

NEW CENTURY MATHS Advanced

9

NSW Stages 5.2/5.3

For the
**australian
curriculum**



Klaas Bootsma
David Badger
Sarah Hamper

Robert Yen Series Editor





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NelsonNet print resources by Megan Boltze



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**New Century Maths Advanced 9 for the Australian Curriculum:
NSW Stages 5.2/5.3**

1st Edition

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Preface

First published in 1996, *New Century Maths Advanced 9 Stages 5.2/5.3* has now been completely rewritten to reflect the spirit of the new Australian curriculum and the NSW syllabus.

For Years 9 and 10, we publish two levels of mathematics books:

- *New Century Maths Stages 5.1/5.2*
- *New Century Maths Advanced Stages 5.2/5.3*

The NSW mathematics syllabus describes Stage 5 (Years 9–10) as a continuum with content classified into three sub-stages, Stage 5.1, Stage 5.2 and Stage 5.3, with each sub-stage including the content of the previous sub-stage. This book has been designed for students and classes progressing along Stages 5.2 or 5.3 of the continuum.

We have retained those familiar features that have made *New Century Maths* a leading series: clear worked examples, graded exercises, multiple-choice questions, Investigation, Technology, Mental skills, SkillCheck pre-chapter exercise, Power Plus extension exercise, Language of Maths, Topic overview with mind map, glossary, video tutorials and worksheets.

New features include:

- Homework sheets
- Worked solutions to selected questions
- *GeoGebra* activities in our Technology sections
- Chapter outlines showing coverage of the Working Mathematically proficiencies
- *NelsonNet* student and teacher websites of print and multimedia resources
- *NelsonNetBook* digital version of this book with interactive features

We always aim to write textbooks to provide flexibility for teachers to plan and teach for a variety of pathways. We thank our families and teaching colleagues for their continued support and patience, and editors Anna Pang and Alan Stewart for transforming our manuscript into this fine text. Finally, we wish all teachers and students using this book every success in embracing the new Australian mathematics curriculum.

About the authors

Klaas Bootsma was head teacher of mathematics at Ambarvale High School in Campbelltown and has taught at Lurnea and Grantham high schools. He was a senior HSC examiner and has worked on the HSC Advice Line and School Certificate marking. Klaas has wide experience teaching all types of students, and his interests are in curriculum and the use of technology (ICT) in learning.

David Badger is principal of Toongabbie Christian School, was deputy principal at Mt Annan Christian College and head teacher of mathematics at Eagle Vale High School in Campbelltown. He has been involved in HSC and School Certificate marking and has worked on the HSC Advice Line. David's passion is to make mathematics interesting, practical and accessible to all students.

Sarah Hamper teaches at Abbotsleigh School in Wahroonga and has taught at Meriden and Tara Anglican schools. She has an interest in gifted and talented (GAT) students and girls education. Sarah's expertise is in using modelling, problem solving and ICT for the effective learning of mathematics, and she has presented workshops for MANSW and nationally.

Series editor **Robert Yen** has taught at Hurlstone Agricultural, Eagle Vale and Ambarvale high schools in southwest Sydney. He co-wrote *New Century Maths Essentials 9–10* and *Mathematics General 2*, writes and presents for MANSW, and has co-edited its journal, *Reflections*. He now works for Cengage Learning as an associate publisher.

Contributing author

Megan Boltze wrote and edited many of the *NelsonNet* print resources (blackline masters) and is head teacher of mathematics at Ashcroft High School.

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Curriculum grid: Australian curriculum

Strand and substrand	New Century Maths Advanced 9 Stages 5.2/5.3 chapter	New Century Maths Advanced 10 Stages 5.2/5.3 chapter
NUMBER AND ALGEBRA		
Real numbers	1 Pythagoras' theorem and surds 2 Working with numbers 5 Indices 11 Coordinate geometry and graphs	1 Surds 7 Graphs 8 Equations and logarithms
Money and financial mathematics	2 Working with numbers	2 Interest and depreciation
Patterns and algebra	3 Products and factors 5 Indices 7 Equations	5 Products and factors 8 Equations and logarithms 14 Polynomials
Linear and non-linear relationships	7 Equations 11 Coordinate geometry and graphs	3 Coordinate geometry 5 Products and factors 7 Graphs 8 Equations and logarithms 10 Simultaneous equations 11 Quadratic equations and the parabola 14 Polynomials
MEASUREMENT AND GEOMETRY		
Using units of measurement	10 Surface area and volume	4 Surface area and volume
Geometric reasoning	6 Geometry 13 Congruent and similar figures	13 Geometry 15 Circle geometry
Pythagoras and trigonometry	1 Pythagoras' theorem and surds 4 Trigonometry	9 Trigonometry
STATISTICS AND PROBABILITY		
Chance	12 Probability	12 Probability
Data representation and interpretation	9 Investigating data	6 Investigating data



Curriculum grid: NSW syllabus

Strand and substrand	New Century Maths Advanced 9 Stages 5.2/5.3 chapter	New Century Maths Advanced 10 Stages 5.2/5.3 chapter
NUMBER AND ALGEBRA		
Financial mathematics (Stage 5.2)	2 Working with numbers 8 Earning money	2 Interest and depreciation
Indices (Stage 5.2)	5 Indices	5 Products and factors
Linear relationships (Stages 5.2, 5.3)	7 Equations 11 Coordinate geometry and graphs	3 Coordinate geometry
Non-linear relationships (Stages 5.2, 5.3)	7 Equations 11 Coordinate geometry and graphs	3 Coordinate geometry 7 Graphs 11 Quadratic equations and the parabola
Ratios and rates (Stages 5.2, 5.3)	2 Working with numbers 11 Coordinate geometry and graphs	7 Graphs
Algebraic techniques (Stages 5.2, 5.3)	3 Products and factors	5 Products and factors
Equations (Stages 5.2, 5.3)	7 Equations	8 Equations and logarithms 10 Simultaneous equations 11 Quadratic equations and the parabola
Surds and indices (Stage 5.3)	1 Pythagoras' theorem and surds 5 Indices	5 Products and factors
Polynomials (Stage 5.3)		14 Polynomials
Logarithms (Stage 5.3)		8 Equations and logarithms
Functions and other graphs (Stage 5.3)		16 Functions
MEASUREMENT AND GEOMETRY		
Area and surface area (Stages 5.2, 5.3)	10 Surface area and volume	4 Surface area and volume
Numbers of any magnitude (Stage 5.1)	5 Indices 10 Surface area and volume	
Right-angled triangles (Trigonometry) (Stages 5.2, 5.3)	1 Pythagoras' theorem and surds 4 Trigonometry	9 Trigonometry
Properties of geometrical figures (Stages 5.2, 5.3)	6 Geometry 13 Congruent and similar figures	4 Surface area and volume 13 Geometry
Volume (Stages 5.2, 5.3)	10 Surface area and volume	4 Surface area and volume
Trigonometry and Pythagoras' theorem (Stage 5.3)		9 Trigonometry
Circle geometry (Stage 5.3)		15 Circle geometry
STATISTICS AND PROBABILITY		
Single variable data analysis (Stages 5.2, 5.3)	9 Investigating data	6 Investigating data
Probability (Stage 5.2)	12 Probability	12 Probability
Double variable data analysis (Stages 5.2, 5.3)		6 Investigating data

Curriculum grid: Year 9 content descriptions

This is an extract from the Australian Curriculum. 

Content description		New Century Maths Advanced 9 Stages 5.2/5.3 chapter
NUMBER AND ALGEBRA		
Real numbers		
ACMNA208: Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems		11 Coordinate geometry and graphs
ACMNA209: Apply index laws to numerical expressions with integer indices		5 Indices
ACMNA210: Express numbers in scientific notation		5 Indices
YEAR 10A ACMNA264: Define rational and irrational numbers and perform operations with surds and fractional indices		1 Pythagoras' theorem and surds 5 Indices
Money and financial mathematics		
ACMNA211: Solve problems involving simple interest		2 Working with numbers
NSW STAGE 5.1: Solve problems involving earning money		8 Earning money
Patterns and algebra		
ACMNA212: Extend and apply the index laws to variables, using positive integer indices and the zero index		5 Indices
ACMNA213: Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate		3 Products and factors
YEAR 10 ACMNA233: Expand binomial products and factorise monic quadratic expressions using a variety of strategies		3 Products and factors
YEAR 10 ACMNA234: Substitute values into formulas to determine an unknown;		7 Equations
YEAR 10A ACMNA269: Factorise monic and non-monic quadratic expressions and solve a wide range of quadratic equations derived from a variety of contexts		3 Products and factors
Linear and non-linear relationships		
ACMNA214: Find the distance between two points located on a Cartesian plane using a range of strategies, including graphing software		11 Coordinate geometry and graphs
ACMNA215: Sketch linear graphs using the coordinates of two points and solve linear equations		7 Equations 11 Coordinate geometry and graphs
YEAR 10 ACMNA235: Solve problems involving linear equations, including those derived from formulas		7 Equations
ACMNA294: Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software		11 Coordinate geometry and graphs
NSW STAGE 5.2: Interpret and graph linear relationships using the gradient-intercept form of the equation of a straight line		11 Coordinate geometry and graphs
ACMNA296: Graph simple non-linear relations with and without the use of digital technologies and solve simple related equations		11 Coordinate geometry and graphs

Content description		New Century Maths Advanced 9 Stages 5.2/5.3 chapter
MEASUREMENT AND GEOMETRY		
Using units of measurement		
ACMMG216: Calculate the areas of composite shapes		10 Surface area and volume
ACMMG217: Calculate the surface area and volume of cylinders and solve related problems		10 Surface area and volume
ACMMG218: Solve problems involving the surface area and volume of right prisms		10 Surface area and volume
ACMMG219: Investigate very small and very large time scales and intervals		10 Surface area and volume
NSW STAGE 5.1: Round numbers to a specified number of significant figures, describe the limits of accuracy of measuring instruments		10 Surface area and volume
Geometric reasoning		
ACMMG220: Use the enlargement transformation to explain similarity and develop the conditions for triangles to be similar		13 Congruent and similar figures
ACMMG221: Solve problems using ratio and scale factors in similar figures		13 Congruent and similar figures
YEAR 10 ACMMG243: Formulate proofs involving congruent triangles and angle properties; ACMMG244: Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes		13 Congruent and similar figures
NSW STAGE 5.2: Apply interior and exterior angle sum results for polygons to find the sizes of unknown angles		6 Geometry
Pythagoras and trigonometry		
ACMMG222: Investigate Pythagoras' theorem and its application to solving simple problems involving right-angled triangles		1 Pythagoras' theorem and surds
ACMMG223: Use similarity to investigate the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles		4 Trigonometry
ACMMG224: Apply trigonometry to solve right-angled triangle problems		4 Trigonometry
YEAR 10 ACMMG245: Solve right-angled triangle problems including those involving direction and angles of elevation and depression		4 Trigonometry
STATISTICS AND PROBABILITY		
Chance		
ACMSP225: List all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Assign probabilities to outcomes and determine probabilities for events		12 Probability
ACMSP226: Calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or'		12 Probability
ACMSP227: Investigate reports of surveys in digital media and elsewhere for information on how data were obtained to estimate population means and medians		12 Probability
Data representation and interpretation		
ACMSP228: Identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly from secondary sources		9 Investigating data
ACMSP282: Construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including 'skewed', 'symmetric' and 'bi-modal'		9 Investigating data
ACMSP283: Compare data displays using mean, median and range to describe and interpret numerical data sets in terms of location (centre) and spread		9 Investigating data

Coverage of the Australian curriculum and NSW syllabus

- *New Century Maths Advanced 9 Stages 5.2/5.3* covers both the Australian curriculum and the NSW syllabus, as shown by the table of contents and curriculum grids on the previous pages. The previous two pages list all of the Year 9 Australian Curriculum content descriptions explicitly.
- This book contains Stages 5.1, 5.2 and 5.3 content, including NSW syllabus content that is not covered by the Australian curriculum. NSW-only content is highlighted in **orange**, while Stage 5.3 content is marked with an asterisk*.
- There are three content strands in Mathematics:
 - NA = Number and Algebra
 - MG = Measurement and Geometry
 - SP = Statistics and Probability
- Each chapter begins with a **chapter outline** that includes the Working Mathematically proficiencies covered in each section:
 - U = Understanding
 - F = Fluency
 - PS = Problem solving
 - R = Reasoning
 - C = Communicating

10ANA264	5-07 Fractional indices $\frac{m}{n}$ *	199
9NA209	5-08 Summary of the index laws	202
NSW	5-09 Significant figures	204
9NA210	5-10 Scientific notation	206

Chapter outline		Proficiency strands			
1-01 Finding the hypotenuse		U	F		
1-02 Finding a shorter side		U	F		
1-03 Surds and irrational numbers*		U	F	R	C
1-04 Simplifying surds*		U	F	R	
1-05 Adding and subtracting surds*		U	F	R	
1-06 Multiplying and dividing surds*		U	F	R	
1-07 Pythagoras' theorem problems			F	PS	C
1-08 Testing for right-angled triangles		U	F	R	C
1-09 Pythagorean triads		U	F		C

Further references to the Working Mathematically proficiencies can be found in the teaching program.

Working Mathematically explained

While the three **content strands** are the 'nouns' of the mathematics curriculum, the five **Working Mathematically proficiencies** are the 'verbs': the doing and thinking processes that go hand-in-hand with the content being taught.

- **Understanding** is 'knowing and relating' maths. It is more than just learning facts. It's deep understanding, seeing how mathematical content is interconnected, knowing 'why' as well as 'how'.
- **Fluency** is 'applying' maths. It is being able to use mathematics competently and effectively. When you are fluent in a language, you have mastered it so that you can improvise and confidently use the correct word or phrase. Fluency in maths is choosing an appropriate skill, method or formula to use at the right place and time.
- **Problem solving** is 'modelling and investigating' with maths. It involves interpreting a rich, elaborate problem, selecting an appropriate strategy or model, solving the problem, then evaluating, communicating and justifying the solution.
- **Reasoning** is 'generalising and proving' with maths, using higher-order thinking to connect specific facts to general principles, using algebra, logic, proof and justification.
- **Communicating** is 'describing and explaining' maths, representing mathematical theory and solutions in words, algebraic symbols, special notations, diagrams, graphs and tables.

In each chapter

- **Wordbank** is a sample glossary of 6–8 terms from the chapter
- There is also a full glossary at the back of the book: in this book, all terms printed in **red** appear in the glossary
- Australian Curriculum content descriptions (printed in **blue** and marked AC ) are listed at the beginning of each chapter
- **SkillCheck** and **StartUp assignment** review prerequisite skills and knowledge for the chapter

- Important facts and formulas are highlighted in a **Summary** box.

In this chapter you will:

- apply index laws to numerical expressions with integer indices
- simplify algebraic products and quotients using index laws
- express numbers in scientific notation
- interpret and use zero and negative indices
- (STAGE 5.3) interpret and use fractional indices
- round numbers to significant figures
- interpret, write and order numbers in scientific notation
- interpret and use scientific notation on a calculator
- solve problems involving scientific notation

SkillCheck

Worksheet

StartUp assignment 5

MAT09NAWK10050

Worksheet

Powers review

MAT09NAWK10051

Worksheet

Indices

MAT09NASS10020

1 For each term:

- state the base
- state the index
- write the expression in words.

2 Express each repeated multiplication in index notation.

a $2 \times 2 \times 2 \times 2 \times 2$	b $3 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7$
c $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 8 \times 8$	d $10 \times x \times x \times x \times x \times x \times x$
e $6 \times 6 \times 6 \times k \times k \times k$	f $x \times y \times x \times y \times x \times y$

Summary

Generally, any number raised to the power of $\frac{1}{n}$ is the n th root of that number:

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

- Pages containing **Stage 5.3 content** are marked by a shaded margin.
- Graded exercises are linked to worked examples and include multiple-choice questions, exam-style problems and realistic applications
- Worked solutions for selected exercise questions are provided.

Stage 5.3

Exercise 1-04 Simplifying surds

See Example 10

See Example 11

1 Simplify each expression.

a $(\sqrt{2})^2$	b $(\sqrt{5})^2$	c $(3\sqrt{3})^2$	d $(5\sqrt{10})^2$
e $(\sqrt{0.09})^2$	f $(-2\sqrt{7})^2$	g $(-3\sqrt{5})^2$	h $(-5\sqrt{2})^2$

2 Simplify each surd.

a $\sqrt{8}$	b $\sqrt{27}$	c $\sqrt{24}$	d $\sqrt{54}$
e $\sqrt{243}$	f $\sqrt{45}$	g $\sqrt{48}$	h $\sqrt{200}$
i $\sqrt{96}$	j $\sqrt{63}$	k $\sqrt{288}$	l $\sqrt{108}$
m $\sqrt{75}$	n $\sqrt{147}$	o $\sqrt{32}$	p $\sqrt{242}$
q $\sqrt{162}$	r $\sqrt{243}$	s $\sqrt{125}$	t $\sqrt{512}$

3 Simplify each expression.

a $5\sqrt{50}$	b $3\sqrt{8}$	c $4\sqrt{27}$	d $8\sqrt{98}$
e $\frac{\sqrt{40}}{2}$	f $\frac{\sqrt{243}}{9}$	g $\frac{\sqrt{28}}{6}$	h $3\sqrt{24}$
i $9\sqrt{68}$	j $\frac{\sqrt{3125}}{10}$	k $\frac{1}{2}\sqrt{72}$	l $\frac{3}{4}\sqrt{48}$
m $10\sqrt{160}$	n $3\sqrt{75}$	o $7\sqrt{68}$	p $\frac{\sqrt{52}}{6}$

4 Decide whether each statement is true (T) or false (F).

a $3\sqrt{7} = \sqrt{21}$	b $\sqrt{12} = 6$	c $(\sqrt{9.4})^2 = 9.4$

- **Investigations** explore the syllabus in more detail, through group work, discovery and modelling activities.

Investigation: A lifetime of heartbeats

How many times does your heart beat in an average lifetime of 80 years?

1 Work in pairs and copy this table.

Name	Trial 1	Trial 2	Average beats per minute

2 Use two fingers to measure your pulse. Have your partner time you for a minute. Do this twice, record your results in the table and find the average.

- **Just for the record** contains interesting facts and applications of the mathematics learnt in the chapter

Just for the record

Hairy numbers



Straight hair
• round follicle



Wavy hair
• oval follicle



Curly hair
• flat follicle

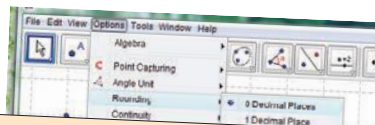
There are about 110 000 hairs on your head. Each hair grows at the rate of about 1.3×10^{-3} cm per hour. A single hair lasts about six years. Every day you lose between 30 and 60 hairs. Each hair grows from a small depression in the skin called a follicle (a gland). After the hair falls out

- **Technology** promotes ICT in the classroom, using spreadsheets, *GeoGebra* and the Internet

Technology Similar right-angled triangles

In this activity you will use *GeoGebra* to measure and calculate trigonometric ratios.

- 1 a Before you start, set angles to measure in degrees. Click **Options, Rounding** and **1 Decimal Place**.



Mental skills 4 Maths without calculators

Estimating answers

A quick way of estimating an answer is to round each number in the calculation.

- 1 Study each example.

a $631 + 280 + 51 + 43 + 96 \approx 600 + 300 + 50 + 40 + 100$
 $= (600 + 300 + 100) + (50 + 40)$
 $= 1000 + 90$
 $= 1090$ (Actual answer = 1101)

- Mental skills reinforce mental calculation strategies ('maths without calculators')

NelsonNet resources

Margin icons link to print (PDF) and multimedia resources found on the NelsonNet website, www.nelsonnet.com.au. These include:

Worksheet Venn diagrams	Worksheets	Video tutorial Negative indices	Video tutorials of worked examples
Puzzle sheet Geometry crossword	Puzzle sheets of matching activities and crosswords	Worked solutions Overtime pay	Worked solutions of selected exercise questions
Skillsheet Indices	Skillsheets of examples and exercises of prerequisite skills and knowledge	Technology worksheet Angle sum of a polygon	Technology worksheets: additional technology activities
Homework sheet Pythagoras' theorem 1	Homework sheets for weekly practice and revision, including mental calculation, numeracy and literacy questions	Quiz Algebra	ExamView quizzes: interactive and self-marking

Each resource has a unique identifier code. For example, the video tutorial 'Pythagoras' theorem' has the code **MAT09MGVT10001**, which stands for Mathematics, Year 9, Measurement and Geometry strand, Video Tutorial 10001.

At the end of each chapter

- **Power plus** is an extension/challenge exercise
- **Language of maths** has a chapter word list and literacy questions
- **Topic overview** has reflection questions and an incomplete mind map
- **Chapter revision** is a review exercise with links to each exercise set of the chapter
- **Mixed revision** is a review exercise after every 3–4 chapters

Language of maths

ascending	base	descending	estimate
expanded form	exponent	fractional power	index
index laws	index notation	indices	negative power
power	product	quotient	reciprocal
scientific notation	significant figures	term	zero power

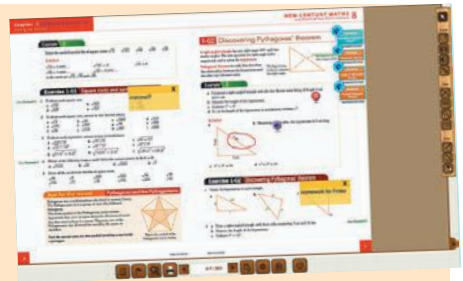
- 1 What does a power of $\frac{1}{2}$ mean?
- 2 Which two words from the list mean 'power'?
- 3 What is the $\times 10^3$ or EXP key on a calculator used for?

At the end of the book

- **General revision** exercise
- **Instructional and Mathematical glossaries** (in this book, words printed in red also appear in the glossary)
- **Answers and Index**

NelsonNetBook

- **NelsonNetBook** is the interactive digital version of this textbook found on NelsonNet, containing margin icons that link directly to NelsonNet resources
- To each page of NelsonNetBook you can add notes, voice and sound bites, highlighting, weblinks and bookmarks
- **Zoom** and **Search** functions
- Chapters can be customised for different groups of students





1

Measurement and geometry

Pythagoras' theorem and surds

Pythagoras was an ancient Greek mathematician who lived in the 5th century BCE. The theorem (or rule) which carries his name was well-known before this time, but Pythagoras may have been the first to prove it. Over 300 proofs for the theorem are known today. Pythagoras' theorem is perhaps the most famous mathematical formula, and it is still used today in architecture, engineering, surveying and astronomy.



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Chapter outline

Proficiency strands

1-01 Finding the hypotenuse	U	F		
1-02 Finding a shorter side	U	F		
1-03 Surds and irrational numbers*	U	F	R	C
1-04 Simplifying surds*	U	F	R	
1-05 Adding and subtracting surds*	U	F	R	
1-06 Multiplying and dividing surds*	U	F	R	
1-07 Pythagoras' theorem problems		F	PS	C
1-08 Testing for right-angled triangles	U	F	R	C
1-09 Pythagorean triads	U	F		C

*STAGE 5.3

Pythagoras' theorem is a Year 9 topic in the Australian Curriculum but a Stage 4 topic in the NSW syllabus, so Pythagoras' theorem has also been covered in Chapter 1 of *New Century Maths 8 for the Australian Curriculum*.

Wordbank

converse A rule or statement turned back-to-front; the reverse statement

hypotenuse The longest side of a right-angled triangle; the side opposite the right angle

irrational number A number such as π or $\sqrt{2}$ that cannot be expressed as a fraction

Pythagoras An ancient Greek mathematician who discovered an important formula about the sides of a right-angled triangle

Pythagorean triad A set of three numbers that follow Pythagoras' theorem, such as 3, 4, 5.

surd A square root (or other root) whose exact value cannot be found

theorem Another name for a formal rule or formula

In this chapter you will:

- investigate Pythagoras' theorem and its application to solving simple problems involving right-angled triangles 
- investigate irrational numbers and surds
- write answers to Pythagoras' theorem problems in decimal or surd form
- (STAGE 5.3) simplify, add, subtract, multiply and divide surds
- test whether a triangle is right-angled
- investigate Pythagorean triads

SkillCheck

Worksheet

StartUp assignment 1

MAT09MGWK10001

- 1 Evaluate each expression.

a 4^2

b 10.3^2

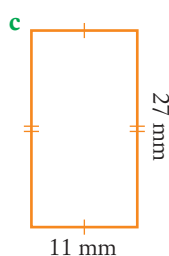
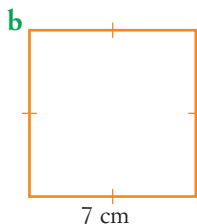
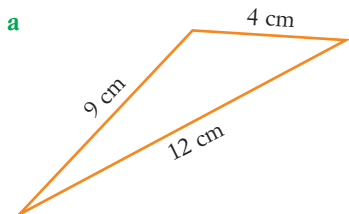
c $3^2 + 5^2$

d $8^2 - 6^2$

e $\sqrt{49}$

f $\sqrt{121}$

- 2 Find the perimeter of each shape.



- 3 Select the square numbers from the following list of numbers.

44 81 25 100 75 72 16 50 64 32

Worksheet

Pythagoras' discovery

MAT09MGWK10002

Worksheet

A page of right-angled triangles

MAT09MGWK10003

Worksheet

Pythagoras 1

MAT09MGWK00055

Skillsheet

Pythagoras' theorem

MAT09MGSS10001

1-01 Finding the hypotenuse

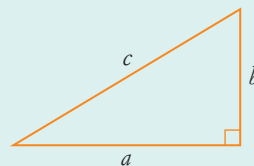
Summary

Pythagoras' theorem

For any right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

If c is the length of the hypotenuse, and a and b are the lengths of the other two sides, then:

$$c^2 = a^2 + b^2$$



Example 1

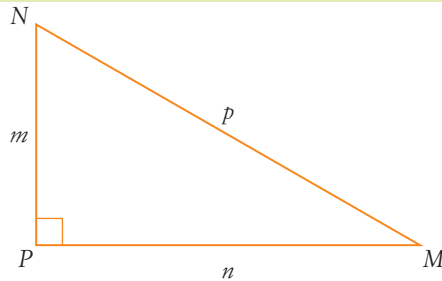
Write Pythagoras' theorem for this triangle.

Solution

p is the hypotenuse, so $p^2 = m^2 + n^2$

OR

NM is the hypotenuse, so $NM^2 = NP^2 + PM^2$



Example 2

Find the value of c in this triangle.

Solution

We want to find the length of the hypotenuse.

Using Pythagoras' theorem:

$$c^2 = 9^2 + 40^2$$

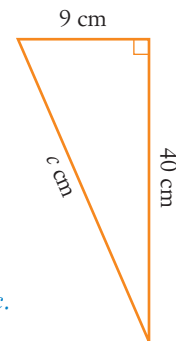
$$= 1681$$

$$c = \sqrt{1681}$$

$$= 41$$

An answer of $c = 41$ looks reasonable because:

- the hypotenuse is the longest side
- from the diagram, the hypotenuse looks a little longer than the side that is 40 cm



Use square root to find c .

Example 3

Find the length of the cable supporting this flagpole:

- a** as a surd
b correct to one decimal place.

Solution

a $AC^2 = 12^2 + 4^2$

$$= 160$$

$$AC = \sqrt{160} \text{ m}$$

This is the answer as a **surd**
(in $\sqrt{\quad}$ form).

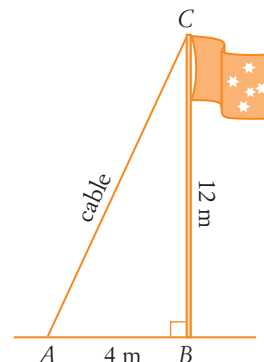
b $AC = \sqrt{160}$

$$= 12.6491 \dots$$

$$\approx 12.6 \text{ m}$$

From part **a**.

Rounded to one decimal place.



Technology

GeoGebra: Pythagoras' theorem

MAT09MGCT00007

Technology worksheet

Finding the hypotenuse

MAT09MGCT10006

Technology worksheet

Excel worksheet: Pythagoras' theorem

MAT09MGCT00024

Technology worksheet

Excel spreadsheet: Pythagoras' theorem

MAT09MGCT00009

Skillsheet

Spreadsheets

MAT09NASS10027

Video tutorial

Pythagoras' theorem

MAT09MGVT10001

Example 4

In $\triangle PQR$, $\angle P = 90^\circ$, $PQ = 25$ cm and $PR = 32$ cm. Sketch the triangle and find the length of the hypotenuse, p , correct to one decimal place.

Solution

$$p^2 = 32^2 + 25^2$$

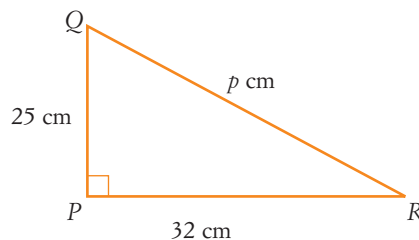
$$= 1649$$

$$p = \sqrt{1649}$$

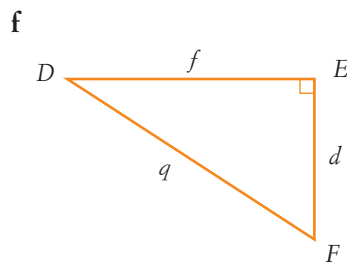
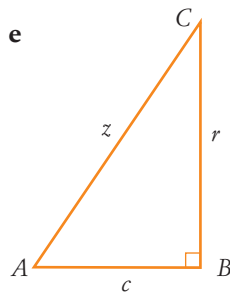
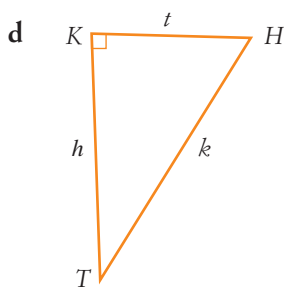
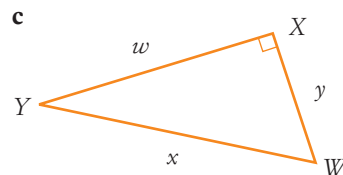
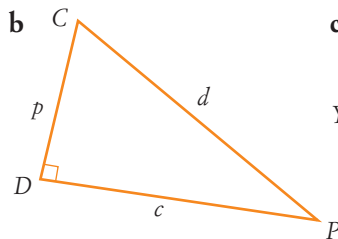
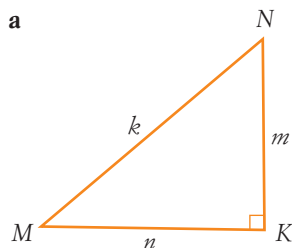
$$= 40.607\ 881\ 01\dots$$

$$\approx 40.6$$

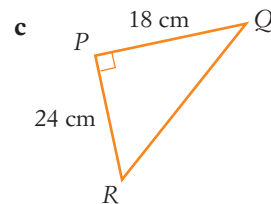
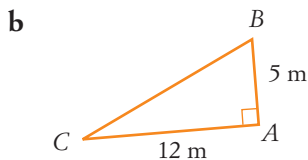
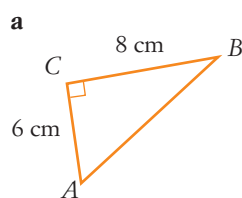
$\therefore$ The length of the hypotenuse is 40.6 cm.

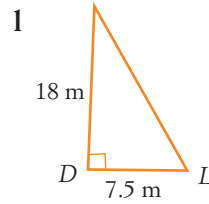
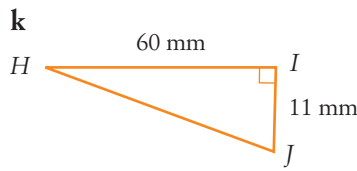
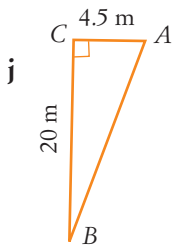
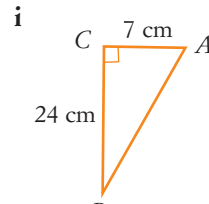
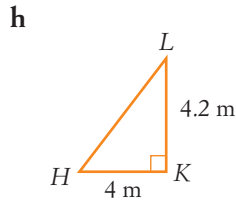
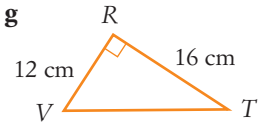
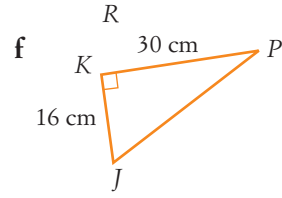
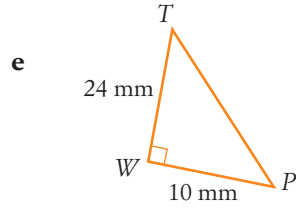
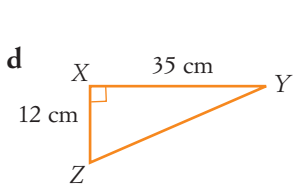
**Exercise 1-01 Finding the hypotenuse**

See Example 1 1 Write Pythagoras' theorem for each right-angled triangle.



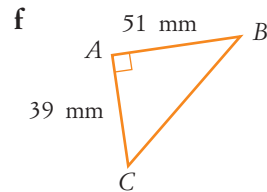
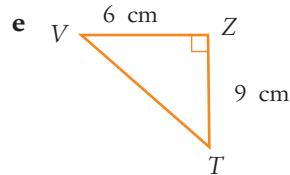
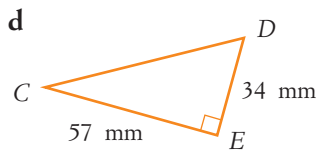
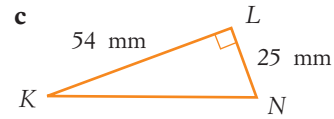
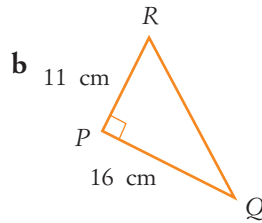
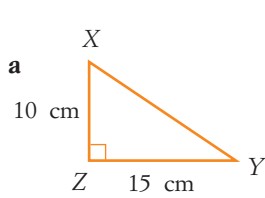
See Example 2 2 Find the length of the hypotenuse in each triangle.



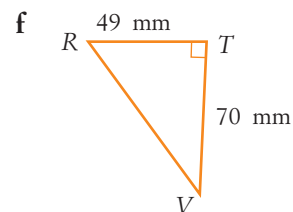
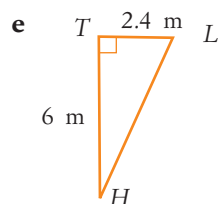
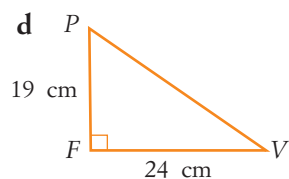
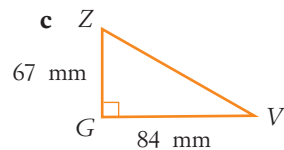
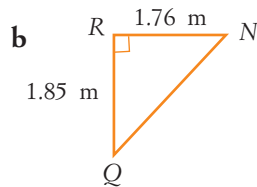
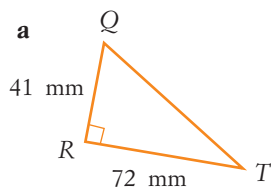


3 Find the length of the hypotenuse in each triangle, as a surd.

See Example 3

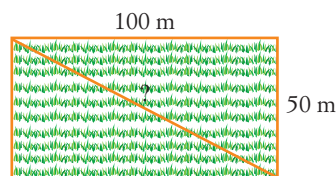


4 Find the length of the hypotenuse in each triangle, correct to one decimal place.



- 5 A rectangular field is 100 m long and 50 m wide. How far is it from one corner to the opposite corner, along the diagonal? Select the correct answer A, B, C or D.

A 150 m B 111.8 m
C 100.2 m D 98.3 m



- 6 A firefighter places a ladder on a window sill 4.5 m above the ground. If the foot of the ladder is 1.6 m from the wall, how long is the ladder? Leave your answer as a surd.



Shutterstock.com/Sergey Rybov

- See Example 4**
- 7 a In $\triangle ABC$, $\angle ABC = 90^\circ$, $AB = 39$ cm and $BC = 57$ cm. Find AC correct to one decimal place.
b In $\triangle MPQ$, $\angle PQM = 90^\circ$, $QM = 2.4$ m and $PQ = 3.7$ m. Find PM correct to one decimal place.
c In $\triangle RVJ$, $\angle J = 90^\circ$, $JV = 12.7$ cm and $JR = 4.2$ cm. Find RV correct to the nearest millimetre.
d In $\triangle EGB$, $EG = EB = 127$ mm and $\angle GEB = 90^\circ$. Find the length of BG correct to one decimal place.
e In $\triangle VZX$, $\angle V = 90^\circ$, $VX = 247$ cm and $VZ = 3.6$ m. Find ZX in metres, correct to the nearest 0.1 m.
f In $\triangle PQR$, $\angle RPQ = 90^\circ$, $PQ = 2.35$ m and $PR = 5.8$ m. Find QR in metres, correct to two decimal places.

Worksheet

Finding an unknown side

MAT09MGWK10004

Homework sheet

Pythagoras' theorem 1

MAT09MGHS10029

Video tutorial

Pythagoras' theorem

MAT09MGVT10001

Technology worksheet

Excel worksheet: Pythagoras' theorem

MAT09MGCT00024

Technology worksheet

Excel spreadsheet: Pythagoras' theorem

MAT09MGCT00009

1-02 Finding a shorter side

Pythagoras' theorem can also be used to find the length of a shorter side of a right-angled triangle, if the hypotenuse and the other side are known.

Example 5

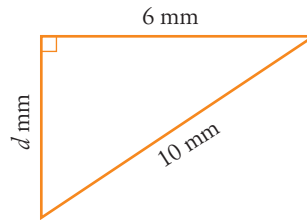
Find the value of d in this triangle.

Solution

We want to find the length of a shorter side.
Using Pythagoras' theorem:

$$\begin{aligned} 10^2 &= d^2 + 6^2 \\ 100 &= d^2 + 36 \\ d^2 + 36 &= 100 \\ d^2 &= 100 - 36 \\ &= 64 \\ d &= \sqrt{64} \\ &= 8 \end{aligned}$$

From the diagram, a length of 8 mm looks reasonable because it must be shorter than the hypotenuse, which is 10 mm.



Puzzle sheet

Pythagoras 1

MAT09MGPS00039

Puzzle sheet

Pythagoras 2

MAT09MGPS00040

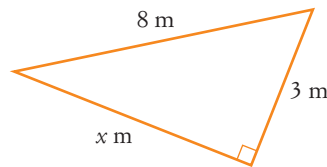
Example 6

Find the value of x as a surd for this triangle.

Solution

$$\begin{aligned} 8^2 &= x^2 + 3^2 \\ 64 &= x^2 + 9 \\ x^2 + 9 &= 64 \\ x^2 &= 64 - 9 \\ &= 55 \\ x &= \sqrt{55} \end{aligned}$$

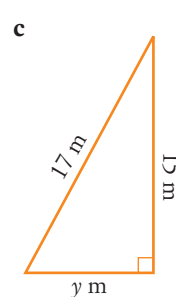
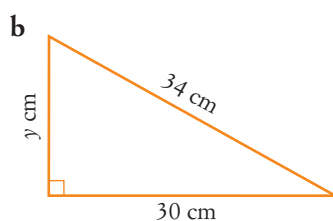
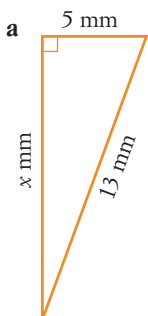
Leave the answer as a surd.

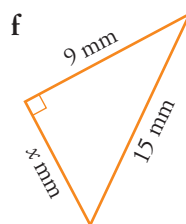
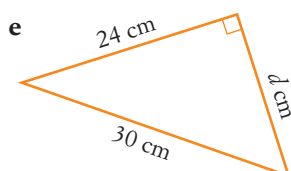
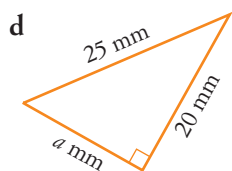


Exercise 1-02 Finding a shorter side

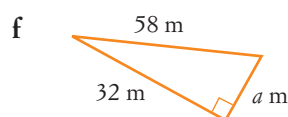
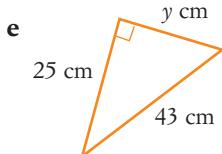
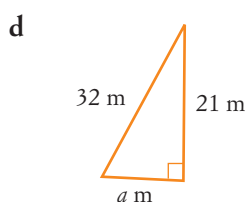
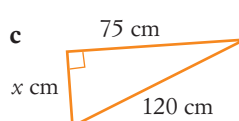
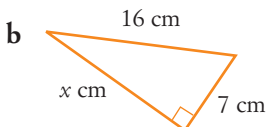
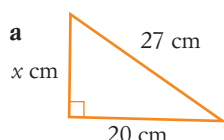
- 1 Find the value of the pronumeral in each triangle.

See Example 5

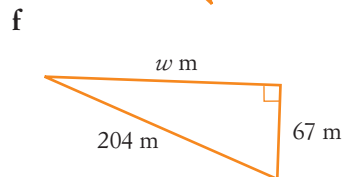
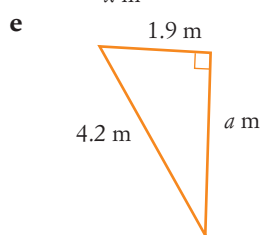
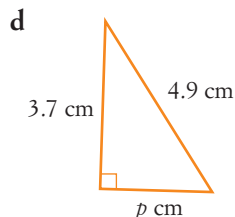
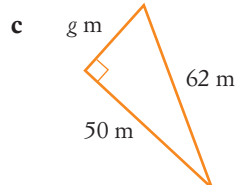
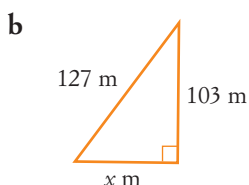
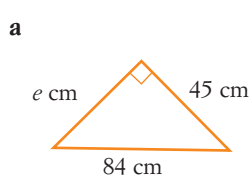




- 2 Find the value of the pronumeral in each triangle correct to one decimal place.

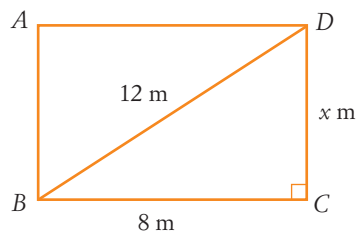


- See Example 6** 3 Find the value of the pronumeral in each triangle as a surd.



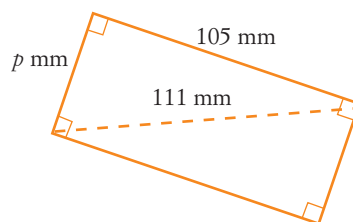
- 4 Find the value of x in this rectangle. Select the correct answer **A**, **B**, **C** or **D**.

- A** $\sqrt{20}$ **B** $\sqrt{80}$
C $\sqrt{136}$ **D** $\sqrt{208}$



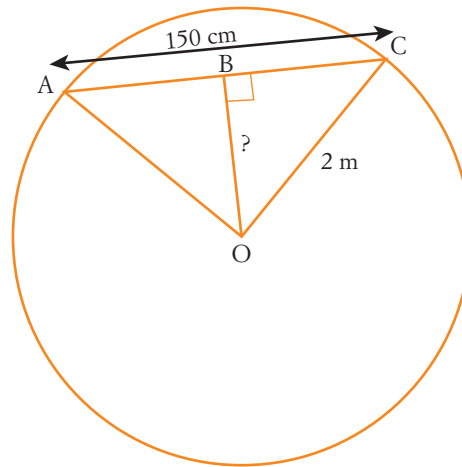
- 5 Find the value of p in this rectangle. Select the correct answer **A**, **B**, **C** or **D**.

- A** 40 **B** 36
C 32 **D** 28



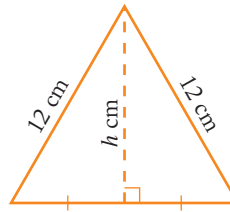
- 6 In this diagram, O is the centre of a circle. A perpendicular line is drawn from O to B such that $OB \perp AC$ and $AB = BC$. Calculate the length of OB . Select the correct answer A, B, C or D.

- A 1.3 m B 1.8 m
C 1.9 m D 3.4 m



Worked solutions
Finding a shorter side
MAT09MGWS10001

- 7 An equilateral triangle has sides of length 12 cm. Find the perpendicular height, h , of the triangle, correct to two decimal places.



- 8 A square has a diagonal of length 30 cm. What is the length of each side of the square, correct to the nearest millimetre? [Hint: Draw a diagram]

1-03 Surds and irrational numbers

- $\sqrt{25} = 5$ because $5^2 = 25$ 'the square root of 25'
- $\sqrt{81} = 9$ because $9^2 = 81$ 'the square root of 81'

Most square roots do not give exact answers like the ones above. For example, $\sqrt{7} = 2.645751311 \dots \approx 2.6$. Such roots are called **surds**.

A surd is a square root ($\sqrt{\quad}$), cube root ($\sqrt[3]{\quad}$), or any type of root whose exact decimal or fraction value cannot be found. As a decimal, its digits run endlessly *without repeating* (like π), so they are neither terminating nor recurring decimals.

Rational numbers such as fractions, decimals and percentages, can be expressed in the form $\frac{a}{b}$ where a and b are integers (and $b \neq 0$), but surds are **irrational numbers** because they cannot be expressed in this form.

Example 7

Select the surds from this list of square roots: $\sqrt{72}$ $\sqrt{121}$ $\sqrt{64}$ $\sqrt{90}$ $\sqrt{28}$

Solution

$$\sqrt{72} = 8.4852 \dots$$

$$\sqrt{121} = 11$$

$$\sqrt{64} = 8$$

$$\sqrt{90} = 9.4868 \dots$$

$$\sqrt{28} = 5.2915 \dots$$

so the surds are $\sqrt{72}$, $\sqrt{90}$ and $\sqrt{28}$.

Example 8

Is each number rational or irrational?

a $4\frac{2}{5}$

b $\sqrt[3]{-8}$

c $-\sqrt{7}$

d $0.\dot{6}$

e 5π

Solution

a $4\frac{2}{5} = \frac{22}{5}$

which is in the form of a fraction $\frac{a}{b}$

$\therefore 4\frac{2}{5}$ is a rational number.

b $\sqrt[3]{-8} = -2$

which can be written as $\frac{-2}{1}$

$\therefore \sqrt[3]{-8}$ is a rational number.

c $-\sqrt{7} = -2.645\ 751\ 311 \dots$

The digits run endlessly without repeating.

$\therefore -\sqrt{7}$ is an irrational number.

d $0.\dot{6} = 0.666 \dots$

which is a recurring decimal

$$= \frac{2}{3}$$

which is a fraction

$\therefore 0.\dot{6}$ is a rational number.

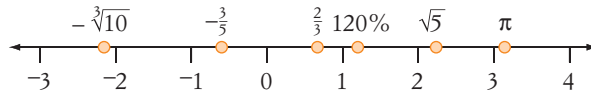
e $5\pi = 15.707\ 963\ 27 \dots$

The digits run endlessly without repeating.

$\therefore 5\pi$ is an irrational number.

Surds on a number line

The rational and irrational numbers together make up the **real numbers**. Any real number can be represented by a point on the number line.



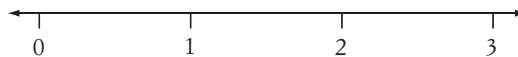
$-\sqrt[3]{10} \approx -2.1544 \dots$	irrational (surd)
$-\frac{3}{5} = -0.6$	rational (fraction)
$\frac{2}{3} \approx 0.6666 \dots$	rational (fraction)
$120\% = 1.2$	rational (percentage)
$\sqrt{5} \approx 2.2360 \dots$	irrational (surd)
$\pi \approx 3.1415 \dots$	irrational (pi)

Example 9

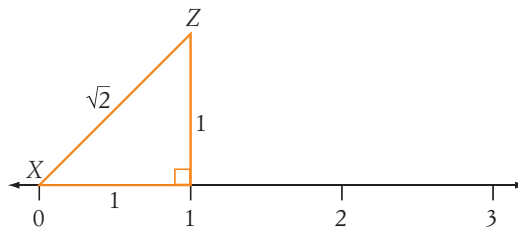
Use a pair of compasses and Pythagoras' theorem to estimate the value of $\sqrt{2}$ on a number line.

Solution

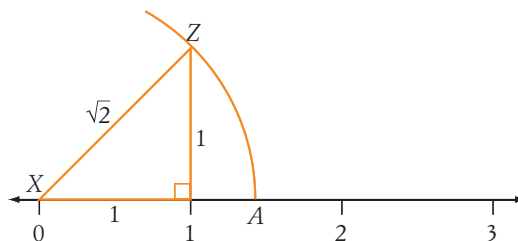
Step 1: Using a scale of 1 unit to 2 cm, draw a number line as shown.



Step 2: Construct a right-angled triangle on the number line with base length and height 1 unit as shown. By Pythagoras' theorem, show that $XZ = \sqrt{2}$ units.



Step 3: With 0 as centre, use compasses with radius $XZ(\sqrt{2})$ to draw an arc to meet the number line at A as shown. The point A represents the value of $\sqrt{2}$ and should be approximately 1.4142...



Worksheet

Surds on the
number line

MAT09MGWK10006

Exercise 1-03 Surds and irrational numbers

See Example 7

- Which one of the following is a surd? Select the correct answer A, B, C or D.
 A $\sqrt{9}$ B $\sqrt{225}$ C $\sqrt{160}$ D $\sqrt{81}$
- Which one of the following is NOT a surd? Select the correct answer A, B, C or D.
 A $\sqrt{77}$ B $\sqrt{144}$ C $\sqrt{18}$ D $\sqrt{200}$
- Select the surds from the following list of square roots.
 $\sqrt{32}$ $\sqrt{33}$ $\sqrt{289}$ $\sqrt{81}$ $\sqrt{4.9}$ $\sqrt{52}$ $\sqrt{121}$ $\sqrt{144}$ $\sqrt{196}$ $\sqrt{200}$

See Example 8

- Is each number rational (R) or irrational (I)?
 a $5.\dot{6}$ b $\sqrt{8}$ c $\sqrt{4}$ d $3\frac{1}{7}$
 e $\sqrt[3]{27}$ f $1.3\dot{5}$ g $\sqrt[3]{-64}$ h $27\frac{1}{2}\%$
 i $\sqrt{5 \times 10^3}$ j $\frac{3}{11}$ k $\frac{\sqrt{50}}{3}$ l $\sqrt{\sqrt{4}}$
- Arrange each set of numbers in descending order.
 a $1\frac{4}{7}, \sqrt{2}, \frac{\pi}{2}$ b $\sqrt[3]{20}, 2.\dot{6}, 2\frac{7}{9}$
- Use the method from Example 9 to estimate the value of $\sqrt{2}$ on a number line.
- Use the method from Example 9 to estimate the value of $\sqrt{5}$ on a number line by constructing a right-angled triangle with base length 2 units and height 1 unit.
 - Use a similar method to estimate the following surds on a number line.
 i $\sqrt{10}$ ii $\sqrt{17}$

Stage 5.3

See Example 9

Investigation: Proof that $\sqrt{2}$ is irrational

A method of proof sometimes used in mathematics is to assume the **opposite** of what is being proved, and to show that it is **impossible**. This is called a **proof by contradiction**, and we will use it to prove that $\sqrt{2}$ is irrational.

First, we assume that $\sqrt{2}$ is rational. So, assume that $\sqrt{2}$ can be written as a simplified fraction $\frac{a}{b}$, where a and b are integers ($b \neq 0$) with no common factors.

$$\sqrt{2} = \frac{a}{b}$$

$$2 = \frac{a^2}{b^2} \quad \text{Squaring both sides}$$

$$a^2 = 2b^2$$

$2b^2$ is an even number because it is divisible by 2, $\therefore a^2$ is even.

$\therefore a$ is even, because an even integer multiplied by itself is always even and an odd integer multiplied by itself is always odd.

$\therefore a = 2m$, where m is another integer.

$$\therefore a^2 = (2m)^2 = 2b^2$$

$$4m^2 = 2b^2$$

$$2m^2 = b^2$$

$$b^2 = 2m^2$$

$\therefore b^2$ is even

$\therefore b$ is even.

But a and b can't **both** be even because this contradicts the assumption that a and b have no common factors. Therefore, the assumption that $\sqrt{2}$ is rational is false, so $\sqrt{2}$ must be irrational.

1 Use proof by contradiction to show that these surds are irrational:

a $\sqrt{3}$

b $\sqrt{5}$

2 Compare your proofs with those of other students.

Just for the record

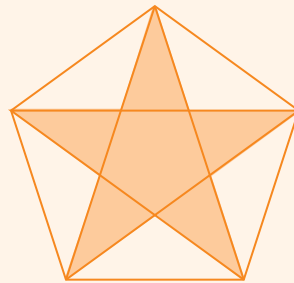
Pythagoras and the Pythagoreans

Pythagoras was a mathematician who lived in ancient Greece. The Pythagoreans were a group of men who followed Pythagoras.

Sometimes when applying Pythagoras' theorem, lengths are found that cannot be expressed as exact rational numbers. Pythagoras encountered this when calculating the diagonal of a square of side length 1 unit.

The Pythagoreans were the first to study the properties of whole numbers. They explained nature, the universe — in fact everything — in terms of whole numbers. Apparently they were so upset about the discovery of surds that they tried to keep the discovery a secret. Hippasus, one of the Pythagoreans, was drowned for revealing the secret to outsiders.

π is an irrational number but it is not a surd. Why? Another such number is e . Investigate the numbers π and e and the meaning of transcendental numbers.



1-04 Simplifying surds

The square of any real number is always positive (except for $0^2 = 0$), so it is not possible to give the square root of a negative number.

The radical symbol ($\sqrt{\quad}$) stands for the positive square root of a number, for example $\sqrt{4} = 2$ (not -2).

Summary

For $x < 0$ (negative), $\sqrt{x}$ is undefined.

For $x = 0$, $\sqrt{x}$ is 0.

For $x > 0$ (positive), $\sqrt{x}$ is the positive square root of x .

$$(\sqrt{x})^2 = \sqrt{x} \times \sqrt{x} = x$$

$$\sqrt{x^2} = x$$

Stage 5.3

Worksheet

Surds

MAT09NAWK10005

Puzzle sheet

Simplifying surds

MAT09NAPS10007

Stage 5.3

Example 10

Simplify each expression.

$$\text{a } (\sqrt{7})^2 \quad \text{b } (3\sqrt{5})^2 \quad \text{c } (-2\sqrt{3})^2$$

Solution

$$\text{a } (\sqrt{7})^2 = 7$$

$$\begin{aligned} \text{b } (3\sqrt{5})^2 &= 3\sqrt{5} \times 3\sqrt{5} && 3\sqrt{5} \text{ means } 3 \times \sqrt{5} \\ &= 3^2 \times (\sqrt{5})^2 \\ &= 9 \times 5 \\ &= 45 \end{aligned}$$

$$\begin{aligned} \text{c } (-2\sqrt{3})^2 &= (-2)^2 \times (\sqrt{3})^2 \\ &= 4 \times 3 \\ &= 12 \end{aligned}$$

Summary

The square root of a product

For $x > 0$ and $y > 0$:

$$\sqrt{xy} = \sqrt{x} \times \sqrt{y}$$

A surd $\sqrt{n}$ can be simplified if n can be divided into two factors where one of them is a square number such as 4, 9, 16, 25, 36, 49, ...

Example 11

Simplify each surd.

$$\text{a } \sqrt{50} \quad \text{b } \sqrt{432} \quad \text{c } 4\sqrt{12} \quad \text{d } \frac{\sqrt{288}}{3}$$

Solution

$$\begin{aligned} \text{a } \sqrt{50} &= \sqrt{25} \times \sqrt{2} && 25 \text{ is a square number.} \\ &= 5 \times \sqrt{2} \\ &= 5\sqrt{2} \end{aligned}$$

b Method 1

$$\begin{aligned}\sqrt{432} &= \sqrt{4} \times \sqrt{108} \\ &= 2 \times \sqrt{108} \\ &= 2 \times \sqrt{36} \times \sqrt{3} \\ &= 2 \times 6 \times \sqrt{3} \\ &= 12 \times \sqrt{3} \\ &= 12\sqrt{3}\end{aligned}$$

Method 2

$$\begin{aligned}\sqrt{432} &= \sqrt{144} \times \sqrt{3} \\ &= 12 \times \sqrt{3} \\ &= 12\sqrt{3}\end{aligned}$$

Method 1 involves simplifying surds *twice* ($\sqrt{432}$ and $\sqrt{108}$).
Method 2 shows that when simplifying surds, it is more efficient to first look for the highest square factor possible.

c
$$\begin{aligned}4\sqrt{12} &= 4 \times \sqrt{4} \times \sqrt{3} \\ &= 4 \times 2 \times \sqrt{3} \\ &= 8\sqrt{3}\end{aligned}$$

d
$$\begin{aligned}\frac{\sqrt{288}}{3} &= \frac{\sqrt{144} \times \sqrt{2}}{3} \\ &= \frac{12\sqrt{2}}{3} \\ &= \frac{\cancel{12}^4 \sqrt{2}}{\cancel{3}^1} \\ &= 4\sqrt{2}\end{aligned}$$

Exercise 1-04 Simplifying surds

1 Simplify each expression.

a $(\sqrt{2})^2$

b $(\sqrt{5})^2$

c $(3\sqrt{3})^2$

d $(5\sqrt{10})^2$

e $(\sqrt{0.09})^2$

f $(-2\sqrt{7})^2$

g $(-3\sqrt{5})^2$

h $(-5\sqrt{2})^2$

2 Simplify each surd.

a $\sqrt{8}$

b $\sqrt{27}$

c $\sqrt{24}$

d $\sqrt{54}$

e $\sqrt{243}$

f $\sqrt{45}$

g $\sqrt{48}$

h $\sqrt{200}$

i $\sqrt{96}$

j $\sqrt{63}$

k $\sqrt{288}$

l $\sqrt{108}$

m $\sqrt{75}$

n $\sqrt{147}$

o $\sqrt{32}$

p $\sqrt{242}$

q $\sqrt{162}$

r $\sqrt{245}$

s $\sqrt{125}$

t $\sqrt{512}$

3 Simplify each expression.

a $5\sqrt{50}$

b $3\sqrt{8}$

c $4\sqrt{27}$

d $8\sqrt{98}$

e $\frac{\sqrt{40}}{2}$

f $\frac{\sqrt{243}}{9}$

g $\frac{\sqrt{28}}{6}$

h $3\sqrt{24}$

i $9\sqrt{68}$

j $\frac{\sqrt{3125}}{10}$

k $\frac{1}{2}\sqrt{72}$

l $\frac{3}{4}\sqrt{48}$

m $10\sqrt{160}$

n $3\sqrt{75}$

o $7\sqrt{68}$

p $\frac{\sqrt{52}}{6}$

4 Decide whether each statement is true (T) or false (F).

a $3\sqrt{7} = \sqrt{21}$

b $\sqrt{12} = 6$

c $(\sqrt{9.4})^2 = 9.4$

d $\sqrt{75} = 5\sqrt{3}$

e $\sqrt{3} \approx 1.7$

f The exact value of $\sqrt{10}$ is 3.162 277 8

See Example 10

See Example 11

Mental skills 1A

Maths without calculators

Multiplying and dividing by a power of 10

Multiplying a number by 10, 100, 1000, etc. moves the decimal point to the right and makes the number bigger. We place zeros at the end of the number if necessary.

- When multiplying a number by **10**, move the decimal point **one** place to the right.
- When multiplying a number by **100**, move the decimal point **two** places to the right.
- When multiplying a number by **1000**, move the decimal point **three** places to the right.

The number of places the decimal is moved to the right matches the number of zeros in the 10, 100 or 1000 we are multiplying by.

1 Study each example.

a $26.32 \times 10 = 26.32 = 263.2$

The point moves one place to the right.

b $8.701 \times 100 = 8.701 = 870.1$

The point moves two places to the right.

c $6.01 \times 1000 = 6.010 = 6010$

The point moves three places to the right after a 0 is placed at the end.

d $17 \times 100 = 17.00 = 1700$

The point moves two places to the right after two zeros are placed at the end.

2 Now evaluate each expression.

a 89.54×10

b 3.7×10

c 0.831×100

d 42×100

e 5.2716×1000

f 156.1×10

g 31.84×1000

h 64.3×100

i 0.0224×1000

j 4.894×10

k 7.389×1000

l 11.42×100

Dividing a number by 10, 100, 1000, etc. moves the decimal point to the left and makes the number smaller. We place zeros at the start of the decimal if necessary.

- When dividing a number by **10**, move the decimal point **one** place to the left.
- When dividing a number by **100**, move the decimal point **two** places to the left.
- When dividing a number by **1000**, move the decimal point **three** places to the left.

3 Study each example.

a $145.66 \div 10 = 145.66 = 14.566$

The point moves one place to the left.

b $2.357 \div 100 = 002.357 = 0.023\ 57$

The point moves two places to the left after two zeros are inserted at the start.

c $14.9 \div 1000 = 0014.9 = 0.0149$

The point moves three places to the left after two zeros are inserted at the start.

d $45 \div 100 = 045. = 0.45$

The point moves two places to the left after one zero is inserted at the start.

4 Now evaluate each expression.

a $733.4 \div 10$

b $9.4 \div 10$

c $652 \div 100$

d $10.4 \div 100$

e $704 \div 1000$

f $198.5 \div 100$

g $2 \div 100$

h $4159 \div 1000$

i $123 \div 10$

j $0.758 \div 100$

k $8.49 \div 100$

l $25.1 \div 1000$

Investigation: A formula for calculating square roots

Calculators and computers use a formula repeatedly to give an approximate decimal answer for the square root. The formula for calculating $\sqrt{M}$ is $x_{n+1} = \frac{1}{2} \left(x_n + \frac{M}{x_n} \right)$ where x_0 is the first guess and each calculation of the formula gives a better approximation than the last one.

For example, to calculate $\sqrt{500}$ using a first guess of $x_0 = 20$:

$$x_1 = \frac{1}{2} \left(20 + \frac{500}{20} \right) = 22.5$$

$$x_2 = \frac{1}{2} \left(22.5 + \frac{500}{22.5} \right) = 22.361\ 111\ 11$$

$$x_3 = \frac{1}{2} \left(22.36 + \frac{500}{22.36} \right) = 22.360\ 679\ 78$$

This process can continue endlessly, with the accuracy increasing each time.

$\therefore \sqrt{500} \approx 22.36$ correct to two decimal places.

- 1 **a** Estimate $\sqrt{55}$.
b Use the formula to evaluate $\sqrt{55}$ to three decimal places, using the iterative formula.
c Evaluate $\sqrt{55}$ on your calculator to check your answer.
- 2 **a** Estimate $\sqrt{700}$.
b Use the formula to evaluate $\sqrt{700}$ to four decimal places.
c Evaluate $\sqrt{700}$ on your calculator to check your answer.
- 3 Design a spreadsheet that uses the formula repeatedly to calculate square roots.

1-05 Adding and subtracting surds

Just as you can only add or subtract 'like terms' in algebra, you can only add or subtract 'like surds'.

Example 12

Simplify each expression.

a $4\sqrt{2} + 5\sqrt{2}$

b $7\sqrt{3} - 2\sqrt{3}$

c $5\sqrt{2} - 3\sqrt{3} + \sqrt{2}$

d $\sqrt{50} + \sqrt{32}$

e $\sqrt{8} - \sqrt{27} + \sqrt{18}$

f $5\sqrt{20} - 3\sqrt{125}$

Solution

a $4\sqrt{2} + 5\sqrt{2} = 9\sqrt{2}$

b $7\sqrt{3} - 2\sqrt{3} = 5\sqrt{3}$

c $5\sqrt{2} - 3\sqrt{3} + \sqrt{2} = 6\sqrt{2} - 3\sqrt{3}$

Stage 5.3

Puzzle sheet

Surds code puzzle

MAT09MGPS10008

Stage 5.3

$$\begin{aligned} \text{d } \sqrt{50} + \sqrt{32} &= \sqrt{25}\sqrt{2} + \sqrt{16}\sqrt{2} \\ &= 5\sqrt{2} + 4\sqrt{2} \\ &= 9\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{e } \sqrt{8} - \sqrt{27} + \sqrt{18} &= \sqrt{4}\sqrt{2} - \sqrt{9}\sqrt{3} + \sqrt{9}\sqrt{2} \\ &= 2\sqrt{2} - 3\sqrt{3} + 3\sqrt{2} \\ &= 5\sqrt{2} - 3\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{f } 5\sqrt{20} - 3\sqrt{125} &= 5\sqrt{4}\sqrt{5} - 3\sqrt{25}\sqrt{5} \\ &= 5 \times 2\sqrt{5} - 3 \times 5\sqrt{5} \\ &= 10\sqrt{5} - 15\sqrt{5} \\ &= -5\sqrt{5} \end{aligned}$$

Simplifying each surd.

Exercise 1-05 Adding and subtracting surds

See Example 12

1 Simplify each expression.

a $5\sqrt{7} + 2\sqrt{7}$

b $3\sqrt{2} - 8\sqrt{2}$

c $7\sqrt{5} - \sqrt{5}$

d $\sqrt{5} + 3\sqrt{5}$

e $5\sqrt{17} - 5\sqrt{17}$

f $3\sqrt{10} - 2\sqrt{10}$

g $4\sqrt{15} - 3\sqrt{15} + 7\sqrt{15}$

h $5\sqrt{6} - 2\sqrt{6} - 4\sqrt{6}$

i $3\sqrt{3} + 4\sqrt{3} - 5\sqrt{3}$

j $4\sqrt{5} + 7\sqrt{5} - \sqrt{5}$

k $8\sqrt{10} - 5\sqrt{10} + 3\sqrt{10}$

l $10\sqrt{3} - 3\sqrt{3} - 12\sqrt{3}$

2 Simplify each expression.

a $3\sqrt{5} - 8 + 2\sqrt{5}$

b $11\sqrt{10} + 3\sqrt{2} + 2\sqrt{10}$

c $-4\sqrt{3} + 5\sqrt{2} - 5\sqrt{3}$

d $3\sqrt{15} + 3\sqrt{2} + 4\sqrt{15} + 5\sqrt{2}$

e $\sqrt{7} - 3\sqrt{5} - 4\sqrt{7} + \sqrt{5}$

f $4\sqrt{6} - 3\sqrt{3} - 2\sqrt{6} - 5\sqrt{3}$

g $10\sqrt{11} - 5\sqrt{3} + 3\sqrt{11} + 4\sqrt{3}$

h $\sqrt{13} + 8\sqrt{7} - 7\sqrt{13} + 3\sqrt{7}$

i $2\sqrt{5} - 3\sqrt{7} - 2\sqrt{5} - 3\sqrt{7}$

j $4\sqrt{10} - 3\sqrt{5} - 4\sqrt{10}$

3 For each expression, select the correct simplified answer A, B, C or D.

a $\sqrt{3} + \sqrt{12}$

A $5\sqrt{3}$

B $\sqrt{15}$

C $2\sqrt{6}$

D $3\sqrt{3}$

b $4\sqrt{5} - 2\sqrt{125}$

A $-6\sqrt{5}$

B $\sqrt{5}$

C $-\sqrt{45}$

D $-46\sqrt{5}$

4 Simplify each expression.

a $\sqrt{8} + \sqrt{32}$

b $\sqrt{108} - \sqrt{27}$

c $\sqrt{20} - \sqrt{80}$

d $\sqrt{28} - \sqrt{63}$

e $3\sqrt{6} + \sqrt{24}$

f $2\sqrt{5} + \sqrt{125}$

g $\sqrt{40} - \sqrt{90}$

h $5\sqrt{11} + \sqrt{99}$

i $3\sqrt{2} + \sqrt{18}$

j $\sqrt{27} + 5\sqrt{3}$

k $\sqrt{200} - 7\sqrt{2}$

l $\sqrt{50} + \sqrt{32}$

m $5\sqrt{3} + 2\sqrt{27}$

n $3\sqrt{20} - \sqrt{245}$

o $7\sqrt{12} - 5\sqrt{48}$

p $4\sqrt{27} + 2\sqrt{243}$

q $3\sqrt{63} - 2\sqrt{28}$

r $2\sqrt{98} + 3\sqrt{162}$

s $-5\sqrt{6} + 2\sqrt{150}$

t $4\sqrt{50} + 3\sqrt{18}$

u $5\sqrt{27} - 6\sqrt{75}$

v $3\sqrt{112} - 2\sqrt{252}$

w $\sqrt{32} + \sqrt{8} + \sqrt{12}$

x $\sqrt{27} + \sqrt{54} + \sqrt{243}$

y $\sqrt{98} - 3\sqrt{20} - 2\sqrt{8}$

z $3\sqrt{96} - 2\sqrt{150} + \sqrt{24}$

1-06 Multiplying and dividing surds

Summary

The square root of products and quotients

For $x > 0$ and $y > 0$:

$$\sqrt{xy} = \sqrt{x} \times \sqrt{y}$$

$$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$$

Example 13

Simplify each expression.

a $\sqrt{3} \times \sqrt{5}$

b $\sqrt{10} \times \sqrt{6}$

c $3\sqrt{7} \times 5\sqrt{7}$

d $5\sqrt{27} \times 3\sqrt{6}$

e $\sqrt{54} \div -\sqrt{2}$

f $\frac{15\sqrt{32}}{5\sqrt{8}}$

Solution

a $\sqrt{3} \times \sqrt{5} = \sqrt{15}$

b $\sqrt{10} \times \sqrt{6} = \sqrt{60}$
 $= \sqrt{4} \times \sqrt{15}$
 $= 2\sqrt{15}$

c $3\sqrt{7} \times 5\sqrt{7} = 3 \times 5 \times \sqrt{7} \times \sqrt{7}$
 $= 15 \times 7$
 $= 105$

d $5\sqrt{27} \times 3\sqrt{6} = 5 \times 3 \times \sqrt{27} \times \sqrt{6}$
 $= 15\sqrt{162}$
 $= 15 \times \sqrt{81} \times \sqrt{2}$
 $= 15 \times 9\sqrt{2}$
 $= 135\sqrt{2}$

e $\sqrt{54} \div -\sqrt{2} = -\frac{\sqrt{54}}{\sqrt{2}}$
 $= -\sqrt{27}$
 $= -\sqrt{9} \times \sqrt{3}$
 $= -3\sqrt{3}$

f $\frac{15\sqrt{32}}{5\sqrt{8}} = 3\sqrt{4}$
 $= 3 \times 2$
 $= 6$

Stage 5.3

Example 14

Simplify $\frac{5\sqrt{2} \times 4\sqrt{12}}{10\sqrt{8}}$.

Solution

$$\begin{aligned}\frac{5\sqrt{2} \times 4\sqrt{12}}{10\sqrt{8}} &= \frac{20\sqrt{24}}{10\sqrt{8}} \\ &= 2\sqrt{3}\end{aligned}$$

Exercise 1-06 Multiplying and dividing surds

See Example 13

1 Simplify each expression.

a $\sqrt{7} \times \sqrt{2}$

b $-\sqrt{5} \times \sqrt{7}$

c $\sqrt{6} \times \sqrt{8}$

d $\sqrt{12} \times \sqrt{3}$

e $\sqrt{10} \times -\sqrt{5}$

f $3\sqrt{3} \times 5\sqrt{3}$

g $5\sqrt{10} \times 3\sqrt{3}$

h $-2\sqrt{7} \times 5\sqrt{3}$

i $7\sqrt{5} \times 4\sqrt{5}$

j $2\sqrt{3} \times -5\sqrt{6}$

k $4\sqrt{3} \times \sqrt{27}$

l $-3\sqrt{5} \times 4\sqrt{10}$

m $-7\sqrt{2} \times 4\sqrt{8}$

n $\sqrt{18} \times 8\sqrt{3}$

o $10\sqrt{2} \times 2\sqrt{8}$

p $3\sqrt{18} \times 5\sqrt{12}$

q $3\sqrt{44} \times -2\sqrt{99}$

r $5\sqrt{8} \times 4\sqrt{40}$

s $8\sqrt{3} \times 3\sqrt{54}$

t $-8\sqrt{32} \times \sqrt{27}$

u $\sqrt{90} \times \sqrt{72}$

v $-5\sqrt{20} \times 3\sqrt{8}$

w $7\sqrt{18} \times 3\sqrt{24}$

x $3\sqrt{48} \times 2\sqrt{42}$

2 Simplify each expression.

a $\sqrt{15} \div \sqrt{3}$

b $\sqrt{18} \div -\sqrt{6}$

c $\frac{6\sqrt{48}}{2\sqrt{8}}$

d $10\sqrt{54} \div 5\sqrt{27}$

e $-3\sqrt{98} \div 6\sqrt{14}$

f $\frac{7\sqrt{18}}{\sqrt{2}}$

g $2\sqrt{24} \div 4\sqrt{6}$

h $\frac{\sqrt{128}}{\sqrt{2}}$

i $15\sqrt{18} \div 3\sqrt{6}$

j $\frac{-20\sqrt{10}}{-4\sqrt{5}}$

k $36\sqrt{24} \div 9\sqrt{8}$

l $16\sqrt{30} \div 8\sqrt{5}$

m $12\sqrt{14} \div 6$

n $\frac{3\sqrt{2}}{-12}$

o $\sqrt{80} \div 4\sqrt{5}$

p $5\sqrt{60} \div \sqrt{15}$

q $6\sqrt{8} \div 3\sqrt{2}$

r $\frac{-42\sqrt{54}}{6\sqrt{3}}$

s $12\sqrt{63} \div 3\sqrt{7}$

t $\frac{8\sqrt{50}}{2\sqrt{200}}$

u $6\sqrt{3} \div \sqrt{243}$

See Example 14

3 Simplify each expression.

a $\frac{3\sqrt{5} \times 4\sqrt{2}}{3\sqrt{40}}$

b $\frac{3\sqrt{12} \times 8\sqrt{6}}{4\sqrt{27}}$

c $\frac{5\sqrt{8} \times 2\sqrt{90}}{10\sqrt{24}}$

d $\frac{4\sqrt{5}}{2\sqrt{15} \times 5\sqrt{27}}$

e $\frac{10\sqrt{686} \times 3\sqrt{12}}{5\sqrt{28} \times \sqrt{18}}$

f $\frac{8\sqrt{80} \times 3\sqrt{2}}{4\sqrt{5} \times 6\sqrt{8}}$

1-07 Pythagoras' theorem problems

Worksheet

Applications of
Pythagoras' theorem

MAT09MGWK100009

When using Pythagoras' theorem to solve problems, it is useful to follow these steps.

- Read the problem carefully
- Draw a diagram involving a right-angled triangle and label any given information
- Choose a variable to represent the length or distance you want to find
- Use Pythagoras' theorem to find the value of the variable
- Answer the question

Example 15

Find the value of y as a surd.

Solution

y is the length of a shorter side.

$$28^2 = y^2 + 12^2$$

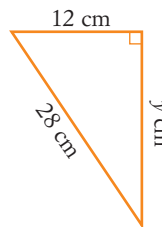
$$784 = y^2 + 144$$

$$y^2 + 144 = 784$$

$$y^2 = 784 - 144$$

$$= 640$$

$$y = \sqrt{640}$$



Example 16

A ship sails 80 kilometres south and then 45 kilometres east.
How far is it from its starting point, correct to one decimal place?

Solution

Let x be the distance the ship is from the starting point.

$$x^2 = 80^2 + 45^2$$

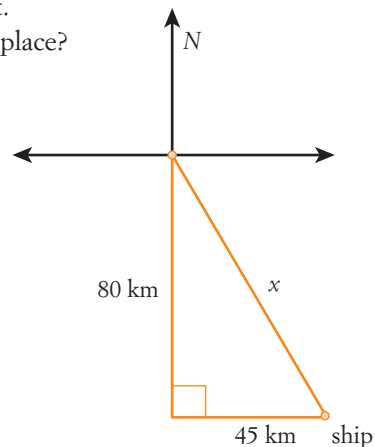
$$= 8425$$

$$x = \sqrt{8425}$$

$$= 91.7877 \dots$$

$$\approx 91.8 \text{ km}$$

From the diagram, this looks like a reasonable answer



Example 17

Find the perimeter of this triangle, correct to one decimal place.

Solution

Let x be the length of the hypotenuse.

$$x^2 = 10^2 + 21^2$$

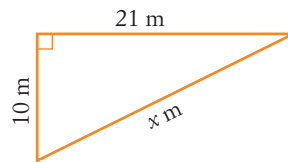
$$= 541$$

$$x = \sqrt{541}$$

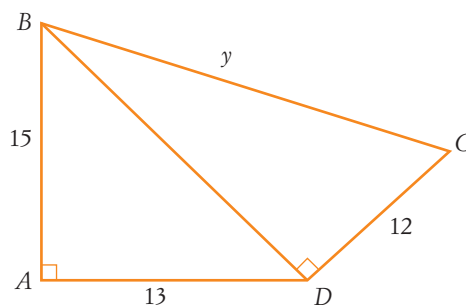
$$\approx 23.3 \text{ m}$$

$$\text{Perimeter} \approx 10 + 21 + 23.3$$

$$= 57.3 \text{ m}$$

**Example 18**

Find the value of y correct to two decimal places.

**Solution**

We need to find BD first.

In $\triangle ABD$,

$$BD^2 = 15^2 + 13^2$$

$$= 394$$

$$BD = \sqrt{394}$$

Leave BD as a surd (don't round) for further working.

In $\triangle BCD$,

$$y^2 = (\sqrt{394})^2 + 12^2$$

$$= 538$$

$$(\sqrt{394})^2 = 394$$

$$y = \sqrt{538}$$

$$= 23.1948 \dots$$

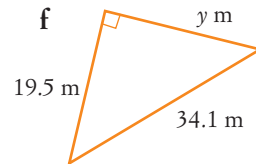
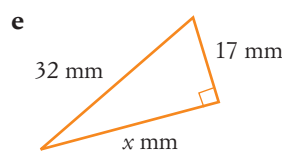
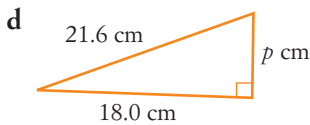
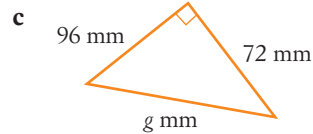
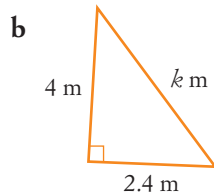
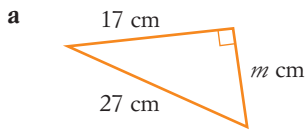
$$\approx 23.19$$

From the diagram, this looks like a reasonable answer.

Exercise 1-07 Pythagoras' theorem problems

- 1 Find the value of the pronumeral in each triangle. Give your answers correct to one decimal place.

See Example 15



- 2 A ladder 5 m long is leaning against a wall. If the base of the ladder is 2 m from the bottom of the wall, how far does the ladder reach up the wall? (Leave your answer as a surd.)



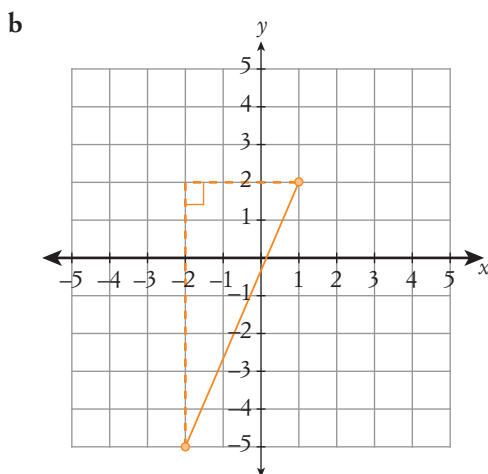
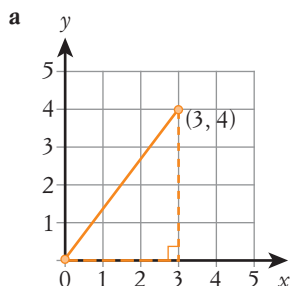
- 3 The size of a television screen is described by the length of its diagonal. If a flat screen TV is 58 cm wide and 32 cm high, what is the size of the screen? Answer to the nearest centimetre.



- 4 A ship sails 70 kilometres west and then 60 kilometres north. How far is it from its starting point, correct to one decimal place?

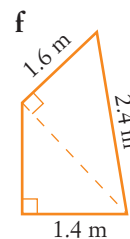
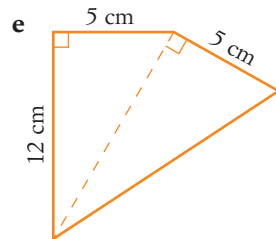
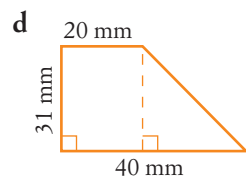
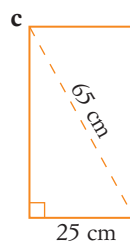
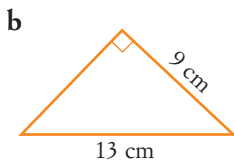
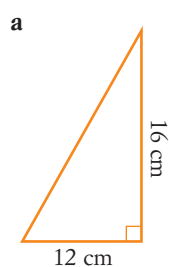
See Example 16

- 5 For each triangle drawn on the number plane, use Pythagoras' theorem to calculate the length of the hypotenuse. In part **b**, write the answer as a surd.

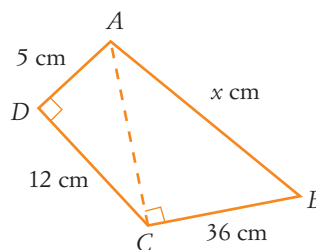


See Example 17

- 6 Calculate the perimeter of each shape, correct to one decimal place where necessary.



- 7 a Find the value of x correct to 1 decimal place.



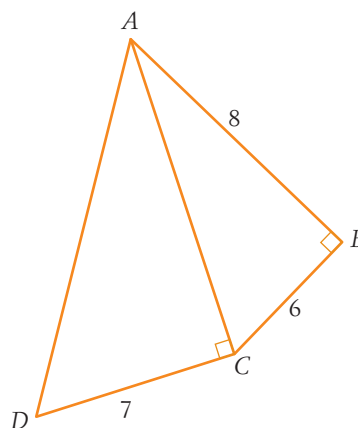
See Example 18

Worked solutions

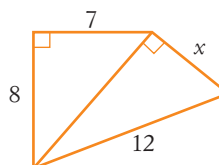
Mixed problems

MAT09MGWS10002

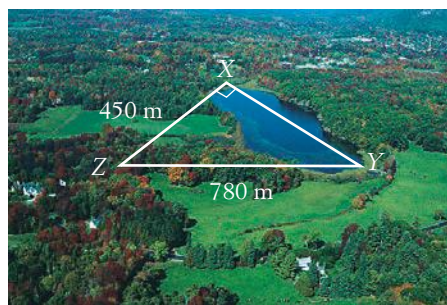
- b Find the length of AD correct to 2 decimal places.



- c Calculate the length of x as a surd.

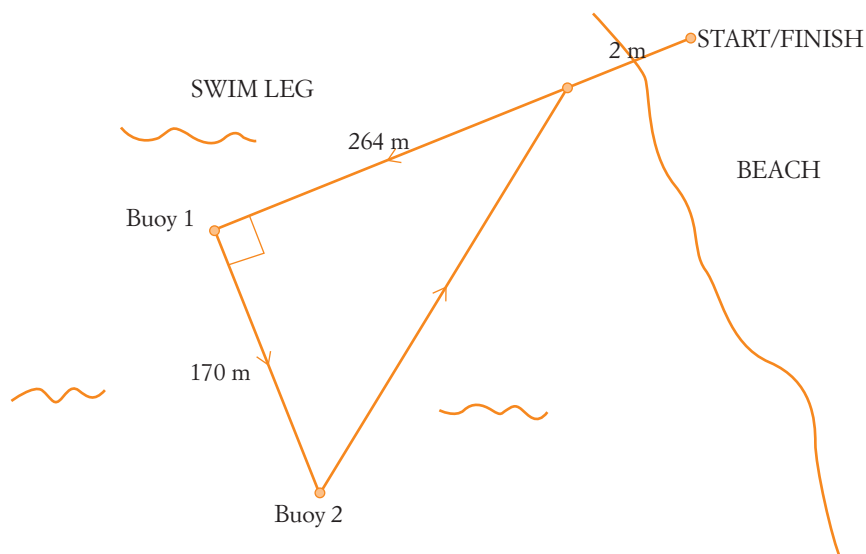


- 8 Cooper wanted to find the width XY of Lake Hartzel shown on the right. He placed a stake at Z so that $\angle YXZ = 90^\circ$. He measured XZ to be 450 m long and ZY to be 780 m long. What is the width of the lake? Answer correct to one decimal place.



Corbis/© Hubert Stadler

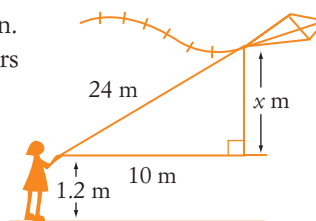
- 9 The swim course of a triathlon race has the shape of a right-angled triangle joined with a 2 metre link to the beach as shown below. Calculate the total distance covered in this course (starting and finishing on the beach).



- 10 A kite is attached to a 24 m piece of rope, as shown. The rope is held 1.2 m above the ground and covers a horizontal distance of 10 m.

Find:

- the value of x correct to one decimal place
- the height of the kite above the ground, correct to the nearest metre.



Worksheet

Pythagorean triads

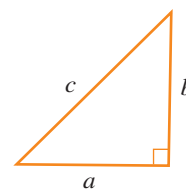
MAT09MGWK00056

1-08 Testing for right-angled triangles

Pythagoras' theorem can also be used to test whether a triangle is right-angled.

Pythagoras' theorem says that if a right-angled triangle has sides of length a , b , and c , then $c^2 = a^2 + b^2$.

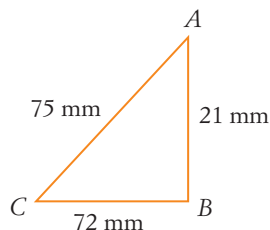
The reverse of this is also true.



If any triangle has sides of length a , b , and c that follow the formula $c^2 = a^2 + b^2$, then the triangle must be right-angled. The right angle is always the angle that is opposite the hypotenuse. This is called the **converse** (or opposite) of Pythagoras' theorem, because it is the 'back-to-front' version of the theorem.

Example 19

Test whether $\triangle ABC$ is right-angled.



Solution

$$75^2 = 5625$$

Square the longest side.

$$21^2 + 72^2 = 5625$$

Square the two shorter sides, then add.

$$\therefore 75^2 = 21^2 + 72^2$$

The sides of this triangle follow $c^2 = a^2 + b^2$

$\therefore \triangle ABC$ is right-angled.

The right angle is $\angle B$.

Video tutorial

Testing for right-angled triangles

MAT09MGVT10002

Example 20

Rahul constructed a triangle with sides of length 37 cm, 12 cm and 40 cm. Show that these measurements do not form a right-angled triangle.

Solution

$$40^2 = 1600$$

Square the longest side.

$$12^2 + 37^2 = 1513 \neq 1600$$

Square the two shorter sides, then add.

$$\therefore 40^2 \neq 12^2 + 37^2$$

The sides of this triangle do not follow $c^2 = a^2 + b^2$

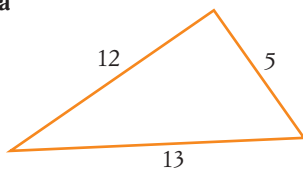
$\therefore$ The triangle is not right-angled.

Exercise 1-08 Testing for right-angled triangles

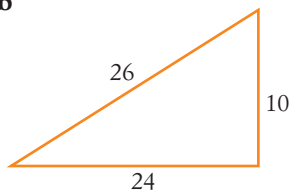
1 Test whether each triangle is right-angled.

See Example 19

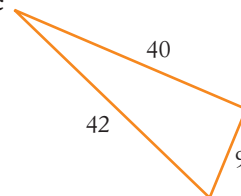
a



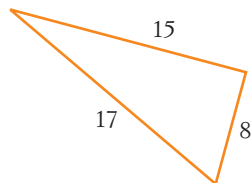
b



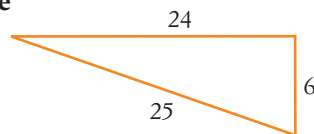
c



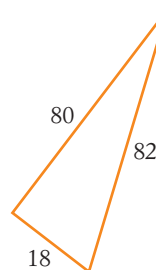
d

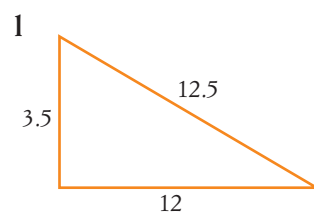
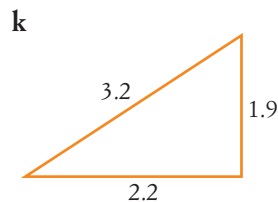
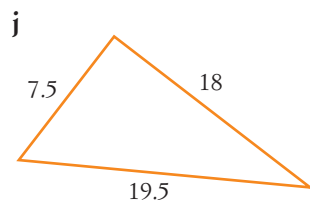
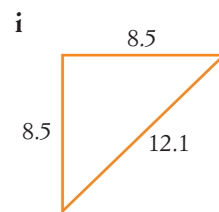
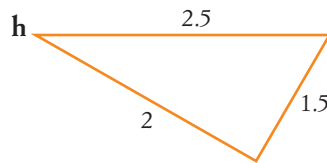
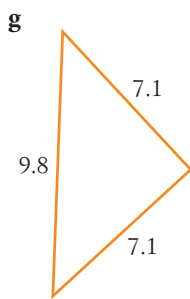


e



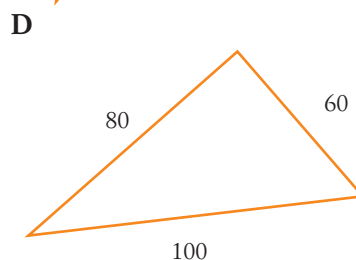
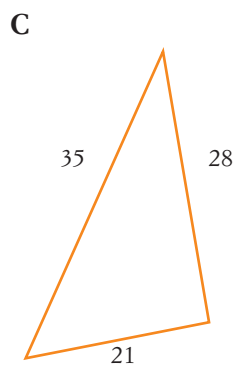
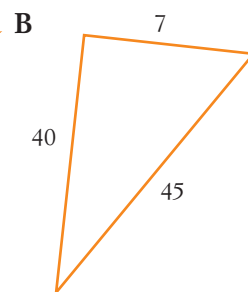
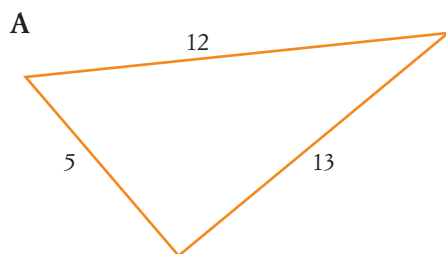
f





- See Example 20**
- Penelope constructed a triangle with sides of length 17 cm, 11 cm and 30 cm. Show that these measurements do *not* form a right-angled triangle.
 - Which set of measurements would make a right-angled triangle? Select the correct answer **A**, **B**, **C** or **D**.

A 2 cm, 3 cm, 4 cm	B 5 mm, 10 mm, 15 mm
C 12 cm, 16 cm, 20 cm	D 7 m, 24 m, 31 m
 - Which one of these triangles is not right-angled? Select **A**, **B**, **C** or **D**.



Mental skills 1B

Maths without calculators

Multiplying and dividing by a multiple of 10

1 Consider each example.

a $4 \times 700 = 4 \times 7 \times 100 = 28 \times 100 = 2800$

b $5 \times 60 = 5 \times 6 \times 10 = 30 \times 10 = 300$

c $12 \times 40 = 12 \times 4 \times 10 = 48 \times 10 = 480$

d $3.2 \times 30 = 3.2 \times 3 \times 10 = 9.6 \times 10 = 96$ (by estimation, $3 \times 30 = 90 \approx 96$)

e $4.6 \times 50 = 4.6 \times 5 \times 10 = 23 \times 10 = 230$ (by estimation, $5 \times 50 = 250 \approx 230$)

f $9.4 \times 200 = 9.4 \times 2 \times 100 = 18.8 \times 100 = 1880$ (by estimation,
 $9 \times 200 = 1800 \approx 1880$)

2 Now evaluate each product.

a 8×2000

b 3×70

c 11×900

d 2×300

e 4×4000

f 5×80

g 7×70

h 1.3×40

i 2.5×600

j 5.8×200

k 3.6×50

l 4.4×3000

3 Consider each example.

a $8000 \div 400 = 8000 \div 100 \div 4 = 80 \div 4 = 20$

b $200 \div 50 = 200 \div 10 \div 5 = 20 \div 5 = 4$

c $6000 \div 20 = 6000 \div 10 \div 2 = 600 \div 2 = 300$

d $282 \div 30 = 282 \div 10 \div 3 = 28.2 \div 3 = 9.4$

e $3520 \div 40 = 3520 \div 10 \div 4 = 352 \div 4 = 88$

f $8940 \div 200 = 8940 \div 100 \div 2 = 89.4 \div 2 = 44.7$

4 Now evaluate each quotient.

a $560 \div 70$

b $2500 \div 500$

c $3200 \div 400$

d $440 \div 20$

e $160 \div 40$

f $1500 \div 30$

g $450 \div 50$

h $744 \div 80$

i $2550 \div 300$

j $846 \div 200$

k $576 \div 60$

l $2160 \div 90$

Worksheet

Pythagorean triads

MAT09MGWK10010

Homework sheet

Pythagoras' theorem 2

MAT09MGHS10030

Homework sheet

Pythagoras' theorem
revision

MAT09MGHS10031

1-09 Pythagorean triads

A **Pythagorean triad** or **Pythagorean triple** is any group of three numbers that follow Pythagoras' theorem, for example, (3, 4, 5) or (2.5, 6, 6.5). The word **triad** means a group of three related items ('tri-' means 3).

Summary

(a, b, c) is a Pythagorean triad if $c^2 = a^2 + b^2$

Any multiple of (a, b, c) is also a Pythagorean triad.

Example 21

Test whether (5, 12, 13) is a Pythagorean triad.

Solution

$$13^2 = 169$$

$$5^2 + 12^2 = 169$$

$$\therefore 13^2 = 5^2 + 12^2$$

$\therefore$ (5, 12, 13) is a Pythagorean triad.

Squaring the largest number.

Squaring the two smaller numbers, and adding.

These three numbers follow Pythagoras' theorem.

Example 22

(3, 4, 5) is a Pythagorean triad. Create other Pythagorean triads by multiplying (3, 4, 5) by:

a 2

b 9

c $\frac{1}{2}$

Solution

a $2 \times (3, 4, 5) = (6, 8, 10)$

Checking: $10^2 = 100$

$$6^2 + 8^2 = 100$$

$$\therefore 10^2 = 6^2 + 8^2$$

$\therefore$ (6, 8, 10) is a Pythagorean triad.

b $9 \times (3, 4, 5) = (27, 36, 45)$

Checking: $45^2 = 2025$

$$27^2 + 36^2 = 2025$$

$$\therefore 45^2 = 27^2 + 36^2$$

$\therefore$ (27, 36, 45) is a Pythagorean triad.

c $\frac{1}{2} \times (3, 4, 5) = (1.5, 2, 2.5)$

Checking: $2.5^2 = 6.25$

$$1.5^2 + 2^2 = 6.25$$

$$\therefore 2.5^2 = 1.5^2 + 2^2$$

$\therefore$ (1.5, 2, 2.5) is a Pythagorean triad.

Exercise 1-09 Pythagorean triads

- Test whether each triad is a Pythagorean triad.
a (8, 15, 17) **b** (10, 24, 26) **c** (30, 40, 50)
d (5, 7, 9) **e** (9, 40, 41) **f** (4, 5, 9)
g (11, 60, 61) **h** (7, 24, 25) **i** (15, 114, 115)
- Which of the following is a Pythagorean triad? Select the correct answer **A**, **B**, **C** or **D**.
A (4, 6, 8) **B** (5, 10, 12) **C** (6, 7, 10) **D** (20, 48, 52)
- Use the spreadsheet you created in **Technology worksheet: Finding the hypotenuse** to check your answers to questions **1** and **2**.
- For each Pythagorean triad, create another Pythagorean triad by multiplying each number in the triad by:
i a whole number **ii** a fraction **iii** a decimal.
a (5, 12, 13) **b** (8, 15, 17) **c** (30, 40, 50) **d** (7, 24, 25)
 Check that each answer follows Pythagoras' theorem.
- Pythagoras developed a formula for finding Pythagorean triads (a , b , c). If one number in the triad is a , the formulas for the other two numbers are $b = \frac{1}{2}(a^2 - 1)$ and $c = \frac{1}{2}(a^2 + 1)$.
a If $a = 5$, use the formulas to find the values of b and c .
b Show that (a , b , c) is a Pythagorean triad.
- Use the formulas to find Pythagorean triads for each value of a .
a $a = 7$ **b** $a = 11$ **c** $a = 15$ **d** $a = 4$
e $a = 9$ **f** $a = 19$ **g** $a = 10$ **h** $a = 51$
- There are many other formulas for creating Pythagorean triads. Use the Internet to research some of them.

See Example 21

Technology worksheet

Finding the hypotenuse

MAT09MGCT10006

See Example 22

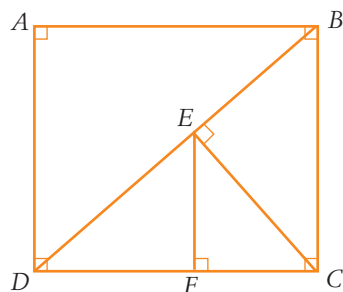
Worked solutions

Pythagorean triads

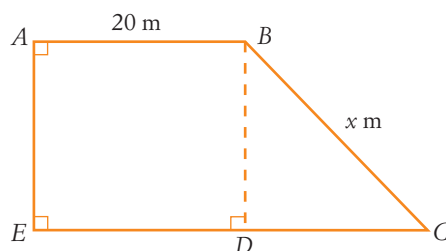
MAT09MGWS10003

Power plus

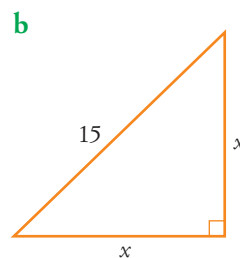
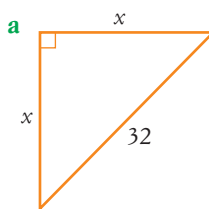
- 1 Name the five hypotenuses on this diagram.



- 2 The rectangle $ABDE$ in the diagram below has an area of 160 m^2 . Find the value of x as a surd if $\triangle BDC$ is an isosceles triangle.



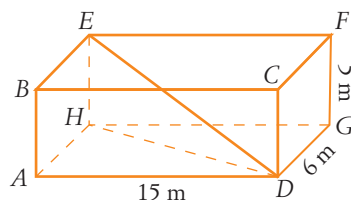
- 3 Find the value of x in each triangle, correct to one decimal place.



- 4 For this rectangular prism, find, correct to one decimal place, the length of diagonal:

a HD

b DE

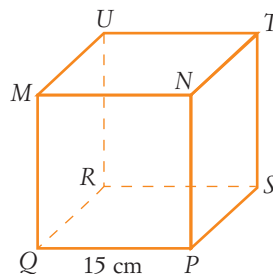


- 5 An interval is drawn between points $A(-2, 1)$ and $B(3, -2)$ on the number plane. Find the length of interval AB correct to one decimal place.

- 6 For this cube, find, correct to two decimal places, the length of diagonal:

a QS

b QT



Language of maths

area	converse	diagonal	formula
hypotenuse	irrational	perimeter	Pythagoras
right-angled	shorter	side	square root
surd	theorem	triad	unknown

Puzzle sheet

Pythagoras' theorem
find-a-word

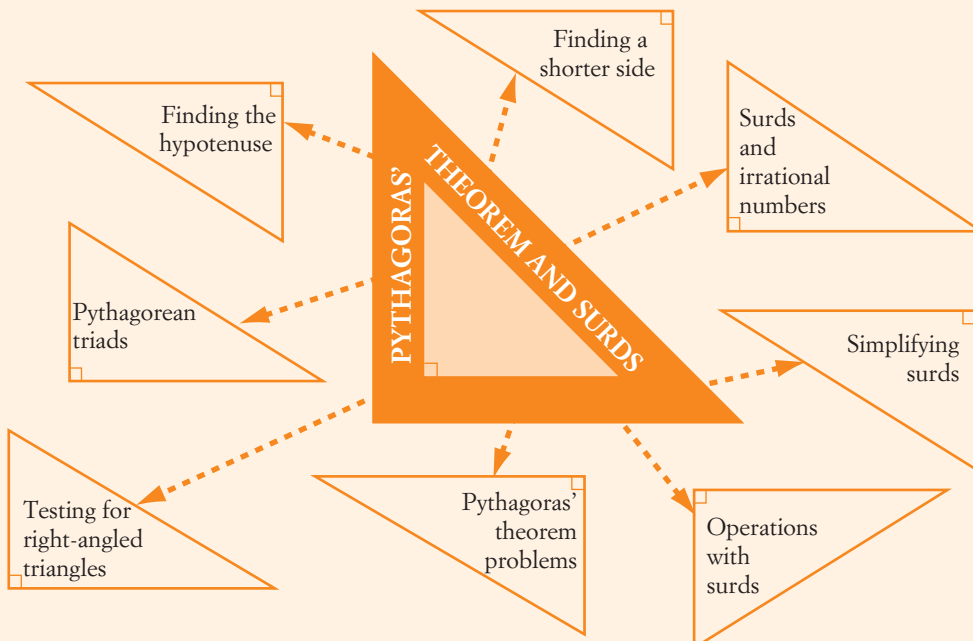
MAT09MGPS10011

- 1 Who was **Pythagoras** and what country did he come from?
- 2 Describe the **hypotenuse** of a right-angled triangle in two ways.
- 3 What is another word for 'theorem'?
- 4 For what type of triangle is **Pythagoras' theorem** used?
- 5 What is a **surd**?
- 6 What is the name given to a set of three numbers that follows Pythagoras' theorem?

Topic overview

- How relevant do you think Pythagoras' theorem is to our world? Give reasons for your answer.
- Give three examples of jobs where Pythagoras' theorem would be used.
- What did you find especially interesting about this topic?
- Is there any section of this topic that you found difficult? Discuss any problems with your teacher or a friend.

Copy (or print) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Worksheet

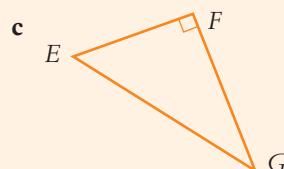
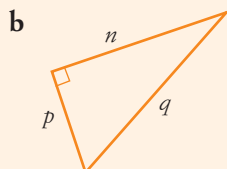
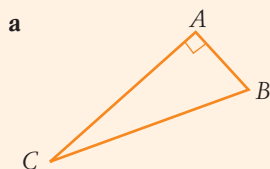
Mind map: Pythagoras'
theorem and surds
(Advanced)

MAT09MGWK10013

Chapter 1 revision

See Exercise 1-01

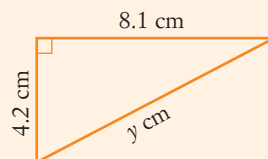
- 1 Write Pythagoras' theorem for each triangle.



See Exercise 1-01

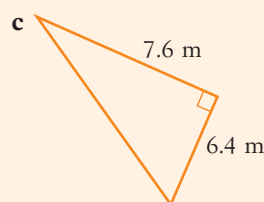
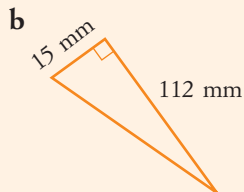
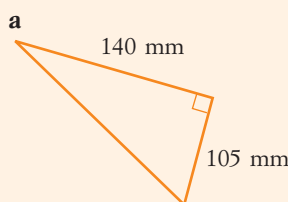
- 2 Find the value of y . Select the closest answer **A**, **B**, **C** or **D**.

A 12.3 **B** 9.1
C 6.9 **D** 3.5



See Exercise 1-01

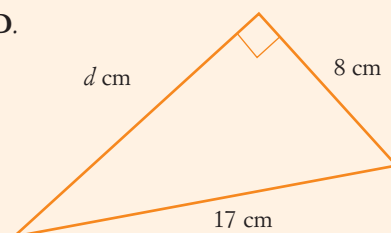
- 3 For each triangle, find the length of the hypotenuse. Give your answer as a surd where necessary.



See Exercise 1-02

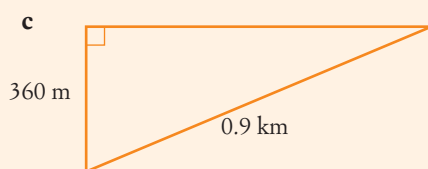
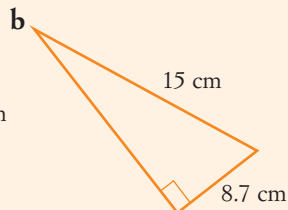
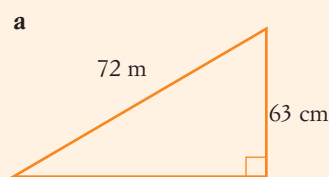
- 4 Find the value of d . Select the closest answer **A**, **B**, **C** or **D**.

A 12 m **B** 15 m
C 16.8 m **D** 18.9 m



See Exercise 1-02

- 5 For each triangle, find the length of the unknown side. Give your answer correct to one decimal place.



Stage 5.3

See Exercise 1-03

- 6 Is each number rational (R) or irrational (I)?

a $\sqrt{8}$ **b** $\frac{22}{7}$ **c** $0.5\dot{7}$ **d** $3\sqrt{5}$ **e** $\sqrt{81}$
f $\sqrt[3]{125}$ **g** $\sqrt[3]{-8}$ **h** $0.\dot{1}2\dot{3}$ **i** $5 + \sqrt{3}$

See Exercise 1-04

- 7 Simplify each surd.

a $\sqrt{72}$ **b** $\sqrt{98}$ **c** $\sqrt{275}$ **d** $\sqrt{128}$ **e** $3\sqrt{150}$
f $7\sqrt{28}$ **g** $4\sqrt{288}$ **h** $5\sqrt{45}$ **i** $7\sqrt{48}$

Stage 5.3

See Exercise 1-05

See Exercise 1-06

See Exercise 1-07

See Exercise 1-07

See Exercise 1-07

See Exercise 1-07

See Exercise 1-08

See Exercise 1-09

8 Simplify each expression.

a $8\sqrt{5} - \sqrt{5}$

c $6 + \sqrt{5} - 4 + 5\sqrt{5}$

e $\sqrt{200} + \sqrt{18}$

g $7\sqrt{32} - \sqrt{27} - 2\sqrt{98} + 4\sqrt{75}$

b $\sqrt{7} - 3\sqrt{2} + 2\sqrt{7}$

d $7\sqrt{7} - \sqrt{7} - 3\sqrt{7}$

f $3\sqrt{5} + \sqrt{50} - 2\sqrt{125}$

h $4\sqrt{45} - 3\sqrt{63} + 5\sqrt{80}$

9 Simplify each expression.

a $\sqrt{3} \times \sqrt{7}$

b $\sqrt{8} \times \sqrt{5}$

c $\sqrt{6} \times \sqrt{8}$

d $\sqrt{5} \times \sqrt{11}$

e $\sqrt{8} \times \sqrt{3}$

f $\sqrt{98} \div \sqrt{7}$

g $8\sqrt{42} \div 2\sqrt{7}$

h $\sqrt{125} \div 5\sqrt{5}$

i $4\sqrt{147} \div 12\sqrt{98}$

j $\frac{\sqrt{18} \times \sqrt{3}}{\sqrt{12}}$

k $\frac{\sqrt{6} \times \sqrt{24}}{\sqrt{27} \times 2\sqrt{3}}$

10 In $\triangle DEF$, $\angle F = 90^\circ$, $DF = 84$ cm, and $EF = 1.45$ m. Find DE correct to the nearest centimetre.

11 Julie walks 6 km due east from a starting point P , while Jane walks 4 km due south from P .

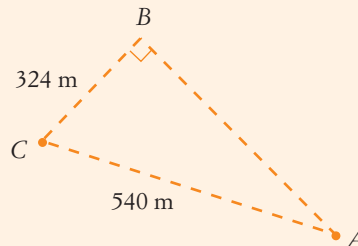
a Draw a diagram showing this information.

b How far are Julie and Jane apart? Give your answer correct to one decimal place.

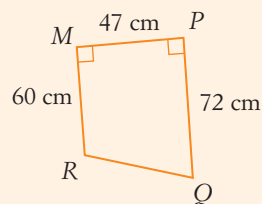
12 On a cross-country ski course, checkpoints C and A are 540 m apart and checkpoints C and B are 324 m apart. Find the distance:

a between checkpoints A and B

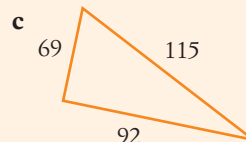
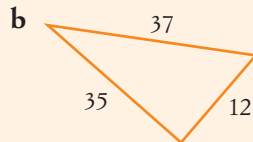
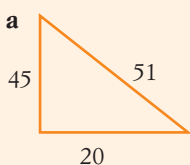
b skied in one lap of the course in kilometres.



13 Find the length of QR in this trapezium, correct to one decimal place.



14 Test whether each triangle is right-angled.



15 Test whether each triad is a Pythagorean triad.

a (7, 24, 25)

b (5, 7, 10)

c (20, 21, 29)

d (1.1, 6, 6.1)

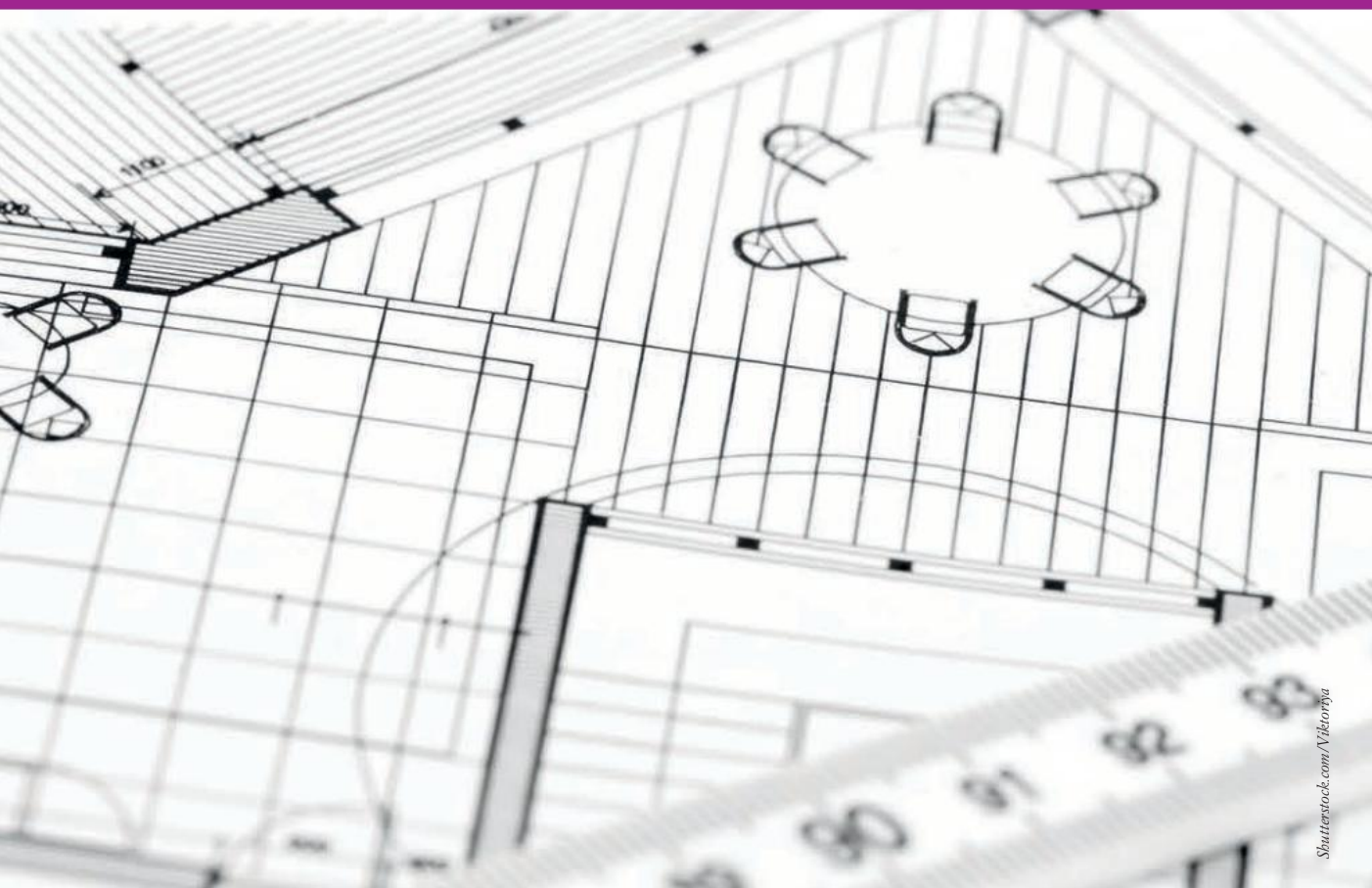


2

Number and algebra

Working with numbers

Numbers are an important part of our lives. We work with numbers when we shop for the best deal and when we borrow or invest money. Calculations are made when using recipes to cook and in the design and construction of houses. Whenever numbers are used to help us solve problems in our lives, calculations will need to be made.



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Chapter outline

	Proficiency strands			
2-01 Terminating and recurring decimals	U	F	R	C
2-02 Converting recurring decimals to fractions*	U	F	R	C
2-03 Operations with percentages		F	PS	C
2-04 Percentages and money		F	PS	C
2-05 Simple interest		F	PS	C
2-06 Ratios and rates	U	F	PS	C
2-07 Converting rates	U	F	R	C

*STAGE 5.3

Wordbank

cost price The price an item costs the retailer

GST Goods and services tax, a 10% tax added to the original price of an item or service

loss The amount lost when selling an item at a lower price than the cost price

per annum (p.a.) Per year

principal An amount of money invested or borrowed, on which interest is given or charged

profit The amount made when selling an item at a higher price than the cost price

recurring decimal A decimal with one or more digits that repeat endlessly, for example, $0.1\dot{6} = 0.1666\dots$

unitary method A method of finding a quantity by finding one part or 1% first

In this chapter you will:

- investigate terminating and recurring decimals
- solve problems involving the use of percentages, including percentage increases and decreases, with and without digital technologies
- solve a range of problems involving ratios and rates, with and without digital technologies 
- (STAGE 5.3) convert recurring decimals to fractions
- solve problems involving profit, loss, discounts, GST and simple interest
- convert between units for rates

SkillCheck

Worksheet

StartUp assignment 2

MAT09NAWK10014

Skillsheet

Integers

MAT09NASS10002

Skillsheet

Integers using diagrams

MAT09NASS10003

Skillsheet

Decimals

MAT09NASS10004

Homework sheet

Integers and decimals

MAT09NAHS10001

Worksheet

Where's the point?

MAT09NAWK10015

1 Evaluate each expression.

a $7 \times (-8)$

b $-120 \div (-6)$

c $7 - (-15)$

d $-10 \times 5 \times (-2)$

e $\frac{27}{-3}$

f $20 - 5 \times 8$

g $(-3)^2 + (-2)^2$

h $(-10 + 7) \times 4$

i $-24 \div 6 \times -5$

j $\frac{-100}{-5} \times 4$

k $-8 - (10 - 15)$

l $(-4)^3$

m $6 - (-30) \div 6$

n $40 + 8 \times (-9)$

o $(-5)^2 + 4 \times (-7)$

p $24 \div (-6) + (-5) \times 3$

q $12 - (4 \times 8)$

r $64 \div (-4)^2$

2 Write each set of decimals in ascending order.

a 0.7, 0.707, 7.007, 0.7007

b 0.202, 0.22, 0.02202, 0.002

3 Evaluate each expression.

a $0.45 \times 0.2 \times 0.1$

b $12.6 + 0.4 \times 7$

c $36.9 \div 3 \times 0.02$

4 Robyn had to calculate $36.024 \div 0.08$ without using a calculator. Which of these calculations will give her the same answer? Select **A**, **B**, **C** or **D**.

A $36\ 024 \div 8$

B $36.024 \div 8$

C $3602.4 \div 8$

D $0.360\ 24 \div 8$

5 Evaluate each expression.

a $\frac{2}{3} + \frac{4}{7}$

b $\frac{3}{5} + \frac{1}{2}$

c $\frac{5}{9} - \frac{1}{4}$

d $\frac{9}{10} - \frac{2}{3}$

e $\frac{5}{7} \times \frac{3}{4}$

f $\frac{1}{2} \times \frac{3}{5}$

g $6 \times \frac{3}{4}$

h $\frac{2}{3} \times 7$

i $\frac{3}{4} \div \frac{1}{2}$

j $\frac{2}{3} \div \frac{4}{5}$

k $\frac{3}{7} \div 5$

l $\frac{6}{7} \div \frac{2}{3}$

6 Express each number as a percentage.

a $\frac{1}{4}$

b 0.3

c 0.07

d $\frac{3}{5}$

7 Arrange $\frac{3}{5}$, 5%, 0.5 and $\frac{5}{9}$ in ascending order.

2-01 Terminating and recurring decimals

Worksheet

Fraction families

MAT09NAWK10016

Technology worksheet

Terminating and
recurring decimals

MAT09NACT10007

Recurring decimals have a pattern of numbers that repeat or recur. We use dots to indicate the digits that repeat. Here are some examples:

$$0.4444 \dots = 0.\dot{4}$$

$$0.588 \ 88 \dots = 0.5\dot{8}$$

$$0.636 \ 363 \dots = 0.\dot{6}\dot{3}$$

$$0.415 \ 415 \ 415 \dots = 0.\dot{4}1\dot{5}$$

Terminating decimals do not repeat (terminating means ‘stopping’). Here are some examples:
0.6 0.875 0.057

Some decimals are neither recurring nor terminating. They have an infinite number of digits but there is no pattern. These are **irrational numbers**. Here are some examples:

$$\pi = 3.141 \ 592 \ 653 \dots \quad \sqrt{3} = 1.732 \ 050 \ 808 \dots$$

To convert a fraction to a decimal, divide its numerator by its denominator.

Example 1

Convert each fraction to a decimal and state whether it is recurring or terminating.

a $\frac{5}{6}$

b $\frac{11}{40}$

Solution

a $\frac{5}{6} = 5 \div 6$

$$= 0.8333 \dots$$

$$= 0.8\dot{3} \quad (\text{a recurring decimal})$$

b $\frac{11}{40} = 11 \div 40$

$$= 0.275 \quad (\text{a terminating decimal})$$

Exercise 2-01 Terminating and recurring decimals

1 Write each recurring decimal using dot notation.

a $0.166 \ 666 \dots$

b $0.273 \ 273 \ 27 \dots$

c $0.888 \ 88 \dots$

d $0.209 \ 99 \dots$

2 Show the repeating pattern for each recurring decimal.

a $0.109\dot{8}$

b $0.1\dot{7}$

c $0.2\dot{5}$

d $8.\dot{3}1\dot{8}$

3 Convert each fraction to a decimal and state whether it is recurring or terminating.

a $\frac{7}{8}$

b $\frac{3}{11}$

c $4\frac{8}{9}$

d $\frac{13}{5}$

e $\frac{19}{25}$

f $\frac{2}{3}$

g $\frac{13}{20}$

h $1\frac{7}{12}$

i $8\frac{13}{40}$

j $\frac{17}{24}$

4 **a** Convert $\frac{1}{3}$ to a recurring decimal.

b Use your answer to part **a** to determine the value of:

i $0.\dot{6}$

ii $0.\dot{9}$

See Example 1

Converting recurring decimals to fractions

Example 2

b 0.54

Multiplying numerator and denominator by 10.

0.545454545454545454 **=** **S↔D**

Exercise 2-02 Converting recurring decimals to fractions

- Convert each recurring decimal to a simple fraction.

a $0.\dot{2}$	b $0.\dot{8}$	c $0.1\dot{5}$	d $0.3\dot{8}$	e $0.4\dot{6}$	f $0.\dot{6}\dot{5}$
g $0.\dot{3}\dot{1}$	h $0.6\dot{2}\dot{4}$	i $0.58\dot{3}$	j $0.\dot{8}3\dot{2}$	k $0.0\dot{3}$	l $0.\dot{7}\dot{5}$
- Convert $0.\dot{3}$ to a simple fraction.
 - Use your answer from part a to write $0.\dot{6}$ as a simple fraction.
- Convert $0.4\dot{5}$ to a simple fraction.
 - Use your answer from part a to write $0.\dot{9}\dot{0}$ as a simple fraction.

Stage 5.3

See Example 2

Skillsheet

Fractions, decimals and percentages

MAT09NASS10009

Homework sheet

Fractions and percentages

MAT09NAHS10002

Puzzle sheet

Percentages to fractions

MAT09NAPS00055

Puzzle sheet

Percentages of an amount

MAT09NAPS00056

Worksheet

Working with percentages

MAT09NAWK00070

Puzzle sheet

Fractions to percentages

MAT09NAPS00054

Skillsheet

Mental percentages

MAT09NASS10010

Technology worksheet

Excel worksheet: Percentages, fractions and decimals

MAT09NACT00029

Technology worksheet

Excel spreadsheet: Percentages, fractions and decimals

MAT09NACT00014

2-03 Operations with percentages

Percentage of a quantity

Example 3

Find:

- 12% of \$90
- $5\frac{1}{2}\%$ of \$540

Solution

$$\text{a } 12\% \times \$90 = \frac{12}{100} \times \$90 \quad \text{or} \quad 0.12 \times \$90 \\ = \$10.80$$

Entering the decimal 0.12 for 12% on a calculator is faster

$$\text{b } 5\frac{1}{2}\% \text{ of } \$540 = 5\frac{1}{2} \div 100 \times \$540 \quad \text{or} \\ 0.055 \times \$540 = \$29.70$$

Percentage increase and decrease

Example 4

- Increase \$140 by 6%.
- Decrease 96 kg by 15%.

Solution

$$\text{a } \text{Increase} = 6\% \times \$140 \\ = \$8.40$$

$$\therefore \text{New amount} = \$140 + \$8.40 \\ = \$148.40$$

$$\text{or New amount} = 106\% \text{ of } \$140 \\ = 1.06 \times \$140 \\ = \$148.40$$

$$100\% + 6\% = 106\%$$

- b Decrease = 15% of 96 kg
 = 14.4 kg
 $\therefore$ New amount = 96 kg – 14.4 kg
 = 81.6 kg
 or New amount = 85% of 96 kg $100\% - 15\% = 85\%$
 = 0.85×96 kg
 = 81.6 kg

Technology worksheet

Converting marks to percentages

MAT09NACT10008

Expressing quantities as percentages

Summary

To **express one quantity as a percentage of another**, write the appropriate fraction and then change it to a percentage:

$$\text{percentage} = \frac{\text{amount}}{\text{whole amount}} \times 100\% \text{ or } \text{amount} \div \text{whole amount} \times 100\%$$

Example 5

- a Jamila scores 43 out of 60 in a History test. What is her percentage mark to the nearest whole number?
 b What percentage (correct to one decimal place) is 35 s of 5 min?

Solution

a Percentage mark = $\frac{43}{60} \times 100\%$
 = 71.666... %
 $\approx 72\%$

b Percentage = $\frac{35 \text{ s}}{5 \text{ min}} \times 100\%$
 = $\frac{35 \text{ s}}{300 \text{ s}} \times 100\%$ $5 \text{ min} = 5 \times 60 \text{ s} = 300 \text{ s}$
 = 11.666... %
 $\approx 11.7\%$

The unitary method

The **unitary method** can be used to find the whole amount (100%) when part of the amount is known. 'Unit' means 'one' and the unitary method involves finding 1% first.

Summary

Given a percentage of an amount, to find the whole amount:

- 1 find 1% of the amount by dividing by the known percentage
- 2 multiply by 100 to find the whole amount (100%).

Example 6

This week, Mario received a bonus of \$86 which was 9% of his weekly wage. Calculate Mario's weekly wage correct to the nearest dollar.

Solution

$$\begin{aligned}
 9\% \text{ of the wage} &= \$86 \\
 \therefore 1\% \text{ of the wage} &= \$86 \div 9 \\
 &= \$9.555 \dots \\
 \therefore \text{Mario's wage} &= \$9.55 \dots \times 100 && \text{100\% of the wage.} \\
 &= \$955.555 \dots \\
 &\approx \$956
 \end{aligned}$$

Exercise 2-03 Operations with percentages

See Example 3

- 1 Find:
 - a 15% of \$85
 - b 120% of 60 kg
 - c 45% of 16 m
 - d 2.4% of \$350
 - e $5\frac{1}{4}\%$ of \$80
 - f 12% of 95 kg
 - g $2\frac{1}{2}\%$ of \$870
 - h 150% of 16 L
 - i 3.75% of \$2850
 - j $33\frac{1}{3}\%$ of \$6540
 - k 6.25% of 25 t
 - l 0.4% of \$3720
- 2 A total of 84 600 people attended the State of Origin match between NSW and Queensland in Sydney. If 71% of the crowd supported NSW:
 - a what percentage of the crowd supported Queensland?
 - b how many people in the crowd supported Queensland?
- 3 980 students attend Marabvale High School. If 19% of them are in Year 9, calculate the number of Year 9 students.
- 4 Buzz Coffee advertises an extra 20% of coffee for the same price. If the normal amount is 700 g, how much extra coffee will you get for the same price?
- 5 Jakub's weight is now 125% of what it was a year ago. If his weight then was 85 kg, what is his weight now?



Newspix/Mark Evans

See Example 4

- 6 Increase:
- | | | |
|-----------------|-----------------|-----------------|
| a \$455 by 10% | b \$80 by 15.5% | c \$680 by 200% |
| d 80 kg by 2.5% | e 35 m by 30% | f 120 L by 140% |
- 7 Decrease:
- | | | |
|----------------|-----------------|------------------|
| a \$830 by 35% | b 120 min by 5% | c 90 L by 12% |
| d 6.8 m by 25% | e 130 kg by 25% | f \$96.50 by 10% |
- 8 Matt buys a bike for \$860 and sells it, making a profit of 5%.
- How much profit did Matt make?
 - For what price did Matt sell the bike?
- 9 T-shirts priced at \$38 are reduced by 45% at an end-of-summer sale. What is the sale price of the T-shirts?
- 10 Due to severe storms in summer, the price of fresh fruit and vegetables increased by 34%. How much will broccoli cost if its original price was \$4.99/kg?
- 11 Lucy's weekly wage of \$895 increased by 5.5%. Find her new weekly wage.
- 12 A car dealer is offering new season sales. What will you pay for a new car marked at \$16 500 if a discount of 12% is given?

See Example 5

- 13 Express each proportion as a percentage, correct to one decimal place if necessary.
- | | | |
|------------------------|---------------------|---------------------|
| a 55 out of 80 | b 80 out of 120 | c 45 out of 70 |
| d 15 out of 18 | e \$8 out of \$12 | f \$9 out of \$60 |
| g 35 s out of 4 min | h 15 min out of 2 h | i 340 mm out of 2 m |
| j 3 hours out of 1 day | k 85 cm out of 5 m | l 140c out of \$10 |
- 14 Jackie achieved the following marks in two tests:
- English: 32 out of 40 Mathematics: 47 out of 60
- Convert each mark to a percentage to decide which is the better mark.
- 15 Chris earns \$840 per week, of which \$280 goes to rent. Find what percentage of Chris' wage goes to rent, correct to one decimal place.
- 16 Bozena earns \$480 per week while she is completing Year 12 at school. If she spends \$95 of it on petrol, what percentage of her wage is this, correct to two decimal places?

See Example 6

- 17 Reece received a 15% discount on the market price of a sound system. If this was a saving of \$165, what was the market price of the sound system, correct to the nearest dollar?
- 18 Alla pays \$254.40 each week towards her home loan. If this amount is 24% of Alla's weekly wage, calculate her weekly wage to the nearest cent.
- 19 A jeweller charges \$105 for valuing a diamond ring. If this charge represents 1.25% of the value of the ring, find this value correct to the nearest dollar.
- 20 A can of energy food drink contains 20% more for the same price as the standard can. If the extra amounts to 150 g, how much does the standard can contain?
- 21 Last year, Ben paid 28% of his salary in tax. If he paid \$23 423 in tax, what was Ben's salary? Answer to the nearest dollar.

Just for the record

Blood: it takes all types

The blood groups in Australia are as follows.

O		A		B		AB	
+	−	+	−	+	−	+	−
40%	9%	31%	7%	8%	2%	2%	1%

- 1 If Australia's population is 26 million, how many people would be in each blood group?
- 2 Do you know what your blood group is? How would you find out?



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2-04 Percentages and money

Profit and loss

- When an item is sold for **more** than it costs, a **profit** is made.
- When an item is sold for **less** than it costs, a **loss** is made.
- The percentage profit and loss are usually calculated as a percentage of the **cost price**.

Example 7

Jackson Digital buys notebook computers for \$550 each and sells them for \$680. Find:

- a the profit on each computer
- b the percentage profit on the cost price, correct to one decimal place.

Solution

$$\begin{aligned} \text{a Profit} &= \$680 - \$550 \\ &= \$130 \end{aligned}$$

$$\begin{aligned} \text{b Percentage profit} &= \frac{\text{profit}}{\text{cost price}} \times 100\% \\ &= \frac{130}{550} \times 100\% \\ &= 23.6363 \dots \\ &\approx 23.6\% \end{aligned}$$

Worksheet

Profit and loss

MAT09NAWK10017

Puzzle sheet

Percentage profit or loss

MAT09NAPS00057

Technology worksheet

Profit and loss

MAT09NACT10001

Technology worksheet

Profit and loss 2

MAT09NACT10009

Worksheet

Discounts and special offers

MAT09NAWK10018

Discounts

Example 8

At *Jeans Minus*' end-of-season sale, jeans are discounted by 25%. What is the discount price of a pair of jeans marked at \$148?

Solution

Method 1

$$\begin{aligned}\text{Discount} &= 25\% \text{ of } \$148 \\ &= \$37\end{aligned}$$

$$\begin{aligned}\text{Discount price} &= \$148 - \$37 \\ &= \$111\end{aligned}$$

Method 2

$$\begin{aligned}\text{Discount price} &= 75\% \text{ of } \$148 \\ &= \$111\end{aligned}$$

Example 9

A TV system costs \$865 after a 24% discount. What was its original price, rounded to the nearest dollar?

Solution

$$\text{Sale price} = \$865, \text{ discount} = 24\%$$

$$\therefore 76\% \text{ of the original price} = \$865$$

$$\begin{aligned}1\% &= \frac{\$865}{76} \\ &= \$11.381\,578\dots\end{aligned}$$

$$\begin{aligned}\therefore \text{Original price} &= \$11.381\,578\dots \times 100 \\ &= \$1138.157\dots \\ &\approx \$1138\end{aligned}$$

$$100\% - 24\% = 76\%$$

Using the unitary method.

To find 100%.

GST

Goods and Services Tax (GST) is a tax paid to the government on most goods (items) and services that we purchase. In Australia, GST is charged at 10% of the original price and is generally included in the marked price of the good or service.

Example 10

The selling price of a lounge suite is \$2695 with GST included. How much of this price is the GST?

Solution

$$\text{Selling price} = 100\% + 10\% = 110\%$$

$$110\% \text{ of the original price} = \$2695$$

$$\begin{aligned} 10\% \text{ GST} &= \$2695 \div 11 \\ &= \$245 \end{aligned}$$

Exercise 2-04 Percentages and money

In this exercise, express all percentages correct to one decimal place and all prices to the nearest cent.

- 1 A home theatre system with a cost price of \$650 is sold for \$1250. Find:
 - a the profit
 - b the percentage profit on the cost price.
- 2 A motorbike originally priced at \$16 000 is sold for \$14 500. Find:
 - a the loss
 - b the percentage loss
- 3 Marcella bought an old car for \$3000, spent another \$1500 on repairs and then sold it for \$6200. Find:
 - a the total amount that Marcella spent on the car
 - b the profit made when she sold the car
 - c her percentage profit.
- 4 A house was bought for \$850 000. A year later, it was sold for \$720 000. Calculate:
 - a the loss
 - b the percentage loss
- 5 Towels with a cost price of \$25 each are sold for \$35. Calculate the percentage profit.
- 6 A car costing \$12 000 is sold for \$8000. Calculate the percentage loss.
- 7 *Home Necessities* store buys cutlery sets for \$120 that are then sold for \$230. Calculate the percentage profit.
- 8 GIA supermarket reduces the price of some items by 35%. Find the discount price of the following items given their original prices.

a 250 g butter, \$2.40	b 2 L milk, \$3
c 1 kg cheese, \$11	d 24 cans of soft drink, \$15.99
- 9 Calculate the selling price of a computer marked at \$760, now discounted by 15%. Select the correct answer **A**, **B**, **C** or **D**.

A \$114	B \$660	C \$646	D \$636
- 10 The price of a punnet of strawberries marked at \$4.95 was reduced by 25%. Calculate its discounted price. Select the correct answer **A**, **B**, **C** or **D**.

A \$4.74	B \$3.74	C \$3.71	D \$1.25
- 11 At a book sale, all books are discounted by 35%. What is the discount price of a book marked at \$25.50?

See Example 7

See Example 8

- 12 A slightly damaged washing machine has its price reduced from \$980 to \$750. What is the percentage discount?
- 13 A TV priced at \$1599 is sold for \$1250. Find the discount as a percentage of the marked price.
- 14 Calculate the percentage discount for each of the following.

a



b



See Example 9

- 15 A jacket discounted by 40% after Christmas sells for \$84. What was the original price of the jacket?
- 16 A bedroom suite reduced by 15% is sold for \$2801. What was its original price? Answer to the nearest dollar.
- 17 Running shoes are sold to make a profit of 85%. If they sold for \$165, what was their cost price? Select the correct answer **A**, **B**, **C** or **D**.
A \$140 **B** \$89 **C** \$76 **D** \$65
- 18 A plane fare from Sydney to Brisbane has been discounted by 45%. If Amy paid \$159 for a discounted ticket, what was the full price?
- 19 For each item, calculate the GST payable and the final price.
a a car priced at \$20 900 **b** a home entertainment unit priced at \$1810
c a theatre ticket priced at \$145 **d** plumber's fees of \$180

See Example 10

- 20 Given the selling price of each item with GST included, calculate the GST and the original price correct to the nearest cent.
a haircut: \$20.90 **b** car repairs: \$739.20
c photo frame: \$31.35 **d** pair of jeans: \$65.89
e refrigerator: \$924 **f** piano lessons: \$198 per term

Worked solutions

Percentages and money

MAT09NAWS10006

- 21 A discount warehouse has a 30% discount on all computers with a further 5% off the discount price if you pay in cash. Carlos wants to buy a computer priced at \$1899.
a What is the discount price of the computer?
b Carlos pays by cash. How much will he need to pay now?
c What is the total discount Carlos received?
d Calculate his total percentage discount (on the original cost).
- 22 Justine buys 20 boxes of paper at \$29.95 a box.
a What is the cost of the 20 boxes of paper?
b Justine receives a business discount of 15%. How much will she need to pay?
c Justine pays by cash and receives a further discount of 5%. How much will she now pay?
d Find the amount Justine would pay if she received a single discount of 20% on the cost of the paper.
e Is the single discount of 20% the same as the two successive discounts of 15% and 5%? Give reasons for your answer.

Mental skills 2A

Maths without calculators

Squaring a number ending in 5, 1 or 9

The square of a number ending in 5 always ends in 25.

For example, $35^2 = 1225$ and $105^2 = 11\,025$.

A mental calculation trick requires three easy steps:

- delete the 5 from the number
- multiply the remaining number by the next consecutive number
- write '25' at the end of the product.

1 Study each example.

a To calculate 35^2 :

- deleting the 5 from 35 leaves 3
- multiply 3 by the next consecutive number: $3 \times 4 = 12$
- write '25' at the end: 1225

$$35^2 = 1225$$

b To calculate 105^2 :

- deleting the 5 from 105 leaves 10
- multiply 10 by the next consecutive number: $10 \times 11 = 110$
- write 25 at the end: 11 025

$$105^2 = 11\,025$$

2 Now calculate each square number.

a 25^2	b 55^2	c 45^2	d 85^2
e 115^2	f 7.5^2	g 95^2	h 195^2
i 1.5^2	j 65^2	k 155^2	l 245^2

The square of a number ending in 1 always ends in 1.

For example, $41^2 = 1681$ and $71^2 = 5041$.

A mental calculation trick requires three steps:

- round the number to the nearest 10 (by subtracting 1) to make a new number
- square the new number
- add to your answer the new number and its next consecutive number

3 Study each example.

a To calculate 41^2 :

- round 41 down to 40
- square 40: $40^2 = 1600$
- add 40 and 41: $1600 + 40 + 41 = 1681$

$$41^2 = 1681$$

b To calculate 71^2 :

- $70^2 = 4900$ Rounding down and squaring
- $4900 + 70 + 71 = 5041$

$$71^2 = 5041$$

4 Now calculate each square number.

a 21^2

b 101^2

c 31^2

d 91^2

e 5.1^2

f 81^2

g 61^2

h 201^2

i 1.1^2

j 4.1^2

The square of a number ending in 9 also ends in 1.

For example, $29^2 = 841$ and $99^2 = 9801$.

A mental calculation trick requires three steps:

- round the number to the nearest 10 (by adding 1) to make a new number
- square the new number
- subtract from your answer the new number and its *previous* consecutive number

5 Study each example.

a To calculate 29^2 :

- round up to 30
- square 30: $30^2 = 900$
- subtract 30 and 29: $900 - 30 - 29 = 841$

$$29^2 = 841$$

b To calculate 99^2 :

- round up to 100
- square 100: $100^2 = 10\,000$
- $10\,000 - 100 - 99 = 9801$

$$99^2 = 9801$$

6 Now calculate each square number.

a 59^2

b 69^2

c 89^2

d 19^2

e 109^2

f 4.9^2

g 79^2

h 11.9^2

i 39^2

j 9.9^2

Worksheet

Simple interest

2-05 Simple interest

MAT09NAWK10019

Banks, credit unions and other financial institutions reward investors by paying them interest on their savings or investments. Conversely, they charge borrowers by making them pay interest on their loans.

- The original amount of money invested or borrowed is called the **principal**.
- **Interest** is calculated as a percentage of the principal.
- This percentage is called the **interest rate**, usually written as a rate **per annum** ('per year'), abbreviated 'p.a.'
- **Simple interest** (or **flat rate interest**) is interest calculated simply on the original principal.

Example 11

Find the simple interest earned on an investment of \$18 000 at 3.5% p.a. for 5 years.

Solution

Principal = \$18 000

Interest rate = 3.5% p.a.

Term = 5 years

Term means the amount of time the investment or loan is for

Over 5 years.

$$\begin{aligned}\text{Interest} &= 3.5\% \times \$18\,000 \times 5 \\ &= \$3150\end{aligned}$$

The simple interest formula

Simple interest is calculated using the following formula.

Summary

$I = PRN$, where

I is the simple interest

P is the principal

R is the interest rate per year, expressed as a decimal and

N is the number of years

Applying this to Example 11 above,

$P = \$18\,000$, $R = 3.5\% = 0.035$ p.a., $N = 5$ years

$$\begin{aligned}I &= PRN \\ &= \$18\,000 \times 0.035 \times 5 \\ &= \$3150\end{aligned}$$

Example 12

Find the simple interest on:

- a** \$8620 at 2.4% p.a. for 7 months **b** \$5600 at 6.25% p.a. for 220 days

Solution

a $P = \$8620$, $R = 2.4\% = 0.024$ p.a., $N = 7$ months = $\frac{7}{12}$ years

$$\begin{aligned}I &= PRN \\ &= \$8620 \times 0.024 \times \frac{7}{12} \\ &= \$120.68\end{aligned}$$

Video tutorial

Simple interest

MAT09NAVT10003

$$\text{b } P = \$5600, R = 6.25\% = 0.0625, N = 220 \text{ days} = \frac{220}{365} \text{ years}$$

$$I = PRN$$

$$= \$5600 \times 0.0625 \times \frac{220}{365}$$

$$= \$210.9589 \dots$$

$$\approx \$210.96$$

Rounded to the nearest cent.

Video tutorial

Simple interest

MAT09NAVT10003

Example 13

After 2 years, an investment of \$1560 has earned \$87.36 in simple interest. What is the annual interest rate?

Solution

$$I = \$87.36, P = \$1560, N = 2 \text{ years}$$

$$I = PRN$$

$$\$87.36 = \$1560 \times R \times 2$$

$$= \$3120R$$

$$R = \frac{\$87.36}{\$3120}$$

$$= 0.028$$

$$= 2.8\%$$

$$\therefore \text{Annual interest rate} = 2.8\%$$

Example 14

For how long will \$12 000 need to be invested to earn \$1350 if the flat rate of interest is 4.5% p.a.?

Solution

$$I = \$1350, P = \$12\,000, N = 4.5\% = 0.045 \text{ p.a.}$$

$$I = PRN$$

$$1350 = 12\,000 \times 0.045 \times N$$

$$1350 = 540N$$

$$N = \frac{1350}{540}$$

$$= 2.5 \text{ years}$$

Exercise 2-05 Simple interest

In this exercise, give all money answers correct to the nearest cent.

- 1 Calculate the simple interest earned on each investment.

a \$18 000 at 7% p.a. for 5 years

c \$3000 at 5.5% p.a. for 4 years

e \$6800 at 6.2% p.a. for 6 years

b \$5280 at 2.85% p.a. for 2 years

d \$9450 at $2\frac{1}{2}\%$ p.a. for 3 years

f \$12 500 at 7.3% p.a. for 2 years
- 2 Steffi invests \$17 000 at 6.45% p.a. simple interest for 4 years. To what final value will her investment grow?
- 3 Calculate the simple interest for each situation.

a \$2100 invested at 6% p.a. for 9 months

b \$3450 invested for 5 months at 3.25% p.a.

c \$10 000 invested at 6.2% p.a. for 150 days

d \$4000 borrowed for 14 months at 6.55% p.a.

e \$280 000 borrowed at 8.3% p.a. for 30 weeks

f \$18 400 borrowed for 160 days at 14.7% p.a.
- 4 Find the annual interest rate if:

a an investment of \$3000 earns \$756 simple interest after 4 years

b an investment of \$17 630 earns \$2715.02 simple interest after 2 years

c a flat rate loan of \$9000 is charged \$2115 after 5 years.
- 5 For how long will \$12 350 need to be invested to earn \$1778.40 if the flat rate of interest is 2.4% p.a.?
- 6 How many months will it take for an investment of \$22 000 to earn \$12 650 in interest if the interest rate is 6.9% p.a.?
- 7 How many days will it take for \$15 600 to earn \$436.97 in interest if the flat rate of interest is 7.1%?
- 8 William took 2 years to pay off a flat rate loan of \$7000. His total loan repayments amounted to \$8400. Calculate:

a the interest charged

b the interest rate
- 9 Simone and Jeff borrowed \$4000 to build a garden in their backyard. They repaid the loan at \$128 per month for 4 years.

a Calculate the total amount they paid for the garden.

b How much interest did they pay on the loan?

c Calculate the flat rate of interest charged.

See Example 11

See Example 12

See Example 13

See Example 14



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Worked solutions

Simple interest

MAT09NAWS10007

Skillsheet

Ratios

MAT09NASS10011

Homework sheet

Percentages, ratios
and rates

MAT09NAHS10003

2-06 Ratios and rates

Simplifying ratios

Example 15

Simplify each ratio.

a $36 : 48 : 18$

b $425 \text{ mL to } 5 \text{ L}$

Solution

$$\begin{aligned} \text{a } 36 : 48 : 18 &= \frac{36}{6} : \frac{48}{6} : \frac{18}{6} \\ &= 6 : 8 : 3 \end{aligned}$$

Dividing all terms by 6, their HCF.

$$\begin{aligned} \text{b } 425 \text{ mL} : 5 \text{ L} &= 425 \text{ mL} : 5000 \text{ mL} \\ &= 425 : 5000 \\ &= \frac{425}{25} : \frac{5000}{25} \\ &= 17 : 200 \end{aligned}$$

Expressing both ratios in the same units.

Dividing both terms by 25, their HCF.

Ratio problems

Ratio problems can be solved using **equivalent ratios** or the **unitary method**. The unitary method requires finding one part first.

Example 16

During summer, the ratio of jeans to shorts sold at a beachside store was $3 : 10$. If 240 pairs of jeans were sold, how many pairs of shorts were sold?

Solution

Method 1: Equivalent ratios

$$\text{Jeans : shorts} = 3 : 10 = 240 : \underline{\hspace{2cm}}$$

$$\text{Since } 3 \times 80 = 240,$$

$$\begin{aligned} \text{Number of shorts} &= 10 \times 80 \\ &= 800 \end{aligned}$$

Method 2: Unitary method

$$\text{Jeans : shorts} = 3 : 10$$

$$\text{Since 240 jeans were sold, 3 parts} = 240$$

$$\begin{aligned} \therefore 1 \text{ part} &= 240 \div 3 \\ &= 80 \end{aligned}$$

$$\begin{aligned} \therefore \text{Number of shorts} &= 80 \times 10 \\ &= 800 \end{aligned}$$

Finding one part first.

Finding 10 parts.

Dividing a quantity in a given ratio

Example 17

Dinesh and Curtis share the rent of their apartment in the ratio 4 : 3. If they pay a total of \$336 in rent each week, calculate each person's share.

Solution

Method 1: Unitary method

Total number of parts = $4 + 3 = 7$

$\therefore 7$ parts = \$336

1 part = $\$336 \div 7 = \$48$

$\therefore$ Dinesh's share = $4 \times \$48 = \$192$

Curtis' share = $3 \times \$48 = \$144$

Checking: $\$192 + \$144 = \$336$

Method 2: Fraction method

Total number of parts = $4 + 3 = 7$

$\therefore$ Dinesh's share = $\frac{4}{7} \times \$48 = \$192$

Curtis' share = $\frac{3}{7} \times \$48 = \$144$

Checking: $\$192 + \$144 = \$336$

Simplifying rates

Example 18

Write each statement as a rate.

a Travelling 600 km in 12 hours.

b A phone call costs \$6.45 for 15 minutes

Solution

a $600 \text{ km in } 12 \text{ h} = 600 \div 12$
 $= 50 \text{ km/h}$

b $\$6.45 \text{ for } 15 \text{ min} = \$6.45 \div 15$
 $= \$0.43/\text{min}$

Worksheet

Rates exercise

MAT09NAWK00041

Rate problems

Problems involving rates can be solved usually by multiplying or dividing. The following strategy may help.

- Write the units of the rate x/y as a fraction: $\frac{x}{y}$
- To find the quantity in the numerator, x , **multiply** by the rate
- To find the quantity in the denominator, y , **divide** by the rate

Example 19

A car travels at 92 km/h.

- a** How far will it travel in $4\frac{1}{2}$ hours?
b How long will it take to travel 800 km? Answer correct to one decimal place.

Solution

- a** Write the units of the rate as a fraction: $\text{km/h} = \frac{\text{km}}{\text{h}}$

To find the distance (km), multiply by the rate.

$$\begin{aligned}\text{Distance} &= 4\frac{1}{2} \times 92 \\ &= 414 \text{ km}\end{aligned}$$

- b** To find the time (h), divide by the rate.

$$\begin{aligned}\text{Time} &= 800 \div 92 \\ &= 8.6956 \dots \\ &\approx 8.7 \text{ h}\end{aligned}$$

Exercise 2-06 Ratios and rates**See Example 15**

- 1** Simplify each ratio.

a 48 : 28

b 15 : 36

c 125 : 60

d 84 : 56

e 124 : 39

f $\frac{3}{5} : \frac{1}{4}$

g 0.68 : 0.24

h 1.25 : 4.5

i $5 : \frac{4}{5}$

j $\frac{5}{8} : \frac{2}{3}$

k 4.8 : 0.64

l 15 : 18 : 6

m 27 : 36 : 9

n 120 : 20 : 80

o $\frac{5}{6} : 3$

p $2\frac{1}{2} : 5$

- 2** Simplify each ratio.

a 75c : \$4

b 5 kg : 800 g

c 2 hours : 45 min

d 600 mm : 2 m

e 8 hours : 1 day

f 8 mm : 4 cm

g 300 mL : 2 L

h \$3.50 : 25c

i 6 min : 45 s

See Example 16

- 3** The ratio of a father's age to the age of his daughter is 10 : 3. If the father is 30, find the age of the daughter.
- 4** Two people invest in a business in the ratio 3 : 5. If the larger investment is \$520 000, find the amount of the smaller investment.
- 5** A survey of car buyers found that they purchased cars with colours in the ratio of white : grey : blue : other colour = 8 : 5 : 11 : 10.
A dealer ordered 20 cars in colours other than white, grey and blue. How many white cars should be ordered?
- 6** A concrete mixture of gravel to sand to cement of 4 : 3 : 1 is needed for strong foundations. How much of each is needed to make 40 cubic metres of concrete?

See Example 17

- 7 At a busy intersection involving a red-light camera, the ratio of cars running a red light to cars stopping was 1 : 79. If 90 000 cars passed through the intersection last week, how many ran the red light?



- 8 Gold jewellery is classified according to its gold content. The ratio of gold to other metals is given in carats. Pure gold is 24 carats, so 10-carat gold has gold mixed with other metals in the ratio 10 : 14.
- Write the ratio of gold to other metals in:
 - a 9-carat bracelet
 - an 18-carat ring
 - Minka purchased a 14-carat gold necklace with a mass of 50 g. How much gold is in the necklace? Answer to the nearest 0.1 g.
- 9 Shahid and Bridie won \$60 000 in a lottery. If they share the prize in the ratio 23 : 27, how much does each person receive?
- 10 An alloy consists of nickel and copper in the ratio 4 : 7. If the alloy weighs 3.41 kg, how much copper was used? Select the correct answer **A**, **B**, **C** or **D**.
- A** 0.21 kg **B** 0.85 kg **C** 1.24 kg **D** 2.17 kg
- 11 Write each statement as a rate using the units in brackets. Round answers to one decimal place if needed. **See Example 18**
- 540 km in 8 h (km/h)
 - 5 kg for \$42 (\$/kg)
 - 2800 words in 50 min (words/min)
 - 14 L for 170 km (km/L)
 - 200 m in 26 s (m/s)
 - 5 kg of seed for 120 m² (g/m²)
 - 40 days to walk 900 km (km/day)
 - \$215 for 3 hours (\$/h)
- 12 A truck maintains an average speed of 65 km/h. Calculate how far it will travel: **See Example 19**
- in 2½ h
 - from 6:30 a.m. to 3:00 p.m.
- 13 Walid earns \$220 for an 8-hour day.
- Express this pay as an hourly rate (\$/h).
 - How much would he earn in a 40-hour week?
 - How many days will it take him to earn \$1760?
- 14 5 kg of lamb loin chops cost \$79.95.
- Express this as a rate (\$/kg).
 - How much would 3 kg cost?
 - How many kilograms (correct to one decimal place) can be bought for \$40?

Worked solutions

Ratios and rates

MAT09NAWS1008

- 15 A utility van has a fuel consumption of 11.6 L/100 km. Calculate how much fuel it will use for a trip of:
a 640 km **b** 58 km **c** 140 km
- 16 Calculate the fuel consumption, in L/100 km, of a Toyota Prius car that uses 35.1 L to travel 900 km.
- 17 Birth rates are given as births/1000 people. A city's birth rate is 16 births/1000 people. If there were 520 births in the city last year, what was its population?
- 18 James' heart beats at 78 beats/min.
a How many times will it beat in one hour?
b How long will it take to beat 1000 times? Answer correct to the nearest second.
- 19 David claims that 5 kg of Greenie grass seed will plant an area of 1200 m². How much grass seed is needed for an area of 60 m²? Select the correct answer **A**, **B**, **C** or **D**.
A 0.25 kg **B** 0.4 kg **C** 1.44 kg **D** 4 kg
- 20 **a** Indonesia has a population of 249 million and an area of 1 919 440 km². Calculate its population density in persons/km² correct to one decimal place.
b Australia has a much smaller population of 23 million but a much larger area of 7 692 000 km². Calculate its population density correct to one decimal place.
c If Australia was as densely populated as Indonesia, what would its population be? Answer to the nearest hundred.

Just for the record

The beat goes on

The heart of an unfit person works harder whether at rest, during activity or even while recovering from activity. A fit person's heart copes with activity better and its beat returns to normal sooner.

	Heartbeats per minute		
	Fit	Unfit	Difference
Before activity	65	80	15
During activity	95	135	40
3 minutes after	80	115	35
6 minutes after	70	100	30
9 minutes after	65	95	30

What patterns do you notice in the differences in heart rates between fit and unfit persons?



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2-07 Converting rates

It is often necessary to convert rates from one set of units to another, for example, from kilometres/hour to metres/second, or from millilitres/minute to litres/day.

Example 20

Convert:

- a** 80 L/kg to L/g **b** 5 m/s to km/h **c** 2 mL/min to L/day

Solution

- a** $80 \text{ L/kg} = \frac{80}{1000} \text{ L/g}$ $1 \text{ kg} = 1000 \text{ g}$
 $= 0.08 \text{ L/kg}$
- b** $5 \text{ m/s} = 5 \times 3600 \text{ m/h}$ $1 \text{ h} = 3600 \text{ s}$
 $= 18\,000 \text{ m/h}$
 $= \frac{18\,000}{1000} \text{ km/h}$ $1 \text{ km} = 1000 \text{ m}$
 $= 18 \text{ km/h}$
- c** $2 \text{ mL/min} = 2 \times 60 \text{ mL/h}$ $1 \text{ h} = 60 \text{ min}$
 $= 120 \text{ mL/h}$
 $= 120 \times 24 \text{ mL/day}$ $1 \text{ day} = 24 \text{ h}$
 $= 2880 \text{ mL/day}$
 $= \frac{2880}{1000} \text{ L/day}$ $1 \text{ L} = 1000 \text{ mL}$
 $= 2.88 \text{ L/day}$

Exercise 2-07 Converting rates

1 Convert:

- a** 5 m/s to m/h **b** 30 m/s to m/h **c** \$10/kg to \$/g
d 20 mL/min to mL/h **e** 40 mL/minute to mL/h **f** 5 mL/s to mL/h
g 5 kg/m² to kg/ha **h** 2.5 tonnes/day to kg/day **i** \$750/week to \$/year
j 0.5 km/min to km/h **k** 15 sheep/h to sheep/day **l** \$60/day to \$/week

2 Convert:

- a** 5 m/s to km/h **b** 35 cm/s to m/h **c** 10 cents/g to \$/kg
d 40 mL/min to L/h **e** 40 L/minute to mL/s **f** 5 mL/s to L/day
g 25 g/m² to kg/ha **h** 2.5 tonnes/h to kg/day **i** 110 km/h to m/s
j 5 kg/m to g/cm **k** 8 km/L to m/mL **l** 70c/min to \$/h

See Example 20

- 3 Convert each speed into kilometres per hour. Round your answers to two decimal places.
- An African cheetah was measured running at 27 m/s.
 - A German peregrine falcon dived at 97 m/s.
 - A Tanzanian snake can travel at a speed of 3.3 m/s.
 - A sailfish off the coast of Florida was estimated to swim 30 m/s.
 - A racing cyclist rode at 23 m/s.
- 4 If Jeff's reaction time is 0.9 seconds, how far will his car travel in this time if its speed is:
- 60 km/h?
 - 80 km/h?
 - 110 km/h?

Mental skills 2B

Maths without calculators

Estimating square roots

We can use the square numbers to estimate square roots.

n	1	2	3	4	5	6	7	8	9	10	11	12
n^2	1	4	9	16	25	36	49	64	81	100	121	144

- 1 Study each example.

- a Estimate $\sqrt{7}$.

7 is between the square numbers 4 and 9 (2^2 and 3^2 respectively).

Because 7 is closer to 9, $\sqrt{7}$ is closer to 3.

An estimate is $\sqrt{7} \approx 2.6$. (Actual answer = 2.645...)

- b Estimate $\sqrt{55}$.

55 is between the square numbers 49 and 64 (7^2 and 8^2 respectively).

Since 55 is closer to 49, $\sqrt{55}$ is closer to 7.

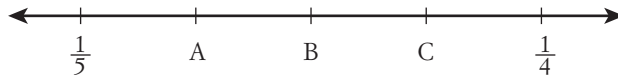
An estimate is $\sqrt{55} \approx 7.4$. (Actual answer = 7.416...)

- 2 Now estimate each square root correct to one decimal place.

a $\sqrt{8}$	b $\sqrt{18}$	c $\sqrt{28}$	d $\sqrt{111}$
e $\sqrt{80}$	f $\sqrt{31}$	g $\sqrt{12}$	h $\sqrt{65}$
i $\sqrt{75}$	j $\sqrt{29}$	k $\sqrt{40}$	l $\sqrt{126}$

Power plus

- 1 What fractions do A , B and C represent on this number line if the intervals between the fractions are equal?



- 2 Helen sold a car for \$14 700, making a loss of 30% on its original price. Find the original price.
- 3 A store marks up the price of a lounge suite by 80%. What percentage discount would bring the price of the lounge suite back to its original price?
- 4 Marie went to a supermarket and bought 5 kg of potatoes. If 3 kg of potatoes cost \$4.99, how much change would Marie receive from \$20?
- 5 Two litres of water are to be shared equally between three people.
- Calculate how much water each would receive (correct to 3 decimal places).
 - Why are three decimal places too accurate for practical purposes? Give reasons.
- 6 Melinda bought a pair of shoes that were reduced by $\frac{1}{3}$. If the shoes sold for \$72, what was their original price?
- 7 After training for six months, Ian reduced his time for running 100 m by 8%. If his new time is 10.2 seconds, what was his old time? Answer to one decimal place.
- 8 At a local football game, the ratio of men to children was 5 : 8 and the ratio of women to children was 7 : 10. What is the ratio of men to women?
- 9 James and Jordan each drove their cars to the beach. James drove his car at a speed of 40 km/h for half the distance and at 60 km/h for the other half. Jordan drove at 50 km/h for the entire distance. Who arrived first, James or Jordan? Give reasons for your answer.
- 10 **a** How long will it take for an investment of \$5000 to double if it is invested at 5% p.a. simple interest?
- b** How long will it take for an investment of \$10 000 to double at the same interest rate?

Chapter 2 review

Language of maths

Puzzle sheet

Numbers crossword

MAT09NAPS10022

convert	cost price	decimal	decrease
discount	flat rate interest	fraction	GST
increase	interest rate	irrational	loss
per	per annum (p.a.)	percentage	principal
profit	rate	ratio	recurring
selling price	simple interest	terminating	unitary method

- 1 What is another name for **flat rate interest**?
- 2 Give an example of an irrational number.
- 3 Find a non-mathematical meaning for:
 - a irrational
 - b recurring
 - c terminating
- 4 What is the difference between the selling price and the cost price called when the selling price is higher?
- 5 What does 'annum' mean, as in 'per annum'?
- 6 Which method involves finding the value of one part when given the value of the whole amount or several parts?

Topic overview

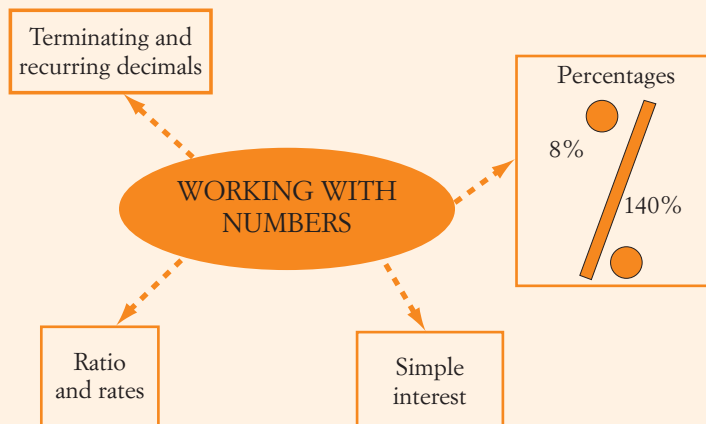
- Write 10 questions (with solutions) that could be used in a test for this chapter. Include some questions that you have found difficult to answer.

Worksheet

Mind map: Working with numbers (Advanced)

MAT09NAWK10024

Copy (or print) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



- 1 Convert $\frac{4}{11}$ to a recurring decimal. See Exercise 2-01
- 2 Convert each recurring decimal to a simple fraction.

a $0.\dot{8}$
b $0.2\dot{7}$
c $0.1\dot{6}$

See Exercise 2-02
- 3 Find: See Exercise 2-03

a 42% of \$580
b $8\frac{1}{4}\%$ of 90 kg
c 2.45% of \$358
- 4 a Increase 120 L by 35% b Decrease \$25 by 12.5% See Exercise 2-03
- 5 What percentage is (correct to one decimal place): See Exercise 2-03

a 5 minutes of 1 hour?
b 75c out of \$4.50?
- 6 In a cricket match, Ricky made 62 runs, which was 21% of the team total. How many runs did the team make? See Exercise 2-03
- 7 Calculate the selling price of a Smartphone marked at \$485 and discounted by 15%. Select the correct answer **A, B, C** or **D**. See Exercise 2-04

A \$72.75
B \$470
C \$412.20
D \$412.25
- 8 A block of land was purchased for \$250 000. Four years later, it was sold for \$630 000. Calculate: See Exercise 2-04

a the profit
b the profit as a percentage of cost price.
- 9 At a sale, Garrett and Kay bought a cutlery set, saving \$150 on the original price. If this discount represented 55% of the price, what was the price of the cutlery? See Exercise 2-04
- 10 Calculate, correct to the nearest cent, the simple interest on: See Exercise 2-05

a \$8050 invested for 5 years at 4.8% p.a.
b \$3890 borrowed for 7 months at 2.4% p.a.
- 11 Calculate the annual simple interest rate if an investment of \$7000 earns \$1820 in interest over 2 years. See Exercise 2-05
- 12 Simplify each ratio. See Exercise 2-06

a 200 : 450
b 35 mm : 2 m
c 5 h : 45 min
- 13 Bill and Ben win \$800 000 in Lotto. If they agree to share the prize in the ratio of 25 : 17, how much will each person receive? See Exercise 2-06
- 14 A truck travels 581 km in 7 hours. See Exercise 2-06

a Calculate its average speed in km/h.
b How far does the truck travel in 4 hours?
c How long will it take to travel 1000 km? Answer correct to one decimal place.
- 15 Convert: See Exercise 2-07

a 0.8 mL/s to mL/h
b 140 g/m^2 to kg/m^2

c 30 km/h to m/s
d 345.6 m/day to cm/min

Mixed revision 1

See Exercise 2-01

- 1 Convert each fraction to a decimal and state whether it is recurring or terminating.

a $\frac{4}{5}$

b $\frac{5}{13}$

c $\frac{4}{9}$

d $4\frac{3}{4}$

See Exercise 3-01

- 2 Simplify each expression.

a $7 + 5m - 9$

b $8p - 3m + 12p + 4m$

c $5n^2 - 3n - 2n^2 - 7n$

Stage 5.3

See Exercise 2-02

- 3 Convert each recurring decimal to a simple fraction.

a $0.\dot{4}\dot{2}$

b $0.2\dot{6}$

c $0.\dot{1}8\dot{0}$

See Exercise 1-04

- 4 Select the surds from this list of square roots.

$\sqrt{98}$

$\sqrt{9}$

$\sqrt{225}$

$\sqrt{160}$

$\sqrt{36}$

$\sqrt{52}$

Stage 5.3

See Exercise 1-04

- 5 Simplify each surd.

a $\sqrt{75}$

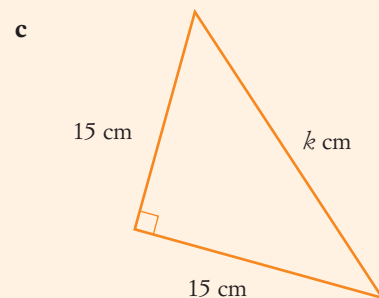
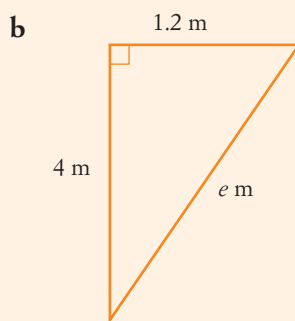
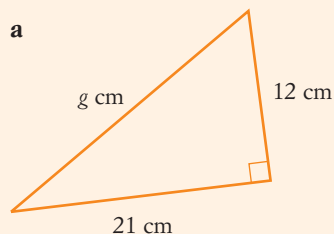
b $(3\sqrt{7})^2$

c $2\sqrt{28}$

d $\sqrt{162}$

See Exercise 1-01

- 6 Find, correct to one decimal place, the value of each pronumeral.



See Exercise 3-07

- 7 Expand each expression.

a $(p + 4)(p - 8)$

b $(2n + 1)(3n + 5)$

c $(7 - y)(7 + y)$

Stage 5.3

See Exercise 3-10

- 8 Expand each expression.

a $(5a + 3)^2$

b $(3r + 4)(3r - 4)$

c $(x - 3)(x + 3) - (x - 3)^2$

See Exercise 1-05

- 9 Simplify each expression.

a $10\sqrt{2} - 3\sqrt{5} + 3\sqrt{2}$

b $\sqrt{18} - \sqrt{8}$

c $\sqrt{50} + 5\sqrt{32}$

See Exercise 1-06

- 10 Simplify each expression.

a $\sqrt{3} \times \sqrt{18}$

b $3\sqrt{2} \times 5\sqrt{2}$

c $4\sqrt{5} \times (-\sqrt{8})$

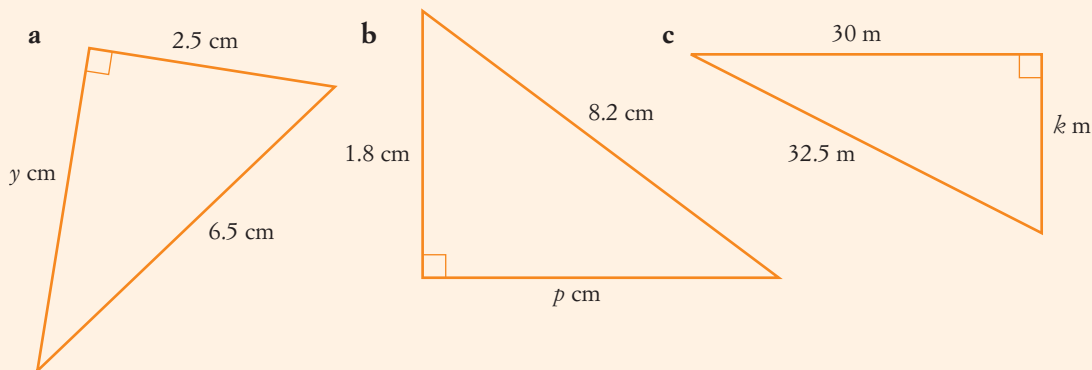
d $\sqrt{27} \div \sqrt{3}$

e $-8\sqrt{15} \div 2\sqrt{5}$

f $\frac{-7\sqrt{12}}{\sqrt{48}}$

- 11 Find the value of each pronumeral.

See Exercise 1-02



- 12 Which expression is $16mn + 24m^2$ completely factorised? Select the correct answer

See Exercise 3-06

A, B, C or D.

A $4(4n + 6m)$

B $8m(2n + 3m)$

C $4m(4n + 6m)$

D $m(16n + 24m)$

- 13 Factorise each expression.

See Exercise 3-06

a $3p - 15$

b $5(m + 4) - t(m + 4)$

c $-8x^2y - 12xy$

- 14 Factorise each expression.

a $36y^2 - 49x^2$

b $m^2 - 5mn + 3m - 15n$

c $8m^2 - 50$

d $2ky^2 - 3ky - 2y + 3$

Stage 5.3

See Exercise 3-11

- 15 Calculate:

See Exercise 2-03

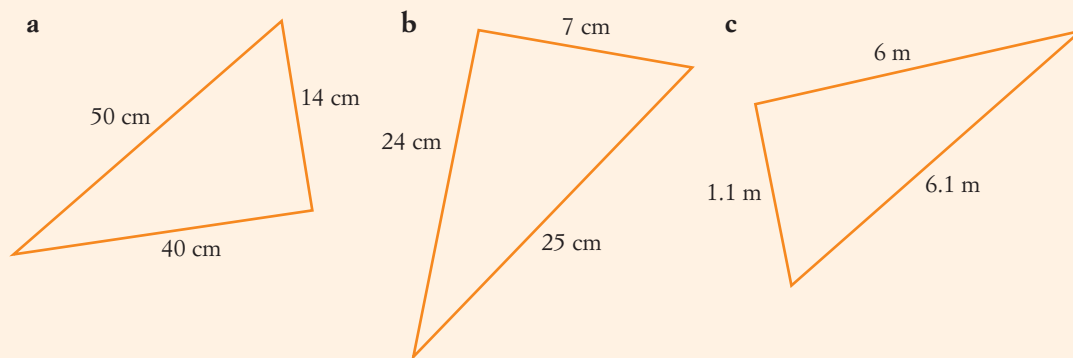
a 170% of \$60

b 14% of \$250

c $3\frac{1}{2}\%$ of \$670

- 16 Test whether each triangle is right-angled.

See Exercise 1-08



- 17 Factorise each expression.

a $y^2 - 2y - 63$

b $k^2 - 7k + 10$

c $w^2 + 5w - 14$

d $m^2 - 17m + 72$

Stage 5.3

See Exercise 3-12

- 18 Factorise each expression.

See Exercise 3-13

a $5k^2 + 14k - 3$

b $3y^2 + y - 24$

c $4h^2 - 7h - 15$

d $8w^2 - 38w + 35$

- 19 Pat bought 55 L of petrol at 146.9 cents per litre.

See Exercise 2-06

a How much did Pat pay (to the nearest cent)?

b How many litres can be bought for \$60? Answer to one decimal place.

Mixed revision 1

See Exercise 2-04 **20** At a sale, the price of a video recorder was reduced by 30%. Find the original price of the recorder if its sale price is \$269.50.

Stage 5.3

See Exercise 3-15

21 Simplify each expression.

a $\frac{7}{7k-21}$

b $\frac{x+3}{x^2-9}$

c $\frac{5y+25}{30}$

d $\frac{d^2-49}{d^2+2d-35}$

See Exercise 3-02

22 Simplify each expression.

a $2a^2 \times 4a$

b $5 \times 4p \times 2pd$

c $9b^2d \times 3bd$

See Exercise 3-02

23 Simplify each expression.

a $\frac{21mn}{7n}$

b $36x \div 12y$

c $\frac{w^2v^3}{2vw}$

Stage 5.3

See Exercise 3-15

24 Simplify each expression.

a $\frac{n+1}{n^2-n} - \frac{2}{n^2-1}$

b $\frac{5y+20}{9} \times \frac{3y}{y+4}$

c $\frac{k^2+4k}{k-3} \div \frac{4k}{k^2-4k+3}$

See Exercise 2-06

25 A dishwasher costing \$190 is sold for \$430. Calculate the percentage profit on the cost price, correct to one decimal place.

See Exercise 2-06

26 Simplify:

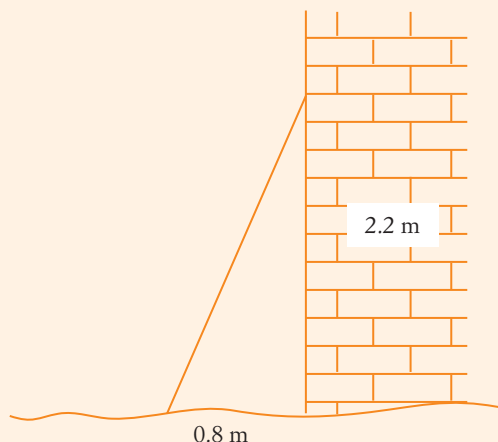
a $160 : 96$

b $3 \text{ h} : 45 \text{ min}$

c $1275 \text{ words in } 15 \text{ minutes (as words/min).}$

See Exercise 1-07

27 The foot of a ladder is placed 0.8 m from the base of a wall. If the ladder reaches 2.2 m up the wall, calculate, correct to one decimal place, the length of the ladder.



See Exercise 2-03

28 **a** Increase 85 kg by 30%.
b Decrease \$6.40 by 18%.

See Exercise 3-07

29 Expand each expression.

a $8(5-x)$

b $3h(2-7h)$

c $9c(2d-3c)$

- 30** Expand and simplify each expression. See Exercise 3-05
a $2(5w - 3) + 4(1 + 3w)$ **b** $3(1 - 4k) - 2(2k + 1)$ **c** $4(3t - 4) - 7(1 - 2t)$
- 31** Divide \$450 between Klaas, David and Sarah in the ratio 3 : 5 : 7. See Exercise 2-06
- 32** The speed of a spacecraft leaving the earth's gravitational pull must reach 11.2 km/s. Express this speed in km/h. See Exercise 2-07
- 33** Simplify each expression. See Exercise 3-03
a $\frac{9d}{11} - \frac{3d}{11}$ **b** $\frac{9t}{20} + \frac{4t}{20}$ **c** $\frac{n}{5} - \frac{n}{6}$
- 34** Simplify each expression. See Exercise 3-04
a $\frac{a}{6} \div \frac{m}{5}$ **b** $\frac{4}{a} \div \frac{7}{a}$ **c** $\frac{3x}{5} \div \frac{9x}{10}$
- 35** A rectangle has side lengths of 5 cm and 7 cm. Find the length of a diagonal as a surd. See Exercise 1-07
- 36** A discount of \$75 represents 4% of the marked price. What is the marked price? See Exercise 2-05



3

Number and algebra

Products and factors

Algebra is the branch of mathematics that uses symbols and formulas to describe number patterns and relationships in our natural and physical world. In 825 CE, the Persian mathematician al-Khwarizmi used the Arabic word 'al-jabr' to describe the process of adding equal quantities to both sides of an equation. When al-Khwarizmi's book was translated into Latin and introduced to Europe, 'al-jabr' became 'algebra' and the word was adopted as the name for the whole subject.



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Chapter outline

	Proficiency strands			
3-01 Adding and subtracting terms	U	F	R	C
3-02 Multiplying and dividing terms	U	F	R	C
3-03 Adding and subtracting algebraic fractions	U	F	R	C
3-04 Multiplying and dividing algebraic fractions	U	F	R	C
3-05 Expanding expressions	U	F	R	C
3-06 Factorising expressions	U	F	R	C
3-07 Expanding binomial products	U	F	R	C
3-08 Perfect squares*	U	F	R	C
3-09 Difference of two squares*	U	F	R	C
3-10 Mixed expansions*		F	R	C
3-11 Factorising special binomial products*	U	F	R	C
3-12 Factorising quadratic expressions*	U	F	R	C
3-13 Factorising quadratic expressions of the form $ax^2 + bx + c$ *	U	F	R	C
3-14 Mixed factorisations*		F	R	C
3-15 Factorising algebraic fractions*	U	F	R	C

*STAGE 5.3

Wordbank

binomial An algebraic expression that consists of two terms, for example, $4a + 9$, $3 - y$, $x^2 - 4x$

factorise To rewrite an expression with grouping symbols, by taking out the highest common factor; factorising is the opposite of expanding; for example, $9r^2 + 36r$ factorised is $9r(r + 4)$

highest common factor (HCF) The largest term that is a factor of two or more terms, for example, the HCF of $9r^2 + 36r$ is $9r$

perfect square A square number or an algebraic expression that represents one, for example, 64, $(x + 9)^2$

quadratic expression An algebraic expression in which the highest power of the variable is 2, for example, $2x^2 + 5x - 3$ or $x^2 + 2$

In this chapter you will:

- simplify algebraic expressions involving the four operations
- apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate 
- add, subtract, multiply and divide simple algebraic fractions
- factorise algebraic expressions
- (STAGE 5.3) expand binomial products
- (STAGE 5.3) factorise quadratic expressions
- (STAGE 5.3) factorise and simplify expressions involving algebraic fractions

SkillCheck

Worksheet

StartUp assignment 3

MAT09NAWK10025

Puzzle sheet

Generalised arithmetic

MAT09NAPS10026

Skillsheet

Algebraic expressions

MAT09NASS10012

Puzzle sheet

Substitution puzzle

MAT09NAPS10027

1 Write an algebraic expression for each statement.

- a** The cost of p books at $\$Q$ each.
b The selling price after a GST of $\$t$ is added to the marked price of $\$40$.
c The number of metres in h km.
d The average of m , p , v and w .
e The number of dollars in y cents.

2 If $f = 5$, $g = -4$ and $r = 2$, then evaluate each expression.

- a** $2f + g$ **b** $8 + rf$ **c** $f^2 - r^2$
d $3g^2 - 5$ **e** $r(f - g)$ **f** $\frac{5r + f}{3}$

3 If the perimeter of a rectangle with length l and width w has the formula $P = 2l + 2w$, find the perimeter of a rectangle with length 4.7 cm and width 2.5 cm.

4 Simplify each expression.

- a** $5n + 6n$ **b** $9h - 3h$ **c** $10pq - qp$ **d** $x^2 + 4x^2$
e $4 \times w$ **f** $8 \times 5x$ **g** $2g \times 3h$ **h** $3y \times 6y$
i $10x \div 2$ **j** $6b \div b$ **k** $\frac{16xy}{8x}$ **l** $\frac{2n^2}{4n}$

5 Find the highest common factor (HCF) of each pair of numbers.

- a** 8 and 12 **b** 18 and 27 **c** 45 and 30

6 Evaluate each expression.

- a** $\frac{2}{3} \times \frac{3}{4}$ **b** $\frac{5}{8} - \frac{1}{4}$ **c** $\frac{2}{5} \div \frac{7}{8}$ **d** $\frac{3}{8} + \frac{4}{5}$

3-01 Adding and subtracting terms

Skillsheet

Algebra using diagrams

MAT09NASS10013

Summary

Only **like terms** can be added or subtracted.

For example, $2x - 5y$ *cannot* be simplified because x and y represent different numbers.

Example 1

Simplify each expression.

a $5r + 4s + 7r - 2s$

b $8u^2 - 4u + 3u - 7u^2$

Solution

a $5r + 4s + 7r - 2s = 5r + 7r + 4s - 2s$
 $= 12r + 2s$

Group the like terms.

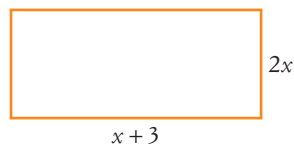
$12r$ and $2s$ cannot be simplified further.

b $8u^2 - 4u + 3u - 7u^2 = 8u^2 - 7u^2 - 4u + 3u$
 $= u^2 - u$

Each $+$ and $-$ sign belongs to the term that follows them.

Example 2

Write a simplified algebraic expression for the perimeter of this rectangle.



Solution

Perimeter $= x + 3 + 2x + x + 3 + 2x$
 $= x + 2x + x + 2x + 3 + 3$
 $= 6x + 6$

Adding the lengths of the 4 sides.
Group the like terms.

Exercise 3-01 Adding and subtracting terms

1 Simplify each algebraic expression.

a $2fg + 3fg - 4fg$

b $6uvw - uvw + 4uvw$

c $3t + 3 + 5t$

d $3x + 2y + 8y$

e $6g + h - 4g$

f $2fg + 3f - 4fg$

g $7p^2 - 3 - 2p^2$

h $4p - q - 3q$

i $-8r + 6r + 3$

j $-3a + 4b - 7a$

k $4v + 3 + 2v + 7$

l $2y + 6x + y - 3x$

m $7p + 8q - p - q$

n $8 - 3w + 7 - 2w$

o $2n - 3 - 12n + 5$

p $8e - 3f - 2e - 4f$

q $2l^2 + 2l + 3l^2 - 6l$

r $9b - 12b^2 + 6b^2 - 15b$

2 Simplify $15a^2 + 12a + 48 - 18a$. Select the correct answer **A**, **B**, **C** or **D**.

A $24a + 48$

B $27a^2 + 30a$

C $15a^2 - 6a + 48$

D $15a^2 + 30a - 48$

See Example 1

- 3 Use the substitutions $a = 2$, $b = 3$ and $c = 4$ to test whether each equation is correct or incorrect.

a $20c - 12c = 8c$

b $9b + b = 9b^2$

c $7a + 3b = 10 + a + b$

d $3a^2 + 5a^2 = 8a^2$

e $4ab + 3ab = 7ab$

f $12c - c = 12$

g $3ac - 5ac = -2ac$

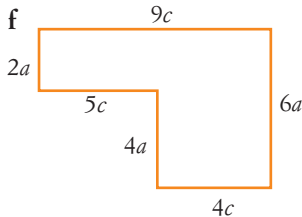
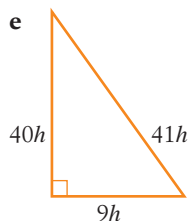
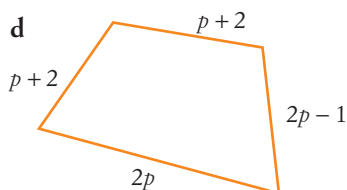
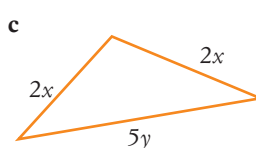
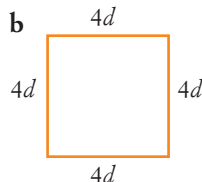
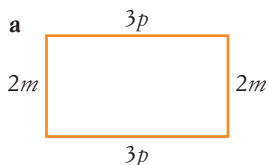
h $5b + 2b^2 = 9b$

i $2c - b + 3b - c = c + 2b$

j $2a + 2b = 2(a + b)$

See Example 2

- 4 Write a simplified algebraic expression for the perimeter of each shape.



- 5 Draw a triangle and write algebraic expressions for its side lengths so that it has a perimeter of $7d + 6$.
- 6 Draw a rectangle and find algebraic expressions for its length and width so that it has a perimeter of $18a - 30$.

Worksheet

Perimeter and area

MAT09NAWK10028

Homework sheet

Algebra 1

MAT09NAHS10009

3-02 Multiplying and dividing terms

Summary

When **multiplying and dividing terms**, multiply or divide the numbers first, then the variables.

Example 3

Simplify each expression.

a $2 \times 5n \times 4m$

b $3d \times 4d$

c $2g \times 3h \times (-5f)$

Solution

a $2 \times 5n \times 4m = 2 \times 5 \times 4 \times n \times m$
 $= 40mn$

Multiply the numbers and multiply the variables.
 Remember to write the variables in alphabetical order.

$$\begin{aligned} \text{b } 3d \times 4d &= 3 \times 4 \times d \times d \\ &= 12d^2 \end{aligned}$$

$$d \times d = d^2$$

$$\begin{aligned} \text{c } 2g \times 3h \times (-5f) &= 2 \times 3 \times (-5) \times g \times h \times f \\ &= -30fgh \end{aligned}$$

Remember to write the variables in alphabetical order.

Example 4

Simplify each expression.

$$\text{a } \frac{15abc}{3ab}$$

$$\text{b } \frac{a^2b}{4ab}$$

$$\text{c } 27cf \div (-3fc)$$

$$\text{d } 5p^2 \div 15p$$

Solution

$$\begin{aligned} \text{a } \frac{15abc}{3ab} &= \frac{15}{3} \frac{\cancel{a}\cancel{b}c}{\cancel{a}\cancel{b}} \\ &= 5c \end{aligned}$$

We can cross out the ab in the numerator and denominator because $\frac{a}{a} = 1$ and $\frac{b}{b} = 1$

Divide the numbers and divide the variables.

$$\begin{aligned} \text{b } \frac{a^2b}{4ab} &= \frac{aab}{4ab} \\ &= \frac{1}{4} \frac{\cancel{a}\cancel{b}a}{\cancel{a}\cancel{b}} \\ &= \frac{a}{4} \end{aligned}$$

$$a^2 = a \times a = aa$$

Divide the numbers and divide the variables.

$$\begin{aligned} \text{c } 27cf \div (-3fc) &= \frac{27cf}{-3fc} \\ &= \frac{27}{-3} \frac{\cancel{c}\cancel{f}}{\cancel{c}\cancel{f}} \\ &= -9 \end{aligned}$$

The variables divide by themselves to give 1.

$$\begin{aligned} \text{d } 5p^2 \div 15p &= \frac{5pp}{15p} \\ &= \frac{5}{15} \frac{\cancel{p}p}{\cancel{p}} \\ &= \frac{1}{3}p \\ &= \frac{p}{3} \end{aligned}$$

$$p^2 = pp$$

Exercise 3-02 Multiplying and dividing terms

1 Simplify each expression.

$$\text{a } 5 \times 8x$$

$$\text{b } 5n \times 3$$

$$\text{c } b \times 5a$$

$$\text{d } t \times 6t$$

$$\text{e } -4 \times 3w$$

$$\text{f } 2 \times (-3p)$$

$$\text{g } 4d \times 3e$$

$$\text{h } 6v \times 4w$$

$$\text{i } 8q \times 8q$$

$$\text{j } (-5a) \times (-4a)$$

$$\text{k } 2y \times 3 \times 4b$$

$$\text{l } -6w \times 3w \times 2$$

$$\text{m } (5n)^2$$

$$\text{n } (-3r)^2$$

$$\text{o } (at)^2$$

See Example 3

See Example 4 2 Simplify each expression.

a $\frac{12x}{3}$

b $\frac{15y}{y}$

c $\frac{36a}{4a}$

d $18mn \div 6m$

e $8vw \div 48vw$

f $\frac{64abc}{16b}$

g $\frac{18g}{-3g}$

h $-25yz \div 5y$

i $\frac{9p}{-45pq}$

j $\frac{-75xyz}{-25x}$

k $10r^2 \div 40r$

l $-12hk^2 \div 2hk$

3 Simplify $\frac{9yx^2}{24xy^2}$. Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{3x^2}{8y^2}$

B $\frac{3x^3}{8y^3}$

C $\frac{3x}{8y}$

D $\frac{3y}{8x}$

4 Simplify each expression.

a $2a \times 3b \times 5c$

b $4p \times 3 \times 2q$

c $40n^2 \div 5n$

d $8b^2 \div 40b^2$

e $6h \times 2k \times (-3)$

f $-2 \times 5k \times (-4k)$

g $\frac{6d}{12}$

h $\frac{7r}{49r}$

i $(-33m) \div 3$

j $\frac{-9w}{-45w}$

k $8ab \div 2bc$

l $2ef \times (-3fe)$

5 Use the substitutions $p = 5$, $q = 4$ and $r = 3$ to test whether each equation is correct or incorrect.

a $3 \times 4p = 12p$

b $q + q = q^2$

c $2r \times 3r = 6r^2$

d $3q^2 \times 5q^2 = 15q^2$

e $\frac{12p}{3p} = 4$

f $\frac{7qr}{qr} = 7$

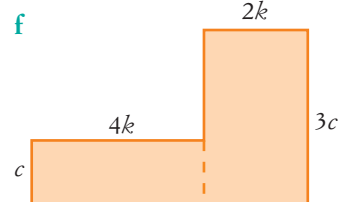
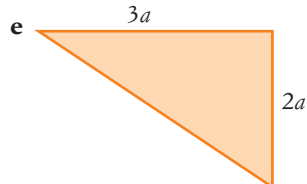
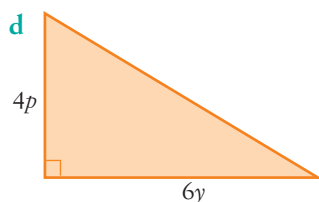
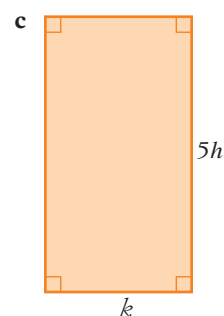
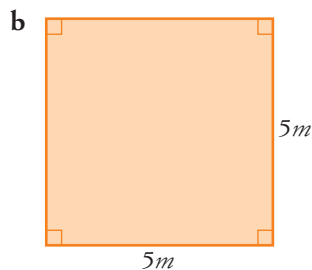
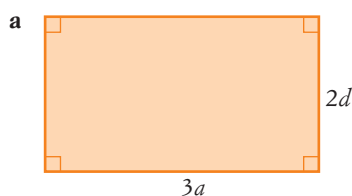
g $3r \times 5q = 15qr$

h $p \times q \times q = 2pq$

i $\frac{14r^2}{7r} = 2r$

j $\frac{5pq}{10p} = \frac{q}{2}$

6 Write a simplified algebraic expression for the area of each shape.

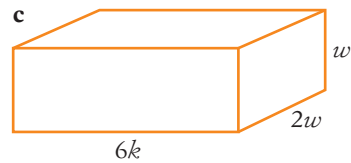
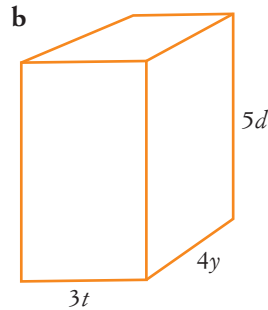
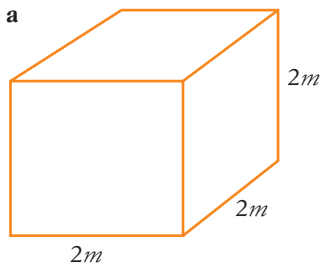


Worked solutions

Multiplying and
dividing terms

MAT09NAWS10009

7 Find an algebraic expression for the volume of each rectangular prism.



3-03 Adding and subtracting algebraic fractions

Summary

To **add or subtract fractions**, convert them (if needed) so that they have the same denominator, then simply add or subtract the numerators.

Example 5

Simplify each expression.

a $\frac{r}{3} + \frac{r}{3}$

b $\frac{7m}{10} - \frac{3m}{10}$

c $\frac{k}{3} + \frac{k}{2}$

d $\frac{3h}{4} + \frac{2h}{5}$

e $\frac{5m}{6} - \frac{2m}{3}$

f $\frac{3}{4} + \frac{7a}{10}$

Solution

a $\frac{r}{3} + \frac{r}{3} = \frac{2r}{3}$

b $\frac{7m}{10} - \frac{3m}{10} = \frac{4m}{10}$
 $= \frac{2m}{5}$

c $\frac{k}{3} + \frac{k}{2} = \frac{2 \times k}{2 \times 3} + \frac{3 \times k}{3 \times 2}$
 $= \frac{2k}{6} + \frac{3k}{6}$
 $= \frac{5k}{6}$

Common denominator = $2 \times 3 = 6$

Puzzle sheet

Algebraic fractions

MAT09NAPS00010

Animated example

Adding and subtracting fractions

MAT09NAAE00004

Video tutorial

Adding and subtracting algebraic fractions

MAT09NAVT10004

$$\begin{aligned} \text{d } \frac{3h}{4} + \frac{2h}{5} &= \frac{5 \times 3h}{5 \times 4} + \frac{4 \times 2h}{4 \times 5} \\ &= \frac{15h}{20} + \frac{8h}{20} \\ &= \frac{23h}{20} \end{aligned}$$

$$\begin{aligned} \text{e } \frac{5m}{6} - \frac{2m}{3} &= \frac{15m}{18} - \frac{12m}{18} \\ &= \frac{3m}{18} \\ &= \frac{m}{6} \end{aligned}$$

$$\begin{aligned} \text{or } \frac{5m}{6} - \frac{2m}{3} &= \frac{5m}{6} - \frac{4m}{6} \\ &= \frac{m}{6} \end{aligned}$$

The lowest common denominator is 6.

$$\begin{aligned} \text{f } \frac{3}{4} + \frac{7a}{10} &= \frac{15}{20} + \frac{14a}{20} \\ &= \frac{15 + 14a}{20} \end{aligned}$$

Exercise 3-03

Adding and subtracting algebraic fractions

See Example 5

1 Simplify each expression.

a $\frac{w}{4} + \frac{2w}{4}$

b $\frac{4k}{8} + \frac{7k}{8}$

c $\frac{7m}{10} - \frac{2m}{10}$

d $\frac{x}{3} + \frac{2x}{3}$

e $\frac{4}{q} + \frac{5}{q}$

f $\frac{5}{3d} - \frac{2}{3d}$

g $\frac{4r}{w} + \frac{r}{w}$

h $\frac{5}{c} - \frac{2}{c}$

i $\frac{4t}{3} + \frac{s}{3}$

j $\frac{11y}{2h} - \frac{7y}{2h}$

k $\frac{6}{a} - \frac{5}{a}$

l $\frac{8}{5e} + \frac{7}{5e}$

m $\frac{p}{z} + \frac{3}{z}$

n $\frac{5u}{8g} - \frac{3u}{8g}$

o $\frac{4}{9f} - \frac{1}{9f}$

p $\frac{7e}{10} - \frac{3e}{10}$

q $\frac{5a}{6} + \frac{5a}{6}$

r $\frac{5}{2d} - \frac{1}{2d}$

s $\frac{7}{5k} + \frac{13}{5k}$

t $\frac{12}{4a} - \frac{8}{4a}$

2 Simplify each expression.

a $\frac{x}{3} + \frac{x}{4}$

b $\frac{s}{3} - \frac{s}{7}$

c $\frac{h}{5} + \frac{h}{3}$

d $\frac{m}{7} - \frac{m}{2}$

e $\frac{w}{4} + \frac{w}{5}$

f $\frac{5t}{4} - \frac{2t}{5}$

g $\frac{2p}{5} + \frac{p}{3}$

h $\frac{5r}{2} - \frac{5r}{3}$

i $\frac{3c}{2} - \frac{c}{5}$

j $\frac{2d}{11} - \frac{r}{3}$

k $\frac{3h}{5} + \frac{2a}{3}$

l $\frac{5}{6} + \frac{4w}{5}$

m $\frac{3}{4} + \frac{2a}{7}$

n $\frac{7e}{8} - \frac{2e}{3}$

o $\frac{m}{2} - \frac{n}{7}$

p $\frac{2k}{5} + \frac{m}{6}$

3-04

Multiplying and dividing algebraic fractions

Puzzle sheet

Algebraic fractions puzzle

MAT09NAPS10029

Puzzle sheet

Upside-down fractions

MAT09NAPS00018

Summary

- To **multiply fractions**, cancel any common factors, then multiply the numerators and denominators separately
- To **divide by a fraction** $\frac{a}{b}$, multiply by its reciprocal $\frac{b}{a}$

Example 6

Simplify each product.

a $\frac{3}{d} \times \frac{4}{c}$

b $\frac{4}{k} \times \frac{3k}{16}$

c $\frac{5}{2r} \times \frac{8r}{11}$

Solution

$$\begin{aligned} \text{a } \frac{3}{d} \times \frac{4}{c} &= \frac{3 \times 4}{d \times c} \\ &= \frac{12}{dc} \end{aligned}$$

$$\begin{aligned} \text{b } \frac{4}{k} \times \frac{3k}{16} &= \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{k}}} \times \frac{3 \overset{1}{\cancel{k}}}{\underset{4}{\cancel{16}}} \\ &= \frac{3}{4} \end{aligned}$$

$$\begin{aligned} \text{c } \frac{5}{2r} \times \frac{8r}{11} &= \frac{5}{\underset{1}{\cancel{2}r}} \times \frac{8 \overset{4}{\cancel{r}}}{11} \\ &= \frac{20}{11} \\ &= 1\frac{9}{11} \end{aligned}$$

Example 7

Simplify each quotient.

a $\frac{3}{h} \div \frac{4}{k}$

b $\frac{xy}{5} \div \frac{3x}{25}$

Solution

$$\begin{aligned} \text{a } \frac{3}{h} \div \frac{4}{k} &= \frac{3}{h} \times \frac{k}{4} \\ &= \frac{3k}{4h} \end{aligned}$$

$$\begin{aligned} \text{b } \frac{xy}{5} \div \frac{3x}{25} &= \frac{\overset{1}{\cancel{x}}y}{\underset{1}{\cancel{5}}} \times \frac{\overset{5}{\cancel{25}}}{\underset{3}{\cancel{x}1}} \\ &= \frac{5y}{3} \end{aligned}$$

Exercise 3-04

Multiplying and dividing algebraic fractions

See Example 6 1 Simplify each product.

a $\frac{w}{3} \times \frac{1}{2}$

d $\frac{4}{m} \times \frac{3}{n}$

g $\frac{2}{x} \times \frac{3}{x}$

j $\frac{4ad}{9} \times \frac{d}{16a}$

m $\frac{2}{3} \times \frac{9u}{10}$

b $\frac{s}{5} \times \frac{t}{4}$

e $\frac{l}{3} \times \frac{5}{f}$

h $\frac{a}{b} \times \frac{c}{d}$

k $\frac{5p}{4} \times \frac{8}{15p}$

n $\frac{u}{3} \times \frac{3}{u}$

c $\frac{3}{h} \times \frac{5}{6k}$

f $\frac{1}{v} \times \frac{2}{3v}$

i $\frac{d}{e} \times \frac{e}{g}$

l $\frac{4}{ak} \times \frac{3a}{5k}$

o $\frac{3z}{r} \times \frac{2r}{9dz}$

See Example 7 2 Simplify each quotient.

a $\frac{r}{2} \div \frac{r}{5}$

d $\frac{q}{4} \div \frac{3}{4}$

g $\frac{3}{e} \div \frac{5}{e}$

j $\frac{h}{k} \div \frac{k}{h}$

m $\frac{t}{3} \div \frac{3t}{5u}$

b $\frac{m}{6} \div \frac{n}{3}$

e $\frac{3y}{5} \div \frac{2y}{d}$

h $\frac{3}{2a} \div \frac{b}{6a}$

k $\frac{8w}{3x} \div \frac{2w}{9x}$

n $\frac{3e}{7g} \div \frac{e}{14g}$

c $\frac{h}{2} \div \frac{h}{8}$

f $\frac{4t}{9} \div \frac{3t}{5}$

i $\frac{5m}{n} \div \frac{2m}{3n}$

l $\frac{3s}{4} \div \frac{6s}{11}$

o $\frac{xh}{5} \div \frac{3h}{15}$

Worked solutions

Multiplying and dividing algebraic fractions

MAT09NAWS100010

3 Simplify each expression.

a $\frac{3p}{2} \div \frac{p^2}{4}$

d $\frac{5}{2g} \div \frac{2}{g}$

g $5 \div \frac{3}{5b}$

j $5p \div \frac{7}{10pt}$

m $\frac{5ty}{k} \times \frac{5ky}{t}$

b $\frac{2w}{7} \div \frac{5w}{6}$

e $\frac{xy}{z} \times \frac{5}{y}$

h $\frac{4t}{9} \div 3$

k $\frac{s}{3} \div \frac{5s}{2} \times \frac{3s}{7}$

n $\frac{6}{r} \times \frac{5r}{9} \div \frac{15}{yh}$

c $\frac{8y}{5} \times \frac{3}{32y^2}$

f $\frac{a}{b} \times \frac{c}{b} \times \frac{b}{3a}$

i $\frac{5mn}{2d} \times \frac{4d}{n} \times \frac{1}{15mn}$

l $\frac{7}{h} \times \frac{1}{42h}$

o $\frac{6cf}{q} \div \frac{5}{qf} \div \frac{10c}{a}$

Mental skills 3A

Maths without calculators

Multiplying and dividing by 5, 15, 25 and 50

It is easier to multiply or divide a number by 10 than by 5. So whenever we multiply or divide a number by 5, we can double the 5 (to make 10) and then adjust the first number.

1 Study each example.

a **To multiply by 5**, halve the number, then multiply by 10.

$$18 \times 5 = 18 \times \frac{1}{2} \times 10 \quad (\text{or } 9 \times 2 \times 10)$$

$$= 9 \times 10$$

$$= 90$$

b To multiply by 50, halve the number, then multiply by 100.

$$\begin{aligned} 26 \times 50 &= 26 \times \frac{1}{2} \times 100 && (\text{or } 13 \times 2 \times 100) \\ &= 13 \times 100 \\ &= 1300 \end{aligned}$$

c To multiply by 25, quarter the number, then multiply by 100.

$$\begin{aligned} 44 \times 25 &= 44 \times \frac{1}{4} \times 100 && (\text{or } 11 \times 4 \times 25) \\ &= 11 \times 100 \\ &= 1100 \end{aligned}$$

d To multiply by 15, halve the number, then multiply by 30.

$$\begin{aligned} 8 \times 15 &= 8 \times \frac{1}{2} \times 30 && (\text{or } 4 \times 2 \times 15) \\ &= 4 \times 30 \\ &= 120 \end{aligned}$$

e To divide by 5, divide by 10 and double the answer. We do this because there are two 5s in every 10.

$$\begin{aligned} 140 \div 5 &= 140 \div 10 \times 2 \\ &= 14 \times 2 \\ &= 28 \end{aligned}$$

f To divide by 50, divide by 100 and double the answer. This is because there are two 50s in every 100.

$$\begin{aligned} 400 \div 50 &= 400 \div 100 \times 2 \\ &= 4 \times 2 \\ &= 8 \end{aligned}$$

g To divide by 25, divide by 100 and multiply the answer by 4. This is because there are four 25s in every 100.

$$\begin{aligned} 600 \div 25 &= 600 \div 100 \times 4 \\ &= 6 \times 4 \\ &= 24 \end{aligned}$$

h To divide by 15, divide by 30 and double the answer. This is because there are two 15s in every 30.

$$\begin{aligned} 240 \div 15 &= 240 \div 30 \times 2 \\ &= 8 \times 2 \\ &= 16 \end{aligned}$$

2 Now evaluate each expression.

a 32×5	b 14×5	c 48×5	d 18×50
e 52×50	f 36×25	g 28×5	h 12×25
i 12×15	j 22×35	k $90 \div 5$	l $170 \div 5$
m $230 \div 5$	n $1300 \div 50$	o $900 \div 50$	p $300 \div 25$
q $1000 \div 25$	r $360 \div 45$	s $210 \div 15$	t $360 \div 15$

Skillsheet

Algebra using diagrams

MAT09NASS10013

Worksheet

Algebra review

MAT09NAWK10034

3-05 Expanding expressions**Summary****The distributive law for expanding an expression**

Multiply each term inside the brackets by the term outside

$$a(b + c) = ab + ac$$

$$a(b - c) = ab - ac$$

Example 8

Expand each expression.

a $4(5 + d)$

b $2w(3w + 10)$

Solution

$$\begin{aligned} \text{a } 4(5 + d) &= 4 \times 5 + 4 \times d \\ &= 20 + 4d \end{aligned}$$

multiply each term inside the brackets by the term outside

$a(b + c) = ab + ac$

$$\begin{aligned} \text{b } 2w(3w + 10) &= 2w \times 3w + 2w \times 10 \\ &= 6w^2 + 20w \end{aligned}$$

Example 9

Expand each expression.

a $3g(h - 2)$

b $r(10 - 4r)$

Solution

$$\begin{aligned} \text{a } 3g(h - 2) &= 3g \times h - 3g \times 2 \\ &= 3gh - 6g \end{aligned}$$

multiply each term inside the brackets by the term outside

$a(b - c) = ab - ac$

$$\begin{aligned} \text{b } r(10 - 4r) &= r \times 10 - r \times 4r \\ &= 10r - 4r^2 \end{aligned}$$

Example 10

Expand each expression.

a $-(y + 4)$ **b** $-3(2m - 7)$

Solution

a $-(y + 4) = -1(y + 4)$
 $= -1 \times y + (-1) \times 4$
 $= -y + (-4)$
 $= -y - 4$

$-()$ is the same as $-1 \times ()$

b $-3(2m - 7) = -3 \times 2m - (-3) \times 7$
 $= -6m - (-21)$
 $= -6m + 21$

Example 11

Expand and simplify by collecting like terms.

a $4(3p + 2) - 5p$ **b** $7(n - 3) + n(n - 1)$

Solution

a $4(3p + 2) - 5p = 4 \times 3p + 4 \times 2 - 5p$
 $= 12p + 8 - 5p$
 $= 7p + 8$

Expanding

Collecting like terms to simplify.

b $7(n - 3) + n(n - 1) = 7 \times n - 7 \times 3 + n \times n - n \times 1$
 $= 7n - 21 + n^2 - n$
 $= n^2 + 7n - n - 21$
 $= n^2 + 6n - 21$

It's neater to place n^2 at the front.
Collecting like terms to simplify.

Exercise 3-05 Expanding expressions

See Example 8 1 Expand each expression.

a $4(h + 6)$

b $5(3 + t)$

c $3(2b + 7)$

d $5(8 + 3x)$

e $x(x + 9)$

f $p(5 + p)$

g $3r(2r + 1)$

h $4y(1 + 3y)$

i $10e(2e + 4f)$

See Example 9 2 Expand each expression.

a $3(t - 2)$

b $7(5 - d)$

c $8(8g - 3)$

d $w(w - 7)$

e $a(a - 1)$

f $6h(3h - 1)$

g $4x(3x - 1)$

h $5x(2x - 4y)$

i $3a(4b - 7a)$

Worked solutions

Expanding expressions

MAT09NAWS10011

3 Use the substitution $x = 5$ to test whether each equation is correct or incorrect.

a $3(x + 3) = 3x + 6$

b $7(x - 2) = 7x - 14$

c $x(4 - 2x) = 4x - 2x^2$

See Example 10 4 Expand each expression.

a $-(a - 5)$

b $-(a + 5)$

c $-2(x + 6)$

d $-2(x - 6)$

e $-(11 + w)$

f $-(11 - w)$

g $-y(y + 9)$

h $-x(8 - z)$

i $-4t(t - 8)$

5 Expand $-5u(8 + 2u)$. Select the correct answer **A**, **B**, **C** or **D**.

A $3u - 3u^2$

B $-40u + 10u^2$

C $-40u - 10u^2$

D $-40u - 3u^2$

See Example 11 6 Expand and simplify by collecting like terms.

a $5(3m + 2) + 4m$

b $3(1 - 5e) + 6e$

c $4w - 2(5 + 2w)$

d $8 - 5(2x - 7)$

e $t(t + 4) + 3(t + 4)$

f $4(3 + h) + h(7 - 2h)$

g $3x(2x + 5) + 4(2x + 5)$

h $v(2v + 3) - 6(v + 1)$

i $3(1 - 2w) - w(2 - w)$

j $2y(3y - 7) - 5(3y - 7)$

Skillsheet

Factorising using
diagrams

MAT09NASS10015

Homework sheet

Algebra 2

MAT09NAHS10010

Puzzle sheet

Factorising puzzle

MAT09NAPS10030

3-06 Factorising expressions

Summary

The **highest common factor (HCF)** of two or more terms is the largest term that is a factor of all of the terms. To find the HCF of algebraic terms:

- find the HCF of the numbers
- find the HCF of the variables
- multiply them together

Example 12

Find the highest common factor (HCF) of each pair of terms.

- a** $20x^2$ and $15xy$ **b** 16 and $12b$

Solution

Find the HCF of the numbers and the HCF of the variables. Their product is the HCF of the expression.

- a** The HCF of 20 and 15 is 5.

The HCF of x^2 and xy is x , since it is the 'largest' common part of x^2 and xy .

$\therefore$ The HCF of $20x^2$ and $15xy = 5 \times x = 5x$

- b** The HCF of 16 and 12 is 4.

There is **no** HCF of the variables because b is in $12b$ but not in 16.

$\therefore$ The HCF of 16 and $12b$ is 4.

Skillsheet

HCF by factor trees

MAT09NASS10014

Skillsheet

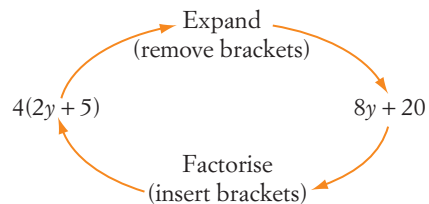
Factorising using
diagrams

MAT09NASS10015

Factorising algebraic expressions

When $4(2y + 5)$ is **expanded**, the answer is $8y + 20$.

Factorising is the reverse of expanding. Factorisation breaks an expression into factors. To factorise $8y + 20$, we take out the greatest common divisor and **insert** brackets. The result is $4(2y + 5)$. The factors are 4 and $(2y + 5)$.



Summary

Factorising an expression

- Find the HCF of the terms and write it outside the brackets
- Divide each term by the HCF and write the answers inside the brackets
 $ab + ac = a(b + c)$
 $ab - ac = a(b - c)$
- To check that the factorised answer is correct, expand it

Video tutorial

Factorising expressions

MAT09NAVT10005

Example 13

Factorise each expression.

a $8y + 16$

b $25b^2 - 20ab$

c $v(4 + w) + 2(4 + w)$

Solution

a The HCF of $8y$ and 16 is 8 .

$$\begin{aligned}\therefore 8y + 16 &= 8 \times y + 8 \times 2 \\ &= 8(y + 2)\end{aligned}$$

Rewrite the expression using the HCF 8 .
Write the HCF at the front of the brackets.

b The HCF of $25b^2$ and $20ab$ is $5b$.

$$\begin{aligned}\therefore 25b^2 - 20ab &= 5b \times 5b - 5b \times 4a \\ &= 5b(5b - 4a)\end{aligned}$$

Rewrite the expression using the HCF $5b$.
Write the HCF at the front of the brackets.

c The HCF of $v(4 + w) + 2(4 + w)$ is $(4 + w)$.

$$\begin{aligned}\therefore v(4 + w) + 2(4 + w) &= (4 + w) \times v + (4 + w) \times 2 \\ &= (4 + w)(v + 2)\end{aligned}$$

Factorising with negative terms

Example 14

Factorise each expression.

a $-x^2 + 3x$

b $-a - ab$

Solution

When factorising expressions that begin with a **negative** term, we use the 'negative' HCF.a The greatest 'negative' common divisor of $-x^2$ and $3x$ is $-x$.

$$\begin{aligned}\therefore -x^2 + 3x &= (-x) \times x + (-x) \times (-3) & (-x) \times (-3) &= +3x \\ &= (-x)(x + (-3)) \\ &= -x(x - 3)\end{aligned}$$

b The greatest 'negative' common divisor of $-a$ and ab is $-a$.

$$\begin{aligned}\therefore -a - ab &= -a \times 1 + (-a) \times b & (-a) \times b &= -ab \\ &= -a(1 + b)\end{aligned}$$

Summary

Factorising with negative terms

- Find the 'negative' HCF of the terms and write it outside the brackets
- Divide each term by the HCF and write the answers inside the brackets

$$-ab - ac = -a(b + c)$$

$$-ab + ac = -a(b - c)$$
- To check that the factorised answer is correct, expand it

Exercise 3-06 Factorising expressions

- 1 Find the HCF of the following terms.

a 12, 6y	b $8a^2$, 24	c 20b, 15
d $6, 9p^2$	e $24mn$, m	f c, c^2
g $18pq, 12p^2$	h $24w^2, 16w$	i $3(x - 5), x(x - 5)$

See Example 12

- 2 Copy and complete each factorisation.

a $18p^2 + 24 = 6(\dots + \dots)$	b $9m^2 + 3m = 3m(\dots + \dots)$
c $20d - 30d^2 = 10d(\dots - \dots)$	d $6x^2y - 8xy = 2xy(\dots - \dots)$
e $x(x - 2) + 5(x - 2) = (x - 2)(\dots + \dots)$	f $n(3 + n) - 3(3 + n) = (3 + n)(\dots - \dots)$
g $h(h + 4) - 2(h + 4) = (\dots + \dots)(h - 2)$	h $v(1 - w) + (1 - w) = (\dots + \dots)(v + 1)$

- 3 Factorise each expression.

a $24x + 30$	b $36 + 27a$	c $16g^2 - 64$	d $xy + y$
e $mn - 3n$	f $6p + pq$	g $x^2 + x$	h $2y - y^2$
i $3d^2 + 6d$	j $16r^2 - 12r$	k $6t^2 + 27t$	l $36p^2 - 108p$
m $12x^2y - 16x$	n $18p^2 + 16pr$	o $4m^2n - 4mn^2$	p $14abc + 21bc$
q $28vw - 21v^2w^2$	r $45rt + 54r^2t$	s $36pq^2r - 144pr$	t $48x^2y + 64xy^2$
u $75g^3h^2 - 125gh$			

See Example 13

- 4 Factorise each expression.

a $a(a - 3) + 6(a - 3)$	b $t(8 + t) - 3(8 + t)$	c $b(b + 5) - 2(b + 5)$
d $x(2 - y) - 6(2 - y)$	e $5(a - 7) + b(a - 7)$	f $s(a + 3) + 4(a + 3)$
g $3p(g + 5) - 2(g + 5)$	h $5(2 - 3m) + 2n(2 - 3m)$	i $r(8 + r) + (8 + r)$
j $(y - 6) - y(y - 6)$		

- 5 Factorise each expression using the 'negative' HCF.

a $-4q - 8$	b $-9u + 18$	c $6 - 3g$	d $-18a + 12$
e $-n - 1$	f $-n + 1$	g $-y^2 - 9y$	h $-6t + 10t^2$
i $-3a^2 - 6ab$	j $-ap + aq$	k $-20e^2 - 22e$	l $-9m + 3m^2$

See Example 14

- 6 Factorise $-10kr + 4rn$. Select the correct answer A, B, C or D.

A $-2r(5k - 4n)$	B $-2r(5k - 2n)$	C $-5r(2k - 2n)$	D $-2r(5k + 2n)$

- 7 Factorise each expression.

a $\frac{1}{2}p + \frac{1}{2}g$	b $7g + 9g^2$	c $\frac{1}{4}x^2 - \frac{1}{4}xy$
d $8a + 12y + 4$	e $20xy - 10x^2 + 5y$	f $t^2m + tm^2 + tm$
g $7y(m + 2) - p(m + 2)$	h $-10p^2 - 10pq$	i $5a(x - y) + 10a^2xy - 10a^2y^2$

Worksheet

Area diagrams

MAT09NAWK10031

Worksheet

Binomial products

MAT09NAWK10032

Homework sheet

Algebra 3

MAT09NAHS10011

Puzzle sheet

Expanding brackets

MAT09NAPS00009

Technology

GeoGebra: Binomial expansion

MAT09NATC00003

Technology worksheet

Excel worksheet: Expanding binomials

MAT09NACT00021

Technology worksheet

Excel spreadsheet: Expanding binomials

MAT09NACT00006

3-07 Expanding binomial products

$(x + 5)$ and $(x - 1)$ are called **binomial expressions** because each expression has exactly two terms (binomial = '2 terms'). $(x + 5)(x - 1)$ is called a **binomial product** because it is a product (multiplication answer) of two binomial expressions.

Expanding binomial products using an area diagram

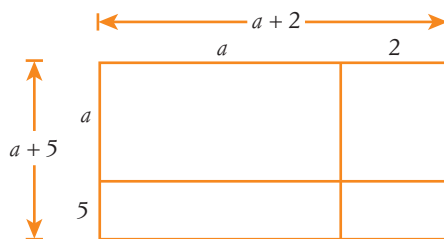
Example 15

Expand each binomial product using an area diagram.

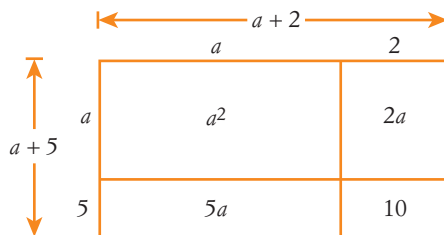
a $(a + 2)(a + 5)$ **b** $(n + 4)(n - 3)$

Solution

- a** Draw an area diagram (rectangle) with length $(a + 2)$ and width $(a + 5)$ and divide the diagram into 4 smaller rectangles.



Find the area of each smaller rectangle.



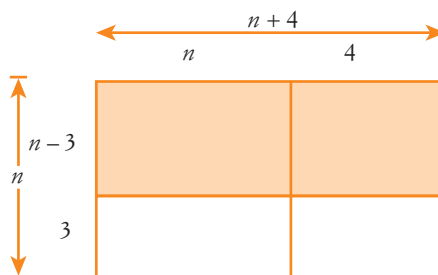
Expand $(a + 2)(a + 5)$ by adding the areas of the 4 rectangles.

$$\begin{aligned}(a + 2)(a + 5) &= a^2 + 2a + 5a + 10 \\ &= a^2 + 7a + 10\end{aligned}$$

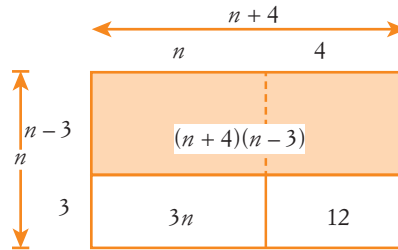
Expanding

Simplifying by collecting like terms.

- b** Draw an area diagram with length $(n + 4)$ and width $(n - 3)$, then increase the width to n by adding 3. This creates 4 rectangles.



The 2 shaded upper rectangles together give the product $(n + 4)(n - 3)$. To find its area, we subtract the areas of the 2 lower rectangles from the area of the whole rectangle.



$$\begin{aligned}(n + 4)(n - 3) &= n(n + 4) - 3n - 12 \\ &= n^2 + 4n - 3n - 12 \\ &= n^2 + n - 12\end{aligned}$$

Whole rectangle – 2 lower rectangles
Expanding
Simplifying by collecting like terms.

Expanding binomial products algebraically

When expanding a binomial product algebraically, each term in the first binomial is multiplied by each term in the second binomial to give **four terms**, which are collected and simplified.

Example 16

Expand each binomial product.

a $(x + 5)(x + 9)$ **b** $(k + 3)(k - 7)$

Solution

$$\begin{aligned}\text{a } (x + 5)(x + 9) &= x(x + 9) + 5(x + 9) \\ &= x^2 + 9x + 5x + 45 \\ &= x^2 + 14x + 45\end{aligned}$$

Each term in $(x + 5)$ is multiplied by $(x + 9)$.
Expanding to make 4 terms.
Adding $9x$ and $5x$.

$$\begin{aligned}\text{b } (k + 3)(k - 7) &= k(k - 7) + 3(k - 7) \\ &= k^2 - 7k + 3k - 21 \\ &= k^2 - 4k - 21\end{aligned}$$

Each term in $(k + 3)$ is multiplied by $(k - 7)$.
Expanding to make 4 terms.
Adding $-7k$ and $3k$.

Video tutorial

Expanding binomial
products

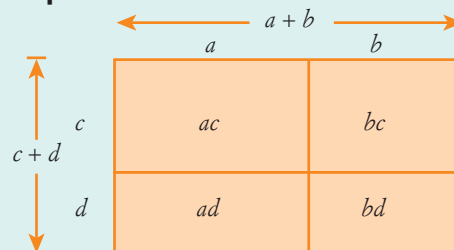
MAT09NAVT10006

Summary

The distributive law for expanding a binomial product

Multiply each term in the first binomial by each term in the second binomial.

$$(a + b)(c + d) = ac + ad + bc + bd$$



One way of remembering which pairs of terms to multiply together in a binomial product is called the FOIL method, as shown below.

$$\begin{array}{c} \text{F} \quad \text{O} \\ \text{I} \quad \text{L} \end{array} \quad (k+3)(k-7) = k^2 - 7k + 3k - 21 \\ = k^2 - 4k - 21$$

- **F** means multiply the first terms: $k \times k = k^2$
- **O** means multiply the outside terms: $k \times (-7) = -7k$
- **I** means multiply the inside terms: $3 \times k = 3k$
- **L** means multiply the last terms: $3 \times (-7) = -21$

Video tutorial

Expanding binomial products

MAT09NAVT10006

Video tutorial

Expanding binomial products

MAT09NAVT00004

Example 17

Expand each binomial product.

a $(x-6)(4x+2)$ **b** $(3t-1)(2t-5)$

Solution

$$\begin{aligned} \text{a } (x-6)(4x+2) &= x(4x+2) - 6(4x+2) \\ &= 4x^2 + 2x - 24x - 12 \\ &= 4x^2 - 22x - 12 \end{aligned}$$

Expanding
Simplifying

$$\begin{aligned} \text{b } (3t-1)(2t-5) &= 3t(2t-5) - 1(2t-5) \\ &= 6t^2 - 15t - 2t + 5 \\ &= 6t^2 - 17t + 5 \end{aligned}$$

Expanding
Simplifying

Exercise 3-07 Expanding binomial products

See Example 15 1 Expand each binomial product using the given area diagram.

a $(x+10)(x+8)$

	x	10
x	---	---
8	---	---

b $(a+3)(a+6)$

	a	3
a	---	---
6	---	---

c $(d+2)(2d+5)$

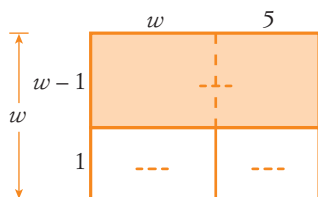
	d	2
$2d$	---	---
5	---	---

d $(4h+1)(3h+7)$

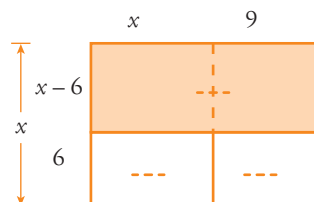
	$4h$	1
$3h$	---	---
7	---	---

2 Expand each binomial product using the given area diagram. (Find the shaded area.)

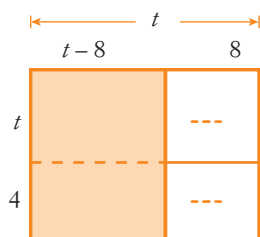
a $(w + 5)(w - 1)$



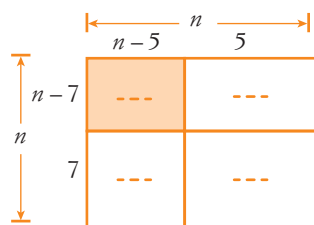
b $(x + 9)(x - 6)$



c $(t - 8)(t + 4)$



d $(n - 5)(n - 7)$



3 Draw an area diagram for each binomial product and use it to expand the product.

a $(2y + 5)(y + 3)$

b $(r + 5)(3r + 2)$

c $(5k + 3)(4k + 1)$

d $(w - 5)(w + 3)$

e $(v + 4)(3v - 1)$

f $(p - 3)(p - 5)$

4 Expand each binomial product algebraically.

a $(n + 6)(n + 3)$

b $(t + 6)(t + 5)$

c $(p + 10)(p - 10)$

d $(x - 8)(x + 3)$

e $(b - 2)(9 + b)$

f $(u - 8)(u - 7)$

g $(15 - r)(r + 1)$

h $(a - 10)(a - 9)$

i $(5 - c)(3 - c)$

j $(t - 1)(t + 2)$

k $(y - 4)(y + 10)$

l $(n - 9)(11 + n)$

5 Expand $(b + 7)^2$. Select the correct answer A, B, C or D.

A $b^2 + 49$

B $b^2 + 49b$

C $b^2 + 7b + 49$

D $b^2 + 14b + 49$

6 Expand each binomial product.

a $(2x + 5)(x + 3)$

b $(3e + 2)(4e + 5)$

c $(10 + 3p)(p - 1)$

d $(7d - 2)(7d - 2)$

e $(2f - 2)(3f + 5)$

f $(4m - 5)(5 + 3m)$

g $(3 - 4h)(2 + 5h)$

h $(4p - 5)(2p - 7)$

i $(2m - 3)(4 - 5m)$

j $(3t + 5)(2t - 1)$

k $(5y - 5)(5y + 5)$

l $(6 - 2a)(2a - 6)$

7 A rectangular barbecue plate has a length of 100 cm and a width of 75 cm. The length and width are both increased by x cm.

a Write an expression for the new length of the plate.

b Write an expression for the new width of the plate.

c Hence find a simplified expression for the new area of the plate.

d By how much has the area of the plate increased?

e If $x = 0.1$, find the increase in the area of the plate.

See Example 16

See Example 17

Worked solutions
Expanding binomial
products

MAT09NAWS10012

- 8 A family room in a house is to be extended. The room is 4 metres long and 3 metres wide. The length is to be increased by x metres and the width by y metres.
- Write down expressions for the new length and width.
 - Write down a binomial expression for the new area of the room.
 - Expand and simplify your expression for the area.
 - By how much has the area of the room increased?

Investigation: Expanding perfect squares

There is a special pattern when you expand a binomial by itself, for example, $(y + 5)(y + 5)$.

This is called the square of a binomial or a perfect square.

- Expand and simplify the perfect square $(y + 5)^2$ if $(y + 5)^2 = (y + 5)(y + 5)$.
 - How many terms are there in your answer?
 - The terms in the binomial $(y + 5)$ are y and 5. The first term of your *answer* in part **a** is y^2 , which is the square of y . How are the other terms in the answer related to y and 5?
- Expand and simplify each perfect square.
 - $(k + 3)^2$
 - $(m + 7)^2$
 - $(p + 2)^2$
- For each of the expansions in question 2:
 - How many terms are there?
 - Describe how each is related to the two terms of the perfect square.
 - Compare and discuss your results with other students.
- Expand and simplify each perfect square.
 - $(t - 1)^2$
 - $(g - 6)^2$
 - $(d - 5)^2$
- For each of the expansions in question 4:
 - How many terms are there?
 - Describe how each is related to the two terms of the perfect square.
 - Compare and discuss your results with other students.
- Describe the expansion of the perfect square $(\text{first term} + \text{second term})^2$ by copying and completing this statement:
The square of a binomial is equal to the square of the first term plus double the product of the two terms plus _____
- Find a formula for $(a + b)^2$.
 - Check this formula by expanding $(a + b)^2$ using an area diagram.
 - Find a formula for $(a - b)^2$.
 - Check this formula by expanding $(a - b)^2$ using an area diagram.

3-08 Perfect squares

16, 49, v^2 and $(y + 5)^2$ are called **perfect squares** because they are square numbers. The rule for expanding the perfect square of a binomial is:

Summary

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

Proof:

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b) \\ &= a(a + b) + b(a + b) \\ &= a^2 + ab + ba + b^2 \\ &= a^2 + 2ab + b^2\end{aligned}$$

$$\begin{aligned}(a - b)^2 &= (a - b)(a - b) \\ &= a(a - b) - b(a - b) \\ &= a^2 - ab - ba + b^2 \\ &= a^2 - 2ab + b^2\end{aligned}$$

Example 18

Copy and complete the expansion of each perfect square.

a $(x + 4)^2 = x^2 + \underline{\hspace{2cm}} + 16$

b $(y - 6)^2 = y^2 - 12y + \underline{\hspace{2cm}}$

c $(5g + 9)^2 = \underline{\hspace{2cm}} + 45g + 81$

d $(3d - 5)^2 = 9d^2 - \underline{\hspace{2cm}} + 25$

Solution

a In the expansion,

$$\begin{aligned}\underline{\hspace{2cm}} &= 2 \times x \times 4 \\ &= 8x\end{aligned}$$

$$\therefore (x + 4)^2 = x^2 + 8x + 16$$

Doubling the product of the two terms.

b In the expansion,

$$\begin{aligned}\underline{\hspace{2cm}} &= 6^2 \\ &= 36\end{aligned}$$

$$\therefore (y - 6)^2 = y^2 - 12y + 36$$

The second term squared.

c $\underline{\hspace{2cm}} = (5g)^2$
 $\hspace{10em} = 25g^2$

$$\therefore (5g + 9)^2 = 25g^2 + 45g + 81$$

The first term squared.

d $\underline{\hspace{2cm}} = 2 \times 3d \times 5$
 $\hspace{10em} = 30d$

$$\therefore (3d - 5)^2 = 9d^2 + 30d + 25$$

Doubling the product of the two terms.

Stage 5.3

Puzzle sheet

Factorominoes

MAT09NAPS10033

Stage 5.3

Example 19

Expand each perfect square.

a $(n - 5)^2$

b $(k + 7)^2$

c $(3y - 8)^2$

Solution

$$\begin{aligned} \text{a } (n - 5)^2 &= n^2 - 2 \times n \times 5 + 5^2 \\ &= n^2 - 10n + 25 \end{aligned}$$

1st term squared – double product +
2nd term squared

$$\begin{aligned} \text{b } (k + 7)^2 &= k^2 + 2 \times k \times 7 + 7^2 \\ &= k^2 + 14k + 49 \end{aligned}$$

$$\begin{aligned} \text{c } (3y - 8)^2 &= (3y)^2 - 2 \times 3y \times 8 + 8^2 \\ &= 9y^2 - 48y + 64 \end{aligned}$$

Exercise 3-08 Perfect squares

See Example 18

1 Copy and complete the expansion of each perfect square.

a $(x + 10)^2 = x^2 + \underline{\hspace{2cm}} + 100$

b $(m - 8)^2 = \underline{\hspace{2cm}} - 16m + 64$

c $(p - t)^2 = p^2 - 2pt + \underline{\hspace{2cm}}$

d $(h + 4)^2 = h^2 \underline{\hspace{2cm}} + 16$

e $(k - 9)^2 = k^2 \underline{\hspace{2cm}} + 81$

f $(8 + 5f)^2 = 64 \underline{\hspace{2cm}} + 25f^2$

g $(2d + 3)^2 = \underline{\hspace{2cm}} + \underline{\hspace{2cm}} + 9$

h $(6a + 1)^2 = \underline{\hspace{2cm}} + 12a + \underline{\hspace{2cm}}$

See Example 19

2 Expand each perfect square.

a $(m + 9)^2$

b $(u + 3)^2$

c $(y - 6)^2$

d $(8 + k)^2$

e $(5 - h)^2$

f $(7 + k)^2$

g $(f + 20)^2$

h $(q - 11)^2$

i $(10 + t)^2$

j $(x - w)^2$

k $(a + g)^2$

l $(2m - 3)^2$

m $(5x - 6)^2$

n $(9a + 2)^2$

o $(3e - 4)^2$

p $(5 + 7b)^2$

q $(4 - 5p)^2$

r $(11 - 2c)^2$

s $(10g + 3)^2$

t $(3k + 11)^2$

u $(5 + 2v)^2$

3 Expand each perfect square.

a $(7h + 2k)^2$

b $(8a - 3y)^2$

c $(xy + z)^2$

d $\left(1 + \frac{1}{y}\right)^2$

e $\left(t - \frac{1}{t}\right)^2$

f $\left(\frac{3}{w} + w\right)^2$

4 Use expansion to evaluate each square number without using a calculator.

a $21^2 = (20 + 1)^2$

b $45^2 = (40 + 5)^2$

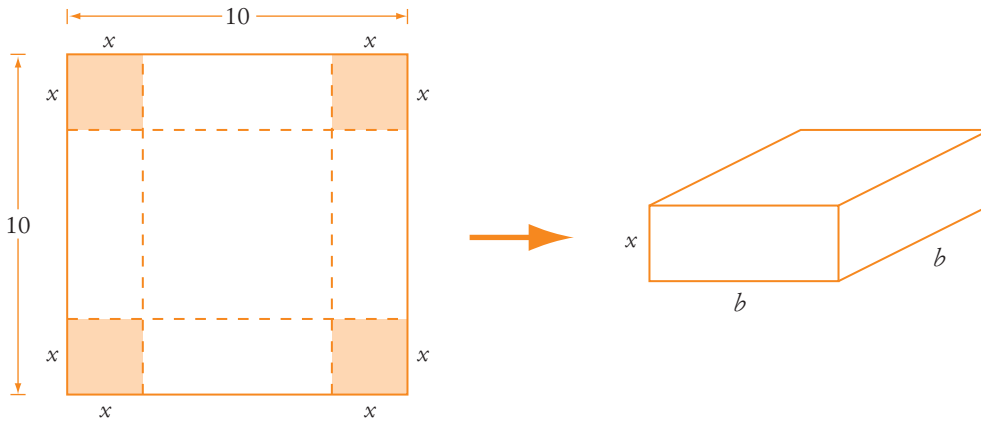
c $29^2 = (30 - 1)^2$

d $59^2 = (60 - 1)^2$

e $102^2 = (100 + 2)^2$

f $98^2 = (100 - 2)^2$

- 5 A 10 cm square sheet of cardboard has a square of length x cm cut from each corner. It is then folded to form a square-based prism.



- Why is the length of the square base (b) equal to $(10 - 2x)$ cm?
- Find the area of the square base.
- Find the area of one side face of the prism.
- Hence show that the surface area of the prism is $(100 - 4x^2)$ cm².

Stage 5.3

Worked solutions

Perfect squares

MAT09NAWS10013

Investigation: Squaring a number ending in 5

Study this mental short cut for squaring a number ending in 5.

- To evaluate 35^2 , calculate $3 \times 4 = 12$, add '25' to the end: $35^2 = 1225$.
- To evaluate 75^2 , calculate $7 \times 8 = 56$, add '25' to the end: $75^2 = 5625$.
- To evaluate 105^2 , calculate $10 \times 11 = 110$, add '25' to the end: $105^2 = 11025$.

Let n stand for the tens digit of the number ending in 5 being squared.
Expand $(10n + 5)^2$ and investigate why the above method works.

Mental skills 3B Maths without calculators

Multiplying by 9, 11, 99 and 101

We can use expansion when multiplying by a number near 10 or near 100.

- 1 Study each example.

$$\begin{aligned} \text{a } 25 \times 11 &= 25 \times (10 + 1) \\ &= 25 \times 10 + 25 \times 1 \\ &= 250 + 25 \\ &= 275 \end{aligned}$$

$$\begin{aligned} \text{b } 14 \times 9 &= 14 \times (10 - 1) \\ &= 14 \times 10 - 14 \times 1 \\ &= 140 - 14 \\ &= 126 \end{aligned}$$

$$\begin{aligned}
 \text{c } 32 \times 12 &= 32 \times (10 + 2) \\
 &= 32 \times 10 + 32 \times 2 \\
 &= 320 + 64 \\
 &= 384
 \end{aligned}$$

$$\begin{aligned}
 \text{d } 7 \times 99 &= 7 \times (100 - 1) \\
 &= 7 \times 100 - 7 \times 1 \\
 &= 700 - 7 \\
 &= 693
 \end{aligned}$$

$$\begin{aligned}
 \text{e } 27 \times 101 &= 27 \times (100 + 1) \\
 &= 27 \times 100 + 27 \times 1 \\
 &= 2700 + 27 \\
 &= 2727
 \end{aligned}$$

$$\begin{aligned}
 \text{f } 18 \times 8 &= 18 \times (10 - 2) \\
 &= 18 \times 10 - 18 \times 2 \\
 &= 180 - 36 \\
 &= 144
 \end{aligned}$$

2 Now evaluate each product.

$$\text{a } 16 \times 11$$

$$\text{b } 33 \times 11$$

$$\text{c } 29 \times 9$$

$$\text{d } 45 \times 9$$

$$\text{e } 62 \times 11$$

$$\text{f } 7 \times 101$$

$$\text{g } 18 \times 101$$

$$\text{h } 36 \times 99$$

$$\text{i } 19 \times 8$$

$$\text{j } 45 \times 12$$

$$\text{k } 21 \times 102$$

$$\text{l } 6 \times 98$$

$$\text{m } 32 \times 9$$

$$\text{n } 7 \times 99$$

$$\text{o } 39 \times 101$$

$$\text{p } 71 \times 12$$

Stage 5.3

Investigation: Expanding sums by differences

There is also a special pattern when you multiply the sum of two terms by the difference of those two terms, for example, $(x + 9)(x - 9)$.

1 a Expand and simplify $(x + 9)(x - 9)$.

b How many terms are there in your answer?

c How are the terms in the answer related to x and 9?

2 Expand and simplify each 'sum by difference'.

$$\text{a } (k + 2)(k - 2)$$

$$\text{b } (m + 5)(m - 5)$$

$$\text{c } (p - 6)(p + 6)$$

3 Describe the expansion of (first term + second term)(first term - second term) by copying and completing this statement:

The product of a 'sum by difference' is equal to the square of the first term _____

4 a Find a formula for $(a + b)(a - b)$.

b Check this formula by expanding $(a + b)(a - b)$ using an area diagram.

Worksheet

Special products

MAT09NAWK00034

Puzzle sheet

Enough time

MAT09NAPS00011

3-09 Difference of two squares

The rule for expanding $(a + b)(a - b)$ is:

Summary

$$(a + b)(a - b) = a^2 - b^2$$

The answer is called the **difference of two squares**.

Proof:

$$\begin{aligned}(a-b)(a+b) &= a(a+b) - b(a+b) \\ &= a^2 + ab - ba - b^2 \\ &= a^2 - b^2 + ab - ba = 0\end{aligned}$$

When the sum of two terms is multiplied by their difference, the answer is the square of the first term minus the square of the second term (the difference of two squares).

Example 20

Expand each expression.

a $(d+3)(d-3)$ **b** $(2+r)(2-r)$ **c** $(7x+2)(7x-2)$ **d** $(4k-5p)(4k+5p)$

Solution

$$\begin{aligned}\mathbf{a} \quad (d+3)(d-3) &= d^2 - 3^2 \\ &= d^2 - 9\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad (2+r)(2-r) &= 2^2 - r^2 \\ &= 4 - r^2\end{aligned}$$

$$\begin{aligned}\mathbf{c} \quad (7x+2)(7x-2) &= (7x)^2 - 2^2 \\ &= 49x^2 - 4\end{aligned}$$

$$\begin{aligned}\mathbf{d} \quad (4k-5p)(4k+5p) &= (4k)^2 - (5p)^2 \\ &= 16k^2 - 25p^2\end{aligned}$$

Exercise 3-09 Difference of two squares

1 Expand each expression.

a $(m+5)(m-5)$	b $(c-10)(c+10)$	c $(a+12)(a-12)$
d $(6-y)(6+y)$	e $(8-m)(8+m)$	f $(p+1)(p-1)$
g $(5+e)(5-e)$	h $(v+11)(v-11)$	i $(w-3)(w+3)$
j $(x-10)(x+10)$	k $(q+7)(q-7)$	l $(9-g)(9+g)$
m $(b-2)(b+2)$	n $(15-r)(15+r)$	o $(d+13)(d-13)$

2 Expand each expression.

a $(2h-3)(2h+3)$	b $(5r+4)(5r-4)$	c $(5b+8)(5b-8)$
d $(4p-7)(4p+7)$	e $(3-8k)(3+8k)$	f $(7x-5)(7x+5)$
g $(2+9m)(2-9m)$	h $(9k-4l)(9k+4l)$	i $(7n+8m)(7n-8m)$
j $(4g-5h)(4g+5h)$	k $(7u+3w)(7u-3w)$	l $(11a+3b)(11a-3b)$
m $\left(t+\frac{1}{t}\right)\left(t-\frac{1}{t}\right)$	n $\left(\frac{w}{3}-2\right)\left(\frac{w}{3}+2\right)$	o $\left(1-\frac{1}{r}\right)\left(1+\frac{1}{r}\right)$

3 Susan's age is p years.

- What was Susan's age last year?
- What will Susan's age be next year?
- Write an expression for (Susan's age last year) $\times$ (Susan's age next year).
- If (Susan's age last year) $\times$ (Susan's age next year) is equal to 48, what is Susan's age?

4 By expressing 31×29 as $(30+1)(30-1)$, use the difference of two squares to find the value of 31×29 .

See Example 20

Worked solutions

Difference of two squares

MAT09NAWS10014

Stage 5.3

- 5 Use the method of question 4 to evaluate each expression.

a 21×19

b 51×49

c 89×91

d 78×82

Worked solutions

- 6 Use 'sum by difference' to evaluate each expression.

a $15^2 - 14^2$

b $24^2 - 23^2$

c $65^2 - 35^2$

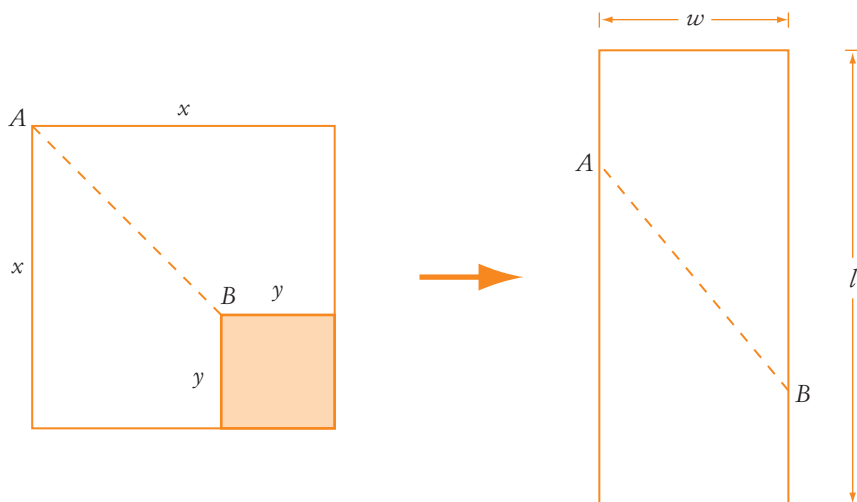
d $101^2 - 99^2$

e $23^2 - 17^2$

f $50^2 - 48^2$

MAT09NAWS10014

- 7 A square sheet of paper of length
- x
- cm has a smaller square of length
- y
- cm cut from one corner. It is then cut on the diagonal
- AB
- and rearranged to form a rectangle as shown.



- a Find an expression for:
- the length (l) of the rectangle
 - the width (w) of the rectangle
 - the area of the rectangle.
- b The area of the rectangle should be the same as the area of the square on the left minus the area of the smaller square. Find an expression for this area. What does this prove?

Worksheet

Mixed expansions

MAT09NAWK00035

3-10 Mixed expansions

Example 21

Expand and simplify each expression.

a $(4r + 5)(1 - 2r)$

b $(7 + 9x)^2$

c $(2d - 10)(2d + 10)$

d $(a + 6)(a - 6) + (a + 12)(a + 3)$

e $(m - 2)^2 - (m - 2)(m + 2)$

Solution

$$\begin{aligned}
 \text{a } (4r + 5)(1 - 2r) &= 4r(1 - 2r) + 5(1 - 2r) \\
 &= 4r - 8r^2 + 5 - 10r \\
 &= -8r^2 - 6r + 5
 \end{aligned}$$

$$\begin{aligned}\text{b } (7 + 9x)^2 &= 7^2 + 2 \times 7 \times 9x + (9x)^2 \\ &= 49 + 126x + 81x^2 \\ &= 81x^2 + 126x + 49 \\ \text{c } (2d - 10)(2d + 10) &= (2d)^2 - 10^2 \\ &= 4d^2 - 100 \\ \text{d } (a + 6)(a - 6) + (a + 12)(a + 3) &= a^2 - 36 + a^2 + 3a + 12a + 36 \\ &= 2a^2 + 15a \\ \text{e } (m - 2)^2 - (m - 2)(m + 2) &= m^2 - 4m + 4 - (m^2 - 4) \\ &= m^2 - 4m + 4 - m^2 + 4 \\ &= -4m + 8\end{aligned}$$

Exercise 3-10 Mixed expansions

1 Expand and simplify each expression.

$$\begin{array}{lll}\text{a } (2m - 1)(2m + 1) & \text{b } (y + 4)(5y - 3) & \text{c } (2k - 7)^2 \\ \text{d } (d + 9)^2 & \text{e } (2e - 1)(e + 1) & \text{f } (5a + 4)(5a - 4) \\ \text{g } (2 - p)(p - 2) & \text{h } (10 - 6y)(10 + 6y) & \text{i } (h - 3z)^2 \\ \text{j } (2x - 3)(y + 3) & \text{k } (11a - 4b)(11a + 4b) & \text{l } \left(u - \frac{1}{u}\right)^2\end{array}$$

2 Expand and simplify each expression.

$$\begin{array}{ll}\text{a } (m - 5)(m + 5) + 25 & \text{b } 6y + (y - 3)^2 + 9 \\ \text{c } (3x + 1)(2 - x) + 2x + 4 & \text{d } (d + 4)^2 - 8d + 5 \\ \text{e } 16 + (4k - 8)(4k + 8) & \text{f } (x - y)^2 - (x + y)^2 \\ \text{g } 20t - (t - 2)(t - 5) + t^2 & \text{h } 2(f - 2)(f + 2) \\ \text{i } (2h + 3)^2 - (2h - 3)(2h + 3) & \text{j } 7xy - (2x - 3)(y + 3)\end{array}$$

3 Expand and simplify each expression.

$$\begin{array}{ll}\text{a } (8a - 1)(8a + 1) - 4a^2 + 1 & \text{b } (n + 1)^2 + 2n + 3 \\ \text{c } 3(4 - t)(4 + t) + (t - 12)(t + 4) & \text{d } (2m - n)^2 + (2m + n)^2 \\ \text{e } (x - 2)(x + 3) - (x - 2)(x + 2) & \text{f } 2(b - 1)^2 - (2b - 1)^2 \\ \text{g } (y + 1)^2 + (y + 2)^2 + (y + 3)^2 & \text{h } (x - 3)(x + 3) + (x + 3)^2 + (x - 3)^2 \\ \text{i } (5n + 3)(5n - 3) + (3n - 5)(3n + 5) & \text{j } 2(a - b)(a + b) - (a + b)^2 - (a - b)^2\end{array}$$

See Example 21

Worked solutions

Mixed expansions

MAT09NAWS10015

3-11 Factorising special binomial products

Factorising by grouping in pairs

An algebraic expression with four terms can often be factorised in pairs, that is, two terms at a time, to make a binomial product.

Stage 5.3

Example 22

Factorise each expression.

a $3ac + 2bd + 2bc + 3ad$ **b** $4km + 6mn - 6kp - 9np$ **c** $10xw - 6yw - 10xt + 6yt$

Solution

a $3ac + 2bd + 2bc + 3ad = 3ac + 3ad + 2bd + 2bc$ Grouping into pairs for factorising.
 $= 3a(c + d) + 2b(d + c)$ Factorising each pair.
 $= (c + d)(3a + 2b)$ Factorising again.

b $4km + 6mn - 6kp - 9np = 2m(2k + 3n) - 3p(2k + 3n)$ Factorising each pair.
 $= (2k + 3n)(2m - 3p)$ Factorising again.

c $10xw - 6yw - 10xt + 6yt = 2(5xw - 3yw - 5xt + 3yt)$ Factorising all terms first.
 $= 2(5xw - 5xt - 3yw + 3yt)$ Grouping into pairs for factorising.
 $= 2[5x(w - t) - 3y(w - t)]$ Factorising each pair.
 $= 2(w - t)(5x - 3y)$ Factorising again.

Factorising the difference of two squares

You should recall the product $(a + b)(a - b) = a^2 - b^2$.

If we use this rule in reverse, then the factors of $a^2 - b^2$ are $(a - b)$ and $(a + b)$.

Summary

$$a^2 - b^2 = (a + b)(a - b)$$

Example 23

Factorise each expression.

a $x^2 - 4$ **b** $9 - 16b^2$ **c** $20d^2 - 5a^2$ **d** $y^3 - y$

Solution

a $x^2 - 4 = x^2 - 2^2$
 $= (x + 2)(x - 2)$

b $9 - 16b^2 = 3^2 - (4b)^2$
 $= (3 + 4b)(3 - 4b)$

c $20d^2 - 5a^2 = 5(4d^2 - a^2)$
 $= 5[(2d)^2 - a^2]$
 $= 5(2d + a)(2d - a)$

d $y^3 - y = y(y^2 - 1)$
 $= y(y + 1)(y - 1)$

Exercise 3-11 Factorising special binomial products

1 Factorise each expression.

a $4ab + 5bc + 4ad + 5cd$	b $2xy - 5wy + 2xt - 5wt$
c $9ac + 6bc + 12ad + 8bd$	d $10x^2 + 30 + x^3 + 3x$
e $3a^2 + 3ab + 3ac + 3bc$	f $6rt - 18wt + 6rp - 18wp$
g $14e - 21 + 2de - 3d$	h $hk - h^2 - 2k + 2h$
i $3mn - 6m + pn - 2p$	j $9p^2 - 27 + qp^2 - 3q$
k $fg - fh - 10g + 10h$	l $9kl - 12ml + 9kn - 12mn$
m $2p - 2c - p^2 + pc$	n $l^3 + lm^2 - 3l^2 - 3m^2$
o $a(x + 1) + y(x + 1) - ka - ky$	p $p(a - b) - 2q(a - b) + 3qp - 6q^2$

2 Factorise each expression.

a $w^2 - 9$	b $y^2 - 36$	c $k^2 - 1$	d $m^2 - 121$
e $p^2 - 64$	f $t^2 - 100$	g $4e^2 - f^2$	h $a^2 - 9b^2$
i $16y^2 - 1$	j $4 - b^2$	k $25 - e^2$	l $1 - 16x^2$
m $y^2 - z^2$	n $49 - 16m^2$	o $b^2 - 121d^2$	p $36c^2 - 25k^2$
q $16 - 81h^2$	r $25a^2 - 64m^2$	s $100 - 49n^2$	t $121p^2 - 144q^2$
u $\frac{1}{4} - 25c^2$	v $4t^2 - \frac{1}{9}$	w $25h^2 - 2\frac{1}{4}$	x $1 - m^2n^2$

3 Factorise each expression.

a $2a^2 - 2b^2$	b $7k^2 - 28$	c $3 - 75u^2$	d $x^3 - 49x$
e $k - 16k^3$	f $50q^2 - 2$	g $3d^2 - 12v^2$	h $5t^5 - 125t^3$
i $2a^2b^2 - 2$	j $x^2y^2 - x^2w^2$	k $192f^2 - 108g^2$	l $45d^2 - \frac{5}{4}$
m $2x^2 - 8a^2$	n $100 - 25w^2$	o $1\frac{1}{4} - 80e^2$	p $9c^2 - 6\frac{1}{4}$

4 Factorise each expression.

a $\frac{p^2}{16} - \frac{x^2}{25}$	b $x^2 - \frac{1}{9}$	c $\frac{v^2}{49} - \frac{u^2}{81}$	d $\frac{2y^2}{9} - \frac{2m^2}{121}$
e $\frac{16a^2}{49} - \frac{25b^2}{4}$	f $t^4 - 81$	g $100 - n^4$	h $(x + y)^2 - x^2$
i $(p - 2q)^2 - (2p + q)^2$	j $\frac{x^2}{4} - \frac{y^2}{36}$	k $(a + b)^2 - (a - b)^2$	

Investigation: Factorising quadratic expressions

- a Show that $(x + 3)(x + 5) = x^2 + 8x + 15$

b The quadratic expression $x^2 + 8x + 15$ has three terms. The coefficient of x is 8, the number in front of the x . How are the 3 and 5 in $(x + 3)(x + 5)$ related to the 8?

c The constant term in $x^2 + 8x + 15$ is 15, the number with no x at the end. How are the 3 and 5 related to the 15?
- a Expand $(x + 9)(x + 2)$.

b What is the coefficient of x ? How are 9 and 2 related to it?

c What is the constant term? How are 9 and 2 related to it?
- a Expand $(x + 8)(x - 3)$.

b What is the coefficient of x ? How are 8 and -3 related to it?

c What is the negative constant term? How are 8 and -3 related to it?

Stage 5.3

- 4 a Expand $(x - 4)(x - 1)$.
 b What is the negative coefficient of x ? How are -4 and -1 related to it?
 c What is the positive constant term? How are -4 and -1 related to it?
- 5 In the expansion of any binomial product, how are the coefficient of x and the constant term related to the numbers in the binomials?
- 6 Copy and complete:
- a $(x + \underline{\quad})(x + \underline{\quad}) = x^2 + 5x + 4$ b $(x + \underline{\quad})(x + \underline{\quad}) = x^2 + 8x + 15$
 c $(x + \underline{\quad})(x + \underline{\quad}) = x^2 + 7x + 12$ d $(x + \underline{\quad})(x - \underline{\quad}) = x^2 - 4x - 32$
 e $(x + \underline{\quad})(x - \underline{\quad}) = x^2 + 2x - 3$ f $(x - \underline{\quad})(x - \underline{\quad}) = x^2 - 9x + 20$

3-12 Factorising quadratic expressions

A quadratic expression such as $x^2 - 5x + 7$ is called a **trinomial** because it has three terms. Other examples of quadratic trinomials are $x^2 + x - 15$, $2x^2 - 3x + 9$ and $-4x^2 + 9x + 20$.

The expansion of $(x + 2)(x + 4)$ is $x^2 + 6x + 8$.

$\therefore$ The factorisation of $x^2 + 6x + 8$ is $(x + 2)(x + 4)$.

Summary

In the factorisation of a quadratic trinomial such as $x^2 + 6x + 8$:

- each factor must have an x term to give x^2

$$x^2 + 6x + 8 = (x + 2)(x + 4)$$

- $2 + 4 = 6$, which is the **coefficient** of x , the number in front of the x

$$x^2 + 6x + 8 = (x + 2)(x + 4)$$

- $2 \times 4 = 8$, which is the **constant term** with no x .

$$x^2 + 6x + 8 = (x + 2)(x + 4)$$

Example 24

Factorise $x^2 + 11x + 24$.

Solution

Find the two numbers that have a sum of 11 and a product of 24.

It is best to test numbers that have a **product of 24** and then check if their sums equal 11:

The correct numbers are 8 and 3.

$$\therefore x^2 + 11x + 24 = (x + 8)(x + 3)$$

Pair of numbers	Product	Sum
6, 4	$6 \times 4 = 24$	$6 + 4 = 10$
2, 12	$2 \times 12 = 24$	$2 + 12 = 14$
8, 3	$8 \times 3 = 24$	$8 + 3 = 11$

Example 25

Factorise $x^2 - 5x - 6$.

Solution

Find two numbers with a sum of -5 and a product of -6 .

Since their product is -6 , one of the numbers must be negative.

Test numbers that have a product of -6 and check their sums.

The correct numbers are -6 and 1 .

$$\therefore x^2 - 5x - 6 = (x - 6)(x + 1)$$

Pair of numbers	Product	Sum
$+3, -2$	$3 \times (-2) = -6$	$3 + (-2) = 1$
$-3, +2$	$-3 \times 2 = -6$	$-3 + 2 = -1$
$+6, -1$	$6 \times (-1) = -6$	$6 + (-1) = 5$
$-6, +1$	$-6 \times 1 = -6$	$-6 + 1 = -5$

Summary

To factorise quadratic trinomials of the form $x^2 + bx + c$:

- find two numbers that have a sum of b and a product of c
- use these two numbers to write a binomial product of the form $(x \text{ \_\_\_\_ })(x \text{ \_\_\_\_ })$

Example 26

Factorise each quadratic expression.

a $x^2 + 6x - 16$ **b** $m^2 - 7m - 18$ **c** $y^2 - 2y + 1$

Solution

a $x^2 + 6x - 16$

Find two numbers that have a product of -16 and a sum of 6 .

Since the product is negative, one of the numbers must be negative.

They are $+8$ and -2 .

$$\therefore x^2 + 6x - 16 = (x + 8)(x - 2)$$

b $m^2 - 7m - 18$

Product = -18 , sum = -7 .

Since the product is negative, one of the numbers must be negative.

They are -9 and $+2$.

$$\therefore m^2 - 7m - 18 = (m - 9)(m + 2)$$

c $y^2 - 2y + 1$

Product = 1 , sum = -2 .

Since the sum is negative, one of the numbers must be negative.

Since the product is positive, both of the numbers must be negative.

They are -1 and -1 .

$$\begin{aligned} \therefore y^2 - 2y + 1 &= (y - 1)(y - 1) \\ &= (y - 1)^2 \end{aligned}$$

Note: $(y - 1)^2$ is a perfect square.

Stage 5.3

Example 27

Factorise each quadratic expression.

a $3g^2 + 12g - 36$

b $48 - 8p - p^2$

Solution

a $3g^2 + 12g - 36 = 3(g^2 + 4g - 12)$
 $= 3(g - 2)(g + 6)$

Taking out the HCF of 3 first.

Product = -12 , sum = 4

b $48 - 8p - p^2 = -p^2 - 8p + 48$
 $= -1(p^2 + 8p - 48)$
 $= -(p + 12)(p - 4)$

Rearranging the terms to make the p^2 term first.

Taking out a common factor of -1 .

Product = -48 , sum = 8

Exercise 3-12 Factorising quadratic expressions

1 Find two numbers whose:

a product is 6 and their sum is -7

b product is -12 and their sum is 1

c product is -15 and their sum is -2

d product is 12 and their sum is 7

e product is 20 and their sum is -9

f product is -14 and their sum is 5

g product is -10 and their sum is 3

h product is -25 and their sum is 0

i product is -2 and their sum is -1

j product is -18 and their sum is -7

See Example 24

2 Factorise each quadratic expression.

a $x^2 + 7x + 12$

b $x^2 + 12x + 35$

c $x^2 + 5x + 4$

d $x^2 + 7x + 10$

e $x^2 + 9x + 20$

f $t^2 + 6t + 5$

g $e^2 + 5e + 6$

h $h^2 + 4h + 4$

i $n^2 + 11n + 10$

j $a^2 + 11a + 30$

k $d^2 + 10d + 24$

l $y^2 + 15y + 44$

See Example 25

3 Factorise each quadratic expression.

a $y^2 - 2y - 3$

b $r^2 - 5r - 14$

c $h^2 - 3h - 4$

d $w^2 - 7w - 18$

e $e^2 - 6e - 27$

f $a^2 - 4a - 12$

See Example 26

4 Factorise each quadratic expression.

a $x^2 + 3x - 4$

b $t^2 + 5t - 24$

c $m^2 + 2m - 15$

d $a^2 + a - 2$

e $k^2 + 5k - 14$

f $w^2 + 4w - 12$

g $m^2 - 5m + 4$

h $w^2 - 6m + 8$

i $x^2 - 12x + 35$

j $p^2 - 10p + 24$

k $n^2 - 3n + 2$

l $a^2 - 6a + 9$

5 Factorise each expression given it is a perfect square.

a $m^2 + 4m + 4$

b $p^2 + 20p + 100$

c $a^2 - 10a + 25$

See Example 27

6 Factorise each quadratic expression. Look for the highest common factor first.

a $3m^2 + 9m + 6$

b $2y^2 + 2y - 4$

c $5t^2 - 10t - 400$

d $5e^4 + 25e^3 - 120e^2$

e $x^3 - x^2 - 110x$

f $4b^2 - 4b - 168$

g $4w^2 + 4w - 48$

h $3a^3 - 9a^2 - 12a$

i $2e^2 + 18e + 40$

j $24 - 5t - t^2$

k $42 + u - u^2$

l $28 + 3x - x^2$

m $12 - b - b^2$

n $7k - 12 - k^2$

o $12x - 35 - x^2$

7 Factorise each quadratic expression.

a $h^2 - 3h - 18$

b $18 - 7t - t^2$

c $w^2 + 8w + 7$

d $k^2 - 12k - 45$

e $v^2 + 8v - 20$

f $3c^2 + 30c + 75$

g $q^2 - 6q + 5$

h $a^2 - 4a + 3$

i $6x^2 + 36x - 96$

j $x^2 - 16x + 64$

k $8u^2 - 24u - 32$

l $b^2 + 11b + 30$

m $y^2 - 12y + 36$

n $5r^2 - 5r - 10$

o $4l^2 - 8l - 32$

p $g^2 - 24g + 80$

q $108 - 6d - 2d^2$

r $26 + 11n - n^2$

Investigation: Factorising quadratic trinomials by grouping in pairs

- 1 We can factorise the quadratic trinomial $6y^2 + 19y + 15$ by rewriting it as $6y^2 + 10y + 9y + 15$, splitting up the middle term $19y$.
 - a** Factorise $6y^2 + 10y + 9y + 15$ by grouping in pairs.
 - b** Factorise $6y^2 + 9y + 10y + 15$ by grouping in pairs. Is your answer the same as that for part **a**?
 - c** In $6y^2 + 19y + 15$, 6 is the coefficient of y^2 while 15 is the constant term. Find the product of 6 and 15 and the product of 10 and 9. What do you notice?
- 2 **a** Factorise $2c^2 + 13c + 21$ by rewriting it as $2c^2 + 6c + 7c + 21$.
b Factorise $2c^2 + 13c + 21$ by rewriting it as $2c^2 + 7c + 6c + 21$.
c Find the product of 2 and 21 and the product of 6 and 7. What do you notice?

3-13

Factorising quadratic expressions of the form $ax^2 + bx + c$

We will now factorise quadratic trinomials such as $6x^2 + 19x + 15$, where x^2 has a coefficient that is not a common factor. We do this by splitting the middle term into two terms and then factorise by grouping in pairs.

Example 28

Factorise $3x^2 + 8x + 4$.

Solution

We need to split up the middle term $8x$.

Find two numbers that have a product of 12 and a sum of 8.

$$3x^2 + 8x + 4 = 3x^2 + 8x + 4$$

Stage 5.3

The two numbers are +6 and +2, so we will split $8x$ into $6x$ and $2x$.

$$\begin{aligned}\therefore 3x^2 + 8x + 4 &= 3x^2 + 6x + 2x + 4 \\ &= 3x(x + 2) + 2(x + 2) \\ &= (x + 2)(3x + 2)\end{aligned}$$

Factorising by grouping in pairs.

Factorising again.

Summary

To factorise quadratic trinomials of the form $ax^2 + bx + c$:

- find two numbers that have a sum of b and a product of ac
- use these two numbers to split the middle term bx into two terms
- factorise by grouping in pairs

Example 29

Factorise each quadratic expression.

a $5k^2 - 12k + 4$

b $9m^2 - 9m - 4$

c $6t^2 + t - 12$

Solution

a $5k^2 - 12k + 4$

$5 \times 4 = 20$. Find two numbers that have a product of 20 and a sum of -12 .

Since the sum is negative, one of the numbers must be negative.

Since the product is positive, both of the numbers must be negative.

They are -10 and -2 . Split $-12k$ into $-10k$ and $-2k$.

$$\begin{aligned}5k^2 - 12k + 4 &= 5k^2 - 10k - 2k + 4 \\ &= 5k(k - 2) - 2(k - 2) \\ &= (k - 2)(5k - 2)\end{aligned}$$

Factorising by grouping in pairs.

b $9m^2 - 9m - 4$

$9 \times (-4) = -36$. Find two numbers with a product of -36 and a sum of -9 .

Since the product is negative, one of the numbers must be negative.

They are -12 and 3 .

$$\begin{aligned}9m^2 - 9m - 4 &= 9m^2 - 12m + 3m - 4 \\ &= 3m(3m - 4) + 1(3m - 4) \\ &= (3m - 4)(3m + 1)\end{aligned}$$

c $6t^2 + t - 12$

$6 \times (-12) = -72$. Find two numbers with a product of -72 and a sum of 1.

They are -8 and $+9$.

$$\begin{aligned} 6t^2 + t - 12 &= 6t^2 - 8t + 9t - 12 \\ &= 2t(3t - 4) + 3(3t - 4) \\ &= (3t - 4)(2t + 3) \end{aligned}$$

Example 30

Factorise each quadratic expression.

a $18a^2 - 18a - 8$ b $10 - 7x - 12x^2$

Solution

$$\begin{aligned} \text{a } 18a^2 - 18a - 8 &= 2(9a^2 - 9a - 4) \\ &= 2(9a^2 - 12a + 3a - 4) \\ &= 2[3a(3a - 4) + 1(3a - 4)] \\ &= 2(3a - 4)(3a + 1) \end{aligned}$$

Taking out the HCF of 2 first.

Product = -36 , sum = -9

$$\begin{aligned} \text{b } 10 - 7x - 12x^2 &= -12x^2 - 7x + 10 \\ &= -(12x^2 + 7x - 10) \\ &= -(12x^2 + 15x - 8x - 10) \\ &= -[3x(4x + 5) - 2(4x + 5)] \\ &= -(4x + 5)(3x - 2) \end{aligned}$$

Rearranging the terms to make the x^2 term first.

Taking out a common factor of -1 .

Product = -120 , sum = -7

Exercise 3-13 Factorising quadratic expressions of the form $ax^2 + bx + c$

1 Factorise each quadratic expression.

a $2x^2 + 11x + 5$	b $4x^2 + 13x + 3$	c $5x^2 + 17x + 6$
d $6x^2 + 19x + 10$	e $2w^2 + 31w + 15$	f $4e^2 + 15e + 9$
g $8f^2 + 14f + 3$	h $3d^2 + 5d + 2$	i $2b^2 + 9b + 7$
j $5y^2 + 16y + 11$	k $8g^2 + 26g + 15$	l $6a^2 + 23a + 21$

2 Factorise each quadratic expression.

a $2y^2 - 11y + 12$	b $10k^2 - 19k + 6$	c $6e^2 - 13e + 6$
d $4b^2 - 13b + 3$	e $6w^2 - 23w + 15$	f $8t^2 + 26t + 15$
g $9x^2 - 12x + 4$	h $12f^2 - 25f + 12$	i $4h^2 - 36h + 81$
j $5y^2 - 6y - 11$	k $4d^2 - d - 5$	l $2m^2 - 3m - 9$
m $3t^2 - t - 30$	n $6h^2 - h - 7$	o $2y^2 - 5y - 12$
p $8a^2 - 2a - 3$	q $15u^2 - 7u - 4$	r $9c^2 - 12c - 5$

See Example 28

See Example 29

Stage 5.3

3 Factorise each quadratic expression.

a $5m^2 + 2m - 7$

b $6g^2 + g - 12$

c $3p^2 + 4p - 4$

d $7w^2 + 6w - 1$

e $5y^2 + 14y - 3$

f $3n^2 + 10n - 8$

g $4b^2 + 9b - 9$

h $8m^2 + 10m - 3$

i $3x^2 + 2x - 16$

4 Factorise each expression given it is a perfect square.

a $81w^2 - 180w + 100$

b $4y^2 + 8y + 4$

c $25h^2 - 40h + 16$

5 Factorise each quadratic expression by first taking out a common factor.

a $6t^2 + 10t - 4$

b $6g^2 + 15g - 36$

c $24e^2 - 28e - 12$

d $8a^2 - 10a - 12$

e $12t^2 + 20t - 8$

f $-25q^2 - 5q + 6$

g $-12m^2 + 14m - 4$

h $20 - h - 12h^2$

i $18 + 48c + 24c^2$

j $15 + 9z - 6z^2$

k $12d^2 + 2d - 30$

l $22x - 12 - 6x^2$

6 Factorise each quadratic expression.

a $2a^2 + 5a + 3$

b $12m^2 - 32m + 5$

c $4x^2 + 11x - 3$

d $7w^2 - 8w + 1$

e $4h^2 - 7h - 15$

f $8x^2 - 2x - 3$

g $5r^2 + 26r + 5$

h $2d^2 - 15d + 7$

i $6n^2 - 7n - 3$

j $8 - 6m - 9m^2$

k $3 - 2c - 5c^2$

l $15g^2 + 19g + 6$

m $15 + 14q - 8q^2$

n $3x^2 - 13x + 14$

o $16 - 8d - 3d^2$

See Example 30

Worked solutions

Factorising quadratic expressions of the form $ax^2 + bx + c$

MAT09NAWS10017

Worksheet

Mixed factorisations

MAT09NAWK00036

Puzzle sheet

Factorominoes

MAT09NAPS10033

Puzzle sheet

Products and factors squaresaw

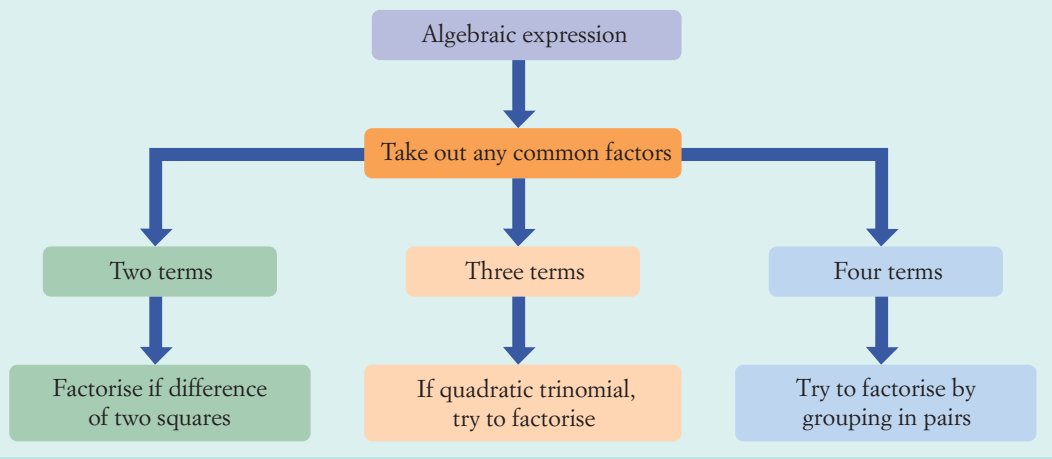
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3-14 Mixed factorisations

Summary

Factorisation strategies

- Look for any common factors and factorise first
- If there are two terms, try factorising using the difference of two squares
- If there are three terms, try factorising as a quadratic trinomial
- If there are four terms, try factorising by grouping in pairs



Example 31

Factorise each quadratic expression.

a $3a^2 - 27$ b $5a^2 + 100$ c $20b^2 - 52b + 24$ d $d^3 - d^2 - d + 1$

Solution

a $3a^2 - 27 = 3(a^2 - 9)$
 $= 3(a + 3)(a - 3)$

b $5a^2 + 100 = 5(a^2 + 20)$

c $20b^2 - 52b + 24 = 4(5b^2 - 13b + 6)$
 $= 4(5b^2 - 10b - 3b + 6)$
 $= 4[5b(b - 2) - 3(b - 2)]$
 $= 4(b - 2)(5b - 3)$

d $d^3 - d^2 - d + 1 = d^2(d - 1) - 1(d - 1)$
 $= (d - 1)(d^2 - 1)$
 $= (d - 1)(d + 1)(d - 1)$
 $= (d - 1)^2(d + 1)$

Taking out the HCF of 3 first.

Difference of two squares.

Two terms but not a difference of two squares.

Factorising by grouping in pairs.

Difference of two squares.

Exercise 3-14 Mixed factorisations

1 Factorise each expression.

a $m^2 - 16m + 64$

d $3k - 15 - 5h + hk$

g $q^2 + 3q - 3pq$

j $25r^2 - 1$

m $4 - d - 5d^2$

p $mn^2 + mnp + 3mn + 3mp$

b $3d^2 - 3d$

e $25y^2 - 64$

h $3 + 2g - g^2$

k $b^3 + b^2 + b + 1$

n $b^3 - b^2 - b + 1$

q $2w^2 - 24w + 72$

c $3d^2 - 4d - 15$

f $100f^2 - 64$

i $24b^2 + 44b - 40$

l $4x^2 - 20x + 25$

o $8 - 2v^2$

r $36h^2 + 12h + 1$

2 Factorise each expression.

a $15r^2 - 31rt - 24t^2$

d $e^3 - 3e^2 - 10e$

g $a^2 - b^2 + 4a - 4b$

j $t^2 - 3t + 5t - 35$

m $9x^2 - 27x + 18x - 54$

p $25u^2 - 10u + 1$

s $3 - 27s^2$

v $m^3n - 4mn$

b $4d^2 + 4d + 1$

e $5(p + q)^2 - 125(p - q)^2$

h $c^3 - 2c^2 - 4c + 8$

k $18p^2 + 24p + 8$

n $2a^2b - 6ab - 3a + 9$

q $4k^2 - 5k - 21$

t $k^3 + 4k^2 - 16k - 64$

w $8 - 2a^2$

c $9g^2 - 36k^2$

f $28x^2 - 7$

i $6a^2 + 13a - 5$

l $1 - 2a - 24a^2$

o $2a^2 + 12a + 18$

r $48 - 3w^2$

u $5y^3 - 10y^2 + 15y$

x $32c^2 - 40c - 12$

See Example 31

Stage 5.3

NSW

3-15 Factorising algebraic fractions

Example 32

Simplify each expression.

a $\frac{10a + 25b}{5}$

b $\frac{9y^2 - 16}{6y + 8}$

c $\frac{x^2 + x}{-4x - 4}$

d $\frac{t^2 - 3t + 2}{3t^2 - 5t - 2}$

Solution

$$\begin{aligned} \text{a } \frac{10a + 25b}{5} &= \frac{5(a + 5b)}{5} \\ &= a + 5b \end{aligned}$$

$$\begin{aligned} \text{b } \frac{9y^2 - 16}{6y + 8} &= \frac{(3y + 4)(3y - 4)}{2(3y + 4)} \\ &= \frac{\cancel{(3y + 4)}(3y - 4)}{2\cancel{(3y + 4)}} \\ &= \frac{3y - 4}{2} \end{aligned}$$

$$\begin{aligned} \text{c } \frac{x^2 + x}{-4x - 4} &= \frac{x(x + 1)}{-4(x + 1)} \\ &= \frac{x\cancel{(x + 1)}}{-4\cancel{(x + 1)}} \\ &= -\frac{x}{4} \end{aligned}$$

$$\begin{aligned} \text{d } \frac{t^2 - 3t + 2}{3t^2 - 5t - 2} &= \frac{(t - 2)(t - 1)}{(t - 2)(3t + 1)} \\ &= \frac{\cancel{(t - 2)}(t - 1)}{\cancel{(t - 2)}(3t + 1)} \\ &= \frac{t - 1}{3t + 1} \end{aligned}$$

Example 33

Simplify each expression.

a $\frac{4}{x^2 + x} - \frac{2}{x^2 - 1}$

b $\frac{3m - 6}{4} \times \frac{8m}{m^2 - 2m}$

c $\frac{d^2 + 3d + 2}{d^2 - 9} \div \frac{d^2 + d}{3d + 9}$

Solution

$$\begin{aligned} \text{a } \frac{4}{x^2 + x} - \frac{2}{x^2 - 1} &= \frac{4}{x(x + 1)} - \frac{2}{(x + 1)(x - 1)} \\ &= \frac{4(x - 1)}{x(x + 1)(x - 1)} - \frac{2x}{x(x + 1)(x - 1)} \\ &= \frac{4x - 4 - 2x}{x(x + 1)(x - 1)} \\ &= \frac{2x - 4}{x(x + 1)(x - 1)} \\ &= \frac{2(x - 2)}{x(x + 1)(x - 1)} \end{aligned}$$

Factorising denominators.

Using common denominators.

$$\begin{aligned} \text{b } \frac{3m-6}{4} \times \frac{8m}{m^2-2m} &= \frac{3(m-2)}{4} \times \frac{8m}{m(m-2)} \\ &= \frac{3\cancel{(m-2)}}{4_1} \times \frac{\overset{2}{\cancel{8}}m}{\cancel{m}(m-2)} \\ &= 6 \end{aligned}$$

$$\begin{aligned} \text{c } \frac{d^2+3d+2}{d^2-9} \div \frac{d^2+d}{3d+9} &= \frac{d^2+3d+2}{d^2-9} \times \frac{3d+9}{d^2+d} \\ &= \frac{(d+2)(d+1)}{(d+3)(d-3)} \times \frac{3(d+3)}{d(d+1)} \\ &= \frac{(d+2)\cancel{(d+1)}}{\cancel{(d+3)}(d-3)} \times \frac{3\cancel{(d+3)}}{d\cancel{(d+1)}} \\ &= \frac{3(d+2)}{d(d-3)} \end{aligned}$$

Exercise 3-15 Factorising algebraic fractions

1 Simplify each expression.

a $\frac{3x+3y}{3}$

b $\frac{5}{10t-10r}$

c $\frac{ab-ac}{a^2}$

d $\frac{y-1}{1-y}$

e $\frac{w^2-16}{w+4}$

f $\frac{5d-5t}{d^2-t^2}$

g $\frac{(k+5)^2}{k^2-25}$

h $\frac{6c^2-6}{2c+2}$

i $\frac{am-an+m-n}{m^2-n^2}$

j $\frac{y^2+9y+20}{2y+10}$

k $\frac{k^2-3k-4}{k^2-16}$

l $\frac{16a^2-25c^2}{4a^2-9ac+5c^2}$

m $\frac{s^2+4s+4}{s^2-s-6}$

n $\frac{1-c-2c^2}{3c^2+2c-1}$

o $\frac{ap+4p-2a-8}{2p^2-8}$

2 Simplify each expression.

a $\frac{5}{m(m+1)} + \frac{2}{(m+1)(m+2)}$

b $\frac{6}{(w+5)(w+3)} - \frac{4}{w(w+3)}$

c $\frac{3}{(b+2)(b-1)} + \frac{1}{(b-1)(b-3)}$

d $\frac{2}{k^2+k} - \frac{3}{k^2-1}$

e $\frac{5}{4h+4} + \frac{3}{h^2+h}$

f $\frac{3}{d^2+3d+2} - \frac{4}{d+2}$

g $\frac{3}{r^2-36} - \frac{5}{4r+24}$

h $\frac{3}{d^2+2d} + \frac{d}{d^2-4}$

i $\frac{5}{k^2-3k-4} - \frac{k}{k^2-1}$

j $\frac{2}{q^2-1} + \frac{3}{q+1}$

See Example 32

See Example 33

Stage 5.3

3 Simplify each expression.

a $\frac{3m+9}{2} \times \frac{4m}{m+3}$

b $\frac{5d-10}{3d-9} \times \frac{5d-15}{8d-16}$

c $\frac{4}{e+2} \times \frac{e^2+2e}{8e}$

d $\frac{3k+6}{5} \times \frac{10k}{k+2}$

e $\frac{5h}{3h+9} \times \frac{6h+18}{h^2+h}$

f $\frac{4}{a^2-b^2} \times \frac{3a+3b}{8}$

g $\frac{r+t}{t^2-r^2} \times \frac{r^2-rt}{5r+5t}$

h $\frac{20m+16}{7m-7} \times \frac{7m}{5m+4}$

i $\frac{p^2+2p+1}{p^2-1} \times \frac{4p-4}{p^2+p}$

j $\frac{y+2}{5y} \div \frac{7y+14}{15y}$

k $\frac{5}{x^2-4} \div \frac{15}{2x+4}$

l $\frac{4n+8}{n+5} \div \frac{6n+12}{5n+25}$

m $\frac{d^2+d}{d+3} \div \frac{6d}{d^2-9}$

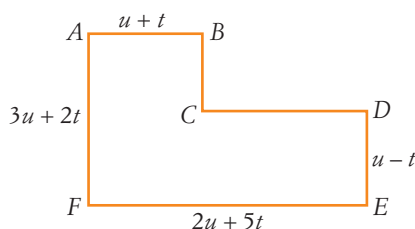
n $\frac{1}{f^2-6f+9} \div \frac{4}{f^2-9}$

o $\frac{3f+6}{f^2+f-6} \div \frac{f^2-2f-8}{f^2-f-12}$

Power plus

1 For this composite shape, write an expression for:

- a the length of CD
- b the length of BC
- c its perimeter
- d its area


2 Find the average of $6r$, $2r+8$, $r-5$, $2r$, $r+7$ and $3r+8$.

3 A rectangular garden has its longer sides each 5 m longer than its shorter sides. If its longer sides each have length y m, then write a simplified expression for:

- a the area of the garden
- b the perimeter of the garden

4 How many hours does it take a car to travel a distance of $20t^2$ kilometres at an average speed of $4t$ km/h?

5 Expand and simplify each expression.

a $(a+b)(a+b+c)$

b $(x-2)(x+y-3)$

c $(p+3)(p-4)(p-5)$

d $(x-1)(y-1)(z-1)$

e $(wt-3r)^2$

f $(x^3+2)^2$

6 Factorise each expression.

a $n^2+4mn+4m^2$

b $x^2-2xy+y^2$

c $25x^2-40xy+16y^2$

d $5a^2-30ab+45b^2$

e c^4+2c^2+1

f $8-7t^2-t^4$

g x^4-1

h $(a+b)^2-c^2$

i $(p+q)^2-(p-q)^2$

7 If $x^2+bx+c=(x+p)(x+q)$, what are the signs of p and q if:

a b and c are both positive?

b b and c are both negative?

c b is positive and c is negative?

d b is negative and c is positive?

Language of maths

area diagram	binomial	binomial product	coefficient
common factor	constant term	difference of two squares	distributive law
expand	expression	factor	factorise
formula	grouping in pairs	highest common factor (HCF)	like terms
perfect square	pronumeral	product	quadratic trinomial
simplify	sum	variable	

Puzzle sheet

Algebra crossword

MAT09NAPS10035

Quiz

Algebra

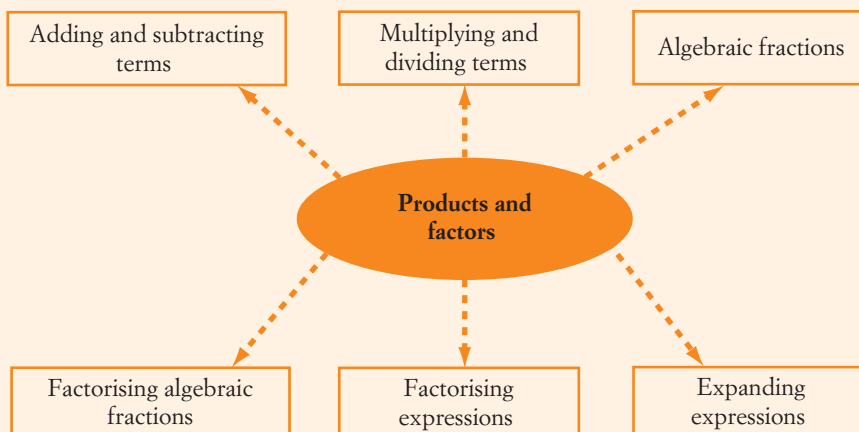
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- Why is it possible to add or subtract terms like $6n$ and $4n$ but not $7x$ and $2y$?
- In the expression $2x^2 - 3x + 6$, what is:
 - the constant term?
 - the coefficient of x^2 ?
- In what type of algebraic expression is the highest power of the variable equal to 2?
- Explain what **product** means.
- In algebra, what is the opposite of **expand**?
- Complete: A binomial is an algebraic expression with ____.

Topic overview

- Write in your own words the new things you have learnt.
- What parts of this topic did you like?
- What parts of the topic did you find difficult or you did not understand?
- Make a list of the skills you have learned in this topic, such as substituting and expanding.

Copy (or print) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Worksheet

Mind map: Products and factors (Advanced)

MAT09NAWK10037

Chapter 3 revision

See Exercise 3-01

1 Simplify each expression.

a $9y + 7y$

d $7p + 2q - 5p$

g $2m + 3n + 5m + 7n$

b $4ab - 7ab$

e $4w - 5 + 2w$

h $5u + 7 - 2u - 3$

c $3x^2 + 5x^2$

f $8x^2 - 13x - 5x^2$

i $12d^2 + 5d - 7d + 4$

See Exercise 3-02

2 Simplify each expression.

a $9 \times 2n$

d $(-3) \times 2r$

g $36a \div 9a$

b $g \times 3h$

e $12y^2 \div 3y$

h $15t^2 \div 5t^2$

c $a \times d$

f $2xy \div y$

i $20pq \div (-5q)$

See Exercise 3-03

3 Simplify each expression.

a $\frac{h}{6} + \frac{4h}{6}$

d $\frac{w}{4} - \frac{2w}{5}$

g $\frac{d}{5k} - \frac{3}{k}$

b $\frac{11k}{8} - \frac{7k}{8}$

e $\frac{4k}{3} - \frac{k}{2}$

h $\frac{8}{m+1} - \frac{3}{m+1}$

c $\frac{7}{m} - \frac{5}{m}$

f $\frac{5}{2d} - \frac{4}{d}$

See Exercise 3-04

4 Simplify each expression.

a $\frac{5}{m} \times \frac{4}{i}$

d $\frac{r}{4} \div \frac{r}{5}$

g $\frac{m}{5} \div \frac{5m}{10d}$

j $\frac{3c}{4d} \div \frac{9c}{16d} \div \frac{6}{8}$

b $\frac{h}{k} \times \frac{3}{k}$

e $\frac{5}{v} \div \frac{2}{v}$

h $\frac{4}{x} \div \frac{12}{xp}$

k $\frac{15a}{4b} \div \frac{4}{3a^2b} \times \frac{a}{bc}$

c $\frac{14}{3y} \times \frac{9y}{2d}$

f $\frac{a}{b} \div \frac{a}{b}$

i $\frac{ad}{6} \times \frac{5a}{2} \div \frac{ad}{8}$

l $\frac{5y}{8} \div \frac{y}{4} \times \frac{15a}{2y}$

See Exercise 3-05

5 Expand and simplify each expression.

a $5(r - 2)$

d $3p(4p - 7q)$

g $b(b + 1) + 5(b + 1)$

b $a(a - 10)$

e $-6n(n - 9)$

h $3x(2x - 7) - 5(2x - 7)$

c $3y(2x - 5y)$

f $-6t(t + 7)$

i $12 - 6(2x - 3)$

See Exercise 3-06

6 Factorise each expression.

a $4m - 12$

d $16w^2 + 24w$

g $3(y - 6) + y(y - 6)$

b $xy + xz$

e $-8q + 16$

h $3x(2x - 1) - 2(2x - 1)$

c $-3g - 30$

f $15k - 10hk^2$

i $p(p + 6) - 2(p + 6)$

See Exercise 3-07

7 Expand each binomial product.

a $(a + 2)(a + 3)$

d $(h - 5)(h - 4)$

g $(5x + 1)(4x - 3)$

b $(y + 3)(y - 7)$

e $(4 - g)(3 + g)$

h $(4r - 5)(3r + 4)$

c $(t - 3)(t + 8)$

f $(3p + 1)(2p + 3)$

i $(5 - 3q)(2q - 1)$

Stage 5.3

See Exercise 3-08

8 Expand each perfect square.

a $(n + 5)^2$

e $(3y + 1)^2$

b $(u - 3)^2$

f $(2x - 5)^2$

c $(9 + x)^2$

g $(6 - 7b)^2$

d $(6 - k)^2$

h $(1 - 4h)^2$

See Exercise 3-09

9 Expand each expression.

a $(n + 6)(n - 6)$

d $(3r + 1)(3r - 1)$

b $(b - 1)(b + 1)$

e $(4w - 5)(4w + 5)$

c $(10 + m)(10 - m)$

f $(1 - 8w)(1 + 8w)$

10 Expand and simplify each expression.

a $(2x + 5)(x - 1) + (x + 3)^2$

c $(3a + 1)(3a - 1) - (3a + 1)^2$

b $(y + 4)^2 - (y - 1)(y + 2)$

d $(2p - 3q)^2 + (2p + 3q)^2$

11 Factorise each expression.

a $5mn - 3n + 10m - 6$

d $10p^2 + 30 + p^3 + 3p$

g $36 - r^2$

j $49a^2 - 25b^2$

m $24 - 150d^2$

b $ab - a - b + 1$

e $w^2 - 9$

h $16a^2 - 9$

k $3q^2 - 27$

c $x^2y + xy^3 - 5x - 5y^2$

f $m^2 - n^2$

i $81x^2 - y^2$

l $50m^2 - 32$

12 Factorise each expression.

a $m^2 - 11m - 12$

d $p^2 + 6p - 27$

g $18 + 3x - x^2$

b $y^2 - 4y + 3$

e $3n^2 + 9n + 6$

h $60 - 5b - 5b^2$

c $t^2 - 8t - 65$

f $4y^2 + 4y - 8$

i $11k - 4 - 6k^2$

13 Factorise each expression.

a $6x^2 + x - 2$

d $3e^2 + 12e + 12$

g $36x - 27x^2 - 12$

b $5s^2 - 7s - 6$

e $6y^2 + 15y - 36$

h $52t - 30 - 16t^2$

c $8w^2 - 14w + 3$

f $24a^2 - 28a - 12$

i $12g^2 - 46g + 14$

14 Factorise each expression.

a $64y^2 - 25$

d $3g - 15 - 5h + hg$

g $4y^2 - 5y - 21$

b $x^2 - 16x + 64$

e $a^3 - a^2 - a + 1$

h $18q^2 + 24q + 8$

c $16r - 48$

f $3b^3 - 3b$

i $1 - 2p - 24p^2$

15 Simplify each expression.

a $\frac{9y^2 - 25}{12y - 20}$

b $\frac{2x^2 + 3x - 5}{6x^2 + 19x + 10}$

c $\frac{3w^2 - 75}{3w^2 - 30w + 75}$

16 Simplify each expression.

a $\frac{4}{x^2 + x} - \frac{3}{x^2 - 1}$

c $\frac{k}{2k + 6} + \frac{5}{k^2 - 9}$

e $\frac{4d}{d^2 - 3d + 2} - \frac{3}{d^2 - 2d}$

g $\frac{3d - 6}{d + 3} \times \frac{3d + 9}{5d - 10}$

i $\frac{4}{d^2 - 4} \div \frac{8}{d + 2}$

b $\frac{4}{x^2 - 9} + \frac{2}{3x + 9}$

d $\frac{1}{(x - 3)(x + 2)} + \frac{4}{(x + 2)(x + 3)}$

f $\frac{3}{m^2 - 2m - 3} - \frac{2}{m^2 + 5m + 4}$

h $\frac{y^2 - 5y + 6}{y^3 - 4y} \times \frac{2y^2 - 8}{y^2 - 2y}$

j $\frac{m}{m^2 + m} \div \frac{4}{5m + 5}$

Stage 5.3

See Exercise 3-10

See Exercise 3-11

See Exercise 3-12

See Exercise 3-13

See Exercise 3-14

See Exercise 3-15

See Exercise 3-15



4

Measurement and geometry

Trigonometry

In the second century BCE, the Greek astronomer Hipparchus could calculate distances to the moon and the Sun and he was the first scientist to chart the positions of over 850 stars. How was he able to achieve this over 2100 years ago? Hipparchus started a new branch of mathematics called **trigonometry**, meaning 'triangle measure', which uses angles, triangles and circles to calculate lengths and distances that cannot be physically measured. Trigonometry is used widely today in engineering, surveying, navigation, astronomy, electronics and construction.



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Chapter outline

	Proficiency strands				
4-01 The sides of a right-angled triangle	U				C
4-02 The trigonometric ratios	U				C
4-03 Similar right-angled triangles	U		R		C
4-04 Trigonometry on a calculator	U	F			
4-05 Finding an unknown side	U	F	PS		
4-06 Finding more unknown sides	U	F	PS		
4-07 Finding an unknown angle	U	F	PS		
4-08 Angles of elevation and depression	U	F	PS	R	C
4-09 Bearings	U	F		R	C
4-10 Problems involving bearings		F	PS	R	C

Wordbank

adjacent side In a right-angled triangle, the side that is next to a given angle and pointing to the right angle

angle of depression The angle of looking **down**, measured from the horizontal

angle of elevation The angle of looking **up**, measured from the horizontal

bearing The angle used to show the direction of one location from a given point

minute (') A unit for measuring angle size, $\frac{1}{60}$ of a degree

opposite side In a right-angled triangle, the side that is facing a given angle and not one of its arms

theta (θ) A letter of the Greek alphabet used as a pronumeral for angles

trigonometric ratio The ratio of two sides in a right angled triangle, for example, sine is the ratio of the opposite side to the hypotenuse

In this chapter you will:

- use similarity to investigate the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles
- apply trigonometry to solve right-angled triangle problems 
- find unknown sides and angles in right-angled triangles where the angle is measured in degrees and minutes
- apply trigonometry to problems involving bearings and angles of elevation and depression

SkillCheck

Worksheet

StartUp assignment 4

MAT09MGWK10038

1 Solve each equation.

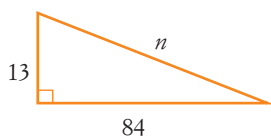
a $\frac{x}{5} = 7$

b $\frac{h}{4} = 8.3$

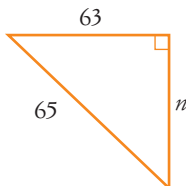
c $\frac{45}{y} = 9$

2 For each triangle, find the value of n .

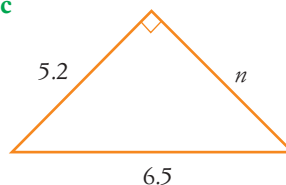
a



b



c



3 Round each time to the nearest hour.

a 8 h 18 min

b 3 h 45 min

c 1 h 30 min

4 Convert each time to hours and minutes.

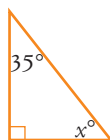
a 4.7 h

b 2.25 h

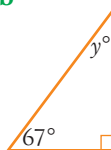
c 6.85 h

5 Find the value of each pronumeral.

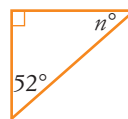
a



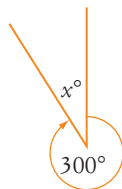
b



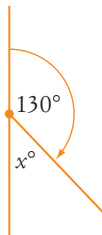
c



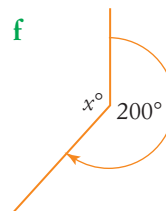
d



e



f



4-01 The sides of a right-angled triangle

The three sides of a **right-angled triangle** have special names. These names depend on the positions of the sides relative to a given angle.

- The **hypotenuse** is the longest side and is always opposite the right angle.
- The **opposite side** directly faces the given angle.
- The **adjacent side** runs from the given angle to the right angle.

We have already learnt about the hypotenuse in **Pythagoras' theorem**.

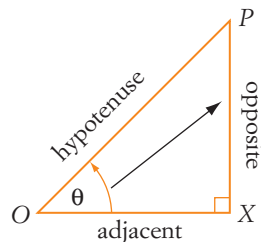
Adjacent means 'next to'.

In the diagram below, the marked $\angle O$ has also been labelled with the Greek letter, θ , ('theta').

The hypotenuse is OP .

The opposite side is XP .

The adjacent side is OX .

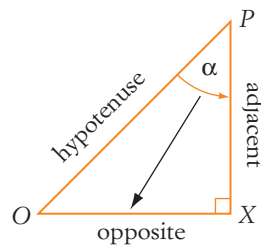


If the marked angle is $\angle P$, also labelled with the Greek letter α ('alpha'), then the opposite and adjacent sides are swapped, but the hypotenuse stays the same.

The hypotenuse is OP .

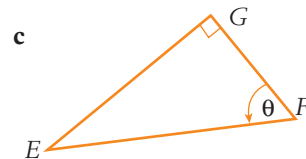
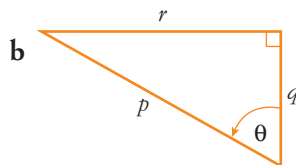
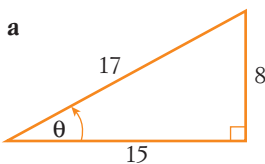
The opposite side is OX .

The adjacent side is XP .



Example 1

For each triangle, name the hypotenuse, opposite and adjacent sides for angle θ .



Solution

a Hypotenuse is 17.
Opposite side is 8.
Adjacent side is 15.

b Hypotenuse is p .
Opposite side is r .
Adjacent side is q .

c Hypotenuse is EF .
Opposite side is EG .
Adjacent side is FG .

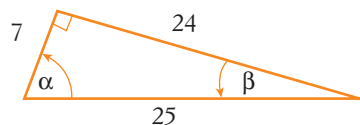
Example 2

For angles α and β , name the adjacent side.

Solution

For α , the adjacent side is 7.

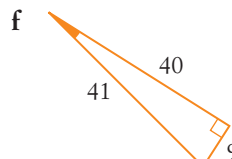
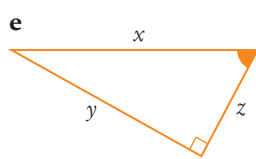
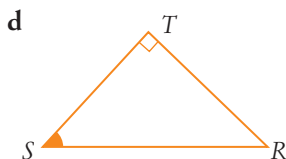
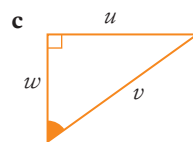
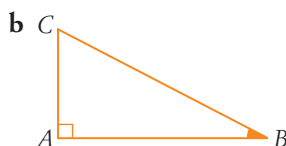
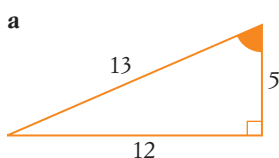
For β , the adjacent side is 24.



Exercise 4-01 The sides of a right-angled triangle

See Example 1

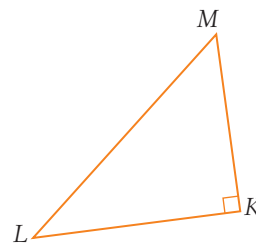
- 1 For the marked angle in each triangle, name the hypotenuse, opposite and adjacent sides.



- 2 For $\triangle LKM$, name the angle:

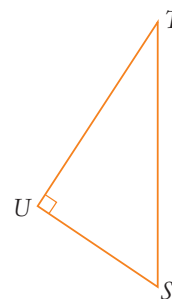
- a** opposite the hypotenuse
c opposite side LK
e adjacent to side LK .

- b** opposite side KM
d adjacent to side KM



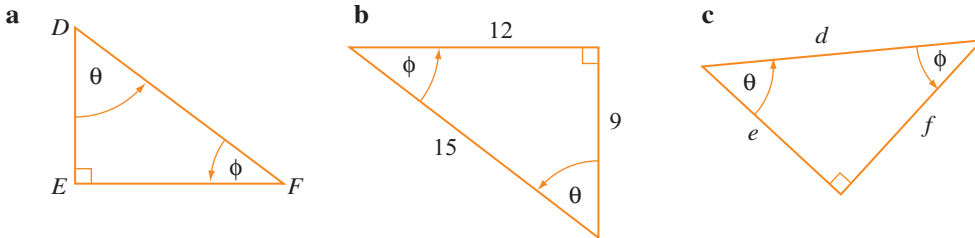
- 3 Which one of these statements is **false** about $\triangle TSU$?

- A** The adjacent side to $\angle S$ is US .
B The adjacent side to $\angle T$ is TU .
C The hypotenuse is UT .
D The opposite side to $\angle T$ is US .

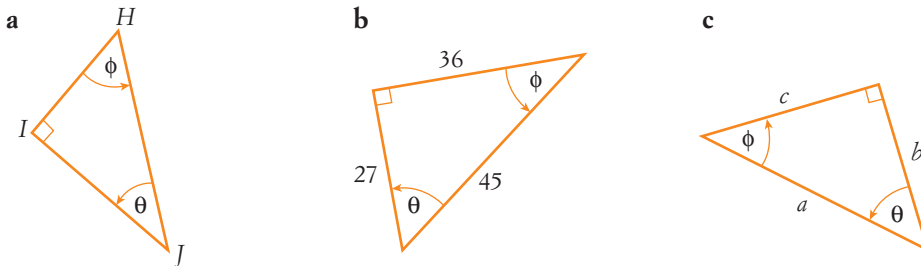


- 4 For each triangle, find the opposite sides for angles θ ('theta') and ϕ ('phi').

See Example 2



- 5 For each triangle, find the adjacent sides for angles θ ('theta') and ϕ ('phi').



- 6 Which side of a right-angled triangle is fixed and does not depend on a given angle? Select the correct answer **A**, **B**, **C** or **D**.

A adjacent **B** hypotenuse **C** opposite **D** shortest

- 7 Given each description of a right-angled triangle, sketch the triangle with the correctly-labelled vertices and angle.

- a** $\triangle ABC$ has hypotenuse AB and side AC opposite angle θ
- b** $\triangle XYZ$ has hypotenuse YZ and side XZ adjacent to angle α
- c** $\triangle PRQ$ has side RQ opposite $\angle P$ and adjacent to $\angle R$
- d** $\triangle DEF$ is right-angled at E , with the opposite and adjacent sides of $\angle D$ equal

Just for the record

The Greek alpha-bet

Here are eight letters (in lower-case and capitals) from the Greek alphabet:

α , A alpha	β , B beta	γ , Γ gamma	δ , Δ delta
θ , Θ theta	π , Π pi	σ , Σ sigma	ω , Ω omega

The ancient Greeks greatly influenced the development of mathematics. It is traditional to use Greek letters as variables, particularly in geometry and trigonometry.

- 1 Find how many letters there are in the Greek alphabet, and name each one.
- 2 Compare the Greek alphabet with our Roman alphabet.
- 3 Can you see where the word alphabet comes from? Explain how it originated.

Puzzle sheet

Trigonometry match-up

MAT09MGPS10040

Technology

GeoGebra:
Trigonometry ratios

MAT09MGTC00006

Technology worksheet

Excel worksheet:
Trigonometry values

MAT09MGCT00026

Technology worksheet

Excel spreadsheet:
Trigonometry values

MAT09MGCT00011

4-02 The trigonometric ratios

There are three special fractions called **trigonometric ratios** that relate the lengths of two sides of a right-angled triangle: **sine**, **cosine** and **tangent**.

Summary

The trigonometric ratios

Ratio	Abbreviation	Meaning
sine	sin	$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$
cosine	cos	$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
tangent	tan	$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$

Example 3

In $\triangle AXP$, find $\sin \theta$, $\cos \theta$ and $\tan \theta$.

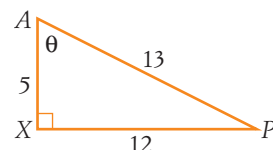
Solution

For angle θ , opposite = 12, adjacent = 5, hypotenuse = 13.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{12}{13}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{5}{13}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{12}{5}$$



Mnemonics for sin, cos and tan

A useful mnemonic (memory aid) for remembering the three ratios is to look at the initials of the words in the ratios:

$$\sin = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{O}{H} \quad \text{S.O.H.}$$

$$\cos = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{A}{H} \quad \text{C.A.H.}$$

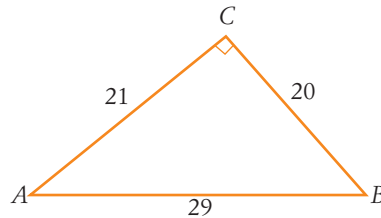
$$\tan = \frac{\text{opposite}}{\text{adjacent}} = \frac{O}{A} \quad \text{T.O.A.}$$

If you remember **SOH-CAH-TOA** (pronounced 'so-car-toe-ah'), then you can remember the ratios for sin, cos and tan. Some students also learn a phrase where the first letter of each word follows the SOH-CAH-TOA sequence, for example, 'Sun Over Head Caused A Huge Tan On Arms'. Find your own mnemonic for the three ratios.

Example 4

For the triangle below, find:

- a $\sin A$ b $\cos B$
c $\tan A$ d $\sin B$



Solution

For angle A , opposite = 20, adjacent = 21, hypotenuse = 29

For angle B , opposite = 21, adjacent = 20, hypotenuse = 29

$$\begin{array}{ll} \text{a } \sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{20}{29} & \text{b } \cos B = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{20}{29} \\ \text{c } \tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{20}{21} & \text{d } \sin B = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{21}{29} \end{array}$$

Video tutorial

The trigonometric ratios

MAT09MGVT10007

Given one ratio, finding another ratio

Example 5

If $\tan R = \frac{8}{15}$, find the value of $\sin R$ and $\cos R$.

Solution

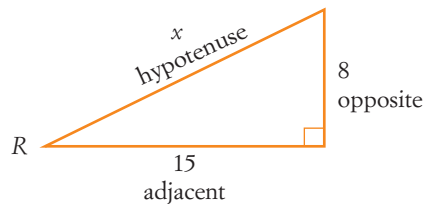
$\tan R = \frac{\text{opposite}}{\text{adjacent}} = \frac{8}{15}$, so draw a right-angled triangle that has an angle R with opposite side 8 and adjacent side 15. Let x be the length of the hypotenuse.

Find x using Pythagoras' theorem.

$$\begin{aligned} x^2 &= 8^2 + 15^2 \\ &= 289 \\ x &= \sqrt{289} \\ &= 17 \end{aligned}$$

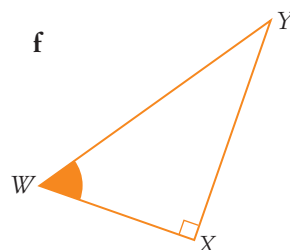
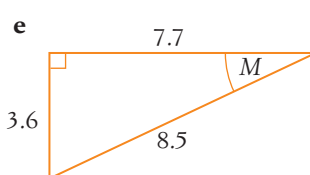
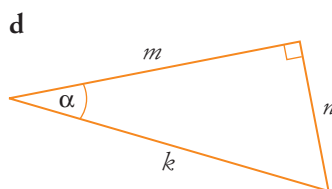
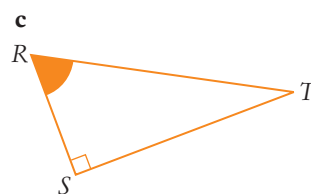
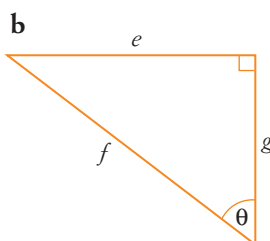
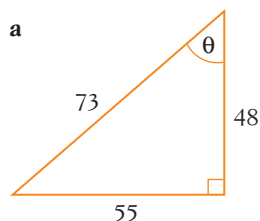
$$\therefore \sin R = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{8}{17}$$

$$\cos R = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{15}{17}$$



Exercise 4-02 The trigonometric ratios

See Example 3 1 For each marked angle, find the sine, cosine and tangent ratios.



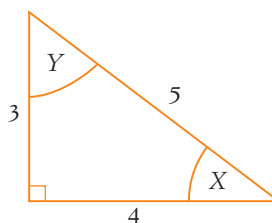
See Example 4 2 For the triangle below, find:

a $\cos X$

b $\tan Y$

c $\sin X$

d $\sin Y$



3 Complete each statement below with the correct angle (α or β).

a $\sin \square = \frac{IJ}{HJ}$

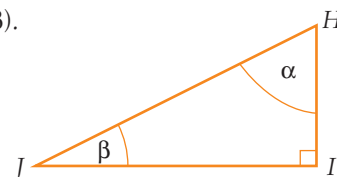
b $\sin \square = \frac{HI}{HJ}$

c $\cos \square = \frac{IJ}{HJ}$

d $\cos \square = \frac{HI}{HJ}$

e $\tan \square = \frac{IJ}{IH}$

f $\tan \square = \frac{IH}{IJ}$



4 For each triangle below, find:

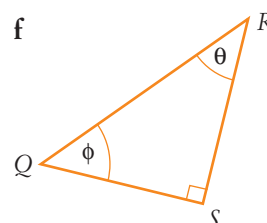
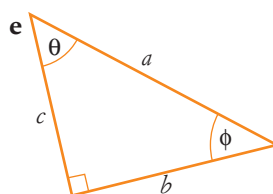
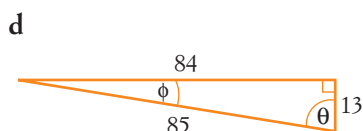
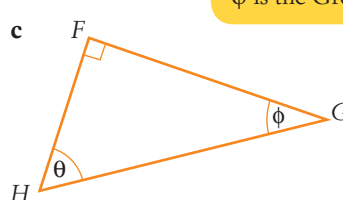
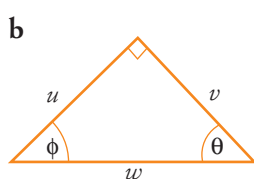
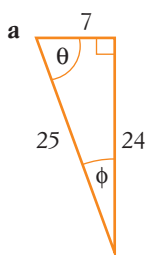
i $\tan \theta$

ii $\cos \theta$

iii $\cos \phi$

iv $\tan \phi$

ϕ is the Greek letter 'phi'



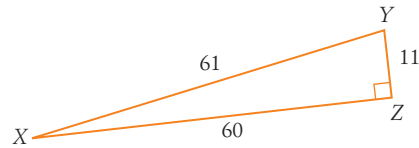
- 5 For each fraction, write a correct trigonometric ratio involving angle X or Y in the triangle.

a $\frac{60}{11}$

b $\frac{11}{60}$

c $\frac{11}{61}$

d $\frac{60}{61}$



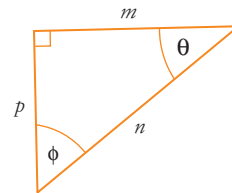
- 6 Which ratio is equal to $\frac{p}{m}$? Select the correct answer **A**, **B**, **C** or **D**.

A $\cos \theta$

B $\cos \phi$

C $\tan \theta$

D $\tan \phi$



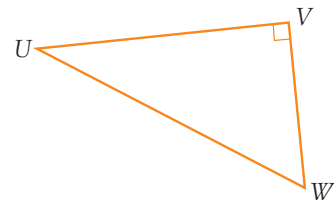
- 7 Which statement is true for this triangle? Select the correct answer **A**, **B**, **C** or **D**.

A $\sin U = \cos W$

B $\tan U = \sin W$

C $\cos U = \tan W$

D $\tan U = \tan W$



- 8 Sketch a right-angled triangle for each trigonometric ratio, then use Pythagoras' theorem to find the length of the unknown side and the other two trigonometric ratios for the same angle.

a $\tan A = \frac{5}{12}$

b $\sin B = \frac{3}{5}$

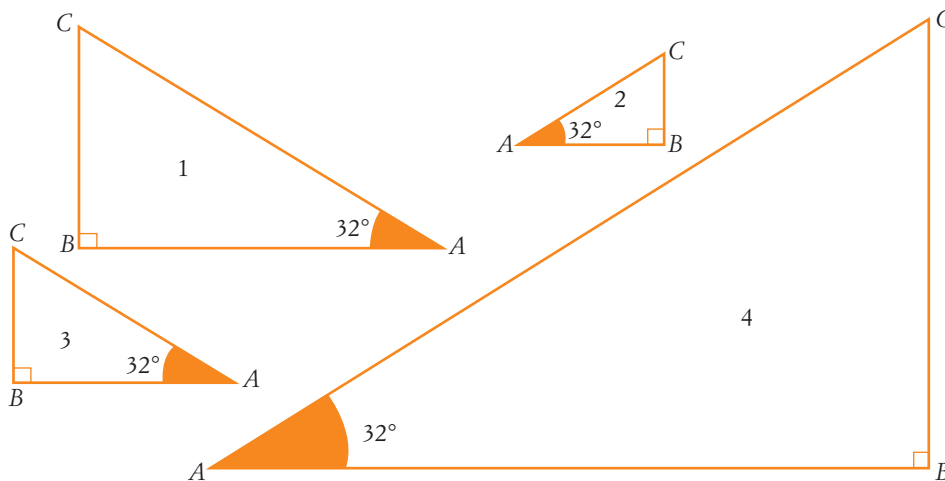
c $\cos X = \frac{9}{41}$

d $\sin Y = \frac{7}{25}$

See Example 5

4-03 Similar right-angled triangles

In each right-angled triangle below, $\angle A = 32^\circ$.



Furthermore, because $\angle B = 90^\circ$, $\angle C = 180^\circ - 90^\circ - 32^\circ = 58^\circ$ because of the angle sum of a triangle. These four triangles are called **similar triangles** because their corresponding angles are equal. They have the same shape but are not the same size. In fact, they are enlargements or reductions of one another.

Worksheet

Investigating the
tangent ratio

MAT09MGWK10039

Technology

GeoGebra:
Trigonometric ratios

MAT09MGTC00006

Example 6

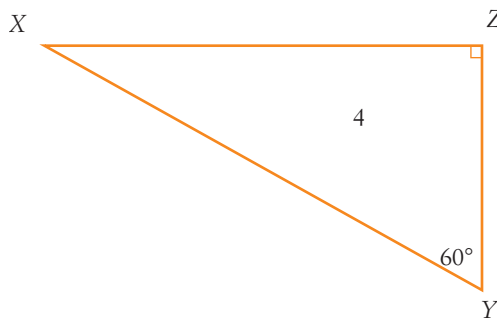
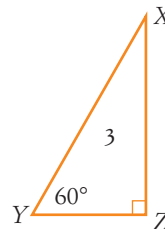
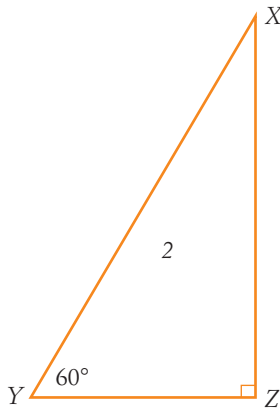
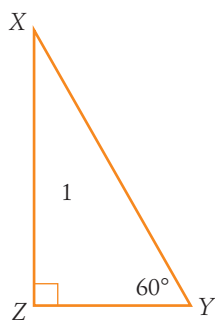
For each triangle on page 125, the length of each side has been measured to the nearest mm, and $\sin A$, $\cos A$ and $\tan A$ has been calculated to two decimal places, as shown in the table below.

	Side length (mm)			Trigonometric ratio		
	BC (opp)	AB (adj)	AC (hyp)	$\sin A = \frac{BC}{AC}$	$\cos A = \frac{AB}{AC}$	$\tan A = \frac{BC}{AB}$
1	29	47	55	$29 \div 55 \approx 0.53$	$47 \div 55 \approx 0.85$	$29 \div 47 \approx 0.62$
2	12	19	22	0.55	0.86	0.63
3	17	28	33	0.52	0.85	0.61
4	57	92	108	0.53	0.85	0.62

Note: See **Technology: Similar right-angled triangles** on page 128 for a *GeoGebra* activity based on this example.

Exercise 4-03 Similar right-angled triangles

- See Example 6** 1 a For each similar right-angled triangle below, measure the length of each side (correct to the nearest mm) and then calculate $\sin Y$, $\cos Y$ and $\tan Y$ as decimals (correct to two decimal places). Copy and complete the table on the opposite page.



	Side length (mm)			Trigonometric ratio		
	XZ (opp)	ZY (adj)	XY (hyp)	$\sin Y = \frac{XZ}{XY}$	$\cos Y = \frac{ZY}{XY}$	$\tan Y = \frac{XZ}{ZY}$
1						
2						
3						
4						

- b What do you notice about the value of $\sin Y$ for all four similar right-angled triangles?
- c Use your calculator to evaluate $\sin 60^\circ$ by pressing $\sin$ 60 $=$. What do you notice about your answer?
- d What do you notice about the value of $\cos Y$ for all four similar triangles?
- e Use your calculator to evaluate $\cos 60^\circ$. What do you notice about your answer?
- f What do you notice about the value of $\tan Y$ for all four similar triangles?
- g Use your calculator to evaluate $\tan 60^\circ$. What do you notice about your answer?
- 2 a Draw four similar right-angled triangles that have an angle of 48° , measure the length of each side (correct to the nearest mm) and then calculate $\sin 48^\circ$, $\cos 48^\circ$ and $\tan 48^\circ$ as decimals (correct to two decimal places). Copy and complete the table below.

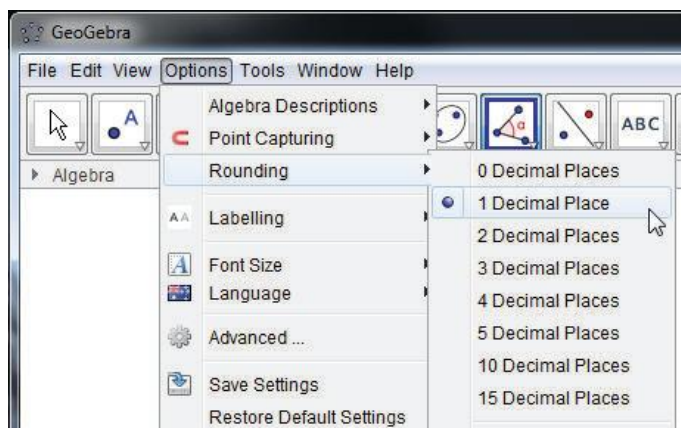
	Side length (mm)			Trigonometric ratio		
	Opp	Adj	Hyp	$\sin 48^\circ = \frac{\text{opp}}{\text{hyp}}$	$\cos 48^\circ = \frac{\text{adj}}{\text{hyp}}$	$\tan 48^\circ = \frac{\text{opp}}{\text{adj}}$
1						
2						
3						
4						

- b Examine the value of $\sin 48^\circ$ for all four similar triangles, then evaluate $\sin 48^\circ$ on a calculator. What do you notice?
- c Examine the values of $\cos 48^\circ$, then evaluate $\cos 48^\circ$ on a calculator.
- d Examine the values of $\tan 48^\circ$, then evaluate $\tan 48^\circ$ on a calculator.
- 3 For each trigonometric ratio, draw a large right-angled triangle with the given angle, then by measurement and calculation, find the value of the ratio correct to three decimal places. Compare your answer to the calculator's answer.
- a $\tan 55^\circ$ b $\cos 39^\circ$ c $\sin 67^\circ$ d $\cos 21^\circ$

Technology Similar right-angled triangles

In this activity you will use *GeoGebra* to measure and calculate trigonometric ratios.

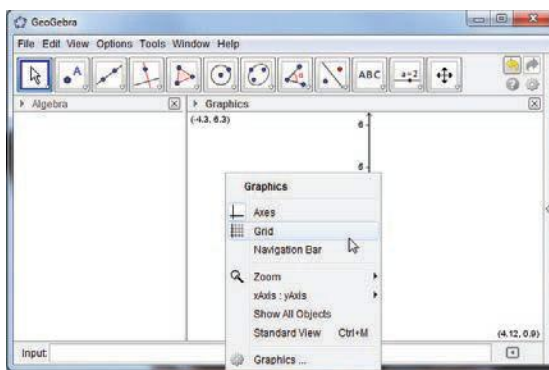
- 1 a** Before you start, set angles to measure in degrees, then click **Options**, **Rounding** and **1 Decimal Place**.



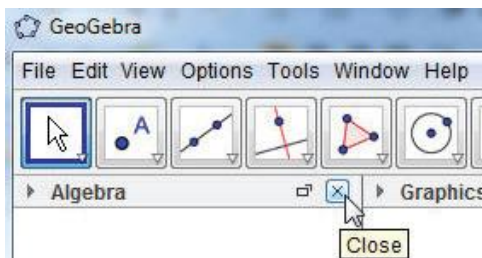
Note: If the angle measure is set to radians, under **Options**, **Advanced** select **Degree**.



- b** In the **Graphics** window, right-click and make sure **Axes** and **Grid** are enabled.

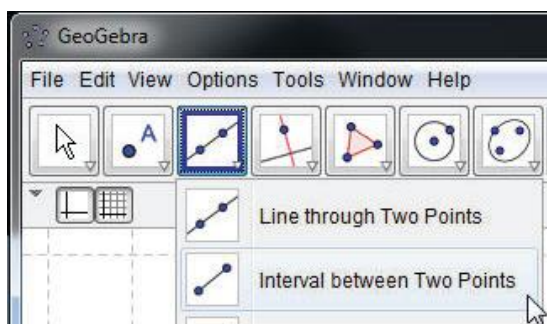


Alternatively, close the **Algebra** window

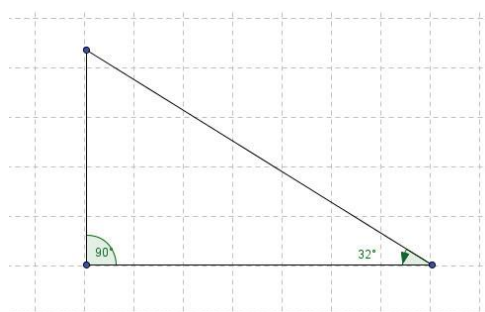


and click on the  icons near the top left-hand side.

Use **Interval between two points** and construct a right-angled triangle.



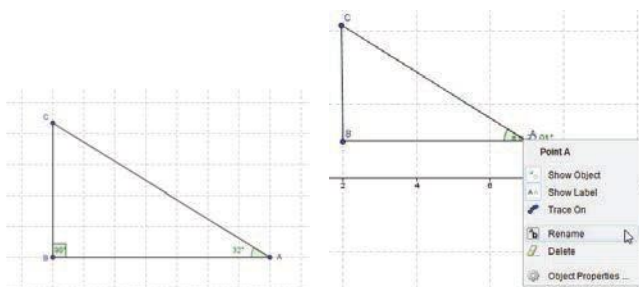
Use **Angle**  to measure the right angle and another angle (as shown below).



c Now keeping the triangle right-angled, use the **Move Tool**  to adjust any vertices so that one angle is 32° .

2 Draw two more similar right-angled triangles with a 32° angle.

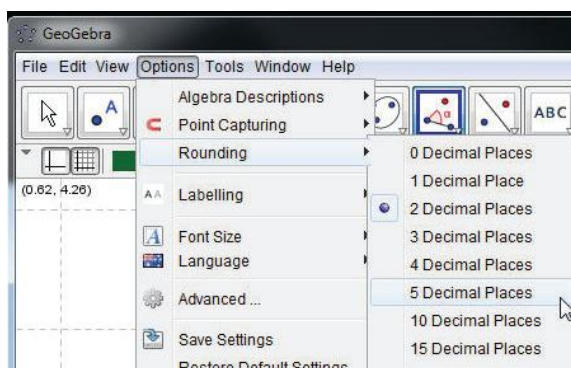
3 Label the vertices of each triangle. Right-click on each vertex and click **Show Label** as shown below. If necessary, relabel the vertices as A, B and C by right-clicking on each vertex and selecting **Rename**. Make sure $\angle A = 32^\circ$ and $\angle B = 90^\circ$.



4 a Copy this table.

	Side length (mm)			Trigonometric ratio		
	BC (opp)	AB (adj)	AC (hyp)	$\sin A = \frac{BC}{AC}$	$\cos A = \frac{AB}{AC}$	$\tan A = \frac{BC}{AB}$
1						
2						
3						
4						

- b Click **Options**, **Rounding** and **5 Decimal Places**.



- c To measure the sides of each triangle, select  **Distance or Length** and click on a side of the triangle. You will see the measurements appear in centimetres. Convert to millimetres for your table.
- d For each triangle, calculate each trigonometric ratio correct to 2 decimal places.
- e What do you notice about the values of each ratio for all of the triangles?

Homework sheet

Trigonometry 1

MAT09MGHS10032

Worksheet

Trigonometry
calculations

MAT09MGWK10041

Puzzle sheet

Trigonometry
squaresaw

MAT09MGPS10042

4-04 Trigonometry on a calculator

In the previous section, we discovered that for any particular angle, the sine, cosine and tangent ratios stay constant (the same) for all right-angled triangles with that angle. For example, $\sin 32^\circ \approx 0.53$ always, no matter what size the similar right-angled triangle.

Summary

For any given angle, the values of the sine, cosine and tangent ratios are constant.

This means that the value of a trigonometric ratio can be easily found on a calculator rather than through constructing and measuring triangles.

Degrees, minutes and seconds

Angles are measured in **degrees**, but one degree can be subdivided into 60 **minutes**.

One minute can be further subdivided into 60 **seconds**. The abbreviations for minutes and seconds are shown below.

Summary

$1^\circ = 60'$ (1 degree = 60 minutes)

$1' = 60''$ (1 minute = 60 seconds)

For example, an angle size of $48^\circ 35' 56''$ is 48 degrees, 35 minutes and 56 seconds, about halfway between 48° and 49° .

When rounding an angle to the nearest degree or minute, use 30 as the halfway mark.

Example 7

Round each angle correct to the nearest degree.

a $73^{\circ}27'$

b $9^{\circ}41'$

Solution

a $73^{\circ}27' \approx 73^{\circ}$

$27' < 30'$, so round down.

b $9^{\circ}41' \approx 10^{\circ}$

$41' \geq 30'$, so round up.

Example 8

Round each angle correct to the nearest minute.

a $33^{\circ}53'30''$

b $44^{\circ}15'40''$

Solution

a $33^{\circ}53'30'' \approx 33^{\circ}54'$

$30'' \geq 30''$, so round up.

b $44^{\circ}15'40'' \approx 44^{\circ}16''$

$40'' \geq 30''$, so round up.

Degrees and minutes on a calculator

To enter degrees and minutes (and seconds) into a scientific calculator, use the $\circ \prime \prime$ or DMS (Degrees-Minutes-Seconds) key.

Example 9

Evaluate each expression correct to two decimal places.

a $\sin 46^{\circ}$

b $\tan 57.4^{\circ}$

c $4 \cos 20^{\circ}$

d $68.3 \sin 38^{\circ}25'$

e $\frac{23}{\cos 18^{\circ}50'}$

Solution

Make sure that your calculator is in the degrees mode (D or DEG) or your answer will be incorrect.

a $\sin 46^{\circ} = 0.71933 \dots$
 ≈ 0.72

On calculator: $\sin 46 =$

b $\tan 57.4^{\circ} = 1.56365 \dots$
 ≈ 1.56

On calculator: $\tan 57.4 =$

↑
This angle is 57.4° (a decimal), not $57^{\circ}4'$.

c $4 \cos 20^{\circ} = 3.75877 \dots$
 ≈ 3.76

On calculator: $4 \cos 20 =$

↑
This means $4 \times \cos 20^{\circ}$.

d $68.3 \sin 38^\circ 25' = 42.43996 \dots$ On calculator: $68.3 \sin 38 \circ ' '' 25 \circ ' '' =$
 ≈ 42.44

e $\frac{23}{\cos 18^\circ 50'} = 24.30103 \dots$ On calculator: $23 \div \cos 18 \circ ' '' 50 \circ ' '' =$
 ≈ 24.30

Example 10

Convert each angle size to degrees and minutes, correct to the nearest minute.

a 82.5°

b 60.81°

Solution

a $82.5^\circ = 82^\circ 30'$

On calculator: $82.5 = \circ ' ''$

b $60.81^\circ = 60^\circ 48' 36''$
 $\approx 60^\circ 49'$

On calculator: $60.81 = \circ ' ''$

Exercise 4-04 Trigonometry on a calculator

See Example 7

1 Round each angle size correct to the nearest degree.

a $27^\circ 54'$

b $40^\circ 30'$

c $19^\circ 18'$

d $33^\circ 7' 25''$

e $33^\circ 41' 5''$

f 56.4°

g 29.75°

h $44^\circ 18'$

See Example 8

2 Round each angle size correct to the nearest minute.

a $68^\circ 39' 42''$

b $54^\circ 22' 21''$

c $68^\circ 39' 30''$

d $18^\circ 30' 27''$

e $9^\circ 10' 55''$

f $47^\circ 59' 9.5''$

g $3^\circ 45' 35''$

h $57^\circ 10' 29''$

See Example 9

3 Evaluate each expression correct to two decimal places.

a $\tan 84^\circ$

b $\cos 15^\circ$

c $\tan 47^\circ$

d $\sin 33^\circ$

e $\sin 77^\circ$

f $\cos 60.1^\circ$

g $\tan 39.55^\circ$

h $\cos 18^\circ$

i $8 \tan 75^\circ$

j $14 \sin 56^\circ$

k $12 \div \tan 20^\circ$

l $\frac{7}{\sin 43^\circ}$

m $50 \times \sin 70.34^\circ$

n $66.2 \cos 81^\circ 42'$

o $18.53 \sin 11.8^\circ$

p $\frac{27}{\cos 35^\circ}$

q $\frac{44.5}{\tan 65^\circ 58'}$

r $\frac{200}{\sin 54.2^\circ}$

s $24.1 \div \tan 63^\circ$

t $\frac{15.7}{\cos 21^\circ 8'}$

See Example 10

4 Convert each angle size to degrees and minutes, correct to the nearest minute.

a 55.5°

b 14.15°

c 72.38°

d 33.77°

e 66.41°

f 7.875°

g 28.123°

h 31.046°

i 34.45°

j 71.087°

k 5.4829°

l 69.4545°

5 By guess-and-checking with your calculator, find the angle size, θ (to the nearest degree), that gives each value.

a $\sin \theta = 0.7880$

b $\tan \theta = 0.2493$

c $\tan \theta = 1.2799$

d $\cos \theta = 0.5$

e $\sin \theta = 0.5446$

f $\cos \theta = 0.8829$

g $\tan \theta = 0.7265$

h $\sin \theta = 0.9998$

Mental skills 4

Maths without calculators

Estimating answers

A quick way of estimating an answer is to round each number in the calculation.

1 Study each example.

$$\begin{aligned} \text{a } 631 + 280 + 51 + 43 + 96 &\approx 600 + 300 + 50 + 40 + 100 \\ &= (600 + 300 + 100) + (50 + 40) \\ &= 1000 + 90 \\ &= 1090 \quad (\text{Actual answer} = 1101) \end{aligned}$$

$$\begin{aligned} \text{b } 55 + 132 - 34 + 17 - 78 &\approx 60 + 130 - 30 + 20 - 80 \\ &= (60 + 20 - 80) + (130 - 30) \\ &= 0 + 100 \\ &= 100 \quad (\text{Actual answer} = 92) \end{aligned}$$

$$\begin{aligned} \text{c } 78 \times 7 &\approx 80 \times 7 \\ &= 560 \quad (\text{Actual answer} = 546) \end{aligned}$$

$$\begin{aligned} \text{d } 510 \div 24 &\approx 500 \div 20 \\ &= 50 \div 2 \\ &= 25 \quad (\text{Actual answer} = 21.25) \end{aligned}$$

2 Now estimate each answer.

$$\text{a } 27 + 11 + 87 + 142 + 64$$

$$\text{b } 55 + 34 - 22 - 46 + 136$$

$$\text{c } 684 + 903$$

$$\text{d } 35 + 81 + 110 + 22 + 7$$

$$\text{e } 517 - 96$$

$$\text{f } 210 - 38 - 71 + 151 - 49$$

$$\text{g } 766 - 353$$

$$\text{h } 367 \times 2$$

$$\text{i } 83 \times 81$$

$$\text{j } 984 \times 16$$

$$\text{k } 828 \div 3$$

$$\text{l } 507 \div 7$$

3 Study each example involving decimals.

$$\begin{aligned} \text{a } 20.91 - 11.3 + 2.5 &\approx 21 - 11 + 3 \\ &= 13 \quad (\text{Exact answer} = 12.11) \end{aligned}$$

$$\begin{aligned} \text{b } 4.78 \times 19.2 &\approx 5 \times 20 \\ &= 100 \quad (\text{Exact answer} = 91.776) \end{aligned}$$

$$\begin{aligned} \text{c } 75.13 \div 8.4 &\approx 75 \div 8 \\ &< 80 \div 8 \\ &< 10 \\ &\approx 9 \quad (\text{Exact answer} = 8.944 \dots) \end{aligned}$$

$$\begin{aligned} \text{d } \frac{37.6 + 9.3}{41.2 - 12.7} &\approx \frac{38 + 9}{40 - 13} \\ &= \frac{47}{27} \\ &\approx \frac{50}{30} \\ &\approx 1.6 \quad (\text{Exact answer} = 1.645 \dots) \end{aligned}$$

4 Now estimate each answer.

a $3.75 + 9.381 + 4.6 + 10.5$

c 18.47×9.61

e $\frac{11.07 + 18.4}{12.2}$

g $54.75 - 18.6 - 14.4$

i $62.13 \div 10.7$

b $14.807 + 6.6 - 7.22$

d 4.27×97.6

f $\frac{38.18}{17.2 - 9.6}$

h $\frac{18.46 \times 4.9}{39.72 - 15.2}$

j $(4.89)^2$

Just for the record

Degrees, minutes and seconds

We are familiar with using base 10 systems in number, measurement and currency: there are 100 centimetres in a metre, 1000 grams in a kilogram and 100 cents in a dollar. So why are there 360° in a revolution, 60 minutes in a degree and 60 seconds in a minute?

In 2000 BCE, the Babylonians used a base 60 or sexagesimal system of numeration, because 60 is a rounder, more convenient number than 10. This is because 60 has more factors and is divisible by 3, 4 and 6. Furthermore, $6 \times 60 = 360$, which was the Babylonian approximation for the number of days in a year, so that each day the Earth would travel 1° around the Sun. As measuring devices and calculations required greater precision, each degree was subdivided into 60 equal parts called **minutes**, and these were further divided into 60 parts called **seconds**. This level of accuracy is essential in navigation and mapping.

1 A minute is a 'small' part of a degree. Investigate how an alternative meaning (and pronunciation) of 'minute' is 'tiny'.

2 A second is the 'second' subdivision of a degree. Explain how there are two different meanings of 'second'.

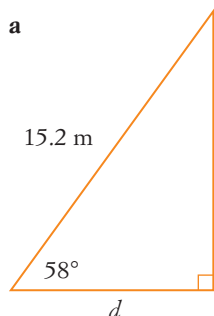
4-05 Finding an unknown side

Since the trigonometric ratio of any angle is a constant number, we can use it to calculate the length of an unknown side in a right-angled triangle if one other side is known. We need to select the correct ratio that links the given angle to the unknown side and known side.

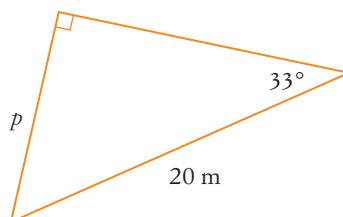
Example 11

Find the value of each pronumeral, correct to two decimal places.

a



b



Solution

a SOH, CAH or TOA?

The marked sides are the adjacent (A) side and the hypotenuse (H), so use cos.

$$\begin{aligned}\cos 58^\circ &= \frac{\text{adjacent}}{\text{hypotenuse}} \\ &= \frac{d}{15.2}\end{aligned}$$

$$\cos 58^\circ \times 15.2 = \frac{d}{15.2} \times 15.2$$

$$15.2 \cos 58^\circ = d$$

$$d = 15.2 \cos 58^\circ$$

$$= 8.05477 \dots$$

$$\approx 8.05$$

From the diagram, a length of 8.05 m looks reasonable.

b SOH, CAH or TOA?

The marked sides are the opposite (O) side and the hypotenuse (H), so use sin.

$$\begin{aligned}\sin 33^\circ &= \frac{\text{opposite}}{\text{hypotenuse}} \\ &= \frac{p}{20}\end{aligned}$$

$$\sin 33^\circ \times 20 = \frac{p}{20} \times 20$$

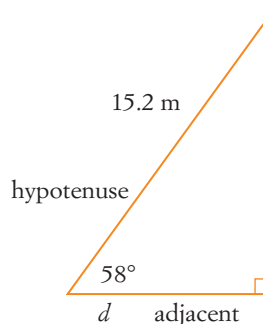
$$20 \sin 33^\circ = p$$

$$p = 20 \sin 33^\circ$$

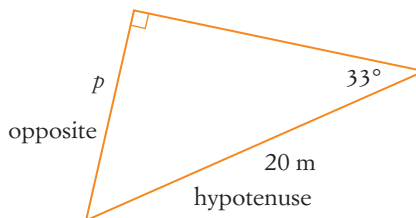
$$= 10.8927 \dots$$

$$\approx 10.89$$

From the diagram, a length of 10.89 m looks reasonable.



Multiply both sides by 15.2.



Video tutorial

Finding an unknown side

MAT09MGVT10008

Video tutorial

Trigonometry

MAT09MGVT00009

Summary

Finding an unknown side in a right-angled triangle

- 1 identify the two labelled sides and decide whether to use sin, cos or tan
- 2 write an equation using the ratio, the given angle and the variable
- 3 solve the equation to find the value of the variable

Example 12

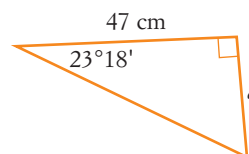
Find the value of q correct to the nearest centimetre.

Solution

q is opposite, 47 cm is adjacent, so use tan.

$$\begin{aligned}\tan 23^\circ 18' &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{q}{47} \\ q &= 47 \tan 23^\circ 18' \\ &= 20.2413 \dots \\ &\approx 20 \text{ cm}\end{aligned}$$

From the diagram, a length of 20 cm looks reasonable.



Example 13

$\triangle JKL$ is right-angled at K , $JK = 35$ m and $\angle J = 63^\circ$. Find the length of LK correct to the nearest metre.

Solution

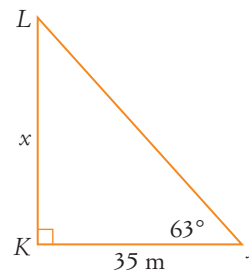
Draw a diagram.

Let the length of LK be x .

x is opposite, 35 m is adjacent, so use tan.

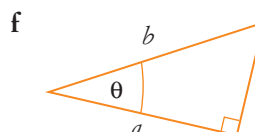
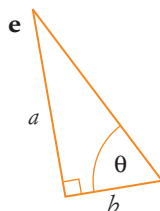
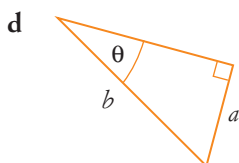
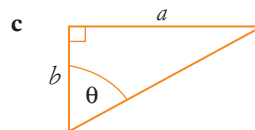
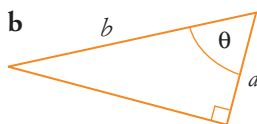
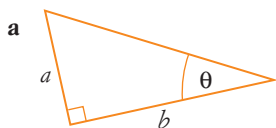
$$\begin{aligned}\tan 63^\circ &= \frac{x}{35} \\ x &= 35 \tan 63^\circ \\ &= 68.6913 \dots \\ LK &\approx 69 \text{ m}\end{aligned}$$

From the diagram, a length of 69 m looks reasonable.



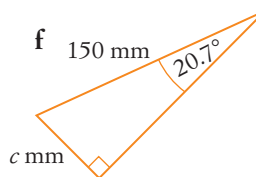
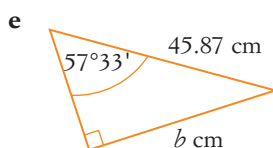
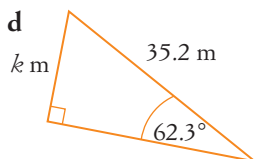
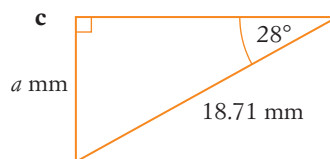
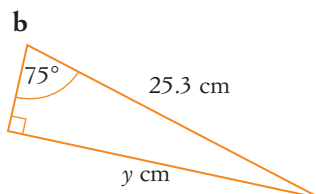
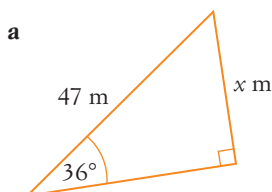
Exercise 4-05 Finding an unknown side

- 1 For each triangle, which trigonometric ratio ($\sin \theta$, $\cos \theta$ or $\tan \theta$) is equal to $\frac{a}{b}$?



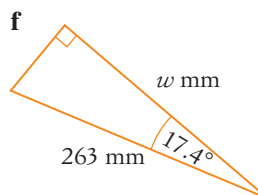
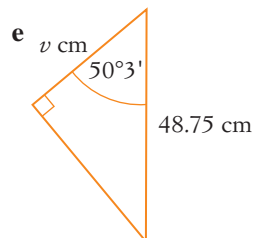
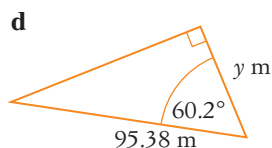
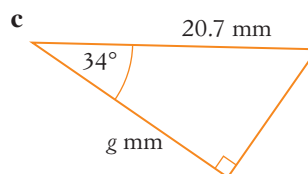
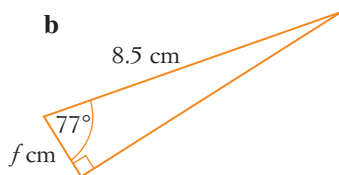
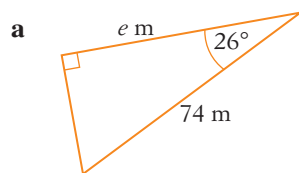
- 2 Find the value of the pronumeral in each triangle, correct to two decimal places.

See Example 11



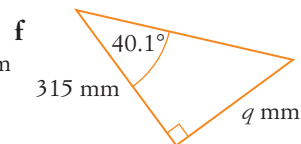
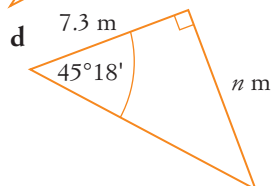
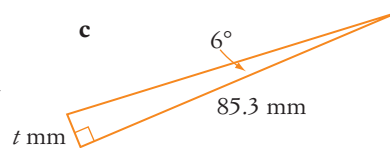
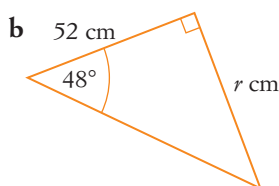
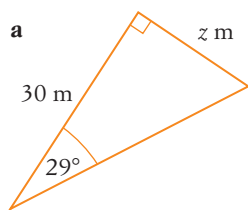
See Example 12

- 3 Find the value of the pronumeral in each triangle, correct to one decimal place.

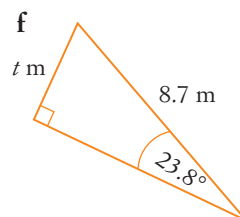
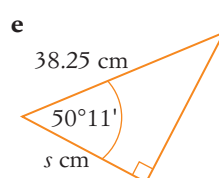
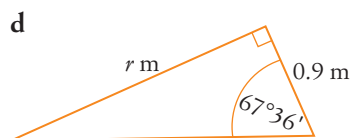
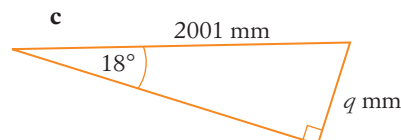
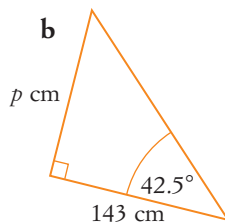
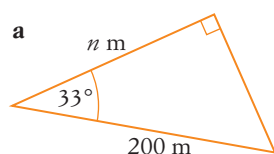


See Example 11

- 4 Find the value of the pronumeral in each triangle, correct to two decimal places.

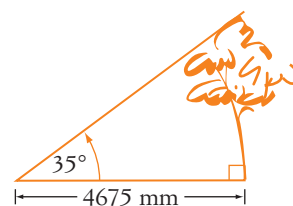


- 5 Find the value of the pronumeral in each triangle, correct to one decimal place.



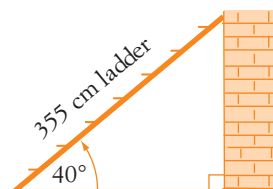
- 6 What is the height of this tree? Select the correct answer **A**, **B**, **C** or **D**.

- A** 2.68 m **B** 3.27 m
C 3.83 m **D** 6.68 m

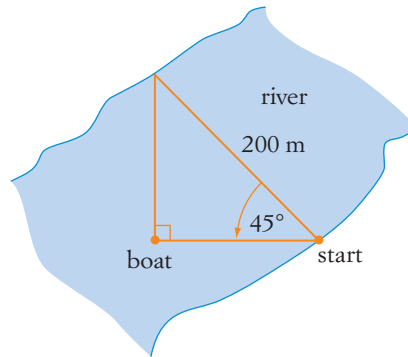


- 7 Find each length or distance correct to one decimal place.

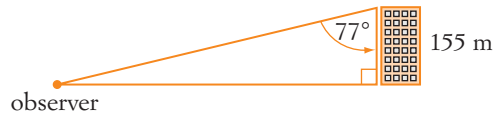
- a** How far up the ladder reaches the wall.



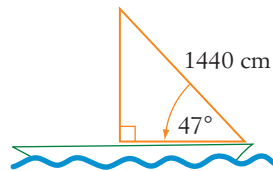
- b The distance between the boat and the start.



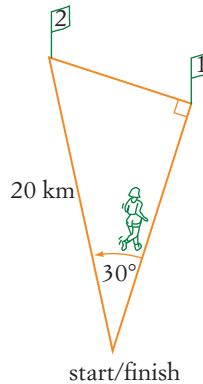
- c The distance from the observer to the base of the building.



- d The height of the boat's mast.



- e The distance between:
i checkpoints 1 and 2
ii checkpoint 1 and the start.



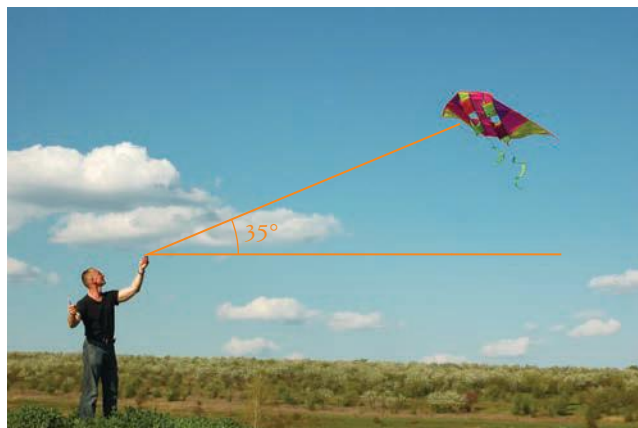
- 8 $\triangle ABC$ is right-angled at B , $AC = 14.8$ m and $\angle C = 56^\circ$. Find the length of side AB , correct to one decimal place.
- 9 $\triangle MNR$ is right-angled at M , $MN = 19$ cm and $\angle N = 27^\circ$. Find the length of MR , correct to the nearest centimetre.
- 10 In $\triangle XYW$, $\angle X = 90^\circ$, $\angle Y = 43.7^\circ$ and $WY = 8.34$ m. Find the length of XW , correct to two decimal places.
- 11 $\triangle AHK$ is right-angled at K , $\angle H = 76^\circ$ and $AH = 13.9$ m. Find the length of HK , correct to one decimal place.
- 12 A tree casts a shadow 20 m long. If the Sun's rays meet the ground at 25° , find the height of the tree, correct to the nearest cm.
- 13 A 6 m ladder is placed against a pole. If the ladder makes an angle of 17° with the pole, how far up the pole does the ladder reach? Answer to the nearest mm.
- 14 A boat is anchored by a rope 5.5 m long. If the anchor rope makes an angle of 23° with the vertical, calculate the depth of the water (correct to one decimal place).

See Example 13

Worked solutions
Finding an unknown side

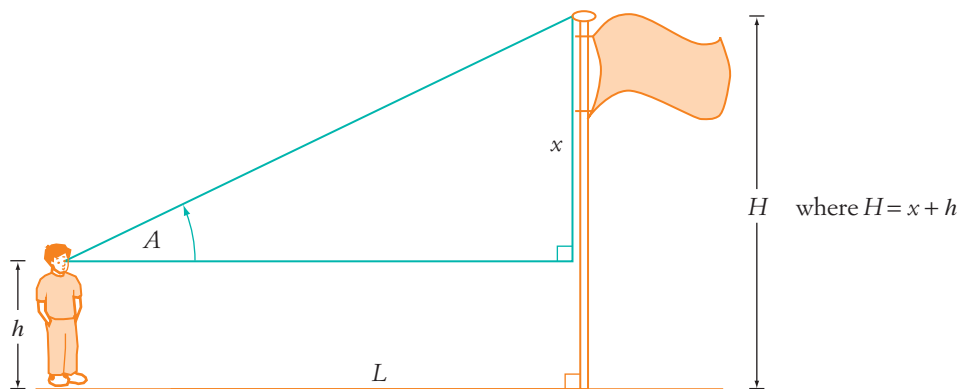
MAT09MGWS10018

- 15 A rectangular gate has a diagonal brace that makes an angle of 60° with the bottom of the gate. If the length of the diagonal brace is 1860 mm, calculate the height of the gate. Select the correct answer **A**, **B**, **C** or **D**.
- A** 2148 mm **B** 930 mm **C** 1610 mm **D** 3221 mm
- 16 Jacob is flying a kite that is attached to a string 155 m long. The string makes an angle of 35° to the horizontal. Calculate, correct to the nearest metre, the height of the kite above Jacob.



Investigation: Calculating the height of an object

You will need: tape measure or trundle wheel, a clinometer (or protractor) to measure the angle. Trigonometry can be used to find the heights of buildings, flagpoles and trees without actually measuring them. This can be done by measuring the distance along the ground from the base of the object to a person. The person then measures the angle to the top of the object. For example, the height of a flagpole can be calculated using the set-up shown in the diagram below.



h is the eye height of the person who measures the angle, A , to the top of the flagpole. L is the distance the person is from the base of the flagpole, x is the height of the flagpole above the person's eye height, $H = x + h$ is the height of the flagpole above the ground.

- 1 Select a tall object outside to measure.
- 2 Work with a partner to measure (in cm) the distance, L , along the ground, the height, h , of the person, and the angle (in degrees) to the top of the object. Copy the table below and record your information in the first row.

Distance, L (cm)	Angle, A°	Height of person, h (cm)	Calculated height, x cm	Height of flagpole, H cm

- Use the tan ratio to calculate the value of x to the nearest whole number.
- Hence find H , the height of the kite to the nearest centimetre. Write your answers in the table.
- Repeat the measurements and calculations three more times from different positions, with different persons measuring the angle. This will help to improve the accuracy of your results and minimise errors. Write your results in the table.
- Did you find similar values for H ? Do they seem reasonable for the height of the object?
- Calculate the average value for H .

4-06 Finding more unknown sides

In the following examples, the unknown appears in the **denominator** of the equation.

Example 14

Find the value of w , correct to two decimal places.

Solution

80 m is the opposite side, w m is the hypotenuse, so use sin.

$$\sin 55^\circ = \frac{80}{w}$$

Note that the variable w appears in the denominator of the equation.

$$\sin 55^\circ \times w = \frac{80}{w} \times w$$

Multiply both sides by w .

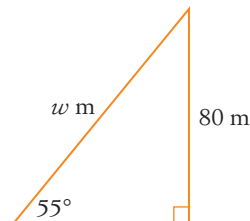
$$w \sin 55^\circ = 80$$

$$\frac{w \sin 55^\circ}{\sin 55^\circ} = \frac{80}{\sin 55^\circ}$$

Divide both sides by $\sin 55^\circ$.

$$\begin{aligned} w &= \frac{80}{\sin 55^\circ} \\ &= 97.66196 \dots \\ &\approx 97.66 \end{aligned}$$

Note that when the unknown appears in the denominator of an equation, it can swap positions with the trigonometric ratio, so that $\sin 55^\circ = \frac{80}{w}$ becomes $w = \frac{80}{\sin 55^\circ}$.



Worksheet

Finding an unknown side

MAT09MGWK10043

Puzzle sheet

Trigonometry equations 1

MAT09MGPS00041

Example 15

Find the length of x , correct to two decimal places.

Solution

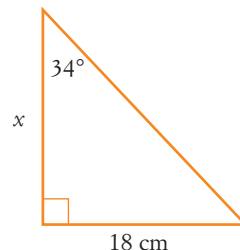
18 cm is the opposite side, x is the adjacent side, so use $\tan$.

$$\tan 34^\circ = \frac{18}{x}$$

x appears in the denominator

$$\begin{aligned} x &= \frac{18}{\tan 34^\circ} \\ &= 26.6860 \dots \\ &\approx 26.69 \text{ cm} \end{aligned}$$

Swap the position of x with $\tan 34^\circ$.



Alternative method

To avoid having x in the denominator, we could use $\tan$ with the third angle of the triangle.

$$\text{Third angle} = 180^\circ - 90^\circ - 34^\circ = 56^\circ$$

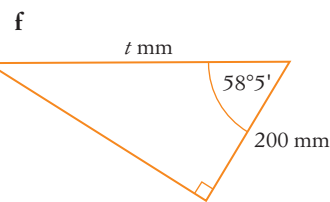
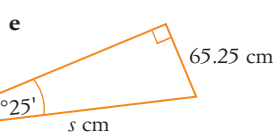
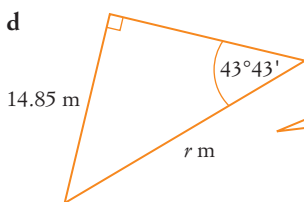
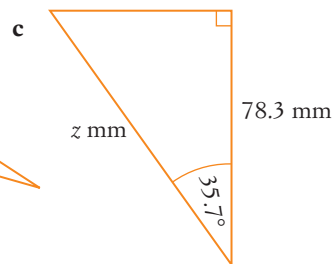
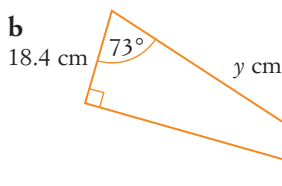
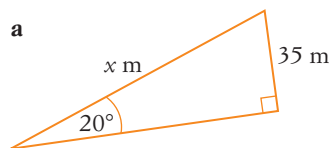
$$\tan 56^\circ = \frac{x}{18}$$

$$\begin{aligned} x &= 18 \tan 56^\circ \\ &= 26.6860 \dots \\ &\approx 26.69 \text{ cm} \end{aligned}$$

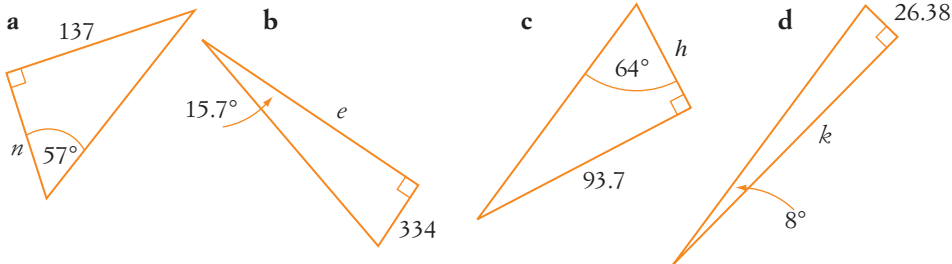
Exercise 4-06 Finding more unknown sides

See Example 14

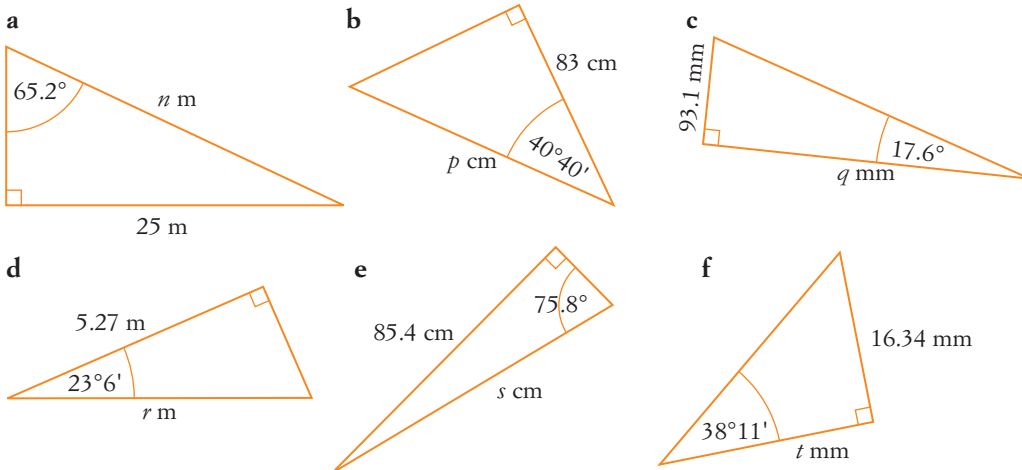
- 1 Find the value of each pronumeral, correct to one decimal place.



- 2 $\triangle XYZ$ is right-angled at Z , $ZY = 230$ mm and $\angle Y = 45^\circ$. Find the length of XY , correct to the nearest millimetre.
- 3 In $\triangle K LW$, $\angle L = 90^\circ$, $KL = 12$ m and $\angle W = 75.2^\circ$. Find KW , correct to the nearest metre.
- 4 Find the value of each pronumeral, correct to two decimal places.

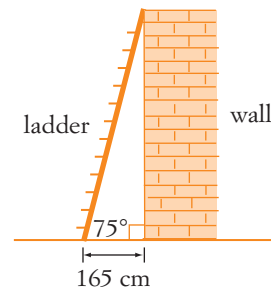


- 5 $\triangle CDE$ is right-angled at D , $\angle E = 36^\circ$ and $CD = 5$ m. Find the length of side DE , correct to two decimal places.
- 6 In $\triangle HMT$, $\angle T = 90^\circ$, $\angle M = 19^\circ 47'$ and side $HT = 18.4$ cm. Find the length of side HM , correct to one decimal place.
- 7 Find the value of each pronumeral, correct to one decimal place.



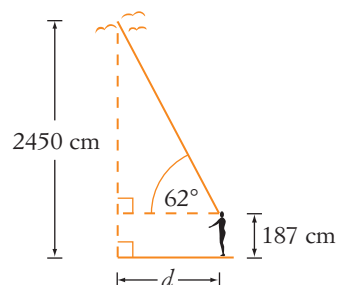
- 8 Find the length of this ladder. Select the correct answer **A**, **B**, **C** or **D**.

A 159 cm	B 171 cm
C 243 cm	D 638 cm

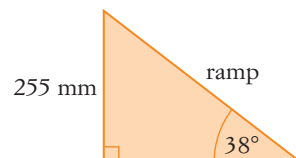


- 9 Find each length or distance correct to one decimal place.

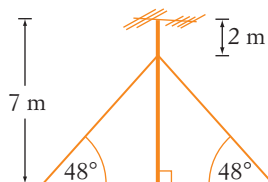
a How far the person is from being directly under the birds.



b The length of the ramp.

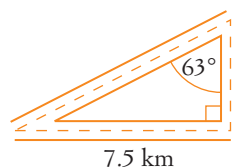


c The length of the support wire.

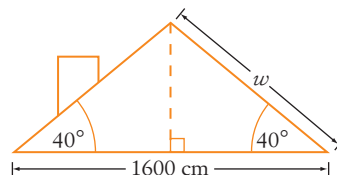


d The length of:

- i the shortest road
- ii the longest road.



e The slant height of the roof.



See Example 15 10 A ladder rests against a wall. The foot of the ladder is 355 cm from the wall and makes an angle of 63° with the ground. How long (to the nearest cm) is the ladder?

- 11 A supporting wire is attached to the top of a flagpole. The wire meets the ground at an angle of 51° and the flagpole is 15 m high. How far from the base of the flagpole is the wire anchored to the ground? (Give your answer to the nearest 0.1 m.)

Worked solutions

Finding more unknown sides

MAT09MGWS10019

- 12 A glider is flying at an altitude (height) of 1.5 km. To land, it descends at an angle of 18° to the ground. How far must the glider travel before landing? (Give your answer to the nearest 0.1 km.)



Shutterstock.com/Pixachi

- 13 A shooter aims directly at a target, but just before firing, the rifle is lifted 1° off target. The shot misses the target by 67 mm. How far is the shooter standing from the target? Select the correct answer **A**, **B**, **C** or **D**.
A 1169 mm **B** 3838 mm **C** 3839 mm **D** 6701 mm
- 14 A hot air balloon is anchored to the ground by a rope. When it drifts 20 m sideways, it makes an angle of 75° with the ground. How long is the rope (correct to one decimal place)?

Worked solutions

Finding more unknown sides

MAT09MGWS10019

Investigation: Finding an angle, given a trigonometric ratio

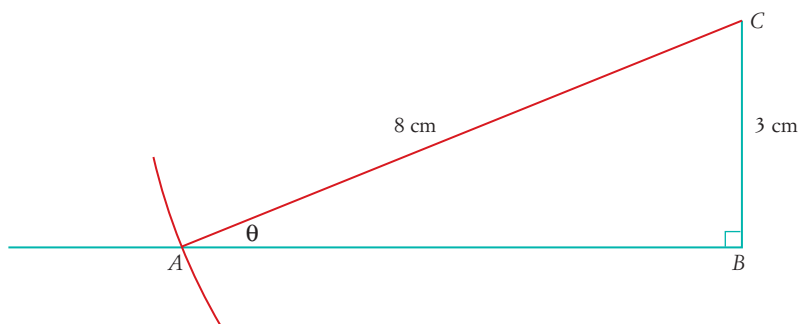
You will need: a ruler, compasses and a calculator.

- 1 Copy and complete this table, calculating each ratio as a decimal correct to three decimal places.

θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
0°			
15°			
30°			
45°			
60°			
75°			
90°			undefined

- 2 Can you work out why there is no answer for $\tan 90^\circ$?
 3 What are the minimum and maximum values of $\sin \theta$?
 4 What are the minimum and maximum values of $\cos \theta$?
 5 Is there a pattern between the values of $\sin \theta$ and $\cos \theta$?
 6 Check the value of $\sin 30^\circ$ by constructing a right-angled triangle with one angle that is 30° , measuring the opposite side and hypotenuse and dividing them.
 7 Check the value of $\tan 45^\circ$ by constructing a right-angled triangle with one angle that is 45° , measuring the opposite and adjacent sides and dividing them.
 8 Use the table to estimate each trigonometric ratio and check your estimate using a calculator.
a $\sin 80^\circ$ **b** $\cos 34^\circ$ **c** $\tan 55^\circ$

- 9 If $\sin \theta = \frac{3}{8}$, find the value of the unknown angle θ to the nearest degree:
- by using the table and estimating (change $\frac{3}{8}$ to a decimal first)
 - using a calculator to guess-and-check
 - constructing a right-angled triangle with one angle θ , opposite side 3 cm and hypotenuse 8 cm, then measuring the size of θ .



- 10 If $\cos \theta = \frac{2}{5}$, find the value of the unknown angle θ to the nearest degree:
- by using the table and estimating (change $\frac{2}{5}$ to a decimal first)
 - using a calculator to guess-and-check
 - constructing a right-angled triangle with one angle θ , adjacent side 2 cm and hypotenuse 5 cm, then measuring the size of θ .
- 11 If $\frac{7}{10}$, find the value of the unknown angle θ to the nearest degree:
- by using the table and estimating (change $\frac{7}{10}$ to a decimal first)
 - using a calculator to guess-and-check
 - constructing a right-angled triangle with one angle θ , opposite side 7 cm and adjacent side 10 cm, then measuring the size of θ .

Worksheet

Finding an unknown angle

MAT09MGWK10044

Puzzle sheet

Trigonometry squaresaw

MAT09MGPS10042

Homework sheet

Trigonometry 2

MAT09MGHS10033

Homework sheet

Trigonometry revision

MAT09MGHS10034

Animated example

Trigonometry

MAT09MGAE00009

Worksheet

Trigonometry problems

MAT09MGWK10045

4-07 Finding an unknown angle

A scientific calculator can be used to evaluate a trigonometric ratio such as $\sin 38^\circ$, but it can also be used to find an unknown angle, θ , if the trigonometric ratio of the angle is known, for example, if $\sin \theta = 0.9063$.

An unknown angle can be found using the $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ keys on the calculator. These are called the **inverse sin**, **inverse cos** and **inverse tan** functions, found by pressing the **SHIFT** or **2ndF** key before the **sin**, **cos** or **tan** keys.

Example 16

- a If $\tan X = 3.754$, find angle X , correct to the nearest minute.
b If $\cos \alpha = \frac{4}{7}$, find angle α , correct to the nearest degree.

Solution

a $\tan X = 3.754$

$$\begin{aligned} X &= 75.0837 \dots^\circ \\ &= 75^\circ 5' 1.62'' \\ &\approx 75^\circ 5' \end{aligned}$$

On calculator: **SHIFT** **tan** **3.754** **=**

On calculator: **° ' "** or **DMS**

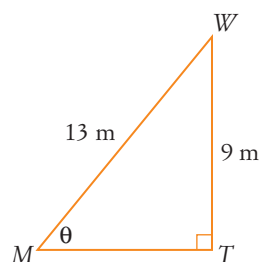
b $\cos \alpha = \frac{4}{7}$

$$\begin{aligned} \alpha &= 55.1500 \dots^\circ \\ &\approx 55^\circ \end{aligned}$$

On calculator: **SHIFT** **cos** **4** **a^{b/c}** **7** **=**

Example 17

Find the size of angle θ , correct to the nearest degree.



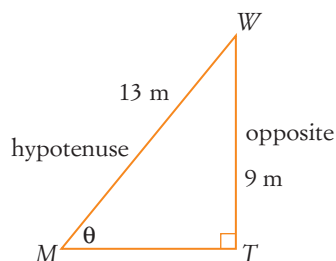
Solution

SOH, CAH or TOA?

The known sides are the opposite (O) side and the hypotenuse (H), so use sin.

$$\sin \theta = \frac{9}{13}$$

$$\begin{aligned} \theta &= 43.8130 \dots^\circ \\ &\approx 44^\circ \end{aligned}$$



On calculator: **SHIFT** **sin** **9** **a^{b/c}** **13** **=**

From the diagram, an angle size of 44° looks reasonable.

Video tutorial

Finding an unknown angle

MAT09MGVT10009

Video tutorial

Trigonometry

MAT09MGVT00009

Puzzle sheet

Trigonometry:
Finding angles

MAT09MGPS00044

Puzzle sheet

Trigonometry
equations 2

MAT09MGPS00042

Summary

Finding an unknown angle in a right-angled triangle

- 1 Identify the two known sides and decide whether to use the sin, cos or tan ratio.
- 2 Write an equation using the ratio, the angle variable and the two sides as a fraction.
- 3 Use the calculator's inverse trigonometric function to find the size of the angle.

Example 18

$\triangle XYZ$ is right-angled at Y , with $XY = 35$ cm and $YZ = 47$ cm. Find $\angle Z$, correct to the nearest minute.

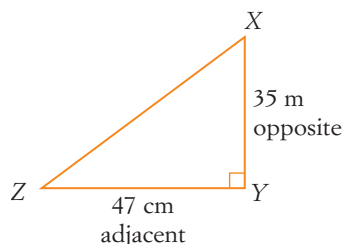
Solution

Sketch a diagram.

SOH, CAH or TOA?

The known sides are the opposite (O) and the adjacent (A), so use tan.

$$\begin{aligned}\tan \theta &= \frac{35}{47} \\ \theta &= 36.6743 \dots^\circ \\ &= 36^\circ 40' 27.66'' \\ &\approx 36^\circ 40'\end{aligned}$$



On calculator: **SHIFT** **tan** **35** **a^{b/c}** **47** **=**

From the diagram, an angle size of $36^\circ 40'$ looks reasonable.

Exercise 4-07 Finding an unknown angle

See Example 16

- 1 Find the size of angle θ correct to the nearest degree.

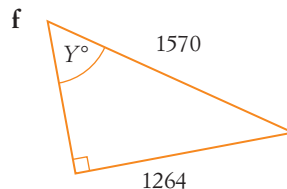
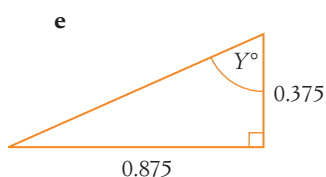
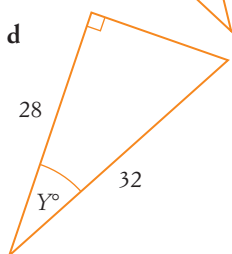
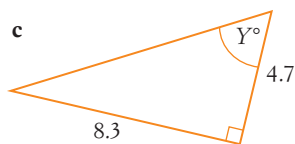
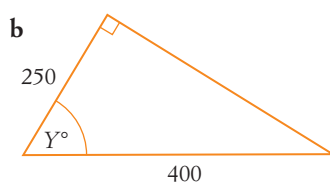
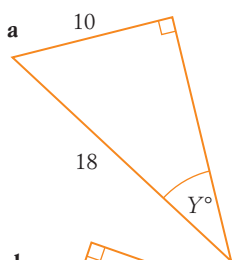
a $\cos \theta = 0.76$	b $\tan \theta = 2.0532$	c $\sin \theta = \frac{\sqrt{3}}{2}$	d $\tan \theta = 6$
e $\sin \theta = \frac{7}{8}$	f $\cos \theta = \frac{13}{15}$	g $\sin \theta = \frac{1}{10}$	h $\cos \theta = \frac{1}{\sqrt{2}}$
i $\tan \theta = \sqrt{3}$	j $\cos \theta = 0.1352$	k $\tan \theta = 8.836$	l $\sin \theta = \frac{1}{4}$

- 2 Find the size of angle A correct to the nearest minute.

a $\tan A = \frac{15}{7}$	b $\sin A = 0.815$	c $\cos A = \frac{4}{5}$	d $\cos A = 0.9387$
e $\tan A = \frac{19}{20}$	f $\cos A = \frac{3}{10}$	g $\sin A = \frac{5}{11}$	h $\sin A = 0.88$
i $\tan A = 15.07$	j $\cos A = \frac{1}{7}$	k $\tan A = \frac{\sqrt{2}}{2}$	l $\sin A = \frac{7}{9}$

3 Find the angle Y , correct to the nearest degree.

See Example 17



4 Find the size of angle α correct to the nearest minute.

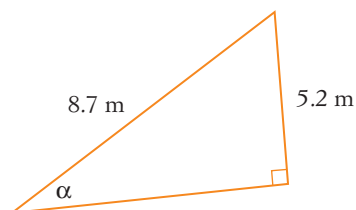
Select the correct answer **A**, **B**, **C** or **D**.

A $30^\circ 52'$

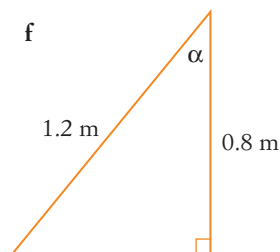
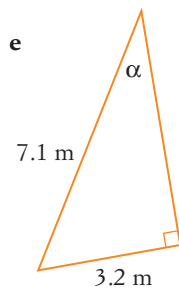
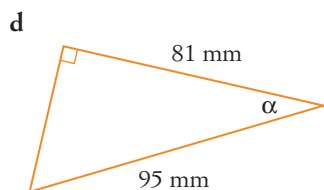
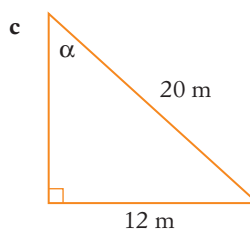
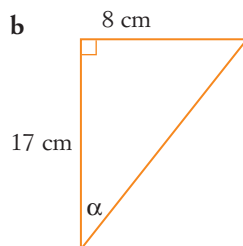
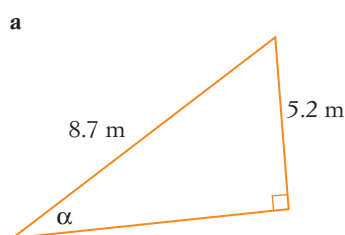
B $30^\circ 53'$

C $36^\circ 42'$

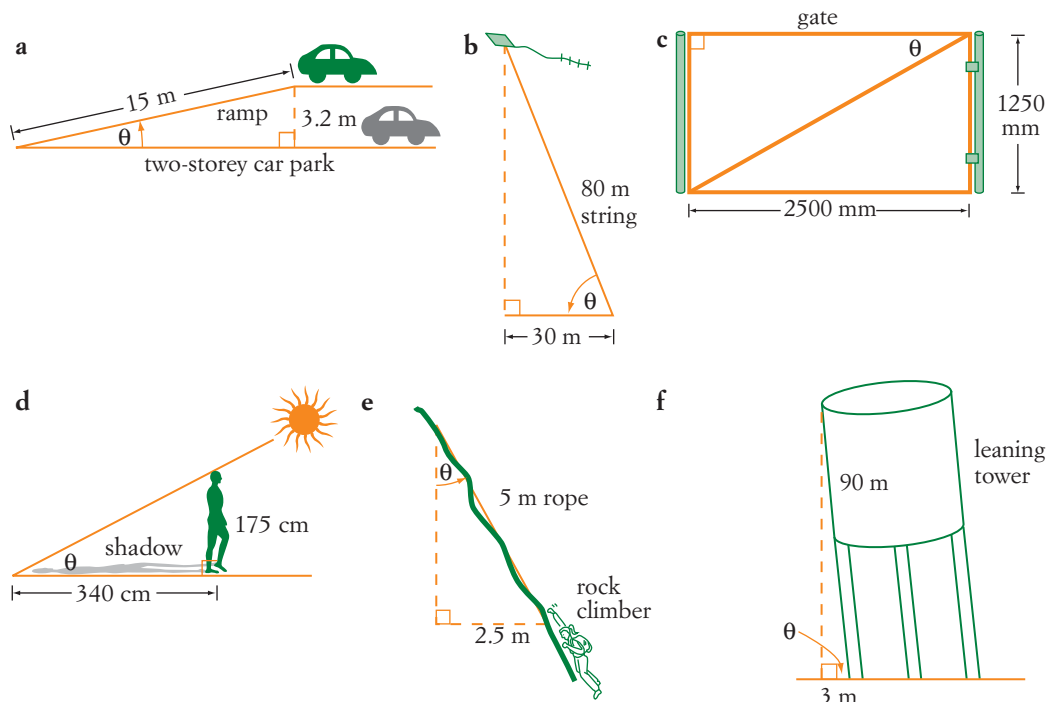
D $36^\circ 43'$



5 Find the size of angle α correct to the nearest minute.



- 6 Find the size of angle θ , correct to the nearest degree.



See Example 18

- 7 In $\triangle XYW$, $\angle X = 90^\circ$, $XY = 8$ cm and $XW = 10$ cm. Find $\angle W$ correct to the nearest degree.
8 In $\triangle FGH$, $\angle G = 90^\circ$, $GH = 3.7$ m and $FH = 19.5$ m. Find the size of angle F , correct to the nearest minute.
9 $\triangle HTM$ is right-angled at T , $HM = 45$ m and $MT = 35$ m. Find $\angle M$, correct to one decimal place.
10 $\triangle TSV$ is right-angled at S , $TV = 9.5$ cm, and $ST = 8.4$ cm. Find $\angle V$, correct to the nearest degree.

For questions 11 to 16, write your answers correct to the nearest degree.

- 11 A stretch of freeway rises 55 m for every 300 m travelled along the road. Find the angle at which the road is inclined to the horizontal.
12 A ladder 20 m long is placed against a building. If the ladder reaches 16 m up the building, find the ladder's angle of inclination to the building.
13 An aircraft is descending in a straight line to an airport. At a height of 1270 m, it is 1500 m horizontally from the airport. Find its angle of descent to the horizontal. Select the correct answer A, B, C or D.
A 32° B 40° C 50° D 58°
14 A tree 8.5 m high casts a shadow 3 m long. What is the angle of the Sun from the ground?
15 At a resort, an artificial beach slopes down at a steady angle. After walking 8.5 m down the slope from the water's edge, the water has a depth of 1.6 m. At what angle is the beach inclined to the horizontal?
16 A pile of wheat is in the shape of a cone that has a diameter of 35 m and measures 27 m up the slope to the apex. Calculate the angle of repose of the wheat (the angle the sloping side makes with the horizontal base).

Worked solutions

Finding an unknown angle

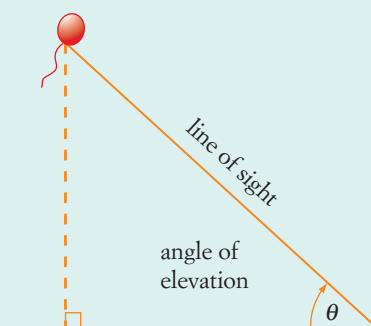
MAT09MGWS10020

4-08 Angles of elevation and depression

Summary

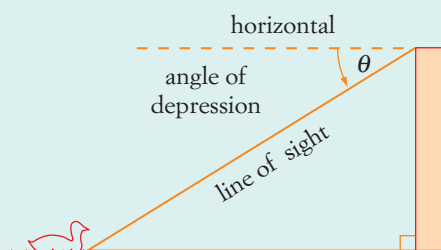
The **angle of elevation** is the angle of looking **up**, measured from the horizontal.

When you feel elevated, things are 'looking up'!



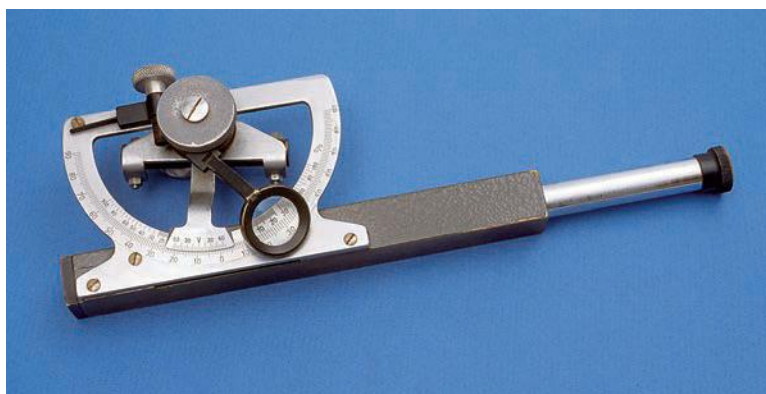
The **angle of depression** is the angle of looking **down**, measured from the horizontal.

When you feel depressed, things are 'looking down'!



Problems involving angles of elevation and depression usually require the tan ratio in their solutions.

An instrument for measuring an angle of elevation or depression is a **clinometer**. It is like a protractor with a sighting tube attached.



The Picture Source/Terry Oakley

Example 19

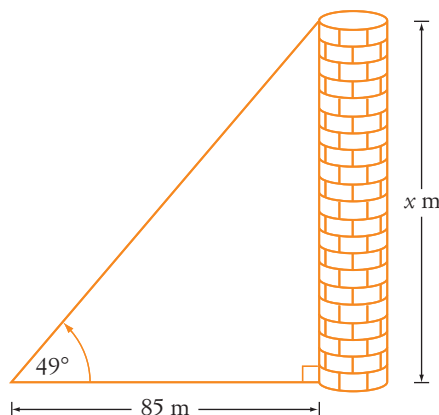
The angle of elevation of a chimney is 49° at a distance of 85 m from its base. Find the height of the chimney, to the nearest metre.

Solution

Let the height be x metres.

$$\begin{aligned}\tan 49^\circ &= \frac{x}{85} \\ x &= 85 \tan 49^\circ \\ &= 97.7813 \dots \\ &\approx 98\end{aligned}$$

The height of the chimney is 98 m.



Example 20

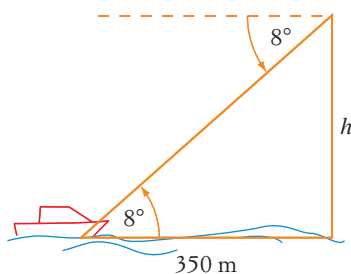
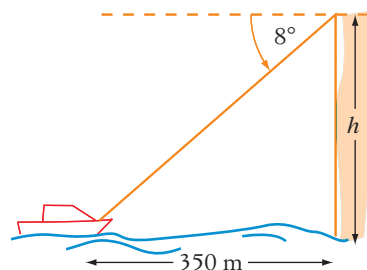
The angle of depression of a boat from the top of a cliff is 8° . If the boat is 350 m from the base of the cliff, calculate the height of the cliff, correct to the nearest metre.

Solution

By alternate angles, the angle of elevation of the top of the cliff from the boat is also 8° .

$$\begin{aligned}\tan 8^\circ &= \frac{h}{350} \\ h &= 350 \tan 8^\circ \\ &= 49.1892 \dots \\ &\approx 49\end{aligned}$$

The height of the cliff is 49 m.



Alternative method

The third angle in the triangle (adjacent to the angle of depression) = $90^\circ - 8^\circ = 82^\circ$.

$$\tan 82^\circ = \frac{350}{h}$$

$$h = \frac{350}{\tan 82^\circ}$$

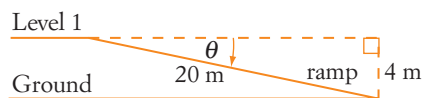
$$= 49.1892 \dots$$

$$\approx 49$$

The height of the cliff is 49 m.

Example 21

The ramp from one level to the next in a car park is 20 m long and drops 4 m. Find the angle of depression of the ramp, to the nearest degree.



Solution

$$\sin \theta = \frac{4}{20}$$

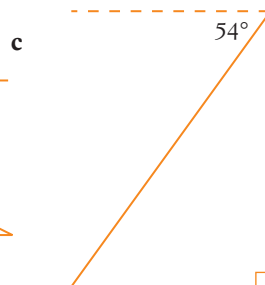
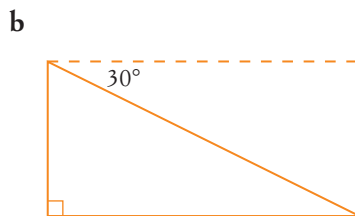
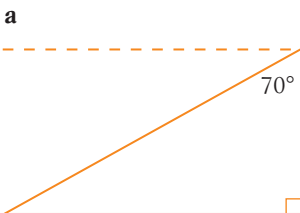
$$\theta = 11.5369 \dots^\circ$$

$$\approx 12^\circ$$

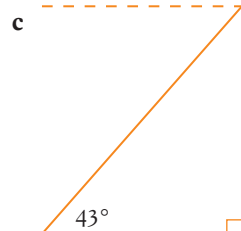
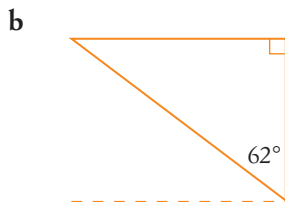
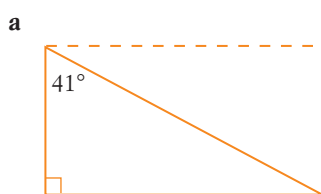
The angle of depression of the ramp is 12° .

Exercise 4-08 Angles of elevation and depression

1 Copy each diagram, mark the angle of elevation θ and find its size.

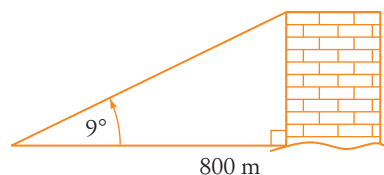


- 2 Copy each diagram, mark the angle of depression θ and find its size.

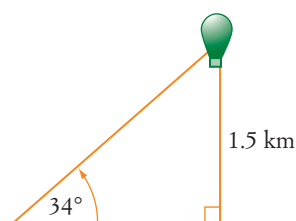


See Example 19

- 3 Marnie stands 800 m from the base of a building. Using a clinometer, she finds that the angle of elevation of the top is 9° . Find the height of the building, to the nearest metre.

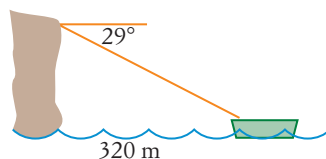


- 4 The angle of elevation of a weather balloon at a height of 1.5 km is 34° . How far (to the nearest metre) is the observer from being directly under the balloon?

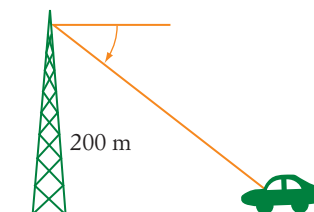


See Example 20

- 5 A raft is 320 m from the base of a cliff. The angle of depression of the raft from the top of the cliff is 29° . Find the height of the cliff, to the nearest metre.



- 6 From the top of a 200 m tower, the angle of depression of a car is 48° . How far is the car from the foot of the tower? Answer to the nearest metre.



See Example 21

- 7 A 275 m radio mast is 1.7 km from a school. Find, correct to the nearest degree, the angle of elevation of the top of the mast from the school.
- 8 In a concert hall, Bill is sitting 20 m from the stage by line of sight. He is also 5 m above the level of the stage. At what angle of depression is the stage? Answer to the nearest minute.
- 9 A monument 24 m high casts a shadow 20 m long. Calculate, correct to the nearest degree, the angle of elevation of the sun at this time of day.
- 10 A plane is 340 m directly above one end of a 1000 m runway. Find, correct to the nearest minute, the angle of depression to the far end of the runway.



Shutterstock.com/ubididook

- 11 An observer 174 cm tall is standing 11.6 m from the base of a flagpole. The angle of elevation to the top of the flagpole is 43° . How high is the flagpole, to the nearest cm?
- 12 A flagpole is mounted on top of a tall building. At a distance of 250 m from the base of the building, the angles of elevation of the bottom and top of the flagpole are 38° and 40° respectively. Calculate the height of the flagpole, correct to one decimal place.
- 13 A news helicopter hovers at a height of 500 m. The angles of depression of a fire moving in the direction of the helicopter are first 10° and then 15° . How far (to the nearest metre) has the fire moved between the two observations?
- 14 The angle of elevation to the bottom of a transmission tower on a hill from an observer 1.8 km away from the base of the hill is 5° . The angle of elevation to the top of the tower from the observer is 6.8° . If the distance from the observer to the base of the hill is 1.8 km, find the height of the tower to the nearest metre.

Worked solutions

Angles of elevation and depression

MAT09MGWS10021

4-09 Bearings

Bearings are used in navigation. A bearing is an angle measurement that is used to precisely describe the direction of one location from a given reference point.

Three-figure bearings

Three-figure bearings, also called true bearings, use angles from 000° to 360° to show the amount of turning measured **clockwise from north 000°** . Note that the angles are always written with three digits.

The **compass rose** below shows the three-figure bearings of eight points on the compass. A bearing of due east is 090° , while a compass direction of southwest (SW) is 225° .

Worksheet

A page of bearings

MAT09MGWK10046

Worksheet

NSW map bearings

MAT09MGWK10047

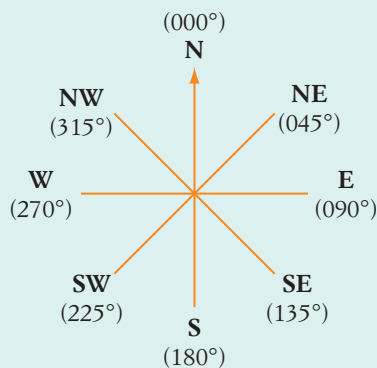
Worksheet

16 points of the compass

MAT09MGWK10048

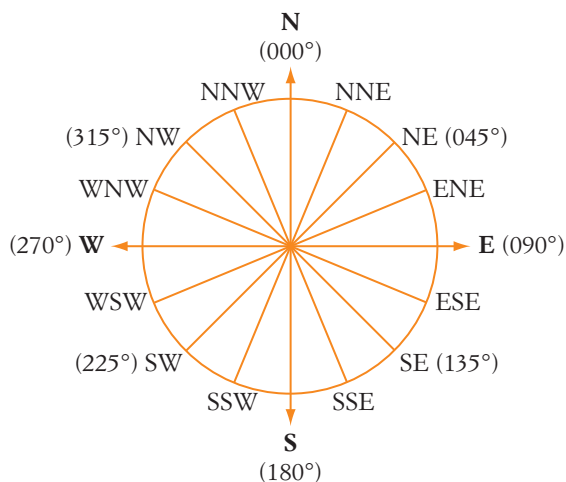
Summary

Bearings from 000° to 090° are in the NE quadrant.
Bearings from 090° to 180° are in the SE quadrant.
Bearings from 180° to 270° are in the SW quadrant.
Bearings from 270° to 360° are in the NW quadrant.



Compass bearings

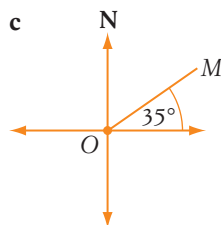
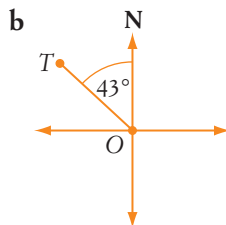
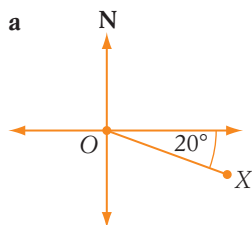
Compass bearings refer to the sixteen points of a mariner's compass.



N = north	NNE = north-northeast	NE = northeast	ENE = east-northeast
E = east	ESE = east-southeast	SE = southeast	SSE = south-southeast
S = south	SSW = south-southwest	SW = southwest	WSW = west-southwest
W = west	WNW = west-northwest	NW = northwest	NNW = north-northwest

Example 22

Write the three-figure bearing of each point from O .



Solution

a Bearing of X from O is $90^\circ + 20^\circ = 110^\circ$.

b Bearing of T from O is $360^\circ - 43^\circ = 317^\circ$.

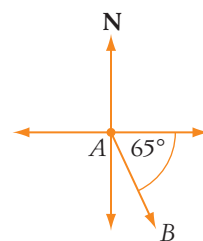
c Bearing of M from O is $90^\circ - 35^\circ = 055^\circ$ Must be written as a three-digit angle.

Example 23

Sketch point B on a compass rose if B has a bearing of 155° from A .

Solution

Draw the compass rose on the point where the bearing is measured *from*. 155° is between 90° and 180° , so B is in the southeast (SE) quadrant. $155^\circ - 90^\circ = 65^\circ$, so B is 65° from east (E).



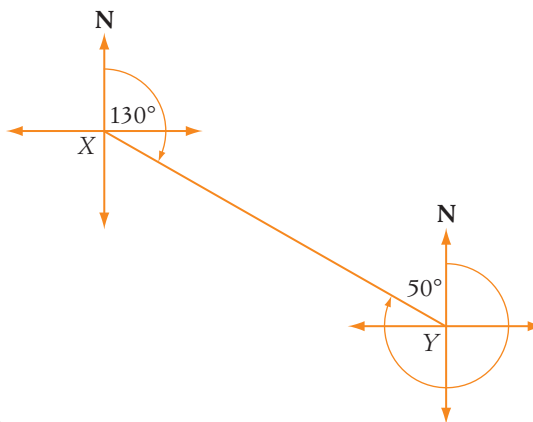
Example 24

The bearing of Y from X is 130° . What is the bearing of X from Y ?

Solution

Sketch the bearing of Y from X .

On the same diagram, draw a compass rose at Y and find $\angle NYX$.



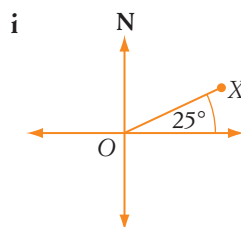
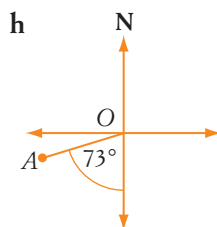
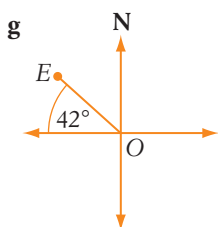
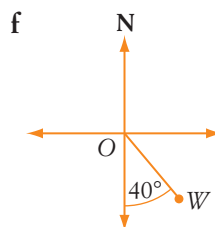
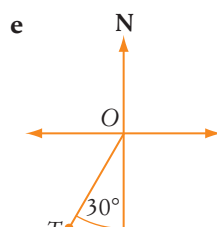
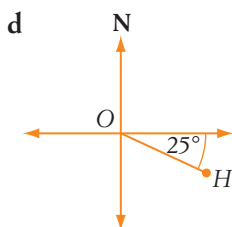
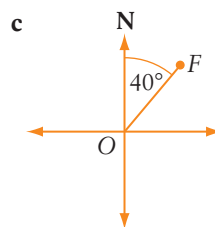
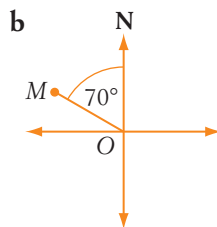
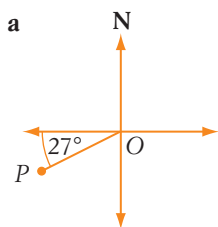
$\angle NYX = 50^\circ$ (co-interior angles, $NX \parallel NY$)

$\therefore$ Bearing of X from $Y = 360^\circ - 50^\circ$
 $= 310^\circ$

Exercise 4-09 Bearings

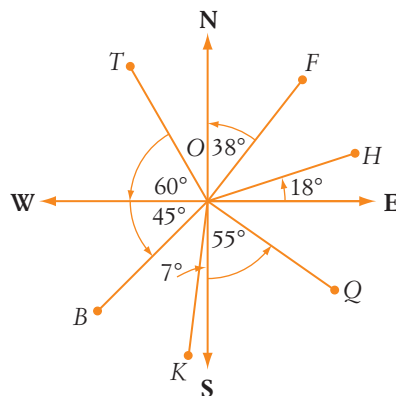
1 Write the bearing of each point from O .

See Example 22



2 What is the bearing of each point from O ?

a N	b E	c S	d W
e F	f Q	g T	h B
i H	j K		



3 What is the compass direction shown by point B in question 2?

4 Sketch each bearing on a compass rose.

a 220°	b 060°	c 260°	d 125°
e 350°	f 267°	g 171°	h 32°

- 5 a What is the compass direction halfway between northwest and west?
b What is the three-figure bearing of this compass direction?

See Example 23

6 Sketch P on a compass rose if P has a bearing of:

a 132° from T	b 260° from M	c 335° from X	d 010° from K

See Example 24

7 If the bearing of P from A is 060° , what is the bearing of A from P ?

8 The bearing of T from Y is 100° . What is the bearing of Y from T ?

9 What is the angle between:

a S and SW?	b NE and SE?	c E and NW?

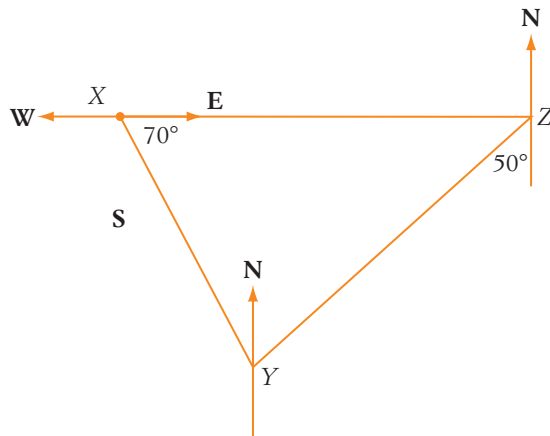
10 The compass bearing of H from M is WNW. Find the compass bearing of M from H .

11 Draw a diagram for each situation described.

- a A plane flies on a bearing of 280° for 150 km and then another 250 km on a bearing of 080° .
b A cyclist travels 15 km due east and then 20 km on a SW bearing.

12 For this diagram, find the bearing of:

- | |
|----------------|
| a Y from Z |
| b X from Z |
| c Y from X |
| d X from Y |
| e Z from Y |
| f Z from X |



Investigation: Compass walks

You need: a directional compass and a tape measure or trundle wheel.

This activity can also be done in the classroom using scale drawings, graph paper, a ruler and a protractor.

A triangular walk

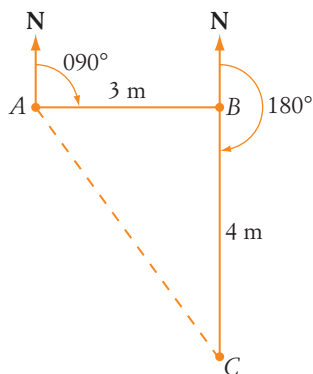
- 1 Starting at A , walk due east for 3 m to B .
- 2 From B , walk due south for 4 m to C .
- 3 How far is C from A ?
- 4 What is the bearing of:
 - a A from C ?
 - b C from A ?

A square walk

- 1 Starting at P , walk a bearing of 045° for 8 m to Q .
- 2 From Q , walk a bearing of 315° for 8 m to R .
- 3 From R , walk a bearing of 225° for 8 m to S .
- 4 How far is S from P ?
- 5 What is the bearing of:
 - a P from S ?
 - b S from P ?

A pentagonal walk

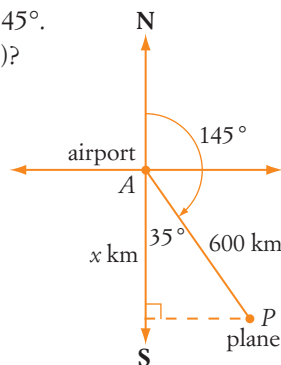
- 1 Starting at U , walk a bearing of 130° for 4 m to V .
- 2 From V , walk a bearing of 40° for 7 m to W .
- 3 From W , walk a bearing of 320° for 4.8 m to X .
- 4 From X , walk a bearing of 270° for 4.5 m to Y .
- 5 How far is Y from U ?
- 6 What is the bearing of:
 - a U from Y ?
 - b Y from U ?



4-10 Problems involving bearings

Example 25

A plane leaves an airport and flies 600 km on a bearing of 145° .
How far south of the airport is the plane (to the nearest km)?



Solution

Let x km = distance south

$$\begin{aligned}\angle SAP &= 180^\circ - 145^\circ \\ &= 35^\circ\end{aligned}$$

$$\cos 35^\circ = \frac{x}{600}$$

$$\begin{aligned}x &= 600 \cos 35^\circ \\ &= 491.4912 \dots \\ &\approx 491\end{aligned}$$

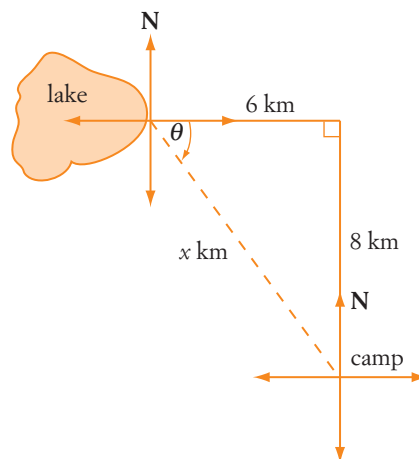
The plane is 491 km south of the airport.

Angles on a straight line

Example 26

From camp, Andie walks due north for 8 km, then 6 km due west to a lake.

- How far is Andie from the camp?
- What is the bearing of the camp from the lake (to the nearest minute)?



Solution

- Let x = distance from camp.

$$\begin{aligned}x^2 &= 6^2 + 8^2 \\ &= 100 \\ x &= \sqrt{100} \\ &= 10\end{aligned}$$

Andie is 10 km from the camp.

- Note angle θ in the diagram.

$$\begin{aligned}\tan \theta &= \frac{8}{6} \\ \theta &= 53.1301 \dots^\circ \\ &= 53^\circ 7' 48.368'' \\ &\approx 53^\circ 8'\end{aligned}$$

$$\begin{aligned}\text{Bearing of camp from lake} &= 90^\circ + 53^\circ 8' \\ &= 143^\circ 8'\end{aligned}$$

by Pythagoras' theorem

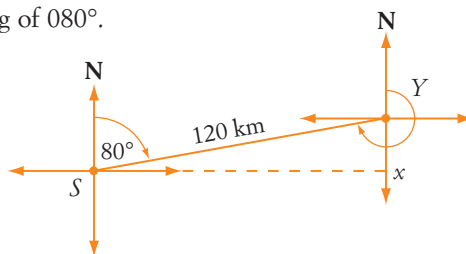
Exercise 4-10 Problems involving bearings

- 1 A yacht leaves Sydney and sails 120 km on a bearing of 080° .

a How far north of Sydney is the yacht?

Answer to the nearest kilometre.

b What is the bearing of Sydney from the yacht?



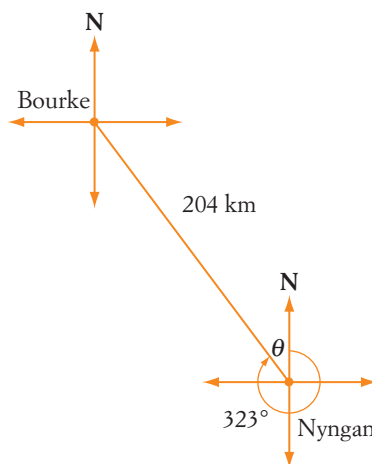
See Example 25

- 2 Colin leaves Nyngan and drives 204 km to Bourke. The bearing of Bourke from Nyngan is 323° .

a Find the value of θ .

b How far north (to the nearest km) of Nyngan is Bourke?

c What is the bearing of Nyngan from Bourke?



- 3 The distance 'as the crow flies' from Sydney to Wollongong is 69 km. If the bearing of Wollongong from Sydney is 205° , calculate:

a how far south Wollongong is from Sydney, correct to the nearest kilometre.

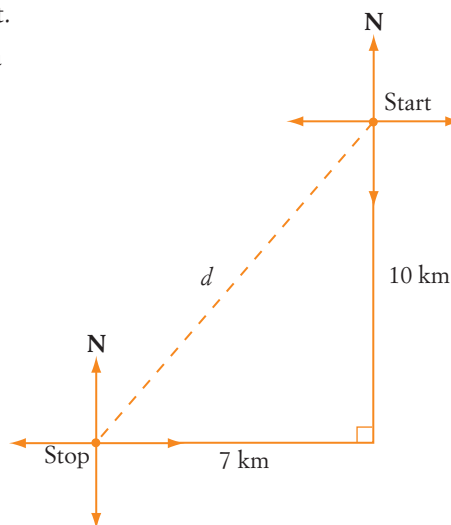
b how far east Sydney is from Wollongong

c the bearing of Sydney from Wollongong.

- 4 Jana cycles 10 km due south, then 7 km due west.

a How far (correct to one decimal place) is Jana from her starting point?

b What is her bearing from the starting point, correct to the nearest degree?



See Example 26

- 5 A triathlete cycles 20 km on a SSE bearing to the finish line.
 - a How far (to the nearest km) has the triathlete travelled in a southerly direction?
 - b What is the compass bearing of the starting point from the finish line?
- 6 A hiking group walks from Sandy Flats to Black Ridge (a distance of 20.9 km) in the direction 078° . They then turn and hike due south to Rivers End, then due west back to Sandy Flats. How far have they hiked altogether (to the nearest 0.1 km)?
- 7 A triangular orienteering run starts at Alpha and passes through the checkpoints of Bravo and Charlie before finishing at Alpha. Bravo is 8.5 km due east of Alpha, and Charlie is 10.5 km due south of Bravo.
 - a Calculate, correct to three decimal places, the distance from Charlie to Alpha.
 - b Find the bearing of Alpha from Charlie to the nearest degree.
- 8 A plane takes off from Darwin at 10:15 a.m. and flies on a bearing of 150° at 700 km/h.
 - a How far (to the nearest km) due south of Darwin is the plane at 1:45 p.m.?
 - b What is the bearing (correct to the nearest degree) of Darwin from the plane?
- 9 A fishing trawler sails 30 km from port on a bearing of 120° until it reaches a submerged reef. How far (to the nearest km) is the port:
 - a north of the reef?
 - b west of the reef?
- 10 Two racing pigeons are set free at the same time. The first bird flies on a course of 040° while the second bird flies on a course of 130° .
 - a The first bird flies 200 km until it is due north of the second bird. Find their distance apart correct to two decimal places.
 - b How far has the second bird flown?
- 11 Two horse riders start from the same stable. The rider of the black horse goes due west for 5.5 km and stops. The rider of the chestnut horse travels in a direction of 303° until he is due north of the black horse. How far did the rider of the chestnut horse travel? Answer correct to three decimal places.
- 12 Two ships leave from the same port. One ship travels on a bearing of 157° at 20 knots. The second ship travels on a bearing of 247° at 35 knots. (1 knot is a speed of 1 nautical mile per hour.)
 - a How far apart are the ships after 8 hours, to the nearest nautical mile?
 - b Calculate the bearing of the second ship from the first, to the nearest minute.

Worked solutions

Problems involving
bearings

MAT09MGWS10022

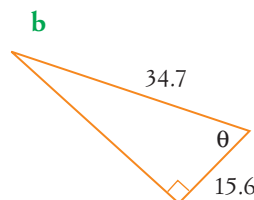
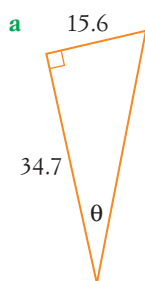
Power plus

- 1 a Copy and complete each pair of trigonometric ratios correct to three decimal places.

i $\sin 20^\circ = \underline{\hspace{2cm}}$, $\cos 70^\circ = \underline{\hspace{2cm}}$	ii $\sin 47^\circ = \underline{\hspace{2cm}}$, $\cos 43^\circ = \underline{\hspace{2cm}}$
iii $\sin 55^\circ = \underline{\hspace{2cm}}$, $\cos 35^\circ = \underline{\hspace{2cm}}$	iv $\sin 85^\circ = \underline{\hspace{2cm}}$, $\cos 5^\circ = \underline{\hspace{2cm}}$
 - b What do you notice about each pair of answers in part a ?
 - c What do you notice about each pair of angles in part a ?
 - d If $\cos 30^\circ \approx 0.8660$ and $\sin \theta \approx 0.8660$, what is the value of θ ?
 - e Copy and complete each equation.

i $\sin 75^\circ = \cos \underline{\hspace{2cm}}$	ii $\underline{\hspace{2cm}} 80^\circ = \cos 10^\circ$	iii $\cos \underline{\hspace{2cm}} = \sin 72^\circ$
iv $\sin 30^\circ = \underline{\hspace{2cm}} 60^\circ$	v $\cos 65^\circ = \sin \underline{\hspace{2cm}}$	vi $\sin \underline{\hspace{2cm}} = \cos 58^\circ$
 - f Copy and complete this general rule: $\sin x = \cos (\underline{\hspace{2cm}})$.
 - g Use a right-angled triangle with one angle x and sides a , b and c to prove that the above rule is true.
- 2 A plane is flying at an angle of 15° inclined to the horizontal.
 - a How far will the plane have to travel along its line of flight to increase its altitude (height) by 500 m?
 - b At what angle must the plane climb to achieve an increase in altitude of 500 m in half the distance needed at an angle of 15° ?
 - 3 If $\sin 30^\circ = \frac{1}{2}$, find, as a surd, the value of:

a $\cos 30^\circ$	b $\tan 30^\circ$
 - 4 Find the value of angle θ , correct to the nearest second.



- 5 By drawing an appropriate triangle, prove that:

a $\tan 45^\circ = 1$	b $\sin 45^\circ = \sin 45^\circ = \frac{1}{\sqrt{2}}$	c $\cos 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$

Chapter 4 review

Language of maths

Puzzle sheet

Trigonometry
crossword

MAT09MGPS10049

Quiz

Trigonometry

MAT09MGQZ00009

adjacent	angle of depression	angle of elevation	bearing
clinometer	compass bearing	cosine (cos)	degree ($^{\circ}$)
denominator	horizontal	hypotenuse	inverse ($^{-1}$)
minute ($'$)	opposite	right-angled	second ($''$)
sine (sin)	tangent (tan)	theta (θ)	three-figure bearing
trigonometry	trigonometric ratio	unknown	vertical

- 1 When measuring angle size, what is a **second** and what is its symbol?
- 2 What word means 'next to'?
- 3 Which side of a right-angled triangle is fixed and does not depend on the position of an angle?
- 4 What are the first two letters of the Greek alphabet?
- 5 The word **minute** has an alternative pronunciation and meaning. What is its alternative meaning?
- 6 What does **inverse** mean and how is it used in trigonometry?

Topic overview

For each statement about the topic, give a rating from 0 to 5 using this scale.



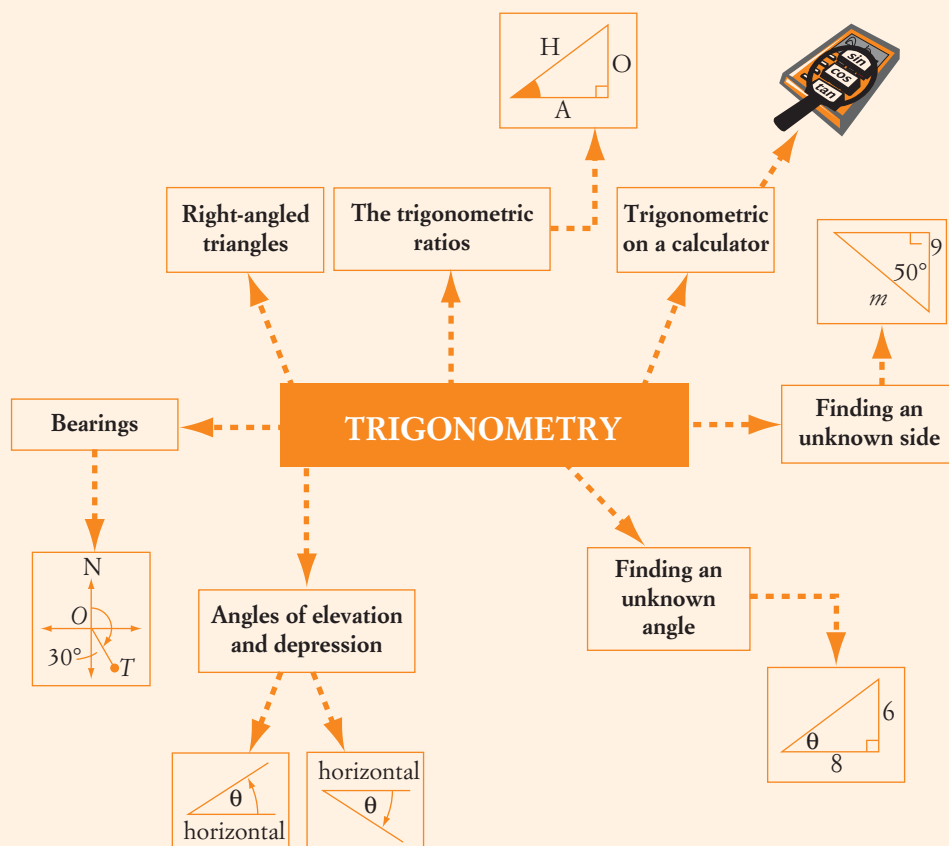
- I understand the meaning of the hypotenuse, opposite and adjacent sides
- I understand the meaning of the sine, cosine and tangent ratios
- I can use the calculator to evaluate trigonometric expressions involving angles stated in degrees and minutes
- I am able to use the trigonometric ratios to find unknown sides
- I am able to use the trigonometric ratios to find unknown angles
- I am able to solve trigonometric problems involving angles of elevation and depression, and bearings

Copy (or print) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.

Worksheet

Mind map:
Trigonometry
(Advanced)

MAT09MGWK10051



Chapter 4 revision

See Exercise 4-02

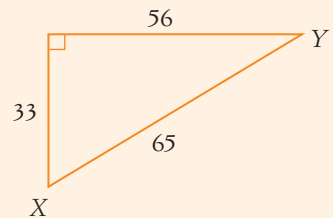
- 1 For this triangle, write as a fraction:

a $\sin Y$

b $\tan Y$

c $\sin X$

d $\cos X$



See Exercise 4-02

- 2 If $\sin \alpha = \frac{36}{85}$, write the values of $\cos \alpha$ and $\tan \alpha$ as fractions. (Draw a diagram.)

See Exercise 4-04

- 3 Round each angle to the nearest degree.

a $64^\circ 27'$

b $25^\circ 43'$

c $12^\circ 8' 50''$

See Exercise 4-04

- 4 Round each angle to the nearest minute.

a $50^\circ 19' 26''$

b $31^\circ 55' 55''$

c $64^\circ 18' 30''$

See Exercise 4-04

- 5 Evaluate each expression, correct to four decimal places.

a $\cos 32^\circ$

b $\sin 50^\circ 9'$

c $\tan 8^\circ 45'$

d $200 \tan 18^\circ$

e $14 \sin 87^\circ 40'$

f $\frac{13}{\cos 18^\circ 27'}$

See Exercise 4-04

- 6 Convert each angle size to degrees and minutes, correct to the nearest minute.

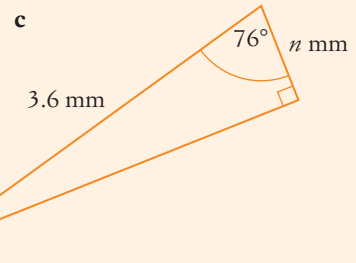
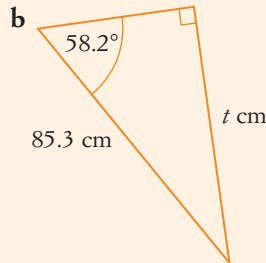
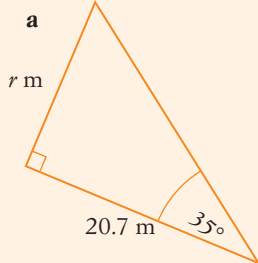
a 45.8°

b 33.175°

c 5.346°

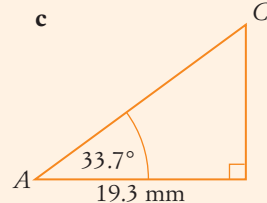
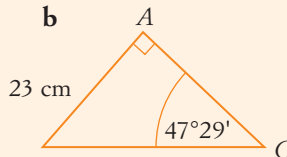
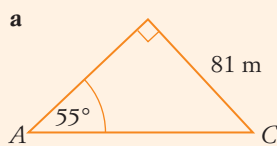
See Exercise 4-05

- 7 Find the value of each pronumeral, correct to two decimal places.



See Exercise 4-06

- 8 For each triangle, find the length of side AC, correct to one decimal place.



See Exercise 4-07

- 9 Find the size of angle θ , correct to the nearest minute.

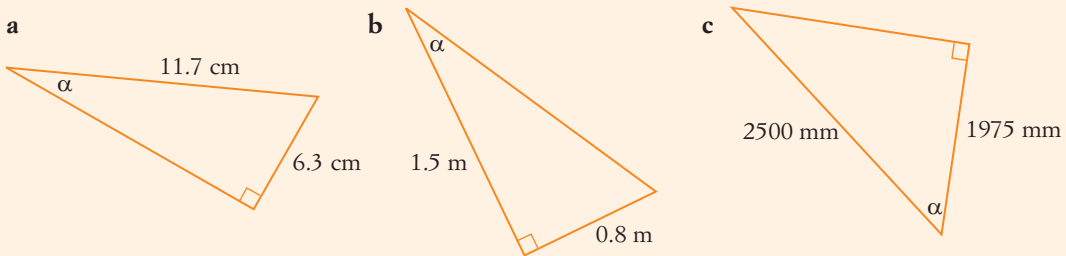
a $\tan \theta = 2.57$

b $\cos \theta = \frac{4}{7}$

c $\sin \theta = \frac{1.5}{1.6}$

- 10 Find the size of angle α , correct to the nearest degree.

See Exercise 4-07



- 11 In $\triangle AEC$, $\angle C = 90^\circ$, $CE = 3.9$ m and $AE = 4.2$ m. Find $\angle A$, correct to the nearest minute.

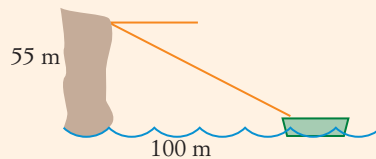
See Exercise 4-07

- 12 The angle of elevation of a tower roof is 26° at a point 400 m from its base. Find the height of the tower correct to the nearest metre.

See Exercise 4-08

- 13 Find the angle of depression (correct to the nearest degree) of a boat that is 100 m from the base of a 55 m cliff.

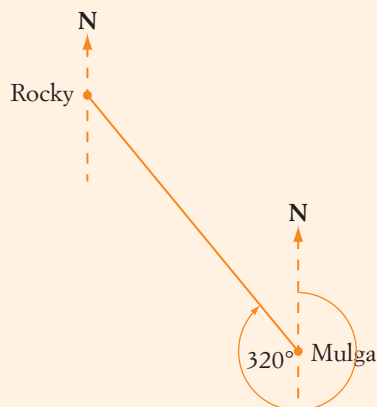
See Exercise 4-08



- 14 What is the bearing of:

See Exercise 4-09

- Rocky from Mulga?
- Mulga from Rocky?



- 15 Two planes leave an airport at the same time. The first travels on a bearing of 063° at 500 km/h. The second travels on a bearing of 153° at 400 km/h.

See Exercise 4-10

- How far apart are the planes after 2 hours, to the nearest km?
- Calculate, correct to the nearest degree, the bearing of the first plane from the second plane.



5

Number and Algebra

Indices

The speed of light is about 300 000 000 metres per second. In one year, light travels approximately 9 460 000 000 000 km. Light from the stars travels for many years before it is seen on Earth. Even light from the Sun takes eight minutes to reach the Earth. Powers or indices provide a way to work easily with very large and very small numbers.



Chapter outline

Proficiency strands

5-01 Multiplying and dividing terms with the same base	U	F		R	C
5-02 Power of a power	U	F		R	C
5-03 Powers of products and quotients	U	F		R	C
5-04 The zero index	U	F		R	C
5-05 Negative indices	U	F		R	C
5-06 Fractional indices $\frac{1}{n}$ *	U	F		R	C
5-07 Fractional indices $\frac{m}{n}$ *	U	F		R	C
5-08 Summary of the index laws	U	F		R	C
5-09 Significant figures	U	F		R	C
5-10 Scientific notation	U	F		R	C
5-11 Scientific notation on a calculator	U	F	PS	R	C

*STAGE 5.3

Wordbank

base A number that is raised to a power, meaning it is multiplied by itself repeatedly, for example, in 2^5 , the base is 2.

index laws Rules for simplifying algebraic expressions involving powers of the same base, for example, $a^m \div a^n = a^{m-n}$.

index notation A way of writing repeated multiplication using indices (powers), for example 2^5 .

negative power A power that is a negative number, as in the term 3^{-2} .

power (or index or exponent) The number of times a base appears in a repeated multiplication, for example, in 2^5 , the power is 5.

scientific notation A shorter way of writing very large or very small numbers using powers of 10. For example, 9 460 000 000 000 in scientific notation is 9.46×10^{12} .

significant figures Meaningful digits in a numeral that tell 'how many'. For example, 28 000 000 has two significant figures: 2 and 8.

In this chapter you will:

- apply index laws to numerical expressions with integer indices
- simplify algebraic products and quotients using index laws
- express numbers in scientific notation 
- interpret and use zero and negative indices
- (STAGE 5.3) interpret and use fractional indices
- round numbers to significant figures
- interpret, write and order numbers in scientific notation
- interpret and use scientific notation on a calculator
- solve problems involving scientific notation

SkillCheck

Worksheet

StartUp assignment 5

MAT09NAWK10052

Worksheet

Powers review

MAT09NAWK10053

Skillsheet

Indices

MAT09NASS10016

1 For each term:

i state the base

ii state the index

iii write the expression in words.

a 8^4

b 4^8

c h^5

d 5^h

2 Express each repeated multiplication in index notation.

a $2 \times 2 \times 2 \times 2 \times 2$

b $3 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7$

c $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 8 \times 8$

d $10 \times x \times x \times x \times x \times x \times x$

e $6 \times 6 \times 6 \times k \times k$

f $x \times y \times x \times y \times x$

g $a \times b \times b \times b \times a$

h $5 \times n \times 5 \times n \times n$

i $q \times p \times q \times p \times q \times q$

3 Write each term in expanded form.

a 9^3

b 7^2

c d^5

d k^2

4 Evaluate each expression.

a $4^2 \times 4^3$

b $10^6 \div 10^2$

c $(3^3)^2$

d 6^0

e 9^1

f $5^5 \times 5$

g $2^4 \div 2$

h $(-8)^2$

5 For each equation, find the missing power.

a $8 = 2^{\square}$

b $81 = 3^{\square}$

c $216 = 6^{\square}$

d $144 = 12^{\square}$

e $4096 = 4^{\square}$

f $2401 = 7^{\square}$

g $64 = 2^{\square}$

h $625 = 5^{\square}$

Investigation: Multiplying and dividing terms with powers

- Write each expression in expanded form, then evaluate it.

a i $2^2 \times 2^3$	ii 2^5	b i $3^4 \times 3^3$	ii 3^7
c i $4^3 \times 4^3$	ii 4^6	d i $5^5 \times 5^3$	ii 5^8
- What do you notice about each pair of answers in question 1?
- Is it true that $2^4 \times 2^6 = 2^{10}$? Give a reason for your answer.
- Determine whether each equation is true (T) or false (F). Justify your answer.

a $2^5 \times 2^5 = 2^{10}$	b $6^3 \times 6^7 = 6^{21}$
c $4^3 \times 4^9 = 4^{27}$	d $3^5 \times 3^{10} = 3^{15}$
- Write in words and as a formula the rule for multiplying a^m and a^n , two terms with the same base.
- Use the rule to copy and complete each equation.

a $5^4 \times 5^2 = 5 \dots$	b $4^5 \times 4^3 = 4 \dots$	c $10^5 \times 10^7 = \dots$
d $9^3 \times 9^2 = \dots$	e $n^3 \times n^8 = \dots$	f $p^3 \times p^7 = \dots$
- Evaluate each expression.

a i $3^6 \div 3^3$	ii 3^3	b i $2^8 \div 2^6$	ii 2^2
c i $5^8 \div 5^3$	ii 5^5	d i $10^8 \div 10^4$	ii 10^4
- What do you notice about each pair of answers in question 7?
- Is it true that $4^8 \div 4^6 = 4^2$? Give a reason for your answer.
- Determine whether each equation is true (T) or false (F). Justify your answer.

a $3^{10} \div 3^6 = 3^4$	b $4^8 \div 4^2 = 4^4$
c $2^{12} \div 2^3 = 2^4$	d $6^{10} \div 6^5 = 6^5$
- Write in words and as a formula the rule for dividing a^m and a^n , two terms with the same base.
- Use the rule to copy and complete each equation.

a $2^6 \div 2^3 = 2 \dots$	b $10^8 \div 10^6 = 10 \dots$	c $3^7 \div 3^2 = \dots$
d $4^{11} \div 4^6 = \dots$	e $x^8 \div x^5 = \dots$	f $g^{12} \div g^{10} = \dots$

5-01 Multiplying and dividing terms with the same base

Video tutorial

Simplifying with the index laws

MAT09NAVT00002

Consider $5^4 \times 5^3 = (5 \times 5 \times 5 \times 5) \times (5 \times 5 \times 5)$

$$= 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$$

$$= 5^7$$

$$\therefore 5^4 \times 5^3 = 5^{4+3}$$

$$= 5^7$$

Summary

When **multiplying terms with the same base**, add the powers:

$$a^m \times a^n = a^{m+n}$$

The rule above is called an **index law**. **Index** is another name for **power**. The plural of index is **indices** (pronounced 'in-de-sees').

Proof:
$$a^m \times a^n = \underbrace{a \times a \times \cdots \times a}_m \times \underbrace{a \times a \times \cdots \times a}_n$$

$$= \underbrace{a \times a \times \cdots \times a}_{(m+n) \text{ factors}}$$

$$= a^{m+n}$$

Example 1

Simplify each expression, writing the answer in index notation.

a $8^4 \times 8^5$ **b** 10×10^3 **c** $d^3 \times d^5$ **d** $4m^2 \times 3m^6$ **e** $3r^2t \times 6r^4t^3$

Solution

a $8^4 \times 8^5 = 8^{4+5}$
 $= 8^9$

b $10 \times 10^3 = 10^1 \times 10^3$
 $= 10^{1+3}$
 $= 10^4$

c $d^3 \times d^5 = d^{3+5}$
 $= d^8$

d $4m^2 \times 3m^6 = (4 \times 3) \times (m^2 \times m^6)$ **e** $3r^2t \times 6r^4t^3 = (3 \times 6) \times (r^2 \times r^4) \times (t^1 \times t^3)$
 $= 12m^{2+6}$ $= 18r^{2+4}t^{1+3}$
 $= 12m^8$ $= 18r^6t^4$

Consider $5^6 \div 5^4 = \frac{5^6}{5^4}$
 $= \frac{\cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5} \times 5 \times 5}{\cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5}}$
 $= 5 \times 5$
 $= 5^2$
 $\therefore 5^6 \div 5^4 = 5^{6-4}$
 $= 5^2$

Summary

When **dividing terms with the same base**, subtract the powers:

$$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$$

This is another **index law**.

Proof: $a^m \div a^n = \frac{a^m}{a^n}$
 $= \frac{\cancel{a} \times \cancel{a} \times \cancel{a} \times a \times a \times \cdots \times \cancel{a}}{\cancel{a} \times \cancel{a} \times \cancel{a} \times \cdots \times \cancel{a}} \quad \begin{matrix} (m \text{ factors}) \\ (n \text{ factors}) \end{matrix}$
 $= a \times a \times \cdots \times a \quad [(m-n) \text{ factors}]$
 $= a^{m-n}$

Example 2

Simplify each expression, writing the answer in index notation.

a $8^5 \div 8^3$

b $\frac{10^8}{10}$

c $d^{20} \div d^4$

d $20w^{10} \div 5w^2$

e $\frac{8x^3y^7}{24x^2y}$

Solution

a $8^5 \div 8^3 = 8^{5-3}$
 $= 8^2$

b $\frac{10^8}{10} = 10^{8-1}$
 $= 10^7$

c $d^{20} \div d^4 = d^{20-4}$
 $= d^{16}$

d $20w^{10} \div 5w^2 = \frac{4\cancel{20}w^{10}}{\cancel{5}w^2}$
 $= 4w^{10-2}$
 $= 4w^8$

e $\frac{8x^3y^7}{24x^2y^1} = \frac{\cancel{8}x^{3-2}y^{7-1}}{\cancel{24}} = \frac{xy^6}{3}$

Exercise 5-01

Multiplying and dividing terms with the same base

- 1 Which expression is equal to $5^{12} \times 5^3$? Select the correct answer A, B, C or D.
A 5^9 B 5^{15} C 25^{15} D 25^{36}

See Example 1

- 2 Simplify each expression, writing the answer in index notation.

a $10^3 \times 10^2$

b 2×2^4

c $3^2 \times 3^5$

d $7^4 \times 7$

e $8 \times 8^3 \times 8^4$

f $5^4 \times 5 \times 5^4$

g $6 \times 6^2 \times 6^3 \times 6^4$

h $4^4 \times 4^4 \times 4^4$

i $3^4 \times 3^0 \times 3^7$

j $x \times x^4$

k $g^4 \times g^4$

l $w^7 \times w$

m $b^3 \times b^{10}$

n $p^{10} \times p^{10}$

o $r \times r$

p $y \times y^3 \times y^2$

q $m^3 \times m \times m^4$

r $n^8 \times n^2$

- 3 Which expression is equal to $10^4 \times 10$? Select the correct answer A, B, C or D.

A 100^5

B 100^4

C 10^4

D 10^5

- 4 Simplify each expression.

a $3p^2 \times 2p^5$

b $4y^{10} \times 3y^2$

c $6m \times 3m^8$

d $h^3 \times 5h^8$

e $3q \times 8q^8$

f $2a^2 \times 5a^5$

g $5n^8t \times 6n^8t^4$

h $2ab^3 \times 15ab$

i $3e^4g^3 \times e^6g^2$

j $8p^4m^5 \times 4p^3m^5$

k $16qr^8 \times 3q^7$

l $9u^3v \times 6uv^2w^8$

- 5 Which expression is equal to $5^{12} \div 5^3$? Select the correct answer A, B, C or D.

A 5^4

B 5^9

C 1^4

D 1^9

See Example 2

- 6 Simplify each expression, writing the answer in index notation.

a $10^7 \div 10^5$

b $8^5 \div 8$

c $20^{15} \div 20^5$

d $\frac{5^8}{5^2}$

e $\frac{9^{12}}{9^3}$

f $\frac{2^{27}}{2^3}$

$$g \quad 7^4 \div 7^3$$

$$j \quad p^{15} \div p^{10}$$

$$m \quad \frac{h^{20}}{h^4}$$

$$p \quad b^{16} \div b^{15}$$

$$h \quad \frac{2^{20}}{2}$$

$$k \quad n^7 \div n$$

$$n \quad \frac{y^8}{y^2}$$

$$q \quad \frac{w^{25}}{w}$$

$$i \quad 11^4 \div 11^4$$

$$l \quad w^{24} \div w^6$$

$$o \quad \frac{a^{12}}{a^4}$$

$$r \quad m^{16} \div m^{16}$$

- 7 Which expression is equal to $10^4 \div 10$? Select the correct answer **A**, **B**, **C** or **D**.
A 10^4 **B** 1^4 **C** 1^3 **D** 10^3

- 8 Simplify each expression.

$$a \quad 10y^{15} \div 5y^3$$

$$b \quad 20w^9 \div 4w^3$$

$$c \quad 24r^8 \div 3r$$

$$d \quad \frac{30x^4}{x^3}$$

$$e \quad \frac{10m^{10}}{2m}$$

$$f \quad \frac{4g^{12}}{8g^6}$$

$$g \quad 14d^4h^{10} \div 7hd^2$$

$$h \quad 15x^6y^8 \div 15xy^4$$

$$i \quad 6e^{25}d^{40} \div 18e^5d^4$$

$$j \quad \frac{12q^5t^4}{16q^4t^3}$$

$$k \quad \frac{45a^{10}b^8}{5a^5}$$

$$l \quad \frac{36pq^3r^5}{24qr}$$

Investigation: Powers of powers

- 1 Write each expression in expanded form, then evaluate it.

$$a \quad i \quad (2^3)^2$$

$$ii \quad 2^6$$

$$b \quad i \quad (3^4)^3$$

$$ii \quad 3^{12}$$

$$c \quad i \quad (5^2)^3$$

$$ii \quad 5^6$$

$$d \quad i \quad (2^5)^4$$

$$ii \quad 2^{20}$$

- 2 What do you notice about each pair of answers in question 1?

- 3 Is it true that: $(2^7)^3 = 2^{21}$? Give a reason for your answer.

- 4 Determine whether each equation is true (T) or false (F). Justify your answer.

$$a \quad (3^5)^3 = 3^{15}$$

$$b \quad (2^3)^2 = 2^5$$

$$c \quad (2^{10})^4 = 2^{14}$$

$$d \quad (4^2)^5 = 4^{10}$$

$$e \quad (3^3)^6 = 3^{18}$$

$$f \quad (5^2)^4 = 5^6$$

- 5 Write in words and as a formula the rule for raising a^m to a power of n , that is, $(a^m)^n$.

- 6 Use the rule to copy and complete each equation.

$$a \quad (3^7)^2 = 3 \dots$$

$$b \quad (5^2)^6 = 5 \dots$$

$$c \quad (4^5)^2 = 4 \dots$$

$$d \quad (a^3)^4 = a \dots$$

$$e \quad (8^3)^7 = \dots$$

$$f \quad (k^4)^6 = \dots$$

Puzzle sheet

Indices puzzle

MAT09NAPS10054

Video tutorial

Simplifying with the index laws

MAT09NAVT00002

5-02 Power of a power

$$\begin{aligned} \text{Consider} \quad (5^3)^4 &= 5^3 \times 5^3 \times 5^3 \times 5^3 \\ &= (5 \times 5 \times 5) \times (5 \times 5 \times 5) \times (5 \times 5 \times 5) \times (5 \times 5 \times 5) \\ &= 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \\ &= 5^{12} \\ \therefore (5^3)^4 &= 5^{3 \times 4} \\ &= 5^{12} \end{aligned}$$

Summary

When **raising a term with a power to another power**, multiply the powers:

$$(a^m)^n = a^{m \times n}$$

Proof:

$$\begin{aligned} (a^m)^n &= \underbrace{a^m \times a^m \times \dots \times a^m}_{n \text{ factors}} \\ &= \underbrace{a \times a \times \dots \times a}_{m \text{ factors}} \times \underbrace{a \times a \times \dots \times a}_{m \text{ factors}} \times \dots \times \underbrace{a \times a \times \dots \times a}_{m \text{ factors}} \\ &\quad \underbrace{\hspace{10em}}_{n \text{ lots of } m \text{ factors}} \\ &= a^{m \times n} \end{aligned}$$

Example 3

Simplify each expression, writing the answer in index notation.

a $(8^5)^2$

b $(d^3)^5$

c $(2g)^4$

d $(5v^4)^3$

e $(-n)^6$

f $(-3t^4)^3$

Solution

a $(8^5)^2 = 8^{5 \times 2}$
 $= 8^{10}$

b $(d^3)^5 = d^{3 \times 5}$
 $= d^{15}$

c $(2g)^4 = 2^4 \times g^4$
 $= 16g^4$

d $(5v^4)^3 = 5^3 \times (v^4)^3$
 $= 125 \times v^{4 \times 3}$
 $= 125v^{12}$

e $(-n)^6 = (-1)^6 \times n^6$
 $= 1 \times n^6$
 $= n^6$

f $(-3t^4)^3 = (-3)^3 \times (t^4)^3$
 $= -27 \times t^{4 \times 3}$
 $= -27t^{12}$

Exercise 5-02 Power of a power

- 1 Which expression is equal to $(10^3)^3$? Select the correct answer A, B, C or D.

A 30^3

B 10^3

C 10^9

D 10^6

See Example 3

- 2 Simplify each expression, writing the answer in index notation.

a $(4^3)^2$

b $(5^2)^8$

c $(3^3)^4$

d $(2^7)^4$

e $(2^1)^2$

f $(9)^3$

g $(10^0)^2$

h $(6^4)^5$

i $(5^3)^5$

j $(e^2)^4$

k $(t^5)^5$

l $(y^3)^7$

m $(c^1)^5$

n $(m^7)^5$

o $(y^4)^4$

p $(h^0)^6$

q $(q^6)^3$

r $(w^4)^1$

s $(2x)^{10}$

t $(5n^3)^8$

u $(4d^3)^3$

v $(-k^5)^9$

w $(-d^3)^4$

x $(2a^8)^8$

- 3 Which expression is equal to $(-3)^5$? Select the correct answer A, B, C or D.

A -3^6

B -3^5

C 3^5

D -15

- 4 Simplify each expression.

a $(2d^3)^4$

b $(5m^3)^2$

c $(4y^5)^2$

d $(3x^2)^4$

e $(5u^6)^5$

f $(2w^5)^3$

g $(10d^5)^4$

h $(3e)^3$

i $(2b^4)^1$

j $(6d^6)^2$

k $(3f^4)^5$

l $(2c^3)^{10}$

m $(-2r)^4$

n $(-5t)^3$

o $(-3m^3)^2$

p $(-y^3)^{12}$

q $(-x)^3$

r $(-m^3)^{10}$

s $(-4w^5)^4$

t $(-3f)^5$

u $(-3p^2)^3$

v $(-3h^5)^4$

w $(-10k)^2$

x $(-8y^3)^1$

Video tutorial

Simplifying with the index laws

MAT09NAVT0002

Homework sheet

Indices 1

MAT09NAHS10005

5-03 Powers of products and quotients

Consider $(2 \times 5)^3 = (2 \times 5) \times (2 \times 5) \times (2 \times 5)$

$$= 2 \times 2 \times 2 \times 5 \times 5 \times 5$$

$$= 2^3 \times 5^3$$

$$\therefore (2 \times 5)^3 = 2^3 \times 5^3$$

Summary

When **raising a product of terms to a power**, raise each term to that power:

$$(ab)^n = a^n b^n$$

Proof: $(ab)^n = \underbrace{ab \times ab \times \cdots \times ab}_{n \text{ factors}}$

$$= \underbrace{a \times a \times \cdots \times a}_{n \text{ factors}} \times \underbrace{b \times b \times \cdots \times b}_{n \text{ factors}}$$

$$= a^n b^n$$

Example 4

Simplify each expression.

a $(-2gh^2)^5$

b $(p^3q^4)^2$

Solution

a $(-2gh^2)^5 = (-2)^5 \times g^5 \times (h^2)^5$

$$= -32 \times g^5 \times h^{2 \times 5}$$

$$= -32g^5h^{10}$$

b $(p^3q^4)^2 = (p^3)^2 \times (q^4)^2$

$$= p^{3 \times 2} \times q^{4 \times 2}$$

$$= p^6q^8$$

Consider $\left(\frac{5}{8}\right)^6 = \frac{5}{8} \times \frac{5}{8} \times \frac{5}{8} \times \frac{5}{8} \times \frac{5}{8} \times \frac{5}{8}$

$$= \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5}{8 \times 8 \times 8 \times 8 \times 8 \times 8}$$

$$= \frac{5^6}{8^6}$$

$$\therefore \left(\frac{5}{8}\right)^6 = \frac{5^6}{8^6}$$

Summary

When raising a quotient of terms to a power, raise each term to that power:

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Proof:

$$\begin{aligned}\left(\frac{a}{b}\right)^n &= \underbrace{\frac{a}{b} \times \frac{a}{b} \times \dots \times \frac{a}{b}}_{n \text{ factors}} \\ &= \frac{a \times a \times \dots \times a \text{ (} n \text{ factors)}}{b \times b \times \dots \times b \text{ (} n \text{ factors)}} \\ &= \frac{a^n}{b^n}\end{aligned}$$

Example 5

Simplify each expression.

a $\left(\frac{7c}{d}\right)^2$

b $\left(\frac{4k^2}{5}\right)^3$

Solution

a $\left(\frac{7c}{d}\right)^2 = \frac{(7c)^2}{d^2}$
 $= \frac{7^2 c^2}{d^2}$
 $= \frac{49c^2}{d^2}$

b $\left(\frac{4k^2}{5}\right)^3 = \frac{(4k^2)^3}{5^3}$
 $= \frac{4^3 (k^2)^3}{125}$
 $= \frac{64k^6}{125}$

Exercise 5-03 Powers of products and quotients

- 1 Which expression is equal to $(-4 \times 5)^2$? Select the correct answer A, B, C or D.

A 16×25

B -16×25

C -8×10

D 8×10

See Example 4

- 2 Simplify each expression.

a $(ab)^3$

b $(x^2y)^5$

c $(l^3m^5)^6$

d $(6dp^2)^4$

e $(-8k^4y^5)^2$

f $(3m^2n)^5$

g $(ek^3)^3$

h $(-w^3x^4)^7$

i $(-8d^3y^5)^2$

j $(4b^2c^3)^4$

k $(-3a^3d)^3$

l $(2p^2q^3)^4$

- 3 Which expression is equal to $\left(-\frac{3}{4}\right)^3$? Select the correct answer A, B, C or D.

A $-\frac{9}{12}$

B $\frac{9}{12}$

C $\frac{27}{64}$

D $-\frac{27}{64}$

See Example 5

- 4 Simplify each expression.

a $\left(\frac{6}{7}\right)^2$

b $\left(\frac{m}{2}\right)^3$

c $\left(\frac{5}{x}\right)^4$

d $\left(\frac{2n^5}{p}\right)^8$

e $\left(\frac{w^2}{t^3}\right)^5$

f $\left(\frac{m^2}{4n}\right)^4$

g $\left(-\frac{2}{3}\right)^4$

h $\left(-\frac{5h}{6}\right)^3$

i $\left(\frac{7k^4}{10}\right)^2$

j $\left(\frac{3r}{t^2}\right)^2$

k $\left(\frac{a^2b}{d^5}\right)^4$

l $\left(-\frac{2}{3c^2}\right)^5$

Worked solutions

5 Simplify each expression.

a $(2x^{10}y^{15})^3 \times 5x^2y^3$

b $(2x^{10}y^{15} \times 5x^2y^3)^3$

c $18q^5r^8 \div (3qr^2)^2$

d $(18q^5r^8 \div 3qr^2)^2$

e $\left(\frac{3a^5x^6}{ax}\right)^3$

f $\frac{3a^5x^6}{(ax)^4}$

g $(4p^3h^{10})^2 \times 2p^2h^9$

h $(4p^3h^{10})^2 \div 2p^2h^9$

i $(4p^3h^{10} \times 2p^2h^9)^2$

MAT09NAWS10023

Investigation: The power of zero

What is the value of a number raised to a power of 0, for example, 2^0 ?

1 Copy and complete each table of decreasing powers. Notice the pattern in your answers.

a

Power of 2	Number
2^5	32
2^4	16
2^3	
2^2	
2^1	
2^0	

b

Power of 3	Number
3^5	243
3^4	
3^3	
3^2	
3^1	
3^0	

2 Simplify each expression in index notation.

a $3^4 \times 3^0$

b $5^2 \times 5^0$

c $2^0 \times 2^7$

d $7^0 \times 7^3$

e $4^5 \times 4^0$

f $5^0 \times 5^7$

g $2^5 \div 2^0$

h $3^5 \div 3^0$

i $4^2 \div 4^0$

j $9^3 \div 9^0$

k $5^6 \div 5^0$

l $8^4 \div 8^0$

3 Any number will remain unchanged when multiplied by what?

4 Any number will remain unchanged when divided by what?

5 What is the answer when any number is raised to the power of 0, that is, a^0 ? Justify your answer.

5-04 The zero index

Consider $5^3 \div 5^3 = \frac{5^3}{5^3}$

$= 1$ Any number divided by itself equals 1.

But also $5^3 \div 5^3 = 5^{3-3}$

$= 5^0$

$\therefore 5^0 = 1$

Summary

Any number raised to the power of zero is equal to 1.

$a^0 = 1$

Proof: $a^m \div a^m = 1$ Any number divided by itself equals 1.

But also $a^m \div a^m = a^{m-m}$
 $= a^0$
 $\therefore a^0 = 1.$

Example 6

Simplify each expression.

a 11^0
d $(3r)^0$

b $(-8)^0$
e $3r^0$

c g^0
f -8^0

Solution

a $11^0 = 1$
d $(3r)^0 = 1$

b $(-8)^0 = 1$
e $3r^0 = 3 \times r^0$
 $= 3 \times 1$
 $= 3$

c $g^0 = 1$
f $-8^0 = -1 \times 8^0$
 $= -1 \times 1$
 $= -1$

Exercise 5-04 The zero index

1 Simplify each expression.

a 2^0

b $(-2)^0$

c -2^0

d $(-m)^0$

e $-m^0$

f $(4a)^0$

g $\left(\frac{2}{3}\right)^0$

h $7x^0$

i -1000^0

j $(p+3)^0$

k $\left(\frac{p}{3}\right)^0$

l $-2b^0$

m $(9k)^0$

n $(x^2y)^0$

o $(xyw)^0$

p $(-ab)^0$

q $(6r)^0$

r $-(6r)^0$

s $-6r^0$

t $6(-r)^0$

u $(cd)^0$

v $-(7x^2)^0$

w $-3(a^2b^3)^0$

x $(-5v^5w^4)^0$

2 Simplify each expression.

a $7^0 + 2^0$

b $7^0 - 2^0$

c $2m^0 + (2m)^0$

d $2m^0 - (2m)^0$

e $(6a)^0 + 6a^0$

f $(6a)^0 - 6x^0$

g $(5y)^0 - 4$

h $(5y)^0 - 4^0$

i $3^0 \times 5^0$

j $3^2 \times 5^0$

k $\left(\frac{1}{2}\right)^0 + \frac{1}{2}y^0$

l $\left(\frac{1}{2}\right)^0 + \left(\frac{1}{2}y\right)^0$

m $2w^0 \times 3p^0$

n $12u^0 \div 3$

o $(5d^0)^3$

p $8b^0 - (3b^0)^2$

q $\frac{12p^0}{(2p)^0}$

r $6n^3 \div 2n^3$

s $\frac{12q^5}{36q^5}$

t $(3x^3)^3 \div x^9$

See Example 6

Worked solutions

The zero index

MAT09NAWS10024

Mental skills 5

Maths without calculators

Adding or multiplying in any order

Numbers can be added or multiplied in any order. We can use this property to make our calculations simpler.

1 Study each example.

$$\begin{aligned}\text{a } 19 + 5 + 5 + 1 &= (19 + 1) + (5 + 5) \\ &= 20 + 10 \\ &= 30\end{aligned}$$

$$\begin{aligned}\text{b } 13 + 8 + 20 + 27 + 80 &= (13 + 27) + (20 + 80) + 8 \\ &= 40 + 100 + 8 \\ &= 148\end{aligned}$$

$$\begin{aligned}\text{c } 2 \times 36 \times 5 &= (2 \times 5) \times 36 \\ &= 10 \times 36 \\ &= 360\end{aligned}$$

$$\begin{aligned}\text{d } 25 \times 11 \times 4 \times 7 &= (25 \times 4) \times (11 \times 7) \\ &= 100 \times 77 \\ &= 7700\end{aligned}$$

2 Now evaluate each sum.

$$\text{a } 45 + 16 + 45 + 4 + 7$$

$$\text{c } 18 + 91 + 9 + 20$$

$$\text{e } 24 + 16 + 80 + 44 + 10$$

$$\text{g } 100 + 36 + 200 + 10 + 90$$

$$\text{i } 70 + 50 + 30 + 25 + 25$$

$$\text{b } 38 + 600 + 50 + 12 + 40$$

$$\text{d } 75 + 33 + 7 + 25$$

$$\text{f } 56 + 5 + 20 + 15 + 4$$

$$\text{h } 54 + 27 + 9 + 16 + 3$$

$$\text{j } 32 + 120 + 40 + 80 + 40$$

3 Now evaluate each product.

$$\text{a } 8 \times 4 \times 5$$

$$\text{d } 5 \times 11 \times 40$$

$$\text{g } 3 \times 20 \times 7 \times 5$$

$$\text{b } 50 \times 7 \times 2$$

$$\text{e } 12 \times 2 \times 3$$

$$\text{h } 6 \times 8 \times 5 \times 2$$

$$\text{c } 3 \times 5 \times 6$$

$$\text{f } 2 \times 4 \times 25 \times 8$$

$$\text{i } 2 \times 3 \times 2 \times 11$$

Investigation: Negative powers

What is the value of a number raised to a negative power, for example, 2^{-1} or 2^{-2} ?

1 Copy and complete each table showing decreasing powers. Notice the pattern in your answers.

a

Power of 2	Number
2^3	8
2^2	4
2^1	
2^0	
2^{-1}	
2^{-2}	
2^{-3}	

b

Power of 10	Number
10^3	1000
10^2	
10^1	
10^0	
10^{-1}	
10^{-2}	
10^{-3}	

- 2 Copy and complete this table showing decreasing powers in expanded form. Notice the pattern in your answers.

a

Power of 5	Expanded form
3^3	$3 \times 3 \times 3$
3^2	3×3
3^1	3
3^0	1
3^{-1}	$\frac{1}{3}$
3^{-2}	$\frac{1}{3 \times 3} = \frac{1}{3^2}$
3^{-3}	$\frac{1}{3 \times 3 \times 3} = \frac{1}{3^3}$
3^{-4}	
3^{-5}	

b

Power of 5	Expanded form
5^3	$5 \times 5 \times 5$
5^2	
5^1	
5^0	
5^{-1}	
5^{-2}	
5^{-3}	
5^{-4}	
5^{-5}	

- 3 If $3^{-2} = \frac{1}{3^2}$ and $5^{-3} = \frac{1}{5^3}$, then write each negative power in a similar way.

a 4^{-1}

b 7^{-4}

c 2^{-6}

- 4 Simplify each expression in index notation.

a $10^4 \div 10^7$

b $2^3 \div 2^8$

c $3^4 \div 3^5$

d $5^2 \div 5^8$

e $a^4 \div a^6$

f $a \div a^4$

5 Consider $\frac{10^4}{10^7} = \frac{10 \times 10 \times 10 \times 10}{10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10}$

$$= \frac{1}{10 \times 10 \times 10}$$

$$= \frac{1}{10^3}$$

But also $\frac{10^4}{10^7} = 10^{4-7}$

$$= 10^{-3}$$

$$\therefore 10^{-3} = \frac{1}{10^3}$$

Use the method above to show that:

a $\frac{2^3}{2^8} = 2^{-5} = \frac{1}{2^5}$

b $\frac{3^4}{3^5} = 3^{-1} = \frac{1}{3}$

c $\frac{5^2}{5^8} = 5^{-6} = \frac{1}{5^6}$

d $\frac{a^4}{a^6} = a^{-2} = \frac{1}{a^2}$

- 6 Write in words and as a formula the rule for raising a to a negative power $-n$, that is, a^{-n} .

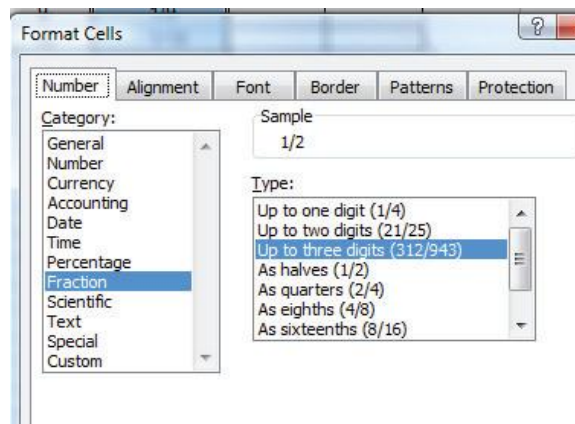
Technology Negative powers

In this activity we will discover the pattern for negative powers. We will consider base values from 2 to 10 shown in column A and indices (powers) -1 , -2 and -3 shown in row 1. On a spreadsheet, the symbol for power is $^$ (called a **carat**, press **SHIFT 6**). For example, 3^{-1} is entered as 3^{-1} .

- 1 Create a spreadsheet as shown below.

	A	B	C	D
1	Index	-1	-2	-3
2				
3	Base			
4	2			
5	3			
6	4			
7	5			
8	6			
9	7			
10	8			
11	9			
12	10			
13				

- 2 We will first examine the power of -1 . In cell B4, enter `=A4^$B$1` to calculate 2^{-1} . `$B$1` is an **absolute cell reference**, which ensures that the cell does not change when a formula is copied. This means that in column B, the power will always refer to cell B1 (-1) only. **Fill Down** from cell B4 to B12.
- 3 Use **Format cells** to set column B decimals to **Fraction** and **Up to three digits**.



- 4 Compare your answers in column B with the original values in column A. Can you describe the pattern when a base is raised to a power of -1 ?
- 5 Now consider powers of -2 . Adapt steps from 1 to 3 for **column C**. Use **Fill Down** from cell C4 to C12.

	A	B	C	D
1	Index	-1	-2	-3
2				
3	Base			
4	2	1/2	=A4^\$C\$1	
5	3	1/3		
6	4	1/4		
7	5	1/5		

- 6 Compare your answers in column C with the original values in column A. Can you describe the pattern when a base is raised to a power of -2 ?
- 7 Now consider powers of -3 . Adapt steps for **column D**. In cell D4, enter the formula `=A4^$D$1`.

Note: D12's fraction is missing as it has 4 digits in the denominator, which the spreadsheet doesn't allow for. Can you figure out what the fraction should be?

- 8 Compare your answers in column D with the original values in column A. Can you describe the pattern when a base is raised to a power of -3 ?
- 9 Write a rule for negative powers, given the answers you have found in this activity. Discuss with other students in your class.

5-05 Negative indices

Worksheet

Power calculations

MAT09NAWK10057

Consider $2^0 \div 2^3 = \frac{2^0}{2^3}$
 $= \frac{1}{2^3}$

But also $2^0 \div 2^3 = 2^{0-3}$
 $= 2^{-3}$
 $\therefore 2^{-3} = \frac{1}{2^3}$

Summary

A number **raised to a negative power** gives a fraction (with a numerator of 1):

$$a^{-n} = \frac{1}{a^n}$$

Proof: $a^0 \div a^n = \frac{a^0}{a^n}$
 $= \frac{1}{a^n}$

But also $a^0 \div a^n = a^{0-n}$
 $= a^{-n}$
 $\therefore a^{-n} = \frac{1}{a^n}$

Example 7

Simplify each expression using a positive index (power).

a 5^{-3}

b $3n^{-2}$

c $(3n)^{-2}$

d $p^{-2}q^3$

Solution

a $5^{-3} = \frac{1}{5^3}$

b $3n^{-2} = 3 \times n^{-2}$
 $= \frac{3}{1} \times \frac{1}{n^2}$
 $= \frac{3}{n^2}$

c $(3n)^{-2} = \frac{1}{(3n)^2}$
 $= \frac{1}{9n^2}$

d $p^{-2}q^3 = \frac{1}{p^2} \times q^3$
 $= \frac{q^3}{p^2}$

Video tutorial

Negative indices

MAT09NAVT10010

The reciprocal as a power

Consider $9^{-1} = \frac{1}{9}$

$\frac{1}{9}$ is the **reciprocal** of 9.

$$\begin{aligned}\text{Consider } \left(\frac{2}{3}\right)^{-1} &= \frac{1}{\left(\frac{2}{3}\right)} \\ &= 1 \div \frac{2}{3} \\ &= 1 \times \frac{3}{2} \\ &= \frac{3}{2} \\ &= 1\frac{1}{2}\end{aligned}$$

Summary

A number **raised to a power of -1** gives its reciprocal.

$$a^{-1} = \frac{1}{a}$$

$$\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$$

Example 8

Simplify each expression.

a $\left(\frac{4}{3}\right)^{-1}$

b $\left(\frac{y}{5}\right)^{-1}$

Solution

a $\left(\frac{4}{3}\right)^{-1} = \frac{3}{4}$

b $\left(\frac{y}{5}\right)^{-1} = \frac{5}{y}$

Stage 5.3

Negative powers of quotients

$$\begin{aligned}\text{Consider } \left(\frac{4}{5}\right)^{-2} &= \frac{1}{\left(\frac{4}{5}\right)^2} \\ &= \frac{1}{\frac{16}{25}} \\ &= 1 \div \frac{16}{25}\end{aligned}$$

$$\begin{aligned}
 &= 1 \times \frac{25}{16} \\
 &= \frac{25}{16} \\
 &= \frac{5^2}{4^2} \\
 &= \left(\frac{5}{4}\right)^2
 \end{aligned}$$

Summary

A number **raised to a power of $-n$** gives its reciprocal raised to the power of n .

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$$

Proof:

$$\begin{aligned}
 \left(\frac{a}{b}\right)^{-n} &= \frac{1}{\left(\frac{a}{b}\right)^n} \\
 &= \frac{1}{\frac{a^n}{b^n}} \\
 &= \frac{b^n}{a^n} \\
 &= \left(\frac{b}{a}\right)^n
 \end{aligned}$$

Example 9

Simplify each expression.

a $\left(\frac{4}{3}\right)^{-3}$

b $\left(2\frac{1}{2}\right)^{-2}$

c $\left(\frac{3a}{b^4}\right)^{-2}$

Solution

a $\left(\frac{4}{3}\right)^{-3} = \left(\frac{3}{4}\right)^3$
 $= \frac{27}{64}$

b $\left(2\frac{1}{2}\right)^{-2} = \left(\frac{5}{2}\right)^{-2}$
 $= \left(\frac{2}{5}\right)^2$
 $= \frac{4}{25}$

c $\left(\frac{3a}{b^4}\right)^{-2} = \left(\frac{b^4}{3a}\right)^2$
 $= \frac{b^8}{9a^2}$

Exercise 5-05 Negative indices

See Example 7

- 1 Simplify each expression using a positive index.
- | | | | |
|------------|------------|------------|-------------|
| a 3^{-2} | b 5^{-7} | c 3^{-1} | d 10^{-2} |
| e g^{-5} | f z^{-1} | g n^{-3} | h t^{-2} |
| i a^{-4} | j 5^{-3} | k y^{-d} | l r^{-m} |
- 2 Evaluate each expression, giving your answers in fraction form.
- | | | | |
|-------------|------------|------------|-------------|
| a 3^{-2} | b 5^{-4} | c 6^{-1} | d 7^{-2} |
| e 25^{-1} | f 2^{-7} | g 4^{-3} | h 10^{-6} |
| i 2^{-10} | j 3^{-3} | k 6^{-2} | l 9^{-4} |
- 3 Write each expression using a negative index.
- | | | | |
|--------------------|-------------------|--------------------|-------------------|
| a $\frac{1}{n^2}$ | b $\frac{1}{n}$ | c $\frac{1}{8^3}$ | d $\frac{1}{8}$ |
| e $\frac{1}{10^5}$ | f $\frac{2}{a^4}$ | g $\frac{1}{3}$ | h $-\frac{1}{b}$ |
| i $\frac{6}{a}$ | j $\frac{4}{t^2}$ | k $-\frac{2}{w^5}$ | l $\frac{5}{d^3}$ |

Worked solutions

Negative indices

MAT09NAWS10025

- 4 Simplify each expression using positive indices.
- | | | | |
|----------------|--------------------|--------------------|------------------|
| a $5h^{-1}$ | b $2b^{-5}$ | c $3e^{-3}$ | d $4n^{-2}$ |
| e pb^{-2} | f r^2s^{-4} | g $w^{-2}y$ | h $d^{-3}y^3$ |
| i $(2m)^{-1}$ | j $(xy)^{-1}$ | k $(4h)^{-2}$ | l $(5k)^{-3}$ |
| m $3m^3p^{-2}$ | n $15k^{-1}w^{-4}$ | o $12x^{-2}y^{-3}$ | p $12x^{-2}y^3$ |
| q $(3h)^{-2}$ | r $(4k)^{-3}$ | s $(2c)^{-4}$ | t $(8y)^{-1}$ |
| u $4pq^{-3}$ | v $4p^{-1}q^{-3}$ | w vm^{-2} | x $v^{-1}m^{-2}$ |

See Example 8

- 5 Simplify each expression.
- | | | | |
|------------------------------------|------------------------------------|------------------------------------|-----------------------------------|
| a $\left(\frac{2}{7}\right)^{-1}$ | b $\left(\frac{8}{5}\right)^{-1}$ | c $\left(\frac{9}{10}\right)^{-1}$ | d $\left(\frac{3}{2}\right)^{-1}$ |
| e $\left(-\frac{3}{4}\right)^{-1}$ | f $\left(\frac{5}{2}\right)^{-1}$ | g $\left(\frac{x}{3}\right)^{-1}$ | h $\left(\frac{5}{a}\right)^{-1}$ |
| i $\left(-\frac{m}{2}\right)^{-1}$ | j $\left(\frac{5r}{4}\right)^{-1}$ | k $\left(\frac{2}{3z}\right)^{-1}$ | l $\left(\frac{1}{v}\right)^{-1}$ |

Stage 5.3

See Example 9

- 6 Simplify each expression.
- | | | | |
|-------------------------------------|---------------------------------------|---|---|
| a $\left(\frac{1}{4}\right)^{-2}$ | b $\left(\frac{2}{3}\right)^{-2}$ | c $\left(\frac{1}{10}\right)^{-6}$ | d $\left(\frac{5}{2}\right)^{-3}$ |
| e $\left(\frac{4}{3}\right)^{-5}$ | f $\left(\frac{5}{4}\right)^{-4}$ | g $\left(2\frac{1}{4}\right)^{-2}$ | h $\left(1\frac{2}{5}\right)^{-3}$ |
| i $\left(\frac{k}{3}\right)^{-2}$ | j $\left(\frac{3}{x}\right)^{-3}$ | k $\left(\frac{a^2}{4}\right)^{-4}$ | l $\left(\frac{4}{3g^3}\right)^{-2}$ |
| m $\left(\frac{2d}{5t}\right)^{-2}$ | n $\left(\frac{h^2}{m^3}\right)^{-5}$ | o $\left(\frac{5d^3}{3p^4}\right)^{-2}$ | p $\left(\frac{3c^3}{4a^2}\right)^{-3}$ |

5-06 Fractional indices $\frac{1}{n}$

We now know the meaning of zero and negative indices, that is, a^0 , a^{-1} and a^{-n} .

What is the meaning of fractional indices, that is, $a^{\frac{1}{2}}$ and $a^{\frac{1}{n}}$?

$$\begin{aligned}\text{Consider } (25^{\frac{1}{2}})^2 &= 25^{\frac{1}{2} \times 2} && \text{Power of a power} \\ &= 25^1 \\ &= 25\end{aligned}$$

$$\text{but } (\sqrt{25})^2 = 25$$

$$\therefore 25^{\frac{1}{2}} = \sqrt{25} = 5$$

Summary

Any number raised to the power of $\frac{1}{2}$ is the square root of that number:

$$a^{\frac{1}{2}} = \sqrt{a}$$

$$\begin{aligned}\text{Proof: } (a^{\frac{1}{2}})^2 &= a^{\frac{1}{2} \times 2} \\ &= a^1 \\ &= a\end{aligned}$$

$$\text{But } (\sqrt{a})^2 = a$$

$$\therefore a^{\frac{1}{2}} = \sqrt{a}$$

$$\begin{aligned}\text{Now consider } (27^{\frac{1}{3}})^3 &= 27^{\frac{1}{3} \times 3} && \text{Power of a power} \\ &= 27^1 \\ &= 27\end{aligned}$$

$$\text{but } (\sqrt[3]{27})^3 = 27$$

$$\therefore 27^{\frac{1}{3}} = \sqrt[3]{27} = 9$$

Summary

Any number raised to the power of $\frac{1}{3}$ is the cube root of that number:

$$a^{\frac{1}{3}} = \sqrt[3]{a}$$

$$\begin{aligned}\text{Proof: } (a^{\frac{1}{3}})^3 &= a^{\frac{1}{3} \times 3} \\ &= a^1 \\ &= a\end{aligned}$$

$$\text{But } (\sqrt[3]{a})^3 = a$$

$$\therefore a^{\frac{1}{3}} = \sqrt[3]{a}$$

$$\begin{aligned}\text{Now consider } (32^{\frac{1}{5}})^5 &= 32^{\frac{1}{5} \times 5} && \text{Power of a power} \\ &= 32^1 \\ &= 32\end{aligned}$$

Stage 5.3

If $(32^{\frac{1}{5}})^5 = 32$, then $32^{\frac{1}{5}}$ is called the **5th root of 32**, written $\sqrt[5]{32}$.

$$2^5 = 32$$

$$\therefore 32^{\frac{1}{5}} = \sqrt[5]{32} = 2$$

Summary

Generally, any number raised to the power of $\frac{1}{n}$ is the n th root of that number:

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

Proof:
$$\begin{aligned} \left(a^{\frac{1}{n}}\right)^n &= a^{\frac{1}{n} \times n} \\ &= a^1 \\ &= a \end{aligned}$$

But $(\sqrt[n]{a})^n = a$
 $\therefore a^{\frac{1}{n}} = \sqrt[n]{a}$

Example 10

Evaluate each expression.

a $900^{\frac{1}{2}}$

b $125^{\frac{1}{3}}$

c $1024^{\frac{1}{10}}$

Solution

a $900^{\frac{1}{2}} = \sqrt{900}$
 $= 30$

b $125^{\frac{1}{3}} = \sqrt[3]{125}$
 $= 5$

c $1024^{\frac{1}{10}} = \sqrt[10]{1024}$
 $= 2$

Enter on calculator: 10  1024 
 because $2^{10} = 1024$

Summary

On a calculator, the n th root key is  or , found by pressing the **SHIFT** or **2ndF** key before pressing  or  respectively.

Example 11

Write each expression using a fractional index.

a $\sqrt{8}$

b $\sqrt[3]{36}$

c $\sqrt[4]{n}$

d $\sqrt[7]{ab}$

Solution

a $\sqrt{8} = 8^{\frac{1}{2}}$

b $\sqrt[3]{36} = 36^{\frac{1}{3}}$

c $\sqrt[4]{n} = n^{\frac{1}{4}}$

d $\sqrt[7]{ab} = (ab)^{\frac{1}{7}}$

Exercise 5-06 Fractional indices $\frac{1}{n}$

- Evaluate each expression.

a $25^{\frac{1}{2}}$	b $343^{\frac{1}{3}}$	c $625^{\frac{1}{2}}$	d $1000^{\frac{1}{3}}$
e $32^{\frac{1}{5}}$	f $(-0.027)^{\frac{1}{3}}$	g $(0.04)^{\frac{1}{2}}$	h $64^{\frac{1}{3}}$
i $(-8)^{\frac{1}{3}}$	j $(-729)^{\frac{1}{3}}$	k $256^{\frac{1}{8}}$	l $3125^{\frac{1}{5}}$
- Write each expression using a radical (root) sign.

a $10^{\frac{1}{2}}$	b $12^{\frac{1}{3}}$	c $g^{\frac{1}{2}}$	d $m^{\frac{1}{4}}$
e $(8r)^{\frac{1}{2}}$	f $(6h)^{\frac{1}{6}}$	g $(5j^8)^{\frac{1}{5}}$	h $90ab^{\frac{1}{3}}$
- Write each expression using a fractional index.

a $\sqrt{5}$	b $\sqrt[3]{49}$	c $\sqrt{20}$	d $\sqrt[5]{400}$
e $\sqrt[6]{66}$	f $\sqrt[4]{64}$	g $\sqrt[8]{144}$	h $\sqrt[10]{1000}$
i $\sqrt{a}$	j $\sqrt[3]{q}$	k $\sqrt[7]{h}$	l $\sqrt[6]{w}$
m $\sqrt[3]{t}$	n $\sqrt{xy}$	o $\sqrt[4]{100f}$	p $\sqrt[3]{2mn}$
- Evaluate each expression correct to 2 decimal places.

a $20^{\frac{1}{3}}$	b $215^{\frac{1}{2}}$	c $\sqrt[3]{144}$	d $\sqrt{2001}$
e $(-666)^{\frac{1}{3}}$	f $\sqrt[4]{1111}$	g $\sqrt[5]{-754}$	h $\sqrt[6]{0.008}$
- Simplify each expression.

a $b^{\frac{1}{2}} \times b^{\frac{1}{2}}$	b $e^{\frac{1}{3}} \times e^{\frac{1}{3}} \times e^{\frac{1}{3}}$	c $y \times y^{\frac{1}{3}}$	d $m^{\frac{3}{5}} \times m^{\frac{2}{5}}$
e $2t^{\frac{1}{3}} \times 5t^{\frac{2}{3}}$	f $(-6a^{\frac{3}{2}})^2$	g $(n^{12}m^4)^{\frac{1}{4}}$	h $(16a^2b^6)^{\frac{1}{2}}$
i $(8v^6w^9)^{\frac{1}{3}}$	j $40a^{\frac{1}{10}} \div 8a^{\frac{1}{10}}$	k $35x \div 5x^{\frac{1}{3}}$	l $36y^{\frac{3}{4}} \div 4y$

5-07 Fractional indices $\frac{m}{n}$

What is the meaning of fractional indices such as $a^{\frac{2}{3}}$ and $a^{\frac{3}{2}}$?

Consider $32^{\frac{3}{5}} = \left(32^{\frac{1}{5}}\right)^3$ Power of a power

$$= \left(\sqrt[5]{32}\right)^3$$

$$= 2^3$$

$$= 8$$

or consider $32^{\frac{3}{5}} = (32^3)^{\frac{1}{5}}$ Power of a power

$$= \sqrt[5]{32^3}$$

$$= \sqrt[5]{32768}$$

$$= 8$$

Summary

$$a^{\frac{m}{n}} = (\sqrt[n]{a})^m \text{ or } \sqrt[n]{a^m}$$

Stage 5.3

Proof: $a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m$ or $(a^m)^{\frac{1}{n}}$
 $= (\sqrt[n]{a})^m$ or $\sqrt[n]{a^m}$

Note: Taking the root first often makes the calculation simpler.

Example 12

Evaluate each expression.

a $8^{\frac{2}{3}}$

b $27^{\frac{4}{3}}$

c $64^{-\frac{1}{3}}$

d $16^{-\frac{3}{4}}$

Solution

$$\begin{aligned} \text{a } 8^{\frac{2}{3}} &= \left(\sqrt[3]{8}\right)^2 \\ &= 2^2 \\ &= 4 \end{aligned}$$

$$\begin{aligned} \text{b } 27^{\frac{4}{3}} &= \left(\sqrt[3]{27}\right)^4 \\ &= 3^4 \\ &= 81 \end{aligned}$$

$$\begin{aligned} \text{c } 64^{-\frac{1}{3}} &= \frac{1}{64^{\frac{1}{3}}} \\ &= \frac{1}{\sqrt[3]{64}} \\ &= \frac{1}{4} \end{aligned}$$

$$\begin{aligned} \text{d } 16^{-\frac{3}{4}} &= \frac{1}{16^{\frac{3}{4}}} \\ &= \frac{1}{(\sqrt[4]{16})^3} \\ &= \frac{1}{2^3} \\ &= \frac{1}{8} \end{aligned}$$

Example 13

Evaluate $300^{\frac{3}{5}}$ correct to two decimal places.

Solution

$$\begin{aligned} 300^{\frac{3}{5}} &= 30.63887063 \dots \\ &\approx 30.64 \end{aligned}$$

Enter on calculator: 300 $x^{\frac{\square}{\square}}$ 3 $\frac{\square}{\square}$ 5 =

Example 14

Write each expression using a fractional index.

a $\sqrt[4]{p^3}$

b $\sqrt{b^7}$

c $\frac{1}{\sqrt[3]{q^4}}$

Solution

$$\begin{aligned} \text{a } \sqrt[4]{p^3} &= (p^3)^{\frac{1}{4}} \\ &= p^{\frac{3}{4}} \end{aligned}$$

$$\begin{aligned} \text{b } \sqrt{b^7} &= (b^7)^{\frac{1}{2}} \\ &= b^{\frac{7}{2}} \end{aligned}$$

$$\begin{aligned} \text{c } \frac{1}{\sqrt[3]{q^4}} &= \frac{1}{(q^4)^{\frac{1}{3}}} \\ &= \frac{1}{q^{\frac{4}{3}}} \text{ or } q^{-\frac{4}{3}} \end{aligned}$$

Example 15

Simplify each expression.

a $\sqrt[4]{(16r^2)^3}$

b $(\sqrt[3]{27k})^2$

c $(32a^5)^{-\frac{3}{5}}$

Solution

a $\sqrt[4]{(16r^2)^3} = (16r^2)^{\frac{3}{4}}$
 $= 16^{\frac{3}{4}} r^{2 \times \frac{3}{4}}$
 $= 8r^{\frac{3}{2}}$

b $(\sqrt[3]{27k})^2 = (27k)^{\frac{2}{3}}$
 $= 27^{\frac{2}{3}} k^{\frac{2}{3}}$
 $= 9k^{\frac{2}{3}}$

c $(32a^5)^{-\frac{3}{5}} = \frac{1}{(32a^5)^{\frac{3}{5}}}$
 $= \frac{1}{32^{\frac{3}{5}} a^{5 \times \frac{3}{5}}}$
 $= \frac{1}{8a^3}$

See Example 12

See Example 13

See Example 14

See Example 15

Exercise 5-07 Fractional indices $\frac{m}{n}$

1 Evaluate each expression.

a $4^{\frac{3}{2}}$

b $8^{\frac{5}{3}}$

c $128^{\frac{5}{7}}$

d $27^{\frac{2}{3}}$

e $1024^{\frac{3}{5}}$

f $64^{\frac{4}{3}}$

g $32^{\frac{3}{5}}$

h $81^{\frac{3}{4}}$

i $1000^{\frac{2}{3}}$

j $125^{\frac{2}{3}}$

k $8^{-\frac{1}{3}}$

l $81^{-\frac{1}{4}}$

m $25^{-\frac{3}{2}}$

n $36^{-\frac{3}{2}}$

o $256^{-\frac{3}{4}}$

p $3125^{-\frac{4}{5}}$

q $1024^{-\frac{4}{5}}$

r $400^{-\frac{3}{2}}$

s $128^{-\frac{4}{7}}$

t $1024^{-\frac{7}{10}}$

2 Evaluate each expression correct to two decimal places.

a $15^{\frac{3}{4}}$

b $8^{-\frac{7}{5}}$

c $50^{\frac{5}{4}}$

d $6^{-\frac{2}{3}}$

e $100^{-\frac{3}{4}}$

f $16^{\frac{3}{5}}$

g $12^{-\frac{3}{2}}$

h $179^{\frac{2}{5}}$

3 Write each expression using a fractional index.

a $\sqrt[7]{g^2}$

b $\sqrt{e^5}$

c $\sqrt[6]{x^{18}}$

d $\frac{1}{\sqrt[4]{y^{16}}}$

e $\sqrt[3]{m^5}$

f $\sqrt[5]{m^3}$

g $\frac{1}{\sqrt[4]{n^3}}$

h $\frac{1}{\sqrt[3]{n^4}}$

4 Simplify each expression.

a $(16n^4)^{\frac{3}{4}}$

b $(8w)^{\frac{2}{3}}$

c $\sqrt[5]{(32d^{10})^3}$

d $(\sqrt{64m^8})^3$

e $(\sqrt[4]{81r^4})^5$

f $\sqrt[4]{(81h^{12})^3}$

g $\frac{1}{(\sqrt[3]{8s^6})^4}$

h $\left(\frac{1}{\sqrt[5]{1024x^{15}}}\right)^2$

i $(125b^6)^{-\frac{2}{3}}$

j $(625t^2)^{-\frac{3}{4}}$

k $(49p^4q^{10})^{\frac{3}{2}}$

l $(1000x^3y^6)^{-\frac{2}{3}}$

Worksheet

Index laws review

MAT09NAWK10055

Puzzle sheet

Indices squaresaw

MAT09NAPS10056

Homework sheet

Indices 2

MAT09NAHS10006

5-08 Summary of the index laws

Summary

$$a^m \times a^n = a^{m+n}$$

$$a^0 = 1$$

$$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$$

$$a^{-1} = \frac{1}{a}$$

$$(a^m)^n = a^{m \times n}$$

$$a^{-n} = \frac{1}{a^n}$$

$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$$

$$a^{\frac{1}{2}} = \sqrt{a}, a^{\frac{1}{3}} = \sqrt[3]{a}, a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$a^{\frac{m}{n}} = (\sqrt[n]{a})^m \text{ or } \sqrt[n]{a^m}$$

Exercise 5-08 Summary of the index laws

1 Simplify each expression.

a $a^4 \times a^3$

b $t^8 \times t$

c $n^8 \div n^2$

d $p^3 \div p$

e $(w^2)^4$

f $(g^3)^6$

g $2b^2 \times 3b^5$

h $4d^7 \times 5d^6$

i $\frac{30c^{12}}{5c^8}$

j $(5b^4)^4$

k $24m^6 \div 8m^4$

l $(3a)^2$

2 Evaluate each expression.

a 4^0

b $(-4)^0$

c 7×2^0

d $(7 \times 2)^0$

e $(-2)^3$

f $(-3)^2$

g $(5^2)^2$

h $2^4 \times 2^3$

i $(7^2)^0$

j $4^5 \div 4^2$

k $4^2 \div 4^5$

l $10^3 \div 10^3$

m $5^2 \div 5^0$

n $10^{-2} \div 10^2$

o $\left(\frac{1}{2}\right)^0$

p $10^{-2} \times 10^2$

3 Evaluate each expression, giving your answers in fraction form.

a 5^{-2}

b 2^{-5}

c 20^{-1}

d 10^{-3}

4 Evaluate each expression.

a $16^{\frac{1}{2}}$

b $27^{\frac{1}{3}} + 4^0$

c $25^{\frac{5}{2}}$

d $8^{\frac{1}{3}} + 4^{\frac{1}{2}}$

e $(8^2)^{\frac{1}{3}}$

f $(9^3)^{\frac{1}{2}}$

g $81^{\frac{3}{4}}$

h $(-32)^{\frac{3}{5}}$

Stage 5.3

5 Simplify each expression.

a $(3mn^3)^2$

b $8a^2w^2 \times 5a^3w^7$

c $(4a^2b^5)^4$

d $\frac{20p^3q^8}{5p^2q^6}$

e $\left(\frac{4}{5}\right)^3$

f $6c^2d^0$

g $\frac{48u^5v^4}{16uv}$

h $\left(\frac{3x}{10}\right)^2$

i $(-4n^2t)^3$

j $\left(-\frac{2}{3}\right)^4$

k $\frac{7x^2y^6}{35x^5y^3}$

l $\left(\frac{p^5}{9y}\right)^2$

m $(2p^3q^2)^5$

n $\left(\frac{1}{7n}\right)^3$

o $-2n^0$

p $\frac{(a^2b)^4 \times a^3}{b^5}$

6 Simplify each expression using a positive index.

a 8^{-7}

b 3^{-5}

c y^{-1}

d x^{-3}

e $(5b)^{-2}$

f $5b^{-2}$

g $(ab)^{-1}$

h ab^{-1}

i $11t^{-3}$

j $(11t)^{-3}$

k p^3q^{-5}

i mw^{-3}

m $8u^{-3}v^{-4}$

n $-2r^6y^{-5}$

o $10e^{-1}f^3$

p $\frac{1}{2}k^{-4}n^7$

7 Simplify each expression.

a $\left(\frac{7}{4}\right)^{-1}$

b $\left(\frac{5}{2}\right)^{-1}$

c $\left(\frac{2}{3}\right)^{-1}$

d $\left(\frac{1}{7}\right)^{-1}$

e $\left(\frac{r}{8}\right)^{-1}$

f $\left(\frac{1}{10p}\right)^{-1}$

g $\left(\frac{6y}{z}\right)^{-1}$

h $\left(\frac{2}{5a}\right)^{-1}$

8 Write each expression using a negative index.

a $\frac{1}{4^3}$

b $\frac{1}{2}$

c $\frac{1}{10^4}$

d $\frac{1}{9^2}$

e $\frac{1}{k}$

f $\frac{9}{k^4}$

g $\frac{-1}{x^7}$

h $\frac{5}{p^3}$

9 Simplify each expression.

a $q^5 \times q^{-2}$

b $d^{-3} \times d^7$

c $m^{-6} \div m^5$

d $t \div t^{-1}$

e $5g^3 \times 6g^{-1}$

f $8a^{-2} \times 3a^3$

g $7x^{-2} \times 4x$

h $\frac{64p^{-1}}{16p^2}$

i $48q \div 3q^{-2}$

j $\frac{5t^3}{10t^{-1}}$

k $2(b^{-1})^4$

l $(3h)^{-2}$

10 Write each expression using a fractional index.

a $\sqrt{5}$

b $\sqrt[3]{d}$

c $\sqrt{3y}$

d $\sqrt[4]{10}$

e $(\sqrt[3]{p})^2$

f $\sqrt{(xy)^5}$

g $\sqrt[4]{(5a)^3}$

h $\left(\sqrt{\frac{y}{6}}\right)^5$

11 Simplify each expression.

a $\left(\frac{4}{5a}\right)^{-3}$

b $(8c^3)^{\frac{2}{3}}$

c $\left(\frac{10}{7m}\right)^{-2}$

d $(25w^5)^{\frac{5}{2}}$

e $\left(\frac{49}{d^2}\right)^{\frac{3}{2}}$

f $\left(\frac{a^3b^9}{c^6}\right)^{\frac{2}{3}}$

g $\sqrt[4]{625m^6}$

h $\left(\frac{64}{y^3}\right)^{-\frac{2}{3}}$

i $\sqrt[5]{32m^{10}}$

j $\left(\frac{1}{3g^2}\right)^{-2}$

k $(16x^8)^{\frac{5}{4}}$

l $\left(\frac{2a^3}{c^2}\right)^4$

Stage 5.3

Worked solutions

Summary of the
index laws

MAT09NAWS10027

NSW

5-09 Significant figures

Worksheet

Significant figures

MAT09NAWK10059

A way of rounding a number is to give the most relevant or important digits of the number. For example, a crowd of 47 321 people can be written as 47 000, which is rounded to the nearest thousand, or to two **significant figures**.

The first significant figure in a number is the first **non-zero** digit. For example, the significant figures are shown in bold in this table:

Number	First significant digit	Number of significant digits
47 321	4	5
47 000	4	2
0.000 159 2	1	4
0.000 2	2	1

- When rounding to significant figures, start counting from the first digit that is not 0.
- If it is a large number, you may need to insert 0s at the end as placeholders.
- Zeros at the end of a whole number or at the beginning of a decimal are **not significant**: they are necessary placeholders.
- Zeros *between* significant figures or at the end of a decimal *are* significant. For example, the significant figures are shown in bold in this table.

Number	First significant digit	Number of significant digits
809 000	8	3
0.0 20 70	2	4

Example 16

State the number of significant figures in each number.

a 63.70

b 0.003 05

c 7600

Solution

- a** The zero after 7 is significant.
 $\therefore$ 63.70 has four significant figures.
- b** The first significant figure is 3, and the zero between 3 and 5 is significant.
 $\therefore$ 0.003 05 has three significant figures.
- c** The zeros after 6 are not significant.
 $\therefore$ 7600 has two significant figures.

Example 17

Round each number to three significant figures.

- a 56.357 b 9.249 c 548 307

Solution

- a $56.357 \approx 56.4$ b $9.249 \approx 9.25$ c $548\,307 \approx 548\,000$

The zeros here are not significant, but they are placeholders that are necessary for showing the place values of the 5, 4 and 8.

Example 18

Write each number correct to one significant figure.

- a 0.007 39 b 0.025 c 0.963

Solution

- a $0.007\,39 \approx 0.007$ b $0.025 \approx 0.03$ c $0.963 \approx 1$

The zeros at the beginning of a decimal are not significant: they are placeholders.

Exercise 5-09 Significant figures

1 State the number of significant figures in each number.

- a 457 b 0.23 c 15 000 d 4.0004 e 0.0005 f 5000
g 0.002 07 h 89 072 i 0.040 j 76 000 000 k 0.000 328 l 169.320

See Example 16

2 Round each number to three significant figures.

- a 37.609 b 9435 c 168.39
d 2.813 e 15.99 f 60 522
g 1 769 000 h 385 764 i 10.2717

See Example 17

3 Write each number correct to two significant figures.

- a 0.0637 b 0.903 c 0.084 55
d 0.000 158 e 0.007 625 f 0.038 71
g 0.2795 h 0.018 944 i 0.3145

See Example 18

4 What is 45 067 853 rounded to 3 significant figures? Select the correct answer A, B, C or D.

- A 45 167 853 B 45 100 000 C 45 067 900 D 45 070 000

5 What is 0.005 605 0 rounded to 2 significant figures? Select the correct answer A, B, C or D.

- A 0.01 B 0.010 000 0 C 0.0056 D 0.005 600 0

6 Round each number to one significant figure.

- a 9.478 b 57.12 c 0.0367
d 0.007 66 e 0.5067 f 10 675
g 1856.78 h 0.000 28 i 56 239 400

7 A company makes a profit of \$35 754 125.

- a Round the profit to the nearest million and state the number of significant figures in the answer.
b Round the profit to the nearest ten million and state the number of significant figures in the answer.

- 8 Australia's population in 2010 was 21 387 000. To how many significant figures has this number been written?
- 9 A total of 21 558 people attended a local football match. Express this number to three significant figures.
- 10 Evaluate each expression, correct to the number of significant figures shown in the brackets.
- a $45.6 \times 8.7 - 2.75 \times 78.32$ (2) b $15.5 - 9.87 \div 0.24 + 8.43 \times 2.4$ (1)
- c $(63.73 - 27.89) \div 5.82$ (3) d $\frac{63.25 + 76.03}{55.89 - 89.24}$ (4)
- e $\frac{9.732 + 2.765}{12.27 \times 15.8}$ (1) f $78.91 \div (23.6 + 94.7)$ (2)
- g $\frac{1}{0.941} + \frac{253}{0.0076}$ (3) h $\sqrt{84.3} \times 0.0715$ (4)

Just for the record

Big numbers

The table below lists the names of some big numbers and their meanings.

Name	Numeral
million	$10^6 = 1\,000\,000$
billion	$10^9 = 1\,000\,000\,000$
trillion	10^{12}
quadrillion	10^{15}
quintillion	10^{18}
sextillion	10^{21}
septillion	10^{24}
octillion	10^{27}
nonillion	10^{30}
decillion	10^{33}

According to the Guinness Book of Records, the largest number for which there is an accepted name is the centillion, first recorded in 1852. It is equal to 10^{303} .

What special name for the number 10^{100} ?

Worksheet

Scientific notation 1

MAT09NAWK00012

Worksheet

Scientific notation 2

MAT09NAWK00019

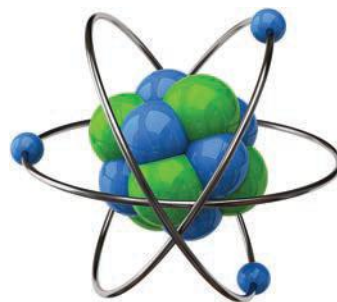
Puzzle sheet

Scientific notation
puzzle

MAT09NAPS10060

5-10 Scientific notation

Scientific notation is a short way of writing very large or very small numbers using powers of 10. It was invented in the early twentieth century when scientists needed to describe very large values, such as astronomical distances and very small values such as the masses of atoms.



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Summary

Numbers written in **scientific notation** are expressed in the form

$$m \times 10^n$$

where m is a number between 1 and 10 and n is an integer.

Example 19

Express each number in scientific notation.

- a 764 000 000 000 b 6000 c 0.0008 d 0.000 000 472

Solution

- a Use the significant figures in the number to write a value between 1 and 10: 7.64

Count how many places the decimal point moves to the right to make 764 000 000 000.
11 places

764 000 000 000

or count the number of
places after the first significant
figure, 7

$$\therefore 764\,000\,000\,000 = 7.64 \times 10^{11}$$

- b Use the significant figures in the number to write a value between 1 and 10: 6

Count how many places the decimal point moves to the right to make 6000.
3 places

6000

or count the number of
places after the first significant
figure, 6

$$\therefore 6000 = 6 \times 10^3$$

- c Use the significant figures in the number to write a value between 1 and 10: 8

Count how many places the decimal point moves to the left to make 0.0008.
4 places

0.0008

or count the number of
decimal places to the first
significant figure, 8

$$\therefore 0.0008 = 8 \times 10^{-4}$$

Note that small numbers are written with negative powers of 10.

- d Use the significant figures in the number to write a value between 1 and 10: 4.72

Count the number of places the decimal point moves to the left to make 0.000 000 472.
7 places

0.000 000 472

or count the number of
decimal places to the first
significant figure, 4

$$0.000\,000\,472 = 4.72 \times 10^{-7}$$

Example 20

Express each number in decimal form.

a 2.7×10^4

b 3.56×10^{-2}

Solution

a $2.7 \times 10^4 = 2.7000$
 $= 27000$

Move the decimal point 4 places to the right.

b $3.56 \times 10^{-2} = 0.0356$
 $= 0.0356$

Move the decimal point 2 places to the left.

Example 21

a Which number is the larger: 3.65×10^{12} or 8.1×10^{12} ?

b Write these numbers in ascending order: 4.3×10^6 , 2.8×10^7 , 1.9×10^7

Solution

To compare numbers in scientific notation, first compare the powers of ten. If the powers of ten are the same, then compare the decimal parts.

a The powers of ten are the same. Compare the decimal parts: $8.1 > 3.65$.
 $\therefore$ The larger number is 8.1×10^{12}

b Compare the powers of ten: $10^6 < 10^7$.
Then compare the two numbers with 10^7 : $1.9 < 2.8$.
 $\therefore$ The numbers in ascending order are 4.3×10^6 , 1.9×10^7 , 2.8×10^7 .

Exercise 5-10 Scientific notation

See Example 19 1 Express each number in scientific notation.

a 2400

b 786 000

c 55 000 000

d 95

e 7.8

f 348 000 000

g 59 670

h 15

i 3 000 000 000

j 80

k 763

l 10

m 0.035

n 0.000 076

o 0.8

p 0.0713

q 0.000 003

r 0.913

s 0.000 007 146

t 0.009

u 0.000 000 1

v 0.000 89

w 0.000 000 078

x 0.1

2 Express each measurement in scientific notation.

a The world's largest mammal is the blue whale, which can weigh up to 130 000 kg.

b The diameter of an oxygen molecule is 0.000 000 29 cm.

c The thickness of a human hair is 0.000 08 m.

d Light travels at a speed of 300 000 000 m/s.



Corbis © Denis Scott

- e The nearest star to Earth, excluding the Sun, is Alpha Centauri, which is 40 000 000 000 000 km away.
- f The thickness of a typical piece of paper is 0.000 12 m.
- g The small intestine of an adult is approximately 610 cm long.
- h The diameter of a hydrogen atom is 0.000 000 0001 m.
- i The diameter of our galaxy, the Milky Way, is 770 000 000 000 000 000 000 m.
- j A microsecond means 0.000 001 s.
- k The Andromeda Galaxy is the most remote body visible to the naked eye, at a distance of 2 200 000 light years away.



Science Photo Library/Robert Gendler

3 Express each number in decimal form.

a 6×10^5	b 7.1×10^3	c 3.02×10^8
d 3.14×10^0	e 6×10^{-5}	f 7.1×10^{-3}
g 3.02×10^{-8}	h 5.9×10^{-10}	i 1.1×10^{12}
j 4×10^{-4}	k 5×10^3	l 4.76×10^{-4}
m 8.03×10^{-1}	n 6.32×10^4	o 1.6×10^{-2}
p 2.2×10^{-7}	q 9.0×10^6	r 1.11×10^{-1}

See Example 20

4 For each pair of numbers, write the larger one.

a 3×10^5 or 4×10^5	b 8.4×10^5 or 2.7×10^6
c 8.4×10^0 or 1.3×10^7	d 3.6×10^{-7} or 6.3×10^{-7}
e 9.3×10^9 or 7.6×10^9	f 3.5×10^{-6} or 9.3×10^2
g 3.04×10^0 or 3.04×10^{-4}	h 4.5×10^{-5} or 3.7×10^{-7}
i 2×10^{-15} or 2×10^{-17}	j 6.23×10^{-5} or 9.7×10^{-5}

See Example 21

- 5 Write each set of numbers in ascending order.
- a 3.8×10^9 , 7.3×10^9 , 5.5×10^9
- b 2.2×10^{-4} , 5.8×10^{-6} , 7×10^{-4}
- c 3.5×10^0 , 5.3×10^2 , 4.9×10^2
- 6 Write each set of numbers in descending order.
- a 6×10^5 , 2.9×10^2 , 1×10^2
- b 1.2×10^{-9} , 6.3×10^2 , 8.1×10^{-4}
- c 4.1×10^{-1} , 9.5×10^{-1} , 6.4×10^{-3}

Worksheet

Scientific notation problems

MAT09NAWK10061

Homework sheet

Indices 3

MAT09NAHS10007

Homework sheet

Indices revision

MAT09NAHS10008

Puzzle sheet

Scientific notation: accomplishing great things

MAT09NAPS00005

5-11 Scientific notation on a calculator

To enter a number in scientific notation on a calculator, use the $\times 10^x$ or **EXP** key.

Example 22

Evaluate each expression using scientific notation.

a $(4.25 \times 10^7) \times (8.2 \times 10^6)$ b $(1.08 \times 10^{-15}) \div (3 \times 10^{11})$ c $(4.9 \times 10^7)^2$

Solution

a Enter 4.25 $\times 10^x$ 7 $\times$ 8.2 $\times 10^x$ 6 $=$
 $(4.25 \times 10^7) \times (8.2 \times 10^6) = 3.485 \times 10^{14}$

b Enter 1.08 $\times 10^x$ - 15 $\div$ 3 $\times 10^x$ 11 $=$
 $(1.08 \times 10^{-15}) \div (3 \times 10^{11}) = 3.6 \times 10^{-27}$

c Enter 4.9 $\times 10^x$ 7 x^2 $=$
 $(4.9 \times 10^7)^2 = 2.401 \times 10^{15}$

Note that with scientific notation on a calculator, there is no need to enter brackets $()$ around the numbers.

Example 23

Estimate the value of each expression in scientific notation, then evaluate it correct to three significant figures.

a $\frac{9.2 \times 10^9}{2.7 \times 10^5}$ b $(8.5 \times 10^4) \times (6.3 \times 10^7)$ c $(6.08 \times 10^3)^2$

Solution

Estimate

a $\frac{9.2 \times 10^9}{2.7 \times 10^5} \approx \frac{9 \times 10^9}{3 \times 10^5}$
 $= \frac{9}{3} \times \frac{10^9}{10^5}$
 $= 3 \times 10^4$

Calculated answer

$\frac{9.2 \times 10^9}{2.7 \times 10^5} = 34\,074.074\,07$
 $\approx 34\,000$
 $= 3.4 \times 10^4$

Estimate

$$\begin{aligned} \text{b } (8.5 \times 10^4) \times (6.3 \times 10^7) \\ \approx (9 \times 10^4) \times (6 \times 10^7) \\ = (9 \times 6) \times (10^4 \times 10^7) \\ = 54 \times 10^{11} \\ = 5.4 \times 10 \times 10^{11} \\ = 5.4 \times 10^{12} \end{aligned}$$

$$\begin{aligned} \text{c } (6.08 \times 10^5)^3 &\approx (6 \times 10^5)^3 \\ &= 6^3 \times (10^5)^3 \\ &= 216 \times 10^{15} \\ &= 2.16 \times 10^2 \times 10^{15} \\ &= 2.16 \times 10^{17} \end{aligned}$$

Calculated answer

$$\begin{aligned} (8.5 \times 10^4) \times (6.3 \times 10^7) &= 5.355 \times 10^{12} \\ &\approx 5.36 \times 10^{12} \end{aligned}$$

$$\begin{aligned} (6.08 \times 10^5)^3 &= 2.24755 \dots \times 10^{17} \\ &\approx 2.25 \times 10^{17} \end{aligned}$$

Exercise 5-11 Scientific notation on a calculator

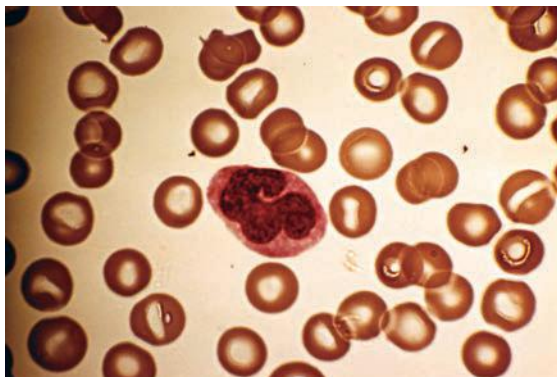
1 Evaluate each expression using scientific notation.

a $(2 \times 10^3) \times (3 \times 10^5)$	b $(8 \times 10^7) \div (4 \times 10^2)$
c $(2 \times 10^5)^3$	d $\sqrt{9 \times 10^{12}}$
e $(4 \times 10^7) \times (6 \times 10^8)$	f $(1 \times 10^8) \div (2 \times 10^3)$
g $(4 \times 10^3)^5$	h $24.08 \div (8 \times 10^6)$
i $\sqrt{3.969 \times 10^{19}}$	j $(2 \times 10^5)^{-2}$
k $\sqrt[3]{8 \times 10^{-9}}$	l $\frac{7.62 \times 10^9}{2 \times 10^{-4}}$

2 Estimate the value of each expression in scientific notation, then evaluate correct to three significant figures.

a $(5.7 \times 10^3) \times (2.3 \times 10^5)$	b $(8 \times 10^5) \times (3.7 \times 10^7)$
c $(9.1 \times 10^{20}) \div (3.2 \times 10^5)$	d $(1.2 \times 10^8)^2$
e $(7.13 \times 10^{10}) \times (9.8 \times 10^8)$	f $(1.9 \times 10^{11}) \div (2.1 \times 10^7)$
g $(5.85 \times 10^4)^3$	h $(6 \times 10^{12}) \div (2.8 \times 10^3)$

3 The human body consists of approximately 6×10^9 cells, and each cell consists of 6.3×10^9 atoms. Roughly how many atoms are there in a human body?



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See Example 22

See Example 23

Worked solutions

Scientific notation
on a calculator

MAT09NAWS10028

- 4 A telephone book 4.5 cm thick has 2000 pages. Find the thickness of one page, in *millimetres*, in scientific notation.
- 5 Evaluate each expression in scientific notation, correct to two significant figures.
- a $(7.4 \times 10^{30}) - (3.59 \times 10^{29})$ b $(1.076 \times 10^{17}) + (2.3 \times 10^{16})$ c $\sqrt{6.6 \times 10^{27}}$
- d $(7.5 \times 10^{23}) \div (3.3 \times 10^{-13})$ e $(8.17 \times 10^{16})^3$ f $\sqrt{2.69 \times 10^{45}}$
- g $(7.05 \times 10^3) \div (3.9 \times 10^7)$ h $\frac{(5.6 \times 10^4) \times (3.9 \times 10^5)}{(2.3 \times 10^7)}$ i 1595×1959
- j 5^{20} k 801^{-1} l 3^{-10}
- m 9^9 n $(0.7)^{-5}$

Express the answers for questions 6 to 10 in scientific notation correct to two significant figures if necessary.

Worked solutions

Scientific notation
on a calculator

MAT09NAWS10028

- 6 The Earth is 1.50×10^8 km from the Sun and the speed of light is 3×10^5 km/s. How long does it take for light to travel from the Sun to Earth? Express your answer in:
- a seconds b minutes.
- 7 The Sun burns 6 million tonnes of hydrogen a second. Calculate how many tonnes of hydrogen it burns in a year (that is, 365.25 days).
- 8 Sound travels at approximately 330 metres per second. If Mach 1 is the speed of sound, how fast is Mach 5? Convert your answer to kilometres per second.
- 9 The distance light travels in one year is called a light year. If the speed of light is approximately 3×10^5 km per second, how far does light travel in a leap year?
- 10 A thunderstorm is occurring 30 km from where you are standing. Use the speed of light (3×10^5 km per second) and the speed of sound (330 metres per second) to calculate in seconds:
- a how long the light from the lightning takes to reach you
- b how long the sound from the thunder takes to reach you.



- 11 a What is the largest number that can be displayed on your calculator?
- b What is the smallest number that can be displayed?

Investigation: A lifetime of heartbeats

How many times does your heart beat in an average lifetime of 80 years?

1 Work in pairs and copy this table.

Name	Trial 1	Trial 2	Average beats per minute

2 Use two fingers to measure your pulse. Have your partner time you for a minute. Do this twice, record your results in the table and find the average.

3 Repeat Step 2 for your partner.

4 Calculate how many times your heart (and your partner's heart) beats in the following periods. Write your answers in scientific notation correct to two significant figures.

- a** an hour **b** a day **c** a week
d a year (use 365.25 days) **e** an average lifetime of 80 years

Just for the record

Hairy numbers



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Straight hair
• round follicle



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Wavy hair
• oval follicle



Shutterstock.com/Jason Stitt

Curly hair
■ flat follicle

There are about 110 000 hairs on your head. Each hair grows at the rate of about 1.3×10^{-3} cm per hour. A single hair lasts about six years. Every day you lose between 30 and 60 hairs. Each hair grows from a small depression in the skin called a follicle (a gland). After the hair falls out, the follicle rests for about three to four months before the next hair starts growing. Hair follicles are either oval, flat or round in shape. How straight, wavy or curly your hair is depends on the shape of your hair follicles.

How many hairs are on all the heads in China if its population is approximately 1.435×10^9 ? Answer in both scientific notation and decimal notation.

Power plus

Worksheet

Binary number system

MAT09NAWK10058

1 Write each number in scientific notation.

a 438.2×10^9

b 0.52×10^{-7}

c 0.0004×10^{12}

d 2013×10^{-3}

e 57.8 thousand

f 57.8 thousandths

g 6.7 millionths

h 3.2 billion

i 3.2 billionths

2 Evaluate each expression.

a $\sqrt{\sqrt{81}}$

b $\sqrt{\sqrt{625}}$

c $\sqrt{\sqrt{\sqrt{256}}}$

d $\sqrt{\sqrt[5]{1024}}$

e $\sqrt{\sqrt{\sqrt{6561}}}$

f $\sqrt[4]{\sqrt{256}}$

g $\sqrt{\sqrt[3]{1\,000\,000}}$

h $\sqrt[3]{\sqrt{1\,000\,000}}$

3 Find values of a , m and n so that each equation is true.

a $a^{\frac{m}{n}} = 2$

b $a^{\frac{m}{n}} = 3$

c $a^{\frac{m}{n}} = 64$

d $a^{\frac{m}{n}} = 125$

4 For how many values of a and b does $a^b = b^a$?

5 The terms in the pattern 3, 5, 17, 257, 65 537, ... can all be generated by a simple method, using only the numbers 1 and 2.

a What is this method?

b What is the next number in the sequence?

Language of maths

ascending	base	descending	estimate
expanded form	exponent	fractional power	index
index laws	index notation	indices	negative power
power	product	quotient	reciprocal
scientific notation	significant figures	term	zero power

Puzzle sheet

Indices crossword

MAT09NAPS10062

Quiz

Numbers and indices

MAT09NAQZ00002

- 1 What does a power of $\frac{1}{2}$ mean?
- 2 Which two words from the list mean 'power'?
- 3 What is the $\times 10^x$ or **EXP** key on a calculator used for?
- 4 What is the index law for dividing terms with the same base?
- 5 Which digits in 0.006 701 are **significant figures**?
- 6 What power is associated with the **reciprocal** of a term or number?
- 7 What type of numbers when written in **scientific notation** have **negative** powers of 10?

Topic overview

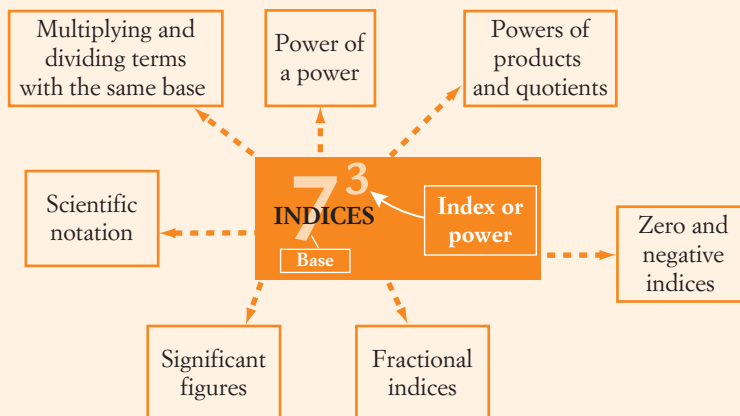
- What was this topic about? What was the main theme?
- What content was new and what was revision?
- What are the index laws?
- Write 10 questions (with solutions) that could be used in a test for this chapter.
- Include some questions that you have found difficult to answer.
- List the sections of work in this chapter that you did not understand. Follow up this work with a friend or your teacher.

Worksheet

Mind map: Indices
(Advanced)

MAT09NAWK10064

Copy (or print) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Chapter 5 revision

See Exercise 5-01

1 Simplify each expression, writing the answer in index notation.

a $10^3 \times 10^7$

b $4^{20} \div 4^4$

c $a^{12} \div a^2$

d $h^8 \times h^2$

e $3n^3 \times 4n$

f $10d^{15} \div 5d^3$

g $20m^9 \div 4m$

h $3v^4w^2 \times 2v^3w^5$

i $5x^5y^2 \times 3xy$

j $\frac{24t^8h^8}{3t^4h^2}$

k $\frac{p^6q^{10}}{p^2q^2}$

l $\frac{100a^2b^4}{5ab^2}$

See Exercise 5-02

2 Simplify each expression, writing the answer in index notation.

a $(2^2)^3$

b $(k^5)^5$

c $(x)^4$

d $(2y^3)^{10}$

e $(5t^2)^2$

f $(10g)^3$

g $(-2)^5$

h $(-2k)^5$

i $(-5m^3)^2$

See Exercise 5-03

3 Simplify each expression.

a $(ab^2)^4$

b $(5x^3y^2)^2$

c $(4t^2)^3$

d $(-4h^2g)^3$

e $\left(\frac{a}{7}\right)^4$

f $(2pqr)^5$

g $\left(\frac{3m}{2}\right)^5$

h $(-3np^2)^4$

i $\left(\frac{2a^7}{b}\right)^4$

j $(4t^4u^5)^3 \times 8t^2u$

k $\left(\frac{b^8y^6}{8b^2y}\right)^3$

l $45c^6d^8 \div (3cd^2)^2$

See Exercise 5-04

4 Simplify each expression.

a 7^0

b $(-7)^0$

c e^0

d $(-e)^0$

e $-e^0$

f g^0h

g $(gh)^0$

h $\left(\frac{2p}{3}\right)^0$

i $\frac{2p^0}{3}$

See Exercise 5-05

5 Simplify each expression using a positive index.

a 8^{-3}

b 19^{-2}

c x^{-1}

d p^{-5}

e $(4m)^{-1}$

f $(4m)^{-2}$

g $(5b)^{-1}$

h $5b^{-1}$

i $-2x^{-4}$

j $\left(\frac{3}{5a}\right)^{-1}$

k $c^{-4}d^2$

l $\left(\frac{100}{9}\right)^{-1}$

See Exercise 5-05

6 Write each expression using a negative index.

a $\frac{1}{10^3}$

b $\frac{1}{r^5}$

c $\frac{1}{r}$

d $\frac{3}{b}$

Stage 5.3

See Exercise 5-05

7 Simplify $\left(-\frac{8}{3x}\right)^{-2}$ using a positive index.

See Exercise 5-06

8 Write each expression using a radical (root) sign.

a $q^{\frac{1}{3}}$

b $u^{\frac{1}{2}}$

c $(2q)^{\frac{1}{3}}$

d $(ar)^{\frac{1}{2}}$

See Exercise 5-07

9 Evaluate each expression.

a $64^{\frac{2}{3}}$

b $(-32)^{\frac{3}{5}}$

c $36^{-\frac{3}{2}}$

See Exercise 5-07

10 Simplify each expression.

a $(125d^{15})^{\frac{4}{3}}$

b $(16y^{20})^{\frac{1}{4}}$

c $(32x^8)^{\frac{2}{5}}$

d $\sqrt[3]{-64n^{12}}$

See Exercise 5-09

11 Round each value correct to the number of significant figures shown in the brackets.

a 8.5678 (2)

b 15 712 (3)

c 476 (1)

d 0.007 126 6 (4)

e 0.9041 (3)

f 301 378 (2)

g 4805.28 (3)

h 0.000 87 (1)

i 67 000 000 (1)

12 Express each number in scientific notation.

See Exercise 5-10

a 37 000

b 0.61

c 250 000

d 0.000 49

e 13

f 0.000 000 000 08

13 Express each number in decimal form.

See Exercise 5-10

a 8.1×10^3

b 6×10^7

c 3.075×10^0

d 8.1×10^{-3}

e 6×10^{-7}

f 3.075×10^{-2}

14 Write these numbers in ascending order: 3×10^3 , 9.1×10^{-8} , 2.4×10^3 .

See Exercise 5-10

15 Evaluate each expression using scientific notation.

See Exercise 5-11

a $(3.65 \times 10^{22}) \times (7.4 \times 10^8)$

b $(1.44 \times 10^{10}) \div (3.6 \times 10^4)$

c $(5 \times 10^5)^3$

d $\sqrt{6.25 \times 10^{-8}}$

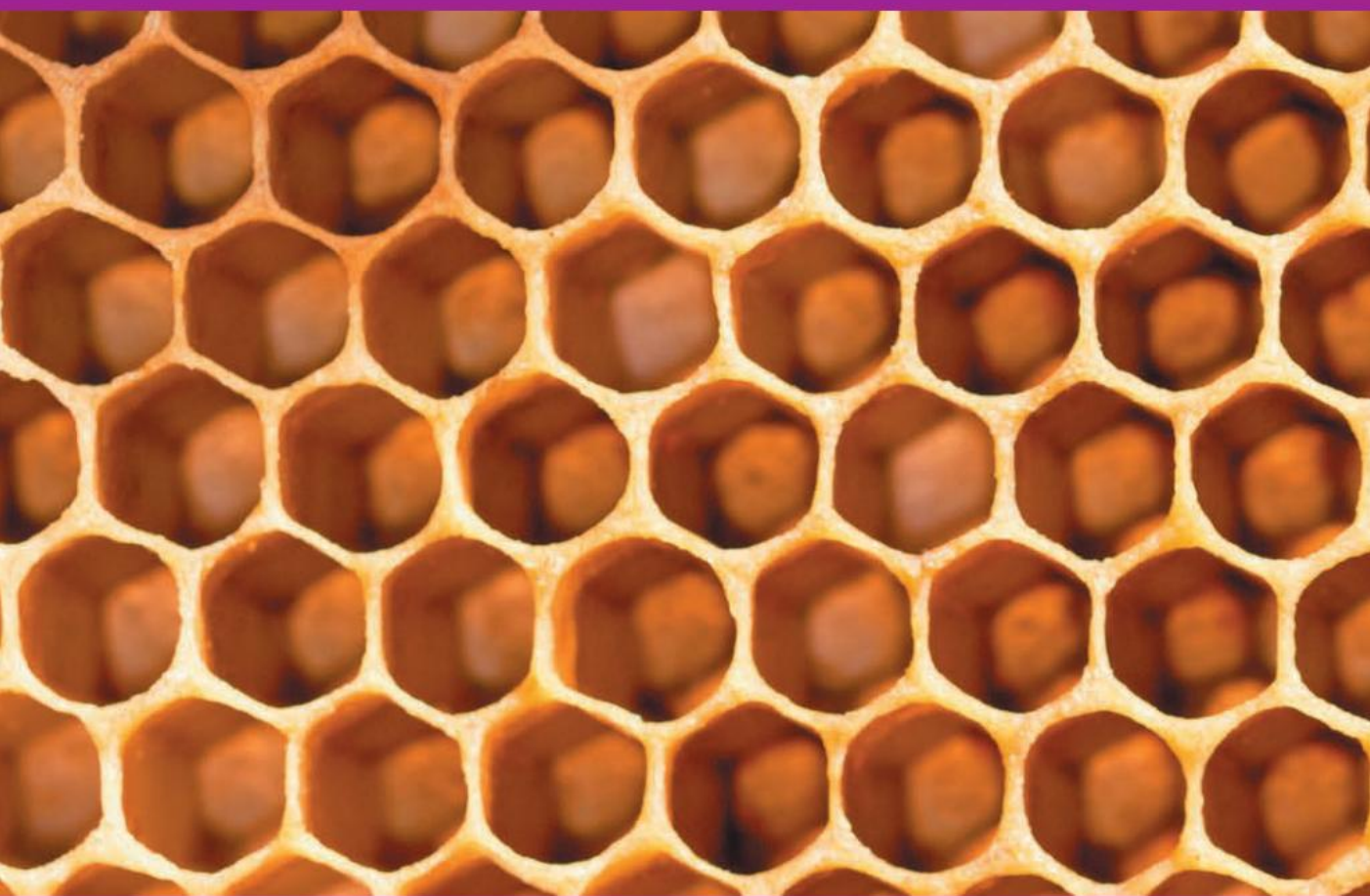
16 Estimate the value of each expression in scientific notation, then evaluate correct to two significant figures.

See Exercise 5-09

a $(8.9 \times 10^9) \times (1.1 \times 10^7)$

b $(9.3 \times 10^{15}) \times (4 \times 10^2)$

c $(3.1 \times 10^4)^2$



6

Measurement and geometry

Geometry

'Geometry' comes from the Greek word *geometria*, which means 'land measuring'. The principles and ideas of geometry are evident everywhere – in road signs, buildings, bridges and patterns for tiles and wallpaper. Many examples of geometric patterns can be seen in nature, such as the hexagonal cells on a honeycomb built by bees to store honey.



Shutterstock.com/Nikolay Dimitrov - ecobo

Chapter outline

	Proficiency strands			
6-01 Triangle geometry	U	F	R	C
6-02 Quadrilateral geometry	U	F	R	C
6-03 Angle sum of a polygon	U	F	R	C
6-04 Exterior angle sum of a convex polygon	U	F	R	C

Wordbank

angle sum The total of the sizes of the angles in a shape, such as a triangle

bisect To cut in half

convex polygon A polygon whose vertices all point outwards

diagonal An interval joining two non-adjacent vertices of a shape

exterior angle of a triangle An 'outside' angle of a triangle formed after extending one of the sides of the triangle

polygon A plane shape with straight sides

regular polygon A polygon with all angles equal and all sides equal, such as a square

In this chapter you will:

- classify triangles and quadrilaterals according to their side and angle properties, and solve related numerical problems using reasoning 
- solve problems involving the angle sum of a triangle and quadrilateral and the exterior angle of a triangle
- solve problems involving the angle sum of a polygon and the exterior angle sum of a convex polygon

SkillCheck

Worksheet

StartUp assignment 6

MAT09MGWK10065

Skillsheet

Types of angles

MAT09MGSS10018

Puzzle sheet

Angles

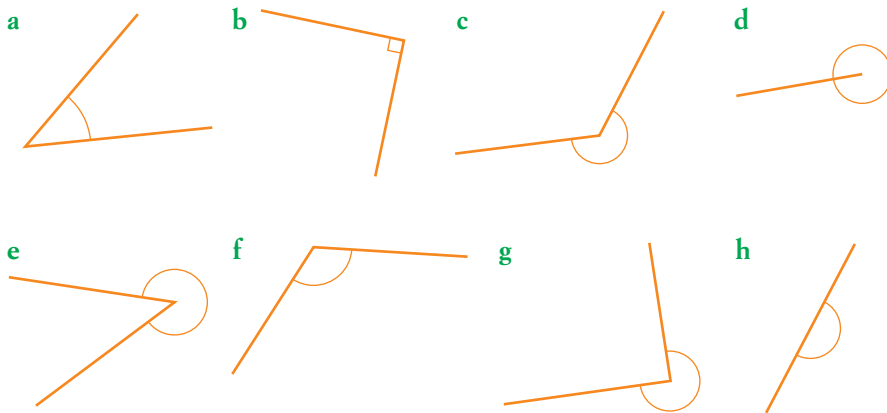
MAT09MGPS00050

Skillsheet

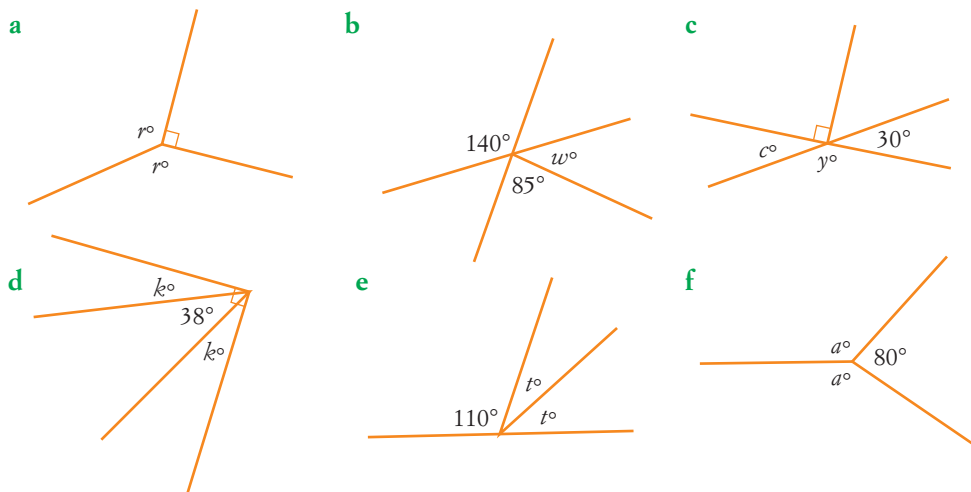
Angles and
parallel lines

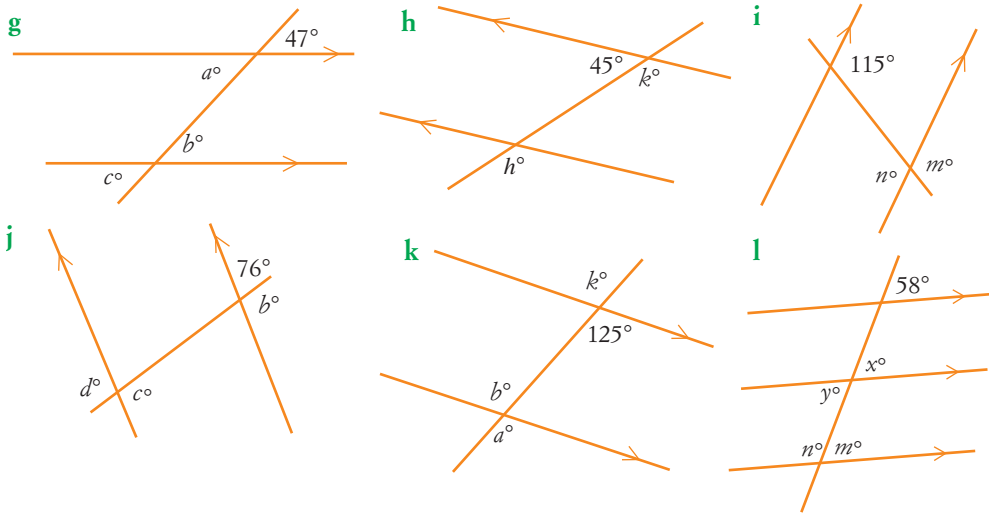
MAT09MGSS10021

1 Classify each angle as acute, obtuse, reflex, right, straight or a revolution.

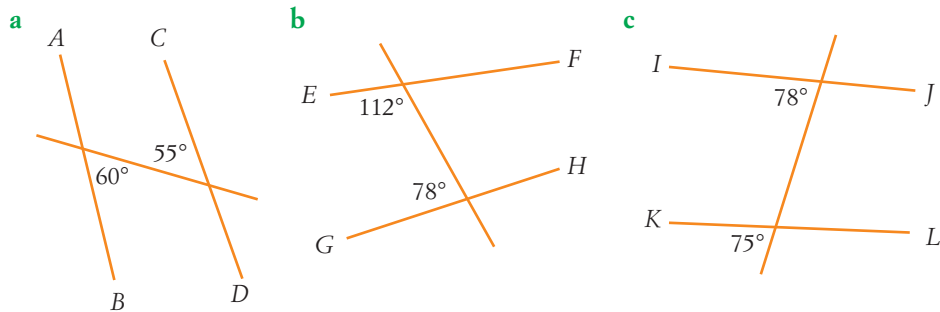


2 Find the value of each pronumeral, giving reasons.





3 Test whether each labelled pair of lines are parallel. Give reasons for each answer.



6-01 Triangle geometry

Homework sheet

Geometry 1

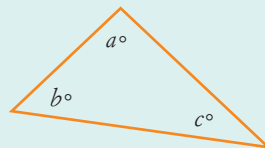
MAT09MGHS10023

Angle sum of a triangle

Summary

The **angle sum of a triangle** is 180° .

$$a + b + c = 180$$



Proof:

Consider any triangle PQR with angles a° , b° and c° .
Construct a line through P parallel to QR .

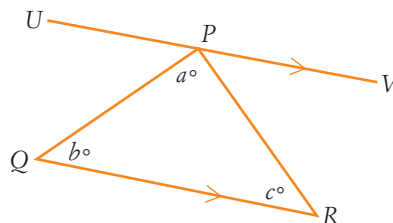
$\therefore \angle UPQ = b^\circ$ (alternate angles, $UV \parallel QR$)

and $\angle VPR = c^\circ$ (alternate angles, $UV \parallel QR$)

But $\angle UPQ + \angle QPR + \angle VPR = 180^\circ$ (angles on a straight line)

$$\therefore a^\circ + b^\circ + c^\circ = 180^\circ$$

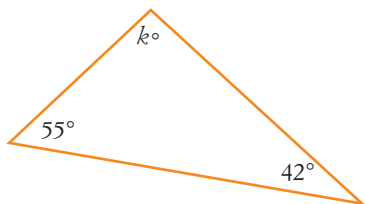
$\therefore$ The sum of the angles of a triangle is 180° .



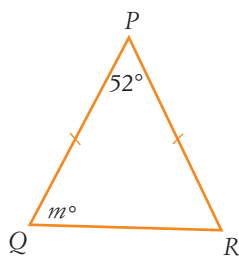
Example 1

Find the value of each pronumeral, giving reasons.

a



b



Solution

a $k + 55 + 42 = 180$ (angle sum of a triangle)

$$k = 180 - 55 - 42$$

$$= 83$$

b $\angle R = \angle Q = m^\circ$ ($\triangle PQR$ is isosceles)

$$m + m + 52 = 180 \quad (\text{angle sum of a triangle})$$

$$2m + 52 = 180$$

$$2m = 128$$

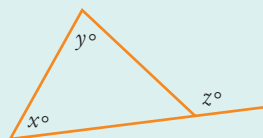
$$m = 64$$

The exterior angle of a triangle

Summary

The **exterior angle of a triangle** is equal to the sum of the two interior opposite angles.

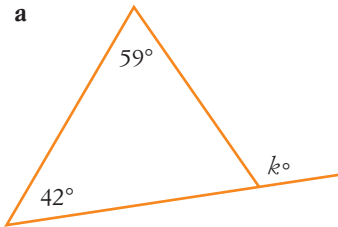
$$z = x + y$$



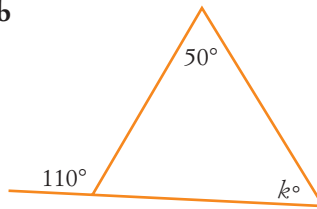
Example 2

Find the value of k in each diagram.

a



b



Solution

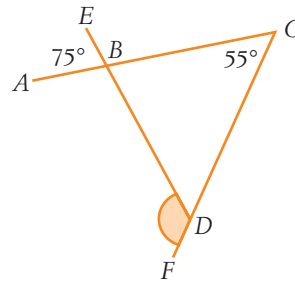
a $k = 42 + 59$ (exterior angle of triangle) $\mathbf{b}$ $k + 50 = 110$ (exterior angle of triangle)
 $= 101$ $k = 60$

Example 3

Find the size of the marked angle $\angle BDF$.

Solution

$\angle DBC = \angle ABE = 75^\circ$ (vertically opposite angles)
 $\angle BDF = 75^\circ + 55^\circ$ (exterior angle of $\triangle BCD$)
 $= 130^\circ$

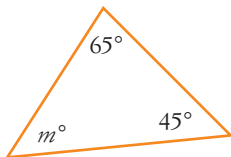


Exercise 6-01 Triangle geometry

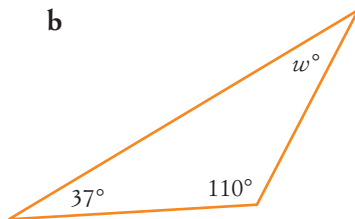
1 Find the value of each pronumeral.

See Example 1

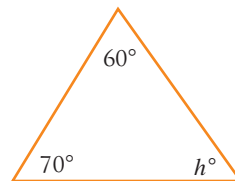
a



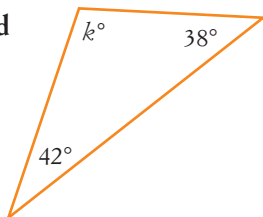
b



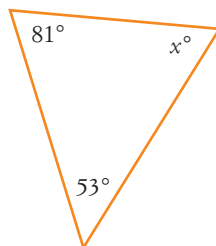
c



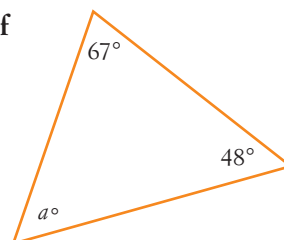
d



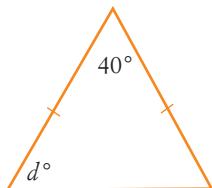
e



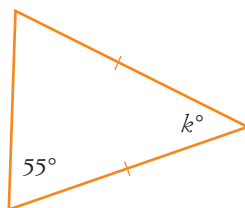
f



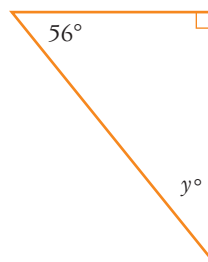
g



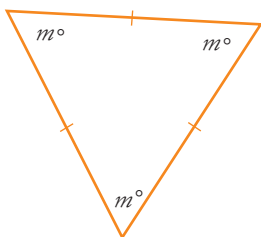
h



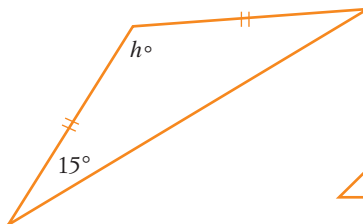
i



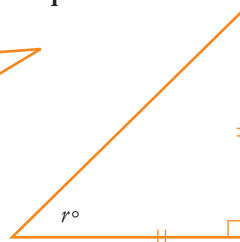
j



k



l



2 What is the size of each angle in an equilateral triangle? Select **A**, **B**, **C** or **D**.

A 30°

B 45°

C 60°

D 90°

3 One angle of an isosceles triangle is equal to 80°. Which *two* of the following could be the sizes of the other angles? Select *two* of **A**, **B**, **C** or **D**.

A 80° and 20°

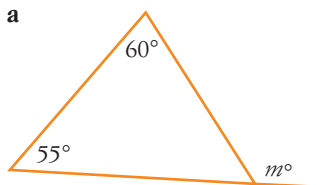
B 40° and 60°

C 40° and 40°

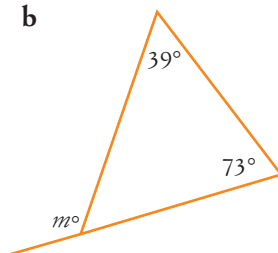
D 50° and 50°

4 Find the value of m .

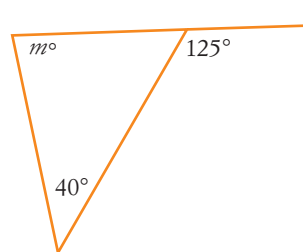
a



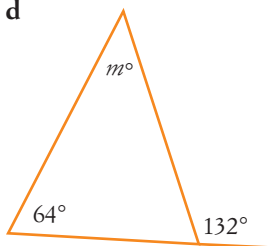
b



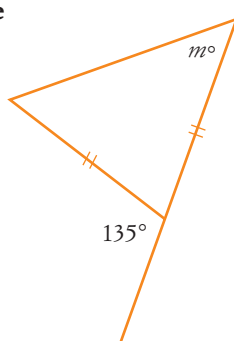
c



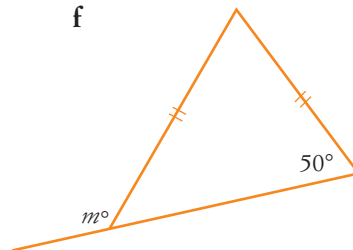
d



e



f



Worked solutions

Triangle geometry

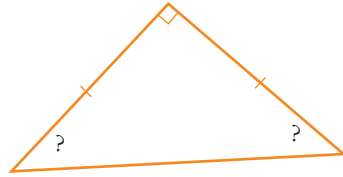
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See Example 2

- 5 What is the size of the unknown angles in this triangle?
Select **A**, **B**, **C** or **D**.

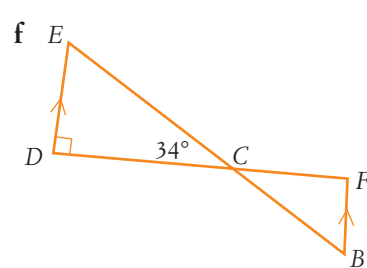
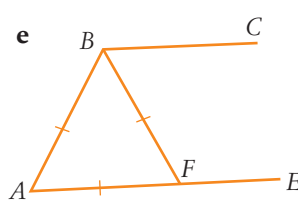
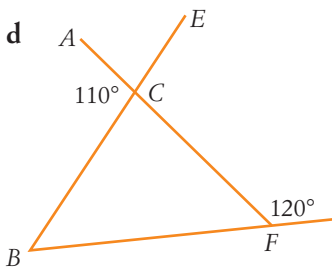
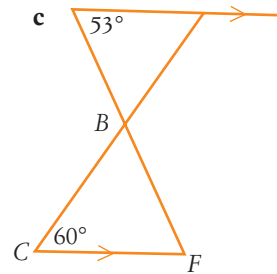
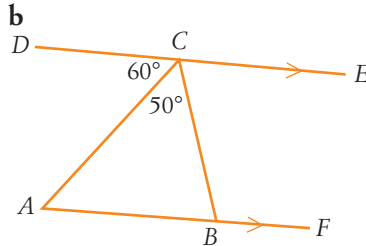
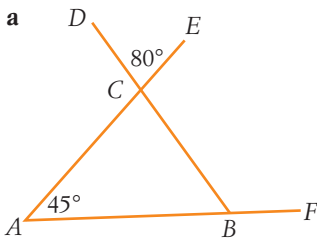
A 30°
C 60°

B 45°
D 90°

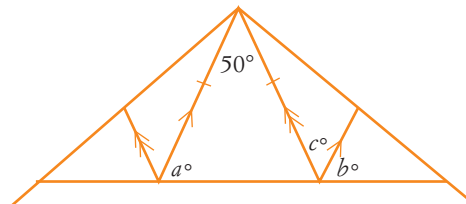


- 6 One angle of an obtuse-angled triangle is 50° . Which of the following could be the sizes of the other angles? Select **A**, **B**, **C** or **D**.
A 80° and 50° **B** 100° and 30° **C** 65° and 65° **D** 60° and 70°
- 7 Find the size of $\angle CBF$ in each diagram.

See Example 3



- 8 An isosceles triangle has a side length of 6 cm and one of its angles equal to 40° . Draw all possible shapes of the triangle.
- 9 The diagram shows the shape of a roof truss. Find the value of each pronumeral.

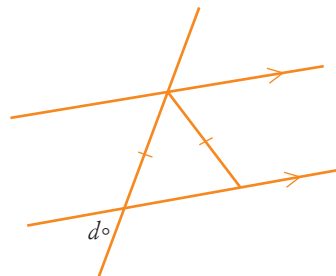


Worked solutions

Triangle geometry

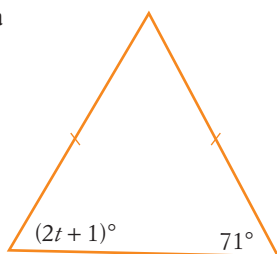
MAT09MGWS10029

- 10 Copy this diagram and mark all angles equal to the angle marked d° .

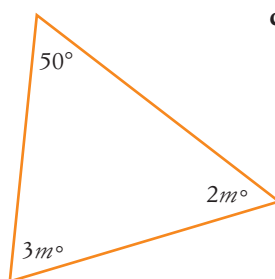


11 Find the value of each pronumeral.

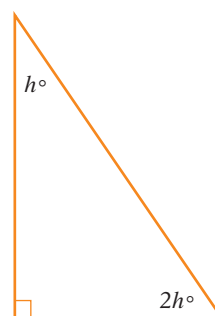
a



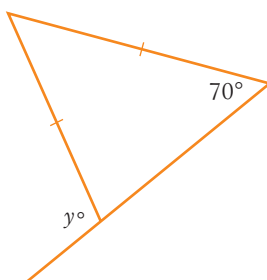
b



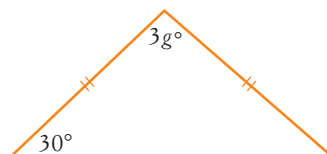
c



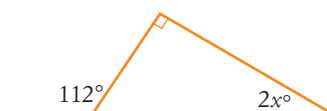
d



e



f



Technology Sketching parallelograms and rectangles

Skillsheet

Starting GeoGebra

MAT09MGSS10019

Skillsheet

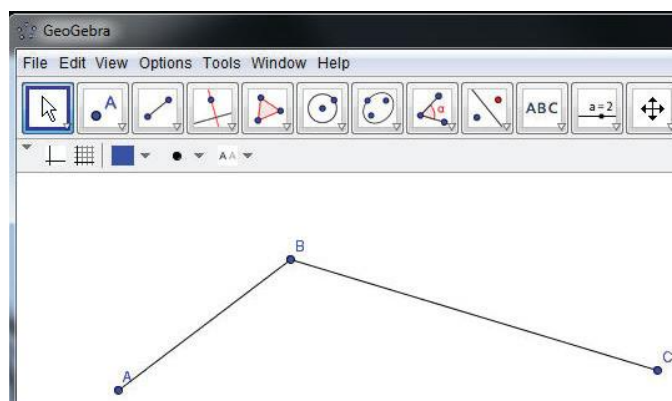
Starting Geometer's Sketchpad

MAT09MGSS10020

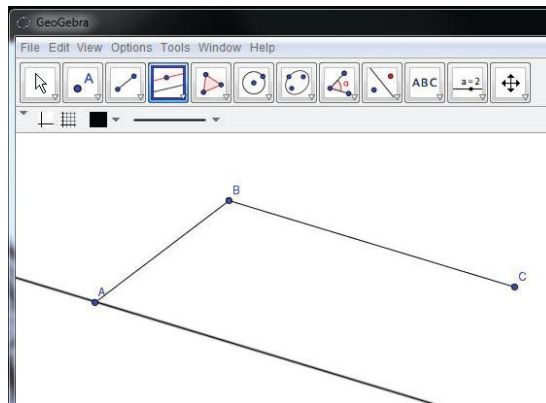
In this activity you will use GeoGebra to construct two quadrilaterals.

Constructing a parallelogram

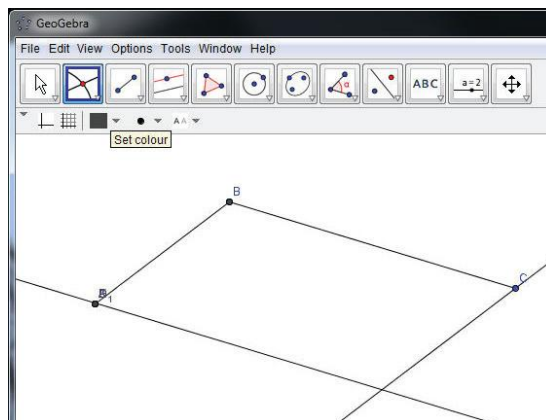
- 1 To construct a parallelogram, use **Interval between two points** and construct two sides of any length (as shown below).
- 2 Label the points (right-click on each point and **Show label**).



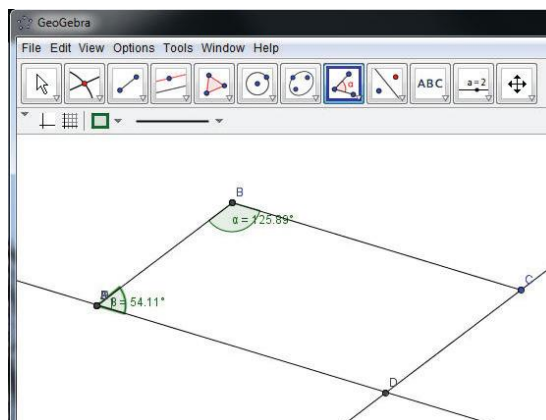
- 3 Select **Parallel line** from the fourth drop-down icon menu and choose side BC and point A.



- 4 Repeat step 2 with side AB and point C. Now select **Intersect Two Objects** from the second drop-down icon menu and the lines through points A and C. Label the point D.

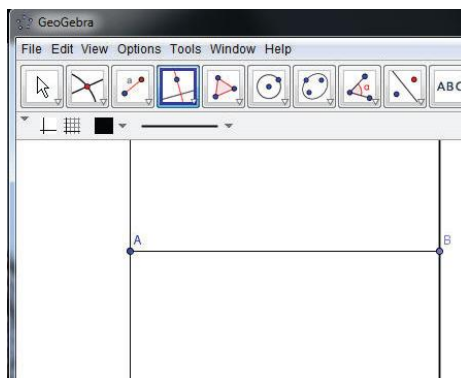


- 5 Use **Angle** to measure the size of $\angle DAB$ and $\angle ABC$. Manipulate the vertices to change the size of the parallelogram.

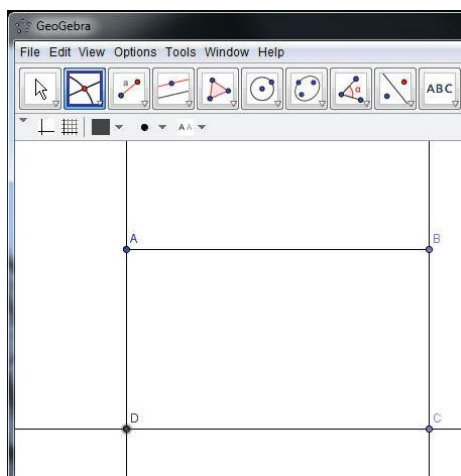


Constructing a rectangle.

- 1 Make sure the axes and grid are removed by right-clicking and disabling them.
- 2 Click  Interval with Fixed Length
Enter a point and 6 (cm). Label the interval endpoints A and B.
- 3 Click  Perpendicular Line
Select point A and 6 cm interval. Repeat for point B.



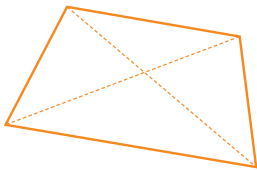
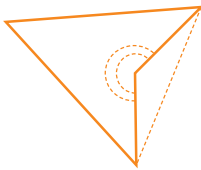
- 4 Click **New point** to insert a point anywhere on the line under point B. Label it point C.
- 5 Construct a perpendicular line through C.
- 6 Click **Intersect two objects** and create a point of intersection at the perpendicular through A and the perpendicular through C. Label the point of intersection, D.



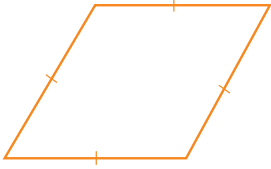
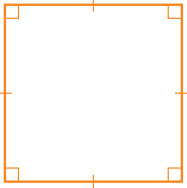
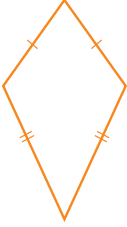
- 7 Select the **Move** tool  and click point C. Manipulate the line through CD.
- 8 Click **Distance or length** and choose points B and C. Use the **Move** tool to manipulate point C until $BC = 3.6$ cm. Click **Distance or length** and points A and B to show the length of AB.
- 9 Now draw at least two other types of quadrilaterals.

6-02 Quadrilateral geometry

A **quadrilateral** is any shape with four straight sides. A quadrilateral may be either **convex** or **non-convex (concave)**.

Convex quadrilateral	Non-convex quadrilateral
	
- All vertices (corners) point outwards. - All diagonals lie within the shape. - All angles are less than 180°.	- One vertex points inwards. - One diagonal lies outside the shape. - One angle is more than 180° (reflex angle).

There are six special types of quadrilaterals.

Trapezium	Parallelogram	Rectangle
		
One pair of parallel sides	Two pairs of parallel sides	Four right angles
Rhombus	Square	Kite
		
Four equal sides	Four equal sides and four right angles	Two pairs of equal adjacent sides 'Adjacent' means 'next to each other'.

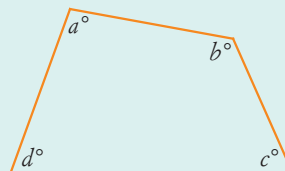
Angle sum of a quadrilateral

Summary

The **angle sum of a quadrilateral** is 360° .

$$a + b + c + d = 360$$

This property is true for both convex and non-convex quadrilaterals.



Worksheet

Classifying quadrilaterals

MAT09MGWS00066

Worksheet

Properties of triangles and quadrilaterals

MAT09MGWS10066

Worksheet

Naming quadrilaterals

MAT09MGWS10067

Worksheet

Deductive geometry

MAT09MGWS10068

Skillsheet

Naming shapes

MAT09MGSS10022

Homework sheet

Geometry 2

MAT09MGHS10024

Technology

GeoGebra: Angle sum of a quadrilateral

MAT09MGTC00010

Animated example

Angles and shapes

MAT09MGAE00011

Worksheet

Diagonal properties of quadrilaterals

MAT09MGWS00067

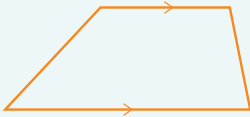
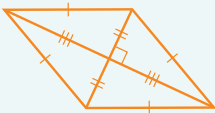
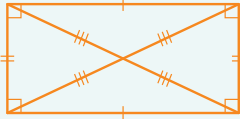
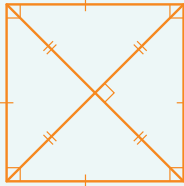
Worksheet

Shapes and angles review

MAT09MGWS00068

Properties of quadrilaterals

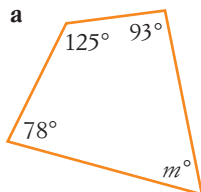
Summary

Trapezium 	• One pair of opposite sides parallel • No axes of symmetry
Kite 	• Two pairs of equal adjacent sides • One pair of opposite angles equal • One axis of symmetry • Diagonals intersect at right angles
Parallelogram 	• Opposite sides are parallel and equal • Opposite angles are equal • No axes of symmetry • Diagonals bisect each other
Rhombus 	• All sides are equal • Two axes of symmetry • A special type of parallelogram • Diagonals bisect each other at right angles • Diagonals bisect the angles of the rhombus
Rectangle 	• All angles are 90° (right angles) • Two axes of symmetry • A special type of parallelogram • Diagonals are equal (in length) • Diagonals bisect each other
Square 	• All sides are equal • All angles are 90° (right angles) • Four axes of symmetry • A special type of rhombus and rectangle • Diagonals are equal • Diagonals bisect each other at right angles • Diagonals bisect the angles of the square

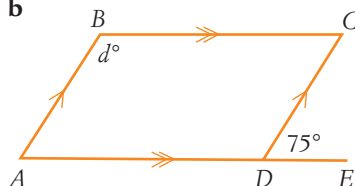
Example 4

Find the value of each pronumeral.

a



b



Solution

a $m + 78 + 125 + 93 = 360$ (angle sum of a quadrilateral)

$$m = 360 - 78 - 125 - 93$$

$$= 64$$

b $\angle CDA = 180^\circ - 75^\circ$ (angles on a straight line)

$$= 105^\circ$$

$$d = 105 \text{ (opposite angles of a parallelogram)}$$

Can you see another method for finding d ?

Puzzle sheet

Mixed angle problems

MAT09MGPS00053

Puzzle sheet

Parallel lines

MAT09MGPS00051

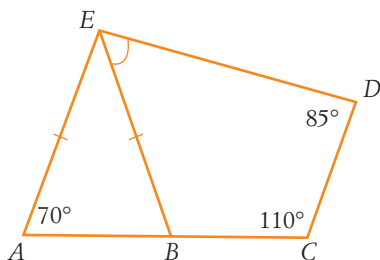
Puzzle sheet

Triangles and quadrilaterals

MAT09MGPS00052

Example 5

Find the size of $\angle BED$.



Solution

$$\angle ABE = 70^\circ \text{ (equal angles in isosceles } \triangle ABE)$$

$$\angle EBC = 180^\circ - 70^\circ \text{ (angles on a straight line)}$$

$$= 110^\circ$$

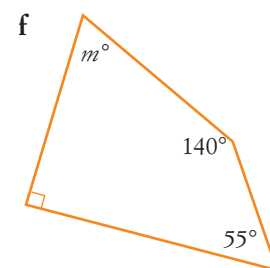
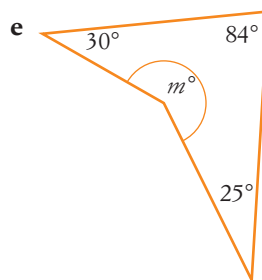
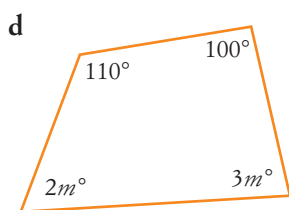
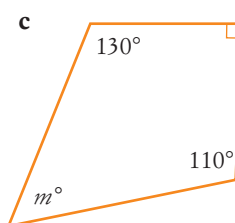
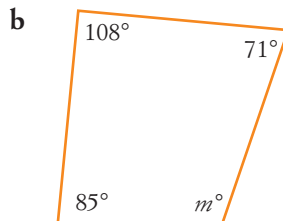
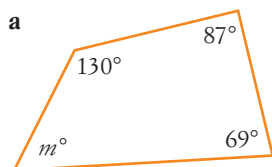
$$\angle BED = 360^\circ - 85^\circ - 110^\circ - 110^\circ \text{ (angle sum of quadrilateral } BCDE)$$

$$= 55^\circ$$

Can you see another method for finding $\angle BED$?

Exercise 6-02 Quadrilateral geometry

See Example 4 1 Find the value of m in each diagram.



2 Copy and complete the table below.

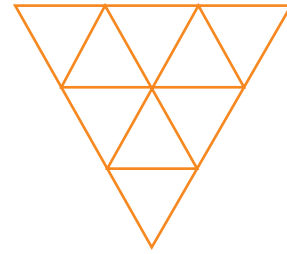
Property	Trapezium	Kite	Parallelogram	Rhombus	Rectangle	Square
One pair of opposite sides parallel						
Opposite sides parallel						
Opposite sides equal			✓			
All sides equal						
Two pairs of adjacent sides equal						
Diagonals equal					✓	
Diagonals bisect each other						
Diagonals meet at right angles						
Diagonals bisect the angles of the shape						
Opposite angles equal						
One pair of opposite angles equal						
All angles 90°						
Axes of symmetry	0					

3 Refer to the table you completed in question 2, and name all quadrilaterals that have:

a no axes of symmetry	b one pair of parallel sides
c four equal sides	d equal diagonals
e opposite sides equal	f four axes of symmetry
g all adjacent sides equal	h one axis of symmetry
i opposite sides parallel	j all angles measuring 90°
k two axes of symmetry	l diagonals which bisect each other
m opposite angles equal	n diagonals meeting at 90°

4 Each triangle in this diagram is an equilateral triangle.

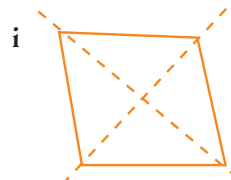
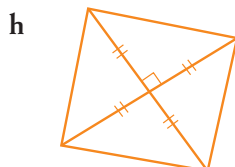
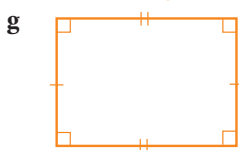
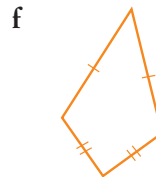
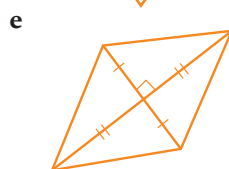
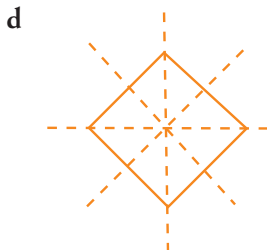
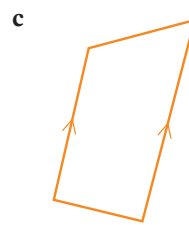
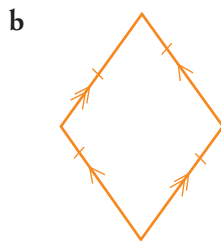
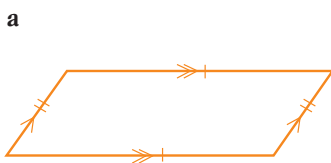
- Name the different types of quadrilaterals you can find in the diagram.
- How many of each type of quadrilateral are there in the diagram?
- How many triangles are there? (It may help to copy the diagram so that you can draw on it using coloured pencils.)



5 Which statements are *always* true?

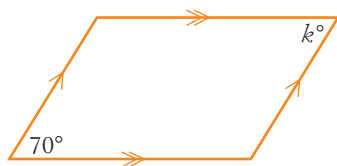
- A rhombus is a parallelogram.
- The diagonals of a parallelogram meet at right angles.
- A square is a rhombus.
- A parallelogram is a quadrilateral with a pair of opposite sides equal and parallel.
- A square is a rectangle.
- The diagonals of an isosceles trapezium bisect each other.
- The opposite angles of a rhombus are equal.
- The diagonals of a rhombus are equal and bisect each other at right angles.
- A rectangle is a parallelogram.

6 Name each quadrilateral using the properties marked on it.

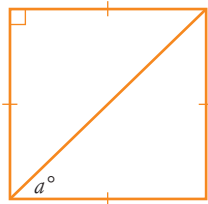


7 Find the value of each pronumeral, giving reasons.

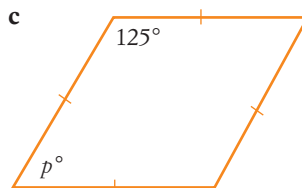
a



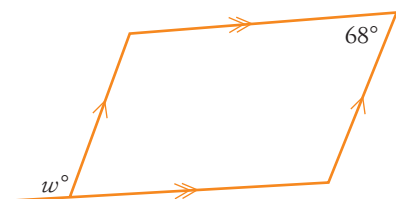
b



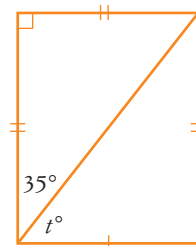
c



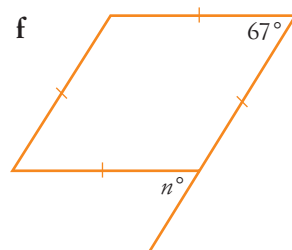
d



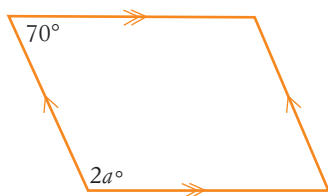
e



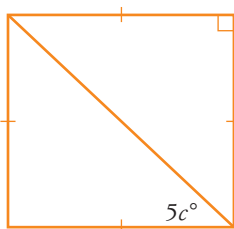
f



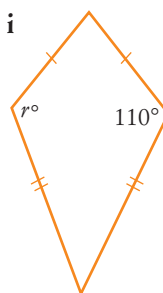
g



h



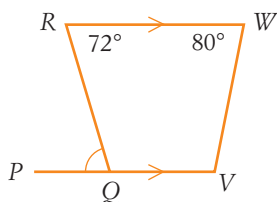
i



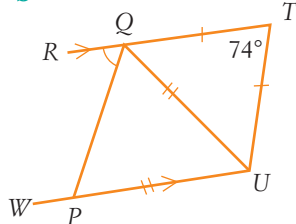
See Example 5

8 Find the size of $\angle PQR$ in each diagram, giving reasons.

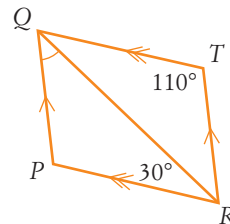
a



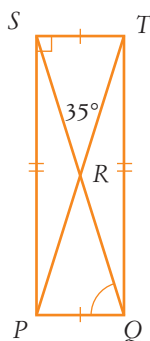
b



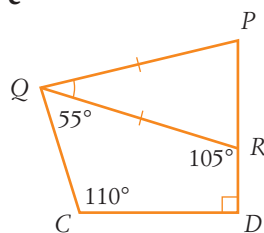
c



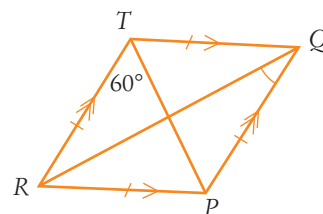
d



e



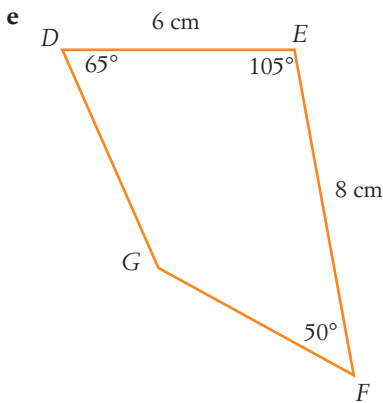
f



Worked solutions
Quadrilateral geometry
MAT09MGWS10030

9 Use geometrical instruments or geometry software such as GeoGebra to construct each quadrilateral described.

- a Parallelogram $TUVW$ with sides $TU = 30$ mm, $UV = 60$ mm and $\angle TUV = 113^\circ$.
- b Rhombus with side length 5 cm.
- c Convex quadrilateral $WXYZ$ with $WX = 65$ mm, $WZ = 40$ mm, $\angle ZWX = 54^\circ$, $\angle WZY = 114^\circ$ and $XY = 24$ mm.
- d Non-convex quadrilateral with one side 4.5 cm and one angle 200° .

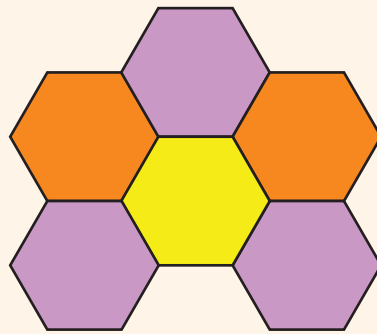


Just for the record

Geometry in art

Geometry has been used in artwork for centuries.

Tessellations can be used to create artwork, such as the regular hexagons used in the tessellation shown below.



Indian and Islamic artworks show incredible intricacy, detail and colour within the geometric images used.



Shutterstock.com/Lenar Musin

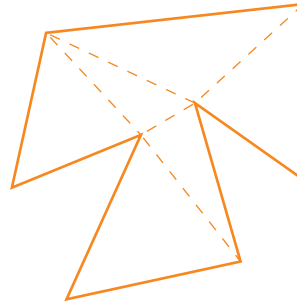
Shutterstock.com/Andrew Park

A diagram of a convex pentagon with vertices labeled A, B, C, D, and E. The vertices are arranged such that A is at the bottom left, B is at the bottom right, C is on the right, D is at the top right, and E is at the top left. Two diagonals are drawn from vertex A: one to vertex D (AD) and one to vertex C (AC). These diagonals divide the pentagon into three triangles: $\triangle ABC$, $\triangle ACD$, and $\triangle ADE$.

Polygon	Number of sides	Number of triangles	Angle sum
triangle	3	1	180°
quadrilateral	4	2	
pentagon	5		
hexagon			
octagon			
decagon			

- 226

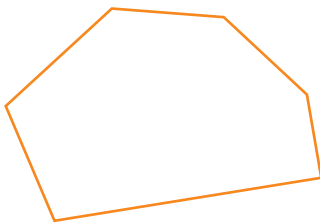
- 6 a Draw a non-convex octagon.
- b Divide the polygon into triangles as shown.
- c How many triangles have been formed?
- d Find the angle sum of this non-convex octagon.
- e Does your rule for the angle sum of a polygon also apply to the non-convex polygon?



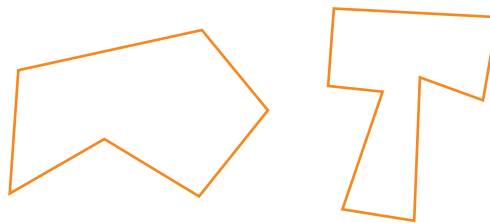
6-03 Angle sum of a polygon

A **polygon** is a general name for any shape with straight sides. The word is derived from the Greek, meaning 'many angles'. Shapes with curved sides, such as circles, ellipses and semicircles, are *not* polygons.

A polygon may be either convex or **non-convex (concave)**.



Convex polygon



Non-convex polygons

In a convex polygon, all vertices point outwards, all diagonals lie within the shape and all angles are less than 180° .

In a non-convex polygon, some vertices point inwards, some diagonals lie outside the shape and some angles are more than 180° (reflex angles).

A polygon's name is determined by the number of sides it has. The images below show the Pentagon building in the USA, a 50p coin from the UK, and a stop sign.



Shutterstock.com/Fontpage



Shutterstock.com/Popartie



Shutterstock.com/Stephen Rees

NSW

Worksheet

Angle sum of a polygon

MAT09MGWK10069

Worksheet

Find the unknown angle

MAT09MGWK10070

Technology worksheet

Angle sum of a polygon

MAT09MGCT10002

Name	Number of sides
Pentagon	5
Hexagon	6
Heptagon	7
Octagon	8
Nonagon	9
Decagon	10
Undecagon	11
Dodecagon	12

Summary

The **angle sum of a polygon** with n sides is given by the formula $A = 180(n - 2)^\circ$. This formula applies to both convex and non-convex polygons.

Example 6

Find the angle sum of a nonagon (9 sides).

Solution

$$\begin{aligned}\text{Angle sum} &= 180(9 - 2)^\circ \\ &= (180 \times 7)^\circ \\ &= 1260^\circ\end{aligned}$$

Example 7

Find the number of sides of a polygon that has an angle sum of 720° .

Solution

$$\begin{aligned}180(n - 2) &= 720 \\ 180n - 360 &= 720 \\ 180n &= 1080 \\ n &= \frac{1080}{180} \\ &= 6\end{aligned}$$

$\therefore$ The polygon has 6 sides (hexagon).

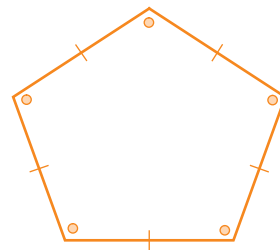
Worksheet

Equal angles

MAT09MGWK00063

Regular polygons

A **regular polygon** has all angles equal and all sides equal. For example, a regular pentagon has 5 equal sides and 5 equal angles. A square is a regular polygon but a rhombus is not.



Summary

The size of each angle in a **regular polygon** with n sides $= \frac{\text{Angle sum}}{\text{No. of sides}} = \frac{180(n - 2)^\circ}{n}$

Example 8

Find the size of one angle in a regular hexagon.

Solution

A hexagon has six sides ($n = 6$).

$$\begin{aligned}\text{Size of one angle} &= \frac{180(6 - 2)^\circ}{6} \\ &= \frac{(180 \times 4)^\circ}{6} \\ &= 120^\circ\end{aligned}$$

Each angle in a regular hexagon is 120° .

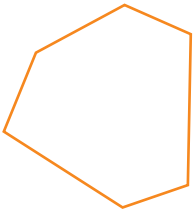
Exercise 6-03 Angle sum of a polygon

1 How many sides has:

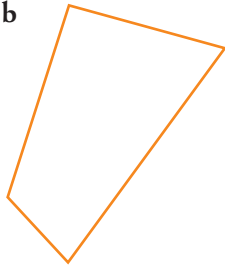
a a hexagon?	b a quadrilateral?	c a nonagon?
d a decagon?	e a heptagon?	f a pentagon?
g a dodecagon?	h an octagon?	i an undecagon?

2 Name each polygon.

a



b



c



d



e



f



g



h



3 Which polygons from question 2 are:

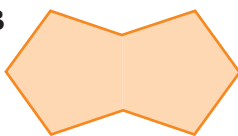
a convex?	b regular?

4 Which shape is a regular octagon? Select A, B, C or D.

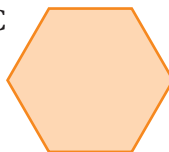
A



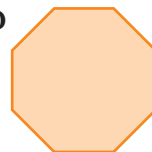
B



C



D



5 What is the more common name for:

a a regular triangle?

b a regular quadrilateral?

See Example 6

6 Find the angle sum of a polygon with:

a 5 sides

b 8 sides

c 15 sides

d 12 sides

e 7 sides

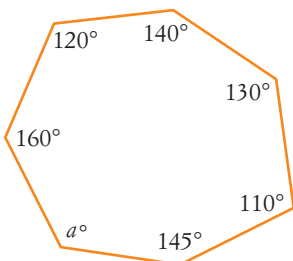
f 10 sides

g 20 sides

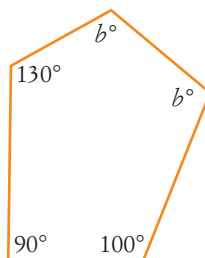
h 11 sides

7 Find the value of each pronumeral.

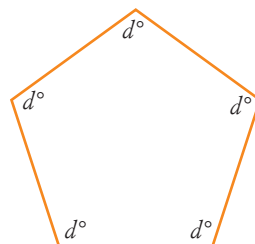
a



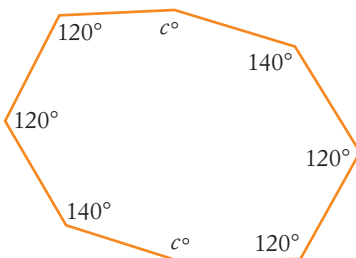
b



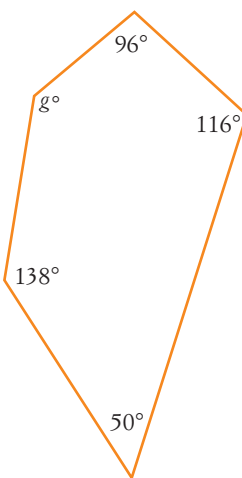
c



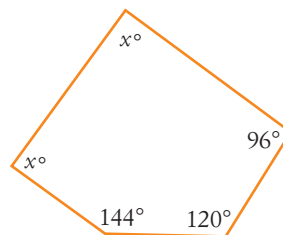
d



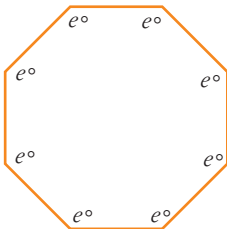
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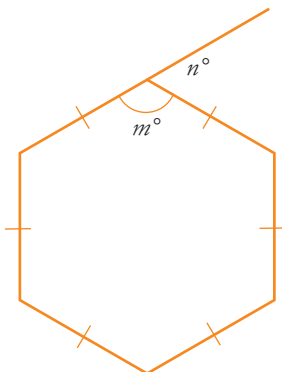
f



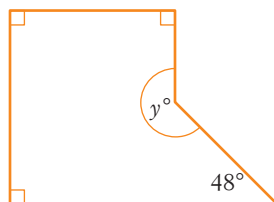
g



h



i



Worked solutions

Angle sum of a polygon

MAT09MGWS10031

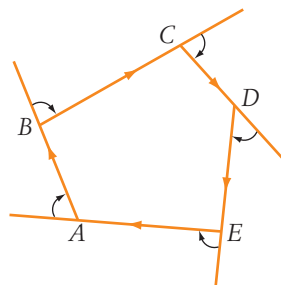
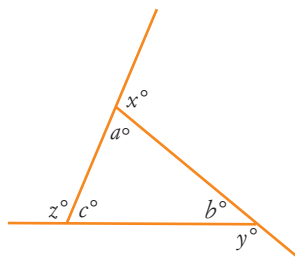
- 8 Find the number of sides of a polygon that has an angle sum of:
- a 2160° b 1620° c 3960° d 2700°
- 9 Which polygon has an angle sum of 1440° ? Select **A**, **B**, **C** or **D**.
- A** pentagon **B** decagon **C** nonagon **D** octagon
- 10 Find the size of one angle in a regular
- a octagon b decagon c dodecagon d hexagon
- 11 The angle sum of a regular polygon is 6840° .
- a How many sides does the polygon have?
b Find the size of each angle.
- 12 How many sides does a regular polygon have if each of its angles is:
- a 165° ? b 170° ? c 144° ?

See Example 7

See Example 8

Investigation: Exterior angle sum of a convex polygon

- 1 a Draw a triangle and extend each side to show the exterior angles of the triangle as shown.
Exterior angles are x° , y° and z° .
Interior angles are a° , b° and c° .
- b Use a protractor to measure angles x° , y° and z° .
- c Find the sum of the exterior angles.
- d Looking at the diagram, what must be the value of $a^\circ + x^\circ + b^\circ + y^\circ + c^\circ + z^\circ$?
- e But what do we know about the value of $a^\circ + b^\circ + c^\circ$?
- f Therefore, what must be the value of $x^\circ + y^\circ + z^\circ$?
- 2 Repeat this procedure for the exterior angles of a convex quadrilateral. What is the sum of the exterior angles of a convex quadrilateral?
- 3 Repeat the procedure for a convex pentagon and a convex hexagon. What do you notice about the sum of the exterior angles of those polygons?
- 4 a Draw any convex polygon and extend the sides. Label the vertices of your polygon A , B , C , etc.
- b Start at A and move around the polygon, turning in the direction indicated at each vertex.
- c Continue until you return to A , facing the same way you started. What must be the sum of the turns in any round trip of a convex polygon?
- d Test whether this rule works for a non-convex polygon.



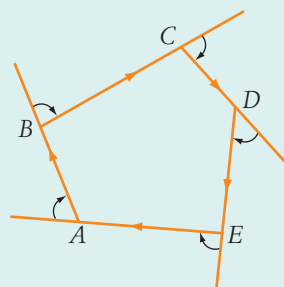
NSW

6-04

Exterior angle sum of a convex polygon

Summary

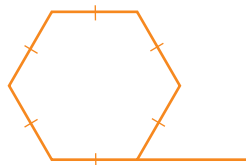
The **sum of the exterior angles** of a convex polygon is 360° .



Example 9

For a regular hexagon, find the size of:

- a** each exterior angle **b** each (interior) angle.



Solution

- a** Sum of exterior angles = 360°

$$\begin{aligned}\text{One exterior angle} &= 360^\circ \div 6 \\ &= 60^\circ\end{aligned}$$

- b** Each angle = $180^\circ - 60^\circ$ (angles on a straight line)
= 120°

$$\begin{aligned}\text{OR: Each angle} &= \frac{180(6-2)}{6} \\ &= 120^\circ\end{aligned}$$

Example 10

Find the number of sides of a regular polygon if:

- a** each exterior angle is 12° **b** each (interior) angle is 160° .

Solution

- a** Sum of exterior angles = 360°

$$\begin{aligned}\text{Number of exterior angles} &= 360 \div 12 \\ &= 30\end{aligned}$$

$\therefore$ The regular polygon has 30 sides.

- b** Exterior angle = $180^\circ - 160^\circ$ (angles on a straight line)
 $= 20^\circ$

$$\text{Sum of exterior angles} = 360^\circ$$

$$\begin{aligned}\text{Number of exterior angles} &= 360 \div 20 \\ &= 18\end{aligned}$$

$\therefore$ The regular polygon has 18 sides.

$$\begin{aligned}\text{OR: } \frac{180(n-2)^\circ}{n} &= 160^\circ \\ 180(n-2) &= 160n \\ 180n - 360 &= 160n \\ 20n - 360 &= 0 \\ 20n &= 360 \\ n &= \frac{360}{20} \\ &= 18\end{aligned}$$

$\therefore$ The regular polygon has 18 sides.

Exercise 6-04 Exterior angle sum of a convex polygon

- Find the size of each exterior angle of a regular:

a octagon **b** decagon **c** 15-sided polygon **d** nonagon
- Find the size of each angle of a regular:

a pentagon **b** dodecagon **c** nonagon **d** 16-sided polygon
- Find the number of sides of a regular polygon if each exterior angle is:

a 24° **b** 36° **c** 40° **d** 10° **e** 18° **f** 60°
- Find the number of sides of a regular polygon if each angle is:

a 150° **b** 175° **c** 162° **d** 140° **e** 108° **f** 176°

See Example 9

See Example 10

Mental skills 6

Maths without calculators

Dividing decimals

To divide one decimal by another, first move the decimal points in both decimals the same number of places to the right so that the second decimal is a whole number.

1 Study each example.

$$\text{a } 0.24 \div 0.06 = 24 \div 6 = 4$$

$$\text{b } 0.45 \div 0.5 = 4.5 \div 5 = 0.9$$

$$\text{c } 0.006 \div 0.3 = 0.06 \div 3 = 0.002$$

$$\text{d } 27 \div 0.9 = 270 \div 9 = 30$$

$$\text{e } 1.6 \div 0.4 = 16 \div 4 = 4$$

$$\text{f } 5.6 \div 0.07 = 560 \div 7 = 80$$

2 Now evaluate each quotient.

$$\text{a } 0.25 \div 0.5$$

$$\text{b } 63 \div 0.7$$

$$\text{c } 3.2 \div 0.4$$

$$\text{d } 0.18 \div 0.2$$

$$\text{e } 2.7 \div 0.03$$

$$\text{f } 0.042 \div 0.06$$

$$\text{g } 4 \div 0.5$$

$$\text{h } 1.2 \div 0.04$$

$$\text{i } 0.072 \div 0.9$$

$$\text{j } 0.35 \div 0.1$$

$$\text{k } 0.28 \div 0.07$$

$$\text{l } 0.033 \div 0.11$$

3 Study each example.

Given that $112 \div 14 = 8$, evaluate each expression.

$$\begin{aligned} \text{a } 112 \div 1.4 &= 112.0 \div 1.4 \\ &= 1120 \div 14 \\ &= 112 \times 10 \div 14 \\ &= 112 \div 14 \times 10 \\ &= 8 \times 10 \\ &= 80 \end{aligned}$$

$$\text{Estimate: } 112 \div 1.4 \approx 112 \div 1 = 112$$

$$\begin{aligned} \text{b } 0.112 \div 0.14 &= 0.112 \div 0.14 \\ &= 11.2 \div 14 \\ &= 112 \div 10 \div 14 \\ &= 112 \div 14 \div 10 \\ &= 8 \div 10 \\ &= 0.8 \end{aligned}$$

$$\text{Estimate: } 0.112 \div 0.14 \approx 0.1 \div 0.1 = 1$$

$$\begin{aligned} \text{c } 1120 \div 1.4 &= 11200 \div 14 \\ &= 112 \times 100 \div 14 \\ &= 112 \div 14 \times 100 \\ &= 8 \times 100 \\ &= 800 \end{aligned}$$

$$\text{Estimate: } 1120 \div 1.4 \approx 1120 \div 1 = 1120$$

$$\begin{aligned} \text{d } 1.12 \div 14 &= 112 \div 100 \div 14 \\ &= 112 \div 14 \div 100 \\ &= 8 \div 100 \\ &= 0.08 \end{aligned}$$

$$\text{Estimate: } 1.12 \div 14 \approx 1.12 \div 10 = 0.112$$

4 Now given that $368 \div 23 = 16$, evaluate each quotient.

$$\text{a } 36.8 \div 2.3$$

$$\text{b } 368 \div 2.3$$

$$\text{c } 3.68 \div 2.3$$

$$\text{d } 0.368 \div 0.23$$

$$\text{e } 36.8 \div 23$$

$$\text{f } 3.68 \div 0.23$$

$$\text{g } 36.8 \div 0.23$$

$$\text{h } 0.368 \div 2.3$$

$$\text{i } 0.368 \div 23$$

$$\text{j } 3.68 \div 0.023$$

$$\text{k } 3.68 \div 23$$

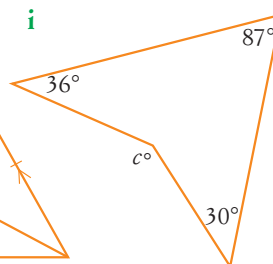
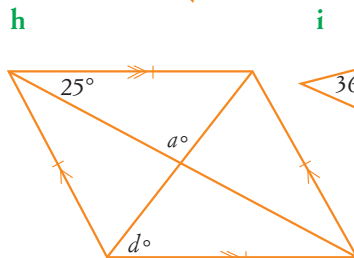
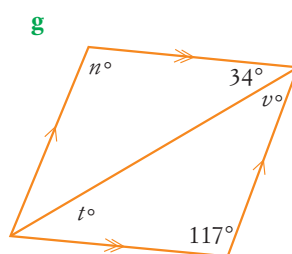
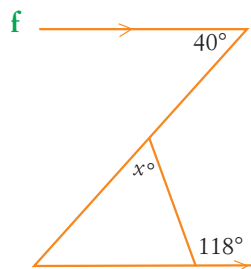
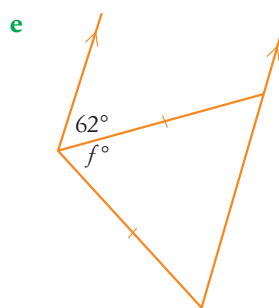
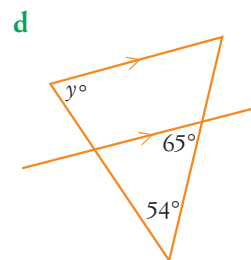
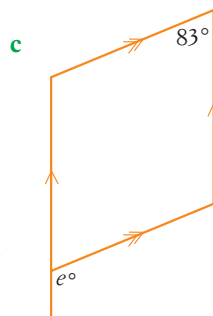
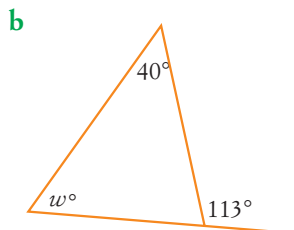
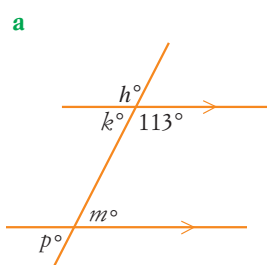
$$\text{l } 0.368 \div 230$$

Power plus

- 1 How many diagonals has:
- a** a quadrilateral? **b** an octagon? **c** a dodecagon?
- 2 **a** By drawing polygons with 3 to 10 sides and counting diagonals, copy and complete this table.

No. of sides, n	3	4	5	6	7	8	9	10
No. of diagonals, d								

- b** Find a formula for the number of diagonals, d , in a polygon with n sides.
- 3 Name all quadrilaterals whose diagonals:
- a** bisect each other at right angles **b** bisect each other
c intersect at right angles **d** have equal length
e bisect the angles of the quadrilateral **f** are equal and bisect each other
- 4 Find the value of each pronumeral, giving reasons.



Chapter 6 review

Language of maths

Puzzle sheet
Geometry crossword
MAT09MGPS10071
Quiz
Shapes and angles
MAT09MGQZ00011

adjacent	angle sum	bisect	co-interior
convex	corresponding	equilateral	exterior angle
interior	isosceles	kite	parallel
parallelogram	polygon	quadrilateral	rectangle
regular polygon	rhombus	right angle	square
supplementary	trapezium	vertex	vertically opposite

- 1 What shape has two pairs of equal adjacent sides?
- 2 Name one property of a convex quadrilateral.
- 3 What type of angle is created when one side of a triangle is extended?
- 4 What is the sum of the exterior angles of any polygon?
- 5 What is a **regular polygon**? Are all regular polygons also **convex**?
- 6 Copy and complete: The exterior angle of a triangle is equal to the _____ of the _____ angles.

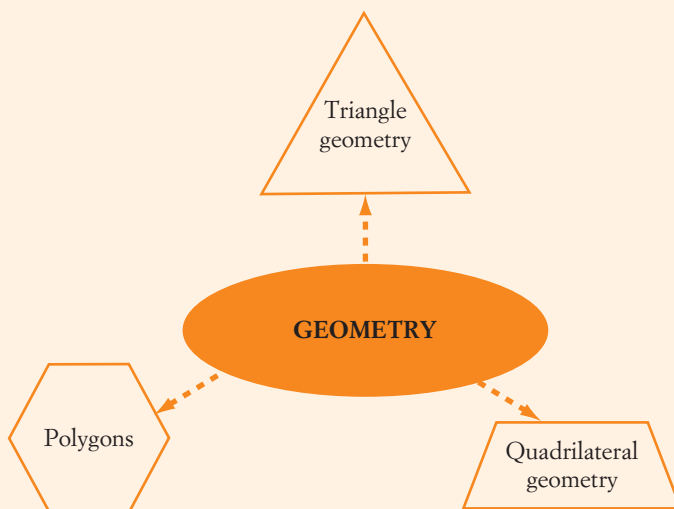
Topic overview

Worksheet
Geometry summary poster
MAT09MGWK10072

- Write three ideas from this topic that were new to you.
- Summarise what you know about the angle sum of a triangle, quadrilateral and polygon.
- Name the three types of triangles, choose one and list all of its properties.
- Name the six special quadrilaterals, choose one and list all of its properties.

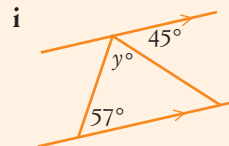
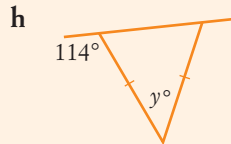
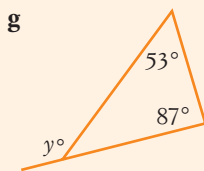
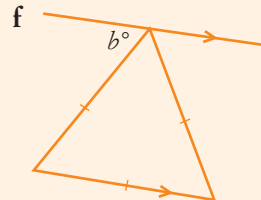
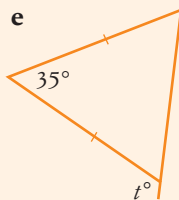
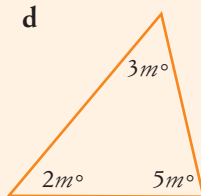
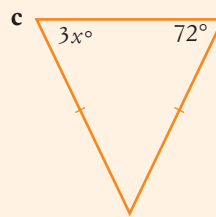
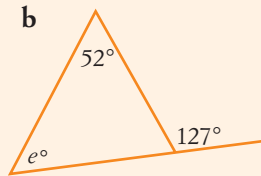
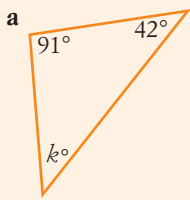
Worksheet
Mind map: Geometry (Advanced)
MAT09MGWK10074

Copy (or print) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



- 1 Find the value of each pronumeral, giving reasons.

See Exercise 6-01



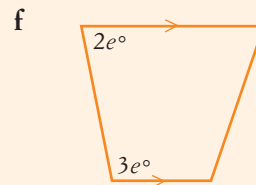
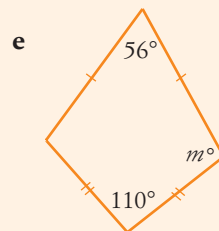
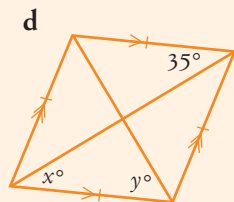
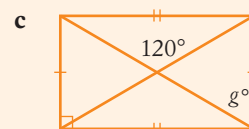
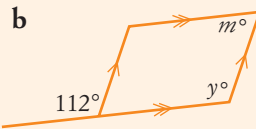
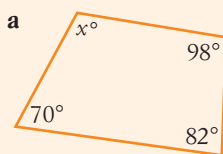
- 2 What are the sizes of the angles in:

See Exercise 6-01

- a** an equilateral triangle? **b** a right-angled isosceles triangle?

- 3 Find the value of each pronumeral, giving reasons.

See Exercise 6-02



- 4 Name all quadrilaterals that have:

See Exercise 6-02

- a** both pairs of opposite sides parallel **b** two equal diagonals
c all sides equal **d** diagonals that bisect each other.

- 5 Draw each pentagon.

See Exercise 6-03

- a** a regular pentagon **b** a non-regular pentagon **c** a non-convex pentagon

- 6 **a** Show that the angle sum of a decagon is 1440°.

See Exercise 6-03

- b** Find the size of one angle in a regular nonagon.

- 7 **a** Find the size of each exterior angle in a regular hexagon.

See Exercise 6-04

- b** If each angle in a regular polygon is 150°, how many sides does it have?

Mixed revision 2

See Exercise 5-09

- 1 State the number of significant figures in each number.

a 1203 **b** 0.008 **c** 1.092 **d** 15 700

See Exercise 5-01

- 2 Simplify each expression.

a $y^9 \times y$ **b** $3g^7 \times 4g^8$ **c** $2p^3q \times 9p^2q^2$
d $\frac{k^{18}}{k^6}$ **e** $\frac{x^{11}}{x^7}$ **f** $h^{10} \div h$
g $54k^7 \div 6k^3$ **h** $\frac{h^3y^7}{hy^4}$ **i** $12m^5n^8 \div 18mn^7$

See Exercise 5-04

- 3 Simplify each expression.

a $6y^0$ **b** $4^0 \times 3^0$ **c** $(4k)^0$ **d** $x \times (3y)^0$ **e** $2q^0 - (2q)^0$

See Exercise 4-09

- 4 Sketch on a compass rose point *A* if it has a bearing of:

a 260° from point *M* **b** 304° from point *W*.

See Exercise 6-03

- 5 **a** Calculate the size of one angle in a regular hexagon.
b Name the polygon that has an angle sum of 1440° .

Stage 5.3

See Exercise 7-08

See Exercise 5-05

- 6 Make *y* the subject of the formula $T = a + py^2$

- 7 Evaluate each expression, giving your answers in fraction form.

a 2^{-1} **b** 5^{-3} **c** 10^{-2}

Stage 5.3

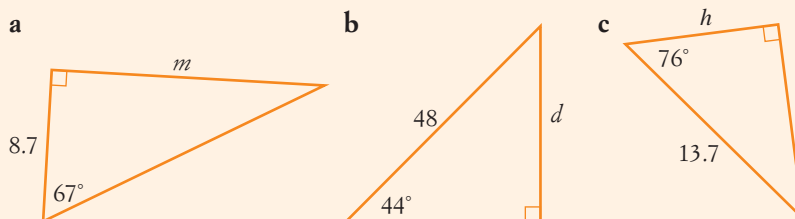
See Exercise 5-05

See Exercise 4-05

- 8 Simplify each expression.

a $\left(\frac{1}{2}\right)^{-1}$ **b** $\left(\frac{8}{3}\right)^{-2}$ **c** $\left(\frac{4d}{5}\right)^{-2}$ **d** $\left(\frac{3}{10x}\right)^{-3}$

- 9 Calculate the value of each pronumeral, correct to two decimal places.



See Exercise 5-02

- 10 Simplify each expression, giving the answers in index notation.

a $(2^3)^5$ **b** $(4^2)^6$ **c** $(3p^7)^2$ **d** $(2w^5)^3$

See Exercise 5-10

- 11 Express each number in decimal form.

a 1.54×10^7 **b** 3.2×10^{-3} **c** 9.837×10^{-1} **d** 2.4×10^4

See Exercise 4-10

- 12 A plane flies on a bearing of SW for 300 km. How far south (correct to the nearest kilometre) from its starting point has the plane flown?

See Exercise 4-07

- 13 Find θ correct to the nearest minute if:

a $\cos \theta = 0.21$ **b** $\sin \theta = \frac{9}{10}$ **c** $\tan \theta = 1.5$

- 14 Write each expression using a negative index.

See Exercise 5-05

a $\frac{1}{n}$ b $\frac{1}{2^3}$ c $\frac{1}{k^5}$ d $\frac{7}{y^2}$

- 15 Write 7 280 000 in scientific notation.

See Exercise 5-10

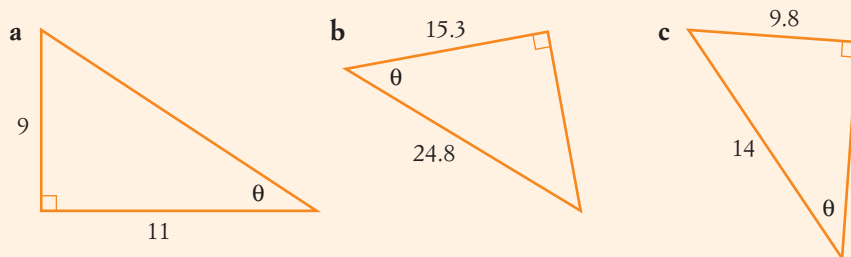
- 16 Evaluate each expression, giving your answers in scientific notation correct to two significant figures.

See Exercise 5-11

a $(7.08 \times 10^7) \times (3.8 \times 10^8)$ b $(4.65 \times 10^{-3}) \div (3.9 \times 10^7)$

- 17 Find θ correct to the nearest degree.

See Exercise 4-07



- 18 Write each expression using a fractional index.

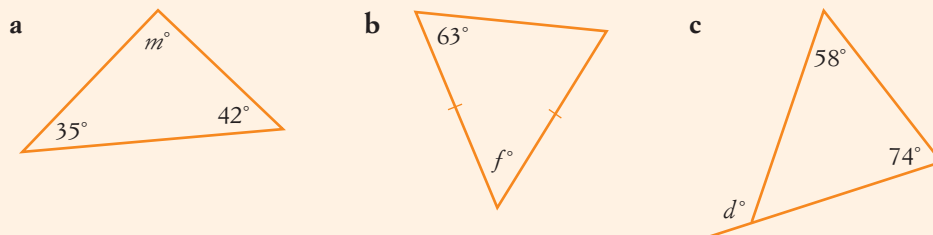
Stage 5.3

See Exercise 5-06, 5-07

a $\sqrt{13}$ b $\sqrt[3]{20}$ c $\sqrt[3]{3p}$ d $(\sqrt{x})^3$

- 19 Find the value of each pronumeral, giving reasons.

See Exercise 6-01



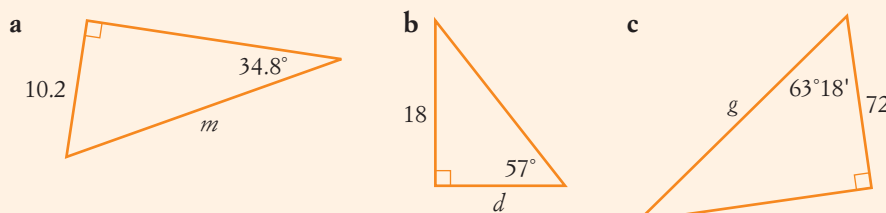
- 20 Solve $12 - 5k = 2k - 9$. Select the correct answer **A**, **B**, **C** or **D**.

See Exercise 7-01

A $k = 3$ **B** $k = 1$ **C** $k = \frac{3}{7}$ **D** $k = 7$

- 21 Find the value of each pronumeral, correct to one decimal place.

See Exercise 4-06



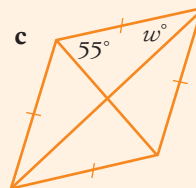
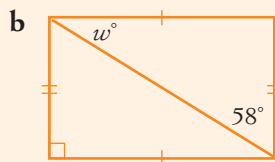
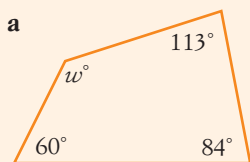
- 22 For each pair of numbers, select the larger number.

See Exercise 5-10

a 5.3×10^9 or 8.6×10^8 b 3.73×10^5 or 3.4×10^5

Mixed revision 2

See Exercise 6-02 **23** Find the value of w in each quadrilateral.

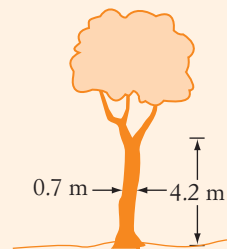


See Exercise 7-07 **24** The volume of useable timber (in cubic metres) in a tree is calculated using the formula:

$$v = 0.5hd^2 + 10.$$

where: d = the diameter of the tree

h = the height to the first branch



Calculate the volume of useable timber in this tree. Select the correct answer **A**, **B**, **C** or **D**.

- A** 16.2 m^3 **B** 11.5 m^3 **C** 11.0 m^3 **D** 18.8 m^3

Stage 5.3

See Exercise 4-06

25 Evaluate each expression.

- a** $25^{\frac{3}{2}}$ **b** $4^{-\frac{5}{2}}$ **c** $(\sqrt[4]{16})^3$ **d** $\sqrt[3]{27^2}$

See Exercise 6-04 **26** How many sides has a regular polygon if:

- a** each exterior angle is 15° ? **b** each interior angle is 135° ?

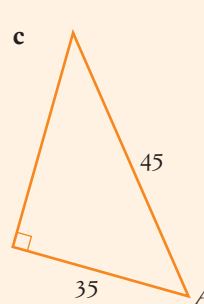
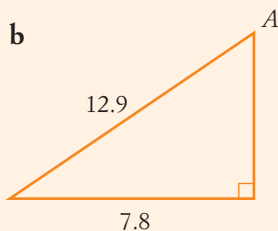
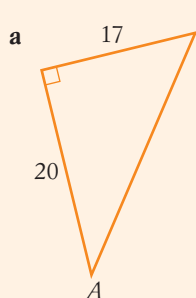
See Exercise 5-03 **27** Simplify each expression.

- a** $(2m^3n^2)^4$ **b** $\left(-\frac{2h}{5}\right)^2$ **c** $(7rt^2)^3$

See Exercise 7-04 **28** Solve each equation.

- a** $\frac{3k-1}{2} = \frac{5k+1}{3}$ **b** $\frac{h}{5} + \frac{h}{2} = 6$

See Exercise 4-07 **29** Find the size of angle A , correct to the nearest degree.



See Exercise 7-07 **30** A formula for cooking roast beef is $T = 40M + 25$, where T is the cooking time in minutes and M is the mass of the beef in kilograms.

- a** How long will it take to cook a 5 kg piece of beef?
b What mass of beef has a cooking time of $2\frac{1}{2}$ hours?

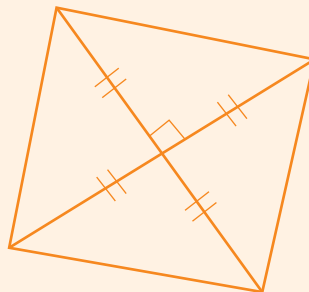
- 31** A 3.5 m ladder leans against a wall, making an angle of 50° with the ground.

See Exercise 4-05

- a** Draw a diagram showing this information.
- b** Calculate, correct to the nearest 0.1 m:
 - i** how far up the wall the ladder reaches
 - ii** how far the foot of the ladder is from the base of the wall.

- 32** What type of quadrilateral is shown here?

See Exercise 6-02



- 33** Solve each equation.

See Exercise 7-02

a $3(p - 3) = 24$ **b** $3(4y - 1) + 5 = 2(9 - 2y)$

- 34** Round:

See Exercise 5-09

- a** 43 759 to two significant figures
- b** 10.7614 to three significant figures
- c** 0.000 038 9 to one significant figure

- 35** **a** The sum of four consecutive numbers is 82. Find the numbers.

See Exercise 7-03

- b** Sean collected \$1 and \$2 coins until he had 275 coins, having a total value of \$402. How many \$2 coins did Sean have?

- 36** Amit is at ground level 72 metres from the base of an apartment building. The angle of elevation to the top of the building is 35° . Find the height of the building, correct to the nearest metre.

See Exercise 4-08

- 37** Solve each equation.

See Exercise 7-05

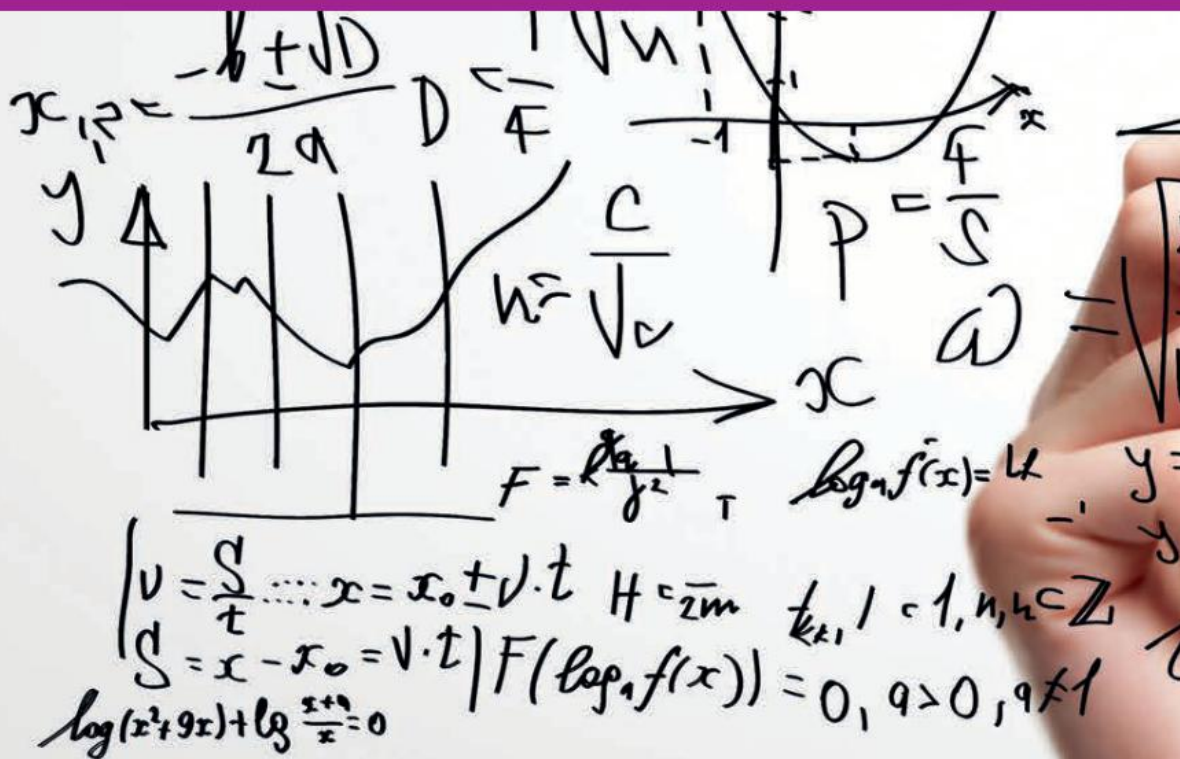
a $5m^2 + 1 = 81$ **b** $\frac{x^2}{3} = 12$

- 38** Solve each equation.

a $2r^3 = 1024$ **b** $\frac{y^3}{6} = -121.5$

Stage 5.3

See Exercise 7-06

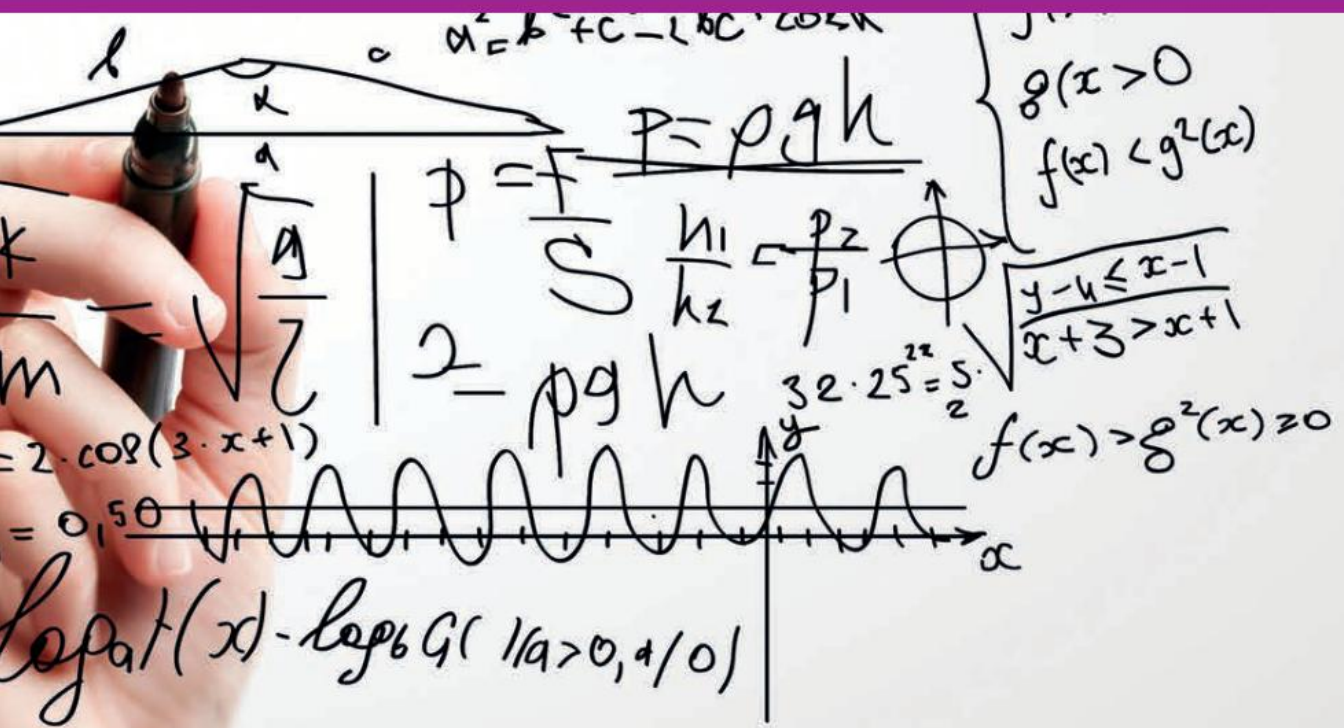


7

Number and algebra

Equations

Equation-solving has been recorded as far back as 1500 BCE. It was first used in ancient Babylon and Egypt and was brought to Europe from India by the Arabs during the 9th century. The word 'algebra' comes from the Arabic word *al-jabr*, meaning restoration, the process of performing the same operation on both sides of an equation to solve the equation.



Chapter outline

	Proficiency strands				
7-01 Equations with variables on both sides	U	F		R	
7-02 Equations with brackets	U	F		R	
7-03 Equation problems	U	F	PS	R	C
7-04 Equations with algebraic fractions	U	F		R	
7-05 Simple quadratic equations $ax^2 = c$	U	F		R	C
7-06 Simple cubic equations $ax^3 = c^*$	U	F		R	C
7-07 Equations and formulas	U	F	PS	R	C
7-08 Changing the subject of a formula*	U	F		R	C

*STAGE 5.3

Wordbank

cubic equation An equation involving a variable cubed (power of 3), such as $2x^3 = 250$.

equation A mathematical statement that two quantities are equal, involving algebraic expressions and an equals sign (=)

formula A rule written as an algebraic equation, using variables.

inverse operation An opposite used in solving an equation, for example, the inverse operation of multiplying is adding

linear equation An equation involving a variable that is not raised to a power, such as $2x + 9 = 17$.

quadratic equation An equation involving a variable squared (power of 2), such as $3x^2 - 6 = 69$.

solution The answer to an equation or problem, the correct value(s) of the variable that makes an equation true

solve (an equation) To find the value of an unknown variable in an equation

In this chapter you will:

- solve linear equations, including equations involving simple algebraic fractions
- (STAGE 5.3) solve equations involving algebraic fractions
- solve simple quadratic equations of the form $ax^2 = c$
- (STAGE 5.3) solve simple cubic equations of the form $ax^3 = c$
- use formulas to solve problems
- (STAGE 5.3) change the subject of a formula

SkillCheck

Worksheet

StartUp assignment 7

MAT09NAWK10075

Puzzle sheet

Solving equations

MAT09NAPS00033

Puzzle sheet

Backtracking

MAT09NAPS00032

Skillsheet

Solving equations
by balancing

MAT09NASS10023

Skillsheet

Solving equations
by backtracking

MAT09NASS10024

Skillsheet

Solving equations
using diagrams

MAT09NASS10025

Puzzle sheet

Equations with
unknowns on
both sides

MAT09NAPS00035

Homework sheet

Equations 1

MAT09NAHS10012

1 Solve each equation. Use substitution to check your solutions.

a $3x - 10 = 5$

b $4 + 2y = 21$

c $12y + 5 = 23$

d $\frac{m}{4} - 6 = 2$

e $\frac{x-5}{4} = 7$

f $\frac{5r}{3} = 10$

g $\frac{-2x}{5} = 3$

h $11 - 2a = 17$

i $20 + 4d = -6$

j $\frac{w}{2} + 3 = 4$

k $\frac{a}{6} - 1 = 7$

l $6 - \frac{n}{3} = 4$

2 Using n to represent 'the number', write an expression for each of these statements.

a The product of the number and 7.

b The square of the number.

c 5 times the sum of the number and 8.

d The number decreased by 20.

e The product of 6 and the number, decreased by nine.

f If the number is even, the *next* even number.

7-01 Equations with variables on both sides

Summary

For **equations with variables on both sides**, perform operations on both sides to move:

- all the variables onto one side of the equation
- all the numbers onto the other side of the equation.

Example 1

Solve each equations.

a $7x + 7 = 2x + 2$

b $9 - 6y = 10 - 2y$

Solution

a $7x + 7 = 2x + 2$

$$7x - 2x + 7 = 2x - 2x + 2$$

Subtracting $2x$ from both sides.

$$5x + 7 = 2$$

$$5x + 7 - 7 = 2 - 7$$

Subtracting 7 from both sides.

$$5x = -5$$

$$\frac{5x}{5} = \frac{-5}{5}$$

Dividing both sides by 5.

$$x = -1$$

Check:

$$\text{LHS} = 7 \times (-1) + 7 = 0$$

$$\text{RHS} = 2 \times (-1) + 2 = 0$$

$$\text{LHS} = \text{RHS}$$

b $9 - 6y = 10 - 2y$

$$9 - 6y + 2y = 10 - 2y + 2y$$

Adding $2y$ to both sides.

$$9 - 4y = 10$$

$$9 - 4y - 9 = 10 - 9$$

Subtracting 9 from both sides.

$$-4y = 1$$

$$\frac{-4y}{-4} = \frac{1}{-4}$$

Dividing both sides by (-4) .

$$y = -\frac{1}{4}$$

Check:

$$\text{LHS} = 9 - 6 \times \left(-\frac{1}{4}\right) = 10\frac{1}{2}$$

$$\text{RHS} = 10 - 2 \times \left(-\frac{1}{4}\right) = 10\frac{1}{2}$$

$$\text{LHS} = \text{RHS}$$

Exercise 7-01 Equations with variables on both sides

1 Solve each equation, and check your solutions.

a $5w + 3 = 2w + 21$

b $2q - 10 = q - 4$

c $13x + 1 = 8x + 26$

d $12n + 3 = 5n - 11$

e $8y - 10 = 10y - 30$

f $3m - 2 = 10 - 3m$

g $9 - 2a = a - 9$

h $9 - 2x = 18 + 7x$

i $12y + 6 = 6 + 9y$

j $-12 - 10u = -20 - 18u$

k $15 - 7x = 22 - 3x$

l $10 - 6x = -15 - 11x$

See Example 1

- 2 For each equation, select the correct solution **A**, **B**, **C** or **D**.
- a $6x - 1 = 2x + 11$
A $x = 12$ **B** $x = 3$ **C** $x = 0$ **D** $x = 2.5$
- b $11 - 4p = 2p + 2$
A $p = 6.5$ **B** $p = 2$ **C** $p = 1.5$ **D** $p = 3$
- 3 Solve each equation.
- a $7w + 15 = w + 3$ b $10 - 3t = 16 + t$ c $4a + 2 = 10 - 4a$
d $50 + 7y = 20 - 3y$ e $8y - 2 = 10y + 1$ f $9y + 3 = 9 - y$
g $9 - t = 7t - 2$ h $5y + 2 = 17 - y$ i $25 - 12k = 15 - 6k$
- 4 Solve $-3n - 8 = -7n - 12$. Select **A**, **B**, **C** or **D**.
A $n = 5$ **B** $n = 2$ **C** $n = -1$ **D** $n = -0.4$

Just for the record

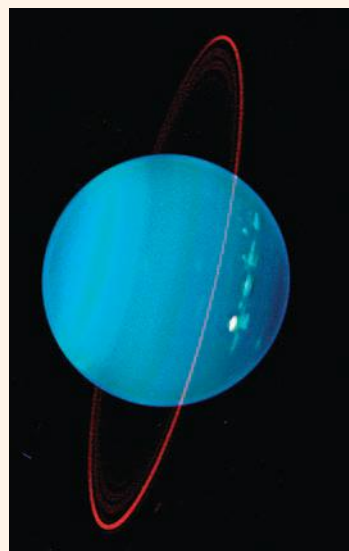
Discovering planets

In 1781, British astronomer William Herschel discovered the planet Uranus. At that time, it was the farthest planet known in our solar system. However, astronomers found that Uranus' orbit around the Sun did not follow the expected path.

Working separately, mathematicians John Couch Adams of England and Urbain Leverrier of France both predicted that this different orbit was caused by an unknown planet. They calculated the position of this undiscovered planet using a number of equations.

In 1846, a German astronomer called Johann Galle located this planet and named it Neptune. The dwarf planet, Pluto, was discovered in a similar manner.

How long ago was Pluto discovered?



Science Photo Library/California Association for Research in Astronomy

Worksheet

Equations 2

MAT09NAWK10076

Worksheet

Checking solutions

MAT09NAWK10078

Puzzle sheet

Equations

MAT09NAPS00036

Puzzle sheet

Equations order activity

MAT09NAPS10077

7-02 Equations with brackets

Summary

For **equations with brackets (grouping symbols)**, expand the expressions and then solve as usual.

Example 2

Solve each equation.

a $3(a + 7) = 6$ **b** $9(m - 5) = 7(m + 1)$ **c** $10y - 3(2y - 5) = 6(8 - 3y)$

Solution

a $3(a + 7) = 6$
 $3a + 21 = 6$

Can you think of another way to solve this equation without expanding?

Expanding the expression to make it a two-step equation.

Subtract 21 from both sides.

$$\begin{aligned} 3a + 21 - 21 &= 6 - 21 \\ 3a &= -15 \\ \frac{3a}{3} &= \frac{-15}{3} \\ a &= -5 \end{aligned}$$

Divide both sides by 3.

Check: $3(-5 + 7) = 3 \times 2 = 6$

b $9(m - 5) = 7(m + 1)$
 $9m - 45 = 7m + 7$
 $9m - 7m - 45 = 7m - 7m + 7$
 $2m - 45 = 7$
 $2m - 45 + 45 = 7 + 45$
 $2m = 52$
 $\frac{2m}{2} = \frac{52}{2}$
 $m = 26$

Expanding brackets on both sides.

Subtracting $7m$ from both sides.

Adding 45 to both sides.

Dividing both sides by 2.

Check:

$$\text{LHS} = 9 \times (26 - 5) = 9 \times 21 = 189$$

$$\text{RHS} = 7 \times (26 + 1) = 7 \times 27 = 189$$

$$\text{LHS} = \text{RHS}$$

c $10y - 3(2y - 5) = 6(8 - 3y)$
 $10y - 6y + 15 = 48 - 18y$
 $4y + 15 = 48 - 18y$
 $4y + 18y + 15 = 48 - 18y + 18y$
 $22y + 15 = 48$
 $22y + 15 - 15 = 48 - 15$
 $22y = 33$
 $\frac{22y}{22} = \frac{33}{22}$
 $y = 1\frac{1}{2}$

Expanding brackets on both sides.

Collecting like terms.

Adding $18y$ to both sides.

Subtracting 15 from both sides.

Dividing both sides by 22.

Video tutorial

Equations with brackets

MAT09NAVT10022

Stage 5.3

Exercise 7-02 Equations with brackets

See Example 2

1 Solve each equation.

a $2(m + 3) = 8$

b $3(x + 1) = 9$

c $5(y - 2) = 15y$

d $35 = 7(k + 1)$

e $4(3 - a) = 16$

f $11 = 9(1 + 2p)$

g $3h = 4(h + 6)$

h $6(m - 10) = -6$

i $8u = 11(u - 3)$

j $27 = 7(2y + 1)$

k $5(2 + 3p) = -8$

l $22x = 9(4x - 3)$

2 In which line has an error been made in solving $5(x - 3) = 25$? Select **A**, **B**, **C** or **D**.

$$5(x - 3) = 25$$

$$5x - 8 = 25 \quad \text{Line 1}$$

$$5x - 8 + 8 = 25 + 8 \quad \text{Line 2}$$

$$5x = 33 \quad \text{Line 3}$$

$$x = \frac{33}{5} \quad \text{Line 4}$$

$$= 6\frac{3}{5}$$

A Line 1

B Line 2

C Line 3

D Line 4

3 Show that $k = 5$ is the solution to $12(k - 1) = 48$.

4 Show that $a = 6$ is the solution to $10 + a = 2(2 + a)$.

5 Solve each equation.

a $8(m + 2) = 5(m + 5)$

b $2(y - 3) = 4(y - 5)$

c $3(2 + x) = 4(1 + x)$

d $5(p + 2) = 3(6 + p)$

e $5n + 6 = 2(2n + 1)$

f $2(4 - 3x) = 4(7 - 3x)$

g $4(3w - 1) = 5(4 + 3w)$

h $-2(x + 1) = 16 - 5x$

i $-8y - 5 = 5(2y - 3)$

6 Show that the solution to $5(2m - 2) = 6(m + 1)$ is $m = 4$.

7 Solve each equation.

a $5(m + 6) + 10 = 3(m + 2) + 20$

b $3(y + 2) - 10 = 2(y - 1) + 5$

c $7y + 2(y + 5) = 4(y - 10)$

d $3x + 4(5 + x) = 6(2 + x) + 20$

e $5y + 2(y - 3) = 4y + 2(2y + 10)$

f $11 - 2(5 + y) = 4(3 + y) - 1$

g $8 - 3(1 - m) = 5(m + 3) + 4$

h $12 - 7(2y - 5) = 6 - 15(2 - 5y)$

Worked solutions

Equations with
brackets

MAT09NAWS10032

Worksheet

Word problems
with equations

MAT09NAWK10079

Worksheet

Angle problems
with algebra

MAT09MGWK000065

Homework sheet

Equations 2

MAT09NAHS10013

Puzzle sheet

Writing and solving
equations

MAT09NAPS00034

7-03 Equation problems

Word problems can often be solved more easily when they are converted into equations. Follow these steps.

- Read the problem carefully and determine what needs to be found: 'What is the question?'
- Use a variable to represent the unknown quantity.
- Write the problem as an equation.
- Solve the equation.
- Answer the problem.

Example 3

When three-quarters of a number is decreased by 8 the result is 46. What is the number?

Solution

Let the number be x .

$$\frac{3x}{4} - 8 = 46$$

Translating from words to algebra.

$$\frac{3x}{4} - 8 + 8 = 46 + 8$$

Adding 8 to both sides.

$$\frac{3x}{4} = 54$$

$$\frac{3x}{4} \times 4 = 54 \times 4$$

Multiplying both sides by 4.

$$3x = 216$$

$$\frac{3x}{3} = \frac{216}{3}$$

Dividing both sides by 3.

$$x = 72$$

$$\text{Check: } \frac{3 \times 72}{4} - 8 = 54 - 8 = 46$$

The number is 72.

Example 4

A rectangle is three times as long as it is wide. If its perimeter is 60 cm, find its dimensions.

Solution

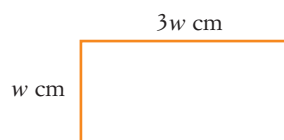
Let the width of the rectangle be w cm. Then the length is $3w$ cm.

The perimeter is $w + 3w + w + 3w$ and this is given as 60.

$$w + 3w + w + 3w = 60$$

$$8w = 60$$

$$w = 7.5$$



$\therefore$ The width of the rectangle is 7.5 cm and the length is $3 \times 7.5 = 22.5$ cm.

Check: The perimeter of a rectangle with dimensions 7.5 cm and 22.5 cm is $7.5 + 22.5 + 7.5 + 22.5 = 60$ cm.

Technology

GeoGebra: Equation problem

MAT09MGTC00005

Example 5

The sum of three consecutive numbers is 150.
Find the numbers.

Consecutive numbers follow each other in order, such as 3, 4, 5.

Solution

Let the first number be x .

The next number is $x + 1$ and the third number is $x + 2$.

Their sum is $x + (x + 1) + (x + 2)$ and this is given as 150.

$$x + x + 1 + x + 2 = 150$$

$$3x + 3 = 150$$

$$3x = 147$$

$$x = \frac{147}{3}$$

$$= 49$$

$\therefore$ The consecutive numbers are 49, 50 and 51.

Check: $49 + 50 + 51 = 150$.

Example 6

Animated example

Applying linear equations

MAT09NAAE00008

Justin is 6 years older than his sister Chelsea.
Their mother is three times Justin's age.

- If the sum of the three ages is 79, write an equation to find Justin's age.
- Solve the equation and find each person's age.

Solution

- Let x = Justin's age.

Chelsea's age is $x - 6$.

Chelsea is 6 years younger than Justin.

The mother's age is $3x$.

$$x + (x - 6) + 3x = 79$$

$$5x - 6 = 79$$

- $5x - 6 = 79$

$$5x = 85$$

$$x = 17$$

Justin is 17 years old.

Chelsea is $17 - 6 = 11$ years old.

Their mother is $3 \times 17 = 51$ years old.

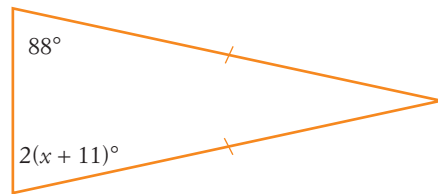
Check: $17 + 11 + 51 = 79$



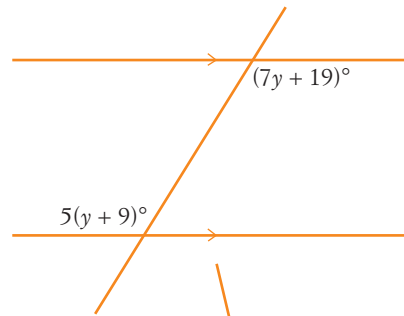
Shutterstock.com/wavebreakmedia

Exercise 7-03 Equation problems

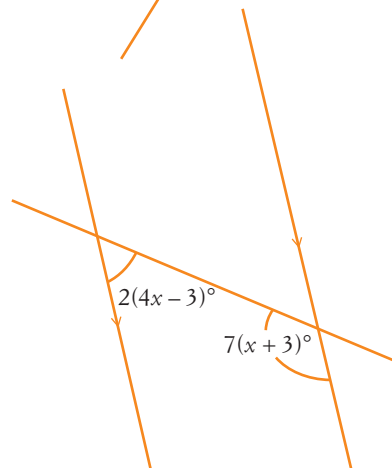
- When 7 is subtracted from four times a certain number, the answer is 37. What is the number?
- If 15 more than a number is 3 more than double the number, what is the number?
- Two-thirds of a number is 16. What is the number?
- When two-fifths of a number is increased by 15 the result is 27. What is the number?
- A rectangle is four times as long as it is wide. The perimeter of the rectangle is 100 cm. Find the dimensions of the rectangle.
- The length of a rectangle is 7 cm longer than its width.
 - Let w be the width of the rectangle. Write an equation for w if the perimeter of the rectangle is 94 cm.
 - Solve the equation and find the dimensions of the rectangle.
- Find the value of x in this triangle.



- Find the value of y .
- What is the size of each alternate angle?



- Calculate the size of each marked angle.



See Example 3

See Example 4

Worked solutions

Equation problems

MAT09NAWS10033

- The sum of two consecutive numbers is 87. Find the numbers.
- The sum of three consecutive numbers is 87. Find the numbers.
- The sum of three consecutive even integers is 168. What are the three integers?
- The sum of three consecutive odd integers is 75. Find the integers.

See Example 5

See Example 6

- 14 Dean's father, Franco, is five times Dean's age. Dean is eight years older than his sister, Helen. The sum of all their ages is 62 years. How old is each person? (*Hint: Let Dean's age be x .*)
- 15 William is three times as old as his daughter, Rebecca. Rebecca is six years younger than her brother, Ben. How old is Rebecca if the sum of their three ages is 76 years?
- 16 In my money box I have only \$1 and \$2 coins. I have 240 coins in total, worth \$318. How many \$2 coins do I have in the money box?
- 17 Aerin bought four ice creams and received \$2.80 change from his \$10 note. How much did each ice cream cost?



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- 18 A boy is twice as tall as his little sister and 30 cm shorter than his father. Their combined height is 3.8 m. Find (in centimetres) the height of each person.
- 19 The length of a rectangle is 9.5 cm longer than its width. The perimeter of the rectangle is 87 cm. Find the dimensions of the rectangle.
- 20 Janine is six years younger than Paul. Paul is three times the age of their son Brett. Brett is five years older than his sister Amanda. The sum of all their ages is 125 years. How old is each person?

Worksheet

Equations 3

MAT09NAWS10080

Puzzle sheet

Equations code puzzle

MAT09NAPS10081

Video tutorial

Solving equations
with fractions

MAT09NAVT00008

7-04 Equations with algebraic fractions

Example 7

Solve each equation.

a $\frac{x+11}{4} + 9 = -3$

b $\frac{a-2}{4} = \frac{2a+5}{3}$

Solution

$$\begin{aligned}
 \text{a} \quad & \frac{x+11}{4} + 9 = -3 \\
 & \frac{x+11}{4} + 9 - 9 = -3 - 9 \\
 & \frac{x+11}{4} = -12 \\
 & \frac{x+11}{4} \times 4 = -12 \times 4 \\
 & x + 11 = -48 \\
 & x + 11 - 11 = -48 - 11 \\
 & x = -59
 \end{aligned}$$

b $\frac{a-2}{4} = \frac{2a+5}{3}$

For equations where all terms are fractions, multiply both sides by a common multiple of the denominators to remove the fractions.

The lowest common multiple (LCM) of 3 and 4 is 12, so multiply both sides by 12.

$$\begin{aligned} 12 \times \frac{(a-2)}{4} &= 12 \times \frac{(2a+5)}{3} \\ \cancel{12}^3 \times \frac{(a-2)}{\cancel{4}_1} &= \cancel{12}^4 \times \frac{(2a+5)}{\cancel{3}_1} \\ 3(a-2) &= 4(2a+5) \\ 3a-6 &= 8a+20 \\ -5a-6 &= 20 \\ -5a &= 26 \\ a &= \frac{26}{-5} \\ a &= -5\frac{1}{5} \end{aligned}$$

Example 8

Solve each equation.

a $\frac{k}{3} + \frac{k}{7} = 4$

b $\frac{4m}{5} - \frac{m}{10} = 4$

Solution

- a Multiply both sides by 21, the LCM of 3 and 7.

$$\begin{aligned} 21\left(\frac{k}{3} + \frac{k}{7}\right) &= 21 \times 4 \\ 21 \times \frac{k}{3} + 21 \times \frac{k}{7} &= 84 \\ \cancel{21}^7 \times \frac{k}{\cancel{3}_1} + \cancel{21}^3 \times \frac{k}{\cancel{7}_1} &= 84 \\ 7k + 3k &= 84 \\ 10k &= 84 \\ k &= \frac{84}{10} \\ k &= 8\frac{2}{5} \end{aligned}$$

- b Multiply both sides by 10, the LCM of 5 and 10.

$$\begin{aligned} 10\left(\frac{4m}{5} - \frac{m}{10}\right) &= 10 \times 4 \\ 10 \times \frac{4m}{5} - 10 \times \frac{m}{10} &= 40 \\ \cancel{10}^2 \times \frac{4m}{\cancel{5}_1} - \cancel{10}^1 \times \frac{m}{\cancel{10}_1} &= 40 \\ 8m - m &= 40 \\ 7m &= 40 \\ m &= \frac{40}{7} \\ m &= 5\frac{5}{7} \end{aligned}$$

Stage 5.3

NSW

Example 9

Solve each equation.

a $\frac{2x-5}{3} - \frac{x+7}{5} = 2$

b $\frac{a+5}{6} + \frac{3a}{4} = 5$

Solution

- a Multiply both sides by 15, the LCM of 3 and 5.

$$\begin{aligned} 15\left(\frac{2x-5}{3} - \frac{x+7}{5}\right) &= 15 \times 2 \\ \cancel{5}^1 \cancel{15}^1 \left(\frac{2x-5}{\cancel{3}_1} - \frac{x+7}{\cancel{5}_1}\right) &= 30 \\ 5(2x-5) - 3(x+7) &= 30 \\ 10x - 25 - 3x - 21 &= 30 \\ 7x - 46 &= 30 \\ 7x &= 76 \\ x &= \frac{76}{7} \\ &= 10\frac{6}{7} \end{aligned}$$

- b Multiply both sides by 12, the LCM of 6 and 4.

$$\begin{aligned} 12\left(\frac{a+5}{6} + \frac{3a}{4}\right) &= 12 \times 5 \\ \cancel{2}^1 \cancel{12}^1 \left(\frac{a+5}{\cancel{6}_1} + \frac{3a}{\cancel{4}_1}\right) &= 60 \\ 2(a+5) + 9a &= 60 \\ 2a + 10 + 9a &= 60 \\ 11a + 10 &= 60 \\ 11a &= 50 \\ a &= \frac{50}{11} \\ &= 4\frac{6}{11} \end{aligned}$$

Exercise 7-04 Equations with algebraic fractions

See Example 7

- 1 Solve each equation.

a $\frac{a+1}{4} + 5 = 15$

b $\frac{x+4}{5} - 10 = 2$

c $\frac{x-5}{5} - 1 = 4$

d $\frac{y-2}{5} - 2 = 6$

e $\frac{p+1}{3} - 11 = 8$

f $5 + \frac{4y+6}{9} = 15$

g $1 + \frac{10x-2}{2} = -10$

h $\frac{3y+5}{4} + 9 = -1$

i $\frac{12-x}{3} - 1 = 6$

j $\frac{15-d}{7} + 10 = 0$

k $\frac{3-2w}{11} + 4 = 5$

l $\frac{6-4p}{9} - 10 = -6$

- 2 Solve each equation.

a $\frac{x+5}{4} = \frac{x+4}{5}$

b $\frac{y+8}{3} = \frac{y-2}{2}$

c $\frac{2p-1}{4} = \frac{p+5}{3}$

d $\frac{2w+5}{3} = \frac{3w+1}{5}$

e $\frac{8x-4}{3} = \frac{4x+5}{2}$

f $\frac{m-6}{4} = \frac{2m-3}{10}$

g $\frac{2y+4}{3} = \frac{5y-2}{5}$

h $\frac{4x-2}{6} = \frac{x+7}{4}$

i $\frac{12-2x}{4} = \frac{9+x}{3}$

j $\frac{8-3w}{5} = \frac{2w+1}{4}$

k $\frac{7-5x}{2} = \frac{1-9x}{4}$

l $\frac{8-x}{2} = \frac{2x+1}{3}$

See Example 8

- 3 Solve $\frac{x}{3} + \frac{x}{2} = 1$. Select A, B, C or D.

A $x = 1\frac{1}{5}$

B $x = 1\frac{1}{2}$

C $x = 1$

D $x = 2$

4 Solve each equation.

a $\frac{d}{4} + \frac{d}{3} = 14$

b $\frac{k}{2} - \frac{k}{3} = 3$

c $\frac{m}{3} + \frac{m}{2} = 10$

d $\frac{x}{8} + \frac{x}{3} = 10$

e $\frac{p}{3} - \frac{p}{5} = 4$

f $\frac{k}{4} - \frac{k}{5} = 1$

g $\frac{4k}{3} - \frac{k}{5} = 34$

h $\frac{3m}{5} + \frac{m}{2} = 11$

i $\frac{4a}{5} + \frac{2a}{3} = 10$

5 What is the solution to $\frac{p-1}{5} - \frac{p+3}{2} = 4$? Select the correct answer A, B, C or D.

A $p = -12$

B $p = -19$

C $p = 14$

D $p = 3$

6 Solve each equation.

a $\frac{x}{7} + \frac{x-1}{4} = 0$

b $\frac{2p}{3} - \frac{p+1}{6} = 2$

c $\frac{m+2}{2} + \frac{m+1}{3} = 12$

d $\frac{c-2}{5} - \frac{c-3}{2} = 4$

e $\frac{2y-1}{4} + \frac{y+1}{2} = 6$

f $\frac{x-1}{3} + \frac{x-4}{6} = -5$

g $\frac{7+2p}{5} - \frac{1-p}{2} = -1$

h $\frac{6y-1}{4} - \frac{y+2}{3} = 8$

i $\frac{a+5}{4} + 3 = \frac{2a}{5}$

j $\frac{3x-1}{4} = 2x + \frac{5}{6}$

k $\frac{12u+3}{7} = 2u - \frac{u+4}{6}$

l $\frac{m-6}{5} + 2m = \frac{3m-1}{4}$

Stage 5.3

NSW

See Example 9

Mental skills 7A

Maths without calculators

Fraction of a quantity

Learn these commonly-used fractions and their decimal equivalents.

Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{2}{5}$
Decimal	0.5	0.25	0.125	0.75	0.2	0.1	0.05	0.4

Now we will use them to find a fraction or decimal of a quantity.

1 Study each example.

a $\frac{1}{4} \times 72 = 72 \div 4$
 $= 18$

b $\frac{2}{5} \times 40 = \left(\frac{1}{5} \times 40\right) \times 2$
 $= 8 \times 2$
 $= 16$

c $\frac{3}{4} \times 32 = \left(\frac{1}{4} \times 32\right) \times 3$
 $= 8 \times 3$
 $= 24$

d $0.5 \times 66 = \frac{1}{2} \times 66$
 $= 33$

e $0.05 \times 80 = \frac{1}{20} \times 80$
 $= 4$

f $0.125 \times 56 = \frac{1}{8} \times 56$
 $= 7$

2 Now simplify each expression.

a $\frac{1}{2} \times 28$

b $\frac{1}{4} \times 36$

c $\frac{1}{10} \times 70$

d $\frac{1}{8} \times 64$

e $\frac{1}{5} \times 15$

f $\frac{1}{10} \times 80$

g $\frac{2}{5} \times 25$

h $\frac{1}{20} \times 100$

i $\frac{3}{4} \times 44$

j $\frac{1}{8} \times 40$

k 0.25×60

l 0.4×45

m 0.1×260

n 0.125×48

o 0.75×48

p 0.05×120

q 0.2×70

r 0.5×320

s 0.25×56

t 0.125×16

Technology Solving equations on a graphics calculator

In this activity, you will solve equations on a Casio graphics calculator.

1 Select **EQUA** (Equation mode) from the main menu and select **Solver**.

2 At 'Eq:', enter the equation $2x + 3 = 15$ by pressing 2 **[X,θ,T]** + 3 **[=]** 15 and **[EXE]**.
(**[=]** is **[SHIFT]** **[.]**)

3 Select **SOLV** to solve the equation.

The solution is $x = 6$. The screen also shows LHS (Lft) = RHS (Rgt) so the solution is true.

4 Select **REPT** to repeat (solve another equation).

5 Enter the equation $4(2x + 7) = 44$ using the **[(]** keys.

6 Select **SOLV** to solve the equation. What is the solution?

7 To enter the equation $\frac{x+5}{2} = \frac{x-1}{3}$, we need to use the **[(]** and **[a^{b/c}]** keys.

Press **[(]** **[X,θ,T]** + 5 **[)]** **[a^{b/c}]** 2 **[=]** **[(]** **[X,θ,T]** - 1 **[)]** **[a^{b/c}]** 3 and **[EXE]**.

8 Select **SOLV** to solve the equation. What is the solution?

9 Now write 5 equations of your own based on the different types studied in this chapter. Use your graphics calculator to solve them, and write down the equations and solutions in your book. Swap the equations with other students in your class and try to solve their equations using the graphics calculator.

Investigation: Solving $x^2 = c$

1 $x^2 = 25$ has two solutions. What are they?

2 What are the possible solutions for each of the following?

a $x^2 = 9$

b $x^2 = 49$

c $x^2 = 100$

3 What is the inverse operation of 'squaring'?

4 Study this example:

$$x^2 = 81$$

$$x = \pm\sqrt{81} \quad \text{which means } \sqrt{81} \text{ or } -\sqrt{81}$$

$$x = \pm 9 \quad \text{which means } 9 \text{ or } -9$$

$$\text{Check: When } x = 9, x^2 = 9^2 = 81$$

$$\text{When } x = -9, x^2 = (-9)^2 = 81$$

Now use the same method to solve each equation and check your answers.

a $m^2 = 1$ **b** $k^2 = 64$

5 How many solutions does each quadratic equation have?

a $m^2 = 1$ **b** $k^2 = 64$ **c** $x^2 = 81$

6 Do the following quadratic equations have solutions? (Give reasons for your answers.)

a $w^2 = -1$ **b** $y^2 = -64$ **c** $h^2 = -81$

7 Write an example of a quadratic equation that has only *one* solution.

7-05 Simple quadratic equations $ax^2 = c$

An equation in which the highest power of the variable is 2 is called a **quadratic equation**, for example, $x^2 = 5$, $3m^2 + 7 = 10$, $d^2 - 4 = 0$ and $4y^2 - 3y = 8$.

Summary

The quadratic equation $x^2 = c$ (where c is a positive number) has two solutions, $x = \pm\sqrt{c}$ (which means $x = \sqrt{c}$ and $x = -\sqrt{c}$).

Example 10

Solve each quadratic equation.

a $y^2 = 16$ **b** $p^2 = 65$ **c** $5a^2 = 245$

Solution

a $y^2 = 16$

$$y = \pm\sqrt{16}$$

$$= \pm 4$$

Finding the square root of both sides.

b $p^2 = 65$

$$p = \pm\sqrt{65}$$

Finding the square root of both sides.

65 is not a square number so leave the answer as a surd.

c $5a^2 = 245$

$$a^2 = \frac{245}{5}$$

Dividing both sides by 5.

$$a^2 = 49$$

$$a = \pm\sqrt{49}$$

$$= \pm 7$$

Example 11

Solve each quadratic equation, writing the solution correct to one decimal place.

a $4x^2 = 600$

b $\frac{3h^2}{5} = 26$

Solution

a $4x^2 = 600$

$$x^2 = \frac{600}{4}$$

$$x^2 = 125$$

$$x = \pm\sqrt{125}$$

$$= \pm 11.1803 \dots$$

$$\approx \pm 11.2$$

b $\frac{3h^2}{5} = 26$

$$3h^2 = 26 \times 5$$

$$3h^2 = 130$$

$$h^2 = \frac{130}{3}$$

$$h^2 = 43\frac{1}{3}$$

$$h = \pm\sqrt{43\frac{1}{3}}$$

$$= \pm 6.5828 \dots$$

$$\approx \pm 6.6$$

Exercise 7-05 Simple quadratic equations $ax^2 = c$

See Example 10 1 Solve each quadratic equation, writing the solutions as surds if necessary.

a $m^2 = 144$

b $x^2 = 400$

c $y^2 = 225$

d $k^2 = 59$

e $y^2 = 10$

f $w^2 = 36$

g $8x^2 = 200$

h $9t^2 = 81$

i $\frac{a^2}{2} = 8$

j $5k^2 = 40$

k $3w^2 = 30$

l $2d^2 = 288$

m $\frac{k^2}{2} = 8$

n $\frac{w^2}{10} = 7$

o $4x^2 = 1$

p $\frac{m^2}{3} = 27$

q $8y^2 = 40$

r $2p^2 + 3 = 21$

s $3k^2 = 48$

t $\frac{y^2}{5} - 2 = 9$

u $6x^2 = 42$

See Example 11 2 Solve each equation, writing the solution correct to one decimal place.

a $m^2 = 20$

b $b^2 = 17$

c $v^2 = 6$

d $2p^2 = 35$

e $9k^2 = 63$

f $\frac{x^2}{5} = 8$

g $\frac{k^2}{16} = 6$

h $\frac{7u^2}{10} = 2$

i $6y^2 = 84$

j $\frac{3w^2}{4} = 20$

k $a^2 + 11 = 28$

l $2y^2 - 14 = 63$

3 Explain why the quadratic equation $k^2 + 25 = 0$ has no solutions.

4 State which of these quadratic equations has no solutions. Give reasons.

a $x^2 = -9$

b $2k^2 + 5 = 9$

c $3m^2 + 8 = 4$

d $\frac{9w^2}{2} - 1 = 1$

e $4 + \frac{d^2}{3} = 8$

f $\frac{5a^2}{2} + 3 = 2$

Investigation: Solving $x^3 = c$

- $x^3 = 27$ has only one solution. What is it?
- What is the solution to each of the following equations?
a $x^3 = 125$ **b** $x^3 = 64$ **c** $x^3 = -8$
- What is the inverse operation of 'cubing'?
- Study this example:
 $x^3 = -27$
 $x = \sqrt[3]{-27}$
 $x = -3$ The cube root of a negative number is also negative.
 Check: When $x = -3$, $x^3 = (-3)^3 = -27$
 Now use the same method to solve each equation and check your answers.
a $r^3 = -1000$ **b** $u^3 = 216$
- How many solutions does each cubic equation have?
a $n^3 = 1$ **b** $t^3 = -343$ **c** $x^3 = -1$

7-06 Simple cubic equations $ax^3 = c$

An equation in which the highest power of the variable is 3 is called a **cubic equation**, for example, $x^3 = 12$, $2m^3 + 1 = 25$, $d^3 - 14 = 4$ and $x^3 - 3x^2 + 5x + 4 = 0$.

Summary

The cubic equation $x^3 = c$ has one solution: $x = \sqrt[3]{c}$

Example 12

Solve each cubic equation.

a $n^3 = 729$ **b** $d^3 = 40$ **c** $-3y^3 = 1029$

Solution

a $n^3 = 729$
 $n = \sqrt[3]{729}$
 $= 9$

Finding the cube root of both sides.

b $d^3 = 40$
 $d = \sqrt[3]{40}$

Finding the cube root of both sides.
 40 is not a cube number so leave the answer as a surd.

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NSW

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c $-3y^3 = 1029$

$$y^3 = \frac{1029}{-3}$$

Dividing both sides by -3 .

$$y^3 = -343$$

$$y = \sqrt[3]{-343}$$

$$= -7$$

Example 13

Solve each cubic equation, writing the solution correct to one decimal place.

a $2h^3 = -18$

b $\frac{z^3}{7} = 11$

Solution

a $2h^3 = -18$

$$h^3 = \frac{-18}{2}$$

$$h^3 = -9$$

$$h = \sqrt[3]{-9}$$

$$= -2.0800 \dots$$

$$\approx -2.1$$

b $\frac{z^3}{7} = 11$

$$z^3 = 11 \times 7$$

$$z^3 = 77$$

$$z = \sqrt[3]{77}$$

$$= 4.2543 \dots$$

$$\approx 4.3$$

Exercise 7-06 Simple cubic equations $ax^3 = c$

See Example 12

1 Solve each cubic equation, writing the solutions as surds if necessary.

a $r^3 = 1$

b $k^3 = 216$

c $d^3 = -27$

d $x^3 = -45$

e $w^3 = 100$

f $f^3 = 64$

g $4m^3 = 32$

h $2t^3 = -250$

i $\frac{c^3}{2} = 108$

j $-3k^2 = 192$

k $7a^3 = 105$

l $\frac{2p^3}{9} = -4$

m $4q^3 = 665.5$

n $\frac{n^3}{5} = -345.6$

o $8s^3 = 150$

p $\frac{e^3}{8} = -1$

q $-6y^3 = 40$

r $2v^3 - 10 = 1014$

s $\frac{4b^3}{5} = 12$

t $\frac{z^3}{6} + 4 = -8$

u $-4x^3 = -144$

See Example 13

2 Solve each equation, writing the solution correct to one decimal place.

a $c^3 = 47$

b $g^3 = 151$

c $y^3 = -450$

d $-8p^3 = 728$

e $3u^3 = 245$

f $\frac{x^3}{5} = -11$

g $-\frac{h^3}{20} = 25$

h $\frac{11a^3}{7} = 80$

i $-6d^3 = 186$

j $\frac{5v^3}{8} = -27$

k $a^3 - 45 = 220$

l $4j^3 + 72 = -166$

- 3 a Does a cubic equation of the form $x^3 = c$ always have a solution?
b When is the solution to $x^3 = c$ positive?
c When does $x^3 = c$ have two solutions?

Stage 5.3

7-07 Equations and formulas

Worksheet

Working with formulas

MAT09NAWK10082

Homework sheet

Equations revision

MAT09NAHS10014

A **formula** is an algebraic equation which shows a relationship between variables. For example, the formula for the area of a circle is $A = \pi r^2$, where A is the area and r is the radius of the circle (π is a constant). Because the formula is for the area, A is called the **subject** of the formula and it is the variable on its own on the left-hand side of the '=' sign.

Example 14

The formula for the perimeter (P) of a rectangle of length l and width w is given by $P = 2(l + w)$. Use the formula to find:

- a the perimeter of a rectangle with length 20 cm and width 9 cm
b the width of a rectangle if its length is 12 m and its perimeter is 70 m
c the length of a rectangle if its width is 42 cm and its perimeter is 1.8 m.

Solution

- a $l = 20, w = 9$:

$$\begin{aligned} P &= 2(l + w) \\ &= 2(20 + 9) \\ &= 2 \times 29 \\ &= 58 \end{aligned}$$

$\therefore$ The perimeter is 58 cm.

- b $l = 12, P = 70$:

$$\begin{aligned} P &= 2(l + w) \\ 70 &= 2(12 + w) \\ 70 &= 24 + 2w \\ 46 &= 2w \\ w &= \frac{46}{2} \\ &= 23 \end{aligned}$$

$\therefore$ The width is 23 m.

- c $w = 42, P = 1.8 \text{ m} = 180 \text{ cm}$ (since w is given in cm).

$$\begin{aligned} P &= 2(l + w) \\ 180 &= 2(l + 42) \\ 180 &= 2l + 84 \\ 96 &= 2l \\ l &= \frac{96}{2} \\ &= 48 \end{aligned}$$

$\therefore$ The length is 48 cm.

Example 15

The cost of hiring a portable sound system for a party is \$80 plus \$15 per hour. The cost can be represented by the formula $C = 80 + 15h$ where C is the total cost (in dollars), and h is the number of hours.

- Find the cost of hiring the sound system for 4 hours.
- A family is willing to spend \$300 for hiring the sound system. What is the maximum number of *whole* rental hours that the family can afford?



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Solution

a $h = 4$:

$$\begin{aligned} C &= 80 + 15h \\ &= 80 + 15 \times 4 \\ &= 140 \\ \therefore \text{The cost is \$140.} \end{aligned}$$

b $C = 300$:

$$\begin{aligned} C &= 80 + 15h \\ 300 &= 80 + 15h \\ 220 &= 15h \\ h &= \frac{220}{15} \\ &= 14\frac{2}{3} \\ \therefore \text{The maximum number of whole hours is 14.} \end{aligned}$$

Exercise 7-07 Equations and formulas

- 1 Given the formula $y = 5x + b$, find:

a y if $x = 5$ and $b = 3$

b y if $x = -1$ and $b = 16$

c b if $y = 40$ and $x = 3$

d b if $y = -6$ and $x = -1$

e x if $y = 27$ and $b = 12$

f x if $y = 64$ and $b = -16$

See Example 14

- 2 A temperature in degrees Celsius ($^{\circ}\text{C}$) can be converted to degrees Fahrenheit ($^{\circ}\text{F}$) using the formula $F = \frac{9C}{5} + 32$. Convert each temperature to $^{\circ}\text{F}$.

a 35°C

b -10°C

c 16°C



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- 3 Use the formula in question 2 to convert each temperature to °C, correct to one decimal place.
a 100°F **b** -45°F **c** 78°F
- 4 The formula $A = \frac{1}{2}(a + b)h$ is used to find the area of a trapezium, where A is the area, a and b are the lengths of the parallel sides, and h is the perpendicular height between them. Use the formula to find:
a the area of a trapezium with height 6 cm and parallel sides of length 9 cm and 15 cm
b the height of a trapezium if its area is 420 cm² and it has parallel sides of length 22 cm and 20 cm
c the length of one side of a trapezium if its parallel side is 20.5 m, its area is 318 m² and its height is 12 m.
- 5 Find the value of S in the formula $M = \frac{kS}{5}$ if $M = 12.6$ and $k = 3.15$. Select **A**, **B**, **C** or **D**.
A 60 **B** 7.938 **C** 20 **D** 0.8
- 6 The cost C (in dollars) of hiring a limousine is given by the formula $C = 180 + 90h$, where h is the number of hours of hire. Find:
a the cost of hiring a limousine for 4 hours
b the cost of hiring a limousine for 2 days
c the number of hours for which you could hire a limousine for \$720
d the maximum number of whole hours for which a limousine could be hired at a cost of \$1000.
- 7 The profit, P , made by a DVD store is given by $P = 5x - 900$, where x represents the number of DVDs sold. Find:
a the profit made when 232 DVDs are sold
b the number of DVDs sold if the profit is \$1635.
- 8 A catering company charges \$ C for a function with P people using the formula $C = 75 + 12.5P$
a How much does the company charge for a function with 10 guests?
b Find the cost of catering for a group of 60 people.
c Diane has \$640 to spend on catering for her next party. What is the maximum number of people she can invite?
- 9 If $P = 52$ and $l = 4$, find w if $P = 2(l + w)$. Select **A**, **B**, **C** or **D**.
A 9 **B** 12 **C** 18 **D** 22
- 10 The temperature T (in °C) of a hot liquid as it cools is given by the formula $T = 100 - 17.5h$, where h is the number of hours it has been cooling. Find:
a the temperature of the liquid after 2 hours
b the temperature of the liquid after 30 minutes
c the number of hours it takes for the temperature of the liquid to reach 30°C.
- 11 Archeologists use the formula $H = 2.52t + 75.8$ to estimate the height H cm of a man when the tibia (shin) bone length t cm is measurable.
a An intact male tibia bone measuring 42 cm long was found. Estimate the height of the male to the nearest centimetre.
b Estimate, correct to the nearest centimetre, the length of the tibia bone of a male of height 174 cm.

See Example 15

Worked solutions
Equations and formulas
MAT09NAWS10034

Stage 5.3

NSW

Puzzle sheet

Formulas squaresaw

MAT09NAPS10083

7-08 Changing the subject of a formula

In the formula $v = u + at$, v is the **subject** of the formula. When the formula is rearranged so that one of the other variables becomes the subject, the process is called **changing the subject of the formula**. To change the subject of a formula, use the same rules as for solving an equation. The answer is not a number but an algebraic expression (another formula).

Example 16

- For the formula $v = u + at$, change the subject to a .
- Make b the subject of the formula $x = \sqrt{b^2 - 4ac}$.
- Given $P = \frac{m-1}{m+1}$, change the subject of the formula to m .

Solution

- To make a the subject of the formula, solve it like an equation for a .

$$v = u + at$$

$$u + at = v$$

$$at = v - u$$

$$a = \frac{v-u}{t}$$

Swapping sides so new subject a appears on the LHS.

Subtracting u from both sides.

Dividing both sides by t .

- $x = \sqrt{b^2 - 4ac}$

$$\sqrt{b^2 - 4ac} = x$$

$$b^2 - 4ac = x^2$$

$$b^2 = x^2 + 4ac$$

$$b = \pm\sqrt{x^2 + 4ac}$$

Swapping sides.

Squaring both sides.

Adding $4ac$ to both sides.

Taking the positive and negative square root of both sides.

- $P = \frac{m-1}{m+1}$

$$\frac{m-1}{m+1} = P$$

$$m-1 = P(m+1)$$

$$= Pm + P$$

$$m - Pm = P + 1$$

$$m(1 - P) = P + 1$$

$$m = \frac{P+1}{1-P}$$

Swapping sides.

Multiplying both sides by $m+1$.

Expanding.

Moving the m -terms to the LHS, the 1 to the RHS.

Factorise m from the LHS.

Dividing both sides by $1 - P$.

Exercise 7-08 Changing the subject of the formula

- Change the subject of each formula to x .
 - $d = x + c$
 - $y = mx + b$
 - $p = ax - y$
 - $y = \frac{x}{m} + p$
 - $k = mrx$
 - $v = \frac{m}{x}$
 - $A = \frac{x+y}{2}$
 - $c = ax + by$
 - $A = \frac{1}{2}h(x+y)$
- The volume of a pyramid has the formula $V = \frac{1}{3}Ah$, where A is the area of the base and h is the perpendicular height. Which of the following is the correct formula for A ? Select the correct answer **A**, **B**, **C** or **D**.
 - $A = \frac{1}{3}Vh$
 - $A = \frac{3}{Vh}$
 - $A = 3Vh$
 - $A = \frac{3V}{h}$
- Make y the subject of each formula.
 - $m = an + ay$
 - $x = y^2$
 - $k = \frac{x}{y}$
 - $Q = P + \sqrt{y}$
 - $x^2 + y^2 = r^2$
 - $M = ny^2$
 - $m = \frac{r}{\sqrt{y}}$
 - $b = c^2 - 2ay$
 - $t = \sqrt{y-x}$
- Change the subject of each formula to the pronumeral shown.
 - $V = lbh$ $h = ?$
 - $A = \pi r^2$ $r = ?$
 - $K = \frac{1}{2}mv^2$ $v = ?$
 - $F = \frac{9C}{5} + 32$ $C = ?$
 - $s = ut + \frac{1}{2}at^2$ $a = ?$
 - $A = \frac{h}{2}(x+y)$ $y = ?$
- What is the correct formula for $p = m(n+x)$ if x is the subject? Select the correct answer **A**, **B**, **C** or **D**.
 - $x = p - mn$
 - $x = \frac{p}{mn}$
 - $x = \frac{p}{m} - n$
 - $x = \frac{p}{n} - m$
- Solve each equation for a .
 - $\frac{a}{5} = ap + q$
 - $ar = 3(a+b)$
 - $Da = D - 2a$
 - $p = \frac{ax}{a+y}$
 - $k = \frac{1-a}{1+a}$
 - $M(a+b) = N(a-b)$
- The cost (\$ C) of a hire car is given by $C = 80 + 4.2d$, where d is the distance travelled in kilometres.
 - Find the cost of hiring the car for a journey of 50 km.
 - Make d the subject of the formula.
 - Find the number of whole kilometres that can be travelled in the hire car for \$402.
- The angle sum of a shape with n sides is A° , where $A = 180(n-2)$.
 - Use the formula to find the angle sum of a shape with 7 sides.
 - Make n the subject of the formula.
 - If the angle sum of a polygon is 1440° , how many sides does it have?
- Make r the subject of the formula $\frac{1}{x} = \frac{1}{r} + \frac{1}{s}$.
- The body-mass index of an adult is given by the formula $B = \frac{m}{h^2}$, where m is the adult's mass in kilograms and h is their height in metres.
 - Change the subject of the formula to h .
 - Hence find, correct to two decimal places, the height of a person with a body mass index of 25 and a mass of 60 kg.

Mental skills 7B

Maths without calculators

Percentage of a quantity

Learn these commonly-used percentages and their fraction equivalents.

Percentage	50%	25%	12.5%	75%	20%	10%	$33\frac{1}{3}\%$	$66\frac{2}{3}\%$
Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{3}$	$\frac{2}{3}$

Now we will use them to find a percentage of a quantity.

1 Study each example.

$$\begin{aligned} \text{a } 20\% \times 25 &= \frac{1}{5} \times 25 \\ &= 5 \end{aligned}$$

$$\begin{aligned} \text{b } 50\% \times 120 &= \frac{1}{2} \times 120 \\ &= 60 \end{aligned}$$

$$\begin{aligned} \text{c } 12.5\% \times 32 &= \frac{1}{8} \times 32 \\ &= 4 \end{aligned}$$

$$\begin{aligned} \text{d } 75\% \times 56 &= \frac{3}{4} \times 60 \\ &= \left(\frac{1}{4} \times 60\right) \times 3 \\ &= 15 \times 3 \\ &= 45 \end{aligned}$$

$$\begin{aligned} \text{e } 33\frac{1}{3}\% \times 27 &= \frac{1}{3} \times 27 \\ &= 9 \end{aligned}$$

$$\begin{aligned} \text{f } 66\frac{2}{3}\% \times 60 &= \frac{2}{3} \times 60 \\ &= \left(\frac{1}{3} \times 60\right) \times 2 \\ &= 20 \times 2 \\ &= 40 \end{aligned}$$

2 Now simplify each expression.

$$\text{a } 25\% \times 44 \quad \text{b } 33\frac{1}{3}\% \times 120$$

$$\text{c } 20\% \times 35 \quad \text{d } 66\frac{2}{3}\% \times 36$$

$$\text{e } 10\% \times 230 \quad \text{f } 12\frac{1}{2}\% \times 48$$

$$\text{g } 50\% \times 86 \quad \text{h } 20\% \times 400$$

$$\text{i } 75\% \times 24 \quad \text{j } 33\frac{1}{3}\% \times 45$$

$$\text{k } 25\% \times 160 \quad \text{l } 10\% \times 650$$

$$\text{m } 12.5\% \times 88 \quad \text{n } 66\frac{2}{3}\% \times 21$$

$$\text{o } 20\% \times 60 \quad \text{p } 75\% \times 180$$

Power plus

1 Solve each equation.

a $\frac{5}{2y} = 14$

b $\frac{7}{r} = \frac{1}{r-3}$

c $\frac{y^2+8}{12} = 4\frac{2}{3}$

d $4m^2 - 3 = m^2 + 72$

e $\frac{3(m+5)}{4} - \frac{2(m-1)}{3} = m - 10 + \frac{m-6}{2}$

2 Given that $W = \frac{X-Y}{X+Y}$, find:

a W when $X = 15$ and $Y = 10$

b W when $X = 6$ and $Y = -12$

c X when $W = 25$ and $Y = 6$

d Y when $W = 5$ and $X = 1$

3 A farmer raises pigs and chickens. From a total of 42 animals she can count 116 legs. Write an equation and solve it to find how many chickens she has.

4 A man is twice as old as his daughter. Ten years ago he was three times as old as her. Write an equation and solve it to determine how old his daughter is now.

5 Consider $x^2 + y^2 = 4$.

a Explain why the smallest value for x is -2 and the largest value for x is 2 .

b Are there any restrictions on the values that y can take? Explain why.

c By making y the subject, show that $y = \pm\sqrt{4-x^2}$.

Chapter 7 review

Language of maths

Puzzle sheet
Equations crossword
MAT09NAPS10084

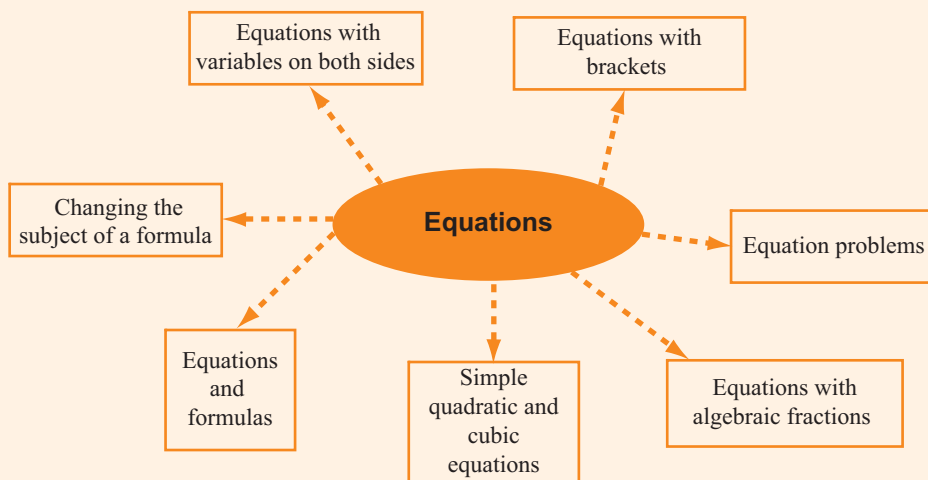
algebraic fraction	brackets	consecutive	cube root
cubic equation	equation	expand	formula
inverse operation	LHS	lowest common multiple (LCM)	linear equation
quadratic equation	RHS	solution'	solve
square root	subject	substitute	surd
undoing	unknown	variable	

- Which method of solving equations involves using inverse operations on both sides of the equation?
- What is the **subject** of the formula $A = \frac{1}{2}(a + b)h$
- What name is given to numbers that follow each other in order, such as 9, 10, 11?
- Write an example of:
 - a quadratic equation
 - a linear equation.
- How many solutions does a linear equation have?
- What does **LHS** stand for?

Topic overview

Worksheet
Mind map: Equations
(Advanced)
MAT09NAWK10086

Copy (or print) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



- 1 Solve each equation. Check your solutions.

See Exercise 7-01

a $2x + 5 = 3x + 4$

b $7 + 4x = x - 8$

c $5 - 6w = 3w - 7$

d $3x + 4 = 2x + 7$

e $5n - 3 = 2n - 15$

f $2d - 8 = -5d - 71$

g $4t = 12 - 4t$

h $8j - 17 = 10j$

i $6 - 3q = 8 - q$

- 2 Write an equation with x on both sides that has the solution $x = 3$.

See Exercise 7-01

- 3 Solve each equation.

See Exercise 7-02

a $2(w - 5) = 4$

b $3(1 + 4n) = 15$

c $5(1 - 3p) = 20$

d $2(3 + x) = 5(x + 1)$

e $3(1 - y) = 4(2 - y)$

f $2(3 - 4x) = -(2x + 3)$

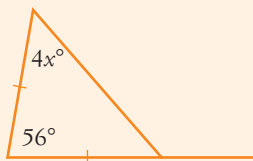
- 4 The length of a rectangle is 6 cm longer than it is wide. The perimeter of the rectangle is 76 cm. Find the dimensions of the rectangle.

See Exercise 7-03

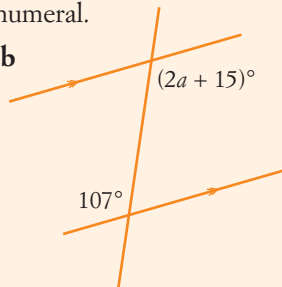
- 5 Find the value of each pronumeral.

See Exercise 7-03

a



b



- 6 If 6 more than a number is the same as 5 more than double the number, what is the number?

See Exercise 7-03

- 7 Solve each equation.

See Exercise 7-04

a $\frac{d-4}{2} + 9 = 4$

b $\frac{8n+6}{3} = \frac{6n+5}{2}$

c $\frac{2p}{3} - \frac{p}{2} = 4$

- 8 Solve each equation.

a $\frac{x-1}{5} + \frac{x-4}{2} = 6$

b $\frac{7n+3}{5} - \frac{5n+4}{3} = 2$

c $\frac{2y-1}{4} + \frac{y+1}{2} = 6$

- 9 Solve each quadratic equation.

a $d^2 = 64$

b $8p^2 = 288$

c $3z^2 = 105$

- 10 Solve each cubic equation.

a $x^3 = 1331$

b $-4h^3 = 864$

c $\frac{t^3}{2} = 62.5$

- 11 The body mass index (BMI) of an adult is $B = \frac{M}{h^2}$, where M is the mass in kilograms and h is the height in metres.

Stage 5.3

See Exercise 7-04

See Exercise 7-05

- a** Find as a whole number the BMI of Dean who is 1.85 m tall and has a mass of 72 kg.

- b** Find the mass of a person with a BMI of 24, who is 2.1 m tall.

Stage 5.3

See Exercise 7-06

See Exercise 7-07

- 12 The cost, C , in dollars, of hiring a taxi is $C = 5 + 2.4d$, where d is the distance travelled in kilometres. Find:

See Exercise 7-07

- a** the cost of a taxi trip if the distance travelled is 15 km

- b** the distance travelled if the cost of a taxi trip was \$78.20.

- 13 Make w the subject of each formula.

Stage 5.3

See Exercise 7-08

a $a = kw + v$

b $p = m(t - w)$

c $x = \frac{1}{w} + y$



8

Number and algebra

Earning money

An Australian person working full-time works an average of 39.9 hours per week. According to the 2011 Census, the median weekly income was \$577 for individuals and \$1481 for families. How much does your family earn each week?



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Chapter outline

	Proficiency strands			
8-01 Wages and salaries	U	F	PS	C
8-02 Overtime pay	U	F	PS	R
8-03 Commission, piecework and leave loading	U	F	PS	C
8-04 Income tax	U	F		C
8-05 PAYG tax and net pay	U	F		C

Note: This Stage 5 (Years 9–10) topic of the NSW syllabus is not part of the Australian curriculum.

Wordbank

allowable deduction A part of a person's yearly income that is not taxed, such as worked related expenses and donations to charities

annual leave loading Extra payment to a worker based on 17.5% of 4 weeks annual leave

income tax A tax paid to the government based on the size of a person's income

net pay Pay received after deductions from gross pay; 'take-home' pay

overtime Time worked beyond normal working hours, such as at night or on weekends, at a higher rate of pay

salary A fixed yearly amount of money that is paid weekly, fortnightly or monthly, not dependent on the number of hours worked

time-and-a-half Overtime pay that is calculated at 1.5 times the normal pay rate

wage An amount of money paid to people for work, calculated on the number of hours worked

In this chapter you will:

- calculate earnings from wages, salaries, overtime, commission and piecework
- calculate weekly, fortnightly, monthly and yearly incomes
- calculate annual leave loading
- use tables to calculate income tax and PAYG tax
- calculate net pay from gross pay

SkillCheck

Worksheet

StartUp assignment 8

MAT09NAWK10087

Worksheet

Time and money
calculations

MAT09NAWK10088

- Write 8.2% as a decimal.
- Find 15% of \$15.
- How many:

a days in one year?	b weeks in one year?	c days in 4 weeks?
d months in 3 years?	e days in a fortnight?	f weeks in a quarter of a year?
g months in 5 years?	h fortnights in one year?	
- Round each amount to the nearest cent.

a \$7.6831	b \$21.4209	c \$524.7373
d \$92.3557	e \$3085.6382	f \$106.3446
- Evaluate (correct to the nearest cent where necessary):

a $38 \times \$12.87 \times 1.5$	b $\$75\,430 \div 12$	c $0.42 \times \$7500$
d $0.13 \times \$9217$	e $\$4758 \times 0.073 \times 2$	f $0.47 \times \$31\,726$
g $\$69\,870 \div 52.18$	h $\$2150 \times 1.09$	i $\$3276 \times 0.175 \times 4$
- Evaluate $\$4650 + 0.3 \times (\$43\,210 - \$37\,000)$.

NSW

8-01 Wages and salaries

Worksheet

Wages and salaries

MAT09NAWK10090

People who work usually earn a wage or a salary. A **wage** is calculated by the number of hours worked and is usually paid weekly. People such as supermarket cashiers, electricians and gardeners earn a wage. Wage earners can earn more income by working extra hours (overtime).



Fairfax/Virginia Star

A **salary** is a fixed annual amount, paid weekly, fortnightly or monthly. People such as architects, software designers and the police earn a salary. Salary earners do not earn overtime pay but can receive benefits such as a company car, expense account, shares in the company or paid medical expenses.

Example 1

Tim's first job pays him \$15.76 per hour. If he works 38 hours each week, calculate his weekly income.

Solution

$$\begin{aligned}\text{Weekly income} &= 38 \times \$15.76 \\ &= \$598.88\end{aligned}$$

Summary

- 1 year = 12 months
- 1 fortnight = 2 weeks
- 1 year = 52 weeks for wage earners
- 1 year = 52.18 weeks for salary earners

To convert a salary to a weekly amount, divide by 52.18 because

$$1 \text{ year} = 365\frac{1}{4} \text{ days} = 365\frac{1}{4} \div 7 \text{ weeks} \approx 52.18 \text{ weeks.}$$

Example 2

Leanne earns a salary of \$62 400 p.a. How much (correct to the nearest cent) does she earn:

← 'per annum' (p.a.) = per year

- a** each week? **b** each fortnight? **c** each month?

Solution

$$\begin{aligned}\text{a Weekly income} &= \$62\,400 \div 52.18 \\ &= \$1195.8604\dots \\ &\approx \$1195.86\end{aligned}$$

$$\begin{aligned}\text{b Fortnightly income} &= 2 \times \$1195.86 & 1 \text{ fortnight} = 2 \text{ weeks} \\ &= \$2391.72\end{aligned}$$

$$\begin{aligned}\text{c Monthly income} &= \$62\,400 \div 12 & 1 \text{ year} = 12 \text{ months} \\ &= \$5200\end{aligned}$$

Example 3

Sarah earns a salary of \$54 706 and works 38 hours per week. Anh earns \$982 per week for working 37 hours. Who has the higher hourly rate of pay?

Solution

$$\begin{aligned}\text{Sarah's hourly pay rate} &= \$54\,706 \div 38 \\ &= \$1439.6315789473684 \dots \\ &\approx \$1439.63\end{aligned}$$

$$\begin{aligned}\text{Anh's hourly pay rate} &= \$982 \div 37 \\ &= \$26.54054054054054 \dots \\ &\approx \$26.54\end{aligned}$$

Sarah has the higher hourly rate of pay.

Exercise 8-01 Wages and salaries

Express all answers correct to the nearest cent where necessary.

See Example 1

- Find the weekly wage for each of the following workers.
 - Alex, who earns \$16.24 per hour and works for 40 hours.
 - Brianna, who earns \$24.60 per hour and works for 38.5 hours.
 - Chris, who works 8 hours a day, Monday to Friday, and is paid \$18.38 per hour.
 - Lionel, who works Monday to Wednesday, 6 hours per day, and is paid \$15.85 per hour.
- What is Chloe's annual salary if she earns \$879 per week?
- If Sunil earns \$3672 per month, what is his annual salary? Select **A**, **B**, **C** or **D**.
A \$95 472 **B** \$88 128 **C** \$36 720 **D** \$44 064
- Rhys earns \$2083.74 per fortnight. How much does he earn in a year?

See Example 2

- Mr Kelly earns a salary of \$68 340 p.a. How much does he earn:
 - each week?
 - each fortnight?
 - each month?
- Mitchell earns \$11.40 per hour before midnight and \$13.75 per hour after midnight for each shift. Calculate his income for working each of these shifts:
 - 7:00 p.m. to 3:00 a.m.
 - 9:00 p.m. to 5:30 a.m.
 - 10:30 p.m. to 6:00 a.m.
- Dana earns \$8.47 an hour and works for 38 hours each week. Darren earns \$9.16 per hour for his 35 hours of work. Who earns more per week and by how much?
- Abdul considers two jobs, one with a salary of \$47 640 p.a. and the other with a fortnightly income of \$1865. Calculate the weekly income for each job to determine which pays more.
- Who earns the higher hourly rate of pay for each pair of workers, and by how much?
 - Mitch worked 40 hours for \$584 pay.
Marisol worked 38 hours for \$501.60 pay.
 - Philip worked 30 hours for \$441 pay.
Paula worked 27 hours for \$384.75 pay.

See Example 3

- c** Faith worked Monday to Friday, 8.5 hours per day, for \$892.50 pay.
Fabian worked Monday to Thursday, 9 hours per day, for \$615.60 pay.
- d** Ranjan earned \$62 760 p.a., working 9:00 a.m. to 5:30 p.m., five days per week.
Rhonda earned \$68 620 p.a., working 6:00 a.m. to 6:00 p.m., Monday to Thursday.
- 10** A printer earns \$22 per hour, working from 8:00 a.m. to 4:30 p.m., Monday to Friday.
Find how much he earns:
a each day **b** each week **c** each month.
- 11** Ziad earns \$8640 per month. Calculate his:
a annual salary **b** weekly income
- 12** Parker earns a salary of \$38 720 p.a. and works 42 hours per week. Find how much Parker earns:
a per month **b** per week **c** per hour
- 13** Liam works 9 hours per day from Tuesday to Friday. He is paid \$17.48 per hour. He also works on Saturday for 6 hours at a special rate of \$19.12 per hour. How much did Liam earn for the week?
- 14** Charlie works from 7:30 a.m. to 3:00 p.m. Monday to Wednesday and from 8:00 a.m. to 4:00 p.m. Thursday and Friday. His hourly pay rate is \$16.38. Calculate Charlie's weekly wage.
- 15** Ross packs shelves at a supermarket and is paid \$9.65 per hour on weekdays and \$11.60 per hour on Saturdays. He worked the following hours last week.

Day	Hours Worked
Monday	6:30 a.m. – 2:00 p.m.
Tuesday	8:00 a.m. – 4:00 p.m.
Wednesday	11:00 a.m. – 4:30 p.m.
Friday	9:30 a.m. – 3:30 p.m.
Saturday	10:00 a.m. – 4:00 p.m.

What was Ross's total income for the week?

- 16** Jack is a café manager and works the following hours in a week.

Day	Hours Worked
Monday	6:30 a.m. – 2:30 p.m.
Tuesday	7:00 a.m. – 4:00 p.m.
Thursday	11:30 a.m. – 8:30 p.m.
Friday	7:30 a.m. – 3:30 p.m.
Saturday	10:30 a.m. – 5:00 p.m.

He is paid at the following rates.

Day	Rate of pay
Monday to Friday	\$10.22 per hour
Thursday after 4:00 p.m. and all day Saturday	\$15.35 per hour

What is Jack's total income for the week?



Alamy/© Radius Images

Worked solutions
Wages and salaries
MAT09NAWS10036

NSW

8-02 Overtime pay

Wage earners receive **overtime** pay when they work more than their standard number of hours.

Summary

The two most common rates of overtime pay are:

- **time-and-a-half** = $1.5 \times$ normal hourly rate
- **double time** = $2 \times$ normal hourly rate

Example 4

Siki works 35 hours at \$12.74 per hour and 4 hours overtime at time-and-a-half. Calculate his total earnings.

Solution

$$\text{Normal pay} = 35 \times \$12.74 = \$445.90$$

$$\text{Overtime pay} = 4 \times \$12.74 \times 1.5 = \$76.44$$

$$\text{Total earnings} = \$445.90 + \$76.44 = \$522.34$$

Alternative method:

$$\begin{aligned} \text{Equivalent number of normal hours} &= 35 + (4 \times 1.5) \\ &= 41 \end{aligned}$$

$$\begin{aligned} \text{Total earnings} &= 41 \times \$12.74 \\ &= \$522.34 \end{aligned}$$

Example 5

Charlotte earns time-and-a-half on Saturdays and double time on Sundays. She works 38 hours from Monday to Friday, 6 hours on Saturday and 4 hours on Sunday. Calculate Charlotte's total earnings if her normal rate of pay is \$14.50 per hour.

Solution

$$\begin{aligned} \text{Normal pay (Monday to Friday)} &= 38 \times \$14.50 \\ &= \$551 \end{aligned}$$

$$\begin{aligned} \text{Saturday pay} &= 6 \times \$14.50 \times 1.5 \\ &= \$130.50 \end{aligned}$$

$$\begin{aligned} \text{Sunday pay} &= 4 \times \$14.50 \times 2 \\ &= \$116 \end{aligned}$$

$$\begin{aligned} \text{Total pay} &= \$551 + \$130.50 + \$116 \\ &= \$797.50 \end{aligned}$$

Example 6

Peter and Vinh are cadet hamburger chefs. Last week, Peter worked for 40 hours and Vinh worked for 43 hours. Calculate their respective total earnings based on the following award.



Award for cadet hamburger chefs	
Normal rate = \$12.80 per hour	
Normal rate	For first 38 hours worked
Time-and-a-half	For the next 3 hours
Double time	For each additional hour after that

Solution

For Peter (40 hours):

$$\begin{aligned}\text{Normal pay} &= 38 \times \$12.80 \\ &= \$486.40\end{aligned}$$

First 38 hours.

$$\begin{aligned}\text{Time-and-a-half pay} &= 2 \times \$12.80 \times 1.5 \\ &= \$38.40\end{aligned}$$

Remaining 2 hours.

$$\begin{aligned}\text{Total earnings} &= \$486.40 + \$38.40 \\ &= \$524.80\end{aligned}$$

For Vinh (43 hours):

$$\text{Normal pay} = \$486.40$$

First 38 hours.

$$\begin{aligned}\text{Time-and-a-half pay} &= 3 \times \$12.80 \times 1.5 \\ &= \$57.60\end{aligned}$$

Next 3 hours.

$$\begin{aligned}\text{Double time pay} &= 2 \times \$12.80 \times 2 \\ &= \$51.20\end{aligned}$$

Remaining 2 hours.

$$\begin{aligned}\text{Total earnings} &= \$486.40 + \$57.60 + \$51.20 \\ &= \$595.20\end{aligned}$$

Example 7

Last week Rani worked her normal 38 hours, then 4 hours at time-and-a-half and 5 hours at double time. She was paid \$909.90 for the week. Find her hourly rate of pay.

Solution

$$\begin{aligned}\text{Equivalent number of normal hours} &= 38 + (4 \times 1.5) + (5 \times 2) \\ &= 54\end{aligned}$$

$$\begin{aligned}\text{Hourly rate of pay} &= \$909.90 \div 54 \\ &= \$16.85\end{aligned}$$

Exercise 8-02 Overtime pay

See Example 4

- 1 The hourly rate of pay for workers in a factory is \$17.65. Calculate the total earnings for each employee.
 - a Soo-jin works 28 hours at normal rates.
 - b Jemima works 33.5 hours at normal rates.
 - c Ethan works 38 hours at normal rates and 3 hours at time-and-a-half.
 - d Stavros works 35 hours at normal rates and 5.5 hours at time-and-a-half.
 - e Lana works 37 hours at normal rates, 6 hours at time-and-a-half and 4 hours at double-time.
 - f Marco works 40 hours at normal rates, 5 hours at time-and-a-half and 4 hours at double-time.
- 2 Pavel works 20 hours at normal rates with an hourly pay rate of \$18.45 per hour and 4 hours at time-and-a-half. Which calculation could be used to find his weekly wage? Select **A**, **B**, **C** or **D**.

A $\$18.45 \times 24$	B $\$18.45 \times 20 + \$8.45 \times 24 \times 1.5$
C $\$18.45 \times 20 + \$18.45 \times 4 \times 1.5$	D $\$18.45 \times 24 \times 1.5$
- 3 Josie works 38 hours at \$14.67 per hour and 3 hours overtime at time-and-a-half. Calculate her total earnings.
- 4 Wayne is a store assistant manager who works 40 hours at \$17.23 per hour, 5 hours overtime at time-and-a-half and 3.5 hours double-time. Calculate his total earnings.



Alamy/© Keith Morris

- 5 Pancake House pays employees normal rates for Monday to Friday, time-and-a-half for Saturday and double-time for Sunday. Mia works 36 hours Monday to Friday, 5.5 hours on Saturday and 4 hours on Sunday. Calculate her total earnings if her normal rate of pay is \$15.40 per hour.
- 6 Anu works at a mobile phone store. She is paid normal rates for shifts worked on Monday to Friday, time-and-a-half for Thursday nights from 5 p.m. onwards, and double-time on Saturdays. In a single week, Anu worked 37 normal hours and on Thursday she worked till 8:00 p.m. On Saturday, she worked from 9:30 a.m. until 2:30 p.m. If her normal pay rate is \$13.74 per hour, what is Anu's total wage for the week?
- 7 Holly works from 9:00 a.m. until 4:00 p.m. each day on Monday, Tuesday and Wednesday at a normal rate of \$11.60 per hour. She works Thursday nights from 3:30 p.m. to 8:30 p.m. at time-and-a-half and Sundays from 8:00 a.m. until 4:00 p.m. at double-time. Which of the following calculations could be used to calculate her weekly wage? Select **A**, **B**, **C** or **D**.
- A** $\$11.60 \times 34 + 11.60 \times 1.5 \times 4 + 11.60 \times 12 \times 2$
- B** $\$11.60 \times 21 + 11.60 \times 1.5 \times 5 + 11.60 \times 8 \times 2$
- C** $\$11.60 \times 7 + 11.60 \times 1.5 \times 6 + 11.60 \times 8 \times 2$
- D** $\$11.60 \times (13 + 1.5 \times 11 + 2 \times 12)$
- 8 Elegant Jewellery pays employees at the following rates.

See Example 5

See Example 6

Weekly Award	
Normal rate = \$15.70/h	
Normal	For the first 35 hours
Time-and-a-half	For the next 5 hours
Double-time	For each hour after that

Use the information above to calculate the weekly wages for each employee listed below.

- a Lionel, who works 33 hours b Tashi, who works 36 hours
- c Kylie, who works 40 hours d Mannix, who works 44.5 hours
- e Noah, who works 38.5 hours f Jennifer, who works 42 hours

9

Nuts and bolts machinists' enterprise agreement	
Normal rate	Applies Monday to Friday
Time-and-a-half	For any hours worked on Saturday
Double time	For any hours worked on Sunday

Use the table above to calculate the total earnings for each employee below.

			Hours worked		
	Name	Hourly rate	Mon–Fri	Sat	Sun
a	Alan Douglas	\$9.60	36	4	8
b	Ava Hawkes	\$23.30	38	0	6
c	Seetha Mehta	\$18.60	38	4	2.5
d	Tarek Mifsud	\$20.00	40	8	0
e	Ivan Vitsic	\$30.00	34	4	6
f	John Hackett	\$16.40	36	4.5	6

See Example 7 10 Calculate the normal hourly rate of pay for each person.

- a Sarah earns \$862.40 by working 36 hours at normal rates and 4 hours at double time.
- b Fiona earns \$969.60 by working 37 hours at normal rates, 3 hours at time-and-a-half and 4 hours at double time.
- c Thierry earns \$240 by working 6 hours at normal rates and 4 hours at double time.
- d Sana earns \$163.80 by working 4 hours at time-and-a-half and 3 hours at double time.
- e Phillip earns \$727.60 by working 30 hours at normal rates, 5 hours at time-and-a-half and 3 hours at double time.
- f Tara earns \$76.50 by working 7 hours at normal rates and 1.5 hours at time-and-a-half.

Worked solutions

Overtime pay

MAT09NAWS10037

Technology Calculating incomes

In this activity, we will use a spreadsheet to calculate the incomes of employees at J-mart department store. The award for all workers is as follows.

- Normal rate for up to 36 hours worked
 - Time-and-a-half for the next 4 hours worked
 - Double-time for any hours worked after that
- 1 Create a spreadsheet as shown below, then for each employee enter the correct number of hours for columns D, E and F.

	A	B	C	D	E	F
1	Name	Hourly Pay	Hours Worked	Normal	Time-and-a-half	Double-time
2	R Jones	\$22.85	38			
3	A Watson	\$19.40	34			
4	Q Tran	\$10.67	40			
5	L Djokovic	\$13.90	42			
6	C Lam	\$12.96	39			
7	J Hampden	\$10.76	41.5			
8	L Ngo	\$10.28	43			
9	K Barnett	\$9.22	45.5			

- 2 In cell G1, enter the heading **Total Pay**.
- 3 To calculate R Jones' total pay, enter in cell G2 the formula **=B2*D2+B2*1.5*E2+B2*2*F2**
- 4 **Fill Down** the formula from G2 to G9. Use **Format Cells** to change the values in column G to **Currency**.

Just for the record

Labour Day holiday

Each year, Australians have a public holiday on Monday called Labour Day or Eight-hours Day. This day celebrates the significant improvement of wages and working conditions in the 19th century.

In the early 1800s, the typical Australian worker was working up to 14 hours per day, six days per week, often under difficult conditions. Sick leave or holidays did not exist, and employees could be sacked immediately without reason.

In 1856, Victorian workers won the right to limit their working time to eight hours per day. This was a world first, allowing workers more time in the day for recreation, reading, education and family, dividing the day into '8 hours labour, 8 hours recreation, 8 hours rest (sleep)'. The Eight-hour Day was adopted in all Australian states by the 1920s. There is an Eight Hour monument that stands in central Melbourne with the numbers '888' featured at the top.



© www.coocepictures.com

Labour Day is celebrated on a different day for each state of Australia. Find out when Labour Day is celebrated in each state.

8-03

Commission, piecework and leave loading

Some workers are not paid by the amount of time they work, but by the number of items they make or process.

Commission

Commission is earned by salespeople or agents of a business or company. It is calculated as a percentage of the value of items sold or income made. A fixed amount called a **retainer** may also be paid. Real estate agents and actors' agents earn a commission.

Example 8

Nathan sells cars and is paid a weekly retainer of \$260 plus 3.5% of the value of the cars he sells. Calculate his income if he sells cars worth a total of \$98 000 this week.

Solution

$$\begin{aligned}\text{Nathan's income} &= \$260 + 3.5\% \times \$98\,000 \\ &= \$3690\end{aligned}$$



iStockphoto/kaif9

NSW

Worksheet

Earning money

MAT09NAWK10091

Worksheet

Percentages without calculators

MAT09NAWK10089

Homework sheet

Earning money

MAT09NAHS10004

Skill sheet

Percentage calculations

MAT09NASS10026

Example 9

Corrina is a real estate agent and is paid according to the following commission rates.

- 2% on the first \$100 000 of property sold
- 1.5% on any value thereafter.

Calculate Corrina's commission for selling an apartment for \$380 000.

Solution

$$\begin{aligned}\text{Commission on 1st \$100 000} &= 2\% \times \$100\,000 \\ &= \$2000\end{aligned}$$

$$\begin{aligned}\text{Remaining amount} &= \$380\,000 - \$100\,000 \\ &= \$280\,000\end{aligned}$$

$$\begin{aligned}\text{Commission on remaining amount} &= 1.5\% \times 280\,000 \\ &= \$4200\end{aligned}$$

$$\begin{aligned}\text{Corrina's commission} &= \$2000 + \$4200 \\ &= \$6200\end{aligned}$$

Piecework

Piecework is earned according to the number of items made or tasks completed. Fruit pickers and dressmakers earn money this way.

Example 10

Sue earns \$245 for each garden she designs. How many gardens must Sue design to earn over \$50 000?

Solution

$$\begin{aligned}\text{Number of gardens} &= \$50\,000 \div 245 \\ &= 204.081\dots \\ &\approx 204\end{aligned}$$

Rounding down.

Sue must design at least 204 gardens to earn over \$50 000.

Bonus

A **bonus** is extra money paid to employees who produce work of high quality or quantity, or for meeting an important goal or deadline. Some businesses reward their employees with a Christmas bonus for a productive year's work.

Example 11

After a successful year, a software company gave each employee a 2.5% bonus on their annual income. Find:

- a the bonus received by Shaina, who earns \$1426.50 per week
- b the annual income of Kahil, whose bonus amounted to \$1175.

Solution

$$\begin{aligned}\text{a} \quad \text{Shaina's annual income} &= 52 \times \$1426.50 \\ &= \$74\,178\end{aligned}$$

$$\begin{aligned}\text{Shaina's bonus} &= 2.5\% \text{ of } \$74\,178 \\ &= \$1854.45\end{aligned}$$

$$\text{b} \quad 2.5\% \text{ of Kahil's income} = \$1175$$

$$\begin{aligned}1\% \text{ of Kahil's income} &= \$1175 \div 2.5 \\ &= \$470\end{aligned}$$

Using the unitary method.

$$\begin{aligned}\therefore \text{Kahil's annual income} &= \$470 \times 100 \\ &= \$47\,000\end{aligned}$$

100%

Annual leave loading

Annual leave loading or holiday loading is extra pay given during annual leave (holidays). It is paid at the rate of 17.5% of four weeks normal pay.

Example 12

For her Christmas holidays, Katrina received 4 weeks normal pay plus 17.5% annual leave loading for the 4 weeks. If Katrina's annual salary is \$51 840, find:

- a her normal weekly pay
- b her leave loading
- c her total pay for the four-week holiday.

Solution

$$\begin{aligned}\text{a} \quad \text{Weekly income} &= \$51\,840 \div 52.18 \\ &= \$993.484\,09\dots \\ &\approx \$993.48\end{aligned}$$

$$\begin{aligned}\text{b} \quad \text{Leave loading} &= 17.5\% \times \$993.48 \times 4 \\ &= \$695.436 \\ &\approx \$695.44\end{aligned}$$

$$\begin{aligned}\text{c} \quad \text{Total holiday pay} &= 4 \times \$993.48 + \$695.44 \\ &= \$4669.36\end{aligned}$$

Exercise 8-03 Commission, piecework and leave loading**See Example 8**

- 1 Anita is paid a commission of 3% on the value of kitchenware she sells. How much will Anita earn if she sells goods to the value of \$18 600 in one week?
- 2 Dan is paid a commission of 4.5% of the value of the book sales he makes in excess of \$5000. In one week Dan sells books to the value of \$37 427. Calculate his income for that week.
- 3 A real estate agent is paid a monthly retainer of \$800 and a commission of 1.5% of the value of properties sold. Her sales in January were \$678 600. Calculate her income for January.
- 4 Consider these two salespeople and their method of payment.
 - Perry: \$250 retainer and 4.5% of the value of goods sold.
 - Amy: no retainer and 7.5% of the value of goods sold.

← 'in excess of' means to go over or above

Calculate the income earned by Perry and Amy if they each sell goods worth \$30 000.

- 5 Lara earns 5% commission on the sale of thoroughbred horses. Last year she sold horses worth \$1 640 000. Calculate Lara's commission last year.



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See Example 9

- 6 A real estate agency pays the following rates of commission.
 - 5% on the first \$100 000
 - 3% on the next \$50 000
 - 2% on any value thereafter.

Calculate the commission earned by selling property valued at:

- a** \$240 000 **b** \$316 000 **c** \$395 000

- 7 Xanthe is paid 5% of the value of used cars she sells and 3.5% of the value of new cars. What commission is earned by selling new cars worth \$78 400 and used cars worth \$36 200?
- 8 Calculate the income for each amount of piecework.
 - a assembling 390 boxes at 42 cents per box
 - b delivering 900 pamphlets at 10 cents per pamphlet
 - c assembling 93 prams at \$1.85 per pram
 - d polishing 16 cars at \$11.75 per car
 - e mowing nine lawns at \$34.20 per lawn
 - f hemming 320 pairs of trousers at 68 cents per pair
 - g polishing 108 shoes at \$1.50 per shoe
 - h laying 64 m^2 of paving at \$24.50 per m^2

- 9 Fiona earns 85 cents for each toy she assembles.
- How much does she earn for assembling:
 - 146 toys?
 - 203 toys?
 - Last week Fiona earned \$459. How many toys did she assemble?
 - Find the number of toys Fiona must assemble to earn (at least) the following amounts:
 - \$200
 - \$620



See Example 10

- 10 Ricky earns \$9.80 for each car he washes. Calculate his income if he washes 16 cars.
- 11 A process worker is paid 28 cents per article assembled, for up to 500 articles, then 35 cents per article assembled after that. Calculate the total income earned for assembling each of the following numbers of articles.
- 346
 - 506
 - 1406
- 12 Kevin works at the SuperGlo Hand Car Wash. He earns \$3.50 per car for a wash only and \$9.50 per car for a wash and polish. His job sheet for last week is given below.

Day	Wash only (number of cars)	Wash and polish (number of cars)
Monday	11	4
Tuesday	12	8
Wednesday	7	10
Thursday	16	3
Friday	18	13
Saturday	10	12

Calculate:

- Kevin's total earnings for the week
 - Kevin's average daily earnings
 - Kevin's average hourly rate of pay if he worked for 8.5 hours each day. (Write your answer to the nearest ten cents.)
- 13 A publishing house gives all employees a 4.75% annual bonus for a profitable year. Calculate the annual bonus for each of these incomes.
- \$42 400 p.a.
 - \$65 600 p.a.
 - \$1095 per week
 - \$794 per week
 - \$6750 per month
 - \$3156 per fortnight
- 14 Zlatko, a construction supervisor, has signed a contract worth \$87 000 to ensure that a factory is built by June 30. He will receive a bonus of 1.5% of his contract for each day the job is completed ahead of time. How much will Zlatko earn if the factory is completed on June 22?
- 15 Frank works 40 hours a week and is paid \$13.40 per hour. Calculate:
- Frank's weekly income
 - the annual leave loading of 17.5% on 4 weeks pay
 - Frank's total pay for the four-week holiday.

See Example 11

See Example 12

- 16 Astra is the manager of a computer megastore. Her salary is \$62 000 p.a. Calculate:
- Astra's fortnightly pay
 - the holiday loading Astra would receive when she takes her 4 weeks annual leave
 - her total holiday pay.
- 17 Jamie, a storeman, works from 8:00 a.m. to 4:00 p.m., Monday to Friday. Jamie's rate of pay is \$15.80 per hour. Calculate:
- Jamie's weekly wage
 - Jamie's holiday pay for 4 weeks (including leave loading).

NSW

8-04 Income tax

Worksheet

Income tax tables

MAT09NAWK10092

Worksheet

Tax return form

MAT09NAWK10093

Anyone who earns more than \$18 200 in a **financial year** (1st July to 30th June the following year) must pay **income tax** to the federal government. Income includes money earned from a job or interest earned from bank accounts. The government uses the money paid in tax to fund public services such as schools, hospitals, roads and the defence forces.

Not all of a person's income is taxed. If we use some of our income for work-related expenses or to donate money to charities, these amounts are called **allowable deductions** (or **tax deductions**) and are not taxed. Other examples of allowable deductions are tools of trade, uniforms, car-related expenses, subscriptions to professional organisations and journals.



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Summary

Income tax is calculated on a person's **taxable income**, which is the **gross income** (total earnings) less all allowable deductions, **rounded down to the nearest dollar**.

Taxable income = gross income – allowable deductions

The more a person earns, the higher the rate of tax to be paid. The table below shows how income tax is calculated.

Tax rates for Australian residents	
Taxable income	Tax on this income
0 – \$18 200	Nil
\$18 201 – \$37 000	19c for each \$1 over \$18 200
\$37 001 – \$80 000	\$3 572 plus 32.5c for each \$1 over \$37 000
\$80 001 – \$180 000	\$17 547 plus 37c for each \$1 over \$80 000
\$180 001 and over	\$54 547 plus 45c for each \$1 over \$180 000

Source: © Australian Taxation Office for the Commonwealth of Australia

Example 13

Mrs Henry earned \$35 660 last financial year and has allowable deductions of \$727 in work-related car expenses and \$259 in donations to charities.

- Calculate her taxable income.
- Use the tax table to calculate the income tax Mrs Henry must pay.

Solution

$$\begin{aligned}\text{a Taxable income} &= \$35\,660 - \$727 - \$259 \\ &= \$34\,674\end{aligned}$$

- According to the table, a taxable income of \$34 674 is in the \$18 201 – \$37 000 tax bracket.

$$\begin{aligned}\text{Income tax} &= 0.19 \times (\$34\,674 - \$18\,200) \\ &= \$3130.06\end{aligned}$$

← '19c for each \$1' means 19% or 0.19

Example 14

Alice is an accountant who earned \$78 642 one financial year. She also earned \$132.51 in interest from her bank savings. Alice has allowable deductions of \$287.20 for office supplies and \$75.88 in books. Calculate:

- Alice's taxable income
- the amount of income tax payable

Solution

$$\begin{aligned}\text{a Taxable income} &= \$78\,642 + \$132.51 - \$287.20 - \$75.88 \\ &= \$78\,411.43 \\ &\approx \$78\,411 \quad \text{Rounded down to the nearest dollar.}\end{aligned}$$

- According to the table, a taxable income of \$78 411 is in the \$37 001 – \$80 000 tax bracket.

$$\begin{aligned}\text{Tax payable} &= \$3572 + 0.325 \times (\$78\,411 - \$37\,000) \\ &= \$17\,030.575 \\ &\approx \$17\,030.58\end{aligned}$$

Exercise 8-04 Income tax

- 1 Jake works as a plumber and earned \$73 219 this year. He is entitled to these deductions: tools \$527.40, training courses \$735.26 and office supplies \$321.55. What is Jake's taxable income, rounded down to the nearest dollar?

See Example 13

- 2 Raina earns \$35 660 in a year and has allowable deductions of \$986.40.
 a Calculate her taxable income, rounded down to the nearest dollar.
 b Use the tax table to calculate the income tax Raina must pay.

See Example 14

- 3 Ali had a yearly income of \$90 157 and earned \$240 in dividends from his shares. He also had allowable deductions of \$540 for car expenses and \$332 in the depreciation of his photocopier. Calculate:

- a Ali's taxable income b the amount of tax payable.

- 4 Kadin earns \$2145 per fortnight and has allowable annual deductions of \$337. Calculate his income tax for the financial year. Select **A**, **B**, **C** or **D**.

- A** \$3412.40 **B** \$6824.60 **C** \$9562.73 **D** \$12 519.90

- 5 Matthew is a diving instructor and earns \$57 300 per year. He has allowable deductions of \$1126 for diving equipment and \$912 for a wetsuit. Calculate:

- a his taxable income
 b his income tax.



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- 6 Rebecca earns \$1756.55 per week and claims deductions of \$455.30 in travel expenses, \$276 depreciation of computer equipment and \$135.85 membership to a professional association. She also donated \$30 each month to a charity. Calculate Rebecca's:

- a gross annual income b taxable income c income tax payable

- 7 Joshua is a swimming pool manager with a yearly income of \$45 958. He also earned \$127.50 in interest from his bank savings account. Joshua also had allowable deductions of \$134 and \$55 in donations to charity. Calculate:

- a Joshua's taxable income b the amount of tax payable

- 8 Addison is a casual dance teacher who earns \$20.08 per hour. She works 18 hours a week for 34 weeks of the year. How much should she pay in income tax? Select **A**, **B**, **C** or **D**.

- A** no tax paid **B** \$1123.10 **C** \$2420.87 **D** \$4693.31

- 9 Sefu earns \$69 160 p.a. His allowable deductions are donations to charity of \$45, uniforms costing \$495 and car-related work expenses worth \$890. Calculate the income tax payable.

- 10 Eerin earns \$2346.87 per fortnight and claims deductions of \$1955.30 in travel expenses, \$572 depreciation of camera equipment, \$180.20 membership to a professional association and \$150 donated to a charity. Calculate Eerin's income tax payable.

Technology Online income tax calculators

The Australian Taxation Office (ATO) website www.ato.gov.au has an online calculator for income tax. Visit the website and search 'Simple Tax Calculator' to find the income tax calculator for individuals.

- 1 Enter the taxable income \$63 000 as '63000' (no spaces).
- 2 Select the current financial year.
- 3 Select 'Resident for full year' and click 'Next'.
- 4 The estimated tax payable will be shown on a new screen.

Repeat for at least two more taxable incomes. Compare the amount of tax payable with the results of other students in your class.

Mental skills 8 Maths without calculators

Percentage increase and decrease

The fraction equivalents of commonly-used percentages can help us when we need to increase or decrease a number by a percentage.

Percentage	1%	5%	10%	$12\frac{1}{2}\%$	20%	25%	$33\frac{1}{3}\%$	50%
Fraction	$\frac{1}{100}$	$\frac{1}{20}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$

- 1 Study each example.

- a Increase 360 by 25%.

$$\begin{aligned}
 25\% \text{ of } 360 &= \frac{1}{4} \times 360 \\
 &= 360 \div 4 \\
 &= 90 \\
 360 + 90 &= 450
 \end{aligned}$$

- b Increase \$80 by 5%.

$$\begin{aligned}
 5\% \text{ of } \$80 &= \frac{1}{20} \times \$80 & \text{ or } & 10\% \text{ of } \$80 = \$8 \\
 &= \$80 \div 20 & \therefore & 5\% \text{ of } \$80 = \$8 \div 2 \\
 &= \$4 & & = \$4 \\
 \$80 + \$4 &= \$84
 \end{aligned}$$

- 2 Now increase:

a \$340 by 20%	b 66 by 50%	c 150 by $33\frac{1}{3}\%$	d \$400 by 1%
e 640 by 5%	f \$72 by $12\frac{1}{2}\%$	g \$470 by 10%	h 180 by 25%
i 420 by $33\frac{1}{3}\%$	j \$80 by 5%	k \$280 by 25%	l 70 by 20%

3 Study each example.

a Decrease \$225 by $33\frac{1}{3}\%$.

$$\begin{aligned} 33\frac{1}{3}\% \text{ of } \$225 &= \frac{1}{3} \times \$225 \\ &= \$225 \div 3 \\ &= \$75 \end{aligned}$$

$$\$225 - \$75 = \$150$$

b Decrease \$70 by 15%.

$$\begin{aligned} 10\% \text{ of } \$70 &= \frac{1}{10} \times \$70 = \$7 \\ \therefore 5\% \text{ of } \$70 &= \frac{1}{2} \times \$7 = \$3.50 \\ \therefore 15\% \text{ of } \$70 &= (10\% \times \$70) + (5\% \times \$70) \\ &= \$7 + \$3.50 \\ &= \$10.50 \end{aligned}$$

$$\$70 - \$10.50 = \$59.50$$

4 Now decrease:

a 440 by 25%

b \$300 by 20%

c 2400 by $33\frac{1}{3}\%$

d \$500 by 15%

e \$250 by 10%

f \$120 by 50%

g \$72 by $12\frac{1}{2}\%$

h 80 by 5%

i \$85 by 20%

j \$3800 by 1%

k \$440 by 15%

l \$150 by $33\frac{1}{3}\%$

NSW

8-05 PAYG tax and net pay

Worksheet

Pay day

MAT09NAWK10094

To avoid paying income tax as a huge sum at the end of the financial year, tax is deducted from your gross pay every payday by your employer. This is called **PAYG (Pay As You Go) tax**. The total amount of PAYG tax paid over the year is usually more than the actual income tax payable, so at the end of the financial year you will receive the difference as a **tax refund**. However, if the PAYG tax paid is less than the income tax payable, you will have a **tax debt** and have to pay more.

Example 15

Weekly earnings (\$)	PAYG tax withheld (\$)
184–188	40
189–192	41
193–197	42
198–201	43
202–206	44

Employers use a PAYG tax table to determine how much tax to deduct from their employees' pays. Part of this table is shown above.

During his uni holidays, Li worked in a factory outlet store and earned \$196 per week for 8 weeks.

a Calculate Li's gross income for the 8 weeks.

b How much PAYG tax was deducted from his pay over the 8 weeks?

Solution

a Gross income = $\$196 \times 8$
= $\$1568$

In the table, $\$196$ falls in the $\$193 - \$197$ range.

Weekly PAYG tax = $\$42$

b Total PAYG tax = $\$42 \times 8$
= $\$336$

Gross and net pay

Gross pay is the total amount a person earns or receives each payday.

Most income earners have a variety of deductions made against their gross pay before they receive it, including PAYG tax, superannuation contributions, union fees and health fund payments. The amount of income left after the deductions is called **net pay**.

Summary

Net pay = gross pay – tax – other deductions

Example 16

Paula earns a gross pay of $\$584.60$ per fortnight. Her deductions are for PAYG tax, $\$38.60$ for private health insurance and $\$54.90$ for superannuation.

- a Use the PAYG tax table to find Paula's PAYG tax per fortnight.

Fortnightly earnings (\$)	PAYG tax withheld (\$)
576–583	126
584–593	128
594–603	130
604–611	132
612–620	134

- b Calculate Paula's net pay.
c What are Paula's total deductions as a percentage of her gross income (correct to one decimal place)?

Solution

- a In the table, $\$584.60$ falls in the $\$584 - \$593$ range.
Weekly PAYG tax = $\$128$

b Net pay = $\$584.60 - \$128 - \$38.60 - \$54.90$
= $\$363.10$

$$\begin{aligned}\text{c Total deductions} &= \$128 + \$38.60 + \$54.90 \\ &= \$221.50\end{aligned}$$

$$\begin{aligned}\text{Deductions percentage} &= \frac{\$221.50}{\$584.60} \times 100\% \\ &= 37.8891\ldots\% \\ &\approx 37.9\%\end{aligned}$$

Exercise 8-05 PAYG tax and net pay

Use the table from Example 15 on page 290 when calculating PAYG tax in questions 1 to 7.

See Example 15

- 1 Xavier earned \$195 per week for 16 weeks when he had a casual job as a childrens' party clown.
 - a Calculate his gross income.
 - b How much PAYG tax was deducted from his pay in total?
- 2 Rose works part-time as a dental nurse. She is paid \$206 per week and works for 42 weeks.
 - a Calculate Rose's gross income over 42 weeks.
 - b Find the total amount she paid in PAYG tax.
- 3 Pushkara earned \$186 per week over the summer holidays working at the local nursery. How much did he pay in tax for the 6 weeks that he worked?

See Example 16

- 4 Rebecca is a music teacher who earns \$195 per week.
 - a How much tax will she pay per week?
 - b What percentage of her gross pay will she pay in tax? Answer correct to 1 decimal place.
 - c Calculate her net pay per week.



Alamy © Ian Shaw

- 5 Calista paid \$215 in tax for 5 weeks work at the local cinema. How much did she earn each week?
- 6 Phoebe earns \$201 per week while Jemima earns \$191 per week. If they both work for 12 weeks in these jobs, who pays more in tax and by how much?
- 7 Huan wants to work during his school holidays over Christmas. He has been offered 2 different jobs for six weeks.
 - Cinema: Earn \$368 per fortnight (fortnightly tax of \$80)
 - Electronics store: Earn \$190 per week

Over six weeks, which job offers the higher net pay? Show all working to justify your answer.

- 8 Mrs Gupta earns \$598 per fortnight. She also pays \$19.20 in union fees and \$84.90 in superannuation per fortnight.
- Use the table from Example 16 on page 291 to calculate how much PAYG tax she pays per fortnight.
 - Calculate Mrs Gupta's fortnightly net pay.
 - What percentage (correct to one decimal place) of her gross pay is made up of deductions?
- 9 Cooper has a gross income of \$1125.86 per week. His deductions are \$219.48 tax, \$64.35 for private health insurance and \$30 for superannuation. Calculate Cooper's:
- net income
 - total deductions as a percentage of his gross income (correct to one decimal place).
- 10 Stefan earns a gross income of \$864.25 per week. His deductions are \$141.94 tax and \$51.33 for private health insurance. Calculate:
- Stefan's net income
 - the percentage (correct to one decimal place) of Stefan's gross income that is paid in tax.
- 11 Lina earns a salary of \$76 800 p.a. Each fortnight she has deductions of \$186.50 for family health insurance and \$65.12 for superannuation taken from her gross income. She also pays tax at the rate of 21.6 cents in the dollar. Calculate:
- Lina's fortnightly gross income
 - the amount of tax Lina pays per fortnight
 - Lina's total deductions each fortnight
 - Lina's net income for the fortnight.
- 12 Balun earns a salary of \$128 900 p.a. Each month he has a deduction of \$43.26 for union fees and he pays \$341.05 per month for health insurance taken from his gross income. He also pays \$36 432 tax for the year. Calculate:
- Balun's annual union fees
 - his annual health insurance payments
 - Balun's annual net income
 - the percentage of Balun's gross income that is paid in tax (correct to one decimal place).
- 13 Copy and complete each pay slip.

a

Employee: Janice Hall		Hourly pay rate: \$16.80	
Hours worked		Deductions	
Normal	38	Tax: \$153.20	Other: \$124.60
Time-and-a-half	0	Gross weekly income	
Double time	0	Total deductions	
		Net weekly income	

b

Employee: Amber Menzies		Hourly pay rate: \$12.50	
Hours worked		Deductions	
Normal	39	Tax: \$167.80	Other: \$229.10
Time-and-a-half	2	Gross weekly income	
Double time	0	Total deductions	
		Net weekly income	

- 14 Jodie earns a gross fortnightly pay of \$1027.60. Apart from tax, she has the following deductions: superannuation (\$26.80), health insurance (\$21.60), union fees (\$5.20) and savings plan (\$65).
- If Jodie's net fortnightly pay is \$657.30, find how much tax is taken out.
 - Express the tax she pays as a percentage of her gross income (correct to one decimal place).

Technology Online PAYG tax calculator

Visit the Australian Tax Office website www.ato.gov.au/taxtables to use the online PAYG tax calculator or to download the PAYG tax tables. It contains the PAYG tax amounts for weekly, fortnightly and monthly incomes. Find the PAYG tax payable and net pay for each of the following gross pays.

- \$505 weekly
- \$1466 fortnightly
- \$2730 monthly

Power plus

- In one year, professional golfer Greg Norman earned \$48.5 million. How much is this per minute? Assume 365 days in a year.
- Danielle is paid \$2500 per month. On average, how much does she earn in 4 hours, if she works 9 hours per day, 5 days per week, for 48 weeks a year?
- A fitness instructor conducts a class with 12 participants who pay \$8 each. The class lasts for 45 minutes. What is the instructor's average hourly income?
- Felicity and Rose have the same net pay. Felicity has \$187.60 deducted from her fortnightly gross pay of \$514. Rose, who is paid weekly, has \$64.20 deducted from her gross pay each week. Calculate the difference between Felicity's and Rose's gross fortnightly pays.
- Mirko earns \$63 for working from 7:30 a.m. to 9:15 a.m. At this rate, how much will he earn for a 38-hour week?
- Anna earned \$903 for working 36 hours at normal rates and 4 hours at time-and-a-half. Find her hourly rate of pay.
- When Barry works 6 hours overtime at time-and-a-half, his gross income increases by \$202.50. Find his weekly income for working 38 hours at normal rates.
- One year, Robert received a holiday loading of \$589.05, which was 17.5% of 4 weeks' pay. What was his normal weekly pay?
- If Ingrid paid income tax of \$34 493 last financial year, what was her taxable income?

Language of maths

allowable deduction	annual leave loading	bonus	commission
deduction	double time	financial year	fortnight
gross pay	holiday loading	income tax	net pay
overtime	PAYG tax	per annum	piecework
retainer	salary	taxable income	time-and-a-half
wage			

Puzzle sheet

Earning money
crossword

MAT09NAPS10095

Quiz

Money and business

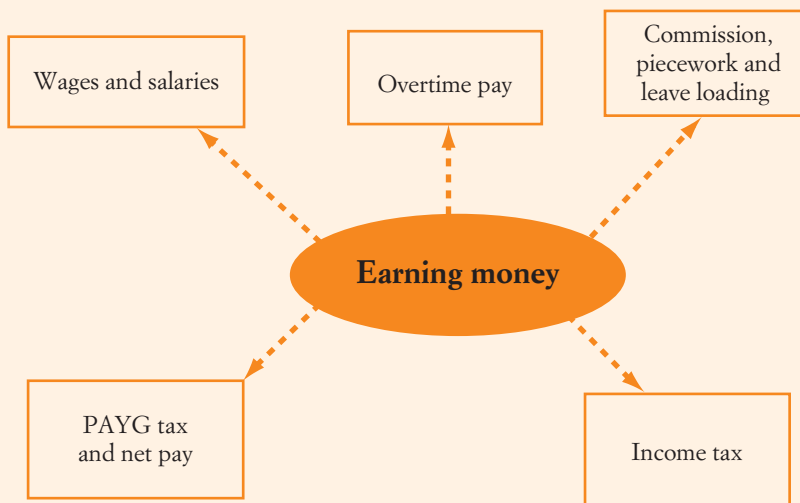
MAT09NAQZ00012

- 1 What is the difference between a **wage** and a **salary**?
- 2 What is an **allowable deduction**? Give an example of one.
- 3 What earnings are calculated by 17.5% of four weeks of normal pay?
- 4 What earnings are calculated as a percentage of the value of items sold?
- 5 What does **PAYG** stand for?
- 6 What is a **financial year**?

Topic overview

- Write 10 questions (with solutions) that could be used in a test for this chapter.
- Swap your questions with another student and check their solutions against yours.
- Write down any part of this topic that you did not understand and discuss it with your teacher.

Copy (or print) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Worksheet

Mind map:
Earning money

MAT09NAWK10096

Chapter 8 revision

See Exercise 8-01

- 1 Goran has an annual salary of \$57 410. How much would he be paid:
a per week? **b** per fortnight? **c** per month?

See Exercise 8-01

- 2 Diane is paid \$13.65 per hour and works a 37-hour week. Find her weekly wage.

See Exercise 8-02

- 3 Riley works 38 hours at \$17.38 per hour and 5 hours overtime at time-and-a-half. Calculate his total earnings.

See Exercise 8-02

- 4 Liam works at a warehouse under the following weekly award.

Award schedule: Normal rate is \$15.75 per hour	
Normal rate	For 0 to 38 hours worked
Time-and-a-half	For the next 3 hours worked
Double time	For each hour worked after that

Calculate his wage for working:

- a** 40 hours **b** 44 hours.

See Exercise 8-03

- 5 Nicole is a bookseller who earns 4% commission on the value of books she sells. She also is paid a monthly retainer of \$900. If Nicole's sales for March were \$82 500, find her total income for March.

See Exercise 8-03

- 6 Sam is paid commission on a sliding scale of 3% on the first \$50 000 of sales and 2.5% on all sales after that. Calculate Sam's income if he sells goods worth \$80 000.

See Exercise 8-03

- 7 Calculate the piecework pay for:
a delivering 1500 pamphlets at 5 cents per pamphlet
b washing 47 cars at \$17.50 per car
c laying 35 m² of tiles at \$28.50 per m².

See Exercise 8-03

- 8 A jockey earns \$100 per race ride and 5% of the value of prize money his horses win. Last week Dan had 30 rides and his horses won prize money worth \$172 500. How much did Dan earn?

See Exercise 8-03

- 9 Sandra earns \$876 per week and is about to begin her annual holidays. Along with her 4 weeks pay, she also receives a 17.5% annual holiday loading on the 4 weeks. Find Sandra's total holiday pay.

See Exercise 8-04

- 10 Emily is a department store manager who earned a gross salary last financial year of \$141 542 and had the following allowable deductions: \$378 uniform cleaning costs, \$218 training course costs and \$1492 travel expenses. Calculate:
a Emily's taxable income
b the amount of income tax Emily should pay using the table on page 286

- 11 Nick earns \$1492.44 per fortnight and pays PAYG tax, \$33.15 for life insurance, \$20.16 in union fees and \$48.17 in superannuation.

See Exercise 8-05

- a Use the PAYG table to find Nick's PAYG tax paid per fortnight.

Fortnightly earnings (\$)	PAYG tax withheld (\$)
1470–1475	174
1476–1481	176
1482–1485	178
1486–1491	180
1492–1497	182

- b Calculate Nick's fortnightly net pay.
 c Calculate, correct to one decimal place, the percentage of Nick's gross pay that was paid in tax.

- 12 For the pay slip below, calculate the:

See Exercise 8-05

- a gross income b total deductions c net income.

Employee: Blair O'Connell		Hourly pay rate: \$22.80	
Hours worked		Deductions	
Normal	37	Tax: \$375.10	Other: \$84.50
Time-and-a-half	4	Gross weekly income	
Double time	2	Total deductions	
		Net weekly income	



9

Statistics and probability

Investigating data

Statistical data that are collected, displayed and analysed are used by governments and businesses to make decisions about health and welfare, the population, production, tourism and the environment. Data collected from a census or through market research (surveys, telephone polls, and so on) can lead to advertising campaigns for health issues, road safety and new products. However, before any action can be taken, the data must be carefully analysed in order to avoid misinterpretation and to check it for relevance and reliability.



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Chapter outline

	Proficiency strands				
9-01 The mean, median, mode and range	U	F	PS	R	C
9-02 Histograms and stem-and-leaf plots	U	F	PS	R	C
9-03 The shape of a distribution	U	F	PS	R	C
9-04 Comparing data sets		F	PS	R	C
9-05 Sampling and types of data	U	F		R	C
9-06 Bias and questionnaires		F	PS	R	C

Wordbank

bias Something unwanted that causes a sample to not truly represent the population

cluster Scores in a data set that are close or 'bunched' together

frequency histogram A special column graph that shows the frequencies of numerical data

outlier An extreme score that is very different from the other scores in a set of data

population All of the items being studied, the entire group

random sample A sample in which every item in the population has an equal chance of being selected

skewed distribution A distribution in which most of the scores are clustered at one end, creating a 'tail' at the other end

symmetrical distribution A distribution in which all scores are distributed equally on both sides of the centre, its shape having line symmetry

In this chapter you will:

- calculate mean, median, mode and range for sets of data, and interpret these statistics in the context of data
- investigate the effect of individual data values, including outliers, on the mean and median
- construct back-to-back stem-and-leaf plots and histograms and describe data using terms such as 'skewed', 'symmetric' and 'bi-modal'
- identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly from secondary sources
- compare data displays using mean, median and range to describe and interpret numerical data sets in terms of location (centre) and spread
- investigate techniques for collecting data, including census, sampling and observation
- investigate reports of surveys in digital media and elsewhere for information on how data was obtained to estimate population means and medians 

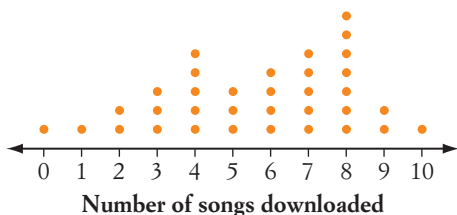
SkillCheck

Worksheet

StartUp assignment 9

MAT09SPWK10097

- 1 This dot plot shows the number of songs downloaded by students last month.



- a How many students were surveyed?
- b What was the most frequent number of songs downloaded?
- c What percentage (correct to one decimal place) of students downloaded more than five songs?
- 2 This stem-and-leaf plot shows the ages of people walking into a newsagent during 8–9 a.m. on a Saturday morning.
- a How many people went to the newsagent?
- b How many of them were aged in their 20s?
- c What was the most common age?
- d What was the youngest age?
- e What percentage (correct to one decimal place) of people were 45 or older?

Stem	Leaf
1	4 6 7 9 9
2	0 4 5 7 8 8
3	1 5 5 6 6 6 7 8 9
4	2 3 3 5 7
5	0 0 1 4 5 6 8 8
6	3 5 6 7 7

- 3 The length of a queue at a supermarket checkout was recorded every five minutes for three hours.
- | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 3 | 6 | 5 | 1 | 4 | 7 | 5 | 4 | 1 | 4 | 4 |
| 4 | 3 | 6 | 3 | 1 | 3 | 4 | 2 | 4 | 5 | 1 | 3 |
| 5 | 4 | 2 | 3 | 5 | 2 | 5 | 2 | 6 | 5 | 1 | 3 |
- Arrange the data in a frequency table.
 - Draw a frequency histogram and a frequency polygon.
 - What was the most common number of people in the queue?
 - How many times was the length of the queue longer than three people?

9-01 The mean, median, mode and range

The **mean**, **median** and **mode** are called **measures of location** because they give an indication of a central value (or average) of a set of data.

The **range** is a measure of spread because it gives an indication of how widely the scores are spread in a set of data.

Summary

$$\begin{aligned}\text{Mean, } \bar{x} &= \frac{\text{sum of scores}}{\text{number of scores}} \\ &= \frac{\Sigma x}{n}\end{aligned}$$

← Σ is the Greek letter **sigma**, and stands for 'the sum of'

where Σx means 'the sum of the scores' and n is the total number of scores.

When the scores are arranged in order, the **median** is:

- the middle score, for an odd number of scores
- the average of the two middle scores, for an even number of scores.

The **mode** is the score (or scores) that occurs most often.

Range = highest score – lowest score

An **outlier** (pronounced 'out-ly-er') is an extremely high or low score that is separated from the other scores in the data set.

Comparing the mean, median and mode

The mean is usually the best measure of location because it takes into account every data score. If there are outliers in the set of data, then the mean may be affected by these extreme scores and will not accurately represent all of the scores. In this case the median is a better measure because it is not affected by outliers.

The mode is useful when the most common score is important, or when the data is categorical (such as make of car or hair colour). With categorical data, it is not possible to have a mean or median.

Worksheet

Mean, median, mode

MAT09SPWK10098

Worksheet

Collecting and
describing data

MAT09SPWK00001

Skillsheet

Statistical measures

MAT09SPSS10028

Video tutorial

Statistics

MAT09SPVT00001

Puzzle sheet

Mode, median and
mean

MAT09SPPS00003

Technology worksheet

Excel worksheet:
Statistical measures
calculator

MAT09SPCT00018

Technology worksheet

Excel spreadsheet:
Statistical measures
calculator

MAT09SPCT00003

Technology worksheet

Most valuable player

MAT09SPCT10010

Example 1

The maximum daily temperatures (in °C) in Sydney for a fortnight in March were:

34	22	32	26	23	24	24
26	31	27	27	28	28	27

Find:

a the mean correct to one decimal place	b the median
c the mode	d the range

Solution

$$\begin{aligned}
 \text{a Mean} &= \frac{34 + 22 + 32 + \cdots + 28 + 27}{14} \\
 &= \frac{379}{14} \\
 &= 27.0714 \dots \\
 &\approx 27.1
 \end{aligned}$$

- b** Arranging the scores from lowest to highest:

22 23 24 24 26 26 26 27 27 28 28 31 32 34

the middle scores

There is an even number of scores (14), so we have two middle scores, the 7th and 8th scores 26 and 27.

$$\begin{aligned}
 \text{Median} &= \frac{26 + 27}{2} \\
 &= 26.5
 \end{aligned}$$

- c** Mode = 26

The most common score, occurring three times.

- d** Range = 34 – 22
= 12

highest score – lowest score

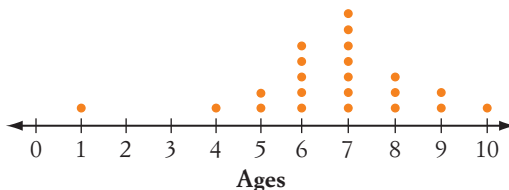
Example 2

This dot plot shows the ages of the children at Soshanna's birthday party. Find:

a the mean (correct to one decimal place)	b the median
c the mode	d the range
	e the outlier

Solution

$$\begin{aligned}
 \text{a Mean} &= \frac{1 + 4 + 5 + 5 + \cdots + 10}{22} \\
 &= 6.636363 \dots \\
 &\approx 6.6
 \end{aligned}$$



- b There are 22 scores, so the median is the average of the 11th and 12th scores.
Counting dots from the left and upwards, these are 7 and 7.

$$\text{Median} = \frac{7+7}{2} = 7$$

- c Mode = 7

The score with the most dots.

- d Range = $10 - 1 = 9$

- e Outlier = 1

This score is much different from the rest.

The mean from a calculator's statistics mode

Scientific and graphics calculators have a statistics mode (SD or STAT). Follow the instructions in the table below to calculate the mean of the scores from Example 1 using your calculator's statistics mode.

Operation	Casio scientific	Sharp scientific
Start statistics mode.	MODE STAT 1-VAR	MODE STAT =
Clear the statistical memory.	SHIFT 1 Edit, Del-A	2ndF DEL
Enter data.	SHIFT 1 Data to get table 34 = 22 = , etc. to enter in column AC to leave table	34 M+ 22 M+ , etc.
Calculate the mean. ($\bar{x} = 27.0714$)	SHIFT 1 Var $\bar{x}$ =	RCL $\bar{x}$
Check the number of scores. ($n = 14$)	SHIFT 1 Var n =	RCL n
Return to normal (COMP) mode.	MODE COMP	MODE 0

Operation	Casio graphics	Texas instruments graphics
Start statistics mode.	MENU STAT for Lists table	Y= and delete any function by highlighting and pressing CLEAR STAT EDIT
Clear the statistical memory.	With cursor in List 1 column EXIT F6 DEL-A Yes	With cursor on L1 CLEAR ENTER
Enter data.	34 EXE 22 EXE , etc. to enter in List 1 column	34 ENTER 22 ENTER , etc. to enter in List 1 column
Calculate the mean. ($\bar{x} = 27.0714$) Calculate the sum of scores. ($\Sigma x = 379$) Check the number of scores. ($n = 14$)	F6 CALC SET to make these settings (if different): 1Var XList: List1 1Var Freq: 1 EXE 1VAR to calculate many statistics (scroll down for more)	STAT CALC 1-Var Stats ENTER to calculate many statistics (scroll down for more)

Worksheet

The mean from a frequency table

The mean and fx tables

MAT09SPWK10100

For a large number of scores, we can make up a frequency distribution table. By adding an fx column, the mean can be easily found.

Score (x)	Frequency (f)
3	2
4	5
5	10
6	36
7	25
8	13
9	4
10	3

Example 3

Video tutorial

Statistics from a frequency table

MAT09SPVT10012

The scores of Year 9 students giving a speech in English (out of 10) are shown in the frequency table.

- How many students were there?
- Use an fx column to calculate the mean score.
- Find the mode.
- What is the range?

Solution

- The total sum of the Frequency column is 98.
There were 98 students.

b

Mark (x)	Frequency (f)	fx
3	2	6
4	5	20
5	10	50
6	36	216
7	25	175
8	13	104
9	4	36
10	3	30
	$\Sigma f = 98$	$\Sigma fx = 637$

fx means $f \times x$, which multiplies each score by its frequency.

Σfx is the sum of the fx column (that is, the sum of all scores).

Σf is the sum of the frequency column (that is, the total number of scores).

$$\begin{aligned}\bar{x} &= \frac{\Sigma fx}{\Sigma f} \\ \text{Mean} &= \frac{637}{98} \\ &= 6.5\end{aligned}$$

Sum of all scores divided by the number of scores.

c Mode = 6

Score with the highest frequency.

d Range = $10 - 3 = 7$

Summary

For data in a frequency table:

- the **mean** is $\bar{x} = \frac{\text{sum of } fx}{\text{sum of } f} = \frac{\Sigma fx}{\Sigma f}$
- the **mode** is the score(s) with the highest frequency, f
- the **range** is the difference between the last and first scores in the score (x) column

The table below contains instructions for finding the mean of data presented in a frequency table. It uses the table of data from Example 3.

Operation	Casio scientific	Sharp scientific
Start statistics mode.	MODE STAT 1-VAR SHIFT MODE scroll down to STAT Frequency? ON	MODE STAT =
Clear the statistical memory.	SHIFT 1 Edit, Del-A	2ndF DEL
Enter data.	SHIFT 1 Data to get table 3 = 4 = , etc. to enter in x column 2 = 5 = , etc. to enter in FREQ column AC to leave table	3 2ndF STO 2 M+ 4 2ndF STO 5 M+ , etc.
Calculate the mean. ($\bar{x} = 6.5$)	SHIFT 1 Var $\bar{x}$ =	RCL $\bar{x}$
Check the number of scores. ($n = 98$)	SHIFT 1 Var n =	RCL n
Return to normal (COMP) mode.	MODE COMP	MODE 0

Operation	Casio graphics	Texas instruments graphics
Start statistics mode.	MENU STAT for Lists table	Y= and delete any function by highlighting and pressing CLEAR STAT EDIT
Clear the statistical memory.	With cursor in List 1 column EXIT F6 DEL-A Yes Repeat for List 2	With cursor on L1 CLEAR ENTER Repeat for L2
Enter data.	3 EXE 4 EXE , etc. to enter in List 1 column 2 EXE 5 EXE , etc. to enter in List 2 column	3 ENTER 4 ENTER , etc. to enter in List 1 column 2 ENTER 5 ENTER , etc. to enter in List 2 column
Calculate the mean. ($\bar{x} = 6.5$) Calculate the number of scores. ($\Sigma x = 637$) Check the number of scores. ($n = 98$)	F6 CALC SET to make these settings (if different): 1Var XList: List1 1Var Freq: List 2 EXE 1VAR to calculate many statistics (scroll down for more)	STAT CALC 1-Var Stats and type 'L1, L2' by pressing 2nd 1 . 2nd 2 ENTER to calculate many statistics (scroll down for more)

Cumulative frequency and the median

To find the median of data presented in a frequency table, add a **cumulative frequency** column, which is a progressive total of the frequency column.

Example 4

Find the median of the Year 9 student speech scores from Example 3.

Solution

The cumulative frequency column is a running total of the frequencies.

$$2 + 5 = 7$$

$$7 + 10 = 17, \text{ etc.}$$

There are 98 scores, so the two middle scores are the 49th and 50th scores. Using the cumulative frequency column, the 17th score is a 5 and the 53rd score is 6, so the 49th and 50th scores must both be 6.

$$\text{Median} = \frac{6 + 6}{2} = 6$$

Mark (x)	Frequency (f)	Cumulative frequency (cf)
3	2	2
4	5	7
5	10	17
6	36	53
7	25	78
8	13	91
9	4	95
10	3	98
$\Sigma f = 98$		

Exercise 9-01

See Example 1

- 1** For each set of data, find:
- i** the mean (correct to one decimal place where necessary)
 - ii** the median
 - iii** the mode
 - iv** the range
- | | | | | | | | | | | | |
|----------|-----|-----|-----|-----|-----|-----|-----|-----|----|----|----|
| a | 23 | 20 | 12 | 15 | 28 | 19 | 15 | 14 | 18 | | |
| b | 5 | 3 | 8 | 7 | 1 | 6 | 7 | 7 | 5 | 15 | 12 |
| c | 32 | 45 | 12 | 46 | 45 | 38 | 32 | | | | |
| d | 8.8 | 8.2 | 8.8 | 8.3 | 8.5 | 8.9 | 9.2 | 8.8 | | | |
| e | -5 | 0 | -2 | 4 | 6 | -2 | 5 | -1 | -2 | 0 | |
- 2** The maximum and minimum daily temperatures (in °C) over a week at Point Perpendicular on the NSW south coast are listed here.

Maximum	23	24	22	23	22	23	23
Minimum	17	17	15	16	15	15	14

- a Calculate correct to one decimal place the average daily maximum and average daily minimum temperature over the week.
 - b What was the difference between the two averages?
 - c Find the mode of daily maximum temperatures and the mode of daily minimum temperatures.
 - d What was the difference between the two modes?
- 3 The monthly rainfall (in mm) in Cairns is shown in the table.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Rainfall	413	435	442	191	94	49	28	27	36	38	90	175

- a What is the range?
 - b Find the mean (correct to the nearest mm).
 - c Find the median.
- 4 Jay played in a golf tournament. His average score for the 4 rounds was 71. If the scores in his first three rounds were 72, 66 and 70, what was his score in the fourth round?



Worked solutions

The mean, median, mode and range

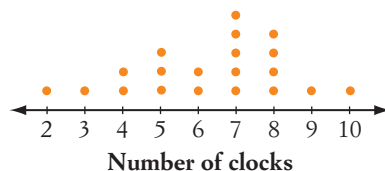
MAT09SPWS10038

- 5 Tasnuba scored a mean of 74% for 5 maths tests that she completed.
- What was the total number of marks Tasnuba scored for the 5 tests?
 - Tasnuba did a sixth test and her mean test mark increased to 77%. What is the total that she scored for the 6 tests?
 - What mark did she achieve in the last test?

- 6 Alice's average mark for 4 Science exams is 68%. Can Alice score enough marks in a fifth exam so that her average mark increases to 75%? Give reasons.

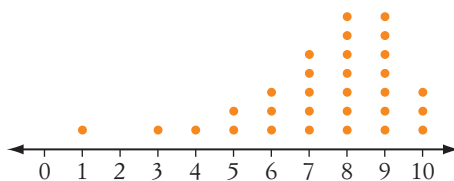
See Example 2

- 7 A number of homes were surveyed on how many clocks they had, with the results shown in the dot plot. Find:



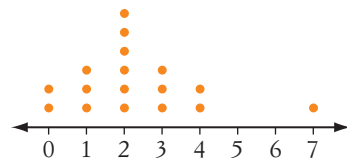
- the mean
- the median
- the mode
- the range

- 8 This dot plot shows the results of a spelling quiz (out of 10).



- Are there any outliers? (Give reasons)
- What is the mode?
- Find the mean correct to one decimal place.
- Find the median.
- What percentage of results (correct to one decimal place) were on or above:
 - the mean?
 - the median?

- 9 The dot plot shows the number of goals scored by a soccer team during the season.



- Are there any outliers?
- Find the mean and median:
 - without the outlier
 - with the outlier
- Which measure (mean or median) is affected by the outlier?

- 10 The salaries of ten people working at a software company are:

\$80 000	\$105 000	\$70 000	\$93 000	\$78 500
\$180 000	\$90 000	\$72 000	\$93 000	\$101 000

- Which salary is an outlier?
- Find the mean and median:
 - without the outlier
 - with the outlier
- Which measure, the mean or the median, represents the salaries better?
- Find the mode and range:
 - without the outlier
 - with the outlier
- Which measure, the mode or the range, is affected by the outlier?

- 11 Use your calculator in the statistics mode to find the mean of each set of data (correct to one decimal place where necessary).

a 5 7 8 3 6 8 5 4 8 3 8 10 6 8 4 9

b 13.5 14.2 11.9 15.7 18.4 17.6 12.0

c 56 72 84 57 49 65 59 79 80 76 57 66 68

- 12 Copy and complete each frequency table, then calculate the mean correct to two decimal places.

See Example 3

a

Score, x	Frequency, f	fx
2	5	
3	3	
4	7	
5	6	
6	4	
	$\Sigma f =$	$\Sigma fx =$

b

x	f	fx
22	3	
23	8	
24	7	
25	9	
26	5	
27	4	
	$\Sigma f =$	$\Sigma fx =$

- 13 Use your calculator in the statistics mode to find the mean of each frequency table (correct to one decimal place where necessary).

a

Score, x	Frequency, f
32	5
33	8
34	12
35	9
36	7
37	4

b

x	f
10	12
11	18
12	27
13	37
14	23
15	15
16	9

- 14 For each frequency table, add a cumulative frequency column and find:

See Example 4

- i the median ii the mode iii the range

a

x	f
2	2
3	5
4	11
5	7
6	10
7	9
8	6

b

x	f
17	3
18	8
19	20
20	15
21	11
22	7

- 15 The number of seedlings that have sprouted in each punnet at a nursery are shown.

10 5 7 7 9 8 8 11 7 8 5 8 4 6 4
 12 11 9 4 6 6 10 7 6 9 10 9 7 8 8
 8 7 4 7 7 11 10 9 8 9 9 8 8 8 9

- a Complete a frequency table and find the median.
 b What is the mode?
 c Find the range.
 d Calculate the mean correct to one decimal place.

Investigation: The effect of outliers

1 Find the mean, median, mode and range of each data set.

a 1 2 2 2 3 4 5 5 5

b 1 2 2 2 3 4 5 5 6

c 1 2 2 2 3 4 5 5 8

d 1 2 2 2 3 4 5 5 17

2 The four sets of scores in question 1 are the same, apart from the last score in each set. If one score is changed, what is the effect on:

a the mean **b** the median **c** the mode **d** the range?

3 Consider the two sets of data:

A: 1 10 11 14 15 12 13 11

B: 3 4 2 6 3 14 6 3 5 4

a Find the mean, median and range of each set of scores.

b Identify the outlier for each set.

c Find the mean, median and range of each set of scores excluding the outlier.

d What effect does the size of the outlier have on:

i the mean **ii** the median **iii** the range?

e Why is the median not affected by the size of the outlier?

Worksheet

Statistics review

MAT09SPWK10099

Worksheet

Analysing data

MAT09SPWK10101

Worksheet

Stem-and-leaf plots

MAT09SPWK00002

Homework sheet

Data 1

MAT09SPHS10038

Puzzle sheet

Highway elephants

MAT09SPPS00002

9-02 Histograms and stem-and-leaf plots

Frequency histograms and polygons

A **frequency histogram** is a column graph of numerical data where the columns stand together without gaps between them.

A **frequency polygon** is a line graph that is constructed by joining the midpoints of the tops of the columns of a frequency histogram, starting and ending on the horizontal axis. It is called a 'polygon' because the graph and the horizontal axis together make a shape with straight sides.

Example 5

45 students were surveyed on the number of hours of homework they did in the past week. The results were as follows.

3	7	5	8	8	7	7	4	8	7	9	8	8	5	7
4	2	5	3	8	6	6	7	9	6	5	4	9	8	4
2	3	6	7	8	2	8	8	4	5	6	8	7	2	5

a Arrange the data in a frequency table.

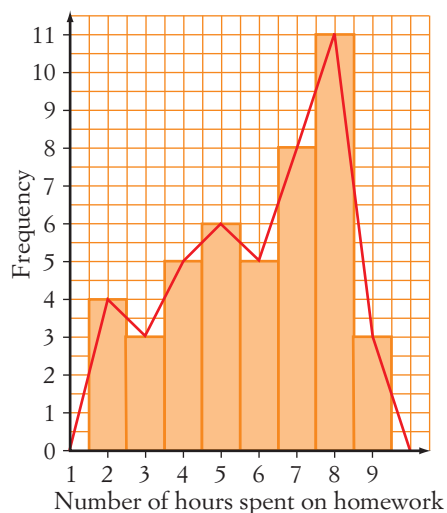
b Draw a frequency histogram and polygon for this data.

Solution

a

Number of hours, x	Tally	Frequency, f
2		4
3		3
4		5
5		6
6		5
7		8
8		11
9		3
Total		45

b Frequency histogram and polygon



Note that each column of the histogram is centred on each score marked on the horizontal axis, so there is a half-column gap at the start between the Frequency axis and the column.

Stem-and-leaf plots

A **stem-and-leaf plot** lists the scores of a data set, usually in order. It shows:

- any **clusters**, where scores are grouped or bunched together
- any **outliers**
- how the scores are spread out

Example 6

This stem-and-leaf plot shows the pulse rate (in heartbeats per minute) of 30 people in a shopping mall.



The stem is formed by the tens digit of the scores.

Stem	Leaf
5	1 5 8
6	0 3 4 4 5 6 8 8 9
7	0 1 2 3 3 4 4 5 7 8 8 9
8	0 3 4 6 7
9	0

The leaves are formed by the units digits of the scores.

The data shown by this row is 80, 83, 84, 86 and 87.

- Are there any outliers?
- Is there any clustering of data?
- Find the mean, correct to one decimal place.
- What is the median?
- What is the mode?
- What is the range?

Solution

- There are no outliers as no scores are 'separate' from the main body of data.
- The scores are bunched in the 60s and 70s so we say that there is clustering in the 60s and 70s.

$$\begin{aligned}
 \text{c Mean} &= \frac{51 + 55 + 58 + \cdots + 87 + 90}{30} \\
 &= 71.8666 \dots \\
 &\approx 71.9
 \end{aligned}$$

- d The 30 scores in the stemplot are in order, so the median is the average of the 15th and 16th scores, 72 and 73 (counting from top to bottom, left to right).

$$\text{Median} = \frac{72 + 73}{2} = 72.5$$

- e Modes = 64, 68, 73, 74, 78

Each occurs two times.

- f The lowest score is 51 and the highest is 90.

$$\begin{aligned}\text{Range} &= 90 - 51 \\ &= 39\end{aligned}$$

Back-to-back stem-and-leaf plots

When two related sets of data are compared, we can use back-to-back stem-and-leaf plots.

Example 7

Buzz batteries and *Neverflat* batteries are to be compared by testing 15 batteries of each brand until they fail. The lengths of the battery lives are recorded to the nearest hour as shown.

Buzz	8	12	22	18	23	20	11	15	18	8	9	19	26	30	31
Neverflat	14	16	24	32	35	36	37	28	31	30	24	26	19	35	32

- a Arrange the data into a back-to-back stem-and-leaf plot to determine which battery is better.
b Compare the median of each brand.
c Compare the mean of each brand.

Solution

a

Buzz	Neverflat
9 8 8	0
9 8 8 5 2 1	1 4 6 9
6 3 2 0	2 4 4 6 8
1 0	3 0 1 2 2 5 5 6 7

From looking at the data in the back-to-back stem-and-leaf plot, it is clear that *Neverflat* is better as its scores are clustered in the 30 hours life while *Buzz* is clustered in the 10 and 20 hours life.

- b Median for Buzz = 18

The 8th score, counting from top to bottom, right to left.

$$\text{Median for Neverflat} = 30$$

The 8th score, counting from top to bottom, left to right.

Neverflat has the higher median of 30 hours.

- c Mean for Buzz = 18
 Mean for Neverflat = 27.933...
 Neverflat has the higher mean of
 approximately 27.9 hours.

Exercise 9-02 Histograms and stem-and-leaf plots

See Example 5

- 1 The results of an Art project (out of 10) for 30 students are shown below.

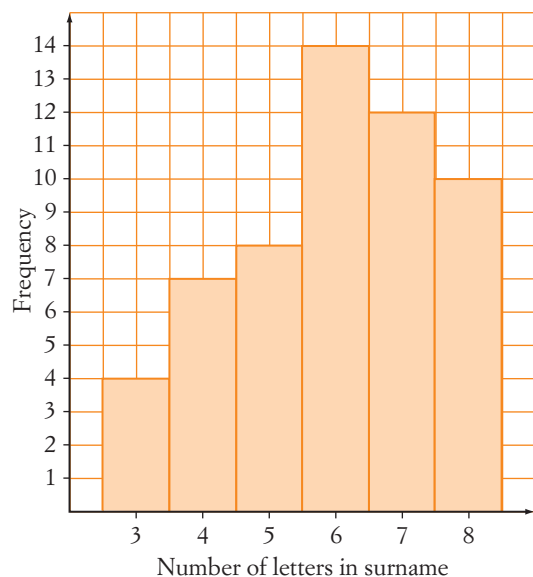
7	9	3	5	6	6	7	5	2	8
7	8	6	5	5	4	3	6	6	7
8	10	7	5	8	9	8	5	4	5

- Arrange the data into a frequency distribution table.
 - Draw a frequency histogram and polygon.
 - Find the mean correct to one decimal place.
 - Find the median.
 - Find the mode.
 - Find the range.
 - Are there any outliers and is there any clustering?
- 2 A sample of students were surveyed on the number of children in their families.
- | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 6 | 5 | 3 | 3 | 2 | 1 | 7 | 4 | 4 | 4 | 7 | 2 | 3 | 5 | 3 | 1 | 2 | 1 |
| 1 | 2 | 7 | 4 | 2 | 2 | 3 | 6 | 3 | 6 | 5 | 3 | 5 | 7 | 9 | 8 | 3 | 2 | 3 | 4 |
- Arrange the data into a frequency table and construct a frequency histogram and polygon.
 - Find the mode and the range.
 - Are there any outliers and is there any clustering?
 - Find the median.
 - Find the mean correct to one decimal place.

- 3 The numbers of letters in the surnames of 55 students are shown in this frequency histogram.

- What is the mode? How is this shown on the frequency histogram?
- Find the mean correct to two decimal places.
- Find the median.
- Find the range.

Frequency histogram



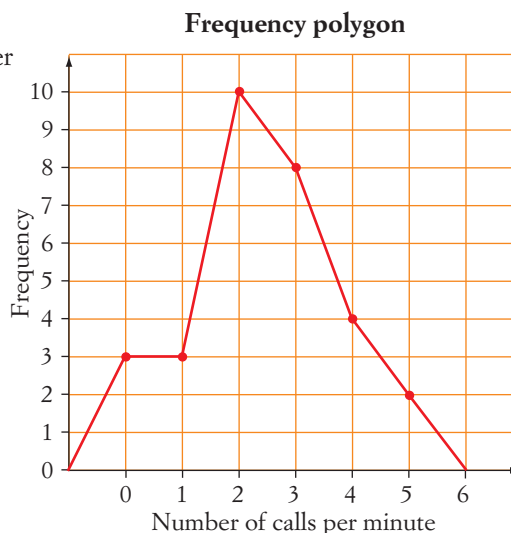
Worked solutions

Histograms and
stem-and-leaf plots

MAT09SPWS10039

- 4 This frequency polygon shows the number of phone calls made per minute to a customer support line over a 30-minute period.

- How many calls were made altogether?
- How many times were there three or more calls received per minute?
- What is the mode? How is this shown on the frequency polygon?
- Find the range.
- Find the median.
- Find the mean correct to two decimal places.
- Is there any clustering of data?



- 5 This unordered stem-and-leaf plot shows the marks of a class of students in a Science test.
- Construct an ordered stem-and-leaf plot.
 - Find the range.
 - Are there any outliers or clusters?
 - How many students are there in the class?
 - Find the mean correct to two decimal places.
 - Find the median.

Stem	Leaf
4	0 1 5
5	3 1 5 6 8
6	7 7 8 6 4 3 5 7
7	9 1 7 4 2
8	8 2 7
9	2 0 4

See Example 6

- 6 This stem-and-leaf plot shows the points scored by a basketball team in different matches.
- Find the range.
 - What is the mode?
 - Find the mean correct to one decimal place.
 - Find the median.

Stem	Leaf
3	0 7
4	1 1 3 5
5	4 6 6 8 9
6	3 5 8 8 8 9
7	2 4 5 5
8	0 2

- 7 When 30 students ran 100 m, their times (in seconds) were as follows.

13.5 15.0 12.1 12.5 17.6 13.7 15.7 18.7 13.1 11.8
 12.6 15.7 14.4 15.4 14.9 15.0 18.3 16.3 15.4 15.1
 11.9 16.1 14.3 14.5 16.3 14.6 14.9 15.0 16.1 13.5

- Copy and complete this stem-and-leaf plot.
- Are there any:
 - outliers?
 - clusters?
- How many students ran the 100 m in less than 13.7 seconds?
- What percentage of students took more than 14 seconds to run the 100 m?
- What percentage (correct to one decimal place) of students ran times that were below:
 - the mean
 - the median?

Stem	Leaf
11	9
12	
13	
14	
15	
16	
17	
18	

- 8 This ordered stem-and-leaf plot shows the marks obtained by students in a History exam.

- One of the marks in the 50s is missing. What could this missing mark be?
- The entry for the lowest mark is missing. If the range of the marks is 63, find the lowest mark.
- Find the median.

Stem	Leaf
2	□
3	0 2 3 4
4	1 5 7 8
5	2 4 □ 6 9 9
6	0 2 3 3 6 8 8 7
7	2 5 6 7 7
8	2 5

See Example 7

- 9 This back-to-back stem-and-leaf plot shows the number of goals scored per match by two netball teams during a season.

Rockets		Blues
7 5 4 3	1	7
8 7 6 5 4 2	2	0 5 6 8
7 6 4 3	3	4 5 7 7 8 9
5 4 0	4	2 3 7 8
2	5	1 3 7

- How many times did each team score more than 30 goals?
- Find the highest number of goals scored by each team.
- Which team had the greater range?
- Which team performed better? Give reasons.
- Which team had the greater median?

- 10 The number of hamburgers sold by Robyn and Anthony at lunchtime over 14 days was:

Robyn	37	23	33	35	17	42	37	54	45	42	37	27	36	41
Anthony	35	28	29	36	43	37	55	42	47	51	36	34	35	39

- Construct an ordered back-to-back stem-and-leaf plot.
 - Who sold more hamburgers?
 - On how many days did Robyn sell more hamburgers than Anthony?
 - Are there any outliers in the sales figures for either Robyn or Anthony?
 - Is there any clustering of sales figures for either person?
 - Which person had the higher mean?
- 11 A sample of commuters were surveyed on how far (in kilometres) they live from the closest railway station.
- | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|-----|-----|
| 1.5 | 4.8 | 6.5 | 1.1 | 3.3 | 3.4 | 3.5 | 4.5 | 4.5 | 12.5 | 3.1 | 3.6 |
| 3.8 | 4.0 | 4.2 | 2.3 | 2.5 | 1.9 | 4.7 | 5.5 | 2.7 | 3.2 | 5.2 | |
- Arrange the data in a stemplot.
 - Are there any outliers? Give reasons.
 - Are there any clusters?
 - Find the mean correct to two decimal places.
 - Find the median.
 - Which is the better measure of location, the mean or median? Give reasons.



Shutterstock.com/Chris Hellyar

12 The results for a writing task for 20 girls and 20 boys are:

Girls:	18	48	56	59	67	73	76	94	76	78
	82	39	45	59	60	85	80	74	51	36
Boys:	50	88	76	72	10	28	40	95	74	63
	67	55	32	29	16	75	26	82	57	50

- Construct a back-to-back stem-and-leaf plot for the data.
- What was the highest mark? Was it scored it by a girl or boy?
- What is the median for:
 - girls
 - boys?
- What percentage of boys scored more than 60?
- What percentage of girls scored more than 60?
- Find the mean mark for:
 - girls
 - boys.
- Is there a significant difference between the median mark and the mean mark for:
 - girls
 - boys?

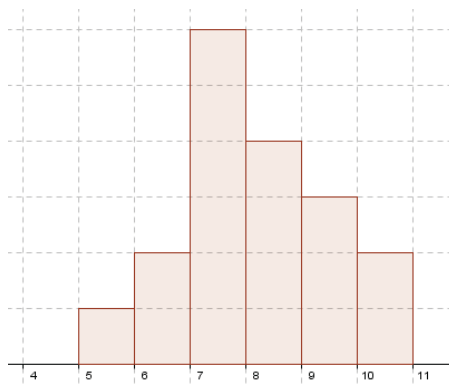
Technology Histograms

In this activity, we will use GeoGebra to construct a histogram of players' scores (out of 10) in a trivia quiz.

Score	Frequency
5	1
6	2
7	6
8	4
9	3
10	2

GeoGebra

- Click **View** and **Spreadsheet**.
Close the Algebra window.
- Enter into cell A1: Histogram
[{5, 6, 7, 8, 9, 10, 11}, {1, 2, 6, 4, 3, 2}].
Make sure the brackets are as shown and note that 11 is not a score but ends the graph. Also make sure that there is a space after each comma.
- You can hold down **ctrl** and use your mouse to drag the screen until you see your entire histogram.



Puzzle sheet

Statistical match-up

MAT09SPPS10104

Technology

GeoGebra:
Symmetrical and
skewed distributions

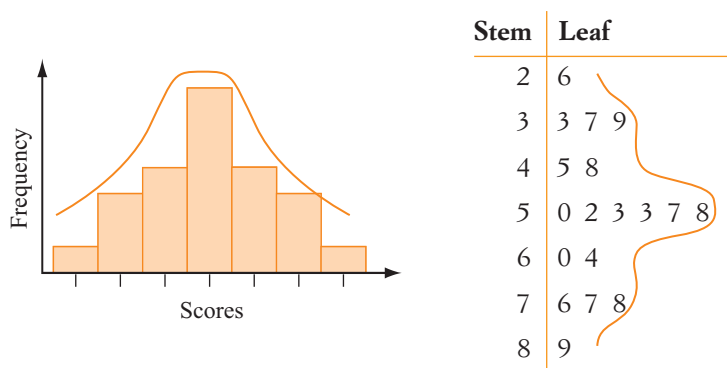
MAT09SPTC00001

9-03 The shape of a distribution

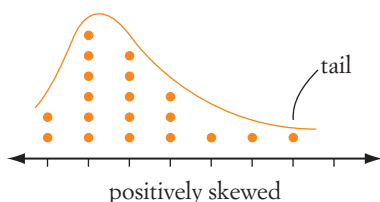
A **statistical distribution** is the way the scores of a data set are arranged, especially when graphed. When looking at histograms, dot plots and stem-and-leaf plots, an overall pattern can be seen from the shape of the display.

The shape of a statistical distribution shows how the data is spread and can be seen by drawing a curve around the graph or display.

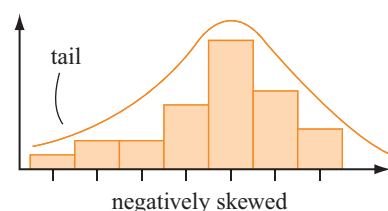
A distribution is **symmetrical** if the data is evenly spread or balanced about the centre. For example, the frequency histogram and stem-and-leaf plot shown below are both symmetrical in shape.



A distribution is **skewed** if most of the data is bunched or clustered at one 'end' of the distribution and the other end has a 'tail', as shown on this dot plot and frequency histogram.



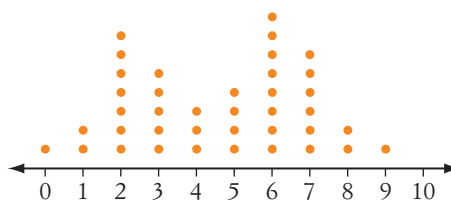
A distribution is **positively skewed** if its tail points to the right.



A distribution is **negatively skewed** if its tail points to the left.

A distribution is **bimodal** if it has two peaks. The higher peak is the mode, while the other peak indicates another score that has a high frequency.

For example, this dot plot has peaks at 2 and 6 so it is bimodal. The mode, however, is 6.

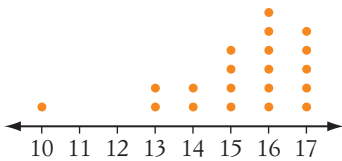


Example 8

For each statistical distribution:

- describe the shape
- identify any outliers and clusters.

a



b Stem | Leaf

Stem	Leaf
3	0 1 2
4	1 3 4 4 5 6
5	0 4 5 7 8
6	3 7 8
7	0 1
8	4
9	8

Solution

- The shape is negatively skewed (tail points towards the lower scores).
 - 10 is an outlier and clustering occurs at 15, 16 and 17.
- The shape is positively skewed (tail points towards the higher scores).
 - 98 is an outlier and clustering occurs in the 40s and 50s.

Video tutorial

The shape of a frequency distribution

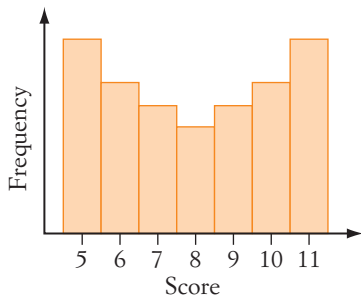
MAT09SPVT10013

Exercise 9-03 The shape of a distribution

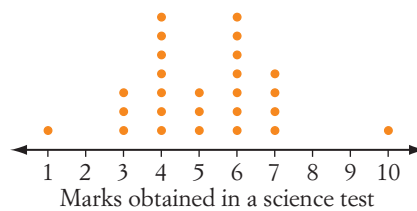
1 For each statistical distribution:

- describe the shape
- identify any outliers and clusters.

a



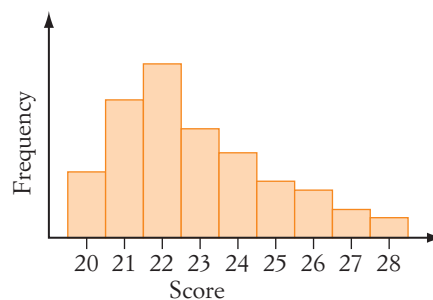
b



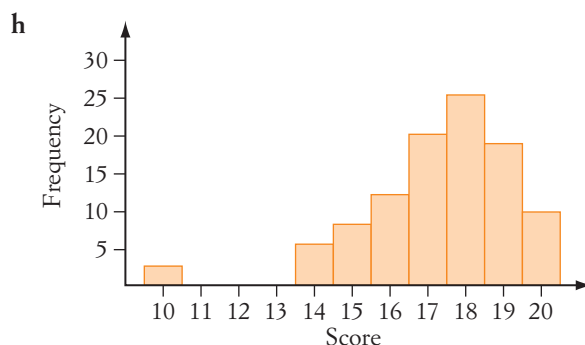
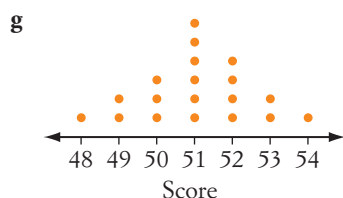
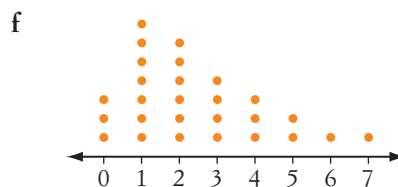
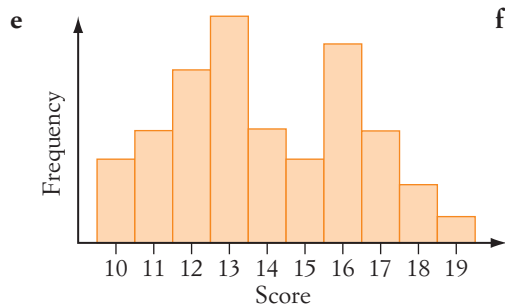
c

Stem	Leaf
2	0 2
3	1 4
4	2 5 6
5	2 3 6 7 8
6	0 4 4 9 9 9
7	1 2 4 5 5 6 7 8 8
8	2 4 5 6 7 7 8
9	0 2 2

d



See Example 8



- 2** The results of a Mathematics test for a Year 10 class are shown in the stem-and-leaf plot.
- How many students are in the class?
 - Where does the clustering occur?
 - Describe the shape of the data.
 - Give a possible reason for the shape of the distribution.
 - Find the mean, median and mode.

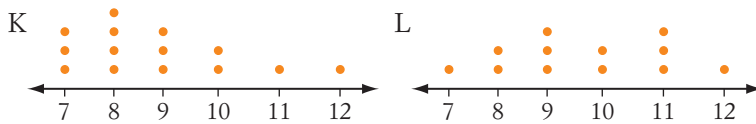
Stem	Leaf
1	8
2	3 5 8 8
3	2 4 6 7 8 8 9
4	0 4 5 5 7 7
5	3 6 7 8
6	0 4 4
7	2 5
8	1 2
9	2

- 3** The number of children in the families of students at Ravambela High School are shown below.

4 4 7 3 3 6 8 6 3 3 4 2 9 5 1 2 3 3 4
7 2 7 2 2 3 5 3 1 4 1 4 1 2 5 6 3 5 2

- Arrange the data into a frequency table and construct a frequency histogram.
- Are there any outliers?
- Describe the shape of the distribution.
- Where does clustering occur?
- Find the mode, the mean and the median and show their position on the histogram.

- 4 Which statement is true about the sets of data below? Select **A**, **B**, **C** or **D**.



- A** K is positively skewed and L is symmetrical.
B K is negatively skewed and L is bimodal.
C L has two peaks and K is positively skewed.
D K is positively skewed and the median of L is 9.

- 5 The results of an entrance exam to select apprentices for an energy supplier are shown in the stem-and-leaf plot.

Stem	Leaf
9	2 3 5 6 7 8
8	3 3 6 7
7	2 4 8
6	0 3 4 4
5	3 5 6 6 8 9
4	
3	
2	6



Stockphoto/Isafix

- a** Describe the shape of the distribution, excluding the outlier.
b What is the mode?
c Find the median.
d Find the mean correct to one decimal place.
e Find the range.
f Is the range a good indicator of the spread of scores? Give reasons.
- 6 The average monthly maximum temperatures for Sydney and Melbourne are shown in the table.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Sydney	26	26	25	22	19	17	16	18	20	22	24	26
Melbourne	26	26	24	20	17	14	13	15	17	20	22	24

- a** Display the data for Sydney and Melbourne on dot plots.
b Find the mean, median, mode and range for each city.
c Which city is generally warmer? Give reasons for your answer.

Worksheet

Comparing word lengths

MAT09SPWK10102

Worksheet

Investigating young drivers

MAT09SPWK10103

Homework sheet

Data 2

MAT09SPHS10039

Technology worksheet

Population polygons

MAT09SPCT10003

Technology worksheet

Excel worksheet: World climate

MAT09SPCT00017

Technology worksheet

Excel spreadsheet: World climate

MAT09SPCT00002

9-04 Comparing data sets

Example 9

This back-to-back stem-and-leaf plot shows the results of a Year 9 class in a Geography exam.

Girls		Boys	
2	2	3	4 8
8 3 1	3	0	3 5 9 9
6 2	4	1	3 7 8 8 8 9 9
7 3 0	5	0	2 3 4 5 5 8 9 9
8 7 7 6 2 1	6	0	1 1 4 7 8
9 8 8 7 5 4 4 3	7	0	2 6 7
9 9 8 7 7 5 3 2 2 0	8	1	3 8
8 5 4 0	9	1	6

- Find the mean (to one decimal place) for:
 - the girls
 - the boys
 - the whole group.
- Find the median for:
 - the girls
 - the boys.
- Where are the scores clustered for:
 - the girls
 - the boys?
- Compare the shapes of the two data sets.
- Which group, girls or boys, performed better? Justify your answer.

Solution

- Mean for girls = $\frac{2600}{37}$
 ≈ 70.3
 - Mean for boys = $\frac{2234}{40}$
 ≈ 55.8
 - Mean for group = $\frac{4834}{77}$
 ≈ 62.8
- Median for girls = 75
19th score
 - Median for boys = $\frac{54 + 55}{2}$
= 54.5
Average of the 20th and 21st scores.
- The girls' scores are clustered at 70s and 80s.
 - The boys' scores are clustered at 40s and 50s.
- The scores for the girls are negatively skewed, while the scores for the boys are positively skewed. The scores for the girls are generally higher than the scores for the boys.
- The girls performed better, which can be seen from their mean of 70.3 being greater than the mean of 55.8 for boys. Also, the median for girls was 75, which was higher than the median of 54.5 for the boys.

Exercise 9-04 Comparing data sets

- 1 The times (in seconds) taken for swimming 100 metres by Brooke and Ryan are:

Brooke: 59.71 59.20 60.21 60.13 59.69 59.50 60.41 59.33 58.95 58.70
Ryan: 55.60 56.20 57.50 56.70 57.15 57.65 58.10 59.70 60.15 56.65

- Find correct to two decimal places the mean for each swimmer.
 - Find correct to one decimal place the median for each swimmer.
 - Find the range for each swimmer.
 - Which swimmer is faster? Give reasons.
 - Which swimmer is more consistent? Give reasons.
- 2 The maximum daily temperatures in Sydney and Adelaide for a 15-day period in March were:

Sydney	34.3	21.9	32.3	30.6	21.2	22.0	25.1	29.2
	31.5	29.2	25.9	30.6	32.1	27.3	24.7	
Adelaide	22.0	22.4	24.3	24.3	26.2	34.6	34.9	23.2
	22.1	21.6	26.8	32.0	28.3	27.1	29.0	

- Find the range for each city.
 - Find the mean for each city.
 - Find the median for each city.
 - On how many days was Sydney warmer than Adelaide?
 - Which city was warmer? Give reasons.
- 3 A class was given a topic test on probability. The results were disappointing and after some more work on the topic the class was given a second test on probability. The results of the two tests are shown in this back-to-back stem-and-leaf plot.

See Example 9

Test 1	Test 2
1 9	0 9
5 2 8	4 8 8
4 3 2 7	0 1 5 5 8 8
7 4 4 1 0 6	0 3 4 4 7 7 7 8
9 9 9 6 5 3 2 2 5	3 4 4 9 9
9 8 7 4 3 0 4	5 7 8
4 2 3	

- Find the median for each test.
- Find correct to one decimal place the mean for each test.
- Find the range for each test.
- Describe the shape of each test.
- Has significant improvement been made from Test 1 to Test 2? Give reasons.

- 4 The dot plots show the temperatures (in $^{\circ}\text{C}$) at 3 p.m. for Logan City, Queensland and Sydney for 16 days in autumn.

Logan City



Sydney



- Which city's temperatures are more skewed?
 - Find the mean for each city.
 - Find the range for each city.
 - Which city had a more consistent temperature? Give reasons.
 - Find the median for each city.
 - Which city had cooler temperatures? Give reasons.
- 5 The reaction times (in seconds) of a group of men and women were measured:
- Men:** 38 29 37 28 30 32 31 53 31 26
49 47 33 55 29 44 32 48 43 29
- Women:** 51 34 40 35 45 40 36 48 57 39
42 35 51 54 36 37 37 39 43 45
- Arrange the two sets of data in a back-to-back stem-and-leaf plot.
 - Would dot plots be suitable for displaying the two sets of data? Give reasons.
 - Are the mean and median the same or different for:
 - men?
 - women?
 - Find the mode for each group.
 - Describe the shape of each distribution. Are there any outliers or clusters?
 - Which group has the faster reaction times?
 - Which group has the greater spread of reaction times?

- 6 Do newspaper articles have bigger words than those in popular magazines?

- Choose a random article from a newspaper and an article from a magazine
- For the first 100 words of each article, count the number of letters in each word and complete a frequency table like the one shown.
- Draw a frequency histogram for the word lengths in each article.
- Describe the shape of each histogram. Are there any clusters or outliers?
- Find the mean word length for each article.
- Find the median word length for each article.
- Find the modal word length for each article.
- Which article has, in general, the longer words?
- Does this relate to the nature of the article (for example, news or lifestyle)?

Word length	Frequency
1	
2	
3	
$\vdots$	

Technology Comparing relative humidities

Relative humidity is the amount of moisture in the atmosphere, measured as a percentage of water vapour per area at a certain temperature. It is reported more often in summer. When presented together with a high temperature, it shows that a day has been particularly warm and ‘muggy’. In this activity, we will compare the relative humidity in a town at 3 p.m. each day for the first fortnight in February in 2010 and 2011.

- 1 Enter the data below into a spreadsheet.

	A	B	C	D	E
		Feb 2010, Relative Humidity @ 3pm (%)			Feb 2011, Relative Humidity @ 3pm (%)
1	Feb-10			Feb-11	
2	1/02/2010	66		1/02/2011	49
3	2/02/2010	60		2/02/2011	70
4	3/02/2010	66		3/02/2011	54
5	4/02/2010	76		4/02/2011	73
6	5/02/2010	88		5/02/2011	22
7	6/02/2010	95		6/02/2011	77
8	7/02/2010	86		7/02/2011	58
9	8/02/2010	70		8/02/2011	52
10	9/02/2010	65		9/02/2011	83
11	10/02/2010	61		10/02/2011	70
12	11/02/2010	60		11/02/2011	60
13	12/02/2010	32		12/02/2011	76
14	13/02/2010	80		13/02/2011	88
15	14/02/2010	92		14/02/2011	59

- 2 Select **Scatter graph with smooth lines and markers** to graph this data on one set of axes. Give the graph a title and label the axes.
- 3 In 2 to 3 sentences, describe any patterns as well as similarities and/or differences in these graphs.
- 4 In B16 and E16, enter formulas to calculate the **average (mean)** relative humidity for each fortnight.
- 5 In B17 and E17, enter formulas to calculate the **maximum** relative humidity.
- 6 In B18 and E18, enter formulas to calculate the **minimum** relative humidity.
- 7 In B19 and E19, enter formulas to calculate the **range**.
- 8 In B20 and E20, enter formulas to calculate the **median**.
- 9 Compare your graphs as well as your statistical values. What do they suggest about the relative humidity for each year? Which year shows a more consistent relative humidity? Justify your answer.
- 10 Visit the Bureau of Meteorology website www.bom.gov.au and select **Climate data online** to find the relative humidity for the past fortnight in your area. Compare the data with this time last year.
 - a Can you see any patterns?
 - b Is the relative humidity consistent?
 - c Are there any outliers?
 - d Analyse the data and describe any similarities or differences you can see.

Mental skills 9

Maths without calculators

Finding a percentage of a multiple of 10

We can use the unitary method to calculate a percentage of a multiple of 10. To do this, we first find 1% of the number by dividing it by 100. This is easily done by moving the decimal point two places to the left.

1 Study each example.

$$\begin{aligned} \text{a } 1\% \times 300 &= 3.00 \\ &= 3 \end{aligned}$$

$$\begin{aligned} \text{b } 4\% \times \$500 \\ 1\% \times \$500 &= \$5 \\ \therefore 4\% \times \$500 &= 4 \times \$5 \\ &= \$20 \end{aligned}$$

$$\begin{aligned} \text{c } 9\% \times 40 \\ 1\% \times 40 &= 0.4 \\ \therefore 9\% \times 40 &= 9 \times 0.4 \\ &= 3.6 \end{aligned}$$

$$\begin{aligned} \text{d } 12\% \times \$700 \\ 1\% \times \$700 &= \$7 \\ \therefore 12\% \times \$700 &= 12 \times \$7 \\ &= \$84 \end{aligned}$$

2 Now evaluate each expression.

$$\begin{array}{ll} \text{a } 1\% \times 800 & \text{b } 1\% \times 30 \\ \text{e } 8\% \times \$600 & \text{f } 3\% \times 40 \\ \text{i } 6\% \times \$50 & \text{j } 8\% \times \$90 \end{array}$$

$$\begin{array}{ll} \text{c } 3\% \times \$400 & \text{d } 4\% \times 20 \\ \text{g } 7\% \times 80 & \text{h } 2\% \times \$900 \\ \text{k } 11\% \times 700 & \text{l } 13\% \times 200 \end{array}$$

3 Study each example.

$$\begin{aligned} \text{a } 4\% \times 320 \\ 1\% \times 320 &= 3.2 \\ \therefore 4\% \times 320 &= 4 \times 3.2 \\ &= 12.8 \end{aligned}$$

$$\begin{aligned} \text{b } 1.5\% \times \$200 \\ 1\% \times \$200 &= \$2 \\ \therefore 1.5\% \times \$200 &= 1.5 \times \$2 \\ &= \$3 \end{aligned}$$

$$\begin{aligned} \text{c } 3\% \times 250 \\ 1\% \times 250 &= 2.5 \\ \therefore 3\% \times 250 &= 3 \times 2.5 \\ &= 7.5 \end{aligned}$$

$$\begin{aligned} \text{d } 4.5\% \times \$800 \\ 1\% \times \$800 &= \$8 \\ \therefore 4.5\% \times \$800 &= 4.5 \times \$8 \\ &= \$36 \end{aligned}$$

4 Now evaluate each expression.

$$\begin{array}{ll} \text{a } 5\% \times 250 & \text{b } 1.5\% \times \$300 \\ \text{e } 2.5\% \times \$600 & \text{f } 3\% \times 160 \\ \text{i } 3.5\% \times \$400 & \text{j } 6\% \times \$120 \end{array}$$

$$\begin{array}{ll} \text{c } 2\% \times \$540 & \text{d } 9\% \times 110 \\ \text{g } 4\% \times \$750 & \text{h } 4.5\% \times \$2000 \\ \text{k } 3\% \times 190 & \text{l } 1.5\% \times \$700 \end{array}$$

9-05 Sampling and types of data

When the marketing department of a large company wishes to launch a new product, it collects information about consumer tastes. Governments collect information about the population so that it can provide for transport, housing, health and education needs. A school may need to collect information regarding school uniforms or food to be sold in the school canteen.

Sample vs census

In statistics, the **population** refers to all the items under investigation or being studied. For example, if the life of AAA-size batteries is being tested, then the population is all the different brands or types of AAA batteries.

A **census** is a survey of the entire population. If the population is too large or difficult to survey, then a **sample** of the population is selected instead. The sample must be representative of the population, by being:

- random: every item in the population should have an equal chance of being selected for the sample
- unbiased: there should not be any factors that may influence the sample and prevent it from being a true indicator of the population
- large: the bigger the sample, the more representative it will be.

Example 10

Determine whether a census or a sample should be used for each investigation.

- Finding the most popular TV program
- Finding the number of primary school children in western Sydney
- Determining people's opinions on public transport

Solution

- Sample, because it would be too expensive and impractical to survey all TV viewers.
- Census, because accurate figures are needed.
- Sample, because it would be too expensive and time-consuming to survey all people.

Types of data

There are two types of data: categorical and numerical.

Categorical data are usually words and can be grouped into categories, such as a person's hair colour or cultural background.

Numerical data (or quantitative data) are numbers describing things that can be counted or measured, like the number of goals scored or a person's height. Numerical data is either discrete or continuous.

Puzzle sheet

Data match-up

MAT09SPPS10105

Technology worksheet

Excel worksheet:
CensusAtSchool

MAT09SPCT00016

Technology worksheet

Excel spreadsheet:
CensusAtSchool

MAT09SPCT00001

- **Discrete data** are counted or measured and can only take on separate, distinct values, with 'gaps' or 'jumps', for example, the number of girls in a family is discrete because it could be 0, 1, 2, etc, but never 2.3 or $1\frac{2}{5}$.
- **Continuous data** are measured on a smooth scale with no 'gaps' or 'jumps' between values, for example, temperature during the day is continuous because it can be 28°C, 29°C, 29.5°C, 30.1°, etc.

Example 11

Classify whether each type of data is categorical or numerical. If numerical, then classify whether it is discrete or continuous.

- School report grades (for example, A, B, C).
- The noise level at a concert.
- People's shoe sizes.
- Marital status (for example, single, married).

Solution

a School report grades is categorical data.	Involves words or categories.
b Noise level is continuous numerical data.	Takes on a smooth scale of values.
c People's shoe sizes is discrete numerical data.	Takes on separate values.
d Marital status is categorical data.	Involves words or categories.

Exercise 9-05 Sampling and types of data

See Example 10

- Determine whether a census or a sample should be used for each investigation.
 - Finding the number of people in Ashfield who were born overseas.
 - Testing the quality of running shoes.
 - Determining whether Australia needs a new flag.
 - Determining the number of people in Newcastle who earn \$50 000 – \$60 000 per year.
 - Determining the average family size in Australia.
- In Australia, a national census was held in 2001, 2006 and 2011.
 - How often is a census held in Australia?
 - When will the next census be held?
- The table shows the number of students in each year at Southwest High.

Year	7	8	9	10	11	12
Students	148	151	160	153	132	128

- What is the school population?
- What percentage (correct to one decimal place) of the school population is formed by:
 - Year 8?
 - Year 11?
- A sample of 100 students is to be surveyed on the amount of time that they spend on the Internet. How many students should be selected from:
 - Year 8?
 - Year 11?

- 4 Students are surveyed about how they travel to school. The first 30 students arriving at school were selected for surveying. Is this a representative sample to use? Give reasons.
- 5 People were asked how often they eat takeaway food. A sample of 100 people was surveyed at a shopping centre. Is this a representative sample to use? Give reasons.
- 6 Classify each sample of data as either categorical or numerical.
 - a The number of people absent from school today.
 - b The distance travelled by a car on a trip.
 - c The amount of petrol bought when filling up the car.
 - d The different type of pies available at the supermarket.
 - e The blood pressure of a patient in hospital.
 - f The amount of time spent on the Internet during the week.
 - g The time for swimming 100 m.
 - h The country of birth of people living in Australia.
 - i The TV shows watched by students.
 - j The age you turn this year.
- 7 Students are asked to rate a movie on a scale of 1 to 5 stars. What type of data is this? Select **A, B, C or D**.
A continuous **B** categorical **C** discrete **D** random
- 8 Classify whether each type of numerical data is discrete or continuous.
 - a The number of people attending the Easter show.
 - b The temperature in Grafton over a 24-hour period.
 - c The time spent doing homework.
 - d The speed of a car passing through an intersection.
 - e The age you turn this year.
 - f The results in a maths test.
 - g The amount of rainfall in a month.
 - h The size of the audience at a concert.

See Example 11

Just for the record

Australia's first census

The first census in Australia was conducted in November 1828. Before this, statistical information was collected by musters on the same day each year. Worried about the accuracy of the figures, Governor Darling and the Legislative Council passed an Act requiring a census to be held at regular intervals. Each citizen needed to disclose his or her name, age, marital status, number of children, the name of the ship on which they arrived, the amount of land owned and the number of cattle and other stock owned. Collectors were employed to gather this information by personal interview.

The first census showed that the total non-Aboriginal population was 36 598 and nearly 30% of that population lived in Sydney.

How many national censuses have there been in Australia?

Investigation: Australian Bureau of Statistics

The Australian Bureau of Statistics (ABS) is the official organisation in charge of collecting data for government departments. The data collected covers many areas, from the height of children in Australia, the alcohol consumption in the country to the number of Internet subscribers and Internet providers. The ABS conducts a census of the entire Australian population every 5 years.

Visit the ABS website www.abs.gov.au to answer the following questions.

- 1
 - a What is a population clock?
 - b What is the current predicted population of Australia?
 - c On average, another baby is born in Australia after how many minutes?
 - d On average, another person dies in Australia after how many minutes?
 - e A new migrant arrives in Australia after how many minutes?
 - f Australia's population increases by one after how many minutes?
- 2 Go to Australian Demographic Statistics and find the populations of the state and territories.
 - a What percentage of Australia's population live in:
 - i NSW
 - ii ACT?
 - b Which state had the greatest percentage increase over the previous year?
- 3 Find the population pyramid for Australia's population.
 - a What is a population pyramid?
 - b What percentage of Australian males are aged 15–19?
 - c What is the modal age?
 - d How many people are aged 100 years and over?
 - e Which group, males or females, have the higher life expectancy? Justify your answer.
- 4
 - a Explain why Australia's population is said to be ageing.
 - b What is the median age of Australia's population?
 - c Find the median ages of all the states and territories and display them in a table. Which has the:
 - i highest median age
 - ii the lowest median age?
- 5 Use the ABS website to find the number of people that live in your local area.

9-06 Bias and questionnaires

Homework sheet

Data 3

MAT09SPHS10040

Technology worksheet

Random numbers

MAT09SPCT10011

Bias in sampling

When taking a sample, it is important that each item in the population has an equal chance of being chosen to be surveyed. This is called **random sampling**. To find a sample of students to survey about school uniform, we could place students' names in a box, mix them up, and then choose 100 names without looking. Alternatively, we could use a computer or calculator to generate random numbers to select students.

A sample that is not random is called a **biased sample**, and is not truly representative of the population. For example, if you survey the first 100 students to arrive at school today, or the first 100 students on the school roll, then not all students in the school have an equal chance of being selected. Your sample would not represent students who arrive at school later or whose names begin with letters at the end of the alphabet.

Example 12

A random sample of students is to be selected for their views on a new school uniform. Will each of the following sampling methods provide a sample that is representative or biased? If biased, explain why.

- a randomly selecting one student from each class
- b selecting twenty students from your mathematics class
- c selecting the students from the school's netball teams
- d randomly selecting students from the school roll by rolling a die and counting down the roll.

Solution

- a Representative. Every student in the school has an equal chance.
- b Biased. Because your mathematics class does not represent the other Year groups (or even the other Year 9 classes) at your school.
- c Biased. Because the netball teams do not represent the whole school and will have mainly female students.
- d Representative. Every student in the school has an equal chance.

Designing a questionnaire

Questionnaires are often used to collect information. They may ask for details about a person's opinions and attitudes, their lifestyle and behaviours such as their recreational activities, TV viewing habits and types of foods eaten.

When designing a questionnaire, it is important that the questions asked are clear, fair and not biased.

- Decide what questions you want answered before you begin.
- Word the questions in plain English so that they are easily understood: make sure they are not vague, too open-ended or have a 'double meaning'.
- Ask for only one piece of information per question.

- Do not allow for too many different answers, provide choices if possible:
 - a tick or a cross beside each response
 - ‘Yes’, ‘No’, ‘Don’t know’ or ‘Other’ responses
 - a rating on a scale
 - a one-word response
 - a sentence
- Determine how you are going to present and use the data you collect.
- Do not try to influence a particular answer unfairly (this is also called bias).
- Do not ask for information that may be too personal, such as phone numbers.

Example 13

Explain what is wrong with each survey question and suggest an alternative.

a ‘How old are you?’

b ‘How much pocket money do you receive each week?’

Solution

a The question is not precise enough. Possible answers could be:

15 15 years, 4 months 14 years 10 months and 3 days $16\frac{1}{2}$

A better question might be:

‘What age do you turn this year?’

14 ☐ 15 ☐ 16 ☐

or

‘What is your date of birth?’

...../...../..... (dd/mm/yy)

b The question is too general and responses could be:

Nothing \$5/day \$40/week Whatever I need

A better question could be:

‘Circle the amount of pocket money you receive in a normal week:’

\$5 or less \$6 to \$10 \$11 to \$15 more than \$15

Exercise 9-06 Bias and questionnaires

- 1 When collecting data it is important to have an unbiased sample. Write, in your own words, what is meant by an 'unbiased sample'.
- 2 List the possible areas of bias in each sampling method.
 - a A survey about the amount of time spent doing homework, taken at a football match.
 - b A survey about violence in sport, taken at a sporting event.
 - c A survey about the use of alternative fuels, taken at a car show.
 - d A survey about types of music listened to, taken at a rock concert.
- 3 A market research company uses a telephone survey to determine the most popular news program on TV. List some of the advantages and disadvantages of using this type of survey to collect data.
- 4 In order to determine what type of music should be played on the school radio station, 20 students were given a questionnaire.
 - a Is the sample suitable for obtaining the required information? Give reasons.
 - b List some of the advantages of using a questionnaire to obtain the data.
 - c List some of the disadvantages of using this method.
 - d What other methods could be used to find out the type of music that should be played?
- 5 Describe how you would collect data about each of the following.
 - a The most popular fast food eaten by students at your school.
 - b The average daily amount of money spent by students at your school canteen.
 - c The most popular TV series watched by students at your school.
 - d The most popular Australian band listened to by students at your school.
 - e The average number of girls in a classroom in NSW.
 - f The most commonly used brand of toothpaste.
- 6 You need to determine how many students at your school have part-time jobs. Which of the following methods would be most suitable? Give reasons.
 - A You ask 15 students in your class at school.
 - B You randomly select 15 students from each Year level at your school.
 - C You ask all students at your school to fill in a questionnaire and return them to you.
- 7 For each survey question, explain what is wrong with the question and suggest a better question.
 - a How much time do you spend studying?
 - b What is your favourite sport?
 - c What is your income?
 - d What do you have for lunch each day?
 - e What do you do in the holidays?
 - f How often do you exercise?
 - g How much time do you spend on the Internet?
 - h How many text messages do you send in one day?

See Example 12

See Example 13

- 8 Select one of the following as the topic for a questionnaire.
- Amount of TV watched at home.
 - How students in Year 9 spend their time on a Sunday.
 - The amount of time spent on homework during the week.
 - The amount of time spent on the internet.
 - Another topic of your choice.
- a Design a questionnaire.
 - b Select a sample of students to answer it.
 - c Collect and present the information.
 - d Did the questions obtain the required data?
 - e Do you need to change any of the questions?
 - f Are there any questions you forgot to ask?
 - g Are there any questions you would omit next time?

Investigation: Media reports of surveys

We live in a world of 24-hour news, often quoting the results of surveys. To detect bias in news reports, we need to always consider where the news item comes from and what types of samples the statistics are based on. Is the information supplied by a reporter, the police, a company executive, a government official or an opinionated blogger? Are the statistics based on a small sample, large sample, unrepresentative sample, or a phone or Internet poll of volunteers? Each may have a particular bias that may influence how the story is reported. Find a report of a statistical survey from a newspaper, magazine or the Internet and investigate the following questions.

- a Where did the article come from?
- b Who wrote the article and what claims does it make about the population?
- c Does it show any bias?
- d What type of sampling was used? Was it representative?
- e How many people were questioned?
- f When was the survey conducted?
- g Was there any bias in the sample?
- h Was there any bias in the survey questions?

Investigation: Year 9 student survey

The survey below is designed to collect data about Year 9 students. Questions can be changed or more can be added, but remember that the data is only useful if good questions are asked in a controlled way.

- 1 What is your gender? ☐ Male ☐ Female
- 2 What is your age (as of last birthday)? _____ years
- 3 How tall are you? _____ cm
- 4 How many people are in your household? _____ people
- 5 What is your position in your household (1 = oldest)? _____
- 6 How many pets do you have? _____ pets
- 7 How far from school do you live (to nearest 0.1 km)? _____ km
- 8 How did you travel to school today?
☐ Car ☐ Bus ☐ Walk ☐ Train
- 9 How long did the journey take? _____ minutes
- 10 On a scale of 0 – 10, rate your enjoyment of school (10 = most). _____
- 11 Which subject do you like best?
☐ Maths ☐ Science ☐ English ☐ Art ☐ PE ☐ History ☐ Other
- 12 Which subject do you study most?
☐ Maths ☐ Science ☐ English ☐ Art ☐ PE ☐ History ☐ Other
- 13 How much home study do you do each week? _____ hours
- 14 How many hours do you study maths each week? _____ hours
- 15 On a scale of 0 – 10, rate your enjoyment of maths (10 = most) _____
- 16 What was your last mark in a maths test? _____ %
- 17 How much competitive sport do you play out of school? _____ hours
- 18 How many competitive sports do you play out of school? _____ sports
- 19 How many hours each week do you spend on getting fit? _____ hours
- 20 Which sport do you prefer to watch?
☐ Football ☐ Soccer ☐ Netball ☐ Tennis ☐ Surfing ☐ Other

Steps in conducting the survey

- 1 Finalise the questions and print the survey sheets.
- 2 Decide how many students you want to respond to the survey.
- 3 Decide how you will select the sample of students.
- 4 Decide whether the survey will be handed to selected students or if an interviewer will record responses.
- 5 Decide whether you will interpret the questions/responses or not.
- 6 Ask the selected students to complete the survey.
- 7 Collect all the sheets and label the first sheet 1, the second sheet 2 and so on.
- 8 Decide on the code you will use for each response, then code the responses and enter the results in a table, either on a spreadsheet or in your workbook.
- 9 Decide what to do if a question is not answered or the answer appears to be a mistake.

Power plus

- 1 The number of sandwiches sold daily in a school canteen were:

25 28 22 35 26 36 33 48 42 37
 25 25 37 44 19 40 23
 38 42 32 28 21 36 22 39 46 28
 45 28 41 43 26 17 28

- a Complete a frequency table by grouping the scores into class intervals like this:
 b Draw a frequency histogram and describe the shape, identifying any outliers or clusters.
 c What is the modal class?
 d Add a cumulative frequency column to your frequency table and find the median class.

Class interval	Frequency
16–21	
22–27	
28–33	
34–39	
40–45	
46–51	

- 2 When would a stem-and-leaf plot be impractical to use?
 3 Seeds were sown in 40 large boxes. The number of seedlings that sprouted per box were:

14 23 18 18 17 28 28 33 4 24
 17 27 11 19 13 18 18 7 25 16
 14 3 31 4 5 12 23 26 11 17
 19 20 24 7 30 16 25 8 19 20

- a Organise the data into a stem-and-leaf plot using intervals of:
 i 10 for the stem ii 5 for the stem (0^0 for 0–4, 0^5 for 5–9, 1^0 for 10–14, ...)
 b Describe the shape of both plots, identifying outliers and clusters.
 c Which plot gives a better indication of how the seedlings have grown? Explain your answer.

Puzzle sheet

Statistics crossword

MAT09SPPS10106

Quiz

Statistics

MAT09SPQZ00001

Language of maths

back-to-back	bias	categorical data	census
cluster	continuous	cumulative frequency	discrete
distribution	dot plot	histogram	mean
measure of location	median	mode	negatively skewed
numerical data	outlier	polygon	population
positively skewed	sample	stem-and-leaf plot	symmetrical

- 1 What is the collective name for the mean, median and mode?
- 2 Is a person's arm length an example of discrete data or continuous data?
- 3 What is the meaning of skewed?
- 4 Explain the difference between a population and a sample.
- 5 What name is given to an extreme score that is very different from the other scores in a data set?
- 6 Name one way of avoiding bias when selecting a sample.

Topic overview

Rate yourself on the work in this chapter by copying and completing the following scales.

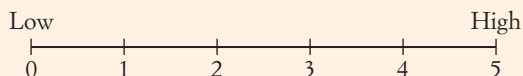
- a** Able to find the mean, median, mode and range and use them to analyse data.



- b** Able to construct and use frequency histograms, polygons and stem-and-leaf plots.



- c** Able to describe the shape of a distribution.



- d** Able to compare two data sets.



- e** Understand sampling and the different types of data.



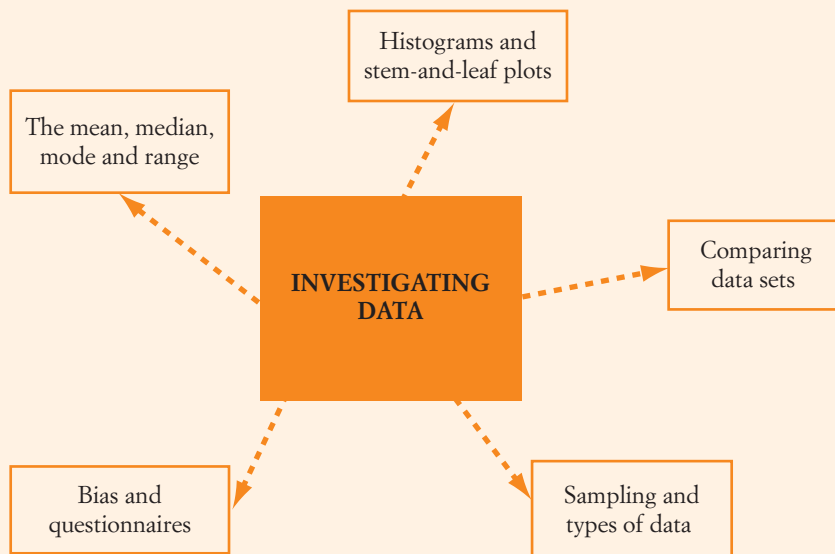
Chapter 9 review

Worksheet

Mind map:
Investigating data

MAT09SPWK10107

Copy (or print) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



- 1 Find the mean, median, mode and range of these scores.

See Exercise 9-01

8 6 9 4 10 8 5 5 8

- 2 A real estate agent sold 10 properties during one week. The prices of these properties are:

\$420 000	\$485 000	\$379 000	\$295 000	\$1 750 000
\$515 500	\$315 500	\$408 000	\$368 000	\$315 000

- Find the mean price of the properties.
- Find the median price of the properties.
- Compare the mean price with the median price. Which is the better measure of location? Give reasons.
- When data on sales of properties are given, the median price is used, not the mean. Explain why.

- 3 The following scores are percentage marks scored by 30 students in a Maths test.

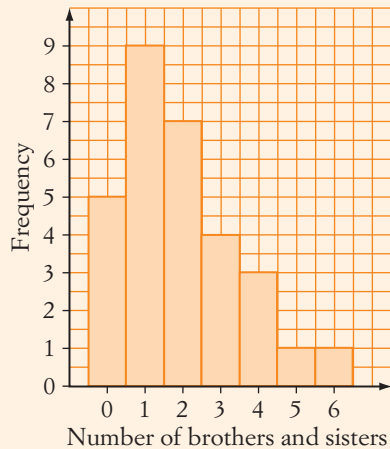
See Exercise 9-02

45	67	78	54	22	38	58	64	73	89
84	70	78	62	56	59	42	47	86	90
64	63	72	77	43	85	75	71	70	83

- Construct an ordered stem-and-leaf plot for the data.
- Find the mean, median, mode and range of these marks.
- Are there any outliers?
- Compare the mean and median. Which is more accurate? Give reasons.

- 4 A group of people were surveyed on the number of brothers and sisters they had. The results are displayed on the histogram.

- What is the range of this data?
- What is the mode?
- Find the mean.
- What is the median?
- Which is the best measure of location?



Chapter 9 revision

- 5 Twenty-four patients were chosen at random from two medical centres. The waiting times (in minutes) for each patient are listed below.

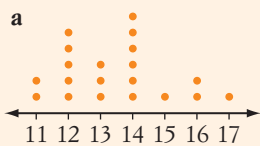


Northbridge:	7	16	28	27	30	30	47	9	9	15	27	11
	38	35	42	53	23	22	45	32	37	37	18	16
Westvale:	44	13	7	9	25	26	37	35	8	18	18	15
	22	24	7	5	24	24	33	37	24	15	36	43

- Display the information in a back-to-back stem-and-leaf plot.
- Find the mean, median and range of both sets of data.
- Identify any outliers or clusters.
- Are there significant differences between the two sets of data? Explain your answer.

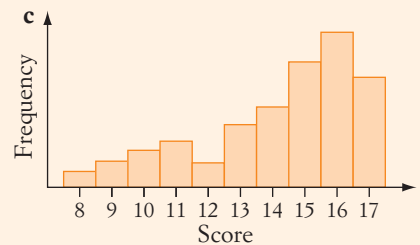
See Exercise 9-03

- 6 For each distribution, describe the shape and identify any outliers or clusters.



b

Stem	Leaf
8	7 8
9	1 3 4 4
10	0 2 7 8 8 9
11	1 2 2
12	0 7



See Exercise 9-04

- 7 The heights (in centimetres) of students in a Year 9 class are:

Girls:	160	153	155	156	142	163	162
	148	150	150	131	138	147	145
Boys:	150	165	164	178	170	171	164
	148	154	176	134	180	176	175

- Arrange the data in a back-to-back stem-and-leaf plot.
- Describe the shape of the display for:
 - the girls
 - the boys.
- Are there any outliers or clusters for either group?
- Find the mean and median for both groups.
- Which measure is the better indicator of the measure of location?
- Which group is shorter?
- Which group has the greater spread of heights?

8 Should a sample or a census be used to investigate each of the following?

See Exercise 9-05

- a the most popular car colour
- b whether energy-saving light globes really use less electricity
- c the number of retired people living on the Gold Coast
- d people's views on which nation will win the rugby World Cup

9 Classify each type of data as categorical or numerical. If numerical, then classify it as discrete or continuous.

- a The temperature at Cooma.
- b The number of viewings of a YouTube video.
- c Suburb or town of home.
- d The make of car passing through a toll booth.
- e The probability of rain today.

10 Explain why each of the following samples is biased.

See Exercise 9-06

- a A sample of *Dolly* magazine readers was surveyed to investigate teenagers' views on road safety.
- b Diners at a restaurant can give feedback about their meal by filling in a survey and posting it to the restaurant.
- c At a factory, a sample of executives was surveyed about safety in the workplace.
- d A sample of audience members at a film premiere was surveyed about their views on the film.
- e Triple J radio listeners were asked to vote for the best song of the year.

11 Explain what is wrong with each survey question and suggest an alternative.

- a Are you happy?
- b How did you enjoy your meal at our restaurant?
- c People have died for our Australian flag, so do you think it should be changed?

Mixed revision 3

See Exercise 10-01

1 Copy and complete:

- a 750 min = _____ h b 1350 mL = _____ L c 512 MB = _____ GB
d 3.005 kg = _____ g e 2.7 km = _____ m f 0.25 MB = _____ kB

See Exercise 8-03

2 Jade is paid 72 cents for each toy she assembles.

- a How much does she earn for assembling 115 toys?
b How many toys did Jade assemble if she earned \$403.20?

See Exercise 8-04

3 Katrina's annual salary is \$62 000 and she earns another \$3500 from investments. Katrina's work-related tax deductions amount to \$1780. Calculate Katrina's:

- a taxable income b income tax payable using the table below.

Taxable income	Tax on this income
0 – \$18 200	Nil
\$18 201 – \$37 000	19c for each \$1 over \$18 200
\$37 001 – \$80 000	\$3572 plus 32.5c for each \$1 over \$37 000
\$80 001 – \$180 000	\$17 547 plus 37c for each \$1 over \$80 000
\$180 001 and over	\$54 547 plus 45c for each \$1 over \$180 000

Source: ato.gov.au, 2012–2013 rates

See Exercise 9-02

4 The shoe sizes of 30 Year 9 students are as follows.

6 10 7 6 8 6 7 7 6 5
5 6 5 8 10 11 9 8 9 5
6 7 9 8 7 6 5 6 7 8

- a Arrange the data in a frequency table.
b Draw a frequency histogram and polygon of this data.
c What percentage (correct to one decimal place) of students had shoe sizes less than 8?
d Find the mode and the range.
e Find the median.
f Calculate the mean correct to one decimal place.

See Exercise 8-01

5 For a salary of \$56 160 p.a., calculate, correct to the nearest cent, the:

- a weekly income b fortnightly income c monthly income.

See Exercise 8-03

6 Ben works 38 hours a week and is paid \$19.80 per hour. Calculate:

- a Ben's weekly income
b the holiday loading of $17\frac{1}{2}\%$ on 4 weeks pay
c Ben's holiday pay for 4 weeks.

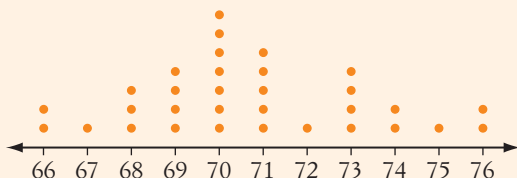
See Exercise 9-01

7 For each set of data, find:

- i the mode ii the mean (correct to one decimal place)
iii the median iv the range

a 47 46 50 51 48 48 52 46 49

b



- 8 Classify whether each type of data is categorical or numerical. If numerical, then classify whether it is discrete or continuous.

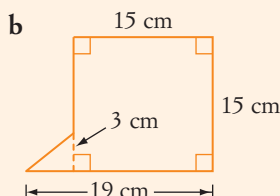
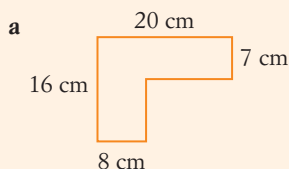
See Exercise 9-05

- a the age that most Year 9 students turn this year b colour of cars in a car park
c travelling time between home and work d amount of rainfall

- 9 For each composite shape, find:

See Exercise 10-03

- i its perimeter ii its area



- 10 Nadia's gross pay for a fortnight is \$2180. She pays \$646.50 in tax and her other deductions amount to \$327.

See Exercise 8-05

- a Calculate Nadia's net pay.
b Express her net pay as a percentage (correct to one decimal place) of her gross pay.

- 11 For the data in this stem-and-leaf plot, which statement is true? Select the correct answer **A**, **B**, **C** or **D**.

See Exercise 9-02

- A** The mean is 82
B The mode is 6
C The range is 46
D The median is 81

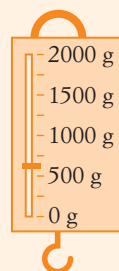
Stem	Leaf
6	0 1 5
7	3 4 4 8 9
8	0 1 2 2 5 6 6 6
9	3 4 7
10	6

- 12 Sefa is paid \$15.60 per hour. Calculate his earnings for working 38 hours at normal time, 8 hours at time-and-a-half and 4 hours at double time.

See Exercise 8-02

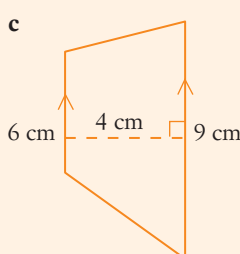
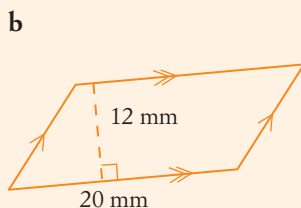
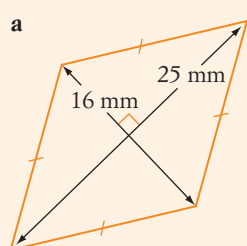
- 13 What are the limits of accuracy for these measuring scales?

See Exercise 10-02



- 14 Find the area of each quadrilateral.

See Exercise 10-04



Mixed revision 3

See Exercise 8-03

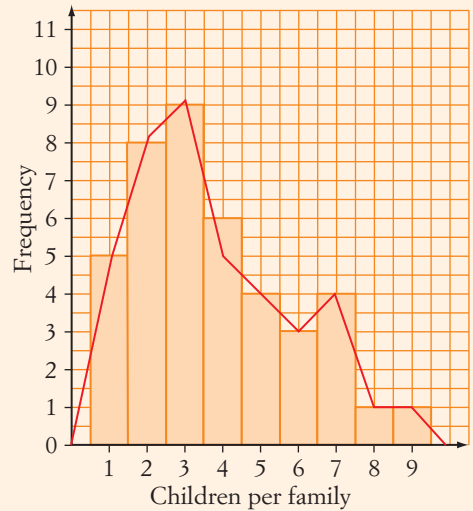
- 15** Trish earns a commission of 3.25% on every property she sells. Calculate her commission for selling an apartment for \$437 500.

See Exercise 9-03

- 16** A sample of students was surveyed on the number of children in their families, and the results were graphed on a frequency histogram and polygon.

See Exercise 9-02

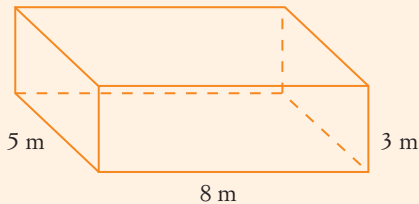
- Describe the shape of the distribution.
- Where does clustering occur?
- Find the mode and range.



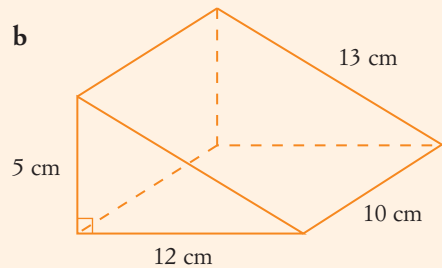
See Exercise 10-06

- 17** Find the surface area of each prism.

a



b



See Exercise 10-08

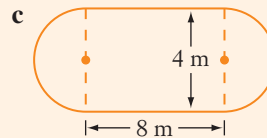
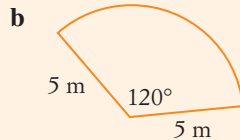
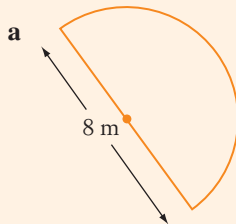
- 18** Find the volume of each prism shown in question 17.

See Exercise 10-05

- 19** For each circular shape, calculate, correct to one decimal place:

i its perimeter

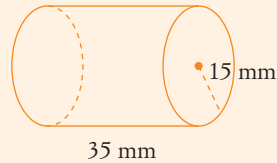
ii its area.



See Exercise 9-06

- 20** In a survey, a sample of people at a park were asked 'Are you employed?'
- Why might this sample of people be biased?
 - What is wrong with the survey question?
 - Suggest a better-worded question.

- 21** Find, correct to three significant figures, the volume of this cylinder.



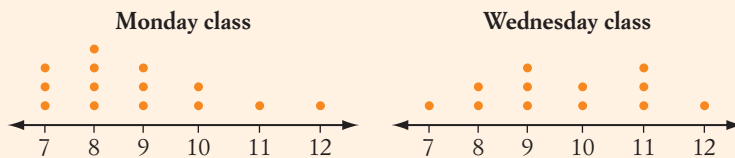
See Exercise 10-08

- 22** Find, correct to three significant figures, the surface area of the cylinder in question **21** if it is open at one end.

See Exercise 10-07

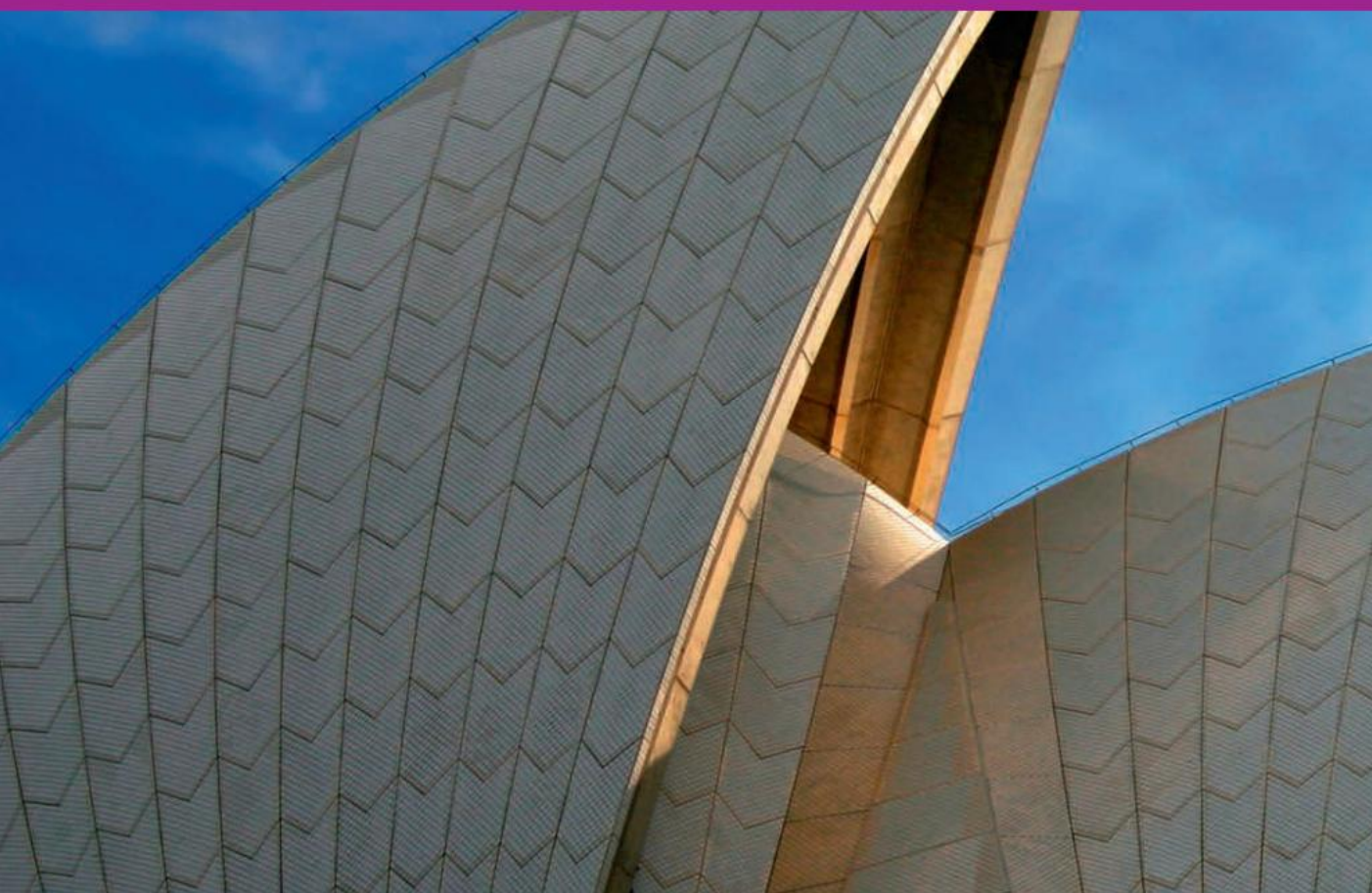
- 23** These dot plots show the ages of students in two Taekwondo classes.

See Exercise 9-04



- Describe the shape of the age distribution for each class.
 - Find correct to two decimal places the mean for each class.
 - Find the median for each class.
 - Which class had more consistent ages?
 - Which class had more older students?
- 24** A cube has a surface area of 384 cm^2 . What is the length of one side of the cube?

See Exercise 10-06



10

Measurement and geometry

Surface area and volume

The Sydney Opera House was designed by Danish architect Jørn Utzon, with construction taking place between 1955 and 1973 at a cost of \$102 million. Its shell design represents a ship under full sail, with each concrete shell being segments of a sphere of radius 75.2 metres. The shells are covered by 1 056 006 white and cream 120 mm square tiles.



Shutterstock.com/cal

Chapter outline

	Proficiency strands			
10-01 Metric units	U	F	R	C
10-02 Limits of accuracy of measuring instruments	U		R	C
10-03 Perimeters and areas of composite shapes	U	F	R	
10-04 Areas of quadrilaterals	U	F	PS	R
10-05 Circumferences and areas of circular shapes	U	F	PS	R
10-06 Surface area of a prism	U	F	PS	R
10-07 Surface area of a cylinder	U	F	PS	R
10-08 Volumes of prisms and cylinders	U	F	PS	R
10-09 Volumes of pyramids and cones*	U	F	PS	R

*STAGE 5.3

Wordbank

capacity The amount of fluid (liquid or gas) in a container

cross-section A 'slice' of a solid, taken across the solid rather than along it

limits of accuracy How accurate a measured value using a measuring instrument can be, equal to half of a unit on the instrument's scale

mega- (abbreviated M) A metric prefix meaning one million

micro- (abbreviated μ) A metric prefix meaning one-millionth

prism A solid shape with identical cross-sections and ends with straight sides

sector A region of a circle cut off by two radii, shaped like a slice of pizza

surface area The total area of all the faces of a solid shape

In this chapter you will:

- investigate very small and very large time scales and intervals
- find areas of parallelograms, trapeziums, rhombuses and kites
- solve problems involving the surface areas and volumes of right prisms
- calculate the surface areas of cylinders and solve related problems 
- learn the metric prefixes for very small and very large units of measurement
- convert between units of digital memory, such as megabytes and gigabytes
- calculate the limits of accuracy of measuring instruments
- calculate the perimeters and areas of composite shapes, including circular shapes involving sectors
- calculate volumes and capacities of right prisms and cylinders
- (STAGE 5.3) calculate volumes of right pyramids and right cones

SkillCheck

Worksheet

StartUp assignment 10

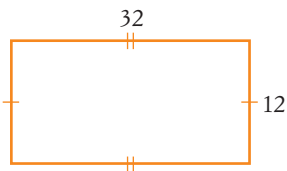
MAT09MGWK10108

- 1 The measurements shown on the shapes below are in centimetres. For each shape, find (correct to two decimal places if necessary):

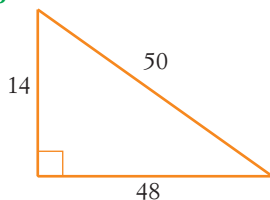
i its perimeter

ii its area

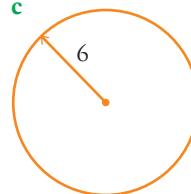
a



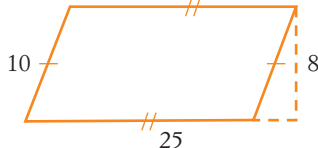
b



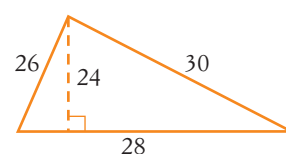
c



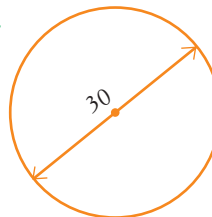
d



e



f



- 2 Convert:

a 34 m to mm

b 925 cm to m

c 1500 m to km

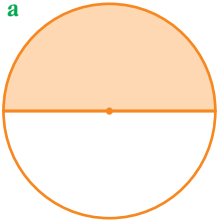
d 3750 mL to L

e 38 cm to mm

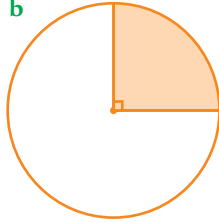
f 7.5 L to mL

3 What fraction of each circle is shaded?

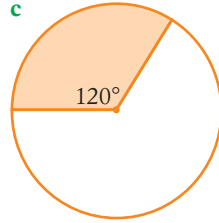
a



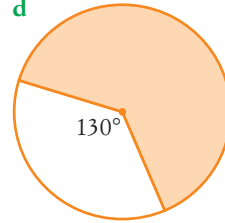
b



c

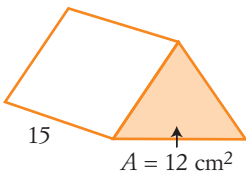


d

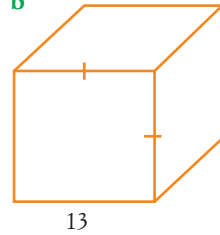


4 Calculate the volume of each prism (all units are in cm).

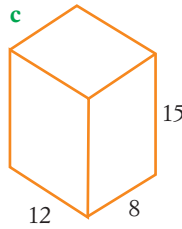
a



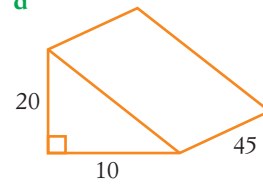
b



c



d



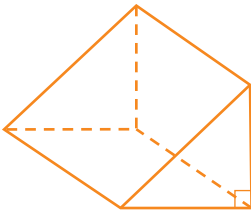
5 For each prism:

i count the number of faces

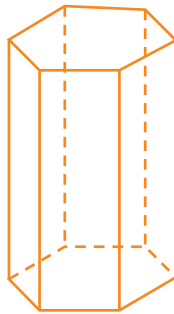
ii name the shapes of the different faces

iii name the prism.

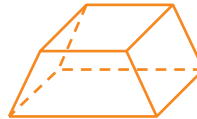
a



b

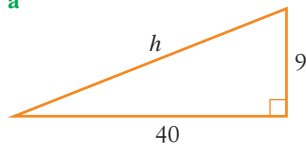


c

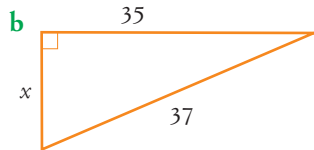


6 Find the value of each pronumeral.

a



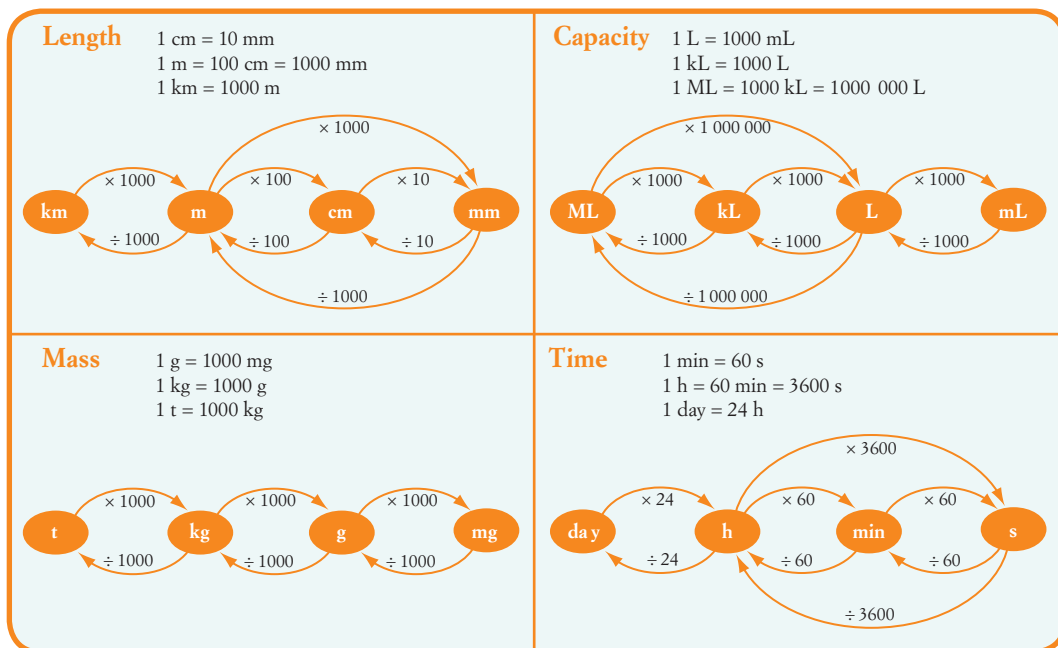
b



c



10-01 Metric units



Other than for time, units of the metric system have prefixes based on powers of 10.

- **kilo-** means 1000 (thousand) or 10^3 1 kilogram = 1000 grams
- **mega-** means 1 000 000 (million) or 10^6 1 megalitre = 1 000 000 litres
- **centi-** means $\frac{1}{100}$ (hundredth) or 10^{-2} 1 centimetre = $\frac{1}{100}$ metre
- **milli-** means $\frac{1}{1000}$ (thousandth) or 10^{-3} 1 millilitre = $\frac{1}{1000}$ litre

Example 1

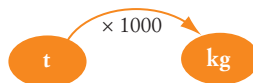
Convert:

- a 5.8 t to kg b 16 km to cm c 23 700 mL to L d 1500 min to h

Solution

a $5.8 \text{ t} = 5.8 \times 1000 \text{ kg}$
 $= 5800 \text{ kg}$

To convert from large to small units, multiply ($\times$).



b $16 \text{ km} = 16 \times 1000 \times 100 \text{ cm}$
 $= 1\,600\,000 \text{ cm}$



c $23\,700 \text{ mL} = 23\,700 \div 1000 \text{ L}$
 $= 23.7 \text{ L}$

To convert from small to large units, divide ($\div$).



d $1500 \text{ min} = 1500 \div 60 \text{ h}$
 $= 25 \text{ h}$



Metric prefixes

This table shows a more detailed list of metric prefixes.

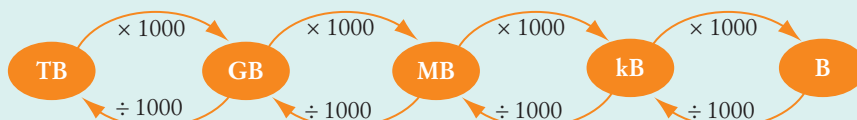
Prefix (abbreviation)	Meaning	Example
pico (p)	10^{-12}	
nano (n)	$10^{-9} = \frac{1}{1\,000\,000\,000}$	nanosecond (ns) = billionth of a second
micro (μ)	$10^{-6} = \frac{1}{1\,000\,000}$	microgram (μg) = millionth of a gram
milli (m)	$10^{-3} = \frac{1}{1000}$	millibar (mbar) = thousandth of a bar (air pressure)
centi (c)	$10^{-2} = \frac{1}{100}$	centilitre (cL) = hundredth of a litre
deci (d)	$10^{-1} = \frac{1}{10}$	decibel (dB) = tenth of a bel (sound level)
deca (da)	$10^1 = 10$	decametre (dam) = ten metres
hecto (h)	$10^2 = 100$	hectopascal (hPa) = hundred pascals (air pressure)
kilo (k)	$10^3 = 1000$	kilojoule (kJ) = thousand joules (energy)
mega (M)	$10^6 = 1\,000\,000$	megahertz (MHz) = million hertz (frequency)
giga (G)	10^9	gigawatt (GW) = billion watts (power)
tera (T)	10^{12}	terabyte (TB) = trillion bytes (computer memory)

Digital memory

Metric prefixes are used in units of digital (computer) memory. A **byte**, abbreviated **B**, is the amount of memory used to store one character, such as @, g, #, 7 or ?.

Summary

Unit	Relationship	Size
kilobyte (kB)	1 kB = 1000 B	About half a page of text.
megabyte (MB)	1 MB = 1000 kB	A million bytes, about the size of a novel.
gigabyte (GB)	1 GB = 1000 MB	A billion bytes, about 600 photos or 7.5 hours of video.
terabyte (TB)	1 TB = 1000 GB	A trillion bytes, about the size of all of the books in a large library.



Some examples of digital memory sizes:

E-mail (without attachment): 50 kB

Web page: 400 kB

Photo: 800 kB

Music file: 6 MB

YouTube video: 7 MB per minute

CD: 750 MB

DVD: 8 GB

Blu-ray disc: 27 GB

Example 2

Convert:

a 3.2 MB to kB

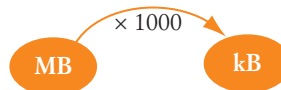
b 147 000 MB to GB

c 0.56 TB to GB

Solution

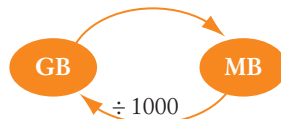
$$\begin{aligned} \text{a } 3.2 \text{ MB} &= 3.2 \times 1000 \text{ kB} \\ &= 3200 \text{ kB} \end{aligned}$$

To convert from large to small units, multiply ($\times$).

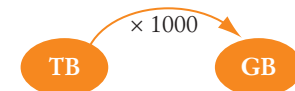


$$\begin{aligned} \text{b } 147\,000 \text{ MB} &= 147\,000 \div 1000 \text{ GB} \\ &= 147 \text{ GB} \end{aligned}$$

To convert from small to large units, divide ($\div$).



$$\begin{aligned} \text{c } 0.56 \text{ TB} &= 0.56 \times 1000 \text{ GB} \\ &= 560 \text{ GB} \end{aligned}$$



Very small and very large units

Some very small and very large units of length, distance and time also use metric prefixes.

Length and distance

- One **nanometre** (nm) = $\frac{1}{1\,000\,000\,000}$ m: your fingernails grown 1 nm every second, the diameter of a helium atom is 0.1 nm
- One **micrometre** (μm) = $\frac{1}{1\,000\,000}$ m: used to measure diameters of bacteria
- One **megametre** (Mm) = 1 000 000 m or 1000 km: the distance between Sydney and Brisbane, or the length of the Bathurst 1000 car race
- One **gigametre** (GM) = 1 000 000 000 m: the diameter of the Sun is 1.4 GM
- One **astronomical unit** (AU) = 149 597 871 km or 149.6 GM: the average distance between the Earth and the Sun, used to measure distances between planets in our solar system
- One **light year** (ly) = 9.46×10^{12} km: the distance light travels in a year, for measuring distances between stars
- One **parsec** (pc) = 3.09×10^{13} km or 3.26 ly: for measuring distances between stars, the nearest star *Proxima Centauri* is 1.30 parsecs away

Time

- One **nanosecond** (ns) = $\frac{1}{1\,000\,000\,000}$ s: the time it takes electricity to travel along a 30 cm wire
- One **microsecond** (μs) = $\frac{1}{1\,000\,000}$ s
- One **millisecond** (ms) = $\frac{1}{1000}$ s: the duration of the flash on a camera
- One **decade** = 10 years
- One **century** = 100 years
- One **millennium** = 1000 years
- One **mega-annum** (Ma) = 1 000 000 years: used in geology and paleontology, the *Tyrannosaurus Rex* dinosaur lived 65 Ma ago
- One **giga-annum** (Ga) = 1 000 000 000 years: the Earth was formed 4.57 Ga ago

Example 3

Convert:

a 0.73 s to μs	b 25 610 000 km to AU
c 4.57 Ga to years	d 1 920 000 nm to m

Solution

a $0.73 \text{ s} = 0.73 \times 1\,000\,000 \mu\text{s}$
 $= 730\,000 \mu\text{s}$

To convert from large to small units, multiply ($\times$).

b $25\,610\,000 \text{ km} = 25\,610\,000 \div 149\,597\,871 \text{ AU}$
 $= 0.17119\dots$
 $\approx 0.17 \text{ AU}$

To convert from small to large units, divide ($\div$).

c $4.57 \text{ Ga} = 4.57 \times 1\,000\,000\,000 \text{ years}$
 $= 4\,570\,000\,000 \text{ years}$

$$\begin{aligned}
 \text{d } 1\,920\,000 \text{ nm} &= 1\,920\,000 \div 1\,000\,000\,000 \text{ m} \\
 &= 1.92 \times 10^{-3} \text{ m} \\
 &= 0.001\,92 \text{ m}
 \end{aligned}$$

Exercise 10-01 Metric units

1 What metric unit would you use for each measurement?

a Width of a computer screen	b Mass of a plane
c Distance between two towns	d Capacity of a car's fuel tank
e Amount of water in a pool	f Time taken to fly overseas

See Example 1

2 Convert:

a 750 g to mg	b 36 kL to L	c 67 kg to g	d 1.5 L to mL
e 6.7 cm to mm	f 3.5 km to m	g 9.4 t to kg	h 3.75 g to mg
i 4.25 m to cm	j 0.2 kg to g	k 26.2 L to mL	l 0.3 km to m
m 3 kL to mL	n 4 days to min	o 15 h to s	p 2.7 km to cm
q 5 kg to mg	r 0.7 m to mm	s 29.375 t to g	t 18.14 km to mm

3 How many mL in 18.9 kL? Select the correct answer A, B, C or D.

A 1 890 000	B 18 900 000	C 189 000	D 18 900

4 Select the most appropriate answer A, B or C for each measurement.

a height of a door		
A 20 cm	B 200 cm	C 20 m
b capacity of a soft drink can		
A 37.5 mL	B 375 mL	C 3.5 L
c time to run 100 metres		
A 12.3 s	B 34.3 s	C 50.3 s
d height of Sydney Tower		
A 30.5 m	B 3050 m	C 305 m
e mass of a family car		
A 100 kg	B 1000 kg	C 10 000 kg



Shutterstock.com/Jose Gil

5 Convert:

a 8000 L to kL	b 90 mm to cm	c 180 s to min
d 240 min to h	e 375 mL to L	f 14 700 kg to t
g 8600 mg to g	h 750 cm to m	i 223 mm to cm
j 800 g to kg	k 125 L to kL	l 3.5 kg to t
m 17 000 000 mg to kg	n 7200 min to days	o 45 000 000 g to t
p 144 000 s to h	q 86 400 s to days	r 2500 cm to km
s 75 000 mL to kL	t 34 700 mm to m	

6 How many km in 408 m? Select the correct answer A, B, C or D.

- A 4.08 B 0.408 C 0.0408 D 0.004 08

7 Convert:

a 4.8 MB to kB	b 2.1 TB to GB	c 8.5 GB to MB
d 10.8 MB to B	e 2910 B to kB	f 740 MB to GB
g 1050 kB to MB	h 5900 GB to TB	i 0.94 kB to MB
j 57 GB to MB	k 0.43 GB to kB	l 21 TB to GB
m 7.8 kB to B	n 2180 MB to GB	o 42 000 B to MB
p 940 MB to kB		

8 Mala made a 3-minute phone call to her friend in Queensland using the Internet application Skype at 1.2 MB per minute. How many kB did she use?

9 Calculate the number of MP3 files of average size 3.7 MB that can be stored on a 16 GB MP3 player.

10 Convert, correct to two significant figures where necessary:

a 3.5 AU to km	b 6000 ms to s	c 7.5 centuries to years
d 12 000 000 nm to m	e 0.64 GM to m	f 2.1 ly to km
g 0.07 s to ms	h 0.25 Ma to years	i 4 800 000 μ s to s
j 5 290 000 km to Mm	k 0.2 pc to km	l 6 mm to μ m
m 0.94 Ma to millennia	n 849 years to centuries	o 8×10^{14} km to ly
p 0.000 008 cm to nm		

See Example 2

Worked solutions

Metric units

MAT09MGWS10040

See Example 3

Just for the record

The metre

The metre is the **standard** or **base** unit of length in the International System of Units.

It was originally defined in France in

1795 as $\frac{1}{10\,000\,000}$ (ten-millionth) of the

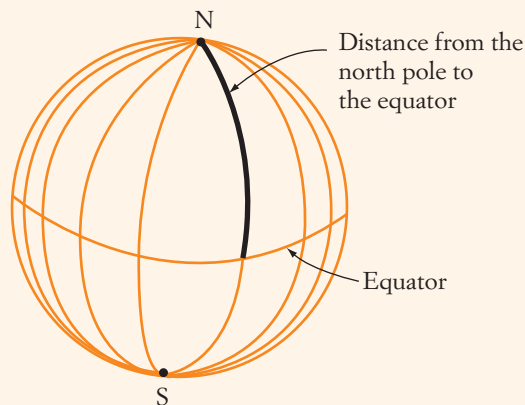
distance between the North Pole and the Equator along a meridian of longitude.

In 1889, it was redefined as the distance between two lines marked on a platinum bar used as the standard.

The current definition was made in 1983, as the distance travelled by light in a

vacuum in $\frac{1}{299\,792\,458}$ of a second.

Investigate the definition of a second.



NSW

Worksheet

Accuracy in measurement

MAT09MGWK10109

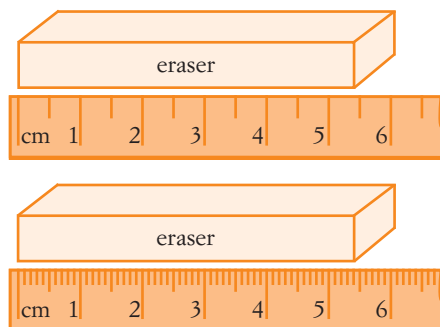
10-02

Limits of accuracy of measuring instruments

All measurements are only approximations.
No measurement is ever **exact**.

For example, we might measure this eraser to be **5.5 cm** long, because the scale on the ruler measures to the nearest 0.5 cm.

If we used a ruler with a more precise scale, such as 0.1 cm markings, then we may find the length to be **5.4 cm**.



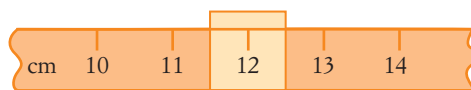
If we could find an instrument such as a micrometer that measured to the nearest 0.01 cm, we may find that the measured length is **5.41 cm**.

Notice that we can always find a more accurate measurement using a more precise measuring instrument. So all measurements are approximations.

A measuring instrument such as a ruler has **limits of accuracy** due to its level of **precision**.

Accuracy means how close a measured value is to the true value, while **precision** means how fine the scale is on the measuring instrument.

For example, this ruler is marked in centimetres, so any length measured with it can only be given to the nearest centimetre.



Any measurement in the shaded region should be recorded as 12 cm. The measured length is 12 cm, but the actual length is 12 ± 0.5 cm, meaning '12 centimetres, give or take 0.5 centimetres' or '11.5 to 12.5 cm'.

The **limits of accuracy** of this ruler are ± 0.5 cm or 'plus or minus half a centimetre'.

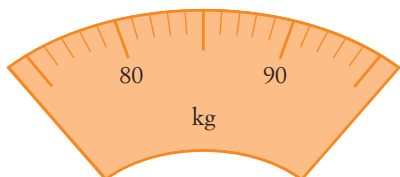
Summary

The **limits of accuracy** of a measuring instrument are ± 0.5 of the unit shown on the instrument's scale.

Example 4

Find the limits of accuracy for each measuring scale.

a



b



Solution

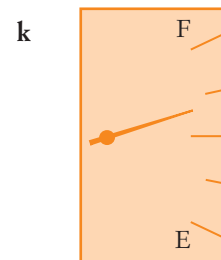
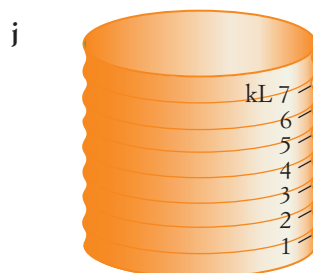
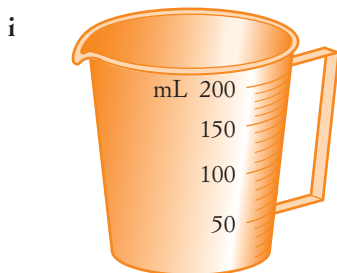
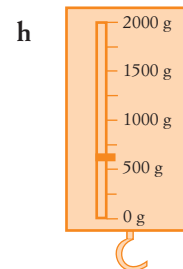
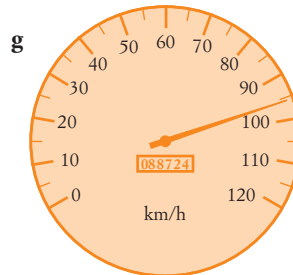
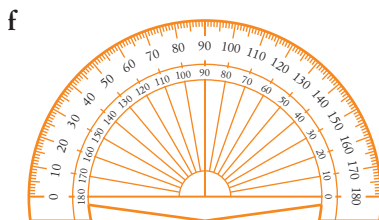
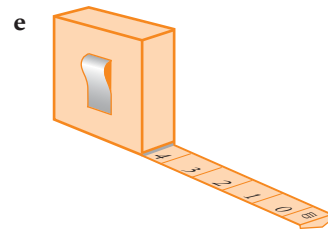
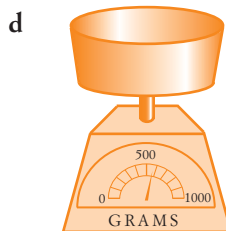
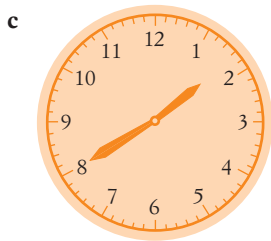
- a The size of one unit on the scale is 1 kg.
The limits of accuracy are $\pm 0.5 \times 1 \text{ kg} = \pm 0.5 \text{ kg}$
- b The size of one unit on the scale is 5 mL.
The limits of accuracy are $\pm 0.5 \times 5 \text{ mL} = \pm 2.5 \text{ mL}$

Exercise 10-02 Limits of accuracy of measuring instruments

1 For each measuring instrument, state:

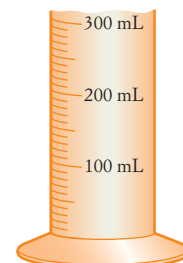
- i the size of one unit on the scale ii its limits of accuracy.

See Example 4



2 What are the limits of accuracy for this measuring cylinder? Select the correct answer A, B, C or D.

- A $\pm 0.5 \text{ mL}$ B $\pm 10 \text{ mL}$ C $\pm 20 \text{ mL}$ D $\pm 5 \text{ mL}$



- 3 Measuring with a ruler, Jade correctly gives a measurement as 26.5 to 27.5 cm.
- What is the size of a unit on the ruler?
 - What are the limits of accuracy of this measurement?
 - Using a ruler marked in millimetres, Marival gives the same measurement as 26.8 cm. What are the limits of accuracy of this measurement?
 - What is the range within which this measurement must lie?

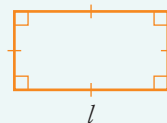
10-03 Perimeters and areas of composite shapes

Summary

The **perimeter** of a shape is the distance around the shape, the sum of the lengths of the sides of the shape.

The **area** of a shape is the amount of surface covered by the shape, measured in **square units**.

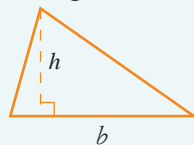
Rectangle



$$A = \text{length} \times \text{width}$$

$$A = lw$$

Triangle



$$A = \frac{1}{2} \times \text{base}$$

$\times$ perpendicular height

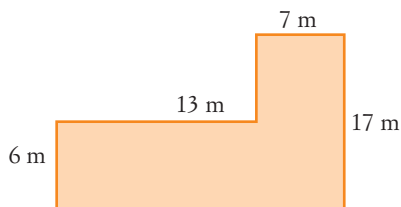
$$A = \frac{1}{2}bh$$

Example 5

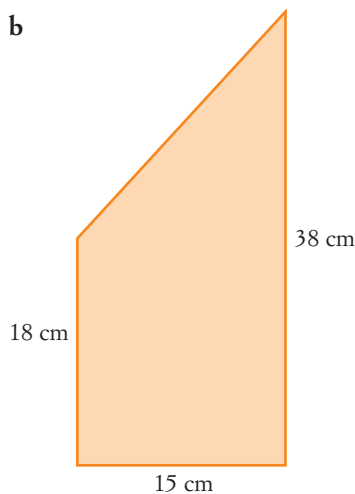
For each composite shape, find:

- its perimeter
- its area.

a



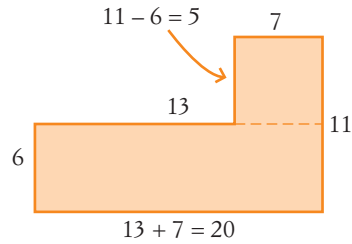
b



Solution

- a i** Find any unknown lengths on the shape.

$$\begin{aligned}\text{Perimeter} &= 6 + 13 + 5 + 7 + 11 + 20 \\ &= 62 \text{ m}\end{aligned}$$



Sum of areas of two rectangles.

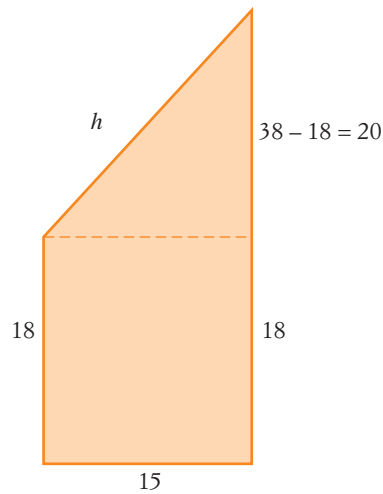
ii $\text{Area} = (7 \times 5) + (20 \times 6)$
 $= 155 \text{ m}^2$

- b i** Divide the shape into a rectangle and a triangle, and find the unknown length, h , using Pythagoras' theorem.

$$\begin{aligned}h^2 &= 15^2 + 20^2 \\ &= 625\end{aligned}$$

$$\begin{aligned}h &= \sqrt{625} \\ &= 25 \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{Perimeter} &= 18 + 25 + 38 + 15 \\ &= 96 \text{ cm}\end{aligned}$$



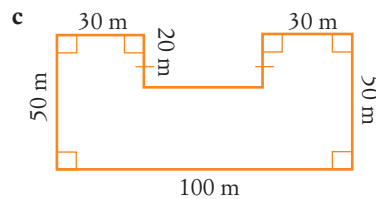
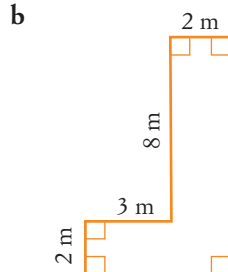
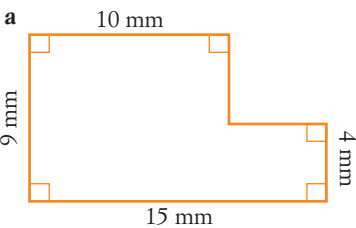
Area of triangle + Area of rectangle

ii $\text{Area} = \left(\frac{1}{2} \times 15 \times 20\right) + (15 \times 18)$
 $= 270 \text{ cm}^2$

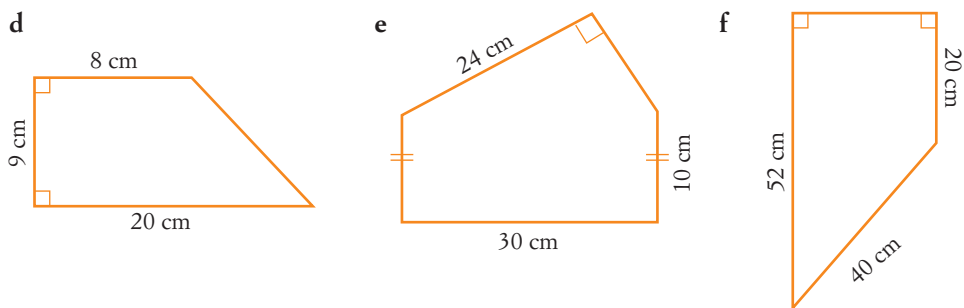
Exercise 10-03 Perimeters and areas of composite shapes

- 1** For each composite shape, find:

- i** its perimeter **ii** its area.



See Example 5

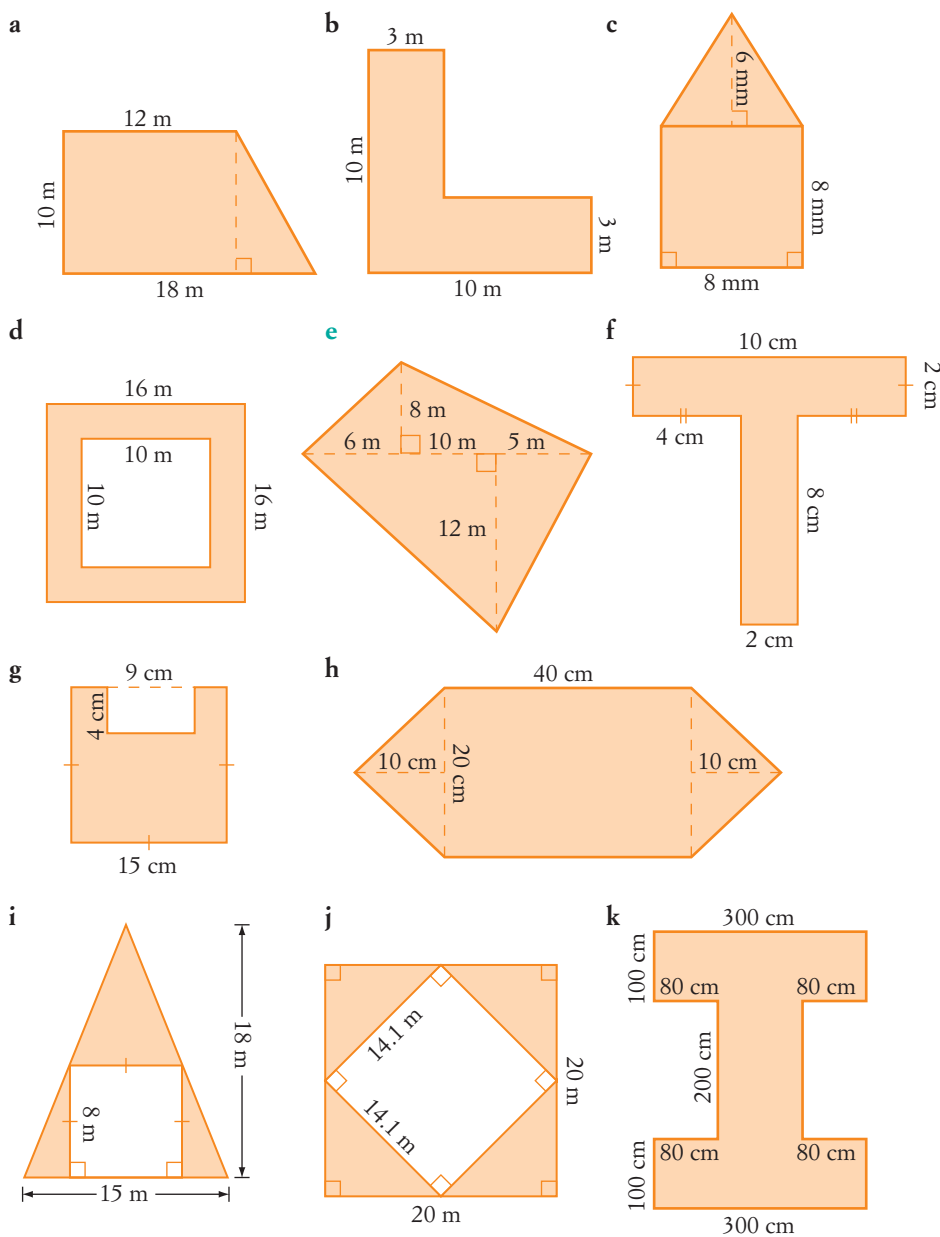


Worked solutions

2 Find the area of each composite shape.

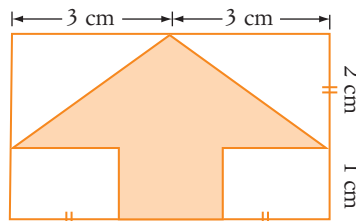
Perimeters and areas of composite shapes

MAT09MGWS10041



- 3 What is the area of the shaded arrow? Select the correct answer **A**, **B**, **C** or **D**.

A 14 cm^2 **B** 8 cm^2 **C** 9 cm^2 **D** 12 cm^2

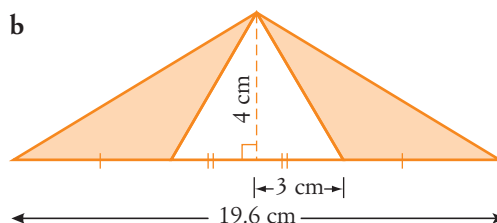
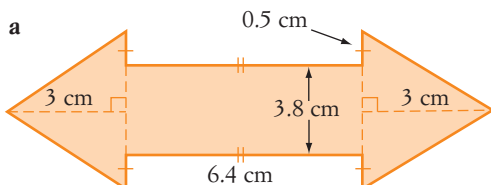


Worked solutions

Perimeters and areas of composite shapes

MAT09MGWS10041

- 4 Find each shaded area.



10-04 Areas of quadrilaterals

Worksheet

Area ID

MAT09MGWK10110

Skillsheet

What is area?

MAT09MGSS10029

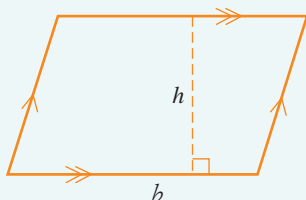
Homework sheet

Area 1

MAT09MGHS10019

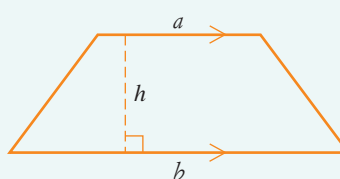
Summary

Parallelogram



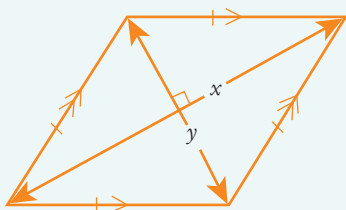
Area = base $\times$ perpendicular height
 $A = bh$

Trapezium



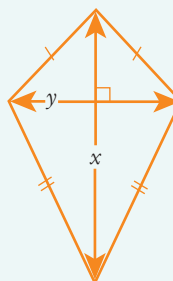
Area = $\frac{1}{2} \times$ (sum of parallel sides)
 $\times$ perpendicular height
 $A = \frac{1}{2}(a + b)h$

Rhombus



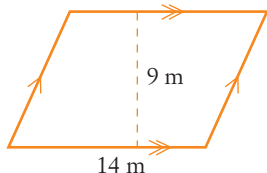
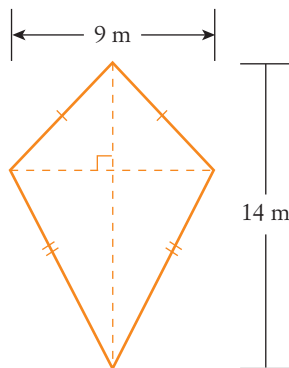
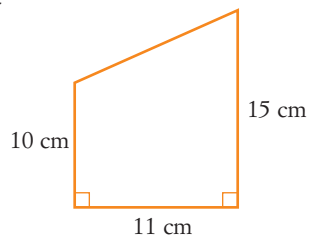
Area = $\frac{1}{2} \times$ (product of diagonals)
 $A = \frac{1}{2}xy$

Kite



Example 6

Find the area of each quadrilateral.

a**b****c****Solution**

$$\begin{aligned} \text{a Area of parallelogram} &= 14 \times 9 \\ &= 126 \text{ m}^2 \end{aligned}$$

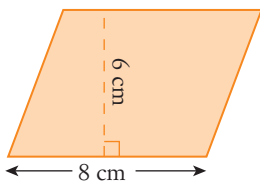
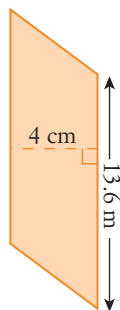
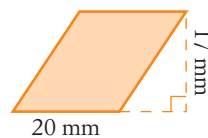
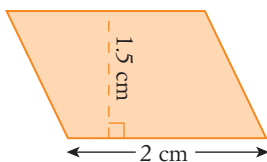
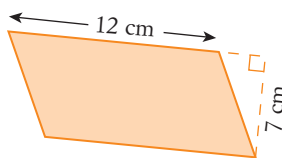
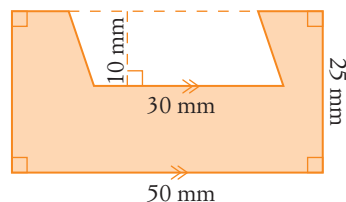
$$A = bh$$

$$\begin{aligned} \text{b Area of kite} &= \frac{1}{2} \times 9 \times 14 \\ &= 63 \text{ m}^2 \end{aligned}$$

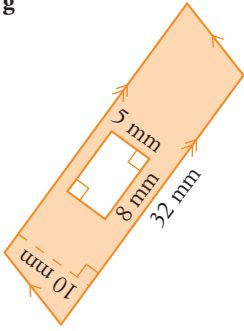
$$A = \frac{1}{2}xy$$

$$\begin{aligned} \text{c Area of trapezium} &= \frac{1}{2} \times (10 + 15) \times 11 \\ &= 137.5 \text{ cm}^2 \end{aligned}$$

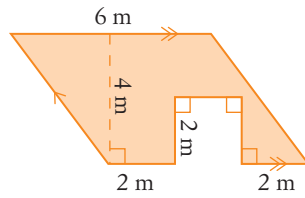
$$A = \frac{1}{2}(a + b)h$$

Exercise 10-04 Areas of quadrilaterals**See Example 6** 1 Find the shaded area of each shape.**a****b****c****d****e****f**

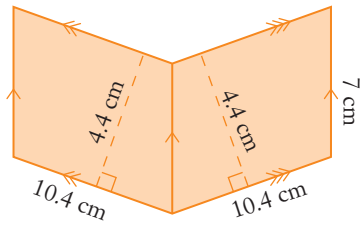
g



h

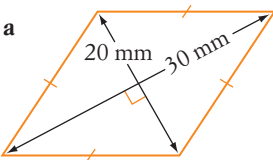


i

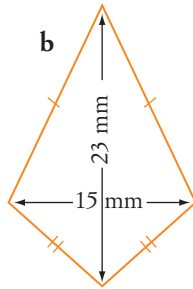


2 Find the area of each shape.

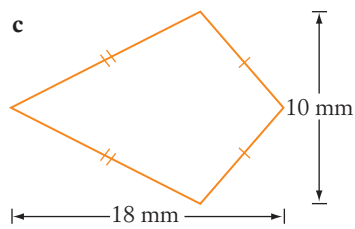
a



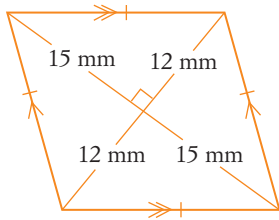
b



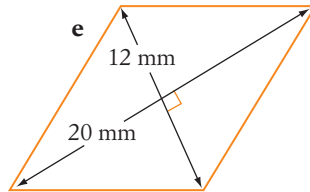
c



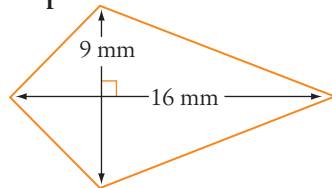
d



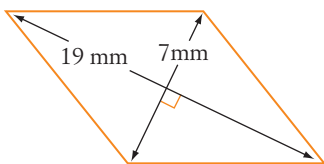
e



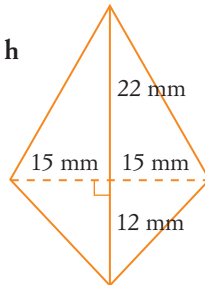
f



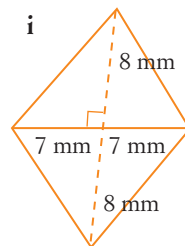
g



h

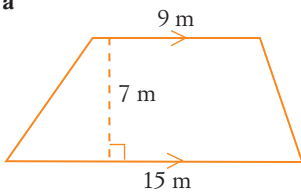


i

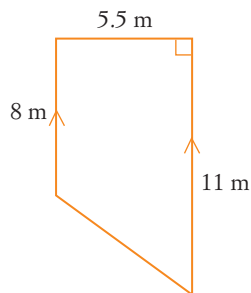


3 Find the area of each shape.

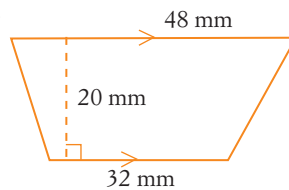
a

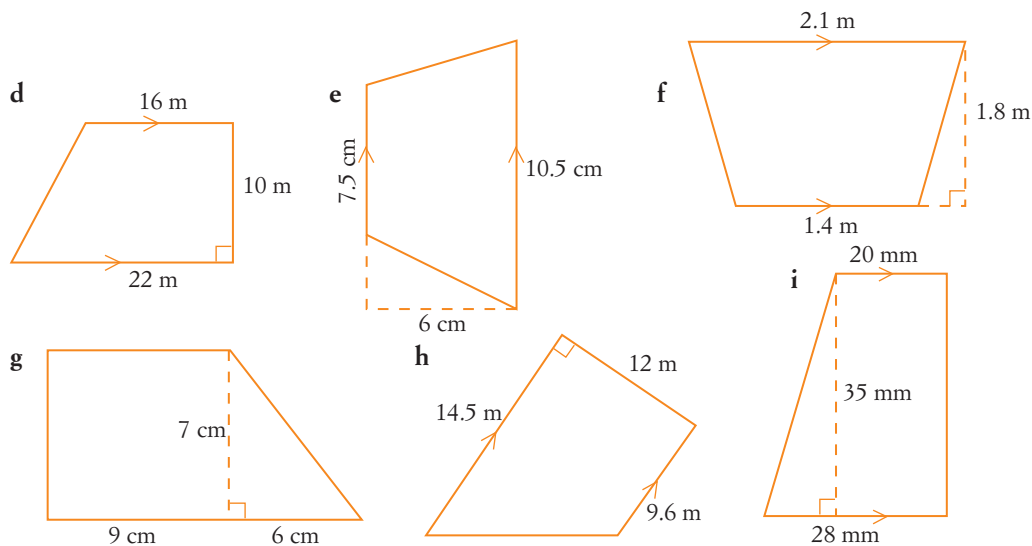


b

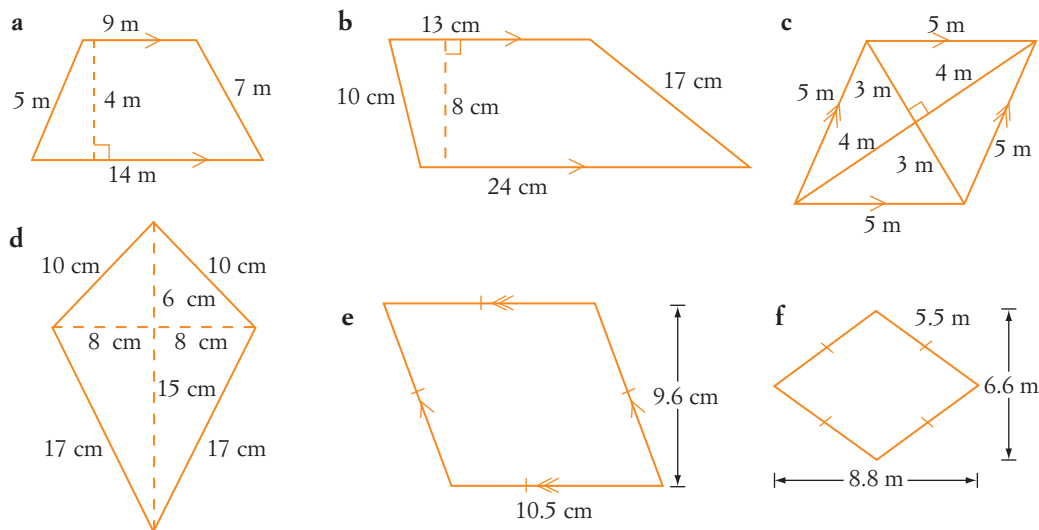


c

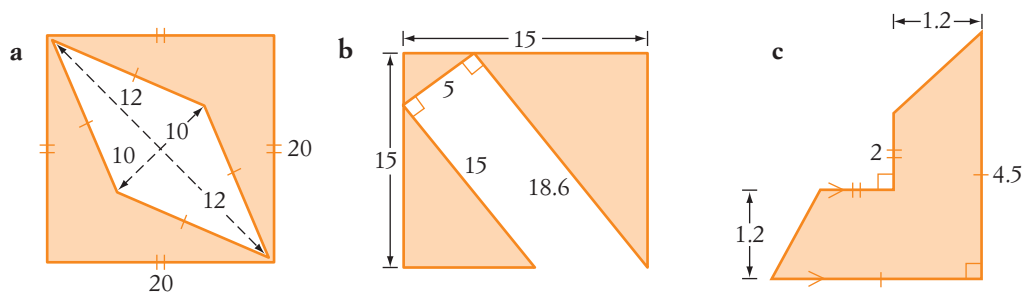




4 Find the area of each shape.



5 Calculate each shaded area (all measurements are in metres).



Mental skills 10A Maths without calculators

Finding 10%, 20% and 5%

To find 10% or $\frac{1}{10}$ of a number, simply divide the number by 10 by moving the decimal point one place to the left.

1 Study each example.

a $10\% \times 150 = 15$

b $10\% \times \$1256.80 = \$125.68$

c $10\% \times 4917 = 491.7 = 491.7$

d $10\% \times \$48.55 = \$4.855 = \$4.885$

2 Now find 10% of each amount.

a 190

b \$75

c 875

d \$202

e \$37.60

f 400

g \$9.25

h 896

i \$2700

j \$3.80

k \$1527.60

l \$72.50

m 3154

n \$10.70

o 426

p \$24 317.60

20% is 10% doubled so to find 20% of a number, first find 10% then double it.

3 Study each example.

a $20\% \times 700$

$10\% \times 700 = 70$

$\therefore 20\% \times 700 = 70 \times 2$
 $= 140$

b $20\% \times \$876$

$10\% \times \$876 = \$87.60$

$\therefore 20\% \times \$876 = \$87.60 \times 2$
 $= \$175.20$

c $20\% \times 325$

$10\% \times 325 = 32.5$

$\therefore 20\% \times 325 = 32.5 \times 2$
 $= 65$

d $20\% \times \$38.50$

$10\% \times \$38.50 = \$3.85$

$\therefore 20\% \times \$38.50 = \$3.85 \times 2$
 $= \$7.70$

4 Now find 20% of each amount.

a 50

b 620

c \$2450

d \$8.60

e 72

f \$12 700

g 390

h \$5.80

i \$45

j \$84

k \$4600

l 320

5% is half of 10%, so to find 5% of a number first find 10% then divide it by 2.

5 Study each example.

a $5\% \times 180$

$10\% \times 180 = 18$

$\therefore 5\% \times 180 = 18 \div 2$
 $= 9$

b $5\% \times \$76$

$10\% \times \$76 = \$7.60$

$\therefore 5\% \times \$76 = \$7.60 \div 2$
 $= \$3.80$

c $5\% \times 120$

$10\% \times 120 = 12$

$\therefore 5\% \times 120 = 12 \div 2$
 $= 6$

d $5\% \times \$142.20$

$10\% \times \$142.20 = \$14.22$

$\therefore 5\% \times \$142.20 = \$14.22 \div 2$
 $= \$7.11$

6 Now find 5% of each amount.

a 2000

b \$12

c 50

d \$27

e \$36.80

f \$84

g 800

h 130

i \$9.60

j \$138

k \$72

l 840

Worksheet

A page of circular shapes

MAT09MGWK10111

Homework sheet

Area 2

MAT09MGHS10020

Worksheet

Applications of area

MAT09MGWK10112

Worksheet

Back-to-front problems

MAT09MGWK10113

Worksheet

Area and perimeter investigations

MAT09MGWK00025

Puzzle sheet

Carpet talk

MAT09MGPS00007

10-05 Circumferences and areas of circular shapes

Summary

The **circumference (perimeter)** of a circle is:

$$C = \pi \times \text{diameter}$$

or

$$C = 2 \times \pi \times \text{radius}$$

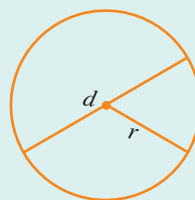
$$C = \pi d$$

$$C = 2\pi r$$

The **area** of a circle is:

$$A = \pi \times (\text{radius})^2$$

$$A = \pi r^2$$



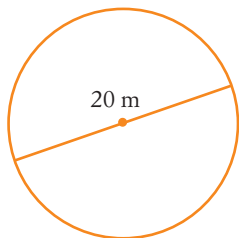
$\pi = 3.141\,592\,653\,897\,93\dots$ and can be found by pressing the π key on your calculator. It has no exact decimal or fraction value, so like the surd $\sqrt{2}$ it is an **irrational number**.

Example 7

For this circle, calculate correct to two decimal places:

a its circumference

b its area.



Solution

a $C = \pi \times 20$

$$C = \pi d$$

$$= 62.8318\dots$$

$$\approx 62.83 \text{ m}$$

b $A = \pi \times 10^2$

$$A = \pi r^2$$

$$= 314.1592\dots$$

$$\approx 314.16 \text{ m}^2$$

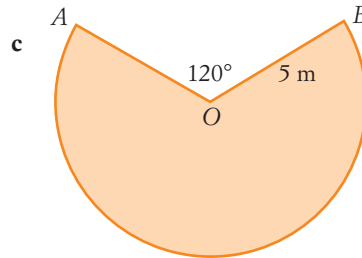
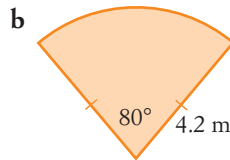
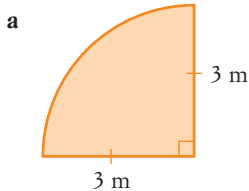
Example 8

For each sector, calculate correct to one decimal place:

i its perimeter

ii its area.

A **sector** is a fraction of a circle 'cut' along two radii, like a pizza slice.



Solution

a i This sector is a **quadrant**, a quarter of a circle.

$$\begin{aligned}\text{Perimeter} &= \frac{1}{4} \times 2 \times \pi \times 3 + 3 + 3 \\ &= 10.71238 \dots \\ &\approx 10.7 \text{ m}\end{aligned}$$

$$\frac{1}{4} \times \text{circumference} + \text{radius} + \text{radius}$$

ii
$$\begin{aligned}\text{Area} &= \frac{1}{4} \times \pi \times 3^2 \\ &= 7.06858 \dots \\ &\approx 7.1 \text{ m}^2\end{aligned}$$

$$\frac{1}{4} \times \text{area of circle}$$

b i
$$\begin{aligned}\text{Perimeter} &= \frac{80}{360} \times 2 \times \pi \times 4.2 + 4.2 + 4.2 \\ &= 14.26430 \dots \\ &\approx 14.3 \text{ m}\end{aligned}$$

$$\frac{80}{360} \times \text{circumference} + \text{radius} + \text{radius}$$

There are 360° in a circle, but a sector is a fraction of a circle.

ii
$$\begin{aligned}\text{Area} &= \frac{80}{360} \times \pi \times 4.2^2 \\ &= 12.31504 \dots \\ &\approx 12.3 \text{ m}^2\end{aligned}$$

$$\frac{80}{360} \times \text{area of circle}$$

c i Sector angle = $360^\circ - 120^\circ = 240^\circ$

$$\begin{aligned}\text{Perimeter} &= \frac{240}{360} \times 2 \times \pi \times 5 + 5 + 5 \\ &= 30.94395 \dots \\ &\approx 30.9 \text{ m}\end{aligned}$$

ii
$$\begin{aligned}\text{Area of sector} &= \frac{240}{360} \times \pi \times 5^2 \\ &= 52.35987 \dots \\ &\approx 52.4 \text{ m}^2\end{aligned}$$

Video tutorial

Perimeter and area of a sector

MAT09MGVT10014

Technology

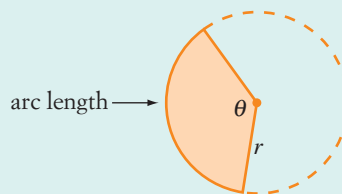
GeoGebra: Area and perimeter of a sector

MAT09MGTC00002

Summary

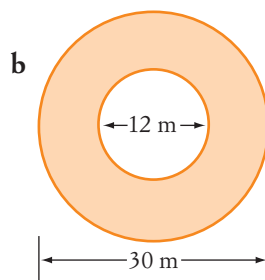
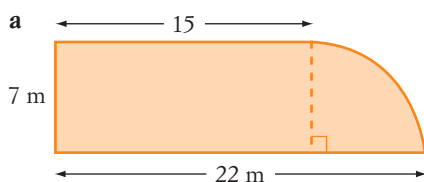
For a sector with angle θ :

- Arc length $= \frac{\theta}{360} \times 2\pi r$
- Perimeter of the sector $= \frac{\theta}{360} \times 2\pi r + r + r$
(arc length + radius + radius)
- Area of the sector $= \frac{\theta}{360} \times \pi r^2$



Example 9

Find, correct to one decimal place, the area of each shape.



Solution

- a** The shape is made up of a rectangle and a quadrant.

Radius of quadrant = 7 m

Length of rectangle = $22 - 7$
= 15 m

Area of shape = area of rectangle
+ quadrant
= $15 \times 7 + \frac{1}{4} \times \pi \times 7^2$
= $143.4845 \dots$
 $\approx 143.5 \text{ m}^2$

- b** This ring shape is called an **annulus**, it is the area enclosed by two circles with the same centre.

Radius of large circle = $\frac{1}{2} \times 30 \text{ m}$
= 15 m

Radius of small circle = $\frac{1}{2} \times 12 \text{ m}$
= 6 m

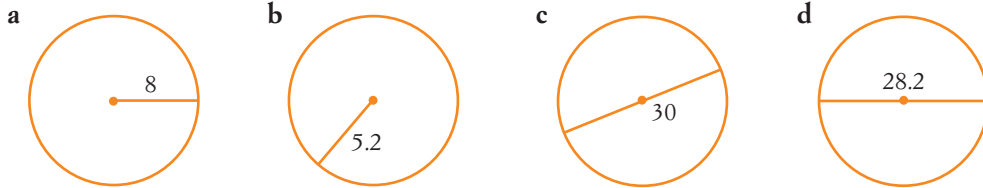
Shaded area = large circle – small circle
= $\pi \times 15^2 - \pi \times 6^2$
= $593.7610 \dots$
 $\approx 593.8 \text{ m}^2$

Exercise 10-05 Circumferences and areas of circular shapes

1 For each circle, calculate correct to two decimal places:

See Example 7

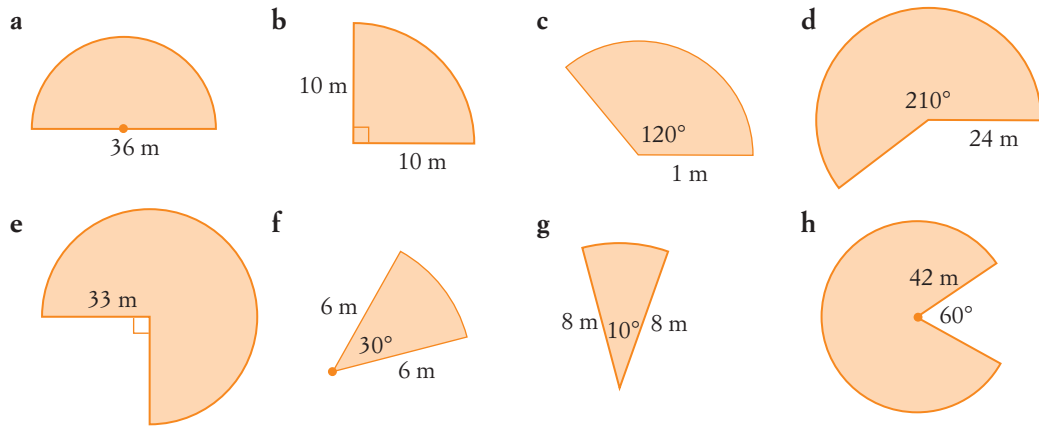
- i its circumference ii its area.



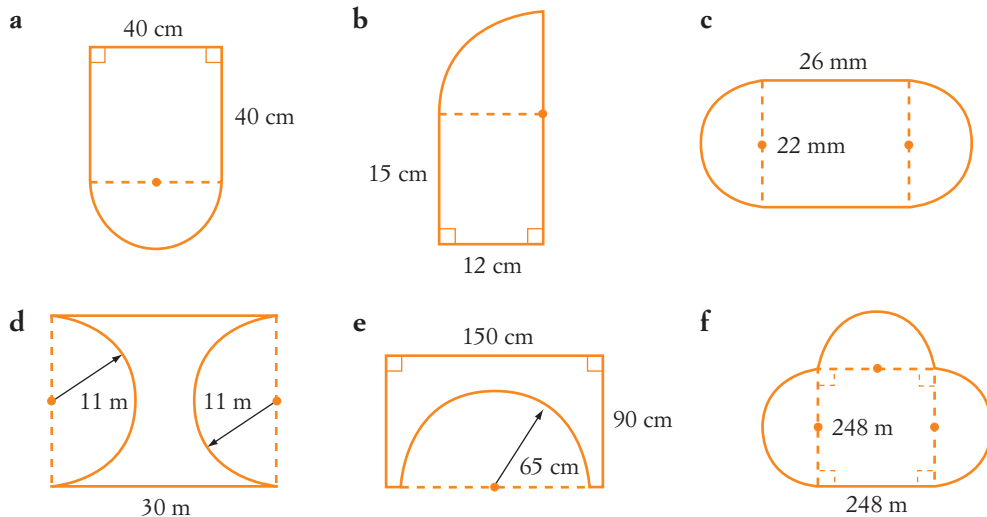
2 For each sector, calculate correct to one decimal place:

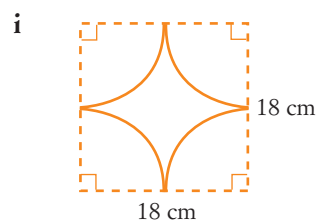
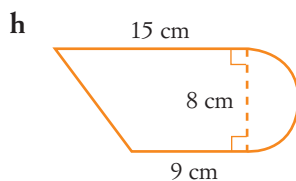
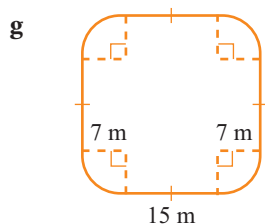
See Example 8

- i its perimeter ii its area.



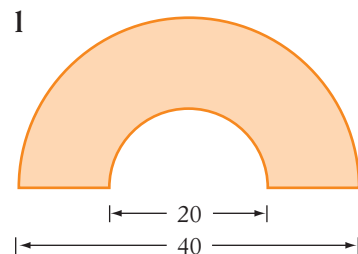
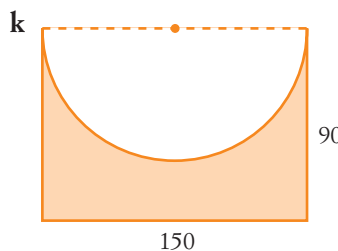
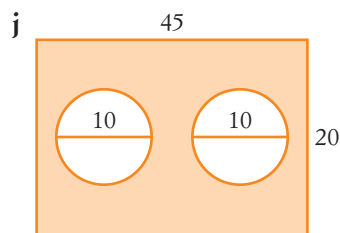
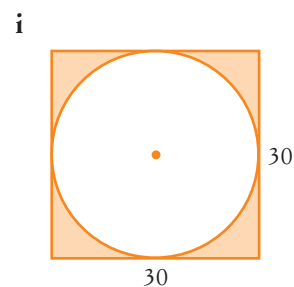
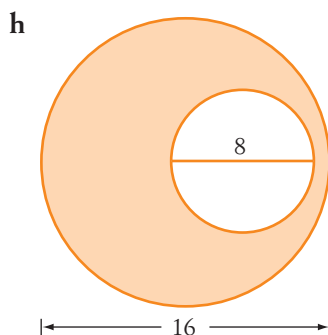
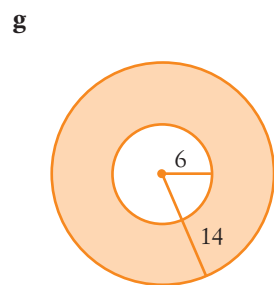
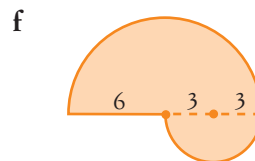
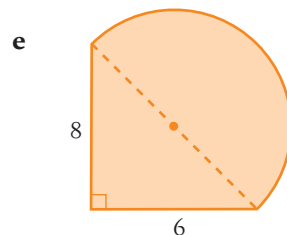
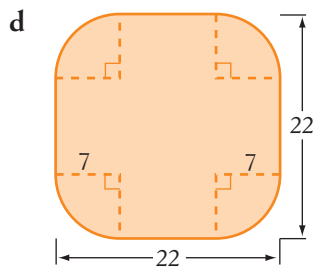
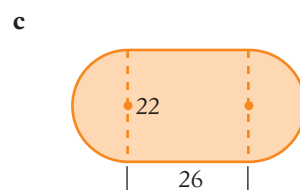
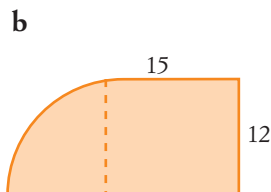
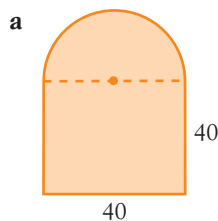
3 Calculate the perimeter of each shape, correct to two decimal places.





See Example 9

- 4 Calculate the area of each shape, correct to the nearest square metre. All measurements are in metres.



- 5 A circular playing field has a radius of 80 m. A rectangular cricket pitch measuring 25 m by 2 m is placed in the middle. The field, excluding the pitch, is to be fertilised.

a Calculate correct to the nearest square metre the area to be fertilised.

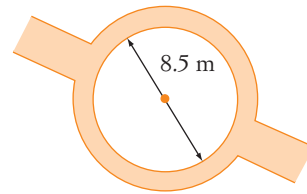
b How much will this cost if the fertiliser is \$19.95 per 100 square metres? Give your answer to the nearest dollar.

Worked solutions

Circumferences and areas of circular shapes

MAT09MGWS10042

- 6 a The diameter of the Earth is 12 756 km. Find the circumference of the Earth, to the nearest kilometre.
- b A space lab orbits the Earth at a height of 1200 km above the Earth's surface. Find the distance it travels in one orbit of the Earth.
- 7 A circular plate of diameter 2 m has 250 holes of diameter 10 cm drilled in it. What is the remaining area of the plate? Answer to the nearest 0.1 m^2 .
- 8 A circular pond of diameter 10 m is surrounded by a path one metre wide.
- a Calculate the area of the path correct to two decimal places.
- b If pavers are \$75 per square metre laid, what is the cost of the path?
- 9 A circular patch of grass has a diameter of 8.5 m. How much further does Nina walk if she walks around the border instead of directly across it? Give your answer correct to the nearest 0.1 m.



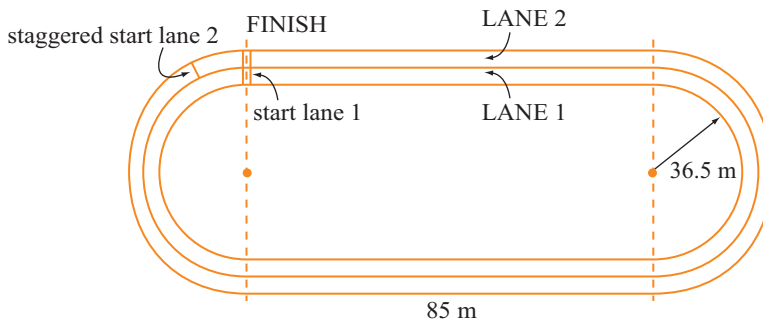
- 10 A new tractor tyre has a diameter of 120 cm, while a worn tyre has a diameter of 115 cm.
- a Calculate the difference in circumference between a new tyre and a worn tyre, correct to three decimal places.
- b Over 1000 revolutions, how much further (to the nearest metre) will a new tyre travel compared to a worn tyre?

Worked solutions

Circumferences and areas of circular shapes

MAT09MGWS10042

- 11 A square courtyard measuring 5 m by 5 m has a semi-circular area added to each side.
- a Calculate the area of the semi-circular additions, correct to the nearest square metre.
- b By what percentage has the area of the courtyard increased?
- (This can be calculated as $\frac{\text{increase in area}}{\text{original area}} \times 100\%$)
- 12 The measurements of a running track are shown below. Each lane is 1 m wide and the athletes run along the insides of the lanes.



- a Zoe runs one lap in lane 1. What distance does she cover, correct to one decimal place?
- b Jaron runs one lap in lane 2. What distance does he cover, correct to one decimal place?
- c By how much should the start line be staggered in lane 2 (from lane 1) so that Zoe and Jaron run the same distance in one lap?

Worksheet

Nets of solids

MAT09MGWK10114

Worksheet

Surface area

MAT09MGWK10115

Skillsheet

Solid shapes

MAT09MGSS10030

Animated example

Surface area

MAT09MGAE00003

Technology worksheet

Excel worksheet:
Surface area calculator

MAT09MGCT00020

Technology worksheet

Excel spreadsheet:
Surface area calculator

MAT09MGCT00005

10-06 Surface area of a prism

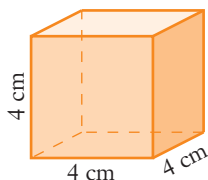
Summary

The **surface area** of a solid is the total area of all the faces of the solid. To calculate the surface area of a solid, find the area of each face and add the areas together.

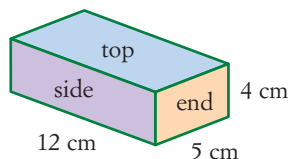
Example 10

Calculate the surface area of each prism.

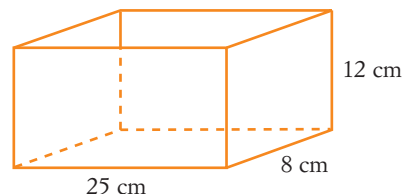
a cube



b rectangular prism



c open rectangular prism



Solution

- a A cube has 6 identical faces in the shape of a square.

$6 \times \text{area of a square}$

$$\begin{aligned}\text{Surface area} &= 6 \times 4^2 \\ &= 96 \text{ cm}^2\end{aligned}$$

- b A rectangular prism has 6 faces that are not all the same. However, opposite faces such as the top and bottom are the same.

$$\begin{aligned}\text{Surface area} &= 2 \text{ top and bottom faces} + 2 \text{ end faces} + 2 \text{ side faces} \\ &= (2 \times 5 \times 12) + (2 \times 5 \times 4) + (2 \times 12 \times 4) \\ &= 256 \text{ cm}^2\end{aligned}$$

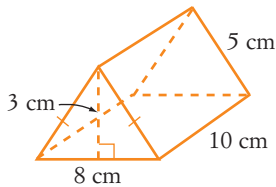
- c The open rectangular prism has no top so it has 5 faces that are not all the same.

$$\begin{aligned}\text{Surface area} &= \text{Bottom face} + 2 \text{ front and back faces} + 2 \text{ side faces} \\ &= (25 \times 8) + (2 \times 25 \times 12) + (2 \times 8 \times 12) \\ &= 992 \text{ cm}^2\end{aligned}$$

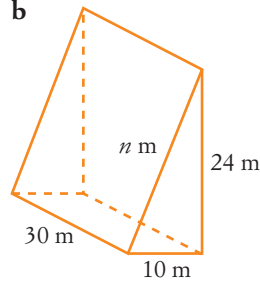
Example 11

Calculate the surface area of each triangular prism.

a



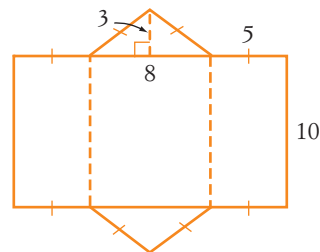
b



Solution

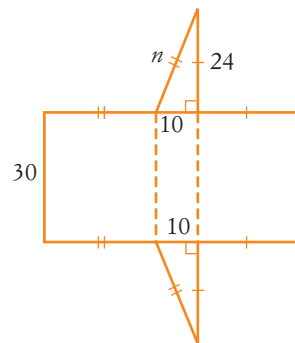
- a** This triangular prism has 5 faces: two identical triangles (front and back), two identical rectangles (sides) and another rectangle (bottom). Drawing the net of the prism may help you see this.

$$\begin{aligned}\text{Surface area} &= (2 \times \frac{1}{2} \times 8 \times 3) + (2 \times 10 \times 5) + (8 \times 10) \\ &= 204 \text{ cm}^2\end{aligned}$$



Front and back faces +
side faces + bottom face

- b** This triangular prism has 5 faces: two identical triangles (front and back), three different rectangles.



To calculate the area of the left face, we need to know the value of n , which can be found using Pythagoras' theorem.

$$\begin{aligned}n^2 &= 10^2 + 26^2 \\ &= 676 \\ n &= \sqrt{676} \\ &= 26\end{aligned}$$

$$\begin{aligned}\text{Surface area} &= (2 \times \frac{1}{2} \times 10 \times 24) + (30 \times 26) \\ &\quad + (30 \times 10) + (30 \times 24) \\ &= 2040 \text{ m}^2\end{aligned}$$

Front and back + left +
bottom + right

Video tutorial

Surface area of a prism

MAT09MGVT10015

Video tutorial

Surface area of prisms

MAT09MGVT00003

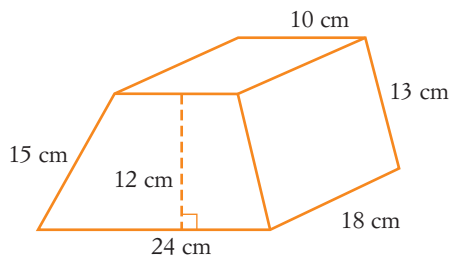
Example 12

Video tutorial

Surface area of a prism

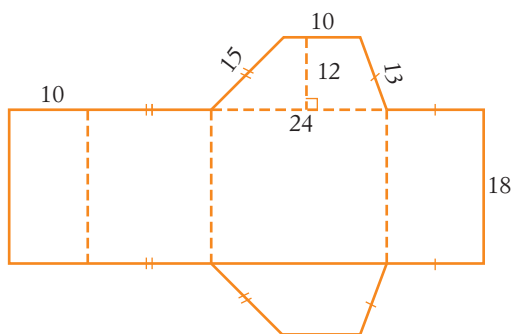
MAT09MGVT10015

Calculate the surface area of this trapezoidal prism.



Solution

This trapezoidal prism has 6 faces: two identical trapeziums (front and back) and four different rectangles.



$$\begin{aligned}\text{Area of each trapezium} &= \frac{1}{2} \times (10 + 24) \times 12 \\ &= 204 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Surface area} &= (2 \times 204) + (18 \times 10) + (18 \times 15) + (18 \times 24) + (18 \times 13) \\ &= 1524 \text{ cm}^2\end{aligned}$$

2 trapeziums + 4 rectangles

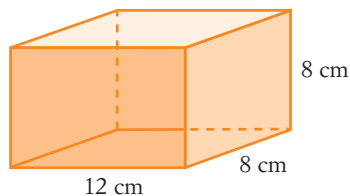
Exercise 10-06 Surface area of a prism

See Example 10

- 1 What is the surface area of this rectangular prism?
Select the correct answer **A**, **B**, **C** or **D**.

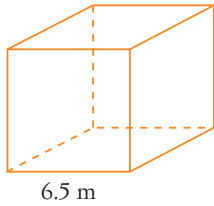
- A** 320 cm^2
B 352 cm^2
C 512 cm^2
D 768 cm^2

- 2 Draw a net of each solid.

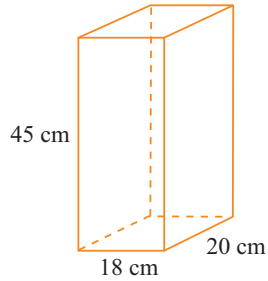
a cube**b** rectangular prism**c** triangular prism**d** cylinder

3 Calculate the surface area of each prism.

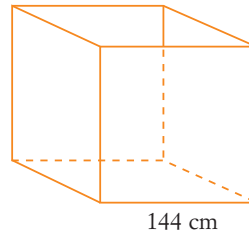
a cube



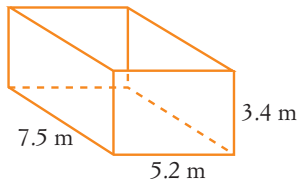
b rectangular prism



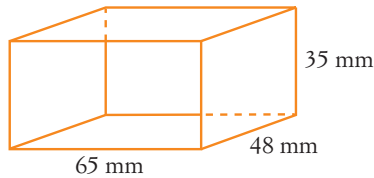
c open cube (no top)



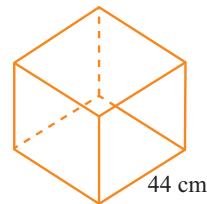
d open rectangular prism



e rectangular prism, open front and back

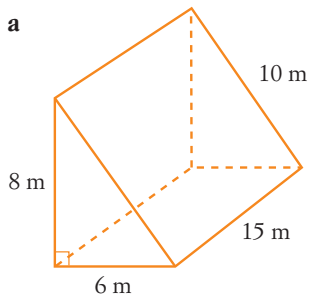


f cube, open one end

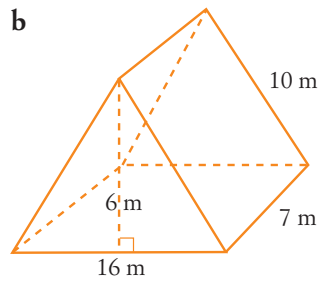


4 Find the surface area of each triangular prism.

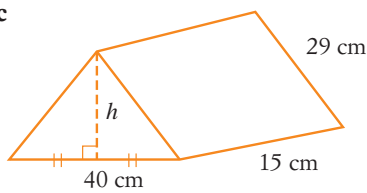
a



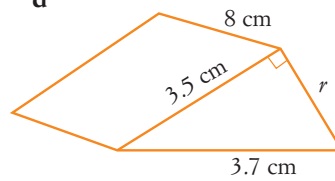
b



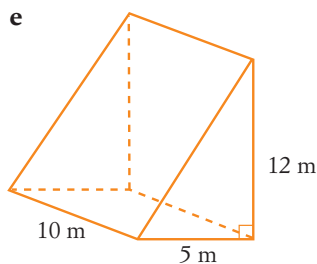
c



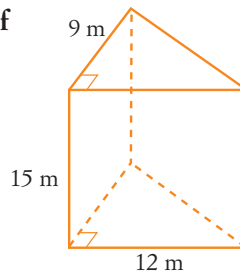
d



e



f



See Example 11

5 Match each net of a solid to the name of the solid.

triangular prism

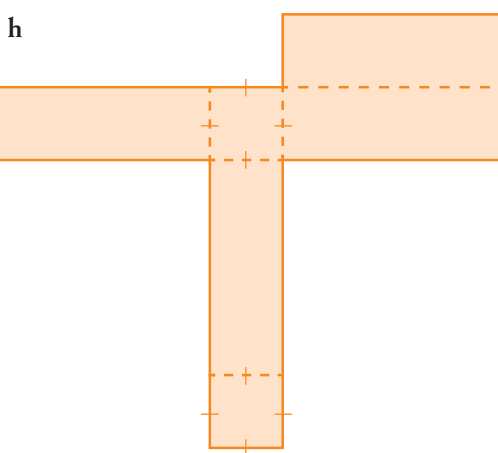
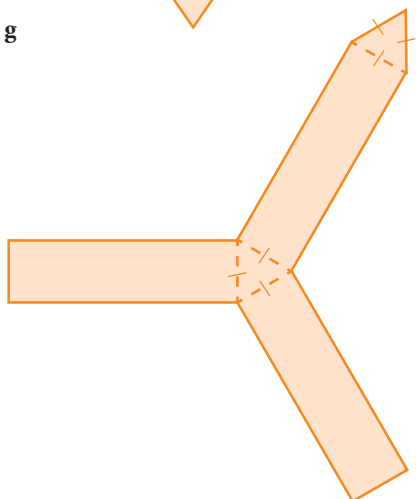
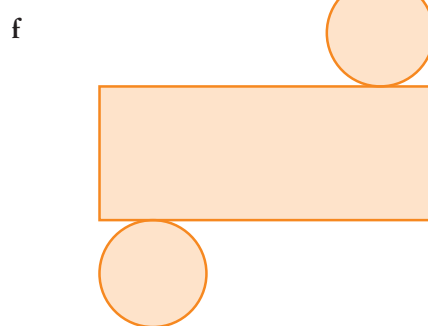
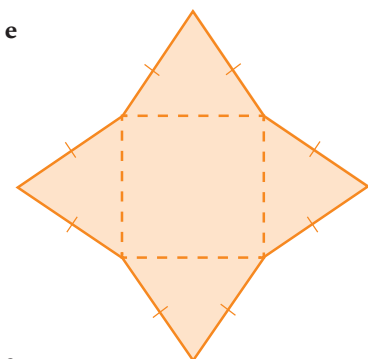
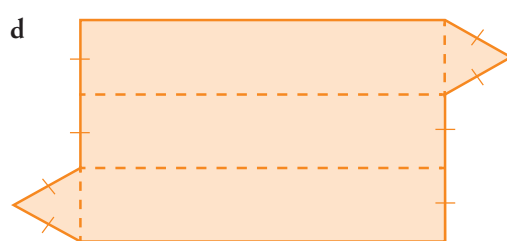
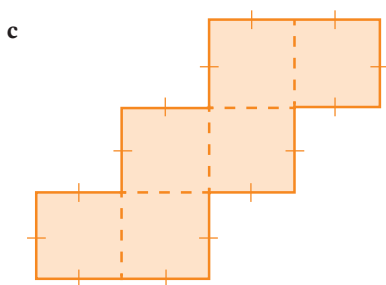
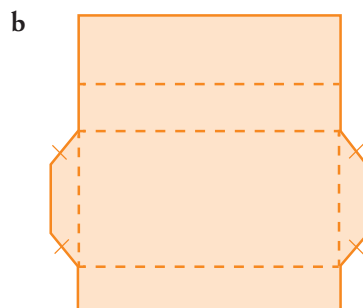
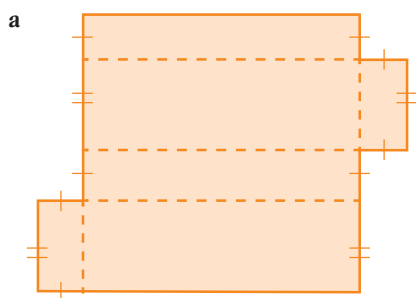
square pyramid

cube

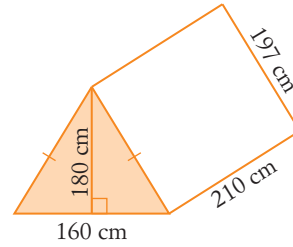
cylinder

rectangular prism

trapezoidal prism

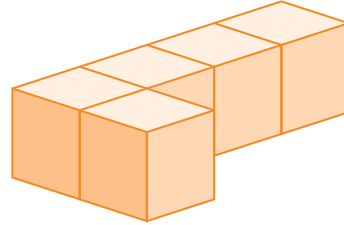


- 6 Caroline made this tent.
- Find the surface area of the tent including the floor.
 - Caroline bought material at \$12 per square metre. How much did it cost Caroline for the material to make the tent?



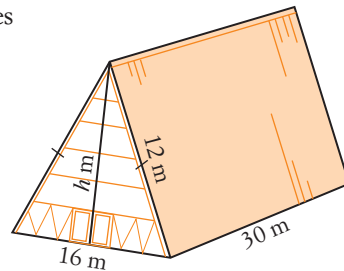
- 7 Five cubes of side length 1 cm are joined to make this solid. What is the surface area of the solid?
Select the correct answer **A**, **B**, **C** or **D**.

- 17 cm^2
- 18 cm^2
- 5 cm^2
- 22 cm^2



- 8 An art gallery has a width of 16 metres, a length of 30 metres and a slant height of 12 metres, as shown in the diagram.

- Find the perpendicular height, h , of the building, correct to one decimal place.
- Find the area of the front wall.
- Find the surface area of the art gallery, without the floor.

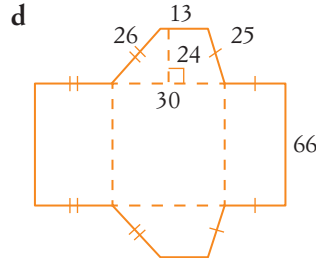
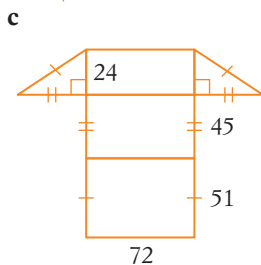
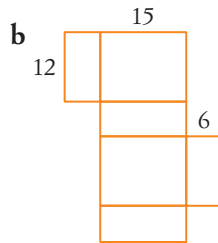
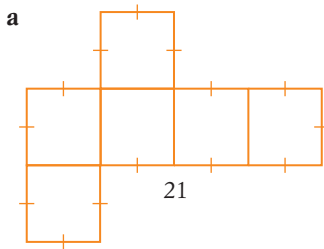


Worked solutions

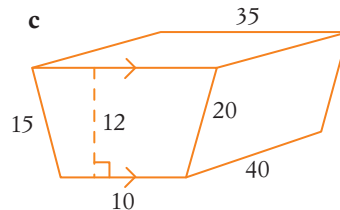
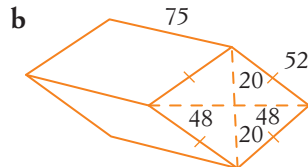
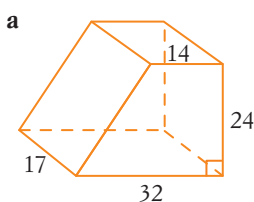
Surface area
of a prism

MAT09MGWS10043

- 9 Name the prism that each net represents, then calculate the surface area of the prism.
All lengths are in metres.



- 10 Calculate the surface area of each prism. Measurements are in centimetres.



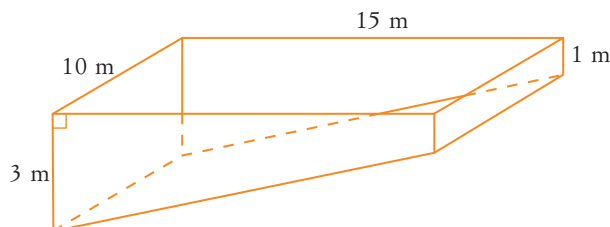
See Example 12

Worked solutions

Surface area of a prism

MAT09MGWS10043

- 11 This swimming pool is 15 m long and 10 m wide. The depth of the water ranges from 1 m to 3 m.



Calculate, correct to two decimal places:

- a the area of the floor of the pool b the total surface area of the pool.

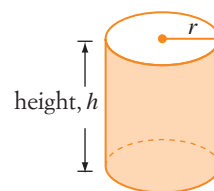
Investigation: Surface area of a cylinder

Collect 5 different cylindrical cans with paper labels.

- 1 Copy this table with rows for Cans 1 to 5 and complete it as you perform the following measurements and calculations.

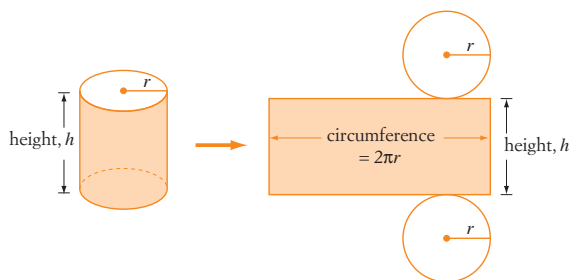
		Circular ends				Curved surface			Surface area
	Height	Diameter	Radius	Area	Circumference	Length	Width	Area	
Can 1									
Can 2									

- 2 Measure the height and diameter of the first can.
 3 Calculate the radius, area and circumference of one circular end, correct to two decimal places.
 4 Cut the label off the first can and lay it out flat. What shape is this curved surface?
 5 Measure the length, width and area of the curved surface (label).
 6 Calculate the surface area of the can by adding the areas of the two circular ends and the area of the label.
 7 Why is the height of each can and the width of its label the same?
 8 Why is the circumference of each circular end and the length of the label the same?
 9 Repeat Steps 2 to 6 for the other four cans.



10-07 Surface area of a cylinder

A closed cylinder has three faces made up of two circles (the circular ends) and a rectangle (the curved surface). The length of the rectangle is the circumference of the circular end, while the width of the rectangle is the height of the cylinder.



$$\begin{aligned}\text{Surface area of a cylinder} &= \text{area of two circles} + \text{area of rectangle} \\ &= 2 \times \pi r^2 + 2\pi r \times h \\ &= 2\pi r^2 + 2\pi rh\end{aligned}$$

Summary

Surface area of a closed cylinder

$$A = 2\pi r^2 + 2\pi rh$$

where r = radius of circular base and h = perpendicular height

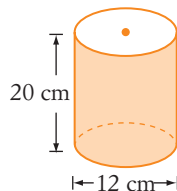
The area of the *two* circular ends $= 2\pi r^2$ and the area of the curved surface $= 2\pi rh$.

Example 13

Find, correct to one decimal place, the surface area of a cylinder with diameter 12 cm and height 20 cm.

Solution

$$\begin{aligned}\text{Radius} &= \frac{1}{2} \times 12 \text{ cm} && \frac{1}{2} \text{ of diameter} \\ &= 6 \text{ cm}\end{aligned}$$



Worksheet

A page of prisms and cylinders

MAT09MGWK10116

Homework sheet

Surface area

MAT09MGHS10021

Technology worksheet

Excel worksheet:
Surface area calculator

MAT09MGCT00020

Technology worksheet

Excel spreadsheet:
Surface area calculator

MAT09MGCT00005

Worksheet

Surface area

MAT09MGWK10115

Worksheet

Surface area

MAT09MGWK00026

Worksheet

Applications of area 3

MAT09MGWK00030

Puzzle sheet

Car song

MAT09MGPS00006

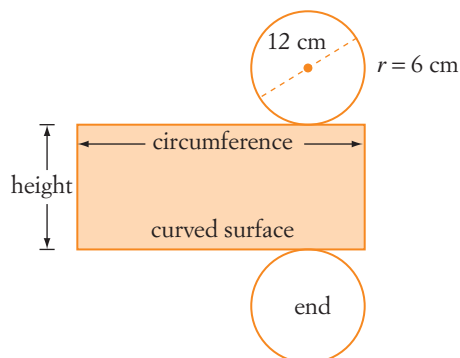
Video tutorial

Surface area of
a cylinder

MAT09MGVT10016

Surface area = area of 2 ends
+ area of the curved surface

$$\begin{aligned} A &= 2\pi r^2 + 2\pi rh \\ &= 2 \times \pi \times 6^2 + 2 \times \pi \times 6 \times 20 \\ &= 980.176 \dots \\ &\approx 980.2 \text{ cm}^2 \end{aligned}$$



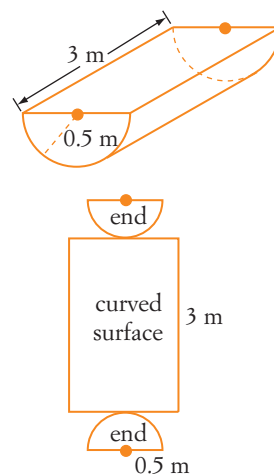
Example 14

Find, correct to two decimal places, the surface area of an open half-cylinder with radius 0.5 m and height 3 m.

Solution

Surface area = 2 semicircle ends + $\frac{1}{2} \times$ curved surface

$$\begin{aligned} A &= 2 \times \left(\frac{1}{2} \times \pi \times 0.5^2 \right) + \frac{1}{2} \times (2 \times \pi \times 0.5 \times 3) \\ &= 5.49778 \dots \\ &\approx 5.50 \text{ cm}^2 \end{aligned}$$



Exercise 10-07 Surface area of a cylinder

See Example 13 1 Calculate, correct to two decimal places, the surface area of a cylinder with:

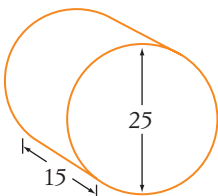
- a radius 7 m, height 10 m b diameter 28 cm, height 15 cm
c diameter 6.2 m, height 7.5 m d radius 0.8 m, height 2.35 m

2 Find, correct to one decimal place, the curved surface area of a cylinder with:

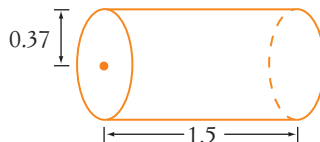
- a diameter 9 cm, height 32 cm b radius 85 mm, height 16 mm

See Example 14 3 Calculate, correct to one decimal place, the surface area of each solid. All lengths shown are in metres.

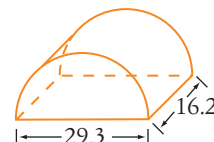
a closed cylinder



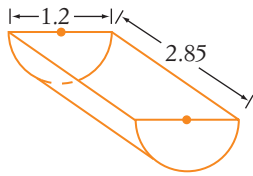
b cylinder with one open end



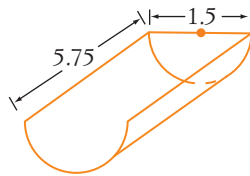
c closed half-cylinder



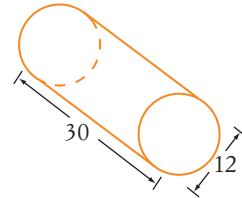
- d half-cylinder with open top



- e half-cylinder with open top, one end open

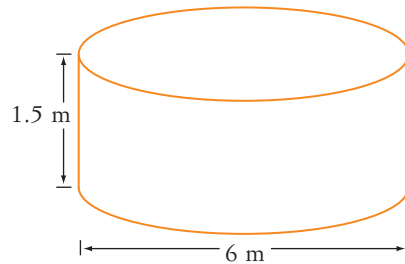


- f cylinder open at both ends



- 4 A swimming pool is in the shape of a cylinder 1.5 m deep and 6 m in diameter. The inside of the pool is to be repainted, including the floor. Find:

- the area to be repainted, correct to one decimal place
- the number of whole litres of paint needed if coverage is 9 m^2 per litre.

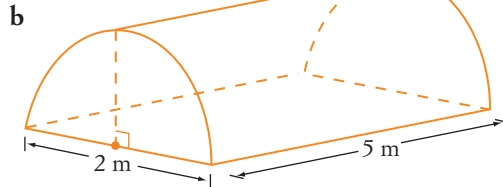
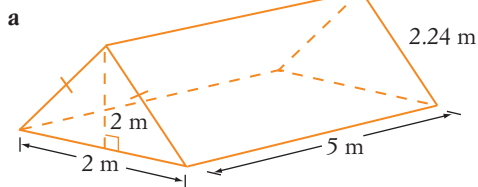


Worked solutions

Surface area of a cylinder

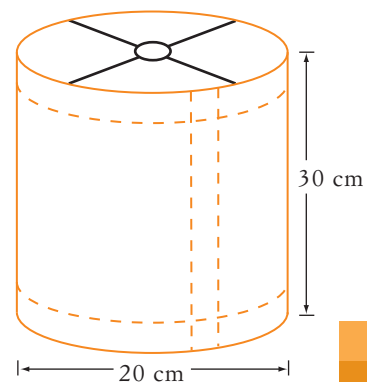
MAT09MGWS10044

- 5 Which tent has the greater surface area?



(Note: the floor is included for both tents)

- 6 This lampshade is to be covered. Find the area of material needed if 10% extra is allowed for seams and overlaps. Note: The lampshade is a cylinder open at both ends.



Worksheet

A page of prisms and cylinders

MAT09MGWK10116

Worksheet

Volume and capacity

MAT09MGWK00028

Skillsheet

What is volume?

MAT09MGSS10031

10-08 Volumes of prisms and cylinders

The **volume** of a solid is the amount of space it occupies. Volume is measured in **cubic units**, for example, cubic metres (m^3) or cubic centimetres (cm^3).

The **capacity** of a container is the amount of fluid (liquid or gas) it holds, measured in millilitres (mL), litres (L), kilolitres (kL) and megalitres (ML).

Homework sheet

Volume

MAT09MGHS10022

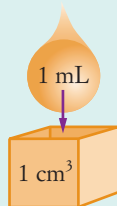
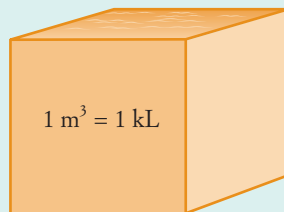
Puzzle sheet

Formula matching
game

MAT09MGPS10117

Summary

 1 cm^3 contains 1 mL.

 1 m^3 contains 1000 L or 1 kL

 $\times 1\,000\,000 =$


Volume of a prism

A **prism** has the same (uniform) cross-section along its length, and each cross-section is a **polygon** (with straight sides). Prisms take their names from their base and cross-section. For example, the prism shown above is a **trapezoidal prism** because its base and cross-sections are trapeziums.

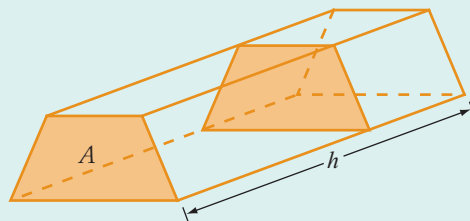
Because a prism is made up of identical cross-sections, its volume can be calculated by multiplying the **area of its base** by its **perpendicular height** (the length or depth of the prism).

Summary

Volume of a prism

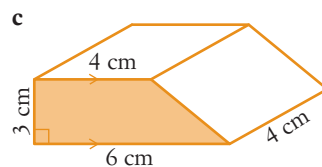
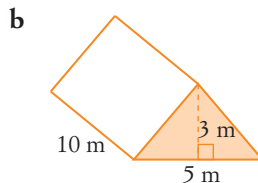
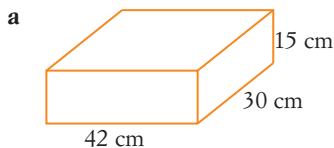
$$V = Ah$$

where A = area of base and

 h = perpendicular height


Example 15

Find the volume of each prism.



Solution

a $V = 42 \times 30 \times 15$
 $= 18\,900 \text{ cm}^3$

For a rectangular prism, volume = length $\times$ width $\times$ height ($V = lwh$).

Video tutorial

Volumes of prisms
and cylinders

MAT09MGVT10017

$$\begin{aligned} \text{b } A &= \frac{1}{2} \times 5 \times 3 \\ &= 7.5 \end{aligned}$$

Area of a triangle.

$$\begin{aligned} V &= 7.5 \times 10 \\ &= 75 \text{ m}^3 \end{aligned}$$

$V = Ah$ where height $h = 10$

$$\begin{aligned} \text{c } A &= \frac{1}{2} \times (4 + 6) \times 3 \\ &= 15 \text{ cm}^2 \end{aligned}$$

Area of a trapezium.

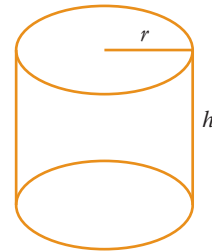
$$\begin{aligned} V &= 15 \times 4 \\ &= 60 \text{ cm}^3 \end{aligned}$$

$V = Ah$, where height $h = 4$

Volume of a cylinder

A cylinder is like a 'circular prism' because its cross-sections are identical circles. Because of this:

$$\text{Volume} = Ah = \pi r^2 \times h = \pi r^2 h$$



Summary

Volume of a cylinder

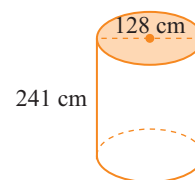
$$V = \pi r^2 h$$

where r = radius of circular base and h = perpendicular height

Example 16

For this cylinder, calculate:

- its volume, correct to the nearest cm^3
- its capacity in kL, correct to 1 decimal place.



Solution

$$\begin{aligned} \text{a } \text{Radius} &= \frac{1}{2} \times 128 \text{ cm} \\ &= 64 \text{ cm} \end{aligned}$$

$\frac{1}{2}$ of diameter

$$\begin{aligned} V &= \pi \times 64^2 \times 241 \\ &= 3\,101\,179.206\dots \\ &\approx 3\,101\,179 \text{ cm}^3 \end{aligned}$$

$$V = \pi r^2 h$$

Video tutorial

Volumes of prisms
and cylinders

MAT09MGVT10017

b Capacity = 3 101 179 mL
 $= (3\ 101\ 179 \div 1000 \div 1000) \text{ kL}$
 $= 3.101\ 179 \text{ kL}$
 $\approx 3.1 \text{ kL}$

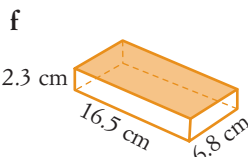
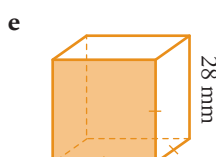
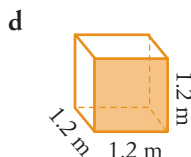
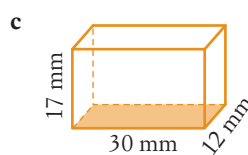
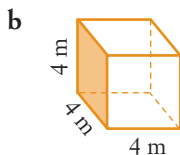
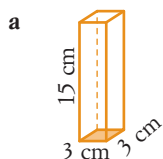
$1 \text{ cm}^3 = 1 \text{ mL}$



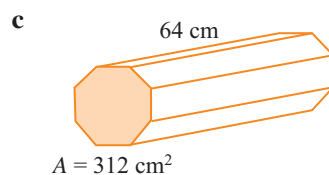
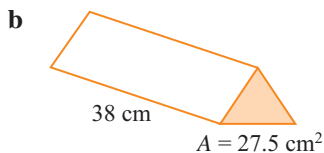
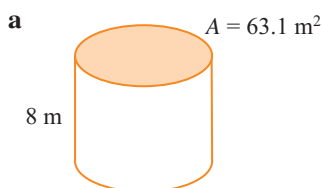
Exercise 10-08 Volumes of prisms and cylinders

See Example 15

- 1 Find the volume of each rectangular prism using the formula $V = lwh$.

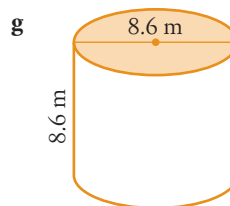
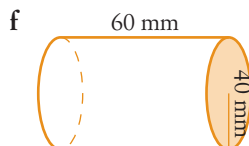
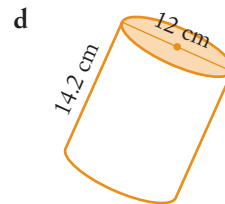
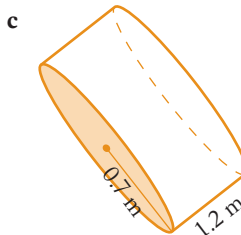
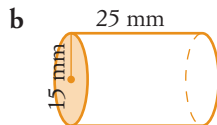
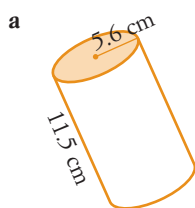


- 2 Find the volume of each solid.

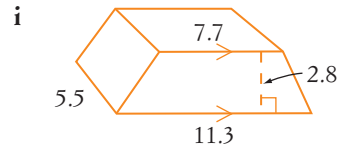
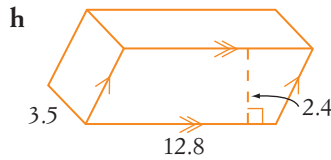
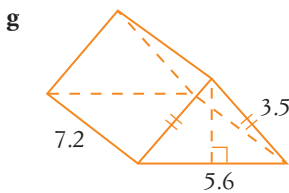
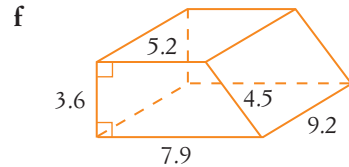
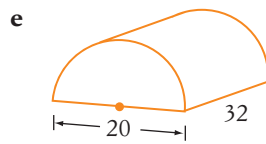
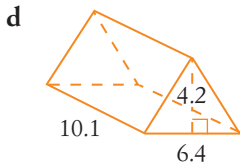
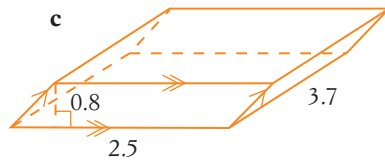
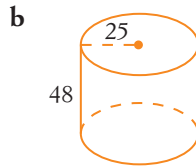
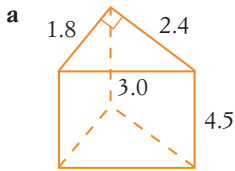


See Example 16

- 3 Find, correct to one decimal place, the volume of each cylinder.

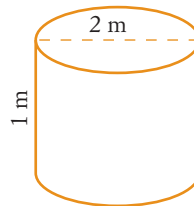


4 Calculate, correct to one decimal place, the volume of each solid. All lengths are in metres.



5 What is the capacity of this cylinder? Select the closest answer **A**, **B**, **C** or **D**.

- A** 15 700 L **B** 12 570 L
C 3140 L **D** 6260 L



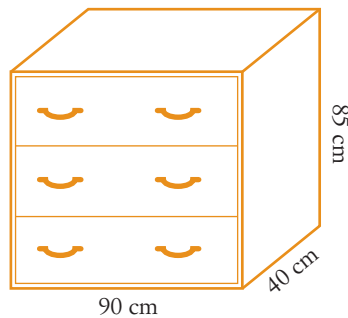
Worked solutions
 Volumes of prisms
 and cylinders

MAT09MGWS10045

6 Find the volume of this chest of drawers in m^3 :

- a** by calculating the volume in cm^3 , then converting to m^3
b by converting each length to metres first, then calculating the volume.

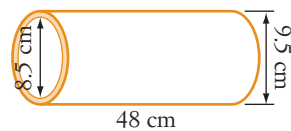
Which method is easier?



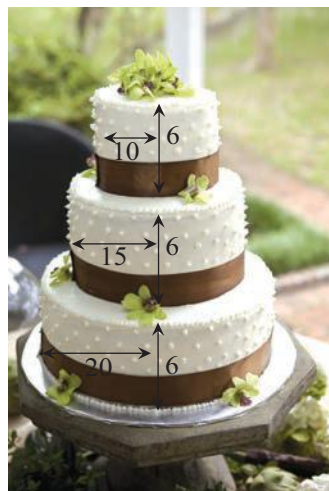
7 A triangular prism has base length 6 m, height 10 m and volume 150 m^3 . What is its length?

8 A fish tank that is 55 cm long, 24 cm wide and 22 cm high is filled to 4 cm below the top. Calculate the capacity of the tank in litres.

9 This metal pipe has an inner diameter of 8.5 cm and an outer diameter of 9.5 cm. Calculate, correct to two decimal places, the volume of metal needed to make the pipe.



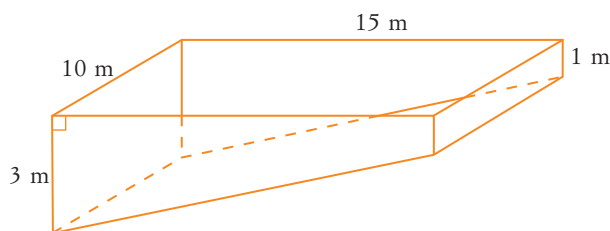
- 10 A cube has a volume of 343 m^3 . What is its side length?
- 11 A wedding cake with three tiers rests on a table. Each tier is 6 cm high. The layers have radii of 20 cm, 15 cm and 10 cm respectively. Find the total volume of the cake, correct to the nearest cm^3 .



Shutterstock.com/John Wollvertb

- 12 This swimming pool is 15 m long and 10 m wide. The depth of the water ranges from 1 m to 3 m.

Calculate the capacity of this pool in kilolitres.



Investigation: Volume vs surface area

You will need: At least 20 centicubes.

- 1 Copy the following table.

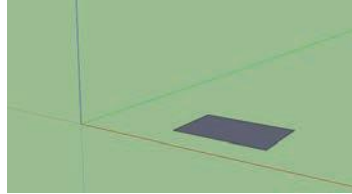
Length	Width	Height	Volume	Surface area
			8	
			8	
			$\vdots$	

- 2 Build as many rectangular prisms as you can with a volume of 8 cubes, but with different dimensions. *Note:* $1 \times 1 \times 8$ is the same as $1 \times 8 \times 1$ and $8 \times 1 \times 1$.
- 3 Record your dimensions in the table and calculate the surface area of each prism.
- 4 What are the dimensions of the solid that has the smallest surface area?
- 5 What name do we give to this three-dimensional solid?
- 6 Repeat the experiment for a volume of:
- a 12 cubes b 20 cubes.

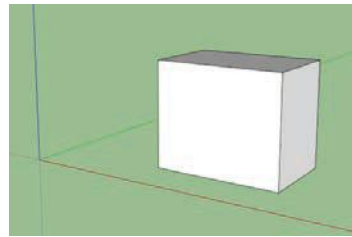
Technology Drawing prisms and cylinders

In this activity you will use Google SketchUp to create your own rectangular prism and cylinder and accurately calculate their surface area and volume.

- 1 Select the **Rectangle** tool  and draw a rectangle in the horizontal plane as shown.

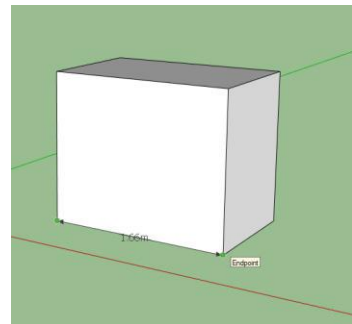


- 2 To make a rectangular prism, select **Push/pull** . Click on the rectangle and pull it up.

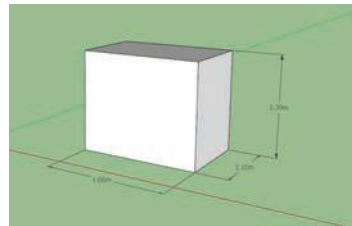


- 3 Measure the dimensions of the rectangular prism by first selecting the **Dimensions** tool .

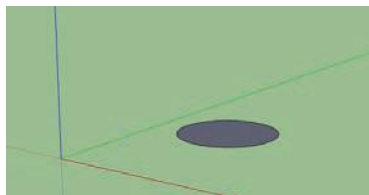
- 4 Click on the bottom edge of the front face as shown below and you should see its length displayed. Drag the length towards you and it will be shown in front of the prism.



- 5 Repeat this step for the width and height of your prism.

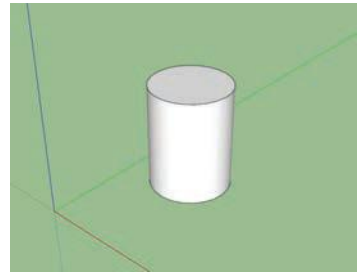


- 6 Now in your own book calculate the surface area and volume of your prism.
- 7 Construct a circle using the **Circle** tool in the horizontal plane.

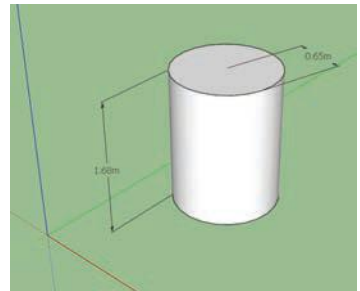


- 8 Use the **Push/pull** tool to create a cylinder by pulling up the circle.

- 9 Use the **Orbit** tool  to change the cylinder to a different orientation. This allows you to view the cylinder in different ways.

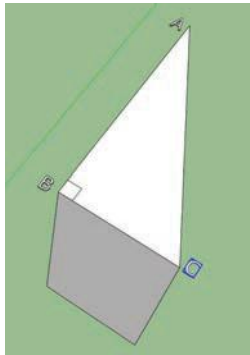


- 10 Use **Dimensions** to measure the radius and height of your cylinder.

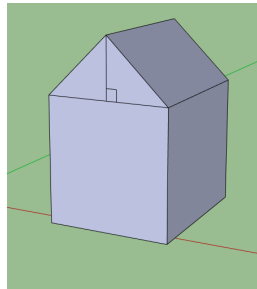


- 11 Now in your own book calculate the surface area and volume of your cylinder.
12 Draw each solid below and find its surface area and volume.

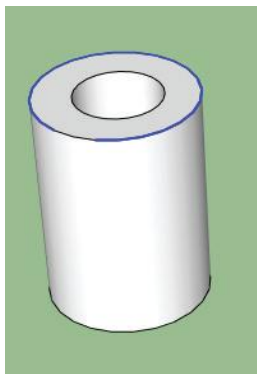
- a triangular prism $\angle ABC = 90^\circ$



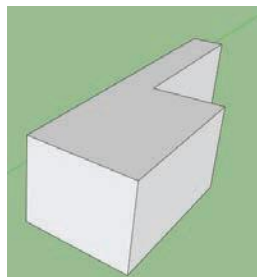
- b a shed with a gabled roof



- c annular cylinder



- d prism



Mental skills 10B Maths without calculators

Finding 15%, $2\frac{1}{2}\%$, 25% and $12\frac{1}{2}\%$

- To find 10% or $\frac{1}{10}$ of a number, divide by 10.
- To find 5% of a number, find 10% first, then halve it (since 5% is half of 10%).
- So to find 15% of a number, find 10% and 5% of the number separately, then add the answers together.

1 Study each example.

$$\begin{array}{ll} \text{a} & 15\% \times 80 = (10\% \times 80) + (5\% \times 80) \\ & = 8 + 4 \\ & = 12 \\ \text{b} & 15\% \times \$170 = (10\% \times \$170) \\ & \quad + (5\% \times \$170) \\ & = \$17 + \$8.50 \\ & = \$25.50 \end{array}$$

$$\begin{array}{ll} \text{c} & 15\% \times 3600 = (10\% \times 3600) \\ & \quad + (5\% \times 3600) \\ & = 360 + 180 \\ & = 540 \\ \text{d} & 15\% \times \$28 = (10\% \times \$28) \\ & \quad + (5\% \times \$28) \\ & = \$2.80 + \$1.40 \\ & = \$4.20 \end{array}$$

2 Now find 15% of each amount.

a 120	b \$840	c 260	d \$202
e \$50	f 72	g \$180	h 400
i \$1600	j \$22	k 6000	l \$350

To find $2\frac{1}{2}\%$ of a number, first find 5%, then halve it.

3 Study each example.

$$\begin{array}{ll} \text{a} & 2\frac{1}{2}\% \times 600 \\ & 10\% \times 600 = 60 \\ & 5\% \times 600 = \frac{1}{2} \times 60 = 30 \\ & 2\frac{1}{2}\% \times 600 = \frac{1}{2} \times 30 = 15 \\ \text{b} & 2\frac{1}{2}\% \times \$820 \\ & 10\% \times \$820 = \$82 \\ & 5\% \times \$820 = \frac{1}{2} \times 82 = \$41 \\ & 2\frac{1}{2}\% \times \$820 = \frac{1}{2} \times \$41 = \$20.50 \end{array}$$

4 Now find $2\frac{1}{2}\%$ of each amount.

a 400	b 6640	c \$2000	d \$880
e 1500	f \$232	g 5400	h \$904

To find 25% of a number, halve the number twice as $25\% = \frac{1}{4}$.

5 Study each example.

$$\begin{array}{ll} \text{a} & 25\% \times 700 \\ & 50\% \times 700 = \frac{1}{2} \times 700 = 350 \\ & 25\% \times 700 = \frac{1}{2} \times 350 = 175 \\ \text{b} & 25\% \times \$86 \\ & 50\% \times \$86 = \frac{1}{2} \times \$86 = \$43 \\ & \therefore 25\% \times \$86 = \frac{1}{2} \times \$43 = \$21.50 \end{array}$$

6 Now find 25% of each of each amount.

a 2000	b \$80	c 18	d \$25
e \$324	f \$140	g 66	h 298
i \$780	j \$1700	k \$126	l 1160

To find $12\frac{1}{2}\%$ of a number, find 25% first, then halve it. In other words, halve three times because $12\frac{1}{2}\% = \frac{1}{8}$.

7 Study each example.

a $12\frac{1}{2}\% \times 400$

$$50\% \times 400 = \frac{1}{2} \times 400 = 200$$

$$25\% \times 400 = \frac{1}{2} \times 200 = 100$$

$$12\frac{1}{2}\% \times 400 = \frac{1}{2} \times 100 = 50$$

b $12\frac{1}{2}\% \times \$144$

$$50\% \times \$144 = \frac{1}{2} \times \$144 = \$72$$

$$25\% \times \$144 = \frac{1}{2} \times \$72 = \$36$$

$$12\frac{1}{2}\% \times \$144 = \frac{1}{2} \times \$36 = \$18$$

8 Now find $12\frac{1}{2}\%$ of each amount.

a 1280	b \$12	c 60	d \$260
e \$540	f \$250	g 304	h 1360

Technology worksheet

Measuring pyramids

MAT09MGCT10004

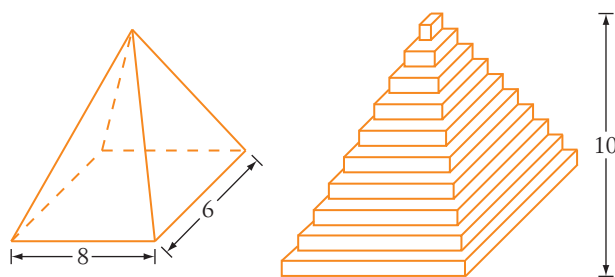
Technology Approximating the volume of a pyramid

Technology worksheet

Approximating the volume of a cone

MAT09MGCT10012

We can approximate the volume of a pyramid by dividing it into layers of prisms as shown below.



We will create a spreadsheet that approximates the volume of a rectangular pyramid with a base of length 8 units and width 6 units, and a perpendicular height of 10 units.

The volume of each layer can be easily calculated using $V = lwh$. Finding the sum of the layers will then give an approximation of the volume of the pyramid.

Let n be the number of layers. Then the height of each layer is $\frac{10}{n}$.

The length and width of each layer decreases from 8 units and 6 units respectively by a constant amount of $\frac{8}{n}$ and $\frac{6}{n}$ respectively from layer to layer.

- 1 Create this spreadsheet for calculating the volume of each layer and the sum of the volumes.

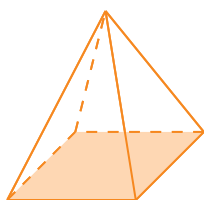
	A	B	C	D	E	F
1				Number of layers		
2						
3	Height	Length	Width	Thickness of layer	Volume of layer	Sum of volumes
4	10	8	6	=A\$4/\$D\$2	=B4*C4*D4	=E4
5		=B4-\$B4/\$D\$2	=C4-\$C4/\$D\$2			=E5+F4
6						
⋮						
13						

- 2 a To divide the volume of the pyramid into 10 layers, enter **10** in cell D2.
b Copy each formula down to row 13.
c Explain the results in cells E13 and F13.
d How accurate was your result in F13? Explain.
e Print out your spreadsheet.
- 3 a To divide the pyramid into 40 layers to calculate a better approximation, enter **40** in cell D2 and copy each formula down to row 43.
b In one or two sentences, compare your volume approximation in F43 with the previous approximation in F13.
- 4 a Enter each of these values in cell D2, copy the formulas down to the appropriate row and write down the approximation for the volume of the pyramid.
i **100** (copy down to row 104)
ii **200** (copy down to row 204)
iii **400** (copy down to row 404)
b Use the formula $V = \frac{1}{3}Ah$ to calculate the exact volume of the pyramid.
c Write a brief report about your results in parts a and b.

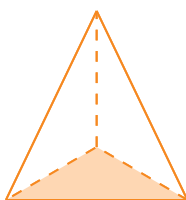
10-09 Volumes of pyramids and cones

Stage 5.3

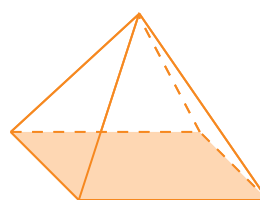
A pyramid's name is determined by the shape of its base.



Square pyramid



Triangular pyramid



Rectangular pyramid

Stage 5.3

Summary

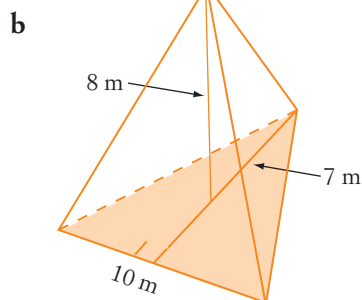
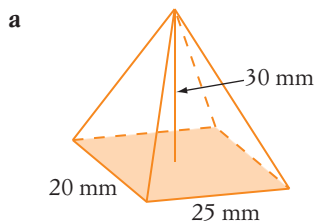
Volume of a pyramid

$$V = \frac{1}{3}Ah$$

where A = area of base and h = perpendicular height

Example 17

Find the volume of each pyramid.



Solution

a $A = 25 \times 20$
 $= 500$

$$V = \frac{1}{3} \times 500 \times 30$$

$$= 5000 \text{ mm}^3$$

Area of rectangular base

$$V = \frac{1}{3}Ah \text{ where height } h = 30$$

b $A = \frac{1}{2} \times 10 \times 7$
 $= 35 \text{ m}^2$

$$V = \frac{1}{3} \times 35 \times 8$$

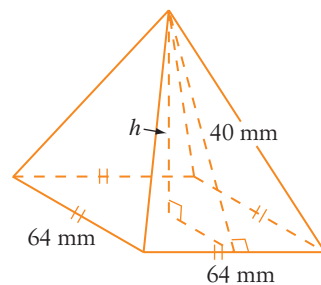
$$= 93\frac{1}{3} \text{ m}^3$$

Area of triangular base.

$$V = \frac{1}{3}Ah, \text{ where height } h = 8$$

Example 18

Find the capacity (to the nearest millilitre) of a square pyramid with a base edge of 64 mm and a slant height of 40 mm.



Solution

First find h , the perpendicular height of the pyramid.

$$h^2 = 40^2 - 32^2$$

$$= 576$$

$$h = \sqrt{576}$$

$$= 24 \text{ mm}$$

$$A = 64^2$$

$$= 4096 \text{ mm}^2$$

$$V = \frac{1}{3} \times 4096 \times 24$$

$$= 32\,768 \text{ mm}^3$$

$$= 32.768 \text{ cm}^3$$

$$\text{Capacity} = 32.768 \text{ mL}$$

$$\approx 33 \text{ mL}$$

Using Pythagoras' theorem.

Area of square base.

$$V = \frac{1}{3}Ah \text{ where height } h = 24$$

$$1 \text{ cm}^3 = 1000 \text{ mm}^3$$

$$1 \text{ cm}^3 = 1 \text{ mL}$$

Example 19

The volume of a square pyramid is 100 cm^3 . If its height is 12 cm, calculate the length of its base.

Solution

Let the length of the square base of the pyramid be x cm.

$$V = \frac{1}{3}Ah$$

$$100 = \frac{1}{3} \times x^2 \times 12$$

$$100 = x^2$$

$$x^2 = \frac{100}{4}$$

$$= 25$$

$$x = \sqrt{25}$$

$$= 5$$

$\therefore$ Length of base = 5 cm.

Volume of a cone

A cone is like a 'circular pyramid' so:

$$\text{Volume} = \frac{1}{3}Ah = \frac{1}{3} \times \pi r^2 \times h = \frac{1}{3}\pi r^2 h$$

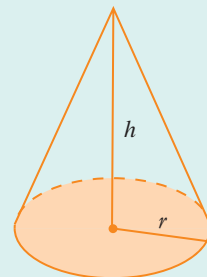
Stage 5.3

Summary

Volume of a cone

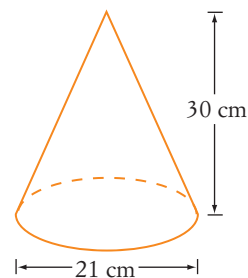
$$V = \frac{1}{3}\pi r^2 h$$

where r = radius of circular base and h = perpendicular height



Example 20

Calculate the volume of the cone correct to the nearest cm^3 .



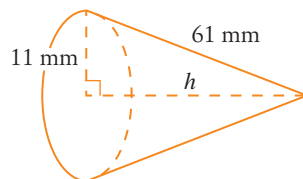
Solution

$$\begin{aligned} V &= \frac{1}{3} \times \pi \times 10.5^2 \times 30 \\ &= 3463.6059 \dots \\ &\approx 3464 \text{ cm}^3 \end{aligned}$$

$$V = \frac{1}{3}\pi r^2 h, \text{ where } r = \frac{1}{2} \times 21 = 10.5$$

Example 21

A cone has a slant edge of 61 mm and a base radius of 11 mm. Find its volume, correct to one decimal place.



Solution

First, find the height, h .

$$\begin{aligned} h^2 &= 61^2 - 11^2 \\ &= 3600 \\ h &= \sqrt{3600} \\ &= 60 \end{aligned}$$

$$\begin{aligned} V &= \frac{1}{3} \times \pi \times 11^2 \times 60 \\ &= 7602.6542 \dots \\ &\approx 7602.7 \text{ mm}^3 \end{aligned}$$

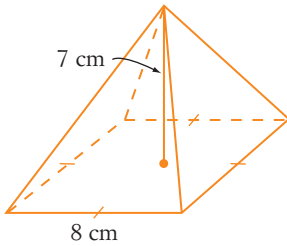
Using Pythagoras' theorem.

$$V = \frac{1}{3}\pi r^2 h$$

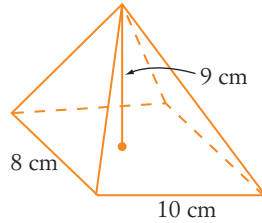
Exercise 10-09 Volumes of pyramids and cones

1 Calculate the volume of each pyramid (correct to one decimal place where necessary).

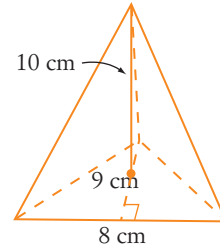
a



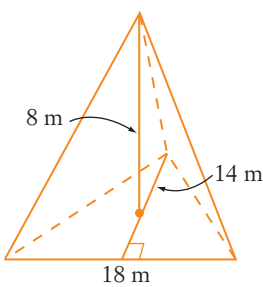
b



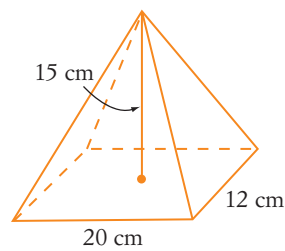
c



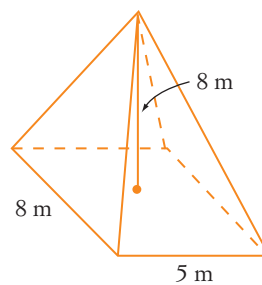
d



e

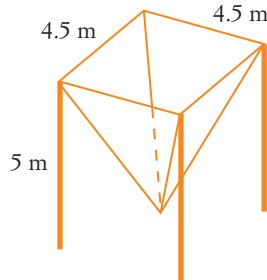


f



2 A grain hopper is in the shape of an inverted square pyramid as shown. What is its volume?
Select the correct answer A, B, C or D.

- A 15 m^3
- B 33.75 m^3
- C 50.625 m^3
- D 101.25 m^3



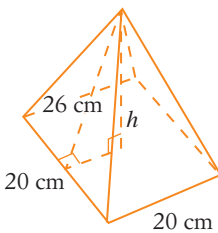
3 For each pyramid, find correct to two decimal places:

i its perpendicular height, h

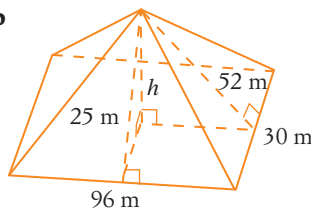
ii its volume

iii its capacity.

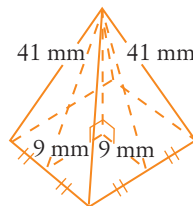
a



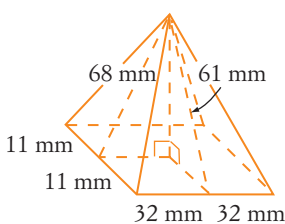
b



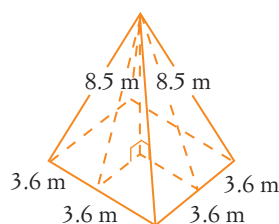
c



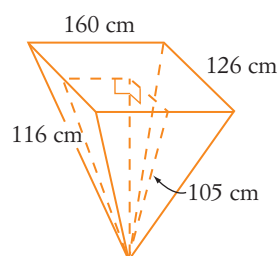
d



e



f



Stage 5.3

- 4 The great pyramid of Khufu (or Cheops) in Egypt was built on a square base with side lengths approximately 230 m.



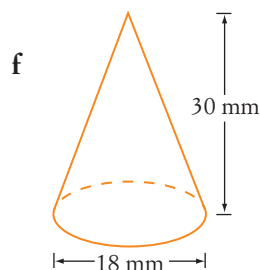
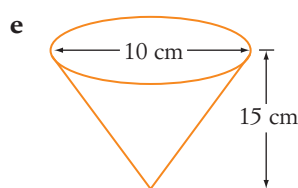
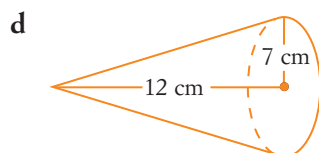
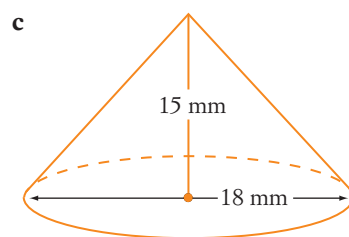
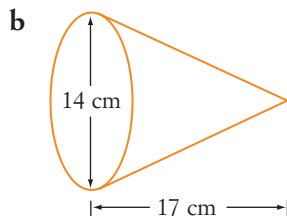
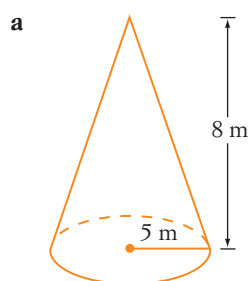
- a Find the volume in cubic metres if the original height of the pyramid was 147 m.
b There are an estimated 2.3 million stone blocks in the pyramid. Calculate the average volume of each block.

See Example 19

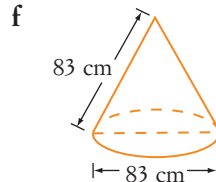
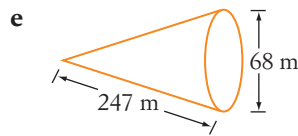
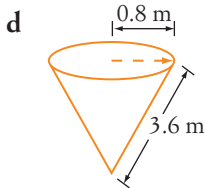
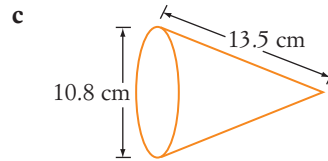
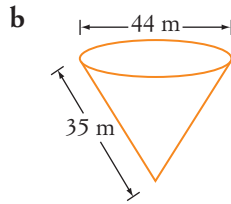
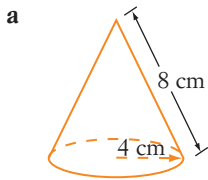
- 5 The area of the base of a pyramid is 40 m^2 . If its volume is 360 m^3 , calculate its perpendicular height.
6 The volume of a square pyramid is 1620 mm^3 . If the length of its base is 8 mm, calculate correct to the nearest millimetre the height of the pyramid.
7 A square pyramid has a volume of 80 cm^3 and a height of 10 cm. Calculate correct to one decimal place the length of the base of the pyramid.

See Example 20

- 8 Calculate the volume of each cone, correct to the nearest whole number.



9 Calculate the volume of each cone, correct to one decimal place.



- 10** A cone has a volume of 1468 cm^3 and a base radius of 12 cm. Find its height correct to one decimal place.
- 11** A cone has a volume of 820 m^3 and a perpendicular height of 10 m. Find its radius correct to one decimal place.
- 12** A cone has a volume of 150 mm^3 . If the height and radius of the cone are equal in length, calculate the radius of the cone. Give your answer to two decimal places.

Stage 5.3

See Example 21

Worked solutions

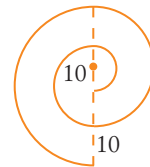
Volumes of pyramids and cones

MAT09MGWS10046

Power plus

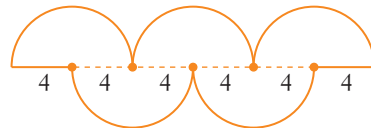
- 1** A spiral is formed from four semicircles as shown. The diameter of the smallest semicircle is 10 cm, and the semicircles are 10 cm apart. Find the total length of the spiral:

a correct to two decimal places **b** in terms of π .

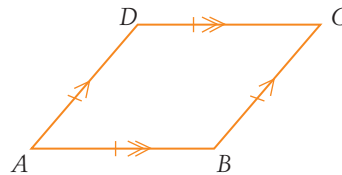


- 2** For this pattern, all lengths in metres. Calculate correct to two decimal places:

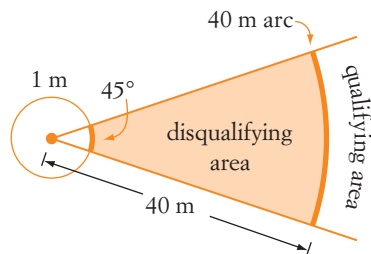
a its perimeter **b** its area.



- 3** $ABCD$ is a rhombus. If $AC = 24 \text{ cm}$ and $BD = 18 \text{ cm}$, find the perimeter of the rhombus.



- 4** To qualify for the next round of the discus trials, Bronte must throw the discus beyond the 40 m arc. Find correct to the nearest m^2 the shaded disqualifying area of the sector, given that the small circle has a radius of 1 m.

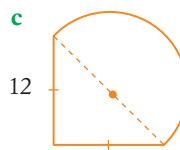
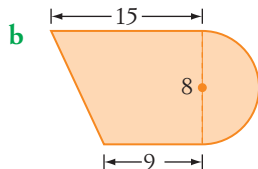
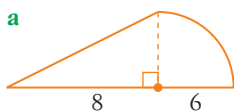


- 5 For each composite shape calculate, correct to one decimal place:

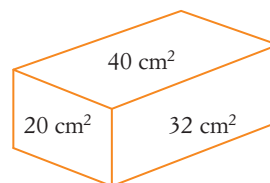
i its area

ii its perimeter.

All measurements are in centimetres.

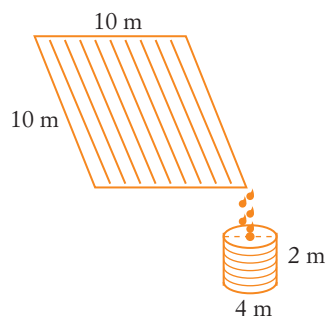


- 6 The three faces of a rectangular prism have areas as shown. Calculate the volume of the prism.



- 7 A cube opened at one end has an external surface area of 1125 cm^2 . Find its volume.

- 8 A 10 m flat square roof drains into a cylindrical rainwater tank with a diameter of 4 m. If 5 mm of rain falls on the roof, by how much (to the nearest mm) does the level of water in the tank rise?



Language of maths

arc length	base	capacity	circumference
cone	cross-section	curved surface	diagonal
diameter	giga-	kilobyte	limits of accuracy
mega-	megabyte	micro- (μ)	net
perimeter	perpendicular height	pi (π)	prism
pyramid	radius/radii	sector	surface area

Puzzle sheet

Surface area and
volume crossword

MAT09MGPS10118

Quiz

Measurement

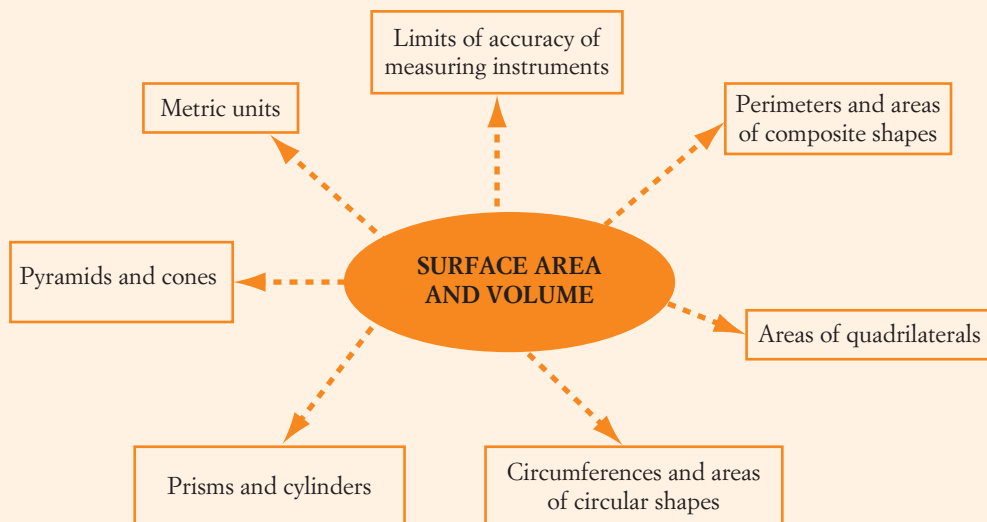
MAT09MGQZ00003

- Draw a circle and label on it:
a a sector **b** a diameter **c** an arc length
- In the formula $V = Ah$, explain what V , A and h stand for.
- Which metric prefix means 'one-millionth'?
- Write the definition of a **prism** using the word **cross-section**.
- How do you find the **surface area** of a solid?
- Name a solid that has a **curved surface**.

Topic overview

- Write 10 questions (with solutions) that could be used in a test for this chapter. Include some questions that you have found difficult to answer.
- Swap your questions with another student and check their solutions against yours.
- List the sections of work in this chapter that you did not understand. Follow up this work with your friend or teacher.

Copy (or print) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Worksheet

Mind map: Surface
area and volume
(Advanced)

MAT09MGWK1020

Chapter 10 revision

See Exercise 10-01

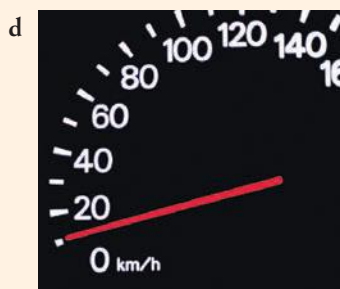
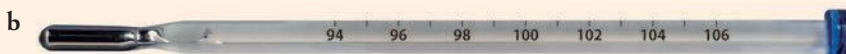
1 Convert:

a 8 km to m	b 15 min to s	c 0.7 m to mm
d 250 000 g to kg	e 8.4 GB to MB	f 9.5 t to kg
g 300 min to h	h 34 000 B to kB	i 125 kL to L
j 2500 cm to km	k 17 000 000 kg to t	l 8900 GB to TB
m 10 500 ms to s	n 9.1 millennia to years	o 2.6 Mm to m
p 3 ly to km	q 0.000 000 4 s to μ s	r 75 000 000 μ m to m

See Exercise 10-02

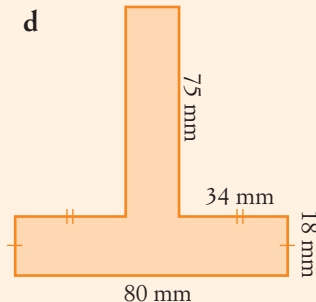
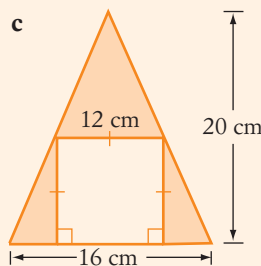
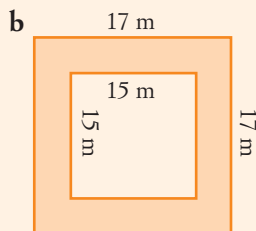
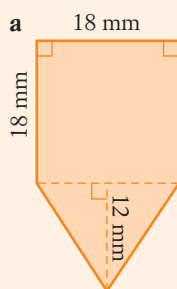
2 For each measuring instrument, state:

- i the size of one unit on the scale ii its limits of accuracy.



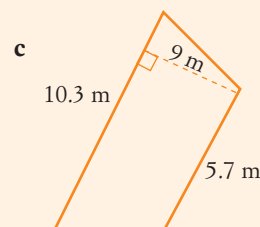
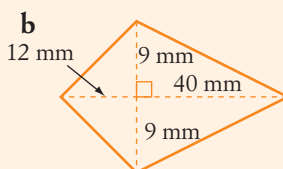
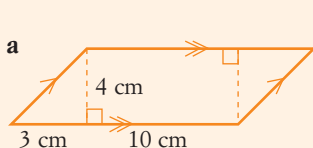
See Exercise 10-03

3 Calculate the area of each shape.

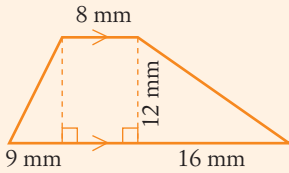


See Exercise 10-04

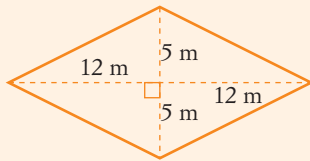
4 Calculate the area of each quadrilateral.



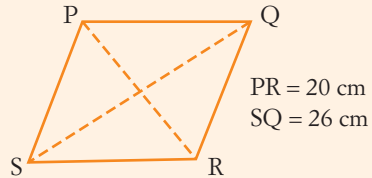
d



e



f



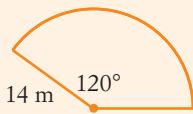
5 For each shape calculate, correct to one decimal place:

i its perimeter

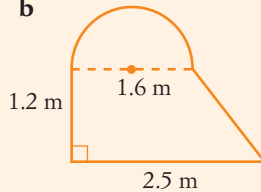
ii its area.

See Exercise 10-05

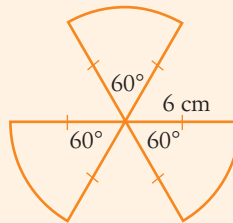
a



b



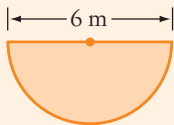
c



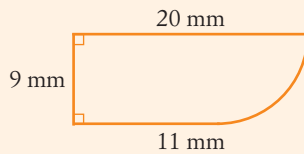
6 Calculate, correct to two decimal places, the area of each shape.

See Exercise 10-05

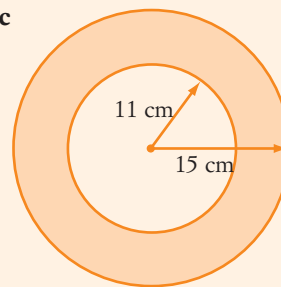
a



b



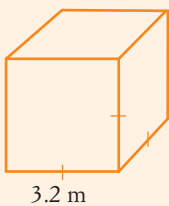
c



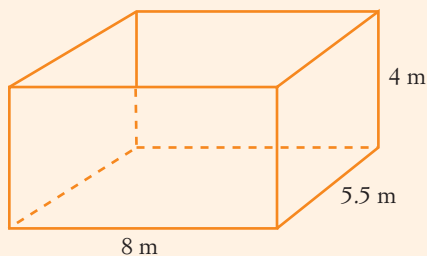
7 Calculate the surface area of each prism.

See Exercise 10-06

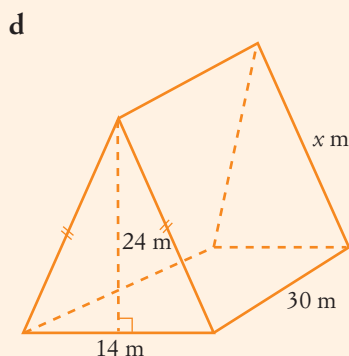
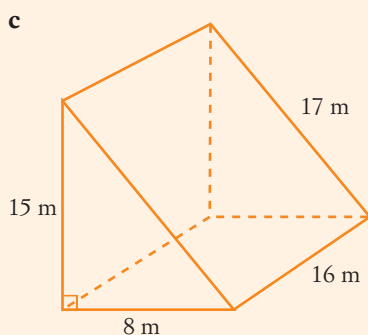
a closed cube



b open rectangular prism

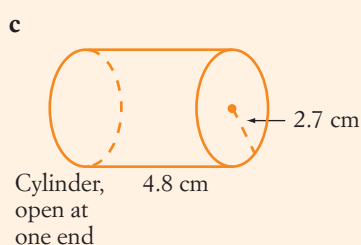
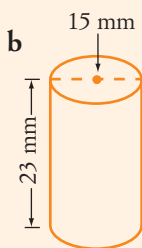
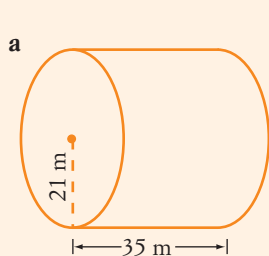


Chapter 10 revision



See Exercise 10-07

- 8 Calculate, correct to two decimal places, the surface area of each cylinder.



See Exercise 10-08

- 9 Calculate the volume of each prism in question 7.

See Exercise 10-08

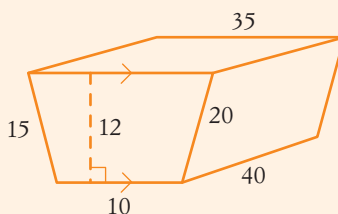
- 10 Calculate, correct to two decimal places, the volume of each cylinder in question 8.

See Exercise 10-08

- 11 A rectangular fish tank is 75 cm long by 55 cm wide by 35 cm deep and is filled to 5 cm from the top. How many litres of water is in the tank?

See Exercise 10-08

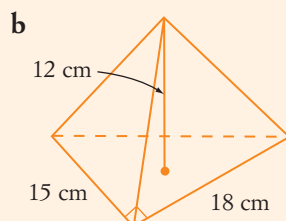
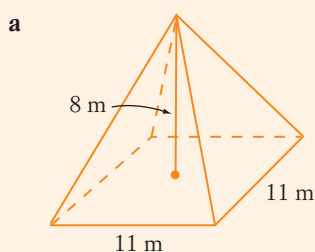
- 12 Calculate the volume of this prism.
Measurements are in centimetres.



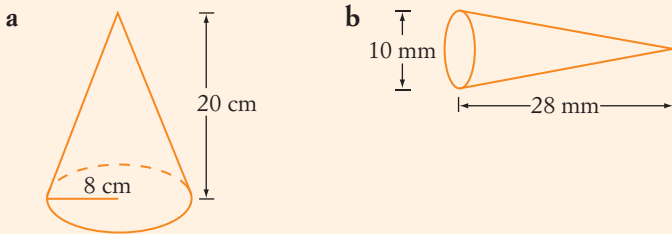
Stage 5.3

See Exercise 10-09

- 13 Find correct to the nearest whole number the volume of each pyramid.



- 14 Find, correct to one decimal place, the volume of each cone.

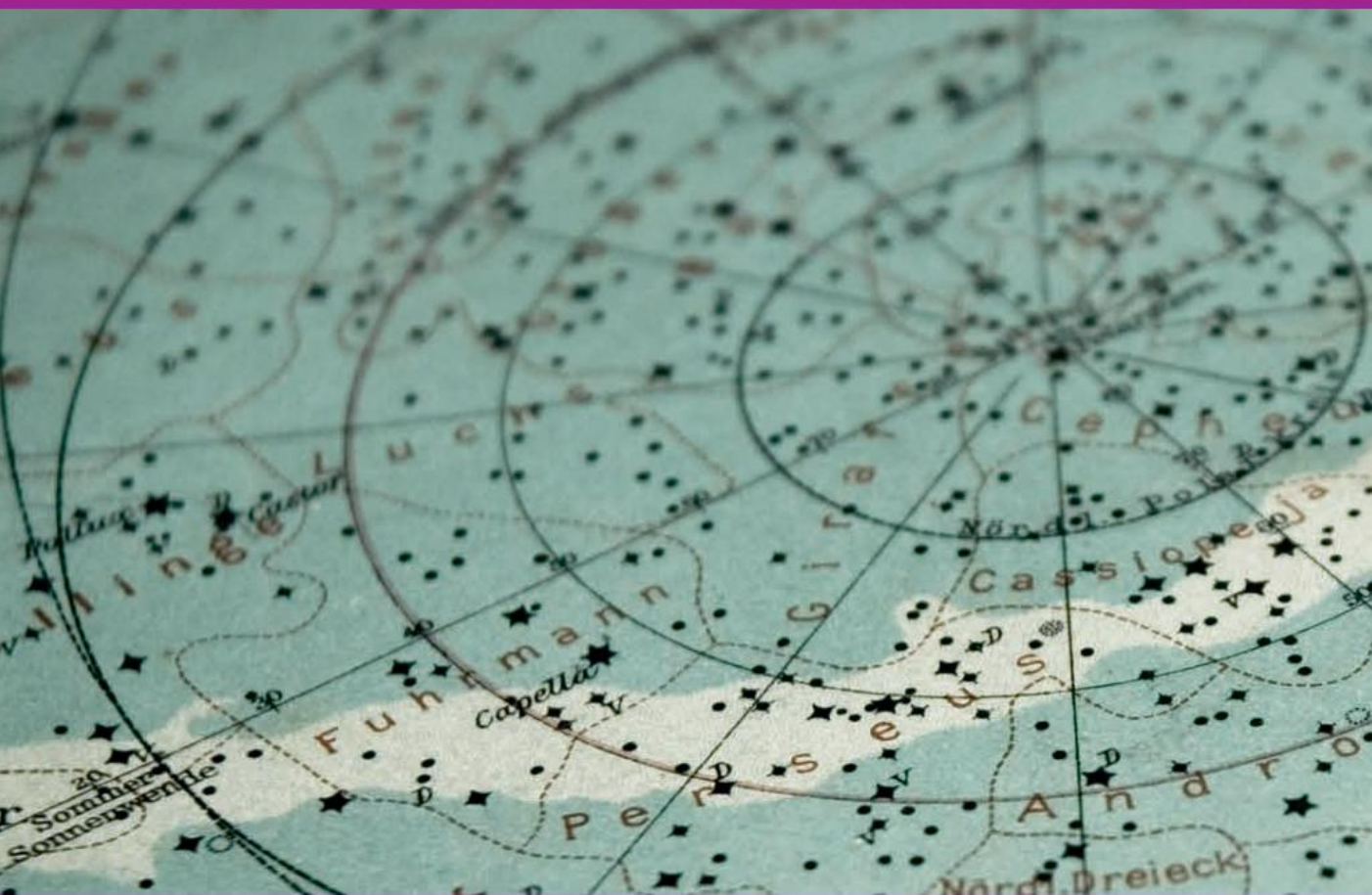


- 15 A cone has a volume of 115 cm^3 and a base radius of 3.2 cm. Find, correct to one decimal place, the height of the cone.

Stage 5.3

See Exercise 10-09

See Exercise 10-09



11

Number and algebra

Coordinate geometry and graphs

The use of a coordinate system allows us to monitor the location of objects on Earth or in space. Whether reading street maps, using global positioning systems or managing road traffic flow, coordinates provide an accurate method for indicating position. Coordinates have important applications in many areas including astronomy, architecture, engineering and art design.



Shutterstock.com/Piotr Sikora

Chapter outline

	Proficiency strands			
11-01 The length of an interval	U	F	R	C
11-02 The midpoint of an interval	U		R	C
11-03 The gradient of a line	U	F	R	C
11-04 Graphing linear equations	U	F	R	C
11-05 The gradient-intercept equation $y = mx + b$	U	F	R	C
11-06 Finding the equation of a line	U	F	R	C
11-07 Solving linear equations graphically	U	F	R	C
11-08 Direct proportion	U	F	PS	R
11-09 Graphing quadratic equations	U	F	R	C
11-10 Graphing circles	U	F	R	C

Wordbank

direct proportion A relationship between two variables in which one variable is a constant multiple of the other. For example, if $y = 8.5x$, then y is directly proportional to x

gradient The steepness of a line or interval, measured by the fraction $\frac{\text{rise}}{\text{run}}$

interval A part of a line with a definite length

linear equation An equation whose graph is a straight line

midpoint The point in the middle of an interval or halfway between two given points

parabola A U-shaped curve that is the graph of a quadratic equation

quadratic equation An equation involving a variable squared (power of 2), such as $y = 3x^2 - 6$, whose graph is a curve called a parabola

y-intercept The y -value at which a graph cuts the y -axis

In this chapter you will:

- find the distance between two points located on the Cartesian plane using a range of strategies, including graphing software
- find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software
- sketch linear graphs using the coordinates of two points
- solve problems involving direct proportion and explore the relationship between graphs and equations corresponding to simple rate problems 
- test whether a point lies on a line
- use the gradient–intercept equation of a straight line $y = mx + b$
- find the equation of a line from its graph
- solve linear equations graphically
- (STAGE 5.3) use the distance, midpoint and gradient formulas for an interval on the number plane
- (STAGE 5.3) graph a linear equation by first finding its x - and y -intercepts
- graph quadratic equations of the form $y = ax^2$ and $y = ax^2 + c$ and circles with equation $x^2 + y^2 = r^2$

SkillCheck

Worksheet

StartUp assignment 11

MAT09NAWK10121

Worksheet

Tables of values

MAT09NAWK10122

1 Complete each table of values using the given equation.

a $y = -2x + 1$

x	-2	-1	0	1	2
y					

b $y = \frac{x}{2} + 1$

x	-2	-1	0	1	2
y					

2 Given the equation $y = 5 - 2x$, find:

a y when $x = 0$

b y when $x = 3$

c x when $y = 0$

d x when $y = 7$

3 Calculate the average of:

a 1 and 3

b -2 and 6

c 5 and 12

11-01 The length of an interval

A line is usually drawn with arrows at both ends to show that it runs infinitely in both directions. However, an **interval** or line segment is part of a line, with endpoints on both sides and a definite length. The interval joining points P and Q is called the interval PQ , shown in green in Example 1 a below. We can calculate the length of an interval (or the distance between two points) on a number plane if we know the coordinates of its endpoints.

Example 1

Find the distance between the points:

- a $P(-4, -2)$ and $Q(2, 6)$ b $A(-2, 5)$ and $B(2, -2)$

Solution

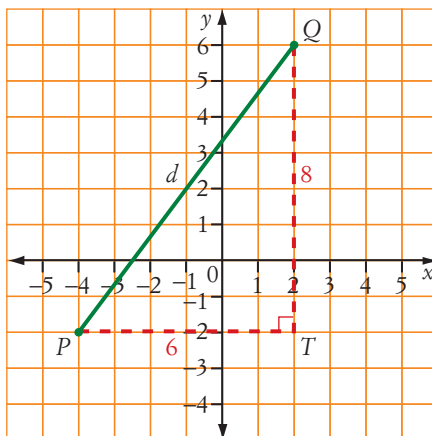
- a With P and Q plotted on a number plane, construct a right-angled triangle PTQ such that PQ is the hypotenuse.

$PT = 6$ units, $TQ = 8$ units.

Let the length of PQ be d , so by Pythagoras' theorem:

$$\begin{aligned} d^2 &= 6^2 + 8^2 \\ &= 100 \\ d &= \sqrt{100} = 10 \end{aligned}$$

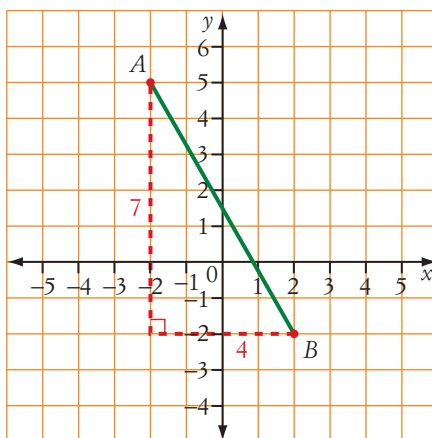
The distance between the points P and Q is 10 units.



- b By plotting A and B and constructing $\triangle ABC$, we have:

$$\begin{aligned} AB^2 &= 7^2 + 4^2 \\ &= 65 \\ AB &= \sqrt{65} \\ &= 8.0622 \dots \\ &\approx 8.06 \text{ units} \end{aligned}$$

We can write the distance AB in **surd** form, $\sqrt{65}$ units as an exact answer, or we can approximate (round) it as a decimal, 8.06 units.



Worksheet

Length of an interval

MAT09NAWK10123

Puzzle sheet

Length of a line segment

MAT09NAPS00047

Skillsheet

The number plane

MAT09NASS10032

Video tutorial

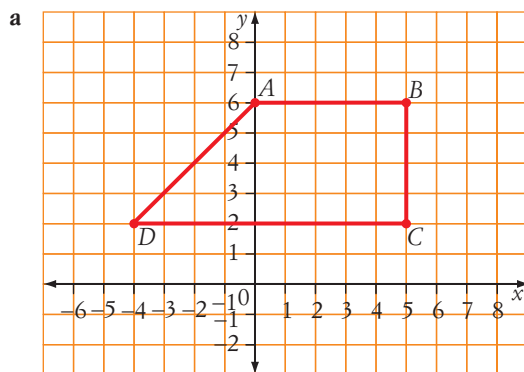
The length of an interval

MAT09NAVT10018

Example 2

- Plot the points $A(0, 6)$, $B(5, 6)$, $C(5, 2)$ and $D(-4, 2)$ on a number plane and join them to make the quadrilateral $ABCD$.
- What type of quadrilateral is $ABCD$?
- Find the exact length of AD .
- Hence find the perimeter of $ABCD$ correct to two decimal places.

Solution



- b $ABCD$ is a trapezium.

c $AD^2 = 4^2 + 4^2$
 $= 32$

$$AD = \sqrt{32} \text{ units}$$

In exact surd form.

- d By counting grid squares,
 $AB = 5$, $BC = 4$, $CD = 9$.

$$\begin{aligned} \text{Perimeter of } ABCD &= 5 + 4 + 9 + \sqrt{32} \\ &= 23.656 \dots \\ &\approx 23.66 \text{ units} \end{aligned}$$

Stage 5.3

The distance formula

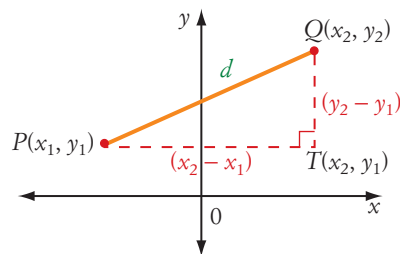
The method for finding the length of an interval can be summarised by a formula. This is called the **distance formula** because it is used to calculate the distance (d) between any two points $P(x_1, y_1)$ and $Q(x_2, y_2)$, in other words, the length of the interval PQ .

$$\text{Length of } PT = x_2 - x_1$$

$$\text{Length of } TQ = y_2 - y_1$$

$$\text{By Pythagoras' theorem: } d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\therefore d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Summary

The distance (d) between the two points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Group the x -coordinates together, group the y -coordinates together.

Example 3

Use the distance formula to find the exact length of the interval joining the following points from Example 1:

- a** $P(-4, -2)$ and $Q(2, 6)$ **b** $A(-2, 5)$ and $B(2, -2)$

Solution

- a** For $P(-4, -2)$ and $Q(2, 6)$:

$$x_1 = -4, y_1 = -2, x_2 = 2, y_2 = 6$$

Length of interval,

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[2 - (-4)]^2 + [6 - (-2)]^2} \\ &= \sqrt{6^2 + 8^2} \\ &= \sqrt{100} \\ &= 10 \text{ units} \end{aligned}$$

Group the x -coordinates together, group the y -coordinates together

- b** For $A(-2, 5)$ and $B(2, -2)$:

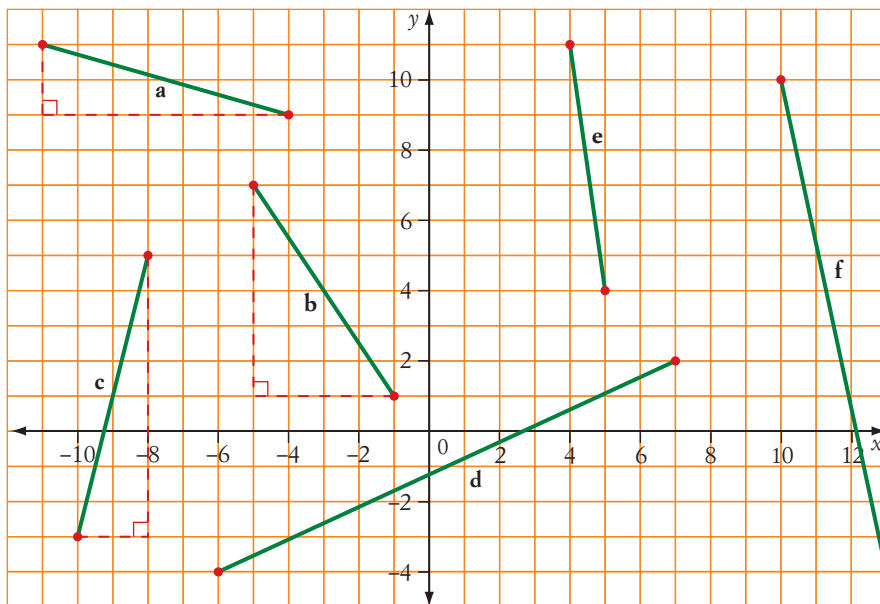
$$x_1 = -2, y_1 = 5, x_2 = 2, y_2 = -2$$

Length of interval,

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[2 - (-2)]^2 + (-2 - 5)^2} \\ &= \sqrt{4^2 + (-7)^2} \\ &= \sqrt{65} \text{ units} \end{aligned}$$

Exercise 11-01 The length of an interval

- See Example 1** 1 Calculate the length of each interval shown on this number plane. Express your answers correct to one decimal place, where necessary.



- 2 Plot each pair of points on a number plane and construct a right-angled triangle to find the distance between the two points. Express your answer:
- as a surd
 - correct to one decimal place
- $A(7, 6)$ and $B(1, 9)$
 - $P(0, -5)$ and $Q(6, 4)$
 - $G(9, -2)$ and $H(1, 6)$
 - $W(-5, -4)$ and $X(-3, 4)$
- See Example 2** 3 A triangle is formed by joining the points $A(5, 2)$, $B(-2, 3)$ and $C(1, 7)$. Find correct to three decimal places:
- the length AB
 - the length BC
 - the length AC
 - the perimeter of $\triangle ABC$
- Stage 5.3** 4 Use the distance formula to calculate the length of the interval joining each pair of points. Express your answer correct to one decimal place where necessary.
- $(6, 3)$ and $(8, 7)$
 - $(0, 10)$ and $(10, 6)$
 - $(9, 2)$ and $(1, -6)$
 - $(-1, -3)$ and $(2, 1)$
 - $(6, 8)$ and $(-10, -10)$
 - $(0, 0)$ and $(5, 12)$
 - $(5, 2)$ and $(-6, -1)$
 - $(3, -8)$ and $(5, -10)$
 - $(6, 12)$ and $(-8, 2)$
- 5 What is the exact distance between the points $Y(6, -3)$ and $Z(-2, 4)$? Select **A**, **B**, **C** or **D**.
- $\sqrt{113}$ units
 - $\sqrt{65}$ units
 - $\sqrt{57}$ units
 - $\sqrt{15}$ units
- 6 The vertices of triangle RST are $R(-1, -1)$, $S(1, 3)$ and $T(3, 1)$.
- Find the exact length of each side of the triangle.
 - What type of triangle is $\triangle RST$?
- 7 A quadrilateral is formed by joining the points $M(-3, 5)$, $N(5, 5)$, $P(8, 1)$ and $T(-3, 1)$.
- What type of quadrilateral is $MNPT$?
 - Find the perimeter of $MNPT$.
 - Find the area of $MNPT$.

Worked solutions

The length of an interval

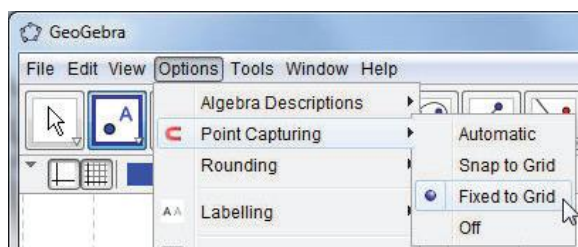
MAT09NAWS10047

- 8 The vertices of a quadrilateral are $H(1, 6)$, $I(7, 2)$, $J(3, -4)$ and $K(-3, 0)$.
- What type of quadrilateral is $HIJK$?
 - Find the exact length of each side of the quadrilateral.
 - Find its perimeter correct to one decimal place.
 - Calculate its area.

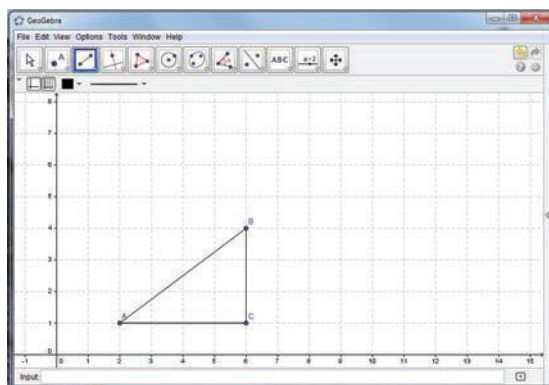
Technology The length of an interval

In this activity you will use GeoGebra to apply Pythagoras' theorem to find the length of an interval.

- Right-click in the Graphics window and make sure **Axes** and **Grid** are enabled. By pressing **ctrl** and holding your mouse down anywhere on the grid (except on the axes), you can move the axes. Do this until they approximately show x -values: 0 to 14 and y -values: 0 to 8.
- Select **Options**, **Point Capturing** and **Fixed to Grid**. This will ensure that you select coordinates with whole numbers only.



- Click **New Point** and plot the points $A(2, 1)$, $B(6, 4)$ and $C(6, 1)$ on the grid. Use  to join AB , BC and AC .



- Select  to find the lengths of AC and BC .
- Calculate $(AC)^2 + (BC)^2$ and $\sqrt{(AC)^2 + (BC)^2}$.
- Now select **Distance or Length** to find the length of AB .
- Compare answer 6 with answer 5. What do you notice?

- 8 Now move the axes until they are centred in the middle of the screen.
[Approx.: x : -10 to 10 and y : -10 to 10]
- 9 Plot each pair of points and calculate the distance between them by repeating all steps above.
- a $X(-4, 2)$ and $Y(4, -4)$ b $P(-7, 5)$ and $Q(-2, -7)$

Just for the record

René Descartes

The number plane is also called the **Cartesian plane**, named after its inventor, the French philosopher and mathematician René Descartes (1596–1650). In an age when Shakespeare and Galileo had made contributions to drama and science respectively, Descartes showed that many relationships and formulas could be represented by simple graphs. In 1637, he made a great contribution to mathematics when he published a book linking algebra and geometry for the first time. As a philosopher, the big question that Descartes spent many years thinking about was whether he, or the world, really existed. He finally concluded that he did exist, using a brilliant argument summarised by his famous quote.

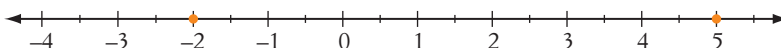


Getty Images/Imagno

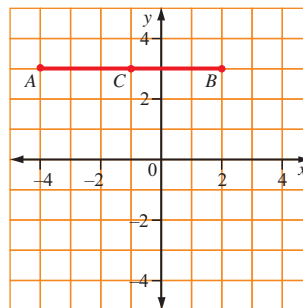
Complete his famous quote: “I _____, therefore _____”

Investigation: The midpoint of an interval

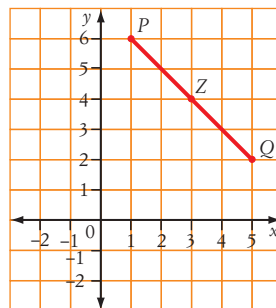
- 1 a What number is halfway between -2 and 5 on this number line?



- b What is the average of -2 and 5 ?
- 2 a What are the coordinates of A and B on this number plane?
- b What is the average of the x -values of A and B ?
- c What is the average of the y -values of A and B ?
- d What are the coordinates of C , the midpoint of AB ?
- e What do you notice about your answers for parts b, c and d?



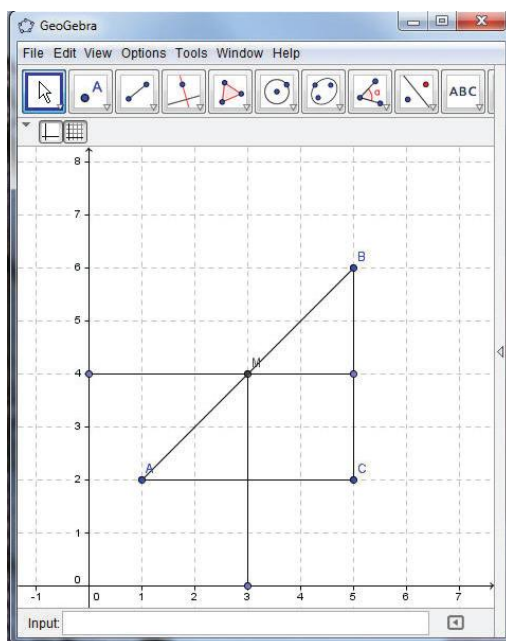
- 3 a What are the coordinates of P and Q ?
 b What is the average of the x -values of P and Q ?
 c What is the average of the y -values of P and Q ?
 d What are the coordinates of Z , the midpoint of PQ ?
 e How do the coordinates of Z relate to your answers for parts b and c?



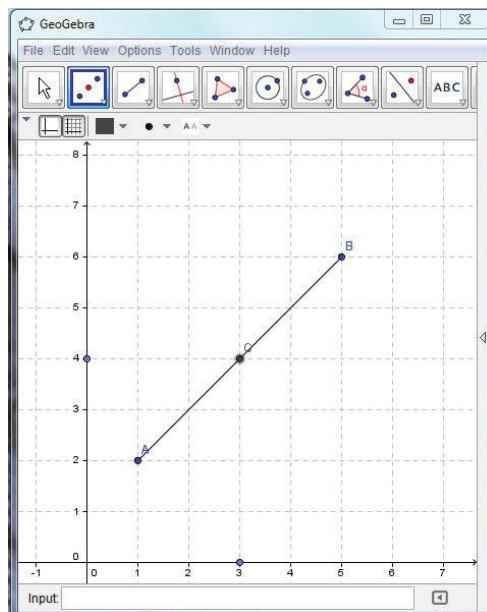
- 4 Describe how to find the midpoint of any interval on a number plane given the coordinates of its endpoints.

Technology The midpoint of an interval

- 1 In GeoGebra, by pressing **ctrl** and holding your mouse down anywhere on the grid (except on the axes), you can move the axes. Do this until the axes show approximately x -values: 0 to 14 and y -values: 0 to 8.
- 2 Select **New Point** and plot the points $A(1, 2)$, $B(5, 6)$ and $C(5, 2)$ on the grid.
- 3 Use **Interval between Two Points** to join AB , BC and AC .
- 4 What is the average x value on AC ?
- 5 What is the average y value on BC ?
- 6 Label the midpoint of AB , M (as shown below). What are its coordinates?



- 7 Redraw interval AB . Select  Midpoint or Centre. Label the midpoint M . Does it have the same coordinates as you found in step 6?



Technology

GeoGebra: Slope,
distance and midpoint

MAT09NATC00008

Technology worksheet

Excel worksheet:
Finding midpoints

MAT09NACT00027

Technology worksheet

Excel spreadsheet:
Finding midpoints

MAT09NACT00012

11-02 The midpoint of an interval

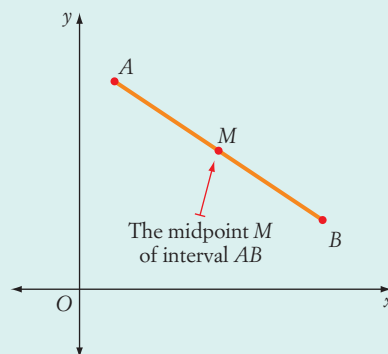
The **midpoint** of an **interval** is the point in the middle of an interval or halfway between two given points.

Summary

To find the **midpoint** of an **interval** joining two points A and B :

- calculate the average of the x -coordinates of A and B
- calculate the average of the y -coordinates of A and B

These two values give the coordinates of the midpoint.



Example 4

Find the midpoint of the interval joining:

- a** $A(5, -2)$ and $B(1, 6)$ **b** $P(-2, 7)$ and $Q(4, 0)$

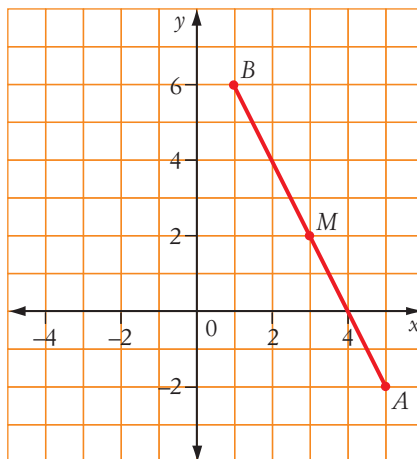
Solution

- a** The x -coordinates are 5 and 1. The average of 5 and 1 $= \frac{5+1}{2} = 3$

The y -coordinates are -2 and 6 . The average of -2 and $6 = \frac{-2+6}{2} = 2$

$\therefore$ The midpoint is the point $(3, 2)$.

The diagram shows that this is true.



- b** The x -coordinates are -2 and 4 . The average of -2 and $4 = \frac{-2+4}{2} = 1$

The y -coordinates are 7 and 0 . The average of 7 and $0 = \frac{7+0}{2} = 3\frac{1}{2}$

$\therefore$ The midpoint is the point $(1, 3\frac{1}{2})$.

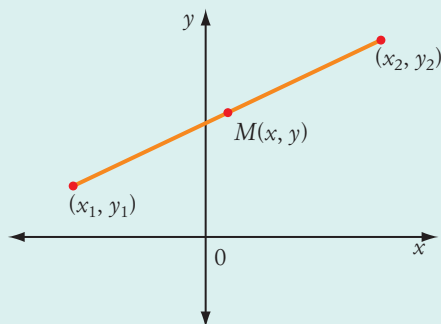
The midpoint formula

The method for finding the midpoint of an interval can be summarised by a formula.

Summary

The midpoint, M , of the interval joining two points (x_1, y_1) and (x_2, y_2) has coordinates:

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$



Stage 5.3

Stage 5.3

Example 5

Use the midpoint formula to find the midpoint of the interval joining $P(12, -8)$ and $Q(7, -2)$.

Solution

For $P(12, -8)$ and $Q(7, -2)$:

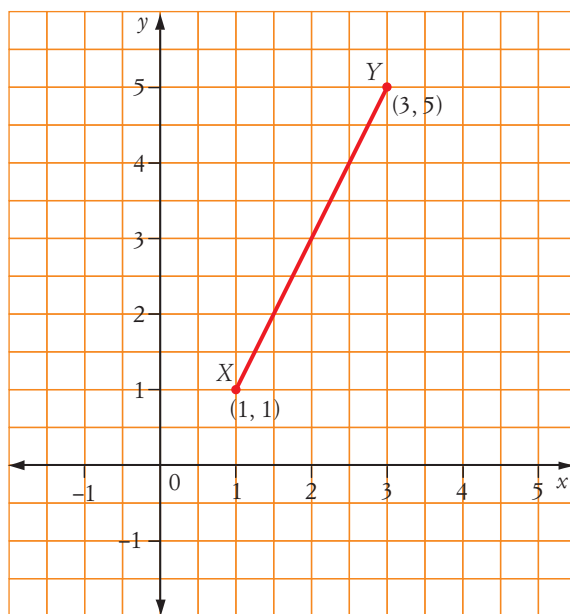
$$x_1 = 12, y_1 = -8, x_2 = 7, y_2 = -2$$

$$\begin{aligned}\text{Midpoint, } M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{12 + 7}{2}, \frac{-8 + (-2)}{2} \right) \\ &= \left(\frac{19}{2}, \frac{-10}{2} \right) \\ &= \left(9\frac{1}{2}, -5 \right)\end{aligned}$$

Group the x -coordinates together, group the y -coordinates together.

Exercise 11-02 The midpoint of an interval

- 1 What is the midpoint of interval XY ? Select the correct answer **A**, **B**, **C** or **D**.



- A** (2, 3) **B** (2, 4) **C** (3, 2) **D** (3, 3)
- 2 On a number plane, plot and join each pair of points, then find and plot the midpoint of each interval.
- a** (1, 3) and (7, 5) **b** (0, 2) and (10, 4) **c** (-3, 3) and (5, -1)

3 Find the midpoint of the interval joining:

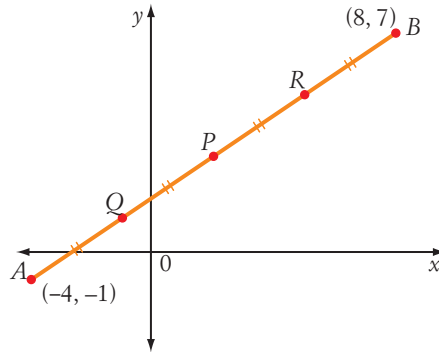
a (10, 10) and (6, 4)	b (2, 7) and (12, 9)	c (0, 5) and (9, 3)
d (2, 2) and (2, 4)	e (-2, 6) and (10, 3)	f (9, -1) and (5, 2)
g (-8, 11) and (9, -4)	h (-6, -1) and (0, 0)	i (3, -3) and (-5, -10)
j (6, -1) and (-8, -1)	k (5, -6) and (9, -10)	l (-8, -7) and (-3, -5)

See Examples 4, 5

4 The interval AB has been subdivided into four equal parts.

Find the coordinates of:

- a P
b Q
c R

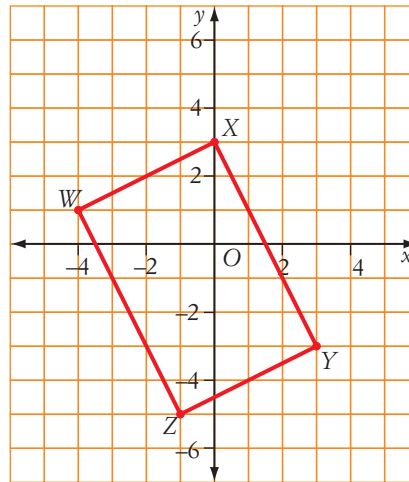


5 What is the midpoint of $K(6, 9)$ and $L(-2, 7)$? Select the correct answer **A**, **B**, **C** or **D**.

- A** (4, 16) **B** (-4, 2) **C** (-2, 1) **D** (2, 8)

6 $WXYZ$ is a rectangle.

- a Find the midpoint of:
i diagonal WY
ii diagonal XZ
- b What do you notice about the 2 midpoints in part a? What property about the diagonals of a rectangle does this show?



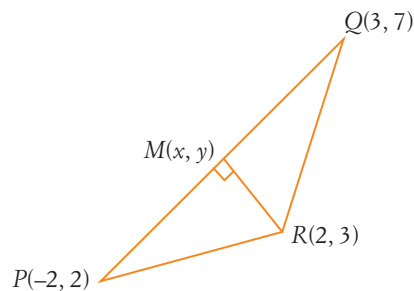
Worked solutions

The midpoint of an interval

MAT09NAWS10048

7 The diagram shows an isosceles triangle, PQR .

- a Find M , the midpoint of PQ .
- b Find, as a surd, the length of:
i PQ ii MR iii QR
- c Calculate, correct to one decimal place
i the area of $\triangle PQR$
ii the perimeter of $\triangle PQR$



- 8 The vertices of a square are $K(1, 6)$, $L(7, 2)$, $M(3, -4)$ and $P(-3, 0)$.
- Find the length of the diagonal KM .
 - Find the length of the diagonal LP .
 - Are the diagonals equal in length?
 - Find the coordinates of the midpoint of KM .
 - Find the coordinates of the midpoint of LP .
 - Do the diagonals bisect each other? Explain.

Worksheet

Drawing gradients

MAT09NAWK10124

Worksheet

A page of intervals

MAT09NAWK10125

Homework sheet

Coordinate geometry 1

MAT09NAHS10015

Worksheet

A page of lines

MAT09NAWK10126

Technology

GeoGebra: Slope, distance and midpoint

MAT09NATC00008

11-03 The gradient of a line

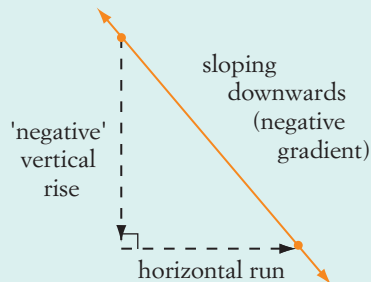
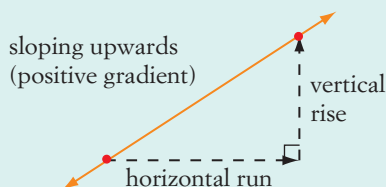
The **gradient** of a line (or interval) is a measure of how steep the line is. It is represented by the variable m and is the ratio (fraction) of the vertical rise to the horizontal run between any two points on a line.

Summary

The gradient of a line or interval

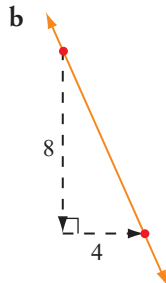
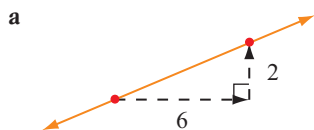
$$m = \frac{\text{vertical rise}}{\text{horizontal run}} = \frac{\text{rise}}{\text{run}}$$

- A line **sloping upwards** has a **positive rise** and a **positive gradient**.
- A line **sloping downwards** has a **negative rise** and a **negative gradient**.
- The **run** is always **positive**.



Example 6

Calculate the gradient of each line.

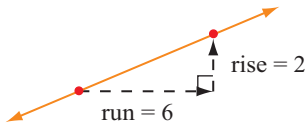


Solution

- a** The line has a positive rise (↑).

$$m = \frac{2}{6} \\ = \frac{1}{3}$$

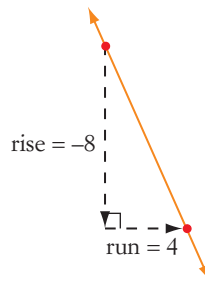
$$m = \frac{\text{rise}}{\text{run}}$$



- b** The line has a negative rise or a drop (↓).

$$m = \frac{-8}{4} \\ = -2$$

$$m = \frac{\text{rise}}{\text{run}}$$



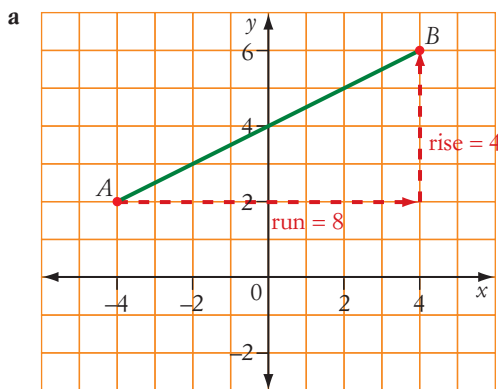
Example 7

Calculate the gradient of the line passing through:

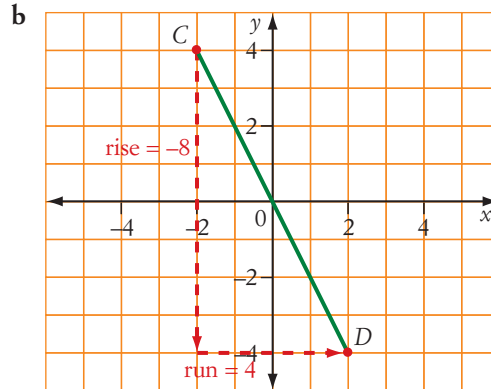
- a** $A(-4, 2)$ and $B(4, 6)$ **b** $C(-2, 4)$ and $D(2, -4)$

Solution

Form a right-angled triangle with the interval as the hypotenuse, as shown.



$$m = \frac{4}{8} \\ = \frac{1}{2}$$



$$m = \frac{-8}{4} \\ = -2$$

Video tutorial

The gradient of a line

MAT09NAVT10019

Video tutorial

Gradient (slope) of a line

MAT09NAVT00010

Worksheet

Gradient between two points

MAT09NAWK00058

Puzzle sheet

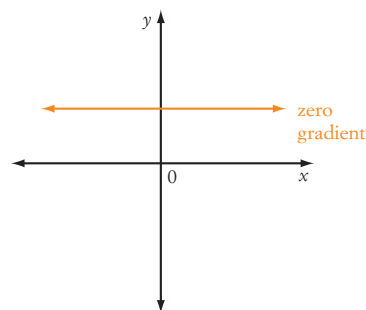
Find the gradient 1

MAT09NAPS00049

The size and sign of the gradient

The **size** of the gradient shows how quickly the line is going up or down: the higher the number, the steeper the line. The **sign** of the gradient shows whether the line is sloping upwards (positive) or downwards (negative) as you look from left to right.

A **horizontal (flat) line** has zero gradient.



Stage 5.3

The gradient formula

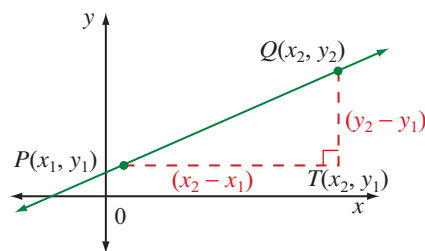
The method for finding the gradient of an interval can be summarised by a formula.

Consider the line passing through the points (x_1, y_1) and (x_2, y_2) .

Vertical rise = difference in y -coordinates = $y_2 - y_1$

Horizontal run = difference in x -coordinates = $x_2 - x_1$

Gradient, $m = \frac{\text{difference in } y}{\text{difference in } x} = \frac{y_2 - y_1}{x_2 - x_1}$



Summary

The gradient (m) of a line or interval joining two points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 8

Use the gradient formula to calculate the gradient of the line going through $A(-4, 2)$ and $B(4, 6)$ (from Example 7a).

Solution

For $A(-4, 2)$ and $B(4, 6)$:

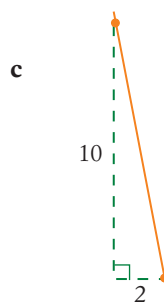
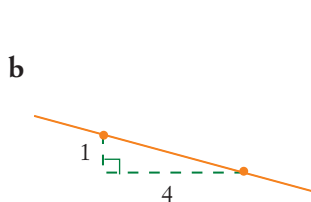
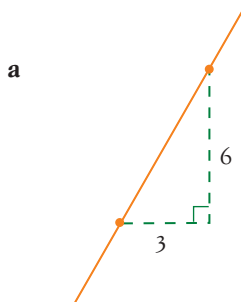
$$x_1 = -4, y_1 = 2, x_2 = 4, y_2 = 6$$

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{6 - 2}{4 - (-4)} \\ &= \frac{4}{8} \\ &= \frac{1}{2} \end{aligned}$$

Group the y -coordinates together **on top**, group the x -coordinates together **below**.

Exercise 11-03 The gradient of a line

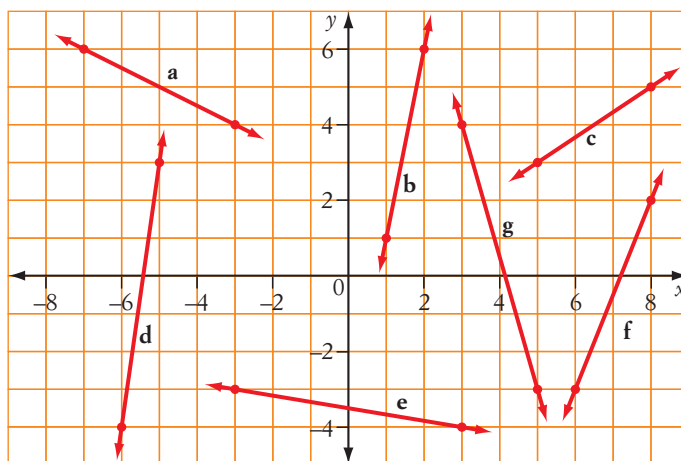
- 1 State whether the gradient of each line is positive (+) or negative (-).



- 2 Calculate the gradient of each line in question 1.
3 Calculate the gradient of each interval.

See Example 6

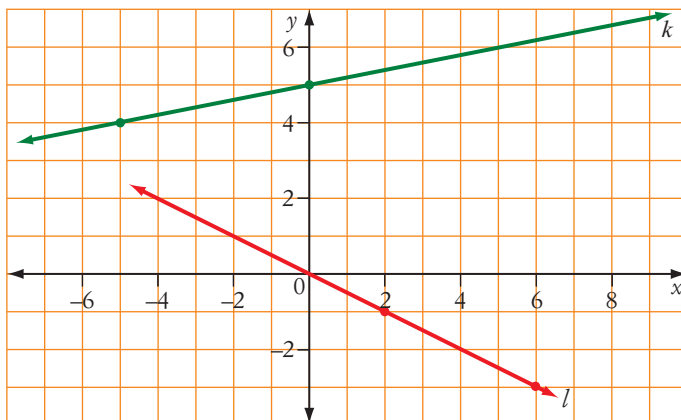
See Example 7



- 4 Which pair of points form an interval that has a gradient of 2? Select the correct answer **A, B, C** or **D**.
A (0, 2) (2, 0) **B** (-5, 2), (5, 7) **C** (-2, -2), (1, 4) **D** (-6, -8), (-2, -6)
- 5 Calculate the gradient of the interval joining each pair of points, either by plotting the points on a number plane or by using the gradient formula.
- | | | |
|---------------------------------|--------------------------------|--------------------------------|
| a A(1, 4) and B(3, 6) | b Z(4, 7) and T(1, 6) | c M(4, -2) and D(3, 3) |
| d A(-2, -1) and P(4, -4) | e D(0, -1) and F(3, -1) | f H(9, -2) and L(-1, 3) |
| g K(4, 9) and M(-4, -5) | h Z(-8, -6) and T(4, 0) | i F(0, 4) and B(3, 4) |
- 6 Which pair of points form an interval that has a gradient of -1? Select **A, B, C** or **D**.
A (1, 2), (2, 1) **B** (-5, 6), (6, 7) **C** (2, 2), (3, -1) **D** (-5, -10), (4, -11)

See Example 8

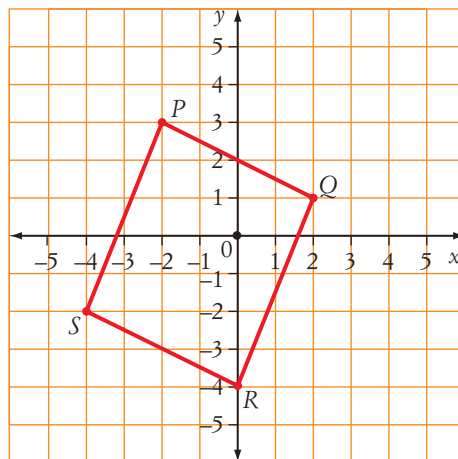
- 7 Find the gradient of the lines labelled k and l .



- 8 Which pair of points lies on a line with a gradient of 0? Select **A**, **B**, **C** or **D**.
A (3, 3) (−3, −3) **B** (−5, 7), (5, 7) **C** (1, −6), (1, 4) **D** (0, 9), (9, 0)

- 9 $PQRS$ is a quadrilateral.

- Find the gradient of each side of $PQRS$.
- Which sides have the same gradient?
What does that mean about those sides?
- What type of quadrilateral is $PQRS$?



- 10 A triangle is formed by joining $P(-8, -2)$, $Q(-2, 10)$ and $R(4, 6)$. T is the midpoint of PQ , and Z is the midpoint of QR .
- Draw a diagram showing all given information.
 - Find the coordinates of T and Z .
 - Calculate the gradient of the interval TZ and the side PR . What do you notice?
 - Calculate the length of the interval TZ and the side PR . What do you notice?

Worked solutions

The gradient of a line

MAT09NAWS10049

Just for the record

Stairs, ramps and hills

- Some stairs are easier to climb than others: the gradient of stairs should not be more than 5 in 6 ($\frac{5}{6}$).
- A ramp for walking should not have a gradient over $\frac{3}{10}$ and the gradient of a wheelchair ramp should be between $\frac{1}{14}$ and $\frac{1}{8}$.
- The maximum slope of a hill for parking your car safely is $\frac{2}{9}$.
- The steepest street in the world is Baldwin Street in Dunedin, New Zealand, with one section having a gradient of 35% or $\frac{7}{20}$.



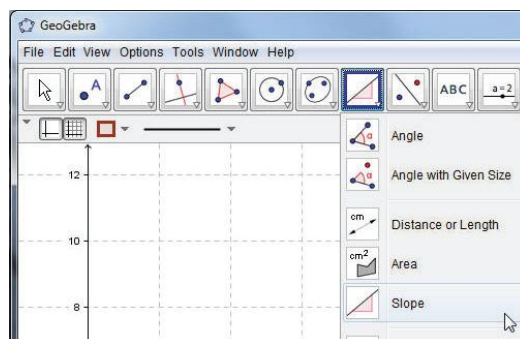
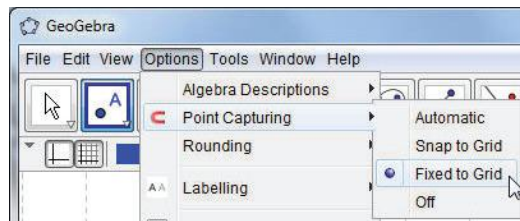
Shutterstock.com/Oleksandr Rozdobukha

Would it be safe to park your car on Bulli Pass, which has a slope of $\frac{1}{6}$?

Technology The gradient of collinear points

In this activity you will use GeoGebra to investigate collinear points.

- Right-click in the Graphics window and make sure **Axes** and **Grid** are enabled.
- In GeoGebra, by pressing **ctrl** and holding your mouse down anywhere on the grid (except on the axes), you can move the axes. Do this until the axes show approximately x values: 0 to 30 and y values: 0 to 16. By pressing **ctrl** and holding your mouse down on the axes, you can move your mouse and adjust the axes scales.
- Select **Options, Point Capturing** and **Fixed to Grid**. This will ensure that you click coordinates with whole numbers only.
- Select **New Point** and plot the points $A(5, 2)$, $B(9, 5)$ and $C(17, 11)$ on the grid. Select **Interval between Two Points** and then click point A and point B . Repeat for points B and C .
- To check if the three points are collinear, select **Slope** and interval AB . Repeat by clicking **Slope** and interval BC . Is the slope (gradient) the same for AB and BC ? If so, these points are called **collinear points**. What do you think **collinear points** are? Discuss with other students and explain in your own words.



6 Now test whether the following sets of points are collinear.

a (0, 3), (2, 4) and (6, 6)

b (8, 0), (−8, 0) and (−6, −10)

c (1, 8), (2, 9) and (4, 10)

d (−2, 8), (−1, 6) and (0, 1)

e (2, 7), (4, 10), (5, 13) and (6, 17)

f (−14, 11), (−8, 9), (1, 6) and (16, 1)

Worksheet

Graphing linear equations

MAT09NAWK10127

Worksheet

A page of intervals

MAT09NAWK10125

Worksheet

A page of lines

MAT09NAWK10126

Worksheet

A page of number planes

MAT09NAWK10159

Worksheet

Number plane grid paper

MAT09NAWK10158

11-04 Graphing linear equations

A relationship between two variables such as x and y whose graph is a **straight line** is called a **linear relationship**. The expression of that relationship as an algebraic formula, such as $y = x + 2$, is called a **linear equation**.

Example 9

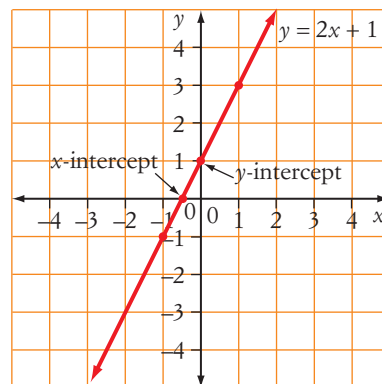
Graph $y = 2x + 1$ on a number plane.

Solution

Complete a table of values. Choose x -values close to 0 for easy calculation and graphing.

x	−1	0	1
y	−1	1	3

Graph the table of values on a number plane. Rule a straight line through the points, place arrows at each end, and label the line with its equation.



Note:

- the **x-intercept** of the line is $-\frac{1}{2}$: this is the x -value where the line cuts the x -axis
- the **y-intercept** of the line is 1: this is the y -value where the line cuts the y -axis
- every point on the line follows the linear equation $y = 2x + 1$: for example, (−4, −7), (−3, −5) and (6, 13) lie on the line and also follow the rule $y = 2x + 1$
- there are an infinite number of points that follow the rule: arrows on both ends of the line indicate that it has infinite length.

Using x - and y -intercepts to graph lines

We can also graph a linear equation by finding its x - and y -intercepts first.

Since any point on the x -axis has a y -coordinate of 0, we can substitute $y = 0$ into the equation to find the x -intercept.

Similarly, any point on the y -axis has an x -coordinate of 0, so we can substitute $x = 0$ into the equation to find the y -intercept.

Summary

- To find the x -intercept, substitute $y = 0$ and solve the equation.
- To find the y -intercept, substitute $x = 0$ and solve the equation.

Example 10

Find the x - and y -intercepts for $y = -4x + 8$ to draw its graph.

Solution

For x -intercept, $y = 0$:

$$0 = -4x + 8$$

$$4x = 8$$

$$x = 2$$

The x -intercept is 2.

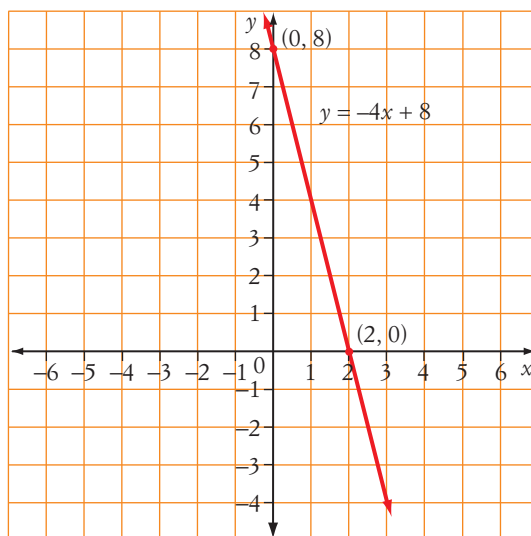
For y -intercept, $x = 0$:

$$y = -4 \times 0 + 8$$

$$y = 8$$

The y -intercept is 8.

Plot both intercepts on the axes, draw a line through the two points and label the line with its equation.



Testing if a point lies on a line

Summary

A point lies on a line if its (x, y) coordinates satisfy the equation of the line.

Example 11

Test whether each point lies on the line $y = 2x + 3$.

a $(3, 9)$

b $(8, -4)$

Solution

- Separate the equation into LHS (left-hand side) and RHS (right-hand side)
- Substitute the coordinates of the point into both sides
- If $\text{LHS} = \text{RHS}$, the point satisfies the equation and lies on the line
- If $\text{LHS} \neq \text{RHS}$, the point does not lie on the line.

a Substitute $x = 3$ and $y = 9$ into $y = 2x + 3$.

$$\begin{array}{rcl} \text{LHS} & = & y \\ & = & 9 \\ \text{RHS} & = & 2x + 3 \\ & = & 2 \times 3 + 3 \\ & = & 9 \end{array}$$

$\text{LHS} = \text{RHS}$ so $(3, 9)$ lies on the line.

b Substitute $x = 8$ and $y = -4$ into $y = 2x + 3$.

$$\begin{array}{rcl} \text{LHS} & = & y \\ & = & -4 \\ \text{RHS} & = & 2x + 3 \\ & = & 2 \times 8 + 3 \\ & = & 19 \end{array}$$

$\text{LHS} \neq \text{RHS}$ so $(8, -4)$ does not lie on the line.

Worksheet

Straight-line equations

MAT09NAWK00059

Technology worksheet

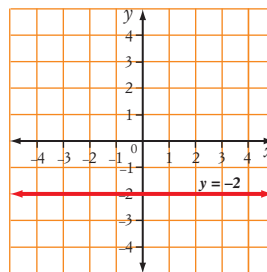
Horizontal and
vertical lines

MAT09NACT10005

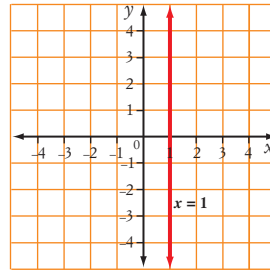
Horizontal and vertical lines

A **horizontal line** is flat and runs across (sideways), parallel to the x -axis.

Every point on the horizontal line above has y -coordinate -2 , and the y -intercept is -2 , so its equation is $y = -2$.



Every point on the vertical line above has x -coordinate 1, and the x -intercept is 1, so its equation is $x = 1$.



Summary

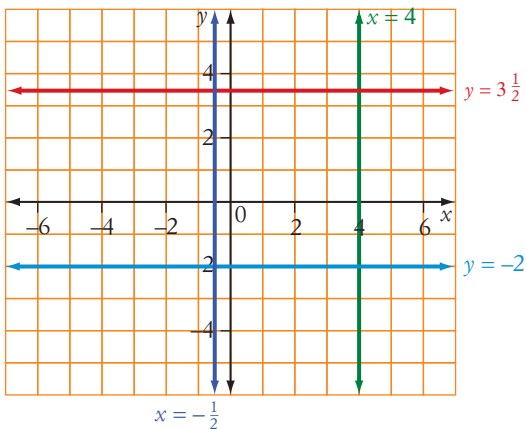
The **equation of a horizontal line** is of the form $y = c$ (where c is a constant number).
The **equation of a vertical line** is of the form $x = c$ (where c is a constant number).

Example 12

Graph each line on the same number plane.

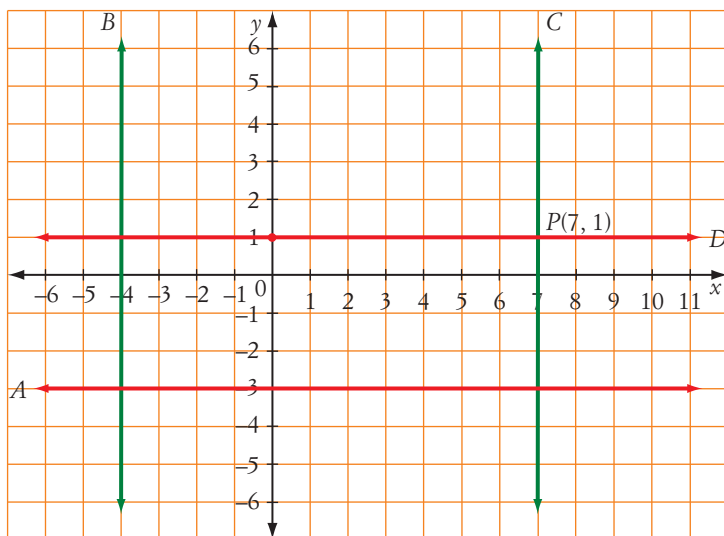
- a $x = 4$ b $y = -2$ c $x = -\frac{1}{2}$ d $y = 3\frac{1}{2}$

Solution



Example 13

Find the equations of lines A , B , C and D .

**Solution**

- Line A is horizontal and has a y -intercept of -3 , so its equation is $y = -3$.
- Line B is vertical and has an x -intercept of -4 , so its equation is $x = -4$.
- Line C is vertical and goes through $(7, 1)$, so its equation is $x = 7$.
- Line D is horizontal and goes through $(7, 1)$, so its equation is $y = 1$.

Exercise 11-04 Graphing linear equations**See Example 9**

1 Graph each linear equation on a number plane, and write:

- i its x -intercept ii its y -intercept.

- a $y = x - 3$ b $y = 2x + 2$ c $y = -x + 5$
 d $y = 3x$ e $y = -x$ f $y = 10 - 4x$
 g $y = \frac{x+2}{2}$ h $y = 2x + 1$ i $y = \frac{x}{3} - 2$

2 What is the x -intercept of the graph of $y = 6 - 3x$? Select the correct answer **A**, **B**, **C** or **D**.

- A** -3 **B** 2 **C** 3 **D** 6

3 Graph each linear equation after finding its x - and y -intercepts.

- a $y = 3x - 6$ b $y = \frac{6-x}{2}$ c $x + y = 7$
 d $x - y = 6$ e $y = \frac{2x}{3} + 1$ f $3x - 4y = 12$
 g $x + y = 4$ h $2x - y = 5$ i $\frac{x}{2} + \frac{y}{3} = 1$

Stage 5.3**See Example 10**

4 Test whether the point $(3, -1)$ lies on each line.

- a $y = 2x - 7$ b $y = -2x + 4$ c $x - y = 4$
d $x + 2y = 1$ e $y = -3x + 8$ f $x - 3y = 0$

See Example 11

5 a Show that the point $(3, 2)$ lies on both the lines $x - 2y = -1$ and $x + y = 5$.

b So at what point do the lines $x - 2y = -1$ and $x + y = 5$ cross?

6 Show that the lines $y = -3x + 6$ and $y = -2x$ intersect at the point $(6, -12)$.

7 Which of these points lies on the line $x - y = -2$? Select the correct answer A, B, C or D.

- A $(1, 1)$ B $(-1, -1)$ C $(0, 2)$ D $(-2, 4)$

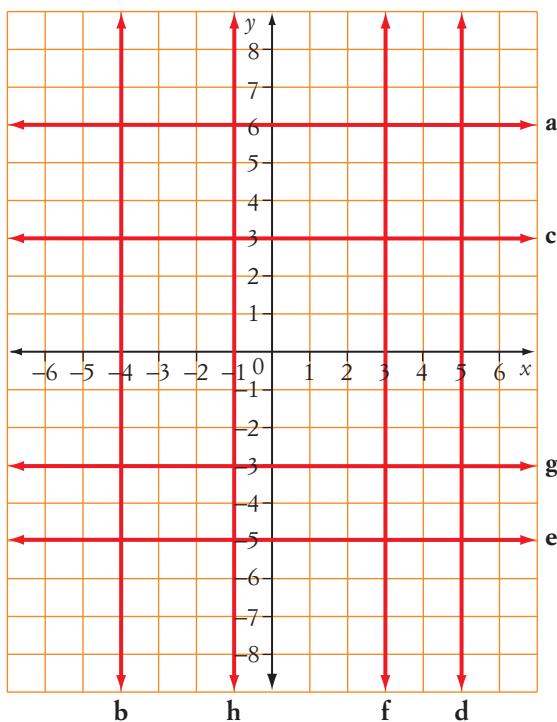
8 Graph each set of lines on a number plane.

- a $x = 2, x = -3$ b $y = 6, y = -2$
c $x = 2\frac{1}{2}, x = 0, y = -5$ d $y = 1.5, y = 3, x = -2$

See Example 12

9 Write down the equation of each line shown below.

See Example 13



10 Find the equation of each line described.

- a A horizontal line that passes through the y -axis at 6.
b A vertical line that goes through $(-5, 2)$.
c Each point on the line is 2 units above the x -axis.
d Each point on the line is 5 units to the left of the y -axis.
e The line is parallel to the x -axis and passes through the point $(0, -2)$.
f A horizontal line that goes through $(4, -1)$.
g The line goes through the points $(-1, 8)$ and $(-1, 2)$.

11 What is the equation of:

- a the y -axis? b the x -axis?

Technology Graphing linear equations

In this activity, we will use GeoGebra to graph a linear equation.

- 1 Right-click in the Graphics window and make sure **Axes** and **Grid** are enabled.
- 2 Select **View, Spreadsheet**. Press **ctrl** and hold your mouse down anywhere on the grid (except on the axes) to can move the axes. Do this until they are centred in the middle of the screen. (Approx.: $x: -10$ to 10 and $y: -10$ to 10).
- 3 Enter the following linear equations in column A of the spreadsheet.

$$A1: y = 3x + 1$$

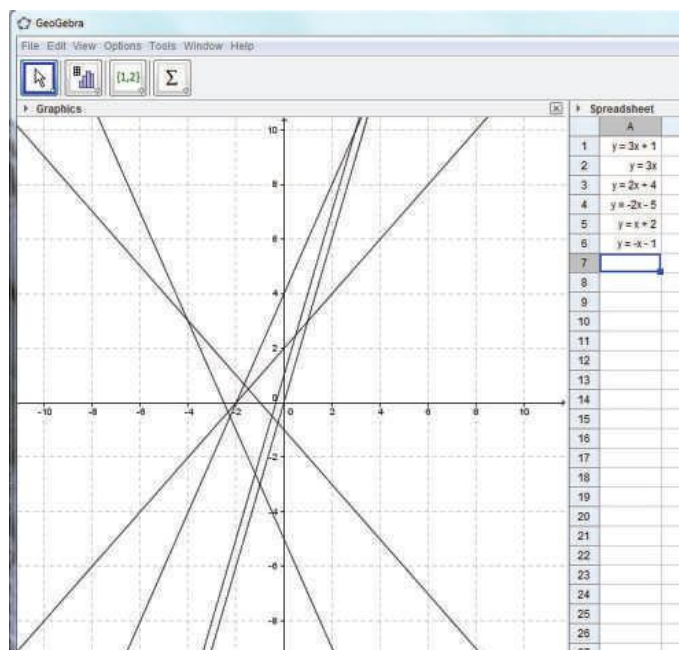
$$A2: y = 3x$$

$$A3: y = 2x + 4$$

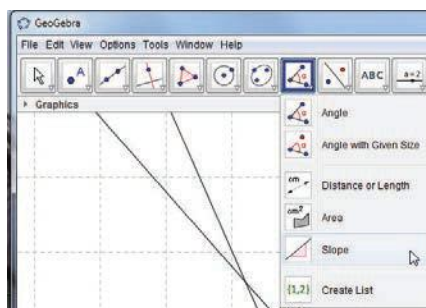
$$A4: y = -2x - 5$$

$$A5: y = x + 2$$

$$A6: y = -x - 1$$



- 4 You can read the gradient of each line by selecting the **Slope tool**. Select a line and you will see the right-angled triangle showing $\frac{\text{rise}}{\text{run}}$.



- 5 Enter each of these linear equations and find the gradient of each line.

$$\mathbf{a} \quad y = x + 1$$

$$\mathbf{b} \quad y = -2x - 4$$

$$\mathbf{c} \quad y = 6x + 3$$

$$\mathbf{d} \quad y = 3x$$

$$\mathbf{e} \quad y = -8x$$

$$\mathbf{f} \quad y = -4x - 6$$

Mental skills 11

Maths without calculators

Divisibility tests

How can you tell if a number is divisible by 2? Look at its last digit. If that digit is 2, 4, 6, 8 or 0, then the number is divisible by 2 (that is, it is even).

How can you tell if a number is divisible by 5? If its last digit is 0 or 5, then the number is divisible by 5.

These are examples of divisibility tests—rules for checking whether a number is divisible by a certain number. The table below shows some common divisibility tests.

A number is divisible by:	if:
2	its last digit is 2, 4, 6, 8 or 0
3	the sum of its digits is divisible by 3
4	its last two digits form a number divisible by 4
5	its last digit is 0 or 5
6	it is even and the sum of its digits is divisible by 3
9	the sum of its digits is divisible by 9
10	its last digit is 0

1 Study each example.

a Test whether 748 is divisible by 2, 3 or 4.

- Last digit is 8 (even). $\therefore$ 748 is divisible by 2.
- Sum of digits = $7 + 4 + 8 = 19$, which is not divisible by 3. $\therefore$ 748 is not divisible by 3.
- 48 is divisible by 4. $\therefore$ 748 is divisible by 4 ($748 \div 4 = 187$).

b Test whether 261 is divisible by 5 or 9.

- Last digit is 1, not 0 or 5. $\therefore$ 261 is not divisible by 5.
- $2 + 6 + 1 = 9$, which is divisible by 9. $\therefore$ 261 is divisible by 9 ($261 \div 9 = 29$).

c Test whether 570 is divisible by 4, 6 or 10.

- 70 is not divisible by 4. $\therefore$ 570 is not divisible by 4.
- 570 is even and $5 + 7 + 0 = 12$, which is divisible by 3. $\therefore$ 570 is divisible by 6 ($570 \div 6 = 95$).
- Last digit is 0. $\therefore$ 570 is divisible by 10 ($570 \div 10 = 57$).

2 Test whether each number is divisible by 2, 3, 5 or 6.

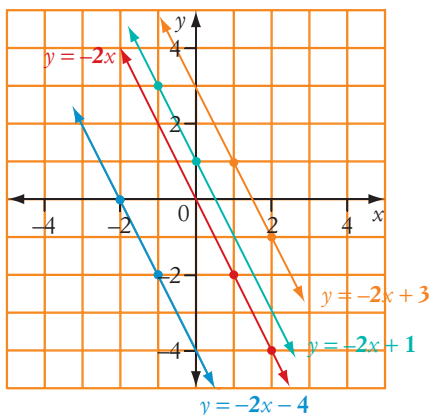
a 250	b 189	c 78	d 465
e 1024	f 840	g 715	h 627

3 Test whether each number is divisible by 4, 9 or 10.

a 144	b 280	c 522	d 4170
e 936	f 726	g 342	h 5580

Investigation: Comparing gradients and y-intercepts

- 1 This number plane shows the graphs of four linear equations.



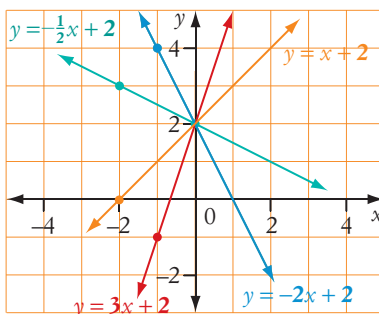
- a Copy and complete this table by finding the gradient and y-intercept of each line.

Equation	Gradient	y-intercept
$y = -2x - 4$	-2	-4
$y = -2x$		
$y = -2x + 1$		
$y = -2x + 3$		

- b How are the four lines:

- i similar? ii different?

- 2 This number plane shows the graphs of another four linear equations.



- a Copy and complete this table.

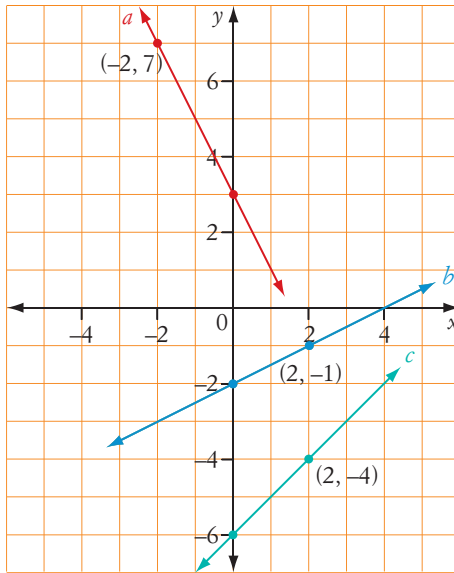
Equation	Gradient	y-intercept
$y = 2x + 2$		
$y = -\frac{1}{2}x + 2$		
$y = x + 2$		
$y = 3x + 2$		

- b How are the four lines:

- i similar? ii different?

3 Copy and complete this table for the three lines graphed below.

Equation	Gradient	y-intercept
$y = -2x + 3$		
$y = \frac{1}{2}x - 2$		
$y = x - 6$		



- b** Explain how the values for the gradient and y-intercept of each line relate to its equation.
c What is the equation of a line that has a gradient of 4 and a y-intercept of -1 ?
d What is the gradient and y-intercept of the line with equation $y = -x + 7$?

11-05 The gradient–intercept equation $y = mx + b$

NSW

Puzzle sheet

Equation of a line

MAT09NAPS00046

Summary

The equation of a straight line is $y = mx + b$, where m is the gradient and b is the y-intercept.
 For this reason, $y = mx + b$ is also called the **gradient–intercept form** of a linear equation.

Example 14

Find the gradient and y -intercept of the line with equation:

a $y = 4x - 7$ **b** $y = \frac{1}{2}x + 3$ **c** $y = 3 - 5x$ **d** $y = \frac{3x}{4} - 1$

Solution

a $y = 4x - 7$ is in the form $y = mx + b$

$\therefore$ Gradient $m = 4$ and y -intercept $b = -7$

b For $y = \frac{1}{2}x + 3$, gradient $m = \frac{1}{2}$ and y -intercept $b = 3$

c $y = 3 - 5x$ can be rewritten as $y = -5x + 3$

$\therefore$ Gradient $m = -5$ and y -intercept $b = 3$

For $y = \frac{3x}{4} - 1$, gradient $m = \frac{3}{4}$ and y -intercept $b = -1$

Example 15

Write the equation of the line that has a gradient of -4 and a y -intercept of -6 .

Solution

$m = -4$, $b = -6$

$\therefore$ The equation of the line is $y = -4x - 6$.

Example 16

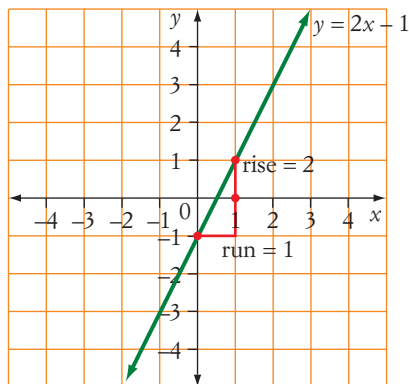
Graph each linear equation by finding the gradient and y -intercept first.

a $y = 2x - 1$ **b** $y = -2x + 5$ **c** $y = \frac{3}{4}x - 2$

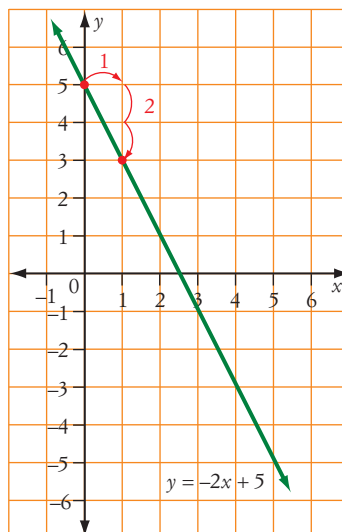
Solution

a $y = 2x - 1$ has a gradient of 2 and a y -intercept of -1 .

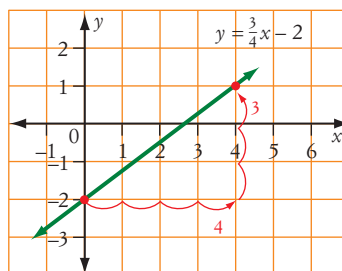
- Plot the y -intercept -1 on the y -axis.
- Make a gradient of 2 by moving **across** 1 unit (**run**) and **up** 2 units (**rise**) and marking the point at $(1, 1)$.
- Rule a line through this point and the y -intercept.
- Label the line with the equation $y = 2x - 1$



- b $y = -2x + 5$ has a gradient of -2 and a y -intercept of 5 .
- Plot the y -intercept 5 on the y -axis.
 - Make a gradient of -2 by moving **across** 1 unit (**run**) and **down** 2 units ('**negative**' **rise**) and marking the point at $(1, 3)$.
 - Rule a line through this point and the y -intercept.



- c $y = \frac{3}{4}x - 2$ has a gradient of $\frac{3}{4}$ and a y -intercept of -2 .
- Plot the y -intercept -2 on the y -axis.
 - Make a gradient of $\frac{3}{4}$ by moving **across** 4 units (**run**) and **up** 3 units (**rise**) and marking the point at $(4, 1)$.
 - Rule a line through this point and the y -intercept.



Exercise 11-05 The gradient-intercept equation $y = mx + b$

- Find the gradient and y -intercept of the line with equation:

a $y = 3x - 2$	b $y = 2x + 7$	c $y = x + 4$	d $y = -x - 9$
e $y = \frac{3x}{4} + 6$	f $y = x$	g $y = \frac{x}{2} - 11$	h $y = \frac{2}{3}x + 6$
i $y = 8 - 2x$	j $y = 5 - x$	k $y = \frac{x}{4}$	l $y = -3x + 11$
m $y = 5$	n $y = \frac{2}{5}x + 1$	o $y = x - \frac{7}{2}$	p $y = \frac{3x}{4} + 1$

See Example 14

- Which equation has a gradient of 5 ? Select the correct answer **A**, **B**, **C** or **D**.
A $y = x + 5$ **B** $y = 5 - x$ **C** $y = 5$ **D** $y = 5x$

- Write the equation of the line with:

a a gradient of 2 and a y -intercept of -1	b a gradient of $\frac{3}{4}$ and a y -intercept of 2
c a gradient of -3 and a y -intercept of 5	d a gradient of $-\frac{2}{5}$ and a y -intercept of 3

See Example 15

- Which equation has a y -intercept of -1 ? Select the correct answer **A**, **B**, **C** or **D**.
A $y = 1 - x$ **B** $y = 1 + x$ **C** $y = x - 1$ **D** $y = -x$

- Graph each linear equation by finding the gradient and y -intercept first.

a $y = 2x + 1$	b $y = x - 4$	c $y = 3x + 2$	d $y = -4x$
e $y = -3x - 3$	f $y = \frac{x}{2} + 2$	g $y = 8 - \frac{3x}{4}$	h $y = \frac{5x}{2} - 1$

See Example 16

NSW

11-06 Finding the equation of a line

Video tutorial

Finding the equation
of a line

MAT09NAVT10020

Worksheet

Finding the equation
of a line

MAT09NAWK10128

Worksheet

A page of intervals

MAT09NAWK10125

Worksheet

A page of lines

MAT09NAWK10126

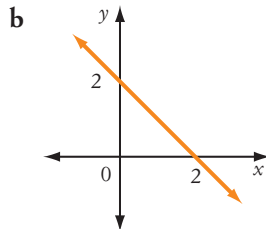
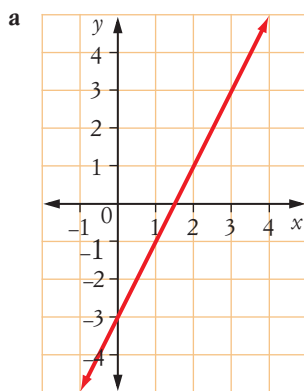
Puzzle sheet

Straight line equations

MAT09NAPS00045

Example 17

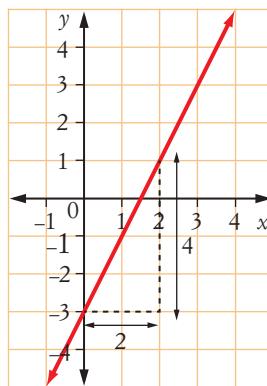
Find the equation of each line.



Solution

- a Select two points on the line to find the gradient, say $(0, -3)$ and $(2, 1)$.

$$\text{Gradient } m = \frac{\text{rise}}{\text{run}} = \frac{4}{2} = 2$$



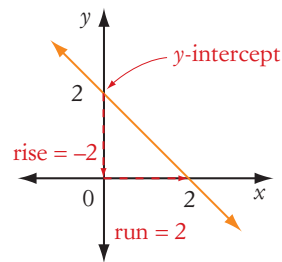
y-intercept $b = -3$ from graph

$\therefore$ The equation of the line is $y = 2x - 3$ $y = mx + b$

We can check that this equation is correct for another point, say $(2, 1)$.

When $x = 2$, $y = 2 \times 2 - 3 = 1$

- b Find the gradient of the line passing through $(0, 2)$ and $(2, 0)$.



$$\text{Gradient } m = \frac{\text{rise}}{\text{run}} = \frac{-2}{2} = -1$$

y-intercept $b = 2$

$\therefore$ The equation of the line is $y = -x + 2$

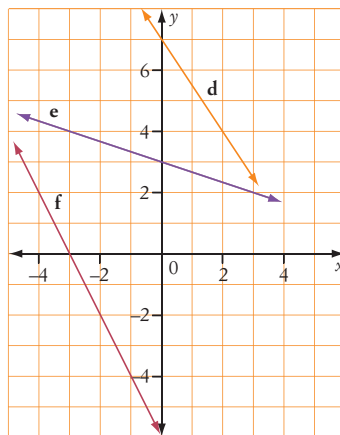
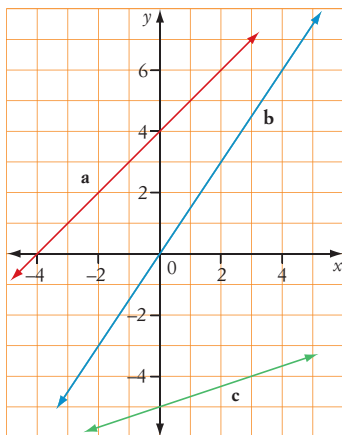
from graph

$y = mx + b$

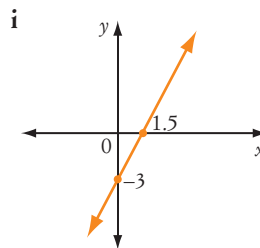
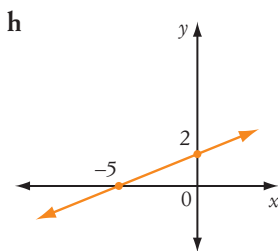
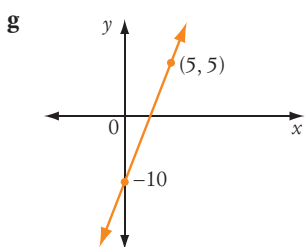
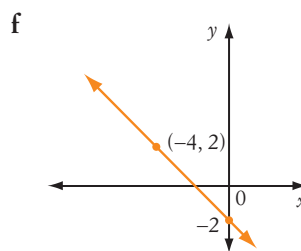
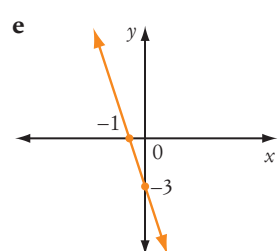
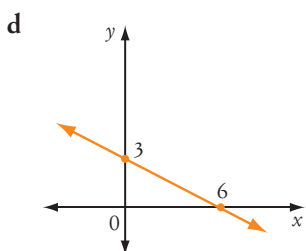
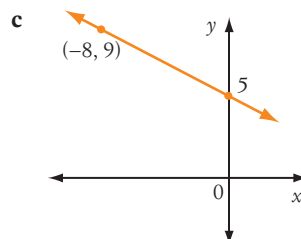
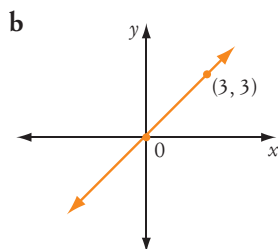
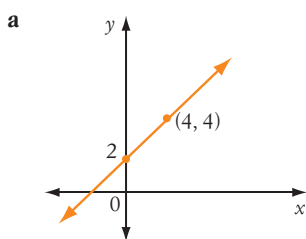
Exercise 11-06 Finding the equation of a line

1 Find the equation of each line.

See Example 17



2 Find the equation of each line.



11-07 Solving linear equations graphically

In Chapter 7 *Equations*, we solved equations **algebraically** using the balancing and backtracking methods. However, we can also solve an equation **graphically** by first graphing it on the number plane.

Example 18

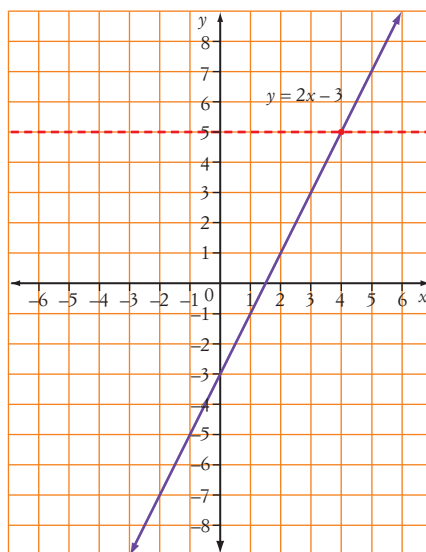
Solve the equation $2x - 3 = 5$ graphically.

Solution

Graph $y = 2x - 3$ (the LHS of the equation) on a number plane.

For every point on the line, the y -coordinate is the value of $2x - 3$. So to solve $2x - 3 = 5$, we need to find the point whose y -coordinate is 5.

Draw a dotted horizontal line at $y = 5$ (shown in red on the diagram) and read off the coordinates of the point on $y = 2x - 3$ crossed by this line.



This point is (4, 5), which means the solution to $2x - 3 = 5$ is $x = 4$.

Check: $2 \times 4 - 3 = 5$

Exercise 11-07 Solving linear equations graphically

This exercise is best completed using *GeoGebra*, a graphics calculator or other graphing software.

See Example 18

- Use the graph in Example 18 above to solve each equation. Check your solutions.
 - $2x - 3 = 3$
 - $2x - 3 = -7$
 - $2x - 3 = 0$
- Graph $y = 2x + 1$ and use it to solve each equation graphically.
 - $2x + 1 = 7$
 - $2x + 1 = 10$
 - $2x + 1 = -5$
- Solve each equation in question 2 algebraically.
- At which point do the graphs of $y = 2x + 3$ and $y = 1$ intersect? Select the correct answer A, B, C or D.

A (2, 1) B (-1, 1) C (4, -2) D (8, -4)

- 5 Graph $y = 3x - 2$ and use it to solve each equation graphically.
 a $3x - 2 = 7$ b $3x - 2 = -11$ c $3x - 2 = -2$
- 6 Graph $y = -x + 1$ and use it to solve each equation graphically.
 a $-x + 1 = -3$ b $-x + 1 = 4$ c $-x + 1 = -1$
- 7 Graph $y = -2x - 1$ and use it to solve each equation graphically.
 a $-2x - 1 = -9$ b $-2x - 1 = 0$ c $-2x - 1 = 3$
- 8 Graph $y = \frac{1}{2}x + 3$ and use it to solve each equation graphically.
 a $\frac{1}{2}x + 3 = 5$ b $\frac{1}{2}x + 3 = 1$ c $\frac{1}{2}x + 3 = 2$

11-08 Direct proportion

Homework sheet

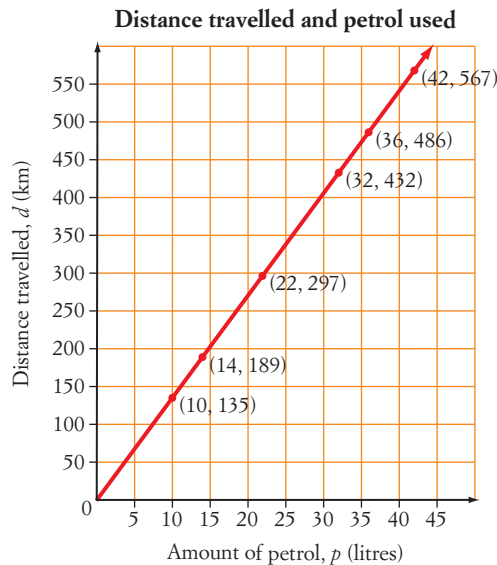
Coordinate geometry 2

MAT09NAHS10016

Ann-Marie noticed a relationship between the amount of petrol used by her car and the distance travelled by the car, as shown in the table of values.

Amount of petrol used (L)	10	14	22	32	36	42
Distance travelled (km)	135	189	297	432	486	567

When she graphed these values, she found a linear relationship.



If p stands for the amount of petrol and d stands for the distance travelled, then it can be shown that d is directly related to p by the formula $d = 13.5p$. This means that the distance travelled is a constant multiple of the amount of petrol used. This is an example of **direct proportion** (or **direct variation**).

Two variables are **directly proportional** to each other if one variable is a constant multiple of the other, and when one variable changes, the other one changes by the same factor.

Summary

If y is **directly proportional** to x , then $y = kx$ where k is a constant (number) called the **constant of proportionality** or **constant of variation**.

- A direct linear relationship exists between x and y .
- If x increases (or decreases), y increases (or decreases).
- If x is doubled (or halved), y is doubled (or halved).
- Another way of saying 'y is directly proportional to x' is y **varies directly** with x'.
- The graph of direct proportion is a straight line going through (0, 0) with gradient k .

Example 19

Video tutorial

Direct proportion

MAT09NAVT10021

The distance (d) in metres travelled by a car is directly proportional to the number of rotations (r) of its tyres. After 540 rotations, a distance of 950 m is travelled.

- What distance (to the nearest metre) will be travelled after 800 rotations?
- How many full rotations will be needed to cover 360 km?

Solution

- d is directly proportional to r .

$$\therefore d = kr$$

To find k , substitute the information given for r and d .

When $r = 540$ and $d = 950$,

$$950 = k \times 540$$

$$\begin{aligned} k &= \frac{950}{540} \\ &= 1.759 \dots \end{aligned}$$

$$\therefore d = 1.759 \dots r$$

When $r = 800$,

$$\begin{aligned} d &= 1.759 \dots \times 800 \\ &= 1407.4074 \dots \\ &\approx 1407 \text{ m} \end{aligned}$$

After 800 rotations, the distance travelled will be 1407 m.

- When $d = 360$,

$$360 = 1.759 \dots \times r$$

$$\begin{aligned} r &= \frac{360}{1.759 \dots} \\ &= 204.661 \dots \\ &\approx 205 \text{ rotations} \end{aligned}$$

For a distance of 360 km, there will be 205 rotations.

Summary

To solve a direct proportion problem:

- 1 identify the two variables (say x and y) and form a proportion equation, $y = kx$
- 2 substitute values for x and y to find k , the constant of proportionality
- 3 rewrite $y = kx$ using the value of k
- 4 substitute a value for x or y into $y = kx$ to solve the problem.

Exercise 11-08 Direct proportion

- 1 The stretch, S cm, of a spring is directly proportional to the mass, M kg, pulling on it.

See Example 19

Mass, M (kg)	0	8	12	20	24	40
Stretch, S (cm)	0	5	7.5	12.5	15	25

- a Graph the relationship between S and M .
 - b Find the equation for S in terms of M .
 - c Find the stretch caused by a mass of 36 kg.
 - d What is the mass that will cause a stretch of 40 cm?
- 2 Y is directly proportional to X . If when $X = 15$, $Y = 75$, find Y when $X = 23$.
- 3 B is directly proportional to h . If when $h = 5$, $B = 16$, what is the constant of proportionality in $B = kh$? Select the correct answer **A**, **B**, **C** or **D**.
- A** 80 **B** 11 **C** 3.2 **D** 0.3125
- 4 The mass, M , in grams of a chemical is directly proportional to its volume, V , in cm^3 .
- a Write the formula for M in terms of V , given that $M = 140$ when $V = 80$.
 - b Calculate the mass of 406 cm^3 of the chemical.
 - c Calculate (correct to 1 decimal place) the volume of the chemical if the mass is:
 - i 28 g
 - ii 0.36 kg
- 5 Find the linear formula for n in terms of m for this table of values.

m	3	6	9	12	15
n	7.5	15	22.5	30	37.5

- 6 The amount of interest, I , earned on an investment account varies directly with the size of the deposit, D .
- a If Hannah earns \$35 interest on an investment of \$1400, write an equation relating I to D .
 - b How much will she earn on an investment of \$2500?
 - c If Hannah doubles the size of her investment in **b**, how much will she earn in interest?
- 7 p varies directly with q . If $q = 7$ when $p = 42$, find p when $q = 18$.
- 8 The distance, D , travelled by Pete, a marathon runner, varies directly with time, t . If Pete runs 8.25 km in 44 minutes:
- a how far will he run in one hour?
 - b how long will it take him to run 24 km?

Worked solutions

Direct proportion

MAT09NAWS10050

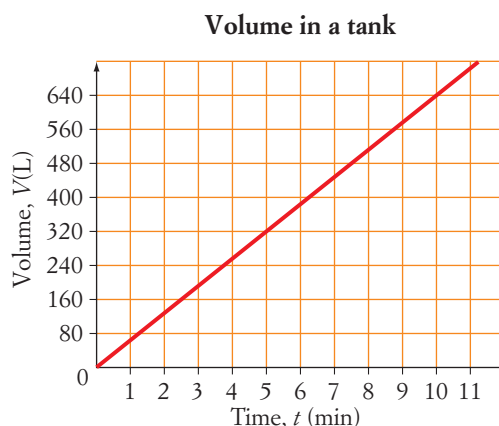
- 9 For an object that is cooling, the drop in temperature varies directly with time. If the temperature drops 8°C in 5 minutes, how long would it take to drop 10°C ?

Select the correct answer **A**, **B**, **C** or **D**.

- A** 6.25 min **B** 7 min **C** 12.8 min **D** 16 min

- 10 The volume V (in litres) of water in a tank being filled is proportional to the time taken t (in minutes) as shown on the graph.

- a** Find the formula for V .
b Calculate the amount of water in the tank after 8 minutes.
c Calculate how long it takes to pump 960 L of water.



- 11 The weight of an astronaut on Mars is proportional to his weight on Earth. A 72 kg astronaut weighs 27.4 kg on Mars.
a Calculate how much a 60 kg astronaut weighs on Mars, correct to 1 decimal place.
b If an astronaut weighs 32 kg on Mars, calculate his weight on Earth, correct to 1 decimal place.
- 12 The pressure experienced by a scuba diver is directly proportional to her depth under the water. The pressure at 25 m is 147 kilopascals (kPa).
a What pressure is experienced at 40 m?
b At what depth correct to one decimal place is the pressure 833 kPa?

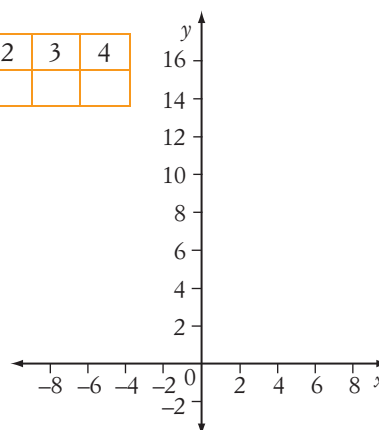
Investigation: Graphing $y = x^2$ and $y = ax^2$

You will need: 1 cm grid paper. This activity can also be completed using a graphics calculator or graphing software.

- 1 Copy and complete this table of values for $y = x^2$

x	-4	-3	-2	-1	-0.5	0	0.5	1	2	3	4
y											

- 2 Draw this number plane on grid paper.
 3 Plot the points from the table of values onto the number plane.
 4 Draw a smooth curve through all the points and label the graph $y = x^2$.
 5 Compare your graph with those of other students.
 6 List as many features as you can about the graph of $y = x^2$ that you have drawn. Compare your list with those of other students.



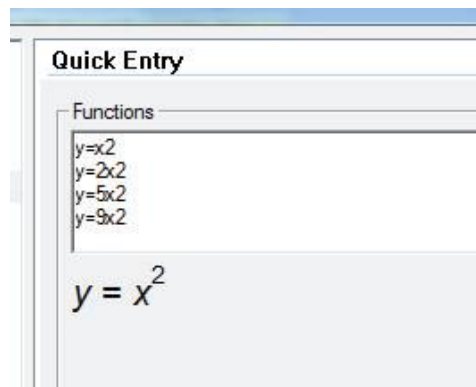
- 7 Copy and complete this table of values for $y = 2x^2$

x	-3	-2	-1	0	1	2	3
y							

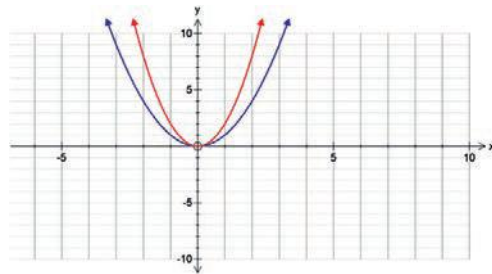
- 8 Graph $y = 2x^2$ on the same number plane as $y = x^2$ and label it.
- 9 How is the graph of $y = 2x^2$ different from the graph of $y = x^2$? Give an explanation for the difference.
- 10 On the same set of axes, draw these graphs:
 - a $y = x^2$
 - b $y = \frac{1}{2}x^2$
 - c $y = \frac{1}{3}x^2$
- 11 What do you notice about the graphs you have drawn? Give an explanation.
- 12 Compare your observations and explanation with those of other students.

Technology Graphing $y = ax^2$

Use graphing software such as Fx-Graph or GeoGebra to compare the graphs of $y = ax^2$

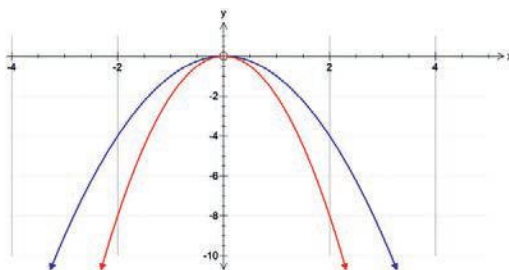
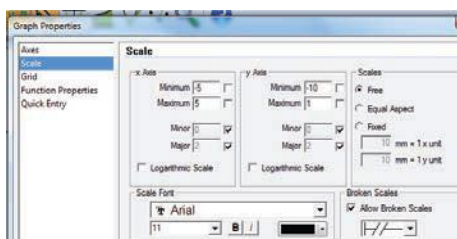


- 1 Using Fx-Graph, enter $y = x^2$ as **y=x2** (as shown below). Then enter the three other equations **y=2x2**, **y=5x2** and **y=9x2**
- 2 The graphs of $y = x^2$ and $y = 2x^2$ are shown. Move the mouse over one of the graphs and its equation should appear.



- 3 For the four graphs you did:
 - a which curve is the widest?
 - b which curve is the narrowest?
 - c which curves had arrows pointing up?
 - d what is their common vertex?
 - e what is their axis of symmetry?
- 4 Enter $y = -x^2$ as **y=-x2**. Then enter the three other equations **y=-2x2**, **y=-3x2** and **y=-5x2**

- 5 Set the scale as shown. The first two graphs are shown below.



- 6 For the four graphs you graphed:
- which curve is the widest?
 - which curve is the narrowest?
 - are they pointing up or down?
 - what is their common vertex?
 - what is their axis of symmetry?
- 7 Enter $y = x^2$ as $y = x^2$. Then enter the three other equations $y = x^2 + 1$, $y = x^2 + 3$ and $y = x^2 + 9$
- 8 Set an appropriate scale.
- 9 For the four parabolas you graphed:
- which curve is the widest?
 - which curve is the narrowest?
 - are they pointing up or down?
 - what is their vertex?
 - what is their axis of symmetry?
- 10 Graph each set of curves.

Quick Entry	
Functions	
$y = x^2$	
$y = x^2 - 1$	
$y = x^2 - 6$	
$y = x^2 - 11$	

Quick Entry	
Functions	
$y = 5x^2 + 1$	
$y = 2x^2 + 1$	
$y = -0.5x^2 + 1$	
$y = -0.8x^2 + 1$	

Quick Entry	
Functions	
$y = -3x^2 - 3$	
$y = -4x^2 - 3$	
$y = x^2 - 3$	
$y = -6x^2 - 3$	

Quick Entry	
Functions	
$y = 0.2x^2$	
$y = 0.6x^2 + 1$	
$y = 0.75x^2 - 1$	
$y = -0.9x^2 - 5$	

Investigation: Graphing $y = ax^2 + c$

You will need: 1 cm grid paper. This activity can also be completed using a graphics calculator or graphing software.

- Graph $y = x^2$ on a number plane for values of x from -3 to 3 .
- Copy and complete this table of values for $y = x^2 + 2$

x	-3	-2	-1	0	1	2	3
y							

- Graph $y = x^2 + 2$ on the same number plane as $y = x^2$ and label it.
- What do the graphs of $y = x^2$ and $y = x^2 + 2$ have in common?
- How are the two graphs different?
- Describe how the graph of $y = x^2 + 2$ could be drawn using the graph of $y = x^2$.

- 7 Graph each equation on the same number plane. Label your graphs clearly.
a $y = x^2 - 1$ **b** $y = x^2 + 4$ **c** $y = x^2 - 3$
- 8 Write a sentence to describe the effect of the constant term (which is added to, or subtracted from, x^2) on the shape and position of the parabola $y = x^2$
- 9 Graph each equation on the same number plane. Label your graphs clearly.
a $y = -x^2$ **b** $y = -x^2 + 2$ **c** $y = -x^2 - 3$ **d** $y = -x^2 - 1$
- 10 What do you notice about the shape and position of each of the graphs in question 9? Compare your results with those of other students.

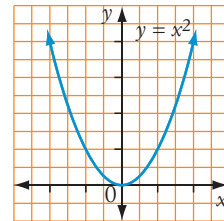
11-09 Graphing quadratic equations

In Chapter 7 *Equations*, we learnt that an equation in which the highest power of the variable is 2 is called a **quadratic equation**, for example, $y = 2x^2 - 5$, $y = x^2 + 7x + 12$ and $y = -5x^2$.

The graph of $y = x^2$

The graph of $y = x^2$ is a smooth U-shaped curve called a **parabola**.

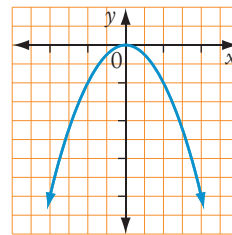
- The axis of symmetry, called the **axis of the parabola**, is the y -axis.
- The **vertex** or **turning point** is $(0, 0)$.
- Because x^2 is always positive or zero, the minimum value of the parabola is 0.
- The parabola's shape is **concave up** (looks like a smile 😊).



The graph of $y = -x^2$

The graph of $y = -x^2$ is the parabola $y = x^2$ reflected in the x -axis, because $-x^2$ is the 'negative' of x^2 .

- The **axis of the parabola** is the y -axis.
- The **vertex** or **turning point** is $(0, 0)$.
- Because $-x^2$ is always negative or zero, the maximum value of the parabola is 0.
- The parabola's shape is **concave down** (looks like a frown ☹️).



Worksheet

Graphing parabolas

MAT09NAWK10129

Technology worksheet

Excel worksheet:
Curve sketcher

MAT09NACT00028

Technology worksheet

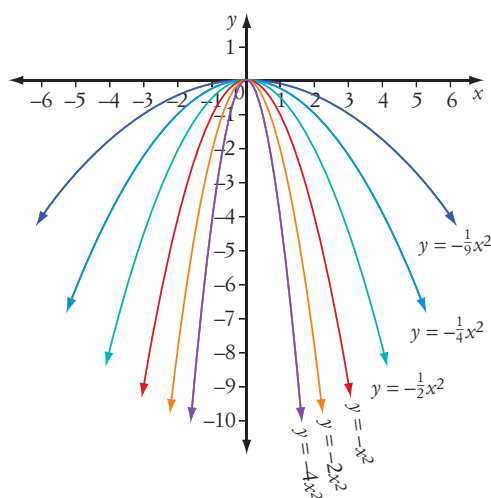
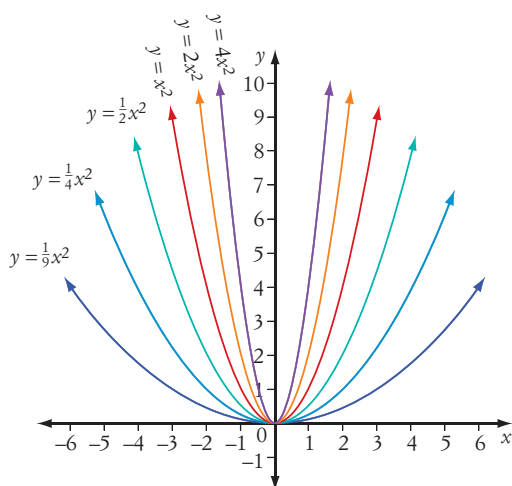
Excel skillsheet:
Curve sketcher

MAT09NACT00013

The graph of $y = ax^2$

For the graph of a quadratic equation in the form $y = ax^2$, where a is a constant (number), the size of a affects whether the parabola is 'wide' or 'narrow'.

As the size of a increases, the parabola becomes 'narrower' and as the size of a decreases, the parabola 'widens'. If a is negative, then the parabola is concave down.



Example 20

- Graph $y = -x^2$ and $y = -2x^2$ on the same set of axes.
- What is the vertex of each parabola?
- Are the parabolas concave up or concave down?
- At what point do the two parabolas meet?
- Which parabola is 'wider'?

Solution

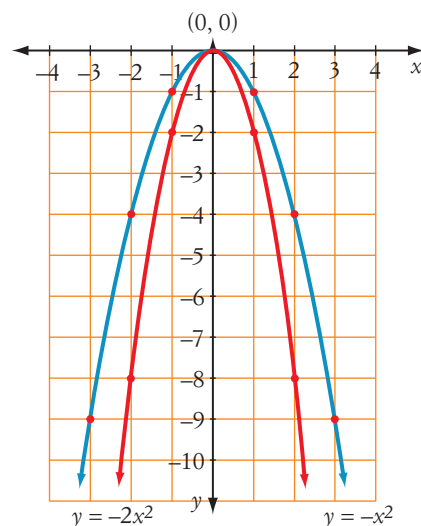
a $y = -x^2$

x	-3	-2	-1	0	1	2	3
y	-9	-4	-1	0	-1	-4	-9

$y = -2x^2$

x	-3	-2	-1	0	1	2	3
y	-27	-8	-2	0	-2	-8	-27

- The vertex of both parabolas is $(0, 0)$.
- The parabolas are both concave down.
- The parabolas meet at $(0, 0)$.
- The parabola $y = -x^2$ is wider.



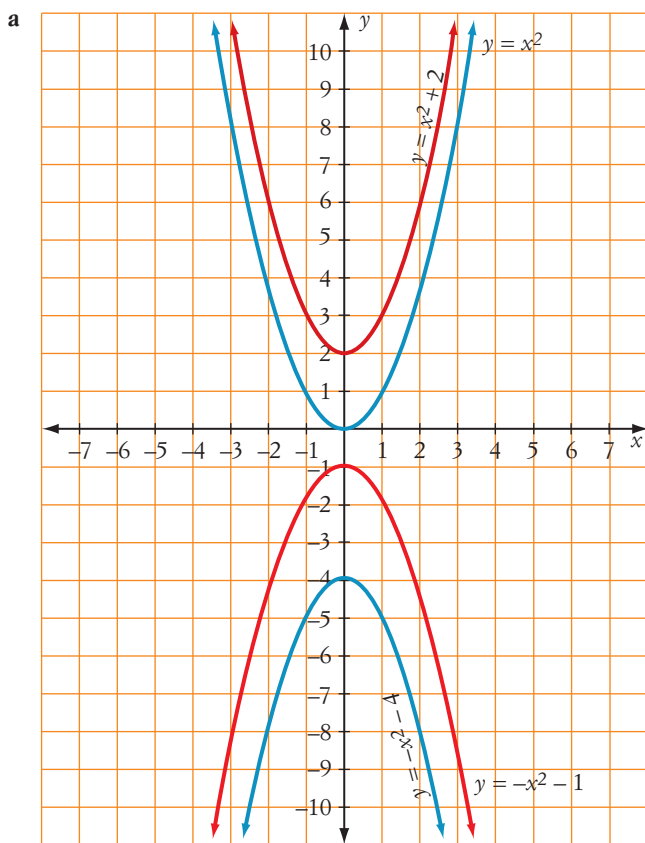
The graph of $y = ax^2 + c$

For the graph of a quadratic equation in the form $y = ax^2 + c$, where a and c are constants, the effect of c is to move the parabola up or down from the origin. Also, c is the y -intercept of the parabola.

Example 21

- Graph $y = x^2$, $y = x^2 + 2$, $y = -x^2 - 1$ and $y = -x^2 - 4$ on the same set of axes.
- Compare each parabola with the parabola $y = x^2$
- What is the vertex of each parabola?
- Is each parabola concave up or concave down?

Solution



- $y = x^2 + 2$ is $y = x^2$ shifted up 2 units.
 $y = -x^2 - 1$ is $y = x^2$ turned upside down and shifted down 1 unit.
 $y = -x^2 - 4$ is $y = x^2$ turned upside down and shifted down 4 units.
- $y = x^2$ has vertex $(0, 0)$.
 $y = x^2 + 2$ has vertex $(0, 2)$.
 $y = -x^2 - 1$ has vertex $(0, -1)$.
 $y = -x^2 - 4$ has vertex $(0, -4)$.
- $y = x^2$ and $y = x^2 + 2$ are concave up.
 $y = -x^2 - 1$ and $y = -x^2 - 4$ are concave down.

Exercise 11-09 Graphing quadratic equations

The questions in this exercise may also be completed using a graphics calculator or graphing software.

See Example 20 1 a Graph $y = 2x^2$ on a number plane after copying and completing this table of values.

x	-2	-1	0	1	2
y					

b Graph $y = \frac{1}{2}x^2$ on the same number plane after copying and completing this table of values.

x	-2	-1	0	1	2
y					

c Compare both parabolas. What features are the same? What is different?

2 Which statement is correct about the parabola for $y = x^2$? Select the correct answer **A**, **B**, **C** or **D**.

A It has a y -intercept of 1

B It is symmetrical about the x -axis

C It is concave down

D Its vertex is (0, 0)

3 a Graph $y = -3x^2$ and $y = -0.1x^2$ on the same number plane after copying and completing this table of values for each equation.

x	-2	-1	0	1	2
y					

b Compare both parabolas. What features are the same? What is different?

4 Graph each pair of parabolas on the same set of axes and state:

i the vertex of each parabola

ii whether each parabola is concave up or concave down

iii which parabola is 'wider'

iv the point at which the two parabolas meet.

a $y = x^2$ and $y = \frac{1}{4}x^2$

b $y = 2x^2$ and $y = -2x^2$

c $y = -3x^2$ and $y = -\frac{1}{2}x^2$

d $y = 5x^2$ and $y = 0.2x^2$

5 Which statement is correct about the parabola for $y = -6x^2$? Select **A**, **B**, **C** or **D**.

A It has a y -intercept of -6.

B It passes through (-1, 6).

C It is concave down.

D Its vertex is (0, -6).

See Example 21 6 Graph each pair of parabolas on the same set of axes and state:

i the vertex of each parabola

ii whether each parabola is concave up or concave down

iii which parabola is 'wider'

iv the y -intercept of each parabola.

a $y = x^2 + 1$ and $y = x^2 + 3$

b $y = -x^2 - 2$ and $y = 2x^2 + 4$

c $y = -5x^2 - 2$ and $y = -2x^2 - 1$

d $y = -3x^2 - 4$ and $y = 2x^2 + 2$

7 Which statement is correct about the parabola for $y = x^2 + 2$? Select **A**, **B**, **C** or **D**.

A It has a maximum value of 2.

B It is symmetrical about the x -axis.

C It is concave up.

D Its vertex is (2, 0).

11-10 Graphing circles

A circle is a completely round shape. Every point on a circle is exactly the same distance from its centre. We can use this fact to find the equation of a circle with radius 2 units.

Let $P(x, y)$ be any point so that the distance $OP = 2$ units and O is the origin.

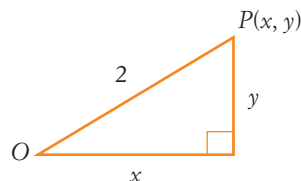
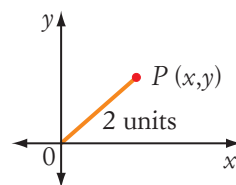
If we plotted every possible position of P , we would graph a circle centred at O with a radius of 2. We can use Pythagoras' theorem to find the equation of this circle by drawing a right-angled triangle with OP as the hypotenuse.

Since P has coordinates (x, y) , the triangle must have a base length of x and a height of y , so by Pythagoras' theorem:

$$x^2 + y^2 = 2^2 = 4$$

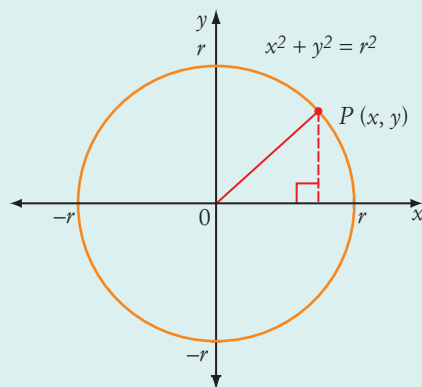
$\therefore$ The equation of a circle with centre $(0, 0)$ and radius 2 is $x^2 + y^2 = 4$.

This can be generalised for a circle of any radius.



Summary

The **equation of a circle** with centre $(0, 0)$ and radius r units is $x^2 + y^2 = r^2$



We can use this formula to find the centre and radius of any circle.

Homework sheet

Coordinate geometry 3

MAT09NAHS10017

Homework sheet

Coordinate geometry
revision

MAT09NAHS10018

Technology worksheet

Excel worksheet:
Curve sketched

MAT09NACT00028

Technology worksheet

Excel spreadsheet:
Curve sketched

MAT09NACT00013

Example 22

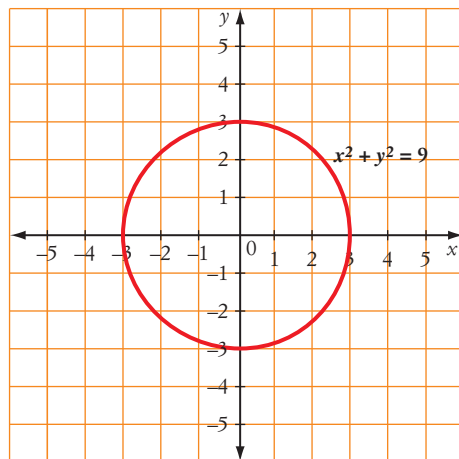
Find the centre and radius of the circle $x^2 + y^2 = 9$ and sketch its graph.

Solution

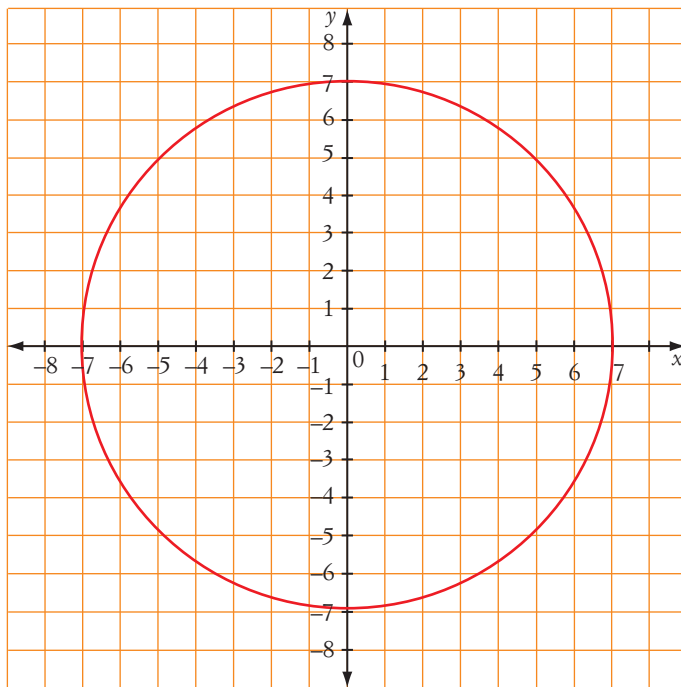
The centre is $(0, 0)$. The radius is r where $r^2 = 9$.

$$r = \sqrt{9}$$

$\therefore$ Radius, $r = 3$ units

**Example 23**

Find the equation of this circle.

**Solution**

The circle has a radius of 7 units and the centre is $(0, 0)$.

The equation of the circle is

$$x^2 + y^2 = 7^2$$

$$x^2 + y^2 = 49$$

Exercise 11-10 Graphing circles

The questions in this exercise may also be completed using a graphics calculator or graphing software.

- 1 Find the centre and radius of the circle given by each equation and sketch its graph.

See Example 22

a $x^2 + y^2 = 4$ **b** $x^2 + y^2 = 1$ **c** $x^2 + y^2 = 81$

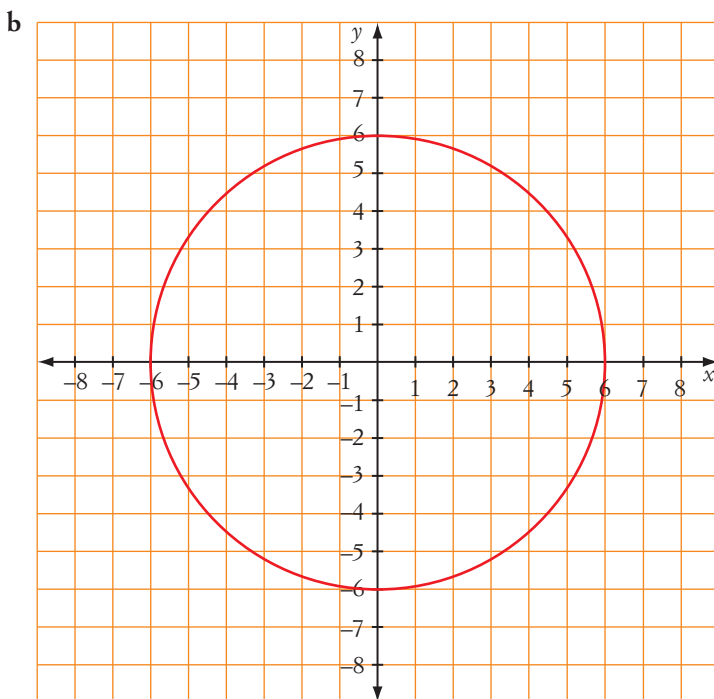
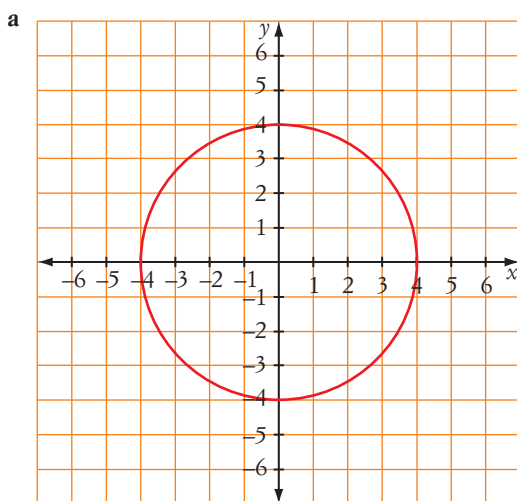
d $x^2 + y^2 = 100$ **e** $x^2 + y^2 = 25$ **f** $x^2 + y^2 = \frac{1}{4}$

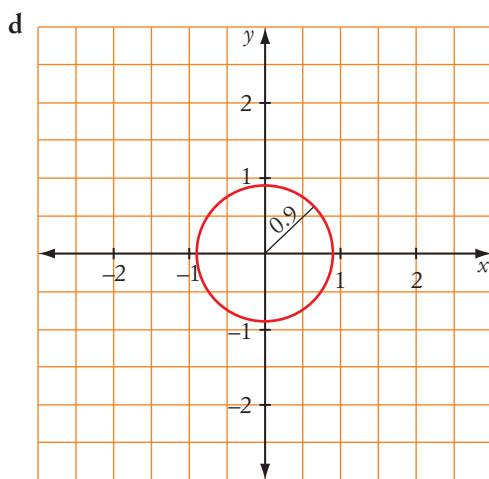
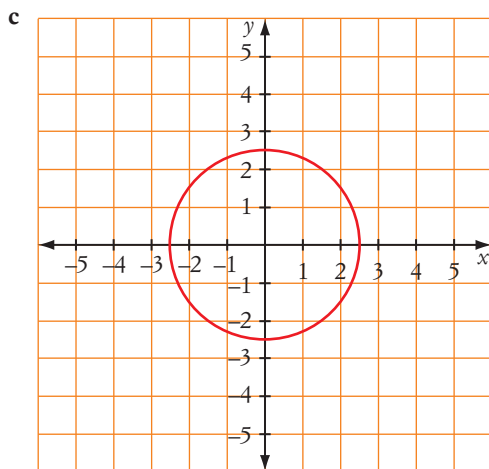
- 2 What is the equation of a circle with centre (0, 0) and radius 6 units? Select the correct answer **A**, **B**, **C** or **D**.

A $x^2 + y^2 = 12$ **B** $x^2 + y^2 = 6$ **C** $x^2 + y^2 = 8$ **D** $x^2 + y^2 = 36$

- 3 Find the equation of each circle shown.

See Example 23





- 4 What is the equation of a circle with centre $(0, 0)$ and radius 1.3 units? Select **A**, **B**, **C** or **D**.
A $x^2 + y^2 = 2.6$ **B** $x^2 + y^2 = 1.69$ **C** $x^2 + y^2 = 1.72$ **D** $x^2 + y^2 = 3.69$
- 5 What is the radius of a circle with equation $x^2 + y^2 = 144$? Select **A**, **B**, **C** or **D**.
A 12 units **B** 14 units **C** 36 units **D** 72 units
- 6 What is the equation of a circle with centre $(0, 0)$ and diameter 16 units?
- 7 What is the radius of a circle with equation $x^2 + y^2 = \frac{9}{4}$? Select **A**, **B**, **C** or **D**.
A $2\frac{1}{4}$ units **B** $1\frac{1}{2}$ units **C** $\frac{3}{4}$ units **D** $\frac{7}{8}$ units
- 8 What is the diameter of the circle whose equation is $x^2 + y^2 = 196$?
- 9 **a** What is the radius of the circle whose equation is $x^2 + y^2 = 12$?
b Sketch the graph of $x^2 + y^2 = 12$

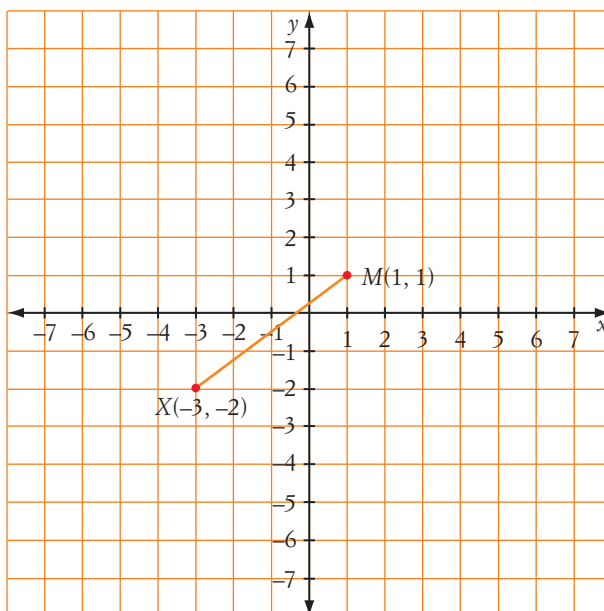
Worked solutions

Graphing circles

MAT09NAWS10051

Power plus

- $ABCD$ is a parallelogram. The coordinates of A , B and C are $(-1, 4)$, $(4, 6)$ and $(2, 7)$ respectively. Find the coordinates of D .
- A line drawn through points $A(0, -2)$ and $B(3, 0)$ also goes through point C with x -coordinate 9. Find:
 - the equation of the line
 - the y -coordinate of C .
- $M(1, 1)$ is the midpoint of the interval XY . If the coordinates of X are $(-3, -2)$, what are the coordinates of Y ?



- A circle is drawn with its centre at $C(2, 3)$. The point $A(-1, -5)$ lies on the circumference.
 - Calculate the radius of the circle.
 - The point P is 10 units from C . Explain why P lies outside the circle.
- The points of $R(2, 19)$, $S(-3, 7)$ and $T(10, 7)$ are joined to form a triangle. Show that the triangle is isosceles.
- The point $A(2, 1)$ is a vertex of a square $ABCD$. $M(5, 1)$ is the point of intersection of the diagonals of the square. Find the coordinates of B , C and D . (*Remember:* The diagonals of a square are equal in length and bisect each other at right angles.)

Chapter 11 review

Language of maths

Puzzle sheet

Coordinate geometry
crossword

MAT09NAPS10130

axis/axes	constant of proportionality	direct proportion	distance
exact	formula	gradient	horizontal
interval	length	linear equation	midpoint
number plane	parabola	quadratic equation	radius
rise	run	surd	varies directly
vertex	vertical	x-intercept	y-intercept

- What is the name given to the graph of:
 - a **quadratic equation**?
 - a **linear equation**?
- Is the line $y = 3$ horizontal or vertical?
- What is another name for the steepness of a line?
- What is the value at which a line crosses the y -axis called?
- What are the coordinates of the **vertex of the parabola** with equation $y = -x^2$?
- If $y = kx$, what is the constant of proportionality?
- When the length of an interval needs to be given as an **exact length**, what does this mean?

Topic overview

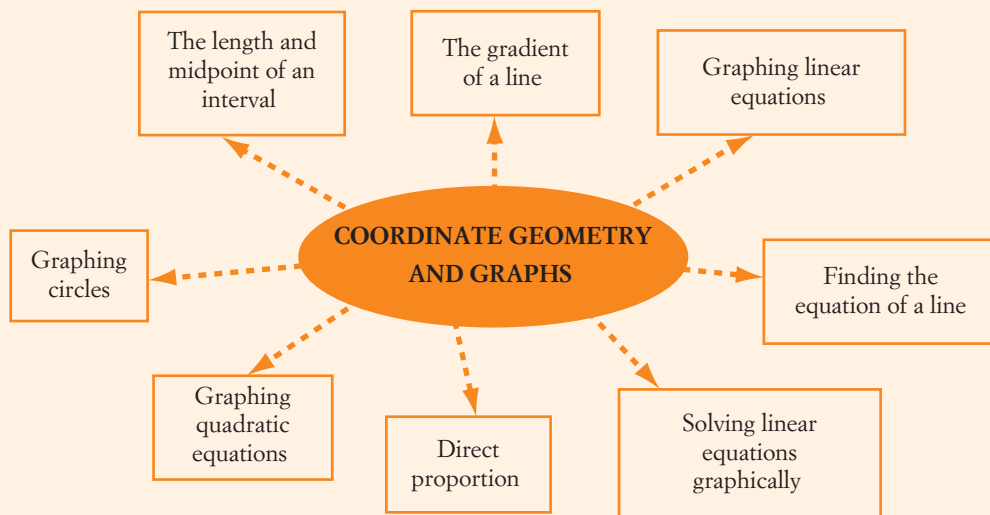
- Write in your own words what a gradient is.
- What parts of this topic did you find difficult?

Worksheet

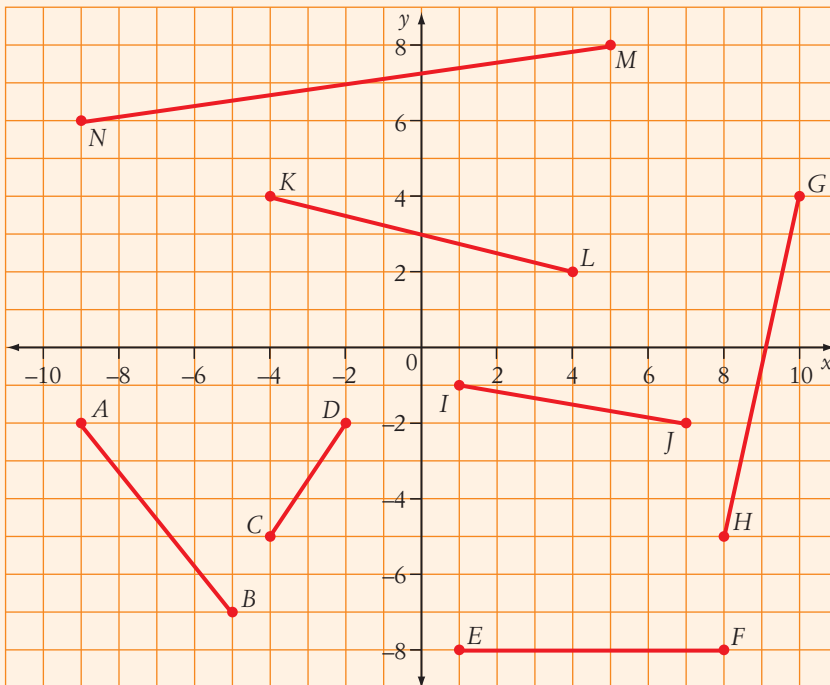
Mind map: Coordinate
geometry and graphs
(Advanced)

MAT09NAWS10131

Copy (or print) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



- 1 Find the length of the interval joining the points $H(7, 2)$ and $T(-2, 6)$. Express your answer:
 - a as a surd
 - b correct to one decimal place.See Exercise 11-01
- 2 Find the coordinates of the midpoint of the interval joining:
 - a $(8, 4)$ and $(-2, 10)$
 - b $(-1, -1)$ and $(-7, 9)$
 - c $(3, -6)$ and $(-5, 10)$See Exercise 11-02
- 3 a Which intervals below have a gradient that is:
 - i positive?
 - ii negative?
 - iii zero?See Exercise 11-03



- b Find the gradient of:
 - i interval NM
 - ii interval AB
 - iii interval KL
- c Find the midpoint of interval HG .
- d Find the exact length of interval CD .
- 4 Find the gradient of the interval joining:
 - a $(1, 9)$ and $(3, 5)$
 - b $(6, 0)$ and $(8, -4)$
 - c $(-1, -1)$ and $(5, -3)$See Exercise 11-03
- 5 Graph $y = -3x + 6$ on a number plane and write:
 - a its x -intercept
 - b its y -intercept.See Exercise 11-04
- 6 Test whether the point $(4, -2)$ lies on the line with equation:
 - a $y = x - 6$
 - b $y = -2x + 6$
 - c $x - 3y = 10$See Exercise 11-04
- 7 a Graph and label each of these lines on the same axes:

$x = -4$
 $x = 3$
 $y = -6$
 $y = 2$

 - b What is the area of the rectangle enclosed by these lines?See Exercise 11-04

Chapter 11 revision

See Exercise 11-04

8 Write the equation of a line which is:

- a horizontal with a y -intercept of 4
- b parallel to the y -axis and passes through $(-1, 7)$
- c the x -axis

See Exercise 11-05

9 Find the gradient and y -intercept of the line with equation:

- a $y = 3x + 1$
- b $y = 8 - x$
- c $y = \frac{x}{5} + 4$
- d $y = -6x$

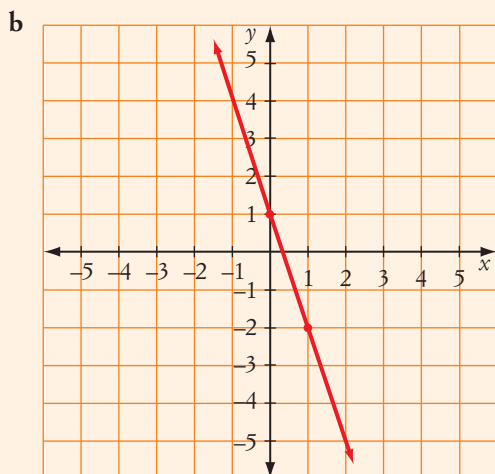
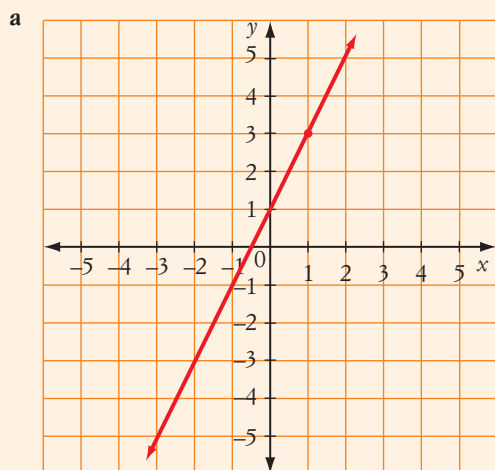
See Exercise 11-05

10 Graph each linear equation by finding the gradient and y -intercept first.

- a $y = -3x - 1$
- b $y = \frac{x}{2} + 6$

See Exercise 11-06

11 Find the equation of each line.



See Exercise 11-07

12 Solve the equation $3x - 4 = 5$ graphically.

- 13** During a thunderstorm, the time, T seconds, between seeing lightning and hearing thunder is directly proportional to the distance, d kilometres, of the observer from the centre of the storm. Thunder is heard 6 seconds after the lightning is seen when a storm is 35 km away.
- a** Find a formula for T in terms of d .
 - b** Calculate, correct to the nearest kilometre, the distance of a storm when thunder is heard 10 seconds after the lightning.
- See Exercise 11-08
- 14** Graph $y = x^2 + 1$ and $y = -2x^2$ on the same set of axes and state:
- a** the vertex of each parabola
 - b** whether each parabola is concave up or concave down
 - c** which parabola is 'wider'.
- See Exercise 11-09
- 15** Which equation below represents a concave up parabola with vertex $(0, 0)$?
Select **A**, **B**, **C** or **D**.
- A** $y = x^2 - 1$ **B** $y = 2x^2$ **C** $y = -x^2$ **D** $y = 3x^2 + 2$
- See Exercise 11-09
- 16** Find the centre and radius of the circle with equation $x^2 + y^2 = 16$ and sketch its graph.
- See Exercise 11-10



12

Statistics and Probability

Probability

In France in 1654, the writer and gambler the Chevalier de Méré asked mathematician Blaise Pascal how many rolls of a pair of dice were required to have a better than 50% chance of rolling a double six. Pascal wrote to another mathematician, Pierre de Fermat, about the problem, and the correspondence between them is regarded to be the beginning of probability theory.

Probability is now used widely many areas, including predicting the sales of a new brand of mobile phone, forecasting the weather on a long weekend, and determining the survival rate of newborn turtles.



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Chapter outline

	Proficiency strands				
12-01 Probability	U	F	PS	R	C
12-02 Relative frequency	U	F	PS	R	C
12-03 Venn diagrams	U	F	PS	R	C
12-04 Two-way tables	U	F	PS	R	C
12-05 Two-step experiments	U	F	PS	R	C

Wordbank

complementary event The 'opposite' event, for example the complementary event to selecting an Ace from a deck of cards is selecting a card that is not an Ace

relative frequency The frequency of an event over repeated trials as a fraction of the total number of trials

tree diagram A diagram of branches for listing all of the possible outcomes of a multi-step chance experiment

trial One go or run of a repeated probability experiment, for example, one roll of a die

two-step experiment A chance experiment with two steps or stages, such as rolling a pair of dice

two-way table A way of grouping items into two overlapping categories, such as gender and the ability of driving a car

Venn diagram A diagram of circles (usually overlapping) for grouping items into categories

In this chapter you will:

- identify complementary events and use the sum of probabilities to solve problems
- describe events using language such as 'at least,' exclusive 'or' (A or B but not both), inclusive 'or' (A or B or both) and 'and'
- represent events in two-way tables and Venn diagrams and solve related problems
- calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or'
- (STAGE 5.2) list all the outcomes for two-step chance experiments, with and without replacement, using tree diagrams or arrays, and assign probabilities to outcomes and determine probabilities for events 

SkillCheck

Worksheet

StartUp assignment 12

MAT09SPWK10132

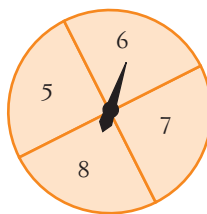
- A bag contains 3 green marbles, 4 red marbles, 5 black and 8 yellow marbles.
 - What fraction of the marbles are red?
 - What percentage of the marbles are yellow?
 - What percentage of the marbles are not yellow?
 - What fraction of the marbles are green or yellow?
- If a die is rolled, which one of the following events is most likely? Select the correct answer **A, B, C or D.**

A an even number	B an odd number
C a number greater than 3	D a number greater than 4
- A card is drawn at random from a deck of cards. What is the probability that it is:

a a 7?	b a heart card?	c a black card?
- Match each description (**a** to **f**) below with the correct set of all counting numbers (**i** to **vi**) that meet that description.

a between 3 and 10	b at least 1	c more than 4
d at most 3	e 5 or less	f not more than 4
i {1, 2, 3, 4, 5}	ii {4, 5, 6, 7, 8, 9}	iii {1, 2, 3}
iv {1, 2, 3, 4}	v {1, 2, 3, ...}	vi {5, 6, 7, ...}
- A die is rolled. Find the probability that the number that comes up is:

a 1	b more than 2	c odd	d composite
- Write the sample space for this spinner.
 - Find, as a percentage, the probability that the number spun is:
 - 5
 - at least 5
 - 5 or less
 - more than 5



12-01 Probability

Worksheet

Games of chance

MAT09SPWK10133

Puzzle sheet

Matching probabilities

MAT09SPPS10134

Summary

$P(E)$ means 'the probability of an event, E (occurring)'. If all possible outcomes are **equally likely**, then:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

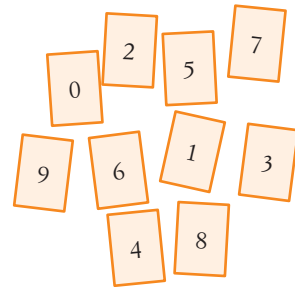
or $P(E) = \frac{\text{number of outcomes matching } E}{\text{number of outcomes in the sample space}}$

A **favourable outcome** is one of the outcomes in the event that you want, whose probability you are calculating.

Example 1

A card is chosen at random from a set of cards numbered 0 to 9.
Find the probability of selecting:

- a the card numbered 8
- b a number less than 4
- c a number 5 or more



Solution

There are 10 outcomes in the sample space.

a $P(8) = \frac{1}{10}$

1 chance in 10.

- b Out of the 10 numbers, there are 4 numbers less than 4: {0, 1, 2, 3}.

$$P(\text{less than 4}) = \frac{4}{10}$$

$$= \frac{2}{5}$$

4 chances in 10.

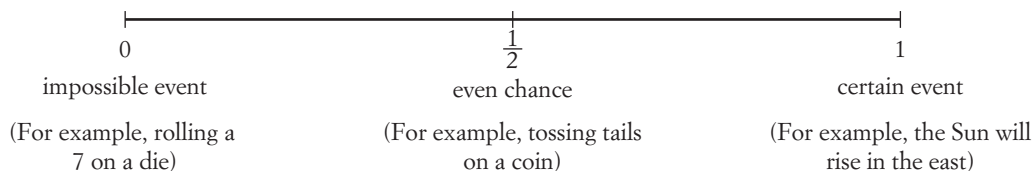
- c Out of the 10 numbers, there are 5 numbers that are 5 or more: {5, 6, 7, 8, 9}.

$$P(5 \text{ or more}) = \frac{5}{10}$$

$$= \frac{1}{2}$$

5 chances in 10.

The range of probability



Complementary events

Complementary events are events that together make up all the possible outcomes, such as a head and a tail when tossing a coin. The **complement** of an event E is made up of those outcomes in the sample space that are ‘not E ’, usually written as $\bar{E}$. Because an event and its complement covers all possible outcomes in the sample space, the sum of their probabilities must equal 1.

Summary

$$P(\text{event}) + P(\text{complementary event}) = 1$$

$$P(E) + P(\overline{E}) = 1$$

$P(\bar{E}) = 1 - P(E)$ where $\bar{E}$ is the complement of E .

Example 2

- List the sample space when this spinner is spun.
- Find $P(\text{red})$.
- What is the complementary event to the spinner landing on red?
- List the outcomes in the complementary event.
- Find $P(\text{not red})$.



Solution

- a** Sample space = {red, yellow, blue, green, white}
- b** $P(\text{red}) = \frac{1}{5}$
- c** The complementary event to landing on red is landing on a colour other than red.
- d** The outcomes in the complementary event are {yellow, blue, green, white}.
- e** $P(\text{not red}) = 1 - P(\text{red})$
 $= 1 - \frac{1}{5}$
 $= \frac{4}{5}$

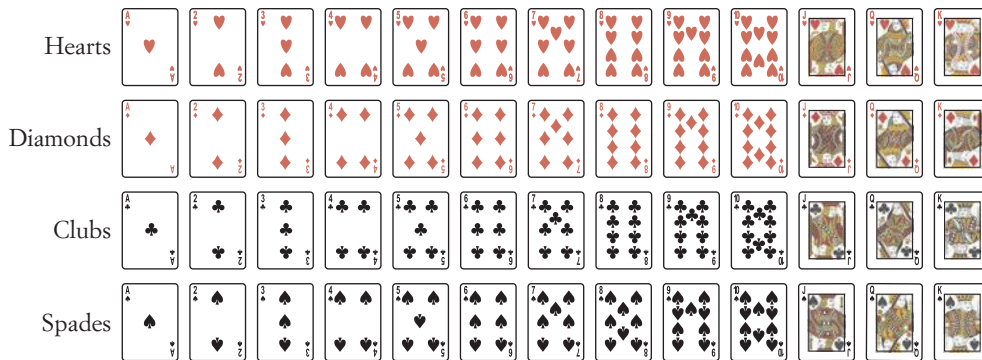
Exercise 12-01 Probability

See Example 1

- A letter is drawn at random from the alphabet.
 - How many outcomes are there in the sample space?
 - Are all the outcomes equally likely?
 - What is the probability of choosing a vowel?
 - What is the probability of choosing a letter from the word PROBABILITY?
- This spinner is spun. What is the decimal probability that the needle points to:
 - yellow?
 - black?
 - red?



- A normal pack of playing cards is divided into four suits as shown below.



A card is selected at random. Find the probability that it is:

- a black card
 - an ace
 - an even-numbered card
 - a picture card
- A die is rolled. What is the probability of rolling:
 - a number greater than 2?
 - a prime number?
 - a factor of 6?
 - a number less than 4?
 - a number greater than 6?
 - a number less than 7?
 - A bag contains 15 red jelly beans and 20 blue jelly beans. One jelly bean is drawn at random from the bag. What is the probability it will be:
 - blue?
 - yellow?
 - red or blue?
 - A student is selected at random from this group: Robert, Samantha, James, Joshua, Heidi, Jasmine, Caitlin, Dianne, Sarah. What is the probability of selecting a student:
 - who is male?
 - who is female?
 - whose name begins with J?
 - whose name has one syllable?
 - whose name contains an E?
 - A bag contains 14 black marbles and 9 white marbles. One marble is drawn at random from the bag. Calculate, as a percentage to one decimal place, the probability that it is:
 - black
 - white
 - black or white
 - red

Worked solutions

Probability

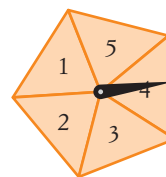
MAT09SPWS10052

- 8 A die with faces marked 1, 1, 2, 3, 4 and 5 is rolled. Find the probability of rolling:
 a 1 b 5 c an odd number d a number greater than 1
- 9 A bag contains 35 blue marbles, 45 green marbles and 20 yellow marbles. A marble is drawn at random. Calculate, as a percentage, the probability that it is:
 a blue b green c not green d neither blue nor yellow
- 10 For each event, write the complementary event.
 a selecting a consonant from the alphabet
 b the weather tomorrow being sunny
 c a traffic light showing red
 d selecting a spade from a normal deck of cards
 e hitting bullseye on a dartboard
 f selecting a blue ball from a bag containing blue, red and yellow balls



See Example 2

- 11 This spinner is spun.
 a List the sample space.
 b Find $P(4)$.
 c What is the complementary event to spinning a 4?
 d List the outcomes in the complementary event.
 e If $P(\text{not } 4) = P(\bar{4})$, find $P(\bar{4})$.
- 12 Dilone buys a ticket in a raffle in which 50 tickets are sold and there is only one prize.
 a What is Dilone's chance of winning the prize?
 Write your answer as a:
 i fraction ii decimal iii percentage
 b Calculate the probability of Dilone not winning the prize. Write your answer as a:
 i fraction ii decimal iii percentage



Worked solutions

Probability

MAT09SPWS10052

- 13 Tracey buys tickets in a raffle in which 200 tickets are sold and there is only one prize. She has a 40% chance of winning the prize.
 a What is the probability that Tracey will not win the prize?
 b How many tickets did Tracey buy?
- 14 Tamin bought a ticket in a raffle in which 500 tickets were sold. If there are 5 prizes, what is the probability that Tamin does not win a prize? Select the correct answer **A**, **B**, **C** or **D**.
 A $\frac{1}{100}$ B 49% C $\frac{499}{500}$ D 0.99
- 15 The probability of rain on Tuesday is 0.35. What is the probability that it will not rain?

- 16 What is the probability that a student chosen at random from your school was not born in a month beginning with the letter M?
- 17 Haylee has a 76% chance of hitting the bullseye when playing darts. What is the probability that she misses the bullseye?
- 18 What is the probability that a mobile phone number selected at random does not begin with a 0?

Just for the record

The origin of playing cards



The earliest European cards were Tarot packs, which contained either 78 or 97 cards, and were used to play a game called Tarocco. The smaller 52-card pack may have become popular for practical reasons because cards were originally drawn and painted by hand. The names and symbols on cards have changed many times but in the 14th century a pack was introduced with the symbols for hearts, spades, diamonds and clubs, the symbols which were eventually adopted by most Western countries. The names for the symbols still differ from country to country.

The pictures used for the Kings, Queens and Jacks have undergone some changes. The four Kings originally represented Charlemagne (hearts), the emperor of most of Europe until 814 CE, David (spades), the Hebrew king mentioned in the Bible, Julius Caesar (diamonds) of ancient Rome and Alexander the Great (clubs) of ancient Greece.

Investigate who the original Queens and Jacks were.

12-02 Relative frequency

Probability calculated using the formula:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

is more specifically called the **theoretical probability** (or calculated probability).

Worksheet

Coins probability

MAT09SPWK10135

Worksheet

Dice probability

MAT09SPWK10136

Worksheet

Probability review

MAT09SPWK10137

Worksheet

Relative frequencies

MAT09SPWK00047

Puzzle sheet

Final 6 investigation

MAT09SPPS00026

We can also determine probability based on the results of a trial that has been repeated many times, such as testing the effectiveness of 100 light globes, or rely on past statistics, such as the number of babies born last year that were girls. This type of probability is called **experimental probability**, which is based on a **relative frequency** that is the number of times an event occurs compared to the total frequency of outcomes.

Summary

Experimental probability

$$P(E) = \frac{\text{number of times the event happened}}{\text{total number of trials}}$$

$$\text{or } P(E) = \frac{\text{frequency of } E}{\text{total frequency}}$$

Expected frequency is the expected number of times an event will occur over repeated trials.

$$\text{Expected frequency} = \text{theoretical probability} \times \text{number of trials}$$

Example 3

Connor tossed a coin 100 times and recorded the results in a table.

- a Calculate, as a decimal,
 - i the experimental probability of tossing a head
 - ii the theoretical probability of tossing a head.
- b Are the experimental and theoretical probabilities similar?
- c For 100 tosses, what is the expected frequency of heads based on the theoretical probability? How does this compare with the actual number of times?

Outcome	Frequency
Head	46
Tail	54
	100

Solution

$$\text{a i Experimental probability } P(H) = \frac{46}{100} = 0.46$$

$$\text{ii Theoretical probability: } P(H) = \frac{1}{2} = 0.5$$

- b By comparing the decimals for the two answers, we see that the experimental and theoretical probabilities are similar.

$$\text{c Expected number of heads} = 0.5 \times 100 = 50$$

probability $\times$ number of trials

From the table, the observed number of heads = 46, which is close to the expected number, 50.

Note: When calculating experimental probability, the results will not always be consistent with what is expected unless a very large number of trials are performed.

Example 4

Briana rolled a die 200 times and recorded her results.

- a** Find the experimental probability of rolling:
- i** an even number
 - ii** an even number or a number greater than 2
 - iii** an even number and a number greater than 2
 - iv** an even number not greater than 2.
- b** Calculate the probability of rolling a 5 or 6
- i** as an experimental probability
 - ii** as a theoretical probability.
- c** What is the expected number of times of rolling a 5 or 6 in 200 rolls of a die? How does this compare with Briana's observed number of times?

Outcome	Frequency
1	38
2	40
3	29
4	24
5	35
6	34

Solution

a i Rolls of even numbers = $40 + 24 + 34$
 $= 98$

Frequencies of 2, 4, 6.

Experimental $P(\text{even}) = \frac{98}{200} = \frac{49}{100}$

ii Rolls of even numbers or numbers greater than 2
 $= 40 + 24 + 34 + 29 + 35$
 $= 162$

Frequencies of 2, 4, 6, 3, 5.

Experimental $P(\text{even or } > 2) = \frac{162}{200} = \frac{81}{100}$

iii Rolls of even numbers greater than 2 = $24 + 34$
 $= 58$

Frequencies of 4 and 6.

Experimental $P(\text{even and } > 2) = \frac{58}{200} = \frac{29}{100}$

iv Rolls of even numbers not greater than 2 = 40

Frequency of 2.

Experimental $P(\text{even and not } > 2) = \frac{40}{200} = \frac{1}{5}$

b i Rolls of 5 or 6 = $35 + 34$
 $= 69$

Frequencies of 5 and 6.

Experimental $P(5 \text{ or } 6) = \frac{69}{200}$

ii Theoretical $P(5 \text{ or } 6) = \frac{2}{6} = \frac{1}{3}$

2 chances out of 6.

c Expected number of rolls of 5 or 6 = $\frac{1}{3} \times 200$
 $= 66.666 \dots$
 ≈ 67

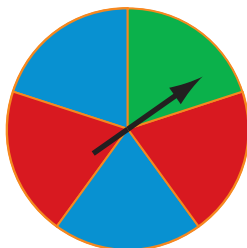
probability $\times$ number of trials

From the table, the observed number of heads = $35 + 34 = 69$, which is close to 67.

Exercise 12-02 Relative frequency

See Example 3

- 1 Fatima spun this spinner 200 times and recorded the results in the table.



Event	Frequency
Red	90
Green	36
Blue	74
	200

- Calculate, as a decimal, the experimental probability that the needle points to:
 - red
 - green
 - Calculate, as a decimal, the theoretical probability that the needle points to:
 - red
 - green
 - Are the experimental and theoretical probabilities similar?
 - For 200 spins, what is the expected frequency of red or blue based on the theoretical probability? How does this compare with the observed frequency?
- 2 Use cardboard and a toothpick to make a square spinner that has four colours as shown in the diagram below. Spin the spinner 50 times and record your results in a table similar to the one shown.



Event	Tally	Frequency
Red		
Blue		
Green		
Yellow		

- Calculate, as a percentage, the relative frequency of spinning:
 - red
 - blue
 - a colour that is not yellow
 - blue or green
- Calculate, as a percentage, the theoretical probability of spinning blue. How does this compare with the relative frequency?
- For 50 spins, what is the expected number of times that blue should come up? How does this compare with the actual number of times?

- 3 A die is rolled.

- Predict the number of times a 2 would appear if the die was rolled 100 times.
- Roll a die 100 times and record your results in a table similar to the one shown.
- What is the relative frequency of rolling a 2?
- How does your predicted frequency compare with the observed frequency?
- What is the relative frequency of rolling an even number?

Event	Tally	Frequency
1		
2		
3		
4		
5		
6		
		100

- 4 Place 4 black counters, 5 red counters and 1 yellow counter in a bag. Select a counter at random, record the colour and return the counter to the bag. Run this experiment 50 times.

- a What is the experimental probability of selecting:
- i a black counter?
 - ii a red counter?
 - iii a yellow counter?

Event	Tally	Frequency
Black		
Red		
Yellow		

- b What is the theoretical probability of selecting:
- i a black counter?
 - ii a red counter?
 - iii a yellow counter?
- c Compare your experimental probabilities to the theoretical probabilities.
- d Repeat the experiment another 50 times and find the relative frequency of each event. Are the results different to the results found in part a?

- 5 A die was repeatedly rolled and the results are shown in the table.

- a What was the total number of trials?
- b Find the experimental probability (as a decimal) of rolling:
- i an even number
 - ii a number greater than 3
 - iii a 3 or a 5
 - iv a number greater than 2 and even
 - v a number greater than 3 or odd.
- c Calculate the probability of rolling a number not less than 3:
- i as an experimental probability
 - ii as a theoretical probability.
- d How do the two probabilities in part c compare?

Outcome	Frequency
1	90
2	84
3	92
4	76
5	83
6	75

See Example 4

- 6 Students at a high school were asked how they travelled to school. The results are presented in the table. Find the probability (correct to two decimal places) that a student chosen at random will:

Outcome	Frequency
Walk	186
Bus	295
Car	103
Bicycle	74

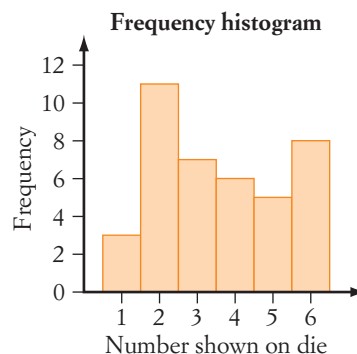
- a walk
- b be driven
- c catch a bus
- d not ride a bicycle
- e be driven or catch a bus.



Newspix/Angelo Soudas

- 7 A die was rolled many times and the results are shown in the frequency histogram.

- a What is the total number of trials?
- b Find the experimental probability of rolling:
- i 2
 - ii 5
 - iii an even number or a number greater than 3.
 - iv a number greater than 4 and odd.



Worked solutions

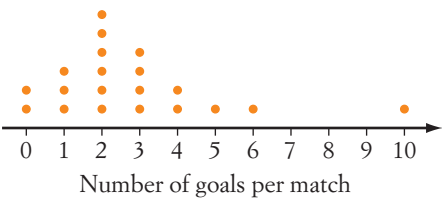
Relative frequency

MAT09SPWS10053

- 8 The table shows the number of babies born per day at a hospital during the month of January. On any particular day, find the probability of the birth of exactly:
- a 1 baby
 - b 3 babies
 - c 1 or 2 babies
 - d at least 2 babies
 - e at most 1 baby
 - f 4 or 5 babies.

Number of babies per day	Frequency
0	1
1	4
2	7
3	10
4	5
5	4

- 9 This dot plot shows the number of goals per match scored by a soccer team during the season.



For a match selected from the season at random, find the probability of the team scoring:

- a 2 goals
 - b 4 goals per game
 - c fewer than 2 goals
 - d 2 or 3 goals
 - e at least 3 goals
 - f not more than 4 goals
- 10 A card is drawn randomly from a normal pack of cards to see if it is an Ace. This trial is run 120 times with each card being replaced in the deck before the next card is drawn. The results are shown in the table.



Outcome	Frequency
Ace	11
Not an Ace	109

- a Calculate, correct to 3 decimal places:
 - i the experimental probability of selecting an Ace
 - ii the theoretical probability of selecting an Ace.
- b Are the experimental and theoretical probabilities similar?
- c If a card is drawn 300 times, then what is the expected number of times an Ace should be drawn? Select the correct answer A, B, C or D.

A 27 B 28 C 23 D 6

Technology Tossing a coin

In this activity, a spreadsheet is used to simulate the tossing of a coin. The computer can quickly generate either a 1 (representing heads) or a 2 (tails).

- 1 In cell A1 enter the label 'Tossing a coin'.
- 2 In cell A2, enter the formula $\text{=INT}(\text{RAND}()*2+1)$, press Enter and either 1 or 2 will appear in the cell.
- 3 Select cell A2 and **Fill Down** to cell A11 to simulate the tossing of a coin 10 times.
- 4 Copy the table below and in the first blank row record your results for Trials 1–10 (the numbers of heads and tails in your first 10 'tosses').

Trial	Number of heads	Number of tails
1–10		
11–20		
21–30		
31–40		
41–50		
51–60		
61–70		
71–80		
81–90		
91–100		
Total		



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- 5 Simulate another 10 tosses of the coin by making the spreadsheet generate another set of random numbers. Do this by placing the cursor between the tops of columns A and B (so that it turns into a 'double arrow') and clicking.
- 6 Enter the results for Trials 11–20 in your table.
- 7 Repeat 8 more times so that you have 100 tosses of the coin recorded in the table.
- 8 **a** For 100 tosses of a coin, what is the expected frequency of heads based on the theoretical probability?
b Compare this with the actual frequency.
- 9 Calculate, as a decimal, the theoretical and experimental probabilities of tossing heads.
- 10 Compare your results with those of other students in your class. Briefly explain any differences and why they may have occurred.
- 11 Combine the results of students in your class to calculate the experimental probability of tossing heads, as a decimal. How does this compare with the theoretical probability?

Worksheet

Coins probability

MAT09SPWK10135

Technology worksheet

Rolling dice

MAT09SPCT10013

Worksheet

Dice probability

MAT09SPWK10136

Homework sheet

Probability 1

MAT09SPHS10035

Worksheet

Venn diagrams

MAT09SPWK10138

Worksheet

Venn diagrams
matching activity

MAT09SPWK10139

Worksheet

Probability review

MAT09SPWK10137

Puzzle sheet

Venn diagrams

MAT09SPPS00028

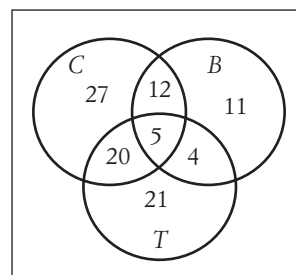
12-03 Venn diagrams

A **Venn diagram** is a diagram of circles (usually overlapping) for grouping items into categories. A rectangle represents the whole group while the circles represent categories. Items common to two or more categories are placed in the intersection (overlapping region) of the circles. The Venn diagram was invented in 1880 by English mathematician and priest, John Venn (1834–1923).

Example 5

This Venn diagram shows the results of a survey on how people travel to work (car (C), bus (B) or train (T)).

- How many people participated in the survey?
- How many people used all three methods: car, bus *and* train? Give an explanation on how this might happen.
- Calculate the probability of selecting a person from this survey who travels by:
 - car only or bus only
 - car and train
 - car and train only
 - bus but not train.
- A person is chosen at random from the people who travel by car or train only (but not bus). What is the probability that the person travels by car only?
- If 110 people actually participated in the survey, what extra information would be needed on the diagram?



Anna Pui Ling Pang

Solution

a Number of people = $27 + 12 + 5 + 20 + 21 + 4 + 11$
 $= 100$

b 5 people travelled by car, bus and train.

They may have driven to the station, caught a train and then a bus to the workplace, or used different methods on different days.

c i People travelling by car only or bus only = $27 + 11$
 $= 38$

$$P(\text{car only or bus only}) = \frac{38}{100}$$

$$= \frac{19}{50}$$

ii People travelling by car and train = $20 + 5$
 $= 25$

$$P(\text{car and train}) = \frac{25}{100}$$

$$= \frac{1}{4}$$

iii People travelling by car and train only = 20

$$P(\text{car and train only}) = \frac{20}{100}$$

$$= \frac{1}{5}$$

iv People travelling by bus but not train = $11 + 12$
 $= 23$

$$P(\text{bus but not train}) = \frac{23}{100}$$

d People travelling by car or train only = $27 + 20 + 21$
 $= 68$

People travelling by car only = 27.

$$P(\text{car only}) = \frac{27}{68}$$

e $110 - 100 = 10$. This means that there were 10 people who did not use a car, bus or train. A '10' would need to be placed in the rectangle, but outside of the three circles.

The region where the 3 circles intersect.

The regions in *C* and *B* that don't overlap.

The region where *C* and *T* intersect.

Where *C* and *T* intersect but excluding *B*.

The circle *B* excluding *T*.

'And' vs 'or'

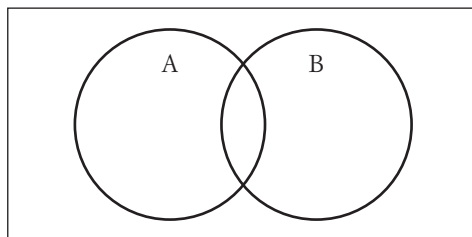
For two categories or events A and B, the phrase '**A and B**' means to have both of them occurring together. For example, 'to drive a car' **and** 'to ride a bus' means to do both things.

If A and B are **overlapping**, the phrase '**A or B**' means to have A or B or both. For example, 'to drive a car' **or** 'to ride a bus' means to drive a car only, or to ride a bus only, or to do both. In this case, 'A or B' actually **includes** 'A and B' so this is an example of an **inclusive** 'or'.

If A and B are **mutually exclusive**, this means that they are **not overlapping** and on a Venn diagram they appear as two separate circles. For mutually exclusive categories or events, the phrase '**A or B**' means to have A only or B only (but not both). For example, 'male' **or** 'female' means to be male, or female, but not both. In this case, 'A or B' **excludes** 'A and B' so this is an example of an **exclusive** 'or'.

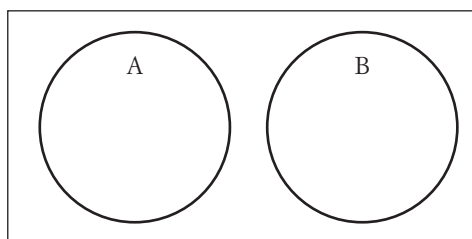
Overlapping events:

'A or B' means A or B or both



Mutually exclusive events:

'A or B' means A or B but not both



Example 6

A survey of 100 students at Rabmavale College showed that 61 students had dark hair, 42 students had brown eyes and 20 students had both dark hair and brown eyes.



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- Represent this information on a Venn diagram.
- How many students had brown eyes or dark hair but not both?
- What is the probability of randomly selecting from this group a student:
 - with dark hair?
 - with dark hair and brown eyes?
 - with dark hair or brown eyes?
 - with neither dark hair nor brown eyes?

Solution

a RC = students at Rabmavale College

DH = students with dark hair

BE = students with brown eyes

When completing a Venn diagram, always begin with the intersection.

There are 20 students with dark hair and brown eyes.

$$\begin{aligned}\therefore \text{Students with dark hair only} &= 61 - 20 \\ &= 41\end{aligned}$$

$$\begin{aligned}\therefore \text{Students with brown eyes but not dark hair} &= 42 - 20 \\ &= 22\end{aligned}$$

$$\begin{aligned}\therefore (\text{Remaining}) \text{ Students with neither dark hair nor brown eyes} &= 100 - 41 - 20 - 22 \\ &= 27\end{aligned}$$

b Number of students with brown eyes or dark hair only = $22 + 41 = 63$

c i Students with dark hair = $41 + 20$
= 61

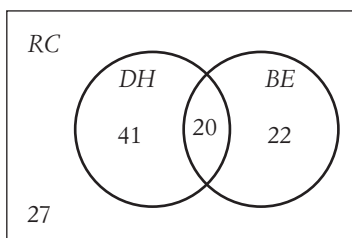
$$P(\text{dark hair}) = \frac{61}{100}$$

$$\begin{aligned}\text{ii } P(\text{dark hair and brown eyes}) &= \frac{20}{100} \\ &= \frac{1}{5}\end{aligned}$$

$$\begin{aligned}\text{iii Number of students with dark hair or brown eyes} &= 41 + 20 + 22 \\ &= 83\end{aligned}$$

$$P(\text{dark hair or brown eyes}) = \frac{83}{100}$$

$$\text{iv } P(\text{neither dark hair or brown eyes}) = \frac{27}{100}$$

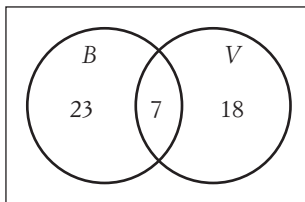


Exercise 12-03 Venn diagrams

See Example 5

- 1 The Venn diagram shows the number of students who play either basketball (B) or volleyball (V).

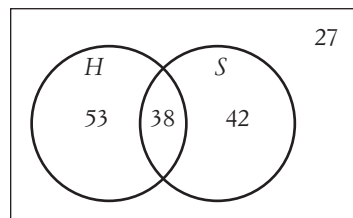
- a Find the total number of students who play basketball or volleyball.
b Find the probability of randomly selecting a student who plays:
i basketball ii volleyball.



- c What is the probability of selecting a student who plays:
i basketball or volleyball?
ii basketball and volleyball?
iii basketball but not volleyball?
iv neither basketball nor volleyball?
d Of the students who play volleyball, find the probability of selecting a student who also plays basketball.

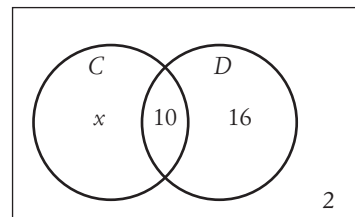
- 2 The Venn diagram shows the number of Year 9 students who achieved Credit grades in their Science (S) and History (H) exams.

- a How many students are in Year 9?
b Find the probability of selecting a student who achieved a Credit in:
i History ii Science
iii History and Science iv History or Science
v History or Science but not both
vi History only vii Science only



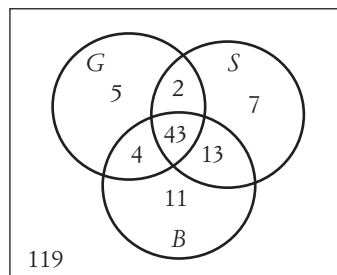
- c What is the percentage probability of selecting a student who did not achieve a Credit in both History and Science?
d Out of the students who achieved a Credit in History, what is the probability that the student also achieved a Credit in Science?
3 Forty people were asked whether they preferred watching movies at the cinema (C) or on DVD (D) at home. The results are shown in the Venn diagram.

- a Describe the category that contains the pronumeral x in the diagram.
b Find the value of x .
c Calculate the decimal probability of randomly selecting a person who likes:
i the cinema and watching DVDs ii DVDs only
iii the cinema or DVDs but not both iv neither movies nor DVDs.



- 4 The Venn diagram describes the number of countries that won gold, silver, bronze or no medals at the 2012 Summer Olympic Games in London.

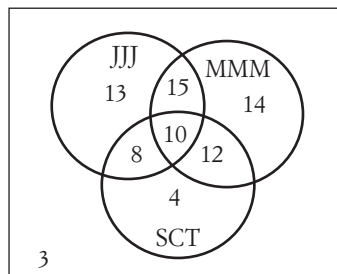
- How many countries competed at the 2012 games?
- How many countries did not win a medal? How is this shown on the Venn diagram?
- What is the probability of randomly selecting a country that:
 - won only gold medals?
 - won gold or silver medals but not both?
 - won gold, silver and bronze medals?
 - won gold and bronze only?
 - won gold only or silver only or bronze only?
 - did not win a gold medal?
 - did not win a gold or silver medal?
- Out of the countries that won medals, what is the probability of selecting a country that:
 - won gold medals?
 - won silver but not bronze?



Getty Images/Popperfoto

- 5 Students were asked what radio stations they listened to and the results are shown in the Venn diagram.

- How many students were surveyed?
- What is the probability of selecting a student who did not listen to any of the three stations?
- What is the probability of selecting a student who listens to:
 - JJJ?
 - MMM?
 - SCT?
- What is the probability of selecting a student who listens to at least two of the stations?
- Find the probability of selecting a student who listens to:
 - JJJ or MMM but not both
 - all three stations
 - MMM only



Worked solutions

Venn diagrams

MAT09SPWS10054

- 6 In a drama group of 30 people, 18 people dance (D), 20 people sing (S) and 8 people dance and sing.

- Show this information on a Venn diagram.
- Find the probability of selecting a person who:
 - dances and sings
 - sings but does not dance
 - dances but does not sing
 - dances or sings
 - dances or sings but not both.

See Example 6

- 7 A class of 30 students were given the option of playing an indoor sport (S) or watching a movie (M) when regular sport was cancelled due to bad weather. 15 students wanted to play an indoor sport and 24 students wanted to watch a movie.
- How many students indicated they would do either?
 - Copy and complete a Venn diagram using the information given.
 - For this Venn diagram there are no values outside the circles. Explain why.
 - What is the probability of selecting a student who prefers to:
 - play an indoor sport or watch a movie but not both?
 - watch a movie but not play an indoor sport?
 - play an indoor sport but not watch a movie?

- 8 80 families in Leeton were surveyed on whether they had a dog (D) or a cat (C) as a pet. 12 families had no pets, 35 had a dog and 47 had a cat

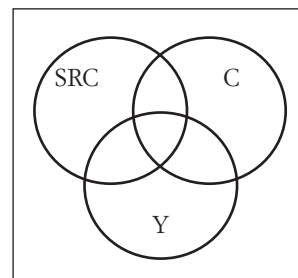


Shutterstock.com/jiri Vachek

- Copy and complete the Venn diagram showing the information given.
 - How many families had:
 - a dog and a cat?
 - a dog but not a cat?
 - at least a dog or a cat?
 - a dog or a cat but not both?
 - Find the decimal probability of selecting a family with:

i a dog only	ii either a dog or a cat but not both
iii at most one pet	iv at least one pet.
 - Given that a family has a pet, what is the probability that they have a cat?
- 9 At Marvalber High School, the Student Representative Council (SRC) has 25 members, the Chess club has 20 members and the Yoga club has 33 members. Five students belong to all three groups, while 13 members belong to the SRC and no other club. Four members of the Yoga club are members of the SRC but not the Chess club, while 10 members belong to both the Yoga and Chess clubs. There are 8 members of the Chess club who are members of the SRC.
- Copy and complete the Venn diagram.
 - How many students belong to the Yoga club only?
 - How many students belong to the Chess club and no others?
 - Find the probability of selecting a student who belongs to exactly 2 clubs.
 - What is the probability of a student belonging to at least 2 clubs?
 - Find the probability of selecting a student who belongs to:

i the Yoga club and the SRC but not the Chess club	ii only one club
iii the SRC or the Chess club but not the Yoga club	iv at most 2 clubs
 - Is it necessary to include the rectangle in this Venn diagram? Give reasons.



12-04 Two-way tables

Worksheet

Two-way tables

MAT09SPWK10140

Worksheet

Two-way probability
tables

MAT09SPWK00048

A **two-way table** is another way of grouping items into overlapping categories, especially when there are many overlaps that cannot easily be represented by Venn diagrams.

Example 7

This table categorises the heights and ages of 200 Year 9 students.

	Short	Tall
Aged 14	45	25
Aged 15	90	40

- a How many students are:
- aged 15 and tall?
 - aged 14 or short?
- b What fraction of students are neither tall nor aged 15?
- c What is the probability of selecting a student at random who is:
- aged 14 and tall?
 - aged 15 but not tall?
 - tall or aged 14 but not both?
- d If a short student is selected at random, what is the probability that he or she is aged 14?



Getty Images/Abvodels

Solution

- a i Students aged 15 and tall = 40.
- ii Students aged 14 or short = $45 + 25 + 90 = 160$
- b Students who are neither tall nor 15 = 45. The short 14-year-olds.
- $\therefore$ Fraction of students neither tall nor 15 = $\frac{45}{200} = \frac{9}{40}$
- c i $P(\text{tall and 14}) = \frac{25}{200} = \frac{1}{8}$
- ii $P(15 \text{ but not tall}) = \frac{90}{200} = \frac{9}{20}$
- iii Students tall or aged 14 but not both = $60 + 45 = 105$
- $P(\text{tall or 14 but not both}) = \frac{105}{200} = \frac{21}{40}$
- d Short students = $90 + 45 = 135$
- Students who are short and 14 = 45
- $P(\text{short student who is 14}) = \frac{45}{135} = \frac{1}{3}$

Exercise 12-04 Two-way tables

See Example 7

- 1 Households were surveyed at random to see whether or not they had dishwashers and clothes dryers. The results are in the table below.

	Dishwasher	No dishwasher
Clothes dryer	75	108
No clothes dryer	49	68

- How many households were surveyed?
 - How many households had a clothes dryer and a dishwasher?
 - How many households had neither?
 - What is the probability of selecting a household at random that has:
 - a dishwasher?
 - a dishwasher and a dryer?
 - a dishwasher or a clothes dryer but not both?
 - at least one of them?
- 2 People at a train station were asked whether they ate breakfast or lunch that day.

	Ate lunch	Did not eat lunch
Ate breakfast	35	26
Did not eat breakfast	54	15

- How many people were surveyed?
- What percentage of people ate lunch?
- Find the probability of picking a person at random who ate:
 - breakfast
 - lunch
 - breakfast but not lunch
 - breakfast or lunch
 - breakfast and lunch
 - breakfast or lunch but not both
- What is the probability of picking a person who has eaten lunch given that the person ate breakfast?
- What is the probability of picking a person who has not eaten lunch given that the person did not eat breakfast?

- 3 People at a shopping mall were asked if they watched football on TV.

	Football	No football
Male	85	27
Female	36	92

- How many people were surveyed?
 - How many people:
 - watched football?
 - did not watch football?
 - What percentage of people surveyed were females?
 - What is the probability of randomly selecting a person who did not watch football and was male?
 - Given that a person watched football, what is the probability that the person was female?
- 4 The results comparing gender and left-handedness of students at a primary school are shown in the table below.

	Male	Female
Left-handed	14	18
Right-handed	86	65

- How many students are at the school?
- What is the probability of selecting a student at random who is:
 - male and left-handed?
 - male or left-handed?

- c Find the probability, expressed as a percentage, of randomly selecting a right-handed female.
- d Given that a student is a male, what is the probability that he is right-handed?
- e What is the probability of selecting a female or a right-handed student?
- 5 After the 2010 Federal election, the members (politicians) of the House of Representatives in the Australian Parliament were as follows.

	Government (Labor)	Opposition (Liberal/National)
Male	53	58
Female	23	14

- a How many members of parliament (MPs) were there in the House of Representatives?
- b Find the percentage probability of randomly selecting an MP who is:
- male
 - female and in Opposition
 - in the Government or female
 - male and in Opposition.
- c Find the probability of selecting an MP who is male and in Government.
- d If a female MP is selected at random, what is the probability that she is in Government?



Getty Images/Pool

- 6 Students at Westvale Christian College were surveyed on whether they played video games. The results are shown in the table.

	Plays video games	Does not play video games
Female	178	267
Male	310	210

- a How many students attended the college?
- b Find the percentage probability that a student selected at random from the college:
- plays video games and is female
 - doesn't play video games
 - is male and doesn't play video games
 - is female and doesn't play video games.
- c If a female student is selected at random, what is the probability that she plays video games?
- d If a male student is selected at random, what is the probability that he plays video games?

7 A group of retired people was surveyed on whether they exercised regularly.

a How many people were surveyed?

b What percentage of people exercised?

c What is the probability of selecting a person at random who doesn't exercise regularly?

d Of the women who were surveyed, what percentage exercised regularly?

e If a man was selected randomly from this survey, what is the probability that he exercises regularly?

	Exercise	Don't exercise
Men	65	106
Women	74	75



Investigation: Using two-way tables

Work in groups of three or four.

Investigate two characteristics, for example, gender and exercise, gender and eating habits, eating habits and exercise, playing sport and going to the movies, eye colour and hair colour.

Conduct a survey to obtain the information required, then summarise your results in a two-way table similar to this one:

	Eat regular meals	Don't eat regular meals
Play sport		
Don't play sport		

Note any patterns in your data. This may include answering questions such as 'What percentage of' or 'What is the probability of picking a person who'.

Mental skills 12

Maths without calculators

The unitary method with percentages

The unitary method is used when you are only given a *percentage* of an amount and you need to find the amount. It is called the unitary method because we find 1% of the amount first, then multiply that by 100 to find the whole (100%).

1 Study each example.

- a** If 8% of a number is 24, what is the number?
 8% of the number = 24
 $\therefore 1\% \text{ of the number} = 24 \div 8 = 3$
 $\therefore 100\% \text{ of the number} = 3 \times 100 = 300$.
 The number is 300. *Check:* $8\% \times 300 = 24$
- b** If 15% of an amount is \$90, what is the whole amount?
 15% of the amount = \$90
 $\therefore 1\% \text{ of the amount} = \$90 \div 15 = \$6$
 $\therefore 100\% \text{ of the amount} = \$6 \times 100 = \$600$.
 The amount is \$600. *Check:* $15\% \times \$600 = \$90$

2 Find the whole amount if:

a 5% of the amount is \$35	b 11% of the amount is \$88
c 20% of the amount is 80	d 6% of the amount is 42
e 90% of the amount is \$270	f 15% of the amount is \$60
g 40% of the amount is 100	h 120% of the amount of \$360
i 25% of the amount is \$75	j 8% of the amount is 40

Just for the record

Lotto in NSW is a game of chance in which 6 balls are selected from a barrel of balls numbered 1 to 45. Players who correctly predict the 6 numbered balls share a first prize of at least one million dollars. The original Lotto game introduced in 1979 used 40 balls, and by increasing to 44 balls in 1989, the chance of winning first prize dropped from 1 in 3 838 380 to 1 in 7 059 052. The addition of four extra balls made Lotto almost twice as difficult to win. Another ball was added in 2004, making the chance of winning first prize even lower.

Lotto



istockphoto/Simon Askeham

Research the probability of winning Lotto today now that it uses 45 balls.

Investigation: Are the Lotto numbers equally likely?

In each Lotto game, 8 numbers are selected at random from the barrel: 6 main numbers plus 2 supplementary numbers. Between 2001 and 2008, there were 3376 Lotto numbers drawn on the Saturday game. The frequency of each number (from 1 to 45) were as follows:

1	77	10	73	19	67	28	75	37	75
2	74	11	85	20	68	29	67	38	72
3	82	12	81	21	97	30	58	39	76
4	72	13	58	22	86	31	70	40	95
5	82	14	68	23	77	32	85	41	79
6	91	15	76	24	71	33	78	42	75
7	94	16	58	25	71	34	78	43	68
8	81	17	69	26	72	35	65	44	63
9	68	18	70	27	71	36	83	45	75

Source: Tatts.com

- If over the 8 years 3376 numbers were drawn, and each number from 1 to 45 was equally likely, how often would you expect each number to come up?
- How many of the frequencies from the above table are equal to this expected number?
- Which Lotto number came up:
 - most often?
 - least often?
- Do you think the Lotto numbers are equally likely, or is the Lotto draw biased? Give a reason for your answer.

Worksheet

Probability review

MAT09SPWK00050

Homework sheet

Probability 2

MAT09SPHS10036

Homework sheet

Probability revision

MAT09SPHS10037

Animated example

Combined events

MAT09SPAE00007

12-05 Two-step experiments

A **two-step experiment** or **two-stage experiment** is a chance experiment that has two parts or stages, for example:

- rolling two dice
- throwing a coin and die together
- drawing two prizes in a raffle
- observing the weather each day over a weekend.

The outcome of the second step or stage may or may not be affected by the outcome of the first step. The sample space for a two-step experiment can be displayed using lists, tables or tree diagrams.

Lists and tables

Example 8

Two coins are thrown together.

a Find all the outcomes in the sample space:

- i** by simply listing them **ii** using a table.

b What is the probability of throwing:

- i** a head and a tail? **ii** two tails? **iii** at least one head?

Solution

a Let H = heads and T = tails.

i The possible outcomes are {HH, HT, TH, TT}. Finding the 4 outcomes by listing.

ii

		1st coin	
		H	T
2nd coin	H	HH	TH
	T	HT	TT

Using a table ensures that all outcomes are counted.

The possible outcomes are {HH, HT, TH, TT}.

b i $P(\text{a head and a tail}) = P(\text{HT or TH}) = \frac{2}{4} = \frac{1}{2}$

2 chances out of 4.

ii $P(2 \text{ tails}) = P(\text{TT}) = \frac{1}{4}$

1 chance out of 4.

iii $P(\text{at least one head}) = P(1 \text{ or } 2 \text{ heads})$
 $= P(\text{HT or TH or HH})$
 $= \frac{3}{4}$

‘At least one’ means ‘1 or more’.

3 chances out of 4.

Example 9

While staying at a hotel, John-Paul can choose one item from each course of a breakfast menu.



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1st course	2nd course
Cereal (C)	Bacon and eggs (B)
Raisin toast (R)	Pancakes (P)
Watermelon (W)	Sausages and hash browns (S)
Yoghurt (Y)	

- a Use a table to list all the different 2-course breakfasts available.
- b If one of these combinations is chosen at random, find the probability that it includes:
- watermelon but not pancakes
 - yoghurt or pancakes
 - yoghurt and pancakes.

Solution

a

		1st course			
		C	R	W	Y
2nd course	B	CB	RB	WB	YB
	P	CP	RP	WP	YP
	S	CS	RS	WS	YS

$4 \times 3 = 12$ different breakfasts possible.

- b i $P(\text{watermelon, not pancakes}) = \frac{9}{12} = \frac{3}{4}$ column W + rows B and S
- ii $P(\text{yoghurt or pancakes}) = \frac{6}{12} = \frac{1}{2}$ column Y + row P
- iii $P(\text{yoghurt and pancakes}) = \frac{1}{12}$ $P(YP)$

Worksheet

Tree diagrams

MAT09SPWK10141

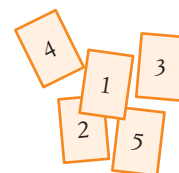
Tree diagrams

A **tree diagram** lists all the possible outcomes of each stage. Branches stretch out to show the possible pathways of outcomes at each step or stage. An outcomes column at the end of the diagram lists the sample space.

Example 10

From a set of cards numbered 1 to 5, two cards are drawn at random (without replacement) to make a two-digit number.

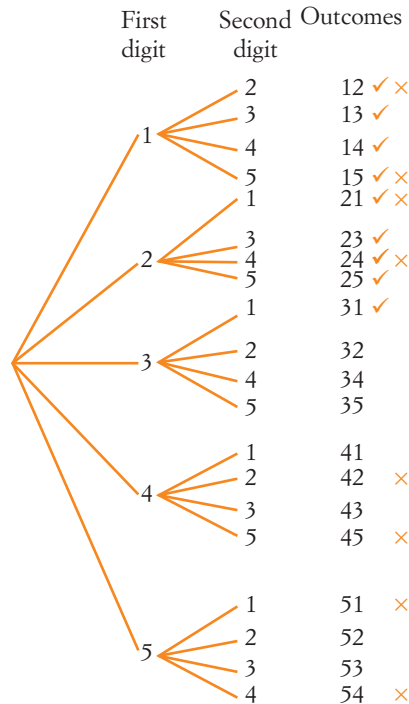
- a Show all outcomes in the sample space using a tree diagram.
- b What is the probability of making a number:
- less than 32?
 - divisible by 3?



Solution

- a** This experiment has two steps: the first digit and the second digit.
For the first digit, there are 5 possible digits: 1, 2, 3, 4 or 5.
For the second digit, there are only 4 possible digits left because the first digit cannot be used again ('without replacement').
There are $5 \times 4 = 20$ possible two-digit numbers in the sample space.

Using a tree diagram ensures that all outcomes are counted.



- b i** There are 9 numbers that are less than 32 (ticked ✓ on the tree diagram).
 $\therefore P(\text{number} < 32) = \frac{9}{20}$
- ii** There are 8 numbers that are divisible by 3 (crossed × on the tree diagram).
 $\therefore P(\text{number divisible by 3}) = \frac{8}{20} = \frac{2}{5}$

Exercise 12-05 Two-step experiments

- 1** Two people out of Ben, Anita, William, Lachlan and Noreen are to be selected for the positions of president and secretary in a tennis club.
- a** List the possible pairings for the positions of president and secretary.
- b** What is the probability of the committee consisting of:
- Ben and Anita?
 - two males?
 - a female president?
 - at least one female?



See Example 8

- 2 The digits 5, 6, 8 and 9 are written on cards and put in a bag. Two cards are drawn at random to make a two-digit number.
- Make a list of all possible outcomes.
 - How many two-digit numbers are possible?
 - Find, as a decimal correct to two decimal places, the probability that the number is:
 - even
 - greater than 60
 - between 60 and 90
 - divisible by 3

See Example 9

- 3 A coin and a die are tossed together.
- Copy and complete the table to display the sample space.

		Die					
		1	2	3	4	5	6
Coin	H						
	T						

- Find the probability of tossing:
 - a head and 4
 - a tail and an even number
 - a tail and a number less than 5.
- 4 Two dice are rolled and the sum of the numbers is calculated.
- Copy and complete this table to show all possible sums.

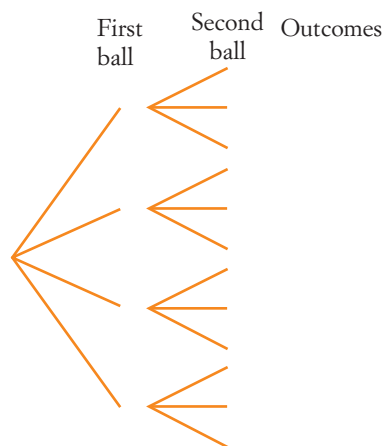
		1st die					
		1	2	3	4	5	6
2nd die	1						
	2					7	
	3						
	4		6				
	5						
	6						

- Find the probability of rolling a sum
 - of 3
 - of 7
 - of at least 6
 - less than 10
 - that is even
 - between 3 and 7.

See Example 10

- 5 A bag contains 2 blue balls, 1 yellow ball and 1 green ball. Two balls are drawn out of the bag at random.

- Copy and complete the tree diagram to list the sample space.
- Find the probability of drawing:
 - a blue and green ball, in that order
 - a yellow ball and a green ball, in any order
 - a blue ball and a yellow ball
 - at least one blue ball.



- 6 The numbers 3, 4, 5 and 8 are written on separate cards and placed in a bag. Two cards are drawn at random, one after the other, to form a two-digit number.
- Use a tree diagram to show all possible outcomes if the cards are drawn *with replacement*, that is, the same card can be used twice.
 - Find the probability that the two-digit number formed:
 - ends in 4
 - is an odd number
 - is less than 45
 - begins with 3 and is even.
- 7 The weather this weekend will be either fine or rain each day, with each outcome being equally likely.
- Draw a tree diagram to show the possible outcomes for Saturday and Sunday.
 - What is the probability that:
 - it is fine on both days?
 - Saturday is fine and it rains on Sunday?
 - it rains on at least one day of the weekend?
- 8 Two cards are chosen at random with replacement from a set of 4 cards consisting of the 2 red Kings and 2 red Queens from a normal deck of cards. Calculate the probability that the cards are:
- both Kings
 - a King and a Queen
 - a King and Queen of the same suit
 - both Queens
 - at least one King
 - no Queens
- 9 Draw a table for the sample space when two dice are rolled. Find the probability of rolling:
- two numbers the same
 - a 3 and a 6 (in any order)
 - at least one 2
 - a 4 or a 5 on only one of the dice.
- 10 While staying at a hotel, John-Paul can choose one item from each course of a breakfast menu.

1st course	2nd course
Cereal (C)	Bacon and eggs (B)
Raisin toast (R)	Pancakes (P)
Watermelon (W)	Sausages and hash browns (S)
Yoghurt (Y)	

- Find all the different 2-course breakfasts available:
 - by listing them
 - using a tree diagram.
 - If one of these combinations is chosen at random, find the probability that it includes raisin toast or bacon and eggs.
- 11 From a set of cards numbered 1 to 5, two cards are drawn at random to make a two-digit number.
- Show all outcomes in the sample space:
 - by simply listing them
 - using a table.
 - What is the probability of making a number:
 - that is odd?
 - greater than 30?

Power plus

- 1 Jenny needs to roll a 6 with a die to start moving on a board game. What is the probability that Jenny will start the game:
a on the first roll of the die? **b** on the second roll? **c** on the third roll?



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- 2 Consider a family with four children. Use a tree diagram to list the 16 possible orders of boys and girls in the family. Find the probability of the family having:
a 3 boys, 1 girl **b** no boys **c** 2 boys and 2 girls
d at least 1 girl **e** at most one girl **f** more than 2 boys.
- 3 A bag contains 5 yellow marbles and 4 red marbles. Two draws are made without replacement. Calculate the probability of drawing:
a two yellow marbles
b a yellow marble first and a red marble second
c a red marble and a yellow marble in any order
d at most one yellow marble.
- 4 The 18th-century naturalist George Louis Leclerc, Comte de Buffon, proposed the following problem.
 'A large piece of paper is ruled with parallel lines exactly as far apart as the length of a needle. The needle is then thrown on to the paper from a distance. What is the probability that the needle will cross one of the lines when it has landed?'
a Perform this experiment over 200 trials with a matchstick or a needle to find the required probability (the exact answer involves π).
b Write a brief report summarising your results.

Language of maths

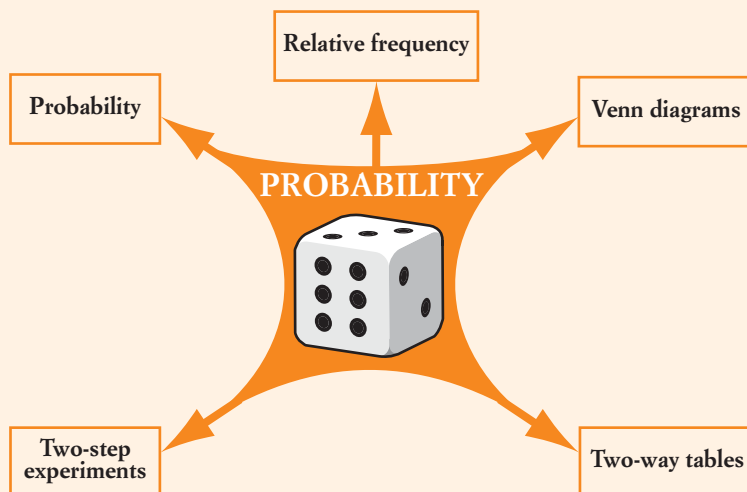
at least	at most	calculated probability	certain
chance experiment	complementary event	die/dice	even chance
event	expected frequency	experimental probability	favourable
frequency	impossible	mutually exclusive events	random
relative frequency	replacement	sample space	theoretical probability
trial	two-step experiment	two-way table	Venn diagram

- Which term from the above list is another name for **theoretical probability**?
- What is the meaning of **relative frequency**?
- What happens to the experimental probability of an event as the number of trials becomes very large?
- Which term describes the set of all possible outcomes of an experiment?
- Name the event that is **complementary** to winning first prize in Lotto.
- What is drawn using two or more circles, often overlapping?

Topic overview

- Copy and complete.
 - The things I understand about probability that I did not understand before are ...
 - The things I am still not confident in doing in this chapter are ... (Give an example of each difficulty.)
 - The steps I will take to overcome my problems with this chapter are ...

Copy (or print) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Chapter 12 revision

See Exercise 12-01

- 1 A jar contains only red and yellow marbles. The probability of selecting a red marble from the jar is 40%.
- What is the probability of selecting a yellow marble?
 - If there are 40 marbles in the bag, how many red marbles are there?

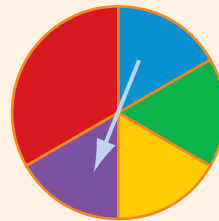
See Exercise 12-01

- 2 A card is drawn randomly from a normal deck of cards. Find the probability that it is:
- a red card
 - a King or a Queen card
 - a card with a number greater than 6

See Exercise 12-02

- 3 For the spinner shown, the red sector is twice as large as each of the other sectors. The spinner was spun 200 times and the results are shown in the table.

Outcome	Frequency
Blue	29
Green	35
Yellow	30
Purple	34
Red	72

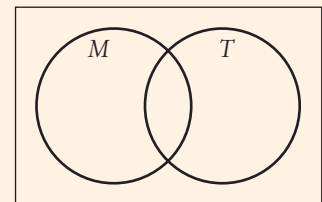


- Find the experimental probability (as a decimal) that the arrow lands on:
 - red
 - blue or green
 - not purple
 - green.
- Find the theoretical probability (as a decimal) that the arrow lands on:
 - red
 - blue or green
 - not purple
 - green.
- Are the experimental probabilities from part **a** similar to the theoretical probabilities from part **b**?
- If the spinner is spun 500 times, calculate the expected frequency of spinning red using the theoretical probability. Select the correct answer **A**, **B**, **C** or **D**.

A 100 **B** 85 **C** 167 **D** 200

See Exercise 12-03

- 4 30 teenagers were surveyed on whether they liked to listen to music (M) or watch TV (T). 21 students liked music and 18 liked TV, while 4 students did not like either.



- How many students liked both music and TV?
- Copy and complete the Venn diagram.
- Find the probability of randomly selecting a teenager who likes:
 - music but not TV
 - TV
 - music or TV but not both
 - both or neither music nor TV.

See Exercise 12-04

- 5 A group of men and women was surveyed on whether they worked full-time or part-time.
- How many people were surveyed?
 - Find the probability (as a percentage to one decimal place) of selecting a person who:

	Full-time work	Part-time work
Male	56	41
Female	45	68

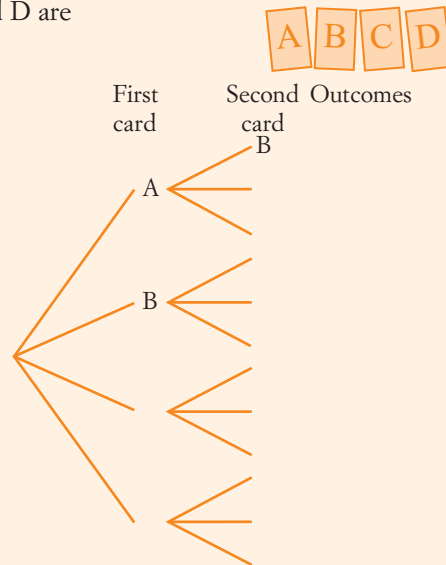
- works full-time
 - is female and works part-time
 - does not work full-time
 - works part-time or is female.
- c If a part-time worker is selected at random, what is the probability that the person is female?

- 6 Four cards labelled with the letters A, B, C and D are placed in a bag.

See Exercise 12-05

Two cards are drawn from the bag at random without replacement.

- List all of the possible outcomes.
- Copy and complete the tree diagram.
- Use the tree diagram to find the probability of drawing two cards with:
 - the same letter
 - the letters A and B
 - the letters A or B or both
 - no letter D.



- 7 Two dice are rolled and the difference between the numbers is calculated.

See Exercise 12-06

- Copy and complete the table to show the sample space.

		Dice 1					
		1	2	3	4	5	6
Dice 2	1						
	2						
	3						3 $6 - 3 = 3$
	4						
	5				1 $5 - 4 = 1$		
	6		4 $6 - 2 = 4$				

- Find the probability of rolling a difference of:

- 0
- at least 2
- 4 or 5.

Mixed revision 4

See Exercise 11-04

- 1 Test whether the point $(2, 3)$ lies on each line.

a $2x + y = 4$

b $y = 2x - 1$

c $x + y = 5$

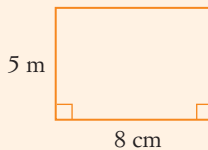
See Exercise 13-05

- 2 The rectangle on the left has been enlarged to make the rectangle on the right.

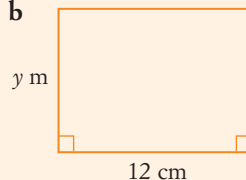
a What is the scale factor?

b Find the value of y .

a



b



See Exercise 12-01

- 3 The probability of Eva hitting a bullseye on a target is 0.2. Find the probability of Eva missing the bullseye.

See Exercise 12-04

- 4 A class was surveyed on whether its students could swim. The results are shown below.

	Can swim	Cannot swim
Boys	13	2
Girls	9	3

a Find the probability of randomly selecting a student who:

i is a boy

ii is a boy or cannot swim

iii is a girl who cannot swim

iv is a girl or can swim but not both.

b If a swimmer is selected at random, what is the probability that it's a boy?

See Exercise 13-06

- 5 The scale on a tourist map of Sydney is $2 \text{ cm} = 500 \text{ m}$. If the scaled distance from Circular Quay station to the Opera House is 2.5 cm, what is the actual distance?

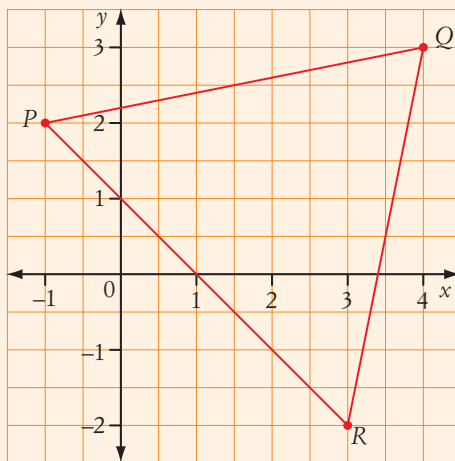
Questions 6 to 9 refer to this triangle whose vertices are $P(-1, 2)$, $Q(4, 3)$ and $R(3, -2)$.

See Exercise 11-01

- 6 Find the length of:

a side PQ as a surd

b side PR correct to one decimal place.



See Exercise 11-01

- 7 Show that $\triangle PQR$ in question 6 is isosceles.

See Exercise 11-02

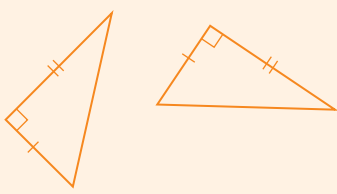
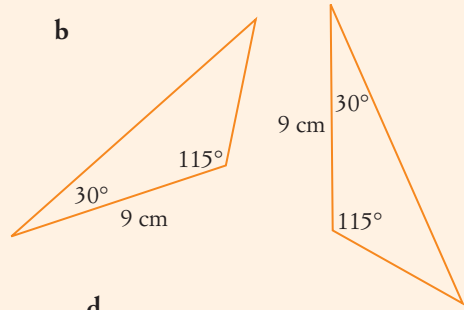
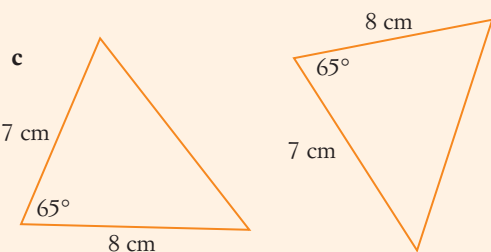
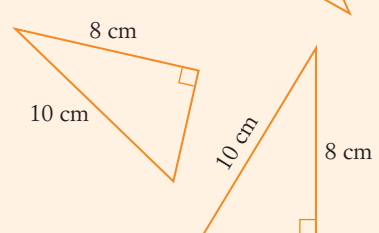
- 8 Find the coordinates of the midpoint of side PR .

See Exercise 11-03

- 9 Find the gradient of:

a PQ

b PR

- 10 Graph $y = 4 - 2x$ on a number plane and write:
a its x -intercept **b** its y -intercept. See Exercise 11-04
- 11 Daniel rolled a biased die 90 times and the number 6 came up 24 times. See Exercise 12-02
a What is the experimental probability of rolling a 6?
b If the same die was rolled 400 times, what is the expected frequency of rolling a 6?
- 12 **a** Sketch the graph of $y = x^2 - 2$ See Exercise 11-09
b Write the coordinates of the vertex.
c Is the graph concave up or concave down?
- 13 Which congruence test (SSS, SAS, AAS or RHS) can be used to prove that each pair of triangles are congruent? See Exercise 13-02
- a**
- 
- b**
- 
- c**
- 
- d**
- 
- 14 What is the equation of the horizontal line passing through $(-2, 4)$? See Exercise 11-04
- 15 The numbers 1 to 30 are written on cards and the cards are placed in a bag. If one card is drawn at random from the bag, what is the probability that it is:
a a multiple of 5? **b** greater than 10? See Exercise 12-01
- 16 **a** Find the gradient and y -intercept of the line with equation $y = \frac{1}{2}x + 3$ See Exercise 11-05
b Graph a line with gradient 2 and y -intercept -5 and write its equation.
- 17 The increase in pressure experienced by a scuba diver is directly proportional to the depth under the water. The increase in pressure at depth 40 m is 235 kPa (kilopascals). See Exercise 11-08
a What increase in pressure is experienced at a depth of 32 m?
b At what depth, to the nearest metre, is the increase in pressure 735 kPa?

Mixed revision 4

See Exercise 12-05

- 18** Two dice are rolled together. Copy and complete the table to show the sample space.

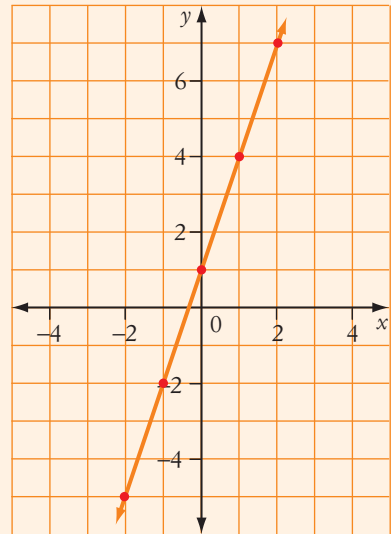
Find the probability of rolling:

- a** a 3 and a 6 (in any order)
- b** at least one 2
- c** two numbers the same
- d** a 4 or a 5 but not both.

		1st die					
		1	2	3	4	5	6
2nd die	1						
	2					5, 2	
	3						
	4		2, 4				
	5						
	6						

See Exercise 11-06

- 19** **a** Find the gradient of this line.
b Find the equation of the line above.



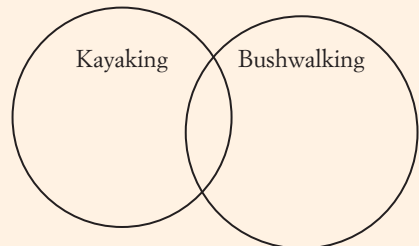
See Exercise 11-10

- 20** What is the equation of a circle with centre (0, 0) and radius 4 units?

See Exercise 12-03

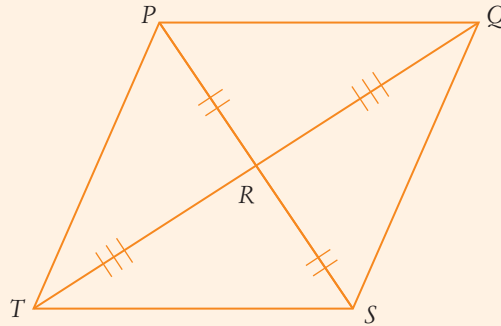
- 21** 40 students at a school camp can select kayaking or bushwalking as an activity, or both. A total of 35 students chose kayaking, 20 chose bushwalking, including some who chose both. There weren't any students who chose neither.

- a** Copy and complete the Venn diagram representing these students.



- b** What is the probability that a student selected at random:
- i** chose kayaking and bushwalking?
 - ii** chose kayaking or bushwalking but not both?

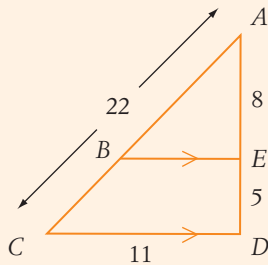
- 22 The diagonals of this rhombus divide the rhombus into four congruent triangles.



See Exercise 13-03

- a Prove that $\triangle PRQ \equiv \triangle PRT$.
 b Hence find the size of $\angle PRQ$.
 c Copy and complete: The diagonals of a rhombus _____ each other at _____.
- 23 Two similar rectangles have their areas in the ratio 1 : 7.84. If the length of the larger rectangle is 5.04 m, calculate the length of the smaller rectangle.

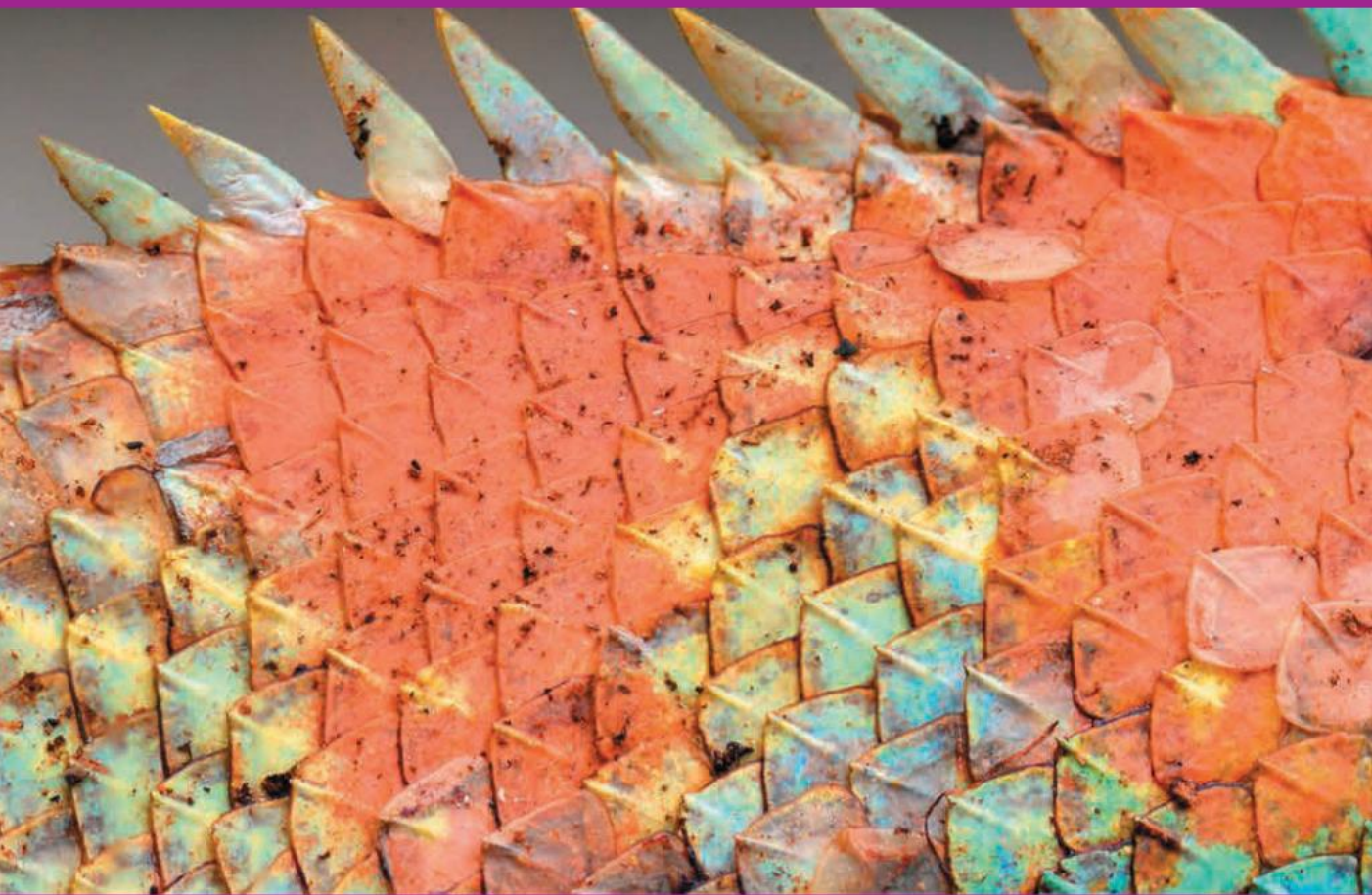
- 24 a Prove that $\triangle ABE \parallel \triangle ACD$.
 b Find the length of:
 i BE ii BC



Stage 5.3

See Exercise 13-07

See Exercise 13-09

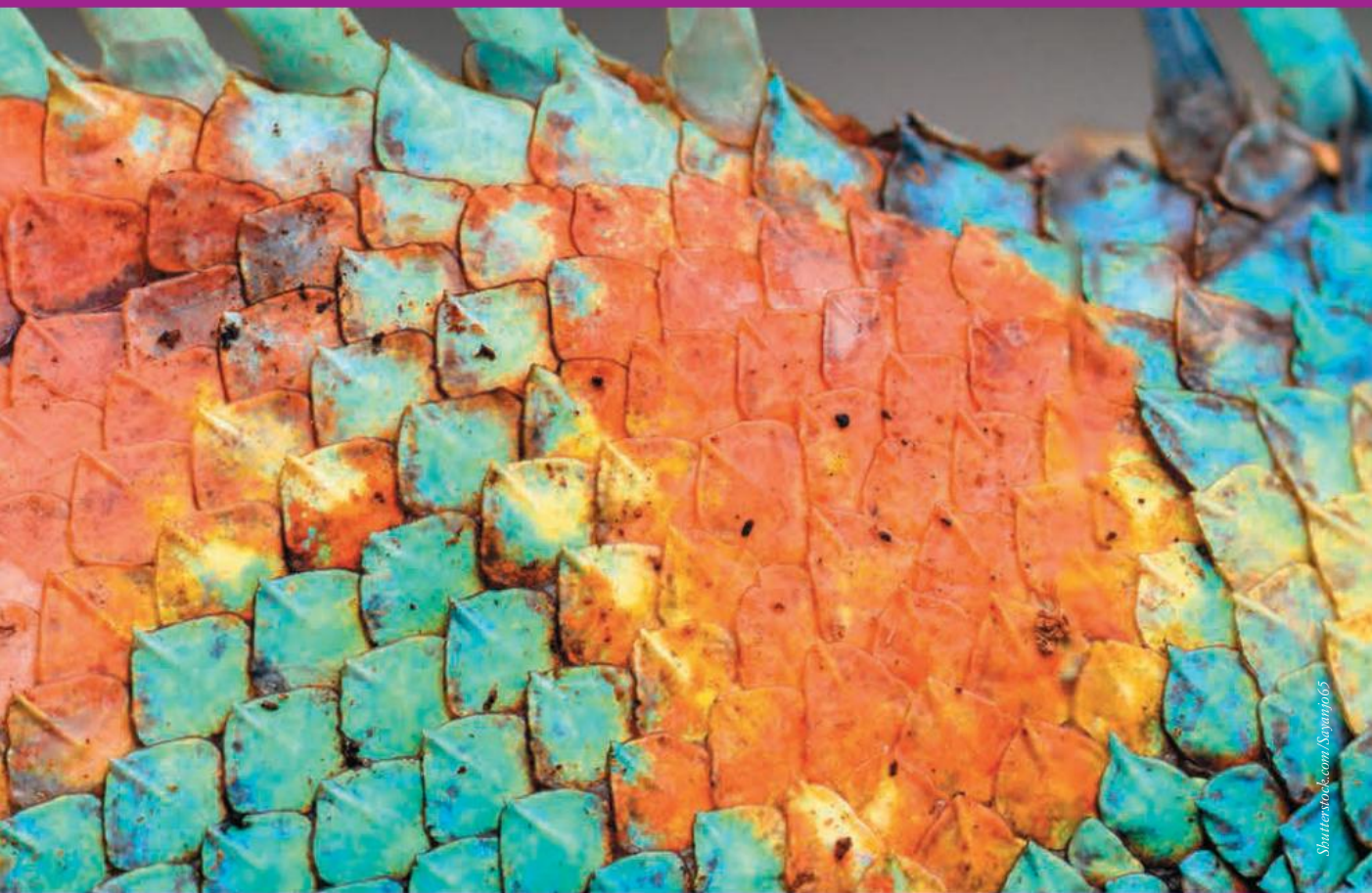


13

Measurement and geometry

Congruent and similar figures

MC (Maurits Cornelis) Escher (1898–1972) was a Dutch graphic artist who used congruent figures to create tessellations (repeated 'tile' patterns). Although he had no mathematical training, Escher was talented in visualising and creating complex geometrical patterns, especially those involving 'impossible' shapes and structures.



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Chapter outline

Proficiency strands

13-01 Tests for congruent triangles	U	F	PS	R	C
13-02 Congruent triangle proofs	U	F	PS	R	C
13-03 Using congruence to prove geometrical properties	U	F	PS	R	C
13-04 Similar figures	U	F		R	C
13-05 Properties of similar figures	U	F		R	C
13-06 Scale diagrams	U	F	PS		C
13-07 Areas of similar figures*	U	F		R	
13-08 Tests for similar triangles	U	F	PS	R	C
13-09 Similar triangle proofs*	U	F	PS	R	C

*STAGE 5.3

Wordbank

congruence test One of four tests for proving that triangles are congruent: SSS, SAS, AAS and RHS

congruent Identical; exactly the same (symbol: $\equiv$)

enlargement An increase in the size of a shape

image A transformed shape after it has been enlarged or reduced

included angle The angle between two given sides of a shape

scale factor The amount by which a shape has been enlarged or reduced, equal to $\frac{\text{image length}}{\text{original length}}$

similar To have the same shape but not necessarily the same size, an enlargement or reduction (symbol: $\sim$)

superimpose To place one figure on top of another figure so that sides and angles match

In this chapter you will:

- develop the conditions for congruence of triangles
- formulate proofs involving congruent triangles and angle properties
- use the enlargement transformation to explain similarity and develop the conditions for triangles to be similar
- solve problems using ratio and scale factors in similar figures 
- identify and use the four tests for congruent triangles
- write formal proofs for congruent triangles
- (STAGE 5.3) investigate ratio of areas of similar figures
- measure and calculate using scale diagrams
- identify and use the four tests for similar triangles
- (STAGE 5.3) write formal proofs for similar triangles

SkillCheck

Worksheet

StartUp assignment 13

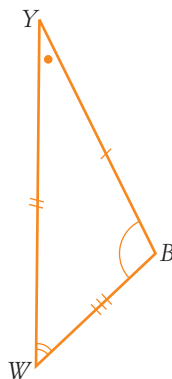
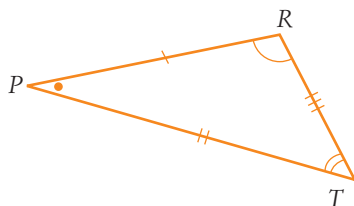
MAT09MGWK10144

1 For each pair of congruent figures, list all pairs of:

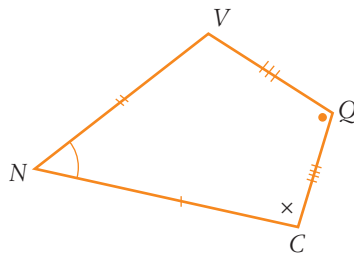
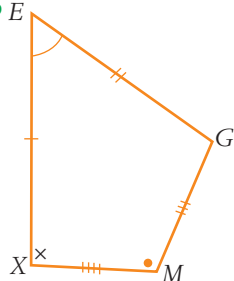
i equal angles

ii equal sides

a



b



Investigation: Congruent triangles

Work in groups of three or four to complete this investigation.

You will need: paper, a ruler, a pencil, a protractor, compasses and scissors.

- 1
 - a Individually, construct a triangle with side lengths 5 cm, 7 cm and 9 cm.
 - b Use your protractor to measure the angles of the triangle.
 - c Compare your results with those of the students in your group. Are all your group's triangles congruent?
 - d Use your results to decide whether each statement is true or false:
 - i *If three sides of one triangle are respectively equal to the three sides of another triangle, then the matching angles of the triangles are equal.*
 - ii *If all sides of one triangle are equal to all sides of another triangle, then the two triangles are congruent.*

- 2
 - a Construct $\triangle ABC$ so that $AB = 5$ cm, $AC = 7$ cm and $\angle A = 65^\circ$.

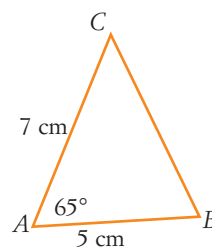
Since $\angle A$ lies between AB and AC , $\angle A$ is called the **included angle**.

- b Measure the third side of the triangle, and the sizes of the other two angles.

- c Compare your results with those of others in your group.

- d Use your results to decide whether this statement is true or false:

If two sides and the included angle of one triangle are respectively equal to two sides and the included angle of another triangle, then the two triangles are congruent.

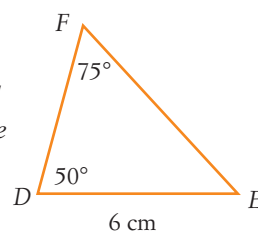
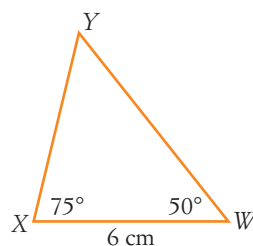


- 3
 - a Construct $\triangle XWY$ with $XW = 6$ cm, $\angle X = 75^\circ$ and $\angle W = 50^\circ$.
 - b Measure $\angle Y$ and the other two sides of the triangle.
 - c Compare your results with those of your group members.
 - d Now construct $\triangle DEF$ with $DE = 6$ cm, $\angle D = 50^\circ$ and $\angle F = 75^\circ$.

- e Are $\triangle XWY$ and $\triangle DEF$ congruent? Give reasons for your answer.

- f Use your results to decide whether the following statement is true or false:

If two angles and one side of one triangle are respectively equal to two angles and the matching side of another triangle, then the two triangles are congruent.



- 4
 - a Construct $\triangle GHK$ with $\angle G = 90^\circ$, $GH = 8$ cm, $GK = 5$ cm.
 - b What is another name for the side GK ?
 - c Measure the other side and angles of the triangle and compare your results with those of other members in your group. Are their triangles congruent to your triangle?
 - d Use your results to decide whether the following statement is true or false:

If the hypotenuse and a second side of one right-angled triangle are respectively equal to the hypotenuse and a second side of another right-angled triangle, then the triangles are congruent.

- 5 a Draw a triangle with angles 40° , 63° and 77° .
 b Compare your triangle with those of other members of your group. Are the triangles congruent? Give reasons.
- 6 Use your results to determine what information is needed to prove two triangles are congruent.

Homework sheet

Congruent figures

MAT09MGHS10025

13-01 Tests for congruent triangles

Two figures are **congruent** if they are identical in shape and size.

For **congruent figures**:

- **matching sides** are equal
- **matching angles** are equal



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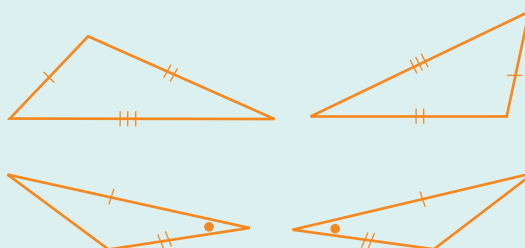
There are four sets of conditions that can be used to determine if **two triangles** are congruent. These are called the **tests for congruent triangles** or **congruence tests**.

Summary

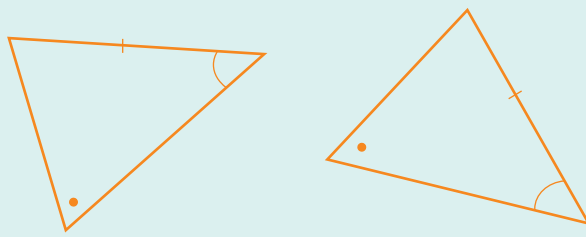
There are four tests for congruent triangles: **SSS**, **SAS**, **AAS** or **RHS**.

Two triangles are congruent if:

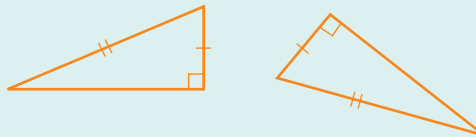
- the three sides of one triangle are respectively equal to the three sides of the other triangle (**SSS rule**)
- two sides and the **included** angle of one triangle are respectively equal to two sides and the **included** angle of the other triangle (**SAS rule**)



- two angles and one side of one triangle are respectively equal to two angles and the matching side of the other triangle (**AAS rule**)

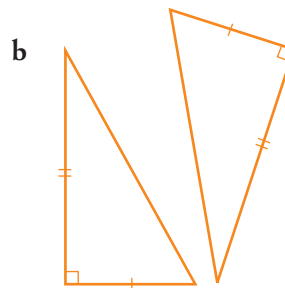
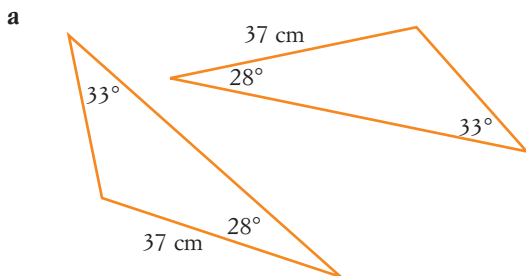


- they are right-angled and the hypotenuse and another side of one triangle are respectively equal to the hypotenuse and another side of the other triangle (**RHS rule**).



Example 1

Which congruence test (SSS, SAS, AAS or RHS) can be used to prove that each pair of triangles are congruent?

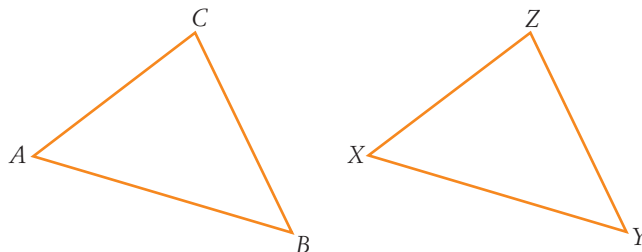


Solution

- a** Two angles and one side of one triangle are equal to two matching angles and one matching side of the other triangle.
 $\therefore$ The congruence test is AAS.
- b** Two sides and the included angle of one triangle are equal to two matching sides and the included angle of the other triangle.
 $\therefore$ The congruence test is SAS.

The congruence symbol $\equiv$

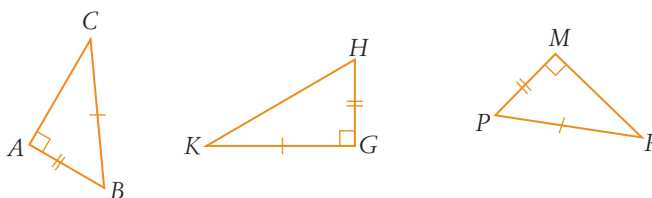
The symbol for 'is congruent to' is a special equals sign, written as ' $\equiv$ ' (which also means 'is identical to'). The two triangles below are congruent, so we can write $\triangle ABC \equiv \triangle XYZ$, which is read 'triangle ABC is congruent to triangle XYZ '.



When using this notation, we must make sure that the vertices (angles) of the congruent figures are written in matching order: $\triangle ABC \equiv \triangle XYZ$ means $\angle A = \angle X$, $\angle B = \angle Y$, $\angle C = \angle Z$.

Example 2

- a Decide which two of these triangles are congruent and state the congruence test used.



- b Use the correct notation to write a congruence statement relating those two triangles.

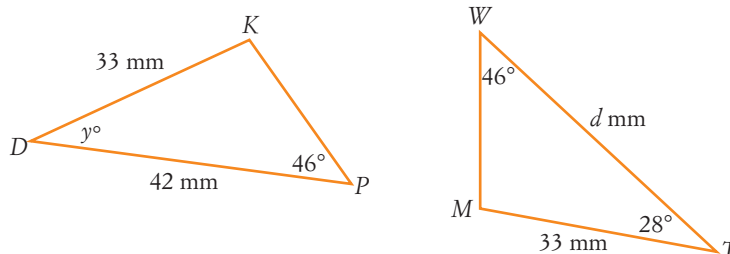
Solution

- a The three right-angled triangles all have two equal sides but BC and PR are both hypotenuses while KG is not.
 $\therefore \triangle CAB$ and $\triangle MPR$ are congruent by the RHS rule.
- b $\angle A$ matches with $\angle M$, $\angle B$ matches with $\angle P$,
 $\angle C$ matches with $\angle R$
 $\therefore \triangle ABC \equiv \triangle MPR$

Matching order of vertices.

Example 3

If the two triangles are congruent, find the value of each pronumeral.



Solution

Since $\triangle DPK \equiv \triangle TWM$,

$$DP = TW$$

$$\therefore d = 42$$

$$\angle T = \angle D$$

$$\therefore y = 28$$

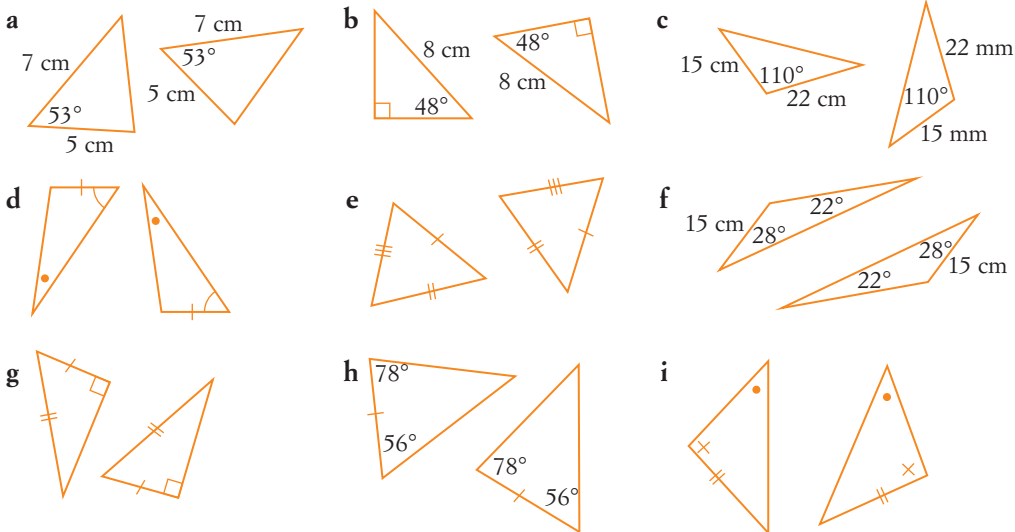
Matching sides of congruent triangles.

Matching angles of congruent triangles.

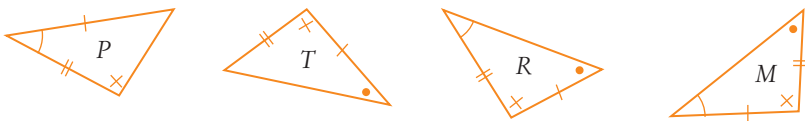
Exercise 13-01 Tests for congruent triangles

- 1 Which congruence test (SSS, SAS, AAS or RHS) can be used to prove that each pair of triangles are congruent?

See Example 1



- 2 Find which two triangles are congruent in each set. Select the correct answer A, B, C or D.



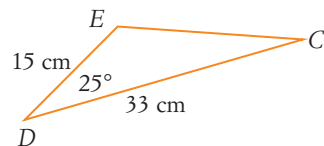
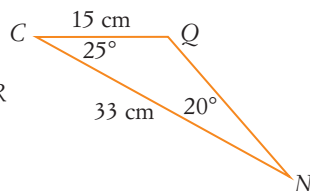
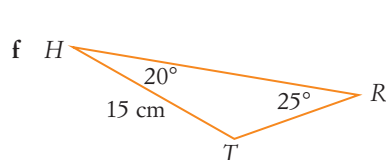
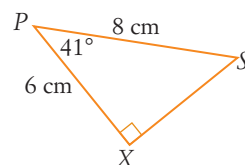
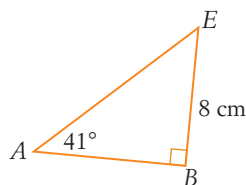
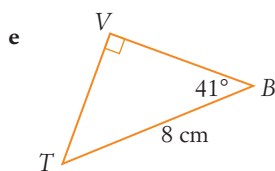
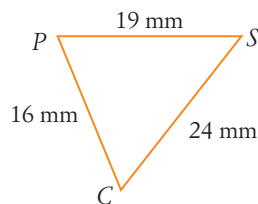
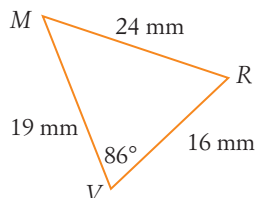
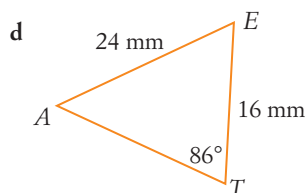
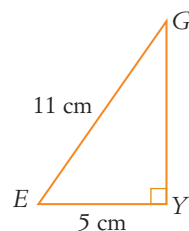
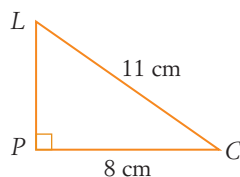
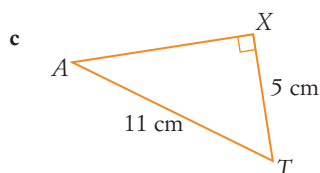
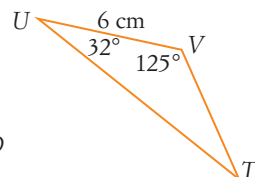
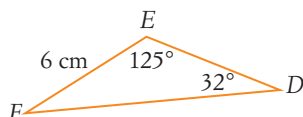
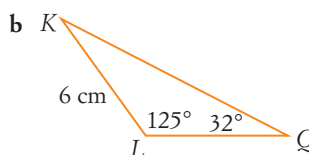
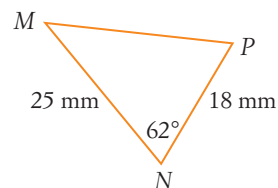
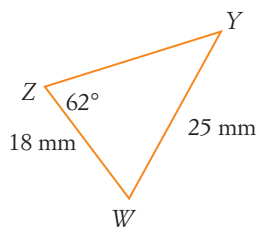
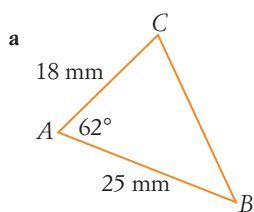
A P and T

B T and R

C M and R

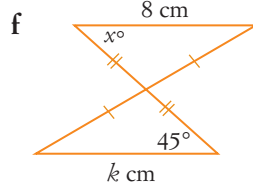
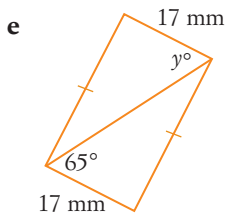
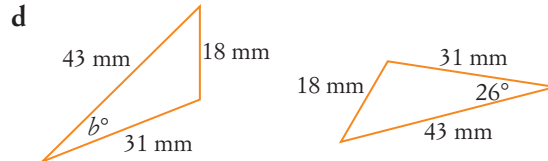
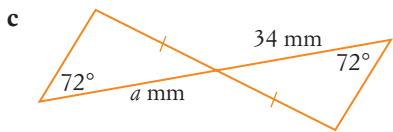
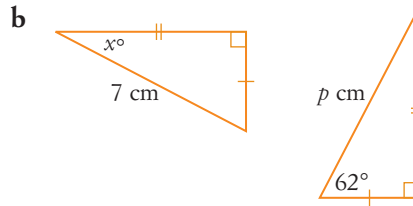
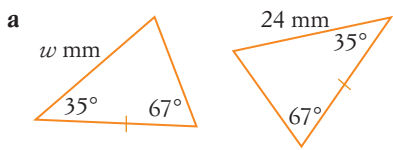
D M and P

- See Example 2** 3 For each set of triangles:
- decide which two are congruent and state the congruence test used
 - use the correct notation to write a congruency statement relating those two triangles.

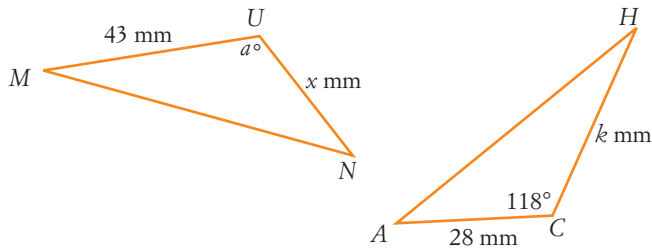


4 Find the value of the pronumeral in each pair of congruent triangles.

See Example 3

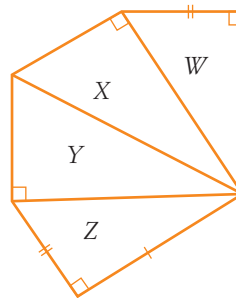


5 If $\triangle NUM \equiv \triangle ACH$, find the value of each pronumeral.



6 The diagram shows four right-angled triangles joined together. State whether each pair of triangles are congruent, and if so, state the test used.

- a** W and X **b** W and Y **c** W and Z
d X and Y **e** X and Z **f** Y and Z



13-02 Congruent triangle proofs

To formally prove that two triangles are congruent, there is a specific format involving using one of the four congruence tests SSS, SAS, AAS or RHS.

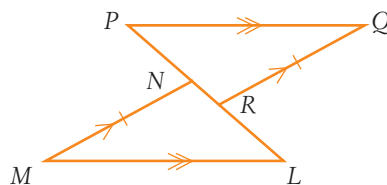
Example 4

In the diagram, $PQ \parallel LM$, $QR \parallel MN$ and $QR = MN$.
Prove that $\triangle PQR \equiv \triangle LMN$.

Solution

In $\triangle PQR$ and $\triangle LMN$:

$$\begin{aligned} QR &= MN && \text{(given)} \\ \angle P &= \angle Q && \text{(alternate angles, } PQ \parallel LM) \\ \angle QRP &= \angle MNL && \text{(alternate angles, } QR \parallel MN) \\ \therefore \triangle PQR &\equiv \triangle LMN && \text{(AAS)} \end{aligned}$$



Identifying the triangles in matching order of vertices.

Stating each part of the congruence test, giving reasons.

Concluding the congruence proof, stating the test used.

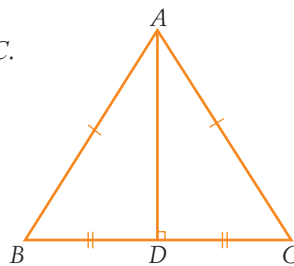
Example 5

In $\triangle ABC$, $AB = AC$ and $AD \perp BC$.
Prove that $\triangle ABD \equiv \triangle ACD$ and hence AD bisects $\angle BAC$.

Solution

In $\triangle ABD$ and $\triangle ACD$:

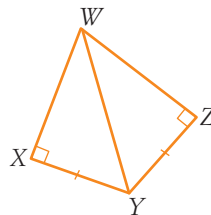
$$\begin{aligned} AB &= AC && \text{(given)} \\ AD &\text{ is common.} \\ \angle ADB &= \angle ADC = 90^\circ && (AD \perp BC) \\ \therefore \triangle ABD &\equiv \triangle ACD && \text{(RHS)} \\ \therefore \angle BAD &= \angle CAD && \text{(matching angles of congruent triangles)} \\ \therefore AD &\text{ bisects } \angle BAC. \end{aligned}$$



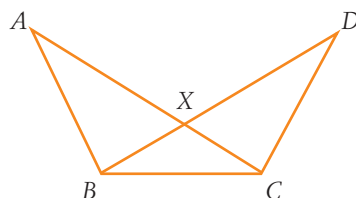
Exercise 13-02 Congruent triangle proofs

See Example 4

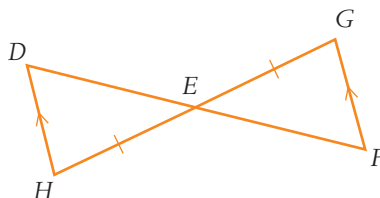
- 1 Prove that $\triangle WXY \equiv \triangle WZY$.



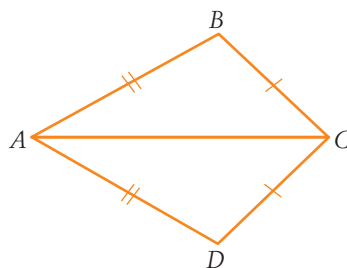
- 2 If $\angle ABC = \angle DCB$ and $AB = DC$
in the diagram, prove that $\triangle ABC \equiv \triangle DCB$.



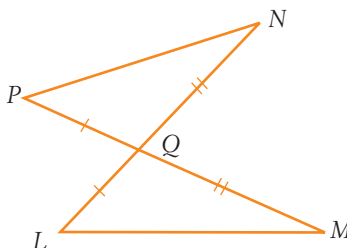
- 3 In the diagram, $EG = EH$ and $DH \parallel FG$.
Show that $\triangle DEH \equiv \triangle FEG$.



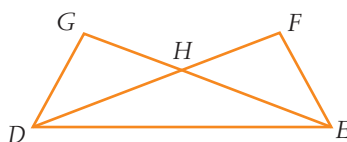
- 4 For this kite $ABCD$, prove that $\triangle ABC \equiv \triangle ADC$.



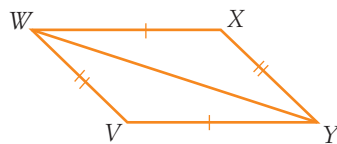
- 5 If $PQ = LQ$ and $NQ = MQ$,
prove that $\triangle PQN \equiv \triangle LQM$.



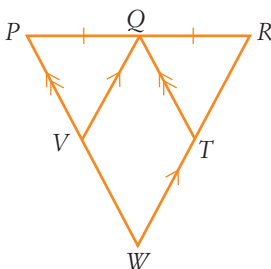
- 6 If $\angle DEG = \angle EDF$ and $GE = FD$,
prove that $\triangle DEG \equiv \triangle EDF$.



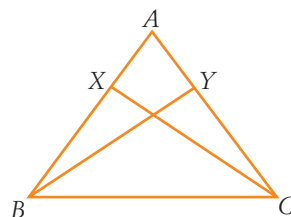
- 7 Prove that $\triangle WXY \equiv \triangle YVW$.



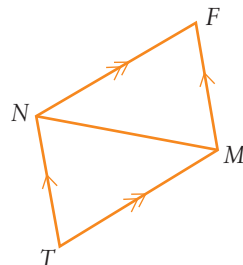
- 8 In this diagram, $PW \parallel QT$, $RW \parallel QV$ and $PQ = QR$.
Prove that $\triangle PVQ \equiv \triangle QTR$.



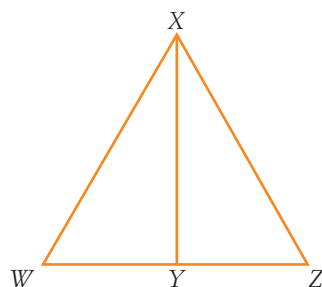
- 9 If $CX \perp AB$, $BY \perp AC$ and $XC = YB$,
 prove that $\triangle BCX \equiv \triangle CBY$.



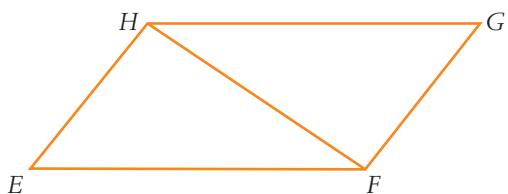
- 10 Prove that $\triangle FNM \equiv \triangle TMN$.



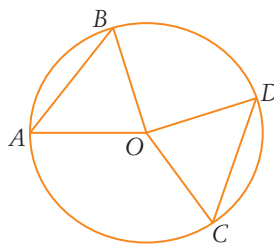
- 11 $XW = YZ$ in the isosceles triangle and Y is the midpoint
 of WZ . Prove that $\triangle WYX \equiv \triangle ZYX$.



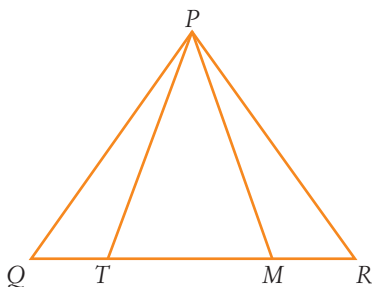
- 12 The opposite sides are equal for
 this quadrilateral. Prove that
 $\triangle EFH \equiv \triangle GHF$.



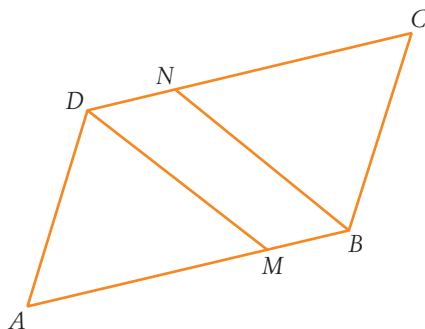
- 13 O is the centre of the circle and
 $AB = CD$. Prove that $\triangle AOB \equiv \triangle COD$.



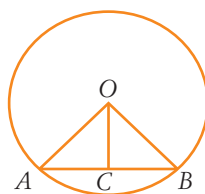
- 14 $\angle Q = \angle R$ in this isosceles triangle.
 If $QT = RM$, prove that $\triangle PQT \equiv \triangle PRM$.



- 15 $ABCD$ is a parallelogram and $DN = BM$.
Prove that $\triangle AMD \equiv \triangle CNB$.

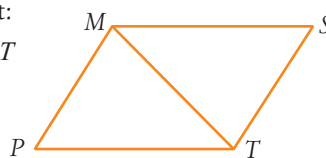


- 16 O is the centre of the circle and $OC \perp AB$.
Prove that $\triangle OAC \equiv \triangle OBC$.

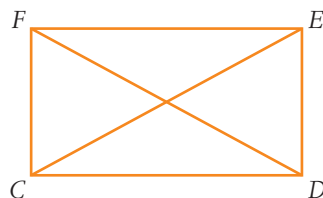


- 17 In the quadrilateral $MSTP$, opposite sides are equal. Prove that:
a $\triangle TPM \equiv \triangle MST$ b $\angle PMT = \angle STM$ c $PM \parallel ST$

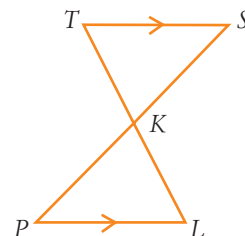
See Example 5



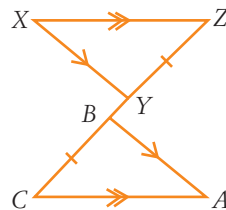
- 18 $CDEF$ is a rectangle. Prove that $\triangle FED \equiv \triangle EFC$ and that the diagonals of the rectangle are equal in length.



- 19 $TS \parallel PL$ and K is the midpoint of TL .
Prove that $\triangle TSK \equiv \triangle LPK$.

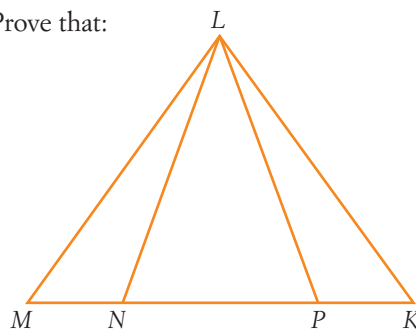


- 20 In the diagram, $AC \parallel XZ$, $XY \parallel AB$ and $BC = YZ$.
a Prove that $\angle XYZ = \angle ABC$.
b Hence prove that $\triangle XYZ \equiv \triangle ABC$.



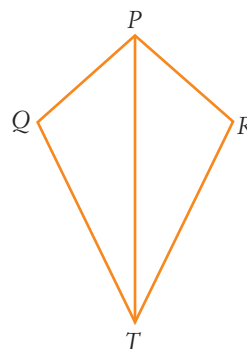
21 $LM = LK$ in the isosceles triangle LMK , and $MN = KP$. Prove that:

- $\triangle LMN \equiv \triangle LKP$
- $\triangle LNP$ is isosceles.



22 For this kite, prove that:

- $\triangle PQT \equiv \triangle PRT$
- $\angle Q = \angle R$
- PT bisects $\angle QTR$.



Worksheet

Proving properties
of quadrilaterals

MAT09MGWK10146

13-03

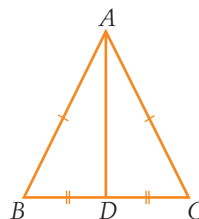
Using congruence to prove geometrical properties

Properties of triangles and quadrilaterals can be proved using the congruence tests.

Example 6

$\triangle ABC$ is an isosceles triangle. AD bisects BC .

- Prove that $\triangle ABD \equiv \triangle ACD$.
- Hence prove that in an isosceles triangle, the angles opposite the equal sides are equal.
- Prove that $AD \perp BC$.



Solution

- In $\triangle ABD$ and $\triangle ACD$:

$$AB = AC$$

AD is common.

$$BD = CD$$

$$\therefore \triangle ABD \equiv \triangle ACD$$

- $\therefore \angle B = \angle C$

$\therefore$ The angles opposite the equal sides in an isosceles triangle are equal.

($\triangle ABC$ is isosceles)

(AD bisects BC)

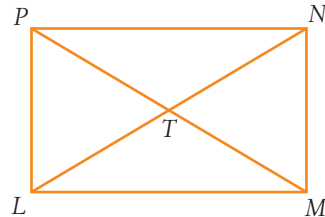
(SSS)

(matching angles of congruent triangles)

- c $\angle ADB = \angle ADC$ (matching angles of congruent triangles)
 $\therefore \angle ADB = \angle ADC = 90^\circ$ (angles on a straight line)
 $\therefore AD \perp BC$.

Example 7

- a If $LMNP$ is a rectangle, prove that $\triangle PNT \equiv \triangle MLT$.
 b Prove that the diagonals of a rectangle bisect each other.

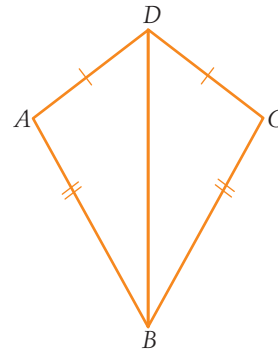


Solution

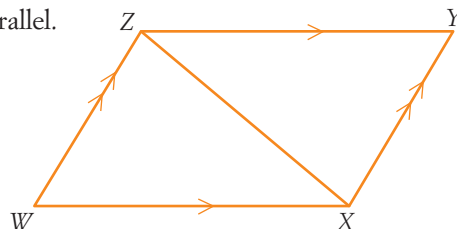
- a In $\triangle PNT$ and $\triangle MLT$:
 $PN = ML$ (opposite sides of a rectangle)
 $\angle PNT = \angle MLT$ (alternate angles, $PN \parallel ML$)
 $\angle PTN = \angle MTL$ (vertically opposite angles)
 $\therefore \triangle PNT \equiv \triangle MLT$ (AAS)
 b $\therefore PT = MT$ and $NT = LT$ (matching sides of congruent triangles)
 $\therefore T$ is the midpoint of the diagonals LN and MP .
 $\therefore$ The diagonals of a rectangle bisect each other.

Exercise 13-03 Using congruence to prove geometrical properties

- 1 $ABCD$ is a kite so adjacent sides are equal.
 a Prove that $\triangle ABD \equiv \triangle CBD$.
 b Prove that $\angle A = \angle C$.
 c Prove that diagonal DB bisects $\angle ADC$ and $\angle ABC$.
 d Copy the diagram and draw the other diagonal AC , intersecting DB at point X .
 e Prove that $\triangle DAX \equiv \triangle DCX$.
 f Prove that diagonal DB bisects diagonal AC .
 g Prove that $DB \perp AC$.
- 2 $WXYZ$ is a parallelogram with opposite sides parallel.
 a Copy the diagram into your book.
 b Prove that $\triangle WXZ \equiv \triangle YZX$.
 c Prove that $\angle W = \angle Y$.
 d Draw the other diagonal WY and prove that $\triangle WXY \equiv \triangle YZW$.
 e Prove that $\angle WXY = \angle YZW$.
 f What angle property of a parallelogram does this prove?

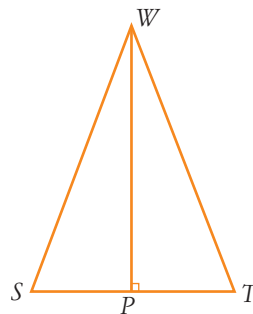


See Example 6

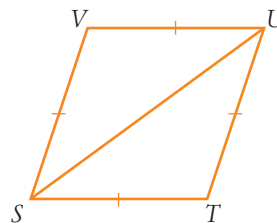


See Example 7

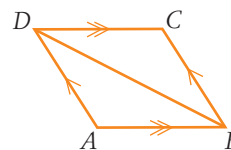
- 3 In the diagram, $\angle S = \angle T$ and $WP \perp ST$.
- Prove that $\triangle SPW \equiv \triangle TPW$.
 - Prove that the sides opposite equal angles in a triangle are equal.



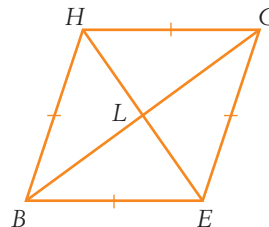
- 4 $STUV$ is a rhombus so all sides are equal.
- Prove that $\triangle VUS \equiv \triangle TUS$.
 - Prove that the diagonal US bisects the angles $\angle VUT$ and $\angle VST$.



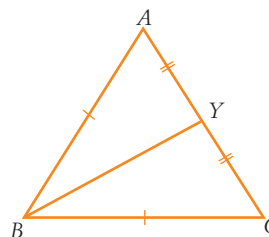
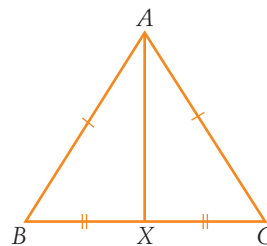
- 5 $ABCD$ is a parallelogram with opposite sides parallel.
- Prove that $\triangle ABD \equiv \triangle CDB$.
 - Prove that opposite sides of a parallelogram are equal.



- 6 $BEGH$ is a rhombus (a parallelogram with equal sides) whose diagonals BG and EH intersect at L .
- Prove that $\triangle BEL = \triangle GHL$.
 - Prove that the diagonals of a rhombus bisect each other.
 - $\triangle BEH$ is isosceles so which two angles are equal?
 - Hence prove that $\triangle BEL \equiv \triangle BHL$.
 - Hence prove that the diagonals of a rhombus cross at right angles.

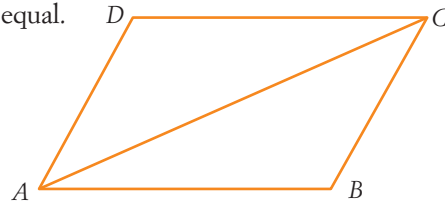


- 7 $\triangle ABC$ is an equilateral triangle ($AB = BC = AC$). X is the midpoint of BC .
- Prove that $\triangle ABX \equiv \triangle ACX$ and $\angle B = \angle C$.
 - In the second diagram, $\triangle ABC$ is redrawn so that Y is the midpoint of AC .
Prove that $\triangle BAY \equiv \triangle BCY$ and $\angle A = \angle C$.
 - Calculate the sizes of the three angles of $\triangle ABC$.



8 $ABCD$ is a quadrilateral in which opposite sides are equal.

- a Prove that $\triangle ACD \equiv \triangle ACB$.
- b Prove that $AB \parallel CD$ and $AD \parallel BC$.
- c What type of quadrilateral is $ABCD$?

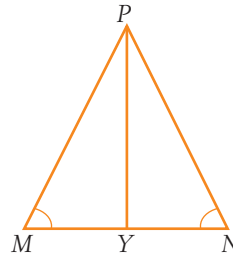


Worked solutions
Using congruence to
prove geometrical
properties

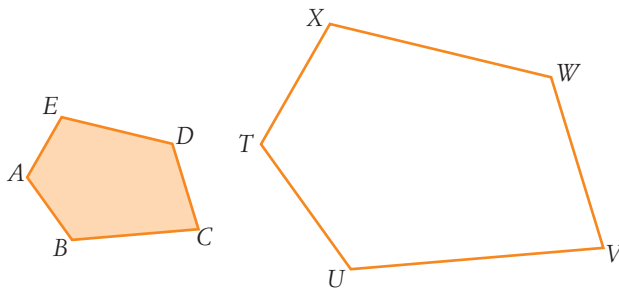
MAT09MGWS10056

9 In $\triangle PMN$, $\angle M = \angle N$ and YP bisects $\angle MPN$.

- a Prove that $\triangle PMY \equiv \triangle PNY$.
- b Prove that $MY = NY$ and $PY \perp MN$.



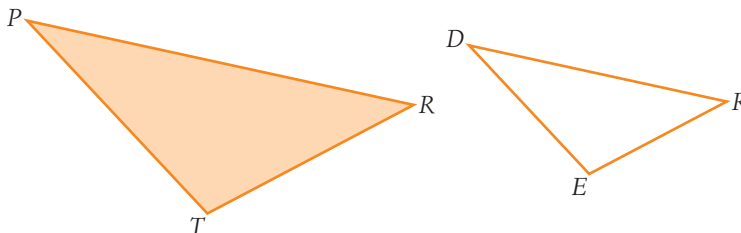
Investigation: Same shape, different size



- a Consider the pentagons $ABCDE$ and $TUVWX$. Do they look similar?
- b What do you think is meant by the word 'similar'?
- c Copy and complete this table by measuring each angle and side in the two 'similar' pentagons.

Angles		Sides (in mm)	
$A =$	$T =$	$AB =$	$TU =$
$B =$	$U =$	$BC =$	$UV =$
$C =$	$V =$	$CD =$	$VW =$
$D =$	$W =$	$DE =$	$WX =$
$E =$	$X =$	$AE =$	$TX =$

- d What do you notice about the sizes of the matching angles?
 - e What do you notice about the sizes of the matching sides?
- 2 $\triangle PTR$ has been reduced to produce the similar $\triangle DEF$.



- a List the three pairs of matching angles and measure the size of each one.
 - b List the three pairs of matching sides and measure the length of each one.
- 3 Copy and complete.
- a For similar figures, m_____ angles are e_____.
 - b Similar figures have the s_____ shape but not the same s_____.

Worksheet

A page of similar figures

MAT09MGWK10147

Worksheet

A page of congruent and similar figures

MAT09MGWK10148

Worksheet

Enlargements and reductions

MAT09MGWK10149

Worksheet

Enlarging a logo

MAT09MGWK10150

Puzzle sheet

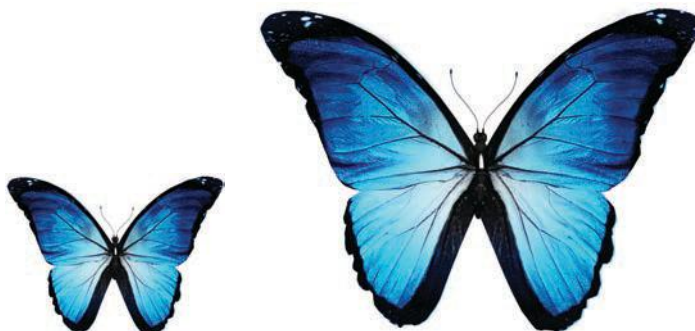
Cartoon enlargement

MAT09MGPS00021

13-04 Similar figures

Similar figures have the same shape but are not necessarily the same size.

When a photograph is enlarged or reduced, all parts of the picture remain the same shape but all lengths and distances will be respectively larger or smaller than the corresponding lengths and distances on the original.

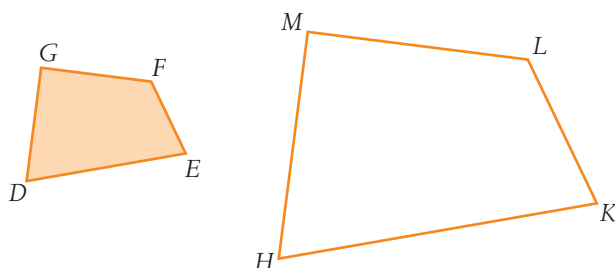


Shutterstock.com/sun07

When a figure is enlarged or reduced, a **similar figure** is created. The original figure is called the **original**, while the enlarged or reduced figure is called the **image**.

The scale factor

The **scale factor** describes by how much a figure has been enlarged or reduced. For example, if the quadrilateral $DEFG$ is enlarged by doubling each side, the scale factor is 2.



$$\begin{aligned} HK &= 2 \times DE \\ KL &= 2 \times EF \\ LM &= 2 \times FG \\ DG &= 2 \times HM \\ \therefore \text{Scale factor} &= 2 \end{aligned}$$

In a similar manner, if the arrow figure is reduced by halving each side, the scale factor is $\frac{1}{2}$.



Scale factor = $\frac{1}{2}$

Summary

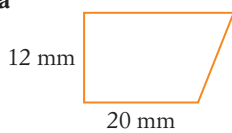
$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$

- If the scale factor is greater than 1, then the image is an enlargement.
- If the scale factor is between 0 and 1, then the image is a reduction.

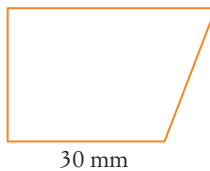
Example 8

Find the scale factor for each pair of similar figures.

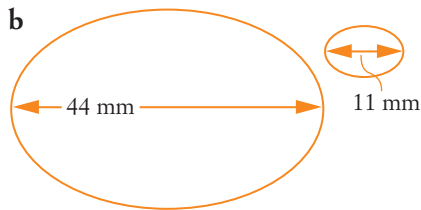
a



18 mm



b



Solution

$$\begin{aligned} \text{a Scale factor} &= \frac{30}{20} \text{ (or } \frac{18}{12}) \\ &= 1\frac{1}{2} \end{aligned}$$

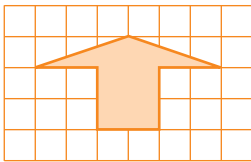
$$\frac{\text{image length}}{\text{original length}}$$

$$\begin{aligned} \text{b Scale factor} &= \frac{11}{44} \\ &= \frac{1}{4} \end{aligned}$$

$$\frac{\text{image length}}{\text{original length}}$$

Example 9

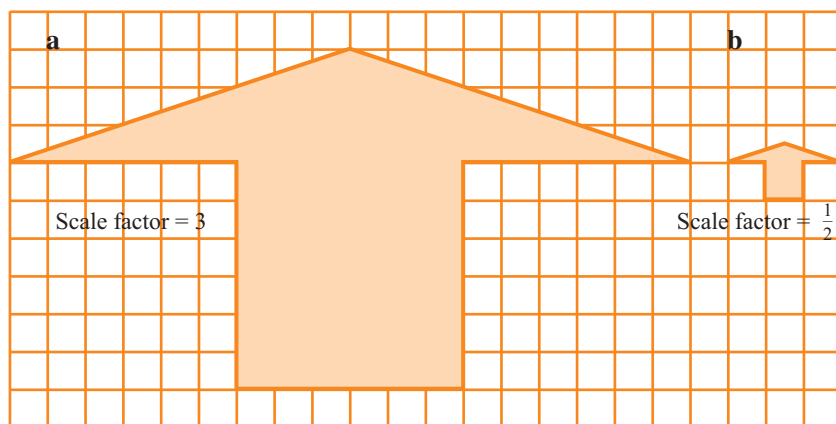
Construct the image of the figure shown if the scale factor is:



a 3

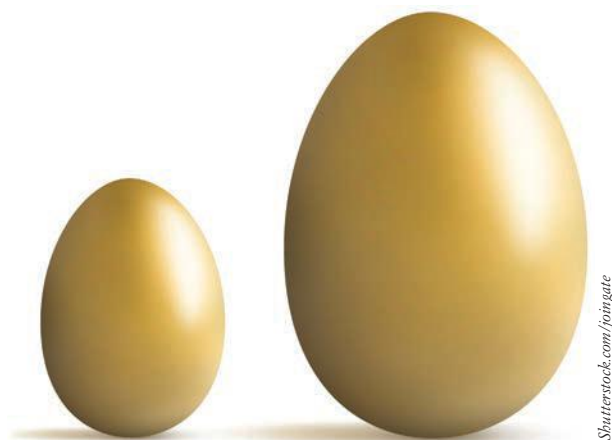
b $\frac{1}{2}$

Solution



The similarity symbol $\sim$

The symbol for 'is similar to' is ' $\sim$ '. As with congruence notation, we must make sure that the vertices (angles) of the similar figures are written in matching order.

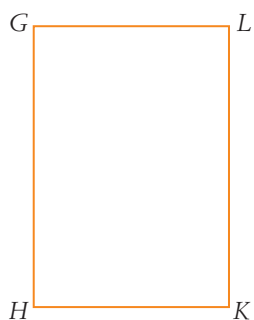
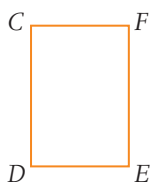


Shutterstock.com/joingate

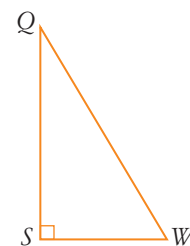
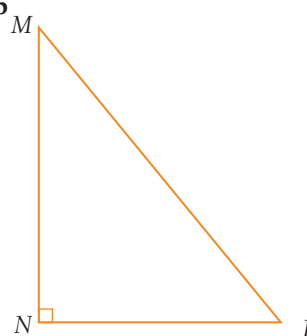
Example 10

Test whether each pair of figures are similar by measuring them.

a



b



Solution

- a Measuring matching sides in millimetres:

$$\frac{GH}{CD} = \frac{38}{19} = 2$$

$$\frac{HK}{DE} = \frac{26}{13} = 2$$

$$\therefore \frac{GH}{CD} = \frac{HK}{DE}$$

$$\therefore CDEF \parallel\parallel GHKL$$

Matching order of vertices.

b $\frac{QS}{MN} = \frac{27}{39} = \frac{9}{13}$

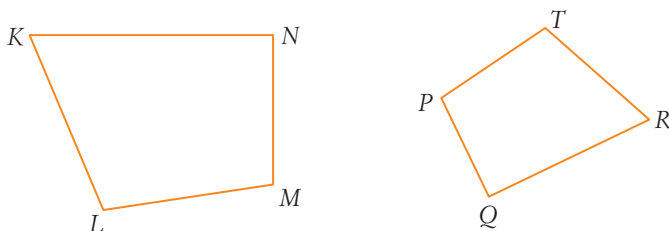
$$\frac{SW}{NP} = \frac{16}{32} = \frac{1}{2}$$

$$\therefore \frac{QS}{MN} \neq \frac{SW}{NP}$$

$\therefore \triangle MNP$ is not similar to $\triangle QSW$.

Example 11

The two quadrilaterals $KLMN$ and $PQRT$ are similar.



- a List all pairs of matching sides and matching angles.
b Use the correct notation to write a similarity statement relating these two quadrilaterals.

Solution

- a By rotating the figure $KLMN$, its shape can be matched with $PQRT$.

The pairs of matching sides are:

KN and QR

MN and PQ

ML and PT

LK and TR .

The pairs of matching angles are:

$\angle K$ and $\angle R$

$\angle N$ and $\angle Q$

$\angle M$ and $\angle P$

$\angle L$ and $\angle T$.

- b $\angle K$ matches with $\angle R$, $\angle L$ matches with $\angle T$, $\angle M$ matches with $\angle P$,
 $\angle N$ matches with $\angle Q$.

$$\therefore KLMN \parallel\parallel RTPQ$$

Matching order of vertices.

Exercise 13-04 Similar figures

See Example 8 1 By measurement, find the scale factor for each pair of similar figures.

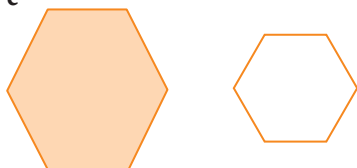
a



b



c



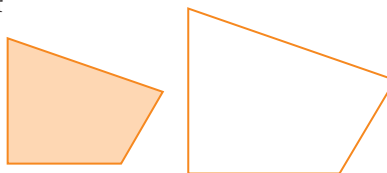
d



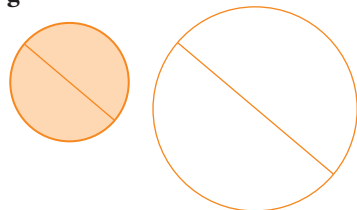
e



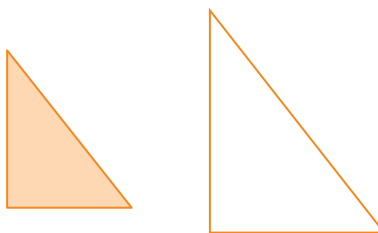
f



g

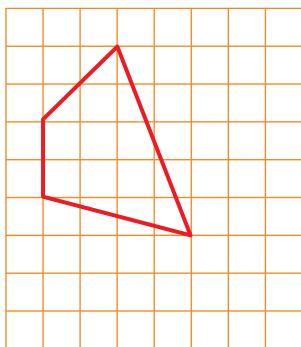


h

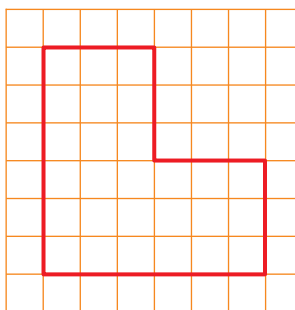


See Example 9 2 Copy each figure onto graph paper and draw its image using the given scale factor.

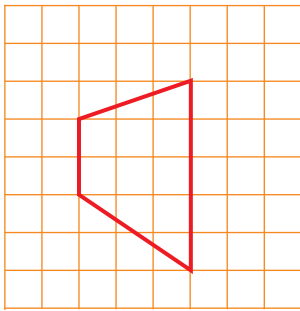
a Scale factor = 2



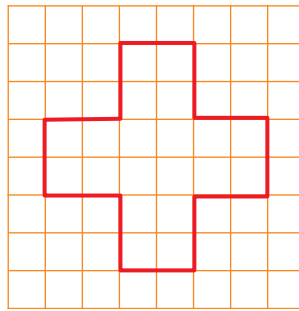
b Scale factor = $\frac{1}{3}$



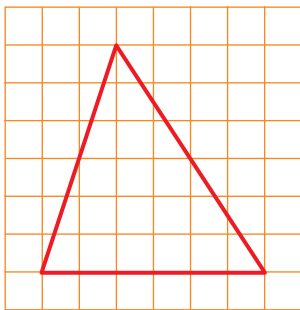
c Scale factor = 2



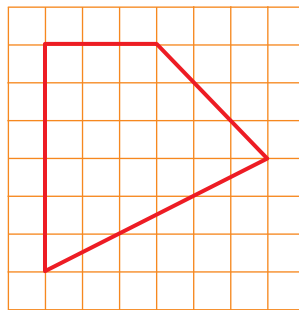
d Scale factor = 2.5



e Scale factor = $\frac{1}{2}$



f Scale factor = $\frac{2}{3}$



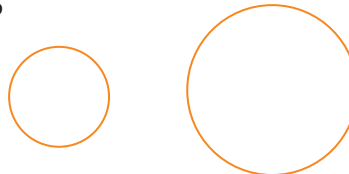
3 Which of the following are pairs of similar figures?

See Example 10

a



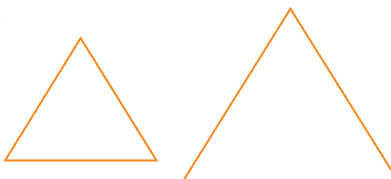
b



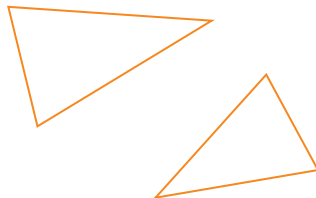
c



d

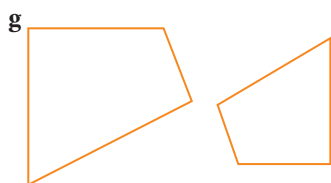


e

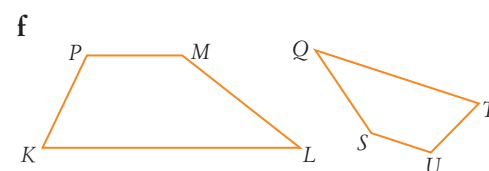
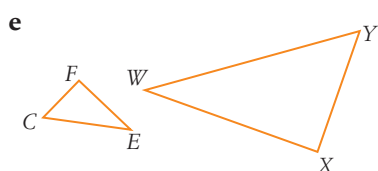
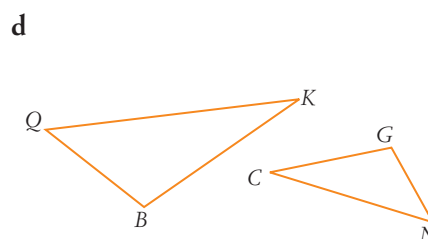
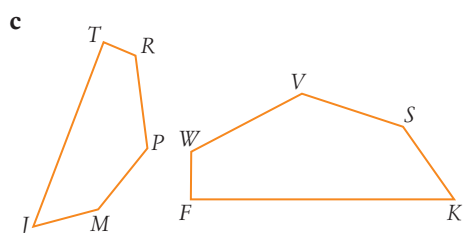
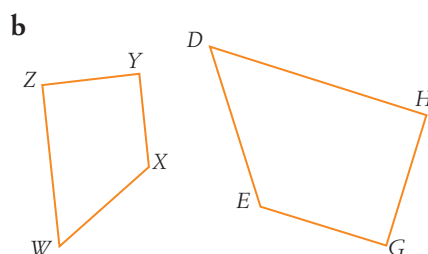
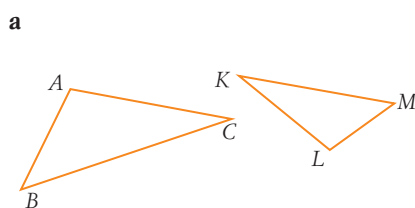


f





- See Example 11** 4 For each pair of similar figures:
- i list all pairs of matching angles
 - ii list all pairs of matching sides
 - iii use the correct notation to write a similarity statement relating them.



Mental skills 13

Maths without calculators

Simplifying fractions and ratios

When simplifying a fraction or a ratio, look for a common factor to divide into both the numerator and the denominator, preferably the highest common factor (HCF). Remember the divisibility tests as shown below.

A number is divisible by ...	if ...
2	it is even (its last digit is 2, 4, 6, 8 or 0)
3	the sum of its digits is divisible by 3
4	its last two digits form a number divisible by 4
5	its last digit is 0 or 5
6	it is even and the sum of its digits is divisible by 3
9	the sum of its digits is divisible by 9
10	its last digit is 0

1 Study each example.

a Simplify $\frac{27}{45}$.

$$\frac{\cancel{27}^9}{\cancel{45}_5} = \frac{9}{15}$$

Dividing numerator and denominator by 3.

$$\frac{\cancel{9}^3}{\cancel{15}_5} = \frac{3}{5}$$

Dividing numerator and denominator by 3 again.

Note: This fraction could be simplified in one step if you divided by 9, the highest common factor (HCF) of 27 and 45.

b Simplify $\frac{160}{400}$.

$$\frac{\cancel{160}^8}{\cancel{400}_8} = \frac{16}{40}$$

Dividing numerator and denominator by 10.

$$\frac{\cancel{16}^2}{\cancel{40}_5} = \frac{2}{5}$$

Dividing numerator and denominator by 8.

Note: This fraction could be simplified in one step if you divided by 80, the HCF of 160 and 400.

c Simplify $24 : 36$.

$$24 : 36 = \cancel{24}^4 : \cancel{36}^6 = 4 : 6$$

Dividing both terms by 6.

$$\cancel{4}^2 : \cancel{6}^3 = 2 : 3$$

Dividing both terms by 2.

Note: This fraction could be simplified in one step if you divided by 12, the HCF of 24 and 36.

d Simplify $135 : 90$.

$$135 : 90 = \cancel{135}^{27} : \cancel{90}^{18} = 27 : 18$$

Dividing both terms by 5.

$$\cancel{27}^3 : \cancel{18}^2 = 3 : 2$$

Dividing both terms by 9.

- e Calculate $\frac{3}{8} \times \frac{2}{15}$ in simplest form.

$$\begin{aligned}\frac{3}{8} \times \frac{2}{15} &= \frac{3}{\cancel{4}^1 \cancel{8}^1} \times \frac{\cancel{2}^1}{15} \\ &= \frac{\cancel{3}^1 \cancel{4}^1}{\cancel{4}^1 \cancel{8}^1} \times \frac{\cancel{2}^1}{\cancel{15}^3 \cancel{5}^1} \\ &= \frac{1}{20}\end{aligned}$$

Dividing 2 and 8 by 2.

Dividing 3 and 15 by 3.

- f What fraction is 36 minutes of 1 hour?

$$\frac{36 \text{ min}}{1 \text{ h}} = \frac{36 \text{ min}}{60 \text{ min}} = \frac{3}{5}$$

- 2 Now simplify each fraction or ratio.

a $\frac{10}{15}$

b $\frac{16}{20}$

c $\frac{30}{42}$

d $\frac{8}{16}$

e $\frac{20}{80}$

f $\frac{6}{36}$

g $\frac{20}{24}$

h $\frac{12}{30}$

i 20 : 36

j 25 : 45

k 18 : 40

l 28 : 35

m 27 : 21

n 16 : 12

o $\frac{5}{6} \times \frac{18}{25}$

p $\frac{12}{50} \times \frac{10}{21}$

- 3 Express each as a simplified fraction.

a 425 g of 1 kg

b 8 months of 1 year

c 64 cm of 1 m

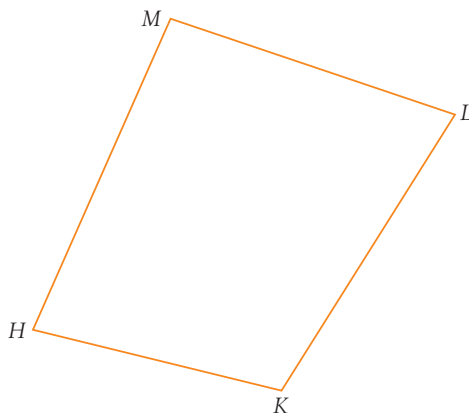
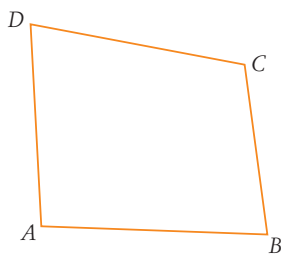
d 750 mL of 3 L

e 10 hours of 2 days

f 80c of \$10

Investigation: Properties of similar figures

- 1 The quadrilaterals $ABCD$ and $HKLM$ are similar.



- a Measure the sizes of each pair of matching angles. What do you notice?
b Measure the lengths of each pair of matching sides.
c Calculate each ratio of matching sides, as a decimal correct to one decimal place.

i $\frac{MH}{AB}$

ii $\frac{HK}{BC}$

iii $\frac{KL}{CD}$

iv $\frac{ML}{AD}$

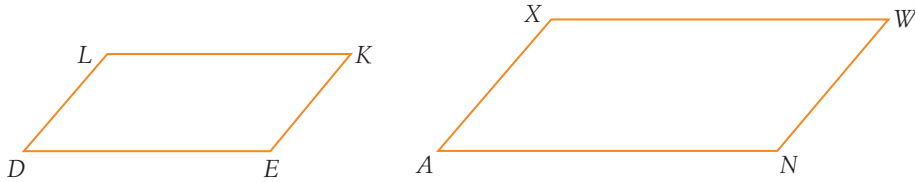
What do you notice?

d Copy and complete.

i The matching angles of similar figures are equal.

ii Matching sides of similar figures are in the same ratio.

2 Consider the parallelograms $DEKL$ and $ANWX$.



a Measure the sizes of each pair of matching angles. Are they equal?

b Measure the lengths of each pair of matching sides.

c Calculate each ratio of matching sides, as a decimal correct to one decimal place.

i $\frac{AN}{DE}$

ii $\frac{AX}{DL}$

Are they equal?

d $DEKL$ is not similar to $ANWX$. Explain why.

13-05 Properties of similar figures

Worksheet

Finding sides in similar figures

MAT09MGWK10151

Skillsheet

Finding sides in similar triangles

MAT09MGSS10033

Puzzle sheet

Similar triangles

MAT09MGPS00024

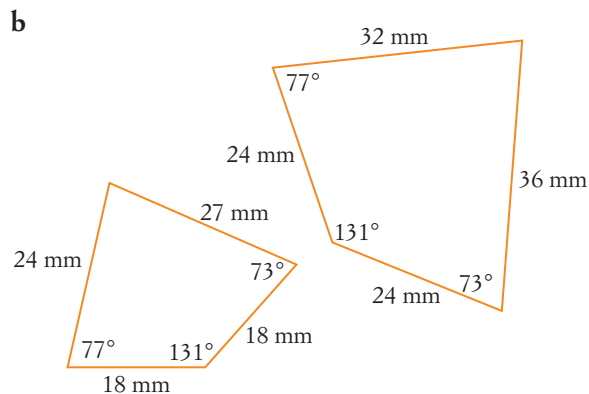
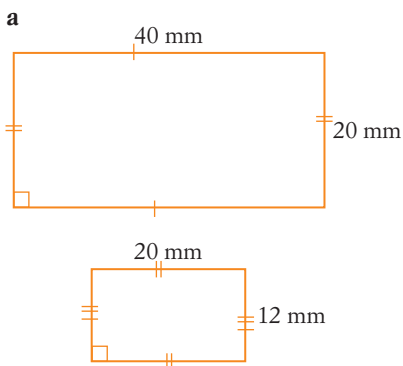
Summary

For **similar figures**:

- **matching angles** are equal
- **matching sides** are in the same ratio

Example 12

Test whether each pair of figures are similar.



Solution

- a For the two rectangles, matching angles are equal (90°) but the ratios of the matching sides are not equal ($\frac{20}{40} = \frac{1}{2}$ but $\frac{12}{20} = \frac{3}{5}$).
 $\therefore$ The rectangles are not similar.
- b For the two quadrilaterals, matching angles are equal and the ratios of matching sides are equal ($\frac{32}{24} = \frac{36}{27} = \frac{24}{18} = \frac{4}{3}$).
 $\therefore$ The quadrilaterals are similar.

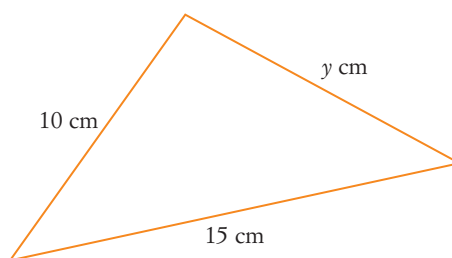
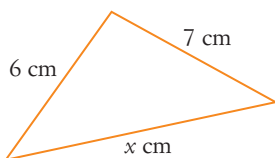
Example 13

Video tutorial

Finding an unknown side in similar figures

MAT09MGVT10023

These two triangles are similar. Find the value of x and y .



Solution

- a Since the triangles are similar, the ratios of matching sides are equal.

$$\frac{x}{15} = \frac{6}{10}$$

$$x = \frac{6}{10} \times 15$$

$$= 9$$

$$\frac{y}{7} = \frac{10}{6}$$

$$y = \frac{10}{6} \times 7$$

$$= 11\frac{2}{3}$$

Alternative method:

$$\text{Scale factor} = \frac{10}{6} = 1\frac{2}{3}$$

$$x = 15 \div 1\frac{2}{3}$$

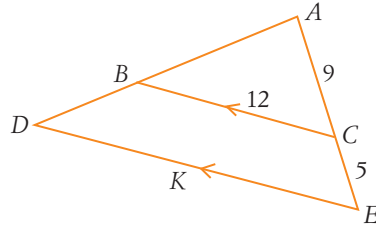
$$= 9$$

$$y = 7 \times 1\frac{2}{3}$$

$$= 11\frac{2}{3}$$

Example 14

$\triangle ABC \parallel \triangle ADE$. Find the value of k .



Solution

$$\frac{DE}{BC} = \frac{AE}{AC}$$

$$\frac{k}{12} = \frac{14}{9}$$

$$k = \frac{14}{9} \times 12$$

$$= 18\frac{2}{3}$$

Ratios of matching sides are equal.

$$AE = 9 + 5 = 14$$

Puzzle sheet

Nested similar triangles

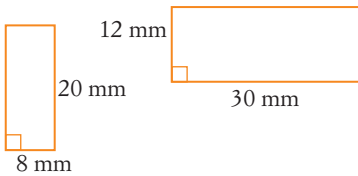
MAT09MGPS00025

Exercise 13-05 Properties of similar figures

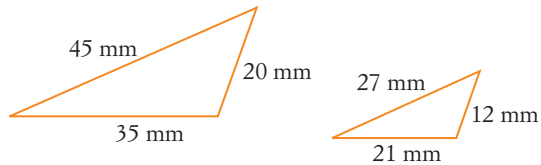
1 Test whether each pair of figures are similar.

See Example 12

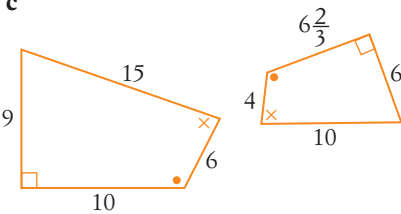
a



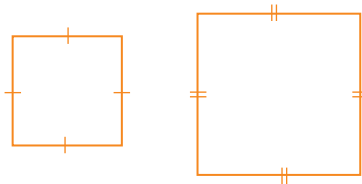
b



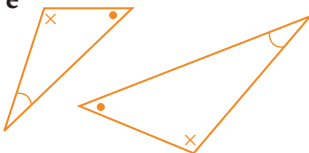
c



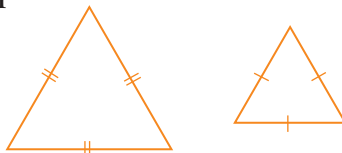
d



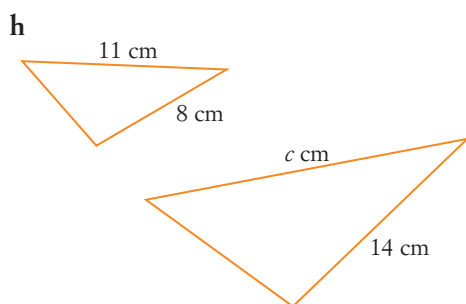
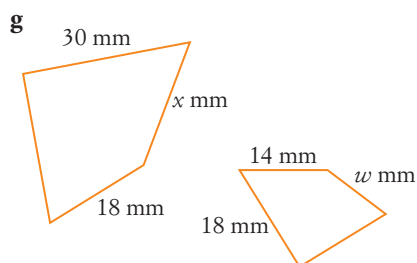
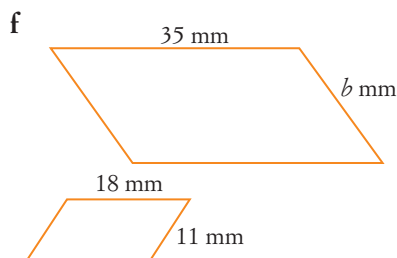
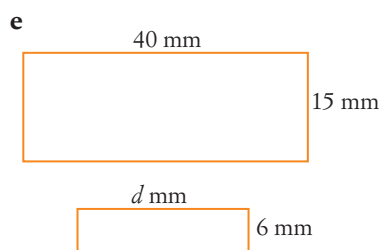
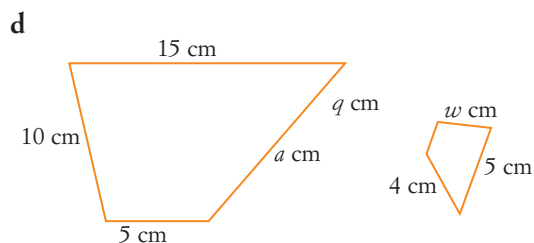
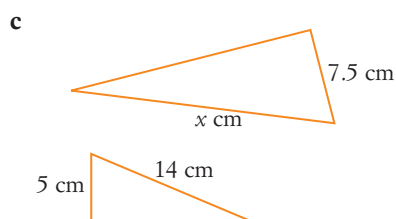
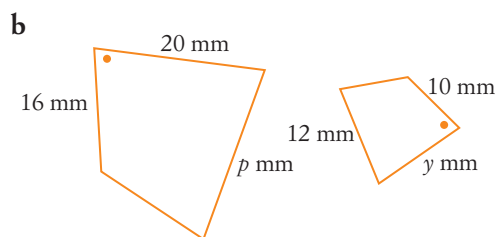
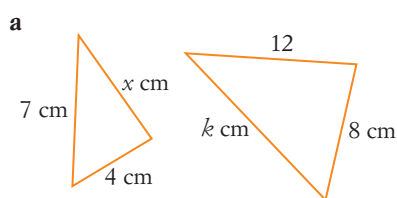
e



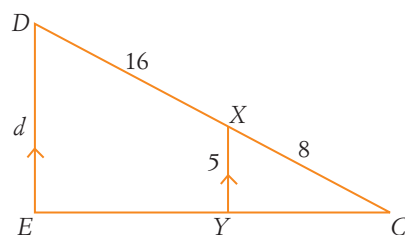
f



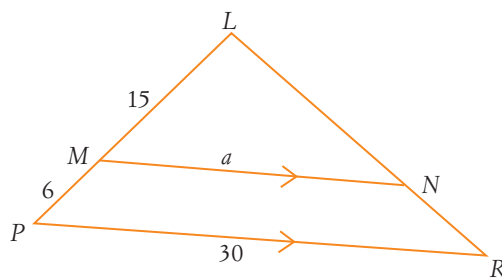
See Example 13 2 Find the value of every pronumeral in each pair of similar figures.



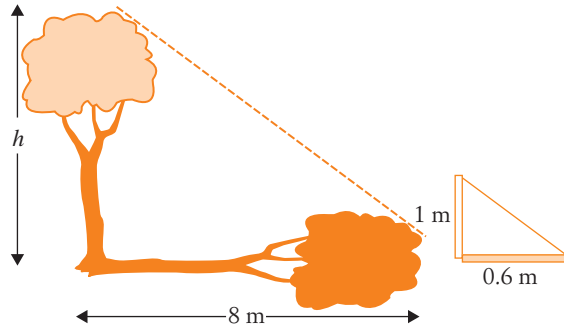
See Example 14 3 $\triangle CED \parallel \triangle CYX$. Find the value of d .



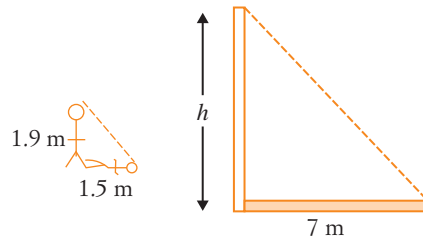
4 $\triangle LMN \parallel \triangle LPR$. Find the value of a .



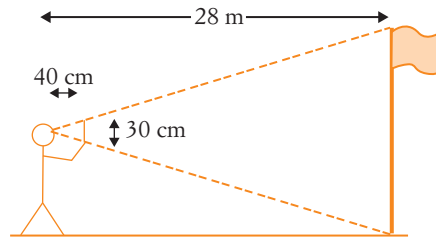
- 5 A metre stick casts a shadow of length 0.6 m when a tree casts a shadow of 8 m. Find the height h of the tree.



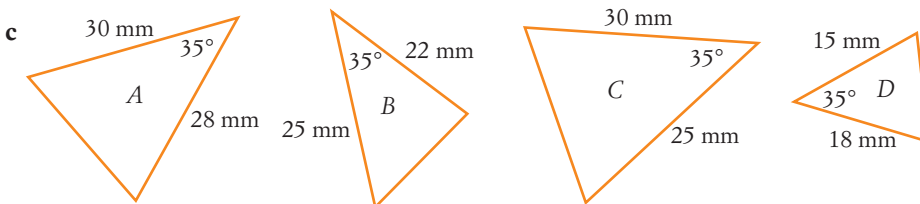
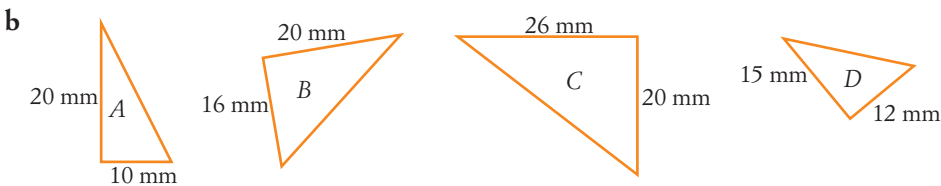
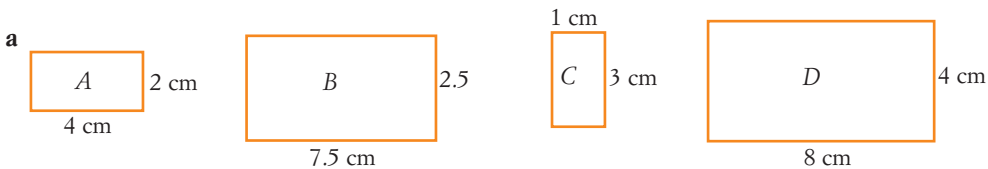
- 6 Ayon, who is 1.9 m tall, casts a 1.5 m long shadow. At the same time, a light pole casts a shadow 7 m long. What is the height of the light pole, correct to one decimal place?

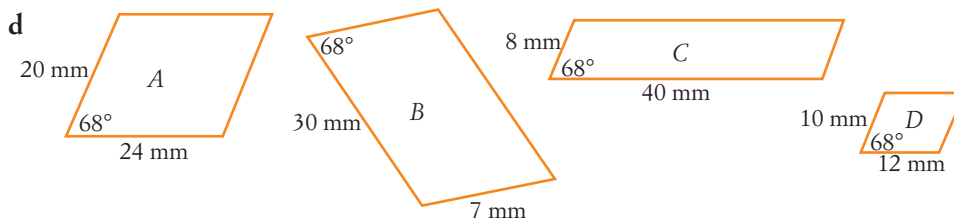


- 7 Emily holds up a small stick 40 cm from her eye so that it appears to be the same height as the flagpole. She is standing 28 m from the flagpole and the stick is 30 cm long. Find the height of the flagpole.



- 8 For each set of figures, find the pair of similar figures and the scale factor between them.





Investigation: Are all shapes similar?

Work in groups of three or four to complete this activity.

You will need: a ruler, a pencil, scissors.

- 1 **a** Draw two equilateral triangles using a different side length for each triangle.
b Are the two equilateral triangles similar? Give reasons.
c Compare your triangles with those of other members of your group. Are all equilateral triangles similar? Give reasons.
- 2 **a** Draw a pair of similar isosceles triangles.
b Are all isosceles triangles similar? Give reasons.
- 3 **a** Draw a pair of rectangles that are similar.
b Draw a pair of rectangles that are *not* similar.
c When are rectangles similar? Discuss your answer with other students.
- 4 State whether the following statements are true or false. Give reasons for your answers. (Drawing a few sketches will help.)
a All parallelograms are always similar. **b** All circles are always similar.
c All squares are always similar. **d** All rhombuses are always similar.

Worksheet

Problems involving
scale drawings

MAT09MGWK10152

Homework sheet

Similar figures 1

MAT09MGHS10026

Technology worksheet

Excel worksheet:
Converting map scales
to ratios

MAT09MGCT00023

Technology worksheet

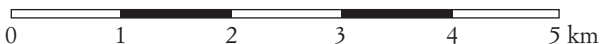
Excel spreadsheet:
Converting map scales
to ratios

MAT09MGCT00008

13-06 Scale diagrams

A **scale diagram** accurately represents a larger or smaller object such that the lengths and distances on it are in the same ratio as the real lengths and distances. This means that the scale diagram is either an enlargement or reduction of the real object. The scale factor used is called the **scale** of the diagram.

A scale may be represented in different ways.

- 1 cm = 10 m or 1 cm to 10 m (a pair of corresponding measurements)
- 1 : 50 (a ratio)
-  (a line drawn to scale)

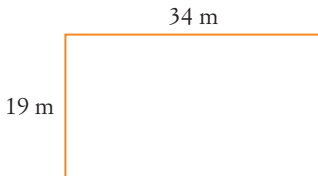
Summary

The scale on a scale diagram is written as a ratio **scaled length : real length** where **scaled length** is the length on the diagram.

For example, a scale of 1 : 100 means that the real lengths are 100 times larger than the lengths on the diagram.

Example 15

A scale diagram of a rectangular block of land has been drawn.
What scale has been used?



Solution

By measurement, the length and width of the scale drawing are 35 mm and 20 mm.

$$\text{Scale} = 34 \text{ mm} : 34 \text{ m (or } 19 \text{ mm} : 19 \text{ m)}$$

$$= 34 \text{ mm} : 34\,000 \text{ mm (or } 19 \text{ mm} : 19\,000 \text{ m)}$$

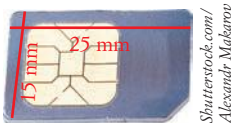
$$= 1 : 1000$$

scaled length : real length

$$1 \text{ m} = 1000 \text{ mm}$$

Example 16

A SIM card for a mobile phone has a length of 25 mm and a width of 15 mm. A scale diagram of the SIM card is shown here.



15 mm



25 mm

By measurement, determine the scale that has been used.

Solution

By measurement the length and width of the scale diagram are 75 mm and 45 mm.

$$\text{Scale} = 75 \text{ mm} : 25 \text{ mm (or } 45 \text{ mm} : 15 \text{ mm)}$$

$$= 3 : 1$$

scaled length : real length

Example 17

The scale on a map of NSW is 1: 500 000. If the distance from Campbelltown to Sydney on the scale drawing is 9 cm, calculate the actual distance from Campbelltown to Sydney.

Solution

Scaled distance = 9 cm

Actual distance = $9 \times 500\,000$ cm

= 4 500 000 cm

= 45 000 m

= 45 km

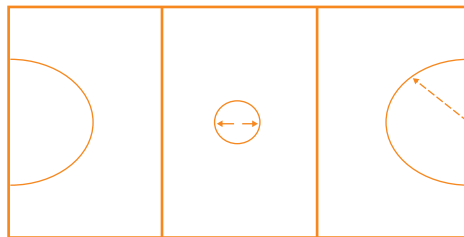
1 m = 100 cm

1 km = 1000 m

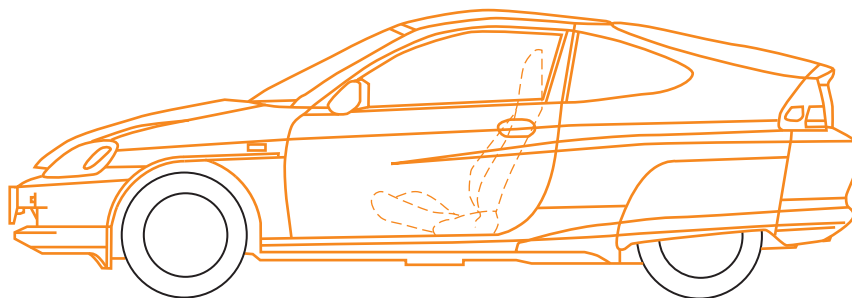
Exercise 13-06 Scale diagrams

See Example 15

- 1 If the length of a netball court is 30.5 m, what scale has been used on this diagram?



- 2 What scale has been used on this diagram of a car if its actual length is 3.92 m?

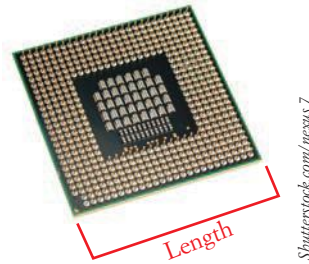


See Example 16

- 3 The picture of the Giant Brown Bull Ant has been enlarged. If its actual length is 25 mm, what scale has been used?



- 4 A scale drawing of a microchip is shown. Determine the scale that has been used if its real length is 8 mm.



Shutterstock.com/nexus 7

- 5 Sydney Tower is 305 m high. What scale has been used in this photo if the scaled height is 41 mm?



Stockphoto/L_rust

- 6 The top of the arch of the Sydney Harbour Bridge is 134 m above the water.
a What scale has been used in this photo?



Shutterstock.com/gaglianifoto

- b Use the scale to calculate the height of the left pylon.
7 This is a photograph of a mosquito, at a scale of 15 : 1. Calculate the length of the wing, correct to the nearest 0.1 mm.



Shutterstock.com/Henrik Larsson

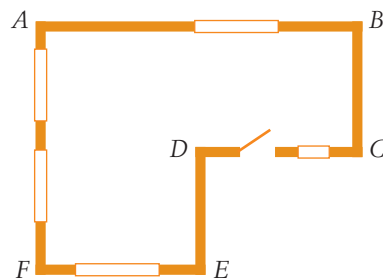
See Example 17

8 The scale on a map of Australia is 1 : 10 000 000. If the distance from Sydney to Melbourne on the map is 7.3 cm, calculate the actual distance in kilometres.

9 This scale plan of a holiday house has a scale of 1 : 200. Find the actual length of:

a AB

b CD



10 The map below shows part of the south coast of New South Wales. Use the scale to find to the nearest kilometre the distance from:

a Nowra to Batemans Bay

b Bega to Moruya

c Mollymook to Mogo

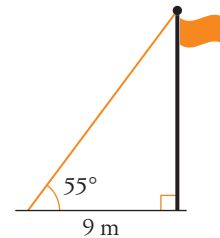
d Nowra to Eden.



- 11 The box of cereal is 16 cm long and 22 cm tall. Construct a scale drawing of the front of the box, using a scale of 1 : 5.



- 12 a Construct a scale drawing of this flagpole using a scale of 1 : 300.
b Use the scale to calculate the actual height of the flagpole.



Investigation: Floor plans

Construct a scale diagram of your bedroom or classroom.

Measure the dimensions of the room and each piece of furniture in it. Remember to measure the location of immovable objects such as windows, doors, wardrobes and whiteboards. Include a list of your measurements and the scale used.

Just for the record

Australia's Big Things

Unique to Australia are a number of tourist attractions known as the 'Bigs' or 'Biggies'.

- the Big Merino (Goulburn)
- the Big Potato (Robertson)
- the Big Banana (Coffs Harbour)
- the Big Bull (Wauchope)
- the Big Oyster (Taree)
- the Big Guitar (Tamworth)
- the Big Pineapple (Nambour, Qld)
- the Big Orange (Berri, SA)
- the Big Crocodile (Wyndham, WA)

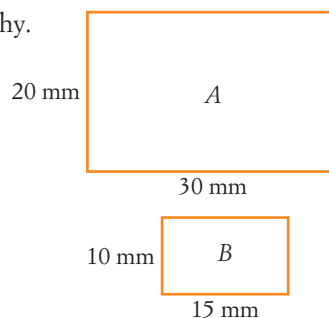


Getty Images/Matthias Gottschalk

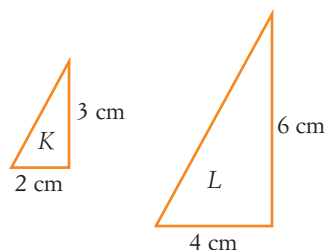
The Big Merino is 15 metres high. If the height of an average ordinary merino sheep is 90 cm, work out the scale of the Big Merino to a real sheep.

Investigation: Areas of similar figures

- 1 a The two rectangles A and B are similar. Explain why.
 b What is the ratio of their matching sides (A to B)?
 c What is the ratio of their areas (A to B)?



- 2 Triangles K and L are similar.
 a What is the ratio of their matching sides (K to L)?
 b Find the areas of both triangles.
 c What is the ratio of their areas (K to L)?
 d Copy and complete: When you double the lengths of a figure, the area is increased _____ times.

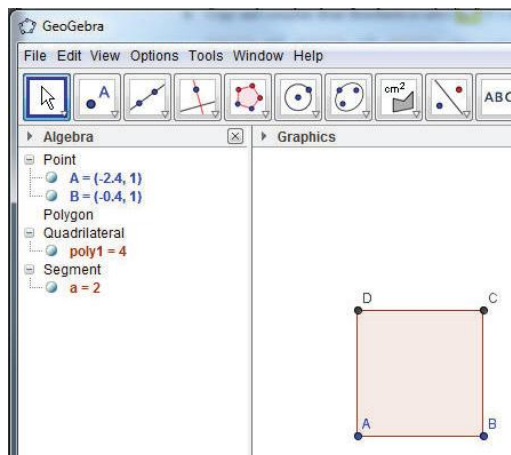


- 3 a What is the area of a rectangle with length 5 cm and width 4 cm?
 b The length and width of the rectangle are both multiplied by 3.
 c What is the area of the enlarged rectangle?
 d What is the ratio of the matching sides of the two rectangles from parts a and b?
 e What is the ratio of the areas of the two rectangles?
 f Copy and complete: When you triple the lengths of a figure, the area is increased _____ times.
- 4 a What is the area of a circle with radius 2 cm?
 b The radius of this circle is multiplied by 4.
 c What is the area of the enlarged circle?
 d What is the ratio of the areas of the two rectangles?
 e Copy and complete: When you multiply the lengths of a figure by 4, the area is increased _____ times.
- 5 If the sides of a square are multiplied by 5, by how many times is the area increased?
- 6 If the sides of a figure are multiplied by k , by how many times is the area increased?

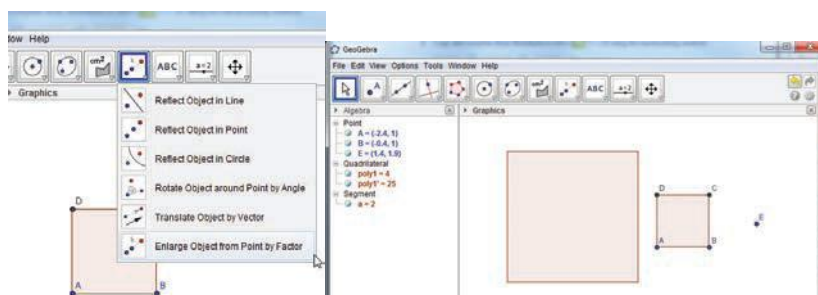
Technology Areas of similar figures

In this activity you will use GeoGebra to draw similar figures and compare their areas.

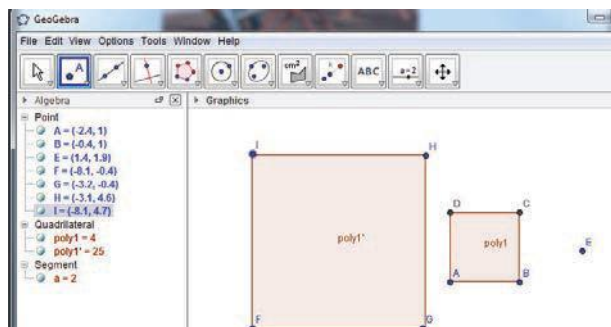
- 1 Right-click in the Graphics window and make sure axes and grid are disabled.
- 2 Select **Regular Polygon** and enter 4 for the number of vertices so that a square is formed.



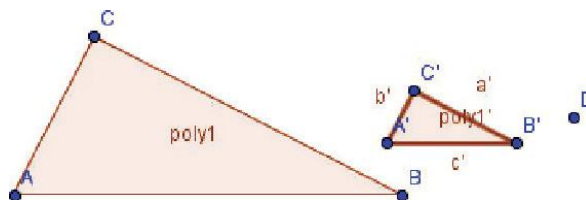
- 3 Move A or B so that $a = 2$ cm. In **Algebra View**, under **Polygon**, **Quadrilateral**, the area of the square?
- 4 Move the square towards the centre by clicking in it, holding ctrl down and using your mouse to drag it.
- 5 To enlarge the square, select **Enlarge Object from Point by Factor** from the 9th drop-down icon menu, click on the square, select a point (the centre of enlargement), enter the scale factor of 2.5 and then click OK.



- 6 In **Algebra View**, the length and area of the enlarged square (poly1') is given. To see the names that GeoGebra has given to the squares, right-click on them and select Show Label.
- 7 Select **New Point** from the second drop-down icon menu and click on each vertex of poly1'.



- 8 a What is the length of the side of the enlarged square poly1'?
- b What is the area of poly1'?
- c What is the ratio of the sides AB : FG?
- d What is the ratio of the areas of the squares poly1 : poly1'?
- e Compare the two ratios. How are they related?
- 9 a Select **Polygon** and draw a rectangle with length 3 cm and width 1.5 cm. What is the area of the rectangle?
- b Select **Enlarge Object from Point by Factor** and use a scale factor of 2 to enlarge the rectangle.
- c Use the values in **Algebra View** to find the ratios of
- i sides AB : FG ii sides BC : GH iii areas of rectangles ABCD : FGHI.
- d Compare the ratio of the sides to the ratio of the areas. How are they related?
- 10 a Select **Polygon** and draw a triangle with AB = 8 cm.
- b Select **Enlarge Object from Point by Factor** and use a scale factor of $\frac{1}{4}$ to reduce the triangle.



- c Use the values in the **Algebra Window** to find the ratios of:
- i sides AB : A'B' ii areas of triangles ABC : A'B'C'.
- d Compare the ratio of the sides to the ratio of the areas. How are they related?
- 11 a Select **Polygon** and draw any shape.
- b Select **Enlarge Object from Point by Factor** to enlarge or reduce the shape.
- c Find the ratios of:
- i matching sides of the similar shapes
- ii the areas of the similar shapes
- d Compare the ratio of the sides to the ratio of the areas. How are they related?

Stage 5.3

13-07 Areas of similar figures

Summary

If the matching sides of two similar figures are in the ratio $1 : k$, then their areas are in the ratio $1 : k^2$.

If the matching sides are in the ratio $m : n$, then their areas are in the ratio $m^2 : n^2$.

Example 18

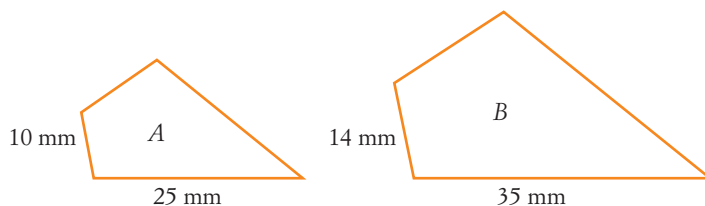
- a** Two similar triangles have sides in the ratio 1 : 5. Find the ratio of their areas.
b Two similar parallelograms have sides in the ratio 3 : 1. Find the ratio of their areas.

Solution

- a** Ratio of sides = 1 : 5
 Ratio of areas = $1 : 5^2$ $1 : k^2$
 $= 1 : 25$
- b** Ratio of sides = 3 : 1
 Ratio of areas = $3^2 : 1$
 $= 9 : 1$

Example 19

What is the ratio of the areas of the similar quadrilaterals shown?



Solution

$$\begin{aligned}\text{Ratio of matching sides (A to B)} &= 25 : 35 && (\text{or } 10 : 14) \\ &= 5 : 7 \\ \text{Ratio of areas (A to B)} &= 5^2 : 7^2 \\ &= 25 : 49\end{aligned}$$

Example 20

Two similar trapeziums have their areas in the ratio 25 : 16. Find the ratio of the lengths of the matching sides.

Solution

$$\begin{aligned}\text{Ratios of areas} &= m^2 : n^2 = 25 : 16 \\ \therefore \text{Ratio of sides} &= m : n = \sqrt{25} : \sqrt{16} = 5 : 4\end{aligned}$$

Video tutorial

Areas of similar
figures

MAT09MGVT10024

Stage 5.3

Example 21

Two similar triangles P and Q have matching sides in the ratio $3 : 4$. If the area of the larger triangle is 288 cm^2 , find the area of the smaller triangle.

Solution

Drawing a diagram of the problem is useful.

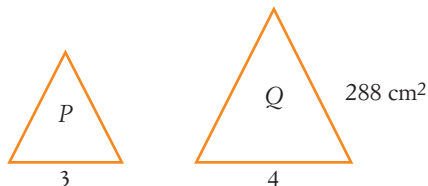
Ratio of matching sides = $3 : 4$

Ratio of areas = $3^2 : 4^2 = 9 : 16$

$$\therefore \frac{\text{Area of } P}{288} = \frac{9}{16}$$

$$\begin{aligned}\text{Area of } P &= \frac{9}{16} \times 288 \\ &= 162 \text{ cm}^2\end{aligned}$$

The area of the smaller triangle is 162 cm^2 .



Exercise 13-07 Areas of similar figures

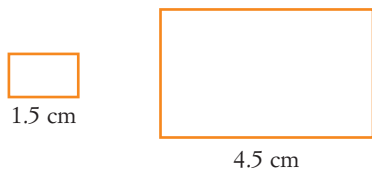
See Example 18

- Two similar rectangles have sides in the ratio $1 : 4$. Find the ratio of their areas.
- Two similar circles have radii in the ratio $1 : 2.5$. Find the ratio of their areas.
- The ratio of the matching sides for pairs of similar figures are as follows. Find the ratio of the areas for each pair.

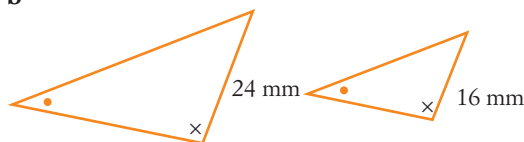
a $10 : 1$	b $5.5 : 1$	c $3 : 4$	d $5 : 2$
e $4 : 7$	f $1 : 10$	g $8 : 5$	h $2 : 9$
- Find the ratio of the areas for each pair of similar figures.

See Example 19

a



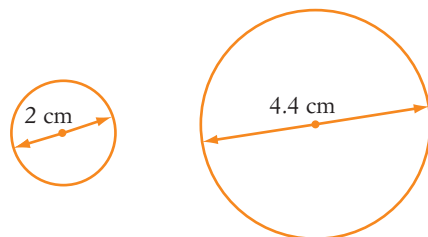
b



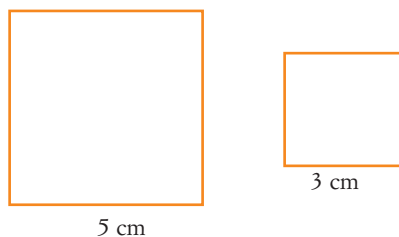
c



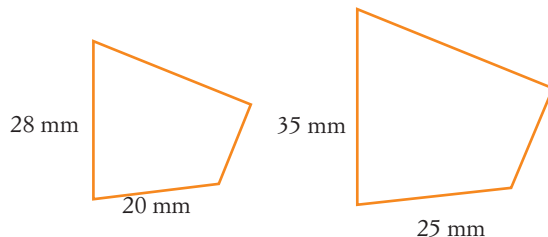
d



e



f

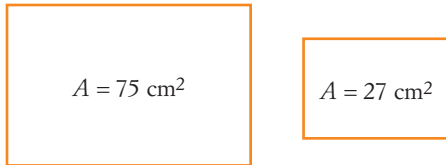


- 5 For each ratio of the areas for a pair of similar figures, find the ratio of the lengths of the matching sides.

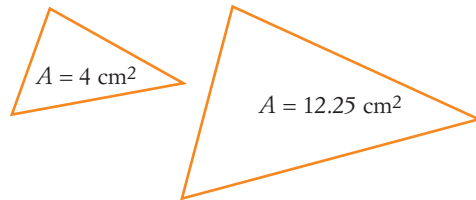
a 64 : 49 b 36 : 1 c 9 : 400 d 1 : 20.25

- 6 Find the ratio of the length of matching sides for each pair of similar figures.

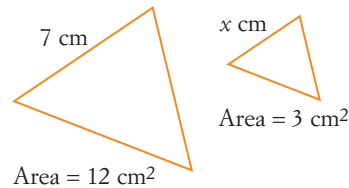
a



b



- 7 Find the value of x if these two triangles are similar.



- 8 Two similar triangles have matching sides in the ratio 4 : 1. If the area of the smaller triangle is 192 cm^2 , find the area of the larger triangle.
- 9 Two squares have sides in the ratio 3 : 4. If the area of the larger square is 288 cm^2 , find the area of the smaller square.
- 10 Two similar quadrilaterals have matching sides in the ratio 8 : 5. If the area of the smaller quadrilateral is 400 mm^2 , find the area of the larger quadrilateral.
- 11 Two circles have radii in the ratio 3 : 7. Find the area of the smaller circle if the area of the larger circle is 154 m^2 .
- 12 Two similar rectangles have areas in the ratio 36 : 121. If the width of smaller rectangle is 84 mm, find the width of the larger rectangle.
- 13 Two similar triangles have areas in the ratio 4 : 9. If the length of the base of the smaller triangle is 15 mm, find the length of the base of the larger triangle.
- 14 Two squares have areas in the ratio 49 : 25. The length of the larger square is 17.5 cm. Find the length of the smaller square.
- 15 If the radius of a circle is tripled, how has its area changed?
- 16 If the area of a square is reduced by a factor of 4, how have the sides changed?
- 17 The area of an equilateral triangle is decreased to $\frac{9}{16}$ of its original amount. How does its new area compare to its original area?
- 18 A rectangle is reduced using a scale factor of $\frac{1}{2}$. If the area of the rectangle was 200 cm^2 , what is the area of the reduced rectangle?

Investigation: Similar triangles

Work in groups of three or four to complete this investigation.

You will need: paper, a ruler, a pencil, a protractor, compasses and scissors.

- 1 a Draw $\triangle ABC$ with $AB = 25$ mm, $AC = 35$ mm and $BC = 20$ mm.

- b How many different triangles can be drawn with sides of 25 mm, 35 mm and 20 mm?
Is this triangle unique?

- c Draw $\triangle DEF$ with $DE = 75$ mm, $DF = 105$ mm and $EF = 60$ mm.

- d Are the matching sides of the triangles in the same ratio?
e Use your protractor to measure the angles of both triangles. Are matching angles equal?
f Is $\triangle ABC$ similar to $\triangle DEF$?

- 2 a Draw $\triangle GHK$ with $\angle G = 72^\circ$ and $\angle H = 45^\circ$.

- b Draw $\triangle TPR$ with $\angle T = 72^\circ$ and $\angle P = 45^\circ$.
Is $\triangle GHK \sim \triangle TPR$?

- c Compare your triangles with those of other students. Are they all similar?

- 3 a Draw $\triangle LMP$ with $LM = 30$ mm, $LP = 40$ mm and $\angle L = 55^\circ$ (the included angle).

- b Draw $\triangle XWY$ with $XW = 45$ mm, $XY = 60$ mm and $\angle X = 55^\circ$.

- c Find the ratio of two pairs of matching sides $\frac{LM}{XW}$ and $\frac{LP}{XY}$. Are they equal?

- d Are the two triangles similar? Explain.

- 4 a Construct $\triangle PQS$ with $PQ = 60$ mm, $\angle P = 90^\circ$ and $QS = 80$ mm (the hypotenuse).

- b Construct $\triangle TXW$ with $TX = 45$ mm, $\angle T = 90^\circ$ and $XW = 60$ mm.

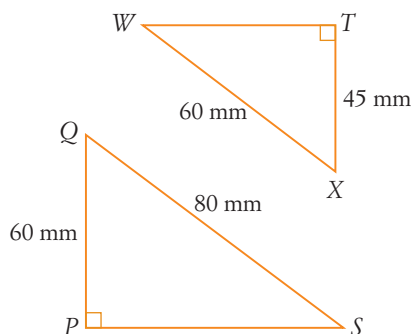
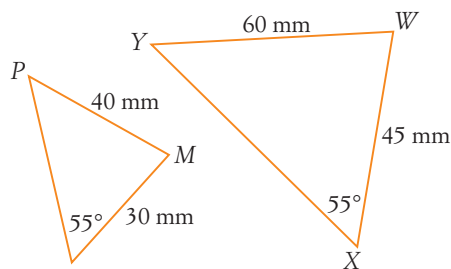
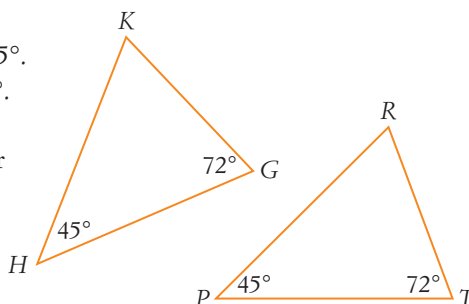
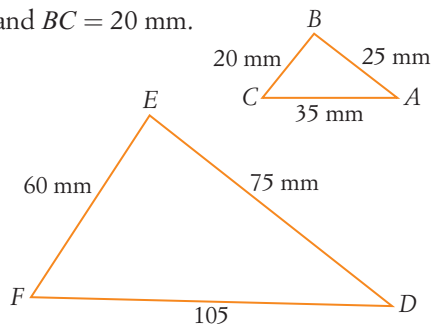
- c Calculate the ratios of matching sides $\frac{PQ}{TX}$ and $\frac{QS}{XW}$

- d Are the triangles similar? Explain.

- 5 a Compare and discuss your results with other students.

- b What tests are necessary for two triangles to be similar?

- c How do these conditions compare to the conditions for congruent triangles?



13-08 Tests for similar triangles

Homework sheet

Similar figures 2

MAT09MGHS10027

Homework sheet

Congruent and similar figures

MAT09MGHS10028

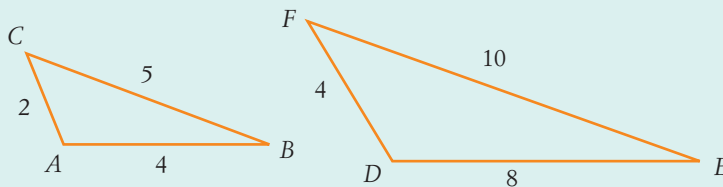
There are four sets of conditions that can be used to determine if two triangles are similar. These are called the **tests for similar triangles** or **similarity tests**.

Summary

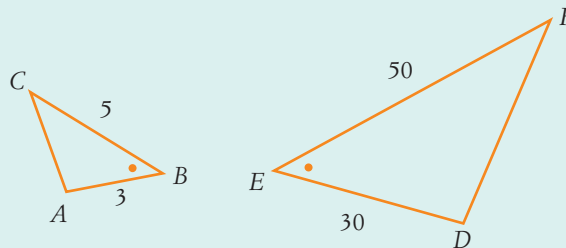
There are four tests for similar triangles.

Two triangles are similar if:

- the three sides of one triangle are proportional to the three sides of the other triangle ('SSS')

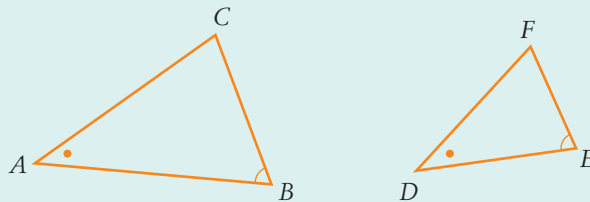


- two sides of one triangle are proportional to two sides of the other triangle, and the **included** angles are equal ('SAS')

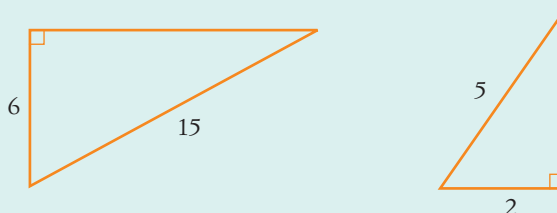


- two angles of one triangle are equal to two angles of the other triangle ('AA' or 'equiangular')

equiangular means 'equal angles'

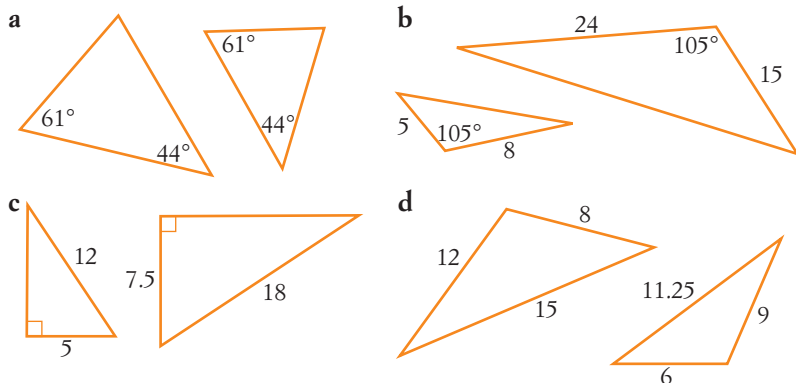


- they are right-angled and the hypotenuse and a second side of one triangle are proportional to the hypotenuse and a second side of the other triangle ('RHS').



Example 22

Which test can be used to prove that each pair of triangles are similar?

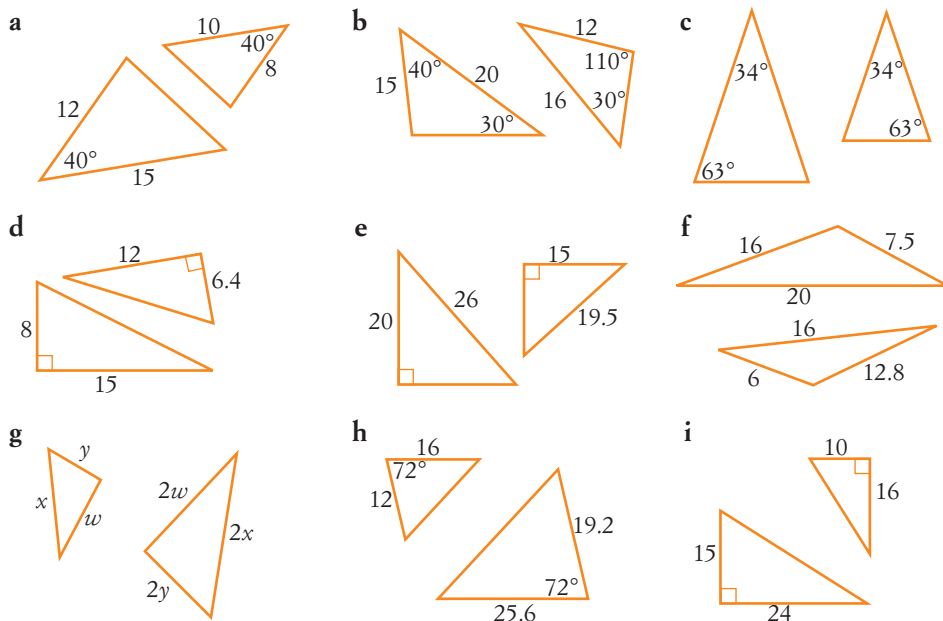


Solution

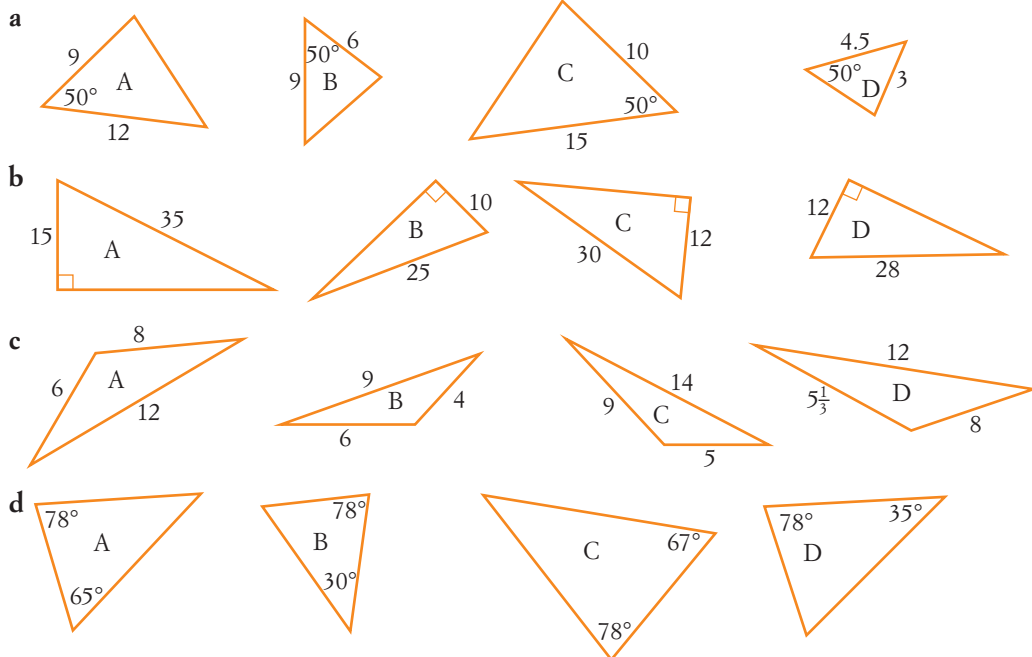
- a** Two pairs of angles are equal.
- b** Two pairs of matching sides are in the same ratio $\frac{15}{5} = \frac{24}{8} = 3$ and the included angles in both triangles are 105° .
- c** In both right-angled triangles, the pairs of hypotenuses and second sides are in the same ratio $\frac{7.5}{5} = \frac{18}{12} = \frac{3}{2}$
- d** All three pairs of matching sides are in the same ratio $\frac{11.25}{15} = \frac{9}{12} = \frac{6}{8} = \frac{3}{4}$

Exercise 13-08 Tests for similar triangles

See Example 22 1 Which test can be used to prove that each pair of triangles are similar?

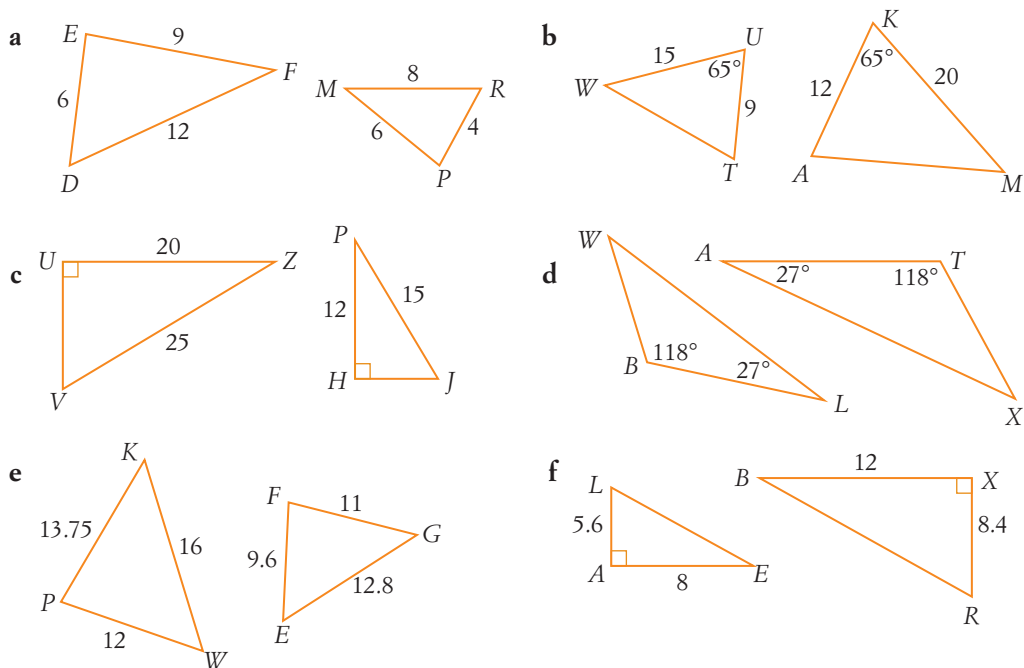


2 For each set of triangles, find the pair of similar triangles and state the similarity test used.



3 Similar triangles have three pairs of matching angles. So why is the similarity test AA and not AAA?

4 Use the correct notation to write a similarity statement relating each pair of similar triangles.



Stage 5.3

Worksheet

Congruent and similar triangle proofs

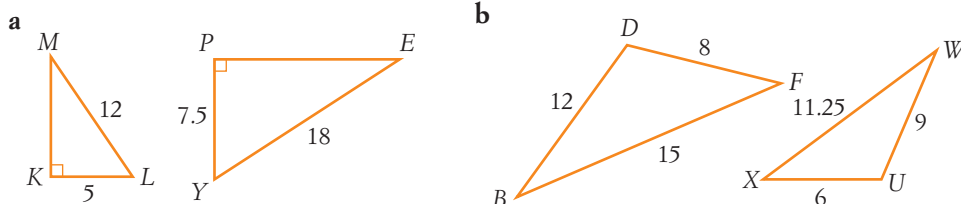
MAT09MGWK10153

13-09 Similar triangle proofs

To formally prove that two triangles are similar, we use a specific format that involves applying one of the four similarity tests SSS, SAS, AAA or RHS.

Example 23

Prove that each pair of triangles are similar.



Solution

a In $\triangle MLK \parallel \triangle EPY$:

$$\frac{ML}{PY} = \frac{12}{7.5} = \frac{2}{3}$$

$$\frac{KL}{YE} = \frac{5}{18} = \frac{2}{3}$$

$$\therefore \frac{ML}{PY} = \frac{KL}{YE}$$

$$\angle K = \angle P = 90^\circ$$

$$\therefore \triangle MLK \parallel \triangle EPY$$

(hypotenuses and another pair of matching sides of right-angled triangles in proportion, or 'RHS')

b In $\triangle DFB \parallel \triangle UXW$:

$$\frac{BF}{WX} = \frac{15}{11.25} = \frac{4}{3}$$

$$\frac{BD}{XW} = \frac{12}{9} = \frac{4}{3}$$

$$\frac{DF}{UX} = \frac{8}{6} = \frac{4}{3}$$

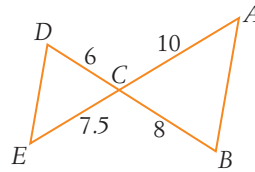
$$\therefore \frac{BF}{WX} = \frac{BD}{XW} = \frac{DF}{UX}$$

$$\therefore \triangle DFB \parallel \triangle UXW$$

(three pairs of matching sides in proportion, or 'SSS')

Example 24

Prove that $\triangle ABC \parallel \triangle EDC$.



Solution

In $\triangle ABC$ and $\triangle EDC$:

$$\frac{AC}{EC} = \frac{10}{7.5} = \frac{4}{3}$$

$$\frac{BC}{DC} = \frac{8}{6} = \frac{4}{3}$$

$$\therefore \frac{AC}{EC} = \frac{BC}{DC}$$

$$\angle ACB = \angle DCE$$

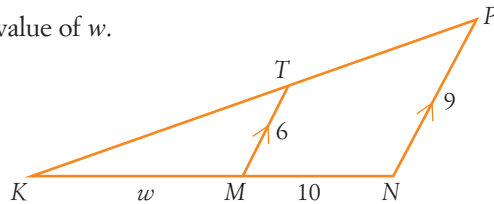
$$\therefore \triangle ABC \parallel \triangle EDC$$

(vertically opposite angles)

(two pairs of matching sides in proportion and the included angles equal, or 'SAS')

Example 25

Prove $\triangle KMT \parallel \triangle KNP$ and hence find the value of w .



Solution

$$\angle KMT = \angle KNP$$

$$\therefore \triangle KMT \parallel \triangle KNP$$

In $\triangle KMT$ and $\triangle KNP$:

$\angle K$ is common.

$$\therefore \frac{w}{w+10} = \frac{6}{9}$$

$$9w = 6(w+10)$$

$$= 6w + 60$$

$$3w = 60$$

$$w = 20$$

(corresponding angles, $MT \parallel NP$)

(equiangular, or 'AAA')

(matching pairs of similar triangles)

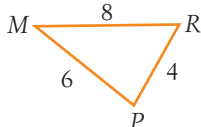
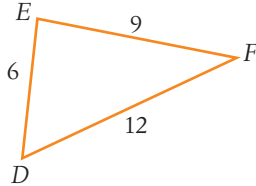
Stage 5.3

Exercise 13-09 Similar triangle proofs

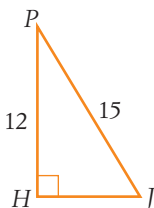
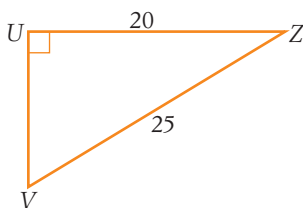
See Example 23

- 1 Prove that each pair of triangles are similar.

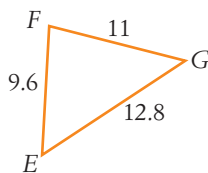
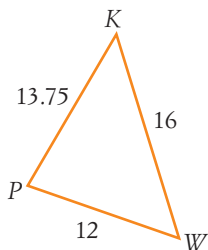
a



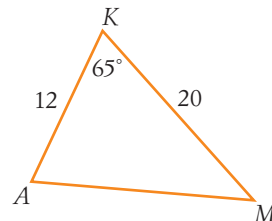
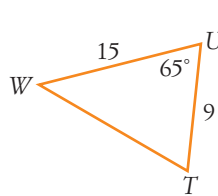
c



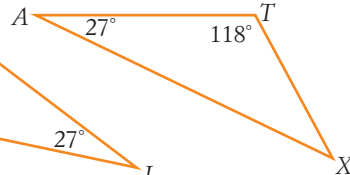
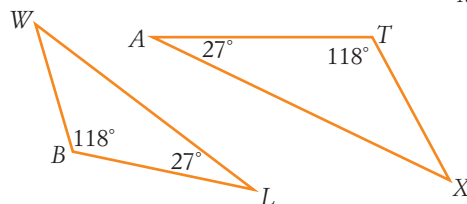
e



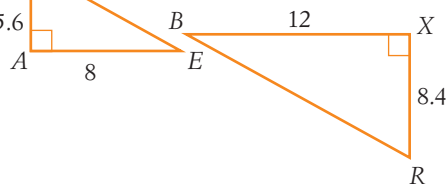
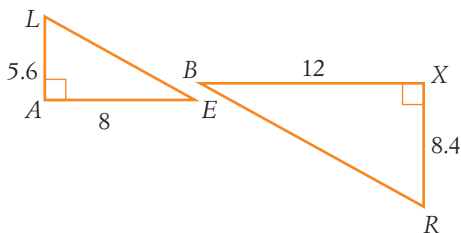
b



d

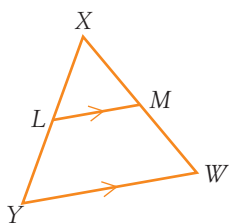
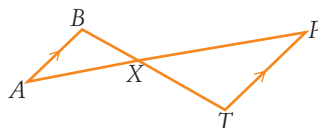
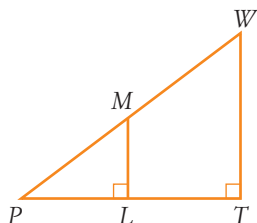
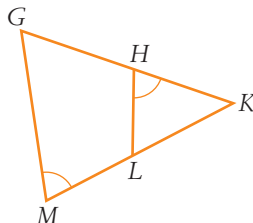


f

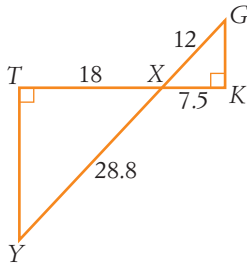


See Example 24

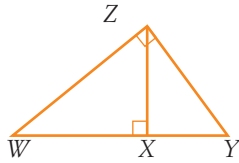
- 2 Prove the following.

a $\triangle XYW \parallel \triangle XLM$.

b $\triangle ABX \parallel \triangle PTX$.

c $\triangle PLM \parallel \triangle PTW$.

d $\triangle GKM \parallel \triangle LKM$.


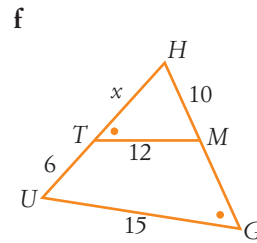
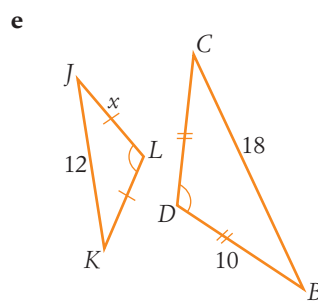
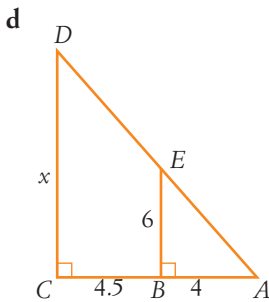
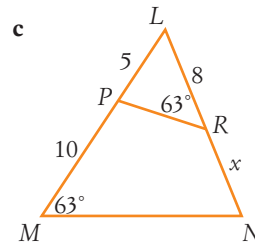
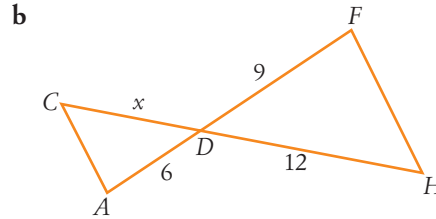
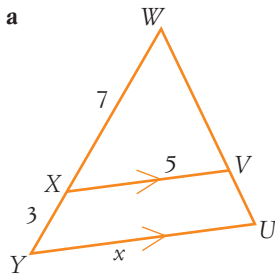
e $\triangle YTX \parallel \triangle GKX$.



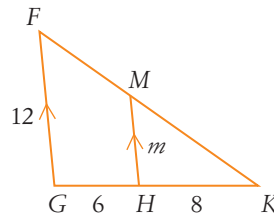
f $\triangle WZY \parallel \triangle WXZ$.



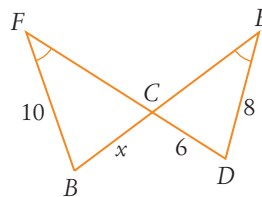
3 Find the value of x in each diagram after proving that two triangles are similar.



- 4 a Prove $\triangle KGF \parallel \triangle KHM$.
b Find the value of m .



- 5 Prove $\triangle BCF \parallel \triangle DCE$ and hence find the value of x .



See Example 25

See Example 24

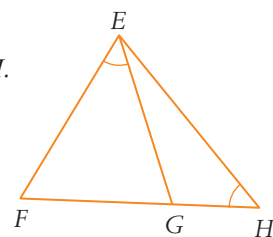
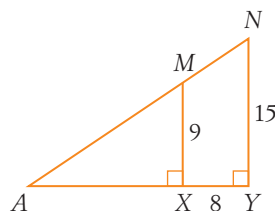
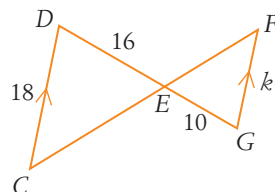
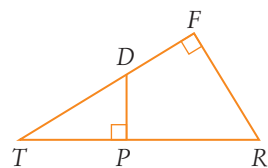
Worked solutions

Similar triangle proofs

MAT09MGWS10057

Stage 5.3

- 6 a Prove that $\triangle TFR \parallel \triangle TPD$.
 b If $FR = 9$ cm, $DP = 5$ cm and $TD = 8$ cm, find the length of TR .
- 7 a Prove that $\triangle CDE \parallel \triangle FGE$.
 b Find the value of k .
- 8 Prove $\triangle AXM \parallel \triangle AYN$ and hence find the length of AX .
- 9 a Prove that $\triangle FGE \parallel \triangle EFH$.
 b If $FG = 16$ cm and $EF = 20$ cm, find the length of FH .



Power plus

Worksheet

Investigating paper
sizes

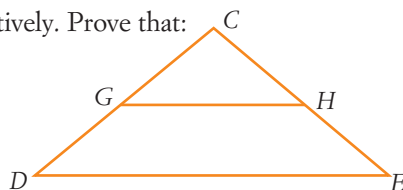
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- 1 G and H are the midpoints of CD and CE respectively. Prove that:

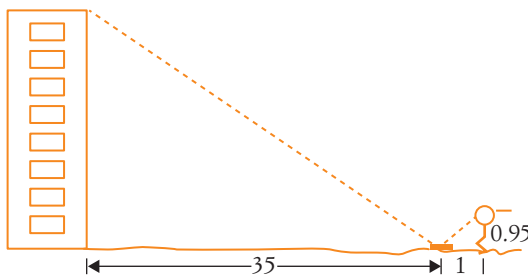
a $\triangle CGH \parallel \triangle CDE$.

b $GH \parallel DE$.

c $GH = \frac{1}{2}DE$.



- 2 Stephen places a mirror on the ground. He stands 1 m from the mirror and then bobs down until he can see the top of a building in the mirror. His eyes are 95 cm above ground level when he can see the building. If the mirror is 35 m from the base of the building, what is the height of the building?



- 3 Is each statement true or false? Give reasons.

a All equilateral triangles are similar.

b All isosceles triangles are similar.

c All circles are similar.

d All rectangles are similar.

e All squares are similar.

f All regular pentagons are similar.

- 4 Given that the diagonals of a quadrilateral bisect each other, prove that the quadrilateral is a parallelogram.

Language of maths

Puzzle sheet
Congruence and
similarity crossword
MAT09MGPS10155

Worksheet
Congruence and
similarity review
MAT09MGWK00046

Quiz
Congruence and
similarity
MAT09MGQZ00006

AAS	bisect	congruence test	congruent ($\equiv$)
diagonal	enlargement	equiangular	hypotenuse
image	included angle	matching	parallel
proportional	ratio	reduction	RHS
SAS	scale	scale diagram	scale factor
similar (III)	similarity test	SSS	vertex/vertices

- 1 When an original figure is enlarged or reduced, what is the new figure called?
- 2 What is the angle between two given sides of a triangle called?
- 3 What is the symbol and meaning of ‘is congruent to’?
- 4 What is the symbol and meaning of ‘is similar to’?
- 5 Which word from the above list is another name for ‘corresponding’?
- 6 What happens to a shape that is changed by a scale factor of 3?
- 7 What are the four tests for congruent triangles?
- 8 What does RHS stand for:

a in congruent triangles?

b in solving equations?

Topic overview

Copy and complete the table below.

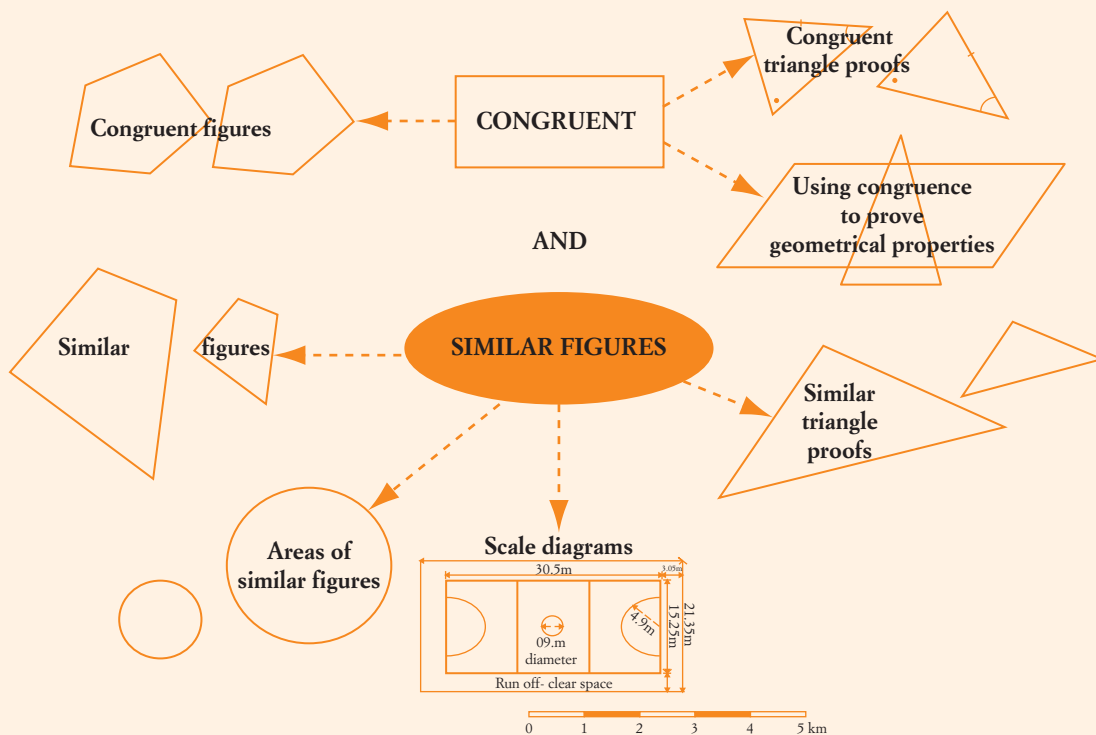
	The best part of this chapter was	
	The worst part was	
	New work	
	I need help with	

Copy (or print) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.

Worksheet

Mind map: Congruent and similar figures (Advanced)

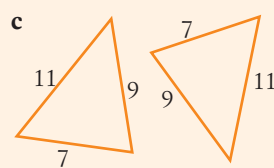
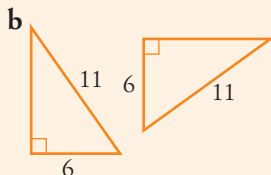
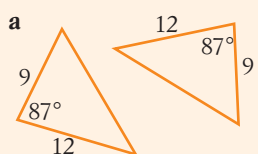
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Chapter 13 revision

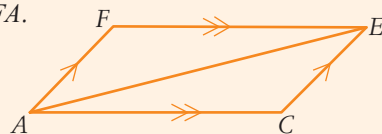
See Exercise 13-01

- 1 Which congruence test (SSS, SAS, AAS or RHS) can be used to prove that each pair of triangles are congruent?



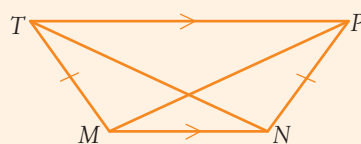
See Exercise 13-02

- 2 If $ACEF$ is a parallelogram, prove that $\triangle ACE \equiv \triangle EFA$.



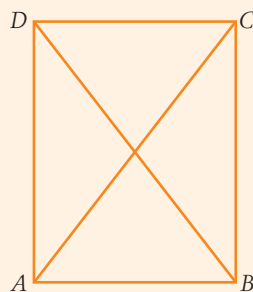
See Exercise 13-02

- 3 If $MN \parallel TP$, $\angle MNP = \angle NMT$ and $TM = PN$, prove that $\triangle MNP \equiv \triangle NMT$.



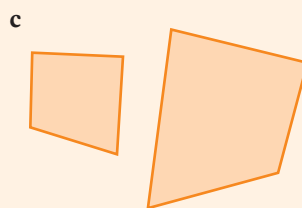
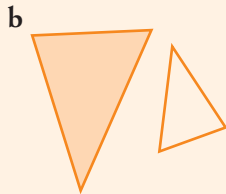
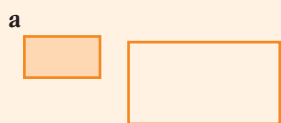
See Exercise 13-03

- 4 $ABCD$ is a rectangle.
a Prove that $\triangle ABD \equiv \triangle BAC$.
b Hence prove that the diagonals of a rectangle are equal in length.



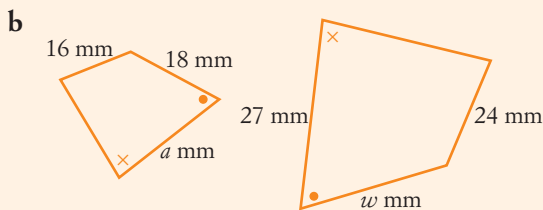
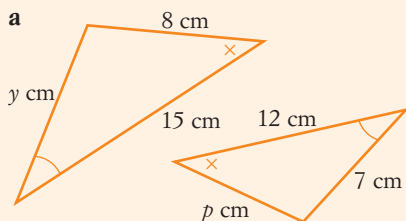
See Exercise 13-04

- 5 By measurement, find the scale factor for each pair of similar figures.



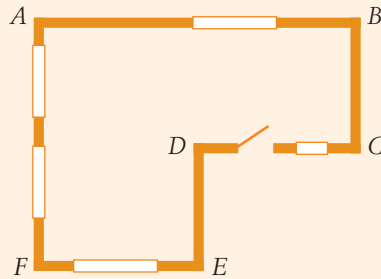
See Exercise 13-05

- 6 Find the value of every pronumeral in each pair of similar figures.



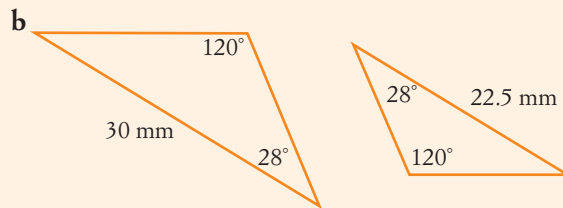
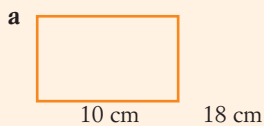
- 7 This floor plan of a holiday house is drawn to a scale of 1: 200. What is the actual length of:

a AF ? b FE ?



See Exercise 13-06

- 8 Find the ratio of the areas for each pair of similar figures.

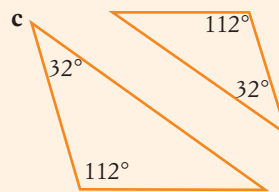
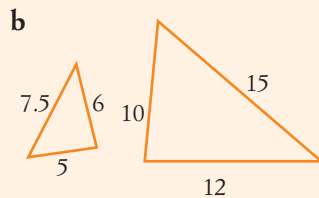
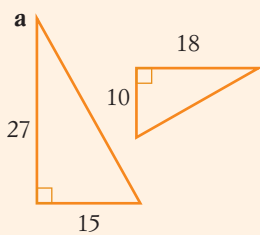


Stage 5.3

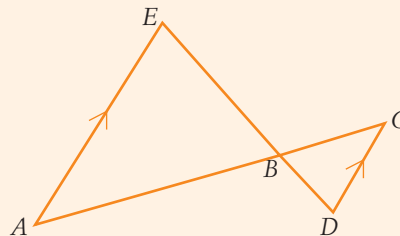
See Exercise 13-07

- 9 Which test can be used to prove that each pair of triangles are similar?

See Exercise 13-08



- 10 a Prove that $\triangle ABE \parallel \triangle CBD$.
b If $DC = 8$ cm, $AE = 14$ cm and $CB = 7$ cm, find the length of AB .

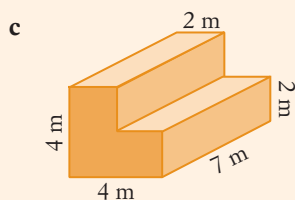
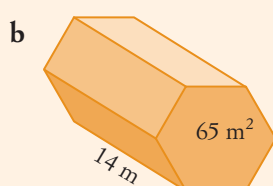
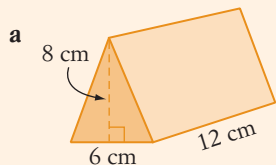


Stage 5.3

See Exercise 13-09

See Chapter 10

- 1 Find the volume of each solid, correct to one decimal place where necessary.



Stage 5.3

See Chapter 1

See Chapter 7

See Chapter 9

- 2 Simplify each expression.

a $\frac{\sqrt{125}}{5}$

b $7\sqrt{3} + \sqrt{12}$

c $-5\sqrt{7} \times 4\sqrt{7}$

d $\frac{9\sqrt{56}}{\sqrt{7}}$

- 3 Make t the subject of the formula $k = my - \frac{1}{2}mt^2$.

- 4 This dot plot shows the project results (out of 10) for a Year 9 class.

a How many students were in the class?

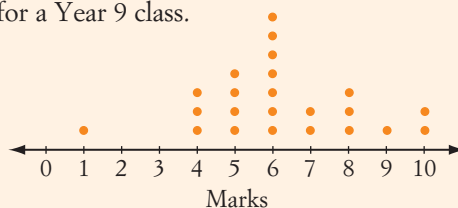
b Find:

i the range

ii the mode

iii the mean, correct to one decimal place

iv the median



c Which of the scores can be considered as an outlier?

d Are there any clusters?

See Chapter 9

- 5 Classify each type of data as categorical (C), quantitative discrete (QD) or quantitative continuous (QC).

a number of taxis on the road

b rating of film (1 star to 5 stars)

c weights of babies in a maternity ward

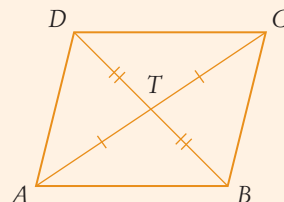
d brands of TVs

See Chapter 13

- 6 The diagonals of the quadrilateral $ABCD$ bisect each other.

a Prove that $\triangle DTC \equiv \triangle BTA$.

b Hence prove that $DC \parallel AB$.

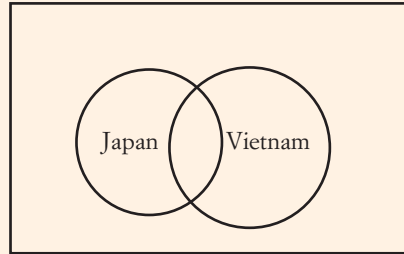


- 7 A survey was held to find whether a sample of 30 Australians had travelled to Japan or Vietnam. The results are shown below:

See Chapter 12

Japan 9 Vietnam 18 Japan and Vietnam 5

- a Copy and complete the Venn diagram to show this information.
b What is the probability that a person chosen at random from this sample:
i has not been to either country?
ii has been to Japan or Vietnam but not both?
iii has been to Vietnam but not Japan?

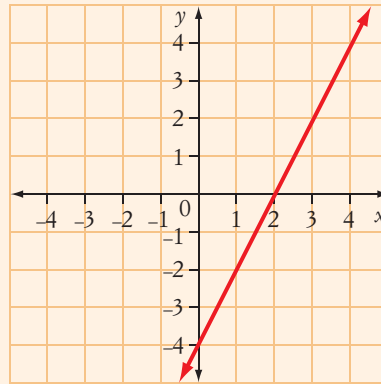


See Chapter 2

- 8 Convert:

a 40 m/s to km/h b 10 g/s to kg/min

- 9 Find the equation of this line.



See Chapter 11

- 10 Jotika earns a salary of \$82 000 p.a. How much (correct to the nearest cent) did she earn:

See Chapter 8

a per week? b per month?

- 11 In $\triangle LKM$, $\angle M = 90^\circ$, $MK = 6$ m and $KL = 9$ m.

See Chapter 4

- a Sketch a diagram of $\triangle LKM$.
b Find the length of ML correct to two decimal places.

- 12 Factorise each expression.

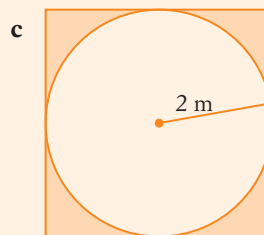
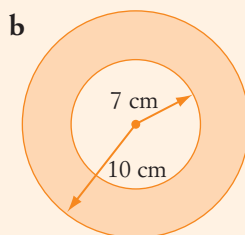
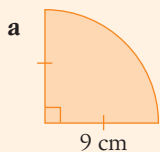
a $y^2 - 4y + 2y - 8$ b $m^2 - 8m + 15$ c $5m^2 - 245$

Stage 5.3

See Chapter 3

- 13 Find each shaded area, correct to one decimal place.

See Chapter 10



See Chapter 8

- 14 George earns \$21.45 per hour. How much does he earn for working 38 hours at normal rates and 6 hours at time-and-a-half?

Stage 5.3

See Chapter 5

- 15 What is the value of $8^{-\frac{3}{2}}$? Select the correct answer **A**, **B**, **C** or **D**.

A $-13\frac{1}{3}$

B -32

C $\frac{1}{32}$

D $\frac{3}{40}$

See Chapter 5

- 16 Simplify each expression.

a $k^7 \times k^5$

b $d^{10} \div d^5$

c $3p^2q \times 5pq^3$

d $20hk^5 \div 5hk$

e $\left(\frac{2r}{5}\right)^3$

f $5c^0$

See Chapter 9

- 17 **a** Complete a frequency table for this data, including a 'fx' column.

53	50	50	52	47	49	49	50	48	50
52	49	47	51	51	52	50	53	50	52
50	48	52	49	50	49	51	45	52	50
51	52	51	50	52	50	51	52	49	51

- b** Find the mean of this data.
c Show this data in a frequency histogram and polygon.
d Find the mode.
e Find the median.

See Chapter 3

- 18 Expand each expression.

a $3(1 - 2y)$

b $x(2x - 7)$

c $-4m(1 - m)$

d $(5x + 7)(x - 4)$

Stage 5.3

See Chapter 7

- 19 Simplify each expression.

a $\left(\frac{5a}{3}\right)^{-3}$

b $(16x)^{\frac{1}{4}}$

c $\sqrt[3]{-8p}$

d $r^{\frac{2}{5}}$

See Chapter 6

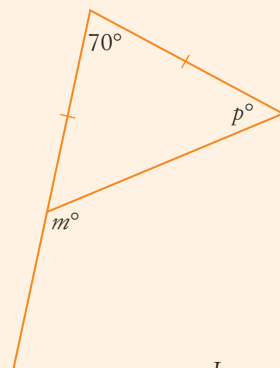
- 20 What are the values of m and p in this diagram? Select the correct answer **A**, **B**, **C** or **D**.

A $m = 110, p = 70$

B $m = 125, p = 70$

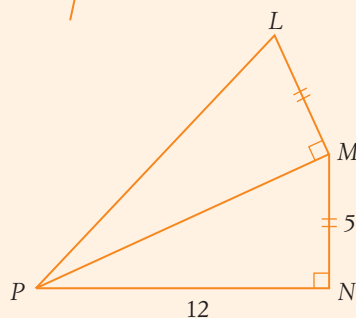
C $m = 110, p = 55$

D $m = 125, p = 55$



See Chapter 10

- 21 For the quadrilateral $LMNP$, calculate its:
a perimeter correct to two decimal places
b area



- 22 The number of goals scored by Damien's hockey team in 18 games were:

See Chapter 9

1	3	1	2	4	1	3	3	2
1	1	1	2	2	3	3	3	4

If 5 goals were scored in the tenth game, which statistical measure will not change? Select the correct answer **A**, **B**, **C** or **D**.

- A** the range **B** the mean **C** the median **D** the mode

- 23 Tahlia is paid a weekly commission of 4.3% on the value of her sales of perfume over \$1500. Calculate her commission if her sales during one week totalled \$19 200.

See Chapter 8

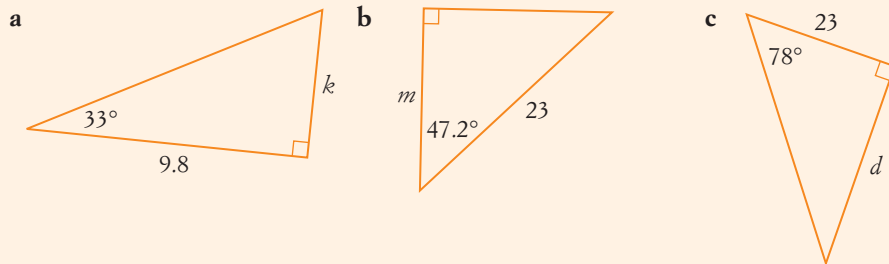
- 24 Factorise each expression.

See Chapter 3

- a** $2y + 8$ **b** $5a - 15ab$ **c** $-8ab + 2b$

- 25 Find, correct to two decimal places, the value of each pronumeral.

See Chapter 4

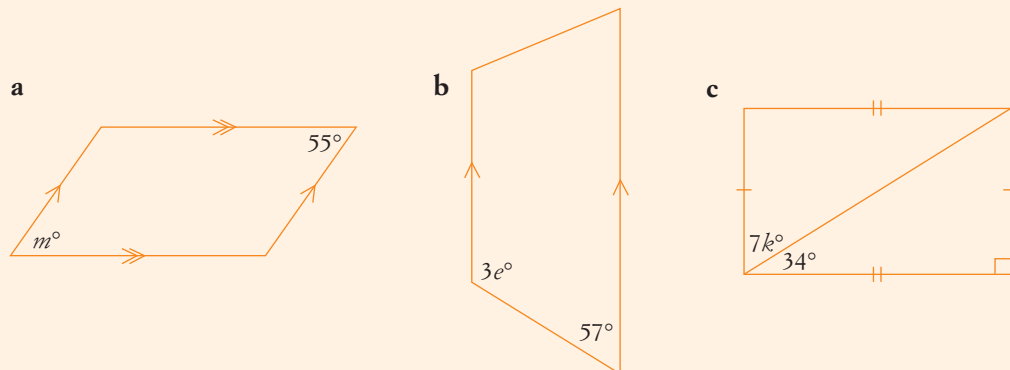


- 26 Romesh buys a car for \$11 000 and sells it for \$5200 two years later. Calculate the percentage loss correct to one decimal place.

See Chapter 2

- 27 Find the value of each pronumeral.

See Chapter 6



- 28 What is the equation of the horizontal line that passes through the point $(-2, 5)$?

See Chapter 11

- 29 For the interval joining the points $(5, 3)$ and $(-4, 2)$, find:

See Chapter 11

- a** the midpoint **b** the length in exact form **c** the gradient

- 30 The scale of a plan is 1 : 2000. What distance in centimetres on the plan would represent a real distance of 50 m?

See Chapter 11

See Chapter 6

- 31** Name the most general shape that matches each description.
- a** A quadrilateral with equal diagonals.
 - b** A triangle with three equal sides.
 - c** A quadrilateral with opposite sides equal and two axes of symmetry.
 - d** A triangle with two equal angles.

See Chapter 9

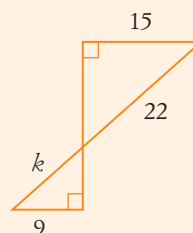
- 32** The median of the data shown in this stem-and-leaf plot is 154.

Stem	Leaf
13	2 3
14	0 5 6 7
15	1 □ 6 8 8 9
16	3 4 4 5

- a** What is the missing value shown by □?
- b** Find the range.
- c** Describe the shape of the distribution.

See Chapter 13

- 33**
- a** Which similarity test can be used to prove that the two triangles are similar?
 - b** Find the value of k .



See Chapter 11

- 34**
- a** Graph each equation on a different number plane.
 - i** $y = 2x - 1$
 - ii** $y = -x^2 + 3$
 - iii** $x^2 + y^2 = 4$
 - b** What is the gradient and y -intercept of the graph of $y = 2x - 1$?
 - c** What are the coordinates of the vertex of the graph of $y = -x^2 + 3$?

See Chapter 2

- 35** Calculate the simple interest on \$8750 invested at 5.5% for 4 years.

See Chapter 2

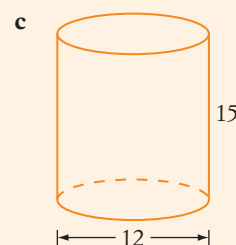
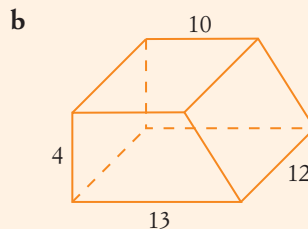
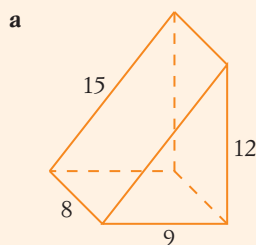
- 36** The cost of a notebook computer, including 10% GST, is \$968. What is the amount of the GST?

See Chapter 11

- 37** Which of the following points lie on the line $y = 2x - 5$? Select the correct answer **A**, **B**, **C** or **D**.
- A** (0, -3) **B** (1, -3) **C** (-3, 0) **D** (-3, 1)

See Chapter 10

- 38** Find the surface area of each solid, correct to three significant figures where necessary. All measurements are in centimetres.



39 Solve each equation.

See Chapter 7

a $\frac{m}{2} - \frac{m}{7} = 3$

b $\frac{3k-1}{5} = \frac{2k+1}{4}$

c $4(3y-1) + 6 = 2(2y+7)$

d $3x^2 = 27$

e $-6(12-x) = 48$

f $\frac{3a-8}{2} = 7$

40 Express each number in scientific notation.

See Chapter 5

a 714 000

b 0.000 35

c 15 763 000

41 Evaluate each expression in scientific notation correct to two significant figures.

See Chapter 5

a $(4.89 \times 10^7)^2$

b $\frac{9.3 \times 10^{-3}}{2.2 \times 10^8}$

42 A die was repeatedly rolled and the results are shown in this table.

See Chapter 12

a Find, correct to two decimal places, the experimental probability of rolling:

i a 1

ii a 4

iii an even number

iv a number less than 5

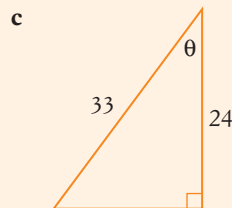
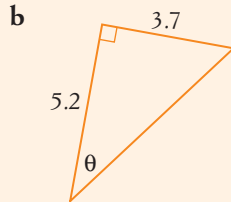
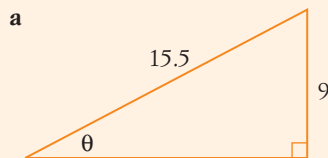
v neither a 2 nor a 6.

b Based on the theoretical probability, what is the expected frequency of rolling a number less than 5? How does the actual frequency compare with the expected frequency?

Outcome	Frequency
1	40
2	35
3	53
4	45
5	50
6	47

43 Find the value of θ correct to the nearest minute.

See Chapter 4



- 44
- What is the angle sum of a regular octagon?
 - What is the size of one angle in a regular octagon?
 - What is the size of one exterior angle in a regular octagon?
 - What is the sum of the exterior angles of any polygon?

See Chapter 6

Instructional glossary

Mathematical 'doing' words

analyse To study something in great detail by breaking it into its parts.

bisect To cut in half.

calculate To find the value of a numerical expression. See also **evaluate**.

classify To sort into categories or types.

complete To fill in detail that makes a statement or diagram correct or whole.

construct To draw a geometrical figure accurately.

convert To change from one form to another. For example, convert a fraction to a decimal, or convert dollars to cents.

decrease To make smaller by subtracting or dividing.

estimate To make an educated guess for a number or answer; to find roughly or approximately.

evaluate To find the value of an expression. For example, evaluate 3×8^2 , or evaluate $4x + 1$ when $x = 5$.

expand To remove the brackets in an algebraic expression; the opposite of **factorise**. Expanding $3(2y + 1)$ gives $6y + 3$.

factorise To take out the highest common factor (HCF) of an expression and insert brackets; the opposite of **expand**. Factorising $5x - 20$ gives $5(x - 4)$.

give reasons When solving a problem, to show the mathematical rules or thinking used.

graph To display on a number line, number plane or statistical graph.

hence find/prove To find an answer or prove a result using previous answers or information supplied.

increase To make larger by adding or multiplying.

measure To find the size of something by using an instrument. For example, to find a length using a ruler.

prove/show that In questions where the answer is given, to use mathematical reasoning to prove that the answer is true.

reduce (a fraction) to its lowest terms See **simplify (a fraction)**.

round (a number) To find the nearest approximation for a number. For example, round 4.3 to the nearest whole number (4), \$12.9598 to the nearest cent (\$12.96), 0.166 66 to three decimal places (0.167).

show working To show the steps you used to find an answer.

simplify To give the answer in its simplest, shortest, neatest form.

simplify (a fraction) To reduce the numerator and denominator of a fraction by dividing by their highest common factor (HCF). When the numerator and denominator are as small as possible, the fraction has been simplified or reduced to its lowest terms.

Glossaries

simplify (a ratio or rate) To reduce the terms or units of a ratio or rate by dividing by a common factor, in a similar way to simplifying a fraction.

sketch To draw a rough diagram. Less accurate than to **construct**.

solve To find the value of the unknown variable in an equation.

substitute To replace a variable with a number, to evaluate an expression. For example, substituting $t = 5$ in the expression $t^2 + 6$ gives 31.

write correct to See **round (a number)**.

write/state To write an answer, formula or result without showing any working or explanation. (This usually means that the answer can be found mentally, or in one step.)

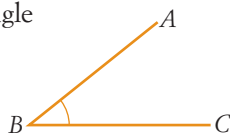
Mathematical glossary

Some common symbols

%	per cent	π	pi, the decimal 3.14159 . . .
()	parentheses, round brackets	$3x$	3 multiplied by x , $3 \times x$
[]	(square) brackets	$\frac{x}{2}$	x divided by 2, $x \div 2$
{ }	braces	x^2	x squared, $x \times x$
=	is equal to	x^3	x cubed, $x \times x \times x$
$\approx$	is approximately equal to	$\bar{x}$	the mean (average)
$\neq$	is not equal to	Σ	the sum of
$<$	is less than	$42^\circ 17' 54''$	42 degrees, 17 minutes, 54 seconds
$>$	is greater than	$\angle ABC$	angle ABC
$\leq$	is less than or equal to	$\triangle ABC$	triangle ABC
$\therefore$	therefore	$\parallel$	is parallel to
-3	negative 3	$\perp$	is perpendicular to
$\sqrt{\quad}$	the square root of	$\equiv$	is congruent (identical) to
$\sqrt[3]{\quad}$	the cube root of	$\sim$	is similar to
± 3	positive or negative 3	$P(E)$	The probability of an event, E
$0.\dot{6}$	the recurring decimal 0.666 666 . . .	$P(\bar{E})$	The probability of the complementary event, $\bar{E}$

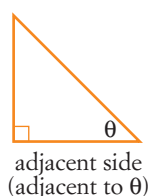
A

acute angle A 'sharp' angle between 0° and 90° , for example, the marked angle in the diagram.



acute-angled triangle A triangle with all three angles acute.

adjacent side In a right-angled triangle, the side 'next to' the given angle, leading to the right angle.

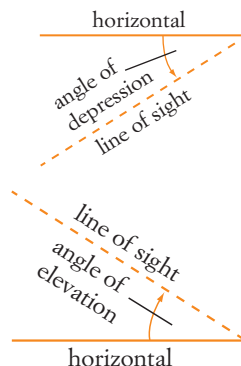


algebraic expression A number written in algebraic form using variables, for example, $2xy + 4y - 5$.

algebraic term See **Terms of an expression**.

allowable (tax) deduction A part of a person's yearly income that is not taxed, such as work-related expenses or donations to charities. All deductions are subtracted from yearly income to determine **taxable income**.

angle of depression The angle of looking down, measured from the horizontal.



angle of elevation The angle of looking up, measured from the horizontal

angle sum The total of the sizes of the angles in a shape. The angle sum of a triangle is 180° .

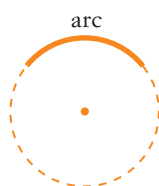
Glossaries

annual leave loading (or holiday loading) Extra payment to a worker during annual leave based on 17.5% of four weeks' pay.

annulus A ring shape between two different-sized circles with the same centre.



arc Part of the circumference of a circle.



area The amount of surface enclosed by a shape, measured in square units.

ascending order Going up, increasing, from smallest to largest (1–2–3). The opposite of **descending order**.

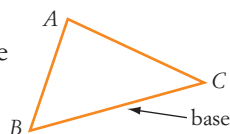
at least Referring to the smallest number, for example, 'at least 2' means 2, 3, 4, . . . , that is, '2 or more'.

average See **Measure of location**.

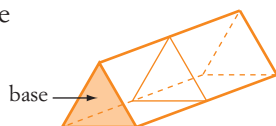
B

base (in index notation) When a number is raised to a power, the number raised is the base. In the expression 3^5 , the 3 is called the base.

base (of a shape) The bottom side of a flat shape such as a triangle.



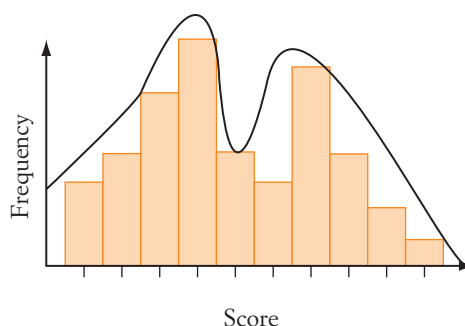
base (of a solid) The bottom or end face of a solid shape, such as a prism, pyramid or cylinder.



bearing The angle used to show the direction of one location from a given point. See also **compass bearing** and **three-figure bearing**.

bias In statistics, something that causes a sample to not truly represent the population.

bimodal distribution A statistical distribution that has two peaks.



binomial expression An algebraic expression with two terms, for example, $x + 9$, $2y - 12$. See also **trinomial**.

bisect To cut in half.

bonus Extra pay for achieving a high quality or volume of work, such as meeting an important quota, goal or deadline.

breadth Another word for 'width'; how wide or broad something is.

byte (B) A unit of digital (computer) memory required to store one character of data, such as the letter 'a'.

C

capacity The amount of material (usually liquid) that a container can hold, measured in millilitres (mL), litres (L), kilolitres (kL) and megalitres (ML). See also **volume**.

cartesian plane Another name for **number plane**.

categorical data Data that can be classified into categories, such as hair colour, favourite radio station or postcode. Data that is not **numerical**.

census A survey of the entire **population** of people or items, not just a survey of a **sample**.

chance experiment An activity or process that involves chance, for example, rolling a die or tossing a coin.

circumference The perimeter of a circle. $C = \pi d$ or $C = 2\pi r$, where C is the circumference, π is pi, d is the diameter and r is the radius.



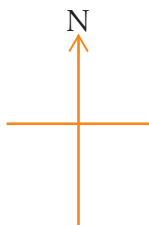
cluster A group of data scores that are bunched or close together.

coefficient The number in front of a variable in an algebraic term. For example, the coefficient of x in $2x - 5$ is 2.

commission Pay earned by salespeople and agents calculated as a percentage of the value of items sold or income made.

compass bearing A bearing that refers one of the sixteen points of a mariner's compass, for example, north-northwest (NNW). See also **bearing** and **three-figure bearing**.

compass rose A cross-shaped diagram that shows the direction of north. See also **compass bearing**.



complementary event All the outcomes that are *not* the event; the 'opposite' event. For example, the complementary event to rolling 1 on a die is rolling a number that is not 1.

composite shape A shape made up of two or more basic shapes.

congruent Identical, exactly the same. The symbol ' $\equiv$ ' means 'is congruent to' or 'is identical to'.

congruence test One of four tests for proving that two triangles are congruent: SSS, SAS, AAS and RHS.

consecutive numbers Any series of integers that follow each other in order, for example, 8, 9 and 10.

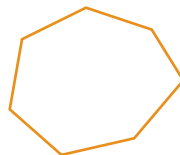
constant of proportionality See **direct proportion**.

constant term The term in an algebraic that is a number only, with no variable. For example, the constant term in $x^2 - 4x + 6$ is 6.

continuous data Numerical data that can be measured on a smooth scale without any gaps, and can take on a full range of values, such as the height of people. Continuous data is measured on a scale without 'gaps', unlike **discrete data**.

converse The 'reverse' of a rule, the rule written 'back-to-front' or 'turned around'. The converse of Pythagoras' theorem is that if the square of the longest side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is right-angled.

convex polygon A polygon whose vertices all point outwards. All diagonals lie within the shape, and all angles are less than 180° .



Convex



Non-convex

cosine A ratio in a right-angled triangle:

$$\cos \theta = \frac{\text{side adjacent to } \theta}{\text{hypotenuse}}$$

where θ is an angle. See also **sine** and **tangent**.

Glossaries

cost price The price it costs a retailer (shop) to buy (to resell). The cost price is usually less than the **selling price**.

cross-section A 'slice' of a solid cut across it rather than along it.



cube root (of a number) The value which, if cubed, gives the number. For example, $\sqrt[3]{8} = 2$ because $2^3 = 2 \times 2 \times 2 = 8$.

cubic equation An equation in which the highest power of the variable is 3, that is, a variable cubed, for example, $x^3 = 12$.

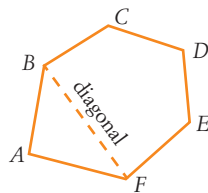
cumulative frequency A progressive or running total of frequencies, the sum of frequencies of a particular score and all scores below it.

D

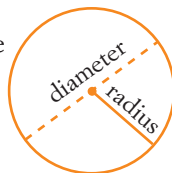
data Information, a collection of facts.

descending order Going down, decreasing, from largest to smallest (3–2–1). The opposite of **ascending order**.

diagonal An interval joining two non-adjacent vertices of a shape.



diameter An interval joining two points on the circumference and passing through the centre of a circle, or the length of that interval. The diameter is double the **radius**.



difference of two squares An algebraic expression of the form $a^2 - b^2$, that can be factorised into $(a + b)(a - b)$. For example, $x^2 - 25 = (x + 5)(x - 5)$.

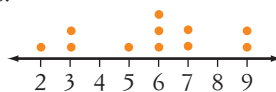
direct proportion (or **direct variation**) A relationship between two variables in which one variable is a constant multiple of the other, that is $y = kx$ where k is a constant called the **constant of proportionality**. For example, if $y = 8.5x$, then y is directly proportional to x .

discount The saving made that is the difference between the original price of an item and the reduced price.

discrete data Numerical data that are counted or measured, that only taking on distinct, separate values, such as the number of children in a family (0, 1, 2, ...). Discrete data has a scale with 'gaps' or jumps, unlike **continuous data**.

distributive law A law of arithmetic that says you can multiply by a number by splitting it into the sum or difference of two other numbers. For example, $27 \times 12 = 27 \times (10 + 2) = 27 \times 10 + 27 \times 2$. Algebraically, $a(b + c) = ab + ac$ for any three numbers a , b and c .

dot plot A graph that uses dots above a number line to represent the frequencies of data scores.



double time Overtime pay that is calculated at 2 times the normal pay rate.

E

equation A mathematical statement that two quantities are equal. For example, $8 + 2 = 10$ or $3b - 7 = 5$.

equilateral triangle A triangle with all three sides equal.

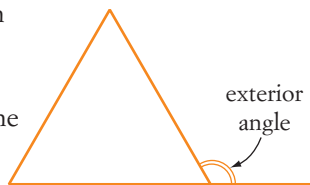


expected frequency The expected number of times an event will occur over repeated trials, calculated by multiplying the probability of the event by the number of trials.

experimental probability An estimate of theoretical probability; the **relative frequency** of an event in repeated trials of an experiment, found using the formula

$$P(E) = \frac{\text{frequency of } E}{\text{total frequency}}$$

exterior angle An 'outside' angle of a shape created by extending one of the sides of the shape.



event In probability, a result involving one or more outcomes. For example, when rolling a die, the event 'rolling an even number' contains the three outcomes {2, 4, 6}.

F

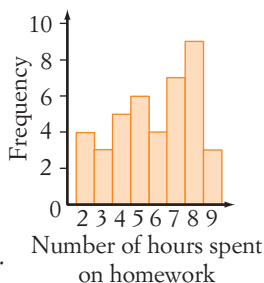
formula (plural: **formulas** or **formulae**) A rule written as an algebraic equation, using variables.

The formula for the area of a triangle is $A = \frac{1}{2}bh$.

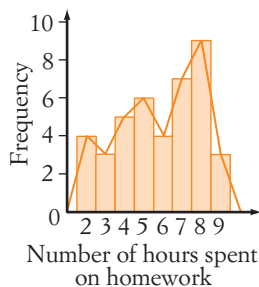
frequency The number of times an event occurs in repeated trials of a probability experiment, or the number of times a value appears in a set of data.

frequency histogram

A column graph that shows the frequencies of numerical data. There are no spaces between the columns, and the graph looks like a row of office buildings.



frequency polygon A line graph that shows the frequencies of numerical data. It can be made by joining the midpoints of the tops of the columns of a histogram. The graph looks like a mountain.



frequency (distribution) table A table listing the frequency of each value in a set of data, with columns for Score (x), Frequency (f) and sometimes Tally and fx .

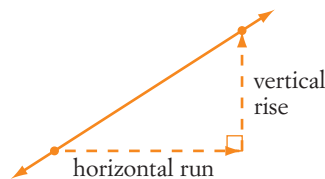
G

giga- A prefix meaning one billion (1 000 000 000), represented by the letter G.

gigabyte (GB) A unit of digital memory equal to one billion bytes. See also **byte**, **kilobyte**, **megabyte**, **terabyte**.

goods and services tax (GST) A 10% tax added to the original price of an item or service.

gradient The steepness of a line or interval, measured by the fraction $\frac{\text{rise}}{\text{run}}$



gradient–intercept form (of a linear equation)

The equation of a straight line $y = mx + b$, where m is the **gradient** and b is the y -intercept.

greatest common divisor (GCD)

See **Highest common factor**.

gross pay Pay received before tax and other deductions are taken out.

GST See **goods and services tax**.

H

hectare A large metric unit of area, equivalent to 10 000 m² or to the area of a square 100 m × 100 m.

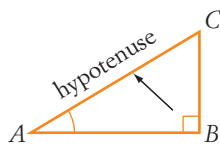
highest common factor (HCF) Also called **greatest common divisor (GCD)**. The largest factor shared by two or more numbers or algebraic terms. For example, the HCF of 36 and 8 is 4 and the HCF of $6xy$ and $12y^2$ is $6y$.

Glossaries

horizontal Going across, sideways, flat.



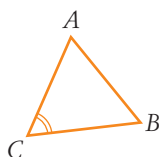
hypotenuse The longest side of a right-angled triangle, opposite the right angle.



I

image A transformed shape after it has been enlarged or reduced.

included angle The angle between two given sides of a shape. For example, the included angle for sides AC and CB in this triangle is $\angle C$.



income tax A tax paid to the government based on the size of a person's income.

index (plural: **indices**) See **Power**.

index notation A way of writing repeated multiplication using indices (powers), in the form a^n . Index notation for $2 \times 2 \times 2 \times 2$ is 2^4 .

index law An algebraic rule for simplifying expressions involving powers of the same base, for example, $a^m \times a^n = a^{m+n}$.

intersect To cross.

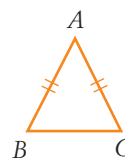
interval A section of a line with a definite length, such as AB below.



inverse operation The opposite or reverse operation, used when solving equations. For example, the inverse operation to adding is subtracting, the inverse operation to dividing is multiplying.

irrational number A number such as π or $\sqrt{2}$ that cannot be expressed as a fraction (rational number). In decimal form, its digits run endlessly without repeating. See also **rational number** and **real number**.

isosceles triangle A triangle with two equal sides.

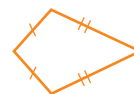


K

kilo- A prefix meaning one thousand (1000), represented by the symbol k. For example, one kilogram is 1000 grams.

kilobyte (kB) A unit of digital memory equal to 1000 bytes. See also **byte**, **gigabyte**, **megabyte**, **terabyte**.

kite A quadrilateral with two pairs of equal adjacent sides.



L

LHS The left-hand side (of an equation).

limits of accuracy (of a measuring instrument) A measure of the precision of an instrument, calculated as ± 0.5 of the smallest unit on the instrument.

linear equation A formula whose graph is a straight line, or an equation involving a variable that is not raised to a power, such as $2x + 9 = 17$.

loss The money lost when selling an item at a lower price, when the selling price is less than the cost price. The opposite of **profit**.

lowest common multiple (LCM) The smallest multiple that is shared by two or more numbers. For example, the LCM of 4 and 10 is 20.

M

mean The average of a set of data, represented by $\bar{x}$, calculated by dividing the sum of the scores by the number of scores.

measure of location An average, middle or typical value of a set of data. The three measures of location are the **mean**, **median** and **mode**.

median The middle score when the scores are arranged in order. If the number of values is even, then the median is the average of the two middle values.

mega- A prefix meaning one million (1 000 000), represented by the letter M. A megalitre is one million litres.

megabyte (MB) A unit of digital memory equal to 1 000 000 bytes. See also **byte**, **gigabyte**, **kilobyte**, **terabyte**.

micro- A prefix meaning one-millionth, represented by the Greek letter μ . One microsecond is one-millionth of a second.

midpoint The point in the middle of an interval or halfway between two given points.

milli- A prefix meaning one-thousandth ($\frac{1}{1000}$), represented by the letter m. One millimetre is one-thousandth of a metre.

minute (symbol ') A measure of angle size. $\frac{1}{60}$ of a degree. $1^\circ = 60'$.

mode The most common or frequent score(s) in a set of data.

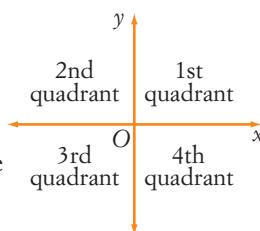
mutually exclusive events Events or categories that have no items in common.

N

nano- A prefix meaning one-billionth, represented by the letter n. One nanosecond is one-billionth of a second.

net pay Pay received after deductions from gross pay; 'take-home' pay.

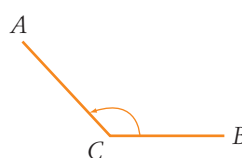
number plane A coordinate grid system based on two number lines that cross at right angles: a horizontal line called the x -axis and a vertical line called the y -axis. Also called **cartesian plane**.



numerical data Data that can be measured or counted, such as a person's height or number of goals scored. Data that is not **categorical**.

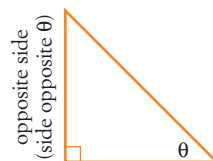
O

obtuse angle A 'wide' angle greater than 90° but less than 180° .

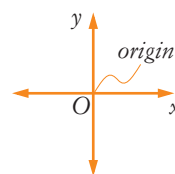


obtuse-angled triangle A triangle with one obtuse angle (between 90° and 180°).

opposite side In a right-angled triangle, the side directly facing the given angle.



origin The point $O(0, 0)$ at the centre of the number plane, where the x -axis and y -axis cross.



outcome In probability, a result of a situation or experiment. For example, when rolling a die, one possible outcome is rolling a 4.

outlier An extreme data value that is very different from the other values in a set.

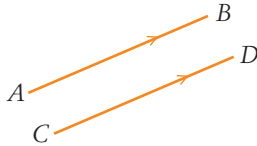
overtime Time worked beyond normal working hours, such as at night or on weekends, at a higher rate of pay.

Glossaries

P

parabola A U-shaped curve that is the graph of a quadratic equation such as $y = x^2$.

parallel lines Lines that point in the same direction and do not intersect.



$AB \parallel CD$ means 'AB is parallel to CD'.

parallelogram A quadrilateral in which the opposite sides are parallel.

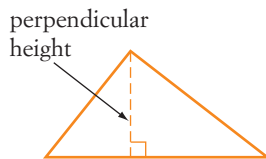


PAYG (Pay As You Go) tax Income tax deducted from your pay each payday by your employer.

per annum (p.a.) Per year.

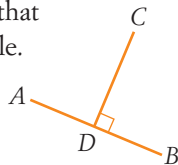
perfect square A square number or an algebraic expression that represents one, for example, 64 , $(x + 9)^2$, $(a - b)^2$.

perpendicular height The height of a shape, measured at right angles to the base.



perpendicular lines Lines that intersect to form a right angle.

$AB \perp CD$ means AB is perpendicular to CD.



pi (π) A special number, approximately 3.1416, used in calculating circle measurements. Pi is a constant value found by dividing the circumference of any circle by the diameter.

piecework Earnings based on the number of items processed, made or delivered, paid at a rate per item rather than on the number of hours worked.

polygon Any flat shape made up of straight sides.

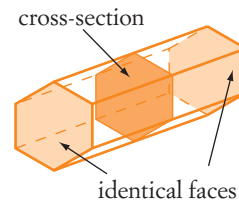


population In statistics, all of the items being studied, the entire group.

power (or index) The number of times a base is multiplied by itself. In 2^5 , the power is 5. Also called the *exponent*.

principal An amount of money invested or borrowed, on which interest is calculated.

prism A solid shape with identical cross-sections with straight sides.



probability The chance of an event occurring, measured as a fraction, decimal or percentage between 0 and 1.

product The result of multiplication. The product of 7 and 3 is 21.

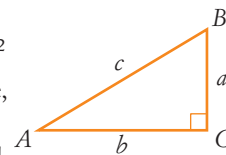
profit The amount made when selling an item at a higher price, when the selling price is more than the cost price. The opposite of **loss**.

pronumeral Another name for **variable**.

proportion See **direct proportion**.

Pythagoras' theorem

The formula $c^2 = a^2 + b^2$ for a right-angled triangle, where c is the length of the hypotenuse and a and b are the lengths of the other two shorter sides.



Pythagorean triad A set of three numbers that follow Pythagoras' theorem, such as 3, 4, 5.

Q

quadrant (of a circle) A sector that is a quarter of a circle, containing a right angle.



quadratic equation An equation in which the highest power of the variable is 2, that is, a variable squared, for example, $3x^2 - 6 = 69$ or an equation such as $y = 3x^2 - 6$ whose graph is a **parabola**.

quadratic trinomial See **trinomial**.

R

radius (plural: **radii**) An interval joining the centre of a circle to the circumference, or the length of that interval. The radius is half of the **diameter**.



random In probability, describing a situation where every possible outcome has an equal chance, or is equally likely.

random sampling In statistics, selecting a sample in which every person or item in the population has an equal chance of being selected. A sample should be random to be truly representative of the population.

range In a set of data, the difference between the highest and lowest scores.

rate A relationship between two quantities measured in different units. For example, a speed of 107 km/h compares distance travelled (in kilometres) with time (in hours).

ratio A relationship between quantities measured in the same units. For example, the ratio of 3 teachers to 40 students is 3 : 40 (read '3 to 40').

rational number A number that can be written as a fraction in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. See also **irrational number** and **real number**.

reciprocal The product of any number and its reciprocal is 1. The reciprocal of any number is found by first writing the number as a fraction and then swapping the numerator with the denominator.

The reciprocal of 5 is $\frac{1}{5}$ and the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

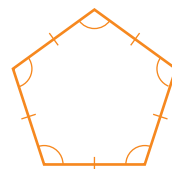
rectangle A quadrilateral with four right angles.



recurring (or repeating) decimal

A decimal with one or more digits that repeat endlessly. For example 0.1666... , abbreviated as 0.1 $\dot{6}$.

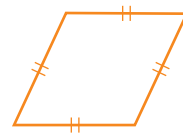
regular polygon A polygon that has all sides equal and all angles equal. For example, this regular pentagon has five equal sides and five equal angles.



relative frequency The number of times an event or score occurred, written as a fraction of the total number of events or scores. See also **experimental probability**.

retainer A fixed amount paid to a salesperson before **commission** is added.

rhombus A quadrilateral with four equal sides.



RHS The right-hand side (of an equation).

right-angled triangle A triangle with one 90° angle.

rise Short for 'vertical rise', this is the change in vertical position between two points on a line or interval, the number of units 'going up', used with the **run** to calculate the **gradient** of a line or interval. See **gradient**.

Glossaries

run Short for 'horizontal run', this is the change in horizontal position between two points on a line or interval, the number of units 'going right', used with the **rise** to calculate the **gradient** of a line or interval. See **gradient**.

S

salary A fixed yearly amount of money that is paid weekly, fortnightly or monthly, not dependent on the number of hours worked.

sample In statistics, a group of people or items selected from a population for study.

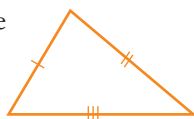
sample space In a probability situation, the set of all possible outcomes.

scale diagram A diagram that is in proportion to the object it represents, such as a house plan or map, that is drawn according to a scale (ratio).

scale factor The amount by which a shape has been enlarged or reduced, equal to
$$\frac{\text{image length}}{\text{original length}}$$

scaled length The length on a scale drawing, map or plan, not the actual length.

scalene triangle A triangle with no equal sides.



scientific notation A shorter way of writing very large or very small numbers using powers of 10, for example, 9 460 000 000 000 in scientific notation is 9.46×10^{12} .

second (") A measure of angle size. $\frac{1}{60}$ of a minute. $1' = 60''$.

sector A region of a circle cut off by two radii, shaped like a slice of pizza.



selling price The price at which an item is sold by the retailer (shop). The shop buys the item for the **cost price**.

shape of a distribution The way the data in a frequency distribution is spread, can be **symmetrical**, positively skewed or negatively skewed.

significant figures The meaningful digits in a number that show its level of accuracy, the first non-zero digits, for example, 31 487 000 has 5 significant figures.

similar To have the same shape but not necessarily the same size, an enlargement or reduction (symbol: |||)

similarity test One of four tests for proving that two triangles are similar.

simple interest Interest that is calculated as a percentage of the original principal.

sine A ratio in a right-angled triangle:

$$\sin \theta = \frac{\text{side opposite to } \theta}{\text{hypotenuse}}$$

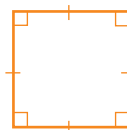
where θ is an angle. See also **cosine** and **tangent**.

skewed distribution A distribution in which most of the scores are clustered at one end, creating a 'tail' at the other end. The tail determines whether the skew is positive or negative. See also **symmetrical distribution**.



solution In algebra, the value that makes an equation true.

square A quadrilateral with four equal sides and four right angles.



stem-and-leaf plot A 'number graph' that lists all the data scores in groups. Each score is split into a 'stem' and a 'leaf'. This stem-and-leaf plot shows 12 test scores, from 42 to 82.

Stem	Leaf
4	2 5
5	0 2 8
6	6 7
7	3 5 7 7
8	2

Key: 5|8 stands for 58

subject of a formula The variable for which a formula is written, the variable on the left-hand-side of a formula.

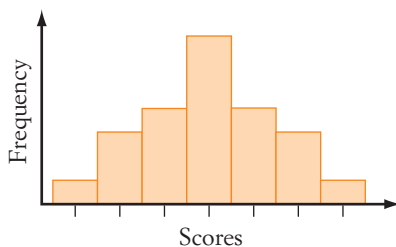
superimpose To place one figure on top of another figure so that sides and angles match.

supplementary angles Two angles whose sum is 180° . The angles 70° and 110° are supplementary.

surd A square root (or other root) whose exact value cannot be found because it is **irrational**, such as $\sqrt{10}$ or $\sqrt[3]{7}$.

surface area The total area of all the faces of a solid shape.

symmetrical distribution A distribution in which all scores are distributed equally on both sides of the centre, its shape having line symmetry. See also **skewed distribution**.



T

tangent A ratio in a right-angled triangle:

$$\tan \theta = \frac{\text{side opposite to } \theta}{\text{side adjacent to } \theta}$$

where θ is an angle. See also **sine** and **cosine**.

tax deduction See **Allowable deduction**.

taxable income The part of a person's income that is taxed, equal to yearly income minus allowable deductions.

terabyte (TB) A unit of digital memory equal to one trillion bytes. See also **byte**, **gigabyte**, **kilobyte**, **megabyte**.

terminating decimal A decimal whose digits do not repeat endlessly, for example, 0.125.

term (of an expression) A part of an algebraic expression. For example, $b^2 + 6b - 9$ has three terms: b^2 , $6b$ and -9 .

theoretical probability (or calculated probability) Probability calculated using the formula:

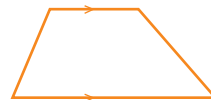
$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

three-figure bearing (or true bearing)

A bearing that uses three-digit angles from 000° to 360° to show the amount of turning measured clockwise from north. See also **bearing** or **compass bearing**.

time-and-a-half Overtime pay that is calculated at 1.5 times the normal pay rate.

trapezium A quadrilateral with one pair of opposite sides parallel.



tree diagram A diagram of branches for listing all of the possible outcomes in a multi-step chance experiment.

trial One go or run of a repeated probability experiment, for example, one roll of a die.

trinomial An algebraic expression with three terms, for example, $3x + 2y - 5$. In a **quadratic** trinomial such as $x^2 + 4x + 6$, the highest power of the pronumeral is 2. See also **binomial** and **quadratic expression**.

Glossaries

two-step experiment (or two-stage experiment)

A chance experiment with two steps or stages, such as rolling a pair of dice.

two-way table A table that shows the number of items belonging to overlapping categories.

	Can swim	Cannot swim
Boys	13	2
Girls	9	3

U

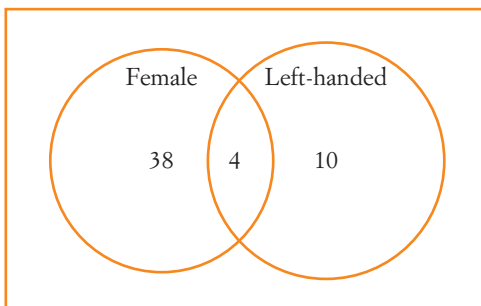
unitary method A method for finding a quantity by finding the size of one part or 1% first.

V

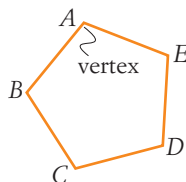
variable A symbol, usually a letter of the alphabet, that stands for a number. Also called a **pronumeral** or **unknown**.

variation See **Direct proportion**.

Venn diagram. A diagram of circles (usually overlapping) for grouping items into categories.

**vertex (plural: vertices)**

A corner of a shape, angle or curve.



vertical Going up and down, at a right angle to the **horizontal**.

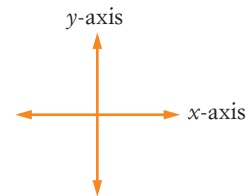
volume The amount of space occupied by a solid object, measured in cubic units.

W

wage An amount of money paid to people for work, calculated on the number of hours worked.

X

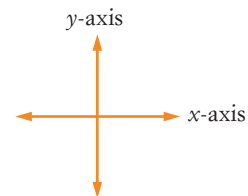
x-axis The horizontal axis of a number plane (running across).



x-intercept The x -value at which a graph cuts the x -axis.

Y

y-axis The vertical axis of a number plane (running up and down).



y-intercept The y -value at which a line cuts the y -axis.

Chapter 1

SkillCheck

- 1 a 16 b 106.09 c 34
d 28 e 7 f 11
2 a 25 cm b 28 cm c 76 mm
3 81, 25, 199, 16, 64

- 3 a $25\sqrt{2}$ b $6\sqrt{2}$ c $12\sqrt{3}$ d $56\sqrt{2}$
e $\sqrt{10}$ f $\sqrt{3}$ g $\frac{\sqrt{7}}{3}$ h $6\sqrt{6}$
i $18\sqrt{17}$ j $\frac{5\sqrt{5}}{2}$ k $3\sqrt{2}$ l $3\sqrt{3}$
m $40\sqrt{10}$ n $15\sqrt{3}$ o $14\sqrt{17}$ p $\frac{\sqrt{13}}{3}$
4 a F b F c T d T
e T f F

Exercise 1-01

- 1 a $k^2 = m^2 + n^2$ b $d^2 = p^2 + c^2$ c $x^2 = w^2 + y^2$
d $k^2 = h^2 + f^2$ e $z^2 = c^2 + r^2$ f $q^2 = f^2 + d^2$
2 a 10 cm b 13 m c 30 cm
d 37 cm e 26 mm f 34 cm
g 20 cm h 5.8 m i 25 cm
j 20.5 m k 61 mm l 19.5 m
3 a $\sqrt{325}$ cm b $\sqrt{377}$ cm c $\sqrt{3541}$ mm
d $\sqrt{4405}$ mm e $\sqrt{117}$ cm f $\sqrt{4122}$ mm
4 a 82.9 mm b 2.6 m c 107.4 mm
d 30.6 cm e 6.5 m f 85.4 mm
5 B 6 $\sqrt{22.81}$ m
7 a 69.1 cm b 4.4 m c 13.4 cm
d 179.6 mm e 4.4 m f 6.26 m

Exercise 1-02

- 1 a 12 mm b 16 cm c 8 m
d 15 mm e 18 cm f 12 mm
2 a 18.1 cm b 14.4 cm c 93.7 cm
d 24.1 m e 35.0 cm f 48.4 m
3 a $\sqrt{5031}$ cm b $\sqrt{5520}$ m or $4\sqrt{345}$ m
c $\sqrt{1344}$ m or $8\sqrt{21}$ m d $\sqrt{10.32}$ cm
e $\sqrt{14.03}$ m f $\sqrt{37\ 127}$ m
4 B 5 B 6 C 7 10.39 cm 8 212 mm or 21.2 cm

Exercise 1-03

- 1 C 2 B
3 $\sqrt{32}$, $\sqrt{33}$, $\sqrt{4.9}$, $\sqrt{52}$, $\sqrt{200}$
4 a R b I c R d R
e R f R g R h R
i I j R k I l I
5 a $1\frac{4}{7}$, $\frac{\pi}{2}$, $\sqrt{2}$ b $2\frac{7}{9}$, $\sqrt[3]{20}$, $2.\dot{6}$
6, 7 Teacher to check, $\sqrt{5} \approx 2.24$, $\sqrt{10} \approx 3.16$, $\sqrt{17} \approx 4.12$

Exercise 1-04

- 1 a 2 b 5 c 27 d 250
e 0.09 f 28 g 45 h 50
2 a $2\sqrt{2}$ b $3\sqrt{3}$ c $2\sqrt{6}$ d $3\sqrt{6}$
e $9\sqrt{3}$ f $3\sqrt{5}$ g $4\sqrt{3}$ h $10\sqrt{2}$
i $4\sqrt{6}$ j $3\sqrt{7}$ k $12\sqrt{2}$ l $6\sqrt{3}$
m $5\sqrt{3}$ n $7\sqrt{3}$ o $4\sqrt{2}$ p $11\sqrt{2}$
q $9\sqrt{2}$ r $7\sqrt{5}$ s $5\sqrt{5}$ t $16\sqrt{2}$

Mental skills 1A

- 2 a 895.4 b 37 c 83.1 d 4200
e 5271.6 f 1561 g 31 840 h 6430
i 22.4 j 48.94 k 7389 l 1142
4 a 73.34 b 0.94 c 6.52 d 0.104
e 0.704 f 1.985 g 0.02 h 4.159
i 12.3 j 0.007 58 k 0.0849 l 0.0251

Exercise 1-05

- 1 a $7\sqrt{7}$ b $-5\sqrt{2}$ c $6\sqrt{5}$ d $4\sqrt{5}$
e 0 f $\sqrt{10}$ g $8\sqrt{15}$ h $-\sqrt{6}$
i $2\sqrt{3}$ j $10\sqrt{5}$ k $6\sqrt{10}$ l $-5\sqrt{3}$
2 a $5\sqrt{5} - 8$ b $13\sqrt{10} + 3\sqrt{2}$ c $-9\sqrt{3} + 5\sqrt{2}$
d $7\sqrt{15} + 8\sqrt{12}$ e $-3\sqrt{7} - 2\sqrt{5}$ f $2\sqrt{6} - 8\sqrt{3}$
g $13\sqrt{11} - \sqrt{3}$ h $-6\sqrt{13} + 11\sqrt{7}$ i $-6\sqrt{7}$
j $-3\sqrt{5}$
3 a D b A
4 a $6\sqrt{2}$ b $3\sqrt{3}$ c $-2\sqrt{5}$
d $-\sqrt{7}$ e $5\sqrt{6}$ f $7\sqrt{5}$
g $-\sqrt{10}$ h $8\sqrt{11}$ i $6\sqrt{2}$
j $8\sqrt{3}$ k $3\sqrt{2}$ l $9\sqrt{2}$
m $11\sqrt{3}$ n $-\sqrt{5}$ o $-6\sqrt{3}$
p $30\sqrt{3}$ q $5\sqrt{7}$ r $41\sqrt{2}$
s $5\sqrt{6}$ t $29\sqrt{2}$ u $-15\sqrt{3}$
v 0 w $6\sqrt{2} + 2\sqrt{3}$ x $12\sqrt{3} + 3\sqrt{6}$
y $3\sqrt{2} - 6\sqrt{5}$ z $4\sqrt{6}$

Exercise 1-06

- 1 a $\sqrt{14}$ b $-\sqrt{35}$ c $4\sqrt{3}$ d 6
e $-5\sqrt{2}$ f 45 g $15\sqrt{30}$ h $-10\sqrt{21}$
i 140 j $-30\sqrt{2}$ k 36 l $-60\sqrt{2}$
m -112 n $24\sqrt{6}$ o 80 p $90\sqrt{6}$
q -396 r $160\sqrt{5}$ s $216\sqrt{2}$ t $-96\sqrt{6}$
u $36\sqrt{5}$ v $-60\sqrt{10}$ w $252\sqrt{3}$ x $72\sqrt{14}$
2 a $\sqrt{5}$ b $-\sqrt{3}$ c $3\sqrt{6}$
d $2\sqrt{2}$ e $-\frac{\sqrt{7}}{2}$ or $-\frac{1}{2}\sqrt{7}$ f 21
g 1 h 8 i $5\sqrt{3}$
j $5\sqrt{2}$ k $4\sqrt{3}$ l $2\sqrt{6}$
m $2\sqrt{14}$ n $-\frac{\sqrt{2}}{4}$ or $-\frac{1}{4}\sqrt{2}$ o 1
p 10 q 4 r $-21\sqrt{2}$
s 12 t 2 u $\frac{2}{3}$
3 a 2 b $4\sqrt{6}$ c $\sqrt{30}$ d $\frac{2}{45}$
e $14\sqrt{3}$ f 2

Exercise 1-07

- 1 a 21.0 cm b 4.7 m c 120.0 mm d 11.9 cm
 e 27.1 mm f 28.0 m g 8.5 cm h 1.9 m
 i 40.1 m j 4.0 m k 11.7 cm l 13.9 m
- 2 $\sqrt{21}$ m 3 66 cm 4 92.2 km
- 5 a 5 b $\sqrt{58}$
- 6 a 48 cm b 31.4 cm c 170 cm
 d 127.9 mm e 35.9 cm f 6.5 m
- 7 a 38.3 cm b 12.21 c $\sqrt{31}$
- 8 637.1 m 9 752 m
- 10 a 21.8 m b 23 m

Exercise 1-08

- 1 a Yes b Yes c No d Yes
 e No f Yes g No h Yes
 i No j Yes k No l Yes
- 2 $17^2 + 21^2 = 730$, $30^2 = 900 \neq 730$
 $\therefore$ triangle is not right-angled
- 3 C 4 B

Mental skills 1B

- 2 a 16 000 b 210 c 9900 d 600
 e 16 000 f 400 g 490 h 52
 i 1500 j 1160 k 180 l 13 200
- 4 a 8 b 5 c 8 d 22
 e 4 f 50 g 9 h 9.3
 i 8.5 j 4.23 k 9.6 l 24

Exercise 1-09

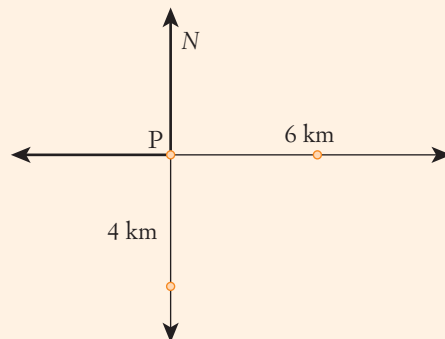
- 1 a Yes b Yes c Yes
 d No e Yes f No
 g Yes h Yes i No
- 2 D 3 Check with a spreadsheet.
- 4 Teacher to check.
- 5 a $b = 12$, $c = 13$
 $b \sqrt{12^2 + 5^2} = \sqrt{169} = 13 \therefore (5, 12, 13)$ is a triad.
- 6 a (7, 24, 25) b (11, 60, 61) c (15, 112, 113)
 d (4, 7.5, 8.5) e (9, 40, 41) f (19, 180, 181)
 g (10, 49.5, 50.5) h (51, 1300, 1301)

Power plus

- 1 BD, BC, CE, DE, CD
- 2 $\sqrt{128}$ m or $8\sqrt{2}$ m
- 3 a 22.6 b 10.6
- 4 a 16.2 m b 16.9 m
- 5 5.8
- 6 a 21.21 cm b 25.98 cm

Chapter 1 revision

- 1 a $(AB)^2 + (AC)^2 = (BC)^2$ b $n^2 + p^2 = q^2$
 c $(EF)^2 + (FG)^2 = (EG)^2$
- 2 B
- 3 a 175 mm b 113 mm c $\sqrt{98.72}$ m
- 4 B
- 5 a 34.9 m b 12.2 cm c 824.9 m
- 6 a I b R c R d I
 e R f R g R h R
 i I
- 7 a $6\sqrt{2}$ b $7\sqrt{2}$ c $5\sqrt{11}$
 d $8\sqrt{2}$ e $15\sqrt{6}$ f $14\sqrt{7}$
 g $48\sqrt{2}$ h $15\sqrt{5}$ i $28\sqrt{3}$
- 8 a $7\sqrt{5}$ b $3\sqrt{7} - 3\sqrt{2}$ c $2 + 6\sqrt{5}$
 d $3\sqrt{7}$ e $13\sqrt{2}$ f $5\sqrt{2} - 7\sqrt{5}$
 g $14\sqrt{2} + 17\sqrt{3}$ h $32\sqrt{5} - 9\sqrt{7}$
- 9 a $\sqrt{21}$ b $2\sqrt{10}$ c $4\sqrt{3}$ d $\sqrt{55}$
 e $2\sqrt{6}$ f $\sqrt{14}$ g $4\sqrt{6}$ h 1
 i $\frac{\sqrt{6}}{6}$ or $\frac{\sqrt{3}}{3\sqrt{2}}$ j $\frac{3\sqrt{2}}{2}$ or $\frac{3}{\sqrt{2}}$ k $\frac{2}{3}$
- 10 168 cm
- 11 a



- b 7.2 km
- 12 a 432 m b 1.296 km
- 13 48.5 cm
- 14 a No b Yes c Yes
- 15 a Yes b No c Yes d Yes

Chapter 2

SkillCheck

- 1 a 56 b 20 c 22 d 100
 e -9 f -20 g 13 h -12
 i 20 j 80 k -3 l -64
 m 11 n -32 o -3 p -19
 q -20 r 4
- 2 a 0.7, 0.7007, 0.707, 7.007 b 0.002, 0.022 02, 0.202, 0.22
- 3 a 0.009 b 15.4 c 0.246
- 4 C
- 5 a $1\frac{5}{21}$ b $1\frac{1}{10}$ c $\frac{11}{36}$ d $\frac{7}{30}$
 e $\frac{15}{28}$ f $\frac{3}{10}$ g $4\frac{1}{2}$ h $4\frac{2}{3}$
 i $1\frac{1}{2}$ j $\frac{5}{6}$ k $\frac{3}{35}$ l $1\frac{2}{5}$
- 6 a 25% b 30% c 7% d 60%
- 7 5%, 0.5, $\frac{5}{9}$, $\frac{3}{5}$

Answers

Exercise 2-01

- 1 a 0.16̇ b 0.273̇ c 0.8̇ d 0.209̇
 2 a 0.109 810 98... b 0.171 717 17...
 c 0.255 55... d 8.318 318...
 3 a 0.875 (T) b 0.27̇ (R) c 4.8̇ (R) d 2.6 (T)
 e 0.76 (T) f 0.6̇ (R) g 0.65 (T) h 1.583̇ (R)
 i 8.325 (T) j 0.7083̇ (R)
 4 a 0.3̇ b i $\frac{2}{3}$ ii $\frac{9}{9} = 1$

Exercise 2-02

- 1 a $\frac{2}{9}$ b $\frac{8}{9}$ c $\frac{7}{45}$ d $\frac{7}{18}$
 e $\frac{7}{15}$ f $\frac{65}{99}$ g $\frac{31}{99}$ h $\frac{103}{165}$
 i $\frac{7}{12}$ j $\frac{832}{999}$ k $\frac{1}{30}$ l $\frac{25}{33}$
 2 a $\frac{1}{3}$ b $\frac{2}{3}$
 3 a $\frac{5}{11}$ b $\frac{10}{11}$

Exercise 2-03

- 1 a \$12.75 b 72 kg c 7.2 m d \$8.40
 e \$4.20 f 11.4 kg g \$21.75 h 24 L
 i \$106.88 j \$2180 k 1.5625 t l \$14.88
 2 a 29% b 24 534
 3 186 4 140 g 5 106.25 kg
 6 a \$500.50 b \$92.40 c \$2040
 d 82 kg e 45.5 m f 288 L
 7 a \$539.50 b 114 min c 79.2 L
 d 5.1 m e 97.5 kg f \$86.85
 8 a \$43 b \$903
 9 \$20.90 10 \$6.69/kg 11 \$944.23 12 \$14 520
 13 a 68.8% b 66.7% c 64.3% d 83.3%
 e 66.7% f 15% g 14.6% h 12.5%
 i 17% j 12.5% k 17% l 14%
 14 English 80%, Maths 78%. English is the better mark.
 15 33.3% 16 19.79% 17 \$1100 18 \$1060
 19 \$8400 20 750 g 21 \$83 654

Exercise 2-04

- 1 a \$600 b 92.3%
 2 a \$1500 b 9.375%
 3 a \$4500 b \$1700 c 37.8%
 4 a \$130 000 b 15.3%
 5 40% 6 $33\frac{1}{3}\%$ 7 $91\frac{2}{3}\%$
 8 a \$1.56 b \$1.95 c \$7.15 d \$10.39
 9 C 10 C 11 \$16.58 12 23.5% 13 21.8%
 14 a 31.25% b 48.7%
 15 \$140 16 \$3295 17 B 18 \$289
 19 a \$2090, \$22 990 b \$181, \$1991
 c \$14.50, \$159.50 d \$18, \$198
 20 a \$1.90, \$19 b \$67.20, \$672 c \$2.85, \$28.50
 d \$5.99, \$59.90 e \$84, \$840 f \$18, \$180
 21 a \$1329.30 b \$1262.84 c \$636.17 d 33.5%
 22 a \$599 b \$509.15 c \$483.69 d \$479.20
 e No, the 15% discount followed by the 5% discount is the same as a 19.25% discount.

Mental skills 2A

- 2 a 625 b 3025 c 2025 d 7225
 e 13 225 f 56.25 g 9025 h 38 025
 i 2.25 j 4225 k 24 025 l 60 025
 4 a 441 b 10 201 c 961 d 8281
 e 26.01 f 6561 g 3721 h 40 401
 i 1.21 j 16.81
 6 a 3481 b 4761 c 7921 d 361
 e 11 881 f 24.01 g 6241 h 141.61
 i 1521 j 98.01

Exercise 2-05

- 1 a \$6300 b \$300.96 c \$660
 d \$708.75 e \$2529.60 f \$1825
 2 \$21 386
 3 a \$94.50 b \$46.72 c \$254.79
 d \$305.67 e \$13 407.69 f \$1185.67
 4 a 6.3% p.a. b 7.7% p.a. c 4.7% p.a.
 5 6 years
 6 100 months
 7 144 days
 8 a \$1400 b 10% p.a.
 9 a \$6144 b \$2144 c 13.4% p.a.

Exercise 2-06

- 1 a 12 : 7 b 5 : 12 c 25 : 12 d 3 : 2
 e 124 : 39 f 12 : 5 g 17 : 6 h 5 : 18
 i 25 : 4 j 15 : 16 k 15 : 2 l 5 : 6 : 2
 m 3 : 4 : 1 n 6 : 1 : 4 o 5 : 18 p 1 : 2
 2 a 3 : 16 b 25 : 4 c 8 : 3
 d 3 : 10 e 1 : 3 f 1 : 5
 g 3 : 20 h 14 : 1 i 8 : 1
 3 9 4 \$312 000 5 16
 6 20 m³ gravel, 15 m³ sand, 5 m³ cement
 7 1125
 8 a i 3 : 5 ii 3 : 1 b 29.2 g
 9 Shahid \$27 600, Bridie \$32 400
 10 D
 11 a 67.5 km/h b \$8.40/kg c 56 words/min
 d 12.1 km/L e 7.7 m/s f 41.7 g/m²
 g 22.5 km/day h \$71.67/h
 12 a 162.5 km b 552.5 km
 13 a \$27.50/h b \$1100 c 8 days
 14 a \$15.99/kg b \$47.97 c 2.5 kg
 15 a 74.24 L b 6.728 L c 16.24 L
 16 3.9 L/100 km 17 32 500
 18 a 4680 beats/hour b 12 min 49 seconds
 19 A
 20 a 129.7 persons/km² b 3.0 persons/km²
 c 997 652 400

Exercise 2-07

1 a 18 000 m/h	b 108 000 m/h	c \$0.01/g
d 1200 mL/h	e 2400 mL/h	f 18 000 mL/h
g 50 000 kg/ha	h 2500 kg/day	i \$39 000/year
j 30 km/h	k 360 sheep/day	l \$420/week
2 a 18 km/h	b 1260 m/h	c \$100/kg
d 2.4 L/h	e 666.7 mL/s	f 432 L/day
g 250 kg/ha	h 60 000 kg/day	i 30.5 m/s
j 50 g/cm	k 8 m/mL	l \$42/h
3 a 97.2 km/h	b 349.2 km/h	c 11.88 km/h
d 108 km/h	e 82.8 km/h	
4 a 15 m	b 20 m	c 27.5 m

Mental skills 2B

2 a 2.8	b 4.2	c 5.3	d 10.5
e 8.9	f 5.6	g 3.5	h 8.1
i 8.7	j 5.4	k 6.3	l 11.2

Power plus

- 1 $A = \frac{17}{80}$, $B = \frac{9}{40}$, $C = \frac{19}{80}$
 2 \$21 000 3 44.4% 4 \$11.68
 5 a 666.667 L
 b Three decimal places mean to the nearest mL and measuring cups do not measure to the nearest mL.
 6 \$108 7 11.1 sec 8 25 : 28
 9 Jordan arrives first. Jordan takes $\frac{D}{50}$ h and James takes $\frac{D}{48}$ h, where D is the distance travelled.
 10 a 20 years b 20 years

Chapter 2 revision

- 1 0.36
 2 a $\frac{8}{9}$ b $\frac{3}{11}$ c $\frac{1}{6}$
 3 a \$243.60 b 7.425 kg c \$8.77
 4 a 162 L b \$21.88
 5 a $8\frac{1}{3}\%$ b $16\frac{2}{3}\%$
 6 295 runs 7 D
 8 a \$380 000 b 152%
 9 \$272.73
 10 a \$1932 b \$54.46
 11 13%
 12 a 4 : 9 b 7 : 400 c 20 : 3
 13 Bill: \$476 190.48, Ben: \$323 809.52
 14 a 83 km/h b 332 km c 12.0 h
 15 a 2880 mL/h b 0.14 kg/m² c $8\frac{1}{3}$ m/s d 24 cm/min

Chapter 3

SkillCheck

1 a $\$pQ$	b $\$(40 + t)$	c $1000h$ m
d $\frac{m+p+v+w}{4}$	e $\$\frac{y}{100}$	

2 a 6	b 18	c 21
d 43	e 18	f 5
3 14.4 cm		
4 a $11n$	b $6h$	c $9pq$
e $4w$	f $40x$	g $6gh$
i $5x$	j 6	k $2y$
5 a 4	b 9	c 15
6 a $\frac{1}{2}$	b $\frac{3}{8}$	c $\frac{16}{35}$
		d $1\frac{7}{40}$

Exercise 3-01

1 a fg	b $9uvw$	c $8t + 3$
d $3x + 10y$	e $2g + h$	f $3f - 2fg$
g $5p^2 - 3$	h $4p - 4q$	i $3 - 2r$
j $-10a + 4b$	k $6v + 10$	l $3x + 3y$
m $6p + 7q$	n $15 - 5w$	o $2 - 10n$
p $6e - 7f$	q $5t^2 - 4l$	r $-6b^2 - 6b$
2 C		
3 a correct	b incorrect	c incorrect
e correct	f incorrect	g correct
i correct	j correct	h incorrect
4 a $4m + 6p$	b $16d$	c $4x + 5y$
d $6p + 3$	e $90h$	f $12a + 18c$
5, 6	Teacher to check.	

Exercise 3-02

1 a $40x$	b $15n$	c $5ab$	d $6t^2$
e $-12w$	f $-6p$	g $12ed$	h $24vw$
i $64q^2$	j $20a^2$	k $24by$	l $-36w^2$
k $25n^2$	l $9r^2$	m a^2t^2	
2 a $4x$	b 15	c 9	d $3n$
e $\frac{1}{6}$	f $4ac$	g -6	h $-5z$
i $-\frac{1}{5q}$	j $3yz$	k $\frac{r}{4}$	l $-6k$
3 C			
4 a $30abc$	b $24pq$	c $8n$	d $\frac{1}{5}$
e $-36hk$	f $40k^2$	g $\frac{d}{2}$	h $\frac{1}{7}$
i $-11m$	j $\frac{1}{5}$	k $\frac{4a}{c}$	l $-6e^2f^2$
5 a correct	b incorrect	c correct	d incorrect
e correct	f correct	g correct	h incorrect
i correct	j correct		
6 a $6ad$	b $25m^2$	c $5hk$	
d $12py$	e $3a^2$	f $10ck$	
7 a $8m^3$	b $60dty$	c $12kw^2$	

Exercise 3-03

1 a $\frac{3w}{4}$	b $\frac{11k}{8}$	c $\frac{m}{2}$	d x
e $\frac{2}{q}$	f $\frac{1}{d}$	g $\frac{2r}{w}$	h $\frac{3}{c}$
i $\frac{4t+s}{3}$	j $\frac{2y}{h}$	k $\frac{1}{a}$	l $\frac{3}{e}$
m $\frac{p+3}{z}$	n $\frac{u}{4g}$	o $\frac{1}{3f}$	p $\frac{2e}{5}$
q $\frac{5a}{3}$	r $\frac{2}{d}$	s $\frac{4}{k}$	t $\frac{1}{a}$
2 a $\frac{7x}{12}$	b $\frac{4s}{21}$	c $\frac{8h}{15}$	d $-\frac{5m}{14}$
e $\frac{9w}{20}$	f $\frac{17t}{20}$	g $\frac{11p}{15}$	h $\frac{5r}{6}$
i $\frac{13c}{10}$	j $\frac{6d-11r}{35}$	k $\frac{9h+10a}{15}$	l $\frac{25+24w}{30}$
m $\frac{21+8a}{28}$	n $\frac{5e}{24}$	o $\frac{7m-2n}{14}$	p $\frac{12k+5m}{30}$

Answers

Exercise 3-04

1 a $\frac{w}{6}$	b $\frac{st}{20}$	c $\frac{5}{2hk}$	d $\frac{12}{mn}$
e $\frac{5l}{3f}$	f $\frac{2}{3v^2}$	g $\frac{6}{x^2}$	h $\frac{ac}{bd}$
i $\frac{d}{g}$	j $\frac{d^2}{36}$	k $\frac{2}{3}$	l $\frac{12}{5k^2}$
m $\frac{3u}{5}$	n 1	o $\frac{2}{3d}$	
2 a $\frac{5}{2}$	b $\frac{m}{2n}$	c 4	d $\frac{q}{3}$
e $\frac{3d}{10}$	f $\frac{20}{27}$	g $\frac{3}{5}$	h $\frac{9}{b}$
i $\frac{15}{2}$	j $\frac{h^2}{k^2}$	k 12	l $\frac{11}{8}$
m $\frac{5u}{9}$	n 6	o x	
3 a $\frac{6}{p}$	b $\frac{12}{35}$	c $\frac{3}{20y}$	d $\frac{5}{4}$
e $\frac{5x}{z}$	f $\frac{c}{3b}$	g $\frac{25b}{3}$	h $\frac{4t}{27}$
i $\frac{2}{3n}$	j $\frac{50p^2t}{7}$	k $\frac{2s}{35}$	l $\frac{1}{6h^2}$
m $25y^2$	n $\frac{2yh}{9}$	o $\frac{3af^2}{25}$	

Mental skills 3A

2 a 160	b 70	c 240	d 900
e 2600	f 900	g 140	h 300
i 180	j 770	k 18	l 34
m 46	n 26	o 18	p 12
q 40	r 8	s 14	t 24

Exercise 3-05

1 a $4h + 24$	b $15 + 3t$	c $6b + 21$
d $40 + 15x$	e $x^2 + 9x$	f $5p + p^2$
g $6r^2 + 3r$	h $4y + 12y^2$	i $20e^2 + 40ef$
2 a $3t - 6$	b $35 - 7d$	c $64g - 24$
d $w^2 - 7w$	e $a^2 - a$	f $18h^2 - 6h$
g $12x^2 - 4x$	h $10x^2 - 20xy$	i $12ab - 21a^2$
3 a incorrect	b correct	c correct
4 a $-a + 5$	b $-a - 5$	c $-2x - 12$
d $-2x + 12$	e $-11 - w$	f $-11 + w$
g $-y^2 - 9y$	h $-8x + xz$	i $-4t^2 + 32t$
5 C		
6 a $19m + 10$	b $3 - 9e$	c -10
d $43 - 10x$	e $t^2 + 7t + 12$	f $12 + 11h - 2h^2$
g $6x^2 + 23x + 20$	h $2v^2 - 3v - 6$	i $w^2 - 8w + 3$
j $6y^2 - 29y + 35$		

Exercise 3-06

1 a 6	b 8	c 5
d 3	e m	f c
g $6p$	h $8w$	i $(x - 5)$
2 a $6(3p^2 + 4)$	b $3m(3m + 1)$	c $10d(2 - 3d)$
d $2xy(3x - 4)$	e $(x - 2)(x + 5)$	f $(3 + n)(n - 3)$
g $(h + 4)(h - 2)$	h $(1 - w)(v + 1)$	
3 a $6(4x + 5)$	b $9(4 + 3a)$	c $16(g^2 - 4)$
d $y(x + 1)$	e $n(m - 3)$	f $p(6 + q)$
g $x(x + 1)$	h $y(2 - y)$	i $3d(d + 2)$

j $4r(4r - 3)$	k $3t(2t + 9)$	l $36p(p - 3)$
m $4x(3xy - 4)$	n $2p(9p + 8r)$	o $4mn(m - n)$
p $7bc(2a + 3)$	q $7vw(4 - 3vw)$	r $9rt(5 + 6r)$
s $36pr(q^2 - 4)$	t $16xy(3x + 4y)$	u $25gh(3g^2h - 5)$
4 a $(a - 3)(a + 6)$	b $(8 + t)(t - 3)$	c $(b + 5)(b - 2)$
d $(2 - y)(x - 6)$	e $(a - 7)(5 + b)$	f $(a + 3)(s + 4)$
g $(g + 5)(3p - 2)$	h $(2 - 3m)(5 + 2n)$	i $(8 + r)(r + 1)$
j $(y - 6)(1 - y)$		
5 a $-4(q + 2)$	b $-9(u - 2)$	c $-3(g - 2)$
d $-6(3a - 2)$	e $-(n + 1)$	f $-(n - 1)$
g $-y(y + 9)$	h $-2t(3 - 5t)$	i $-3a(a + 2b)$
j $-a(p - q)$	k $-2e(10e + 11)$	l $-3m(3 - m)$
6 B		
7 a $\frac{1}{2}(p + g)$	b $g(7 + 9g)$	
c $\frac{1}{4}x(x - y)$	d $4(2a + 3y + 1)$	
e $5(4xy - 2x^2 + y)$	f $tm(t + m + 1)$	
g $(m + 2)(7y - p)$	h $-10p(p + q)$	
i $5a(1 + 2ay)(x - y)$		

Exercise 3-07

1	a	$x^2 + 18x + 80$	b	$a^2 + 9a + 18$		
	c	$2d^2 + 9d + 10$	d	$12h^2 + 31h + 7$		
2	a	$w^2 + 4w - 5$	b	$x^2 + 3x - 54$		
	c	$t^2 - 4t - 32$	d	$n^2 - 12n + 35$		
3	a	$4y^2 + 11y + 15$	b	$3r^2 + 17r + 10$	c	$20k^2 + 17k + 3$
	d	$w^2 - 2w - 15$	e	$3v^2 + 11v - 4$	f	$p^2 - 8p + 15$
4	a	$n^2 + 9n + 18$	b	$t^2 + 11t + 30$	c	$p^2 - 100$
	d	$x^2 - 5x - 24$	e	$b^2 + 7b - 18$	f	$u^2 - 15u + 56$
	g	$15 + 14r - r^2$	h	$a^2 - 19a + 90$	i	$c^2 - 8c + 15$
	j	$t^2 + t - 2$	k	$y^2 + 6y - 40$	l	$n^2 + 2n - 99$
5	D					
6	a	$2x^2 + 11x + 15$	b	$12e^2 + 23e + 10$		
	c	$3p^2 + 7p - 10$	d	$49d^2 - 28d + 4$		
	e	$6f^2 + 4f - 10$	f	$12m^2 + 5m - 25$		
	g	$6 + 7h - 20h^2$	h	$8p^2 - 38p + 35$		
	i	$-10m^2 + 23m - 12$	j	$6t^2 + 7t - 5$		
	k	$25y^2 - 25$	l	$-4a^2 + 24a - 36$		
7	a	$(100 + x)$ cm	b	$(75 + x)$ cm		
	c	$A = x^2 + 175x + 7500$ cm ²	d	$x^2 + 175x$		
	e	17.51 cm ²				
8	a	$l = 4 + x, w = 3 + y$	b	$A = (4 + x)(3 + y)$		
	c	$A = 12 + 4y + 3x + xy$	d	$3x + 4y + xy$		

Exercise 3-08

1	a	$20x$	b	m^2	c	t^2	d	$+8h$
	e	$-18k$	f	$+80f$	g	$4d^2+12d$	h	$36a^2, 1$
2	a	$m^2+18m+81$	b	u^2+6u+9				
	c	$y^2-12y+36$	d	$64+16k+k^2$				
	e	$25-10h+h^2$	f	$49+14k+k^2$				
	g	$f^2+40f+400$	h	$q^2-22q+121$				
	i	$100+20t+t^2$	j	$x^2-2xw+w^2$				
	k	$a^2+2ag+g^2$	l	$4m^2-12m+9$				
	m	$25x^2-60x+36$	n	$81a^2+36a+4$				

- o $9e^2 - 24e + 16$ p $25 + 70b + 49b^2$
 q $16 - 40p + 25p^2$ r $121 - 44c + 4c^2$
 s $100g^2 + 60g + 9$ t $9k^2 + 66k + 121$
 u $25 + 20v + 4v^2$
 3 a $49h^2 + 28hk + 4k^2$ b $64a^2 - 48ay + 9y^2$
 c $x^2y^2 + 2xyz + z^2$ d $1 + \frac{2}{y} + \frac{1}{y^2}$
 e $t^2 - 2 + \frac{1}{t^2}$ f $\frac{9}{w^2} + 6 + w^2$
 4 a 441 b 2025 c 841
 d 3481 e 10 404 f 9604
 5 a Two squares of length x cm have been cut from the ends,
 so the length of the base is $(10 - 2x)$ cm.
 b $100 - 40x + 4x^2$ c $10x - 2x^2$
 e $102 - 4 \times x^2 = 100 - 4x^2$

Mental skills 3B

- 2 a 176 b 363 c 261 d 405
 e 682 f 707 g 1818 h 3564
 i 152 j 540 k 2142 l 588
 m 288 n 693 o 3939 p 852

Exercise 3-09

- 1 a $m^2 - 25$ b $c^2 - 100$ c $a^2 - 144$
 d $36 - y^2$ e $64 - m^2$ f $p^2 - 1$
 g $25 - e^2$ h $v^2 - 121$ i $w^2 - 9$
 j $x^2 - 100$ k $q^2 - 49$ l $81 - g^2$
 m $b^2 - 4$ n $225 - r^2$ o $d^2 - 169$
 2 a $4h^2 - 9$ b $25r^2 - 16$ c $25b^2 - 64$
 d $16p^2 - 49$ e $9 - 64k^2$ f $49x^2 - 25$
 g $4 - 81m^2$ h $81k^2 - 16l^2$ i $49n^2 - 64m^2$
 j $16g^2 - 25h^2$ k $49u^2 - 9w^2$ l $121a^2 - 9b^2$
 m $t^2 - \frac{1}{t^2}$ n $\frac{w^2}{9} - 4$ o $1 - \frac{1}{t^2}$
 3 a $p - 1$ b $p + 1$ c $p^2 - 1$ d 7
 4 899
 5 a 399 b 2499 c 8099 d 6396
 6 a 29 b 47 c 3000 d 400
 e 240 f 196
 7 a i $1 = x + y$ ii $w = x - y$
 iii $(x + y)(x - y)$ or $x^2 - y^2$
 b $x^2 - y^2, (x + y)(x - y) = x^2 - y^2$

Exercise 3-10

- 1 a $4m^2 - 1$ b $5y^2 + 17y - 12$
 c $4k^2 - 28k + 49$ d $d^2 + 18d + 81$
 e $2e^2 + e - 1$ f $25a^2 - 16$
 g $-p^2 + 4p - 4$ h $100 - 36y^2$
 i $h^2 - 6hz + 9z^2$ j $2xy + 6x - 3y - 9$
 k $121a^2 - 16b^2$ l $u^2 - 2 + \frac{1}{u^2}$
 2 a m^2 b $y^2 + 18$ c $-3x^2 + 7x + 6$
 d $d^2 + 21$ e $16k^2 - 48$ f $-4xy$
 g $27t - 10$ h $2f^2 - 8$ i $12h + 18$
 j $5xy - 6x + 3y + 9$

- 3 a $60a^2$ b $n^2 + 4n + 4$ c $-2t^2 - 8t$
 d $8m^2 + 2n^2$ e $x - 2$ f $1 - 2b^2$
 g $3y^2 + 12y + 14$ h $3x^2 + 9$ i $34n^2 - 34$
 j $-4b^2$

Exercise 3-11

- 1 a $(b + d)(4a + 5c)$ b $(y + t)(2x - 5w)$
 c $(3c + 4d)(3a + 2b)$ d $(10 + x)(x^2 + 3)$
 e $3(a + b)(a + c)$ f $6(t + p)(r - 3w)$
 g $(7 + d)(2e - 3)$ h $(h - 2)(k - h)$
 i $(3m + p)(n - 2)$ j $(9 + q)(p^2 - 3)$
 k $(f - 10)(g - h)$ l $3(l + n)(3k - 4m)$
 m $(2 - p)(p - c)$ n $(l - 3)(l^2 + m^2)$
 o $(a + y)(x + 1 - k)$ p $(a - b + 3q)(p - 2q)$
 2 a $(w + 3)(w - 3)$ b $(y + 6)(y - 6)$
 c $(k + 1)(k - 1)$ d $(m + 11)(m - 11)$
 e $(p + 8)(p - 8)$ f $(t + 10)(t - 10)$
 g $(2e + f)(2e - f)$ h $(a + 3b)(a - 3b)$
 i $(4y + 1)(4y - 1)$ j $(2 + b)(2 - b)$
 k $(5 + e)(5 - e)$ l $(1 + 4x)(1 - 4x)$
 m $(y + z)(y - z)$ n $(7 + 4m)(7 - 4m)$
 o $(b + 11d)(b - 11d)$ p $(6c + 5k)(6c - 5k)$
 q $(4 + 9h)(4 - 9h)$ r $(5a + 8m)(5a - 8m)$
 s $(10 + 7n)(10 - 7n)$ t $(11p + 12q)(11p - 12q)$
 u $(\frac{1}{2} + 5c)(\frac{1}{2} - 5c)$ v $(2t + \frac{1}{3})(2t - \frac{1}{3})$
 w $(5h + \frac{3}{2})(5h - \frac{3}{2})$ x $(1 + mn)(1 - mn)$
 3 a $2(a + b)(a - b)$ b $7(k + 2)(k - 2)$
 c $3(1 + 5u)(1 - 5u)$ d $x(x + 7)(x - 7)$
 e $k(1 + 4k)(1 - 4k)$ f $2(5q + 1)(5q - 1)$
 g $3(d + 2v)(d - 2v)$ h $5t^3(t + 5)(t - 5)$
 i $2(ab + 1)(ab - 1)$ j $x^2(y + w)(y - w)$
 k $12(4f + 3g)(4f - 3g)$
 l $5(3d + \frac{1}{2})(3d - \frac{1}{2})$ or $\frac{5}{4}(6d + 1)(6d - 1)$
 m $2(x + 2a)(x - 2a)$ n $25(2 + w)(2 - w)$
 o $5(\frac{1}{2} + 4e)(\frac{1}{2} - 4e)$ or $\frac{5}{4}(1 + 8e)(1 - 8e)$
 p $(3c + 2\frac{1}{2})(3c - 2\frac{1}{2})$ or $\frac{1}{4}(6c + 5)(6c - 5)$
 4 a $(\frac{p}{4} + \frac{x}{5})(\frac{p}{4} - \frac{x}{5})$ b $(x + \frac{1}{3})(x - \frac{1}{3})$
 c $(\frac{y}{3} + \frac{u}{9})(\frac{y}{3} - \frac{u}{9})$ d $2(\frac{v}{3} - \frac{m}{11})(\frac{v}{3} + \frac{m}{11})$
 e $(\frac{4a}{7} + \frac{5b}{2})(\frac{4a}{7} - \frac{5b}{2})$ f $(t + 3)(t - 3)(t^2 + 9)$
 g $(10 + n^2)(10 - n^2)$ h $y(2x + y)$
 i $-(p + 4q)(3p - q)$ j $(\frac{x}{2} + \frac{y}{6})(\frac{x}{2} - \frac{y}{6})$
 k $4ab$

Exercise 3-12

- 1 a $-1, -6$ b $-3, 4$ c $-5, 3$ d $3, 4$
 e $-5, -4$ f $-2, 7$ g $-2, 5$ h $-5, 5$
 i $-2, 1$ j $-9, 2$
 2 a $(x + 3)(x + 4)$ b $(x + 7)(x + 5)$
 c $(x + 4)(x + 1)$ d $(x + 5)(x + 2)$
 e $(x + 5)(x + 4)$ f $(t + 1)(t + 5)$
 g $(e + 2)(e + 3)$ h $(h + 2)^2$

Answers

- i $(n+10)(n+1)$
 k $(d+4)(d+6)$
 3 a $(y-3)(y+1)$
 c $(h-4)(h+1)$
 e $(e-9)(e+3)$
 4 a $(x+4)(x-1)$
 c $(m+5)(m-3)$
 e $(k+7)(k-2)$
 g $(m-4)(m-1)$
 i $(x-7)(x-5)$
 k $(n-2)(n-1)$
 5 a $(m+2)^2$ b $(p+10)^2$ c $(a-5)^2$
 6 a $3(m+1)(m+2)$
 c $5(t-10)(t+8)$
 e $x(x-11)(x+10)$
 g $4(w+4)(w-3)$
 i $2(e+5)(e+4)$
 k $-(u-7)(u+6)$
 m $-(b+4)(b-3)$
 o $-(x-5)(x-7)$
 7 a $(h-6)(h+3)$
 c $(w+1)(w+7)$
 e $(v+10)(v-2)$
 g $(q-1)(q-5)$
 i $6(x+8)(x-2)$
 k $8(u-4)(u+1)$
 m $(y-6)^2$
 o $4l(l-4)(l+2)$
 q $-2(d-6)(d+9)$
 j $(a+6)(a+5)$
 l $(y+4)(y+11)$
 b $(r-7)(r+2)$
 d $(w-9)(w+2)$
 f $(a-6)(a+2)$
 b $(t+8)(t-3)$
 d $(a+2)(a-1)$
 f $(w+6)(w-2)$
 h $(w-2)(w-4)$
 j $(p-6)(p-4)$
 l $(a-3)^2$
 b $2(y+2)(y-1)$
 d $5e^2(e+8)(e-3)$
 f $4(b-7)(b+6)$
 h $3a(a-4)(a+1)$
 j $-(t+8)(t-3)$
 l $-(x-7)(x+4)$
 n $-(k-3)(k-4)$
 b $-(t+9)(t-2)$
 d $(k+3)(k-15)$
 f $3(c+5)^2$
 h $(a-3)(a-1)$
 j $(x-8)^2$
 l $(b+5)(b+6)$
 n $5(r-2)(r+1)$
 p $(g-20)(g-4)$
 r $-(n+2)(n-13)$

Exercise 3-13

- 1 a $(x+5)(2x+1)$ b $(x+3)(4x+1)$
 c $(x+3)(5x+2)$ d $(2x+5)(3x+2)$
 e $(w+15)(2w+1)$ f $(e+3)(4e+3)$
 g $(2f+3)(4f+1)$ h $(d+1)(3d+2)$
 i $(b+1)(2b+7)$ j $(y+1)(5y+11)$
 k $(4g+3)(2g+5)$ l $(3a+7)(2a+3)$
 2 a $(y-4)(2y-3)$ b $(2k-3)(5k-2)$
 c $(2e-3)(3e-2)$ d $(b-3)(4b-1)$
 e $(w-3)(6w-5)$ f $(2t+5)(4t+3)$
 g $(3x-2)^2$ h $(3f-4)(4f-3)$
 i $(2h-9)^2$ j $(y+1)(5y-11)$
 k $(4d-5)(d+1)$ l $(2m+3)(m-3)$
 m $(3t-10)(t+3)$ n $(6h-7)(h+1)$
 o $(y-4)(2y+3)$ p $(2a+1)(4a-3)$
 q $(5u-4)(3u+1)$ r $(3c+1)(3c-5)$
 3 a $(5m+7)(m-1)$ b $(3g-4)(2g+3)$
 c $(3p-2)(p+2)$ d $(7w-1)(w+1)$
 e $(5y-1)(y+3)$ f $(3n-2)(n+4)$
 g $(4b-3)(b+3)$ h $(4m-1)(2m+3)$
 i $(3x+8)(x-2)$
 4 a $(9w-10)^2$ b $4(y+1)^2$ c $(5h-4)^2$
 5 a $2(t+2)(3t-1)$ b $3(g+4)(2g-3)$
 c $4(2e-3)(3e+1)$ d $2(a-2)(4a+3)$

- e $4(t+2)(3t-1)$ f $-(5q+3)(5q-2)$
 g $-2(2m-1)(3m-2)$ h $-(3h+4)(4h-5)$
 i $6(2c+3)(2c+1)$ j $-3(z+1)(2z-5)$
 k $2(2d-3)(3d+5)$ l $-2(x-3)(3x-2)$
 6 a $(a+1)(2a+3)$ b $(2m-5)(6m-1)$
 c $(4x-1)(x+3)$ d $(w-1)(7w-1)$
 e $(h-3)(4h+5)$ f $(4x-3)(2x+1)$
 g $(r+5)(5r+1)$ h $(d-7)(2d-1)$
 i $(3n+1)(2n-3)$ j $-(3m-2)(3m+4)$
 k $-(5c-3)(c+1)$ l $(3g+2)(5g+3)$
 m $-(4q+3)(2q-5)$ n $(x-2)(3x-7)$
 o $-(3d-4)(d+4)$

Exercise 3-14

- 1 a $(m-8)^2$ b $3d(d-1)$
 c $(d-3)(3d+5)$ d $(3+h)(k-5)$
 e $(5y+8)(5y-8)$ f $4(5f+4)(5f-4)$
 g $q(q+3-3p)$ h $-(g-3)(g+1)$
 i $4(2b+5)(3b-2)$ j $(5r+1)(5r-1)$
 k $(b^2+1)(b+1)$ l $(2x-5)^2$
 m $-(5d-4)(d+1)$ n $(b-1)^2(b+1)$
 o $2(2+v)(2-v)$ p $m(n+3)(n+p)$
 q $2(w-6)^2$ r $(6h+1)^2$
 2 a $(3r-8t)(5r+3t)$ b $(2d+1)^2$
 c $9(g+2k)(g-2k)$ d $e(e-5)(e+2)$
 e $-20(2p-3q)(3p-2q)$ f $7(2x+1)(2x-1)$
 g $(a-b)(a+b+4)$ h $(c-2)^2(c+2)$
 i $(3a-1)(2a+5)$ j $(t+7)(t-5)$
 k $2(3p+2)^2$ l $-(6a-1)(4a+1)$
 m $9(x+2)(x-3)$ n $(a-3)(2ab-3)$
 o $2(a+3)^2$ p $(5u-1)^2$
 q $(k-3)(4k+7)$ r $3(4+w)(4-w)$
 s $3(1+3s)(1-3s)$ t $(k+4)^2(k-4)$
 u $5y(y^2-2y+3)$ v $mn(m+2)(m-2)$
 w $-2(a+2)(a-2)$ x $4(2c-3)(4c+1)$

Exercise 3-15

- 1 a $x+y$ b $\frac{1}{2(t-r)}$ c $\frac{b-c}{a}$
 d -1 e $w-4$ f $\frac{5}{d+t}$
 g $\frac{k+5}{k-5}$ h $3(c-1)$ i $\frac{a+1}{m+n}$
 j $\frac{y+4}{2}$ k $\frac{k+1}{k+4}$ l $\frac{4a+5c}{a-c}$
 m $\frac{s+2}{s-3}$ n $\frac{1-2c}{3c-1}$ o $\frac{a+4}{2(p+2)}$
 2 a $\frac{7m+10}{m(m+1)(m+2)}$ b $\frac{2w-20}{w(w+3)(w+5)}$
 c $\frac{4b-7}{(b-1)(b+2)(b-3)}$ d $\frac{-k-2}{k(k+1)(k-1)}$
 e $\frac{5h+12}{4h(h+1)}$ f $\frac{-4d-1}{(d+2)(d+1)}$
 g $\frac{42-5r}{4(r+6)(r-6)}$ h $\frac{d^2+3d-6}{d(d+2)(d-2)}$
 i $\frac{-k^2+9k-5}{(k+1)(k-1)(k-4)}$ j $\frac{3q-1}{(q+1)(q-1)}$
 3 a $6m$ b $1\frac{1}{24}$ c $\frac{1}{2}$ d $6k$
 e $\frac{10}{h+1}$ f $\frac{3}{2(a-b)}$ g $\frac{-r}{5(r+t)}$ h $\frac{4m}{m-1}$
 i $\frac{4}{p}$ j $\frac{3}{7}$ k $\frac{2}{3(x-2)}$ l $3\frac{1}{3}$
 m $\frac{(d+1)(d-3)}{6}$ n $\frac{f+3}{4(f-3)}$ o $\frac{3}{j-2}$

Power plus

- 1 a $u + 4t$ b $2u + 3t$
 c $10u + 14t$ d $4u^2 + 8ut - 2t^2$
 2 $\frac{5t+6}{2}$
 3 a $A = y^2 - 5y$ b $P = 4y - 10$
 4 $5t$ hours
 5 a $a^2 + b^2 + 2ab + ac + bc$
 b $x^2 + xy - 5x - 2y + 6$
 c $p^3 - 6p^2 - 7p + 60$
 d $xyz - xy - yz - xz + x + y + z - 1$
 e $w^2t^2 - 6rwt + 9r^2$
 f $x^6 + 4x^3 + 4$
 6 a $(n + 2m)^2$ b $(x - y)^2$
 c $(5x - 4y)^2$ d $5(a - 3b)^2$
 e $(c^2 + 1)^2$ f $-(r^2 + 8)(t + 1)(t - 1)$
 g $(x^2 + 1)(x + 1)(x - 1)$ h $(a + b - c)(a + b + c)$
 i $4pq$
 7 a both positive
 b the larger of p, q is negative; the smaller of p, q is positive
 c the larger of p, q is positive; the smaller of p, q is negative
 d both negative

Chapter 3 revision

- 1 a $16y$ b $-3ab$ c $8x^2$
 d $2p + 2q$ e $6w - 5$ f $3x^2 - 13x$
 g $7m + 10n$ h $3u + 4$ i $12d^2 - 2d + 4$
 2 a $18n$ b $3gh$ c ad
 d $-6r$ e $4y$ f $2x$
 g 4 h 3 i $-4p$
 3 a $\frac{5h}{6}$ b $\frac{k}{2}$ c $\frac{2}{m}$ d $-\frac{3w}{20}$
 e $\frac{5k}{6}$ f $\frac{-3}{2d}$ g $\frac{d-15}{5k}$ h $\frac{5}{m+1}$
 4 a $\frac{20}{im}$ b $\frac{3h}{k^2}$ c $\frac{21}{d}$ d $\frac{5}{4}$
 e $\frac{5}{2}$ f 1 g $\frac{2d}{5}$ h $\frac{p}{3}$
 i $\frac{10a}{3}$ j $\frac{16}{9}$ k $\frac{45a^4}{16bc}$ l $\frac{75a}{4y}$
 5 a $5r - 10$ b $a^2 - 10a$ c $6xy - 15y^2$ d $9\sqrt{2}$
 d $12p^2 - 21pq$ e $-6n^2 + 54n$ f $-6t^2 - 42t$
 g $b^2 + 6b + 5$ h $6x^2 - 31x + 35$ i $30 - 12x$
 6 a $4(m - 3)$ b $x(y + z)$ c $-3(g + 10)$
 d $8w(2w + 3)$ e $-8(q - 2)$ f $5k(3 - 2hk)$
 g $(y - 6)(3 + y)$ h $(2x - 1)(3x - 2)$ i $(p + 6)(p - 2)$
 7 a $a^2 + 5a + 6$ b $y^2 - 4y - 21$ c $t^2 + 5t - 24$
 d $h^2 - 9h + 20$ e $12 + g - g^2$ f $6p^2 + 11p + 3$
 g $20x^2 - 11x - 3$ h $12r^2 + r - 20$ i $-6q^2 + 13q - 5$
 8 a $n^2 + 10n + 25$ b $u^2 - 6u + 9$ c $81 + 18x + x^2$
 d $36 - 12k + k^2$ e $9y^2 + 6y + 1$ f $4x^2 - 20x + 25$
 g $36 - 84b + 49b^2$ h $1 - 8h + 16h^2$
 9 a $n^2 - 36$ b $b^2 - 1$ c $100 - m^2$
 d $9r^2 - 1$ e $16w^2 - 25$ f $1 - 64w^2$
 10 a $3x^2 + 9x + 4$ b $7y + 18$ c $-6a - 2$ d $8p^2 + 18q^2$

- 11 a $(5m - 3)(n + 2)$ b $(b - 1)(a - 1)$
 c $(x + y^2)(xy - 5)$ d $(p^2 + 3)(10 + p)$
 e $(w + 3)(w - 3)$ f $(m + n)(m - n)$
 g $(6 + r)(6 - r)$ h $(4a + 3)(4a - 3)$
 i $(9x + y)(9x - y)$ j $(7a + 5b)(7a - 5b)$
 k $3(q + 3)(q - 3)$ l $2(5m + 4)(5m - 4)$
 m $6(2 + 5d)(2 - 5d)$
 12 a $(m - 12)(m + 1)$ b $(y - 3)(y - 1)$
 c $(t - 13)(t + 5)$ d $(p + 9)(p - 3)$
 e $3(n + 1)(n + 2)$ f $4(y + 2)(y - 1)$
 g $-(x - 6)(x + 3)$ h $-5(b + 4)(b - 3)$
 i $-(2k - 1)(3k - 4)$
 13 a $(3x + 2)(2x - 1)$ b $(5s + 3)(s - 2)$
 c $(4w - 1)(2w - 3)$ d $3(e + 2)^2$
 e $3(2y - 3)(y + 4)$ f $4(3a + 1)(2a - 3)$
 g $-3(3x - 2)^2$ h $-2(4t - 3)(2t - 5)$
 i $2(3g - 1)(2g - 7)$
 14 a $(8y + 5)(8y - 5)$ b $(x - 8)^2$
 c $16(r - 3)$ d $(g - 5)(3 + h)$
 e $(a - 1)^2(a + 1)$ f $3b(b + 1)(b - 1)$
 g $(4y + 7)(y - 3)$ h $2(3q + 2)^2$
 i $-(6p - 1)(4p + 1)$
 15 a $\frac{3y+5}{4}$ b $\frac{x-1}{3x+2}$ c $\frac{w+5}{w-5}$
 16 a $\frac{x-4}{x(x+1)(x-1)}$ b $\frac{2}{3(x-3)}$
 c $\frac{k^2-3k+10}{2(k+3)(k-3)}$ d $\frac{5x-9}{(x+2)(x+3)(x-3)}$
 e $\frac{4d^2-3d+3}{d(d-1)(d-2)}$ f $\frac{m+18}{(m+1)(m+4)(m-3)}$
 g $\frac{9}{5}$ h $\frac{2(y-3)}{y^2}$
 i $\frac{1}{2(d-2)}$ j $\frac{5}{4}$

Mixed revision 1

- 1 a 0.8, terminating b $0.\dot{3}8461\dot{5}$, recurring
 c 0.4, recurring d 4.75, terminating
 2 a $5m - 2$ b $20p + m$ c $3n^2 - 10n$
 3 a $\frac{14}{33}$ b $\frac{4}{15}$ c $\frac{20}{111}$
 4 $\sqrt{98}, \sqrt{160}, \sqrt{52}$
 5 a $5\sqrt{3}$ b 63 c $4\sqrt{7}$ d $9\sqrt{2}$
 6 a $g = 24.2$ b $e = 4.2$ c $k = 21.2$
 7 a $p^2 - 4p - 32$ b $6n^2 + 13n + 5$ c $49 - y^2$
 8 a $25a^2 + 30a + 9$ b $9r^2 - 16$ c $6x$
 9 a $13\sqrt{2} - 3\sqrt{5}$ b $\sqrt{2}$ c $25\sqrt{2}$
 10 a $3\sqrt{6}$ b 30 c $-8\sqrt{10}$
 d 3 e $-4\sqrt{3}$ f $-\frac{7}{2}$ (or $-3\frac{1}{2}$)
 11 a 6 b 8 c 12.5
 12 B
 13 a $3(p - 5)$ b $(m + 4)(5 - t)$ c $-4xy(2x + 3)$
 14 a $(6y + 7x)(6y - 7x)$ b $(m - 5n)(m + 3)$
 c $2(2m + 5)(2m - 5)$ d $(2y - 3)(ky - 1)$
 15 a \$102 b \$35 c \$23.45
 16 a No, $50^2 \neq 40^2 + 14^2$ b Yes, $25^2 = 7^2 + 24^2$
 c Yes, $6.1^2 = 1.1^2 + 6^2$

Answers

- 17 a $(y-9)(y+7)$ b $(k-2)(k-5)$
 c $(w+7)(w-2)$ d $(m-9)(m-8)$
 18 a $(k+3)(5k-1)$ b $(y+3)(3y-8)$
 c $(h-3)(4h+5)$ d $(2w-7)(4w-5)$
 19 a \$80.80 b 40.8 L
 20 \$385
 21 a $\frac{1}{k-3}$ b $\frac{1}{x-3}$ c $\frac{v+5}{6}$ d $\frac{d-7}{d-5}$
 22 a $8a^3$ b $40p^2d$ c $27b^3d^2$
 23 a $3m$ b $\frac{3x}{y}$ c $\frac{wy^2}{2}$
 24 a $\frac{n^2+1}{n(n+1)(n-1)}$ b $\frac{5y}{3}$ c $\frac{(k+4)(k-1)}{4}$
 25 126.3%
 26 a 5 : 3 b 4 : 1 c 85 words/min
 27 2.3 m
 28 a 110.5 kg b \$5.25
 29 a $40-8x$ b $6h-21h^2$ c $18cd-27c^2$
 30 a $22w-2$ b $1-16k$ c $26t-23$
 31 \$90, \$150, \$210
 32 40 320 km/h
 33 a $\frac{6d}{11}$ b $\frac{13t}{20}$ c $\frac{n}{30}$
 34 a $\frac{5a}{6m}$ b $\frac{4}{7}$ c $\frac{2}{3}$
 35 a $\sqrt{74}$ cm
 36 \$1875

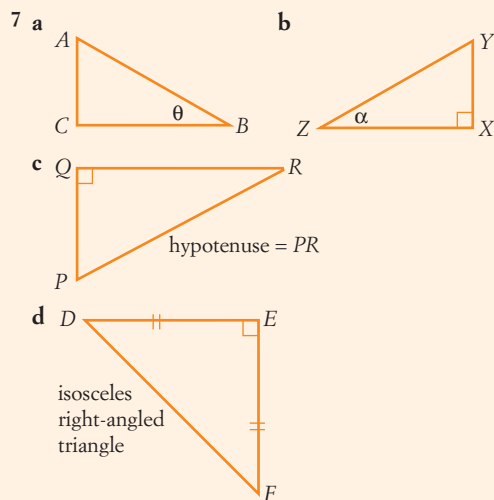
Chapter 4

SkillCheck

- 1 a 35 b 33.2 c 5
 2 a 85 b 16 c 3.9
 3 a 8 h b 4 h c 2 h
 4 a 4h 42min b 2h 15min c 6h 51min
 5 a $x=55$ b $y=23$ c $n=38$
 d $x=60$ e $x=50$ f $x=160$

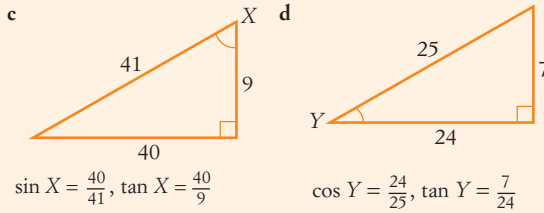
Exercise 4-01

- 1 a opp = 12, adj = 5, hyp = 13
 b opp = AC, adj = AB, hyp = BC
 c opp = u , adj = w , hyp = v
 d opp = TR, adj = ST, hyp = SR
 e opp = y , adj = z , hyp = x
 f opp = 9, adj = 40, hyp = 41
 2 a $\angle K$ b $\angle L$ c $\angle M$ d $\angle M$ e $\angle L$
 3 C
 4 a $\angle \theta$ EF, $\angle \phi$ DE b $\angle \theta$ 12, $\angle \phi$ 9 c $\angle \theta$ f , $\angle \phi$ e
 5 a $\angle \theta$ IJ, $\angle \phi$ HI b $\angle \theta$ 27, $\angle \phi$ 36 c $\angle \theta$ b , $\angle \phi$ c
 6 B



Exercise 4-02

- 1 a $\sin \theta = \frac{55}{73}$ $\cos \theta = \frac{48}{73}$ $\tan \theta = \frac{55}{48}$
 b $\sin \theta = \frac{e}{f}$ $\cos \theta = \frac{g}{f}$ $\tan \theta = \frac{e}{g}$
 c $\sin R = \frac{ST}{RT}$ $\cos R = \frac{RS}{RT}$ $\tan R = \frac{ST}{RS}$
 d $\sin \alpha = \frac{n}{k}$ $\cos \alpha = \frac{m}{k}$ $\tan \alpha = \frac{n}{m}$
 e $\sin M = \frac{3.6}{8.5}$ $\cos M = \frac{7.7}{8.5}$ $\tan M = \frac{3.6}{7.7}$
 f $\sin W = \frac{XY}{WY}$ $\cos W = \frac{WX}{WY}$ $\tan W = \frac{XY}{WX}$
 2 a $\frac{4}{5}$ b $\frac{4}{3}$ c $\frac{3}{5}$ d $\frac{4}{5}$
 3 a α b β c β
 d α e α f β
 4 a i $\frac{24}{7}$ ii $\frac{7}{25}$ iii $\frac{24}{25}$ iv $\frac{7}{24}$
 b i $\frac{u}{v}$ ii $\frac{v}{w}$ iii $\frac{u}{w}$ iv $\frac{v}{u}$
 c i $\frac{EG}{FH}$ ii $\frac{EH}{HG}$ iii $\frac{EG}{HG}$ iv $\frac{EH}{FG}$
 d i $\frac{84}{13}$ ii $\frac{13}{85}$ iii $\frac{84}{85}$ iv $\frac{13}{84}$
 e i $\frac{b}{c}$ ii $\frac{c}{a}$ iii $\frac{b}{a}$ iv $\frac{c}{b}$
 f i $\frac{QS}{RS}$ ii $\frac{RS}{QR}$ iii $\frac{QS}{QR}$ iv $\frac{RS}{QS}$
 5 a $\tan Y = \frac{60}{11}$ b $\tan X = \frac{11}{60}$
 c $\sin X = \frac{11}{61}$ or $\cos Y = \frac{11}{61}$ d $\cos X = \frac{60}{61}$ or $\sin Y = \frac{60}{61}$
 6 C
 7 A
 8 a
- b



Exercise 4-03

- Teacher to check.
 - All are approximately the same (0.9).
 - The table answer is an approximation of the calculator answer.
 - All are approximately the same (0.5).
 - The table answer is an approximation of the calculator answer.
 - All are approximately the same (1.7).
 - The table answer is an approximation of the calculator answer.
- Teacher to check.
 - The table answer is an approximation of the calculator answer.
 - The table answer is an approximation of the calculator answer.
 - The table answer is an approximation of the calculator answer.
- Teacher to check.

Exercise 4-04

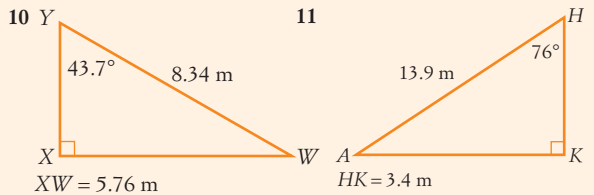
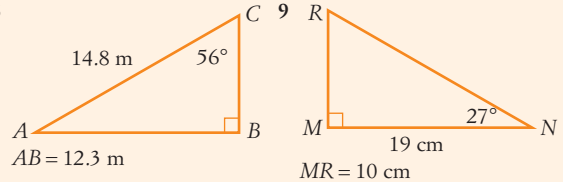
1 a 28°	b 41°	c 19°	d 33°
e 34°	f 56°	g 30°	h 44°
2 a 68°40'	b 54°22'	c 68°40'	d 18°30'
e 9°11'	f 47°59'	g 3°46'	h 57°10'
3 a 9.51	b 0.97	c 1.07	d 0.54
e 0.97	f 0.50	g 0.83	h 0.95
i 29.86	j 11.61	k 32.97	l 10.26
m 47.09	n 9.56	o 3.79	p 32.96
q 19.84	r 246.59	s 12.28	t 16.83
4 a 55°30'	b 14°9'	c 72°23'	d 33°46'
e 66°25'	f 7°53'	g 28°7'	h 31°3'
i 34°27'	j 71°5'	k 5°29'	l 69°27'
5 a 52°	b 14°	c 52°	d 60°
e 33°	f 28°	g 36°	h 89°

Mental skills 4

2 a 330	b 160	c 1580	d 260
e 420	f 200	g 420	h 740
i 6400	j 16 000	k 300	l 70
4 a 29	b 15	c 180	d 400
e 3	f 4	g 20	h 4
i 6	j 25		

Exercise 4-05

1 a $\tan \theta$	b $\cos \theta$	c $\tan \theta$
d $\sin \theta$	e $\tan \theta$	f $\cos \theta$
2 a 27.63	b 24.44	c 8.78
d 31.17	e 38.71	f 53.02
3 a 66.5	b 1.9	c 17.2
d 47.4	e 31.3	f 251.0
4 a 16.63	b 57.75	c 8.97
d 7.38	e 114.80	f 265.25
5 a 167.7	b 131.0	c 618.3
d 2.2	e 24.5	f 3.5
6 B		
7 a 228.2 cm	b 141.4 m	c 671.4 m
d 1053.1 cm	e i 10 km	ii 17.3 km
8		



- 12 9.33 m or 933 cm 13 5.738 m or 5738 mm
 14 5.1 m 15 C 16 89 m

Exercise 4-06

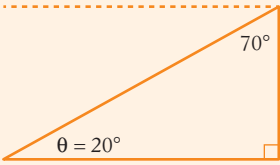
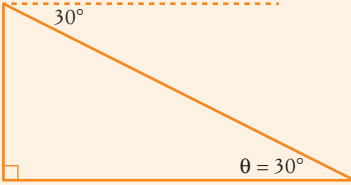
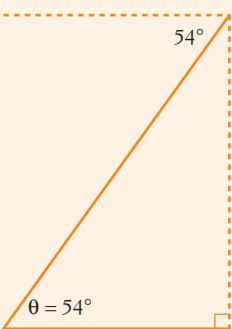
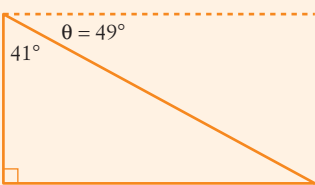
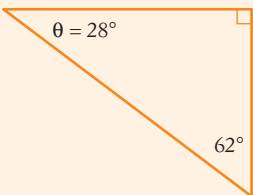
1 a 102.3	b 62.9	c 96.4
d 21.5	e 245.5	f 378.3
2 325 mm	3 12 m	
4 a 88.97	b 1188.24	c 45.70 d 187.70
5 6.88 m	6 54.4 mm	
7 a 27.5	b 109.4	c 293.5
d 5.7	e 88.1	f 20.8
8 D		
9 a 1203.3 cm	b 414.2 mm	c 6.7 m
d i 3.8 km	ii 8.4 km	e 1044.3 cm
10 782 m	11 12.1 m	12 4.9 km 13 B 14 77.3 m

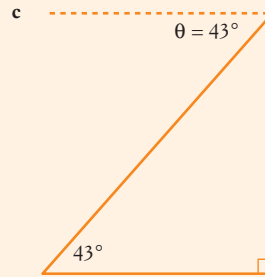
Exercise 4-07

1 a 41°	b 64°	c 60°	d 81°
e 61°	f 30°	g 6°	h 45°
i 60°	j 82°	k 84°	l 14°
2 a 64°59'	b 54°35'	c 36°52'	d 20°10'
e 43°32'	f 72°33'	g 27°2'	h 61°39'
i 86°12'	j 81°47'	k 35°16'	l 51°3'
3 a 34°	b 51°	c 60°	
d 29°	e 67°	f 54°	

- 4 C
 5 a $36^{\circ}42'$ b $25^{\circ}12'$ c $36^{\circ}52'$
 d $31^{\circ}30'$ e $26^{\circ}47'$ f $48^{\circ}11'$
 6 a 12° b 68° c 27°
 d 27° e 30° f 88°
 7 39° 8 $10^{\circ}56'$ 9 38.9° 10 62° 11 11°
 12 37° 13 B 14 71° 15 11° 16 50°

Exercise 4-08

- 1 a 
 b 
 c 
 2 a 
 b 



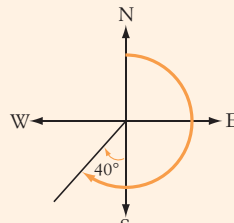
- 3 127 m 4 2224 m or 2.224 km 5 177 m
 6 180 m 7 9° 8 $14^{\circ}29'$ 9 50° 10 $18^{\circ}47'$
 11 1256 cm or 12.56 m 12 14.5 m 13 970 m 14 58 m

Exercise 4-09

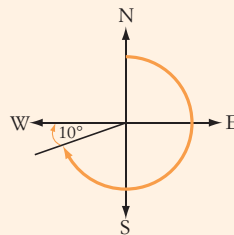
- 1 a 243° b 290° c 040°
 d 115° e 210° f 140°
 g 312° h 253° i 065°
 2 a 000° b 090° c 180°
 d 270° e 038° f 125°
 g 330° h 225° i 072°
 j 187°

3 SW

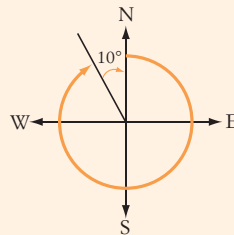
4 a



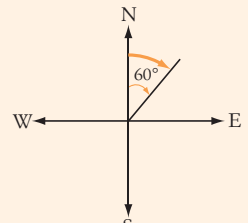
c



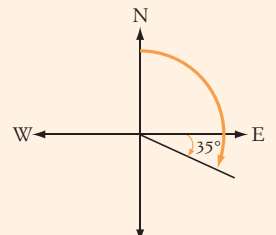
e



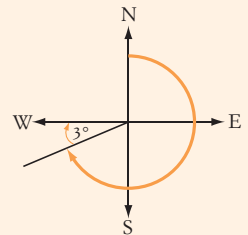
b

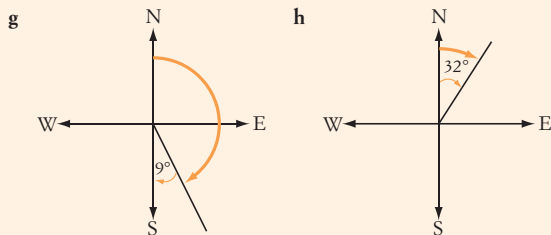


d

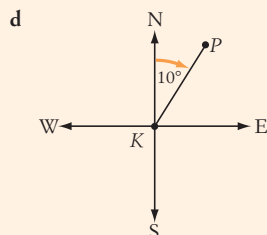
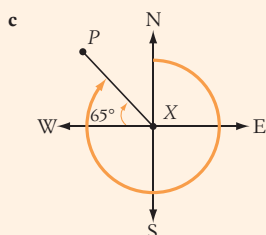
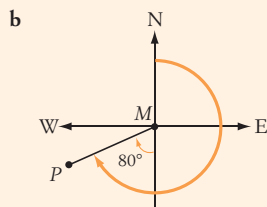
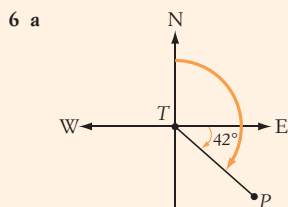


f





5 a NNW b 337.5°



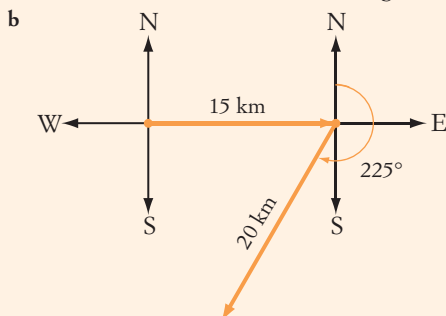
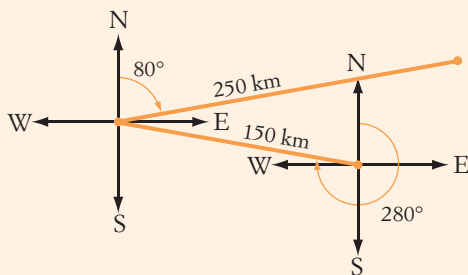
7 240° 8 280°

9 a 45° b 90°

c 135° or 225°

10 ESE

11 a



12 a 230° b 270° c 160°
d 340° e 050° f 090°

Exercise 4-10

1 a 21 km	b 260°	
2 a 37°	b 163 km	c 143°
3 a 63 km	b 29 km	c 025°
4 a 12.2 km	b 035°	
5 a 18 km	b NNW	
6 45.7 km		
7 a 13.509 km	b 321°	
8 a 2122 km	b 330°	
9 a 15 km	b 26 km	
10 a 261.08 km	b 167.82 km	
11 6.558 km		
12 a 322 nautical miles	b 276°45''	

Power plus

- 1 a i 0.342, 0.342 ii 0.731, 0.731
iii 0.819, 0.819 iv 0.996, 0.996
b Each pair of trigonometric ratios has the same value.
c The pairs of angles are complementary (add to 90°).
d 60°
e i 15° ii sin iii 18°
iv cos v 25° vi 32°
f $\sin x = \cos(90^\circ - x)$ g Teacher to check.
2 a 1932 m b 31°
3 a $\cos 30^\circ = \frac{\sqrt{3}}{2}$ b $\tan 30^\circ = \frac{1}{\sqrt{3}}$
4 a 24°12'26'' b 63°17'3''
5 Teacher to check.

Chapter 4 revision

1 a $\frac{33}{65}$	b $\frac{33}{56}$	c $\frac{56}{65}$	d $\frac{33}{65}$
2 $\cos \alpha = \frac{77}{85}$	$\tan \alpha = \frac{36}{77}$		
3 a 64°	b 26°	c 12°	
4 a 50°19'	b 31°56'	c 64°19'	
5 a 0.8480	b 0.7677	c 0.1539	
d 64.9839	e 13.9884	f 13.7044	
6 a 45°48'	b 33°11'	c 5°21'	
7 a 14.49	b 72.50	c 0.87	
8 a 98.9	b 21.1	c 23.2	
9 a 68°44''	b 55°9'	c 69°38'	
10 a 33°	b 28°	c 38°	
11 68°13'			
12 195 m			
13 29°			
14 a 320°	b 140		
15 a 1281 km	b 024°		

Answers

Chapter 5

SkillCheck

1 a i 8	ii 4	iii eight to the power of four
b i 4	ii 8	iii four to the power of eight
c i h	ii 5	iii h to the power of five
d i 5	ii h	iii five to the power of h
2 a 2^5	b $3^4 \times 7^3$	c $5^6 \times 8^2$
d $10^1 \times x^5$	e $6^3 \times k^2$	f $x^3 \times y^2$
g $a^2 \times b^3$	h $5^2 \times n^3$	i $p^2 \times q^4$
3 a $9 \times 9 \times 9$	b 7×7	c $d \times d \times d \times d \times d$
d $k \times k$		
4 a 1024	b 10 000	c 729	d 1
e 9	f 15 625	g 8	h 64
5 a 3	b 4	c 3	d 2
e 6	f 4	g 6	h 4

Exercise 5-01

1 B			
2 a 10^5	b 2^5	c 3^7	d 7^5
e 8^8	f 5^9	g 6^{10}	h 4^{12}
i 3^{11}	j x^5	k g^8	l w^8
m b^{13}	n p^{20}	o r^2	p y^6
q m^8	r n^{10}		
3 D			
4 a $6p^7$	b $12y^{12}$	c $18m^9$	
d $5h^{11}$	e $24q^9$	f $10a^7$	
g $30n^{16}r^5$	h $30a^2b^4$	i $3e^{10}g^5$	
j $32p^7m^{10}$	k $48q^8r^8$	l $54u^4v^3w^8$	
5 B			
6 a 10^2	b 8^4	c 20^{10}	d 5^6
e 9^9	f 2^{24}	g 7	h 2^{19}
i 11^0	j p^5	k n^6	l w^{18}
m h^{16}	n y^6	o a^8	p b
q w^{24}	r m^0		
7 D			
8 a $2y^{12}$	b $5w^6$	c $8r^7$	d $30x$
e $5m^9$	f $\frac{g^6}{2}$	g $2d^2h^9$	h x^5y^4
i $\frac{e^{20}d^{16}}{3}$	j $\frac{3qt}{4}$	k $9a^5b^8$	l $\frac{3pq^2r^4}{2}$

Exercise 5-02

1 C			
2 a 4^6	b 5^{16}	c 3^{12}	d 2^{28}
e 2^2	f 9^3	g 10^0	h 6^{20}
i 5^{15}	j e^8	k t^{25}	l y^{21}
m c^5	n m^{35}	o y^{16}	p h^0
q q^{18}	r w^4	s $2^{10}x^{10}$	t 5^8n^{24}
u 4^3d^9	v $-k^{45}$	w d^{12}	x 2^8d^{64}
3 B			
4 a $16d^{12}$	b $25m^6$	c $16y^{10}$	d $81x^8$
e $3125u^{30}$	f $8w^{15}$	g $10\,000d^{20}$	h $27e^3$
i $2b^4$	j $36d^{12}$	k $243f^{20}$	l $1024c^{30}$

m $16r^4$	n $-125t^3$	o $9m^6$	p y^{36}
q $-x^3$	r m^{30}	s $256w^{20}$	t $-243f^5$
u $-27p^6$	v $81h^{20}$	w $100k^2$	x $-8y^3$

Exercise 5-03

1 A			
2 a a^3b^3	b $x^{10}y^5$	c $l^{18}m^{30}$	
d $1296d^4p^8$	e $64k^8y^{10}$	f $243m^{10}n^5$	
g e^3k^9	h $-w^{21}x^{28}$	i $64d^6y^{10}$	
j $256b^8c^{12}$	k $-27a^9d^3$	l $16p^8q^{12}$	
3 D			
4 a $\frac{36}{49}$	b $\frac{m^3}{8}$	c $\frac{625}{x^4}$	d $\frac{256m^{40}}{p^8}$
e $\frac{w^{10}}{r^{15}}$	f $\frac{m^8}{256m^4}$	g $\frac{16}{81}$	h $-\frac{125h^3}{216}$
i $\frac{49k^8}{100}$	j $\frac{9r^2}{r^4}$	k $\frac{d^8b^4}{d^{20}}$	l $-\frac{32}{243c^{10}}$
5 a $40x^{32}y^{48}$	b $1000x^{36}y^{54}$	c $2q^3r^4$
d $36q^8r^{12}$	e $27a^{12}x^{15}$	f $3ax^2$
g $32p^8h^{29}$	h $8p^4h^{11}$	i $64p^{10}h^{38}$

Exercise 5-04

1 a 1	b 1	c -1	d 1
e -1	f 1	g 1	h 7
i -1	j 1	k 1	l -2
m 1	n 1	o 1	p 1
q 1	r -1	s -6	t 6
u 1	v -1	w -3	x 1
2 a 2	b 0	c 3	d 1
e 7	f -5	g -3	h 0
i 1	j 9	k $1\frac{1}{2}$	l 2
m 6	n 4	o 125	p -1
q 12	r 3	s $\frac{1}{5}$	t 27

Mental skills 5

2 a 117	b 740	c 138	d 140
e 174	f 100	g 436	h 109
i 200	j 312		
3 a 160	b 700	c 90
d 2200	e 72	f 1600
g 2100	h 480	i 132

Exercise 5-05

1 a $\frac{1}{6^2}$	b $\frac{1}{5^7}$	c $\frac{1}{3^1}$	d $\frac{1}{10^2}$
e $\frac{1}{8^3}$	f $\frac{1}{2^7}$	g $\frac{1}{n^7}$	h $\frac{1}{r^7}$
i $\frac{1}{a^4}$	j $\frac{1}{5^3}$	k $\frac{1}{y^8}$	l $\frac{1}{r^m}$
2 a $\frac{1}{9}$	b $\frac{1}{625}$	c $\frac{1}{6}$	d $\frac{1}{49}$
e $\frac{1}{25}$	f $\frac{1}{128}$	g $\frac{1}{64}$	h $\frac{1}{1\,000\,000}$
i $\frac{1}{1024}$	j $\frac{1}{27}$	k $\frac{1}{36}$	l $\frac{1}{6561}$
3 a n^{-2}	b n^{-1}	c 8^{-3}	d 8^{-1}
e 10^{-5}	f $2a^{-4}$	g 3^{-1}	h $-b^{-1}$
i $6a^{-1}$	j $4t^{-2}$	k $-2w^{-5}$	l $5d^{-3}$

a	$\frac{5}{h}$	b	$\frac{2}{b^5}$	c	$\frac{3}{e^3}$	d	$\frac{4}{n^2}$	2 a	1	b	1	c	7	d	1
e	$\frac{p}{b^2}$	f	$\frac{r^2}{s^4}$	g	$\frac{y}{w^2}$	h	$\frac{y^3}{d^4}$	e	-8	f	9	g	625	h	128
i	$\frac{1}{2m}$	j	$\frac{1}{xy}$	k	$\frac{1}{16h^2}$	l	$\frac{1}{125k^3}$	i	1	j	64	k	$\frac{1}{64}$	l	1
m	$\frac{3m^3}{p^2}$	n	$\frac{15}{kvw^4}$	o	$\frac{12}{x^2y^3}$	p	$\frac{12y^3}{x^2}$	m	25	n	$\frac{1}{1000}$	o	1	p	1
q	$\frac{1}{9h^2}$	r	$\frac{1}{64k^3}$	s	$\frac{1}{16c^4}$	t	$\frac{1}{8y}$	3 a	$\frac{1}{25}$	b	$\frac{1}{32}$	c	$\frac{1}{20}$	d	$\frac{1}{1000}$
u	$\frac{4p}{q^2}$	v	$\frac{4}{pq^2}$	w	$\frac{v}{m^2}$	x	$\frac{1}{vm^2}$	4 a	4	b	4	c	3125	d	4
5 a	$\frac{7}{2}$	b	$\frac{5}{8}$	c	$\frac{10}{9}$	d	$\frac{2}{3}$	e	4	f	27	g	27	h	-8
e	$-\frac{4}{3}$	f	$\frac{2}{5}$	g	$\frac{3}{x}$	h	$\frac{a}{5}$	5 a	$9m^2n^6$	b	$40a^5w^9$	c	$256a^8b^{20}$	d	$4pq^2$
i	$-\frac{2}{m}$	j	$\frac{4}{5r}$	k	$\frac{3z}{2}$	l	v	e	$\frac{64}{125}$	f	$6c^2$	g	$3u^4v^3$	h	$\frac{9x^2}{100}$
6 a	16	b	$2\frac{1}{4}$	c	1 000 000			i	$-64n^6t^3$	j	$\frac{16}{81}$	k	$\frac{y^3}{5x^3}$	l	$\frac{p^{10}}{81y^2}$
d	$\frac{8}{125}$	e	$\frac{243}{1024}$	f	$\frac{256}{625}$			m	$32p^{15}q^{10}$	n	$\frac{1}{343n^3}$	o	-2	p	$\frac{a^{11}}{b}$
g	$\frac{16}{81}$	h	$\frac{125}{343}$	i	$\frac{9}{k^2}$			6 a	$\frac{1}{8^7}$	b	$\frac{1}{3^3}$	c	$\frac{1}{y}$	d	$\frac{1}{x^3}$
j	$\frac{x^3}{27}$	k	$\frac{256}{a^8}$	l	$\frac{9c^6}{16}$			e	$\frac{1}{25b^2}$	f	$\frac{5}{b^2}$	g	$\frac{1}{ab}$	h	$\frac{a}{b}$
m	$\frac{25t^2}{4d^2}$	n	$\frac{m^{15}}{h^{10}}$	o	$\frac{9p^8}{25d^6}$			i	$\frac{11}{t^3}$	j	$\frac{1}{1331t^3}$	k	$\frac{p^3}{q^3}$	l	$\frac{m}{w^3}$
p	$\frac{64a^6}{27c^9}$							m	$\frac{-2r^6}{w^3v^4}$	n	$\frac{10t^3}{y^3}$	o	$\frac{10t^3}{e}$	p	$\frac{n^7}{2k^4}$
								7 a	$\frac{4}{7}$	b	$\frac{2}{5}$	c	$\frac{3}{2}$	d	7
									$\frac{8}{7}$				$\frac{7}{2}$		$\frac{5q}{2}$

Exercise 5-06

1	a 5	b 7	c 25	d 10	e q^3	f d^4	g $\frac{-x}{m^{11}}$	h t^2
	e 2	f -0.3	g 0.2	h 4			c $\frac{28}{x}$	d t^2
	i -2	j -9	k 2	l 5	e $30g^2$	f $24a$	g $\frac{2}{b^4}$	h $\frac{4}{p^7}$
2	a $\sqrt{10}$	b $\sqrt[3]{12}$	c $\sqrt{g}$	d $\sqrt[4]{m}$	i $16q^3$	j $\frac{r^4}{2}$	k $(3y)^{\frac{1}{2}}$	l $\frac{1}{9h^2}$
	e $\sqrt{8r}$ or $2\sqrt{2r}$	f $\sqrt[3]{6h}$	g $\sqrt[3]{5j^8}$	h $90a\sqrt{b}$	10 a $5^{\frac{1}{2}}$	b $d^{\frac{1}{3}}$	c $(3y)^{\frac{1}{2}}$	d $10^{\frac{1}{4}}$
3	a $5^{\frac{1}{2}}$	b $49^{\frac{1}{3}}$	c $20^{\frac{1}{2}}$	d $400^{\frac{1}{3}}$	e $p^{\frac{3}{2}}$	f $(xy)^{\frac{5}{2}}$	g $(5a)^{\frac{1}{3}}$	h $(\frac{y}{6})^{\frac{2}{3}}$
	e $66^{\frac{1}{6}}$	f $64^{\frac{1}{4}}$	g $144^{\frac{1}{8}}$	h $1000^{\frac{1}{10}}$	11 a $\frac{125a^3}{64}$	b $4c^2$	c $\frac{49m^2}{100}$	d $3125w^{\frac{25}{3}}$
	i $a^{\frac{1}{2}}$	j $q^{\frac{1}{3}}$	k $h^{\frac{1}{4}}$	l $w^{\frac{1}{6}}$	e $\frac{343}{d^3}$	f $\frac{a^2b^6}{c^4}$	g $5m^{\frac{1}{2}}$	h $\frac{y^2}{16}$
	m $t^{\frac{1}{3}}$	n $(xy)^{\frac{1}{2}}$	o $(100f)^{\frac{1}{4}}$	p $(2mn)^{\frac{1}{3}}$	i $2m^2$	j $9g^4$	k $32x^{10}$	l $\frac{16a^{12}}{c^3}$
4	a 2.71	b 14.66	c 5.24	d 44.73				
	e -8.73	f 5.77	g -3.76	h 0.45	Exercise 5-09			
5	a b	b e	c $y^{\frac{6}{5}}$	d m				
	e 10t	f $36a^3$	g n^3m	h $4ab^3$	1 a 3	b 2	c 2	d 5
	i $2v^2w^3$	j 5	k $7x^{\frac{2}{3}}$	l $9y^{-\frac{1}{4}}$	e 1	f 1	g 3	h 5

Exercise 5-07

1	a 8	b 32	c 32	d 9	d 2.81	e 16.0	f 60 500
e	64	f 256	g 8	h 27	g 1 770 000	h 386 000	i 10.3
i	100	j 25	k $\frac{1}{2}$	l $\frac{1}{3}$	3 a 0.064	b 0.90	c 0.085
m $\frac{1}{125}$	n $\frac{1}{216}$	o $\frac{1}{64}$	p $\frac{1}{625}$	d 0.000 16	e 0.0076	f 0.039	
q $\frac{1}{256}$	r $\frac{1}{8000}$	s $\frac{1}{16}$	t $\frac{1}{128}$	g 0.28	h 0.019	i 0.31	
2	a 7.62	b 0.05	c 132.96	d 0.30	4 B	5 C	
e 0.03	f 5.28	g 0.02	h 7.96	6 a 9	b 60	c 0.04	
3	a $g^{\frac{2}{3}}$	b $e^{\frac{2}{3}}$	c x^3	d y^{-4}	d 0.008	e 0.5	f 10 000
e $m^{\frac{5}{3}}$	f $m^{\frac{3}{3}}$	g $n^{-\frac{3}{4}}$	h $n^{-\frac{4}{3}}$	g 2000	h 0.0003	i 60 000 000	
4	a $8n^3$	b $4\sqrt[3]{w^2}$	c $8d^6$	d $512m^{12}$	7 a \$36 000 000, 2	b \$40 000 000, 1	
e $243r^5$	f $3h^9$	g $\frac{1}{16s^8}$	h $\frac{1}{16x^6}$	8 5 9 21 600	10 a 180	b -5	c 6.16
i $\frac{1}{25b^4}$	j $\frac{1}{125y^3}$	k $343p^6q^{15}$	l $\frac{100}{100x^2y^4}$	e 0.06	f 0.67	g 33 300	d -4.176
							h 0.6565

Exercise 5-08

1 a	a^7	b	t^9	c	n^6	d	p^2	1 a	2.4×10^3	b	7.86×10^5	c	5.5×10^7
e	w^8	f	g^{18}	g	$6b^7$	h	$20d^{13}$	d	9.5×10^1	e	7.8×10^0	f	3.48×10^8
i	$6c^4$	j	$625b^{16}$	k	$3m^2$	l	$9a^2$	g	5.967×10^4	h	1.5×10^1	i	3×10^9

Exercise 5-09

1	a	3	b	2	c	2	d	5
	e	1	f	1	g	3	h	5
	i	2	j	2	k	3	l	6
2	a	37.6	b	9440	c	168		
	d	2.81	e	16.0	f	60 500		
	g	1 770 000	h	386 000	i	10.3		
3	a	0.064	b	0.90	c	0.085		
	d	0.000 16	e	0.0076	f	0.039		
	g	0.28	h	0.019	i	0.31		
4	B		5	C				
6	a	9	b	60	c	0.04		
	d	0.008	e	0.5	f	10 000		
	g	2000	h	0.0003	i	60 000 000		
7	a	\$36 000 000, 2	b	\$40 000 000, 1				
8	5	9 21 600						
10	a	180	b	−5	c	6.16	d	−4.176
	e	0.06	f	0.67	g	33 300	h	0.6565

Exercise 5-10

1 a	2.4×10^3	b	7.86×10^5	c	5.5×10^7
d	9.5×10^1	e	7.8×10^0	f	3.48×10^8
g	5.967×10^4	h	1.5×10^1	i	3×10^9

Answers

- j 8×10^1 k 7.63×10^2 l 1×10^1
 m 3.5×10^{-2} n 7.6×10^{-5} o 8×10^{-1}
 p 7.13×10^{-2} q 3×10^{-6} r 9.13×10^{-1}
 s 7.146×10^{-6} t 9×10^{-3} u 1×10^{-7}
 t 8.9×10^{-4} w 7.8×10^{-3} x 1×10^{-1}
 2 a 1.3×10^5 kg b 2.9×10^{-7} cm c 8×10^{-5} m
 d 3×10^8 m/s e 4×10^{13} km f 1.2×10^{-4} m
 g 6.1×10^2 cm h 1×10^{-10} m i 7.7×10^{20} s
 j 1×10^{-6} s k 2.2×10^6 ly
 3 a 600 000 b 7100
 c 302 000 000 d 3.14
 e 0.000 06 f 0.0071
 g 0.000 000 030 2 h 0.000 000 000 59
 i 1 100 000 000 000 j 0.0004
 k 5000 l 0.000 476
 m 0.803 n 63 200
 o 0.016 p 0.000 000 22
 q 9 000 000 r 0.111
 4 a 4×10^5 b 2.7×10^6 c 1.3×10^7
 d 6.3×10^{-7} e 9.3×10^9 f 9.3×10^2
 g 3.04×10^0 h 4.5×10^{-5} i 2×10^{-15}
 j 9.7×10^{-5}
 5 a $3.8 \times 10^9, 5.5 \times 10^9, 7.3 \times 10^9$
 b $5.8 \times 10^{-6}, 2.2 \times 10^{-4}, 7 \times 10^{-4}$
 c $3.5 \times 10^0, 4.9 \times 10^2, 5.3 \times 10^2$
 6 a $6 \times 10^5, 2.9 \times 10^2, 1 \times 10^2$
 b $6.3 \times 10^2, 8.1 \times 10^{-4}, 1.2 \times 10^{-9}$
 c $9.5 \times 10^{-1}, 4.1 \times 10^{-1}, 6.4 \times 10^{-3}$

Exercise 5-11

- 1 a 6×10^8 b 2×10^5 c 8×10^{15}
 d 3×10^6 e 2.4×10^{16} f 5×10^4
 g 1.024×10^{18} h 3.01×10^{-6} i 6.3×10^9
 j 2.5×10^{-11} k 2×10^{-3} l 3.81×10^{13}
 2 a 1.31×10^9 b 2.96×10^{13} c 2.84×10^{15}
 d 1.44×10^{16} e 6.99×10^{19} f 9.05×10^3
 g 2.00×10^{14} h 2.14×10^9
 3 3.78×10^{19} atoms 4 2.25×10^{-2} mm
 5 a 7.0×10^{30} b 1.3×10^{17} c 8.1×10^{13}
 d 2.3×10^{36} e 5.5×10^{50} f 5.2×10^{22}
 g 1.8×10^{-4} h 9.5×10^2 i 3.1×10^6
 j 9.5×10^{13} k 1.2×10^{-3} l 1.7×10^{-5}
 m 3.9×10^8 n 5.9×10^0
 6 a 500 s b 8 mins 20 s
 7 1.9×10^{14} t 8 1.7×10^0 km/s 9 9.5×10^{12} km
 10 a 0.000 1 s b 91 s
 11 a $9.999\,999\,999 \times 10^{99}$ b $1.000\,000\,000 \times 10^{-99}$

Power plus

- 1 a 4.382×10^{11} b 5.2×10^{-8} c 4×10^8
 d 2.013×10^0 e 5.78×10^4 f 5.78×10^{-2}
 g 6.7×10^{-6} h 3.2×10^9 i 3.2×10^{-9}

- 2 a 3 b 5 c 2 d 2
 e 3 f 2 g 10 h 10

3 Different answers possible.

	a	m	n	
a	4	1	2	$4^{\frac{1}{2}} = 2$
b	9	1	2	$9^{\frac{1}{2}} = 3$
c	16	3	2	$16^{\frac{3}{2}} = 64$
d	625	3	4	$625^{\frac{3}{4}} = 125$

- 4 If $a = b$ there are an infinite number of solutions, otherwise
 $a = 2, b = 4$
 5 a $2^n + 1$ where $n = 1, 2, 4, 8, \dots$ (doubling the power each time)
 b 4 294 967 297

Chapter 5 revision

- 1 a 10^{10} b 4^{16} c a^{10} d h^{10}
 e $12n^4$ f $2d^{12}$ g $5m^8$ h $6v^7w^5$
 i $15x^6y^3$ j $8t^4h^6$ k p^6q^8 l $20ab^2$
 2 a 2^6 b k^{25} c x^4
 d $1024y^{30}$ e $25t^4$ f $1000g^3$
 g -32 h $-32k^5$ i $25m^6$
 3 a a^4b^8 b $25x^6y^4$ c $64t^6$
 d $-64h^6g^3$ e $\frac{a^4}{2401}$ f $32p^5q^5r^5$
 g $\frac{243m^3}{32}$ h $81n^4p^8$ i $\frac{16a^{28}}{b^4}$
 j $512t^{14}u^{16}$ k $\frac{b^{18}y^{15}}{512}$ l $5c^4d^4$
 4 a 1 b 1 c 1 d 1 e -1
 f $\frac{1}{h}$ g 1 h 1 i $\frac{2}{3}$
 5 a $\frac{1}{8^3}$ b $\frac{1}{19^2}$ c $\frac{1}{x}$ d $\frac{1}{p^3}$
 e $\frac{1}{4m}$ f $\frac{1}{16m^2}$ g $\frac{1}{5b}$ h $\frac{2}{b}$
 i $\frac{-2}{x^4}$ j $\frac{5a}{3}$ k $\frac{d^2}{c^4}$ l $\frac{9}{100}$
 6 a 10^{-3} b r^{-5} c r^{-1} d $3b^{-1}$
 7 $\frac{9x^2}{64}$
 8 a $\sqrt[3]{q}$ b $\sqrt{u}$ c $\sqrt[3]{2q}$ d $\sqrt{ar}$
 9 a 16 b -8 c $\frac{1}{216}$
 10 a $625d^{20}$ b $2y^5$ c $4x^{\frac{16}{5}}$ d $-4n^4$
 11 a 8.6 b 15 700 c 500
 d 0.007 127 e 0.904 f 300 000
 g 4810 h 0.0009 i 70 000 000
 12 a 3.7×10^4 b 6.1×10^{-1} c 2.5×10^5
 d 4.9×10^{-4} e 1.3×10^1 f 8×10^{-11}
 13 a 8100 b 60 000 000 c 3.075
 d 0.0081 e 0.000 000 6 f 0.030 75
 14 $9.1 \times 10^{-8}, 2.4 \times 10^3, 3 \times 10^3$
 15 a 2.701×10^{31} b 4×10^5
 c 1.25×10^{17} d 2.5×10^{-4}
 16 a 9.8×10^{16} b 3.7×10^8 c 9.6×10^8

Chapter 6

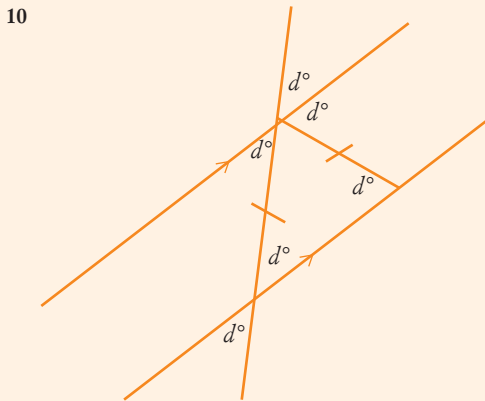
SkillCheck

- 1 a acute b right c reflex d revolution
e reflex f obtuse g reflex h straight
- 2 a $r = 135$ (angles at a point)
b $w = 55$ (vertically opposite angles)
c $c = 30$ (vertically opposite angles),
 $y = 150$ (angles on a straight line)
d $k = 26$ (angles in a right angle)
e $t = 35$ (angles on a straight line)
f $a = 140$ (angles at a point)
g $a = 47$ (vertically opposite angles), $b = 47$ (corresponding angles on parallel lines), $c = 47$ (vertically opposite angles) (other reasons possible)
h $k = 135$ (angles on a straight line), $h = 135$ (corresponding angles on parallel lines)
i $m = 115$ (corresponding angles on parallel lines),
 $n = 115$ (vertically opposite angles)
j $b = 104$ (angles on a straight line), $c = 104$ (corresponding angles on parallel lines), $d = 104$ (vertically opposite angles)
k $k = 125$ (vertically opposite angles),
 $a = 125$ (corresponding angles on parallel lines),
 $b = 125$ (vertically opposite angles)
l $x = m = 58$ (corresponding angles on parallel lines),
 $y = 58$ (vertically opposite angles), $n = 122$ (angles on a straight line) (other reasons possible)
- 3 a AB is not parallel to CD since alternate angles are not equal
b EF is not parallel to GH since co-interior angles are not supplementary
c IJ is not parallel to KL since corresponding angles are not equal

Exercise 6-01

- 1 a $m = 70$ b $w = 33$ c $h = 50$
d $k = 100$ e $x = 46$ f $a = 65$

- g $d = 70$ h $k = 70$ i $y = 34$
j $m = 60$ k $h = 150$ l $r = 45$
- 2 C
- 3 A, D
- 4 a 115 b 112 c 85
d 68 e 67.5 f 130
- 5 B
- 6 B
- 7 a 125° b 110° c 67°
d 50° e 60° f 56°
- 8 Teacher to check.
- 9 $a = 65$, $c = 50$, $b = 65$



- 10
- 11 a $t = 35$ b $m = 26$ c $h = 30$
d $y = 110$ e $g = 40$ f $x = 11$

Exercise 6-02

- 1 a $m = 74$ b $m = 96$ c $m = 30$
d $m = 30$ e $m = 221$ f $m = 75$
- 2 See table below.
- 3 a trapezium, parallelogram
b trapezium
c square, rhombus
d rectangle, square

Exercise 6-02

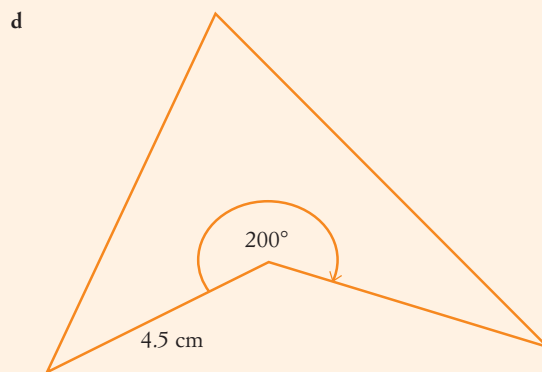
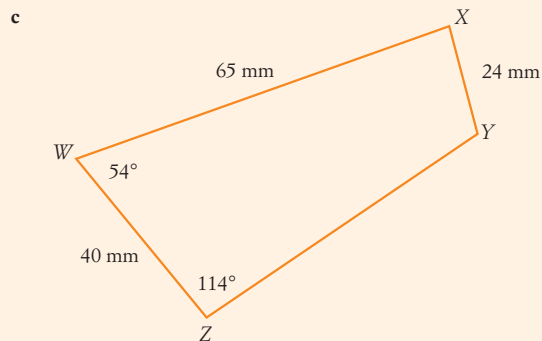
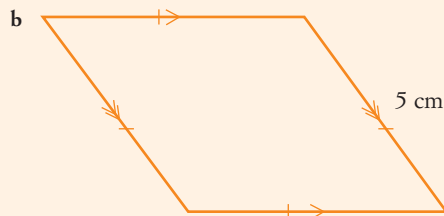
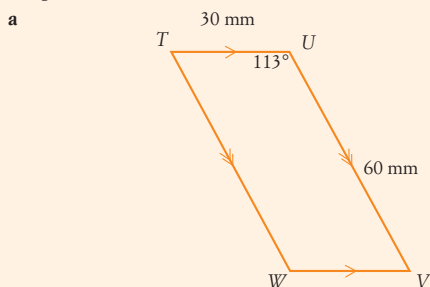
2

Properties	Trapezium	Parallelogram	Rectangle	Square	Rhombus	Kite
One pair of opposite sides parallel	✓					
Opposite sides parallel		✓	✓	✓	✓	
Opposite sides equal		✓	✓	✓	✓	
All sides equal				✓	✓	
Two pairs of adjacent sides equal						✓
Diagonals equal			✓	✓		
Diagonals bisect each other		✓	✓	✓	✓	
Diagonals meet at right angles				✓	✓	✓
Diagonals bisect the angles of the shape				✓	✓	
Opposite angles equal		✓	✓	✓	✓	
One pair of opposite angles equal						✓
All angles 90°			✓	✓		
Axes of symmetry	0	0	2	4	2	1

Answers

- e parallelogram, rectangle, square, rhombus
 f square
 g rhombus, square
 h kite
 i parallelogram, rectangle, square, rhombus
 j rectangle, square
 k rectangle, rhombus
 l parallelogram, rectangle, square, rhombus
 m parallelogram, rectangle, square, rhombus
 n rhombus, kite, square
- 4 a rhombus, parallelogram, trapezium
 b 9 rhombuses, 6 parallelograms, 18 trapeziums
 c 13
- 5 A, C, D, E, G, I
- 6 a parallelogram b rhombus c trapezium
 d square e rhombus f kite
 g rectangle h square i rhombus
- 7 a $k = 70$ (opposite angles of parallelogram are equal)
 b $a = 45$ (angles of square bisected by diagonals)
 c $p = 55$ (co-interior angles, opposite sides of rhombus parallel)
 d $w = 112$ (opposite angles of parallelogram are equal, then angles on a straight line)
 e $t = 55$ (angles of a rectangle equal 90°)
 f $n = 67$ (corresponding angles, opposite sides of a rhombus are parallel)
 g $a = 55$ (co-interior angles)
 h $c = 9$ (diagonals bisect angles of a square)
 i $r = 110$ (angles between unequal sides of a kite are equal)
- 8 a $\angle PQR = 72^\circ$ (alternate angles, $RW \parallel PV$)
 b $\angle TQU = \angle TUQ = 53^\circ$ (angles in an isosceles triangle)
 $\angle QUP = 53^\circ$ (alternate angles, $QT \parallel PU$)
 $\angle UPQ = \angle PQU = 63.5^\circ$ (angles in an isosceles triangle)
 $\therefore \angle PQR = 63.5^\circ$ (alternate angles, $RQ \parallel PU$)
 c $(30^\circ + \angle QRT) + 110^\circ = 180^\circ$ (co-interior angles, $QT \parallel PR$)
 $\angle QRT = 40^\circ$
 $\therefore \angle PQR = 40^\circ$ (alternate angles, $PQ \parallel RT$)
 d $\angle PRQ = 35^\circ$ (vertically opposite angles)
 $\therefore \angle PQR = 72.5^\circ$ (angles in isosceles $\triangle PQR$, diagonals of rectangle are equal and bisect each other)
 e $\angle QRP = 75^\circ$ (angle sum of straight line)
 $\angle QPR = 75^\circ$ (angles in isosceles triangle)
 $\therefore \angle PQR = 30^\circ$ (angle sum of triangle)
 f $\angle TPQ = 60^\circ$ (alternate angles, $RT \parallel PQ$)
 $\therefore \angle PQR = 30^\circ$ (angle sum of triangle, diagonals of rhombus meet at 90°)

9 sample answers



Exercise 6-03

1 a 6	b 4	c 9
d 10	e 7	f 5
g 12	h 8	i 11
2 a non-regular hexagon	b quadrilateral	
c non-regular pentagon	d scalene triangle	
e regular decagon	f non-regular heptagon	
g regular octagon	h non-regular nonagon	
3 a a, b, d, e, f, g	b e, g	
4 D		
5 a equilateral triangle	b square	
6 a 540°	b 1080°	c 2340°
d 900°	e 1440°	f 3240°
7 a $a = 95$	b $b = 110$	c $d = 108$
d $c = 160$	e $g = 140$	f $x = 90$
g $e = 135$	h $m = 120, n = 60$	i $y = 222$
8 a 14	b 11	c 24
d 17		
9 B		

- 10 a 135° b 144° c 150° d 120°
 11 a 40 b 171°
 12 a 24 b 36 c 10

Exercise 6-04

- 1 a 45° b 36° c 24° d 40°
 2 a 108° b 150° c 140° d 157.5°
 3 a 15 b 10 c 9
 d 36 e 20 f 6
 4 a 12 b 72 c 20
 d 9 e 5 f 90

Mental skills 6

- 2 a 0.5 b 90 c 8 d 0.9
 e 90 f 0.7 g 8 h 30
 i 0.08 j 3.5 k 4 l 0.3
 4 a 16 b 160 c 1.6 d 1.6
 e 1.6 f 16 g 160 h 0.16
 i 0.016 j 160 k 0.16 l 0.0016

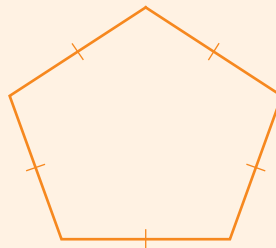
Power plus

- 1 a 2 b 20 c 54
 2 a 0, 2, 5, 9, 14, 20, 27, 35 b $d = \frac{n(n-3)}{2}$
 3 a square, rhombus
 b parallelogram, rhombus, rectangle, square
 c kite, rhombus, square
 d rectangle, square
 e rhombus, square
 f square, rectangle
 4 Some answers are abbreviated to fully show the reasons required to reach the final answer. Other answers possible.
 a $h = 113$ (vertically opposite angles)
 $m = 67$ (co-interior angles on parallel lines)
 $p = 67$ (vertically opposite angles)
 $k = 67$ (corresponding angles)
 b $w = 73$ (exterior angle of triangle is equal to the sum of the two opposite interior angles)
 c $e = 97$ (co-interior angles on parallel lines, then alternate angles on parallel lines)
 d $y = 61$ (angle sum of a triangle, then corresponding angles on parallel lines)
 e $f = 56$ (alternate angles on parallel lines, then angle sum of isosceles triangle)
 f $x = 78$ (alternate angles, then exterior angle of triangle result)
 g $t = 34$ (alternate angles on parallel lines)
 $n = 117$ (opposite angles of parallelogram)
 $v = 29$ (angle sum of a triangle)
 h $a = 90$ (diagonals of rhombus meet at right angles)
 $d = 65$ (alternate angles on parallel lines, then angle sum of a triangle)
 i $c = 153$ (angle sum of quadrilateral, then angles at a point)

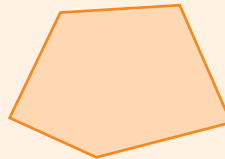
Chapter 6 revision

- 1 a $k = 47$ (angle sum of triangle)
 b $e = 75$ (exterior angle of a triangle)
 c $x = 24$ (equal angles in an isosceles triangle)
 d $m = 18$ (angle sum of triangle)
 e $t = 107.5$ (equal angles in an isosceles triangle, exterior angle of a triangle)
 f $b = 60$ (equilateral triangle, alternate angles on parallel lines)
 g $y = 140$ (exterior angle of a triangle)
 h $y = 48$ (angles on a straight line, isosceles triangle)
 i $y = 78$ (co-interior angles on parallel lines)
 2 a 60° b $90^\circ, 45^\circ, 45^\circ$
 3 a $x = 110$ (angle sum of quadrilateral)
 b $y = 112$ (corresponding angles on parallel lines),
 $m = 68$ (co-interior angles on parallel lines)
 c $g = 60$ (angles on a straight line, isosceles triangle)
 d $x = 35, y = 55$ (alternate angles on parallel lines, diagonals of a rhombus bisect at right angles, angle sum of a triangle)
 e $m = 97$ (one pair of opposite angles equal in a kite, angle sum of a quadrilateral)
 f $e = 36$ (co-interior angles on parallel lines)
 4 a parallelogram, rectangle, square, rhombus
 b square, rectangle
 c square, rhombus
 d square, rectangle, rhombus, parallelogram

5 a



b



c



- 6 a $(10 - 2) \times 180 = 1440^\circ$ b 140°
 7 a 60° b 12

Chapter 7

SkillCheck

- 1 a $x = 5$ b $y = 8\frac{1}{2}$ c $y = 1\frac{1}{2}$ d $m = 32$
 e $x = 33$ f $r = 6$ g $x = -7\frac{1}{2}$ h $a = -3$
 i $d = -6\frac{1}{2}$ j $w = 2$ k $a = 48$ l $n = 6$
 2 a $7n$ b n^2 c $5(n+8)$
 d $n-20$ e $6n-9$ f $n+2$

Exercise 7-01

- 1 a $w = 6$ b $q = 6$ c $x = 5$ d $n = -2$
 e $y = 10$ f $m = 2$ g $a = 6$ h $x = -1$
 i $y = 0$ j $u = -1$ k $x = -1\frac{3}{4}$ l $x = -5$
 2 a B b C
 3 a $w = -2$ b $t = -1\frac{1}{2}$ c $a = 1$
 d $y = -3$ e $y = -1\frac{1}{2}$ f $y = \frac{3}{5}$
 g $t = 1\frac{3}{8}$ h $y = 2\frac{1}{2}$ i $k = 1\frac{2}{3}$
 4 C

Exercise 7-02

- 1 a $m = 1$ b $x = 2$ c $y = -1$ d $k = 4$
 e $a = -1$ f $p = \frac{1}{9}$ g $h = -24$ h $m = 9$
 i $u = 11$ j $y = 1\frac{3}{7}$ k $p = -1\frac{1}{5}$ l $x = 1\frac{13}{14}$
 2 A
 3 LHS = $12(5-1)$
 $= 12 \times 4$
 $= 48$
 $= \text{RHS}$
 $\therefore k = 5$ is a solution
 4 LHS = RHS = 16
 5 a $m = 3$ b $y = 7$ c $x = 2$
 d $p = 4$ e $n = -4$ f $x = 3\frac{1}{3}$
 g $w = -8$ h $x = 6$ i $y = \frac{5}{9}$
 6 LHS = RHS = 30
 7 a $m = -7$ b $y = 7$ c $y = -10$ d $x = 12$
 e $y = -26$ f $y = -1\frac{2}{3}$ g $m = -7$ h $y = \frac{71}{89}$

Exercise 7-03

- 1 11 2 12 3 24 4 30 5 40 cm $\times$ 10 cm
 6 a $4w + 14 = 94$ b $27 \text{ cm} \times 20 \text{ cm}$
 7 $x = 33$
 8 a $y = 13$ b 110°
 9 $82^\circ, 98^\circ$ 10 43, 44 11 28, 29, 30 12 54, 56, 58
 13 23, 25, 27 14 Dean is 10, Franco is 50, Helen is 2.
 15 Rebecca is 14. 16 78 17 \$1.80
 18 Boy: 140 cm, sister: 70 cm, father: 170 cm.
 19 $26.5 \text{ cm} \times 17 \text{ cm}$
 20 Brett is 17, Paul is 51, Amanda is 12, Janine is 45.

Exercise 7-04

- 1 a $a = 39$ b $x = 56$ c $x = 30$ d $y = 42$
 e $p = 56$ f $y = 21$ g $x = -2$ h $y = -15$
 i $x = -9$ j $d = 85$ k $w = -4$ l $p = -7.5$
 2 a $x = -9$ b $y = 22$ c $p = 11.5$ d $w = -22$
 e $x = 5.75$ f $m = 24$ g $y = 5.2$ h $x = 5$
 i $x = 0$ j $w = 1\frac{5}{22}$ k $x = 13$ l $x = 3\frac{1}{7}$
 3 A
 4 a $d = 24$ b $k = 18$ c $m = 12$
 d $x = 21\frac{9}{11}$ e $p = 30$ f $k = 20$
 g $k = 30$ h $m = 10$ i $a = 6\frac{9}{11}$
 5 B
 6 a $x = \frac{7}{11}$ b $p = 4\frac{1}{3}$ c $m = 12\frac{4}{5}$
 d $c = -9\frac{2}{3}$ e $y = 5\frac{3}{4}$ f $x = -8$
 g $p = -2\frac{1}{9}$ h $y = 7\frac{9}{14}$ i $a = 28\frac{1}{3}$
 j $x = -\frac{13}{15}$ k $u = 9\frac{1}{5}$ l $m = \frac{19}{29}$

Mental skills 7A

- 2 a 14 b 9 c 7 d 8
 e 3 f 8 g 10 h 5
 i 33 j 5 k 15 l 18
 m 26 n 6 o 36 p 6
 q 14 r 160 s 14 t 2

Exercise 7-05

- 1 a $m = \pm 12$ b $x = \pm 20$ c $y = \pm 15$
 d $k = \pm\sqrt{59}$ e $y = \pm\sqrt{10}$ f $w = \pm 6$
 g $x = \pm 5$ h $t = \pm 3$ i $a = \pm 4$
 j $k = \pm\sqrt{8}$ k $w = \pm\sqrt{10}$ l $d = \pm 12$
 m $k = \pm 4$ n $w = \pm\sqrt{70}$ o $x = \pm\frac{1}{2}$
 p $m = \pm 9$ q $y = \pm\sqrt{5}$ r $p = \pm 3$
 s $k = \pm 4$ t $y = \pm\sqrt{55}$ u $x = \pm\sqrt{7}$
 2 a $m = \pm 4.5$ b $b = \pm 4.1$ c $v = \pm 2.4$ d $p = \pm 4.2$
 e $k = \pm 2.6$ f $x = \pm 6.3$ g $k = \pm 9.8$ h $u = \pm 1.7$
 i $y = \pm 3.7$ j $w = \pm 5.2$ k $a = \pm 4.1$ l $y = \pm 6.2$
 3 $\sqrt{-25}$ doesn't exist for real numbers; there isn't a real number whose square is -25 .
 4 a, c and f. They all involve square roots of negative numbers.

Exercise 7-06

- 1 a $r = 1$ b $k = 6$ c $d = -3$
 d $x = \sqrt[3]{-45}$ e $w = \sqrt[3]{100}$ f $f = 4$
 g $m = 2$ h $t = -5$ i $c = 6$
 j $k = -4$ k $a = \sqrt[3]{15}$ l $p = \sqrt[3]{-18}$
 m $q = 5.5$ n $n = -\frac{12}{25}$ o $s = \sqrt[3]{18.75}$
 p $e = -2$ q $y = \sqrt[3]{-6\frac{2}{3}}$ r $v = 8$
 s $b = \sqrt[3]{15}$ t $z = \sqrt[3]{-72}$ u $x = \sqrt[3]{36}$
 2 a $c \approx 3.6$ b $g \approx 5.3$ c $y \approx -7.7$ d $p \approx -4.5$
 e $u \approx 4.3$ f $x \approx -3.8$ g $h \approx -7.9$ h $a \approx 3.7$
 i $d \approx -3.1$ j $v = -3.5$ k $a \approx 6.4$ l $j \approx -3.9$

- 3 a Yes b When c is positive.
c Never; it always has exactly one solution.

Exercise 7-07

- 1 a $y = 28$ b $y = 11$ c $b = 25$
d $b = -1$ e $x = 3$ f $x = 16$
2 a 95°F b 14°F c 60.8°F
3 a 37.8°C b -42.8°C c 25.6°C
4 a $A = 72\text{ cm}^2$ b $h = 20\text{ cm}$ c $p = 32.5\text{ m}$
5 C
6 a \$540 b \$4500 c 6 hours d 9 hours
7 a \$260 b 507
8 a \$200 b \$825 c 45 people
9 D
10 a 65°C b 91.25°C c 4 hours
11 a 182 cm b 39 cm

Exercise 7-08

- 1 a $x = d - c$ b $x = \frac{y-b}{m}$
c $x = \frac{p+y}{a}$ d $x = my - mp$ or $m(y - p)$
e $x = \frac{k}{mr}$ f $x = \frac{m}{p}$
g $x = 2A - y$ h $x = \frac{c-bv}{a}$
i $x = \frac{2A}{h} - y$
2 D
3 a $y = \frac{m-an}{a}$ or $\frac{m}{a} - n$ b $y = \pm\sqrt{x}$
c $y = \frac{x}{k}$ d $y = (Q - P)^2$
e $y = \pm\sqrt{r^2 - x^2}$ f $y = \pm\sqrt{\frac{M}{n}}$
g $y = \frac{r^2}{m^2}$ h $y = \frac{c^2-b}{2a}$
i $y = r^2 + x$
4 a $h = \frac{V}{lb}$ b $r = \pm\sqrt{\frac{A}{\pi}}$ c $V = \pm\sqrt{\frac{2K}{m}}$
d $C = \frac{5}{9}(F - 32)$ e $a = \frac{2(s-ut)}{t^2}$ f $y = \frac{2A}{h} - x$
5 C
6 a $a = \frac{5q}{1-5p}$ b $a = \frac{3b}{r-3}$ c $a = \frac{D}{D+2}$
d $a = \frac{-pv}{p-x}$ or $\frac{pv}{x-p}$ e $a = \frac{1-k}{k+1}$
f $a = \frac{-b(M+N)}{M-N}$ or $\frac{b(M+N)}{N-M}$
7 a \$290 b $n = \frac{C-80}{4.2}$ c 76 km
8 a 900° b $n = \frac{A}{180} + 2$ or $\frac{A+360}{180}$ c 10
9 $r = \frac{sx}{s-x}$
10 a $h = \sqrt{\frac{m}{B}}$ (h is positive) b 1.55

Mental skills 7B

- 2 a 11 b 40 c 7 d 24
e 23 f 6 g 43 h 80
i 18 j 15 k 40 l 65
m 11 n 14 o 12 p 135

Power plus

- 1 a $y = \frac{5}{28}$ b $r = 3.5$ c $y = \pm\sqrt{48}$ or $\pm 4\sqrt{3}$
d $m = \pm 5$ e $m = 12\frac{5}{17}$

- 2 a $W = \frac{1}{5}$ b $W = -3$ c $X = -6\frac{1}{2}$ d $Y = -\frac{2}{3}$
3 26

4 Daughter is 20.

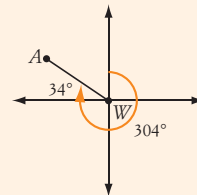
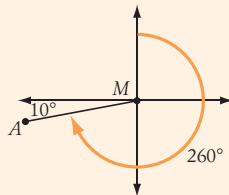
- 5 a If $x^2 + y^2 = 4$, then x^2 , which is always positive or zero, must be from 0 and 4. So the smallest value of x must be $-\sqrt{4} = -2$ and its largest value must be $\sqrt{4} = 2$.
b If $x^2 + y^2 = 4$, then y^2 must also be from 0 and 4. So y must be from -2 and 2 .

Chapter 7 revision

- 1 a $x = 1$ b $x = -5$ c $w = 1\frac{1}{3}$
d $x = 3$ e $n = -4$ f $d = -9$
g $t = 1\frac{1}{2}$ h $j = -8.5$ i $q = -1$
2 Teacher to check.
3 a $w = 7$ b $n = 1$ c $p = -1$
d $x = \frac{1}{3}$ e $y = 5$ f $x = 1.5$
4 $22\text{ cm} \times 16\text{ cm}$
5 a $x = 15.5$ b $a = 46$
6 1
7 a $d = -6$ b $n = -1.5$ c $p = 24$
8 a $x = 11\frac{5}{7}$ b $n = -10\frac{1}{4}$ c $y = 5\frac{3}{4}$
9 a $d = \pm 8$ b $p = \pm 6$ c $z = \pm\sqrt{35}$
10 a $x = 11$ b $h = -6$ c $t = 5$
11 a BMI = 21.0 b 105.84 kg
12 a \$41 b $d = 30.5\text{ km}$
13 a $w = \frac{a-v}{k}$ b $w = t - \frac{p}{m}$ c $w = \frac{1}{x-y}$

Mixed revision 2

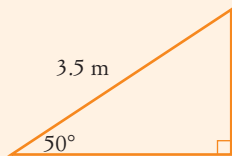
- 1 a 4 b 1 c 4 d 3
2 a y^{10} b $12g^{15}$ c $18p^5q^3$
d k^{12} e x^4 f h^9
g $9k^4$ h h^2y^3 i $\frac{2m^4n}{3}$
3 a 6 b 1 c 1 d x e 1
4 a



- 5 a 120° b decagon
6 $y = \pm\sqrt{\frac{T-a}{p}}$
7 a $\frac{1}{2}$ b $\frac{125}{125}$ c $\frac{100}{16d^2}$ d $\frac{1000x^3}{27}$
8 a 2 b $\frac{9}{64}$ c 3.31
9 a 20.50 b 33.34 c 3^2p^{14} d 2^3w^{15}
10 a 2^{15} b 4^{12} c 0.9837 d 24 000
11 a 15 400 000 b 0.0032
12 212 km
13 a $77^\circ 53'$ b $64^\circ 9'$ c $56^\circ 19'$
14 a n^{-1} b 2^{-3} c k^{-5} d $7y^{-2}$
15 7.28×10^6

Answers

- 16 a 2.7×10^{16} b 1.2×10^{-10}
 17 a 39° b 52° c 44°
 18 a $13^{\frac{1}{2}}$ b $20^{\frac{1}{3}}$ c $(3p)^{\frac{1}{5}}$ d $x^{\frac{3}{2}}$
 19 a $m = 103$ (angle sum of triangle)
 b $f = 54$ (equal angles of isosceles triangle, angle sum of triangle)
 c $d = 132$ (exterior angle of triangle)
 20 A
 21 a $m = 17.9$ b $d = 11.7$ c $g = 160.2$
 22 a 5.3×10^9 b 3.73×10^5
 23 a $w = 103$ b $w = 32$ c $w = 35$
 24 C
 25 a 125 b $\frac{1}{32}$ c 8 d 9
 26 a 24 b 8
 27 a $16m^{12}n^8$ b $\frac{4h^2}{25}$ c $343r^3t^6$
 28 a $k = -5$ b $h = 8\frac{4}{7}$
 29 a 40° b 37° c 39°
 30 a 225 min or 3.75 h b 3.125 kg
 31 a



- b i 2.7 m ii 2.2 m
 32 square
 33 a $p = 11$ b $y = 1$
 34 a 44 000 b 10.8 c 0.000 04
 35 a 19, 20, 21, 22 b 127 \$2 coins
 36 50 m
 37 a $m = \pm 4$ b $x = \pm 6$
 38 a $r = 8$ b $y = -9$

Chapter 8

SkillCheck

- 1 0.082 2 \$2.25
 3 a 365 b 52 c 28 d 36
 e 14 f 13 g 60 h 26
 4 a \$7.68 b \$21.42 c \$524.74
 d \$92.36 e \$3085.64 f \$106.34
 5 a \$733.59 b \$6285.83 c \$3150
 d \$1198.21 e \$694.67 f \$14 911.22
 g \$1339.02 h \$2343.50 i \$2293.20
 6 \$6513

Exercise 8-01

- 1 a \$649.60 b \$947.10 c \$735.20 d \$285.30
 2 \$45 866.22
 3 D 4 \$54 177.24
 5 a \$1309.70 b \$2619.39 c \$5695

- 6 a \$98.25 b \$109.83 c \$99.60
 7 Dana, by \$1.26.
 8 fortnightly income
 9 a Mitch by \$1.40/h b Philip by \$0.45/h
 c Faith by \$3.90/h d Ranjan by \$0.90/h
 10 a \$187 b \$935 c \$4051.67
 11 a \$103 680 b \$1986.97
 12 a \$3226.67 b \$742.05 c \$17.67
 13 \$744 14 \$630.63 15 \$330.15
 16 \$470.34

Exercise 8-02

- 1 a \$494.20 b \$591.28 c \$750.13
 d \$763.36 e \$953.10 f \$979.58
 2 C 3 \$623.48 4 \$939.04
 5 \$804.65 6 \$707.61 7 B
 8 a \$518.10 b \$573.05 c \$667.25
 d \$808.55 e \$631.93 f \$730.05
 9 a \$556.80 b \$1165 c \$911.40
 d \$1040 e \$1560 f \$897.90
 10 a \$19.60/h b \$19.59/h c \$17.14/h
 d \$13.65/h e \$16.73/h f \$8.27/h

Exercise 8-03

- 1 \$558 2 \$1459.22 3 \$10 979
 4 Perry: \$1600, Amy: \$2250 5 \$820 000
 6 a \$8300 b \$9820 c \$11 400
 7 \$4554
 8 a \$163.80 b \$90 c \$172.05 d \$188
 e \$307.80 f \$217.60 g \$162 h \$1568
 9 a i \$124.10 ii \$172.55 b 540
 c i 236 ii 730
 10 \$156.80
 11 a \$96.88 b \$142.10 c \$457.10
 12 a \$734 b \$122.33 c \$14.40/h
 13 a \$2014 b \$3116 c \$2704.65
 d \$1961.18 e \$3847.50 f \$3897.66
 14 \$10 440
 15 a \$536 b \$375.20 c \$2519.20
 16 a \$2376.39 b \$831.74 c \$5584.52
 17 a \$632 b \$2970.40

Exercise 8-04

- 1 \$71 634
 2 a \$34 673 b \$3129.87
 3 a \$89 525 b \$21 071.25
 4 C
 5 a \$55 262 b \$9507.15
 6 a \$91 340.60 b \$90 113 c \$21 288.81
 7 a \$45 896 b \$6463.20
 8 A 9 \$13 559.25 10 \$10 449.33

Mental skills 8

- 2 a \$408 b 99 c 200
 d \$404 e 672 f \$81
 g \$517 h 225 i 560
 j \$84 k \$350 l 84
 4 a 330 b \$240 c 1600 d \$425
 e \$225 f 60 g \$63 h 76
 i \$68 j \$3762 k \$374 l \$100

Exercise 8-05

- 1 a \$3120 b \$672
 2 a \$8652 b \$1848
 3 \$240
 4 a \$42 b 21.5% c \$153
 5 \$198–\$201
 6 Phoebe, by \$24.
 7 Cinema: \$864, Electronics store: \$894.
 Electronics store offers higher net pay.
 8 a \$130 b \$363.90 c 39.1%
 9 a \$812.03 b 27.9%
 10 a \$670.98 b 16.4%
 11 a \$2943.66 b \$635.83 c \$887.45 d \$2056.21
 12 a \$519.12 b \$4092.60 c \$87 856.28 d 28.3%
 13 a gross weekly income = \$638.40, total deductions = \$277.80, net weekly income = \$360.60
 b gross weekly income = \$525, total deductions = \$396.90, net weekly income = \$128.10
 14 a \$251.70 b 24.5%

Power plus

- 1 \$92.28/min 2 \$55.56 3 \$128/h
 4 Felicity: \$514; Rose: \$454.80; Difference: \$59.20
 5 \$1368 6 \$21.50/h 7 \$855
 8 \$841.50 9 \$125 800

Chapter 8 revision

- 1 a \$1100.23 b \$2200.46 c \$4784.17
 2 \$505.05
 3 \$790.79
 4 a \$645.75 b \$763.88
 5 \$4200
 6 \$2250
 7 a \$75 b \$822.50 c \$997.50
 8 \$11 625
 9 \$4117.20
 10 a \$139 454 b \$39 544.98
 11 a \$182 b \$1208.96 c 12.2%
 12 a \$1071.60 b \$459.60 c \$612

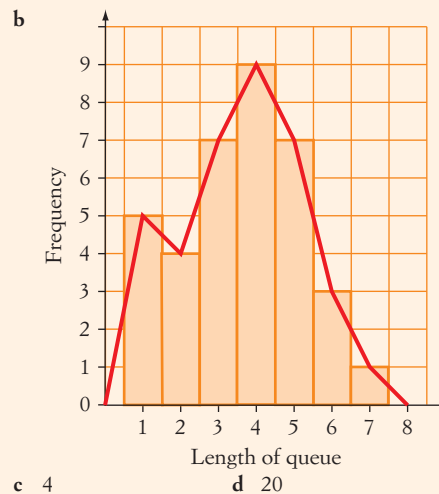
Chapter 9

SkillCheck

- 1 a 34 b 8 c 55.9%
 2 a 38 b 6 c 36 d 14 e 39.5%

3 a

Length of queue (Score)	Frequency
1	5
2	4
3	7
4	9
5	7
6	3
7	1



c 4

d 20

Exercise 9-01

- 1 a i 18.2 ii 18 iii 15 iv 16
 b i 6.9 ii 7 iii 7 iv 14
 c i 35.7 ii 38 iii 32, 45 iv 34
 d i 8.7 ii 8.8 iii 8.8 iv 1
 e i 0.3 ii -0.5 iii -2 iv 11
 2 a max $\bar{x}$ = 22.9°C, min $\bar{x}$ = 15.6°C b 7.3°
 c max mode = 23°C, min mode = 15°C d 8°
 3 a 415 mm b 168 mm c 92 mm
 4 76
 5 a 370 b 462 c 92
 6 No, as she needs to score 103%.
 7 a 6.3 b 7 c 7 d 8
 8 a 1 is an outlier as it is away from the main body of data.
 b 8, 9 c 7.3 d 8
 e i 56.7% ii 33.3%
 9 a Yes, 7
 b i $\bar{x}$ = 2, median = 2 ii $\bar{x}$ = 2.3, median = 2
 c The mean.
 10 a \$180 000
 b i $\bar{x}$ = \$86 944, median = \$90 000
 ii $\bar{x}$ = \$96 250, median = \$91 500

Answers

c The median, as it is only \$1500 higher with the outlier included, while the mean is \$9306 higher.

d i mode = \$93 000, range = \$35 000

ii mode = \$93 000, range = \$110 000

e Range

11 a 6.4 b 14.8 c 66.8

12 a 10, 9, 28, 30, 24 $\Sigma f = 25$, $\Sigma fx = 101$, $\bar{x} = 4.04$

b 66, 184, 168, 225, 130, 108 $\Sigma f = 36$, $\Sigma fx = 881$, $\bar{x} = 24.47$

13 a 34.4 b 12.9

14 a 2, 7, 18, 25, 35, 44, 50 i 5.5 ii 4 iii 6

b 3, 11, 31, 46, 57, 64 i 20 ii 19 iii 5

15 a

Number of seedlings per punnet	Frequency
4	4
5	2
6	4
7	8
8	11
9	8
10	4
11	3
12	1

b 8

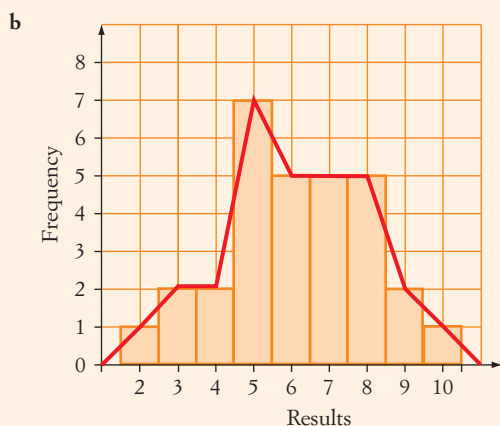
c 8

d 7.8

Exercise 9-02

1 a

Results (Score)	Frequency
2	1
3	2
4	2
5	7
6	5
7	5
8	5
9	2
10	1



c 6.1

d 6

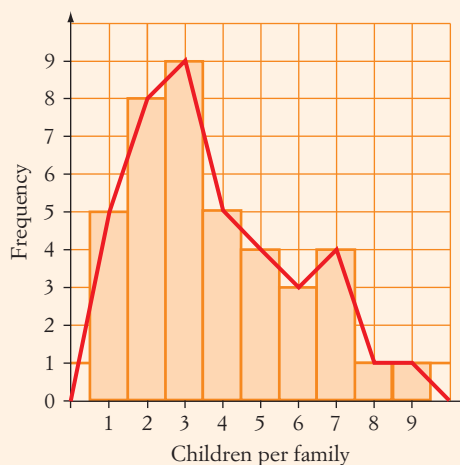
e 5

f 8

g No outliers but clustering from 5–8.

2 a

Children per family	Frequency
1	5
2	8
3	9
4	5
5	4
6	3
7	4
8	1
9	1



b mode = 3, range = 8

c no outliers, clustered near 2–3 children

d 3 e 3.8

3 a 6 It's the highest column.

b 5.96 c 6 d 5

4 a 73 b 14

c 2 It's the score with the greatest frequency.

d 5 e 2

f 2.43 g clustering at 2–3 calls per minute

5 a

Stem	Leaf
4	0 1 5
5	1 3 5 6 8
6	3 4 5 6 7 7 7 8
7	1 2 4 7 9
8	2 7 8
9	0 2 4

b 54 c Clustering occurs in the 60s, no outliers.

d 27 e 67.85 f 67

6 a 52 b 68 c 60.0 d 63

7 a

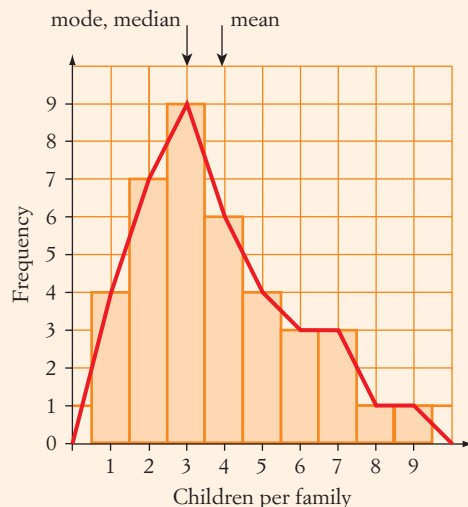
Stem	Leaf
11	8 9
12	1 5 6
13	1 5 5 7
14	3 4 5 6 9 9
15	0 0 0 1 4 4 7 7
16	1 1 3 3
17	6
18	3 7

- b i no outliers
ii clustering occurs near 14–15 seconds
- c 8 d 70% e i 43.3% ii 50%
- 8 a 54, 55 or 56 b 22 c 59.5
- 9 a Rockets: 8, Blues: 13 b Rockets: 52, Blues: 57
c The Blues, 40.
d The Blues performed better as they had more games in which they scored more than 30 points.
e The Blues' median was higher – 37.5 compared to Rockets' median of 27.5.
- 10 a
- | Robyn | Stem | Anthony |
|-------------|------|---------------|
| 7 | 1 | |
| 7 3 | 2 | 8 9 |
| 7 7 7 6 5 3 | 3 | 4 5 5 6 6 7 9 |
| 5 2 2 1 | 4 | 2 3 7 |
| 4 | 5 | 1 5 |
- b Anthony (547) sold more than Robyn (506).
c 7 d Robyn, 54
e Clustering is in the 30s for both.
f Anthony
- 11 a
- | Stem | Leaf |
|------|---------------|
| 1 | 1 5 9 |
| 2 | 3 5 7 |
| 3 | 1 2 3 4 5 6 8 |
| 4 | 0 2 5 5 7 8 |
| 5 | 2 5 |
| 6 | 5 |
| 12 | 5 |
- b Yes, 12.5 – this score is away from the main body of data.
c Clustering occurs in the 3–4 km distances.
d 4.01 e 3.6
f The median is the better as it is not affected by the outlier score of 12.5 km.
- 12 a
- | Girls | Stem | Boys |
|-----------|------|---------|
| 8 | 1 | 0 6 |
| | 2 | 6 8 9 |
| 9 6 | 3 | 2 |
| 8 5 | 4 | 0 |
| 9 9 6 1 | 5 | 0 0 5 7 |
| 7 0 | 6 | 3 7 |
| 8 6 6 4 3 | 7 | 2 4 5 6 |
| 5 2 0 | 8 | 2 8 |
| 4 | 9 | 5 |
- b 95, boy c i 63.5 ii 56
d 45% e 50%
f i 62.8 ii 54.25
g i no ii no

- c i negatively skewed
ii no outliers, clustering in the 70s and 80s
- d i positively skewed
ii no outlier, clustering near 21, 22
- e i not symmetrical, bimodal
ii no outliers, some clustering near 13, 16
- f i positively skewed
ii no outliers, clustering near 1, 2
- g i symmetrical
ii no outliers, some clustering near 51
- h i negatively skewed
ii 10 is an outlier, clustering near 18
- 2 a 36 b in the 30s and 40s
c positively skewed
d Test was too hard or students did not prepare for the test.
e $\bar{x} = 48.6$, median = 45, mode is 28, 38, 45, 47, 64

3 a

Children per family	Frequency
1	4
2	7
3	9
4	6
5	4
6	3
7	3
8	1
9	1



- b No c positively skewed d 2 to 4
e mode = 3, median = 3, $\bar{x} = 3.8$
- 4 C
- 5 a symmetrical, bimodal b 56, 64, 83
c 76 d 72.8 e 72
f No, because the range has been affected by the outlier of 26. Without the outlier, the range is 45.

Exercise 9-03

- 1 a i symmetrical and bimodal
ii no outliers, some clustering near 5, 11
b i symmetrical and bimodal
ii 10 is an outlier, clustering near 4, 6

Answers

6 a



- b Sydney: $\bar{x} = 21.75$, median = 22, mode = 26, range = 10
 Melbourne: $\bar{x} = 19.8$, median = 20, modes = 17, 20, 24, 26, range = 13
- c Sydney is generally warmer – higher mean, median and lower range.

f Men, as their mean and median are less than that for women.

g Men. Their range is 29 compared to a range of 23 for women.

6 Teacher to check.

Mental skills 9

2 a 8	b 0.3	c \$12	d 0.8
e \$48	f 1.2	g 5.6	h \$18
i \$3	j \$7.20	k 77	l 26
4 a 12.5	b \$4.50	c \$10.80	d 9.9
e \$15	f 4.8	g \$30	h \$90
i \$14	j \$7.20	k 5.7	l \$10.50

Exercise 9-04

- 1 a B: 59.58 s R: 57.54 s b B: 59.60 s R: 57.33 s
 c B: 1.71 s R: 4.55 s
 d Ryan, his mean and median time is lower.
 e Brooke, as her range is 1.71 s compared to Ryan's range of 4.55 s.
- 2 a S: 13.1°C A: 13.3°C b S: 27.9°C A: 26.6°C
 c S: 29.2°C A: 26.2°C d 8 days
 e Sydney. Its mean and median were higher.
- 3 a Test 1: 59 Test 2: 67
 b Test 1: 58.3 Test 2: 68.0
 c Test 1: 59 Test 2: 54
 d Test 1 is positively skewed. Test 2 is slightly positively skewed.
 e Yes, the mean and median have increased significantly.
- 4 a Logan City
 b L: 27.25°C S: 26.13°C
 c L: 11°C S: 12°C
 d Logan City, slightly lower range and temperatures clustered from 26°C to 28°C.
 e L: 27°C S: 26°C
 f Sydney, its mean and median are slightly lower and it had a temperature of 26°C or more on only 8 days compared to Logan City's 14 days.
- 5 a
- | Men | Stem | Women |
|---------------|------|---------------------|
| 9 9 9 8 6 | 2 | |
| 8 7 2 2 1 1 0 | 3 | 3 4 5 5 6 6 7 7 9 9 |
| 9 8 7 4 3 | 4 | 0 0 2 3 5 5 8 |
| 5 3 | 5 | 1 1 4 7 |
- b No, the range is too high.
 c i The mean and median for men is different ($\bar{x} = 37.2$, median = 32.5).
 ii The mean and median for women is also different ($\bar{x} = 42.2$, median = 40), but the difference is less.
 d Men: 29 Women: 35, 36, 37, 38, 40, 45, 51
 e Both are positively skewed. Clustering occurs in 30s for both.

Exercise 9-05

- 1 a census b sample c sample
 d census e census
- 2 a every 5 years
 b 2016, 2021, 2026, ... – every year ending in a 1 or 6
- 3 a 872 b i 17.3% ii 15.1%
 c i 17 ii 15
- 4 No, as these students may live close to school or they may be driven to school rather than use public transport.
- 5 No, as they were shoppers, and from a particular shopping centre.
- 6 a N b N c N d C e N
 f N g N h C i C j N
- 7 B
- 8 a D b C c C d C
 e D f D g C h D

Exercise 9-06

- 1 Teacher to check.
- 2 a People at a football match would not be thinking and/or interested in the amount of time spent on homework.
 b Many sports are/have body contact to some degree.
 c Most of the cars at the car show will use petrol.
 d People at a rock concert will usually favour rock music.
- 3 Advantages – high accessibility as most people have a telephone
 – good quality control as questions are asked the same way
 – anonymity of people being surveyed
 – data can be processed quickly
- Disadvantages – time taken for survey is limited so questions are restricted to short answer type ones
 – people who screen calls may not answer when called
 – if the survey is about a product, participants cannot see, taste or feel the product
- 4 Teacher to check.
 a Depends on the number of students in the school. For a large school, more students need to be surveyed.

- b Can be given out at random to students in a school, may only take a short time to compete.
- c May take too much time to complete, some of the questions may require a lengthy response.
- d Personal interview – students are asked at random at recess or lunch.
- 5 a Students from each year group are chosen at random (stratified sample).
- b Choose students at random as they leave the canteen.
- c, d Choose students from each year group (stratified sample).
- e Randomly select some classes from your school or from different schools in NSW.
- f Telephone survey or personal interview in a shopping centre.
- 6 C as all students in the school are asked. Answer A means only year 9 students are asked and Answer B would only provide an estimate.
- 7 a The question is not precise enough. A better question might be:
'Circle the amount of time you spend studying per week:
0 – less than 2 hours 2 – less than 4 hours 4 hours or more'
- b The question is too general. A better question could be:
'Select your favourite sport from the list below' and include 'Other' in the list.
- c The question is too general. A better question could be:
'What is your income?
a \$15 000 – <\$25 000 b \$25 000 – < \$40 000
c \$40 000 – < \$60 000
- d, e The question is too general. It should include:
'Select from the list below' and include 'Other' in the list.
- f, g The question is not precise enough. A better question might be:
'Circle the amount of time you spend exercising/
on the Internet per day or week:
0 – less than 1 hours 1 hour – less than 2 hours
2 hours or less than 4 hours more than 4 hours'
- h The question is too general. A better one might be:
'How many text messages do you send in one day?
☐ 0 – 10 ☐ 11 – 20 ☐ 21 – 40 ☐ more than 40
- 8 Teacher to check.

Power plus

1 a

Class interval	Frequency	Cumulative frequency
16–21	3	3
22–27	8	11
28–33	7	18
34–39	7	25
40–45	7	32
46–51	2	34

- b Shape is nearly symmetrical or slightly positively skewed. No outliers or clusters.

c 22–27 d 28–33

2 If the range is too large.

3 a i

Stem	Seedlings
0	3 4 4 5 7 7 8
1	1 1 2 3 4 4 6 6 7 7 7 8 8 8 8 9 9 9
2	0 0 3 3 4 4 5 5 6 7 8 8
3	0 1 3

ii Stem Seedlings

0 ⁰	3 4 4
0 ⁵	5 7 7 8
1 ⁰	1 1 2 3 4 4
1 ⁵	6 6 7 7 7 8 8 8 8 9 9 9
2 ⁰	0 0 3 3 4 4
2 ¹	5 5 6 7 8 8
3 ⁰	0 1 3

- b i Positively skewed, no outliers, clustered in the 10s stem.

ii Symmetrical, no outliers, clustered in 15–19 stem.

- c Plot ii gives the better indication. It gives a better breakdown of how many seedlings grow per punnet (modal group is 15–19).

Chapter 9 revision

1 $\bar{x} = 7$, median = 8, mode = 8, range = 6

2 a \$525 100 b \$393 500

- c The median is the better measure as it is not affected by the high price of \$1 750 000.

If \$1 750 000 is excluded, the mean is \$389 000.

- d Because the mean is affected by either very high or very low prices (outliers).

3 a Stem Leaf

2	2
3	8
4	2 3 5 7
5	4 6 8 9
6	2 3 4 4 7
7	0 0 1 2 3 5 7 8 8
8	3 4 5 6 9
9	0

- b $\bar{x} = 65.5$, median = 68.5, mode = 64, 70 and 78, range = 68

c Yes, 22

- d The median, as the mean is affected by the outlier 22.

4 a 6 b 1 c 1.93 d 2

- e The mean as there are no outliers.

Answers

5 a	Northbridge	Stem	Westvale
	9 9 7	0	5 7 7 8 9
	8 6 6 5 1	1	3 5 5 8 8
	8 7 7 3 2	2	2 4 4 4 4 5 6
	8 7 7 5 2 0 0	3	3 5 6 7 7
	7 5 2	4	3 4
	3	5	

- b Northbridge: 27.25 min, 27.5 min, 46 min
Westvale: 22.875 min, 24 min, 39 min
- c No outliers, some clustering for Northbridge in the 30s and for Westvale in the 20s.
- d The waiting times for Westvale are less than for Northbridge and more consistent.
- 6 a no symmetry or skewness, no outliers, clustering at 12, 14.
b symmetrical, no outliers, clustering near 100 – 109.
c negatively skewed, no outliers, clustering near 15 – 17.

7 a	Girls	Stem	Boys
	8 1	13	4
	8 7 5 2	14	8
	6 5 3 0 0	15	0 4
	3 2 0	16	4 4 5
		17	0 1 5 6 6 8
		18	0

- b i Nearly symmetrical ii negatively skewed
c No outliers for girls, 134 is an outlier for the boys.
d Girls: 150, 150; Boys: 164.64, 167.5
e For the boys, the median is the better measure. For the girls, both measures are good.
f Girls
g Boys. Their range is 46 cm compared to the range for girls, which is 32 cm.
- 8 a sample b sample c census d sample
- 9 a numerical, continuous b numerical, discrete
c categorical d categorical
e numerical, continuous
- 10 a Not all teens read *Dolly* magazine.
b Only those diners interested/volunteering to complete the survey are sampled.
c Need to sample workers as well, not only management who may have a biased view.
d People at a premiere are not the usual moviegoers.
e It is restricted to one radio station playing a specific type of music.
- 11 a The question is too general and many responses could be given. Instead, give choices such as always, most of the time, sometimes, half the time, not often, rarely, never.
b The question is too general. Give choices such as excellent, good, satisfactory, poor.
c The question encourages bias towards saying yes because people have died for the flag. A better question might be 'Do you think Australia's flag should be changed?' with a Yes/No response and then a choice of reasons such as
☐ Australia is independent of Great Britain
☐ Australia is a multicultural nation
☐ Australia is 'part' of Asia
☐ Other

Chapter 10

SkillCheck

- 1 a i 88 cm ii 384 cm²
b i 112 cm ii 336 cm²
c i 37.70 cm ii 113.10 cm²
d i 70 cm ii 200 cm²
e i 84 cm ii 336 cm²
f i 94.25 cm ii 706.86 cm²
- 2 a 34 000 mm b 9.25 m c 1.5 km
d 3.75 L e 380 mm f 7 500 mL
- 3 a $\frac{1}{2}$ b $\frac{1}{4}$ c $\frac{1}{3}$ d $\frac{23}{36}$
- 4 a 180 cm³ b 2197 cm³ c 1440 cm³ d 4500 cm³
- 5 a i 5
ii Three rectangles and two right triangles
iii Right triangular prism
b i 8 ii Six rectangles and two hexagons
iii Hexagonal prism
c i 6 ii Four rectangles and two trapeziums
iii Trapezoidal prism
- 6 a 41 b 12 c 3.9

Exercise 10-01

- 1 a cm b t c km
d L e kL f hour
- 2 a 0.75 mg b 36 000 L c 67 000 g
d 1500 mL e 67 mm f 3500 m
g 9400 kg h 3750 mg i 425 cm
j 200 g k 26 200 mL l 300 m
m 3000 000 mL n 5760 min o 54 000 s
p 270 000 cm q 5 000 000 mg r 700 mm
s 29 375 000 g t 18 140 000 mm
- 3 B
- 4 a B b B c A d C e B
- 5 a 8 kL b 9 cm c 3 min d 4 h
e 0.375 L f 14.7 t g 8.6 g h 7.5 m
i 22.3 cm j 0.8 kg k 0.125 kL l 0.0035 t
m 17 kg n 5 days o 45 t p 40 h
q 1 days r 0.025 km s 0.075 kL t 34.7 m
- 6 B
- 7 a 4800 kB b 2100 GB c 8500 MB
d 10 800 000 B e 2.91 kB f 0.74 GB
g 1.05 MB h 5.9 TB i 0.000 94 MB
j 57 000 MB k 430 000 kB l 21 000 GB
m 7800 B n 2.18 GB o 0.042 MB
p 940 000 kB
- 8 3600 kB 9 4324 files
- 10 a 520 000 000 = 5.2×10^8 km b 6 s
c 750 y d 0.012 m
e 6.4×10^8 m f 2.0×10^{13} km
g 70 ms h 250 000 y
i 4.8 s j 5290 Mm
k 6.2×10^{12} km l 6000 μ m
m 940 millennia n 8.5 centuries
o 85 ly p 80 nm

Exercise 10-02

- 1 a i 1 mm ii ± 0.5 mm
 b i 1°C ii $\pm 0.5^\circ\text{C}$
 c i 1 min ii ± 0.5 min = ± 30 s
 d i 100 g ii ± 50 g
 e i 1 cm ii ± 0.5 cm = ± 5 mm
 f i 1° ii $\pm 0.5^\circ$
 g i 5 km/h ii ± 2.5 km/h
 h i 250 g ii ± 125 g
 i i 10 mL ii ± 5 mL
 j i 1 kL ii ± 0.5 kL = ± 500 L
 k i $\frac{1}{4}$ tank ii $\pm \frac{1}{8}$ tank
- 2 D
- 3 a 1 cm b ± 0.5 cm
 c ± 0.5 mm d 26.75 to 26.85 mm

Exercise 10-03

- 1 a i 48 mm ii 110 mm²
 b i 30 m ii 26 m²
 c i 340 m ii 4200 m²
 d i 52 cm ii 126 cm²
 e i 92 cm ii 516 cm²
 f i 136 cm ii 864 cm²
- 2 a 150 m² b 51 m² c 88 mm²
 d 156 m² e 210 m² f 36 cm²
 g 189 cm² h 1000 cm² i 71 m²
 j 201.19 m² k 88 000 cm²
- 3 B
- 4 a 38.72 cm² b 27.2 cm²

Exercise 10-04

- 1 a 48 cm² b 54.4 m² c 340 mm²
 d 3 cm² e 84 cm² f 950 mm²
 g 280 mm² h 20 m² i 91.52 cm²
 2 a 300 mm² b 172.5 mm² c 90 mm²
 d 360 mm² e 120 mm² f 72 mm²
 g 66.5 mm² h 510 mm² i 112 mm²
 3 a 84 m² b 52.25 m² c 800 mm²
 d 190 m² e 54 cm² f 3.15 m²
 g 84 cm² h 144.6 m² i 840 mm²
 4 a 46 m² b 148 cm² c 24 m²
 d 168 cm² e 100.8 cm² f 29.04 m²
 5 a 160 m² b 141 m² c 7.8 m²

Mental skills 10A

- 2 a 19 b \$7.50 c 87.5 d \$20.20
 e \$3.76 f 40 g \$0.925 h 89.6
 i \$270 j \$0.38 k \$152.76 l \$7.25
 m 315.4 n \$1.07 o 42.6 p \$2431.76
- 4 a 10 b 124 c 490 d \$1.72
 e 14.4 f \$2540 g 78 h \$1.16
 i \$9 j \$16.80 k \$920 l 64

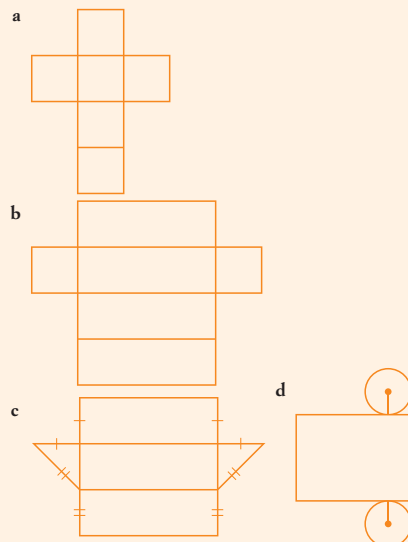
- 6 a 100 b \$0.60 c 2.5 d \$1.35
 e \$1.84 f \$4.20 g 40 h 6.5
 i \$0.48 j \$6.90 k \$3.60 l 42

Exercise 10-05

- 1 a i 50.27 cm ii 201.06 cm²
 b i 32.67 cm ii 84.95 cm²
 c i 94.25 cm ii 706.86 cm²
 d i 88.59 cm ii 624.58 cm²
- 2 a i 92.5 m ii 508.9 m²
 b i 35.7 m ii 78.5 m²
 c i 4.1 m ii 1.0 m²
 d i 136.0 m ii 1055.6 m²
 e i 221.5 m ii 2565.9 m²
 f i 15.1 m ii 9.4 m²
 g i 17.4 m ii 5.6 m²
 h i 303.9 m ii 4618.1 m²
- 3 a 182.83 cm b 72.85 cm c 121.12 mm
 d 129.12 m e 554.20 cm f 1416.67 m
 g 103.98 m h 46.57 m i 56.55 cm
- 4 a 2228 m² b 293 m² c 952 m² d 442 m²
 e 63 m² f 71 m² g 503 m² h 151 m²
 i 193 m² j 743 m² k 4664 m² l 471 m²
- 5 a 20 056 m² b \$4001
- 6 a 40 074 km b 47 614 km
- 7 1.2 m²
- 8 a 34.56 m² b \$2592
- 9 4.9 m
- 10 a difference = 15.708 cm b extra = 157 m
- 11 a 39 m² b 156% increase
- 12 a Zoe 399.3 m b Jaron 405.6 m c 6.3 m

Exercise 10-06

- 1 C
 2



Answers

- 3 a 253.5 m^2 b 4140 cm^2 c $103\,680 \text{ cm}^2$
 d 125.36 cm^2 e 9600 mm^2 f 9680 cm^2
 4 a 408 m^2 b 348 m^2 c 2310 cm^2
 d 71.4 cm^2 e 360 m^2 f 648 m^2
 5 a rectangular prism b trapezoidal prism
 c cube d triangular prism
 e square pyramid f cylinder
 g triangular prism h rectangular prism
 6 a $145\,140 \text{ cm}^2$ b $\$174.17$
 7 D
 8 a $h = 8.9 \text{ m}$ b 71.2 m^2 c 862.4 m^2
 9 a cube $A = 2646 \text{ m}^2$
 b rectangular prism, $A = 684 \text{ m}^2$
 c right triangular prism, $A = 9720 \text{ m}^2$
 d trapezoidal prism, $A = 6378 \text{ m}^2$
 10 a 2804 cm^2 b $19\,440 \text{ cm}^2$ c 3740 cm^2
 11 a 151.33 m^2 b 251.33 m^2

Exercise 10-07

- 1 a 747.70 m^2 b 2550.97 cm^2
 c 206.47 m^2 d 15.83 m^2
 2 a 904.8 cm^2 b 8545.1 mm^2
 3 a 2159.8 m^2 b 3.9 m^2 c 1894.5 m^2
 d 6.5 m^2 e 14.4 m^2 f 1131.0 m^2
 4 a $A = 56.5 \text{ m}^2$ b 7 L of paint
 5 The triangular prism tent has 7.6 m^2 more surface area.
 6 2073.5 cm^2

Exercise 10-08

- 1 a 135 cm^3 b 64 m^3 c 6120 mm^3
 d 1.728 m^3 e $21\,952 \text{ mm}^3$ f 258.06 cm^3
 2 a 504.8 m^3 b 1045 cm^3 c $19\,968 \text{ cm}^3$
 3 a 1133.0 cm^3 b $17\,671.5 \text{ mm}^3$ c 1.8 m^3
 d 1606.0 cm^3 e $17\,583.1 \text{ cm}^3$ f $301\,593.0 \text{ mm}^3$
 g 500.0 m^3
 4 a 9.7 m^3 b $94\,247.8 \text{ m}^3$ c 7.4 m^3
 d 135.7 m^3 e 5026.5 m^3 f 216.9 m^3
 g 42.3 m^3 h 107.5 m^3 i 146.3 m^3
 5 C
 6 a $306\,000 \text{ cm}^3 = 0.306 \text{ m}^3$
 b $l = 0.90 \text{ m}$, $w = 0.40 \text{ m}$, $h = 0.85 \text{ m}$, $V = 0.306 \text{ m}^3$
 It is easier to convert each measurement to the required units before doing the calculation.
 7 5 m 8 23.76 L 9 678.58 cm^3
 10 7 m 11 $13\,666 \text{ cm}^3$ 12 300 kL

Mental skills 10B

- 2 a 18 b $\$126$ c 39 d $\$30.30$
 e $\$7.50$ f 10.8 g $\$27$ h 60
 i $\$240$ j $\$3.30$ k 900 l $\$52.50$
 4 a 10 b 166 c $\$50$ d $\$22$
 e 37.5 f $\$5.80$ g 135 h $\$22.60$

- 6 a 500 b $\$20$ c 4.5 d $\$6.25$
 e $\$81$ f $\$35$ g 16.5 h 74.5
 i $\$195$ j $\$425$ k $\$31.50$ l 290
 8 a 160 b $\$1.50$ c 7.5 d $\$32.50$
 e $\$67.50$ f $\$31.25$ g 38 h 170

Exercise 10-09

- 1 a 149.3 cm^3 b 240 cm^3 c 120 cm^3
 d 336 m^3 e 1200 cm^3 f 106.7 m^3
 2 B
 3 a i 24 cm ii 3200 cm^3 iii 3200 mL or 3.2 L
 b i 20 m ii $19\,200 \text{ m}^3$ iii $19\,200 \text{ kL}$
 c i 40 mm ii 4320 mm^3 iii 4.32 mL
 d i 60 mm ii $28\,160 \text{ mm}^3$ iii 28.16 mL
 e i 7.7 m ii 133.06 m^3 iii 133.06 kL
 f i 84 cm ii $564\,480 \text{ cm}^3$ iii $56\,448 \text{ mL}$ or 56.448 L
 4 a $2\,592\,100 \text{ m}^3$ b 1.127 m^3
 5 27 m 6 76 mm 7 4.9 cm
 8 a 209 m^3 b 872 cm^3 c 1272 mm^3
 d 616 cm^3 e 393 cm^3 f 2545 mm^3
 9 a 116.1 cm^3 b $13\,796.9 \text{ m}^3$ c 377.8 cm^3
 d 2.4 m^3 e $296\,162.1 \text{ m}^3$ f $129\,638.4 \text{ cm}^3$
 10 9.7 cm
 11 8.8 m
 12 5.23 mm

Power plus

- 1 a 157.08 cm b 50π
 2 a 70.83 cm b 125.66 m^2
 3 60 cm c 628 m^2
 5 a i $A = 52.3 \text{ cm}^2$ ii $P = 33.4 \text{ cm}$
 b i $A = 121.1 \text{ cm}^2$ ii $P = 46.6 \text{ cm}$
 c i $A = 185.1 \text{ cm}^2$ ii $P = 50.7 \text{ cm}$
 6 $V = 160 \text{ cm}^3$
 7 side length = 15 cm, volume = 3375 cm^3
 8 40 mm

Chapter 10 revision

- 1 a 8000 m b 900 s c 700 mm
 d 250 kg e 8400 MB f 9500 kg
 g 5 h h 34 kB i $125\,000 \text{ L}$
 j 0.025 km k $17\,000 \text{ t}$ l 8.9 TB
 m 10.5 s n 9100 y o $2\,600\,000 \text{ m}$
 p $2.838 \times 10^{13} \text{ km}$ q $0.4 \mu\text{s}$ r 75 m
 2 a i 1 mm ii $\pm 0.5 \text{ mm}$
 b i 1° ii $\pm 0.5^\circ$
 c i 1 min or 1 s ii $\pm 0.5 \text{ min} = \pm 30 \text{ s}$, or $\pm 0.5 \text{ s}$
 d i 10 km/h ii $\pm 5 \text{ km/h}$
 3 a 432 mm^2 b 64 m^2 c 16 cm^2 d 2340 mm^2

- 4 a 52 cm^2 b 468 mm^2 c 72 m^2
d 198 mm^2 e 120 m^2 f 260 cm^2
- 5 a i $P = 57.3 \text{ m}$ ii $A = 205.3 \text{ m}^2$
b i $P = 7.7 \text{ m}$ ii $A = 3.5 \text{ m}^2$
c i 54.8 cm ii 56.5 cm^2
- 6 a 14.14 m^2 b 162.62 mm^2 c 326.73 cm^2
7 a 61.44 m^2 b 152 m^2 c 760 m^2 d 2256 m^2
8 a 7389.03 m^2 b 1437.28 mm^2 c 104.33 cm^2
9 a 32.768 m^3 b 176 m^3 c 960 m^3 d 5040 m^3
10 a $48\,490.48 \text{ m}^3$ b 4064.44 mm^3 c 109.93 cm^3
11 123.75 L
12 $10\,800 \text{ cm}^3$
13 a 323 m^3 b 540 cm^3
14 a 1340.4 cm^3 b 733.0 mm^3
15 10.7 cm

Mixed revision 3

- 1 a 12.5 h b 1.35 L c 0.512 GB
d 3005 g e 2700 m f 250 kB
- 2 a $\$82.80$ b 560
3 a $\$63\,720$ b $\$12\,256$
- 4 a

Shoe size	Frequency
5	5
6	8
7	6
8	5
9	3
10	2
11	1

b Shoe sizes



- c 63.3% d mode = 6, range = 6
e 7 f 7.1
- 5 a $\$1076.27$ b $\$2152.55$ c $\$4680$
6 a $\$752.40$ b $\$526.68$ c $\$3536.28$
7 a i 46, 48 ii 48.6 iii 48 iv 6
b i 70 ii 70.7 iii 70 iv 10
- 8 a numerical, discrete b categorical
c numerical, continuous d numerical, continuous

- 9 a Perimeter = 72 cm , Area = 212 cm^2
b Perimeter = 66 cm , Area = 231 cm^2
- 10 a $\$1206.50$ b 55.3%
- 11 C 12 $\$904.80$ 13 $\pm 125 \text{ g}$
- 14 a 200 mm^2 b 240 mm^2 c 30 cm^2
- 15 $\$14\,218.75$
- 16 a Positively skewed b 2 and 3
c mode = 3, range = 8
- 17 a 158 m^2 b 360 cm^2
- 18 a 120 m^3 b 300 cm^3
- 19 a i 20.6 m ii 25.1 m^2
b i 20.5 m ii 26.2 m^2
c i 28.6 m ii 44.6 m^2
- 20 a 'People at a park' does not represent everybody, as employed people are less likely to be at the park.
b Not enough information obtained as responses will just be Yes or No.
c Which of the following applies to you?
☐ going to school/TAFE/University ☐ working part time
☐ working full time ☐ retired
☐ carer for children and/or other person
☐ on unemployment benefits
- 21 $24\,700 \text{ mm}^3$ 22 4710 mm^2
- 23 a Mon: positively skewed Wed: almost symmetrical
b Mon: 8.79 Wed: 9.58
c Mon: 8.5 Wed: 9.5
d Monday
e Wednesday
- 24 8 cm

Chapter 11

SkillCheck

- 1 a
- | | | | | | |
|---|----|----|---|----|----|
| x | -2 | -1 | 0 | 1 | 2 |
| y | 5 | 3 | 1 | -1 | -3 |
- b
- | | | | | | |
|---|----|---------------|---|----------------|---|
| x | -2 | -1 | 0 | 1 | 2 |
| y | 0 | $\frac{1}{2}$ | 1 | $1\frac{1}{2}$ | 2 |
- 2 a $y = 5$ b $y = -1$ c $x = 2\frac{1}{2}$ d $x = -1$
3 a 2 b 2 c 8.5

Exercise 11-01

- 1 a 7.3 b 7.2 c 8.2
d 14.3 e 7.1 f 14.3
- 2 a i $\sqrt{45}$ or $3\sqrt{5}$ ii 6.7
b i $\sqrt{117}$ ii 10.8
c i $\sqrt{128}$ or $8\sqrt{2}$ ii 11.3
d i $\sqrt{68}$ or $2\sqrt{17}$ ii 8.2
- 3 a 7.071 b 5 c 6.403 d 18.474
4 a 4.5 b 10.8 c 11.3 d 5 e 24.1
f 13 g 11.4 h 2.8 i 17.2
- 5 A

Answers

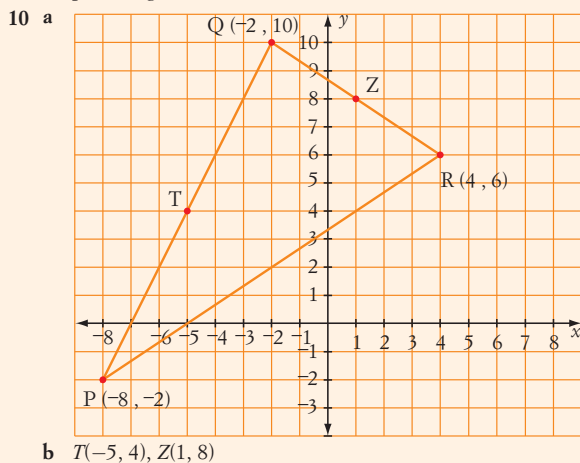
- 6 a $RS = \sqrt{20}$, $ST = \sqrt{8}$, $RT = \sqrt{20}$
 b isosceles triangle
 7 a trapezium b $P = 28$ units c $A = 38$ sq units
 8 a square b $\sqrt{52}$ or $2\sqrt{13}$ units c 28.8 units
 d 52 sq units

Exercise 11-02

- 1 A
 2 a (4, 4) b (5, 3) c (1, 1)
 3 a (8, 7) b (7, 8) c $(4\frac{1}{2}, 4)$
 d (2, 3) e $(4, 4\frac{1}{2})$ f $(7, \frac{1}{2})$
 g $(\frac{1}{2}, 3\frac{1}{2})$ h $(-3, -\frac{1}{2})$ i $(-1, -6\frac{1}{2})$
 j $(-1, -1)$ k (7, -8) l $(-5\frac{1}{2}, -6)$
 4 a $P(2, 3)$ b $Q(-1, 1)$ c $R(5, 5)$
 5 D
 6 a i $(-\frac{1}{2}, -1)$ ii $(-\frac{1}{2}, -1)$
 b They are the same point. The diagonals bisect each other.
 7 a $M(\frac{1}{2}, 4\frac{1}{2})$
 b i $\sqrt{50}$ or $5\sqrt{2}$ ii $\sqrt{45}$ iii $\sqrt{17}$
 c i 7.5 units² ii $P = 15.3$ units
 8 a $2\sqrt{26}$ units b $2\sqrt{26}$ units c Yes
 d (2, 1) e (2, 1)
 f Yes, their midpoints are the same.

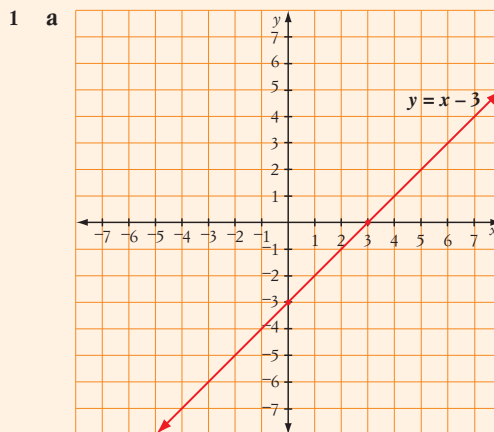
Exercise 11-03

- 1 a + b - c -
 2 a 2 b $-\frac{1}{4}$ c -5
 3 a $-\frac{1}{2}$ b 5 c $\frac{2}{3}$ d 7
 e $-\frac{1}{6}$ f $\frac{5}{2}$ g $-\frac{7}{2}$
 4 C
 5 a 1 b $\frac{1}{3}$ c -5
 d $-\frac{1}{2}$ e 0 f $-\frac{1}{2}$
 g $\frac{7}{4}$ h $\frac{1}{2}$ i 0
 6 A 7 k: $\frac{1}{5}$, l: $-\frac{1}{2}$ 8 B
 9 a $m_{PQ} = -\frac{1}{2}$, $m_{QR} = \frac{5}{2}$, $m_{RS} = -\frac{1}{2}$, $m_{PS} = \frac{5}{2}$
 b QR and PS , PQ and RS . They are parallel.
 c parallelogram

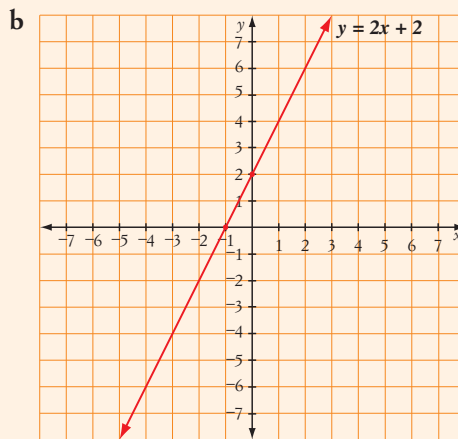


- c Both gradients are $\frac{2}{5}$.
 d $TZ = 2\sqrt{13}$ units, $PR = 4\sqrt{13}$ units, PR is double the length of TZ .

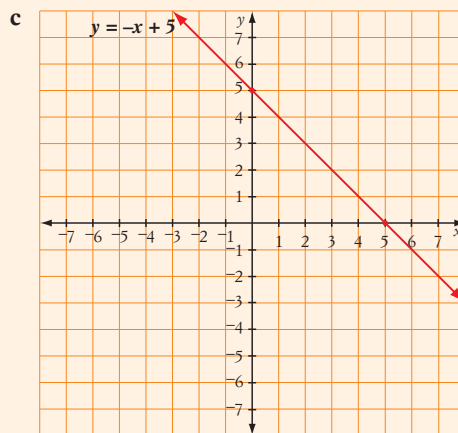
Exercise 11-04



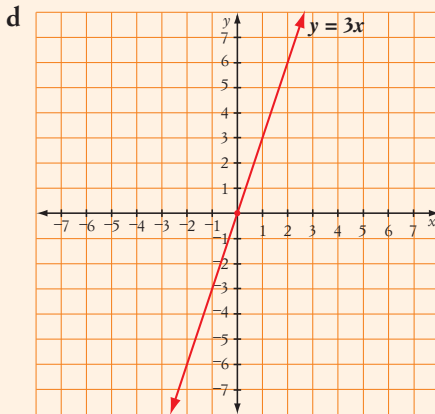
- i 3 ii -3



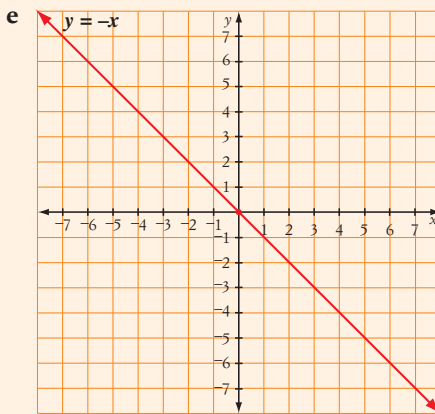
- i -1 ii 2



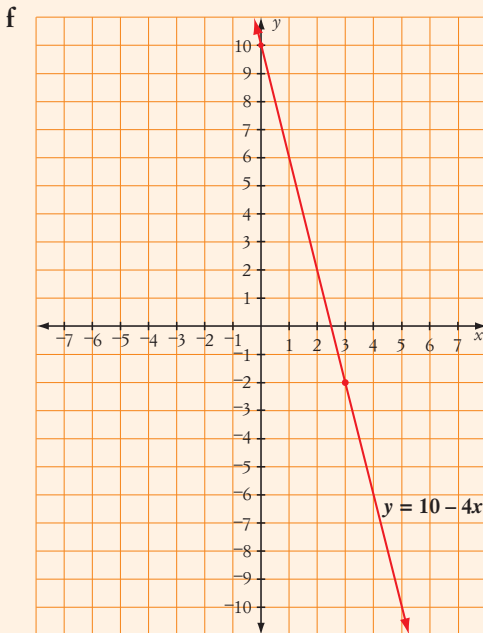
- i 5 ii 5



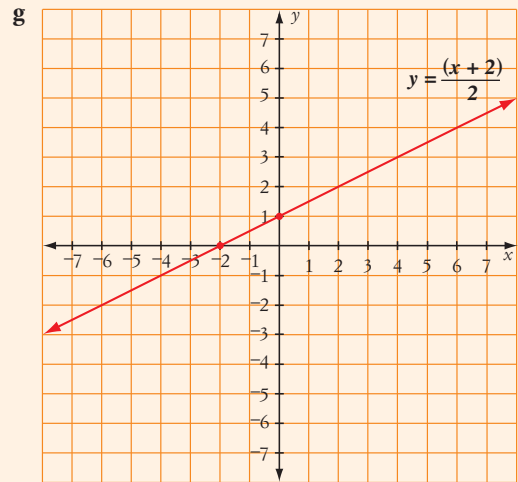
i 0 ii 0



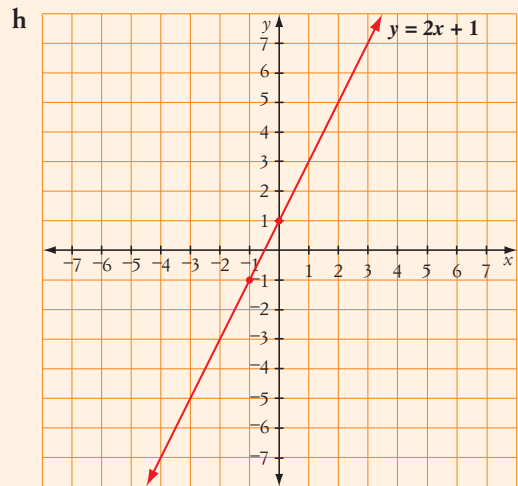
i 0 ii 0



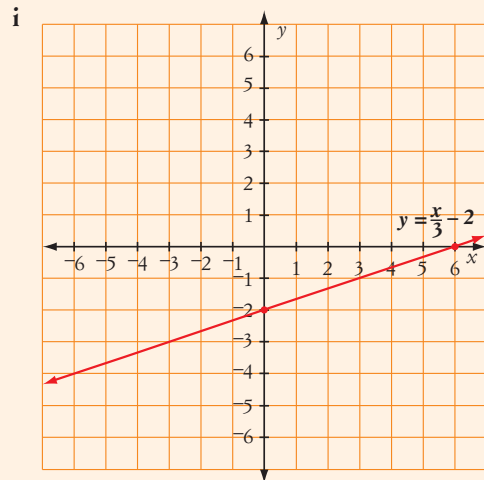
i $2\frac{1}{2}$ ii 10



i -2 ii 1



i $-\frac{1}{2}$ ii 1

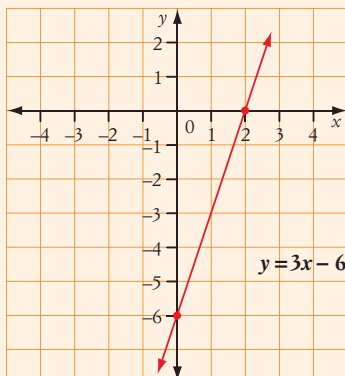


i 6 ii -2

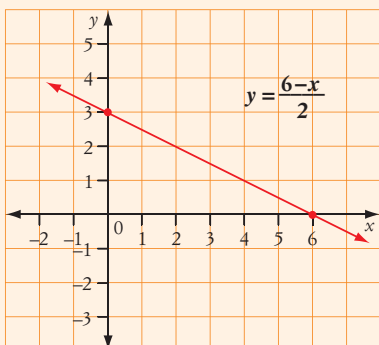
Answers

2 B

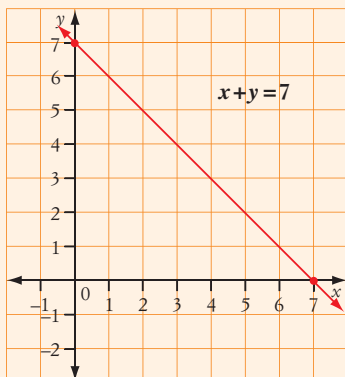
3 a



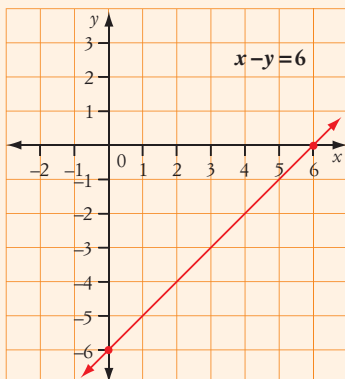
b



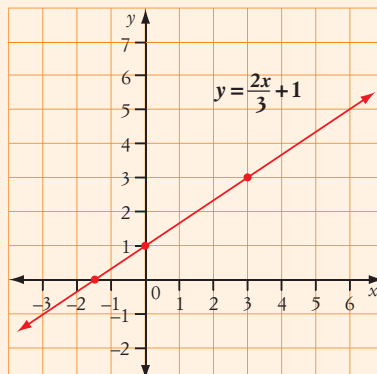
c



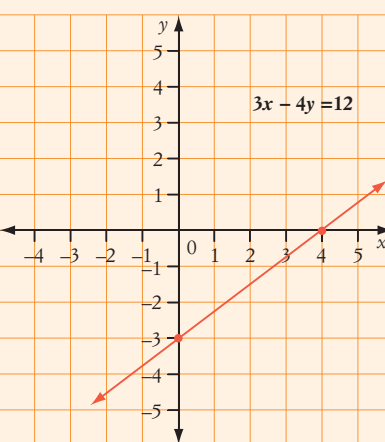
d



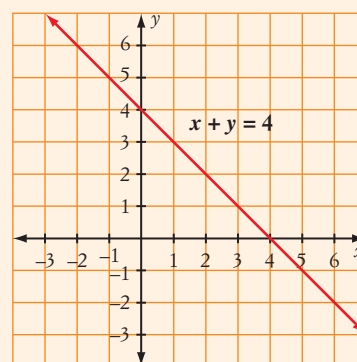
e



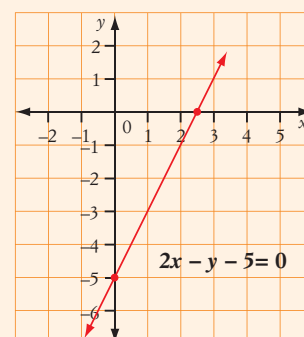
f

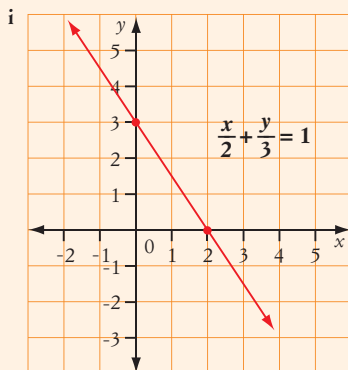


g



h





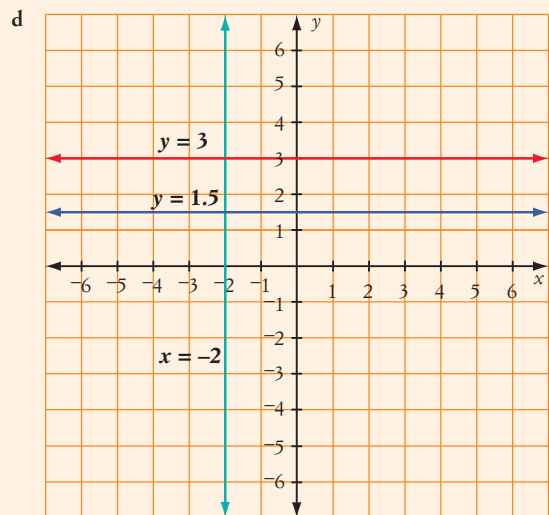
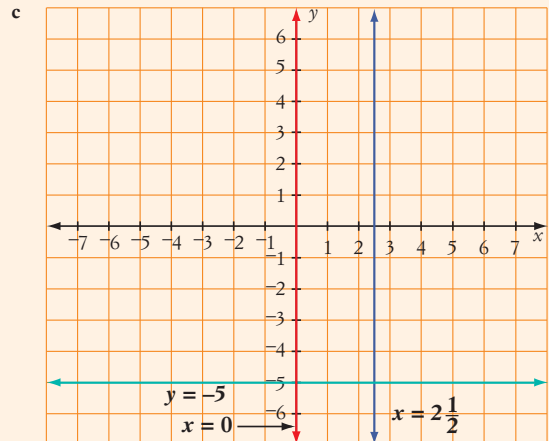
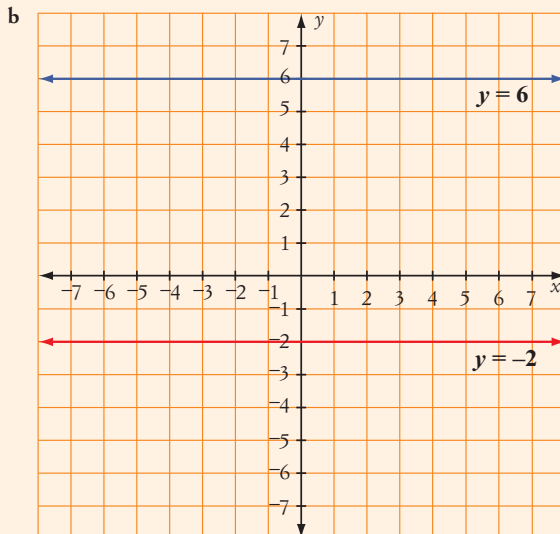
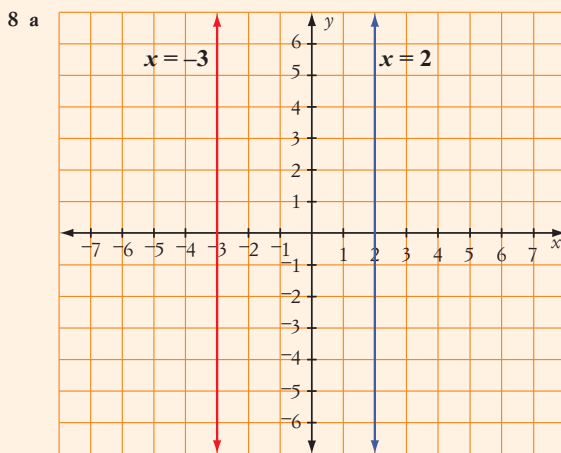
- 4 a yes b no c yes
d yes e yes f no

5 a LHS = RHS = -1 for $x - 2y = -1$;
LHS = RHS = 5 for $x + y = 5$.

b (3, 2)

6 LHS = RHS = -12 for $y = -3x + 6$;
LHS = RHS = -12 for $y = -2x$.

7 C



- 9 a $y = 6$ b $x = -4$ c $y = 3$ d $x = 5$
e $y = -5$ f $x = 3$ g $y = -3$ h $x = -1$
10 a $y = 6$ b $x = -5$ c $y = 2$ d $x = -5$
e $y = -2$ f $y = -1$ g $x = -1$
11 a $x = 0$ b $y = 0$

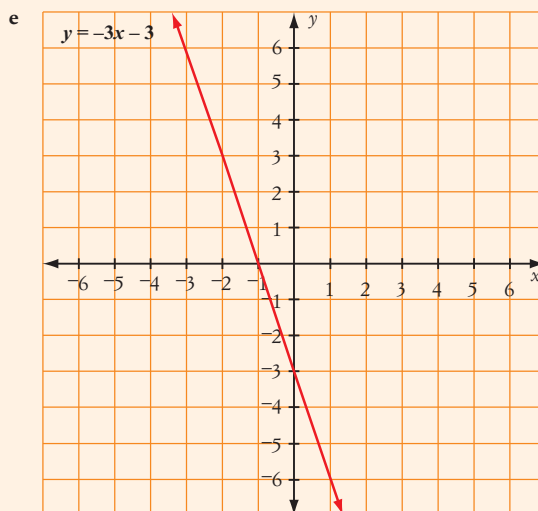
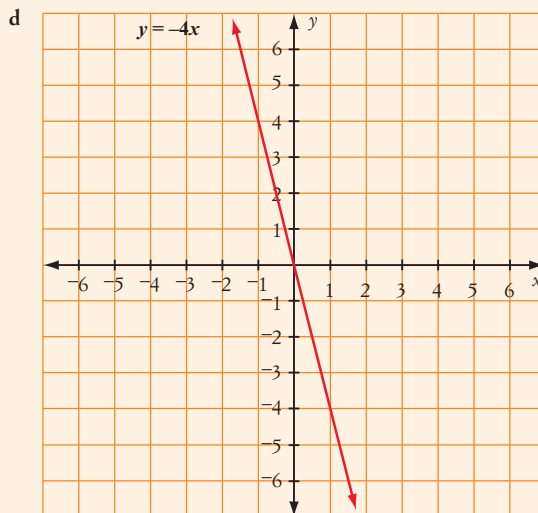
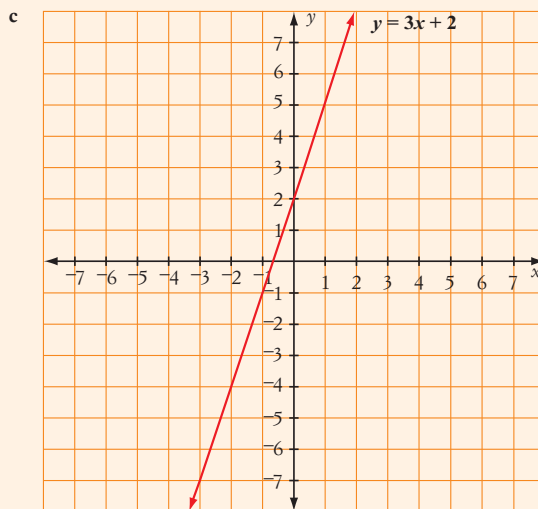
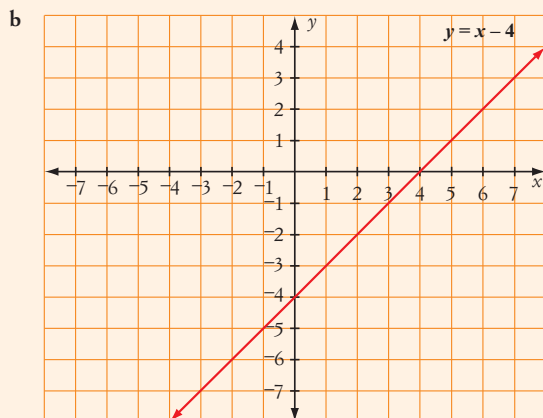
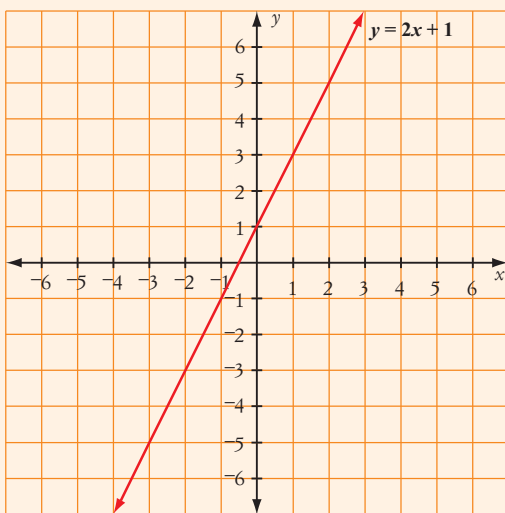
Mental skills 11

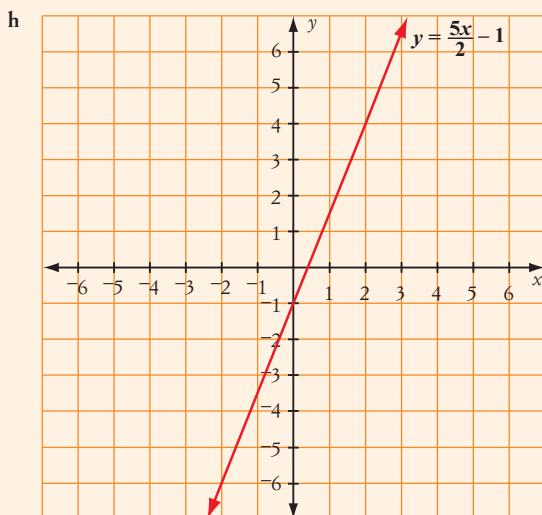
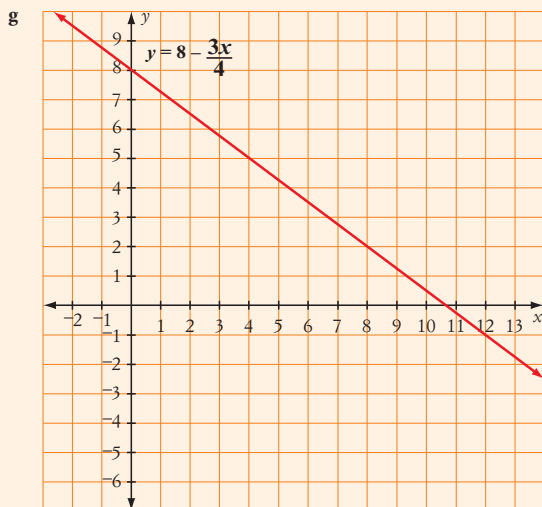
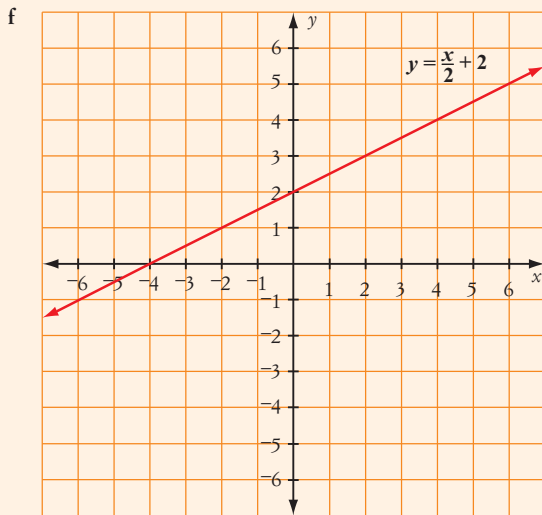
- 2 a 2, 5 b 3 c 2, 3, 6 d 3, 5
e 2 f 2, 3, 5, 6 g 5 h 3
3 a 4, 9 b 4, 10 c 9 d 10
e 4, 9 f None g 9 h 4, 9, 10

Answers

Exercise 11-05

- 1 a $m = 3, b = -2$ b $m = 2, b = 7$
 c $m = 1, b = 4$ d $m = -1, b = -9$
 e $m = \frac{3}{4}, b = 6$ f $m = 1, b = 0$
 g $m = \frac{1}{2}, b = -11$ h $m = \frac{2}{3}, b = 6$
 i $m = -2, b = 8$ j $m = -1, b = 5$
 k $m = \frac{1}{4}, b = 0$ l $m = -3, b = 11$
 m $m = 0, b = 5$ n $m = \frac{2}{5}, b = 1$
 o $m = 1, b = -\frac{7}{2}$ p $m = \frac{3}{4}, b = 1$
- 2 D
- 3 a $y = 2x - 1$ b $y = \frac{3}{4}x + 2$
 c $y = -3x + 5$ d $y = -\frac{2}{5}x + 3$
- 4 C
- 5 a



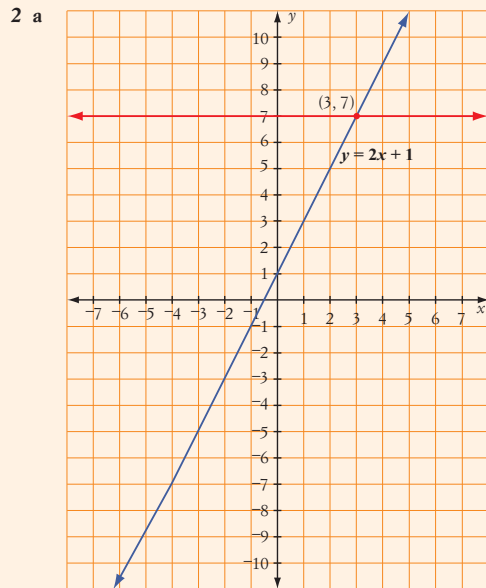


Exercise 11-06

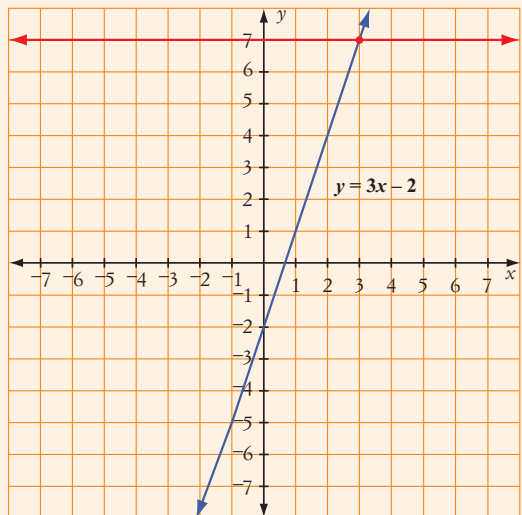
1 a $y = x + 4$	b $y = \frac{3x}{2}$	c $y = \frac{x}{3} - 5$
d $y = -\frac{3x}{2} + 7$	e $y = -\frac{x}{3} + 3$	f $y = -2x - 6$
2 a $y = \frac{x}{2} + 2$	b $y = x$	c $y = -\frac{x}{2} + 5$
d $y = -\frac{x}{2} + 3$	e $y = -3x - 3$	f $y = -x - 2$
g $y = 3x - 10$	h $y = \frac{2x}{5} + 2$	i $y = 2x - 3$

Exercise 11-07

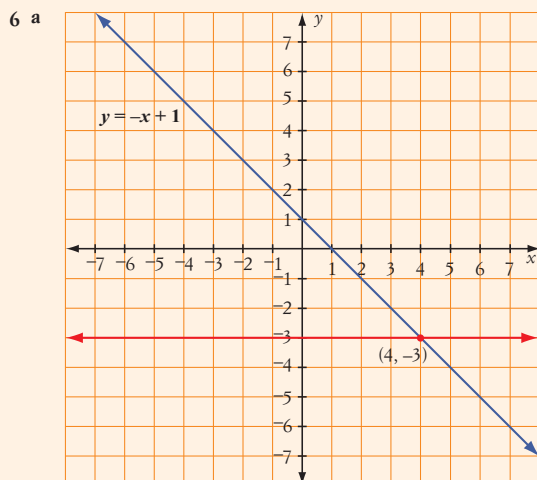
- 1 a $x = 3$ b $x = -2$ c $x = 1\frac{1}{2}$



- $x = 3$
 b $x = 4.5$ c $x = -3$
 3 a $x = 3$ b $x = 4\frac{1}{2}$ c $x = -3$
 4 B
 5 a



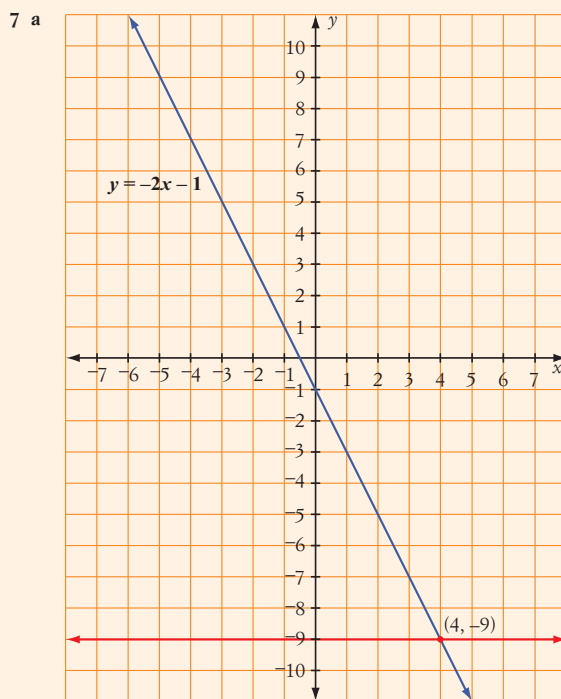
- $x = 3$
 b $x = -3$ c $x = 0$



$x = 4$

b $x = -3$

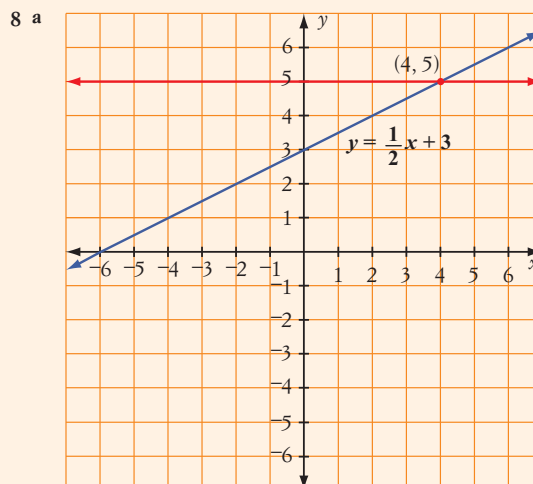
c $x = 2$



$x = 4$

b $x = -\frac{1}{2}$

c $x = -2$

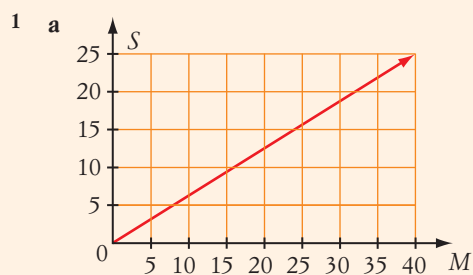


$x = 4$

b $x = -4$

c $x = -2$

Exercise 11-08



b $S = \frac{5}{8}M$

c 22.5 cm

d 64 kg

2 $Y = 115$

3 C

4 a $M = \frac{7}{4}V$

b 710.5 g

c i 16 cm³

ii 205.7 cm³

5 $n = 2.5m$

6 a $I = 0.025D$

b \$62.50

c \$125

7 $p = 108$

8 a 11.25 km

b 2h 8 min

9 A

10 a $V = 64t$

b 512 L

c 15 min

11 a 22.8 kg

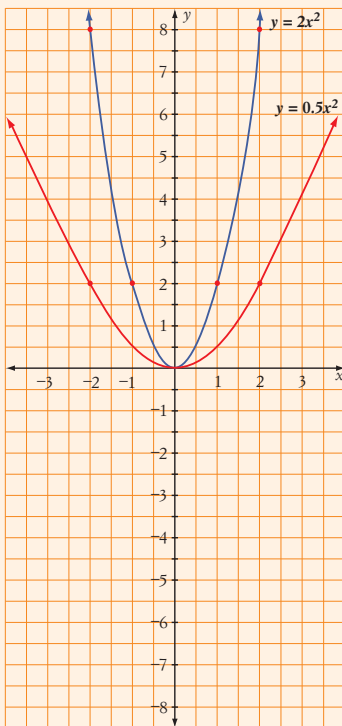
b 84.1 kg

12 a 235.2 kPa

b 141.7 m

Exercise 11-09

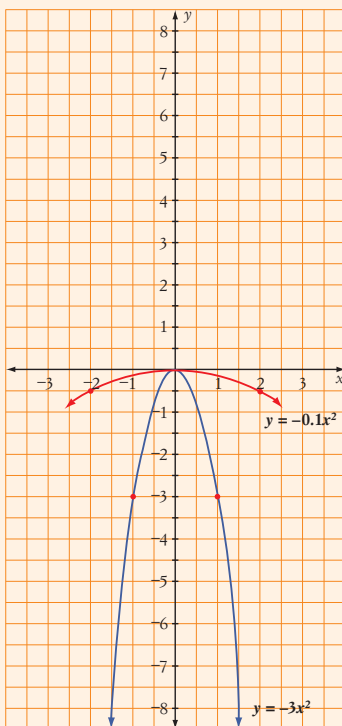
1 a, b



c Same: both concave up, vertex (0, 0).
Different: $y = 2x^2$ is narrower than $y = \frac{1}{2}x^2$.

2 D

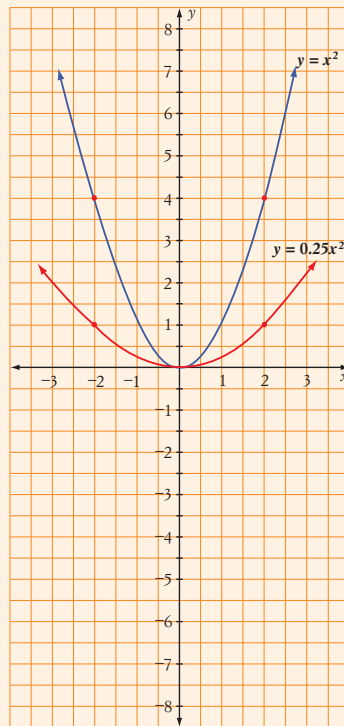
3 a



b Same: concave down, vertex (0, 0).

Different: $y = -3x^2$ narrower than $y = -0.1x^2$.

4 a i vertex (0, 0)

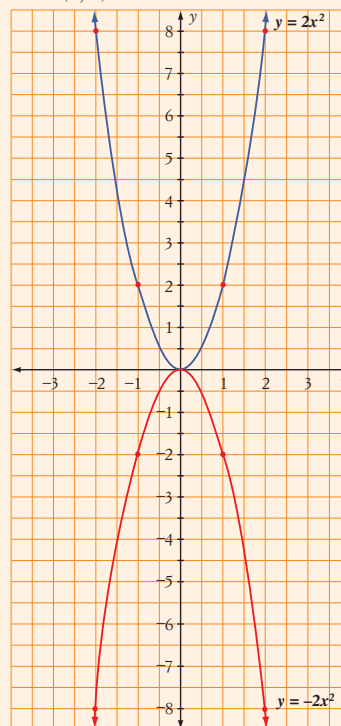


ii Both concave up.

iii $y = \frac{1}{4}x^2$

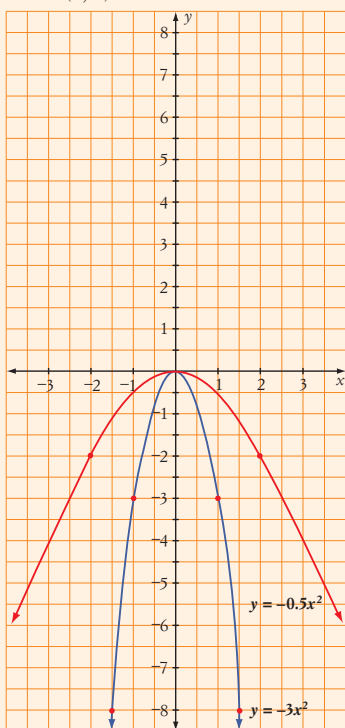
iv (0, 0)

b i vertex (0, 0)

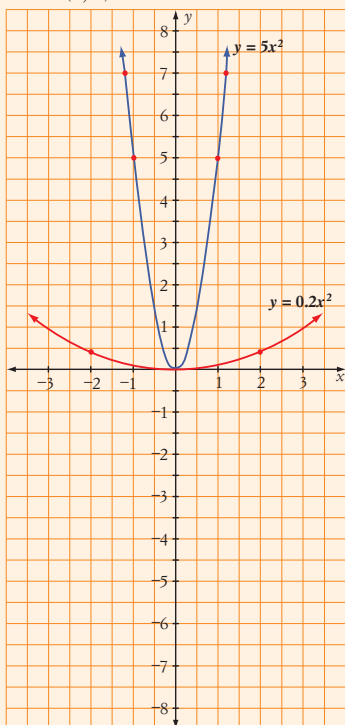


Answers

- ii $y = 2x^2$ concave up, $y = -2x^2$ concave down
 iii same width iv (0, 0)
 c i vertex (0, 0)



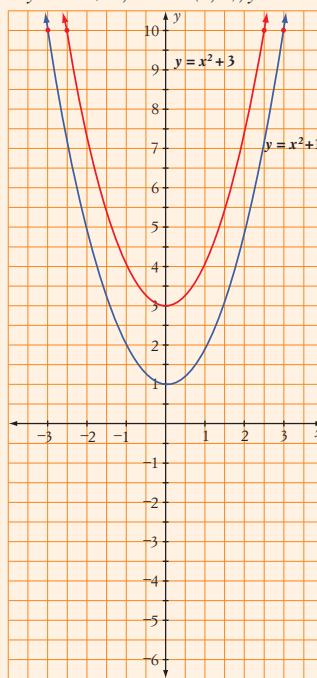
- ii both concave down iii $y = -\frac{1}{2}x^2$ wider iv (0, 0)
 d i vertex (0, 0)



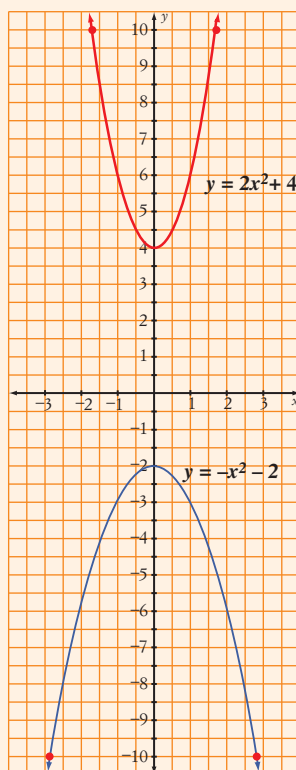
- ii both concave up iii $y = 0.2x^2$ is wider iv (0, 0)

5 C

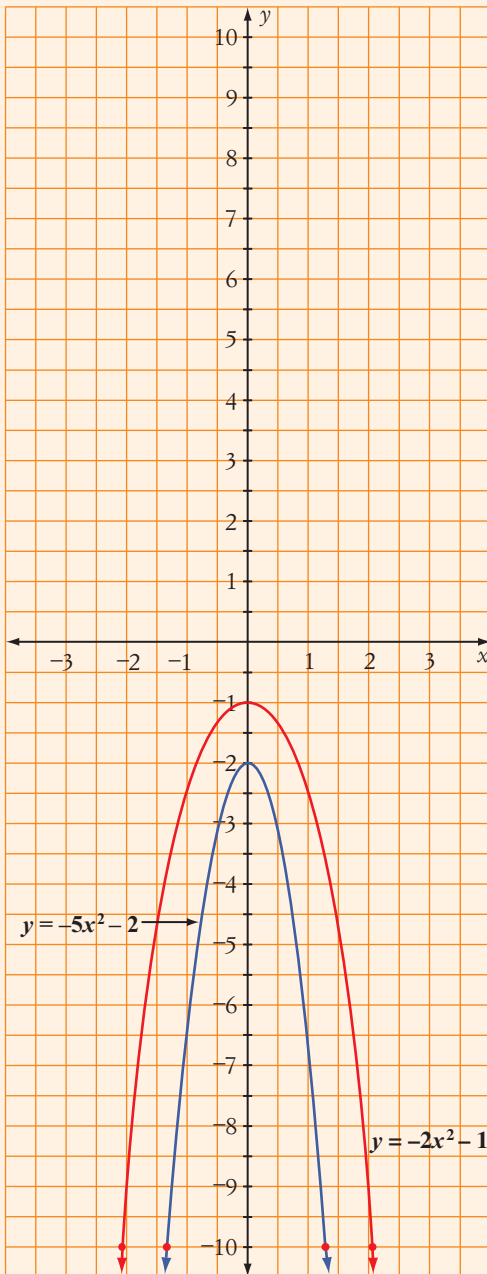
- 6 a i $y = x^2 + 1$, vertex (0, 1); $y = x^2 + 3$, vertex (0, 3).



- ii both concave up
 iii both same width
 iv $y = x^2 + 1$, y-int.: 1; $y = x^2 + 3$, y-int.: 3.
 b i $y = -x^2 - 2$, vertex (0, -2); $y = 2x^2 + 4$, vertex (0, 4).

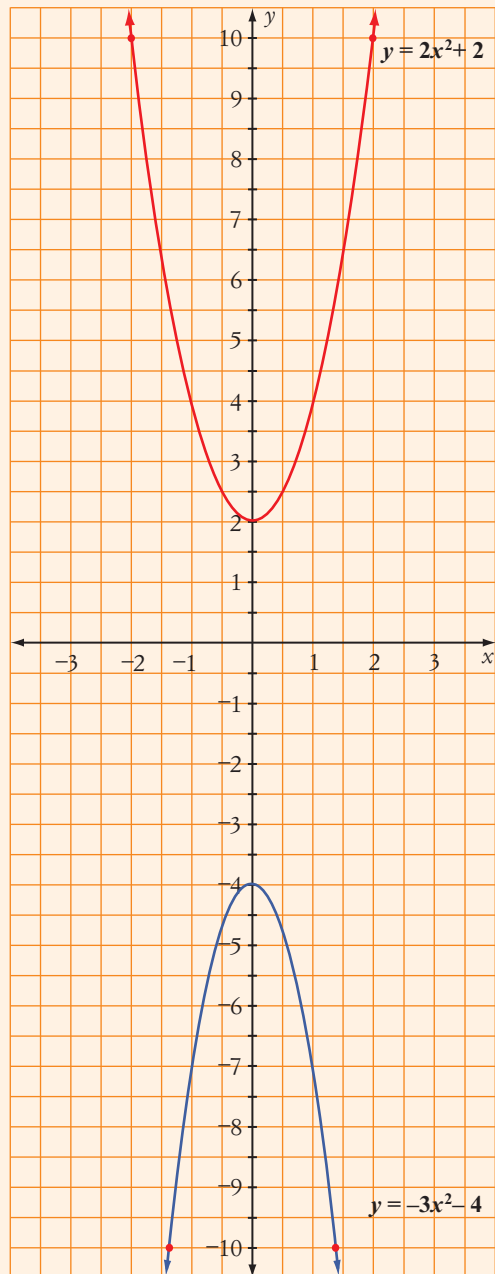


- ii $y = 2x^2 + 4$ concave up; $y = -x^2 - 2$ concave down.
 iii $y = -x^2 - 2$
 iv $y = -x^2 - 2$, y -int.: -2 ; $y = 2x^2 + 4$, y -int.: 4 .
 c i



$y = -5x^2 - 2$, vertex $(0, -2)$; $y = -2x^2 - 1$, vertex $(0, -1)$

- ii Both concave down.
 iii $y = -2x^2 - 1$
 iv $y = -5x^2 - 2$, y -int.: -2 ; $y = -2x^2 - 1$, y -int.: -1 .
 d i $y = -3x^2 - 4$, vertex $(0, -4)$; $y = 2x^2 + 2$, vertex $(0, 2)$.



- ii $y = 2x^2 + 2$ concave up; $y = -3x^2 - 4$ concave down.
 iii $y = 2x^2 + 2$
 iv $y = 2x^2 + 2$, y -int.: 2 ; $y = -3x^2 - 4$, y -int.: -4 .

7 C

Exercise 11-10

1 a centre $(0, 0)$, $r = 2$	b centre $(0, 0)$, $r = 1$
c centre $(0, 0)$, $r = 9$	d centre $(0, 0)$, $r = 10$
e centre $(0, 0)$, $r = 5$	f centre $(0, 0)$, $r = \frac{1}{2}$

Answers

2 D

3 a $x^2 + y^2 = 16$

c $x^2 + y^2 = 6.25$

4 B

5 A

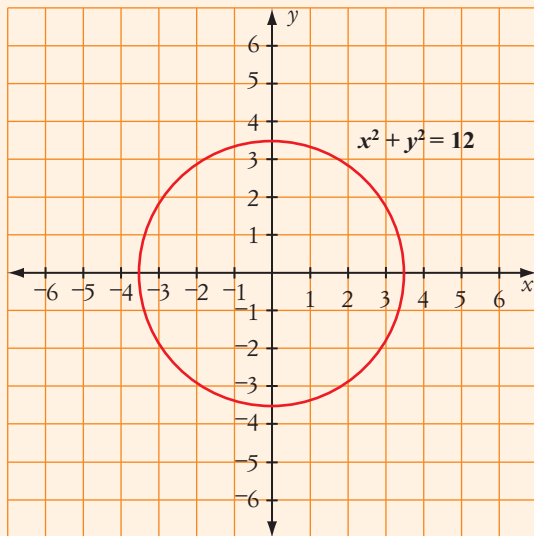
6 $x^2 + y^2 = 64$

7 B

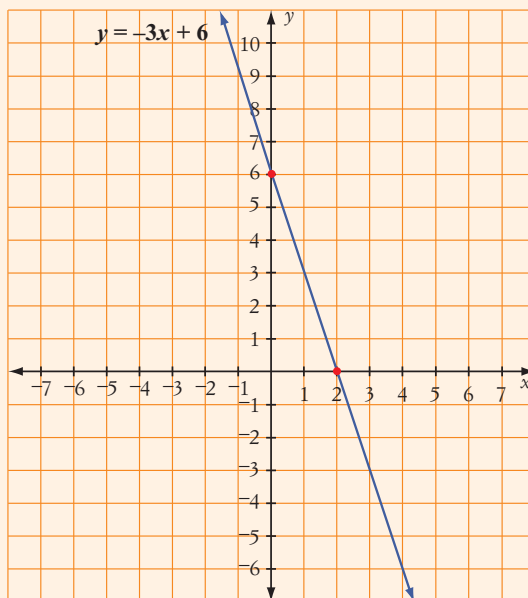
8 28 units

9 a $\sqrt{12}$ or $2\sqrt{3}$

b



5



a 2

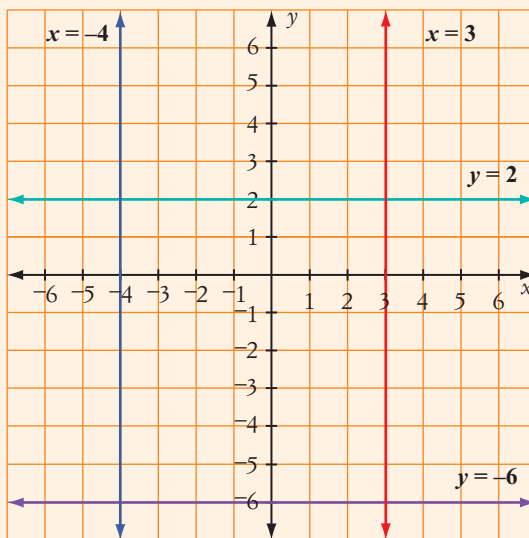
b 6

6 a yes

b yes

c yes

7 a



b 56 units²

8 a $y = 4$

b $x = -1$

c $y = 0$

9 a $m = 3, b = 1$

b $m = -1, b = 8$

c $m = \frac{1}{5}, b = 4$

d $m = -6, b = 0$

Power plus

1 $D(-3, 5)$

2 a $y = \frac{2}{3}x - 2$

b 4

3 $Y(5, 4)$

4 a $r = \sqrt{73}$

b $r = \sqrt{73} \approx 8.54$; $10 > 8.54$, $\therefore P$ lies outside the circle as it is further from the origin.

5 $RS = 13, ST = 13, RT = \sqrt{208}$ $\therefore RS = ST = 13$ units

$\therefore$ isosceles triangle

6 $B(5, 4), C(8, 1), D(5, -2)$

Chapter 11 revision

1 a $\sqrt{97}$

b 9.8

2 a $(3, 7)$

b $(-4, 4)$

c $(-1, 2)$

3 a i MN, CD, GH

ii KL, IJ, AB

iii EF

b i $\frac{1}{7}$

ii $-\frac{5}{4}$

iii $-\frac{1}{4}$

c $(9, -\frac{1}{2})$

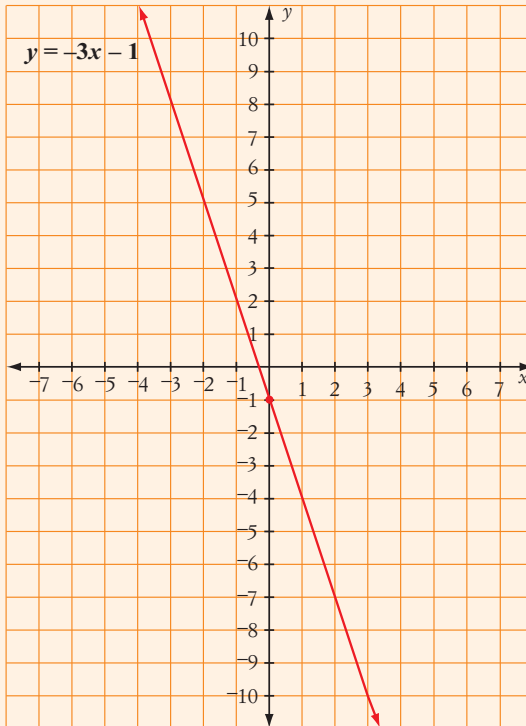
d $\sqrt{13}$

4 a $m = -2$

b $m = -2$

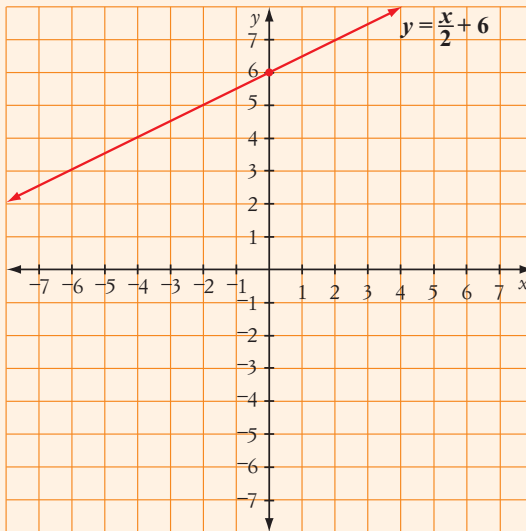
c $m = -\frac{1}{3}$

10 a



$$m = -3, b = -1$$

b



$$m = \frac{1}{2}, b = 6$$

11 a $y = 2x + 1$

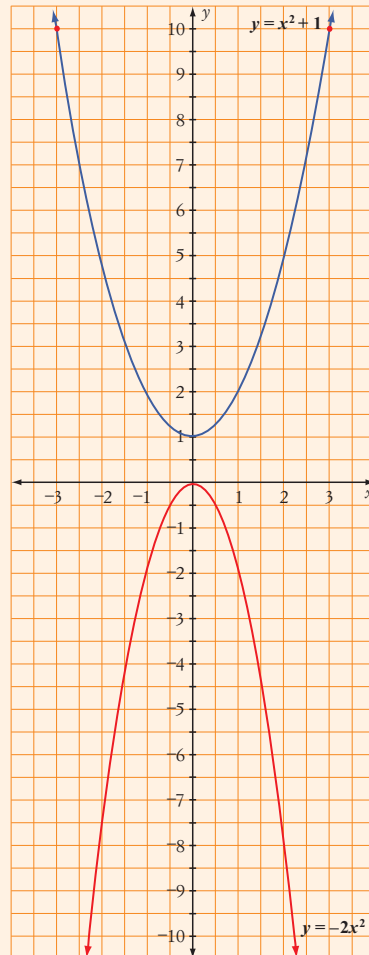
b $y = -3x + 1$

12 $x = 3$

13 a $T = \frac{6}{35}d$

b $d = 58 \text{ km}$

14



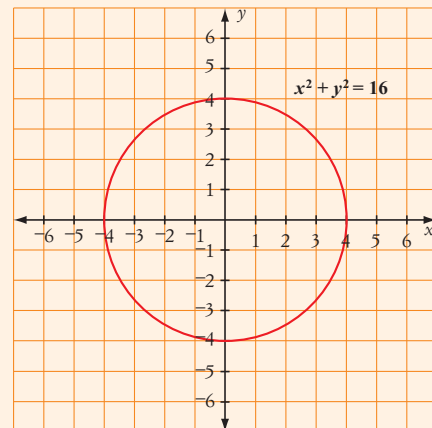
a $y = x^2 + 1$, vertex $(0, 1)$; $y = -2x^2$, vertex $(0, 0)$.

b $y = x^2 + 1$ concave up; $y = -2x^2$ concave down.

c $y = x^2 + 1$

15 B

16 centre $(0, 0)$, radius 4



Chapter 12

SkillCheck

- 1 a $\frac{1}{5}$ b 40% c 60% d $\frac{11}{20}$
 2 A, B, C
 3 a $\frac{1}{13}$ b $\frac{1}{4}$ c $\frac{1}{2}$
 4 a ii b v c vi
 d iii e i f iv
 5 a $\frac{1}{6}$ b $\frac{2}{3}$ c $\frac{1}{2}$ d $\frac{1}{3}$
 6 a 5, 6, 7, 8
 b i 25% ii 100% iii 25% iv 75%

Exercise 12-01

- 1 a 26 b yes c $\frac{5}{26}$ d $\frac{9}{26}$
 2 a 0.375 b 0.25 c 0.25
 3 a $\frac{1}{2}$ b $\frac{1}{13}$ c $\frac{5}{13}$ d $\frac{3}{13}$
 4 a $\frac{2}{3}$ b $\frac{1}{2}$ c $\frac{2}{3}$
 d $\frac{1}{2}$ e 0 f 1
 5 a $\frac{4}{7}$ b 0 c 1
 6 a $\frac{1}{3}$ b $\frac{2}{3}$ c $\frac{1}{3}$ d $\frac{1}{9}$ e $\frac{5}{9}$
 7 a 60.9% b 39.1% c 100% d 0%
 8 a $\frac{1}{3}$ b $\frac{1}{6}$ c $\frac{2}{3}$ d $\frac{2}{3}$
 9 a 35% b 45% c 55% d 45%
 10 a selecting a vowel
 b weather not being sunny or weather being overcast
 c traffic light showing amber or green
 d selecting a heart, diamond or club
 e not hitting the bullseye
 f selecting a red or yellow ball
 11 a 1, 2, 3, 4, 5 b $\frac{1}{5}$
 c Not spinning a 4 or spinning a 1, 2, 3 or 5.
 d 1, 2, 3, 5 e $\frac{4}{5}$
 12 a i $\frac{1}{50}$ ii 0.02 iii 2%
 b i $\frac{49}{50}$ ii 0.98 iii 98%
 13 a 60% b 80
 14 D 15 0.65 16 $\frac{5}{6}$ 17 24%
 18 0 as all mobile numbers begin with 0.

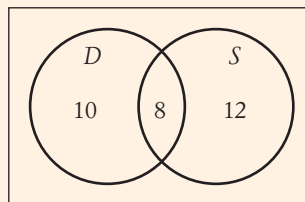
Exercise 12-02

- 1 a i 0.45 ii 0.18
 b i 0.4 ii 0.2
 c They are similar but not equal.
 d 160, which is close to the observed frequency of 164.
 2 a Teacher to check b 25%, teacher to check.
 c 12 or 13, teacher to check.
 3 a 16 or 17 b-e Teacher to check.
 4 a Teacher to check.
 b i 0.4 ii 0.5 iii 0.1
 c Teacher to check. d Teacher to check.

- 5 a 500
 b i 0.47 ii 0.468 iii 0.35
 iv 0.302 v 0.832
 c i 0.652 ii 0.667
 d The probabilities are close.
 6 a 0.28 b 0.16 c 0.45
 d 0.89 e 0.60
 7 a 40
 b i $\frac{11}{40}$ ii $\frac{1}{8}$ iii $\frac{3}{4}$ iv $\frac{1}{8}$
 8 a $\frac{4}{31}$ b $\frac{10}{31}$ c $\frac{11}{31}$
 d $\frac{26}{31}$ e $\frac{5}{31}$ f $\frac{9}{31}$
 9 a $\frac{3}{10}$ b $\frac{1}{10}$ c $\frac{1}{4}$
 d $\frac{1}{2}$ e $\frac{9}{20}$ f $\frac{17}{20}$
 10 a i 0.092 ii 0.077
 b They are similar. The experimental probability is just higher.
 c C

Exercise 12-03

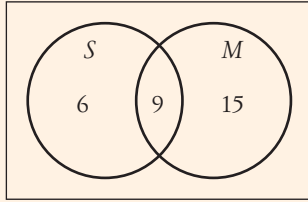
- 1 a 48 b i $\frac{5}{8}$ ii $\frac{25}{48}$
 c i 1 ii $\frac{7}{48}$ iii $\frac{23}{48}$ iv 0
 d $\frac{7}{25}$
 2 a 160
 b i $\frac{91}{160}$ ii $\frac{1}{2}$ iii $\frac{19}{80}$ iv $\frac{133}{160}$
 v $\frac{19}{32}$ vi $\frac{53}{160}$ vii $\frac{21}{80}$
 c 76.25% d $\frac{38}{91}$
 3 a People that preferred watching movies just at the cinema.
 b 12
 c i 0.25 ii 0.4 iii 0.7 iv 0.05
 4 a 204
 b 119 – inside the rectangle but outside of the circles.
 c i $\frac{5}{204}$ ii $\frac{29}{204}$ iii $\frac{43}{204}$ iv $\frac{1}{51}$
 v $\frac{23}{204}$ vi $\frac{25}{34}$ vii $\frac{65}{102}$
 d i $\frac{54}{85}$ ii $\frac{9}{85}$
 5 a 79
 b $\frac{3}{79}$
 c i $\frac{46}{79}$ ii $\frac{51}{79}$ iii $\frac{34}{79}$
 d $\frac{45}{79}$
 e i $\frac{47}{79}$ ii $\frac{10}{79}$ iii $\frac{14}{79}$
 6 a



- b i $\frac{4}{15}$ ii $\frac{2}{5}$ iii $\frac{1}{5}$
 iv 1 v $\frac{11}{15}$

7 a 9

b



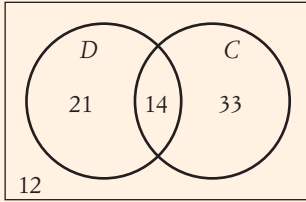
c Because neither playing sport nor watching a movie was not an option.

d i $\frac{7}{10}$

ii $\frac{1}{2}$

iii $\frac{1}{5}$

8 a



b i 14

ii 21

iii 68

iv 54

c i 0.2625

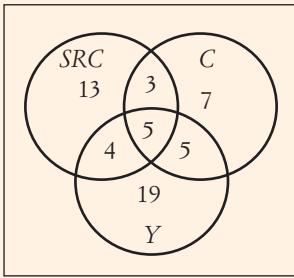
ii 0.675

iii 0.825

iv 0.85

d $\frac{47}{68}$

9 a



b 19

c 7

d $\frac{3}{14}$

e $\frac{17}{56}$

f i $\frac{1}{14}$

ii $\frac{39}{56}$

iii $\frac{23}{56}$

iv $\frac{51}{56}$

g No, because information is only given about numbers belonging to the clubs and not about the numbers attending the school.

Exercise 12-04

1 a 300

b 75

c 68

iv $\frac{58}{75}$

d i $\frac{31}{75}$

ii $\frac{1}{4}$

iii $\frac{157}{300}$

iv $\frac{58}{75}$

2 a 130

b 68.5%

iii $\frac{1}{5}$

iv $\frac{8}{13}$

c i $\frac{61}{130}$

ii $\frac{89}{130}$

iii $\frac{1}{5}$

iv $\frac{8}{13}$

d $\frac{35}{61}$

v $\frac{7}{26}$

vi $\frac{8}{13}$

v $\frac{8}{13}$

3 a 240

b i 121

ii 119

iii $\frac{36}{121}$

c 53.3%

d $\frac{9}{80}$

e $\frac{36}{121}$

ii $\frac{118}{183}$

4 a 183

b i $\frac{14}{183}$

ii $\frac{118}{183}$

iii $\frac{169}{183}$

c 35.5%

d $\frac{43}{50}$

e $\frac{169}{183}$

iv $\frac{169}{183}$

5 a 148

b i 75%

ii 9.5%

iii 60.8%

iv 39.2%

c $\frac{53}{148} = 35.8\%$

d $\frac{23}{37} = 62.2\%$

iii 60.8%

iv 39.2%

6 a 965

b i 18.4%

ii 49.4%

iii 21.8%

iv 27.7%

c $\frac{2}{5}$

d $\frac{31}{52}$

7 a 320

b 43.4%

c $56.6\% = \frac{181}{320}$

d 49.7%

e 38%

Mental skills 12

2 a \$700

b \$800

c \$400

d \$700

e \$300

f \$400

g \$250

h \$300

i \$300

j \$500

Exercise 12-05

1 a BA, BW, BL, BN, AB, AW, AL, AN, WB, WA, WL, WN, LB, LA, LW, LN, NB, NA, NW, NL

b i $\frac{2}{20} = \frac{1}{10}$

ii $\frac{6}{20} = \frac{3}{10}$

iii $\frac{2}{20} = \frac{1}{10}$

iv $\frac{14}{20} = \frac{7}{10}$

2 a 56, 58, 59, 65, 68, 69, 85, 86, 89, 95, 96, 98,

b 12

c i $\frac{6}{12} = \frac{1}{2}$

ii $\frac{9}{12} = \frac{3}{4}$

iii $\frac{6}{12} = \frac{1}{2}$

iv $\frac{2}{12} = \frac{1}{6}$

3 a

		Die					
		1	2	3	4	5	6
Coin	H	H1	H2	H3	H4	H5	H6
	T	T1	T2	T3	T4	T5	T6

b i $\frac{1}{12}$

ii $\frac{3}{12} = \frac{1}{4}$

iii $\frac{4}{12} = \frac{1}{3}$

4 a

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b i $\frac{2}{36} = \frac{1}{18}$

ii $\frac{6}{36} = \frac{1}{6}$

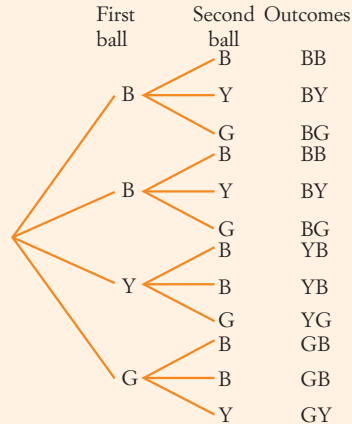
iii $\frac{26}{36} = \frac{13}{18}$

iv $\frac{30}{36} = \frac{5}{6}$

v $\frac{18}{36} = \frac{1}{2}$

vi $\frac{12}{36} = \frac{1}{3}$

5 a



b i $\frac{2}{12} = \frac{1}{6}$

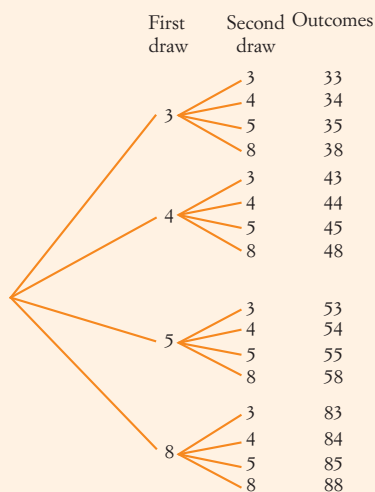
ii $\frac{2}{12} = \frac{1}{6}$

iii $\frac{4}{12} = \frac{1}{3}$

iv $\frac{10}{12} = \frac{5}{6}$

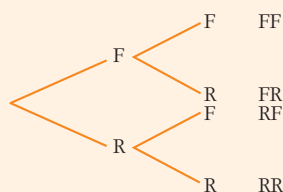
Answers

6 a



b i $\frac{4}{16} = \frac{1}{4}$ ii $\frac{8}{16} = \frac{1}{2}$ iii $\frac{6}{16} = \frac{3}{8}$ iv $\frac{2}{16} = \frac{1}{8}$

7 a

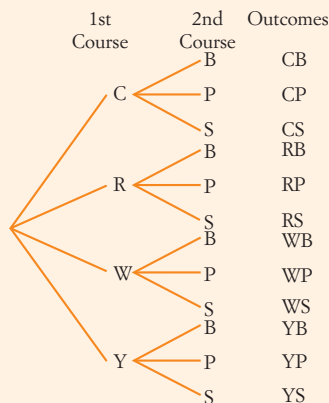


b i $\frac{1}{4}$ ii $\frac{1}{4}$ iii $\frac{3}{4}$
 8 a $\frac{1}{4}$ b $\frac{1}{4}$ c $\frac{1}{4}$
 d $\frac{1}{4}$ e $\frac{3}{4}$ f $\frac{1}{4}$

9

		1st die					
		1	2	3	4	5	6
2nd die	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

a $\frac{1}{6}$ b $\frac{1}{18}$ c $\frac{11}{36}$ d $\frac{4}{9}$
 10 a i CB, CP, CS, RB, RP, RS, WB, WP, WS, YB, YP, YS
 ii



b $\frac{1}{2}$

11 a i 12, 13, 14, 15, 21, 23, 24, 25, 31, 32, 34, 35, 41, 42, 43, 45, 51, 52, 53, 54

ii

		2nd digit				
		1	2	3	4	5
1st digit	1	–	12	13	14	15
	2	21	–	23	24	25
	3	31	32	–	34	35
	4	41	42	43	–	45
	5	51	52	53	54	–

b $\frac{3}{5}$

ii $\frac{3}{5}$

Power plus

1 a $\frac{1}{6}$

b $\frac{5}{36}$

c $\frac{25}{216}$

2 Teacher to check the tree diagram.

a $\frac{1}{4}$

b $\frac{1}{16}$

c $\frac{3}{8}$

d $\frac{15}{16}$

e $\frac{5}{16}$

f $\frac{5}{16}$

3 a $\frac{5}{18}$

b $\frac{5}{18}$

c $\frac{5}{9}$

d $\frac{13}{18}$

Chapter 12 revision

1 a 60%

b 16

2 a $\frac{1}{2}$

b $\frac{2}{13}$

c $\frac{4}{13}$

3 a i 0.36

ii 0.32

iii 0.83

iv 0.175

b i 0.3

ii 0.3

iii 0.83

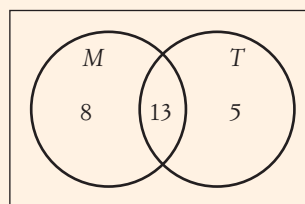
iv 0.16

c They are similar but not equal.

d C

4 a 13

b



c i $\frac{4}{15}$

ii $\frac{9}{15}$

iii $\frac{13}{30}$

iv $\frac{17}{30}$

5 a 210

b i 48.1%

ii 32.4%

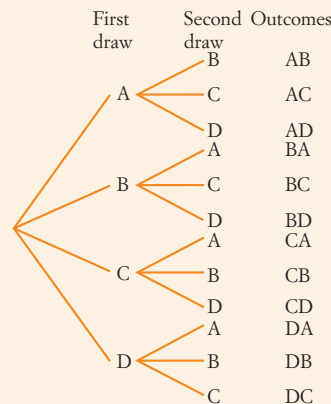
iii 51.9%

iv 73.3%

c i $\frac{68}{109} = 62.4\%$

6 a AB, AC, AD, BA, BC, BD, CA, CB, CD, DA, DB, DC

b



c i 0

ii $\frac{1}{6}$

iii $\frac{5}{6}$

iv $\frac{1}{2}$

7 a

Dice 1 \ Dice 2	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

b i $\frac{1}{6}$

ii $\frac{5}{9}$

iii $\frac{1}{6}$

Chapter 13

SkillCheck

- 1 a i $\angle P$ and $\angle Y$, $\angle R$ and $\angle B$, $\angle T$ and $\angle W$
 ii PT and YW , PR and YB , RT and BW
 b i $\angle E$ and $\angle N$, $\angle G$ and $\angle V$, $\angle M$ and $\angle Q$, $\angle X$ and $\angle C$
 ii EX and NC , XM and CQ , MG and QV , GE and VN

Exercise 13-01

- 1 a SAS b AAS c SAS d AAS
 e SSS f AAS g RHS h AAS
 i AAS
 2 B
 3 a $\triangle ABC \equiv \triangle NMP$ (SAS) b $\triangle KLQ \equiv \triangle FED$ (AAS)
 c $\triangle AXT \equiv \triangle GYE$ (RHS) d $\triangle MVR \equiv \triangle SPC$ (SSS)
 e $\triangle BTV \equiv \triangle PSX$ (AAS) f $\triangle CNQ \equiv \triangle DCE$ (SAS)
 4 a $w = 24$ b $x = 28, p = 7$ c $a = 34$
 d $b = 26$ e $y = 65$ f $x = 45, k = 8$
 5 $a = 118, k = 43, x = 28$
 6 a No b No c Yes, SAS
 d Yes, RHS e No f No

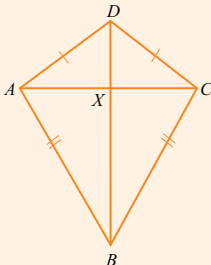
Exercise 13-02

- 1 In $\triangle WXY$ and $\triangle WZY$
 $\angle X = \angle Z = 90^\circ$ (given)
 $XY = ZY$ (given)
 WY is common
 $\therefore \triangle WXY \equiv \triangle WZY$ (RHS)
 2 In $\triangle ABC$ and $\triangle DCB$
 $\angle ABC = \angle DCB$ (given)
 $AB = DC$ (given)
 BC is common
 $\therefore \triangle ABC \equiv \triangle DCB$ (SAS)
 3 In $\triangle DEH$ and $\triangle FEG$
 $\angle D = \angle F$ (alternate angles, $HD \parallel GF$)
 $\angle H = \angle G$ (alternate angles, $HD \parallel GF$)
 $EG = EH$ (given)
 $\therefore \triangle DEH \equiv \triangle FEG$ (AAS)

- 4 In $\triangle ABC$ and $\triangle ADC$
 $BC = DC$ (given)
 $AB = AD$ (given)
 AC is common
 $\therefore \triangle ABC \equiv \triangle ADC$ (SSS)
 5 In $\triangle PQN$ and $\triangle LQM$
 $\angle PQN = \angle LQM$ (vertically opposite angles)
 $PQ = LQ$ (given)
 $NQ = MQ$ (given)
 $\therefore \triangle PQN \equiv \triangle LQM$ (SAS)
 6 In $\triangle DEG$ and $\triangle EDF$
 $\angle DEG = \angle EDF$ (given)
 $GE = FD$ (given)
 ED is common
 $\therefore \triangle DEG \equiv \triangle EDF$ (SAS)
 7 In $\triangle WXY$ and $\triangle YVW$
 $XY = WV$ (given)
 $WX = VY$ (given)
 WY is common
 $\therefore \triangle WXY \equiv \triangle YVW$ (SSS)
 8 In $\triangle PVQ$ and $\triangle QTR$
 $\angle VPQ = \angle TQR$ (corresponding angles, $PW \parallel QT$)
 $\angle VQP = \angle TRQ$ (corresponding angles, $RW \parallel VQ$)
 $PQ = QR$ (given)
 $\therefore \triangle PVQ \equiv \triangle QTR$ (AAS)
 9 In $\triangle BCX$ and $\triangle CBY$
 $\angle BXC = \angle CYB$ ($CX \perp AB$, $BY \perp AC$)
 $XC = YB$ (given)
 BC is common
 $\therefore \triangle BCX \equiv \triangle CBY$ (RHS)
 10 In $\triangle FNM$ and $\triangle TMN$
 $\angle FNM = \angle TMN$ (alternate angles, $FN \parallel TM$)
 $\angle FMN = \angle TNM$ (alternate angles, $TN \parallel FM$)
 NM is common
 $\therefore \triangle FNM \equiv \triangle TMN$ (AAS)
 11 In $\triangle WYX$ and $\triangle ZYX$
 $XW = XZ$ (given)
 $WY = ZY$ (Y is midpoint of WZ)
 XY is common
 $\therefore \triangle WYX \equiv \triangle ZYX$ (SSS)
 12 In $\triangle EFH$ and $\triangle GHF$
 $EF = GH$ (given)
 $EH = GF$ (given)
 FH is common
 $\therefore \triangle EFH \equiv \triangle GHF$ (SSS)
 13 In $\triangle AOB$ and $\triangle COD$
 $OA = OC$ (equal radii)
 $OB = OD$ (equal radii)
 $AB = CD$ (given)
 $\therefore \triangle AOB \equiv \triangle COD$ (SSS)
 14 In $\triangle PQT$ and $\triangle PRM$
 $\angle Q = \angle R$ (given)
 $PQ = PR$ (sides opposite equal angles of isosceles triangle)
 $QT = RM$ (given)
 $\therefore \triangle PQT \equiv \triangle PRM$ (SAS)

- 15 In $\triangle AMD$ and $\triangle CNB$
 $\angle A = \angle C$ (opposite angle of a parallelogram are equal)
 $AD = CB$ (opposite sides of a parallelogram are equal)
 $AM = AB - BM$
 But $AB = CD$ (opposite sides of a parallelogram) and
 $BM = DN$ (given)
 $\therefore AM = CD - DN$
 $= CN$
 $\therefore \triangle AMD \equiv \triangle CNB$ (SAS)
- 16 In $\triangle OAC$ and $\triangle OBC$
 $OA = OB$ (equal radii)
 OC is common
 $\angle ACO = \angle BCO = 90^\circ$ ($OC \perp AB$)
 $\therefore \triangle OAC \equiv \triangle OBC$ (RHS)
- 17 a In $\triangle TPM$ and $\triangle MST$
 $PT = SM$ (given)
 $PM = ST$ (given)
 MT is common
 $\therefore \triangle TPM \equiv \triangle MST$ (SSS)
- b $\angle PMT = \angle STM$ (matching angles of congruent triangles)
- c $PM \parallel ST$ (alternate angles proved equal in b)
- 18 In $\triangle FED$ and $\triangle EFC$
 $\angle FED = \angle EFC = 90^\circ$ (angles in a rectangle)
 $ED = FC$ (opposite sides of a rectangle are equal)
 FE is common
 $\therefore \triangle FED \equiv \triangle EFC$ (SAS)
 $\therefore FD = EC$ (matching sides of congruent triangles)
- 19 In $\triangle TSK$ and $\triangle LPK$
 $\angle T = \angle L$ (alternate angles, $TS \parallel PL$)
 $\angle S = \angle P$ (alternate angles, $TS \parallel PL$)
 $TK = LK$ (K is the midpoint of TL)
 $\therefore \triangle TSK \equiv \triangle LPK$ (AAS)
- 20 a $\angle XYB = \angle ABY$ (alternate angles, $XY \parallel AB$)
 $\angle XYZ = 180^\circ - \angle XYB$ (angles on a line)
 $= 180^\circ - \angle ABY$ ($\angle XYB = \angle ABY$)
 $= \angle ABC$
- b In $\triangle XYZ$ and $\triangle ABC$
 $\angle XYZ = \angle ABC$ (proved in a)
 $\angle XZY = \angle ACB$ (alternate angles, $XZ \parallel AC$)
 $ZY = CB$ (given)
 $\therefore \triangle XYZ \equiv \triangle ABC$ (AAS)
- 21 a In $\triangle LMN$ and $\triangle LKP$
 $LM = LK$ (given)
 $MN = KP$ (given)
 $\angle M = \angle K$ (angles opposite equal sides of isosceles triangle LMK)
 $\therefore \triangle LMN \equiv \triangle LKP$ (SAS)
- b $LN = LP$ (matching sides of congruent triangles)
 $\therefore \triangle LNP$ is isosceles (two sides equal)
- 22 a In $\triangle PQT$ and $\triangle PRT$
 $PQ = PR$ (equal adjacent sides of a kite)
 $TQ = TR$ (equal adjacent sides of a kite)
 PT is common
 $\therefore \triangle PQT \equiv \triangle PRT$ (SSS)
- b $\angle Q = \angle R$ (matching angles of congruent triangles)
- c $\angle QTP = \angle RTP$ (matching angles of congruent triangles)
 $\therefore PT$ bisects $\angle QTR$

Exercise 13-03

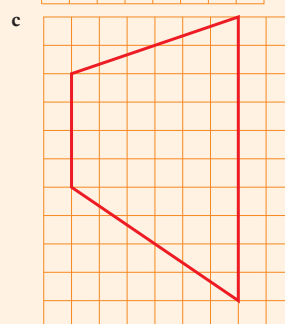
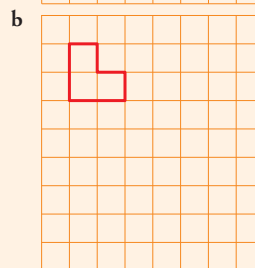
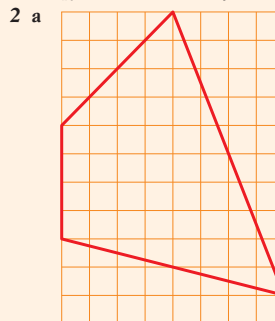
- 1 a In $\triangle ABD$ and $\triangle CBD$
 $AB = CB$ (given)
 $AD = CD$ (given)
 BD is common
 $\therefore \triangle ABD \equiv \triangle CBD$ (SSS)
- b $\angle A = \angle C$ (matching angles of congruent triangles)
- c $\angle ADB = \angle CDB$ (matching angles of congruent triangles)
 $\angle ABD = \angle CBD$ (matching angles of congruent triangles)
 $\therefore DB$ bisects $\angle ADB$ and $\angle ABC$
- d
- 
- e In $\triangle DAX$ and $\triangle DCX$
 $AD = CD$ (given)
 DX is common
 $\angle ADX = \angle CDX$ (Matching angles of congruent triangles proved in a)
 $\therefore \triangle DAX \equiv \triangle DCX$ (SAS)
- f $AX = CX$ (matching sides of congruent triangles)
 $\therefore X$ is midpoint of AC
- g $\angle AXD = \angle CXD$ (matching angles of congruent triangles)
 and $\angle AXD + \angle CXD = 180^\circ$ (angles on a line)
 $\therefore \angle AXD = \angle CXD = 90^\circ$
 $\therefore DX \perp AC$
 $\therefore BD \perp AC$
- 2 b In $\triangle WXZ$ and $\triangle YXZ$
 $\angle WXZ = \angle YXZ$ (alternate angles, $WX \parallel YZ$)
 $\angle WZX = \angle YXZ$ (alternate angles, $WZ \parallel YX$)
 XZ is common
 $\therefore \triangle WXZ \equiv \triangle YXZ$ (AAS)
- c $\angle W = \angle Y$ (matching angles of congruent triangles)
- d In $\triangle WXY$ and $\triangle YZW$
 $\angle XYW = \angle ZWY$ (alternate angles, $XY \parallel ZW$)
 $\angle XWY = \angle ZYW$ (alternate angles, $WX \parallel YZ$)
 WY is common
 $\therefore \triangle WXY \equiv \triangle YZW$ (AAS)
- e $\angle WXY = \angle YZW$ (matching angles of congruent triangles)
- f Opposite angles of a parallelogram are equal.
- 3 a In $\triangle SPW$ and $\triangle TPW$
 $\angle S = \angle T$ (given)
 WP is common
 $\angle WPS = \angle WPT = 90^\circ$ ($WP \perp ST$)
 $\therefore \triangle SPW \equiv \triangle TPW$ (AAS)
- b $SW = TW$ (matching sides of congruent triangles)
 $\therefore$ sides opposite equal angles in a triangle are equal (SW is opposite to $\angle T$, TW is opposite to $\angle S$)

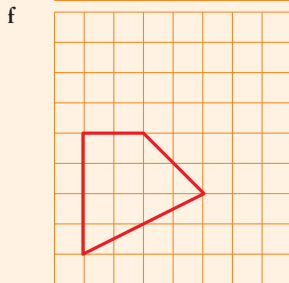
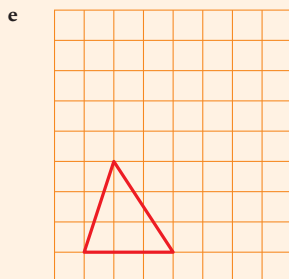
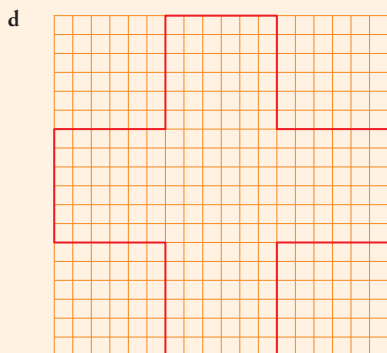
- 4 a In $\triangle VUS$ and $\triangle TUS$
 $VU = TU$ (given)
 $VS = TS$ (given)
 SU is common
 $\therefore \triangle VUS \equiv \triangle TUS$ (SSS)
- b $\angle VUS = \angle TUS$ (matching angles of congruent triangles)
 $\angle VSU = \angle TSU$ (matching angles of congruent triangles)
 $\therefore VS$ bisects $\angle VUT$ and $\angle VST$.
- 5 a In $\triangle ABD$ and $\triangle CDB$
 $\angle ABD = \angle CDB$ (alternate angles, $AB \parallel CD$)
 $\angle ADB = \angle CBD$ (alternate angles, $AD \parallel CB$)
 BD is common
 $\therefore \triangle ABD \equiv \triangle CDB$ (AAS)
- b $AD = CB$ (matching sides of congruent triangles)
 $AB = CD$ (matching sides of congruent triangles)
 $\therefore$ opposite sides of a parallelogram are equal.
- 6 a In $\triangle BEL$ and $\triangle GHL$
 $\angle BEL = \angle GHL$ (alternate angles, $BE \parallel GH$)
 $\angle BLE = \angle GLH$ (vertically opposite angles are equal)
 $BE = GH$ (given)
 $\therefore \triangle BEL \equiv \triangle GHL$ (AAS)
- b $BL = GL$ (matching sides of congruent triangles)
 $EL = HL$ (matching sides of congruent triangles)
 $\therefore$ diagonals of a rhombus bisect each other.
- c $\angle BEH = \angle BHE$ (angles opposite equal sides in isosceles triangle)
 $\therefore \angle BEL = \angle BHL$
- d In $\triangle BEL$ and $\triangle BHL$
 $\angle BEL = \angle BHL$ (proven in c)
 $BE = BH$ (given)
 $EL = HL$ (proven in b)
 $\therefore \triangle BEL \equiv \triangle BHL$ (SAS)
- e $\angle BLE = \angle BLH$ (matching angles of congruent triangles)
and $\angle BLE + \angle BLH = 180^\circ$ (angles on a line)
 $\therefore \angle BLE = \angle BLH = 90^\circ$
 $\therefore$ The diagonals of a rhombus at right angles
- 7 a In $\triangle ABX$ and $\triangle ACX$
 $AB = AC$ (given)
 $BX = CX$ (X is the midpoint of BC)
 AX is common
 $\therefore \triangle ABX \equiv \triangle ACX$ (SSS)
 $\angle B = \angle C$ (matching angles of congruent triangles)
- b In $\triangle BAY$ and $\triangle BCY$
 $BA = BC$ (given)
 $AY = CY$ (Y is the midpoint of AC)
 BY is common
 $\therefore \triangle BAY \equiv \triangle BCY$ (SSS)
 $\angle A = \angle C$ (matching angles of congruent triangles)
- c Since $\angle B = \angle C$, $\angle A = \angle C$,
 $\angle A = \angle B = \angle C$
 $\angle A + \angle B + \angle C = 180^\circ$ (angle sum of a triangle)
 $\therefore \angle A = \angle B = \angle C = 60^\circ$

- 8 a In $\triangle ACD$ and $\triangle CAB$
 $AD = CB$ (given)
 $AB = CD$ (given)
 AC is common
 $\therefore \triangle ACD \equiv \triangle CAB$ (SSS)
- b $\angle DAC = \angle BCA$ (matching angles of congruent triangles)
 $\therefore AD \parallel BC$ (alternate angles are equal)
 $\angle DCA = \angle BAC$ (matching angles of congruent triangles)
 $\therefore AB \parallel CD$ (alternate angles are equal)
- c $ABCD$ is a parallelogram
- 9 a In $\triangle PMY$ and $\triangle PNY$
 $\angle M = \angle N$ (given)
 $\angle MPY = \angle NPY$ (PY bisects $\angle MPN$)
 PY is common
 $\therefore \triangle PMY \equiv \triangle PNY$ (AAS)
- b $MY = NY$ (matching sides of congruent triangles)
 $\angle PYM = \angle PYN$ (matching angles of congruent triangles)
and $\angle PYM + \angle PYN = 180^\circ$ (angles on a line)
 $\therefore \angle PYM = \angle PYN = 90^\circ$
 $\therefore PY \perp MN$

Exercise 13-04

- 1 a 2 b $\frac{2}{3}$ c $\frac{4}{5}$ d $2\frac{2}{3}$
e $\frac{7}{10}$ f $1\frac{1}{3}$ g $1\frac{4}{5}$ h $1\frac{2}{5}$





- 3 a Yes b Yes
 c No d Yes
 e No f Yes
 g No h No
- 4 a i $\angle A$ and $\angle L$, $\angle B$ and $\angle M$, $\angle C$ and $\angle K$
 ii AC and LK , AC and LM , BC and MK
 iii $\triangle ACB \parallel \triangle LKM$
- b i $\angle W$ and $\angle D$, $\angle X$ and $\angle E$, $\angle Y$ and $\angle G$, $\angle Z$ and $\angle H$
 ii WX and DE , XY and EG , YZ and GH , WZ and DH
 iii $ZYXW \parallel HGED$
- c i $\angle T$ and $\angle F$, $\angle R$ and $\angle W$, $\angle P$ and $\angle V$, $\angle M$ and $\angle S$, $\angle J$ and $\angle K$
 ii TR and FW , RP and WV , PM and VS , MJ and SK , TJ and FK
 iii $TRPMJ \parallel FWVSK$
- d i $\angle Q$ and $\angle N$, $\angle K$ and $\angle C$, $\angle B$ and $\angle G$
 ii QB and NG , BK and GC , KQ and CN
 iii $\triangle QKB \parallel \triangle NCG$
- e i $\angle F$ and $\angle X$, $\angle E$ and $\angle W$, $\angle C$ and $\angle Y$
 ii FE and XW , CE and YW , CF and YX
 iii $\triangle FEC \parallel \triangle XWY$

- f i $\angle K$ and $\angle T$, $\angle L$ and $\angle Q$, $\angle M$ and $\angle S$, $\angle P$ and $\angle U$
 ii KL and TQ , KP and TU , PM and US , LM and QS
 iii $PMLK \parallel USQT$

Mental skills 13

2 a $\frac{2}{5}$	b $\frac{4}{5}$	c $\frac{5}{7}$	d $\frac{1}{2}$
e $\frac{1}{4}$	f $\frac{1}{6}$	g $\frac{5}{6}$	h $\frac{2}{5}$
i 5 : 9	j 5 : 9	k 9 : 20	l 4 : 5
m 9 : 7	n 4 : 3	o $\frac{3}{5}$	p $\frac{4}{35}$
4 a $\frac{17}{40}$	b $\frac{2}{3}$	c $\frac{16}{25}$	d $\frac{1}{4}$
e $\frac{5}{24}$	f $\frac{2}{25}$		

Exercise 13-05

- 1 a Yes ($\frac{8}{12} = \frac{20}{30} = \frac{2}{3}$) b Yes ($\frac{12}{20} = \frac{27}{45} = \frac{21}{35} = \frac{3}{5}$)
 c Yes ($\frac{4}{6} = \frac{6}{9} = \frac{64}{10} = \frac{10}{15} = \frac{2}{3}$) d Yes, all squares are similar.
 e Yes, matching angles are equal.
 f Yes, all equilateral triangles are similar.
- 2 a $k = 14$, $x = 6$ b $y = 12.5$, $p = 19.2$
 c $x = 21$ d $a = 12$, $q = 1\frac{2}{3}$, $w = 3\frac{1}{5}$
 e $d = 16$ f $b = 21\frac{7}{18}$
 g $w = 10\frac{4}{5} = 10.8$, $x = 23\frac{1}{3}$ h $c = 19\frac{1}{4}$
- 3 d = 15 4 a = $21\frac{2}{3}$ 5 $13\frac{1}{3}$ m 6 8.9 m 7 21 m
- 8 a A and D , scale factor = 2 b B and D , scale factor = $\frac{3}{4}$
 c C and D , scale factor = $\frac{3}{5}$ d A and D , scale factor = $\frac{1}{2}$

Exercise 13-06

- 1 1 : 500 2 1 : 35 3 8 : 5 4 3 : 1 5 1 : 7500
- 6 a 1 : 5000 b 70 m
 7 2 mm 8 730 km
- 9 a 8 m b 4 m
- 10 a 100 km b 80 km c 54 km d 224 km
- 11 length = 3.2 cm, height = 4.4 cm
- 12 a Scaled length = 30 mm, scaled height = 43 mm b 12.9 m

Exercise 13-07

- 1 1 : 16
- 2 1 : 6.25 or 4 : 25
- 3 a 100 : 1 b 30.25 : 1 or 121 : 4 c 9 : 16 d 25 : 4
 e 16 : 49 f 1 : 100 g 64 : 25 h 4 : 81
- 4 a 1 : 9 b 9 : 4 c 49 : 16 d 25 : 121
 e 25 : 9 f 16 : 25
- 5 a 8 : 7 b 6 : 1 c 3 : 20 d 1 : 4.5
- 6 a 5 : 3 b 4 : 7
- 7 3.5 8 3072 cm² 9 162 cm² 10 1024 mm²
- 11 $28\frac{2}{3}$ m² 12 154 mm 13 22.5 mm 14 12.5 cm
- 15 Increased 9 times 16 reduced by a factor of 2
- 17 9 : 16
- 18 50 cm²

Exercise 13-08

- 1 a Two pairs of matching sides are in the same ratio
 $\frac{12}{8} = \frac{15}{10} = \frac{3}{2}$ and the included angles are equal (SAS).
 b Two pairs of angles are equal (AA).
 c Two pairs of angles are equal (AA).
 d Two pairs of matching sides are in the same ratio
 $\frac{8}{6.4} = \frac{15}{12} = \frac{5}{4}$ and the included angles are equal (SAS).
 e In both right-angled triangles, the pairs of hypotenuses and second sides are in the same ratio $\frac{20}{15} = \frac{26}{19.5} = \frac{4}{3}$ (RHS).
 f All three pairs of matching sides are in the same ratio
 $\frac{20}{16} = \frac{16}{12.8} = \frac{7.5}{6} = \frac{5}{4}$ (SSS).
 g All three pairs of matching sides are in the same ratio
 $\frac{x}{2x} = \frac{y}{2y} = \frac{w}{2w} = \frac{1}{2}$ (SSS).
 h Two pairs of matching sides are in the same ratio
 $\frac{12}{19.2} = \frac{16}{25.6} = \frac{5}{8}$ and the included angles are equal (SAS).
 i Two pairs of matching sides are in the same ratio
 $\frac{15}{10} = \frac{24}{16} = \frac{3}{2}$ and the included angles are equal (SAS).
- 2 a B and C (SAS) b B and C (RHS)
 c B and D (SSS) d C and D (AA)
- 3 If two angles of one triangle are equal to two angles of another triangle, then the third angles must be equal as the angle sum of both triangles is 180° .
- 4 a $\triangle EFD \parallel \triangle PMR$ (SSS)
 b $\triangle UTW \parallel \triangle KAM$ (SAS)
 c $\triangle UZV \parallel \triangle HPZ$ (RHS)
 d $\triangle WLB \parallel \triangle XAT$ (AA)
 e $\triangle KWP \parallel \triangle GEF$ (SSS)
 f $\triangle LEA \parallel \triangle RBX$ (SAS)

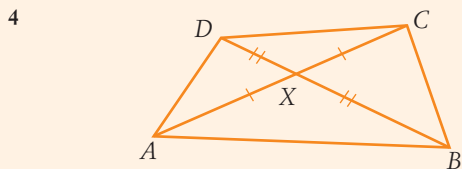
Exercise 13-09

- 1 a In $\triangle DEF$ and $\triangle RPM$
 $\frac{DE}{RP} = \frac{6}{4} = \frac{3}{2}$
 $\frac{EF}{PM} = \frac{9}{6} = \frac{3}{2}$
 $\frac{FD}{MR} = \frac{12}{8} = \frac{3}{2}$
 $\therefore \triangle DEF \parallel \triangle RPM$ (SSS)
 b In $\triangle WUT$ and $\triangle MKA$
 $\frac{WU}{MK} = \frac{15}{20} = \frac{3}{4}$
 $\frac{UT}{KA} = \frac{9}{12} = \frac{3}{4}$
 $\angle U = \angle K = 65^\circ$
 $\therefore \triangle WUT \parallel \triangle MKA$ (SAS)
- c In $\triangle UVZ$ and $\triangle HJP$
 $\frac{UZ}{HP} = \frac{20}{12} = \frac{5}{3}$
 $\frac{VZ}{JP} = \frac{25}{15} = \frac{5}{3}$
 $\angle U = \angle H = 90^\circ$
 $\therefore \triangle UVZ \parallel \triangle HJP$ (RHS)
- e In $\triangle KPW$ and $\triangle GFE$
 $\frac{KP}{GF} = \frac{13.75}{11} = \frac{5}{4}$
 $\frac{PW}{FE} = \frac{12}{9.6} = \frac{5}{4}$
 $\frac{WK}{EG} = \frac{16}{12.8} = \frac{5}{4}$
 $\therefore \triangle KPW \parallel \triangle GFE$ (SSS)
- d In $\triangle WBL$ and $\triangle XTA$
 $\angle B = \angle T = 118^\circ$
 $\angle L = \angle A = 27^\circ$
 $\therefore \triangle WBL \parallel \triangle XTA$
 (equiangular or AA)
- f In $\triangle LAE$ and $\triangle RXB$
 $\frac{LA}{RX} = \frac{5.6}{8.4} = \frac{2}{3}$
 $\frac{AE}{XB} = \frac{8}{12} = \frac{2}{3}$
 $\angle A = \angle X = 90^\circ$
 $\therefore \triangle LAE \parallel \triangle RXB$ (SAS)

- 2 a In $\triangle XLM$ and $\triangle XYW$
 $\angle X$ is common
 $\angle XLM = \angle XYW$
 corresponding angles,
 $LM \parallel YW$
 $\therefore \triangle XLM \parallel \triangle XYW$
 (AA)
 b In $\triangle ABX$ and $\triangle PTX$
 $\angle A = \angle P$ (alternate angles,
 $AB \parallel PT$)
 $\angle B = \angle T$ (alternate angles,
 $AB \parallel PT$)
 $\therefore \triangle ABX \parallel \triangle PTX$ (AA)
- c In $\triangle PLM$ and $\triangle PTW$
 $\angle P$ is common
 $\angle PLM = \angle PTW = 90^\circ$ (given)
 $\therefore \triangle PLM \parallel \triangle PTW$ (AA)
 d In $\triangle KHL$ and $\triangle KMG$
 $\angle K$ is common
 $\angle KHL = \angle KMG$ (given)
 $\therefore \triangle KHL \parallel \triangle KMG$ (AA)
- e In $\triangle TXY$ and $\triangle KXG$
 $\angle T = \angle K = 90^\circ$ (given)
 $\angle TXY = \angle KXG$
 (vertically opposite angles)
 $\therefore \triangle TXY \parallel \triangle KXG$ (AA)
 f In $\triangle WZY$ and $\triangle WXZ$
 $\angle W$ is common
 $\angle WZY = \angle WXZ = 90^\circ$ (given)
 $\therefore \triangle WZY \parallel \triangle WXZ$ (AA)
- 3 a $x = 7\frac{1}{2}$ b $x = 8$ c $x = 1\frac{3}{8}$ d $x = 12\frac{3}{4}$
 e $x = 6\frac{2}{3}$ f $x = 6\frac{1}{2}$
- 4 a In $\triangle KGF$ and $\triangle KHM$
 $\angle K$ is common
 $\angle KGF = \angle KHM$ (corresponding angles, $GF \parallel HM$)
 $\therefore \triangle KGF \parallel \triangle KHM$ (AA)
 b $m = 6\frac{6}{7}$
- 5 $\triangle BCF \parallel \triangle DCE$ (AA)
 $m = 7\frac{1}{2}$ or 7.5
- 6 a In $\triangle TFR$ and $\triangle TPD$
 $\angle T$ is common
 $\angle TFR = \angle TPD = 90^\circ$ (given)
 $\therefore \triangle TFR \parallel \triangle TPD$ (AA)
 b $TR = 14.4$ cm
- 7 a In $\triangle CDE$ and $\triangle FGE$
 $\angle C = \angle F$ (alternate angles, $CD \parallel FG$)
 $\angle D = \angle G$ (alternate angles, $CD \parallel FG$)
 $\therefore \triangle CDE \parallel \triangle FGE$ (AA)
 b $k = 11\frac{1}{4}$ or 11.25
- 8 $\triangle AXM \parallel \triangle AYN$ (AA)
 $AX = 12$
- 9 a In $\triangle FGE$ and $\triangle EFH$
 $\angle F$ is common
 $\angle FEG = \angle EHF$ (given)
 $\therefore \triangle FGE \parallel \triangle EFH$ (AA)
 b $FH = 25$ cm

Power plus

- 1 a In $\triangle CGH$ and $\triangle CDE$
 $\frac{CG}{CD} = \frac{1}{2}$ (G is the midpoint of CD)
 $\frac{CH}{CE} = \frac{1}{2}$ (H is the midpoint of CE)
 $\angle C$ is common
 $\therefore \triangle CGH \parallel \triangle CDE$ (SAS)
- b Since $\triangle CGH \parallel \triangle CDE$
 $\angle CGH = \angle CDE$ (matching angles of similar triangles)
 $\therefore GH \parallel DE$ (corresponding angles are equal)
- c $\frac{CG}{CD} = \frac{CH}{CE}$ (pairs of matching sides of similar triangles in the same ratio)
 But $\frac{CG}{CD} = \frac{1}{2}$
 $\therefore \frac{CH}{CE} = \frac{1}{2}$
 $\therefore GH = \frac{1}{2}DE$
- 2 33.25 m
- 3 a True, angles in all equilateral triangles equal 60° .
 b False, since two isosceles triangles may have different angles such as 50° , 50° , 80° and 70° , 70° , 40°
 c True, since all circles have the same shape.
 d False, as the ratio of lengths does not necessarily have to be equal to the ratio of the widths.
 e True, since all squares have the same shape.
 f True, since all regular pentagons have the same shape.



Consider quadrilateral $ABCD$ where the diagonals AC and BD bisect each other.

In $\triangle AXB$ and $\triangle CXD$

$\angle AXB = \angle CXD$ (vertically opposite angles)

$AX = CX$ (diagonal AC is bisected)

$BX = DX$ (diagonal BD is bisected)

$\therefore \triangle AXB \equiv \triangle CXD$ (SAS)

$\therefore AB = CD$ (matching sides of congruent triangles)

Similarly, $\triangle AXD \equiv \triangle CXB$ (SAS)

$\therefore AD = CB$ (matching sides of congruent triangles)

$\therefore ABCD$ is a parallelogram (opposite sides are equal).

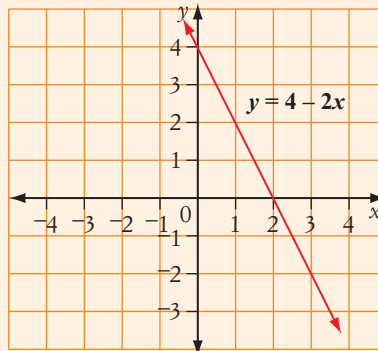
Chapter 13 revision

- 1 a SAS b RHS c SSS
- 2 In $\triangle ACE$ and $\triangle EFA$
 $\angle AEC = \angle EAF$ (alternate angles, $AF \parallel EC$)
 $\angle CAE = \angle FEA$ (alternate angles, $AC \parallel EF$)
 AE is common
 $\therefore \triangle AEC \equiv \triangle EFA$ (AAS)

- 3 In $\triangle MNP$ and $\triangle NMT$
 $NP = MT$ (given)
 $\angle MNP = \angle NMT$ (given)
 MN is common
 $\therefore \triangle MNP \equiv \triangle NMT$ (SAS)
- 4 In $\triangle ABD$ and $\triangle BAC$
 a $\angle BAD = \angle ABC = 90^\circ$ (angles in a rectangle)
 $AD = BC$ (opposite sides of a rectangle)
 AB is common
 $\therefore \triangle ABD \equiv \triangle BAC$ (SAS)
 b $\therefore BD = AC$ (matching sides of congruent triangles)
 $\therefore$ the diagonals of a rectangle are equal.
- 5 a 2 b $\frac{3}{5}$ c 1.5 or $\frac{3}{2}$
- 6 a $y = 8.75$, $p = 6.4$ b $a = 18$, $w = 27$
- 7 a 10 m b 6.4 m
- 8 a 25 : 81 b 16 : 9
- 9 a SAS b SSS c AA
- 10 a In $\triangle ABE$ and $\triangle CBD$
 $\angle A = \angle C$ (alternate angles, $AE \parallel CD$)
 $\angle E = \angle D$ (alternate angles, $AE \parallel CD$)
 $\therefore \triangle ABE \parallel \triangle CBD$ (AA)
 b $k = 12.25$ cm

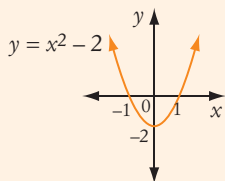
Mixed revision 4

- 1 a no b yes c yes
- 2 a $\frac{3}{2}$ b 7.5 m
- 3 0.8
- 4 a i $\frac{5}{9}$ ii $\frac{2}{27}$ iii $\frac{2}{3}$ iv $\frac{25}{27}$
 b $\frac{5}{9}$
- 5 625 m
- 6 a $\sqrt{26}$ units b 5.7 units
- 7 $PQ = \sqrt{26} = QR$ Since 2 sides are equal, it is an isosceles triangle.
- 8 (1, 0)
- 9 a $\frac{1}{5}$ b -1
- 10



- a 2 b 4
- 11 a $\frac{4}{15}$ b 107

12 a



b (0, -2) c concave up

13 a SAS

b AAS

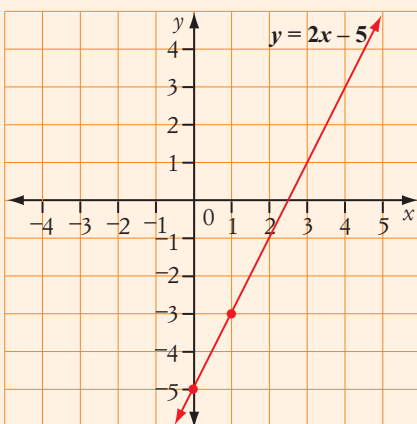
14 $y = 4$

15 a $\frac{1}{5}$

b $\frac{2}{3}$

16 a gradient $\frac{1}{2}$, y-intercept 3

b $y = 2x - 5$



17 a 188 kPa

b 125 m

18

		1st die					
		1	2	3	4	5	6
2nd die	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

a $\frac{1}{18}$

b $\frac{11}{36}$

c $\frac{1}{6}$

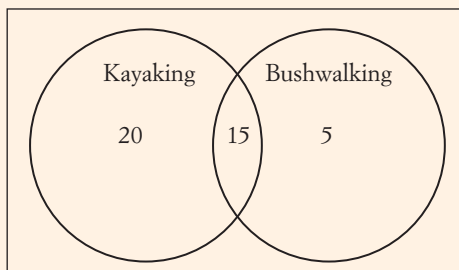
d $\frac{1}{2}$

19 a 3

b $y = 3x + 1$

20 $x^2 + y^2 = 16$

21 a



b i $\frac{3}{8}$

ii $\frac{5}{8}$

22 a Teacher to check (SSS).

b 90°

c bisect, right angles

23 1.8 m

24 a Teacher to check (equiangular or AA).

b i $6\frac{10}{13}$

ii $8\frac{6}{13}$

General Revision

1 a 288 cm^3 b 910 m^3 c 84 m^3 d 4536.5 mm^3

2 a $\sqrt{5}$ b $9\sqrt{3}$ c -140 d $18\sqrt{2}$

3 $t = \pm\sqrt{\frac{2my-2k}{m}}$ or $t = \pm\sqrt{2y - \frac{2k}{m}}$

4 a 23

b i 9

ii 6

iii 6.2

iv 6

c 1

d Yes, around 5 and 6.

5 a QD

b QD

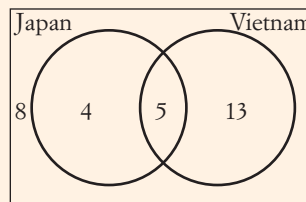
c QC

d C

6 a Teacher to check (SAS).

b $\angle CDT = \angle ABT$ (matching angles in congruent triangles, $\therefore DC \parallel AB$ (alternate angles are equal))

7 a



b i $\frac{4}{15}$

ii $\frac{17}{30}$

iii $\frac{13}{30}$

8 a 144 km/h

b 0.6 kg/min

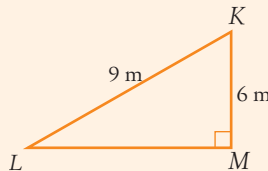
9 $y = 2x - 4$

10 a \$1571.48

b \$6833.33

11 a

b 6.71 m



12 a $(y-4)(y+2)$ b $(m-3)(m-5)$ c $5(m+7)(m-7)$

13 a 63.6 cm^2 b 160.2 cm^2 c 3.4 m^2

14 \$1008.15

15 C

16 a k^{12}

b d^5

c $15p^3q^4$

d $4k^4$

e $\frac{8p^3}{125}$

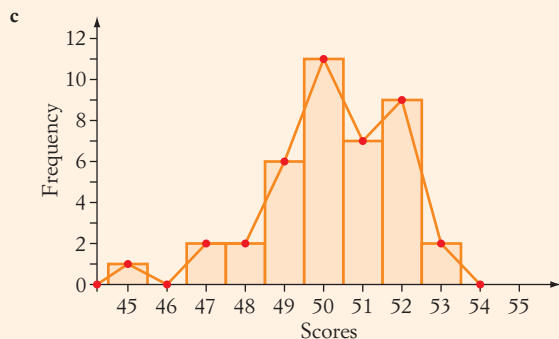
f 5

17 a

Score	Tally	f	fx
45		1	45
47		2	94
48		2	96
49		6	294
50		11	550
51		7	357
52		9	468
53		2	106
Total		40	2010

b 50.25

Answers



d 50

e 50

18 a $3 - 6y$

b $2x^2 - 7x$

c $-4m + 4m^2$

d $5x^2 - 13x - 28$

19 a $\frac{27}{125a^3}$ b $2\sqrt[4]{x}$ c $-2\sqrt[3]{p}$ d $(\sqrt[3]{r})^2$ or $\sqrt[3]{r^2}$

20 D

21 a 35.93 m

b 62.5 m²

22 D

23 \$761.10

24 a $2(y + 4)$

b $5a(1 - 3b)$

c $-2b(4a - 1)$

25 a 6.36

b 15.63

c 108.21

26 52.7%

27 a $m = 55$

b $e = 41$

c $k = 8$

28 $y = 5$

29 a $(\frac{1}{2}, 2\frac{1}{2})$

b $\sqrt{82}$

c $\frac{1}{9}$

30 2.5 cm

31 a rectangle

b equilateral triangle

c rectangle

d isosceles triangle

32 a 2

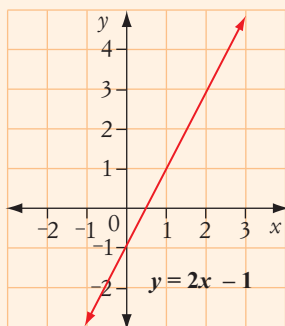
b 33

c negatively skewed

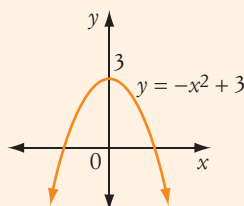
33 a equiangular (AA)

b $k = 13\frac{1}{5}$

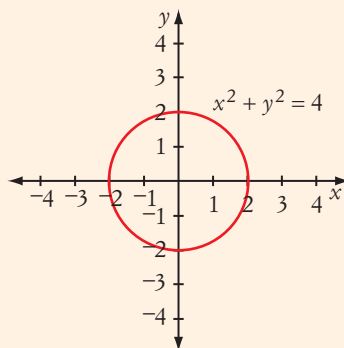
34 a i



ii



iii



b gradient 2, y-intercept -1

c (0, 3)

35 \$1925

36 \$88

37 B

38 a 396 cm²

b 476 cm²

c 792 cm²

39 a $m = 8\frac{2}{5}$

b $k = 4\frac{1}{2}$

c $y = 1\frac{1}{2}$

d $x = \pm 3$

e $x = 20$

f $a = 7\frac{1}{3}$

40 a 7.14×10^5

b 3.5×10^{-4}

c 1.5763×10^7

41 a 2.4×10^{15}

b 4.2×10^{-11}

42 a i 0.15

ii 0.17

iii 0.47

iv 0.64

v 0.70

b 180; the actual frequency of 173 is close to this.

43 a 35°30'

b 35°26'

c 43°21'

44 a 1080°

b 135°

c 45°

d 360°

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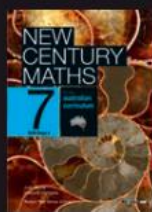
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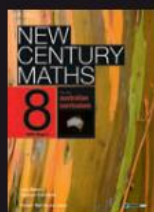
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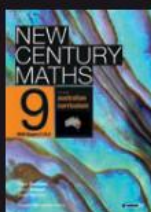
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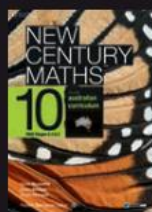
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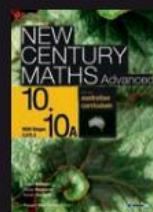
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