	42.00
43	–	–	7.00	113	1.00	–	19.00	183	11.00	11.00	30.00	253	22.00	21.00	42.00
44	–	–	7.00	114	1.00	1.00	19.00	184	11.00	11.00	30.00	254	22.00	22.00	42.00
45	–	–	7.00	115	1.00	1.00	19.00	185	11.00	11.00	31.00	255	22.00	22.00	42.00
46	–	–	8.00	116	1.00	1.00	19.00	186	12.00	11.00	31.00	256	22.00	22.00	42.00
47	–	–	8.00	117	1.00	1.00	19.00	187	12.00	11.00	31.00	257	22.00	22.00	42.00
48	–	–	8.00	118	1.00	1.00	19.00	188	12.00	12.00	31.00	258	23.00	22.00	43.00
49	–	–	8.00	119	1.00	1.00	20.00	189	12.00	12.00	31.00	259	23.00	22.00	43.00
50	–	–	8.00	120	2.00	1.00	20.00	190	12.00	12.00	31.00	260	23.00	22.00	43.00
51	–	–	8.00	121	2.00	2.00	20.00	191	12.00	12.00	32.00	261	23.00	23.00	43.00
52	–	–	9.00	122	2.00	2.00	20.00	192	13.00	12.00	32.00	262	23.00	23.00	44.00
53	–	–	9.00	123	2.00	2.00	20.00	193	13.00	12.00	32.00	263	23.00	23.00	44.00
54	–	–	9.00	124	2.00	2.00	20.00	194	13.00	13.00	32.00	264	23.00	23.00	44.00
55	–	–	9.00	125	2.00	2.00	21.00	195	13.00	13.00	32.00	265	24.00	23.00	45.00
56	–	–	9.00	126	3.00	2.00	21.00	196	13.00	13.00	32.00	266	24.00	23.00	45.00
57	–	–	9.00	127	3.00	2.00	21.00	197	13.00	13.00	33.00	267	24.00	23.00	45.00
58	–	–	10.00	128	3.00	3.00	21.00	198	13.00	13.00	33.00	268	24.00	24.00	46.00
59	–	–	10.00	129	3.00	3.00	21.00	199	14.00	13.00	33.00	269	24.00	24.00	46.00
60	–	–	10.00	130	3.00	3.00	21.00	200	14.00	13.00	33.00	270	24.00	24.00	46.00
61	–	–	10.00	131	3.00	3.00	22.00	201	14.00	14.00	33.00	271	25.00	24.00	47.00
62	–	–	10.00	132	3.00	3.00	22.00	202	14.00	14.00	33.00	272	25.00	24.00	47.00
63	–	–	10.00	133	4.00	3.00	22.00	203	14.00	14.00	33.00	273	25.00	24.00	47.00
64	–	–	11.00	134	4.00	4.00	22.00	204	14.00	14.00	34.00	274	25.00	25.00	47.00
65	–	–	11.00	135	4.00	4.00	22.00	205	14.00	14.00	34.00	275	25.00	25.00	48.00
66	–	–	11.00	136	4.00	4.00	22.00	206	15.00	14.00	34.00	276	25.00	25.00	48.00
67	–	–	11.00	137	4.00	4.00	23.00	207	15.00	14.00	34.00	277	25.00	25.00	48.00
68	–	–	11.00	138	4.00	4.00	23.00	208	15.00	15.00	34.00	278	26.00	25.00	49.00
69	–	–	11.00	139	4.00	4.00	23.00	209	15.00	15.00	34.00	279	26.00	25.00	49.00
70	–	–	12.00	140	5.00	4.00	23.00	210	15.00	15.00	35.00	280	26.00	25.00	49.00

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	Amount to be withheld		
Weekly earnings	With tax-free threshold with leave loading	With tax-free threshold no leave loading	No tax-free threshold
1	2	3	4
281	26.00	26.00	50.00
282	26.00	26.00	50.00
283	26.00	26.00	50.00
284	26.00	26.00	51.00
285	27.00	26.00	51.00
286	27.00	26.00	51.00
287	27.00	26.00	52.00
288	27.00	27.00	52.00
289	27.00	27.00	52.00
290	27.00	27.00	52.00
291	28.00	27.00	53.00
292	28.00	27.00	53.00
293	28.00	27.00	53.00
294	28.00	28.00	54.00
295	28.00	28.00	54.00
296	28.00	28.00	54.00
297	28.00	28.00	55.00
298	29.00	28.00	55.00
299	29.00	28.00	55.00
300	29.00	28.00	56.00
301	29.00	29.00	56.00
302	29.00	29.00	56.00
303	29.00	29.00	57.00
304	30.00	29.00	57.00
305	30.00	29.00	57.00
306	30.00	29.00	58.00
307	30.00	29.00	58.00
308	30.00	30.00	58.00
309	30.00	30.00	58.00
310	30.00	30.00	59.00
311	31.00	30.00	59.00
312	31.00	30.00	59.00
313	31.00	30.00	60.00
314	31.00	31.00	60.00
315	31.00	31.00	60.00
316	31.00	31.00	61.00
317	31.00	31.00	61.00
318	32.00	31.00	61.00
319	32.00	31.00	62.00
320	32.00	31.00	62.00
321	32.00	32.00	62.00
322	33.00	32.00	63.00
323	33.00	32.00	63.00
324	33.00	32.00	63.00
325	33.00	33.00	64.00
326	34.00	33.00	64.00
327	34.00	33.00	64.00
328	34.00	33.00	64.00
329	34.00	34.00	65.00
330	35.00	34.00	65.00
331	35.00	34.00	65.00
332	35.00	34.00	66.00
333	35.00	35.00	66.00
334	36.00	35.00	66.00
335	36.00	35.00	67.00
336	36.00	35.00	67.00
337	36.00	36.00	67.00
338	37.00	36.00	68.00
339	37.00	36.00	68.00
340	37.00	36.00	68.00
341	37.00	37.00	69.00
342	38.00	37.00	69.00
343	38.00	37.00	69.00
344	38.00	37.00	69.00
345	38.00	38.00	70.00
346	39.00	38.00	70.00
347	39.00	38.00	70.00
348	39.00	38.00	71.00
349	40.00	39.00	71.00
350	40.00	39.00	71.00
351	40.00	39.00	72.00
352	40.00	39.00	72.00
353	41.00	40.00	72.00
354	41.00	40.00	73.00
355	41.00	40.00	73.00
356	41.00	40.00	73.00
357	42.00	41.00	74.00
358	42.00	41.00	74.00
359	42.00	41.00	74.00
360	42.00	41.00	75.00
361	43.00	42.00	75.00
362	43.00	42.00	75.00
363	43.00	42.00	75.00
364	43.00	42.00	76.00
365	44.00	43.00	76.00
366	44.00	43.00	76.00
367	44.00	43.00	77.00
368	44.00	43.00	77.00
369	45.00	44.00	77.00
370	45.00	44.00	78.00
371	45.00	44.00	78.00
372	45.00	44.00	78.00
373	46.00	45.00	79.00
374	46.00	45.00	79.00
375	46.00	45.00	79.00
376	46.00	45.00	80.00
377	46.00	46.00	80.00
378	46.00	46.00	80.00
379	47.00	46.00	81.00
380	47.00	46.00	81.00
381	47.00	46.00	81.00
382	47.00	46.00	81.00
383	47.00	47.00	82.00
384	47.00	47.00	82.00
385	48.00	47.00	82.00
386	48.00	47.00	83.00
387	48.00	47.00	83.00
388	48.00	47.00	83.00
389	48.00	48.00	84.00
390	48.00	48.00	84.00
391	49.00	48.00	84.00
392	49.00	48.00	85.00
393	49.00	48.00	85.00
394	49.00	48.00	85.00
395	49.00	49.00	86.00
396	49.00	49.00	86.00
397	50.00	49.00	86.00
398	50.00	49.00	87.00
399	50.00	49.00	87.00
400	50.00	49.00	87.00
401	50.00	50.00	87.00
402	50.00	50.00	88.00
403	51.00	50.00	88.00
404	51.00	50.00	88.00
405	51.00	50.00	89.00
406	51.00	50.00	89.00
407	51.00	51.00	89.00
408	51.00	51.00	90.00
409	52.00	51.00	90.00
410	52.00	51.00	90.00
411	52.00	51.00	91.00
412	52.00	51.00	91.00
413	52.00	52.00	91.00
414	52.00	52.00	92.00
415	53.00	52.00	92.00
416	53.00	52.00	92.00
417	53.00	52.00	92.00
418	53.00	52.00	93.00
419	53.00	53.00	93.00
420	53.00	53.00	93.00
421	54.00	53.00	94.00
422	54.00	53.00	94.00
423	54.00	53.00	94.00
424	54.00	53.00	95.00
425	54.00	54.00	95.00
426	54.00	54.00	95.00
427	55.00	54.00	96.00
428	55.00	54.00	96.00
429	55.00	54.00	96.00
430	55.00	54.00	97.00
431	55.00	55.00	97.00
432	55.00	55.00	97.00
433	56.00	55.00	98.00
434	56.00	55.00	98.00
435	56.00	55.00	98.00
436	56.00	55.00	98.00
437	56.00	56.00	99.00
438	56.00	56.00	99.00
439	57.00	56.00	99.00
440	57.00	56.00	100.00
441	57.00	56.00	100.00
442	57.00	56.00	100.00
443	57.00	57.00	101.00
444	57.00	57.00	101.00
445	58.00	57.00	101.00
446	58.00	57.00	102.00
447	58.00	57.00	102.00
448	58.00	57.00	102.00
449	58.00	58.00	103.00
450	58.00	58.00	103.00
451	59.00	58.00	103.00
452	59.00	58.00	104.00
453	59.00	58.00	104.00
454	59.00	58.00	104.00
455	59.00	59.00	104.00
456	59.00	59.00	105.00
457	60.00	59.00	105.00
458	60.00	59.00	105.00
459	60.00	59.00	106.00
460	60.00	59.00	106.00
461	60.00	59.00	106.00
462	60.00	60.00	107.00
463	61.00	60.00	107.00
464	61.00	60.00	107.00
465	61.00	60.00	108.00
466	61.00	60.00	108.00
467	61.00	60.00	108.00
468	61.00	61.00	109.00
469	62.00	61.00	109.00
470	62.00	61.00	109.00
471	62.00	61.00	110.00
472	62.00	61.00	110.00
473	62.00	61.00	110.00
474	62.00	62.00	110.00
475	63.00	62.00	111.00
476	63.00	62.00	111.00
477	63.00	62.00	111.00
478	63.00	62.00	112.00
479	63.00	62.00	112.00
480	63.00	63.00	112.00
481	64.00	63.00	113.00
482	64.00	63.00	113.00
483	64.00	63.00	113.00
484	64.00	63.00	114.00
485	64.00	63.00	114.00
486	64.00	64.00	114.00
487	65.00	64.00	115.00
488	65.00	64.00	115.00
489	65.00	64.00	115.00
490	65.00	64.00	115.00
491	65.00	64.00	116.00
492	65.00	65.00	116.00
493	66.00	65.00	116.00
494	66.00	65.00	117.00
495	66.00	65.00	117.00
496	66.00	65.00	117.00
497	66.00	65.00	118.00
498	66.00	66.00	118.00
499	67.00	66.00	118.00
500	67.00	66.00	119.00
501	67.00	66.00	119.00
502	67.00	66.00	119.00
503	67.00	66.00	120.00
504	67.00	67.00	120.00
505	68.00	67.00	120.00
506	68.00	67.00	121.00
507	68.00	67.00	121.00
508	68.00	67.00	121.00
509	68.00	67.00	121.00
510	68.00	68.00	122.00
511	69.00	68.00	122.00
512	69.00	68.00	122.00
513	69.00	68.00	123.00
514	69.00	68.00	123.00
515	69.00	68.00	123.00
516	69.00	69.00	124.00
517	70.00	69.00	124.00
518	70.00	69.00	124.00
519	70.00	69.00	125.00
520	70.00	69.00	125.00
521	70.00	69.00	125.00
522	70.00	70.00	126.00
523	71.00	70.00	126.00
524	71.00	70.00	126.00
525	71.00	70.00	127.00
526	71.00	70.00	127.00
527	71.00	70.00	127.00
528	71.00	71.00	127.00
529	72.00	71.00	128.00
530	72.00	71.00	128.00
531	72.00	71.00	128.00
532	72.00	71.00	129.00
533	72.00	71.00	129.00
534	72.00	72.00	129.00
535	73.00	72.00	130.00
536	73.00	72.00	130.00
537	73.00	72.00	130.00
538	73.00	72.00	131.00
539	73.00	72.00	131.00
540	73.00	73.00	131.00
541	74.00	73.00	132.00
542	74.00	73.00	132.00
543	74.00	73.00	132.00
544	74.00	73.00	132.00
545	74.00	73.00	133.00
546	74.00	74.00	133.00
547	75.00	74.00	133.00
548	75.00	74.00	134.00
549	75.00	74.00	134.00
550	75.00	74.00	134.00
551	75.00	74.00	135.00
552	75.00	75.00	135.00
553	76.00	75.00	135.00
554	76.00	75.00	136.00
555	76.00	75.00	136.00
556	76.00	75.00	136.00
557	76.00	75.00	137.00
558	76.00	76.00	137.00
559	77.00	76.00	137.00
560	77.00	76.00	138.00
561	77.00	76.00	138.00
562	77.00	76.00	138.00
563	77.00	76.00	138.00
564	77.00	76.00	139.00
565	78.00	77.00	139.00
566	78.00	77.00	139.00
567	78.00	77.00	140.00
568	78.00	77.00	140.00
569	78.00	77.00	140.00
570	79.00	77.00	141.00
571	79.00	78.00	141.00
572	79.00	78.00	141.00
573	79.00	78.00	142.00
574	80.00	78.00	142.00
575	80.00	78.00	142.00
576	80.00	78.00	143.00
577	81.00	79.00	143.00
578	81.00	79.00	143.00
579	81.00	79.00	144.00
580	82.00	80.00	144.00
581	82.00	80.00	144.00
582	82.00	80.00	144.00
583	83.00	81.00	145.00
584	83.00	81.00	145.00
585	83.00	81.00	145.00
586	84.00	82.00	146.00
587	84.00	82.00	146.00
588	84.00	82.00	146.00
589	84.00	83.00	147.00
590	85.00	83.00	147.00
591	85.00	83.00	147.00
592	85.00	84.00	148.00
593	86.00	84.00	148.00
594	86.00	84.00	148.00
595	86.00	84.00	149.00
596	87.00	85.00	149.00
597	87.00	85.00	149.00
598	87.00	85.00	150.00
599	88.00	86.00	150.00
600	88.00	86.00	150.00

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Weekly earnings	Amount to be withheld: With tax-free threshold with leave loading	With tax-free threshold no leave loading	No tax-free threshold
1	2	3	4
601	88.00	86.00	150.00
602	89.00	87.00	151.00
603	89.00	87.00	151.00
604	89.00	87.00	151.00
605	90.00	88.00	152.00
606	90.00	88.00	152.00
607	90.00	88.00	152.00
608	90.00	89.00	153.00
609	91.00	89.00	153.00
610	91.00	89.00	153.00
611	91.00	90.00	154.00
612	92.00	90.00	154.00
613	92.00	90.00	154.00
614	92.00	90.00	155.00
615	93.00	91.00	155.00
616	93.00	91.00	155.00
617	93.00	91.00	155.00
618	94.00	92.00	156.00
619	94.00	92.00	156.00
620	94.00	92.00	156.00
621	95.00	93.00	157.00
622	95.00	93.00	157.00
623	95.00	93.00	157.00
624	96.00	94.00	158.00
625	96.00	94.00	158.00
626	96.00	94.00	158.00
627	96.00	95.00	159.00
628	97.00	95.00	159.00
629	97.00	95.00	159.00
630	97.00	95.00	160.00
631	98.00	96.00	160.00
632	98.00	96.00	160.00
633	98.00	96.00	161.00
634	99.00	97.00	161.00
635	99.00	97.00	161.00
636	99.00	97.00	161.00
637	100.00	98.00	162.00
638	100.00	98.00	162.00
639	100.00	98.00	162.00
640	101.00	99.00	163.00
641	101.00	99.00	163.00
642	101.00	99.00	163.00
643	101.00	100.00	164.00
644	102.00	100.00	164.00
645	102.00	100.00	164.00
646	102.00	101.00	165.00
647	103.00	101.00	165.00
648	103.00	101.00	165.00
649	103.00	101.00	166.00
650	104.00	102.00	166.00
651	104.00	102.00	166.00
652	104.00	102.00	167.00
653	105.00	103.00	167.00
654	105.00	103.00	167.00
655	105.00	103.00	167.00
656	106.00	104.00	168.00
657	106.00	104.00	168.00
658	106.00	104.00	168.00
659	107.00	105.00	169.00
660	107.00	105.00	169.00
661	107.00	105.00	169.00
662	107.00	106.00	170.00
663	108.00	106.00	170.00
664	108.00	106.00	170.00
665	108.00	107.00	171.00
666	109.00	107.00	171.00
667	109.00	107.00	171.00
668	109.00	107.00	172.00
669	110.00	108.00	172.00
670	110.00	108.00	172.00
671	110.00	108.00	173.00
672	111.00	109.00	173.00
673	111.00	109.00	173.00
674	111.00	109.00	173.00
675	112.00	110.00	174.00
676	112.00	110.00	174.00
677	112.00	110.00	174.00
678	113.00	111.00	175.00
679	113.00	111.00	175.00
680	113.00	111.00	175.00

Weekly earnings	Amount to be withheld: With tax-free threshold with leave loading	With tax-free threshold no leave loading	No tax-free threshold
1	2	3	4
681	113.00	112.00	176.00
682	114.00	112.00	176.00
683	114.00	112.00	176.00
684	114.00	113.00	177.00
685	115.00	113.00	177.00
686	115.00	113.00	177.00
687	115.00	113.00	178.00
688	116.00	114.00	178.00
689	116.00	114.00	178.00
690	116.00	114.00	178.00
691	117.00	115.00	179.00
692	117.00	115.00	179.00
693	117.00	115.00	179.00
694	118.00	116.00	180.00
695	118.00	116.00	180.00
696	118.00	116.00	180.00
697	119.00	117.00	181.00
698	119.00	117.00	181.00
699	119.00	117.00	181.00
700	119.00	118.00	182.00
701	120.00	118.00	182.00
702	120.00	118.00	182.00
703	120.00	118.00	183.00
704	121.00	119.00	183.00
705	121.00	119.00	183.00
706	121.00	119.00	184.00
707	122.00	120.00	184.00
708	122.00	120.00	184.00
709	122.00	120.00	184.00
710	123.00	121.00	185.00
711	123.00	121.00	185.00
712	123.00	121.00	185.00
713	124.00	122.00	186.00
714	124.00	122.00	186.00
715	124.00	122.00	186.00
716	124.00	123.00	187.00
717	125.00	123.00	187.00
718	125.00	123.00	187.00
719	125.00	124.00	188.00
720	126.00	124.00	188.00
721	126.00	124.00	188.00
722	126.00	124.00	189.00
723	127.00	125.00	189.00
724	127.00	125.00	189.00
725	127.00	125.00	190.00
726	128.00	126.00	190.00
727	128.00	126.00	190.00
728	128.00	126.00	190.00
729	129.00	127.00	191.00
730	129.00	127.00	191.00
731	129.00	127.00	191.00
732	130.00	128.00	192.00
733	130.00	128.00	192.00
734	130.00	128.00	192.00
735	130.00	129.00	193.00
736	131.00	129.00	193.00
737	131.00	129.00	193.00
738	131.00	130.00	194.00
739	132.00	130.00	194.00
740	132.00	130.00	194.00
741	132.00	130.00	195.00
742	133.00	131.00	195.00
743	133.00	131.00	195.00
744	133.00	131.00	195.00
745	134.00	132.00	196.00
746	134.00	132.00	196.00
747	134.00	132.00	196.00
748	135.00	133.00	197.00
749	135.00	133.00	197.00
750	135.00	133.00	197.00
751	136.00	134.00	198.00
752	136.00	134.00	198.00
753	136.00	134.00	198.00
754	136.00	135.00	199.00
755	137.00	135.00	199.00
756	137.00	135.00	199.00
757	137.00	135.00	200.00
758	138.00	136.00	200.00
759	138.00	136.00	200.00
760	138.00	136.00	201.00

Weekly earnings	Amount to be withheld: With tax-free threshold with leave loading	With tax-free threshold no leave loading	No tax-free threshold
1	2	3	4
761	139.00	137.00	201.00
762	139.00	137.00	201.00
763	139.00	137.00	201.00
764	140.00	138.00	202.00
765	140.00	138.00	202.00
766	140.00	138.00	202.00
767	141.00	139.00	203.00
768	141.00	139.00	203.00
769	141.00	139.00	203.00
770	142.00	140.00	204.00
771	142.00	140.00	204.00
772	142.00	140.00	204.00
773	142.00	141.00	205.00
774	143.00	141.00	205.00
775	143.00	141.00	205.00
776	143.00	141.00	206.00
777	144.00	142.00	206.00
778	144.00	142.00	206.00
779	144.00	142.00	207.00
780	145.00	143.00	207.00
781	145.00	143.00	207.00
782	145.00	143.00	207.00
783	146.00	144.00	208.00
784	146.00	144.00	208.00
785	146.00	144.00	208.00
786	147.00	145.00	209.00
787	147.00	145.00	209.00
788	147.00	145.00	209.00
789	147.00	146.00	210.00
790	148.00	146.00	210.00
791	148.00	146.00	210.00
792	148.00	147.00	211.00
793	149.00	147.00	211.00
794	149.00	147.00	211.00
795	149.00	147.00	212.00
796	150.00	148.00	212.00
797	150.00	148.00	212.00
798	150.00	148.00	213.00
799	151.00	149.00	213.00
800	151.00	149.00	213.00
801	151.00	149.00	213.00
802	152.00	150.00	214.00
803	152.00	150.00	214.00
804	152.00	150.00	214.00
805	153.00	151.00	215.00
806	153.00	151.00	215.00
807	153.00	151.00	215.00
808	153.00	152.00	216.00
809	154.00	152.00	216.00
810	154.00	152.00	216.00
811	154.00	153.00	217.00
812	155.00	153.00	217.00
813	155.00	153.00	217.00
814	155.00	153.00	218.00
815	156.00	154.00	218.00
816	156.00	154.00	218.00
817	156.00	154.00	218.00
818	157.00	155.00	219.00
819	157.00	155.00	219.00
820	157.00	155.00	219.00
821	158.00	156.00	220.00
822	158.00	156.00	220.00
823	158.00	156.00	220.00
824	159.00	157.00	221.00
825	159.00	157.00	221.00
826	159.00	157.00	221.00
827	159.00	158.00	222.00
828	160.00	158.00	222.00
829	160.00	158.00	222.00
830	160.00	158.00	223.00
831	161.00	159.00	223.00
832	161.00	159.00	223.00
833	161.00	159.00	224.00
834	162.00	160.00	224.00
835	162.00	160.00	224.00
836	162.00	160.00	224.00
837	163.00	161.00	225.00
838	163.00	161.00	225.00
839	163.00	161.00	225.00
840	164.00	162.00	226.00

Weekly earnings	Amount to be withheld: With tax-free threshold with leave loading	With tax-free threshold no leave loading	No tax-free threshold
1	2	3	4
841	164.00	162.00	226.00
842	164.00	162.00	226.00
843	164.00	163.00	227.00
844	165.00	163.00	227.00
845	165.00	163.00	227.00
846	165.00	164.00	228.00
847	166.00	164.00	228.00
848	166.00	164.00	228.00
849	166.00	164.00	229.00
850	167.00	165.00	229.00
851	167.00	165.00	229.00
852	167.00	165.00	230.00
853	168.00	166.00	230.00
854	168.00	166.00	230.00
855	168.00	166.00	230.00
856	169.00	167.00	231.00
857	169.00	167.00	231.00
858	169.00	167.00	231.00
859	170.00	168.00	232.00
860	170.00	168.00	232.00
861	170.00	168.00	232.00
862	170.00	169.00	233.00
863	171.00	169.00	233.00
864	171.00	169.00	233.00
865	171.00	170.00	234.00
866	172.00	170.00	234.00
867	172.00	170.00	234.00
868	172.00	170.00	235.00
869	173.00	171.00	235.00
870	173.00	171.00	235.00
871	173.00	171.00	236.00
872	174.00	172.00	236.00
873	174.00	172.00	236.00
874	174.00	172.00	236.00
875	175.00	173.00	237.00
876	175.00	173.00	237.00
877	175.00	173.00	237.00
878	176.00	174.00	238.00
879	176.00	174.00	238.00
880	176.00	174.00	238.00
881	176.00	175.00	239.00
882	177.00	175.00	239.00
883	177.00	175.00	239.00
884	177.00	176.00	240.00
885	178.00	176.00	240.00
886	178.00	176.00	240.00
887	178.00	176.00	241.00
888	179.00	177.00	241.00
889	179.00	177.00	241.00
890	179.00	177.00	241.00
891	180.00	178.00	242.00
892	180.00	178.00	242.00
893	180.00	178.00	242.00
894	181.00	179.00	243.00
895	181.00	179.00	243.00
896	181.00	179.00	243.00
897	182.00	180.00	244.00
898	182.00	180.00	244.00
899	182.00	180.00	244.00
900	182.00	181.00	245.00
901	183.00	181.00	245.00
902	183.00	181.00	245.00
903	183.00	181.00	246.00
904	184.00	182.00	246.00
905	184.00	182.00	246.00
906	184.00	182.00	247.00
907	185.00	183.00	247.00
908	185.00	183.00	247.00
909	185.00	183.00	247.00
910	186.00	184.00	248.00
911	186.00	184.00	248.00
912	186.00	184.00	248.00
913	187.00	185.00	249.00
914	187.00	185.00	249.00
915	187.00	185.00	249.00
916	187.00	186.00	250.00
917	188.00	186.00	250.00
918	188.00	186.00	250.00
919	188.00	187.00	251.00
920	189.00	187.00	251.00

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Weekly earnings	Amount to be withheld: With tax-free threshold with leave loading	With tax-free threshold no leave loading	No tax-free threshold
1	2	3	4
921	189.00	187.00	251.00
922	189.00	187.00	252.00
923	190.00	188.00	252.00
924	190.00	188.00	252.00
925	190.00	188.00	253.00
926	191.00	189.00	253.00
927	191.00	189.00	253.00
928	191.00	189.00	253.00
929	192.00	190.00	254.00
930	192.00	190.00	254.00
931	192.00	190.00	254.00
932	193.00	191.00	255.00
933	193.00	191.00	255.00
934	193.00	191.00	255.00
935	193.00	192.00	256.00
936	194.00	192.00	256.00
937	194.00	192.00	256.00
938	194.00	193.00	257.00
939	195.00	193.00	257.00
940	195.00	193.00	257.00
941	195.00	193.00	258.00
942	196.00	194.00	258.00
943	196.00	194.00	258.00
944	196.00	194.00	258.00
945	197.00	195.00	259.00
946	197.00	195.00	259.00
947	197.00	195.00	259.00
948	198.00	196.00	260.00
949	198.00	196.00	260.00
950	198.00	196.00	260.00
951	199.00	197.00	261.00
952	199.00	197.00	261.00
953	199.00	197.00	261.00
954	199.00	198.00	262.00
955	200.00	198.00	262.00
956	200.00	198.00	262.00
957	200.00	198.00	263.00
958	201.00	199.00	263.00
959	201.00	199.00	263.00
960	201.00	199.00	264.00
961	202.00	200.00	264.00
962	202.00	200.00	264.00
963	202.00	200.00	264.00
964	203.00	201.00	265.00
965	203.00	201.00	265.00
966	203.00	201.00	265.00
967	204.00	202.00	266.00
968	204.00	202.00	266.00
969	204.00	202.00	266.00
970	205.00	203.00	267.00
971	205.00	203.00	267.00
972	205.00	203.00	267.00
973	205.00	204.00	268.00
974	206.00	204.00	268.00
975	206.00	204.00	268.00
976	206.00	204.00	269.00
977	207.00	205.00	269.00
978	207.00	205.00	269.00
979	207.00	205.00	270.00
980	208.00	206.00	270.00
981	208.00	206.00	270.00
982	208.00	206.00	270.00
983	209.00	207.00	271.00
984	209.00	207.00	271.00
985	209.00	207.00	271.00
986	210.00	208.00	272.00
987	210.00	208.00	272.00
988	210.00	208.00	272.00
989	210.00	209.00	273.00
990	211.00	209.00	273.00
991	211.00	209.00	273.00
992	211.00	210.00	274.00
993	212.00	210.00	274.00
994	212.00	210.00	274.00
995	212.00	210.00	275.00
996	213.00	211.00	275.00
997	213.00	211.00	275.00
998	213.00	211.00	276.00
999	214.00	212.00	276.00
1000	214.00	212.00	276.00

Weekly earnings	Amount to be withheld: With tax-free threshold with leave loading	With tax-free threshold no leave loading	No tax-free threshold
1	2	3	4
1001	214.00	212.00	276.00
1002	215.00	213.00	277.00
1003	215.00	213.00	277.00
1004	215.00	213.00	277.00
1005	216.00	214.00	278.00
1006	216.00	214.00	278.00
1007	216.00	214.00	278.00
1008	216.00	215.00	279.00
1009	217.00	215.00	279.00
1010	217.00	215.00	279.00
1011	217.00	216.00	280.00
1012	218.00	216.00	280.00
1013	218.00	216.00	280.00
1014	218.00	216.00	281.00
1015	219.00	217.00	281.00
1016	219.00	217.00	281.00
1017	219.00	217.00	281.00
1018	220.00	218.00	282.00
1019	220.00	218.00	282.00
1020	220.00	218.00	282.00
1021	221.00	219.00	283.00
1022	221.00	219.00	283.00
1023	221.00	219.00	283.00
1024	222.00	220.00	284.00
1025	222.00	220.00	284.00
1026	222.00	220.00	284.00
1027	222.00	221.00	285.00
1028	223.00	221.00	285.00
1029	223.00	221.00	285.00
1030	223.00	221.00	286.00
1031	224.00	222.00	286.00
1032	224.00	222.00	286.00
1033	224.00	222.00	287.00
1034	225.00	223.00	287.00
1035	225.00	223.00	287.00
1036	225.00	223.00	287.00
1037	226.00	224.00	288.00
1038	226.00	224.00	288.00
1039	226.00	224.00	288.00
1040	227.00	225.00	289.00
1041	227.00	225.00	289.00
1042	227.00	225.00	289.00
1043	227.00	226.00	290.00
1044	228.00	226.00	290.00
1045	228.00	226.00	290.00
1046	228.00	227.00	291.00
1047	229.00	227.00	291.00
1048	229.00	227.00	291.00
1049	229.00	227.00	292.00
1050	230.00	228.00	292.00
1051	230.00	228.00	292.00
1052	230.00	228.00	293.00
1053	231.00	229.00	293.00
1054	231.00	229.00	293.00
1055	231.00	229.00	293.00
1056	232.00	230.00	294.00
1057	232.00	230.00	294.00
1058	232.00	230.00	294.00
1059	233.00	231.00	295.00
1060	233.00	231.00	295.00
1061	233.00	231.00	295.00
1062	233.00	232.00	296.00
1063	234.00	232.00	296.00
1064	234.00	232.00	296.00
1065	234.00	233.00	297.00
1066	235.00	233.00	297.00
1067	235.00	233.00	297.00
1068	235.00	233.00	298.00
1069	236.00	234.00	298.00
1070	236.00	234.00	298.00
1071	236.00	234.00	299.00
1072	237.00	235.00	299.00
1073	237.00	235.00	299.00
1074	237.00	235.00	299.00
1075	238.00	236.00	300.00
1076	238.00	236.00	300.00
1077	238.00	236.00	300.00
1078	239.00	237.00	301.00
1079	239.00	237.00	301.00
1080	239.00	237.00	301.00

Weekly earnings	Amount to be withheld: With tax-free threshold with leave loading	With tax-free threshold no leave loading	No tax-free threshold
1	2	3	4
1081	239.00	238.00	302.00
1082	240.00	238.00	302.00
1083	240.00	238.00	302.00
1084	240.00	239.00	303.00
1085	241.00	239.00	303.00
1086	241.00	239.00	303.00
1087	241.00	239.00	304.00
1088	242.00	240.00	304.00
1089	242.00	240.00	304.00
1090	242.00	240.00	304.00
1091	243.00	241.00	305.00
1092	243.00	241.00	305.00
1093	243.00	241.00	305.00
1094	244.00	242.00	306.00
1095	244.00	242.00	306.00
1096	244.00	242.00	306.00
1097	245.00	243.00	307.00
1098	245.00	243.00	307.00
1099	245.00	243.00	307.00
1100	245.00	244.00	308.00
1101	246.00	244.00	308.00
1102	246.00	244.00	308.00
1103	246.00	244.00	309.00
1104	247.00	245.00	309.00
1105	247.00	245.00	309.00
1106	247.00	245.00	310.00
1107	248.00	246.00	310.00
1108	248.00	246.00	310.00
1109	248.00	246.00	310.00
1110	249.00	247.00	311.00
1111	249.00	247.00	311.00
1112	249.00	247.00	311.00
1113	250.00	248.00	312.00
1114	250.00	248.00	312.00
1115	250.00	248.00	312.00
1116	250.00	249.00	313.00
1117	251.00	249.00	313.00
1118	251.00	249.00	313.00
1119	251.00	250.00	314.00
1120	252.00	250.00	314.00
1121	252.00	250.00	314.00
1122	252.00	250.00	315.00
1123	253.00	251.00	315.00
1124	253.00	251.00	315.00
1125	253.00	251.00	316.00
1126	254.00	252.00	316.00
1127	254.00	252.00	316.00
1128	254.00	252.00	317.00
1129	255.00	253.00	317.00
1130	255.00	253.00	318.00
1131	255.00	253.00	318.00
1132	256.00	254.00	319.00
1133	256.00	254.00	319.00
1134	256.00	254.00	319.00
1135	256.00	255.00	320.00
1136	257.00	255.00	320.00
1137	257.00	255.00	321.00
1138	257.00	256.00	321.00
1139	258.00	256.00	321.00
1140	258.00	256.00	322.00
1141	258.00	256.00	322.00
1142	259.00	257.00	323.00
1143	259.00	257.00	323.00
1144	259.00	257.00	323.00
1145	260.00	258.00	324.00
1146	260.00	258.00	324.00
1147	260.00	258.00	325.00
1148	261.00	259.00	325.00
1149	261.00	259.00	326.00
1150	261.00	259.00	326.00
1151	262.00	260.00	326.00
1152	262.00	260.00	327.00
1153	262.00	260.00	327.00
1154	262.00	261.00	328.00
1155	263.00	261.00	328.00
1156	263.00	261.00	328.00
1157	263.00	261.00	329.00
1158	264.00	262.00	329.00
1159	264.00	262.00	330.00
1160	264.00	262.00	330.00

Weekly earnings	Amount to be withheld: With tax-free threshold with leave loading	With tax-free threshold no leave loading	No tax-free threshold
1	2	3	4
1161	265.00	263.00	331.00
1162	265.00	263.00	331.00
1163	265.00	263.00	331.00
1164	266.00	264.00	332.00
1165	266.00	264.00	332.00
1166	266.00	264.00	333.00
1167	267.00	265.00	333.00
1168	267.00	265.00	333.00
1169	267.00	265.00	334.00
1170	268.00	266.00	334.00
1171	268.00	266.00	335.00
1172	268.00	266.00	335.00
1173	268.00	267.00	336.00
1174	269.00	267.00	336.00
1175	269.00	267.00	336.00
1176	269.00	267.00	337.00
1177	270.00	268.00	337.00
1178	270.00	268.00	338.00
1179	270.00	268.00	338.00
1180	271.00	269.00	338.00
1181	271.00	269.00	339.00
1182	271.00	269.00	339.00
1183	272.00	270.00	340.00
1184	272.00	270.00	340.00
1185	272.00	270.00	341.00
1186	273.00	271.00	341.00
1187	273.00	271.00	341.00
1188	273.00	271.00	342.00
1189	273.00	272.00	342.00
1190	274.00	272.00	343.00
1191	274.00	272.00	343.00
1192	274.00	273.00	343.00
1193	275.00	273.00	344.00
1194	275.00	273.00	344.00
1195	275.00	273.00	345.00
1196	276.00	274.00	345.00
1197	276.00	274.00	345.00
1198	276.00	274.00	346.00
1199	277.00	275.00	346.00
1200	277.00	275.00	347.00
1201	277.00	275.00	347.00
1202	278.00	276.00	348.00
1203	278.00	276.00	348.00
1204	278.00	276.00	348.00
1205	279.00	277.00	349.00
1206	279.00	277.00	349.00
1207	279.00	277.00	350.00
1208	279.00	278.00	350.00
1209	280.00	278.00	350.00
1210	280.00	278.00	351.00
1211	280.00	279.00	351.00
1212	281.00	279.00	352.00
1213	281.00	279.00	352.00
1214	281.00	279.00	353.00
1215	282.00	280.00	353.00
1216	282.00	280.00	353.00
1217	282.00	280.00	354.00
1218	283.00	281.00	354.00
1219	283.00	281.00	355.00
1220	283.00	281.00	355.00
1221	284.00	282.00	355.00
1222	284.00	282.00	356.00
1223	284.00	282.00	356.00
1224	285.00	283.00	357.00
1225	285.00	283.00	357.00
1226	285.00	283.00	358.00
1227	285.00	284.00	358.00
1228	286.00	284.00	358.00
1229	286.00	284.00	359.00
1230	286.00	284.00	359.00
1231	287.00	285.00	360.00
1232	287.00	285.00	360.00
1233	287.00	285.00	360.00
1234	288.00	286.00	361.00
1235	288.00	286.00	361.00
1236	288.00	286.00	362.00
1237	289.00	287.00	362.00
1238	289.00	287.00	363.00
1239	289.00	287.00	363.00
1240	290.00	288.00	363.00

Weekly earnings	Amount to be withheld: With tax-free threshold with leave loading	Amount to be withheld: With tax-free threshold no leave loading	Amount to be withheld: No tax-free threshold
1	2	3	4
1241	290.00	288.00	364.00
1242	290.00	288.00	364.00
1243	290.00	289.00	365.00
1244	291.00	289.00	365.00
1245	291.00	289.00	365.00
1246	291.00	290.00	366.00
1247	292.00	290.00	366.00
1248	292.00	290.00	367.00
1249	292.00	290.00	367.00
1250	293.00	291.00	367.00
1251	293.00	291.00	368.00
1252	293.00	291.00	368.00
1253	294.00	292.00	369.00
1254	294.00	292.00	369.00
1255	294.00	292.00	370.00
1256	295.00	293.00	370.00
1257	295.00	293.00	370.00
1258	295.00	293.00	371.00
1259	296.00	294.00	371.00
1260	296.00	294.00	372.00
1261	296.00	294.00	372.00
1262	296.00	295.00	372.00
1263	297.00	295.00	373.00
1264	297.00	295.00	373.00
1265	297.00	296.00	374.00
1266	298.00	296.00	374.00
1267	298.00	296.00	375.00
1268	298.00	296.00	375.00
1269	299.00	297.00	375.00
1270	299.00	297.00	376.00
1271	299.00	297.00	376.00
1272	300.00	298.00	377.00
1273	300.00	298.00	377.00
1274	300.00	298.00	377.00
1275	301.00	299.00	378.00
1276	301.00	299.00	378.00
1277	301.00	299.00	379.00
1278	302.00	300.00	379.00
1279	302.00	300.00	380.00
1280	302.00	300.00	380.00
1281	302.00	301.00	380.00
1282	303.00	301.00	381.00
1283	303.00	301.00	381.00
1284	303.00	302.00	382.00
1285	304.00	302.00	382.00
1286	304.00	302.00	382.00
1287	304.00	302.00	383.00
1288	305.00	303.00	383.00
1289	305.00	303.00	384.00
1290	305.00	303.00	384.00
1291	306.00	304.00	384.00
1292	306.00	304.00	385.00
1293	306.00	304.00	385.00
1294	307.00	305.00	386.00
1295	307.00	305.00	386.00
1296	307.00	305.00	387.00
1297	308.00	306.00	387.00
1298	308.00	306.00	387.00
1299	308.00	306.00	388.00
1300	308.00	307.00	388.00
1301	309.00	307.00	389.00
1302	309.00	307.00	389.00
1303	309.00	307.00	389.00
1304	310.00	308.00	390.00
1305	310.00	308.00	390.00
1306	310.00	308.00	391.00
1307	311.00	309.00	391.00
1308	311.00	309.00	392.00
1309	311.00	309.00	392.00
1310	312.00	310.00	392.00
1311	312.00	310.00	393.00
1312	312.00	310.00	393.00
1313	313.00	311.00	394.00
1314	313.00	311.00	394.00
1315	313.00	311.00	394.00
1316	313.00	312.00	395.00
1317	314.00	312.00	395.00
1318	314.00	312.00	396.00
1319	314.00	313.00	396.00
1320	315.00	313.00	397.00

Weekly earnings	Amount to be withheld: With tax-free threshold with leave loading	Amount to be withheld: With tax-free threshold no leave loading	Amount to be withheld: No tax-free threshold
1	2	3	4
1321	315.00	313.00	397.00
1322	315.00	313.00	397.00
1323	316.00	314.00	398.00
1324	316.00	314.00	398.00
1325	316.00	314.00	399.00
1326	317.00	315.00	399.00
1327	317.00	315.00	399.00
1328	317.00	315.00	400.00
1329	318.00	316.00	400.00
1330	318.00	316.00	401.00
1331	318.00	316.00	401.00
1332	319.00	317.00	402.00
1333	319.00	317.00	402.00
1334	319.00	317.00	402.00
1335	319.00	318.00	403.00
1336	320.00	318.00	403.00
1337	320.00	318.00	404.00
1338	320.00	319.00	404.00
1339	321.00	319.00	404.00
1340	321.00	319.00	405.00
1341	321.00	319.00	405.00
1342	322.00	320.00	406.00
1343	322.00	320.00	406.00
1344	322.00	320.00	406.00
1345	323.00	321.00	407.00
1346	323.00	321.00	407.00
1347	323.00	321.00	408.00
1348	324.00	322.00	408.00
1349	324.00	322.00	409.00
1350	324.00	322.00	409.00
1351	325.00	323.00	409.00
1352	325.00	323.00	410.00
1353	325.00	323.00	410.00
1354	325.00	324.00	411.00
1355	326.00	324.00	411.00
1356	326.00	324.00	411.00
1357	326.00	324.00	412.00
1358	327.00	325.00	412.00
1359	327.00	325.00	413.00
1360	327.00	325.00	413.00
1361	328.00	326.00	414.00
1362	328.00	326.00	414.00
1363	328.00	326.00	414.00
1364	329.00	327.00	415.00
1365	329.00	327.00	415.00
1366	329.00	327.00	416.00
1367	330.00	328.00	416.00
1368	330.00	328.00	416.00
1369	330.00	328.00	417.00
1370	331.00	329.00	417.00
1371	331.00	329.00	418.00
1372	331.00	329.00	418.00
1373	331.00	330.00	419.00
1374	332.00	330.00	419.00
1375	332.00	330.00	419.00
1376	332.00	330.00	420.00
1377	333.00	331.00	420.00
1378	333.00	331.00	421.00
1379	333.00	331.00	421.00
1380	334.00	332.00	421.00
1381	334.00	332.00	422.00
1382	334.00	332.00	422.00
1383	335.00	333.00	423.00
1384	335.00	333.00	423.00
1385	335.00	333.00	424.00
1386	336.00	334.00	424.00
1387	336.00	334.00	424.00
1388	336.00	334.00	425.00
1389	336.00	335.00	425.00
1390	337.00	335.00	426.00
1391	337.00	335.00	426.00
1392	337.00	336.00	426.00
1393	338.00	336.00	427.00
1394	338.00	336.00	427.00
1395	338.00	336.00	428.00
1396	339.00	337.00	428.00
1397	339.00	337.00	428.00
1398	339.00	337.00	429.00
1399	340.00	338.00	429.00
1400	340.00	338.00	430.00

Weekly earnings	Amount to be withheld: With tax-free threshold with leave loading	Amount to be withheld: With tax-free threshold no leave loading	Amount to be withheld: No tax-free threshold
1	2	3	4
1401	340.00	338.00	430.00
1402	341.00	339.00	431.00
1403	341.00	339.00	431.00
1404	341.00	339.00	431.00
1405	342.00	340.00	432.00
1406	342.00	340.00	432.00
1407	342.00	340.00	433.00
1408	342.00	341.00	433.00
1409	343.00	341.00	433.00
1410	343.00	341.00	434.00
1411	343.00	342.00	434.00
1412	344.00	342.00	435.00
1413	344.00	342.00	435.00
1414	344.00	342.00	436.00
1415	345.00	343.00	436.00
1416	345.00	343.00	436.00
1417	345.00	343.00	437.00
1418	346.00	344.00	437.00
1419	346.00	344.00	438.00
1420	346.00	344.00	438.00
1421	347.00	345.00	438.00
1422	347.00	345.00	439.00
1423	347.00	345.00	439.00
1424	348.00	346.00	440.00
1425	348.00	346.00	440.00
1426	348.00	346.00	441.00
1427	348.00	347.00	441.00
1428	349.00	347.00	441.00
1429	349.00	347.00	442.00
1430	349.00	347.00	442.00
1431	350.00	348.00	443.00
1432	350.00	348.00	443.00
1433	350.00	348.00	443.00
1434	351.00	349.00	444.00
1435	351.00	349.00	444.00
1436	351.00	349.00	445.00
1437	352.00	350.00	445.00
1438	352.00	350.00	446.00
1439	353.00	350.00	446.00
1440	353.00	351.00	446.00
1441	353.00	351.00	447.00
1442	354.00	351.00	447.00
1443	354.00	352.00	448.00
1444	355.00	352.00	448.00
1445	355.00	353.00	448.00
1446	356.00	353.00	449.00
1447	356.00	353.00	449.00
1448	356.00	354.00	450.00
1449	357.00	354.00	450.00
1450	357.00	355.00	450.00
1451	358.00	355.00	451.00
1452	358.00	355.00	451.00
1453	358.00	356.00	452.00
1454	359.00	356.00	452.00
1455	359.00	357.00	453.00
1456	360.00	357.00	453.00
1457	360.00	358.00	453.00
1458	361.00	358.00	454.00
1459	361.00	358.00	454.00
1460	361.00	359.00	455.00
1461	362.00	359.00	455.00
1462	362.00	360.00	455.00
1463	363.00	360.00	456.00
1464	363.00	360.00	456.00
1465	363.00	361.00	457.00
1466	364.00	361.00	457.00
1467	364.00	362.00	458.00
1468	365.00	362.00	458.00
1469	365.00	363.00	458.00
1470	365.00	363.00	459.00
1471	366.00	363.00	459.00
1472	366.00	364.00	460.00
1473	367.00	364.00	460.00
1474	367.00	365.00	460.00
1475	368.00	365.00	461.00
1476	368.00	365.00	461.00
1477	368.00	366.00	462.00
1478	369.00	366.00	462.00
1479	369.00	367.00	463.00
1480	370.00	367.00	463.00

Weekly earnings	Amount to be withheld: With tax-free threshold with leave loading	Amount to be withheld: With tax-free threshold no leave loading	Amount to be withheld: No tax-free threshold
1	2	3	4
1481	370.00	368.00	463.00
1482	370.00	368.00	464.00
1483	371.00	368.00	464.00
1484	371.00	369.00	465.00
1485	372.00	369.00	465.00
1486	372.00	370.00	465.00
1487	373.00	370.00	466.00
1488	373.00	370.00	466.00
1489	373.00	371.00	467.00
1490	374.00	371.00	467.00
1491	374.00	372.00	467.00
1492	375.00	372.00	468.00
1493	375.00	373.00	468.00
1494	375.00	373.00	469.00
1495	376.00	373.00	469.00
1496	376.00	374.00	470.00
1497	377.00	374.00	470.00
1498	377.00	375.00	470.00
1499	378.00	375.00	471.00
1500	378.00	375.00	471.00
1501	378.00	376.00	472.00
1502	379.00	376.00	472.00
1503	379.00	377.00	472.00
1504	380.00	377.00	473.00
1505	380.00	377.00	473.00
1506	380.00	378.00	474.00
1507	381.00	378.00	474.00
1508	381.00	379.00	475.00
1509	382.00	379.00	475.00
1510	382.00	380.00	475.00
1511	383.00	380.00	476.00
1512	383.00	380.00	476.00
1513	383.00	381.00	477.00
1514	384.00	381.00	477.00
1515	384.00	382.00	477.00
1516	385.00	382.00	478.00
1517	385.00	382.00	478.00
1518	385.00	383.00	479.00
1519	386.00	383.00	479.00
1520	386.00	384.00	480.00
1521	387.00	384.00	480.00
1522	387.00	385.00	480.00
1523	387.00	385.00	481.00
1524	388.00	385.00	481.00
1525	388.00	386.00	482.00
1526	389.00	386.00	482.00
1527	389.00	387.00	482.00
1528	390.00	387.00	483.00
1529	390.00	387.00	483.00
1530	390.00	388.00	484.00
1531	391.00	388.00	484.00
1532	391.00	389.00	485.00
1533	392.00	389.00	485.00
1534	392.00	390.00	485.00
1535	392.00	390.00	486.00
1536	393.00	390.00	486.00
1537	393.00	391.00	487.00
1538	394.00	391.00	487.00
1539	394.00	392.00	487.00
1540	395.00	392.00	488.00
1541	395.00	392.00	488.00
1542	395.00	393.00	489.00
1543	396.00	393.00	489.00
1544	396.00	394.00	489.00
1545	397.00	394.00	490.00
1546	397.00	395.00	490.00
1547	397.00	395.00	491.00
1548	398.00	395.00	491.00
1549	398.00	396.00	492.00
1550	399.00	396.00	492.00
1551	399.00	397.00	492.00
1552	400.00	397.00	493.00
1553	400.00	397.00	493.00
1554	400.00	398.00	494.00
1555	401.00	398.00	494.00
1556	401.00	399.00	494.00
1557	402.00	399.00	495.00
1558	402.00	399.00	495.00
1559	402.00	400.00	496.00
1560	403.00	400.00	496.00

	Amount to be withheld		
Weekly earnings	With tax-free threshold with leave loading	With tax-free threshold no leave loading	No tax-free threshold
1	2	3	4
1561	403.00	401.00	497.00
1562	404.00	401.00	497.00
1563	404.00	402.00	497.00
1564	404.00	402.00	498.00
1565	405.00	402.00	498.00
1566	405.00	403.00	499.00
1567	406.00	403.00	499.00
1568	406.00	404.00	499.00
1569	407.00	404.00	500.00
1570	407.00	404.00	500.00
1571	407.00	405.00	501.00
1572	408.00	405.00	501.00
1573	408.00	406.00	502.00
1574	409.00	406.00	502.00
1575	409.00	407.00	502.00
1576	409.00	407.00	503.00
1577	410.00	407.00	503.00
1578	410.00	408.00	504.00
1579	411.00	408.00	504.00
1580	411.00	409.00	504.00
1581	412.00	409.00	505.00
1582	412.00	409.00	505.00
1583	412.00	410.00	506.00
1584	413.00	410.00	506.00
1585	413.00	411.00	507.00
1586	414.00	411.00	507.00
1587	414.00	412.00	507.00
1588	414.00	412.00	508.00
1589	415.00	412.00	508.00
1590	415.00	413.00	509.00
1591	416.00	413.00	509.00
1592	416.00	414.00	509.00
1593	417.00	414.00	510.00
1594	417.00	414.00	510.00
1595	417.00	415.00	511.00
1596	418.00	415.00	511.00
1597	418.00	416.00	511.00
1598	419.00	416.00	512.00
1599	419.00	416.00	512.00
1600	419.00	417.00	513.00
1601	420.00	417.00	513.00
1602	420.00	418.00	514.00
1603	421.00	418.00	514.00
1604	421.00	419.00	514.00
1605	422.00	419.00	515.00
1606	422.00	419.00	515.00
1607	422.00	420.00	516.00
1608	423.00	420.00	516.00
1609	423.00	421.00	516.00
1610	424.00	421.00	517.00
1611	424.00	421.00	517.00
1612	424.00	422.00	518.00
1613	425.00	422.00	518.00
1614	425.00	423.00	519.00
1615	426.00	423.00	519.00
1616	426.00	424.00	519.00
1617	426.00	424.00	520.00
1618	427.00	424.00	520.00
1619	427.00	425.00	521.00
1620	428.00	425.00	521.00
1621	428.00	426.00	521.00
1622	429.00	426.00	522.00
1623	429.00	426.00	522.00
1624	429.00	427.00	523.00
1625	430.00	427.00	523.00
1626	430.00	428.00	524.00
1627	431.00	428.00	524.00
1628	431.00	429.00	524.00
1629	431.00	429.00	525.00
1630	432.00	429.00	525.00
1631	432.00	430.00	526.00
1632	433.00	430.00	526.00
1633	433.00	431.00	526.00
1634	434.00	431.00	527.00
1635	434.00	431.00	527.00
1636	434.00	432.00	528.00
1637	435.00	432.00	528.00
1638	435.00	433.00	529.00
1639	436.00	433.00	529.00
1640	436.00	434.00	529.00

	Amount to be withheld		
Weekly earnings	With tax-free threshold with leave loading	With tax-free threshold no leave loading	No tax-free threshold
1	2	3	4
1641	436.00	434.00	530.00
1642	437.00	434.00	530.00
1643	437.00	435.00	531.00
1644	438.00	435.00	531.00
1645	438.00	436.00	531.00
1646	439.00	436.00	532.00
1647	439.00	436.00	532.00
1648	439.00	437.00	533.00
1649	440.00	437.00	533.00
1650	440.00	438.00	533.00
1651	441.00	438.00	534.00
1652	441.00	438.00	534.00
1653	441.00	439.00	535.00
1654	442.00	439.00	535.00
1655	442.00	440.00	536.00
1656	443.00	440.00	536.00
1657	443.00	441.00	536.00
1658	444.00	441.00	537.00
1659	444.00	441.00	537.00
1660	444.00	442.00	538.00
1661	445.00	442.00	538.00
1662	445.00	443.00	538.00
1663	446.00	443.00	539.00
1664	446.00	443.00	539.00
1665	446.00	444.00	540.00
1666	447.00	444.00	540.00
1667	447.00	445.00	541.00
1668	448.00	445.00	541.00
1669	448.00	446.00	541.00
1670	448.00	446.00	542.00
1671	449.00	446.00	542.00
1672	449.00	447.00	543.00
1673	450.00	447.00	543.00
1674	450.00	448.00	543.00
1675	451.00	448.00	544.00
1676	451.00	448.00	544.00
1677	451.00	449.00	545.00
1678	452.00	449.00	545.00
1679	452.00	450.00	546.00
1680	453.00	450.00	546.00
1681	453.00	451.00	546.00
1682	453.00	451.00	547.00
1683	454.00	451.00	547.00
1684	454.00	452.00	548.00
1685	455.00	452.00	548.00
1686	455.00	453.00	548.00
1687	456.00	453.00	549.00
1688	456.00	453.00	549.00
1689	456.00	454.00	550.00
1690	457.00	454.00	550.00
1691	457.00	455.00	550.00
1692	458.00	455.00	551.00
1693	458.00	456.00	551.00
1694	458.00	456.00	552.00
1695	459.00	456.00	552.00
1696	459.00	457.00	553.00
1697	460.00	457.00	553.00
1698	460.00	458.00	553.00
1699	461.00	458.00	554.00
1700	461.00	458.00	554.00
1701	461.00	459.00	555.00
1702	462.00	459.00	555.00
1703	462.00	460.00	555.00
1704	463.00	460.00	556.00
1705	463.00	460.00	556.00
1706	463.00	461.00	557.00
1707	464.00	461.00	557.00
1708	464.00	462.00	558.00
1709	465.00	462.00	558.00
1710	465.00	463.00	558.00
1711	466.00	463.00	559.00
1712	466.00	463.00	559.00
1713	466.00	464.00	560.00
1714	467.00	464.00	560.00
1715	467.00	465.00	560.00
1716	468.00	465.00	561.00
1717	468.00	465.00	561.00
1718	468.00	466.00	562.00
1719	469.00	466.00	562.00
1720	469.00	467.00	563.00

	Amount to be withheld		
Weekly earnings	With tax-free threshold with leave loading	With tax-free threshold no leave loading	No tax-free threshold
1	2	3	4
1721	470.00	467.00	563.00
1722	470.00	468.00	563.00
1723	470.00	468.00	564.00
1724	471.00	468.00	564.00
1725	471.00	469.00	565.00
1726	472.00	469.00	565.00
1727	472.00	470.00	565.00
1728	473.00	470.00	566.00
1729	473.00	470.00	566.00
1730	473.00	471.00	567.00
1731	474.00	471.00	567.00
1732	474.00	472.00	568.00
1733	475.00	472.00	568.00
1734	475.00	473.00	568.00
1735	475.00	473.00	569.00
1736	476.00	473.00	569.00
1737	476.00	474.00	570.00
1738	477.00	474.00	570.00
1739	477.00	475.00	570.00
1740	478.00	475.00	571.00
1741	478.00	475.00	571.00
1742	478.00	476.00	572.00
1743	479.00	476.00	572.00
1744	479.00	477.00	572.00
1745	480.00	477.00	573.00
1746	480.00	478.00	573.00
1747	480.00	478.00	574.00
1748	481.00	478.00	574.00
1749	481.00	479.00	575.00
1750	482.00	479.00	575.00
1751	482.00	480.00	575.00
1752	483.00	480.00	576.00
1753	483.00	480.00	576.00
1754	483.00	481.00	577.00
1755	484.00	481.00	577.00
1756	484.00	482.00	577.00
1757	485.00	482.00	578.00
1758	485.00	482.00	578.00
1759	485.00	483.00	579.00
1760	486.00	483.00	579.00
1761	486.00	484.00	580.00
1762	487.00	484.00	580.00
1763	487.00	485.00	580.00
1764	487.00	485.00	581.00
1765	488.00	485.00	581.00
1766	488.00	486.00	582.00
1767	489.00	486.00	582.00
1768	489.00	487.00	582.00
1769	490.00	487.00	583.00
1770	490.00	487.00	583.00
1771	490.00	488.00	584.00
1772	491.00	488.00	584.00
1773	491.00	489.00	585.00
1774	492.00	489.00	585.00
1775	492.00	490.00	585.00
1776	492.00	490.00	586.00
1777	493.00	490.00	586.00
1778	493.00	491.00	587.00
1779	494.00	491.00	587.00
1780	494.00	492.00	587.00
1781	495.00	492.00	588.00
1782	495.00	492.00	588.00
1783	495.00	493.00	589.00
1784	496.00	493.00	589.00
1785	496.00	494.00	590.00
1786	497.00	494.00	590.00
1787	497.00	495.00	590.00
1788	497.00	495.00	591.00
1789	498.00	495.00	591.00
1790	498.00	496.00	592.00
1791	499.00	496.00	592.00
1792	499.00	497.00	592.00
1793	500.00	497.00	593.00
1794	500.00	497.00	593.00
1795	500.00	498.00	594.00
1796	501.00	498.00	594.00
1797	501.00	499.00	594.00
1798	502.00	499.00	595.00
1799	502.00	499.00	595.00
1800	502.00	500.00	596.00

	Amount to be withheld		
Weekly earnings	With tax-free threshold with leave loading	With tax-free threshold no leave loading	No tax-free threshold
1	2	3	4
1801	503.00	500.00	596.00
1802	503.00	501.00	597.00
1803	504.00	501.00	597.00
1804	504.00	502.00	597.00
1805	505.00	502.00	598.00
1806	505.00	502.00	598.00
1807	505.00	503.00	599.00
1808	506.00	503.00	599.00
1809	506.00	504.00	599.00
1810	507.00	504.00	600.00
1811	507.00	504.00	600.00
1812	507.00	505.00	601.00
1813	508.00	505.00	601.00
1814	508.00	506.00	602.00
1815	509.00	506.00	602.00
1816	509.00	507.00	602.00
1817	509.00	507.00	603.00
1818	510.00	507.00	603.00
1819	510.00	508.00	604.00
1820	511.00	508.00	604.00
1821	511.00	509.00	604.00
1822	512.00	509.00	605.00
1823	512.00	509.00	605.00
1824	512.00	510.00	606.00
1825	513.00	510.00	606.00
1826	513.00	511.00	607.00
1827	514.00	511.00	607.00
1828	514.00	512.00	607.00
1829	514.00	512.00	608.00
1830	515.00	512.00	608.00
1831	515.00	513.00	609.00
1832	516.00	513.00	609.00
1833	516.00	514.00	609.00
1834	517.00	514.00	610.00
1835	517.00	514.00	610.00
1836	517.00	515.00	611.00
1837	518.00	515.00	611.00
1838	518.00	516.00	612.00
1839	519.00	516.00	612.00
1840	519.00	517.00	612.00
1841	519.00	517.00	613.00
1842	520.00	517.00	613.00
1843	520.00	518.00	614.00
1844	521.00	518.00	614.00
1845	521.00	519.00	614.00
1846	522.00	519.00	615.00
1847	522.00	519.00	615.00
1848	522.00	520.00	616.00
1849	523.00	520.00	616.00
1850	523.00	521.00	616.00
1851	524.00	521.00	617.00
1852	524.00	521.00	617.00
1853	524.00	522.00	618.00
1854	525.00	522.00	618.00
1855	525.00	523.00	619.00
1856	526.00	523.00	619.00
1857	526.00	524.00	619.00
1858	527.00	524.00	620.00
1859	527.00	524.00	620.00
1860	527.00	525.00	621.00
1861	528.00	525.00	621.00
1862	528.00	526.00	621.00
1863	529.00	526.00	622.00
1864	529.00	526.00	622.00
1865	529.00	527.00	623.00
1866	530.00	527.00	623.00
1867	530.00	528.00	624.00
1868	531.00	528.00	624.00
1869	531.00	529.00	624.00
1870	531.00	529.00	625.00
1871	532.00	529.00	625.00
1872	532.00	530.00	626.00
1873	533.00	530.00	626.00
1874	533.00	531.00	626.00
1875	534.00	531.00	627.00
1876	534.00	531.00	627.00
1877	534.00	532.00	628.00
1878	535.00	532.00	628.00
1879	535.00	533.00	629.00
1880	536.00	533.00	629.00

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Weekly earnings	Amount to be withheld: With tax-free threshold with leave loading	With tax-free threshold no leave loading	No tax-free threshold
1	2	3	4
1881	536.00	534.00	629.00
1882	536.00	534.00	630.00
1883	537.00	534.00	630.00
1884	537.00	535.00	631.00
1885	538.00	535.00	631.00
1886	538.00	536.00	631.00
1887	539.00	536.00	632.00
1888	539.00	536.00	632.00
1889	539.00	537.00	633.00
1890	540.00	537.00	633.00
1891	540.00	538.00	633.00
1892	541.00	538.00	634.00
1893	541.00	539.00	634.00
1894	541.00	539.00	635.00
1895	542.00	539.00	635.00
1896	542.00	540.00	636.00
1897	543.00	540.00	636.00
1898	543.00	541.00	636.00
1899	544.00	541.00	637.00
1900	544.00	541.00	637.00
1901	544.00	542.00	638.00
1902	545.00	542.00	638.00
1903	545.00	543.00	638.00
1904	546.00	543.00	639.00
1905	546.00	543.00	639.00
1906	546.00	544.00	640.00
1907	547.00	544.00	640.00
1908	547.00	545.00	641.00
1909	548.00	545.00	641.00
1910	548.00	546.00	641.00
1911	549.00	546.00	642.00
1912	549.00	546.00	642.00
1913	549.00	547.00	643.00
1914	550.00	547.00	643.00
1915	550.00	548.00	643.00
1916	551.00	548.00	644.00
1917	551.00	548.00	644.00
1918	551.00	549.00	645.00
1919	552.00	549.00	645.00
1920	552.00	550.00	646.00
1921	553.00	550.00	646.00
1922	553.00	551.00	646.00
1923	553.00	551.00	647.00
1924	554.00	551.00	647.00
1925	554.00	552.00	648.00
1926	555.00	552.00	648.00
1927	555.00	553.00	648.00
1928	556.00	553.00	649.00
1929	556.00	553.00	649.00
1930	556.00	554.00	650.00
1931	557.00	554.00	650.00
1932	557.00	555.00	651.00
1933	558.00	555.00	651.00
1934	558.00	556.00	651.00
1935	558.00	556.00	652.00
1936	559.00	556.00	652.00
1937	559.00	557.00	653.00
1938	560.00	557.00	653.00
1939	560.00	558.00	653.00
1940	561.00	558.00	654.00
1941	561.00	558.00	654.00
1942	561.00	559.00	655.00
1943	562.00	559.00	655.00
1944	562.00	560.00	655.00
1945	563.00	560.00	656.00
1946	563.00	561.00	656.00
1947	563.00	561.00	657.00
1948	564.00	561.00	657.00
1949	564.00	562.00	658.00
1950	565.00	562.00	658.00
1951	565.00	563.00	658.00
1952	566.00	563.00	659.00
1953	566.00	563.00	659.00
1954	566.00	564.00	660.00
1955	567.00	564.00	660.00
1956	567.00	565.00	660.00
1957	568.00	565.00	661.00
1958	568.00	565.00	661.00
1959	568.00	566.00	662.00
1960	569.00	566.00	662.00

Weekly earnings	Amount to be withheld: With tax-free threshold with leave loading	With tax-free threshold no leave loading	No tax-free threshold
1	2	3	4
1961	569.00	567.00	663.00
1962	570.00	567.00	663.00
1963	570.00	568.00	663.00
1964	570.00	568.00	664.00
1965	571.00	568.00	664.00
1966	571.00	569.00	665.00
1967	572.00	569.00	665.00
1968	572.00	570.00	665.00
1969	573.00	570.00	666.00
1970	573.00	570.00	666.00
1971	573.00	571.00	667.00
1972	574.00	571.00	667.00
1973	574.00	572.00	668.00
1974	575.00	572.00	668.00
1975	575.00	573.00	668.00
1976	575.00	573.00	669.00
1977	576.00	573.00	669.00
1978	576.00	574.00	670.00
1979	577.00	574.00	670.00
1980	577.00	575.00	670.00
1981	578.00	575.00	671.00
1982	578.00	575.00	671.00
1983	578.00	576.00	672.00
1984	579.00	576.00	672.00
1985	579.00	577.00	673.00
1986	580.00	577.00	673.00
1987	580.00	578.00	673.00
1988	580.00	578.00	674.00
1989	581.00	578.00	674.00
1990	581.00	579.00	675.00
1991	582.00	579.00	675.00
1992	582.00	580.00	675.00
1993	583.00	580.00	676.00
1994	583.00	580.00	676.00
1995	583.00	581.00	677.00
1996	584.00	581.00	677.00
1997	584.00	582.00	677.00
1998	585.00	582.00	678.00
1999	585.00	582.00	678.00
2000	585.00	583.00	679.00
2001	586.00	583.00	679.00
2002	586.00	584.00	680.00
2003	587.00	584.00	680.00
2004	587.00	585.00	680.00
2005	588.00	585.00	681.00
2006	588.00	585.00	681.00
2007	588.00	586.00	682.00
2008	589.00	586.00	682.00
2009	589.00	587.00	682.00
2010	590.00	587.00	683.00
2011	590.00	587.00	683.00
2012	590.00	588.00	684.00
2013	591.00	588.00	684.00
2014	591.00	589.00	685.00
2015	592.00	589.00	685.00
2016	592.00	590.00	685.00
2017	592.00	590.00	686.00
2018	593.00	590.00	686.00
2019	593.00	591.00	687.00
2020	594.00	591.00	687.00
2021	594.00	592.00	687.00
2022	595.00	592.00	688.00
2023	595.00	592.00	688.00
2024	595.00	593.00	689.00
2025	596.00	593.00	689.00
2026	596.00	594.00	690.00
2027	597.00	594.00	690.00
2028	597.00	595.00	690.00
2029	597.00	595.00	691.00
2030	598.00	595.00	691.00
2031	598.00	596.00	692.00
2032	599.00	596.00	692.00
2033	599.00	597.00	692.00
2034	600.00	597.00	693.00
2035	600.00	597.00	693.00
2036	600.00	598.00	694.00
2037	601.00	598.00	694.00
2038	601.00	599.00	695.00
2039	602.00	599.00	695.00
2040	602.00	600.00	695.00

Weekly earnings	Amount to be withheld: With tax-free threshold with leave loading	With tax-free threshold no leave loading	No tax-free threshold
1	2	3	4
2041	602.00	600.00	696.00
2042	603.00	600.00	696.00
2043	603.00	601.00	697.00
2044	604.00	601.00	697.00
2045	604.00	602.00	697.00
2046	605.00	602.00	698.00
2047	605.00	602.00	698.00
2048	605.00	603.00	699.00
2049	606.00	603.00	699.00
2050	606.00	604.00	699.00
2051	607.00	604.00	700.00
2052	607.00	604.00	700.00
2053	607.00	605.00	701.00
2054	608.00	605.00	701.00
2055	608.00	606.00	702.00
2056	609.00	606.00	702.00
2057	609.00	607.00	702.00
2058	610.00	607.00	703.00
2059	610.00	607.00	703.00
2060	610.00	608.00	704.00
2061	611.00	608.00	704.00
2062	611.00	609.00	704.00
2063	612.00	609.00	705.00
2064	612.00	609.00	705.00
2065	612.00	610.00	706.00
2066	613.00	610.00	706.00
2067	613.00	611.00	707.00
2068	614.00	611.00	707.00
2069	614.00	612.00	707.00
2070	614.00	612.00	708.00
2071	615.00	612.00	708.00
2072	615.00	613.00	709.00
2073	616.00	613.00	709.00
2074	616.00	614.00	709.00
2075	617.00	614.00	710.00
2076	617.00	614.00	710.00
2077	617.00	615.00	711.00
2078	618.00	615.00	711.00
2079	618.00	616.00	712.00
2080	619.00	616.00	712.00
2081	619.00	617.00	712.00
2082	619.00	617.00	713.00
2083	620.00	617.00	713.00
2084	620.00	618.00	714.00
2085	621.00	618.00	714.00
2086	621.00	619.00	714.00
2087	622.00	619.00	715.00
2088	622.00	619.00	715.00
2089	622.00	620.00	716.00
2090	623.00	620.00	716.00
2091	623.00	621.00	716.00
2092	624.00	621.00	717.00
2093	624.00	622.00	717.00
2094	624.00	622.00	718.00
2095	625.00	622.00	718.00
2096	625.00	623.00	719.00
2097	626.00	623.00	719.00
2098	626.00	624.00	719.00
2099	627.00	624.00	720.00
2100	627.00	624.00	720.00
2101	627.00	625.00	721.00
2102	628.00	625.00	721.00
2103	628.00	626.00	721.00
2104	629.00	626.00	722.00
2105	629.00	626.00	722.00
2106	629.00	627.00	723.00
2107	630.00	627.00	723.00
2108	630.00	628.00	724.00
2109	631.00	628.00	724.00
2110	631.00	629.00	724.00
2111	632.00	629.00	725.00
2112	632.00	629.00	725.00
2113	632.00	630.00	726.00
2114	633.00	630.00	726.00
2115	633.00	631.00	726.00
2116	634.00	631.00	727.00
2117	634.00	631.00	727.00
2118	634.00	632.00	728.00
2119	635.00	632.00	728.00
2120	635.00	633.00	729.00

Weekly earnings	Amount to be withheld: With tax-free threshold with leave loading	With tax-free threshold no leave loading	No tax-free threshold
1	2	3	4
2121	636.00	633.00	729.00
2122	636.00	634.00	729.00
2123	636.00	634.00	730.00
2124	637.00	634.00	730.00
2125	637.00	635.00	731.00
2126	638.00	635.00	731.00
2127	638.00	636.00	731.00
2128	639.00	636.00	732.00
2129	639.00	636.00	732.00
2130	639.00	637.00	733.00
2131	640.00	637.00	733.00
2132	640.00	638.00	734.00
2133	641.00	638.00	734.00
2134	641.00	639.00	734.00
2135	641.00	639.00	735.00
2136	642.00	639.00	735.00
2137	642.00	640.00	736.00
2138	643.00	640.00	736.00
2139	643.00	641.00	736.00
2140	644.00	641.00	737.00
2141	644.00	641.00	737.00
2142	644.00	642.00	738.00
2143	645.00	642.00	738.00
2144	645.00	643.00	738.00
2145	646.00	643.00	739.00
2146	646.00	644.00	739.00
2147	646.00	644.00	740.00
2148	647.00	644.00	740.00
2149	647.00	645.00	741.00
2150	648.00	645.00	741.00
2151	648.00	646.00	741.00
2152	649.00	646.00	742.00
2153	649.00	646.00	742.00
2154	649.00	647.00	743.00
2155	650.00	647.00	743.00
2156	650.00	648.00	743.00
2157	651.00	648.00	744.00
2158	651.00	648.00	744.00
2159	651.00	649.00	745.00
2160	652.00	649.00	745.00
2161	652.00	650.00	746.00
2162	653.00	650.00	746.00
2163	653.00	651.00	746.00
2164	653.00	651.00	747.00
2165	654.00	651.00	747.00
2166	654.00	652.00	748.00
2167	655.00	652.00	748.00
2168	655.00	653.00	748.00
2169	656.00	653.00	749.00
2170	656.00	653.00	749.00
2171	656.00	654.00	750.00
2172	657.00	654.00	750.00
2173	657.00	655.00	751.00
2174	658.00	655.00	751.00
2175	658.00	656.00	751.00
2176	658.00	656.00	752.00
2177	659.00	656.00	752.00
2178	659.00	657.00	753.00
2179	660.00	657.00	753.00
2180	660.00	658.00	753.00
2181	661.00	658.00	754.00
2182	661.00	658.00	754.00
2183	661.00	659.00	755.00
2184	662.00	659.00	755.00
2185	662.00	660.00	756.00
2186	663.00	660.00	756.00
2187	663.00	661.00	756.00
2188	663.00	661.00	757.00
2189	664.00	661.00	757.00
2190	664.00	662.00	758.00
2191	665.00	662.00	758.00
2192	665.00	663.00	758.00
2193	666.00	663.00	759.00
2194	666.00	663.00	759.00
2195	666.00	664.00	760.00
2196	667.00	664.00	760.00
2197	667.00	665.00	760.00
2198	668.00	665.00	761.00
2199	668.00	665.00	761.00
2200	668.00	666.00	762.00

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	Amount to be withheld		
Weekly earnings	With tax-free threshold with leave loading	With tax-free threshold no leave loading	No tax-free threshold
1	2	3	4
2201	669.00	666.00	762.00
2202	669.00	667.00	763.00
2203	670.00	667.00	763.00
2204	670.00	668.00	763.00
2205	671.00	668.00	764.00
2206	671.00	668.00	764.00
2207	671.00	669.00	765.00
2208	672.00	669.00	765.00
2209	672.00	670.00	765.00
2210	673.00	670.00	766.00
2211	673.00	670.00	766.00
2212	673.00	671.00	767.00
2213	674.00	671.00	767.00
2214	674.00	672.00	768.00
2215	675.00	672.00	768.00
2216	675.00	673.00	768.00
2217	675.00	673.00	769.00
2218	676.00	673.00	769.00
2219	676.00	674.00	770.00
2220	677.00	674.00	770.00
2221	677.00	675.00	770.00
2222	678.00	675.00	771.00
2223	678.00	675.00	771.00
2224	678.00	676.00	772.00
2225	679.00	676.00	772.00
2226	679.00	677.00	773.00
2227	680.00	677.00	773.00
2228	680.00	678.00	773.00
2229	680.00	678.00	774.00
2230	681.00	678.00	774.00
2231	681.00	679.00	775.00
2232	682.00	679.00	775.00
2233	682.00	680.00	775.00
2234	683.00	680.00	776.00
2235	683.00	680.00	776.00
2236	683.00	681.00	777.00
2237	684.00	681.00	777.00
2238	684.00	682.00	778.00
2239	685.00	682.00	778.00
2240	685.00	683.00	778.00
2241	685.00	683.00	779.00
2242	686.00	683.00	779.00
2243	686.00	684.00	780.00
2244	687.00	684.00	780.00
2245	687.00	685.00	780.00
2246	688.00	685.00	781.00
2247	688.00	685.00	781.00
2248	688.00	686.00	782.00
2249	689.00	686.00	782.00
2250	689.00	687.00	782.00
2251	690.00	687.00	783.00
2252	690.00	687.00	783.00
2253	690.00	688.00	784.00
2254	691.00	688.00	784.00
2255	691.00	689.00	785.00
2256	692.00	689.00	785.00
2257	692.00	690.00	785.00
2258	693.00	690.00	786.00
2259	693.00	690.00	786.00
2260	693.00	691.00	787.00
2261	694.00	691.00	787.00
2262	694.00	692.00	787.00
2263	695.00	692.00	788.00
2264	695.00	692.00	788.00
2265	695.00	693.00	789.00
2266	696.00	693.00	789.00
2267	696.00	694.00	790.00
2268	697.00	694.00	790.00
2269	697.00	695.00	790.00
2270	697.00	695.00	791.00
2271	698.00	695.00	791.00
2272	698.00	696.00	792.00
2273	699.00	696.00	792.00
2274	699.00	697.00	792.00
2275	700.00	697.00	793.00

	Amount to be withheld		
Weekly earnings	With tax-free threshold with leave loading	With tax-free threshold no leave loading	No tax-free threshold
1	2	3	4
2276	700.00	697.00	793.00
2277	700.00	698.00	794.00
2278	701.00	698.00	794.00
2279	701.00	699.00	795.00
2280	702.00	699.00	795.00
2281	702.00	700.00	795.00
2282	702.00	700.00	796.00
2283	703.00	700.00	796.00
2284	703.00	701.00	797.00
2285	704.00	701.00	797.00
2286	704.00	702.00	797.00
2287	705.00	702.00	798.00
2288	705.00	702.00	798.00
2289	705.00	703.00	799.00
2290	706.00	703.00	799.00
2291	706.00	704.00	799.00
2292	707.00	704.00	800.00
2293	707.00	705.00	800.00
2294	707.00	705.00	801.00
2295	708.00	705.00	801.00
2296	708.00	706.00	802.00
2297	709.00	706.00	802.00
2298	709.00	707.00	802.00
2299	710.00	707.00	803.00
2300	710.00	707.00	803.00
2301	710.00	708.00	804.00
2302	711.00	708.00	804.00
2303	711.00	709.00	804.00
2304	712.00	709.00	805.00
2305	712.00	709.00	805.00
2306	712.00	710.00	806.00
2307	713.00	710.00	806.00
2308	713.00	711.00	807.00
2309	714.00	711.00	807.00
2310	714.00	712.00	807.00
2311	715.00	712.00	808.00
2312	715.00	712.00	808.00
2313	715.00	713.00	809.00
2314	716.00	713.00	809.00
2315	716.00	714.00	809.00
2316	717.00	714.00	810.00
2317	717.00	714.00	810.00
2318	717.00	715.00	811.00
2319	718.00	715.00	811.00
2320	718.00	716.00	812.00
2321	719.00	716.00	812.00
2322	719.00	717.00	812.00
2323	719.00	717.00	813.00
2324	720.00	717.00	813.00
2325	720.00	718.00	814.00
2326	721.00	718.00	814.00
2327	721.00	719.00	814.00
2328	722.00	719.00	815.00
2329	722.00	719.00	815.00
2330	722.00	720.00	816.00
2331	723.00	720.00	816.00
2332	723.00	721.00	817.00
2333	724.00	721.00	817.00
2334	724.00	722.00	817.00
2335	724.00	722.00	818.00
2336	725.00	722.00	818.00
2337	725.00	723.00	819.00
2338	726.00	723.00	819.00
2339	726.00	724.00	819.00
2340	727.00	724.00	820.00
2341	727.00	724.00	820.00
2342	727.00	725.00	821.00
2343	728.00	725.00	821.00
2344	728.00	726.00	821.00
2345	729.00	726.00	822.00
2346	729.00	727.00	822.00
2347	729.00	727.00	823.00
2348	730.00	727.00	823.00
2349	730.00	728.00	824.00
2350	731.00	728.00	824.00

	Amount to be withheld		
Weekly earnings	With tax-free threshold with leave loading	With tax-free threshold no leave loading	No tax-free threshold
1	2	3	4
2351	731.00	729.00	824.00
2352	732.00	729.00	825.00
2353	732.00	729.00	825.00
2354	732.00	730.00	826.00
2355	733.00	730.00	826.00
2356	733.00	731.00	826.00
2357	734.00	731.00	827.00
2358	734.00	731.00	827.00
2359	734.00	732.00	828.00
2360	735.00	732.00	828.00
2361	735.00	733.00	829.00
2362	736.00	733.00	829.00
2363	736.00	734.00	829.00
2364	736.00	734.00	830.00
2365	737.00	734.00	830.00
2366	737.00	735.00	831.00
2367	738.00	735.00	831.00
2368	738.00	736.00	831.00
2369	739.00	736.00	832.00
2370	739.00	736.00	832.00
2371	739.00	737.00	833.00
2372	740.00	737.00	833.00
2373	740.00	738.00	834.00
2374	741.00	738.00	834.00
2375	741.00	739.00	834.00
2376	741.00	739.00	835.00
2377	742.00	739.00	835.00
2378	742.00	740.00	836.00
2379	743.00	740.00	836.00
2380	743.00	741.00	836.00
2381	744.00	741.00	837.00
2382	744.00	741.00	837.00
2383	744.00	742.00	838.00
2384	745.00	742.00	838.00
2385	745.00	743.00	839.00
2386	746.00	743.00	839.00
2387	746.00	744.00	839.00
2388	746.00	744.00	840.00
2389	747.00	744.00	840.00
2390	747.00	745.00	841.00
2391	748.00	745.00	841.00
2392	748.00	746.00	841.00
2393	749.00	746.00	842.00
2394	749.00	746.00	842.00
2395	749.00	747.00	843.00
2396	750.00	747.00	843.00
2397	750.00	748.00	843.00
2398	751.00	748.00	844.00
2399	751.00	748.00	844.00
2400	751.00	749.00	845.00
2401	752.00	749.00	845.00
2402	752.00	750.00	846.00
2403	753.00	750.00	846.00
2404	753.00	751.00	846.00
2405	754.00	751.00	847.00
2406	754.00	751.00	847.00
2407	754.00	752.00	848.00
2408	755.00	752.00	848.00
2409	755.00	753.00	848.00
2410	756.00	753.00	849.00
2411	756.00	753.00	849.00
2412	756.00	754.00	850.00
2413	757.00	754.00	850.00
2414	757.00	755.00	851.00
2415	758.00	755.00	851.00
2416	758.00	756.00	851.00
2417	758.00	756.00	852.00
2418	759.00	756.00	852.00
2419	759.00	757.00	853.00
2420	760.00	757.00	853.00
2421	760.00	758.00	853.00
2422	761.00	758.00	854.00
2423	761.00	758.00	854.00
2424	761.00	759.00	855.00
2425	762.00	759.00	855.00

	Amount to be withheld		
Weekly earnings	With tax-free threshold with leave loading	With tax-free threshold no leave loading	No tax-free threshold
1	2	3	4
2426	762.00	760.00	856.00
2427	763.00	760.00	856.00
2428	763.00	761.00	856.00
2429	763.00	761.00	857.00
2430	764.00	761.00	857.00
2431	764.00	762.00	858.00
2432	765.00	762.00	858.00
2433	765.00	763.00	858.00
2434	766.00	763.00	859.00
2435	766.00	763.00	859.00
2436	766.00	764.00	860.00
2437	767.00	764.00	860.00
2438	767.00	765.00	861.00
2439	768.00	765.00	861.00
2440	768.00	766.00	861.00
2441	768.00	766.00	862.00
2442	769.00	766.00	862.00
2443	769.00	767.00	863.00
2444	770.00	767.00	863.00
2445	770.00	768.00	863.00
2446	771.00	768.00	864.00
2447	771.00	768.00	864.00
2448	771.00	769.00	865.00
2449	772.00	769.00	865.00
2450	772.00	770.00	865.00
2451	773.00	770.00	866.00
2452	773.00	770.00	866.00
2453	773.00	771.00	867.00
2454	774.00	771.00	867.00
2455	774.00	772.00	868.00
2456	775.00	772.00	868.00
2457	775.00	773.00	868.00
2458	776.00	773.00	869.00
2459	776.00	773.00	869.00
2460	776.00	774.00	870.00
2461	777.00	774.00	870.00
2462	777.00	775.00	870.00
2463	778.00	775.00	871.00
2464	778.00	775.00	871.00
2465	778.00	776.00	872.00
2466	779.00	776.00	872.00
2467	779.00	777.00	873.00
2468	780.00	777.00	873.00
2469	780.00	778.00	873.00
2470	780.00	778.00	874.00
2471	781.00	778.00	874.00
2472	781.00	779.00	875.00
2473	782.00	779.00	875.00
2474	782.00	780.00	875.00
2475	783.00	780.00	876.00
2476	783.00	780.00	876.00
2477	783.00	781.00	877.00
2478	784.00	781.00	877.00
2479	784.00	782.00	878.00
2480	785.00	782.00	878.00
2481	785.00	783.00	878.00
2482	785.00	783.00	879.00
2483	786.00	783.00	879.00
2484	786.00	784.00	880.00
2485	787.00	784.00	880.00
2486	787.00	785.00	880.00
2487	788.00	785.00	881.00
2488	788.00	785.00	881.00
2489	788.00	786.00	882.00
2490	789.00	786.00	882.00
2491	789.00	787.00	882.00
2492	790.00	787.00	883.00
2493	790.00	788.00	883.00
2494	790.00	788.00	884.00
2495	791.00	788.00	884.00
2496	791.00	789.00	885.00
2497	792.00	789.00	885.00
2498	792.00	790.00	885.00
2499	793.00	790.00	886.00
2500	793.00	790.00	886.00

Chapter answers

Chapter 1

Exercise 1.1

1 **a** $a = 150$ m **b** $b \approx 141.8$ mm
c $c \approx 1.54$ **d** $d \approx 8630$ mm
e $h = 0.7$ m **f** $f \approx 19.4$ mm

2 **a** 4187 mm **b** 4.795 m **c** 23.537 m
d 23.28 mm **e** 118 cm

3 $a \approx 57.2$ km, $b \approx 4.50$ cm, $c \approx 39$ m, $d \approx 289$ cm, $e \approx 248$ mm

4 $a = 17$ m, $b \approx 19.2$ mm, $c \approx 4.7$ cm, $d = 35$ km, $e \approx 10$ m, $f \approx 19.1$ cm

5 **a** $a = 52$ **b** $b = 25$ **c** $c = 12$
d $d = 1.5$ **e** $e = 50$

6 **a** Obtuse **b** Acute **c** Acute
d Acute **e** Right-angled

7 4082 mm **8** 6.325 m

9 About 382 m **10** About 248 m

Exercise 1.2

1 **a** $x = 36$ m **b** $y = 43.4$ cm **c** $z = 2.5$ m
d $m = 16.8$ m **e** $n = 11.2$ m **f** $p = 5.04$ km

2 22.5 m **3** 34.3 m

4 **a** They omitted the metre ruler shadow length for the gym.
b 8.1 m **c** 8.0 m **d** About 4.7 m

5 18.75 m **6** $143 - 8 = 135$ m

7 9 m **8** 120 m **9** 24.5 m **10** 174 cm

Exercise 1.3

1

	Opposite	Adjacent	Hypotenuse
a	ST	SU	TU
b	j	k	i
c	CD	CE	DE
d	a	c	b
e	VW	VX	WX

2 **a** $\tan U \approx 0.89$ **b** $\tan P \approx 2.85$
c $\tan G \approx 1.44$ **d** $\tan M \approx 0.65$
e $\tan R \approx 1.5$

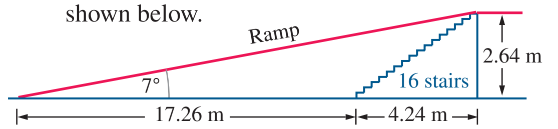
3 **a** 4.2635 **b** 0.0875 **c** 0.6249
d 10.5789 **e** 0.3096

4 **a** 38.7° **b** 44.3° **c** 50.2°
d 73.6° **e** 8.5°

5 **a** $\angle C = 71.0°$ and $\angle M = 19.0°$
b $\angle Z = 47.4°$ and $\angle K = 42.6°$
c $\angle M = 40.9°$ and $\angle N = 49.1°$
d $\angle J = 57.4°$ and $\angle K = 32.6°$
e $\angle L = 37.6°$ and $\angle T = 52.4°$

6 **a** $a \approx 80$ cm **b** $b \approx 168$ m **c** $c \approx 910$ mm
d $d \approx 0.121$ km **e** $e \approx 5.39$ cm

7 **a** 31.9° to horizontal
b The ramp would need to rest on the top stair and begin 17.26 m from the bottom riser, as shown below.



8 17.1° and 12.5° to horizontal

9 23.8 m **10** 62 m **11** 1.1 m

Exercise 1.4

1 **a** $\sin C \approx 0.84$ **b** $\sin K = 0.65$ **c** $\sin Q \approx 0.39$
d $\sin N \approx 0.71$ **e** $\sin A \approx 0.911$

2 **a** $FG \approx 6.5$, $\sin H \approx 0.79$
b $XZ = 37$, $\sin Z \approx 0.32$
c $EF \approx 12.1$, $\sin D \approx 0.782$

3 **a** 0.8910 **b** 0.2588 **c** 0.9986 **d** 0.2924
e 0.7071 **f** 0.5270 **g** 0.3762 **h** 0.5105
i 0.8616 **j** 0.9641

4 **a** 35.5° **b** 71.8° **c** 3.4° **d** 21.1° **e** 56.1°

5 **a** $\angle K \approx 35.9°$ and $\angle L \approx 54.1°$
b $\angle Q \approx 48.2°$ and $\angle J \approx 41.8°$
c $\angle P \approx 19.7°$ and $\angle F \approx 70.3°$
d $\angle A \approx 45.2°$ and $\angle B \approx 44.8°$
e $\angle P \approx 29.5°$ and $\angle R \approx 60.5°$

6 **a** $\angle L = 21.7°$, $l = 81$ km, $m = 32$ km
b $\angle C = 31°$, $k = 4114$ m, $c = 2472$ m
c $\angle D = 62.2°$, $l = 45.0$ cm, $d = 85.4$ cm
d $\angle W = 58.8°$, $s = 27.28$ mm, $w = 45.04$ mm
e $\angle C = 70.9°$, $d = 39.9$ mm, $c = 115$ mm

7 **a** $RU \approx 507$ cm, $UZ \approx 105$ cm
b $IL \approx 28$ mm, $BI \approx 24$ mm
c $MT \approx 29.6$ km, $PT \approx 26.1$ km
d $AC \approx 138.4$ cm, $AB \approx 98.8$ cm
e $RC \approx 989$ mm, $RW \approx 346$ mm

8 23.3 cm

9 **a** 1.2 m **b** 5.9 m
c A single run up 3 floors would need 7 planks. For runs from floor to floor, 9 planks would be needed altogether.

10 30.5 m

11 **a** 16.4 km **b** 26.5 km

12 Line ≈ 20.6 m long, mast ≈ 19.9 m high

13 Rope ≈ 32.3 m, distance ≈ 32.0 m

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Exercise 1.5

1 a $\cos M \approx 0.47$ b $\cos T \approx 0.59$
c $\cos W \approx 0.83$ d $\cos I \approx 0.40$
e $\cos T \approx 0.59$

2 a 0.5299 b 0.8870 c 0.2940
d 0.6794 e 0.9265

3 a 62.7° b 8.9° c 66.4° d 89.8° e 78.6°

4 a $\angle C = 20.2°$ and $\angle K = 69.8°$
b $\angle D = 61.9°$ and $\angle U = 28.1°$
c $\angle M = 18.9°$ and $\angle C = 71.1°$
d $\angle V = 60°$ and $\angle W = 30°$
e $\angle H = 40.0°$ and $\angle G = 50.0°$

5 a $a \approx 17.0$ mm b $b \approx 6180$ mm
c $c \approx 20$ m d $d \approx 216$ mm
e $e \approx 7.29$ cm

6 a $a \approx 14.4$ m b $b \approx 17.4$ mm
c $c \approx 3218$ mm d $d \approx 478$ cm
e $e \approx 548$ km

7 a $a \approx 525$ m b $b \approx 0.048$ mm
c $c \approx 10.08$ m d $d \approx 12.8$ cm
e $e \approx 8481$ km

8 26.8°

9 a 4.65° b Yes

10 4907 m 11 570 mm 12 30.0 m

Exercise 1.6

1 a $M = 41.5°$, $l \approx 154$ m, $m \approx 136$ m
b $A = 54°$, $a \approx 65.5$ mm, $c = 81.0$ mm
c $C \approx 51°$, $K \approx 39°$, $k \approx 11.6$ m
d $V \approx 62.1°$, $n \approx 0.051$ m, $p \approx 0.109$ m
e $C = 77.4°$, $c \approx 17.11$ cm, $d \approx 3.82$ cm

2 a $M = 8°$, $c \approx 150.8$ m, $l \approx 152.3$ m
b $P = 58.4°$, $c \approx 76$ mm, $p \approx 124$ mm
c $P \approx 26°$, $Q \approx 64°$, $r \approx 9.6$ mm
d $D \approx 44.7°$, $V \approx 45.3°$, $d \approx 0.95$ cm
e $O = 32.2°$, $o \approx 44.5$ cm, $r \approx 83.4$ cm

3 a $B = 62°$, $a \approx 9.06$ cm, $c \approx 4.25$ cm
b $P \approx 35.5°$, $G \approx 54.5°$, $d \approx 8.60$ m
c $K = 56°$, $k \approx 8.90$ mm, $m \approx 10.73$ mm
d $M = 32.8°$, $r \approx 8.2$ km, $s \approx 9.8$ km
e $F \approx 48.6°$, $G \approx 41.4°$, $g \approx 50$ m

4 a 58.4075° b 82.905° c 32.2125°
d 17.2667° e 4.4714°

5 a 14°39′54″ b 48°33′ c 78°15′27″
d 68°54′ e 26°43′28″

6 37 km 7 108 m

8 632 m 9 40.6 m 10 11.3 km

11 104 km south, 108 km east

12 742 km, 371 km/h 13 82 km

14 149 km from road, 155 km from cyclist

15 28 km

Chapter 2

Answers to this chapter may vary.

Exercise 2.1

1 a Discrete b Ordinal c Continuous
d Discrete e Categorical f Ordinal
g Categorical h Continuous

2 **1**, **2** and **3** are continuous; **4** and **6** are categorical; **5** is discrete.

3 **1** is categorical; **2**, **3** and **6** are continuous; **4** and **5** are discrete.

4 a 50–2500 students b 400 kg–2500 kg
c \$10 000–\$200 000 d 5c–\$2
e 1c–\$1000

5 a Answers vary. b 15–19 or 15–45
c 15–45

6 The data is categorical, so it doesn't make sense to average the responses, even when they are coded as numbers.

7 The scales are ordinal and it doesn't make sense to average ordinal scales.

8 The distance is continuous, even though it is written discretely, so Deirdre is technically correct.

9 a Age is continuous, but it is normally rounded down to the nearest year for convenience.
b The number of students is discrete because it must be a whole number.

10 The lowest price is about \$1/kg, direct from the orchard but the cheapest fruit shop price is about \$2/kg. The most expensive prices are about \$10/kg for high quality imported fruit in the off-season.

Exercise 2.2

1 a Circle your height category (in cm).

Under 130	130–139	140–149
150–159	160–169	170–179
180–189	190–199	200+

b Circle the hours you spent watching TV last week.

Under 6	6–10	11–15
16–20	21–25	Over 25

c Circle the time (hours) you spent on homework last night.

Less than $\frac{1}{2}$	$\frac{1}{2}$–1	1–$1\frac{1}{2}$
$1\frac{1}{2}$–2	2–$2\frac{1}{2}$	More than $2\frac{1}{2}$

d Circle the number of DVDs there are at home.

0–9	10–19	20–29	30–39
40–49	50–59	60–69	70–79
80–89	90–99	100+	

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e Circle the time in minutes it takes you to get to school.

Less than 6	6–10	11–15	16–20
21–25	26–30	More than 30	

2 a Which soft-drink do you prefer?

b What brand is your calculator?

c How many storeys is the building you live in?

d Which TV channel do you watch most?

e What is your favourite subject?

3 a Circle the number of brothers and sisters you have.

0 1 2 3 4 5 6 7 8 9 10+

b Circle the number of your grandparents who are still alive.

0 1 2 3 4

c Circle the radio station you listen to most.

4BC	4BH	4KQ	B105	97.3	FM104
NOVA	612	4RN	4JJJ	ABC	

(Answers will vary by city.)

d Circle your main breakfast food.

Toast	Cereal	Eggs
Sausages	None	Other

e Circle the drink you have with your night meal.

Water	Milk	Soft-drink	Tea
Coffee	Cordial	Juice	Other

4 a What kind of sport do you play the most? (Could be ‘watch’ instead.)

b What time do you take to run 100 m? (Or similar question.)

c Families can have at home:

Both parents	Mother only
Father only	Parent and Step-parent
Other relative	Guardian.

What kind of family do you have?

d Spare money is what you have after you have paid for things you must buy. Do you have any spare money in most weeks?

e How many hours of homework do you usually do in a week?

5 a What soap opera do you watch the most on TV? ___________________________

b List the two careers you are most interested in.

1 _______________ **2** _______________

c Please indicate your age category.

0–10	11–20	21–30	31–40	41–50
51–60	61–70	71–80	81+	

d What is your occupation? _______________

e Please circle the highest standard of education you have completed.

Under Year 10	Year 10	Year 11
Year 12	Trade certificate	
TAFE certificate	University degree	

6

Homework is work set by your teacher.
Study is time in addition to homework time spent on a subject.

1 How many hours do you spend on homework each week? __________

2 How many hours do you spend on study each week? __________

3 List your subjects and your results last term (A–E) in each subject.

7

Circle your answers to these questions.

1 What is your sex? *M* *F*

2 What is your age category?

Under 11 11–20 21–40 Over 40

3 What do you most prefer for holidays?

Beach Farmstay Sightseeing Resort
Fishing Bushwalking Climbing Other

8 Age and sex should be in categories. Leisure activities on weekdays are excluded. ‘A lot’ is unclear. The concept of ‘leisure’ is biased towards TV and sport.

Circle or state your answers.

1 What is your sex? *M* *F*

2 What is your age group?

Under 11 11–20 21–40 Over 40

3 Leisure time is the time left after you have done the things you must do, like eating, sleeping, working and going to school. What do you most prefer to do in your leisure time? ___________

9 Age should be in categories. ‘A lot’ is unclear: it should be how many times in a particular period. Kinds of films are not explained. The last two questions are biased because they assume the respondent has a VCR or DVD player.

10 Questions are imprecise and could be coded as follows.

Circle your answers to these questions.

1 What year are you in? *8 9 10 11 12*

2 What is your sex? *M* *F*

3 How many hours did you spend on homework last week?

0 1 2 3 4 5 6 7 8 9 10
More than 10

4 How many As, Bs, Cs, Ds and Es did you get for last term’s level of achievement? (Write the number.)

A___ B___ C___ D___ E___

> *5 How many more hours of homework would you need to do in a week to improve your results?*
> *1 2 3 4 5 6 7 8 9 10*
> *More than 10*

Exercise 2.3

1 *Advantages:* The survey is cheap and easy to administer.
Disadvantages: Only a few people will send the form back; the survey is biased by choosing only one suburb; the form could be filled in by someone who doesn't buy the groceries; and there is no check on accuracy.

2 People who think it is a telemarketer or don't like having a meal interrupted may hang up.
Advantages: The survey does target people in the area affected, and it is relatively cheap and quick to administer.
Disadvantages: The survey is biased towards those who have a landline telephone and could be biased by the page chosen (for example, they could all have the surname Lee). It also has a non-response bias.

3 The survey only asks Optus users who use SMS, so is biased towards younger people. People who respond are likely to want things cheap, so may say a lower price than they would pay.

4 There will be a high non-response bias because a lot of people will throw the form away without reading it. The survey may still give good results because people going through the Mall at lunchtime may be going to buy their lunch, but many will not go past the same place the next day, so the overall response may be very low.

5 People reading a health and fitness magazine will probably be at a high level of fitness, so will not be typical of the population. The bias will be made worse by the prize offered.

6 The survey is likely to get a response only from people who feel that bans should be imposed. Pressure groups may influence results by sending in multiple copies of the form.

7 The cost will deter some viewers, but does provide money to help pay for the show. The survey doesn't stop people voting more than once. It may make people more interested in the show because they can take part by voting.

8 By being given a chance to influence the results, readers may be interested in buying the next magazine to see what happens. Since it is a celebrity magazine, it is not likely to represent the whole population, so results will be biased.

9 The proximity of the survey method to the story almost guarantees that most people will vote for option 2. This survey method (called push-polling) is actually trying to influence people's opinions and is sometime used by politicians.

10 The result will only represent people who use the website, and these are likely to be biased towards younger age groups.

Exercise 2.4

1 **a** 13 people **b** Listening to a neighbour

2 **a** 10 people **b** Sugar
c Sugar (14), Sucralose (19), Xylitol (34), Aspartame (37), Saccharin (46)

3 Total time of observation =

1 Cars	2 Trucks	3 Motorbikes
4 Taxis	5 Other	
Place a tally mark for each vehicle.		

4 Total time of observation =

1 Writing on board	2 Explaining	3 Asking questions
4 Telling what to do	5 Helping individuals	6 Other
Place a tally mark for each minute.		

5 Ask each student in your class to place their books on a scale and record the masses to the nearest 100 g.

6 Measure every student in one class from each grade, noting the students' grades and ages. Use the same scales for each student, and measure their masses in school uniform, without shoes.

7 Total time of observation =

1 Looking	2 Using computer	3 Listening
4 Reading	5 Writing	6 Other
Place a tally mark for each person for each minute. Number of people =		

8 Total time of observation =

1 Crossing at green lights	2 Crossing at red or amber lights	3 Running straight across the road
4 Running halfway, then running the rest	5 Walking halfway, then walking the rest	6 Walking straight across the road
Place a tally mark for each person. Number of people =		

9 Buy different flavours and brands of chocolates and give each kind a different number. Break the chocolates into squares. Wrap each square separately and number it by kind. Ask volunteers to taste the different kinds and rate them. Compile the results.

10 Repackage the facial creams in similar containers labelled with 1, 2, 3, etc. Have a group of volunteers test each cream for a week, in different orders. Ask them to rate the creams in order. You could ask for comments about greasiness, ease of application, etc.

Exercise 2.5

1

	A	B	C	D	E	F
1	Survey of MP3 use					
2	Respondent	Q1: Age	Q2: Sex	Q3: MP3	Q4: Use	Q5: Songs
3	1					
4	2					

2

	A	B	C	D	E	F	G	H
1	Survey of CDs/DVDs							
2	Res'nt	Q1: Grade	Q2: Sex	Q3: Music type	Q4: No. music type	Q5: MP3	Q6: Fav	Q7: No. fav
3	1							
4	2							

3

	A	B	C	D	E	F	G
1	Survey of fitness and activity						
2	Res'nt	Q1: Grade	Q2: Sex	Q3: TV	Q4: Height	Q5: Weight	Q6: Exercise
3	1						
4	2						

4 a

Grade	Frequency
8	4
9	8
10	9
11	5
12	4
Total	**30**

b

As	Frequency
0	10
1	12
2	3
3	3
4	2
5	0
6	0
Total	**30**

c

Passes	Frequency
0	0
1	3
2	0
3	3
4	4
5	7
6	13
Total	**30**

5

Vehicle	Frequency
Car	93
Motorbike	9
Truck	30
Taxi	10
Other	42
Total	**184**

6 a

Cream	Frequency
1	4
2	8
3	5
4	8
Total	**25**

Creams 2 and 4 were equally popular by first preferences.

b For each cream, multiply the number of times the cream is rated 1st × 1, 2nd × 2, 3rd × 3, 4th × 4, and add the products.

Cream	Frequency
1	70
2	57
3	67
4	56
Total	**250**

According to this, the preference order would be 4-2-3-1 (best first).

7 The lowest mass is 51 kg and the highest is 121 kg, so categories of 10 kg will give a reasonable number of groups and reasonable frequencies in each group. Since the masses have been rounded to the nearest kg, the groups should show this using 49.5–59.5 and so on.

Mass	Frequency
49.5–59.5	5
59.5–69.5	7
69.5–79.5	7
79.5–89.5	4
89.5–99.5	7
99.5–109.5	10
109.5–119.5	2
119.5–129.5	2
Total	**44**

8 a

BMI	Frequency
10.5–15.5	1
15.5–20.5	1
20.5–25.5	15
25.5–30.5	9
30.5–35.5	1
35.5–40.5	1
Total	**28**

b

Exercise (hours)	Frequency
0–5.5	7
5.5–10.5	12
10.5–15.5	8
15.5–20.5	1
Total	**28**

c The person with a BMI of 13 may be too thin and doing too much exercise. Those with BMIs of 29 and 36 may be too heavy and not doing enough exercise.

Chapter 3

Answers that involve the use of π or measurement of diagrams may vary a little from the printed answers, depending on the value of π used or the measurements used.

Exercise 3.1

1 a 3.2 m **b** 5230 m **c** 146 cm **d** 0.58 m **e** 0.537 km **f** 3270 cm **g** 54 mm **h** 0.4735 km **i** 640 mm
2 a 16.1 cm **b** 151 mm **c** 6.7 m **d** 115 cm **e** 142.7 m
3 10.28 m ≈ 10.3 m
4 $2261.1 (for 58 posts, 223 m of railing, 930 full palings, gate)
5 a $2577.33 **b** $597.08 (including gate)
6 $152 744.23
7 a 85 posts, 423 m of rail **b** 1208 palings **c** $5208

Exercise 3.2

1 a 25 m^2 **b** 240 mm^2 **c** 0.0058 km^2 **d** 25.85 ha **e** 36 000 m^2 **f** 21 m^2 **g** 6.72 m^2 **h** 0.568 cm^2
2 a 400.05 cm^2 ≈ 400 cm^2 **b** 126.3 m^2 **c** 63.9 mm^2 **d** 0.475 km^2 **e** 2392 cm^2 ≈ 2400 cm^2 **f** 0.002 227 5 m^2 ≈ 0.0022 m^2
3 a 2.6546 m^2 ≈ 2.7 m^2 **b** 401.035 m^2 ≈ 401 m^2 **c** 25.33 cm^2 **d** 32 200 mm^2 **e** 4.427 m^2 **f** 0.238 14 m^2
4 a i 1.087 152 m^2 **ii** 10 871.52 cm^2 **b i** 0.3477 m^2 **ii** 3477 cm^2 **c i** 0.067 419 m^2 **ii** 674.19 cm^2
5 a 22.1 m^2 **b** 12.92 km^2 **c** 2952 cm^2 ≈ 3000 cm^2 **d** 227 000 mm^2 **e** 24.52 m^2 **f** 2375 cm^2 ≈ 2400 cm^2

Exercise 3.3

1 a 3384 cm^2 **b** 301.6 cm^2 **c** 254 cm^2
2 a 13.3 m^2 **b** 10.2 m^2 **c** 6.51 m^2
3 a 70.48 m^2 **b** $684.60
4 a 179 m^2 **b** $2431.50
5 989 m^2 **6** 22.6 m^2

Exercise 3.4

1 a 228 tiles **b** 5.76 L, so 6 L **c** 3 bags **d** $1467.90
2 $4008
3 22.6 L with wastage
4 Ceiling area ≈ 171 m^2, so about 36 L with wastage
5 About 681 tiles with wastage
6 32.4 m (no wastage)
7 a 188 tiles **b** $827.20 with wastage
8 a 20.7 m, $7348 **b** 21.3 m, $7543
9 11.8 L, so 2 buckets costing $236
10 $332.40 with wastage
11 328.8 m^2 ≈ 330 m^2
12 a 226.8 m^2 **b** $6804
13 $5323
14 a Court 30.48 m × 15.24 m; centre circle 0.9144 m diameter; goal circles 9.7536 m diameter
b Goal attack and goal defence 310 m^2, 66.7%; goal shooter and goal keeper 155 m^2, 33.3%; wing attack and wing defence 272 m^2, 58.6%; centre 390 m^2, 83.9%
15 36.6%

Exercise 3.5

1 a cm^3 (mL) **b** cm^3 (mL) **c** m^3 (kL) **d** ML **e** cm^3 (mL) **f** L **g** cm^3 (mL) **h** L **i** m^3 **j** cm^3 (mL)
2 a 26 000 000 cm^3 **b** 0.32 m^3 **c** 280 ML **d** 0.8 L **e** 56 m^3
3 a 2000 cm^3 **b** 2000 mL **c** 0.002 m^3

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4 **a** $1480\ cm^3$ **b** $18\ 200\ cm^3$
c $285\ cm^3$ **d** $1.17\ m^3$
e $1.59\ m^3$ **f** $248\ cm^3$
g $2.744\ m^3$ **h** $22.6\ cm^3$
i $27\ m^3$

5 **a** $0.074\ 88\ m^3$ **b** $0.3532\ m^3$
c $5.747\ m^3$ **d** $73.31\ m^3$
e $43\ 769\ mm^3$ **f** $0.8333\ m^3$

6 **a** $1\ 243\ 547\ mm^3$ **b** $1437\ cm^3$
c $8369\ mm^3$

7 **a** $7.07\ m^3$ **b** $18.1\ m^3$
c $297\ 000\ cm^3$

8 **a** $1\ 922\ 800\ mm^3$ **b** $1922.8\ cm^3$

9 **a** **i** 50 225 L **ii** About 105 h
b **i** 55 930 L **ii** About 117 h
c **i** 145 300 L **ii** About 303 h

10 2.083 m

Exercise 3.6

1 **a** 13 kL **b** 140 kL **c** 160 kL

2 23 cm

3 1.46 m

4 **a** Table below **b** 9000 L

Month	Rain collected (L)	Water consumed (L)	Monthly surplus (+) or deficit (–) (L)	Cum. surplus or deficit (L)
Jan.	12 120	10 700	+1420	+1420
Feb.	9 460	11 400	–1940	–520
Mar.	7 080	10 300	–3220	–3740
Apr.	8 350	9 800	–1450	–5190
May	7 130	10 600	–3470	–8660
Jun.	10 240	8 700	+1540	–7120
Jul.	11 350	8 900	+2450	–4670
Aug.	9 900	10 800	–900	–5570
Sep.	8 960	10 400	–1440	–7010
Oct.	10 730	10 200	+530	–6480
Nov.	13 450	10 800	+2650	–3830
Dec.	14 760	12 200	+2560	–1270
Totals	**123 530**	**124 800**	**–1270**	

5 587 mm

6 **a** $21\ 120\ m^3$ **b** 2024 loads

7 $25\ 196\ m^3$

8 **a** $13\ 600\ m^3$ **b** $43\ 300\ m^3$
c $235\ 700\ m^3$ **d** $494\ 400\ m^3$

9 $3.32\ m^3$

10 About $224\ 000\ m^3$ using 30 m grid squares ($208\ 000$ to $240\ 000\ m^3$)

Chapter 4

Variations of about 1 mm should be allowed for in answers that depend on measurements.

Exercise 4.1

1 **a** 1 : 100 **b** 1 : 100 000
c 1 : 1000 **d** 1 : 10 000
e 1 : 50 **f** 1 : 200 000

2 **a** 1 m **b** 1 m **c** 5 m
d 100 m **e** 6.5 km **f** 50 m

3 **a** 5.6 m **b** 160 m **c** 13 m
d 150 m **e** 2.9 km **f** 920 m

4 **a** 13.6 m **b** 1.9 m **c** 0.9 m
d 3.7 m **e** 1.1 m, 0.6 m

5 6.875 km

6 **a** 10.5 m **b** About 234 m and 53 km
c 24.5 m each

7 **a** 2000 mm **b** 900 mm
c 800 mm **d** 1200 mm
e 1600 mm **f** 475 mm

8 **a** 4.1 m **b** 1.7 m
c 53 cm **d** 12 cm

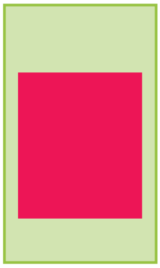
9 **a** 84 km **b** 139 km
c 30 km **d** 29 km

Exercise 4.2

1 **a** 1 : 400 **b** 1 : 20 000
c 1 : 25 **d** 1 : 100
e 1 : 25 000 **f** 1 : 100 000

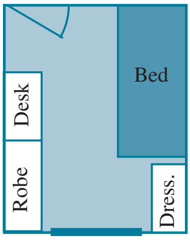
2 **a** 30 cm × 40 cm **b** 8 cm × 5.5 cm
c 6 cm × 9 cm **d** 18 mm × 42 mm
e 56 cm × 120 cm **f** 90 cm × 160 cm

3 Choose scale 1 : 200. Your drawing should look like this, but will be bigger.



Front

4 Choose scale 1 : 20. Your drawing should look like this, but will be bigger. Other arrangements are possible.



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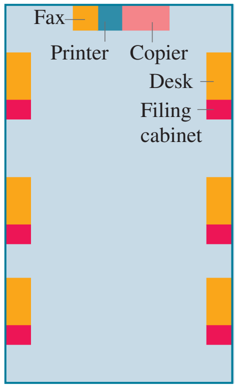
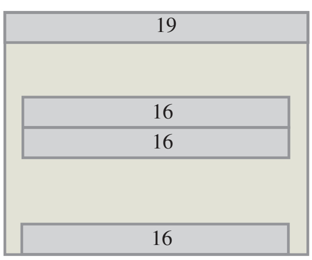
5 Your drawing should look like this, but will be bigger.
A lot more people could work in the office. Other arrangements are possible.
a Choose scale 1 : 50.
b Choose scale 1 : 50.



Exercise 4.3

Variations of about 1 mm should be allowed for in answers that depend on measurements.

1 **a** 620 m **b** 900 m **c** 480 m
d 1.22 km **e** 84 000 m^2 (8.4 ha)
2 **a** 13 m^3, $2912 (rounding up to nearest m^3)
b 7 m^3, $1407
3 **a** 5406 bricks **b** 3308 bricks
4 **a** 4422 ≈ 4500 bricks, $3780
b 2338 ≈ 2400 bricks, $2016
5 **a** 410 m, $2583 **b** 221 m, $1392.30
6 37 m **7** 19 rolls **8** $707.40
9 There may be other methods.
a 2 × 2.4 m, 9 × 3 m sheets
b 8 × 2.4 m, 8 × 3 m, 2 × 3.6 m, 5 × 4.2 m sheets
c 2 × 2.4 m, 11 × 6 m sheets
10 **a** 50 sheets **b** 672 tiles
11 **a** 154 sheets, $6009.08 **b** 2250 tiles, $11 700
12 67 cars



Chapter 5

Exercise 5.1

1 **a** $12 927.20 **b** $38 833.60
c $26 540.80 **d** $24 112.40
2 **a** $23 280.40 **b** $25 285
c $29 265.60 **d** $36 041.20
3 $40 320 **4** $1919.73
5 $1071.38 **6** $1401.57 **7** $19 257
8 $35 724 **9** $5708.33
10 **a** Permanent $17.34, casual $21.33
b $533.25
11 **a** $15.52 **b** Sally $534.80, Max $434.56
12 $20.31/h
13 **a** 51.25 h **b** 13.25 h **c** $1025.26
14 $525.03 **15** 31 h
16 **a** $33 243.60 **b** $36 801.28
c Shelly is financially better off working as a casual, but may prefer the security of permanence.

Exercise 5.2

1 $117 **2** $517.12 **3** $950
4 $1040 **5** $4250
6 **a** $232 **b** $19.33/h
7 **a** $55.25 **b** 2769 houses **c** 2011 deliveries
8 $180 **9** $425.50
10 **a** $124.80 **b** $28.41
c Real earnings $96.39, real rate $16.07/h
11 At least 77 CDs

Exercise 5.3

*Answers to questions **4c** and **7c** assume uniform allowance of $159 ÷ 365 per day or part-day worked.*

1 3 h **2** 12.5% extra
3 Time-and-a-quarter, i.e. 25% extra
4 **a** $389.40 **b** $395.31 **c** $916.18
5 $13.34/h **6** $21.57/h
7 **a** $341.58 **b** $260.83 **c** $953.76
d $614.22 **e** $449.00 **f** $671.70
g $918.40 **h** $616.62

Exercise 5.4

1 $126
2 **a** $323.06 **b** $118
3 **a** $111 **b** $63
4 $38 **5** $314 **6** $14
7 **a** $62 **b** $3224
8 $194
9 **a** $139 **b** $164
10 Tax $188, total deductions $277.20, net pay $959.70
11 Tax $338, total deductions $578.72, net pay $1135.98
12 Gross pay $977.40, tax $205, total deductions $379.97, net pay $597.43
13 Gross pay $732.09, tax $128, total deductions $133.30, net pay $598.79

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Exercise 5.5

1 a $1500 b $32 c $4.50
d $35 e $22 f $18
2 a $800 b $300 c $14
d $0.25 e $2.65 f $18.18
3 a $124 b $54.36 c $32
d $42 e $40.43 f $10.90
4 a $518 b $1700 c $144
d $13 280 e $5650 f $20 775
5 a Julie $3, customer $6.50
b Wendy $3, Julie $3.50
6 a John $84, Mandy $210
b Tran $84, John $126
7 Retail price (including GST) $250.31, GST $22.76
8 $51

Chapter 6

Exercise 6.1

1 a No—they become smaller as you move away from the Equator.
b The Equator
c The South Pole
d Yes—they are all great semicircles.
2 East
3 a 20°S b 50°N c 20°N d 0°
e 45°N f 5°S g 25°S h 40°S
i 65°N j 50°N
4 a 80°E b 20°W c 140°E d 40°E
e 90°W f 100°E g 170°E h 60°E
i 70°W j 15°E
5 a 0° 82°E b 41°S 0°
c 48°S 110°W d 70°N 25°W
e 53°N 67°E f 15°N 37°W
g 84°N 46°W h 37°S 102°E
6 a Alice Springs b Brussels
c Rio de Janeiro d Rangoon
e Phnom Penh f Santiago, Chile
g Shanghai h Havana
i Prague j Copenhagen
7 a 30°N 31°E b 60°N 25°E
c 27°S 153°E d 36°N 5°W
e 53°N 6°W f 22°N 114°E
g 12°S 131°E h 18°N 66°W
i 19°N 99°W j 35°S 56°W
8 a $27\frac{1}{2}$°S b 41°
9 a 4° b $6\frac{1}{2}$°
10 a 51° b $40\frac{1}{2}$°

Exercise 6.2

1 3781 km 2 8340 km
3 10 342 km; about 17 h 14 min
4 9314 km 5 1149 km
6 14 118 km 7 About 7 h 12 min
8 19°S 147°E (Townsville)
9 About 26 days 7 h
10 a 5677 km, 35 667 km, 1486 km/h
b 6024 km, 37 849 km, 1577 km/h
c 5625 km, 35 345 km, 1473 km/h
d 6254 km, 39 295 km, 1637 km/h

Exercise 6.3

1 a 9.1 h b 18.4 h c 13.2 h d 9.1 h
e 13.2 h f 10.1 h g 18.4 h
2 a 10 h 41 min and 13 h 39 min
b About 12:23 pm and 5:43 pm
3 a 9 h 32 min and 14 h 48 min
b 8:36 pm
4 10 h 47 min and 13 h 33 min
5 5 h 42 min
6 824 min = 13 h 44 min
7 The sunlight is more intense at Quebec as it is in the Northern Hemisphere, having summer, because the Earth is tilted so that the Sun is more directly overhead there.
8 The seasons would be more pronounced. Latitudes greater than 50° would be in 24 hours of darkness at the winter solstice or sunlight at the summer solstice.
9 The seasons would be reversed. The winter and summer solstices would occur on 22 December and 21 June respectively in the Southern Hemisphere.
10 a Friday b Monday c Wednesday
d Thursday e Thursday

Exercise 6.4

1 a 4:00 am (4:30 am by world map)
b 1:20 pm c 12:45 am next day
d 7:15 pm e 12:15 pm
2 a 1:00 pm Wednesday b 8:00 am Wednesday
c 8:00 pm Tuesday d 3:00 am Wednesday
e 2:00 am Wednesday f 1:00 am Wednesday
g 8:00 pm Tuesday h 9:00 am Wednesday
3 a 4:30 pm Saturday b 3:30 pm Saturday
4 a 3:05 pm Wednesday b 5:05 am Wednesday
5 2:00 am Tuesday 6 3:15 pm Wednesday
7 7:10 am Tuesday 8 4:25 pm next day
9 4:45 pm Thursday 10 7:15 am Wednesday

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Chapter 7

Exercise 7.1

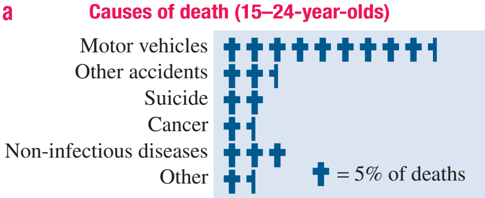
1 **a** Passenger vehicles sold in Queensland each year from 2000 to 20006
b 10 000 vehicles **c** 2001, 2006
d 121 000 vehicles
e Trucks, trailers, bulldozers, motorcycles

2 **a** 1 000 000 passengers/cm
b 3 400 000 passengers **c** 2005
d More ferries, bus strike

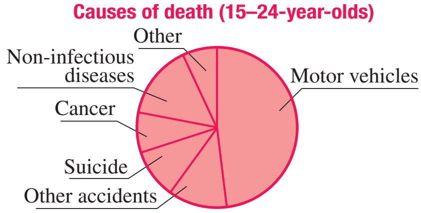
3 **a** Australia (all except 654 000 people)
b Northwest Europe
c 26.1% **d** 27 250 people
e 54 500 people **f** 16.4%

4 **a** Falls **b** Suicide **c** Falls
d About 1680 men **e** About 620 women

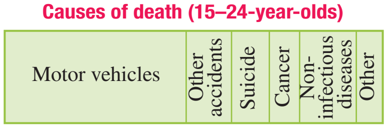
5 **a**



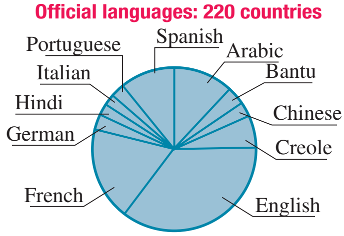
b



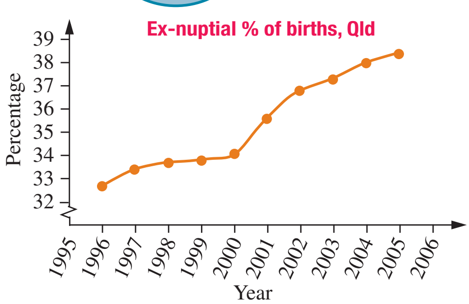
c



6

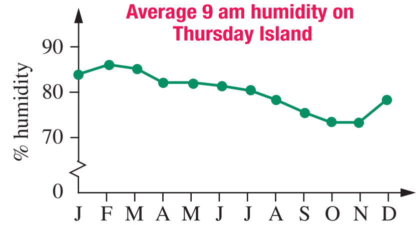


7 **a**



b There was a fairly steady increase over the period.
c Because 2015 is too far away. The percentage can never be more than 100%, so the graph must start to level off at some time.

8



Exercise 7.2

1 **Gross turnover ($)**

Key: 4 | 13 = 413

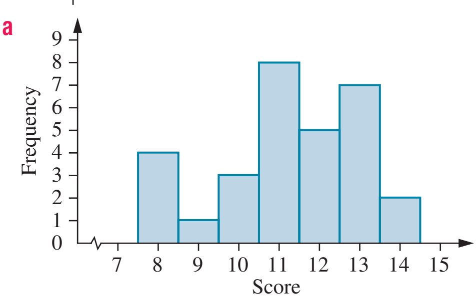
Stem	Leaf
2	99
3	44
4	13
5	02 78 92
6	
7	38 54 89
8	01 80 86
9	23 56
10	98
11	
12	03

2 **Times to run 400 m (s)**

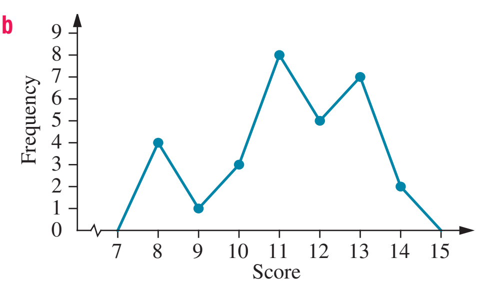
Key: 49 | 3 = 49.3

Stem	Leaf
48	6 8
49	1 3 4 7 9
50	3 3 4 4 7 8
51	1 2
52	6 8
53	7
54	6 7

3 **a**



b



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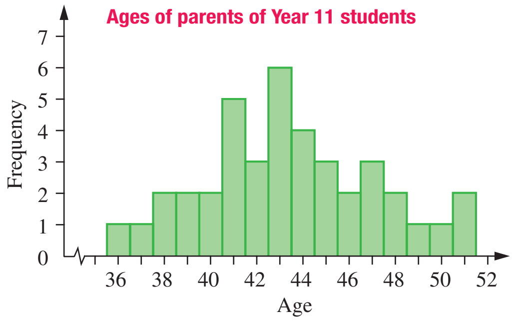
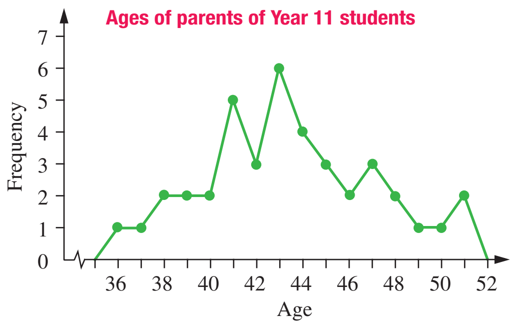
4 a

Age	Frequency
36	1
37	1
38	2
39	2
40	2
41	5
42	3
43	6
44	4
45	3
46	2
47	3
48	2
49	1
50	1
51	2

b



c



5 a House rental prices in Tarragindi ($/week)

Stem	Leaf
2	40 70 90
3	00 00 00 10 20 20 25 30 40 40 50 50
4	00 00 50
5	00 50 50

Key: 4 | 50 = 450

b Prices are between $200 and $600, with most in the $300s. The range should be given slightly bigger than shown in this data to allow for other properties that may become available for rent.

6 a Motorcycle prices ($)

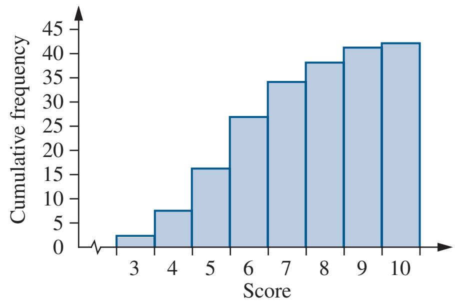
Stem	Leaf
1	400 500 900
2	000 140 400 400 700 900
3	000 000 400 500 500 500 500 800
4	000 200 250 800
5	000 500 500
6	000 800
7	200 500 750
8	000 400 500
9	000
10	
11	
12	
13	000 990
14	500
15	000

Key: 4 | 520 = 4250

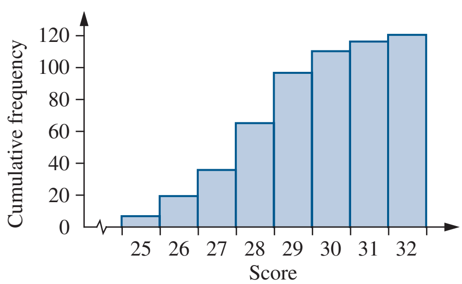
b Prices range up to $15 000, but there are none in the range $10 000–$12 000, and most are around $3000–$4000. There are likely to be more expensive bikes like Harleys advertised elsewhere.

Exercise 7.3

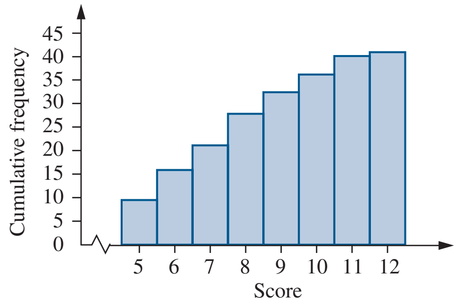
1 a



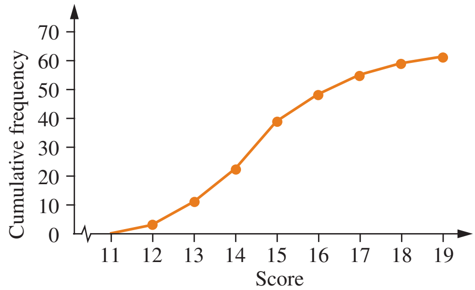
b



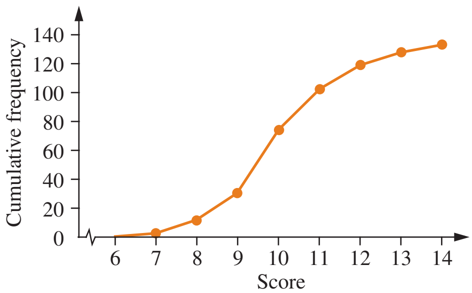
c



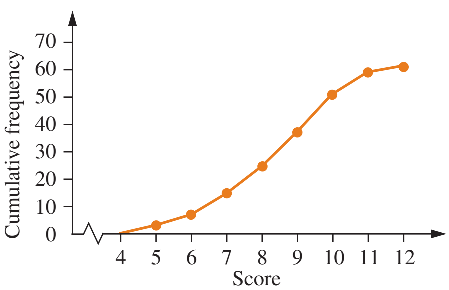
2 a



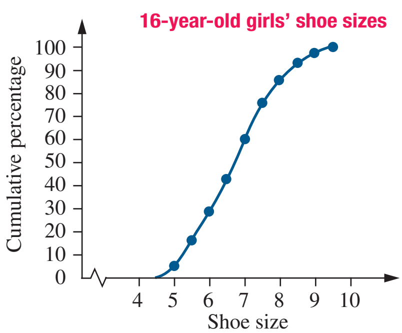
b



c



3 a



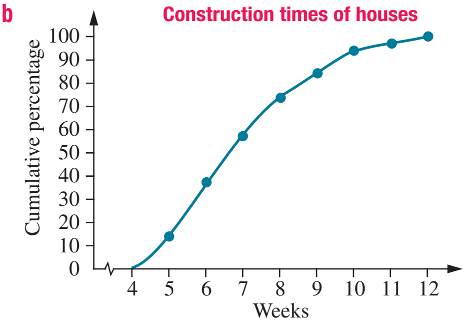
b 43% **c** 34 girls

d Probably, as the data could include unusual sizes

4 a

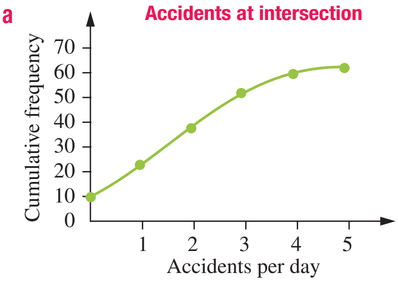
Weeks	Frequency	Cumulative %
5	4	13.3
6	7	36.7
7	6	56.7
8	5	73.3
9	3	83.3
10	3	93.3
11	1	96.7
12	1	100

b



c 37% **d** No, most take longer.

5 a

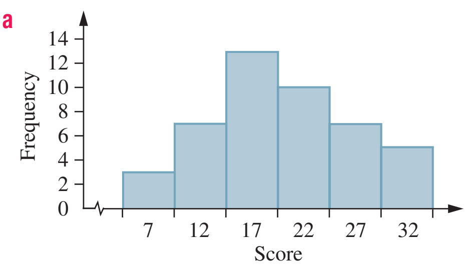


b 24 days **c** 36%

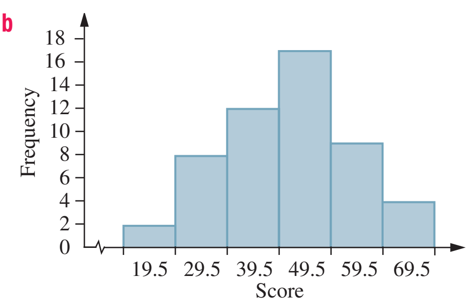
d No, as the data was for one busy intersection, not all intersections

Exercise 7.4

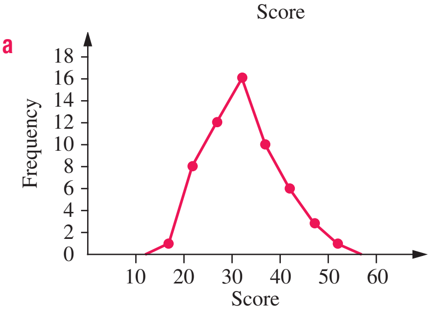
1 a



b



2 a



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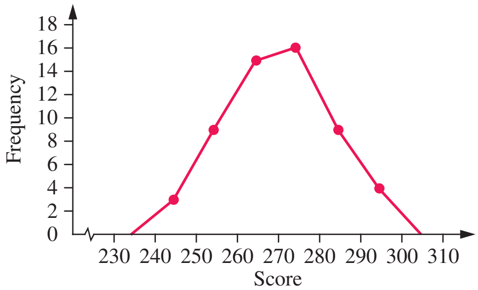
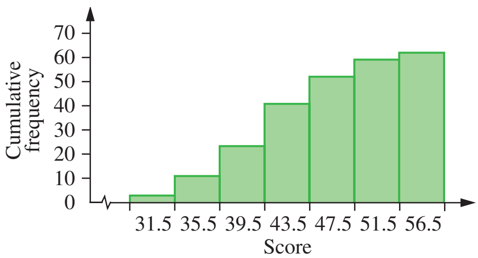
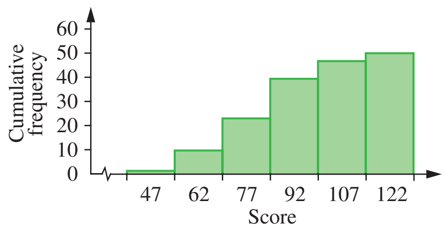


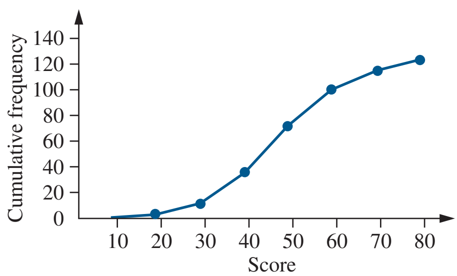
3 a



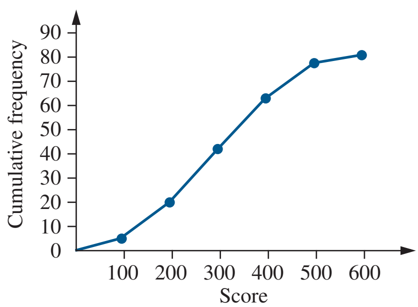
b



4 a



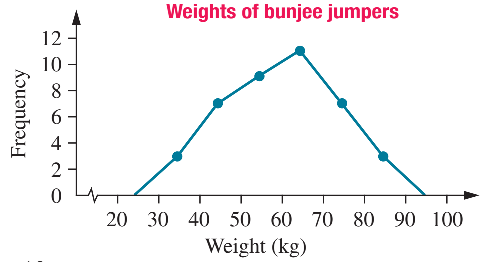
b



5 a

Weight (kg)	Frequency
30–39	3
40–49	7
50–59	9
60–69	11
70–79	7
80–89	3
Total	**40**

Weights of bunjee jumpers

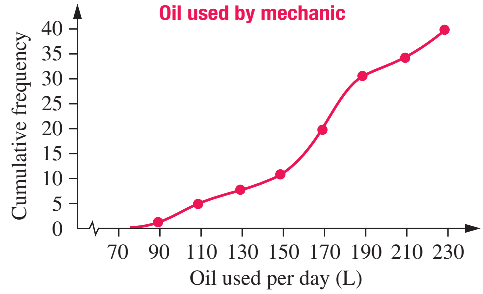


b $\frac{10}{40} \times 200 = 50$ jumpers

6 a

Oil used (L)	Frequency	Cumulative frequency
70–89	1	1
90–109	4	5
110–129	3	8
130–149	3	11
150–169	9	20
170–189	11	31
190–209	3	34
210–229	6	40

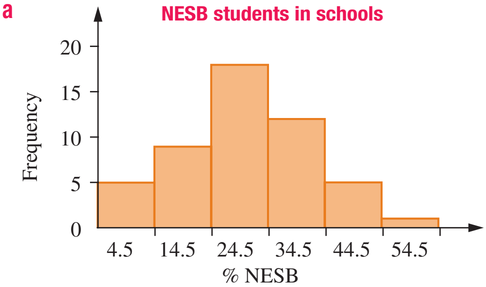
Oil used by mechanic



b Average $\times$ 10 days $= \frac{6599}{40} \times 10 \approx 1650$ L

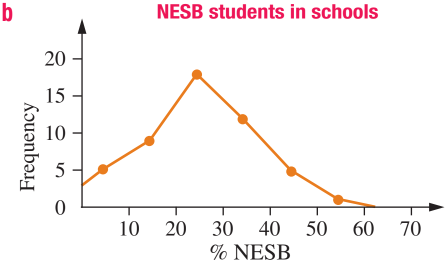
7 a

NESB students in schools



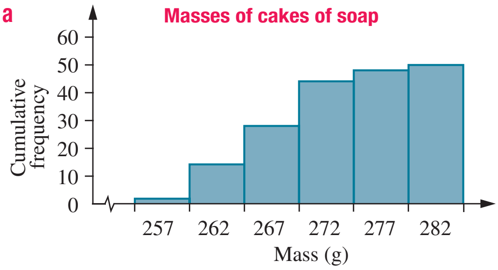
b

NESB students in schools



c Less than 26% (average of SE Queensland), as most migrants are in SE Queensland

8 a



b 58% **c** $\frac{14}{50} \times 800 = 224$ cakes of soap

Exercise 7.5

1 a 0–4, 5–9, 10–14, 25–29, 30–34

b There was a greater proportion of babies in 1955, but this does not necessarily mean that there was a greater number.

c The population has aged, as there are greater proportions in all age groups above 34.

d There is a greater proportion of older women, indicating that women generally live longer than men.

2 a **Words per sentence**

News article		Computer article
Leaf	**Stem**	**Leaf**
9 8 8 3 2	1	0 1 4 5 5 7 7 8 9
8 7 7 7 4 3 2 2	2	0 1 2 3 4 4 5 6 7 7 7 8 9
9 8 5 3 3 2 2 2	3	1 6
6 1 1	4	

Key: 3 | 1 = 13 = 1 | 3

b The computer magazine article generally has shorter sentences than the newspaper article.

3 a **Relative humidity at 3 pm (%)**

Town A		Town B
Leaf	**Stem**	**Leaf**
5 3	3	
8 8 5 5	4	3 5 8
8 7 6 6	5	8 9
8 7 7	6	5 7 9 9
	7	8 8 9
4	8	4 9

Key: 5 | 4 = 45 = 4 | 5

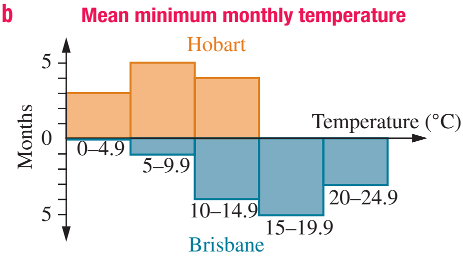
b Town B's relative humidity is generally higher. It probably got more storms.

4 a **Mean minimum monthly temperature (°C)**

Brisbane		Hobart
Leaf	**Stem**	**Leaf**
9.6	0	3.9 4.5 4.5 5.8 6.4 7.3 8.7 9.0
9.9 9.6 8.1 7.1 5.7 3.7 2.6 1.1 0.1	1	0.6 0.6 1.7 1.8
0.9 0.8	2	

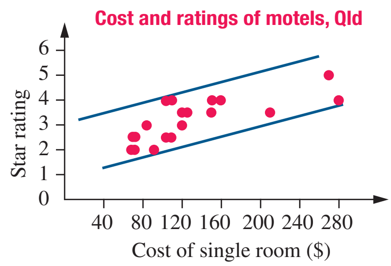
Key: 4.3 | 1 = 14.3 = 1 | 4.3

b



c The summer temperatures are warmer in both places, but the variation from summer to winter is greater in Brisbane.

5



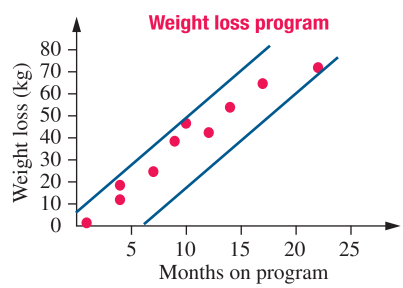
There is a weak positive relationship.

6 a

There does not appear to be a relationship.

b There is unequal participation in different industries.

7



There is a clear positive relationship.

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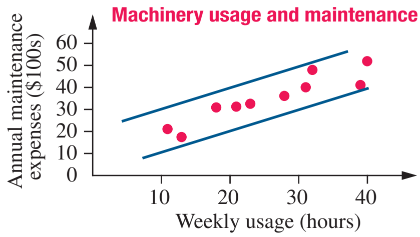
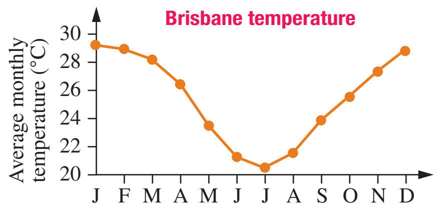
8 a



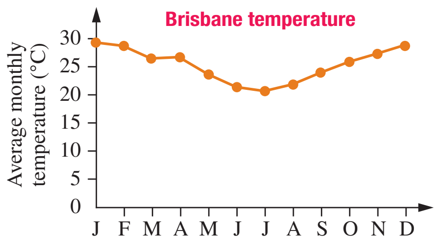
b Yes, higher usage goes with higher maintenance expenses.

Exercise 7.6

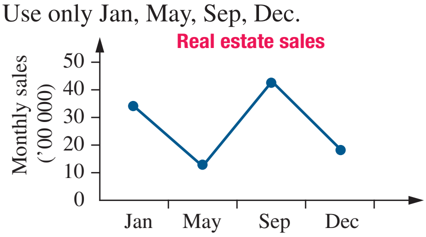
1 a Start the vertical scale at 20°C.



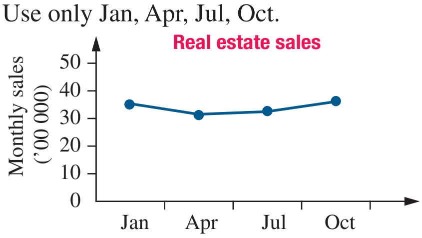
b Start the vertical scale at 0°C.



2 a Use only Jan, May, Sep, Dec.



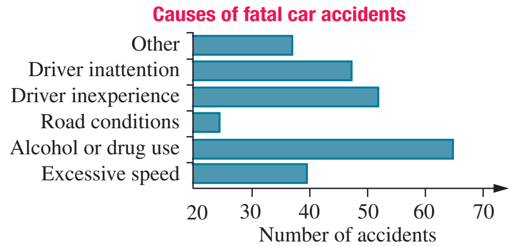
b Use only Jan, Apr, Jul, Oct.



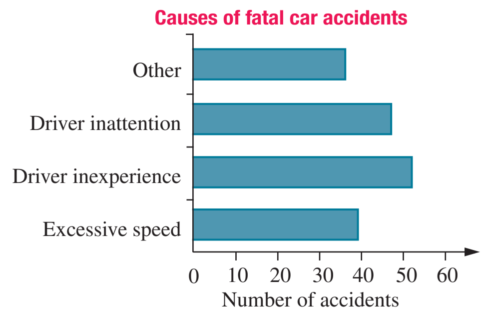
3 a Use only 2003, 2005, 2007.

b Use only 2004, 2006, 2007.

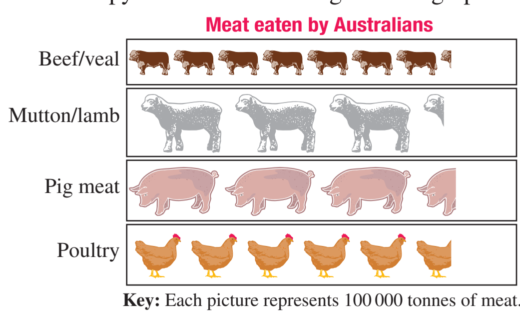
4 a Start the graph from 20 and stretch the horizontal axis.



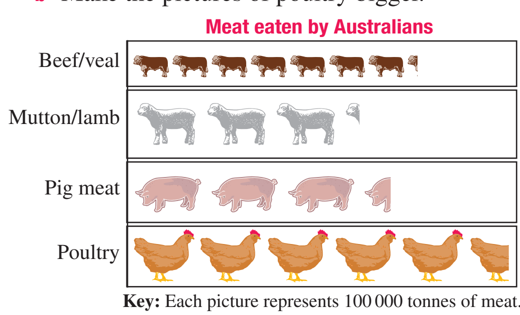
b Omit the extreme values and start from zero.



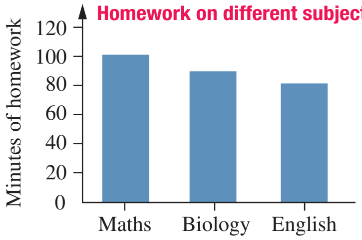
5 a Make the pictures different sizes so they occupy about the same length in the graph.



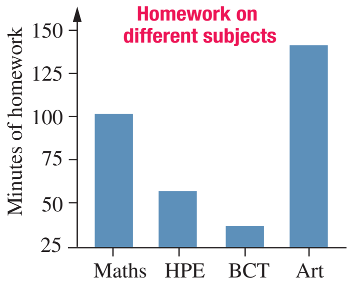
b Make the pictures of poultry bigger.



6 a Leave out the extremes and start the vertical scale from zero.



b Omit the middle values, stretch the vertical axis and start the vertical scale from 25.

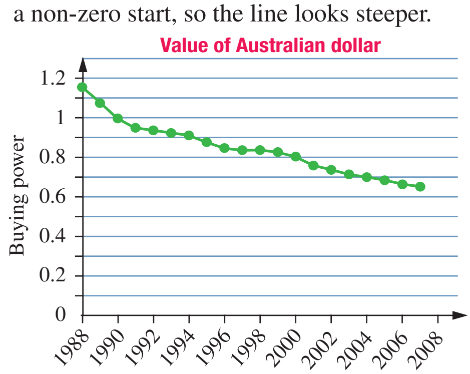


7 a The pictures are different sizes, so they don't give an accurate impression.
b Make the pictures all the same size.

8 a The 3D shape makes the proportions for 17% and 15% look almost as big as the 32% and 28%. The larger %s are at the back.
b The brighter colours of the smaller %s make them seem more important.
c Make the graph a flat pie chart and make the colours all the same tone.

9 a The graph omits some important diseases, like heart attacks and cancer.
b Include all important causes of death.

10 a The vertical axis is stretched and the scale has a non-zero start, so the line looks steeper.
b



Chapter 8

For answers that depend on single measurements, a variation of about 1 mm or 1° is acceptable.
For answers that result from multiple measurements, a variation of about 3 mm or 3° is acceptable.

Exercise 8.1

1 a 143°, 079°, 299°, 078°, 185°
b 323°, 259°, 119°, 258°, 005°

2 a 125° **b** 354° **c** 270° **d** 063° **e** 212°

3 a 304° **b** 111° **c** 063° **d** 301° **e** 020°

4 a 196° **b** 073° **c** 069° **d** 311° **e** 227°

5 a 2.0 km **b** 4.2 km **c** 3.95 km
d 6.55 km **e** 5.0 km

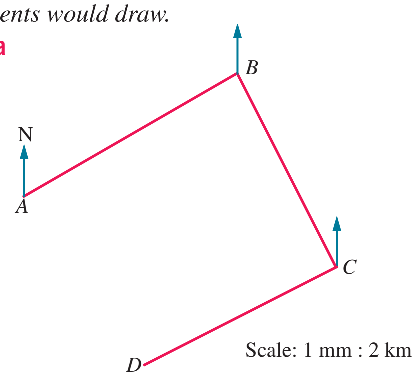
6 a 9.2 km **b** 8.0 km **c** 3.2 km
d 4.0 km **e** 3.0 km

7 a Eagle Farm Racecourse **b** Jubilee Creek
c Geebung railway station
d Wynnum golf course
e Sewage treatment plant **f** Cannery
g Radio mast **h** Luggage Point

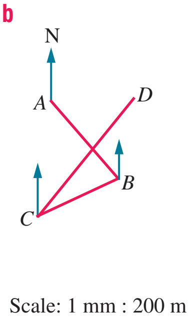
Exercise 8.2

Scale drawings shown here are smaller than students would draw.

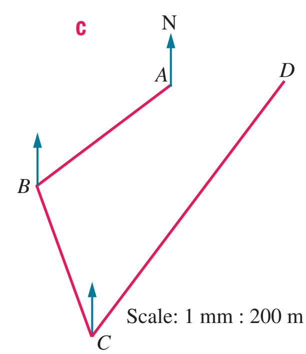
1 a



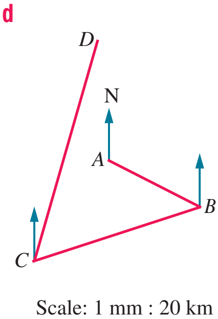
b



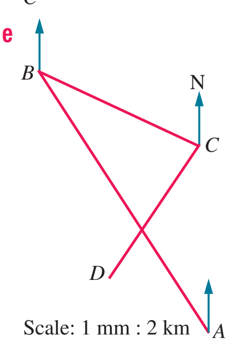
c



d



e



2 **a** 54 km, 145° **b** 2200 m, 088°
c 3.0 km, 088° **d** 310 km, 354°
e 30 km, 299° (Answers need not be exact.)

3 **a** 064.2°T, 069.35°M **b** 126.2°T, 131.35°M
c 247.2°T, 252.35°M **d** 197.2°T, 202.35°M
e 356.2°T, 001.35°M

4 **a** 207.6°T, 192.95°M **b** 092.6°T, 077.95°M
c 164.6°T, 149.95°M **d** 009.6°T, 354.95°M
e 327.60°T, 312.95°M

5 **a** 270°T, 270.9°G **b** 060°T, 060.9°G
c 115°T, 115.9°G **d** 356°T, 356.9°G
e 289°T, 289.9°G

6 **a** 009°29′T, 010°26′G **b** 041°29′T, 042°26′G
c 130°29′T, 131°26′G **d** 217°29′T, 218°26′G
e 359°29′T, 000°26′G

Exercise 8.3

1 **a** 305°M **b** 057°M
c 216°M **d** 267°M
e 313°M **f** 113°M
g 131°M

2 **a** 088°M **b** 339°M
c 178°M **d** 042°M
e 273°M **f** 062°M

3 **a** Yard **b** Windmill
c Hill 160 high **d** Reservoir
e Bridge

4 **a** Lake Eacham **b** Malanda
c Yungaburra **d** Caravan park
e Oval

5 **a** Mt D'Aguilar **b** Mt Samson
c 'Glenidol' **d** Sawmill ruin
e 'Mataranka'

6 3700 m at 046°M, 3100 m at 086°M, 3200 m at 206°M, 3200 m at 119°M, 4500 m at 253°M, 4000 m at 317°M

7 6400 m at 017°M, 3200 m at 081°M, 3400 m at 108°M, 2600 m at 198°M, 2700 m at 148°M, 9000 m at 265°M

8 'Glenidol' **9** 1743 m

10 **a** 300 m **b** 196°

Chapter 9

For answers that depend on single measurements, a variation of about 1 mm or 1° is acceptable.
For answers that result from multiple measurements, a variation of about 3 mm or 3° is acceptable.

Exercise 9.1

The answers shown are obtained by rounding paced length before calculation. Answers obtained using exact values are in brackets.

1 **a** 0.8 m **b** About 0.9 m
c About 0.8 m **d** About 0.7 m

2 **a** About 25 m **b** About 11 m
c About 66 m **d** About 412 m

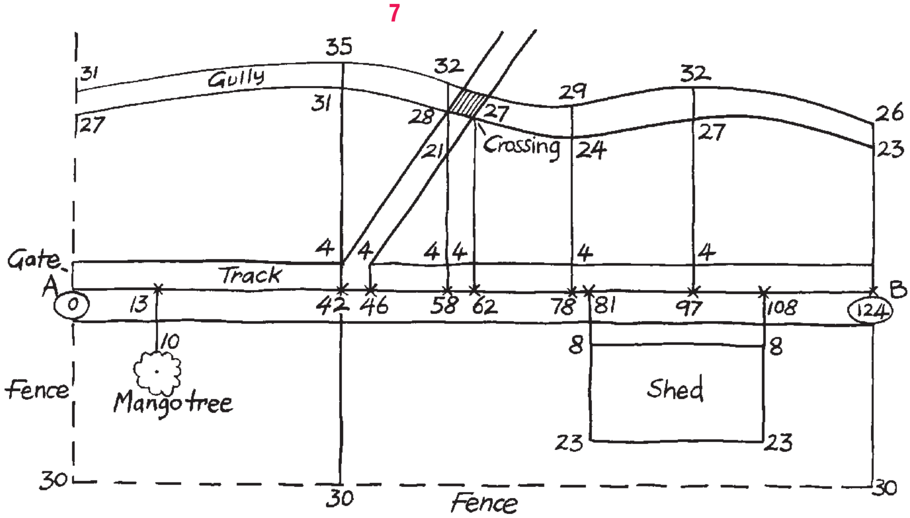
3 **a** 308 m^2 (304.6 m^2) **b** 486 m^2 (478.7 m^2)
c 1020 m^2 (1017.9 m^2) **d** 560 m^2 (565.5 m^2)

4 **a** 31 m by 27 m **b** 15 m by 9 m
c 837 m^2 (832 m^2) **d** 135 m^2 (129 m^2)
e 97 m

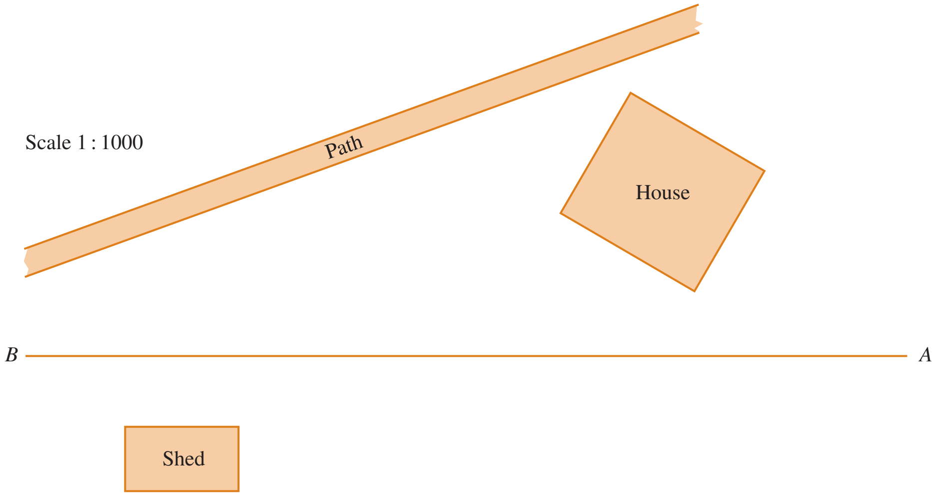
5 **a** 0.87 m
b *A* 35 m (height), 55 m (base); *B* 28 m (height), 73 m (base)
c *A* 963 m^2 (947 m^2), *B* 1022 m^2 (1008 m^2)

6 **a** 0.69 m **b** 41 m, 30 m, 17 m
c 964 m^2 (868 m^2)

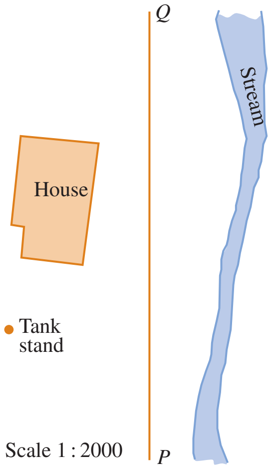
7



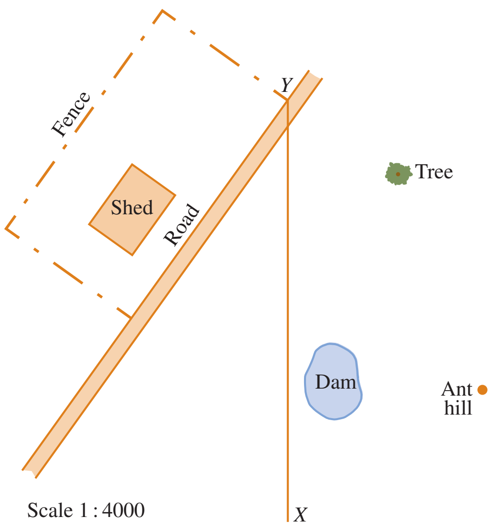
8



9



10



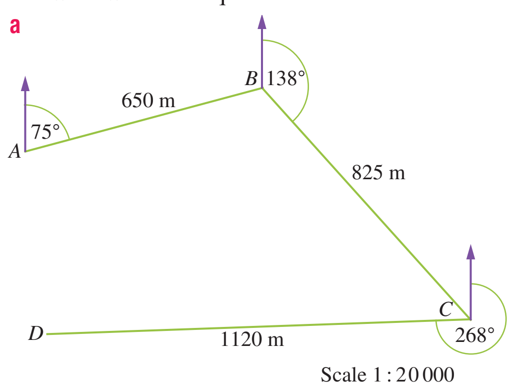
Exercise 9.2

1 **a** PQ 077°, QR 157°, RS 053°, ST 199°
b PQ 257°, QR 337°, RS 233°, ST 019°

2 **a** 125° **b** 354° **c** 270°
d 063° **e** 212°

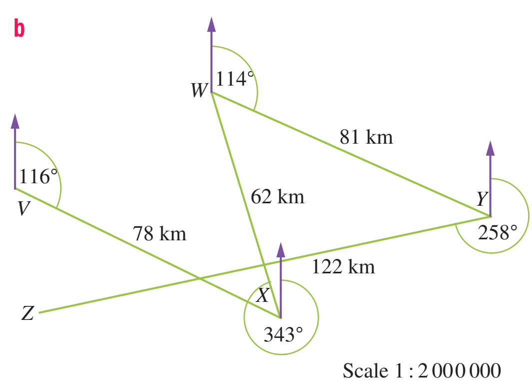
3 Answers will be unique.

4 **a**



$AD = 480$ m at 173°

b



$VZ = 34$ km at 169°.

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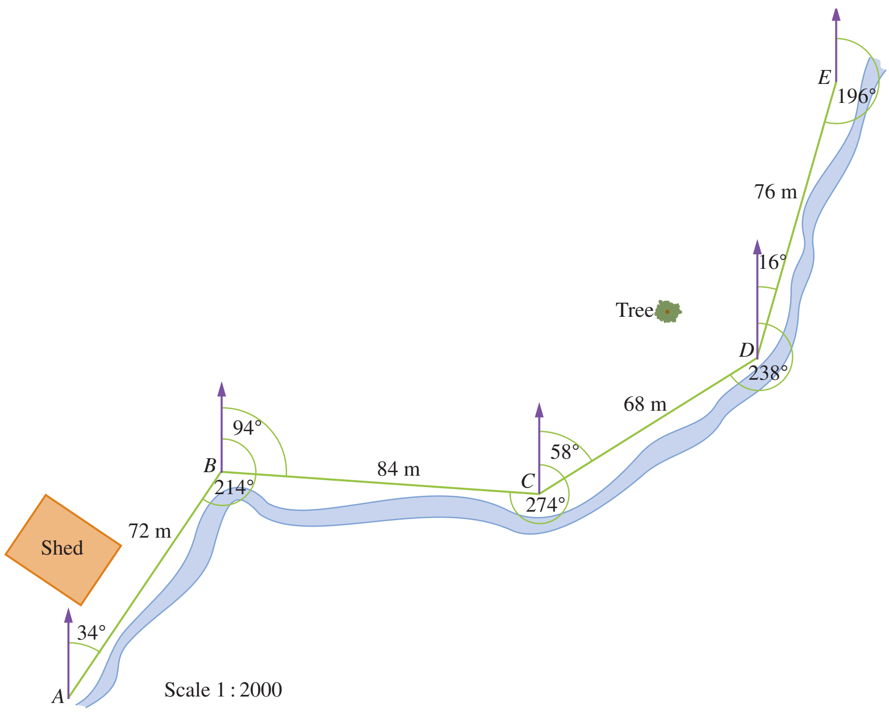
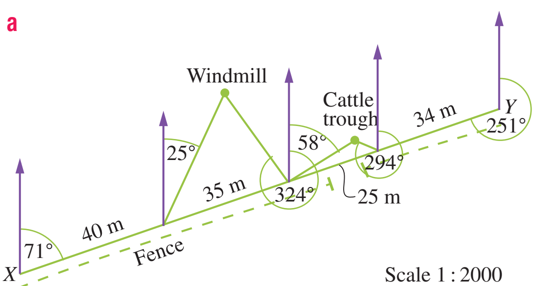
5

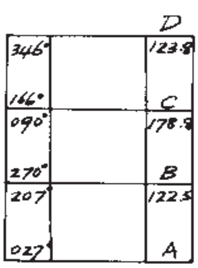


6 a

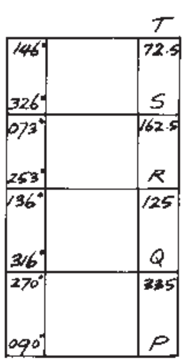


b Cattle trough 5 m, windmill 27.5 m

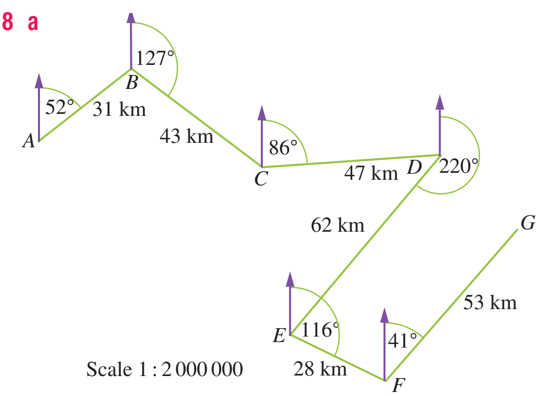
7 a



b



8 a



b AG = 128 km at 100°

Exercise 9.3

1 **a** 305°M **b** 057°M **c** 216°M
d 267°M **e** 313°M **f** 113°M
g 131°M

2 **a** 088°M **b** 339°M **c** 178°M
d 042°M **e** 273°M **f** 062°M

3 **a** Yard **b** Windmill
c Hill 160 high **d** Reservoir
e Bridge

4 **a** Lake Eacham **b** Malanda
c Yungaburra **d** Caravan park
e Oval

5 **a** Mt D'Aguilar **b** Mt Samson
c 'Glenidol' **d** Sawmill ruin
e 'Mataranka'

6 3700 m at 046°M, 3100 m at 086°M, 3200 m at 206°M, 3200 m at 119°M, 4500 m at 253°M, 4000 m at 317°M

7 6400 m at 017°M, 3200 m at 081°M, 3400 m at 108°M, 2600 m at 198°M, 2700 m at 148°M, 9000 m at 265°M

8 'Glenidol'

9 a 300 m b 196°

Exercise 9.4

1 a 895 m^2 b 12 789 m^2 c 1983 m^2 d 32 776 m^2

2 1440 m^2 3 880 m^2

4 1340 m^2 5 960 m^2

Chapter 10

Exercise 10.1

1

	Item	Original (cost) price	Marked price	Up or down?	Amount of markup/ markdown
a	DVD player	$240	$310	Mark up	$70
b	Table	$560	$785	Mark up	$225
c	Lamp	$154	$140	Mark down	$14
d	Software bundle	$235	$345	Mark up	$110
e	Bottle of drink	$0.90	$2.10	Mark up	$1.20
f	Maga-zine	$4.05	$7.50	Mark up	$3.45
g	Travel bag	$28	$62	Mark up	$34
h	Digital TV	$1700	$2900	Mark up	$1200

2 a $19.80 b $231 c $99 d $4.95 e $462

3 a 81.2% b 125.8% c 84.2%

4 a $1.00 b $1.91 c $0.50 d $1.30

5 a $148.75 b $204 c $280 d $80

6 35% 7 80% 8 28.6%

9 a $84 b $224

10 150%

Exercise 10.2

1 a $16.84 b $23.66 c $29.25 d $6.47 e $9.72

2 a $9460 b $43 050

3 a $4000 b $6000 c $5600 d $6400

4 a $3990 b 26 May c 15 June d $4116 e 15 July

5 a $28 000 a $26 891.20 b 16 January c $27 440

6 a $940 b $37.60 c 13 November d 23 December

7 a $4250 b $4037.50 c 8 December d 18 December e $4165 f 27 January

8 a $53, $19 b About 26.4%

9 a $182.88 b $188.65 c $192.50

10 a $1040 b $938.60 c $1456 d $1383.20 e $1182.64 (early payment)

Exercise 10.3

1 a 43.75% b 59.3% c 97.7% d 58.6% e 53.7%

2 Profits: fishing reel 33.3%, video camera 31.8%, filing cabinet 62.5%
Losses: jewellery box 28.6%, printer 25%

3 a 68.5% b $67 000 c 7.4%

4 42.8%

5 a $1.50 b $2.00 c $2.50

6 a $187 b $484 c $127.60 d $319

7 Buying $320, selling $384

8 $5757 9 $240 10 $1200

Exercise 10.4

1

Hot dogs

Item	Quantity	Cost	Unit cost
Wieners	100	$60.00	$0.60
Rolls	100	$55.00	$0.55
Sauce	200	$10.80	$0.054
Total			**$1.204**

Cost of 1000 hot dogs = $1204

2

Hamburgers

Item	Quantity	Cost	Unit cost
Patties	100	$95.00	$0.95
Rolls	100	$55.00	$0.55
Sauce	200	$10.80	$0.054
Tomatoes	20	$6.80	$0.34
Onions	10	$4.60	$0.46
Lettuce	20	$4.20	$0.21
Total			**$2.56**

Cost of 1000 hamburgers = $2560

3 Income from food stall = \$4070

4

Income	
Hot dog sales	\$1610.00
Hamburger sales	\$1440.00
Soft drink sales	\$1020.00
Leftover roll sales	\$44.00
Leftover wiener sales	\$28.00
Leftover pattie sales	\$32.00
Total	**\$4174.00**
Expenses	
Hot dog costs	\$722.40
Hamburger costs	\$1024.00
Soft drink costs	\$408.00
Total	**\$2154.40**

Profit from food stall = \$2019.60

5 a \$1155 b 26 h 40 min
c \$800 d \$1955
e 65.2 cents

6 a \$3068.50 b \$1310
c \$1758.50

7

Weekly income	
Wages (÷ 2)	\$536
Total	**\$536**
Weekly expenses	
Rent (÷ 4)	\$87.50
Power (÷ 4 ÷ 12)	\$8.75
Food etc.	\$220.00
Car payments (÷ 4)	\$40.00
Running car (÷ 2)	\$37.50
Clothes (÷ 50)	\$20.00
Total	**\$413.75**

He could save \$122.25 × 50 = \$6112.50 a year.

8

Weekly income	
Wages	\$510
Total	\$510
Weekly expenses	
Rent and power (÷ 4)	\$102.50
Food etc.	\$95.00
Train	\$24.00
Clothes (÷ 50)	\$38.00
Total	**\$259.50**

She could save \$250.50 × 50 = \$12 525 a year.

Exercise 10.5

1

Item	Associated cost	Yearly cost
Registration	\$580/year	\$580
Insurance	\$640/year	\$640
Services	\$320/3 mths	\$320 × 4 = \$1280
Tyres	\$620/4 years	620 ÷ 4 = \$155
Repairs	\$900/year	\$900
Petrol	\$1.38/L	\$1.38 × 10.5 × 4 × 26 ≈ \$1507
Total		**\$5062**

Weekly cost = \$102.24 ≈ \$102

2

Item	Associated cost	Yearly cost
Repairs	\$1500/year	\$1500
Services	\$335/10 000 km	\$335 × 12 = \$4020
Tyres	\$675/$\frac{1}{2}$ year	\$675 × 2 = \$1350
Petrol	\$1.36/L	\$1.36 × 10.5 × 1200 = \$17 136
Total		**\$24 006**

Weekly cost = \$480.12 ≈ \$480

3

Item	Associated cost	Yearly cost
Repayments	\$260/month	\$260 × 12 = \$3120
Registration	\$600/year	\$600
Insurance	\$2010/year	\$2010
Services	\$285/5000 km	\$285 × 6 = \$1710
Tyres	\$550/40 000 km	\$550 × 0.75 ≈ \$413
Repairs	\$1125/year	\$1125
Petrol	\$1.28/L	\$1.28 × 14 × 300 ≈ \$5376
Total		**\$14 354**

Weekly cost = \$287.08 ≈ \$287

4

Item	Associated cost	Yearly cost
Repayments	\$105/week	\$105 × 52 = \$5460
Registration	\$660/year	\$660
Insurance	\$1950/year	\$1950
Services	\$68/5000 km	\$68 × 6.5 = \$442
Tyres	\$556/year	\$556
Petrol	\$1.33/L	\$1.33 × 15 × 325 ≈ \$6484
Total		**\$15 552**

a Annual cost = \$15 552
b Weekly cost = \$311.04 ≈ \$311
c \$45 656 ≈ \$46 000 (if you deduct the sale of the car)

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5

Item	Cost
Airfares Transfers Accommodation and breakfast Other meals Spending money	$449 × 2 = $898 $165 $185 × 7 = $1295 $65 × 2 × 7 = $910 $25 × 2 × 7 = $350
Total	**$3618**

6

Item	Cost
Petrol Admission Lunch Ice-creams Drinks	$64 $65 × 2 + $45 + $25 × 2 = $225 $28 × 2 + $17 × 3 = $107 $4.50 × 2 × 5 = $45 $4 × 3 × 5 = $60
Total	**$501**

7 a

Item	Cost
Cabins Catering Buses	$100 × 18 × 5 = $9000 $82 × 98 = $8036 $482 × 2 × 2 = $1928
Total	**$18 964**

b $203.91 c $215

8

Item	Cost
Tickets Buses (25-seat)	$10.50 × 110 = $1155 $275 × 5 = $1375
Total	**$2530**

Cost per student = $23

Item	Cost
Tickets Buses (45-seat)	$10.50 × 110 = $1155 $482 × 3 = $1446
Total	**$2601**

Cost per student = $23.65

Item	Cost
Tickets Buses (2 × 45-seat, 1 × 25-seat)	$10.50 × 110 = $1155 $482 × 2 + $275 × 1 = $1239
Total	**$2394**

Cost per student = $21.80

Item	Cost
Tickets Buses (1 × 45-seat, 3 × 25-seat)	$10.50 × 110 = $1155 $482 × 1 + $275 × 3 = $1307
Total	**$2462**

Cost per student = $22.40

Exercise 10.6

1

Code	Midrate
CAD	0.9045
CNY	6.4192
DKK	4.8004
EUR	0.6293
FJD	1.3508
HKD	6.6970
INR	33.835
JPY	105.225
MYR	2.8424
NZD	1.1308
NOK	5.1097

Code	Midrate
GPK	2.5169
PHP	38.794
SGD	1.3164
ZAR	6.1320
LKR	91.7
SEK	5.9555
CHF	1.0451
THB	28.56
GBP	0.4258
USD	0.8455

2 a 3392.40 NZD b 711.08 GBP
c 61 030.50 JPY d 32 815.30 HKD
e 6356.26 AUD f 2720.28 AUD
g 539.44 AUD h 475.17 AUD

3 a 3245.40 NZD b 686.70 GBP
c 57 994.20 JPY d 31 254.65 HKD
e 6066.12 AUD f 2659.26 AUD
g 517.03 AUD h 452.65 AUD

4 62 226.49 JPY (using midrates)

5 505.68 FJD

6 3621.22 AUD

7 1652.20 USD, 1198.40 EUR, 50 660 THB

8 1387.44 AUD

9 1816.53 AUD

Chapter 11

Exercise 11.1

1 a Mean ≈ 6.44, median = 6, mode = 6
b Mean ≈ 13.29, median = 14, mode = 14
c Mean = 214.1 km/h, median = 213.5 km/h, modes = 212, 217 km/h

2 a Mean = 8.85, median = 9, mode = 9
b Mean ≈ 4.15, median = 4, mode = 5
c Mean ≈ 49.18, median = 49, mode = 49

3 a Mean ≈ 11.56, median = 12, mode = 12
b Mean ≈ 4.07, median = 4, mode = 4
c Mean ≈ 7.41, median = 7, mode = 7
d Mean ≈ 21.23, median = 21, mode = 21

4 a Mean = 7.8, median = 8, mode = 8
b Size 8

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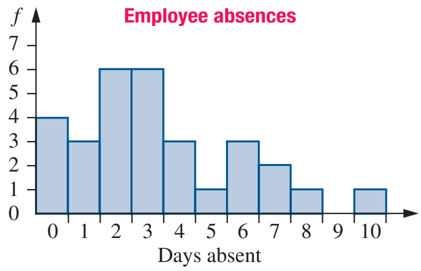
5 a



b Mean ≈ 3.3 days, modes = 2, 3 days, median = 3 days

c $\frac{100}{30} \times 125 \approx 417$ days **d** 3 days

6 a Mode = 18 cars, median = 18 cars, mean = 17.75 cars

b 18 cars

Exercise 11.2

1 a Range = 7, interquartile range = 5, SD ≈ 2.28

b Range = 7, interquartile range = 3.5, SD ≈ 2.19

c Range = 20, interquartile range = 12, SD ≈ 6.63

2 a Range = 9, interquartile range = 4

b Range = 10, interquartile range = 2

c Range = 8, interquartile range = 3

3 a Range = 8, interquartile range = 3.5, SD ≈ 2.12

b Range = 8, interquartile range = 4, SD ≈ 2.32

c Range = 10, interquartile range = 3, SD ≈ 2.34

d Range = 10, interquartile range = 3, SD ≈ 2.30

4 Tanya is more consistent; her range, interquartile range and SD are all smaller than Juan's.

5 15 s

6 a 1 year **b** 15 births (median and mode)

c It is fairly evenly spread with a small range and interquartile range.

d No

Exercise 11.3

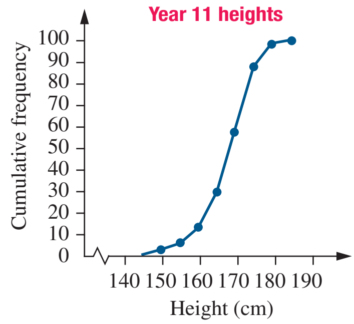
1 a Mean = 14, median ≈ 15.04, modal class = 15–19, range = 24.5, interquartile range ≈ 6.55

b Mean ≈ 71.1, median ≈ 72.54, modal class = 70–79, range = 60, interquartile range ≈ 16.87

c Mean ≈ 112.26, median ≈ 111.83, modal class = 110–114, range = 25, interquartile range ≈ 6.01

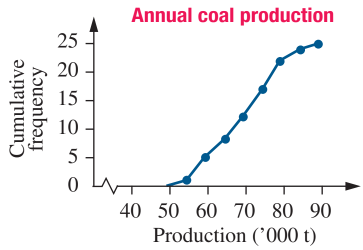
d Mean ≈ 227.28, median ≈ 240.31, modal class = 230–269, range = 240, interquartile range ≈ 71.17

2 a



Median ≈ 168 cm, interquartile range ≈ 9 cm

b



Median ≈ 70 000 t, interquartile range ≈ 15 000 t

3 a Mean ≈ 58.46, SD ≈ 16.55

b Mean ≈ 58.63, SD ≈ 11.85

4 a Mean ≈ \$4433, median ≈ \$4333, modal class = \$3500–\$3999

b \$4333 (median), because the mean is affected by a few employees having very high incomes

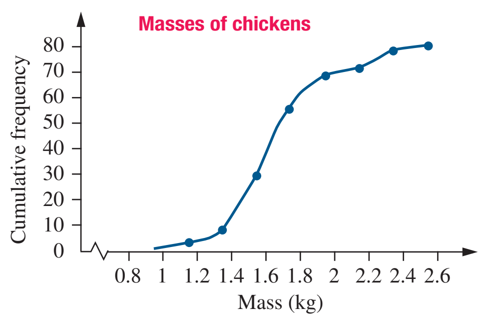
c Range = \$5500, interquartile range ≈ \$1425, SD ≈ \$1020

d \$1425, from \$3664 to \$5090 (rounded)

5 a

Mass (kg)	Frequency	Cumulative frequency
1.0–1.1	3	3
1.2–1.3	5	8
1.4–1.5	21	29
1.6–1.7	26	55
1.8–1.9	13	68
2.0–2.1	3	71
2.2–2.3	7	78
2.4–2.5	2	80

b



c Mean ≈ 1.67 kg, modal class = 1.6–1.7 kg, median ≈ 1.63 kg

d Range = 1.5 kg, interquartile range ≈ 0.44 kg, SD ≈ 0.31 kg

Exercise 11.4

1 a 120 s

b $P_{82} = 127$ s, $P_{35} = 116$ s

c $Q_1 = 111$ s, $Q_3 = 125$ s

d $D_6 = 122$ s, $D_9 = 130$ s

e 37 drivers

2 a $Q_1 = 173$ cm, Q_2 (median) = 177 cm, $Q_3 = 181.5$ cm

b 186.5 cm **c** 55%

d $P_{15} = 170.5$ cm, $P_{42} = 176.5$ cm, $P_{85} = 184$ cm

e $D_2 = 172$ cm, $D_6 = 178.5$ cm, $D_4 = 176$ cm

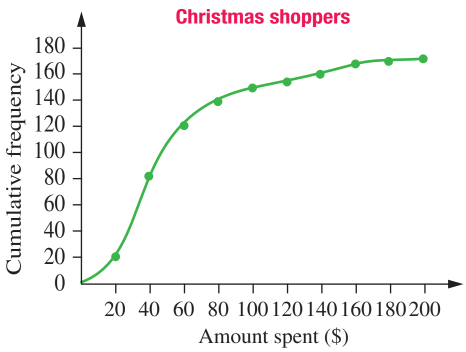
3 a $P_{35} = 8{:}04$ am, $P_{45} = 8{:}09$ am, $P_{78} = 8{:}28$ am

b $D_1 = 7{:}46$ am, $D_4 = 8{:}06$ am, $D_9 = 8{:}39$ am

4 a $P_{32} = 63.8$ kg, $P_{65} = 78.6$ kg, $P_{18} = 56.2$ kg

b $D_2 = 57.5$ kg, $D_4 = 67.6$ kg, $D_8 = 88.3$ kg

5 a



b 51 people **c** 19%

d $41.80 **e** $32.80

f $92.70

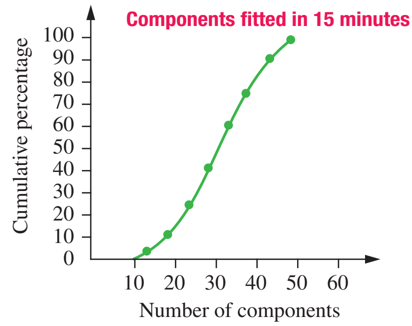
6 a 110 students **b** 4%

c 53.8 h **d** 33.7 h

7 a

Components fitted	Frequency	Cum. frequency
10–14	2	2
15–19	5	7
20–24	9	16
25–29	11	27
30–34	13	40
35–39	9	49
40–44	10	59
45–49	6	65

b



c 58%

d 40, 45 or 50 components, as $P_{90} = 44$ and $P_{95} = 47$

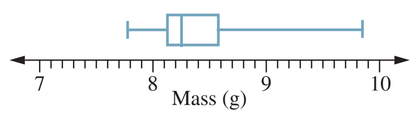
8 a 36 years 7 months

b $P_{35} = 25$ y 8 m, $P_{65} = 46$ y 9 m, so 21 years 1 month

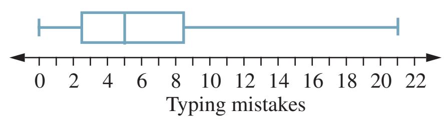
c 73 years 6 months (from 3 y 7 m to 77 y 1 m)

Exercise 11.5

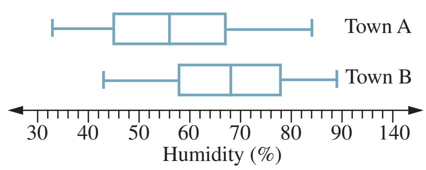
1



2

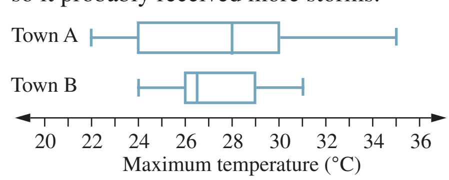


3 a



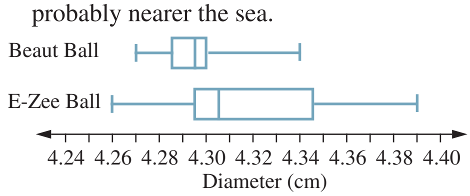
b Humidity was generally higher in Town B, so it probably received more storms.

4 a



b Town B has more even temperatures, so is probably nearer the sea.

5



Beaut Ball seems more efficient because the balls are more consistent in size.

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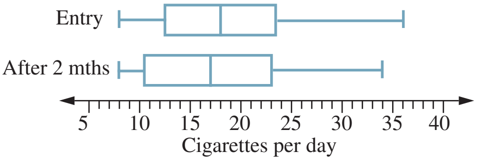
6



The shift in the boxplot indicates the program had some success.

7 a Mean = 89.4%, median = 90%, mode = 90%
b 89.4% (mean of continuous data)

8 a 2 accidents
b 3.27 accidents
c 3 accidents (discrete)
d Median, because it is asymmetrical discrete data

9 a 3°C
b 4.2°C (mean of continuous data)

10 a Mean = 87.25 kg, median = 87.54 kg, modal group = 85–89 kg
b Mean

11 a Mean = 53.25 kg, mode = 52 kg, median = 53.5 kg
b 53.25 kg
c Mode and median

12 Hasim is more SDs below the Chemistry mean than David is below the Physical Recreation mean, so if their abilities in Maths A were the same as their respective abilities in Chemistry and Physical Recreation, it could be true. However, this also assumes that the classes are of comparable ability, so there are a lot of assumptions in Hasim's argument.

13 Peter is 0.67 SDs above the height mean, but only 0.47 SDs above the weight mean, so he would appear to be a little thin.

Chapter 12

Exercise 12.1

1 15.310 m
2 800 mm
3 a 2900 mm b 1560 mm
c 3893 mm
4 11 180 mm
5 No, it could be in the shape of a parallelogram.
6 No, they can only be rectangle or square.
7 They should be moved across (to the right) a little less than 0.4 m (use trial and error).
8 a 26 624 mm b 16 227 mm

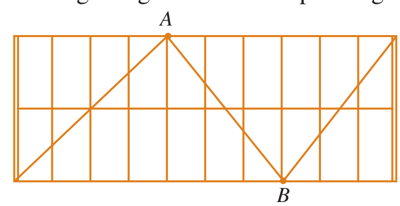
Exercise 12.2

1 243 mm
2 4.5 cm
3 1.48 m
4 8.16 m
5 a 5.68 m b 13.9°
6 a 1.21 m b 3.3° c About 3 m
7 a i 11.00 m ii 11.75 m iii 12.40 m
b About 2.5 m
c About 190 m^3 (using grid squares)
8 a Between 3.8 cm and 7.5 cm (say 5 cm)
b About 0.4°

Exercise 12.3

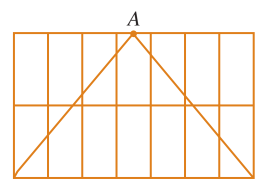
1 a 1531 mm
b Yes, the brace is fitted correctly to prevent the latch–hinge diagonal from compressing.

2 a i



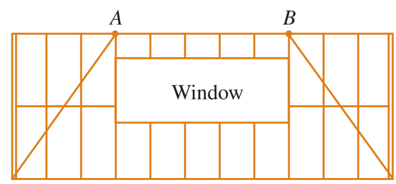
ii 9718 mm

b i



ii 6248 mm

c i



ii 6000 mm

3 a 3842 mm b 4653 mm
4 5532 mm
5 a 180 m b 9.5 m

Exercise 12.4

1 a 139.35 m^2 b 176.51 m^2
c 222.96 m^2 d 297.28 m^2
e 157.93 m^2
2 a $129 150 b $205 410
c $169 125 d $131 407
e $102 840

3 **a** \$216 383 **b** \$219 980
c \$168 220 **d** \$174 308
e \$126 223

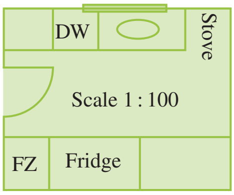
4 \$560 **5** \$3225

6 \$1218 **7** \$179 786

8 \$186 477 **9** \$12 920

10 \$1080 **11** \$5859

12



Other designs are possible.

Exercise 12.5

1 23.6 m^2

2 **a** Entrance/exit end bays = 13.5 m^2, other end bays = 16 m^2, all other bays = 15.3 m^2 (15.25 m^2)
b 303 m^2 **c** 55.4%

3 **a** $x \approx 2.5$ m **b** $y \approx 7.9$ m
c 19.6 m^2 **d** 13.5 m^2

4 147%

5 **a** About 48% **b** 64.8 m^2
c About 324 m^2 **d** 61 pairs of lights
e About 40.5 kW

6 **a** About 93 m^2 **b** 35 lights
c About 11.6 kW

7 **a** 2.4 m **b** 11.6 m^2

8 10.13 m^2

Review answers

Chapter 1

1 a 13.4 m b 10.1 cm c 8.2 km
 d 7.79 m e 15 000 mm
2 a Obtuse b Acute
 c Right-angled d Acute
3 18.75 m
4 a Opposite *KM*, adjacent *LM*, hypotenuse *KL*
 b Opposite *DQ*, adjacent *BD*, hypotenuse *BQ*
 c Opposite *CR*, adjacent *RS*, hypotenuse *CS*
5 a 0.8 b 0.786 c 0.385
 d 0.923 e 0.6 f 0.8
 g 0.786 h 0.385 i 1.333
 j 0.417
6 a 0.7314 b 0.9978 c 11.4301
 d 1 e 0.3090 f 0.8181
 g 0.9421 h 0.7071 i 0.9770
 j 0.5389
7 a 33.4° b 75.7° c 74.1°
 d 18.8° e 36.6° f 11.5°
8 a $A \approx 46.1°$, $B = 43.9°$
 b $D \approx 45.6°$, $F \approx 44.4°$
 c $I \approx 52.5°$, $J \approx 37.5°$
 d $K \approx 28.8°$, $M \approx 61.2°$
 e $O \approx 55.2°$, $P \approx 34.8°$
 f $Q \approx 47.6°$, $R \approx 42.4°$
9 a $K = 49°$, $LM = 32$ m, $KM = 43$ m
 b $A = 57.8°$, $C = 32.2°$, $BC = 25$ m
 c $N = 28°$, $KR = 9.2$ mm, $KN = 17.3$ mm
 d $T = 32.1°$, $U = 57.9°$, $TU = 16.8$ cm
 e $M = 68°$, $IM = 792$ mm, $MV = 2114$ mm
 f $R = 42.7°$, $CP = 38.90$ km, $CR = 42.15$ km
 g $E = 21.6°$, $EG = 18.03$ mm, $FG = 7.14$ mm
 h $Q = 14.7°$, $P = 75.3°$, $PQ = 18.1$ mm
 i $J = 50°$, $JH = 14.8$ cm, $GH = 17.6$ cm
10 6.65 m 11 5.4 km
12 Cliff = 28.7 m, lighthouse = 29.1 m
13 833 mm 14 40°
15 389 km

Chapter 2

1 a Discrete b Continuous
 c Categorical
2 a 0–3 m b \$100–\$1000
 c 5–13
3 a List the TV soap operas that you watch.
 b How many times in a fortnight do you eat chips, lollies or takeaways?
 c Circle the time that best shows how long your parents have been/were married:
 0–5 6–10 11–15 16–20 21–25
 26–30 31+ years

4

	A	B	C	D	E
1	Survey of football				
2	Respondent	Q1: Age	Q2: Sex	Q3: Team	Q4: Time
3	1				
4	2				

5 Provided that the people were going to watch that picture and most of the people entering were asked, this should work. Also provided the sampling occurred spread over the different times the movie was shown.

6

1 Circle your age group:
0–10 11–20 21–30 831–40
41–50 51–60 61–70 71–80 81+
2 What is your sex? M F
3 Write the name of the radio station you listened to most in the last 7 days.
____
4 Write the names of the 3 programs you listen to most.
1 ____________________
2 ____________________
3 ____________________

7 **1** assumes that a drivers licence is meant and that the person has one.
2 and **3** do not say what is meant by long trips or state the period over which the driving is meant.
4 would be better in categories because it is sensitive information. It does not define whether serious accidents or 'bingles' or both should be included.
5 does not say what is meant by 'safe'.

8 There is no way to tell if people have voted multiple times. It is likely to get responses only from people who have strong views so may not be representative. The TV program itself introduces bias.

9 Prepare cups of coffee using the same amount of instant coffee in identical cups, labelled without the brand names. Have tastings in different orders for different people to rate the coffees, allowing for preferences such as milk and sugar.

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10 **Assault with weapon**

Sentence (years)	Frequency
2	2
3	1
4	2
5	1
6	4
7	2
8	2
9	1
10	2
11	3

Actual bodily harm

Sentence (years)	Frequency
3	1
4	1
5	0
6	2
7	3
8	1
9	3
10	1
11	5
12	0
13	0
14	1
15	2

Chapter 3

1 D

2 a 308.5 cm b 19.65 m c 41.7 cm

3 a $A = \pi r^2$

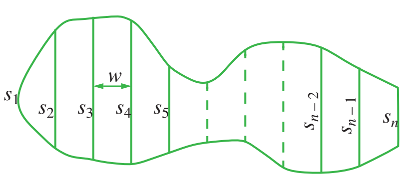
b $A = \frac{1}{2} b \times h$ or $A = \sqrt{s(s-a)(s-b)(s-a)}$

where $s = \frac{1}{2}(a + b + c)$

c $A = \frac{1}{2}(a + b) \times h$

a 147.9 cm^2 b 3855.5 cm^2 c 3.08 m^2

5



Area $\approx \frac{w}{2}(s_1 + 2s_2 + 2s_3 + \ldots + 2s_{n-1} + s_n)$

6 44.01 m^2

7 a 15 000 L b 0.28 L

c 570 mL d 0.000 57 m^3

8 a $V = A \times h$ b $V = \frac{1}{3} A \times h$

c $V = \pi r^2 h$ d $V = \frac{4}{3} \pi r^3$

9 a $V = 88584.3$ cm^3, $SA = 11144.8$ cm^2

b $V = 9065$ mm^3, $SA = 2960.37$ mm^2

c $V = 18.12$ m^3

10 C

11 a 38.18 m^2 b 48.76 m^2 c 133.58 m^2

d 66.685 m^2 (assuming the office is halfway between the house and the footpath)

12 $2889.52

13 33 sheets

14 8777 W

15 $3620

16 a 27.9 m^2 b $1326

c 235 d 26

17 487 m^3

18 About 9 kL

19 9.7 m^3

Chapter 4

1 a 1:200 b 1:500 c 1:500

2 a 5 m b 2 m c 50 m d 8 km

3 a 1.7 m b 172 m c 1.25 km d 72 m

4 1.06 km

5 a 1 : 250 b 1 : 20 000

c 1 : 25 d 1 : 100

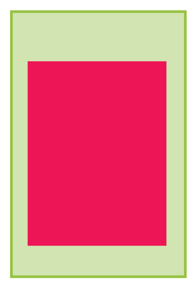
6 a 15 cm × 20 cm b 17.5 cm × 10.5 cm

c 48 mm × 70 mm d 48 mm × 72 mm

7 a 650–750 m b 245 m via St c 375 m

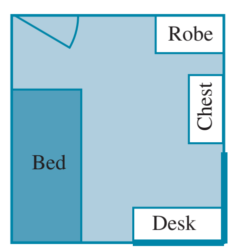
d 8.5 ha (not including streets) or 11.28 ha (including streets)

8 Choose scale 1 : 200. Your drawing should look like the following, but will be bigger.



Front

9 Choose scale 1 : 20. Your drawing should look like this, but will be bigger. Other arrangements are possible.



10 10.62 m^3 ≈ 11 m^3, about $2000

11 3152 bricks: use area of sides less doors and windows. Allow 53 bricks/m^2 which includes mortar and wastage.

12 1 × 2.4 m, 4 × 4.2 m, 15 × 6 m

13 21 sheets each 3.8 m long: each side is 11.2 m long and each sheet does 1.1 m with overlap. So 11.2 ÷ 1.1 = 10.18… or 21 sheets if one is cut lengthways.

Chapter 5

1 $32 520.80 **2** $2226.54
3 **a** $1065.06 **b** $869.77
4 $1120 **5** $167.70
6 **a** $635.34 **b** $621.57 **c** $1120.07
7 $113 **8** $88 **9** $$546.85
10 **a** $19.60 **b** $377.30 **c** $34.30
11 $390 **12** $4344.50
13 $18 055 **14** About $4167
15 Gross pay = $1245.50
Tax = $291
Deductions = $559.85
Net pay = $685.65
16 Retail price = $1881.25
GST = $171.02

Chapter 6

1 **a** 40°N, 112°E **b** 70°N, 36°W
c 42°S, 75°E **d** 37°S, 0°W
2 **a** 34°S, 151°E **b** 42°N, 12°E
c 3°N, 102°E **d** 35°N, 136°E
3 **a** 5907 km, 37 115 km, 1546 km/h
b 5282 km, 33 187 km, 1383 km/h
c 6208 km, 39 004 km, 1625 km/h
4 **a** 9118 km **b** 3395 km
5 **a** 1893, Sunday (1900 was not a leap year); 1907, Tuesday
6 12.6 h
7 Longest day is 13 h 31 min, shortest day is 10 h 49 min
8 6 pm **9** 9 am
10 New Zealand, Fiji, Vanuatu
11 6:19 am Thursday the next week
12 12:05 pm on the same day
13 2:10 am Tuesday
14 3:20 am Thursday

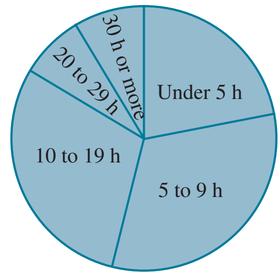
Chapter 7

1 **a**

Homicide	700
Assault	5200
Sexual assault	9000
Armed robbery	3700
Other robbery	5000

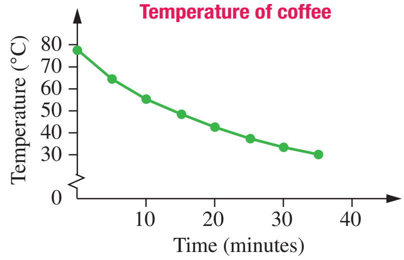
b 38% **c** About 9500 assaults

2 **a** 5 to 9 h
b About 100 000 children
c **Formal childcare in Australia—455 000 children**



3 **a** July 2005–June 2007
b Monthly
c The youth unemployment rate is usually about 5% higher than the full employment rate. Both rates change in the same way but youth unemployment has a much greater deviation.
d 12%

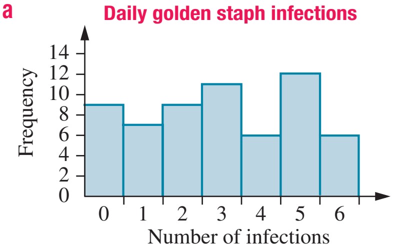
4 **Temperature of coffee**



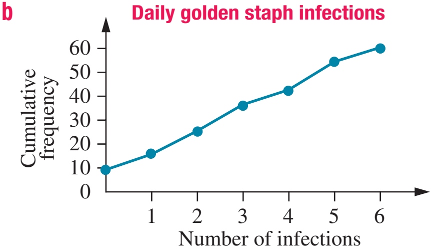
5 **Response times for 1000 calls (s)**

Stem	Leaf
1	2 5 8
2	2 2 3
3	1 8 9
4	0 1 5 5 5 7 9
5	1 2 2 3 4 4 5 8
6	0 1 1 4 6 8 9
7	5 9

6 **a** **Daily golden staph infections**



b **Daily golden staph infections**



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7 **Student results**

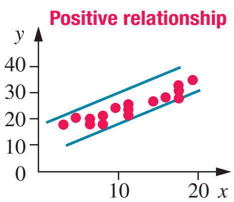
Maths A		English
Leaf	**Stem**	**Leaf**
5 5	3	
2 1	4	4 4 5
5 5 4 2 1 1 1	5	0 0 2 5 5 7 9 9
7 6 4 2	6	2 3 3 6
7 4	7	1 1 2 7 8
7 4 2	8	

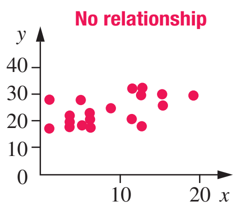
Most students scored in the 50–59% range on both tests. The Maths results were more spread out than the English results.

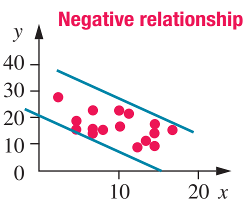
8 By finding the average of the true class limits:
Class midpoint

$$= \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

9



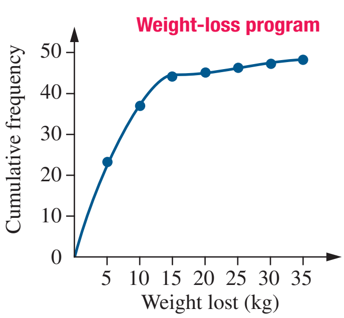




10 a

Weight lost (kg)	Frequency
0–4.9	23
5–9.9	14
10–14.9	7
15–19.9	1
20–24.9	1
25–29.9	1
30–34.9	1

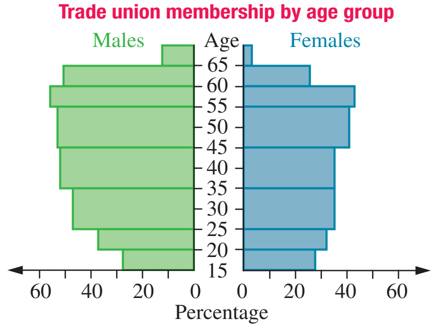
b



c 64.6%

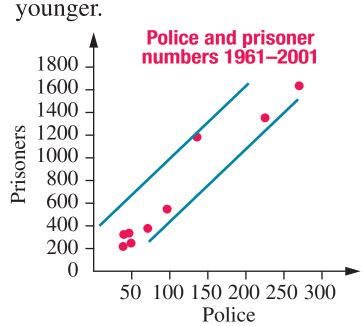
d The general weight loss was 5–10 kg, which is quite good for a 3-month program, considering that greater weight losses are unlikely to be sustained over longer periods.

11 a



b Except for the 15–19 age group, a greater proportion of males than females were members of trade unions. Also, membership was greater among older age groups than younger.

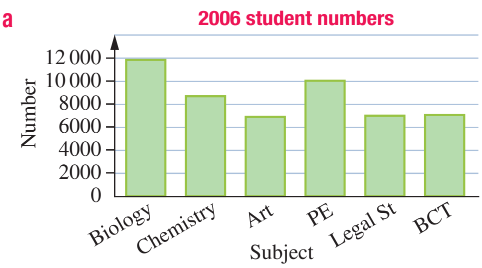
12 a



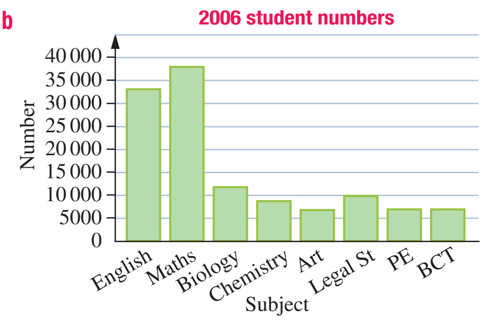
b Yes, it suggests a strong relationship.

c No, the population and hence the number of police and prisoners have increased over the 40-years. The crime rate has also increased resulting in more police and prisoners.

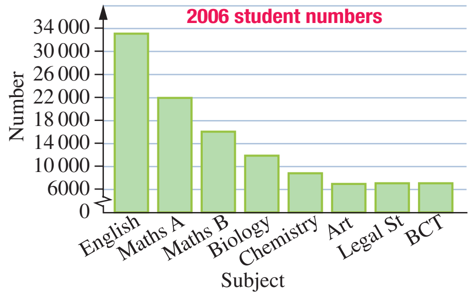
13 a



b



c



14 a WA has a brighter colour and its placement in the 3D pie graph makes it look bigger than Queensland. A 3D graph at that angle is misleading and bright colours accentuate certain sections of the graph.

b Make it a normal pie graph or normal column graph.

Chapter 8

1 a $AB = 135°$, $BC = 038°$, $CD = 086°$, $DE = 211°$, $EF = 290°$

b $BA = 315°$, $CB = 218°$, $DC = 266°$ $ED = 031°$, $FE = 110°$

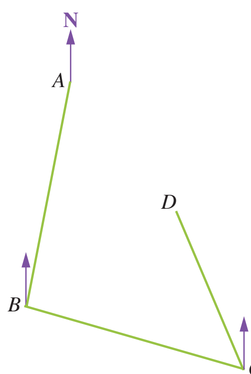
2 a 058° **b** 319° **c** 242° **d** 194° **e** 040°

3 a 4.1 km **b** 11.5 km **c** 4.7 km **d** 7.0 km **e** 8.7 km

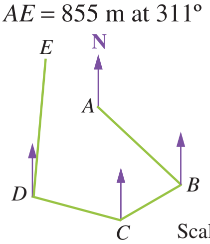
4 a Packers Camp **b** Green Hill **c** Quarry **d** Radio Masts **e** Walshs Pyramid

5 Scale drawings shown here are smaller than those expected of students.

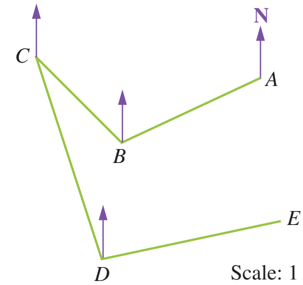
a $AD = 4.4$ km at 140°



b $AE = 855$ m at 311°



c $AE = 95$ km at 172°



6 a 061.5°T, 047.4°M **b** 006.5°T, 352.4°M **c** 352.5°T, 338.4°M **d** 196.5°T, 182.4°M **e** 239.5°T, 225.4°M

7 a 215°47′T, 215°2′G **b** 012°47′T, 012°2′G **c** 162°47′T, 162°2′G **d** 004°47′T, 004°2′G **e** 094°47′T, 094°2′G

8 a 205°M **b** 245°M **c** 030°M **d** 008°M **e** 142°M

9 a Mt Jim Crow **b** Dunrobin **c** Alton Downs **d** Rockhampton Airport **e** Etna Creek Prison Farm

10 1575 m at 080°M, 1525 m at 151°M, 900 m at 232°M, 1025 m at 316°M, 750 m at 003°M, 925 m at 283°M

11 He is at the bend in the railway track north of Aloomba.

12 1564 m

13 a 200 m **b** 293°

Chapter 9

1 a 0.87 m/pace **b** 0.91 m/pace

2 a About 54 m **b** About 1337 m

3 a 1420 m^2 (1425.2 m^2) **b** 1700 m^2 (1714 m^2)

4

Leg	Forward bearing	Back bearing
OA	030°	210°
OB	078°	258°
OC	135°	315°
OD	242°	062°
OE	350°	170°

5 a $AB = 135°$, $BC = 038°$, $CD = 086°$, $DE = 211°$, $EF = 289°$

b $AB = 315°$, $BC = 218°$, $CD = 266°$, $DE = 031°$, $EF = 109°$

6 a 040° **b** 007° **c** 227° **d** 141° **e** 302°

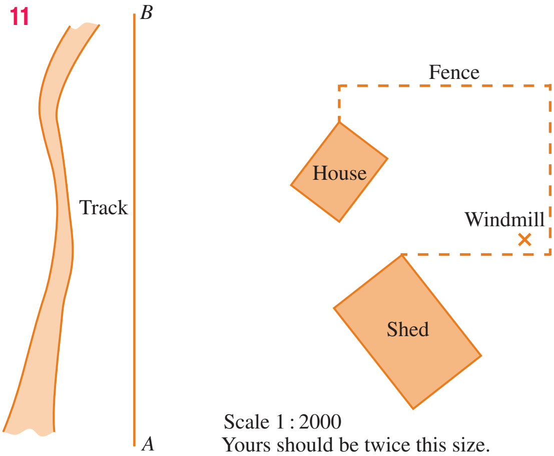
7 a 205°M **b** 245°M **c** 030°M **d** 008°M **e** 142°M

8 **a** Mt Jim Crow **b** Dunrobin
c Alton Downs
d Rockhampton Airport
e Etna Creek Prison Farm

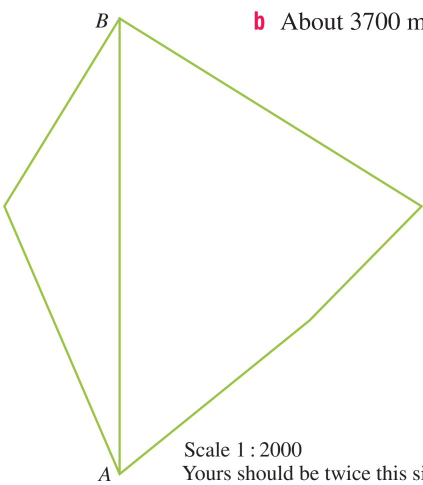
9 1575 m at 080ºM, 1525 m at 151ºM, 900 m at 232ºM, 1025 m at 316ºM, 750 m at 003ºM, 925 m at 283ºM

10 990 m^2 (992.25 m^2)

11

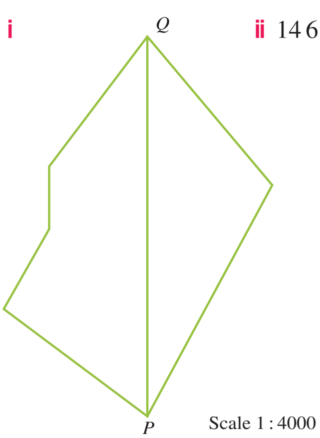


12 **a**



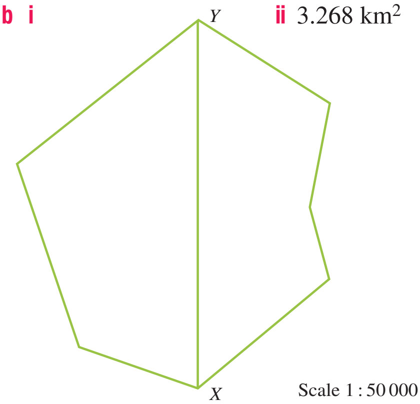
b About 3700 m^2

13 **a** **i**



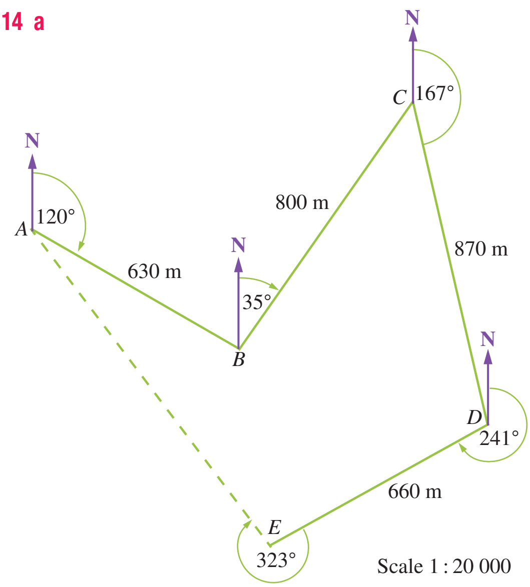
ii 14 614 m^2

b **i**



ii 3.268 km^2

14 **a**



b AE = 1040 m at 143°

15 Near the junction of Limestone Creek and the creek downstream from the junction with Cabbage Tree Creek.

16 **a** 200 m **b** 293º

17 **a** 11 002 m^2 **b** 5984 m^2

18 2568 m^2

Chapter 10

1 **a** $87 **b** $80
2 60% **3** $256
4 $380 **5** $2240
6 $480
7 **a** $8400 **b** 4 November
c $7980 **d** 24 November
8 **a** $5557.50 **b** 38.25%
9 $9900 **10** 28%
11 **a** $300 **b** 100%
c 66.7% **d** 40%
12 $35

13 Cost price = \$11 750, profit = \$7050

14 Cost price = \$16 000, selling price = \$23 200

15 **a** 35.188 INR **b** 47 503.80 INR
c 45 073.80 INR **d** 67.59 AUD

16 \$660.23, 154%

17 **a** 255 containers **b** 63.75 kg
c 85 kg **d** \$432.50

e

Income	
Sales	\$1020
Total	**\$1020**
Expenses	
Fruit Containers	\$432.50 \$25.50
Total	**\$458.00**

Profit = \$562

18

Income	
Wages	\$880.00
Total	**\$880.00**
Expenses	
Rent (÷ 2) Food etc. Transport Repayments (÷ 4) Clothes (÷ 4) Aerobics class	\$190.00 \$320.00 \$70.00 \$28.00 \$95.00 \$18.00
Total	**\$721.00**

She could save \$159 × 50 = \$7950 a year.

19

Item	Associated cost	Yearly cost
Repayments	\$245/month	\$245 × 12 = \$2940
Registration	\$690/year	\$690
Insurance	\$1675/year	\$1675
Services	\$280/5000 km	\$280 × 6 = \$1680
Tyres	\$560/2 years	\$560 ÷ 2 = \$280
Repairs	\$1600/year	\$1600
Petrol	\$1.30/L	\$1.30 × 13 × 300 ≈ \$5070
Total		**\$13 935**

Weekly cost = \$278.70 ≈ \$280

20

Item	Cost
Concert Food Petrol Cabin	4 × \$150 = \$600 4 × \$110 = \$440 \$1.32 × 15 × 15 = \$297 \$225
Total	**\$1562**

Cost for each person = \$390.50

21 **a** 288.18AUD **b** 392.25AUD
c 43.23AUD **d** 28.82AUD
e 464.30AUD

Chapter 11

1 $\bar{x} = \dfrac{\Sigma fx}{\Sigma f}$ (grouped data) or

$\bar{x} = \dfrac{\Sigma x}{n}$ (ungrouped data)

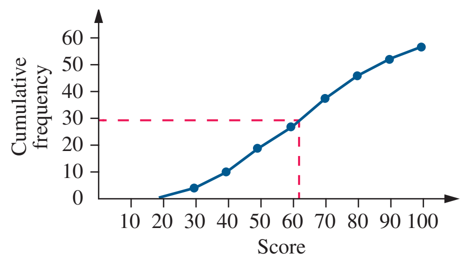
2 Order the data and find the median. Divide the data into lower and an upper halves: if there is an odd number of scores, exclude the median. The first quartile, Q_1, is the median of the lower half, the third quartile, Q_3, is the the median of the upper half. $IQR = Q_1 - Q_1$

3 **a** Mean ≈ 26.12, median = 26, mode = 24, range = 11, interquartile range = 4.5, SD ≈ 2.99

b Mean = 15.5, median = 15, mode = 15, range = 8, interquartile range = 3, SD ≈ 2.04

c Mean ≈ 32.66, median ≈ 32.76, modal class = 30–34, range = 55, interquartile range ≈ 17.90, SD ≈ 11.82

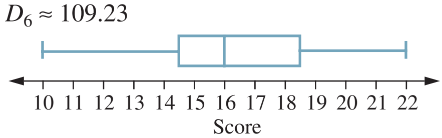
4



Median ≈ 61, interquartile range ≈ 31

5 $P_{23} \approx 97.45$, $P_{86} \approx 119.54$, $D_1 \approx 91.93$, $D_6 \approx 109.23$

6



7 True class limits for, say , age 15–24 may be treated as 14.5–24.5 or 15 –24.999 (in brackets)

a Mean ≈ 41.76 (42.26), median ≈ 39.48 (40.0), modal class = 35–44

b 39.5 (40) , because ages are unevenly spread.

8 Mean = 127, standard deviation = 11.6

9 **a**

Mass (kg)	Frequency	Cumulative frequency
75–79	2	2
80–84	1	3
85–89	0	3
90–94	6	9
95–99	12	21
100–104	13	34
105–109	6	40
110–114	6	46
115–119	3	49
120–124	1	50

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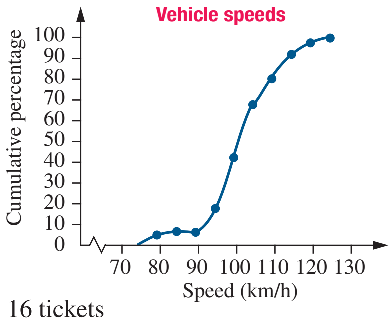
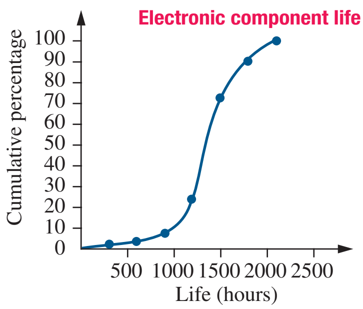


b 16 tickets

c **i** 95.3 km/h **ii** 107.4 km/h

iii 101 km/h

10 **a**



$P_{10} \approx 957$ h, $P_{21} \approx 1160$ h, $P_{83} \approx 1683$ h

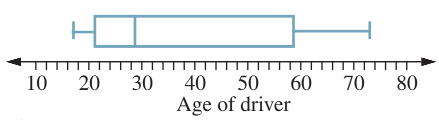
b 1683 h (or 1650 h) because 83% of the components will last at least this long.

11 **a**

Age of driver	Frequency	Cumulative frequency
10–19	12	12
20–29	21	33
30–39	4	37
40–49	5	42
50–59	4	46
60–69	10	56
70–79	4	60

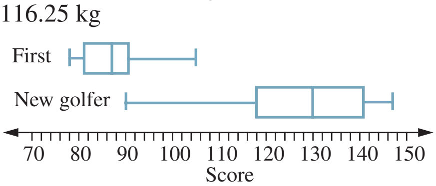
b Range = 56, interquartile range ≈ 37.1, SD ≈ 20.0, mean ≈ 36.8, median ≈ 28.3

c



12 116.25 kg

13



The new golfer is not as good as the first golfer.

14 *Machine 1:* mean ≈ 2.059 kg, SD ≈ 0.034 kg
Machine 2: mean ≈ 2.070 kg, SD ≈ 0.0053 kg
Machine 1 is slightly closer to the 2 kg required, and is more consistent.

Chapter 12

1 3–4–5 **2** 17.718 m

3 53.3 cm ≈ 54 cm **4** 1.96 m

5 **a** Yes **b** No **c** Yes

6 **a** 74.32 m^2 **b** 250.83 m^2

7 **a** \$114 750 **b** \$102 654.50

8 \$578

9 **a** \$6848 **b** 3.6%

10 \$10 440

11 **a** Type A: 8.16 m^2, type B: 12.58 m^2
b About double for type A and triple for type B

12 **a** Family = 14.56 m^2, disabled = 16.64 m^2 standard = 12.48 m^2
b About 2.08 m^2 **c** About 4.16 m^2

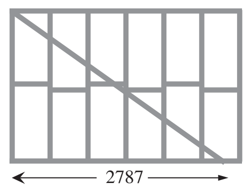
13 About 49% of the office floor

14 **a** Not square **b** Square **c** Not square

15 **a** 24 697 mm **b** 6185 mm **c** 1453 mm

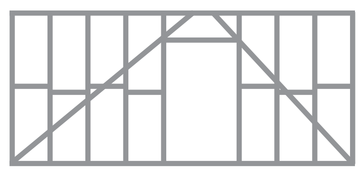
16 **a** 2380 mm
b 2.9°
$\tan\theta = \frac{\text{rise}}{\text{run}} = \frac{2380}{46\,700} \approx 0.050\,96\ldots$, so $\theta \approx 2.9°$

17 **a** **i**



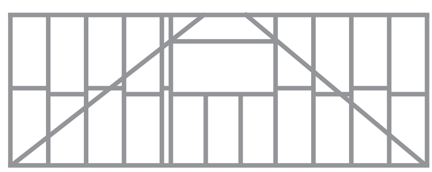
ii 3489 mm **iii** 2787 mm

b **i**



ii 3352 mm and 3111 mm
iii 2340 mm and 1980 mm

c **i**



ii 3721 mm and 3645 mm
iii 2450 mm and 2333 mm

18 \$274 489 **19** \$19 101

20 14.7 m^2

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