9

CambridgeMATHS NSW

STAGE 5 – CORE AND STANDARD PATHS

THIRD EDITION

Stuart Palmer, Karen McDaid
David Greenwood, Sara Woolley
Jenny Goodman, Jennifer Vaughan

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About the authors

Stuart Palmer was born and educated in NSW. He is a fully qualified high school mathematics teacher with more than 25 years' experience teaching students from all walks of life in a variety of schools. He has been Head of Mathematics in two schools. He is well known by teachers throughout the state for the professional learning workshops he delivers. Stuart also assists thousands of Year 12 students every year as they prepare for their HSC Examinations. At the University of Sydney, Stuart spent more than a decade running tutorials for pre-service mathematics teachers.

David Greenwood is the Head of Mathematics at Trinity Grammar School in Melbourne and has 25+ years' experience teaching mathematics from Years 7 to 12. He has run numerous workshops within Australia and overseas regarding the implementation of the Australian Curriculum and the use of technology for the teaching of mathematics. He has written more than 30 mathematics titles and has a particular interest in the sequencing of curriculum content and working with the Australian Curriculum proficiency strands.

Sara Woolley was born and educated in Tasmania. She completed an Honours degree in Mathematics at the University of Tasmania before completing her education training at the University of Melbourne. She has taught mathematics in Victoria from Years 7 to 12 since 2006 and is currently a Head of Mathematics. She has written more than 15 mathematics titles and specialises in lesson design and differentiation.

Jennifer Vaughan has taught secondary mathematics for more than 30 years in New South Wales, Western Australia, Queensland and New Zealand, and has tutored and lectured in mathematics at Queensland University of Technology. She is passionate about providing students of all ability levels with opportunities to understand and to have success in using mathematics. She has had extensive experience in developing resources that make mathematical concepts more accessible and hence, facilitating student confidence, achievement and an enjoyment of maths.

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Jenny Goodman has taught in schools for more than 25 years and is currently teaching at a selective high school in Sydney. Jenny has an interest in the importance of literacy in mathematics education, and in teaching students of differing ability levels. She was awarded the Jones Medal for Education at Sydney University and the Bourke Prize for Mathematics. She has written for *CambridgeMATHS NSW* and was involved in the *Spectrum* and *Spectrum Gold* series.

Karen McDaid has more that 20 years' experience teaching mathematics in primary and secondary schools, and as a lecturer teaching mathematics education to primary pre-service teachers at university. As an executive member and Past President of the Mathematics Association of NSW, she has been heavily involved in Mathematics K–10 Curriculum Consultation in NSW and across Australia. Karen co-authored the *CambridgeMATHS NSW GOLD* books Years 7 to 10. Karen is currently working in primary mathematics education at The Australian Catholic University.

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Introduction

The third edition of CambridgeMATHS NSW Stages 4 and 5 has been carefully prepared for the NSW Syllabus implemented from 2024 onwards. The series and is packed with new features. It is intended for students across a wide range of ability levels and has been designed to provide the best possible preparation for students as they make their way from the end of Stage 3 towards Stage 6 mathematics.

Coverage of Core and optional Path topics in Stage 5

As with the previous editions, Stage 5 is covered with two differentiated pairs of books:

- The *Core & Standard Paths* books replace the old Stage 5.1/5.2 books,
- The *Core & Advanced/Extension Paths* books replace the Stage 5.1/5.2/5.3 books.

Care has been taken to choose the content in each book carefully and to present it in a way that reflects the spirit of the new syllabus as well as the practicalities of the classroom. 'Paths', in the plural, refers to the fact that the Path topics are optional, and that a student's journey through Stage 5 may incorporate the core content in combination with none, some or all the Path topics in the books, depending upon the student's needs. Detailed teaching programs and Scope and Sequence documents are available via the Online Teaching Suite.

Beyond Stage 5

Students who have used these *Core & Standard Paths* books in Years 9 and 10 will be thoroughly prepared for Year 11 Mathematics Standard. These books are not recommended for students who have aspirations of studying Mathematics Advanced in Stage 6.

Learning Intentions, Success Criteria checklist, and 'Past, present and future learning'

Every lesson now includes a set of Learning Intentions that describe what the student can expect to learn in the lesson. At the end of the chapter, these appear again in the form of a checklist of Success Criteria; students can use this to check their progress through the chapter. Every criterion is listed with an example question to remind students of what the mathematics mentioned looks like. These checklists can also be downloaded and printed so that students can physically check them off as they accomplish their goals.

Also included at the beginning of each section, under the Learning Intentions, is a short set of dot points that place the topic at hand in the context of **past, present and future learning.**

Now you try

Every worked example now contains additional questions, without solutions, called 'Now you try', which give students immediate practice at the same type of question as the example. We also anticipate these questions will be useful for the teacher to do in front of the class, given that students will not have seen the solution beforehand.

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Working Mathematically tasks

A Working Mathematically activity now accompanies an Investigation in each chapter, with the goal of familiarising students with using the Working Mathematically process to define, solve, verify and then communicate their solutions when solving non-routine problems.

Maths@Work

An activity called 'Maths@Work' is now included at the end of every chapter. The intention of these activities is to show students how the maths learnt in the chapter is relevant to the workplace in a range of industries. The first part of the activity involves students doing the work by hand, while the second part involves the use of technology – often spreadsheets – to allow for greater efficiency, power and accuracy.

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Guide to the working programs in exercises

The suggested working programs in the exercises in the Year 9 and 10 Core & Standard Paths books provide two pathways to allow differentiation for Building and Progressing students (schools will have their own names for different ability levels or streams).

Each exercise is structured in subsections that match the working mathematically components of Understanding, Fluency, Problem-solving and Reasoning, as well as Enrichment (Challenge). In the exercises, the questions suggested for each pathway are listed in two columns at the top of each subsection:

- The left column (lightest shaded colour) is the Building pathway
- The right column (darkest shaded colour) is the Progressing pathway.

Building	Progressing
UNDERSTANDING	
1–4	4
FLUENCY	
5–7(½)	5–8(½)
PROBLEM-SOLVING AND REASONING	
9–10	10–12
ENRICHMENT: The missing y-intercept	
–	13

The working program for Exercise 5G in Year 9. The questions recommended for a Building student are: 1, 2, 3, 4, 5, 6, 7(½), 9 and 10.

Gradients within exercises and proficiency strands

The working programs make use of the two difficulty gradients contained within exercises. A gradient runs through the overall structure of each exercise – where there is an increasing level of mathematical sophistication required from Understanding to Fluency through to Problem-solving and Reasoning, and Enrichment – but also within each Working Mathematically component; the first few questions in Fluency, for example, are easier than the last Fluency question.

The right mix of questions

Questions in the working programs are selected to give the most appropriate mix of *types* of questions for each learning pathway. Students going through the Building pathway should use the left tab, which includes all but the hardest Understanding and Fluency questions as well as the easiest Problem-solving and Reasoning questions. The Progressing pathway, while not challenging, spends a little less time on basic Understanding questions and a little more time on Fluency and Problem-solving and Reasoning questions.

Choosing a pathway

There are a variety of ways to determine the appropriate pathway for students through the course. Schools and individual teachers should follow the method that works for them. If required, the Warm-up quiz at the start of each chapter can be used as a tool for helping students select a pathway. The following are recommended guidelines:

- A student who gets 40% or lower should heavily revise core concepts before attempting the Buildings questions.
- A student who gets above 40% and below 75% should complete the Building questions
- A student who gets 75% or higher should complete the Progressing questions.

∗ The nomenclature used to list questions is as follows:

- 3, 4: complete all parts of questions 3 and 4
- 1–4: complete all parts of questions 1, 2, 3 and 4
- 10(½): complete half of the parts from question 10 (a, c, e, ... or b, d, f, ...)
- 2–4(½): complete half of the parts of questions 2, 3 and 4
- 4(½), 5: complete half of the parts of question 4 and all parts of question 5
- — : complete none of the questions in this section.

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Guide to this resource

PRINT TEXTBOOK FEATURES

1 **NSW Syllabus:** content strands, outcome groups and outcomes are listed at the beginning of the chapter to assist with course planning (see teaching programs and Syllabus mapping grids for more detailed guidance)

2 **Chapter introduction:** sets context for students about how the topic connects with the real world and the history of mathematics

3 **Warm-up quiz:** a quiz for students on the prior knowledge and essential skills required before beginning the chapter

4 NEW **Learning intentions:** sets out what a student will be expected to learn in the lesson

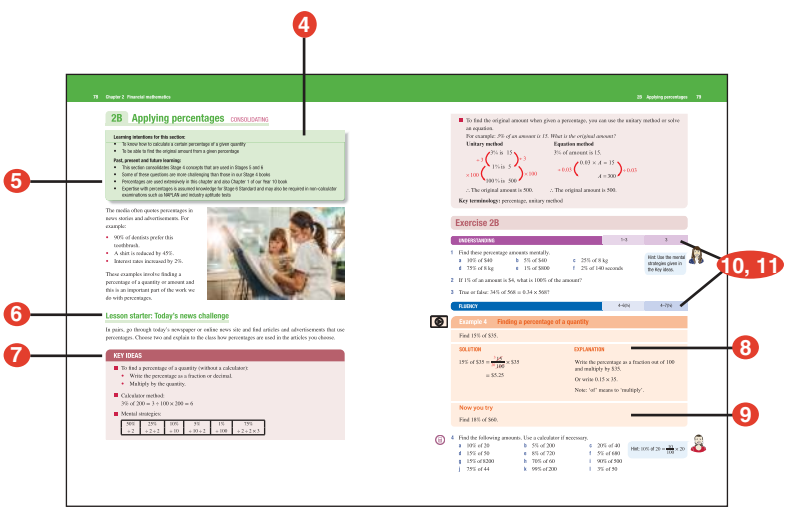
5 NEW **Past, present and future learning**: shows how the lesson builds on previous or extends to future lessons in the context of the syllabus

6 **Lesson starter:** an activity, which can often be done in groups, to start the lesson

7 **Key ideas:** summarises the knowledge and skills for the lesson

8 **Worked examples:** solutions and explanations of each line of working, along with a description that clearly describes the mathematics covered by the example

9 NEW **Now you try:** try-it-yourself questions provided after every worked example in exactly the same style as the worked example to give immediate practice



10 **Gentle start to exercises:** the exercise begins at Understanding to ensure comprehension of the lesson's Key Ideas before progressing to the other Working Mathematically components

11 **Working programs:** differentiated question sets for two ability levels in exercises

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12 NEW **Progress quiz** about two-thirds of the way through each chapter provides a check-in to establish how students are going with the topic so far

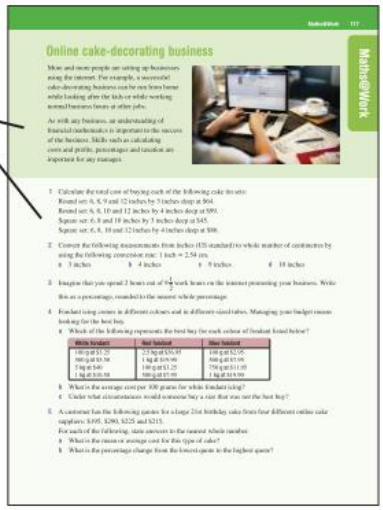
13 NEW **Maths@Work:** a set of questions that gives practice at applying the mathematics of the chapter to real-life contexts

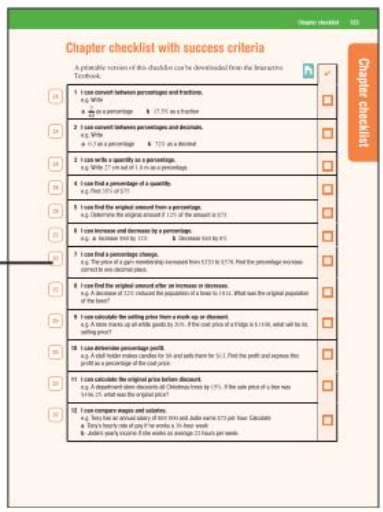
14 NEW **Working Mathematically investigative tasks:** in each chapter that apply the Working Mathematically process to the topic to give students guided practice

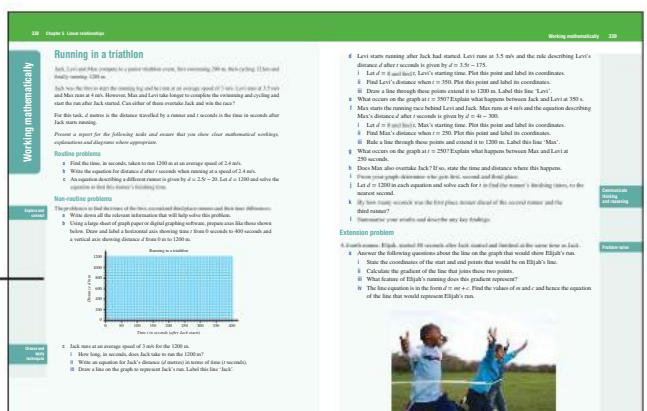
15 **Puzzles and challenges:** in each chapter provide problem-solving practice in the context of puzzles and challenges connected with the topic

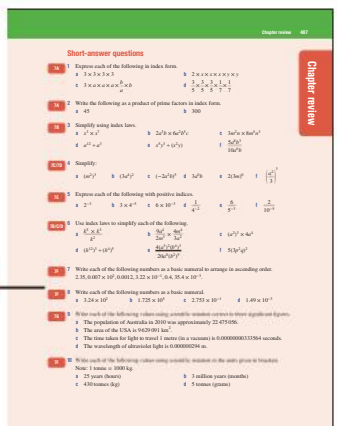
16 NEW **Chapter checklist with success criteria:** a checklist of the learning intentions for the chapter, with example questions

17 **Chapter reviews:** with short-answer, multiple-choice and extended-response questions; questions that are extension are clearly signposted









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INTERACTIVE TEXTBOOK FEATURES

18 NEW **Targeted Skillsheets**, one for each lesson, focus on a small set of related Fluency-style skills for students who need extra support, with questions linked to worked examples

19 **Workspaces:** almost every textbook question – including all working-out – can be completed inside the Interactive Textbook by using either a stylus, a keyboard and symbol palette, or uploading an image of the work

20 **Self-assessment:** students can then self-assess their own work and send alerts to the teacher. See the Introduction on page xi for more information.

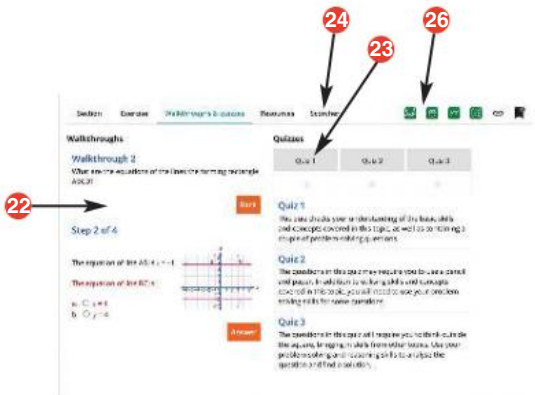
21 **Interactive working programs** can be clicked on so that only questions included in that working program are shown on the screen

22 **HOTmaths resources:** a huge catered library of widgets, HOTsheets and walkthroughs seamlessly blended with the digital textbook

23 A revised set of **differentiated auto-marked practice quizzes** per lesson with saved scores

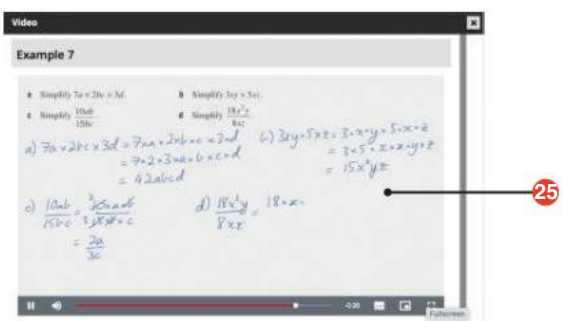
24 **Scorcher:** the popular competitive game

25 **Worked example videos:** every worked example is linked to a high-quality video demonstration, supporting both in-class learning and the flipped classroom



26 **Desmos graphing calculator**, scientific calculator and geometry tool are always available to open within every lesson

27 **Desmos interactives:** a set of Desmos activities written by the authors allow students to explore a key mathematical concept by using the Desmos graphing calculator or geometry tool



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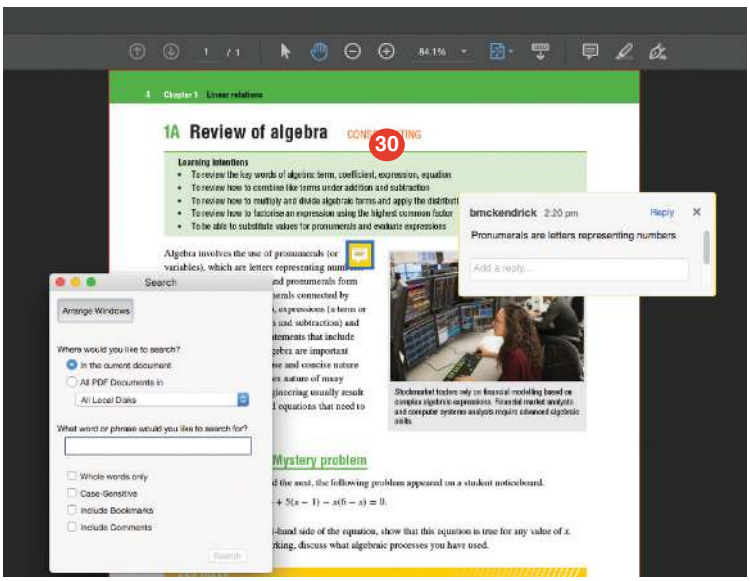
28 **Auto-marked warm-up quiz** for testing the knowledge that students will need before starting the chapter

29 **Auto-marked progress quizzes and chapter review multiple-choice questions** in the chapter reviews can now be completed online

DOWNLOADABLE PDF TEXTBOOK

30 In addition to the Interactive Textbook, a **PDF version of the textbook** has been retained for times when users cannot go online. PDF search and commenting tools are enabled.



ONLINE TEACHING SUITE

31 NEW **Diagnostic Assessment Tool** included with the Online Teaching Suite allows for flexible diagnostic testing, reporting and recommendations for follow-up work to assist you to help your students to improve

32 NEW **PowerPoint lesson** summaries contain the main elements of each lesson in a form that can be annotated and projected in front of class

33 **Learning Management System** with class and student analytics, including reports and communication tools

34 **Teacher view of student's work and self-assessment** allows the teacher to see their class's workout, how students in the class assessed their own work, and any 'red flags' that the class has submitted to the teacher

35 **Powerful test generator** with a huge bank of levelled questions as well as ready-made tests

36 **Revamped task manager** allows teachers to incorporate many of the activities and tools listed above into teacher-controlled learning pathways that can be built for individual students, groups of students and whole classes

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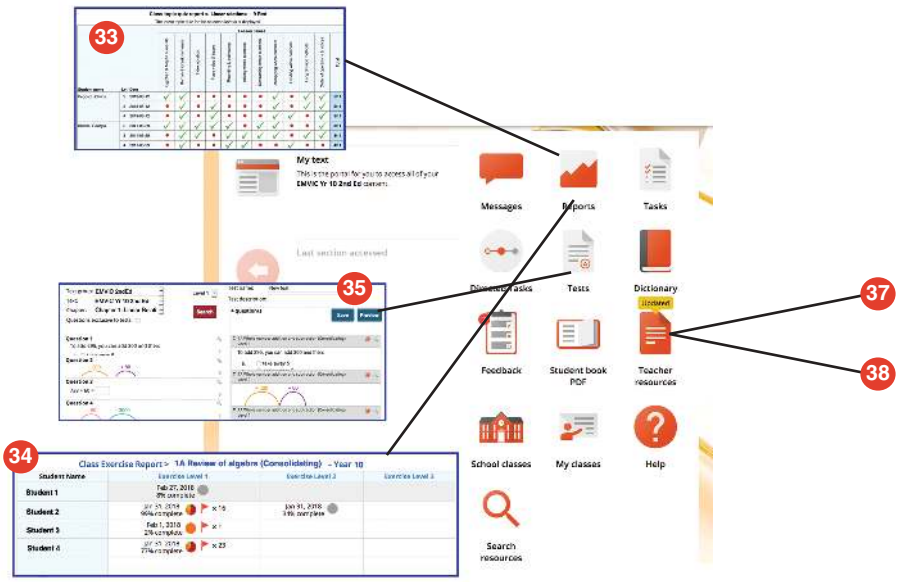
37 **Worksheets and two differentiated chapter tests in every chapter,** provided in editable Word documents

38 **More printable resources:** all warm-up quizzes, Progress quizzes and Working Mathematically tasks are provided in printable worksheet versions



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1

Integers, decimals, fractions, ratios and rates

Maths in context: Number skills in cooking, cycling and manufacturing

Number skills are essential for trades, professions and many practical tasks. Negative numbers are used for temperatures and in financial calculations. Skills with rates, ratios and fractions are applied in numerous occupations, for example:

- Cooks and chefs regularly make calculations using rates, fractions and ratios when adapting recipes.
- Competitive cyclists select gear ratios to allow the maximum possible acceleration and speed for various race conditions.
- Fishermen and power boat owners mix petrol and oil in a ratio of 50 : 1 for outboard motor fuel.
- Jewellers can mix gold, copper and silver in a ratio of 15 : 4 : 1 for a rose gold ring, necklace or bracelet.
- Builders mix cement, sand and gravel in a ratio of 1 : 2 : 4 for driveway concrete and they mix cement and sand in a ratio of 1 : 3 for swimming pool concrete.

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Rate calculations are essential for farmers to efficiently manage water usage. Rates include pump flow rates (litres per second); drip irrigation rates (litres per hour); travelling irrigator rates (acres per hour); and irrigation frequency rates (number of times irrigation occurs per week).

Chapter contents

1A Adding and subtracting positive and negative integers (CONSOLIDATING)
1B Multiplying and dividing positive and negative integers (CONSOLIDATING)
1C Decimal places and significant figures
1D Rational numbers and irrational numbers (CONSOLIDATING)
1E Adding and subtracting fractions (CONSOLIDATING)
1F Multiplying and dividing fractions (CONSOLIDATING)
1G Ratios (CONSOLIDATING)
1H Rates (CONSOLIDATING)

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- compares, orders and calculates with integers to solve problems (MA4-INT-C-01)
- represents and operates with fractions, decimals and percentages to solve problems (MA4-FRC-C-01)
- solves problems involving ratios and rates, and analyses distance–time graphs (MA4-RAT-C-01).

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Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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Warm-up quiz

1 Arrange the following mathematical terms under four headings: 'Addition', 'Subtraction', 'Multiplication' and 'Division'.

a Sum	**b** Total	**c** Less than	**d** Lots of
e Product	**f** Into	**g** Take away	**h** Difference
i Add	**j** Times	**k** Minus	**l** More than
m Quotient	**n** Less	**o** Share	**p** Increase

2 Without using a calculator, find an answer to each of the following.

a 16 less 12	**b** 24 more than 8
c the difference between 12 and 8	**d** increase 45 by 7
e the total of 40, 34 and 0	**f** 9 into 45
g the quotient of 63 and 7	**h** 480 shared between 12

3 Evaluate the following.

a $9 + 47$	**b** $135 - 35$	**c** $19 - 19$	**d** $56 + 89 - 12$
e 9×7	**f** $320 \div 4$	**g** 17×60	**h** $200 - 47 - 100$

4 Use a number line to find:

a $-5 - 7$	**b** $12 - 15$	**c** $-6 + 9$	**d** $-12 + 12$
e $16 - 17$	**f** $-4 + 3$	**g** $-7 + 4 + 3$	**h** $-4 - 4 - 4$

5 Copy and complete each of the following statements.

a $5 + 5 + 5 = \square \times 5$	**b** $-6 - 6 - 6 = \square \times (-6)$
c $9 - (+10) = 9 \square 10$	**d** $12 - (-2) = 12 \square 2$

6 The population of Australia in 2050 is projected to be 26 073 258. Round this number to the nearest:

a ten	**b** hundred	**c** thousand	**d** million

7 Write down the place value of the 5 in each of the following numbers.

a 1256	**b** 345	**c** 5049
d 0.56	**e** 0.15	**f** 9.005

8 Arrange the numbers in each of the following sets in descending order (largest to smallest).

a 2.645, 2.654, 2.465 and 2.564	**b** 0.456, 0.564, 0.0456 and 0.654

9 Evaluate each of the following.

a $4.26 + 3.73$	**b** $3.12 + 6.99$	**c** $10.89 - 3.78$

10 Evaluate:

a 7×0.2	**b** 0.3×0.2	**c** 2.3×1.6
d 4.2×3.9	**e** $14.8 \div 4$	**f** $12.6 \div 0.07$

11 Evaluate each of the following.

a 0.345×100	**b** $3.74 \times 100\,000$	**c** $37.54 \div 1000$	**d** $3.7754 \div 100\,000$

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12 Complete these equivalent fractions.

a $\frac{1}{2} = \frac{\square}{12}$ **b** $\frac{3}{4} = \frac{\square}{16}$ **c** $\frac{5}{6} = \frac{25}{\square}$ **d** $\frac{\square}{9} = \frac{2}{18}$

13 Find the lowest common denominator for these pairs of fractions.

a $\frac{1}{3}$ and $\frac{1}{5}$ **b** $\frac{1}{6}$ and $\frac{1}{4}$ **c** $\frac{1}{5}$ and $\frac{1}{10}$

14 Find:

a $\frac{3}{7} + \frac{2}{7}$ **b** $2 - \frac{3}{4}$ **c** $4 \div \frac{1}{2}$ **d** $\frac{3}{4} \times \frac{1}{2}$

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1A Adding and subtracting positive and negative integers CONSOLIDATING

Learning intentions for this section:

- To review the use of the number line for addition and subtraction
- To evaluate expressions involving addition and subtraction of integers

Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- In Chapter 3 of this book, we apply this learning to algebraic expressions
- Expertise with integers is assumed knowledge for Stage 6 Standard and may also be required in non-calculator examinations such as NAPLAN and industry aptitude tests

Integers are the set of positive and negative whole numbers, as well as the number zero.

Being able to work with whole numbers is very important, since whole numbers are used every day for counting, ordering, calculating, measuring and computing.

Lesson starter: Naming groups

Here are some groups of numbers. In groups of two or three, use the correct mathematical terms to describe each group. (Suggestions include 'multiples of', 'factors of', 'integers', 'squares' and 'cubes'.)

- 2, 4, 6, 8, …
- 1, 4, 9, 16, …
- 1, 3, 5, 7, 9, …
- 1, 2, 3, 4, 5, 6, …
- −1, −2, −3, −4, −5, …
- 1, 8, 27, 64, …
- 1, 2, 3, 4, 6 and 12

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KEY IDEAS

- **Integers** are the positive and negative whole numbers, including zero: $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$
- To add a negative number, you subtract its opposite: $a + (-b) = a - b$
  e.g. $5 + (-7) = 5 - 7$
  $-6 + (-2) = -6 - 2$
- To subtract a negative number, you add its opposite: $a - (-b) = a + b$
  e.g. $5 - (-7) = 5 + 7$
  $-6 - (-2) = -6 + 2$
- Adding or subtracting zero leaves a number unchanged.
  $a + 0 = a$ e.g. $5 + 0 = 5$
  $a - 0 = a$ e.g. $5 - 0 = 5$

Key terminology: integer, positive, negative

Exercise 1A

UNDERSTANDING 1–3 3

1 Match each sentence to the correct expression.

a	The sum of 5 and 7	**i**	$5 + (-7)$
b	The total of negative 5 and 7	**ii**	$5 - (-7)$
c	The difference between negative 5 and 7	**iii**	$5 + 7$
d	The sum of 5 and negative 7	**iv**	$7 - (-5)$
e	The difference between 5 and negative 7	**v**	$-5 + 7$

2 Match each question on the left to an expression or answer on the right.

a	$10 + (-7)$	**i**	$-10 - 7$
b	$-10 - (-7)$	**ii**	$-10 + 7$
c	$10 - (-7)$	**iii**	0
d	$-10 + (-7)$	**iv**	$10 - 7$
e	$-10 + 10$	**v**	$10 + 7$

3 Are the following true (T) or false (F)?

a $18 + 0 = 18$
b $6 - (-4) = 6 + 4$
c $4 - (-2) = 4 - 2$
d $9 + 8 = 8 + 9$
e $6 - (-4) = 6 + 4$
f $4 - (-2) = 4 - 2$

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FLUENCY 4–8(½) 5–8(½)

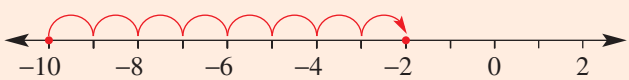
Example 1 Using a number line for addition and subtraction of integers

Use a number line to find:

a $-10 + 8$ **b** $-3 - 5$

SOLUTION

a



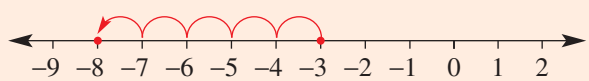
$-10 + 8 = -2$

EXPLANATION

Draw a number line showing -10 on it.

Start at -10 and count up (for addition) eight places to finish at -2.

b



$-3 - 5 = -8$

Draw a number line showing -3 on it.

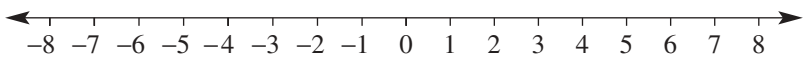
Count down (for subtraction) five places to -8.

Now you try

Use a number line to find:

a $-7 + 4$ **b** $-6 - 4$

4 Use the number line below to help you find the answers to the following.



Hint: Move right for addition and left for subtraction.

a $-6 + 4$ **b** $-6 + 8$ **c** $0 - 3$

d $-1 + 5$ **e** $6 - 10$ **f** $-2 - 3$

g $-1 - 1$ **h** $-3 + 3$ **i** $-3 - 3$

5 Use a number line to evaluate the following.

a $9 - 6$ **b** $9 - 7$ **c** $9 - 8$

d $9 - 9$ **e** $9 - 10$ **f** $-9 - 0$

g $-9 + 8$ **h** $-9 - 8$ **i** $-6 - 4$

j $-3 + 12$ **k** $-8 + 6$ **l** $-1 + 12$

m $-12 + 5$ **n** $-10 - 8$ **o** $18 - 19$

6 Find:

Hint: Work from left to right.

a $4 + 8 - 7$ **b** $5 + 6 + 9$ **c** $-19 - 1$

d $-9 + 8 + 1$ **e** $-12 - 3 + 8$ **f** $8 + 3 - 5$

g $-12 - 12$ **h** $15 + 5 - 15$ **i** $-6 - 5 - 4$

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Example 2 Adding a negative integer

Find $17 + (-12)$

SOLUTION

$17 + (-12) = 17 - 12$
$= 5$

EXPLANATION

Adding a negative is the same as subtraction:
$17 + (-12) = 17 - 12$

Now you try

Find $12 + (-8)$

7 Find:

a $9 + (-5)$ **b** $12 + (-16)$ **c** $3 + (-7)$
d $15 + (-24)$ **e** $-9 + (-23)$ **f** $-13 + (-25)$
g $-100 + (-89)$ **h** $56 + (-80)$ **i** $-9 + (-9)$
j $18 + (-18)$ **k** $-245 + (-560)$ **l** $98 + (-155)$
m $-89 + (-78)$ **n** $145 + (-3)$ **o** $-567 + (-237)$

Hint:
$9 + (-5) = 9 - 5$
$-9 + (-23) = -9 - 23$

Example 3 Subtracting a negative integer

Find $-13 - (-9)$

SOLUTION

$-13 - (-9) = -13 + 9$
$= -4$

EXPLANATION

Subtracting a negative is the same as addition:
$-13 - (-9) = -13 + 9$

Now you try

Find $-18 - (-10)$

8 Find:

a $-8 - (-6)$ **b** $-5 - (-9)$ **c** $7 - (-6)$
d $2 - (-7)$ **e** $12 - (-12)$ **f** $-34 - (-34)$
g $-35 - (-7)$ **h** $-90 - (-9)$ **i** $90 - (-90)$
j $-68 - (-70)$ **k** $-90 - (-87)$ **l** $234 - (-6)$
m $670 - (-85)$ **n** $-6 - (-100)$ **o** $-230 - (-240)$

Hint: $-5 - (-9) = -5 + 9$

PROBLEM-SOLVING AND REASONING 9, 10 | 9–10(½), 11, 12

9 Simplify each of these using the previous rules. Check your answers with a calculator.

a $-9 - 9 - 9$ **b** $-23 - (-8) + 12$
c $50 - 46 - (-6)$ **d** $24 - (-8) + (-6)$
e $-20 - (-5) - (-15)$ **f** $-18 - (-6) + (-12)$
g $-90 - (-89) - 90$ **h** $125 - 35 - (-35)$

Hint: Work from left to right.

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10 Copy and complete each of the following statements.

a $\square + 6 = 8$
b $-6 + \square = 5$
c $12 - \square = 15$
d $-9 + \square = -10$
e $13 - \square = 20$
f $10 - \square = -1$

11 What must be added to each of the following to obtain a final result of zero?

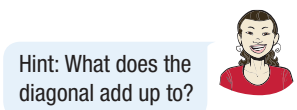
a 8 **b** $-7 + 3$ **c** $-8 - 5$
d -124 **e** 19 **f** 0
g $12 - 8 + 18$ **h** -98 **i** $12 - (-14)$

12 The temperature on a mountain top reaches a maximum of $-2°C$ during the day. By night it has dropped $10°C$. What is the night-time temperature?

ENRICHMENT: Magic squares with negatives — 13

13 In a magic square, each row, column and diagonal add to the same number.

a Copy and complete this magic square.



-8		
	-2	-6
		4

b Arrange these 9 integers into a 3 by 3 magic square.
$-13, -10, -7, -4, -1, 2, 5, 8$ and 11

c Complete this 4 by 4 magic square.

5			
0	10		6
-1	4	2	8
		7	-4

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1B Multiplying and dividing positive and negative integers CONSOLIDATING

Learning intentions for this section:

- To review the rules for multiplying and dividing integers
- To evaluate expressions involving multiplication, division and powers of integers
- To be able to apply order of operations to expressions involving integers

Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- In Chapter 3 of this book, we apply this learning to algebraic expressions
- Expertise with integers is assumed knowledge for Stage 6 Standard and may also be required in non-calculator examinations such as NAPLAN and industry aptitude tests

Multiplication is a way of writing repeated addition. It is possible to develop rules for multiplying and dividing with negative integers.

Lesson starter: Repeated addition

1 Write each of the following as a multiplication before finding the answer.
   a $4 + 4 + 4$
   b $(-4) + (-4) + (-4)$
   c $5 + 5 + 5 + 5$
   d $(-5) + (-5) + (-5) + (-5)$
   e $(-7) + (-7) + (-7) + (-7) + (-7)$
   f $(-8) + (-8)$

2 Use your results from Question 1 to answer these divisions.
   a $12 \div 4$
   b $-12 \div (-4)$
   c $-12 \div 3$
   d $-20 \div 4$
   e $-20 \div (-5)$
   f $20 \div 5$
   g $-35 \div (-7)$
   h $-35 \div 5$
   i $-16 \div (-8)$

3 What can you conclude about dividing a negative number by:
   a a positive number?
   b a negative number?

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KEY IDEAS

- When multiplying two numbers with the same sign, the product is a positive number. (If the signs are the same, the product is positive.)
  positive × positive = positive
  negative × negative = positive
  e.g. $-5 \times (-4) = 20$ and $-18 \div (-6) = 3$
- When multiplying two numbers with different signs, the product is a negative number.
  positive × negative = negative
  negative × positive = negative
  e.g. $-5 \times 4 = -20$ and $18 \div (-6) = -3$
- The same rules apply for division:
  positive ÷ positive = positive
  negative ÷ negative = positive
  positive ÷ negative = negative
  negative ÷ positive = negative
- The **square** of a number is the number multiplied by itself, e.g. $5^2 = 5 \times 5$. The **cube** of a number is the product of a number multiplied by itself twice, e.g. 7^3 is $7 \times 7 \times 7$.
- On your calculator, you may need to use brackets when finding powers of negative numbers. For example, $(-13)^2 = 169$ (not -169)
- For cube numbers $-2^3 = (-2)^3 = -8$
  since $-2 \times (-2) \times (-2) = -8$
- Order of operations
  - Deal with brackets first
  - Do multiplication and division next, working from left to right
  - Do addition and subtraction last, working from left to right

Key terminology: integer, square, cube

Exercise 1B

UNDERSTANDING 1–3 3

1 Complete the following statements.

a A negative number multiplied by a negative number equals a _____ number.

b A negative number multiplied by a positive number equals a _____ number.

c A negative number divided by a negative number equals a _____ number.

d A positive number divided by a negative number equals a _____ number.

e A negative number divided by a positive number equals a _____ number.

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2 Would these products be positive or negative?

a $2 \times (-8)$ **b** $78 \times (-1)$ **c** $56 \div (-2)$

d $-90 \div (-10)$ **e** 8×12 **f** $-18 \times (-9)$

3 Copy and complete this table

×	−4	−3	−2	−1	0	1	2	3	4
−4									
−3									
−2									
−1									
0									
1									
2									
3									
4									

FLUENCY 4–7(½) 4–7(½)

Example 4 Multiplying with negatives

Find the value of:

a $-6 \times (-7)$ **b** $8 \times (-12)$

SOLUTION

a $-6 \times (-7) = 42$

EXPLANATION

The product of two negative numbers gives a positive answer.

$- \times - = +$

b $8 \times (-12) = -96$

The product of a positive number and a negative number results in a negative answer.

$+ \times - = -$

Now you try

Find the value of:

a $-8 \times (-4)$ **b** -5×7

4 Find these products. Some may need to be done with a calculator.

a -8×6 **b** -10×2 **c** -9×8

d $-9 \times (-8)$ **e** $-15 \times (-3)$ **f** $45 \times (-2)$

g $100 \times (-6)$ **h** $-9 \times (-9)$ **i** -12×12

j -13×11 **k** $11 \times (-13)$ **l** $34 \times (-1)$

m $-24 \times (-3)$ **n** $-18 \times (-5)$ **o** $36 \times (-3)$

Hint: $+ \times + = +$
$- \times - = +$
$+ \times - = -$
$- \times + = -$

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Example 5 Finding squares and cubes of negative numbers

Find the value of:

a $(-5)^2$ **b** $(-6)^3$

SOLUTION

a $(-5)^2 = 25$

EXPLANATION

$(-5)^2$ is the same as $-5 \times (-5)$

The product of two negative numbers gives a positive answer.

$- \times - = +$

b $(-6)^3 = -216$

$(-6)^3 = -6 \times (-6) \times (-6)$

Multiply the first two numbers together: $-6 \times (-6) = 36$

Now: $36 \times (-6) = -216$

Now you try

Find the value of:

a $(-8)^2$ **b** $(-3)^3$

5 Find the answers to each of the following. Use a calculator to check your answers.

a $(-6)^2$ **b** $(-8)^2$ **c** $(-12)^2$

d $(-3)^2$ **e** $(-10)^2$ **f** $(-13)^2$

g $(-2)^3$ **h** $(-1)^3$ **i** $(-4)^3$

Hint: $(-6)^2 = -6 \times (-6)$
$(-2)^3 = -2 \times (-2) \times (-2)$

Example 6 Dividing with a negative number

Find the value of:

a $-24 \div 8$ **b** $-45 \div (-3)$

SOLUTION

a $-24 \div 8 = -3$

EXPLANATION

The division of two numbers with different signs gives a negative answer.

$- \div + = -$

b $-45 \div (-3) = 15$

The division of two numbers with the same sign gives a positive answer.

$- \div - = +$

Now you try

Find the value of:

a $-30 \div 6$ **b** $-28 \div (-7)$

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6 Perform the following divisions. Use a calculator if necessary.

a $18 \div (-2)$	**b** $-36 \div (-6)$	**c** $-100 \div (-10)$
d $-50 \div 5$	**e** $-6 \div (-2)$	**f** $-10 \div (-2)$
g $-50 \div (-25)$	**h** $16 \div (-16)$	**i** $24 \div (-3)$
j $-100 \div (-4)$	**k** $312 \div (-3)$	**l** $-45 \div (-9)$
m $185 \div (-5)$	**n** $-428 \div 2$	**o** $-156 \div (-12)$

Hint: $+ \div + = +$
$- \div - = +$
$+ \div - = -$
$- \div + = -$

Example 7 Applying the order of operations

Find the value of $9 \times (-8) + (5 - 12)$

SOLUTION	EXPLANATION
$9 \times (-8) + (5 - 12)$	Work out the value of the brackets first: $(5 - 12) = (-7)$
$= 9 \times (-8) + (-7)$	Multiplication is next: $9 \times (-8) = -72$
$= -72 + (-7)$	The addition of a negative is the same as subtraction:
$= -79$	$-72 + (-7) = -72 - 7$

Now you try

Find the value of $8 \div (-2) \times (-8 + 5)$

7 Follow the order of operations and evaluate:

a $(7 - 3) \times 4$	**b** $(4 + 9) \times (-2)$
c $(6 - 2) \times (-3)$	**d** $-6 + 4 \times (-2)$
e $10 \div (-2) \times 4$	**f** $(-5 - 2) \times 4$
g $6 \times (-3) \times 4$	**h** $-21 \div 7 + (-5)$
i $-45 - 9 \times 5$	**j** $(4 - 6) \times (7 - 12)$
k $(15 - 9) \times (3 - 7)$	**l** $9 + 9 \times (-3)$
m $-(3 - 12)$	**n** $10 - 9 \times 4$

Hint: The order of operations:
- brackets first, following the order of operations within them
- then $\times$ and $\div$, working from left to right
- then $+$ and $-$, working from left to right.

PROBLEM-SOLVING AND REASONING 8, 9 | 10–12

8 Copy and complete:

a $-5 \times \square = -35$	**b** $\square \div (-4) = 8$	**c** $-10 \times \square = 200$
d $17 \times \square = -68$	**e** $34 \div \square = -34$	**f** $-6 \times \square = -36$
g $\square \div 9 \times (-3) = 3$	**h** $15 \div \square \div 3 = -1$	**i** $-15 \times \square = 225$

9 The sum of two numbers is -3 and their product is -10. What are the two numbers?

10 Give the value of two different numbers that when squared each produce an answer of:

a 144	**b** 64	**c** 10 000

11 Explain why $-5 \times 4 \div (-3)$ produces a positive answer.

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12 Decide whether each of the following would produce a positive or a negative answer.

a $-6 \times (-4) \times 5$

b $-12 \div 4 \times 9 \div (-3)$

c $-8 \div (-2) \times 5 \times (6 - 9)$

d $(-3)^4$

e $(-1)^{201}$

f $-(-5)^3$

ENRICHMENT: Substitutions involving negatives — 13, 14

13 Evaluate these expressions by substituting $a = -2$, $b = 6$ and $c = -3$.
Check that you can get the same answer with a calculator.

a $a^2 - b$

b $a - b^2$

c $2c + a$

d $b^2 - c^2$

e $a^3 + c^2$

f $3b + ac$

g $c - 2ab$

h $abc - (ac)^2$

14 Use a calculator to evaluate the following, where $x = -45$ and $y = -15$.

a $x + y$ **b** $x - y$ **c** xy **d** $x - 3y$

e $x^2 + y^2$ **f** $x^2 - y^2$ **g** $(x + y)^2$ **h** $3y^2$

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1C Decimal places and significant figures

Learning intentions for this section:

- To consolidate the process of rounding off decimals to any number of decimal places
- To introduce and understand the concept of significant figures
- To be able to round a decimal or whole number to any number of significant figures

Past, present and future learning:

- This section consolidates Stage 4 decimals and introduces Stage 5 significant figures
- Section 7G of this book extends significant figures to scientific notation
- From here onwards, you may be required to round off numbers in two different ways
- This concept will be revised briefly in Stage 6 Standard and may also be required in non-calculator examinations such as NAPLAN and industry aptitude tests

Numbers with and without decimal places can be rounded. The time for a 100 m sprint race, for example, might be 9.94 seconds, correct to two decimal places.

Another way of rounding numbers is to use significant figures. The number of cubic metres of gravel required for a road, for example, might be calculated as 3485 but is rounded to 3500. This number is written using two significant figures.

Lesson starter: In the middle

For each number given on the left, decide which of the two numbers on the right it is closest to.

a	84	80 or 90
b	856	800 or 900
c	856	850 or 860
d	0.599	0.5 or 0.6
e	1.2099	1.2 or 1.3
f	1.2099	1.20 or 1.21
g	89 543	89 000 or 90 000
h	0.035	0.03 or 0.04

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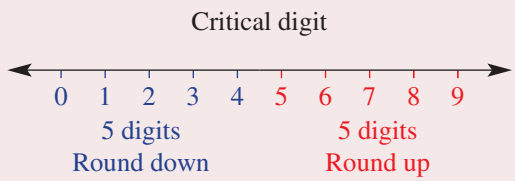
KEY IDEAS

- **Rounding** involves approximating a decimal number using fewer digits
- To round a number to a required number of **decimal places** (number of digits after the decimal point):
  - Locate the digit in the required decimal place.
  - Round down (leave as is) if the next digit (**critical digit**) is 4 or less.
  - Round up (increase by 1) if the next digit is 5 or more.



For example:
  - To two decimal places, 1.54③ rounds to 1.54 and 32.92⑧3 rounds to 32.93.
  - To one decimal place, 0.2④8 rounds to 0.2 and 0.2⑤3 rounds to 0.3.
- The symbols ≒ and ≈ mean 'approximately equal to'
- To round a number to a required number of **significant figures**:
  - Locate the first *non-zero* digit counting from left to right.
  - From this first significant digit, count out the number of digits, including zeros.
  - Stop at the required number of digits and round this last digit.
  - Replace any non-significant digits to the left of a decimal point with a zero.

For example, these numbers are all rounded to three significant figures:
2.53⑨1 ≈ 2.54 0.00271③ ≈ 0.00271 568⑧10 ≈ 569 000

Key terminology: decimal places, critical digit, significant figure

Exercise 1C

UNDERSTANDING | 1–4 | 4

1 How many decimal places do each of these numbers have?
a 1.467 **b** 146.1 **c** 0.08 **d** 47 900

2 How many significant figures do each of these numbers have?
a 0.46 **b** 21.46 **c** 0.08 **d** 4906

Hint: Count the digits starting from the first non-zero digit. These are the significant ones.

3 The critical digit is circled. Would you round the next digit up or down?
a 2.75④37 **b** 0.87⑦2 **c** 21.5⑤4

4 If the given number is rounded to three significant figures:
a is 32 124 rounded to 321, 3210 or 32 100? **b** is 431.92 rounded to 431, 432 or 430?
c is 5.8871 rounded to 5.887, 5.88 or 5.89? **d** is 0.44322 rounded to 0.44, 0.443 or 0.44302?
e is 0.0019671 rounded to 0.002, 0.00197 or 0.00196?

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FLUENCY 5–6(½), 7, 8 | 5–6(½), 7, 8, 9(½)

Example 8 Rounding to a number of decimal places

Round each of these to two decimal places.

a 256.1793 b 0.04459 c 4.8972

SOLUTION	EXPLANATION
a 256.1793 ≈ 256.18	256.1793 The number after the second decimal place is 9, so round up (increase the 7 by 1).
b 0.04459 ≈ 0.04	0.04459 The number after the second decimal place is 4, so round down.
c 4.8972 ≈ 4.90	4.8972 The number after the second decimal place is 7, so round up. 89 rounds to 90

Now you try

Round each of these to two decimal places.

a 107.3874 b 0.0321 c 2.7956

5 Round each of the following numbers to two decimal places.

a 17.962	b 11.082	c 72.986
d 47.859	e 63.925	f 23.807
g 804.5272	h 500.5749	i 821.2749
j 5810.2539	k 1004.9981	l 2649.9974

Hint: The critical digit will be the third decimal place.

Example 9 Rounding to a number of significant figures

Round each of these numbers to two significant figures.

a 2567 b 23 067.453 c 0.04059

SOLUTION	EXPLANATION
a 2567 ≈ 2600	**25**67 Significant digits The next digit is 6, so round up. Replace last 2 digits with zeros to maintain place value.

Continued on next page

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b $23\,067.453 \approx 23\,000$

23067.453

Significant digits

The next digit is 0, so round down.

Replace the remaining digits, in front of the decimal point, with zeros.

c $0.04059 \approx 0.041$

Locate the first non-zero digit and count two digits:

0.0**40**59

Significant digits

The next digit is 5, so round up.

No extra zeros are needed, as they are after the decimal point.

Now you try

Round each of these numbers to two significant figures.

a 3472 **b** 1792.41 **c** 0.00226

6 Round each of these numbers to two significant figures.

a 2436	**b** 35 057.4	**c** 0.06049
d 34.024	**e** 107 892	**f** 0.00245
g 2.0745	**h** 0.7070	**i** 4706
j 59 134	**k** 0.4567	**l** 1.0631

Hint: Start at the first non-zero digit on the left.

7 Copy and complete this table.

	Number	Rounded to two decimal places	Rounded to two significant figures
a	1.4638		
b	0.0936		
c	23.7124		
d	0.00783		
e	100.465		

8 Round these numbers to the nearest whole number.

a 6.814 **b** 73.148 **c** 129.94 **d** 36 200.49

9 Round these numbers to one significant figure.

a 32 000	**b** 194.2	**c** 0.0492
d 0.0006413	**e** 4793	**f** 890
g 0.89	**h** 0.000304	**i** 0.95

Hint: $48.06 = 50$ to one significant figure.
$4730 = 5000$ to one significant figure.
$0.638 = 0.6$ to one significant figure.

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PROBLEM-SOLVING AND REASONING | 10–12 | 11–14

Example 10 Estimating using significant figures

Estimate the answer to $27 + 1329.5 \times 0.0064$ by rounding each number in the problem to one significant figure. Use your calculator to check how reasonable your answer is.

SOLUTION

$27 + 1329.5 \times 0.0064$

$\approx 30 + 1000 \times 0.006$

$= 30 + 6$

$= 36$

The estimated answer is reasonable.

EXPLANATION

Round each number to one significant figure and evaluate.

Recall multiplication occurs before addition.

By calculator (to 1 d.p.):

$27 + 1329.5 \times 0.0064 = 35.5$

Now you try

Estimate the answer to $32 - 117 \div 5.2$ by rounding each number in the problem to one significant figure. Use your calculator to check how reasonable your answer is.

10 Estimate the answers to the following by rounding each number in the expression to one significant figure. Check how reasonable your answer is with a calculator.

a $567 + 3126$ **b** $795 - 35.6$ **c** 97.8×42.2

d $965.98 + 5321 - 2763.2$ **e** $4.23 - 1.92 \times 1.827$ **f** $17.43 - 2.047 \times 8.165$

g $0.0704 + 0.0482$ **h** 0.023×0.98 **i** $0.38 \div 1.9$

11 An electronic timer records the time for a running relay between two teams, A and B. Team A's time is 54.283 seconds and team B's time is 53.791 seconds. Find the difference in the times for teams A and B if the times were written down using:

a one decimal place **b** four significant figures

c two significant figures **d** one significant figure

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12 One tonne (1000 kg) of soil is to be equally divided between 7 garden beds. How much soil does each garden bed get? Write your answer in tonnes rounded to the nearest kilogram.

13 A scientific experiment uses very small amounts of magnesium (0.0025 g) and potassium (0.0062 g). Why does it make sense to use two significant figures instead of two decimal places when recording numbers in a situation like this?

14 Should 2.14999 be rounded down or up if it is to be rounded to one decimal place? Give reasons.

ENRICHMENT: *n*th decimal place and π — 15, 16

15 $\frac{2}{11}$ can be written as 0.18181818 correct to eight decimal places.

a Using the decimal pattern described, find the digit in the:

i 20th decimal place

ii 45th decimal place

iii 1000th decimal place

b Express $\frac{1}{7}$ as a decimal correct to 13 decimal places.

c Using the decimal pattern from part **b** find the digit in the:

i 20th decimal place

ii 45th decimal place

iii 1000th decimal place

16 π is a decimal that is non-terminating and has no pattern. $\pi = 3.141592653\ldots$

a How many decimal places can you remember?

b Quiz yourself and your classmates.

c Use the internet to find π correct to 100 decimal places.

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1D Rational numbers and irrational numbers CONSOLIDATING

Learning intentions for this section:

- To know and understand the difference between rational numbers and irrational numbers
- To be able to convert between fractions and decimals (terminating and recurring)
- To be able to use both notations for recurring decimals
- To be able to compare fractions with different denominators

Past, present and future learning:

- This section is mostly revision of Stage 4 but the concept of irrational numbers may be new
- Some of these questions are more challenging than those in our Stage 4 books
- Irrational numbers will appear in various topics in the future, such as trigonometry
- Expertise with fractions is assumed knowledge for Stage 6 Standard and may also be required in non-calculator examinations such as NAPLAN and industry aptitude tests

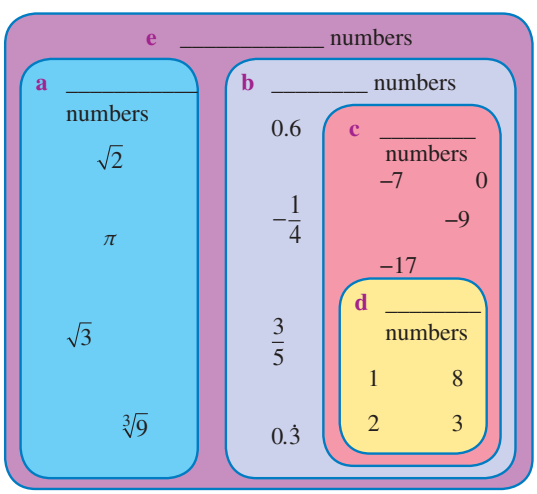
Rational numbers are any numbers that can be written as a fraction in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. Pythagoras and his followers discovered around 500 BCE that not all numbers are rational. Pi (π) and $\sqrt{2}$ are examples of these numbers. When written as decimals, they do not terminate or repeat. We call them irrational numbers.

This is $\sqrt{2}$ to 100 decimal places:

1. 41421356237309504880168872420969807856967187537694
807317667973799073247846210703885038753432764157273

Lesson starter: The real number system

Copy the diagram below and insert these words correctly: rational, irrational, real, integers, counting.



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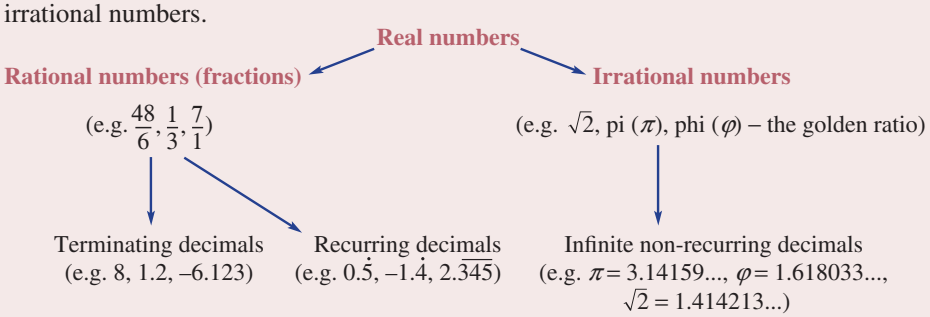
KEY IDEAS

- The **real numbers** (any positive or negative number or zero) can be classified as rational or irrational numbers.



- An infinite decimal is one where the decimal places continue forever.
- **Equivalent fractions** have the same value. For example: $\frac{2}{3} = \frac{6}{9}$
- Fractions can be simplified by dividing the **numerator** and **denominator** by their **highest common factor** (HCF).
- If $\frac{a}{b}$ is a **proper fraction**, then $a < b$. For example: $\frac{2}{7}$
- If $\frac{a}{b}$ is an **improper fraction**, then $a \geq b$. For example: $\frac{10}{3}$ or $\frac{3}{3}$
- A **mixed number** is written as a whole number plus a proper fraction. For example: $2\frac{3}{5}$

  As a mixed number $\frac{11}{3}$ is $3\frac{2}{3}$ and as an improper fraction $4\frac{1}{5}$ is $\frac{21}{5}$
- Fractions can be compared using a common denominator. This should be the lowest common multiple (LCM) of the denominators.
- A dot or bar is used to show a pattern in a **recurring decimal** number.

  For example: $\frac{1}{6} = 0.16666... = 0.1\dot{6}$ or $0.1\overline{6}$ and $\frac{3}{11} = 0.272727... = 0.\overline{27}$ or $0.\dot{2}\dot{7}$
- A calculator can be used to convert a fraction to a recurring decimal, but it won't show the dot(s) or the bar.
- Some calculators will also convert a recurring decimal to a fraction.

  For example, type 0.272727 until the screen is full and overflowing. Then press [=], then press the button that converts decimals to fractions.

Key terminology: real numbers, rational numbers, irrational numbers, numerator, denominator, proper fraction, improper fraction, mixed number, highest common factor, recurring decimal, equivalent fractions

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Exercise 1D

UNDERSTANDING | 1–3 | 1

1 Are the following true (T) or false (F)?

a $\sqrt{3}$ is an irrational number.

b $\sqrt{4}$ is an irrational number.

c $0.\dot{3}$ is a rational number.

d $\frac{4}{5}$ is a rational number.

Hint: An irrational number as a decimal is never-ending and has no pattern.

2 State whether each of the following is a proper fraction, improper fraction or mixed number.

a $\frac{2}{3}$ **b** $4\frac{1}{7}$ **c** $\frac{11}{6}$ **d** $\frac{1}{5}$

3 Write each of these in the form $\frac{a}{b}$.

a 0.7 **b** 0.3 **c** $1\frac{1}{2}$ **d** $2\frac{1}{3}$

FLUENCY | 4, 5, 6–9(½) | 4, 5, 6–9(½)

Example 11 Identifying rational numbers

Choose the rational numbers in the following list.

$\sqrt{9}, \sqrt{2}, 0.6, \pi, -\frac{3}{4}$

SOLUTION

$\sqrt{9}, 0.6$ and $-\frac{3}{4}$ are rational numbers.

EXPLANATION

$\sqrt{9} = 3 = \frac{3}{1}$ $\therefore$ rational (a fraction)

$0.6 = \frac{6}{10}$ $\therefore$ rational

$-\frac{3}{4}$ is also rational.

π and $\sqrt{2}$ are infinite non-recurring decimals.

Now you try

Choose the rational numbers in the following list.

$2\pi, \frac{5}{8}, 0.32, \sqrt{3}, -\sqrt{25}$

4 Which of the following are rational numbers?

a 0.1 **b** 20% **c** $6\frac{1}{4}$ **d** $\frac{2}{5}$

e $\frac{2}{3}$ **f** 0.001 **g** $\sqrt{64}$ **h** $\sqrt{7}$

i 4^2 **j** π **k** $0.\dot{2}$ **l** $\sqrt{11}$

Hint: Rational numbers can be written as a fraction.

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Example 12 Converting between mixed numbers and improper fractions

Express

a $\frac{11}{4}$ as a mixed number

b $2\frac{3}{5}$ as an improper fraction

SOLUTION

a $\frac{11}{4} = 2\frac{3}{4}$

b $2\frac{3}{5} = \frac{13}{5}$

EXPLANATION

4 divides into 11, 2 whole times. $2 \times 4 = 8$, leaving a remainder of 3.

To obtain the numerator $2 \times 5 + 3 = 13$.

Now you try

Express

a $\frac{17}{5}$ as a mixed number

b $2\frac{1}{4}$ as an improper fraction

5 Complete the following conversions.

a Express each of the following improper fractions as mixed numbers.

i $\frac{12}{5}$ **ii** $\frac{17}{6}$ **iii** $\frac{23}{4}$ **iv** $\frac{9}{8}$

b Express each of the following mixed numbers as improper fractions.

i $3\frac{1}{5}$ **ii** $6\frac{2}{7}$ **iii** $4\frac{4}{5}$ **iv** $10\frac{3}{8}$

Example 13 Writing fractions as decimals

Write $4\frac{3}{8}$ as a decimal.

SOLUTION

$$8\overline{)3.^{3}0^{6}0^{4}0}\ \ 0.3\ 7\ 5$$

$4\frac{3}{8} = 4.375$

EXPLANATION

Find a decimal for $\frac{3}{8}$ by dividing 8 into 3 using the short division algorithm.

Now you try

Write $2\frac{5}{16}$ as a decimal.

6 Write these fractions as decimals.

a $\frac{11}{4}$ **b** $\frac{7}{20}$ **c** $3\frac{2}{5}$ **d** $\frac{15}{8}$

e $2\frac{5}{8}$ **f** $3\frac{4}{5}$ **g** $\frac{37}{16}$ **h** $\frac{7}{28}$

Hint: Use short division. $\frac{11}{4}$ means $11 \div 4$

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Example 14 Recurring decimals from fractions

Write $\frac{5}{13}$ as a recurring decimal.

SOLUTION

$$13\overline{)5.^{5}0^{11}0^{6}0^{8}0^{2}0^{7}0^{5}0}\quad 0.\,3\;\;8\;4\;6\;1\;5\;3$$

$$\frac{5}{13} = 0.\overline{384615}$$

EXPLANATION

Divide 13 into 5 and continue until the pattern repeats.

Add a bar over the repeating pattern.

Writing $0.\dot{3}84615\dot{3}$ is an alternative.

Now you try

Write $\frac{5}{12}$ as a recurring decimal.

7 Write these fractions as recurring decimals. Check your answers using a calculator.

a $\frac{3}{11}$ **b** $\frac{7}{9}$ **c** $\frac{9}{7}$ **d** $\frac{7}{12}$

e $\frac{10}{9}$ **f** $3\frac{5}{6}$ **g** $7\frac{4}{15}$ **h** $\frac{29}{11}$

Example 15 Writing decimals as fractions

Write these decimals as fractions.

a 0.24 **b** 2.385

SOLUTION

a $0.24 = \frac{24}{100}$

$= \frac{6}{25}$

b $2.385 = \frac{2385}{1000}$

$= \frac{477}{200}$

$= 2\frac{77}{200}$

OR

$2\frac{385}{1000}$

$= 2\frac{77}{200}$

EXPLANATION

Write as a fraction using the smallest place value (hundredths) and then simplify by dividing by the HCF of 4.

The smallest place value is thousandths.

Simplify to an improper fraction or a mixed number.

Now you try

Write these decimals as fractions.

a 0.45 **b** 3.225

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8 Write these decimals as fractions.

a 0.35	**b** 0.06	**c** 3.7	**d** 0.56
e 1.07	**f** 0.075	**g** 3.32	**h** 7.375
i 2.005	**j** 10.044	**k** 6.45	**l** 2.101

Hint: Divide by 100 when there are two decimal places.

Example 16 Comparing fractions

Decide which is the larger fraction of the following.

$\frac{7}{12}$ or $\frac{8}{15}$

SOLUTION

LCM of 12 and 15 is 60.

$\frac{7}{12} = \frac{35}{60}$ and $\frac{8}{15} = \frac{32}{60}$

$\therefore \frac{7}{12} > \frac{8}{15}$

EXPLANATION

Find the lowest common multiple of the two denominators (lowest common denominator).

Write each fraction as an equivalent fraction using the common denominator. Then compare numerators (i.e. 35 > 32) to determine the larger fraction.

Now you try

Decide which is the larger fraction of the following.

$\frac{4}{9}$ or $\frac{5}{12}$

9 Decide which is the larger fraction in the following pairs.

a $\frac{3}{4}, \frac{5}{6}$	**b** $\frac{13}{20}, \frac{3}{5}$	**c** $\frac{7}{10}, \frac{8}{15}$	**d** $\frac{5}{12}, \frac{7}{18}$
e $\frac{7}{16}, \frac{5}{12}$	**f** $\frac{26}{35}, \frac{11}{14}$	**g** $\frac{7}{12}, \frac{19}{30}$	**h** $\frac{7}{18}, \frac{11}{27}$

Hint: First write both fractions using a common denominator.

PROBLEM-SOLVING AND REASONING 10–12 | 11, 13, 14

10 Express the following quantities as simplified fractions.

a $45 out of $100

b 12 kg out of 80 kg

c 64 baskets out of 90 shots in basketball

d 115 mL out of 375 mL

11 These sets of fractions form a pattern. Find the next two fractions in the pattern.

a $\frac{1}{3}, \frac{5}{6}, \frac{4}{3}, \frac{\square}{\square}, \frac{\square}{\square}$

b $\frac{6}{5}, \frac{14}{15}, \frac{2}{3}, \frac{\square}{\square}, \frac{\square}{\square}$

c $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{\square}{\square}, \frac{\square}{\square}$

d $\frac{1}{2}, \frac{4}{7}, \frac{9}{14}, \frac{\square}{\square}, \frac{\square}{\square}$

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12 The 'Weather forecast' website says there is a 0.45 chance that it will rain tomorrow. The 'Climate control' website says that the chance of rain is $\frac{14}{30}$. Which website gives the least chance that it will rain?

13 A jug has 400 mL of $\frac{1}{2}$ strength orange juice. The following amounts of full strength juice are added to the mix. Find a fraction to describe the strength of the orange drink after the full strength juice is added.

a 100 mL **b** 50 mL **c** 120 mL **d** 375 mL

14 If x is an integer, determine the values that x can take in the following.

a The fraction $\frac{x}{3}$ is a number between (and not including) 10 and 11

b The fraction $\frac{x}{7}$ is a number between (and not including) 5 and 8

c The fraction $\frac{34}{x}$ is a number between 6 and 10

Hint: You may have more than one answer.

ENRICHMENT: Converting recurring decimals to fractions — 15

15 Here are two examples of how to convert recurring decimals to fractions.

$0.\dot{6} = 0.6666\ldots$

Let $x = 0.6666\ldots$ (1)

$10x = 6.6666\ldots$ (2)

(2) − (1) $9x = 6$

$x = \frac{6}{9} = \frac{2}{3}$

$\therefore 0.\dot{6} = \frac{2}{3}$

$1.\overline{27} = 1.272727\ldots$

Let $x = 1.272727\ldots$ (1)

$100x = 127.2727\ldots$ (2)

(2) − (1) $99x = 126$

$x = \frac{126}{99}$

$\therefore 1.\overline{27} = \frac{126}{99} = 1\frac{27}{99} = 1\frac{3}{11}$

Convert these recurring decimals to fractions using the above method.

a $0.\dot{8}$ **b** $1.\dot{2}$ **c** $0.\overline{81}$ **d** $3.\overline{43}$

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1E Adding and subtracting fractions CONSOLIDATING

Learning intentions for this section:

- To be able to find the lowest common denominator of two or more fractions
- To revise the addition and subtraction of fractions, including mixed numbers

Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- In Chapter 3 of this book, we apply this learning to algebraic fractions
- Expertise with fractions is assumed knowledge for Stage 6 Standard and may also be required in non-calculator examinations such as NAPLAN and industry aptitude tests

As you will remember, fractions represent parts of a whole.

Fractions, like other numbers, can be added and subtracted.

To add or subtract fractions they need to be the same type; that is, they both need to be eighths, quarters, thirds or tenths etc. They need to have a common denominator.

Lesson starter: Shade the fraction

Shade the fraction suggested by each of these additions and subtractions.

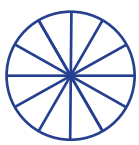
a $\frac{1}{4}+\frac{1}{4}=$

b $\frac{1}{4}+\frac{1}{2}=$

c $\frac{3}{4}-\frac{1}{2}=$

d $\frac{5}{8}+\frac{1}{4}=$

e $\frac{1}{2}+\frac{1}{3}=$

f $\frac{1}{3}-\frac{1}{4}=$ 

KEY IDEAS

- Equivalent fractions are created by multiplying or dividing both the numerator and the denominator by the same factor.

  For example:

$$\frac{3}{4}\ \overset{\times 3}{=}\ \frac{9}{12} \qquad \frac{10}{15}\ \overset{\div 5}{=}\ \frac{2}{3}$$

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■ To add or subtract fractions, the denominators need to be the same.

- If the denominators are the same, add/subtract the numerators, keeping the denominator unchanged.

  For example: $\frac{3}{5}+\frac{1}{5}=\frac{4}{5}$

- If the denominators are different, find the **lowest common denominator** (LCD) using equivalent fractions. Then add/subtract the numerators, remembering to simplify your answer where possible.

  For example: $\frac{1}{2}+\frac{2}{3}=\frac{3}{6}+\frac{4}{6}$ The LCD of halves and thirds is sixths.

  $=\frac{7}{6}$

  $=1\frac{1}{6}$

Key terminology: equivalent fractions, numerator, denominator, lowest common denominator

Exercise 1E

UNDERSTANDING 1–4 4

1 Copy and complete these equivalent fractions.

a $\frac{3}{4}=\frac{\square}{16}$ **b** $\frac{3}{5}=\frac{\square}{10}$ **c** $\frac{5}{6}=\frac{\square}{18}$

d $\frac{3}{7}=\frac{21}{\square}$ **e** $\frac{4}{5}=\frac{12}{\square}$ **f** $\frac{5}{7}=\frac{25}{\square}$

Hint: What you do to the numerator must also be done to the denominator, and vice versa.

2 Are the following true (T) or false (F)?

a $\frac{7}{9}+\frac{1}{9}=\frac{8}{9}$ **b** $\frac{6}{9}-\frac{1}{3}=\frac{5}{6}$

3 Copy and complete.

a $\frac{3}{10}+\frac{2}{5}$

$=\frac{3}{10}+\frac{\square}{10}$

$=\frac{\square}{10}$

b $\frac{1}{2}-\frac{3}{8}$

$=\frac{\square}{8}-\frac{3}{8}$

$=\frac{\square}{8}$

c $1\frac{1}{2}+\frac{1}{4}$

$=1\frac{2}{\square}+\frac{1}{4}$

$=1\frac{\square}{\square}$

4 What is the LCD needed if these fractions are to be added or subtracted?

a $\frac{1}{2}, \frac{1}{3}$ **b** $\frac{3}{7}, \frac{5}{9}$ **c** $\frac{3}{10}, \frac{8}{15}$

d $\frac{1}{2}, \frac{3}{8}$ **e** $\frac{9}{11}, \frac{4}{33}$ **f** $\frac{5}{12}, \frac{7}{30}$

Hint: The LCD is the lowest common multiple of both denominators.

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FLUENCY 5–7(½) 5–7(½)

Example 17 Adding and subtracting with same denominators

Find:

a $\frac{5}{11}+\frac{8}{11}$ b $1\frac{3}{8}-\frac{5}{8}$

SOLUTION

a $\frac{5}{11}+\frac{8}{11}=\frac{13}{11}$

$=1\frac{2}{11}$

b $1\frac{3}{8}-\frac{5}{8}=\frac{11}{8}-\frac{5}{8}$

$=\frac{6}{8}$

$=\frac{3}{4}$

EXPLANATION

Denominators are the same (elevenths).

Keep the denominator and add the numerators.

Write as a mixed number.

Write $1\frac{3}{8}$ as an improper fraction.

Denominators are the same (eighths). Subtract the numerators.

Simplify the answer by dividing by the HCF (2).

Now you try

Find:

a $\frac{3}{7}+\frac{5}{7}$ b $1\frac{1}{5}-\frac{4}{5}$

5 Find:

a $\frac{2}{5}+\frac{1}{5}$ b $\frac{3}{8}+\frac{2}{8}$ c $\frac{9}{10}-\frac{2}{10}$ d $\frac{2}{7}+\frac{2}{7}$

e $\frac{13}{17}-\frac{8}{17}$ f $\frac{3}{5}-\frac{2}{5}$ g $\frac{11}{6}-\frac{1}{6}$ h $\frac{13}{8}-\frac{3}{8}$

i $\frac{4}{5}+\frac{3}{5}$ j $1\frac{1}{5}+\frac{3}{5}$ k $2\frac{2}{3}-\frac{1}{3}$ l $1\frac{1}{8}-\frac{5}{8}$

m $1\frac{1}{10}-\frac{7}{10}$ n $3\frac{1}{4}+1\frac{1}{4}$ o $2\frac{3}{10}-\frac{9}{10}$ p $3\frac{7}{8}+2\frac{5}{8}$

Hint: Writing mixed numbers as improper fractions may help.

Example 18 Adding with different denominators

Evaluate $\frac{1}{2}+\frac{3}{5}$.

SOLUTION

$\frac{1}{2}+\frac{3}{5}=\frac{5}{10}+\frac{6}{10}$

$=\frac{11}{10}$ or $1\frac{1}{10}$

EXPLANATION

The lowest common multiple of the denominators of 2 and 5 is 10. Rewrite as equivalent fractions using a denominator of 10: $\frac{1\times5}{2\times5}=\frac{5}{10}$ and $\frac{3\times2}{5\times2}=\frac{6}{10}$.

Add the numerators.

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Now you try

Evaluate $\frac{4}{7}+\frac{1}{3}$.

6 Evaluate the following. Check your answers with a calculator.

a $\frac{1}{2}+\frac{1}{8}$	b $\frac{1}{2}-\frac{1}{6}$	c $\frac{3}{10}+\frac{1}{5}$	d $\frac{4}{3}-\frac{5}{6}$
e $\frac{3}{2}-\frac{3}{5}$	f $\frac{1}{3}+\frac{1}{4}$	g $\frac{3}{5}+\frac{1}{10}$	h $\frac{3}{4}-\frac{3}{5}$
i $\frac{5}{6}-\frac{3}{8}$	j $\frac{2}{3}+\frac{1}{2}$	k $\frac{5}{3}-\frac{2}{7}$	l $\frac{3}{8}+\frac{1}{3}$
m $\frac{2}{9}+\frac{1}{12}$	n $\frac{9}{21}+\frac{1}{7}$	o $\frac{27}{50}-\frac{7}{20}$	p $\frac{11}{30}-\frac{7}{45}$

Hint: First express each fraction using the LCD.

Example 19 Adding and subtracting mixed numbers

Evaluate:

a $1\frac{2}{3}+4\frac{5}{6}$ b $3\frac{2}{5}-2\frac{3}{4}$

SOLUTION

a $1\frac{2}{3}+4\frac{5}{6}=\frac{5}{3}+\frac{29}{6}$

$=\frac{10}{6}+\frac{29}{6}$

$=\frac{39}{6}$

$=\frac{13}{2}$ or $6\frac{1}{2}$

EXPLANATION

Change each mixed number to an improper fraction.

Remember that the lowest common denominator of 3 and 6 is 6. Change $\frac{5}{3}$ to an equivalent fraction with denominator 6 then add the numerators and simplify.

b $3\frac{2}{5}-2\frac{3}{4}=\frac{17}{5}-\frac{11}{4}$

$=\frac{68}{20}-\frac{55}{20}$

$=\frac{13}{20}$

Convert to improper fractions and then express as equivalent fractions with the same denominator.

Subtract the numerators.

Now you try

Evaluate:

a $2\frac{3}{4}+1\frac{5}{8}$ b $4\frac{1}{3}-3\frac{4}{5}$

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7 Evaluate the following.

a $1\frac{1}{3}+2\frac{1}{2}$ **b** $1\frac{1}{3}-\frac{3}{4}$ **c** $5\frac{1}{2}-2\frac{7}{10}$

d $2\frac{1}{3}+\frac{1}{4}$ **e** $4\frac{1}{2}+2\frac{1}{4}$ **f** $6\frac{1}{3}-2\frac{1}{2}$

g $1\frac{3}{8}-1\frac{1}{4}$ **h** $2\frac{3}{4}-1\frac{1}{3}$ **i** $4\frac{1}{10}-2\frac{2}{5}$

j $1\frac{7}{12}-\frac{2}{3}$ **k** $3\frac{1}{5}+1\frac{1}{2}$ **l** $2\frac{5}{6}-1\frac{1}{4}$

Hint: First rewrite using improper fractions.

PROBLEM-SOLVING AND REASONING

8–10 | 9–12

8 To remove impurities a mining company filters $3\frac{1}{2}$ tonnes of raw material. If $2\frac{5}{8}$ tonnes are removed, what quantity of material remains?

9 When a certain raw material is processed it produces $3\frac{1}{7}$ tonnes of mineral and $2\frac{3}{8}$ tonnes of waste. How many tonnes of raw material were processed?

10 The ingredients to make a batch of punch are $2\frac{1}{4}$ L of apple juice, $1\frac{1}{2}$ L of guava juice and $1\frac{1}{5}$ L of lemonade.

a How much punch is produced?

b If a cup holds 150 mL, how many cups can this punch serve?

11 Liv owns $\frac{3}{10}$ of a company, Sienna owns another $\frac{1}{4}$ of the company and Zac owns $\frac{2}{5}$ of it. The bank owns the rest. How much of the company does the bank own?

12 Here is an example involving the subtraction of fractions where improper fractions are not used.

$$2\frac{1}{2}-1\frac{1}{3}=2\frac{3}{6}-1\frac{2}{6}$$

$$=1\frac{1}{6}$$

Try this technique on the following problem and explain the difficulty that you find.

$2\frac{1}{3}-1\frac{1}{2}$

ENRICHMENT: Which fractions do you choose?

— | 13

13 The six fractions listed below are each used exactly once in part **a**.

$\frac{1}{8}, \frac{5}{12}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$

a Arrange the six fractions correctly so that each equation produces the required answer. Use each fraction only once.

i $\square+\square=\frac{7}{8}$ **ii** $\square-\square=\frac{1}{4}$ **iii** $\square-\square=\frac{1}{3}$

b Which of the fractions, when added, produce an answer of 2?

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Progress quiz

1A **1** Find the value of:

a $-15 + 7$ **b** $-10 - 4$ **c** $18 - (-7)$

d $12 + (-4)$ **e** $-8 + 5 - (-7)$ **f** $-9 - (-6)$

1B **2** Find the value of:

a $-7 \times (-6)$ **b** $12 \times (-5)$ **c** $-56 \div 7$

d $-48 \div (-6)$ **e** $(-4)^3$ **f** $(-9)^2$

1B **3** Apply the order of operations to find the value of:

a $3 \times (4 - 9) + 6$ **b** $7 - 3 \times (-4)$ **c** $(6 - 10) \div (-2) - 9$

1C **4** **a** Round each of the following to two decimal places.

i 24.187 **ii** −6.143988 **iii** 7.595699

b Round each of the following to two significant figures.

i 3.268 **ii** 237 **iii** 7932.8 **iv** 0.0078812

1C **5** Estimate the following using one significant figure rounding of each number. Check how reasonable your answer is with a calculator.

a $242 - 121$ **b** $93.4 - 9.7 \times 0.0234$

1D **6** Write these fractions as decimals, parts **c** and **d** are recurring decimals.

a $\frac{5}{8}$ **b** $3\frac{8}{25}$ **c** $\frac{7}{12}$ **d** $2\frac{7}{11}$

1D **7** Write these decimals as simplified fractions.

a 0.8 **b** 0.36 **c** 0.004 **d** 4.205

1D **8** Arrange these fractions in ascending (smallest to largest) order.

$\frac{11}{24}, \frac{3}{8}, \frac{5}{12}$

1E **9** Evaluate the following.

a $\frac{3}{8} + \frac{2}{8}$ **b** $\frac{2}{3} - \frac{4}{7}$ **c** $2\frac{3}{10} + 1\frac{1}{4}$ **d** $4\frac{1}{5} - 2\frac{3}{4}$

1E **10** A recipe requires $\frac{3}{4}$ cup of sugar, $2\frac{1}{3}$ cups of flour and $\frac{1}{2}$ cup of cocoa. How many cups of ingredients are needed in total?

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1F Multiplying and dividing fractions CONSOLIDATING

Learning intentions for this section:

- To know when it is acceptable to cancel common factors to make expressions simpler
- To revise the multiplication and division of fractions, including mixed numbers

Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- In Chapter 3 of this book, we apply this learning to algebraic fractions
- Expertise with fractions is assumed knowledge for Stage 6 Standard and may also be required in non-calculator examinations such as NAPLAN and industry aptitude tests

A series of steps can be followed to make multiplying and dividing fractions easier. This includes cancelling, which reduces the size of the numbers inside the fractions.

Lesson starter: How much is left?

How much is left for Kimberly to eat if:

- Tom eats half the tart
- then Sarah eats half of what's left
- then Zara eats a third of the remaining section?

KEY IDEAS

- To multiply fractions (proper or improper), multiply the numerators together and multiply the denominators together.
  - In general:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

For example:

$$\frac{2}{5} \times \frac{3}{7} = \frac{2 \times 3}{5 \times 7}$$

$$= \frac{6}{35}$$

- The word 'of' usually means multiply.
For example:

$$\frac{1}{3} \text{ of } 24 = \frac{1}{3} \times 24$$

- Cancel any common factors in the numerator with any common factors in the denominator before multiplying.

$$\frac{{}^{1}\cancel{5}}{\cancel{4}} \times \frac{{}^{1}\cancel{2}}{\cancel{15}_{3}} = \frac{1}{4} \times \frac{1}{3}$$

$$= \frac{1}{12}$$

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- To find the **reciprocal** of a fraction, you simply invert it (that is, flip the proper or improper fraction upside down).
  - The reciprocal of 2 is $\frac{1}{2}$ since $2 = \frac{2}{1}$.
  - The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$.
  - The reciprocal of $1\frac{1}{5}$ $\left(\text{or } \frac{6}{5}\right)$ is $\frac{5}{6}$.
- To divide a number by a fraction, multiply by its reciprocal.
  - In general:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c}$$

For example:

$$\frac{2}{3} \div \frac{5}{6} = \frac{2}{3} \times \frac{6}{5}$$

$$= \frac{2}{\cancel{3}_{1}} \times \frac{\cancel{6}^{2}}{5}$$

$$= \frac{4}{5}$$

Key terminology: numerator, denominator, highest common factor, reciprocal, proper fraction, improper fraction, mixed number

Exercise 1F

UNDERSTANDING 1–3 3

1 Are the following true (T) or false (F)?

a $\frac{2}{5} \times \frac{1}{3} = \frac{2}{15}$ **b** $\frac{1}{6}$ of $30 = 6 \times 30$ **c** $\frac{3}{7} \div \frac{2}{5} = \frac{3}{7} \times \frac{5}{2}$

2 Find:

a $\frac{1}{2}$ of 24 **b** $\frac{1}{3}$ of 12 **c** $\frac{1}{5}$ of 20

d $\frac{3}{4}$ of 16 **e** $\frac{9}{10}$ of 200 **f** $\frac{2}{3}$ of 24

3 Find the reciprocal of:

a $\frac{3}{4}$ **b** $\frac{1}{7}$ **c** 6 **d** $1\frac{2}{3}$

Hint: Whole numbers can be written with a denominator of 1. $6 = \frac{6}{1}$

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FLUENCY 4–7(½) 4–7(½)

Example 20 Multiplying with proper fractions

Evaluate the following.

a $\frac{2}{3} \times \frac{5}{7}$ b $\frac{4}{5} \times \frac{25}{32}$

SOLUTION

a $\frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$

b $\frac{\cancel{4}^{1}}{\cancel{5}_{1}} \times \frac{\cancel{25}^{5}}{\cancel{32}_{8}} = \frac{1}{1} \times \frac{5}{8}$

$= \frac{5}{8}$

EXPLANATION

No common factors therefore multiply the numerators together and multiply the denominators together.

$$\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7}$$

Cancel the common factors between the numerators and the denominators.

Multiply the numerators and then multiply the denominators.

$$\frac{1 \times 5}{1 \times 8}$$

Now you try

Evaluate the following.

a $\frac{3}{7} \times \frac{4}{5}$ b $\frac{3}{8} \times \frac{20}{27}$

4 Evaluate the following.

a $\frac{3}{5} \times \frac{1}{4}$ b $\frac{2}{3} \times \frac{2}{5}$ c $\frac{3}{4} \times \frac{5}{7}$

d $\frac{1}{2} \times \frac{1}{3}$ e $\frac{1}{4} \times \frac{1}{5}$ f $\frac{1}{9} \times \frac{2}{3}$

g $\frac{5}{8} \times \frac{16}{17}$ h $\frac{1}{2} \times \frac{6}{7}$ i $\frac{2}{3} \times \frac{3}{5}$

j $\frac{8}{9} \times \frac{3}{4}$ k $\frac{4}{5} \times \frac{1}{2}$ l $\frac{5}{6} \times \frac{24}{25}$

m $\frac{6}{7} \times \frac{7}{8}$ n $\frac{7}{9} \times \frac{18}{21}$ o $\frac{8}{21} \times \frac{7}{12}$

p $\frac{5}{14} \times \frac{21}{40}$

Hint: First cancel, if possible.

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Example 21 Multiplying with mixed numbers

Evaluate $1\frac{2}{3} \times 2\frac{1}{10}$.

SOLUTION

$$1\frac{2}{3} \times 2\frac{1}{10} = \frac{\cancel{5}^{1}}{\cancel{3}_{1}} \times \frac{\cancel{21}^{7}}{\cancel{10}_{2}}$$

$$= \frac{7}{2} \text{ or } 3\frac{1}{2}$$

EXPLANATION

Rewrite as improper fractions.

Cancel common factors between numerators and denominators and then multiply numerators and denominators.

Now you try

Evaluate $1\frac{4}{5} \times 2\frac{1}{3}$.

5 Evaluate, then check using a calculator.

a $1\frac{1}{2} \times \frac{1}{3}$ **b** $1\frac{2}{3} \times \frac{3}{4}$ **c** $2\frac{1}{3} \times \frac{9}{10}$ **d** $1\frac{1}{2} \times 1\frac{1}{3}$ **e** $2\frac{1}{3} \times 1\frac{4}{5}$

f $\frac{3}{4} \times 1\frac{1}{7}$ **g** $2\frac{1}{2} \times 2\frac{1}{2}$ **h** $4\frac{1}{2} \times \frac{8}{9}$ **i** $1\frac{3}{4} \times 1\frac{2}{3}$ **j** $3\frac{3}{4} \times \frac{8}{5}$

Example 22 Dividing fractions

Evaluate the following.

a $\frac{4}{15} \div \frac{12}{25}$ **b** $1\frac{17}{18} \div 1\frac{1}{27}$

SOLUTION

a $\frac{4}{15} \div \frac{12}{25} = \frac{\cancel{4}^{1}}{\cancel{15}_{3}} \times \frac{\cancel{25}^{5}}{\cancel{12}_{3}}$

$$= \frac{5}{9}$$

EXPLANATION

To divide by $\frac{12}{25}$ we multiply by its reciprocal $\frac{25}{12}$.

Cancel common factors between numerators and denominators then multiply fractions.

b $1\frac{17}{18} \div 1\frac{1}{27} = \frac{35}{18} \div \frac{28}{27}$

$$= \frac{\cancel{35}^{5}}{\cancel{18}_{2}} \times \frac{\cancel{27}^{3}}{\cancel{28}_{4}}$$

$$= \frac{15}{8} \text{ or } 1\frac{7}{8}$$

Rewrite mixed numbers as improper fractions.

Multiply by the reciprocal of the second fraction.

The reciprocal of $\frac{28}{27}$ is $\frac{27}{28}$.

Now you try

Evaluate the following.

a $\frac{5}{12} \div \frac{5}{18}$ **b** $1\frac{7}{8} \div 1\frac{1}{24}$

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6 Evaluate the following (recall that the reciprocal of 8 is $\frac{1}{8}$).

Hint: Multiply by the reciprocal of the second fraction.

a $\frac{4}{7} \div \frac{3}{5}$ **b** $\frac{3}{4} \div \frac{2}{3}$ **c** $\frac{5}{8} \div \frac{7}{9}$

d $\frac{3}{7} \div \frac{4}{9}$ **e** $\frac{3}{4} \div \frac{9}{16}$ **f** $\frac{4}{5} \div \frac{8}{15}$

g $\frac{8}{9} \div \frac{4}{27}$ **h** $\frac{15}{42} \div \frac{20}{49}$ **i** $15 \div \frac{5}{6}$

j $6 \div \frac{2}{3}$ **k** $\frac{4}{5} \div 8$ **l** $\frac{8}{9} \div 6$

7 Evaluate the following.

Hint: First change any mixed numbers to improper fractions.

a $14 \div 4\frac{1}{5}$ **b** $6 \div 1\frac{1}{2}$ **c** $1\frac{1}{3} \div 8$

d $2\frac{1}{4} \div 1\frac{1}{2}$ **e** $4\frac{2}{3} \div 5\frac{1}{3}$ **f** $7\frac{1}{5} \div \frac{4}{15}$

PROBLEM-SOLVING AND REASONING

8(½), 9, 10 | 8(½), 9, 11, 12

8 Find the following amounts.

Hint: 'Of' means multiply and 20 as a fraction is $\frac{20}{1}$.

a $\frac{1}{2}$ of \$20 **b** $\frac{1}{2}$ of \$7 **c** $\frac{1}{3}$ of \$18

d $\frac{3}{4}$ of \$24 **e** $\frac{4}{5}$ of \$8000 **f** $\frac{9}{10}$ of \$1

9 Evaluate these mixed-operation expressions.

a $\frac{2}{3} \times \frac{1}{3} \div \frac{7}{9}$ **b** $\frac{4}{5} \times \frac{3}{5} \div \frac{9}{10}$ **c** $\frac{4}{9} \times \frac{6}{25} \div \frac{1}{150}$

10 In a $1\frac{1}{2}$ hour maths exam, $\frac{1}{6}$ of the time is allocated as reading time. How long is reading time?

11 A car's fuel gauge shows that it has $\frac{1}{4}$ of a tank of petrol remaining. The petrol tank holds 64 litres of fuel when full. The car can travel 10 km on 1 litre. How many kilometres can you travel on the amount of petrol that remains in the tank?

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12 Thomas, Ahn and Oscar agree to equally share the job of cleaning the house after a party. They estimate the job will take $4\frac{1}{2}$ hours to complete. How many minutes of work should they each contribute?

ENRICHMENT: Electronics and reciprocals — 13

13 Reciprocals are used in the study of electronics. On the calculator, the x^{-1} button can be used to find the reciprocal of a number.

a Use this button to find reciprocals of the following numbers.

i 2 **ii** 5 **iii** 6.2 **iv** 1.5 **v** -1

b If $\frac{1}{6.4} + \frac{1}{7.2} = A$, find the value of $\frac{1}{A}$ (to 2 decimal places).

c If $\frac{1}{50} + \frac{1}{50} = B$, what is $\frac{1}{B}$?

d Investigate, using your calculator, the types of numbers for which the reciprocal is bigger than the original number.

e What positive number is its own reciprocal?

f Are there any numbers that do not have a reciprocal?

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1G Ratios CONSOLIDATING

Learning intentions for this section:

- To revise the meaning, use and applications of ratios
- To be able to simplify ratios, recognise equivalent ratios and divide a quantity in a given ratio
- To be able to solve problems involving ratios, including the use of the unitary method

Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- Scale factor and similarity are in Chapter 8 of this book and also in our Year 10 book
- Ratio concepts are revisited in Stage 6 Standard and may also be required in non-calculator examinations such as NAPLAN and industry aptitude tests

Fractions, ratios and rates are used to compare quantities. A lawn mower, for example, might require $\frac{1}{6}$ of a litre of oil to make a petrol mix of 2 parts oil to 25 parts petrol, which is an oil to petrol ratio of 2 to 25 or 2 : 25.

Lesson starter: The lottery win

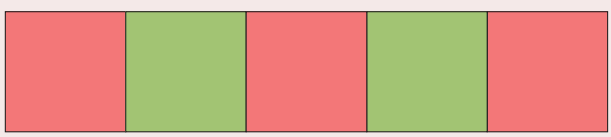
$100 000 is to be divided up for three lucky people into a ratio of 2 to 3 to 5 (2 : 3 : 5). Work out how the money is to be divided.

- Clearly write down your method and answer. There may be many different ways to solve this problem.
- Write down and discuss the methods suggested by other students in the class.

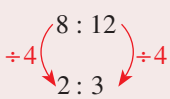
KEY IDEAS

■ **Ratios** are used to compare quantities with the same units.
- The ratio of a is to b is written $a : b$.
- Ratios in simplest form use whole numbers that have no common factor.

In the diagram below the ratio can be written as: green squares to red squares = 2 : 3



■ A ratio can be reduced to its simplest form by dividing by the highest common factor (HCF).



■ The **unitary method** involves finding the value of one part of a total.
- Once the value of one part is found, then the value of several parts can be easily determined.

Key termionology: ratio, unitary method, highest common factor

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Exercise 1G

UNDERSTANDING 1–3 3

1 State if each of the following ratios is in simplest form (yes or no).

a 4 : 10 **b** 3 : 7 **c** 20 : 39 **d** 48 : 64

2 Draw up a table with the following headings.

Diagram	Ratio of shaded parts to unshaded parts	Ratio of shaded parts to parts in the whole

Enter information about these diagrams to complete the table.

a

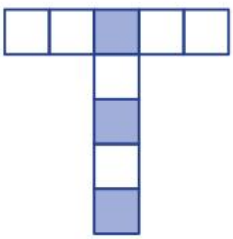
b

c

d

e

f



3 At a school presentation assembly there were five times as many students as teachers and twice as many guests as students. Write the ratio of:

a students to teachers
b students to guests
c guests to students
d guests to teachers
e students to the total number of people at the assembly

Hint: Remember that, with a ratio, the order matters.

FLUENCY 4–8(½) 4–8(½)

Example 23 Simplifying ratios using HCFs

Simplify the ratio 38 : 24.

SOLUTION

38 : 24 = 19 : 12

EXPLANATION

The HCF of 38 and 24 is 2 so divide both parts by 2.

Now you try

Simplify the ratio 45 : 27.

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4 Simplify these ratios.

a 6 : 30	**b** 8 : 20	**c** 40 : 50
d 2 : 20	**e** 15 : 18	**f** 9 : 27
g 18 : 6	**h** 24 : 36	**i** 52 : 39
j 144 : 36	**k** 48 : 96	**l** 30 : 10
m 2000 : 5600	**n** 3 : 6 : 12	**o** 15 : 30 : 10

Hint: Divide by the HCF.

Example 24 Simplifying ratios involving fractions and decimals

Simplify these ratios.

a 0.2 : 0.14 **b** $2\frac{1}{2} : 1\frac{1}{3}$

SOLUTION

a 0.2 : 0.14 = 20 : 14
= 10 : 7

b $2\frac{1}{2} : 1\frac{1}{3} = \frac{5}{2} : \frac{4}{3}$
$= \frac{15}{6} : \frac{8}{6}$
= 15 : 8

EXPLANATION

Multiply by 100 to remove all the decimal places and simplify.

Write as improper fractions using the same denominator.

Multiply both parts by 6 to write as whole numbers.

Now you try

Simplify these ratios.

a 0.5 : 0.32 **b** $3\frac{1}{4} : 1\frac{7}{8}$

5 Simplify these ratios.

a 0.1 : 0.2	**b** 0.3 : 4.1	**c** 0.5 : 3.2
d 0.3 : 0.9	**e** 0.7 : 3.5	**f** 0.4 : 0.12
g 8 : 0.2	**h** 15 : 0.01	**i** 1.6 : 0.56

6 Simplify:

a $\frac{1}{4} : \frac{1}{3}$	**b** $\frac{2}{3} : \frac{1}{4}$	**c** $1\frac{1}{2} : 3\frac{1}{3}$
d $2\frac{1}{4} : 1\frac{2}{5}$	**e** $\frac{3}{8} : 1\frac{3}{4}$	**f** $1\frac{5}{6} : 3\frac{1}{4}$

Hint: A simplified ratio must include only whole numbers (with no common factors).

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Example 25 Simplifying ratios involving units

Simplify the ratio of 560 metres to 2 kilometres.

SOLUTION

560 m : 2 km
= 560 m : 2000 m
= 560 : 2000
= 7 : 25

EXPLANATION

Write using ratio notation.
Convert the units so that both are the same:
2 km = 2000 m
Cancel the units and divide both numbers by their HCF. HCF of 560 and 2000 is 80

Now you try

Simplify the ratio of 35 cents to $2.

7 Write each of the following as a ratio in simplest form.

a 80c : $8
b 90c : $4.50
c 80 cm : 1.2 m
d 0.7 kg : 800 g
e 2.5 kg : 400 g
f 30 min : 2 hours
g 45 min : 3 hours
h 4 hours : 50 min
i 40 cm : 2 m : 50 cm
j 80 cm : 600 mm : 2 m
k 2.5 hours : 1.5 days
l 0.09 km : 300 m : 1.2 km

Hint: Convert to the same units first.

Example 26 Dividing into a given ratio

$300 is to be divided into the ratio 2 : 3.
Find the value of the larger portion using the unitary method.

SOLUTION

Total number of parts is 2 + 3 = 5
5 parts = $300

1 part = $\frac{1}{5}$ of $300
= $60

Larger portion = 3 × $60
= $180

EXPLANATION

Use the ratio 2 : 3 to get the total number of parts.

Calculate the value of each part. ($300 ÷ 5)

Calculate the value of 3 parts.

Now you try

$600 is to be divided into the ratio 5 : 7.
Find the value of the larger portion using the unitary method.

8 Use the unitary method to divide:

a $500 in the ratio of 1 : 4
b $36 in the ratio of 4 : 5
c 88 kg in the ratio of 3 : 8
d $96 in the ratio of 7 : 5
e $500 in the ratio of 2 : 3
f 2000 g in the ratio of 3 : 5
g $100 in the ratio of 7 : 3
h $600 in the ratio of 1 : 1
i $70 in the ratio of 2 : 7 : 1
j 420 g in the ratio of 8 : 2

Hint: First find the value of one part.

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PROBLEM-SOLVING AND REASONING

9–11 | 10–13

9 420 g of flour is to be divided into a ratio of 7 : 3 for two different recipes. Find the smaller amount.

10 Kirsty manages a restaurant. Each day she buys watermelons and mangoes in the ratio of 3 : 2. How many watermelons did she buy if, on one day, the total number of watermelons and mangoes was 200?

11 If a prize of $6000 was divided among Georgia, Leanne and Maya in the ratio of 5 : 2 : 3, how much did each girl get?

12 The dilution ratio for a particular chemical with water is 2 : 3 (chemical to water). If you have 72 litres of chemical, how much water is needed to dilute the chemical?

Hint: If 72 litres is two parts, what is one part?

13 Amy, Belinda, Candice and Diane invested money in the ratio of 2 : 3 : 1 : 4 in a publishing company. If the profit was shared according to their investment, and Amy's profit was $2400, find the profit each investor made.

ENRICHMENT: Mixing drinks

— | 14

14 Four jugs of cordial have a cordial to water ratio as shown and a given total volume.

Jug	Cordial to water ratio	Total volume
1	1 : 5	600 mL
2	2 : 7	900 mL
3	3 : 5	400 mL
4	2 : 9	330 mL

a How much cordial is in:
- **i** Jug 1?
- **ii** Jug 2?

b How much water is in:
- **i** Jug 3?
- **ii** Jug 4?

c If Jug 1 and 2 were mixed together to give 1500 mL of drink:
- **i** how much cordial is in the drink?
- **ii** find the ratio of cordial to water in the drink.

d Find the ratio of cordial to water if the following jugs are mixed.
- **i** Jug 1 and 3
- **ii** Jug 2 and 3
- **iii** Jug 2 and 4
- **iv** Jug 3 and 4

e Which combination of two jugs gives the strongest cordial to water ratio?

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1H Rates CONSOLIDATING

Learning intentions for this section:

- To revise the meaning, use and applications of rates
- To be able to simplify rates and recognise equivalent rates
- To be able to solve problems involving speed, best buys and other rates

Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- The Path topic *Variation and rates of change* can be found in Chapter 6 of our Year 10 book
- Rates concepts are revisited in Stage 6 Standard and may also be required in non-calculator examinations such as NAPLAN and industry aptitude tests

It is often necessary to compare two quantities with different units. When this occurs it is called a rate.

Rates are used to describe speed (m/s, km/h), pay, lap times in formula one racing, and even the prices at the supermarket. Using rates is important in our everyday lives.

Lesson starter: Fastest animals on Earth

In pairs, arrange these twelve animals from fastest to slowest. Then see if you can write their speeds in metres per second (m/s).

Zebra 64.37 km/h

Ostrich 70 km/h

Giraffe 52 km/h

African bush elephant 40.7 km/h

Lion 80 km/h

Cheetah 120 km/h

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Dog 72.4 km/h

Galapagos tortoise 0.32 km/h

Kangaroo 70.01 km/h

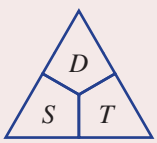
Wildebeest 64.05 km/h

Hyena 65 km/h

Fox 32.03 km/h

KEY IDEAS

- A **rate** compares related quantities with different units.
  - The rate is usually written with one quantity compared to a single unit of the other quantity.
    For example: 50 km per 1 hour or 50 km/h
- Rates can be used to determine best buys when purchasing products.
- Speed (S) = distance (D) ÷ time (T)
  For example:
  80 km in 2 hours = 40 km/h



$S = \frac{D}{T}$, $D = S \times T$, $T = \frac{D}{S}$

Key terminology: rate

Exercise 1H

UNDERSTANDING 1–4 3, 4

1 Write the following as simple rates using the / symbol.

a Tom travelled 60 km in 1 hour

b $84 in 1 hour

c 600 km in 10 hours

Hint: Compare to 1 hour in part **c**.

2 Find the cost of 1 kg if:

a 2 kg costs $8 **b** 5 kg costs $15 **c** 4 kg costs $10

Hint: Use units m/s or km/h.

3 Determine the speed of the following trips

a 100 m in 10 seconds **b** 120 km in 2 hours **c** 42 km in 3 hours

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4 Callum's daily wage is directly proportional to the number of hours he works. If he works for 2 hours, he earns $42. How much does he earn if the number of hours worked:

a doubles? **b** halves? **c** triples (×3)?

FLUENCY 5–6(½), 7, 8–9(½) | 5–6(½), 8–9(½)

Example 27 Simplifying rates

Write these rates in simplest form.

a 120 km every 3 hours **b** 5000 revolutions in $2\frac{1}{2}$ minutes

SOLUTION

a 120 km per 3 hours $= \frac{120}{3}$ km/h

$= 40$ km/h

b 5000 revolutions per $2\frac{1}{2}$ minutes

$= 10\,000$ revolutions per 5 minutes

$= \frac{10\,000}{5}$ revs/min

$= 2000$ revs/min

EXPLANATION

Divide by 3 to write the rate compared to 1 hour.

First multiply by 2 to remove the fraction.

Then divide by 5 to write the rate using 1 minute.

Now you try

Write these rates in simplest form.

a $150 in 5 hours **b** 1800 revolutions in $1\frac{1}{2}$ minutes

5 Write these as rates in their simplest form.

a $84 in 3 hours

b $200 in 4 hours

c 12 kg every 2 minutes

d $3.50 for $\frac{1}{2}$ kg

e 64 runs in 16 overs

f 623 points in 7 games

g 76 cm in 4 years

h 56 metres in 4 seconds

i 207 heart beats in $2\frac{1}{4}$ minutes

j 180 mL in 22.5 seconds

Hint: Work out each per one unit.

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Example 28 Determining distance, speed and time

A family took 3 hours to complete the 210 km from their home to their holiday house.

a Find their average speed for the trip.

b If they increased their average speed to 90 km/h, how many hours and minutes would the trip have taken?

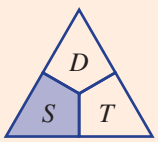
SOLUTION

a Average speed = distance ÷ time
= 210 km ÷ 3 hours
= 70 km/h

EXPLANATION

Use the *DST* triangle and cover the *S* to provide the formula for finding speed:

$S = \frac{D}{T}$ or $D \div T$



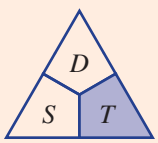
Substitute in your values and calculate the speed.

Remember to put units in your answer.

b Time = distance ÷ speed
= 210 km ÷ 90 km/h
$= 2\frac{1}{3}$ hours
= 2 hours 20 minutes

Use the *DST* triangle and cover the *T* to find the formula for time:

$T = \frac{D}{S}$ or $D \div S$



Substitute in your values and find the time:

$\frac{1}{3}$ hour $= \frac{1}{3} \times 60$ min $= 20$ minutes

Now you try

A rally driver took 5 hours to complete a 300 km course.

a Find the average speed for the rally.

b If the average speed was increased to 72 km/h, how many hours and minutes would the course have taken?

6 Find the average speed (km/h) of:

a a car travelling 140 km in 2 hours

b a bike travelling 60 km in 4 hours

c a walker travelling 12 km in 3 hours

d a horse galloping 2.7 km in 3 minutes ($\frac{1}{20}$ hour)

e a truck travelling 760 km in 9.5 hours

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7 Rick drives the 1040 km from Sydney to Melbourne to watch the Sydney Swans play the Western Bulldogs in the AFL. He averaged 80 km/h.

a How many hours did the trip take?

b How many hours and minutes would the trip take if the average speed was reduced to 64 km/h?

Example 29 Finding best buys

Which is better value?
5 kg of potatoes for $3.80 or 3 kg for $2.20

SOLUTION

Price per kg.

5 kg bag.
1 kg costs $3.80 ÷ 5 = $0.76

3 kg bag.
1 kg costs $2.20 ÷ 3 = $0.73
∴ the 3 kg bag is cheaper

EXPLANATION

Divide each price by the number of kilograms to find the price per kilogram.

Then compare. Choose the cheapest.

Now you try

Which is better value?
2 L of chocolate ice cream for $4.50 or 3 L of the same ice cream for $6.45.

8 Determine the best buy in each of the following.

a 2 kg of washing powder for $11.70 or 3 kg for $16.20

b 1.5 kg of red delicious apples for $4.80 or 2.2 kg of royal gala apples for $7.92

c 2.4 litres of orange juice for $4.20 or 3 litres of orange juice for $5.40

d 0.7 GB of internet usage for $14 or 1.5 GB for $30.90 with different service providers

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Example 30 Using rates

A company claims that one 8 kg bag of its fertiliser covers 10 m^2.
How many kilograms would be needed to cover 25 m^2?

SOLUTION

8 kg per 10 m^2
$= 0.8$ kg/m^2
0.8×25
$= 20$ kg are needed for 25 m^2

EXPLANATION

Write the rate: 8 kg/10 m^2

Divide by 10 to simplify the rate to kg per m^2.

Multiply by 25 to find the kilograms needed for 25 m^2.

Now you try

A 4 litre tin of paint covers 50 m^2.
How many litres would be needed to cover 20 m^2?

9 Consider the following.

a If Shute types 65 words a minute, how many words can he type in:
i 2 minutes? **ii** 5 minutes? **iii** 1 hour?

b A jeweller uses 3 grams of copper per 22 grams of gold. Find the gold needed for:
i 6 grams of copper **ii** 30 grams of copper

c Lucy and Tom both pay the same percentage of their income in tax. When Lucy earns $25 000 she pays $5000 in tax. How much tax does Tom pay if he earns four times what Lucy does?

d A carpet cleaner charges $80 per 20 m^2 of carpet cleaned. How much would it cost to clean 4 rooms with a total of 45 m^2 of carpet?

PROBLEM-SOLVING AND REASONING

10–12 | 11, 13, 14

10 Hamish rides his bike at an average speed of 22 km/h. How far does he ride in:

a $2\frac{1}{2}$ hours? **b** $\frac{3}{4}$ hours? **c** 15 minutes?

11 Find the cost of 100 g of each product below then decide which is the best buy.

a 300 g of coffee A at $10.80 or 220 g of coffee B at $8.58.
b 600 g of pasta A for $7.50 or 250 g of pasta B for $2.35
c 1.2 kg of cereal A for $4.44 or 825 g of cereal B for $3.30

12 A shearer sheared 80 sheep in 2 hours.

a Express this as a rate.
b If the shearer is able to continue at this rate for $5\frac{1}{2}$ hours, how many sheep can be sheared?
c How long would it take, at this rate, to shear all 1000 sheep on the property?

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13 Light travels at approximately 300 000 km/s.

- **a** Express this speed as km/h.
- **b** How far does light travel in 2 minutes?
- **c** How long, in minutes, does light from the Sun take to travel the 149 million kilometres to Earth?

14 Exchange rates between currencies vary every day. If 1 euro equals 1.58 Australian dollars, but you must pay a 1% conversion fee:

- **a** how many euros (to 2 decimal place) can you get for 100 Australian dollars?
- **b** what does it cost to buy 100 euros?

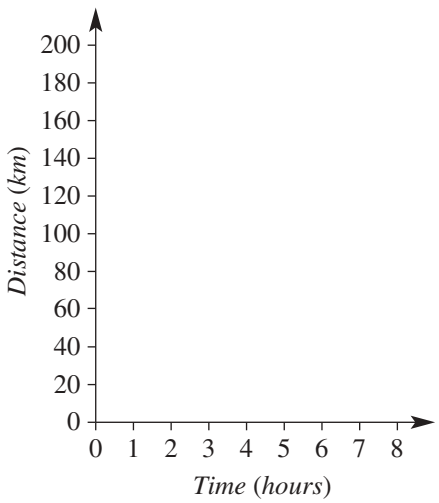
ENRICHMENT: Rates of progress — 15

15 A cyclist's progress over a journey of 160 km is recorded. The ratio of time (in hours) to distance (in kilometres) is 1 : 20.

- **a** Copy and complete the table showing the progress of the cyclist.

Time in hours	0	1	2	3	4	5	6	7	8
Distance in km	0	20							

- **b** What is the cyclist's average speed?
- **c** Draw a set of axes like the ones shown and record the information from the table onto the axes.
- **d** What conclusions can you make about the shape of the graph? Why is this the case?
- **e** If the cyclist doubles their speed, on the same set of axes plot a graph that shows the journey for this new speed.



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Maths@Work

Cooks and chefs

Being a professional cook or chef is a physically demanding job. Long hours in hot kitchens and being on your feet all day are challenging. Most chefs have a passion for food and flavours and are seen as creative people who create not just a meal but a masterpiece on a plate.

Chefs need to have good communication skills, a practical sense of measurement and excellent number skills. Their job involves ordering stock, understanding cooking temperatures and adapting recipes using ratios and direct proportion.

1 A recipe for a classic pavlova uses 6 egg whites and 270 grams ($1\frac{1}{4}$ cups) of superfine sugar or caster sugar and serves 8. (A pavlova also contains cornflour and white vinegar.)

- **a** What is the simplified ratio of egg whites to servings?
- **b** How many egg whites are needed for a pavlova serving 12 people?
- **c** How many whole egg whites are needed for a pavlova serving 6 people?
- **d** How many servings are in a pavlova using 15 egg whites?
- **e** How many grams of sugar are equivalent to 1 cup?
- **f** How many grams of sugar are needed per egg white for this pavlova recipe?

2 A recipe for café-style banana bread indicates that it would make 3 large loaves. The recipe is as follows:

9 cups of plain flour
$4\frac{1}{2}$ cups sugar
3 teaspoons of bicarb of soda
1125 grams of melted butter
9 eggs
10 mashed bananas

- **a** If each loaf can be cut into 18 single slices, how many loaves need to be baked per day for 50 single-slice serves at a café?
- **b** How many cartons of a dozen eggs are needed for a week where the cook makes this recipe 8 times?
- **c** If 1 egg weighs 75 grams, what is the simplified ratio of eggs to butter in this recipe?
- **d** If 1 cup of plain flour is equivalent to 150 grams of flour, how many grams of flour is needed for this recipe?
- **e** What is the simplified ratio of flour to melted butter for this recipe?
- **f** If only one loaf is to be made, write down the amounts of ingredients required.

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3 Many chefs start their careers mastering the basics such as mayonnaise.
The recipe for 1 cup of classic mayonnaise is:

2 egg yolks
2 teaspoons of lemon juice
1 teaspoon of mustard
$\frac{1}{2}$ teaspoon of salt
pinch of pepper
250 mL of olive oil

a What is the simplified ratio of:
- **i** lemon juice to mustard?
- **ii** lemon juice to olive oil (1 teaspoon = 5 mL)?
- **iii** mustard to salt?

b If a litre is the same as 4 cups, how many eggs are needed for 1 L of mayonnaise?

c If one lemon yielded 25 mL of fresh juice, how many cups of mayonnaise can be made using all of the juice in this one lemon?

4 Scaling a recipe means that the ingredient quantities are each multiplied by a conversion factor so that the recipe makes more or fewer serves. If a cake recipe makes 12 cupcakes, what conversion factor is required to scale up the recipe quantities to make 42 cupcakes?

Using technology

Hint: Format all formula cells to 'number'/one decimal place.
Use $ signs to fix (i.e. anchor) a cell reference, e.g. C4.

5 Set up an Excel spreadsheet as shown below for scaling cookie recipe quantities. Enter formulas in the shaded cells.

	A	B	C	D
1	Bella's Bakery			
2	Chocloate chip cookies recipe			
3	Recipe number of cookies	Desired number of cookies	Conversion factor	
4	24	60		
5	**Ingredient**	**Recipe quantities**	**Unit**	**Scaled quantities**
6	Eggs	1	egg	
7	Macadamia nut oil	60	mL	
8	Apple sauce	2	teaspoon	
9	Brown sugar	160	g	
10	Vanilla extract	1	teaspoon	
11	Plain flour	1	cup	
12	Baking powder	1	teaspoon	
13	Rolled oats	1	cup	
14	Dark chocolate	100	g	

a For a recipe to make 60 cookies, state the quantities required of the following ingredients:
- **i** oil
- **ii** flour
- **iii** chocolate

b Bella has a school tuckshop order for 32 bags with 3 cookies in each. State the quantities of the following ingredients that Bella will need:
- **i** eggs
- **ii** apple sauce
- **iii** sugar

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Working mathematically

Painting makeover

Archer and Emily are mixing their own paint to form a special colour for their loungeroom and bedroom makeover. They have tested the following ratios and feel that these two options, A and B, suit their tastes.

	White	Yellow	Red	Blue
Colour ratio A	18	2	1	3
Colour ratio B	14	3	2	1

Archer and Emily want to know the amount and cost of paint required to cover the walls and ceiling of the loungeroom and bedroom using either ratio A mix or ratio B mix.

The white paint costs \$8 per litre and the colours all cost \$24 per litre.
The paint covers 10 m^2 per litre.
The loungeroom is 6 m long, 5 m wide and 2.5 m high.
The bedroom is 5 m long, 4 m wide and 2.5 m high.

Present a report for the following tasks and ensure that you show clear mathematical workings, explanations and diagrams where appropriate.

Routine problems

a Using the table above, copy and complete the colour ratios for:
- **i** Colour ratio A, W : Y : R : B = ☐ : ☐ : ☐ : ☐
- **ii** Colour ratio B, W : Y : R : B = ☐ : ☐ : ☐ : ☐

b What fraction of:
- **i** white paint is in ratio A?
- **ii** blue paint is in ratio A?
- **iii** white paint is in ratio B?
- **iv** red paint is in ratio B?

c If 36 L of paint is required in total, find the volume of the following amounts, giving each answer as a mixed number.
- **i** white paint in ratio A
- **ii** blue paint in ratio A
- **iii** white paint in ratio B
- **iv** red paint in ratio B

Non-routine problems

Explore and connect

The problem is to compare the cost of painting the loungeroom and bedroom using the colour mixes A and B.

a Write down all the relevant information that will help solve this problem.

b For the loungeroom and bedroom:
- **i** Draw a labelled diagram of each rectangular ceiling and each room's 4 rectangular walls.
- **ii** For each room, calculate the total area of the walls and ceiling in m^2.
- **iii** If 2 coats of paint are applied, calculate the total area to be painted.
- **iv** Determine the total volume of paint needed given that each litre covers 10 m^2.

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Choose and apply techniques

c For colour ratio A, determine:
  i The fraction that is white paint and the fraction that is coloured paint.
  ii The volume of white paint needed.
  iii The cost of white paint at $8 per litre.
  iv The volume of coloured paint needed.
  v The cost of coloured paint at $24 per litre.

d For colour ratio B, determine:
  i The fraction that is white paint and the fraction that is coloured paint.
  ii The volume of white paint needed.
  iii The cost of white paint at $8 per litre.
  iv The volume of coloured paint needed.
  v The cost of coloured paint at $24 per litre.

Communicate thinking and reasoning

e Determine the total cost of the paint for colour ratio A and for colour ratio B.
f Which colour ratio will be the cheaper option and by how much?
g Compare the percentages of white and coloured paint in ratios A and B.
h Explain why one ratio produces a more expensive paint.
i Summarise your results and describe any key findings.

Extension problems

Problem-solve

Emily paints the bedroom at the same time as Archer paints the loungeroom. Emily paints at a rate of 13 m^2 per hour and Archer paints at a rate of 17 m^2 per hour.

a How long does it take for one coat of paint to be applied to each room?
b If there is a 3 hour gap between coats, find the total time needed for this painting makeover.

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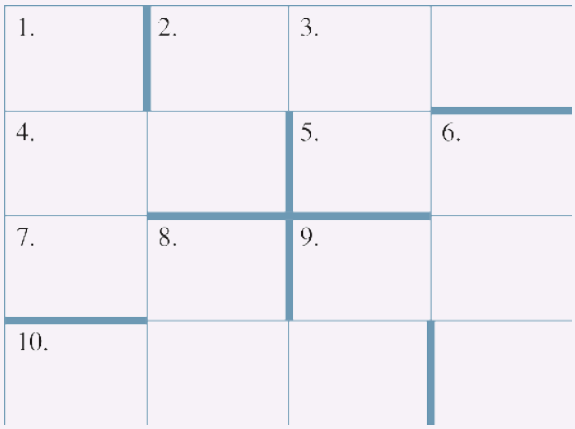
Puzzles and challenges

1 Can you arrange the first nine counting numbers using signs (+, −, ×, ÷) and brackets to give an answer of 100?

2 In how many ways can four girls and two boys be seated in a row if the two boys are to sit at the ends?

3 Complete the number cross below.



Clues

Across

2. The sum of 100, 150 and 3
4. The product of 5 and the first prime number after 6
5. Reverse the digits of the product of 13 and 5
7. One less than the square of 10
9. Half of 6 times 8
10. 2 to the power of 7

Down

1. A palindromic number
2. $5 \times 10 - 5 \times 5$
3. A multiple of 11
6. 639 written correct to two significant figures
8. The next even integer after 90
9. A multiple of 7

4 **a** Perfect numbers are positive integers that are equal to the sum of all their factors, excluding the number itself.

i Show that 6 is a perfect number.

ii There is one perfect number between 20 and 30. Find the number.

iii The next perfect number is 496. Show that 496 is a perfect number.

b Triangular numbers are the numbers of dots required to form triangles, as shown in this table.

i Complete this table.

Number of rows	1	2	3	4	5	6
Diagram	•	• • •	• • • • • •			
Number of dots (triangular number)	1	3				

ii Find the 7th and 8th triangular numbers.

c Fibonacci numbers are a sequence of numbers where each number is the sum of the two preceding numbers. The first two numbers in the sequence are 0 and 1.

i Write down the first 10 Fibonacci numbers.

ii If the Fibonacci numbers were to be extended in the negative direction, what would the first four negative Fibonacci numbers be?

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Chapter summary

Integers, decimals, fractions, ratios and rates

Decimals
Decimals are a way of writing fractions with denominators of 10, 100, 1000.

$\frac{1}{10} = 0.1$, $\frac{1}{100} = 0.01$,

$\frac{1}{1000} = 0.001$

Rounding decimals
If the critical digit (the next one in the number) is 0, 1, 2, 3 or 4, round down. If it is 5, 6, 7, 8 or 9, round up.

$4.763 = 4.8$ (1 d.p.)

$= 4.76$ (2 d.p.)

Integers
Integers are the set of positive and negative whole numbers including zero.

0, ±1, ±2, ±3, ±4, ...

Working with negatives
- To add a negative number, you subtract.
  $a + (-b) = a - b$
- To subtract a negative number, you add.
  $a - (-b) = a + b$

× and ÷
- Two numbers of the same sign multiply or divide to give a positive answer. $-4 \times (-5) = 20$, $-10 \div (-2) = 5$
- Two numbers with different signs multiply or divide to give a negative answer. $-4 \times 5 = -20$, $10 \div (-2) = -5$

Rational numbers
Rational numbers can be written as a fraction.

$3, -4, \frac{2}{3}, 0.\dot{3}, \sqrt{4}, 0.416$ are all rationals.

π is an example of an irrational number. It cannot be written as a fraction.

Ratios
Ratios compare two or more quantities with the same units in a definite order.

5 cents to \$1 = 5 : 100 = 1 : 20

You can simplify ratios by using the HCF.

24 : 36 = 2 : 3 (dividing both by 12)

Use whole numbers:

0.3 : 7 = 3 : 70

$\frac{3}{5} : \frac{8}{5} = 3 : 8$

Reciprocals
The reciprocal of a proper or improper fraction is the fraction turned upside down (inverted).

The reciprocal of:

6 is $\frac{1}{6}$

$1\frac{1}{2}$ is $\frac{2}{3}$

$\frac{4}{5}$ is $\frac{5}{4}$ or $1\frac{1}{4}$

Rates
Rates compare two quantities with different units.

50 km in 2 hours = 25 km/h

Fractions
- To add or subtract fractions, you need to use the lowest common denominator.
  $\frac{3}{4} + \frac{1}{6} = \frac{9}{12} + \frac{2}{12} = \frac{11}{12}$
- To multiply fractions, multiply the numerators and multiply the denominators after cancelling common factors.
  $\frac{8}{9} \times \frac{3}{7} = \frac{8 \times \cancel{3}^{1}}{\cancel{9}_{3} \times 7} = \frac{8}{21}$
- To divide by a fraction, multiply by the reciprocal.
  $\frac{3}{2} \div \frac{4}{5} = \frac{3}{2} \times \frac{5}{4}$
- Convert mixed numbers to improper fractions to apply operations.

Fractions and decimals
- Divide to express a fraction as a decimal.
  $\frac{11}{4} = 3.75$, $\frac{8}{3} = 2.\dot{6}$
- Use a denominator of 10, 100 etc. to write a decimal as a fraction.
  $0.37 = \frac{37}{100}$, $2.4 = 2\frac{4}{10} = 2\frac{2}{5}$

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Chapter checklist

Chapter checklist with success criteria

A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

		✔
1A	**1 I can add and subtract integers.** e.g. Find **a** $-15 + 5$ **b** $-4 - 6$	☐
1A	**2 I can add a negative integer.** e.g. Find $4 + (-9)$	☐
1A	**3 I can subtract a negative integer.** e.g. Find $3 - (-7)$	☐
1B	**4 I can multiply and divide with negatives.** e.g. Find the value of **a** $-7 \times (-4)$ **b** $48 \div (-6)$	☐
1B	**5 I can find squares and cubes of negative numbers.** e.g. Find the value of **a** $(-4)^2$ **b** $(-5)^3$	☐
1B	**6 I can apply order of operations.** e.g. Find the value of $3 \times (-4 + 2) - 5$	☐
1C	**7 I can round to a required number of decimal places.** e.g. Round each of these to two decimal places: **a** 32.2389 **b** 3.712 **c** 1.4954	☐
1C	**8 I can round to a number of significant figures.** e.g. Round each of these numbers to two significant figures: **a** 7842 **b** 0.0375	☐
1C	**9 I can estimate using significant figures.** e.g. Estimate the answer to $1067 - 506 \times 0.052$ by rounding each number in the problem to one significant figure. Use your calculator to check how reasonable your answer is.	☐
1D	**10 I can identify rational numbers.** e.g. Choose the rational numbers in the following list: $\sqrt{10}, \pi, 1.4, \sqrt{36}, -\frac{3}{5}$	☐
1D	**11 I can convert between mixed numbers and improper fractions.** e.g. Express **a** $\frac{18}{5}$ as a mixed number **b** $2\frac{3}{7}$ as an improper fraction	☐
1D	**12 I can write fractions as decimals, including recurring decimals.** e.g. Write **a** $2\frac{7}{8}$ as a decimal **b** $\frac{4}{11}$ as a recurring decimal	☐
1D	**13 I can write decimals as fractions.** e.g. Write these decimals as fractions: **a** 0.36 **b** 2.215	☐

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Chapter checklist with success criteria

1D	**14 I can compare fractions.** e.g. Decide which is the larger fraction of the following: $\frac{7}{9}$ or $\frac{11}{15}$	☐
1E	**15 I can add and subtract fractions with the same denominator.** e.g. Find **a** $\frac{8}{17}-\frac{3}{17}$ **b** $2\frac{1}{8}+\frac{5}{8}$	☐
1E	**16 I can add and subtract fractions with different denominators.** e.g. Evaluate $\frac{5}{6}+\frac{3}{5}$	☐
1E	**17 I can add and subtract mixed numbers.** e.g. Evaluate: **a** $2\frac{3}{4}+1\frac{5}{6}$ **b** $2\frac{1}{3}-1\frac{3}{7}$	☐
1F	**18 I can multiply proper fractions.** e.g. Evaluate: **a** $\frac{5}{6}\times\frac{1}{4}$ **b** $\frac{6}{7}\times\frac{14}{27}$	☐
1F	**19 I can divide proper fractions.** e.g. Evaluate $\frac{5}{8}\div\frac{15}{28}$	☐
1F	**20 I can multiply and divide with mixed numbers.** e.g. Evaluate $4\frac{4}{7}\div 2\frac{2}{3}$	☐
1G	**21 I can simplify ratios.** e.g. Simplify these ratios: **a** $40:32$ **b** $0.3:0.21$ **c** $\frac{5}{8}:\frac{9}{20}$	☐
1G	**22 I can simplify ratios by converting to common units.** e.g. Simplify the ratio of 80 cm to 5 metres	☐
1G	**23 I can divide into a given ratio using the unitary method.** e.g. $\$480$ is to be divided into the ratio $3:5$. Find the value of the smaller portion using the unitary method.	☐
1H	**24 I can simplify a rate.** e.g. Write these rates in simplest form: **a** 120 L in 2 hours **b** 108 strokes in $1\frac{1}{2}$ minutes	☐

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Chapter checklist

Chapter checklist with success criteria

1H	**25 I can determine distance, speed and time.** e.g. A driver took 3 hours to complete a 180 km trip. What was their average speed and what was the time taken, in hours and minutes, on the return trip if the average speed was 75 km/h?	☐
1H	**26 I can determine a best buy.** e.g. Which is better value? 4 L of paint for $84 or 7 L of the same paint for $126?	☐
1H	**27 I can solve problems involving rates.** e.g. A gardener charges $15 per 12 m^2 of grass mown. How much would it cost to mow 32 m^2 of lawn?	☐

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Short-answer questions

1A/1B **1** Evaluate the following.

a $-4-(-12)$ b $18-9\times 7$ c $-90\times(-3)$

d $(-4)^3$ e $-15\div 5\times(-2)$ f $(10-12)\times(7-11)$

1C **2** Round these numbers to three significant figures.

a 21.483 b 29 130 c 0.15271 d 0.002414

1C **3** Round to two decimal places:

a 14.97683 b 0.7149871498 … c 1.999

1D **4** Write these fractions as decimals.

a $2\frac{1}{8}$ b $\frac{5}{6}$ c $\frac{13}{7}$

1D **5** Write these decimals as fractions.

a 0.75 b 1.6 c 2.55

1F **6** State the reciprocal of:

a $\frac{3}{4}$ b $\frac{1}{5}$ c 8 d $1\frac{2}{7}$

1E/1F **7** Simplify the following.

a $\frac{5}{6}-\frac{1}{3}$ b $1\frac{1}{2}+\frac{2}{3}$ c $\frac{13}{8}-\frac{4}{3}$

d $3\frac{1}{2}\times\frac{4}{7}$ e $5\div\frac{4}{3}$ f $3\frac{3}{4}\div 1\frac{2}{5}$

1G **8** Simplify these ratios.

a 30 : 12 b 1.6 : 0.9 c $7\frac{1}{2} : 1\frac{2}{5}$

1G **9** Divide 80 into the given ratio.

a 5 : 3 b 5 : 11 c 1 : 2 : 5

1H **10** Dry dog food can be bought from store A for \$18 for 8 kg or from store B for \$14.19 for 5.5 kg.

a Determine the cost per kilogram at each store and state which is the best buy.

b Determine to the nearest whole number how many grams of each brand you get per dollar.

1G **11** Share \$660 in the ratio of 2 : 3 : 5.

1H **12** Complete these rates.

a \$96 in 3 hours = \$_____/h

b 14 g in 2 minutes = _____g/min

c 32 m in 10 seconds = _____m/min

d \$7.40 in 30 minutes = \$_____/h

e 660 runs in 8 matches = _____runs/match

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Chapter review

1H **13** Write the following speeds in km/h.

a 780 km in 5 hours **b** 90 km in $2\frac{1}{2}$ hours

1G **14** Tom and Claire share their lottery win in the ratio of 3 : 2. If Tom got $9000, how much did Claire get?

1H **15** In an electrical wire the resistance is proportional to the length of the wire. A length of 6 m has a resistance of 5 ohms. What is the length if the resistance is measured to be 30 ohms?

1H **16** In a rectangle what happens to the perimeter when the side lengths are doubled?

Multiple-choice questions

1D **1** $\frac{2}{7}$ written as a decimal is:

A 0.29 **B** 0.286 **C** $0.28\dot{5}$ **D** $0.\overline{285714}$ **E** $0.285714\dot{}$

1C **2** 3.0456 written to three significant figures is:

A 3.04 **B** 3.05 **C** 3.045 **D** 3.046 **E** 3.45

1D **3** 2.25 written as a fraction in simplest form is:

A $2\frac{1}{2}$ **B** $\frac{5}{4}$ **C** $\frac{9}{4}$ **D** $9\frac{1}{4}$ **E** $\frac{225}{100}$

1E **4** $1\frac{1}{2} - \frac{5}{6}$ is equal to:

A $\frac{2}{3}$ **B** $\frac{5}{6}$ **C** $-\frac{1}{2}$ **D** $\frac{2}{6}$ **E** $\frac{1}{2}$

1F **5** $\frac{2}{7} \times \frac{3}{4}$ is equivalent to:

A $\frac{8}{11}$ **B** $\frac{3}{7}$ **C** $\frac{5}{11}$ **D** $\frac{8}{12}$ **E** $\frac{3}{14}$

1F **6** $\frac{3}{4} \div \frac{5}{6}$ is equivalent to:

A $\frac{5}{8}$ **B** 1 **C** 21 **D** $\frac{4}{5}$ **E** $\frac{9}{10}$

1G **7** Simplifying the ratio 50 cm : 4 m gives:

A 50 : 4 **B** 8 : 1 **C** 25 : 2 **D** 1 : 8 **E** 5 : 40

1D **8** 0.28 as a fraction in its simplest form is:

A 0.28 **B** $\frac{28}{100}$ **C** $\frac{0.28}{100}$ **D** $\frac{2.8}{100}$ **E** $\frac{7}{25}$

1G **9** $2.50 divided in the ratio of 4 : 1 is:

A 50 : 200 **B** 50c **C** $2 : 50c **D** $200 **E** $2

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1G **10** A childcare centre requires the ratio of carers to children to be 1 : 5. How many carers are needed for 30 children?

A 5 **B** 6 **C** 150 **D** 30 **E** 36

1H **11** Michael earns $460 in 25 hours. His hourly rate of pay is:

A $11500/h **B** $0.05/h **C** $18.40/h **D** $18.40/day **E** $5/h

1H **12** 36 km in 40 minutes is the same as:

A 40 km/h **B** 36 km/h **C** 54 km/h **D** 90 km/h **E** 56 km/h

1G **13** To mix a particular shade of pink, red paint is mixed with white paint. The ratio is 8 to 3. If 20 cans of red paint are used, how many cans of white paint will be needed to mix the correct shade of pink?

A 15 **B** 7.5 **C** 7 **D** 8 **E** 24

Extended-response questions

1 A class of 28 students is split into groups A and B in a ratio of 3 : 4.

- **a** How many students are in group A?
- **b** Two students are moved from group A to group B. How many are now in each group?
- **c** What is the new ratio of group A to group B students in part **b**?

2 The Tomslin family plans to drive the 856 km from Perth to Monkey Mia in Western Australia.

- **a** At what speed should they travel, on average, if they hope to only take 10 hours?
- **b** The family left home at 6 a.m. and arrived at Monkey Mia at 8 p.m. How long did the trip take?
- **c** Calculate their average speed in part **b**. Answer to one decimal place.
- **d** How far could they have travelled at the speed in part **c** if they only drove for the original 10 hours as planned? Answer to the nearest km.

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2

Financial mathematics

Maths in context: Calculating wages, taxes and discounts

Skills in financial mathematics are essential for successful business management and for achieving personal financial independence. Percentage skills are needed for the financial calculations performed by individuals, accountants, bookkeepers, small business managers and employers.

Percentages are used to calculate mark-up and discount amounts, cost prices and selling prices, profits, losses, insurance payments, GST, business tax, wage tax, wage increases and the interest on loans.

Personal income can be earned in various ways:

- a fixed salary p.a. ('per annum', i.e. per year), e.g. accountants, business owners, teachers, engineers, pharmacists and surveyors
- a wage calculated from an hourly rate, e.g. chefs, cleaners, florists, dental assistants, nurses, receptionists, retail salespeople, construction workers, panel beaters and auto mechanics
- by the job, e.g. personal trainers, fruit pickers, bricklayers, fishers, tailors, hairdressers, piano tuners and carpet layers
- commission, e.g. car salespeople, real estate agents
- royalties, e.g. songwriters and book authors.

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Chapter contents

2A Percentages, fractions and decimals (CONSOLIDATING)
2B Applying percentages (CONSOLIDATING)
2C Percentage increase and decrease (CONSOLIDATING)
2D Profits and discounts (CONSOLIDATING)
2E Income
2F Taxation
2G Simple interest
2H Applications of simple interest

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- solves financial problems involving simple interest, earning money and spending money (MA5-FIN-C-01).

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Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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Warm-up quiz

1 Simplify:

a $\frac{12}{100}$ **b** $\frac{20}{100}$ **c** $\frac{35}{100}$ **d** $\frac{75}{100}$ **e** $\frac{60}{100}$ **f** $\frac{50}{100}$

2 Multiply these decimals by 100.

a 0.99 **b** 0.58 **c** 0.9 **d** 1.22 **e** 0.08 **f** 1.5

3 Which is larger in each of the following pairs?

a $\frac{1}{2}$ or 55% **b** $\frac{3}{4}$ or 70% **c** 0.89 or 98%

4 Copy and complete.

a $61\% = \frac{\square}{100}$ **b** $9\% = \frac{\square}{100}$ **c** $37\% = \frac{\square}{100}$

d $121\% = \frac{\square}{100}$ **e** $1\% = \frac{\square}{100}$ **f** $75\% = \frac{\square}{100} = \frac{3}{\square}$

5 Copy and complete the following table.

Fraction	Decimal	Percentage
$\frac{1}{100}$		
	0.1	
	0.25	
		50%
$\frac{3}{4}$		

6 Write down 10% of these amounts.

a 100 g **b** 70 km **c** \$450

d \$8000 **e** \$5 **f** 90c

7 Find:

a 25% of \$400 **b** 75% of 80 m

c 50% of \$3 **d** 10% of \$678

e 1% of 600 days **f** $\frac{1}{2}$% of 600 days

8 Complete the following.

a 1% of 60 = 60 divided by $\square$ **b** 10% of 50 = 50 divided by $\square$

c 5% of 100 = 100 divided by $\square$ **d** $33\frac{1}{3}$% of 963 = 963 divided by $\square$

e 25% of 88 = 88 divided by $\square$

9 Find $33\frac{1}{3}$% of 6300 km.

10 Which is larger: 40% of 50 or 25% of 100?

11 Copy and complete.

a 1 week = $\square$ days **b** 1 year = $\square$ days

c 1 year = $\square$ weeks **d** 1 year = $\square$ months

12 How many hours are there from:

a 5 a.m. to 7 p.m.? **b** 9 a.m. to 3 p.m.? **c** 8:30 a.m. to 9 p.m.?

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2A Percentages, fractions and decimals CONSOLIDATING

Learning intentions for this section:

- To review the conversions between percentages, fractions and decimals
- To memorise the most useful fraction/decimal/percentage conversions
- To be able to express one quantity as a percentage of another

Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- Percentages are used extensively in this chapter and also Chapter 1 of our Year 10 book
- Expertise with percentages is assumed knowledge for Stage 6 Standard and may also be required in non-calculator examinations such as NAPLAN and industry aptitude tests

We use percentages for many different things in our daily lives. Some examples include home loans, credit cards, sales and profits.

We know from our previous work on percentages that they represent a fraction with a denominator of 100. 'Per cent' comes from the Latin words *per centum*, and means 'out of 100'.

Lesson starter: Ordering with percentages

Ten different values are given below.

$$\frac{1}{2}, 0.8, 0.05, 15\%, \frac{7}{20}, 0.9, 9\%, \frac{3}{5}, \frac{1}{3}, 0.3$$

In pairs, decide:

a which of the numbers is the smallest

b which of the numbers is the largest.

Write the 10 numbers in ascending order.

KEY IDEAS

- The table below contains some commonly used fractions, decimals and percentages. Memorise these.

Fraction	Decimal	Percentage	Therefore
1	1	100%	$3 = 300\%$
$\frac{1}{2}$	0.5	50%	$1.5 = 150\%$
$\frac{1}{3}$	0.333 … or $0.\dot{3}$	$33\frac{1}{3}\%$	$\frac{2}{3} = 0.\dot{6} = 66\frac{2}{3}\%$
$\frac{1}{4}$	0.25	25%	$\frac{3}{4} = 0.75 = 75\%$
$\frac{1}{5}$	0.2	20%	$\frac{3}{5} = 0.6 = 60\%$
$\frac{1}{10}$	0.1	10%	$\frac{7}{10} = 0.7 = 70\%$
$\frac{1}{100}$	0.01	1%	$\frac{21}{100} = 0.21 = 21\%$

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■ There are six **conversions** involving fractions, decimals and percentages.

Conversion	Without a calculator	With a calculator
Decimal to fraction $0.6 = \frac{6}{10} = \frac{3}{5}$	Use tenths, hundredths, etc. Simplify the fraction.	Type the decimal. Press =. Press the button that converts fractions to decimals.
Fraction to decimal $\frac{13}{50} = \frac{26}{100} = 0.26$ $\frac{5}{8} = 5 \div 8 = 0.625$	Convert fraction to tenths or hundredths or Divide numerator by denominator.	Type the fraction. Press =. Press the button that converts fractions to decimals.
Decimal to percentage $0.24 \times 100 = 24$ $\therefore 0.24 = 24\%$	Multiply by 100. Add % symbol. (The decimal point appears to move two places to the right.)	Multiply by 100. Add % symbol.
Percentage to decimal $24 \div 100 = 0.24$ $\therefore 24\% = 0.24$	Remove % symbol. Divide by 100. (The decimal point appears to move two places to the left.)	Remove % symbol. Divide by 100.
Fraction to percentage $\frac{13}{50} = \frac{26}{100} = 26\%$ $\frac{5}{8} = 5 \div 8 = 0.625$ $0.625 \times 100 = 62.5$ $\therefore \frac{5}{8} = 62.5\%$	Convert the fraction to hundredths. Add % symbol or Divide numerator by denominator. Multiply by 100. Add % symbol.	Multiply by 100. Add % symbol.
Percentage to fraction $60\% = \frac{60}{100} = \frac{3}{5}$	Remove % symbol. Write as 'hundredths'. Simplify fraction.	Remove % symbol. Divide by 100. Press the button that converts fractions to decimals.

■ Expressing one quantity as a percentage of another (percentage composition):

- Form a fraction, then convert to a percentage.

  For example: 4 goals from 10 attempts $= \frac{4}{10} = \frac{40}{100} = 40\%$

Key terminology: percentage, fraction, decimal

Exercise 2A

UNDERSTANDING 1–4 1, 4

1 Complete the following using the words multiply or divide.

a To convert a decimal into a percentage _____ by 100.

b To convert a percentage into a fraction _____ by 100.

c To convert a fraction into a percentage _____ by 100.

d To convert a percentage into a decimal _____ by 100.

2 Copy and complete this table of common percentages.

Percentage	10%			50%			
Fraction			$\frac{1}{4}$			$\frac{1}{3}$	$\frac{2}{3}$
Decimal		0.2			0.75		

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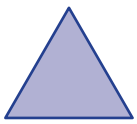
3 What percentage of each of the following diagrams has been shaded?

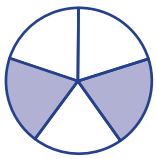
a

b

c



d



Hint: Consider common fractions and percentages for the shaded areas.

4 Scott scored 38 out of 50 on a maths quiz and Sarah scored 79% on the same test. Who scored the higher mark?

FLUENCY 5–9(½) 5–9(½)

Example 1 Percentages and fractions

a Write $\frac{12}{25}$ as a percentage.

b Write 7.5% as a fraction.

SOLUTION

a $\frac{12}{25} = \frac{12}{25} \times 100$

$= 48\%$

b $7.5\% = \frac{7.5}{100}$

$= \frac{15}{200}$

$= \frac{3}{40}$

EXPLANATION

Multiply the fraction by 100.

$\frac{12}{25} \times 100 = \frac{12}{\cancel{25}_{1}} \times \frac{\cancel{100}^{4}}{1}$

Write the percentage as a fraction, using a denominator of 100, $\frac{7.5}{100}$.

Multiply this fraction by $\frac{2}{2}$ so that we don't have a decimal in the fraction and it will be easier to simplify. $\frac{7.5}{100} \times \frac{2}{2} = \frac{15}{200}$

Simplify: $\frac{15}{200} = \frac{3}{40}$ by cancelling a common factor of 5.

Now you try

a Write $\frac{11}{20}$ as a percentage.

b Write 12.5% as a fraction.

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5 Express the following fractions as percentages.

a $\frac{1}{5}$ **b** $\frac{4}{5}$ **c** $\frac{8}{10}$

d $\frac{3}{10}$ **e** $\frac{1}{4}$ **f** $\frac{1}{8}$

g $\frac{3}{4}$ **h** $\frac{12}{20}$ **i** $\frac{14}{25}$

j $\frac{7}{20}$ **k** $\frac{9}{100}$ **l** $\frac{3}{40}$

Hint: Multiply by 100.

6 Express the following percentages as simplified fractions.

a 19% **b** 23% **c** 99%

d 5% **e** 22% **f** 45%

g 74% **h** 75% **i** 2.5%

j 17.25% **k** 1% **l** 125%

Hint: Divide by 100.

Example 2 Converting between percentages and decimals

a Write 0.45 as a percentage. **b** Write 25% as a decimal.

SOLUTION

a $0.45 = 0.45 \times 100 = 45\%$

b $25\% = 25 \div 100 = 0.25$

EXPLANATION

Multiply by 100.

Divide by 100.

Now you try

a Write 0.23 as a percentage. **b** Write 48% as a decimal.

7 Express the following decimals as percentages.

a 0.78 **b** 0.95

c 0.65 **d** 0.48

e 0.75 **f** 1.42

g 0.07 **h** 0.3

i 0.03 **j** 1.04

k 0.12 **l** 0.1225

Hint: Move the decimal point two places to the right.

8 Express the following percentages as decimals.

a 12% **b** 83%

c 57% **d** 88%

e 99% **f** 100%

g 120% **h** 5%

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Example 3 Writing a quantity as a percentage

Write 50c out of $2.50 as a percentage.

SOLUTION

$$50\text{c out of } \$2.50 = \frac{{}^{1}\cancel{50}}{{}_{5}\cancel{250}} \times 100$$
$$= 20\%$$

EXPLANATION

Convert to the same units ($2.50 = 250c) and write as a fraction.

Reduce the fraction to simplest form.
Multiply by 100.

Now you try

Write 90c out of $3.60 as a percentage.

9 In each of the following cases, express the first quantity as a percentage of the second.

a 5 g out of 200 g
b 40c out of $4
c 10 km out of 200 km
d 3 s out of 1 minute
e 200 m out of 1 km
f 100 mL out of $\frac{1}{2}$ L
g 200c out of $1
h 45 marks out of a possible 60 marks

Hint: Write as a fraction and then multiply by 100.

PROBLEM-SOLVING AND REASONING 10–12 11–14

10 Copy and complete the table of the favourite summer sports of Year 9 students.

Sport	Number of students who chose sport	Fraction of the total	Percentage of the total
Swimming	44		
Golf	12		
Volleyball	58		
Cricket	36		
Total			

11 Toni pays 31.5 cents in the dollar in tax. Express this as a percentage.

12 Bad weather stopped a cricket game for 35 minutes of a scheduled $3\frac{1}{2}$ hour match. What percentage of the scheduled time was lost?

13 Joe lost 4 kg and now weighs 60 kg. What percentage of his original weight did he lose?

Hint: What was Joe's original weight?

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14 A company claims that the apple pies it makes are 97% fat free. If the nutritional information on the side of the pack states that total fat is 7 grams of the 250 gram pie, is the claim correct?

ENRICHMENT: Today's timeline — 15

15 Complete this table by filling in estimated times. In the first two columns, insert the times that you did each activity. Use that information to help you fill in the other columns. Round percentages to one decimal place.

a	Time you went to bed last night:	Time you woke up this morning:	Hours and minutes spent in bed:	Percentage of a day spent in bed:
b	Time you started breakfast today:	Time you finished breakfast today:	Minutes spent eating breakfast:	Percentage of the day you spent at breakfast:
c	Time school started today:	Time school is due to finish today:	Hours and minutes spent at school:	Percentage of the day spent at school:
d	Time this maths lesson started:	Time this maths lesson will finish:	Minutes spent in the maths lesson:	Percentage of the day spent in the maths lesson:
e	Time school will finish today:	Time you will arrive home:	Minutes spent travelling home:	Percentage of the day spent travelling home:
f	Time you started your homework yesterday:	Time you finished your homework yesterday:	Minutes spent on homework:	Percentage of your day spent on homework:
g	Time you started watching TV or playing games yesterday:	Time you finished watching TV or playing games:	Minutes spent at this activity:	Percentage of the day spent at this activity:
h	Time you woke up today:	Time you will go to bed tonight:	Hours and minutes spent awake today:	Percentage of the day spent awake:

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2B Applying percentages CONSOLIDATING

Learning intentions for this section:

- To know how to calculate a certain percentage of a given quantity
- To be able to find the original amount from a given percentage

Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- Percentages are used extensively in this chapter and also Chapter 1 of our Year 10 book
- Expertise with percentages is assumed knowledge for Stage 6 Standard and may also be required in non-calculator examinations such as NAPLAN and industry aptitude tests

The media often quotes percentages in news stories and advertisements. For example:

- 90% of dentists prefer this toothbrush.
- A shirt is reduced by 45%.
- Interest rates increased by 2%.

These examples involve finding a percentage of a quantity or amount and this is an important part of the work we do with percentages.

Lesson starter: Today's news challenge

In pairs, go through today's newspaper or online news site and find articles and advertisements that use percentages. Choose two and explain to the class how percentages are used in the articles you choose.

KEY IDEAS

- To find a percentage of a quantity (without a calculator):
  - Write the percentage as a fraction or decimal.
  - Multiply by the quantity.
- Calculator method:
  3% of $200 = 3 \div 100 \times 200 = 6$
- Mental strategies:

50%	25%	10%	5%	1%	75%
$\div 2$	$\div 2 \div 2$	$\div 10$	$\div 10 \div 2$	$\div 100$	$\div 2 \div 2 \times 3$

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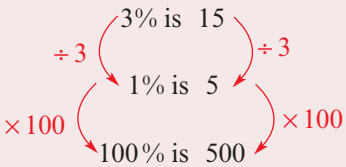
■ To find the original amount when given a percentage, you can use the unitary method or solve an equation.

For example: *3% of an amount is 15. What is the original amount?*

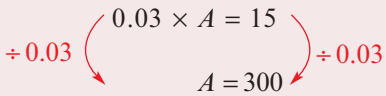
Unitary method



∴ The original amount is 500.

Equation method

3% of amount is 15.



∴ The original amount is 500.

Key terminology: percentage, unitary method

Exercise 2B

UNDERSTANDING	1–3	3

1 Find these percentage amounts mentally.

a 10% of $40 **b** 5% of $40 **c** 25% of 8 kg

d 75% of 8 kg **e** 1% of $800 **f** 2% of 140 seconds

Hint: Use the mental strategies given in the Key ideas.

2 If 1% of an amount is $4, what is 100% of the amount?

3 True or false: 34% of 568 = 0.34 × 568?

FLUENCY	4–6(½)	4–7(½)

Example 4 Finding a percentage of a quantity

Find 15% of $35.

SOLUTION

$$15\% \text{ of } \$35 = \frac{\cancel{15}^{3}}{\cancel{100}_{20}} \times \$35$$

$$= \$5.25$$

EXPLANATION

Write the percentage as a fraction out of 100 and multiply by $35.

Or write 0.15 × 35.

Note: 'of' means to 'multiply'.

Now you try

Find 18% of $60.

4 Find the following amounts. Use a calculator if necessary.

a 10% of 20 **b** 5% of 200 **c** 20% of 40

d 15% of 50 **e** 8% of 720 **f** 5% of 680

g 15% of 8200 **h** 70% of 60 **i** 90% of 500

j 75% of 44 **k** 99% of 200 **l** 3% of 50

Hint: 10% of 20 = $\frac{10}{100} \times 20$

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5 Use a calculator, if necessary, to find:

a 10% of $360 **b** 50% of $420 **c** 75% of 64 kg

d 12.5% of 240 km **e** 37.5% of 40 apples **f** 87.5% of 400 m

g $33\frac{1}{3}$% of 750 people **h** $66\frac{2}{3}$% of 300 cars **i** $8\frac{3}{4}$% of $560

Example 5 Finding the original amount

Determine the original amount if 5% of the amount is $45.

SOLUTION

5% of the amount = $45
1% of the amount = $9
100% of the amount = $900
So the original amount is $900

Alternative method: 5% of A = $45
$0.05 \times A = \$45$
$A = \$45 \div 0.05$
$= \$900$

EXPLANATION

To use the unitary method, find the value of 1 part or 1% then multiply by 100 to find 100%.

Write an equation using the pronumeral A to represent the amount.

Solve for A by dividing both sides by 0.05.

Now you try

Determine the original amount if 8% of the amount is $40.

6 Determine the original amount if:

a 10% of the amount is $12
b 6% of the amount is $42
c 3% of the amount is $9
d 40% of the amount is $2.80
e 90% of the amount is $0.18
f 6% of the amount is $27
g 12% of the amount is $96
h 15% of the amount is $54

Hint: First find the value of 1%.

7 Determine the value of x in the following if:

a 10% of x is $54 **b** 15% of x is $90

c 25% of x is $127 **d** 18% of x is $225

e 105% of x is $126 **f** 110% of x is $44

Hint: x is the original amount.

PROBLEM-SOLVING AND REASONING 8–11 8(½), 11–14

8 Without a calculator, evaluate the following.

a 10% of $58 **b** 5% of $84

c 1% of $46 **d** $2\frac{1}{2}$% of $20

e $33\frac{1}{3}$% of $132 **f** $66\frac{2}{3}$% of $60

Hint: $33\frac{1}{3}\% = \frac{1}{3}$

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9 If $\frac{1}{3}$ of 96 = 32, what is $66\frac{2}{3}$% of 96?

10 If 10% of \$800 is \$80, explain how you can use this result to find:

- **a** 1% of \$800
- **b** 5% of \$800
- **c** $2\frac{1}{2}$% of \$800

11 About 80% of the mass of the human body is water. If Carla weighs 60 kg, how many kilograms of water make up her body weight?

12 In a class of 25 students, 40% have been to England. How many students have not been to England?

13 Explain why 10% of 24 = 24% of 10.

14 10% of 1 day is the same as x hours and y minutes. What is the value of x and y?

ENRICHMENT: More than 100% — 15

15 **a** Find 120% of 60.

- **b** Determine the value of x if 165% of x = 1.5.
- **c** Write 2.80 as a percentage.
- **d** Write 325% as a fraction.
- **e** \$2000 in a bank account increases to \$5000 over a period of time. By how much has the amount increased as a percentage?

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2C Percentage increase and decrease CONSOLIDATING

Learning intentions for this section:

- To be able to increase or decrease an amount by a given percentage
- To be able to calculate the percentage by which an amount has increased or decreased

Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- Percentages are used extensively in this chapter and also Chapter 1 of our Year 10 book
- Expertise with percentages is assumed knowledge for Stage 6 Standard and may also be required in non-calculator examinations such as NAPLAN and industry aptitude tests

Percentages are often used to describe by how much a quantity has increased or decreased. The price of a car in the new year might be increased by 5%. On a $70 000 car, this is a $3500 increase. The price of a shirt might be marked down by 30%. If the shirt originally cost $60, this provides an $18 discount. It is important to note that the increase or decrease is calculated on the original amount.

Lesson starter: The quicker method

Two students, Nicky and Mila, consider the question: $250 is increased by 15%. What is the final amount?

Nicky puts his solution on the board with two steps.

Step 1: $15\% \text{ of } \$250 = 0.15 \times \$250$
$= \$37.50$

Step 2: $\text{Final amount} = \$250 + \$37.50$
$= \$287.50$

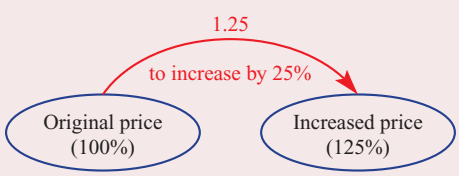
Mila says that the same problem can be solved with only one step using the number 1.15.

- Can you describe Mila's method? Write it down.
- What if the question was altered so that $250 is decreased by 15%. How would Nicky's and Mila's methods work in this case?
- Which of the two methods do you prefer and why?

KEY IDEAS

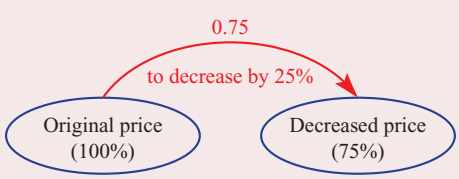
■ To increase an amount by a given percentage:
- Add the percentage increase to 100%.
- Multiply the amount by this new percentage.

For example, to increase by 25%, multiply by $100\% + 25\% = 125\% = 1.25$



■ To decrease an amount by a given percentage:
- Subtract the percentage from 100%.
- Multiply the amount by this new percentage.

For example, to decrease by 25%, multiply by $100\% - 25\% = 75\% = 0.75$.



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■ To find a percentage change, use:

$$\text{Percentage change} = \frac{\text{change in price}}{\text{original price}} \times 100\%$$

For example, if a price changes from \$40 to \$50:

$$\text{Percentage change} = \frac{10}{40} \times 100\% = 25\%$$

Key terminology: percentage

Exercise 2C

UNDERSTANDING | 1–3 | 3

1 Write the missing number for these increases.

a To increase a number by 40%, multiply by _____.

b To increase a number by 26%, multiply by _____.

c To increase a number by _____, multiply by 1.6.

d To increase a number by _____, multiply by 1.21.

Hint: To increase by 40%, need 140% of amount

2 Write the missing number for these decreases.

a To decrease a number by 20%, multiply by _____.

b To decrease a number by 73%, multiply by _____.

c To decrease a number by _____, multiply by 0.94.

d To decrease a number by _____, multiply by 0.31.

Hint: To decrease by 20%, need 80% of amount

3 Find the percentage change if

a Original amount = \$120, change = \$30

b Original amount = \$35, change = \$70

FLUENCY | 4–5(½), 6, 7 | 4–5(½), 6, 7(½), 8, 9

Example 6 Increasing by a percentage

Increase \$70 by 15%

SOLUTION

$100\% + 15\% = 115\%$

$= 1.15$

$\$70 \times 1.15 = \$80.50$

EXPLANATION

First add 15% to 100%

Note that $15\% = 0.15$ and $100\% = 1$

Multiply by 1.15 to give \$70 plus the increase in one step.

Now you try

Increase \$120 by 30%

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4 Complete the following, using a calculator if necessary.

a Increase 56 by 10%
b Increase 980 by 20%
c Increase 100 by 12%
d Increase 890 by 5%
e Increase 180 by 15%
f Increase 450 by 20%
g Increase 8 by 50%
h Increase 98 by 100%

Example 7 Decreasing by a percentage

Decrease \$5.20 by 40%

SOLUTION

$100\% - 40\% = 60\%$
$= 0.6$

$\$5.20 \times 0.6 = \$3.12$

EXPLANATION

First subtract the 40% from 100% to find the percentage remaining.

Multiply by $60\% = 0.6$ to get the result.

Now you try

Decrease \$64 by 15%

5 Complete the following.

a Decrease 80 by 5%
b Decrease 600 by 10%
c Decrease 45 by 50%
d Decrease 700 by 12%
e Decrease 8000 by 8%
f Decrease 450 by 25%
g Decrease 68 by 75%
h Decrease 9000 by 1%
i Decrease 7000 by 100%
j Decrease 10 000 by 1.5%

Example 8 Finding a percentage change

a The price of a mobile phone increased from \$250 to \$280. Find the percentage increase.
b The population of a town decreases from 3220 to 2985. Find the percentage decrease and round to one decimal place.

SOLUTION

a Increase $= \$280 - \$250$
$= \$30$

Percentage increase $= \frac{30}{250} \times \frac{100}{1}$
$= 12\%$

b Decrease $= 3220 - 2985$
$= 235$

Percentage decrease $= \frac{235}{3220} \times \frac{100}{1}$
$= 7.3\%$ (to 1 d.p.)

EXPLANATION

First find the actual increase.

Divide the increase by the original amount and multiply by 100.

First find the actual decrease.

Divide the decrease by the original population and multiply by 100. Round as indicated.

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Now you try

a The height of a plant increased from 20 cm to 28 cm. Find the percentage increase.
b The price of a washing machine decreased from $649 to $545. Find the percentage decrease and round to one decimal place.

6 The price of a flight increased from $125 to $150 overnight. Find the percentage increase.

7 Copy and complete the tables showing percentage change. Round to one decimal place where necessary.

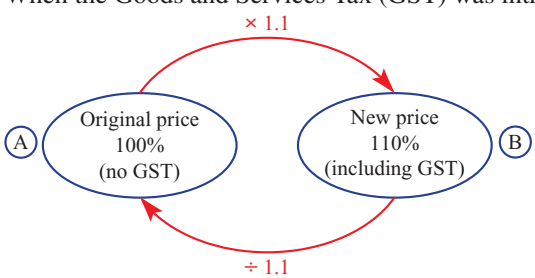
a

Original amount	New amount	Increase	Percentage change
40	60		
12	16		
100	125		
24	30		
88	100		

b

Original amount	New amount	Decrease	Percentage change
90	81		
100	78		
20	15		
24	18		
150	50		

8 When the Goods and Services Tax (GST) was introduced, all prices increased by 10%.



Hint: GST = 10% of (A)
or
GST = (B) − (A)

Copy and complete the table.

	Old price (excluding GST)	New price (including GST)	GST amount
a	$40		
b	$120		
c		$55	
d		$154	
e			$7

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Example 9 Finding the original amount from an increase or decrease

After rain, the volume of water in a tank increased by 24% to 2200 L. How much water was in the tank before it rained? Round to the nearest litre.

SOLUTION

$100\% + 24\% = 124\%$

124% of original value $= 2200$ L

1% of original value $= 17.74$ L

100% of original value $= 1774$ L

Alternative method:

$100\% + 24\% = 124\%$

124% of $V = 2200$ L

$1.24 \times V = 2200$

$V = 2200 \div 1.24$

$= 1774$

EXPLANATION

Write the total percentage.

The original volume is increased by 24% to give 2200 litres

Divide by 124 to find 1%

Multiply by 100 to find the original amount.

Write the total percentage.

Write an equation using the pronumeral V to represent the volume.

Divide both sides by 1.24 to solve for V.

Round to the nearest litre.

Now you try

At the start of a new year, daily public transport tickets increase by 4% to \$9.20. What was the cost of a daily ticket before the increase? Round to the nearest cent.

9 Find the original cost for each of the following if:

- **a** an increase of 10% on the cost of a can of cola drink increased it to \$3.30
- **b** an increase of 10% on the cost of a meal increased the cost to \$88
- **c** after an increase of 5%, the cost of a pair of running shoes came to \$210
- **d** a decrease of 30% made the cost of car insurance \$350
- **e** a decrease of 60% brought the price of a used car down to \$5000

PROBLEM-SOLVING AND REASONING 10–13 14–17

10 The price of a computer was decreased by 15% in a sale. What is the sale price, if the original price was \$2100?

11 Plumbers on a salary of \$82 570 were given a $2\frac{1}{2}\%$ pay increase. Find their new annual salary.

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12 A car manufacturer intends to increase sales by 14.7% next year. If the company sold 21 390 new cars this year, how many does it expect to sell next year?

13 The length of a bike sprint race is increased from 800 m to 1200 m. Find the percentage increase.

Hint:
$\%\text{ increase} = \frac{\text{increase}}{\text{original amount}} \times 100$

14 The number of people on a bus decreased from 25 to 18 after one stop. Find the percentage decrease in the number of people on the bus.

15 After a price increase of 20%, the cost of entry to a museum rose to $25.80. Find the original price.

16 The total price of an item including GST (at 10%) is $120. How much GST is paid, to the nearest cent?

Hint: $120 represents 110% of the price before GST is added.

17 An investor starts with $1000.

- **a** After a bad day the initial investment is reduced by 10%. Find the balance at the end of the day.
- **b** The next day is better and the balance is increased by 10%. Find the balance at the end of the second day.
- **c** The initial amount decreased by 10% on the first day and increased by 10% on the second day. Explain why the balance on the second day didn't return to $1000.

ENRICHMENT: Repeated increase and decrease — 18, 19

18 If the cost of a pair of shoes was increased twice, by 10% from an original price of $80 and then another 15% from this new price, the final price would be

$$\$80 \times 1.10 \times 1.15 = \$101.20$$

Use a similar technique to find the final price of these items. Round to the nearest cent.

- **a** Skis starting at $450 and increasing by 20% and 10%
- **b** A computer starting at $2750 and increasing by 6% and 11%
- **c** A DVD player starting at $280 and decreasing by 10% and 25%
- **d** A circular saw starting at $119 and decreasing by 18% and 37%

19 If an amount is increased by the same percentage each time, powers can be used.
For example, 50 kg increased by 12% three times would increase to

$$50\text{ kg} \times 1.12 \times 1.12 \times 1.12$$
$$= 50\text{ kg} \times (1.12)^3$$
$$= 70.25\text{ kg (to 2 d.p.)}$$

Hint: You have a power/index key on your calculator.

Use a similar technique to find the final value in these situations. Round to two decimal places.

- **a** The mass of a rat initially at 60 grams grows at a rate of 10% every month for 3 months.
- **b** The cost of a new car initially at $80 000 increases by 5% every year for 4 years.
- **c** The value of an apartment initially at $380 000 decreases by 4% per year for 3 years.
- **d** The length of a pencil initially at 16 cm decreases through being sharpened by 15% every week for 5 weeks.

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2D Profits and discounts CONSOLIDATING

Learning intentions for this section:

- To know and understand the terminology associated with profits, losses and discounts
- To be able to solve problems involving percentage profits, losses and discounts

Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- Percentages are used extensively in this chapter and also Chapter 1 of our Year 10 book
- Expertise with percentages is assumed knowledge for Stage 6 Standard and may also be required in non-calculator examinations such as NAPLAN and industry aptitude tests

Percentages are widely used in the world of finance. Profits, losses, commissions, discounts and taxation are often expressed and calculated using percentages.

Lesson starter: The best discount

Two book shops are selling the same book at a discounted price. The recommended retail price for the book is the same for both shops. Each shop has a sign near the book with the given details:

- Shop A. Discounted by 25%
- Shop B. Reduced by 20% then take a further 10% off that.

Which shop offers the bigger discount?

Is the difference equal to 5% of the retail price?

Examples of percentages used in the media.

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KEY IDEAS

- **Profit** is the amount of money made on a sale.
  Profit = **selling price** – **cost price**
- A **loss** is made when the selling price is less than the cost price.
  Loss = cost price – selling price
- **Mark-up** is the amount added to the cost price to produce the selling price.
  Selling price = cost price + mark-up
- The percentage profit or loss can be found by dividing the profit or loss by the cost price and multiplying by 100.

$$\%\ \text{profit/loss} = \left(\frac{\text{profit or loss}}{\text{cost price}} \times 100\right)\%$$

- **Discount** is the amount by which an item is marked down.
  Discount = % discount × original price
  New price = original price – discount amount

Key terminology: profit, selling price, cost price, loss, mark-up, percentage, discount

Exercise 2D

UNDERSTANDING 1–4 1

1 Match the definition in the left column with the correct word in the right-hand column.

a	The amount of money made on a sale	**A**	Loss
b	The amount by which an item is marked down	**B**	Mark-up
c	The result when the selling price is less than the cost price	**C**	Profit
d	The amount added to the cost price	**D**	Discount

2 Copy and complete the table of profits and losses.

Cost price ($)	7	18	24.80	7.30	460.95	3250
Selling price ($)	10	15.50	26.20	11.80	395	4430
Profit/loss ($)						

Hint: Profit/loss is the difference between cost price and selling price. Mark up increases price. Discount reduces price.

3 Copy and complete the table of mark-ups.

Cost price ($)	30.95	99.95	199.95			18 000
Mark-up ($)	10	80	395.95	700	16 700	
Selling price ($)				1499.95	35 499	26 995

4 Copy and complete the table of discounts.

Original price ($)	100	49.95	29.95			2215
New price ($)	72	40.90	22.70	176	299.95	
Discount ($)				23	45.55	178

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FLUENCY 5–8 5–8

Example 10 Calculating selling price from mark-up

An electrical store marks up all entertainment systems by 30%.
If the cost price of one entertainment system is $8000, what will be its selling price?

SOLUTION

Mark-up = 30% of $8000
= 0.3 × 8000
= $2400

Selling price = $8000 + $2400
= $10 400

Alternatively, multiply cost price by 130% or 1.3.

EXPLANATION

Change percentage to a decimal or fraction and multiply by the cost price.

Selling price = cost price + mark-up

130% is a 30% increase on the cost price

Now you try

A computer store marks up all notebook computers by 24%.
If the cost price of one notebook is $1200, what will be its selling price?

5 Copy and complete this table by calculating the selling price of each item.

Item	Cost price	% mark-up	Selling price
Jeans	$60	28%	
Toaster	$40	80%	
Car	$22 000	45%	
Can of drink	$1.20	140%	
Loaf of bread	$1.80	85%	
Handbag	$80	70%	
Electronic tablet	$320	35%	

Example 11 Finding the discount amount

An electrical store advertises a 15% discount on all equipment as a holiday special. Find the sale price on a projection system that has a marked price of $18 000.

SOLUTION

Discount = 15% of $18 000
= 0.15 × 18 000
= $2700

The new price = $18 000 − $2700
= $15 300

Alternatively, multiply the original price by 85% or 0.85.

EXPLANATION

Change the percentage to a decimal and evaluate.

New price is original price minus discount.

A 15% discount leaves 85%.

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Now you try

A department store offers a post-Christmas discount of 35% on all decorations. Find the sale price on a wreath that has a marked price of $80.

6 Copy and complete the table by writing in the missing values.

Item	Cost price	% discount	Selling price
Camera	$900	15%	
Car	$24 000	20%	
Bike	$600	25%	
Shoes	$195	30%	
Blu-ray player	$245	50%	
Electric razor	$129	20%	
Lawn mower	$880	5%	

Example 12 Determining profit

A manufacturer produces an item for $400 and sells it for $540.

a Determine the profit made.

b Express this profit as a percentage of the cost price.

SOLUTION

a Profit $= \$540 - \$400$
$= \$140$

b % profit $= \frac{140}{400} \times 100$
$= 35\%$

EXPLANATION

Profit = selling price − cost price

$$\% \text{ profit} = \frac{\text{profit}}{\text{cost price}} \times 100$$

Now you try

A jeweller produces a necklace for $36 and sells it for $49.50.

a Determine the profit made.

b Express this profit as a percentage of the cost price.

7 Find the missing values in these tables by finding the profit or loss and expressing this as a percentage of the cost price. Round to two decimal places where necessary.

a

Cost price ($)	Selling price ($)	Profit ($)	Profit (%)
10	15		
24	30		
100	150		
250	255		
17.50	20		

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b

Cost price ($)	Selling price ($)	Loss ($)	Loss (%)
10	8		
16	12		
100	80		
34	19		
95	80.75		

Example 13 Calculating the original price

A toy store discounts a toy by 10% in a sale. If the sale price was $10.80, what was the original price?

SOLUTION

90% of the original = $10.80

1% of the original = 10.80 ÷ 90

1% of the original = 0.12

100% of the original = $12

The original price was $12.

Alternative method: 90% of P = $10.80

$0.9 \times P = \$10.80$

$P = \$10.80 \div 0.9$

$= \$12$

EXPLANATION

The discount factor = 100% − 10% = 90%. Thus, $10.80 is 90% of the original price.

Use the unitary method to find 1%.

Multiply by 100 to find the original amount.

Write the answer in words.

Write an equation using the pronumeral P to represent the original price.

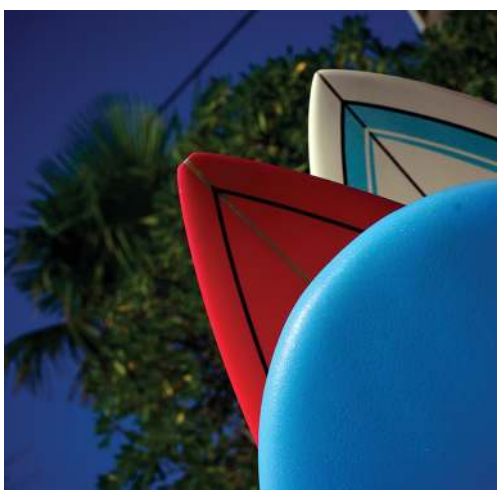
Solve the equation for P by dividing both sides by 0.9

Now you try

An outdoor equipment store discounts a tent by 20% in a sale. If the sale price was $176, what was the original price?

8 Answer the following questions relating to finding the original price (cost price).

- **a** Find the original price if a coffee mug was discounted by 20% and sold for $4.40.
- **b** Find the cost price of a pair of shoes that sold for $250 after a mark-up of 25%.
- **c** Find the cost price after a discount of 10% was given on a surfboard that sold for $1350.
- **d** Find the original price on a concert ticket for a major recording artist if it was marked up by 100% and sold for $250.



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PROBLEM-SOLVING AND REASONING 9–11 11–13

9 A manufacturer produces and sells items for the prices shown.

i Determine the profit made.

ii Express this profit as a percentage of the cost price.

a Cost price $10, selling price $12

b Cost price $20, selling price $25

c Cost price $120, selling price $136.80

d Cost price $1400, selling price $3850

10 Lenny marks up all computers in his store by 12.5%. If a computer cost him $890, what will be the selling price of the computer?

Hint: What is the increase as a percentage of the original price?

11 A used-car dealer purchases a vehicle for $13 000 and sells it for $18 500. Determine the percentage mark-up on the vehicle to one decimal place.

12 A refrigerator is discounted by 25%. If Paula pays $460 for it, what was the original price? Round to the nearest cent.

13 An electrical store buys a computer from the wholesaler for $500. The store marks up the computer by 80%.

a What is the amount of the mark-up in dollars?

b What is the retail price of the computer after the store's mark-up?

c The store offers the computer on sale for a discount of 15%. What is the price now?

d If the 15% discount was calculated on the original $500 cost price, and then the computer was marked up 80% after that, would it make a difference to the sale price?

ENRICHMENT: House value — 14

14 The graph shows the changes in the value of a particular house.

a How much was the house worth in 2004?

b Find the percentage increase, to one decimal place, in the value of the house from 2004 to:

i 2008 ii 2010 iii 2012

c In 2024 the owners wished to sell for 1.25 million dollars. What percentage increase was needed in the two years from 2022 to 2024 so they could sell for that price?

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2E Income

Learning intentions for this section:

- To know and understand the different ways that workers can be paid for their labour
- To use rates and percentages to calculate salaries, wages, commissions, royalties etc.
- To be able to make conversions between pay per hour, day, week, fortnight, month and year

Past, present and future learning:

- This section introduces the Stage 5 Core topic called *Financial mathematics A*
- Section 1C of our Year 10 book revises and further extends these concepts
- *Financial mathematics* is a major topic in Stage 6 Standard

There are many different ways of earning a living. You can be self-employed or work for someone else. Your income is usually related to the skills you have. It can be calculated and paid in different ways. You can earn, for example, a salary, a wage, a commission or possibly a royalty.

Lesson starter: Types of income

As a class, write down one example of a job that earns each of the following types of income.

- Salary
- Wage (i.e. hourly rate of pay)
- Overtime
- Commission

In the newspaper classifieds or online, find a job advertisement for each of the income types above.

KEY IDEAS

- Workers who earn a **wage** (for example, a casual waiter) are paid a fixed rate per hour. Hours outside the normal working hours (public holidays etc.) are paid at a higher rate called **overtime**. This can occur in a couple of common ways:
  - **Time and a half:** pay is 1.5 times the usual hourly rate
  - **Double time:** pay is twice the usual hourly rate
- Workers who earn a **salary** (for example, an engineer) are paid a fixed amount per year, say, $125 000. This is paid monthly, fortnightly or weekly.
  - 12 months in a year and approximately 52 weeks in a year = 26 fortnights

Annual salary $\xrightarrow{\div 12}$ Monthly pay (reverse: $\times 12$)

Annual salary $\xrightarrow{\div 26}$ Fortnightly pay (reverse: $\times 26$)

Annual salary $\xrightarrow{\div 52}$ Weekly pay (reverse: $\times 52$)

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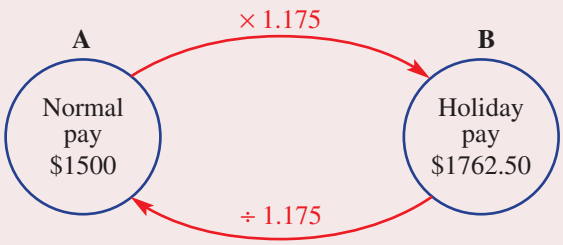
- Some wage and salary earners are paid **leave loading**. When they are on holidays, they earn their normal pay plus a bonus called leave loading. This is usually 17.5% of their normal pay.

$$100\% + 17.5\% = 117.5\%$$
$$= 1.175$$



Leave loading = 17.5% of A = $262.50

- **Commission** is a percentage of the overall sales amount. Salespeople may receive a commission on their sales as well as a set weekly or monthly fee called a **retainer**.
  - Commission = % commission × total sales
- Some people who write books or music are paid **royalties**, which could be 10% of the sales price of every book that is sold.
- Some people who make or do things are paid **piecework**. For example, $4 for every sheep shorn.

Key terminology: wage, overtime, time and a half, double time, salary, commission, retainer, royalties, piecework

Exercise 2E

UNDERSTANDING 1–4 | 1, 4

1 Match the pay description (**a**–**d**) with the income type (**A**–**D**).

a	Sally earns $22.70 per hour working in a shop	**A**	Overtime
b	Maths teacher Stephanie earns $72 000 per year	**B**	Commission
c	Ross earns 5% of all sales he makes	**C**	Wage
d	Matt earns time and a half pay on Saturdays	**D**	Salary

2 Tom earns $12.70 an hour. How much does he earn for:
a 2 hours of work? **b** 8 hours of work? **c** 38 hours of work?

3 Sela's hourly rate of pay is $24. Calculate her overtime rate at:
a time and a half **b** double time

4 William earns a salary of $136 875 each year.
Approximately how much is this:
a each month?
b each week (to the nearest cent)?
c each day?

Hint: To find approximate monthly salary, divide annual salary by 12.
To find approximate weekly salary, divide annual salary by 52.
To find approximate daily salary, divide annual salary by 365.

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FLUENCY 5–7, 8(½), 9 | 5, 6, 8(½), 9

Example 14 Comparing wages and salaries

Ken earns an annual salary of $90 000 and works a 38-hour week. His wife Brooke works part time in retail and earns $61.80 per hour.

a Calculate how much Ken earns per week.
b Determine who has the higher hourly rate of pay.
c If Brooke works on average 18 hours per week, what is her yearly income?

SOLUTION	EXPLANATION
a Weekly rate $= \$90\,000 \div 52$ $= \$1730.77$	$90 000 pay in a year.
$\therefore$ Ken earns $1730.77 per week	There are approximately 52 weeks in a year. Divide by 52 to find the weekly wage.
b Brooke: $61.80/h	Ken works 38 hours in the week.
Ken: $\$1730.77 \div 38$ $= \$45.55/h$	Hourly rate = weekly rate ÷ number of hours. Round to the nearest cent.
$\therefore$ Brooke is paid more per hour.	Compare hourly rates.
c In one week: $\$61.80 \times 18$ $= \$1112.40$	Weekly income = hourly rate × number of hours
Yearly income $= \$1112.40 \times 52$ $= \$57\,844.80$	Multiply by 52 weeks to get yearly income.

Now you try

Mali earns an annual salary of $77 200 and works a 38-hour week. Her partner Ben works part time as a photographer and earns $75 per hour.

a Calculate how much Mali earns per week.
b Determine who has the higher hourly rate of pay.
c If Ben works on average 15 hours per week, what is his yearly income from photography?

5 Talib earns $58 000 per year at a fast food restaurant. His sister works part time as a waitress and earns $33.20 per hour.

a How much does Talib earn each week?
b Each week Talib works 38 hours. Calculate if his hourly rate of pay is higher than his sister's.
c If his sister averages 10 hours of work per week, what is her yearly income?

6 Paul earns $790 each week. How much does he earn each:

a year?
b month?
c hour, if he worked 40 hours each week?

Hint: There are 12 months in a year.

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7 Copy and complete the table.

Employee	Hourly rate	Hours worked	Income
Adam	$20.40	8	
Betty	$15.50	$8\frac{1}{2}$	
Ceanna	$19.70	15	
David	$24.30	38	
Edward	$57.85	42	
Francis	$30	27	
George	$35.20	7.25	

Example 15 Calculating overtime

Georgio works some weekends and late nights, and earns overtime for that work. His hourly rate of pay is $32 per hour.

a Calculate Georgio's time and a half rate of pay per hour.

b Calculate Georgio's double time rate of pay per hour.

c Calculate Georgio's weekly wage for a week where he works 18 hours at his normal rate, 2 hours at time and a half, and 1 hour at double time.

SOLUTION

a Time and a half $= \$32 \times 1.5$
$= \$48$

b Double time $= \$32 \times 2$
$= \$64$

c $\$32 \times 18 = \$576$
$\$48 \times 2 = \$96$
$\$64 \times 1 = \$64$
Total wage $= \$736$

Alternatively,
$18\text{ h} + (2\text{ h} \times 1.5) + (1\text{ h} \times 2)$
$= 18 + 3 + 2$
$= 23$
Wage $= 23 \times 32 = \$736$

EXPLANATION

Time and a half is 1.5 times the hourly rate.

Double time is 2 times the hourly rate.

Find the sum of:

- the normal hourly rate ($32) multiplied by the number of hours worked at the normal rate (18)
- the time and a half hourly rate ($48) multiplied by the number of hours worked at that rate (2)
- the double time hourly rate ($64) multiplied by the number of hours worked at that rate (1).

Calculate the number of 'normal' hours.

Multiply by the hourly rate.

Now you try

Kane works some weekends and some public holidays and earns overtime for that work. His hourly rate of pay is $24 per hour.

a Calculate Kane's time and a half rate of pay per hour.

b Calculate Kane's double time rate of pay per hour.

c Calculate Kane's weekly wage for a week where he works 15 hours at his normal rate, 6 hours at time and a half and 3 hours at double time.

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8 A job has a normal working hours pay rate of $29.20 per hour. Calculate the pay, including overtime, from the following hours worked.

Hint: For time and a half: $\times 1.5$
For double time: $\times 2$

- **a** 3 hours at the normal rate and 4 hours at time and a half
- **b** 4 hours at the normal rate and 6 hours at time and a half
- **c** 14 hours at the normal rate and 3 hours at double time
- **d** 20 hours at the normal rate and 5 hours at double time
- **e** 10 hours at the normal rate and 8 hours at time and a half and 3 hours at double time
- **f** 34 hours at the normal rate and 4 hours at time and a half and 2 hours at double time

Example 16 Calculating commission

A part-time saleswoman is paid a retainer of $1500 per month. She also receives a commission of 5% on the value of goods she sells. If she sells goods worth $5600 during the month, calculate her earnings for that month.

SOLUTION

Commission = 5% of $5600
= $0.05 \times \$5600$
= $280

Earnings = $1500 + $280
= $1780

EXPLANATION

Calculate the commission on sales. Change the percentage to a decimal and evaluate.

Earnings = retainer + commission

Now you try

A real estate agent is paid a retainer of $2200 per month. She also receives a commission of 0.4% on the value of houses she sells. If she sells houses worth $1 575 000 during the month, calculate her earnings for that month.

9 Copy and complete the table.

Person	Weekly retainer	Rate of commission	Commission earned ($)	Weekly wage ($)
Adina	$0	12% on $7000		
Byron	$160	8% on $600		
Cindy	$300	5% on $680		
Deanne	$260	5% on $40 000		
Elizabeth	$500	8% on $5600		
Faruq	$900	2% on $110 000		
Gary	$1000	1.5% on $45 000		

PROBLEM-SOLVING AND REASONING 10–12 | 11, 13–15

10 Calculate how many hours at the standard hourly rate the following working hours are the same as:

- **a** 3 hours and 2 hours at double time
- **b** 6 hours and 8 hours at time and a half
- **c** 15 hours and 12 hours at time and a half

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11 Jim, a part-time gardener, earned $522 in a week. If he worked 12 hours during normal working hours and 4 hours overtime at time and a half, what was his hourly rate of pay?

Hint: Calculate the number of hours at the standard hourly rate.

12 Sally earned $658.80 in a week. She worked 10 hours during the week (Monday – Friday), 6 hours on Saturday at time and a half and 4 hours on Sunday at double time. What was her hourly rate of pay?

13 Amy works at Best Bookshop. During one week she sells books valued at $800. If she earns $450 per week plus 5% commission, how much does she earn in this week?

14 Jason works for a campervan company. If he sells $84 000 worth of campervans in a month, and he earns $1650 per month plus 4% commission on sales, how much does he earn that month?

15 Stephen earns an hourly rate of $34.60 for the first 38 hours, time and a half for the next 3 hours and double time for each extra hour above that. Calculate his earnings if he works 44 hours in a week.

ENRICHMENT: Pay slips — 16

16 Workers at a fast food restaurant are paid $21.63 per hour for working Monday to Friday up until 7 p.m. and time and a half after 7 p.m. They earn time and a half on Saturdays and double time on Sundays. They are given an unpaid 30 minute meal break for any shift over 5 hours.
Donna's shifts for the week are given below.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
–	4.30 p.m.–	4.30 p.m.–	5 p.m.–	5 p.m.–	10 a.m.–	10 a.m.–
–	7 p.m.	7.30 p.m.	10 p.m.	8 p.m.	6.30 p.m.	1.30 p.m.

a How much can Donna expect to earn, to the nearest cent, if she works the hours she is rostered for?

b The restaurant decides on a new workplace deal, gets rid of all overtime and creates a flat hourly rate of $24 per hour. How much worse off will Donna be for the week above?

c How many extra hours a week does Donna need to work during the week to make up the extra income? Answer in a whole number of hours.

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Progress quiz

2A **1** Express the following in the required form.

a 32% as a simplified fraction
b 7% as a decimal
c 0.23 as a percentage
d 0.125 as a percentage
e $\frac{7}{20}$ as a percentage
f 4.5% as a simplified fraction

2A **2** Express the following as percentages.

a 20c out of $2
b 32 marks out of 40
c 5 kg out of 40 kg

2B **3** Find the following amounts.

a 20% of 80 km
b 35% of $120
c $33\frac{1}{3}$% of 180 g
d 4.2% of $500

2B **4** Determine the original amount if:

a 25% of the amount is $100
b 12% of the amount is $36
c 8% of the amount is $60

2C **5** Complete the following.

a Increase 82 by 10%
b Increase 220 by 8%
c Decrease 110 by 15%
d Decrease 250 by 6%

2C **6** **a** The number of students in a school increased from 450 to 540. Find the percentage increase.

b The cost of petrol dropped from 121c per litre to 103c per litre overnight. Find the percentage decrease correct to one decimal place.

2C **7** After offering a special ticket deal for students, a concert's ticket sales rose by 24% to 961 ticket sales. How many tickets had been sold before the ticket deal?

2D **8** Find the selling price of the following items.

a A $1200 tablet computer marked up by 16%
b A pair of $126 sneakers discounted by 20%

2D **9** Jane has a stall at the market selling bracelets that she makes. If each one costs $6.20 to produce and she sells them for $10, find her percentage profit correct to one decimal place.

2D **10** Find the original price if:

a a dress was discounted by 15% and sold for $187
b a computer game was marked up by 12% and sold for $72.80

2E **11** Find the weekly earnings in the following work situations.

a Seb earns an annual salary of $74 100. (Assume 52 weeks in the year.)
b Jodie works 22 hours in a week at $18.20 per hour.
c Ned works in a fast food restaurant and has a normal hourly rate of $15.10 per hour. In a week he works 4 hours at the normal rate, 3 hours at time and a half and 2 hours at double time.
d Harper is paid a weekly retainer of $200 and she also earns a 5% commission on her sales. In a particular week she makes $8400 worth of sales.

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2F Taxation

Learning intentions for this section:

- To know and understand the terminology gross income, net income and taxable income
- To be able to calculate net income from gross income and deductions
- To be able to use the Australian Taxation Office table to calculate tax payable

Past, present and future learning:

- This section continues the Stage 5 Core topic called *Financial mathematics A*
- Section 1D of our Year 10 book revises and further extends these concepts
- *Financial mathematics* is a major topic in Stage 6 Standard

All wage and salary earners have some deductions taken out of their pay. Deductions usually include income tax. Tax is paid to the government. The government uses it to pay for community welfare, education and a number of other services.

PAY SLIP

Payment date: 14 November 2023
Pay period: 1 November – 14 November 2023

Employee's name:	Jason Smith
Employer's name:	BSJ Clothing
Job Title:	Sales assistant
Hourly rate:	$22.00

Wages: Hours worked: 18 @ $22.00		$396.00
	Gross payment	$396.00
Deductions		
Tax		$75.24
	Net payment	$320.76

Lesson starter: Deductions from pay

What types of deductions can you and your class think of that might be taken out of someone's pay?

Discuss, as a class, what superannuation and the Medicare levy is, and who pays it.

KEY IDEAS

- **Gross income** = the total of all income from all sources
- **Net income** = gross income minus **deductions** such as tax, union fees and superannuation
- **Taxable income** = gross income minus work-related expenses and donations to charity

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- **Income tax** is paid to the government. It is based on a person's taxable income.
- **Income tax payable** can be calculated using the tax table published every year by the Australian Taxation Office (ATO). Every row in the table is called a **tax bracket**. The following table shows the tax rates for the 2022/2023 financial year.
  Answers to the questions in the following exercise are based upon this table, but the actual rates may vary from year to year.

Taxable income	Tax on this income
$0 – $18 200	Nil
$18 201 – $45 000	19 cents for each $1 over $18 200
$45 001 – $120 000	$5 092 plus 32.5 cents for each $1 over $45 000
$120 001 – $180 000	$29 467 plus 37 cents for each $1 over $120 000
$180 001 and over	$51 667 plus 45 cents for each $1 over $180 000

The above rates **do not** include the Medicare levy of 2%.

The **Medicare levy** is an additional tax which the government uses to support funding for medicines, doctors, hospitals and other aspects of the health care system.

Key terminology: gross income, net income, taxable income, income tax, deductions, income tax payable, tax bracket, Medicare levy

Exercise 2F

UNDERSTANDING 1–4 4

1 Copy and complete the table by inserting the net income amounts.

Gross income	– Deductions	= Net income
$5600	$450	$5150
$87 000	$28 000	
$50 000	$6700	
$890	$76	
$84 650	$24 790	

Hint: Recall that net income = gross income minus deductions.

2 Find Hoang's gross weekly income if he earns:
- $1200 a week as a teacher
- $60 an hour for tutoring, when he tutors for 3 hours a week
- $25 interest on his bank account per week

3 John has a gross income of $45 000 and a net income of $20 000. How much did his deductions come to?

4 Pamela pays 31.5% of her gross income in tax. How much tax does she pay on a gross income of $85 790?

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FLUENCY 5–6(½), 7, 8 | 5–6(½), 8(½)

Example 17 Calculating tax to find net income

Liam starts a new job with an annual salary of $52 800. His payslip each month shows deductions for taxation of $968.

a Calculate Liam's net income each month.
b What percentage of Liam's monthly pay is being paid to the government by his employer for taxation?
c Liam's salary is increased to $65 000 and the taxation rate for Liam's salary changes to 24% with the first $18 200 tax free. Calculate Liam's net income for the year.

SOLUTION

a Monthly pay $= \$52\,800 \div 12$
$= \$4400$

$\therefore$ net monthly income $= \$4400 - \$968$
$= \$3432$

b $\%\text{ tax} = \dfrac{968}{4400} \times 100$
$= 22\%$

c Salary for tax purposes $= \$65\,000 - \$18\,200$
$= \$46\,800$

Tax amount $= 24\%$ of $\$46\,800$
$= 0.24 \times \$46\,800$
$= \$11\,232$

$\therefore$ net income $= \$65\,000 - \$11\,232$
$= \$53\,768$

EXPLANATION

Calculate gross income per month.

Net income = gross income – taxation

Calculate what fraction $968 is of the monthly income $4400. Multiply by 100 to convert to a percentage.

First $18 200 is not taxed.

Calculate tax amount on $46 800. Convert percentage to a decimal and evaluate.

Net income = gross income – tax amount.

Now you try

Poppy has a job with an annual salary of $74 400. Her payslip each month shows deductions for taxation of $1488.

a Calculate Poppy's net income each month.
b What percentage of Poppy's monthly pay is being paid to the government by her employer for taxation?
c Poppy's salary is increased to $82 800 and the taxation rate for Poppy's salary changes to 28% with the first $18 200 tax free. Calculate Poppy's net income for the year.

5 For each of the following find:
i the annual net income
ii the percentage of gross income paid as tax. Round to one decimal place where necessary.

a Gross annual income = $48 241, tax withdrawn = $8206
b Gross annual income = $67 487, tax withdrawn = $13 581.20
c Gross monthly income = $4041, tax withdrawn = $606.15
d Gross monthly income = $3219, tax withdrawn = $714.62

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6 Calculate the amount of tax to be paid using the following annual salaries and tax rates if the first $18 200 is tax free.

- **a** salary = $30 400, tax rate = 15%
- **b** salary = $56 500, tax rate = 21%
- **c** salary = $69 700, tax rate = 24.5%
- **d** salary = $96 400, tax rate = 30.4%

Hint: Remember to subtract the $18 200 to find the salary to calculate tax on.

7 Ed earns $1400 per week and pays 27% of his annual income in tax.

- **a** Calculate the amount of income tax that Ed pays in one year.
- **b** Find Ed's annual net income.

Example 18 Using the tax table to calculate tax payable

Use the tax table below to find the income tax payable for a taxable income of $124 000.

Taxable income	Tax on this income
$0 – $18 200	Nil
$18 201 – $45 000	19 cents for each $1 over $18 200
$45 001 – $120 000	$5 092 plus 32.5 cents for each $1 over $45 000
$120 001 – $180 000	$29 467 plus 37 cents for each $1 over $120 000
$180 001 and over	$51 667 plus 45 cents for each $1 over $180 000

The above rates **do not** include the Medicare levy of 2%.

SOLUTION

Tax = $29 467 + 0.37 × (124 000 − 120 000)
= $30 947

EXPLANATION

Find the tax bracket in which the amount $124 000 lies. This is ($120 001 − $180 000).

Write down the values in this bracket, remembering that 37 cents in the dollar is 37%, or 0.37.

Subtract 120 000 from 124 000 to find the amount of income that the 37% applies to.

Use your calculator to find the answer, working from left to right.

Now you try

Use the tax table above to find the income tax for an income of $75 000.

8 Use the tax table in Example 18 to find the income tax payable on each of these incomes.

- **a** $10 000
- **b** $30 000
- **c** $50 000
- **d** $80 000
- **e** $129 000
- **f** $156 000
- **g** $200 000
- **h** $500 000
- **i** $1 000 000

Hint: Look carefully for the correct tax bracket

PROBLEM-SOLVING AND REASONING 9, 10 | 9–12

9 Mel has a net annual income of $53 246 after 21% of her income is withdrawn for tax purposes. What was her gross income?

Hint: What % of gross income does $53 246 represent?

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10 Fred earns $50 000 each year as a shop clerk and an extra $1200 each year for doing gardening on some weekends.

- **a** Calculate his annual gross salary.
- **b** Use his gross salary to find his income tax, using the tax table in Example 18.
- **c** What is his annual net income, after tax?
- **d** What is his approximate fortnightly net income, to the nearest cent, after tax?

11 Sophie earns $1600 per week. Using the tax table in Example 18, how much tax is deducted from her pay each week? Assume 52 weeks in a year.

12 John pays $10 000 in tax a year.

- **a** Which tax bracket does John fall into?
- **b** Work backwards from this amount to work out how much John earns per year. Answer to the nearest dollar.

ENRICHMENT: Other types of deductions and taxable income — 13–15

The lower a person's income, the less tax they must pay. People therefore try to lower their taxable income by claiming allowable tax deductions.

Work-related expenses such as uniforms, stationery and job-related travel expenses are all examples of allowable tax deductions. The income tax is therefore calculated on what we call a person's taxable income.

Taxable income = gross income − allowable tax deductions

Use the tax table from Example 18 to compare the following jobs.

13 An young accountant earns $3120 a fortnight, with no allowable tax deductions.

- **a** What is the accountant's taxable income?
- **b** How much tax does the accountant owe in a year?
- **c** What is the accountant's annual net pay?
- **d** What is the accountant's fortnightly net pay?

14 A young lawyer earns $3120 a fortnight with allowable tax deductions totalling $2560 a year.

- **a** What is the lawyer's taxable income?
- **b** How much tax does the lawyer owe in a year?
- **c** What is the lawyer's annual net pay?
- **d** What is the lawyer's fortnightly net pay?

15 How much less money does the lawyer in Question **14** have each week compared to the accountant in question **13**?

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2G Simple interest

Learning intentions for this section:

- To understand the concept of simple interest and how it is calculated
- To be able to use the simple interest formula to perform calculations

Past, present and future learning:

- This section continues the Stage 5 Core topic called *Financial mathematics A*
- The following section, 2H, further extends these concepts
- *Financial mathematics* is a major topic in Stage 6 Standard
- You may be expected to recall the formula from memory in the HSC examination

Interest is charged when a person or institution borrows money. The interest is an extra amount that must be paid back, on top of the borrowed amount. Interest can also be earned, when a person or institution invests money. Simple or flat rate interest is usually charged or earned each year. It is calculated on the full amount borrowed or invested at the beginning of the loan.

Lesson starter: Developing the rule

$5000 is invested in a bank and 5% simple interest is paid to the investor every year. In the table, the amount of interest paid is shown for Year 1, and the total interest is shown for Years 1 and 2.

- Complete the table.
- How much interest would the investor earn in 10 years?

Year	Interest paid that year	Total interest
0	$0	$0
1	$\frac{5}{100} \times \$5000 = \$250$	$1 \times \$250 = \$250$
2		$2 \times \$250 = \$500$
3		
4		
5		

KEY IDEAS

- **Simple interest** is interest calculated each time period on the initial amount.
- To calculate simple interest, we apply the formula:

$$I = Prn$$

I is the amount of simple interest (in $)
P is the **principal** amount; the money borrowed or invested (in $), also known as present value
r is the interest rate per period, expressed as a decimal, not a percentage
n is the number of time periods, which could be days, months, years etc.

- The **future value** ($\$A$) equals the principal plus interest

$$A = P + I$$

- p.a. means '**per annum**' or 'per year'.

Key terminology: simple interest, principal, future value, per annum

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Exercise 2G

UNDERSTANDING	1–3	1, 2

1 $12 000 is invested at 6% p.a. for 42 months.

a What is the principal amount?

b What is the interest rate?

c What is the time period in years?

2 Jann earns $560 p.a. in simple interest on an investment. How much would he earn on the investment in:

a 2 years? **b** 5 years? **c** 10 years?

3 Use the rule $I = Prn$ to find the simple interest (I) earned in these financial situations.

a $10 000, 10% p.a., 3 years

b $6000, 12% p.a., 5 years

c $5200, 4% p.a., 2 years

FLUENCY	4–6, 8	4, 6–8

Example 19 Using the simple interest formula

Calculate the simple interest earned if the principal is $1000, the rate is 5% p.a. and the time is 3 years.

SOLUTION	EXPLANATION
$P = 1000, r = 0.05, n = 3$	List the information given, noting that $5\% = 0.05$.
$I = Prn$ $= 1000 \times 0.05 \times 3$ $= 150$	Write the formula and substitute the given values.
Interest = $150	Answer the question.

Now you try

Calculate the simple interest earned if the principal is $2000, the rate is 4% p.a and the time is 5 years.

4 Find the simple interest earned on:

a $5000 at 6% p.a. for 1 year

b $5000 at 6% p.a. for 3 years

c $8000 at 4% p.a. for 5 years

d $15 000 at 3% p.a. for 7 years

e $7250 at 5.5% p.a. for 3 years

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5 Wally invests $15 000 at a rate of 6% p.a. for 3 years. Calculate the simple interest and the amount available at the end of 3 years.

Hint:
Amount = principal + interest

Example 20 Using other time periods

Calculate the simple interest on $7000 invested at $6\frac{1}{4}\%$ p.a. for 18 months.

SOLUTION

$P = 7000, r = 6\frac{1}{4} \div 100 = 0.0625$

$n = 18$ months $= \frac{18}{12}$ or 1.5 years

$I = Prn$

$I = 7000 \times 0.0625 \times 1.5$

$= 656.25$

Interest = $656.25

EXPLANATION

List the information.

Convert 18 months into years by dividing by 12.

Write the formula.

Substitute in the values and evaluate.

Now you try

Calculate the simple interest on $4500 invested at $5\frac{1}{2}\%$ for 30 months.

6 Calculate the simple interest earned on:

- **a** $500 at 7% p.a. for 18 months
- **b** $1000 at 5% p.a. for 24 months
- **c** $2000 at 4% p.a. for 6 months
- **d** $4700 at $4\frac{1}{2}\%$ p.a. for 15 months (Round to the nearest cent.)
- **e** $50 000 at 3.75% p.a. for 200 days (Round to the nearest cent.)

Hint: 365 days = 1 year

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Example 21 Calculating the future value

Allan and Rachel plan to invest some money for their child Kaylan. They invest $4000 for 30 months in a bank that pays 4.5% p.a. Calculate the simple interest and the future value at the end of the 30 months.

SOLUTION

$P = 4000$, $r = \frac{4.5}{100} = 0.045$, $n = \frac{30}{12} = 2.5$

$I = Prn$
$= 4000 \times 0.045 \times 2.5$
$= 450$

Interest = $450

Total amount = $4000 + $450 = $4450

EXPLANATION

n is written in years since interest rate is per annum.

Write the formula, substitute and evaluate.

Total amount = principal + interest

Now you try

Hannah wins some money which she decides to invest. She invests the $5000 for 36 months in a bank account that pays 3.8% p.a. Calculate the simple interest and the amount available at the end of the 36 months.

7 Annie invests $22 000 at a rate of 4% p.a. for 27 months. Calculate the simple interest and the amount available at the end of 27 months.

8 Copy and complete the table.

	Principal	Annual interest rate	Time period	Interest	Future value
a	$7000	3%	4 years		
b	$1500	7%	8 years		
c	$40 000	2.5%	18 months		
d	$70 000	$3\frac{1}{4}\%$	2 years		
e	$2000	4%	30 months		

PROBLEM-SOLVING AND REASONING 9, 10 | 10, 11

9 A finance company charges 14% p.a. simple interest. If Lyn borrows $2000 to be repaid over 2 years, calculate her total repayment.

10 Markus borrows $20 000 to buy a car. He is charged simple interest at 18% p.a. for a period of 5 years.

- **a** How much interest is Markus charged each year?
- **b** Calculate the total interest Markus will pay on this loan.
- **c** What is the total amount that Markus will have paid at the end of the loan period?

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11 Wendy wins $5000 during a chess tournament. She wishes to invest her winnings, and has the two choices given below. Which one gives her the greater total at the end of the time?
Choice 1: 8.5% p.a. simple interest for 4 years
Choice 2: 8% p.a. simple interest for 54 months

ENRICHMENT: Simple interest tables and graphs — 12, 13

12 The table shows the amount of simple interest payable (in $, to the nearest $) on loans at a certain interest rate.

Amount of loan	1 year	2 years	3 years	5 years	10 years
50	6	13	19	32	65
100	13	26	39	65	129
500	65	129	194	323	645
1000	129	258	387	645	1290
5000	645	1290	1935	3225	6450
10 000	1290	2580	3870	6450	12 900
50 000	6450	12 900	19 350	32 250	64 500
100 000	12 900	25 800	38 700	64 500	129 000

Use the table to find the interest payable on the following loans.

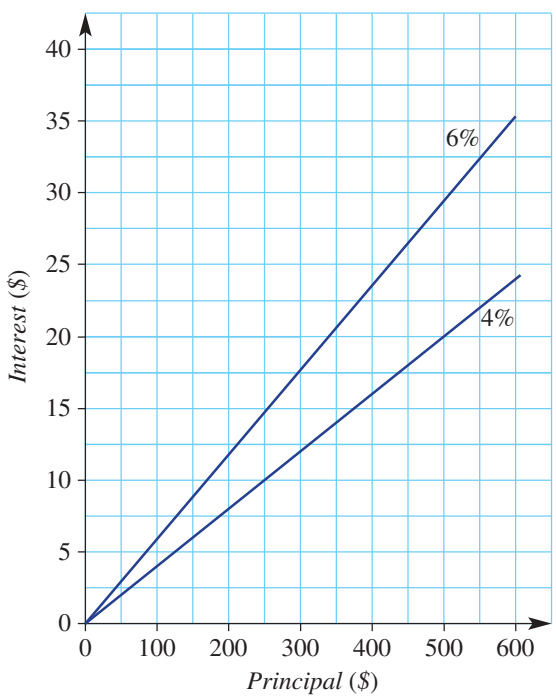
a $5000 for 1 year
b $500 for 3 years
c $100 for 10 years
d $150 for 1 year
e $85 500 for 5 years
f $9550 for 10 years
g $5000 for 4 years
h $50 000 for 9 years
i $100 000 for 10 years

13 The graph on the right shows the annual interest earned on investments for interest rates of 4% p.a. and 6% p.a. Use the graph to answer the following.

a Find the annual interest earned on an investment of:
  i $300 at 4% p.a.
  ii $520 at 6% p.a.
  iii $250 at 4% p.a.

b What investments would earn annual interest of (to the nearest $5):
  i $20 at 6% p.a.?
  ii $20 at 4% p.a.?
  iii $14 at 6% p.a.?

Annual interest earned at 4% and 6%



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2H Applications of simple interest

Learning intentions for this section:

- To be able to use the simple interest formula to solve problems about investments and loans
- To be able to calculate repayment amounts given the terms of a loan

Past, present and future learning:

- This section continues the Stage 5 Core topic called *Financial mathematics A*
- Section 1F of our Year 10 book revises and further extends these concepts
- *Financial mathematics* is a major topic in Stage 6 Standard
- You may be expected to recall the formula from memory in the HSC examination

Financial calculations are a critical component of the thinking behind the decisions people make as to where to borrow or invest money.

Lesson starter: Where do I invest?

Bank A: $4000 at 5% p.a. for 8 years

Bank B: $5000 at 8% p.a. for 4 years

Bank C: $8000 at 4% p.a. for 5 years

Calculate the simple interest earned on each investment option. What do you notice?

Which bank would you choose and why?

KEY IDEAS

- The simple interest formula, $I = Prn$, contains four variables:
  - I is amount of interest
  - P is principal (i.e. amount invested)
  - r is interest rate, expressed as a decimal
  - n is number of time periods.

  If values are given for three of these, then they can be used to find the fourth.
- To repay a loan, you must repay the amount borrowed (the principal) and the interest.
- **Repayments** are the amount of money, usually the same amount each time period, required to repay a loan.

Key terminology: simple interest, principal, repayment

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Exercise 2H

UNDERSTANDING	1–4	4

1 If Cooper earns \$100 in simple interest in one year, how long would it take him to earn:

a \$200 **b** \$400 **c** \$5000 **d** \$250

2 How many years does it take for \$100 to earn \$50 in interest if the simple interest rate is 10% p.a.?

Hint: How much interest is earned on \$100 at 10% p.a. for 1 year?

3 Jodie repays \$200 a month for 12 months to pay back her loan and interest. How much does she repay?

4 A loan of \$4000 has interest of \$500 added to it. Calculate the size of each of the 10 repayments needed to repay the loan.

FLUENCY	5, 6, 8, 9	5, 7, 8, 9

Example 22 Determining the investment period

Remy invests \$2500 at 8% p.a. simple interest, for a period of time, to produce \$50 interest. For how long did she invest the money?

SOLUTION

$I = 50, P = 2500, r = \frac{8}{100} = 0.08$

$I = Prn$

$50 = 2500 \times 0.08 \times n$

$50 = 200 \times n$

$200n = 50$

$n = 0.25$

Time $= 0.25$ years

$= 0.25 \times 12$ months

$= 3$ months

EXPLANATION

List the information.

Write the formula.

Substitute the known information and simplify.

Solve the remaining equation for t by dividing both sides by 200.

Convert decimal time to months where appropriate.

Now you try

Jas invests \$3000 at 6% p.a. simple interest, for a period of time, to produce \$450 interest. For how long did he invest the money?

5 Alvi invests \$5000 at 8% p.a. simple interest and wants to earn \$1200 in interest. For how many years should Alvi invest his money?

6 Sam earns \$288 interest on his \$1600 investment. If the interest was calculated at 4% p.a., how many years did Sam invest the money for?

7 How many months does it take an investment of \$8000 to earn \$600 in interest at 5% p.a.?

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Example 23 Determining the interest rate

Bank East advertises $450 interest a year on an investment of $7500. Calculate the simple interest rate for this investment.

SOLUTION

$I = 450, P = 7500, n = 1, r = ?$

$I = Prn$

$450 = 7500 \times r \times 1$

$450 = 7500r$

$7500r = 450$

$r = \frac{450}{7500}$

$r = 0.06$

Interest rate is 6% p.a.

EXPLANATION

List the information.

Write the formula and substitute $I = 450$, $P = 7500$ and $t = 1$.

Simplify and solve for r.

Write r as a percentage.

Now you try

Oz Loans offers $610 interest a year on an investment of $12 200. Calculate the simple interest rate for this investment.

8 Find the annual simple interest rate needed for each of the following situations.

Hint: Set up the formula $I = Prn$ where r is the unknown.

a $4000 earns $500 in 2 years

b $500 earns $120 in 12 years

c $18 000 earns $3510 in 3 years

d $950 earns $470.25 in 9 years

e $3000 earns $945 in 18 months

f $2500 earns $393.75 in 4.5 years

Example 24 Calculating repayments

'Deals 4 You' offers a loan of $24 000 at 16% p.a. simple interest if the loan is repaid in equal monthly repayments over 5 years.

a How much interest is charged on the loan?

b What is the total amount of the loan and the interest?

c Calculate the size of each repayment.

SOLUTION

a $I = Prn$

$P = 24\,000, r = 0.16, n = 5$

$I = 24\,000 \times 0.16 \times 5$

$= \$19\,200$

b Total $= \$24\,000 + \$19\,200$

$= \$43\,200$

EXPLANATION

Write down the simple interest formula and list the information.

Substitute in the values and evaluate.

Total = principal + interest

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c Repayments $= \frac{43\ 200}{60}$

$= 720$

Repayments come to \$720 per month.

Divide the total amount by the number of months in 5 years ($5 \times 12 = 60$).

Now you try

A loan of \$35 000 is repaid in equal monthly instalments over 6 years at 8% p.a. simple interest.

a How much interest is charged on the loan?

b What is the total amount of the loan and the interest?

c Calculate the size of each repayment.

9 Copy and complete this table and find the monthly repayment for each loan.

Amount borrowed	Annual simple interest rate	Number of years	Interest	Total amount to be repaid	Monthly repayment
\$5000	21%	5			
\$14 000	15%	5			
\$10 000	6%	4			
\$55 000	8%	10			
\$250 000	7%	30			

PROBLEM-SOLVING AND REASONING

10, 11 | 10, 12, 13

10 Calculate the principal amount which earns \$500 simple interest over 3 years at a rate of 8% p.a. Round to the nearest cent.

Hint: Substitute into $I = Prn$ and solve for P.

11 Charlotte borrows \$9000 to buy a second-hand car. The loan must be repaid over 5 years at 12% p.a. simple interest. Calculate:

a the total amount to be repaid

b the monthly repayment amount if the repayments are spread equally over the 5 years

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12 If $5000 grows to $11 000 in 12 years, find the simple interest rate.

13 An investor invests $\$P$ and wants to double this amount of money.

- **a** How much interest must be earned to double this initial amount?
- **b** What simple interest rate is required to double the initial amount in 8 years?
- **c** If the simple interest rate is 5% p.a.:
  - **i** how many years will it take to double the investment?
  - **ii** how many years will it take to triple the investment amount?
  - **iii** how do the investment periods in parts **i** and **ii** compare?

ENRICHMENT: Introducing compound interest — 14, 15

With simple interest, the principal and the interest earned each year remain the same for the period of the investment.

However, with compound interest, each time the interest is calculated it is added to the principal to give a new value. This means that the next time the interest is calculated, it is done so using a larger amount.

In the following questions you will be asked to do repeated applications of simple interest to find the final compounded amount.

14 $500 is invested for 4 years at 10% p.a. interest compounded annually.

- **a** Complete the table to find the final value of the investment at the end of this time and the total.

Time (years)	Amount (A)	Interest (I)	New amount ($A + I$)
1	500	$500 \times 0.1 = 50$	$500 + 50 = 550$
2	550	$550 \times 0.1 =$	
3			
4			

- **b** How much interest did the investment earn over the 4 years?

15 **a** Complete the following table to find the final value of an investment of $4500 compounded at 5% p.a. annually for 5 years.

Time (years)	Amount (A)	Interest (I)	New amount ($A + I$)
1	4500	225	4725
2	4725		
3			
4			
5			

- **b** Type 4500×1.05^5 into your calculator. What do you notice about this answer?
- **c** Can you explain how the answers to part **a** and part **b** relate?

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Online cake-decorating business

More and more people are setting up businesses using the internet. For example, a successful cake-decorating business can be run from home while looking after the kids or while working normal business hours at other jobs.

As with any business, an understanding of financial mathematics is important to the success of the business. Skills such as calculating costs and profits, percentages and taxation are important for any manager.

1 Calculate the total cost of buying each of the following cake tin sets:
Round set: 6, 8, 9 and 12 inches by 3 inches deep at $64.
Round set: 6, 8, 10 and 12 inches by 4 inches deep at $99.
Square set: 6, 8 and 10 inches by 3 inches deep at $45.
Square set: 6, 8, 10 and 12 inches by 4 inches deep at $86.

2 Convert the following measurements from inches (US standard) to whole number of centimetres by using the following conversion rate: 1 inch = 2.54 cm.

a 3 inches **b** 4 inches **c** 9 inches **d** 10 inches

3 Imagine that you spend 2 hours out of $9\frac{1}{2}$ work hours on the internet promoting your business. Write this as a percentage, rounded to the nearest whole percentage.

4 Fondant icing comes in different colours and in different-sized tubes. Managing your budget means looking for the best buy.

a Which of the following represents the best buy for each colour of fondant listed below?

White fondant	Red fondant	Blue fondant
100 g at $3.25	2.5 kg at $36.95	100 g at $2.95
500 g at $5.50	1 kg at $19.90	500 g at $7.95
5 kg at $40	100 g at $3.25	750 g at $11.95
1 kg at $10.50	500 g at $7.95	1 kg at $19.90

b What is the average cost per 100 grams for white fondant icing?

c Under what circumstances would someone buy a size that was not the best buy?

5 A customer has the following quotes for a large 21st birthday cake from four different online cake suppliers: $195, $290, $225 and $215.
For each of the following, state answers to the nearest whole number.

a What is the mean or average cost for this type of cake?

b What is the percentage change from the lowest quote to the highest quote?

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c If it costs each supplier $50 in product to make the cake, calculate the percentage profit for each of the four quotes.

d If it takes each supplier on average $5\frac{1}{2}$ hours to make and decorate the cake, what is each person charging per hour, excluding the $50 product costs?

Using technology

6 Imagine that you have started an online cake-making business. To analyse possible profits, set up an Excel spreadsheet as shown below and enter formulas in the shaded cells. Note that this is a simplified analysis and excludes power, gas and equipment costs.

Hint: Format all $ cells as 'currency' with zero decimal places.
Format profit % cells as 'percentage' with zero decimal places.

	A	B	C	D	E	F	G
1	CELEBRATION CAKES						
2	Cakes (30 serves, decorated)	Product costs in $	Selling price in $	Profit in $	Profit %	Prep time in hours	Hourly rate
3	**Birthday** - caramel mud	$35	$105			3.25	
4	**Birthday** - choc fudge mud	$38	$112			3	
5	**Birthday** - choc Jaffa	$35	$108			3.5	
6	**Birthday** - child's theme	$58	$295			5.5	
7	**21st Birthday** - 4 tiers cupcakes	$52	$220			4	
8	**Valentine's Day** - red velvet choc	$36	$140			3	
9	**Wedding** - 2 tiers 70 serves	$85	$360			6	
10	**Wedding** - 3 tiers 120 serves	$120	$545			9	

a How much profit would be made from selling 2 Valentines' Day cakes and 3 choc Jaffa birthday cakes?

b List, in ascending order, the 3 cakes that bring the lowest percentage profit.

c List, in descending order, the 3 cakes that pay the highest hourly rates.

d Suggest a reason why the cakes in **c** cost the customer more in hourly rates.

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Saving for a holiday

Sally is an accounts clerk earning a salary of $70 348. She plans to save 20% of her fortnightly after tax take home pay, for any extra expenses and to put towards a holiday. In fact, Sally is trying to save for an overseas holiday in 12 months and she estimates she will need $12 000 for her planned itinerary.

To reach her goal she decides to get a second job working at her local late night supermarket. They offer her three 2½ hour shifts a week paying $25 an hour. She plans to work here for a total of 20 weeks, thinking that this will give her enough for her trip.

Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate. Give answers to the nearest cent.

Routine problems

a At her job at the supermarket, how much does Sally earn per shift?

b How much would she earn per week at the supermarket?

c Over the 20 weeks, calculate Sally's total gross income from the supermarket.

d As an accounts clerk, Sally earns a salary. What is the gross fortnightly income Sally earns from this job?

e What is her total annual taxable income from both jobs, assuming she only works for the 20 weeks at the supermarket?

Non-routine problems

Explore and connect

Choose and apply techniques

a The problem is to determine if Sally has saved enough to afford to go on her trip after tax. Write down all relevant information that will help you solve this problem.

b The firm that Sally works for as an accounts clerk take out income tax from her fortnightly pay on her behalf, they do this by using her salary and finding the tax owed by using the current personal income tax table found on the ATO website.

By using the personal income tax table below, calculate the annual tax that the firm deducts from Sally's pay.

Taxable income	Tax on this income
$0 – $18 200	Nil
$18 201 – $45 000	19 cents for each $1 over $18 200
$45 001 – $120 000	$5 092 plus 32.5 cents for each $1 over $45 000
$120 001 – $180 000	$29 467 plus 37 cents for each $1 over $120 000
$180 001 and over	$51 667 plus 45 cents for each $1 over $180 000

The above rates **do not** include the Medicare levy of 2%.

c Calculate the fortnightly net pay (take home pay) that Sally receives from her firm.

d Sally aims to save 20% of her take home pay from her firm each fortnight. How much will Sally have saved at the end of 12 months?

e By considering her total taxable income from both jobs, calculate her income tax for both jobs for the year.

f Hence, calculate Sally's annual net income from her second job.

g Calculate her total tax including the Medicare levy of 2%.

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Communicate thinking and reasoning

h When Sally takes into account all her finances and tax, will she have enough left from her savings to take her proposed trip?

i Summarise your findings and make a recommendation about Sally's savings and proposed trip at the end of the year.

Extension problems

Problem-solve

a What percentage pay rise is needed from her firm for Sally to no longer need her second job?

b Investigate what happens to Sally's tax on her salary from her firm if she has allowable tax deductions of:

i $2000

ii $4000

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1 The answers to the clues are hidden in the wordfind. Can you find all 16 words?

S	A	G	R	O	S	S	W	H	K	L	O	M
A	P	E	R	C	E	N	T	A	G	E	C	O
L	A	R	C	O	M	M	I	S	S	I	O	N
A	S	E	R	V	I	N	T	E	R	E	S	T
R	T	Y	H	E	T	S	L	O	S	S	T	H
Y	E	F	O	R	T	N	I	G	H	T	I	L
W	S	M	O	T	A	X	A	T	I	O	N	Y
E	I	O	L	I	D	I	D	N	P	Y	I	E
K	M	N	Q	M	O	N	T	R	N	I	S	C
E	P	Y	U	E	D	I	S	C	O	U	N	T
M	L	R	E	P	A	Y	M	E	N	T	A	H
D	E	D	U	C	T	I	O	N	S	I	X	L

Clues

Down

a. A fixed annual income
b. A percentage of the value of goods sold, which you earn as an income
c. Working longer than normal working hours
d. Money from your income given to the government
e. Two weeks
f. The total of all income
g. Money taken from total pay
h. Yearly

Across

i. 12 times a year
j. Flat-rate interest
k. Meaning 'out of 100'
l. Money given to repay a loan
m. Money earned on an investment
n. An item offered for a sale price has had this happen
o. The original price of an item
p. You incur this when you sell an item for less than you paid for it

2 Find out the four classical elements of the world by answering the following simple interest problems. Match the letter beside each question to its corresponding answer in each grid.

E = \$600 at 6% p.a. for 1 year	**H** = \$796 at 5% p.a. for 4 months
R = \$12 500 at $6\frac{1}{4}$% p.a. for 2 years	**A** = \$7000 at 5% p.a. for 3 years
I = \$1000 at 1% p.a. for 100 years	**F** = \$576.50 at 19% p.a. for 18 months
W = \$36 000 at 2% p.a. for 5 years	**T** = \$550 at 10% p.a. for 6 months

\$36	\$1050	\$1562.50	\$27.50	\$13.27

\$1050	\$1000	\$1562.50

\$164.30	\$1000	\$1562.50	\$36

\$3600	\$1050	\$27.50	\$36	\$1562.50

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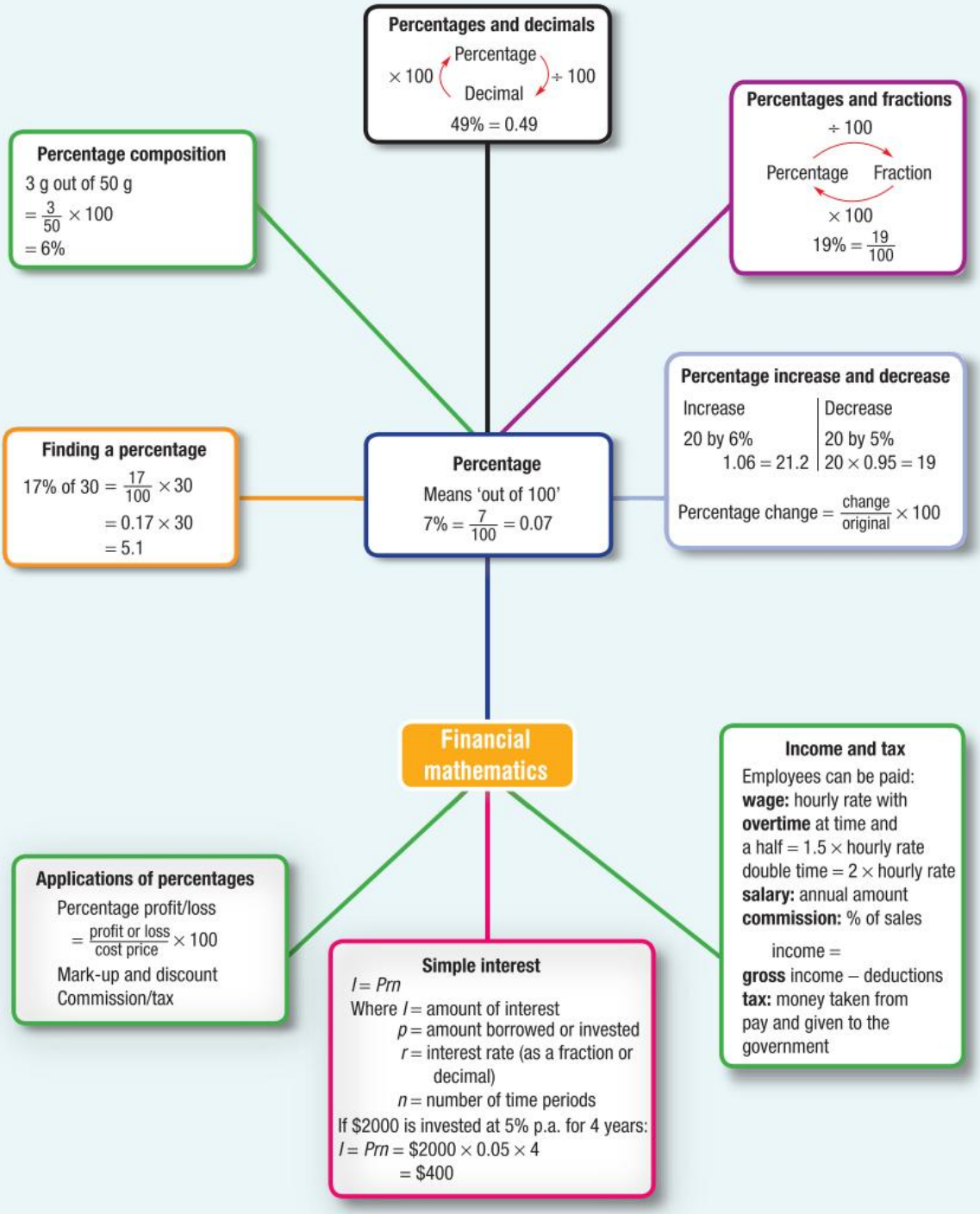
Chapter summary



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Chapter checklist with success criteria

A printable version of this checklist can be downloaded from the Interactive Textbook.

		✔
2A	**1 I can convert between percentages and fractions.** e.g. Write **a** $\frac{7}{40}$ as a percentage **b** 17.5% as a fraction	☐
2A	**2 I can convert between percentages and decimals.** e.g. Write **a** 0.3 as a percentage **b** 72% as a decimal	☐
2A	**3 I can write a quantity as a percentage.** e.g. Write 27 cm out of 1.8 m as a percentage.	☐
2B	**4 I can find a percentage of a quantity.** e.g. Find 35% of $\$75$	☐
2B	**5 I can find the original amount from a percentage.** e.g. Determine the original amount if 12% of the amount is $\$72$	☐
2C	**6 I can increase and decrease by a percentage.** e.g. **a** Increase $\$80$ by 12% **b** Decrease $\$40$ by 8%	☐
2C	**7 I can find a percentage change.** e.g. The price of a gym membership increased from $\$320$ to $\$370$. Find the percentage increase correct to one decimal place.	☐
2C	**8 I can find the original amount after an increase or decrease.** e.g. A decrease of 22% reduced the population of a town to 1014. What was the original population of the town?	☐
2D	**9 I can calculate the selling price from a mark-up or discount.** e.g. A store marks up all white goods by 20%. If the cost price of a fridge is $\$1100$, what will be its selling price?	☐
2D	**10 I can determine percentage profit.** e.g. A stall holder makes candles for $\$8$ and sells them for $\$13$. Find the profit and express this profit as a percentage of the cost price.	☐
2D	**11 I can calculate the original price before discount.** e.g. A department store discounts all Christmas trees by 15%. If the sale price of a tree was $\$106.25$, what was the original price?	☐
2E	**12 I can compare wages and salaries.** e.g. Tony has an annual salary of $\$88\,000$ and Jodie earns $\$72$ per hour. Calculate **a** Tony's hourly rate of pay if he works a 38-hour week **b** Jodie's yearly income if she works on average 22 hours per week.	☐

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Chapter checklist

Chapter checklist with success criteria

2E	**13 I can calculate overtime.** e.g. Calculate Julian's weekly wage for a week where he works 12 hours at his normal hourly rate of \$28, 4 hours at time and a half and 3 hours at double time.	☐
2E	**14 I can calculate commission.** e.g. A salesperson is paid a retainer of \$2200 per month and receives an 8% commission on sales. If one month he makes sales worth \$10 400, calculate his earnings for that month.	☐
2F	**15 I can calculate net income.** e.g. Danielle has an annual salary of \$64 800. She has monthly tax deductions on her payslip of \$1296. Calculate her net income each month.	☐
2F	**16 I can calculate net income using income tax rates.** e.g. Anna has an annual salary of \$78 000 and a taxation rate of 26% with the first \$18 200 tax free. Calculate Anna's net income for the year.	☐
2F	**17 I can use the tax table to calculate income tax.** e.g. Use the tax table from Example 18 to find the income tax for an income of \$90 000.	☐
2G	**18 I can use the simple interest formula and find the future value.** e.g. Calculate the simple interest earned if the principal is \$4000, the rate is 3% p.a. and the time is 4 years. Hence, what is the amount at the end of the 4 years?	☐
2G	**19 I can work with simple interest using other time periods.** e.g. Calculate the simple interest on \$6000 invested at $4\frac{1}{2}\%$ p.a. for 42 months.	☐
2H	**20 I can determine the investment period or interest rate for simple interest.** e.g. Joshua invests \$3500 at 6% p.a. simple interest, for a period of time, to produce \$315 interest. For how long did he invest the money?	☐
2H	**21 I can calculate repayments.** e.g. Syd takes out a loan to purchase a yacht. The loan is for \$32 000 at 8% p.a. simple interest if the loan is repaid in equal monthly instalments over 4 years. Calculate the interest charged on the loan and hence the total amount owing and the size of the required monthly repayments.	☐

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Short-answer questions

2A **1** Copy and complete the table shown.

Decimal	Fraction	Percentage
0.6		
	$\frac{1}{3}$	
		$3\frac{1}{4}\%$
	$\frac{3}{4}$	
1.2		
		200%

2B **2** Find:

a 25% of $310

b 110% of 1.5

2B **3** Determine the original amount if:

a 20% of the amount is 30.

b 72% of the amount is 18.

2C **4**

a Increase 45 by 60%.

b Decrease 1.8 by 35%.

c Find the percentage change if $150 is reduced by $30.

2C **5** The mass of a cat increased by 12% to 14 kg over a 12-month period. What was its previous mass?

2D **6** Determine the discount given on a $15 000 car if it is discounted by 12%.

2D **7** A couch at a cost price of $3500 is to be marked up by 25%. Find the selling price.

2D **8** The cost price of an article is $150. If it is sold for $175:

a determine the profit made

b express the profit as a percentage of the cost price.

2E **9** Determine the hourly rate of pay for each of the following cases:

a a person with an annual salary of $76 076 working a 38-hour week

b a person who earns $429 working 18 hours at the hourly rate and 8 hours at time and a half.

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Chapter review

2F 10 Jo's monthly income is $5270; however, 20% of this is paid straight to the government in taxes. What is Jo's net yearly income?

2G 11 Find the simple interest earned on $1500 at 7% p.a. for 5 years.

2H 12 Bill invests $6000 at 4% simple interest, for a period of time, to produce $720. For how long did he invest the money?

2F 13 Use the tax table below to find the income tax payable on an income of $78 000.

Taxable income	Tax on this income
$0 – $18 200	Nil
$18 201 – $45 000	19 cents for each $1 over $18 200
$45 001 – $120 000	$5 092 plus 32.5 cents for each $1 over $45 000
$120 001 – $180 000	$29 467 plus 37 cents for each $1 over $120 000
$180 001 and over	$51 667 plus 45 cents for each $1 over $180 000

The above rates **do not** include the Medicare levy of 2%.

Multiple-choice questions

2A 1 2.8% as a decimal is:

A 2.8 B 0.28 C 0.028 D 0.0028 E 280

2A 2 What percentage of $2 is 50 cents?

A 4% B 40% C 25% D $2\frac{1}{2}\%$ E 400%

2A 3 $12\frac{1}{2}\%$ as a simple fraction is:

A $\frac{12}{100}$ B $\frac{1}{8}$ C $\frac{3}{25}$ D 0.125 E 12.5

2B 4 $33\frac{1}{3}\%$ of $660 is the same as:

A $\$660 \div 2$ B $\$660 \times 0.3$ C $\$660 \times 0.03$ D $\$660 \div 3$ E $\$660 \div \frac{1}{3}$

2B 5 15% of $1600 is equal to:

A 24 B 150 C $240 D $24 E 240

2B 6 If 110% of a number is 528, then the number is:

A 475.2 B 52.8 C 480 D 580.8 E 475

2C 7 9670 increased by 12% becomes:

A 9682 B 9658 C 10830.4 D 1160.4 E 8509.6

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2E

8 Sura is paid a wage of $27.80 per hour. She works 12 hours at this rate during a week, plus 4 hours on a public holiday where she gets paid at time and half. Her earnings in the week are:

A $500.40 B $444.80 C $556 D $667.20 E $278

2E

9 Simon earns a weekly retainer of $370 and 12% commission of any sales he makes. If he makes $2700 worth of sales in a particular week, he will earn:

A $595 B $652 C $694 D $738.40 E $649.60

2G/2H

10 $1200 is invested with a simple interest rate of 10% for two years. The future value at the end of the two years is:

A $252 B $1452 C $1450 D $240 E $1440

Extended-response questions

1 Pauline buys a debutante dress at cost price from her friend Tila. Pauline paid $420 for the dress which is normally marked up by 55%.

a How much did she save?

b What is the normal selling price of the dress?

c If Tila gets a commission of 15%:

i how much commission did she get?

ii how much commission did Tila lose by selling the dress at cost price rather than the normal selling price?

2 Adam starts a new job and works a 38-hour week for a wage of $975.84.

a Calculate his hourly rate of pay.

b If overtime is calculated at time and a half, what is Adam's overtime rate?

c How much does Adam earn for 4 hours of overtime work?

d How many hours of overtime did Adam work in a week if his wage for that week was $1226.22?

e If Adam usually works the amount of overtime in part d in the 52 weeks of the year he works, and he pays 27% of his pay in tax, what is his net annual income?

f If Adam invests 10% of his net income in an account earning 8% p.a. simple interest for 18 months, how much extra income will he have earned?

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3

Expressions and equations

Maths in context: Algebra in the workplace

Skills using algebraic formulas and solving algebraic equations are important in agriculture, animal management, manufacturing, technology, business, computer programming, finance, science (including sports science), and widely applied in the food industry, trades and other professional occupations.

- Training for a scuba diving certificate includes solving the equations that show with increasing depth the water pressure increases and the volume of air in a diver's lungs decreases. When the diver is ascending, the air in the lungs expands.
- Vets use algebraic formulas relating drug dosage to an animal's weight. Vets calculate a horse's weight in kg using the formula: weight $= \frac{G^2 L}{11880}$, where G is the girth in cm and L is body length in cm.
- Computer programmers often need to solve linear equations, such as when writing software code for various applications; developing websites; troubleshooting network problems; calculating data upload times; and adjusting security settings.

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Chapter contents

3A Algebraic expressions (CONSOLIDATING)
3B Adding and subtracting algebraic expressions (CONSOLIDATING)
3C Multiplying and dividing algebraic expressions (CONSOLIDATING)
3D Expanding algebraic expressions
3E Linear equations with a pronumeral on one side (CONSOLIDATING)
3F Linear equations involving fractions
3G Linear equations involving brackets
3H Equations with pronumerals on both sides
3I Using linear equations to solve problems
3J Using formulas

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- generalises number properties to operate with algebraic expressions including expansion and factorisation (MA4-ALG-C-01)
- solves linear equations of up to 3 steps, limited to one algebraic fraction (MA5-EQU-C-01).

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Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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Warm-up quiz

1 Write in simplified form.

a $3 \times x$ **b** $4 \times a \times b$

c $2 \times 5 \times x$ **d** $3 \times b \times 7$

2 If $x = 3$, find the value of:

a $x + 5$ **b** $10 - x$ **c** $4x$

d $\frac{18}{x}$ **e** $2x + 3$ **f** $2(x + 4)$

3 Write algebraic expressions for:

a 3 more than x

b the product of a and b

c 2 lots of y less 3

d the sum of x and 2, all divided by 3

4 Calculate:

a -6×3 **b** -7×4

c $-8 \times (-5)$ **d** $-1 \times (-3)$

e $-2 + 5$ **f** $-8 + 3$

g $7 - 11$ **h** $-2 - 6$

5 Simplify by collecting like terms.

a $2x + 5x - 4x$ **b** $7y + 5 - y$

c $4x - (-4x)$ **d** $3x + 12y - 3x + 5y$

e $3a + 4a^2 + 7a + 5a^2$ **f** $3xy - 4y + 2yx - 3y$

6 Simplify the following.

a $3 \times 2a$ **b** $7x \times (-3y)$

c $\frac{8b}{2}$ **d** $\frac{9mn}{6n}$

7 Expand the following.

a $2(x + 3)$ **b** $3(a - 5)$ **c** $4(2x + 1)$

8 Evaluate the following if $a = 3$ and $b = -2$.

a $2a - 5$ **b** $ab + 4$

c $\frac{9}{a} - b$ **d** $2a(b + 1)$

9 To which of the following equations is $x = 4$ a solution?

a $2x + 3 = 9$ **b** $\frac{x}{2} + 3 = 5$

c $\frac{2x + 1}{3} = 3$ **d** $5 - 2x = -1$

10 Find the value of a that makes the following true.

a $a + 4 = 13$ **b** $a - 3 = 7$

c $2a + 1 = 9$ **d** $\frac{a - 1}{4} = 5$

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3A Algebraic expressions CONSOLIDATING

Learning intentions for this section:

- To review the terminology associated with algebraic expressions
- To identify terms, coefficients and constant terms in expressions
- To multiply and divide expressions involving pronumerals
- To be able to convert worded scenarios into algebraic expressions
- To be able to substitute numbers into algebraic expressions

Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- Expertise with substitution is assumed knowledge for Stage 6 Standard

Algebra is central to the study of mathematics and is commonly used to solve problems in many everyday situations and in more complex problems. Algebra involves working with unknown values in maths. Pronumerals (or variables) are used to represent these unknown values.

Lesson starter: What does the letter mean?

Consider the following situations and how they could be written as algebraic expressions.

1 Fidel has 7 marbles and Jack has an unknown number of marbles, which we will call m. They then each get 4 more marbles.
   - **a** How many marbles does Fidel have now?
   - **b** What mathematical operation ($+, -, \times, \div$) did you use to answer part **a**?
   - **c** Using the same operation, how could you describe how many marbles Jack has now?

2 Fidel can buy a chocolate bar for $2. Jack can buy a different chocolate bar but he will pay an unknown amount for it. We'll call the amount Jack pays p (dollars).
   - **a** If Fidel buys 3 chocolate bars, how much will he pay?
   - **b** If Fidel buys 10 chocolate bars, how much will he pay?
   - **c** What mathematical operation did you use to answer parts **a** and **b**?
   - **d** Using the same operation, how could you write how much Jack will pay for 5 chocolate bars?

KEY IDEAS

- In algebra, letters are used to represent numbers. These letters are called **pronumerals** or **variables**.
- In algebra, the multiplication and division signs are generally not shown.
  For example: $3 \times x$ is written as $3x$ and $x \div 4$ is written as $\frac{x}{4}$.
- An **expression** is a combination of numbers and pronumerals.

Expression

Term → $5x + \frac{4}{y}$ ← Pronumeral

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- A **term** is a combination of numbers and variables connected only by multiplication and/or division. The expression $5x + \frac{4}{y}$ contains two terms:

  $5x$ and $\frac{4}{y}$.

- **Coefficients** are the numbers being multiplied by pronumerals.
  For example, in $3x + 4y$:
  3 is the coefficient of x.
  $+4$ is the coefficient of y.

- **Constant terms** consist of a number only.
  For example: $4x - 2 \leftarrow -2$ is a constant term. (The sign must be included.)

- Expressions can be evaluated by **substituting** a number for a pronumeral.
  For example: When $a = 2$, then
  $a + 6 = 2 + 6$
  $= 8$ (substitute)

- Order of operations should be followed when **evaluating** expressions:
  1. Brackets
  2. Powers
  3. Multiplication and division, from left to right
  4. Addition and subtraction, from left to right

Key terminology: pronumeral, variable, expression, term, coefficient, constant term, substitution, evaluate

Exercise 3A

UNDERSTANDING | 1–4 | 4

1 Use the words from the list below to complete the following.
expression, *coefficient*, *constant term*, *terms*, *pronumeral*

a $3x - 2y + 7$ is an example of an algebraic _____.

b In $2a + b - 3$, -3 is the _____.

c In $4m + 3$, m is called a _____.

d In $3 - 2y + 4z$, 4 is the _____ of z.

e In the expression $3s + 7t$, $3s$ and $7t$ are the _____ .

2 Express in simplified form without the $\times$ or $\div$ symbols.

a $7 \times y$

b $-2 \times x$

c $a \times b$

d $y \div 2$

e $x \div y$

f $2 \div a$

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3 Substitute $x = 2$ into the following and evaluate.

a $x + 3$ **b** $10 - x$ **c** $7x$ **d** $\frac{8}{x}$

Hint: Substitute means to replace the pronumeral with the given numeral.

4 State, yes or no, if 2 is the coefficient of x in the following.

a $2x + 7$ **b** $y + 2x$ **c** $3xy - 4y + 2$ **d** $\frac{x}{2} - 3$

FLUENCY 5, 6, 7–8(½) 5, 6, 7–8(½)

Example 1 Identifying parts of algebraic expressions

Consider the expression $5a + 2ab + 7 - 4b$.

a How many terms are in the expression?

b State the coefficient of:

i a **ii** b

c What is the constant term?

SOLUTION	EXPLANATION
a There are 4 terms.	The terms in the expression are separated by + or −.
b i 5 is the coefficient of a.	$5a$ is $5 \times a$; 5 is the number being multiplied by a.
ii −4 is the coefficient of b.	$-4b$ is $-4 \times b$; the negative sign is included.
c 7 is the constant term.	7 is the number with no pronumeral part.

Now you try

Consider the expression $3ab + 5 - 2a + 6b$.

a How many terms are in the expression?

b State the coefficient of:

i a **ii** b

c What is the constant term?

5 Complete each of the parts **i–iii** for the algebraic expressions below.

i How many terms are in the expression?

ii What is the coefficient of y?

iii What is the constant term?

a $5x + 2y + 3$

b $2x - 3y$

c $3xy + 7y - 4$

d $2x^2 - 1 + 4x + \frac{y}{2}$

Hint: Remember: the coefficient of a pronumeral includes the sign before it; in $3 - 2x$, -2 is the coefficient of x.

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Example 2 Writing algebraic expressions for word problems

Write an algebraic expression for the following.

a The number of tickets needed for 3 children and r adults
b The cost in dollars of P pies at \$3 each
c The amount for each person if a \$300 prize is shared equally among m friends

SOLUTION	EXPLANATION
a $3 + r$	3 tickets plus the number of girls, r.
b $\$3P$	2 pies would cost $\$3 \times 2 = \$6$. The cost is 3 × the number of pies, so P pies costs $\$3 \times P = \$3P$
c $\frac{\$300}{m}$	\$300 shared between 3 people is \$300 divided into 3 parts, so \$300 divided into m parts is $\$300 \div m = \frac{\$300}{m}$.

Now you try

Write an algebraic expression for the following.

a The number of tickets required for 5 children and g adults
b The cost of 4 tickets at $\$x$ each
c The amount for each child if 600 mL of juice is shared equally among y children

6 Write an algebraic expression for the following.

a The number of tickets required for:
- **i** 4 children and r adults
- **ii** t children and 2 adults
- **iii** x children, y adults and z pets

b The cost in dollars of:
- **i** P pies at \$6 each
- **ii** 10 pies at $\$n$ each
- **iii** D drinks at \$2 each
- **iv** P pies at \$5 and D drinks at \$2

c The number of grams of lollies for one child if 500 g of lollies is shared equally among C children.

Hint: For part **b i**:
1 pie costs $\$6 \times 1 = \$6$
2 pies cost $\$6 \times 2 = \$12$
5 pies cost $\$6 \times 5 = \$30$
P pies cost …

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Example 3 Converting words to expressions

Write an algebraic expression for the following.

a Five less than x
b Three more than twice x
c The sum of a and b is divided by 4
d The square of the sum of x and y

SOLUTION	EXPLANATION
a $x - 5$	5 below x is 5 subtracted from x.
b $2x + 3$	Twice x is $2 \times x$, then add 3.
c $\frac{a+b}{4}$	The sum of a and b is done first $(a + b)$ and the result divided by 4.
d $(x + y)^2$	The sum of x and y is done first and then the result is squared (recall that the square of a is $a^2 = a \times a$).

Now you try

Write an algebraic expression for the following.

a Seven more than y
b Six less than twice x
c The sum of m and n is divided by 3
d The square root of the sum of a and b

7 Write an algebraic expression for each of the following.

Hint: Sum is +
Product is ×
Twice is × 2
Square of x is x^2

a The sum of 2 and x
b The sum of ab and y
c 5 less than x
d 7 subtracted from $2y$
e The product of x and 3
f Three times the value of p
g Four more than twice x
h The sum of x and y is divided by 5
i 10 less than the product of 4 and x
j 3 lots of x subtracted from 1
k The sum of 3 and y is divided by 2
l Half of 1 more than x
m The square of the sum of m and n
n The sum of the squares of m and n

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Example 4 Substituting values into expressions

Evaluate these expressions if $a = 5$, $b = 6$ and $c = -3$.

a $3a + b$ **b** $2a - (b + c)$ **c** $a^2 - bc$

SOLUTION

a $3a + b = 3 \times 5 + 6$
$= 15 + 6$
$= 21$

b $2a - (b + c) = 2 \times 5 - (6 + (-3))$
$= 10 - 3$
$= 7$

c $a^2 - bc = (5)^2 - 6 \times (-3)$
$= 25 - (-18)$
$= 25 + 18$
$= 43$

EXPLANATION

Substitute the value 5 for a and the value 6 for b. Recall $3a = 3 \times a$.

Substitute the values for a, b and c. Then apply order of operations, with brackets evaluated first, before moving onto multiplication and division, then addition and subtraction.
$6 + (-3) = 6 - 3 = 3$

Evaluate powers before the other operations: $5^2 = 5 \times 5$.

A positive (6) times a negative (-3) gives a negative (-18).

To subtract a negative means to add.

Now you try

Evaluate these expressions if $a = 4$, $b = -2$ and $c = 5$.

a $a + 4c$ **b** $2b - (a + c)$ **c** $c^2 - ab$

8 Evaluate these expressions if $a = 4$, $b = -3$ and $c = 8$.

a $3a + c$ **b** $ac - 7$ **c** $2c - (a + b)$

d $a^2 - 2c$ **e** $\frac{a + c}{2}$ **f** $b + 2(c - a)$

g $2c - ab$ **h** abc **i** $b - c + a^2$

Hint: When adding or subtracting a negative number:
$8 + (-3) = 8 - 3 = 5$
$8 - (-3) = 8 + 3 = 11$

PROBLEM-SOLVING AND REASONING 9–11 10–13

9 A rectangular garden bed is 12 m long and 5 m wide.

a Find the area of the garden bed.

b The length is increased by x m and the width is decreased by y m. Find the new length and width of the garden.

c One length and one width of the rectangular garden will be lined with paving. Write an expression for the total distance around the garden that will be paved.

Hint: Draw a diagram to help.

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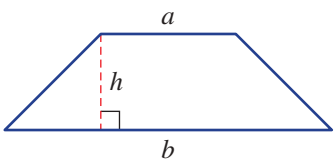
10 Jelena earns $(10 + 8t)$ dollars for each shift that she works, where t is the number of hours worked in the shift.

- **a** How much does Jelena earn if $t = 2$?
- **b** How much does Jelena earn from a 5-hour shift?
- **c** Will she earn \$100 from a 10-hour shift?
- **d** How many whole hours must she work to earn more than \$100?

11 The expression for the area of a trapezium is $\frac{h}{2}(a + b)$, where a, b and h represent the lengths shown.



- **a** Find the area of the trapezium with $a = 5$, $b = 7$ and $h = 3$.
- **b** A trapezium has $h = 4$ and area 12. If a and b are whole numbers, what possible values can the pronumeral a have?

12 Christina earns \$$A$ from selling 20 glasses of lemonade. Write an expression for:

- **a** the cost of one glass of lemonade
- **b** the cost of three glasses of lemonade
- **c** the cost of n glasses of lemonade

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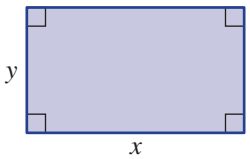
13 For each of these shapes, write an expression for:

i the perimeter **ii** the area

a



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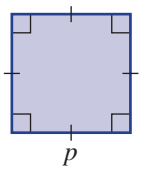

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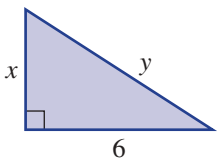

Hint: Perimeter is the distance around the outside of the shape. Area of a triangle is $\frac{1}{2} \times$ base $\times$ height.

ENRICHMENT: Should there be brackets? — 14

14 Brackets are used to ensure that the mathematics inside the brackets occurs first.

a By substituting values for x, decide whether the following are the same.

i $2(x+1)$ and $2x+1$

ii $3+\frac{x}{2}$ and $\frac{3+x}{2}$

b Write down two different possible expressions that could describe each of the following.

i 3 lots of x plus 1

ii The sum of 5 and x divided by 3

iii Half of x plus y

c Write down a clear statement that describes the following expressions.

i $4(x+2)$ **ii** $3+\frac{x}{2}$ **iii** $\frac{1}{3}(m+n)$

iv $5x+7$ **v** $\frac{x+y}{2}$ **vi** $\frac{1}{2}a+b$

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3B Adding and subtracting algebraic expressions CONSOLIDATING

Learning intentions for this section:

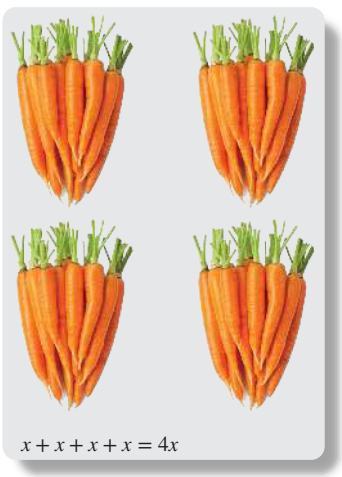
- To be able to identify like terms
- To understand that only like terms can be combined under addition or subtraction
- To be able to collect and combine like terms using addition or subtraction

Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- Expertise with like terms is assumed knowledge for Stage 6 Standard

Just as $2 + 2 + 2 + 2 = 4 \times 2$, so $x + x + x + x = 4 \times x$ or $4x$. We say that the expression $x + x + x + x$ is simplified to $4x$; i.e. 4 lots of x. Similarly, $3x + 2x = x + x + x + x + x = 5x$ and $4x - 3x = x + x + x + x - (x + x + x) = x$.

All these expressions have like terms and can be simplified to an expression with a smaller number of terms by adding or subtracting their coefficients.


$x + x + x + x = 4x$

Lesson starter: Two groups

Place the terms in the following four sets into groups of similar terms on either side of the dividing line. The first set has been started for you.

Set 1	Set 2	Set 3	Set 4
~~$3x$~~ $7y$ $2x$ $8x$	$4ab$ $2a$ $5ab$	2 $5y$ 7 1	$3ab$ $4a^2b$ $2ab$ $10ab$
y $5x$ ~~$2y$~~	$7a$ ab $6ab$	$3y$ $2y$ 8 25	$5a^2b$ $12ab$ $9a^2b$
$3x$ | $2y$			

- Describe how you classified each set into two groups.
- What do the terms in each group have in common?
- Simplify the sum of all the terms in each of the eight groups.

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KEY IDEAS

- $x + x + x + x$ is '4 lots of x', which is $4 \times x$ or just $4x$.
  - $x \times 1$ and $1 \times x$ are $1x$ but we leave off the 1 and express it simply as x.
  - $x \times 0$ and $0 \times x$ are $0x$, which is 0.
- **Like terms** have the same pronumerals and powers.
  For example: $5x$ and $7x$ are like terms
  $3ab$ and $-8ab$ are like terms
  - Since $a \times b = b \times a$ then ab and ba are also like terms
- The pronumeral part of a term is often written in alphabetical order; e.g. xy rather than yx.
- Like terms can be collected to form a single term by adding or subtracting the **coefficients**.
  For example: $4x + 7x = 11x$
  $10ab - 2ab = 8ab$
- Unlike terms do not have the same pronumeral factors.
  For example: $5x$, x^2, xy, $2xy^2$ and 3 are all unlike terms.

Key terminology: like terms, pronumeral, coefficient

Exercise 3B

UNDERSTANDING 1–3 3

1 Use the words from the list below to complete the following statements.
one, *pronumerals*, *coefficients*, *like*

a $4a$ and $7a$ are an example of _____ terms.

b To add like terms, we add the _____.

c Like terms have the same _____ and powers.

d The coefficient of y in $2x + y$ is _____.

2 Decide whether the following pairs of terms are like terms (Yes or No).

a $4ab$ and $3ab$ **b** $2x$ and $7xy$ **c** 5 and $4m$

d $7yz$ and $-yz$ **e** $2mn$ and $9nm$ **f** $3x^2y$ and $7xy^2$

Hint: Like terms must have exactly the same pronumerals and powers.

3 Which of the following represent the same expression as $w - x + y - z$?

a $w - z + y - x$ **b** $w - y + x - z$

c $y + w - z - x$ **d** $-x + w - z + y$

e $y - w - x + z$ **f** $y - z - x + w$

Hint: Make sure the correct pronumerals are being added or subtracted.

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FLUENCY 4, 5–6(½) 4–6(½)

Example 5 Identifying like terms

Choose the pair(s) of like terms from the following sets.

a $3x,\ 3,\ 4xy,\ 5x,\ y$

b $4b,\ 4bc,\ b,\ 2abc,\ -7bc,\ 2bc^2$

SOLUTION

a $3x$ and $5x$ are like terms.
The rest are not.

b $4b$ and b are like terms.
$4bc$ and $-7bc$ are like terms.

EXPLANATION

$3x$ and $5x$ have the same pronumeral; i.e. x.

$4b$ and b have the same pronumeral.

$4bc$ and $-7bc$ have the same pronumerals.

$2bc^2$ does not, due to the power of 2.

All other terms do not have the same pronumerals and powers.

Now you try

Choose the pair(s) of like terms from the following sets.

a $5m, 3mn, 7, n, 2m$

b $7y, 4xz, 5xy^2, -2xyz, zx, -y$

4 Choose the pair(s) of like terms from the following sets.

a $4y,\ 5,\ 3xy,\ 2y,\ 7xy$

b $3x,\ 3y,\ 3,\ xy,\ 7x$

c $7ab,\ 2a,\ -3ab,\ b^2,\ 5a$

d $2a^2,\ 4a,\ 3ab,\ a,\ -3a^2$

e $2xy,\ 3x^2y,\ -5x,\ 7yx^2$

f $5ab^2,\ 3ab,\ 2b^2,\ 4ab^2,\ 7ba$

Hint: st and ts are like terms since $s \times t = t \times s$.

Example 6 Collecting like terms

Simplify the following by collecting like terms.

a $4a + 7a$

b $3x + 4 - 2x$

SOLUTION

a $4a + 7a = 11a$

b $3x + 4 - 2x = 3x - 2x + 4$
$= x + 4$

EXPLANATION

Since $4a$ and $7a$ are like terms they can be simplified to one term by adding their coefficients: $4 + 7 = 11$.

Collect like terms ($3x$ and $-2x$). The sign belongs to the term that follows. Combine their coefficients: $3 - 2 = 1$. Recall that $1x$ is written as x.

Now you try

Simplify the following by collecting like terms.

a $5t + 3t$

b $9y + 3 - 5y$

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5 Simplify the following by collecting like terms.

a $3a + 7a$	**b** $4n + 3n$	**c** $12y - 4y$
d $5x + 2x + 4x$	**e** $6ab - 2ab$	**f** $7mn + 2mn - mn$
g $4y + 3y + 8$	**h** $7x + 5 - 4x$	**i** $5m + 2 - m$
j $5ab + 3a + 7ab$	**k** $6xy + xy + 4y$	**l** $5bc - 4 - 2bc$

Hint: In order to add or subtract coefficients, the terms must be like terms.

Example 7 Combining like terms

Simplify the following by collecting like terms.

a $3x + 2y + 4x + 7y$

b $3xy + 4x + xy - 6x$

c $8ab^2 - 9ab - 5ab^2 + 3ba$

SOLUTION

a $3x + 2y + 4x + 7y = 3x + 4x + 2y + 7y$
$= 7x + 9y$

b $3xy + 4x + xy - 6x = 3xy + xy + 4x - 6x$
$= 4xy - 2x$

c $8ab^2 - 9ab - 5ab^2 + 3ba$
$= 8ab^2 - 5ab^2 - 9ab + 3ab$
$= 3ab^2 - 6ab$

EXPLANATION

Collect like terms ($3x$ and $4x$ and $2y$ and $7y$) and combine their coefficients: $3 + 4 = 7$ and $2 + 7 = 9$.

Collect the like terms ($3xy$ and xy and $4x$ and $-6x$). Combine their coefficients (recall $xy = 1xy$): $3 + 1 = 4$ and $4 - 6 = -2$

Collect like terms in ab^2 and ab. Remember that $ba = ab$ and the + or − sign belongs to the term that follows; i.e. $-5ab^2$.

Combine coefficients: $8 - 5 = 3$ and $-9 + 3 = -6$.

Now you try

Simplify the following by collecting like terms.

a $4a + 3b + 2a + 10b$

b $5st + 3s - 2st + s$

c $2x^2y - 5xy + 3x^2y - yx$

6 Simplify the following by collecting like terms.

a $2a + 4b + 3a + 5b$	**b** $4x + 3y + 2x + 2y$	**c** $xy + 8x + 4xy - 4x$
d $6t + 5 - 2t + 1$	**e** $5x + 1 + 6x + 3$	**f** $3mn + 4 + 4nm - 5$
g $4ab + 2a + ab - 3a$	**h** $3st - 8ts + 2st + 3ts$	**i** $8ac - 6c + 2ac - 3c$
j $5xy^2 - 2xy^2$	**k** $8m^2n - 6nm^2 + m^2n$	**l** $2x^2y - 4xy + 5yx^2$
m $x^2 - 7x + 3x^2 + 4x$	**n** $a^2b - 4ab + 3a^2b + ba$	**o** $10pq^2 - 2qp - 3pq^2 - 6pq$

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PROBLEM-SOLVING AND REASONING

7, 8 | 8(½), 9, 10

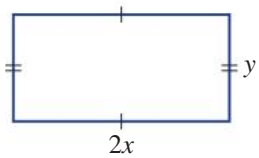
7 A farmer has x pigs and y chickens.

a Write an expression for the total number of heads.

b Write an expression for the total number of legs.

8 Write simplified expressions for the perimeter of the following shapes.

a



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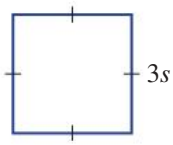

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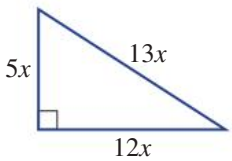

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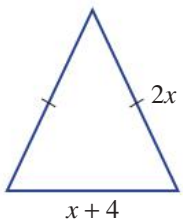

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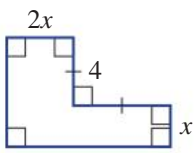

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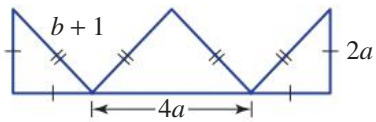
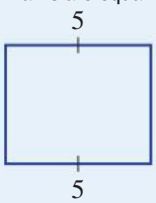


Hint: Sides with matching dash marks are equal in length.



9 A rectangular pool's length is three times its width, x metres. Write an expression for its perimeter.

10 Fill in the missing term to make each of the following correct.

a $11t + 3 - \square = 7t + 3$

b $\square + 3y - 2x = 7x + 3y$

c $3x + 4y - \square + 5y = x + 9y$

d $4a + 7b - \square - 2b = 5b - 3a$

e $3mn - \square + nm + 4n = 4mn - n$

f $2pq + 2p - \square - 5pq = 2p - 7pq$

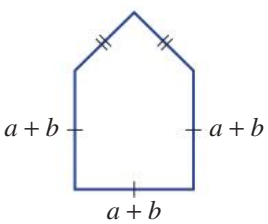
g $3x^2y - 11xy + 3yx^2 - \square = 6x^2y - 13xy$

h $4b^2 - 3b + 2b^2 + \square = 6b^2$

ENRICHMENT: Around the edge

— | 11

11 Consider the shape shown.



a If the unknown sides each have length $2b$, what is the perimeter in simplified form?

b If $a = 3$ and $b = 2$ and the perimeter is 23 units, what is the length of the unknown sides?

c If the perimeter is $(5a + 3b)$, what is the length of each unknown side?

d If the perimeter is $(7a + b)$, what is the length of each unknown side?

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3C Multiplying and dividing algebraic expressions CONSOLIDATING

Learning intentions for this section:

- To understand that like terms and order do not matter when multiplying
- To be able to multiply algebraic terms by multiplying coefficients and pronumerals
- To be able to divide algebraic terms by cancelling the highest common factor

Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- Expertise with these concepts is assumed knowledge for Stage 6 Standard

We have seen that $3x + 3x$ can be simplified to $6x$ and that $3x + 3x$ can also be written as $2 \times 3x$. Thus, $2 \times 3x = 6x$. Also, $4 \times 5a = 4 \times 5 \times a = 20a$. Any two or more terms can be simplified using multiplication; we do not need the terms to be like terms.

A single term such as $2 \times 5 \times x \div 10$ can also be simplified using multiplication and division, so $2 \times 5 \times x \div 10 = \frac{10x}{10} = x$, since $\frac{10}{10}$ simplifies to 1 by cancelling the common factor.

Lesson starter: Are they equivalent?

These expressions can be separated into two groups. Group them so that the expressions in each group are equivalent.

$$4x \qquad 2x - y \qquad 6x - x - x \qquad 10x - y - 8x$$
$$\frac{24x}{6} \qquad y + 2x - y + 2x \qquad 2 \times x - 1 \times y \qquad -y + 2x$$
$$\frac{1}{2} \times 8x \qquad 2 \times 2x \qquad \frac{8x^2}{2x} \qquad -1 \times y + \frac{2x^2}{x}$$

KEY IDEAS

- The symbols for multiplication ($\times$) and division ($\div$) are usually not shown in simplified algebraic expressions.
  For example:
  $5 \times a \times b = 5ab$
  $-7 \times x \div y^2 = -\frac{7x}{y^2}$
- Order does not matter in multiplication;
  For example:
  $2 \times 3 = 3 \times 2$
  $a \times 2 \times b = 2 \times a \times b$
- Algebraic terms are multiplied by multiplying coefficients with the pronumeral parts.
  For example: $4a \times 3b = 4 \times 3 \times a \times b = 12ab$.

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- The factors of a number are the set of numbers that divide evenly into it. For example, the **factors** of 6 are 1, 2, 3 and 6.
- When dividing algebraic expressions, common factors can be cancelled. Cancel the highest common factor of the numerals and then the pronumerals.
  For example:
  $\frac{{}^{1}\cancel{7}x}{{}^{2}\cancel{14}} = \frac{x}{2}$
  $\frac{{}^{2}\cancel{8}xy^{1}}{{}^{3}\cancel{12}y_{1}} = \frac{2x}{3}$
  $\frac{a^2b}{a} = \frac{{}^{1}\cancel{a} \times a \times b}{{}_{1}\cancel{a}} = ab$

Key terminology: pronumeral, coefficient, factor, highest common factor

Exercise 3C

UNDERSTANDING	1, 2	2

1 Complete the simplified form of the following.

a $7 \times x = \square$

b $a \times b = \square$

c $3 \times 2 \times m = \square\, m$

d $8 \times 3 \times y = 24\, \square$

e $6 \times 7 \times b = \square$

f $4 \times x \times 5 = \square$

g $a \times a = \square$

h $9 \times y \times y = 9\, \square$

2 Answer true (T) or false (F) to the following statements.

a $2 \times x \times 3 = 2 \times 3 \times x$

b $7a \times 3b = 7 \times 3 \times a \times b$

c $4x \times 5xy = 4 \times 5 \times x \times y$

d $\frac{8x}{16} = 2x$

e $5a^2 \div (10a) = \frac{10a}{5a^2}$

f $\frac{3xy^2}{y} = 3xy$

Hint: Simplify by cancelling the highest common factor of the numerator and denominator:
$\frac{6}{10} = \frac{{}^{1}\cancel{2} \times 3}{{}_{1}\cancel{2} \times 5} = \frac{3}{5}$

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FLUENCY 3–5(½) 3–5(½)

Example 8 Multiplying algebraic terms

Simplify the following.

a $3 \times 2b$ **b** $-2a \times 3b$

SOLUTION

a $3 \times 2b = 3 \times 2 \times b$
$= 6b$

b $-2a \times 3b = -2 \times 3 \times a \times b$
$= -6ab$

EXPLANATION

Multiply the coefficients.

Recall that multiplication can occur in any order; i.e. $2 \times 3 \times 4 = 2 \times 4 \times 3$ etc.
Multiply the coefficients and simplify.

Now you try

Simplify the following.

a $4 \times 5x$ **b** $-3m \times 6p$

3 Simplify the following.

a $5 \times 2m$	**b** $2 \times 6b$	**c** $3 \times 5p$	**d** $3x \times 2$
e $3p \times 6r$	**f** $4m \times 4n$	**g** $-2x \times 7y$	**h** $5m \times (-3n)$
i $-4c \times 3d$	**j** $2a \times 3b \times 5$	**k** $-4r \times 3 \times 2s$	**l** $5j \times (-4) \times 2k$
m $4m \times 5n \times (-3p)$	**n** $-8m \times (-5n)$	**o** $9pq \times -5$	**p** $8 \times (-w) \times 4 \times (-y)$

Example 9 Dividing algebraic terms

Simplify the following.

a $\frac{4x}{8}$ **b** $10ab \div (15b)$

SOLUTION

a $\frac{{}^{1}\cancel{4}x}{{}_{2}\cancel{8}} = \frac{x}{2}$

b $10ab \div (15b) = \frac{10ab}{15b}$
$= \frac{{}^{2}\cancel{10} \times a \times \cancel{b}_{1}}{{}_{3}\cancel{15} \times \cancel{b}_{1}}$
$= \frac{2a}{3}$

EXPLANATION

Deal with numerals and pronumerals separately, cancelling any common factors.

The highest common factor of 4 and 8 is 4.

Write division as a fraction first. Write numerator and denominator as a product and cancel the common factors 5 and b.

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Now you try

Simplify the following.

a $\frac{9x}{27}$ b $18pq \div (4p)$

4 Simplify the following by cancelling.

a $\frac{8b}{2}$ b $\frac{2a}{6}$ c $\frac{4ab}{6}$ d $\frac{3mn}{6n}$

e $\frac{5xy}{20y}$ f $\frac{10st}{6t}$ g $\frac{3xy}{xy}$ h $\frac{27pq}{6p}$

i $2x \div 4$ j $12ab \div (2a)$ k $7mn \div (3n)$ l $8y \div (20xy)$

m $6p \div 3$ n $3p \div 6$ o $12pq \div (18p)$ p $\frac{12a - 3a}{6}$

q $\frac{2m + 5m}{m}$ r $\frac{2w \times 3x}{6x}$

Hint: Make sure they are in fraction form before cancelling; e.g. $2x \div 4 = \frac{2x}{4}$

Example 10 Multiplying and dividing with squared pronumerals

Simplify the following.

a $6x \times 5xy$ b $\frac{12a^2b}{3ab}$

SOLUTION

a $6x \times 5xy = 6 \times 5 \times x \times x \times y$
$= 30x^2y$

b $\frac{12a^2b}{3ab} = \frac{^{4}\cancel{12} \times {}^{1}\cancel{a} \times a \times \cancel{b}^{1}}{{}^{1}\cancel{3} \times \cancel{a}^{1} \times \cancel{b}^{1}}$
$= 4a$

EXPLANATION

Multiply the coefficients and the pronumerals. Recall that $x \times x$ is written as x^2 (x squared).

Write numerator and denominator as a product, with $a^2 = a \times a$. Cancel common factor (3) for numerals, then cancel common factors for pronumerals.

Now you try

Simplify the following.

a $7ab \times 6a$ b $\frac{20xy^2}{4xy}$

5 Simplify the following.

a $4n \times 6n$ b $-3q \times q$ c $5s \times 2s$

d $7a \times 3ab$ e $5mn \times (-3n)$ f $-3gh \times (-6h)$

g $\frac{24ab^2}{8ab}$ h $\frac{25x^2y}{5xy}$ i $\frac{9m^2n}{18mn}$

j $\frac{2xy}{8xy^2}$ k $\frac{6a^2b}{10a}$ l $\frac{45p^2q^2}{15pq}$

Hint: Deal with numerals and pronumerals separately.
$\frac{\cancel{24}^{3}}{\cancel{8}_{1}}$ and $\frac{ab^2}{ab} = \frac{\cancel{a}_{1} \times b \times \cancel{b}_{1}}{\cancel{a}^{1} \times \cancel{b}_{1}} = b$

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PROBLEM-SOLVING AND REASONING

6(½), 7 | 6(½), 7–9

6 Simplify the following by first writing in fraction form.

a $4 \times x \div y$ **b** $3 \times m \div 9$ **c** $4 \times a \div (2b)$

d $5x \times 4 \div (2y)$ **e** $6 \times 4mn \div (3m)$ **f** $8a \times 5b \div (8a)$

g $10m \times 4n \div (8mn)$ **h** $4x \times 3xy \div (2x)$

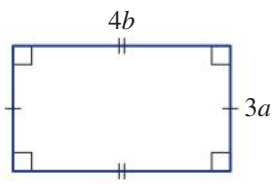
i $3pq \times p \div (6q)$ **j** $5ab^2 \times 4 \div (10b)$

Hint: Apply one operation at a time from left to right in these; e.g. $4 \times x \div y = 4x \div y$

$= \frac{4x}{y}$

7 Write a simplified expression for the area of the following shapes.

a



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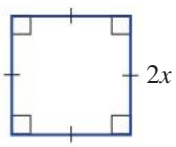

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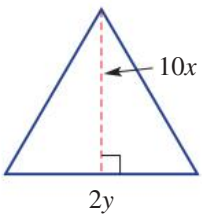

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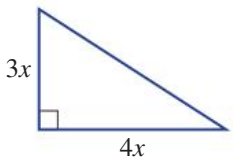


Hint: Area of a triangle = $\frac{1}{2} \times$ base $\times$ height

8 A bag of lollies contains x jelly beans. Jessica buys 6 bags of lollies to share equally among her three grandchildren. Write a simplified expression for how many jelly beans each grandchild receives.

9 Fill in the missing term to make each of the following correct.

a $3m \times \square = 12mn$ **b** $4x \times \square = 28xy$ **c** $-2xy \times \square = -18xy^2$

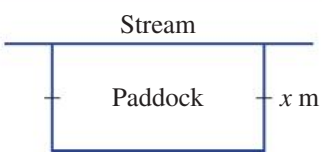
d $6a \times \square = 30a^2b$ **e** $\frac{\square}{7} = 2x$ **f** $\frac{15}{\square} = \frac{3}{2y}$

g $\frac{\square}{18ab} = \frac{2}{3b}$ **h** $\frac{\square}{4x} = 2xy$ **i** $\frac{\square}{2x} = 5x$

ENRICHMENT: Rectangular paddocks

— | 10, 11

10 The length of a rectangular paddock is three times its width. Its width is x metres. It is bordered on one side by a stream.



a Write a simplified expression for the:

i area of the paddock

ii length of fencing needed for the three sides of the paddock

b If $x = 50$, use your answers to part **a** to give the:

i area of the paddock **ii** length of fencing material required.

11 Another rectangular paddock has its area given by the expression $4ab^2$. Find the width of the paddock if the length is:

a ab **b** $2b$ **c** $4ab$

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3D Expanding algebraic expressions

Learning intentions for this section:

- To know how the distributive law is used to expand brackets in expressions
- To be able to simplify expressions involving brackets and like terms

Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- In Section 3G of this book you will use this skill to solve equations
- Expertise with these concepts is assumed knowledge for Stage 6 Standard

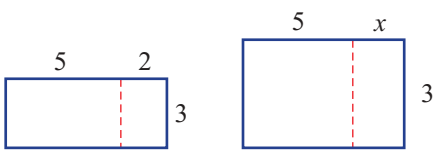
The expression $2(3 + 4)$ means $2 \times (3 + 4)$. It can be simplified in two ways:

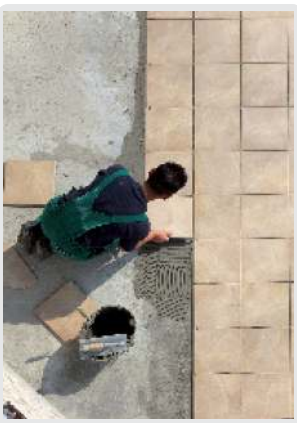
$$2 \times (3 + 4) = 2 \times 7 \quad \text{OR} \quad 2 \times (3 + 4) = 2 \times 3 + 2 \times 4$$
$$= 14 \qquad\qquad = 6 + 8$$
$$= 14$$

Lesson starter: Rectangular distributions

This rectangle has the dimensions as shown. Given that the area of a rectangle is length × breadth:

- Describe two different ways to find the area of the rectangle.
- Substitute x for 2 and rewrite the expressions.





Examples of the distributive law can often be found in everyday tasks.

KEY IDEAS

- Some expressions can be written in two ways.
  For example:
  $5 \times (3 + 4)$ can also be written $5 \times 3 + 5 \times 4$
  Similarly:
  $5 \times (x + 4)$ can also be written $5 \times x + 5 \times 4$ or $5x + 20$
- The **distributive law** is used to **expand** and remove brackets.
  - It states that adding numbers in brackets, *then* multiplying the total, gives the same answer as multiplying each number in the brackets separately first, *then* adding the products.

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■ To expand brackets:

- The term on the outside of the brackets is multiplied by each term inside the brackets.

$$a(b + c) = ab + ac \quad \text{or} \quad a(b - c) = ab - ac$$

For example:

$$\begin{aligned} 2(x + 4) &= 2 \times x + 2 \times 4 \\ &= 2x + 8 \end{aligned}$$

- If the number in front of the brackets is negative, the sign of each of the terms inside the brackets will change when expanded.
For example: $-2(x - 3) = -2x + 6$ because $-2 \times x = -2x$ and $-2 \times (-3) = +6$

Key terminology: distributive law, expand, like terms

Exercise 3D

UNDERSTANDING 1–4 | 4

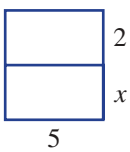
1 Are the following true (T) or false (F)?

a $a(b + c) = ab + c$ **b** $a(b + c) = abc$ **c** $a(b + c) = ab + ac$

2 Complete the following.

The total area of the rectangles can be written as:

$5 \times (\square + \square)$ or $5 \times \square + 5 \times \square$



3 Using the distributive law, is $x(y - z)$ equivalent to A, B, C or D?

A $xy - z$ **B** $xy + z$ **C** $xy - xz$ **D** $-xyz$

4 Fill in the boxes to complete the following.

a $3(x + 5) = \square \times x + \square \times 5$
$= 3x + \square$

b $-4(x + 2) = \square \times x + \square \times 2$
$= \square - \square$

FLUENCY 5–7(½) | 5–8(½)

Example 11 Expanding simple expressions with brackets

Expand the following.

a $3(x + 4)$ **b** $5(x - 11)$

SOLUTION

a $3(x + 4) = 3x + 12$

EXPLANATION

The 3 on the outside is multiplied by each term inside the brackets; i.e.

$3(x + 4) = 3 \times x + 3 \times 4$

b $5(x - 11) = 5x - 55$

$5 \times x = 5x$ and $5 \times (-11) = -55$

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Now you try

Expand the following.

a $4(x + 2)$ **b** $6(x - 10)$

5 Expand the following.

a $2(x + 3)$ **b** $5(x + 12)$
c $2(x + 7)$ **d** $7(x + 9)$
e $3(x - 7)$ **f** $6(x - 11)$
g $5(x - 9)$ **h** $10(x - 1)$
i $3(2 + x)$ **j** $7(3 + x)$
k $4(7 - x)$ **l** $5(8 - x)$

Hint: Remember that the number in front of the brackets is multiplied by each term inside the brackets when expanding:

$2(x + 3) = \ldots$

Example 12 Expanding brackets with a negative number in front

Expand the following.

a $-2(x + 5)$
b $-3(x - 4)$

SOLUTION

a $-2(x + 5) = -2x - 10$

b $-3(x - 4) = -3x + 12$

EXPLANATION

-2 is multiplied by each term inside the brackets.

$-2 \times x = -2x$ and $-2 \times 5 = -10$

Multiplying by a negative changes the sign of each term in the brackets.

$-3 \times x = -3x$ and $-3 \times (-4) = +12$

Now you try

Expand the following.

a $-3(x + 7)$
b $-5(x - 9)$

6 Expand the following.

a $-3(x + 2)$ **b** $-2(x + 11)$
c $-5(x - 3)$ **d** $-6(x - 6)$
e $-4(2 - x)$ **f** $-13(3 + x)$
g $-8(9 + x)$ **h** $-300(1 - x)$

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Example 13 Expanding brackets and simplifying

Expand the following.

a $4(x + 3y)$ **b** $2x(4x - 3)$

SOLUTION

a $4(x + 3y) = 4 \times x + 4 \times 3y$
$= 4x + 12y$

b $2x(4x - 3) = 2x \times 4x + 2x \times (-3)$
$= 8x^2 - 6x$

EXPLANATION

Multiply each term inside the brackets by 4.

$4 \times x = 4x$ and $4 \times 3 \times y = 12y$.

Each term inside the brackets is multiplied by $2x$.

$2x \times 4x = 2 \times 4 \times x \times x = 8x^2$ and
$2x \times (-3) = 2 \times (-3) \times x = -6x$.

Now you try

Expand the following.

a $5(a + 4b)$ **b** $3y(5y - 4)$

7 Expand the following.

a $2(a + b)$ **b** $5(a + 2b)$ **c** $3(2m + y)$
d $8(2x - 5)$ **e** $-3(4x + 5)$ **f** $4x(x - 2y)$
g $t(2t - 3)$ **h** $a(3a + 4)$ **i** $d(2d - 5)$
j $2b(3b - 5)$ **k** $2x(4x + 1)$ **l** $5y(1 - 3y)$

Hint: $x \times x$ is written as x^2.

Example 14 Simplifying by removing brackets

Expand the following and collect like terms.

a $4(x + 5) - 10$ **b** $2 - 3(x - 4)$

SOLUTION

a $4(x + 5) - 10 = 4x + 20 - 10$
$= 4x + 10$

b $2 - 3(x - 4) = 2 - (3x - 12)$
$= 2 - 3x + 12$
$= 14 - 3x$

EXPLANATION

Expand brackets first: $4(x + 5) = 4 \times x + 4 \times 5$.
Then collect like terms and simplify.

Expand brackets.

$3(x - 4) = 3x - 12$.

$-(3x - 12) = -1(3x - 12)$, so multiplying by negative 1 changes the sign of each term inside the brackets;

i.e. $-1 \times 3x = -3x$ and $-1 \times (-12) = +12$.

Now you try

Expand the following and collect like terms.

a $7(x + 3) - 15$ **b** $8 - 2(y - 5)$

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8 Expand the following and collect like terms.

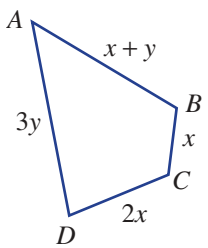
a $2(x+4)+3$	**b** $6(x+3)+4$	**c** $5(x+2)-4$
d $3(x+4)-2$	**e** $3+4(x-2)$	**f** $7+2(x-3)$
g $2-3(x-2)$	**h** $1-5(x-4)$	**i** $5-2(x+3)$
j $12-3(x+4)$	**k** $7-(x+4)$	**l** $4-(3x-2)$
m $6(2x-1)+5$	**n** $4(2x+7)-12$	**o** $2x(x-3)+2x$
p $m(2m-3)-5m$	**q** $12+3(3x-5)$	**r** $5x-6(x-1)$
s $2x(x-1)+x^2+5x$	**t** $2x+3(7x-2y)$	**u** $5x-(2x-7)-11$

Hint: Expand the brackets first then collect like terms.

PROBLEM-SOLVING AND REASONING

9–11 | 10–13

9 The diagram shows the route taken by a salesperson who travels from A to D via B and C.



a If the salesperson then returns directly to A, write an expression (in simplest form) for the total distance travelled.

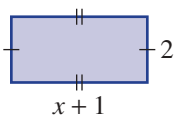
b If $y = x + 1$, write an expression for the total distance the salesperson travels in terms of x only. Simplify your expression.

c When $y = x + 1$, how much would the distance have been reduced by (in terms of x) if the salesperson had travelled directly from A to D and straight back to A?

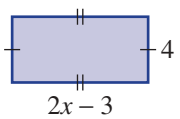
Hint: Subtract the distance from A to D and back again from the total distance.

10 Find the area of these basic shapes in expanded form. All angles are 90°.

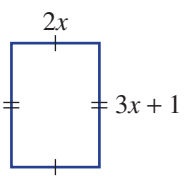
a



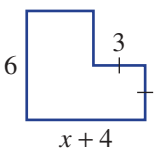
b



c



d



Hint: Remember to include brackets when multiplying length by breadth.

11 Liam has \$60 in savings. One Saturday, he does some odd jobs for a neighbour and earns \$$x$, which he adds to his savings. Liam's parents think he's doing a good job of saving money, so they decide to double his savings. Write an expanded expression for the amount of savings Liam has now.

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12 Identify the errors in these expressions then write out the correct expansion.

a $2(x + 6) = 2x + 6$

b $x(x - 4) = 2x - 4x$

c $-3(x + 4) = -3x + 12$

d $-7(x - 7) = -7x - 49$

e $5(x + 2) + 4 = 5x + 6$

f $5 - 2(x - 7) = 5 - 2x - 14$

$= -9 - 2x$

Hint: Watch out for the negative numbers.

13 Chloe pays tax at 20c in the dollar for every dollar earned over \$10 000. Chloe earns $\$x$ and $x > 10\,000$.

a Write an expression for the amount that Chloe pays tax on.

b Write an expression for the amount Chloe pays in tax. Expand your answer.

Hint: 20c in the dollar is 0.2 of each dollar. In \$10, this is $0.2 \times 10 = \$2$.

ENRICHMENT: Pairs of brackets — 14, 15

14 Expand each pair of brackets first, then collect like terms.

a $2(x + 3) + 3(x + 2)$

b $2(x + 4) + 2(x - 1)$

c $3(2x + 4) + 5(x - 1)$

d $4(3x + 2) + 5(x - 3)$

e $-2(x + 2) + 3(x - 1)$

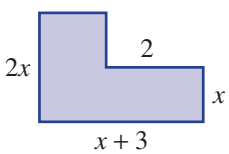
f $2(4x - 3) - 2(3x - 1)$

g $x(x + 2) + 2x(x + 4)$

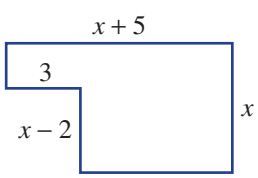
h $3(x - 1) + x(x + 2)$

15 Find the area of these shapes in expanded form. All angles are 90°.

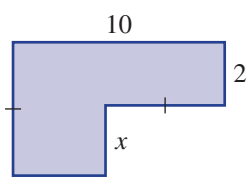
a



b



c



Hint: Split these shapes into rectangles and add or subtract areas.

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3E Linear equations with a pronumeral on one side CONSOLIDATING

Learning intentions for this section:

- To understand the difference between expressions and equations
- To understand that equivalent equations can be created by applying the same operation to both sides of the equation
- To review the steps for solving simple linear equations
- To be able to solve simple linear equations with a pronumeral on one side only

Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- Solving equations is an important component of Stage 6 Standard

An equation is a mathematical statement containing an equals sign, a left-hand side and a right-hand side.

$5 = 10 \div 2, 3x = 9, x^2 + 1 = 10$ and $\frac{1}{x} = 4$ are examples of equations.

Linear equations can be written in the form $ax + b = c$, where the power of x is 1.

$4x - 1 = 6, 3 = 2(x + 1)$ and $5x = 2x + 1$ are all linear equations.

Equations are solved by finding the value of the pronumeral that makes the equation true. This can be done by inspection for very simple linear equations (for example, if $3x = 15$ then $x = 5$, since $3 \times 5 = 15$). More complex linear equations can be solved through a series of steps where each step produces an equivalent equation.

Lesson starter: Is $x = 4$ a solution?

State which of the following equations has the solution $x = 4$.

$5x = 20$	$2x - 1 = 5$	$10 - x = 14$
$x + 9 = 13$	$3x + 2 = 14$	$-4x = 16$

- How did you find the equations for which $x = 4$ is a solution?
- The equations you have listed should be equivalent equations to $x = 4$. Starting from $x = 4$, describe what you would do to produce each equivalent equation; for example, 'add 1 to both sides'.

KEY IDEAS

- An **equation** is a mathematical statement containing an equals sign.
  For example: $3x + 1 = 7$.
- **Equivalent equations** are created by:
  - adding a number to or subtracting a number from both sides of the equation
  - multiplying or dividing both sides of the equation by the same number (not including 0)
  - swapping the left-hand side (LHS) with the right-hand side (RHS).
    For example: $13 = 2x + 1$ is equivalent to $2x + 1 = 13$

$$\begin{array}{l} 5x + 4 = 14 \quad (-4, -4) \\ 5x = 10 \end{array}$$

$$\begin{array}{l} 5x = 10 \quad (\div 5, \div 5) \\ x = 2 \end{array}$$

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- Linear equations can be **solved** by creating equivalent equations, then using opposite operations
- The **solution** to an equation can be checked by **substituting** the solution into the original equation and checking that both sides are equal.
  Equation: $5x + 4 = 14$
  Possible solution: $x = 2$

LHS	RHS
$= 5x + 4$	$= 14$
$= 5 \times 2 + 4$	
$= 14$	

  $\therefore x = 2$ *is* a solution.
- The pronumeral in an equation is sometimes called 'the unknown'.

Key terminology: equation, equivalent equation, solve, solution, substitution

Exercise 3E

UNDERSTANDING 1–4 3, 4

1 Which equations have the solution $x = 3$?

a $x + 5 = 9$ **b** $x - 1 = 2$ **c** $4 - x = 1$
d $1 - x = -4$ **e** $2x + 4 = 10$ **f** $3x - 2 = 7$
g $2 = 7 - 2x$ **h** $-5 = 4 - 3x$ **i** $1 = -x + 2$

Hint: If $x = 3$ is a solution, the left-hand side must equal the value that is on the right-hand side of the = sign.

2 What value of x makes these equations true?

a $x + 4 = 10$
b $x - 3 = 5$
c $2 = x + 5$
d $2x = 8$
e $20 = 5x$
f $-3x = 9$

Hint: What value plus 4 equals 10?

3 Obtain an equivalent equation by completing the step given in brackets.

a $2x + 5 = 11$ (subtract 5 from both sides)
b $3x - 4 = 17$ (add 4 to both sides)
c $4x = 36$ (divide both sides by 4)

Hint:

$$2x + 5 = 11 \quad (-5 \text{ on both sides})$$
$$2x = \ldots$$

4 State the single operation that has been performed to obtain these pairs of equivalent equations; for example, 'add 3 to both sides'.

a $x + 9 = 11$ / $x = 2$ **b** $2x - 1 = 5$ / $2x = 6$ **c** $3x = 27$ / $x = 9$

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FLUENCY 5(½), 6 5–7(½)

Example 15 Solving simple linear equations

Solve each of the following equations and check that the solution is correct.

a $2x + 5 = 9$ **b** $11 = 3x - 7$

SOLUTION

a $2x + 5 = 9$

$2x = 4$

$x = 2$

Check:

LHS $= 2x + 5$ $= 2 \times (2) + 5$ $= 4 + 5$ $= 9$ ✓	RHS $= 9$ ✓

b $11 = 3x - 7$

$3x - 7 = 11$

$3x = 18$

$x = 6$

Check:

LHS $= 11$ ✓	RHS $= 3x - 7$ $= 3 \times (6) - 7$ $= 18 - 7$ $= 11$ ✓

EXPLANATION

Subtract 5 from both sides to undo the +5.

Divide both sides by 2, since $2x = 2 \times x$ and $\frac{{}^{1}\cancel{2} \times x}{\cancel{2}^{1}} = x$.

Check the answer by substituting $x = 2$ into the original equation to check that the LHS = RHS.

Swap the LHS and RHS.

Add 7 to both sides to undo the −7, since $-7 + 7 = 0$.

$3x = 3 \times x$, so divide both sides by 3.

Check your solution by substituting $x = 6$.

Now you try

Solve each of the following equations.

a $3x + 8 = 23$ **b** $4x - 2 = 10$

5 Solve each of the following equations. Check your answers using substitution.

a $2x + 7 = 15$ **b** $5a + 6 = 11$

c $16 = 3m + 4$ **d** $2x + 4 = 12$

e $2n + 13 = 17$ **f** $-7 = 3x + 5$

g $5b - 3 = 12$ **h** $3y - 2 = 19$

i $20 = 7a - 8$ **j** $4b - 7 = 25$

k $2x - 4 = -6$ **l** $-15 = 10y - 35$

Hint: Think carefully about each step to make sure you do produce an equivalent equation.

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Example 16 Solving equations with negative coefficients

Solve the following equations and verify that the solutions are correct.

a $9 - 2x = 15$ **b** $7 - 4x = 10$

SOLUTION

a $9 - 2x = 15$

$-2x = 6$

$x = -3$

Check:

LHS $= 9 - 2x$	RHS $= 15$ ✓
$= 9 - 2 \times (-3)$	
$= 9 - (-6)$	
$= 15$ ✓	

EXPLANATION

Subtract 9 from both sides, leaving $-2x$ on the LHS.

Divide both sides by -2; recall that a positive number divided by a negative number is negative.

Check your solution using substitution.

b $7 - 4x = 10$

$-4x = 3$

$x = -\frac{3}{4}$

Subtract 7 from both sides.

Divide both sides by -4. Leave your answer in fraction form; i.e. $3 \div (-4) = -\frac{3}{4}$.

Check:

LHS $= 7 - 4x$	RHS $= 10$ ✓
$= 7 - 4 \times -\frac{3}{4}$	
$= 7 - (-3)$	
$= 10$ ✓	

Substitute $x = -\frac{3}{4}$ into LHS and RHS.

Now you try

Solve the following equations.

a $10 - 3x = 16$ **b** $11 - 5x = 15$

6 Solve each of the following equations.

a $12 - 2x = 18$ **b** $2 - 7x = 9$

c $15 - 5x = 25$ **d** $19 = 3 - 2x$

e $2 - 5x = -8$ **f** $24 - 7x = 10$

g $14 - 8x = -2$ **h** $-21 = 3 - 4x$

7 Solve these equations. Leave your answer in fraction form.

a $2x + 5 = 8$ **b** $5x - 1 = 2$ **c** $7 = 11x - 3$

d $6 - 2x = 5$ **e** $4 - 3x = 5$ **f** $4 = 8 - 5x$

Hint: If $5x = 3$, $x = \frac{3}{5}$ (divide both sides by 5). The answer can be left as a fraction.

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PROBLEM-SOLVING AND REASONING

8, 9	8(½), 9, 10

8 For each of the following, write an equation and solve it to find the unknown value. Use x as the unknown value.

Hint: Form the algebraic expression (e.g. $x + 8$) and set it equal to the result to make the equation.

- **a** If 8 is added to a certain number, the result is 34.
- **b** Seven less than a certain number is 21.
- **c** I think of a number, double it and add 4. The result is 10.
- **d** I think of a number, triple it and subtract 4. The result is 11.
- **e** Four less than three times a number is 20.
- **f** I start with the number 25 and subtract twice a certain number. The result is 7.

9 Describe the error made in each of these incorrect solutions.

a
$$3x + 2 = 8$$
$$3x = 10$$
$$x = \frac{10}{3}$$

b
$$2x - 1 = 5$$
$$2x = 6$$
$$x = 12$$

c
$$5 - x = 12$$
$$x = 7$$

d
$$2x + 3 = 8$$
$$x + 3 = 4$$
$$x = 1$$

10 Solve these equations.

a $-3 + 4x = 17$	**b** $-5 - 2x = 13$	**c** $-10 - 3x = -4$
d $12 - 4x = 0$	**e** $14 + 2x = -3$	**f** $12 = 8m - 4$
g $-10 = 2 - 4x$	**h** $22 = 6x - 2$	**i** $-3x - 9 = 0$
j $-5m - 9 = 1$	**k** $3m - 7 = 1$	**l** $-9 - 7x = -1$
m $6h - 5 = -7$	**n** $7 - 3w = 4$	**o** $-15 - 9d = 0$

ENRICHMENT: More than one solution

—	11, 12

11 These equations have exactly two solutions. Can you guess both?

Hint: They are whole numbers between −5 and 5.

- **a** $x^2 = 9$
- **b** $x^2 = x$
- **c** $x^2 = 2x$

12 Find three solutions for the equation $x^3 - x = 0$.

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3F Linear equations involving fractions

Learning intentions for this section:

- To know how to write pronumerals with fractional coefficients in fraction form
- To know the order in which steps need to be applied to solve equations involving fractions
- To be able to solve linear equations involving fractions

Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- Solving equations is an important component of Stage 6 Standard

We saw in the previous section that to solve an equation such as $2x = 8$, we divide each side of the equation by 2.

What happens if the equation involves a fraction? For the equation $\frac{1}{2}x = 2$, we could similarly divide both sides by $\frac{1}{2}$ to solve it. But we can also write the equation in a different way, to make it easier to solve.

A half of x is equivalent to dividing x by 2, so $\frac{1}{2}x = 5$ can be written as $\frac{x}{2} = 5$.

To solve this new equation, we can multiply both sides by 2, since $2 \times \frac{x}{2} = x$. This gives us $x = 10$.

From this process, we can see that we could solve this equation by either dividing both sides by $\frac{1}{2}$ *or* multiplying both sides by 2.
For more complex equations we'll also need to consider the order of operations.

Lesson starter: Fractions of *x*

Consider these problems involving fractions.

1. If half of x is 8, what is x?
2. If a third of x is 5, what is x?
3. If three-quarters of x is 30, what is x?
   - Discuss the methods you used to find the value of x.
   - Write an equation to represent each case.
   - Discuss how your method for finding the answer relates to the fraction in the equation.

KEY IDEAS

- Pronumerals with a fraction coefficient can be expressed in fraction form.
  For example: 'One-third of a number' can be written as $x \div 3$ or $\frac{1}{3}x$ or $\frac{x}{3}$.
- Equations involving fractions can be solved by creating a series of simpler equivalent equations.
- At some stage, both sides of the equation can be multiplied by the number in the denominator of the fraction. The order of when this happens is important.

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For example

$\frac{x}{5} + 3 = 7$

-3 -3

$\frac{x}{5} = 4$

$\times 5$ $\times 5$

$x = 20$

$\frac{x+3}{5} = 7$

$\times 5$ $\times 5$

$x + 3 = 35$

-3 -3

$x = 32$

- It is always a good idea to check your solution using substitution.
  For example:

  Equation: $\frac{x}{5} + 3 = 7$

  Possible solution: $x = 20$

LHS	RHS
$= \frac{20}{5} + 3$	
$= 7$ ✓	

Key terminology: coefficient, equation, equivalent equation

Exercise 3F

UNDERSTANDING 1–4 4

1 Fill in the boxes.

a 'One-fifth of x' can be written as $x \div \square$ or $\frac{1}{\square} x$ or $\frac{\square}{5}$

b 'Three-quarters of x' can be written as $\frac{\square}{4} x$ or $\frac{3x}{\square}$

2 Write down the value of x that is the solution to these equations. No written working is required.

a $\frac{x}{2} = 10$ **b** $\frac{x}{4} = 6$ **c** $\frac{x+1}{3} = 4$ **d** $\frac{x}{5} - 2 = 1$

3 Which one of the following (A, B, C or D) is an equivalent equation to $\frac{x}{4} + 1 = 8$?

A $x + 1 = 32$ **B** $x = 36$ **C** $\frac{x}{4} = 7$ **D** $\frac{x+1}{4} = 2$

4 Match each equation in the first column with its equivalent equation in the second column and state the single operation that generates the equivalent equation (for example, multiply both sides by 5).

a $\frac{x}{3} - 1 = 2$ **i** $x + 2 = 3$

b $\frac{x}{4} = 2$ **ii** $x - 1 = 6$

c $\frac{x+2}{3} = 1$ **iii** $x = 8$

d $\frac{x-1}{2} = 3$ **iv** $\frac{x}{3} = 3$

Hint: Think about what step you would apply first to produce a simple equivalent equation.

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FLUENCY 5–6(½) 5–7(½)

Example 17 Solving linear equations with fractional coefficients

Solve each of the following equations and verify that they are correct.

a $\frac{2x}{3} = 8$

b $\frac{x}{4} - 3 = 7$

SOLUTION

a $\frac{2x}{3} = 8$

$2x = 24$

$x = 12$

Check:

$\text{LHS} = \frac{2x}{3}$	$\text{RHS} = 8$ ✓
$= \frac{2 \times (12)}{3}$	
$= \frac{24}{3}$	
$= 8$ ✓	

EXPLANATION

Multiply both sides by 3 to remove the fraction on the left-hand side. Then divide both sides by 2.

Check that $x = 12$ makes LHS = RHS.

b $\frac{x}{4} - 3 = 7$

$\frac{x}{4} = 10$

$x = 40$

Add 3 to both sides first to undo the -3.

Multiply both sides by 4.

Check the answer by substituting $x = 40$ into $\frac{x}{4} - 3$.

Check:

$\text{LHS} = \frac{x}{4} - 3$	$\text{RHS} = 7$ ✓
$= \frac{40}{4} - 3$	
$= 10 - 3$	
$= 7$ ✓	

Since this equals 7, $x = 40$ is the solution.

Now you try

Solve each of the following equations.

a $\frac{3x}{4} = 6$

b $\frac{x}{2} - 5 = 10$

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5 Solve each of the following equations.

a $\frac{x}{5} = 3$ **b** $\frac{y}{2} = 7$ **c** $\frac{2y}{5} = 4$

d $6 = \frac{3x}{2}$ **e** $\frac{x}{4} + 3 = 5$ **f** $\frac{x}{2} + 4 = 5$

g $\frac{b}{3} + 5 = 9$ **h** $2 = \frac{t}{2} + 5$ **i** $\frac{a}{3} + 4 = 2$

j $\frac{y}{5} - 4 = 2$ **k** $\frac{x}{3} - 7 = -2$ **l** $-1 = \frac{s}{2} - 3$

m $-2 = \frac{x}{3} + 5$ **n** $\frac{m}{4} - 2 = -3$ **o** $\frac{y}{5} + 4 = -3$

Hint: In part **d**, swap LHS and RHS.

Example 18 Solving simple fractional equations

Solve the equation $\frac{x+4}{6} = 2$ and check your solution.

SOLUTION

$\frac{x+4}{6} = 2$

$x + 4 = 12$

$x = 8$

Check:

LHS $= \frac{x+4}{6}$	RHS $= 2$ ✓
$= \frac{(8)+4}{6}$	
$= \frac{12}{6}$	
$= 2$ ✓	

EXPLANATION

Multiply both sides by 6 first, since all of $(x + 4)$ is divided by 6.

Subtract 4 from both sides and check the answer.

Check your solution using substitution.

Now you try

Solve the equation $\frac{x-5}{3} = 4$.

6 Solve each of the following equations. Check your answers.

a $\frac{x+1}{3} = 4$ **b** $\frac{x+4}{2} = 5$ **c** $\frac{y+4}{3} = 2$

d $\frac{b+6}{2} = 3$ **e** $\frac{y-2}{3} = 6$ **f** $\frac{t-4}{5} = 7$ **g** $\frac{k-1}{7} = 8$

h $\frac{x-7}{9} = 7$ **i** $\frac{x+2}{4} = -2$ **j** $\frac{b+3}{7} = -3$ **k** $\frac{y-3}{5} = -5$

Hint: Think carefully about the order of the steps here.

7 Solve this mixed selection of equations.

a $2t + 1 = 7$ **b** $\frac{m}{4} - 2 = 8$ **c** $5 - 2y = 9$

d $\frac{x-4}{5} = 3$ **e** $3 + \frac{y}{6} = 2$ **f** $4t - 5 = -8$

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PROBLEM-SOLVING AND REASONING

8, 9 | 8(½), 9, 10, 11(½)

8 For each of the following, write an equation and solve it to find the unknown value. Use x as the unknown value.

a If a certain number is divided by 3, the result is 12.

b A certain number is doubled, then divided by 5. The result is 4.

c I think of a number, halve it and subtract 4. The result is 10.

d I think of a number, add 3 and divide this by 4. The result is 6.

e A number is multiplied by 7 and the product is divided by 3. The final result is 8.

f 5 is added to a third of a certain number. The result is 16.

Hint: x halved is $\frac{x}{2}$.

A third of x is $\frac{x}{3}$.

9 A bag of chocolate eggs containing x eggs is shared equally between India and her two brothers. After India eats 2, she has 5 left. How many eggs were in the bag?

Hint: Start with an expression for the number of eggs India first receives.

10 Describe the error made in each of these incorrect solutions. Then write out the correct solution.

a $\frac{x+2}{3} = 7$

$\frac{x}{3} = 5$

$x = 15$

b $\frac{x}{3} - 4 = 2$

$x - 4 = 6$

$x = 10$

11 Solve these equations involving more than two steps.

a $\frac{2x}{3} - 1 = 7$ **b** $\frac{3x}{4} - 2 = 7$ **c** $\frac{x}{2} + 3 = 1$

d $\frac{3b-8}{2} = 5$ **e** $\frac{2x+2}{3} = 4$ **f** $\frac{7m-8}{3} = 9$

g $\frac{5-y}{3} = 2$ **h** $\frac{4-2t}{6} = 3$ **i** $1 - \frac{4x}{3} = 5$

Hint: Keep operating on both sides of the equation until you find the solution.

ENRICHMENT: Fractional coefficients

— | 12, 13

12 Solve these equations involving fractional coefficients.

a $\frac{2}{3}x = 6$ **b** $\frac{5}{6}x = 30$ **c** $-\frac{2}{7}x = 2$

d $\frac{3}{4}x - 1 = 5$ **e** $\frac{7}{5}x - 3 = 4$ **f** $1 - \frac{3}{5}x = 7$

Hint: $\frac{2}{3}x$ can be written as $\frac{2x}{3}$.

13 If $\frac{7}{5}x = 14$, does it matter whether you multiply by 5 first or divide by 7 first? Which method would be best here?

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Progress quiz

3A **1** Consider the expression $4x - 5xy - 3y + 8$.

a How many terms are in the expression?

b What is the coefficient of y?

c What is the constant term?

3A **2** Write algebraic expressions for the following.

a The cost of 3 drinks at $\$d$ each

b \$500 winnings shared equally between x people

c Four less than three lots of y

d The sum of m and n doubled

3A **3** Evaluate these expressions if $x = 3$, $y = -4$ and $z = 7$.

a $2y + z$ b $xy - y$ c $y^2 + x - z$

3B **4** Simplify the following by collecting like terms.

a $5x + 7x$ b $8b - b$

c $12a + 5 - 3a$ d $6xy - 4x + 2xy$

e $4a + 7b - a + 3b$ f $5mn + 2n + mn - 5n$

g $4xy^2 - 5xy - 2xy^2 + 3yx$ h $3a^2b + 2ab^2 - a^2b + 5ab^2$

3C **5** Simplify the following.

a $6 \times 4b$ b $3a \times 8c$

c $-5x \times 4y$ d $7ab \times 4b$

e $\dfrac{3x}{12}$ f $\dfrac{6xy}{8x}$

g $\dfrac{9x^2y}{6x}$ h $6x \times 4y \div (3y)$

3D **6** Expand the following.

a $3(x + 5)$ b $4(y - 1)$

c $-2(x + 5)$ d $-5(a - 2)$

e $6(2x - 5y)$ f $3m(4m + n)$

3D **7** Expand the following and collect like terms.

a $3(2x - 5) + 7$ b $8 - 4(x + 3)$ c $1 - (2x - 1)$

3E **8** Solve the following equations.

a $3x + 2 = 11$ b $5x - 3 = 17$

c $8 - 2x = 26$ d $12 - 5x = 15$

3F **9** Solve the following equations involving fractions.

a $\dfrac{x}{7} = 4$ b $\dfrac{x}{3} + 6 = 22$

c $\dfrac{x - 7}{2} = 10$ d $\dfrac{x + 1}{3} = -7$

3E/3F **10** Write an equation to find the unknown number x in the following and solve it.

a Five more than three times the number is 32.

b Three less than the number is divided by 4 to give 5.

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3G Linear equations involving brackets

Learning intentions for this section:

- To know that equations involving brackets can be solved by first expanding the brackets
- To know that some equations can be simplified by dividing both sides by a common factor
- To be able to solve equations involving brackets

Past, present and future learning:

- Equations involving brackets is a Stage 5 Core concept, introduced in our Year 8 book
- Some of these questions are more challenging than those in the Year 8 book
- Solving equations is an important component of Stage 6 Standard

We know that $3(x + 2)$ is the same as $3x + 6$. Likewise the equation $3(x + 2) = 10$ has the same solutions as $3x + 6 = 10$.

Lesson starter: To expand or not to expand?

The steps to solve two problems involving brackets are listed here in the incorrect order.

Method 1

$3(x + 4) = 14$

$x = \frac{2}{3}$

$3x + 12 = 14$

$3x = 2$

Method 2

$3(x + 2) = 12$

$x = 2$

$x + 2 = 4$

- Arrange the steps in the correct order, working from top to bottom.
- By considering all the steps in the correct order, explain what has happened in each step.
- Describe the two different approaches to solving the problems.
- Can you see why these methods have been used in each case?
- Apply Method 1 to the second equation.

KEY IDEAS

- Equations with brackets can be solved by first expanding the brackets and then solving the remaining equation.
  For example:
  $3(x + 1) = 2$ becomes $3x + 3 = 2$.

- You can check if a solution is correct by substituting the solution.
  For example:
  Equation: $2(x + 3) = 18$
  Possible solution: $x = 5$

  Check:

LHS	RHS
$= 2(x + 3)$	$= 18$ ✗
$= 2 \times (5 + 3)$	
$= 2 \times 8$	
$= 16$ ✗	

  $\therefore x = 5$ is *not* a solution because LHS $\neq$ RHS.

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- Sometimes it is wise to swap the LHS and RHS.
  For example:
  $12 = 3(x + 1)$ becomes $3(x + 1) = 12$.
- An alternative method is to deal with the common factor first.
  For example: $5(x - 1) = 15$ becomes $x - 1 = 3$ by first dividing both sides by 5.
- For equations like $2(x + 4) + 3x = 28$ always expand the brackets first, then collect any like terms on the LHS and continue to solve as usual.

Key terminology: expand, equation

Exercise 3G

UNDERSTANDING 1–3 | 3

1 Complete the missing parts to produce an equation without brackets.

a
$$3(x + 7) = 25$$
$$3 \times \square + \square \times 7 = 25$$
$$3x + \square = 25$$

b
$$4(2x - 1) = 15$$
$$\square \times 2x + 4 \times \square = 15$$
$$\square - 4 = 15$$

2 Which equations have $x = 2$ as a solution?

a $3(x + 1) = 3$ **b** $3(x + 1) = 6$ **c** $3(x + 1) = 9$
d $5(x + 6) = 40$ **e** $4(x - 3) = 4$ **f** $2(x - 2) = 2$

3 Which equations are showing a correct solution?

a $2(x + 3) = 20$, $x = 7$ **b** $4(x + 3) = 20$, $x = 2$ **c** $4(x - 3) = 20$, $x = 7$

FLUENCY 4–5(½) | 4–6(½)

Example 19 Solving equations with brackets

Solve $2(3x - 4) = 11$.

SOLUTION

$2(3x - 4) = 11$
$6x - 8 = 11$
$6x = 19$
$x = \frac{19}{6}$ or $3\frac{1}{6}$

EXPLANATION

Expand the brackets:
$2(3x - 4) = 2 \times 3x + 2 \times (-4)$

Solve the remaining equation by adding 8 to both sides, then dividing both sides by 6.

Leave your answer in fraction form.

Now you try

Solve $3(2x - 5) = 7$.

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4 Solve each of the following equations by first expanding the brackets.

a $2(x + 3) = 11$
b $5(a + 3) = 18$
c $3(m + 4) = 31$
d $5(y - 7) = 12$
e $4(p - 5) = 15$
f $2(k - 5) = 9$
g $4(5 - b) = 21$
h $13 = 2(1 - m)$
i $5(3 - x) = 19$
j $8 = 7(2a + 1)$
k $30 = 4(3x - 2)$
l $0 = 3(3n - 2)$
m $5(3 - 2x) = 16$
n $8 = 6(1 - 2y)$

Hint: Once you have expanded the brackets, the equation can be solved in the usual way.

Example 20 Solving equations with brackets and collecting like terms

Solve $6(x + 3) - 4x = 32$.

SOLUTION

$6(x + 3) - 4x = 32$
$6x + 18 - 4x = 32$
$2x + 18 = 32$
$2x = 14$
$x = 7$

EXPLANATION

Expand the brackets first, then collect any like terms: $6x - 4x = 2x$. Solve the remaining equation by subtracting 18 from both sides, and then divide both sides by 2.

Now you try

Solve $5(x + 4) - 2x = 35$.

5 Expand and simplify, then solve each of the following equations.

a $2(x + 4) + 2x = 12$
b $2(x - 3) + 3x = 4$
c $6(x + 3) - 2x = 26$
d $5(x + 2) - 2x = 46$
e $3(2x - 3) + x = 12$
f $4(3x + 1) - 3x = 19$
g $2(3x + 5) - 8x = 20$
h $4(1 - x) - x = 9$

Hint: Expand the brackets and then combine like terms before solving.

6 Solve these equations.

a $4(x - 1) + x - 1 = 0$
b $3(x + 2) - 2 + 4x = 18$
c $3(2x + 3) - 1 - 4x = 10$
d $2(4x - 1) - 5x + 6 = 31$
e $2(4x + 2) - 10x - 6 = 6$
f $5(2x + 3) - 15x - 7 = 3$

PROBLEM-SOLVING AND REASONING

7, 8 | 7(½), 9, 10

7 Using x for the unknown number, write down an equation then solve it to find the number. The first one is done for you.

a Three times 1 more than a number is 4. ($3(x + 1) = 4$; $x = \frac{1}{3}$)
b Twice 2 less than a number is 19.
c The product of 2 and 3 more than a number is 7.
d The product of 3 and 4 less than a number is 8.
e When 2 less than 3 lots of a number is doubled, the result is 5.
f When 5 more than 2 lots of a number is tripled, the result is 10.

Hint: You will need to use brackets when setting up the expressions in these equations.

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8 Since Tara started her job, her original hourly wage ($\$x$) has been tripled, then decreased by \$16. Her pay is now to be doubled so that she earns \$58 an hour. What was her original hourly wage?

Hint: Tripling Tara's pay of $\$x$ is $3 \times x = 3x$.

9 Consider the equation $3(x - 2) = 9$.

- **a** Solve the equation by first dividing both sides by 3.
- **b** Solve the equation by first expanding the brackets.
- **c** Which of the above two methods is preferable and why?

Hint: You may recall this idea from the 'Lesson starter' activity at the beginning of the section.

10 Consider the equation $3(x - 2) = 7$.

- **a** Solve the equation by first dividing both sides by 3.
- **b** Solve the equation by first expanding the brackets.
- **c** Which of the above two methods is better and why?

ENRICHMENT: Brackets or no brackets? — 11

11 Decide if brackets need to be inserted into the following equations to make $x = 4$ the solution. If so, insert the brackets where required.

- **a** $3x - 2 = 10$
- **b** $6x - 12 = 12$
- **c** $4x + 1 = 20$
- **d** $2x - 4 + 3x + 1 = 13$
- **e** $3x + 2 + 2x - 5 = 12$
- **f** $2x - 3 - 3x - 1 = -4$

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3H Equations with pronumerals on both sides

Learning intentions for this section:

- To be able to collect pronumerals on one side of the equation
- To be able to solve equations with pronumerals on both sides of the equation

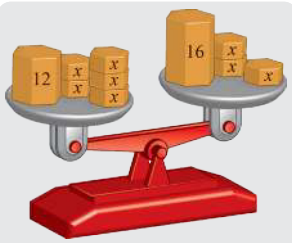
Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- Solving equations is an important component of Stage 6 Standard

Some linear equations have pronumerals on both sides of the equation.

In the diagram:

- 12 could be subtracted from both sides.
- $3x$ could be subtracted from both sides.



The equation has numerals on both sides and pronumerals on both sides.

Lesson starter: Which method? Same solution?

There are two methods to solve $5x + 12 = 3x + 16$.

Method 1

Start by subtracting 3x from both sides

$5x + 12 = 3x + 16$

$-3x$: $\square + 12 = 16$; $-3x$

-12: $\square = \square$; -12

$\div 2$: $x = \square$; $\div 2$

Method 2

Start by subtracting 12 from both sides

$5x + 12 = 3x + 16$

-12: $5x = 3x + \square$; -12

$-3x$: $\square = \square$; $-3x$

$\div 2$: $x = \square$; $\div 2$

Are both of these correct? Check by substitution.

KEY IDEAS

- If an equation has pronumerals and numbers on both sides, add or subtract one of the terms. For example:

$-2x$: $6x + 4 = 2x - 3$; $-2x$: $4x + 4 = -3$ OR -4: $6x + 4 = 2x - 3$; -4: $6x = 2x - 7$

- At any time, the LHS (left-hand side) and RHS (right-hand side) can be swapped. For example:

$2x + 3 = 5x - 2$

$5x - 2 = 2x + 3$

Solving is easier if the larger coefficient is on the LHS.

Key terminology: pronumeral, expand, equation, coefficient

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Exercise 3H

UNDERSTANDING 1–3 | 3

1 State which side (left or right) you would collect the x terms on to keep the x coefficient positive.

a $5x = 2x + 3$ **b** $6x + 1 = 2x + 5$

c $3x - 5 = 7x + 3$ **d** $1 - 2x = 3x - 9$

Hint: The x coefficient is the numeral multiplied by the x.

2 Copy and complete the following.

a

$7x = 3x + 4$

$-3x$ $-3x$

$\square = 4$

$\div 4$ $\div 4$

$x = \square$

b

$9x + 2 = 5x + 10$

$-5x$ $-5x$

$\square + 2 = 10$

-2 -2

$\div \square$ $\div \square$

$\square = \square$

$x =$

3 Copy and complete the following.

a

$4x + 2 = 8 - 2x$

$+2x$ $+2x$

$\square + 2 = 8$

-2 -2

$\div \square$ $\div \square$

$\square = \square$

$x = \square$

b

$4x + 2 = 8 - 2x$

-2 -2

$\square = \square - 2x$

$+2x$ $+2x$

$\div \square$ $\div \square$

$\square = \square$

$x = \square$

FLUENCY 4–7(½) | 4–7(½)

Example 21 Solving equations with pronumerals on both sides

Solve these equations with pronumerals on each side.

a $4x = 2x + 10$ **b** $3x = 16 - 5x$

SOLUTION

a $4x = 2x + 10$

$2x = 10$

$x = 5$

Check:

LHS = $4x$	RHS = $2x + 10$
$= 4 \times (5)$	$= 2 \times (5) + 10$
$= 20$ ✓	$= 20$ ✓

b $3x = 16 - 5x$

$8x = 16$

$x = 2$

Check:

LHS = $3x$	RHS = $16 - 5x$
$= 3 \times (2)$	$= 16 - 5 \times (2)$
$= 6$ ✓	$= 16 - 10$
	$= 6$ ✓

EXPLANATION

Collect x terms on LHS (since the x coefficient is larger on that side) by subtracting $2x$ from both sides: $4x - 2x = 2x$.

Solve by dividing both sides by 2 and check that $x = 5$ gives the same value for $4x$ and $2x + 10$.

Check your solution.

Collect x terms by adding $5x$ to both sides (collect on LHS since $3x$ coefficient is positive). Divide both sides by 8 and check the solution.

Check your solution.

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Now you try

Solve these equations with pronumerals on each side.

a $8x = 5x + 12$ **b** $2x = 18 - 4x$

4 Solve these equations by first collecting the pronumeral on one side.

a $7x = 5x + 8$ **b** $8x = 3x + 15$
c $9x = 5x + 12$ **d** $10x = 9x + 7$
e $4x = x - 9$ **f** $6x = 2x - 8$
g $3x = 10 - 2x$ **h** $x = 12 - 2x$
i $x = 12 - x$ **j** $4x = 14 - 3x$

Hint: Remember: It is easier to collect on the side that will keep the x coefficient positive.

Example 22 Solving equations with pronumerals and numerals on both sides

Solve each of the following equations.

a $5x + 2 = 3x + 6$ **b** $3 - 2x = 5x - 4$

SOLUTION

a $5x + 2 = 3x + 6$
$2x + 2 = 6$
$2x = 4$
$x = 2$

Check:

LHS	RHS
$= 5 \times (2) + 2$	$= 3 \times (2) + 6$
$= 12$ ✓	$= 12$ ✓

b $3 - 2x = 5x - 4$
$3 = 7x - 4$
$7 = 7x$
$1 = x$
$\therefore x = 1$

Check:

LHS	RHS
$= 3 - 2 \times (1)$	$= 5 \times (1) - 4$
$= 1$ ✓	$= 1$ ✓

Alternative method:
$3 - 2x = 5x - 4$
$5x - 4 = 3 - 2x$
$7x - 4 = 3$
$7x = 7$
$x = 1$

EXPLANATION

Collect x terms on one side by subtracting $3x$ from both sides: $5x - 3x = 2x$. Collect on the side with the larger x term to keep it positive. Solve the remaining equation.

Check by substituting $x = 2$ into $5x + 2$ and $3x + 6$ to see that it gives the same result for both.

Add $2x$ to both sides to collect x terms on the side with the positive x coefficient. Solve the remaining equation by adding 4 to both sides and dividing both sides by 7.

Rewrite with x on the left.

Check, using substitution.

Swap LHS and RHS.

Add $2x$ to both sides.

Add 4 to both sides.

Divide both sides by 7.

Don't forget to check your solution using substitution.

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Now you try

Solve each of the following equations.

a $7x + 3 = 4x + 15$

b $8 - x = 4x - 2$

5 Solve each of the following equations.

a $5x - 3 = 4x + 5$ **b** $9a + 3 = 8a + 6$

c $3m - 8 = 2m$ **d** $12x - 3 = 10x + 5$

e $8x + 4 = 12x - 16$ **f** $3x + 7 = 8x - 8$

g $2x + 6 = 9 - x$ **h** $3y + 6 = 14 - y$

i $5m - 18 = 15 - 6m$ **j** $4 - 7x = 2x - 23$

k $8 - 2b = 4b + 14$ **l** $3 - 4m = 3m + 24$

Hint: Collect the pronumerals on one side first.

Example 23 Solving equations with brackets and pronumerals on both sides

Solve these equations involving brackets.

a $7(x + 2) = 3x + 2$ **b** $3(2x + 4) = 8(x + 1)$

SOLUTION

a $7(x + 2) = 3x + 2$

$7x + 14 = 3x + 2$

$4x + 14 = 2$

$4x = -12$

$x = -3$

EXPLANATION

Expand the brackets first, then subtract $3x$ from both sides to collect x terms.

Subtract 14 from both sides ($2 - 14 = -12$) and divide by 4. Recall that a negative number divided by a positive number is negative ($-12 \div 4 = -3$).

b $3(2x + 4) = 8(x + 1)$

$6x + 12 = 8x + 8$

$12 = 2x + 8$

$4 = 2x$

$2 = x$

$\therefore x = 2$

Expand the brackets on each side.

Subtract $6x$ from both sides. (Subtracting $6x$ keeps the x-coefficient positive, as alternatively subtracting $8x$ would end up with $-2x + 12 = 8$.)

Solve the equation and make x the subject.

Now you try

Solve these equations involving brackets.

a $5(x + 3) = 3x + 7$ **b** $4(2x - 1) = 6(x + 2)$

6 Solve each of the following equations.

a $5(x - 2) = 2x + 11$ **b** $3(a + 1) = a + 13$

c $3(y + 4) = y + 16$ **d** $2(x + 5) = x - 4$

e $5b - 4 = 6(b + 2)$ **f** $2(4m - 5) = 4m + 2$

g $3(2a - 3) = 5(a + 2)$ **h** $4(x - 3) = 3(3x + 1)$

i $3(x - 2) = 5(x + 4)$ **j** $3(n - 2) = 4(n - 5)$

k $2(a + 5) = 2(2a + 3)$ **l** $4(x + 2) = 3(2x + 1)$

Hint: Expand the brackets before collecting the pronumerals on one side.

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7 By substituting into the left-hand side and the right-hand side of the equation, decide whether it is true (T) or false (F) that the given value of x is the correct solution of the equation.

a $7x - 3 = x + 9$ $[x = 2]$
b $6 - x = x + 5$ $[x = 1]$
c $3x - 5 = x + 4$ $[x = 9]$
d $2 - 3x = 5 - x$ $[x = 5]$
e $10x + 15 = x - 9$ $[x = -2]$
f $5 - 3x = x + 9$ $[x = -1]$
g $9 - 5x = 1 + 3x$ $[x = 1]$
h $5x = 3x - 8$ $[x = 2]$

PROBLEM-SOLVING AND REASONING

8, 9 | 8(½), 9, 10

8 Using x for the unknown number, write down an equation then solve it to find the number. The first one has been done for you as an example.

a Twice a number is equal to 4 less than 3 times a number. ($2x = 3x - 4$; $x = 4$)
b 4 more than 2 lots of a number is equal to 5 times the number.
c 10 more than 3 lots of a number is equivalent to 5 lots of the number.
d 2 more than 3 times the number is equivalent to 6 less than 5 times the number.
e 1 less than a doubled number is equivalent to 5 more than 3 lots of the number.
f 4 more than 2 lots of a number is equivalent to the number subtracted from 13.

9 Mardy is x years old. He correctly tells his mother that she is 6 years older than 3 times his age. His mother replies that she is also 10 years younger than 5 times his age. How old is Mardy?

Hint: Build the equation step by step. You should get two expressions for Mardy's mother's age.

10 Consider the equation $3x + 1 = 5x - 7$.

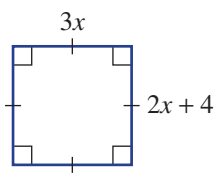
a Solve the equation by first subtracting $3x$ from both sides.
b Solve the equation by first subtracting $5x$ from both sides.
c Which method above do you prefer and why? Describe the differences.

ENRICHMENT: Matching sides

— | 11

11 Use the properties of these basic shapes to find their perimeters.

a



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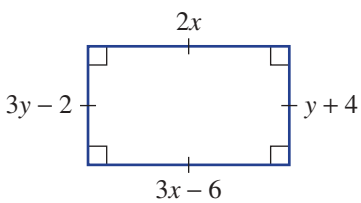

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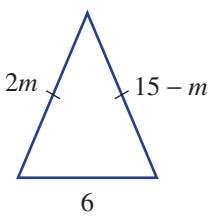

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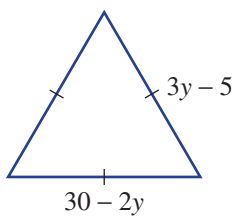

Hint: Find the value of each pronumeral first by setting up and solving an equation.

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3I Using linear equations to solve problems

Learning intentions for this section:

- To be able to define a variable to represent the unknown in a worded problem
- To be able to write an equation to represent a worded problem
- To be able to solve the problem by solving the equation

Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- This concept is further developed in Stage 6 Standard

Many types of problems can be solved by writing and solving linear equations. Often problems are expressed only in words. Reading and understanding the problem, defining a pronumeral and writing an equation become important steps in solving the problem.

Lesson starter: Two pieces of rope

A 30-metre rope breaks into two pieces. One is 8 metres longer than the other. How long are the pieces?

- In how many different ways can your classmates solve this problem?
- Can you solve it using an equation?

KEY IDEAS

To solve a problem, follow these steps:

1	Define pronumerals to stand for unknown numbers.	1	Let x = Julian's age.
2	Write an equation to describe the situation.	2	$x + x + 4 = 32$
3	Solve the equation by inspection or systematically.	3	$2x + 4 = 32$ -4, -4 $2x = 28$ $\div 2$, $\div 2$ $x = 14$
4	Make sure you answer the original question, including the correct units. (e.g. dollars, years, cm).	4	Julian is 14 years old and his sister is 18.

Key terminology: pronumeral, variable, equation

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Exercise 3I

UNDERSTANDING	1–3	3

1 Write an equation for the following scenario.
The mass of a bag is x kg.
Another bag weighs twice as much.
Together they weigh 30 kg.

2 Match the written scenario with its possible equation.

a	7 is 3 more than a number	**i**	$3x + 1 = 16$
b	2 less than a number is 10	**ii**	$\frac{a}{2} = 3$
c	16 is 1 more than 3 lots of a number	**iii**	$n + 3 = 7$
d	Half of a number is 3	**iv**	$y - 2 = 10$

3 The sum of the ages of Sam and his brother Bernard is 34. If Sam is 4 years older than Bernard, fill in the following to find their ages.

Let x be the ☐ of Bernard.
The age of Sam is ☐.
The sum of their ages is 34.
$\therefore x +$ ☐ $= 34$
☐ $+ 4 = 34$
$2x =$ ☐
$x =$ ☐
$\therefore$ Bernard is _____ years old and Sam is _____ years old.

Hint: Start by underlining the key information.

FLUENCY	4–8	5–9

Example 24 Turning a word problem into an equation

Five less than three times a certain number is 13. Write an equation and solve it to find the number.

SOLUTION

Let x be the number.
$3x - 5 = 13$
$3x = 18$
$x = 6$

The number is 6.
Check: $3 \times (6) - 5 = 13$ ✓

EXPLANATION

Define the unknown using a pronumeral.

Interpret the wording bit by bit to construct the equation.

3 times the number is $3x$, 5 less than this is $3x - 5$ and this equals 13.

Solve the equation by adding 5 to both sides, then dividing both sides by 3.

Write the answer in words and check your solution.

Now you try

Four more than five times a certain number is 29. Write an equation and solve it to find the number.

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4 Write an equation and solve it to find the unknown number in the following.

- **a** 4 less than 2 times a number is 10.
- **b** When a certain number is doubled, it results in a number that is 5 more than the original number.
- **c** I think of a number, divide it by 3 and add 5. The result is 12.
- **d** I think of a number, take away 2 and multiply the result by 4. This gives 24.
- **e** 3 less than a certain number is 9 less than 4 times the number.

Example 25 Applying algebra to a word problem

A bicycle shop hires out bikes. It charges an initial fee of \$10 for hiring a bike and then \$8 per hour. Leah returns her bike and is charged \$42. For how many hours did she hire the bike?

SOLUTION	EXPLANATION
Let h be the number of hours of hire.	Define the unknown value using a pronumeral.
$10 + 8h = 42$	Write an equation from the information: the cost is \$10 plus \$8 per hour ($8 \times h$), which equals \$42.
$8h = 32$	Solve the equation for h.
$h = 4$	
$\therefore$ Leah had the bike for 4 hours.	Answer the question in words.
Check: $\$10 + \$8 \times 4 = \$42$ ✓	Check that the answer is reasonable.

Now you try

A babysitter, Jess, charges an initial fee of \$20 and then \$15 per hour. If Jess earns \$95, for how many hours did she babysit?

5 Toby rented a car for a total cost of \$290. The rental company charged \$40 per day, plus a hiring fee of \$50.

- **a** Define a pronumeral for the number of days Toby rented the car.
- **b** Write an equation in terms of your pronumeral in part **a** to represent the problem.
- **c** Solve the equation in part **b** to find the unknown value.
- **d** For how many days did Toby rent the car?

6 A jeweller earns a weekly amount of \$200 plus \$10 per item she sells. If in one week she earned \$680, how many items did she sell?

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Example 26 Solving word problems with more complex equations

David and Usman made 254 runs between them in a cricket match. If Usman made 68 more runs than David, how many runs did each of them make?

SOLUTION

Let the number of runs for David be r.
Number of runs Usman made is $r + 68$.

$$r + (r + 68) = 254$$
$$2r + 68 = 254$$
$$2r = 186$$
$$r = 93$$

David made 93 runs and Usman made $93 + 68 = 161$ runs.

EXPLANATION

Define the unknown value as a pronumeral.

Write all other unknown values in terms of r.

Usman made 68 more runs than David: $r + 68$.

Write an equation: number of runs for David + number of runs for Usman = 254.

Collect like terms then solve the equation.

Answer the question in words.

Now you try

Max and Kim kicked 11 goals between them in a football match. If Kim kicked 3 more goals than Max, how many goals did each of them kick?

7 Leonie and Emma scored 28 goals between them in a netball match. Leonie scored 8 more goals than Emma.

- **a** Define a pronumeral for the number of goals scored by Emma.
- **b** Write the number of goals scored by Leonie in terms of the pronumeral in part **a**.
- **c** Write an equation in terms of your pronumeral to represent the problem.
- **d** Solve the equation in part **c** to find the unknown value.
- **e** How many goals did each of them score?

8 A rectangle is four times as long as it is wide and its perimeter is 560 cm.

- **a** Define a pronumeral for the unknown width.
- **b** Write an expression for the length in terms of your pronumeral in part **a**.
- **c** Write an equation involving your pronumeral and the perimeter to represent the problem.
- **d** Solve the equation in part **c**.
- **e** What is the length and width of the rectangle?

9 A prize of $1000 is divided between Adele and Benita so that Adele receives $280 more than Benita. How much did each person receive?

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PROBLEM-SOLVING AND REASONING

10–12 | 11–14

10 Andrew, Brooke and Cammi all work part-time at a supermarket. Cammi earns $20 more than Andrew's wage. Brooke earns $30 less than twice Andrew's wage. If their total combined wage is $400, find how much each of these workers earns.

11 Ed walked a certain distance, and then ran twice as far as he walked. He then caught a bus for the last 2 km. If he travelled a total of 32 km, find how far Ed walked and ran.

Hint: Remember to follow these steps
1. Define a pronumeral for the unknown.
2. Set up an equation using the information from the question.
3. Solve the equation.
4. Answer the question in words.

12 Kate is three times as old as her son. If Kate is 30 years older than her son, what are their ages?

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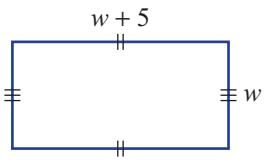
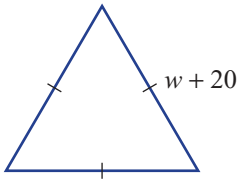
13 Two paddocks in the shapes shown below are to be fenced with wire. If the same total amount of wire is used for each paddock, what are the side lengths of each paddock, in metres?





Hint: Solve for w first.

14 Consecutive integers can be represented algebraically as x, $x + 1$, $x + 2$ etc.

a Find three consecutive numbers that add to 84.

b **i** Write three consecutive even numbers starting with x.

ii Find three consecutive even numbers that add to 18.

Hint: Make sure that you are still setting up an equation.

ENRICHMENT: Profit and revenue — 15

15 Tedco produces a teddy bear which sells for \$24. Each teddy bear costs the company \$8 to manufacture and there is an initial start-up cost of \$7200.

a Write a rule for the total cost, \$$T$, of producing x teddy bears.

b If the cost of a particular production run was \$9600, how many teddy bears were manufactured in that run?

c If x teddy bears are sold, write a rule for the revenue, \$$R$, received by the company.

d How many teddy bears were sold if the revenue was \$8400?

e If they want to make an annual profit of \$54 000, how many teddy bears do they need to sell?

Hint: Revenue is the total amount of money collected. Profit is the amount left over from the revenue after the costs have been taken out.
Profit = revenue − costs

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3J Using formulas

Learning intentions for this section:

- To know which variable is the subject of a formula
- To be able to work with a formula by substituting known values
- To be able to find the unknown value in a formula by solving an equation
- (Extending) To be able to change the subject of a formula

Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- This concept is further developed in Stage 6 Standard

A formula is an equation that relates two or more variables. The following are some examples of formulas.

- $A = \pi r^2$ is the formula for finding the area, A, of a circle given its radius, r.
- $V = \pi r^2 h$ is the formula for finding the volume of a cylinder given its radius, r, and height, h.
- $F = \frac{9}{5}C + 32$ is the formula for converting degrees Celsius, C, to degrees Fahrenheit, F.

The letters A, V and F are the subjects of the formulas given above, because they are alone on the left-hand side.

Lesson starter: What are these formulas?

Search the internet to find out:

- What each formula represents.
- The meaning of each letter in the formula.

$E = mc^2$	$v = u + at$	$V = \frac{4}{3}\pi r^3$

KEY IDEAS

- The **subject** of a **formula** is a pronumeral that usually sits on its own on the LHS.
  For example: In $A = lb$, A is the subject of the formula.
- A pronumeral in a formula can be evaluated by substituting numbers for all other pronumerals.
- If the unknown is not the subject, a simple equation will need solving.
- A formula can be rearranged to make another pronumeral the subject. Similar steps are used to those applied when solving an equation.
  For example: $A = lb$ can be rearranged to give $l = \frac{A}{b}$ with l now the subject.

Key terminology: subject, formula, variable, substitute, transpose, equation

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Exercise 3J

UNDERSTANDING	1–3	3

1 Match each formula (**a-e**) with its description (**A-E**).

a $A = lb$

b $A = \frac{1}{2}bh$

c $A = s^2$

d $C = 2\pi r$

e $A = \pi r^2$

A circumference of a circle

B area of a rectangle

C area of a triangle

D area of a circle

E area of a square

2 Write down the letter that is the subject of these formulas.

a $A = \frac{1}{2}bh$

b $D = b^2 - 4ac$

c $M = \frac{a+b}{2}$

d $A = \pi r^2$

Hint: In $V = lbh$, V is the subject.

3 The formula for the perimeter of a rectangle is $P = 2l + 2b$.

a What does l represent?

b What does b represent?

c Why does the number 2 appear twice in this formula?

d Could the formula be written as $P = 2(l + b)$?

FLUENCY	4, 5, 6(½)	4–6(½)

Example 27 Substituting values into formulas

Substitute the given values into the formula to find the value of the subject.

a $A = \frac{h}{2}(a + b)$, when $a = 3$, $b = 7$ and $h = 5$

b $E = \frac{1}{2}mv^2$, when $m = 4$ and $v = 5$

SOLUTION

a $A = \frac{h}{2}(a + b)$

$A = \frac{5}{2} \times (3 + 7)$

$= \frac{5}{2} \times 10$

$= 25$

EXPLANATION

Substitute $a = 3$, $b = 7$ and $h = 5$. Work out the sum in the brackets first, then multiply out to find the value of A.

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b $E = \frac{1}{2}mv^2$

$E = \frac{1}{2} \times 4 \times 5^2$

$= \frac{1}{2} \times 4 \times 25$

$= 50$

Substitute $m = 4$ and $v = 5$.
$5^2 = 5 \times 5 = 25$, then work out E by multiplying.

Now you try

Substitute the given values into the formula to find the value of the subject.

a $M = \frac{a+b}{2}$, when $a = 5$ and $b = 9$

b $F = \frac{mn}{r^2}$, when $m = 12$, $n = 6$ and $r = 3$

4 Substitute the given values into each of the following formulas to work out the subject. Round to two decimal places where appropriate.

a $A = bh$, when $b = 3$ and $h = 7$

b $F = ma$, when $m = 4$ and $a = 6$

c $m = \frac{a+b}{4}$, when $a = 14$ and $b = -6$

d $t = \frac{d}{v}$, when $d = 18$ and $v = 3$

e $A = \pi r^2$, when $r = 12$

f $V = \frac{4}{3}\pi r^3$, when $r = 2$

g $c = \sqrt{a^2 + b^2}$, when $a = 12$ and $b = 22$

h $Q = \sqrt{2gh}$, when $g = 9.8$ and $h = 11.4$

i $I = \frac{MR^2}{2}$, when $M = 12.2$ and $R = 6.4$

j $x = ut + \frac{1}{2}at^2$ when $u = 0$, $t = 4$ and $a = 10$

Example 28 Finding the unknown value in a formula

Substitute the given values into each formula, then solve to find the unknown value. Round to one decimal place in part **b**.

a $S = \frac{d}{t}$ given $S = 15$ and $d = 60$

b $C = 2\pi r$ given $C = 30$

SOLUTION

a $S = \frac{d}{t}$

$15 = \frac{60}{t}$

$15t = 60$

$t = 4$

EXPLANATION

Write the formula and substitute the given values of S and d.

Solve for t by first multiplying both sides by t.

Divide both sides by 15.

Continued on next page

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b $C = 2\pi r$

$30 = 2\pi r$

$\frac{30}{2\pi} = r$

$r = 4.8$ (to 1 d.p.)

Substitute $C = 30$. Solve for r by dividing both sides by 2π in one step. $\frac{30}{2\pi} = 4.77464\ldots$

Note: On a calculator you will need to insert brackets, $30 \div (2\pi)$, or set up using the fraction template, $\frac{\square}{\square}$.

Now you try

Substitute the given values into each formula, then solve to find the unknown value. Round to one decimal place in part **b**.

a $R = \frac{pl}{A}$ given $R = 16$, $p = 6$ and $l = 8$

b $S = 2\pi rh$ given $S = 100$ and $h = 5$

5 Substitute the given values into each of the following formulas. Solve the equations to find the unknown value. Round to two decimal places where appropriate.

a $m = \frac{F}{a}$, when $m = 12$ and $a = 3$

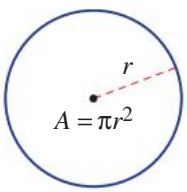
b $A = lb$, when $A = 30$ and $l = 6$

c $v = u + at$, when $v = 20$, $u = 5$ and $a = 2$

d $p = \frac{m}{v}$, when $p = 2$ and $m = 50$

e $A = \pi r^2$, when $A = 48$ and $(r > 0)$

f $A = \frac{h}{2}(a + b)$, when $A = 64$, $b = 12$ and $h = 4$



Example 29 Transposing formulas (Extending)

Transpose each of the following to make the variable in red the subject.

a $ax + by = c$

b $A = \pi r^2$ $(r > 0)$

SOLUTION

a $ax + by = c$

$by = c - ax$

$y = \frac{c - ax}{b}$

EXPLANATION

We need to obtain y on one side by itself.

Follow steps as if you are solving the equation for y.

Subtract ax from both sides.

by is $b \times y$. Just as for $2y$ you would divide both sides by 2, divide both sides by b.

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b $A = \pi r^2$ — Divide both sides by π.

$\frac{A}{\pi} = r^2$

$r^2 = \frac{A}{\pi}$ — Make r^2 the subject.

$r = \sqrt{\frac{A}{\pi}}$ — Take the square root of both sides, since $\sqrt{r^2} = r$ if $r > 0$.

Now you try

Transpose each of the following to make the variable in red the subject.

a $v = u + at$ **b** $E = \frac{mv^2}{2}, v > 0$

Ext **6** Transpose each of the following formulas to make the pronumeral in red the subject.

a $C = \pi d$ **b** $a + bx = d$

c $p = m(x + n)$ **d** $I = Prn$

e $P = \frac{v^2}{R} \quad (v > 0)$ **f** $A = 2\pi rh$

g $V = \pi r^2 h \quad (r > 0)$ **h** $\sqrt{A} + B = 4C$

i $c^2 = a^2 + b^2 \quad (a > 0)$ **j** $\sqrt{b + c} = a$

Hint: Follow similar steps as if you were solving to find the letter in red, such as $d = \ldots$

PROBLEM-SOLVING AND REASONING

7, 8 | 8–10

7 The formula $s = \frac{d}{t}$ gives the speed s km/h of a car which has travelled a distance of d km in t hours.

a Find the speed of a car that has travelled 400 km in 4.5 hours. Round to two decimal places.

b Find the distance covered if a car travels at 75 km/h for 3.8 hours.

8 The velocity, v m/s, of an object is described by the rule $v = u + at$, where u is the initial velocity in m/s, a is the acceleration in m/s^2 and t is the time in seconds.

a Find the velocity after 3 seconds if the initial velocity is 5 m/s and the acceleration is 10 m/s^2.

b Find the time taken for a body to reach a velocity of 20 m/s if its acceleration is 4 m/s^2 and its initial velocity is 12 m/s.

Hint: Write down the value of the known variables first; i.e. $t = 3$, $u = 5$ etc.

Hint: Velocity is another word for speed.

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9 The volume of water (V litres) in a tank is given by $V = 4000 - 0.1t$, where t is the time in seconds after a tap is turned on.

- **a** Over time, does the water volume increase or decrease according to the formula?
- **b** Find the volume after 2 minutes. (Note: t is in seconds.)
- **c** Find the time it takes for the volume to reach 1500 litres. Round to the nearest minute.

10 The formula $F = \frac{9}{5}C + 32$ converts degrees Celsius, C, to degrees Fahrenheit, F.

- **a** Find what each of the following temperatures is in degrees Fahrenheit.
  - **i** 100°C
  - **ii** 38°C
- **b** Calculate what each of the following temperatures is in degrees Celsius. Round to one decimal place where necessary.
  - **i** 14°F
  - **ii** 98°F

Hint: $\frac{9}{5}C$ can also be written as $\frac{9C}{5}$.

ENRICHMENT: Basketball formulas — 11, 12

11 The formula $T = 3x + 2y + f$ can be used to calculate the total number of points made in a basketball game where:

x = number of three-point goals
y = number of two-point goals
f = number of free throws made
T = total number of points

- **a** Find the total number of points for a game where 12 three-point goals, 15 two-point goals and 7 free throws were made.
- **b** Find the number of three-point goals made if the total number of points was 36, with 5 two-point goals made and 5 free throws made.

12 The formula $V = \dfrac{\left(p + \frac{1.5r}{2} + 2a + \frac{1.5s}{2} + 2b\right) - (1.5t + 2f + m - o)}{g}$ can be used to calculate the value, V, of a basketball player where:

p = points earned
a = number of assists
b = number of blocks
f = number of personal fouls
o = number of offensive rebounds
r = number of rebounds
s = number of steals
t = number of turnovers
m = number of missed shots
g = number of games played

Calculate the value of a player with 350 points earned, 2 rebounds, 14 assists, 25 steals, 32 blocks, 28 turnovers, 14 personal fouls, 24 missed shots, 32 offensive rebounds and 10 games.

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Plumber

Plumbers use many skills in their day-to-day jobs. The training course is up to 5 years of on-the-job experience via an apprenticeship and TAFE courses.

Mathematics is an important skill for a plumber. Plumbers need to measure, cut, bend and connect pipes according to project specifications. Equations and formulas play an important role in the life of a plumber, especially if working on big construction sites, because calculations need to be accurate for buildings to satisfy Australian regulations.

1 Water expands when heated. Plumbers need to allow for this additional volume when selecting the most suitable water storage unit for a project. The volume E (litres) of water expansion is calculated with this formula:

$E = L \times (T - t) \times 0.000375$ where

L = initial volume of water in litres

T = highest temperate in °C

t = lowest temperature in °C

0.000375 is the coefficient of expansion for water.

Calculate the volume of water expansion in litres (to 2 decimal places) that would occur when:

a 600 L of water is heated from 22° to 50°

b 500 L of water is heated from 18° to 60°

c 1000 L of water is heated from 20° to 70°

d 800 L of water increased in temperature by 50°

2 Plumbers need to calculate water pressure, P, measured in kPa (kilopascals) for any given height, h (m), above an outlet. The following formula is used:

$P = h \times 9.81$ where 9.81 is the gravitational pressure exerted by 1 m height of water.

a Calculate the water pressure for a pressure head of:

i 10 m **ii** 20 m **iii** 30 m **iv** 40 m

b Rewrite the above formula with h as the subject and calculate how many metres (to 2 decimal places) of water are in a non-pressurised tank if the pressure gauge reads:

i 24.6 kPa **ii** 24.1 kPa **iii** 13.5 kPa **iv** 27.9 kPa

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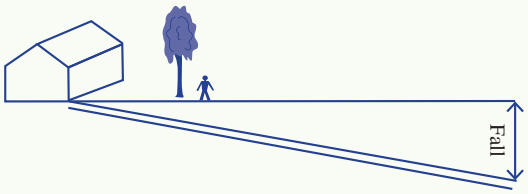
3 A plumber needs to install the pipes that connect a house to the underground mains water supply and to the council sewage disposal system. Water flows under pressure up to the house and the sewerage pipe is sloped downwards to allow waste to flow away under the force of gravity. The difference in level between each end of a sloping pipe is called the fall measurement.



$\text{Fall (mm)} = \dfrac{\text{pipe length in mm} \times x}{100}$, where the gradient $x\%$ = the fall in mm per 100 mm of pipe.

Calculate the total fall in mm required to install the following pipes. Round to the nearest mm.

a A water pipe is 6 m long and installed at a 2.5% gradient.

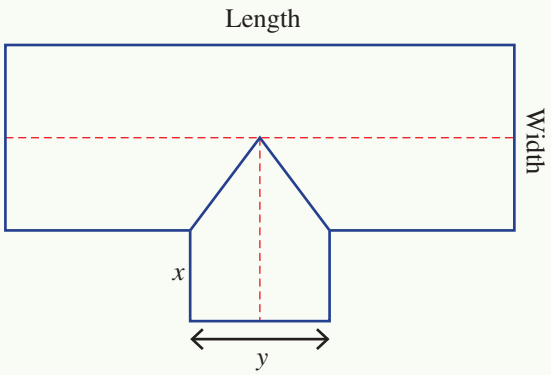
b A water pipe is 17 m long and installed at a 2.1% gradient.

c A sewerage pipe is 9 m in length installed at a 1.67% gradient.

d A sewerage pipe is 18 m in length installed at a 1.85% gradient.

Using technology

4 The number of downpipes required for a house depends on the roof area and the size of the downpipes. Each size of downpipe can drain a maximum area of roof.
The number of downpipes needed can be estimated using the rain catchment area of a roof, which is the equivalent flat area of the roof, not the actual sloping area.



$\text{Number of downpipes} = \dfrac{\text{total roof catchment area}}{\text{maximum area per downpipe}}$

Hint: The whole number of downpipes is manually determined by rounding up the decimal number of downpipes.

Set up an Excel spreadsheet as shown below. Use the given roof dimensions applied to the roof plan above and determine the number of downpipes required for each house.

	A	B	C	D	E	F	G	H	I
1	The number of downpipes for a house								
2	House	Length m	Width m	x m	y m	Total roof catchment area in m^2	Maximum area in m^2 per downpipe	Decimal number of downpipes	Whole number of downpipes
3	i	14	8.5	4	3		47		
4	ii	15	9	3	4		45		
5	iii	17	10	3	3.5		46		
6	iv	22	11	3	5		47		

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Theme parks

Theme parks and amusement parks all over the world offer a vast array of exhilarating and thrilling rides, such as Buzzsaw and The Giant Drop.

Hugo and Alice live near two theme parks called Extreme Thrill and Adventure World. They plan to run tourist bus trips to the theme parks and want to advertise the best deal at each theme park depending on the number of rides.

The theme parks charge different entry prices and cost for each ride.

Theme park	Entry cost	Fee per ride
Extreme Thrill	$42	$3
Adventure World	$30	$5

Present a report for the following tasks and ensure that you show clear mathematical workings, explanations and diagrams where appropriate.

Routine problems

Tickets are bought at the entry for a certain number of rides.

a Find the cost of a ticket at Extreme Thrill for
  i 4 rides **ii** 10 rides

b Find the cost of a ticket at Adventure World for
  i 4 rides **ii** 10 rides

Non-routine problems

The problem is to determine the cheapest theme park depending on the number of rides taken.

a Write down all the relevant information that will help solve this problem.

b Let T = total ticket price and n = number of rides.
Construct a formula for the total ticket price T for n rides at
  i Extreme Thrill **ii** Adventure World

c Hugo spends $75 for a ticket into Extreme Thrill.
  i Write the formula for the ticket price T, for n rides at Extreme Thrill.
  ii Set up an equation by substituting $T = 75$
  iii Solve this equation algebraically for n.
  iv How many rides were included in Hugo's $75 ticket?

d Alice spends $75 for a ticket into Adventure World.
  i Write the formula for the ticket price T, for n rides at Adventure World.
  ii Set up an equation by substituting $T = 75$
  iii Solve this equation algebraically for n.
  iv How many rides were included in Alice's $75 ticket?

e Compare your answers to **c** and **d** above, and state which theme park offers the best deal for a $75 ticket. Give a reason for your answer.

f Hugo and Alice buy 2 identical tickets for Extreme Thrill costing a total of $138.
  i Write the formula for 2 Extreme Thrill tickets and form an equation by substituting $T = 138$.
  ii Solve this equation for n and state how many rides they each paid for.

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g Hugo decides to calculate the number of rides that make the ticket prices equal at each theme park.

- **i** Set up the equation by equating the ticket price formulas.
  i.e. Extreme Thrill ticket formula = Adventure World ticket formula
- **ii** Solve this equation algebraically for n.
- **iii** For how many rides does each theme park have the same ticket price?
- **iv** Calculate this ticket price for each theme park.

Communicate thinking and reasoning

h For how many rides is Extreme Thrill the cheaper option? Justify your answer with a mathematical calculation.

i For how many rides is Adventure World the cheaper option? Justify your answer with a mathematical calculation.

j Summarise your results using a table like the one below and describe any key findings.

Ticket prices							
Number of rides *n*							
Extreme Thrill							
Adventure world							

Extension problems

Problem-solve

A new theme park called Monster Rides charges $22 entry plus $7 per ride.

a Construct a formula for the total ticket price T for n rides at Monster Rides.

b Set up equations by equating ticket price formulas and then solve each equation to find the number of rides n that make the Monster Rides ticket price equal to the:

- **i** Extreme Thrill ticket price
- **ii** Adventure World ticket price

c If, on average, people buy tickets for 7 or more rides, which theme park out of the three would be expected to make the largest profit? Justify your answer with some mathematical calculations.

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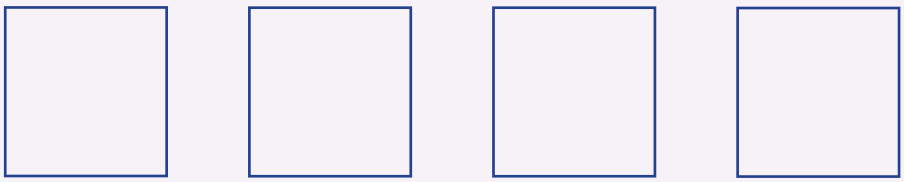
1 In a magic square, all the rows, columns and main diagonals add to the same total. The total for this magic square is 15. Find the value of x then complete the magic square.

$2x - 2$	$3x$	$x - 1$
x		

2 Simplify $2xy - 3x(y - 2x) - 8x^2y \div (2y) + 2x \times 6xy \div (4y)$.

3 Reveal the word below by solving each equation for x. The value of x gives the number of the letter in the alphabet; e.g. $x = 1$ would be A, $x = 2$ would be B etc.



$\frac{x-3}{4} = 5$ $\quad 6x = 4x + 30$ $\quad 2(x - 10) = 16$ $\quad 1 - 2x = -7$

4 Twelve years ago Eric's father was seven times as old as Eric was. If Eric's father is now 54 years old, how old is Eric now?

5 In a yacht race the second leg was half the length of the first leg. The third leg was two-thirds of the length of the second leg. The fourth leg was twice the length of the second leg. If the total distance was 153 km, find the length of each leg.

6 A group of office workers had some prize money to distribute among themselves. When all but one took \$9 each, the last person only received \$5. When they all took \$8 each, there was \$12 left over. How much had they won?

7 Try finding a solution to these more complex linear equations.

a $\frac{x}{3} + \frac{x}{2} = 2$ $\qquad$ **b** $\frac{x+1}{5} - \frac{x-1}{7} = 1$

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Chapter summary

Expressions and equations

Expanding brackets

E.g. $2(x+3) = 2 \times x + 2 \times 3$
$= 2x + 6$
$-3(4-2x) = -3 \times 4 + (-3) \times (-2x)$
$= -12 + 6x$

Expressions

E.g. $2x - 3y + 5$
term, coefficient, constant
E.g. 2 more than 3 lots of m is $3m + 2$
3 less than x is $x - 3$

Substitution

The process of replacing a pronumeral with a given value;
E.g. if $x = 3$, $2x + 4 = 2 \times (3) + 4$
$= 6 + 4$
$= 10$
E.g. if $x = 2$, $y = -4$,
then $xy = 2 \times (-4)$
$= -8$

Solving word problems

1 Define a pronumeral for the unknown.
2 Set up an equation.
3 Solve the equation.
4 Check the answer and write in words.

Addition/subtraction

Only like terms can be combined under addition or subtraction.
Like terms have the same pronumerals and powers; i.e. $4xy$ and $2xy$.
E.g. $5a + 2 + 3a$
$= 5a + 3a + 2$
$= 8a + 2$
E.g. $3x + 2y - x + 4y$
$= 3x - x + 2y + 4y$
$= 2x + 6y$

Solving linear equations

The solution of a linear equation is the value that makes the equation true.
E.g. $2x + 5 = 9$
$2x = 4$ (subtract 5)
$x = 2$ (divide by 2)
E.g. $5(x-1) = 2x + 7$
$5x - 5 = 2x + 7$ (expand)
$3x - 5 = 7$ (subtract $2x$)
$3x = 12$ (add 5)
$x = 4$ (divide by 3)

Multiplication/ Division

In simplified form the multiplication sign ($\times$) and division sign ($\div$) are not shown.
That is, $2 \times x = 2x$ and $x \div 2 = \frac{x}{2}$.
Multiplying:
$2a \times 3b = 2 \times 3 \times a \times b$
$= 6ab$
Dividing:
$4x \div 2 = \frac{\cancel{4}^{2}x}{\cancel{2}_{1}} = 2x$
$\frac{6ab^2}{9b} = \frac{\cancel{6}^{2} \times a \times \cancel{b}^{1} \times \cancel{b}^{1}}{\cancel{9}_{3} \times \cancel{b}_{1}}$
$= \frac{2ab}{3}$

Equations involving fractions

E.g. $\frac{x}{3} + 1 = 5$
$\frac{x}{3} = 4$ (subtract 1)
$x = 12$ (multiply by 3)
E.g. $\frac{x-2}{4} = 1$
$x - 2 = 4$ (multiply by 4)
$x = 6$ (add 2)
E.g. $\frac{5x}{2} = 10$
$5x = 20$ (multiply both sides by 2)
$x = 4$ (divide both sides by 5)

Formulas

Some common formulas are
$A = lb$, $C = 2\pi r$
An unknown value can be found by substituting values for the other variables.
Extension: A formula can be transposed to make a different variable the subject.
E.g. make r the subject in $A = \pi r^2$
$A = \pi r^2$
$\frac{A}{\pi} = r^2$
$\sqrt{\frac{A}{\pi}} = r$
$\therefore r = \sqrt{\frac{A}{\pi}}$ where $r > 0$

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Chapter checklist with success criteria

A printable version of this checklist can be downloaded from the Interactive Textbook.

		✔
3A	**1 I can identify parts of an algebraic expression.** e.g. Consider the expression $3x - 5y + 6$. State: **a** the number of terms **b** the coefficient of y **c** the constant term	☐
3A	**2 I can convert words and word problems to expressions.** e.g. Write an algebraic expression for **a** the cost of n movie tickets at \$15 each **b** four less than three times x **c** the square of y is divided by 4	☐
3A	**3 I can substitute values into expressions and evaluate.** e.g. Evaluate these expressions if $p = -4$, $q = 6$ and $r = 5$: **a** $4q - r$ **b** $3p - (q + r)$	☐
3B	**4 I can identify like terms.** e.g. Choose the pair(s) of like terms from the following set: $4x, 5, 3xy, -5x, 2xy^2, yx$	☐
3B	**5 I can collect like terms.** e.g. Simplify the following by collecting like terms: **a** $5a - 4 + 3a$ **b** $4xy + 6y - xy + 4y$	☐
3C	**6 I can multiply algebraic terms.** e.g. Simplify the following: **a** $4s \times 5t$ **b** $-3y \times 8xy$	☐
3C	**7 I can divide algebraic terms.** e.g. Simplify the following: **a** $\frac{15x}{12}$ **b** $16a^2b \div (24ab)$	☐
3D	**8 I can expand expressions with brackets.** e.g. Expand the following and simplify: **a** $4(x + 5)$ **b** $-3(x - 6)$ **c** $2x(3x - y)$	☐
3D	**9 I can expand and collect like terms.** e.g. Expand the following and collect like terms: **a** $3(x + 2) - 4$ **b** $5 - 2(x + 1)$	☐
3E	**10 I can solve simple linear equations.** e.g. Solve the following equations: **a** $2x - 5 = 7$ **b** $8 - 3x = 15$	☐
3F	**11 I can solve linear equations with fractional coefficients.** e.g. Solve the following equations: **a** $\frac{3x}{4} = 9$ **b** $\frac{x}{5} - 4 = 2$	☐

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Chapter checklist

Chapter checklist with success criteria

3F	**12 I can solve simple fractional equations.** e.g. Solve $\frac{x-5}{3} = 6$	☐
3G	**13 I can solve equations with brackets.** e.g. Solve $3(3x + 5) = 22$	☐
3G	**14 I can solve equations with brackets and a common factor.** e.g. Solve $4(2x + 3) = 32$ by first dividing by the common factor	☐
3G	**15 I can solve equations with brackets and like terms.** e.g. Solve $2(4x + 7) - 5x = 20$	☐
3H	**16 I can solve equations with pronumerals on both sides.** e.g. Solve these equations: **a** $8x = 3x + 25$ **b** $10 - 3x = x + 2$	☐
3H	**17 I can solve equations with brackets and pronumerals on both sides.** e.g. Solve $4(2x - 1) = 2(x - 8)$	☐
3I	**18 I can turn a word problem into an equation.** e.g. 8 more than two times a certain number is 30. Write an equation and solve it to find the number.	☐
3I	**19 I can apply algebra to a word problem.** e.g. A surf shop hires out surfboards. It charges an initial fee of \$15 for hiring a board and then a charge of \$12 per hour. Jett returns his board and is charged \$51. For how many hours did he hire the board?	☐
3I	**20 I can solve more complex word problems.** e.g. Toni and Trey earn \$370 between them walking dogs. If Toni earned \$140 more than Trey, how much money did each of them make?	☐
3J	**21 I can substitute values into formulas and find the unknown.** e.g. Substitute the given values into the formula to find the value of the unknown. Round to one decimal place where necessary. **a** $A = \frac{1}{2}xy$ when $x = 7$ and $y = 10$ **b** $V = \pi r^2 h$ when $V = 120$ and $r = 3$	☐
3J	**22 (Extension) I can transpose a formula.** e.g. Transpose the formula $v^2 = u^2 + 2as$ to make u the subject $(u > 0)$.	☐

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Short-answer questions

3A **1** Write algebraic expressions to represent the following.

a The sum of x and y

b The product of m and 7

c The cost of 3 movie tickets at m dollars each

d 3 less than n, all divided by 4

3A **2** Evaluate the following if $x = 2$, $y = -1$ and $z = 5$.

a $xz + 1$ b $4x + y$ c $x - 2yz$ d $x(y + z)$ e $x^2 - 3z$

3C **3** Simplify.

a $2 \times 4n$ b $3x \times 2y$ c $8a \div 2$ d $\dfrac{4x^2y}{12x}$

3B **4** Simplify by collecting like terms.

a $2b + 4b + b$ b $6x + 3 - 2x$

c $4p - 3q - p + 5q$ d $3mn + 2m - 6mn + n$

3D **5** Expand and simplify the following.

a $2(x + 7)$ b $-3(2x + 5)$ c $2x(3x - 4)$

d $-2a(5 - 4a)$ e $4(x + 2) + 5$ f $3(x - 2) - 1$

3E **6** Solve the following linear equations for x.

a $5x + 6 = 51$ b $7x - 4 = 10$ c $4 - x = 7$ d $3 - 2x = 21$

3F **7** Solve these linear equations involving fractions.

a $\dfrac{2x}{5} = 4$ b $\dfrac{x}{2} - 1 = 3$ c $\dfrac{x + 2}{4} = 7$ d $\dfrac{1}{3}x + 2 = 4$

3I **8** Write an equation to represent each of the following and then solve it for the pronumeral.

a A number, n, is doubled and increased by 3 to give 21.

b A number of lollies, l, is decreased by 5 and then shared equally among 3 friends so that they each get 7.

c 5 less than the result of Toni's age, x, divided by 4 is 0.

3G **9** Solve the following linear equations by first expanding the brackets.

a $2(x + 4) = 18$ b $3(2x - 3) = 2$

3H **10** Solve these equations with variables on both sides.

a $8x = 2x + 24$

b $5x - 2 = 3x + 2$

c $3 - 4x = 7x - 8$

d $5(2x + 4) = 7x + 5$

3I **11** Nick makes an initial bid of $\$x$ in an auction for an old cricket bat. By the end of the auction he has paid \$550, which is \$30 more than twice his initial bid, x. Set up and solve an equation to determine Nick's initial bid.

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Chapter review

3J **12** Find the value of the unknown in each of the following formulas.

a $E = \sqrt{PR}$ when $P = 90$ and $R = 40$

b $v = u + at$ when $v = 20$, $u = 10$, $t = 2$

c $V = \frac{1}{3}Ah$ when $V = 20$, $A = 6$

3J **13** (Extension) Rearrange the following formulas to make the variable in brackets the subject.

a $v^2 = u^2 + 2ax$ (x)

b $P = RI^2, I > 0$ (I)

Multiple-choice questions

3A **1** The algebraic expression that represents 2 less than 3 lots of n is:

A $3(n - 2)$ **B** $2 - 3n$ **C** $3n - 2$ **D** $3 + n - 2$ **E** n

3C **2** The fully simplified form of $2ab \div (8a)$ is:

A $\frac{2ab}{8a}$ **B** $\frac{4}{b}$ **C** $\frac{b}{4}$ **D** $4b$ **E** $4a$

3B **3** The simplified form of $6ab + 14a - 2ab + 3a$ is:

A $21ab$ **B** $4ab + 11a$ **C** $ab + 7a$ **D** $4ab + 17a$ **E** $4 + 17a$

3E **4** The solution to the equation $2x - 3 = 7$ is:

A $x = 2$ **B** $x = -5$ **C** $x = 5$ **D** $x = 6.5$ **E** $x = 0.5$

3F **5** The solution to $\frac{x}{3} - 1 = 4$ is:

A $x = 13$ **B** $x = 7$ **C** $x = 9$ **D** $x = 15$ **E** $x = \frac{5}{3}$

3D **6** The expanded form of $3(2y - 1)$ is:

A $6y - 3$ **B** $5y - 1$ **C** $6y - 1$ **D** $6y + 2$ **E** $3y - 3$

3H **7** An equivalent equation to $5x = 2x + 9$ is:

A $2x = 5$ **B** $3x = 9$ **C** $0 = 3x + 9$ **D** $7x = 9$ **E** $x = 9$

3G **8** The solution to the equation $2(x - 3) = 7$ is:

A $x = 5$ **B** $x = \frac{13}{2}$ **C** $x = 2$ **D** $x = \frac{1}{2}$ **E** $x = \frac{7}{3}$

3I **9** Eli is x years old. His sister is 2 years older. The sum of their ages is 22. A simplified equation to represent this is:

A $x + 2 = 22$ **B** $2x + 2 = 22$ **C** $2(x + 2) = 22$ **D** $x(x + 2) = 22$ **E** $2x = 22$

3J **10** If $A = \frac{1}{2}bh$ with $A = 360$ and $h = 40$, then the value of b is:

A 90 **B** 4.5 **C** 20 **D** 18 **E** 9

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Extended-response questions

1 Julie hires a jumping castle for her daughter's birthday party. It costs $60 for the set-up, plus $25 for each hour that it is hired.

a What is the cost for:

i 1 hour of hire? **ii** 2 hours of hire?

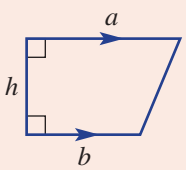
b Julie is charged $210 for the hire of the jumping castle.

i Define a variable to represent the number of hours for which Julie hired the castle.

ii Set up an equation using your variable.

iii Solve the equation to find how many hours Julie hired the jumping castle for.

2 A new backyard deck is being designed in the shape of the trapezium shown.



The area of a trapezium is given by $A = \frac{h}{2}(a + b)$.

Currently the dimensions are set such that $a = 12$ m and $b = 8$ m.

a Substitute the given values and rearrange the equation to make h the subject.

b Use your answer to part **a** to find the width of the deck (h m) required to have a deck area of 100 m^2.

c It is decided to extend the width (h) to 12 m to increase the area to 150 m^2. If b is fixed at 8 m, find the value of a that gives the required area.

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4

Right-angled triangles

Maths in context: Building houses, creating maps and designing clothing

Pythagoras' theorem and trigonometry are essential skills for the accurate calculations of lengths and angles. These methods are widely applied, including by builders, carpenters, plumbers, electricians, surveyors, navigators, engineers, architects and designers. Everyday users of Pythagoras' theorem and trigonometry include:

- Designers and builders of house frames, roof trusses, verandas or decks, kitchen cabinets, stairs, carports, holiday cabins, awnings and wheel chair ramps.
- Plumbers and electricians who calculate placement angles and lengths of water pipes and conduit (plastic protection tubing for electrical cables).
- Fashion designers who use trigonometry to calculate dart angles in clothing patterns and interior designers who apply trigonometry and geometry for the placement of lamps, furniture and loud speakers.
- Surveyors who calculate the height of mountains and the placement on maps of streets, roads, bridges and buildings.

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Chapter contents

4A Exploring Pythagoras' theorem (CONSOLIDATING)
4B Finding the length of the hypotenuse (CONSOLIDATING)
4C Finding the lengths of the shorter sides (CONSOLIDATING)
4D Using Pythagoras' theorem to solve two-dimensional problems (CONSOLIDATING)
4E Introducing the trigonometric ratios
4F Finding unknown sides
4G Solving for the denominator
4H Finding unknown angles
4I Using trigonometry to solve problems

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- applies Pythagoras' theorem to solve problems in various contexts (MA4-PYT-C-01)
- applies trigonometric ratios to solve right-angled triangle problems (MA5-TRG-C-01).

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Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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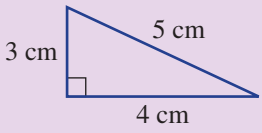
Warm-up quiz

1 What is the length of the longest side of these triangles?

a



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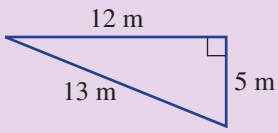

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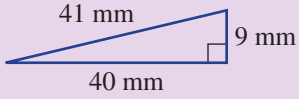


2 Calculate each of the following.

a 3^2 **b** 7^2 **c** $2^2 + 4^2$ **d** $9^2 - 6^2$

3 Round off each number correct to four decimal places.

a 0.45678 **b** 0.34569 **c** 0.04562 **d** 0.27997

4 Round off each number correct to two decimal places.

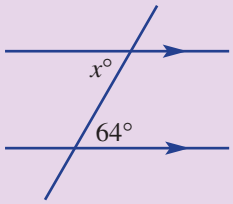
a 4.234 **b** 5.678 **c** 76.895 **d** 23.899

5 Use a calculator to calculate each of the following correct to two decimal places.

a $\sqrt{35}$ **b** $\sqrt{236}$ **c** $\sqrt{23.6}$ **d** $\sqrt{65.4}$

6 Find the value of x in these diagrams.

a



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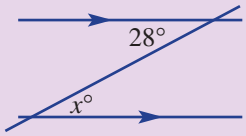

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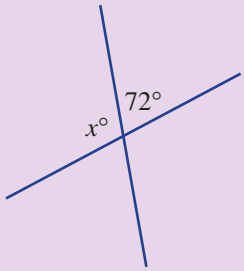

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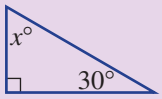

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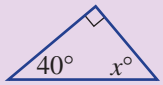

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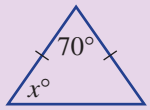

7 Solve each of the following equations to find the value of x. Multiply or divide both sides to start.

a $3x = 6$ **b** $4x = 12$ **c** $5x = 60$ **d** $8x = 48$

e $\frac{x}{3} = 4$ **f** $\frac{x}{5} = 6$ **g** $\frac{x}{7} = 4$ **h** $\frac{x}{13} = 14$

8 Solve each of the following equations to find the value of x correct to one decimal place. First multiply both sides by x, then divide to finish.

a $\frac{3}{x} = 5$ **b** $\frac{4}{x} = 7$ **c** $\frac{32}{x} = 15$ **d** $\frac{14}{x} = 27$

9 Given that x is a positive number, find its value in each of the following equations. Note, for example, that if $x^2 = 9$ then $x = \sqrt{9} = 3$ (provided that $x > 0$).

a $x^2 = 4$ **b** $x^2 = 16$ **c** $x^2 = 3^2 + 4^2$ **d** $x^2 = 12^2 + 5^2$

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4A Exploring Pythagoras' theorem CONSOLIDATING

Learning intentions for this section:

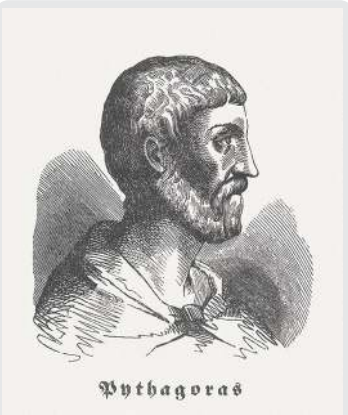
- To be able to identify the hypotenuse in a right-angled triangle
- To know the relationship between the square of the sides of a right-angled triangle
- To be able to write Pythagoras' theorem for a given triangle using variables or numbers

Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of these questions are more challenging than those in the Year 8 book
- These concepts will be revised, briefly, in Stage 6 Standard
- You may be expected to recall the formula from memory in the HSC Examination

The philosopher Pythagoras was born in Greece in the 6th century BCE. He travelled to Egypt and Persia where he developed his ideas in mathematics and philosophy. His students and followers were called the Pythagoreans. They made many advances in mathematics.

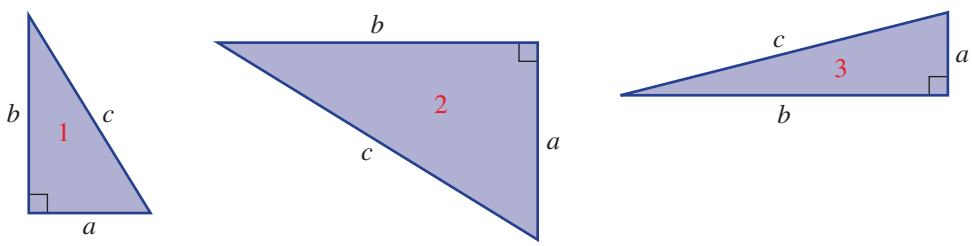
The Pythagoreans discovered the famous theorem, which is named after Pythagoras, and the existence of irrational numbers such as $\sqrt{2}$, which cannot be written down as a fraction or terminating decimal. The Pythagoreans called these numbers 'unutterable' numbers because any member who mentioned these numbers in public might be put to death.



A woodcut engraving of Pythagoras

Lesson starter: Discovering Pythagoras' theorem

Use a ruler to measure the sides of these right-angled triangles to the nearest mm. Then complete the table.



Triangle	a	b	c	a^2	b^2	c^2
1						
2						
3						

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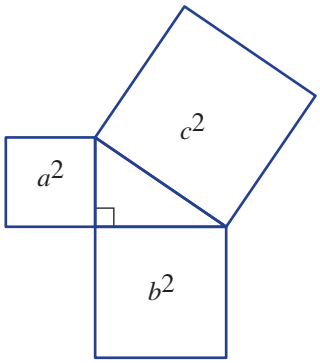
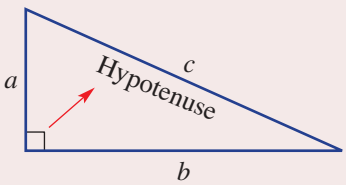
- Can you see any relationship between the numbers in the columns for a^2 and b^2 and the number in the column for c^2?
- Can you write down this relationship as an equation?
- Explain how you might use this relationship to calculate the value of c if it was unknown.
- Research how you can cut the two smaller squares in the diagram on the right (a^2 and b^2) to fit the pieces into the largest square (c^2).



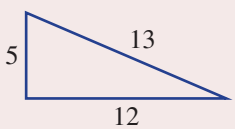
KEY IDEAS

■ The **hypotenuse**:
- is the longest side of a **right-angled** (90°) triangle.
- is opposite the right angle.



■ **Pythagoras' theorem** states:
- the square of the hypotenuse is equal to the sum of the squares of the other two shorter sides.
  $a^2 + b^2 = c^2$ or $c^2 = a^2 + b^2$

■ If you know all three sides of a triangle, you can prove that a triangle is right-angled.
For example, $5^2 + 12^2 = 13^2$ is a true statement, so the triangle with sides of these lengths is right-angled.



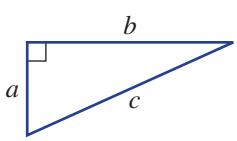
Key terminology: hypotenuse, Pythagoras' theorem, right-angled, square

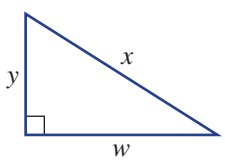
Exercise 4A

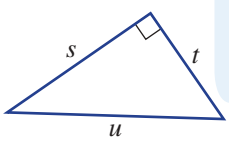
UNDERSTANDING 1–3 3

1 Write the missing words in this sentence.
The _____ is the longest side of a right-angled _____.

2 Which letter marks the length of the hypotenuse in these triangles?







Hint: The hypotenuse is the longest side of a right-angled triangle.

3 Decide if these equations are true (T) or false (F).
a $2^2 + 3^2 = 4^2$ **b** $6^2 + 8^2 = 10^2$
c $3^2 + 4^2 = 5^2$ **d** $5^2 + 5^2 = 10^2$

Hint: Does LHS = RHS?

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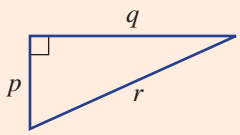
FLUENCY 4, 5 4, 5

Example 1 Writing Pythagoras' theorem

Write down Pythagoras' theorem for this triangle.



SOLUTION

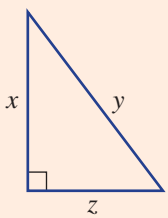
$p^2 + q^2 = r^2$

EXPLANATION

r is the length of the hypotenuse and p and q are the lengths of the two shorter sides.
Alternatively, $r^2 = p^2 + q^2$.

Now you try

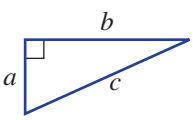
Write down Pythagoras' theorem for this triangle.



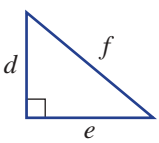
4 Write down Pythagoras' theorem for these triangles using the given pronumerals.

Hint: Replace the letters a, b and c in $a^2 + b^2 = c^2$ with the given letters.

a



b



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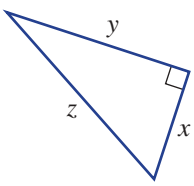
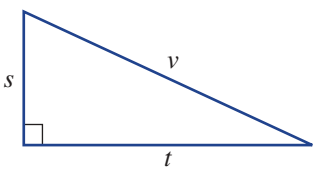


d



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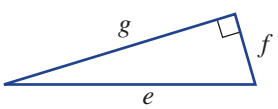

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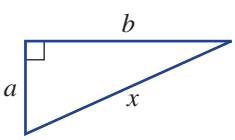

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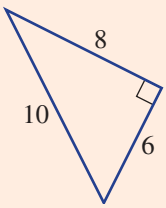
Example 2 Proving that a triangle is right-angled

Substitute the numbers on this triangle into Pythagoras' theorem $a^2 + b^2 = c^2$.



SOLUTION

$6^2 + 8^2 = 36 + 64$
$= 100$ ✓

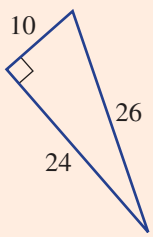
$10^2 = 100$ ✓

EXPLANATION

10 is the length of the hypotenuse and 6 and 8 are the lengths of the two shorter sides.

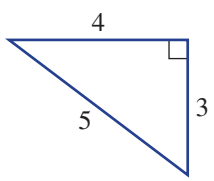
Now you try

Substitute the numbers on this triangle into Pythagoras' theorem $a^2 + b^2 = c^2$.



5 Prove that these triangles are right-angled.

a



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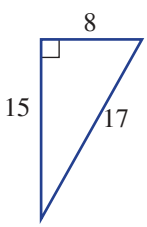

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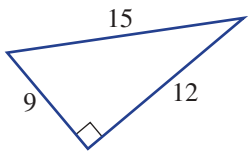

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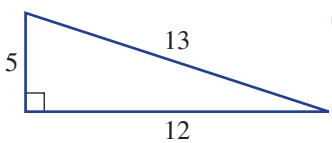

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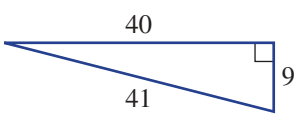

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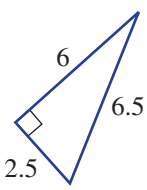
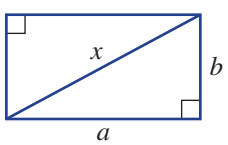


PROBLEM-SOLVING AND REASONING 6–8 7–10

6 Write down Pythagoras' theorem using the pronumerals given in these diagrams.

a



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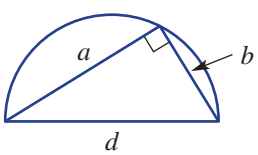

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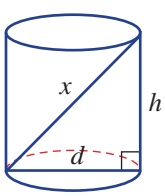

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7 Complete this table and answer the questions below.

a	b	c	a^2	b^2	$a^2 + b^2$	c^2
3	4	5				
6	8	10				
8	15	17				

a Which two columns give equal results?

b What would be the value of c^2 if $a^2 + b^2 = c^2$ and:

i $a^2 = 4$ and $b^2 = 9$?

ii $a^2 = 7$ and $b^2 = 13$?

c What would be the value of $a^2 + b^2$ if $a^2 + b^2 = c^2$ and:

i $c^2 = 25$?

ii $c^2 = 110$?

8 A cable connects the top of a 30 m mast to a point on the ground. The cable is 40 m long and connects to a point 20 m from the base of the mast.

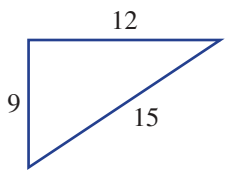
a Using $c = 40$, decide if $a^2 + b^2 = c^2$.

b Do you think the triangle formed by the mast and the cable is right-angled? Give a reason.

9 If $a^2 + b^2 = c^2$, we know that the triangle must have a right angle. Which of these triangles must have a right angle?

Hint: Substitute into $a^2 + b^2 = c^2$ to see if it's true.

a



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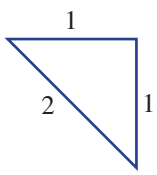

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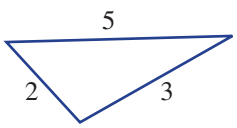

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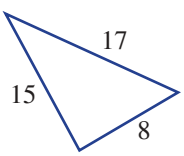

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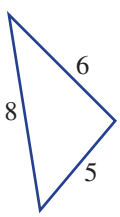

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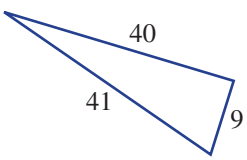

10 (3, 4, 5) and (5, 12, 13) are Pythagorean triads, since $3^2 + 4^2 = 5^2$ and $5^2 + 12^2 = 13^2$. Find 10 more Pythagorean triads using whole numbers less than 100.

Hint: If (3, 4, 5) is a Pythagorean triad, then (6, 8, 10) is also a triad.

ENRICHMENT: Pythagorean triads — 11

11 Find the total number of Pythagorean triads with whole numbers of less than 100. Include any found from Question **10** above.

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4B Finding the length of the hypotenuse CONSOLIDATING

Learning intentions for this section:

- To be able to use Pythagoras' theorem to find the length of the hypotenuse given two sides
- To be able to find hypotenuse lengths as rounded decimals or exact values

Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of these questions are more challenging than those in the Year 8 book
- These concepts will be revised, briefly, in Stage 6 Standard
- You may be expected to recall the formula from memory in the HSC examination

If you know the lengths of the two shorter sides, you can use Pythagoras' theorem to find the length of the hypotenuse.

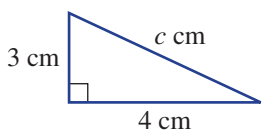
Lesson starter: Correct substitution

Here are three attempts at using Pythagoras' theorem for the given triangle.

$c^2 = a^2 + b^2$
$= 3^2 + 4^2$
$= 25$
$\therefore c = \sqrt{25}$
$= 5$

$c^2 = a^2 + b^2$
$= 4^2 + 3^2$
$= 25$
$\therefore c = \sqrt{25}$
$= 5$

$c^2 = a^2 + b^2$
$= (3 + 4)^2$
$= 49$
$\therefore c = 7$



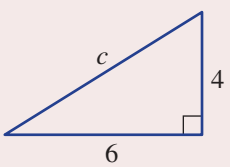
- Which set of working is incorrect? Give reasons.
- Why are the other two sets of working correct? Are they identical?

KEY IDEAS

- Using Pythagoras' theorem to find the length of the hypotenuse:
  - $c^2 = a^2 + b^2$
  - Substitute the shorter lengths for a and b.
  - Find the value of $a^2 + b^2$.
  - Take the square root.
  - This is called the exact value.
  - Calculate the square root of 52 to find the value of c.
  - Round to one decimal place.

$c^2 = a^2 + b^2$
$c^2 = 4^2 + 6^2$
$c^2 = 16 + 36$
$c^2 = 52$
$\therefore c = \sqrt{52}$

$c = 7.211\ldots$
$c = 7.2$ (to 1 d.p.)



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- Lengths can be expressed with exact values using **surds**. $\sqrt{2}$, $\sqrt{28}$ and $2\sqrt{3}$ are examples of surds.
- When expressed as a decimal, a surd is an infinite non-recurring decimal with no pattern; e.g. $\sqrt{2} = 1.4142135623\ldots$

  $\sqrt{2}$ cannot be expressed as a fraction. It is an example of an irrational number.

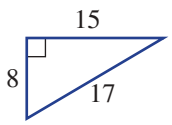
Key terminology: hypotenuse, Pythagoras' theorem, right-angled, substitute, surds, square root

Exercise 4B

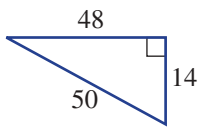
UNDERSTANDING 1–4 3, 4

1 Write down the length of the hypotenuse (c) in these right-angled triangles.

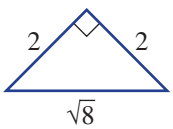
a



b



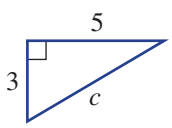
c



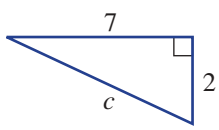
2 Write down Pythagoras' theorem using the given pronumerals and values for these right-angled triangles.

Hint: Write a statement such as $c^2 = 2^2 + 3^2$.

a



b



3 Identify the surds from the following list.

a $\sqrt{3}$ **b** 7 **c** 6.4

d $\sqrt{25}$ **e** $0.\dot{3}$ **f** $\sqrt{7}$

4 Find the value of c ($c > 0$) in the following using the square root. Round to one decimal place in parts **a** and **b** and answer as a surd in parts **c** and **d**.

a $c^2 = 45$

b $c^2 = 10 + 30$

c $c^2 = 22$

d $c^2 = 3^2 + 5^2$

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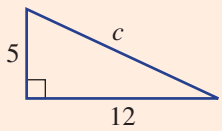
FLUENCY 5(½), 6 5–7(½)

Example 3 Finding the length of the hypotenuse

Find the length of the hypotenuse in this right-angled triangle.



SOLUTION

$c^2 = a^2 + b^2$

$= 5^2 + 12^2$

$= 169$

$\therefore c = \sqrt{169}$

$= 13$

Check:

$c^2 = 13^2$	$a^2 + b^2 = 5^2 + 12^2$
$= 169$ ✓	$= 169$ ✓

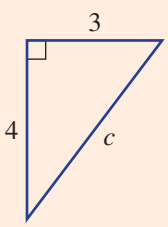
EXPLANATION

Write the rule and substitute the lengths of the two shorter sides. Any order is okay.

If $c^2 = 169$ then $c = \sqrt{169} = 13$.

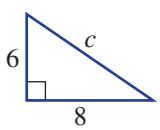
Now you try

Find the length of the hypotenuse in this right-angled triangle.

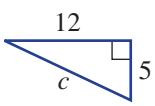


5 Find the length of the hypotenuse in each of the following right-angled triangles.

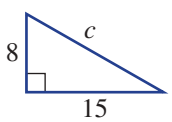
a



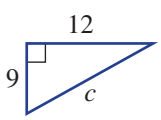
b



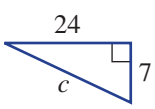
c



d



e



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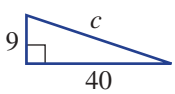
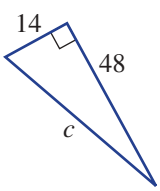


g



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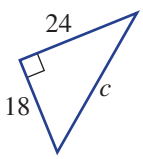
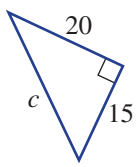


i



Hint: Substitute the two shorter sides for a and b in the equation $c^2 = a^2 + b^2$.

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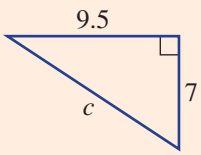
Example 4 Using rounding with Pythagoras' theorem

Find the length of the hypotenuse in this right-angled triangle, rounding to two decimal places.



SOLUTION

$c^2 = a^2 + b^2$

$= 7^2 + 9.5^2$

$= 139.25$

$\therefore c = \sqrt{139.25}$

$= 11.80$ (to 2 d.p.)

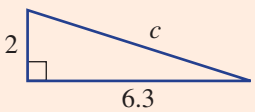
EXPLANATION

The order for a and b does not matter since $7^2 + 9.5^2 = 9.5^2 + 7^2$.

$\sqrt{139.25} = 11.8004\ \ldots$ and the third decimal place is zero, so round down.

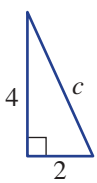
Now you try

Find the length of the hypotenuse in this right-angled triangle, rounding to two decimal places.

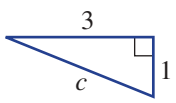


6 Find the length of the hypotenuse in each of these right-angled triangles, correct to two decimal places.

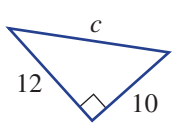
a



b



c



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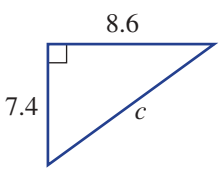

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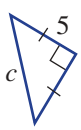

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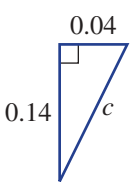

Hint: Use a calculator to find the value of c and round your answer to two decimal places.

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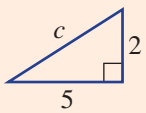
Example 5 Finding the length of the hypotenuse using exact values

Find the length of the hypotenuse in this right-angled triangle, leaving your answer as an exact value.



SOLUTION

$c^2 = a^2 + b^2$

$= 5^2 + 2^2$

$= 29$

$\therefore c = \sqrt{29}$

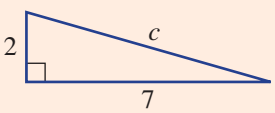
EXPLANATION

Apply Pythagoras' theorem to find the value of c.

Express the answer exactly using a surd.

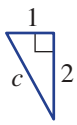
Now you try

Find the length of the hypotenuse in this right-angled triangle, leaving your answer as an exact value.



7 Find the length of the hypotenuse in these triangles, leaving your answer as an exact value.

a



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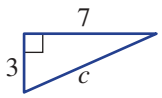
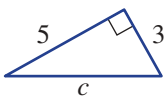


c



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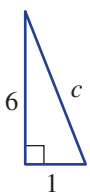

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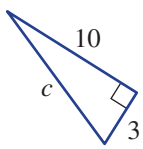

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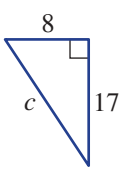
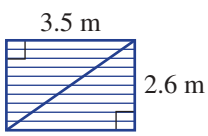


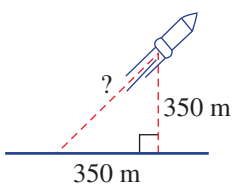
Hint: Leave your answer as a surd; e.g. $\sqrt{3}$ or $\sqrt{26}$.

PROBLEM-SOLVING AND REASONING 8–10, 11(½) | 10, 11(½), 12–14

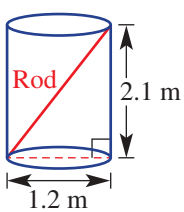
8 Find the length of the diagonal steel brace required to support a wall of length 3.5 m and height 2.6 m. Give your answer correct to one decimal place.



9 A miniature rocket blasts off at an angle of 45° and, after a few seconds, reaches a height of 350 m above the ground. At this point it has also covered a horizontal distance of 350 m. How far has the rocket travelled to the nearest metre?



10 Find the length of the longest rod, as shown, that will fit inside a cylinder of height 2.1 m and with circular end surface of 1.2 m diameter. Give your answer correct to one decimal place.



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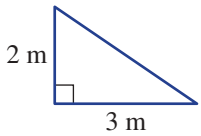
11 For each of these triangles, first calculate the length of the hypotenuse then find the perimeter, correct to two decimal places.

a



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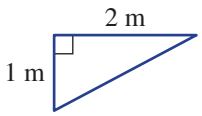

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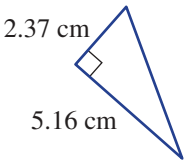

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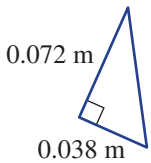

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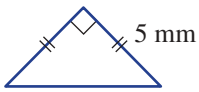

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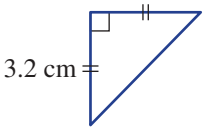
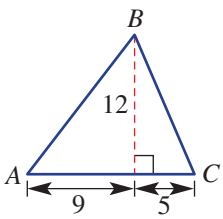


12 A helicopter hovers at a height of 150 m above the ground and is a horizontal distance of 200 m from a beacon on the ground. Find the direct distance of the helicopter from the beacon.

Hint: First draw a right-angled triangle.

13 Find the perimeter of this triangle.



Hint: You will need to find AB and BC first by considering two different triangles.

14 One way to check whether a four-sided figure is a rectangle is to make sure that both its diagonals are the same length. What should the length of the diagonals be if a rectangle has side lengths 3 m and 5 m? Answer to two decimal places.

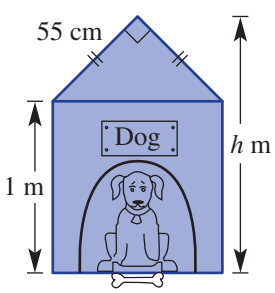
ENRICHMENT: The dog kennel — 15

15 A dog kennel has the dimensions shown in the diagram on the right. Give your answers to each of the following, correct to two decimal places.

a What is the width of the kennel?

b What is the total height, h m, of the kennel?

c If the sloping height of the roof was to be reduced from 55 cm to 50 cm, what difference would this make to the total height of the kennel? (Assume that the width is the same as in part **a**.)



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4C Finding the lengths of the shorter sides CONSOLIDATING

Learning intentions for this section:

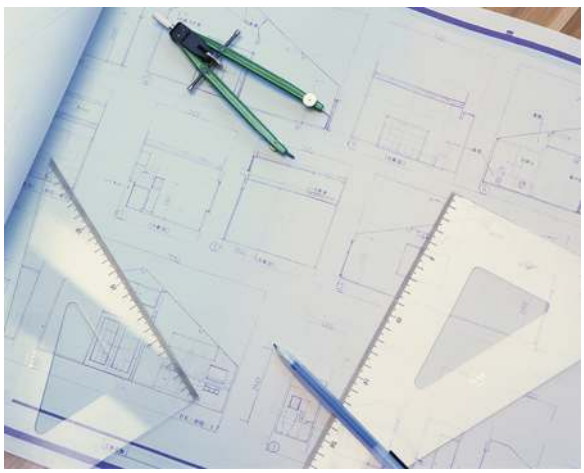
- To be able to set up Pythagoras' theorem where the unknown is a shorter side
- To be able to solve the resulting equation to find the length of a shorter side

Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of these questions are more challenging than those in the Year 8 book
- These concepts will be revised, briefly, in Stage 6 Standard
- You may be expected to recall the formula from memory in the HSC examination

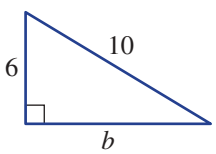
We know that the sum $7 = 3 + 4$ can be written as a difference: $3 = 7 - 4$ or $4 = 7 - 3$. Likewise, if $c^2 = a^2 + b^2$ then $a^2 = c^2 - b^2$ or $b^2 = c^2 - a^2$.

Applying this idea to a right-angled triangle means that we can now find the length of one of the shorter sides if the other two sides are known.



Lesson starter: True or false?

Below are some mathematical statements relating to a right-angled triangle with hypotenuse 10 and the two shorter sides 6 and b.



Some of these mathematical statements are true and some are false. Can you sort them into true and false groups?

$6^2 + b^2 = 10^2$ $\quad$ $6 = \sqrt{10^2 - b^2}$ $\quad$ $10^2 - 6^2 = b^2$ $\quad$ $6^2 - 10^2 = b^2$

$10 = \sqrt{6^2 + b^2}$ $\quad$ $b = \sqrt{6^2 - 10^2}$ $\quad$ $10 = \sqrt{6^2 - b^2}$ $\quad$ $10^2 - b^2 = 6^2$

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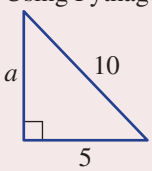
KEY IDEAS

■ Using Pythagoras' theorem to find a shorter side:



- Use $a^2 + b^2 = c^2$.
- Substitute the known lengths for b and c.
- Solve the equation.

$$a^2 + b^2 = c$$
$$a^2 + 5^2 = 10^2$$
$$a^2 + 25 = 100 \quad (-25 \text{ on both sides})$$
$$a^2 = 75$$
$$\therefore a = \sqrt{75}$$
$$a = 8.66...$$
$$a = 8.7 \text{ (to 1 d.p.)}$$

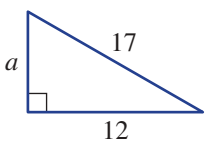
- This is called the exact value.
- Calculate the square root to find the value of a.
- Round to one decimal place.

Key terminology: Pythagoras' theorem, hypotenuse, equation, square root, surd

Exercise 4C

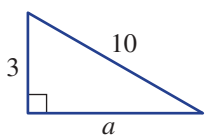
UNDERSTANDING 1–3 | 3

1 For the diagram shown, are the following true (T) or false (F)?



- **a** $a^2 + 12^2 = 17^2$
- **b** $a^2 = 12^2 + 17^2$
- **c** $12^2 + 17^2 = a^2$

2 For the diagram shown, are the following true (T) or false (F)?



- **a** $a^2 = 3^2 + 10^2$
- **b** $a^2 + 10^2 = 3^2$
- **c** $a^2 + 3^2 = 10^2$

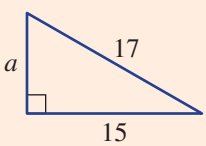
3 Find the positive value of a.

- **a** $a^2 = 144$
- **b** $a^2 = 400$
- **c** $a^2 + 9 = 25$
- **d** $a^2 + 49 = 625$
- **e** $a^2 + 36 = 100$
- **f** $a^2 + 15^2 = 17^2$

FLUENCY 4–5(½) | 4-5(½)

Example 6 Finding the length of a shorter side

Find the value of the pronumeral.



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SOLUTION

$a^2 + 15^2 = 17^2$

$a^2 + 225 = 289$

$a^2 = 64$

$\therefore a = \sqrt{64}$

$a = 8$

EXPLANATION

Write the rule and substitute the known sides.

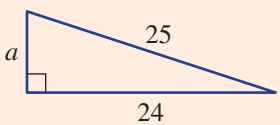
Square 15 and 17.

Subtract 225 from both sides.

Take the square root of both sides.

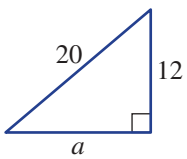
Now you try

Find the value of the pronumeral.

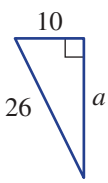


4 Find the value of the pronumeral.

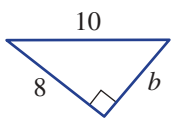
a



b

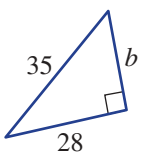


c

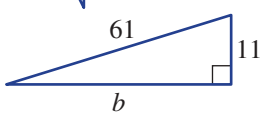


Hint: Substitute for a, b and c, then solve for the remaining pronumeral.

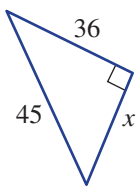
d



e

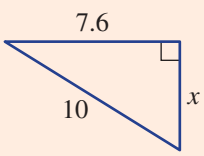


f



Example 7 Using rounding to approximate the length of a shorter side

Find the value of x, rounding to two decimal places.



SOLUTION

$a^2 + b^2 = c^2$

$x^2 + 7.6^2 = 10^2$

$x^2 + 57.76 = 100$

$x^2 = 42.24$

$\therefore x = \sqrt{42.24}$

$x = 6.50$ (to 2 d.p.)

EXPLANATION

Write the rule.

Substitute $c = 10$ and $b = 7.6$

Subtract 57.76 from both sides.

Take the square root of both sides.

Round to two decimal places.

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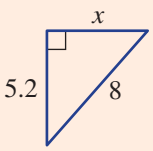
Now you try

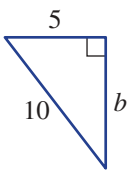
Find the value of x, rounding to two decimal places.



5 Find the value of the pronumeral. Express your answers correct to two decimal places.

Hint: Use your calculator to get your answer, then round to two decimal places.

a



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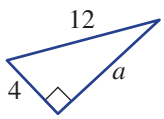
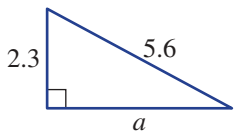


c



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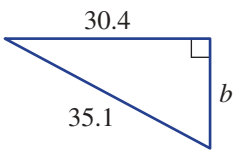

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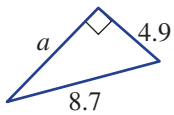

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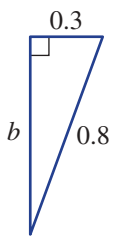
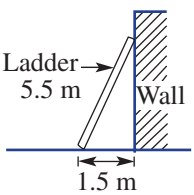


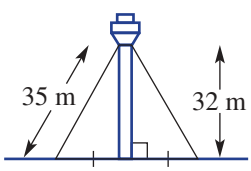
PROBLEM-SOLVING AND REASONING

6–8 | 7–10

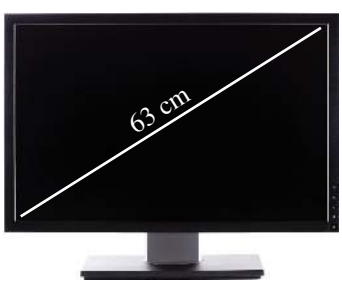
6 The base of a ladder leaning against a wall is 1.5 m from the base of the wall. The ladder is 5.5 m long. Find how high the top of the ladder is above the ground, correct to one decimal place.



7 A 32 m communication tower is supported by 35 m cables stretching from the top of the tower to a position at ground level. Find the distance from the base of the tower to the point where the cable reaches the ground, correct to one decimal place.



8 The diagonal of a rectangular computer screen is 63 cm long. If the vertical side is 39 cm long, how wide is the screen, correct to the nearest centimetre?



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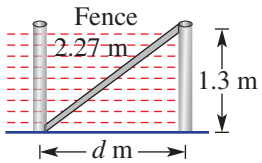
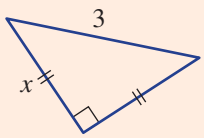
9 A 1.3 m vertical fence post is supported by a 2.27 m bar, as shown in the diagram on the right. Find the distance (d metres) from the base of the post to where the support enters the ground. Give your answer correct to two decimal places.



Example 8 Using Pythagoras' theorem and surds

Find the value of x, giving your answer as a surd.



SOLUTION

$x^2 + x^2 = 3^2$

$2x^2 = 9$

$x^2 = \frac{9}{2}$

$\therefore x = \sqrt{\frac{9}{2}}$ or $\frac{3}{\sqrt{2}}$

EXPLANATION

Two sides are of length x.

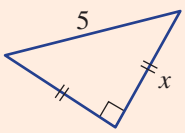
Add like terms.

Divide both sides by 2.

Take the square root of both sides. To express as an exact answer, do not calculate the square root.

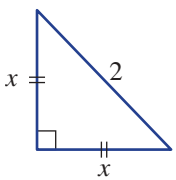
Now you try

Find the value of x, giving your answer as a surd.

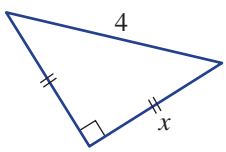


10 Find the value of x as an exact answer. Note that the triangles are isosceles.

a



b



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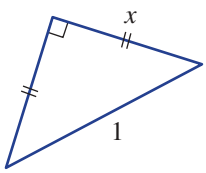
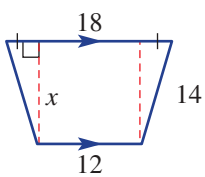


ENRICHMENT: First find a missing length — 11

11 Find the value of x. Give an exact answer each time.

a



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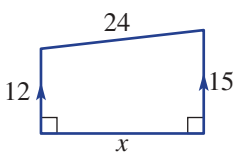

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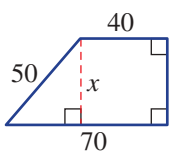

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4D Using Pythagoras' theorem to solve two-dimensional problems CONSOLIDATING

Learning intentions for this section:

- To be able to identify right-angled triangles in a two-dimensional situation
- To be able to use Pythagoras' theorem to solve a problem

Past, present and future learning:

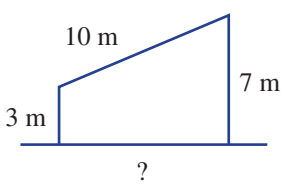
- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of these questions are more challenging than those in the Year 8 book
- These concepts will be revised, briefly, in Stage 6 Standard
- You may be expected to recall the formula from memory in the HSC examination

To apply Pythagoras' theorem to solve a real problem, it can help to first draw a right-angled triangle. This makes it easier to identify the unknown side. As long as two sides of the right-angled triangle are known, the length of the third side can be found.

The length of each cable on the Anzac Bridge, Sydney, can be calculated using Pythagoras' theorem.

Lesson starter: But where is the right-angled triangle?

A shed with two walls of length 3 m and 7 m has a sloping 10 m roof. We need to determine the distance between the two walls.



- Can you identify a right-angled triangle?
- What two side lengths on the right-angled triangle do you know?
- Show how Pythagoras' theorem can be used to find the unknown side.
- How would you put the answer in words?

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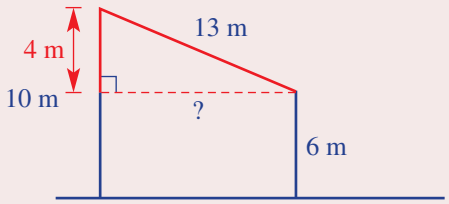
KEY IDEAS

- When applying Pythagoras' theorem to real-world problems:
  - Identify and draw right-angled triangles that may help to solve the problem (shown in red on diagram).
  - Label the sides with their lengths or with a letter (pronumeral) if the length is unknown.
  - Use Pythagoras' theorem to solve for the unknown.
  - Solve the problem by making any further calculations and answering in words.
  - Check that the answer that you have appears to be reasonable in the given situation.



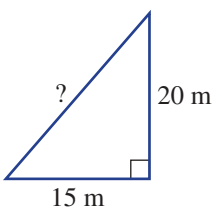
Key terminology: Pythagoras' theorem, right-angled

Exercise 4D

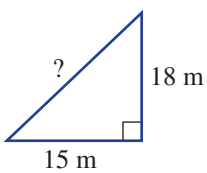
UNDERSTANDING 1, 2 | 2

1 Two vertical posts are connected at the top by a wire as shown. Which of the following triangles could be used to help find the cable length?

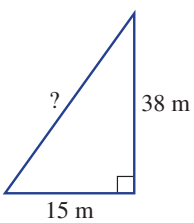
A

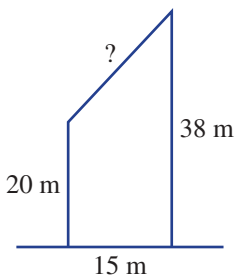


B



C

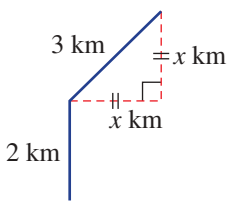




2 Match each problem (**a**, **b** or **c**) with both a diagram (**A**, **B** or **C**) and its solution (**I**, **II**, **III**).

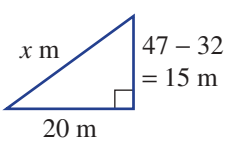
a Two trees stand 20 m apart. They are 32 m and 47 m tall. What is the distance between the tops of the two trees?

A



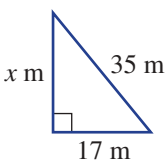
b A kite is flying with a kite string of length 35 m. Its horizontal distance from its anchor point is 17 m. How high is the kite flying?

B



c A man walks due north for 2 km then north-east for 3 km. How far north is he from his starting point?

C



I The kite is flying at a height of 30.59 m.

II The man has walked a total of $2 + 2.12 = 4.12$ km north from his starting point.

III The distance between the top of the two trees is 25 m.

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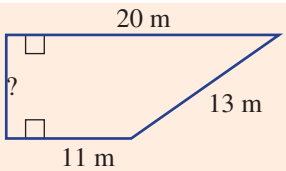
FLUENCY 3, 4, 6 | 3, 5, 6

Example 9 Identifying right-angled triangles in a diagram

An area of lawn is in the shape of a trapezium, as shown.

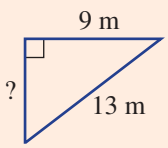
a Draw a right-angled triangle to help find the unknown side length.

b Find the unknown side length correct to two decimal places.



SOLUTION

a



b $a^2 + b^2 = c^2$

$a^2 + 9^2 = 13^2$

$a^2 + 81 = 169$

$a^2 = 88$

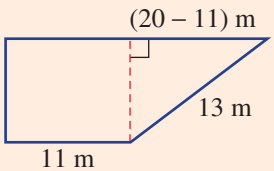
$\therefore a = \sqrt{88}$

$= 9.38$ (to 2 d.p.)

The unknown length is 9.38 m.

EXPLANATION

The top side of the triangle is 20 m − 11 m = 9 m.



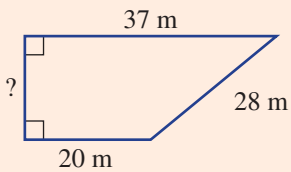
Use Pythagoras' theorem with the hypotenuse as 13 m. Solve for a and round as required.

Now you try

A paved area is in the shape of a trapezium, as shown.

a Draw a right-angled triangle to help find the unknown side length.

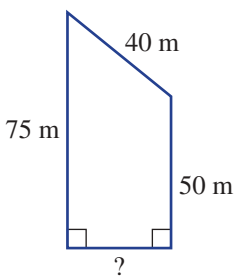
b Find the unknown side length correct to two decimal places.



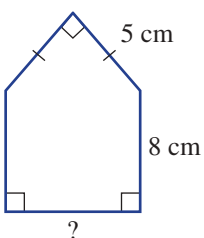
3 The side of a sculpture is in the shape of a trapezium, as shown.

a Draw a right-angled triangle to help find the unknown side length.

b Find the unknown length correct to two decimal places.



4 A scale drawing of the front of a dog kennel is shown. Find the unknown side length correct to two decimal places.



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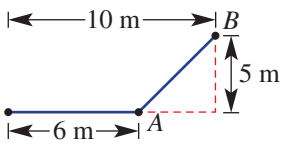
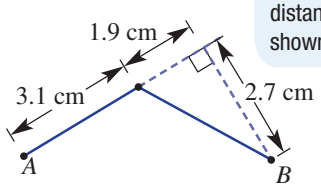
5 Find the direct distance between the points A and B in each of the following, correct to one decimal place.

Hint: Draw in the right-angled triangle that will help to find the distance AB. The first one is shown for you.

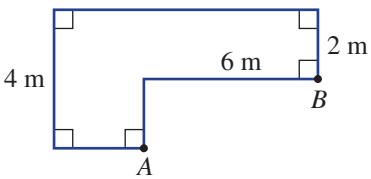
a



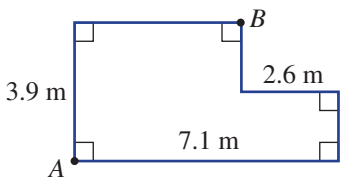
b



c

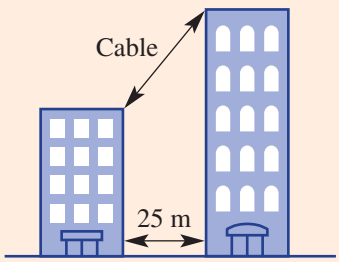


d



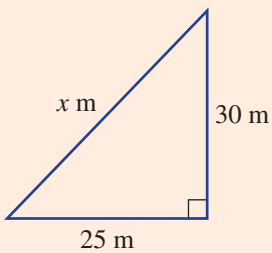
Example 10 Applying Pythagoras' theorem

Two skyscrapers are located 25 m apart and a cable links the tops of the two buildings. Find the length of the cable if the buildings are 50 m and 80 m in height. Give your answer correct to two decimal places.



SOLUTION

Let x m be the length of the cable.



$c^2 = a^2 + b^2$

$x^2 = 25^2 + 30^2$

$x^2 = 625 + 900$

$= 1525$

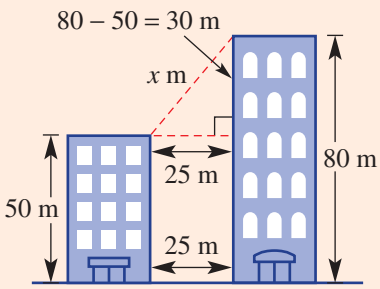
$\therefore x = \sqrt{1525}$

$= 39.05$ (to 2 d.p.)

$\therefore$ The cable is 39.05 m long.

EXPLANATION

Draw a right-angled triangle and label the measurements and pronumerals.



Set up an equation using Pythagoras' theorem and solve for x.

Answer the question in words.

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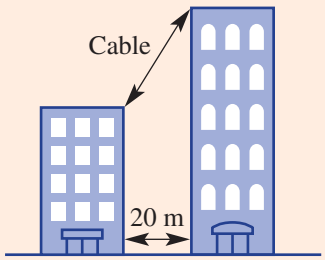
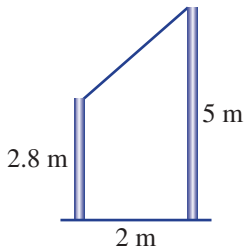
Now you try

Two buildings are 20 m apart. A cable links the top of the two buildings. Find the length of the cable if the buildings are 40 m and 65 m in height. Give your answer correct to two decimal places.



6 Two poles are located 2 m apart. A wire links the tops of the two poles. Find the length of the wire if the poles are 2.8 m and 5 m in height. Give your answer correct to one decimal place.



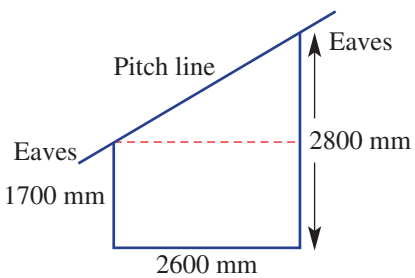
PROBLEM-SOLVING AND REASONING

7, 8 | 7, 9, 10

7 Two skyscrapers are located 25 m apart and a cable of length 62.3 m links the tops of the two buildings. If the taller building is 200 metres tall, what is the height of the shorter building? Give your answer correct to one decimal place.

Hint: First draw a right-angled triangle just like in Example 10. Then find the difference in their heights.

8 A garage is to be built with a skillion roof (a roof with a single slope). The measurements are given in the diagram. Calculate the pitch line length, to the nearest millimetre. Allow 500 mm for each of the eaves.



9 Two bushwalkers are standing on different mountain sides. According to their maps, one of them is at a height of 2120 m and the other is at a height of 1650 m. If the horizontal distance between them is 950 m, find the direct distance between the two bushwalkers. Give your answer correct to the nearest metre.

Hint: You will need to find the hypotenuse length of a right-angled triangle. Use a diagram like in Example 10.

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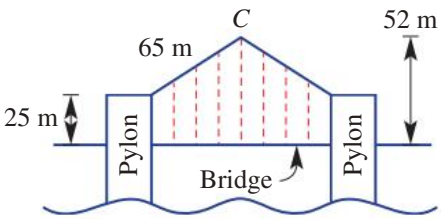
10 A 100 m radio mast is supported by six cables in two sets of three cables. They are anchored to the ground at an equal distance from the mast.

The top set of three cables is attached at a point 20 m below the top of the mast. Each cable in the lower set of three cables is 60 m long and is attached at a height of 30 m above the ground. If all the cables have to be replaced, find the total length of cable required. Give your answer correct to two decimal places.

Hint: First draw a diagram and add all the given information.

ENRICHMENT: The suspension bridge — 11

11 A suspension bridge is built with two vertical pylons and two straight beams of equal length. They extend from the top of the pylons to meet at a point, C, above the centre of the bridge, as shown in the diagram on the right.



a Calculate the vertical height of the point C above the tops of the pylons.

b Calculate the distance between the pylons; that is, the length of the span of the bridge correct to one decimal place.

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4E Introducing the trigonometric ratios

Learning intentions for this section:

- To know how to label the sides of a right-angled triangle as hypotenuse, opposite, adjacent
- To understand that the opposite and adjacent sides are relative to the angle involved
- To know the trigonometric ratios for sine, cosine and tangent
- To know that the trigonometric ratios remain constant when a triangle is enlarged
- To be able to use two sides, an angle and a trigonometric ratio to write an equation

Past, present and future learning:

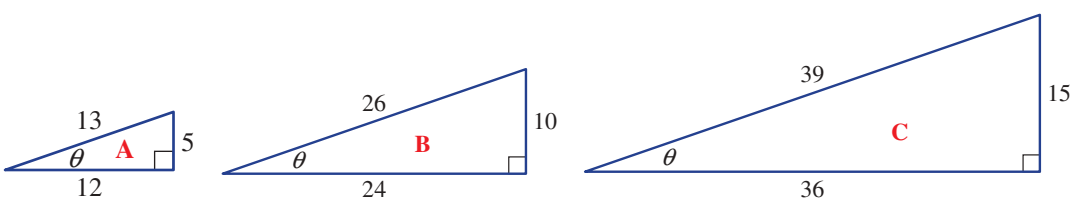
- This section introduces the Stage 5 Core Topic called *Trigonometry A*
- These concepts will be revised, briefly, in Stage 6 Standard
- You may be expected to recall the three ratios from memory in the HSC examination

Trigonometry deals with the relationship between the side lengths and angles in triangles. Trigonometry dates back to the ancient Egyptian and Babylonian civilisations when it was used in the building of pyramids and in the study of astronomy. In the first century CE, Claudius Ptolemy developed tables of values linking the sides and angles of a triangle. The Greek letter theta, θ, is often used to represent an unknown angle.

A basic form of trigonometry was used in the building of the pyramids.

Lesson starter: What is the sine ratio?

The triangles below are similar. They contain the same angles but triangles B and C are enlargements of triangle A.



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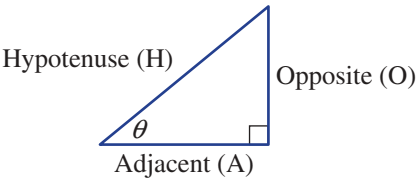
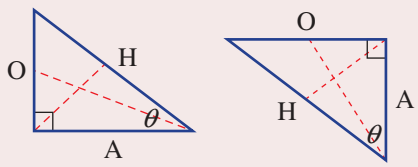


- In triangle A, the ratio $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{\square}{\square}$ (in simplest form)
- In triangle B, the ratio $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{\square}{\square} = \frac{\square}{\square}$ (in simplest form)
- In triangle C, the ratio $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{\square}{\square} = \frac{\square}{\square}$ (in simplest form)

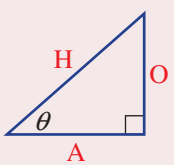
∴ When a right-angled triangle is enlarged, the angles stay the same and the sine ratio stays the same.

KEY IDEAS

- In this topic the Greek letter θ (pronounced theta) is often used for the size of an unknown angle.
- If a right-angled triangle has another angle θ, then the:
  - longest side is called the **hypotenuse** (H); it is opposite the right angle
  - side opposite θ is called the **opposite** (O)
  - remaining side is called the **adjacent** (A).



- For a right-angled triangle with a given angle θ, the three ratios **sine** (sin), **cosine** (cos) and **tangent** (tan) are given by the:
  - sine ratio: $\sin\theta = \frac{\text{length of the opposite side}}{\text{length of the hypotenuse}}$
  - cosine ratio: $\cos\theta = \frac{\text{length of the adjacent side}}{\text{length of the hypotenuse}}$
  - tangent ratio: $\tan\theta = \frac{\text{length of the opposite side}}{\text{length of the adjacent side}}$
- For any **right-angled** triangle with the same angles, these ratios are always the same.



- Memory aid: The letters **SOHCAHTOA** are useful for remembering the three ratios.

SOH CAH TOA

$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$ $\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$ $\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$

Key terminology: trigonometric ratio, opposite, adjacent, hypotenuse, sine, cosine, tangent, right-angled

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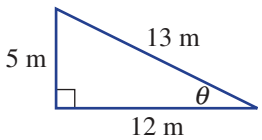
Exercise 4E

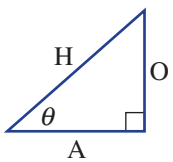
UNDERSTANDING	1, 2	2

1 For this triangle:



- **a** What is the length of the hypotenuse?
- **b** What is the length of the side opposite θ?
- **c** What is the length of the side adjacent to θ?

2 Write the missing word in these sentences.

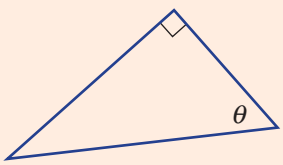


- **a** H stands for the word _____.
- **b** O stands for the word _____.
- **c** A stands for the word _____.
- **d** $\sin\theta =$ _____ ÷ Hypotenuse.
- **e** $\cos\theta =$ Adjacent ÷ _____.
- **f** $\tan\theta =$ Opposite ÷ _____.

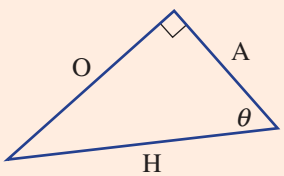
FLUENCY	3–5	3–4(½), 5, 6

Example 11 Labelling the sides of triangles

Copy this triangle and label the sides as opposite to θ (O), adjacent to θ (A) and hypotenuse (H).

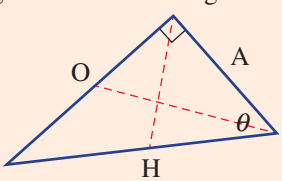


SOLUTION



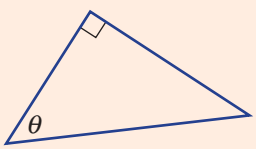
EXPLANATION

Draw the triangle and label the side opposite the right angle as hypotenuse (H), the side opposite the angle θ as opposite (O) and the remaining side next to the angle θ as adjacent (A).



Now you try

Copy this triangle and label the sides as opposite to θ (O), adjacent to θ (A) and hypotenuse (H).



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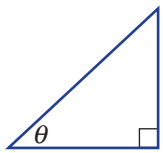
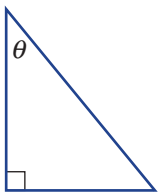
3 Copy each of these triangles and label the sides as opposite to θ (O), adjacent to θ (A) and hypotenuse (H).

a

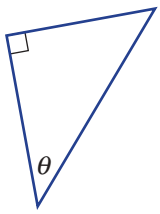


b

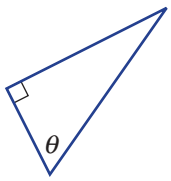


Hint: Alternatively, pencil the letters O, A and H onto this page.

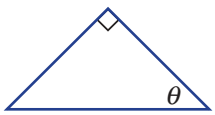
c



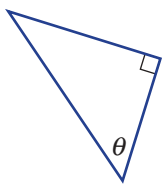
d



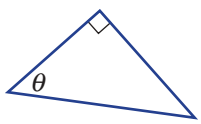
e



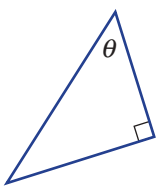
f



g



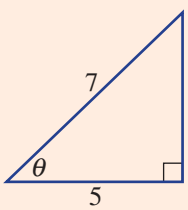
h



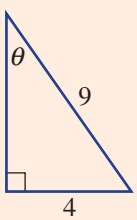
Example 12 Writing trigonometric ratios

Write a trigonometric ratio (in fraction form) for each of the following triangles.

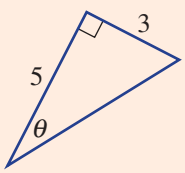
a



b



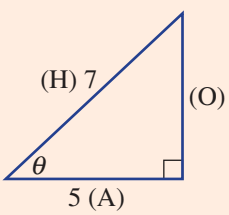
c



SOLUTION

a $\cos\theta = \frac{\text{A}}{\text{H}}$

$\cos\theta = \frac{5}{7}$

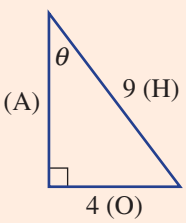


EXPLANATION

Side length 7 is opposite the right angle so it is the hypotenuse (H).
Side length 5 is adjacent to angle θ so it is the adjacent (A).
Use $\cos\theta$ since we have A and H.

b $\sin\theta = \frac{\text{O}}{\text{H}}$

$\sin\theta = \frac{4}{9}$



Side length 9 is opposite the right angle so it is the hypotenuse (H).
Side length 4 is opposite angle θ so it is the opposite (O).
Use $\sin\theta$ since we have *O* and H.

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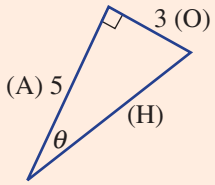
c $\tan\theta = \frac{O}{A}$

$\tan\theta = \frac{3}{5}$

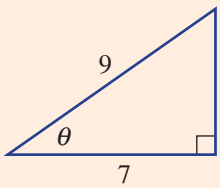


Side length 3 is opposite angle θ so it is the opposite (O).
Side length 5 is the adjacent side to angle θ so it is the adjacent (A).
Use $\tan\theta$ since we have O and A.

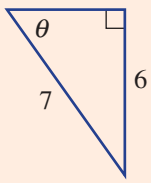
Now you try

Write a trigonometric ratio (in fraction form) for each of the following triangles.

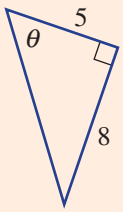
a



b



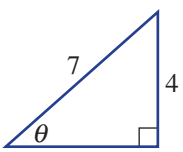
c



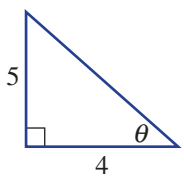
4 Write a trigonometric ratio (in fraction form) for each of the following triangles and simplify where possible.

Hint: Use $\sin\theta$ if you have O and H, use $\cos\theta$ if you have A and H, and use $\tan\theta$ if you have O and A.

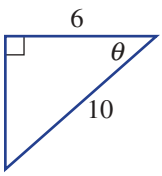
a



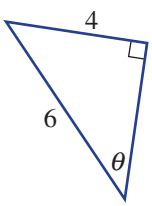
b



c



d



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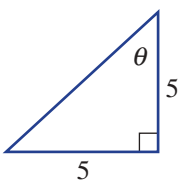

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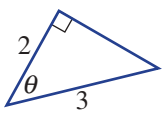
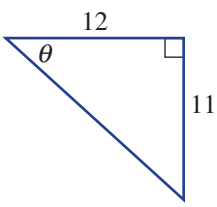
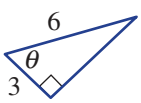


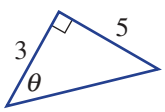
g



h

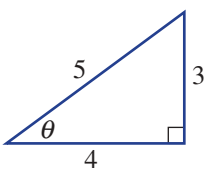


i

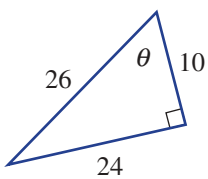


5 For each of these triangles, write a ratio (in simplified fraction form) for $\sin\theta$, $\cos\theta$ and $\tan\theta$.

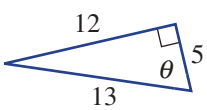
a



b



c



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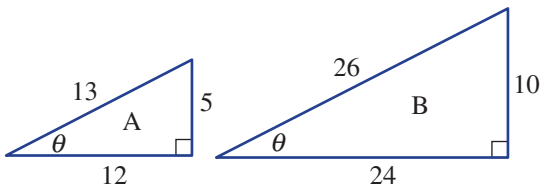
6 Here are two similar triangles, A and B.



Hint: Simplify your fractions from the larger triangle to help see the connection.

a i Write the ratio $\sin\theta$ (as a fraction) for triangle A.

ii Write the ratio $\sin\theta$ (as a fraction) for triangle B.

iii What do you notice about your two answers from parts **a i** and **a ii** above?

b i Write the ratio $\cos\theta$ (as a fraction) for triangle A.

ii Write the ratio $\cos\theta$ (as a fraction) for triangle B.

iii What do you notice about your two answers from parts **b i** and **b ii** above?

c i Write the ratio $\tan\theta$ (as a fraction) for triangle A.

ii Write the ratio $\tan\theta$ (as a fraction) for triangle B.

iii What do you notice about your two answers from parts **c i** and **c ii** above?

PROBLEM-SOLVING AND REASONING

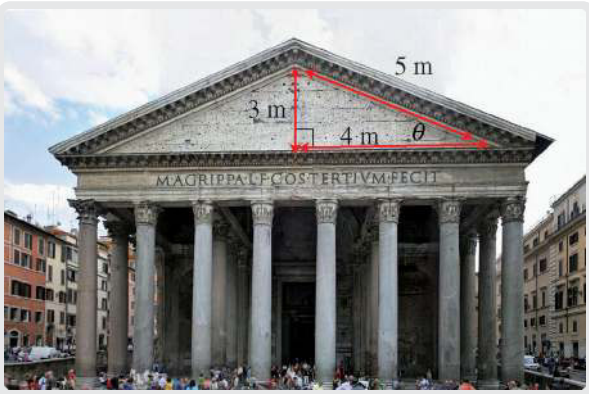
7–9 | 8–11

7 The facade of a Roman temple has the given measurements. Write down the ratio for:

a $\sin\theta$

b $\cos\theta$

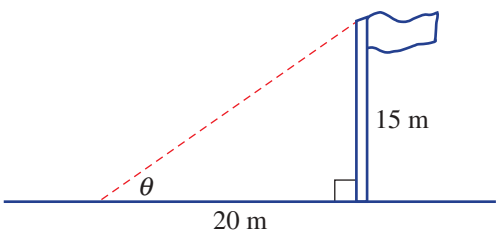
c $\tan\theta$



The Pantheon, a Roman temple that was built in 126 CE.

8 A vertical flag pole casts a shadow 20 m long. If the pole is 15 m high, find the ratio for $\tan\theta$.

Hint: Decide which side is the opposite to θ and which side is adjacent to θ.



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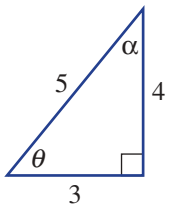
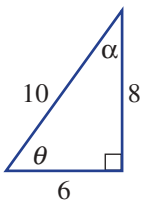
9 For the triangle shown on the right, state the length of the side that corresponds to:



- **a** the hypotenuse
- **b** the side opposite angle θ
- **c** the side opposite angle α
- **d** the side adjacent to angle θ
- **e** the side adjacent to angle α.

10 For the triangle shown on the right, write a ratio (in fraction form) for:



- **a** $\sin\theta$ **b** $\sin\alpha$ **c** $\cos\theta$
- **d** $\tan\alpha$ **e** $\cos\alpha$ **f** $\tan\theta$

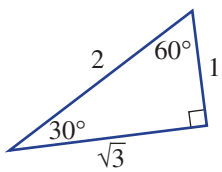
11 **a** Draw a right-angled triangle and mark one of the angles as θ. Mark in the length of the opposite side as 15 units and the length of the hypotenuse as 17 units.

Hint: Use $a^2 + b^2 = c^2$ for part **b** with $c = 17$ and $b = 15$.

- **b** Using Pythagoras' theorem, find the length of the adjacent side.
- **c** Determine the ratios for $\sin\theta$, $\cos\theta$ and $\tan\theta$.

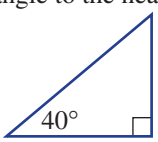
ENRICHMENT: The complementary connection — 12, 13

12 This triangle has angles 90°, 60° and 30° and side lengths 1, 2 and $\sqrt{3}$.



- **a** Write a ratio for:
  - **i** $\sin 30°$ **ii** $\cos 30°$ **iii** $\tan 30°$
  - **iv** $\sin 60°$ **v** $\cos 60°$ **vi** $\tan 60°$
- **b** What do you notice about the following pairs of ratios?
  - **i** $\cos 30°$ and $\sin 60°$ **ii** $\sin 30°$ and $\cos 60°$

13 **a** Measure all the side lengths of this triangle to the nearest millimetre.



- **b** Use your measurements from part **a** to find an approximate ratio for:
  - **i** $\cos 40°$ **ii** $\sin 40°$ **iii** $\tan 40°$ **iv** $\sin 50°$ **v** $\tan 50°$ **vi** $\cos 50°$
- **c** Do you notice anything about the trigonometric ratios for 40° and 50°?

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4F Finding unknown sides

Learning intentions for this section:

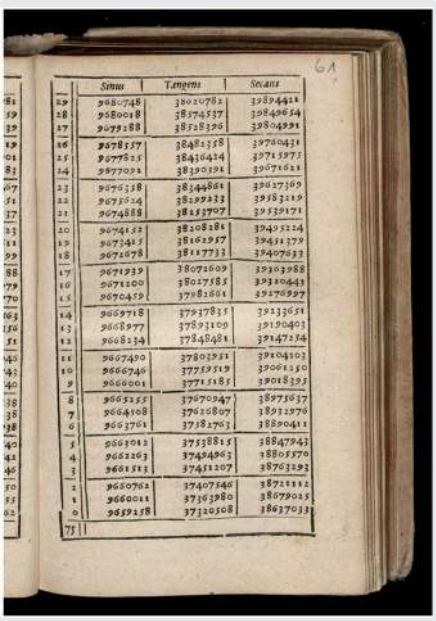
- To know how to use a calculator to evaluate the sine, cosine or tangent of an angle
- To be able to use a trigonometric ratio to set up an equation to find a missing side length
- To know that one angle and one side must be known to find another side length
- To be able to use a trigonometric ratio to find an unknown side length

Past, present and future learning:

- This section introduces the Stage 5 Core Topic called *Trigonometry A*
- These concepts will be revised, briefly, in Stage 6 Standard
- You may be expected to recall the three ratios from memory in the HSC examination

Since ancient times, mathematicians have attempted to tabulate the three trigonometric ratios for varying angles. Here are the ratios for some angles in a right-angled triangle, correct to three decimal places.

Angle (θ)	sin θ	cos θ	tan θ
0°	0	1	0
15°	0.259	0.966	0.268
30°	0.5	0.866	0.577
45°	0.707	0.707	1
60°	0.866	0.5	1.732
75°	0.966	0.259	3.732
90°	1	0	undefined



Trigonometric tables in a 400-year-old European book

In modern times these values can be evaluated using calculators to a high degree of accuracy and can be used to help solve problems involving triangles with unknown side lengths.

Angles can be expressed in:

- Whole degrees, such as 36°
- Decimals, such as 36.5°
- Degrees and minutes, just like hours and minutes, such as 36°30′, where every degree contains 60 minutes.

Lesson starter: Calculator start up

Calculators can produce accurate values of $\sin\theta$, $\cos\theta$ and $\tan\theta$.

- Ensure that your calculator is in degree mode.
- Find the value of the following, correct to three decimal places.
  - $\sin 50°$
  - $\cos 81.5°$
  - $\tan 36°15′$
- Use trial and error to find (to the nearest degree) an angle, θ, which satisfies these conditions.
  - $\sin\theta = 0.454$
  - $\cos\theta = 0.588$
  - $\tan\theta = 9.514$
- Try to get a more accurate answer to the nearest minute.

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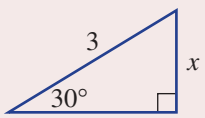
KEY IDEAS

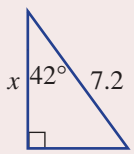
- If θ is in degrees, the ratios for $\sin\theta$, $\cos\theta$ and $\tan\theta$ can accurately be found using a calculator in degrees mode. For this topic, ensure that your calculator is in 'degrees' mode.
- Your calculator has a button for degrees, minutes, seconds. Find it and learn how to use it.
  Note: $25.5° = 25°30'$ (because half of $1°$ is $30'$)
  $25.25° = 25°15'$ (because one quarter of $1°$ is $15'$)
- If the angles and one side length of a right-angled triangle are known, then the other side lengths can be found using the $\sin\theta$, $\cos\theta$ or $\tan\theta$ ratios.



$\sin 30° = \frac{x}{3}$

$x = 3 \times \sin 30°$

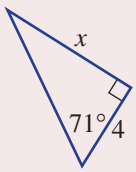
$= 1.5$



$\cos 42° = \frac{x}{7.2}$

$x = 7.2 \times \cos 42°$

≈ 5.35 (to 2 d.p.)



$\tan 71° = \frac{x}{4}$

$x = 4 \times \tan 71°$

≈ 11.62 (to 2d.p.)

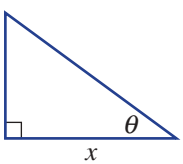
Key terminology: trigonometric ratio, opposite, adjacent, hypotenuse, sine, cosine, tangent, right-angled, numerator

Exercise 4F

UNDERSTANDING 1–3 2, 3

1 For the marked angle θ, decide if x represents the length of the opposite (O), adjacent (A) or hypotenuse (H) side.

a



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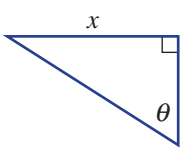

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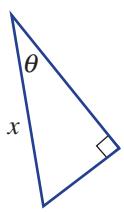
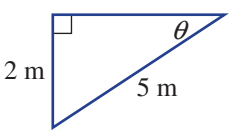


2 Which two sides (choose a pair from O, A and H) are given in these triangles?

a



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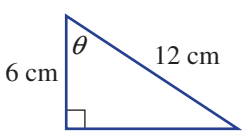

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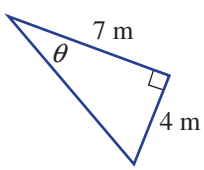

Hint: Choose either O, A or O, H or A,H.

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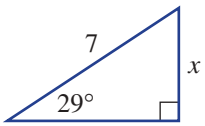
3 Decide whether you would use $\sin\theta = \frac{O}{H}$, $\cos\theta = \frac{A}{H}$ or $\tan\theta = \frac{O}{A}$ to help find the value of x in these triangles. Do not find the value of x; just state which ratio you would use.

a



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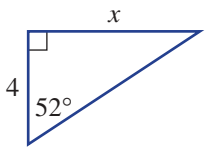

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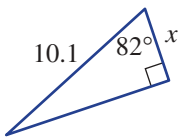

FLUENCY 4–6(½) 4–6(½)

Example 13 Using a calculator

Use a calculator to evaluate the following, correct to two decimal places.

a $\sin 50°$ **b** $\cos 16.1°$ **c** $\tan 77°18'$

SOLUTION

a $\sin 50° = 0.766044\ldots$
$= 0.77$ (to 2 d.p.)

b $\cos 16.1° = 0.9607\ldots$
$= 0.96$ (to 2 d.p.)

c $\tan 77°18' = 4.4373\ldots$
$= 4.44$ (to 2 d.p.)

EXPLANATION

The 3rd decimal place is greater than 4, so round up.

The 3rd decimal place is less than 5, so round down.

Your calculator has a special button for degrees and minutes. One degree is equal to 60 minutes.

Now you try

Use a calculator to evaluate the following, correct to two decimal places.

a $\sin 20°$
b $\cos 38.5°$
c $\tan 67°25'$

4 Use a calculator to evaluate the following, correct to two decimal places.

a $\sin 20°$
b $\cos 37°$
c $\tan 64°$
d $\sin 47°$
e $\cos 84°$
f $\tan 14.1°$
g $\sin 27°24'$
h $\cos 76°12'$
i $\sin 81°$
j $\cos 56°10'$
k $\tan 34°15'$
l $\cos 24°12'$

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Example 14 Solving for x in the numerator of a trigonometric ratio

Find the value of x in the equation $\cos 20° = \frac{x}{3}$, correct to two decimal places.

SOLUTION

$\cos 20° = \frac{x}{3}$

$x = 3 \times \cos 20°$

$= 2.82$ (to 2 d.p.)

EXPLANATION

Multiply both sides of the equation by 3 and round as required.

Swap sides to put x on the left.

Now you try

Find the value of x in the equation $\cos 40° = \frac{x}{5}$, correct to two decimal places.

5 Find the value of x correct to two decimal places.

a $\sin 50° = \frac{x}{4}$ **b** $\tan 81° = \frac{x}{3}$ **c** $\cos 33° = \frac{x}{6}$

d $\cos 75° = \frac{x}{3.5}$ **e** $\sin 24° = \frac{x}{4.2}$ **f** $\tan 42° = \frac{x}{10}$

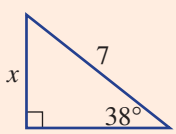
g $\frac{x}{7.1} = \tan 18.4°$ **h** $\frac{x}{5.3} = \sin 64°42'$ **i** $\frac{x}{12.6} = \cos 52°54'$

Hint: For part **a**, for example, multiply by 4 and use a calculator to evaluate $4 \times \sin 50°$.

Example 15 Finding side lengths

For each triangle, find the value of x correct to two decimal places.

a



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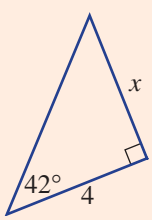

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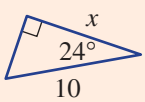


SOLUTION

a $\sin 38° = \frac{O}{H}$

$\sin 38° = \frac{x}{7}$

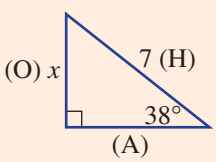
$x = 7 \sin 38°$

$= 4.3096\ldots$

$= 4.31$ (to 2 d.p.)

EXPLANATION

Since the opposite side (O) and the hypotenuse (H) are involved, the sin θ ratio must be used. Use $\theta = 38°$.



Multiply both sides by 7 and evaluate using a calculator.

Continued on next page

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b $\tan 42° = \frac{O}{A}$

$\tan 42° = \frac{x}{4}$

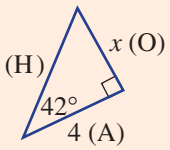
$x = 4 \tan 42°$

$= 3.6016 \ldots$

$= 3.60$ (to 2 d.p.)

Since the opposite side (O) and the adjacent side (A) are involved, the $\tan \theta$ ratio must be used. Use $\theta = 42°$.

Multiply both sides by 4 and evaluate.



c $\cos 24° = \frac{A}{H}$

$\cos 24° = \frac{x}{10}$

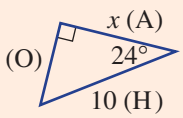
$x = 10 \cos 24°$

$= 9.135 \ldots$

$= 9.14$ (to 2 d.p.)

Since the adjacent side (A) and the hypotenuse (H) are involved, the $\cos \theta$ ratio must be used.

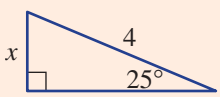
Multiply both sides by 10.



Now you try

For each triangle, find the value of x correct to two decimal places.

a



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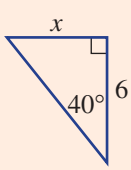

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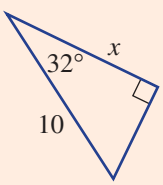
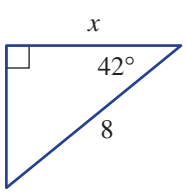


6 Find the value of x correct to two decimal places.

Hint: First decide which sides you have, including the unknown (O, A or H), then choose the correct ratio ($\sin \theta$, $\cos \theta$ or $\tan \theta$).

a



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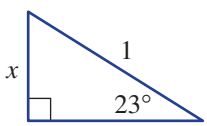

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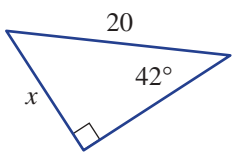

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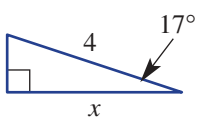

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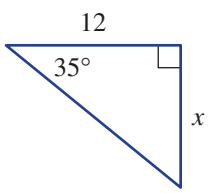

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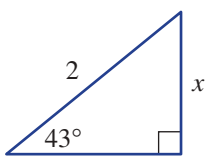

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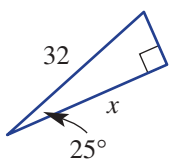

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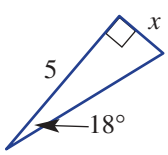

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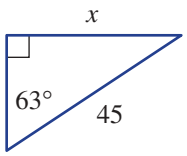

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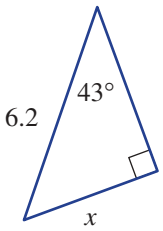

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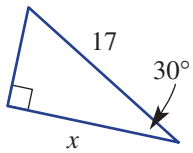

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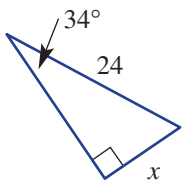

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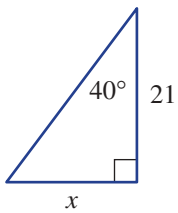

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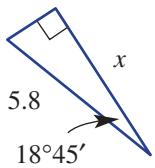
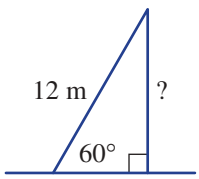


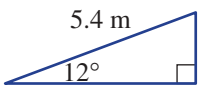
PROBLEM-SOLVING AND REASONING

7–9 | 8–11

7 A pole is supported by a 12 m cable that makes a 60° angle with the ground. How tall is the pole, correct to two decimal places?

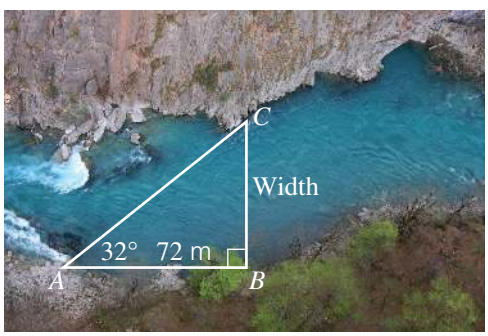


8 Amy walks 5.4 m up a ramp that is inclined at 12° to the horizontal. How high (correct to two decimal places) is she above her starting point?



9 Kane wanted to measure the width of a river at a certain point. He placed two markers, *A* and *B*, 72 m apart along the bank. *C* is a point directly opposite marker *B*. Kane measured angle *CAB* to be 32°. Find the width of the river correct to two decimal places.

Hint: The 'width' is opposite (O) the 32° angle and the 72 m length is adjacent (A) to the 32° angle.



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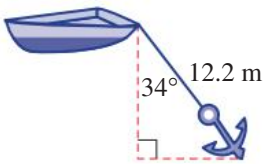
10 One end of a 12.2 m rope is tied to a boat. The other end is tied to an anchor, which is holding the boat steady in the water. If the anchor is making an angle of 34° with the vertical, how deep is the water? Give your answer correct to two decimal places.



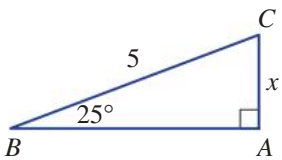
Hint: The depth length is adjacent (A) to the 34° angle.

11 For this right-angled triangle:

a Find the value of angle $\angle C$.

b Calculate the value of x correct to three decimal places using the sine ratio.

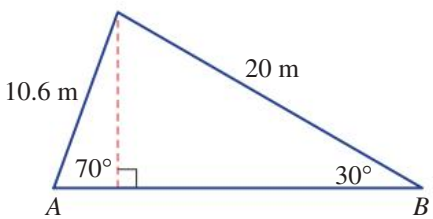
c Calculate the value of x correct to three decimal places but instead use the cosine ratio.



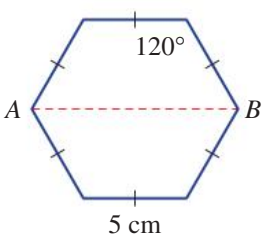
ENRICHMENT: Combining multiple trigonometric calculations — 12

12 Find the length AB in these diagrams. A combination of calculations is required. Round to two decimal places where necessary.

a



b



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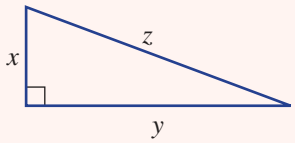
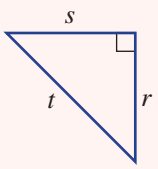
Progress quiz

4A **1** Write down Pythagoras' theorem for these triangles using the pronumerals and numbers given.

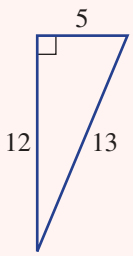
a



b

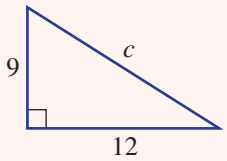


c

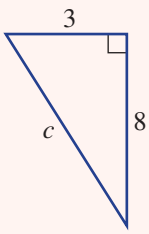


4B **2** Find the length of the hypotenuse in these right-angled triangles. Give exact answers for parts **a** and **b** and round to one decimal place in part **c**.

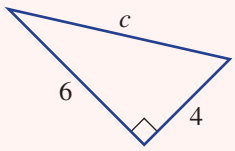
a



b

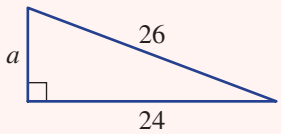


c

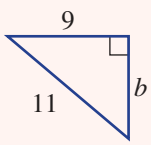


4C **3** Find the value of the pronumeral in these right-angled triangles. Round to two decimal places where necessary.

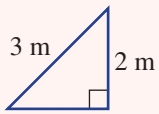
a



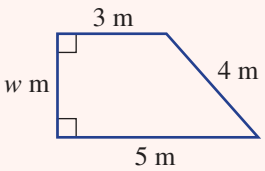
b



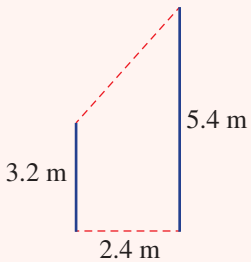
4C **4** A 3 m wooden plank is supporting an advertising billboard that is 2 m high. What is the distance of the plank from the base of the billboard? Round to one decimal place.



4D **5** A backyard deck is in the shape of a trapezium as shown. By drawing an appropriate right-angled triangle, find the width of the deck, w metres, correct to two decimal places.



4D **6** Two telegraph poles 3.2 m and 5.4 m high are 2.4 m apart. A wire connects the top of the two poles. Find the length of the wire correct to one decimal place.



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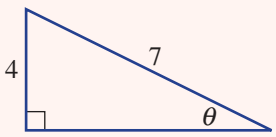
Progress quiz

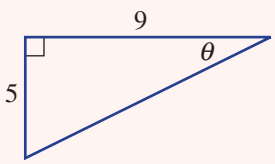
4E

7 Write a trigonometric ratio (in fraction form) for each of the following triangles.

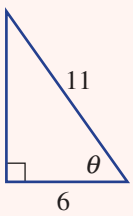
a



b



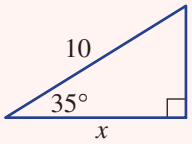
c



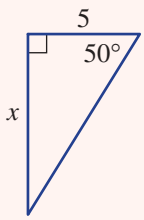
4F

8 Find the value of x, correct to two decimal places, in each of the following triangles.

a

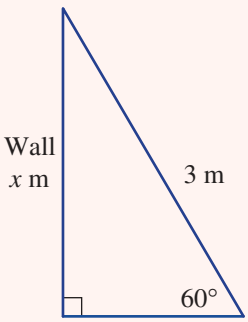


b



4F

9 A ladder leaning against a wall makes a 60° angle with the ground as shown. If the ladder is 3 m long, how far up the wall (x metres) does it reach? Round to one decimal place.



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4G Solving for the denominator

Learning intentions for this section:

- To be able to use a trigonometric ratio, then find an unknown in the denominator

Past, present and future learning:

- This section introduces the Stage 5 Core Topic called *Trigonometry A*
- These concepts will be revised, briefly, in Stage 6 Standard
- You may be expected to recall the three ratios from memory in the HSC examination

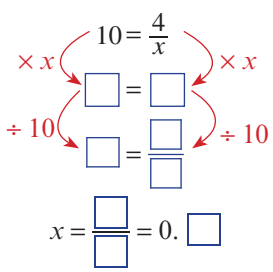
So far we have constructed trigonometric ratios using a pronumeral that has always appeared in the **numerator**: for example, $\sin 40° = \frac{x}{5}$. When the pronumeral appears in the **denominator**, there are a number of algebraic steps that can be taken to find the solution.

Lesson starter: Unknown in the denominator

In the equation $10 = \frac{4}{x}$, the unknown is in the denominator.

Read the instructions and fill in the blanks.

- Write the equation.
- Multiply both sides by x.
- Divide both sides by 10.
- Simplify the answer.



Solve these equations.

1 $8 = \frac{5}{x}$

2 $3 = \frac{4}{x}$

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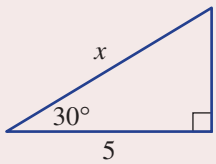
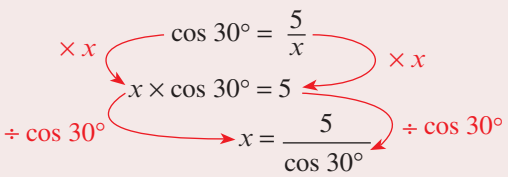
KEY IDEAS

- If the unknown value of a trigonometric ratio is in the **denominator**, you need to rearrange the equation.

For example, for the triangle shown:

- Multiply both sides by x (removes the fraction).
- Divide both sides by $\cos 30°$ ($\cos 30°$ is just a number).





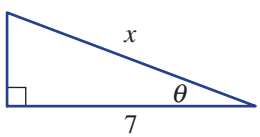
Key terminology: trigonometric ratio, opposite, adjacent, hypotenuse, sine, cosine, tangent, right-angled, denominator

Exercise 4G

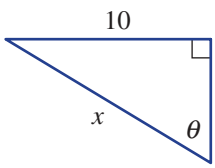
UNDERSTANDING 1–3 | 3

1 Which trigonometric ratio ($\sin\theta$, $\cos\theta$ or $\tan\theta$) would be used to find the value of x in these triangles?

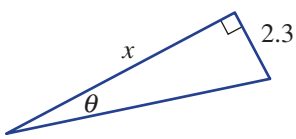
a



b



c



2 Solve these simple equations for x.

a $\frac{4}{x} = 2$ **b** $\frac{20}{x} = 4$

c $\frac{15}{x} = 5$ **d** $25 = \frac{100}{x}$

Hint: For part **a**, first multiply by x, then divide by 2.

3 Which of these equations has x in the denominator of the fraction?

a $\frac{x}{2} = \sin 40°$ **b** $\frac{3}{x} = \cos 70°$

c $\frac{7.1}{x} = \tan 34°$ **d** $\cos 47° = \frac{x}{5}$

FLUENCY 4–5(½) | 4–5(½)

Example 16 Solving for x in the denominator

Solve for x in the equation $\cos 35° = \frac{2}{x}$, correct to two decimal places.

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SOLUTION

$\cos 35° = \frac{2}{x}$

$x \cos 35° = 2$

$x = \frac{2}{\cos 35°}$

$= 2.4415 \ldots$

$= 2.44$ (to 2 d.p.)

EXPLANATION

Multiply both sides of the equation by x.

Divide both sides of the equation by $\cos 35°$.

Evaluate and round off to two decimal places.

Now you try

Solve for x in the equation $\tan 40° = \frac{3}{x}$, correct to two decimal places.

4 Find the value of x correct to two decimal places.

a $\cos 43° = \frac{3}{x}$ **b** $\sin 36° = \frac{4}{x}$ **c** $\tan 9° = \frac{6}{x}$

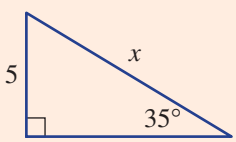
d $\tan 64° = \frac{2}{x}$ **e** $\cos 67° = \frac{5}{x}$ **f** $\sin 12° = \frac{3}{x}$

g $\sin 38.3° = \frac{5.9}{x}$ **h** $\frac{45}{x} = \tan 21°24'$ **i** $\frac{18.7}{x} = \cos 32°15'$

Hint: First multiply both sides by x.

Example 17 Finding side lengths

Find the value of x correct to two decimal places.



SOLUTION

$\sin 35° = \frac{O}{H}$

$\sin 35° = \frac{5}{x}$

$x \sin 35° = 5$

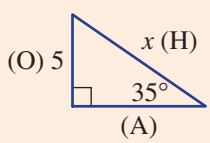
$x = \frac{5}{\sin 35°}$

$= 8.717 \ldots$

$= 8.72$ (to 2 d.p.)

EXPLANATION

Since the opposite side (O) is given and we require the hypotenuse (H), use $\sin \theta$.

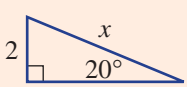


Multiply both sides of the equation by x then divide both sides of the equation by $\sin 35°$.

Evaluate on a calculator and round off to two decimal places.

Now you try

Find the value of x correct to two decimal places.



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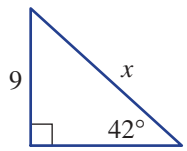
5 Find the value of x correct to two decimal places using the sine, cosine or tangent ratios.

a



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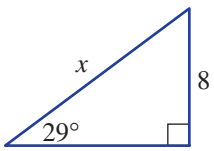

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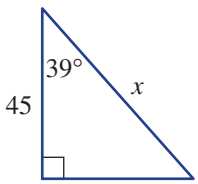
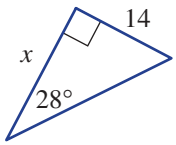
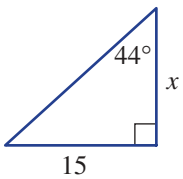


Hint: Are you going to use $\sin\theta = \frac{O}{H}$, $\cos\theta = \frac{A}{H}$ or $\tan\theta = \frac{O}{A}$?

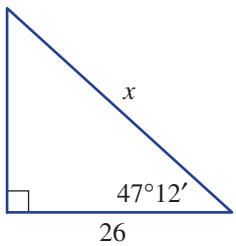
d



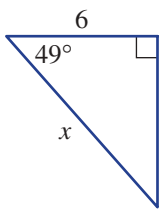
e



f



g



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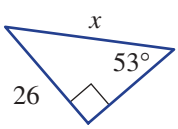
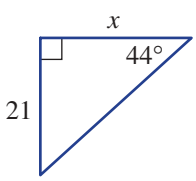


i



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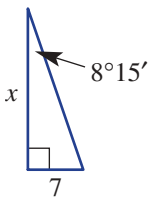

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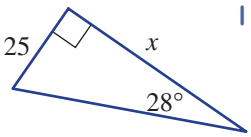

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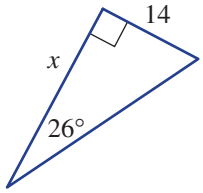
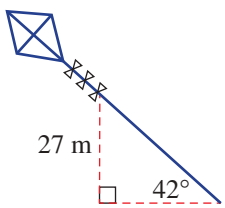


PROBLEM-SOLVING AND REASONING

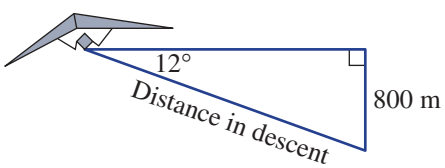
6–8 | 7–10

6 A kite is flying at a height of 27 m above the anchor point. If the string is inclined at 42° to the horizontal, find the length of the string correct to the nearest metre.



Hint: First decide if you are going to use $\sin\theta$ (SOH), $\cos\theta$ (CAH) or $\tan\theta$ (TOA).

7 A hang glider flying at a height of 800 m descends at an angle of 12° to the horizontal. How far (to the nearest metre) has it travelled in descending to the ground?



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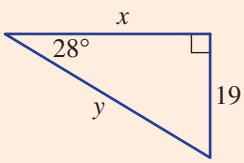
Example 18 Finding two side lengths on one triangle

Find the value of x and y correct to two decimal places.



SOLUTION

$$\tan 28° = \frac{O}{A}$$

$$\tan 28° = \frac{19}{x}$$

$$x \tan 28° = 19$$

$$x = \frac{19}{\tan 28°}$$

$$= 35.7338\ldots$$

$$= 35.73 \text{ (to 2 d.p.)}$$

$$\sin 28° = \frac{19}{y}$$

$$y \sin 28° = 19$$

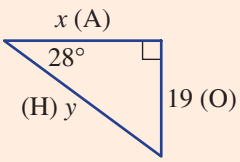
$$y = \frac{19}{\sin 28°}$$

$$= 40.4710\ldots$$

$$= 40.47 \text{ (to 2 d.p.)}$$

EXPLANATION

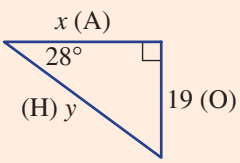
Since the opposite side (O) is given and the adjacent (A) is required, use $\tan\theta$.



Multiply both sides of the equation by x.

Divide both sides of the equation by $\tan 28°$ and round answer to two decimal places.

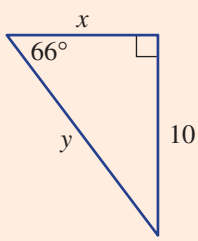
Use the opposite (19) as it is an exact length, so use $\sin\theta = \frac{O}{H}$.



Alternatively, y can be found by Pythagoras' theorem.

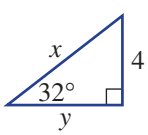
Now you try

Find the value of x and y correct to two decimal places.



8 Find the value of each pronumeral correct to one decimal place.

a



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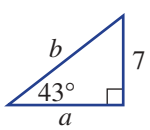

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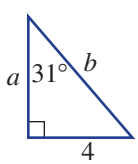

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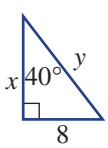
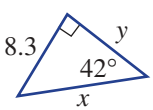


e



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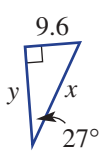

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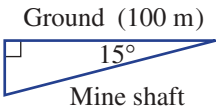
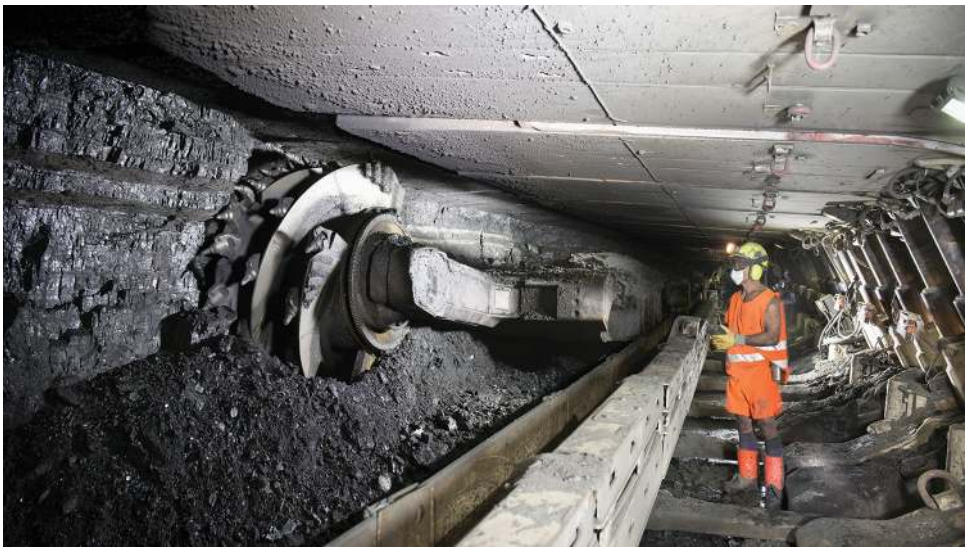
9 A mine shaft is dug at an angle of 15° to the horizontal. The horizontal length of the mine is 100 m. Answer the questions to the nearest metre.



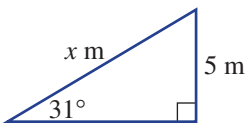
a How far below ground level is the end of the shaft?

b How long is the mine shaft?



10 In calculating the value of x for this triangle, correct to two decimal places, two students came up with these answers.

A $x = \dfrac{5}{\sin 31°} = \dfrac{5}{0.52} = 9.62$ B $x = \dfrac{5}{\sin 31°} = 9.71$



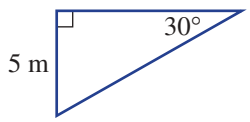
a Which of the above two answers is more correct and why?

b What advice would you give to the student whose answer is not accurate?

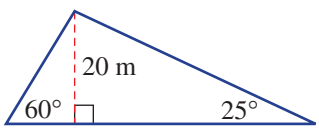
ENRICHMENT: Trigonometry and perimeters — 11

11 Find the perimeter of these triangles correct to one decimal place. You will first need to find the lengths of all the sides.

a



b



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4H Finding unknown angles

Learning intentions for this section:

- To know that two sides must be known to find an unknown angle in a right-angled triangle
- To be able to evaluate the unknown angle using a calculator

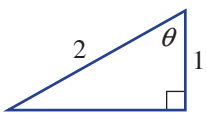
Past, present and future learning:

- This section introduces the Stage 5 Core Topic called *Trigonometry A*
- These concepts will be revised, briefly, in Stage 6 Standard
- You may be expected to recall the three ratios from memory in the HSC examination

Given two sides of a right-angled triangle, we can use trigonometry to calculate one of the angles.

By using one of the three trigonometric ratios, we can use inverse trigonometric functions to find a value of θ.

For example, if $\cos\theta = 0.5$, then
$\theta = \cos^{-1}(0.5) = 60°$

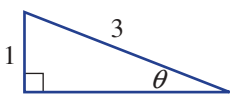


Learn how to do this on your calculator.

Surveyors use trigonometry to measure the height and angles of land.

Lesson starter: Trial and error can be slow

We know that for this triangle, $\sin\theta = \frac{1}{3}$

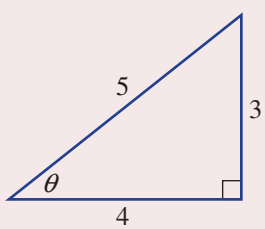


- Guess the angle θ.
- For your guess, use a calculator to see if $\sin\theta = \frac{1}{3}$ $(= 0.333\ldots)$ for your chosen value of θ.
- Update your guess and use your calculator to check once again.
- Repeat this trial-and-error process until you think you have the angle θ correct to three decimal places.
- Now use your calculator to evaluate $\sin^{-1}\left(\frac{1}{3}\right)$ and check your guess.

KEY IDEAS

- Your calculator has functions for solving an equation such as $\sin\theta = \frac{3}{5}$.
  'sin^{-1}' is written above the 'sin' button.
  You must learn how to use this function.

- Use your calculator to check the following solutions:
  - $\sin\theta = \frac{3}{5}$ means $\theta = 37°$ (to nearest degree)
  - $\cos\theta = \frac{4}{5}$ means $\theta = 37°$ (to nearest degree)
  - $\tan\theta = \frac{3}{4}$ means $\theta = 37°$ (to nearest degree)



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■ When finding an unknown angle, you can express your answer in many different ways.

$\sin\theta = \frac{3}{5}$ or $\sin\theta = \frac{3}{5}$ or $\sin\theta = \frac{3}{5}$

$\theta = 37°$ (nearest degree) $\theta = 36.9°$ (1 d.p.) $\theta = 37°52'$ (nearest minute)

Key terminology: trigonometric ratio, opposite, adjacent, hypotenuse, sine, cosine, tangent

Exercise 4H

UNDERSTANDING 1–3 3

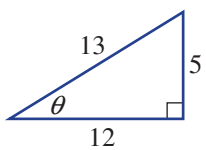
1 Are these statements true (T) or false (F)?

a If $\cos\theta = \frac{1}{2}$ then $\theta = 60°$.

b If $\tan\theta = 1$ then $\theta = 45°$.

c If $\sin\theta = \frac{1}{2}$ then $\theta = 60°$.

2 Refer to the diagram on the right to complete the following:



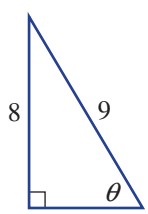
a $\sin\theta = \frac{\square}{\square}$

b $\cos\theta = \frac{\square}{\square}$

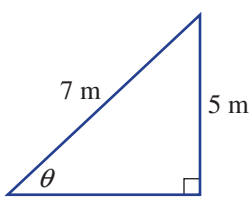
c $\tan\theta = \frac{\square}{\square}$

3 Which trigonometric ratio should be used to solve for θ?

a

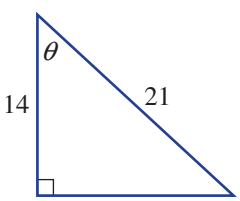


b

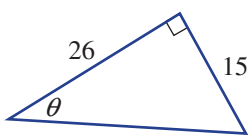


Hint: Choose $\sin\theta$ if using O and H, $\cos\theta$ if using A and H, and $\tan\theta$ if using O and A.

c



d



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FLUENCY 4–6(½) 4–6(½)

Example 19 Using inverse trigonometric ratios

Find the value of θ to the nearest degree if $\sin\theta = 0.3907$.

SOLUTION

$\sin\theta = 0.3907$

$\theta = 22.998\ldots$

$= 23°$ (to nearest degree)

EXPLANATION

Use the $\sin^{-1}$ key on your calculator

Round off to the nearest whole number.

Now you try

Find the value of θ to the nearest degree if $\sin\theta = 0.464$.

4 Find the value of θ to the nearest degree.

a $\sin\theta = 0.5$ **b** $\cos\theta = 0.5$

c $\tan\theta = 1$ **d** $\cos\theta = 0.8660$

e $\sin\theta = 0.7071$ **f** $\tan\theta = 0.5774$

g $\sin\theta = 1$ **h** $\tan\theta = 1.192$

i $\cos\theta = 0$ **j** $\cos\theta = 0.5736$

k $\cos\theta = 1$ **l** $\sin\theta = 0.9397$

Hint: For part **a**, press $\sin^{-1}(0.5)$ on your calculator.

Example 20 Using inverse trigonometric ratios with fractions

Find the value of θ correct to two decimal places if $\tan\theta = \frac{1}{2}$.

SOLUTION

$\tan\theta = \frac{1}{2}$

$\theta = 26.56505\ldots$

$= 26.57°$ (to 2 d.p.)

EXPLANATION

Use the $\tan^{-1}$ key on your calculator and round the answer to two decimal places.

Now you try

Find the value of θ correct to two decimal places if $\cos\theta = \frac{2}{3}$.

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5 Find the angle θ correct to two decimal places.

a $\sin\theta = \frac{4}{7}$ **b** $\sin\theta = \frac{1}{3}$ **c** $\sin\theta = \frac{9}{10}$

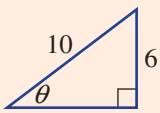
d $\cos\theta = \frac{1}{4}$ **e** $\cos\theta = \frac{4}{5}$ **f** $\cos\theta = \frac{7}{9}$

g $\tan\theta = \frac{3}{5}$ **h** $\tan\theta = \frac{8}{5}$ **i** $\tan\theta = 12$

Hint: For part **a**, type $\sin^{-1}\left(\frac{4}{7}\right)$ on your calculator.

Example 21 Finding an angle correct to the nearest minute

Find the value of θ to the nearest minute.



SOLUTION

$\sin\theta = \frac{O}{H}$

$= \frac{6}{10}$

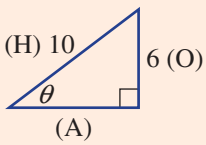
$\theta = 36.869\ldots°$

$\theta = 36°52'11.63''$

$\theta = 36°52'$ (nearest minute)

EXPLANATION

Since the opposite side (O) and the hypotenuse (H) are given, use $\sin\theta$.

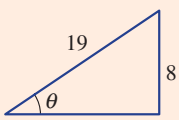


Use the $\sin^{-1}$ key on your calculator. Use your calculator to convert the angle into degrees, minutes and seconds.

Round off to the nearest minute. Since there are 60 seconds in a minute, round up for 30 seconds or more, or round down for less than 30 seconds.

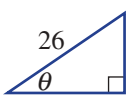
Now you try

Find the value of θ to the nearest minute.



6 Find the value of θ to the nearest minute.

a



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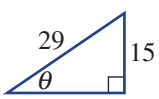

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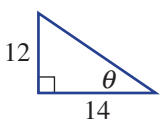
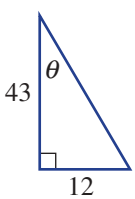
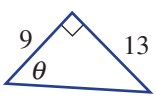


Hint: First set up an equation (like those in Question **5**), then use a calculator to find θ.

d



e



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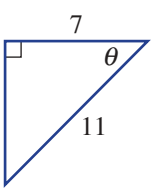

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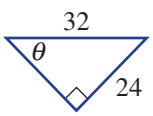

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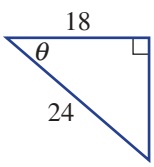

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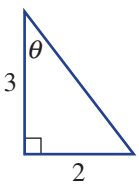

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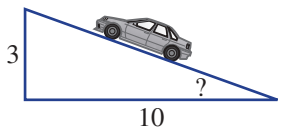
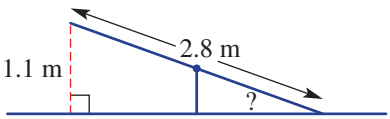
PROBLEM-SOLVING AND REASONING

7–9 | 9–12

7 A road rises at a grade of 3 in 10. Find the angle (to the nearest degree) the road makes with the horizontal.



8 When a 2.8 m long seesaw is at its maximum height, it is 1.1 m off the ground. What angle (correct to two decimal places) does the seesaw make with the ground?

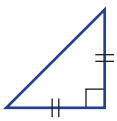


Hint: The 1.1 m length is opposite the angle and the 2.8 m length forms the hypotenuse.

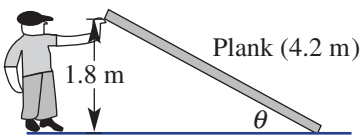
9 Consider the triangle on the right.

a Calculate $\tan 45°$ on your calculator.

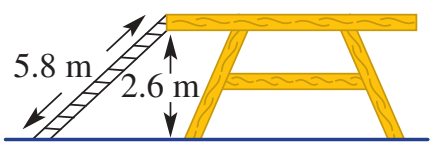
b Explain why $\tan 45°$ is exactly 1.



10 Adam holds one end of a plank of wood 1.8 m above the ground. The other end of the plank rests on the ground. The plank of wood is 4.2 m long. Find the angle that the plank makes with the ground (θ), correct to one decimal place.



11 A ladder has a length of 5.8 m. The ladder is resting against the top of a platform that is 2.6 m high. Find the angle the ladder makes with the ground, correct to one decimal place.



12 For what value of θ is $\sin\theta = \cos\theta$? Choose a value of θ between 0° and 90°.

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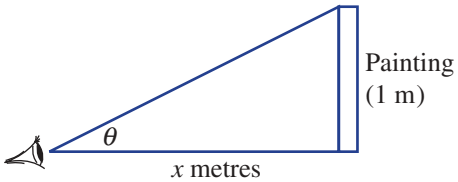
ENRICHMENT: Viewing angle — 13

13 Joe has trouble with his eyesight but every Sunday he goes to view his favourite painting at the gallery. His eye level is at the same level as the base of the painting and the painting is 1 metre tall.



Answer the following to the nearest degree for angles and to two decimal places for lengths.

a If $x = 3$, find the viewing angle θ.

b If $x = 2$, find the viewing angle θ.

c If Joe can stand no closer than 1 metre to the painting, what is Joe's largest viewing angle?

d When the viewing angle is 10°, Joe has trouble seeing the painting. How far is he from the painting at this viewing angle?

e What would be the largest viewing angle if Joe could go as close as he would like to the painting?

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4I Using trigonometry to solve problems

Learning intentions for this section:

- To be able to solve word problems by drawing a diagram then using a trigonometric ratio
- To know that angles of elevation and depression are measured from the horizontal
- To be able to work with angles of elevation and depression in diagrams and word problems

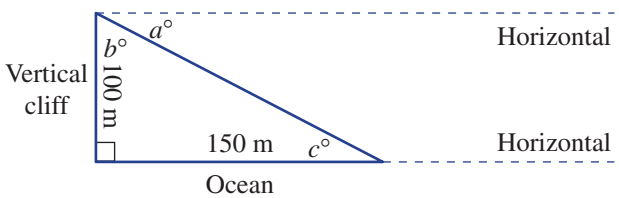
Past, present and future learning:

- This section includes Stage 5 Core concepts from *Trigonometry A* and *Trigonometry B*
- In Chapter 8 of our Year 10 book, we extend this topic to include non-right-angled triangles
- You may be expected to recall the three ratios from memory in the HSC examination

In many practical problems, trigonometry can be used to find unknown lengths and angles. Two special angles are called the angle of elevation and the angle of depression. These angles are measured from the horizontal.

Lesson starter: Draw and measure

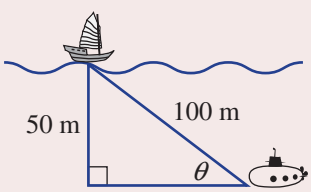
Use a scale where 1 millimetre represents 1 metre to draw a neat, accurate scale drawing of this diagram.



Use a protractor to estimate the values of a, b and c. Do a and c have the same value? Do you have the same values for a, b and c as other people in your class?

KEY IDEAS

- To solve problems involving trigonometry:
  - Draw a diagram and label the key information.
  - Identify and draw the appropriate right-angled triangles.
  - Use trigonometry to find the unknown measurements.
  - Check that your answer is reasonable.
  - Express your answer in words.



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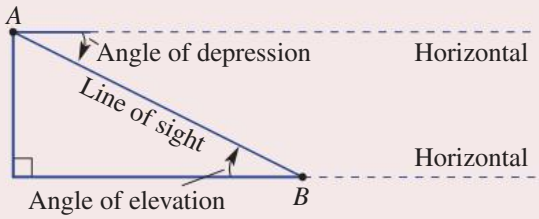
■ **Angles of elevation** and **depression** are always measured from the horizontal.



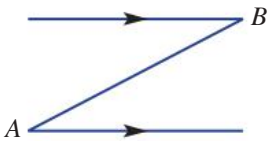
■ The **horizontal** lines are parallel. Therefore, the angle of depression and the angle of elevation are equal alternate angles.

Key terminology: angle of elevation, angle of depression and horizontal

Exercise 4I

UNDERSTANDING | 1–4 | 3, 4

1 Draw this diagram and complete these tasks.



a Mark in the following.

i The angle of elevation (θ) of B from A

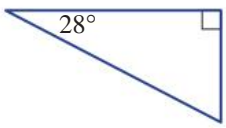
ii The angle of depression (α) of A from B

b Is $\theta = \alpha$ in your diagram? Why?

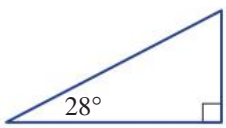
2 Choose the diagram (**A**, **B** or **C**) that matches this situation. A boy views a kite at an angle of elevation of 28°.

A

B



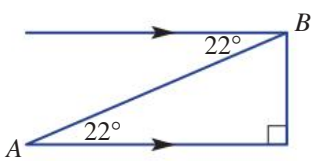
C



3 For this diagram:

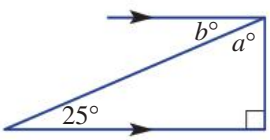
a What is the angle of elevation of B from A?

b What is the angle of depression of A from B?



Hint: Why do you think the two angles are equal in this diagram?

4 Find the values of the pronumerals in this diagram.



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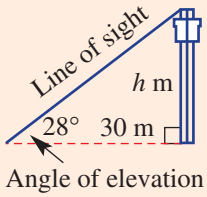
FLUENCY 5, 7, 9, 10 | 5, 6, 8–10

Example 22 Using angles of elevation

The angle of elevation of the top of a tower from a point on the ground 30 m away from the base of the tower is 28°. Find the height of the tower to the nearest metre.



SOLUTION

Let the height of the tower be h m.

$$\tan 28° = \frac{\text{O}}{\text{A}}$$

$$= \frac{h}{30}$$

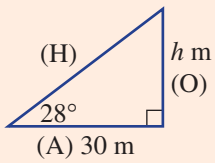
$$h = 30 \tan 28°$$

$$= 15.95\ldots$$

The height is 16 m, to the nearest metre.

EXPLANATION

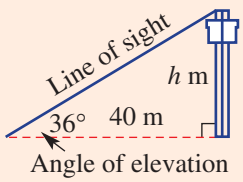
Since the opposite side (O) is required and the adjacent (A) is given, use $\tan\theta$.



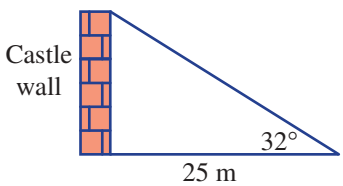
Multiply both sides by 30 and evaluate, rounding to the nearest metre.
Write the answer in words.

Now you try

The angle of elevation of the top of a tower from a point on the ground 40 m from the base of the tower is 36°. Find the height of the tower to the nearest metre.



5 The angle of elevation of the top of a castle wall from a point on the ground 25 m from the base of the castle wall is 32°. Find the height of the castle wall to the nearest metre.



6 From a point on the ground, Emma measures the angle of elevation of an 80 m tower to be 27°. Find how far Emma is from the base of the tower, correct to the nearest metre.

Hint: First draw a simple triangle and label your side lengths and given angle.

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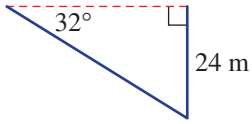
7 From a pedestrian overpass, Chris spots a landmark at an angle of depression of 32°. How far away (to the nearest metre) is the landmark from the base of the 24 m high overpass?



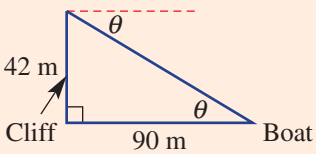
8 From a lookout tower, David spots a bushfire at an angle of depression of 25°. If the lookout tower is 42 m high, how far away (to the nearest metre) is the bushfire from the base of the tower?

Hint: Be sure to mark in the angle of depression below the horizontal.

Example 23 Finding an angle of depression

From the top of a vertical cliff, Andrea spots a boat out at sea. If the top of the cliff is 42 m above sea level and the boat is 90 m away from the base of the cliff, find Andrea's angle of depression to the boat to the nearest degree.

SOLUTION



$\tan\theta = \frac{O}{A}$

$\tan\theta = \frac{42}{90}$

$\theta = 25.0168\ldots$

$\theta = 25°$ (to nearest degree)

The angle of depression is 25°.

EXPLANATION

Draw a diagram and label all the given measurements. The angle of depression is below the horizontal.
Use alternate angles in parallel lines to mark θ inside the triangle.

Since the opposite (O) and adjacent sides (A) are given, use $\tan\theta$.

Use the $\tan^{-1}$ key on your calculator and round off to the nearest degree.

Express the answer in words.

Now you try

From the top of a lighthouse, a yacht is spotted out at sea. If the top of the lighthouse is 34 m above sea level and the yacht is 120 m away from the base of the lighthouse (at sea level), find the angle of depression from the top of the lighthouse to the yacht to the nearest degree.

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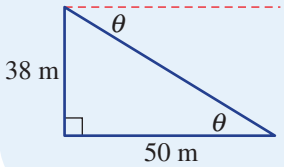
9 From the top of a vertical cliff, Josh spots a swimmer out at sea If the top of the cliff is 38 m above sea level and the swimmer is 50 m away from the base of the cliff, find the angle of depression from Josh to the swimmer, to the nearest degree.

Hint: Start with a diagram like this.



10 From a ship, a person is spotted floating in the sea 200 m away. If the viewing position on the ship is 20 m above sea level, find the angle of depression from the ship to the person in the sea. Give your answer to the nearest degree.

Hint: Draw a diagram first. The angle of depression is below the horizontal.

PROBLEM-SOLVING AND REASONING

11, 12 | 11–14

11 A power line is stretched from a pole to the top of a house. The house is 4.1 m high and the power pole is 6.2 m high. The horizontal distance between the house and the power pole is 12 m. Find the angle of elevation of the top of the power pole from the top of the house, to the nearest degree.

12 A road has a steady gradient of 1 in 10. It rises 1 m for every 10 m across.

a What angle does the road make with the horizontal? Give your answer to the nearest degree.

b A car starts from the bottom of the inclined road and drives 2 km along the road. How high, vertically, has the car climbed? Use your rounded answer from part **a** and give your answer correct to the nearest metre.

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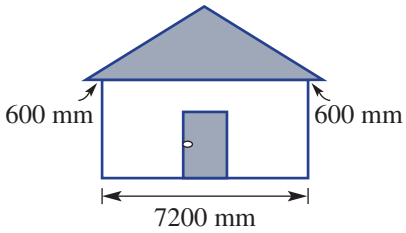
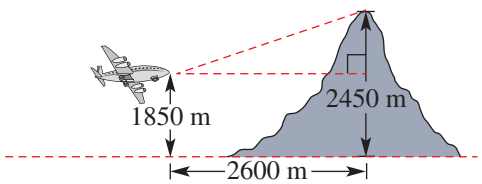
13 A house is to be built using the design shown on the right. The eaves are 600 mm and the house is 7200 mm wide, excluding the eaves. Calculate the length (to the nearest mm) of a sloping edge of the roof, which is pitched at 25° to the horizontal.



14 A plane flying at 1850 m starts to climb at an angle of 18° to the horizontal when the pilot sees a mountain peak 2450 m high, 2600 m away from him in a horizontal direction. Will the pilot clear the mountain?



ENRICHMENT: Plane trigonometry — 15

15 An aeroplane takes off and climbs at an angle of 20° to the horizontal, at 190 km/h along its flight path for 15 minutes.

a Find:

i the distance the aeroplane travels in 15 minutes

ii the height the aeroplane reaches after 15 minutes correct to two decimal places.

b If the angle at which the plane climbs is twice the original angle but its speed is halved, will it reach a greater height after 15 minutes?

c If the plane's speed is doubled and its climbing angle is halved, will the plane reach a greater height after 15 minutes?

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Maths@Work

Carpenter

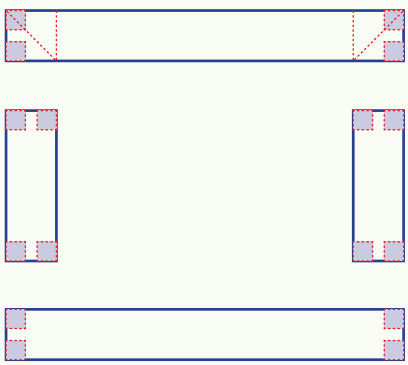
Carpenters work with wood, building house frames, verandahs or decks, kitchen cabinets, built-in wardrobes and so on. They often work outdoors and have to be proficient with tools such as augers, drills, saws, chisels, rulers, squares, compasses and levels.

As with most trades, mathematical skills are used regularly. Carpenters must be able to measure, convert between units, order material and calculate lengths using measuring devices, as well as apply Pythagoras' theorem and trigonometry rules.

1 Carpenters work in millimetres as their base unit for measurement of length. Convert each of the following into millimetres.

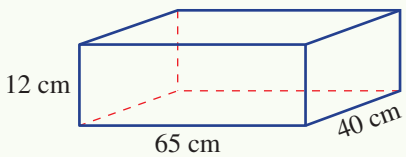
a 45 cm **b** 2.4 m **c** 9.01 m **d** 270 cm **e** 46.0 m

2 A carpenter needs to cut mitred joints to join 4 pieces of wood of equal width to construct a picture frame. A mitre joint is formed when cuts at exactly 45° are made so that two pieces of timber fit together perfectly. Copy the diagram and show how the mitre cuts can be marked out by using isosceles triangles instead of a set square. Shade the final shape of the wood after the cuts are made. The top piece is done for you.

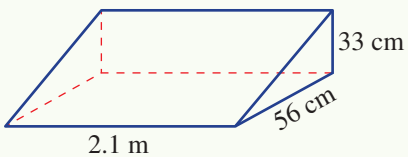


3 For each box below, find the minimum dimensions of a single rectangular sheet of plywood required to construct that closed box. Draw and label the least wasteful arrangement of the sides and ignore any overlap for joining. State the answers in millimetres. Diagrams are not to scale. You will need Pythagoras' theorem in part **b**.

a



b



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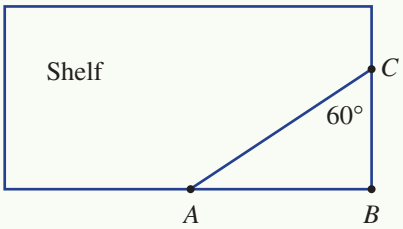
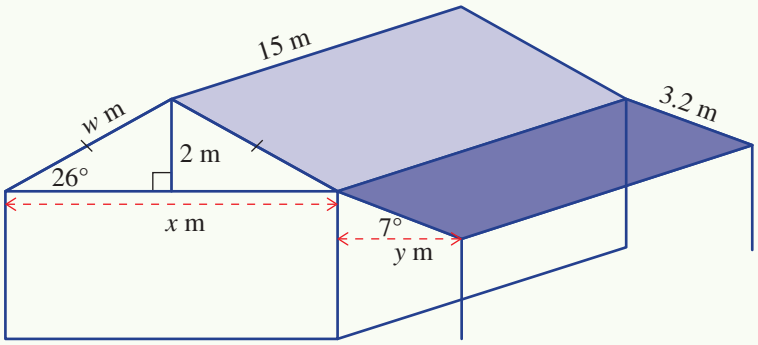
4 A carpenter is constructing a set of shelves near the kitchen doorway. To avoid sharp corners and allow more walking space, a 60° triangle is cut from each shelf as shown in this diagram. The carpenter carefully measures AB and BC to achieve the same 60° angle on each shelf.



Calculate the unknown length (BC or AB) to the nearest mm, given these measurements.

a $AB = 125$ mm **b** $AB = 184$ mm **c** $BC = 108$ mm **d** $BC = 210$ mm

Many tradespeople work together constructing a house. Use the measurements on this diagram of a basic house for questions **5**, **6** *and* **7**. *It is not drawn to scale.*



5 The house roof is tiled and the verandah has a steel roof. If the builder uses 10 tiles/m^2, calculate the number of tiles needed for this house, to the nearest 10 tiles. You will need to calculate the value of w.

6 The rainfall catchment area of any roof is equal to the ground area that the roof covers. Calculate the total possible rainfall catchment area of this house, to the nearest 10 m^2.

7 The litres of rainfall that a house roof can collect per year is given by the formula:

volume of rain in litres = annual rainfall in mm × roof catchment area in m^2 × percentage of run-off

Copy and complete the following Excel spreadsheet to calculate the average annual volume of water available from this house situated in various locations around Australia. Use your answer to Question **6** for the catchment area.

Recall that 1 mm of rain over an area of 1 m^2 equals 1 litre of rain.

	A	B	C	D	E
1	**Average annual volume of water saved from house roof catchments**				
2	Location	Annual average rainfall in mm	Rainfall catchment area in m^2	percentage run-off	Volume of rain in litres
3	Strahan, TAS	1521		90	
4	Melbourne, VIC	620		90	
5	Port Lincoln, SA	508		90	
6	Perth, WA	728		90	
7	Newcastle, NSW	1200		90	
8	Tully, QLD	4105		90	
9	Alice Springs, NT	243		90	

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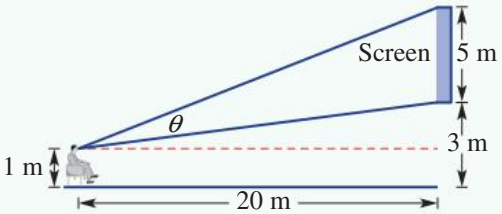
Viewing angle

Lucas is keen to maximise his experience at the outdoor cinema and usually sits at a distance of 20 metres horizontally from the screen. The base of the screen is 3 metres above the ground and the screen is 5 metres in height. Lucas' eye level is 1 metre above ground level as shown.

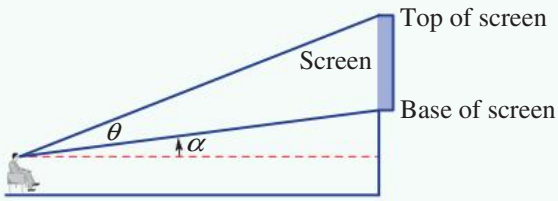


Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate. Use two decimal places for all rounding.

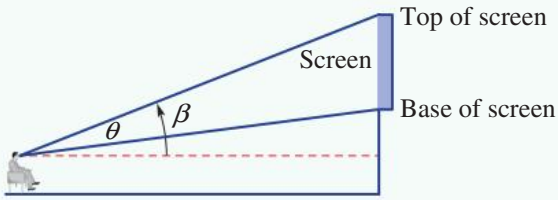
Routine problems

a What is the vertical distance between Lucas' eye level and the base of the screen?

b Find the angle of elevation (α) from Lucas' eye level to the base of the screen.



c Find the angle of elevation (β) from Lucas' eye level to the top of the screen.



d Find Lucas' viewing angle (θ).

Non-routine problems

The problem is to determine how close Lucas should sit to maximise his viewing angle.

a Write down all the relevant information that will help solve this problem, and illustrate using diagrams.

b Determine Lucas' viewing angle if the 20 metre horizontal distance is changed to:

i 14 metres **ii** 8 metres **iii** 2 metres.

c Choose integer values for the horizontal distance between Lucas and the screen and determine which integer horizontal distance delivers the largest viewing angle and state the corresponding viewing angle.

Explore and connect

Choose and apply techniques

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Communicate thinking and reasoning

d Find the viewing angle for the following horizontal distances:
i your answer from part **c** plus 0.1 metres **ii** your answer from part **c** minus 0.1 metres.

e Do your results from part **d** confirm that you have found a distance which maximises the viewing angle? Explain why.

f Refine your choices for the horizontal distance and determine a distance correct to one decimal place that maximises Lucas' viewing angle.

g Summarise your results into a spreadsheet, like the one below.

	A	B	C	D	E	F
1	distance	h1	α	h2	β	θ
2	20	2		7		
3						
4						

h Draw a scale diagram illustrating where Lucas should sit to maximise his viewing angle. Decide if this position for Lucas is reasonable.

Extension problem

Problem-solve

a If the screen is lowered to 2 metres above the ground, rather than the current height of 3 metres, investigate how this changes the position that Lucas should sit to maximise his viewing angle.

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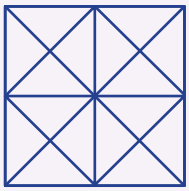
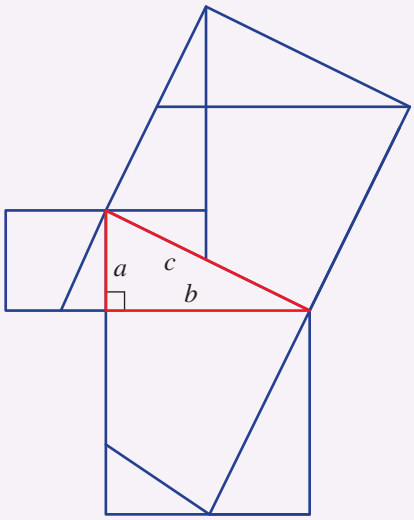
1 How many right-angled triangles are there in this diagram?



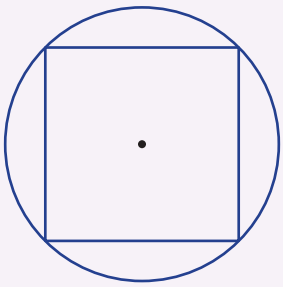
2 Look at this right-angled triangle and the squares drawn on each side. Each square is divided into smaller sections.

- **a** Can you see how the parts of the two smaller squares would fit into the larger square?
- **b** What is the area of each square if the side lengths of the right-angled triangle are a, b and c as marked?
- **c** What do the answers to the above two questions suggest about the relationship between a, b and c?



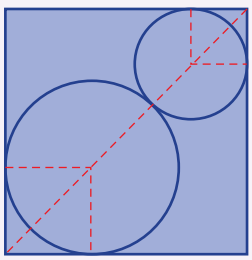
3 Imagine trying to cut the largest square from a circle of a certain size and calculating the side length of the square.

- **a** If the circle has a diameter of 2 cm, can you find a good position to draw the diameter that also helps to form a right-angled triangle?
- **b** Can you determine the side length of the largest square if the circle has a diameter of 2 cm?
- **c** What percentage of the area of a circle does the largest square occupy?



4 Which is a better fit: a square peg in a round hole or a round peg in a square hole? Use area calculations and percentages to investigate.

5 Two circles of radius 10 cm and 15 cm respectively are placed inside a square. Find the perimeter of the square to the nearest centimetre.



Hint: First find the diagonal length of the square using the diagram shown.

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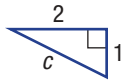
Chapter summary

Right-angled triangles

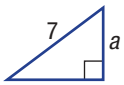
Finding the hypotenuse

$c^2 = 2^2 + 1^2$
$= 5$
$c = \sqrt{5}$
$= 2.24$ (to 2 d.p.)

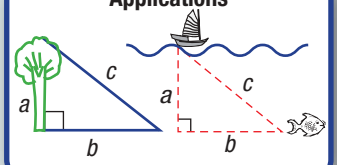


Shorter sides

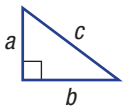
$a^2 + 5^2 = 7^2$
$a^2 = 7^2 - 5^2$
$= 24$
$a = \sqrt{24}$ (exact)



Applications

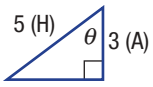


Pythagoras' theorem



$c^2 = a^2 + b^2$

SOHCAHTOA

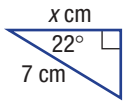


$\sin\theta = \frac{O}{H} = \frac{4}{5}$

$\cos\theta = \frac{A}{H} = \frac{3}{5}$

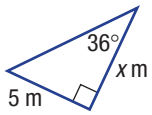
$\tan\theta = \frac{O}{A} = \frac{4}{3}$

Finding side lengths (x in the numerator)



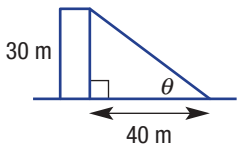
$\cos 22° = \frac{x}{7}$
$x = 7 \times \cos 22°$
$= 6.49$ (to 2 d.p.)

Finding side lengths (x in the denominator)



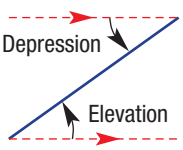
$\tan 36° = \frac{5}{x}$
$x \times \tan 36° = 5$
$x = \frac{5}{\tan 36°}$
$= 6.88$ (to 2 d.p.)

Applying trigonometry

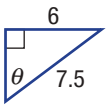


$\tan\theta = \frac{30}{40}$
$\theta = 36.86...$
$\theta = 37°$
(to the nearest degree)

Elevation and depression



Finding angles



$\sin\theta = \frac{6}{7.5}$
$\theta = 53.13...$
$\theta = 53°$
(to the nearest degree)

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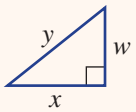
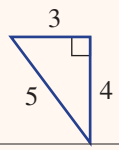
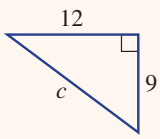
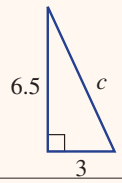
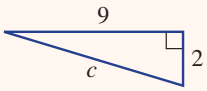
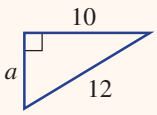
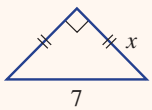
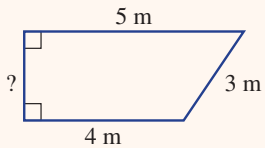
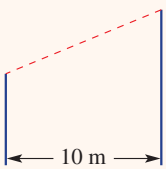
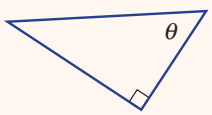
Chapter checklist with success criteria

A printable version of this checklist can be downloaded from the Interactive Textbook.

		✔
4A	**1 I can write Pythagoras' theorem for a right-angled triangle.** e.g. Write down Pythagoras' theorem for these triangles **a**  **b** 	☐
4B	**2 I can find the length of the hypotenuse.** e.g. Find the length of the hypotenuse in these right-angled triangles, rounding to two decimal places where necessary. **a**  **b** 	☐
4B	**3 I can find the length of the hypotenuse as a surd.** e.g. Find the length of the hypotenuse in this right-angled triangle, leaving your answer as an exact value. 	☐
4C	**4 I can find the length of a shorter side.** e.g. Find the value of the pronumeral, rounding to two decimal places. 	☐
4C	**5 I can find the exact length of a shorter side in a right isosceles triangle.** e.g. Find the value of x, giving your answer as a surd. 	☐
4D	**6 I can identify right-angled triangles in a diagram to find unknown lengths.** e.g. An apartment courtyard is in the shape of a trapezium, as shown. Find the unknown side length correct to two decimal places by first drawing a suitable right-angled triangle. 	☐
4D	**7 I can apply Pythagoras' theorem.** e.g. Two poles are 10 m apart. The heights of the poles are 8 m and 12 m. If a bird flies from the top of one pole in a direct line to the top of the other pole, how far does the bird fly correct to two decimal places? 	☐
4E	**8 I can label the sides of a triangle.** e.g. Copy this triangle and label the sides as opposite to θ (O), adjacent to θ (A) and hypotenuse (H). 	☐

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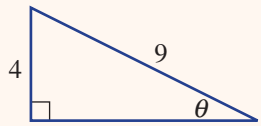
Chapter checklist

Chapter checklist with success criteria

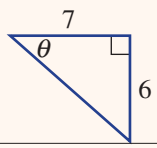
4E

9 I can write trigonometric ratios for right-angled triangles.
e.g. Write a trigonometric ratio (in fraction form) for each of the following triangles.

a



b



☐

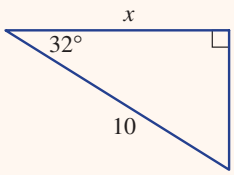
4F

10 I can find trigonometric values on a calculator.
e.g. Use a calculator to evaluate tan $35°$ correct to two decimal places.

☐

4F

11 I can find an unknown side length using trigonometry.
e.g. Find the value of x in the triangle correct to two decimal places.



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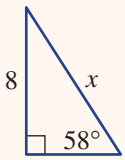
4G

12 I can find an unknown side length in the denominator of a trigonometric ratio.
e.g. Find the value of x correct to two decimal places.



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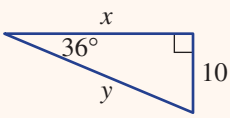
4G

13 I can find two side lengths using trigonometry.
e.g. Find the value of x and y correct to two decimal places.



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4H

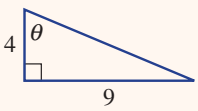
14 I can use my calculator to find an unknown angle.
e.g. Find the value of θ to the nearest degree if:

a $\cos\theta = 0.6261$ **b** $\sin\theta = \frac{3}{7}$.

☐

4H

15 I can find an angle using trigonometry.
e.g. Find the value of θ to the nearest degree.



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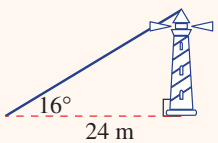
4I

16 I can use angles of elevation.
e.g. The angle of elevation of the top of a light tower from a point on the ground 24 m away from the base of the tower is $16°$. Find the height of the light tower to the nearest metre.



☐

4I

17 I can work with angles of depression.
e.g. From the surface of the water, a snorkeller spots a ray below the water. If the surface of the water is 3 m above the ray and the ray is 5 m horizontally from the snorkeller, find the angle of depression from the snorkeller to the ray to the nearest degree.

☐

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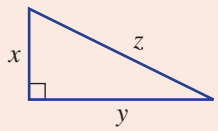
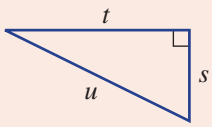
Short-answer questions

4A **1** Write down Pythagoras' theorem for these triangles, using the given pronumerals.

a



b



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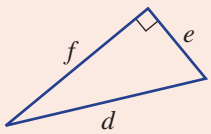
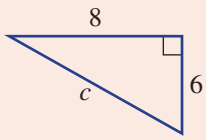
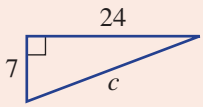


4B **2** Find the length of the hypotenuse in these triangles. Round to two decimal places where appropriate.

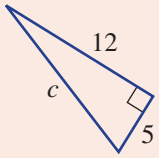
a



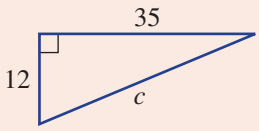
b



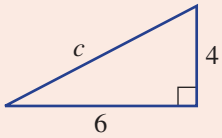
c



d



e



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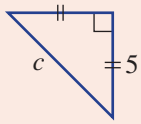
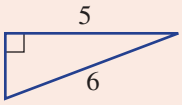
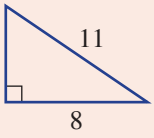


4C **3** Find the length of the unknown side in these right-angled triangles. Round to two decimal places.

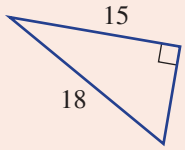
a



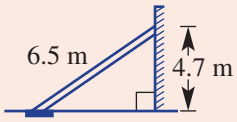
b



c



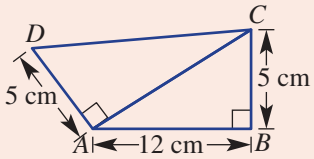
4D **4** A steel support beam of length 6.5 m is connected to a wall at a height of 4.7 m from the ground. Find the distance (to two decimal places) between the base of the building and the point where the beam is joined to the ground.



4D **5** For this double triangle, find:

a the length AC

b the length CD (correct to two decimal places)



4D **6** Two different cafés on opposite sides of an atrium in a shopping centre are respectively 10 m and 15 m above the ground floor. If the cafés are linked by a 20 m escalator, find the horizontal distance (to the nearest metre) across the atrium, between the two cafés.

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Chapter review

4E

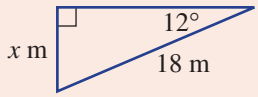
7 Find the value of each of the following, correct to two decimal places.

a $\sin 40°$ **b** $\tan 66°$ **c** $\cos 44°$

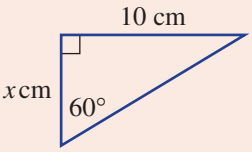
4E

8 Which ratio $\left(\sin\theta = \frac{O}{H},\ \cos\theta = \frac{A}{H} \text{ or } \tan\theta = \frac{O}{A}\right)$ would be used to find the value of the unknown in these triangles?

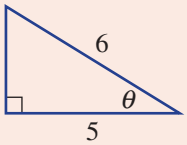
a



b



c

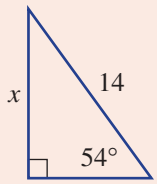


4F/G/H

9 Find the value of each pronumeral, correct to two decimal places where necessary.

Use $\sin\theta = \frac{O}{H}$, $\cos\theta = \frac{A}{H}$ or $\tan\theta = \frac{O}{A}$.

a



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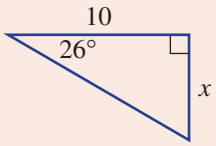

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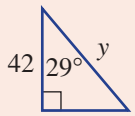
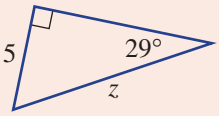


d



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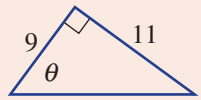

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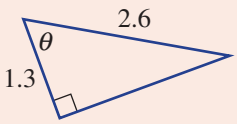
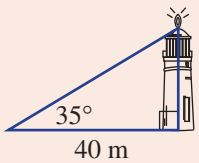


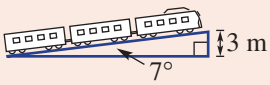
4I

10 The angle of elevation of the top of a lighthouse from a point on the ground 40 m from its base is 35°. Find the height of the lighthouse to two decimal places.



4I

11 A train travels up a slope, making an angle of 7° with the horizontal. When the train is at a height of 3 m above its starting point, find the distance it has travelled up the slope, to the nearest metre.



4I

12 From a point on the ground, Ahbed measures the angle of elevation of a 120 m tower to be 34°. How far from the base of the tower is Ahbed, correct to two decimal places?

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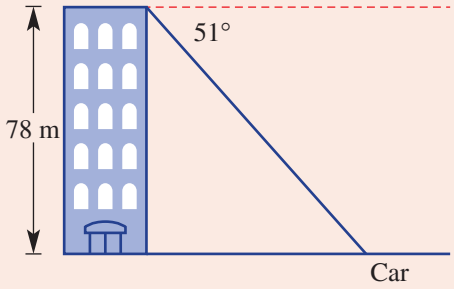
4I

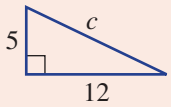
13 Aisha is standing on the roof of a skyscraper when she spots a car at an angle of depression of 51° below her. If the skyscraper is 78 m high, how far away is the car from the base of the skyscraper, correct to one decimal place?



Multiple-choice questions

4A

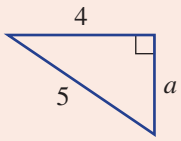
1 For the right-angled triangle shown, which is a correct statement?



A $c^2 = 5^2 + 12^2$ B $c^2 = 5^2 - 12^2$ C $c^2 = 12^2 - 5^2$
D $c^2 = 5^2 \times 12^2$ E $(5 + 12)^2$

4A

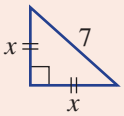
2 Which of the following is true for this triangle?



A $a = 5 - 4$ B $a^2 = 5^2 - 4^2$ C $a^2 = 5^2 + 4^2$
D $a^2 = 4^2 - 5^2$ E $a^2 - 4^2 = 5^2$

4C

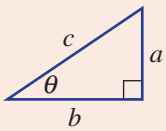
3 For the right-angled triangle shown, which is a correct statement?



A $2x^2 = 49$ B $7x^2 = 2$ C $2x^2 = 7$
D $x^2 + 7^2 = x^2$ E $x^2 = \frac{2}{7}$

4E

4 For the triangle shown, which is a correct statement?



A $\sin\theta = \frac{a}{b}$ B $\sin\theta = \frac{c}{a}$ C $\sin\theta = \frac{a}{c}$
D $\sin\theta = \frac{b}{c}$ E $\sin\theta = \frac{c}{b}$

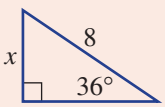
4E

5 The value of cos 46°, correct to four decimal places, is:

A 0.7193
B 0.6947
C 0.594
D 0.6532
E 1.0355

4F

6 In the diagram, the value of x, correct to two decimal places, is:



A 40
B 13.61
C 4.70
D 9.89
E 6.47

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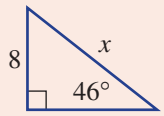
Chapter review

4G **7** The value of x in the triangle is given by:



A $8 \sin 46°$

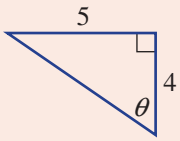
B $8 \cos 46°$

C $\dfrac{8}{\cos 46°}$

D $\dfrac{8}{\sin 46°}$

E $\dfrac{\cos 46°}{8}$

4H **8** Which equation could be used to find the value of θ?



A $\theta = \tan\left(\dfrac{5}{4}\right)$

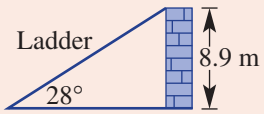
B $\theta = \tan\left(\dfrac{4}{5}\right)$

C $\tan\theta = \dfrac{5}{4}$

D $\sin\theta = \dfrac{4}{5}$

E $\tan\theta = \dfrac{4}{5}$

4I **9** A ladder is inclined at an angle of 28° to the horizontal. If the ladder reaches 8.9 m up the wall, the length of the ladder correct to the nearest metre is:



A 19 m

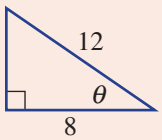
B 4 m

C 2 m

D 10 m

E 24 m

4H **10** The value of θ in the diagram, correct to two decimal places, is:



A 0.73°

B 48.19°

C 41.81°

D 33.69°

E 4.181°

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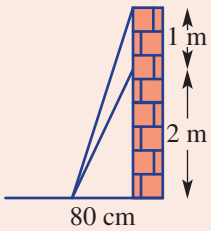
Extended-response questions

1 From the top of a 100 m cliff, Skevi sees a boat out at sea at an angle of depression of 12°.

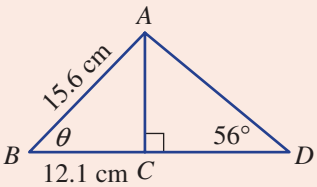
- **a** Draw a simple diagram for this situation.
- **b** Find how far out to sea the boat is to the nearest metre.
- **c** A swimmer is 2 km away from the base of the cliff and in line with the boat. What is the angle of depression to the swimmer to the nearest degree?
- **d** How far away is the boat from the swimmer, to the nearest metre?

2 An extension ladder is initially placed so that it reaches 2 metres up a wall. The foot of the ladder is 80 centimetres from the base of the wall.

- **a** Find the length of the ladder, to two decimal places, in its original position.
- **b** Without moving the foot of the ladder, it is extended so that it reaches 1 m further up the wall. How far (to two decimal places) has the ladder been extended?
- **c** The ladder is placed so that its foot is now 20 cm closer to the base of the wall. How far up the wall can the ladder length found in part **b** reach? Round to two decimal places.



3 Consider triangle ABD, where AC meets BD at 90°.



- **a** Use Pythagoras' theorem to find the height AC, correct to 2 decimal places.
- **b** Find θ, correct to the nearest degree.
- **c** Find the length of
  - **i** AD
  - **ii** CD, each to 1 decimal place.
- **d** Find the perimeter of triangle ABD.
- **e** Calculate the area of triangle ABD, correct to the nearest m^2.

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5

Linear relationships

Maths in context: Linear relationships in industry

Skills using linear relationships are widely applied to solve problems in many trades, scientific research, manufacturing and finance.

- Linear relationships involving direct variation have graphs through $(0, 0)$. For example:
  - weekly pay $= \$/h \times$ number of hours
  - crop spray in kg $=$ kg/acre $\times$ number of acres
  - \$AUD $=$ exchange rate $\times$ foreign currency amount
- Builders of wheelchair ramps apply the gradient regulations of the *Design for access and mobility* standards. This requires a maximum gradient $= \frac{1}{14}$, i.e. a 1 m rise for every 14 m of horizontal run.
- Industrial robots are programmed to calculate the length and midpoint of a virtual straight line segment joining two points in 3D having coordinates (x, y, z).
- A second-hand car bought for \$12 000 can cost \$4000 p.a. to run, e.g. for fuel, repairs, registration and insurance. The total cost, C, over n years, is the modelled with the equation $C = 4000n + 12\,000$.

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5C Graphing straight lines using intercepts (EXTENDING)
5D Lines with only one intercept
5E Gradient from rise and run
5F Gradient and direct variation
5G Gradient–intercept form
5H Finding the equation of a line using $y = mx + c$
5I Midpoint and length of a line segment from diagrams
5J Linear relationships in real-life contexts

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- determines the midpoint, gradient and length of an interval, and graphs linear relationships, with and without digital tools (MA5-LIN-C-01)
- graphs and interprets linear relationships using the gradient/slope-intercept form (MA5-LIN-C-02).

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Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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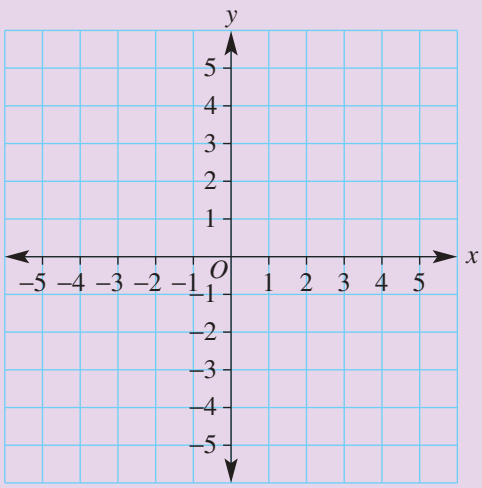
Warm-up quiz

1 Plot and label the following points.

a $(3, 1)$ **b** $(-2, 4)$ **c** $(0, 0)$ **d** $(5, -3)$



2 Given $y = x - 2$, find the value of y when:

a $x = 3$ **b** $x = 1$ **c** $x = -1$ **d** $x = -2$

3 Solve the following equations for x.

a $3x = 15$ **b** $-2x = 4$ **c** $-x = 3$

d $0 = x - 4$ **e** $0 = 3x + 6$ **f** $0 = 2x - 8$

4 Find the value of y in each of the following when $x = 0$.

a $y = 2x + 3$ **b** $y = 3x - 4$ **c** $x + 2y = 8$ **d** $3x - 4y = 12$

5 Complete the table of values for these equations.

a $y = x + 3$

x	−2	−1	0	1	2
y					

b $y = 2x - 1$

x	−2	−1	0	1	2
y					

6 If $x = 2$ and $y = 1$, decide whether the following equations are true (T) or false (F).

a $y = 2x + 1$ **b** $y = 3x - 5$ **c** $y = -2x + 5$ **d** $5x - 2y = 6$

7 Simplify:

a $\frac{4 + 10}{2}$ **b** $\frac{2 + 11}{2}$ **c** $\frac{-1 + 7}{2}$ **d** $\frac{-3 + (-7)}{2}$

8 Find the vertical distance between the following pairs of points.

a $(3, 2)$ and $(3, 7)$ **b** $(-1, 1)$ and $(-1, -3)$

c $(2, 3)$ and $(2, -2)$ **d** $(1, -4)$ and $(1, -1)$

9 Find the horizontal distance between the following pairs of points.

a $(1, 3)$ and $(5, 3)$ **b** $(-1, 2)$ and $(4, 2)$

c $(-2, -3)$ and $(5, -3)$ **d** $(-4, 1)$ and $(-1, 1)$

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5A Points and lines on the Cartesian plane

Learning intentions for this section:

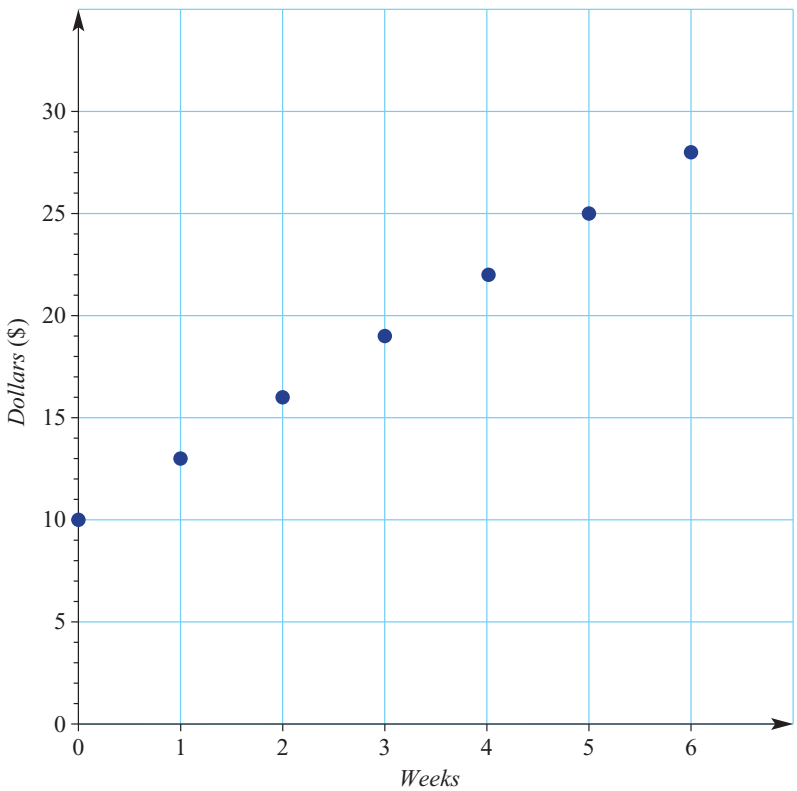
- To review the features of the Cartesian plane and plot points using coordinates
- To know that a linear relationship gives a set of points that form a straight line
- To be able to complete a table of values and plot points to form a linear graph
- To be able to arrange the equation of a line in various ways, including general form
- To be able to decide if a point is on a line by substituting into the equation of the line

Past, present and future learning:

- *Linear relationships* is a major topic in Stage 4, Stage 5 Core and Stage 6 Standard
- This section addresses concepts from the Stage 5 Core topics called *Linear relationships A* and *Linear relationships B*
- In Stage 6 Standard these concepts will be revised, briefly, then extended

In the table below, \$3 is added every week. This is an example of a linear relationship because the points form a straight line.

Weeks	0	1	2	3	4	5	6
Dollars	10	13	16	19	22	25	28



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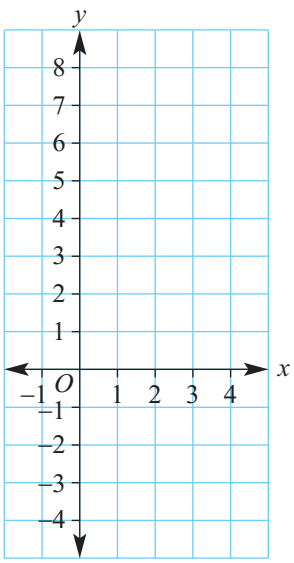
Lesson starter: Which equation is linear?

Here are three equations and their tables of values. Copy and complete the tables of values and plot the points on grid paper with the axes shown. (Use a different colour for each set of points.)



1 $y = 2^x - 1$

x	0	1	2	3
y				

2 $y = x^2 - 1$

x	0	1	2	3
y				

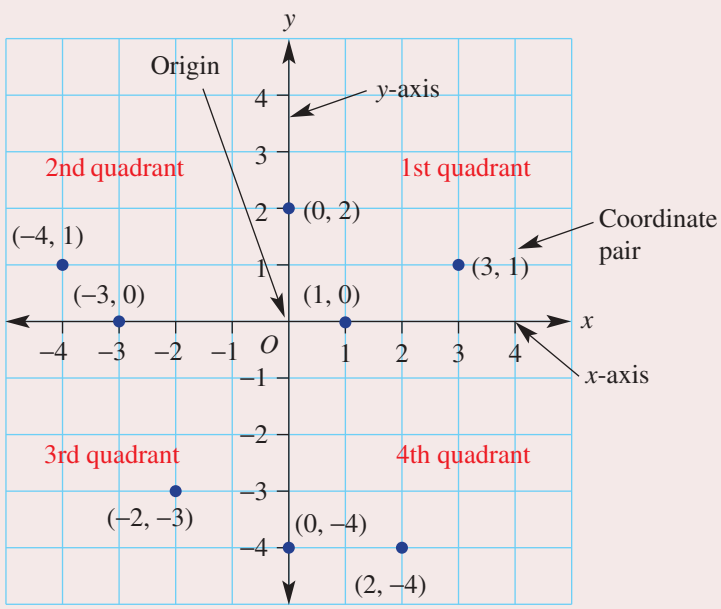
3 $y = 3x - 4$

x	0	1	2	3
y				

KEY IDEAS

■ The **Cartesian plane** (or number plane) consists of two axes that divide the number plane into four quadrants.

- The ***x*-axis** is the horizontal axis.
- The ***y*-axis** is the vertical axis.
- The x-axis and y-axis intersect at right angles at the **origin** (O).
- A point is precisely positioned on a Cartesian plane using the **coordinate pair** (x, y).
  x (the ***x*-coordinate**) describes the horizontal position of the point from the origin.
  y (the ***y*-coordinate**) describes the vertical position of the point from the origin.



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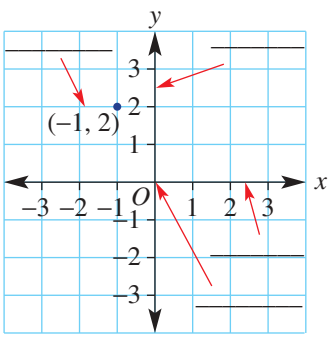
- In the coordinate pair $(2, -4)$, 2 is the x-coordinate and -4 is the y-coordinate. The point is 2 units to the right of the origin (horizontal direction) and 4 units below the origin (vertical direction).
- A **linear relationship** is a set of ordered pairs (x, y) that, when graphed, give a straight line.
- Every linear relationship has an equation (rule), which may be of the form:
  - $y = mx + c$, for example $y = 2x + 1$ (gradient–intercept form)
  - $ax + by = d$, for example $2x - 3y = 4$
  - $ax + by + c = 0$, for example $2x - 3y - 4 = 0$ (general form)
- Each point that is on the line satisfies the equation for the linear relationship. For example, on the line with rule $y = 2x$, each y-coordinate will be two times the x-coordinate. So, $(3, 6)$ will be on the line but $(4, 10)$ will not.

Key terminology: Cartesian plane, x-axis, y-axis, linear relationship, coordinates (or coordinate pair), origin, x-coordinate, y-coordinate

Exercise 5A

UNDERSTANDING 1–4 2, 4

1 **a** For the Cartesian plane shown, label the following parts.

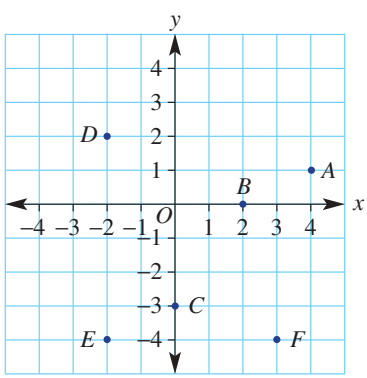


- **i** x-axis
- **ii** origin
- **iii** y-axis
- **iv** coordinate pair

b For the point $(3, -1)$:

- **i** 3 is the _____.
- **ii** -1 is the _____.
- **iii** From the origin, the point is 3 units to the _____ and 1 unit _____ .

2 Refer to the Cartesian plane shown.



a Draw this set of axes (without A–F) and plot the following points.

- **i** $(3, 1)$
- **ii** $(-4, -2)$
- **iii** $(0, 2)$
- **iv** $(-1, 2)$

b Give the coordinates of all the points A–F.

Hint: List the x-coordinate followed by the y-coordinate; e.g. $(2, 4)$

3 Write down the coordinates of these points.

- **A** F is the point 4 units above the origin.
- **B** G is the point 4 units below the origin.
- **C** H is the point 4 units left of the origin.

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4 On the straight line $y = 3x$, the y-coordinate of each point is 3 times the x-coordinate. Which two points lie on this line?

A $I(1, 3)$ **B** $J(3, 6)$ **C** $K(-3, -9)$ **D** $L(-1, 3)$

FLUENCY 5, 6 5, 6

Example 1 Filling in tables from equations

Complete the tables for the given equations.

a $y = 2x - 1$

x	−2	−1	0	1	2	3
y						

b $y = -3x + 2$

x	−2	−1	0	1	2
y					

SOLUTION

a $y = 2x - 1$

x	−2	−1	0	1	2	3
y	−5	−3	−1	1	3	5

EXPLANATION

Substitute each x value in the table into the rule $y = 2x - 1$ to find its corresponding y value.

For $x = -2$, $y = 2 \times (-2) - 1$
$= -4 - 1$
$= -5$

For $x = 1$, $y = 2 \times 1 - 1$
$= 2 - 1$
$= 1$

b $y = -3x + 2$

x	−2	−1	0	1	2
y	8	5	2	−1	−4

Substitute each x value into the rule $y = -3x + 2$. Recall that negative × negative = positive.

For $x = -2$, $y = -3 \times (-2) + 2$
$= 6 + 2$
$= 8$

For $x = 0$, $y = -3 \times 0 + 2$
$= 0 + 2$
$= 2$

Now you try

Complete the table for the given equations.

a $y = 3x - 2$

b $y = -2x + 4$

x	−2	−1	0	1	2
y					

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5 Complete the tables for the given equations.

a $y = x + 2$

x	−2	−1	0	1	2	3
y						

b $y = 2x + 3$

x	−2	−1	0	1	2
y					

c $y = -2x$

x	−2	−1	0	1	2
y					

d $y = -3x - 1$

x	−2	−1	0	1	2	3
y						

Hint: A negative × positive is negative.
A negative × negative is positive.

Example 2 Plotting points to graph straight lines

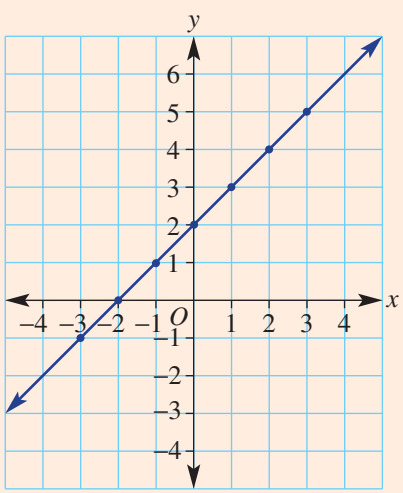
Construct a table of values from $x = -3$ to $x = 3$ and plot a graph for these linear equations.

a $y = x + 2$ **b** $y = -2x + 2$

SOLUTION

a

x	−3	−2	−1	0	1	2	3
y	−1	0	1	2	3	4	5



EXPLANATION

Construct a table of x values as instructed. Substitute each value of x into the rule $y = x + 2$.

The coordinates of the points are read from the table; i.e. $(-3, -1)$, $(-2, 0)$ etc.

Draw a set of axes with an appropriate scale to show the points from the table.

Plot each point and join to form a straight line.

Extend the line to show it continues in either direction.

Continued on next page

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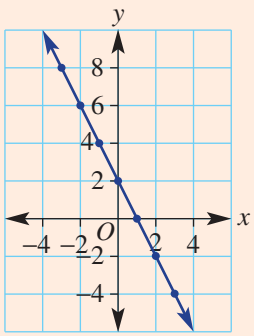
b

x	−3	−2	−1	0	1	2	3
y	8	6	4	2	0	−2	−4



Substitute each value of x into the rule $y = -2x + 2$.

Plot each point and join to form a straight line.

Extend the line beyond the plotted points.

Now you try

Construct a table of values from $x = -3$ to $x = 3$ and plot a graph for these equations.

a $y = x - 2$ **b** $y = -3x + 4$

6 Construct a table of values from $x = -3$ to $x = 3$ and plot a graph for these equations.

a $y = x - 1$ **b** $y = x + 3$ **c** $y = 2x - 3$

d $y = -3x$ **e** $y = -2x - 1$ **f** $y = -x + 4$

PROBLEM-SOLVING AND REASONING 7–9 8–11

Example 3 Deciding whether a point is on a line

Decide whether the point (3, 7) is on the line with the given equation.

a $y = 3x - 4$ **b** $y = 2x + 1$

SOLUTION

a $y = 3x - 4$
Substitute $x = 3$
$y = 3 \times 3 - 4$
$= 5$ (not 7)
$\therefore$ the point (3, 7) is not on the line

b $y = 2x + 1$
Substitute $x = 3$
$y = 2 \times 3 + 1$
$= 7$
$\therefore$ the point (3, 7) is on the line

EXPLANATION

Find the value of y on the graph of the equation for $x = 3$.

To fit the equation, the y value needs to be 5, not 7, so (3, 7) is not on the line.

By substituting $x = 3$ into the equation for the line, we find $y = 7$.

Since (3, 7) fits the equation, it is on the line.

Now you try

Decide whether the point (2, 10) is on the line with the given equation.

a $y = 2x + 6$ **b** $y = 5x - 4$

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7 Is the point $(2, 8)$ on these lines? (Answer yes or no.)

a $y = 2x + 6$ **b** $y = 3x + 2$ **c** $y = -x + 6$.

Hint: When $x = 2$, does $y = 8$?

8 Is the point $(-1, 4)$ on these lines? (Answer yes or no.)

a $y = x + 5$ **b** $y = -2x + 2$ **c** $y = 4x$

9 What equation in the form $y = mx + c$ (e.g. $y = 2x - 1$) matches these tables of values?

Hint: Think about how you can find the y value from the x value. For example, is it double the x value each time or 3 more than the x value each time, etc.?

a

x	0	1	2	3	4
y	2	3	4	5	6

b

x	0	1	2	3
y	0	2	4	6

c

x	-2	-1	0	1	2	3
y	-3	-1	1	3	5	7

d

x	-2	-1	0	1	2	3
y	-7	-4	-1	2	5	8

e

x	-1	0	1	2	3	4
y	-3	-2	-1	0	1	2

10 The table below shows the recorded height, y cm, of a seedling x days after it sprouts. Find a linear rule in the form $y = \ldots$ that matches the data in the table.

x (day)	1	2	3	4
y (height)	3	6	9	12

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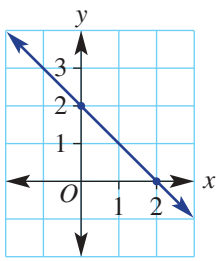
11 By considering the relationship between the coordinates, match these rules **i**, **ii**, **iii** and **iv** to the graphs **a**, **b**, **c** and **d**.

i $y = 2x + 1$ **ii** $y = -x - 1$

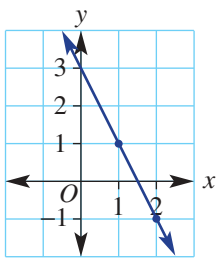
iii $y = -2x + 3$ **iv** $x + y = 2$

Hint: Each rule shows the relationship between the x- and y-coordinates for each point on the line. In part **i**, each y-coordinate has to be 1 more than twice its x-coordinate.

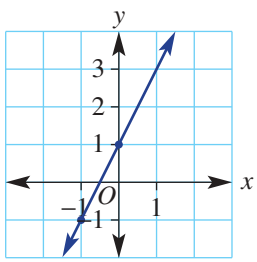
a



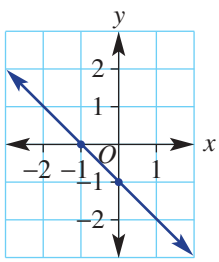
b



c



d



ENRICHMENT: Tougher rule finding — 12

12 Find the linear rule linking x and y in these tables.

a

x	−1	0	1	2	3
y	5	7	9	11	13

b

x	−2	−1	0	1	2
y	22	21	20	19	18

c

x	1	3	5	7	9
y	1	5	9	13	17

d

x	6	7	8	9	10
y	4	4.5	5	5.5	6

e

x	0	2	4	6	8
y	4	−2	−8	−14	−20

f

x	−5	−4	−3	−2	−1
y	−21	−16	−11	−6	−1

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5B The x-intercept and y-intercept

Learning intentions for this section:

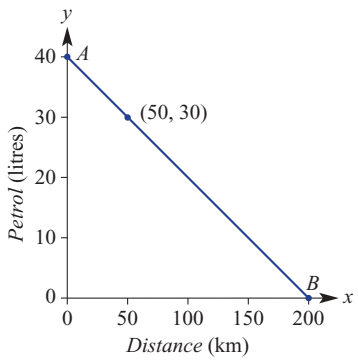
- To know which points are represented by the x-intercept and y-intercept
- To be able to identify the x-intercept and y-intercept from a table or graph
- To know how to find the x-intercept and the y-intercept of a line

Past, present and future learning:

- *Linear relationships* is a major topic in Stage 4, Stage 5 Core and Stage 6 Standard
- This section addresses concepts from the Stage 5 Core topics called *Linear relationships A* and *Linear relationships B*
- In Stage 6 Standard these concepts will be revised, briefly, then extended

When a linear relationship is a model for a real situation, each point gives a piece of information.

For example, the following graph for the linear relationship $y = -0.2x + 40$ models how much petrol is in a tank.



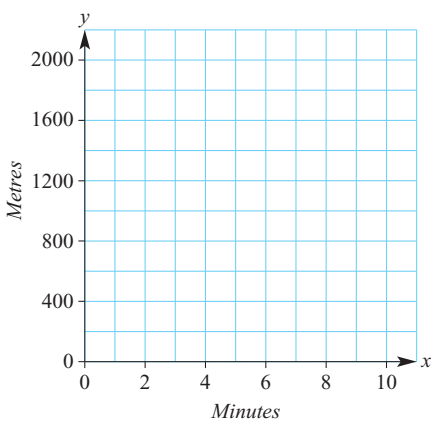
- The y-axis shows the amount of petrol in the tank.
- The x-axis shows the number of kilometres travelled.
- The coordinate pair (50, 30) shows that after travelling 50 km, 30 litres of petrol are left in the tank.
- Point A shows us how much petrol was in the tank before the car had gone anywhere. The point is (0, 40). The y-intercept is 40.
- We also want to know when the car will run out of petrol. This point is (200, 0). This is where the graph cuts the x-axis. The x-intercept is 200.

Lesson starter: Are we there yet?

This table shows the distance (y metres) a student is from their home (at x minutes) when they ride their bike home from school.

x	0	2	4	6	8	10
y	2000	1600	1200	800	400	0

- Copy the axes shown and plot these points to form a line.
- How far is school from home?
- How long does it take to get home?
- Circle the two points above on the graph.
- Which point is the x-intercept?
- Which point is the y-intercept?



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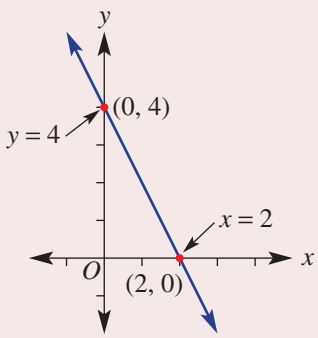
KEY IDEAS

- The ***y*-intercept** is the y-value of the point where the line cuts (intersects) the y-axis. It always has an x-coordinate of 0.
- The ***x*-intercept** is the x-value of the point where the line cuts (intersects) the x-axis. It always has a y-coordinate of 0.

x-intercept

x	−2	−1	0	1	2	3
y	8	6	4	2	0	−2

y-intercept



- The equation can be used to find the x- and y-intercepts.
  - To find the y-intercept, substitute $x = 0$ into the equation.
  - To find the x-intercept, substitute $y = 0$ into the equation.

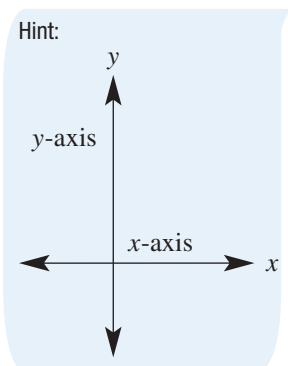
Key terminology: x-intercept, y-intercept, coordinates, x-axis, y-axis

Exercise 5B

UNDERSTANDING 1–3 1, 3

1 Plot the points $A(3, 0)$ and $B(0, 2)$ on a Cartesian plane. Draw a line through A and B. What is the value of the:

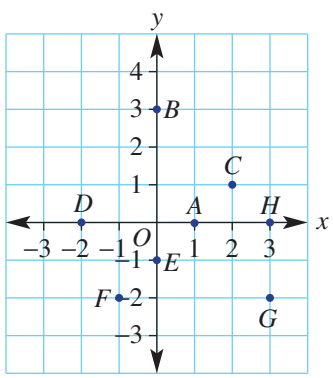
a x-intercept? **b** y-intercept?



2 For the Cartesian plane shown:

a give the coordinates of each point that is on the x-axis

b give the coordinates of each point that is on the y-axis



3 Copy and complete the following

a The x-intercept is found by substituting _____ into the equation.

b The y-intercept is found by substituting _____ into the equation.

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FLUENCY 4, 5, 6(½) 4–6(½)

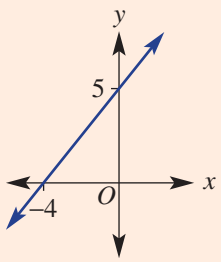
Example 4 Reading off the x-intercept and y-intercept

Read off the x-intercept and y-intercept from this table and graph.

a

x	−2	−1	0	1	2	3
y	12	9	6	3	0	−3

b



SOLUTION

a The x-intercept is 2.
The y-intercept is 6.

b The x-intercept is −4.
The y-intercept is 5.

EXPLANATION

The x-intercept is at the point where $y = 0$ (on the x-axis).
The y-intercept is at the point where $x = 0$ (on the y-axis).

The x-intercept is at the point on the x axis ($y = 0$).
The y-intercept is at the point on the y axis ($x = 0$).

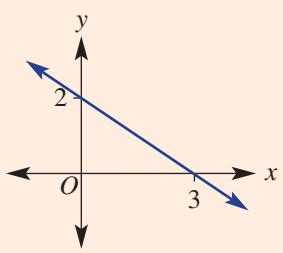
Now you try

Read off the x-intercept and y-intercept from this table and graph.

a

x	−2	−1	0	1	2	3
y	−2	0	2	4	6	8

b



4 Read off the x- and y-intercepts from these tables and graphs.

Hint: The x-intercept is where $y = 0$. The y-intercept is where $x = 0$.

a

x	−3	−2	−1	0	1	2	3
y	4	3	2	1	0	1	2

b

x	−3	−2	−1	0	1	2	3
y	−1	0	1	2	3	4	5

c

x	−1	0	1	2	3	4
y	10	8	6	4	2	0

d

x	−5	−4	−3	−2	−1	0
y	0	2	4	6	8	10

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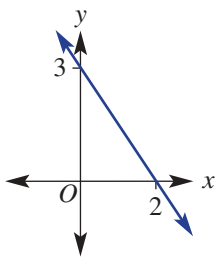
e



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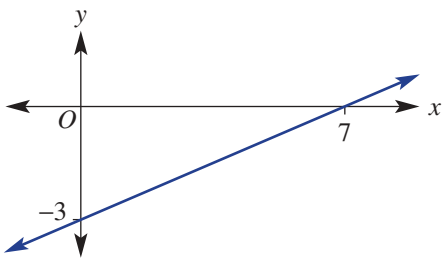

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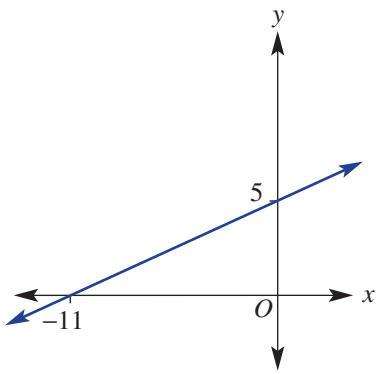

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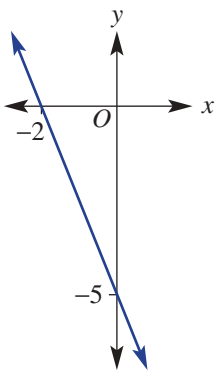

Example 5 Finding the *y*-intercept

Find the *y*-intercept of these lines.

a $y = 2x - 1$ **b** $2x + 3y = 6$

SOLUTION

a $y = 2x - 1$
y-intercept (let $x = 0$):
$y = 2 \times (0) - 1$
$= -1$
the *y*-intercept is -1

b $2x + 3y = 6$
y-intercept (let $x = 0$):
$2 \times (0) + 3y = 6$
$3y = 6$
$y = 2$
the *y*-intercept is 2

EXPLANATION

Substitute $x = 0$ into the rule to find the *y* value of the *y*-intercept.

Recall that anything multiplied by 0 is 0.

The line cuts the *y*-axis at $(0, -1)$.

Substitute $x = 0$ to find the *y*-intercept.

Solve the equation $3y = 6$ by dividing both sides by 3.

The line cuts the *y*-axis at $(0, 2)$.

Now you try

Find the *y*-intercept of these lines.

a $y = 3x + 5$ **b** $3x + 4y = 12$

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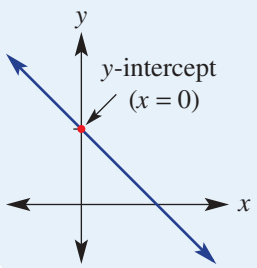
5 Find the y-intercept of these lines.

a	$y = 2x + 5$	**b**	$y = 3x + 1$
c	$y = x - 7$	**d**	$y = 2x - 3$
e	$y = -4x + 2$	**f**	$y = -5x - 4$
g	$y = -x - 2$	**h**	$y = -2x + 10$
i	$2x + y = 11$	**j**	$2x + 3y = 9$
k	$x + 4y = 8$	**l**	$-3x + 2y = 12$
m	$4x - 2y = 6$	**n**	$3x - 4y = 4$
o	$-2x - y = 3$	**p**	$-2x - 3y = -9$

Hint:



Example 6 Finding the x-intercept

Find the x-intercept of these lines.

a $y = 2x + 4$ **b** $3x - 2y = 12$

SOLUTION

a $y = 2x + 4$

x-intercept (let $y = 0$):

$0 = 2x + 4$

$-4 = 2x$

$-2 = x$

$x = -2$

the x-intercept is -2

EXPLANATION

Substitute $y = 0$ into the rule to find the x-coordinate of the x-intercept.

Solve the equation by subtracting 4 from both sides. Then divide both sides by 2.

Using coordinates the x-intercept is at $(-2, 0)$.

b $3x - 2y = 12$

x-intercept (let $y = 0$):

$3x - 2 \times (0) = 12$

$3x = 12$

$x = 4$

the x-intercept is 4

Substitute $y = 0$ to find the x-intercept.

$2 \times 0 = 0$ and then solve the remaining equation for x.

Using coordinates the x-intercept is at $(4, 0)$.

Now you try

Find the x-intercept of these lines.

a $y = 3x - 6$ **b** $5x + 4y = -15$

6 Find the x-intercept of these lines.

a	$y = x + 6$	**b**	$y = x - 4$	**c**	$y = 2x - 8$
d	$y = 3x + 6$	**e**	$y = -2x - 8$	**f**	$y = -4x + 12$
g	$x + 2y = 7$	**h**	$2x - y = 4$	**i**	$3x + 4y = 12$
j	$4x - 6y = 24$	**k**	$-2x + 3y = -2$	**l**	$-5x - 7y = 15$

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PROBLEM-SOLVING AND REASONING

7, 8 | 7–10

7 Find the x-intercept and y-intercept of these lines. Leave coordinates in fraction form.

a $y = 2x - 1$

b $y = \frac{1}{2}x - 2$

c $y = -\frac{1}{3}x + 4$

d $3x + 2y = 7$

Hint: Recall that another way of writing $\frac{1}{2}x$ is $\frac{x}{2}$.

8 The height, h, in metres, of a lift above ground after t seconds is given by $h = 100 - 5t$.

a What height does the lift start at?

b How long does it take for the lift to reach the ground?

Hint: In part **a**, let $t = 0$
In part **b**, let $h = 0$

9 Water is being drained from a fish tank. The amount of water in the tank, V litres, after t minutes is given by the rule $V = 80 - 4t$.

a How long will it be until the tank is empty ($V = 0$)?

b How much water was in the tank to begin with ($t = 0$)?

10 The line $y = 3x$ has both its x-intercept and y-intercept with coordinates $(0, 0)$. Can you explain how this is the case, and what it means for the graph of $y = 3x$?

ENRICHMENT: Making money

— | 11

11 Ana makes badges to sell at the market. The rule for the amount of money, A dollars, she makes from the sale of b badges is given by $A = 2b - 60$.

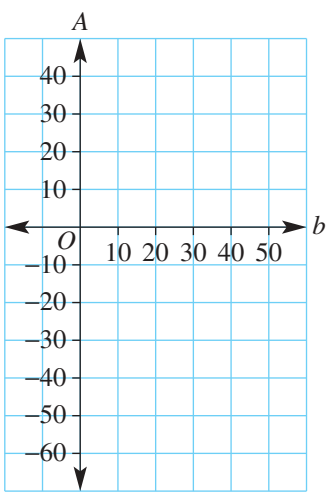
a What is the value of A if she sells no badges ($b = 0$)?

b Can you explain your answer to part **a**?

c How many badges must she sell to cover her initial costs ($A = 0$)?

d What happens when she sells more badges than in your answer to part **c**?

e Use the information from your answers above to draw a graph for the rule on the axes shown.
The A-axis is the y-axis and the b-axis is the x-axis.



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5C Graphing straight lines using intercepts EXTENDING

Learning intentions for this section:

- To understand that only two points are required to graph a straight line
- To be able to graph a line by finding the x-intercept and y-intercept

Past, present and future learning:

- Extending: This section goes beyond the syllabus but it is very useful
- *Linear relationships* is a major topic in Stage 4, Stage 5 Core and Stage 6 Standard
- In Stage 6 Standard these concepts will be revised, briefly, then extended

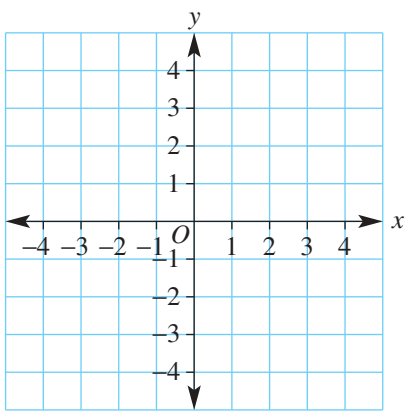
A straight line passes through an infinite number of points. If the x-intercept and y-intercept are not 0, we have enough information to draw one line passing through all the points on that line.

Straight lines are frequently used in business to illustrate the relationship between variables

Lesson starter: Two key points

Consider the equation $y = 2x - 2$.

- Write a set of steps for another student, explaining to them how to find the y-intercept using the equation.
- Write another set of steps, this time explaining how to find the x-intercept using the equation.
- Now that you have these two points, describe how you would use them to sketch the graph of $y = 2x - 2$ on the axes shown, without drawing up a table of values.



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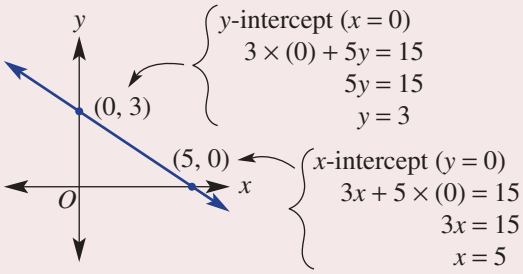
KEY IDEAS

- A straight line can be drawn using only two points.
- The x-intercept and y-intercept give us two key points we can use to draw the graph.
- To graph a straight line using intercepts:
  - find the y-intercept by substituting $x = 0$ into the equation
  - find the x-intercept by substituting $y = 0$ into the equation
  - plot these two points on the axes
  - draw a straight line passing through the two intercepts.

For example: $3x + 5y = 15$



Key terminology: x-intercept, y-intercept

Exercise 5C

UNDERSTANDING 1–4 4

1 Insert the missing number.

a A minimum of _____ points are required to sketch a straight line.

b To find the y-intercept, substitute $x =$ _____.

c To find the x-intercept, substitute $y =$ _____.

2 For these equations, find the y-intercept by letting $x = 0$.

a $2x + 3y = 9$

b $y = 2x - 4$

3 For these equations, find the x-intercept by letting $y = 0$.

a $2x - y = -4$

b $y = 3x - 6$

4 Plot each of the following pairs of points on a set of axes and join the points to form a straight line.

a $(3, 0)$ and $(0, -2)$

b $(0, 4)$ and $(2, 0)$

c $(-3, 0)$ and $(0, 6)$

d $(1, 0)$ and $(0, 3)$

Hint: $(3, 0)$ is on the x-axis as it is 3 to the right of the origin and 0 up or down.

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FLUENCY 5–6(½) 5–6(½)

Example 7 Sketching lines of the form $ax + by = d$ using intercepts

Sketch the graph of $2x + 3y = 6$, showing the x- and y-intercepts.

SOLUTION

$2x + 3y = 6$

y-intercept (let $x = 0$):

$2 \times (0) + 3y = 6$

$3y = 6$

$y = 2$

$\therefore$ the y-intercept is 2

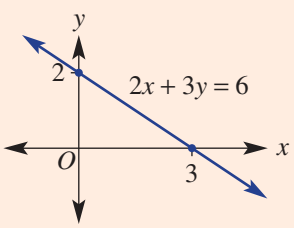
x-intercept (let $y = 0$):

$2x + 3 \times (0) = 6$

$2x = 6$

$x = 3$

$\therefore$ the x-intercept is 3



EXPLANATION

Only two points are required to generate a straight line.
For the y-intercept, substitute $x = 0$ into the rule and solve for y by dividing each side by 3.

State the y-intercept. Plot the point $(0, 2)$.

To find the x-intercept, substitute $y = 0$ into the rule and solve for x.

State the x-intercept. Plot the point $(3, 0)$.

Mark and label the intercepts on the axes and sketch the graph by joining the two intercepts. Continue the line past these points.

Now you try

Sketch the graph of $3x + 4y = 12$, showing the x- and y-intercepts.

5 Sketch the graph of the following relationships, by finding the x- and y-intercepts.

a $x + y = 2$
b $x + y = 5$
c $x - y = 3$
d $x - y = -2$
e $2x + y = 4$
f $3x - y = 9$
g $4x - 2y = 8$
h $3x + 2y = 6$
i $3x - 2y = 6$
j $3x - y + 12 = 0$
k $2x - 5y + 10 = 0$
l $x - 7y + 21 = 0$

Hint: For the x-intercept, let $y = 0$.
For the y-intercept, let $x = 0$.

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Example 8 Sketching lines of the form $y = mx + c$ using intercepts

Sketch the graph of $y = 2x - 6$, showing the x- and y-intercepts.

SOLUTION

$y = 2x - 6$

y-intercept (let $x = 0$):

$y = 2 \times (0) - 6$

$y = -6$

$\therefore$ the y-intercept is -6

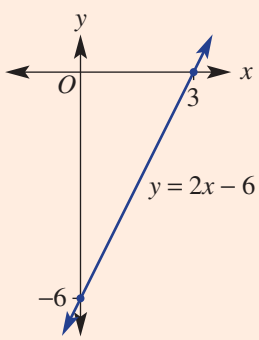
x-intercept (let $y = 0$):

$0 = 2x - 6$

$6 = 2x$

$x = 3$

$\therefore$ the x-intercept is 3



EXPLANATION

Substitute $x = 0$ for the y-intercept.
Simplify to find the y-coordinate.

Plot the point $(0, -6)$.

Substitute $y = 0$ for the x-intercept. Solve the remaining equation for x by adding 6 to both sides. Then divide both sides by 2.

Plot the point $(3, 0)$.

Mark in the two intercepts and join to sketch the graph.

Now you try

Sketch the graph of $y = 2x + 4$, showing the x- and y-intercepts.

6 Sketch the graph of the following lines, showing the x- and y-intercepts.

a $y = 3x + 3$
b $y = 2x + 2$
c $y = x - 5$
d $y = -x - 6$
e $y = -2x - 2$
f $y = -3x - 6$
g $y = -2x + 4$
h $y = 2x - 3$
i $y = -x + 1$
j $y = -2x + 1$

Hint: To solve:
$3x + 3 = 0$
$3x = -3$ (subtract 3)
$x = -1$ (divide by 3)

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PROBLEM-SOLVING AND REASONING

7, 8 | 7–9

7 The distance, d metres, of a remote controlled car from an observation point after t seconds is given by the rule $d = 8 - 2t$.

Hint: The axes will have d on the vertical axis and t on the horizontal axis.

a Find the distance from the observation point initially (at $t = 0$).

b Find after what time, t, the distance, d, is equal to 0 (substitute $d = 0$).

c Sketch a graph of d versus t between the points from part **a** (the d-intercept) and part **b** (the t-intercept).

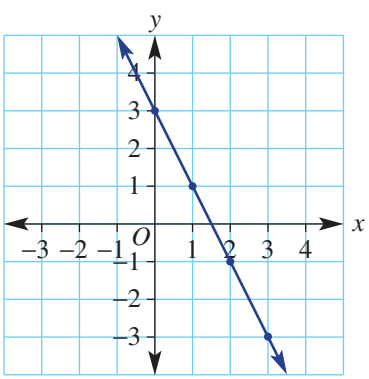
8 Use your algebra and fraction skills to help sketch graphs for these relationships by finding x- and y-intercepts.

Hint: Recall: $\frac{0}{2} = 0$

a $\frac{x}{2} + \frac{y}{3} = 1$ **b** $y = \frac{8 - x}{4}$ **c** $\frac{y}{2} = \frac{2 - 4x}{8}$

9 Give reasons why the x-intercept on these graphs has the exact coordinates $(1.5, 0)$.

a



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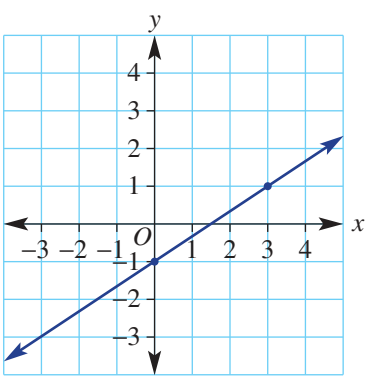

ENRICHMENT: Points to rules

— | 10

10 Write down the rule for the graph with these axes intercepts. Write the rule in the form $ax + by = d$.

a $(0, 4)$ and $(4, 0)$ **b** $(0, 2)$ and $(2, 0)$

c $(0, -3)$ and $(3, 0)$ **d** $(0, 1)$ and $(-1, 0)$

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5D Lines with only one intercept

Learning intentions for this section:

- To be able to use equations to graph vertical lines and horizontal lines
- To know that every line of the form $y = mx$ passes through the origin
- To be able to graph lines of the form $y = mx$ using the origin and one other point

Past, present and future learning:

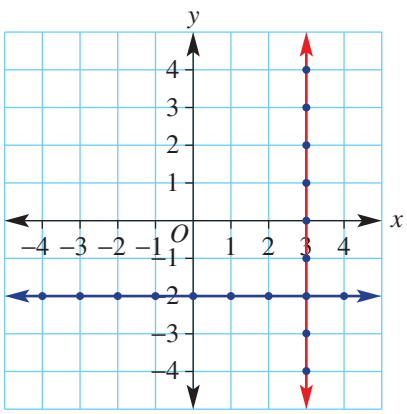
- *Linear relationships* is a major topic in Stage 4, Stage 5 Core and Stage 6 Standard
- This section addresses concepts from the Stage 5 Core topics called *Linear relationships A* and *Linear relationships B*
- In Stage 6 Standard these concepts will be revised, briefly, then extended

In this section we will work with:

- Vertical lines
- Horizontal lines
- Lines that pass through the origin.

Lesson starter: What rule satisfies all points?

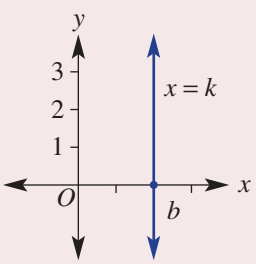
Here is one vertical (red) and one horizontal (blue) line.



- For the vertical line shown, write down the coordinates of all the points shown as dots.
- What is always true for each coordinate pair?
- Can you think of a simple equation that describes every point on the line?
- For the horizontal line shown, write down the coordinates of all the points shown as dots.
- What is always true for each coordinate pair?
- Can you think of a simple equation that describes every point on the line?

KEY IDEAS

- **Vertical** line: $x = k$
  - **Parallel** to the y-axis, the x-coordinate is the same for every point on the line.
  - The equation is of the form $x = k$, where k is a constant (fixed number).
  - The x-intercept is k.
  - There is no y-intercept.



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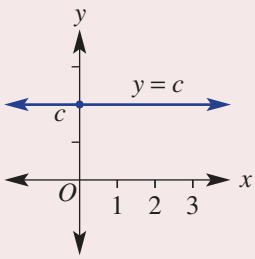
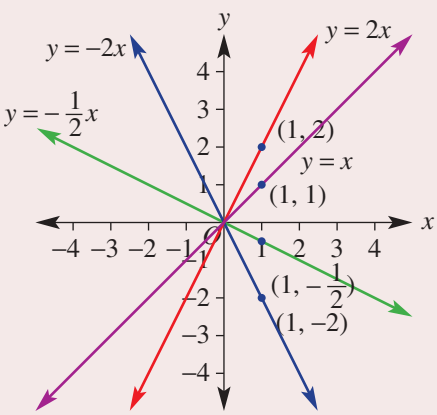
■ **Horizontal** line: $y = c$

- Parallel to the x-axis, the y-coordinate is the same for every point on the line.
- The equation is of the form $y = c$, where c is a constant (fixed number).
- The y-intercept is c.
- There is no x-intercept.



■ Lines through the origin $(0, 0)$: $y = mx$

- The y-intercept is 0.
- The x-intercept is 0.
- Since the x- and y-intercepts give only one point, a second point is required. Substitute any x value into the rule to give a second point. Use $x = 1$ for ease of calculation.



Key terminology: origin, vertical, horizontal, parallel, x-intercept, y-intercept

Exercise 5D

UNDERSTANDING 1–4 2, 4

1 Classify the following lines as either *horizontal* or *vertical*.

a $y = 3$
b $y = -1$
c $x = 2$
d $y = 0$
e $x = -5$
f $x = 0$

Hint: Will they be parallel to the x-axis or y-axis?

2 Select the features from the list (**i**–**v**) that apply to:

a horizontal lines **b** vertical lines

i parallel to the y-axis
ii has a y-intercept
iii has an x-intercept
iv parallel to the x-axis
v always pass through the origin

Hint: Draw a horizontal and vertical line to help.

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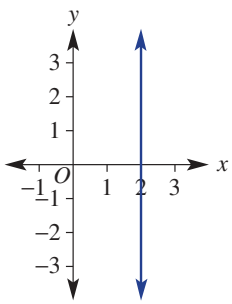
3 a What is the x-coordinate of each point on these graphs?

i



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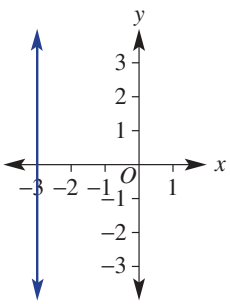
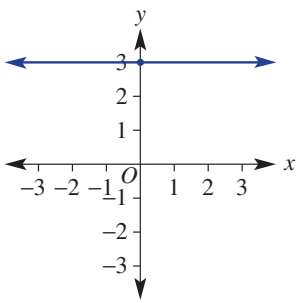
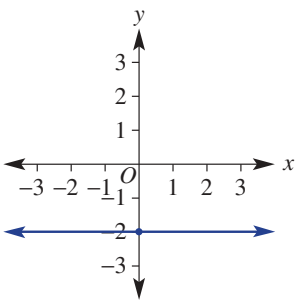


b What is the y-coordinate of each point on these graphs?

i



ii



4 Which lines pass through $(0, 0)$?

a $y = 4$ **b** $y = 2x$ **c** $y = 3x + 2$

d $x = 2$ **e** $y = -x$ **f** $y = \frac{1}{2}x$

Hint: Equations of the form $y = mx$ pass through the origin.

FLUENCY 5–7(½), 8 5–8(½)

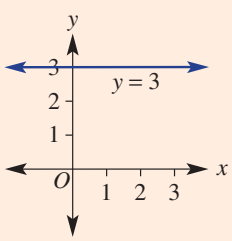
Example 9 Graphing vertical and horizontal lines

Sketch the graph of the following vertical and horizontal lines.

a $y = 3$ **b** $x = -4$ **c** $y = 0$

SOLUTION

a



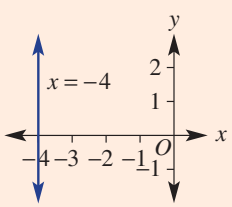
EXPLANATION

Each point on the line has a y-coordinate of 3, so the y-intercept is 3.

Sketch a horizontal line through all points where $y = 3$.

Memory aid: $y = 3$ hits the y-axis at 3

b



Each point on the line has an x-coordinate of -4, so the x-intercept is -4.

Sketch a vertical line through all points where $x = -4$.

Memory aid: $x = -4$ hits the x-axis at -4

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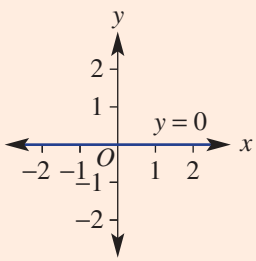
c



Each point on the line has a y-coordinate of 0.
Sketch a horizontal line through all points where $y = 0$; this line is the x-axis.

Note: The equation of the x-axis is $y = 0$

Now you try

Sketch the graph of the following vertical and horizontal lines.

a $y = -1$ **b** $x = 3$ **c** $x = 0$

5 Draw these vertical and horizontal lines on a Cartesian plane.

a $x = 2$ **b** $x = 5$ **c** $y = 4$
d $y = 1$ **e** $x = -3$ **f** $x = -2$
g $y = -1$ **h** $y = -3$

Hint: Do they have an x-intercept or a y-intercept?

Example 10 Sketching lines that pass through the origin

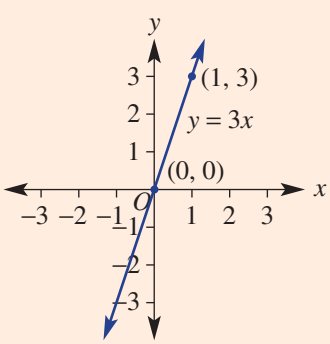
Sketch the graph of $y = 3x$.

SOLUTION

$y = 3x$
The x- and y-intercept are both 0.
Another point (let $x = 1$):

$$y = 3 \times (1)$$
$$y = 3$$

Another point is at (1, 3).



EXPLANATION

The equation is of the form $y = mx$; i.e. when you substitute $x = 0$, $y = 3 \times 0 = 0$, giving 0 as both the x- and y-intercepts.

Two points are required to generate the straight line. Find another point by substituting $x = 1$.

Any other x value could be used but the calculation is simplest for $x = 1$.

Plot and label both points and sketch the graph by joining the points in a straight line.

Now you try

Sketch the graph of $y = 4x$.

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6 Graph these straight lines.

a $y = 2x$ **b** $y = 5x$ **c** $y = 6x$

d $y = x$ **e** $y = -4x$ **f** $y = -3x$

g $y = -2x$ **h** $y = -x$ **i** $y = \frac{1}{2}x$

Hint: Substitute $x = 1$ to obtain a second point.

7 Graph these straight lines.

a $x = -2$ **b** $y = -3$ **c** $y = 2$

d $x = 4$ **e** $y = 4x$ **f** $y = -\frac{1}{2}x$

g $y = -1.5x$ **h** $x = 0.5$ **i** $x = 0$

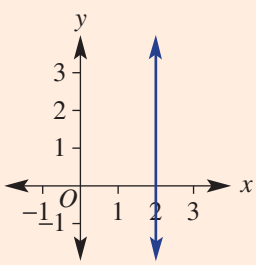
j $y = 0$ **k** $y = 2x$ **l** $y = 1.5x$

Hint: There are vertical lines, horizontal lines and lines through the origin.

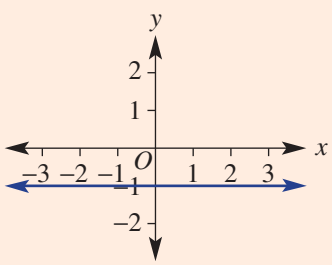
Example 11 Finding the equation of horizontal and vertical lines

Give the equation of the following vertical and horizontal lines.

a



b



SOLUTION

a $x = 2$

b $y = -1$

EXPLANATION

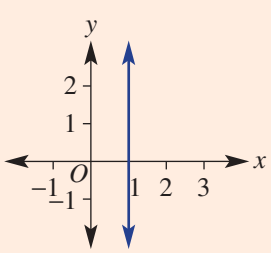
Line is vertical, with x-coordinate always 2.
$x = 2$ hits the x-axis at 2

Line is horizontal, with y-coordinate always -1.
$y = -1$ hits the y-axis at -1

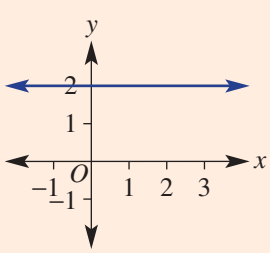
Now you try

Give the equation of the following vertical and horizontal lines.

a



b



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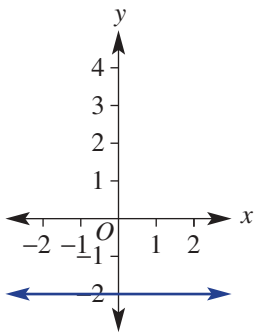
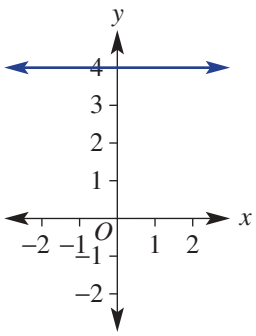
8 Give the equation of the following vertical and horizontal lines.

a



b



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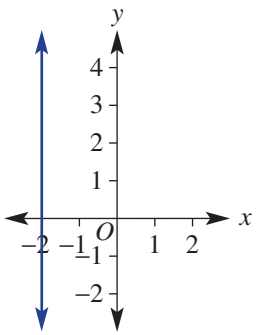
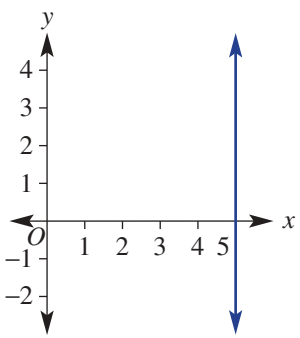


d



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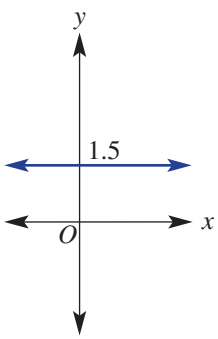

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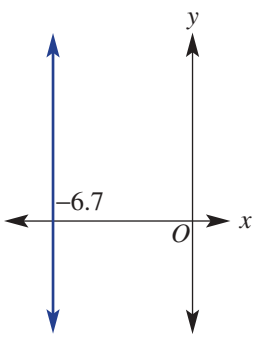

PROBLEM-SOLVING AND REASONING

9, 10 | 9–11

9 Find the equation of the straight line that is:

- **a** parallel to the x-axis and passes through the point $(1, 3)$
- **b** parallel to the y-axis and passes through the point $(5, 4)$
- **c** parallel to the y-axis and passes through the point $(-2, 4)$
- **d** parallel to the x-axis and passes through the point $(0, 0)$

Hint: First decide whether the line is horizontal or vertical, then use the point.

10 If the x-axis represents the surface of the sea, state the equation of the following paths.

- **a** A plane flies horizontally at 250 m above sea level. One unit is 1 metre.
- **b** A submarine travels horizontally 45 m below sea level. One unit is 1 metre.

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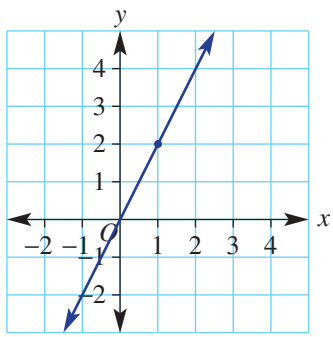
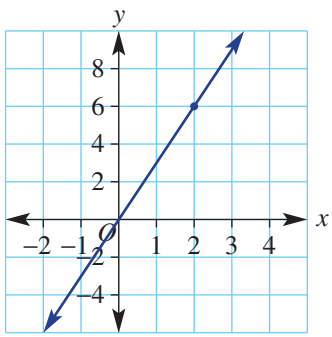
11 The equations of the following graphs are of the form $y = mx$. Use the points marked with a dot to find m and hence state the equation.

a

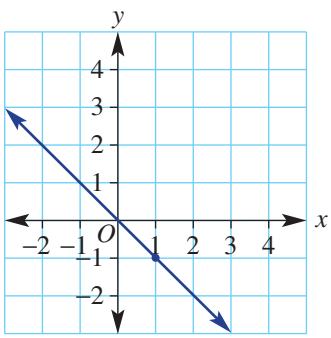


b



Hint: In an equation such as $y = 3x$, when $x = 1$, $y = 3 \times 1 = 3$.

c



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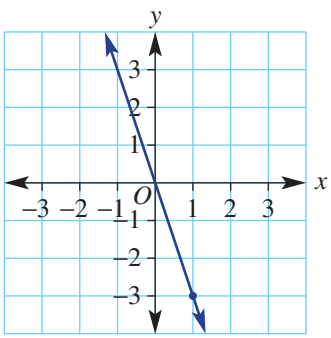
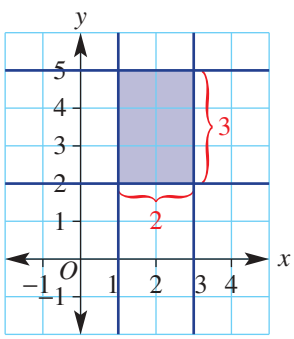


ENRICHMENT: Rectangular areas — 12, 13

The four lines $x = 1$, $x = 3$, $y = 2$ and $y = 5$ drawn on the one set of axes form a rectangle, as shown (shaded). The area of this rectangle is length $\times$ width $= 3 \times 2 = 6$ square units.



12 Find the area of the rectangle contained within the following four lines.

a $x = 1, x = -2, y = -3, y = 2$ **b** $x = 0, x = 17, y = -5, y = -1$

13 The lines $x = -1, x = 3$ and $y = -2$ form three sides of a rectangle. Find the possible equation of the fourth line if:

a the area of the rectangle is:

i 12 square units **ii** 8 square units **iii** 22 square units

b the perimeter of the rectangle is:

i 14 units **ii** 26 units **iii** 31 units

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5E Gradient from rise and run

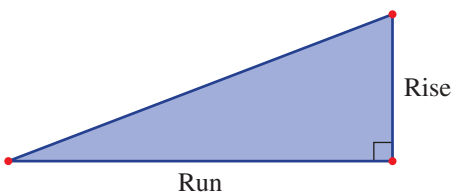
Learning intentions for this section:

- To understand what is meant by the gradient of a line
- To know that the gradient of a line can be positive, negative, zero or undefined
- To be able to find the gradient of a line from a graph or using two given points

Past, present and future learning:

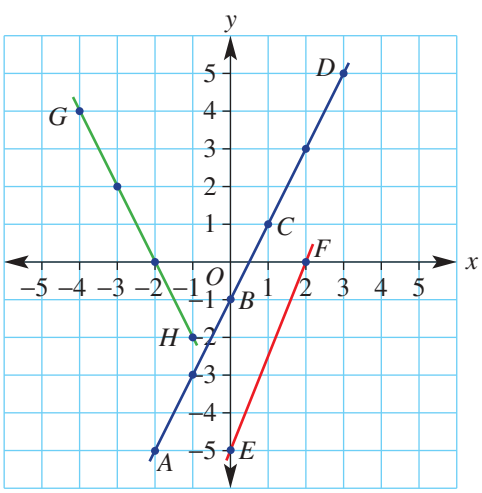
- *Linear relationships* is a major topic in Stage 4, Stage 5 Core and Stage 6 Standard
- This section addresses concepts from the Stage 5 Core topics called *Linear relationships A* and *Linear relationships B*
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall the formula from memory in the HSC Examination

The gradient of a line is a measure of its slope. It is a number that describes the steepness of a line. It is calculated by considering how far a line rises or falls between two points within a given horizontal distance. The horizontal distance between two points is called the *run*. The vertical distance is called the *rise*.



Lesson starter: Which line is the steepest?

The three lines here connect the points A, B, C, D, E, F, G and H.



- Calculate the rise and run (working from left to right) and also the fraction $\frac{\text{rise}}{\text{run}}$ for these segments.
  - i AB ii BC iii BD
  - iv EF v GH
- What do you notice about the fractions $\left(\frac{\text{rise}}{\text{run}}\right)$ for parts i, ii and iii?
- How does the $\frac{\text{rise}}{\text{run}}$ for EF compare with the $\frac{\text{rise}}{\text{run}}$ for parts i, ii and iii? Which of the two lines is the steepest?
- Your $\frac{\text{rise}}{\text{run}}$ for GH should be negative. Why is this the case?
- Discuss whether or not GH is steeper than AD.

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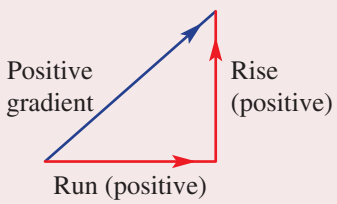
Use computer software (dynamic geometry) to produce a set of axes and grid.

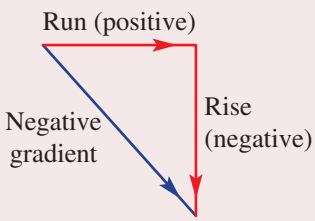
- Construct a line segment with endpoints on the grid. Show the coordinates of the endpoints.
- Calculate the rise (vertical distance between the endpoints) and the run (horizontal distance between the endpoints).
- Calculate the gradient as the *rise* divided by the *run*.
- Now drag the endpoints and explore the effect on the gradient.
- Can you drag the endpoints but retain the same gradient value? Explain why this is possible.
- Can you drag the endpoints so that the gradient is zero or undefined? Describe how this can be achieved.

KEY IDEAS

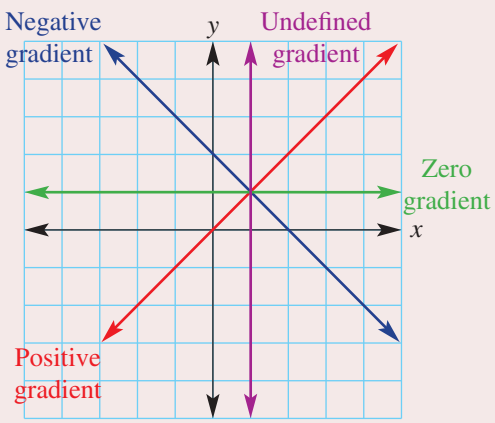
- The **gradient** of a line is a measure of its steepness or slope. It is usually expressed as a fraction, such as $m = \frac{5}{4}$

- The gradient of a line can be found using the formula $m = \frac{\text{rise}}{\text{run}}$.
  - We work from left to right, so the run is always positive.





- The gradient can be positive, negative, zero or undefined.
  - The gradient is positive if the graph increases from left to right.
  - The gradient is negative if the graph decreases from left to right.
  - The gradient is zero if the line is horizontal.
  - The gradient is undefined if the line is vertical. It does not have a numerical value.



Key terminology: gradient, rise, run

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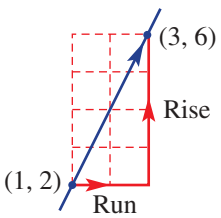
Exercise 5E

UNDERSTANDING | 1, 2 | 2

1 For these lines, calculate the value for: **i** run **ii** rise

a



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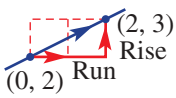

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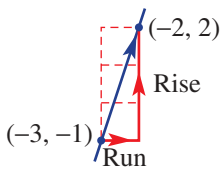
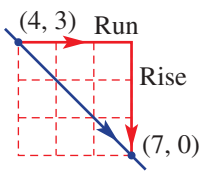
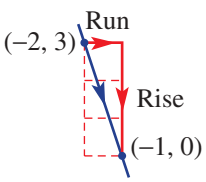


Hint: Remember that the rise is negative if the line slopes downward from left to right.

d



e



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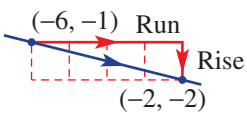


2 Use the words *positive*, *negative*, *zero* or *undefined* to complete each sentence.

a The gradient of a horizontal line is _____.

b The gradient of the line joining (0, 3) with (5, 0) is _____.

c The gradient of the line joining (−6, 0) with (1, 1) is _____.

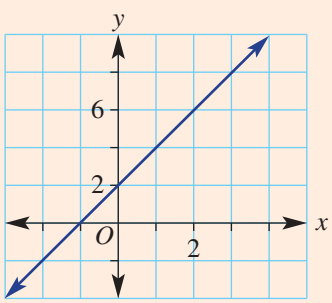
d The gradient of a vertical line is _____.

FLUENCY | 3(½), 5(½) | 3(½), 4, 5(½)

Example 12 Finding the gradient of a line from a graph

For each graph, state whether the gradient is positive, negative, zero or undefined, then find the gradient, where possible.

a



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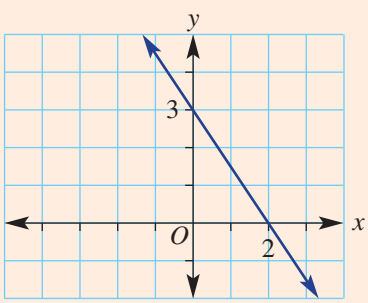

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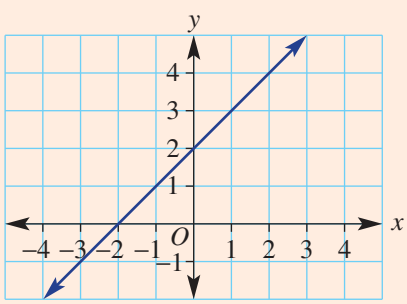
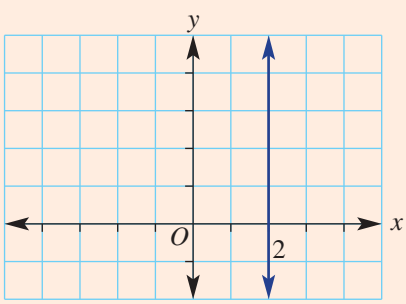


d



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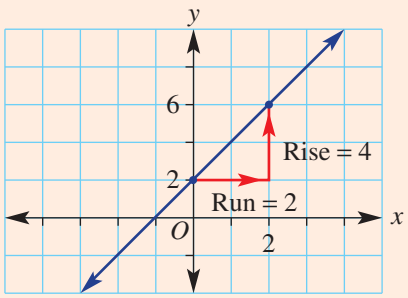
SOLUTION

a The gradient is positive.



$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{4}{2}$$
$$= 2$$

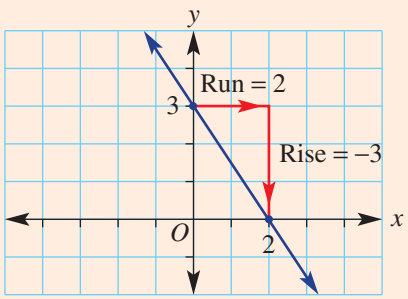
EXPLANATION

By inspection, the gradient will be positive since the graph increases from left to right.

Select any two points and create a right-angled triangle between them to determine the rise and run.

Substitute rise = 4 and run = 2, and simplify.

b The gradient is negative.



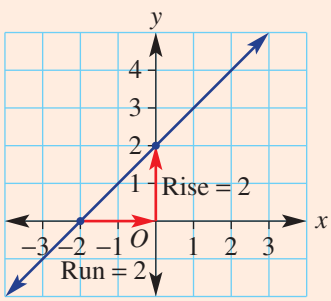
$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{-3}{2}$$
$$= -\frac{3}{2}$$

By inspection, the gradient will be negative since y values decrease from left to right.

Create a right-angled triangle between two points.

Rise = −3 since it 'falls' 3 units, and run = 2.

c The gradient is positive.



$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{2}{2}$$
$$= 1$$

A rising graph indicates a positive gradient.

For this triangle, rise = 2. The run is from left to right, from −2 to 0, so the run is +2.

d The gradient is undefined.

The line is vertical.

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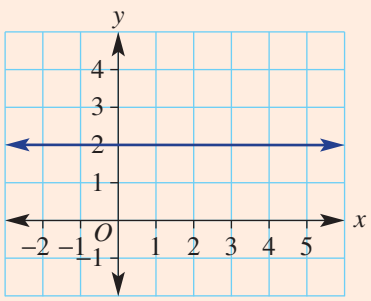
Now you try

For each graph, state whether the gradient is positive, negative, zero or undefined, then find the gradient, where possible.

a



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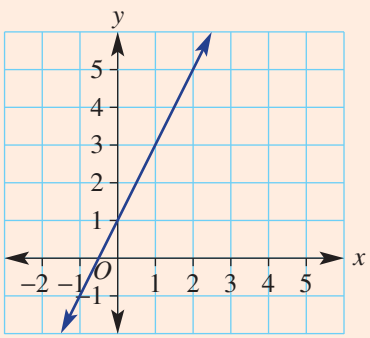

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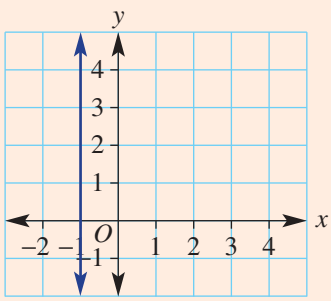

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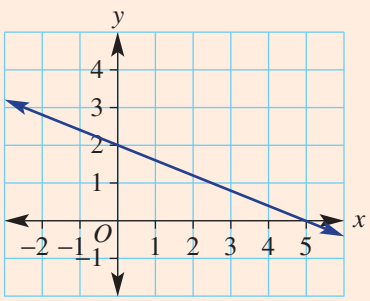
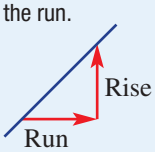


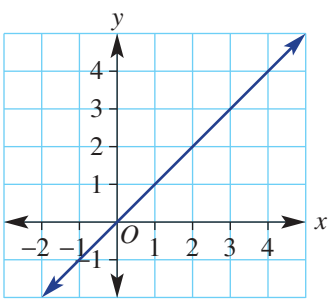
3 For each graph state whether the gradient is positive, negative, zero or undefined, then find the gradient where possible.

Hint: Create a right-angled triangle to find the rise and the run.



$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$

a



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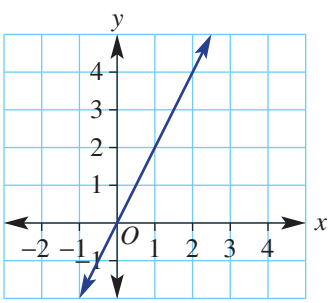

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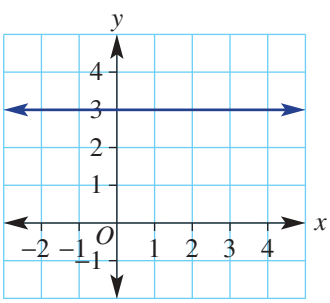

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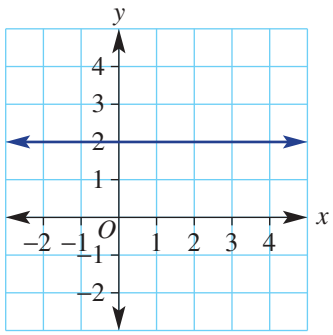
d



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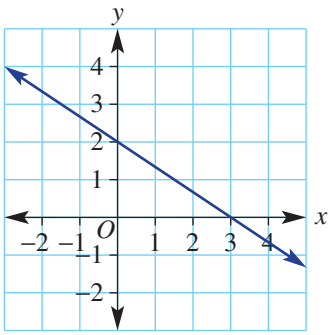

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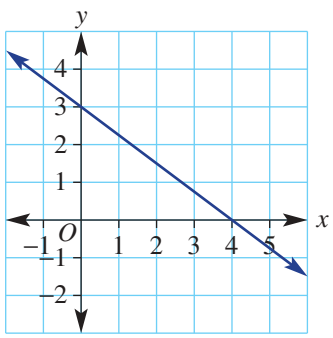

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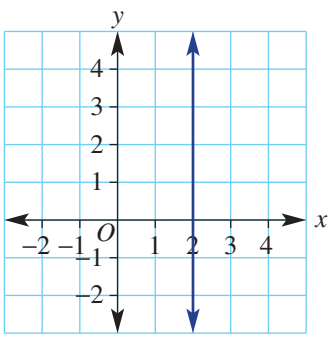

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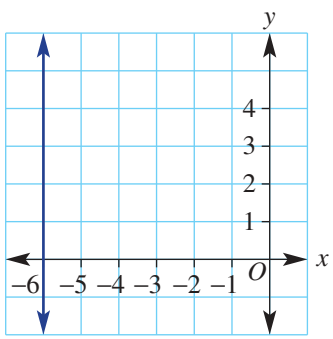

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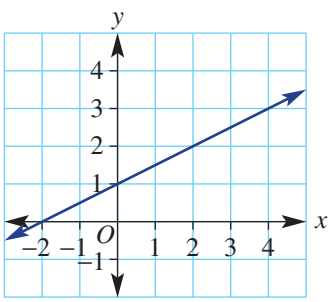

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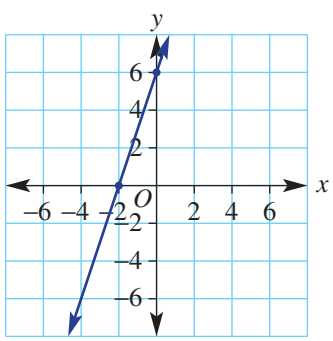

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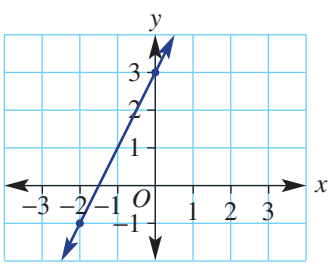

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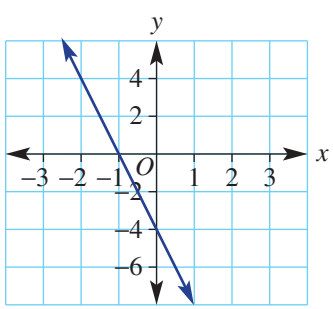

Hint: In part **k**, from -1 to 3 is a rise of 4.

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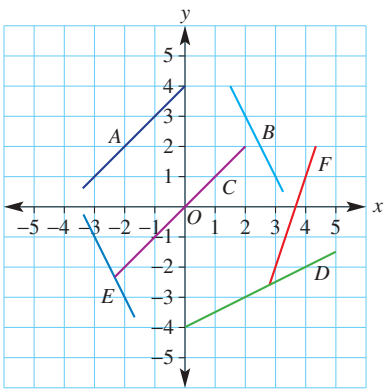
4 Find the gradient of each line $A-F$ on this graph and grid.



Hint: Pick two known points on each line to find the gradient between them.

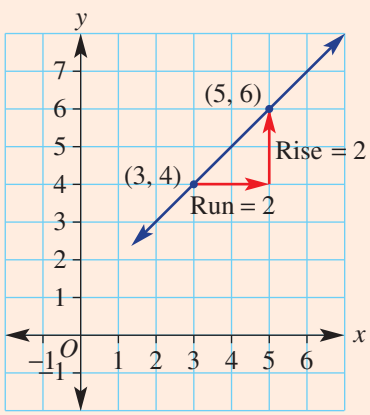
Example 13 Finding the gradient using two points

Find the gradient (m) of the line joining the given points.

a $A(3, 4)$ and $B(5, 6)$ **b** $A(-3, 6)$ and $B(1, -3)$

SOLUTION

a



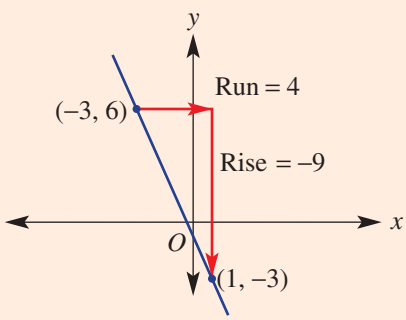
$$m = \frac{\text{rise}}{\text{run}}$$
$$= \frac{2}{2}$$
$$= 1$$

EXPLANATION

Plot the two points on a set of axes and construct a right-angled triangle between them.

Observe from the graph that the gradient will be positive. Calculate the gradient by finding the rise and run.

b



$$m = \frac{\text{rise}}{\text{run}}$$
$$= \frac{-9}{4} \text{ or } -\frac{9}{4}$$

Plot points A and B and join in a line. Observe that the gradient will be negative.

From left to right, -3 to 1, run $= 4$.

Rise from 6 to -3 is a 'fall' of 9 units, so rise $= -9$.

Note: Leave the gradient as $-\frac{9}{4}$, not $-2\frac{1}{4}$ or -2.25.

Now you try

Find the gradient (m) of the line joining the given points.

a $A(2, 3)$ and $B(4, 7)$ **b** $A(-2, 2)$ and $B(3, -6)$

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5 Find the gradient of the lines joining the following pairs of points.

Hint: Plot the points to help visualise the rise and run.

- **a** $A(2, 3)$ and $B(3, 5)$
- **b** $C(0, 8)$ and $D(2, 6)$
- **c** $E(0, 4)$ and $F(2, 0)$
- **d** $G(2, 1)$ and $H(5, 4)$
- **e** $A(1, 5)$ and $B(2, 7)$
- **f** $C(-2, 4)$ and $D(1, -2)$
- **g** $E(-3, 4)$ and $F(2, -1)$
- **h** $G(-1, 5)$ and $H(1, 6)$
- **i** $A(-4, -2)$ and $B(-2, 1)$
- **j** $C(1, 1)$ and $D(3, -4)$
- **k** $E(3, 2)$ and $F(0, 1)$
- **l** $G(-1, 1)$ and $H(-3, -4)$

PROBLEM-SOLVING AND REASONING

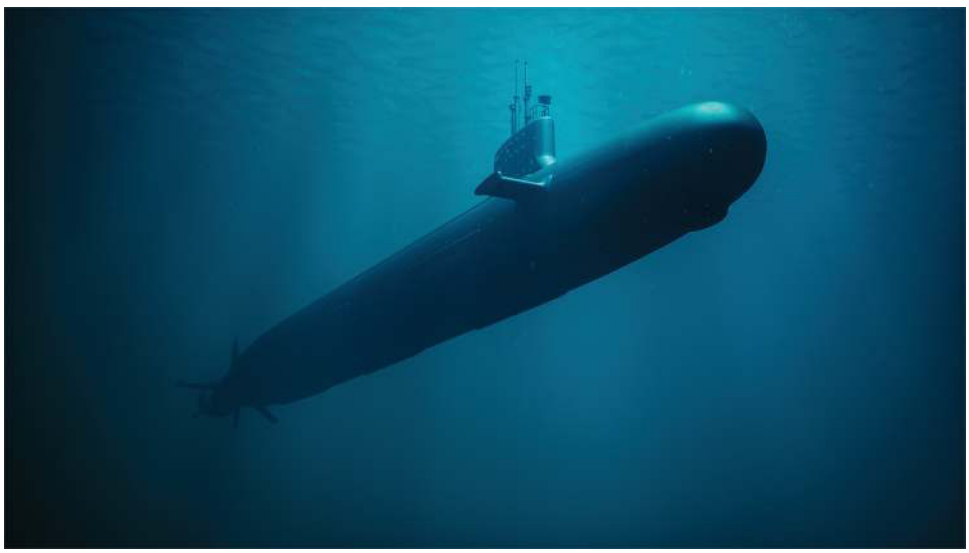
6, 7 | 6, 8, 9

6 Find the gradient, using $\frac{\text{rise}}{\text{run}}$, corresponding to the following slopes.

Hint: The horizontal distance is the run.

- **a** A road falls 10 m for every 200 horizontal metres.
- **b** A cliff rises 35 metres for every 2 metres horizontally.
- **c** A plane descends 2 km for every 10 horizontal kilometres.
- **d** A submarine ascends 150 m for every 20 horizontal metres.

7 A firework ascends with a gradient of 2. How far, horizontally, has the firework travelled after rising 80 m?



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8 Sally runs up a sand dune that has a gradient of $\frac{1}{3}$. How far, horizontally, has Sally moved after rising 30 m?

9 Find the missing number.

a The gradient joining the points $(0, 2)$ and $(1, ?)$ is 4.

b The gradient joining the points $(?, 5)$ and $(3, 9)$ is 2.

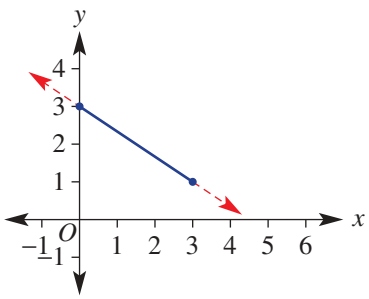
c The gradient joining the points $(-3, 4)$ and $(?, 1)$ is -1.

d The gradient joining the points $(-4, ?)$ and $(-2, -12)$ is -4.

Hint: A gradient of $4 = \frac{4}{1}$: a rise of 4 units for every 1 unit to the right.

ENRICHMENT: Where does it hit? — 10

10 The line here has gradient $\frac{-2}{3}$, which means that it falls 2 units for every 3 units across. The y-intercept is $(0, 3)$.



a Use the gradient to find the y-coordinate on the line where:

i $x = 6$ **ii** $x = 9$

b What will be the coordinates of the x-intercept?

c What would be the x-intercept if the gradient was changed to:

i $\frac{-1}{2}$? **ii** -2? **iii** $\frac{-5}{4}$?

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5F Gradient and direct variation

Learning intentions for this section:

- To understand that gradient is the rate of change of one variable with respect to another
- To understand what it means for two variables to be directly proportional
- To be able to form and work with equations involving direct variation

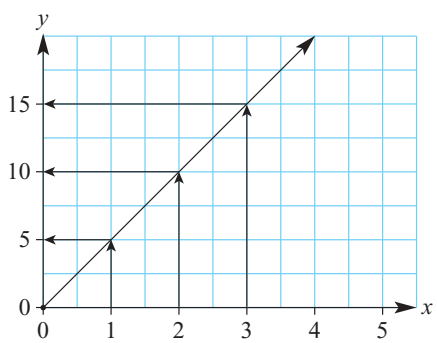
Past, present and future learning:

- *Linear relationships* is a major topic in Stage 4, Stage 5 Core and Stage 6 Standard
- This section addresses concepts from the Stage 5 Core topics called *Linear relationships A* and *Linear relationships B*
- This section also introduces the Stage 5 Path topic called *Variation and rates of change A*
- Direct variation is consolidated and extended in Chapter 6 of our Year 10 book
- Inverse variation and hyperbolas are also in Chapter 6 of our Year 10 book
- In Stage 6 Standard these concepts will be revised and extended

The table below and graph opposite are showing that:

- x starts from 0.
- y starts from 0.
- Every time x increases by 1, y increases by 5.

x	0	1	2	3
y	0	5	10	15

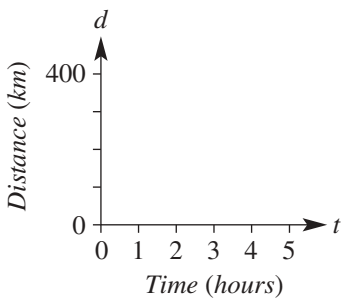


In situations like this we say 'y is in direct proportion to x' or 'y is directly proportional to x' or 'y varies directly with x'.

Lesson starter: Average speed

Over 5 hours, Sandy travels 400 km in a car.

- What is Sandy's average speed for the trip?
- Draw a graph of distance versus time for the journey, assuming a constant speed.
- Where does your graph intersect the axes and why?
- Find the gradient of your graph. What do you notice?
- Find a rule linking distance (d) and time (t).



KEY IDEAS

- A **rate** is the change in one **variable** compared with another (often time). For example, 60 km/h (60 kilometres per hour) is a rate. It states a 60 km change in distance for each hour that passes.
- **Direct variation** occurs when the rate of change is constant. In the diagram below, y varies directly with x. The two variables are **directly proportional**.

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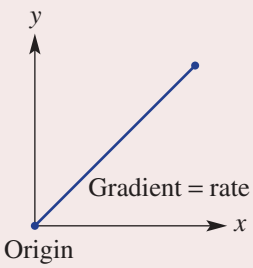
- If two variables are directly proportional, the:
  - rate of change is constant
  - graph is a straight line passing through the origin
  - equation is of the form $y = mx$
  - gradient (m) of the graph equals the rate of change.



- In the diagram, y varies directly as x. This can be written as $y \propto x$ and $y = kx$.
- As x increases, y increases at a constant rate. The letter k, which replaces m, is called the constant of variation, which is the gradient.

Key terminology: rate, variable, gradient, directly proportional, direct variation

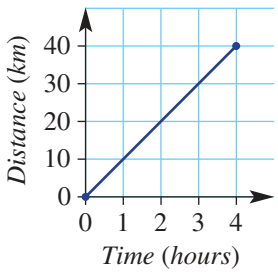
Exercise 5F

UNDERSTANDING 1–3 3

1 Are these rates? (Answer yes or no.)

a \$120 **b** 12 cm/s **c** 150 mL/min **d** 80 km

2 This graph shows how far Caroline travels on her bike over 4 hours.



a State how far Caroline has travelled after:

i 1 hour **ii** 2 hours **iii** 3 hours

b Write down the speed of the bike in km/h (rate of change of distance over time).

c Find the gradient of the graph.

d What do you notice about your answers from parts **b** and **c**?

3 The rule describing the height of a plant over time is given by $h = 5t$, where h is in millimetres and t is in days.

a Find the height of the plant after 3 days ($t = 3$).

b Find the time for the plant to reach:

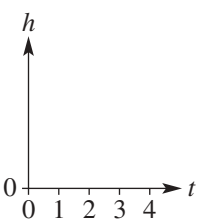
i 30 mm **ii** 10 cm

c Copy and complete this table.

t	0	1	2	3	4
h					

Hint: Add an appropriate scale to the h-axis and plot and join the points from the table.

d Copy and complete this graph.



e Find the gradient of the graph.

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FLUENCY 4–6 4–7

Example 14 Forming equations

Write an equation linking the variables A dollars and t hours if \$60 is earned in 5 hours of work.

SOLUTION

$60 \div 5 = 12$
Rate is \$12/h.

$A = 12 \times t$
$\therefore A = 12t$

EXPLANATION

$\div 5$ (\$60 in 5 hours / \$12 in 1 hour) $\div 5$

The amount earned, A, is \$12 for each hour worked.
t hours of work earns $12 \times t$ dollars.

Now you try

Write an equation linking the variables V litres and t seconds if a bucket contains 8 L of water from a hose after 4 seconds.

4 Write an equation linking the given variables.

a I travel 720 km in 12 hours. Use d for distance and t for time.

b A calf grows 12 cm in 6 months. Use g for growth height and t for time.

c The cost of petrol is \$120 for 80 litres. Use C for cost and n for the number of litres.

d The profit is \$10 000 for 500 tonnes. Use P for profit and t for the number of tonnes.

Hint: First write the rate in simplest form.

Example 15 Exploring direct variation

Water is poured into an empty tank. It takes 3 hours to fill the tank with 6000 litres.

a What is the rate at which water is poured into the tank?

b Draw a graph of volume (V litres) vs time (t hours) from $t = 0$ to $t = 3$.

c Find:

i the gradient of your graph **ii** the equation for V

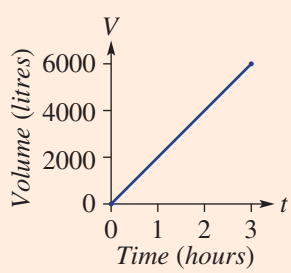
d Use your equation to find:

i the volume after 1.5 hours **ii** the time to fill 5000 litres

SOLUTION

a 6000 L in 3 hours = 2000 L/hour

b



EXPLANATION

$\div 3$ (6000 L per 3 hours / 2000 L per 1 hour) $\div 3$

Draw axes with volume on the vertical axis and time on the horizontal axis, since volume depends on time.

Plot the two endpoints:
(0, 0) since the tank starts empty and
(3, 6000). Join these points with a straight line.

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c i gradient $= \frac{6000}{3} = 2000$

Using the endpoints, rise = 6000, run = 3.

Note that the gradient is the same as the rate.

ii $V = 2000t$

2000 L are filled for each hour.

d i $V = 2000t$

$= 2000 \times 1.5$

$= 3000$

$\therefore$ 3000 L after 1.5 hours

Substitute $t = 1.5$ into your equation.

ii $V = 2000t$

$5000 = 2000t$

$2.5 = t$

$\therefore$ it takes $2\frac{1}{2}$ hours to fill 5000 L

Substitute $V = 5000$ into the equation and solve for t by dividing each side by 2000.

Now you try

Water is poured into an empty inflatable pool. It takes 50 minutes to fill the pool with 2500 litres.

a What is the rate at which water is poured into the tank?

b Draw a graph of volume (V litres) vs time (t minutes) from $t = 0$ to $t = 50$.

c Find:

i the gradient of your graph

ii the equation for V

d Use your equation to find:

i the volume after 20 minutes

ii the time to fill 2000 litres

5 A 300 litre fish tank takes 3 hours to fill using a hose.

a What is the rate at which water is poured into the tank?

Hint: 300 litres in 3 hours; how many litres in 1 hour?

b Draw a graph of volume (V litres) vs time (t hours) from $t = 0$ to $t = 3$.

c Find:

i the gradient of your graph

ii the equation for V

Hint: Place time on the horizontal axis.

d Use your equation to find:

i the volume after 1.5 hours

ii the time to fill 200 litres

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6 A solar powered car travels 100 km in 4 hours.

a What is the rate of change of distance over time (i.e. speed)?

b Draw a graph of distance (d km) vs time (t hours) from $t = 0$ to $t = 4$.

c Find:

i the gradient of your graph

ii the equation for d

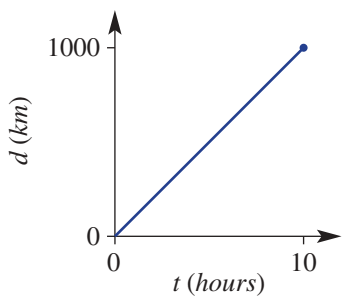
d Use your equation to find:

i the distance after 2.5 hours

ii the time to travel 40 km

7 Use the gradient to find the rate of change of distance over time (speed) for these graphs. Use the units given on each graph.

a



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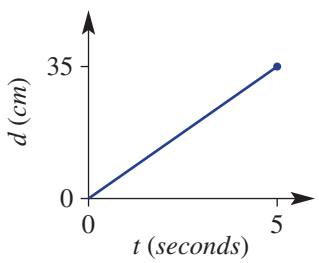

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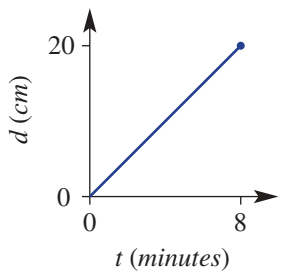

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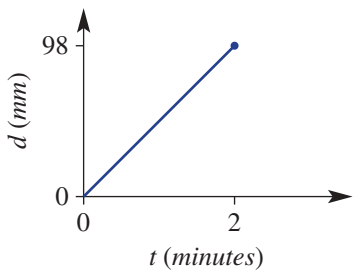

PROBLEM-SOLVING AND REASONING 8, 9 9–11

8 Who is travelling the fastest?

- Henry runs 720 m in 2 minutes.
- Isla rides 550 m in 1 minute.
- Udhav jogs 2000 m in 5 minutes.

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9 Which animal is travelling the slowest?

- A leopard runs 400 m in 30 seconds.
- A jaguar runs 2700 m in 3 minutes.
- A panther runs 60 km in $1\frac{1}{3}$ hours.

Hint: To compare rates, they will need to be in the same units.

10 A car's trip computer says that the fuel economy for a trip is 8.5 L per 100 km.

a How many litres would be used for 120 km?

b How many litres would be used for 850 km?

c How many kilometres could be travelled if the car's petrol tank capacity was 68 L?

Hint: Consider how many litres are used in 1 kilometre.

11 The area of a particular rectangle is given by $A = lb$. Its length is fixed at 12 cm but its breadth, b cm, can vary.

a Write an equation for the area of this rectangle.

b Draw a graph of A against b from $b = 0$ to $b = 4$.

c Decide if the area of this rectangle is directly proportional to its breadth. Explain.

Hint: Equations of the form $y = mx$ show that y is directly proportional to x.

ENRICHMENT: Rate challenge — 12, 13

12 Hose A can fill a bucket in 2 minutes and hose B can fill the same bucket in 4 minutes.

a What fraction of a bucket does hose A fill in 1 minute?

b What fraction of a bucket does hose B fill in 1 minute?

c If both hoses were used at the same time, what fraction of a bucket could they fill in 1 minute?

d If both hoses were used at the same time, how long would it take to fill the bucket?

13 Vonda can vacuum an office building in 2 hours and her husband Chris can vacuum the same office building in 3 hours. How long would it take to vacuum the office building if they both vacuumed at the same time?

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5G Gradient–intercept form

Learning intentions for this section:

- To know that $y = mx + c$ is the gradient–intercept form of a straight line
- To be able to rearrange a linear equation into gradient–intercept form
- To be able to determine the gradient and y-intercept of a line from the equation
- To be able to sketch a graph using the y-intercept and the gradient

Past, present and future learning:

- *Linear relationships* is a major topic in Stage 4, Stage 5 Core and Stage 6 Standard
- This section addresses concepts from the Stage 5 Core topics called *Linear relationships A* and *Linear relationships B*
- In Stage 6 Standard these concepts will be revised and extended
- You may be expected to recall the formula from memory in the HSC Examination

When an equation is written like this:

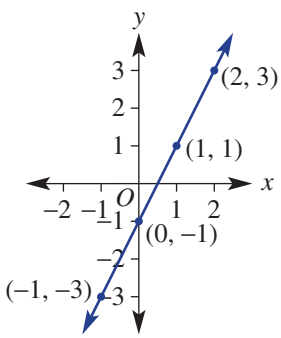
$y = 2x - 1$

it is written in gradient–intercept form.

Equation: $y = 2x - 1$

Gradient is 2 y-intercept is -1

Graph of $y = 2x - 1$:



On the graph, the gradient is 2 and the y-intercept is -1.

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Lesson starter: What's in common?

Sketch the following lines on the same set of axes. Plot points or use the x- and y-intercepts.

a $y = 2x$ **b** $y = 2x + 2$ **c** $y = 2x - 1$

- What do these graphs have in common?
- Calculate the gradient of each line. What do you notice?
- How do the gradients relate to the equations?

Now, sketch the following lines on the same set of axes. Plot points or use the x- and y-intercepts.

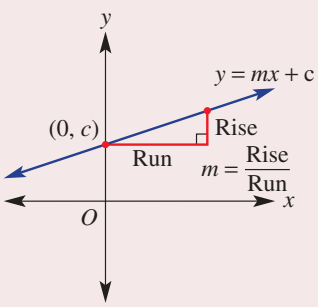
a $y = x + 1$ **b** $y = 2x + 1$ **c** $y = -\frac{1}{2}x + 1$

- What do these graphs have in common?
- How does the common feature relate to the rules?
- Can you form the equation of a line with a gradient of 3 and a y-intercept at 2?

KEY IDEAS

m = gradient c = y-intercept

- $y = mx + c$ is the **gradient–intercept form** of a straight line equation.
- If the y-intercept is zero, the equation becomes $y = mx$ and these graphs will pass through the origin.
- Any equation can be rearranged to be written in gradient–intercept form by making y the subject.
- To sketch a graph using the gradient–intercept method, find the y-intercept and use the gradient to find a second point.
  - For example, if $m = \frac{2}{5}$, move 5 right and 2 up from the y-intercept.
  - For the gradient of $-2 = \frac{-2}{1}$ move 1 right and 2 down from the y-intercept.



- Graphs can be drawn quickly on a computer using computer software such as Desmos or Geogebra
- If a line is written in general form, for example $3x + 2y - 5 = 0$, it can be rearranged into gradient–intercept form, as follows:

$3x + 2y - 5 = 0$

$2y = -3x + 5$

$y = -\frac{3}{2}x + \frac{5}{2}$

Key terminology: gradient–intercept form, y-intercept, gradient

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Exercise 5G

UNDERSTANDING 1–4 | 4

1 Choose from the words *y-intercept*, *gradient* and *subject* to complete the following. In the gradient–intercept form, $y = mx + c$:

a m is the _____

b c is the _____

c y is the _____ of the equation

2 Match the given gradient and y-intercept in the first column with the rule in the left column.

a	Gradient $= 2$, y-intercept $= 5$	**i**	$y = 5x + 2$
b	Gradient $= 5$, y-intercept $= 2$	**ii**	$y = -2x + 3$
c	Gradient $= -2$, y-intercept $= 3$	**iii**	$y = 2x + 5$
d	Gradient $= -1$, y-intercept $= -2$	**iv**	$y = -x - 2$

3 **a** From the point $(0, 2)$, give the coordinates of the point that is:

i 2 right and 4 up **ii** 3 right and 1 down

b From the point $(0, -1)$, give the coordinates of the point that is:

i 1 right and 2 up **ii** 3 right and 4 down

Hint: Plot the point to help you visualise, if required.

4 Fill in the missing numbers.

a The gradient $\frac{5}{3}$ describes a run of _____ for a rise of _____.

b The gradient 4 describes a run of _____ for a rise of _____.

c The gradient $\frac{-1}{2}$ describes a run of _____ for a rise of _____.

Hint: Gradient $= \frac{\text{rise}}{\text{run}}$
Run is always positive
Rise can be positive or negative

FLUENCY 5–7(½) | 5–8(½)

Example 16 Stating the gradient and *y*-intercept

State the gradient and the y-intercept for these straight lines.

a $y = 2x - 1$ **b** $y = -3x$

SOLUTION

a $y = 2x - 1$
The gradient $= 2$
y-intercept $= -1$

b $y = -3x$
The gradient $= -3$
y-intercept $= 0$

EXPLANATION

The equation is given in gradient–intercept form, $y = mx + c$.

The gradient is the coefficient of x (the numeral multiplied by x).

The constant term is the y-intercept.

The gradient is the coefficient of x including the negative sign.

There is no constant term so the y-intercept $= 0$.

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Now you try

State the gradient and the y-intercept for these straight lines.

a $y = 3x - 2$ **b** $y = -x$

5 State the gradient and y-intercept for these straight lines.

a $y = 3x - 4$ **b** $y = -5x - 2$ **c** $y = -2x + 3$

d $y = \frac{1}{3}x + 4$ **e** $y = -4x$ **f** $y = 2x$

g $y = 2.3x$ **h** $y = -0.7x$ **i** $y = x$

Hint: $y = mx + c$

Gradient y-intercept

Example 17 Rearranging linear equations into the form $y = mx + c$

Rearrange these linear equations into the form $y = mx + c$.

a $2x + y = 7$ **b** $4x + 2y - 10 = 0$

SOLUTION

a $2x + y = 7$

$y = 7 - 2x$

$y = -2x + 7$

EXPLANATION

To have y by itself, subtract $2x$ from both sides.

$7 - 2x$ is the same as $-2x + 7$, which is in the form $mx + c$.

b $4x + 2y - 10 = 0$

$2y = -4x + 10$

$y = -2x + 5$

Solve for y by first subtracting $4x$ from both sides and adding 10 to both sides.

Divide both sides of the equation by 2:

$\frac{-4x + 10}{2} = \frac{-4}{2}x + \frac{10}{2} = -2x + 5$

Now you try

Rearrange these linear equations into the form $y = mx + c$.

a $3x + y = 5$ **b** $6x + 3y - 9 = 0$

6 Rearrange these linear equations into the form $y = mx + c$.

a $3x + y = 4$

b $2x + y = 7$

c $y - 3x = 2$

d $y - 2x = 7$

e $5x - y = 3$

f $4x + 2y = 8$

g $3x + 2y - 6 = 0$

h $4x - 2y - 5 = 0$

i $5x - 3y + 6 = 0$

Hint: Follow steps as if you were solving an equation for y.

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Example 18 Sketching linear graphs using the gradient and y-intercept

Find the value of the gradient and y-intercept for these relationships and sketch their graphs.

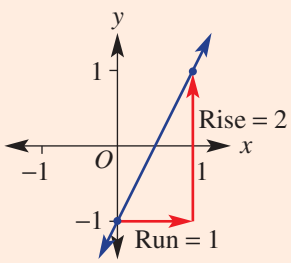
a $y = 2x - 1$ **b** $y = \frac{-3}{4}x + 2$

SOLUTION

a $y = 2x - 1$

y-intercept $= -1$

Gradient $= 2 = \frac{2}{1}$



EXPLANATION

The rule is in gradient–intercept form so we can read off the gradient (coefficient of x) and the y-intercept (constant term).

Gradient $= \frac{2\text{ (rise)}}{1\text{ (run)}}$; for every 1 right, move 2 up.

Label the y-intercept at $(0, -1)$.

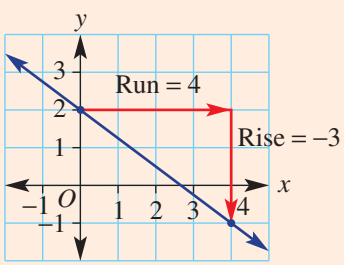
From $(0, -1)$ move 1 to the right and 2 up to give a second point at $(1, 1)$.

Mark and join the points to form a line.

b $y = \frac{-3}{4}x + 2$

y-intercept $= 2$

Gradient $= \frac{-3}{4}$



Read off the y-intercept and gradient from the gradient–intercept form.

Rise $= -3$, run $= 4$.

Label the y-intercept and from here move 4 across to the right (run) and 3 down (due to -3) to give a second point at $(4, -1)$.

Join points to form a line.

Now you try

Find the value of the gradient and y-intercept for these relationships and sketch their graphs.

a $y = 3x - 2$ **b** $y = \frac{-5}{4}x + 1$

7 Find the gradient and y-intercept for these relationships and sketch their graphs.

a $y = x - 2$ **b** $y = 2x + 1$ **c** $y = 3x + 2$

d $y = \frac{1}{2}x + 2$ **e** $y = -3x + 3$ **f** $y = \frac{3}{2}x + 1$

g $y = 2x$ **h** $y = \frac{-4}{3}x$ **i** $y = \frac{-2}{3}x$

Hint: Plot the y-intercept first, then use the rise and run of the gradient.

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Example 19 Rearranging equations to sketch using the gradient and *y*-intercept

Rearrange the equation $x + 2y = 6$ to find the gradient and y-intercept, and sketch its graph.

SOLUTION

$x + 2y = 6$

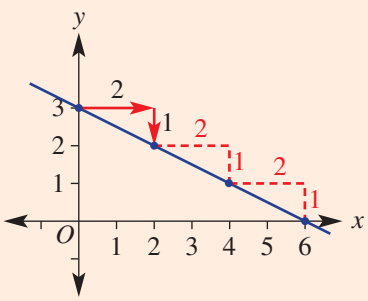
$2y = 6 - x$

$y = 3 - \frac{1}{2}x$

$= -\frac{1}{2}x + 3$

y-intercept is 3

Gradient $= -\frac{1}{2} = \frac{-1}{2}$



EXPLANATION

Make y the subject by subtracting x from both sides and then dividing both sides by 2.

$\frac{6-x}{2} = \frac{6}{2} - \frac{x}{2} = 3 - \frac{1}{2}x$

Rewrite in the form $y = mx + c$ to read off the gradient and y-intercept.

Link the negative sign to the rise (-1) so the run is positive $(+2)$.

Mark the y-intercept and then from this point move 2 right and 1 down to give a second point at $(2, 2)$.

Note that the x-intercept will be 6. If the gradient is $\frac{-1}{2}$ then a fall of 3 needs a run of 6.

Now you try

Rearrange the equation $2x + 3y = 12$ to find the gradient and y-intercept, and sketch its graph.

8 Rearrange these equations to find the gradient and y-intercept and sketch their graphs.

Hint: Solve for y to make y the subject with equations in the form $y = mx + c$.

a $x + y = 4$	**b** $y - x = 6$	**c** $4x + 2y = 6$
d $x + 2y = 8$	**e** $2x + 3y = 6$	**f** $4x + 3y = 12$
g $x - 2y - 4 = 0$	**h** $2x - 3y - 6 = 0$	**i** $3x - 4y - 12 = 0$
j $x + 4y = 0$	**k** $x - 5y = 0$	**l** $8x - 2y = 0$

PROBLEM-SOLVING AND REASONING 9, 10 | 10–12

9 During a heavy rainstorm, a rain gauge is filling with water. The rule for the volume of water in the gauge (V mL), t hours after the start of the storm, is given by $V = 4t + 2$.

a State the gradient and V-intercept of this relationship and sketch its graph.

b In this scenario, what do the gradient and the V-intercept represent?

Hint: Here, the V-intercept is like the y-intercept.

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10 Vera is trying to convince her friend Li that $\frac{4x+6}{2} = 2x+6$. She offers to pay Li \$2 if she is wrong. Does Vera lose her money? Explain.

11 Which of these lines have a gradient of 2 and y-intercept of -3?

a $y = 3 - 2x$ **b** $y = \frac{2x-6}{2}$ **c** $y = \frac{4x-6}{2}$

d $y = \frac{3-2x}{-1}$ **e** $2y = 4x - 3$ **f** $-2y = 6 - 4x$

Hint: Rewrite each in the form $y = mx + c$ and look for $y = 2x - 3$.

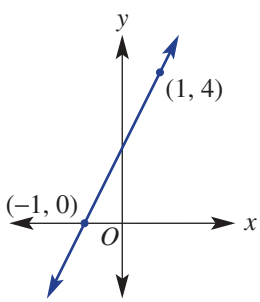
12 Jeremy says that the graph of the rule $y = 2(x + 1)$ has gradient 2 and y-intercept 1.

a Explain his error.

b What can be done to the rule to help show the y-intercept?

ENRICHMENT: The missing y-intercept — 13

13 This graph shows two points $(-1, 0)$ and $(1, 4)$, with a gradient of 2. By considering the gradient (2 up for every 1 right), the y-intercept can be calculated to be 2, so $y = 2x + 2$.



Use this approach to find the rule of the line passing through these points.

a $(-1, 1)$ and $(1, 5)$ **b** $(-2, 4)$ and $(2, 0)$

c $(-2, -1)$ and $(3, 9)$ **d** $(-4, 1)$ and $(2, 4)$

e $(-1, 3)$ and $(1, 4)$

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Progress quiz

5A **1** For the rules listed below:

i Complete a table of values.

x	−3	−2	−1	0	1	2	3
y							

ii Plot a graph.

a $y = 3x + 2$ **b** $y = -2x + 4$

5A **2** Does the point $(2, -3)$ lie on these lines?

a $y = 2x - 5$

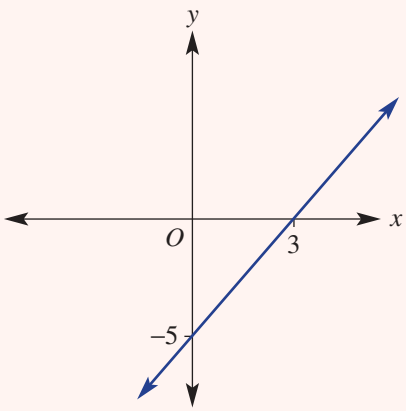
b $y = -2x + 1$

5B **3** State the coordinates of the x- and y-intercepts from the following table and graph.

a

x	−2	−1	0	1	2
y	−6	−4	−2	0	2

b



5B/5C **4** For the lines listed below:

i Find the y-intercept.

ii Find the x-intercept.

iii Sketch the graph showing the x- and y-intercepts.

a $y = 3x + 6$ **b** $y = -2x - 8$ **c** $2x - 5y = 10$

5C **5** A cyclist rides from school to home, with his distance from home given by the rule $d = -\frac{1}{2}t + 10$, where d is in kilometres and t in minutes.

a Find how far school is from home ($t = 0$).

b Find how long it takes to cycle home ($d = 0$).

c Sketch a graph of the cycle from school to home, labelling intercepts.

5D **6** Sketch the graph of the following horizontal and vertical lines.

a $x = 5$ **b** $y = -2$ **c** $x = 0$

5D **7** On the one set of axes, sketch the graph of the following lines, which pass through the origin.

a $y = 6x$ **b** $y = -x$

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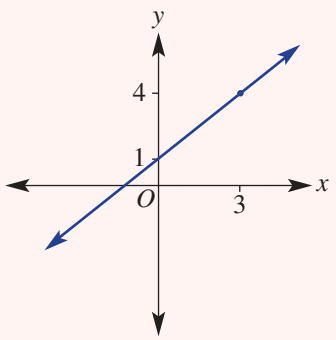
Progress quiz

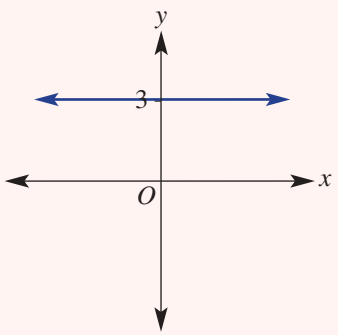
5E

8 Find the gradient (m) of the following lines (where possible).

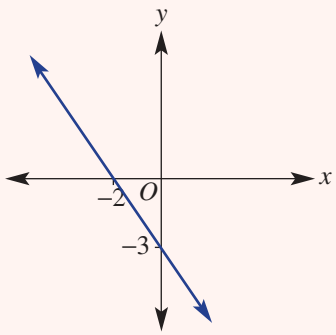
a



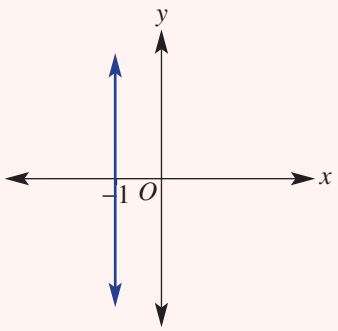
b



c



d



e the line joining the points:

- **i** $A(2, 5)$ and $B(5, 11)$
- **ii** $C(-2, -4)$ and $D(1, 3)$
- **iii** $E(-1, 3)$ and $F(1, -7)$
- **iv** $G(2, 6)$ and $H(-1, 6)$

5F

9 Write an equation linking the given variables.

a I travel 150 km in 3 hours. Use d for distance and t for time.

b I earn \$80 in 5 hours. Use E for earnings and t for time.

c A cricket team makes 160 runs off 20 overs. Use r for runs and o for overs.

5F

10 A birdbath is being filled with water. It takes 5 minutes to fill the bath with 300 litres.

a What is the rate at which the birdbath is filled with water?

b Draw a graph of volume (V litres) vs time (t minutes) from $t = 0$ to $t = 5$.

c Find:

- **i** the gradient of your graph
- **ii** the equation for V

d Use your equation to find:

- **i** the volume after 2 minutes
- **ii** the time to fill 210 litres

5G

11 State the gradient and y-intercept of these straight lines. For parts **d** and **e** you will need to first rearrange the equations into the form $y = mx + c$.

a $y = 2x - 3$ **b** $y = -4x + 1$ **c** $y = \frac{1}{2}x$

d $y - 2x = 3$ **e** $3y - 6x = 12$

5G

12 Find the gradient and y-intercept for these lines and sketch their graphs.

a $y = 3x - 2$ **b** $y = -\frac{1}{2}x + 3$ **c** $3x + 2y = 8$

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5H Finding the equation of a line using $y = mx + c$

Learning intentions for this section:

- To understand that all straight lines can be expressed in the form $y = mx + c$
- To be able to find the equation of a line given the y-intercept and the gradient
- To be able to find the equation of a line given the y-intercept and another point
- To be able to find the equation of a line given the gradient and a point

Past, present and future learning:

- *Linear relationships* is a major topic in Stage 4, Stage 5 Core and Stage 6 Standard
- This section addresses concepts from the Stage 5 Core topics called *Linear relationships A* and *Linear relationships B*
- In Stage 6 Standard these concepts will be revised and extended
- You may be expected to recall the formula from memory in the HSC Examination

When data points from an experiment are plotted, they may form a straight line. This shows a linear relationship between the two variables, such as time and growth, involved in the experiment.

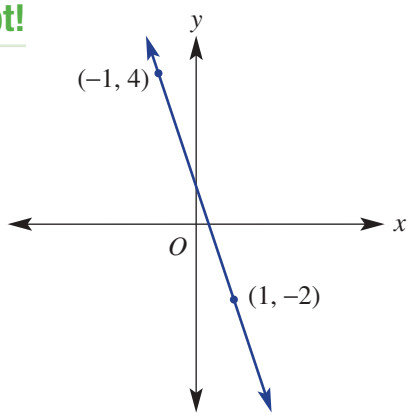
By finding the equation of this line, we can develop a rule to relate the two variables. This rule can be used to make further predictions about the data, and could be applied to a larger scale experiment.

Using gradient–intercept form, the rule (or equation) of a line can be found by calculating the value of the gradient and the y-intercept.

Lesson starter: But we don't know the y-intercept!

A line with the rule $y = mx + c$ passes through two points $(-1, 4)$ and $(1, -2)$.

- Using the information given, is it possible to find the value of m?
  If so, calculate its value.
- The y-intercept is not given on the graph. Discuss what information could be used to find the value of the constant, c, in the rule. Is there more than one way you can find the y-intercept?
- Write the rule for the line.



KEY IDEAS

- To find the equation of a line in gradient–intercept form, $y = mx + c$, you need to find:
  - the value of the gradient (m) using $m = \frac{\text{rise}}{\text{run}}$
  - the value of the constant (c), by observing the y-intercept or by substituting another point.
- All the points (x, y) on a line satisfy the equation of the line.
  For example, $(2, 5)$ is on the line with equation $y = 2x + 1$, because substituting $x = 2$ and $y = 5$ gives a true equation: $5 = 2 \times (2) + 1$.

Key terminology: equation, gradient–intercept form, gradient, y-intercept

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Exercise 5H

UNDERSTANDING 1–4 | 4

1 Fill in the missing words.
To find the equation of a line in the form $y = mx + c$, the _____ (m) is required as well as the _____ (c) or another _____.

2 Substitute the given values of m and c into $y = mx + c$ to write the equation.

a $m = 2, c = 5$ **b** $m = 4, c = -1$

c $m = -2, c = 5$ **d** $m = -1, c = -\frac{1}{2}$

3 Which line does the point $(3, 1)$ lie on?

A $y = 3x + 1$ **B** $y = x - 3$ **C** $y = -2x + 7$

D $y = 2x + 1$ **E** $y = -x - 2$

Hint: Which rule gives $y = 1$ when $x = 3$?

4 Substitute the point into the given equation and solve to find the value of c.
For example, using $(3, 4)$, substitute $x = 3$ and $y = 4$ into the rule.

a $(3, 4), y = x + c$ **b** $(1, 5), y = 2x + c$

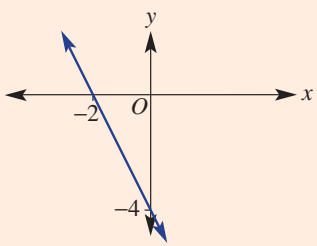
c $(3, -1), y = -2x + c$

Hint: In part **a**, $4 = 3 + c$.
$\therefore c = 1$

FLUENCY 5(½), 7(½) | 5–7(½)

Example 20 Finding the equation of a line given the y-intercept and another point

Determine the equation of the straight line shown here.



SOLUTION

The equation will have the form $y = mx + c$.

$$\text{gradient, } m = \frac{\text{rise}}{\text{run}}$$

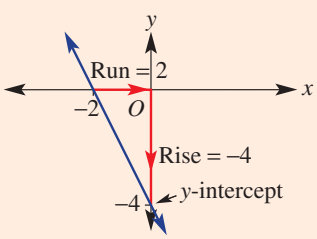
$$= \frac{-4}{2}$$

$$= -2$$

y-intercept $= -4$
$\therefore y = -2x - 4$

EXPLANATION

For the gradient–intercept form you need to find the gradient (m) and the y-intercept (c).



The y-intercept (-4) can be read from the graph.
In $y = mx + c$, substitute $m = -2$ and $c = -4$.

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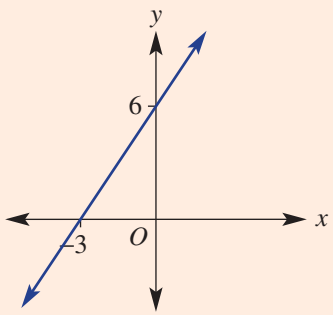
Now you try

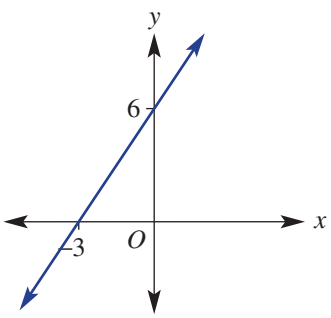
Determine the equation of the straight line shown here.



5 Determine the equation of the following straight lines.

Hint: Write in the form $y = mx + c$. You will need the gradient and the y-intercept (or another point if the y-intercept is not given).

a



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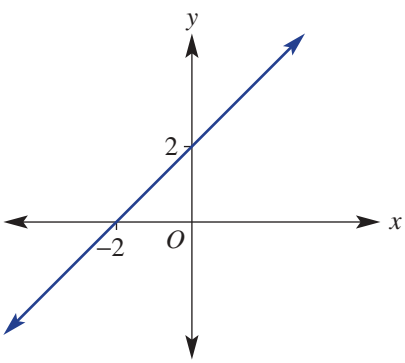

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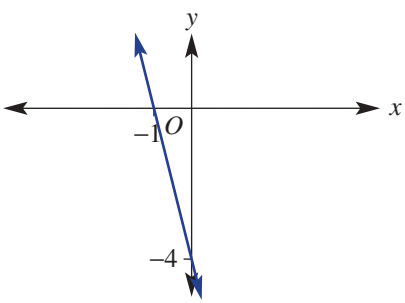

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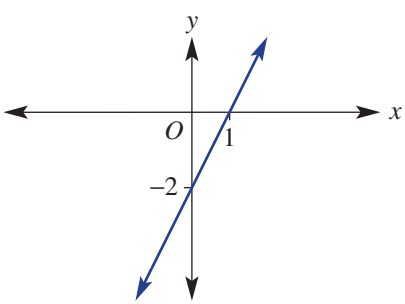

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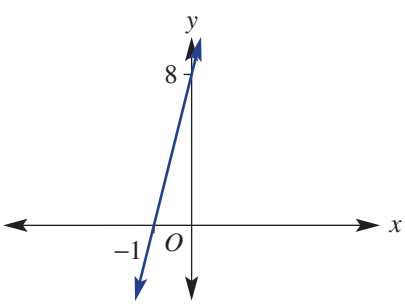

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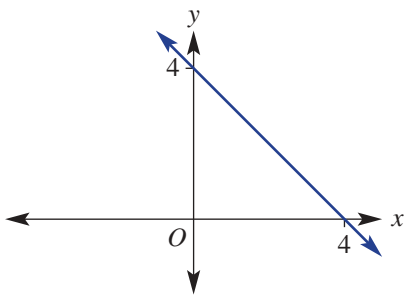

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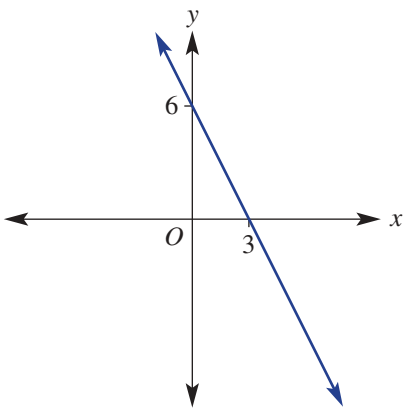
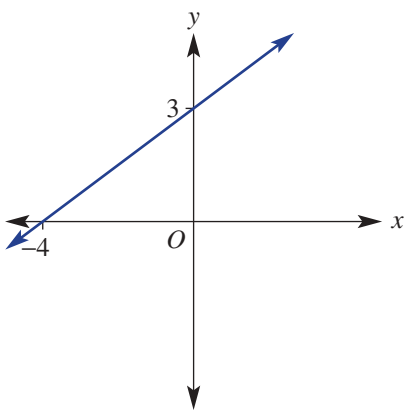


6 Find the equation of these straight lines, which have fractional gradients. Simplify any fractions.

a



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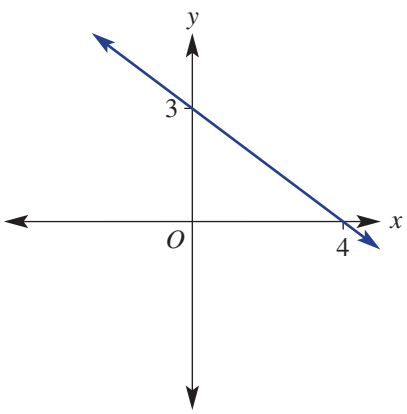

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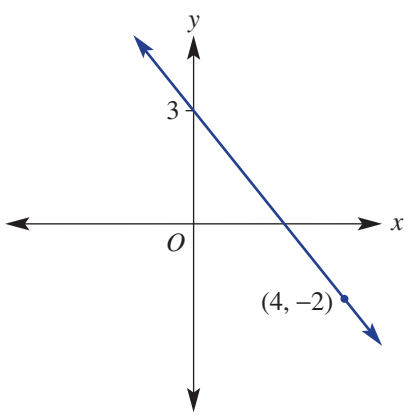

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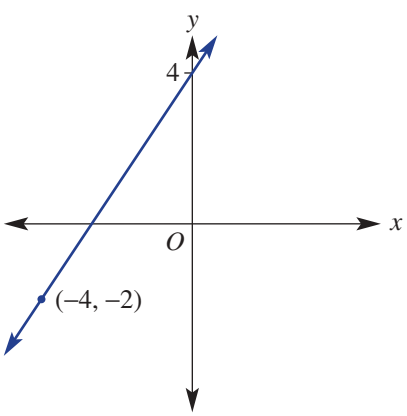

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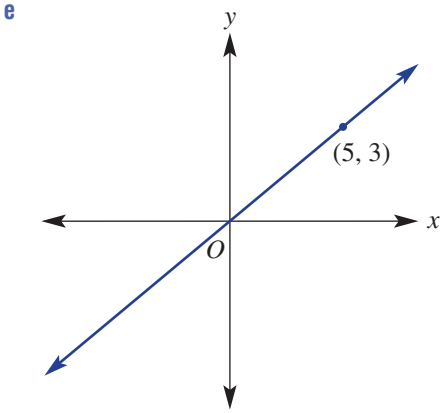

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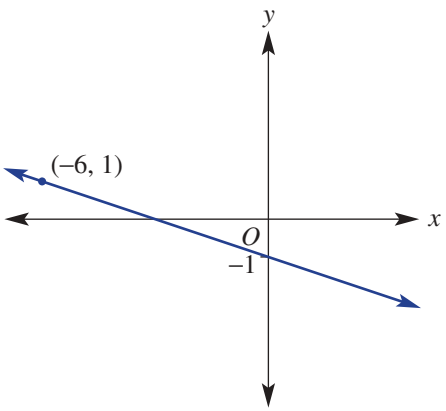

Example 21 Finding the equation of a line given the gradient and a point

Find the equation of the line that has a gradient, m, of 3 and passes through the point $(2, -1)$.

SOLUTION	EXPLANATION
$y = mx + c$	Substitute $m = 3$ into $y = mx + c$.
$y = 3x + c$	
$-1 = 3 \times (2) + c$	Since $(2, -1)$ is on the line, it must satisfy the equation $y = 3x + c$, hence substitute the point $(2, -1)$ where $x = 2$ and $y = -1$ to find c. Simplify and solve for c by subtracting 6 from both sides.
$-1 = 6 + c$	
$-7 = c$	
$\therefore y = 3x - 7$	Write the equation in the form $y = mx + c$.

Now you try

Find the equation of the line that has a gradient, m, of 2 and passes through the point $(2, -1)$.

7 Find the equation of the line that:

- **a** has a gradient of 3 and passes through the point $(1, 8)$
- **b** has a gradient of -2 and passes through the point $(2, -5)$
- **c** has a gradient of -3 and passes through the point $(2, 2)$
- **d** has a gradient of 1 and passes through the point $(1, -2)$
- **e** has a gradient of -3 and passes through the point $(-1, 6)$
- **f** has a gradient of 5 and passes through the point $(2, 9)$
- **g** has a gradient of -1 and passes through the point $(4, 4)$
- **h** has a gradient of -3 and passes through the point $(3, -3)$
- **i** has a gradient of -2 and passes through the point $(-1, 4)$
- **j** has a gradient of -4 and passes through the point $(-2, -1)$

Hint: In $y = mx + c$, insert the gradient value for m, then substitute the point for x and y to find c.

PROBLEM-SOLVING AND REASONING 8, 9 8–11

8 The coordinates $(0, 1)$ mark the take-off point 1 metre above the ground for a rocket constructed as part of a science class. The positive x direction is considered to be east. Find the equation of the rocket's path if it rises at a rate of 5 m vertically for every 1 m it travels east.

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9 A line has gradient -2 and y-intercept 5. Find its x-intercept.

Hint: First write the rule of the line.

10 Brad starts the year with \$80 in his bank account. He adds money into his account each week and does not take money out. After 4 weeks he has \$220 in his account. If the relationship is linear:

a find a rule for the amount of money in the account after x weeks

b use your rule to find when he will have \$500

Hint: Plot the linear information on a graph to help.

11 For the line connecting the following pairs of points:

i find the gradient **ii** find the equation using $y = mx + c$

a $(2, 6)$ and $(4, 10)$ **b** $(1, 7)$ and $(3, -1)$

c $(-3, 6)$ and $(5, -2)$ **d** $(-4, -8)$ and $(1, -3)$

Hint: Compare with Example 21.

ENRICHMENT: Two pieces of information make a linear rule — 12, 13

Assume that the relationships between the variables in these questions are linear.

12 Water is leaking from a tank. The volume of water in the tank after 1 hour is 100 L and after 5 hours the volume is 20 L.

a Find the equation of the line joining the points to find a rule. Use y for volume and x for time.

b Use the rule to state the amount of water that was in the tank to start with.

13 At 1 minute before midnight, the temperature inside a house in winter was $-5°C$. The heater was switched on at this time. Three minutes after midnight, the temperature had reached $3°C$.

a Find the rule to represent this. (Plot the points on axes and find the equation of the line joining them. Use y for temperature and x for time.)

b Find the x-intercept and the y-intercept.

c Describe what the points in part **b** represent in a practical sense.

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5I Midpoint and length of a line segment from diagrams

Learning intentions for this section:

- To be able to find the midpoint of a line segment given the endpoints
- To be able to use Pythagoras' theorem to find the length of a line segment

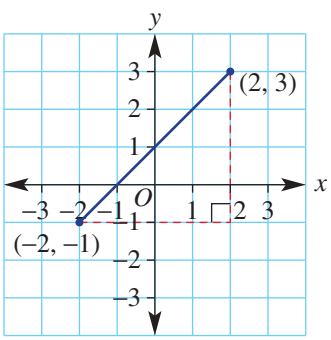
Past, present and future learning:

- *Linear relationships* is a major topic in Stage 4, Stage 5 Core and Stage 6 Standard
- This section addresses concepts from the Stage 5 Core topics called *Linear relationships A* and *Linear relationships B*
- This section has limited relevance to Stage 6 Standard

A line continues on forever, but a line segment (or line interval) has endpoints; this means it has a defined length and therefore must have a midpoint. Both the midpoint and length can be found by using the coordinates of the endpoints.

Lesson starter: Choosing a method

This graph shows a line segment between the points at $(-2, -1)$ and $(2, 3)$.



- What is the horizontal distance between the two points?
- What is the vertical distance between the two points?
- Discuss and explain a method for finding the length of a line segment, using the right-angled triangle formed.
- What is the x-coordinate of the point halfway along the line segment?
- What is the y-coordinate of the point halfway along the line segment?
- Discuss and explain a method for finding the midpoint of a line segment.

Using graphing software or dynamic geometry software, produce a line segment like the one shown above. Label the coordinates of the endpoints and the midpoint. Also find the length of the line segment. Now drag one or both of the endpoints to a new position.

- Describe how the coordinates of the midpoint relate to the coordinates of the endpoints. Is this true for all positions of the endpoints that you choose?
- Now use your software to calculate the vertical distance and the horizontal distance between the two endpoints. Then square these lengths. Describe these squared lengths compared to the square of the length of the line segment. Is this true for all positions of the endpoints that you choose?

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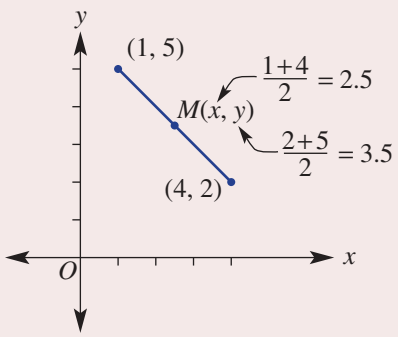
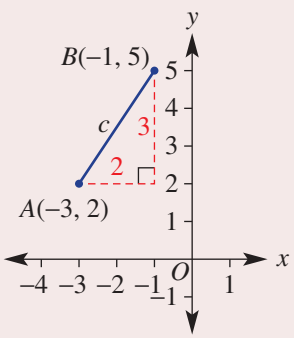
KEY IDEAS

- A **line segment** is a section of a straight line between two points (**endpoints**).
- The **midpoint** (M) of a line segment is the halfway point between the two endpoints.
  - The x-coordinate is the **average** (mean) of the x-coordinates of the two endpoints.
  - The y-coordinate is the average (mean) of the y-coordinates of the two endpoints.
  - Mid-point is (average of x-values, average of y-values)



- The **length** of a line segment (or line interval) is found using Pythagoras' theorem.
  - The line segment is the hypotenuse (longest side) of a right-angled triangle.
  - The coordinates can be used to find the horizontal distance between them, and the vertical distance.



Horizontal distance from -3 to -1 is 2.
Vertical distance from 2 to 5 is 3.
Pythagoras' theorem:

$$c^2 = 2^2 + 3^2$$
$$= 13$$
$$\therefore c = \sqrt{13} \text{ units}$$

This is the distance from A to B.

Key terminology: midpoint, line segment, coordinates, endpoints, average, Pythagoras' theorem, hypotenuse, horizontal, vertical, length

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Exercise 5I

UNDERSTANDING 1–3 | 3

1 Find the number that is halfway between these pairs of numbers.

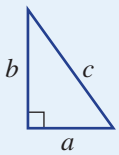
a 5, 11 **b** −2, 4 **c** −6, 0 **d** 4, 7

Hint: If in doubt, add them and divide by 2.

2 Use Pythagoras' theorem to find the length of the hypotenuse in these right-angled triangles. Round to one decimal place where required.

Hint: Pythagoras' theorem:

$c^2 = a^2 + b^2$



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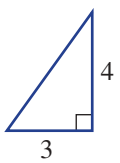

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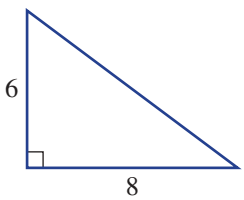

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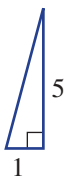

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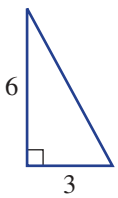

3 **a** Find the horizontal distance between the points:

i (1, 4) and (4, 6)

ii (−2, 3) and (2, −1)

b Find the vertical distance between the points:

i (−2, 3) and (0, 5)

ii (2, 5) and (6, −2)

Hint: For horizontal distance, consider the x-coordinates. For vertical distance, consider the y-coordinates.

FLUENCY 4–5(½) | 4–5(½)

Example 22 Finding a midpoint

Find the midpoint $M(x, y)$ of the line segment joining these pairs of points.

a (1, 0) and (5, 4)

b (−3, −2) and (5, 3)

SOLUTION

a $x = \frac{1+5}{2} = 3$

$y = \frac{0+4}{2} = 2$

$\therefore M = (3, 2)$

EXPLANATION

The x-values are 1 and 5
The y-values are 0 and 4

Find the average (mean) of the x-coordinates and y-coordinates for both points.

Continued on next page

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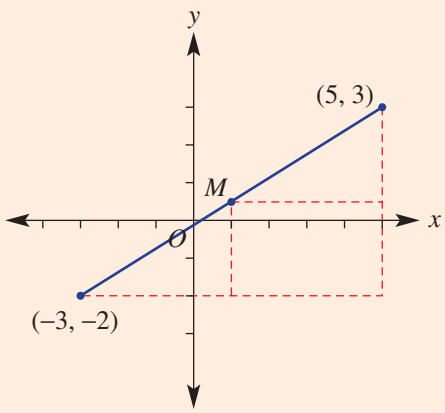
b $x = \dfrac{-3+5}{2} = 1$

$y = \dfrac{-2+3}{2} = 0.5$

$\therefore M = (1, 0.5)$

The x-values are -3 and 5
The y-values are -2 and 3

By plotting the points to form the line segment, you can check that the coordinates you find for the midpoint appear to be the halfway point of the line segment.



Now you try

Find the midpoint $M(x, y)$ of the line segment joining these pairs of points.

a (2, 1) and (6, 9)

b (−4, −1) and (0, 6)

4 Find the midpoint $M(x, y)$ of the line segment joining these pairs of points.

a (0, 0) and (6, 6)
b (0, 0) and (4, 4)
c (0, 2) and (2, 8)
d (3, 0) and (5, 2)
e (−2, 0) and (0, 6)
f (−4, −2) and (2, 0)
g (1, 3) and (2, 0)
h (−1, 5) and (6, −1)
i (−3, 7) and (4, −1)
j (−2, −4) and (−1, −1)
k (−7, −16) and (1, −1)
l (−4, −3) and (5, −2)

Hint: Check that your coordinates for the midpoint appear to lie halfway along the line segment.

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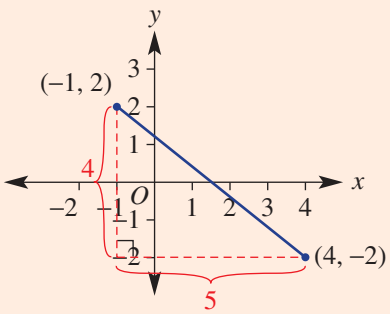
Example 23 Finding the length of a segment

Find the length of the segment joining $(-1, 2)$ and $(4, -2)$, correct to two decimal places.

SOLUTION



EXPLANATION

Plot the points and form the line segment to visualise the problem.

Form the right-angled triangle.

Use the coordinates to find the lengths of the horizontal and vertical sides.

Horizontal length $= 5$

Vertical length $= 4$

Horizontal length: from -1 to 4 is 5 units.
Vertical length: from 2 to -2 is 4 units.

$\therefore c^2 = 5^2 + 4^2$

$= 25 + 16$

$= 41$

Apply Pythagoras' theorem $c^2 = a^2 + b^2$.

From $c^2 = 41$, take the square root of both sides.

$\therefore c = \sqrt{41}$

$\therefore$ length $= 6.40$ (to 2 d.p.)

Round to two decimal places on a calculator as required.

Now you try

Find the length of the segment joining $(-2, 3)$ and $(1, -3)$, correct to two decimal places.

5 Find the length of the segment joining these pairs of points, correct to two decimal places.

a $(1, 1)$ and $(2, 6)$
b $(1, 2)$ and $(3, 4)$
c $(0, 2)$ and $(5, 0)$
d $(-2, 0)$ and $(0, -4)$
e $(-1, 3)$ and $(2, 1)$
f $(-2, -2)$ and $(0, 0)$
g $(-1, 7)$ and $(3, -1)$
h $(-4, -1)$ and $(2, 3)$
i $(-3, -4)$ and $(3, -1)$

PROBLEM-SOLVING AND REASONING 6–8 | 7–10

6 A fence line on the plans for a property shows fence posts at points with coordinates $(-4, 8)$, $(2, 3)$ and $(8, -2)$. To reinforce the fence, two posts are to be placed halfway between the current posts. At what coordinates will these posts be placed?

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7 Find the missing coordinates in this table if M is the midpoint of points A and B.

A	B	M
(4, 2)		(6, 1)
	(0, −1)	(−3, 2)
	(4, 4)	(−1, 6.5)

8 A circle has its centre at (2, 1). Find the coordinates of the endpoint of a diameter if the other endpoint has these coordinates.

a (7, 1) **b** (3, 6) **c** (−4, −1)

Hint: A diameter of a circle is shown: It passes through the centre.

9 Find the perimeter of these shapes correct to one decimal place.

a A triangle with vertices (−2, 0), (−2, 5) and (1, 3).

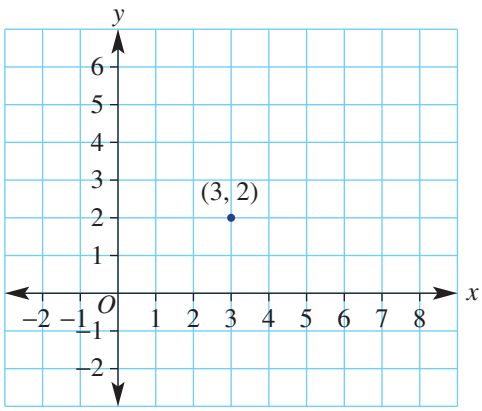
b A trapezium with vertices (−6, −2), (1, −2), (0, 4) and (−5, 4).

Hint: Start by plotting the points to form the line segments that make up the shape.

10 For the point (3, 2) shown:

a list four points that are 3 units from this point and add them to the axes

b describe the shape that would be formed by all the points that are 3 units from (3, 2).



ENRICHMENT: Division by ratio — 11

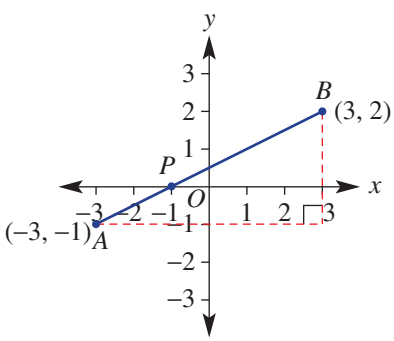
11 Looking from left to right, this line segment shows the point $P(-1, 0)$ which divides the segment in the ratio 1 : 2.

a What fraction of the horizontal distance between the endpoints is P from A?

b What fraction of the vertical distance between the endpoints is P from A?

c Find the coordinates of point P on the segment AB if it divides the segment in these ratios.

i 2 : 1 **ii** 1 : 5 **iii** 5 : 1



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5J Linear relationships in real-life contexts

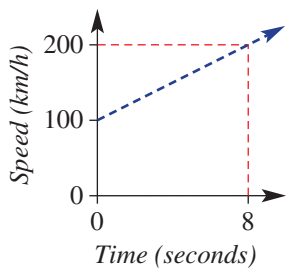
Learning intentions for this section:

- To understand that many real-life situations can be modelled using a straight line
- To be able to find the equation linking two variables
- To know where the dependent and independent variable are positioned on a set of axes
- To be able to use an equation or graph to estimate the value of one variable given the other

Past, present and future learning:

- *Linear relationships* is a major topic in Stage 4, Stage 5 Core and Stage 6 Standard
- This section addresses concepts from the Stage 5 Core topics called *Linear relationships A* and *Linear relationships B*
- In Stage 6 Standard these concepts will be revised and extended

If a relationship between two variables is linear, the graph will be a straight line. The equation linking the two variables can therefore be written in gradient–intercept form. The area of mathematics that uses line graphs and rules to explore the relationship between two variables is called linear modelling. A test car, for example, increasing its speed from 100 km/h to 200 km/h in 8 seconds with constant acceleration, could be modelled by the rule $s = 12.5t + 100$. This rule could then be used to calculate the speed at different times in the test run.



Lesson starter: The test car

The above graph describes the speed of a racing car over an 8-second period.

- Explain how the rule $s = 12.5t + 100$ for the graph is found by describing what the 100 means and what the 12.5 means.
- Why might negative values of t not be considered for the graph?
- How could you accurately calculate the speed at the 6.5 second mark?
- If the car continued to accelerate at the same rate, how could you accurately predict the car's speed at the 13.2 second mark?

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KEY IDEAS

- Many real-life situations can be modelled by using a linear equation and graph.
  The key elements of linear modelling include:
  - Finding the equation linking the two variables
  - Sketching a graph
  - Using the graph or equation to predict or estimate the value of one variable given the other
  - Finding the rate of change of one variable with respect to the other variable. This is the same as finding the gradient.
- On a set of axes the **dependent variable** goes on the vertical axis and the **independent variable** goes on the horizontal axis.
  - The dependent variable changes as a result of changes that occur in the independent variable.
  - If the variables are speed and time, then speed is on the vertical axis, since the speed reached depends on the time passed.
  - Time is usually on the horizontal axis.

Key terminology: dependent variable, independent variable, linear, gradient, rate

Exercise 5J

UNDERSTANDING 1–3 3

1 A person gets paid \$50 plus \$20 per hour. Decide which equation describes the relationship between the pay, \$$P$, and number of hours, n.

A $P = 50 + n$
B $P = 50 + 20n$
C $P = 50n + 20$
D $P = 20 + 50$

2 The amount of money in a bank account is \$1000 and it is increasing by \$100 per month.

a Find the amount of money in the account after:
i 2 months **ii** 5 months **iii** 12 months

b Write an equation for the amount of money, A dollars, after n months.

3 **a** Insert the word *independent* or *dependent* in the following.
i The _____ variable goes on the vertical axis.
ii The _____ variable goes on the horizontal axis.

b Choose the dependent variable in the following cases.
i The volume of water, V, in a tank t minutes after turning on the hose
ii The height, h, of a tree t days after it was planted
iii The amount of money earned from the sale of m items

Hint: Which variable changes in response to changes in the other variable?

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FLUENCY 4–6 4–7

Example 24 Applying linear relationships

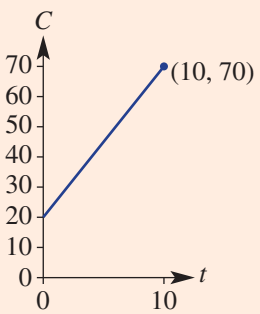
Netshare is an internet provider. It charges $20 up front plus $5 per week of use.

a Write an equation for the total cost, $C, of using Netshare for t weeks.
b Sketch the graph of C versus t from $t = 0$ to $t = 10$.
c What is the total cost when Netshare is used for 4 weeks?
d If the total cost was $50, for how many weeks was Netshare used?

SOLUTION

a $C = 20 + 5t$

b C-intercept is 20
At $t = 10$, $C = 20 + 5 \times (10) = 70$
$\therefore$ endpoint is (10, 70)



c $C = 20 + 5t$
$= 20 + 5 \times (4)$
$= 40$

The cost is $40

d $C = 20 + 5t$
$50 = 20 + 5t$
$30 = 5t$
$\therefore t = 6$

Netshare was used for 6 weeks.

EXPLANATION

A fixed amount of $20 plus $5 for each week. This is $5 \times t$.

The cost, C, is the dependent variable and so it goes on the vertical (y) axis.

Let $t = 0$ to find the C-intercept (this is the point on the vertical axis).

Letting $t = 10$ gives $C = 70$ and this gives the other endpoint.

Sketch the graph using the points (0, 20) and (10, 70).

Substitute $t = 4$ into the equation.

Answer the question using the correct units.

Write the equation and substitute $C = 50$. Solve the resulting equation for t by subtracting 20 from both sides then dividing both sides by 5.

Answer the question in words.

Now you try

A gardener charges $60 upfront plus $30 per hour of work.

a Write the equation for the total cost, $C, of hiring the gardener for n hours.
b Sketch the graph of C versus n from $n = 0$ to $n = 6$.
c What is the total cost for a job that takes 3 hours?
d If the total cost was $210, how many hours was the gardener hired for?

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4 A sales representative earns \$400 a week plus \$20 for each sale she makes.

Hint: To sketch over $0 \leq x \leq 40$, find the endpoints; that is, the point where $x = 0$ and where $x = 40$.

- **a** Write the equation that gives the total weekly wage, \$$W$, if the sales representative makes x sales.
- **b** Draw a graph of W versus x from $x = 0$ to $x = 40$.
- **c** How much does the sales representative earn if, in a particular week, she makes 12 sales?
- **d** If, in a particular week, the sales representative earns \$1000, how many sales did she make?

5 A plumber charges a \$40 fee upfront per job, and \$50 for each hour he works.

- **a** What is the equation for the total charge, \$$C$, for n hours of work?
- **b** What will a 4-hour job cost?
- **c** If the plumber works on a job for two days and averages 6 hours per day, what will be the total cost?

6 A catering company charges \$500 for the hire of a marquee (a giant tent), plus \$25 per guest.

- **a** What is the equation for the cost, \$$C$, of hiring a marquee and catering for n guests?
- **b** Draw a graph of C versus n from $n = 0$ to $n = 100$.
- **c** How much would a party catering for 40 guests cost?
- **d** If a party cost \$2250, how many guests were catered for?

7 The cost, \$$C$, of hiring a recording studio is \$300, plus \$120 per hour.

- **a** What is the equation for the cost, \$$C$, of hiring the studio for t hours?
- **b** Draw a graph of C versus t from $t = 0$ to $t = 10$.
- **c** How much does a recording using 6 hours of studio time cost?
- **d** If a recording cost \$660 to make, for how long was the studio used?

PROBLEM-SOLVING AND REASONING

8, 9 | 9–11

8 A petrol tank holds 66 litres of petrol. If it starts with 12 litres of petrol and the petrol pump fills it at 3 litres every 10 seconds, find:

Hint: Read units carefully. 3 L in 10 seconds equals 18 L in 1 minute.

- **a** a linear equation for the amount of fuel (F litres) in the tank after t minutes
- **b** how long it will take to fill the tank.

9 A tank starts with 4000 L of water. Water from it is used at a rate of 20 L per minute.

Hint: If the water is being used, think about what should be happening to the volume of water in the tank.

- **a** Write the equation for the volume, V litres, of water after t minutes.
- **b** Calculate the volume after 1.5 hours.
- **c** How long will it take for the tank to be emptied?
- **d** How long will it take for the tank to have only 500 L?

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10 The rule for distance travelled, d km, over a given time, t hours, for a moving vehicle is given by $d = 50 + 80t$.

- **a** What is the speed of the vehicle?
- **b** If the speed was actually 70 km per hour, how would this change the rule? Write the new equation.

11 The altitude, h metres, of a helicopter t seconds after it begins its descent is given by $h = 350 - 20t$.

Hint: Altitude is the height above ground at which the helicopter is flying.

- **a** Use the equation to find:
  - **i** the rate at which the helicopter's altitude is decreasing
  - **ii** the helicopter's initial height ($t = 0$)
  - **iii** how long it will take for the helicopter to reach the ground.
- **b** If the equation was $h = 350 + 20t$, describe what the helicopter would be doing.

ENRICHMENT: Where will they meet? — 12

12 Car A heads out of town on the open highway travelling at an average speed of 65 km/h.

- **a** Write an equation for the distance, d km, of the car from the town after t hours.
- **b** Sketch a graph of distance (d) vs time (t) from $t = 0$ to $t = 5$.

At the same time as Car A leaves town, Car B sets out on the highway from the opposite direction, travelling towards the town. The equation for the distance, d km, of Car B from the town after t hours is given by $d = 350 - 75t$.

- **c** How many kilometres from town does Car B begin?
- **d** How long will it take for Car B to reach the town?
- **e** Sketch the graph of the rule for Car B on the same axes that you used in part **b**.
- **f** Using your graph, find:
  - **i** how many hours into the journey the two cars will pass each other
  - **ii** how many kilometres out of town they will be when they pass each other.

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Maths@Work

Trading in foreign currencies

Many professionals use linear relationships to model aspects of their jobs. Foreign exchange (forex) traders or currency traders use their knowledge of trends and future growth to buy and sell currencies from around the world.

It is the biggest market in the world. People often trade in shares, futures and overseas currencies from the privacy of their home.

1 The currency of one country is directly proportional to the currency of another country. Each currency exchange can be written as an equation in the form $y = mx$.
The top line of this table states that 1 euro (EUR) = 1.5687 Australian dollars (AUD).
Writing this as an equation gives: $y = 1.5687 \times x$ where x = EUR, y = AUD.

	From (x: input currency)	To (y: output currency)	Exchange rate
i	The EURO (EUR)	Australian dollars (AUD)	1.5687
ii	British pounds (GBP)	AUD	2.0306
iii	USD	Chinese Yuan (CNY)	6.5181
iv	EUR	GBP	0.7725
v	AUD	EUR	0.6375
vi	AUD	United states dollar (USD)	0.7158
vii	AUD	Singapore dollar (SGD)	1.0038
viii	AUD	New Zealand dollar (NZD)	1.0753

Use the exchange rate table above to complete a table with these column headings. Work with four decimal places for the linear equation and three decimal places for y when $x = 10$ in the 'Two points' column.

x: input currency	y: output currency	Linear equation	Two points (0, _) and (10, _)

2 Use the information in your answer to Question **1** for the following tasks.

a For exchange rates **i** to **iv**, sketch each line on a separate number plane. Label the currency on each axis and label each point where $x = 10$.

b What feature of the straight line does the exchange rate represent in each graph?

c For exchange rates **v** to **viii**, sketch each line on the same number plane. Label each point where $x = 10$.

d What do you notice about the slope of the line as the exchange rate increases?

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Using technology

3 Currency rates fluctuate over time. Follow the Excel spreadsheet instructions below to graph a trendline and determine the trendline equation for the exchange rate of Australian dollars to Japanese yen (JPY).

May	AUD to JPY
1	80.48545
2	80.98463
3	81.15554
4	82.53038
5	82.53038
6	83.17927
7	84.55183
8	84.55183

a i Enter the data from this table into an Excel spreadsheet.

ii Select both columns.

iii Choose Insert and Scatter graph and click on the icon of unconnected points.

iv Click on one of the graphed points, right click, and select 'Add trend line'.

v From the trendline options that appear, select 'linear' and at the bottom of the callout tick 'display equation on chart'.

vi Label the graph.

b Copy the trendline equation. Define the x and y for the trendline.

c What does the gradient of this trendline tell us?

d Use the trendline equation to predict the exchange rate for 10 May.

e If the exchange rate continues to follow this trend, how many Japanese yen would you expect to receive when exchanging 1500 AUD on 15 May?

f Give one reason why a currency trendline equation can't be relied on to make accurate predictions.

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Working mathematically

Running in a triathlon

Jack, Levi and Max compete in a junior triathlon event, first swimming 200 m, then cycling 12 km and finally running 1200 m.

Jack was the first to start the running leg and he runs at an average speed of 3 m/s. Levi runs at 3.5 m/s and Max runs at 4 m/s. However, Max and Levi take longer to complete the swimming and cycling and start the run after Jack started. Can either of them overtake Jack and win the race?

For this task, d metres is the distance travelled by a runner and t seconds is the time in seconds after Jack starts running.

Present a report for the following tasks and ensure that you show clear mathematical workings, explanations and diagrams where appropriate.

Routine problems

a Find the time, in seconds, taken to run 1200 m at an average speed of 2.4 m/s.

b Write the equation for distance d after t seconds when running at a speed of 2.4 m/s.

c An equation describing a different runner is given by $d = 2.5t - 20$. Let $d = 1200$ and solve the equation to find this runner's finishing time.

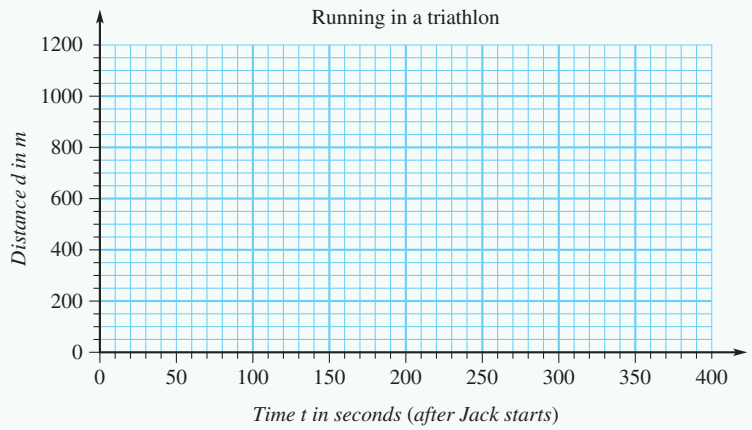
Non-routine problems

Explore and connect

The problem is to find the times of the first, second and third place runners and their time differences.

a Write down all the relevant information that will help solve this problem.

b Using a large sheet of graph paper or digital graphing software, prepare axes like those shown below. Draw and label a horizontal axis showing time t from 0 seconds to 400 seconds and a vertical axis showing distance d from 0 m to 1200 m.



Choose and apply techniques

c Jack runs at an average speed of 3 m/s for the 1200 m.

i How long, in seconds, does Jack take to run the 1200 m?

ii Write an equation for Jack's distance (d metres) in terms of time (t seconds).

iii Draw a line on the graph to represent Jack's run. Label this line 'Jack'.

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d Levi starts running after Jack had started. Levi runs at 3.5 m/s and the rule describing Levi's distance d after t seconds is given by $d = 3.5t - 175$.
- **i** Let $d = 0$ and find t, Levi's starting time. Plot this point and label its coordinates.
- **ii** Find Levi's distance when $t = 350$. Plot this point and label its coordinates.
- **iii** Draw a line through these points extend it to 1200 m. Label this line 'Levi'.

e What occurs on the graph at $t = 350$? Explain what happens between Jack and Levi at 350 s.

f Max starts the running race behind Levi and Jack. Max runs at 4 m/s and the equation describing Max's distance d after t seconds is given by $d = 4t - 300$.
- **i** Let $d = 0$ and find t, Max's starting time. Plot this point and label its coordinates.
- **ii** Find Max's distance when $t = 250$. Plot this point and label its coordinates.
- **iii** Rule a line through these points and extend it to 1200 m. Label this line 'Max'.

g What occurs on the graph at $t = 250$? Explain what happens between Max and Levi at 250 seconds.

h Does Max also overtake Jack? If so, state the time and distance where this happens.

i From your graph determine who gets first, second and third place.

j Let $d = 1200$ in each equation and solve each for t to find the runner's finishing times, to the nearest second.

k By how many seconds was the first place runner ahead of the second runner and the third runner?

l Summarise your results and describe any key findings.

Communicate thinking and reasoning

Extension problem

Problem-solve

A fourth runner, Elijah, started 80 seconds after Jack started and finished at the same time as Jack.

a Answer the following questions about the line on the graph that would show Elijah's run.
- **i** State the coordinates of the start and end points that would be on Elijah's line.
- **ii** Calculate the gradient of the line that joins these two points.
- **iii** What feature of Elijah's running does this gradient represent?
- **iv** The line equation is in the form $d = mt + c$. Find the values of m and c and hence the equation of the line that would represent Elijah's run.

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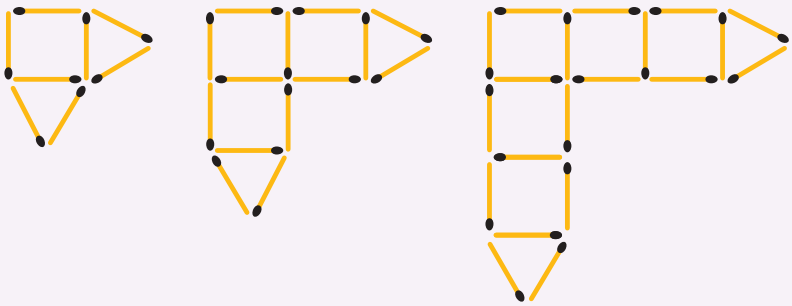
1 A student arranges matches such that the first three diagrams in the pattern are:

How many matches are in the 50th diagram of the pattern?

2 The first three shapes in a pattern made with matchsticks are:

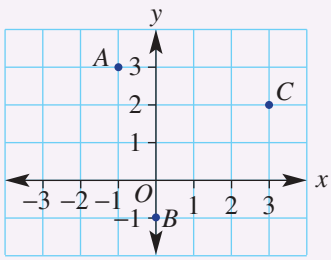


How many matchsticks make up the 100th shape?

3 A tank with 520 L of water begins to leak at a rate of 2 L per day. At the same time, a second tank is being filled at a rate of 1 L per hour starting at 0 L. How long does it take for the tanks to have the same volume?

4 The points $(-1, 4)$, $(4, 6)$, $(2, 7)$ and $(-3, 5)$ are the vertices of a parallelogram. Find the midpoints of its diagonals. What do you notice?

5 Prove that the triangle with vertices at the points $A(-1, 3)$, $B(0, -1)$ and $C(3, 2)$ is isosceles.



6 Bill takes 3 days to paint a house. Rashid takes 4 days to paint a house. Lucy takes 5 days to paint a house. How long would it take to paint a house (to the nearest hour) if all three of them worked together?

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Chapter summary

Linear relationships

Direct variation (proportion)

$y = mx$ means y is directly proportional to x.

m = *gradient*; the rate of change of y with respect to x.

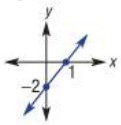
e.g. travelling at 60 km/h for t hours, distance $d = 60t$.

Finding the equation of a line

Gradient–intercept form ($y = mx + c$)

Require gradient and y-intercept or any other point to substitute.

E.g.



$m = \frac{\text{rise}}{\text{run}} = \frac{2}{1} = 2$

y-int is -2

$\therefore y = 2x - 2$

Midpoint and length of line segment

Midpoint is halfway between segment endpoints.

$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

E.g. midpoint of segment joining $(-1, 2)$ and $(3, 6)$

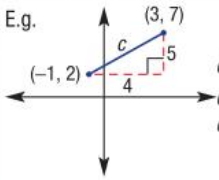
$x = \frac{-1 + 3}{2} = 1$

$y = \frac{2 + 6}{2} = 4$, i.e. $(1, 4)$

Length of segment

Use Pythagoras' theorem $c^2 = a^2 + b^2$

E.g.



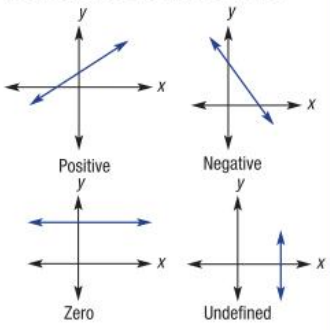
$c^2 = 4^2 + 5^2$

$c = \sqrt{41}$

$c = 6.4$ (to 1 d.p.)

Gradient

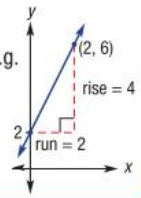
This measures the slope of a line.



Gradient $m = \frac{\text{rise}}{\text{run}}$

$m = \frac{4}{2} = 2$

e.g.



Linear modelling

Define variables to represent the problem and write a rule relating the two variables.

Straight line

$y = mx + c$

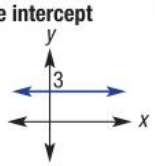
gradient, y-intercept

In the form $ax + by = d$, rearrange to $y = mx + c$ form to read off gradient and y-intercept.

Lines with one intercept

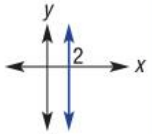
Horizontal line $y = c$

e.g. $y = 3$

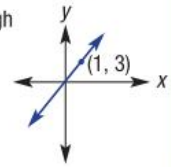


Vertical line $x = b$

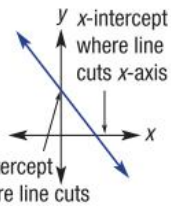
e.g. $x = 2$



$y = mx$ passes through the origin, substitute $x = 1$ to find another point, e.g. $y = 3x$



x- and y-intercepts



E.g. for $2x + 3y = 6$

y-int ($x = 0$)

$2(0) + 3y = 6$

$3y = 6$

$y = 2$

y-int is 2

x-int ($y = 0$)

$2x + 3(0) = 6$

$2x = 6$

$x = 3$

x-int is 3

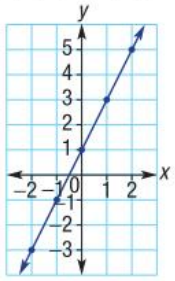
x-intercept has y-coordinate 0.

y-intercept has x-coordinate 0.

A linear relationship is made up of points (x, y) that form a straight line when plotted,

e.g. $y = 2x + 1$

x	–2	–1	0	1	2
y	–3	–1	1	3	5



Sketching linear graphs

Sketch with two ponts. Often we find the x-intercept ($y = 0$) and y-intercept ($x = 0$)

E.g. $y = 2x - 4$

y-int: $x = 0$, $y = 2(0) - 4$

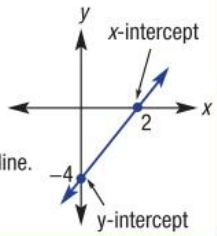
$= -4$

x-int: $y = 0$, $0 = 2x - 4$

$2x = 4$

$x = 2$

Plot these points and join in a line.



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Chapter checklist

Chapter checklist with success criteria

A printable version of this checklist can be downloaded from the Interactive Textbook.

✔

5A

1 I can plot points to graph a straight line.
e.g. Complete the table of values below for the rule $y = -2x + 1$ and plot a graph.

x	-2	-1	0	1	2
y					

☐

5A

2 I can decide whether a point is on a line.
e.g. Decide whether the point $(2, -1)$ is on the line with the given rules:
a $y = 2x - 3$ **b** $y = 3x - 7$

☐

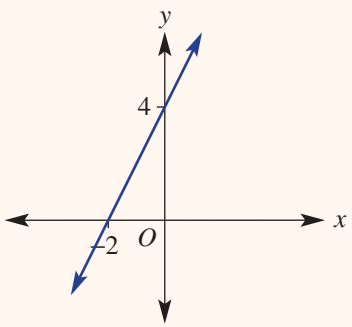
5B

3 I can identify *x*- and *y*-intercepts from a table or graph.
e.g. Read off the x-intercept and y-intercept from this table and graph

a

x	-2	-1	0	1	2
y	12	8	4	0	-4

b



☐

5B

4 I can find the *y*-intercept.
e.g. Find the y-intercept of these lines:
a $y = 3x - 4$ **b** $4x - 5y = 20$

☐

5B

5 I can find the *x*-intercept.
e.g. Find the x-intercept of these lines:
a $y = 3x - 4$ **b** $4x - 5y = 20$

☐

5C

6 I can sketch lines of the form $ax + by = d$ or $ax + by + c = 0$ using intercepts.
e.g. Sketch the graph of $2x + 7y = 14$, showing the x- and y-intercepts

☐

5C

7 I can sketch lines of the form $y = mx + c$ using intercepts.
e.g. Sketch the graph of $y = 3x - 9$ showing the x- and y-intercepts

☐

5D

8 I can graph vertical and horizontal lines.
e.g. Sketch the graphs of the following vertical and horizontal lines
a $y = -2$ **b** $x = 5$ **c** $y = 0$

☐

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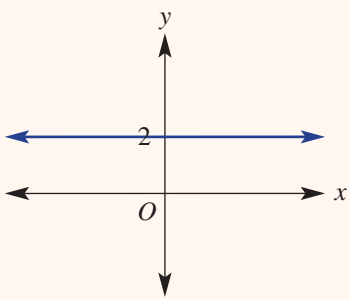
Chapter checklist with success criteria

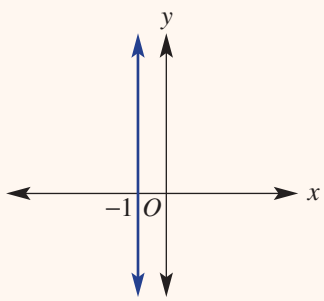
5D **9 I can sketch lines that pass through the origin.**
e.g. Sketch the graph of $y = 5x$

5D **10 I can find the equation of horizontal and vertical lines.**
e.g. Give the equation of the following vertical and horizontal lines.

a

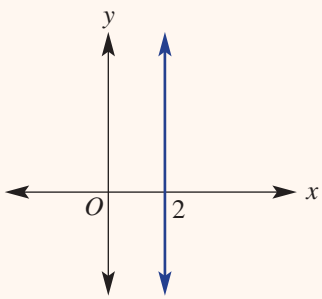


b

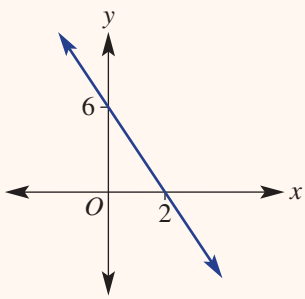


5E **11 I can find the gradient of a line.**
e.g. For each graph, state whether the gradient is positive, negative, zero or undefined then find the gradient where possible.

a



b



5E **12 I can find the gradient between two points.**
e.g. Find the gradient (m) of the line joining the points: $A(-2, 3)$ and $B(1, 7)$

5F **13 I can form a direct variation equation (rule).**
e.g. Write an equation linking the variables d km and t hours if I travelled 420 km in 7 hours.

5F **14 I can work with direct variation.**
e.g. A 400 L aquarium is filled with water. It takes 40 minutes to fill the tank.
a Draw a graph of volume (V litres) vs time (t minutes) from $t = 0$ to $t = 40$.
b Find the gradient of the graph and form a rule for V.
c Use the rule for V to find the time to fill 250 L.

5G **15 I can state the gradient and *y*-intercept from $y = mx + c$.**
e.g. State the gradient and the y-intercept for the graph of $y = 3x - 2$.

5G **16 I can rearrange linear equations into the form $y = mx + c$.**
e.g. Rearrange these linear equations into the form $y = mx + c$:
a $3x + y = 4$ **b** $6x + 2y - 12 = 0$

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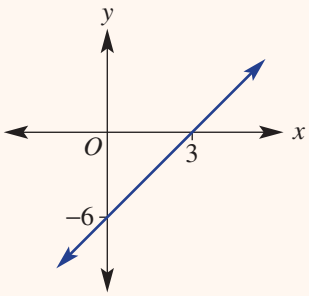
Chapter checklist

Chapter checklist with success criteria

5G	**17 I can sketch linear graphs using the gradient and *y*-intercept.** e.g. Find the value of the gradient and y-intercept for these relationships and sketch their graphs: **a** $y = 4x - 3$ **b** $3x + 2y = 10$ (rearrange first)	☐
5H	**18 I can find the equation of a line given the *y*-intercept and another point.** e.g. Determine the equation of the straight line shown here. 	☐
5H	**19 I can find the equation of a line given the gradient and a point.** e.g. Find the equation of the line that has a gradient, m, of 2 and passes through the point $(3, -2)$.	☐
5I	**20 I can find the midpoint of a line segment.** e.g. Find the midpoint $M(x, y)$ of the line segment joining $(1, -1)$ and $(5, 6)$.	☐
5I	**21 I can find the length of a line segment (the distance from point to point).** e.g. Find the length of the segment joining $(-2, 1)$ and $(3, 5)$, correct to two decimal places.	☐
5J	**22 I can apply linear relationships to real-life contexts.** e.g. A wedding photographer charges a $\$200$ upfront fee plus \$80 per hour. **a** Write an equation for the total cost, $\$C$, of hiring the photographer for t hours. **b** Sketch the graph of C vs t from $t = 0$ to $t = 6$. **c** Determine the total cost if the photographer is hired for 4 hours, and how many hours the photographer is hired for if the cost is $\$400$.	☐

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Short-answer questions

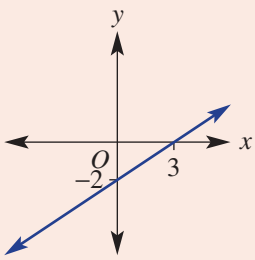
5A **1** Construct a table of values and draw a graph for $y = 2x - 3$.

5B **2** Read off the x- and y-intercepts from the table and graph.

a

x	−2	−1	0	1	2
y	0	2	4	6	8

b



5C **3** Sketch the following linear graphs, labelling x- and y-intercepts.

a $y = 2x - 4$ **b** $y = 3x + 9$ **c** $y = -2x + 5$ **d** $y = -x + 4$

e $2x + 4y = 8$ **f** $4x - 2y = 10$ **g** $2x - y = 7$ **h** $3x - 6y + 12 = 0$

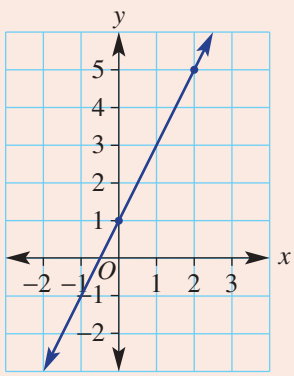
5D **4** Sketch the following lines with one intercept and classify their gradient as positive, negative, zero or undefined.

a $y = 3$ **b** $y = -2$ **c** $x = -4$

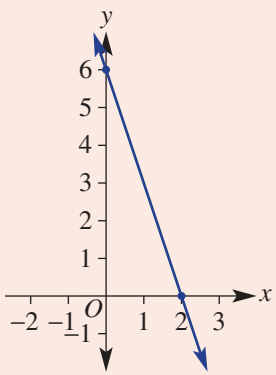
d $x = 5$ **e** $y = 3x$ **f** $y = -2x$

5E **5** Find the gradient of the following lines.

a



b



5E **6** Find the gradient of the line passing through these points.

a (3, 1) and (5, 5) **b** (2, 5) and (4, 3)

c (−3, −2) and (1, 6) **d** (−2, 6) and (1, −4)

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Chapter review

5F **7** An empty swimming pool is being filled with water by a hose. It takes 4 hours to fill 8000 L.

a What is the rate at which water is poured into the pool?

b Draw a graph of volume (V litres) vs time (t hours) from $t = 0$ to $t = 4$.

c By finding the gradient of your graph, give the equation for V in terms of t.

d Use your equation to find the time to fill 5000 L.

5G **8** For each of the following lines, state the value of the gradient and the y-intercept and then sketch using the gradient–intercept method.

a $y = 2x + 3$

b $y = -3x + 7$

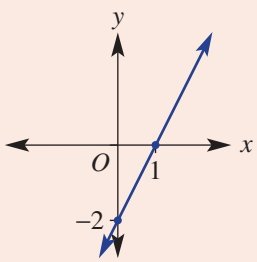
5G **9** Rearrange the following equations into the form $y = mx + c$ and state the gradient and y-intercept.

a $2x + y = 6$

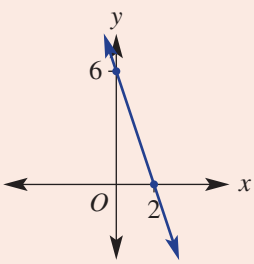
b $x + 2y - 4 = 0$

5H **10** Find the equations of these linear graphs in the form $y = mx + c$.

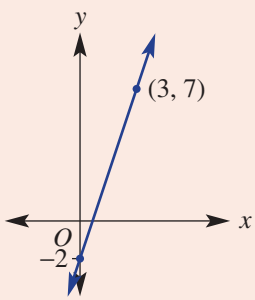
a



b



c



5H **11** Give the equation of the straight line that:

a has gradient 3 and passes through the point $(1, 4)$

b has gradient -2 and passes through the point $(2, -1)$

5I **12** For the line segment joining the following pairs of points, find:

i the midpoint

ii the length (to two decimal places)

a $(2, 4)$ and $(6, 8)$

b $(5, 2)$ and $(10, 7)$

c $(-2, 1)$ and $(2, 7)$

d $(-5, 7)$ and $(-1, -2)$

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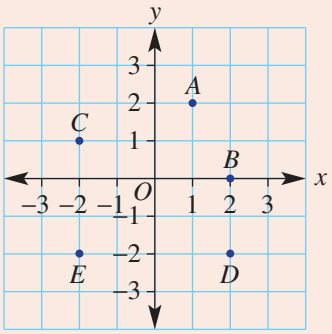
Multiple-choice questions

5A **1** The point(s) on the Cartesian plane shown with an x-coordinate of 2 are:



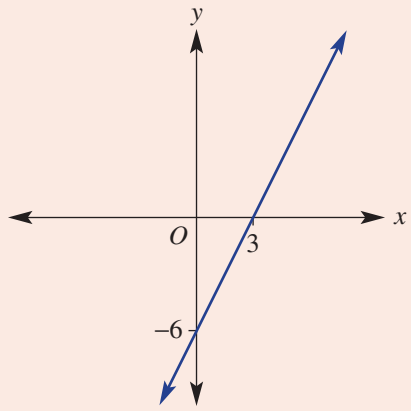
A B only **B** A only
C C, D and E **D** B and D
E E and D

5B **2** The line $2x + 4y = 12$ hits the x-axis at:

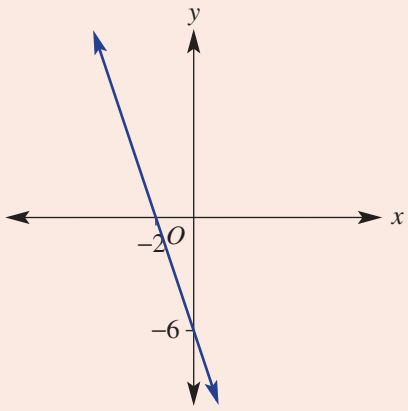
A $(4, 0)$ **B** $(0, 3)$ **C** $(6, 0)$
D $(2, 0)$ **E** $(0, 2)$

5C **3** The graph of $y = 3x - 6$ is represented by:

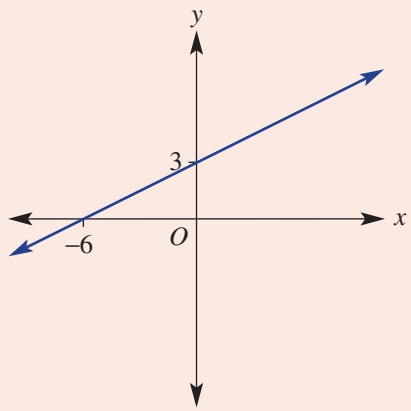
A



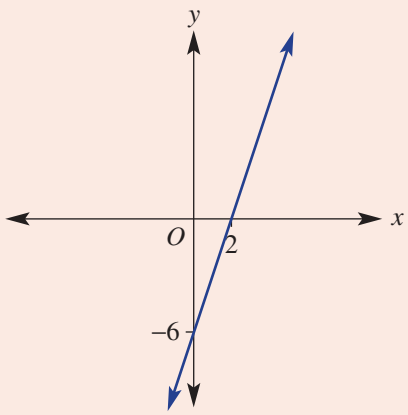
B



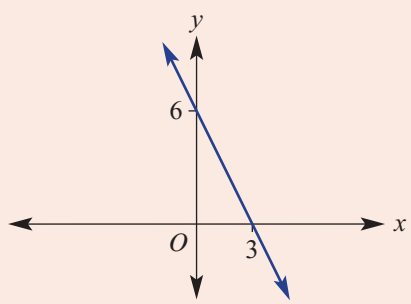
C



D



E



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Chapter review

5C **4** The point that is not on the straight line $y = 2x - 1$ is:

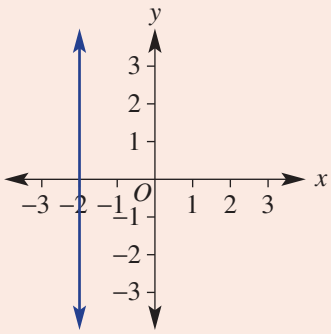
A $(1, 1)$ **B** $(0, -1)$ **C** $(3, 5)$ **D** $(-2, 7)$ **E** $(-1, -3)$

5E **5** The gradient of the line joining the points $(1, -2)$ and $(5, 6)$ is:

A 2 **B** 1 **C** 3 **D** -1 **E** $\frac{1}{3}$

5D **6** The graph shown has equation:

A $y = x - 2$
B $y = -2x$
C $y = -2$
D $x = -2$
E $x + y = -2$

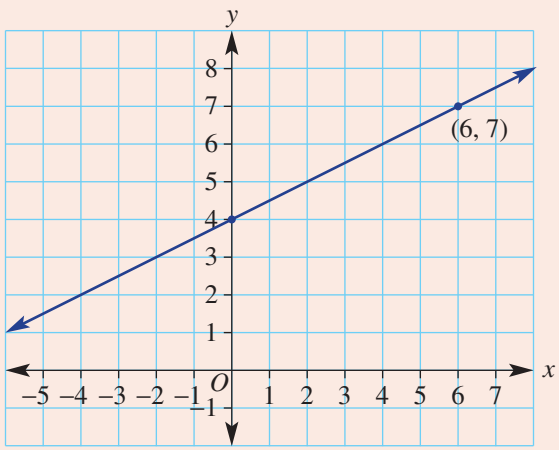


5H **7** A straight line has a gradient of -2 and passes through the point $(0, 5)$. Its equation is:

A $2y = -2x + 10$ **B** $y = -2x + 5$ **C** $y = 5x - 2$
D $y - 2x = 5$ **E** $y = -2(x - 5)$

5H **8** The equation of the graph shown is:

A $y = x + 1$
B $y = \frac{1}{2}x + 4$
C $y = 2x + 4$
D $y = -2x - 1$
E $y = \frac{1}{2}x + 2$



5I **9** The distance from $A(1, 2)$ to $B(4, 6)$ is:

A 5 **B** $\frac{4}{3}$ **C** 7
D $\sqrt{5}$ **E** $\sqrt{2}$

5J **10** A satellite phone call costs a 20c connection fee plus 60c per minute. The equation for the cost, C cents, of a call lasting n minutes is:

A $C = 20n + 60$ **B** $n = 60C + 20$ **C** $C = 60n + 20$
D $n = 20C + 60$ **E** $20C + 60n = 1$

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Extended-response questions

1 Justin is hiring a clown for his son's birthday party. The clown charges a booking fee of $100 as well as $50 per hour at the party.

- a Write the equation for the cost (C dollars) of hiring the clown for t hours.
- b Sketch a graph of cost (C) vs time (t) from $t = 0$ to $t = 5$.
- c How much will it cost to hire the clown for 4 hours?
- d At the end of the party, Justin pays $225. How long was the clown at the party for?

2 Ava runs a marathon at a constant speed. The distance, d km, she has remaining in the marathon t hours after the start is given by $d = 42 - 14t$.

- a Find the t- and d-intercepts and sketch the graph between these points.
- b What is the distance of the marathon?
- c How long does Ava take to run the marathon?
- d Find the gradient of the graph.
- e What average speed is Ava running at?

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Chapter 1: Integers, decimals, fractions, ratios and rates

Short-answer questions

1 Evaluate the following.

a $\frac{3}{4}+\frac{1}{4}$ **b** $\frac{13}{10}-\frac{3}{10}$ **c** $2\frac{1}{4}+\frac{1}{2}$ **d** $2\frac{1}{3}-1\frac{5}{9}$

2 Evaluate the following.

a $\frac{9}{10}\times\frac{5}{12}$ **b** $3\frac{3}{4}\div 2\frac{1}{12}$

3 Round the following to three decimal places.

a 4.1255 **b** 21.00241 **c** 0.0096

4 Divide $800 into the following ratios.

a 1 : 4 **b** 2 : 3 **c** 7 : 1

5 Write these rates and ratios in simplest form.

a Prize money is shared between two people in the ratio 60 : 36.

b Jodie travels 165 km in 3 hours.

c 30 mL of rain falls in $1\frac{1}{4}$ hours.

6 A car averages 68 km/h on a journey. How far will it travel at this speed for 2 hours and 40 minutes? Round to one decimal place.

7 A tree that is 2 metres tall has a width of 0.6 metres. Assuming the same ratio, how tall is a tree with width 1.2 metres?

Multiple-choice questions

1 $7\times(-5)+3\times(-4)$ is equal to:

A 23 **B** −47 **C** 47 **D** −23 **E** 420

2 45.5 ÷ 9.5, by first using rounding to one significant figure, gives:

A 4.9 **B** 5.35 **C** 5.4 **D** 5 **E** 4.95

3 $450 is divided in the ratio 4 : 5. The value of the smaller portion is:

A $210 **B** $250 **C** $90 **D** $200 **E** $220

4 Con runs 4.5 km in 20 minutes. His speed is:

A 1.5 km/h **B** 1.5 km/min **C** 13.5 km/h **D** 90 km/h **E** 22.5 km/h

5 $\frac{7}{10\,000}+\frac{13}{100}$ is equal to:

A 0.713 **B** 0.0713 **C** 0.1307 **D** 0.83 **E** 7.13

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Extended-response questions

Thomas walks, on average, 6 km an hour. Phillip walks at an average speed of 8 km/h.

a How far does Thomas walk in 4 hours?

b Phillip starts at 8 a.m.

i If he stops at 9.45 a.m, how far has he walked?

ii What time will Phillip finish his walk if he walks twice as far as Thomas did in part **a**?

Chapter 2: Financial mathematics

Short-answer questions

1 Convert each of the following to a percentage.

a 0.6 **b** $\frac{16}{50}$ **c** 2 kg out of 20 kg **d** 75c out of $3

2 Find:

a 10% of $96 **b** 85% of $900 **c** $5\frac{1}{2}$% of $200

3 **a** Increase 80 m by 5%. **b** Decrease $54 by 6%.

4 Evie spends 20% of her day (24 hours) reading a novel. How many hours and minutes did she spend reading?

5 Jamal earns a weekly retainer of $400 plus 6% of the sales he makes. If he sells $8200 worth of goods, how much will he earn for the week?

6 Ivy's salary is $84 800 and the taxation rate for her salary is 26% for the amount above $18 200, which is tax free. Calculate Ivy's net income for the year.

7 Sean invests $800 at 6% p.a. for 18 months.

a Calculate his simple interest.

b What is the final balance on Sean's account after the interest has been paid?

Multiple-choice questions

1 $2\frac{1}{2}$% is the same as:

A 0.25 **B** $\frac{1}{40}$ **C** $\frac{25}{100}$ **D** 2.5 **E** $\frac{5}{2}$

2 If cost price = $36 and selling price = $28.80, the percentage loss is:

A 7.2% **B** 80% **C** 92.8% **D** 20% **E** 2%

3 The simple interest earned on $660 invested at 5% p.a. for $1\frac{1}{2}$ years is:

A $49.50 **B** $40.50 **C** $594 **D** $33 **E** $709.50

4 A book that costs $27 is discounted by 15%. The new price is:

A $20.25 **B** $31.05 **C** $4.05 **D** $22.95 **E** $25.20

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5 Anna is paid a normal rate of $12.10 per hour. If in a week she works 6 hours at the normal rate, 2 hours at time and a half and 3 hours at double time. How much does she earn?

A $181.50 **B** $145.20 **C** $193.60 **D** $175.45 **E** $163.35

Extended-response questions

Riley plans to trial a simple interest plan. Before investing her money she increases the amount in her account by 20% to $21 000.

a What was the original amount in her account?

b She invests the $21 000 for 4 years at a simple interest rate of 3% p.a. How much does she have in her account at the end of the four years?

c She continues with the plan in part **b** and after a certain number of years has obtained $5670 interest. For how many years has she had the money invested?

d What percentage increase does this interest in part **c** represent on her initial investment?

Chapter 3: Expressions and equations

Short-answer questions

1 Simplify the following.

a $2x + 6y - 4x + y$ **b** $-3m \times 5n$ **c** $\frac{6xy}{18x}$

2 Solve the following equations.

a $3x + 7 = 25$ **b** $\frac{x-1}{4} = 2$ **c** $4(2m + 3) = 15$ **d** $5a - 8 = 3a - 2$

3 Noah receives m dollars pocket money per week. His older brother Jake gets 1 dollar more than twice Noah's amount. If Jake receives $15:

a write an equation to represent the problem using the variable m

b solve the equation in part **a** to determine how much Noah receives each week.

4 The formula $S = \frac{n}{2}(a + l)$ gives the sum, S, of a sequence of n numbers with first term a and last term l.

a Find the sum of the sequence of 10 terms $2, 5, 8, \ldots, 29$.

b If a sequence of 8 terms has a sum of 88 and a first term equal to 4, use the formula to find the last term of this sequence.

Multiple-choice questions

1 The expression that represents 3 more than half a certain number, n, is:

A $\frac{1}{2}n + 1.5$ **B** $2n + 3$ **C** $\frac{3}{2}n$ **D** $\frac{n}{2} + 3$ **E** $\frac{n+3}{2}$

2 The simplified form of $5ab + 3a + 2ab - a$ is:

A $9ab$ **B** $10ab + 3a$ **C** $13ab - a$ **D** $7ab + 3$ **E** $7ab + 2a$

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3 The expanded form of $-2(3m - 4)$ is:

A $-6m + 8$ **B** $-6m + 4$ **C** $-6m - 8$ **D** $-5m - 6$ **E** $5m + 8$

4 The solution to $\frac{d}{4} - 7 = 2$ is:

A $d = -20$ **B** $d = 15$ **C** $d = 36$ **D** $d = 1$ **E** $d = 30$

5 The formula $m = \sqrt{\frac{b-1}{a}}$, with $b = 9$ and $a = 2$, gives m equals:

A 3 **B** $\frac{3}{2}$ **C** 2 **D** $\sqrt{3}$ **E** $\frac{\sqrt{3}}{2}$

Extended-response questions

Chris and his brother Michael play basketball. In the last game, Chris scored 12 more points than Michael. Between them they scored 38 points. Let p be the number of points scored by Michael.

a Write an expression for the number of points scored by Chris.

b Write an equation involving the unknown, p, to represent the problem.

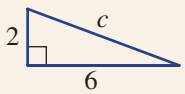
c Solve the equation to find the number of points scored by each of Chris and Michael.

Chapter 4: Right-angled triangles

Short-answer questions

1 Find the value of each pronumeral, correct to one decimal place.

a



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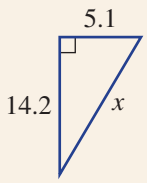

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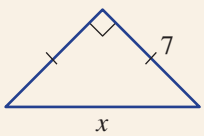

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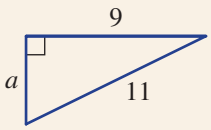

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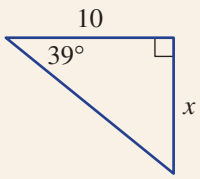
2 Find the value of each pronumeral correct to one decimal place.

a



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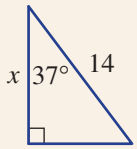

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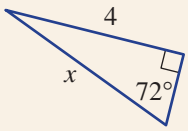

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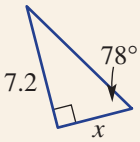

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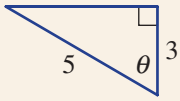

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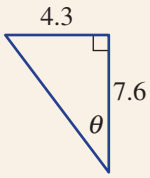
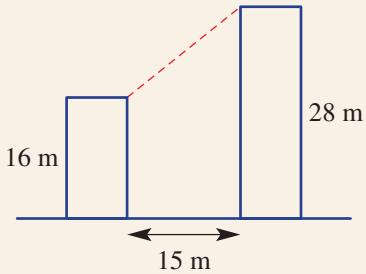


3 A wire is to be connected from the edge of the top of a 28 m high building to the edge of the top of a 16 m high building. The buildings are 15 m apart.



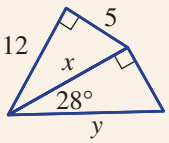
a What length of wire is required? Round to two decimal places.

b What is the angle of depression from the top of the taller building to the top of the smaller building? Round to one decimal place.

4 For this diagram:

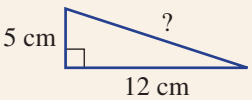
a use Pythagoras' theorem to find the value of x

b use trigonometry to find the value of y correct to two decimal places.



Multiple-choice questions

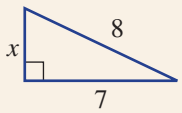
1 The length of the hypotenuse in this triangle is:



A 11 cm **B** 13 cm **C** 14.8 cm

D 169 cm **E** $\frac{1}{\sqrt{119}}$ cm

2 The exact value of x in the triangle shown is:



A 3.9 **B** $\sqrt{113}$ **C** $\sqrt{57}$

D $\sqrt{15}$ **E** 2.7

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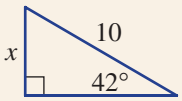
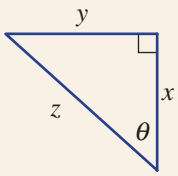
3 The value of x in the triangle shown is closest to:



A 6.7 **B** 7.4 **C** 14.9
D 13.5 **E** 9.0

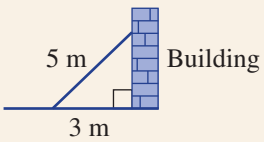
4 A correct ratio for this triangle is:



A $\tan\theta = \frac{x}{y}$ **B** $\cos\theta = \frac{y}{z}$ **C** $\tan\theta = \frac{y}{x}$

D $\sin\theta = \frac{y}{x}$ **E** $\cos\theta = \frac{x}{y}$

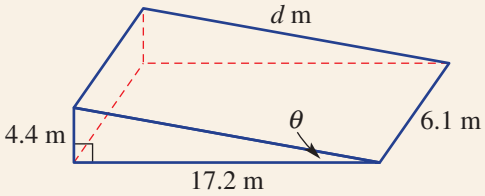
5 A 5-metre plank of wood is leaning up against a side of a building as shown. If the wood touches the ground 3 m from the base of the building, the angle the wood makes with the building is closest to:



A 36.9° **B** 59° **C** 53.1° **D** 31° **E** 41.4°

Extended-response questions

A skateboard ramp is constructed as shown.



a Calculate the distance d metres up the ramp correct to two decimal places. Use Pythagoras' theorem.

b What is the angle of elevation (θ) between the ramp and the ground correct to one decimal place?

c If the skateboarder rides from one corner of the ramp diagonally to the other corner, what distance do they travel? Round to one decimal place.

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Chapter 5: Linear relationships

Short-answer questions

1 Sketch the following linear graphs labelling x- and y-intercepts.

a $y = 2x - 6$

b $3x + 4y = 24$

c $y = 4x$

2 Find the gradient of each of the following.

a The line passing through the points $(-1, 2)$ and $(2, 4)$

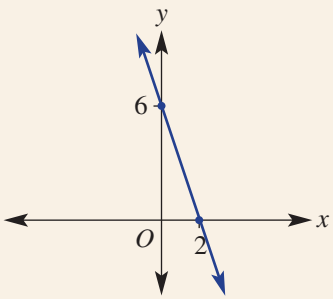
b The line passing through the points $(-2, 5)$ and $(1, -4)$

c The line with equation $y = -2x + 5$

d The line with equation $-4x + 3y = 9$

3 Give the equation of the following lines in gradient–intercept form.

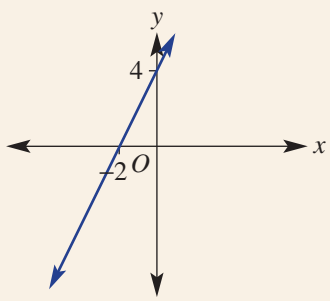
a The line with the given graph



b The line with gradient 3 and passing through the point $(2, 5)$

Multiple-choice questions

1 At what points does this line meet the x-axis and y-axis?



A $(-2, 4)$ and $(4, -2)$

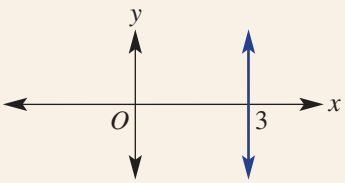
B $(0, 4)$ and $(2, 0)$

C $(-2, 0)$ and $(0, 4)$

D $(4, 0)$ and $(0, -2)$

E $(2, 0)$ and $(0, -4)$

2 The graph shown has equation:



A $y = 3x$ **B** $y = 3$

C $y = x + 3$ **D** $x = 3$

E $x + y = 3$

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3 The line passing through the points $(-3, -1)$ and $(1, 1)$ has gradient:

A $\frac{1}{2}$ **B** -3 **C** 1 **D** 2 **E** -2

4 If the point $(-1, 3)$ is on the line $y = 2x + c$, the value of c is:

A 1 **B** 5 **C** -7 **D** -5 **E** -1

5 The midpoint and length (to one decimal place) of the line segment joining the points $(-2, 1)$ and $(4, 6)$ are:

A $(1, 3.5)$ and 7.8 **B** $(3, 5)$ and 5.4 **C** $(3, 3.5)$ and 6.1
D $(1, 3.5)$ and 3.3 **E** $(3, 3.5)$ and 3.6

Extended-response questions

Doug works as a labourer. He is digging a trench and has 180 kg of soil to remove. He has taken 3 hours to remove 36 kg.

a What is the rate at which he is removing the soil?

b If he maintains this rate, write an equation for the amount of soil, S kg, remaining after t hours.

c Draw a graph of your equation.

d How long will it take to remove all of the soil?

e Doug is paid \$40 for the job plus \$25 per hour worked.

i Write an equation for his pay P dollars for working h hours.

ii How much will he be paid to remove all the soil?

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6

Length, area, surface area and volume

Maths in context: Measurement skills in many occupations

It's hard to overstate the importance of measurement skills, which are found in almost every type of practical work, including the work of:

- bakers, boilermakers, bricklayers, builders, carpenters, carpet layers, concreters, cooks;
- engineers, farmers, forestry workers, furniture makers, glaziers, graphic artists;
- hairdressers, house painters, landscapers, machinists, mechanics, pipelayers, painters;
- plumbers, plasterers, sheet-metal workers, surveyors, tailors, tilers and welders.

To give just a few examples:

- Sports courts and athletics tracks have official dimensions applied around the world. An Olympic running track has 84.39 m for the straight sections and a 36.50 m radius for each semicircle.
- The rectangular prism that totally changed global cargo transport is the shipping container. The largest container ship (in 2023) can carry more than 24 000 containers.
- Cylinder surface area and volume calculations include for food cans, water storage tanks, fuel and milk tankers, and fuel station tanks (buried underground).

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Chapter contents

6A Length and perimeter (CONSOLIDATING)
6B Circumference of circles and perimeter of sectors (CONSOLIDATING)
6C Area of quadrilaterals and triangles (CONSOLIDATING)
6D Area of circles (CONSOLIDATING)
6E Perimeter and area of composite shapes
6F Surface area of prisms
6G Surface area of cylinders
6H Volume of prisms
6I Volume of cylinders

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- applies knowledge of the perimeter of plane shapes and the circumference of circles to solve problems (MA4-LEN-C-01)
- applies knowledge of area and composite area involving triangles, quadrilaterals and circles to solve problems (MA4-ARE-C-01)
- solves problems involving the surface area of right prisms and practical problems involving the area of composite shapes and solids (MA5-ARE-C-01)
- solves problems involving the volume of composite solids consisting of right prisms and cylinders (MA5-VOL-C-01).

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Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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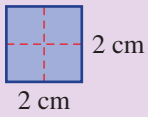
Warm-up quiz

1 Count the number of squares in these shapes to find the area.

a



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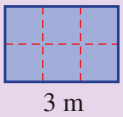


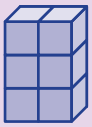
c

2 Find the distance (perimeter) around the shapes in Question **1**.

3 Count the number of cubes in these solids to find the volume.

a

b



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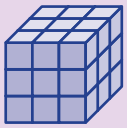
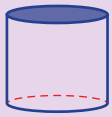


4 **a** Name the solid shown at right.

b What is the name of the shape (shaded) on top of the solid?

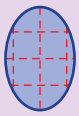


5 Calculate the following by moving the decimal point right (for ×) or left (for ÷).

a 2.3×10 **b** 0.048×1000 **c** $270 \div 100$

d $52134 \div 10\,000$ **e** $0.0005 \times 100\,000$ **f** $72\,160 \div 1000$

6 Estimate the area (number of squares) in this ellipse.



7 Convert the following to the units shown in the brackets. Remember: 1 km = 1000 m, 1 m = 100 cm and 1 cm = 10 mm.

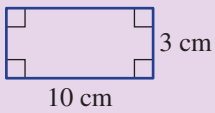
a 3 cm (mm) **b** 20 m (cm) **c** 1.6 km (m)

d 23 mm (cm) **e** 3167 m (km) **f** 72 cm (m)

g 20 000 m (km) **h** 0.03 km (m) **i** 75.6 m (km)

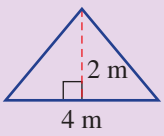
8 Find the area of these basic shapes using the given formulas.

a



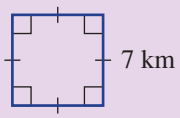
$A = lb$

b



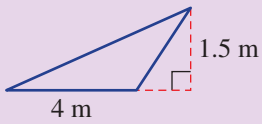
$A = \frac{1}{2}bh$

c



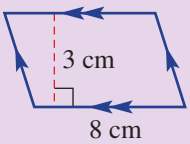
$A = l^2$

d



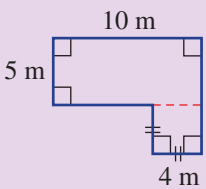
$A = \frac{1}{2}bh$

e



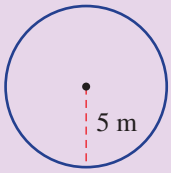
$A = bh$

f



$A = l \times b + l^2$

9 Find the circumference ($C = 2\pi r$) and area ($A = \pi r^2$) of this circle, rounding to two decimal places.



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6A Length and perimeter CONSOLIDATING

Learning intentions for this section:

- To review the metric units of length and be able to convert between them
- To be able to find the perimeter of closed shapes

Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall formulas from memory in the HSC examination

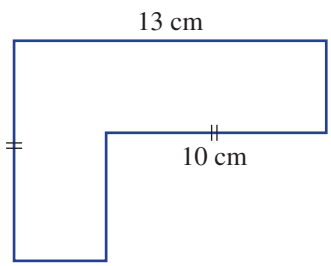
Measurement is the branch of mathematics that includes the consideration of length, area and volume. This includes units of measurement, perimeter, circumference, surface area, composite shapes and capacity.

We use measurement when we design buildings, water our gardens, control satellites, fill our cars with petrol or participate in school athletics days.

Lesson starter: Not enough information?

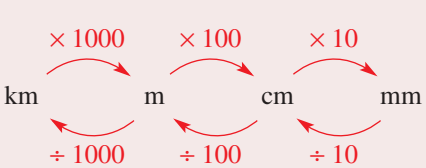
All the angles at each vertex in this shape are 90° and the two given lengths are 10 cm and 13 cm.

- Is there enough information to find the perimeter of the shape?
- If there is enough information, find the perimeter and discuss your method.



KEY IDEAS

- Units of length in the metric system include kilometre (km), metre (m), centimetre (cm) and millimetre (mm)
- To convert between metric units of length, multiply or divide by the appropriate power of 10.



1 km = 1000 m
1 m = 100 cm = 1000 mm
1 cm = 10 mm

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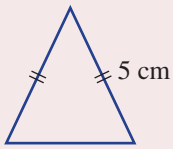
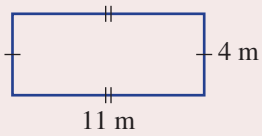
■ **Perimeter** is the distance around the outside of a closed shape.

- Sides with the same markings are of equal **length**



$P = 2 \times 5 + 3$
$= 13$ cm



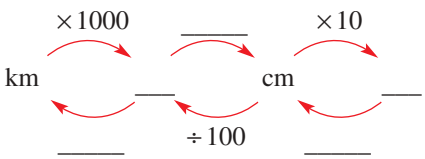
$P = 2 \times 11 + 2 \times 4$
$= 30$ m

Key terminology: perimeter, length

Exercise 6A

UNDERSTANDING	1–3	3

1 Fill in the gaps on this flow chart.

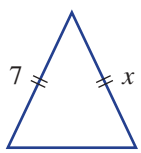


2 Choose from the words *multiply*, *divide*, *add* or *subtract* to complete this sentence.

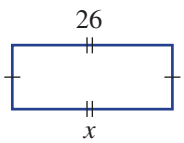
To find the perimeter of a shape you would _____ all the side lengths.

3 Write down the value of x in these diagrams.

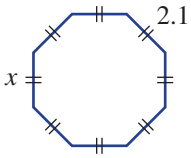
a



b



c



FLUENCY	4(½), 5, 6(½)	4(½), 6(½), 7

Example 1 Converting units of length

Convert to the units shown in the brackets.

a 5.41 cm (mm) **b** 3200 m (km)

SOLUTION

a $5.41 \text{ cm} = 5.41 \times 10 \text{ mm}$
$= 54.1 \text{ mm}$

EXPLANATION

1 cm = 10 mm and you are moving to a smaller unit, so multiply.

Move the decimal point one place to the right.

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b 3200 m = 3200 ÷ 1000 km
= 3.2 km

1 km = 1000 m and you are moving to a larger unit, so divide.

Move the decimal point three places to the left.

Now you try

Convert to the units shown in the brackets.

a 2.4 m (cm)

b 420 mm (cm)

4 Convert the following length measurements into the units given in the brackets.

a 5 cm (mm)
b 41 cm (mm)
c 2.8 m (cm)
d 0.4 m (cm)
e 4.6 km (m)
f 0.9 km (m)
g 521 mm (cm)
h 36 mm (cm)
i 240 cm (m)
j 83.7 cm (m)
k 7000 m (km)
l 2170 m (km)

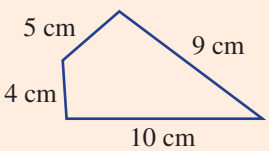
Hint: Multiply when changing to a smaller unit and divide when changing to a larger unit.

5 A steel beam is 8.25 m long and 22.5 mm wide. Write down the length and the width of the beam in centimetres.

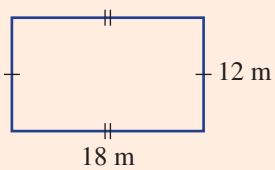
Example 2 Finding perimeters of simple shapes

Find the perimeter of each of the following shapes.

a



b



SOLUTION

a $P = 10 + 9 + 5 + 4$
$= 28$ cm

b $P = 2 \times 12 + 2 \times 18$
$= 24 + 36$
$= 60$ m

EXPLANATION

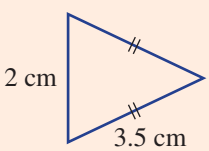
To find the perimeter, add all the side lengths together.

Two lengths of 12 m and two lengths of 18 m.

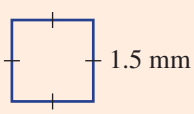
Now you try

Find the perimeter of each of the following shapes.

a



b



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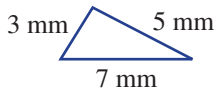
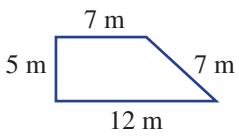
6 Find the perimeter of each of the following shapes.

a



b



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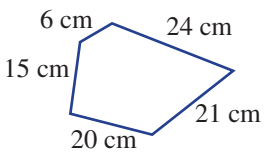
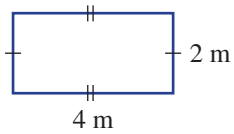


d



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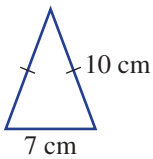

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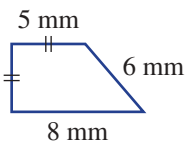

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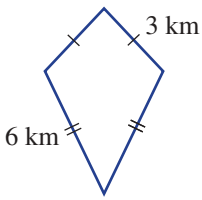

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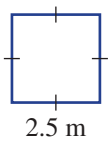
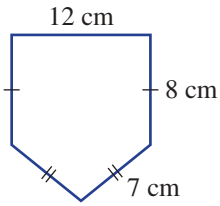
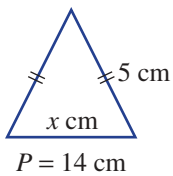


i



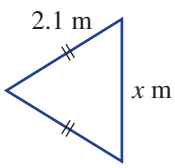
7 Find the unknown side length in these shapes with the given perimeters.

a



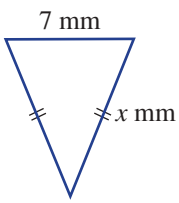
$P = 14$ cm

b



$P = 6.4$ m

c



$P = 35$ mm

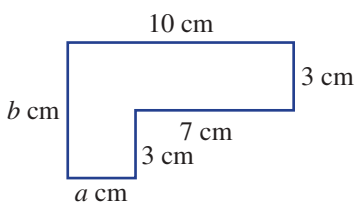
Hint: Use trial and error, if you like, to find the value of x.

PROBLEM-SOLVING AND REASONING

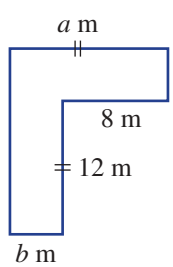
8–10 | 9–12

8 Write down the values of the pronumerals in these hexagons. All angles are 90°.

a



b



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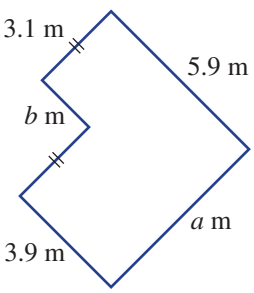
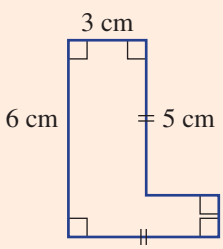


Example 3 Finding the perimeter of composite shapes

Find the perimeter of this composite shape.



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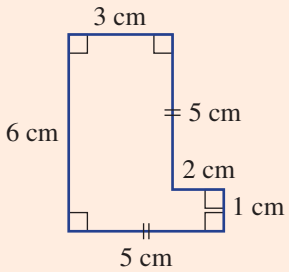
SOLUTION

Perimeter $= (2 \times 5) + 6 + 3 + 2 + 1$
$= 22$ cm

EXPLANATION

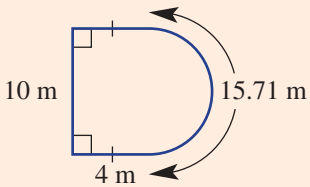
Missing sides are:
$5\text{ cm} - 3\text{ cm} = 2\text{ cm}$
$6\text{ cm} - 5\text{ cm} = 1\text{ cm}$

Alternatively:
$2 \times 6 + 2 \times 5 = 22\text{ cm}$



Now you try

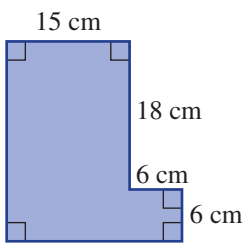
Find the perimeter of this composite shape.



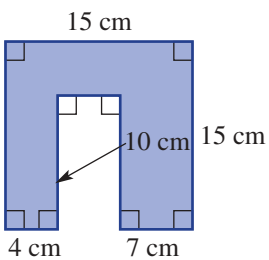
9 Find the perimeter of each of the following composite shapes.

Hint: First label all the missing side lengths and then find the perimeter.

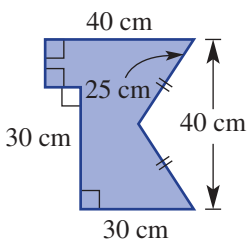
a



b



c



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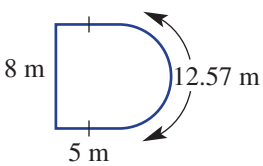

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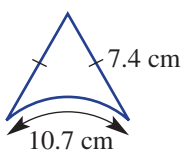

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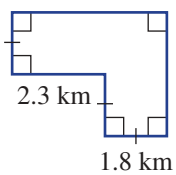

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10 A lion cage is made up of five straight fence sections. Three sections are 20 m in length and the other two sections are 15.5 m and 32.5 m. Find the perimeter of the cage.

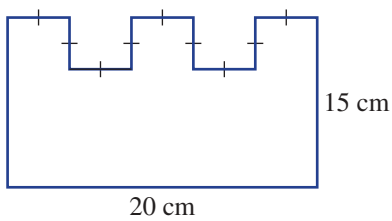
11 Convert the following measurements into the units given in the brackets.

a 8 m (mm)
b 110 000 mm (m)
c 0.00001 km (cm)
d 0.02 m (mm)
e 28 400 cm (km)
f 62 743 000 mm (km)

Hint: In part **a**, first convert to cm and then to mm.

12 Find the perimeter of these shapes. All angles are right angles.

a



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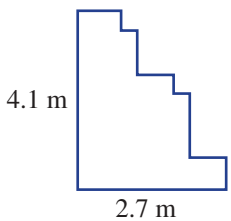

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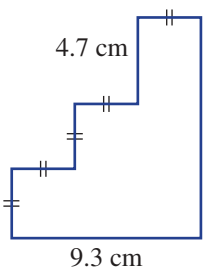
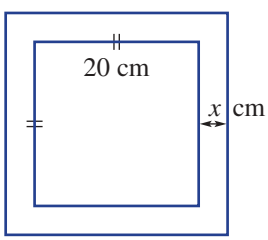
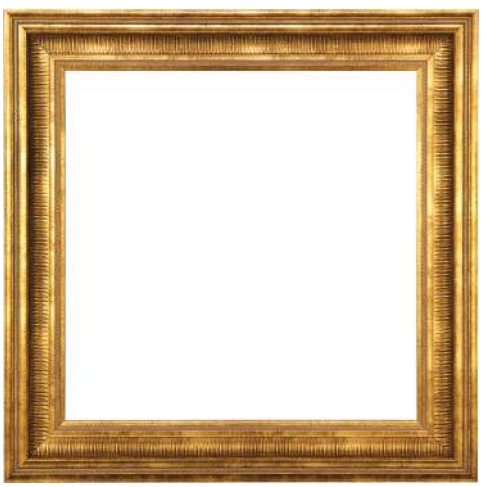


ENRICHMENT: Picture framing — 13, 14

13 A photo 12 cm wide and 20 cm long is surrounded with a picture frame 3 cm thick. Find the outside perimeter of the framed picture.

14 A square picture of side length 20 cm is inside a frame of width x cm.





a Find the perimeter of the framed picture if:
i $x = 2$ **ii** $x = 3$ **iii** $x = 5$

b Write a rule for the perimeter, P, of the framed picture in terms of x.

c Use your rule to find the perimeter if $x = 3.7$.

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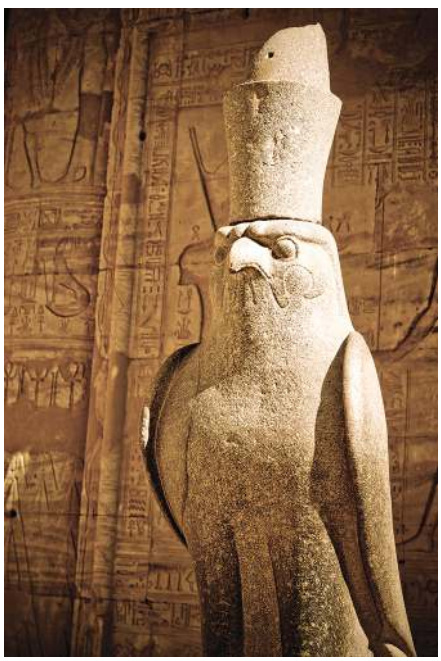
6B Circumference of circles and perimeter of sectors CONSOLIDATING

Learning intentions for this section:

- To know the formula for the circumference of a circle
- To be able to calculate circumference of a circle and perimeter of semicircles and quadrants

Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of the questions are more challenging than those in our Stage 4 books
- You will find arc length in Section 2D of our Year 10 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall formulas from memory in the HSC examination



For thousands of years, mathematicians have known about a special number that links a circle's diameter to its circumference.

The Egyptians, Babylonians and ancient Indians knew about this special number, and thought its value was about 3.15.

Today we call this number pi (π) and know it to be 3.14159, correct to five decimal places.

An exact value of pi cannot be written down as a decimal, as it has an infinite number of decimal places with no pattern.

Lesson starter: Working with string

Use a pair of compasses, a ruler and a piece of string for this activity.

The table below contains some approximate circle measurements.

Circumference (C)	Diameter (d)	$C \div d$
18.8	6.0	
12.6	4.0	
…	…	
…	…	

- Copy the table, leaving two blank rows.
- Use your compasses to draw a circle, then measure its diameter using a ruler and its circumference using your string (try to be as accurate as possible). Add this information to the table.
- Do this again, with a different-sized circle.
- Calculate $C \div d$ in the third column for each of the four circles. What do you notice?
- What does this say about how to calculate the circumference of a circle if you know the diameter?

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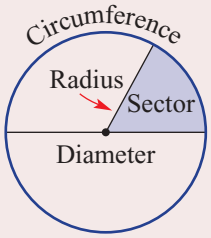
KEY IDEAS

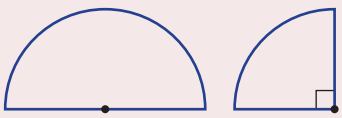
- π (pi) is the ratio of circumference to diameter. It is approximately 3.14. For a more accurate value, use the button on your calculator.
- Features of a **circle**:
  - **Diameter** (d) is the distance across the centre.
  - **Radius** (r) is the distance from the centre to the circumference. The radius is half of the diameter.
  - **Circumference** of a circle is

    $C = 2\pi r$ or $C = \pi d$

    Use the π button on your calculator.



- A **sector** is a fraction of a circle bounded by two radii. The angle at the centre is between 0° and 360°.
- Special circle sectors:
  - A half circle is called a **semicircle**.
  - A quarter circle is called a **quadrant**.

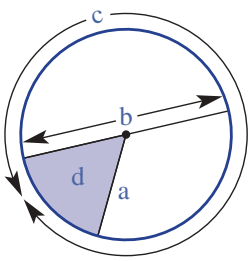


Key terminology: circumference, pi, radius, diameter, circle, sector, semicircle, quadrant

Exercise 6B

UNDERSTANDING 1–4 4

1 Name features **a–d** shown in the circle at right.



2 **a** What is the radius of a circle if its diameter is 5.6 cm?

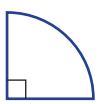
b What is the diameter of a circle if its radius is 48 mm?

3 Write down the rule for the circumference of a circle using:

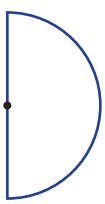
a r, the radius **b** d, the diameter

4 Determine the **fraction** of a circle shown in these sectors. Write the fraction in simplest form.

a



b



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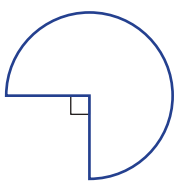

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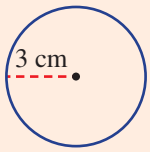
FLUENCY 5–7(½) 5–7(½)

Example 4 Finding the circumference of a circle

Find the circumference of this circle correct to two decimal places.



SOLUTION

$C = 2\pi r$
$= 2 \times \pi \times 3$
$= 6\pi$
$= 18.85$ cm (to 2 d.p.)

EXPLANATION

Use the formula $C = 2\pi r$ or $C = \pi d$ and substitute $r = 3$ (or $d = 6$).

6π would be the exact answer and 18.85 is the rounded answer.

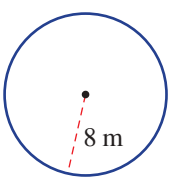
Now you try

Find the circumference of this circle correct to two decimal places.

5 Find the circumference of these circles correct to two decimal places. Use a calculator for the value of π.

Hint: Write the rule $C = 2\pi r$, then substitute the radius length.

a



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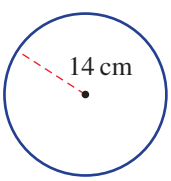

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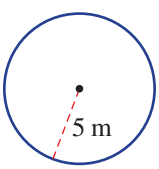

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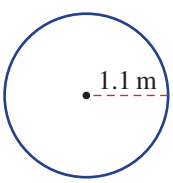

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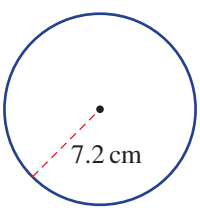

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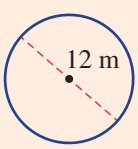
Example 5 Finding circumference using the diameter

Find the circumference of this circle correct to two decimal places.



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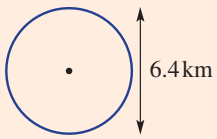
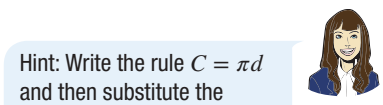
SOLUTION	EXPLANATION
$C = \pi d$	Write the formula. $C = \pi d$ is preferred since d is given.
$= \pi \times 12$	Substitute $d = 12$ and multiply by π. Use a calculator and round.
$= 37.70$ m (to 2 d.p.)	Note: 37.6991 rounds to 37.70 for two decimal places.

Now you try

Find the circumference of this circle correct to two decimal places.

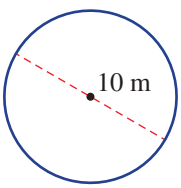


6 Find the circumference of these circles, correct to two decimal places.



Hint: Write the rule $C = \pi d$ and then substitute the diameter length.

a



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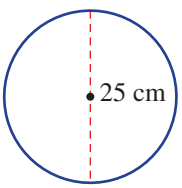

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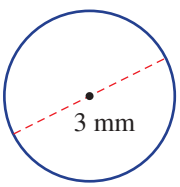

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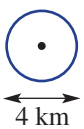

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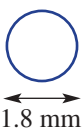

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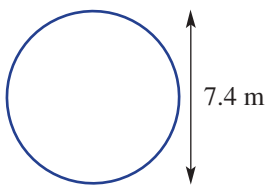
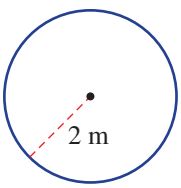


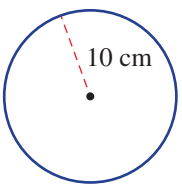
7 Find the circumference of these circles without a calculator using the given approximation of π.

a



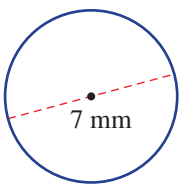
$\pi = 3.14$

b



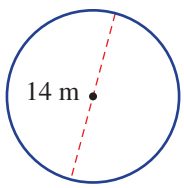
$\pi = 3.14$

c



$\pi = \frac{22}{7}$

d

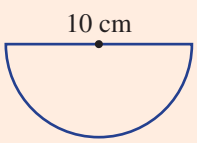


$\pi = \frac{22}{7}$

PROBLEM-SOLVING AND REASONING	8(½), 9, 10	8, 10, 11

Example 6 Working with a semicircle or quadrant

Find the perimeter of this semicircle correct to two decimal places.



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SOLUTION

$P = \frac{1}{2} \times \pi d + 10$

$= \frac{1}{2} \times \pi \times 10 + 10$

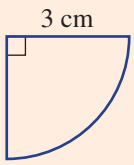
$= 25.71$ cm (to 2 d.p.)

EXPLANATION

The perimeter consists of half the circumference of a circle (with diameter 10 cm) plus the 10 cm diameter across the top.

Now you try

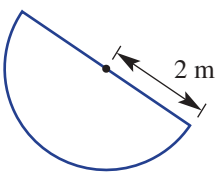
Find the perimeter of this quadrant correct to two decimal places.



8 Find the perimeter of these sectors correct to two decimal places.

Hint: Decide what fraction of the circumference you want and don't forget to add the straight sides.

a



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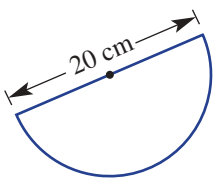

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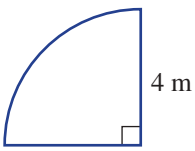

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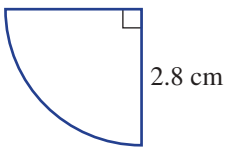

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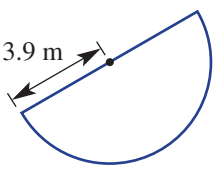

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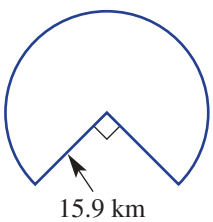

9 Find the distance around the outside of a circular pool of radius 4.5 m correct to two decimal places.

10 Find the length of string required to surround the circular trunk of a tree that has a diameter of 1.3 m, correct to one decimal place.

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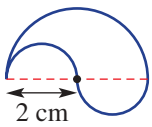
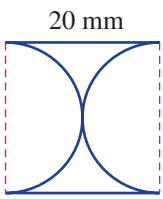
11 Find the perimeter of these shapes correct to two decimal places.

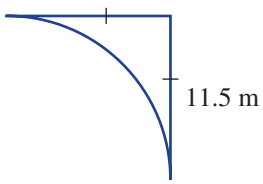
a



b



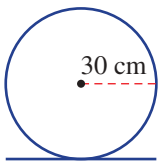
c



Hint: Two semicircles of the same size make a full circle. When finding the perimeter, imagine you are walking around the edge of the shape.

ENRICHMENT: The rolling wheel — 12

12 A wheel of radius 30 cm is rolled in a straight line.



a Find the circumference of the wheel correct to two decimal places.

b How far, correct to two decimal places, has the wheel rolled after completing:

i 2 rotations? **ii** 10.5 rotations?

c Can you find how many rotations would be required to cover at least 1 km in length? Round to the nearest whole number.

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6C Area of quadrilaterals and triangles CONSOLIDATING

Learning intentions for this section:

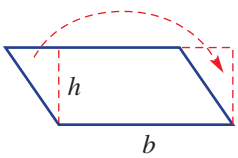
- To be able to convert between metric units for area
- To know the formulas for the area of common shapes
- To be able to find the area of common shapes

Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall area formulas from memory in the HSC examination

For many common shapes, such as the parallelogram and trapezium, the rules for finding their area can be found using simple rectangles and triangles.

The parallelogram, for example, can be seen as a 'pushed over' rectangle. Its area can be calculated in the same way as a rectangle.



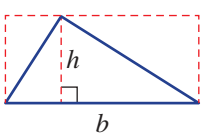
So $A = b \times h$, where h is the perpendicular height.

Lesson starter: Build the formulas

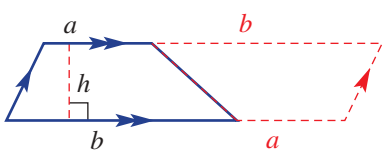
From the introduction, you can see how the rule for the area of a parallelogram is given by $A = b \times h$.

Use the given diagrams to explain the rules for the areas of each shape.

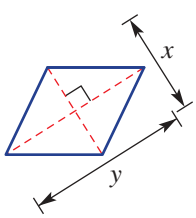
- Triangle: $A = \frac{1}{2}b \times h$



- Trapezium: $A = \frac{h}{2}(a + b)$



- Rhombus: $A = \frac{1}{2}xy$



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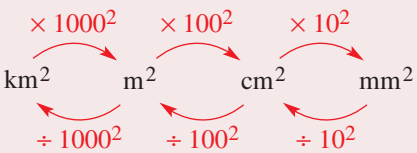
KEY IDEAS

- The metric units for area are
  - square kilometre (km^2) $1\ km^2 = 1000\,000\ m^2$
  - hectare (ha) $1\ ha = 10\,000\ m^2$
  - square cm (cm^2) $1\ cm^2 = 100\ mm^2$
  - square mm (mm^2)

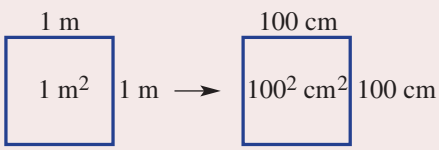
- Conversion of area units



$10^2 = 10 \times 10 = 100$

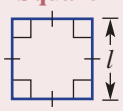
$100^2 = 100 \times 100 = 10\,000$

$1000^2 = 1000 \times 1000 = 1\,000\,000$



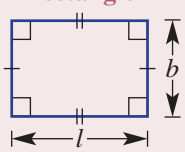
- The **area** of a two-dimensional shape is a measure of the space enclosed within its boundaries.
  - The 'height' (h) in the formulas for the area of a triangle, parallelogram and trapezium must be **perpendicular** (at 90°) to the base.

Square



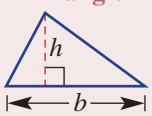
Area $= l^2$

Rectangle



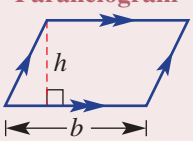
Area $= lb$

Triangle



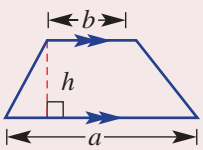
Area $= \frac{1}{2}bh$

Parallelogram



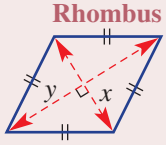
Area $= bh$
(Can also be used for a rhombus)

Trapezium



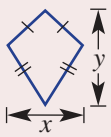
Area $= \frac{h}{2}(a + b)$

Rhombus



Area $= \frac{1}{2}xy$
(Can also be used for a square)

Kite



$A = \frac{1}{2}xy$

Key terminology: area, triangle, square, rectangle, rhombus, parallelogram, trapezium, kite, perpendicular

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Exercise 6C

UNDERSTANDING | 1–3 | 3

1 Write the missing number.

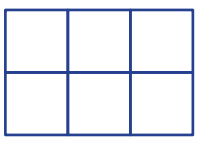
a $1\text{ cm}^2 = 10^2\text{ mm}^2 =$ _____ mm^2

b $1\text{ m}^2 = 100^2\text{ cm}^2 =$ _____ cm^2

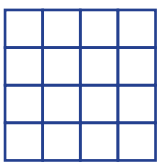
c $1\text{ km}^2 = 1000^2\text{ m}^2 =$ _____ m^2

2 Count the number of squares to find the area of these shapes. Each square in each shape represents one square unit.

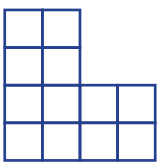
a



b

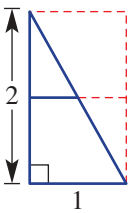


c

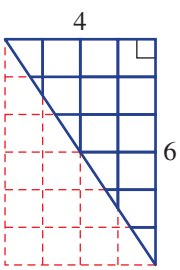


Hint: For parts **d** and **e**, note that each triangle is half of a rectangle.

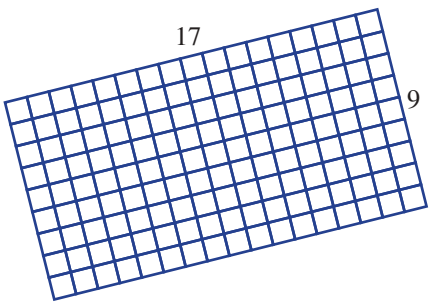
d



e



f



3 Name the shape(s) that usually has the given area formula.

a $A = lb$

b $A = \frac{1}{2}xy$ (2 shapes)

c $A = \frac{1}{2}bh$

d $A = \frac{h}{2}(a + b)$

e $A = bh$

f $A = l^2$

FLUENCY | 4(½), 5, 6 | 4–6(½)

Example 7 Converting units of area

Convert the following area measurements into the units given in the brackets.

a 859 mm^2 (cm^2)

b 2.37 m^2 (cm^2)

SOLUTION

a $859\text{ mm}^2 = 859 \div 10^2\text{ cm}^2$
$= 8.59\text{ cm}^2$

b $2.37\text{ m}^2 = 2.37 \times 100^2\text{ cm}^2$
$= 23\,700\text{ cm}^2$

EXPLANATION

cm^2 ← mm^2 859.
$\div 10^2 = 100$

$\times 100^2 = 10\,000$
m^2 → cm^2 2.3700

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Now you try

Convert the following area measurements into the units given in the brackets.

a 32 000 cm^2 (m^2) **b** 0.4 km^2 (m^2)

4 Convert the following area measurements into the units given in the brackets.

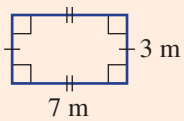
a 2 cm^2 (mm^2) **b** 0.4 cm^2 (mm^2) **c** 500 mm^2 (cm^2)
d 310 mm^2 (cm^2) **e** 2.1 m^2 (cm^2) **f** 0.2 m^2 (cm^2)
g 210 000 cm^2 (m^2) **h** 3700 cm^2 (m^2) **i** 0.001 km^2 (m^2)
j 4.3 km^2 (m^2) **k** 3 200 000 m^2 (km^2) **l** 39 400 m^2 (km^2)

Hint: Multiply or divide by:
$10^2 = 100$
$100^2 = 10\,000$
or $1000^2 = 1\,000\,000$

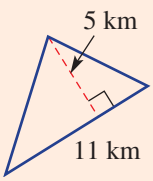
Example 8 Finding areas of rectangles, triangles and parallelograms

Find the area of each of the following plane figures.

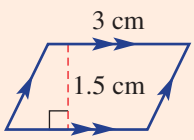
a



b



c

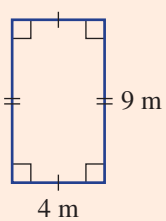


SOLUTION	EXPLANATION
a Area $= lb$	Use the area formula for a rectangle. Substitute $l = 7$ and $b = 3$.
$= 7 \times 3$	Include the correct units.
$= 21\ m^2$	
b Area $= \frac{1}{2}bh$	Use the area formula for a triangle.
$= \frac{1}{2} \times 11 \times 5$	Substitute $b = 11$ and $h = 5$, where h is the height perpendicular (at 90°) to the base.
$= 27.5\ km^2$	
c Area $= bh$	Use the area formula for a parallelogram.
$= 3 \times 1.5$	Multiply the base length by the perpendicular height.
$= 4.5\ cm^2$	

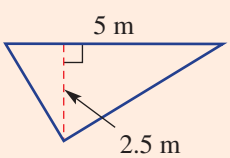
Now you try

Find the area of each of the following plane figures.

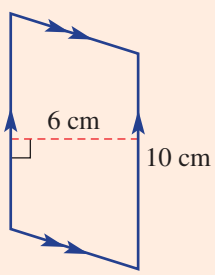
a



b



c



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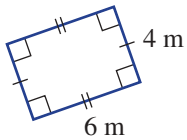
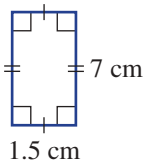
5 Find the area of each of the following plane figures.

a

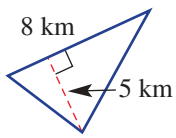


b

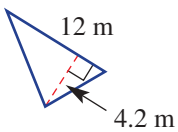


Hint: Choose from
$A = lb$,
$A = \frac{1}{2}bh$
or $A = bh$

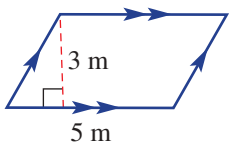
c



d



e



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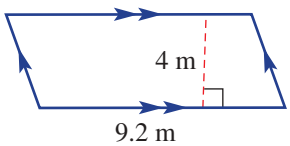
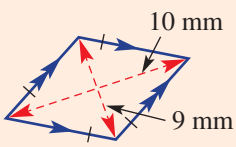


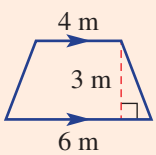
Example 9 Finding areas of rhombuses and trapeziums

Find the area of each of the following plane figures.

a



b



SOLUTION

a Area $= \frac{1}{2}xy$

$= \frac{1}{2} \times 10 \times 9$

$= 45 \text{ mm}^2$

b Area $= \frac{h}{2}(a + b)$

$= \frac{3}{2}(4 + 6)$

$= 15 \text{ m}^2$

EXPLANATION

Use the area formula for a rhombus.

x and y are the lengths of the diagonals.

$\frac{1}{2} \times 10 \times 9 = 5 \times 9 = 45$

Use the area formula for a trapezium.

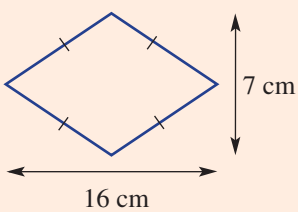
Substitute $a = 4$, $b = 6$ and $h = 3$.

Include the correct units.

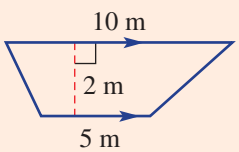
Now you try

Find the area of each of the following plane figures.

a



b



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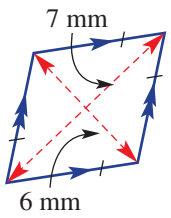
6 Find the area of each of the following plane figures.

a



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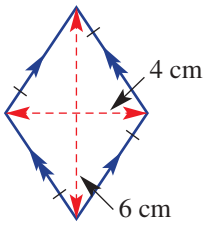

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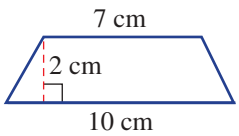

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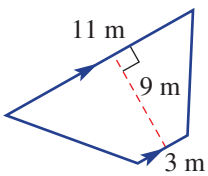

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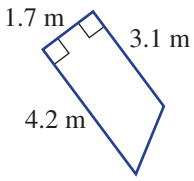

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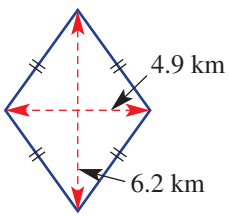


Hint: Choose from
$A = \frac{1}{2}xy$ or
$A = \frac{h}{2}(a + b)$

PROBLEM-SOLVING AND REASONING

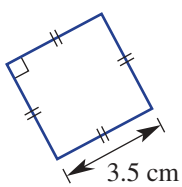
7, 8, 9(½) | 8–11

7 A piece of land has an area of one-half a square kilometre (0.5 km^2). How many square metres (m^2) is this?

8 A rectangular park covers an area of 175 000 m^2. Give the area of the park in km^2.

9 Find the area of each of the following plane figures.

a



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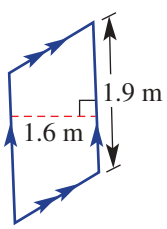

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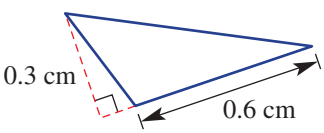

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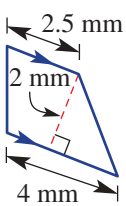

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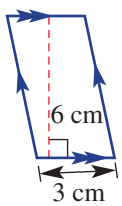

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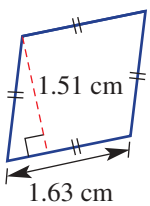

Hint: Choose from
$A = l^2$, $A = lb$,
$A = \frac{1}{2}bh$
$A = \frac{h}{2}(a + b)$

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10 An old picture frame that was once square now leans to one side to form a rhombus. If the distances between pairs of opposite corners are 85 cm and 1.2 m, find the area inside the frame in m^2.

11 Convert the following measurements into the units given in the brackets.

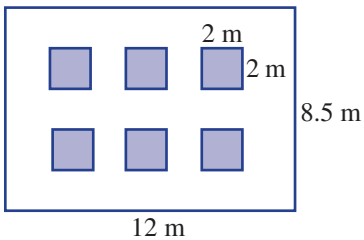
a 1.5 km^2 (cm^2)

b 0.000005 m^2 (mm^2)

c 75 000 mm^2 (m^2)

ENRICHMENT: Windows — 12

12 Six square windows of side length 2 m are placed into a 12 m wide by 8.5 m rectangular high wall, as shown. The windows are positioned so that the vertical spacing between the windows and the wall edges are equal. The horizontal spacings are also equal.



a **i** Find the horizontal distance between the windows.

ii Find the vertical distance between the windows.

b Find the area of the wall, not including the window spaces.

c If the wall included 3 rows of 4 windows (instead of 2 rows of 3), would it be possible to space all the windows so that the horizontal and vertical spacings are the same? (Horizontal doesn't have to be the same as vertical.)

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6D Area of circles CONSOLIDATING

Learning intentions for this section:

- To know the formula for the area of a circle
- To be able to find the area of circles, semicircles and quadrants

Past, present and future learning:

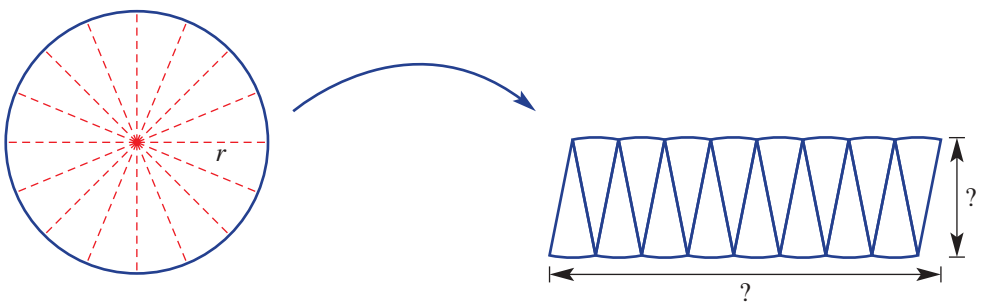
- This section consolidates Stage 4 concepts that are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall area formulas from memory in the HSC examination

We know that the circumference of a circle is connected to the diameter by the special number pi (π).

Similarly, the area of a circle is also connected to pi. This time it is the product of pi and the square of the radius that gives the area; so, $A = \pi r^2$.

Lesson starter: How does a circle become a rectangle?

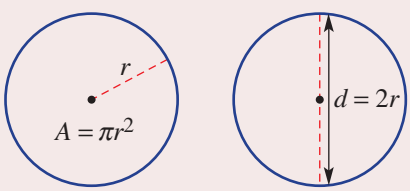
Consider a circle cut into small sectors, as shown, then rearranged to form a rectangular-style shape.



- Compared to the circle, what is the height of the rectangle close to?
- Compared to the circle, what is the length of the rectangle close to?
- What does this say about the area of the rectangle and hence the area of a circle?
- What could be done with the cutting up of sectors so that the rectangular arrangement is closer to a true rectangle?

KEY IDEAS

- The area of a circle is given by $A = \pi r^2$.
  - If the diameter is given, halve it to find the radius. For example, if the diameter is 10, the radius is 5.
  - $\pi r^2 = \pi \times r \times r$



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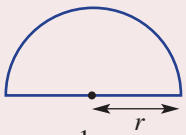
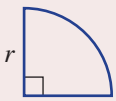
■ For the area of a **semicircle** or **quadrant**, use the appropriate fraction.

Semicircle



$A = \frac{1}{2}\pi r^2$

Quadrant



$A = \frac{1}{4}\pi r^2$

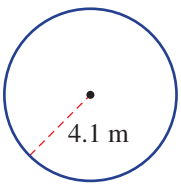
Key terminology: circle, sector, radius, diameter, semicircle, quadrant, pi

Exercise 6D

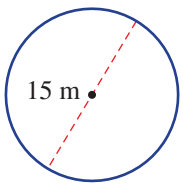
UNDERSTANDING 1–3 3

1 What is the radius of these circles?

a



b



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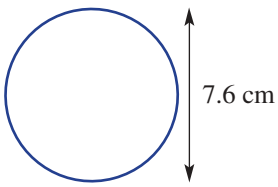


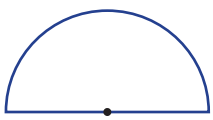
2 Which is the correct formula for the area of a circle?

A $A = \pi d^2$ **B** $A = \frac{1}{2}\pi r^2$ **C** $A = \pi r^2$ **D** $A = (\pi r)^2$

3 What fraction of a full circle is shown by these sectors?

a

b



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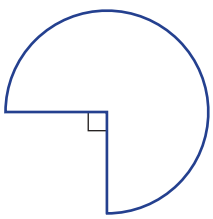
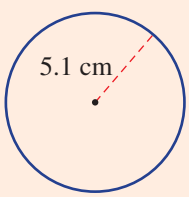


FLUENCY 4(½), 5 4(½), 5, 6

Example 10 Finding areas of circles

Find the area of this circle correct to two decimal places.



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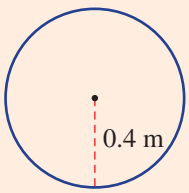
SOLUTION	EXPLANATION
$A = \pi r^2$	Write the rule and substitute $r = 5.1$.
$= \pi \times (5.1)^2$	
$= 81.7128\ldots$	
$= 81.71\ \text{cm}^2$ (to 2 d.p.)	81.7128 rounds to 81.71 since the third decimal place is 2.

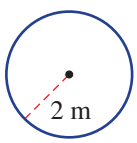
Now you try

Find the area of this circle correct to two decimal places.

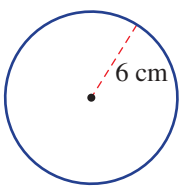


4 Find the area of these circles correct to two decimal places.

a

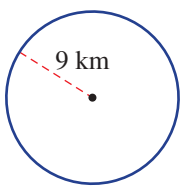


b



Hint: Substitute the radius into $A = \pi r^2$.

c



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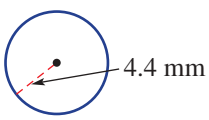

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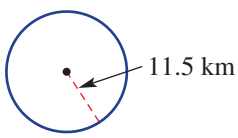

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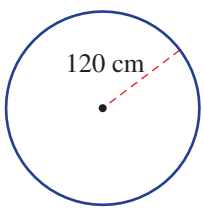
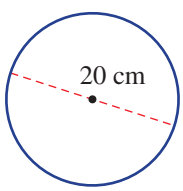


5 Find the area of these circles correct to two decimal places.

a



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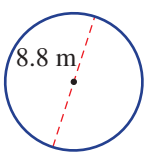


c

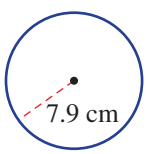
Hint: First work out the radius.

6 Find the area of these circles using the given value for pi (π). Round to two decimal places.

a $\pi = 3.14$

b $\pi = \frac{22}{7}$

c π (from calculator)



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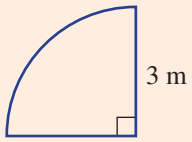
PROBLEM-SOLVING AND REASONING	7–9	7, 9–11

Example 11 Finding the area of a quadrant or semicircle

Find the area of this quadrant correct to two decimal places.



SOLUTION

$A = \frac{1}{4} \times \pi r^2$

$= \frac{1}{4} \times \pi (3)^2$

$= 7.0685 \ldots$

$= 7.07 \text{ m}^2$ (to 2 d.p.)

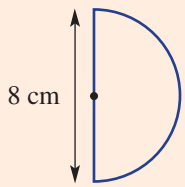
EXPLANATION

A quadrant is one quarter of the area of a full circle.

Substitute $r = 3$ and evaluate.

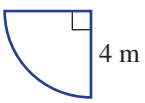
Now you try

Find the area of this semicircle correct to two decimal places.



7 Find the area of these sectors correct to two decimal places.

a



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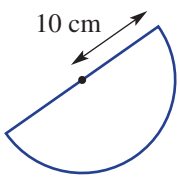

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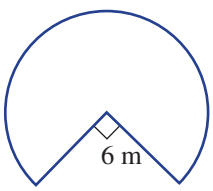


8 A pizza of radius 15 cm is divided into quarters.
Find the area of each quarter correct to the nearest cm^2.

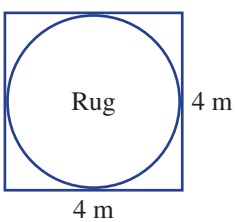
9 A circular rug touches the edges of a square room of side length 4 m.

a What is the radius of the rug?

b Find the area of the rug correct to two decimal places.

c Find the area not covered by the rug correct to two decimal places.

d Find the percentage of the floor that is not covered by the rug, correct to one decimal place.



Hint:
$\%\text{ non-rug area} = \frac{\text{non-rug area}}{\text{total area}} \times \frac{100}{1}$

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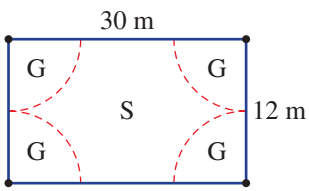
10 A pizza shop is considering increasing the diameter of its family pizza tray from 32 cm to 34 cm.
Find the percentage increase in area, correct to two decimal places, from the 32 cm tray to the 34 cm tray.

11 You can rearrange $A = \pi r^2$ to give $r = \sqrt{\frac{A}{\pi}}$. Use this new rule to find the radius of a circle for these areas. Round to one decimal place where necessary.

a $10\ \text{cm}^2$ **b** $117.8\ \text{m}^2$ **c** $4\pi\ \text{km}^2$

ENRICHMENT: Tennis lights — 12

12 A tennis court area is lit by 4 corner lights. The area close to each light is considered to be good (G) while the remaining area is lit satisfactorily (S).
What percentage of the area is 'good'? Round to the nearest per cent.



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6E Perimeter and area of composite shapes

Learning intentions for this section:

- To be able to recognise the basic shapes that make up a composite shape
- To be able to find the perimeter and area of composite shapes

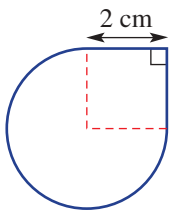
Past, present and future learning:

- This section consolidates Stage 4 concepts and introduces the Stage 5 Core topic called *Area and surface area A*
- Some of these questions are more challenging than those in our Stage 4 books
- These concepts are important in Stage 6 Standard
- You may be expected to recall area formulas from memory in the HSC examination

Composite shapes are made up of a number of simple shapes. You can find the perimeters and areas of composite shapes by first identifying the simple shapes and then using their perimeter and area formulas.

Lesson starter: A fraction of a circle plus a square

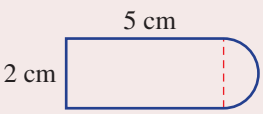
This diagram shows a fraction of a circle plus a square.



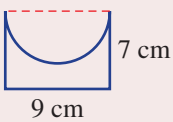
- How could you find the area and perimeter of the shape?
- See if you can write down a full solution for finding the area and perimeter of the shape.
- See if your teacher or another student can easily follow your solution.

KEY IDEAS

- **Composite shapes** are made up of more than one basic shape.
- Addition and/or subtraction can be used to find areas and perimeters of composite shapes.
  - Use addition



  - Use subtraction (for area)



- The layout of the relevant mathematical working needs to make sense so that the reader of your work understands each step.

Key terminology: composite shape, area, perimeter

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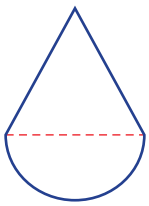
Exercise 6E

UNDERSTANDING 1–3 3

1 Name the two different shapes that make up these composite shapes; e.g. square and semicircle.

a

b



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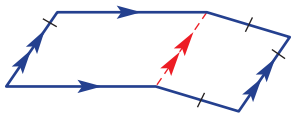
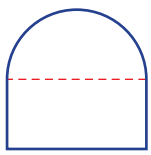


2 What fraction of a circle would be considered when finding the perimeter of these shapes?

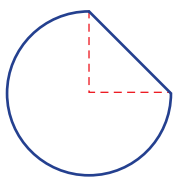
a



b

Hint: Choose from $\frac{1}{4}$, $\frac{1}{2}$ or $\frac{3}{4}$.

c



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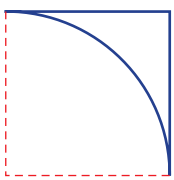
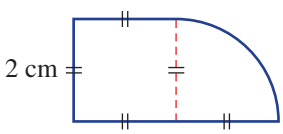


3 This composite shape includes a square and a quadrant $\left(\frac{1}{4} \text{ circle}\right)$.



a Find the area of the square.

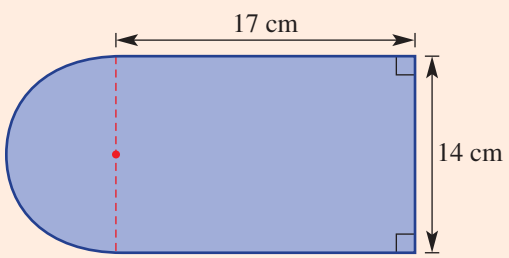
b Find the area of the quadrant correct to two decimal places.

c Find the total area correct to two decimal places.

FLUENCY 4(½), 5 4(½), 5, 6(½)

Example 12 Finding perimeters and areas of composite shapes

Find the perimeter and area of this composite shape, rounding answers to two decimal places.



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SOLUTION

$P = 2 \times l + b + \frac{1}{2} \times 2\pi r$

$= 2 \times 17 + 14 + \frac{1}{2} \times 2\pi \times 7$

$= 69.99\text{ cm}$ (to 2 d.p.)

Area (rectangle) $= l \times b$

$= 17 \times 14$

$= 238\text{ cm}^2$

Area (semicircle) $= \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 7^2$

$= 76.969 \ldots \text{ cm}^2$

Total area $= 238 + 76.969 \ldots$

$= 314.97\text{ cm}^2$ (to 2 d.p.)

EXPLANATION

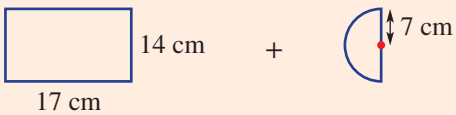
3 straight sides ⊐ + semicircle arc ☾

Recall that $C = 2\pi r$.

Substitute $l = 17, b = 14$ and $r = 7$.

Calculate and round to two decimal places.

The total area consists of a rectangle plus a semicircle.

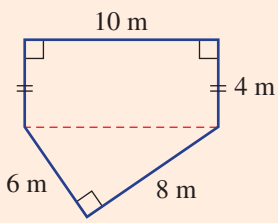


$A = l \times b$ $A = \frac{1}{2}\pi r^2$

Round to two decimal places.

Now you try

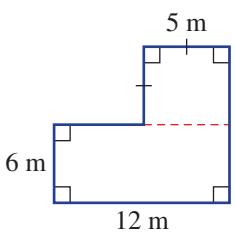
Find the perimeter and area of this composite shape.



4 Find the perimeter and the area of each of these composite shapes, rounding answers to two decimal places where necessary.

Hint: First decide which two shapes you will be dealing with in each case.

a



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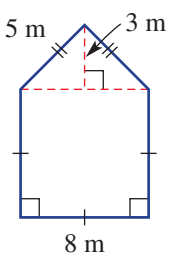

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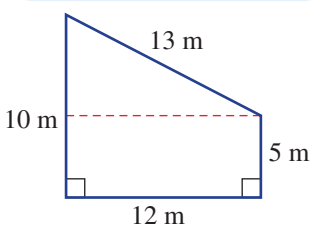

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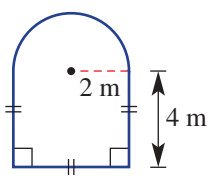

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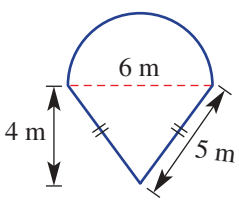

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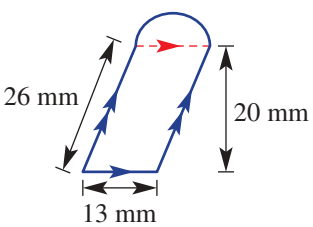

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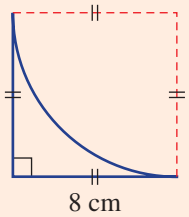
Example 13 Finding more perimeters and areas

This shape is a square with a quadrant (quarter circle) subtracted. Find its perimeter and area correct to two decimal places.



SOLUTION

$P = 2 \times l + \frac{1}{4} \times 2\pi r$

$= 2 \times 8 + \frac{1}{4} \times 2\pi \times 8$

$= 28.57$ cm (to 2 d.p.)

EXPLANATION

Includes 2 sides and a quarter of the circumference of a circle of radius 8 cm.

$A = l^2 - \frac{1}{4} \times \pi r^2$

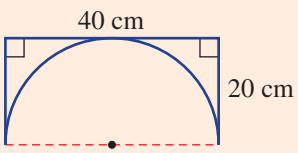
$= 8^2 - \frac{1}{4} \times \pi \times 8^2$

$= 13.73\text{ cm}^2$ (to 2 d.p.)

Includes a square minus a quarter of the area of a circle of radius 8 cm.

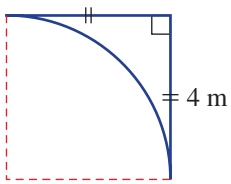
Now you try

This shape is a rectangle with a semicircle subtracted. Find its perimeter and area correct to two decimal places.



5 Find the perimeter and area of these shapes correct to two decimal places.

a



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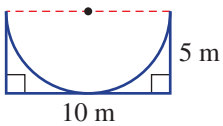

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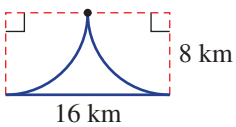

Hint: Use subtraction to find the area.

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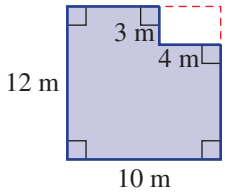
6 Find the area of the shaded part of these composite shapes.

Hint: Use subtraction in each case.

a



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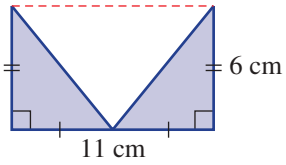

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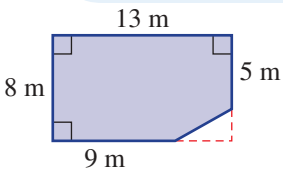

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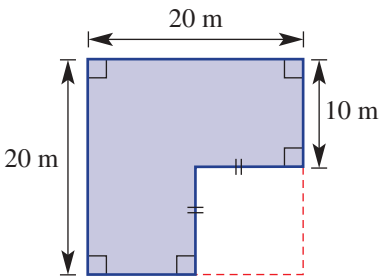

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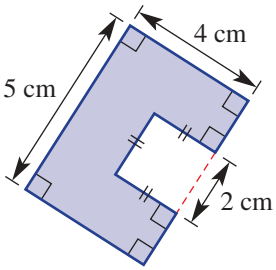

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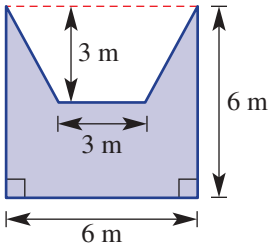

PROBLEM-SOLVING AND REASONING

7, 8 | 8, 9(½), 10

7 An area of lawn is made up of a rectangle measuring 10 m by 15 m and a semicircle of radius 5 m. Find the total area of lawn correct to two decimal places.

8 Twenty circular pieces of pastry, each of diameter 4 cm, are cut from a rectangular layer of pastry 20 cm long and 16 cm wide. What is the area, correct to two decimal places, of pastry remaining after the 20 pieces are removed?

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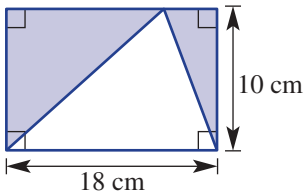
9 Find the area of the shaded region of each of the following shapes by subtracting the area of the clear shape from the total area. Round to two decimal places where necessary.

a



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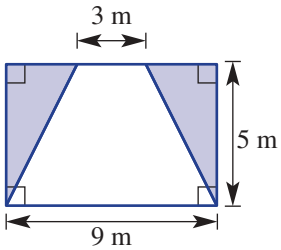

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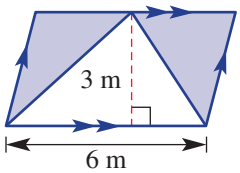

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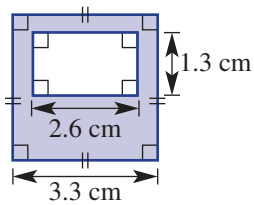

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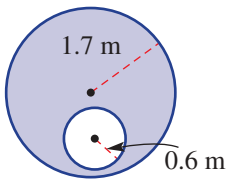

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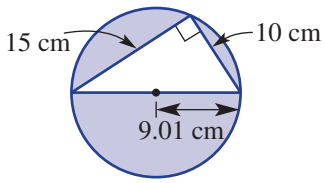

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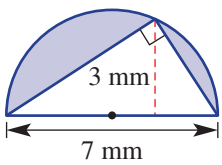

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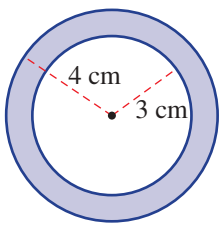

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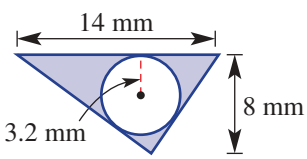
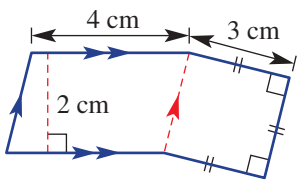


10 Find the area of each of the following composite shapes.

a



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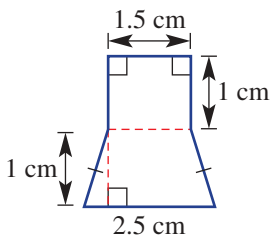

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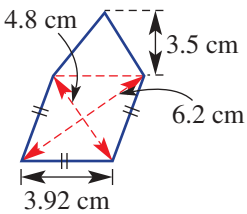
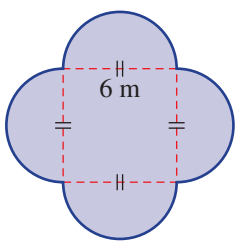


ENRICHMENT: Circular challenges — 11

11 Find the perimeter and the area of each of the following composite shapes correct to two decimal places where necessary.

a



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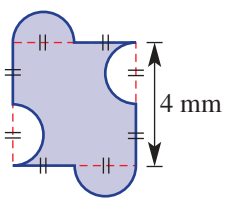

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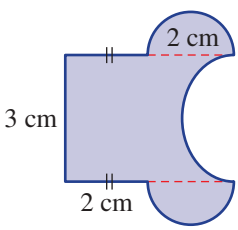

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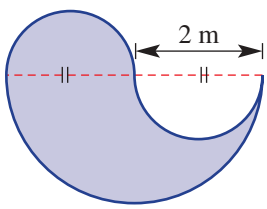

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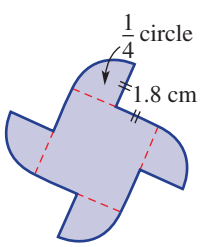

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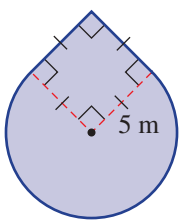

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6F Surface area of prisms

Learning intentions for this section:

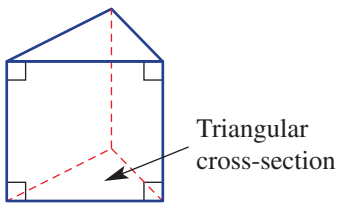
- To understand how the surface area of a solid can be represented using a net
- To be able to calculate the surface area of a prism

Past, present and future learning:

- Area concepts from Stage 4 are applied to three-dimensional solids from Year 9 onwards
- This section includes concepts from the Stage 5 Core topic called *Area and surface area A*
- Chapter 2 of our Year 10 book revises and further extends surface area concepts
- Surface area concepts are important in Stage 6 Standard
- You may be expected to recall surface area formulas from memory in the HSC examination

Three-dimensional objects or solids have outside surfaces that together form the surface area. Nets are very helpful for determining the number and shape of the surfaces of a three-dimensional object.

For this section we will deal with right prisms. A right prism has a uniform cross-section with two identical ends and the remaining sides are rectangles at right angles to the base and top.



Right triangular prism

Lesson starter: Drawing prisms

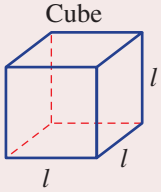
Prisms are named by the shape of their cross-section.

- Try to draw as many different right prisms as you can.
- Describe the different types of shapes that make up the surface of your solids.
- Which solids are the most difficult to draw and why?

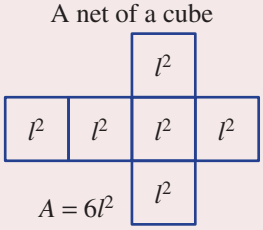
KEY IDEAS

- The **surface area** of a solid is the sum of the areas of all the surfaces.
- A **net** is a two-dimensional illustration of all the surfaces of a solid.

Cube



A net of a cube



$A = 6l^2$

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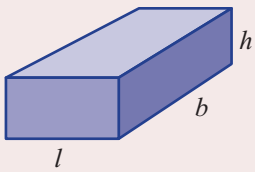
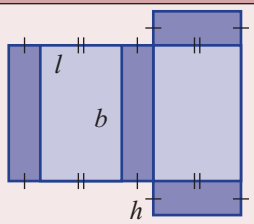
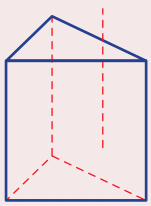
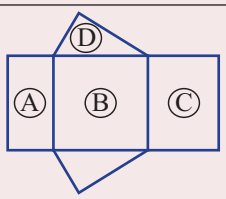
■ A **right prism** is a solid with a **uniform** (constant) **cross-section** and with remaining sides as rectangles.
- Prisms are named by the shape of their cross-section.

■ The nets for a rectangular prism (cuboid) and triangular prism are shown here.

Solid	Net	Surface area
Rectangular prism 		$A = 2(lb + bh + lh)$
Triangular prism 		$A =$ area Ⓐ + area Ⓑ + area Ⓒ + 2 × area Ⓓ

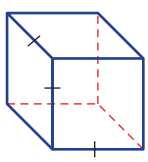
Key terminology: surface area, net, right prism, cross-section, uniform

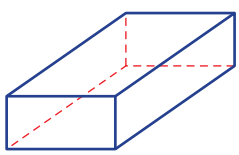
Exercise 6F

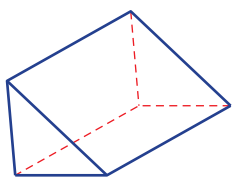
UNDERSTANDING 1–3 3

1 How many faces do the following solids have?
- **a** Rectangular prism
- **b** Cube
- **c** Triangular prism

2 Draw a suitable net for these prisms and name each solid.

a 

b 

c 

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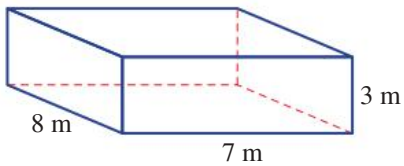
3 Copy and complete the working to find the surface area of these solids.

a

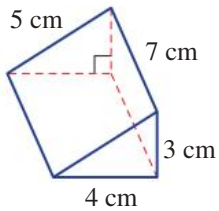


$A = 2 \times (8 \times 7) + 2 \times (8 \times _____) + 2 \times (_____ \times _____)$

$= _____ + _____ + _____$

$= _____ \text{ m}^2$

b



$A = 2 \times \frac{1}{2} \times 4 \times _____ + 5 \times 7 + 4 \times _____ + _____ \times _____$

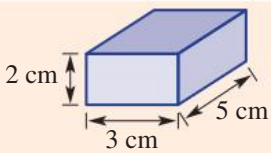
$= _____ + _____ + _____ + _____$

$= _____ \text{ cm}^2$

FLUENCY 4–5(½) 4–5(½), 6

Example 14 Finding the surface area of a rectangular prism

Find the surface area of this rectangular prism.



SOLUTION

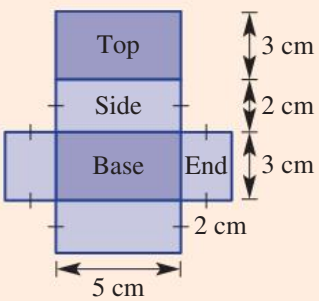
$A = 2 \times \text{top} + 2 \times \text{side} + 2 \times \text{end}$

$= 2 \times (5 \times 3) + 2 \times (5 \times 2) + 2 \times (2 \times 3)$

$= 30 + 20 + 12$

$= 62 \text{ cm}^2$

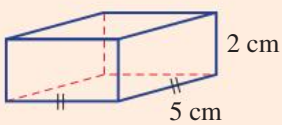
EXPLANATION



Top $= 5 \times 3$
Side $= 5 \times 2$
End $= 2 \times 3$

Now you try

Find the surface area of this rectangular prism.



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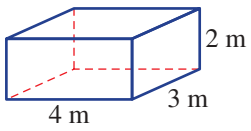
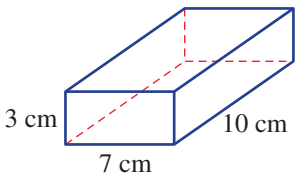
4 Find the surface area of the following rectangular prisms. Draw a net of the solid to help you.

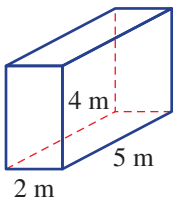
a



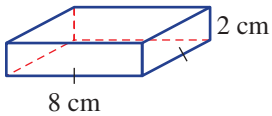
b



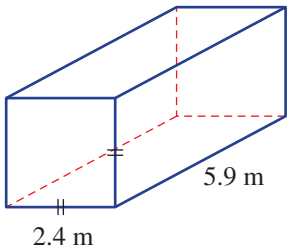
c



d



e



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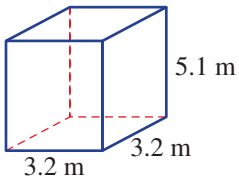
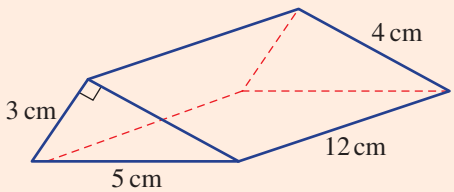


Hint: Add up all the pairs of rectangles, using $A = l \times b$.

Example 15 Finding the surface area of a right triangular prism

Find the surface area of this triangular prism.



SOLUTION

A = area A + area B + area C + 2 × area D

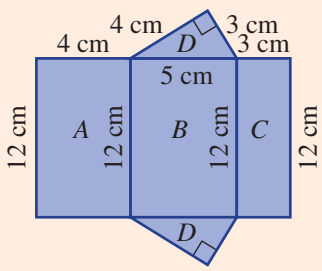
$= 12 \times 4 + 12 \times 5 + 12 \times 3 + 2 \times (\frac{1}{2} \times 4 \times 3)$

$= 48 + 60 + 36 + 12$

$= 156 \text{ cm}^2$

EXPLANATION

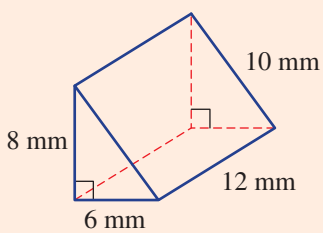
Draw and label the surface area net.



The surface is made up of three rectangles and two identical triangles. Use 3 cm and 4 cm for the base and height, respectively, of the triangles.

Now you try

Find the surface area of this triangular prism.



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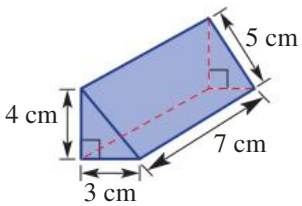
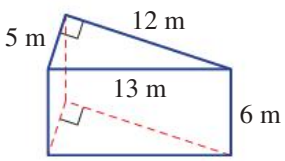
5 Find the surface area of each of the following triangular prisms.

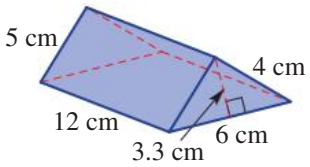
a



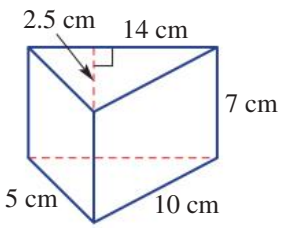
b



c



d



Hint: Each prism consists of three rectangles and two identical triangles.

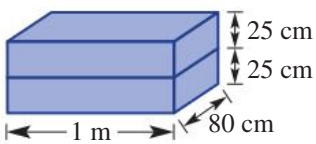
6 Find the surface area of a cube of side length 1 metre.

PROBLEM-SOLVING AND REASONING

7, 8 | 8–10

7 A rectangular box is to be covered in material. How much is required to cover the entire box if it has the dimensions 1.3 m, 1.5 m and 1.9 m?

8 Two wooden boxes, both with dimensions 80 cm, 1 m and 25 cm, are placed on the ground, one on top of the other, as shown. The entire outside surface, including the underside of the bottom box, is then painted. Find the area of the painted surface in cm^2.

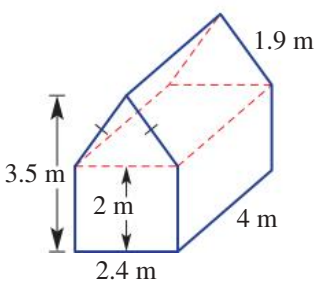


Hint: Only include the outside surfaces.

9 The four walls and roof of a barn (shown) are to be painted.

a Find the surface area of the barn, not including the floor.

b If 1 litre of paint covers 10 m^2, find how many litres are required to complete the job.



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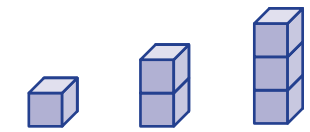
10 Cubes of side length one unit are stacked as shown.



1 cube 2 cubes 3 cubes etc.

a Complete this table.

Number of cubes (n)	1	2	3	4	5	6	7	8	9
Surface area (S)									

b Can you find the rule for the surface area (S) for n cubes stacked in this way? Write down the rule for S in terms of n.

c Use your rule to find the surface area if there are 100 cubes.

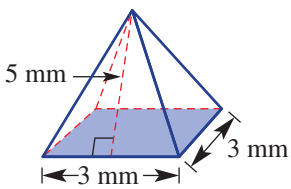
Hint: An example of a rule is $S = 2n + 5$, but this is not the rule for this question.

ENRICHMENT: Pyramids — 11

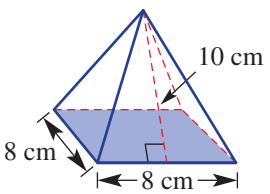
11 Pyramids consist of a base and a number of triangular faces. Surface area can be calculated by adding up all the face areas, similar to that of prisms. Remember that the area of a triangle is given by $A = \frac{1}{2}bh$.

Find the surface area of each of these pyramids. Draw a net of the solid to help you.

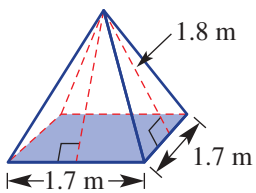
a



b



c



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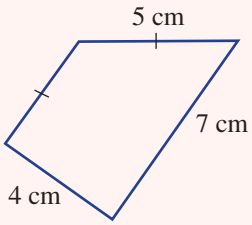
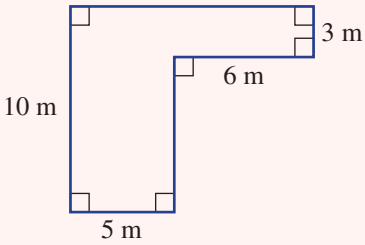
Progress quiz

6A **1** Find the perimeter of the following shapes.

a

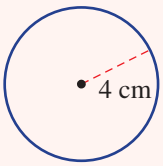


b

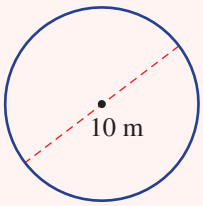


6B **2** Find the circumference of these circles correct to two decimal places.

a



b



6A/6C **3** Convert the following measurements into the units given in brackets.

a 824 cm (m)

b 30 cm (mm)

c 1.24 km (m)

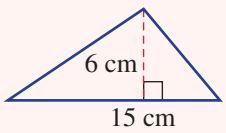
d 3.6 m^2 (cm^2)

e 450 cm^2 (mm^2)

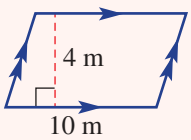
f 832 cm^2 (m^2)

6C **4** Find the area of the following plane figures.

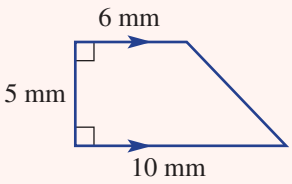
a



b



c



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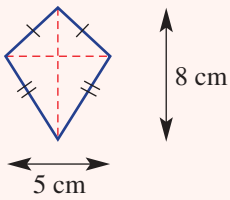
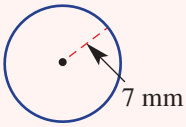


6D **5** Find the area of the following circles correct to one decimal place.

a



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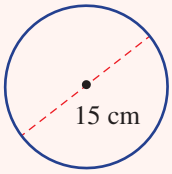

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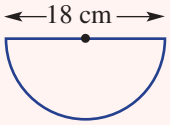
Progress quiz

6B/6D

6 For the semicircle shown, find (correct to one decimal place):

a its perimeter **b** its area

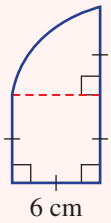


6E

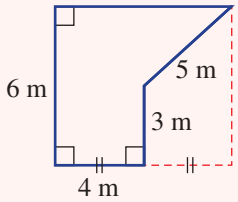
7 For each of the composite shapes, find (correct to one decimal place where necessary):

i its perimeter **ii** its area

a



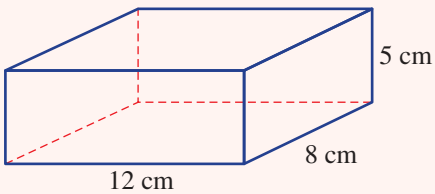
b



6F

8 Find the surface area of these prisms.

a



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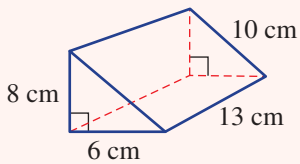

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6G Surface area of cylinders

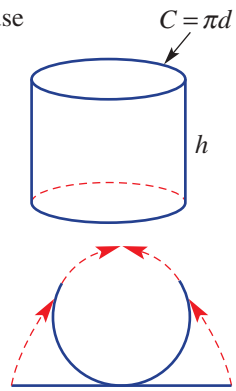
Learning intentions for this section:

- To understand how the net of a cylinder can be drawn to show the surface area
- To know the formula for the surface area of a cylinder
- To be able to calculate the surface area of a cylinder and sections of cylinders

Past, present and future learning:

- This section includes concepts from the Stage 5 Core topic called *Area and surface area A*
- Chapter 2 of our Year 10 book revises and further extends surface area concepts
- Surface area concepts are important in Stage 6 Standard
- You may be expected to recall surface area formulas from memory in the HSC examination

Like a rectangular prism, a cylinder has a uniform cross-section (a circle); but because it doesn't have rectangular sides, it isn't classified as a right prism. A cylinder's surface area can be found by considering the curved part as a rectangle with length $2\pi r$ (or πd) and height h.



Lesson starter: Curved area

- Roll a piece of paper to form the curved surface of a cylinder.
- Do not stick the ends together, so that you can allow the paper to return to a flat surface.
- What shape is the paper when lying flat on a table?
- When curved to form the cylinder, what do the sides of the rectangle represent on the cylinder? How does this help to find the surface area of a cylinder?

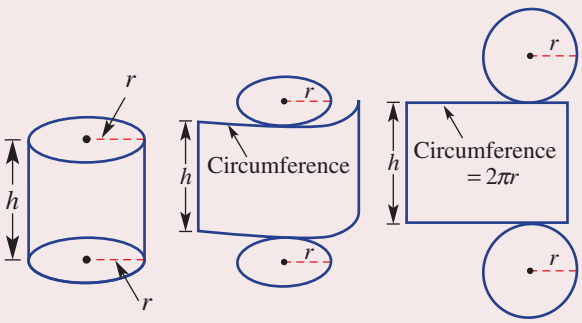
KEY IDEAS

- A **cylinder** is a solid with two parallel, **congruent** (identical) circular faces connected by a curved surface.
- Surface area of a cylinder $=$ 2 circles + rectangle

$$= 2 \times \pi r^2 + 2\pi r \times h$$

$$A = 2\pi r^2 + 2\pi rh$$

2 circular ends — Curved area



Key terminology: cylinder, area, prism, circumference, net, congruent, cross-section

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Exercise 6G

UNDERSTANDING	1–4	4

1 The formula for the surface area of a cylinder is $A = 2\pi r^2 + 2\pi rh$.

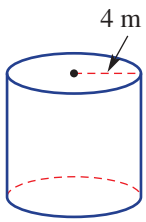
a Which part of the formula works out the area of the curved surface?

b Which part of the formula works out the area of the two ends?

2 Find the circumference of the circular end of these cylinders.
Round to two decimal places.

Hint: Use $C = \pi d$ or $C = 2\pi r$.

a



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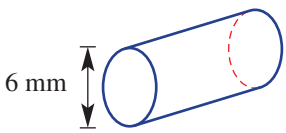
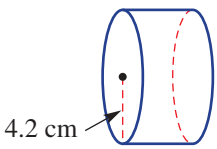
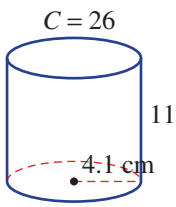


c



3 Draw a net suited to these cylinders. Label the sides using the given measurements.

a



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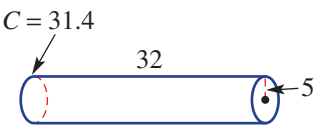
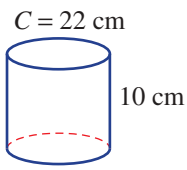


4 The curved surface of these cylinders is allowed to flatten out to form a rectangle. What would be the length and width of the rectangles?
Round to two decimal places where necessary.

Hint: Length = $2\pi r$

a



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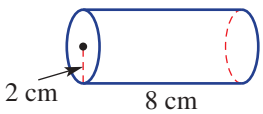

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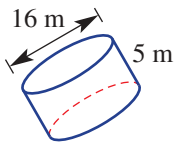
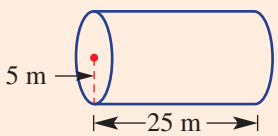


FLUENCY	5, 6	5, 7

Example 16 Finding the surface area of a cylinder

Find the surface area of this cylinder, rounding to two decimal places.



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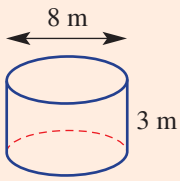
SOLUTION	EXPLANATION
$A = 2\pi r^2 + 2\pi rh$	In the diagram, $r = 5$ and $h = 25$.
$= 2 \times \pi \times 5^2 + 2\pi \times 5 \times 25$	
$= 50\pi + 250\pi$	
$= 300\pi$	300π is the *exact* answer.
The surface area is 924.48 m^2 (to 2 d.p.).	Calculate and round off.

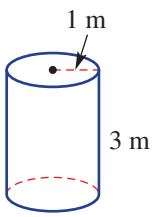
Now you try

Find the surface area of this cylinder, rounding to two decimal places.



5 Find the surface area of these cylinders, rounding to two decimal places.

a



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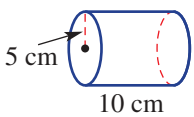

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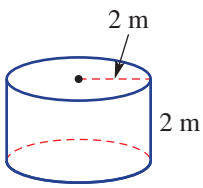
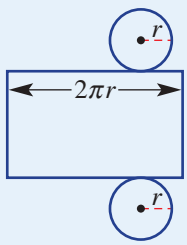


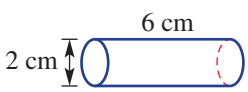
Hint: Draw a net to help:



6 Find the surface area of these cylinders, rounding to one decimal place.

Hint: Remember that the radius of a circle is half the diameter.

a



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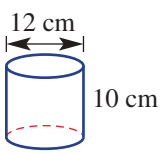

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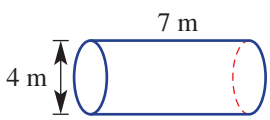
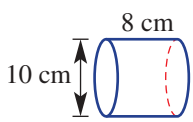


7 Find the area of the curved surface only for these cylinders correct to two decimal places.

Hint: For the curved part only, use $A = 2\pi rh$.

a



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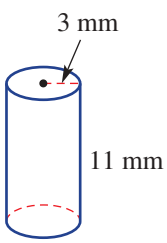

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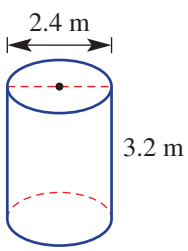

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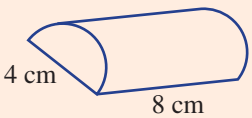
PROBLEM-SOLVING AND REASONING | 8, 9(½) | 9(½), 10, 11

8 Find the surface area of a cylindrical plastic container of height 18 cm and with a circle of radius 3 cm at each end, correct to two decimal places.

Example 17 Finding surface areas of cylindrical portions

Find the surface area of this half cylinder, rounding to two decimal places.



SOLUTION

Area (two semicircles) $= \pi r^2$

$= \pi \times 2^2$

$= 4\pi \text{ cm}^2$

Area (rectangles) $= 8 \times 4 + 8 \times \frac{1}{2} \times 2\pi \times 2$

$= 32 + 16\pi \text{ cm}^2$

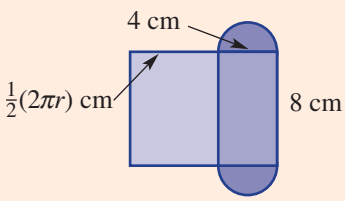
$A = 4\pi + 32 + 16\pi$

$= 94.831\ldots$

$= 94.83 \text{ cm}^2$ (to 2 d.p.)

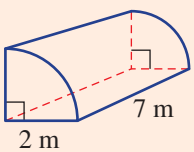
EXPLANATION

The two semicircles combine to form a full circle.



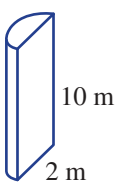
Now you try

Find the surface area of this quarter cylinder, rounding to two decimal places.



9 Find the surface area of these solids, rounding to two decimal places.

a



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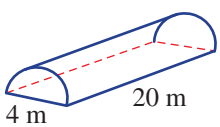

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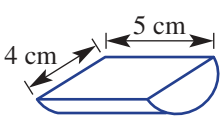

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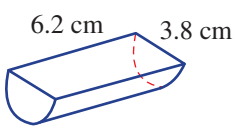

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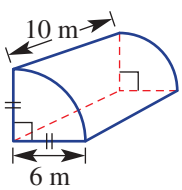

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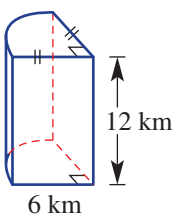

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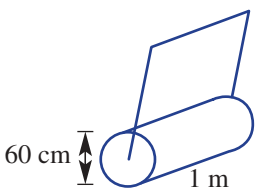
10 A water trough is in the shape of a half cylinder. Its semicircular ends have diameter 40 cm and the trough length is 1 m. Find the outside surface area, in cm^2, of the curved surface plus the two semicircular ends, correct to two decimal places.

11 A solid cylinder cut in half gives half the volume but not half the surface area. Explain why.

ENRICHMENT: Roller revolutions — 12

12 A cylindrical roller is used to press crushed rock in preparation for a tennis court. The entire rectangular tennis court area is 30 m long and 15 m wide. The roller has a width of 1 m and diameter 60 cm.



a Find the surface area of the curved part of the roller, in cm^2, correct to three decimal places.

b Find the area, in m^2 to two decimal places, of crushed rock that can be pressed after:

i 1 revolution **ii** 20 revolutions

c Find the minimum number of complete revolutions required to press the entire tennis court area.

Hint: $1\ m^2 = (100 \times 100)\ cm^2$

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6H Volume of prisms

Learning intentions for this section:

- To be able to convert between metric units of volume and capacity
- To understand how the volume of a solid relates to its constant cross-section and height
- To be able to calculate the volume of a solid with uniform cross-section

Past, present and future learning:

- This section consolidates Stage 4 concepts and introduces the Stage 5 Core topic called *Volume A*
- Chapter 2 of our Year 10 book revises and further extends volume concepts
- Volume concepts are important in Stage 6 Standard
- You may be expected to recall volume formulas from memory in the HSC examination

We use cubic units to describe the amount of space inside a three-dimensional object, such as:

- cubic kilometres for the volume of the Earth
- cubic metres for the space inside a room
- cubic centimetres for the volume of space occupied by this book
- cubic millimetres for the volume of metal in a pin.

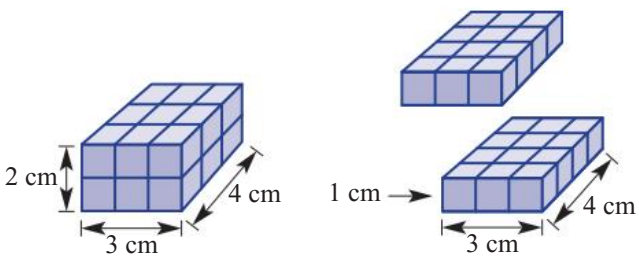
Units for capacity (millilitres, litres, kilolitres and megalitres) are used for liquids and gases.

Lesson starter: Why length × breadth × height?

For most people, the first thing that comes to mind when dealing with volume is length × breadth × height. This rule only applies to finding the volume of rectangular prisms.

Let's look at a rectangular prism split into two layers.

- How many cubes sit on one layer?
- What is the area of the base? What do you notice?
- What is the height and how many layers are there?
- Why is the volume rule given by $V = lbh$ in this case?



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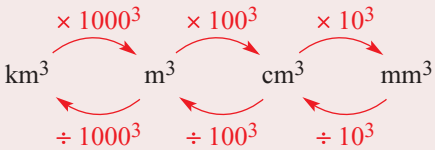
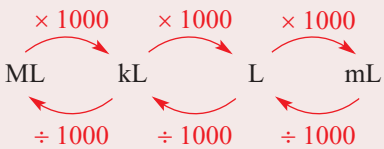
KEY IDEAS

- Common metric units for **volume** include cubic kilometres (km^3), cubic metres (m^3), cubic centimetres (cm^3) and cubic millimetres (mm^3).

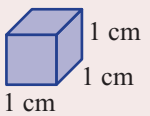
$1000^3 = 1000\,000\,000$
$100^3 = 1\,000\,000$
$10^3 = 1000$



- For **capacity**, common units include:
  - megalitres (ML) 1 ML = 1000 kL
  - kilolitres (kL) 1 kL = 1000 L
  - litres (L) 1 L = 1000 mL
  - millilitres (mL)

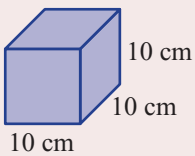


- A cubic centimetre holds 1 mL of liquid



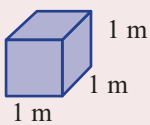
1 cm^3 holds 1 mL.

- This container holds 1 litre of liquid



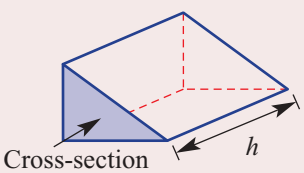
1000 cm^3 holds 1 L.

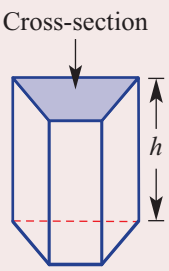
- A cubic metre holds 1000 litres or 1 kilolitre



1 m^3 holds 1000 L or 1 kL.

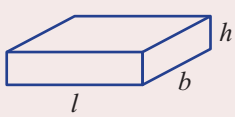
- Volume of **solids** with a uniform **cross-section** is equal to area of cross-section (A) × height (h).
  $V = Ah$





- The 'height' is the length of the edge that runs perpendicular to the cross-section in any solid.

- Volume of a rectangular prism:
  $V = lbh$



Key terminology: solid, volume, cross-section, uniform, prism, perpendicular, capacity, litres, millilitres

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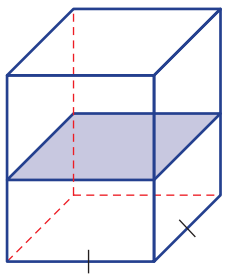
Exercise 6H

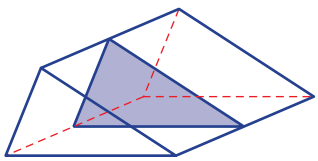
UNDERSTANDING 1–3 3

1 What is the name given to the shape of the shaded cross-section of each of the following solids?

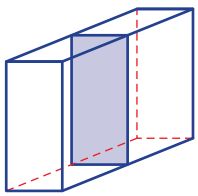
a



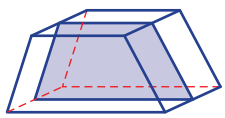
b



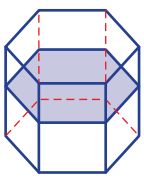
c



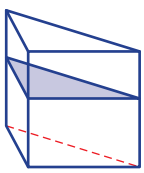
d



e



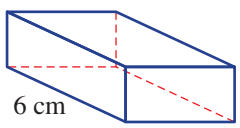
f



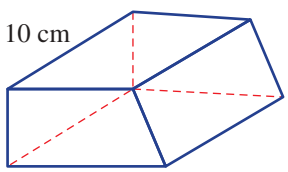
2 Draw the cross-sectional shape for these prisms and state the given 'height' (perpendicular to the cross-section).

Hint: 'Perpendicular' means 'at a right angle (90°)'.

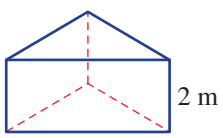
a



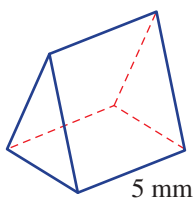
b



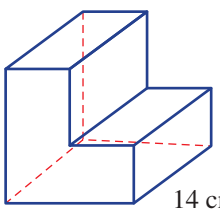
c



d



e



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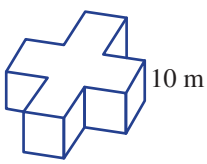

3 Write the missing number.

a The number of mm in 1 cm is _____.

b The number of mm^2 in 1 cm^2 _____.

c The number of mm^3 in 1 cm^3 is _____.

d There are _____ cm^3 in 1 m^3.

e There are _____ m^3 in 1 km^3.

f There are _____ mL in 1 L.

g There are _____ L in 1 kL.

h There is _____ cm^3 in 1 mL.

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FLUENCY 4–5(½), 6, 8 | 4–5(½), 7, 8(½), 9

Example 18 Converting units of volume

Convert the following volume measurements into the units given in the brackets.

a $2.5\ m^3$ (cm^3) **b** $458\ mm^3$ (cm^3)

SOLUTION

a $2.5\ m^3 = 2.5 \times 100^3\ cm^3$
$= 2\,500\,000\ cm^3$

b $458\ mm^3 = 458 \div 10^3\ cm^3$
$= 0.458\ cm^3$

EXPLANATION

$\times 100^3 = 1\,000\,000$
m^3 cm^3 2.500000

cm^3 mm^3
$\div 10^3 = 1000$ 458.

Now you try

Convert the following volume measurements into the units given in the brackets.

a $0.6\ cm^3$ (mm^3) **b** $4\,520\,000\ cm^3$ (m^3)

4 Convert the following volume measurements into the units given in brackets.

a $3\ cm^3$ (mm^3) **b** $0.3\ cm^3$ (mm^3) **c** $2000\ mm^3$ (cm^3)
d $0.001\ m^3$ (cm^3) **e** $8.7\ m^3$ (cm^3) **f** $5900\ cm^3$ (m^3)
g $0.00001\ km^3$ (m^3) **h** $21\,700\ m^3$ (km^3) **i** $430\,000\ cm^3$ (m^3)

Hint: $1\ km^3 = 1000^3\ m^3$
$1\ m^3 = 100^3\ cm^3$
$1\ cm^3 = 10^3\ mm^3$

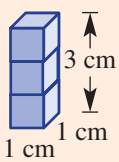
5 Convert these units of capacity to the units given in brackets.

a 3 L (mL) **b** 0.2 kL (L) **c** 3500 mL (L)
d 0.021 L (mL) **e** 37 000 L (kL) **f** 42 900 kL (ML)
g $2\ cm^3$ (mL) **h** 2 L (cm^3) **i** $1\ m^3$ (L)

Hint:
1 ML = 1000 kL
1 kL = 1000 L
1 L = 1000 mL
$1\ cm^3 = 1$ mL

Example 19 Finding the volume of a rectangular prism

Find the volume of this rectangular prism.



SOLUTION

Volume $= l \times b \times h$
$= 1 \times 1 \times 3$
$= 3\ cm^3$

EXPLANATION

The solid is a rectangular prism.

Length = 1 cm, breadth = 1 cm and height = 3 cm

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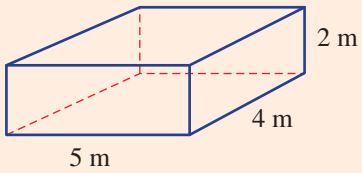
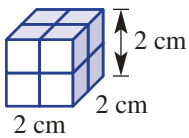
Now you try

Find the volume of this rectangular prism.

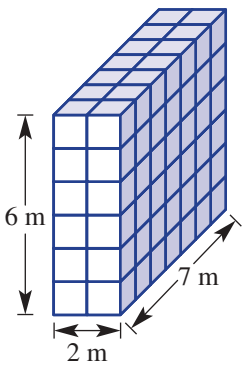


6 Find the volume of these three-dimensional rectangular prisms.

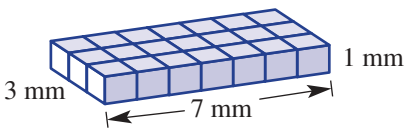
a



b

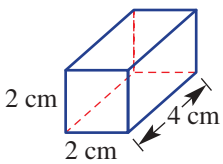


c

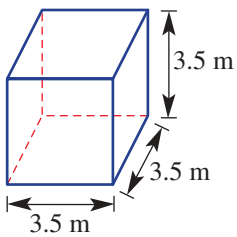


7 Find the volume of these rectangular prisms (cuboids).

a



b



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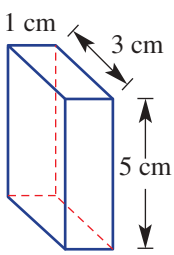
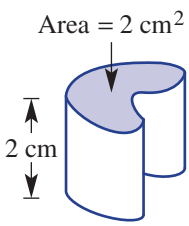
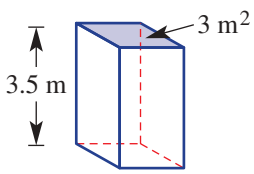


8 Find the volume of each of these three-dimensional objects. The cross-sectional area has been given.

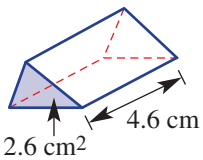
a



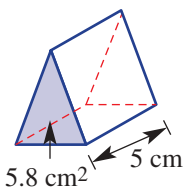
b



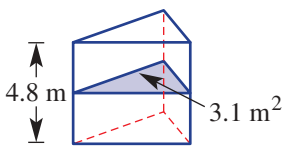
c



d



e



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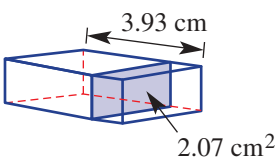

Hint: Simply use $V = Ah$, since the area of the cross-section is given.

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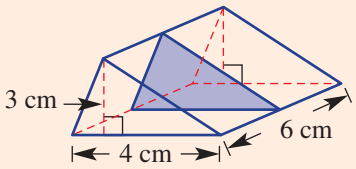
Example 20 Finding the volume of a triangular prism

Find the volume of this triangular prism.



SOLUTION

Area of cross-section $= \frac{1}{2}bh$

$= \frac{1}{2} \times 4 \times 3$

$= 6\,\text{cm}^2$

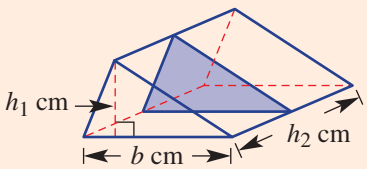
Volume $= Ah$

$= 6 \times 6$

$= 36\,\text{cm}^3$

EXPLANATION

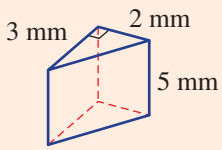
The cross-section is a triangle.



Multiply the area of the cross-section by the height.

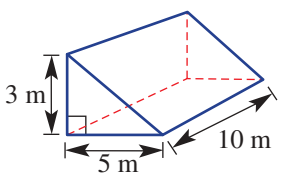
Now you try

Find the volume of this triangular prism.

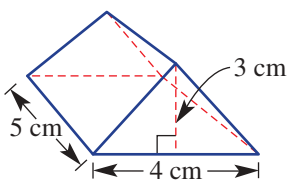


9 Find the volume of these prisms.

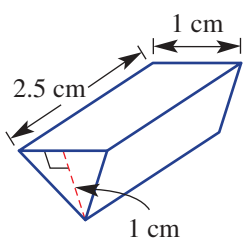
a



b



c



Hint: First find the area of the triangular cross-section.
$A = \frac{1}{2}bh$

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PROBLEM-SOLVING AND REASONING

10, 11 | 11–13

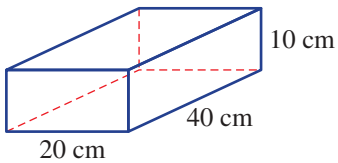
10 A rectangular container is 10 cm wide, 20 cm long and 8 cm high. What is the capacity of the container in litres?

Hint: There are 1000 cm^3 in 1 L.

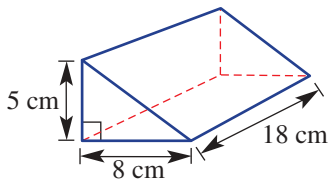
11 25 L of water is poured into a rectangular fish tank that is 50 cm long, 20 cm wide and 20 cm high. Will it overflow?

12 Find the volume of these solids, converting your answer to litres.

a



b



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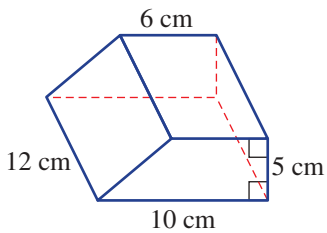
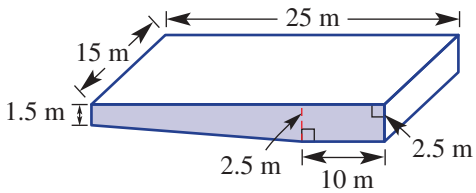


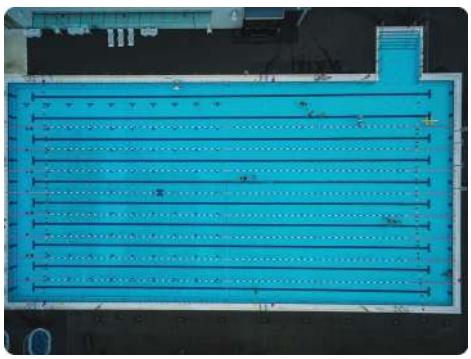
Hint: Area of a trapezium: $A = \frac{h}{2}(a + b)$

13 This diagram is a sketch of a new 25 m swimming pool to be installed in a school sports complex.

a Find the area of one side of the pool (shaded).

b Find the volume of the pool in litres.
Use 1 m^3 = 1000 L.





ENRICHMENT: Volume of a pyramid

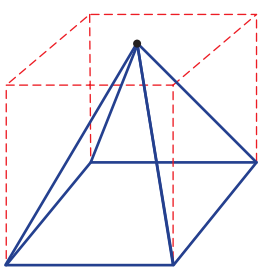
— | 14

14 Someone tells you that the volume of a pyramid is half of the volume of a rectangular prism with the same base. Do you think this is true?

a Make an educated guess as to what fraction of the prism's volume is the pyramid's volume.

b Use the internet to find the actual answer to part **a**.

c Draw some pyramids and find their volume using the results from part **b**.



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6I Volume of cylinders

Learning intentions for this section:

- To know the formula for the volume of a cylinder
- To be able to find the volume and capacity of a cylinder

Past, present and future learning:

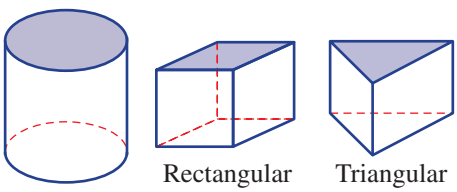
- This section addresses Stage 4 concepts and the Stage 5 Core topic called *Volume A*
- Chapter 2 of our Year 10 book revises and further extends volume concepts
- Volume concepts are important in Stage 6 Standard
- You may be expected to recall volume formulas from memory in the HSC examination

A cylinder with its sides at right angles to its base is called a right cylinder. It has a uniform cross-section (a circle), so its volume can be calculated in a similar way to that of a prism. Cylindrical objects are often used to store gases and liquids, so working out the volume and capacity of a cylinder is an important measurement calculation.

Lesson starter: How is a cylinder like a prism?

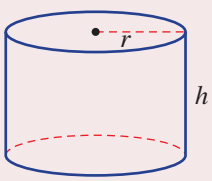
Here is a cylinder and two prisms.

- What do they all have in common?
- Do you think you can work out the volume of the cylinder in the same way as for a prism? Why?
- What do you think the formula would be for the volume of a cylinder?



KEY IDEAS

- The volume of a cylinder is given by $V = \pi r^2 h$, where:
  - r is the radius of the circular ends
  - h is the length or distance between the circular ends



Key terminology: volume, cylinder, capacity, radius, diameter

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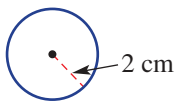
Exercise 6I

UNDERSTANDING 1–3(½) 2–3(½)

1 Find the area of these circles correct to two decimal places.

a



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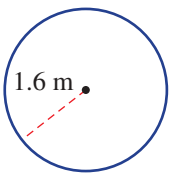

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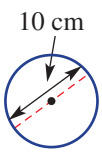

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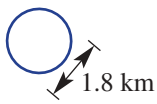


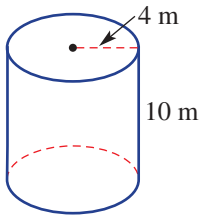
2 Convert the following into the units given in the brackets. Remember: $1\text{ L} = 1000\text{ cm}^3$ and $1\text{ m}^3 = 1000\text{ L}$.

a 2000 cm^3 (L) **b** 4.3 cm^3 (mL) **c** 3.7 L (cm^3)

d 1 m^3 (L) **e** 38 000 L (m^3) **f** 0.0002 m^3 (mL)

3 State the radius (r) and the height (h) of these cylinders.

a



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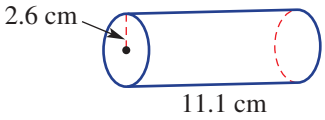
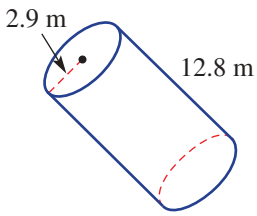


Hint: The height is perpendicular (at 90°) to the circular ends.

c



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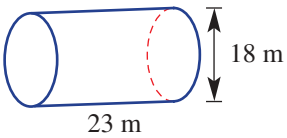

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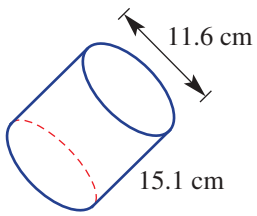

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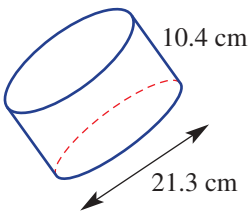

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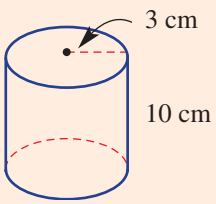
FLUENCY 4 4(½)

Example 21 Finding the volume of a cylinder

Find the volume of these cylinders correct to two decimal places.

a



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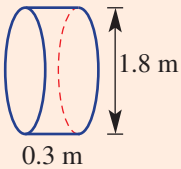


SOLUTION

a $V = \pi r^2 h$

$= \pi \times (3)^2 \times 10$

$= 90\pi \text{ cm}^3$

$= 282.74 \text{ cm}^3$ (to 2 d.p.)

b $V = \pi r^2 h$

$= \pi \times (0.9)^2 \times 0.3$

$= 0.7634 \ldots$

$= 0.76 \text{ m}^3$ (to 2 d.p.)

EXPLANATION

πr^2 gives the area of the circular cross-section and h is the distance between the ends.

Substitute $r = 3$ and $h = 10$ into the rule.

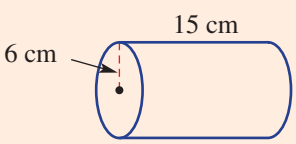
The diameter is 1.8 m so $r = 0.9$ m.

The distance between the ends is 0.3 m, so $h = 0.3$.

Now you try

Find the volume of these cylinders correct to two decimal places.

a



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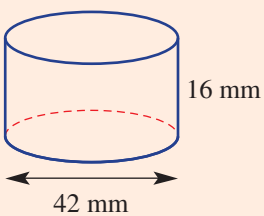
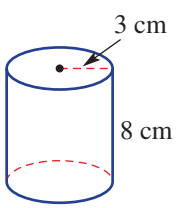


4 Find the volume of these cylinders correct to two decimal places.

Hint: Substitute the values of r and h into $V = \pi r^2 h$. Note that $r = d \div 2$ for parts **d**, **e** and **f**.

a



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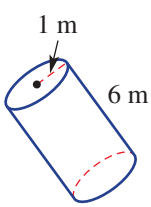

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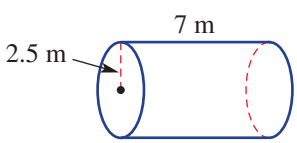

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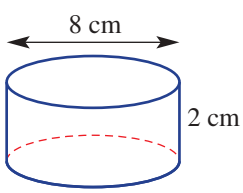

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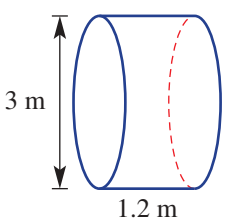

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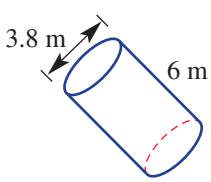

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PROBLEM-SOLVING AND REASONING | 5(½), 6, 7 | 5(½), 7–10

Example 22 Finding the capacity of a cylinder

Find the capacity, in litres, of a cylinder with radius 30 cm and height 90 cm. Round to the nearest litre.

SOLUTION

$V = \pi r^2 h$

$= \pi \times (30)^2 \times 90$

$= 81\,000\pi$

$= 254\,469.004\ldots \text{ cm}^3$

$= 254\,469.004\ldots \text{ mL}$

$= 254.469\text{L}$

$= 254 \text{ L (nearest litre)}$

EXPLANATION

Substitute $r = 30$ and $h = 90$.

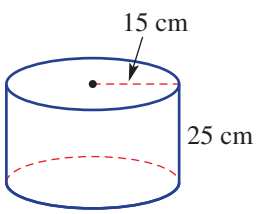
There are 1000 cm^3 in 1 L, so divide by 1000 to convert to litres.

Now you try

Find the capacity, in litres, of a cylinder with diameter 20 cm and height 45 cm. Round to the nearest litre.

5 Find the capacity, in litres, of these cylinders. Round to the nearest litre.

a



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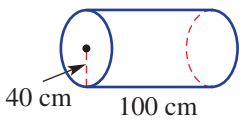

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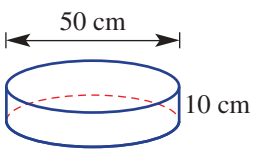

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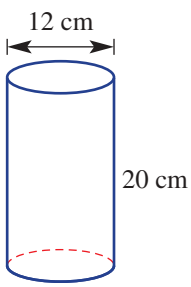

Hint: First work out the volume in cm^3 and then use $1 \text{ L} = 1000 \text{ cm}^3$ to find the capacity in litres.

6 A cylindrical storage drum has a radius of 0.5 m and a height of 2 m.

a Find its volume, in m^3, correct to three decimal places.

b Find its volume, in L, correct to the nearest litre ($1 \text{ m}^3 = 1000$ L).

7 Which has a greater capacity: a 10 cm by 10 cm by 10 cm cube or a cylinder with radius 6 cm and height 10 cm?

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8 A cylindrical water tank has a radius of 2 m and a height of 2 m.

a Find its capacity, in m^3, rounded to three decimal places.

b Find its capacity, in L, rounded to the nearest litre.

Hint:
$1\ m^3 = 1000\ L$.

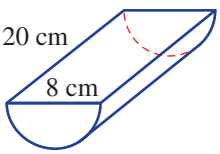
9 How many litres of gas can a tanker carry if its tank is cylindrical with a 2 m diameter and is 12 m in length? Round to the nearest litre.

10 Draw a cylinder with its circumference equal to its height. Try to draw it to scale.

ENRICHMENT: Cylinder portions — 11

11 Find the volume of these cylindrical portions correct to two decimal places.

a



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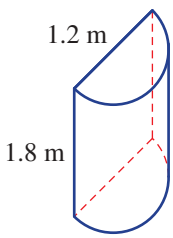

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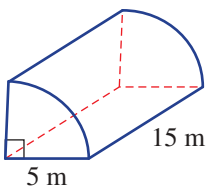

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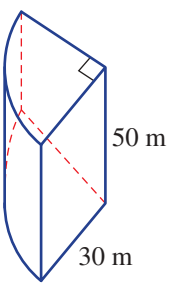

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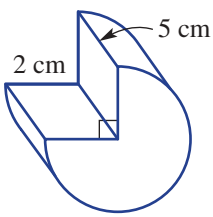

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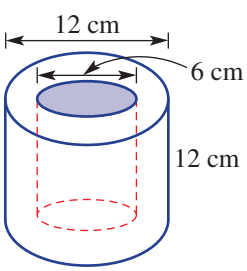

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Maths@Work

Vegetable and fruit growers

The importance of farming in Australia can never be underestimated. From small scale gardens to commercial agriculture, farmers and their crops are essential for our food supply. Mathematics plays a major role in the profitable management of farms, including analysis of the optimum planting times, weather cycles, irrigation rates, pricing and production costs.

The issue of water usage and storage is very important in Australia. Farmers regularly measure soil moisture content to find out what is happening around the roots of crops. Irrigation is managed according to plant watering needs, rainfall and evaporation rates.

1 Farmers need to be able to convert the volume of their dams from cubic metres (m^3) to megalitres (ML) by dividing by 1000.

a Convert each of the following into ML.

i $890\ m^3$ **ii** $60\,000\ m^3$ **iii** $800\,000\ m^3$ **iv** $380\,000\ m^3$ **v** $190\,000\,000\ m^3$

b Convert the following into cubic metres.

i 9 ML **ii** 1.2 ML **iii** 120 ML **iv** 0.98 ML **v** 12 000 ML

2 The volume of a property dam is estimated using the formula volume $= A \times d \times 0.4$ where A = the surface area of the top of the dam; d = maximum depth of the dam; and the factor of 0.4 allows for the dam's sloping sides.

Calculate the volume of the following farm dams.

a Surface area $= 600\ m^2$, maximum depth 3 m

b Surface area $= 400\ m^2$, maximum depth 2.5 m

c Surface area $= 800\ m^2$, maximum depth 4 m

d Surface area $= 5000\ m^2$, maximum depth 5 m

e Surface area $= 16\,000\ m^2$, maximum depth 6 m

3 Concrete water storage tanks can be in the shape of cylinders. Calculate the capacity in litres and kilolitres for each water tank illustrated below. Recall that $1\ m^3 = 1000\ L = 1\ kL$.

a Height 1.8 m, diameter 8.2 m **b** Height 1.5 m, diameter 5 m

Hint: Recall that the volume of a cylinder is $V = \pi r^2 h$.

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Using technology

4 A soil moisture probe is a device that measures the percentage of water in soil.

- Maximum % of soil moisture is when soil is drenched.
- Minimum % of soil moisture is when plants wilt.

The difference between these percentages is the percentage of soil water that a plant uses. The depth of watering needed for plants is measured in mm, like rainfall.

Plant watering depth = percentage moisture used by plant × root depth in mm

Set up the Excel spreadsheet below to calculate watering depth for the given plants.

Hint: Format columns B, C and D as 'percentage' and enter the decimal values in each cell; e.g. enter 0.43 for 43%.

	A	B	C	D	E	F
1	Calculation of watering depths for various plants					
2	Plant	Maximum moisture percentage	Minimum moisture percentage	Percentage moisture used by plant	Root depth in mm	Plant watering depth in mm
3	Onions	41%	37%		400	
4	Carrots	43%	38%		600	
5	Tomatoes	47%	43%		1200	
6	Apple trees	47%	41%		1500	
7	Banana trees	49%	38%		600	

5 Fruit trees can be micro-irrigated by dripping or spraying water in a circular area near the trunk. The volume of water required for each tree is calculated using the formula for volume of a cylinder:

$V = \pi r^2 h = \pi r^2 \times$ plant watering depth

Note that an area of 1 m^2 covered by 1 mm of water = 1 L of water; hence area in m^2 multiplied by depth in mm gives volume in litres.

Set up the Excel spreadsheet shown below to calculate the litres of irrigation water required per 100 trees.

In the area formula, enter pi() for π.

Hint: Use the plant watering depth values calculated in Question 4.

	A	B	C	D	E	F
1	Volume of irrigation water required for 100 trees					
2	Plant	Radius in m of micro-irrigation	Area of irrigation in m^2	Plant watering depth in mm	Litres of water per plant per irrigation	Litres of water per 100 trees
3	Apple trees	0.21				
4	Banana trees	0.32				

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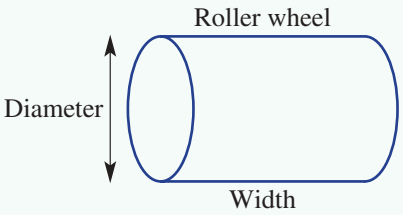
Working mathematically

Pressing a new road

The construction of a new asphalt road involves pressing the surface with a roller. Two rollers are available, each with main roller wheels of different diameter and width. The options are:

- Roller 1: Wheel diameter 1 metre and width 2 metres
- Roller 2: Wheel diameter 1.2 metres and width 2.5 metres.



The cost of hiring each roller depends on the number of revolutions, in the following way:

- Roller 1: $2.20 per revolution
- Roller 2: $3 per revolution.

The rectangular section of road being built is 1 km long and 10 metres wide.

Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate.

Routine problems

a Write the formula for finding the curved surface area of a cylinder using the pronumerals r and h.

b What is the value of r and h for:

- **i** Roller 1?
- **ii** Roller 2?

c For Roller 1 find,

- **i** the surface area of the roller wheel (the curved surface area only)
- **ii** the area of ground pressed after 10 revolutions.

d Find the surface area, in square metres, of a section of road which is 1 km long and 10 metres wide.

e How many rotations of the wheel would need to occur if Roller 1 was to travel a single length of the 1 km length of road? Round to one decimal place.

Non-routine problems

Explore and connect

Choose and apply techniques

The problem is to determine the cost of hiring each roller to press the entire 1 km stretch of road.

a Write down all the relevant information that will help solve this problem with the aid of diagrams.

b Using Roller 1, determine the total number of revolutions that the wheel needs to rotate if the entire 10 metre-wide road is to be pressed. Use the following assumptions.

- It is not necessary to overlap one pressed strip with the next.
- No revolutions are counted while the roller is manoeuvring into position at the start of each strip.

c Calculate the cost of hiring Roller 1 to press the 1 km stretch of road.

d Repeat parts **b** and **c** for Roller 2.

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Communicate thinking and reasoning

e Assume that each strip rolled must now overlap the previous pressed strip by 10 cm. Determine the total number of revolutions required for each of the two rollers under these conditions.

f Compare and comment on the cost of hiring the rollers if:

- **i** no overlap is required between strips
- **ii** a 10 cm overlap is required between strips.

g Summarise your results in a table like the one below, and decide which roller should be hired and why.

Roller	Surface area of 1 revolution	Number of revolutions needed for road with no overlap	Cost of hiring with no overlap	Number of revolutions needed for road with 10 cm overlap	Cost of hiring with 10 cm overlap
1					
2					

Extension problem

Problem-solve

a If the overlap is changed to 0.5 m instead, which roller is the cheapest to hire? Give reasons and show calculations.

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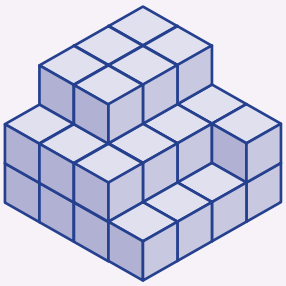
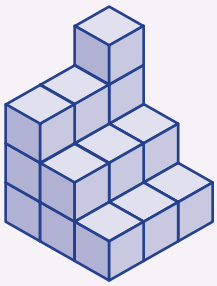
Puzzles and challenges

1 These towers are made by stacking cubes. How many cubes are needed in each case?

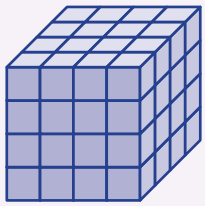
a



b

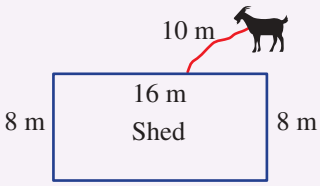


2 A large cube of side length 4 cm is painted then cut into 64 single 1 cm cubes. How many 1 cm cubes are not painted on any face?

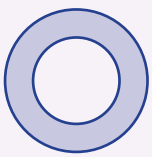


3 To the nearest metre, how far will a wheel of diameter 1 m travel after 100 revolutions?

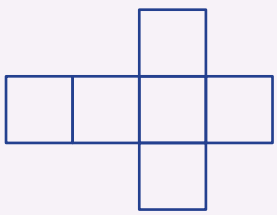
4 A goat is tethered to the centre of one side of a shed with a 10 m length of rope. In what area of grass can the goat graze?



5 A circle of radius 10 cm has a hole cut out of its centre to form a ring. Find the radius of the hole if the remaining area is 50% of the original area. Round to one decimal place.



6 Here is one net for a cube. How many different nets are possible?
Do not count nets that can be rotated or reflected (flipped) to give the same net.



7 A 100 m^2 factory flat roof feeds all the water collected into a rainwater tank. If there is 1 mm of rainfall, how many litres of water go into the tank?

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Chapter summary

Length, area, surface area and volume

Length and perimeter

Units

$\times 1000^3$ $\times 100^3$ $\times 10^3$

km^3 m^3 cm^3 mm^3

$\div 1000^3$ $\div 100^3$ $\div 10^3$

$1\text{ L} = 1000\text{ mL} = 1000\text{ cm}^3$

Circle circumference

$C = 2\pi r$ or πd

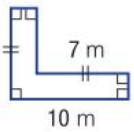
Units

$\times 1000$ $\times 100$ $\times 10$

km m cm mm

$\div 1000$ $\div 100$ $\div 10$

Perimeter



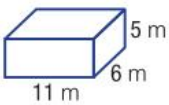
$P = 2 \times 10 + 2 \times 7$
$= 34\text{ m}$

Volume

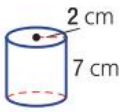
Prisms and cylinders

$V = Ah$
A = area of cross-section
h = height

$V = Ah$
$= 66 \times 5$
$= 330\text{ m}^3$



$V = \pi r^2 h$
$= \pi(2)^2 \times 7$
$= 87.96\text{ cm}^3$



Area

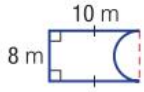
Triangle

$A = \frac{1}{2}bh$

Quadrilaterals

Square $A = l^2$
Rectangle $A = lb$
Parallelogram $A = bh$
Trapezium $A = \frac{h}{2}(a + b)$
Rhombus or kite $A = \frac{1}{2}xy$

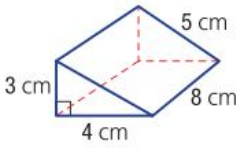
Composite shapes



$A = lb - \frac{1}{2}\pi r^2$
$= 10 \times 8 - \frac{1}{2}\pi \times 4^2$
$= 54.87\text{ m}^2$ (to 2 d.p.)
$P = 2 \times 10 + 8 + \frac{1}{2}2\pi(4)$
$= 40.57\text{ m}$ (to 2 d.p.)

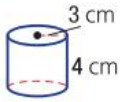
Surface area

Prisms



$A = 2 \times$ triangles $+ 3 \times$ rectangles
$= 2 \times \frac{1}{2} \times 4 \times 3 + 8 \times 5$
$+ 8 \times 4 + 8 \times 3$
$= 108\text{ cm}^2$

Cylinder



$A = 2\pi r^2 + 2\pi rh$
$= 2\pi(3)^2 + 2\pi(3)(4)$
$= 131.95\text{ cm}^2$ (to 2 d.p.)

Circle

$A = \pi r^2$
$A = \pi \times 6^2$
$= 113.10\text{ m}^2$

Units

$\times 1000^2$ $\times 100^2$ $\times 10^2$

km^2 m^2 cm^2 mm^2

$\div 1000^2$ $\div 100^2$ $\div 10^2$

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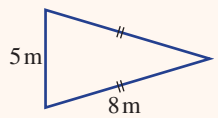
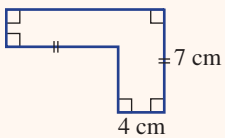
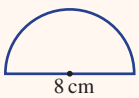
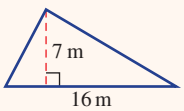
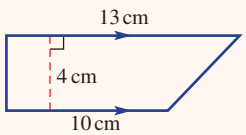
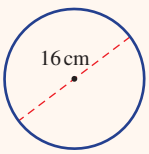
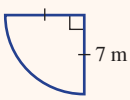
Chapter checklist

Chapter checklist with success criteria

A printable version of this checklist can be downloaded from the Interactive Textbook.

		✔
6A	**1 I can convert between metric units of length.** e.g. Convert these measurements to the units shown in the brackets. **a** 9.2 cm (mm) **b** 61 000 cm (m)	☐
6A	**2 I can find the perimeter of basic shapes, including composite shapes.** e.g. Find the perimeter of these shapes. **a**  **b** 	☐
6B	**3 I can find the circumference of a circle.** e.g. Find the circumference of a circle with a diameter of 5 m, correct to two decimal places.	☐
6B	**4 I can find the perimeter of a semicircle or a quadrant.** e.g. Find the perimeter of this semicircle correct to two decimal places. 	☐
6C	**5 I can convert between metric units of area.** e.g. Convert these measurements to the units shown in the brackets. **a** 5.32 cm^2 (mm^2) **b** 728 000 cm^2 (m^2)	☐
6C	**6 I can find the area of rectangles, triangles and parallelograms.** e.g. Find the area of this triangle. 	☐
6C	**7 I can find the area of rhombuses and trapeziums.** e.g. Find the area of this trapezium. 	☐
6D	**8 I can find the area of a circle.** e.g. Find the area of this circle correct to two decimal places. 	☐
6D	**9 I can find the area of a quadrant or semicircle.** e.g. Find the area of this quadrant correct to two decimal places. 	☐

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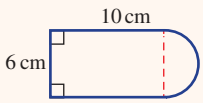
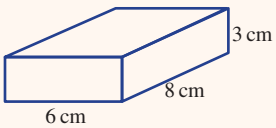
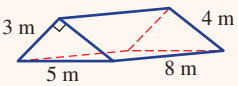
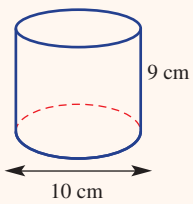
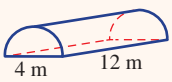
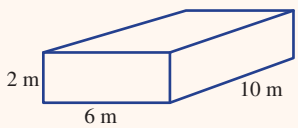
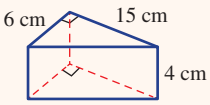
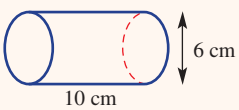
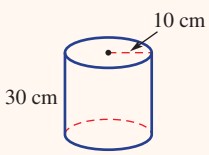
Chapter checklist with success criteria

6E	**10 I can find the perimeter and area of simple composite shapes.** e.g. Find the perimeter and area of this composite shape correct to two decimal places. 	☐
6F	**11 I can find the surface area of a rectangular prism.** e.g. Find the surface area of this rectangular prism. 	☐
6F	**12 I can find the surface area of a triangular prism.** e.g. Find the surface area of this triangular prism. 	☐
6G	**13 I can find the surface area of a cylinder.** e.g. Find the surface area of this cylinder correct to two decimal places. 	☐
6G	**14 I can find the surface area of a cylindrical portion.** e.g. Find the surface area of this half cylinder correct to two decimal places. 	☐
6H	**15 I can convert between metric units of volume and capacity.** e.g. Convert these measurements to the units shown in the brackets. **a** $0.003\ m^3$ (cm^3) **b** $640\,000\ mm^3$ (cm^3)	☐
6H	**16 I can find the volume of a rectangular prism.** e.g. Find the volume of this rectangular prism. 	☐
6H	**17 I can find the volume of a triangular prism.** e.g. Find the volume of this triangular prism. 	☐
6I	**18 I can find the volume of a cylinder.** e.g. Find the volume of this cylinder correct to two decimal places. 	☐
6I	**19 I can find the capacity of a cylinder giving an answer in L or mL.** e.g. Find the volume of this cylinder in litres correct to the nearest litre. 	☐

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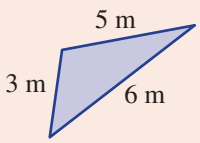
Chapter review

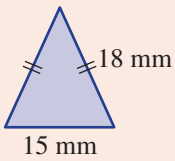
Short-answer questions

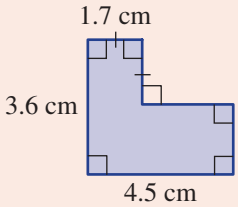
6A/C/H **1** Convert the following measurements into the units given in brackets.

a 3.8 m (cm)
b 1.27 km (m)
c 48 mm (cm)
d 273 mm^2 (cm^2)
e 5.2 m^2 (cm^2)
f 0.01 m^3 (cm^3)
g 53 100 mm^3 (cm^3)
h 3100 mL (L)
i 0.043 L (mL)
j 2.83 kL (L)
k 4000 cm^3 (L)
l 1 m^3 (L)

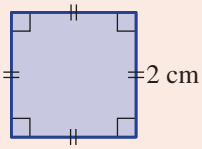
6A **2** Find the perimeter of each of the following shapes.

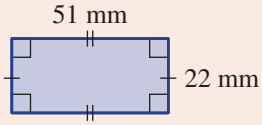
a 

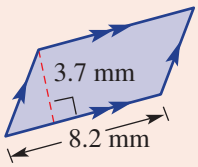
b 

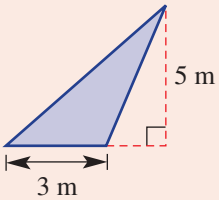
c 

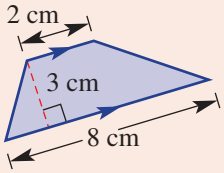
6C **3** Find the area of each of the following plane figures.

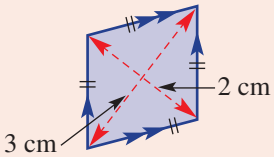
a 

b 

c 

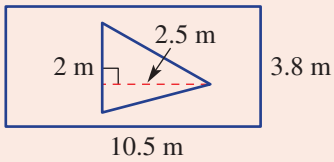
d 

e 

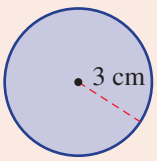
f 

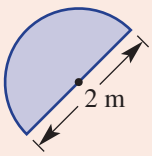
6E **4** Inside a rectangular lawn area of length 10.5 m and width 3.8 m, a new garden bed is to be constructed. The garden bed is to be the shape of a triangle with base 2 m and height 2.5 m. Find:

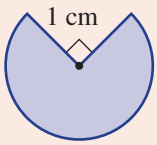
a the area of the garden bed
b the area of the lawn remaining around the garden bed.



6B/6D **5** Find the area and circumference/perimeter of each of the following shapes correct to two decimal places.

a 

b 

c 

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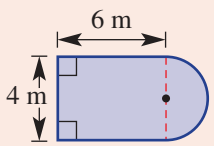
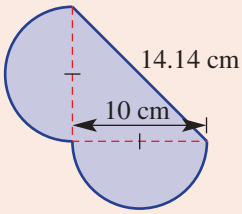
6E

6 Find the perimeter and area of each of the following composite shapes correct to two decimal places.

a



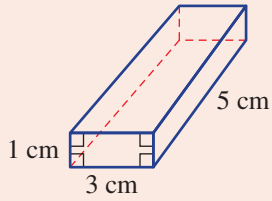
b



6F

7 Find the surface area of each of the following solid objects.

a



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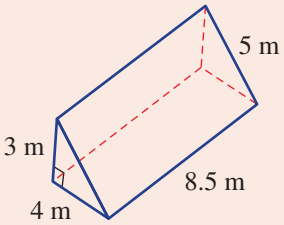
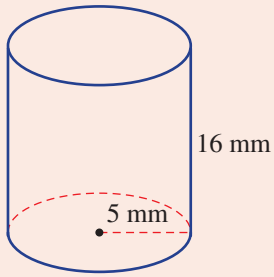


6G

8 Find the surface area of each of these cylinders correct to two decimal places.

a



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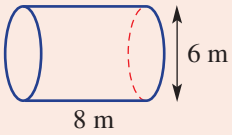
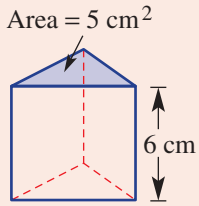


6H/6I

9 Find the volume of each of these solid objects, rounding to two decimal places where necessary.

a



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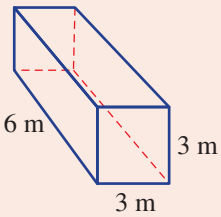

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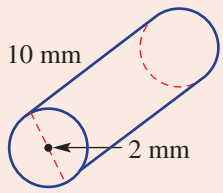

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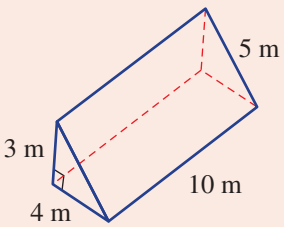

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Chapter review

Multiple-choice questions

6A/6C **1** If the area of a square field is 25 km^2, the length of one of its sides is:

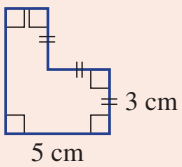
A 10 km **B** 5 km **C** 4 km **D** 50 km **E** 25 km

6C **2** If 1 m^2 = 100^2 cm^2, 2.7 m^2 is the same as:

A 270 cm^2 **B** 0.0027 km^2 **C** 27 000 cm^2 **D** 2700 mm^2 **E** 27 cm^2

6A **3** The perimeter of this shape is:

A 21 cm **B** 14 cm **C** 24 cm
D 20 cm **E** 22 cm

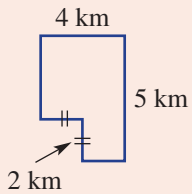


6C **4** A parallelogram has area 10 m^2 and base 5 m. Using $A = bh$, its perpendicular height is:

A 50 m **B** 2 m **C** 50 m^2
D 2 m^2 **E** 0.5 m

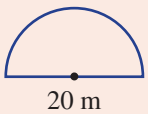
6E **5** This composite shape could be considered as a rectangle with an area in the shape of a square removed. The shape's area is:

A 16 km^2 **B** 12 km^2 **C** 20 km^2
D 24 km^2 **E** 6 km^2



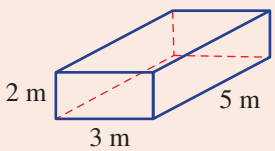
6B **6** A semicircular goal area has diameter 20 m. Its perimeter, correct to the nearest metre, is:

A 41 m **B** 36 m **C** 83 m
D 51 m **E** 52 m



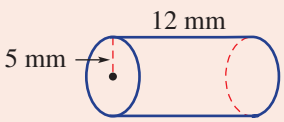
6F **7** The surface area of the rectangular prism shown is:

A 30 m^2 **B** 62 m^2 **C** 31 m^2
D 60 m^2 **E** 100 m^2



6G **8** The area of the *curved surface* of a cylinder with radius 5 mm and height 12 mm is closest to:

A 534 mm^2 **B** 94 mm^2 **C** 188 mm^2
D 754 mm^2 **E** 377 mm^2



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6H **9** A prism's cross-sectional area is 100 m^2. If its volume is 6500 m^3, the prism's total height would be:

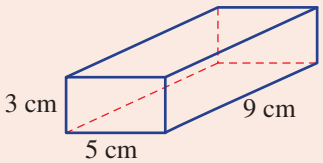
A 0.65 m **B** 650 000 m **C** 65 m **D** 6.5 m **E** 650 m

6I **10** The volume of a cylinder with radius 3 cm and height 10 cm is closest to:

A 188 cm^2 **B** 251 cm **C** 141 cm^3 **D** 94 cm^3 **E** 283 cm^3

Extended-response questions

1 A kindergarten teacher collects some blocks of wood for a painting activity. Each block is a rectangular prism, as shown.



a Find the volume of each block.

b Find the surface area to be painted for each block.

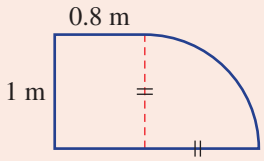
c If the paint costs \$2.50 per 100 cm^2, find the cost of painting 10 blocks.

d Another wood block is a cylinder with radius 3 cm and height 9 cm. Compared to the rectangular block:

i does it have a greater volume?

ii does it have a greater surface area?

2 An office receives five new desks with a bench shape made up of a rectangle and quarter circle as shown.



The edge of the bench is lined with a rubber strip at a cost of \$2.50 per metre.

a Find the length of the rubber edging strip for one desk correct to two decimal places.

b Find the total cost of the rubber strip for the five desks. Round to the nearest dollar.

The manufacturer claims that the desk top area space is more than 1.5 m^2.

c Is the manufacturer's claim correct?

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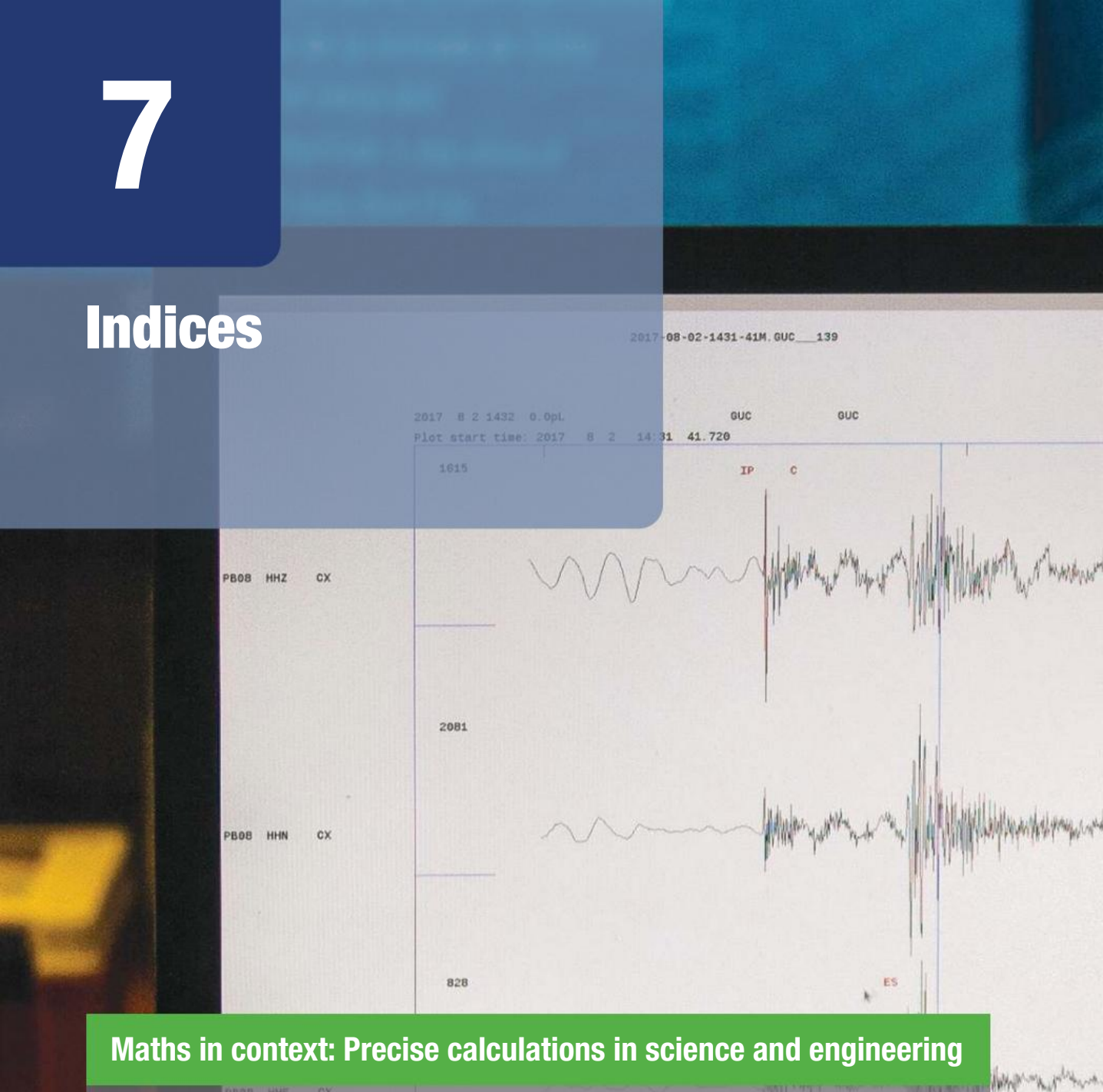
7

Indices



Maths in context: Precise calculations in science and engineering

People who regularly use index laws and scientific notation include computer programmers, scientists, chemists, radiographers, financial planners, economists, astronomers, geologists, and medical, electrical, sound and aerospace engineers.

Index laws and scientific notation are essential skills in numerous occupations, including when:

- scientists calculate the shaking intensity of earthquakes and the loudness of sound;
- food scientists predict the time for a food bacteria population to reach food-poisoning levels;
- accountants and financial advisors calculate the possible future values of an investment or the amount remaining of a debt;
- chemists determine the acidity of swimming pool water, which needs enough chlorine to kill bacteria but must remain comfortable for the swimmers;
- sound engineers design music synthesisers that electronically reproduce the vibrations that make sound;
- medical scientists calculate the decay time of the radioactive isotopes that are injected for disease diagnosis.

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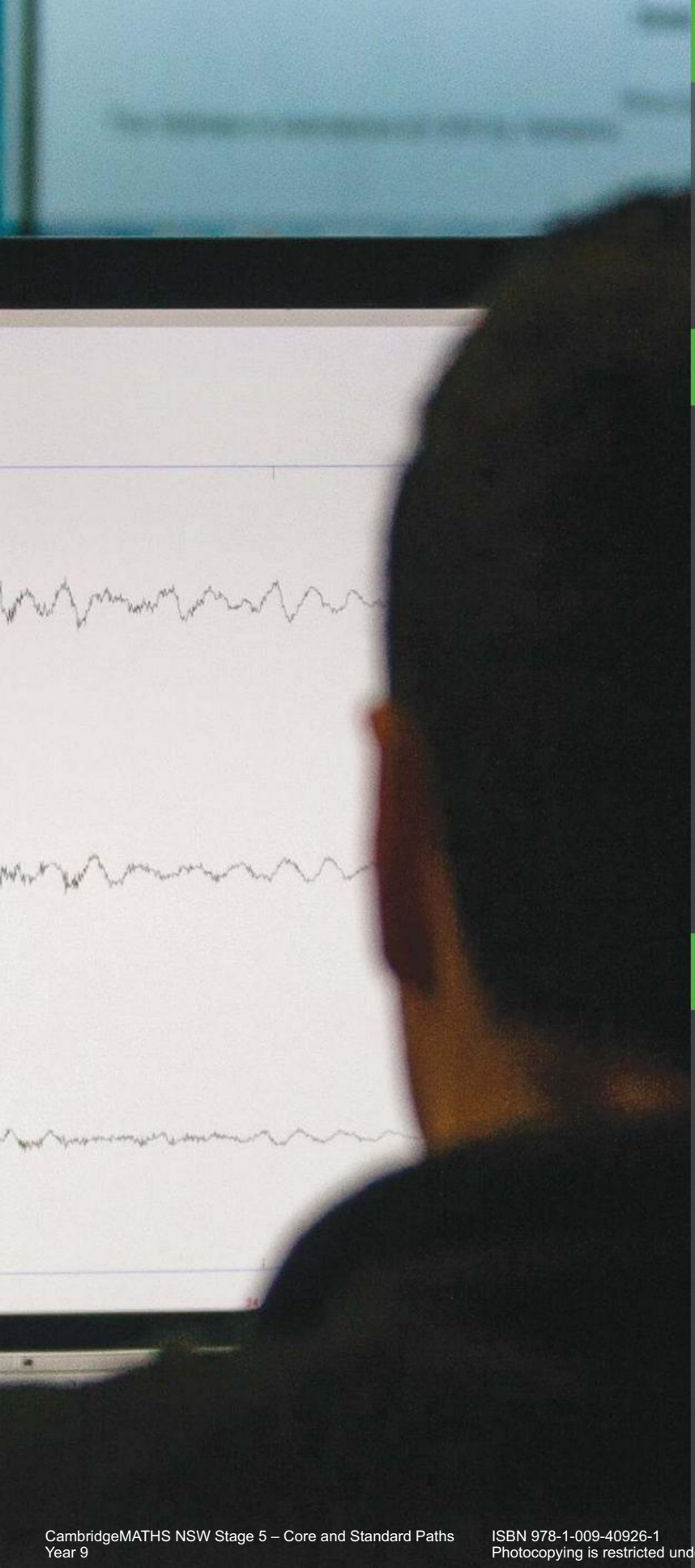

Chapter contents

7A Index notation
7B Index laws for multiplying and dividing
7C The zero index and power of a power
7D Index laws extended
7E Negative indices
7F Scientific notation
7G Scientific notation using significant figures

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- simplifies algebraic expressions involving positive-integer and zero indices, and establishes the meaning of negative indices for numerical bases (MA5-IND-C-01)
- solves measurement problems by using scientific notation to represent numbers and rounding to a given number of significant figures (MA5-MAG-C-01).

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Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 9
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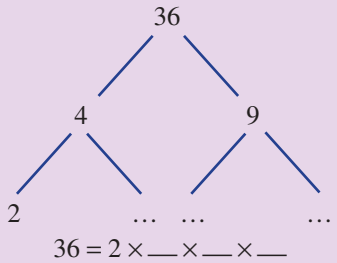
Warm-up quiz

1 Draw a factor tree for 36 and then write 36 as a product of prime factors.

2 **a** List the factors of 24.
b List the factors of 45.
c List the prime factors of 24.
d List the prime factors of 45.



3 Write each of the following in index form (as a power; e.g. 6^4).
a $7 \times 7 \times 7$
b $5 \times 5 \times 2 \times 2 \times 2$
c $(3 \times 3 \times 3 \times 3) \div (3 \times 3)$
d $(4 \times 4 \times 4) \div (4 \times 4)$

4 Write each of the following as 2 raised to a single power.
a $2^2 \times 2$ **b** $2^4 \times 2^2$ **c** $2^3 \times 2^2$

5 Evaluate:
a 5^2 **b** 10^2 **c** 1^4 **d** 5^1 **e** 4^0

6 Write each of the following in expanded form and evaluate.
a 3×2^2 **b** 4×3^2 **c** 5×2^3 **d** 3×10^4

7 Simplify the following by removing brackets.
a $(3^2)^2$ **b** $(2^4)^3$ **c** $(-3)^2$

8 Write each of these fractions as decimals.
a $\frac{3}{1000}$ **b** $\frac{4}{100}$ **c** $\frac{2}{1\,000\,000}$ **d** $\frac{8}{10\,000}$

9 Round the following to two decimal places.
a 3.732 **b** 24.6174 **c** 18.3654 **d** 4.3971

10 State the place value of the 4 in each of these numbers.
a 246 **b** 0.0043 **c** 4 320 000 **d** 32.48

11 State the number of significant figures in each of the following.
a 23.102 **b** 30.05 **c** 0.0012 **d** 6001

12 Complete the following.
a $3.8 \times 10 =$ _____
b $2.31 \times 1000 =$ _____
c $17.2 \div 100 =$ _____
d $0.18 \div 100 =$ _____
e $3827 \div$ _____ $= 3.827$
f $6.49 \times$ _____ $= 64\,900$

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7A Index notation

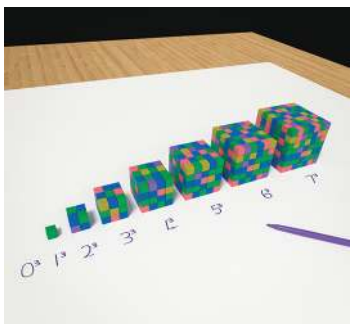
Learning intentions for this section:

- To understand index notation and know that it is a way of writing repeated multiplication
- To be able to convert between expanded form and index form
- To be able to evaluate expressions written in index form
- To be able to express a number as a product of prime factors

Past, present and future learning:

- This section consolidates and extends Stage 4 concepts and introduces the Stage 5 Core topic called *Indices A*
- Some of these questions are more challenging than those in our Stage 4 books
- *Indices* is a Stage 5 Core topic that has limited relevance to Stage 6 Standard

Consider the expression $5 \times 5 \times 5$.
It is the product of three fives, which is 125.
It can be written as 5^3, using index notation.
The number 5 is the base.
The number 3 is the index, sometimes called the power or exponent.
The number 125 is a power of 5. The powers of 5 are
5, 25, 125, 625, ….



Lesson starter: Who has the most?

A person offers you one of two prizes.

- Which offer would you take?
- Try to calculate the final amount for prize B.
- How might you use index notation to help calculate the value of prize B?
- How can a calculator help to find the amount for prize B using the power button $\boxed{\wedge}$?

KEY IDEAS

■ When a number is multiplied by itself many times, that product can be written using **index form**. For example,

Expanded form: $\overbrace{2 \times 2 \times 2 \times 2 \times 2}$ Index form: $= 2^5 = 32$

Base: 2, Index: 5, Basic numeral: 32

$x \times x \times x \times x = x^4$

Base: x, Index: 4

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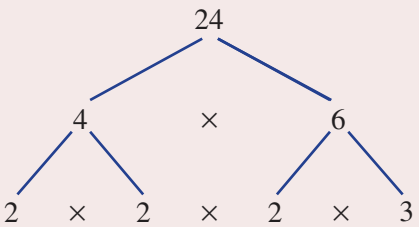
- The **base** is the factor in the product.
- The **index** is the number of times the factor (base number) appears.
  - 2^2 reads '2 to the power of 2' or '2 squared', where $2^2 = 4$.
  - 2^3 reads '2 to the power of 3' or '2 cubed', where $2^3 = 8$.
  - 2^5 reads '2 to the power of 5', where $2^5 = 32$.

  Note that $a^1 = a$. For example: $5^1 = 5$.
  - 3^2 does not mean $3 \times 2 = 6$.
- **Prime factorisation** involves writing a number as a product of its prime factors.
  - A prime number has only two factors: 1 and itself.



Prime factor form

$24 = 2 \times 2 \times 2 \times 3$

$= 2^3 \times 3$

Index form

Key terminology: index form, expanded form, power, exponent, base, index, prime, factor, substitution

Exercise 7A

UNDERSTANDING | 1, 2–5(½) | 1, 5

1 **a** The product $3 \times 3 \times 3 \times 3$ is called the _____ form of 81.

b 3^4 is called the _____ form of 81.

c 3^4 reads '3 to the _____ of 4'.

d In 3^4 the special name for the 3 is the _____ number.

e In 3^4 the special name for the 4 is the _____ number or _____.

f A _____ number has only two factors, itself and 1.

g Prime factorisation involves writing a number as a product of its _____ _____.

2 Evaluate:

a 5^2 **b** 2^3 **c** 3^3 **d** $(-4)^2$

3 Write the number or pronumeral that is the base in these expressions.

a 3^7 **b** 6^4 **c** $(1.2)^5$ **d** $(-7)^3$

e $\left(\frac{2}{3}\right)^4$ **f** y^{10} **g** w^6 **h** t^2

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4 Write the number that is the index in these expressions.

a 4^3 **b** 10^8 **c** $(-3)^7$ **d** $\left(\frac{1}{2}\right)^4$

e x^{11} **f** $(xy)^{13}$ **g** $\left(\frac{x}{2}\right)^9$ **h** $(1.3x)^2$

5 Write the prime factors of these numbers.

a 6 **b** 15 **c** 30 **d** 77

FLUENCY 6–11(½) 6–11(½)

Example 1 Writing in expanded form

Write in expanded form:

a 5^4 **b** a^3 **c** $(xy)^4$ **d** $2a^3b^2$

SOLUTION

a $5^4 = 5 \times 5 \times 5 \times 5$

b $a^3 = a \times a \times a$

c $(xy)^4 = xy \times xy \times xy \times xy$

d $2a^3b^2 = 2 \times a \times a \times a \times b \times b$

EXPLANATION

Factor 5 appears four times.

Factor a appears three times.

Factor xy appears four times.

Factor a appears three times and factor b appears twice. Factor 2 only appears once.

Now you try

Write in expanded form:

a 6^3 **b** b^4 **c** $(mn)^2$ **d** $3x^2y$

6 Write each of the following in expanded form.

a 4^3 **b** 7^4

c 3^5 **d** 5^3

e a^4 **f** b^3

g x^3 **h** $(xp)^6$

i $(5a)^4$ **j** $(3y)^3$

k $4x^2y^5$ **l** $(pq)^2$

m $-3s^3t^2$ **n** $6x^3y^5$

o $5(yz)^6$ **p** $4(ab)^3$

Hint:

Factor → a^5 ← Number of times the factor appears

$a^5 = a \times a \times a \times a \times a$

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Example 2 Expanding and evaluating

Write each of the following in expanded form and then evaluate.

a 5^3 **b** $(-2)^5$ **c** $\left(\frac{2}{5}\right)^3$

SOLUTION

a $5^3 = 5 \times 5 \times 5$
$= 125$

b $(-2)^5 = (-2) \times (-2) \times (-2) \times (-2) \times (-2)$
$= -32$

c $\left(\frac{2}{5}\right)^3 = \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$
$= \frac{8}{125}$

EXPLANATION

Write in expanded form with 5 appearing three times and evaluate.

Write in expanded form with -2 appearing five times and evaluate.

Write in expanded form.

Evaluate by multiplying numerators and denominators.

Now you try

Write each of the following in expanded form and then evaluate.

a 2^3 **b** $(-3)^3$ **c** $\left(\frac{3}{5}\right)^2$

7 Write each of the following in expanded form and then evaluate.

a 6^2 **b** 2^4 **c** 3^5

d 12^1 **e** $(-2)^3$ **f** $(-1)^7$

g $(-3)^4$ **h** $(-5)^2$ **i** $\left(\frac{2}{3}\right)^3$

j $\left(\frac{3}{4}\right)^2$ **k** $\left(\frac{1}{6}\right)^3$ **l** $\left(\frac{5}{2}\right)^2$

m $\left(\frac{2}{-3}\right)^3$ **n** $\left(\frac{-3}{4}\right)^4$ **o** $\left(\frac{-1}{4}\right)^2$ **p** $\left(\frac{5}{-2}\right)^5$

Hint:
$-2 \times (-2) \times (-2)$
$= +4 \times (-2)$
$= -8$

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Example 3 Writing in index form

Write each of the following in index form.

a $5 \times 5 \times 5 \times 5$ **b** $6 \times x \times x \times x \times x$ **c** $4 \times a \times 4 \times a \times 4 \times a$

SOLUTION

a $5 \times 5 \times 5 \times 5 = 5^4$

b $6 \times x \times x \times x \times x = 6x^4$

c $4 \times a \times 4 \times a \times 4 \times a$
$= 4 \times 4 \times 4 \times a \times a \times a$
$= 4^3a^3$

EXPLANATION

Factor 5 appears 4 times.

Factor x appears 4 times; 6 appears only once.

Group the factors of 4 together and the factors of a together.

Write in index form.

Now you try

Write each of the following in index form.

a $9 \times 9 \times 9$ **b** $2 \times a \times a$ **c** $11 \times b \times 11 \times b \times b$

8 Write each of the following in index form.

a $3 \times 3 \times 3$ **b** $8 \times 8 \times 8 \times 8 \times 8 \times 8$
c $y \times y$ **d** $3 \times x \times x \times x$
e $4 \times c \times c \times c \times c \times c$ **f** $5 \times 5 \times 5 \times d \times d$
g $x \times x \times y \times y \times y$ **h** $7 \times b \times 7 \times b \times 7$

Hint: The index is the number of times the factor appears.

Example 4 Writing in index form with fractions

Write each of the following in index form.

a $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}$ **b** $\frac{3}{7} \times \frac{3}{7} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}$

SOLUTION

a $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \left(\frac{3}{4}\right)^3$

b $\frac{3}{7} \times \frac{3}{7} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = \left(\frac{3}{7}\right)^2 \times \left(\frac{4}{5}\right)^3$

EXPLANATION

The fraction $\frac{3}{4}$ appears 3 times.

$\frac{3}{7}$ appears twice and $\frac{4}{5}$ three times.

Now you try

Write each of the following in index form.

a $\frac{5}{7} \times \frac{5}{7} \times \frac{5}{7} \times \frac{5}{7}$ **b** $\frac{2}{5} \times \frac{4}{7} \times \frac{2}{5} \times \frac{4}{7} \times \frac{4}{7}$

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9 Write each of the following in index form.

a $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$

b $\frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}$

c $\frac{4}{7} \times \frac{4}{7} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}$

d $\frac{7x}{9} \times \frac{7x}{9} \times \frac{y}{4} \times \frac{y}{4} \times \frac{y}{4}$

Example 5 Writing in index form with a combination of pronumerals

Write each of the following in index form.

a $8 \times a \times a \times 8 \times b \times b \times a \times b$

b $3a \times 2m \times 3a \times 2m$

c $4am(4am)(4am)$

SOLUTION

a $8 \times a \times a \times 8 \times b \times b \times a \times b$
$= 8 \times 8 \times a \times a \times a \times b \times b \times b$
$= 8^2a^3b^3$

EXPLANATION

Group the numerals and like pronumerals and write in index form.

$64a^3b^3$ or $64(ab)^3$ are alternative answers.

b $3a \times 2m \times 3a \times 2m$
$= 2 \times 2 \times 3 \times 3 \times a \times a \times m \times m$
$= 2^2 3^2 a^2 m^2$

Rearrange so like factors are grouped together and write in index form.

$36a^2m^2$ or $36(am)^2$ are alternative answers.

c $4am(4am)(4am)$
$= 4 \times 4 \times 4 \times a \times a \times a \times m \times m \times m$
$= 4^3a^3m^3$

Rearrange and write in index form.

$64a^3m^3$ or $64(am)^3$ are alternative answers.

Now you try

Write each of the following in index form.

a $7 \times x \times x \times 7 \times y \times 7 \times x \times y$

b $2m \times 7s \times 2m \times 7s \times 2m$

c $3ab(3ab)(3ab)(3ab)$

10 Write each of the following in index form.

a $3 \times x \times y \times x \times 3 \times x \times 3 \times y$

b $3x \times 2y \times 3x \times 2y$

c $4d \times 2e \times 4d \times 2e$

d $6by(6by)(6y)$

e $3pq(3pq)(3pq)(3pq)$

f $7mn \times 7mn \times mn \times 7$

Hint: First rearrange the factors with numbers first, then form groups of like bases.

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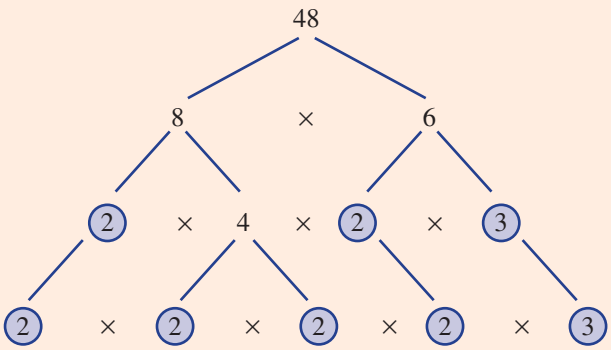
Example 6 Finding the prime factor form

Express 48 as a product of prime factors in index form. Prime numbers are divisible only by 1 and themselves.

SOLUTION

Factor tree diagram



$48 = 2 \times 2 \times 2 \times 2 \times 3$

$= 2^4 \times 3$

EXPLANATION

Choose a pair of factors of 48; for example, 8 and 6.

Choose a pair of factors of 8; i.e. 2 and 4.

Choose a pair of factors of 6; i.e. 2 and 3.

Continue this process until the factors are all prime numbers.

Write the prime factors of 48. Express in index notation.

Repeated division method

2	48
2	24
2	12
2	6
3	3
	1

$48 = 2 \times 2 \times 2 \times 2 \times 3$

$= 2^4 \times 3$

Start by dividing by 2, 3, 5, 7, etc. (prime numbers).

$48 \div 2 = 24$

$24 \div 2 = 12$

$12 \div 2 = 6$

$6 \div 2 = 3$

$3 \div 3 = 1$

Write prime factors in ascending order.

Express in index notation.

Now you try

Express 60 as a product of prime factors.

11 Express each of the following as a product of prime factors in index form.

a 10 **b** 8

c 144 **d** 75

e 147 **f** 500

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PROBLEM-SOLVING AND REASONING | 12, 13 | 12–14(½)

12 Copy and fill in the missing numbers or symbols.

a $3 \times 3 \times a \times a \times a = 3^{\square}a^{\square}$

b $\square \times \square \times k \times k \times k = 5^2k^{\square}$

c $\frac{2}{7} \times \frac{2}{7} \times \frac{2}{7} = \left(\frac{2}{7}\right)^{\square}$

d $3p^2q = 3 \times \square \times \square \times q$

e $(abc)^1 = a \square b \square c$

f $4ab^2 = \square \times (-2) \times \square \times \square \times b$

Example 7 Evaluating expressions in index form by using substitution

If $a = 5$, $b = 2$ and $c = -5$, evaluate these expressions.

a $(ab)^2$ **b** $\left(\frac{a}{c}\right)^3$ **c** $\left(\frac{b}{c}\right)^2$

SOLUTION

a $(ab)^2 = (5 \times 2)^2$
$= 10^2$
$= 10 \times 10$
$= 100$

EXPLANATION

Replace a with 5 and b with 2. Include the $\times$ sign. $5 \times 2 = 10$
Base of 10 appears twice.

b $\left(\frac{a}{c}\right)^3 = \left(\frac{5}{-5}\right)^3$
$= (-1)^3$
$= -1 \times (-1) \times (-1)$
$= -1$

Replace a with 5 and c with -5.
$5 \div (-5) = -1$
$-1 \times (-1) = +1, \quad +1 \times (-1) = -1$

c $\left(\frac{b}{c}\right)^2 = \left(\frac{2}{-5}\right)^2$
$= \frac{2}{-5} \times \frac{2}{-5}$
$= \frac{4}{25}$

Replace b with 2 and c with -5.

Write in expanded form.

$2 \times 2 = 4, \quad -5 \times (-5) = 25$

Now you try

If $a = 3$, $b = -2$ and $c = 7$, evaluate these expressions.

a $(ab)^2$ **b** $\left(\frac{b}{a}\right)^3$ **c** $ab^2 + ac$

13 If $a = 3$, $b = 2$ and $c = -3$, evaluate these expressions.

a $(ab)^2$ **b** $(bc)^3$ **c** $\left(\frac{a}{c}\right)^4$ **d** $\left(\frac{b}{c}\right)^3$

e $(abc)^1$ **f** $c^2 + ab$ **g** ab^2c **h** c^2ab^3

Hint:
$abc = a \times b \times c$

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14 Find the missing number.

a $3^{\square} = 81$ **b** $2^{\square} = 256$ **c** $\square^{3} = 125$

d $\square^{5} = 32$ **e** $\square^{3} = -64$ **f** $\square^{7} = -128$

g $\square^{3} = \frac{1}{8}$ **h** $\left(\frac{2}{3}\right)^{\square} = \frac{16}{81}$ **i** $\left(\frac{\square}{4}\right)^{3} = \frac{1}{64}$

Hint: For part **a**, ask 'How many times should 3 appear so that the product is 81?'

ENRICHMENT: Splitting cells — 15

15 Certain bacteria cells split in 2 every 1 minute. New cells also continue splitting in the same way. So, after each minute, the number of bacteria cells has doubled.

a Copy and complete this table showing the number of bacteria after each minute for 10 minutes.

Time in minutes	Number of bacteria	Number in index form
0	1	2^0
1	$1 \times 2 = 2$	2^1
2	$2 \times 2 = 4$	2^2
3	$2 \times 2 \times 2 = 8$	2^3

b How long will it take for 1 cell to divide into:

i 4 cells? **ii** 16 cells? **iii** 64 cells?

c A single cell is set aside to divide for 24 minutes. Use index form to quickly find how many cells there will be after this time.

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7B Index laws for multiplying and dividing

Learning intentions for this section:

- To know the index laws for multiplication and division involving a common base
- To be able to apply the index laws for multiplication and division to simplify expressions

Past, present and future learning:

- This section addresses Stage 4 concepts and the Stage 5 Core topic called *Indices A*
- Some of these questions are more challenging than those in our Stage 4 books
- *Indices* is a Stage 5 Core topic that has limited relevance to Stage 6 Standard

When multiplying or dividing numbers with the same base, index laws can be used to simplify the expression.

Consider $5^{18} \times 5^{10}$:

Using expanded form: $5^{18} \times 5^{10} = \overbrace{5 \times 5 \times 5 \times \ldots \times 5}^{\text{18 factors of 5}} \times \overbrace{5 \times 5 \times \ldots \times 5}^{\text{10 factors of 5}}$

$= \underbrace{5 \times 5 \times 5 \times \ldots \times 5 \times 5 \times 5 \times \ldots \times 5}_{\text{18 + 10 factors of 5}}$

$= 5^{18+10}$

$= 5^{28}$

So the total number of factors of 5 is $18 + 10 = 28$.

Also $5^{18} \div 5^{10} = \dfrac{\overbrace{5 \times 5 \times \ldots \times \cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5}}^{\text{18 factors of 5}}}{\underbrace{\cancel{5} \times \ldots \times \cancel{5} \times \cancel{5}}_{\text{10 factors of 5}}}$

$= 5^{18-10}$

$= 5^{8}$

So the total number of factors of 5 is $18 - 10 = 8$.

Lesson starter: Discovering the index laws

Consider the two expressions $2^3 \times 2^5$ and $6^8 \div 6^6$.

Complete this working. $2^3 \times 2^5 = 2 \times \square \times \square \times 2 \times \square \times \square \times \square \times \square$

$= 2^{\square}$

$6^8 \div 6^6 = \dfrac{6 \times \square \times \square \times \square \times \square \times \square \times \square \times \square}{6 \times \square \times \square \times \square \times \square \times \square}$

$= \dfrac{6 \times 6}{1}$

$= 6^{\square}$

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1 What do you notice about the given expression and the answer in each case? Can you express this as a rule or law in words?

2 Repeat the type of working given above and test your laws on these expressions.

a $3^2 \times 3^7$ b $4^{11} \div 4^8$

KEY IDEAS

■ **Index law** for multiplication: $a^m \times a^n = a^{m+n}$

- When multiplying terms with the same base, add the powers.
  For example, $7^3 \times 7^2 = 7^{3+2} = 7^5$

■ Index law for division: $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$

- When dividing terms with the same base, subtract the powers.
  For example, $8^5 \div 8^3 = \frac{8^5}{8^3} = 8^{5-3} = 8^2$

Key terminology: index law, base

Exercise 7B

UNDERSTANDING 1–4 4

1 Write the missing words.

a When you multiply two terms with the same _____ you _____ the powers.

b When you divide two terms with the same _____ you _____ the powers.

2 Copy and complete to give an answer in index form.

Hint: The index shows how many times the factor should appear.

a $3^2 \times 3^4 = 3 \times \square \times 3 \times \square \times \square \times \square$
$= 3^{\square}$

b $k^3 \times k^2 = k \times \square \times \square \times \square \times \square$
$= k^{\square}$

3 Copy and complete to give an answer in index form.

Hint: Show cancelling, e.g. $\frac{5 \times \cancel{5}^{1} \times \cancel{5}^{1}}{\cancel{5}^{1} \times \cancel{5}^{1}}$

a $5^5 \div 5^3 = \frac{5 \times \square \times \square \times \square \times \square}{5 \times \square \times \square}$
$= 5^{\square}$

b $a^6 \div a^2 = \frac{a \times \square \times \square \times \square \times \square \times \square}{\square \times \square}$
$= a^{\square}$

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4 Copy and complete.

a $6^5 \times 6^7 = 6^{\square+\square} = 6^{\square}$

b $a^{13} \times a^2 = a^{\square+\square} = a^{\square}$

c $5^{12} \div 5^4 = 5^{\square-\square} = 5^{\square}$

d $\dfrac{m^{16}}{m^2} = m^{\square-\square} = m^{\square}$

Hint: When dividing, subtract the indices.

FLUENCY 5–8(½) 5–8(½)

Example 8 Multiplying and dividing with a numerical base

Simplify, giving your answer in index form.

a $3^6 \times 3^4$ **b** $4^5 \times 4$ **c** $7^9 \div 7^5$ **d** $6^8 \div 6$

SOLUTION

a $3^6 \times 3^4 = 3^{6+4}$
$= 3^{10}$

b $4^5 \times 4 = 4^{5+1}$
$= 4^6$

c $7^9 \div 7^5 = 7^{9-5}$
$= 7^4$

d $6^8 \div 6 = 6^{8-1}$
$= 6^7$

EXPLANATION

$a^m \times a^n = a^{m+n}$
Add powers: $6 + 4 = 10$. The base 3 is unchanged.

$4 = 4^1$
Add powers: $5 + 1 = 6$. The base 4 is unchanged.

$a^m \div a^n = a^{m-n}$
Subtract powers: $9 - 5 = 4$. The base 7 is unchanged.

$6 = 6^1$
Subtract powers: $8 - 1 = 7$. The base 6 is unchanged.

Now you try

Simplify, giving your answer in index form.

a $2^4 \times 2^7$ **b** $7^5 \times 7$ **c** $11^{12} \div 11^4$ **d** $5^{10} \div 5$

5 Simplify, giving your answers in index form.

a $2^4 \times 2^3$ **b** $5^6 \times 5^3$
c $7^2 \times 7^4$ **d** $8^9 \times 8$
e $3^4 \times 3^4$ **f** $6^5 \times 6^9$
g $3^7 \div 3^4$ **h** $6^8 \div 6^3$
i $5^4 \div 5$ **j** $10^6 \div 10^5$
k $9^9 \div 9^6$ **l** $(-2)^5 \div (-2)^3$

Hint:
Remember: $8 = 8^1$

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Example 9 Multiplying non-numerical bases

Simplify each of the following using the multiplication index law.

a $x^4 \times x^5 \times x^2$ **b** $x^3y^4 \times x^2y$

SOLUTION

a $x^4 \times x^5 \times x^2 = x^{4+5+2}$
$= x^{11}$

b $x^3y^4 \times x^2y = x^3x^2y^4y$
$= x^{3+2}y^{4+1}$
$= x^5y^5$

EXPLANATION

Add the indices, since all terms have base x.

Regroup so like bases are together. Add the indices corresponding to each different base.

Recall that $y = y^1$.

Now you try

Simplify each of the following using the index law for multiplication.

a $a^3 \times a \times a^4$ **b** $m^2n^3 \times mn^4$

6 Simplify each of the following.

a $x^4 \times x^3$ **b** $a^6 \times a^3$ **c** $t^5 \times t^3$
d $y \times y^4$ **e** $d^2 \times d$ **f** $y^2 \times y \times y^4$
g $b \times b^5 \times b^2$ **h** $q^6 \times q^3 \times q^2$ **i** $a^2m^2 \times a^3m^2$
j $k^3p^2 \times k^2p$ **k** $x^2y^3 \times x^4y^5$ **l** $m^5e^3 \times m^2e$

Hint: Check that the bases are the same before adding the indices.

Example 10 Using the index law for division

Simplify $x^{10} \div x^2$ using the index law for division.

SOLUTION

$x^{10} \div x^2 = x^{10-2}$
$= x^8$

EXPLANATION

Subtract the indices: $10 - 2 = 8$.

The base x is unchanged.

Now you try

Simplify $y^{11} \div y^5$ using the law for division.

7 Simplify each of the following.

a $a^6 \div a^4$ **b** $x^5 \div x^2$ **c** $\frac{q^{12}}{q^2}$ **d** $\frac{d^7}{d^6}$

e $\frac{b^{10}}{b^5}$ **f** $\frac{d^9}{d^4}$ **g** $\frac{a^{14}}{a^7}$ **h** $\frac{y^{15}}{y^7}$

Hint:
Recall: $\frac{q^{12}}{q^2} = q^{12} \div q^2$

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Example 11 Simplifying using index laws

Simplify each of the following using the index laws for multiplication or division.

a $3m^4 \times 2m^5$ **b** $12y^7 \div (4y^3)$ **c** $\dfrac{8a^6b^3}{12a^2b^2}$

SOLUTION

a $3m^4 \times 2m^5 = 3 \times 2 \times m^4 \times m^5$
$= 6 \times m^{4+5}$
$= 6m^9$

EXPLANATION

Regroup with numbers first then like bases together. Multiply the numbers then add the indices of the base m.

b $12y^7 \div (4y^3) = \dfrac{12y^7}{4y^3}$
$= 3y^{7-3}$
$= 3y^4$

$12 \div 4 = 3$
Subtract indices.

c $\dfrac{8a^6b^3}{12a^2b^2} = \dfrac{8}{12} \times \dfrac{a^6}{a^2} \times \dfrac{b^3}{b^2}$
$= \dfrac{2}{3}a^4b$ or $\dfrac{2a^4b}{3}$

$\dfrac{8}{12} = \dfrac{2}{3}$ in simplest form.

Subtract the indices for each different base:
$6 - 2 = 4, 3 - 2 = 1$.

Recall: $b = b^1$.

Now you try

Simplify each of the following.

a $4a^2 \times 5a^3$ **b** $20b^8 \div (5b^3)$ **c** $\dfrac{6x^4y^2}{8x^2y}$

8 Simplify, using the index laws.

a $7x^3y^3 \times x^4y^2$ **b** $3x^7y^3 \times x^2y$ **c** $5x^3y^5 \times xy^4$

d $xy^4z \times 4xy$ **e** $3m^3 \times 5m^2$ **f** $4e^4f^2 \times 2e^2f^2$

g $5c^4d \times 4c^3d$ **h** $9yz^2 \times 2yz^5$ **i** $9m^3 \div (3m^2)$

j $14x^4 \div (2x)$ **k** $5y^4 \div y^2$ **l** $6a^6 \div (2a^5)$

m $\dfrac{36m^7}{12m^2}$ **n** $\dfrac{5w^2}{25w}$ **o** $\dfrac{4a^4}{20a^3}$ **p** $\dfrac{7x^5}{63x}$

q $\dfrac{16x^8y^6}{12x^2y^3}$ **r** $\dfrac{6s^6t^3}{14s^5t}$ **s** $\dfrac{8m^5n^4}{6m^4n^3}$ **t** $\dfrac{5x^2y}{xy}$

Hint: Rearrange first, and group numbers and like bases together.

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PROBLEM-SOLVING AND REASONING | 9–10(½) | 9–12(½)

Example 12 Combining index laws

Simplify each of the following using the index laws.

a $x^2 \times x^3 \div x^4$

b $\dfrac{2a^3b \times 8a^2b^3}{4a^4b^2}$

SOLUTION

a $x^2 \times x^3 \div x^4 = x^5 \div x^4$
$= x$

b $\dfrac{2a^3b \times 8a^2b^3}{4a^4b^2} = \dfrac{2 \times 8 \times a^3a^2 \times bb^3}{4 \times a^4 \times b^2}$
$= \dfrac{16}{4} \times \dfrac{a^5}{a^4} \times \dfrac{b^4}{b^2}$
$= 4 \times a \times b^2$
$= 4ab^2$

EXPLANATION

Add the indices for $x^2 \times x^3$.

Subtract the indices for $x^5 \div x^4$.

Rearrange the question with numbers first, then like bases grouped together.

$2 \times 8 = 16, a^3a^2 = a^{3+2} = a^5, bb^3 = b^{1+3} = b^4$

$\dfrac{16}{4} = 4, \dfrac{a^5}{a^4} = a^{5-4} = a^1 = a, \dfrac{b^4}{b^2} = b^{4-2} = b^2$

Now you try

Simplify each of the following using the index laws.

a $a^3 \times a^6 \div a^8$

b $\dfrac{3m^3n^2 \times 4mn^3}{2m^2n^3}$

9 Simplify each of the following using the index laws.

a $b^5 \times b^2 \div b$ **b** $y^5 \times y^4 \div y^3$ **c** $c^4 \div c \times c^4$

d $x^4 \times x^2 \div x^5$ **e** $\dfrac{t^4 \times t^3}{t^6}$ **f** $\dfrac{p^2 \times p^7}{p^3}$

g $\dfrac{d^5 \times d^3}{d^2}$ **h** $\dfrac{x^9 \times x^2}{x}$ **i** $\dfrac{3x^3y^4 \times 8xy}{6x^2y^2}$ **j** $\dfrac{9b^4}{2g^3} \times \dfrac{4g^4}{3b^2}$

Hint: Write pronumerals in alphabetical order.

10 Simplify each of the following.

a $\dfrac{m^4}{n^2} \times \dfrac{m}{n^3}$ **b** $\dfrac{x}{y} \times \dfrac{x^3}{y}$ **c** $\dfrac{a^4}{b^3} \times \dfrac{b^6}{a}$

d $\dfrac{12a}{3c^3} \times \dfrac{6a^4}{4c^4}$ **e** $\dfrac{3f^2 \times 8f^7}{4f^3}$ **f** $\dfrac{4x^2b \times 9x^3b^2}{3xb}$

11 Write the missing number.

a $2^7 \times 2^{\square} = 2^{19}$ **b** $6^{\square} \times 6^3 = 6^{11}$ **c** $11^6 \div 11^{\square} = 11^3$

d $19^{\square} \div 19^2 = 19$ **e** $x^6 \times x^{\square} = x^7$ **f** $a^{\square} \times a^2 = a^{20}$

g $b^{13} \div b^{\square} = b$ **h** $y^{\square} \div y^9 = y^2$ **i** $\square \times x^2 \times 3x^4 = 12x^6$

j $15y^4 \div (\square y^3) = y$ **k** $\square a^9 \div (4a) = \dfrac{a^8}{2}$ **l** $13b^6 \div (\square b^5) = \dfrac{b}{3}$

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12 Evaluate without using a calculator.

a $7^7 \div 7^5$ **b** $10^6 \div 10^5$ **c** $13^{11} \div 13^9$

d $2^{20} \div 2^{17}$ **e** $101^5 \div 101^4$ **f** $200^{30} \div 200^{28}$

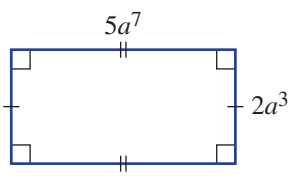
g $7 \times 31^{16} \div 31^{15}$ **h** $3 \times 50^{200} \div 50^{198}$

Hint: Simplify using index laws first.

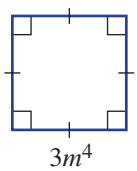
ENRICHMENT: Areas and index notation — 13

13 a Write the area of each of these shapes using index notation.

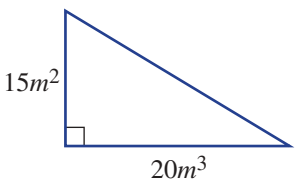
i



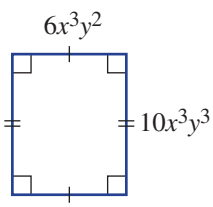
ii



iii

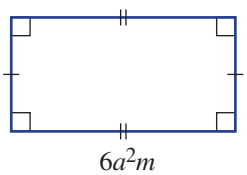


iv

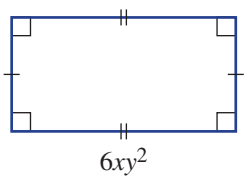


b Find the breadth of each of these shapes using index notation.

i Area $= 24a^3m^2$



ii Area $= 36x^2y^4$



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7C The zero index and power of a power

Learning intentions for this section:

- To know the index law for raising a power to another power
- To know the rule for the zero index
- To be able to simplify expressions using the index laws and zero index

Past, present and future learning:

- This section addresses Stage 4 concepts and the Stage 5 Core topic called *Indices A*
- Some of these questions are more challenging than those in our Stage 4 books
- *Indices* is a Stage 5 Core topic that has limited relevance to Stage 6 Standard

Sometimes we find that expressions already written in index form are raised to another power, such as $(2^3)^4$ or $(a^2)^5$.

Consider $(a^2)^5$.

$$\begin{aligned}(a^2)^5 &= \underbrace{a^2} \times \underbrace{a^2} \times \underbrace{a^2} \times \underbrace{a^2} \times \underbrace{a^2} \\ &= a \times a \times a \times a \times a \times a \times a \times a \times a \times a \\ &= a^{10}\end{aligned}$$

When a^2 appears 5 times there are a total of $5 \times 2 = 10$ factors of a. So when a^m appears n times, there will be a total of $m \times n = mn$ factors of a.

The power of 0 has a special property. Consider $\frac{a^3}{a^3}$.

Simplify using expanded form:

$$\frac{a^3}{a^3} = \frac{\cancel{a}^1 \times \cancel{a}^1 \times \cancel{a}^1}{\cancel{a}_1 \times \cancel{a}_1 \times \cancel{a}_1} = 1$$

Simplify using an index law:

$$\frac{a^3}{a^3} = a^{3-3} = a^0$$

So $a^0 = 1$

Lesson starter: Power of a power and the zero index

Use the expanded form of 5^3 to simplify $(5^3)^2$ as shown. $(5^3)^2 = 5 \times \square \times \square \times 5 \times \square \times \square$

$= 5^{\square}$

- Repeat these steps to also simplify $(3^2)^4$ and $(x^4)^2$.
- What do you notice about the given expression and answer in each case? Can you express this as a law or rule in words?

  Now copy and complete this table.

Index form	3^5	3^4	3^3	3^2	3^1	3^0
Basic numeral	243	81				

- What pattern do you notice in the basic numerals?
- What conclusion do you come to regarding 3^0?

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KEY IDEAS

- Index law for power of a power: $(a^m)^n = a^{m \times n} = a^{mn}$
  - When raising a term in index form to another power, retain the base and multiply the indices. For example: $(x^2)^3 = x^{2 \times 3} = x^6$.
  - A power outside brackets only applies to the expression inside those brackets. For example: $5(a^3)^2 = 5a^{3 \times 2} = 5a^6$.
- The **zero index**: $a^0 = 1$, where $a \neq 0$
  - Any term except 0 raised to the power of zero is 1. For example: $5^0 = 1$, $m^0 = 1$ and $(2a)^0 = 1$.

Key terminology: index law, zero index

Exercise 7C

UNDERSTANDING 1–4 | 4

1 Write the missing word or number in these sentences.

a When raising a term or numbers in index form to another power, _____ the indices.

b Any number (except 0) raised to the power 0 is equal to _____.

Hint: Choose from 0, 1, 2, *divide*, *add*, *subtract* or *multiply*.

2 Write the missing numbers in these tables.

a

Index form	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Basic numeral	64	32					

b

Index form	4^5	4^4	4^3	4^2	4^1	4^0
Basic numeral	1024	256				

3 Copy and complete this working.

a $(4^2)^3 = 4^2 \times 4^2 \times 4^2$

$= \left(4 \times \square\right) \times \left(4 \times \square\right) \times \left(4 \times \square\right)$

$= 4^{\square}$

b $(12^3)^2 = 12^{\square} \times 12^{\square}$

$= \left(12 \times \square \times \square\right) \times \left(12 \times \square \times \square\right)$

$= 12^{\square}$

c $(x^4)^2 = x^{\square} \times x^{\square}$

$= \left(x \times \square \times \square \times \square\right) \times \left(x \times \square \times \square \times \square\right)$

$= x^{\square}$

d $(a^2)^5 = a^{\square} \times a^{\square} \times a^{\square} \times a^{\square} \times a^{\square}$

$= \left(a \times \square\right) \times \left(a \times \square\right) \times \left(a \times \square\right) \times \left(a \times \square\right) \times \left(a \times \square\right)$

$= a^{\square}$

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4 Find the value of each of the following.

a 6^0 **b** 21^0
c 2^0 **d** 1^0
e $(3.7)^0$ **f** 582^0
g $\left(\frac{3}{4}\right)^0$ **h** 2760^0

Hint: $a^0 = 1$

FLUENCY 5–7(½) 5–9(½)

Example 13 Simplifying a power of a power

Simplify each of the following.

a $(x^5)^4$ **b** $3(y^5)^2$

SOLUTION

a $(x^5)^4 = x^{5\times 4}$
$= x^{20}$

b $3(y^5)^2 = 3y^{5\times 2}$
$= 3y^{10}$

EXPLANATION

Keep x as the base and multiply the indices.

Keep y and multiply the indices. The power of 2 is only applied to what is inside the brackets.

The 3 is unchanged.

Now you try

Simplify each of the following.

a $(a^3)^7$ **b** $2(b^2)^4$

5 Simplify each of the following. Leave your answers in index form.

a $(y^6)^2$ **b** $(m^3)^6$
c $(x^2)^5$ **d** $(b^3)^4$
e $(3^2)^3$ **f** $(4^3)^5$
g $(3^5)^6$ **h** $(7^5)^2$
i $5(m^8)^2$ **j** $4(q^7)^4$
k $-3(c^2)^5$ **l** $2(j^4)^6$

Hint: Keep the base. Multiply the indices.

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Example 14 Using the zero index

Apply the zero index rule to evaluate each of the following.

a $(-3)^0$ **b** $3(5x)^0$ **c** $2y^0 - (3y)^0$

SOLUTION

a $(-3)^0 = 1$

b $3(5x)^0 = 3 \times 1$
$= 3$

c $2y^0 - (3y)^0 = 2 \times 1 - 1$
$= 2 - 1$
$= 1$

EXPLANATION

Any number raised to the power of 0 is 1.

Everything in the brackets is to the power of 0 so $(5x)^0$ is 1. The 3 is not to the power of 0.

$2y^0$ has no brackets so the power applies to the y only, so $2y^0 = 2 \times y^0 = 2 \times 1$ while $(3y)^0 = 1$.

Now you try

Apply the zero index rule to evaluate each of the following.

a $\left(\frac{2}{3}\right)^0$ **b** $2(7y)^0$ **c** $(6y)^0 + 2y^0$

6 Evaluate each of the following.

a 5^0 **b** 9^0 **c** $(-6)^0$ **d** $(-3)^0$

e $(4)^0$ **f** $\left(\frac{3}{4}\right)^0$ **g** $\left(-\frac{1}{7}\right)^0$ **h** $(4y)^0$

i $5m^0$ **j** $-3p^0$ **k** $6x^0 - 2x^0$ **l** $-5n^0 - (8n)^0$.

m $(3x^4)^0$ **n** $1^0 + 2^0 + 3^0$ **o** $3a^0 + (3a)^0$ **p** $100^0 - a^0$

Hint: Any number (except zero) raised to the power 0 equals 1.

Example 15 Combining index laws

Simplify $(x^2)^3 \times (x^3)^5$ by applying the various index laws.

SOLUTION

$(x^2)^3 \times (x^3)^5 = x^{2\times3} \times x^{3\times5}$
$= x^6 \times x^{15}$
$= x^{21}$

EXPLANATION

Use power of a power to remove brackets first by multiplying indices. Then use the index law for multiplication to add indices.

Now you try

Simplify $(a^3)^3 \times (a^2)^4$ by applying the various index laws.

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7 Simplify each of the following by combining various index laws.

a $4 \times (4^3)^2$ **b** $(3^4)^2 \times 3$ **c** $x \times (x^0)^5$

d $y^5 \times (y^2)^4$ **e** $b^5 \times (b^3)^3$ **f** $(a^2)^3 \times a^4$

g $(d^3)^4 \times (d^2)^6$ **h** $(y^2)^6 \times (y)^4$ **i** $z^4 \times (z^3)^2 \times (z^5)^3$

Hint: First remove brackets by multiplying indices. Remember that $4 = 4^1$.

Example 16 Combining index laws

Simplify $\frac{(m^3)^4}{m^7}$ by applying index laws.

SOLUTION

$$\frac{(m^3)^4}{m^7} = \frac{m^{3 \times 4}}{m^7}$$
$$= \frac{m^{12}}{m^7}$$
$$= m^5$$

EXPLANATION

Remove brackets by multiplying indices then simplify using the index law for division.

$12 - 7 = 5$

Now you try

Simplify $\frac{a^8}{(a^2)^3}$ by applying index laws.

8 Simplify each of the following.

a $\frac{(b^2)^5}{b^4}$ **b** $\frac{(x^4)^3}{x^7}$ **c** $\frac{(y^3)^3}{y^3}$

d $7^8 \div (7^3)^2$ **e** $(4^2)^3 \div 4^5$ **f** $(3^6)^3 \div (3^5)^2$

g $(m^3)^6 \div (m^2)^9$ **h** $(y^5)^3 \div (y^6)^2$ **i** $(h^{11})^2 \div (h^5)^4$

Hint: First remove brackets by multiplying the powers.

Example 17 Combining index laws with the zero index

Simplify $\frac{4x^2 \times 3x^3}{6x^5}$ by applying index laws.

SOLUTION

$$\frac{4x^2 \times 3x^3}{6x^5} = \frac{4 \times 3 \times x^2 \times x^3}{6x^5}$$
$$= \frac{12x^5}{6x^5}$$
$$= 2x^0$$
$$= 2 \times 1$$
$$= 2$$

EXPLANATION

Regroup, then simplify the numerator first by multiplying numbers and adding indices of base x.
$12 \div 6 = 2$, and subtract indices: $5 - 5 = 0$

The zero index says $x^0 = 1$.

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Now you try

Simplify $\frac{5y^4 \times 3y^2}{10y^6}$ by applying index laws.

9 Simplify each of the following using various index laws.

a $\frac{3x^4 \times 6x^8}{9x^{12}}$ **b** $\frac{5x^5 \times 4x^6}{2x^{10}}$ **c** $\frac{24(x^4)^4}{8(x^4)^2}$

d $\frac{4(d^4)^3 \times (e^4)^2}{8(d^2)^5 \times e^7}$ **e** $\frac{6(m^3)^2(n^5)^3}{15(m^5)^0(n^2)^7}$ **f** $\frac{2(a^3)^4(b^2)^6}{16(a)^0(b^6)^2}$

Hint: Remove brackets, then rearrange with numbers first and then like bases grouped together.

PROBLEM-SOLVING AND REASONING

10 | 10, 11

10 If m and n are positive integers, in how many ways can $(a^m)^n = a^{16}$? Show each possibility.

11 Explain the error made in the following problems, then give the correct answer.

a $(a^4)^5 = a^9$ **b** $3(x^4)^2 = 9x^8$ **c** $(2x)^0 = 2$

ENRICHMENT: Rabbits!

— | 12

12 There are 100 rabbits on Mt Burrow at the start of the year 2015. The rule for the number of rabbits, N, after t years (from the start of the year 2015) is $N = 100 \times 2^t$.

a Find the number of rabbits at:

i $t = 2$

ii $t = 6$

iii $t = 0$

b Find the number of rabbits at the beginning of:

i 2018

ii 2022

iii 2025

c How many years will it take for the population to first rise to more than 500 000? Give a whole number of years.

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7D Index laws extended

Learning intentions for this section:

- To know the index laws for removing brackets
- To be able to expand and simplify expressions using the index laws
- To be able to combine index laws to simplify complex expressions

Past, present and future learning:

- This section addresses Stage 4 concepts and the Stage 5 Core topic called *Indices A*
- Some of these questions are more challenging than those in our Stage 4 books
- *Indices* is a Stage 5 Core topic that has limited relevance to Stage 6 Standard

It is common to find expressions such as $(2x)^3$ and $\left(\frac{x}{3}\right)^4$ in mathematical problems. These are different to most of the expressions in previous sections. They contain more than one single number or pronumeral, connected by multiplication or division, raised to a power. These expressions can also be simplified using two index laws that remove the brackets.

Consider $(a \times b)^6$.

Using expanded form: $(a \times b)^6 = \overbrace{ab \times ab \times ab \times ab \times ab \times ab}^{\text{6 factors of } ab}$

$$= \overbrace{a \times a \times a \times a \times a \times a}^{\text{6 factors of } a} \times \overbrace{b \times b \times b \times b \times b \times b}^{\text{6 factors of } b}$$

$$= a^6 \times b^6$$

So this becomes a product of 6 factors of a and 6 factors of b.

Also, $\left(\frac{a}{b}\right)^6 = \overbrace{\frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b}}^{\text{6 factors of } \frac{a}{b}}$

$$= \frac{\overbrace{a \times a \times a \times a \times a \times a}^{\text{6 factors of } a}}{\underbrace{b \times b \times b \times b \times b \times b}_{\text{6 factors of } b}}$$

$$= \frac{a^6}{b^6}$$

So to remove the brackets we can raise each of a and b to the power 6.

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Lesson starter: Can you see a short-cut?

Use the expanded form of $(2x)^3$ and $\left(\frac{x}{3}\right)^4$ to help simplify the expressions.

$(2x)^3 = 2x \times \square \times \square$

$= 2 \times 2 \times 2 \times \square \times \square \times \square$

$= 2^{\square} \times \square^{\square}$

$\left(\frac{x}{3}\right)^4 = \frac{x}{3} \times \square \times \square \times \square$

$= \frac{x \times \square \times \square \times \square}{3 \times \square \times \square \times \square}$

$= \frac{\square^{\square}}{\square^{\square}}$

- Repeat these steps to also simplify the expressions $(3y)^4$ and $\left(\frac{x}{2}\right)^5$.
- What do you notice about the given expressions and the answer in each case? Can you express this as a rule or law in words?

KEY IDEAS

- Power of a product: $(a \times b)^m = (ab)^m = a^m b^m$
  - If the product of two or more numbers is raised to the power of m, raise each number in the brackets to the power of m. For example: $(2x)^2 = 2^2 x^2 = 4x^2$.
- Power of a fraction: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ and $b \neq 0$
  - If the quotient of two numbers is raised to the power of m, raise each number in the brackets to the power of m. For example: $\left(\frac{y}{3}\right)^3 = \frac{y^3}{3^3} = \frac{y^3}{27}$.

Key terminology: index law

Exercise 7D

UNDERSTANDING 1–4 | 4

1 Copy and complete the following.

a $(a \times b)^m = a^m \times \square$

b $\left(\frac{a}{b}\right)^m = \frac{a^m}{\square}$.

2 Copy and complete this working.

a $(ab)^4 = ab \times \square \times \square \times \square$

$= a \times \square \times \square \times \square \times b \times \square \times \square \times \square$

$= a^4 \times \square$

b $\left(\frac{x}{6}\right)^3 = \frac{x}{6} \times \square \times \square$

$= \frac{x \times \square \times \square}{6 \times \square \times \square}$

$= \frac{x^3}{\square}$

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3 Copy and complete these bracket expansions.

a $(xy)^5 = x^{\square}y^5$ **b** $(2m)^3 = 2^{\square}m^{\square}$

c $(abc)^4 = a^4b^{\square}c^{\square}$ **d** $\left(\frac{5}{k}\right)^3 = \frac{5^3}{k^{\square}}$

e $\left(\frac{a}{m}\right)^5 = \frac{a^{\square}}{m^{\square}}$ **f** $\left(\frac{2}{3}\right)^4 = \frac{2^{\square}}{3^{\square}} = \frac{16}{\square}$

Hint: Raise each number or pronumeral in the brackets to the power.

4 Copy and complete these bracket expansions.

a $6(ab)^2 = 6a^{\square}b^2$ **b** $4(xy)^3 = \square x^{\square}y^{\square}$

c $7(2m)^2 = 7 \times 2^{\square}m^{\square}$
$= 7 \times \square \times m^2$
$= \square m^2$

d $4(3ab)^2 = \square \times 3^{\square}a^{\square}b^{\square}$
$= 4 \times \square a^{\square}b^{\square}$
$= \square a^{\square}b^{\square}$

Hint: The index outside the brackets only applies to the values inside the brackets.

FLUENCY 5–7(½) 5–7(½)

Example 18 Applying a power to a product

Expand each of the following.

a $(3a)^2$ **b** $(9a^3)^2$

SOLUTION

a $(3a)^2 = 3^2 \times a^2$
$= 9a^2$

b $(9a^3)^2 = 9^2(a^3)^2$
$= 81a^{3\times 2}$
$= 81a^6$

EXPLANATION

Apply the power of 2 to each factor in the brackets.

Raise 9 and (a^3) to the power of 2.
$9^2 = 9 \times 9 = 81, (a^3)^2 = a^{3\times 2} = a^6$

Now you try

Expand each of the following.

a $(4x)^3$ **b** $(2y^4)^4$

5 Expand each of the following. Evaluate numbers raised to a power.

a $(2x)^3$ **b** $(5y)^2$ **c** $(4a)^3$

d $(3r)^2$ **e** $(3b)^4$ **f** $(7r)^3$

g $(2h^2)^4$ **h** $(5c^2)^4$ **i** $(3a^2)^3$

j $(7p^4)^2$ **k** $(5m^3)^2$ **l** $(3y^{10})^2$

Hint: The index outside the brackets must be applied to each number or pronumeral inside the brackets.

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Example 19 Applying a power to a fraction

Expand each of the following and write with index notation.

a $\left(\frac{x}{2}\right)^3$ b $\left(\frac{m^3}{5}\right)^2$

SOLUTION

a $\left(\frac{x}{2}\right)^3 = \frac{x^3}{2^3}$

$= \frac{x^3}{8}$

EXPLANATION

Raise each factor in the brackets to the power of 3.

b $\left(\frac{m^3}{5}\right)^2 = \frac{(m^3)^2}{5^2}$

$= \frac{m^{3\times 2}}{5^2}$

$= \frac{m^6}{25}$

Raise m^3 and 5 to the power of 2.
$(m^3)^2 = m^{3\times 2} = m^6$

Now you try

Expand each of the following.

a $\left(\frac{a}{b}\right)^2$ b $\left(\frac{3a}{b^2}\right)^3$

6 Expand the following. Evaluate numbers raised to a power.

a $\left(\frac{p}{q}\right)^3$ b $\left(\frac{x}{y}\right)^4$ c $\left(\frac{4}{y}\right)^3$

d $\left(\frac{5}{p^2}\right)^4$ e $\left(\frac{2}{r^3}\right)^2$ f $\left(\frac{s^3}{7}\right)^2$

g $\left(\frac{7}{x^3}\right)^2$ h $\left(\frac{w^4}{10}\right)^2$ i $\left(\frac{a^2}{5}\right)^3$

j $\left(\frac{m^2}{2}\right)^4$ k $\left(\frac{2m}{n}\right)^5$ l $\left(\frac{2a^2}{3}\right)^3$

Hint: Recall:
$4^3 = 4 \times 4 \times 4 = 64$

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Example 20 Applying the power of a product

Expand each of the following using the power of a product.

a $(-2x^3y)^4$ **b** $4(c^2d^3)^5$

SOLUTION

a $(-2x^3y)^4 = (-2)^4(x^3)^4y^4$
$= 16x^{12}y^4$

b $4(c^2d^3)^5 = 4(c^2)^5(d^3)^5$
$= 4c^{10}d^{15}$

EXPLANATION

Raise each value in the brackets to the power of 4.

Evaluate $(-2)^4 = -2 \times (-2) \times (-2) \times (-2)$ and simplify $(x^3)^4$ using power of a power.

Raise each value in the brackets to the power of 5.

Note that the coefficient (4) is not raised to the power of 5 since it is not in the brackets. Simplify using index laws.

Now you try

Expand each of the following.

a $(-4a^2b)^2$ **b** $6(a^3b^2)^4$

7 Expand each of the following using the power of a product.

a $(2x^3y^2)^5$ **b** $9(p^2q^4)^3$ **c** $2(x^3y)^2$

d $(8t^2u^9v^4)^0$ **e** $(-3w^3y)^3$ **f** $-4(p^4qr)^2$

g $(-5s^7t)^2$ **h** $-(-2x^4yz^3)^3$ **i** $-3(-2ab^2)^3$

Hint: Powers only apply to numbers or pronumerals inside the brackets.

PROBLEM-SOLVING AND REASONING 8(½) 8–9(½)

Example 21 Applying the power to a fraction

Expand $\left(\frac{-2a^2}{3bc^3}\right)^4$.

SOLUTION

$\left(\frac{-2a^2}{3bc^3}\right)^4 = \frac{(-2)^4a^8}{3^4b^4c^{12}}$
$= \frac{16a^8}{81b^4c^{12}}$

EXPLANATION

Raise each value in the brackets to the power of 4.
Evaluate $(-2)^4$ and 3^4.

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Now you try

Expand $\left(\frac{-3x^2}{2yz^3}\right)^3$.

8 Expand the following. Evaluate numbers that are raised to a power.

a $\left(\frac{3n^3}{2m^4}\right)^3$ **b** $\left(\frac{-2r}{n}\right)^4$ **c** $\left(\frac{-3f}{2^3g^5}\right)^2$ **d** $\left(\frac{5w^4y}{2x^3}\right)^2$

e $\left(\frac{-3x}{2y^3g^5}\right)^2$ **f** $\left(\frac{3km^3}{4n^7}\right)^3$ **g** $-\left(\frac{-5w^4y}{2zx^3}\right)^2$ **h** $\left(-\frac{3x^2y^3}{2a^5b^3}\right)^2$

Example 22 Using a variety of index laws

Simplify each of the following by applying various index laws.

a $a(-2a^2b)^3$ **b** $\left(\frac{x^2y^3}{c}\right)^3 \times \left(\frac{xc}{y}\right)^4$

SOLUTION

a $a\left(-2a^2b\right)^3 = a(-2)^3(a^2)^3b^3$
$= a \times (-8a^6b^3)$
$= -8a^7b^3$

EXPLANATION

Apply the power 3 to the values inside the brackets.

$(-2)^3 = -2 \times (-2) \times (-2) = -8$

$(a^2)^3 = a^{2\times 3} = a^6$

Combine powers with a common base:
$a \times a^6 = a^{1+6} = a^7$

b $\left(\frac{x^2y^3}{c}\right)^3 \times \left(\frac{xc}{y}\right)^4 = \frac{x^6y^9}{c^3} \times \frac{x^4c^4}{y^4}$
$= \frac{x^{10}y^9c^4}{c^3y^4}$
$= cx^{10}y^5$

Raise each value in the brackets to the power. Multiply the numerators, then divide.

Write the answer in alphabetical order.

Now you try

Simplify each of the following by applying various index laws.

a $2(-3ab^2)^2$ **b** $\left(\frac{xy^2}{b}\right)^2 \times \left(\frac{b^2}{xy^4}\right)$

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9 Simplify each of the following by applying the various index laws.

a $a(3b)^2$ **b** $a(3b^2)^3$

c $-3(2a^3b^4)^2a^2$ **d** $2(3x^2y^3)^3$

e $(-4b^2c^5d)^3$ **f** $a(2a)^3$

g $a(3a^2)^2$ **h** $5a^3(-2a^4b)^3$

i $-5(-2m^3pt^2)^5$ **j** $\left(\dfrac{-3x^2y^0}{5a^5b^3}\right)^3$

k $\left(\dfrac{a^3b}{c}\right)^3 \times \left(\dfrac{ac^4}{b}\right)^2$ **l** $\left(\dfrac{x^2z}{y}\right)^4 \times \left(\dfrac{xy^2}{z}\right)^3$

Hint: Remove brackets. Combine numerator. Divide for parts **j** to **l**.

ENRICHMENT: Germinating seeds — 10

10 The number of seeds germinating in a glass house over a two-week period is given by the equation $N = \left(\dfrac{t}{2}\right)^3$, where N is the number of germinating seeds and t is the number of days.

a Find the number of germinating seeds after:

i 4 days **ii** 10 days

b Expand to rewrite the equation without brackets.

c Use your equation in part **b** to find the number of seeds germinating after:

i 6 days **ii** 4 days

d Find the number of days required to germinate:

i 64 seeds **ii** 1 seed

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7E Negative indices

Learning intentions for this section:

- To know how negative indices can be equivalently expressed using positive indices
- To be able to express negative indices in terms of positive indices
- Extending: To be able to use the index laws with negative indices

Past, present and future learning:

- This section addresses the Stage 5 Core topic called *Indices A*
- *Indices* is a Stage 5 Core topic that has limited relevance to Stage 6 Standard

We know that $2^3 = 8$ and $2^0 = 1$ but what about 2^{-1} or 2^{-6}? Numbers written in index form using negative indices also have meaning in mathematics.

Consider the expression $= \frac{a^2}{a^5}$

Method 1: Using the index law

$$\frac{a^2}{a^5} = a^{2-5}$$
$$= a^{-3}$$

Method 2: By cancelling

$$\frac{a^2}{a^5} = \frac{a^{\cancel{1}} \times a^{\cancel{1}}}{a \times a \times a \times \cancel{a}^{\cancel{1}} \times \cancel{a}^{\cancel{1}}}$$
$$= \frac{1}{a^3}$$

So we can see that $a^{-3} = \frac{1}{a^3}$

Lesson starter: Continuing the pattern

Explore the use of negative indices by completing this table.

Index form	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
Whole number or fraction	16	8					$\frac{1}{4} = \frac{1}{2^2}$	

÷ 2 ÷ 2 ÷ 2 ÷ 2 ÷ 2 ÷ 2 ÷ 2

- What do you notice about the numbers with negative indices in the top row and the fractions in the second row?
- Can you describe this connection in words?
- What might be another way of writing 2^{-7} or 5^{-4}?

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KEY IDEAS

- To express a negative index as a positive index use the following rules:

  $a^{-m} = \frac{1}{a^m}$ and $a^m = \frac{1}{a^{-m}}$

  a raised to the power $-m$ is equal to the reciprocal of a raised to the power m. ($a \neq 0$)

  For example: $5^{-2} = \frac{1}{5^2}$, $\frac{1}{3^{-4}} = 3^4$.

  - Recall: the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

- All the index laws also apply to expressions with negative indices.

Key terminology: negative, index/indices, positive

Exercise 7E

UNDERSTANDING 1–4 | 4

1 Write the following using positive indices. For example: $\frac{1}{8} = \frac{1}{2^3}$.

a $\frac{1}{4}$ **b** $\frac{1}{9}$ **c** $\frac{1}{125}$ **d** $\frac{1}{27}$

Hint: Write the denominator as a power of a prime number.

2 Copy and complete these tables.

a

Index form	3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}
Whole number or fraction	81	27					$\frac{1}{9} = \frac{1}{3^2}$	

÷ 3 ÷ 3 ÷ 3 ÷ 3 ÷ 3 ÷ 3 ÷ 3

b

Index form	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
Whole number or fraction	10 000							$\frac{1}{1000} = \frac{1}{10^3}$

÷ 10 ÷ 10 ÷ 10 ÷ 10 ÷ 10 ÷ 10 ÷ 10

3 Copy and complete each of the following.

a $10^{-4} = \frac{1}{10^{\square}}$ **b** $3^{-2} = \frac{1}{3^{\square}}$ **c** $7^{-3} = \frac{1}{7^{\square}}$

Hint:

$a^{-m} = \frac{1}{a^m}$

So $10^{-4} = \frac{1}{10^4}$

4 Copy and complete each of the following.

a $\frac{1}{3^{-4}} = 3^{\square}$ **b** $\frac{1}{7^{-6}} = 7^{\square}$ **c** $\frac{1}{8^{-3}} = 8^{\square}$

Hint:

$\frac{1}{a^{-m}} = a^m$

So $\frac{1}{3^{-4}} = 3^4$

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FLUENCY 5–7(½) 5–8(½)

Example 23 Using $a^{-m} = \frac{1}{a^m}$

Rewrite the following using positive indices.

a 3^{-2} **b** 5×4^{-3}

SOLUTION

a $3^{-2} = \frac{1}{3^2}$

b $5 \times 4^{-3} = \frac{5}{1} \times \frac{1}{4^3}$

$= \frac{5}{4^3}$

EXPLANATION

Use $a^{-m} = \frac{1}{a^m}$.

The 5 does not have a negative power so it remains unchanged.

$4^{-3} = \frac{1}{4^3}, 5 \times 1 = 5, 1 \times 4^3 = 4^3$

Now you try

Rewrite the following using positive indices.

a 6^{-4} **b** 7×2^{-4}

5 Write each of the following with positive indices.

a 5^{-2} **b** 7^{-4} **c** 8^{-3} **d** 3^{-5}

e 9^{-2} **f** 10^{-3} **g** 4^{-5} **h** 2^{-3}

Hint:
Remember: $a^{-m} = \frac{1}{a^m}$

So $5^{-2} = \frac{1}{5^2}$

6 Rewrite the following with positive indices only.

a 3×2^{-4} **b** 5×4^{-3} **c** 7×5^{-6}

d 2×3^{-4} **e** 4×3^{-5} **f** 9×5^{-2}

g 8×7^{-3} **h** 6×5^{-6} **i** 1×4^{-2}

Hint:

$5 \times 2^{-3} = \frac{5}{2^3}$

Don't change

Example 24 Using $\frac{1}{a^{-m}} = a^m$

Rewrite the following with positive indices.

a $\frac{1}{2^{-3}}$ **b** $\frac{7}{3^{-2}}$

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SOLUTION	EXPLANATION
a $\frac{1}{2^{-3}} = 2^3$	Use $\frac{1}{a^{-m}} = a^m$.
b $\frac{7}{3^{-2}} = 7 \times \frac{1}{3^{-2}}$	The 7 remains unchanged.
$= 7 \times 3^2$	$\frac{1}{3^{-2}} = 3^2$

Now you try

Rewrite the following with positive indices.

a $\frac{1}{10^{-3}}$ **b** $\frac{5}{9^{-4}}$

7 Write each of the following with positive powers.

a $\frac{1}{2^{-4}}$ **b** $\frac{1}{3^{-2}}$ **c** $\frac{1}{4^{-3}}$ **d** $\frac{1}{6^{-5}}$

e $\frac{1}{5^{-3}}$ **f** $\frac{1}{8^{-5}}$ **g** $\frac{1}{7^{-3}}$ **h** $\frac{1}{9^{-4}}$

Hint:
Remember: $\frac{1}{a^{-m}} = a^m$

So $\frac{1}{2^{-4}} = 2^4$

8 Rewrite the following with positive indices only.

a $\frac{6}{4^{-3}}$ **b** $\frac{5}{8^{-2}}$ **c** $\frac{4}{7^{-5}}$ **d** $\frac{3}{2^{-5}}$

e $\frac{12}{5^{-4}}$ **f** $\frac{6}{10^{-3}}$ **g** $\frac{7}{9^{-4}}$ **h** $\frac{7}{10^{-6}}$

Hint:
Don't change

$\frac{7}{3^{-4}} = 7 \times 3^4$

PROBLEM-SOLVING AND REASONING 9–10(½) 9–11(½), 12

Example 25 Changing to fractions

Rewrite 3×2^{-4} with a positive index and then as a fraction.

SOLUTION	EXPLANATION
$3 \times 2^{-4} = 3 \times \frac{1}{2^4}$	$2^{-4} = \frac{1}{2^4}$
$= \frac{3}{2^4}$	$2^4 = 2 \times 2 \times 2 \times 2 = 16$
$= \frac{3}{16}$	

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Now you try

Rewrite 7×5^{-2} with a positive index and then as a fraction.

9 Rewrite each of these with positive indices and then as a fraction.

a 6×5^{-2} **b** 2×3^{-2} **c** 4×5^{-3}
d 6×7^{-1} **e** 4×10^{-3} **f** 2×10^{-1}
g 5×2^{-4} **h** 4×5^{-1} **i** 1×3^{-2}

Hint:
Remember: $5^1 = 5$

So $3 \times 5^{-1} = 3 \times \frac{1}{5^1}$

$= \frac{3}{5}$

Example 26 Changing to fractions and decimals

Rewrite 5×10^{-3} with a positive index and then as a fraction and a decimal.

SOLUTION

$5 \times 10^{-3} = 5 \times \frac{1}{10^3}$

$= \frac{5}{10^3}$

$= \frac{5}{1000}$

$= 0.005$

EXPLANATION

$10^{-3} = \frac{1}{10^3}$

$10^3 = 10 \times 10 \times 10 = 1000$

5 thousandths $= 0.005$

Now you try

Rewrite 3×10^{-2} with a positive index and then as a fraction and a decimal.

10 Rewrite each of the following with positive indices and then as a fraction and a decimal.

a 2×10^{-3} **b** 5×10^{-2}
c 7×10^{-1} **d** 3×10^{-4}
e 5×10^{-4} **f** 8×10^{-5}
g 2×10^{-6} **h** 4×10^{-8}

Hint:

$\frac{1}{10} = 0.1, \frac{1}{100} = 0.01$

$\frac{1}{1000} = 0.001$

$\frac{1}{10\,000} = 0.0001$

$\frac{1}{100\,000} = 0.00001$

Example 27 Evaluating without a calculator

Express using positive indices only, then evaluate without using a calculator.

a 3^{-4} **b** $\frac{5}{3^{-2}}$ **c** $\left(\frac{2}{3}\right)^{-4}$

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SOLUTION	EXPLANATION
a $3^{-4} = \frac{1}{3^4}$	Express 3^{-4} as a positive power.
$= \frac{1}{81}$	$3^4 = 3 \times 3 \times 3 \times 3 = 81$
b $\frac{5}{3^{-2}} = 5 \times \frac{1}{3^{-2}}$	$\frac{1}{3^{-2}} = 3^2$
$= 5 \times 3^2$	$3^2 = 3 \times 3 = 9$
$= 5 \times 9$	
$= 45$	
c $\left(\frac{2}{3}\right)^{-4} = \frac{2^{-4}}{3^{-4}}$	Apply the power to each numeral in the brackets.
$= 2^{-4} \times \frac{1}{3^{-4}}$	$\frac{a}{b} = a \times \frac{1}{b}$
$= \frac{1}{2^4} \times 3^4$	$2^{-4} = \frac{1}{2^4}, \frac{1}{3^{-4}} = 3^4$
$= \frac{3^4}{2^4}$	$3^4 = 3 \times 3 \times 3 \times 3 = 81$
$= \frac{81}{16}$	$2^4 = 2 \times 2 \times 2 \times 2 = 16$
	Give answer as an improper fraction.

Now you try

Express using positive indices only, then evaluate without using a calculator.

a 2^{-5} **b** $\frac{2}{3^{-3}}$ **c** $\left(\frac{4}{5}\right)^{-2}$

Hint: First write expressions using positive indices.

11 Evaluate without the use of a calculator. Express in simplified form.

a 5^{-1} **b** 3^{-2} **c** 4×10^{-2} **d** -5×10^{-3}

e $8 \times (2^2)^{-2}$ **f** $6^4 \times 6^{-6}$ **g** $8^{-7} \times (8^2)^3$ **h** $\frac{1}{8^{-1}}$

i $\frac{2}{2^{-3}}$ **j** $\frac{2^3}{2^{-3}}$ **k** $\left(\frac{3}{8}\right)^{-2}$ **l** $\left(\frac{4}{3}\right)^{-3}$

12 Describe the error made in these problems, then give the correct answer.

a $2x^{-2} = \frac{1}{2x^2}$ **b** $\frac{5}{a^4} = \frac{a^{-4}}{5}$ **c** $\frac{2}{(3b)^{-2}} = \frac{2b^2}{9}$

ENRICHMENT: Kilograms and grams — 13

13 **a** The mass of a small insect is 2^{-9} kg. How many grams is this? Round to two decimal places.

b The mass of an asteroid is 3^{20} kg. How many tonnes is this?

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Progress quiz

7A **1** Write the following in expanded form and evaluate parts **d–f**.

a b^4 **b** $2x^2y^3$ **c** $(3x)^3$ **d** 4^3 **e** $(-5)^4$ **f** $\left(\frac{3}{5}\right)^3$

7A **2** Write each of the following in index form.

a $5 \times 5 \times y \times y \times y \times y$

b $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{5}{7} \times \frac{5}{7}$

c $4 \times a \times b \times 4 \times b \times b \times a \times b \times 4$

7A **3** Express the following as a product of their prime factors in index form.

a 45 **b** 120

7A **4** If $x = 3$, $y = 4$ and $z = -1$, evaluate these expressions.

a $(xz)^3$ **b** $\left(\frac{z}{y}\right)^2$ **c** $x(yz)^2$

7B **5** Simplify using the first two index laws; give your answer in index form.

a $4^3 \times 4^4$ **b** $9^6 \div 9^2$ **c** $x^5 \times x^3$

d $a^2b \times a^3b^4$ **e** $y^9 \div y^3$ **f** $18a^5 \div (6a^2)$

g $3mn^4 \times 4m^2n^3$ **h** $\frac{2x^5y^3}{4x^2y}$ **i** $\frac{-6a^3b^2}{3a^2b}$

7C **6** Simplify the following.

a $(x^4)^2$ **b** $5(y^3)^3$ **c** 4^0

d $(5m)^0$ **e** $(3x)^0 - 2x^0$ **f** $5(x^0)^3$

7B/7C **7** Combine index laws to simplify the following.

a $\frac{3m^2 \times 4m^3n^5}{m^4n^2}$ **b** $\frac{2(x^2)^3}{12x^4}$ **c** $\frac{(3a)^0 \times 4a^3b^2}{6ab^2}$

7D **8** Expand each of the following using the index laws.

a $(2x)^2$ **b** $(2m^3)^4$ **c** $\left(\frac{x^2}{3}\right)^4$

d $4(-2ab^2)^3$ **e** $\left(\frac{3a^2}{5}\right)^3$ **f** $\left(\frac{-2a^2c}{7b^3}\right)^2$

7B/C/D **9** Combine index laws to simplify the following.

a $\frac{(4y^3)^2}{xy^2 \times 8(xy)^2}$ **b** $\left(\frac{3x^2}{4}\right)^3 \times \frac{2y^0}{x^4}$

7E **10** Rewrite the following with positive powers.

a 5^{-2} **b** 3×4^{-4} **c** $\frac{1}{3^{-2}}$ **d** $\frac{5}{2^{-3}}$

7E **11** Evaluate the following without a calculator. Express part **d** as a decimal.

a 9^{-2} **b** 5×2^{-2} **c** $\frac{6}{4^{-2}}$ **d** 6×10^{-4} **e** $\left(\frac{4}{5}\right)^{-3}$

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7F Scientific notation

Learning intentions for this section:

- To know that scientific notation is a way of representing very large and very small numbers
- To know the form of numbers written in scientific notation
- To be able to express numbers using scientific notation and as a basic numeral

Past, present and future learning:

- This section relates to the Stage 5 Core topic called *Numbers of any magnitude*
- This may be the first time that students have worked with scientific notation
- In Stage 6 Standard this concept will be revised, briefly

It is common in practical situations to work with very large or very small numbers. For example, the number of cubic metres of concrete used to build the Hoover Dam in the United States was 3 400 000 m^3. The mass of a molecule of water is 0.00000000000000000000000299 grams. Numbers like this can be written more efficiently using powers of 10 with positive or negative indices. This is called scientific notation. The number is written using a number between 1 and 10 that is multiplied by a power of 10. This notation is also used to show very large and very small time intervals.

Lesson starter: Equal times

Work in pairs and help each other to complete this 'matching puzzle'.

For each time in the first column, find the equal time in the second column and in the third column.

Column 1	Column 2	Column 3
a 6 thousand seconds	6 000 000 s	6×10^2 s
b 6 millionths of a second	0.006 s	6×10^3 s
c 6 hundred seconds	0.06 s	6×10^6 s
d 6 million seconds	6000 s	6×10^{-2} s
e 6 thousandths of a second	0.000006 s	6×10^{-3} s
f 6 hundredths of a second	600 s	6×10^{-6} s

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KEY IDEAS

- Numbers written in **scientific notation** are expressed in the form $a \times 10^m$, where a is a number from 1 to less than 10 (or from greater than -10 to -1) and m is an **integer**.
  - To write numbers using scientific notation, place the decimal point after the first non-zero digit then multiply by a power of 10.
- Large numbers are written in scientific notation using positive powers of 10.
  For example: 4 million years $= 4\,000\,000$ years
  $= 4 \times 1\,000\,000$ years
  $= 4 \times 10^6$ years
- Small numbers are written in scientific notation using negative powers of 10.
  For example: 5 thousandths of a metre $= 0.005$ metres
  $= \frac{5}{1000}$ metres
  $= \frac{5}{10^3}$ metres
  $= 5 \times 10^{-3}$ metres

Key terminology: scientific notation, integer

Exercise 7F

UNDERSTANDING 1–4 3, 4

1 Copy and complete this table. The first one has been done for you.

Scientific notation	Power of 10 expanded	Basic numeral
5×10^3	5×1000	5 000
3×10^4		
2×10^5		
7×10^2		
		70 000
		400 000

Hint: The power of 10 is equal to the number of zeros.

2 Copy and complete this table. The first one has been done for you.

Scientific notation	Positive power	Fraction	Basic numeral
2×10^{-4}	$\frac{2}{10^4}$	$\frac{2}{10\,000}$	0.0002
3×10^{-2}			
5×10^{-3}			
7×10^{-6}			
			0.009
			0.08

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3 Which of the numbers 1000, 10 000 or 100 000 completes each equation?

a $6.2 \times$ _____ $= 62\,000$
b $9.41 \times$ _____ $= 9410$
c $1.03 \times$ _____ $= 103\,000$
d $3.2 \div$ _____ $= 0.0032$
e $5.16 \div$ _____ $= 0.0000516$
f $1.09 \div$ _____ $= 0.000109$

Hint: The number of zeros tells you how many places to move the decimal point.

4 If these numbers were written using scientific notation, would positive or negative indices be used?

a 2000 **b** 0.0004 **c** 19 300 **d** 0.00101431

FLUENCY 5–9(½) 5–9(½)

Example 28 Writing large numbers using scientific notation

Write 4 500 000 using scientific notation.

SOLUTION

$4\,500\,000 = 4.5 \times 10^6$

EXPLANATION

Place the decimal point after the first non-zero digit (4).
Multiply by 10^6 since decimal point has been moved six places to the left.

Now you try

Write 71 000 using scientific notation.

5 Write the following using scientific notation.

a 40 000 **b** 2 300 000 000 000 **c** 16 000 000 000
d 7 200 000 **e** 3500 **f** 8 800 000
g 52 hundreds **h** 3 million **i** 21 thousands

Hint: Large numbers: use 10 to a positive power.

Example 29 Writing small numbers using scientific notation

Write 0.0000004 using scientific notation.

SOLUTION

$0.0000004 = 4 \times 10^{-7}$

EXPLANATION

The first non-zero digit is 4. Multiply by 10^{-7} since decimal point has been moved seven places to the right.

Now you try

Write 0.0000275 using scientific notation.

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6 Write the following using scientific notation.

a 0.000003	**b** 0.0004	**c** 0.00876
d 0.00000000073	**e** 0.00003	**f** 0.000000000125
g 0.00000000809	**h** 0.000000024	**i** 0.0000345

Hint: Small numbers: use 10 to a negative power.

7 Write each of the following numbers using scientific notation.

a 6000	**b** 720 000	**c** 324.5	**d** 7869.03
e 8459.12	**f** 0.2	**g** 0.000328	**h** 0.00987
i 0.00001	**j** 460 100 000	**k** 17 467	**l** 128

Hint: Place the decimal point after the first non-zero digit.

Example 30 Writing basic numerals using positive powers

Express 9.34×10^6 as a basic numeral.

SOLUTION

$9.34 \times 10^6 = 9\,340\,000$

EXPLANATION

Move the decimal point six places to the right, so that the 9 increases in place value by six places from the units position to the millions position.

Now you try

Express 2.4×10^4 as a basic numeral.

8 Express each of the following as a basic numeral.

a 5.7×10^4	**b** 3.6×10^6	**c** 4.3×10^8
d 3.21×10^7	**e** 4.23×10^5	**f** 9.04×10^{10}
g 1.97×10^8	**h** 7.09×10^2	**i** 6.357×10^5

Hint: Move the decimal point right to increase the place value of each digit.

Example 31 Writing basic numerals using negative powers

Express 4.71×10^{-6} as a basic numeral.

SOLUTION

$4.71 \times 10^{-6} = 0.00000471$

EXPLANATION

Move the decimal point six places to the left and insert zeros where necessary. The 4 decreases in place value by six places from the units position to the millionths position.

Now you try

Express 9.4×10^{-3} as a basic numeral.

9 Express each of the following as a basic numeral.

a 1.2×10^{-4}	**b** 4.6×10^{-6}	**c** 8×10^{-10}
d 3.52×10^{-5}	**e** 3.678×10^{-1}	**f** 1.23×10^{-7}
g 9×10^{-5}	**h** 5×10^{-2}	**i** 4×10^{-1}

Hint: Move the decimal point left to decrease the place value of each digit.

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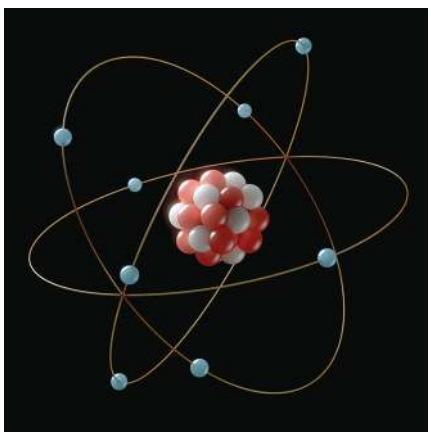
PROBLEM-SOLVING AND REASONING

10–11(½) | 10–11(½), 12

10 Express each of the following approximate numbers using scientific notation.

- **a** The mass of Earth is 6 000 000 000 000 000 000 000 000 kg.
- **b** The diameter of Earth is 40 000 000 m.
- **c** The diameter of a gold atom is 0.0000000001 m.
- **d** The radius of Earth's orbit around the Sun is 150 000 000 km.
- **e** The universal constant of gravitation is $0.0000000000667\ \text{Nm}^2/\text{kg}^2$.
- **f** The half-life of polonium-214 is 0.00015 seconds.
- **g** Uranium-238 has a half-life of 4 500 000 000 years.

11 Express each of the following numbers as a basic numeral.



- **a** Neptune is approximately 4.6×10^9 km from Earth.
- **b** A population of bacteria contained 8×10^{12} organisms.
- **c** Earth is approximately 3.84×10^5 km from the Moon.
- **d** A 50c coin is approximately 3.8×10^{-3} m thick.
- **e** The diameter of the nucleus of an atom is approximately 1×10^{-14} m.
- **f** The population of a city is 7.2×10^5.

12 Write the answers to each of these problems using scientific notation.

- **a** Two planets are 2.8×10^8 km and 1.9×10^9 km from their sun. What is the difference between these two distances?
- **b** Two particles weigh 2.43×10^{-2} g and 3.04×10^{-3} g. Find the difference in their weights.

ENRICHMENT: Scientific notation with numbers larger than 10

— | 13

13 The number 47×10^4 is not written using scientific notation, since 47 is not a number between 1 and 10. The following shows how to convert to scientific notation.

$$47 \times 10^4 = 4.7 \times 10 \times 10^4$$
$$= 4.7 \times 10^5$$

Write these numbers using scientific notation.

- **a** 32×10^3
- **b** 41×10^5
- **c** 0.13×10^5
- **d** 0.092×10^3
- **e** 61×10^{-3}
- **f** 424×10^{-2}
- **g** 0.02×10^{-3}
- **h** 0.0004×10^{-2}

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7G Scientific notation using significant figures

Learning intentions for this section:

- To know how significant figures are counted
- To be able to round off a number to a given number of significant figures
- To be able to write a number in scientific notation to a given number of significant figures

Past, present and future learning:

- This section relates to the Stage 5 Core topic called *Numbers of any magnitude*
- The concept of significant figures was introduced in Section 1C of this book
- From here onwards, you may be required to round off numbers in two different ways
- This concept will be revised briefly in Stage 6 Standard and may also be required in non-calculator examinations such as NAPLAN and industry aptitude tests

The number of digits used to record measurements depends on how accurately the measurements can be recorded. The volume of Earth, for example, has been calculated as 1 083 210 000 000 km^3 using six significant figures and could be written using scientific notation as 1.08321×10^{12}. A more accurate calculation may include more non-zero digits in the last seven places.

The mass of a single oxygen molecule is known to be 0.000000000000000000000000053 g. This shows two significant figures and is written using scientific notation as 5.3×10^{-26}.

On many calculators you will notice that very large or very small numbers are automatically converted to scientific notation using a certain number of significant figures. Numbers can also be entered into a calculator using scientific notation.

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Lesson starter: Significant discussions

Work in pairs and sort these numbers into groups according to how many significant figures are in each number.

23	502	0.0018
7208	46	0.0001073
0.000907	1609	91
6182	6	0.0000741

KEY IDEAS

- **Significant figures** are the important digits of a number which indicate how accurate it is.
- Significant figures are counted from left to right starting at the first non-zero digit. For example:
  - 38 041 shows five significant figures
  - 6.034 shows four significant figures
  - 0.0016 shows two significant figures
  - 0.00160 shows three significant figures.
- Zeros at the end of a number are counted for decimals (e.g. 0.00160) but not necessarily for whole numbers (e.g. 38 041 000).
- When using **scientific notation**, the first significant figure sits to the left of the decimal point. For example:
  3.217×10^4 shows four significant figures.
- Calculators can be used to work with scientific notation.
  - EXP or $\times 10^x$ are common key names on calculators.
  - Pressing 2.37 $\times 10^x$ 5 gives 2.37×10^5.
  - 2.37E5 means 2.37×10^5.

Key terminology: scientific notation, significant figures

Exercise 7G

UNDERSTANDING 1–4 3, 4

1 **a** Round each of these numbers to the nearest hundred.
i 267 **ii** 32 740 **iii** 18 350

b Round each of these numbers to the nearest tenth.
i 0.063 **ii** 0.1902 **iii** 21.04

c Round each of these numbers to the nearest thousand.
i 267 540 **ii** 38 290 **iii** 4 060 990

Hint:
Rounding rules:
Locate the first digit to the right of the required digit.
- Round down (leave it as it is) for a 4 or less
- Round up (increase by 1) for a 5 or more

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2 Which of these numbers have two significant figures?
62 100, 30 500, 42, 0.0071, 0.0805, 201 000

3 Complete the tables, rounding each number to the given number of significant figures.

a 57 263

Significant figures	Rounded number
4	
3	57 300
2	
1	

b 0.0036612

Significant figures	Rounded number
4	
3	
2	
1	0.004

Hint: In 57 263, 6 is the fourth significant figure so round to the nearest 10 for four significant figures.

4 Are the following numbers written using scientific notation with three significant figures? (Answer yes or no.)

a 4.21×10^4 **b** 32×10^{-3}
c 1800×10^6 **d** 0.04×10^2
e 1.89×10^{-10} **f** 9.04×10^{-6}

Hint: The number to the left of the decimal point must be from 1 to less than 10.

FLUENCY 5–8(½) 5–9(½)

Example 32 Stating the number of significant figures

State the number of significant figures given in these numbers.

a 401 **b** 0.005012 **c** 3.2×10^7

SOLUTION	EXPLANATION
a Three significant figures	All three digits are significant.
b Four significant figures	Start counting at the first non-zero digit (5).
c Two significant figures	With scientific notation the first significant figure is to the left of the decimal point.

Now you try

State the number of significant figures given in these numbers.

a 7105 **b** 0.00016 **c** 4.21×10^{-2}

5 State the number of significant figures given in these numbers.

a 272 **b** 1007
c 30 101 **d** 19
e 0.0183 **f** 0.20
g 0.706 **h** 0.00109
i 4.21×10^3 **j** 2.905×10^{-2}
k 1.07×10^{-6} **l** 5.90×10^5

Hint: Start counting from the first non-zero digit.

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Example 33 Writing numbers using scientific notation and significant figures

Write these numbers using scientific notation and three significant figures.

a 2 183 000 **b** 0.0019482

SOLUTION

a $2\,183\,000 = 2.18 \times 10^6$

EXPLANATION

Put the decimal point after the first non-zero digit. The decimal point has moved six places so multiply by 10^6. Round the third significant figure down (i.e. leave it as is) since the following digit (3) is less than 5.

b $0.0019482 = 1.95 \times 10^{-3}$

Move the decimal point three places to the right and multiply by 10^{-3}. Round the third significant figure up to 5 since the following digit (8) is greater than 4.

Now you try

Write these numbers using scientific notation and three significant figures.

a 472 000 **b** 0.00032

6 Write these numbers using scientific notation and three significant figures.

a 242 300 **b** 171 325 **c** 2829
d 3 247 000 **e** 0.00034276 **f** 0.006859
g 0.01463 **h** 0.001031 **i** 23.41
j 326.042 **k** 19.618 **l** 0.172046

7 Write each number using scientific notation, rounding to the number of significant figures given in the brackets.

a 47 760 (3) **b** 21 610 (2)
c 4 833 160 (4) **d** 37.16 (2)
e 99.502 (3) **f** 0.014427 (4)
g 0.00201 (1) **h** 0.08516 (1)
i 0.0001010 (1)

Hint: First round the number to the required number of significant figures.

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Example 34 Using a calculator with scientific notation

Use a calculator to evaluate $3.67 \times 10^5 \times 23.6 \times 10^4$. Answer using scientific notation correct to four significant figures.

SOLUTION

$3.67 \times 10^5 \times 23.6 \times 10^4$
$= 8.661 \times 10^{10}$

EXPLANATION

Use a calculator with an appropriate key sequence.

Write using scientific notation with four significant figures.

Look for a $\boxed{\times 10^x}$ or $\boxed{\text{EXP}}$ button or similar.

Now you try

Use a calculator to evaluate $62.14 \times 10^2 \times 1.6 \times 10^4$. Answer using scientific notation correct to four significant figures.

8 Use a calculator to evaluate each of the following. Write your answers using scientific notation correct to four significant figures.

a 4^{-6}

b 78^{-3}

c $(-7.3 \times 10^{-4})^{-5}$

d $\dfrac{3.185}{7 \times 10^4}$

e $2.13 \times 10^4 \times 9 \times 10^7$

f $5.671 \times 10^2 \times 3.518 \times 10^5$

g $9.419 \times 10^5 \times 4.08 \times 10^{-4}$

h $2.85 \times 10^{-9} \times 6.33 \times 10^{-3}$

i $12\,345^2$

j 87.14^8

k $\dfrac{1.83 \times 10^{26}}{4.5 \times 10^{22}}$

l $\dfrac{-4.7 \times 10^{-2} \times 6.18 \times 10^7}{3.2 \times 10^6}$

Example 35 Using a calculator with scientific notation and negative indices

Use a calculator to evaluate $7.6 \times 10^{-3} + \sqrt{2.4 \times 10^{-2}}$. Write your answer using scientific notation correct to four significant figures.

SOLUTION

$7.6 \times 10^{-3} + \sqrt{2.4 \times 10^{-2}}$
$= 0.1625$ (to 4 sig. figs)
$= 1.625 \times 10^{-1}$

EXPLANATION

Use a calculator with an appropriate key sequence.

Write using scientific notation with a number between 1 and 10.

Now you try

Use a calculator to evaluate $\dfrac{\sqrt{5.1 \times 10^{-7}}}{4.32 \times 10^{-2}}$. Write your answer using scientific notation correct to four significant figures.

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9 Use a calculator to evaluate each of the following. Write answers using scientific notation correct to five significant figures.

Hint: For fraction calculations, insert brackets so that the numerator and denominator are calculated before division.

a $\sqrt{8756}$

b $\sqrt{634 \times 7.56 \times 10^{7}}$

c $8.6 \times 10^{5} + \sqrt{2.8 \times 10^{-2}}$

d $-8.9 \times 10^{-4} + \sqrt{7.6 \times 10^{-3}}$

e $\dfrac{5.12 \times 10^{21} - 5.23 \times 10^{20}}{2 \times 10^{6}}$

f $\dfrac{8.942 \times 10^{47} - 6.713 \times 10^{44}}{2.5 \times 10^{19}}$

g $\dfrac{2 \times 10^{7} + 3 \times 10^{8}}{5}$

h $\dfrac{4 \times 10^{8} + 7 \times 10^{9}}{6}$

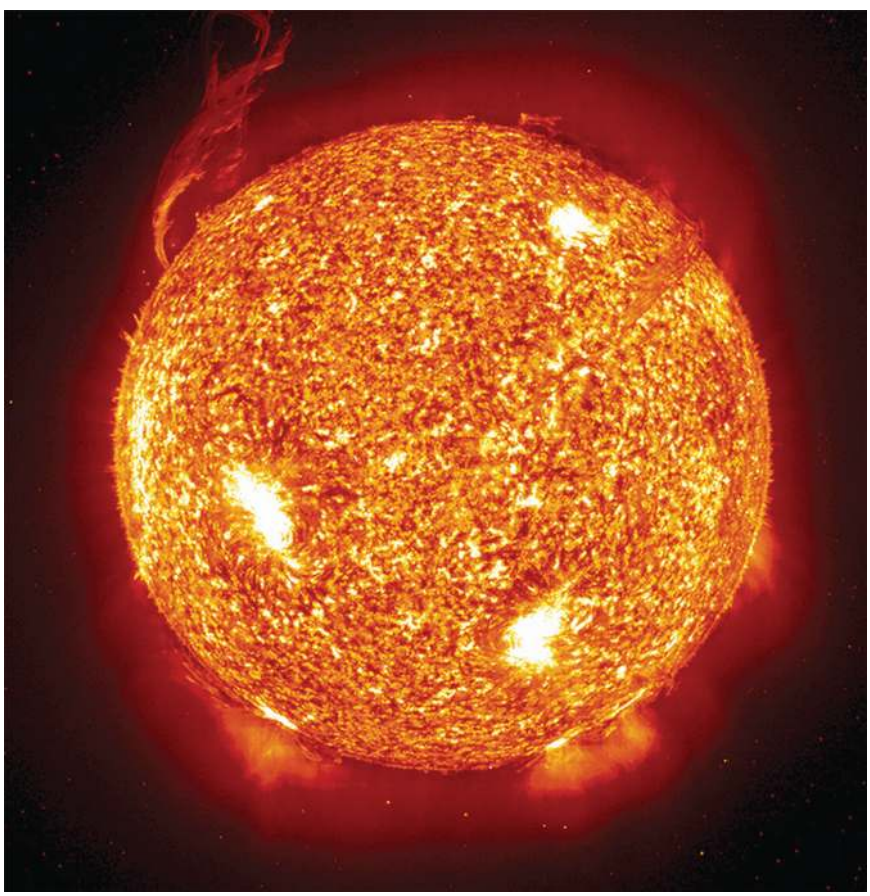
i $\dfrac{6.8 \times 10^{-8} + 7.5 \times 10^{27}}{4.1 \times 10^{27}}$

j $\dfrac{2.84 \times 10^{-6} - 2.71 \times 10^{-9}}{5.14 \times 10^{-6} + 7 \times 10^{-8}}$

PROBLEM-SOLVING AND REASONING

10, 11 | 10, 12, 13

10 The mass of Earth is approximately 6 000 000 000 000 000 000 000 000 kg. The mass of the Sun is 330 000 times the mass of Earth. Find the mass of the Sun. Express your answer using scientific notation correct to three significant figures.



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11 The diameter of Earth is approximately 12 756 000 m. If the Sun's diameter is 109 times that of Earth, compute its diameter in kilometres. Express your answer using scientific notation correct to three significant figures.

12 Write these numbers from largest to smallest.
$2.41 \times 10^6, 24.2 \times 10^5, 0.239 \times 10^7, 2421 \times 10^3, 0.02 \times 10^8$

Hint: First write each number using scientific notation.

13 The following output is common on a number of different calculators and computers. Write down the number that you think they represent.

a 4.26E6 **b** 9.1E−3
c 5.04EE11 **d** 1.931EE−1
e 2.1 ∧ 06 **f** 6.14 ∧ −11

ENRICHMENT: Combining bacteria — 14

14 A flask of type A bacteria contains 5.4×10^{12} cells and a flask of type B bacteria contains 4.6×10^8 cells. The two types of bacteria are combined in the same flask.

Hint: Set up a table to show the number of each type of bacteria after every 8 hours.

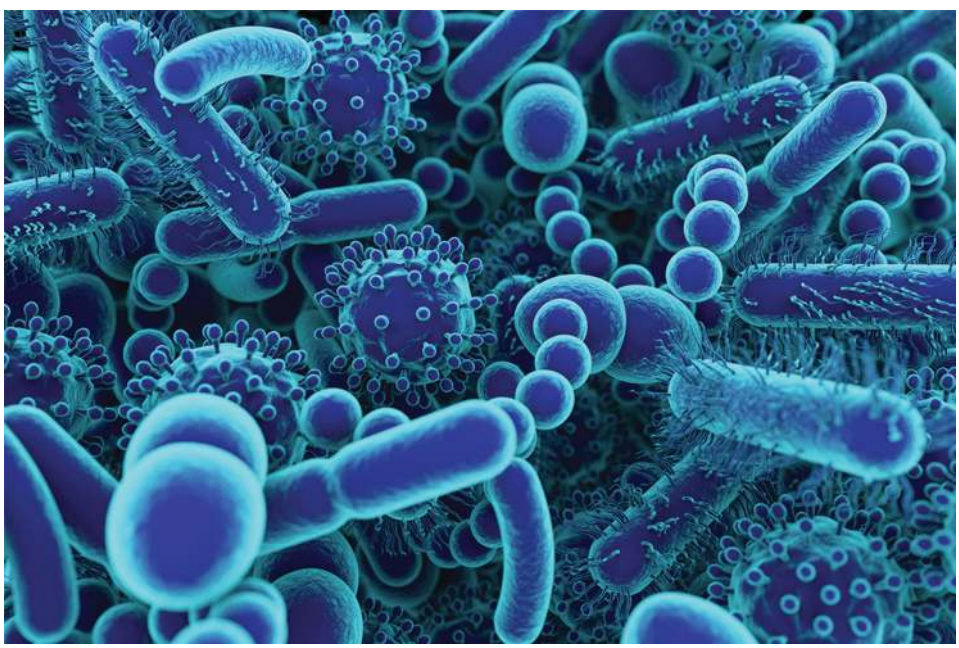
a How many bacterial cells are there now in the flask?

b If type A bacterial cells double every 8 hours, and type B bacterial cells triple every 8 hours, how many cells are in the flask after:

i 8 hours?

ii one day?

Express your answers to part **b** using scientific notation correct to three significant figures.



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Lab technician

Lab technicians work in laboratories supporting scientists by helping to carry out tests for research and to report their findings. Research can be done in medical, clinical and forensic labs as well as pharmaceutical and brand development labs.

Lab technicians require competent practical and technical skills as well as good communication skills, both verbal and written. An enquiring and analytical mind also helps. All these qualities are important in modern labs.

Mathematical skills are vital in this role. Determining growth rates of cultures can form part of a lab technician's work. Working in and with scientific notation is also required.

1 Various types of cells each have different volumes and are measured in cubic micrometres (μm^3). Write the following volumes in μm^3 using scientific notation.

- **a** Red blood cell 100 μm^3
- **b** Ear hair cell 4000 μm^3
- **c** Lymphocyte 130 μm^3
- **d** Beta cell 1000 μm^3
- **e** Fat cell 600 000 μm^3

2 The diameter of a human egg is 120 µm. Recall that $1\ \mu m = 1 \times 10^{-6}$ m.

- **a** Write this diameter in metres as a basic numeral.
- **b** Write this diameter in metres using scientific notation.

3 The mass of one red blood cell is 1 picogram or 1×10^{-12} grams. Calculate the mass of these blood cell counts using grams and write the result using scientific notation.

- **a** 2000 red blood cells
- **b** 1 000 000 red blood cells
- **c** 3×10^{12} red blood cells

4 A human being is said to be made up of $(3.7 \pm 0.8) \times 10^{13}$ cells.

- **a** What is the upper limit given here as a basic numeral?
- **b** What is the lower limit as a basic numeral?

5 A half-life is the time taken for a quantity to halve in size. Write each of the following half-lives using scientific notation.

- **a** Carbon-14 has a half-life of 5700 years. (It is used by archaeologists to date historic objects.)
- **b** Uranium-238 has a half-life of 4 500 000 000 years.
- **c** Iodine-129 has a half-life of 15.7 million years.
- **d** Polonium-214 has a half-life of 150 microseconds (state the answer in seconds).

Hint:
1 microsecond = 1 millionth of a second.

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6 Convert the half-lives in Question **5a**, **b** and **c** into days using 365.25 days per year. State answers using scientific notation to three significant figures.

7 Yeast doubles every 90–120 minutes. Taking the median value, how long would it take for a food technician to grow 1 g of yeast to $\frac{1}{2}$ kg of yeast?

8 *E. coli* bacteria can double every 20–30 minutes. Using the median time, how many grams of *E. coli* has grown from 1 mg using the following times? Write each answer in scientific notation to three significant figures.

i 50 minutes

ii $8\frac{1}{3}$ hours

9 After each 'half-life' period, 1 g of radioactive isotope halves in quantity. List the 10 fractions of radioactive isotope remaining (in grams) after each of 10 half-lives. First list these fractions with positive powers of 2 and then with negative powers of 2.

Using technology

10 Radioactive isotopes have various medical applications.

a Set up the following spreadsheet to calculate quantities remaining after multiples of half-lives for the radioactive isotopes listed. Enter formulas in the shaded cells and format cells as 'number' to four decimal places.

	A	B	C	D	E	F	G
1	**Calculation of time for multiple half-lives**						
2				**Quantity remaining in mg**			
3	Application	Radioactive isotope	**Initial quantity in mg**	After 1 half-life	After 2 half-lives	After 4 half-lives	After 10 half-lives
4	Biochemical tracer	hydrogen-3	8				
5	Biomedical imaging	carbon-11	64.128				
6	Heart system tracer	sodium-24	24.73				
7	Red blood cell lifetime tracer	iron-59	7.86				
8	Radiation therapy for cancer	radium-226	4.25				

b To help determine the correct dose, medical scientists need to calculate the length of time that a radioactive isotope lasts. Set up the following spreadsheet to calculate the total times for multiple half-lives. Enter formulas in the shaded cells and format cells as number to four decimal places.

	A	B	C	D	E	F	G	H
1	**Calculation of time for multiple half-lives**							
2			**Half-life**		**Total time**			
3	Application	Radioactive isotope	Value	Time unit	For 2 half-lives	For 4 half-lives	For 6 half-lives	For 10 half-lives
4	Biochemical tracer	hydrogen-3	12.32	hours				
5	Biomedical imaging	carbon-11	20.3	minutes				
6	Heart system tracer	sodium-24	14.951	hours				
7	Red blood cell lifetime tracer	iron-59	44.495	days				
8	Radiation therapy for cancer	radium-226	1600	years				

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Rabbits and hares

The population of rabbits and hares in a particular part of the countryside are booming because fox numbers are low.

Present a report for the following tasks and ensure that you show clear mathematical workings, explanations and diagrams where appropriate.

Routine problems

a A population of a certain colony of hares is modelled by the rule $P = 3^t$, where P is the population and t is in months. Find:
- **i** the population after 6 months
- **ii** the number of whole months for the population to reach at least 1000.

b A population of a certain colony of rabbits is modelled by the rule $P = 200 \times 2^t$, where P is the population and t is in months. Find:
- **i** the population after 6 months
- **ii** the number of whole months for the population to reach at least 20 000.

Non-routine problems

Explore and connect

The problem is to determine the number of months required for the hare population to overtake the population of rabbits in a particular colony with the given data.

Time in months	0	1	2	3			
Month	**Dec**	**Jan**	**Feb**	**Mar**	**Apr**	**May**	**June**
Rabbit population	500	1000	2000	4000			
Hare population	1	4	16	64			

a Write down all the relevant information that will help solve this problem.

b Copy and extend the given table showing the population of rabbits and hares in the first 6 months.

c By what factor does the rabbit population increase each month?

d The rule for the population of rabbits is of the form $P = 500 \times ?^t$. State the missing number and rewrite the rule for rabbits.

e By what factor does the hare population increase each month?

f The rule for the population of hares is of the form $P = ?^t$. State the missing number and rewrite the rule for hares.

Choose and apply techniques

g Use the rules outlined in parts **d** and **f** to confirm the population of:
- **i** rabbits initially (at $t = 0$)
- **ii** rabbits after 6 months
- **iii** hares after 6 months.

h Use the rules outlined in parts **d** and **f** to estimate the population of:
- **i** rabbits after 12 months
- **ii** hares after 12 months

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i Extend your table and use your rules to calculate the population numbers for the months from August to December.

Time in months	7					
Month	July	Aug	Sep	Oct	Nov	Dec
Rabbit population						
Hare population						

Communicate thinking and reasoning

j State the first month in which the hare population overtakes the population of rabbits.

k In which month is the hare population more than twice the rabbit population?

l In which month does the rabbit population exceed 1 million?

m Summarise your results and describe any key findings.

Extension problems

Problem-solve

A new population of 100 rabbits breed in a different part of the country. The population is modelled by the rule $P = 100 \times 4^t$ where t is in years from the beginning of 2021.

a What would you expect the population of rabbits to be at the beginning of 2025?

b During what year will this rabbit population exceed:

i 1 million

ii 25 million?

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1 A population of bacteria doubles every 5 minutes. What is this type of growth called? Solve this puzzle to find the answer.

Write the basic numeral for each of the following. Write the letter corresponding with each answer in the boxes below to form a word.

T	$\frac{1}{10^3}$	**L**	2.15×10^3	**I**	$\frac{9^5}{9^3}$
A	5×10^{-2}	**O**	2^{-2}	**N**	2^4
P	$3^2 \times 2^2 - 3^0$	**X**	$5^2 + 2^4 - 3^2 \times 4$	**E**	$\frac{4^3}{8}$

8	5	35	0.25	16	8	16	0.001	81	0.05	2150

2 Find the answer to these calculations without using a calculator:

a $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

b $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 4 \times 4 \times 4 \times 4 \times 4 \times 5 \times 5 \times 5 \times 5 \times 5$

3 Show that this expression is equal to 1.

$$\frac{1}{2} \times \frac{a^2b}{3am} \times \frac{6a^3}{b^2} \times \frac{10(ab)^2}{2bm^3} \times \frac{m^4}{5a^6}$$

4 Insert brackets in this expression to make the given answer correct.

$$4a \times 3m^3 \times \frac{a}{6m} = 3a^3m$$

5 Nick and Maddy have a big bag of 10-cent coins. They decide to make 30 piles of coins, starting with one coin in the first pile, then 2 coins in the second pile, 4 coins in the third pile, 8 in the fourth pile and so on, continuing to double the number of coins for each new pile. Given that each 10-cent coin is 2 mm thick, find the heights of the 1st, 5th, 10th, 15th, 20th, 25th and 30th piles and the value in dollars of each of these piles.

6 Each day that a bushfire burns in a national park, the total area burnt is twice as big an area as the day before. If a national park was totally burnt out in 15 days, on which day was the park only half burnt out?

7 It is thought that the game of chess was invented by an ancient Indian mathematician. There is a story that the king at that time was so pleased with the game that he offered the inventor any reward of his choice: rare jewels, bags of gold or even a large property.

To the king's surprise the Indian mathematician asked for some wheat! He asked for 1 grain for the first square of the chess board, 2 grains for the second square, 4 grains for the third square, 8 grains for the fourth square etc. continuing this way right up to the 64th square.

a If one grain of wheat weighs 2×10^{-8} tonnes, what weight of wheat would the inventor have received for the 64th square? Answer using scientific notation with three significant figures.

b How much money would this wheat be worth at the Australian price of \$275/tonne?

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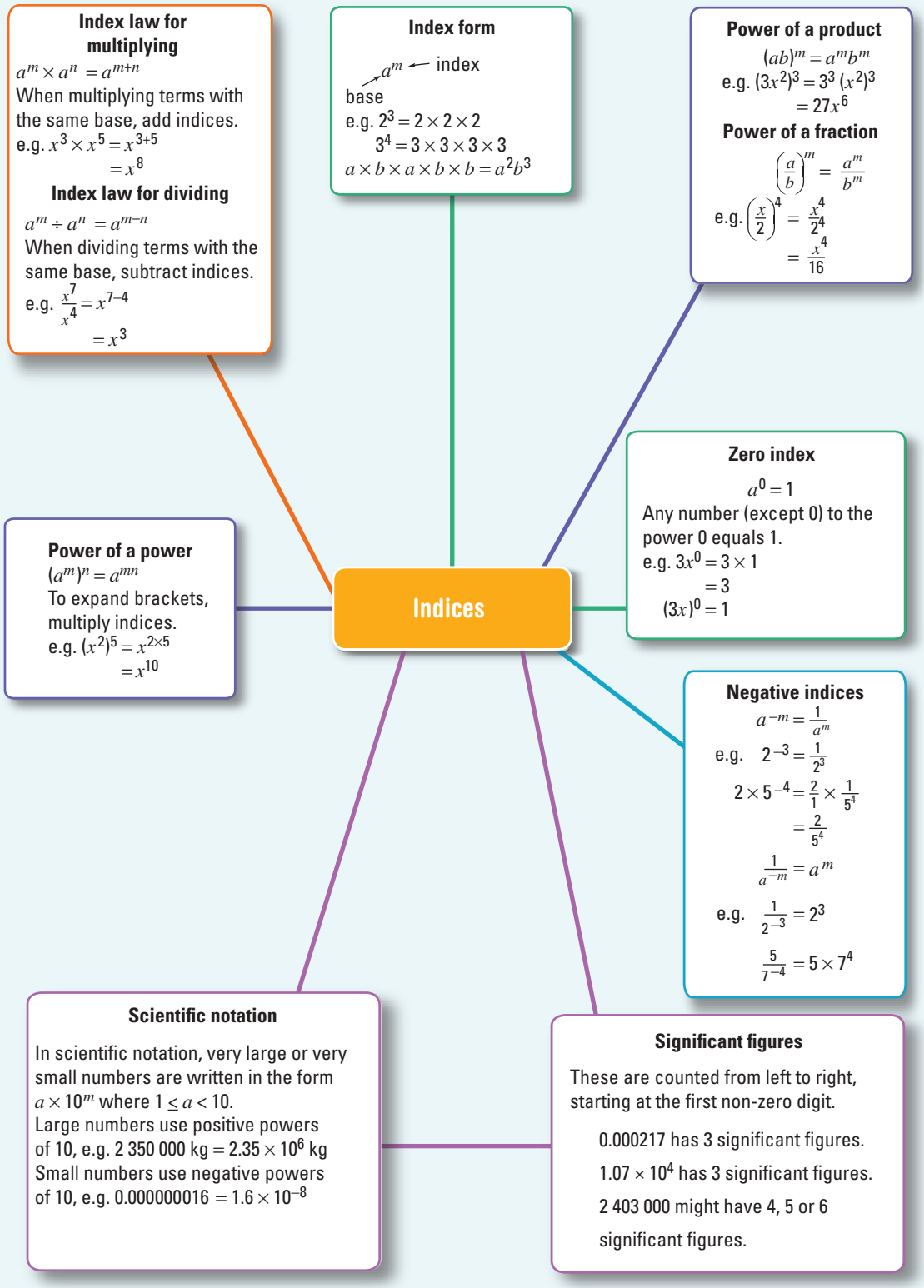
Chapter summary



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Chapter checklist with success criteria

A printable version of this checklist can be downloaded from the Interactive Textbook.

		✔
7A	**1 I can write expressions in expanded form and simplify.** e.g. Write the following in expanded form and simplify. **a** $(2ab)^3$ **b** $\left(\frac{2}{7}\right)^2$	☐
7A	**2 I can write numbers and expressions using index form.** e.g. Write the following in index form: **a** $\frac{7}{9}\times\frac{7}{9}\times\frac{7}{9}\times\frac{7}{9}$ **b** $2\times a\times a\times b\times a\times b$	☐
7A	**3 I can express a number as a product of its prime factors.** e.g. Express 92 as a product of its prime factors.	☐
7A	**4 I can evaluate expressions involving indices using substitution.** e.g. If $a = 2$, $b = -3$ and $c = 11$, evaluate the following. **a** $(ab)^3$ **b** $\left(\frac{b}{c}\right)^2$	☐
7B	**5 I can simplify expressions with numerical bases using index laws.** e.g. Simplify, giving your answer in index form. **a** $4^3\times4^4$ **b** $7^5\div7$	☐
7B	**6 I can use the index law for multiplication.** e.g. Simplify the following. **a** $a^3\times a^7$ **b** $9x^2\times3x^3$	☐
7B	**7 I can use the index law for division.** e.g. Simplify $y^6\div y^2$.	☐
7B	**8 I can combine index laws for multiplication and division.** e.g. Simplify: $\frac{3a^2b\times4ab^3}{8a^2b^3}$.	☐
7C	**9 I can simplify expressions containing the zero index.** e.g. Evaluate using the zero index: 3^0+3a^0	☐
7C	**10 I can use the index law for power of a power.** e.g. Simplify $5(x^3)^6$	☐
7C	**11 I can combine index laws.** e.g. Simplify the following. **a** $(x^2)^3\times(x^4)^2\div x^{14}$ **b** $\frac{7a^3\times2a^4}{4a^5}$	☐
7D	**12 I can use index laws to rewrite expressions.** e.g. Simplify, using index laws. **a** $(2a^4)^3$ **b** $\left(\frac{-2b^2}{a^3}\right)^4$	☐

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Chapter checklist

Chapter checklist with success criteria

7D	**13 I can combine index laws to simplify expressions.** e.g. Simplify the following. **a** $x(-xy^2)^3$ **b** $\left(\frac{a^2b^3}{2}\right)^2 \times \frac{4}{(ab)^2}$	☐
7E	**14 I can express negative indices in positive index form.** e.g. Rewrite the following with positive indices only: **a** $4x^{-2}$ **b** $\frac{5}{2^{-3}}$	☐
7E	**15 I can evaluate expressions involving negative indices.** e.g. Write the following with a positive index and then as a fraction. **a** 5×2^{-3} **b** 4×10^{-2}	☐
7F	**16 I can convert from scientific notation to a basic numeral.** e.g. Write these numbers as a basic numeral: **a** 4.9×10^3 **b** 3.01×10^{-6}	☐
7F	**17 I can write numbers using scientific notation.** e.g. Write these numbers using scientific notation: **a** 27 000 **b** 0.0000375	☐
7G	**18 I can write numbers using scientific notation and rounding to a given number of significant figures.** e.g. Write these numbers in scientific notation using three significant figures: **a** 9 143 000 **b** 0.00032	☐
7G	**19 I can use a calculator to evaluate expressions involving numbers expressed with scientific notation.** e.g. Use a calculator to evaluate $\frac{\sqrt{5.3 \times 10^{-3}}}{8.32 \times 10^{-2}}$ and express your answer using four significant figures.	☐

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Short-answer questions

7A **1** Express each of the following in index form.

a $3 \times 3 \times 3 \times 3$

b $2 \times x \times x \times x \times y \times y$

c $3 \times a \times a \times a \times \frac{b}{a} \times b$

d $\frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{1}{7} \times \frac{1}{7}$

7A **2** Write the following as a product of prime factors in index form.

a 45

b 300

7B **3** Simplify using index laws.

a $x^3 \times x^7$

b $2a^3b \times 6a^2b^5c$

c $3m^2n \times 8m^5n^3$

d $a^{12} \div a^3$

e $x^5y^3 \div (x^2y)$

f $\frac{5a^8b^3}{10a^6b}$

7C/7D **4** Simplify:

a $(m^2)^3$

b $(3a^4)^2$

c $(-2a^2b)^5$

d $3a^0b$

e $2(3m)^0$

f $\left(\frac{a^2}{3}\right)^3$

7E **5** Express each of the following with positive indices.

a 2^{-3}

b 3×4^{-5}

c 6×10^{-3}

d $\frac{1}{4^{-2}}$

e $\frac{6}{5^{-3}}$

f $\frac{2}{10^{-5}}$

7B/C/D **6** Use index laws to simplify each of the following.

a $\frac{k^5 \times k^3}{k^2}$

b $\frac{9a^4}{2m^3} \times \frac{4m^4}{3a^2}$

c $(a^2)^3 \times 4a^4$

d $(h^{12})^3 \div (h^4)^5$

e $\frac{4(a^3)^2(b^4)^3}{20a^0(b^2)^5}$

f $5(3p^2q)^2$

7F **7** Write each of the following numbers as a basic numeral to arrange in ascending order.
$2.35, 0.007 \times 10^2, 0.0012, 3.22 \times 10^{-1}, 0.4, 35.4 \times 10^{-3}$.

7F **8** Write each of the following numbers as a basic numeral.

a 3.24×10^2

b 1.725×10^5

c 2.753×10^{-1}

d 1.49×10^{-3}

7G **9** Write each of the following values using scientific notation correct to three significant figures.

a The population of Australia in 2010 was approximately 22 475 056.

b The area of the USA is 9 629 091 km^2.

c The time taken for light to travel 1 metre (in a vacuum) is 0.00000000333564 seconds.

d The wavelength of ultraviolet light is 0.000000294 m.

7F **10** Write each of the following values using scientific notation in the units given in brackets.
Note: 1 tonne = 1000 kg.

a 25 years (hours)

b 3 million years (months)

c 430 tonnes (kg)

d 5 tonnes (grams)

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Chapter review

Multiple-choice questions

7B **1** $3x^7 \times 4x^4$ is equivalent to:

A $12x^7$ B $12x^{28}$ C $7x^{11}$ D $12x^{11}$ E $7x^3$

7C **2** $3(2y^2)^0$ simplifies to:

A 6 B 3 C $6y^2$ D $3y$ E 12

7D **3** $(2x^2)^3$ expands to:

A $2x^5$ B $2x^6$ C $6x^6$ D $8x^5$ E $8x^6$

7B **4** $\dfrac{x^6y^2}{4x^2y}$ simplifies to:

A $-4x^4y$ B $\dfrac{x^3y^2}{4}$ C $\dfrac{x^4y}{4}$ D $\dfrac{x^4y^2}{4}$ E $\dfrac{x^3y}{4}$

7D **5** $\left(\dfrac{2a}{k}\right)^3$ is equal to:

A $\dfrac{2a^3}{k}$ B $\dfrac{2a^3}{k^3}$ C $\dfrac{6a^3}{k^3}$ D $\dfrac{8a^3}{k^3}$ E $\dfrac{6a}{3k}$

7D **6** $\left(\dfrac{5^2}{7^3}\right)^4$ is equal to:

A $\dfrac{4 \times 5^2}{4 \times 7^3}$ B $\dfrac{5^8}{7^{12}}$ C $\dfrac{5^6}{7^3}$ D $\dfrac{5^8}{7^3}$ E $\dfrac{5^6}{7^7}$

7D **7** $5(2am^3)^3$ is equal to:

A $30a^3m^9$ B $10a^3m^9$ C $40a^3m^6$ D $40a^3m^9$ E $10am^6$

7E **8** 4×7^{-3} expressed with positive indices is:

A 28^3 B 4^37^3 C $\dfrac{4}{7^3}$ D $\dfrac{-4}{7^3}$ E $\dfrac{1}{4 \times 7^3}$

7F **9** The weight of a cargo crate is 2.32×10^4 kg. In expanded form this weight in kilograms is:

A 2 320 000 B 232 C 23 200 D 0.000232 E 2320

7G **10** 0.00032761 using scientific notation rounded to three significant figures is:

A 328×10^{-5} B 3.27×10^{-4} C 3.28×10^4 D 3.30×10^4 E 3.28×10^{-4}

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Extended-response questions

1 Use a calculator to evaluate the following, giving your answer using scientific notation correct to two significant figures.

a $m_s \times m_e$ where m_s (mass of Sun) $= 1.989 \times 10^{30}$ kg and m_e (mass of Earth) $= 5.98 \times 10^{24}$ kg.

b The speed, v, in m/s of an object of mass $m = 2 \times 10^{-3}$ kg and kinetic energy $E = 1.88 \times 10^{-12}$ joules where $v = \sqrt{\frac{2E}{m}}$.

2 Use the table of solar system data below to answer these questions. Express each answer using scientific notation rounded to three significant figures.

Approximate data for some planets in the solar system			
Planet	**Distance from the Sun in millions of km**	**Length (in Earth days) of one revolution around the Sun**	**Mass compared to Earth**
Mercury	57.9	88.0	0.553
Venus	108.2	224.7	0.815
Earth	149.6	365.25	1.00
Mars	227.9	687.0	0.1074
Jupiter	778.3	4331	317 896
Saturn	1427	10 760	95 185

a Find the distance from the Sun, in km, for:

i Mercury **ii** Earth **iii** Jupiter **iv** Saturn

b How many Earth years will have passed for these planets to complete one full revolution around the Sun?

i Mercury **ii** Mars **iii** Saturn

c The mass of Earth is approximately 5.98×10^{24} kg. Determine the mass of:

i Venus **ii** Mars **iii** Jupiter

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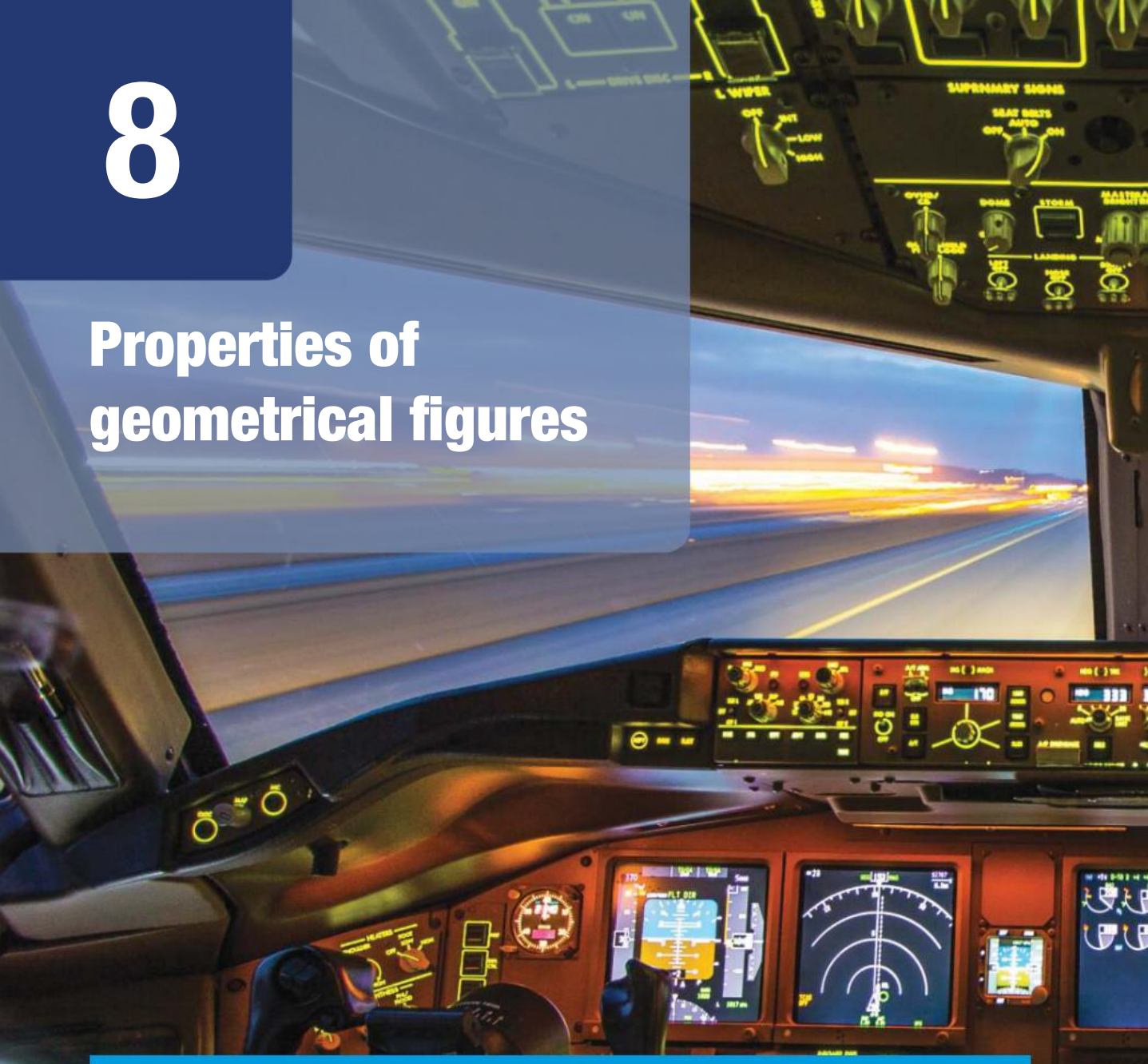
8

Properties of geometrical figures



Maths in context: Geometry for engineering, construction and navigation

Geometry is an essential skill for builders, carpenters, engineers, plumbers, electricians, bricklayers, jewellers, architects, desingers, surveyors, navigators, astronomers and animators.

- Geometry is widely applied by trade workers, including plumbers who construct clean water supplies and sanitation systems, electricians who install power supplies, construction workers who lay bricks in parallel rows, and builders of house and roof frames.
- Triangles are the strongest form of support; similar triangle geometry is applied when designing and building electricity pylons, communication towers and bridge trusses.
- Surveyors can apply similar triangle geometry to determine the width of a gorge.
- Pilot and navigator training includes applying geometry, trigonometry and Pythagoras' theorem to solve problems of speed and direction.

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Chapter contents

8A Angles and triangles (CONSOLIDATING)
8B Parallel lines (CONSOLIDATING)
8C Quadrilaterals (CONSOLIDATING)
8D Polygons (EXTENDING)
8E Enlargement and similar figures
8F Applying scale factor to similar triangles

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- applies angle relationships to solve problems, including those related to transversals on sets of parallel lines (MA4-ANG-C-01)
- identifies and applies the properties of triangles and quadrilaterals to solve problems (MA4-GEO-C-01)
- identifies and applies the properties of similar figures and scale drawings to solve problems (MA5-GEO-C-01)

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Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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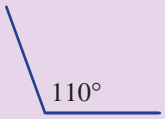
Warm-up quiz

1 Name the following types of angles. Choose from *acute*, *right*, *obtuse*, *straight*, *reflex* or *revolution*.

a



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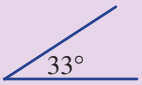

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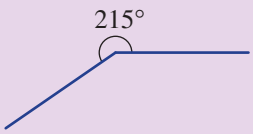

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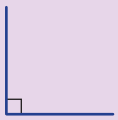

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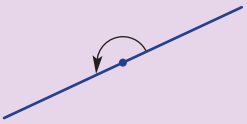
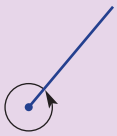
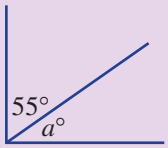


f



2 Find the value of a.

a



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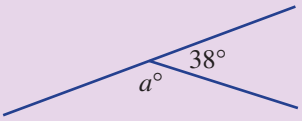

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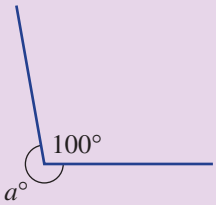
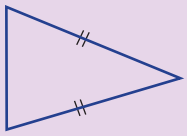


3 Identify the following triangles as *equilateral*, *isosceles* or *scalene*.

a



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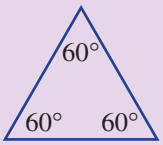

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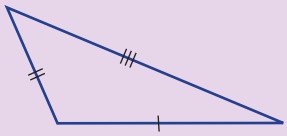
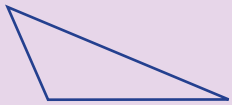


4 Identify the following as *right*, *acute* or *obtuse* triangles.

a



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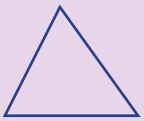
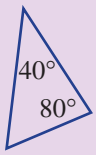


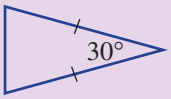
c

5 Find any missing angles in these triangles.

a



b



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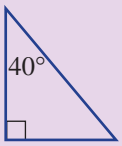


6 Of the six special quadrilaterals (square, rectangle, parallelogram, rhombus, kite and trapezium), which ones match the following descriptions?

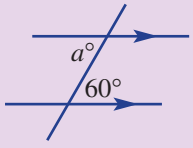
a Two pairs of equal length sides

b Two pairs of parallel sides

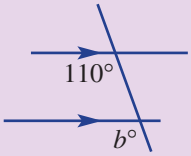
c Equal length diagonals

7 Find the value of the pronumerals in each of the following.

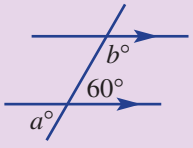
a



b

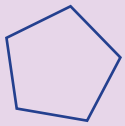


c



8 Name the following shapes.

a



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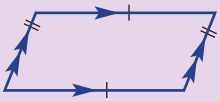

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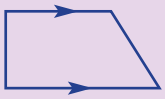

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8A Angles and triangles CONSOLIDATING

Learning intentions for this section:

- To review the various types of angles and triangles
- To know the relationships between angles formed at a point
- To be able to find unknown angles in geometric diagrams
- To be able to use the exterior angle theorem for a triangle

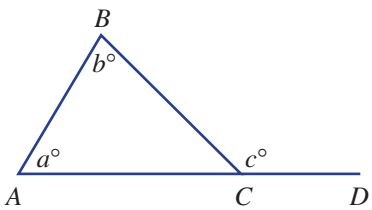
Past, present and future learning:

- This section consolidates Stage 4 concepts
- Some of these questions are more challenging than those in our Stage 4 books
- These concepts are assumed knowledge for Stage 6 Standard and are occasionally useful

When looking at a building or structure, we see angles formed where two lines meet. We also see triangles being used for their strength and rigidity. Angles associated with triangles and with lines that meet at a point will be revised in this section.

Lesson starter: Exterior angle discovery

Here is a triangle with one side extended to form the exterior angle $\angle BCD$ with size $c°$.

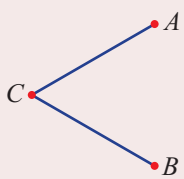


1. If $a = 50$ and $b = 85$, find $\angle ACB$. Then find $\angle BCD$. What do you notice?
2. Repeat for $a = 60$ and $b = 95$. What do you notice?
3. What is the relationship between a, b and c? This is called the exterior angle theorem.

KEY IDEAS

■ A **point** represents a position.
- It is shown using a dot and labelled with an upper case letter.
- The diagram shows points A, B and C.

■ This diagram shows **intervals** AC and CB. These are sometimes called **line segments**.
- AC and CB form two **angles**. One is *acute* and one is *reflex*.
- C is called the **vertex**. The plural of vertex is **vertices**.



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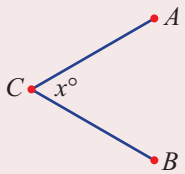
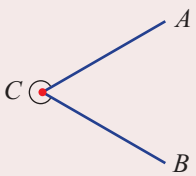
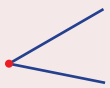
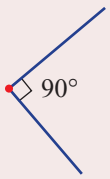
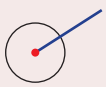
- This diagram shows acute angle ACB. It can be written as:
  $\angle C$ or $\angle ACB$ or $\angle BCA$ or $A\hat{C}B$ or $B\hat{C}A$
  - CA and CB are sometimes called **arms**.
  - The pronumeral x represents the number of degrees in the angle.

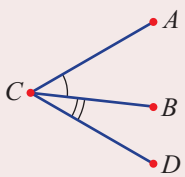


- This diagram shows reflex angle $\angle ACB$.



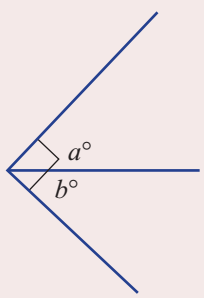
Type of angle	Size of angle	Diagram
Acute	Greater than 0° but less then 90°	
Right	Exactly 90°	
Obtuse	Greater than 90° but less than 180°	
Straight	Exactly 180°	
Reflex	Greater than 180° but less than 360°	
Revolution	Exactly 360°	

- This diagram shows two angles sharing a vertex and an arm. They are called **adjacent** angles.



- This diagram shows two angles in a right angle. They are *adjacent complementary angles*. $a°$ is the **complement** of $b°$.

$$a + b = 90$$



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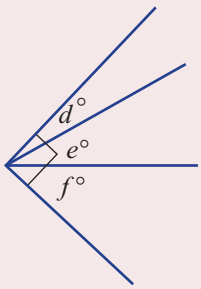
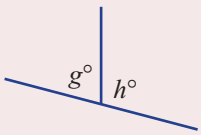
- It is possible to have three or more angles in a right angle. They are not complementary.

$$d + e + f = 90$$



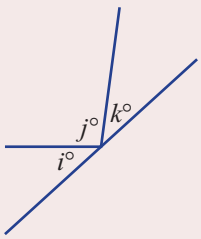
- This diagram shows two angles on a straight line. They are *adjacent supplementary* angles. $g°$ is the **supplement** of $h°$.

$$g + h = 180$$



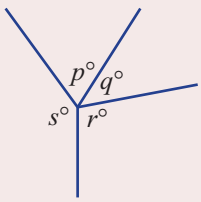
- It is possible to have three or more angles on a straight line. They are not supplementary.

$$i + j + k = 180$$



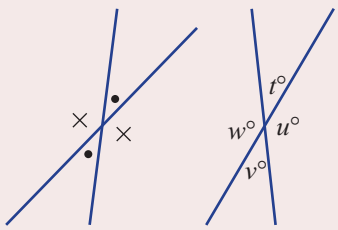
- This diagram shows *angles at a point* and *angles in a revolution*.

$$p + q + r + s = 360$$



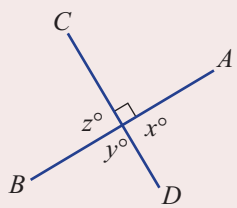
- When two straight lines meet they form two pairs of **vertically opposite angles**.
  Vertically opposite angles are equal.

$$t = v$$
$$u = w$$



- If one of the four angles in vertically opposite angles is a right angle, then all four angles are right angles.
  - AB and CD are **perpendicular lines**. This is written as $AB \perp CD$.

$$x = 90$$
$$y = 90$$
$$z = 90$$



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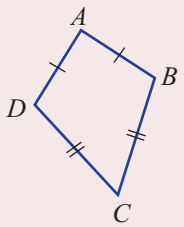
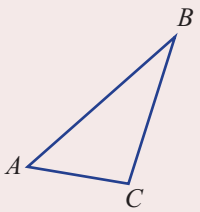
- The markings in this diagram indicate that:
  - $AB = AD$
  - $BC = CD$
  - $\angle ABC = \angle ADC$

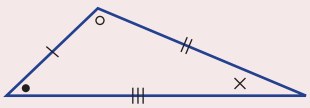


- A *triangle* has
  - 3 sides: AB, BC and AC
  - 3 vertices (the plural of vertex): A, B and C
  - 3 interior angles.



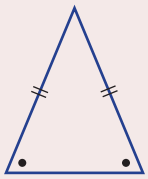
- Triangles classified by side lengths
  - Sides with the same number of dashes are of equal length.

Scalene



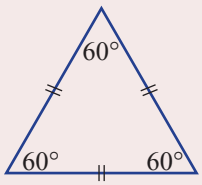
3 unequal sides
3 unequal angles

Isosceles



2 equal sides
2 equal angles
The equal sides are opposite the equal angles.

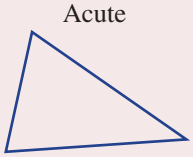
Equilateral



3 equal sides
3 equal 60° angles

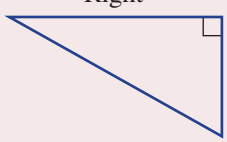
- Triangles classified by interior angles

Acute



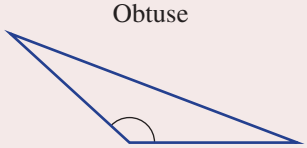
All angles acute

Right



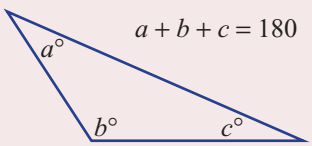
One right angle

Obtuse



One obtuse angle

- The sum of the interior angles of a triangle is 180°.



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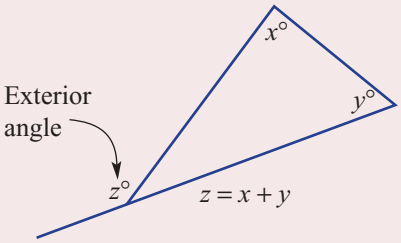
- An exterior angle of a triangle is formed by extending one of the sides.
- The exterior angle of a triangle is equal to the sum of the two opposite interior angles.



Key terminology: line, line segment, angle, complementary, supplementary, revolution, vertically opposite, acute, right, obtuse, reflex, straight, angle sum, scalene, isosceles, equilateral, exterior angle, triangle

Exercise 8A

UNDERSTANDING 1–3 2, 3

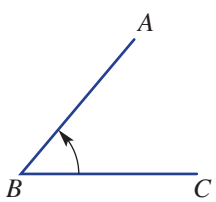
1 Choose a word or number to complete each sentence.

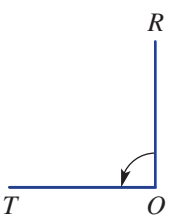
a A 90° angle is called a _____ angle.
b A _____ angle is called a straight angle.
c A 360° angle is called a _____.
d _____ angles are between 90° and 180°.
e _____ angles are between 0° and 90°.
f Reflex angles are between _____ and 360°.
g Complementary angles sum to _____.
h _____ angles sum to 180°.
i The three angles in a triangle sum to _____.
j Vertically opposite angles are _____ .

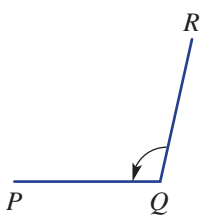
2 What type of triangle has:

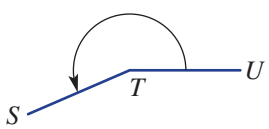
a a pair of equal length sides?
b one obtuse angle?
c all angles 60°?
d one pair of equal angles?
e all angles acute?
f all sides of different length?
g one right angle?
h two 45° angles?

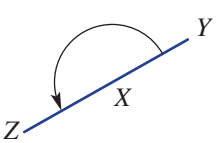
3 Estimate the size of each of the following angles and use your protractor to determine an accurate measurement.

a


b


c


d


e


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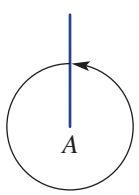

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FLUENCY 4(½), 5, 6(½) 4–6(½)

Example 1 Finding supplementary and complementary angles

For the angle 47°, determine the:

a supplementary angle **b** complementary angle

SOLUTION

a $180° - 47° = 133°$

b $90° - 47° = 43°$

EXPLANATION

Supplementary angles sum to 180°.

Complementary angles sum to 90°.

Now you try

For the angle 74°, determine the:

a supplementary angle **b** complementary angle

4 For each of the following angles determine:

i the supplementary angle

ii the complementary angle

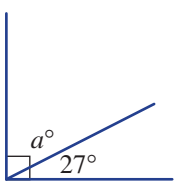
a 55°	**b** 31°	**c** 74°	**d** 10°	**e** 89°
f 22°	**g** 38°	**h** 65°	**i** 47°	**j** 77°

Hint:
- Supplementary angles add to 180°.
- Complementary angles add to 90°.

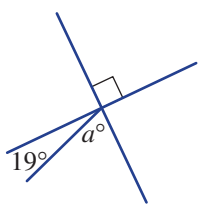
5 Find the value of a in these diagrams.

Hint:
- A straight angle is 180°.
- A revolution is 360°.

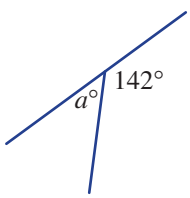
a



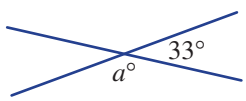
b



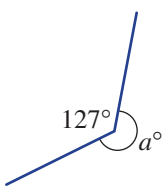
c



d



e



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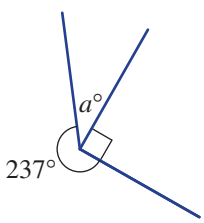

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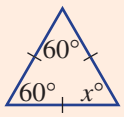
Example 2 Finding unknown angles in triangles

Name the types of triangles shown here. Determine the values of the pronumerals.

a



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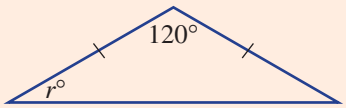

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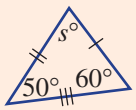


SOLUTION

a Equilateral triangle
$x = 60$

b Obtuse isosceles triangle
$2r + 120 = 180$
$2r = 60$
$r = 30$

c Acute scalene triangle
$s + 50 + 60 = 180$
$s + 110 = 180$
$s = 70$

EXPLANATION

All sides are equal, therefore all angles are equal.

One angle is more than 90° and two sides are equal.

Angles in a triangle add to 180° and since it is isosceles the two base angles are equal.

Subtract 120 from both sides and then divide both sides by 2.

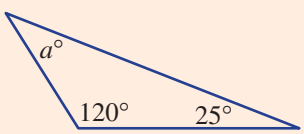
All angles are less than 90° and all sides are of different length.

Angles in a triangle add to 180°. Simplify and solve for s.

Now you try

Name the types of triangles shown here. Determine the values of the pronumerals.

a



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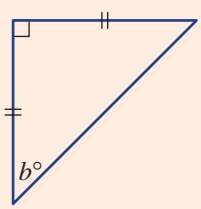

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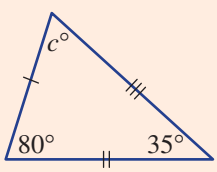
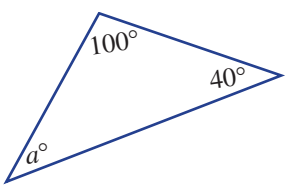


6 Name the types of triangles shown here. Determine the values of the pronumerals.

a



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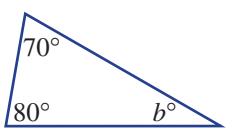
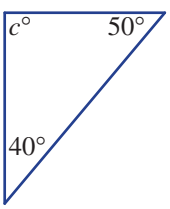


Hint: The angle sum of a triangle is 180°.

c



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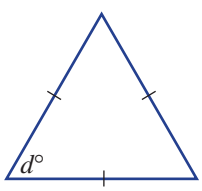

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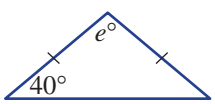

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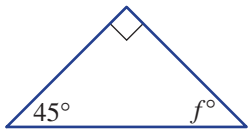

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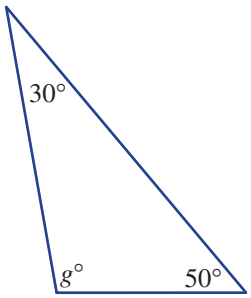

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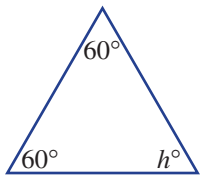

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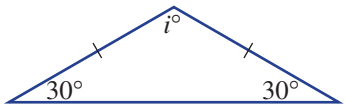

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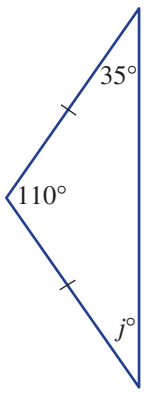

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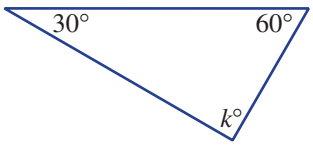

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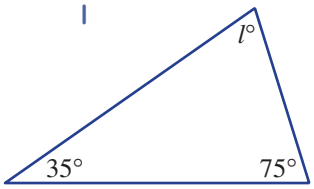


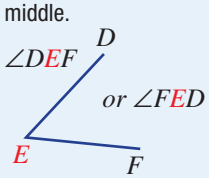
PROBLEM-SOLVING AND REASONING

7, 8 | 7–8(½), 9

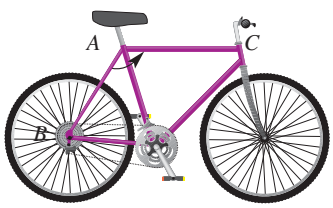
7 For each diagram:

- **i** name the angle shown (e.g. $\angle ABC$)
- **ii** state the type of angle given
- **iii** estimate the size of the angle
- **iv** measure the angle using a protractor.

Hint: When naming an angle, put the letter at the vertex in the middle.



a



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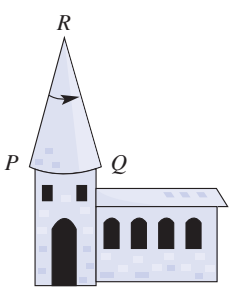

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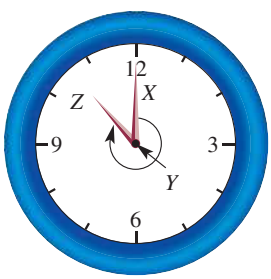

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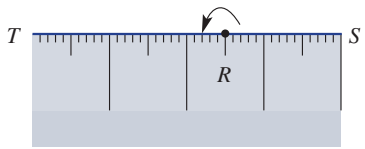

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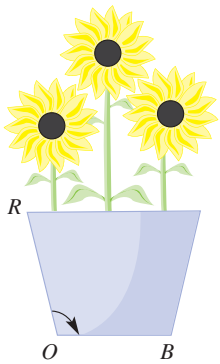

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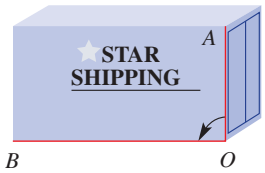
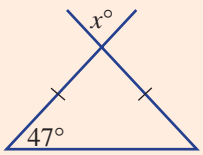


Example 3 Finding angles outside a triangle

Find the value of each pronumeral. Give reasons for your answers.

a



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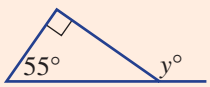


SOLUTION

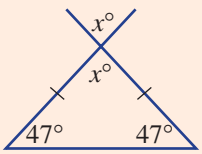
a $x + 47 + 47 = 180$ (angle sum of triangle)

$x + 94 = 180$

$x = 86$

EXPLANATION

Note the isosceles triangle and vertically opposite angles and mark in those angles.



Angles in a triangle add to 180° and vertically opposite angles are equal. Simplify and solve for x.

b Let a be the unknown angle.

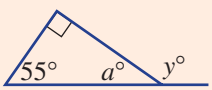
$a + 90 + 55 = 180$ (angle sum of triangle)

$a = 35$

$y + 35 = 180$ (angles on a straight line)

$y = 145$

Angles in a triangle sum of 180°.



Angles in a straight line are supplementary (sum to 180°).

Alternative method:

$y = 90 + 55$ (exterior angle of triangle)

$= 145$

Alternatively, use the exterior angle theorem, which says that the exterior angle is equal to the sum of the two opposite interior angles.

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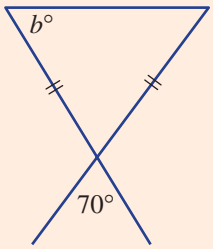
Now you try

Find the value of each pronumeral. Give reasons for your answers.

a



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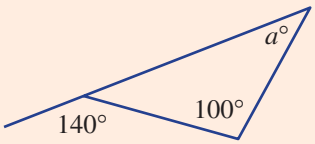
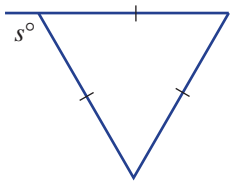


8 Find the value of each pronumeral.

a



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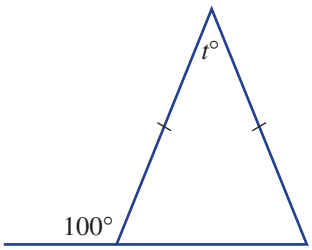

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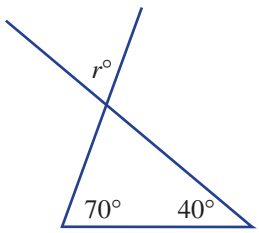

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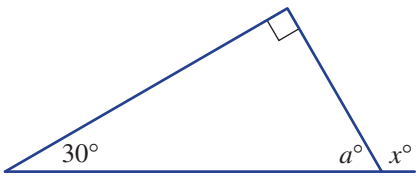

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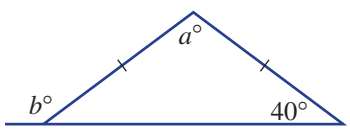

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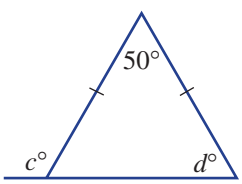

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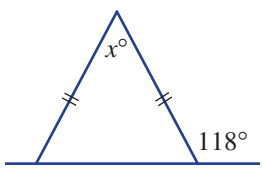

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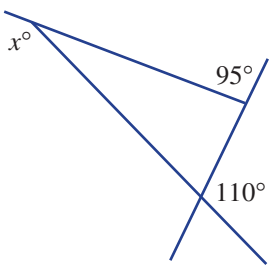

Hint: If you can't find the value of the pronumeral straight away, try finding one of the other unknown angles.

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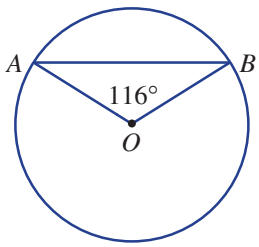
9 Explain why $\angle OAB$ is 32° in this circle if O marks the centre of the circle.



ENRICHMENT: Clock geometry — 10, 11

10 Calculate how many degrees the minute hand of a clock rotates in:

- **a** 1 hour
- **b** $\frac{1}{4}$ of an hour
- **c** 10 minutes
- **d** 15 minutes
- **e** 72 minutes
- **f** 1 minute
- **g** 2 hours
- **h** 1 day

11 Find the non-reflex angle between the hour and minute hands at these times. Remember to consider how the hour hand moves between each whole number. For example, at 9:30, the hour hand is halfway between the 9 and 10.

- **a** 3 p.m.
- **b** 5 a.m.
- **c** 6:30 p.m.
- **d** 11:30 p.m.
- **e** 3:45 a.m.
- **f** 1:20 a.m.
- **g** 4:55 a.m.
- **h** 2:42 a.m.

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8B Parallel lines CONSOLIDATING

Learning intentions for this section:

- To know the special pairs of angles formed when a transversal cuts two other lines
- To know the relationships between angles formed by parallel lines and a transversal
- To be able to calculate unknown angles associated with parallel lines

Past, present and future learning:

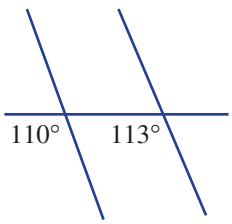
- This section consolidates Stage 4 concepts
- Some of these questions are more challenging than those in our Stage 4 books
- These concepts are assumed knowledge for Stage 6 Standard and are occasionally useful

When two lines are crossed by a third line, called a transversal, many special pairs of angles are formed. If the two original lines are parallel, then the pairs of angles either are equal or they add to 180°.

Lesson starter: Are they parallel?

Here are three diagrams that show a transversal crossing two other lines.

1



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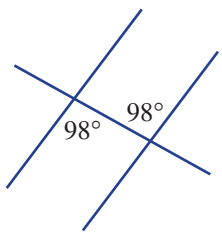
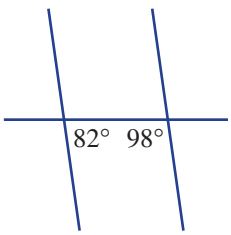


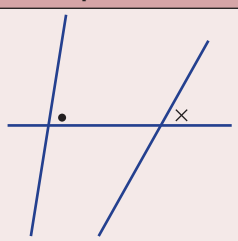
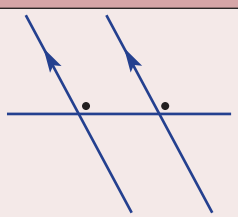
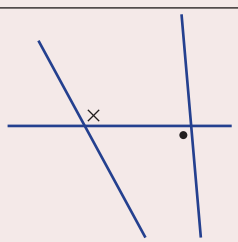
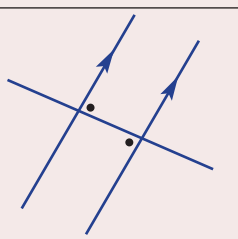
3



Decide whether each diagram contains a pair of parallel lines. Give reasons for your answer.

KEY IDEAS

- Parallel lines never meet.
  - Arrows indicate that lines are parallel.
- A **transversal** is a line crossing two or more other lines.

	Non-parallel lines	Parallel lines
Corresponding angles • If lines are parallel, corresponding angles are equal.		
Alternate angles • If lines are parallel, alternate angles are equal.		

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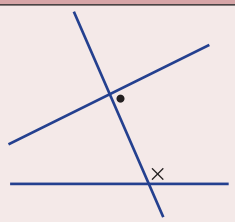
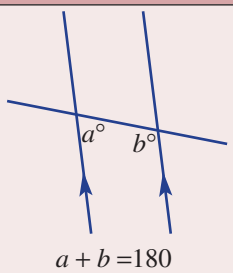
	Non-parallel lines	Parallel lines
Co-interior angles • If lines are parallel, co-interior angles are supplementary (sum to 180°).		

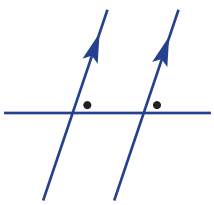
Key terminology: parallel, transversal, corresponding, alternate, co-interior, vertically opposite, supplementary

Exercise 8B

UNDERSTANDING 1, 2 2

1 Decide whether the pair of marked angles are corresponding, alternate or co-interior.

a



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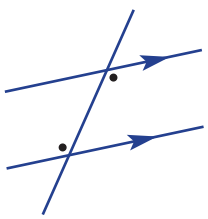

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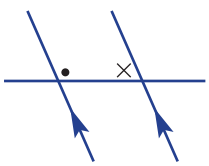

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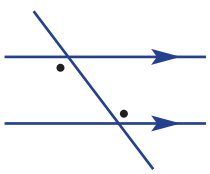
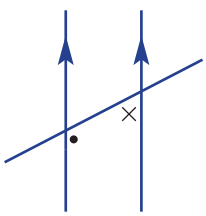


e



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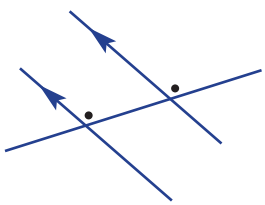

2 Use the words *equal* or *supplementary* to complete these sentences.

Hint: Supplementary angles add to 180°.

a If two lines are parallel, corresponding angles are _____.

b If two lines are parallel, alternate angles are _____.

c If two lines are parallel, cointerior angles are _____.

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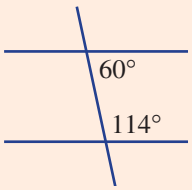
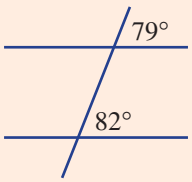
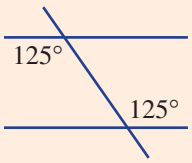
FLUENCY 3–4(½), 5 3–5(½)

Example 4 Deciding whether lines are parallel

Decide whether each diagram contains a pair of parallel lines. Give a reason.

a 

b 

c 

SOLUTION

a No. The two cointerior angles do not add to 180°.

b No. The two corresponding angles are not equal.

c Yes. The two alternate angles are equal.

EXPLANATION

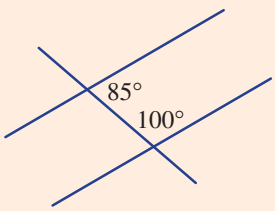
$60° + 114° = 174° \neq 180°$

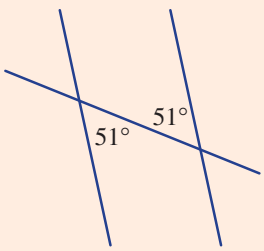
$79° \neq 82°$

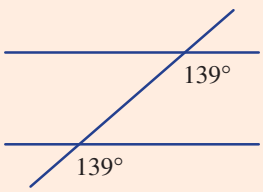
If alternate angles are equal then the lines are parallel.

Now you try

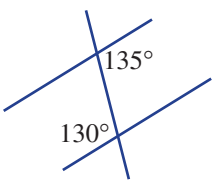
Decide whether each diagram contains a pair of parallel lines. Give a reason.

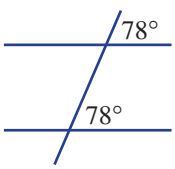
a 

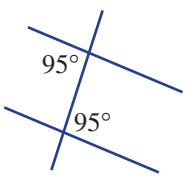
b 

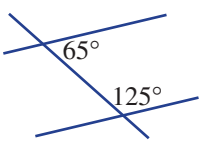
c 

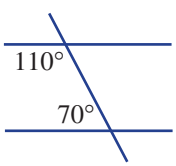
3 Decide if each diagram contains a pair of parallel lines. Give a reason.

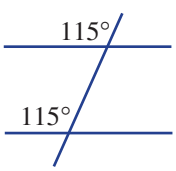
a 

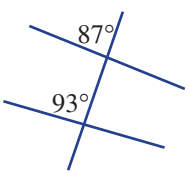
b 

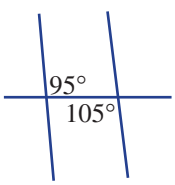
c 

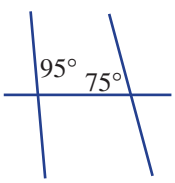
d 

e 

f 

g 

h 

i 

Hint: The reasons why lines could be parallel include:
- Corresponding angles are equal.
- Alternate angles are equal.
- Co-interior angles add to 180° (are supplementary).

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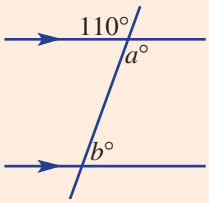
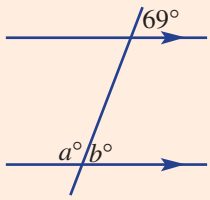
Example 5 Finding angles in parallel lines

Find the value of each of the pronumerals. Give reasons for your answers.

a



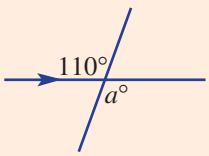
b



SOLUTION

a $a = 110$ (vertically opposite angles)

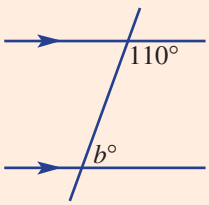
EXPLANATION



Vertically opposite angles are equal.

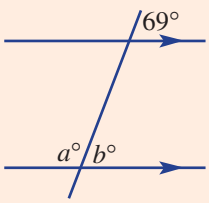
$b + 110 = 180$ (co-interior angles in parallel lines)

$b = 70$



Co-interior angles add to 180° in parallel lines.

b $b = 69$ (corresponding angles in parallel lines)



Corresponding angles are equal in parallel lines.

$a + 69 = 180$ (angles on a straight line)

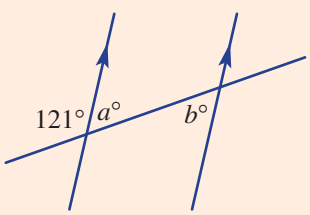
$a = 111$

Angles on a straight line add to 180°, so $a + b = 180$.

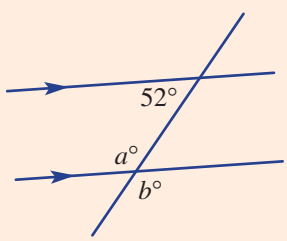
Now you try

Find the value of each of the pronumerals. Give reasons for your answers.

a



b



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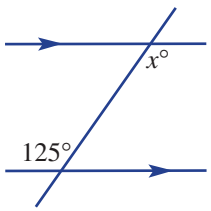
4 Find the values of the pronumerals in these pairs of parallel lines. Give a reason for each answer.

a



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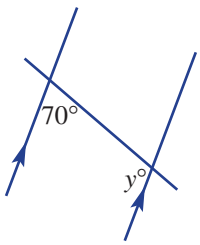
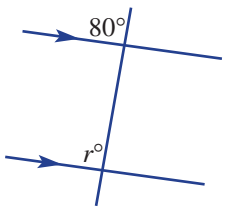


Hint: For part **a**, $x = 125$.
Reason: Alternate angles in parallel lines.

c



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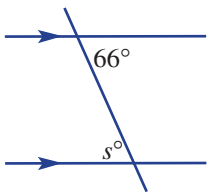

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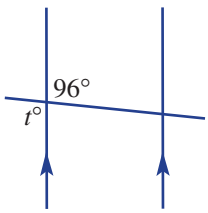

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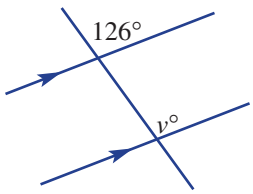

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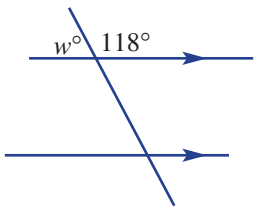

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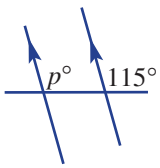

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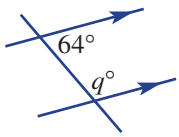
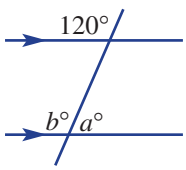
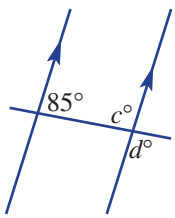


5 Find the value of each pronumeral. Give reasons for your answers.

a



b



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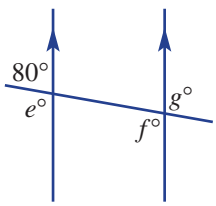

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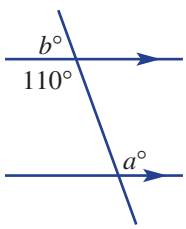

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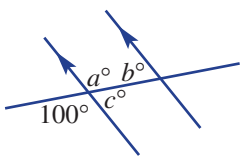

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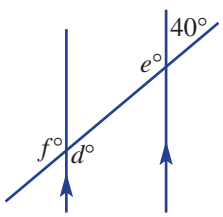

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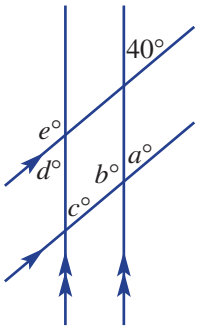
PROBLEM-SOLVING AND REASONING

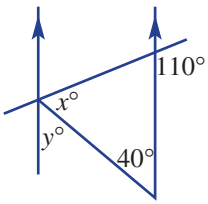
6, 7 | 7, 8

6 Two pairs of straight train tracks meet at 40°, as shown on the right. Find the value of a, b, c, d and e.

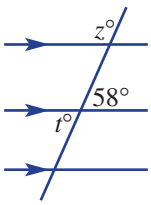


7 Find the value of each pronumeral.

a

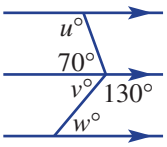


b

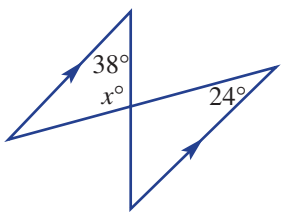


Hint: Look for pairs of angles inside parallel lines. Mark in all known missing angles to help find the value of the pronumeral.

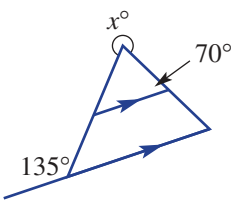
c



d



e



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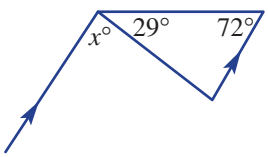
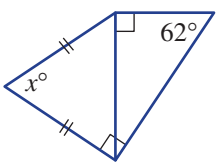


8 Find the value of x.

a



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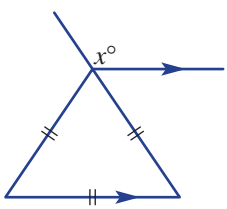
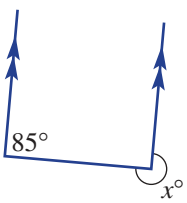


Hint: Use all the given information to mark in any missing angles.

c



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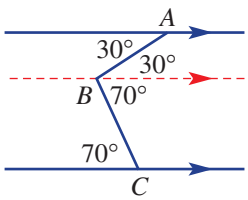
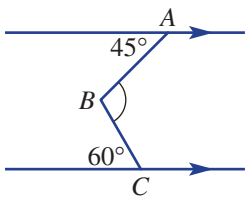
ENRICHMENT: Add a new line to help — 9

9 Sometimes you can add a third parallel line to a diagram to help you find an angle. For example, to find $\angle ABC$ in this diagram, you can draw a parallel line through B, then find the two alternate angles (30° and 70°). So $\angle BC = 30° + 70° = 100°$. Add a third parallel line to help fine $\angle ABC$ in these diagrams.



a



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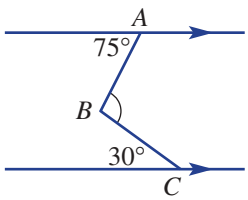
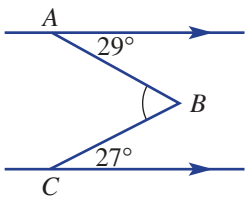


c



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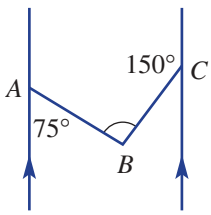

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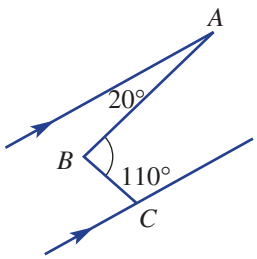

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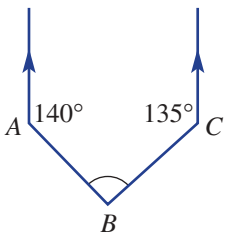

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8C Quadrilaterals CONSOLIDATING

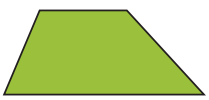
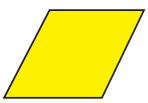
Learning intentions for this section:

- To know the properties of special quadrilaterals
- To know the angle sum of a quadrilateral
- To be able to calculate unknown angles inside quadrilaterals

Past, present and future learning:

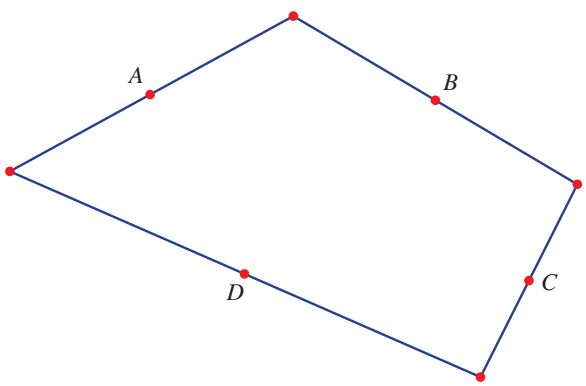
- This section consolidates Stage 4 concepts
- Some of these questions are more challenging than those in our Stage 4 books
- These concepts are assumed knowledge for Stage 6 Standard and are occasionally useful

In this section we will revise the properties of the six special quadrilaterals. An example of each is shown below.

Special quadrilaterals		
Kite	Trapezium 	Square
Rhombus 	Parallelogram	Rectangle

Lesson starter: Random quadrilateral

- On a sheet of paper, draw a random quadrilateral with sides more than 5 cm long.
- Use a ruler to carefully find the midpoint of each side.
- Join A to B, B to C, C to D and D to A.
- What type of quadrilateral is $ABCD$?
- Compare yours to others in your class.



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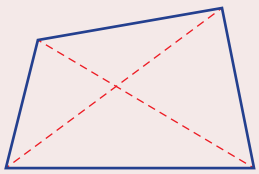
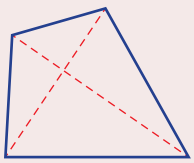
KEY IDEAS

- Every quadrilateral has two diagonals.
  - In some quadrilaterals the diagonals bisect each other (i.e. cut each other in half).



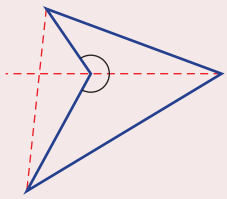
- **Quadrilaterals** can be convex or non-convex.
  - **Convex** quadrilaterals have all vertices pointing outwards.
  - **Non-convex** quadrilaterals have one vertex pointing inwards.
  - Both diagonals of convex quadrilaterals lie inside the figure.

Convex



All interior angles less than 180°

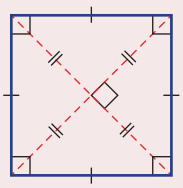
Non-convex



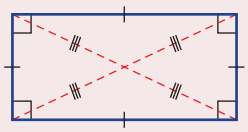
One reflex interior angle

- Special quadrilaterals

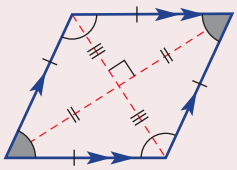
Square



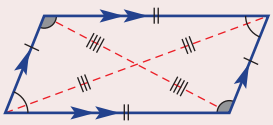
Rectangle



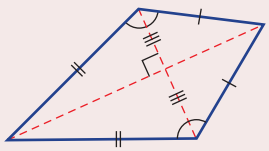
Rhombus



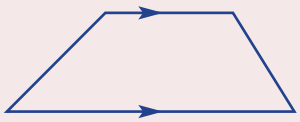
Parallelogram



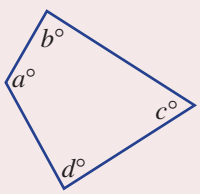
Kite



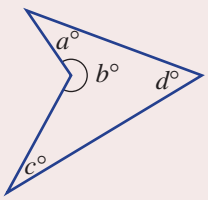
Trapezium



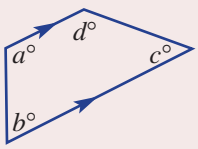
- The angle sum of a quadrilateral is 360°.



$a + b + c + d = 360$



- Quadrilaterals with parallel sides contain two pairs of cointerior angles.



$c + d = 180$
$a + b = 180$

Key terminology: quadrilateral, convex, non-convex, parallelogram, square, rectangle, rhombus, trapezium, kite, diagonal, parallel

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Exercise 8C

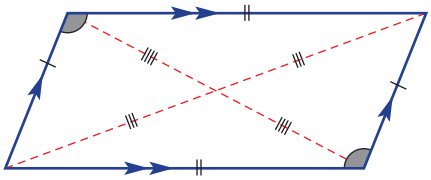
UNDERSTANDING	1–3	3

1 Name the six special quadrilaterals.

2 Using the diagram of a parallelogram given below, which of the following sentences are *definitely* true.

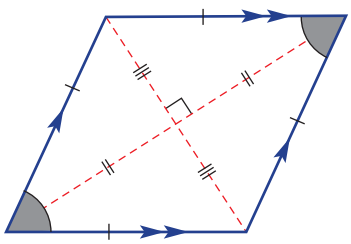
a All sides are equal in length.
b Opposite sides are equal.
c Opposite sides are parallel.
d All angles are 90°.
e Opposite angles are equal.
f The diagonals are equal in length.
g The diagonals bisect each other.
h The diagonals meet at right angles.

Parallelogram



3 Repeat Question **2**, using this diagram of a rhombus.

Rhombus



FLUENCY	4, 5–6(½)	4, 5–6(½)

Example 6 Working with properties of quadrilaterals

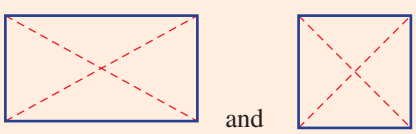
Name all the types of quadrilaterals that have these properties.

a Diagonals of equal length **b** Diagonals intersecting at right angles

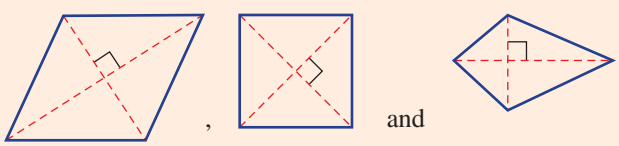
SOLUTION

a Rectangle and square

EXPLANATION



b Rhombus, square and kite



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Now you try

Name all the types of quadrilaterals that have these properties.

a Equal opposite side lengths

b Exactly one pair of equal opposite angles.

Hint: Refer to the diagrams in the Key Ideas for help.

4 Which of the six 'special' quadrilaterals *definitely* have these properties?

a Opposite sides equal

b All interior angles 90°

c Diagonals of equal length

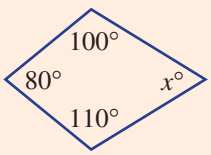
d Diagonals intersecting at right angles

e Exactly 1 pair of parallel sides

f Diagonals of different length

Example 7 Finding angles in quadrilaterals

Find the value of the pronumeral in this quadrilateral.



SOLUTION

$x + 80 + 100 + 110 = 360$ (angle sum of a quadrilateral)

$x + 290 = 360$

$x = 70$

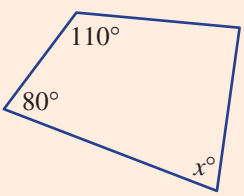
EXPLANATION

The angles in a quadrilateral add to 360°.

Simplify and solve for x.

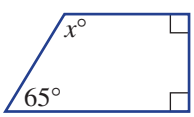
Now you try

Find the value of the pronumeral in this quadrilateral.

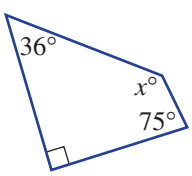


5 Find the value of the pronumeral x in these quadrilaterals.

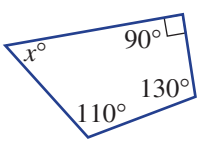
a



b

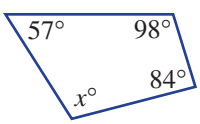


c

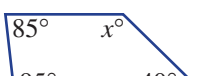


Hint: Use the angle sum of a quadrilateral, which is 360°.

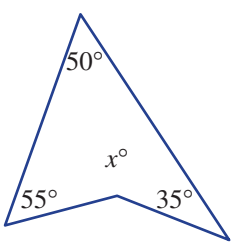
d



e



f



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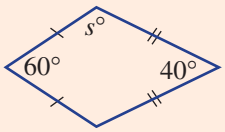
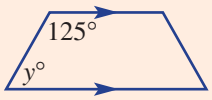
Example 8 Finding angles in special quadrilaterals

Find the value of the pronumeral in this kite and trapezium. Give a reason.

a



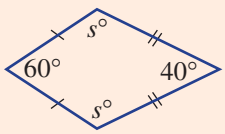
b



SOLUTION

a $2s + 60 + 40 = 360$ (angle sum of a quadrilateral)
$2s + 100 = 360$
$2s = 260$
$s = 130$

EXPLANATION



The angles in a quadrilateral add to 360° and the opposite angles ($s°$) are equal in a kite.

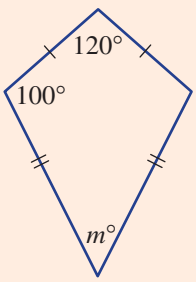
b $y + 125 = 180$ (co-interior angles in parallel lines)
$y = 55$

Co-interior angles inside parallel lines are supplementary (add to 180°).

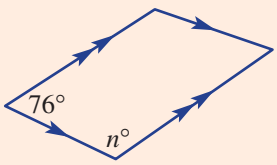
Now you try

Find the value of the pronumeral in this kite and trapezium.

a

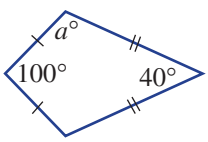


b



6 Find the value of the pronumerals.

a



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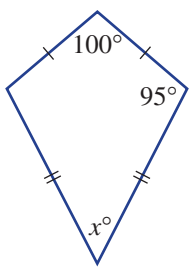

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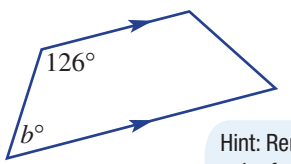
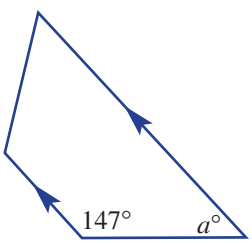


Hint: Remember: A kite has one pair of equal opposite angles. Cointerior angles inside parallel lines are supplementary (add to 180°).

d



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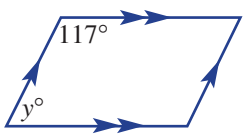

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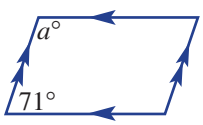

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PROBLEM-SOLVING AND REASONING

7, 8 | 7, 9, 10

7 Decide whether it is possible to draw a quadrilateral with:

a 1 interior reflex angle

b 2 interior reflex angles

c 3 acute angles

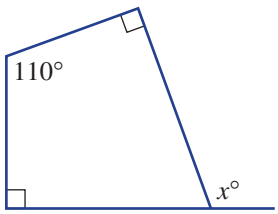
d 4 acute angles

8 A shopping centre's floor is a quadrilateral with one 120° angle and two 90° angles. What is the size of the fourth angle?

9 Explain why a rectangle, square and rhombus are all parallelograms.

10 These quadrilaterals include exterior angles. Find the value of x in each.

a



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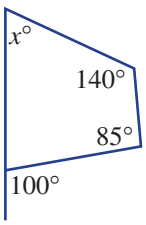
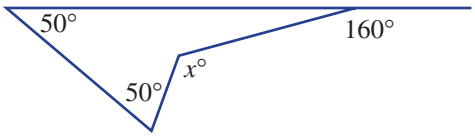


c

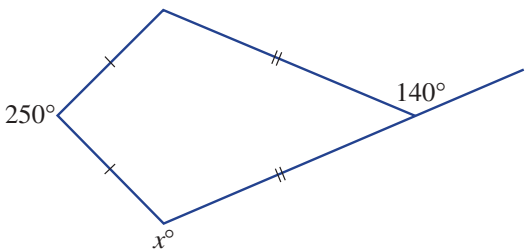


ENRICHMENT: Quad *x* challenge

— | 11

11 Find the value of x in these diagrams.

a



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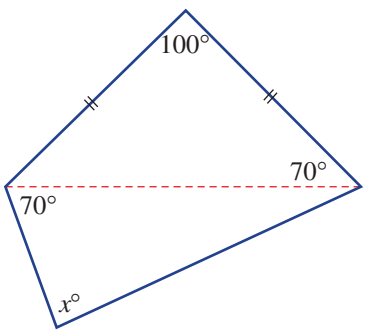

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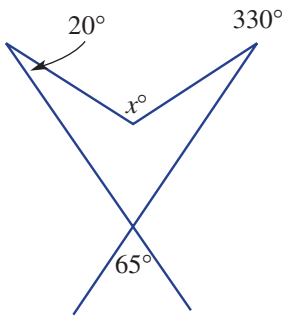

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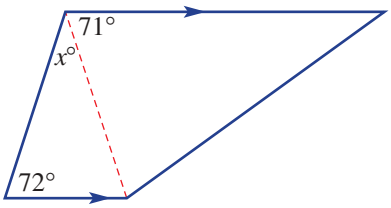
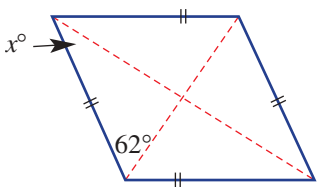


e



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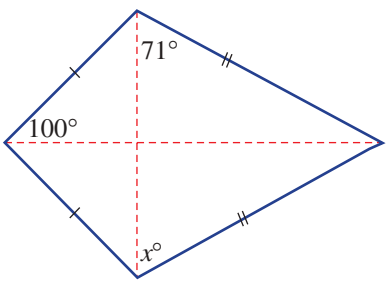

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8D Polygons EXTENDING

Learning intentions for this section:

- To know the formula for the interior angle sum of a polygon
- To be able to calculate unknown angles inside a polygon
- To be able to calculate the size of each interior angle of a regular polygon

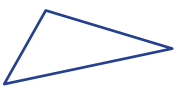
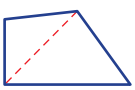
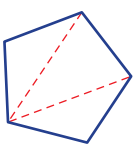
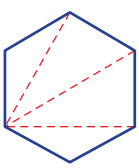
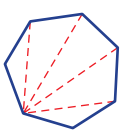
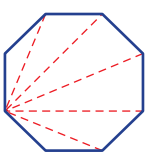
Past, present and future learning:

- This section includes some Stage 5 Path concepts that are relevant to Stage 6 Standard
- It may be necessary to recall the formula from memory in the HSC examination

Closed two-dimensional shapes with straight sides are called polygons. They are classified by their number of sides. We have already seen triangles (3 sides) and quadrilaterals (4 sides), which have been classified further by their special properties.

Lesson starter: Developing the rule

The following procedure uses the fact that the angle sum of a triangle is 180°. Copy and complete the table and try writing the general rule in the final row.

Shape	Number of sides	Number of triangles	Angle sum
Triangle 	3	1	$1 \times 180° = 180°$
Quadrilateral 	4	2	$___ \times 180° = ___$
Pentagon 	5		
Hexagon 	6		
Heptagon 	7		
Octagon 	8		
n-sided polygon	n		$(___) \times 180°$

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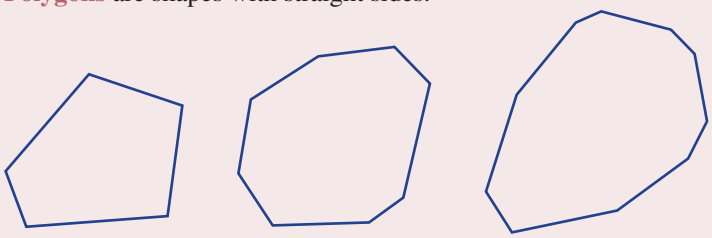
KEY IDEAS

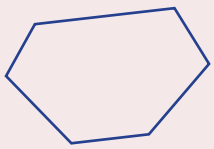
- **Polygons** are shapes with straight sides.



Pentagon Octagon Decagon

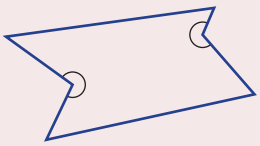
- Polygons can be convex or non-convex.
  - Convex polygon have all vertices pointing outwards.
  - Non-convex polygons have at least one vertex pointing inwards.

Convex



This means all interior angles are less than 180°.

Non-convex

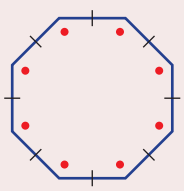


This means there is one or more reflex interior angles.

- Polygons are named according to their number of sides.

Number of sides	Name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
11	Undecagon
12	Dodecagon

- The angle sum, S, of a polygon with n sides is given by the rule $S = (n - 2) \times 180°$.
- The sum of the exterior angles of a convex polygon is 360°.
- A **regular polygon** has sides of equal length and equal interior angles.



A regular octagon

Key terminology: polygon, regular polygon, convex, non-convex

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Exercise 8D

UNDERSTANDING | 1–4 | 4

1 Write the missing word/rule from the table to complete the sentences.

regular	$S = (n - 2) \times 180°$
non-convex	hexagon

a A _____ has 6 sides.
b A _____ polygon has sides of equal length and equal angles.
c A _____ polygon has at least one reflex angle.
d The rule for the angle sum of a polygon is _____.

2 How many sides do these polygons have?

a Pentagon **b** Heptagon
c Quadrilateral **d** Undecagon
e Nonagon **f** Dodecagon

Hint: Refer to the Key Ideas for help.

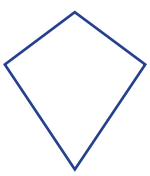
3 Use $S = (n - 2) \times 180°$ to find the angle sum, S, of these polygons. n is the number of sides.

a Hexagon **b** Octagon
c Undecagon **d** Heptagon

Hint: A convex polygon has all internal angles less than 180°.

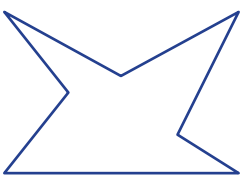
4 Name each of these shapes as convex or non-convex, and write their polygon name.

a

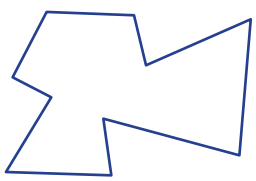


b

c



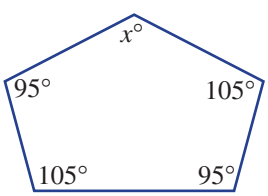
d



FLUENCY | 5–7(½) | 5–7(½)

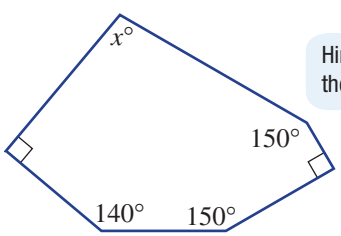
5 Find the value of x using the given angle sum.

a



Angle sum = 540°

b



Angle sum = 720°

Hint: All the angles must sum to the given angle sum.

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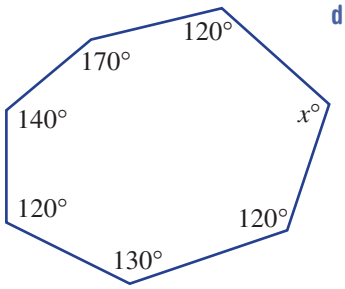
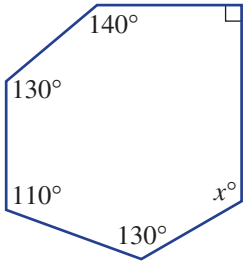
c



Angle sum = 900°

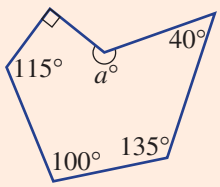
d



Angle sum = 720°

Example 9 Finding angles in polygons

Find the angle sum using $S = (n - 2) \times 180°$, then find the value of a.



SOLUTION

$n = 6$ and $S = (n - 2) \times 180°$
$= (6 - 2) \times 180°$
$= 720°$

$a + 90° + 115° + 100° + 135° + 40° = 720°$
$a + 480° = 720°$
$a = 240°$

EXPLANATION

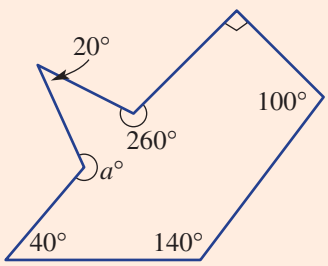
The shape is a hexagon with 6 sides so $n = 6$.

$4 \times 180° = 720°$

The sum of all angles is 720°. Simplify and solve for a.

Now you try

Find the angle sum using $S = (n - 2) \times 180°$, then find the value of a.



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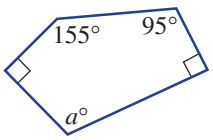
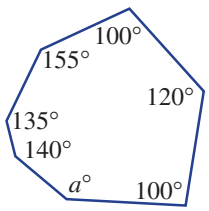
6 For each polygon, find the angle sum using $S = (n - 2) \times 180°$. Then find the value of a.

a



b



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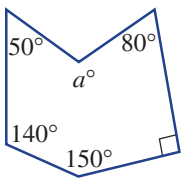

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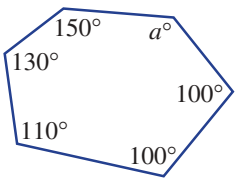

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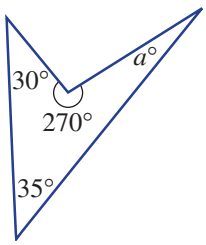
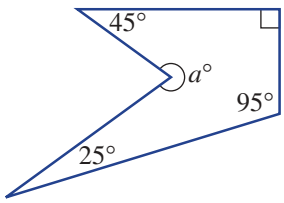


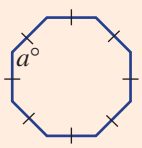
f



Hint: First find the angle sum using $S = (n - 2) \times 180°$, where n is the number of sides.

Example 10 Working with regular polygons

Find the size of an interior angle of a regular octagon.



SOLUTION

$n = 8$ and $S = (n - 2) \times 180°$

$= (8 - 2) \times 180°$

$= 1080°$

$\therefore a = 1080° \div 8$

$= 135°$

EXPLANATION

The regular octagon has 8 sides so use $n = 8$.

Each interior angle in a regular polygon is equal so divide by 8 to get a single angle.

Alternative method:

The exterior angle sum is 360°.

This is true for *all* polygons.

The exterior angles are $360° \div 8 = 45°$

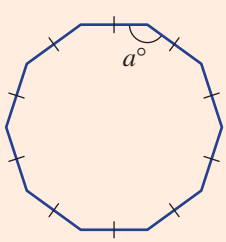
The interior angles are $180° - 45° = 135°$

Interior angle + exterior angle = 180°

$\therefore a = 135°$

Now you try

Find the size of an interior angle of a regular decagon.



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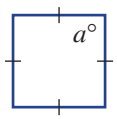
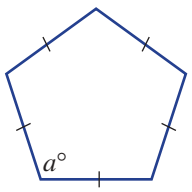
7 Find the size of an interior angle of these regular polygons. Round to two decimal places in part **e**.

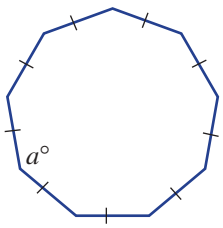
a



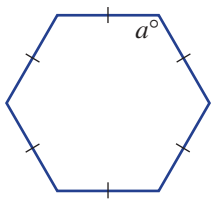
b



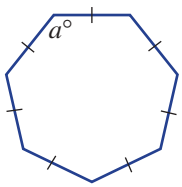
c



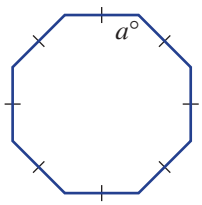
d



e



f



PROBLEM-SOLVING AND REASONING

8, 9 | 8–10

8 Calculate the number of sides if a polygon has the given angle sum.
Suggestion: Use the rule $S = (n - 2) \times 180°$.

a 2520°

b 4140°

c 18 000°

9 Decide whether the following are possible. If so, make a drawing.

a A hexagon with all angles equal but not all sides equal.

b A hexagon with all sides equal but not all angles equal.

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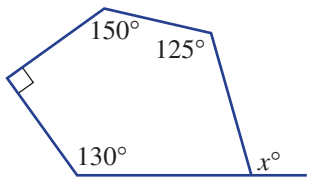
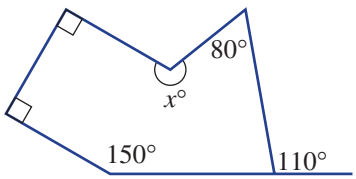
10 These polygons include exterior angles. Find the value of x in each.

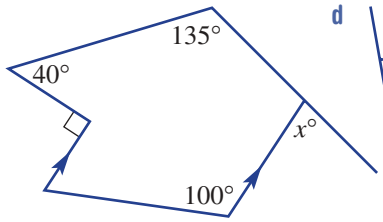
a



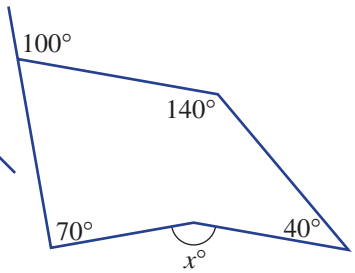
b

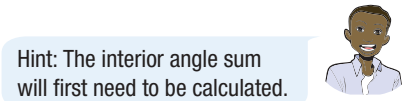


c



d





Hint: The interior angle sum will first need to be calculated.

ENRICHMENT: Maximum reflex — 11

11 Recall that a non-convex polygon has at least one reflex interior angle.

a Use drawings to decide whether the following are possible.

- **i** A quadrilateral with 2 reflex angles.
- **ii** A pentagon with 2 reflex angles
- **iii** A hexagon with 4 reflex angles

b What is the maximum number of interior reflex angles possible for these polygons?

- **i** Quadrilateral
- **ii** Pentagon
- **iii** Octagon

c Write a rule for the maximum number of interior reflex angles for a polygon with n sides.

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Progress quiz

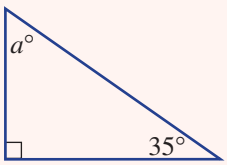
8A **1** For the angle 63°, determine the:

a complementary angle

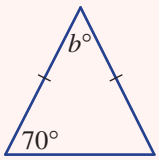
b supplementary angle

8A **2** Name the types of triangles shown and find the value of each pronumeral. Part **c** includes an exterior angle.

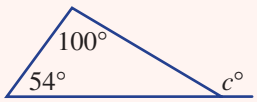
a



b

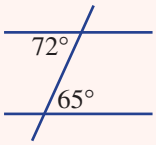


c

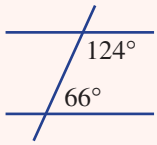


8B **3** Explain why the pairs of lines shown below are *not* parallel.

a

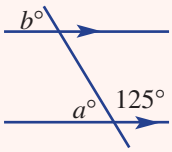


b

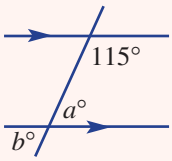


8B **4** Find the values of the pronumerals in the parallel lines below. Give reasons for your answers.

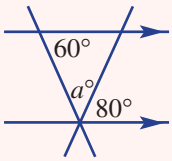
a



b

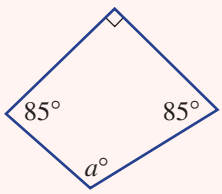


c

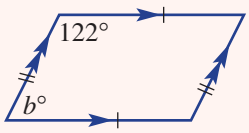


8C **5** Find the values of the pronumerals in these quadrilaterals and name the type of quadrilateral in parts **b** and **c**.

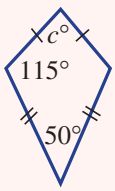
a



b

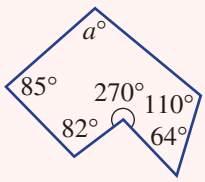


c

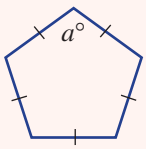


8D **6** Find the angle sum of each of the following polygons using $S = (n - 2) \times 180°$ and then find the value of a. The polygon in part **b** is a regular polygon.

a



b



8D **7** What is the angle sum of a

a triangle?

b quadrilateral?

c pentagon?

d hexagon?

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8E Enlargement and similar figures

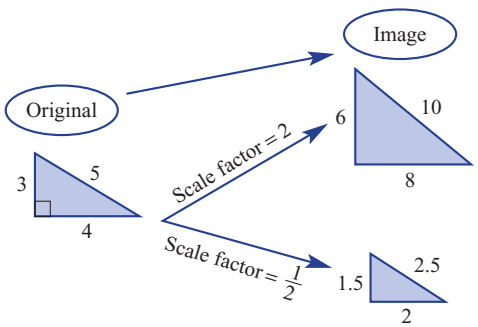
Learning intentions for this section:

- To know that two similar figures are connected by a scale factor
- To be able to construct similar figures using the enlargement transformation
- To be able to calculate a scale factor from similar figures
- To be able to find unknown side lengths in similar figures

Past, present and future learning:

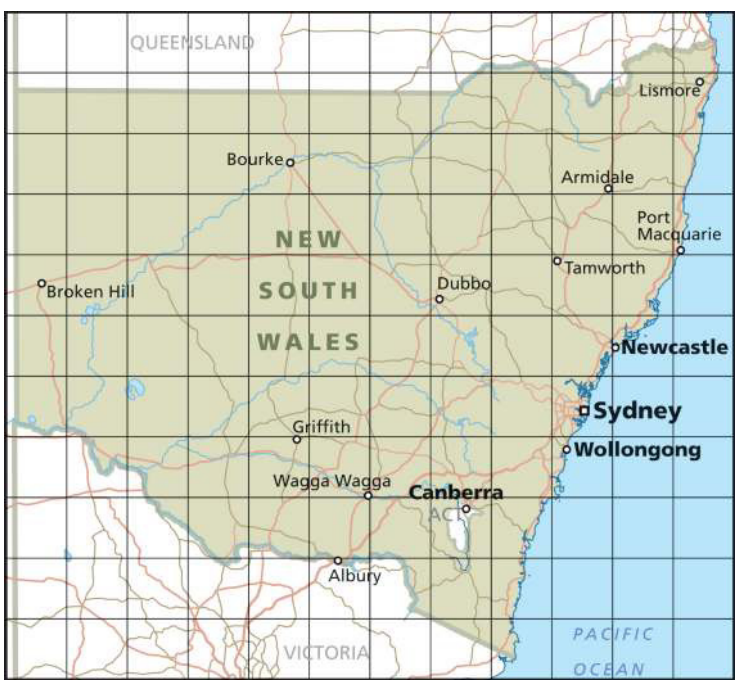
- This is an Extending topic in our Year 8 book, but it may be new to students
- Similar figures is part of a Stage 5 Core topic and also appears in Stage 6 Standard
- Students are not required to know or use the four tests for proving similarity

Similar figures have the same shape but not necessarily the same size. Architects and artists use scale factors to create small images and models of larger objects.



Lesson starter: Enlarging an image (the old-fashioned way)

The image below shows a map of NSW and ACT with a grid overlay.



Enlarge the map of New South Wales onto a larger grid and indicate the position of all the towns on your larger map. Your teacher may have a prize for the most accurate enlargement.

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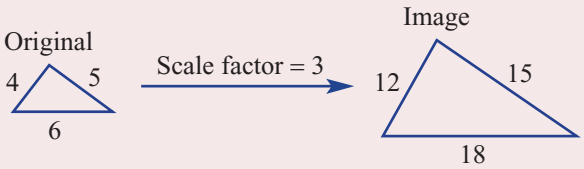
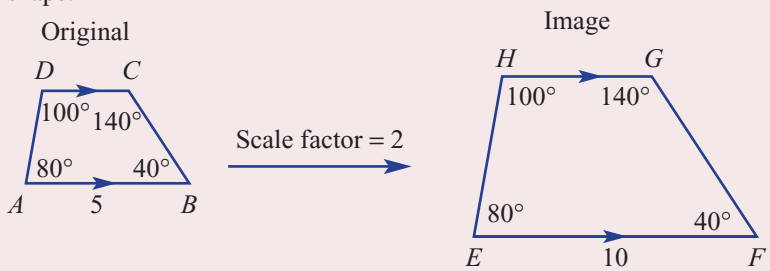
KEY IDEAS

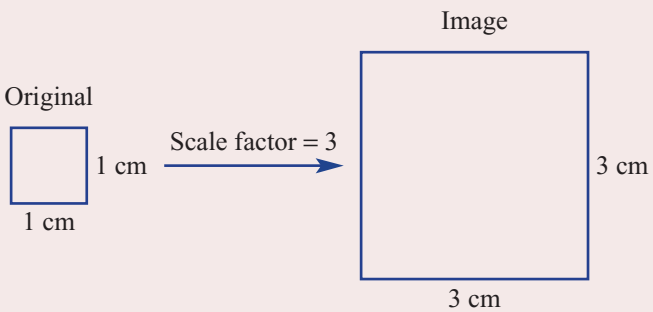
- **Enlargement** is a transformation that involves the increase or decrease in size of an object.
  - The shape of the object and the angles within the shape are unchanged.
  - Enlargement is done by multiplying all dimensions of a shape by a **scale factor**.



- The scale factor $= \dfrac{\text{image length}}{\text{original length}}$
- If the scale factor is greater than 1, the image will be larger than the original. If the scale factor is between 0 and 1, the image will be smaller. If the scale factor is 1, the image will be congruent.
- Two figures are **similar** if one can be enlarged to be congruent to the other.
  - Matching angles are equal.
  - Pairs of matching sides are in the same proportion or ratio.
- The symbol ||| is used to describe similarity.
  For example: $ABCD \mid\mid\mid EFGH$
  - The letters are usually written in matching order.
  - Scale factor $= \dfrac{EF}{AB} = \dfrac{10}{5} = 2$, so all the sides of the image are twice the length of the original shape.



- All squares are similar.
- All circles are similar.
- All equilateral triangles are similar.



Note: If the scale factor is 1, the original and the image are identical or **congruent**.

Key terminology: similar figures, enlargement, scale factor, ratio, corresponding sides, corresponding angles, image

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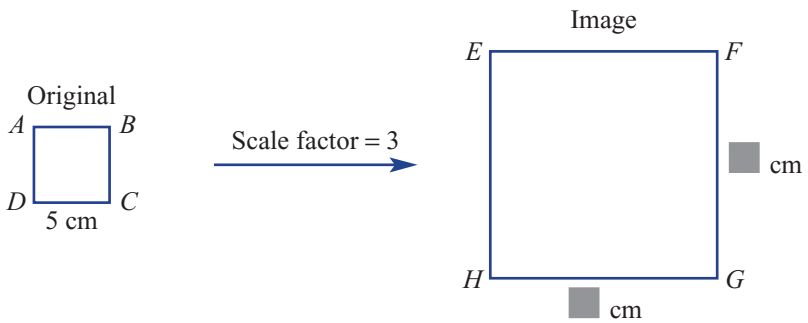
Exercise 8E

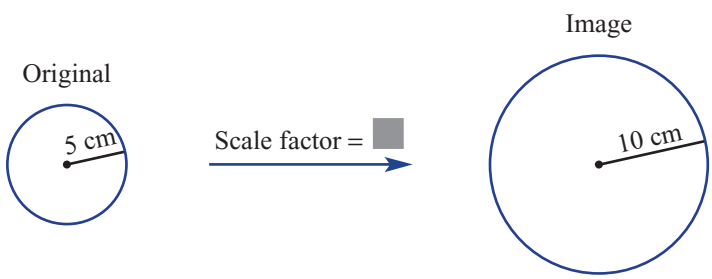
UNDERSTANDING | 1–4 | 1, 2, 4

1 Write the missing information, represented by the grey square (■) in your exercise book.
Note: Diagrams are not to scale.

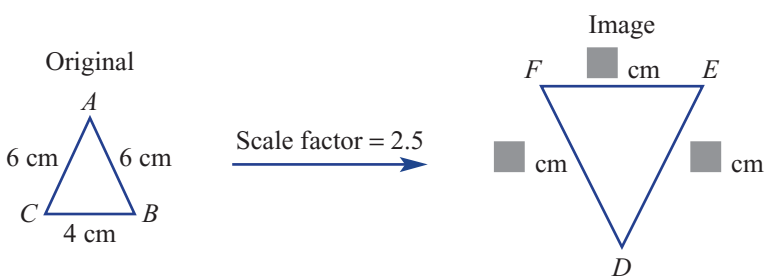
a *ABCD* is a square.



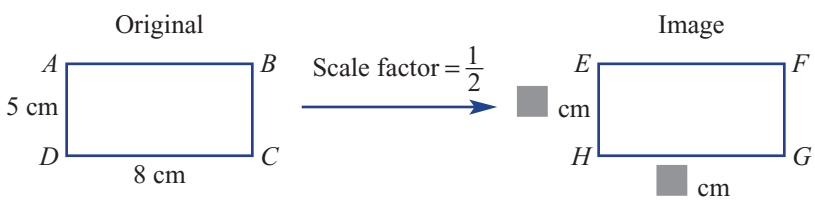
b



c



d *ABCD* is a rectangle.



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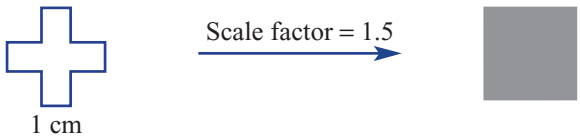
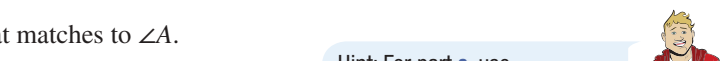
e Original

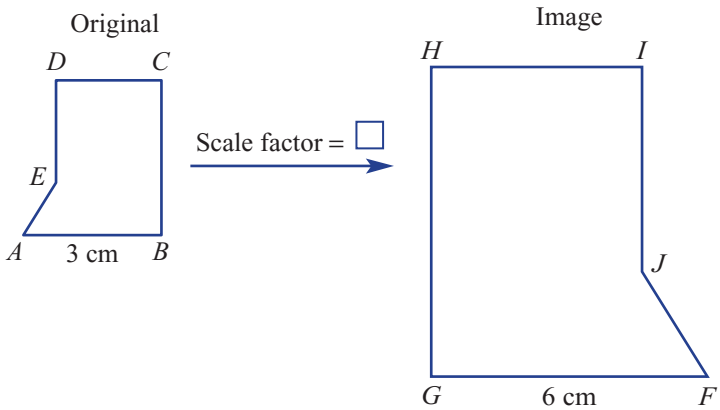


All sides are 1 cm.
All angles are 90°.

2 The two figures below are similar.

a Name the angle in the larger figure that matches to $\angle A$.
b Name the angle in the smaller figure that matches to $\angle I$.
c Name the side in the larger figure that matches to BC.
d Name the side in the smaller figure that matches to FJ.
e Use FG and AB to find the scale factor.


Hint: For part **e**, use
scale factor = $\frac{\text{image length}}{\text{original length}}$



3 Draw a rectangle with base 6 cm and height 4 cm. Use the following scale factors to draw similar rectangles.

a 3 **b** 2 **c** 1.5 **d** 1.25 **e** 0.5 **f** 0.75

4 Answer true (T) or false (F).

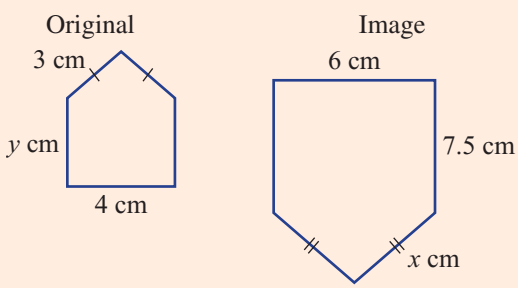
a All squares are similar.
b All rectangles are similar.
c All equilateral triangles are similar.
d All isosceles triangles are similar.
e All circles are similar.

FLUENCY 5, 6 5(½), 6, 7

Example 11 Using the scale factor

These figures are similar.

a Find a scale factor.
b Find the value of x.
c Find the value of y.



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SOLUTION

a Scale factor $= \frac{\text{image}}{\text{original}}$

$= \frac{6}{4}$

$= 1.5$ or $\frac{3}{2}$

b 3 cm on original

$= 3 \text{ cm} \times 1.5$ on image

$\therefore x = 4.5$

c 7.5 cm on image

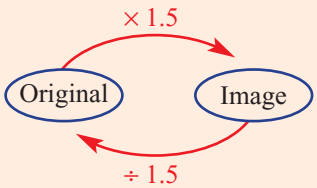
$= 7.5 \div 1.5$ on original

$\therefore y = 5$

EXPLANATION

Choose two matching sides are use

Scale factor $= \frac{\text{image length}}{\text{original length}}$



Multiply by the scale factor to get the length of the side of the larger image.

Divide by the scale factor to get the length of the side of the smaller image.

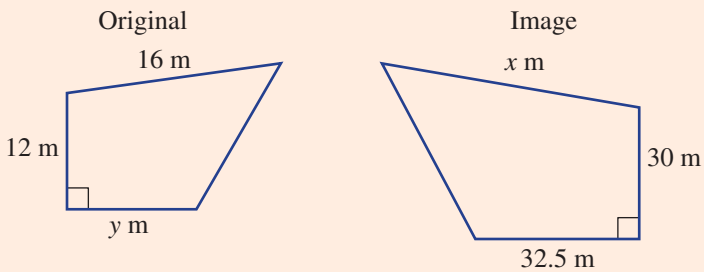
Now you try

These figures are similar.

a Find the scale factor.

b Find the value of x.

c Find the value of y.



5 Each of the pairs of figures shown here are similar. For each pair:

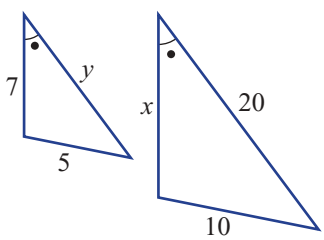
i Find the scale factor.

ii Find the value of x.

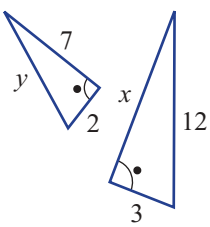
iii Find the value of y.

Hint: Scale factor $= \frac{\text{image}}{\text{original}}$
Make sure you choose a matching pair of sides.

a



b



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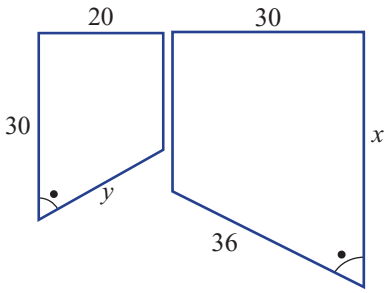
c



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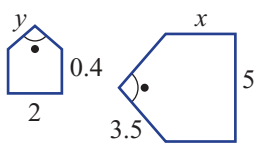

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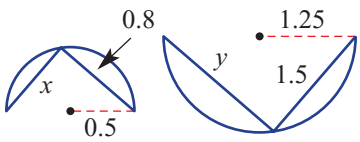

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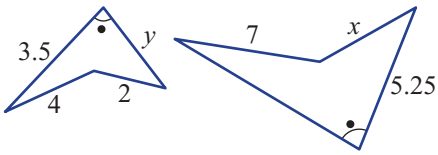
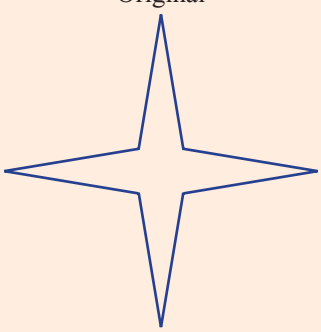


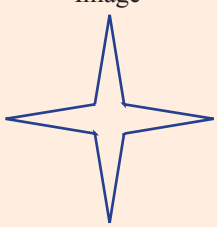
Example 12 Measuring lengths to estimate a scale factor

a Measure the original and the image to estimate the scale factor.

Original



Image



b On grid paper, use a ruler and a protractor to make another image of the original, using a scale factor of 1.5.

SOLUTION

a Each side of the original is approximately 18 mm.

Each side of the image is approximately 12 mm.

$$\text{Scale factor} = \frac{\text{image}}{\text{original}}$$

$$\approx \frac{12}{18}$$

$$\approx 0.7 \text{(to 1d.p.)}$$

EXPLANATION

Measure a line segment in the original, in mm.

Measure the matching line segment in the image.

$$\text{Scale factor} = \frac{\text{image}}{\text{original}}$$

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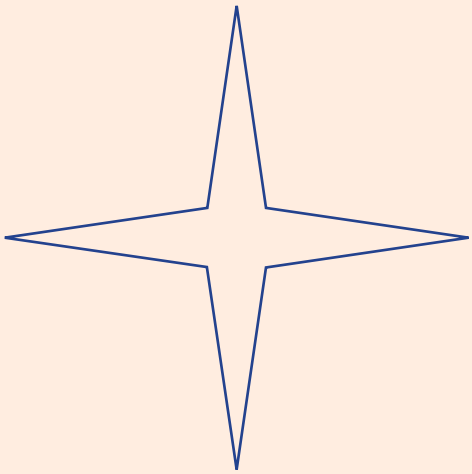
b



The edges of the original are approximately 18 mm.

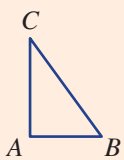
$18 \times 1.5 = 27$

The edges of the image will be 27 mm. The angles will not change in size.

Now you try

Copy the given diagram using plenty of space, and use these scale factors to enlarge ΔABC.

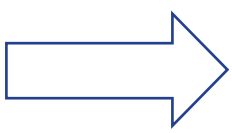
a Scale factor 2 **b** Scale factor $\frac{1}{2}$



6 Use a ruler, calculator and grid paper to enlarge these shapes, using a scale factor of 1.5.

a

b



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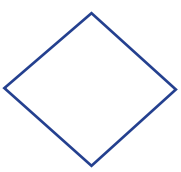

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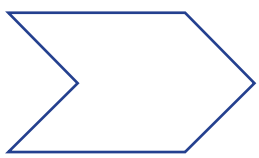


7 Repeat Question **6**, using a scale factor of 0.5.

PROBLEM-SOLVING AND REASONING

8, 9 | 10, 11

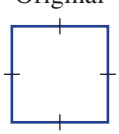
8 A square is enlarged by a scale factor of 4.

a Are the internal angles the same for both the original and the image?

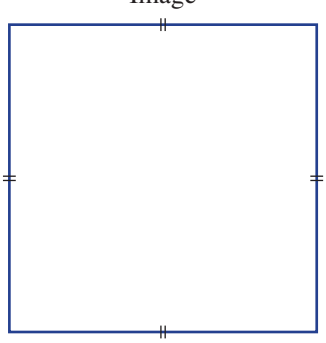
b If the side length of the original square was 2 cm, what would be the side length of the image square?

c If the side length of the image square was 100 cm, what would be the side length of the original square?

Original



Image



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9 A photo of a sunflower is enlarged by a scale factor of 2. If the flower in the larger image has a diameter of 7.4 cm, what is the diameter of the flower in the smaller image?

10 An A4 sheet of paper is 210 mm by 297 mm. An A3 sheet of paper is 297 mm by 420 mm. What is the scale factor, correct to two decimal places?

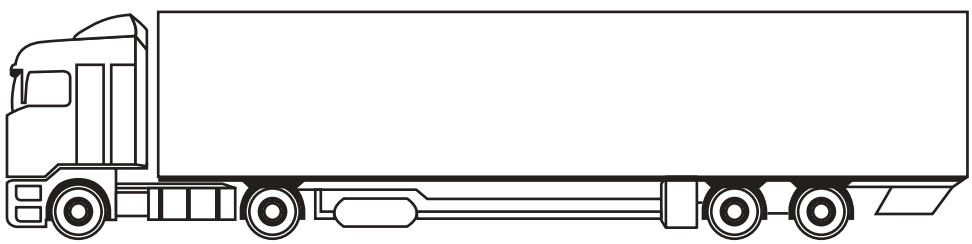
11 Explain why:

- **a** any two squares are similar
- **b** any two equilateral triangles are similar
- **c** any two rectangles are not necessarily similar
- **d** any two isosceles triangles are not necessarily similar

ENRICHMENT: Actual lengths from drawings and maps — 12, 13

12 The container of this truck is 12.5 metres long.

- **a** Convert 12.5 metres to millimetres.
- **b** Measure the length of the container on the truck in the drawing, in millimetres.
- **c** If the drawing is the original, what scale factor will be used to build the real truck?
- **d** Measure the height of the container on the truck in the drawing, in millimetres.
- **e** Use the scale factor to estimate the height of the container on the real truck, in millimetres.



13 A map has a scale ratio of 1 : 50 000.

- **a** What length on the ground is represented by 2 cm on the map?
- **b** What length on the map is represented by 12 km on the ground?

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8F Applying scale factor to similar triangles

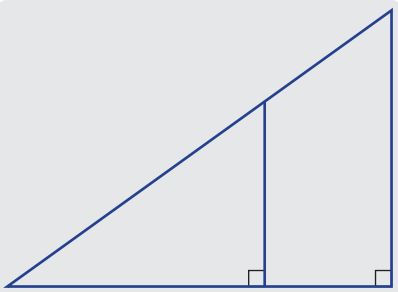
Learning intentions for this section:

- To be able to identify a pair of similar triangles in a given context
- To be able to calculate the scale factor and use it to find unknown lengths

Past, present and future learning:

- This is an Extending topic in our Year 8 book, but it may be new to students
- Similar triangles is part of a Stage 5 Core topic and also appears in Stage 6 Standard
- Students are not required to know or use the four tests for proving similarity

Similar triangles can be used in many mathematical and practical problems. If two triangles are known to be similar, then the properties of similar triangles can be used to find missing lengths or unknown angles. The approximate height of a tall object or the width of a projected image can be found using similar triangles.



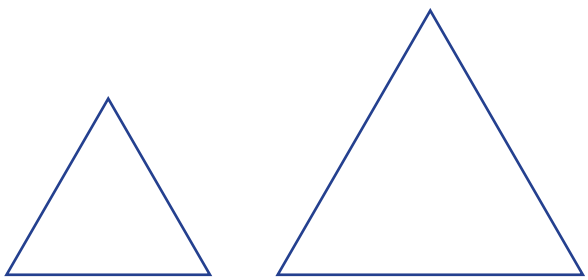
This diagram shows two similar triangles.

Lesson starter: Why is sine of 30° exactly 0.5?

When you were doing trigonometry, you may have noticed that your calculator usually gives long, non-repeating decimals. For example, calculate these:

- sin 28°
- sin 29°
- sin 31°

Now try sin 30°. It is exactly 0.5. The following experiment will explain why this is true. Trace the triangles (right) and follow these steps:



1. Choose a triangle.
2. Fold it down the middle along the axis of symmetry.
3. Cut it in half and throw half away.
4. Measure the three angles and write them on your triangle.
5. Measure the three sides and write them on your triangle.
6. Divide the side opposite the 30° angle by the hypotenuse.
7. Repeat the experiment using another equilateral triangle.

- Copy and complete:
  In any right-angled triangle containing an angle of 30°, the _____ side is exactly _____ of the _____.
  Therefore, sine of 30° is exactly _____.

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KEY IDEAS

- To apply similarity in practical problems involving triangles that you know to be similar:
  - Calculate the scale factor.
  - Use the scale factor to find the value of any unknowns.

Key terminology: similar figures, scale factor

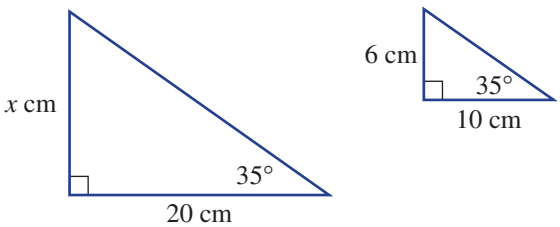
Exercise 8F

UNDERSTANDING 1–3 3

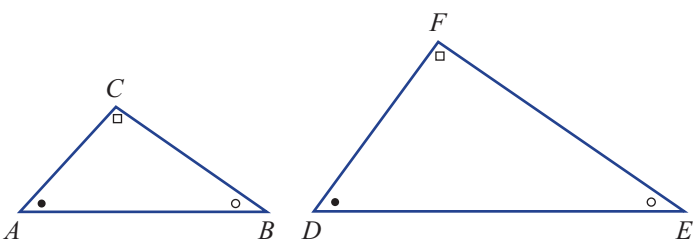
1 Here are two triangles.

a What is the scale factor if the smaller triangle is enlarged to the size of the larger triangle?

b Find the value of x.



2 These two triangles are similar.



Hint: You can name the angle like this: $\angle S$.

a Which vertex on ΔDEF matches vertex B on ΔABC?

b Which vertex on ΔABC matches vertex F on ΔDEF?

c Which side on ΔDEF matches side AC on ΔABC?

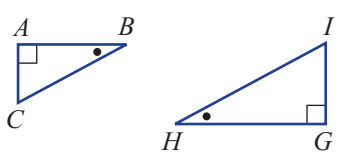
d Which side on ΔABC matches side EF on ΔDEF?

e Which angle on ΔABC matches ΔD on ΔDEF?

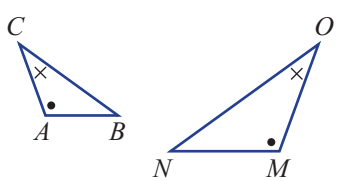
f Which angle on ΔDEF matches ΔB on ΔABC?

3 Write similarity statements for these pairs of similar triangles. Write letters in matching order.

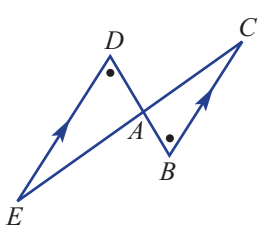
a



b



c



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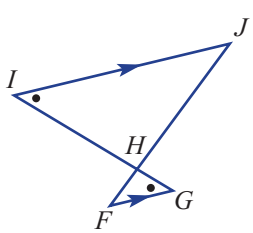

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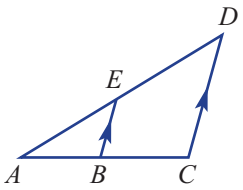
e



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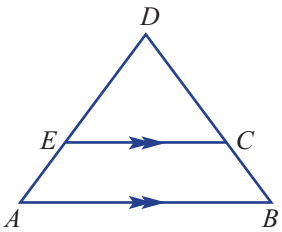
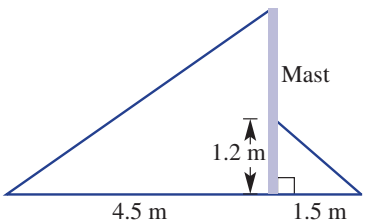


FLUENCY 4–7 5–8

4 Two similar triangles are created by cables supporting a yacht's mast.

a Find a scale factor for the two triangles.

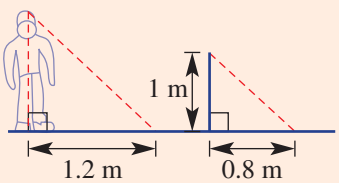
b Find the height of the mast.



Example 13 Applying similarity

Chris' shadow is 1.2 m long while a 1 m vertical stick has a shadow 0.8 m long.

Determine Chris' height.



SOLUTION

$$\text{Scale factor} = \frac{1.2}{0.8} = 1.5$$

$$\therefore \text{Chris' height} = 1 \times 1.5$$
$$= 1.5 \text{ m}$$

EXPLANATION

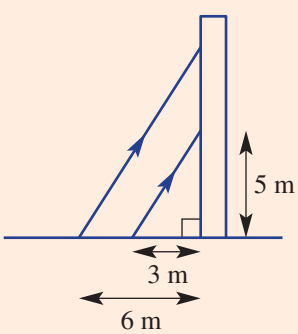
First find the scale factor.

Multiply the height of the stick by the scale factor to find Chris' height.

Now you try

Two ladders lean against a vertical wall at the same angle as shown. The distances that the base of the ladders are from the wall are 3 m and 6 m.

Find how high the longer ladder reaches up the wall if the shorter ladder reaches up 5 m.



5 A tree's shadow is 20 m long while a 2 m vertical stick has a shadow 1 m long.

a Give a reason why the two triangles contained within the objects and their shadows are similar.

b Find the height of the tree.

Hint: Construct a diagram as in Example 13.

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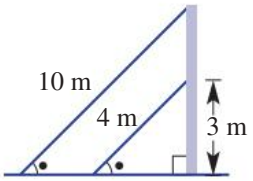
6 Jai stands 6 m from a lamp post and casts a 2 m shadow. The shadow from the pole and from Jai end at the same place. If Jai is 1.5 m tall, what is the height of the lamp post?

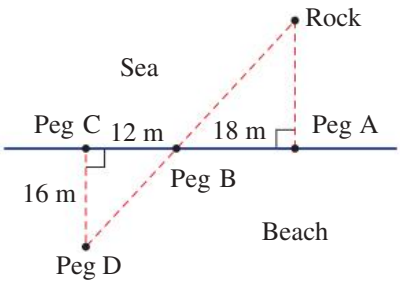
7 Two cables support a steel pole at the same angle as shown. The two cables are 4 m and 10 m in length and the shorter cable reaches 3 m up the pole.

a Give a reason why the two triangles are similar.

b Find the height of the pole.



8 Ali is at the beach and decides to estimate how far an exposed rock is from the seashore. He places four pegs in the sand as shown, and measures the distance between them. How far is the rock from the beach?

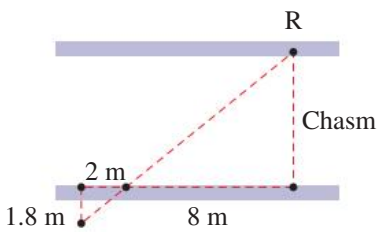


PROBLEM-SOLVING AND REASONING

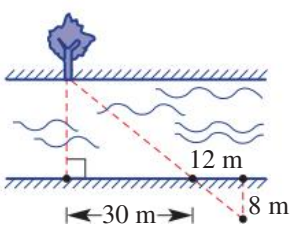
9, 10 | 9, 11

9 A deep chasm has a large rock (R) sitting on its side, as shown. Find the width of the chasm.

Hint: Don't forget to find the scale factor.



10 Joanne wishes to determine the width of the river shown without crossing it. She places four pegs as shown. Calculate the river's width.



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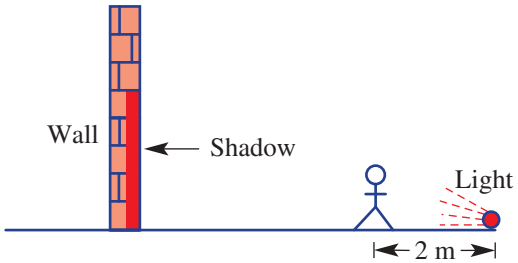
11 A person 1.8 m tall stands in front of a light that sits on the floor.
The person casts a shadow on the wall behind them.



a Find how tall the shadow will be if the distance between the wall and the light is:
- **i** 4 m
- **ii** 10 m
- **iii** 3 m

b How tall will the shadow be if the distance between the wall and the person is:
- **i** 4 m?
- **ii** 5 m?

c Find the distance from the wall to the person if the shadow is:
- **i** 5.4 m
- **ii** 7.2 m

ENRICHMENT: Enlargement of photographs — 12

12 When you take some photos to a shop to be 'enlarged' there are several options, which are sometimes expressed using inches. The standard size is 4 inches by 6 inches, which is usually called a '4 by 6'.

Here are some other sizes that are sold in Australia.

5 by 7	6 by 8	8 by 10
8 by 12	11 by 14	12 by 16
16 by 20	20 by 24	20 by 30

a '4 by 6' can be simplified to the ratio 2 : 3. Express the other nine sizes as simplified ratios.

b A '4 by 6' is similar to the '8 by 12'. The scale factor is 2. What other size is similar to the '4 by 6'? What is the scale factor?

c What size is similar to the '6 by 8'? What is the scale factor?

d What size is similar to the '8 by 10'? What is the scale factor?

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Maths@Work

Animator

Animators are creative people who use their skills to hand-draw, model or computer-generate images to create animated effects.

While animators are artistic, many other skills are necessary to excel in this field. Animators must be patient, have an eye for detail, and be able to work in groups as well as independently. They need excellent communication skills and IT skills.

Geometry and linear programming are important in this field as they allow the animator to illustrate how an image can be rotated and shifted.

Animating is not just restricted to the movies. Computer and console computer games, television series, advertising and even corporate training organisations often directly employ or out-source animators.

1 Being able to accurately copy drawings by hand is a skill used in some forms of animation. Use your knowledge of angles and proportion to carefully copy these logos into your workbook.

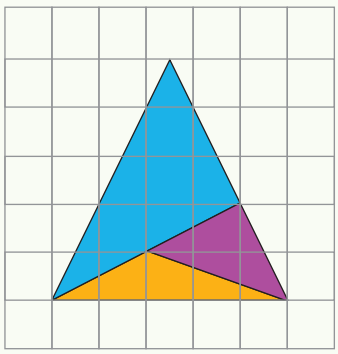
a

b

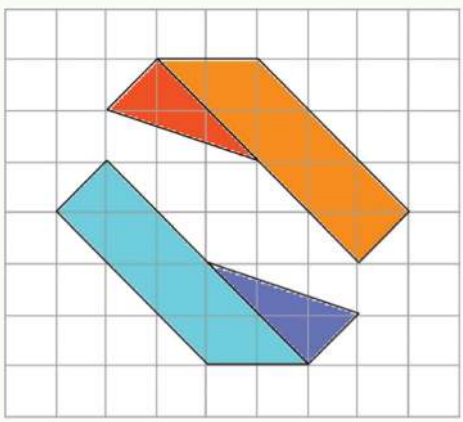
c

2 Using graph paper, enlarge each of the following logos by a factor of two.

a



b



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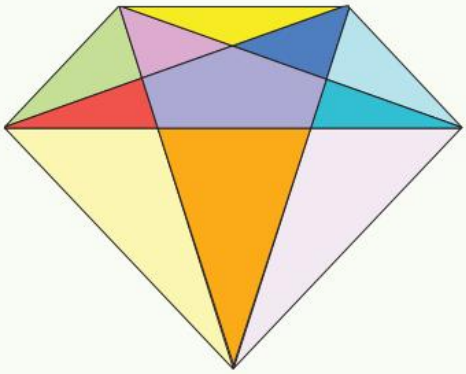
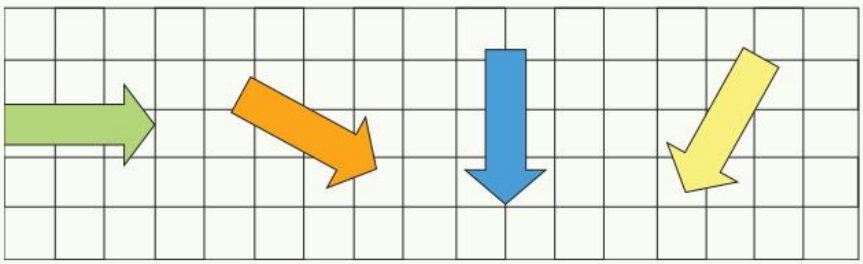
3 How many pairs of congruent (identical) triangles are in the symmetrical logo below? Can you see any pairs of similar triangles?



4 Compare each of the following figures and state how each has been rotated relative to the preceding one. You will need to use a protractor.



Using technology

5 Animators enjoy creating original designs. Use computer drawing software to design a geometrical logo from the initials of your name. Some ideas for 'D and W' logos are shown below, but your design can be original if you wish.

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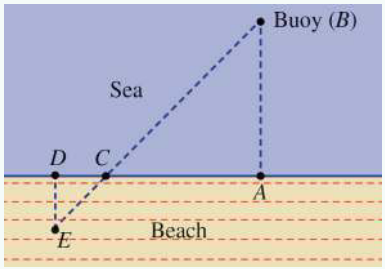
Working mathematically

Ocean ironman

The organisers of an ocean ironman event want to estimate the distance from the beach to a floating buoy in the sea. Flags are placed on the beach by the organisers in an effort to create similar triangles as shown.



The flag positions are at A, C, D and E and the line segments AB and DE are assumed to be at right angles with AD at the beach.

Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate.

Routine problems

Once the flags are in position, one of the organisers measures the following;

- $AC = 200$ metres
- $CD = 5$ metres
- $DE = 4$ metres

a Find the scale factor between the two similar triangles.

b Hence, estimate the distance from the beach to the buoy (distance AB).

Non-routine problems

Explore and connect

Choose and apply techniques

The problem is to determine the distance between the beach and the buoy using similar triangles.

a Write down all the relevant information that will help solve this problem, and illustrate using diagrams.

b Two more attempts at positioning flags and measuring distances are undertaken by the organisers.

Jimmy's placement:

- $AC = 360$ metres
- $CD = 8$ metres
- $DE = 3$ metres.

Olivia's placement:

- $AC = 90$ metres
- $CD = 5$ metres
- $DE = 8$ metres.

i Use Jimmy's model to estimate the distance between the beach and the buoy.

ii Use Olivia's model to estimate the distance between the beach and the buoy.

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Communicate thinking and reasoning

c Another organiser attempts to place the flags such that:

- $AC = 100$ metres
- $CD = 50$ centimetres
- $DE = 1.2$ metres.

Do you think that arranging the flags in this way is a good idea? Explain why.

d If the actual distance from the beach to the buoy is 200 metres, determine an arrangement for the flags that would deliver the correct distance. Illustrate with a diagram including the measurements.

e Summarise your results in a table.

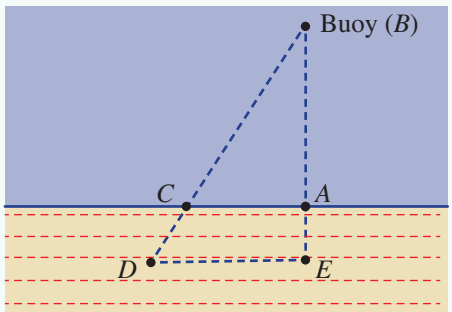
	AC	*CD*	*DE*	*AB*
Person 1	200	5	4	
Jimmy				
Olivia				
Person 4				

Comment on any findings and recommendations.

Extension problems

Problem-solve

a Another way to arrange the flags is to use the model shown here. Using this model, identify the pair of similar triangles.



b Using the following measurements, estimate the distance from the beach to the buoy.

- $AC = 100$ metres
- $AE = 60$ metres
- $DE = 140$ metres

c Compare the two models studied in this task using the different diagrams. Do you have a preferred model? Explain why.

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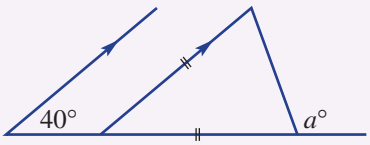
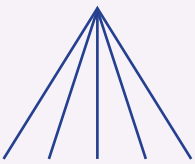
Puzzles and challenges

1 Use 12 matchsticks to make 6 equilateral triangles.

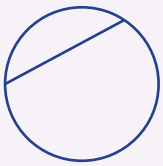
2 Find the value of a in the diagram.



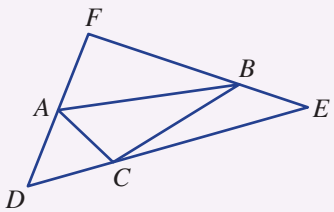
3 How many acute angles are there in this diagram?



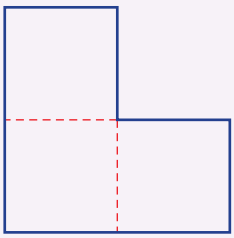
4 A circle is divided using chords (one chord is shown here, giving two regions). What is the maximum number of regions that can be formed if the circle is divided with 4 chords?



5 Explore (using dynamic geometry software) where the points A, B and C should be on the sides of ΔDEF so that the perimeter of ΔABC is a minimum.

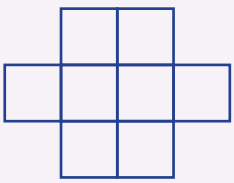


6 Draw this shape on square grid paper.



Divide the shape into four congruent (i.e. identical) shapes.

7 Write the numbers 1 to 8 into the squares, so that the squares with consecutive numbers do not touch (neither edges nor corners). For example, the square with 2 cannot touch squares with 3 or 4.



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Chapter summary

Properties of geometrical figures

Parallel lines

If the lines are parallel:

- Corresponding angles are equal
- Alternate angles are equal
- Co-interior angles are supplementary $a + b = 180$

Quadrilaterals

Angle sum = 360°

Parallelogram

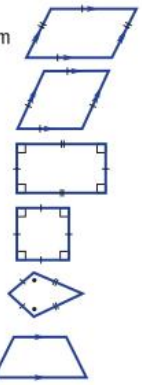
Rhombus

Rectangle

Square

Kite

Trapezium



Polygons

The sum of the interior angles in a polygon with n sides is $S = (n - 2) \times 180°$. Regular polygons have all sides and angles equal.

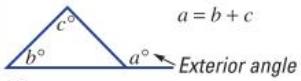
Angles

Acute $0° < \theta < 90°$
Right 90°
Obtuse $90° < \theta < 180°$
Straight 180°
Reflex $180° < \theta < 360°$
Revolution 360°
Complementary angles sum to 90°.
Supplementary angles sum to 180°.
Vertically opposite angles are equal.

Triangles

Sum of angles is 180°.
Exterior angle equals sum of two opposite interior angles

$a = b + c$



Types:
Acute angled – all angles < 90°
Obtuse angled – 1 angle > 90°
Right angled – 1 angle 90°
Equilateral – all angles 60° – all sides are equal
Isosceles – 2 angles and 2 sides are equal
Scalene – all sides and angles are different sizes

Similar figures

These are the same shape but different size.

Scale factor = $\frac{\text{Image length}}{\text{Original length}}$

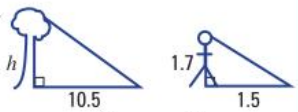
Applying similar triangles

In practical problems:

- Look to identify and prove pairs of similar triangles
- Find a scale factor
- Use this to find the value of any unknowns.

e.g. The shadow cast by a tree is 10.5 m while a person 1.7 m tall has a 1.5 m shadow. How tall is the tree?

The triangles are similar.
Scale factor = $\frac{10.5}{1.5} = 7$
$\therefore h = 1.7 \times 7$
$= 11.9$ m
Tree is 11.9 m tall.



(Not to scale)

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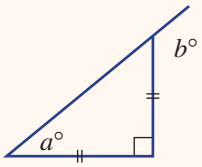
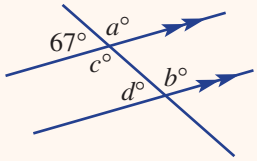
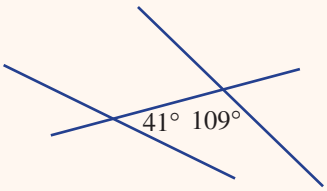
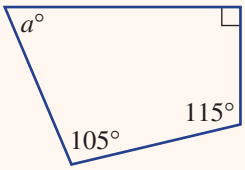
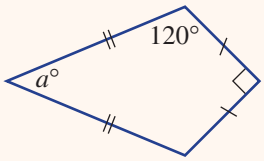
Chapter checklist

Chapter checklist with success criteria

A printable version of this checklist can be downloaded from the Interactive Textbook.

		✔
8A	**1 I can find supplementary and complementary angles.** e.g. State the supplementary and complementary angle to 25°.	☐
8A	**2 I can name triangles and find the size of unknown angles.** e.g. Name the type of triangle in this diagram and find the values of the pronumerals. 	☐
8B	**3 I can find unknown angles in parallel lines.** e.g. Find the values of the pronumerals in this diagram and give a reason. 	☐
8B	**4 I can explain why two lines are parallel.** e.g. Decide, with reasons, whether the given pair of lines is parallel. 	☐
8C	**5 I can identify properties of various quadrilaterals.** e.g. Name the special quadrilaterals that have diagonals that intersect at 90°.	☐
8C	**6 I can find an unknown angle in a quadrilateral.** e.g. Find the value of a in this quadrilateral. 	☐
8C	**7 I can find an unknown angle in a special quadrilateral.** e.g. Find the value of a in this kite. 	☐

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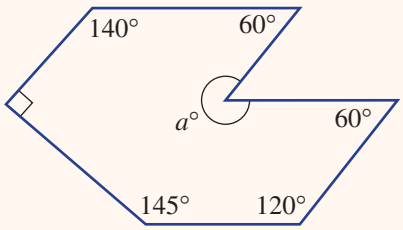
Chapter checklist with success criteria

8D Ext

8 I can find an angle sum of a polygon and an unknown angle in a polygon.
e.g. Find the value of a in this heptagon after finding the angle sum.

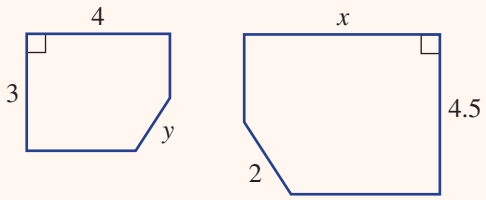


8D Ext

9 I can find the internal angle in a regular polygon.
e.g. Find the size of an internal angle inside a regular octagon.

8E

10 I can find the scale factor for a pair of similar figures and find the length of unknown sides.
e.g. Find the scale factor for these similar figures then find the value of the pronumerals.

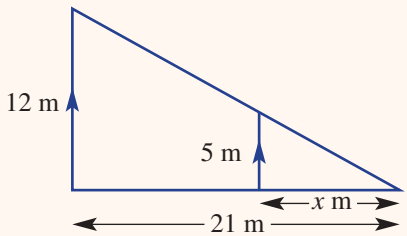


8E

11 I can draw a similar figure using the enlargement transformation and a given scale factor.
e.g. Copy the given diagram and enlarge the given shape using the scale factors of 3 and also $\frac{1}{2}$.

8F

12 I can find a scale factor and use this to find an unknown length.
e.g. If the given pair of triangles is known to be similar, find the value of x.



8F

13 I can identify similar triangles in a real context and use a scale factor to find an unknown length.
e.g. A tree's shadow is 20 m long and a 4 m high vertical stick has a 5 m shadow at the same time of day.
a Use a diagram to form a pair of similar triangles and give a reason why they are similar.
b Use the scale factor to find the height of the tree.

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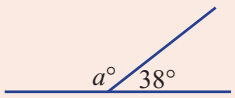
Chapter review

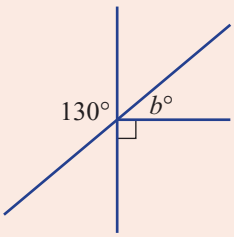
Short-answer questions

8A **1** Find the value of the pronumerals.

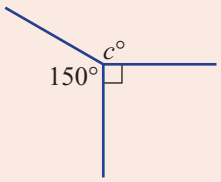
a



b

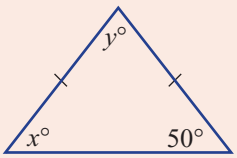


c

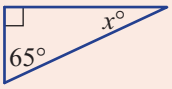


8A **2** Name the following triangles and find the value of the pronumerals.

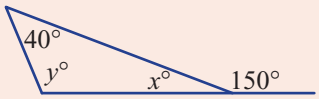
a



b

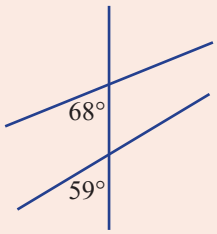


c

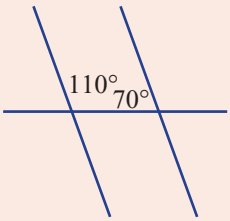


8B **3** Decide whether the following contain a pair of parallel lines. Give a reason.

a



b



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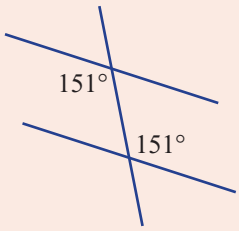
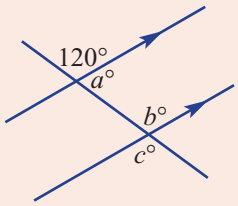
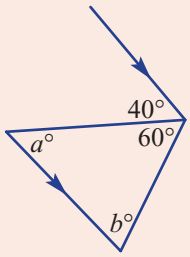


8B **4** State the values of the pronumerals.

a



b



8C **5** Which of the six 'special' quadrilaterals *definitely* have:

kite	trapezium	rectangle
square	rhombus	parallelogram

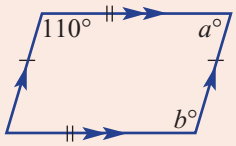
a diagonals of equal length?

b diagonals intersecting at right angles?

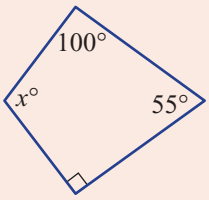
c equal opposite sides?

8C **6** Find the value of each pronumeral in the following quadrilaterals.

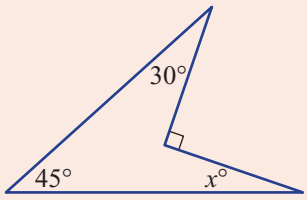
a



b



c



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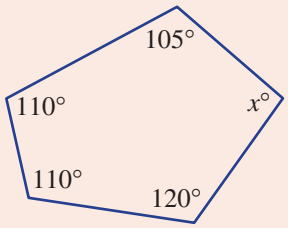
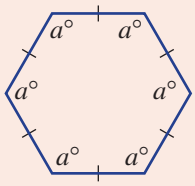
8D **7** Find the value of the pronumerals in these polygons. Use $S = (n - 2) \times 180°$ for the angle sum.

Ext

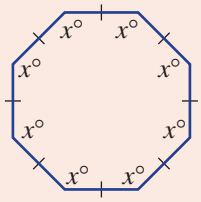
a



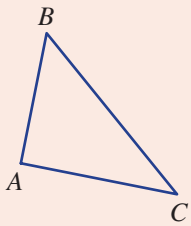
b



c

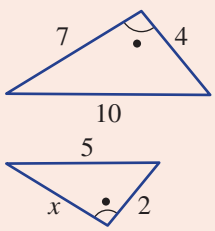


8E **8** Copy the given diagram using plenty of space. Using the scale factor of 3, enlarge ΔABC.



8F **9** For the following pairs of similar triangles, find the value of x.

a



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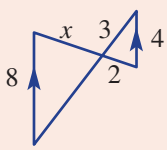

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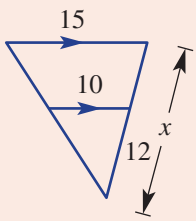
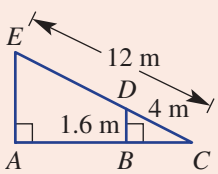


8F **10** A conveyor belt loading luggage onto a plane is 12 m long. A vertical support 1.6 m high is placed under the belt. It is 4 m along the conveyor belt, as shown in the diagram.

Ext

a Find the scale factor for the two similar triangles.

b Find the height (AE) of the luggage door above the ground.



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Chapter review

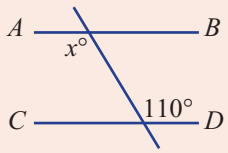
Multiple-choice questions

8A **1** The supplementary angle of 55° is:

A 35° **B** 55° **C** 95° **D** 125° **E** 305°

8B **2** AB is parallel to CD. The value of x is:

A 110 **B** 70 **C** 20
D 130 **E** 120

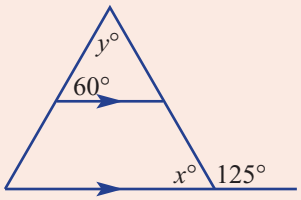


8A **3** If two angles in a triangle are 30° and 80°, then the third angle is:

A 110° **B** 70° **C** 90° **D** 80° **E** 30°

8B **4** The values of x and y in the diagram are:

A $x = 55, y = 85$
B $x = 45, y = 65$
C $x = 50, y = 60$
D $x = 55, y = 65$
E $x = 65, y = 55$



8D Ext **5** Using $S = (n - 2) \times 180°$, the sum of the interior angles in a hexagon is:

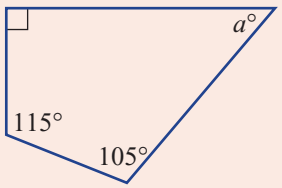
A 1078° **B** 360° **C** 720° **D** 900° **E** 540°

8C **6** The quadrilateral that can have no parallel sides is a:

A kite **B** trapezium **C** parallelogram
D rhombus **E** square

8C **7** The value of a in the diagram is:

A 45 **B** 310 **C** 75
D 60 **E** 50

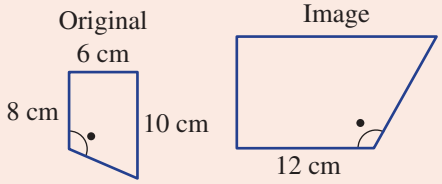


8E **8** If a square is enlarged by a factor of 2, what happens to the area?

A It stays the same
B It doubles
C It is multiplied by a factor of 4
D It depends on the size of the original square
E None of the above

8E **9** The scale factor in the two similar figures shown that enlarges the original figure to its image is:

A $\frac{2}{3}$ **B** 2 **C** 1.2
D 1.5 **E** 0.5



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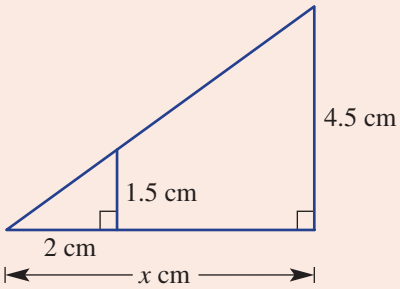
8F **10** The value of x in the diagram is:

A 6 **B** 9 **C** 10
D 8 **E** 7.5

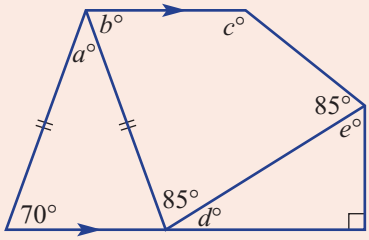


Extended-response questions

1 In this diagram, there are two triangles and one quadrilateral.

a Name the types of triangles.

b Find the values of the pronumerals, in alphabetical order. Give a reason at each step.

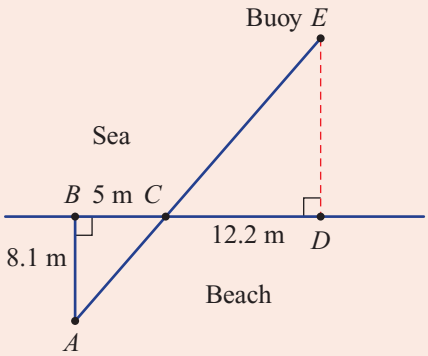


2 A buoy (E) is floating in the sea at some unknown distance from the beach, as shown. The points A, B, C and D are measured and marked out on the beach, as shown. Triangles ABC and EDC are similar.

a Name the angle that is vertically opposite to $\angle ACB$.

b What is the scale factor?

c Find the distance from the buoy to the beach (ED), to one decimal place.



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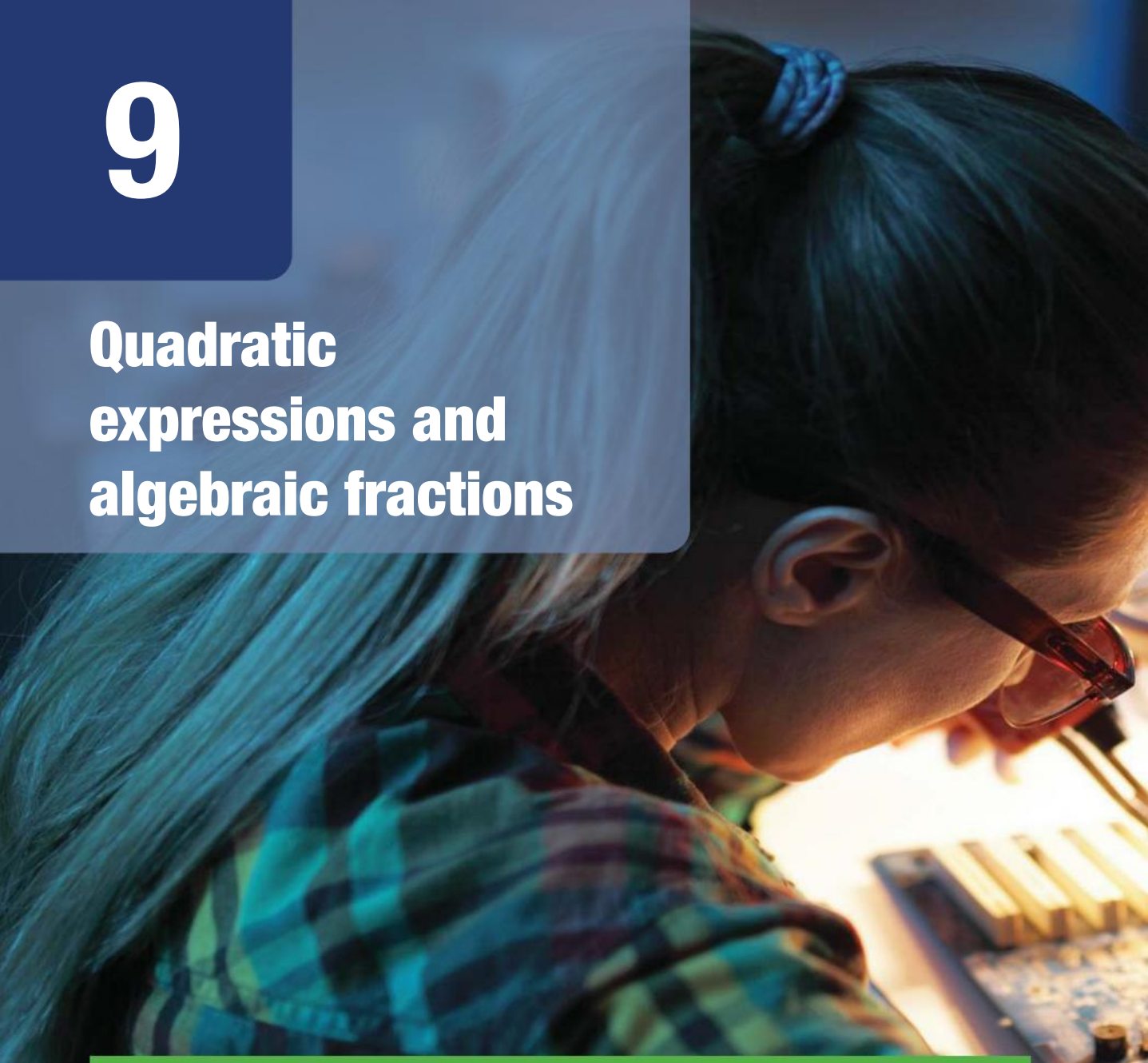
9

Quadratic expressions and algebraic fractions



Maths in context: Financial analysis in business and other applications of algebra

Algebra skills are essential when applying formulas in business, the professions and the trades, such as the air-conditioning, aviation, construction, electrical, electronic, manufacturing, mechanical, metal working, plumbing, retail and welding trades.

- Apprenticeship training for the electrical trades involves using many formulas requiring algebraic techniques including fractions. Nurses use algebraic techniques including fractions to determine medical dosage amounts and intravenous fluid quantities.
- Financial mathematical analysis is a key to success for businesses. Algebraic skills are required to apply formulas that calculate the financial aspects of a business, including expenses, revenue, losses, profits, GST, wages, tax amounts, insurance and any loan repayments.
- Examples of small businesses that require financial analysis include hairdressers, bakers, cake makers, café or food truck owners, dog groomers, fashion designers, florists, food delivery services, personal fitness trainers and numerous technology start-ups.

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Chapter contents

9A Reviewing algebra (CONSOLIDATING)
9B Expanding binomial products
9C Expanding perfect squares
9D Difference of two squares
9E Using HCF to factorise algebraic expressions (CONSOLIDATING)
9F Simplifying algebraic fractions: multiplication and division
9G Simplifying algebraic fractions: addition and subtraction

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- simplifies algebraic fractions with numerical denominators and expands algebraic expressions (MA5-ALG-C-01).

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Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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Warm-up quiz

1 Write down the coefficient of x in these expressions.

a $4x - 1$ **b** $3y + 6x$ **c** $3xy - 2x$ **d** $-4x^2 - 7x$

2 What is the constant term in these expressions?

a $5 - 2x$ **b** $-16a + 2$

c $5xy - 2x + 1$ **d** $101x - 6$

3 Evaluate the following if $a = 2$ and $b = -5$.

a ab **b** $-3b + 1$ **c** $4 - b + a$ **d** $5a^2b$

e $\sqrt{a^2}$ **f** $2b^2 - a$ **g** $5 - b^2$ **h** $a^2b - 2b^2$

4 Expand the following using $a(b + c) = ab + ac$.

a $2(x + 3)$ **b** $3(a - 5)$ **c** $4x(3 - 2y)$ **d** $-3(2b - 1)$

5 Write down the highest common factor of:

a 4 and 6 **b** 12 and 18 **c** $2x$ and $4x$

d $3xy$ and $9y$ **e** $10x$ and $15x^2$ **f** $3a^2b$ and $4ab^2$

6 Factorise by taking out the highest common factor.

a $2a + 6$ **b** $3x + 12y$ **c** $5x^2 - 15x$ **d** $4m - 6mn$

7 Add or subtract these fractions. You will need a common denominator for parts **c** and **d**.

a $\frac{3}{7} + \frac{2}{7}$ **b** $\frac{4}{9} - \frac{5}{9}$ **c** $\frac{1}{2} + \frac{2}{3}$ **d** $\frac{3}{8} - \frac{1}{2}$

8 Multiply or divide these fractions.

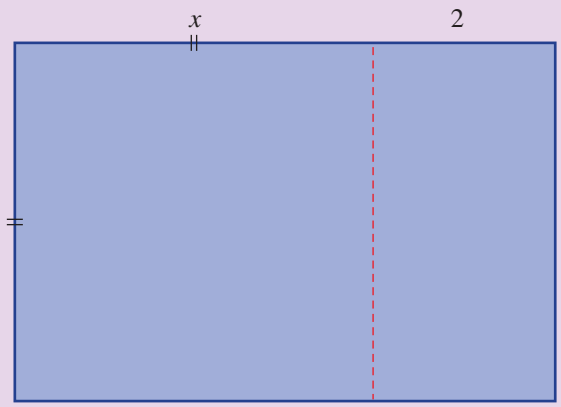
a $\frac{2}{3} \times \frac{4}{5}$ **b** $\frac{3}{4} \times \frac{2}{3}$ **c** $\frac{7}{14} \times \frac{2}{3}$ **d** $\frac{6}{11} \times \frac{22}{12}$

e $\frac{2}{3} \div \frac{1}{3}$ **f** $\frac{7}{8} \div \frac{14}{24}$ **g** $\frac{3}{2} \div \frac{4}{3}$ **h** $\frac{7}{9} \div \frac{4}{3}$

9 Expand and simplify.

a $3(x - 1) + 5$ **b** $4(1 - x) + 5x$ **c** $-2(5 + x) - x$

10 Write two expressions for the area of this rectangle, one with brackets and one without.



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9A Reviewing algebra CONSOLIDATING

Learning intentions for this section:

- To know the terminology associated with algebraic expressions
- To be able to form algebraic expressions from worded situations
- To know that only like terms can be combined under addition and subtraction
- To be able to simplify algebraic expressions using the four basic operations
- To be able to expand expressions involving brackets
- To be able to evaluate expressions by substituting given values

Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stage 5 and Stage 6 Standard
- Some of these questions are more challenging than those in our Stage 4 books

A high level of skill in algebra is required to solve more complex mathematical problems. Skills include adding, subtracting, multiplying and dividing algebraic expressions, as well as expanding and factorising expressions.

In this section we will review some basic concepts in algebra.

Lesson starter: Vocab review

Consider the expression $3x^2 + 4xy - 2 - xy$.

- How many terms are given in the expression?
- What letters are used as pronumerals?
- What is the coefficient of x^2?
- What is the constant term?
- Are there any like terms?
- Can the expression be simplified? If so, how?
- What would be the value of the expression if $x = 2$ and $y = -1$?

KEY IDEAS

- Terms: 3-term expression with pronumerals x and y

$$\overbrace{7y^2 + 2xy - 4}$$

Coefficient of y^2 is 7 **Constant term** is -4

- **Like terms** have the same pronumeral part.
  - They can be collected using addition and subtraction.
    For example: $5a - 7a = -2a$ and $3xy^2 + 2y^2x = 5xy^2$

- For multiplication and division, the symbols $\times$ and $\div$ are not usually shown.
  For example: $-3ab \times 2b = -6ab^2$

$$14a^2b \div (7ab) = \frac{14a^2b}{7ab} = 2a$$

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- The **distributive law** is used to expand brackets.
  For example:

$$-3(x-2) = -3x + 6$$

$$4x + 2(1-x) = 4x + 2 - 2x$$
$$= 2x + 2$$

- To evaluate an expression, substitute a value for each pronumeral and simplify.
  For example: if $a = 2$ and $b = -3$ then

$$b^2 - a = (-3)^2 - 2$$
$$= 9 - 2$$
$$= 7$$

Key terminology: expression, pronumeral, variable, term, like terms, constant term, coefficient, distributive law, evaluate

Exercise 9A

UNDERSTANDING 1–3 3

1 Write expressions for each of the following.

a The cost of:

i x movie passes at \$11 each

ii n apples at 50 cents each

b The number of tickets purchased for:

i x adults and y children

ii b boys and g girls

c The cost of hiring an electrician for n hours if the callout fee is \$50 and the cost of labour is \$60 per hour

2 Consider the expression $5x - 2xy - y^2 - 7$.

Hint: Remember to include the negative sign in the coefficient of xy.

a What is the coefficient of x?

b What is the coefficient of xy?

c What is the constant term?

d Are there any like terms?

e Evaluate the expression if $x = 1$ and $y = 2$.

f Evaluate the expression if $x = 2$ and $y = -1$.

3 Write expressions for the following.

Hint: 'Product' means multiply.

a The sum of 5 and $2x$.

b 7 less than $4a$.

c 1 less than the square of y.

d The product of x and z.

e The sum of the squares of a and b.

f The square root of 8 more than x.

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FLUENCY 4–6(½) 4–6(½)

Example 1 Collecting like terms

Collect like terms to simplify the following.

a $5y - x + 2y$ **b** $4a^2b + ab - ba^2$

SOLUTION

a $5y - x + 2y = 7y - x$

b $4a^2b + ab - ba^2 = 3a^2b + ab$

EXPLANATION

$5y$ and $2y$ are like terms.

$4a^2b$ and ba^2 (or a^2b) are like terms and $4 - 1 = 3$.

Now you try

Collect like terms to simplify the following.

a $7m + 2n - 5m - 3n$ **b** $2x^2y - xy - 4yx^2$

4 Simplify by collecting like terms.

a $3a - 4a$ **b** $4x + 7x$ **c** $6ab + 2ab$

d $5xy - 9xy$ **e** $3a - 1 - 2a$ **f** $7y + 2x - 11y$

g $5x - y - 3y + x$ **h** $ab + ba$ **i** $a^2b - 5a^2b - 3$

j $11xy - 14yx$ **k** $10r^2a + 2ar^2$ **l** $9st^2 - st - 8t^2s$

Hint: Only collect 'like' terms. Remember that $ab = ba$ and that the + or − belongs only to the term that immediately follows.

Example 2 Multiplying and dividing terms

Simplify these expressions.

a $-4a \times 2ab$ **b** $21a^2b \div (3abc)$

SOLUTION

a $-4a \times 2ab = -8a^2b$

b $21a^2b \div (3abc) = \dfrac{21a^2b}{3abc}$

$= \dfrac{7a}{c}$

EXPLANATION

$-4 \times 2 = -8$ and $a \times a = a^2$

Write as a fraction then cancel where possible.

Note: $\dfrac{a^2}{a} = \dfrac{a \times a^1}{a^1} = a$

Now you try

Simplify these expressions.

a $5xy \times (-3y)$ **b** $6a^2b \div (12ab)$

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5 Simplify the following.

a $5 \times 3x$ **b** $7a \times 2$ **c** $-4x \times 2y$

d $6a \times (-7a)$ **e** $-3ab \times b$ **f** $-6a^2b \times 2b$

g $4x \div 2$ **h** $7a \div a$ **i** $12a \div (4a)$

j $22a^2 \div (11a)$ **k** $40a \div (4a^2)$ **l** $100x^2y \div (25xy)$

m $\dfrac{7ab^2}{7b}$ **n** $\dfrac{48a^2bc}{16abc^2}$ **o** $\dfrac{12xy}{36xy^2}$

Example 3 Expanding brackets

Use the distributive law to expand and simplify:

a $-x(2 + x)$ **b** $4x - 2(x - 1)$

SOLUTION

a $-x(2 + x) = -2x - x^2$

b $4x - 2(x - 1) = 4x - 2x + 2$
$= 2x + 2$

EXPLANATION

$-x \times 2 = -2x$ and $-x \times x = -x^2$

First expand the brackets then collect like terms.

Note: $-2 \times (-1) = 2$, not -2.

Now you try

Use the distributive law to expand and simplify:

a $-3(x + 2)$ **b** $-4x(1 - x) + 2x^2$

6 Use the distributive law to expand and simplify:

a $3(x + 2)$ **b** $2(4 + x)$

c $-3(x + 4)$ **d** $-6(x + 1)$

e $-2(x - 3)$ **f** $-x(x + 1)$

g $-3y(2 - 3y)$ **h** $-3(5x^2 + 2y + 4)$

i $3 + 2(x - 1)$ **j** $3x + 4(1 - x)$

k $3(x + 1) - 7x$ **l** $4(x + 2) - 2(x + 1)$

m $7(x - 3) - 3(x - 4)$ **n** $-4(1 - x) - 21x + 1$

o $-6(2 - x) - 7(4 - x)$ **p** $-2(3 - x) - (5 - x)$

q $2y(y - 5) + 4(y - 5)$ **r** $2y(y - 5) - 4(y - 5)$

s $4(3x + 2y + 2) + 2(x + 1)$ **t** $4(3x + 2y + 2) - 2(x + 1)$

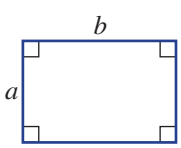
u $18 + 4(x - 2)$ **v** $18 - 4(x - 2)$

Hint: Remember: a negative times a negative is a positive.

PROBLEM-SOLVING AND REASONING 7, 8(½), 9 8(½), 9–11

7 For these shapes, write expressions for **i** perimeter and **ii** area.

a



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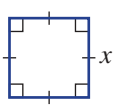

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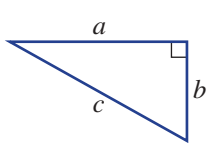

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8 Evaluate the following if $a = 3$, $b = -2$ and $c = -4$.

a $2a + b$ **b** abc **c** $2a \div (3b)$

d $-3c + b$ **e** $a^2 + b^2$ **f** $c^2 \div b$

g $\dfrac{a^2 + c^2}{2}$ **h** $\dfrac{a - c}{7}$ **i** $\dfrac{1}{a}(b + c)$

j $a^3 - b^3$ **k** $2b^3 - c$ **l** $\sqrt{a^2 + b^2}$

Hint: Substitute pronumeral values and work out the answer.

9 You can hire a sports car for an upfront fee of $100 plus $80 per hour after that.

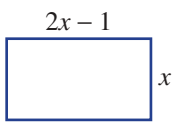
a What is the cost of hiring a sports car for:

i 2 hours? **ii** 9 hours?

b Write an expression for the cost of hiring a sports car for n hours.

10 Find the area of these shapes in expanded form. All angles are 90°.

a



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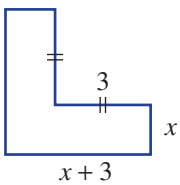
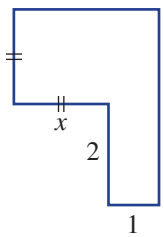


c



11 Identify the errors, then correct to find the answer.

a $-3(x - 1)$
$= -3x - 3$

b $\dfrac{3ab}{6b} = 2a$

c $\dfrac{4a}{2a^2} = 2a$

d $3(x + 2) - 3(x - 1)$
$= 3x + 6 - 3x - 3$
$= 3$

ENRICHMENT: Jake vs Lucas — 12

12 Jake and Lucas operate separate computer consulting services.

- Jake charges $60 per hour.
- Lucas charges a $40 callout fee plus $50 per hour after that.

a What is the cost of hiring Jake for:

i 2 hours? **ii** 10 hours?

b What is the cost of hiring Lucas for:

i 2 hours? **ii** 10 hours?

c Write an expression for the cost of hiring Jake for n hours.

d Write an expression for the cost of hiring Lucas for n hours.

e After what time is the cost of hiring Jake or Lucas the same?

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9B Expanding binomial products

Learning intentions for this section:

- To understand the distributive law for expanding binomial products
- To be able to expand and simplify binomial products

Past, present and future learning:

- This concept may be new to students
- This section addresses the Stage 5 Core topic called *Algebraic techniques A*
- This section has very limited relevance to Stage 6 Standard

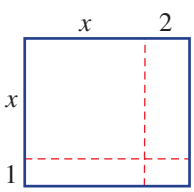
A binomial is an expression with two terms such as $x + 5$ or $x^2 + 3$. You will recall from the previous section that we looked at the product of a single term with a binomial expression; e.g. $2(x - 3)$ or $x(3x - 1)$.

The product of two binomial expressions can also be expanded using the distributive law. This involves multiplying every term in one expression by every term in the other expression.

Expanding the product of two expressions can be applied to problems involving the expansion of rectangular areas, such as a farmer's paddocks.

Lesson starter: Rectangular expansions

If $(x + 1)$ and $(x + 2)$ are the side lengths of a rectangle as shown, the total area can be found as an expression in two different ways.



- Write an expression for the total area of the rectangle using length $= (x + 2)$ and width $= (x + 1)$.
- Now find the area of each of the four parts of the rectangle and combine to give an expression for the total area.
- Compare your two expressions above and complete this equation:

$$(x + 2)(___) = x^2 + ___ + ___.$$

- Can you explain a method for expanding the left-hand side to give the right-hand side?

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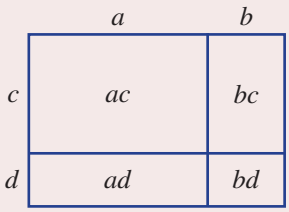
KEY IDEAS

- A binomial expression contains two terms.
  For example: $2x + 3$
- Expanding **binomial products** uses the distributive law.

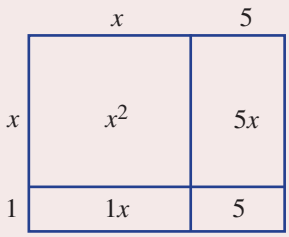
$$(a + b)(c + d) = a(c + d) + b(c + d)$$
$$= ac + ad + bc + bd$$



- Diagrammatically

$(a + b)(c + d) = ac + ad + bc + bd$

For example: $(x + 3)(x + 5) = x^2 + 5x + x + 5$
$= x^2 + 6x + 5$

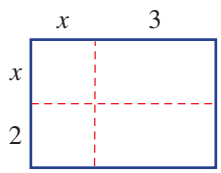


Key terminology: binomial product, distributive law, expand

Exercise 9B

UNDERSTANDING 1–3 2, 3

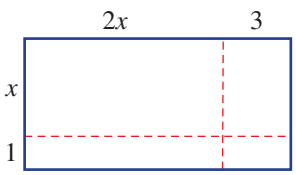
1 The given diagram shows the area $(x + 2)(x + 3)$.



a Write down an expression for the area of each of the four regions inside the rectangle.

b Copy and complete: $(x + 2)(x + 3) = _____ + 3x + _____ + 6$
$= _____ + 5x + _____$

2 The given diagram shows the area $(2x + 3)(x + 1)$.



a Write down an expression for the area of each of the four regions inside the rectangle.

b Copy and complete: $(2x + 3)(_____) = 2x^2 + _____ + 3x + _____$
$= _____ + _____ + _____$

3 Copy and complete these expansions

a $(x + 1)(x + 5) = _____ + 5x + _____ + 5$
$= _____ + 6x + _____$

b $(x - 3)(x + 2) = _____ + _____ - 3x - _____$
$= _____ - x - _____$

c $(3x - 2)(7x + 2) = _____ + 6x - _____ - _____$
$= _____ - _____ - _____$

d $(4x - 1)(3x - 4) = _____ - _____ - 3x + _____$
$= _____ - 19x + _____$

Hint: The product is negative if there are opposite signs $(+, -)$ or $(-, +)$ and positive if they are the same sign $(+, +)$ or $(-, -)$.

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FLUENCY 4–5(½) 4–5(½)

Example 4 Expanding binomial products

Expand $(x + 3)(x + 5)$.

SOLUTION

$$(x + 3)(x + 5) = x^2 + 5x + 3x + 15$$
$$= x^2 + 8x + 15$$

EXPLANATION

Use the distributive law to expand the brackets and then collect the like terms $5x$ and $3x$.

Now you try

Expand $(x + 2)(x + 9)$.

4 Expand the following.

a $(x + 2)(x + 5)$ **b** $(b + 3)(b + 4)$ **c** $(t + 8)(t + 7)$
d $(p + 6)(p + 6)$ **e** $(x + 9)(x + 6)$ **f** $(d + 15)(d + 4)$
g $(a + 1)(a + 7)$ **h** $(y + 10)(y + 2)$ **i** $(m + 4)(m + 12)$
j $(a + 9)(a + 1)$ **k** $(2 + a)(3 + a)$ **l** $(n + 6)(n + 6)$

Hint: First expand with the distributive law then collect the two like terms.

Example 5 Expanding binomial products involving subtraction signs

Expand the following.

a $(x - 4)(x + 7)$ **b** $(2x - 1)(x - 6)$ **c** $(5x - 2)(3x + 7)$

SOLUTION

a $(x - 4)(x + 7) = x^2 + 7x - 4x - 28$
$= x^2 + 3x - 28$

EXPLANATION

After expanding to get the four terms, collect the like terms $7x$ and $-4x$.

Note: $x \times 7 = 7x$ and $-4 \times x = -4x$.

b $(2x - 1)(x - 6) = 2x^2 - 12x - x + 6$
$= 2x^2 - 13x + 6$

Remember: $2x \times x = 2x^2$ and $-1 \times (-6) = 6$.

c $(5x - 2)(3x + 7) = 15x^2 + 35x - 6x - 14$
$= 15x^2 + 29x - 14$

Recall: $5x \times 3x = 5 \times 3 \times x \times x = 15x^2$.

Now you try

Expand the following.

a $(x + 3)(x - 6)$ **b** $(3x - 2)(x - 4)$ **c** $(6x + 5)(3x - 4)$

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5 Expand the following.

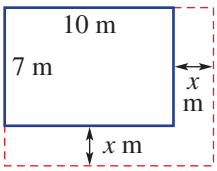
a $(x+3)(x-4)$	**b** $(x+5)(x-2)$	**c** $(x+4)(x-8)$
d $(x-6)(x+2)$	**e** $(x-1)(x+10)$	**f** $(x-7)(x+9)$
g $(x-2)(x+7)$	**h** $(x-1)(x-2)$	**i** $(x-4)(x-5)$
j $(4x+3)(2x+5)$	**k** $(3x+2)(2x+1)$	**l** $(3x+1)(5x+4)$
m $(2x-3)(3x+5)$	**n** $(8x-3)(3x+4)$	**o** $(3x-2)(2x+1)$
p $(5x+2)(2x-7)$	**q** $(2x+3)(3x-2)$	**r** $(4x+1)(4x-5)$
s $(3x-2)(6x-5)$	**t** $(5x-2)(3x-1)$	**u** $(7x-3)(3x-4)$

Hint: First expand with the distributive law then collect the two like terms.

PROBLEM-SOLVING AND REASONING

6–8, 9(½) | 7–11

6 A 10 m by 7 m rectangular factory shed is expanded by x metres on two sides.

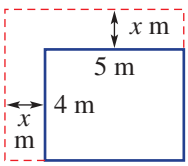


a Write expressions for:
- **i** the new length of the shed (horizontal length)
- **ii** the new width of the shed (vertical length)

b Using your results from part **a**, expand brackets to find an expression for the new total area.

c What would be the new area if $x = 2$?

7 A rectangular room in a house with dimensions 4 m by 5 m is to be extended. Both the length and width are to be increased by x m.

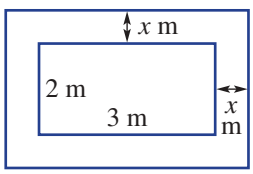


a Find an expanded expression for the area of the new room.

b
- **i** If $x = 3$, find the area of the new room.
- **ii** By how much has the area increased?

Hint: First label the length and width of the new room.

8 A rectangular trampoline of length 3 m and width 2 m is to be surrounded by padding of width x metres.



a Write expressions for:
- **i** the total length of the trampoline and padding
- **ii** the total width of the trampoline and padding

b Find an expression for the total area by expanding brackets.

c What would be the total area if $x = 1$?

Hint: Don't forget to count the x on both sides.

9 Write the missing terms in these expansions.

a $(x+2)(x+___) = x^2+5x+6$	**b** $(x+___)(x+5) = x^2+7x+10$
c $(x+1)(x+___) = x^2+7x+___$	**d** $(x+___)(x+9) = x^2+11x+___$
e $(x+3)(x-___) = x^2+x-___$	**f** $(x-5)(x+___) = x^2-2x-___$

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10 Expand these binomial products.

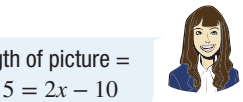
a $(a + b)(a + c)$ **b** $(a - b)(a + c)$ **c** $(b - a)(a + c)$

d $(2x + y)(x - 2y)$ **e** $(2a + b)(a - b)$ **f** $(3x - y)(2x + y)$

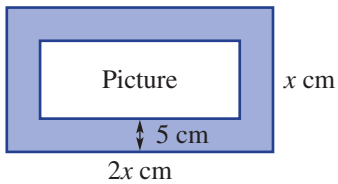
11 A picture frame 5 cm wide has a length that is twice the width, x cm.

a Find an expression for the total area of the frame and picture.

b Find an expression in expanded form for the area of the picture only.

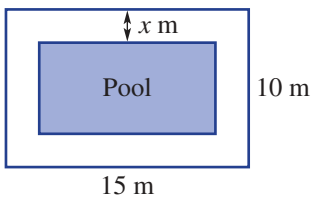


Hint: Length of picture = $2x - 5 - 5 = 2x - 10$



ENRICHMENT: Paving the pool — 12

12 The outside edge of a path around a rectangular swimming pool is 15 m long and 10 m wide. The path is x metres wide.



a Write expressions for:

i the length of the pool

ii the width of the pool

b Find an expression for the area of the pool in expanded form.

c Find the area of the pool if $x = 2$.

d What value of x makes the pool area 50 m^2? Use trial and error.

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9C Expanding perfect squares

Learning intentions for this section:

- To be able to recognise a perfect square
- To understand that expressions that are perfect squares can be expanded
- To be able to expand and simplify perfect squares

Past, present and future learning:

- This concept may be new to students
- This section addresses the Stage 5 Core topic called *Algebraic techniques A*
- This section has very limited relevance to Stage 6 Standard

A special type of binomial product involves perfect squares. Examples of perfect squares are $2^2 = 4$, $15^2 = 225$, x^2 and $(a+b)^2$. To expand $(a+b)^2$ we multiply $(a+b)$ by $(a+b)$ and use the distributive law:

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= a(a+b) + b(a+b)\\ &= a^2 + ab + ba + b^2\\ &= a^2 + 2ab + b^2\end{aligned}$$

A similar result is obtained for the square of $(a-b)$:

$$\begin{aligned}(a-b)^2 &= (a-b)(a-b)\\ &= a(a-b) - b(a-b)\\ &= a^2 - ab - ba + b^2\\ &= a^2 - 2ab + b^2\end{aligned}$$

Lesson starter: Seeing the pattern

Using $(a+b)(c+d) = ac + ad + bc + bd$, expand and simplify the binomial products below.

$(x+1)(x+1) = x^2 + x + x + 1$
$=$

$(x+3)(x+3) =$
$=$

$(x-5)(x-5) =$
$=$

$(x-7)(x-7) =$
$=$

- Describe the patterns you see in the expansions above.
- Generalise your observations by completing the following expansions.

$(a+b)(a+b) = a^2 + ___ + ___ + ___$
$= a^2 + ___ + ___$

$(a-b)(a-b) =$
$=$

KEY IDEAS

- $3^2 = 9, a^2, (2y)^2, (x-1)^2$ and $(3-2y)^2$ are all examples of **perfect squares**. They are expressions that can be written as a single square.

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■ Expanding perfect squares:

$(a+b)^2 = (a+b)(a+b)$
$= a(a+b) + b(a+b)$
$= a^2 + ab + ba + b^2$
$= a^2 + 2ab + b^2$

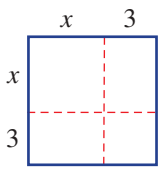
$(a-b)^2 = (a-b)(a-b)$
$= a(a-b) - b(a-b)$
$= a^2 - ab - ba + b^2$
$= a^2 - 2ab + b^2$

Key terminology: perfect square, distributive law, expand

Exercise 9C

UNDERSTANDING 1–3 3

1 The side lengths of this square are $(x + 3)$ units.



a What are the areas of each of the four regions? Write expressions each time.

b Add up all the area expressions to find an expression for the total area.

c Complete the following: $(x + 3)(x + 3) = x^2 + 3x + __ + __$
$= x^2 + 6x + __$

2 Complete these expansions.

Hint: Expand using the distributive law: $(a + b)(c + d) = ac + ad + bc + bd$

a $(x + 4)(x + 4) = x^2 + 4x + __ + __$
$= _____$

b $(x + 5)(x + 5) = x^2 + 5x + __ + __$
$= _____$

c $(x - 2)(x - 2) = x^2 - 2x - __ + __$
$= _____$

d $(x - 7)(x - 7) = x^2 - 7x - __ + __$
$= _____$

3 **a** Substitute the given value of b into $x^2 + 2bx + b^2$ and simplify.

i $b = 3$ **ii** $b = 11$

b Substitute the given value of b into $x^2 - 2bx + b^2$ and simplify.

i $b = 2$ **ii** $b = 9$

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FLUENCY 4–5(½) 4–6(½)

Example 6 Expanding perfect squares

Expand each of the following.

a $(x+3)^2$ **b** $(x-2)^2$

SOLUTION

a $(x+3)^2 = (x+3)(x+3)$
$= x^2 + 3x + 3x + 9$
$= x^2 + 6x + 9$

Alternative solution:
$(x+3)^2 = x^2 + 2 \times x \times 3 + 3^2$
$= x^2 + 6x + 9$

EXPLANATION

Write in expanded form.

Use the distributive law.

Collect like terms.

Expand using $(a+b)^2 = a^2 + 2ab + b^2$ where $a = x$ and $b = 3$.

b $(x-2)^2 = (x-2)(x-2)$
$= x^2 - 2x - 2x + 4$
$= x^2 - 4x + 4$

Alternative solution:
$(x-2)^2 = x^2 - 2 \times x \times 2 + 2^2$
$= x^2 - 4x + 4$

Write in expanded form.

Use the distributive law.

Collect like terms.

Expand using $(a-b)^2 = a^2 - 2ab + b^2$ where $a = x$ and $b = 2$.

Now you try

Expand each of the following.

a $(x+8)^2$ **b** $(x-5)^2$

4 Expand each of the following perfect squares.

a $(x+1)^2$ **b** $(x+3)^2$
c $(x+2)^2$ **d** $(x+5)^2$
e $(x+4)^2$ **f** $(x+9)^2$
g $(x+7)^2$ **h** $(x+10)^2$
i $(x-2)^2$ **j** $(x-6)^2$
k $(x-1)^2$ **l** $(x-3)^2$
m $(x-9)^2$ **n** $(x-7)^2$
o $(x-4)^2$ **p** $(x-12)^2$

Hint: Recall:
$(a+b)^2 = (a+b)(a+b)$
$(a-b)^2 = (a-b)(a-b)$

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Example 7 Expanding more perfect squares

Expand $(2x + 3)^2$.

SOLUTION

$(2x + 3)^2 = (2x + 3)(2x + 3)$
$= 4x^2 + 6x + 6x + 9$
$= 4x^2 + 12x + 9$

Alternative solution:
$(2x + 3)^2 = (2x)^2 + 2 \times 2x \times 3 + 3^2$
$= 4x^2 + 12x + 9$

EXPLANATION

Write in expanded form.

Use the distributive law.

Collect like terms.

Expand using $(a + b)^2 = a^2 + 2ab + b^2$

where $a = 2x$ and $b = 3$.

Recall $(2x)^2 = 2x \times 2x = 4x^2$.

Now you try

Expand $(5x - 3)^2$.

5 Expand each of the following perfect squares.

a $(2x + 1)^2$	**b** $(2x + 5)^2$	**c** $(3x + 2)^2$
d $(3x + 1)^2$	**e** $(5x + 2)^2$	**f** $(4x + 3)^2$
g $(7 + 2x)^2$	**h** $(5 + 3x)^2$	**i** $(2x - 3)^2$
j $(3x - 1)^2$	**k** $(4x - 5)^2$	**l** $(2x - 9)^2$

Hint:
$(2x)^2 = 2x \times 2x$
$= 4x^2$

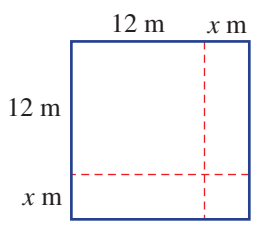
6 Expand each of the following perfect squares.

a $(3 - x)^2$	**b** $(5 - x)^2$	**c** $(1 - x)^2$
d $(6 - x)^2$	**e** $(11 - x)^2$	**f** $(4 - x)^2$
g $(7 - x)^2$	**h** $(12 - x)^2$	**i** $(8 - 2x)^2$
j $(2 - 3x)^2$	**k** $(9 - 2x)^2$	**l** $(10 - 4x)^2$

Hint:
$-x \times (-x) = x^2$

PROBLEM-SOLVING AND REASONING 7, 8 8–10

7 A farmer extends a 12 m square sheep pen on two sides by x metres.



a Write the expression for the side length of the new pen.

b Write an expression for the area of the new pen and expand this expression.

c Use your result to find the new area if:

i $x = 2$ **ii** $x = 5$

Hint: An example of an expression is $10 - x$ or $x^2 - 20x + 100$.

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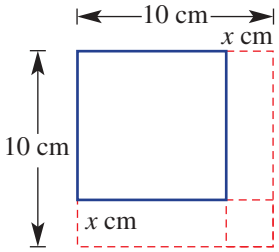
8 A child at pre-school cuts off a strip of width x cm from two sides of a square piece of paper of side length 10 cm.



a Write an expression for the new side length of the remaining paper.

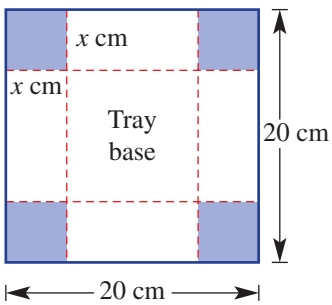
b Find the area of the new piece of paper in expanded form.

c Use your result to find the new area if:

i $x = 2$ **ii** $x = 6$

d What value of x makes the new area one quarter of the original?

9 A square piece of tin of side length 20 cm has four squares of side length x cm removed from each corner. The sides are folded up to form a tray. The centre square forms the tray base.



Hint: For part **d**, what is the height of the tray?

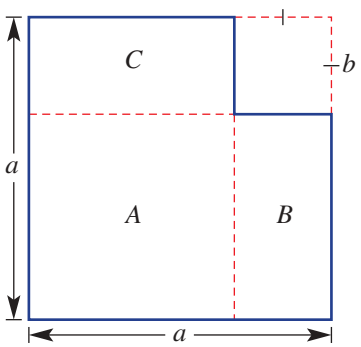
a Write an expression for the side length of the base of the tray.

b Write an expression for the area of the base of the tray. Expand your answer.

c Find the area of the tray base if $x = 3$.

d Find the volume of the tray if $x = 3$.

10 A square of side length b is removed from a square of side length a.



a Using subtraction, write down an expression for the remaining area.

b Write expressions for the area of the regions in expanded form:

i A **ii** B **iii** C

c Add all the expressions from part **b** to see if you get your answer from part **a**.

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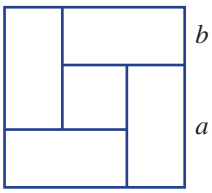
ENRICHMENT: Arranging tennis courts — 11

11 Four tennis courts are arranged as shown below, with a square storage space in the centre. Each court area has the same dimensions, $a \times b$.



a Write an expression for the side length of the total area.

b Write an expression for the area of the total area.

c Write an expression for the side length of the inside storage space.

d Write an expression for the area of the storage space in expanded form.

e Subtract your answer to part **d** from your answer to part **b** to find the area of the four courts.

f Find the area of one court. Does your answer confirm that your answer to part **e** is correct?

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9D Difference of two squares

Learning intentions for this section:

- To understand how a difference of two squares is formed
- To be able to expand and simplify to form a difference of two squares

Past, present and future learning:

- This concept may be new to students
- This section addresses the Stage 5 Core topic called *Algebraic techniques A*
- This section has very limited relevance to Stage 6 Standard

Another type of expansion deals with the product of the sum and difference of the same two terms. The result is the difference of two squares:

$$(a+b)(a-b) = a(a-b) + b(a-b)$$
$$= a^2 - ab + ba - b^2$$
$$= a^2 - b^2$$

or

$$(a+b)(a-b) = a^2 - ab + ba - b^2$$
$$= a^2 - b^2$$

Lesson starter: How is 16×14 a difference of two squares?

Using the fact that $15^2 = 225$, follow these steps to show how 16×14 can be calculated mentally using a difference of two squares.

- $16 \times 14 = (15 + ___) \times (15 - ___)$ Rewrite.
  $= 15^2 - ___ + ___ - ___$ Expand.
  $= ___ - 1$ Simplify.
  $= ___$ Evaluate.
- Now try this technique on 17×13 and 19×21.

KEY IDEAS

- A **difference of two squares** is formed when one square is subtracted from another.
- This is formed when $(a+b)(a-b)$ is expanded and simplified.

$$(a+b)(a-b) = a^2 - ab + ba - b^2$$
$$= a^2 - b^2$$

$(a-b)(a+b)$ also expands to $a^2 - b^2$
The result is a difference of two squares.

Key terminology: difference of two squares, distributive law, expand

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Exercise 9D

UNDERSTANDING	1–3	2, 3

1 Why do the two middle terms in an expansion of $(x + a)(x - a)$ (i.e. $x^2 + ax - xa - a^2$) cancel out?

2 Decide whether each of the following shows a single square or a difference of two squares.

a 4^2 **b** $7^2 - 3^2$ **c** $a^2 - b^2$ **d** x^2

3 Complete these expansions.

a $(x + 4)(x - 4) = x^2 - 4x + ___ - ___$

$= _____$

b $(2x - 1)(2x + 1) = 4x^2 + ___ - ___ - ___$

$= _____$

Hint:
$(a + b)(a - b) = a^2 - ab + ab - b^2$
$= a^2 - b^2$

FLUENCY	4-5(½)	4-5(½)

Example 8 Forming a difference of two squares

Expand and simplify the following.

a $(x + 2)(x - 2)$ **b** $(x - 7)(x + 7)$

SOLUTION

a $(x + 2)(x - 2) = x^2 - 2x + 2x - 4$

$= x^2 - 4$

Alternative solution:

$(x + 2)(x - 2) = (x)^2 - (2)^2$

$= x^2 - 4$

EXPLANATION

Expand using the distributive law.

$-2x + 2x = 0$.

$(a + b)(a - b) = a^2 - b^2$. Here $a = x$ and $b = 2$.

b $(x - 7)(x + 7) = x^2 + 7x - 7x - 49$

$= x^2 - 49$

Alternative solution:

$(x - 7)(x + 7) = (x)^2 - (7)^2$

$= x^2 - 49$

Expand, then note that $7x - 7x = 0$.

$(a - b)(a + b) = a^2 - b^2$, with $a = x$ and $b = 7$.

Now you try

Expand and simplify the following.

a $(x + 5)(x - 5)$ **b** $(x - 15)(x + 15)$

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4 Expand and simplify the following to form a difference of two squares.

a $(x+1)(x-1)$ **b** $(x+3)(x-3)$
c $(x+8)(x-8)$ **d** $(x+4)(x-4)$
e $(x+12)(x-12)$ **f** $(x+11)(x-11)$
g $(x-9)(x+9)$ **h** $(x-5)(x+5)$
i $(x-6)(x+6)$ **j** $(5-x)(5+x)$
k $(2-x)(2+x)$ **l** $(7-x)(7+x)$

Hint: Recall:
$(a+b)(a-b) = a^2 - b^2$

Example 9 Forming more differences of two squares

Expand and simplify $(3x-2y)(3x+2y)$.

SOLUTION

$$(3x-2y)(3x+2y) = 9x^2 + \cancel{6xy} - \cancel{6xy} - 4y^2$$
$$= 9x^2 - 4y^2$$

Alternative solution:
$$(3x-2y)(3x+2y) = (3x)^2 - (2y)^2$$
$$= 9x^2 - 4y^2$$

EXPLANATION

Expand using the distributive law.

$6xy - 6xy = 0$.

$(a+b)(a-b) = a^2 - b^2$, with $a = 3x$ and $b = 2y$ here. Recall that $(3x)^2 = 3x \times 3x$.

Now you try

Expand and simplify $(12x-5y)(12x+5y)$.

5 Expand and simplify the following.

a $(3x-2)(3x+2)$ **b** $(5x-4)(5x+4)$
c $(4x-3)(4x+3)$ **d** $(7x-3y)(7x+3y)$
e $(9x-5y)(9x+5y)$ **f** $(11x-y)(11x+y)$
g $(8x+2y)(8x-2y)$ **h** $(10x-9y)(10x+9y)$
i $(7x-5y)(7x+5y)$ **j** $(6x-11y)(6x+11y)$
k $(8x-3y)(8x+3y)$ **l** $(9x-4y)(9x+4y)$

Hint:
$(3x)^2 = 3x \times 3x$
$= 9x^2$

PROBLEM-SOLVING AND REASONING 6(½), 7 6(½), 8

6 To calculate 21×19, here is a method using the difference of two squares.

$21 \times 19 = (20+1) \times (20-1)$
$= 20^2 - 1^2$
$= 400 - 1$
$= 399$

Use this technique to evaluate the following (mentally if you can).

a 31×29 (Note: $30^2 = 900$) **b** 41×39 (Note: $40^2 = 1600$)
c 26×24 (Note: $25^2 = 625$) **d** 51×49 (Note: $50^2 = 2500$)
e 22×18 (Use $20^2 = 400$) **f** 23×17 (Use $20^2 = 400$)
g 35×25 (Use $30^2 = 900$) **h** 54×46 (Use $50^2 = 2500$)

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7 Lara is x years old and her two best friends are $(x - 2)$ and $(x + 2)$ years old.

a Write an expression for:

i the square of Lara's age

ii the product of the ages of Lara's best friends

Hint: Expand in part **a ii**.

b Are the answers from parts **a i** and **ii** equal? If not, by how much do they differ?

8 A square of side length x has one side reduced by 1 unit and the other increased by 1 unit.

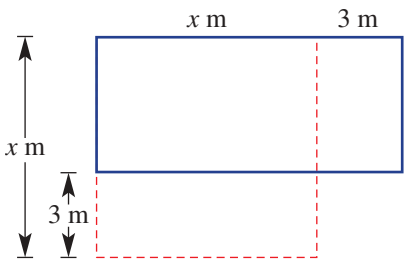
a Find an expanded expression for the area of the resulting rectangle.

b Is the area of the original square the same as the area of the resulting rectangle? Explain why/why not.

Hint: Determine expressions for the length and width of the rectangle first.

ENRICHMENT: Classroom renovation — 9

9 A square classroom is to be shortened on the south side by 3 m and extended on the east side by 3 m. The original side length of the classroom was x m.



a Write expressions for:

i the original area of the classroom

ii the new length of the classroom

iii the new width of the classroom

b Expand to find an expression for the new area of the classroom.

c Is the new area the same as the original area? If not, by how much do they differ?

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9E Using HCF to factorise algebraic expressions CONSOLIDATING

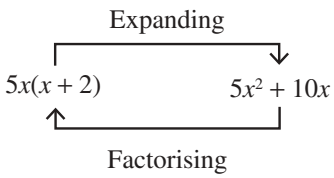
Learning intentions for this section:

- To understand that factorising and expanding are reverse processes
- To be able to identify the highest common factor
- To be able to factorise algebraic expressions involving a common factor

Past, present and future learning:

- This concept may be new to students
- This section addresses the Stage 5 Core topic called *Algebraic techniques A*
- This section has very limited relevance to Stage 6 Standard

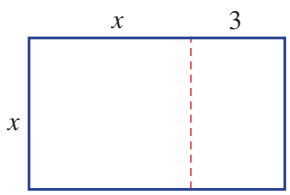
The process of factorisation is a key step in the simplification of many algebraic expressions and in the solution of equations. It is the reverse process of expansion and involves writing an expression as a product of its factors.



Lesson starter: Factorised areas

Here is a rectangle of length $(x + 3)$ and width x.

- Write an expression for the total area using the given length and width.
- Write an expression for the total area by adding up the area of the two smaller regions.
- Are your two expressions equivalent? How could you work from your second expression (expanded) to the first expression (factorised)?



KEY IDEAS

- To **factorise** means to write an expression as a product of its factors.
- When factorising expressions with common factors, take out the **highest common factor** (HCF). The HCF could be:
  - a number
    For example: $2x + 10 = 2(x + 5)$
  - a pronumeral
    For example: $x^2 + 5x = x(x + 5)$
  - the product of numbers and pronumerals
    For example: $2x^2 + 10x = 2x(x + 5)$

HCF of $2x$ and 10

$2x + 10 = 2(x + 5)$

Expanded form Factorised form

- A factorised expression can be checked by using expansion.
  - For example: $2x(x + 5) = 2x^2 + 10x$.

Key terminology: highest common factor, factorise

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Exercise 9E

UNDERSTANDING 1–3 | 3

1 Write down the highest common factor (HCF) of these pairs of numbers.

a 8, 12 **b** 10, 20 **c** 5, 60 **d** 24, 30

2 Write down the missing factor.

a $5 \times ___ = 5x$
b $7 \times ___ = 7x$
c $2a \times ___ = 2a^2$
d $5a \times ___ = 10a^2$
e $___ \times 3y = -6y^2$
f $___ \times 12x = -36x^2$

3 **a** Write down the missing factor in each part.

i $___(x^2 + 2x) = 6x^2 + 12x$
ii $___(2x + 4) = 6x^2 + 12x$
iii $___(x + 2) = 6x^2 + 12x$

b Which equation above uses the HCF of $6x^2$ and $12x$?

FLUENCY 4–6(½) | 4–6(½)

Example 10 Finding the highest common factor (HCF)

Determine the HCF of the following.

a $6a$ and $8ab$ **b** $3x^2$ and $6xy$

SOLUTION

a HCF of $6a$ and $8ab$ is $2a$

b HCF of $3x^2$ and $6xy$ is $3x$

EXPLANATION

HCF of 6 and 8 is 2.

HCF of a and ab is a.

HCF of 3 and 6 is 3.

HCF of x^2 and xy is x.

Now you try

Determine the HCF of the following.

a $10ab$ and $5b$ **b** $7xy^2$ and $21xy$

4 Determine the HCF of the following.

a $6x$ and $14xy$ **b** $12a$ and $18a$ **c** $10m$ and 4
d $12y$ and 8 **e** $15t$ and $6s$ **f** 15 and p
g $9x$ and $24xy$ **h** $6n$ and $21mn$ **i** $10y$ and $2y$
j $8x^2$ and $14x$ **k** $4x^2y$ and $18xy$ **l** $5ab^2$ and $15a^2b$

Hint: HCF stands for highest common factor.

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Example 11 Factorising expressions

Factorise the following.

a $4x + 12$ **b** $10y - 25y^2$

SOLUTION

a $4x + 12 = 4(x + 3)$

EXPLANATION

4 is the HCF of $4x$ and 12.

Place 4 in front of the brackets and divide each term by 4: $4x \div 4 = x$ and $12 \div 4 = 3$.

Check your answer using expansion.

b $10y - 25y^2 = 5y(2 - 5y)$

The HCF of $10y$ and $25y^2$ is $5y$. Place $5y$ in front of the brackets and divide each term by $5y$.

Now you try

Factorise the following.

a $15a + 20$ **b** $2x^2 - 6x$

5 Factorise the following.

a $7x + 7$	**b** $3x + 3$	**c** $4x - 4$
d $5x - 5$	**e** $4 + 8y$	**f** $10 + 5a$
g $3 - 9b$	**h** $6 - 2x$	**i** $12a + 3b$
j $6m + 6n$	**k** $10x - 8y$	**l** $4a - 20b$
m $x^2 + 2x$	**n** $a^2 - 4a$	**o** $y^2 - 7y$
p $x - x^2$	**q** $3p^2 + 3p$	**r** $8x - 8x^2$
s $4b^2 + 12b$	**t** $6y - 10y^2$	

Hint: Always take out the highest common factor (HCF).

Example 12 Using a negative number as a factor

Factorise $-8x^2 - 12x$.

SOLUTION

$-8x^2 - 12x = -4x(2x + 3)$

EXPLANATION

The HCF of the terms is $-4x$, including the common negative. Place the factor in front of the brackets and divide each term by $-4x$.

Note: $-8x^2 \div (-4x) = 2x$ and $-12x \div (-4x) = 3$.

Now you try

Factorise $-11a - 22a^2$.

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6 Factorise the following by including the negative sign in the common factor.

a $-8x - 4$
b $-4x - 2$
c $-10x - 5y$
d $-7a - 14b$
e $-9x - 12$
f $-6y - 8$
g $-10x - 15y$
h $-4m - 20n$
i $-3x^2 - 18x$
j $-8x^2 - 12x$
k $-16y^2 - 6y$
l $-5a^2 - 10a$

Hint: Take a negative factor out of both terms.

PROBLEM-SOLVING AND REASONING

7, 8(½) | 8(½), 9–11

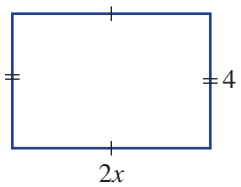
7 Write the missing number or expression.

a $3x + 9 = ___(x + 3)$
b $xy + x = x(___ + 1)$
c $a^2 - a = ___(a - 1)$
d $5xy + 10x = ___(y + 2)$
e $-7a - 14 = ___(a + 2)$
f $-24a^2 - 36a = ___(2a + 3)$

8 Write down the perimeter of these shapes in factorised form.

Hint: First find an expression for the perimeter, then factorise it.

a



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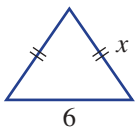

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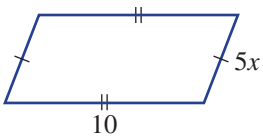

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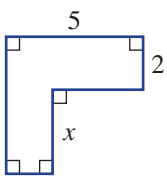

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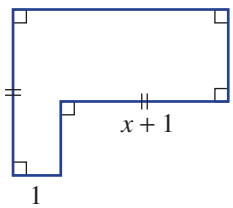
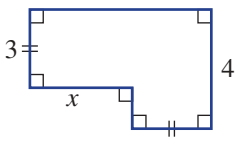
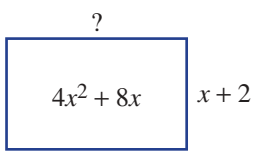


f



9 The expression for the area of a rectangle is $(4x^2 + 8x)$. Find an expression for its breadth if the length is $(x + 2)$.



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10 The height, in metres, of a ball thrown in the air is given by $5t - t^2$, where t is the time in seconds.

a Write an expression for the ball's height in factorised form.

b Find the ball's height at these times:

i $t = 0$ **ii** $t = 2$ **iii** $t = 4$

c How long does it take for the ball's height to return to 0 metres? Use trial and error if required.

11 $7 \times 9 + 7 \times 3$ can be evaluated by first factorising to $7(9 + 3)$. This gives $7 \times 12 = 84$. Use a similar technique to evaluate the following.

a $9 \times 2 + 9 \times 5$

b $6 \times 3 + 6 \times 9$

c $-2 \times 4 - 2 \times 6$

d $-5 \times 8 - 5 \times 6$

e $23 \times 5 - 23 \times 2$

f $63 \times 11 - 63 \times 8$

ENRICHMENT: Further factorisation — 12, 13

12 Common factors can also be removed from expressions with more than two terms.

For example: $2x^2 + 6x + 10xy = 2x(x + 3 + 5y)$

Factorise these expressions by taking out the HCF.

a $3a^2 + 9a + 12$

b $5z^2 - 10z + zy$

c $x^2 - 2xy + x^2y$

d $4by - 2b + 6b^2$

e $-12xy - 8yz - 20xyz$

f $3ab + 4ab^2 + 6a^2b$

13 You can factorise some expressions by taking out a binomial factor.

For example: $3(x - 2) + x(x - 2) = (x - 2)(3 + x)$

Factorise the following by taking out a binomial common factor.

Hint: $x - 2$ is the common factor.

a $4(x + 3) + x(x + 3)$

b $3(x + 1) + x(x + 1)$

c $7(m - 3) + m(m - 3)$

d $x(x - 7) + 2(x - 7)$

e $8(a + 4) - a(a + 4)$

f $5(x + 1) - x(x + 1)$

g $y(y + 3) - 2(y + 3)$

h $a(x + 2) - x(x + 2)$

i $t(2t + 5) + 3(2t + 5)$

j $m(5m - 2) + 4(5m - 2)$

k $y(4y - 1) - (4y - 1)$

l $(7 - 3x) + x(7 - 3x)$

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Progress quiz

9A **1** Simplify the following.

a $3x - 2y + 5x - 4y$ **b** $3ab^2 - ab + 4ab^2$

c $4a \times 5ab$ **d** $\dfrac{3xy^2}{9xy}$

9A **2** Expand and simplify the following.

a $3(x - 4)$ **b** $2y(3y + 5)$ **c** $5 - 2(3x + 1)$

9B **3** Expand the following binomial products.

a $(x + 4)(x + 6)$ **b** $(x + 3)(x - 2)$

c $(2x - 3)(x + 4)$ **d** $(3x - 2)(2x - 5)$

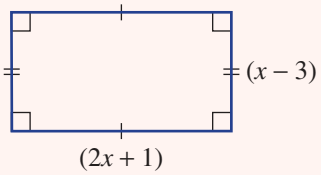
9C **4** Expand the following perfect squares.

a $(x + 6)^2$ **b** $(x - 8)^2$

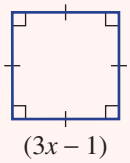
c $(3x + 7)^2$ **d** $(5 - x)^2$

9B/9C **5** Find the area of these figures in expanded form.

a



b



9D **6** Expand and simplify the following.

a $(x + 10)(x - 10)$ **b** $(x - 1)(x + 1)$

c $(3x + 2)(3x - 2)$ **d** $(4x - 5y)(4x + 5y)$

9C/9D **7** Expand and simplify $(x + 1)^2 - (x + 1)(x - 1)$.

9E **8** Determine the HCF of the following.

a $8x$ and 12 **b** $6a$ and $15a$

c $5ab$ and $10ab^2$ **d** $12x^2$ and $9x$

9E **9** Factorise the following. Take a negative factor out in parts **e** and **f**.

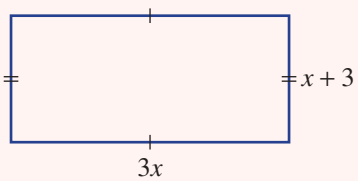
a $6x + 12$ **b** $15y - 20$

c $8ab + 20a$ **d** $4x^2 - 2x$

e $-10x - 5$ **f** $-4x^2 - 10x$

9E **10** Give the perimeter of these shapes in factorised form

a



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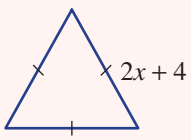

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9F Simplifying algebraic fractions: multiplication and division

Learning intentions for this section:

- To know that expressions must be factorised before common factors can be cancelled
- To be able to simplify algebraic fractions by cancelling common factors
- To be able to multiply and divide algebraic fractions

Past, present and future learning:

- This concept may be new to students
- This section addresses the Stage 5 Core topic called *Algebraic techniques A*
- This section has very limited relevance to Stage 6 Standard

Fractions such as $\frac{4}{6}$ and $\frac{24}{16}$ can be simplified by cancelling common factors. This is also true of algebraic fractions such as $\frac{3x}{6}$, $\frac{5a^2}{10a}$ and $\frac{2x+4}{2}$.

Lesson starter: Does $\frac{2x+6^{3}}{2^{1}} = 2x + 3$?

For the expression $\frac{2x+6}{2}$, a student attempts to cancel the 6 with the 2 in the denominator.

They write $\frac{2x+6^{3}}{2^{1}} = 2x + 3$.

- Is this a correct method?
- Substitute $x = 1$ into the left-hand side and then the right-hand side. Are they equal?
- Can you show a correct method? Does it involve factorisation?

KEY IDEAS

- Simplify **algebraic fractions** by cancelling **common factors**.

$$\frac{3^{1}x}{6^{2}} = \frac{x}{2} \qquad \frac{5a^{1}}{3a} = \frac{5a}{3} \qquad \frac{2x+4}{2} = \frac{2^{1}(x+2)}{2^{1}} = x + 2$$

  - This is an incorrect cancellation: $\frac{2x+4^{2}}{2^{1}} = 2x + 2$

- To multiply algebraic fractions, first cancel any numerator with any denominator.

$$\frac{2^{1}x^{1}}{5_{1}} \times \frac{15^{3}}{4_{2}x_{1}y} = \frac{1 \times 3}{1 \times 2y} = \frac{3}{2y}$$

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■ To divide algebraic fractions, multiply by the reciprocal of the fraction following the division sign.

- The **reciprocal** of $\frac{a}{b} = \frac{b}{a}$.

$$\frac{3x}{4} \div \frac{9}{8y} = \frac{\cancel{3}^{1}x}{\cancel{4}_{1}} \times \frac{\cancel{8}^{2}y}{\cancel{9}^{3}} = \frac{2xy}{3}$$

Key terminology: algebraic fraction, common factor, factorise, numerator, denominator, reciprocal

Exercise 9F

UNDERSTANDING 1–3 | 3

1 Cancel to simplify these fractions.

a $\frac{4}{2}$ **b** $\frac{4x}{2}$ **c** $\frac{4(x+2)}{2}$ **d** $\frac{4(x-1)}{2}$

2 Write down the reciprocal of these fractions.

a $\frac{2}{3}$ **b** $\frac{4}{3}$ **c** $\frac{7x}{2}$

d $\frac{5}{4a^2}$ **e** 7 **f** $6x$

Hint:
Recall: $7 = \frac{7}{1}$ for part **e**.

3 Divide these simple fractions.

a $\frac{2}{3} \div \frac{3}{4}$ **b** $\frac{7}{8} \div \frac{3}{4}$ **c** $\frac{6}{7} \div \frac{4}{7}$

d $\frac{5}{14} \div \frac{10}{7}$ **e** $\frac{21}{4} \div \frac{7}{8}$ **f** $\frac{4}{9} \div \frac{4}{9}$

Hint:
- Copy the first fraction.
- Change the ÷ to ×.
- Flip over the second fraction.

FLUENCY 4–7(½) | 4–7(½)

Example 13 Cancelling common factors

Simplify by cancelling common factors.

a $\frac{7x}{21}$ **b** $\frac{5(x+2)(x-1)}{x+2}$

SOLUTION

a $\frac{\cancel{7}^{1}x}{\cancel{21}_{3}} = \frac{x}{3}$

b $\frac{5\cancel{(x+2)}^{1}(x-1)}{\cancel{x+2}^{1}} = 5(x-1)$

EXPLANATION

The HCF of 7 and 21 is 7.

Treat $(x + 2)$ as a common factor to both the numerator and denominator.

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Now you try

Simplify by cancelling common factors.

a $\frac{-2ab^2}{4b}$ **b** $\frac{-3(x-2)(x+7)}{3(x-2)}$

4 Simplify by cancelling common factors.

a $\frac{4x}{8}$ **b** $\frac{8x}{16}$ **c** $\frac{-4ab}{12}$

d $\frac{7a}{14b}$ **e** $\frac{4(x-1)}{4}$ **f** $\frac{-6(2x+1)}{3}$

g $\frac{-7x}{21x}$ **h** $\frac{4a^2}{40a}$ **i** $\frac{3(x+2)}{x+2}$

j $\frac{-2(x-7)}{x-7}$ **k** $\frac{5(x+3)}{10(x+3)}$ **l** $\frac{-16(x-14)}{24(x-14)}$

m $\frac{(x+2)(x-4)}{x+2}$ **n** $\frac{2(x-1)(x+3)}{x+3}$ **o** $\frac{-4(x+2)(x-5)}{2(x-5)}$

p $\frac{-7(x+1)(x-9)}{7(x+1)(x-9)}$

Example 14 Simplifying by factorising

Simplify these fractions by factorising first.

a $\frac{2x+6}{2}$ **b** $\frac{-3x-9}{x+3}$

SOLUTION

a $\frac{2x+6}{2} = \frac{\cancel{2}^{1}(x+3)}{\cancel{2}^{1}}$

$= x+3$

EXPLANATION

First factorise the numerator, then cancel the 2.

b $\frac{-3x-9}{x+3} = \frac{-3\cancel{(x+3)}^{1}}{\cancel{x+3}^{1}}$

$= -3$

-3 is common to both $-3x$ and -9. $(x+3)$ can now be cancelled.

Now you try

Simplify these fractions by factorising first.

a $\frac{5a-30}{5}$ **b** $\frac{-7x-14}{x+2}$

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5 Simplify these fractions by factorising first.

a $\frac{4x+8}{4}$ **b** $\frac{2x-6}{2}$ **c** $\frac{3x-12}{3}$

d $\frac{3}{3x+3}$ **e** $\frac{-7}{14x+21}$ **f** $\frac{5a}{15a-10}$

g $\frac{2x+8}{x+4}$ **h** $\frac{7x+14}{x+2}$ **i** $\frac{-5x-15}{x+3}$

j $\frac{-4x-10}{2x+5}$ **k** $\frac{18+12x}{3+2x}$ **l** $\frac{-25-15x}{5+3x}$

Hint: First factorise the numerator or denominator, then cancel.

Example 15 Multiplying algebraic fractions

Simplify these products.

a $\frac{3x}{4} \times \frac{8}{9x}$ **b** $\frac{4(x+1)}{3} \times \frac{12}{x+1}$

SOLUTION

a $\frac{\cancel{3x}^{1}}{\cancel{4}^{1}} \times \frac{\cancel{8}^{2}}{\cancel{9x}^{3}} = \frac{1 \times 2}{1 \times 3}$

$= \frac{2}{3}$

EXPLANATION

Cancel common factors from any numerator to any denominator, then multiply.

b $\frac{\cancel{4(x+1)}^{1}}{\cancel{3}_{1}} \times \frac{\cancel{12}^{4}}{\cancel{x+1}^{1}} = \frac{4 \times 4}{1 \times 1}$

$= 16$

Cancel all common factors before multiplying.

Now you try

Simplify these products.

a $\frac{2a^2}{3} \times \frac{9}{a}$ **b** $\frac{3x-1}{5} \times \frac{20}{6x-2}$

6 Simplify these products.

a $\frac{4x}{5} \times \frac{10}{8x}$ **b** $\frac{3x}{7} \times \frac{7}{6x}$ **c** $\frac{15a}{4} \times \frac{8}{25}$

d $\frac{16}{3a} \times \frac{9}{32}$ **e** $\frac{4y^2}{3} \times \frac{7x}{4y}$ **f** $\frac{8}{3x} \times \frac{5x^2}{16}$

g $\frac{2(x-1)}{3} \times \frac{6}{x-1}$ **h** $\frac{5(2x+1)}{4} \times \frac{4}{2x+1}$ **i** $\frac{3(x-7)}{7} \times \frac{21}{2(x-7)}$

j $\frac{x-3}{4} \times \frac{x+1}{x-3}$ **k** $\frac{3x+7}{x-2} \times \frac{x-2}{3x+7}$ **l** $\frac{5(x-2)}{3(x+7)} \times \frac{3(x+6)}{10(x-2)}$

Hint: Remember to cancel first before multiplying.

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Example 16 Dividing algebraic fractions

Simplify the following.

a $\frac{3x}{4} \div \frac{6}{7}$

b $\frac{5(x+1)}{6} \div \frac{2(x+1)}{9}$

SOLUTION

a $\frac{3x}{4} \div \frac{6}{7} = \frac{3^{1}x}{4} \times \frac{7}{6^{2}}$

$= \frac{7x}{8}$

EXPLANATION

Multiply by the reciprocal of the second fraction.

Cancel before multiplying.

b $\frac{5(x+1)}{6} \div \frac{2(x+1)}{9} = \frac{5(x+1)^{1}}{6_{2}} \times \frac{9^{3}}{2(x+1)^{1}}$

$= \frac{15}{4}$

Invert the second fraction and multiply.

Cancel before multiplying.

Now you try

Simplify the following.

a $\frac{-2x}{3} \div \frac{4}{9}$

b $\frac{2}{3(4a-1)} \div \frac{5}{4a-1}$

7 Simplify the following.

a $\frac{2x}{3} \div \frac{4}{7}$

b $\frac{5x}{4} \div \frac{15}{8}$

c $\frac{-3a}{4} \div \frac{2a}{7}$

d $\frac{-6a}{7} \div \frac{12}{7a}$

e $\frac{4(x+1)}{3} \div \frac{2(x+1)}{9}$

f $\frac{2(x-1)}{5} \div \frac{3(x-1)}{10}$

g $\frac{11(x+4)}{x+1} \div \frac{x+4}{x+1}$

h $\frac{-3(2x-1)}{2x+1} \div \frac{2(2x-1)}{2x+1}$

i $\frac{-7(4x+3)}{2x-7} \div \frac{14(4x+3)}{2x-4}$

Hint: Multiply by the reciprocal of the fraction following the division sign.

PROBLEM-SOLVING AND REASONING 8–9(½) 8–10(½)

8 Find the error in these problems and then find the correct answer.

a $\frac{2x+2^{1}}{2^{1}} = 2x+1$

b $\frac{-3x-6}{3} = \frac{-3(x-2)}{3}$

$= -(x-2)$

c $\frac{2x}{3} \div \frac{4x}{9} = \frac{8x^2}{27}$

d $\frac{x+2^{1}}{3^{1}} \times \frac{6^{2}x}{2^{1}} = 2x^2+1$

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9 Simplify the following.

a $\frac{2x+4}{3} \times \frac{6}{x+2}$

b $\frac{5x-15}{7} \times \frac{7}{x-3}$

c $\frac{-6x-12}{x} \times \frac{3x}{x+2}$

d $\frac{5x-5}{2} \div \frac{x-1}{6}$

e $\frac{-17x-34}{x+1} \div \frac{x+2}{2x+2}$

f $\frac{-7x+14}{3x+2} \div \frac{2-x}{6x+4}$

Hint: Factorise expressions before cancelling; e.g. $2x + 4$ can be written as $2(x + 2)$.

10 We know that $\frac{a^2}{a}$ cancels to $\frac{a}{1} = a$. Similarly, $\frac{(x+1)^2}{x+1} = \frac{(x+1)\cancel{(x+1)}}{\cancel{x+1}} = x + 1$. Use this idea to cancel the following.

a $\frac{(x+3)^2}{x+3}$

b $\frac{(2x-1)^2}{2x-1}$

c $\frac{-5(x-3)^2}{x-3}$

d $\frac{-2(x-4)}{6(x-4)^2}$

ENRICHMENT: The negative 1 factor — 11

11 Note that $3 - 2x = -1(2x - 3)$. Use this idea to cancel the following.

a $\frac{3-2x}{2x-3}$

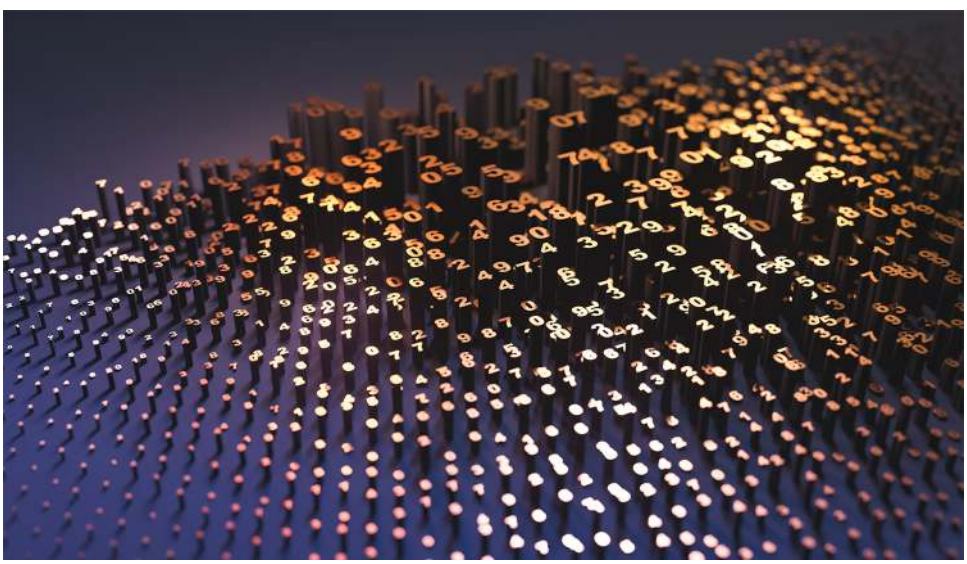
b $\frac{5-7x}{7x-5}$

c $\frac{4(2-x)}{3(x-2)}$

d $\frac{-2(x-5)}{5-x}$

e $\frac{2-x}{(x-2)^2}$

f $\frac{16(x-6)^2}{4(6-x)}$



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9G Simplifying algebraic fractions: addition and subtraction

Learning intentions for this section:

- To know the steps for adding and subtracting algebraic fractions
- To be able to find the lowest common denominator of fractions
- To be able to add and subtract algebraic fractions

Past, present and future learning:

- This concept may be new to students
- This section addresses the Stage 5 Core topic called *Algebraic techniques A*
- This section has very limited relevance to Stage 6 Standard

The process required for adding or subtracting algebraic fractions is similar to that used for fractions without pronumerals.

To simplify $\frac{2}{3}+\frac{4}{5}$, for example, you would find the lowest common multiple of the denominators (15) then express each fraction with this denominator. Adding the numerators completes the task.

Lesson starter: Compare the working

Here is the working for the simplification of the sum of a pair of numerical fractions and the sum of a pair of algebraic fractions.

$$\frac{2}{5}+\frac{3}{4}=\frac{8}{20}+\frac{15}{20}$$
$$=\frac{8+15}{20}$$
$$=\frac{23}{20}$$

$$\frac{2x}{5}+\frac{3x}{4}=\frac{8x}{20}+\frac{15x}{20}$$
$$=\frac{8x+15x}{20}$$
$$=\frac{23x}{20}$$

- What steps taken to simplify the algebraic fractions were the same as those used for the numerical fractions?
- Write down the steps required to add (or subtract) algebraic fractions.

KEY IDEAS

- To add or subtract algebraic fractions:
  - determine the **lowest common denominator** (LCD)
  - express each fraction using the LCD
  - add or subtract the numerators
  - simplify if necessary.

$$\frac{2x}{3}-\frac{x}{5}=\frac{10x}{15}-\frac{3x}{15}$$
$$=\frac{10x-3x}{15}$$
$$=\frac{7x}{15}$$

Key terminology: lowest common denominator, equivalent fraction, algebraic fraction, numerator, denominator

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Exercise 9G

UNDERSTANDING | 1–4 | 3, 4

1 Find the lowest common multiple (LCM) of these pairs of numbers.

a 6 and 8 **b** 3 and 5

c 11 and 13 **d** 12 and 18

2 Evaluate:

a $\frac{1}{2}+\frac{1}{3}$ **b** $\frac{3}{5}+\frac{1}{4}$

c $\frac{7}{8}+\frac{3}{4}$ **d** $\frac{9}{10}-\frac{1}{5}$

e $\frac{2}{3}-\frac{4}{5}$ **f** $\frac{11}{12}-\frac{5}{6}$

Hint: First find the LCM. Recall that the LCM of 8 and 4 is 8, not 32.

3 Write equivalent fractions by stating the missing expression.

a $\frac{2x}{5}=\frac{\square}{10}$ **b** $\frac{7x}{3}=\frac{\square}{9}$

c $\frac{x+1}{4}=\frac{\square(x+1)}{12}$ **d** $\frac{3x+5}{11}=\frac{\square(3x+5)}{22}$

e $\frac{4}{x}=\frac{\square}{2x}$ **f** $\frac{30}{x+1}=\frac{\square}{3(x+1)}$

4 Copy and complete these simplifications.

a $\frac{x}{4}+\frac{2x}{3}=\frac{\square}{12}+\frac{\square}{12}=\frac{\square}{12}$

b $\frac{5x}{7}-\frac{2x}{5}=\frac{\square}{35}-\frac{\square}{35}=\frac{\square}{35}$

Hint: Remember to multiply the numerator by the same number that you multiplied the denominator.

FLUENCY | 5, 6(½) | 5, 6–7(½)

5 Write down the LCD for these pairs of fractions.

a $\frac{x}{3}, \frac{2x}{5}$ **b** $\frac{3x}{7}, \frac{x}{2}$ **c** $\frac{-5x}{4}, \frac{x}{8}$

d $\frac{2x}{3}, \frac{-5x}{6}$ **e** $\frac{7x}{10}, \frac{-3x}{5}$

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Example 17 Adding and subtracting with a numeral in the denominator

Simplify:

a $\frac{7x}{3}+\frac{x}{6}$ b $\frac{x}{4}-\frac{2x}{5}$

SOLUTION

a $\frac{7x}{3}+\frac{x}{6}=\frac{7x\times 2}{3\times 2}+\frac{x}{6}$

$=\frac{14x}{6}+\frac{x}{6}$

$=\frac{14x+x}{6}$

$=\frac{15x}{6}$

$=\frac{5x}{2}$

EXPLANATION

Note that the LCM of 3 and 6 is 6, not $3\times 6=18$.

For $\frac{7x}{3}$, multiply the numerator and denominator by 2. $\frac{x}{6}$ already has a denominator of 6.

$\frac{15}{6}=\frac{5}{2}$ after cancelling.

b $\frac{x}{4}-\frac{2x}{5}=\frac{5x}{20}-\frac{8x}{20}$

$=\frac{5x-8x}{20}$

$=-\frac{3x}{20}$

Determine the LCM of 4 and 5; i.e. 20.

Express each fraction as an equivalent fraction with a denominator of 20. $2x\times 4=8x$.

Then subtract numerators.

Now you try

Simplify:

a $\frac{2a}{5}+\frac{3a}{10}$ b $\frac{7x}{8}-\frac{2x}{3}$

6 Simplify:

a $\frac{x}{7}+\frac{x}{2}$ b $\frac{x}{3}+\frac{x}{15}$ c $\frac{x}{4}-\frac{x}{8}$

d $\frac{x}{9}+\frac{x}{5}$ e $\frac{y}{7}-\frac{y}{8}$ f $\frac{a}{2}+\frac{a}{11}$

g $\frac{b}{3}-\frac{b}{9}$ h $\frac{m}{3}-\frac{m}{6}$ i $\frac{m}{6}+\frac{3m}{4}$

j $\frac{a}{4}+\frac{2a}{7}$ k $\frac{2x}{5}+\frac{x}{10}$ l $\frac{p}{9}-\frac{3p}{7}$

m $\frac{b}{2}-\frac{7b}{9}$ n $\frac{9y}{8}+\frac{2y}{5}$ o $\frac{4x}{7}-\frac{x}{5}$

p $\frac{3x}{4}-\frac{x}{3}$

Hint:
The LCM of 7 and 2 is 14, so write $\frac{x}{7}+\frac{x}{2}=\frac{2x}{14}+\frac{7x}{14}$

$=\frac{2x+7x}{14}$

$=\frac{9x}{14}$

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Example 18 Adding more algebraic fractions

Simplify: $\frac{x+3}{2}+\frac{x-2}{5}$

SOLUTION

$$\frac{(x+3)}{2}+\frac{(x-2)}{5}=\frac{5(x+3)}{10}+\frac{2(x-2)}{10}$$
$$=\frac{5x+15+2x-4}{10}$$
$$=\frac{7x+11}{10}$$

EXPLANATION

The LCM of 2 and 5 is 10. Write as equivalent fractions with denominator 10.

Expand the brackets and simplify the numerator by adding and collecting like terms.

Now you try

Simplify: $\frac{x-3}{4}+\frac{x-2}{3}$

7 Simplify:

a $\frac{x+1}{2}+\frac{x+3}{5}$ **b** $\frac{x+3}{3}+\frac{x-4}{4}$

c $\frac{a-2}{7}+\frac{a-5}{8}$ **d** $\frac{y+4}{5}+\frac{y-3}{6}$

e $\frac{m-4}{8}+\frac{m+6}{5}$ **f** $\frac{x-2}{12}+\frac{x-3}{8}$

g $\frac{2b-3}{6}+\frac{b+2}{8}$ **h** $\frac{3x+8}{6}+\frac{2x-4}{3}$

i $\frac{2y-5}{7}+\frac{3y+2}{14}$ **j** $\frac{2t-1}{8}+\frac{t-2}{16}$

k $\frac{4-x}{3}+\frac{2-x}{7}$ **l** $\frac{2m-1}{4}+\frac{m-3}{6}$

Hint: Remember to include brackets when multiplying a numeral by these binomial numerators.

PROBLEM-SOLVING AND REASONING 8, 9 9–11

8 A student thinks that the LCD to use when simplifying $\frac{x}{2}+\frac{3x}{4}$ is 8.

a Complete the simplification using a common denominator of 8.

b Now complete the simplification using the actual LCD of 4.

c How does your working for parts **a** and **b** compare? Which method is preferable and why?

9 Find and describe the error in each set of working. Then give the correct answer.

a $\frac{4x}{5}-\frac{x}{3}=\frac{3x}{2}$

b $\frac{x+1}{5}+\frac{x}{2}=\frac{2x+1}{10}+\frac{5x}{10}$
$=\frac{7x+1}{10}$

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10 Simplify the following.

a $x + \frac{x}{2}$

b $x + \frac{x}{4}$

c $x + \frac{3x}{4}$

d $2x + 2x + \frac{x}{2}$

Hint: Recall that x can be written as $\frac{x}{1}$.

Ext

11 Simplify by first finding the LCD:

a $\frac{2x}{5} - \frac{3x}{2} - \frac{x}{3}$

b $\frac{x}{4} - \frac{2x}{3} + \frac{5x}{6}$

c $\frac{5x}{8} - \frac{5x}{6} + \frac{3x}{4}$

d $\frac{x+1}{4} + \frac{2x-1}{3} - \frac{x}{5}$

e $\frac{2x-1}{3} - \frac{2x}{7} + \frac{x-3}{6}$

f $\frac{1-2x}{5} - \frac{3x}{8} + \frac{3x+1}{2}$

Hint: Choose a common denominator that works for all three fractions.

ENRICHMENT: The missing algebraic fractions — 12

12 Find the missing algebraic fraction. The fraction should be in simplest form.

a $\frac{x}{2} + \frac{\square}{\square} = \frac{5x}{6}$

b $\frac{\square}{\square} + \frac{x}{4} = \frac{3x}{8}$

c $\frac{2x}{5} + \frac{\square}{\square} = \frac{9x}{10}$

d $\frac{2x}{3} - \frac{\square}{\square} = \frac{7x}{15}$

e $\frac{\square}{\square} - \frac{x}{3} = \frac{5x}{9}$

f $\frac{2x}{3} - \frac{\square}{\square} = \frac{5x}{12}$

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Automotive technology

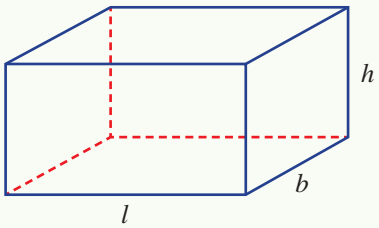
The automotive industry is now more technically complex with the increased use of computer technology in cars. Training for automotive trades requires students to have mechanical ability as well as problem-solving and mathematical skills.

The machinists and technicians constructing and installing parts must have an eye for detail and excellent measurement skills.

Formulas are widely used in all trades to calculate quantities such as area, volume and maximum loads and friction. The universal tool for simplifying such formulas is algebra.

1 Write the algebraic formula for the surface area (A) of each solid below using the given pronumerals. Simplify each formula by taking out any common factors.

a



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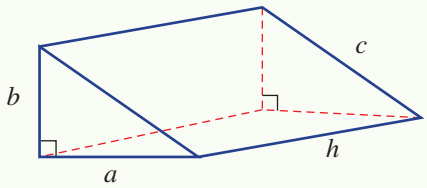

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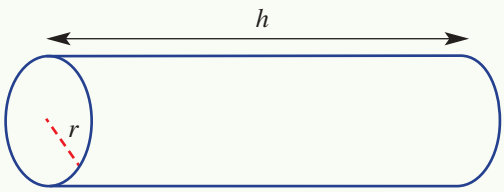

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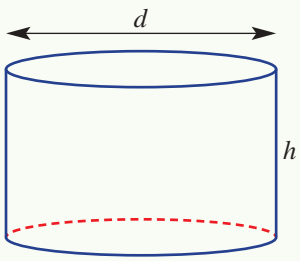

2 Match each of the following volume formulae to one of the solids shown in Question **1**.

i $V = \pi r^2 h$

ii $V = lbh$

iii $V = \frac{1}{2}abh$

iv $V = \frac{\pi d^2 h}{4}$

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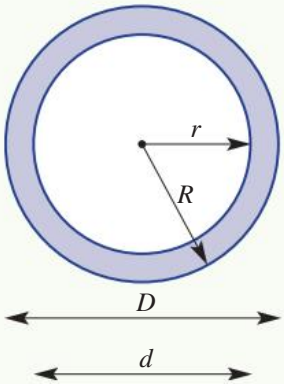
3 The cross-sectional area of a pipe is used when calculating the weight of a given length of pipe. To simplify the area formula, the algebraic difference of two squares rule is used.

a Copy and complete this procedure.



$$\text{Area of cross-section} = \pi R^2 - \square$$
$$= \pi(\square - r^2)$$
$$= \pi\left(\frac{\square}{2^2} - \frac{d^2}{2^2}\right)$$
$$= \frac{\pi}{4}(D^2 - \square)$$
$$= \frac{\pi}{4}(D + d)(\square - \square)$$

b Use the above formula to calculate the exact area of cross-section, as a multiple of π, given the following diameters of various pipes.

i $D = 40$ mm and $d = 36$ mm

ii $D = 32$ mm and $d = 30$ mm

iii $D = 2.7$ cm and $d = 2.3$ cm

iv $D = 6.5$ cm and $r = 5.5$ cm

Using technology

4 The approximate weight of a length of pipe can be determined by this formula:

$$\text{weight} = \text{cross-section area (cm}^2\text{)} \times \text{length (cm)} \times \text{density (g/cm}^3\text{)}$$

Develop the Excel spreadsheet shown below to calculate the weights of various lengths of pipe. Enter appropriate formulas into the shaded cells.

Hint: When entering the formula for cross-sectional area use *pi()* for π. Link all the length cells' formulae to cell F2 by using $ signs, i.e. = F2. Format area and weight cells to 'number' with two decimal places.

	A	B	C	D	E	F	G	H
1	Weight of pipe lengths							
2						100		
3	Pipe material	Outer diameter in cm	Inner diameter in cm	Area of cross-section in cm^2	Density in g/cm^3	Length in cm	Weight in g	Weight in kg
4	Poly pipe a	4.3	3.7		0.955			
5	Poly pipe b	12.5	11.9		0.955			
6	Poly pipe c	31.5	28.5		0.955			
7	Copper pipe a	1.2	1		8.94			
8	Copper pipe b	2	1.9		8.94			
9	Copper pipe c	5	4.8		8.94			
10	Steel pipe a	4.8	4.4		7.85			
11	Steel pipe b	6	5		7.85			
12	Steel pipe c	16.5	15.5		7.85			

a Find the difference in kg/m between copper pipe c and steel pipe b.

b Find the difference in weight between 3.5 m of poly pipe a and 3.5 m of steel pipe a.

c Find the difference in weight between 2.75 m of poly pipe c and 5 m of steel pipe c.

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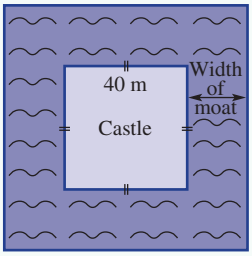
Working mathematically

Square castle moats

Moats were constructed around castles to help protect their occupants from invaders. A square castle of side length 40 metres is to be surrounded by a square moat of a certain width as shown.



Present a report for the following tasks and ensure that you show clear mathematical workings, explanations and diagrams where appropriate.

Routine problems

a Find the area of land occupied by the square castle of side 40 metres.

b If the width of the moat is 10 metres, find the surface area of the water in the moat.

c Find the outside perimeter of the moat if the width of the moat is 8.5 metres.

Non-routine problems

Explore and connect

The problem is to determine a width for the moat so that the surface area of water is at least 10 times the area of the castle.

a Write down all the relevant information that will help solve this problem with the aid of a diagram.

b If the width of the moat is x metres, find expressions in terms of x for the following:

i the perimeter of the outside of the moat

ii the total area occupied by the moat and the castle.

Choose and apply techniques

c Use your expressions from part **b** to show that the area occupied by the moat, not including the castle, can be written in the form $4x(40 + x)$.

d Using the expression from part **c**, find the area occupied by the moat for the following values of x.

i 10 **ii** 40 **iii** 50

e For each value of x in part **d**, calculate the value of $\frac{\text{Moat area}}{\text{Castle area}}$.

f Using trial and error, determine the **smallest integer** value of x such that the area of the moat is at least 10 times the area of the castle.

Communicate thinking and reasoning

g Assuming that the lord of the castle wants the area of the moat to be as close as possible to 10 times the area of the castle, investigate if this is possible using non-integer values of x to one decimal place.

h Summarise your results using a table like the one below and describe any key findings.

Moat width x metres					
Area of moat in m^2					
Area of castle in m^2					
$\frac{\text{Moat area}}{\text{Castle area}}$					

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Extension problem

Problem-solve

a Instead, the lord wants to build a square castle of side length x metres inside a square lake of side length 50 metres.

i Draw a labelled diagram showing the square castle within the square moat.

ii Show that the remaining lake area forming the moat is given by the expression $(50 + x)(50 - x)$.

iii Calculate the value of $\frac{\text{Moat area}}{\text{Castle area}}$ to one decimal place, when:

I $x = 13$

II $x = 17$

iv Determine the smallest integer width of the castle that delivers a moat area which is at least 10 times the area of the castle.

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Puzzles and challenges

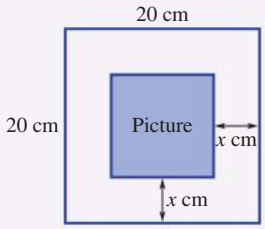
1 'You use me when dividing fractions.' Simplify each expression, then match the answers to the letters to solve the riddle.

P	L	C	R
$-2x \times 3x$	$(x+1)(x-1)$	$\frac{2x-6}{2}$	$\frac{5x^2}{10x}$

O	A	E	I
$\frac{x}{2} - \frac{x}{3}$	$\frac{x+3}{2} \times \frac{6}{x+3}$	$\frac{2x}{3} \div \frac{4}{9}$	$7ab - 6ba$

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
$\frac{x}{2}$	$\frac{3x}{2}$	$x-3$	ab	$-6x^2$	$\frac{x}{2}$	$\frac{x}{6}$	$x-3$	3	x^2-1

2 A square framed photograph has side length 20 cm and frame width x cm.

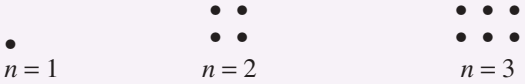


a Find an expanded expression for the area of the picture.

b What value of x makes the picture $\frac{1}{4}$ of the total area?

3 Write an expression that gives the total number of dots required for the nth diagram.

a



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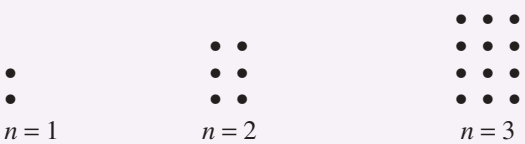

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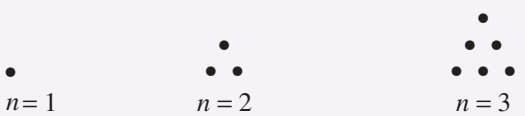

4 Simplify these algebraic fractions. Don't be fooled by the double negative.

a $\frac{x+1}{3} - \frac{x-2}{4}$ **b** $\frac{3x-4}{5} - \frac{4x-2}{3}$

5 **a** The difference between the squares of two consecutive numbers is 97. What are the two numbers?

b The difference between the squares of two consecutive odd numbers is 136. What are the two numbers?

c The difference between the squares of two consecutive multiples of 3 is 81. What are the two numbers?

6 In a race over 4 km, Ryan ran at a constant speed. Sophie ran the first 2 km at a speed 1 km/h faster than Ryan. She ran the second 2 km at a speed 1 km/h slower than Ryan. Who won the race?

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Chapter summary

Quadratic expressions and algebraic fractions

Multiplying and dividing terms

$-3ab \times 2a = -6a^2b$

$15xy \div (3x^2y) = \frac{15xy}{3x^2y} = \frac{5}{x}$

Factorisation

The reverse process of expansion. Always remove the highest common factor first.

E.g. $2ab + 8b$ HCF $= 2b$
$= 2b(a + 4)$

E.g. $3x - 9 = 3(x - 3)$ HCF $= 3$

$-4x - 8 = -4(x + 2)$ HCF $= -4$

Expansion

The process of removing brackets

E.g. $2(x + 5) = 2x + 10$

	5
	x
2	

$(x + 3)(2x - 4) = 2x^2 - 4x + 6x - 12$
$= 2x^2 + 2x - 12$

Review

3-term expression

$7xy - 4yx + 6$

like terms (can collect) — constant term

The coefficient of x in $4 - 2x$ is -2.

Difference of two squares

$(a - b)(a + b) = a^2 - b^2$

E.g. $(x - 5)(x + 5) = x^2 + 5x - 5x - 25$
$= x^2 - 25$

Simplifying by cancelling

E.g. $\frac{7(x+1)}{2(x+1)} = \frac{7}{2}$

E.g. $\frac{5x - 10}{5} = \frac{{}^{1}\cancel{5}(x - 2)}{\cancel{5}_{1}} = x - 2$

Perfect squares

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

E.g. $(2x + 3)^2 = (2x + 3)(2x + 3)$
$= 4x^2 + 6x + 6x + 9$
$= 4x^2 + 12x + 9$

Algebraic fractions

These involve algebraic expressions in the numerator and/or denominator.

E.g. $\frac{2x}{x - 7}$, $\frac{3}{5x}$, $\frac{7}{(x - 1)^2}$

Add/subtract

Must find lowest common denominator (LCD) before applying operation.

E.g. $\frac{2x}{3} - \frac{x}{4} = \frac{8x}{12} - \frac{3x}{12}$
$= \frac{5x}{12}$

E.g. $\frac{x - 1}{2} + \frac{x - 3}{3} = \frac{3(x - 1)}{6} + \frac{2(x - 3)}{6}$
$= \frac{3x - 3 + 2x - 6}{6}$
$= \frac{5x - 9}{6}$

Multiplying

Cancel any numerator with any denominator.

E.g. $\frac{{}^{1}\cancel{3x + 2}}{\cancel{5}_{1}} \times \frac{{}^{3}\cancel{15}}{2\cancel{(3x + 2)}_{1}}$

$= \frac{3}{2}$

Dividing

To divide, multiply by the reciprocal. Factorise all expressions, cancel and then multiply.

E.g. $\frac{3x}{4} \div \frac{9x}{8}$

$= \frac{{}^{1}\cancel{3x}}{\cancel{4}_{1}} \times \frac{{}^{2}\cancel{8}}{{}_{3}\cancel{9x}}$

$= \frac{2}{3}$

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Chapter checklist

Chapter checklist with success criteria

A printable version of this checklist can be downloaded from the Interactive Textbook.

		✔
9A	**1 I can identify and collect like terms.** e.g. Simplify: **a** $4a^2 - ab - 2a^2 + 1$ **b** $5x^2y - xy - 6yx^2$	☐
9A	**2 I can evaluate an algebraic expression using substitution.** e.g. If $x = 2$ and $y = -5$, evaluate $y^2 - xy$.	☐
9A	**3 I can multiply and divide algebraic terms.** e.g. Simplify: **a** $2ab \times 7a$ **b** $8xy \div (24x)$	☐
9A	**4 I can expand expressions with brackets.** e.g. Expand: **a** $-3(x - 2)$ **b** $-2a(4a - 5)$	☐
9B	**5 I can expand binomial products.** e.g. Expand: **a** $(x - 3)(2x + 5)$ **b** $(3x - 7)(2x - 5)$	☐
9C	**6 I can expand perfect squares.** e.g. Expand: **a** $(x + 7)^2$ **b** $(4x - 3)^2$	☐
9D	**7 I can expand to form a difference of two squares.** e.g. Expand: **a** $(x - 4)(x + 4)$ **b** $(3x + 5)(3x - 5)$	☐
9E	**8 I can determine the HCF.** e.g. Determine the HCF of the terms $6xy$ and $18xy^2$.	☐
9E	**9 I can factorise expressions with common factors.** e.g. Factorise the following: **a** $5a + 15$ **b** $11x^2 - 33xy$ **c** $-14mn - 7m$ (including common negative)	☐
9F	**10 I can simplify algebraic fractions.** e.g. Simplify this fraction by factorising first: $\frac{2x - 4}{x - 2}$.	☐
9F	**11 I can multiply algebraic fractions.** e.g. Simplify $\frac{3x}{4} \times \frac{2x}{9}$.	☐
9F	**12 I can divide algebraic fractions.** e.g. Simplify $\frac{3x}{4} \div \frac{9x}{2}$.	☐
9G	**13 I can add and subtract simple algebraic fractions.** e.g. Simplify: **a** $\frac{a}{2} + \frac{a}{3}$ **b** $\frac{2x}{5} - \frac{x}{3}$	☐

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Short-answer questions

9A **1** Simplify these expressions.

a $3x \times 2x$ **b** $-3ab \times 6b$ **c** $7ab \div (14a)$ **d** $-40x^2y \div (10xy)$

9A **2** Expand the following.

a $3(x-4)$ **b** $-2(x+6)$ **c** $-x(x-1)$ **d** $-4x(2x-3)$

9A **3** Collect like terms to simplify.

a $5x - 1 + 2x$ **b** $7ab - 2ba$ **c** $4a^2b - 7a^2b$ **d** $5x^2 - xy + 3x^2$

9B **4** Expand the following binomial products.

a $(x-3)(x+4)$ **b** $(x-7)(x-2)$ **c** $(2x-3)(3x+2)$ **d** $(x-1)(3x+4)$

9C/9D **5** Expand the following.

a $(x+3)^2$ **b** $(x-4)^2$ **c** $(3x-2)^2$

d $(x-5)(x+5)$ **e** $(7-x)(7+x)$ **f** $(11x-4)(11x+4)$

9E **6** Write the following in fully factorised form by removing the highest common factor.

a $4a + 12b$ **b** $-3 - 9x$ **c** $x^2 + x$

d $6x - 9x^2$ **e** $-5x^2y - 10xy$ **f** $11a^2b - 33ab$

9F **7** Simplify the following.

a $\dfrac{3(x-1)}{6}$ **b** $\dfrac{(x+1)(x-3)}{x-3}$ **c** $\dfrac{3x+12}{3}$

d $\dfrac{-5x-15}{10}$ **e** $\dfrac{2x-16}{3x-24}$ **f** $\dfrac{2(x+2)(x-1)}{x+2}$

9F **8** Simplify the following algebraic fractions by first factorising and cancelling where possible.

a $\dfrac{3}{2x} \times \dfrac{x}{6}$ **b** $\dfrac{7a^2}{x+1} \times \dfrac{x+1}{14a}$ **c** $\dfrac{x(x-4)}{8(x+1)} \times \dfrac{4(x+1)}{x}$

d $\dfrac{3a}{5} \div \dfrac{9}{10}$ **e** $\dfrac{2(x-1)}{3} \div \dfrac{x-1}{6}$ **f** $\dfrac{2x}{5x+20} \div \dfrac{x}{x+4}$

9G **9** Simplify the following by first finding the lowest common denominator.

a $\dfrac{x}{4} + \dfrac{2x}{3}$ **b** $\dfrac{5x}{6} - \dfrac{7x}{8}$ **c** $\dfrac{3a}{4} - \dfrac{a}{2}$

d $\dfrac{x}{2} + \dfrac{x-1}{3}$ **e** $\dfrac{2x-3}{4} + \dfrac{x+1}{3}$ **f** $\dfrac{x-1}{6} + \dfrac{x+3}{8}$

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Chapter review

Multiple-choice questions

9A **1** The constant term in the expression $2x^2 - 3$ is:

A 3 **B** 2 **C** -2 **D** $2x^2$ **E** -3

9A **2** The expanded form of $-2(x + 3)$ is:

A $2x + 6$ **B** $-2x - 3$ **C** $-2x - 6$ **D** $-2x + 6$ **E** $-2x + 3$

9A **3** $x^2 - 2xy + 2yx$ is equal to:

A xy **B** x^2 **C** $x^2 - 4xy$ **D** 0 **E** $4xy$

9A **4** $-3ab \times 4b$ is equal to:

A $-7ab^2$ **B** $-12ab^2$ **C** $-7a^2b$ **D** $-12a^2b$ **E** $12ab^2$

9B **5** $(x - 3)(x - 4)$ expands to:

A 12 **B** $x^2 + 12$ **C** $x^2 - 7x + 12$ **D** $x^2 - x - 12$ **E** $x^2 + 7x - 12$

9B **6** $(2x - 1)(x + 5)$ in expanded and simplified form is:

A $2x^2 + 9x - 5$ **B** $x^2 + 11x - 5$ **C** $4x^2 - 5$ **D** $3x^2 - 2x + 5$ **E** $2x^2 + 4x - 5$

9C **7** $(3a + 2b)^2$ is equivalent to:

A $9a^2 + 6ab + 4b^2$ **B** $9a^2 + 4b^2$ **C** $3a^2 + 6ab + 2b^2$
D $3a^2 + 12ab + 2b^2$ **E** $9a^2 + 12ab + 4b^2$

9F **8** $\frac{5x}{3} \div \frac{15x}{12}$ is equal to:

A $\frac{4}{3}$ **B** $\frac{4x}{3}$ **C** $\frac{4}{3x}$ **D** $\frac{25x^2}{12}$ **E** $\frac{25x}{12}$

9G **9** $\frac{4x}{3} - \frac{2x}{9}$ is equal to:

A $\frac{10x}{27}$ **B** $\frac{30x}{9}$ **C** $\frac{2x}{9}$ **D** $\frac{2x}{3}$ **E** $\frac{10x}{9}$

9G **10** $\frac{x + 2}{5} + \frac{2x - 1}{3}$ written as a single fraction is:

A $\frac{11x + 1}{15}$ **B** $\frac{11x + 9}{8}$ **C** $\frac{3x + 1}{8}$ **D** $\frac{11x + 7}{15}$ **E** $\frac{13x + 1}{15}$

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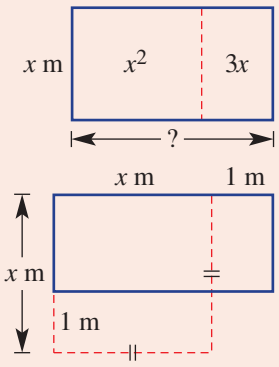
Extended-response questions

1 A pig pen for a small farm is being redesigned. It is originally a square of side length x m.



a In the planning, the length is initially kept as x m. The breadth is altered such that the area of the pen is $(x^2 + 3x)$ square metres. What is the new breadth? (Consider factorising.)

b Instead, it is decided that the original length will be increased by 1 metre and the original breadth will be decreased by 1 metre.

i What effect does this have on the perimeter of the pig pen compared with the original size?

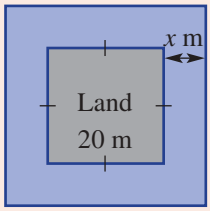
ii Work out an expression for the new area of the pig pen in expanded form. How does this compare to the original area?

c For the pig pen described in part **b**:

i what would be the area if $x = 2$?

ii what value of x gives an area of 15 m^2? Use trial and error.

2 The security tower for a palace is on a small square piece of land 20 m by 20 m. It has a moat of width x metres the whole way around it, as shown.



a State the area of the piece of land.

b **i** Give an expression for the side length of the combined moat and land.

ii Find an expression, in expanded form, for the entire area occupied by the moat and the land.

c Use your answers to parts **a** and **b** to give an expression for the area occupied by the moat alone. Answer in factorised form.

d Use trial and error to find the value of x such that the area of the moat alone is 500 m^2.

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10

Probability and data analysis

Maths in context: Data skills in meteorology, medicine and insurance

Statistical measures, displays and probabilities are essential for making sense of huge amounts of data. Statistics and probability are widely used for planning, including by farmers, sports clubs, businesses, medical researchers, governments, insurance and marketing agents.

- The probabilities of rain, storms, wind strengths, temperatures, fires, floods and droughts given in weather forecasts are important for planning by emergency services, air traffic control, farmers and holiday makers.
- Venn diagrams and arrays help us to analyse data, such as for the number of people who smoke, die of cancer or both; and the number of vehicle accidents caused by speed, alcohol or both.
- Car insurance premiums are based on past proportions of accidents with drivers of various ages. Hence car insurance is a lot more expensive for car owners aged under 25.

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Chapter contents

10A Probability review (CONSOLIDATING)
10B Venn diagrams and two-way tables (EXTENDING)
10C Using arrays for two-step experiments
10D Using tree diagrams
10E Using relative frequencies to estimate probabilities
10F Mean, median, mode and range (CONSOLIDATING)
10G Interpreting data from tables and graphs
10H Stem-and-leaf plots
10I Grouping data into classes

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- solves problems involving probabilities in multistage chance experiments and simulations (MA5-PRO-C-01)
- compares and analyses datasets using summary statistics and graphical representations (MA5-DAT-C-01).

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Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 9
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Warm-up quiz

1 Write as decimals:

a $\frac{1}{10}$ **b** $\frac{2}{8}$ **c** 30% **d** 85% **e** 23.7%

2 Express in simplest form:

a $\frac{4}{8}$ **b** $\frac{7}{21}$ **c** $\frac{20}{30}$ **d** $\frac{100}{100}$ **e** $\frac{0}{4}$

f $\frac{12}{144}$ **g** $\frac{36}{72}$ **h** $\frac{36}{58}$ **i** $\frac{72}{108}$ **j** $\frac{2}{7}$

3 A six-sided die is tossed.

a List the possible outcomes.

b How many of the outcomes are:

i even? **ii** less than 3? **iii** less than or equal to 3?

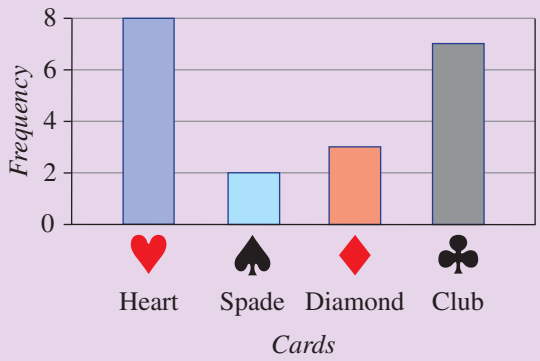
iv at least 2? **v** not a 6? **vi** not odd?

4 From the group of the first 10 integers $\{1, 2, \ldots, 10\}$, how many of the numbers are:

a odd? **b** less than 8? **c** greater than or equal to 5?

d no more than 7? **e** prime? **f** not prime?

5 Several cards were randomly selected from a pack of playing cards. The suit of each card was noted, the card was replaced and the pack was shuffled. The frequency of each suit is shown in the column graph.



a How many times was a heart selected?

b How many times was a card selected in total?

c In what fraction of the trials was a diamond selected?

6 Consider this simple dataset: 1, 2, 3, 5, 5, 7, 8, 10, 13.

a State the number of scores, n.

b Find the mean (the sum of scores divided by n).

c Find the median (the middle value).

d Find the mode (the most common value).

e Find the range (the difference between the highest and lowest value).

f Find the probability of randomly selecting:

i a 5

ii a number that is not 5

iii a number that is no more than 5

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10A Probability review CONSOLIDATING

Learning intentions for this section:

- To understand the concept of probability and how it is described numerically
- To know the terminology of probability
- To be able to find the probability of an event in an experiment with equally likely outcomes

Past, present and future learning:

- This section consolidates Stage 4 concepts that are used in Stage 5 and Stage 6 Standard
- Some of these questions are more challenging than those in our Stage 4 books

Probability is the study of chance. The probability of an event occurring is a number between 0 and 1. An impossible event has a probability of 0. An event that is certain to occur has a probability of 1.

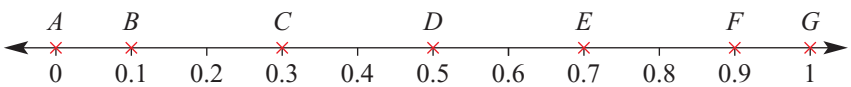
We can use probability to find the likelihood of rolling a total of 7 with two dice.

Lesson starter: Events and probabilities

As a class group, write down and discuss at least three events that have the following chance of occurring.

- no chance
- very low chance
- medium to low chance
- even (50–50) chance
- medium to high chance
- very high chance
- certainty

For each decimal marked on the number line below, choose the probability description from the list above that best matches it.



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KEY IDEAS

- There is a large amount of terminology in this topic. The following two pages include most of the key words and phrases.

Terminology	Example	Definition
Chance experiment	Rolling a fair six-sided die	A chance experiment is an activity that may produce a variety of different results that occur randomly. The example given is a single-step experiment.
Trials	Rolling a die 50 times	When an experiment is performed one or more times, each occurrence is called a trial. The example given indicates 50 trials of a single-step experiment.
Outcome	Rolling a 5	An outcome is one of the possible results of a chance experiment.
Equally likely outcomes	Rolling a 5 Rolling a 6	Equally likely outcomes are two or more results that have the same chance of occurring.
Sample space	$\{1, 2, 3, 4, 5, 6\}$	The sample space is the set of all possible outcomes of an experiment. It is usually written inside braces, as shown in the example.
Event	E.g. 1: rolling a 2 E.g. 2: rolling an even number	An event is either one outcome or a collection of outcomes. It is a subset of the sample space.
Compound event	Rolling an even number (e.g. 2 or 4 or 6)	A compound event is a collection of two or more outcomes from the sample space of a chance experiment.
Mutually exclusive events	Rolling a 5 Rolling an even number	Two or more events are mutually exclusive if they share no outcomes.
Non-mutually exclusive events	Rolling a 5 Rolling an odd number	Events are non-mutually exclusive if they share one or more outcomes. In the example given, the outcome 5 is shared.
Complementary events	Rolling a 2 or 3 Rolling a 1, 4, 5 or 6	If *all* the outcomes in the sample space are divided into two events, they are complementary events.
Complement	Rolling 2, 3, 4 or 5 is an event. Rolling a 1 or 6 is the complement.	If an experiment was performed and an event did not occur, then the complement definitely occurred.
Favourable outcome(s)	In some games you must roll a 6 before you can start moving your pieces.	Outcomes are favourable when they are part of some desired event.

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Terminology	Example	Definition
Theoretical probability or likelihood or chance	The probability of rolling an odd number is written as: $P(\text{odd}) = \frac{3}{6} = \frac{1}{2} = 0.5 = 50\%$ Probabilities can be expressed as fractions, decimals and percentages.	Theoretical or expected probability is the actual chance or likelihood that an event will occur when an experiment takes place. $P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$ Probabilities range from 0 to 1 or 0% to 100%.
Experimental or observed probability	A die was rolled 600 times and showed a 5 on 99 occasions. The experimental probability of rolling a 5 on this die is: $P(5) = \frac{99}{600} = 0.165 = 16.5\%$	Sometimes it is difficult or impossible to calculate a theoretical probability, so an estimate can be found using a large number of trials. This is called the experimental or observed probability. If the number of trials is large, then the experimental probability should be very close to the theoretical.
Certain Likely Even chance Unlikely Impossible	Rolling a number below 7 Rolling a number below 6 Rolling a 1, 2 or 3 Rolling a 2 Rolling a 7	The probability is 100% or 1. The probability is 50% or 0.5 or $\frac{1}{2}$. The probability is 0% or 0.
The sum of all probabilities in an experiment	$P(1) = \frac{1}{6}$ $P(2) = \frac{1}{6}$ $P(3) = \frac{1}{6}$ $P(4) = \frac{1}{6}$ $P(5) = \frac{1}{6}$ $P(6) = \frac{1}{6}$ $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{6}{6} = 1$	The sum of the probablities of all the outcomes of a chance experiment is 1.
The sum of the probabilities of an event and its complement	$P(\text{rolling 1 or 6}) = \frac{2}{6}$ $P(\text{rolling 2, 3, 4 or 5}) = \frac{4}{6}$ $\frac{2}{6} + \frac{4}{6} + \frac{6}{6} = 1$	The sum of the probabilities of an event and its complement is 1. $P(\text{event}) = P(\text{complementary event}) = 1$

Key terminology: probability, experiment, sample space, outcome, event, complement, chance

Exercise 10A

UNDERSTANDING 1–4 3,4

1 Write the missing words in each statement.

a The _____ _____ is the list of all possible outcomes in an experiment.

b An _____ is a collection of the outcomes from an experiment.

c *P* stands for _____.

d An _____ event has a probability of 0, an event that is _____ to occur has a probability of 1.

e An event with probability of 0.5 has _____ or 50 : 50 chance of occurring.

f The _____ of an event, A, is the event where A does not occur.

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2 Nathan believes that there is a 1 in 4 chance that the flower on his prize rose will bloom tomorrow.

a Write the chance '1 in 4' as:

i a fraction **ii** a decimal **iii** a percentage

b Draw a number line from 0 to 1 and mark the level of chance described by Nathan.

Hint: '1 in 4' means $\frac{1}{4}$.

Hint: Percentage ÷ 100 = decimal

3 Copy and complete this table.

	Percentage	Decimal	Fraction	Number line
a	50%	0.5	$\frac{1}{2}$	0, 0.5, 1
b	25%			
c			$\frac{3}{4}$	
d				0, 0.2, 0.5, 1

4 Ten people make the following guesses of the chance that they will get a salary bonus this year.

$0.7, \frac{2}{5}, 0.9, \frac{1}{3}$, 2 in 3, $\frac{3}{7}$, 1 in 4, 0.28, $\frac{2}{9}$, 0.15

Order their chances from lowest to highest.

Hint: First write each chance as a decimal.

FLUENCY 5(½), 6–9 5–7(½), 8, 9(½)

Example 1 Finding probabilities of events

This spinner has five equally divided sections.

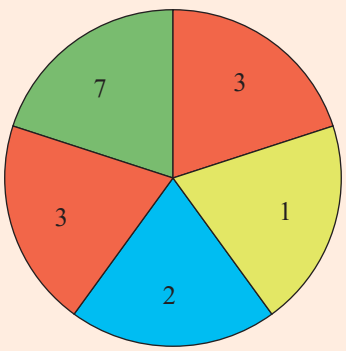
a List the sample space, using the given numbers.

b Find $P(3)$.

c Find $P(\text{not a } 3)$.

d Find $P(\text{a 3 or a 7})$.

e Find $P(\text{a number that is at least a 3})$.



SOLUTION

a {1, 2, 3, 3, 7}

b $P(3) = \frac{2}{5}$ or 0.4

EXPLANATION

Use braces, { }, and list all the possible outcomes in any order. Include 3 twice to represent its increased chance.

$P(3) = \frac{\text{number of sections labelled 3}}{\text{number of equal sections}}$

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c $P(\text{not a 3}) = 1 - P(3)$
$= 1 - \frac{2}{5}$ or $1 - 0.4$
$= \frac{3}{5}$ or 0.6

'Not a 3' is the complementary event of obtaining a 3.

Alternatively, count the number of sectors that are not 3.

d $P(\text{a 3 or a 7}) = \frac{2}{5} + \frac{1}{5}$
$= \frac{3}{5}$

There are two 3s and one 7 in the five sections.

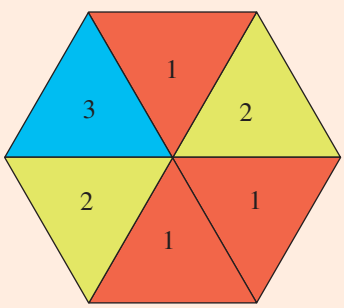
e $P(\text{at least a 3}) = \frac{3}{5}$

Three of the sections have the numbers 3 or 7, which are at least 3; i.e. 3 or more.

Now you try

This spinner has six equally divided sections.

a List the sample space using the different numbers.
b Find $P(2)$.
c Find $P(\text{not a 1})$.
d Find $P(\text{not a 2 or a 3})$.
e Find $P(\text{a number that is at most 2})$.

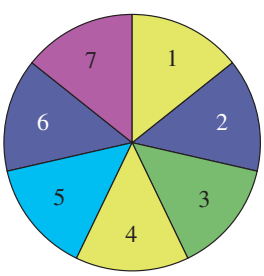


5 The spinners below have equally divided sections. Complete the following for each spinner. Write all probabilities as simplified fractions.

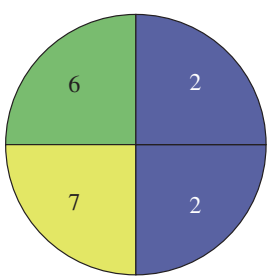
i List the sample space using the given numbers.
ii Find $P(2)$.
iii Find $P(\text{not a 2})$.
iv Find $P(\text{a 2 or a 3})$.
v Find $P(\text{a number that is at least a 2})$.

Hint: $P(\text{event})$ means 'find the probability of the event'. 'At least 2' means '2 or more'.

a



b



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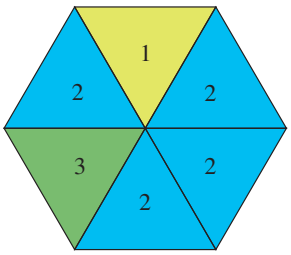
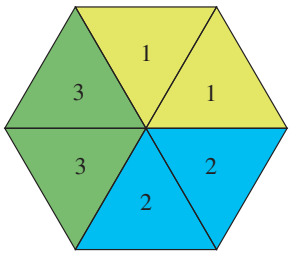
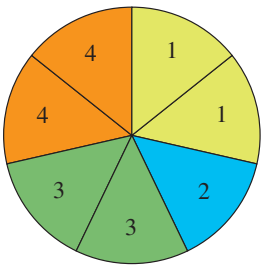
c



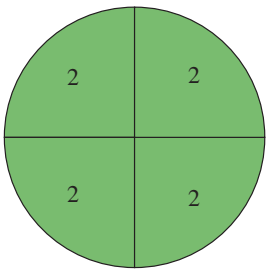
d



e



f



6 Find the probability of obtaining a blue ball if a ball is selected at random from a box that contains:

- **a** 4 blue balls and 4 red balls
- **b** 3 blue balls and 5 red balls
- **c** 1 blue ball, 3 red balls and 2 white balls
- **d** 8 blue balls, 15 black balls and 9 green balls
- **e** 15 blue balls only
- **f** 5 yellow balls and 2 green balls

Hint: $P(\text{blue}) = \frac{\text{number of blue balls}}{\text{total number of balls}}$

7 Find the probability of *not* selecting a blue ball if a ball is selected at random from a box containing the balls described in Question **6**, parts **a** to **f**, above.

Hint: $P(\text{not blue}) = 1 - P(\text{blue})$

8 If a swimming pool has eight lanes and each of eight swimmers has an equal chance of being placed in lane 1, find the probability that a particular swimmer:

- **a** will swim in lane 1
- **b** will not swim in lane 1

Example 2 Choosing letters from a word

A letter is randomly chosen from the word PROBABILITY. Find the following probabilities.

- **a** $P(\text{L})$
- **b** $P(\text{not L})$
- **c** $P(\text{vowel})$
- **d** $P(\text{consonant})$
- **e** $P(\text{vowel or B})$
- **f** $P(\text{vowel or consonant})$

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SOLUTION	EXPLANATION
a $P(\text{L}) = \frac{1}{11}$	One of the 11 letters is an L.
b $P(\text{not L}) = 1 - \frac{1}{11}$ $= \frac{10}{11}$	The event 'not L' is the complement of the event 'selecting an L'. Complementary events sum to 1. $P(\text{not L}) = 1 - P(\text{L})$.
c $P(\text{vowel}) = \frac{4}{11}$	The vowels of the alphabet are A, E, I, O and U. There are 4 vowels in PROBABILITY: O, A and two letter Is.
d $P(\text{consonant}) = 1 - \frac{4}{11}$ $= \frac{7}{11}$	The events 'vowel' and 'consonant' are complementary. Alternatively, the other 7 letters are consonants; thus $\frac{7}{11}$.
e $P(\text{vowel or a B}) = \frac{6}{11}$	There are 4 vowels and 2 letter Bs.
f $P(\text{vowel or consonant}) = 1$	This event includes all possible outcomes, since a letter is either a vowel or a consonant.

Now you try

A letter is chosen from the word EXPERIMENT. Find the following probabilities.

a $P(\text{M})$
b $P(\text{not an E})$
c $P(\text{vowel})$
d $P(\text{consonant})$
e $P(\text{E or consonant})$
f $P(\text{letter from the first half of the alphabet})$

9 A letter is chosen at random from the word ALPHABET. Find the following probabilities.

a $P(\text{L})$
b $P(\text{A})$
c $P(\text{A or L})$
d $P(\text{vowel})$
e $P(\text{consonant})$
f $P(\text{vowel or consonant})$
g $P(\text{Z})$
h $P(\text{A or Z})$
i $P(\text{not an A})$
j $P(\text{letter from the first half of the alphabet})$

Hint: Recall that vowels are A, E, I, O and U.

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PROBLEM-SOLVING AND REASONING

10, 11 | 11–13

10 The school captain is to be chosen at random from four candidates. Hayley, Alisa, Rocco and Stuart.

a List the sample space.

b Find the probability that the school captain will be:

i Hayley

ii Rocco or Hayley

iii neither Stuart nor Alisa

Hint: The sample space is a list of the possible students.

11 A card is drawn at random from a pack of 52 playing cards. Find the probability that the selected card will be:

a the queen of diamonds

b an ace

c a red king

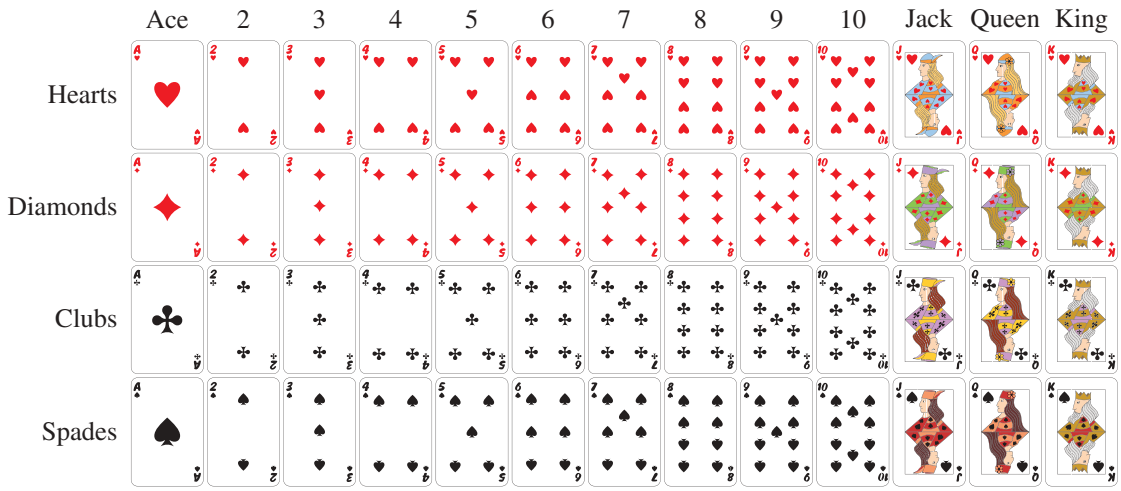
d a red card

e a jack or a queen

f any card except a 2

g any card except a jack or a black queen

h not a black ace



12 A six-sided die is rolled and the uppermost face is observed and recorded. Find the following probabilities.

a $P(6)$

b $P(3)$

c $P(\text{not } 3)$

d $P(1 \text{ or } 2)$

e $P(\text{number less than } 5)$

f $P(\text{even number or odd number})$

g $P(\text{square number})$

h $P(\text{not a prime number})$

Hint: 'Less than 5' means that 5 is not included. 1 is not a prime. Square numbers are $1^2 = 1, 2^2 = 4, 3^2 = 9$ etc.

13 A letter is chosen at random from the word PROBABILITY. Find the probability that the letter will be:

a a B

b not a B

c a vowel

d not a vowel

e a consonant

f a letter belonging to one of the first five letters in the alphabet

g a letter from the word RABBIT

h a letter that is not in the word RABBIT

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ENRICHMENT: Faulty CD player — 14

14 A CD contains eight tracks. The time length for each track is shown in the table on the right.

Track	Time (minutes)
1	3
2	4
3	4
4	5
5	4
6	3
7	4
8	4

The CD is placed in a faulty CD player, which begins playing randomly at an unknown place somewhere on the CD, not necessarily at the beginning of a track.

a Find the total number of minutes of music available on the CD.

b Find the probability that the CD player will begin playing somewhere on track 1.

c Find the probability that the CD player will begin somewhere on:

- **i** track 2
- **ii** track 3
- **iii** a track that is 4 minutes long
- **iv** track 4
- **v** track 7 or 8
- **vi** a track that is not 4 minutes long

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10B Venn diagrams and two-way tables EXTENDING

Learning intentions for this section:

- To know how Venn diagrams and two-way tables show all combinations of two attributes
- To be able to use a Venn diagram to display the distribution of two sets
- To be able to fill out a two-way table either from a problem or from a Venn diagram
- To be able to use a Venn diagram or two-way table to calculate probabilities of events

Past, present and future learning:

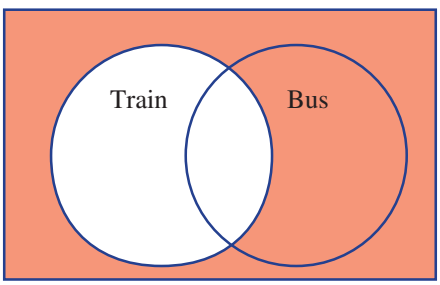
- This is an Extending topic in our Year 8 book, but it may be new to students
- Two-way tables are mentioned in Stage 6 Standard, but Venn diagrams are not
- Venn diagrams are in a Stage 5 Path Topic for students intending to do Stage 6 Advanced

When the results of an experiment involve overlapping categories it can be very helpful to organise the information into a Venn diagram or two-way table. Probabilities can easily be calculated from these types of diagrams.

Lesson starter: Solving puzzles

Work with a partner to find the answer to each puzzle. For Puzzles 1–4, draw some Venn diagrams, like the ones shown, to help you. For Puzzle 5, use a two-way table.

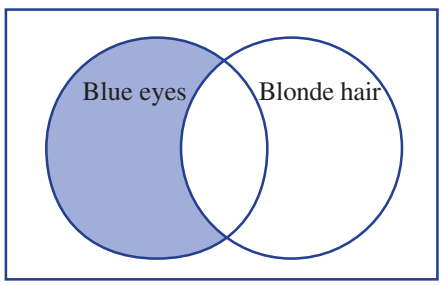
Puzzle 1



12 students travel to school by bus only and 10 students don't travel by either bus or train.

- How many students don't travel by train?

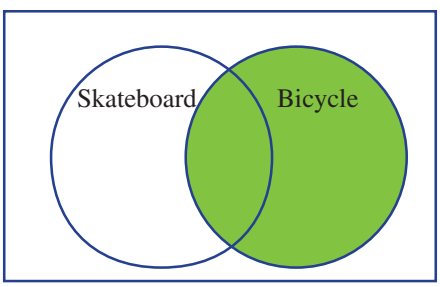
Puzzle 2



14 students in total have blue eyes, and of these students 5 have both blue eyes and blonde hair.

- How many have blue eyes but not blonde hair?

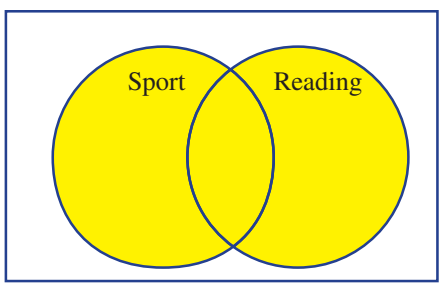
Puzzle 3



11 students own a bicycle only and 4 students own both a skateboard and a bicycle.

- How many students in total own a bicycle?

Puzzle 4



12 students in total like sport, 14 students in total like reading, and 9 students like both sport and reading.

- How many students altogether like either sport or reading or both?

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Puzzle 5

A survey of 40 students found that a total of 22 play basketball, 9 play both volleyball and basketball and 6 do not play either basketball or volleyball.

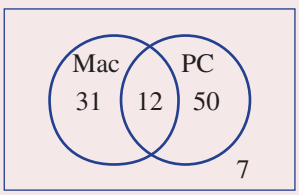
- How many basketball players don't play volleyball?
- How many students in total don't play volleyball?
- How many students in total do play volleyball?

	Basketball	Not basketball	Total
Volleyball			
Not volleyball			
Total			

KEY IDEAS

■ **Venn diagrams** and **two-way tables** help to organise outcomes into different categories. This example shows the types of computers owned by 100 people.

Venn diagram



Two-way table

	Mac	No Mac	Total
PC	12	50	62
No PC	31	7	38
Total	43	57	100

These diagrams show, for example, that:

- 12 people own both a Mac and a PC
- 62 people own a PC
- 57 people do not own a Mac
- $P(\text{Mac}) = \frac{43}{100}$
- $P(\text{only Mac}) = \frac{31}{100}$
- $P(\text{Mac or PC}) = \frac{93}{100}$
- $P(\text{Mac and PC}) = \frac{12}{100} = \frac{3}{25}$

Key terminology: Venn diagram, two-way table, probability

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Exercise 10B

UNDERSTANDING | 1–4 | 3, 4

1 This Venn diagram shows the number of people who enjoy riding and running.

a How many people in total are represented by this Venn diagram?

b Find how many people enjoy:

i riding only? **ii** riding (in total)?

iii running only? **iv** running (in total)?

v both riding and running?

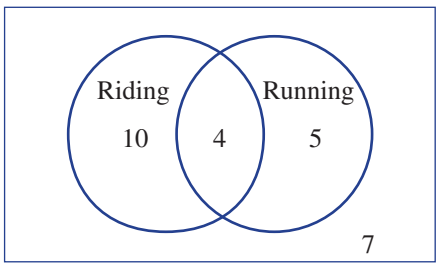
vi neither riding nor running?

vii riding or running or both?

c How many people do not enjoy:

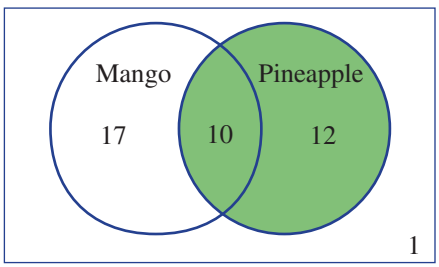
i riding? **ii** running?

Hint: 'Riding only' does not include the overlapping section.

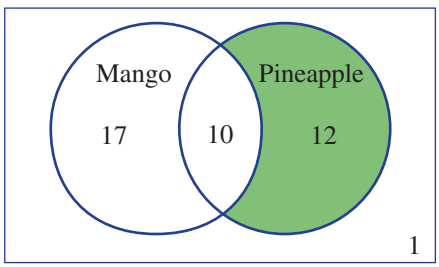


2 Each statement is about the shaded area in the Venn diagram. State the missing number in each statement.

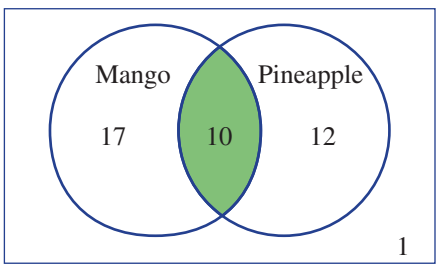
a _____ people in total like pineapple.



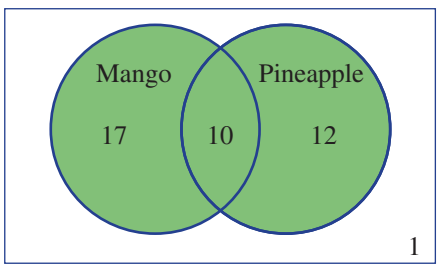
b _____ people like pineapple only.



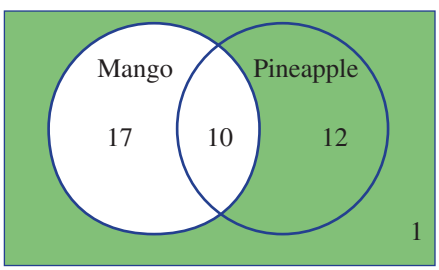
c _____ people like both pineapple and mango.



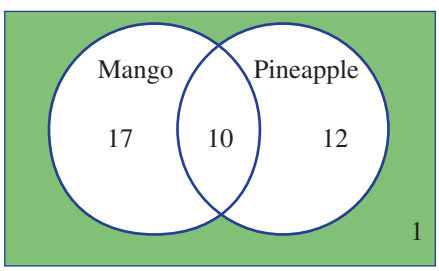
d _____ people like pineapple or mango or both.



e _____ people don't like mango.



f _____ person likes neither mango nor pineapple.



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3 Match the diagrams A, B, C or D with the given description.

a own a cat

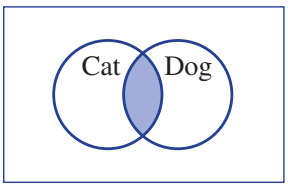
b own a cat only

c own both a cat and a dog

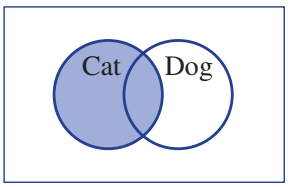
d own a cat or a dog or both

Hint: 'Both a cat and a dog' is where the circles overlap.

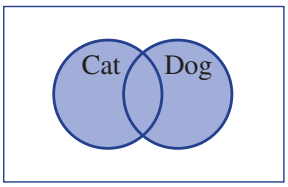
A



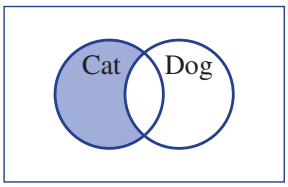
B



C



D



4 Fill in the missing numbers in these two-way tables.

Hint: Start with the column or row that has two numbers in it.

a

	A	Not A	Total
B	7	8	
Not B		1	
Total	10		

b

	A	Not A	Total
B	2		7
Not B		4	
Total			20

FLUENCY 5–8 5–9

Example 3 Constructing a Venn diagram

A survey of 30 people found that 21 like Australian Rules football and 12 like soccer. Also, 7 people like both football and soccer and 4 like neither football nor soccer.

a Construct a Venn diagram for the survey results.

b How many people:

i like football or soccer?

ii do not like soccer?

iii like only football?

c If one of the 30 people was randomly selected, find:

i P(like football and soccer)

ii P(like neither football nor soccer)

iii P(like only soccer)

Continued on next page

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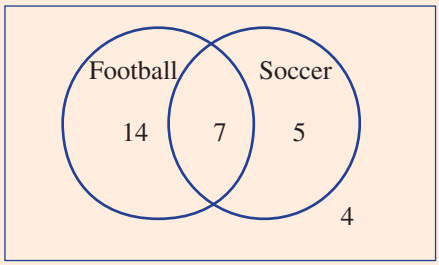
SOLUTION

a



EXPLANATION

Place the appropriate number in each category. Place the 7 in the overlap first, then ensure that:

- the total that like football is 21 (14 + 7 = 21).
- the total that like soccer is 12 (5 + 7 = 12).

The 4 people that like neither are placed outside the circles.

b i 26 like football or soccer or both

The total number of people who like football, soccer or both is 14 + 7 + 5 = 26.

ii 30 − 12 = 18 don't like soccer

12 like soccer, so 18 do not.

iii 14 like football but not soccer

21 like football but 7 of these also like soccer.

c i $P(\text{like football and soccer}) = \frac{7}{30}$

7 out of 30 people like football and soccer.

ii $P(\text{like neither football nor soccer})$

$= \frac{4}{30}$

$= \frac{2}{15}$

The 4 people who like neither football nor soccer sit outside both categories.

iii $P(\text{like soccer only})$

$= \frac{5}{30}$

$= \frac{1}{6}$

5 people like soccer but not football.

Now you try

A survey of 50 people found that 25 watch live TV and 32 people watch streamed shows. Also 14 people watch both live TV and streamed shows and 7 watch neither.

a Construct a Venn diagram for the survey results.

b How many people:

i watch live TV or streamed shows?

ii do not watch live TV?

iii watch only streamed shows?

c If one of the 50 people was randomly selected, find:

i $P(\text{watch live TV and streamed shows})$

ii $P(\text{watch neither live TV nor streamed shows})$

iii $P(\text{watch only live TV})$

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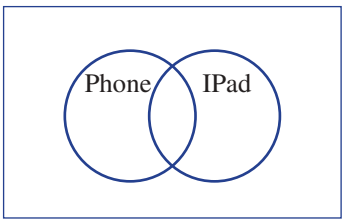
5 In a class of 30 students, 22 carried a phone and 9 carried an iPad. Three carried both a phone and an iPad and 2 students carried neither.

a Copy and complete this Venn diagram.



b How many students:

- **i** carried a phone or an iPad (includes carrying both)?
- **ii** do not carry an iPad?
- **iii** carry only an iPad?

c If one of the 30 students was selected at random, find the following probabilities.

- **i** P(carry a phone and an iPad)
- **ii** P(carry neither a phone nor an iPad)
- **iii** P(carry only a phone)

Hint: Start by writing a number in the overlapping section. '22 with a phone' is the total for the phone circle, including the overlap.

6 Match each diagram with the correct statement.

- **i** Not A
- **ii** A or B
- **iii** A and B
- **iv** B
- **v** B only
- **vi** Neither A nor B

Hint: 'A or B' means 'either A or B or both'. 'A and B' means 'both A and B'.

Diagram P

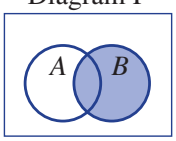


Diagram Q

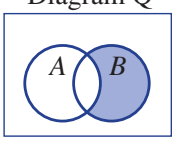


Diagram R

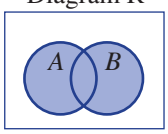


Diagram S

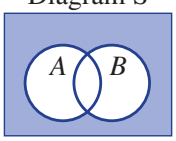


Diagram T

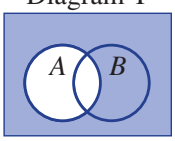
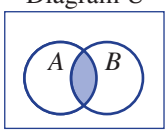
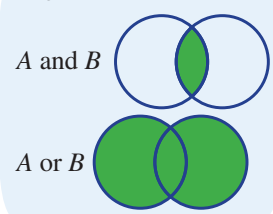


Diagram U



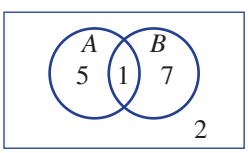
Hint:



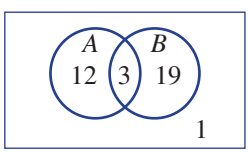
7 For each Venn diagram, find the following probabilities.

- **i** $P(A)$
- **ii** $P(A$ only$)$
- **iii** P(not B)
- **iv** $P(A$ and $B)$
- **v** $P(A$ or $B)$
- **vi** P(neither A nor B)

a



b



Hint: First calculate the total number in each sample.

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Example 4 Constructing a two-way table

At a car yard, 24 cars are tested for fuel economy. Eighteen of the cars run on petrol, 8 cars run on gas and 3 cars can run on both petrol and gas.

a Illustrate the situation using a two-way table.

b How many of the cars:

i do not run on gas?

ii run on neither petrol nor gas?

c Find the probability that a randomly selected car:

i runs on gas

ii runs on only gas

iii runs on gas or petrol

SOLUTION

a

	Gas	Not gas	Total
Petrol	3	15	18
Not petrol	5	1	6
Total	8	16	24

EXPLANATION

Set up a table as shown and enter the numbers (in black) from the given information.

Fill in the remaining numbers (in red) ensuring that each column and row adds to the correct total.

b i 16

The total at the base of the 'Not gas' column is 16.

ii 1

The number at the intersection of the 'Not gas' column and the 'Not petrol' row is 1.

c i $P(\text{gas}) = \frac{8}{24}$

$= \frac{1}{3}$

Of the 24 cars, 8 cars run on gas.

ii $P(\text{only gas}) = \frac{5}{24}$

Of the 8 cars that run on gas, 5 of them do not also run on petrol.

iii $P(\text{gas or petrol}) = \frac{15 + 5 + 3}{24}$

$= \frac{23}{24}$

Of the 24 cars, some run on petrol only (15), some run on gas only (5) and some run on gas and petrol (3).

Now you try

Of 100 clothing items in a small shop, 65 contained natural fibres, 45 contained artificial fibres and 20 contained both types of fibres.

a Illustrate the situation using a two-way table.

b How many items:

i do not have artificial fibres?

ii have neither type of fibre?

c Find the probability that a randomly selected item:

i has natural fibres

ii has only natural fibres

iii has artificial or natural fibres

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8 Of 50 desserts served at a restaurant one evening, 25 were served with ice-cream, 21 were served with cream and 5 were served with both cream and ice-cream.

Hint: 25 is the total for the ice-cream row.

a Copy and complete this two-way table.

	Cream	Not cream	Total
Ice-cream			
Not ice-cream			
Total			

b How many of the desserts:

i did not have cream?

ii had neither cream nor ice-cream?

c Find the probability that a chosen dessert:

i had cream

ii had only cream

iii had cream or ice-cream

9 Find the following probabilities using each of the given tables. First copy and complete each two-way table.

i $P(A)$ **ii** $P(\text{not } A)$ **iii** $P(A \text{ and } B)$

iv $P(A \text{ or } B)$ **v** $P(B \text{ only})$ **vi** $P(\text{neither } A \text{ nor } B)$

Hint: 'Neither A nor B' is the same as 'Not A and not B'.

a

	A	Not A	Total
B	3	1	
Not B	2		4
Total			

b

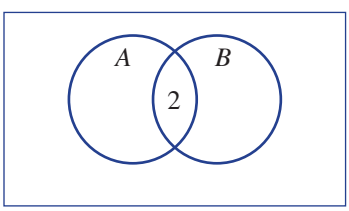
	A	Not A	Total
B		4	15
Not B	6		
Total			26

PROBLEM-SOLVING AND REASONING 10–12 11–14

10 For each two-way table, fill in the missing numbers then transfer the information to a Venn diagram.

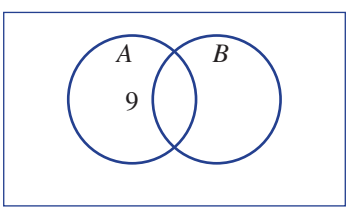
a

	A	Not A	Total
B	2		8
Not B			
Total		7	12



b

	A	Not A	Total
B		4	
Not B	9		13
Total	12		



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11 In a group of 17 people, 13 rented their house, 6 rented a car and 3 did not rent either a car or their house.

a Draw a Venn diagram, showing circles for 'Rents house' and 'Rents car'.

b How many people rented both a car and their house?

c Find the probability that one of them rented only a car.

Hint: 14 people will be in the circles, but $13 + 6 = 19$. How many must be in the overlap?

12 One hundred citizens were surveyed regarding their use of water in their garden. Of these, 23 said that they used tank water, 48 said that they used tap water and 41 said that they did not use water on their garden at all.

a Copy and complete this two-way table.

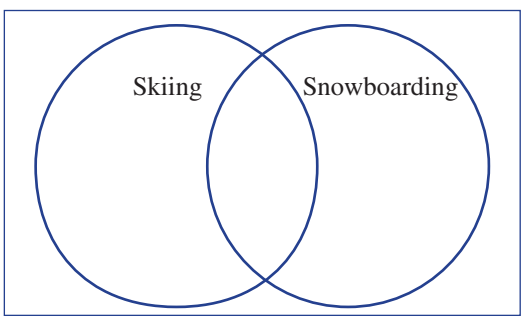
	Tank water	Not tank water	Total
Tap water			
Not tap water			
Total			

b How many people used both tank and tap water?

c What is the probability that one of the people uses only tap water?

d What is the probability that one of the people uses tap water or tank water?

13 All members of a ski club enjoy either skiing and/or snowboarding.
Seven enjoy only snowboarding, 16 enjoy skiing and 4 enjoy both snowboarding and skiing.



a Copy and complete the Venn diagram shown.

b How many people are in the club?

c If a person from the club is randomly selected, what is the probability of choosing a snowboarder?

d If a person is randomly selected out of the group that likes skiing, what is the probability of choosing a snowboarder?

e What is the probability of choosing a skier out of the group that likes snowboarding?

Hint: For part **d**, what fraction of skiers also like snowboarding?

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14 Of a group of 30 cats, 24 like either tinned or dry food or both, 10 like only dry food and 5 like both tinned and dry food.

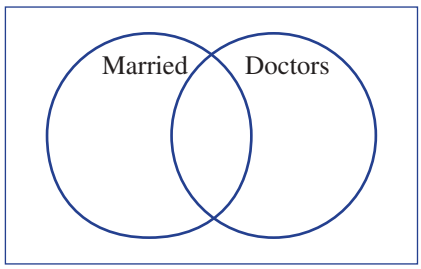
Hint: Use a Venn diagram to record the information.

a Find the probability that a selected cat likes only tinned food.

b Out of the group of cats that eat dry food, what is the probability of selecting a cat that also likes tinned food?

ENRICHMENT: Numbers challenge — 15

15 One hundred people were surveyed and it was found that 55 were married and 30 were doctors. The number of married doctors was 17. Copy and complete this Venn diagram and then determine the number of people in each question below.



a The number who are neither married nor a doctor

b The number who are not married

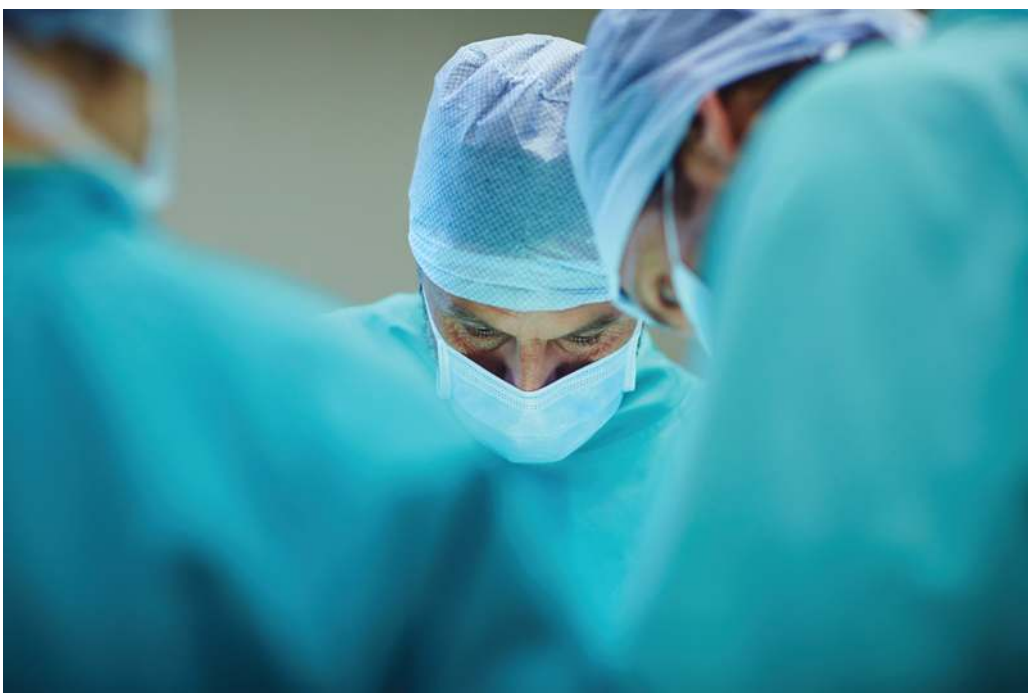
c The number who are not doctors

d The number who are married but not a doctor

e The number who are a doctor but not married

f The number who are unmarried and a doctor

g The number who are unmarried or a doctor



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10C Using arrays for two-step experiments

Learning intentions for this section:

- To know how to list the sample space of a two-step experiment in an array
- To understand the difference between 'with replacement' and 'without replacement'
- To be able to construct arrays and then find associated probabilities

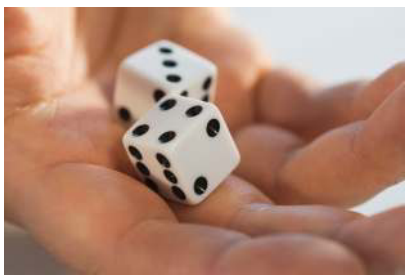
Past, present and future learning:

- This section addresses the Stage 5 Core topic called *Probability A*
- Chapter 3 of our Year 10 book revises and further extends arrays
- Arrays are important in Stage 6 Standard

Sometimes an experiment consists of two steps, such as tossing a single coin twice or rolling two dice. Or perhaps a card is pulled from a hat and then a spinner is spun. We can use tables to list the sample space for such experiments.

Lesson starter: The maths cup

This activity can be run in small groups or as a class.



a Draw the table below on the whiteboard. Each student should also have a copy.
b Each student selects one horse as their 'own' (choose a winner!).
c Students take turns to roll 2 dice and state the sum of the uppermost faces of the dice.
d The total of each roll refers to the horse number. When its number is rolled, that horse moves another 100 m towards the finish line. A cross is placed in the cell to show the move.
e The winning horse is the first to reach the finish at 1000 m.
f Keep rolling the dice until first, second and third places are decided.

	Horse	100 m	200 m	300 m	400 m	500 m	600 m	700 m	800 m	900 m	1000 m
1	SCRATCHED										
2	Greased Lightning										
3	Flying Eagle										
4	Quick Stix										
5	Break a Leg										
6	Slow and Steady										
7	The Donkey										
8	The Wombat										
9	Tooting Tortoise										
10	Ripper Racer										
11	Speedy Gonzo										
12	Pharaoh										

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Discussion questions

- Do you think that the horse that won your maths cup race would always win?
- In this game, do all the horses have the same chance of winning?
- Why is Horse 1 scratched?
- When two dice are thrown, what are all the possible outcomes for the sum of the two uppermost faces?
- What are the horse numbers that are highly likely to win this race?
- What are the horse numbers that are unlikely to win this race?

KEY IDEAS

- A table called an **array** can be used to list the sample space of a two-step experiment.
- When listing outcomes, it is important to be consistent with the order for each outcome. For example: the outcome (heads, tails) is different from the outcome (tails, heads). So when two coins are tossed, the outcome of coin 1 is written first in each cell of the table.

		Coin 2	
		H	T
Coin 1	H	HH	HT
	T	TH	TT

The sample space is shown in the body of the table; i.e. {HH, HT, TH, TT}.

- The probability is still given by:

$$P(\text{event}) = \frac{\text{number of outcomes where the event occurs}}{\text{total number of possible outcomes}}$$

For example, when two coins are tossed, $P(\text{a tail and a head}) = \frac{2}{4} = \frac{1}{2}$

- Some experiments are conducted **without replacement**, which means that each individual trial outcome cannot be achieved again.
For example: Two letters are chosen from the word CAT. 'Without replacement' means that the first letter chosen is removed and it is not possible to choose it again.

With replacement

		1st		
		C	A	T
2nd	C	(C, C)	(A, C)	(T, C)
	A	(C, A)	(A, A)	(T, A)
	T	(C, T)	(A, T)	(T, T)

9 outcomes

Without replacement

		1st		
		C	A	T
2nd	C	×	(A, C)	(T, C)
	A	(C, A)	×	(T, A)
	T	(C, T)	(A, T)	×

6 outcomes

Key terminology: array, with replacement, without replacement, two-step experiment, sample space

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Exercise 10C

UNDERSTANDING | 1–4 | 3, 4

1 A letter is chosen from the word HI and a letter is chosen from the word BYE.
Copy and complete this table to show the sample space.

	B	Y	E
H	HB		
I			

Hint: Write 'H' in each cell of the top row.
Write 'B' in each cell of the first column.

2 A coin and a four-sided die are tossed.

a Copy and complete this table to show the sample space.

	1	2	3	4
H				
T				

Hint: The sample space is all the outcomes in the central part of the table.

b How many outcomes are in the sample space?

c What is the probability of tossing a 'H3'?

d What is the probability of tossing a 'T2'?

3 A coin is flipped and then a spinner with numbers 1 to 5 is spun. The possible outcomes are listed in the table below.

	1	2	3	4	5
H	H1	H2	H3	H4	H5
T	T1	T2	T3	T4	T5

a How many outcomes are possible?

b List the four outcomes in which an even number is displayed on the spinner.

c Hence, state the probability that an even number is displayed.

d List the outcomes for which the coin shows tails and the spinner shows an odd number.

e What is $P(\text{T, odd number})$?

4 Two coins are tossed and the four possible outcomes are shown below.

		20-cent coin	
		H	T
50-cent coin	H	HH	HT
	T	TH	TT

a What is the probability that the 50-cent coin will be heads and the 20-cent coin will be tails?

b For which outcomes are the two coins displaying the same face?

c What is the probability of the two coins displaying the same face?

Hint: The same face means either two heads or two tails.

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FLUENCY 5, 6 5–7

Example 5 Using a table for two-step experiments

A spinner with the numbers 1, 2 and 3 is spun, and then a card is chosen at random from cards containing the letters of ATHS.

a Draw a table to list the sample space of this experiment.
b How many outcomes does the experiment have?
c Find the probability of the combination 2S.
d Find the probability of an odd number being spun and the letter H being chosen.

SOLUTION

a

	A	T	H	S
1	1A	1T	1H	1S
2	2A	2T	2H	2S
3	3A	3T	3H	3S

b There are 12 outcomes.

c $P(2S) = \frac{1}{12}$

d $P(\text{odd, H}) = \frac{2}{12} = \frac{1}{6}$

EXPLANATION

The sample space of the spinner (1, 2, 3) is put into the left column.

The sample space of the cards (A, T, H, S) is put into the top row.

The table has $4 \times 3 = 12$ items in it.

All 12 outcomes are equally likely. Spinning 2 and choosing an S is one of the 12 outcomes.

Possible outcomes are 1H and 3H, so probability $= 2 \div 12$.

Now you try

A coin is flipped and an eight-sided die is rolled.

a Draw a table to list the sample space of this experiment.
b How many outcomes does the experiment have?
c Find the probability of the outcome H7.
d Find the probability of obtaining a tail and a number less than 4.

5 A coin is flipped and then a regular die is rolled.

a Copy and complete this table to list the sample space for this experiment.

	1	2	3	4	5	6
H						
T						

b How many possible outcomes are there?

c Find the probability of the pair H3.

d Find the probability of 'heads' on the coin with an odd number on the die.

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6 A letter is chosen from the word LINE and another is chosen from the word RIDE.

a Copy and complete this table to list the sample space.

	R	I	D	E
L				
I				
N				
E				

b How many possible outcomes are there?

c Find P(N, R); i.e. the probability that N is chosen from LINE and R is chosen from RIDE.

d Find P(L, D).

e Find the probability that two vowels are chosen.

f Find the probability that two consonants are chosen.

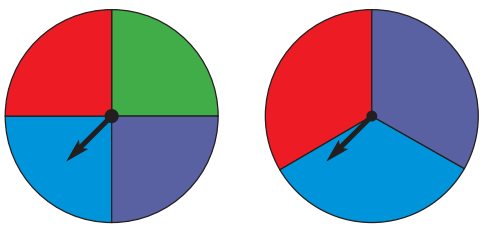
g Find the probability that the two letters chosen are the same.

Hint: Recall that the vowels are A, E, I, O and U.

7 The spinners shown on the right are each spun.

a Copy and complete this table to list the sample space. Use R for red, P for purple and so on.

		Spinner 2		
		R	P	B
Spinner 1	R			
	P			
	G			
	B			



Spinner 1 Spinner 2

b Find the probability that spinner 1 will display red and spinner 2 will display blue.

c Find the probability that both spinners will display red.

d What is the probability that one of the spinners displays red and the other displays blue?

e What is the probability that both spinners display the same colour?

Hint: List the colour from spinner 1 first in each cell.

PROBLEM-SOLVING AND REASONING 8–10 8, 10, 11

8 Two dice are rolled for a board game. The numbers showing are then added together to get a number between 2 and 12.

a Copy and complete a table like the one given to show the sample space. State the number of outcomes.

b Find the probability that the two dice add to 5.

c Find the probability that the two dice add to an even number.

d What is the most likely sum to occur?

e What are the two least likely sums to occur between 2 and 12?

Hint: Write the sum in each cell. For example, for (1, 1), write 2.

		Die 1					
		1	2	3	4	5	6
Die 2	1						
	2						
	3						
	4						
	5						
	6						

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9 In Ruby's left pocket she has two orange marbles and one white marble. In her right pocket she has a yellow marble, a white marble and 3 blue marbles. She chooses a marble at random from each pocket.

a Draw a table to describe the sample space.

b Find the probability that she will choose an orange marble and a yellow marble.

c What is the probability that she chooses a white marble and a yellow marble?

d What is the probability that she chooses a white marble and an orange marble?

e Find the probability that a white marble and a blue marble are selected.

f What is the probability that the two marbles selected are the same colour?

Example 6 Finding the sample space for events without replacement

The letters T R E K are written on cards. A letter is chosen and not replaced so that this letter is now unavailable. Then a second letter is chosen.

a In a table, list the outcomes from choosing two letters without replacement.

b Find the probability that the two letters chosen are (E, R), in that order.

c Find the probability of obtaining an outcome with an E in it.

SOLUTION

a

		1st choice			
		T	**R**	**E**	**K**
2nd choice	**T**	×	(R, T)	(E, T)	(K, T)
	R	(T, R)	×	(E, R)	(K, R)
	E	(T, E)	(R, E)	×	(K, E)
	K	(T, K)	(R, K)	(E, K)	×

EXPLANATION

List all the outcomes, writing the '1st choice' letter first in each cell. Note that the same letter cannot be chosen twice.

b $P(\text{E, R}) = \frac{1}{12}$

Without replacement there are 12 outcomes.

c $P(\text{includes E}) = \frac{6}{12} = \frac{1}{2}$

6 of the 12 outcomes contain an E.

Now you try

Two digits are selected from the set $\{3, 5, 6\}$ without replacement.

a Draw a table to list the sample space.

b Find the probability that the outcome (5, 6) will be the result.

c Find the probability that (5, 6) or (6, 5) will be the result.

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10 Two letters are chosen from the word DOG *without replacement*.

a Complete the given table.

		1st		
		D	**O**	**G**
2nd	**D**	×	(O, D)	(G, D)
	O		×	
	G			×

Hint: 'Without replacement' means the first letter chosen is removed, so it is not possible to choose it again.

b Find the probability of obtaining the (G, D) outcome.

c Find the probability of obtaining an outcome with an O in it.

11 Two digits are selected *without replacement* from the set $\{1, 2, 3, 4\}$.

a Draw a table to show the sample space.

b Find:

i $P(1, 2)$

ii $P(4, 3)$

Hint: Remember that doubles such as (1, 1), (2, 2) etc. are not allowed.

c Find the probability that:

i both numbers will be at least 3

ii the outcome will contain a 1 or a 4

iii the outcome will contain a 1 and a 4

iv the outcome will not contain a 3

Hint: 'At least 3' includes 3.

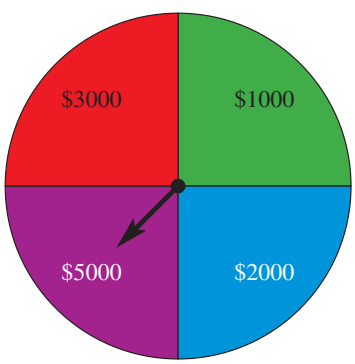
ENRICHMENT: Game show — 12

12 A wheel is spun during a game show to determine the prize money, and then a six-sided die is rolled. The prize money shown on the wheel is multiplied by the number on the die to give the total winnings.

a What is the probability that a contestant will win \$6000?

b What is the probability that they win more than \$11 000?

Hint: Draw a table to show all outcomes.
List the prize in brackets under each outcome.



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10D Using tree diagrams

Learning intentions for this section:

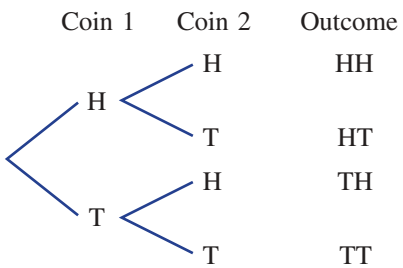
- To know how to use a tree diagram to list the sample space from multistage experiments
- To be able to use a tree diagram to find the probability of certain events

Past, present and future learning:

- This section addresses the Stage 5 Core topic called *Probability A*
- Experiments can be 'with replacement' or 'without replacement'
- Chapter 3 of our Year 10 book revises and further extends tree diagrams
- Tree diagrams are important in Stage 6 Standard
- You may be expected to draw and use a tree diagram in the HSC examination

When two coins are flipped, we can draw a table to list the sample space. Yet if three coins are flipped, then we would need a three-dimensional table to list all outcomes. Imagine trying to find probabilities when five coins are flipped!

Another tool that mathematicians use for probability is the tree diagram. This tree diagram describes the four outcomes when two coins are flipped.



It is important to be able to read a tree diagram correctly. The first row (HH) represents the outcome where the first coin flipped was heads and the second coin flipped was heads. The third row (TH) represents the outcome where the first coin was tails and the second was heads.

Lesson starter: Coin puzzle

Two coins are flipped. Rank these outcomes from most likely to least likely:

- Exactly two heads are obtained.
- Exactly one head and exactly one tail are obtained.
- At least one coin shows tails.
- Three tails are shown.

How might the order change if three coins are flipped? Compare your answers with other students.

KEY IDEAS

- **Tree diagrams** are diagrams used to list the sample space for multistage experiments with two or more steps.
  - The outcomes for each stage of the experiment are listed vertically and each stage is connected with branches.

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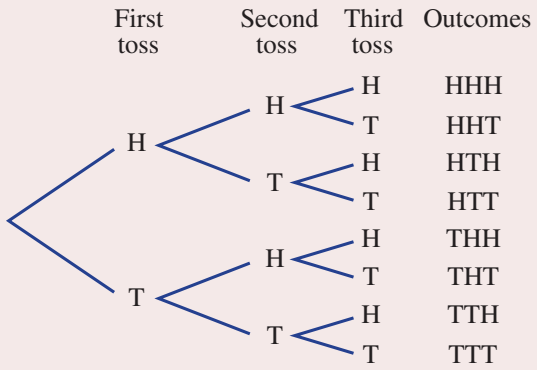
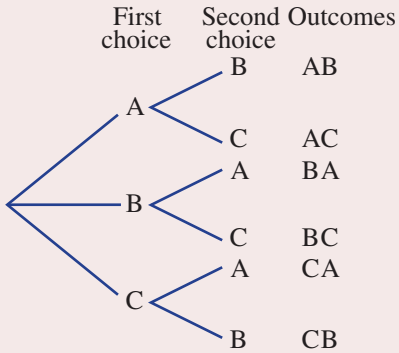
For example:

Tossing a coin 3 times (with replacement)



Selecting 2 letters from {A, B, C} (without replacement)



In these examples, each set of branches produces outcomes that are all equally likely. Tree diagrams can also be used where all outcomes are not equally likely.

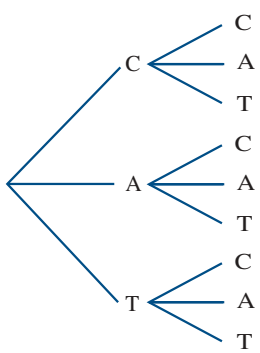
Key terminology: tree diagram, with replacement, without replacement, sample space

Exercise 10D

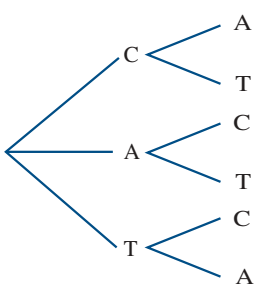
UNDERSTANDING 1–3 1, 3

1 These two tree diagrams show the selection of two letters from the word CAT.

A



B



a Which tree diagram shows selection 'with replacement'?

b Which tree diagram shows selection 'without replacement'?

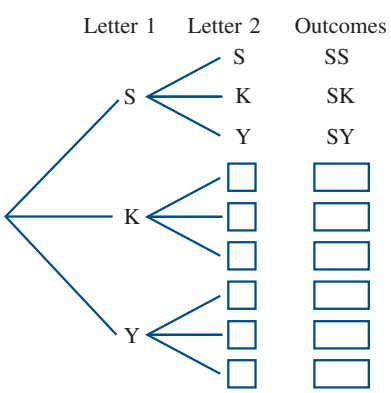
2 The letters S, K and Y are written on cards and placed in a hat. Two letters are randomly selected one at a time.

a Write the missing word or number from each statement about this event.

i 'With _____' means the first letter is returned to the hat before the second letter is selected.

ii When the first letter chosen is replaced, there are still _____ letters to choose from for the second letter.

b Copy and complete this tree diagram for selecting two letters *with replacement*.



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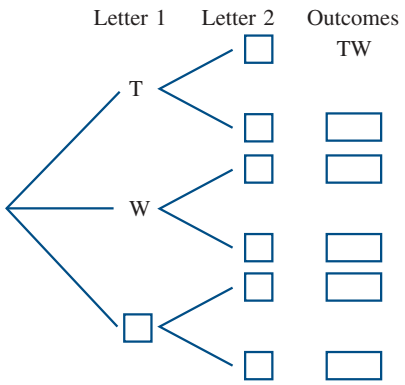
3 The letters T, W and O are written on cards and placed in a container. One letter is randomly selected and not returned to the container. A second letter is then selected.

a Write the missing word or number from each statement about this event.

i _____ _____ means the first letter chosen is placed aside and the second letter is selected from the remaining letters.

ii Since the first letter chosen is not replaced, there will be only _____ letters to choose from for the second letter.

b Complete the tree diagram for selecting two letters *without replacement* from the letters T, W and O.



FLUENCY 4–6 5–7

Example 7 Constructing a tree diagram

An experiment involves tossing two coins.

a Complete a tree diagram to show all possible outcomes.

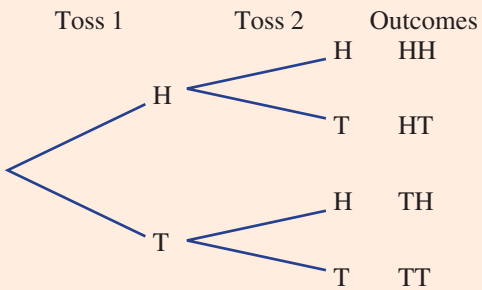
b What is the total number of outcomes?

c Find the probability of tossing:

i two tails **ii** one tail **iii** at least one head

SOLUTION

a



EXPLANATION

Tree diagram shows two coin tosses one after the other, resulting in $2 \times 2 = 4$ outcomes.

b The total number of outcomes is 4.

There are four possibilities in the outcomes column.

c i $P(\text{TT}) = \frac{1}{4}$

One out of the four outcomes is TT.

ii $P(\text{1 tail}) = \frac{2}{4} = \frac{1}{2}$

Two outcomes have one tail: {HT, TH}

iii $P(\text{at least 1 head}) = \frac{3}{4}$

Three outcomes have at least one head: {HH, HT, TH}

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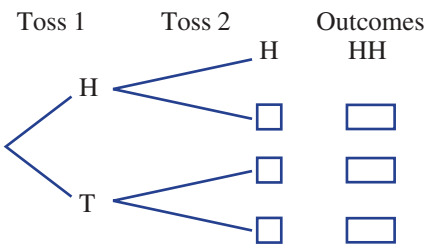
Now you try

An experiment involves tossing two 4-sided dice.

a Complete a tree diagram to show all possible outcomes.
b What is the total number of outcomes?
c Find the probability of tossing:
i two 3's **ii** one 4 **iii** at most one 2

4 A coin is tossed twice.

a Complete this tree diagram to show all the possible outcomes.

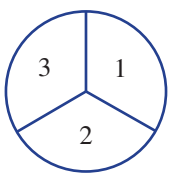


b What is the total number of outcomes?

c Find the probability of obtaining:
i two heads
ii exactly one head
iii at least one head
iv at least one tail

Hint: 'At least one' means one or two in this case.

5 A spinner with numbers 1, 2 and 3 is spun twice.



a Draw a tree diagram and list the outcomes.
b Find P(1 then 1).
c Find P(1 then 2).
d Find P(1 and 2 spun in either order).
e Find P(both show the same number).
f Find P(numbers add to 4).

Hint: Tree diagram headings: 'First spin', 'Second spin', 'Outcomes'.

Example 8 Constructing a tree diagram without replacement

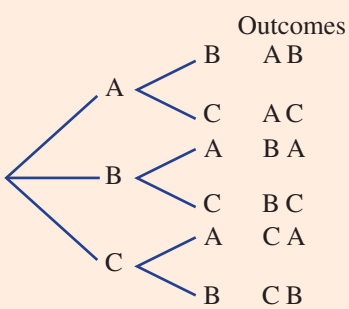
Two people are selected without replacement from a group of three: Annabel (A), Brodie (B) and Chris (C).

a List all the possible combinations for the selection using a tree diagram.

b Find the probability that the selection will contain:
i Annabel and Brodie **ii** Chris **iii** Chris or Brodie

SOLUTION

a



EXPLANATION

On the first choice there are three options (A, B or C) but on the second choice there are only two remaining.

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b i $P(\text{Annabel and Brodie}) = \frac{2}{6}$
$= \frac{1}{3}$

2 of the 6 outcomes contain Annabel and Brodie (AB) and (BA).

ii $P(\text{Chris}) = \frac{4}{6}$
$= \frac{2}{3}$

4 out of the 6 outcomes contain Chris.

iii $P(\text{Chris or Brodie}) = \frac{6}{6}$
$= 1$

All of the outcomes contain at least one of Chris or Brodie.

Now you try

A drawer contains two red and one blue sock. Two socks are selected at random without replacement.

a List all the possible combinations for the selection using a tree diagram.

b Find the probability that the selection will contain:

i two red socks

ii two socks of different colour

iii at least one red sock.

6 Two people are selected without replacement from a group of three: Donna (D), Elle (E) and Fernando (F).

a List all the possible combinations for the selection using a tree diagram.

b Find the probability that the selection will contain:

i Donna and Elle

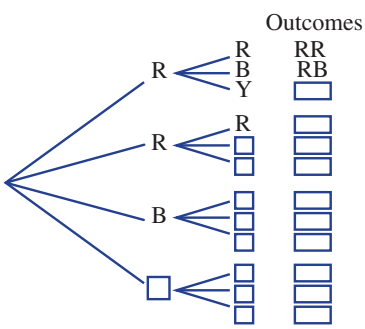
ii Fernando

iii Fernando or Elle

Hint: The combinations are the outcomes.

7 A drawer contains two red socks (R), one blue sock (B) and one yellow sock (Y), and two socks are selected at random without replacement.

a Complete this tree diagram.



b Find the probability of obtaining:

i a red sock and a blue sock

ii two red socks

iii any pair of socks of the same colour

iv any pair of socks of different colour

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PROBLEM-SOLVING AND REASONING

8 | 8, 9

8 A coin is tossed three times.

a Draw a tree diagram and list the outcomes.

b Find $P(3 \text{ tails})$.

c Find $P(2 \text{ tails then } 1 \text{ head})$.

d Find $P(2 \text{ tails and } 1 \text{ head, in any order})$.

e Which is more likely: getting exactly 3 tails or getting exactly 2 tails?

Hint: Use headings: Coin 1', 'Coin 2', 'Coin 3', 'Outcomes'.

9 There are three bottles of white wine and two bottles of red wine on a shelf in a cellar. It is too dark to read the labels so two bottles are randomly selected, one at a time. Find the probability that:

a two bottles of different colour wine are selected

b two bottles of the same colour wine are selected

c one or more bottles of red wine are selected

Hint: Draw a tree diagram using the 'without replacement' method.

ENRICHMENT: Selecting matching clothes

— | 10

10 A person randomly selects a tie from their collection of one green and two red ties. They select a shirt from a collection of one red and two white. They then select either a red or black hat. Use a tree diagram to help find the probability that the person selects a tie, shirt and hat according to the following descriptions:

a a red tie, red shirt and black hat

b all three items red

c one item red

d two items red

e at least two items red

f a green hat

g a green tie and a black hat

h a green tie or a black hat

i not a red item

j a red tie or white shirt or black hat

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10E Using relative frequencies to estimate probabilities

Learning intentions for this section:

- To understand how relative frequency can be used to estimate probability
- To be able to calculate observed probability
- To be able to calculate the expected number of occurrences

Past, present and future learning:

- This section consolidates and extends Stage 4 concepts that are used in Stages 5 and 6
- Using relative frequency to estimate probability is an important concept in Stage 6 Standard

The theoretical probability of tossing 'heads' is 50%. In some real-life scenarios it is impossible to calculate the theoretical probability, so relative frequencies are used.

Tossing a coin 100 times does not mean it will come up heads 50 times.

Lesson starter: Heads or tails?

Ella tossed a fair coin 10 times and recorded heads 8 out of the 10 tosses.
Jonah tossed the same coin 100 times and recorded heads 49 out of the 100 tosses.

- According to Ella's data, what is the observed probability of her tossing a head?
- According to Jonah's data, what is the observed probability of him tossing a head?
- Which of the two datasets is more reliable? Why?
- Ella said that she had a higher chance of rolling a head than Jonah. Is she right?

KEY IDEAS

- The results of repeated trials of a probability experiment can be used to generate **relative frequencies**, which are sometimes called observed probabilities.
- Observed probabilities are sometimes called experimental probabilities.
- **Observed probability** is calculated using the results of an experiment or survey.

$$\text{Observed probability} = \frac{\text{number of times the outcome occurs}}{\text{total number of trials in the experiment}}$$

- The **expected number of occurrences** = probability × number of trials

$$E = P \times n$$

Key terminology: relative frequency, theoretical probability, observed probability, expected number of occurrences, experiment, outcome

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Exercise 10E

UNDERSTANDING	1, 2	2

1 Insert the word *observed*, *theoretical* or *expected* to complete each statement.

a _____ probability is calculated from the results of an experiment or survey.

b _____ probability is calculated from the sample space of an event.

c When a coin is tossed, there are 2 possible equally likely outcomes. So the _____ probability of obtaining a head is $\frac{1}{2}$.

d Jayce tossed a coin 20 times and obtained a head 12 times. So the _____ probability of obtaining a head was $\frac{12}{20} = \frac{3}{5}$.

e A survey found that 3 out of every 5 students have a sister. Out of 1000 students, the _____ number of students with a sister would be $\frac{3}{5} \times 1000 = 600$.

2 Write the observed probability from each set of results listed here.

a Out of 20 people surveyed, 18 preferred cereal for breakfast.

b 50 students were surveyed and it was found that 40 had pets.

c There were 25 juniors and 20 seniors at the school bus stop one afternoon. If one student is selected from those at the bus stop, state the observed probability that the student is a junior.

Hint: In part **c**, first find the total number of students at the bus stop.

FLUENCY	3–7	4–8

Example 9 Using observed probability

Kris plays rugby and has a record of kicking a goal from a penalty kick 4 times out of every 7 attempts.

a State the observed probability for Kris to achieve a penalty goal.

b Calculate the expected number of goals from 28 penalty kicks that Kris takes.

SOLUTION	EXPLANATION
a $P(\text{goal}) = \frac{4}{7}$	Experimentally, we know that Kris can kick 4 goals out of every 7 kicks.
b Expected number $= \frac{4}{7} \times 28 = 16$ goals	Expected number = probability × number of trials

Now you try

In the last 10 school days Mia caught the correct bus in the morning 8 times.

a State the observed probability of Mia catching the correct bus in the morning on the next school day.

b Calculate the expected number of times Mia will catch the correct morning bus in the next 50 school days.

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3 Ashleigh has found that they can shoot a basketball through the hoop 4 times out of 10 from the '3 point' area.

a State the observed probability for Ashleigh to shoot a basketball through the hoop from the '3 point' area.

b Calculate the expected number of times the ball would go through the hoop from 100 shots Ashleigh makes from the '3 point' area.

4 The observed probability of Jess hitting a bullseye on a dartboard is 0.05 (or $\frac{5}{100}$). How many bullseyes would you expect Jess to get if he threw the following number of darts?

a 100 darts **b** 200 darts

c 1000 darts **d** 80 darts

Hint: Expected number = probability × number of trials

5 A bus company surveyed a random selection of 90 people in one suburb. They found that 35 of these people regularly used a bus service from that suburb to the centre of the city.

a State the observed probability for a person in that suburb to regularly use a bus service from that suburb to the city.

b If there are 2700 residents in that suburb, find the expected number who would regularly use a bus service from that suburb to the city.

Example 10 Finding the observed probability

A box contains an unknown number of coloured balls and a ball is drawn from the box and then replaced. The procedure is repeated 100 times and the colour of the ball drawn is recorded each time. Twenty-five red balls were recorded.

a Find the observed probability for selecting a red ball.

b Find the expected number of red balls if the box contained 500 balls in total.

SOLUTION

a $P(\text{red balls}) = \frac{25}{100}$

$= 0.25$

b Expected number of red balls in 500

$= 0.25 \times 500$

$= 125$

EXPLANATION

$P(\text{red balls}) = \frac{\text{number of red balls drawn}}{\text{total number of balls drawn}}$

There are 25 red balls and 100 balls in total.

Expected number of occurrences = probability × number of trials

Now you try

120 people were surveyed and asked if they had contracted the flu in the last 12 months.
25 responded Yes, 90 responded No and the remainder responded Not sure.

a Find the observed probability that a person contracted the flu in the last 12 months.

b Find the expected number of people who contracted the flu in the last 12 months if 960 people are surveyed.

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6 A bag contains an unknown number of counters.
A counter is selected from the bag and then replaced.
The procedure is repeated 100 times and the colour of the counter is recorded each time. Sixty of the counters drawn were blue.

Hint: Observed probability $= \frac{\text{number of blue counters}}{\text{total number of counters selected}}$

a Find the observed probability for selecting a blue counter.

b Find the expected number of blue counters if the bag contained:
i 100 counters **ii** 200 counters **iii** 600 counters

7 In an experiment involving 200 people chosen at random, 175 people said that they owned a home computer.

a Calculate the observed probability of choosing a person who owns a home computer.

b Find the expected number of people who would own a home computer in the following group sizes:
i 400 people **ii** 5000 people **iii** 40 people

Hint: Expected number = probability × number of trials

8 By calculating the observed probability, estimate the chance that each of the following events will occur.

a Nat will walk to work today, given that she walked to work five times in the last 75 working days.

b Mike will win the next game of cards if, in the last 80 games, he has won 32.

c Brett will hit the bullseye on the dartboard with his next attempt if, in the last 120 attempts, he was successful 22 times.

PROBLEM-SOLVING AND REASONING

9–11 | 10–13

9 This table shows the results of three different surveys of people in Perth about their use of public transport (PT).

Survey	Number who use PT	Survey size	Observed probability
A	2	10	$\frac{2}{10} = 0.2$
B	5	20	_____
C	30	100	_____

Hint: A large survey size makes observed probability calculations more accurate.

a What are the two missing numbers in the observed probability list?

b Which survey should be used to estimate the probability that a person uses public transport and why?

10 The results of tossing a drawing pin and observing how many times the pin lands with the spike pointing up are shown in the table. Results are recorded at different stages of the experiment.

Which observed probability would you choose to best represent the probability that the pin will land spike up? Why?

Number of throws	Frequency (spike up)	Observed probability
1	1	1.00
5	2	0.40
10	5	0.50
20	9	0.45
50	18	0.36
100	41	0.41

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11 A six-sided die is rolled 120 times. State how many times you would expect the following events to occur?

a a 6 **b** a 1 or a 2

c a number less than 4 **d** a number which is at least 5

Hint: Use the theoretical probability to calculate the expected numbers.

12 The colour of cars along a highway was noted over a short period of time and summarised in this frequency table.

Colour	White	Silver	Blue	Green
Frequency	7	4	5	4

a How many cars had their colour recorded?

b Find the observed probability that a car's colour is:

i blue

ii white

c If the colour of 100 cars was recorded, find the expected number of:

i blue cars

ii green cars

iii blue or green cars

13 A spinner is divided into three regions not necessarily of equal size. The regions are numbered 1, 2 and 3 and the spinner is spun 50 times. The table shows the results.

Number	1	2	3
Frequency	26	11	13

Hint: The angles of the spinner regions are multiples of 90°.

a Find the observed probability of obtaining:

i a 1 **ii** at least a 2 **iii** a 1 or a 3

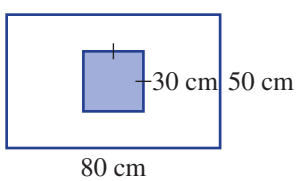
b Based on these results, how many 3s would you expect if the spinner is spun 300 times?

c In fact, the spinner is divided up using simple and common fractions. Draw and label how you think the spinner regions are divided up.

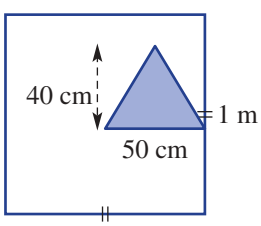
ENRICHMENT: Different-shaped dart boards — 14

14 One hundred darts are randomly thrown at the given dartboards. No darts miss the dartboard entirely. How many darts do you expect to hit the blue shaded region? Give reasons.

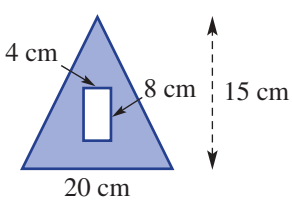
a



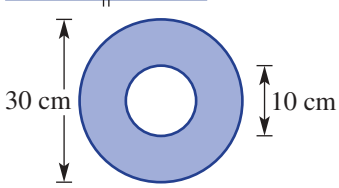
b



c



d



Hint: First find the proportion of the entire dartboard taken up by each blue region.

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10F Mean, median, mode and range CONSOLIDATING

Learning intentions for this section:

- To know that mean, median, mode and range are used to summarise a dataset
- To be able to find the mean, median, mode and range of a dataset
- To know how the median is found for datasets with an odd or even number of values
- To understand that an outlier is a value much smaller or larger than the rest of the data

Past, present and future learning:

- This section consolidates and extends Stage 4 concepts that are used in Stages 5 and 6
- These are very important concepts in Stage 6 Standard

Statistics involves collecting and summarising data. It also involves drawing conclusions and making predictions. The type and amount of product stocked on supermarket shelves, for example, is determined by the sales statistics and other measures such as average cost and price range.

Lesson starter: Mean, median or mode!

Joshua scored the following scores out of 100 on five Maths tests.

51, 62, 98, 55, 55

- Recall and discuss the meanings of the words mean, median and mode.
- Which one of the three measures of centre (i.e. mean, median or mode) would Joshua prefer to use as an 'average score' when telling his parents the results of his Maths tests?
- Why is the mean greater than the median in this case?

KEY IDEAS

- Statisticians use **summary statistics** to highlight important aspects of a dataset. These are summarised below.
- Some summary statistics are called **measures of location**.
  - The two most commonly used measures of location are the **mean** and the **median**. These are also called '**measures of centre**' or '**measures of central tendency**'. Mean and median can only be applied to numerical data.
  - The **mean** is sometimes called the '**arithmetic mean**' or the '**average**'.
    The formula used for calculating the mean, $\bar{x}$, is:

$$\bar{x} = \frac{\text{sum of data values}}{\text{number of data values}}$$

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For example, in the following dataset

5	7	2	5	1

the mean is $\frac{5+7+2+5+1}{5} = 20 \div 5 = 4$

- The **median** divides an ordered dataset into two sets, each of which contain the same number of data values. It is often called the 'middle value'. The median is found by firstly ensuring the data values are in ascending order, then selecting the 'middle' value.

  If the number of values is odd, simply choose the one in the middle.

  2 2 3 **4** 6 9 9

  Median = 4

  If the number of values is even, find the average of the two in the middle.

  2 2 3 **4** 7 9 9 9

  Median = $(4 + 7) \div 2 = 5.5$

- The **mode** of a dataset is the most frequently occurring data value. There can be more than one mode. When there are two modes, the data is said to be **bimodal**. The mode can be very useful for categorical data. It can also be used for numerical data, but it may not be an accurate measure of centre. For example, in the dataset below the mode is 10, but it is the largest data value.

1	2	3	10	10

- An **outlier** is a data value that is significantly greater than or less than the other data values in the dataset. The inclusion of an outlier in a dataset can inflate or deflate the mean and give a false impression of the 'average'.
  For example:
  In the dataset 1, 2, 3, 4, 5, the mean is 3 and median is 3.
  If the data value 5 is changed to 20 it is now an outlier.
  In the dataset 1, 2, 3, 4, 20, the median is still 3, but the mean is now 6, which is greater than four of the five data values. In this case the median is a better measure of centre than the mean.

■ Some summary statistics are called **measures of spread**

- The **range** is a measure of spread.
- The range of a dataset is given by the formula:

  range = maximum data value − minimum data value

  For example, in the dataset below the range is $7 - 1 = 6$

5	7	2	5	1

Maximum Minimum

- Later, you will encounter some other measures of spread such as interquartile range and standard deviation.

Key terminology: mode, mean, range, median, outlier, bimodal, dataset

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Exercise 10F

UNDERSTANDING | 1–5 | 1–5

1 **a** To calculate the _____, you add up all the values and divide by the number of values.

b Find the mean of these datasets (round each answer to one decimal place).

i 3 6 8 10 12

ii 6 8 4 12 9 11 16 13

Hint: First find the sum of the scores, then divide by the number of scores. Mean $= \frac{39}{5}$

2 **a** The mode is the most _____ value.

b State the mode of these datasets.

i 1 2 2 2 4 5 6

ii 1 4 8 8 9 10 10 10 12

3 **a** The median is the _____ value when the scores are listed in order.

b Find the median of these datasets.

i 1 3 5 6 8

ii 4 5 7 9 10 11

4 **a** The _____ is the difference between the highest and lowest values.

b Find the range of these datasets.

i 3 6 7 12 15 24

ii 5 12 14 18 23 27

Hint: Range = highest value − lowest value

5 **a** An _____ is a score that is much larger or smaller than the rest of the data.

b For each of these datasets, if there is an outlier, state its value.

i 2 4 5 4 6 2 3 36

ii 21 25 3 27 28 24 29 30

FLUENCY | 6(½), 7, 8 | 6(½), 7–9

Example 11 Finding measures of centre and the range

For the given datasets, find the following:

i the mean **ii** the median **iii** the mode **iv** the range

a 5 2 4 10 6 1 2 9 6

b 17 13 26 15 9 10

SOLUTION	EXPLANATION
a i Mean $= \frac{45}{9}$	$5+2+4+10+6+1+2+9+6=45$
$= 5$	Find the sum of all the numbers and divide by the number of values.
ii 1 2 2 4 (5) 6 6 9 10	First, order the data.
Median = 5	The median is the middle value.

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iii Mode = 2 and 6

The dataset is bimodal since there are two numbers with the highest frequency.

iv Range = 10 − 1
= 9

The range is the highest score minus the lowest score.

b i Mean $= \frac{90}{6}$
$= 15$

$17 + 13 + 26 + 15 + 9 + 10 = 90$
There are 6 values.

ii 9 10 13 15 17 26
14

First, order the data.

Since there are two values in the middle, find the mean of them.

Median $= \frac{13 + 15}{2}$
$= 14$

iii No mode

None of the values are repeated so there is no mode.

iv Range = 26 − 9
= 17

The highest score is 26 and the lowest score is 9.

Now you try

For the given datasets, find the following:

i the mean **ii** the median
iii the mode **iv** the range

a 8 4 2 7 7 3 4 7 3

b −4 2 0 1 −1 0 3 5

6 For the given datasets, find:

i the mean **ii** the median
iii the mode **iv** the range

a 7 2 3 8 5 9 8

b 6 13 5 4 16 10 3 5 10

c 12 9 2 5 8 7 2 3

d 10 17 5 16 4 14

e 3.5 2.1 4.0 8.3 2.1

f 0.7 3 2.9 10.4 6 7.2 1.3 8.5

g 6 0 −3 8 2 −3 9 5

h 3 −7 2 3 −2 −3 4

Hint: Recall that to subtract a negative, you add a positive: e.g. $7 - (-2) = 7 + 2 = 9$

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7 These datasets include an outlier. Calculate the mean and the median and write down the outlier. Include the outlier in your calculations.

Hint: An outlier is much larger or smaller than the other scores.

- **a** 5 7 7 8 12 33
- **b** 2 40 42 45 47
- **c** 1.3 1.1 1.0 1.7 1.5 1.6 −1.1 1.5
- **d** −58 −60 −59 −4 −64

8 Decide whether the following datasets are bimodal.

Hint: Bimodal means that there are two modes.

- **a** 2 7 9 5 6 2 8 7 4
- **b** 1 6 2 3 3 1 5 4 1 9
- **c** 10 15 12 11 18 13 9 16 17
- **d** 23 25 26 23 19 24 28 26 27

9 This ordered dataset shows the number of fish Daniel caught in the 12 weekends that he went fishing during the year.

1 2 3 4 4 5 6 7 7 9 11 13

- **a** Find the range.
- **b** Find the median.
- **c** Find the mean.
- **d** Find the mode.

PROBLEM-SOLVING AND REASONING 10–12 11–13

10 In three running races, Pippa recorded the times 25.1 seconds, 24.8 seconds and 24.1 seconds.

- **a** What is the mean time of the races? Round to two decimal places.
- **b** Find the median time.

11 This is a dataset of six house prices in Darwin.

$324 000 $289 000 $431 000 $295 000 $385 000 $1 700 000

Hint: To find the median, list the prices in order, then find the middle value.

- **a** Which price would be considered the outlier?
- **b** If the outlier was removed from the dataset, by how much would the median change? (First work out the original median house price.)
- **c** If the outlier was removed from the dataset, by how much would the mean change, to the nearest dollar? (First work out the original mean house price.)

Example 12 Finding a data value for a required mean

The hours a shop assistant spends cleaning the store in eight successive weeks are:
8, 9, 12, 10, 10, 8, 5, 10

- **a** Calculate the mean for this set of data.
- **b** Determine the number of hours that needs to be added to this data to make the mean equal to 10.

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SOLUTION	EXPLANATION
a Mean $= \frac{72}{8}$ $= 9$	$8 + 9 + 12 + 10 + 10 + 8 + 5 + 10 = 72$. Sum of the 8 data values is 72.
b Let a be the new score. Require $\frac{72 + a}{8 + 1} = 10$ $\frac{72 + a}{9} = 10$ $72 + a = 90$ $a = 18$	$72 + a$ is the total of the new data and $8 + 1$ is the new total number of scores. Set this equal to the required mean of 10. Solve for a. 9 scores have a mean of 10, so the sum of the scores $= 9 \times 10 = 90$.
The new score would need to be 18.	Write the answer.

Now you try

Mia received scores of 85, 91, 94 and 78 on her last four maths tests.

a Calculate the mean for this set of data.

b Determine the score that needs to be added to this data to make the mean equal to 89.

12 A netball player scored the following number of goals in her 10 most recent games:

15 14 16 14 15 12 16 17 16 15

a What is her mean score?

b What number of goals does she need to score in the next game for the mean of her scores to be 16?

13 Stevie obtained the following scores on her first five Maths tests: 92 89 94 82 93

a What is her mean test score?

b If there is one more test left to complete, and she wants to achieve an average of at least 85, what is the lowest score Stevie can obtain for her final test?

ENRICHMENT: Aiming for an A — 14

14 A school gives grades in Mathematics each semester according to this table. Raj has scored the following results for four topics this semester, and has one topic to go: 75 68 85 79

Average score	Grade
90–100	A+
80–	A
70–	B+
60–	B
50–	C+
0–	C

a What is Raj's mean score so far?

b What grade will Raj get for the semester if his fifth score is:

i 50? **ii** 68? **iii** 94?

c Find the maximum average score Raj can receive for the semester. Is it possible for him to get an A+?

d Find the least score that Raj needs in his fifth topic for him to receive an average of:

i B+ **ii** A

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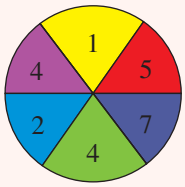
Progress quiz

10A **1** This spinner has six equally divided sections.



a List the sample space using the given numbers.

b Find:

i $P(4)$ **ii** $P(\text{not a } 4)$

iii $P(4 \text{ or a } 5)$ **iv** $P(\text{at most } 5)$

10A **2** A letter is randomly chosen from the word PROGRESS. Find the following probabilities.

a $P(\text{G})$ **b** $P(\text{S})$ **c** $P(\text{not S})$

d $P(\text{vowel})$ **e** $P(\text{S or R})$

10B **3** **a** Complete the following two-way table.

	A	Not A	Total
B	4		
Not B		7	11
Total		12	

b Use your two-way table to find the following probabilities.

i $P(\text{A})$ **ii** $P(\text{A and B})$ **iii** $P(\text{B only})$ **iv** $P(\text{A or B})$

10B **4** A survey of 20 people found that 8 people attended fitness classes and 12 people ran for fitness. 5 of these people did both.

a Construct a Venn diagram for the survey results.

b How many people:

i do not run?

ii run or attend a fitness class?

iii only attend a fitness class?

c If one of the 20 people was randomly selected, find:

i $P(\text{run and attend a class})$

ii $P(\text{neither run nor attend a class})$

iii $P(\text{only run})$

d Turn your Venn diagram into a two-way table.

10C **5** A coin is tossed and then a four-sided die is rolled.

a Copy and complete the table to list the sample space of this experiment.

	1	2	3	4
H				
T				

b How many possible outcomes are there?

c Find the probability of:

i the combination H4

ii tails on the coin and a factor of 6 on the die

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10C **6** The letters of the word BEE are written on cards. One card (letter) is chosen and not replaced and then a second card is chosen.

a Complete the table below to list the possible outcomes.

		1st choice		
		B	E	E
2nd choice	B	×		
	E		×	
	E			×

b Find the probability that the word BE is formed from the cards chosen (in order)

c Find the probability of choosing a B.

10D **7** Two snake lollies are selected without replacement from a bag containing three snakes: 1 red (R), 1 blue (B) and 1 orange (O).

a List all the possible combinations for the selection using a tree diagram.

b Find the probability that the selection will contain:

i red and orange **ii** blue

10E **8** In a survey involving 80 people chosen at random, 48 said they catch public transport to work.

a Calculate the observed probability of choosing a person who catches public transport to work.

b Find the expected number of people who would catch public transport to work in a group of 400 people.

10F **9** For the dataset below, find:

i the mean **ii** the median **iii** the mode **iv** the range

a 8 14 6 21 17 8 10

b 22 16 8 15 25 10

10F **10** The temperatures for the past six days have been: 22°, 28°, 26°, 30°, 25°, 19°.

a Calculate the mean of this set of temperatures.

b Determine the temperature that would need to occur on the seventh day to make the mean temperature 26° for the week.

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10G Interpreting data from tables and graphs

Learning intentions for this section:

- To know the common types of statistical graphs and be able to interpret them
- To be able to recognise positive skew, negative skew and symmetry in a graph
- To be able to recognise when a graph may be misleading or drawn to give a false impression

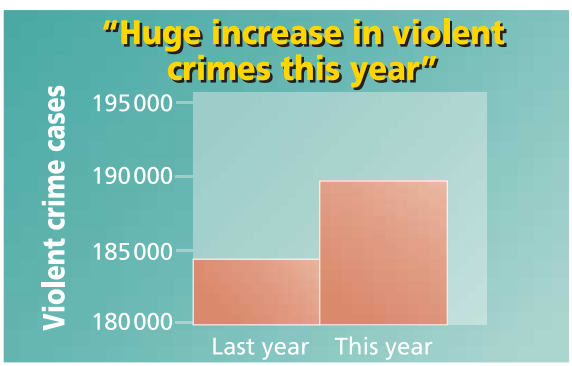
Past, present and future learning:

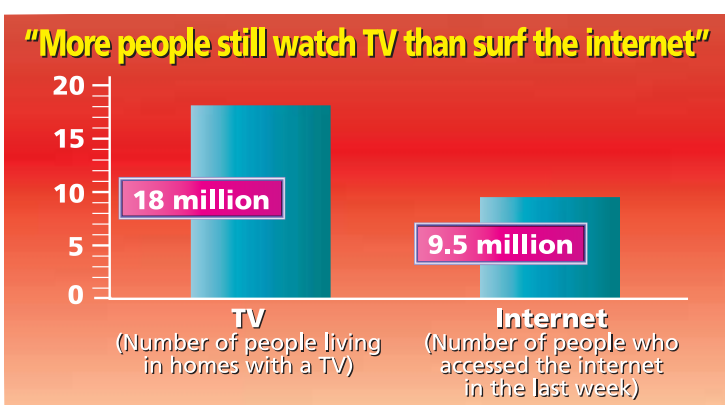
- This section consolidates and extends Stage 4 concepts that are used in Stages 5 and 6
- These are very important concepts in Stage 6 Standard

In our everyday lives, it is important to be able to understand many forms of information. Data that is presented in a table or a graph is much easier to interpret than data in a long list. The headings in a table add detail and graphs give a visual comparison between values or categories.

Lesson starter: Misleading?

Data can be presented in tables, lists or graphs. The headings and labels used on a graph help us to know what the data represent. However, sometimes graphs can be misleading. Look at the graphs below and on the next page, and discuss why each one of them is misleading.





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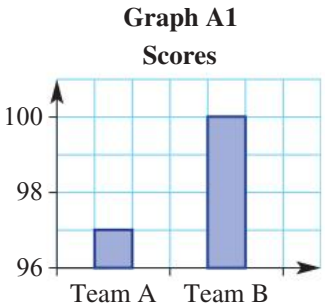
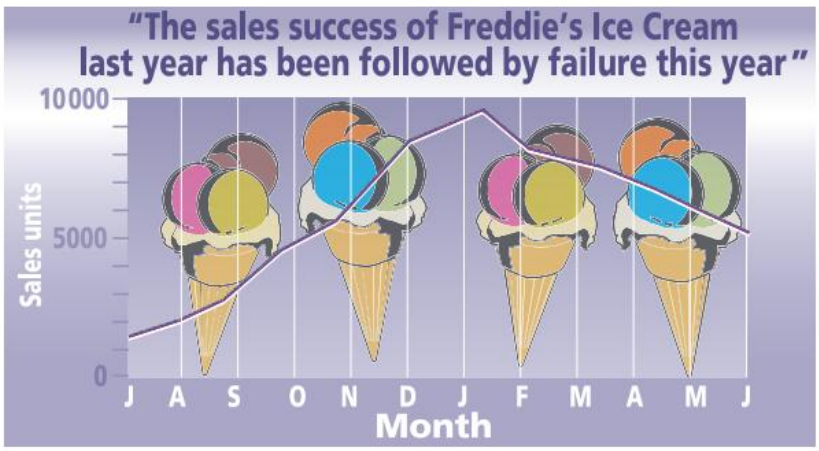
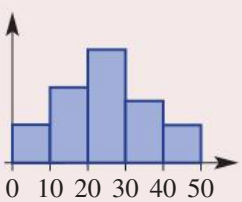




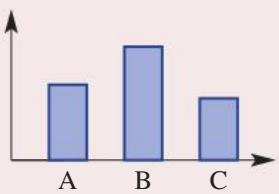
KEY IDEAS

- Graphs can be presented in different forms. Some common examples are **histogram**, **column graph**, **dot plot** and **pie chart**.

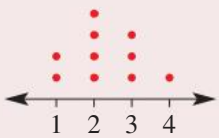
Histogram



Column graph



Dot plot



Pie chart

- The titles, scales and column heights or sector angles tell us something about the data that is used to make the graphs.

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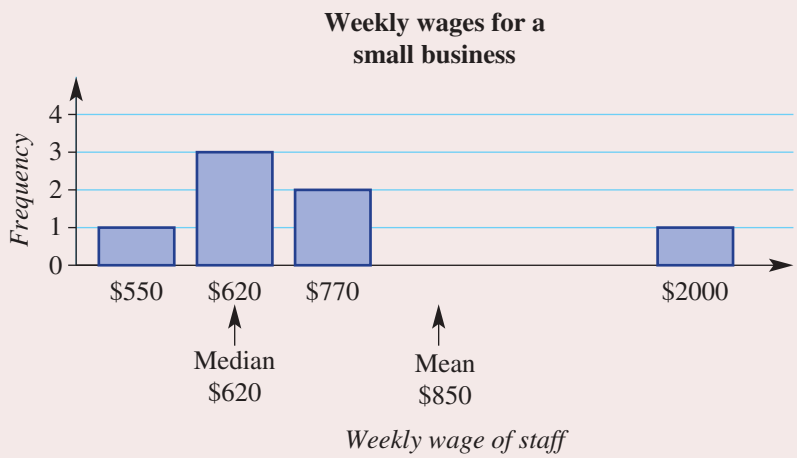
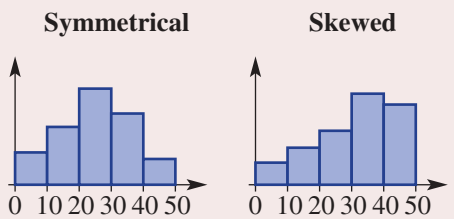
- Misleading graphs give a false impression about data.
  - When only part of the scale is shown, the difference between results may be exaggerated.
  - If one column has a different size or colour, it may appear to have greater value than the other columns.
  - When pictograph symbols are not the same size, the larger symbol appears to have a higher value.
  - A measure of centre can be misleading if it is not a fair representation of the centre of the data values.
    For example, for the data in the graph below, the mean wage of $850 is actually larger than all other wages except the outlier of $2000. So the median (middle) wage of $620 is a better representation of the 'average' wage in this case.



- Graphs can help to determine if data is symmetrical or skewed.



  - Symmetrical data will appear like the data has a mirror line at its centre.
  - Skewed data will have values bunched to one side of the middle.

Key terminology: histogram, dot plot, pie chart, symmetrical data, skewed data, mean, median, mode, range

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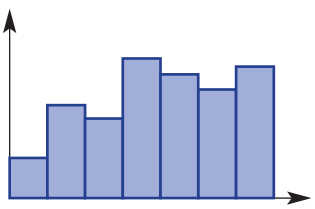
Exercise 10G

UNDERSTANDING	1–3	2, 3

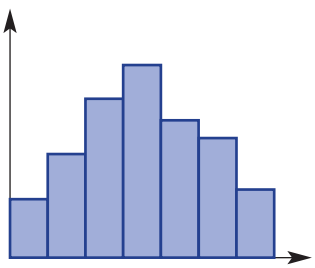
1 Which of the following are not examples of graphs?
dot plot, mean, column graph, pie chart, range and histogram.

2 Which of the following graphs is symmetrical and which is skewed?

A

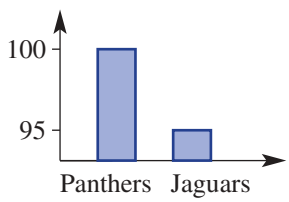


B

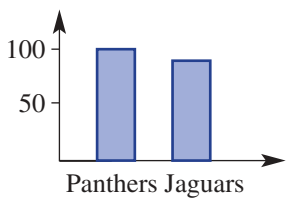


3 Which of the two graphs might a newspaper use if it wanted to give the impression that the Panthers beat the Jaguars by a large margin? Give a reason.

A



B

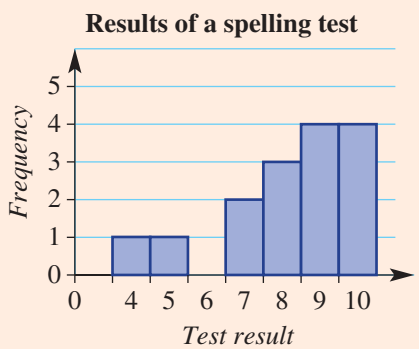


FLUENCY	4–6, 8, 9	4, 6, 7, 9, 10

Example 13 Interpreting histograms or column graphs

This column graph shows the results of a spelling test out of 10.

a How many results are shown in this histogram?
b List the results in ascending order.
c Calculate the mean.
d Calculate the median.
e What is the range of results from this test?
f Is this data skewed or symmetrical?
g What fraction of results is greater than or equal to 7?



SOLUTION

a $1 + 1 + 2 + 3 + 4 + 4 = 15$

EXPLANATION

The frequency shows how many times each score occurred.

Add the frequency values to find the total number of scores.

b 4, 5, 7, 7, 8, 8, 8, 9, 9, 9, 9, 10, 10, 10, 10

1 lot of 4, 1 lot of 5, 2 lots of 7, 3 lots of 8, 4 lots of 9, 4 lots of 10.

Continued on next page

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c Mean $= \frac{123}{15}$
$= 8.2$

$4 + 5 + 7 + 7 + 8 + 8 + 8 + 9 + 9 + 9 + 9 + 10 + 10 + 10 + 10 = 123$

d Median = 9

The 8th score is the middle score.

e Range $= 10 - 4$
$= 6$

The range is the highest score minus the lowest score.

f Skewed

Results are bunched to the higher scores.

g $\frac{13}{15}$

13 scores are greater than or equal to 7 out of a total of 15 scores.

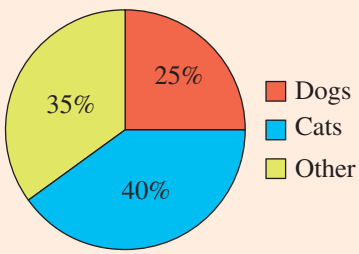
Now you try

This pie chart shows the percentage of households in a particular town that have a type of pet. There are 500 households in the town.

a How many of the households owned:
i dogs? **ii** cats? **iii** other?

b How many more households own cats compared to dogs?

c What is the sector angle for cats?



4 This graph shows the results of a number of maths tests out of 10.

a How many results are shown in this graph?

b List the results in ascending order.

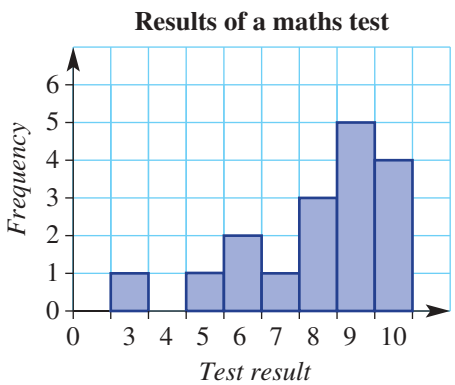
c Calculate the mean.

d Calculate the median.

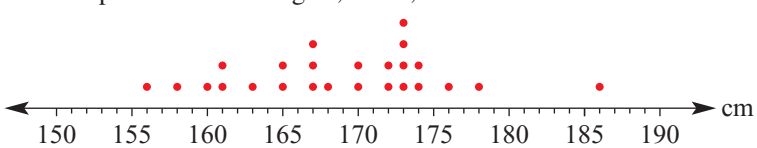
e What is the range of results from this test?

f Is this data skewed or symmetrical?

g What fraction of results is greater than or equal to 8?



5 This dot plot shows the heights, in cm, of the students in a Year 9 class.



Hint: Each dot represents a student.

a How many students have their heights recorded on this dot plot?

b What is the range of heights?

c What is the mode of these heights?

d What is the median height for this class?

e What is the mean height for this class?

f What value is the outlier?

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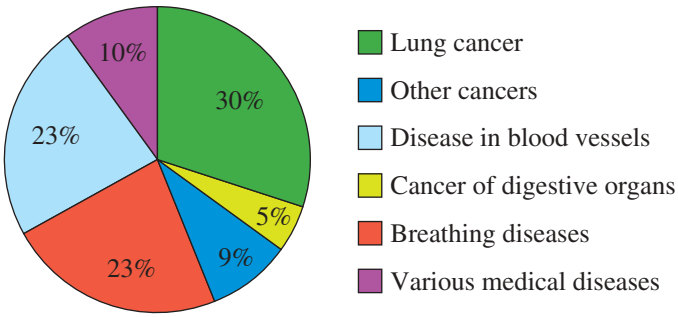
6 This pie chart shows the percentages of deaths in Australia from diseases caused by smoking.

Deaths from smoking-related diseases



Hint: Number of deaths

$$= \frac{\text{percentage}}{100} \times \text{total deaths}$$

Sector angle = percentage × 360°

Around 24 000 Australians in total die from smoking-related illness per year.

a What is the total percentage of smoking deaths caused by cancer?

b How many Australian smokers die of cancer in a year?

c How many Australian smokers die of cancer each day? Round to the nearest whole number.

d How many Australian smokers die from various breathing diseases each day? Round to one decimal place.

e Calculate the sector angle for lung cancer. Recall that 100% represents 360°.

7 The following table shows tide times and heights in December for Yamba, on the NSW north coast. The tide heights shown are red for low tide and blue for high tide. The times are in 24-hour time.

Saturday 17		*Sunday* 18		*Monday* 19		*Tuesday* 20		*Wednesday* 21	
Time	**Height**	**Time**	**Height**	**Time**	**Height**	**Time**	**Height**	**Time**	**Height**
0038	1.14	0145	1.18	0255	1.26	0401	1.37	0.502	1.49
0619	0.45	0729	0.49	0847	0.50	1008	0.46	1123	0.38
1243	1.42	1340	1.34	1445	1.26	1556	1.21	1703	1.18
1925	0.29	2018	0.29	2115	0.28	2211	0.26	2307	0.22

Hint: 1520 in 24-hour time is 3:20 p.m. in 12-hour time.

Write all time answers in both 24-hour time and 12-hour time.

a How high is the second high tide on Saturday 17 December?

b What time is the first low tide on Monday 19 December?

c How much later in the morning is the low tide on Tuesday 20 December than the low tide on Monday 19 December?

d What is the difference in height between the two high tides on Wednesday 21 December?

e How long is it between the two high tides on Sunday 18 December?

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8 The following is a table of life expectancy estimates for people in various countries.

Life expectancy in years			
Country	**Overall average**	**Male**	**Female**
Japan	83	79	86
Switzerland	82	80	84
Australia	81	79	84
Malaysia	74	72	77
Vietnam	74	72	77
Indonesia	71	69	73
Papua New Guinea	57	55	60
South Africa	49	49	50
Mozambique	39	38	39

a How many years more do Australian males expect to live compared to Mozambique males?

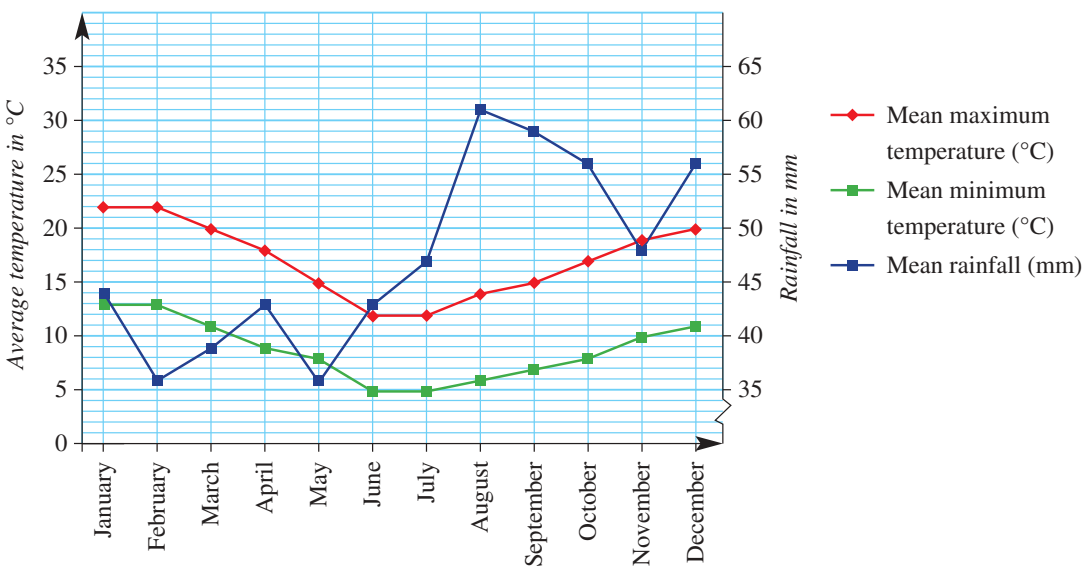
b How many years more do Australian females expect to live compared to Papua New Guinea females?

c If the overall world average of life expectancy is 67 years, how far above the average is Japan's overall average life expectancy?

d Find the increase in life expectancy between Indonesian males and Australian males. Now calculate this increase as a percentage of Indonesian male life expectancy. Round to one decimal place.

9 The line graphs below show the mean monthly minimum and maximum temperatures and also the monthly rainfall in mm for Hobart.
The temperature values are read from the scale on the left and the rainfall values are read from the scale on the right. For example, in January the mean maximum temperature is 22°C and the rainfall is 44 mm.

Hobart monthly temperature and rainfall



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a What was the mean maximum temperature in April?

b What was the rainfall in April?

c What was the mean minimum temperature in July?

d What was the rainfall in July?

e During what months was the mean maximum temperature greater than 19°C?

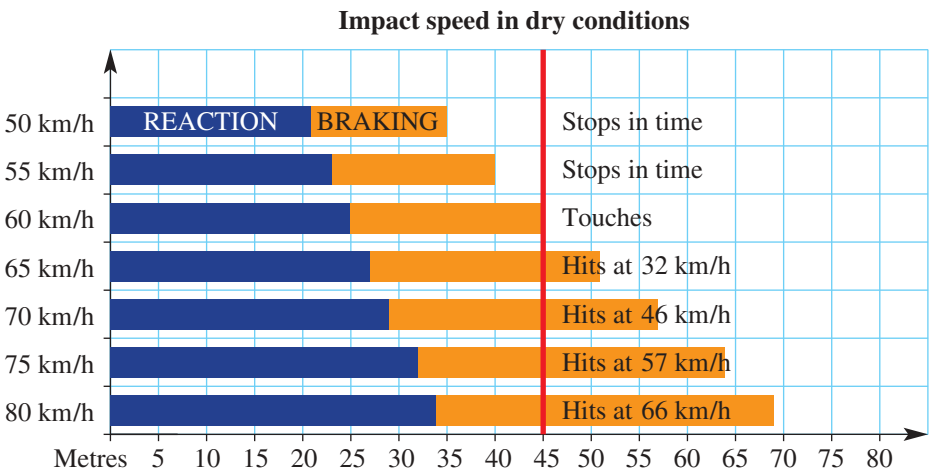
f During what months was the rainfall less than 40 mm per month?

g Which were the wettest two months in Hobart?

h What was the temperature range in December?

10 The following graph shows impact speeds (reaction time and distance travelled while braking) for cars when driving at various speeds on a dry road.

Hint: Reaction time is shown by the purple bar, braking time is shown by the orange bar.



a The red line on the graph represents an object or person in front of the braking car. How far in front of the car is the object or person at the start of the driver's reaction time?

b How many metres are travelled during *reaction time* when driving at 60 km/h?

c How many metres are travelled during *braking time* when driving at 60 km/h?

d How much distance is needed, overall, to stop when driving at 80 km/h?

e By how much does the braking distance increase when driving at 80 km/h compared to 50 km/h?

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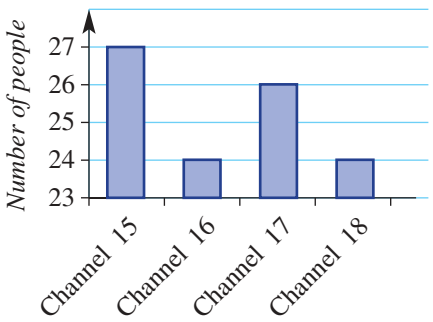
PROBLEM-SOLVING AND REASONING

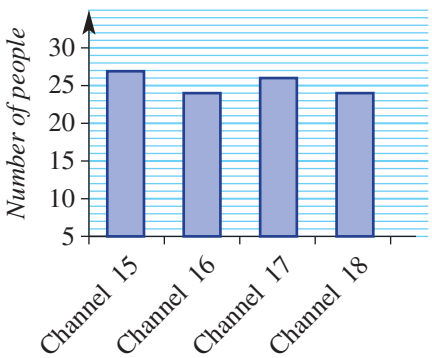
11, 12 | 11, 13, 14

11 Here are two column graphs, each showing the same results of a survey that asked people which TV channel they preferred.

Graph A



Graph B



a From Graph A, write down how many viewers preferred each channel.

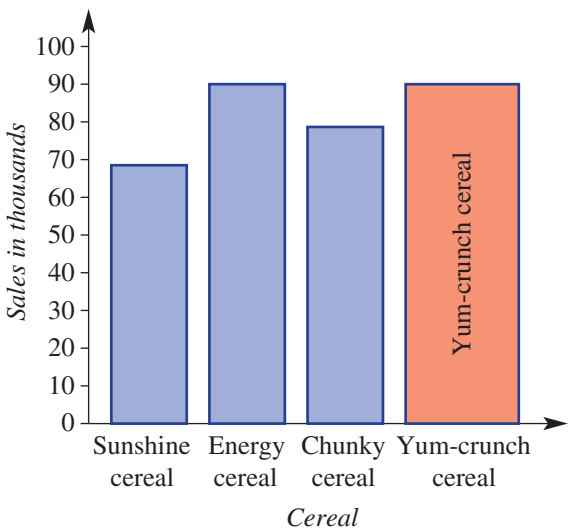
b Which graph could be titled 'Channel 15 is clearly most popular'?

c Which graph could be titled 'All TV channels have similar popularity'?

d What is the difference between the two graphs?

e Which graph is misleading, and why?

12 This column graph shows the number of sales for one month of four popular breakfast cereals. The height of each column represents the sales.



a List the breakfast cereals in order of sales.

b Did Yum-crunch have more sales than the other cereals?

c List 3 changes that have been made to the Yum-crunch column to make it look like it has better sales than any of the other cereals.

d How should the columns be drawn so the graph is not misleading?

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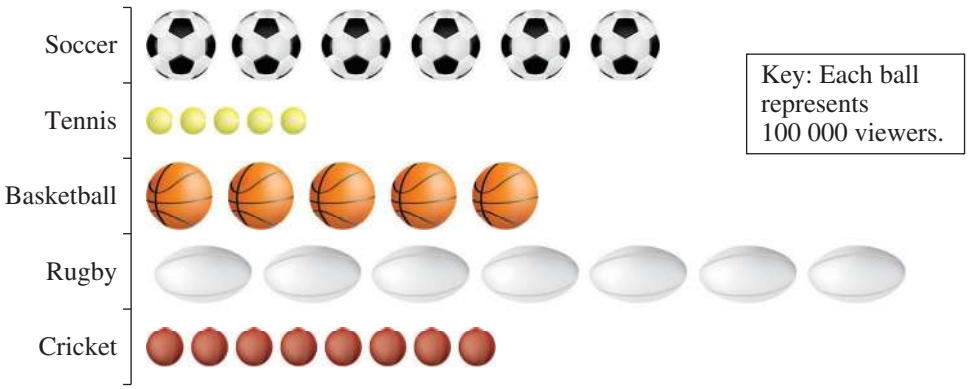
13 This pictograph shows the number of TV viewers for the final matches of various sports.

Size of TV audience for finals matches



a Which sport *appears* to have had the greatest TV audience?

b List the sports in order, according to their *length* on the pictograph.

c Using the key, determine the audience sizes for the rugby and soccer finals.

d Which sport actually had the largest TV audience? What was the size of its audience?

e List the sports in order according to the audience size calculated using the key.

f In what way is this graph misleading?

g How should a pictograph be drawn so it is not misleading?

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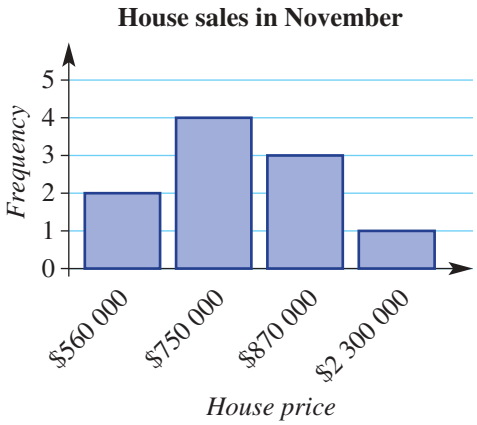
14 This column graph shows the price of some houses that were sold during one November in a Sydney suburb.



a How many house sales are shown in this column graph?

b List the price of each house in ascending order.

c Find the mean house price.

d Find the median house price.

e A newspaper headline read 'House prices now average over $900 000'. Is this 'average' referring to the mean or median price?

f What fraction of house prices were less than the mean value?

g Do you think the mean or the median would better represent the 'average' house price? Give a reason for your answer.

ENRICHMENT: Viewing distance for TV — 15

15 a Draw a line graph showing the minimum and maximum viewing distances for each TV screen size. Include a key.

TV screen size (length of diagonal in inches)	Maximum viewing distance (cm)	Minimum viewing distance (cm)
26	200	100
30	220	120
34	260	130
42	320	160
47	360	180
50	380	200
55	420	220
60	470	230
65	490	250

b If a lounge chair is 1.5 m in front of the TV, which sizes of TV would be suitable?

c If a lounge chair is 3 m in front of the TV, which sizes of TV would be suitable?

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10H Stem-and-leaf plots

Learning intentions for this section:

- To be able to construct and interpret stem-and-leaf plots, both single and back-to-back
- To be able to interpret the shape of a stem-and-leaf plot in terms of symmetry and skewness

Past, present and future learning:

- This section consolidates and extends Stage 4 concepts that are used in Stages 5 and 6
- These are very important concepts in Stage 6 Standard

Stem-and-leaf plots (or stem plots) are commonly used to display a single dataset or two related datasets. They help to show how the data is distributed. They retain all the individual data elements so no detail is lost. The median and mode can be easily read from a stem-and-leaf plot because all the data sits in order.

Lesson starter: Ships vs Chops

At a school, Ms Ships' class and Mr Chops' class sit the same exam. The scores are displayed using this back-to-back stem-and-leaf plot. Discuss the following.

- Which class had the most students?
- What were the lowest and highest scores from each class?
- What were the median scores from each class?
- Which class could be described as symmetrical and which as skewed?
- Which class had the better results?

Leaf Ms Ships' class	Stem	Leaf Mr Chops' class
3 1	5	0 1 1 3 5 7
8 8 7 5	6	2 3 5 5 7 9 9
6 4 4 2 1	7	8 9 9
7 4 3	8	0 3
6	9	1

7 | 8 means 78

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KEY IDEAS

- A **stem-and-leaf plot** is a way to display numerical data.
- Each number is split into a stem (the first digit or digits) and a leaf (the last digit).

For example:

	Stem	Leaf
The number 7 is	0	7
The number 31 is	3	1
The number 152 is	15	2

- Leaves are aligned vertically, getting bigger as you move away from the stem.
- **Back-to-back stem-and-leaf plots** can be used to compare two sets of data. The stem is drawn in the middle, with the leaves on either side.

Scores for the last 30 AFL games

Winning scores		Losing scores
	7	4 5 8 8 9
1	8	0 0 3 3 6 7
8 7 5	9	1 2 3 6
8 4 4 1	10	3 9
9 5 0	11	1
1	12	

Lowest winning score was 81

Highest losing score was 111

10 | 9 means 109

- Symmetrical data will produce a graph that is symmetrical about the centre. The 'winning scores' are symmetrical.
- Skewed data will produce a graph that lacks symmetry. The 'losing scores' are skewed.

Scores for the last 30 AFL games

Winning scores		Losing scores
	7	4 5 8 8 9
1	8	0 0 3 3 6 7
8 7 5	9	1 2 3 6
8 4 4 1	10	3 9
9 5 0	11	1
1	12	

'Shape ' of the data is symmetrical

'Shape' of the data is skewed, not symmetrical

10 | 9 means 109

Key terminology: stem-and-leaf plot, symmetrical data, skewed data, back-to-back stem-and-leaf plot

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Exercise 10H

UNDERSTANDING | 1–3 | 3

1 List the statistical data that would produce these stem-and-leaf plots. Use the key to help.

a

Stem	Leaf
3	5 7
4	1 3 8

3 | 1 means 3.1

b

Stem	Leaf
5	2
6	0 1 7
7	3 5

6 | 3 means 63

Hint: The same stem goes with each leaf along each row.

2 This stem-and-leaf plot shows the number of minutes Alexis spoke on her phone for a number of calls.

Stem	Leaf
0	8
1	5 9
2	1 1 3 7
3	4 5

2 | 1 means 21 minutes

Hint: Count the leaves to find the number of calls.

a How many calls are represented by the stem-and-leaf plot?

b What is the length of the:

i shortest phone call?

ii longest phone call?

c What is the mode (the most common) call time?

d What is the median call time (middle value)?

3 This back-to-back stem-and-leaf plot shows the thickness of tyre tread on a selection of cars from the city and country.

City		Country
8 7 3 1 0 0 0	0	6 8 8 9
8 6 3 1 0	1	0 4 5 5 6 9
1	2	3 4 4

1 | 3 means 13 mm

Hint: 06 is written just as 6.

a How many car tyres were tested altogether?

b What was the smallest tyre tread thickness in:

i the city? **ii** the country?

c What was the largest tyre tread thickness in:

i the city? **ii** the country?

d Find the median tyre tread thickness for tyres in:

i the city **ii** the country

Hint: Skewed distributions are not symmetrical.

e Is the distribution of tread thickness for city cars more symmetrical or skewed?

f Is the distribution of tread thickness for country cars more symmetrical or skewed?

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FLUENCY | 4(½), 5, 6 | 4(½), 6, 7

Example 14 Constructing and using a stem-and-leaf plot

For this set of data:

0.3 2.5 4.1 3.7 2.0 3.3 4.8 3.3 4.6 0.1 4.1 7.5 1.4 2.4
5.7 2.3 3.4 3.0 2.3 4.1 6.3 1.0 5.8 4.4 0.1 6.8 5.2 1.0

a Organise the data into an ordered stem-and-leaf plot.
b Find the median.
c Find the mode.
d Describe the data as symmetrical or skewed.

SOLUTION

a

Stem	Leaf
0	1 1 3
1	0 0 4
2	0 3 3 4 5
3	0 3 3 4 7
4	1 1 1 4 6 8
5	2 7 8
6	3 8
7	5

3 | 4 means 3.4

EXPLANATION

The minimum is 0.1 and the maximum is 7.5 so stems range from 0 to 7.

Place leaves in order from smallest to largest. Some numbers appear more than once; e.g. two instances of 0.1 means that the leaf 1 appears twice: 0.1, 0.1, …

b $\text{Median} = \frac{3.3 + 3.4}{2}$

$= 3.35$

There are 28 data values. The median is the average of the two middle values (the 14th and 15th values).

c Mode is 4.1.

The most common value is 4.1.

d Data is approximately symmetrical.

The distribution of numbers is approximately symmetrical about the stem containing the median.

Now you try

For this set of data:
24 37 52 16 11 29 13 26
34 42 9 29 21 35 46 50

a Organise the data into an ordered stem-and-leaf plot.
b Find the median.
c Find the mode.
d Describe the data as symmetrical or skewed.

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4 For each of the following datasets:

- **i** organise the data into an ordered stem-and-leaf plot
- **ii** find the median
- **iii** find the mode
- **iv** describe the data as symmetrical or skewed

Hint: Draft version: Enter the leaves in the same order as in the question.
Final version: Rewrite the stem-and-leaf plot with leaves in order. Remember to include the key.

Hint: In part **d**, use 16 as the stem for 167.

a 41 33 28 24 19 32 54 35
19 23 32 26 28 26 28

b 31 33 23 35 15 23 48 50 35 42 45 15 21
51 31 34 23 42 50 26 30 45 37 39 45

c 34.5 34.9 33.7 34.5 35.8 33.8 34.3 35.2 37.0 34.7
35.2 34.4 35.5 36.5 36.1 33.3 35.4 32.0 36.3 34.8

d 167 159 159 193 161 164 167 157 158 175 177 185
177 202 185 187 159 189 167 159 173 198 200

5 The number of vacant rooms in a motel each week over a 20-week period is shown below.

12 8 11 10 21 12 6 11 12 16
14 22 5 15 20 6 17 8 14 9

a Draw a stem-and-leaf plot of this data.

b In how many weeks were there fewer than 12 vacant rooms?

c Find the median number of vacant rooms.

Example 15 Constructing back-to-back stem-and-leaf plots

A shop owner has two jeans shops. The daily sales in each shop over a 16-day period are monitored and recorded as follows.

Shop A
3 12 12 13 14 14 15 15 21 22 24 24 24 26 27 28

Shop B
4 6 6 7 7 8 9 9 10 12 13 14 14 16 17 27

a Draw a back-to-back stem-and-leaf plot.

b Compare and comment on differences between the sales made by the two shops.

SOLUTION

a

Shop A		Shop B
3	0	4 6 6 7 7 8 9 9
5 5 4 4 3 2 2	1	0 2 3 4 4 6 7
8 7 6 4 4 4 2 1	2	7

1|3 means 13

EXPLANATION

The data for each shop is already ordered. Stems are in intervals of 10. Record leaf digits for Shop A on the left and Shop B on the right, ordered from the middle to the outside.

Continued on next page

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b Shop A sales are generally between 12 and 28, with one low value of 3. Shop B sales are generally between 4 and 17, with one high value of 27. Shop A has a lot more high values than Shop B. Shop B has more low values than Shop A.

Look at both sides of the plot for the highest and lowest values and whether there are a few or many of the small and large numbers.

Now you try

At a school, two reading classes in Year 6 read books for homework one night. The number of pages read by the students are recorded as follows.

Class A	21	36	19	15	19	31	17	24	29
Class B	3	16	14	20	11	18	12	13	

a Draw a back-to-back stem-and-leaf plot.
b Compare and comment on the differences between the amount of reading completed by the different class groups.

6 For each of the following sets of data:
i Draw a back-to-back stem-and-leaf plot.
ii State the smallest and largest value in each set and compare the numbers of small and large values in each set.

Hint: Order the leaves with the smallest on the inside and largest on the outside. State whether each set has a few or many of the small numbers and large numbers.

a Set A: 46 32 40 43 45 47 53
33 48 39 43 54 40 54

Set B: 48 49 31 40 43 47 48
44 46 53 44 41 49 51

b Set A: 0.7 0.8 1.4 8.8 9.1 2.6 3.2 0.3 1.7 1.9 2.5 4.1 4.3 3.3 3.4
3.6 3.9 3.9 4.7 1.6 0.4 5.3 5.7 2.1 2.3 1.9 5.2 6.1 6.2 8.3

Set B: 0.1 0.9 0.6 1.3 0.9 0.1 0.3 2.5 0.6 3.4 4.8 5.2 8.8 4.7 5.3
2.6 1.5 1.8 3.9 1.9 0.1 0.2 1.2 3.3 2.1 4.3 5.7 6.1 6.2 8.3

7 a Draw a back-to-back stem-and-leaf plot for the final scores of St Kilda and Collingwood in the 24 AFL games given here.

St Kilda:	126	68	78	90	87	118
	88	125	111	117	82	82
	80	66	84	138	109	113
	122	80	94	83	106	68
Collingwood:	104	80	127	88	103	95
	78	118	89	82	103	115
	98	77	119	91	71	70
	63	89	103	97	72	68

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b In what percentage of their games did each team score more than 100 points?

c Comment on the symmetry of the distribution of the scores for each team.

d Which team has scores that are more consistent? Which team has more higher scores?

Hint: Percentage = fraction × 100

PROBLEM-SOLVING AND REASONING

8, 9 | 8–10

8 This stem-and-leaf plot shows the time taken, in seconds, by Helena to run 100 m in her last 25 races.

Stem	Leaf
14	9
15	4 5 6 6 7 7 7 8 9
16	0 0 1 1 2 2 3 4 4 5 5 5 7 7
17	2

14 | 9 means 14.9 seconds

a Find Helena's median time.

b What is the difference between the slowest and fastest time?

c If in her 26th race her time was 14.8 seconds and this was added to the stem-and-leaf plot, would her median time change? If so, by how much?

9 Two brands of batteries were tested to determine their lifetime in hours. The data below shows the lifetime of 20 batteries of each brand.

Brand A: 7.3 8.2 8.4 8.5 8.7 8.8 8.9 9.0 9.1 9.2
9.3 9.4 9.4 9.5 9.5 9.6 9.7 9.8 9.9 9.9

Brand B: 7.2 7.3 7.4 7.5 7.6 7.8 7.9 7.9 8.0 8.1
8.3 9.0 9.1 9.2 9.3 9.4 9.5 9.6 9.8 9.8

Hint: The stems will be the 'units' value and each leaf the 'tenths' value.

a Draw a back-to-back stem-and-leaf plot for this data.

b How many batteries from each brand lasted more than 9 hours?

c Which brand shows the best performance?

10 Find the median if all the data in each back-to-back stem-and-leaf plot was combined.

a

5	3	8 9
9 7 7 1	4	0 2 2 3 6 8
8 6 5 2 2	5	3 3 7 9
7 4 0	6	1 4

4 | 2 means 42

b

3	16	0 3 3 6 7 9
9 6 6 1	17	0 1 1 4 8 8
8 7 5 5 4 0	18	2 2 6 7
2	19	0 1

16 | 3 means 16.3

Hint: First combine each back-to-back plot into just one stem-and-leaf plot.

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ENRICHMENT: Birth weights and smoking — 11

11 The back-to-back stem-and-leaf plot below shows the birth weight in kilograms of babies born to mothers who do or don't smoke.

Birth weight of babies

Smoking mothers		Non-smoking mothers
4 3 2 2	2	4
9 9 8 7 6 6 5 5	2*	8 9
4 3 2 1 1 1 0 0 0	3	0 0 1 2 2 3
6 5 5	3*	5 5 5 6 6 7 7 8
1	4	
	4*	5 5 6

2 | 4 means 2.4 kg

2* | 5 means 2.5 kg

a What percentage of babies born to smoking mothers have a birth weight of less than 3 kg?

b What percentage of babies born to non-smoking mothers have a birth weight of less than 3 kg?

c Compare and comment on the differences between the birth weights of babies born to mothers who smoke and those born to mothers who don't smoke.

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10I Grouping data into classes

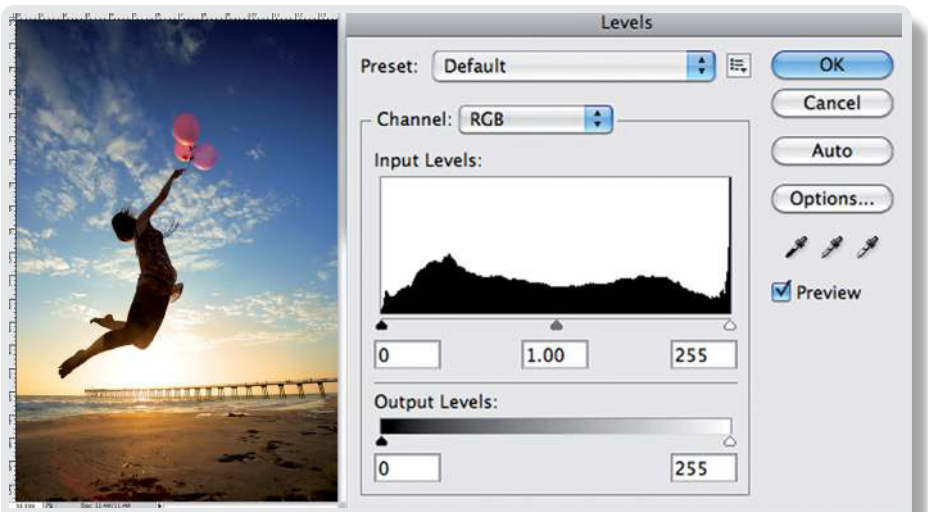
Learning intentions for this section:

- To be able to construct a frequency table from a set of data
- To be able to construct and analyse a histogram or percentage frequency histogram

Past, present and future learning:

- This section consolidates and extends Stage 4 concepts that are used in Stages 5 and 6
- These are very important concepts in Stage 6 Standard

For some data, especially large sets, it makes sense to group the data and then record the frequency for each group to produce a frequency table. For numerical data, a graph generated from a frequency table gives a histogram. Like a stem-and-leaf plot, a histogram shows how the data is distributed across the full range of values. A histogram, for example, is used to display the level of exposure of the pixels in an image in digital photography. It uses many narrow columns to show how the light values are distributed across the scale from black to white.

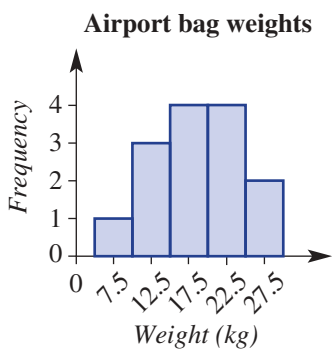


A luminance value histogram used in digital photography software.

Lesson starter: Baggage check

This histogram shows the distribution of the weight of a number of bags checked at an airport.

- How many bags had a weight in the range 10–15 kg?
- How many bags were checked in total?
- Is it possible to determine the exact mean, median or mode of the weight of the bags by looking at the histogram? Discuss.
- Describe the distribution of checked bag weights for the given graph.



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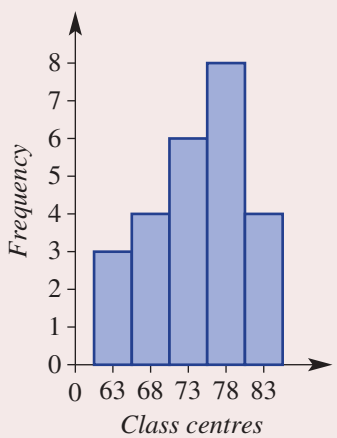
KEY IDEAS

- A **frequency table** shows the number of values within a set of categories or **class intervals**.
- Grouped numerical data can be illustrated using a **histogram**.
  - The height of a column corresponds to the frequency of values in that class interval.
  - There are usually no gaps between columns.
  - The scales are evenly spread, with each bar spreading across the boundaries of the class interval.
  - A **percentage frequency histogram** shows the frequencies as percentages of the total.
- Like a stem-and-leaf plot, a histogram can show if the data is **skewed** or **symmetrical**.

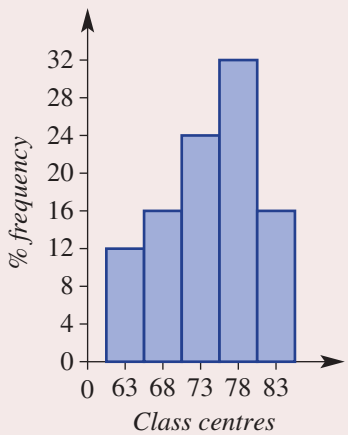
Frequency table

Class	Class centre	Frequency	Percentage frequency
61–65	63	3	12
66–70	68	4	16
71–75	73	6	24
76–80	78	8	32
81–85	83	4	16
	Total	25	100

Frequency histogram



Percentage frequency histogram



Key terminology: frequency table, percentage frequency histogram, histogram, class interval

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Exercise 10I

UNDERSTANDING | 1–3 | 2–4

1 Write the missing word for each of these statements.

a A group of scores such as 10–15 kg is called a _____ _____.

b A _____ _____ shows the number of values in each class interval.

c A column graph that shows frequencies and scores is called a _____.

d There are _____ _____ between the columns in a histogram.

e A _____ _____ histogram shows each frequency as a percentage of the total number of scores.

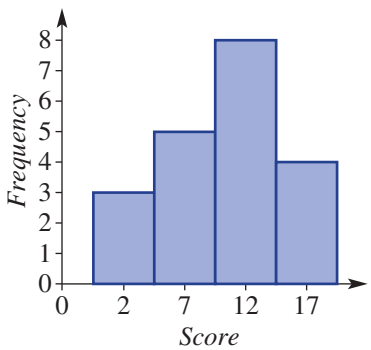
2 Write the total number of scores in each of these displays.

Hint: The sum of the frequencies equals the number of scores.

a

Class interval	Class centre	Frequency
0–4	2	3
5–9	7	5
10–14	12	8
15–19	17	4

b

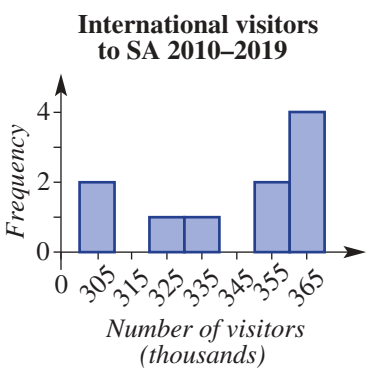


Hint: The frequency shows the number of years.

3 The frequency histogram shows the number of years for which the number of international visitors to South Australia was within a given range for the decade from 2010 to 2019.

a How many years in the decade were there fewer than 330 000 international visitors?

b Which range of visitor numbers had the highest frequency?



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4 Some Year 9 students selected a sample of textbooks from the library and recorded the number of pages in each book. The frequency histogram on the right shows their results.

a How many textbooks had between 100 and 200 pages?

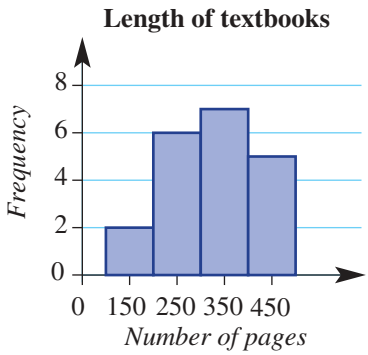
b How many textbooks were selected from the library?

c What percentage of textbooks had between:

i 200 and 300 pages?

ii 200 and 400 pages?

Hint: The frequency shows the number of textbooks.
Percentage $= \frac{\text{frequency}}{\text{total}} \times 100$



FLUENCY 5–7 6–8

Example 16 Constructing frequency tables and histograms

The data below shows the number of hamburgers sold each hour by a 24-hour fast-food store during a 50-hour period.

1	10	18	14	20	11	19	10	17	21
5	16	7	15	21	15	10	22	11	18
12	12	3	12	8	12	6	5	14	14
14	4	9	15	17	19	6	24	16	17
14	11	17	18	19	19	19	18	18	20

a Set up and complete a grouped frequency table, using class intervals 0–4, 5–9 etc. Include a percentage frequency column.

b Construct a frequency histogram.

c For how many hours did the fast-food store sell:

i fewer than 10 hamburgers? ii at least 15 hamburgers?

SOLUTION

a

Class interval	Class centre	Frequency	Percentage frequency
0–4	2	3	6
5–9	7	7	14
10–14	12	15	30
15–19	17	19	38
20–24	22	6	12
Total		50	100

EXPLANATION

Create class intervals of 5 from 0 up to 25, since 24 is the maximum number. Record the number of data values in each interval in the frequency column. Convert to a percentage by dividing by the total (50) and multiplying by 100.

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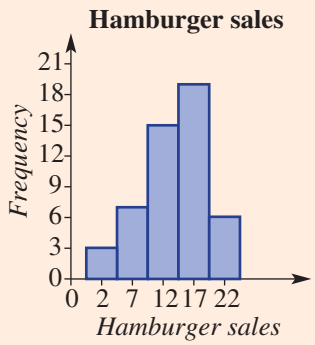
b



Create a frequency histogram with frequency on the vertical axis and the class intervals on the horizontal axis. The height of the column shows the frequency of that interval.

c **i** $3 + 7 = 10$ hours

Fewer than 10 hamburgers covers the 0–4 and 5–9 intervals.

ii $19 + 6 = 25$ hours

At least 15 hamburgers covers the 15–19 and 20–24 intervals.

Now you try

The data below shows the number of cars sold each month at a large city car yard.

18	26	31	12	52	46
38	19	24	40	57	54

a Set up and complete a grouped frequency table, using class intervals 10–19, 20–29 etc. Include a percentage frequency column.

b Construct a frequency histogram.

c For how many months did the car yard sell more than 50 cars?

d What percentage of months were less than 20 cars sold?

5 The data below shows the number of ice-creams sold from an ice-cream van over a 50-day period.

0	5	0	35	14	15	18	21	21	36	45	2	8
2	2	3	17	3	7	28	35	7	21	3	46	47
1	1	3	9	35	22	7	18	36	3	9	2	
11	37	37	45	11	12	14	17	22	1	2	2	

a Set up and complete a grouped frequency table using class intervals 0–9, 10–19, 20–29 etc. Include a percentage frequency column.

b Construct a frequency histogram.

c How many days did the ice-cream van sell:

i fewer than 20 ice-creams?

ii at least 30 ice-creams?

d What percentage of days were 20 or more ice-creams sold?

Hint: 'Fewer than 20' doesn't include 20. 'At least 30' includes 30.

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6 The data below shows the mark out of 100 on the Science exam for 60 Year 9 students.

50	67	68	89	82	81	50	50	89	52	60	82	52	60	87
89	71	73	75	83	86	50	52	71	80	95	87	87	87	74
60	60	61	63	63	65	82	86	96	88	50	94	87	64	64
72	71	72	88	86	89	69	71	80	89	92	89	89	60	83

Hint: The frequency for '50–59' will be the number of scores from 50–59.

a Set up and complete a grouped frequency table, using class intervals 50–59, 60–69, 70–79 etc. Include a percentage frequency column.

b Construct a frequency histogram.

c **i** How many marks were less than 70 out of 100?

ii What percentage of marks were at least 80 out of 100?

7 The number of goals kicked by a country footballer in each of his last 30 football matches is given below.

8	9	3	6	12	14	8	3	4	5	2	5	6	4	13
8	9	12	11	7	12	14	10	9	8	12	10	11	4	5

Hint: The frequency for '0–2' will be the number of scores that are 0, 1 or 2. Label the horizontal scale 1, 4, 7, 10 etc.

a Organise the data into a grouped frequency table using class intervals of 0–2, 3–5 etc.

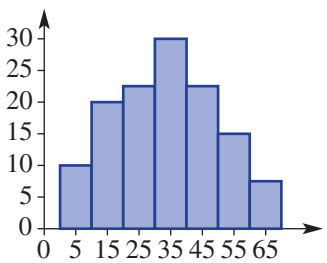
b Draw a frequency histogram for the data.

c In how many games did the player kick fewer than six goals?

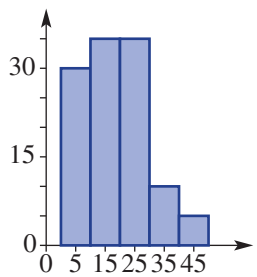
d In how many games did he kick more than 11 goals?

8 Which one of these histograms illustrates a symmetrical dataset and which one shows a skewed dataset?

a



b



Hint: Skewed data is bunched either below or above the middle.

PROBLEM-SOLVING AND REASONING

9,10 | 9, 11, 12

9 Write down the missing numbers in these frequency tables; i.e. find the values of the pronumerals.

a

Class interval	Class centre	Frequency	Percentage frequency
0–4	2	1	10
5–9	7	3	c
10–14	12	4	d
a–19	17	b	e
Total		10	f

b

Class interval	Class centre	Frequency	Percentage frequency
40–49	44.5	20	20
a–59	54.5	28	b
60–69	64.5	12	c
70–79	74.5	d	40
Total		100	e

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10 The data below shows the length of overseas phone calls (in minutes) made by a particular household over a six-week period.

1.5	1	1.5	1	4.8	4	4	10.1	9.5	1	3
8	5.9	6	6.4	7	3.5	3.1	3.6	3	4.2	4.3
4	12.5	10.2	10.3	4.5	4.5	3.4	3.5	3.5	5	3.5
3.6	4.5	4.5	12	11	12	14	14	12	13	10.8
12.1	2.4	3.8	4.2	5.6	10.8	11.2	9.3	9.2	8.7	8.5

Hint: How many calls were 3 minutes or less?

Hint: Percentage $= \frac{\text{frequency}}{\text{total}} \times 100$

What percentage of phone calls were more than 3 minutes in length? Answer to one decimal place.

11 This percentage frequency histogram shows the heights of office towers in a city.

a What percentage of office towers have the following heights?

- **i** Between 50 m and 100 m
- **ii** Less than 150 m
- **iii** No more than 200 m
- **iv** At least 100 m
- **v** Between 100 m and 150 m or greater than 200 m

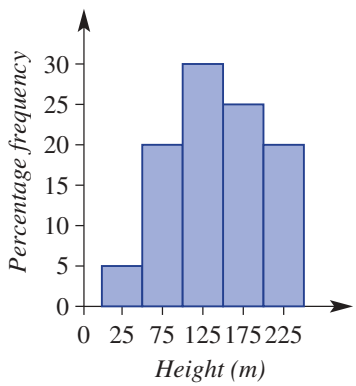
Hint: In parts **b** and **c**, number of office towers = percentage × total

b If the city had 100 office towers, how many would have a height of:

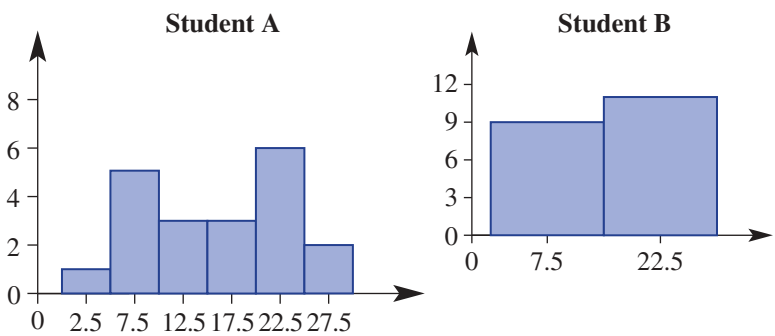
- **i** between 100 m and 150 m?
- **ii** at least 150 m?

c If the city had 40 office towers, how many would have a height of:

- **i** between 0 m and 50 m?
- **ii** no more than 150 m?



12 Two students show different histograms for the same set of data.



Hint: A histogram that shows a lot of detail is more useful.

a Which histogram is more useful in helping to analyse the data?

b What would you advise student B to do differently when constructing the histogram?

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ENRICHMENT: The distribution of weekly wages — 13

13 The data below shows the weekly wages (in dollars) of 50 people.

400 500 552 455 420 424 325 204 860 894 464 379 563 230
940 384 370 356 345 380 720 540 654 678 628 656 670 725
740 750 730 766 760 700 700 768 608 576 890 920 874 860
450 674 725 612 605 600 548 670

a What is the minimum weekly wage and the maximum weekly wage?

b **i** Organise the data into about 10 class intervals: 200–299, 300–399 etc.

ii Draw a frequency histogram for the data.

c **i** Organise the data into about four class intervals.

ii Draw a frequency histogram for the data.

d Discuss the shapes of the two graphs. Which graph represents the data better and why?

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Maths@Work

Personal trainer

Personal training is a growing trend in our society. Personal trainers, while being fit themselves, must have an understanding of mathematics. The mathematics involved in personal training includes calculations of body mass index (BMI) using body mass and weight, calories and kilojoules, business-related mathematics and clients' pulse rates.

1 Calculate the mean pulse rate (measured in beats per minute, bpm) for each of the following clients, over their first five training sessions. Round to a whole number of bpm.

- **a** 85, 92, 86, 90, 88
- **b** 78, 90, 79, 82, 91
- **c** 98, 100, 102, 94, 98
- **d** 120, 122, 134, 116, 104

2 The resting heart rate of a person aged from 6 to 15 is 70 bpm to 100 bpm. Decide if each of the following children aged between 6 and 15 years has a healthy resting heart rate.

- **a** Ella 88 bpm
- **b** Caleb 102 bpm
- **c** Whitney 72 bpm
- **d** Mia 120 bpm
- **e** Kevin 92 bpm

3 As a personal trainer, monitoring a client's resting pulse rate over time is important. Luke's resting pulse rates are displayed in the following stem-and-leaf plot.

Find the mean, mode, median and range to one decimal place of Luke's resting heart rates for the month of February.

Resting heart rate in bpm

Stem	Leaf
7	8 9
8	2 4 4 6 7 8 8
9	0 0 0 1 3 4 5 5 8 8 9
10	4 5 6 6 7 9
11	0 3

8|4 means 84

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4 A personal trainer needs to be able to compare resting pulse rates to pulse rates after exercise. Look at the back-to-back stem-and-leaf plot below for Luke, and find the mean, mode and median bpm after exercise and compare to Question 3.

Heart rate in bpm

Leaf (after exercise)	Stem	Leaf (resting)
8 6 4	7	8 9
6 6 5	8	2 4 4 6 7 8 8
9 8 8 8 8 7 6	9	0 0 0 1 3 4 5 5 8 8 9
6 6 5 4	10	4 5 6 6 7 9
8 6 2	11	0 3
7 6 6 5 4	12	
3 2 2	13	

Using technology

5 Wal, a personal fitness trainer, keeps a monthly record of the PBs (personal best times) for his clients. One popular routine is performing the following three exercises:

- 20 step-ups
- 10 burpees
- 10 squats.

Here is a list of PBs, in seconds, for Wal's clients when doing this routine one October.

94 63 103 84 98 89 73 105 85 115
77 96 87 107 64 90 88 102 91 78
99 119 82 76 82 71 83 80 92 117

Enter these 30 times into a graphics calculator and construct a histogram using interval widths of 10, starting with 60 seconds.

Note: Refer to the PDF in the More Resources section of the Interactive Textbook, which has instructions for graphing grouped data using a graphics calculator.

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The missing data

Gregory is a marine biologist and is collecting and analysing data regarding shark and dolphin numbers in a particular area of the Great Barrier Reef.

He collects data for 14 consecutive days, but unfortunately two pieces of data ($\boldsymbol{D}_1$ and $\boldsymbol{D}_2$) are lost due to a computer glitch as shown in the table below.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Sharks	15	12	4	D_1	15	11	19	5	1	10	12	6	24	11
Dolphins	4	7	2	3	2	6	4	2	D_2	4	7	2	4	6

Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate.

Routine problems

a If Gregory assumes that $D_1 = 10$, find the following:

- **i** The mean and median number of sharks recorded, correct to one decimal place where necessary.
- **ii** The range for the number of sharks recorded over the two weeks.

b If Gregory assumes that $D_2 = 5$, find the following:

- **i** The mean and median number of dolphins recorded, correct to one decimal place where necessary.
- **ii** The range for the number of dolphins recorded over the two weeks.

c Which set of data for the two species (sharks or dolphins) appears to be more inconsistent and spread out?

Non-routine problems

Explore and connect

The problem is to determine the possible values of D_1 and D_2 so that Gregory can complete the report based on the statistics he can remember.

a Write down all the relevant information that will help solve this problem, and illustrate using diagrams.

Choose and apply techniques

b Gregory seems to remember that the range for the number of dolphins was 7. Determine the value of D_2 assuming Gregory remembers the range correctly.

c From memory, Gregory thought that the mean number of sharks over the 14-day period was approximately 12. Based on this value, determine the likely value of D_1.

d From memory, Gregory thought that the mean number of dolphins over the 14-day period was approximately 5. Based on this value, determine the likely value of D_2.

e If the median number of sharks was 11, find the possible value(s) of D_1.

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Communicate thinking and reasoning

f If in fact the mean number of dolphins over the 14-day period was 4 after rounding to the nearest integer, find all possible values of D_2.

g If the mean of the first 7 days was the same as the mean of the next 7 days for the dolphin data collected, determine the value of D_2.

h Is it possible for the mean of the first 7 days to be the same as the mean of the next 7 days for the shark data collected? Give reasons.

i Summarise your results, showing the calculations that lead to each possible value of D_1 and D_2 and decide which values you believe are most likely to represent the missing data.

Extension problems

Problem-solve

a Draw a dot plot for each species and decide on a set of likely values for D_1 and D_2.

b Decide if it is possible for the median of either set of data to be anything other than an integer. If so, determine the values of D_1 and D_2 to allow this to happen.

c Estimate the probability that on any given day the shark numbers exceed 12 at the area of the reef Gregory is studying.

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1 One coin says to another coin: *How does my face appear?*
Answer these questions for each scenario, then match the letter in bold to the answers listed below to find the answer to the riddle.

2 coins are tossed 12 times	**1 die is tossed 12 times**	**Choosing one letter from the word PUZZLES**
$P(\text{HH}) =$ **D**	$P(5) =$ **A**	$P(\text{Z}) =$ **M**
$P(\text{HT or TH}) =$ **I**	$P(\text{greater than } 2) =$ **N**	$P(\text{consonant}) =$ **O**
Expected number of TT = **R**	Expected number of 6s = **S**	
Expected number of one or more tails = **T**		

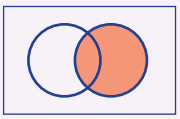
$\square\ \square\ \square$, $\square\ \square\ \square\ \square\ \square\ \square$

$\frac{1}{2}$ 9 2 3 $\frac{1}{6}$ $\frac{2}{3}$ $\frac{1}{4}$ $\frac{5}{7}$ $\frac{2}{7}$

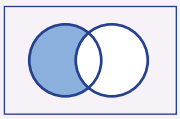
2 Match each description (**1–5**) with the most suitable diagram (**A–E**). Note: There is only one description for each diagram.

1 Don't own an iPhone
2 Own an iPhone only
3 Own both an iPad and an iPhone
4 Own an iPad or an iPhone or both
5 Own an iPad

A



B



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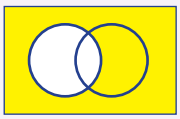
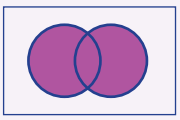
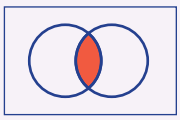


D



E

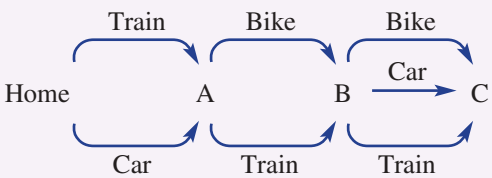


3 In a class of 28, each student owns a cat or a dog or both. If 18 students own cats and 16 students own dogs, how many students own both a cat and a dog?

4 Write down the set of five positive integers that has a mean of 5, a mode of 8 and a range of 6.

5 A 6-sided die and a 10-sided die are tossed simultaneously. What total sums have the highest chance of occurring? (A total sum means the sum of the two uppermost faces.)

6 Michael needs to deliver parcels to three places (A, B and C, in order) in the city. This diagram shows the different ways that he can travel.



a Draw a tree diagram showing all the possible combinations of transportation.
b What is the total number of possible outcomes?
c Find the probability that Michael will use a different transport each time.

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Chapter summary

Probability and data analysis (single variable)

Probability review

The sample space is the list of all possible outcomes of an experiment.

For all possible outcomes:

$$P(\text{event}) = \frac{\text{number of outcomes where event occurs}}{\text{total number of outcomes}}$$

e.g. roll a normal six-sided die

$P(>4) = \frac{2}{6} = \frac{1}{3}$

$P(\text{not } A) = 1 - P(A)$

Two-step experiments

These can be represented by tables or tree diagrams. They involve more than one component and can occur with or without replacement.

e.g. a bag contains 2 blue counters and 1 green, 2 are selected at random.

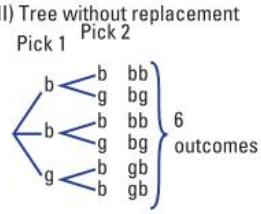
(I) Table with replacement.

Pick 2 \ Pick 1	b	b	g
b	bb	bb	gb
b	bb	bb	gb
g	bg	bg	gg

Sample space of 9 outcomes

$P(2 \text{ of same colour}) = \frac{5}{9}$

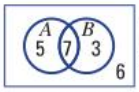
(II) Tree without replacement



$P(2 \text{ of same colour}) = \frac{2}{6} = \frac{1}{3}$

Venn diagrams and two-way tables

These organise data from two or more categories



Venn diagram

Two-way table

	A	Not A	Totals
B	7	3	10
Not B	5	6	11
Totals	12	9	21

i.e. 6 in neither category, 7 in both categories

$P(A) = \frac{12}{21} = \frac{4}{7}$ $P(A \text{ and } B) = \frac{7}{21} = \frac{1}{3}$

$P(B \text{ only}) = \frac{3}{21} = \frac{1}{7}$ $P(A \text{ or } B \text{ or both}) = \frac{15}{21} = \frac{5}{7}$

Stem-and-leaf plots

These display all the data values using a stem and a leaf. An ordered back-to-back stem-and-leaf plot.

Leaf	Stem	Leaf	
9 8 7 2	1	0 3	Symmetrical
7 4 3 3	2	2 2 4	
5 2 1	3	3 6 7 8	
7	4	4 5 9	
0	5	0	

Skewed

3 | 5 means 35 ← Key

Observed probability (Experimental probability)

This is calculated from results of experiment or survey.

$$\text{Observed probability} = \frac{\text{number of times event occurs}}{\text{total number of trials}}$$

$$\text{Expected number of occurrences} = \text{probability} \times \text{number of trials}$$

Measure of spread

Range = maximum value − minimum value

Summarising data: Measures of centre

Mode is the most common value (there can be more than one). Two modes means data is bimodal.

Mean is the average of the data values.

$$\text{Mean } (\bar{x}) = \frac{\text{sum of data values}}{\text{number of values}}$$

Median is the middle value of data that is ordered.

Odd dataset: 2 4 (7) 10 12 — Median

Even dataset: 2 4 6 10 15 18 — Median $= \frac{6+10}{2} = 8$

An outlier is a value that is not near the rest of the data.

Grouped data

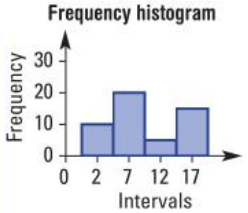
Data values can be grouped into class intervals, e.g. 0–4, 5–9, etc. and recorded in a frequency table.

The frequency or percentage frequency of each interval can be recorded in a histogram.

Frequency table

Class interval	Class centre	Frequency	Percentage frequency
0–4	2	10	20
5–9	7	20	40
10–14	12	5	10
15–19	17	15	30
Total		50	100

Frequency histogram



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Chapter checklist with success criteria

A printable version of this checklist can be downloaded from the Interactive Textbook.

		✔
10A	**1 I can calculate a probability from experiments with equally likely outcomes.** e.g. A letter is chosen from the word COMPLEMENT. Find the probability that the letter is: **a** an E **b** not a vowel **c** an E or a consonant	☐
10B	**2 I can use a Venn diagram to calculate probabilities.** e.g. From a group of 30 tennis players, 16 do a slice backhand, 18 do a top-spin backhand and 6 do both types of backhand. Illustrate this information in a Venn diagram and **a** state the number of tennis players who do a slice backhand only **b** find the probability that a randomly selected tennis player does a slice or a top-spin backhand.	☐
10B	**3 I can use a two-way table to calculate probabilities.** e.g. From a group of 20 single car owners, 15 have a car that runs on petrol, 7 have a car that runs on electricity and 4 people have a car that runs on both types of fuel. Illustrate this information using a two-way table and **a** find the number of people who have a car that runs on neither type of fuel **b** find the probability that a randomly selected car owner has a car that runs on petrol only.	☐
10C	**4 I can construct an array (table) for a two-step experiment with replacement.** e.g. A fair four-sided die is rolled twice. List all the outcomes (the sum total from the two dice) using a table and find: **a** P(a double) **b** P(sum of at least 5) **c** P(sum not equal to 7).	☐
10C	**5 I can construct an array (table) for a two-step experiment without replacement.** e.g. Two letters are chosen from the word KEY, without replacement. Construct a table to list the sample space and find the probability of: **a** obtaining the outcome (K, E) **b** selecting an E or a Y in any order.	☐
10D	**6 I can construct and use a tree diagram to find probabilities for multistage experiments with replacement.** e.g. Two marbles are selected from a container including 2 clear and 2 coloured marbles. Two marbles are randomly chosen with replacement. Represent the selections using a tree diagram that shows all possible outcomes. Find the probability of selecting: **a** one clear marble **b** at least one coloured marble.	☐
10D	**7 I can construct and use a tree diagram to find probabilities for multistage experiments without replacement.** e.g. A box contains 2 dark and 3 white chocolates. Draw a tree diagram to show the outcomes and probabilities of the selection of two chocolates without replacement. Find the probability of selecting: **a** one white **b** at least one dark chocolate.	☐
10E	**8 I can find an observed probability and expected number of occurrences.** e.g. 40 people were surveyed regarding their main mode of transport to work. 25 said *Public transport*, 10 said *Drive* and 5 said *Walk*. Find: **a** the observed probability that a person drives to work **b** the expected number of people who use public transport from a group of 1000.	☐

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Chapter checklist

Chapter checklist with success criteria

10F

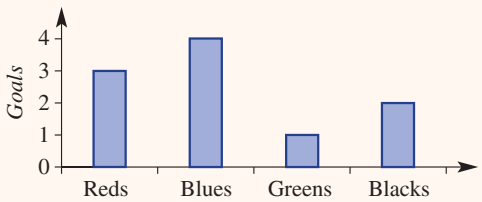
9 I can find the mean, median, mode and range of a simple dataset.
e.g. For the datasets: 3, 4, 5, 2, 3, 3, 1 and 12, 21, 21, 25, 18, 14 find:
a the mean **b** median **c** mode **d** range

10F

10 I can solve problems associated with data means.
e.g. Hilbert received 78, 59, 66 and 69 on his previous four maths tests. What is his current mean and what does he need to score in his next test to raise his average to 70?

10G

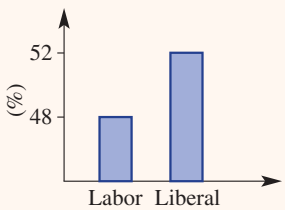
11 I can interpret a common statistical graph.
e.g. This column graph shows how many goals four teams scored in a round of soccer.
a How many goals did the Blues score?
b What was the mean and median score for the four teams?
c What combined percentage of the goals were scored by the Reds and the Blues?



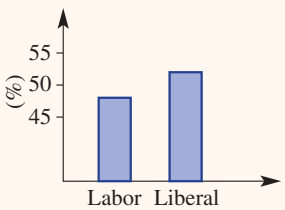
10G

12 I can decide if a graph may be misleading.
e.g. The following two graphs shows the two-party preferred position of the Labor and Liberal parties after an election. Which one is misleading about the margin of difference and why?

a



b



10H

13 I can construct a stem-and-leaf plot from a dataset and calculate summary statistics.
e.g. For this dataset construct a stem-and-leaf plot and find the mode and median.
26 19 32 42 49 16 35 29 22 25 26 37 15 28

10H

14 I can construct a back-to-back stem-and-leaf plot and compare the two datasets.
e.g. The total games scores of two tennis teams for a season of 9 matches is recorded as follows.
Team A
31 45 69 40 76 55 49 12 28
Team B
23 34 16 29 27 12 25 29 31
Construct an ordered back-to-back stem-and-leaf plot, and compare and comment on the differences between the performance of the two teams.

10I

15 I can construct and analyse a frequency table and a histogram.
e.g. The age of 20 tagged sea turtles is recorded as follows.
45 63 79 66 38 97 82 44 59 31
58 62 69 53 85 93 85 77 62 41
Organise the data into a frequency table, using class intervals of 10, and include a percentage frequency column. Construct a frequency histogram and a percentage frequency histogram and determine the percentage of turtles older than 50 years.

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Short-answer questions

10A **1** Determine the probability of each of the following.

- **a** Rolling more than 2 on a normal six-sided die
- **b** Selecting a vowel from the word EDUCATION
- **c** Selecting a pink or white jelly bean from a packet containing 4 pink, 2 white and 4 black jelly beans

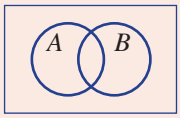
10B **2** From a survey of 50 people, 30 have the newspaper delivered, 25 read it online, 10 do both and 5 do neither.

- **a** Construct a Venn diagram for the survey results.
- **b** How many people only read the newspaper online?
- **c** If one of the 50 people were randomly selected, find:
  - **i** P(have paper delivered and read it online)
  - **ii** P(don't have it delivered)
  - **iii** P(only read it online)

10B **3** **a** Copy and complete this two-way table.

	A	Not A	Total
B		16	
Not B	8		20
Total	17		

- **b** Convert the information into a Venn diagram, as shown.



- **c** Find the following.
  - **i** P(not B)
  - **ii** P(both A and B)
  - **iii** number of 'A only'
  - **iv** number in either A or B or both A and B

10C **4** A spinner with equal areas of red, green and blue is spun and a four-sided die numbered 1 to 4 is rolled.

		Die			
		1	2	3	4
Spinner	Red	(Red, 1)	(Red, 2)		
	Green				
	Blue				

- **a** Complete a table like the one shown and state the number of outcomes in the sample space.
- **b** Find the probability that:
  - **i** the outcome is red and an even number
  - **ii** the outcome is blue or green and a 4
  - **iii** the outcome does not involve blue

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Chapter review

10D **5** Libby randomly selects two coins from her pocket *without replacement*. Her pocket contains a $1 coin and two 10-cent coins.

a List all the possible combinations using a tree diagram.

b If a chocolate bar costs $1.10, find the probability that she can hand over the two coins to pay for it.

10E **6** A quality controller records the frequency of the types of chocolates from a sample of 120 off its production line.

Centre	Soft	Hard	Nut
Frequency	50	22	48

a What is the observed probability of randomly selecting a nut centre?

b In a box of 24 chocolates, how many would be expected to have a non-soft centre?

10F **7** Claudia records the number of emails she receives each weekday for two weeks as follows.

30 31 33 23 29 31 21 15 24 23

Find:

a the mean

b the median

c the mode

d the range

10H **8** Two mobile phone salespeople are both aiming for a promotion to be the new assistant store manager. The best salesperson over a 15-week period will achieve the promotion. The number of mobile phones they sold each week is recorded below.

Employee 1: 21 34 40 38 46 36 23 51 35 25 39 19 35 53 45
Employee 2: 37 32 29 41 24 17 28 20 37 48 42 38 17 40 45

a Draw an ordered back-to-back stem-and-leaf plot for the data.

b For each employee, find:

i the median number of sales

ii the mean number of sales

c By comparing the two sets of data, state, with reasons, who you think should get the promotion.

d Describe each employee's data as approximately symmetrical or skewed.

10I **9** The data below represents the finish times, in minutes, of 25 competitors in a local car rally race.

134 147 162 164 145 159 151 143 136 155 163 157 168
171 152 128 144 161 158 136 178 152 167 154 161

a Record the above data in a frequency table in class intervals of 120–, 130– etc. Include a percentage frequency column.

b Construct a frequency histogram.

c Determine:

i the number of competitors that finished in less than 140 minutes

ii the percentage of competitors that finished between 130 and 160 minutes

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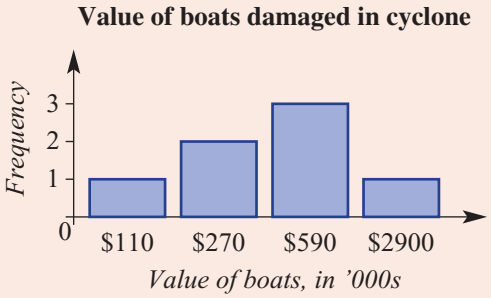
10G

10 This column graph shows the value of some of the boats that were badly damaged in cyclone Yasi in North Queensland in 2011.

a Find the mean and median boat value.

b Do you think the mean or the median would better represent the 'average' of these boat values? Give a reason for your answer.



Multiple-choice questions

10A

1 A letter is randomly chosen from the word XYLOPHONE. The probability that it is an O is:

A $\frac{1}{8}$ **B** $\frac{2}{9}$ **C** $\frac{1}{4}$ **D** $\frac{1}{9}$ **E** $\frac{1}{3}$

10B

2 The values of x and y in the two-way table are:

A $x = 12, y = 8$ **B** $x = 12, y = 11$
C $x = 16, y = 4$ **D** $x = 10, y = 1$
E $x = 14, y = 6$

	A	Not A	Total
B		5	9
Not B	8	y	
Total	x		25

10B

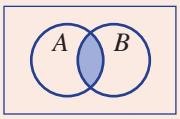
3 **i** Which shaded region represents both A and B?

ii Which shaded region represents A only?

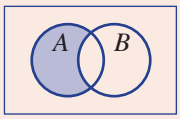
iii Which shaded region represents A or B or both?

iv Which shaded region represents the complement of A?

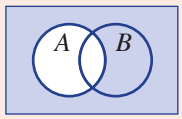
A



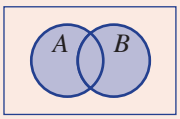
B



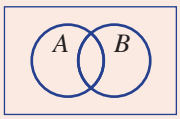
C



D



E



10D

4 A bag contains 2 green balls and 1 red ball. Two balls are randomly selected *without replacement*. Using a tree diagram, the probability of selecting one of each colour is:

A $\frac{1}{2}$ **B** $\frac{2}{3}$ **C** $\frac{5}{6}$ **D** $\frac{1}{3}$ **E** $\frac{3}{4}$

10E

5 From rolling a biased die, a class finds an observed probability of 0.3 of rolling a 5. From 500 rolls of the die, the expected number of 5s would be:

A 300 **B** 167 **C** 180 **D** 150 **E** 210

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Chapter review

10H

6 **i** The median of the data in this stem-and-leaf plot is:

A 74 **B** 71 **C** 86
D 65 **E** 70

Stem	Leaf
5	3 5 8
6	1 4 7
7	0 2 4 7 9
8	2 6 6

7|4 means 74

ii The range of the data in the stem-and-leaf plot is:

A 3 **B** 8 **C** 33
D 86 **E** 14

10F

7 If Jacob achieved scores of 12, 9, 7 and 12 on his last four language vocabulary tests, what score must he get on the fifth test to have a mean of 11?

A 16 **B** 14 **C** 11 **D** 13 **E** 15

Extended-response questions

1 The local Sunday market has a number of fundraising activities.

a For $1 you can spin a spinner numbered 1–5 twice. If you spin two even numbers you receive $2 (your dollar back plus an extra dollar), if you spin two odd numbers you receive your dollar back and for any other result you lose your dollar.

i Complete the table shown to list the sample space.

		First spin				
		1	2	3	4	5
Second spin	1	(1, 1)	(2, 1)			
	2					
	3					
	4					
	5					

ii What is the probability of losing your dollar (i.e. spinning one odd and one even number)?

iii What is the probability of making a dollar profit (i.e. spinning two even numbers)?

iv In 50 attempts, how many times would you expect to lose your dollar?

v If you start with $100 and have 100 attempts, how much money would you expect to end up with?

b Forty-five people were surveyed as they walked through the market as to whether they bought a sausage and/or a drink from the sausage sizzle. Twenty-five people bought a sausage and 30 people bought a drink, with 15 buying both.

i Construct a Venn diagram to represent this information.

ii How many people bought neither a drink nor a sausage?

iii How many people bought a sausage only?

iv If a person was randomly selected from the 45, what is the probability they bought a drink but not a sausage?

v Find P(didn't buy a sausage).

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2 The delay time (in minutes) of the flight departure of the same evening flight of two rival airlines was recorded over 30 consecutive days. The data is shown below.

Airline A	2	11	6	14	18	1	7	4	12	14	9	2	13	4	19
	13	17	3	52	24	19	12	14	0	7	13	18	1	23	8
Airline B	6	12	9	22	2	15	10	5	10	19	5	12	7	11	18
	21	15	10	4	10	7	18	1	18	8	25	4	22	19	26

a Copy and complete this ordered back-to-back stem-and-leaf plot for the data. Use 2 lines for each stem, starting with '0' for leaves 0–4, and then '0*' for leaves 5–9 etc.

b Does the data for airline A appear to have any outliers (numbers not near the majority of data elements)?

c By removing any outliers listed in part **b**, find the following for each airline, rounding to one decimal place where necessary.

i the median

ii the mean

d Airline A reports that half its flights for that month had a delay time of less than 10 minutes. Is this claim correct? Explain.

Airline A		Airline B
	0	
	0*	
	1	
	1*	
	2	
	2*	
	3	
	3*	
	4	
	4*	
	5	

1|2 means 12

1*|5 means 15

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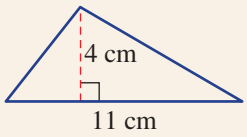
Chapter 6: Length, area, surface area and volume

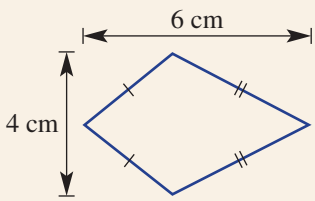
Short-answer questions

1 Find the area of these shapes.

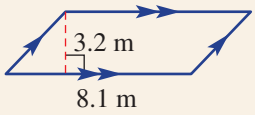
a



b



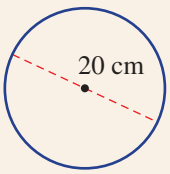
c



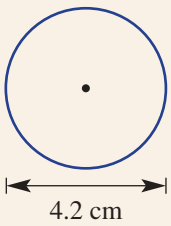
2 Find the circumference and area of these circles, correct to two decimal places.

a

b

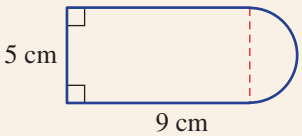


c

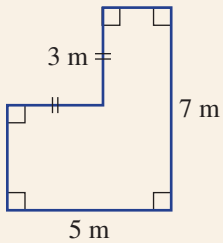


3 Find the area of each of the figures below. Round to two decimal places where necessary.

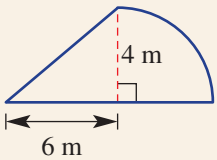
a



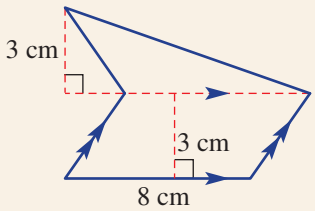
b



c

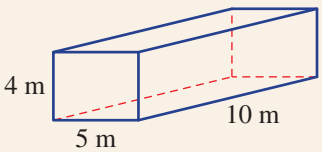


d

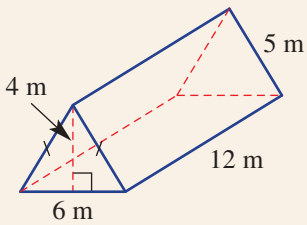


4 Find the surface area of these solid objects. Round to two decimal places where necessary.

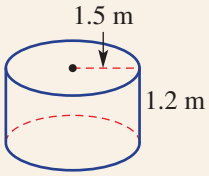
a



b



c



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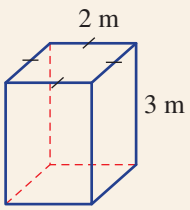
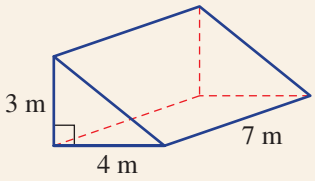
5 Find the volume of these solids. Round to two decimal places for part **c**.

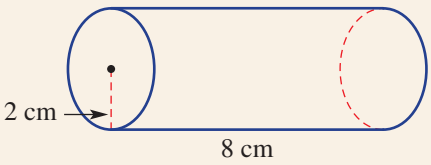
a



b

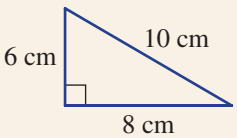


c



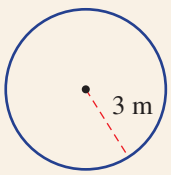
Multiple-choice questions

1 The perimeter and area of this triangle, respectively, are:



A 48 cm, 24 m^2 **B** 68 cm, 80 cm^2
C 24 cm, 24 cm^2 **D** 24 cm, 12 cm^3
E 24 cm, 48 cm^2

2 The circumference and area for this circle, correct to two decimal places, respectively, are:

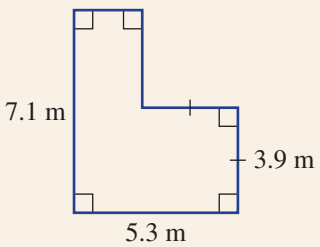


A 37.70 m, 18.85 m^2 **B** 9.42 m, 56.55 m^2
C 18.85 m, 28.27 m^2 **D** 18.85 m, 18.85 m^2
E 9.42 m, 28.27 m^2

3 420 cm^2 is equal to:

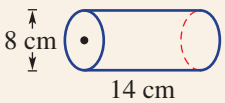
A 4.2 m^2 **B** 0.42 m^2 **C** 42 000 m^2 **D** 0.042 m^2 **E** 0.0042 m^2

4 The perimeter and area of the figure shown, respectively, are:



A 20.2 m, 22.42 m^2 **B** 24.8 m, 22.42 m^2
C 24.8 m, 25.15 m^2 **D** 20.2 m, 25.15 m^2
E 21.6 m, 24.63 m^2

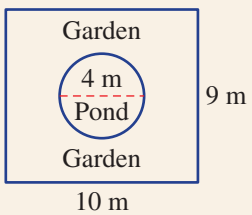
5 The volume of the cylinder shown is closest to:



A 703.7 cm^3 **B** 351.9 cm^3
C 2814.9 cm^3 **D** 452.4 cm^3
E 1105.8 cm^3

Extended-response question

A pond is to be built inside a rectangular garden, as shown.



a Find the area of the pond, correct to two decimal places.

b What is the area of the garden, not including the pond?

c The pond is to be surrounded by a low wall that costs $50 per metre. What will it cost to build this wall? Round to the nearest dollar.

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Chapter 7: Indices

Short-answer questions

1 Use index laws to simplify the following.

a $5p^2q \times 3pq$

b $\dfrac{9a^6b^3}{18a^4b^2}$

c $(-3x^4y^2)^2 \times 6xy^2$

d $-3x^0 + (5x)^0$

2 Write each of the following using positive indices and then write as a basic numeral.

a $\dfrac{3}{4^{-2}}$ **b** 4×10^{-3}

3 Write these numbers as basic numerals.

a 2.4×10^3 **b** 1.08×10^6 **c** 7.1×10^{-3} **d** 2.06×10^{-5}

4 Write these numbers using scientific notation.

a 60 300 **b** 2 700 000 **c** 0.004 **d** 0.000703

5 Convert these numbers to the units given in brackets. Write your answer using scientific notation using three significant figures.

a 30.71 g (kg)

b 4236 tonnes (kg)

c 3.4 hours (seconds)

d 235 seconds (years)

Multiple-choice questions

1 $3a^2b^3 \times 4ab^2$ is equivalent to:

A $12a^2b^6$ **B** $7a^3b^5$ **C** $12a^3b^5$ **D** $12a^4b^5$ **E** $7a^2b^6$

2 $\left(\dfrac{2x}{5}\right)^3$ is equivalent to:

A $\dfrac{6x^3}{5}$ **B** $\dfrac{8x^3}{125}$ **C** $\dfrac{2x^3}{5}$ **D** $\dfrac{2x^4}{15}$ **E** $\dfrac{2x^3}{125}$

3 4^{-2} can be expressed as:

A $\dfrac{1}{4^{-2}}$ **B** $\dfrac{1}{8}$ **C** -16 **D** $\dfrac{1}{16}$ **E** -8

4 3×10^{-4} written with positive indices is:

A -3×10^4 **B** $\dfrac{1}{3 \times 10^4}$ **C** $\dfrac{-3}{10^4}$ **D** $\dfrac{1}{3 \times 10^{-4}}$ **E** $\dfrac{3}{10^4}$

5 0.00371 in scientific notation is:

A 0.371×10^{-3} **B** 3.7×10^{-2} **C** 3.71×10^{-3} **D** 3.71×10^3 **E** 371×10^3

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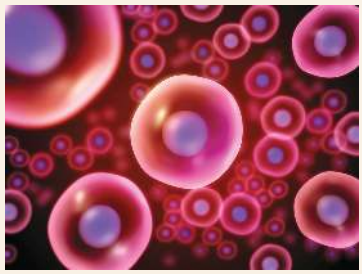
Extended-response question

The average human body contains about 74 billion cells.
(Note:1 billion = 1 thousand million)



a Write this number of cells:
  i as a basic numeral
  ii using scientific notation

b If the population of a particular city is 2.521×10^6, how many human cells are there in the city?
Give your answer using scientific notation correct to three significant figures.

c If the average human weighs 64.5 kilograms, what is the average mass of one cell in grams?
Give your answer using scientific notation correct to three significant figures.

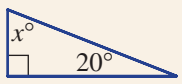
Chapter 8: Properties of geometrical figures

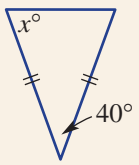
Short-answer questions

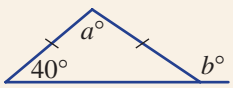
Ext 1 State the angle sums for these shapes.

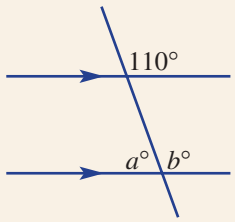
a Triangle b Quadrilateral c Pentagon
d Heptagon e Octagon f Decagon

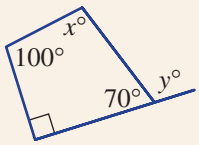
2 Find the value of each pronumeral in the following.

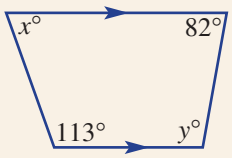
a 

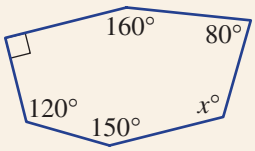
b 

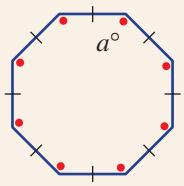
c 

d 

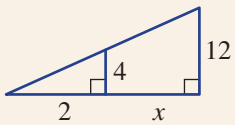
e 

f 

Ext g 

h 

3 Find the value of x in these similar triangles.



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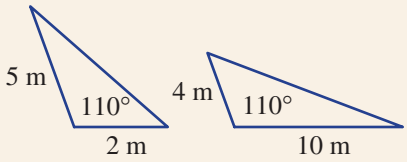
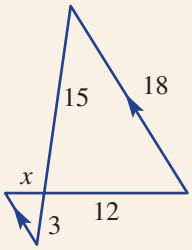
4 For the given pair of similar triangles (not to scale), find the scale factor.



5 For this pair of similar triangles find the value of x.



Multiple-choice questions

1 The supplement of 55° is:

A 55° **B** 35° **C** 125° **D** 135° **E** 70°

2 A quadrilateral with all four sides equal and opposite sides parallel is best described as a:

A parallelogram
B rhombus
C rectangle
D trapezium
E kite

Ext **3** The size of the interior angle in a regular pentagon is:

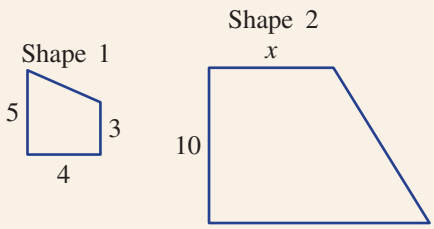
A 108° **B** 120° **C** 96° **D** 28° **E** 115°

4 In which shape might you find opposite sides which are *not* equal:

A trapezium **B** rectangle **C** square **D** rhombus **E** parallelogram

5 What is the scale factor that enlarges shape 1 to shape 2 in these similar figures, and what is the value of x?

A 2 and $x = 8$
B 2.5 and $x = 7.5$
C 3.33 and $x = 13.33$
D 2.5 and $x = 12.5$
E 2 and $x = 6$



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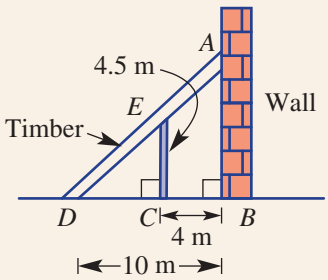
Extended-response question

A vertical wall is being supported by a piece of timber that touches the ground 10 metres from the base of the wall. A vertical metal support 4.5 m high is placed under the timber support 4 m from the wall to form a pair of similar triangles.



a What is the length DC?

b What is the scale factor for the two triangles formed by the timber and support?

c Find how far the timber reaches up the wall.

d How far above the ground is the point halfway along the timber support?

Chapter 9: Quadratic expressions and algebraic fractions

Short-answer questions

1 Expand these expressions.

a $-3(x+2)$ b $x(1-x)$ c $(x+1)(x-1)$

d $(x-4)^2$ e $(5x-2)(5x+2)$ f $(3x-4)^2$

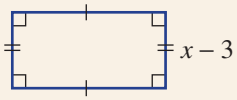
g $(4x-1)(2x+3)$ h $5x-3(x-1)$ i $(2-x)^2$

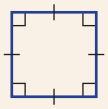
2 Simplify these expressions.

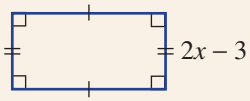
a $-3x \times 2yx$ b $7a \times 6ab^2$ c $4ab \div (8b)$

d $-6x^2y \div (xy^2)$ e $a^2b \div (ab^2)$ f $2xy \div (4xy^2)$

3 Find the area of the following shapes in expanded form.

a 

b 

c 

4 Factorise the following.

a $3x-12$ b $-7x-14$ c $5x^2+2x$ d $14x^2-21x$

5 Simplify these expressions.

a $\frac{5x^2}{3} \times \frac{9}{10x}$ b $\frac{x-1}{2} \times \frac{3}{2(x-1)}$ c $\frac{4x-8}{x-2}$

d $\frac{-6x^2-x}{6x+1}$ e $\frac{4x}{3} \div \frac{8x}{9}$ f $\frac{3(x-2)}{5a} \div \frac{x(x-2)}{15a^2}$

6 Simplify these algebraic fractions.

a $\frac{x}{3}-\frac{x}{4}$ b $\frac{5x}{4}+\frac{7x}{5}$ c $\frac{x+1}{2}+\frac{x}{5}$ d $\frac{2x-1}{5}+\frac{x+2}{3}$

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Multiple-choice questions

1 $-3(x-4)$ is equal to:

A $-3x+7$ **B** $-3x+4$ **C** $-3x-4$ **D** $-3x-12$ **E** $-3x+12$

2 The expanded form of $(x+4)(3x-2)$ is:

A $3x^2+12x-8$ **B** $4x^2+14x-8$ **C** $3x^2+12x-10$
D $3x^2-8$ **E** $3x^2+10x-8$

3 $(2x-1)^2$ expands to:

A $4x^2-4x+1$ **B** $2x^2-2x-1$ **C** $2x^2-4x+1$ **D** $4x^2-2x+1$ **E** $4x^2-1$

4 $\dfrac{2x-6}{2}$ simplifies to:

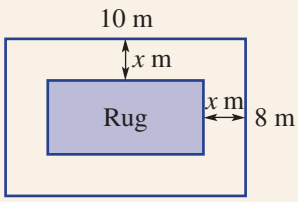
A $2x-3$ **B** $x-3$ **C** $2x-4$ **D** $x-6$ **E** $x-4$

5 $\dfrac{x+1}{3}+\dfrac{x}{4}$ simplifies to:

A $\dfrac{7x+1}{12}$ **B** $\dfrac{7x+4}{7}$ **C** $\dfrac{5x+4}{7}$ **D** $\dfrac{7x+4}{12}$ **E** $\dfrac{5x-3}{8}$

Extended-response question

A room that is 10 metres long and 8 metres wide has a rectangular rug in the middle of it that leaves a border, x metres wide, all the way around it as shown.



a Write expressions for the length and the breadth of the rug.

b Write an expression for the area of the rug in expanded form.

c What is the area of the rug when $x = 1$?

Chapter 10: Probability and data analysis

Short-answer questions

Ext

1 In a survey of 30 people, 18 people drink coffee during the day, 14 people drink tea and 8 people drink both. Let C be the set of people who drink coffee and T the set of people who drink tea.

a Construct a Venn diagram for the survey results.

b Find:

i the number who drink either coffee or tea or both

ii the number who do not drink tea

c If one of the 30 people was randomly selected, find:

i P(drinks neither coffee nor tea) **ii** $P(C$ only)

2 A coin and a four-sided die are tossed.

a Draw a table to show the sample space.

b How many outcomes are in the sample space?

c What is the probability of tossing T4?

d What is the probability of tossing tails and an odd number?

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3 A spinner has equal areas for the colours blue, orange and yellow. It is spun twice.

- **a** Draw a tree diagram and list the outcomes.
- **b** Find the probability of orange then blue.
- **c** Find the probability that both outcomes are the same colour.
- **d** Find the probability that blue and yellow show in any order.

4 Two ice creams are randomly selected without replacement from a box containing one vanilla (V), two strawberry (S) and one chocolate (C) flavoured ice creams.

- **a** Draw a tree diagram to show each of the possible outcomes.
- **b** What is the probability of selecting:
  - **i** a vanilla and a strawberry flavoured ice cream?
  - **ii** two strawberry flavoured ice creams?
  - **iii** no vanilla-flavoured ice creams?

5 The data below shows the number of aces served by a player in each of their grand slam tennis matches for the year.

15 22 11 17 25 25 12 31 26 18 32 11 25 32 13 10

- **a** Construct a stem-and-leaf plot for the data.
- **b** From the stem-and-leaf plot, find the mode and median number of aces.
- **c** Is the data symmetrical or skewed?

6 The frequency table shows the number of visitors, in intervals of 50, to a theme park each day in April.

Class interval	Class centre	Frequency	Percentage frequency (%)
0–49	24.5	2	
50–99	74.5	4	
100–149	124.5	5	
150–199	174.5	9	
200–249	224.5		
250–299	274.5	3	
	Total	30	

- **a** Copy and complete the frequency table shown. Round to one decimal place where necessary.
- **b** Construct a frequency histogram.
- **c** **i** On how many days were there fewer than 100 visitors?
  - **ii** What percentage of days had between 50 and 199 visitors?

Multiple-choice questions

1 The probability of rolling a number less than five on a normal six-sided die is:

A $\frac{1}{3}$ **B** 4 **C** $\frac{1}{2}$ **D** $\frac{2}{3}$ **E** 3

2 From the two-way table, P(both A and B) is:

	A	Not A	Total
B		7	
Not B	5		
Total		11	20

A $\frac{1}{5}$ **B** 4 **C** $\frac{9}{20}$ **D** $\frac{1}{4}$ **E** 16

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3 In a bag of 40 marbles, 28 are blue.

a The probability of selecting a blue marble is:

A 0.28 **B** 0.4 **C** 0.7 **D** 0.54 **E** 0.75

b If a marble is selected 50 times *with replacement*, the number of blue marbles you would expect to select is:

A 28 **B** 38 **C** 70 **D** 35 **E** 40

4 The median, mean and range of the dataset

12 3 1 6 10 1 5 18 11 15

are respectively:

A 5.5, 8.2, 1 – 18 **B** 8, 8.2, 17 **C** 5.5, 8, 17

D 8, 8.2, 1 – 18 **E** 8, 74.5, 18

5 The median of this stem-and-leaf plot is:

A 55 **B** 67 **C** 7 **D** 69 **E** 4

Stem	Leaf
5	1 3 5 5 5 5
6	2 4 5 6 7 7 9
7	1 3 4 8 9
8	3 4
9	1 2 6

5|1 means 51

Extended-response question

A game at the school fair involves randomly selecting a green ball and a red ball, each numbered 1, 2 or 3.

a List the outcomes in a table.

b What is the probability of getting an odd and an even number?

c Participants win $1 when they draw each ball showing the same number.

i What is the probability of winning $1?

ii If someone wins six times, how many games are they likely to have played?

d The ages of those playing the game in the first hour are recorded and are shown below.

12 16 7 24 28 9 11 17 18 18 37 9 40 16 32 42 14

i Approximately 50% of the participants are below what age?

ii If this data is used as a model for the 120 participants throughout the day, how many would be expected to be aged less than 30?

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Chapter 1

Warm-up quiz

1 Addition: **a** Sum, **b** Total, **i** Add, **l** More than, **p** Increase
Subtraction: **c** Less than, **g** Take away, **h** Difference, **k** Minus, **n** Less
Multiplication: **d** Lots of, **e** Product, **j** Times
Division: **f** Into, **m** Quotient, **o** Share

2 **a** 4 **b** 32 **c** 4 **d** 52
e 74 **f** 5 **g** 9 **h** 40

3 **a** 56 **b** 100 **c** 0 **d** 133
e 63 **f** 80 **g** 1020 **h** 53

4 **a** −12 **b** −3 **c** 3 **d** 0
e −1 **f** −1 **g** 0 **h** −12

5 **a** 3 **b** 3 **c** − **d** +

6 **a** 26 073 260 **b** 26 073 300 **c** 26 073 000
d 26 000 000

7 **a** 50 **b** 5 **c** 5000
d $\frac{5}{10}$ **e** $\frac{5}{100}$ **f** $\frac{5}{1000}$

8 **a** 2.654, 2.645, 2.564, 2.465
b 0.654, 0.564, 0.456, 0.0456

9 **a** 7.99 **b** 10.11 **c** 7.11

10 **a** 1.4 **b** 0.06 **c** 3.68 **d** 16.38
e 3.7 **f** 180

11 **a** 34.5 **b** 374 000 **c** 0.03754
d 0.000037754

12 **a** 6 **b** 12 **c** 30 **d** 1

13 **a** 15 **b** 12 **c** 10

14 **a** $\frac{5}{7}$ **b** $1\frac{1}{4}$ **c** 8 **d** $\frac{3}{8}$

1A

Now you try

Example 1
a −3 **b** −10

Example 2
4

Example 3
−8

Exercise 1A

1 **a** iii **b** v **c** iv **d** i **e** ii

2 **a** iv **b** ii **c** v **d** i **e** iii

3 **a** T **b** T **c** F **d** T
e T **f** F

4 **a** −2 **b** 2 **c** −3 **d** 4
e −4 **f** −5 **g** −2 **h** 0
i −6

5 **a** 3 **b** 2 **c** 1 **d** 0
e −1 **f** −9 **g** −1 **h** −17
i −10 **j** 9 **k** −2 **l** 11
m −7 **n** −18 **o** −1

6 **a** 5 **b** 20 **c** −20 **d** 0
e −7 **f** 6 **g** −24 **h** 5
i −15

7 **a** 4 **b** −4 **c** −4 **d** −9
e −32 **f** −38 **g** −189 **h** −24
i −18 **j** 0 **k** −805 **l** −57
m −167 **n** 142 **o** −804

8 **a** −2 **b** 4 **c** 13 **d** 9
e 24 **f** 0 **g** −28 **h** −81
i 180 **j** 2 **k** −3 **l** 240
m 755 **n** 94 **o** 10

9 **a** −27 **b** −3 **c** 10 **d** 26
e 0 **f** −24 **g** −91 **h** 125

10 **a** 2 **b** 11 **c** −3 **d** −1
e −7 **f** 11

11 **a** −8 **b** 4 **c** 13 **d** 124
e −19 **f** 0 **g** −22 **h** 98
i −26

12 −12°C

13 **a**

−8	6	−4
2	−2	−6
0	−10	4

b Example answer (other answers are possible):

2	−13	8
5	−1	−7
−10	11	−4

c

5	−2	7	3
0	10	−3	6
−1	4	2	8
9	1	7	−4

1B

Now you try

Example 4
a 32 **b** −35

Example 5
a 64 **b** −27

Example 6
a −5 **b** 4

Example 7
12

Exercise 1B

1 **a** Positive **b** Negative **c** Positive **d** Negative
e Negative

2 **a** Negative **b** Negative **c** Negative **d** Positive
e Positive **f** Positive

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3

×	−4	−3	−2	−1	0	1	2	3	4
−4	16	12	8	4	0	−4	−8	−12	−16
−3	12	9	6	3	0	−3	−6	−9	−12
−2	8	6	4	2	0	−2	−4	−6	−8
−1	4	3	2	1	0	−1	−2	−3	−4
0	0	0	0	0	0	0	0	0	0
1	−4	−3	−2	−1	0	1	2	3	4
2	−8	−6	−4	−2	0	2	4	6	8
3	−12	−9	−6	−3	0	3	6	9	12
4	−16	−12	−8	−4	0	4	8	12	16

4 **a** −48 **b** −20 **c** −72 **d** 72
e 45 **f** −90 **g** −600 **h** 81
i −144 **j** −143 **k** −143 **l** −34
m 72 **n** 90 **o** −108

5 **a** 36 **b** 64 **c** 144 **d** 9
e 100 **f** 169 **g** −8 **h** −1
i −64

6 **a** −9 **b** 6 **c** 10 **d** −10
e 3 **f** 5 **g** 2 **h** −1
i −8 **j** 25 **k** −104 **l** 5
m −37 **n** −214 **o** 13

7 **a** 16 **b** −26 **c** −12 **d** −14
e −20 **f** −28 **g** −72 **h** −8
i −90 **j** 10 **k** −24 **l** −18
m 9 **n** −26

8 **a** 7 **b** −32 **c** −20 **d** −4
e −1 **f** 6 **g** −9 **h** −5
i −15

9 −5 and 2

10 **a** 12 and −12
b 8 and −8
c 100 and −100

11 Two negatives with × and ÷ produce a positive.

12 **a** Positive **b** Positive **c** Negative **d** Positive
e Negative **f** Positive

13 **a** −2 **b** −38 **c** −8 **d** 27
e 1 **f** 24 **g** 21 **h** 0

14 **a** −60 **b** −30 **c** 675 **d** 0
e 2250 **f** 1800 **g** 3600 **h** 675

1C

Now you try

Example 8
a 107.39 **b** 0.03 **c** 2.80

Example 9
a 3500 **b** 1800 **c** 0.0023

Example 10
10, reasonable as real answer is 9.5

Exercise 1C

1 **a** 3 **b** 1 **c** 2 **d** 0
2 **a** 2 **b** 4 **c** 1 **d** 4
3 **a** Down **b** Up **c** Up
4 **a** 32 100 **b** 432 **c** 5.89 **d** 0.443
e 0.00197

5 **a** 17.96 **b** 11.08 **c** 72.99 **d** 47.86
e 63.93 **f** 23.81 **g** 804.53 **h** 500.57
i 821.27 **j** 5810.25 **k** 1005.00 **l** 2650.00

6 **a** 2400 **b** 35 000 **c** 0.060 **d** 34
e 110 000 **f** 0.0025 **g** 2.1 **h** 0.71
i 4700 **j** 59 000 **k** 0.46 **l** 1.1

7 **a** 1.46, 1.5 **b** 0.09, 0.094 **c** 23.71, 24
d 0.01, 0.0078 **e** 100.47, 100

8 **a** 7 **b** 73 **c** 130 **d** 36 200

9 **a** 30 000 **b** 200 **c** 0.05 **d** 0.0006
e 5000 **f** 900 **g** 0.9 **h** 0.0003
i 1

10 **a** 3600, 3693
b 760, 759.4
c 4000, 4127.16
d 3000, 3523.78
e 0, 0.72216
f 4, 0.716245
g 0.12, 0.1186
h 0.02, 0.02254
i 0.2, 0.2

11 **a** A: 54.3, B: 53.8; 0.5 sec
b A: 54.28, B: 53.79; 0.49 sec **c** A: 54, B: 54; 0 sec
d A: 50, B: 50; 0 sec

12 0.143 tonnes

13 As magnesium in this case would be zero if rounded to two decimal places rather than two significant figures

14 2.14999 is closer to 2.1 correct to one decimal place,
∴ round down

15 **a** **i** 8 **ii** 1 **iii** 8
b 0.1428571428571
c **i** 4 **ii** 2 **iii** 8

16 Teacher to check

1D

Now you try

Example 11
$\frac{5}{8}$, 0.32 and $-\sqrt{25}$ are rational numbers

Example 12
a $3\frac{2}{5}$ **b** $\frac{9}{4}$

Example 13
2.3125

Example 14
$0.41\dot{6}$

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Example 15

a $\frac{9}{20}$ **b** $3\frac{9}{40}$

Example 16

$\frac{4}{9} > \frac{5}{12}$

Exercise 1D

1 **a** T **b** F **c** T **d** T

2 **a** Proper **b** Mixed number
c Improper **d** Proper

3 **a** $\frac{7}{10}$ **b** $\frac{3}{10}$ **c** $\frac{3}{2}$ **d** $\frac{7}{3}$

4 a, b, c, d, e, f, g, i, k

5 **a** **i** $2\frac{2}{5}$ **ii** $2\frac{5}{6}$ **iii** $5\frac{3}{4}$ **iv** $1\frac{1}{8}$

b **i** $\frac{16}{5}$ **ii** $\frac{44}{7}$ **iii** $\frac{24}{5}$ **iv** $\frac{83}{8}$

6 **a** 2.75 **b** 0.35 **c** 3.4 **d** 1.875
e 2.625 **f** 3.8 **g** 2.3125 **h** 0.25

7 **a** $0.\overline{27}$ **b** $0.\dot{7}$ **c** $1.\overline{285714}$ **d** $0.58\dot{3}$
e $1.\dot{1}$ **f** $3.8\dot{3}$ **g** $7.2\dot{6}$ **h** $2.\overline{63}$

8 **a** $\frac{7}{20}$ **b** $\frac{3}{50}$ **c** $3\frac{7}{10}$ **d** $\frac{14}{25}$

e $1\frac{7}{100}$ **f** $\frac{3}{40}$ **g** $3\frac{8}{25}$ **h** $7\frac{3}{8}$

i $2\frac{1}{200}$ **j** $10\frac{11}{250}$ **k** $6\frac{9}{20}$ **l** $2\frac{101}{1000}$

9 **a** $\frac{5}{6}$ **b** $\frac{13}{20}$ **c** $\frac{7}{10}$ **d** $\frac{5}{12}$

e $\frac{7}{16}$ **f** $\frac{11}{14}$ **g** $\frac{19}{30}$ **h** $\frac{11}{27}$

10 **a** $\frac{9}{20}$ **b** $\frac{3}{20}$ **c** $\frac{32}{45}$ **d** $\frac{23}{75}$

11 **a** $\frac{11}{6}, \frac{7}{3}$ **b** $\frac{2}{5}, \frac{2}{15}$ **c** $\frac{11}{12}, 1$ **d** $\frac{5}{7}, \frac{11}{14}$

12 Weather forecast

13 **a** $\frac{3}{5}$ **b** $\frac{5}{9}$ **c** $\frac{8}{13}$ **d** $\frac{23}{31}$

14 **a** 31, 32 **b** 36, 37, 38, … , 55 **c** 4, 5

15 **a** $\frac{8}{9}$ **b** $1\frac{2}{9}$ **c** $\frac{81}{99} = \frac{9}{11}$ **d** $3\frac{43}{99}$

1E

Now you try

Example 17

a $1\frac{1}{7}$ **b** $\frac{2}{5}$

Example 18

$\frac{19}{21}$

Example 19

a $4\frac{3}{8}$ **b** $\frac{8}{15}$

Exercise 1E

1 **a** 12 **b** 6 **c** 15 **d** 49
e 15 **f** 35

2 **a** T **b** F

3 **a** $\frac{3}{10} + \frac{4}{10} = \frac{7}{10}$ **b** $\frac{4}{8} - \frac{3}{8} = \frac{1}{8}$

c $1\frac{2}{4} + \frac{1}{4} = 1\frac{3}{4}$

4 **a** 6 **b** 63 **c** 30 **d** 8
e 33 **f** 60

5 **a** $\frac{3}{5}$ **b** $\frac{5}{8}$ **c** $\frac{7}{10}$ **d** $\frac{4}{7}$

e $\frac{5}{17}$ **f** $\frac{1}{5}$ **g** $\frac{10}{6} = 1\frac{2}{3}$ **h** $\frac{10}{8} = 1\frac{1}{4}$

i $\frac{7}{5} = 1\frac{2}{5}$ **j** $1\frac{4}{5}$ **k** $2\frac{1}{3}$ **l** $\frac{1}{2}$

m $\frac{2}{5}$ **n** $4\frac{1}{2}$ **o** $1\frac{2}{5}$ **p** $6\frac{1}{2}$

6 **a** $\frac{5}{8}$ **b** $\frac{1}{3}$ **c** $\frac{1}{2}$ **d** $\frac{1}{2}$

e $\frac{9}{10}$ **f** $\frac{7}{12}$ **g** $\frac{7}{10}$ **h** $\frac{3}{20}$

i $\frac{11}{24}$ **j** $1\frac{1}{6}$ **k** $1\frac{8}{21}$ **l** $\frac{17}{24}$

m $\frac{11}{36}$ **n** $\frac{4}{7}$ **o** $\frac{19}{100}$ **p** $\frac{19}{90}$

7 **a** $3\frac{5}{6}$ **b** $\frac{7}{12}$ **c** $2\frac{4}{5}$ **d** $2\frac{7}{12}$

e $6\frac{3}{4}$ **f** $3\frac{5}{6}$ **g** $\frac{1}{8}$ **h** $1\frac{5}{12}$

i $1\frac{7}{10}$ **j** $\frac{11}{12}$ **k** $4\frac{7}{10}$ **l** $1\frac{7}{12}$

8 $\frac{7}{8}$ tonnes

9 $5\frac{29}{56}$ tonnes

10 **a** $4\frac{19}{20}$ litres **b** 33 cups

11 $\frac{1}{20}$

12 $\frac{5}{6}$, problem is the use of negatives in the method, since $\frac{1}{3} < \frac{1}{2}$

13 **a** **i** $\frac{1}{8} + \frac{3}{4}$ **ii** $\frac{2}{3} - \frac{5}{12}$ **iii** $\frac{5}{6} - \frac{1}{2}$

b $\frac{1}{2} + \frac{2}{3} + \frac{5}{6} = 2$

Progress quiz

1 **a** −8 **b** −14 **c** 25 **d** 8
e 4 **f** −3

2 **a** 42 **b** −60 **c** −8 **d** 8
e −64 **f** 81

3 **a** −9 **b** 19 **c** −7

4 **a** **i** 24.19 **ii** −6.14 **iii** 7.60

b **i** 3.3 **ii** 240 **iii** 7900
iv 0.0079

5 **a** 100, 121 **b** 89.8, 93.17302

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6 **a** 0.625 **b** 3.32 **c** $0.58\dot{3}$ **d** $2.\overline{63}$

7 **a** $\frac{4}{5}$ **b** $\frac{9}{25}$ **c** $\frac{1}{250}$ **d** $4\frac{41}{200}$

8 $\frac{3}{8}, \frac{5}{12}, \frac{11}{24}$

9 **a** $\frac{5}{8}$ **b** $\frac{2}{21}$ **c** $3\frac{11}{20}$ **d** $1\frac{9}{20}$

10 $3\frac{7}{12}$

1F

Now you try

Example 20

a $\frac{12}{35}$ **b** $\frac{5}{18}$

Example 21

$4\frac{1}{5}$

Example 22

a $\frac{3}{2}$ or $1\frac{1}{2}$ **b** $\frac{9}{5}$ or $1\frac{4}{5}$

Exercise 1F

1 **a** T **b** F **c** T

2 **a** 12 **b** 4 **c** 4 **d** 12
e 180 **f** 16

3 **a** $\frac{4}{3}$ **b** 7 **c** $\frac{1}{6}$ **d** $\frac{3}{5}$

4 **a** $\frac{3}{20}$ **b** $\frac{4}{15}$ **c** $\frac{15}{28}$ **d** $\frac{1}{6}$
e $\frac{1}{20}$ **f** $\frac{2}{27}$ **g** $\frac{10}{17}$ **h** $\frac{3}{7}$
i $\frac{2}{5}$ **j** $\frac{2}{3}$ **k** $\frac{2}{5}$ **l** $\frac{4}{5}$
m $\frac{3}{4}$ **n** $\frac{2}{3}$ **o** $\frac{2}{9}$ **p** $\frac{3}{16}$

5 **a** $\frac{1}{2}$ **b** $1\frac{1}{4}$ **c** $2\frac{1}{10}$ **d** 2
e $4\frac{1}{5}$ **f** $\frac{6}{7}$ **g** $6\frac{1}{4}$ **h** 4
i $2\frac{11}{12}$ **j** 6

6 **a** $\frac{20}{21}$ **b** $1\frac{1}{8}$ **c** $\frac{45}{56}$ **d** $\frac{27}{28}$
e $1\frac{1}{3}$ **f** $1\frac{1}{2}$ **g** 6 **h** $\frac{7}{8}$
i 18 **j** 9 **k** $\frac{1}{10}$ **l** $\frac{4}{27}$

7 **a** $3\frac{1}{3}$ **b** 4 **c** $\frac{1}{6}$ **d** $1\frac{1}{2}$
e $\frac{7}{8}$ **f** 27

8 **a** \$10 **b** \$3.50 **c** \$6
d \$18 **e** \$6400 **f** 90c

9 **a** $\frac{2}{7}$ **b** $\frac{8}{15}$ **c** 16

10 $\frac{3}{12}$ hour = 15 minutes

11 160 km

12 90 minutes each

13 **a** **i** 0.5 **ii** 0.2 **iii** 0.16129 … **iv** $0.\dot{6}$
v −1
b 3.39
c 25
d Numbers greater than zero but less than one
e 1
f 0

1G

Now you try

Example 23

5 : 3

Example 24

a 25 : 16 **b** 26 : 15

Example 25

7 : 40

Example 26

\$350

Exercise 1G

1 **a** No **b** Yes **c** Yes **d** No

2

a	1 : 3	1 : 4
b	1 : 2	1 : 3
c	2 : 3	2 : 5
d	1 : 2	1 : 3
e	1 : 3	1 : 4
f	1 : 2	1 : 3

3 **a** 5 : 1 **b** 1 : 2 **c** 2 : 1 **d** 10 : 1 **e** 5 : 16

4 **a** 1 : 5 **b** 2 : 5 **c** 4 : 5 **d** 1 : 10
e 5 : 6 **f** 1 : 3 **g** 3 : 1 **h** 2 : 3
i 4 : 3 **j** 4 : 1 **k** 1 : 2 **l** 3 : 1
m 5 : 14 **n** 1 : 2 : 4 **o** 3 : 6 : 2

5 **a** 1 : 2 **b** 3 : 41 **c** 5 : 32 **d** 1 : 3
e 1 : 5 **f** 10 : 3 **g** 40 : 1 **h** 1500 : 1
i 20 : 7

6 **a** 3 : 4 **b** 8 : 3 **c** 9 : 20 **d** 45 : 28
e 3 : 14 **f** 22 : 39

7 **a** 1 : 10 **b** 1 : 5 **c** 2 : 3 **d** 7 : 8
e 25 : 4 **f** 1 : 4 **g** 1 : 4 **h** 24 : 5
i 4 : 20 : 5 **j** 4 : 3 : 10 **k** 5 : 72 **l** 3 : 10 : 40

8 **a** \$100, \$400 **b** \$16, \$20
c 24 kg, 64 kg **d** \$56, \$40
e \$200, \$300 **f** 750 g, 1250 g
g \$70, \$30 **h** \$300, \$300
i \$14, \$49, \$7 **j** 336 g, 84 g

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9 126 g
10 120 watermelons
11 Georgia : $3000, Leanne : $1200, Maya : $1800 respectively
12 108 L
13 Amy : $2400, Belinda : $3600, Candice : $1200, Diane : $4800
14 a i 100 mL **ii** 200 mL
b i 250 mL **ii** 270 mL
c i 300 mL **ii** 1 : 4
d i 1 : 3 **ii** 7 : 19 **iii** 26 : 97 **iv** 21 : 52
e Jug 3 and 4

1H

Now you try

Example 27
a $30/h **b** 1200 revs/min

Example 28
a 60 km/h **b** 4 hours 10 minutes

Example 29
The 3 L is cheaper

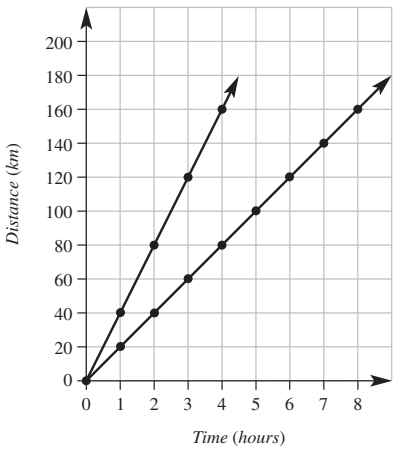
Example 30
1.6 L

Exercise 1H

1 a 60 km/h **b** $84/h **c** 60 km/h
2 a $4 **b** $3 **c** $2.50
3 a 10 m/s **b** 60 km/h **c** 14 km/h
4 a $84 **b** $21 **c** $126
5 a $28/h **b** $50/h **c** 6 kg/min
d $7/kg **e** 4 runs/over **f** 89 pts/game
g 19 cm/y **h** 14 m/s **i** 92 beats/min
j 8 mL/s
6 a 70 km/h **b** 15 km/h **c** 4 km/h
d 54 km/h **e** 80 km/h
7 a 13 hours **b** 16 hours 15 minutes
8 a 3 kg deal **b** 1.5 kg of Red delicious
c 2.4 L of orange juice **d** 0.7 GB
9 a i 130 words **ii** 325 words **iii** 3900 words
b i 44 g **ii** 220 g
c $20 000
d $180
10 a 55 km **b** $16\frac{1}{2}$ km **c** $5\frac{1}{2}$ km
11 a Coffee A: $3.60, coffee B: $3.90. Therefore, coffee A is the best buy.
b Pasta A: $1.25, pasta B: $0.94. Therefore, pasta B is the best buy.
c Cereal A: $0.37, cereal B: $0.40. Therefore, cereal A is the best buy.
12 a 40 sheep/h **b** 220 sheep **c** 25 hours
13 a 1 080 000 000 km/h **b** 36 000 000 km
c $8\frac{5}{18}$ min
14 a 62.66 **b** 159.58 Australian dollars
15 a

Time in hours	0	1	2	3	4	5	6	7	8
Distance in km	0	20	40	60	80	100	120	140	160

b 20 km/h
c, e



d Straight line, speed was constant

Maths@Work: Cooks and chefs

1 a 3 : 4 **b** 9 egg whites **c** 5 egg whites
d 20 servings **e** 216 g **f** 45 g
2 a 3 loaves **b** 6 cartons **c** 3 : 5
d 1350 grams **e** 6 : 5
f Recipe for 1 large loaf of banana bread:
3 cups of plain flour
$1\frac{1}{2}$ cups sugar
1 teaspoon of bicarb of soda
375 grams of melted butter
3 eggs
$3\frac{1}{3}$ mashed bananas
3 a i 2 : 1 **ii** 1 : 25 **iii** 2 : 1
b 8 eggs
c $2\frac{1}{2}$ cups
4 3.5
5 a i 150 mL oil
ii 2.5 cups flour
iii 250 g chocolate
b i 4 eggs
ii 8 tablespoons apple sauce
iii 640 g brown sugar

Puzzles and challenges

1 $1 + 2 + 3 + 4 + 5 + 6 + 7 + (8 \times 9) = 100$
2 48 ways

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3

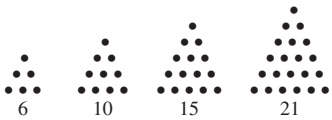
1. **9**	2. **2**	3. **5**	**3**
4. **3**	**5**	5. **5**	6. **6**
7. **9**	8. **9**	9. **2**	**4**
10. **1**	**2**	**8**	**0**

4 a i $1 + 2 + 3 = 6$

ii $28\,(1 + 2 + 4 + 7 + 14 = 28)$

iii $1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248 = 496$

b i



ii 28, 36

c i 0, 1, 1, 2, 3, 5, 8, 13, 21, 34

ii $-21, 13, -8, 5, -3, 2, -1, 1, 0, 1, \ldots, \therefore -21, -8, -3, -1$

Chapter checklist with success criteria

1 a -10 **b** -10

2 -5

3 10

4 a 28 **b** -8

5 a 16 **b** 25

6 -11

7 a 32.24 **b** 3.71 **c** 1.50

8 a 7800 **b** 0.038

9 975

10 $1.4, \sqrt{36}, -\frac{3}{5}$

11 a $3\frac{3}{5}$ **b** $\frac{17}{7}$ or $2\frac{3}{17}$

12 a 2.875 **b** $0.\overline{36}$

13 a $\frac{9}{25}$ **b** $\frac{443}{200}$ or $2\frac{43}{200}$

14 $\frac{7}{9}$

15 a $\frac{5}{17}$ **b** $\frac{22}{8}$ or $2\frac{6}{8}$

16 $\frac{43}{30}$ or $1\frac{13}{30}$

17 a $\frac{55}{12}$ or $4\frac{7}{12}$ **b** $\frac{19}{21}$

18 a $\frac{5}{24}$ **b** $\frac{4}{9}$

19 $\frac{7}{6}$ or $1\frac{1}{6}$

20 $\frac{12}{7}$ or $1\frac{5}{7}$

21 a 5 : 4 **b** 10 : 7 **c** 25 : 18

22 4 : 25

23 \$180

24 a 60 L/h **b** 72 strokes/min

25 60 Km/h, 2 hours and 24 minutes

26 7 L

27 \$40

Chapter 1 review

Short-answer questions

1 a 8 **b** -45 **c** 270

d -64 **e** 6 **f** 8

2 a 21.5 **b** 29 100 **c** 0.153 **d** 0.00241

3 a 14.98 **b** 0.71 **c** 2.00

4 a 2.125 **b** $0.8\dot{3}$ **c** $1.\overline{857142}$

5 a $\frac{3}{4}$ **b** $1\frac{3}{5}$ **c** $2\frac{11}{20}$

6 a $\frac{4}{3}$ **b** $\frac{5}{1} = 5$ **c** $\frac{1}{8}$ **d** $\frac{7}{9}$

7 a $\frac{1}{2}$ **b** $2\frac{1}{6}$ **c** $\frac{7}{24}$ **d** 2

e $3\frac{3}{4}$ **f** $2\frac{19}{28}$

8 a 5 : 2 **b** 16 : 9 **c** 75 : 14

9 a 50, 30 **b** 25, 55 **c** 10, 20, 50

10 a Store A: \$2.25/kg; store B: \$2.58/kg. Store A is the best buy.

b Store A: 444 g/\$; store B: 388 g/\$

11 \$132, \$198, \$330

12 a 32 **b** 7 **c** 192 **d** \$14.80

e 82.5

13 a 156 km/h **b** 36 km/h

14 \$6000

15 36 m

16 Perimeter is doubled

Multiple-choice questions

1 D **2** B **3** C **4** A

5 E **6** E **7** D **8** E

9 C **10** B **11** C **12** C

13 B

Extended-response questions

1 a 12 boys **b** 10 boys, 18 girls **c** 5 : 9

2 a 85.6 km/h **b** 14 hours **c** 61.1 km/h **d** 611 km

Chapter 2

Warm-up quiz

1 a $\frac{3}{25}$ **b** $\frac{1}{5}$ **c** $\frac{7}{20}$ **d** $\frac{3}{4}$ **e** $\frac{3}{5}$ **f** $\frac{1}{2}$

2 a 99 **b** 58 **c** 90 **d** 122 **e** 8 **f** 150

3 a 55% **b** $\frac{3}{4}$ **c** 98%

4 a 61 **b** 9 **c** 37 **d** 121 **e** 1 **f** 75, 4

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5

Fraction	Decimal	Percentage
$\frac{1}{100}$	0.01	1%
$\frac{1}{10}$	0.1	10%
$\frac{1}{4}$	0.25	25%
$\frac{1}{2}$	0.5	50%
$\frac{3}{4}$	0.75	75%

6 a 10 g b 7 km c \$45 d \$800
e 50 c f 9 c

7 a \$100 b 60 m c \$1.50 d \$67.80
e 6 days f 3 days

8 a 100 b 10 c 20 d 3 e 4

9 2100 km

10 25% of 100

11 a 7 b 365 c 52 d 12

12 a 14 b 6 c 12.5

2A

Now you try

Example 1

a 55% b $\frac{1}{8}$

Example 2

a 23% b 0.48

Example 3

25%

Exercise 2A

1 a multiply b divide c multiply d divide

2

%	10%	20%	25%	50%	75%	$33\frac{1}{3}\%$	$66\frac{2}{3}\%$
Fraction	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{2}{3}$
Decimal	0.1	0.2	0.25	0.5	0.75	$0.\dot{3}$	$0.\dot{6}$

3 a 25% b 50% c 100% d 40%

4 Sarah, $79\% > \frac{38}{50} = 76\%$

5 a 20% b 80% c 80% d 30%
e 25% f $12\frac{1}{2}\%$ g 75% h 60%
i 56% j 35% k 9% l $7\frac{1}{2}\%$

6 a $\frac{19}{100}$ b $\frac{23}{100}$ c $\frac{99}{100}$ d $\frac{1}{20}$
e $\frac{11}{50}$ f $\frac{9}{20}$ g $\frac{37}{50}$ h $\frac{3}{4}$
i $\frac{1}{40}$ j $\frac{69}{400}$ k $\frac{1}{100}$ l $1\frac{1}{4}$

7 a 78% b 95% c 65% d 48%
e 75% f 142% g 7% h 30%
i 3% j 104% k 12% l 12.25%

8 a 0.12 b 0.83 c 0.57 d 0.88
e 0.99 f 1.0 g 1.2 h 0.05

9 a 2.5% b 10% c 5% d 5%
e 20% f 20% g 200% h 75%

10

Sport	Number of students who chose sport	Fraction of the total	Percentage of the total
Swimming	44	$\frac{22}{75}$	$29\frac{1}{3}\%$
Golf	12	$\frac{2}{25}$	8%
Volleyball	58	$\frac{29}{75}$	$38\frac{2}{3}\%$
Cricket	36	$\frac{6}{25}$	24%
Total	150	1	100%

11 31.5%

12 $16\frac{2}{3}\%$

13 $6\frac{1}{4}\%$

14 Yes, 2.8% fat content

15 Answers will vary.

2B

Now you try

Example 4

\$10.80

Example 5

\$500

Exercise 2B

1 a \$4 b \$2 c 2 kg d 6 kg
e \$8 f 2.8 seconds

2 \$400

3 True

4 a 2 b 10 c 8 d 7.5
e 57.6 f 34 g 1230 h 42
i 450 j 33 k 198 l 1.5

5 a \$36 b \$210 c 48 kg d 30 km
e 15 apples f 350 m g 250 people h 200 cars
i \$49

6 a \$120 b \$700 c \$300 d \$7
e \$0.20 f \$450 g \$800 h \$360

7 a \$540 b \$600 c \$508 d \$1250
e \$120 f \$40

8 a \$5.80 b \$4.20 c \$0.46 d \$0.50
e \$44 f \$40

9 64

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10 a Divide by 10.
b Divide by 2.
c Divide by 2 and then 2 again (or just 4).
11 48 kg
12 15 students
13 $\frac{10}{100} \times 24 = \frac{24}{100} \times 10$; can multiply in any order
14 $x = 2,\ y = 24$
15 a 72
b $\frac{10}{11}$
c 280%
d $3\frac{1}{4}$
e 150%

2C

Now you try

Example 6
$156

Example 7
$54.40

Example 8
a 40% **b** 16.0%

Example 9
$8.85

Exercise 2C

1 a 1.4 **b** 1.26 **c** 60% **d** 21%
2 a 0.8 **b** 0.27 **c** 6% **d** 69%
3 a 25% **b** 200%
4 a 61.6 **b** 1176 **c** 112 **d** 934.5
e 207 **f** 540 **g** 12 **h** 196
5 a 76 **b** 540 **c** 22.5 **d** 616
e 7360 **f** 337.5 **g** 17 **h** 8910
i 0 **j** 9850
6 20%
7 a

Original amount	New amount	Increase	Percentage change
40	60	20	50%
12	16	4	33.3%
100	125	25	25%
24	30	6	25%
88	100	12	13.6%

b

Original amount	New amount	Decrease	Percentage change
90	81	9	10%
100	78	22	22%
20	15	5	25%
24	18	6	25%
150	50	100	66.6%

8

Old price (excluding GST)	New price (including GST)	GST amount
$40	$44	$4
$120	$132	$12
$50	$55	$5
$140	$154	$14
$70	$77	$7

9 a $3 **b** $80 **c** $200 **d** $500
e $12 500
10 $1785
11 $84 634.25
12 24 534 cars
13 50%
14 28%
15 $21.50
16 $10.91
17 a $900 **b** $990
c As 10% of 1000 = 100 but 10% of 900 = 90
18 a $594 **b** $3235.65 **c** $189 **d** $61.48
19 a 79.86 g **b** $97 240.50 **c** $336 199.68 **d** 7.10 cm

2D

Now you try

Example 10
$1488

Example 11
$52

Example 12
a $13.50
b 37.5%

Example 13
$220

Exercise 2D

1 a C
b D
c A
d B

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2 *See Table at bottom of this page*

3 *See Table at bottom of this page*

4 *See Table at bottom of this page*

5

Item	Cost price	% mark-up	Selling price
Jeans	$60	28%	$76.80
Toaster	$40	80%	$72
Car	$22 000	45%	$31 900
Can of drink	$1.20	140%	$2.88
Loaf of bread	$1.80	85%	$3.33
Handbag	$80	70%	$136
Tablet	$320	35%	$432

6

Item	Cost price	% discount	Selling price
Camera	$900	15%	$765
Car	$24 000	20%	$19 200
Bike	$600	25%	$450
Shoes	$195	30%	$136.50
Blu-ray player	$245	50%	$122.50
Electric razor	$129	20%	$103.20
Lawn mower	$880	5%	$836

7 a

Cost price ($)	Selling price ($)	Profit ($)	Profit (%)
10	15	5	50%
24	30	6	25%
100	150	50	50%
250	255	5	2%
17.50	20	2.50	14.29%

b

Cost price ($)	Selling price ($)	Loss ($)	Loss (%)
10	8	2	20%
16	12	4	25%
100	80	20	20%
34	19	15	44.12%
95	80.75	14.25	15%

8 **a** $5.50 **b** $200 **c** $1500 **d** $125

9 **a** **i** $2 **ii** 20%
b **i** $5 **ii** 25%
c **i** $16.80 **ii** 14%
d **i** $2450 **ii** 175%

10 $1001.25

11 42.3%

12 $613.33

13 **a** $400 **b** $900 **c** $765 **d** No

14 **a** $350 000
b **i** 71.4% **ii** 157.1% **iii** 114.3%
c 25%

2E

Now you try

Example 14
a $1484.62 **b** Ben **c** $58 500

Example 15
a $36 **b** $48 **c** $720

Example 16
$8500

2

Cost price ($)	7	18	24.80	7.30	460.95	3250
Selling price ($)	10	15.50	26.20	11.80	395	4430
Profit/loss ($)	$3 profit	$2.50 loss	$1.40 profit	$4.50 profit	$65.95 loss	$1180 profit

3

Cost price ($)	30.95	99.95	199.95	799.95	18 799	18 000
Mark-up ($)	10	80	395.95	700	16 700	8995
Selling price ($)	40.95	179.95	595.90	1499.95	35 499	26 995

4

Original price ($)	100	49.95	29.95	199	345.50	2215
New price ($)	72	40.90	22.70	176	299.95	2037
Discount ($)	28	9.05	7.25	23	45.55	178

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Exercise 2E

1 **a** C **b** D **c** B **d** A
2 **a** $25.40 **b** $101.60 **c** $482.60
3 **a** $36 **b** $48
4 **a** $11 406.25 **b** $2632.21 **c** $375
5 **a** $1115.38 **b** $29.35/h, therefore less **c** $17 264
6 **a** $41 080 **b** $3423.33 **c** $19.75
7

Employee	Hourly rate	Hours worked	Income
Adam	$20.40	8	$163.20
Betty	$15.50	$8\frac{1}{2}$	$131.75
Ceanna	$19.70	15	$295.50
David	$24.30	38	$923.40
Edward	$57.85	42	$2429.70
Francis	$30	27	$810
George	$35.20	7.25	$255.20

8 **a** $262.80 **b** $379.60 **c** $584 **d** $876
e $817.60 **f** $1284.80
9

Person	Weekly retainer	Rate of commission	Commission earned	Weekly wage
Andrew	$0	12% on $7000	$840	$840
Byron	$160	8% on $600	$48	$208
Cindy	$300	5% on $680	$34	$334
Deanne	$260	5% on $40 000	$2000	$2260
Elizabeth	$500	8% on $5600	$448	$948
Faruq	$900	2% on $110 000	$2200	$3100
Gary	$1000	1.5% on $45 000	$675	$1675

10 **a** 7 **b** 18 **c** 33
11 $29 per hour
12 $24.40 per hour
13 $490
14 $5010
15 $1678.10
16 **a** $751.64 **b** $151.64 **c** 7 hours

Progress quiz

1 **a** $\frac{8}{25}$ **b** 0.07 **c** 23% **d** 12.5%
e 35% **f** $\frac{9}{200}$
2 **a** 10% **b** 80% **c** 12.5%
3 **a** 16 km **b** $42 **c** 60 g **d** $21
4 **a** $400 **b** $300 **c** $750
5 **a** 90.2 **b** 237.6 **c** 93.5 **d** 235
6 **a** 20% **b** 14.9%
7 775
8 **a** $1392 **b** $100.80
9 61.3%
10 **a** $220 **b** $65
11 **a** $1425 **b** $400.40 **c** $188.75 **d** $620

2F

Now you try

Example 17
a $4712 **b** 24% **c** $64 712

Example 18
$14 482

Exercise 2F

1

Gross income	Deductions	Net income
$5600	$450	$5150
$87 000	$28 000	$59 000
$50 000	$6700	$43 300
$890	$76	$814
$84 650	$24 790	$59 860

2 $1405
3 $25 000
4 $27 023.85
5 **a** **i** $40 035 **ii** 17.0%
b **i** $53 905.80 **ii** 20.1%
c **i** $41 218.20 **ii** 15%
d **i** $30 052.56 **ii** 22.2%
6 **a** $1830 **b** $8043 **c** $12 617.50
d $23 772.80
7 **a** $19 656 **b** $53 144
8 **a** $0 **b** $2242 **c** $6717 **d** $16 467
e $32 797 **f** $42 787 **g** $60 667 **h** $195 667
i $420 667
9 $67 400
10 **a** $51 200 **b** $7107 **c** $44 093 **d** $1695.88
11 $336.67
12 **a** $45 001 – $120 000 **b** $60 101.54
13 **a** $81 120
b $16 831
c $64 289
d $2472.65
14 **a** $78 560
b $15 999
c $62 561
d $2406.19
15 $33.23

2G

Now you try

Example 19
$400

Example 20
$618.75

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Example 21
Interest = $570
Total = $5570

Exercise 2G

1 **a** $12 000 **b** 6% p.a. **c** 3.5 years
2 **a** $1120 **b** $2800 **c** $5600
3 **a** $3000 **b** $3600 **c** $416
4 **a** $300 **b** $900 **c** $1600
d $3150 **e** $1196.25
5 $2700, $17 700
6 **a** $52.50 **b** $100 **c** $40
d $264.38 **e** $1027.40
7 $1980, $23 980
8

a	$840	$7840
b	$840	$2340
c	$1500	$41 500
d	$4550	$74 550
e	$200	$2200

9 $2560
10 **a** $3600 **b** $18 000 **c** $38 000
11 Interest: $1800 vs $1700 ∴ choice 2
12 **a** $645 **b** $194 **c** $129
d $19 **e** $55 148 **f** $12 320
g $2580 **h** $58 050 **i** $129 000
13 **a** **i** $12 **ii** $31 **iii** $10
b **i** $335 **ii** $500 **iii** $235

2H

Now you try

Example 22
2.5 years or 2 years 6 months

Example 23
5% p.a.

Example 24
a $16 800 **b** $51 800 **c** $719.44

Exercise 2H

1 **a** 2 years **b** 4 years **c** 50 years **d** $2\frac{1}{2}$ years
2 5 years
3 $2400
4 $450
5 3 years
6 $4\frac{1}{2}$ years
7 18 months
8 **a** 6.25% **b** 2% **c** 6.5%
d 5.5% **e** 21% **f** 3.5%
9

Interest	Total amount to be repaid	Monthly repayment
$5250	$10 250	$170.83
$10 500	$24 500	$408.33
$2400	$12 400	$258.33
$44 000	$99 000	$825
$525 000	$775 000	$2152.78

10 $2083.33
11 **a** $14 400 **b** $240
12 10%
13 **a** $\$P$
b 12.5%
c **i** 20 years **ii** 40 years **iii** Double
14 **a**

Time (years)	Amount (A)	Interest (I)	New amount ($A + I$)
1	500	$500 \times 0.1 = 50$	$500 + 50 = 550$
2	550	$550 \times 0.1 = 55$	$550 + 55 = 605$
3	605	$605 \times 0.1 = 60.5$	$605 + 60.5 = 665.5$
4	665.5	$665.5 \times 0.1 = 66.55$	$665.5 + 66.55 = 732.05$

15 **a**

Time (years)	Amount (A)	Interest (I)	New amount ($A + I$)
1	4500	225	4725
2	4725	236.55	4961.25
3	4961.25	248.06	5209.31
4	5209.31	260.47	5469.78
5	5469.78	273.49	5743.27

b $5743.27, same as final answer in part **a**.
c Increases the amount by 5% five times, giving the same result

Maths@Work: Facebook cake-decorating business

1 $294
2 **a** 8 cm **b** 10 cm **c** 23 cm **d** 25 cm
3 21%
4 **a** White: 5 kg at $40; Red: 2.5 kg at $36.95; Blue: 500g at $7.95
b $1.55/100g
c When a specific quantity was required and any extra would be wasted
5 **a** $231
b 49%
c 290%, 480%, 350%, 330%
d $26/h; $44/h; $32/h; $30/h
6 **a** $427
b Birthday – choc fudge mud; Birthday – caramel mud; Birthday – choc Jaffa
c $47/h wedding cake 3 tiers; $46/h wedding cake 2 tiers; $43/h child's theme birthday cake
d The higher rate of pay for these cakes is due to their complexity requiring a higher level of skill for baking and decorating.

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Puzzles and challenges

1

S	A	G	R	O	S	S	W	H	K	L	O	M
A	P	E	R	C	E	N	T	A	G	E	C	O
L	A	R	C	O	M	M	I	S	S	I	O	N
A	S	E	R	V	I	N	T	E	R	E	S	T
R	T	Y	H	E	T	S	L	O	S	S	T	H
Y	E	F	O	R	T	N	I	G	H	T	I	L
W	S	M	O	T	A	X	A	T	I	O	N	Y
E	I	O	L	I	D	I	D	N	P	Y	I	E
K	M	N	Q	M	O	N	T	R	N	I	S	C
E	P	Y	U	E	D	I	S	C	O	U	N	T
M	L	R	E	P	A	Y	M	E	N	T	A	H
D	E	D	U	C	T	I	O	N	S	I	X	L

2 EARTH, AIR, FIRE, WATER

Chapter checklist with success criteria

1 **a** 17.5%
b $\frac{7}{40}$
2 **a** 30%
b 0.72
3 15%
4 $26.25
5 $600
6 **a** $89.6
b $36.8
7 15.6%
8 1300
9 $1320
10 $5, 62.5%
11 $125
12 **a** $44.53
b $82 368
13 $672
14 $3032
15 $4104
16 $62 452
17 $68 258
18 $480, $4480
19 $945
20 1.5 years
21 $10 240, $42 240, $880

Chapter 2 review

Short-answer questions

1

Decimal	Fraction	Percentage
0.6	$\frac{3}{5}$	60%
$0.\dot{3}$	$\frac{1}{3}$	$33\frac{1}{3}\%$
0.0325	$\frac{13}{400}$	$3\frac{1}{4}\%$
0.75	$\frac{3}{4}$	75%
1.2	$1\frac{1}{5}$	120%
2	2	200%

2 **a** $77.50 **b** 1.65
3 **a** 150 **b** 25
4 **a** 72 **b** 1.17 **c** 20%
5 12.5 kg
6 $1800
7 $4375
8 **a** $25 **b** $16\frac{2}{3}\%$
9 **a** $38.50 **b** $14.30
10 $50 592
11 $525
12 3 years
13 $15 817

Multiple-choice questions

1 C **2** C **3** B **4** D
5 C **6** C **7** C **8** A
9 C **10** E

Extended-response questions

1 **a** $231
b $651
c **i** $63 **ii** $34.65
2 **a** $25.68 **b** $38.52 **c** $154.08
d 6.5 **e** $46 547.31 **f** $558.57

Chapter 3

Warm-up quiz

1 **a** $3x$ **b** $4ab$ **c** $10x$ **d** $21b$
2 **a** 8 **b** 7 **c** 12 **d** 6
e 9 **f** 14
3 **a** $x + 3$ **b** ab **c** $2y - 3$ **d** $\frac{x+2}{3}$
4 **a** -18 **b** -28 **c** 40 **d** 3
e 3 **f** -5 **g** -4 **h** -8
5 **a** $3x$ **b** $6y + 5$ **c** $8x$ **d** $17y$
e $10a + 9a^2$ **f** $5xy - 7y$
6 **a** $6a$ **b** $-21xy$ **c** $4b$ **d** $\frac{3m}{2}$

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7 **a** $2x + 6$ **b** $3a - 15$ **c** $8x + 4$
8 **a** 1 **b** -2 **c** 5 **d** -6
9 b, c
10 **a** 9 **b** 10 **c** 4 **d** 21

3A

Now you try

Example 1

a 4
b **i** -2 **ii** 6
c 5

Example 2

a $g + 5$ **b** $\$4x$ **c** $\frac{600}{y}$ mL

Example 3

a $y + 7$ **b** $2x - 6$ **c** $\frac{m+n}{3}$ **d** $\sqrt{a+b}$

Example 4

a 24 **b** -13 **c** 33

Exercise 3A

1 **a** expression **b** constant term
c pronumeral **d** coefficient **e** terms
2 **a** $7y$ **b** $-2x$ **c** ab **d** $\frac{y}{2}$
e $\frac{x}{y}$ **f** $\frac{2}{a}$
3 **a** 5 **b** 8 **c** 14 **d** 4
4 **a** Yes **b** Yes **c** No **d** No
5 **a** **i** 3 **ii** 2 **iii** 3
b **i** 2 **ii** -3 **iii** 0
c **i** 3 **ii** 7 **iii** -4
d **i** 4 **ii** $\frac{1}{2}$ **iii** -1
6 **a** **i** $4 + r$ **ii** $t + 2$ **iii** $x + y + z$
b **i** $\$6P$ **ii** $\$10n$ **iii** $\$2D$
iv $\$(5P + 2D)$
c $\frac{500}{C}$
7 **a** $2 + x$ **b** $ab + y$ **c** $x - 5$ **d** $2y - 7$
e $3x$ **f** $3p$ **g** $2x + 4$ **h** $\frac{x+y}{5}$
i $4x - 10$ **j** $1 - 3x$ **k** $\frac{3+y}{2}$ **l** $\frac{1}{2}(x + 1)$
m $(m + n)^2$ **n** $m^2 + n^2$
8 **a** 20 **b** 25 **c** 15 **d** 0 **e** 6 **f** 5
g 28 **h** -96 **i** 5
9 **a** 60 m^2 **b** Length $= 12 + x$, width $= 5 - y$
c $17 + x - y$
10 **a** \$26 **b** \$50 **c** No **d** 12 hours
11 **a** 18 square units **b** 1, 2, 3, 4, 5
12 **a** $\frac{A}{20}$ **b** $\frac{3A}{20}$ **c** $\frac{nA}{20}$
13 **a** **i** $P = 2x + 2y$ **ii** $A = xy$
b **i** $P = 4p$ **ii** $A = p^2$
c **i** $P = 6 + x + y$ **ii** $A = 3x$
14 **a** **i** No **ii** No
b **i** $3x + 1$ or $3(x + 1)$ **ii** $5 + \frac{x}{3}$ or $\frac{5+x}{3}$
iii $\frac{1}{2}(x + y)$ or $\frac{1}{2}x + y$
c Answers may vary.
i The sum of 2 and x is multiplied by 4.
ii 3 is added to a half of x.
iii A third of the sum of m and n
iv 7 more than 5 lots of x
v The sum of x and y is divided by 2.
vi The sum of b and a half of a

3B

Now you try

Example 5

a $5m$ and $2m$ are like terms
b $7y$ and $-y$ are like terms
$4xz$ and zx are like terms

Example 6

a $8t$ **b** $4y + 3$

Example 7

a $6a + 13b$ **b** $3st + 4s$ **c** $5x^2y - 6xy$

Exercise 3B

1 **a** like **b** coefficients
c pronumerals **d** 1
2 **a** Y **b** N **c** N **d** Y **e** Y **f** N
3 a, c, d, f
4 **a** $4y$ and $2y$, $3xy$ and $7xy$ **b** $3x$ and $7x$
c $7ab$ and $-3ab$, $2a$ and $5a$ **d** $2a^2$ and $-3a^2$, $4a$ and a
e $3x^2y$ and $7yx^2$
f $5ab^2$ and $4ab^2$, $3ab$ and $7ba$
5 **a** $10a$ **b** $7n$ **c** $8y$ **d** $11x$
e $4ab$ **f** $8mn$ **g** $7y + 8$ **h** $3x + 5$
i $4m + 2$ **j** $12ab + 3a$ **k** $7xy + 4y$ **l** $3bc - 4$
6 **a** $5a + 9b$ **b** $6x + 5y$ **c** $5xy + 4x$ **d** $4t + 6$
e $11x + 4$ **f** $7mn - 1$ **g** $5ab - a$ **h** 0
i $10ac - 9c$ **j** $3xy^2$ **k** $3m^2n$
l $7x^2y - 4xy$ **m** $4x^2 - 3x$ **n** $4a^2b - 3ab$
o $7pq^2 - 8pq$
7 **a** $x + y$ **b** $4x + 2y$
8 **a** $4x + 2y$ **b** $12s$ **c** $30x$ **d** $5x + 4$
e $6x + 16$ **f** $12a + 4b + 4$
9 $8x$ metres
10 **a** $4t$ **b** $9x$ **c** $2x$ **d** $7a$
e $5n$ **f** $4pq$ **g** $2xy$ **h** $3b$
11 **a** $3a + 7b$ **b** 4 units each
c a **d** $2a - b$

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3C

Now you try

Example 8
a $20x$ **b** $-18mp$

Example 9
a $\frac{x}{3}$ **b** $\frac{9q}{2}$

Example 10
a $42a^2b$ **b** $5y$

Exercise 3C

1 **a** $7x$ **b** ab **c** 6 **d** y
e $42b$ **f** $20x$ **g** a^2 **h** y^2
2 **a** T **b** T **c** F **d** F **e** F **f** T
3 **a** $10m$ **b** $12b$ **c** $15p$ **d** $6x$
e $18pr$ **f** $16mn$ **g** $-14xy$ **h** $-15mn$
i $-12cd$ **j** $30ab$ **k** $-24rs$ **l** $-40jk$
m $-60mnp$ **n** $40mn$ **o** $-45pq$ **p** $32wy$
4 **a** $4b$ **b** $\frac{a}{3}$ **c** $\frac{2ab}{3}$ **d** $\frac{m}{2}$
e $\frac{x}{4}$ **f** $\frac{5s}{3}$ **g** 3 **h** $\frac{9q}{2}$
i $\frac{x}{2}$ **j** $6b$ **k** $\frac{7m}{3}$ **l** $\frac{2}{5x}$
m $2p$ **n** $\frac{p}{2}$ **o** $\frac{2q}{3}$ **p** $\frac{3a}{2}$
q 7 **r** w
5 **a** $24n^2$ **b** $-3q^2$ **c** $10s^2$ **d** $21a^2b$
e $-15mn^2$ **f** $18gh^2$ **g** $3b$ **h** $5x$
i $\frac{m}{2}$ **j** $\frac{1}{4y}$ **k** $\frac{3ab}{5}$ **l** $3pq$
6 **a** $\frac{4x}{y}$ **b** $\frac{m}{3}$ **c** $\frac{2a}{b}$ **d** $\frac{10x}{y}$
e $8n$ **f** $5b$ **g** 5 **h** $6xy$
i $\frac{p^2}{2}$ **j** $2ab$
7 **a** $12ab$ **b** $4x^2$ **c** $10xy$ **d** $6x^2$
8 $2x$
9 **a** $4n$ **b** $7y$ **c** $9y$ **d** $5ab$
e $14x$ **f** $10y$ **g** $12a$ **h** $8x^2y$
i $10x^2$
10 **a** **i** $3x^2$ m^2 **ii** $5x$ metres
b **i** 7500 m^2 **ii** 250 m
11 **a** $4b$ **b** $2ab$ **c** b

3D

Now you try

Example 11
a $4x + 8$ **b** $6x - 60$

Example 12
a $-3x - 21$ **b** $-5x + 45$

Example 13
a $5a + 20b$ **b** $15y^2 - 12y$

Example 14
a $7x + 6$ **b** $18 - 2y$

Exercise 3D

1 **a** F **b** F **c** T
2 $5 \times (x + 2)$ or $5 \times x + 5 \times 2$
3 C
4 **a** 3, 3, 15 **b** $-4, -4, -4x, 8$
5 **a** $2x + 6$ **b** $5x + 60$ **c** $2x + 14$ **d** $7x + 63$
e $3x - 21$ **f** $6x - 66$ **g** $5x - 45$ **h** $10x - 10$
i $6 + 3x$ **j** $21 + 7x$ **k** $28 - 4x$ **l** $40 - 5x$
6 **a** $-3x - 6$ **b** $-2x - 22$ **c** $-5x + 15$
d $-6x + 36$ **e** $-8 + 4x$ **f** $-39 - 13x$
g $-72 - 8x$ **h** $-300 + 300x$
7 **a** $2a + 2b$ **b** $5a + 10b$ **c** $6m + 3y$ **d** $16x - 40$
e $-12x - 15$ **f** $4x^2 - 8xy$ **g** $2t^2 - 3t$ **h** $3a^2 + 4a$
i $2d^2 - 5d$ **j** $6b^2 - 10b$ **k** $8x^2 + 2x$ **l** $5y - 15y^2$
8 **a** $2x + 11$ **b** $6x + 22$ **c** $5x + 6$ **d** $3x + 10$
e $4x - 5$ **f** $2x + 1$ **g** $8 - 3x$ **h** $21 - 5x$
i $-1 - 2x$ **j** $-3x$ **k** $3 - x$ **l** $6 - 3x$
m $12x - 1$ **n** $8x + 16$ **o** $2x^2 - 4x$ **p** $2m^2 - 8m$
q $9x - 3$ **r** $6 - x$ **s** $3x^2 + 3x$ **t** $23x - 6y$
u $3x - 4$
9 **a** $4x + 4y$ **b** $8x + 4$ **c** $2x - 2$
10 **a** $2x + 2$ **b** $8x - 12$ **c** $6x^2 + 2x$ **d** $6x + 15$
11 $2x + 120$
12 **a** $2x + 12$ **b** $x^2 - 4x$ **c** $-3x - 12$
d $-7x + 49$ **e** $5x + 10 + 4 = 5x + 14$
f $5 - 2x + 14 = 19 - 2x$
13 **a** $x - 10\,000$ **b** $0.2(x - 10\,000) = 0.2x - 2000$
14 **a** $5x + 12$ **b** $4x + 6$ **c** $11x + 7$ **d** $17x - 7$
e $x - 7$ **f** $2x - 4$ **g** $3x^2 + 10x$
h $5x - 3 + x^2$
15 **a** $2x^2 + 4x$ **b** $x^2 + 2x + 6$ **c** $20 + 8x - x^2$

3E

Now you try

Example 15
a $x = 5$ **b** $x = 3$

Example 16
a $x = -2$ **b** $x = -\frac{4}{5}$

Exercise 3E

1 b, c, e, f, h
2 **a** 6 **b** 8 **c** -3 **d** 4
e 4 **f** -3
3 **a** $2x = 6$ **b** $3x = 21$ **c** $x = 9$
4 **a** Subtract 9 from both sides.
b Add 1 to both sides.
c Divide both sides by 3.

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5 **a** $x = 4$ **b** $a = 1$ **c** $m = 4$ **d** $x = 4$
e $n = 2$ **f** $x = -4$ **g** $b = 3$ **h** $y = 7$
i $a = 4$ **j** $b = 8$ **k** $x = -1$ **l** $y = 2$

6 **a** $x = -3$ **b** $x = -1$ **c** $x = -2$ **d** $x = -8$
e $x = 2$ **f** $x = 2$ **g** $x = 2$ **h** $x = 6$

7 **a** $x = \frac{3}{2}$ **b** $x = \frac{3}{5}$ **c** $x = \frac{10}{11}$ **d** $x = \frac{1}{2}$
e $x = -\frac{1}{3}$ **f** $x = \frac{4}{5}$

8 **a** $x + 8 = 34,\ x = 26$ **b** $x - 7 = 21,\ x = 28$
c $2x + 4 = 10,\ x = 3$ **d** $3x - 4 = 11,\ x = 5$
e $3x - 4 = 20,\ x = 8$ **f** $25 - 2x = 7,\ x = 9$

9 **a** Added 2 on the right hand side rather than subtracting 2.
b Multiplied the right hand side by 2 in the last step rather than dividing by 2.
c The negative was dropped, $-x = 7$ so $x = -7$.
d Needed to subtract 3 from both sides before dividing both sides by 2.

10 **a** $x = 5$ **b** $x = -9$ **c** $x = -2$ **d** $x = 3$
e $x = -\frac{17}{2}$ **f** $m = 2$ **g** $x = 3$ **h** $x = 4$
i $x = -3$ **j** $m = -2$ **k** $m = \frac{8}{3}$ **l** $x = -\frac{8}{7}$
m $h = -\frac{1}{3}$ **n** $w = 1$ **o** $d = -\frac{5}{3}$

11 **a** $x = -3, x = 3$ **b** $x = 0, x = 1$
c $x = 0, x = 2$

12 $x = -1, x = 0, x = 1$

3F

Now you try

Example 17
a $x = 8$ **b** $x = 30$

Example 18
$x = 17$

Exercise 3F

1 **a** $x \div 5, \frac{1}{5}x, \frac{x}{5}$ **b** $\frac{3}{4}x$ or $\frac{3x}{4}$

2 **a** 20 **b** 24 **c** 11 **d** 15

3 C

4 **a** iv, add 1 to both sides
b iii, multiply both sides by 4
c i, multiply both sides by 3
d ii, multiply both sides by 2

5 **a** $x = 15$ **b** $y = 14$ **c** $y = 10$ **d** $x = 4$
e $x = 8$ **f** $x = 2$ **g** $b = 12$ **h** $t = -6$
i $a = -6$ **j** $y = 30$ **k** $x = 15$ **l** $s = 4$
m $x = -21$ **n** $m = -4$ **o** $y = -35$

6 **a** $x = 11$ **b** $x = 6$ **c** $y = 2$ **d** $b = 0$
e $y = 20$ **f** $t = 39$ **g** $k = 57$ **h** $x = 70$
i $x = -10$ **j** $b = -24$ **k** $y = -22$

7 **a** $t = 3$ **b** $m = 40$ **c** $y = -2$
d $x = 19$ **e** $y = -6$ **f** $t = -\frac{3}{4}$

8 **a** $\frac{x}{3} = 12,\ x = 36$ **b** $\frac{2x}{5} = 4,\ x = 10$
c $\frac{x}{2} - 4 = 10,\ x = 28$ **d** $\frac{x+3}{4} = 6,\ x = 21$
e $\frac{7x}{3} = 8,\ x = \frac{24}{7}$ **f** $\frac{x}{3} + 5 = 16,\ x = 33$

9 21

10 **a** Needed to $\times 3$ before $-2,\ x + 2 = 21,\ x = 19$
b Need to $+4$ before $\times 3,\ \frac{x}{3} = 6,\ x = 18$

11 **a** $x = 12$ **b** $x = 12$ **c** $x = -4$ **d** $b = 6$
e $x = 5$ **f** $m = 5$ **g** $y = -1$ **h** $t = -7$
i $x = -3$

12 **a** $x = 9$ **b** $x = 36$ **c** $x = -7$ **d** $x = 8$
e $x = 5$ **f** $x = -10$

13 No, order does not matter for $\times$ or $\div$. By dividing by 7 first you deal with smaller numbers, since 7 is a factor of 14.

Progress quiz

1 **a** 4 **b** -3 **c** 8

2 **a** $\$3d$ **b** $\frac{\$500}{x}$ **c** $3y - 4$ **d** $2(m + n)$

3 **a** -1 **b** -8 **c** 12

4 **a** $12x$ **b** $7b$ **c** $9a + 5$ **d** $8xy - 4x$
e $3a + 10b$ **f** $6mn - 3n$ **g** $2xy^2 - 2xy$
h $2a^2b + 7ab^2$

5 **a** $24b$ **b** $24ac$ **c** $-20xy$ **d** $28ab^2$
e $\frac{x}{4}$ **f** $\frac{3y}{4}$ **g** $\frac{3xy}{2}$ **h** $8x$

6 **a** $3x + 15$ **b** $4y - 4$ **c** $-2x - 10$ **d** $-5a + 10$
e $12x - 30y$ **f** $12m^2 + 3mn$

7 **a** $6x - 8$ **b** $-4x - 4$ **c** $2 - 2x$

8 **a** $x = 3$ **b** $x = 4$ **c** $x = -9$ **d** $x = -\frac{3}{5}$

9 **a** $x = 28$ **b** $x = 48$ **c** $x = 27$ **d** $x = -22$

10 **a** $3x + 5 = 32$, number is 9 **b** $\frac{x-3}{4} = 5$, number is 23

3G

Now you try

Example 19
$x = \frac{11}{3}$ or $3\frac{2}{3}$

Example 20
$x = 5$

Exercise 3G

1 **a** $3 \times \underline{x} + \underline{3} \times 7 = 25$
$3x + \underline{21} = 25$
b $\underline{4} \times 2x + 4 \times \underline{(-1)} = 15$
$\underline{8x} - 4 = 15$

2 c and d

3 a and b

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4 **a** $x = \frac{5}{2}$ or $2\frac{1}{2}$ **b** $a = \frac{3}{5}$
c $m = \frac{19}{3}$ or $6\frac{1}{3}$ **d** $y = \frac{47}{5}$ or $9\frac{2}{5}$
e $p = \frac{35}{4}$ or $8\frac{3}{4}$ **f** $k = \frac{19}{2}$ or $9\frac{1}{2}$
g $b = -\frac{1}{4}$ **h** $m = -\frac{11}{2}$ or $-5\frac{1}{2}$ **i** $x = -\frac{4}{5}$
j $a = \frac{1}{14}$ **k** $x = \frac{19}{6}$ or $3\frac{1}{6}$ **l** $n = \frac{2}{3}$
m $x = -\frac{1}{10}$ **n** $y = -\frac{1}{6}$

5 **a** $x = 1$ **b** $x = 2$ **c** $x = 2$ **d** $x = 12$
e $x = 3$ **f** $x = 1\frac{2}{3}$ **g** $x = -5$ **h** $x = -1$

6 **a** $x = 1$ **b** $x = 2$ **c** $x = 1$ **d** $x = 9$
e $x = -4$ **f** $x = 1$

7 **a** $3(x + 1) = 4,\ x = \frac{1}{3}$ **b** $2(x - 2) = 19,\ x = 11\frac{1}{2}$
c $2(x + 3) = 7,\ x = \frac{1}{2}$ **d** $3(x - 4) = 8,\ x = 6\frac{2}{3}$
e $2(3x - 2) = 5,\ x = 1\frac{1}{2}$ **f** $3(2x + 5) = 10,\ x = -\frac{5}{6}$

8 $15/hour

9 **a** $x = 5$ **b** $x = 5$
c Dividing both sides by 3 is faster because $9 \div 3$ is a whole number

10 **a** $x = 4\frac{1}{3}$ **b** $x = 4\frac{1}{3}$
c Expanding the brackets is easier because $7 \div 3$ gives a fraction answer

11 **a** No **b** No **c** Yes, $4(x + 1) = 20$
d Yes, $2(x - 4) + 3x + 1 = 13$
e Yes, $3x + 2 + 2(x - 5) = 12$
f Yes, $2x - 3 - 3(x - 1) = -4$

3H

Now you try

Example 21
a $x = 4$ **b** $x = 3$

Example 22
a $x = 4$ **b** $x = 2$

Example 23
a $x = -4$ **b** $x = 8$

Exercise 3H

1 **a** Left **b** Left **c** Right **d** Right

2 **a** $4x = 4$
$x = 1$
b $4x + 2 = 10$
$4x = 8$
$x = 2$

3 **a** $6x + 2 = 8$
$6x = 6$
$x = 1$
b $4x = 6 - 2x$
$6x = 6$
$x = 1$

4 **a** $x = 4$ **b** $x = 3$ **c** $x = 3$ **d** $x = 7$
e $x = -3$ **f** $x = -2$ **g** $x = 2$ **h** $x = 4$
i $x = 6$ **j** $x = 2$

5 **a** $x = 8$ **b** $a = 3$ **c** $m = 8$ **d** $x = 4$
e $x = 5$ **f** $x = 3$ **g** $x = 1$ **h** $y = 2$
i $m = 3$ **j** $x = 3$ **k** $b = -1$ **l** $m = -3$

6 **a** $x = 7$ **b** $a = 5$ **c** $y = 2$ **d** $x = -14$
e $b = -16$ **f** $m = 3$ **g** $a = 19$ **h** $x = -3$
i $x = -13$ **j** $n = 14$ **k** $a = 2$ **l** $x = 2\frac{1}{2}$

7 **a** T **b** F **c** F **d** F
e F **f** T **g** T **h** F

8 **a** $2x = 3x - 4, x = 4$ **b** $2x + 4 = 5x, x = 1\frac{1}{3}$
c $3x + 10 = 5x, x = 5$ **d** $3x + 2 = 5x - 6, x = 4$
e $2x - 1 = 3x + 5, x = -6$ **f** $2x + 4 = 13 - x, x = 3$

9 Mardy is 8 years old.

10 **a** $x = 4$ **b** $x = 4$
c Method a: don't have to deal with negatives

11 **a** 48 **b** 38 **c** 26 **d** 48

3I

Now you try

Example 24
$5x + 4 = 29; x = 5$

Example 25
5 hours

Example 26
Max kicked 4 goals and Kim, 7 goals

Exercise 3I

1 $x + 2x = 30$

2 **a** iii **b** iv **c** i **d** ii

3 age, $x + 4$, $x + 4$, $2x$, 30, 15, Bernard is 15 and Sam is 19.

4 **a** $2x - 4 = 10; x = 7$ **b** $2x = x + 5; x = 5$
c $\frac{x}{3} + 5 = 12; x = 21$ **d** $4(x - 2) = 24; x = 8$
e $x - 3 = 4x - 9; x = 2$

5 **a** Let d be the number of days the car is rented for.
b $50 + 40d = 290$ **c** $d = 6$
d The car was rented for 6 days

6 48 items

7 **a** Let e be the number of goals for Emma. **b** $e + 8$
c $e + e + 8 = 28$ **d** $e = 10$
e Emma scored 10 goals, Leonie scored 18 goals

8 **a** Let w be the width in centimetres.
b Length $= 4w$ **c** $2w + 2(4w) = 560$
d $w = 56$ **e** Length = 224 cm, width = 56 cm

9 Benita $360, Adele $640

10 Andrew $102.50, Brooke $175, Cammi $122.50

11 Walked 10 km, ran 20 km

12 Kate is 45, Her son is 15

13 Rectangle $L = 55$ m, $W = 50$ m. Triangle side = 70 m

14 **a** 27, 28, 29
b **i** $x, x + 2, x + 4$ **ii** 4, 6, 8

15 **a** $T = 8x + 7200$ **b** 300 **c** $R = 24x$
d $x = 350$ **e** 3825

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3J

Now you try

Example 27

a 7 **b** 8

Example 28

a $A = 3$ **b** $r = 3.2$

Example 29

a $a = \frac{v-u}{t}$ **b** $v = \sqrt{\frac{2E}{m}}$

Exercise 3J

1 **a** B **b** C **c** E **d** A **e** D

2 **a** A **b** D **c** M **d** A

3 **a** The length of the rectangle.

b The breadth of the rectangle.

c Because there are two sides for length and two sides for breadth.

d Yes

4 **a** 21 **b** 24 **c** 2 **d** 6

e 452.39 **f** 33.51 **g** 25.06 **h** 14.95

i 249.86 **j** 80

5 **a** $F = 36$ **b** $b = 5$ **c** $t = 7.5$ **d** $v = 25$

e $r = 3.91$ **f** $a = 20$

6 **a** $d = \frac{C}{\pi}$ **b** $x = \frac{d-a}{b}$ **c** $n = \frac{p}{m} - x$ or $\frac{p-mx}{m}$

d $r = \frac{I}{Pn}$ **e** $v = \sqrt{PR}$ **f** $h = \frac{A}{2\pi r}$

g $r = \sqrt{\frac{V}{\pi h}}$ **h** $A = (4C - B)^2$

i $a = \sqrt{c^2 - b^2}$ **j** $b = a^2 - c$

7 **a** 88.89 km/h

b 285 km

8 **a** 35 m/s **b** 2 s

9 **a** Decrease **b** 3988 L **c** 417 min

10 **a** **i** 212°F **ii** 100.4°F

b **i** −10°C **ii** 36.7°C

11 **a** 73 **b** 7

12 40.025

Maths@Work: Plumber

1 **a** 6.3 L **b** 7.88 L **c** 18.75 L **d** 15 L

2 **a** **i** 98.1 kPa **ii** 196.2 kPa **iii** 294.3 kPa **iv** 392.4 kPa

b $h = \frac{P}{9.81}$

i 2.51 m **ii** 2.46 m **iii** 1.38 m **iv** 2.84 m

3 **a** 150 mm

b 357 mm

c 150 mm

d 333 mm

4

The number of drain pipes for a house			
House	**Total roof catchment area m²**	**Decimal number of downpipes**	**Whole number of downpipes**
i	131	2.79	3
ii	147	3.27	4
iii	180.5	3.92	4
iv	257	5.47	6

Puzzles and challenges

1 $x = 3$

4	9	2
3	5	7
8	1	6

2 $5x^2 - xy$

3 WORD

4 Eric is 18 years old now.

5 1st leg = 54 km, 2nd leg = 27 km, 3rd leg = 18 km, 4th leg = 54 km

6 $140

7 **a** $x = \frac{12}{5}$

b $x = \frac{23}{2}$

Chapter checklist with success criteria

1 **a** 3 **b** −5 **c** 6

2 **a** $15n **b** $3x - 4$ **c** $\frac{y^2}{4}$

3 **a** 19 **b** −23

4 $4x$ and $-5x$, $3xy$ and yx

5 **a** $8a - 4$ **b** $3xy + 10y$

6 **a** 20st **b** $-24xy^2$

7 **a** $\frac{5x}{4}$ **b** $\frac{2a}{3}$

8 **a** $4x + 20$ **b** $-3x + 18$ **c** $6x^2 - 2xy$

9 **a** $3x + 2$ **b** $-2x + 3$

10 **a** $x = 6$ **b** $x = -\frac{7}{3}$

11 **a** $x = 12$ **b** $x = 30$

12 $x = 23$

13 $x = \frac{7}{9}$

14 $x = \frac{5}{2}$

15 $x = 2$

16 **a** $x = 5$

b $x = 2$

17 $x = -2$

18 $2x + 8 = 30$, $x = 11$

19 3 hours

20 $115 and $255

21 **a** $A = 35$ **b** $h = 4.2$

22 $u = \sqrt{v^2 - 2as}$

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Chapter 3 review

Short-answer questions

1 **a** $x + y$ **b** $7m$ **c** $\$3m$ **d** $\frac{n-3}{4}$

2 **a** 11 **b** 7 **c** 12 **d** 8 **e** −11

3 **a** $8n$ **b** $6xy$ **c** $4a$ **d** $\frac{xy}{3}$

4 **a** $7b$ **b** $4x + 3$ **c** $3p + 2q$
d $2m - 3mn + n$

5 **a** $2x + 14$ **b** $-6x - 15$ **c** $6x^2 - 8x$
d $-10a + 8a^2$ **e** $4x + 13$ **f** $3x - 7$

6 **a** $x = 9$ **b** $x = 2$ **c** $x = -3$ **d** $x = -9$

7 **a** $x = 10$ **b** $x = 8$ **c** $x = 26$ **d** $x = 6$

8 **a** $2n + 3 = 21, n = 9$ **b** $\frac{l-5}{3} = 7, l = 26$

c $\frac{x}{4} - 5 = 0, x = 20$

9 **a** $x = 5$ **b** $x = \frac{11}{6}$ or $1\frac{5}{6}$

10 **a** $x = 4$ **b** $x = 2$ **c** $x = 1$ **d** $x = -5$

11 $30 + 2x = 550; x = \$260$

12 **a** $E = 60$ **b** $a = 5$ **c** $h = 10$

13 **a** $x = \frac{v^2 - u^2}{2a}$

b $I = \sqrt{\frac{P}{R}}$

Multiple-choice questions

1 C	**2** C	**3** D	**4** C
5 D	**6** A	**7** B	**8** B
9 B	**10** D		

Extended-response questions

1 **a** **i** \$85 **ii** \$110

b **i** Let h be the number of hours of hire.

ii $60 + 25h = 210$ **iii** 6 hours

2 **a** $h = \frac{A}{10}$ **b** 10 m **c** 17 m

Chapter 4

Warm-up quiz

1 **a** 5 cm **b** 13 m **c** 41 mm

2 **a** 9 **b** 49 **c** 20 **d** 45

3 **a** 0.4568 **b** 0.3457 **c** 0.0456 **d** 0.2800

4 **a** 4.23 **b** 5.68 **c** 76.90 **d** 23.90

5 **a** 5.92 **b** 15.36 **c** 4.86 **d** 8.09

6 **a** $x = 64$ **b** $x = 28$ **c** $x = 108$ **d** $x = 60$
e $x = 50$ **f** $x = 55$

7 **a** $x = 2$ **b** $x = 3$ **c** $x = 12$ **d** $x = 6$
e $x = 12$ **f** $x = 30$ **g** $x = 28$ **h** $x = 182$

8 **a** $x = 0.6$ **b** $x = 0.6$ **c** $x = 2.1$ **d** $x = 0.5$

9 **a** $x = 2$ **b** $x = 4$ **c** $x = 5$ **d** $x = 13$

4A

Now you try

Example 1

$x^2 + z^2 = y^2$ or $y^2 = x^2 + z^2$

Example 2

$10^2 + 24^2 = 26^2$

Exercise 4A

1 hypotenuse, triangle

2 **a** c **b** x **c** u

3 **a** F **b** T **c** T **d** F

4 **a** $a^2 + b^2 = c^2$ **b** $d^2 + e^2 = f^2$
c $x^2 + y^2 = z^2$ **d** $s^2 + t^2 = v^2$
e $f^2 + g^2 = e^2$ **f** $a^2 + b^2 = x^2$

5 **a** $3^2 + 4^2 = 5^2$ **b** $8^2 + 15^2 = 17^2$
c $9^2 + 12^2 = 15^2$ **d** $5^2 + 12^2 = 13^2$
e $9^2 + 40^2 = 41^2$ **f** $2.5^2 + 6^2 = 6.5^2$

6 **a** $a^2 + b^2 = x^2$ **b** $a^2 + b^2 = d^2$
c $d^2 + h^2 = x^2$

7

a	b	c	a^2	b^2	$a^2 + b^2$	c^2
3	4	5	9	16	25	25
6	8	10	36	64	100	100
8	15	17	64	225	289	289

a The last two columns

b **i** 13 **ii** 20

c **i** 25 **ii** 110

8 **a** No

b No, $a^2 + b^2 = c^2$ should be true for a right-angled triangle.

9 **a** Yes **b** No **c** No **d** Yes
e No **f** Yes

10/11 {(3, 4, 5), (6, 8, 10), (9, 12, 15), (12, 16, 20), (15, 20, 25), (18, 24, 30), (21, 28, 35), (24, 32, 40), (27, 36, 45), (30, 40, 50), (33, 44, 55), (36, 48, 60), (39, 52, 65), (42, 56, 70), (45, 60, 75), (48, 64, 80), (51, 68, 85), (54, 72, 90), (57, 76, 95)}, {(5, 12, 13), (10, 24, 26), (15, 36, 39), (20, 48, 52), (25, 60, 65), (30, 72, 78), (35, 84, 91)}, {(7, 24, 25), (14, 48, 50), (21, 72, 75)}, {(8, 15, 17), (16, 30, 34), (24, 45, 51), (32, 60, 68), (40, 75, 85)}, {(9, 40, 41), (18, 80, 82)}, {(11, 60, 61)}, {(20, 21, 29), (40, 42, 58), (60, 63, 87)}, {(12, 35, 37), (24, 70, 74)}, {(28, 45, 53)}, {(33, 56, 65)}, {(16, 63, 65)}, {(48, 55, 73)}, {(13, 84, 85)}, {(36, 77, 85)}, {(39, 80, 89)}, {(65, 72, 97)};

50 Pythagorean triads

4B

Now you try

Example 3

$c = 5$

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Example 4
6.61

Example 5
$\sqrt{53}$

Exercise 4B

1 **a** 17 **b** 50 **c** $\sqrt{8}$
2 **a** $c^2 = 3^2 + 5^2$ **b** $c^2 = 2^2 + 7^2$
3 a, f
4 **a** $c = 6.7$ **b** $c = 6.3$ **c** $c = \sqrt{22}$ **d** $c = \sqrt{34}$
5 **a** $c = 10$ **b** $c = 13$ **c** $c = 17$ **d** $c = 15$
e $c = 25$ **f** $c = 41$ **g** $c = 50$ **h** $c = 30$
i $c = 25$
6 **a** 4.47 **b** 3.16 **c** 15.62 **d** 11.35
e 7.07 **f** 0.15
7 **a** $\sqrt{5}$ **b** $\sqrt{58}$ **c** $\sqrt{34}$ **d** $\sqrt{37}$
e $\sqrt{109}$ **f** $\sqrt{353}$
8 4.4 m
9 495 m
10 2.4 m
11 **a** 8.61 m **b** 5.24 m **c** 13.21 cm **d** 0.19 m
e 17.07 mm **f** 10.93 cm
12 250 m
13 42 units
14 5.83 m
15 **a** 77.78 cm
b 1.39 m
c Reduce by 7.46 cm

4C

Now you try

Example 6
$a = 7$

Example 7
$x = 6.08$

Example 8
$x = \sqrt{\frac{25}{2}}$ or $\frac{5}{\sqrt{2}}$

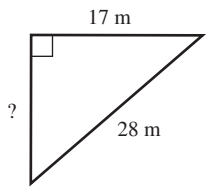
Exercise 4C

1 **a** T **b** F **c** F
2 **a** F **b** F **c** T
3 **a** 12 **b** 20 **c** 4 **d** 24
e 8 **f** 8
4 **a** 16 **b** 24 **c** 6 **d** 21
e 60 **f** 27
5 **a** 8.66 **b** 11.31 **c** 5.11 **d** 17.55
e 7.19 **f** 0.74
6 5.3 m
7 14.2 m
8 49 cm
9 1.86 m
10 **a** $\sqrt{2}$ **b** $\sqrt{8}$ **c** $\sqrt{\frac{1}{2}}$
11 **a** $\sqrt{187}$ **b** $\sqrt{567}$ **c** 40

4D

Now you try

Example 9
a

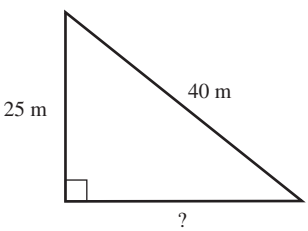


b 22.25 m

Example 10
The cable is 32.02 m long

Exercise 4D

1 B
2 **a** B, III **b** C, I **c** A, II
3 **a**

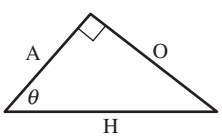


b 31.22 m
4 7.07 cm
5 **a** 6.4 m **b** 5.7 cm **c** 6.3 m **d** 6.0 m
6 3.0 m
7 142.9 m
8 3823 mm
9 1060 m
10 466.18 m
11 **a** 27 m **b** 118.3 m

4E

Now you try

Example 11



Example 12
a $\cos\theta = \frac{7}{9}$ **b** $\sin\theta = \frac{6}{7}$ **c** $\tan\theta = \frac{8}{5}$

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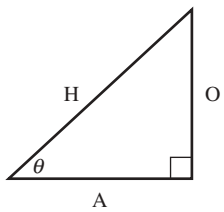
Exercise 4E

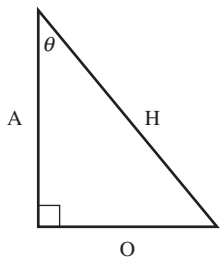
1 **a** 13 m **b** 5 m **c** 12 m

2 **a** hypotenuse **b** opposite **c** adjacent
d opposite **e** hypotenuse **f** adjacent

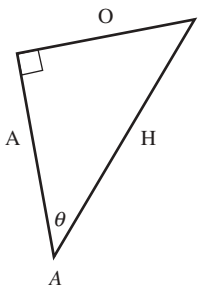
3 **a**



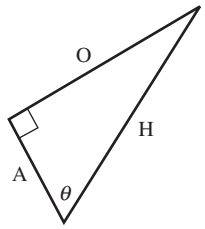
b



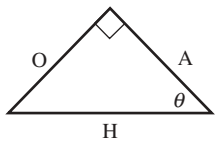
c



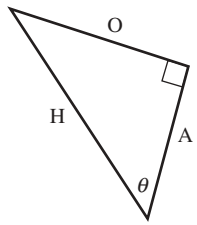
d



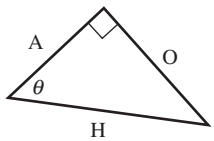
e



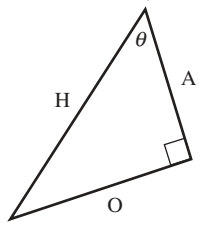
f



g



h



4 **a** $\sin\theta = \frac{4}{7}$ **b** $\tan\theta = \frac{5}{4}$ **c** $\cos\theta = \frac{3}{5}$ **d** $\sin\theta = \frac{2}{3}$

e $\tan\theta = 1$ **f** $\cos\theta = \frac{2}{3}$ **g** $\tan\theta = \frac{11}{12}$

h $\cos\theta = \frac{1}{2}$ **i** $\tan\theta = \frac{5}{3}$

5 **a** $\sin\theta = \frac{3}{5}$ $\cos\theta = \frac{4}{5}$ $\tan\theta = \frac{3}{4}$

b $\sin\theta = \frac{12}{13}$ $\cos\theta = \frac{5}{13}$ $\tan\theta = \frac{12}{5}$

c $\sin\theta = \frac{12}{13}$ $\cos\theta = \frac{5}{13}$ $\tan\theta = \frac{12}{5}$

6 **a** **i** $\frac{5}{13}$ **ii** $\frac{5}{13}$ **iii** The same

b **i** $\frac{12}{13}$ **ii** $\frac{12}{13}$ **iii** The same

c **i** $\frac{5}{12}$ **ii** $\frac{5}{12}$ **iii** The same

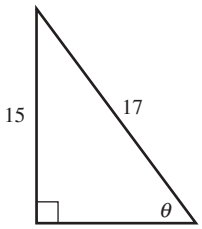
7 **a** $\frac{3}{5}$ **b** $\frac{4}{5}$ **c** $\frac{3}{4}$

8 $\frac{3}{4}$

9 **a** 5 **b** 4 **c** 3 **d** 3 **e** 4

10 **a** $\frac{4}{5}$ **b** $\frac{3}{5}$ **c** $\frac{3}{5}$ **d** $\frac{3}{4}$ **e** $\frac{4}{5}$ **f** $\frac{4}{3}$

11 **a**



b 8 units

c $\sin\theta = \frac{15}{17}$, $\cos\theta = \frac{8}{17}$, $\tan\theta = \frac{15}{8}$

12 **a** **i** $\frac{1}{2}$ **ii** $\frac{\sqrt{3}}{2}$ **iii** $\frac{1}{\sqrt{3}}$

iv $\frac{\sqrt{3}}{2}$ **v** $\frac{1}{2}$ **vi** $\sqrt{3}$

b **i** They are equal. **ii** They are equal.

13 **b** **i** 0.766 **ii** 0.643 **iii** 0.839
iv 0.766 **v** 1.192 **vi** 0.643

c $\sin 40° = \cos 50°$, $\sin 50° = \cos 40°$, $\tan 50° = \frac{1}{\tan 40°}$

$\tan 40° = \frac{1}{\tan 50°}$

4F

Now you try

Example 13

a 0.34 **b** 0.78 **c** 2.40

Example 14

3.83

Example 15

a 1.69 **b** 5.03 **c** 8.48

Exercise 4F

1 **a** A **b** O **c** H

2 **a** O, H **b** A, H **c** O, A

3 **a** sin **b** tan **c** cos

4 **a** 0.34 **b** 0.80 **c** 2.05 **d** 0.73
e 0.10 **f** 0.25 **g** 0.46 **h** 0.24
i 0.99 **j** 0.56 **k** 0.68 **l** 0.91

5 **a** 3.06 **b** 18.94 **c** 5.03 **d** 0.91
e 1.71 **f** 9.00 **g** 2.36 **h** 4.79
i 7.60

6 **a** 5.95 **b** 0.39 **c** 13.38 **d** 3.83
e 8.40 **f** 1.36 **g** 29.00 **h** 1.62
i 40.10 **j** 4.23 **k** 14.72 **l** 13.42
m 17.62 **n** 5.49

7 10.39 m

8 1.12 m

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9 44.99 m
10 10.11 m
11 **a** 65° **b** 2.113 **c** 2.113
12 **a** 20.95 m **b** 10 cm

Progress quiz

1 **a** $x^2 + y^2 = z^2$ **b** $r^2 + s^2 = t^2$
c $5^2 + 12^2 = 13^2$
2 **a** 15 **b** $\sqrt{73}$ **c** 7.2
3 **a** $a = 10$ **b** $b = 6.32$
4 2.2 m
5 3.46 m
6 3.3 m
7 **a** $\sin\theta = \frac{4}{7}$ **b** $\tan\theta = \frac{5}{9}$ **c** $\cos\theta = \frac{6}{11}$
8 **a** 8.19 **b** 5.96
9 2.6 m

4G

Now you try

Example 16
$x = 3.58$

Example 17
$x = 5.85$

Example 18
$x = 4.45$, $y = 10.95$

Exercise 4G

1 **a** $\cos\theta = \frac{7}{x}$ **b** $\sin\theta = \frac{10}{x}$ **c** $\tan\theta = \frac{2.3}{x}$
2 **a** $x = 2$ **b** $x = 5$ **c** $x = 3$ **d** $x = 4$
3 b, c
4 **a** 4.10 **b** 6.81 **c** 37.88 **d** 0.98
e 12.80 **f** 14.43 **g** 9.52 **h** 114.83
i 22.11
5 **a** 13.45 **b** 16.50 **c** 57.90 **d** 26.33
e 15.53 **f** 38.27 **g** 9.15 **h** 32.56
i 21.75 **j** 48.28 **k** 47.02 **l** 28.70
6 40 m
7 3848 m
8 **a** $x = 7.5$, $y = 6.4$
b $a = 7.5$, $b = 10.3$
c $a = 6.7$, $b = 7.8$
d $x = 9.5$, $y = 12.4$
e $x = 12.4$, $y = 9.2$
f $x = 21.1$, $y = 18.8$
9 **a** 27 m **b** 104 m
10 **a** B as student B did not use an approximation in their working out.
b Use your calculator and do not round sin 31° during working.
11 **a** 23.7 m **b** 124.9 m

4H

Now you try

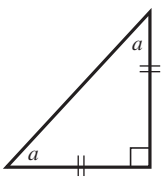
Example 19
28°

Example 20
48.19°

Example 21
25°

Exercise 4H

1 **a** T **b** T **c** F
2 **a** $\frac{5}{13}$ **b** $\frac{12}{13}$ **c** $\frac{5}{12}$
3 **a** sin **b** sin
c cos **d** tan
4 **a** 30° **b** 60°
c 45° **d** 30°
e 45° **f** 30°
g 90° **h** 50°
i 90° **j** 55°
k 0° **l** 70°
5 **a** 34.85° **b** 19.47° **c** 64.16°
d 75.52° **e** 36.87° **f** 38.94°
g 30.96° **h** 57.99° **i** 85.24°
6 **a** 43°3′ **b** 31°9′ **c** 40°36′
d 15°36′ **e** 55°18′ **f** 50°29′
g 48°35′ **h** 41°25′ **i** 33°41′
7 17°
8 23.13°
9 **a** $\tan 45° = 1$



b This triangle is isoceles; therefore, 2 angles are equal
$a + a = 90$
$\therefore a = 45°$
$\tan 45° = \frac{O}{A}$
The O and A are equal
$\therefore \tan 45° = 1$
10 25.4°
11 26.6°
12 45°
13 **a** 18°
b 27°
c 45°
d 5.67 m
e up to 90°

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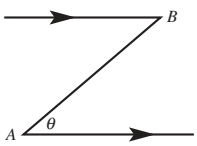
4I

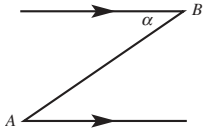
Now you try

Example 22
29 m

Example 23
16°

Exercise 4I

1 a i 

ii 

b Yes, $\theta = \alpha$, alternate angles are equal on parallel lines.

2 C

3 a 22° **b** 22°

4 $a = 65, b = 25$

5 16 m

6 157 m

7 38 m

8 90 m

9 37°

10 6°

11 10°

12 a 6° **b** 209 m

13 4634 mm

14 Yes, by 244.8 m

15 a i 47.5 km ii 16.25 km

b No, it will only reach 12.86 km

c Yes, it will reach 16.50 km

Maths@Work: Carpenter

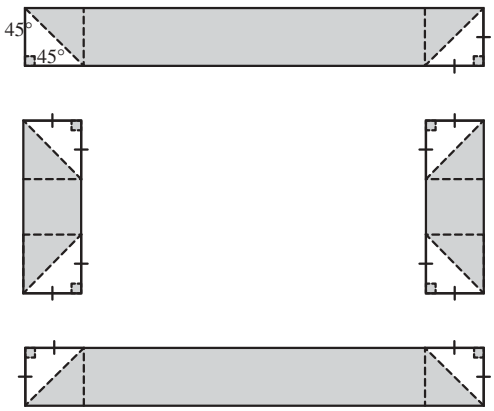
1 a 450 mm

b 2400 mm

c 9010 mm

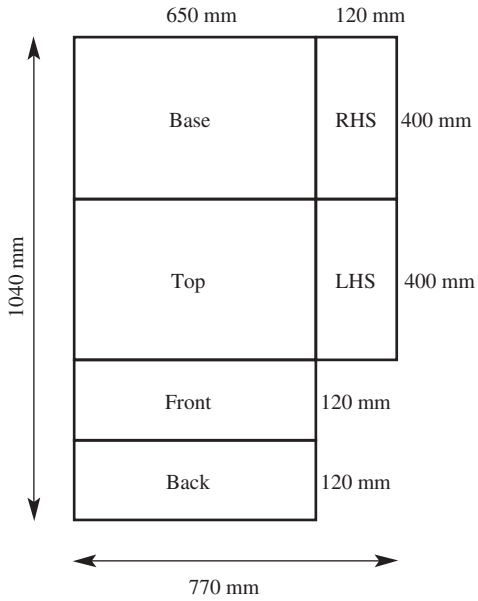
d 2700 mm

e 46 000 mm

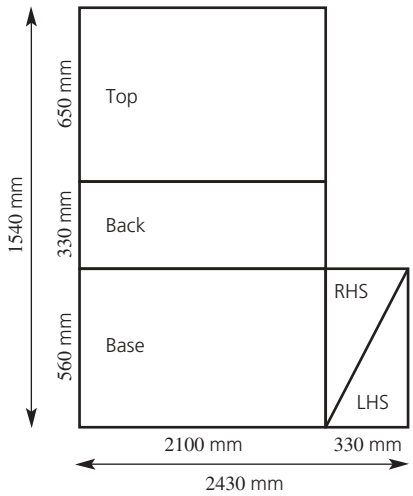
2 

3 Diagrams are not to scale.

a 770 mm by 1040 mm



b 2430 mm by 1540 mm



4 a $BC = 72$ mm **b** $BC = 106$ mm

c $AB = 187$ mm **d** $AB = 364$ mm

5 1370 tiles

6 170 m^2

7

Location	Volume of rain in litres
Strahan, TAS	232 713
Melbourne, VIC	94 860
Port Lincoln, SA	77 724
Perth, WA	111 384
Newcastle, NSW	183 600
Tully, QLD	628 065
Alice Springs, NT	37 179

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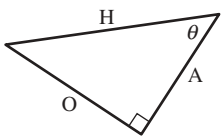
Puzzles and challenges

1 44
2 a the sum of the areas of the two smaller squares is equal to the area of the large square
b a^2, b^2, c^2 c $a^2 + b^2 = c^2$
3 a A diagonal of the square b $\sqrt{2}$ cm c 63.7%
4 63.7% vs 78.5% ∴ Round peg square hole
5 171 cm

Chapter checklist with success criteria

1 a $y^2 = w^2 + x^2$ b $5^2 = 3^2 + 4^2$
2 a 15 b 7.16
3 $c = \sqrt{85}$
4 6.63
5 $x = \sqrt{\frac{49}{2}}$
6 2.83
7 10.77 m
8



9 a $\sin\theta = \frac{4}{9}$ b $\tan\theta = \frac{6}{7}$
10 0.70
11 8.19
12 9.43
13 $x = 13.76$, $y = 17.01$
14 a 51° b 25°
15 66°
16 7 m
17 31°

Chapter 4 review

Short-answer questions

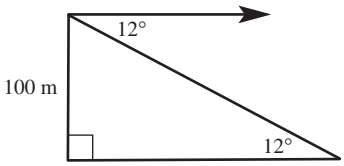
1 a $x^2 + y^2 = z^2$ b $s^2 + t^2 = u^2$ c $e^2 + f^2 = d^2$
2 a 10 b 25 c 13 d 37 e 7.21 f 7.07
3 a 3.32 b 7.55 c 9.95
4 4.49 m
5 a 13 cm b 13.93 cm
6 19 m
7 a 0.64 b 2.25 c 0.72
8 a $\sin\theta$ b $\tan\theta$ c $\cos\theta$
9 a 11.33 b 4.88 c 48.02
d 10.31 e 50.71° f 60°
10 28.01 m
11 25 m
12 177.91 m
13 63.2 m

Multiple-choice questions

1 A 2 B 3 A 4 C 5 B 6 C
7 D 8 C 9 A 10 B

Extended-response questions

1 a

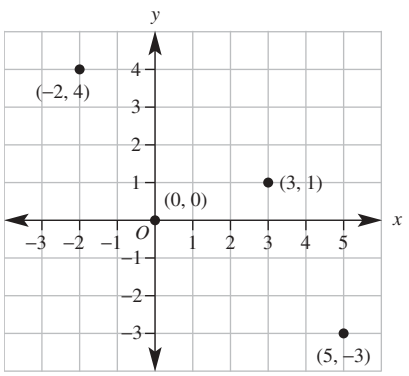


b 470 m c 3° d 1530 m
2 a 2.15 m b 0.95 m c 3.05 m
3 a 9.85 cm b 39°
c i 11.9 ii 6.6 cm
d 46.2 cm e 92 cm^2 (nearest m^2)

Chapter 5

Warm-up quiz

1



2 a 1 b –1 c –3 d –4
3 a $x = 5$ b $x = -2$ c $x = -3$ d $x = 4$
e $x = -2$ f $x = 4$
4 a 3 b –4 c 4 d –3
5 a

x	–2	–1	0	1	2
y	1	2	3	4	5

b

x	–2	–1	0	1	2
y	–5	–3	–1	1	3

6 a F b T c T d F
7 a 7 b 6.5 c 3 d –5
8 a 5 b 4 c 5 d 3
9 a 4 b 5 c 7 d 3

5A

Now you try

Example 1

a

x	–2	–1	0	1	2
y	–8	–5	–2	1	4

b

x	–2	–1	0	1	2
y	8	6	4	2	0

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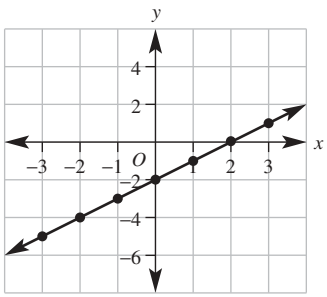
Example 2

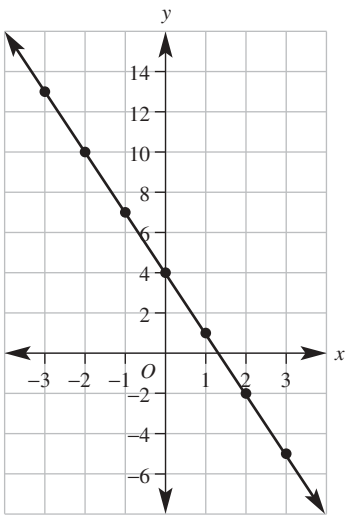
a

x	−3	−2	−1	0	1	2	3
y	−5	−4	−3	−2	−1	0	1



b

x	−3	−2	−1	0	1	2	3
y	13	10	7	4	1	−2	−5

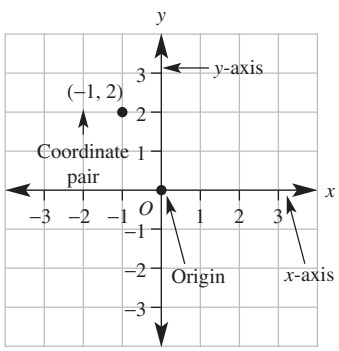


Example 3

a Yes **b** No

Exercise 5A

1 a

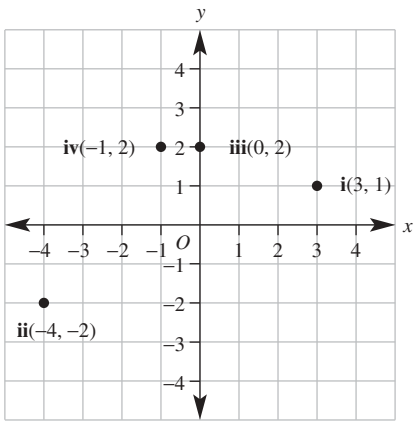


b **i** x-coordinate

ii y-coordinate

iii right, down

2 a



b $A(4, 1)$, $B(2, 0)$, $C(0, -3)$, $D(-2, 2)$, $E(-2, -4)$, $F(3, -4)$

3 a $F(0, 4)$ **b** $G(0, -4)$ **c** $H(-4, 0)$

4 $I(1, 3)$, $K(-3, -9)$

5 a

x	−2	−1	0	1	2	3
y	0	1	2	3	4	5

b

x	−2	−1	0	1	2
y	−1	1	3	5	7

c

x	−2	−1	0	1	2
y	4	2	0	−2	−4

d

x	−2	−1	0	1	2	3
y	5	2	−1	−4	−7	−10

6 a

x	−3	−2	−1	0	1	2	3
y	−4	−3	−2	−1	0	1	2

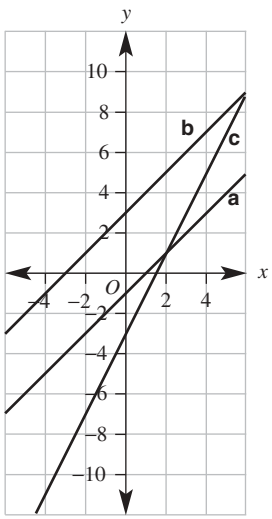
b

x	−3	−2	−1	0	1	2	3
y	0	1	2	3	4	5	6

c

x	−3	−2	−1	0	1	2	3
y	−9	−7	−5	−3	−1	1	3

a $y = x - 1$ **b** $y = x + 3$ **c** $y = 2x - 3$



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d

x	−3	−2	−1	0	1	2	3
y	9	6	3	0	−3	−6	−9

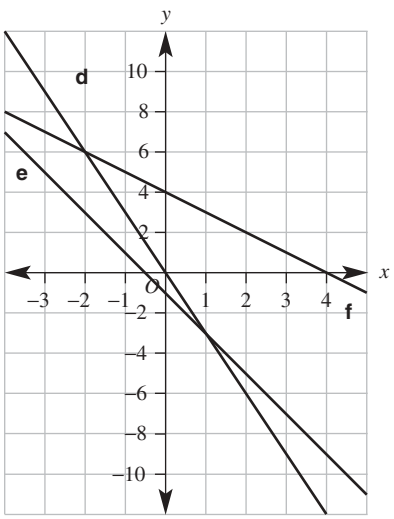
e

x	−3	−2	−1	0	1	2	3
y	5	3	1	−1	−3	−5	−7

f

x	−3	−2	−1	0	1	2	3
y	7	6	5	4	3	2	1

d $y = -3x$ **e** $y = -2x - 1$ **f** $y = -x + 4$



7 **a** No **b** Yes **c** No
8 **a** Yes **b** Yes **c** No
9 **a** $y = x + 2$ **b** $y = 2x$ **c** $y = 2x + 1$
d $y = 3x - 1$ **e** $y = x - 2$
10 $y = 3x$
11 **a** iv **b** iii **c** i **d** ii
12 **a** $y = 2x + 7$ **b** $y = 20 - x$ **c** $y = 2x - 1$
d $y = \frac{1}{2}x + 1$ **e** $y = -3x + 4$
f $y = 5x + 4$

5B

Now you try

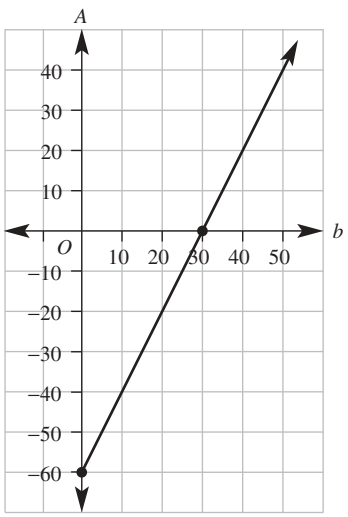
Example 4
a x-intercept is -1, y-intercept is 2
b x-intercept is 3, y-intercept is 2

Example 5
a 5 **b** 3

Example 6
a 2 **b** -3

Exercise 5B

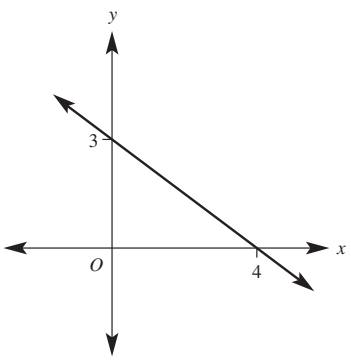
1 **a** (3,0) **b** (0,2)
2 **a** A(1, 0), D(−2, 0), H(3, 0) **b** B(0, 3), E(0, −1)
3 **a** $y = 0$ **b** $x = 0$
4 **a** (1, 1) **b** (−2, 2) **c** (4, 8) **d** (−5, 10)
e (2, 3) **f** (7, −3) **g** (−11, 5) **h** (−2, −5)
5 **a** 5 **b** 1 **c** −7 **d** −3 **e** 2 **f** −4
g −2 **h** 10 **i** 11 **j** 3 **k** 2 **l** 6
m −3 **n** −1 **o** −3 **p** 3
6 **a** −6 **b** 4 **c** 4 **d** −2 **e** −4 **f** 3
g 7 **h** 2 **i** 4 **j** 6 **k** 1 **l** −3
7 **a** $(\frac{1}{2}, -1)$ **b** (4, −2) **c** (12, 4) **d** $(2\frac{1}{3}, 3\frac{1}{2})$
8 **a** 100 m **b** 20 s
9 **a** 20 minutes **b** 80 litres
10 When you substitute $x = 0$ into the rule, $y = 3 \times (0) = 0$. It means the graph passes through the origin.
11 **a** $A = -60$ **b** She makes a loss of $60.
c 30 badges
d She then starts to make a profit from her stall.
e



5C

Now you try

Example 7



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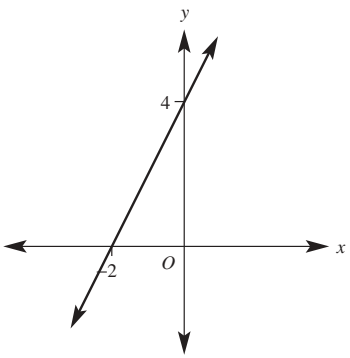
Example 8



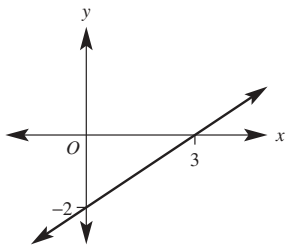
Exercise 5C

1 **a** 2 **b** 0 **c** 0

2 **a** 3 **b** −4

3 **a** −2 **b** 2

4 **a**



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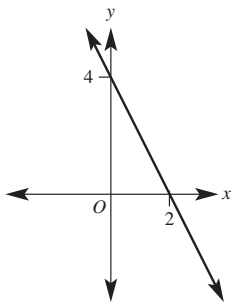

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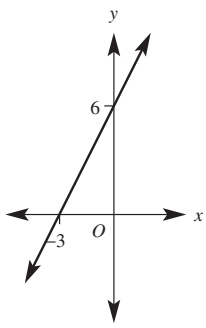

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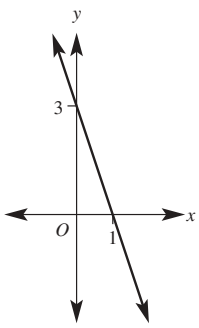
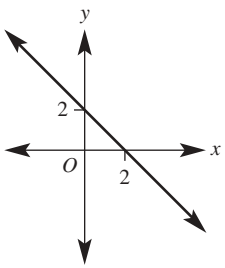


5 **a**



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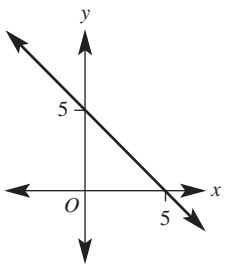

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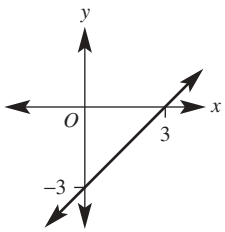

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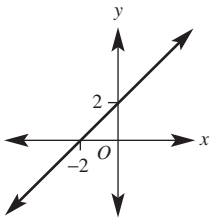

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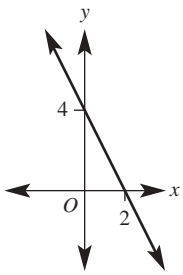

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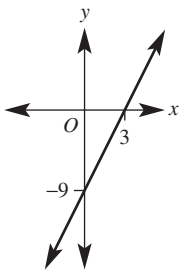

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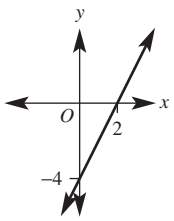

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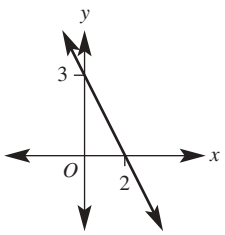

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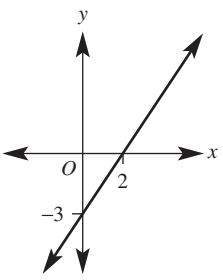

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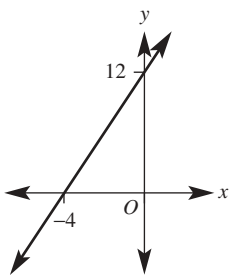

5C

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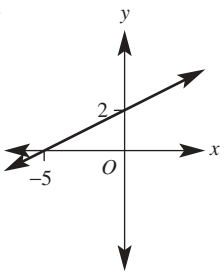

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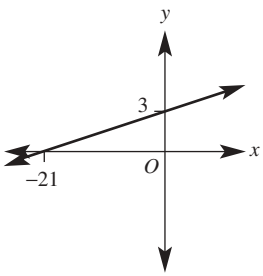
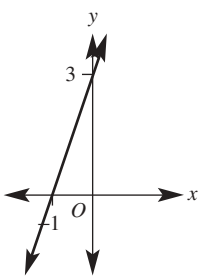


6 a



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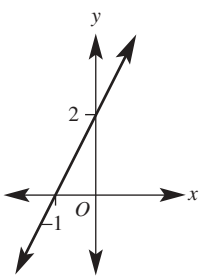

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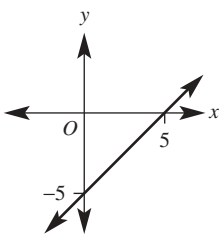

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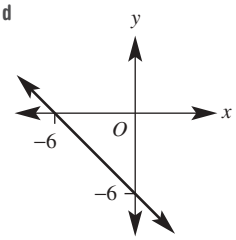

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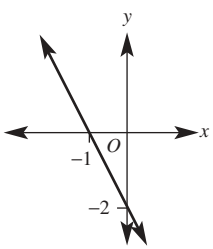

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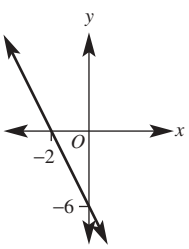

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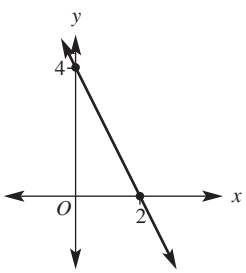

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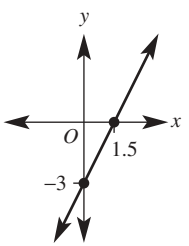

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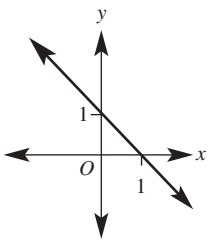

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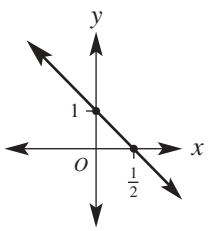
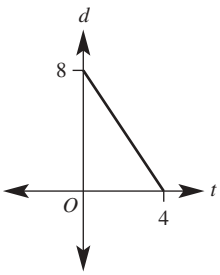


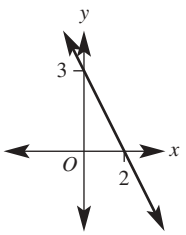
7 a 8 m

b 4 seconds

c



8 a



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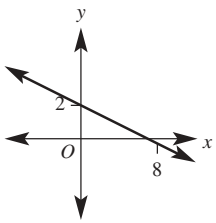

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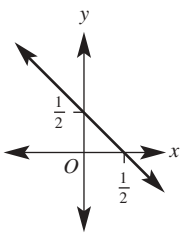


9 a For 0.5 across, graph moves 1 down.

b For 1.5 across, graph moves one up.

10 a $x + y = 4$ **b** $x + y = 2$ **c** $x - y = 3$

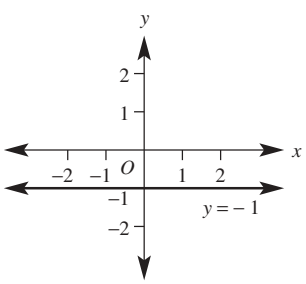
d $x - y = -1$

5D

Now you try

Example 9

a



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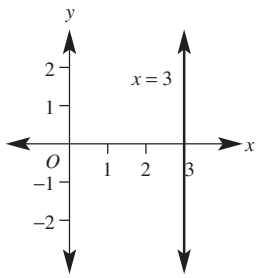
b



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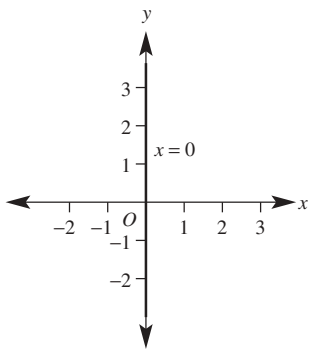
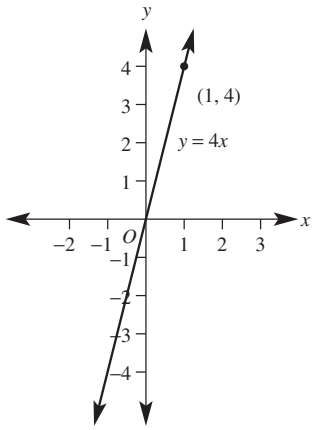


Example 10



Example 11

a $x = 1$ **b** $y = 2$

Exercise 5D

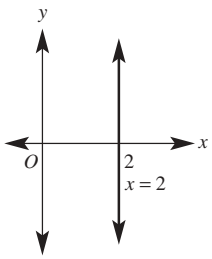
1 **a** Horizontal
b Horizontal
c Vertical
d Horizontal
e Vertical
f Vertical

2 **a** ii, iv
b i, iii

3 **a** **i** 2 **ii** −3
b **i** 3 **ii** −2

4 b, e, f

5 **a**



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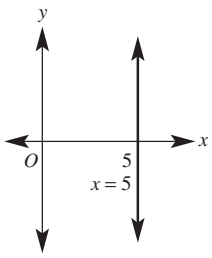

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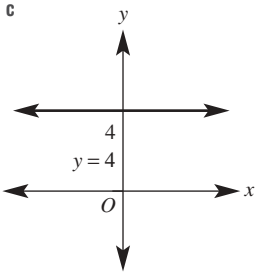

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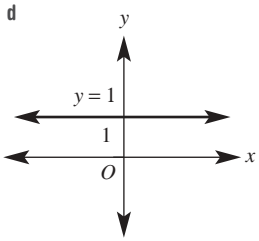

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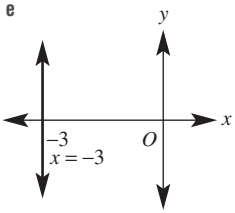

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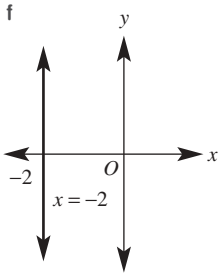

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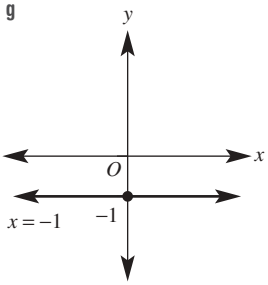

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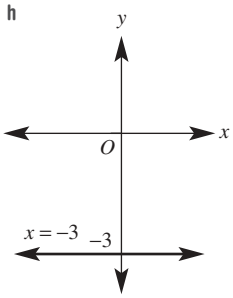
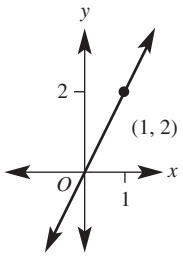


6 **a**



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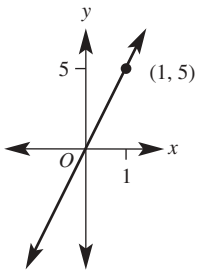

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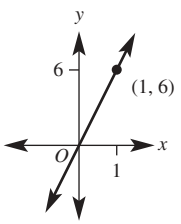
c



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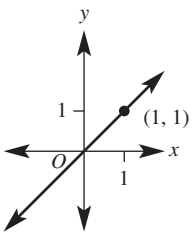

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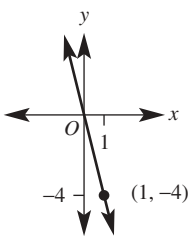

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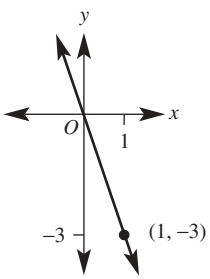

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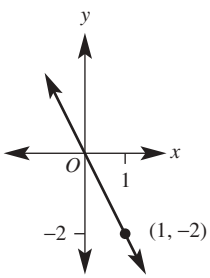

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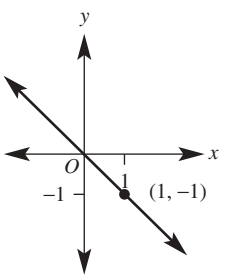

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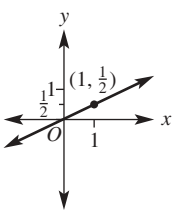

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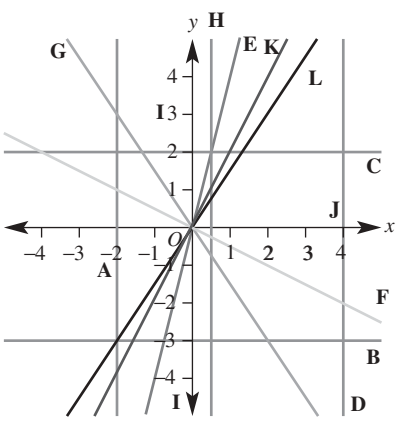

8 **a** $y = -2$ **b** $y = 4$ **c** $x = -2$ **d** $x = 5$
e $y = 1.5$ **f** $x = -6.7$

9 **a** $y = 3$ **b** $x = 5$ **c** $x = -2$ **d** $y = 0$

10 **a** $y = 250$ **b** $y = -45$

11 **a** $y = 2x$ **b** $y = 3x$ **c** $y = -x$ **d** $y = -3x$

12 **a** $A = 15$ square units **b** $A = 68$ square units

13 **a** **i** $y = 1$ or $y = -5$ **ii** $y = 0$ or $y = -4$
iii $y = 3\frac{1}{2}$ or $y = -7\frac{1}{2}$
b **i** $y = 1$ or $y = -5$ **ii** $y = 7$ or $y = -11$
iii $y = 9\frac{1}{2}$ or $y = -13\frac{1}{2}$

5E

Now you try

Example 12

a Zero **b** Positive, 2 **c** Undefined
d Negative, $-\frac{2}{5}$

Example 13

a $m = 2$ **b** $m = -\frac{8}{5}$

Exercise 5E

1 **a** **i** 2 **ii** 4
b **i** 2 **ii** 1
c **i** 1 **ii** 3
d **i** 3 **ii** –3
e **i** 1 **ii** –3
f **i** 4 **ii** –1

2 **a** Zero **b** Negative **c** Positive
d Undefined

3 **a** Positive, 1 **b** Positive, 2 **c** Zero **d** Zero
e Negative, $-\frac{2}{3}$ **f** Negative $-\frac{3}{4}$
g Undefined **h** Undefined **i** Positive, $\frac{1}{2}$
j Positive, 3 **k** Positive, 2 **l** Negative, –4

4 **A** 1 **B** –2 **C** 1 **D** $\frac{1}{2}$
E –2 **F** 3

5 **a** 2 **b** –1 **c** –2 **d** 1
e 2 **f** –2 **g** –1 **h** $\frac{1}{2}$
i $\frac{3}{2}$ **j** $-\frac{5}{2}$ **k** $\frac{1}{3}$ **l** $\frac{5}{2}$

6 **a** $-\frac{1}{20}$ **b** 17.5 or $\frac{35}{2}$ **c** $-\frac{1}{5}$ **d** 7.5 or $\frac{15}{2}$

7 40 metres

8 90 metres

9 **a** 6 **b** 1 **c** 0 **d** –4

10 **a** **i** –1 **ii** –3
b (4.5, 0)
c **i** (6, 0) **ii** (1.5, 0) **iii** (2.4, 0)

5F

Now you try

Example 14

$V = 2t$

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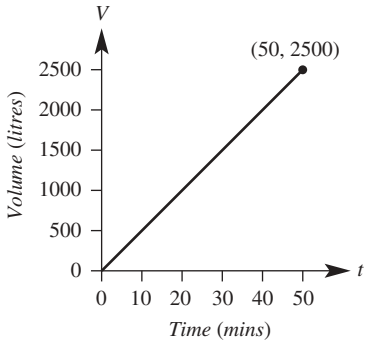
Example 15

a 50 L/min

b



c **i** 50 **ii** $V = 50t$

d **i** 1000 L **ii** 40 mins

Exercise 5F

1 **a** No **b** Yes **c** Yes **d** No

2 **a** **i** 10 km **ii** 20 km **iii** 30 km

b 10 km/h

c 10

d They are the same.

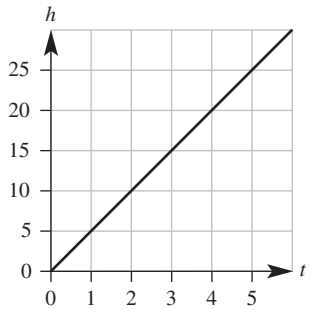
3 **a** 15 mm

b **i** 6 days **ii** 20 days

c

t	0	1	2	3	4
h	0	5	10	15	20

d



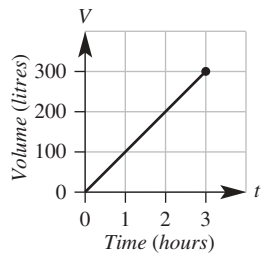
e 5

4 **a** $d = 60t$ **b** $g = 2t$

c $C = 1.5n$ **d** $P = 20t$

5 **a** 100 L/h

b

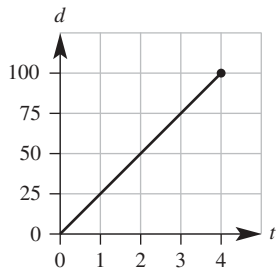


c **i** 100 **ii** $V = 100t$

d **i** 150 L **ii** 2 h

6 **a** 25 km/h

b



c **i** 25 **ii** $d = 25t$

d **i** 62.5 km **ii** 1.6 h

7 **a** 100 km/h **b** 7 cm/s **c** 2.5 cm/min

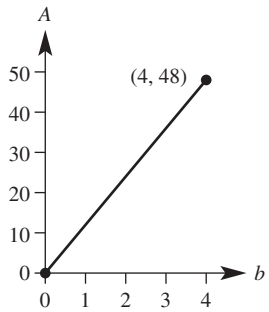
d 49 mm/min

8 Isla

9 The panther

10 **a** 10.2 L **b** 72.25 L **c** 800 km

11 **a** $A = 12b$ **b**



c Yes, the rule for the graph is in the form $y = mx$.

12 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{3}{4}$

d 1 min 20 s or $1\frac{1}{3}$ min

13 1.2 h or 1 h 12 min

5G

Now you try

Example 16

a Gradient = 3, y-intercept = -2

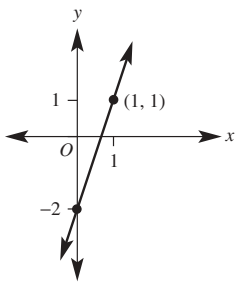
b Gradient = -1, y-intercept = 0

Example 17

a $y = -3x + 5$ **b** $y = -2x + 3$

Example 18

a Gradient = 3, y-intercept = -2



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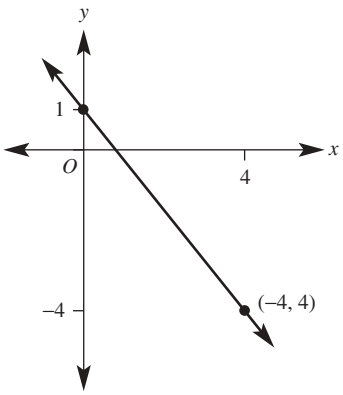
b Gradient $= \frac{-5}{4}$, y-intercept $= 1$

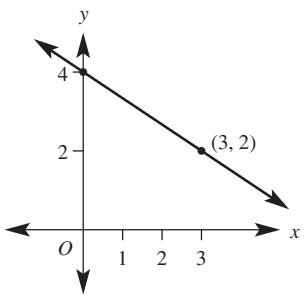


Example 19

$y = -\frac{2}{3}x + 4$

Gradient $= -\frac{2}{3} = \frac{-2}{3}$

y-intercept $= 4$



Exercise 5G

1 **a** gradient **b** y-intercept **c** subject

2 **a** iii **b** i **c** ii **d** iv

3 **a** **i** (2, 6) **ii** (3, 1)

b **i** (1, 1) **ii** (3, −5)

4 **a** 3, 5 **b** 1, 4 **c** 2, −1

5 **a** Gradient $= 3$, y-intercept $= -4$

b Gradient $= -5$, y-intercept $= -2$

c Gradient $= -2$, y-intercept $= 3$

d Gradient $= \frac{1}{3}$, y-intercept $= 4$

e Gradient $= -4$, y-intercept $= 0$

f Gradient $= 2$, y-intercept $= 0$

g Gradient $= 2.3$, y-intercept $= 0$

h Gradient $= -0.7$, y-intercept $= 0$

i Gradient $= 1$, y-intercept $= 0$

6 **a** $y = -3x + 4$ **b** $y = -2x + 7$

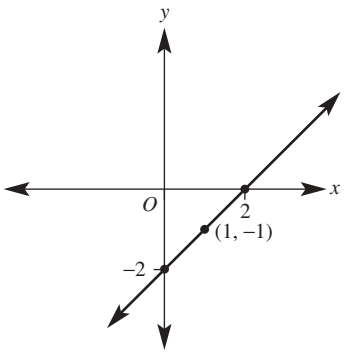
c $y = 3x + 2$ **d** $y = 2x + 7$

e $y = 5x - 3$ **f** $y = -2x + 4$

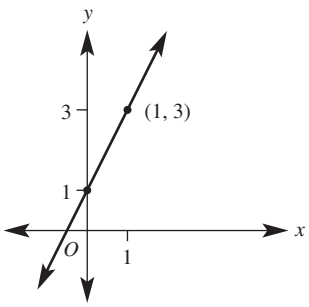
g $y = -\frac{3}{2}x + 3$ **h** $y = 2x - \frac{5}{2}$

i $y = \frac{5}{3}x + 2$

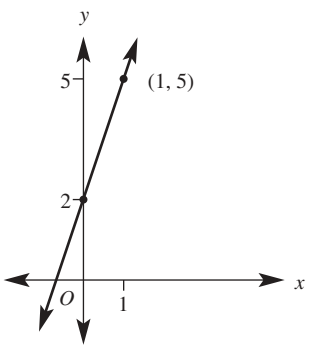
7 **a** $m = 1$, y-int $= -2$



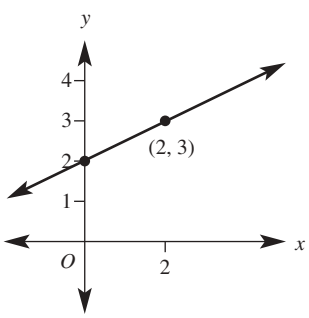
b $m = 2$, y-int $= 1$



c $m = 3$, y-int $= 2$



d $m = \frac{1}{2}$, y-int $= 2$



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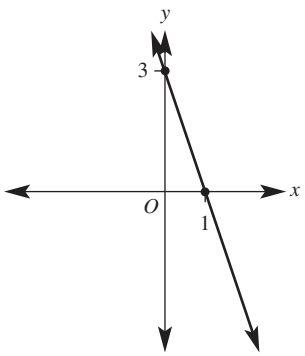
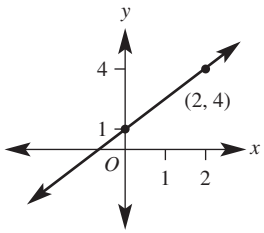
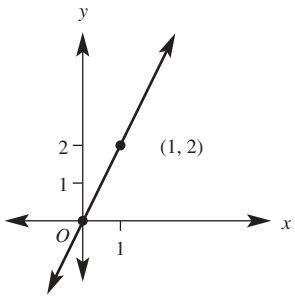
e $m = -3$, y-int $= 3$



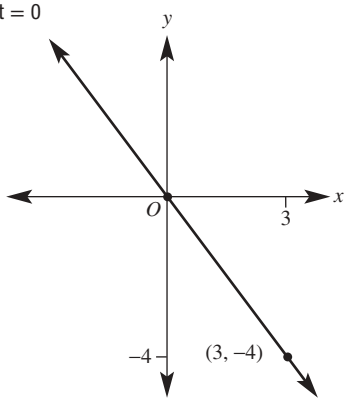
f $m = \frac{3}{2}$, y-int $= 1$



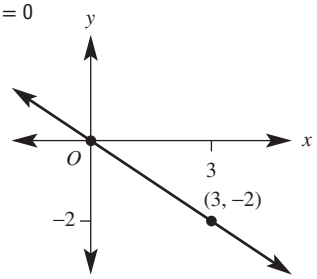
g $m = 2$, y-int $= 0$



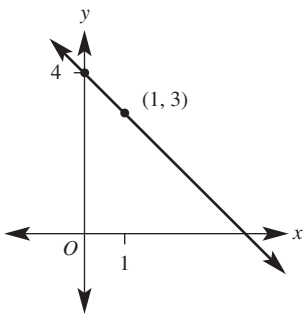
h $m = \frac{-4}{3}$, y-int $= 0$



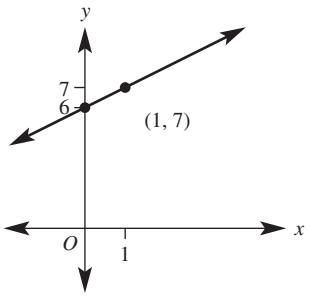
i $m = \frac{-2}{3}$, y-int $= 0$



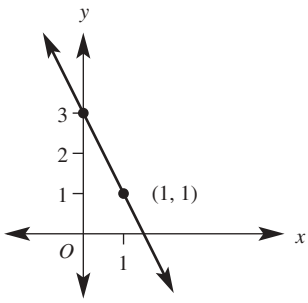
8 a $m = -1$, y-int $= 4$



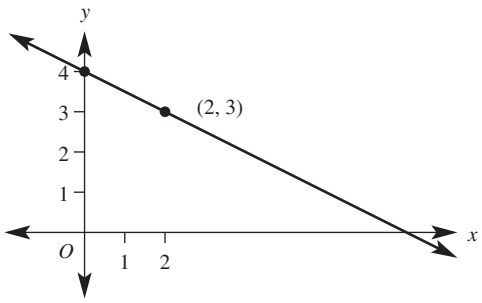
b $m = 1$, y-int $= 6$



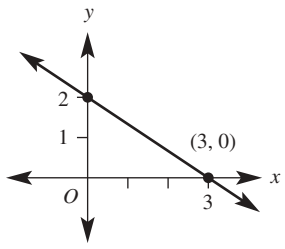
c $m = -2$, y-int $= 3$



d $m = \frac{-1}{2}$, y-int $= 4$



e $m = \frac{-2}{3}$, y-int $= 2$



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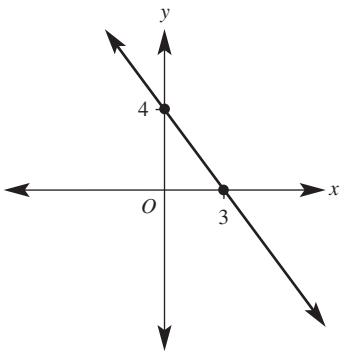
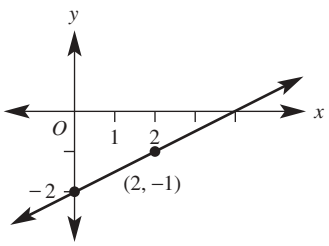
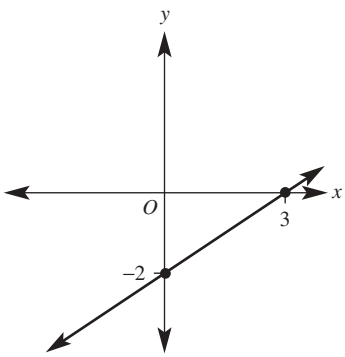
f $m = \frac{-4}{3}$, y-int $= 4$



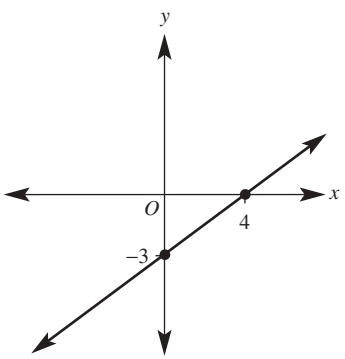
g $m = \frac{1}{2}$, y-int $= -2$



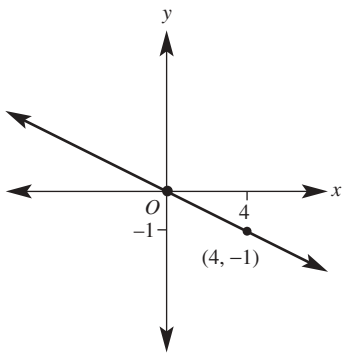
h $m = \frac{2}{3}$, y-int $= -2$



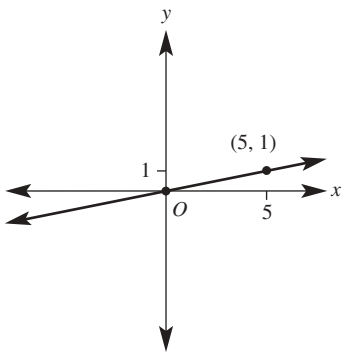
i $m = \frac{3}{4}$, y-int $= -3$



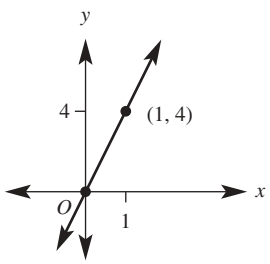
j $m = \frac{-1}{4}$, y-int $= 0$



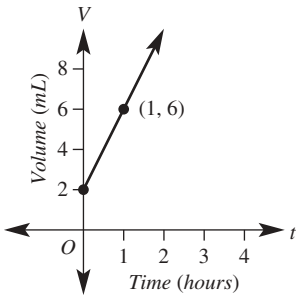
k $m = \frac{1}{5}$, y-int $= 0$



l $m = 4$, y-int $= 0$



9 a Gradient $= 4$, V-intercept $= 2$



b Gradient is the rate the water level is increasing (the rate of rain fall in mL/h); the V-intercept is the amount of water in the gauge at the start of the storm.

10 Yes, she loses the bet. 6 needs to be divided by 2 also to give $2x + 3$.

11 c, d, f

12 a Jeremy needed to multiply both terms inside the brackets by 2, so that $y = 2x + 2$, y-intercept is 2.

b Expand brackets

13 a $y = 2x + 3$ **b** $y = -x + 2$ **c** $y = 2x + 3$

d $y = \frac{1}{2}x + 3$ **e** $y = \frac{1}{2}x + 3\frac{1}{2}$

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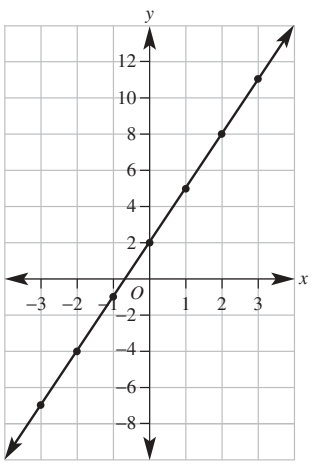
Progress quiz

1 a i

x	−3	−2	−1	0	1	2	3
y	−7	−4	−1	2	5	8	11

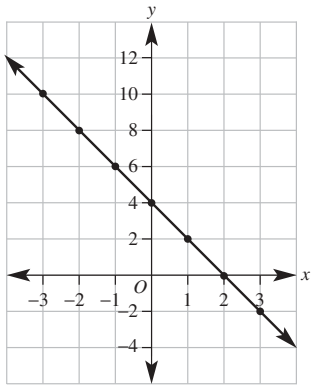
ii



b i

x	−3	−2	−1	0	1	2	3
y	10	8	6	4	2	0	−2

ii



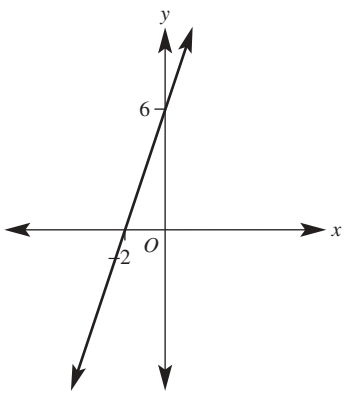
2 a No **b** Yes

3 a x-int $(1, 0)$ and y-int $(0, -2)$

b x-int $(3, 0)$ and y-int $(0, -5)$

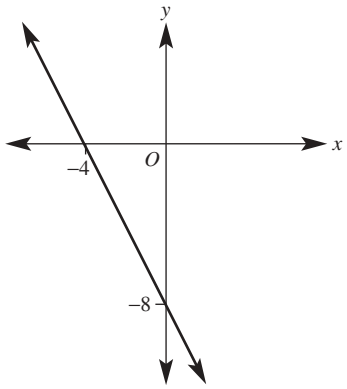
4 a i y-int $= 6$ **ii** x-int $= -2$

iii



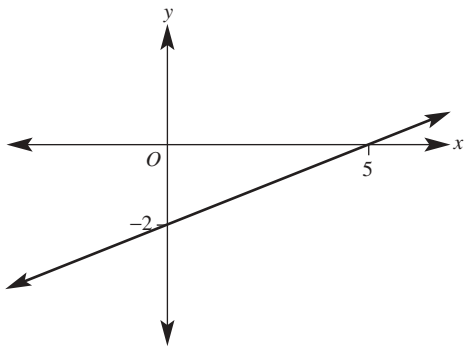
b i y-int $= -8$ **ii** x-int $= -4$

iii



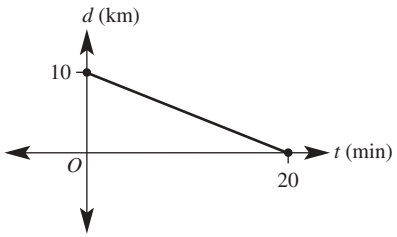
c i y-int $= -2$ **ii** x-int $= 5$

iii

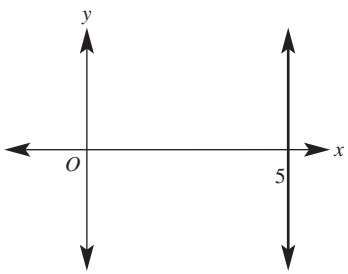


5 a 10 km **b** 20 min

c



6 a



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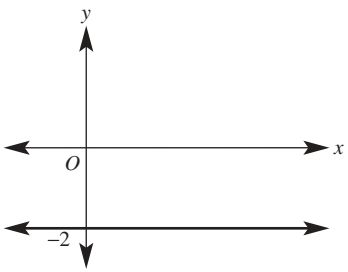
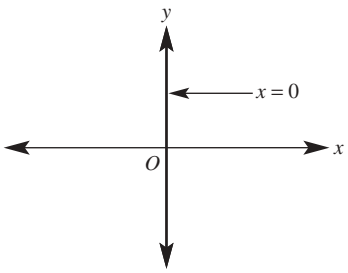
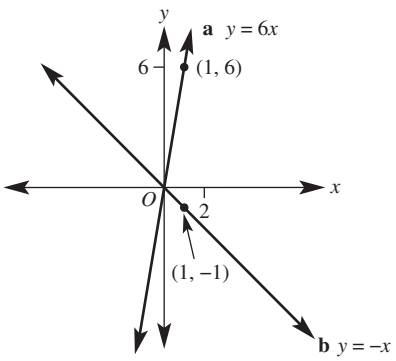
b



c



7



8 **a** $m = 1$

b $m = 0$

c $m = \frac{-3}{2}$

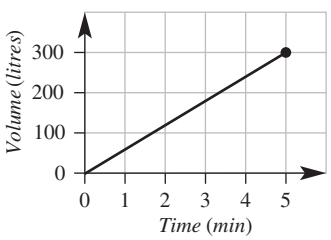
d Undefined

e **i** $m = 2$ **ii** $m = \frac{7}{3}$ **iii** $m = -5$ **iv** $m = 0$

9 **a** $d = 50t$ **b** $E = 16t$ **c** $r = 8o$

10 **a** 60 L/min

b



c **i** 60 **ii** $V = 60t$

d **i** 120 L **ii** 3.5 min

11 **a** Gradient = 2, y-intercept = −3

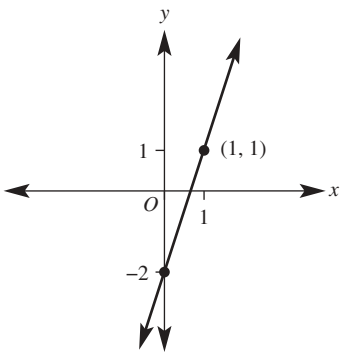
b Gradient = −4, y-intercept = 1

c Gradient $= \frac{1}{2}$, y-intercept = 0

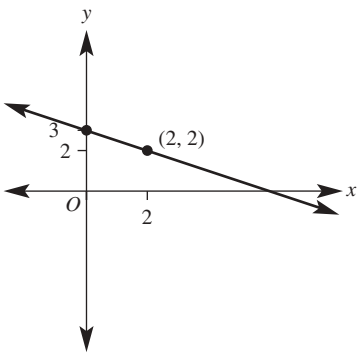
d Gradient = 2, y-intercept = 3

e Gradient = 2, y-intercept = 4

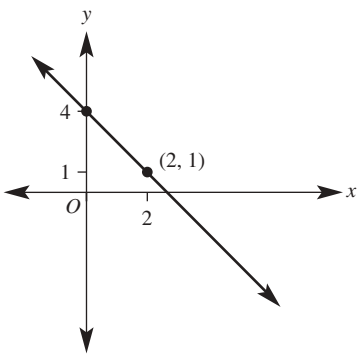
12 **a** Gradient = 3, y-intercept = −2



b Gradient $= \frac{-1}{2}$, y-intercept = 3



c Gradient $= \frac{-3}{2}$, y-intercept = 4



5H

Now you try

Example 20

$y = 2x + 6$

Example 21

$y = 2x - 5$

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Exercise 5H

1 gradient, y-intercept, point

2 **a** $y = 2x + 5$ **b** $y = 4x - 1$
c $y = -2x + 5$ **d** $y = -x - \frac{1}{2}$

3 C

4 **a** 1 **b** 3 **c** 5

5 **a** $y = 2x + 6$ **b** $y = x + 2$ **c** $y = -4x - 4$
d $y = 2x - 2$ **e** $y = 8x + 8$ **f** $y = -x + 4$
g $y = -2x + 6$

6 **a** $y = \frac{3}{4}x + 3$ **b** $y = -\frac{3}{4}x + 3$
c $y = -\frac{5}{4}x + 3$ **d** $y = \frac{3}{2}x + 4$
e $y = \frac{3}{5}x$ **f** $y = -\frac{1}{3}x - 1$

7 **a** $y = 3x + 5$ **b** $y = -2x - 1$
c $y = -3x + 8$ **d** $y = x - 3$
e $y = -3x + 3$ **f** $y = 5x - 1$
g $y = -x + 8$ **h** $y = -3x + 6$
i $y = -2x + 2$ **j** $y = -4x - 9$

8 $y = 5x + 1$

9 $(2.5, 0)$

10 **a** $y = 35x + 80$ **b** After 12 weeks

11 **a** **i** 2 **ii** $y = 2x + 2$
b **i** -4 **ii** $y = -4x + 11$
c **i** -1 **ii** $y = -x + 3$
d **i** 1 **ii** $y = x - 4$

12 **a** $y = -20x + 120$ **b** 120 L

13 **a** $y = 2x - 3$
b $(1.5, 0), (0, -3)$
c The x-intercept is when the temperature reaches 0°C and the y-intercept represents the temperature at midnight.

5I

Now you try

Example 22
a $(4, 5)$ **b** $(-2, 2.5)$

Example 23
6.71

Exercise 5I

1 **a** 8 **b** 1 **c** -3 **d** 5.5

2 **a** 5 **b** 10 **c** 5.1 **d** 6.7

3 **a** **i** 3 **ii** 4
b **i** 2 **ii** 7

4 **a** $(3, 3)$ **b** $(2, 2)$ **c** $(1, 5)$
d $(4, 1)$ **e** $(-1, 3)$ **f** $(-1, -1)$
g $(1.5, 1.5)$ **h** $(2.5, 2)$ **i** $(0.5, 3)$
j $(-1.5, -2.5)$ **k** $(-3, -8.5)$
l $(0.5, -2.5)$

5 **a** 5.10 **b** 2.83 **c** 5.39 **d** 4.47
e 3.61 **f** 2.83 **g** 8.94 **h** 7.21
i 6.71

6 $(-1, 5.5)$ and $(5, 0.5)$

7 $B(8, 0), A(-6, 5), A(-6, 9)$

8 **a** $(-3, 1)$ **b** $(1, -4)$ **c** $(8, 3)$

9 **a** 12.8 **b** 24.2

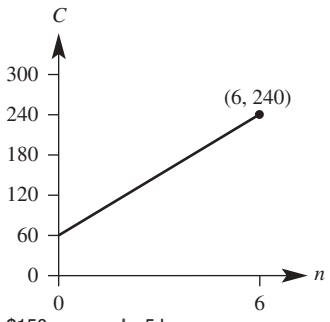
10 **a** $(3, 5), (3, -1), (0, 2), (6, 2)$ are the obvious points
b A circle

11 **a** $\frac{1}{3}$
b $\frac{1}{3}$
c **i** $(1, 1)$ **ii** $(-2, -0.5)$ **iii** $(2, 1.5)$

5J

Now you try

Example 24
a $C = 60 + 30n$
b



c \$150 **d** 5 hours

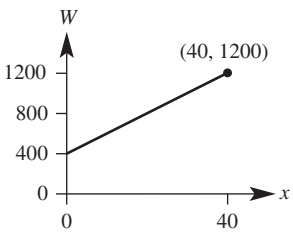
Exercise 5J

1 B

2 **a** **i** \$1200 **ii** \$1500 **iii** \$2200
b $A = 1000 + 100n$

3 **a** **i** dependent **ii** independent
b **i** Volume **ii** Height **iii** Amount of money

4 **a** $W = 20x + 400$
b



c \$640
d 30

5 **a** $C = 50n + 40$ **b** \$240 **c** \$640

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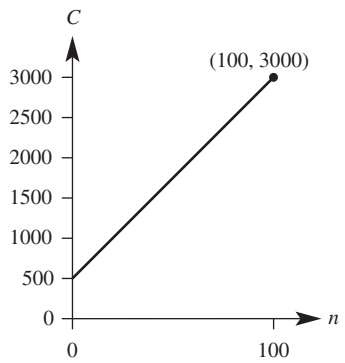
6 a $C = 25n + 500$

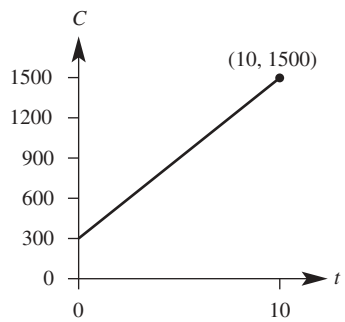
b



c \$1500 **d** 70

7 a $C = 120t + 300$

b



c \$1020 **d** 3 h

8 a $F = 18t + 12$ **b** 3 min

9 a $V = 4000 - 20t$ **b** 2200 L

c 200 min **d** 175 min

10 a 80 km/h

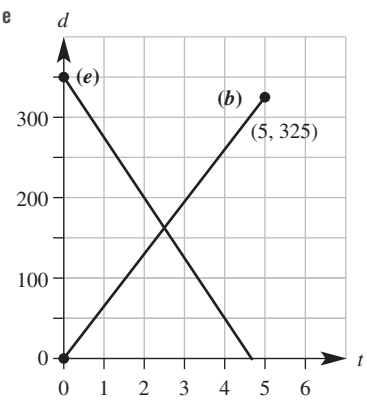
b Rate changes; i.e. new gradient = 70, $d = 50 + 70t$

11 a i 20 m/s **ii** 350 m **iii** 17.5 s

b Increasing altitude (i.e. rising) at rate of 20 m/s

12 a $d = 65t$

b and **e**



c 350 km

d 4 h and 40 min ($4\frac{2}{3}$ hours)

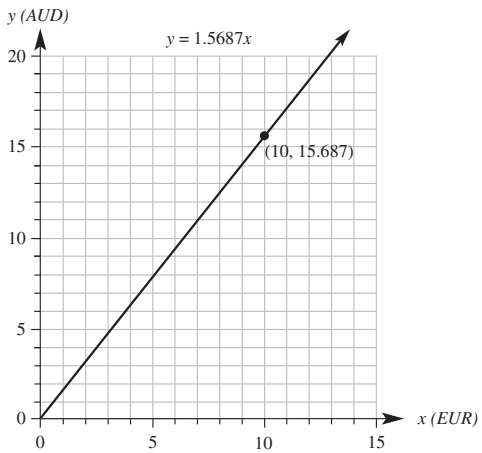
f i 2.5 h **ii** 162.5 km

Maths@Work: Trading in foreign currencies

1

	x (input)	*y* (output)	Linear equation	Points
i	EUR	AUD	$y = 1.5687x$	(0, 0)(10, 15.687)
ii	GBP	AUD	$y = 2.0306x$	(0, 0)(10, 20.306)
iii	USD	CNY	$y = 6.5181x$	(0, 0)(10, 65.181)
iv	EUR	GBP	$y = 0.7725x$	(0, 0)(10, 7.725)
v	AUD	EUR	$y = 0.6375x$	(0, 0)(10, 6.375)
vi	AUD	USD	$y = 0.7158x$	(0, 0)(10, 7.185)
vii	AUD	SGD	$y = 1.0038x$	(0, 0)(10, 10.038)
viii	AUD	NZD	$y = 1.0753x$	(0, 0)(10, 10.753)

2 a i



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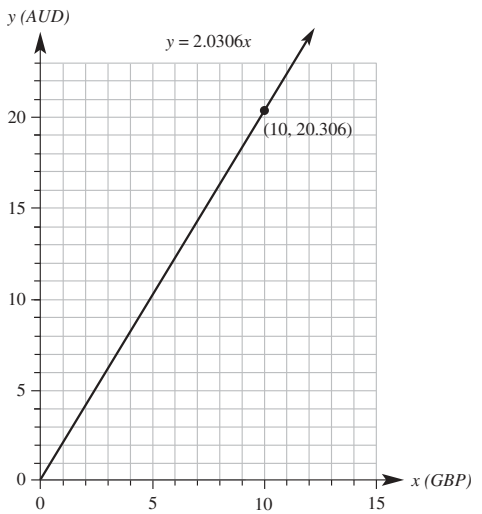

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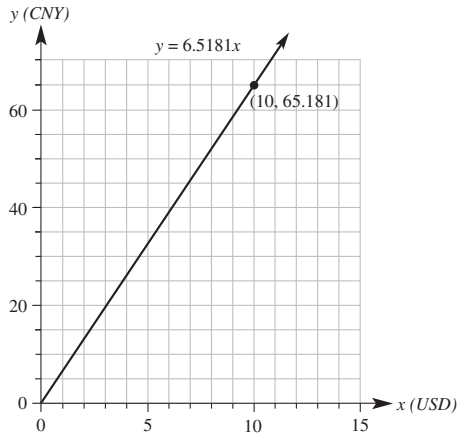
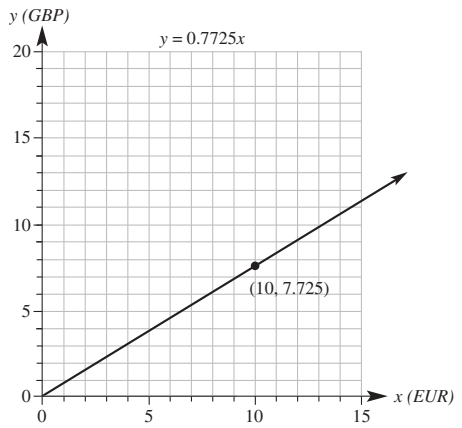
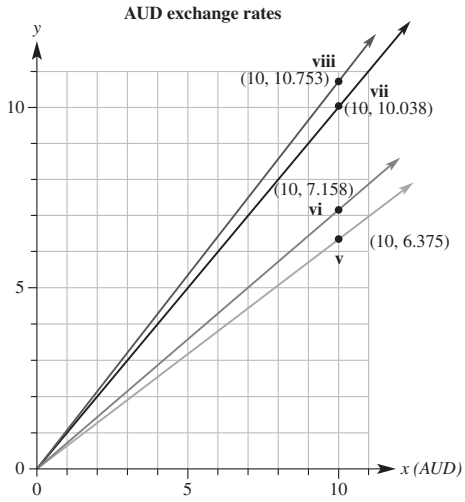
iii 

iv 

b The exchange rate is the gradient of each graph

c 

d The slope of the line increases as the exchange rate increases.

3 a *See figure at bottom of this page.*

b $y = 0.6235x + 79.691$; x is the date in May; y is the Japanese exchange rate for 1 Australian dollar on that date.

c The gradient tells us that the exchange rate is increasing by 0.6235 Japanese yen per day in May.

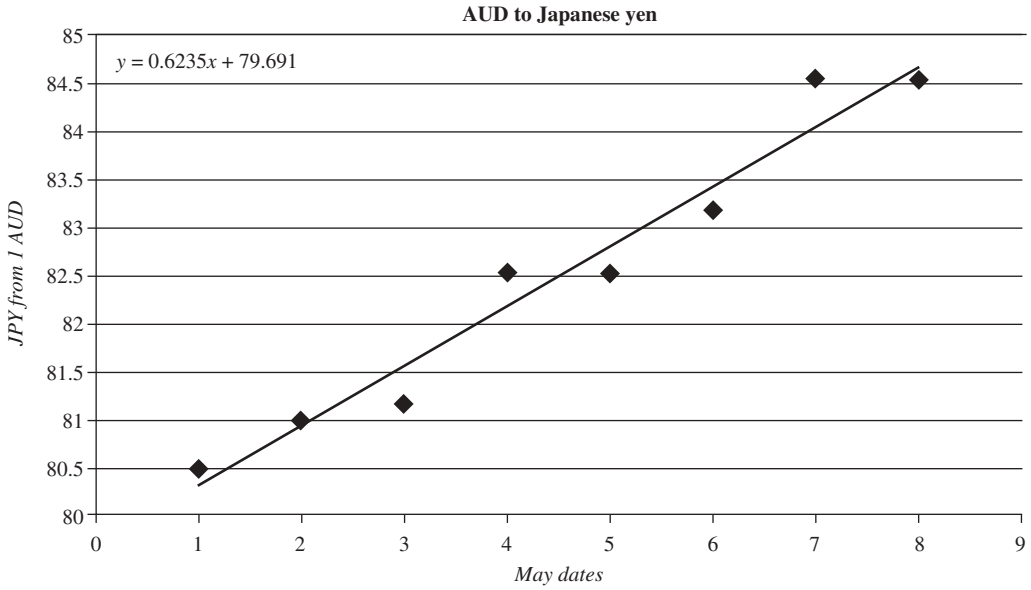
d 85.926 Japanese yen

e 133565.25 Japanese yen

f Exchange rates can vary at any time and don't necessarily follow a trend line.

Puzzles and challenges

1 101
2 602
3 20 days
4 (0.5, 5.5), diagonals intersect at their midpoint
5 Length AB = length $AC = \sqrt{17}$
6 31 hours

3a 

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Chapter checklist with success criteria

1

x	−2	−1	0	1	2
y	5	3	1	−1	−3

2 **a** no **b** yes

3 **a** x-intercept is 1, y-intercept is 4

b x-intercept is −2, y-intercept is 4

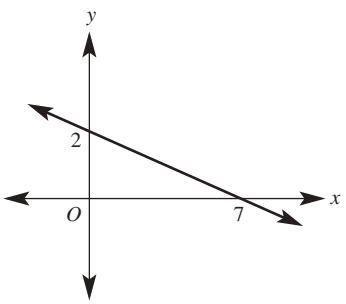
4 **a** −4

b −4

5 **a** $\frac{4}{3}$

b 5

6



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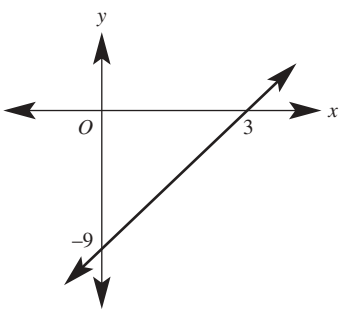
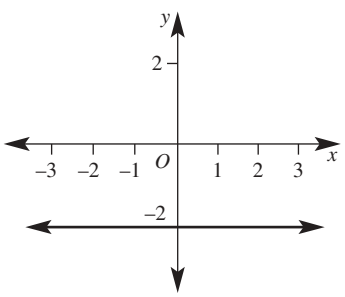
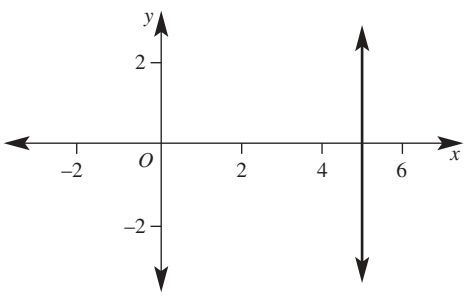


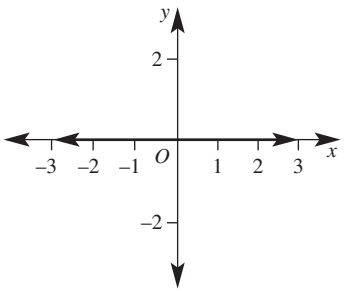
8 **a**



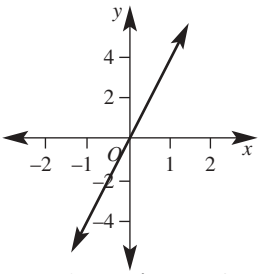
b



c



9



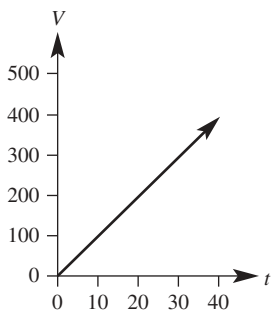
10 **a** $y = 2$ **b** $x = -1$

11 **a** undefined **b** negative

12 $\frac{4}{3}$

13 $d = 60t$

14 **a**



b 10 **c** 25 minutes

15 gradient = 3 and y-intercept = −2

16 **a** $y = -3x + 4$ **b** $y = -3x + 6$

17 **a** gradient = 4 and y-intercept = −3

b gradient = $-\frac{3}{2}$ and y-intercept = 5

18 $y = 2x - 6$

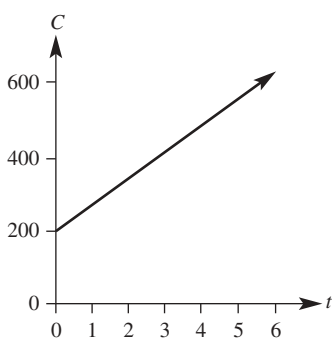
19 $y = 2x - 8$

20 (3, 2.5)

21 6.40

22 **a** $\$C = \$80t + \$200$

b



c \$520, 2.5 hours

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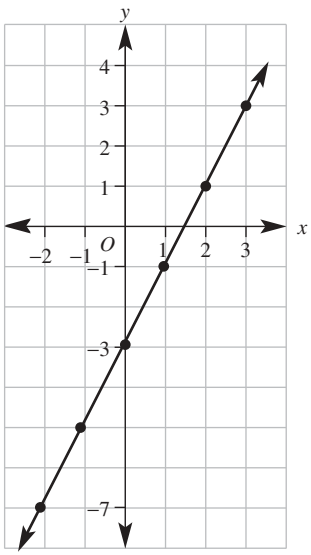
Chapter 5 review

Short-answer questions

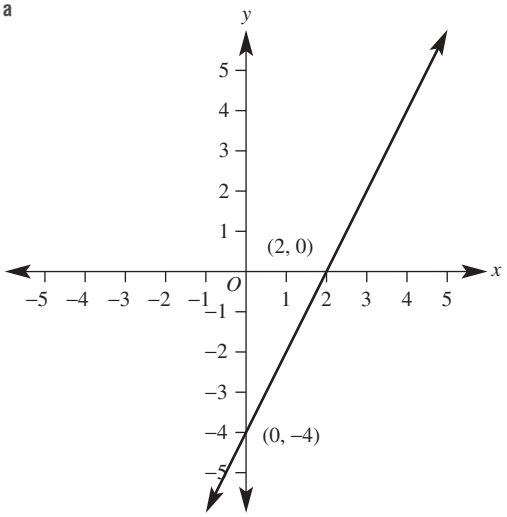
1

x	−2	−1	0	1	2	3
y	−7	−5	−3	−1	1	3

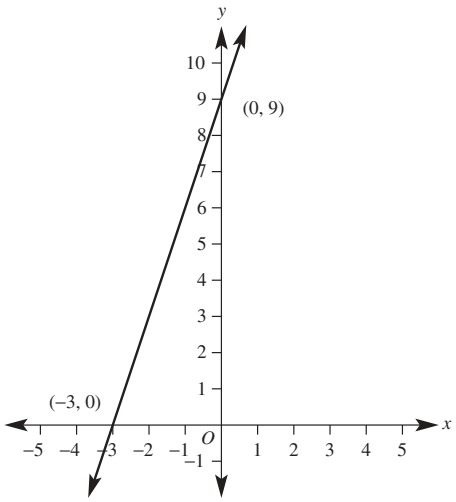


2 **a** x-int −2 and y-int 4 **b** x-int 3 and y-int −2

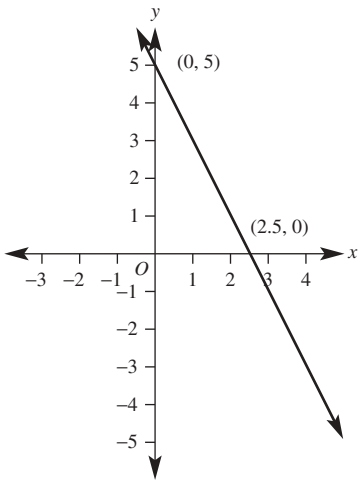
3 **a**



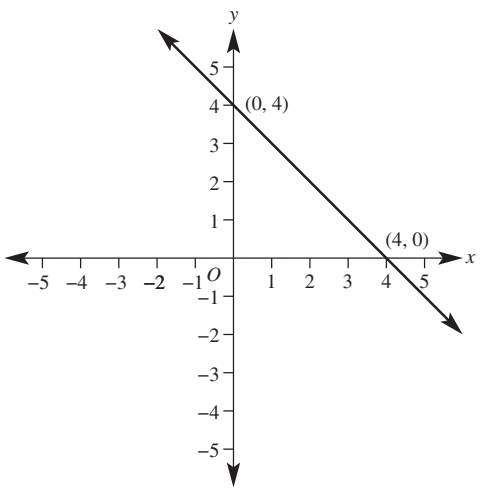
b



c



d



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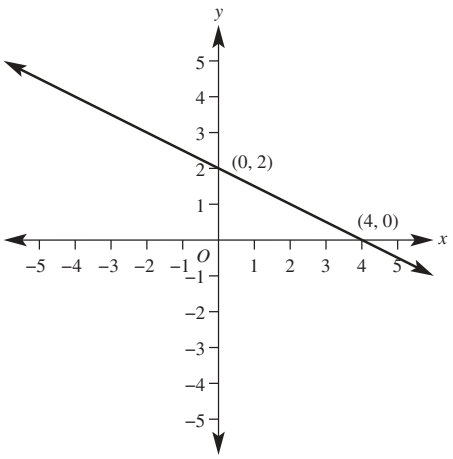
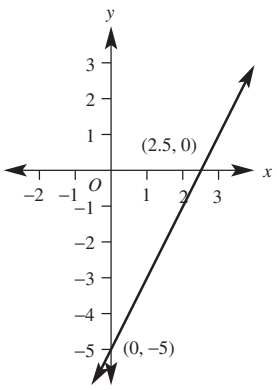
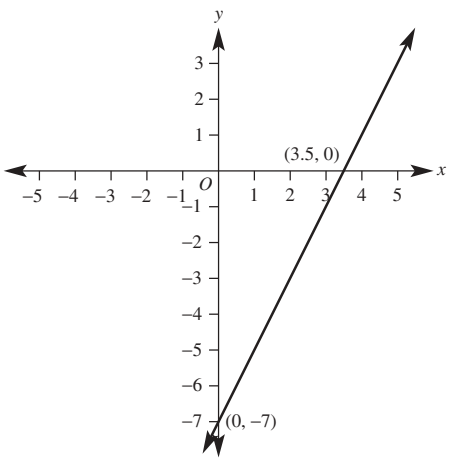
e



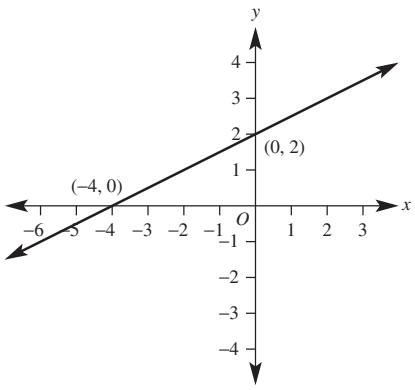
f



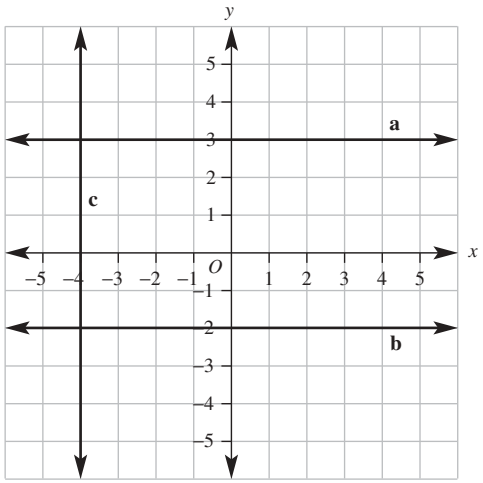
g



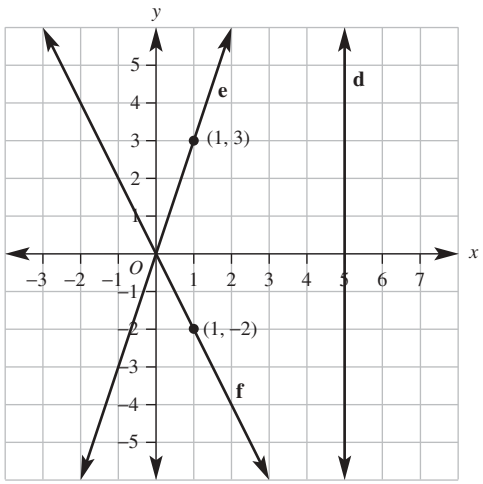
h



4 **a** Zero **b** Zero **c** Undefined



d Undefined **e** Positive **f** Negative



5 **a** 2 **b** −3

6 **a** 2 **b** −1 **c** 2 **d** $\frac{-10}{3}$

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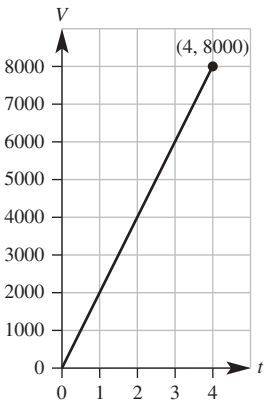
7 a 2000 L/h

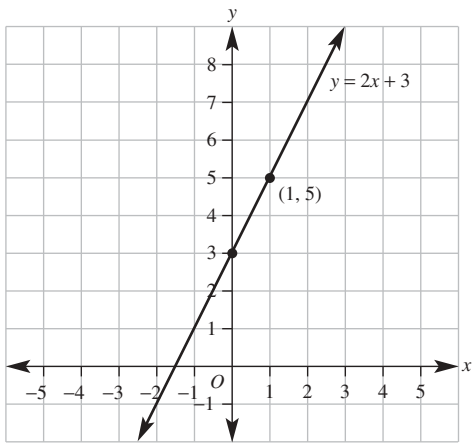
b



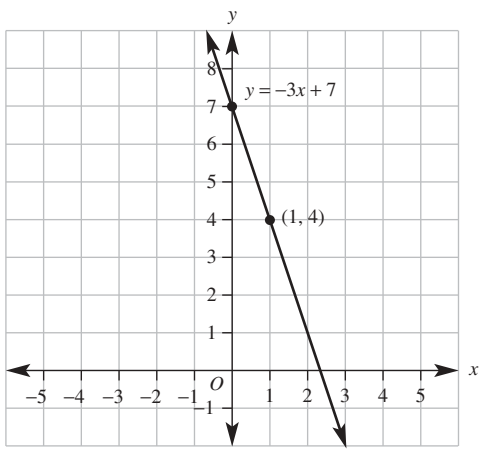
c $V = 2000t$

d 2.5 hours

8 a Gradient = 2, y-intercept = 3



b Gradient = −3, y-intercept = 7



9 a $y = -2x + 6$, gradient = −2, y-intercept = 6

b $y = -\frac{1}{2}x + 2$, gradient = $\frac{-1}{2}$, y-intercept = 2

10 a $y = 2x - 2$ **b** $y = -3x + 6$

c $y = 3x - 2$

11 a $y = 3x + 1$ **b** $y = -2x + 3$

12 a $M(4, 6)$, 5.66 **b** $M(7.5, 4.5)$, 7.07

c $M(0, 4)$, 7.21 **d** $M(-3, 2.5)$, 9.85

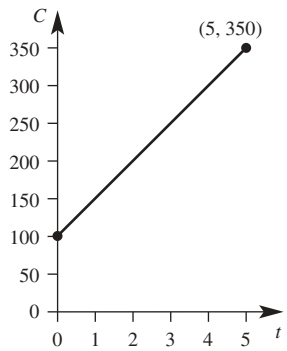
Multiple-choice questions

1 D **2** C **3** D **4** D **5** A **6** D
7 B **8** B **9** A **10** C

Extended-response questions

1 a $C = 50t + 100$

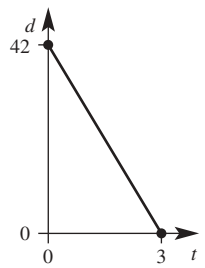
b



c $300

d $2\frac{1}{2}$ hours

2 a



b 42 km **c** 3 hours **d** −14 **e** 14 km/h

Semester review 1

Chapter 1: Integers, decimals, fractions, ratios and rates

Short-answer questions

1 a 1 **b** $\frac{10}{10} = 1$ **c** $2\frac{3}{4}$ **d** $\frac{7}{9}$

2 a $\frac{3}{8}$ **b** $1\frac{4}{5}$

3 a 4.126 **b** 21.002 **c** 0.010

4 a \$160, \$640 **b** \$320, \$480 **c** \$700, \$100

5 a 5 : 3 **b** 55 km/h **c** 24 mL/h

6 181.3 km

7 4 m

Multiple-choice questions

1 B **2** D **3** D **4** C **5** C

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Extended-response question

a 24 km
b **i** 14 km **ii** 2 p.m.

Chapter 2: Financial mathematics

Short-answer questions

1 **a** 60% **b** 32% **c** 10% **d** 25%
2 **a** \$9.60 **b** \$765 **c** \$11
3 **a** 84 m **b** \$50.76
4 4 h 48 min
5 \$892
6 \$67 484
7 **a** \$72 **b** \$872

Multiple-choice questions

1 B **2** D **3** A **4** D **5** A

Extended-response question

a \$17 500 **b** \$23 520 **c** 9 years **d** 27%

Chapter 3: Expressions and equations

Short-answer questions

1 **a** $-2x + 7y$ **b** $-15mn$ **c** $\frac{y}{3}$
2 **a** $x = 6$ **b** $x = 9$ **c** $m = \frac{3}{8}$ **d** $a = 3$
3 **a** $2m + 1 = 15$ **b** \$7
4 **a** 155 **b** 18

Multiple-choice questions

1 D **2** E **3** A **4** C **5** C

Extended-response question

a $p + 12$
b $p + (p + 12) = 38$; i.e. $2p + 12 = 38$
c Michael scored 13 points and Chris 25 points

Chapter 4: Right-angled triangles

Short-answer questions

1 **a** 6.3 **b** 15.1 **c** 9.9 **d** 6.3
2 **a** 8.1 **b** 11.2 **c** 4.2 **d** 1.5
e 53.1° **f** 29.5°
3 **a** 19.21 m **b** 38.7°
4 **a** $x = 13$ **b** $y = 14.72$

Multiple-choice questions

1 B **2** D **3** A **4** C **5** A

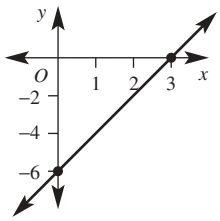
Extended-response question

a 17.75 m **b** 14.3° **c** 18.8 m

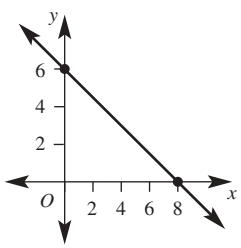
Chapter 5: Linear relationships

Short-answer questions

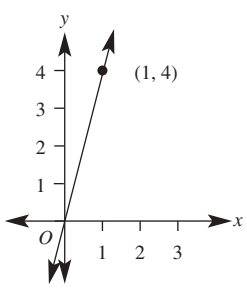
1 **a**



b



c



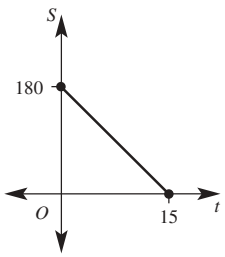
2 **a** $\frac{2}{3}$ **b** -3 **c** -2 **d** $\frac{4}{3}$
3 **a** $y = -3x + 6$ **b** $y = 3x - 1$

Multiple-choice questions

1 C **2** D **3** A **4** B **5** A

Extended-response question

a 12 kg/h
b $S = 180 - 12t$
c



d 15 hours
e **i** $P = 40 + 25h$ **ii** \$415

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Chapter 6

Warm-up quiz

1 **a** 4 cm^2 **b** 6 m^2 **c** 8 cm^2
2 **a** 8 cm **b** 10 m **c** 18 cm
3 **a** 3 **b** 6 **c** 27
4 **a** Cylinder **b** Circle
5 **a** 23 **b** 48 **c** 2.7
d 5.2134 **e** 50 **f** 72.16
6 7
7 **a** 30 mm **b** 2000 cm **c** 1600 m **d** 2.3 cm
e 3.167 km **f** 0.72 m **g** 20 km **h** 30 m
i 0.0756 km
8 **a** 30 cm^2 **b** 4 m^2 **c** 49 km^2
d 3 m^2 **e** 24 cm^2 **f** 66 m^2
g $C = 31.42$ m, $A = 78.54$ m^2

6A

Now you try

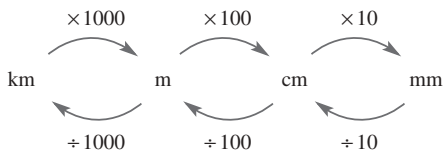
Example 1
a 240 cm **b** 42 cm

Example 2
a 9 cm **b** 6 mm

Example 3
33.71 m

Exercise 6A

1



2 add
3 **a** 7 **b** 26 **c** 2.1
4 **a** 50 mm **b** 410 mm **c** 280 cm **d** 40 cm
e 4600 m **f** 900 m **g** 52.1 cm **h** 3.6 cm
i 2.4 m **j** 0.837 m **k** 7 km **l** 2.17 km
5 825 cm, 2.25 cm
6 **a** 15 mm **b** 31 m **c** 86 cm **d** 12 m **e** 27 cm
f 24 mm **g** 18 km **h** 10 m **i** 42 cm
7 **a** $x = 4$ **b** $x = 2.2$ **c** $x = 14$
8 **a** $a = 3,\ b = 6$ **b** $a = 12,\ b = 4$
c $a = 6.2,\ b = 2$
9 **a** 90 cm **b** 80 cm **c** 170 cm **d** 30.57 m
e 25.5 cm **f** 15.4 km
10 108 m
11 **a** 8000 mm **b** 110 m **c** 1 cm **d** 20 mm
e 0.284 km **f** 62.743 km
12 **a** 86 cm **b** 13.6 m **c** 40.4 cm
13 88 cm
14 **a** **i** 96 cm **ii** 104 cm **iii** 120 cm
b $P = 4(20 + 2x)$ or $P = 80 + 8x$
c 109.6 cm

6B

Now you try

Example 4
23.88 m

Example 5
20.11 km

Example 6
10.71 cm

Exercise 6B

1 **a** Radius **b** Diameter **c** Circumference
d Sector
2 **a** 2.8 cm **b** 96 mm
3 **a** $C = 2\pi r$ **b** $C = \pi d$
4 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{3}{4}$
5 **a** 50.27 m **b** 87.96 cm **c** 31.42 m **d** 6.91 m
e 45.24 cm **f** 101.79 mm
6 **a** 31.42 m **b** 78.54 cm **c** 9.42 mm **d** 12.57 km
e 5.65 mm **f** 23.25 m
7 **a** 12.56 m **b** 62.8 cm **c** 22 mm **d** 44 m
8 **a** 10.28 m **b** 51.42 cm **c** 14.28 m **d** 10.00 cm
e 20.05 m **f** 106.73 km
9 28.27 m
10 4.1 m
11 **a** 12.57 cm **b** 102.83 mm **c** 41.06 m
12 **a** 188.50 cm
b **i** 376.99 cm **ii** 1979.20 cm
c 531

6C

Now you try

Example 7
a 3.2 m^2 **b** 400 000 m^2

Example 8
a 36 m^2 **b** 6.25 m^2 **c** 60 cm^2

Example 9
a 56 cm^2 **b** 15 m^2

Exercise 6C

1 **a** 100 **b** 10 000 **c** 1 000 000
2 **a** 6 **b** 16 **c** 12 **d** 1
e 12 **f** 153
3 **a** Rectangle **b** Rhombus/kite **c** Triangle
d Trapezium **e** Parallelogram **f** Square
4 **a** 200 mm^2 **b** 40 mm^2 **c** 5 cm^2
d 3.1 cm^2 **e** 21 000 cm^2 **f** 2000 cm^2
g 21 m^2 **h** 0.37 m^2 **i** 1000 m^2
j 4 300 000 m^2 **k** 3.2 km^2 **l** 0.0394 km^2

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5 **a** 24 m^2 **b** 10.5 cm^2 **c** 20 km^2
d 25.2 m^2 **e** 15 m^2 **f** 36.8 m^2
6 **a** 21 mm^2 **b** 12 cm^2 **c** 17 cm^2
d 63 m^2 **e** 6.205 m^2 **f** 15.19 km^2
7 500 000 m^2
8 0.175 km^2
9 **a** 12.25 cm^2 **b** 3.04 m^2 **c** 0.09 cm^2
d 6.5 mm^2 **e** 18 cm^2 **f** 2.4613 cm^2
10 0.51 m^2
11 **a** 1.5×10^{10}(15 000 000 000) cm^2
b 5 mm^2
c 0.075 m^2
12 **a** **i** 1.5 m **ii** 1.5 m
b 78 m^2
c Yes

6D

Now you try

Example 10
0.50 m^2

Example 11
25.13 cm^2

Exercise 6D

1 **a** 4.1 m **b** 7.5 m **c** 3.8 cm
2 C
3 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{3}{4}$
4 **a** 12.57 m^2 **b** 113.10 cm^2
c 254.47 km^2 **d** 60.82 mm^2
e 415.48 km^2 **f** 45 238.93 cm^2
5 **a** 314.16 cm^2 **b** 60.82 m^2 **c** 17.35 cm^2
6 **a** 21.23 m^2 **b** 216.51 km^2 **c** 196.07 cm^2
7 **a** 12.57 m^2 **b** 157.08 cm^2 **c** 84.82 m^2
8 177 cm^2
9 **a** 2 m **b** 12.57 m^2
c 3.43 m^2 **d** 21.5%
10 12.89%
11 **a** 1.8 cm **b** 6.1 m **c** 2 km
12 31%

6E

Now you try

Example 12
$P = 32$ m
$A = 64$ m^2

Example 13
$P = 142.83$ cm
$A = 171.68$ cm^2

Exercise 6E

1 **a** Semicircle and rectangle
b Triangle and semicircle
c Rhombus and parallelogram
2 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{3}{4}$ **d** $\frac{1}{4}$
3 **a** 4 cm^2 **b** 3.14 cm^2 **c** 7.14 cm^2
4 **a** 46 m, 97 m^2 **b** 34 m, 76 m^2
c 40 m, 90 m^2 **d** 18.28 m, 22.28 m^2
e 19.42 m, 26.14 m^2 **f** 85.42 mm, 326.37 mm^2
5 **a** $P = 14.28$ m, $A = 3.43$ m^2
b $P = 35.71$ m, $A = 10.73$ m^2
c $P = 41.13$ km, $A = 27.47$ km^2
6 **a** 108 m^2 **b** 33 cm^2
c 98 m^2 **d** 300 m^2
e 16 cm^2 **f** 22.5 m^2
7 189.27 m^2
8 68.67 cm^2
9 **a** 90 cm^2 **b** 15 m^2 **c** 9 m^2 **d** 7.51 cm^2
e 7.95 m^2 **f** 180.03 cm^2 **g** 8.74 mm^2
h 21.99 cm^2 **i** 23.83 mm^2
10 **a** 17 cm^2 **b** 3.5 cm^2 **c** 21.74 cm^2
11 **a** 37.70 m, 92.55 m^2
b 20.57 mm, 16 mm^2
c 18.00 cm, 11.61 cm^2
d 12.57 m, 6.28 m^2
e 25.71 cm, 23.14 cm^2
f 33.56 m, 83.90 m^2

6F

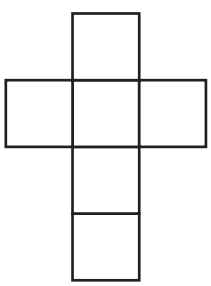
Now you try

Example 14
$A = 90$ cm^2

Example 15
$A = 336$ mm^2

Exercise 6F

1 **a** 6 **b** 6 **c** 5
2 **a**



Cube

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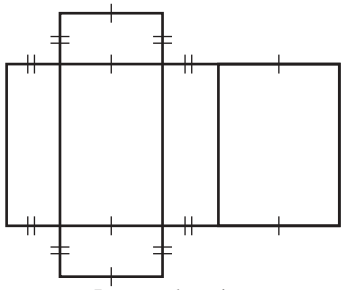
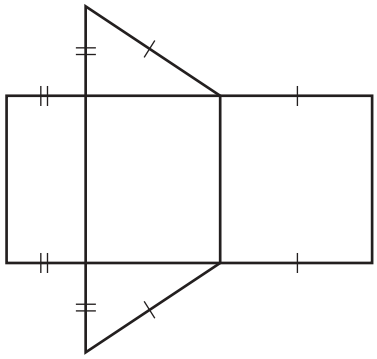
b



Rectangular prism

c



Triangular prism

3 a $A = 2 \times 8 \times 7 + 2 \times 8 \times 3 + 2 \times 7 \times 3$

$= 112 + 48 + 42$

$= 202$ m^2

b $A = 2 \times \frac{1}{2} \times 4 \times 3 + 5 \times 7 + 4 \times 7 + 3 \times 7$

$= 12 + 35 + 28 + 21$

$= 96$ cm^2

4 a 52 m^2 **b** 242 cm^2
c 76 m^2 **d** 192 cm^2
e 68.16 m^2 **f** 85.76 m^2

5 a 96 cm^2 **b** 240 m^2 **c** 199.8 cm^2 **d** 238 cm^2

6 6 m^2

7 14.54 m^2

8 34 000 cm^2

9 a 44.4 m^2 **b** 4.44 L

10 a [6, 10, 14, 18, 22, 26, 30, 34, 38]
b $S = 4n + 2$ **c** 402

11 a 39 mm^2 **b** 224 cm^2 **c** 9.01 m^2

Progress quiz

1 a 21 cm **b** 42 m

2 a 25.13 cm **b** 31.42 m

3 a 8.24 m **b** 300 mm **c** 1240 m **d** 36 000 cm^2
e 45 000 mm^2 **f** 0.0832 m^2

4 a 45 cm^2 **b** 40 m^2 **c** 40 mm^2 **d** 20 cm^2

5 a 153.9 mm^2 **b** 176.7 cm^2

6 a 46.3 cm **b** 127.2 cm^2

7 a **i** 33.4 cm **ii** 64.3 cm^2
b **i** 26 m **ii** 30 m^2

8 a 392 cm^2 **b** 360 cm^2

6G

Now you try

Example 16
175.93 m^2

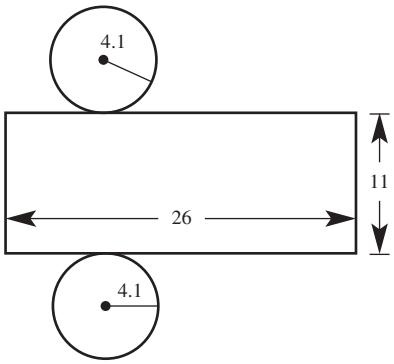
Example 17
56.27 m^2

Exercise 6G

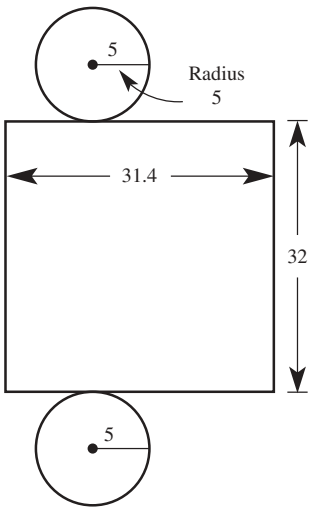
1 a $2\pi rh$ **b** $2\pi r^2$

2 a 25.13 m **b** 18.85 mm **c** 26.39 cm

3 a



b



4 a 22 cm by 10 cm **b** 12.57 cm by 8 cm
c 50.27 m by 5 m

5 a 25.13 m^2 **b** 471.24 cm^2 **c** 50.27 m^2

6 a 44.0 cm^2 **b** 603.2 cm^2 **c** 113.1 m^2

7 a 251.33 cm^2 **b** 207.35 mm^2 **c** 24.13 m^2

8 395.84 cm^2

9 a 54.56 m^2 **b** 218.23 m^2 **c** 63.98 cm^2
d 71.91 cm^2 **e** 270.80 m^2 **f** 313.65 km^2

10 7539.82 cm^2

11 Half cylinder is more than half surface area as it includes new rectangular surface.

12 a 18 849.556 cm^2
b **i** 1.88 m^2 **ii** 37.70 m^2
c 239

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6H

Now you try

Example 18
a 600 mm^3 **b** 4.52 m^3

Example 19
40 m^3

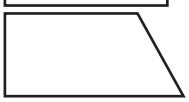
Example 20
15 mm^3

Exercise 6H

1 **a** Square **b** Triangle **c** Rectangle
d Trapezium **e** Hexagon **f** Triangle

2 **a**

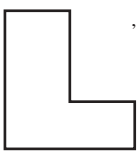
b



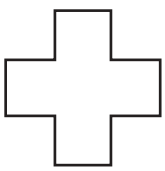
c

d

e



f



3 **a** 10 **b** 100 **c** 1000
d 1 000 000 **e** 1 000 000 000
f 1000 **g** 1000 **h** 1

4 **a** 3000 mm^3 **b** 300 mm^3 **c** 2 cm^3
d 1000 cm^3 **e** 8 700 000 cm^3
f 0.0059 m^3 **g** 10 000 m^3 **h** 0.0000217 km^3
i 0.43 m^3

5 **a** 3000 mL **b** 200 L **c** 3.5 L **d** 21 mL
e 37 kL **f** 42.9 ML **g** 2 mL
h 2000 cm^3 **i** 1000 L

6 **a** 8 cm^3 **b** 84 m^3 **c** 21 mm^3

7 **a** 16 cm^3 **b** 42.875 m^3 **c** 15 cm^3

8 **a** 4 cm^3 **b** 10.5 m^3
c 11.96 cm^3 **d** 29 cm^3
e 14.88 m^3 **f** 8.1351 cm^3

9 **a** 75 m^3 **b** 30 cm^3 **c** 1.25 cm^3

10 1.6 L

11 Yes, the tank only holds 20 L

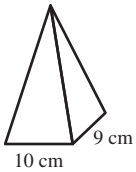
12 **a** 8 L **b** 0.36 L **c** 0.48 L

13 **a** 55 m^2 **b** 825 000 L

14 **a** Answers will vary.

b The volume of the pyramid is $\frac{1}{3}$ the volume of the prism

c one example



Volume pyramid $= \frac{1}{3} \times 10 \times 9$
$= 30\text{ cm}^3$

6I

Now you try

Example 21
a 1696.46 cm^3 **b** 22 167.08 mm^3

Example 22
14 litres

Exercise 6I

1 **a** 12.57 cm^2 **b** 8.04 m^2 **c** 78.54 cm^2 **d** 2.54 km^2

2 **a** 2 L **b** 4.3 mL **c** 3700 cm^3 **d** 1000 L
e 38 m^3 **f** 200 mL

3 **a** $r = 4$ m, $h = 10$ m **b** $r = 2.6$ cm, $h = 11.1$ cm
c $r = 2.9$ m, $h = 12.8$ m **d** $r = 9$ m, $h = 23$ m
e $r = 5.8$ cm, $h = 15.1$ cm
f $r = 10.65$ cm, $h = 10.4$ cm

4 **a** 226.19 cm^3 **b** 18.85 m^3 **c** 137.44 m^3
d 100.53 cm^3 **e** 8.48 m^3 **f** 68.05 m^3

5 **a** 18 L **b** 503 L **c** 20 L **d** 2 L

6 **a** 1.571 m^3 **b** 1571 L

7 Cylinder by 131 cm^3

8 **a** 25.133 m^3 **b** 25 133 L

9 37 699 L

10 A number of answers. Require $h = 2\pi r$.

11 **a** 502.65 cm^3 **b** 1.02 m^3
c 294.52 m^3 **d** 35 342.92 m^3
e 47.12 cm^3 **f** 1017.88 cm^3

Maths@Work: Vegetable and fruit growers

1 **a** **i** 0.89 ML **ii** 60 ML **iii** 800 ML
iv 380 ML **v** 190 000 ML
b **i** 9000 m^3 **ii** 1200 m^3 **iii** 120 000 m^3
iv 980 m^3 **v** 12 000 000 m^3

2 **a** 720 m^3 **b** 400 m^3 **c** 1280 m^3
d 10 000 m^3 **e** 38 400 m^3

3 **a** 95058 L; 95 kL **b** 29452 L; 29 kL

4

Plant	Percentage moisture used by plant	Plant watering depth in mm
Onions	4%	16
Carrots	5%	30
Tomatoes	4%	48
Apple trees	6%	90
Banana trees	11%	66

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5

Plant	Area of irrigation in m^2	Litres of water per plant per irrigation	Litres of water per 100 trees
Apple trees	0.1385	12	1247
Banana trees	0.3217	21	2123

Puzzles and challenges

1 **a** 35 **b** 19
2 8
3 314 m
4 163.4 m^2
5 7.1 cm
6 11
7 100 L

Chapter checklist with success criteria

1 **a** 92 mm **b** 610 m
2 **a** 21 m **b** 36 cm
3 15.71 m
4 20.57 cm
5 **a** 532 mm^2 **b** 72.8 m^2
6 56 m^2
7 46 cm^2
8 201.06 cm^2
9 38.48 m^2
10 35.42 cm, 74.14 cm^2
11 180 cm^2
12 108 m^2
13 439.82 cm^2
14 148.53 m^2
15 **a** 3000 cm^3 **b** 640 cm^3
16 120 m^3
17 180 cm^3
18 282.74 cm^2
19 9 L

Chapter 6 review

Short-answer questions

1 **a** 380 cm **b** 1270 m **c** 4.8 cm **d** 2.73 cm^2
e 52 000 cm^2 **f** 10 000 cm^3 **g** 53.1 cm^3 **h** 3.1 L
i 43 mL **j** 2830 L **k** 4 L **l** 1000 L
2 **a** 14 m **b** 51 mm **c** 16.2 cm
3 **a** 4 cm^2 **b** 1122 mm^2 **c** 30.34 mm^2 **d** 7.5 m^2
e 15 cm^2 **f** 3 cm^2
4 **a** 2.5 m^2 **b** 37.4 m^2
5 **a** $A = 28.27$ cm^2, $P = 18.85$ cm
b $A = 1.57$ m^2, $P = 5.14$ m
c $A = 2.36$ cm^2, $P = 6.71$ cm
6 **a** $P = 22.28$ m, $A = 30.28$ m^2
b $P = 45.56$ cm, $A = 128.54$ cm^2
7 **a** 46 cm^2 **b** 114 m^2
8 **a** 659.73 mm^2 **b** 207.35 m^2
9 **a** 30 cm^3 **b** 54 m^3 **c** 31.42 mm^3 **d** 60 m^3

Multiple-choice questions

1 B **2** C **3** E **4** B
5 A **6** D **7** B **8** E
9 C **10** E

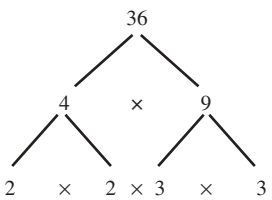
Extended-response questions

1 **a** 135 cm^3
b 174 cm^2
c $43.50
d **i** Yes, by $\sim$ 119 cm^3 **ii** Yes, by $\sim$ 52 cm^2
2 **a** 5.17 m
b $65
c 1.59 m^2, claim is correct

Chapter 7

Warm-up quiz

1



$36 = 2 \times 2 \times 3 \times 3$

2 **a** 24, 1, 12, 2, 8, 3, 6, 4 **b** 45, 1, 15, 3, 9, 5
c 2, 3 **d** 3, 5
3 **a** 7^3 **b** $5^2 \times 2^3$ **c** 3^2 **d** 4^1 or 4
4 **a** 2^3 **b** 2^6 **c** 2^5
5 **a** 25 **b** 100 **c** 1 **d** 5 **e** 1
6 **a** $3 \times 2 \times 2 = 12$ **b** $4 \times 3 \times 3 = 36$
c $5 \times 2 \times 2 \times 2 = 40$
d $3 \times 10 \times 10 \times 10 \times 10 = 30\,000$
7 **a** 3^4 **b** 2^{12} **c** 9
8 **a** 0.003 **b** 0.04 **c** 0.000 002 **d** 0.0008
9 **a** 3.73 **b** 24.62 **c** 18.37 **d** 4.40
10 **a** 40 **b** $\frac{4}{1000}$ or 0.004
c 4 000 000 **d** $\frac{4}{10}$ or 0.4
11 **a** 5 **b** 4 **c** 2 **d** 4
12 **a** 38 **b** 2310 **c** 0.172
d 0.0018 **e** 1000 **f** 10 000

7A

Now you try

Example 1
a $6 \times 6 \times 6$ **b** $b \times b \times b \times b$
c $mn \times mn$ **d** $3 \times x \times x \times y$

Example 2
a 8 **b** -27 **c** $\frac{9}{25}$

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Example 3

a 9^3 **b** $2a^2$ **c** 11^2b^3

Example 4

a $\left(\frac{5}{7}\right)^4$ **b** $\left(\frac{2}{5}\right)^2 \times \left(\frac{4}{7}\right)^3$

Example 5

a $7^3x^3y^2$ **b** $2^37^2m^3s^2$ **c** $3^4a^4b^4$

Example 6

$2^2 \times 3 \times 5$

Example 7

a 36 **b** $-\frac{8}{27}$ **c** 33

Exercise 7A

1 **a** expanded **b** index **c** power
d base **e** index, power or exponent
f prime **g** prime factors

2 **a** 25 **b** 8 **c** 27 **d** 16

3 **a** 3 **b** 6 **c** 1.2 **d** -7
e $\frac{2}{3}$ **f** y **g** w **h** t

4 **a** 3 **b** 8 **c** 7 **d** 4
e 11 **f** 13 **g** 9 **h** 2

5 **a** 2, 3 **b** 3, 5 **c** 2, 3, 5 **d** 7, 11

6 **a** $4 \times 4 \times 4$
b $7 \times 7 \times 7 \times 7$
c $3 \times 3 \times 3 \times 3 \times 3$
d $5 \times 5 \times 5$
e $a \times a \times a \times a$
f $b \times b \times b$
g $x \times x \times x$
h $xp \times xp \times xp \times xp \times xp \times xp$
i $5a \times 5a \times 5a \times 5a$
j $3y \times 3y \times 3y$
k $4 \times x \times x \times x \times y \times y \times y \times y \times y$
l $pq \times pq$
m $-3 \times s \times s \times s \times s \times t \times t$
n $6 \times x \times x \times x \times x \times y \times y \times y \times y \times y \times y$
o $5 \times yz \times yz \times yz \times yz \times yz \times yz$
p $4 \times ab \times ab \times ab$

7 **a** 36 **b** 16 **c** 243 **d** 12
e -8 **f** -1 **g** 81 **h** 25
i $\frac{8}{27}$ **j** $\frac{9}{16}$ **k** $\frac{1}{216}$ **l** $\frac{25}{4}$
m $-\frac{8}{27}$ **n** $\frac{81}{256}$ **o** $\frac{1}{16}$ **p** $-\frac{3125}{32}$

8 **a** 3^3 **b** 8^6 **c** y^2 **d** $3x^3$
e $4c^5$ **f** 5^3d^2 **g** x^2y^3 **h** 7^3b^2

9 **a** $\left(\frac{2}{3}\right)^4$ **b** $\left(\frac{3}{5}\right)^5$ **c** $\left(\frac{4}{7}\right)^2 \times \left(\frac{1}{5}\right)^4$
d $\left(\frac{7x}{9}\right)^2 \times \left(\frac{y}{4}\right)^3$

10 **a** $3^3x^3y^2$ **b** $(3x)^2(2y)^2$ or $3^22^2x^2y^2$
c $(4d)^2(2e)^2$ or $4^22^2d^2e^2$ **d** $(6by)^3$ or $6^3b^3y^3$
e $(3pq)^4$ or $3^4p^4q^4$ **f** $(7mn)^3$ or $7^3m^3n^3$

11 **a** 2×5 **b** 2^3 **c** $2^4 \times 3^2$
d 3×5^2 **e** 3×7^2 **f** $2^2 \times 5^3$

12 **a** $3 \times 3 \times a \times a \times a = 3^2a^3$ **b** $5 \times 5 \times k \times k \times k = 5^2k^3$
c $\frac{2}{7} \times \frac{2}{7} \times \frac{2}{7} = \left(\frac{2}{7}\right)^3$ **d** $3p^2q = 3 \times p \times p \times q$
e $(abc) = a \times b \times c$
f $4ab^2 = -2 \times (-2) \times a \times b \times b$

13 **a** 36 **b** -216 **c** 1 **d** $-\frac{8}{27}$
e -18 **f** 15 **g** -36 **h** 216

14 **a** 4 **b** 8 **c** 5 **d** 2 **e** -4
f -2 **g** $\frac{1}{2}$ **h** 4 **i** 1

15 **a**

Time in minutes	Number of bacteria	Number in index form
0	1	2^0
1	$1 \times 2 = 2$	2^1
2	$2 \times 2 = 4$	2^2
3	$2 \times 2 \times 2 = 8$	2^3
4	$2 \times 2 \times 2 \times 2 = 16$	2^4
5	$2 \times 2 \times 2 \times 2 \times 2 = 32$	2^5
6	$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$	2^6
7	$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 128$	2^7
8	$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256$	2^8
9	$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 512$	2^9
10	$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 1024$	2^{10}

b **i** 2 min **ii** 4 min **iii** 6 min
c $2^{24} = 16\,777\,216$ cells

7B

Now you try

Example 8

a 2^{11} **b** 7^6 **c** 11^8 **d** 5^9

Example 9

a a^8 **b** m^3n^7

Example 10

y^6

Example 11

a $20a^5$ **b** $4b^5$ **c** $\frac{3}{4}x^2y = \frac{3x^2y}{4}$

Example 12

a a **b** $6m^2n^2$

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Exercise 7B

1 **a** multiply, base, add
b divide, base, subtract

2 **a** $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$
b $k \times k \times k \times k \times k \times k = k^5$

3 **a** $\frac{5 \times 5 \times \cancel{5}^{1} \times \cancel{5}^{1} \times \cancel{5}^{1} \times \cancel{5}^{1}}{\cancel{5}^{1} \times \cancel{5}^{1} \times \cancel{5}^{1}} = 5^2$

b $\frac{a \times a \times a \times a \times \cancel{a}^{1} \times \cancel{a}^{1}}{\cancel{a}^{1} \times \cancel{a}^{1}} = a^4$

4 **a** $6^{5+7} = 6^{12}$ **b** $a^{13+2} = a^{15}$ **c** $5^{12-4} = 5^8$
d $m^{16-2} = m^{14}$

5 **a** 2^7 **b** 5^9 **c** 7^6 **d** 8^{10}
e 3^8 **f** 6^{14} **g** 3^3 **h** 6^5
i 5^3 **j** 10 **k** 9^3 **l** $(-2)^2$

6 **a** x^7 **b** a^9 **c** t^8 **d** y^5
e d^3 **f** y^7 **g** b^8 **h** q^{11}
i a^5m^4 **j** k^5p^3 **k** x^6y^8 **l** m^7e^4

7 **a** a^2 **b** x^3 **c** q^{10} **d** d
e b^5 **f** d^5 **g** a^7 **h** y^8

8 **a** $7x^7y^5$ **b** $3x^9y^4$ **c** $5x^4y^9$ **d** $4x^2y^5z$
e $15m^5$ **f** $8e^6f^4$ **g** $20c^7d^2$ **h** $18y^2z^7$
i $3m$ **j** $7x^3$ **k** $5y^2$ **l** $3a$
m $3m^5$ **n** $\frac{w}{5}$ **o** $\frac{a}{5}$ **p** $\frac{x^4}{9}$
q $\frac{4x^6y^3}{3}$ **r** $\frac{3st^2}{7}$ **s** $\frac{4mn}{3}$ **t** $5x$

9 **a** b^6 **b** y^6 **c** c^7 **d** x
e t **f** p^6 **g** d^6 **h** x^{10}
i $4x^2y^3$ **j** $6b^2g$

10 **a** $\frac{m^5}{n^5}$ **b** $\frac{x^4}{y^2}$ **c** a^3b^3 **d** $\frac{6a^5}{c^7}$
e $6f^6$ **f** $12x^4b^2$

11 **a** 12 **b** 8 **c** 3 **d** 3
e 1 **f** 18 **g** 12 **h** 11
i 4 **j** 15 **k** 2 **l** 39

12 **a** $7^2 = 49$ **b** 10 **c** $13^2 = 169$ **d** $2^3 = 8$
e 101 **f** $200^2 = 40\,000$
g $7 \times 31 = 217$ **h** $3 \times 50^2 = 7500$

13 **a** **i** $10a^{10}$ **ii** $9m^8$ **iii** $150m^5$
iv $60x^6y^5$
b **i** $4am$ **ii** $6xy^2$

7C

Now you try

Example 13
a a^{21} **b** $2b^8$

Example 14
a 1 **b** 2 **c** 3

Example 15
a^{17}

Example 16
a^2

Example 17
$\frac{3}{2}$

Exercise 7C

1 **a** multiply **b** 1

2 **a** 16, 8, 4, 2, 1 **b** 64, 16, 4, 1

3 **a** $4^2 \times 4^2 \times 4^2 = 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6$
b $12^3 \times 12^3 = 12 \times 12 \times 12 \times 12 \times 12 \times 12 = 12^6$
c $x^4 \times x^4 = x \times x \times x \times x \times x \times x \times x \times x = x^8$
d $a^2 \times a^2 \times a^2 \times a^2 \times a^2 = a \times a \times a \times a \times a \times a \times a \times a \times a \times a = a^{10}$

4 **a** 1 **b** 1 **c** 1 **d** 1
e 1 **f** 1 **g** 1 **h** 1

5 **a** y^{12} **b** m^{18} **c** x^{10} **d** b^{12}
e 3^6 **f** 4^{15} **g** 3^{30} **h** 7^{10}
i $5m^{16}$ **j** $4q^{28}$ **k** $-3c^{10}$ **l** $2j^{24}$

6 **a** 1 **b** 1 **c** 1 **d** 1
e 1 **f** 1 **g** 1 **h** 1
i 5 **j** -3 **k** 4 **l** -6
m 1 **n** 3 **o** 4 **p** 0

7 **a** 4^7 **b** 3^9 **c** x **d** y^{13}
e b^{14} **f** a^{10} **g** d^{24} **h** y^{16}
i z^{25}

8 **a** b^6 **b** x^5 **c** y^6 **d** 7^2 **e** 4
f 3^8 **g** 1 **h** y^3 **i** h^2

9 **a** 2 **b** $10x$ **c** $3x^8$ **d** $\frac{d^2e}{2}$
e $\frac{2m^6n}{5}$ **f** $\frac{a^{12}}{8}$

10 5 ways: $(a^{16})^1 = a^{16}$, $(a^1)^{16} = a^{16}$, $(a^2)^8 = a^{16}$, $(a^8)^2 = a^{16}$, $(a^4)^4 = a^{16}$

11 **a** 4×5 not $4 + 5$, a^{20}
b Power of 2 only applies to x^4, $3x^8$
c Power zero applies to whole bracket, 1

12 **a** **i** 400 **ii** 6400 **iii** 100
b **i** 800 **ii** 12 800 **iii** 102 400
c 13 years

7D

Now you try

Example 18
a $64x^3$ **b** $16y^{16}$

Example 19
a $\frac{a^2}{b^2}$ **b** $\frac{27a^3}{b^6}$

Example 20
a $16a^4b^2$ **b** $6a^{12}b^8$

Example 21
$-\frac{27x^6}{8y^3z^9}$

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Example 22

a $18a^2b^4$ **b** x

Exercise 7D

1 a $a^m \times b^m$ **b** $\dfrac{a^m}{b^m}$

2 a $ab \times ab \times ab \times ab$

$= a \times a \times a \times a \times b \times b \times b \times b$

$= a^4 \times b^4$

b $\dfrac{x}{6} \times \dfrac{x}{6} \times \dfrac{x}{6}$

$= \dfrac{x \times x \times x}{6 \times 6 \times 6}$

$= \dfrac{x^3}{6^3}$

3 a x^5y^5 **b** 2^3m^3 **c** $a^4b^4c^4$

d $\dfrac{5^3}{k^3}$ **e** $\dfrac{a^5}{m^5}$ **f** $\dfrac{2^4}{3^4} = \dfrac{16}{81}$

4 a $6a^2b^2$

b $4x^3y^3$

c $7 \times 2^2m^2 = 7 \times 4 \times m^2 = 28m^2$

d $4 \times 3^2a^2b^2 = 4 \times 9a^2b^2 = 36a^2b^2$

5 a $8x^3$ **b** $25y^2$ **c** $64a^3$ **d** $9r^2$

e $81b^4$ **f** $343r^3$ **g** $16h^8$ **h** $625c^8$

i $27a^6$ **j** $49p^8$ **k** $25m^6$ **l** $9y^{20}$

6 a $\dfrac{p^3}{q^3}$ **b** $\dfrac{x^4}{y^4}$ **c** $\dfrac{64}{y^3}$ **d** $\dfrac{625}{p^8}$

e $\dfrac{4}{r^6}$ **f** $\dfrac{s^6}{49}$ **g** $\dfrac{49}{x^6}$ **h** $\dfrac{w^8}{100}$

i $\dfrac{a^6}{125}$ **j** $\dfrac{m^8}{16}$ **k** $\dfrac{32m^5}{n^5}$ **l** $\dfrac{8a^6}{27}$

7 a $32x^{15}y^{10}$ **b** $9p^6q^{12}$ **c** $2x^6y^2$

d 1 **e** $-27w^9y^3$ **f** $-4p^8q^2r^2$

g $25s^{14}t^2$ **h** $8x^{12}y^3z^9$ **i** $24a^3b^6$

8 a $\dfrac{27n^9}{8m^{12}}$ **b** $\dfrac{16r^4}{n^4}$ **c** $\dfrac{9f^2}{64g^{10}}$ **d** $\dfrac{25w^8y^2}{4x^6}$

e $\dfrac{9x^2}{4y^6g^{10}}$ **f** $\dfrac{27k^3m^9}{64n^{21}}$ **g** $-\dfrac{25w^8y^2}{4z^2x^6}$ **h** $\dfrac{9x^4y^6}{4a^{10}b^6}$

9 a $9ab^2$ **b** $27ab^6$ **c** $-12a^8b^8$ **d** $54x^6y^9$

e $-64b^6c^{15}d^3$ **f** $8a^4$ **g** $9a^5$

h $-40a^{15}b^3$ **i** $160m^{15}p^5t^{10}$

j $\dfrac{-27x^6}{125a^{15}b^9}$ **k** $a^{11}bc^5$ **l** $x^{11}y^2z$

10 a i 8 **ii** 125

b $N = \dfrac{t^3}{8}$

c i 27 **ii** 8

d i 8 **ii** 2

7E

Now you try

Example 23

a $\dfrac{1}{6^4}$ **b** $\dfrac{7}{2^4}$

Example 24

a 10^3 **b** 5×9^4

Example 25

$\dfrac{7}{5^2} = \dfrac{7}{25}$

Example 26

$\dfrac{3}{100} = 0.03$

Example 27

a $\dfrac{1}{32}$ **b** 54 **c** $\dfrac{25}{16}$

Exercise 7E

1 a $\dfrac{1}{2^2}$ **b** $\dfrac{1}{3^2}$ **c** $\dfrac{1}{5^3}$ **d** $\dfrac{1}{3^3}$

2 a

Index form	3^4	3^3	3^2	3^1
Whole number or fraction	81	27	9	3

Index form	3^0	3^{-1}	3^{-2}	3^{-3}
Whole number or fraction	1	$\dfrac{1}{3}$	$\dfrac{1}{9} = \dfrac{1}{3^2}$	$\dfrac{1}{27} = \dfrac{1}{3^3}$

b

Index form	10^4	10^3	10^2	10^1
Whole number or fraction	10 000	1000	100	10

Index form	10^0	10^{-1}	10^{-2}	10^{-3}
Whole number or fraction	1	$\dfrac{1}{10}$	$\dfrac{1}{100} = \dfrac{1}{10^2}$	$\dfrac{1}{1000} = \dfrac{1}{10^3}$

3 a $10^{-4} = \dfrac{1}{10^4}$ **b** $3^{-2} = \dfrac{1}{3^2}$ **c** $7^{-3} = \dfrac{1}{7^3}$

4 a $\dfrac{1}{3^{-4}} = 3^4$ **b** $\dfrac{1}{7^{-6}} = 7^6$ **c** $\dfrac{1}{8^{-3}} = 8^3$

5 a $\dfrac{1}{5^2}$ **b** $\dfrac{1}{7^4}$ **c** $\dfrac{1}{8^3}$ **d** $\dfrac{1}{3^5}$

e $\dfrac{1}{9^2}$ **f** $\dfrac{1}{10^3}$ **g** $\dfrac{1}{4^5}$ **h** $\dfrac{1}{2^3}$

6 a $\dfrac{3}{2^4}$ **b** $\dfrac{5}{4^3}$ **c** $\dfrac{7}{5^6}$ **d** $\dfrac{2}{3^4}$ **e** $\dfrac{4}{3^5}$

f $\dfrac{9}{5^2}$ **g** $\dfrac{8}{7^3}$ **h** $\dfrac{6}{5^6}$ **i** $\dfrac{1}{4^2}$

7 a 2^4 **b** 3^2 **c** 4^3 **d** 6^5

e 5^3 **f** 8^5 **g** 7^3 **h** 9^4

8 a 6×4^3 **b** 5×8^2 **c** 4×7^5 **d** 3×2^5

e 12×5^4 **f** 6×10^3 **g** 7×9^4 **h** 7×10^6

9 a $\dfrac{6}{25}$ **b** $\dfrac{2}{9}$ **c** $\dfrac{4}{125}$ **d** $\dfrac{6}{7}$ **e** $\dfrac{1}{250}$

f $\dfrac{1}{5}$ **g** $\dfrac{5}{16}$ **h** $\dfrac{4}{5}$ **i** $\dfrac{1}{9}$

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10 a $\frac{2}{1000} = 0.002$ **b** $\frac{5}{100} = 0.05$

c $\frac{7}{10} = 0.7$ **d** $\frac{3}{10\,000} = 0.0003$

e $\frac{5}{10\,000} = 0.0005$ **f** $\frac{8}{100\,000} = 0.00008$

g $\frac{2}{1\,000\,000} = 0.000002$ **h** $\frac{4}{100\,000\,000} = 0.00000004$

11 a $\frac{1}{5}$ **b** $\frac{1}{9}$ **c** $\frac{1}{25}$ **d** $-\frac{1}{200}$

e $\frac{1}{2}$ **f** $\frac{1}{36}$ **g** $\frac{1}{8}$ **h** 8

i 16 **j** 64 **k** $\frac{64}{9}$ **l** $\frac{27}{64}$

12 a Negative power only applies to x, $\frac{2}{x^2}$

b $5 = 5^1$ has a positive power, $5a^{-4}$

c $\frac{2}{3^{-2}b^{-2}} = 2 \times 3^2 \times b^2 = 18b^2$

13 a 1.95 g **b** $3\,486\,784.401t$

Progress quiz

1 a $b \times b \times b \times b$

b $2 \times x \times x \times y \times y \times y$

c $3x \times 3x \times 3x$

d $4 \times 4 \times 4 = 64$

e $-5 \times (-5) \times (-5) \times (-5) = 625$

f $\frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} = \frac{27}{125}$

2 a 5^2y^4 **b** $\left(\frac{2}{3}\right)^3 \times \left(\frac{5}{7}\right)^2$ **c** $4^3a^2b^4$

3 a $45 = 3^2 \times 5$ **b** $120 = 2^3 \times 3 \times 5$

4 a -27 **b** $\frac{1}{16}$ **c** 48

5 a 4^7 **b** 9^4 **c** x^8 **d** a^5b^5

e y^6 **f** $3a^3$ **g** $12m^3n^7$ **h** $\frac{x^3y^2}{2}$

i $-2ab$

6 a x^8 **b** $5y^9$ **c** 1 **d** 1

e -1 **f** 5

7 a $12mn^3$ **b** $\frac{x^2}{6}$ **c** $\frac{2a^2}{3}$

8 a $4x^2$ **b** $16m^{12}$ **c** $\frac{x^8}{81}$

d $-32a^3b^6$ **e** $\frac{27a^6}{125}$ **f** $\frac{4a^4c^2}{49b^6}$

9 a $\frac{2y^2}{x^3}$ **b** $\frac{27x^2}{32}$

10 a $\frac{1}{5^2}$ **b** $\frac{3}{4^4}$

c 3^2 **d** 5×2^3

11 a $\frac{1}{81}$ **b** $\frac{5}{4}$ **c** 96

d 0.0006 **e** $\frac{125}{64}$

7F

Now you try

Example 28
7.1×10^4

Example 29
2.75×10^{-5}

Example 30
24 000

Example 31
0.0094

Exercise 7F

1

Scientific notation	Power of 10 expanded	Basic numeral
5×10^3	5×1000	5000
3×10^4	$3 \times 10\,000$	30 000
2×10^5	$2 \times 100\,000$	200 000
7×10^2	7×100	700
7×10^4	$7 \times 10\,000$	70 000
4×10^5	$4 \times 100\,000$	400 000

2

Scientific notation	Positive power	Fraction	Basic numeral
2×10^{-4}	$\frac{2}{10^4}$	$\frac{2}{10\,000}$	0.0002
3×10^{-2}	$\frac{3}{10^2}$	$\frac{3}{100}$	0.03
5×10^{-3}	$\frac{5}{10^3}$	$\frac{5}{1000}$	0.005
7×10^{-6}	$\frac{7}{10^6}$	$\frac{7}{1\,000\,000}$	0.000007
9×10^{-3}	$\frac{9}{10^3}$	$\frac{9}{1000}$	0.009
8×10^{-2}	$\frac{8}{10^2}$	$\frac{8}{100}$	0.08

3 a 10 000 **b** 1000 **c** 100 000 **d** 1000

e 100 000 **f** 10 000

4 a Positive **b** Negative

c Positive **d** Negative

5 a 4×10^4 **b** 2.3×10^{12} **c** 1.6×10^{10} **d** 7.2×10^6

e 3.5×10^3 **f** 8.8×10^6 **g** 5.2×10^3 **h** 3×10^6

i 2.1×10^4

6 a 3×10^{-6} **b** 4×10^{-4} **c** 8.76×10^{-3}

d 7.3×10^{-10} **e** 3×10^{-5} **f** 1.25×10^{-10}

g 8.09×10^{-9} **h** 2.4×10^{-8} **i** 3.45×10^{-5}

7 a 6×10^3 **b** 7.2×10^5 **c** 3.245×10^2

d 7.86903×10^3 **e** 8.45912×10^3

f 2×10^{-1} **g** 3.28×10^{-4} **h** 9.87×10^{-3} **i** 1×10^{-5}

j 4.601×10^8 **k** 1.7467×10^4

l 1.28×10^2

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8 **a** 57 000 **b** 3 600 000 **c** 430 000 000
d 32 100 000 **e** 423 000 **f** 90 400 000 000
g 197 000 000 **h** 709 **i** 635 700

9 **a** 0.00012 **b** 0.0000046 **c** 0.0000000008
d 0.0000352 **e** 0.3678 **f** 0.000000123
g 0.00009 **h** 0.05 **i** 0.4

10 **a** 6×10^{24} kg **b** 4×10^{7} m **c** 1×10^{-10} m
d 1.5×10^{8} km **e** 6.67×10^{-11} Nm^2/kg^2
f 1.5×10^{-4} s **g** 4.5×10^{9} years

11 **a** 4 600 000 000 km
b 8 000 000 000 000 organisms **c** 384 000 km **d** 0.0038 m
e 0.00000000000001 m **f** 720 000 people

12 **a** 1.62×10^{9} km **b** 2.126×10^{-2} g

13 **a** 3.2×10^{4} **b** 4.1×10^{6} **c** 1.3×10^{4} **d** 9.2×10^{1}
e 6.1×10^{-2} **f** 4.24 **g** 2×10^{-5} **h** 4×10^{-6}

7G

Now you try

Example 32
a 4 **b** 2 **c** 3

Example 33
a 4.72×10^{5} **b** 3.2×10^{-4}

Example 34
9.942×10^{7}

Example 35
1.653×10^{-2}

Exercise 7G

1 **a** **i** 300 **ii** 32 700 **iii** 18 400
b **i** 0.1 **ii** 0.2 **iii** 21.0
c **i** 268 000 **ii** 38 000 **iii** 4 061 000

2 42, 0.0071

3 **a** 57 260, 57 300, 57 000, 60 000
b 0.003661, 0.00366, 0.0037, 0.004

4 **a** Yes **b** No **c** No **d** No
e Yes **f** Yes

5 **a** 3 **b** 4 **c** 5 **d** 2
e 3 **f** 2 **g** 3 **h** 3
i 3 **j** 4 **k** 3 **l** 3

6 **a** 2.42×10^{5} **b** 1.71×10^{5} **c** 2.83×10^{3}
d 3.25×10^{6} **e** 3.43×10^{-4} **f** 6.86×10^{-3}
g 1.46×10^{-2} **h** 1.03×10^{-3} **i** 2.34×10^{1}
j 3.26×10^{2} **k** 1.96×10^{1} **l** 1.72×10^{-1}

7 **a** 4.78×10^{4} **b** 2.2×10^{4} **c** 4.833×10^{6}
d 3.7×10^{1} **e** 9.95×10^{1} **f** 1.443×10^{-2}
g 2×10^{-3} **h** 9×10^{-2} **i** 1×10^{-4}

8 **a** 2.441×10^{-4} **b** 2.107×10^{-6}
c -4.824×10^{15} **d** 4.550×10^{-5}
e 1.917×10^{12} **f** 1.995×10^{8}
g 3.843×10^{2} **h** 1.804×10^{-11}
i 1.524×10^{8} **j** 3.325×10^{15}
k 4.067×10^{3} **l** -9.077×10^{-1}

9 **a** 9.3574×10^{1} **b** 2.1893×10^{5}
c 8.6000×10^{5} **d** 8.6288×10^{-2}
e 2.2985×10^{15} **f** 3.5741×10^{28}
g 6.4000×10^{7} **h** 1.2333×10^{9}
i 1.8293 **j** 5.4459×10^{-1}

10 1.98×10^{30} kg

11 1.39×10^{6} km

12 2421×10^{3}, 24.2×10^{5}, 2.41×10^{6}, 0.239×10^{7}, 0.02×10^{8}

13 **a** 4.26×10^{6} **b** 9.1×10^{-3}
c 5.04×10^{11} **d** 1.931×10^{-1}
e 2.1×10^{6} **f** 6.14×10^{-11}

14 **a** $5.400\,46 \times 10^{12}$
b **i** 1.08×10^{13} **ii** 4.32×10^{13}

Maths@Work: Lab technician

1 **a** $1 \times 10^{2}\mu m^3$ **b** $4 \times 10^{3}\mu m^3$ **c** $1.3 \times 10^{2}\mu m^3$
d $1 \times 10^{3}\mu m^3$ **e** $6 \times 10^{5}\mu m^3$

2 **a** 0.00012 m **b** 1.2×10^{-4} m

3 **a** 2×10^{-9} g **b** 1×10^{-6} g **c** 3 g

4 **a** 45 000 000 000 000 cells **b** 29 000 000 000 000 cells

5 **a** 5.7×10^{3} years **b** 4.5×10^{9} years
c 1.57×10^{7} years **d** 1.5×10^{-4} seconds

6 **a** 2.08×10^{6} days **b** 1.64×10^{12} days
c 5.73×10^{9} days

7 Just under 15.7 hours

8 **a** 4.00×10^{-3} g **b** 1.05×10^{3} g

9 $\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}, \frac{1}{2^5}, \frac{1}{2^6}, \frac{1}{2^7}, \frac{1}{2^8}, \frac{1}{2^9}, \frac{1}{2^{10}}$ g

$2^{-1}, 2^{-2}, 2^{-3}, 2^{-4}, 2^{-5}, 2^{-6}, 2^{-7}, 2^{-8}, 2^{-9}, 2^{-10}$ g

10 **a**

	Quantity remaining in mg			
Radioactive isotope	**After 1 half-life**	**After 2 half-lives**	**After 4 half-lives**	**After 10 half-lives**
Hydrogen-3	4.0000	2.0000	0.5000	0.0078
Carbon-11	32.0640	16.0320	4.0080	0.0626
Sodium-24	12.3650	6.1825	1.5456	0.0242
Iron-59	3.9300	1.9650	0.4913	0.0077
Radium-226	2.1250	1.0625	0.2656	0.0042

b

	Total time			
Radioactive isotope	**For 2 half-lives**	**For 4 half-lives**	**For 6 half-lives**	**For 10 half-lives**
Hydrogen-3	24.64	49.28	73.92	123.2
Carbon-11	40.6	81.2	121.8	203
Sodium-24	29.902	59.804	89.706	149.51
Iron-59	88.99	177.98	266.97	444.95
Radium-226	3200	6400	9600	16000

Puzzles and challenges

1 EXPONENTIAL

2 **a** $10^{8} = 100\,000\,000$ **b** $100^{5} = 10\,000\,000\,000$

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3 Answers will vary

4 $4a \times (3m)^3 \times (\frac{a}{6m})^2 = 3a^3m$

5

Pile	Height	Value
1	2 mm	$0.10
5	3.2 cm	$1.60
10	1.024 m	$51.20
15	32.77 m	$1638.40
20	1.05 km	$52 428.80
25	33.55 km	$1 677 721.60
30	1073.74 km	$53 687 091.20

6 14th day

7 **a** 1.84×10^{11} tonnes

b $50 729 000 000 000 or more than 50 trillion dollars

Chapter checklist with success criteria

1 **a** $8a^3b^3$ **b** $\frac{4}{49}$

2 **a** $\left(\frac{7}{9}\right)^4$ **b** $2a^3b^2$

3 $2^2 \times 23$

4 **a** -216 **b** $\frac{9}{121}$

5 **a** 4^7 **b** 7^4

6 **a** a^{10} **b** $27x^5$

7 y^4

8 $\frac{3ab}{2}$

9 4

10 $5x^{18}$

11 **a** 1 **b** $\frac{7a^2}{2}$

12 **a** $8a^{12}$ **b** $\frac{16b^8}{a^{12}}$

13 **a** $-x^4y^6$ **b** a^2b^4

14 **a** $\frac{4}{x^2}$ **b** 5×2^3 or 40

15 **a** $\frac{5}{2^3}$ or $\frac{5}{8}$ **b** $\frac{4}{10^2}$ or $\frac{4}{100}$

16 **a** 4900 **b** 0.00000301

17 **a** 2.7×10^4 **b** 3.75×10^{-5}

18 **a** 9.14×10^6 **b** 3.20×10^{-4}

19 8.750×10^{-1}

Chapter 7 review

Short-answer questions

1 **a** 3^4 **b** $2x^3y^2$ **c** $3a^2b^2$

d $\left(\frac{3}{5}\right)^3 \times \left(\frac{1}{7}\right)^2$

2 **a** $3^2 \times 5$ **b** $2^2 \times 3 \times 5^2$

3 **a** x^{10} **b** $12a^5b^6c$ **c** $24m^7n^4$

d a^9 **e** x^3y^2 **f** $\frac{a^2b^2}{2}$

4 **a** m^6 **b** $9a^8$ **c** $-32a^{10}b^5$

d $3b$ **e** 2 **f** $\frac{a^6}{27}$

5 **a** $\frac{1}{2^3}$ **b** $\frac{3}{4^5}$ **c** $\frac{6}{10^3}$ **d** 4^2

e 6×5^3 **f** 2×10^5

6 **a** k^6 **b** $6a^2m$ **c** $4a^{10}$

d h^{16} **e** $\frac{a^6b^2}{5}$ **f** $45p^4q^2$

7 $0.0012, 35.4 \times 10^{-3}, 3.22 \times 10^{-1}, 0.4, 0.007 \times 10^2, 2.35$

8 **a** 324 **b** 172 500 **c** 0.2753 **d** 0.00149

9 **a** 2.25×10^7 people **b** 9.63×10^6 km^2

c 3.34×10^{-9} s **d** 2.94×10^{-7} m

10 **a** 2.19×10^5 h **b** 3.6×10^7 months

c 4.3×10^5 kg **d** 5×10^6 g

Multiple-choice questions

1 D **2** B **3** E **4** C **5** D **6** B

7 D **8** C **9** C **10** E

Extended-response questions

1 **a** 1.2×10^{55} kg^2 **b** 4.3×10^{-5} m/s

2 **a** **i** 5.79×10^7 km **ii** 1.50×10^8 km

iii 7.78×10^8 km **iv** 1.43×10^9 km

b **i** 2.41×10^{-1} **ii** 1.88

iii 2.95×10^1

c **i** 4.87×10^{24} kg **ii** 6.42×10^{23} kg

iii 1.90×10^{30} kg

Chapter 8

Warm-up quiz

1 **a** Obtuse **b** Acute **c** Reflex **d** Right

e Straight **f** Revolution

2 **a** 35 **b** 142 **c** 260

3 **a** Isosceles **b** Equilateral **c** Scalene

4 **a** Obtuse **b** Acute **c** Right

5 **a** 60° **b** 75° each **c** 50°

6 **a** Parallelogram, rectangle, kite

b Parallelogram, rectangle, square, rhombus

c Square, rectangle

7 **a** $a = 60$ **b** $b = 110$ **c** $a = 60, b = 120$

8 **a** Pentagon **b** Parallelogram

c Trapezium

8A

Now you try

Example 1

a 106° **b** 16°

Example 2

a Obtuse scalene, $a = 35$ **b** Right isosceles, $b = 45$

c Acute scalene, $c = 65$

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Example 3
a $b = 55$ **b** $a = 40$

Exercise 8A

1 **a** right **b** 180° **c** revolution **d** Obtuse
e Acute **f** 180° **g** 90°
h Supplementary **i** 180° **j** equal
2 **a** Isosceles triangle **b** Obtuse-angled triangle
c Equilateral triangle **d** Isosceles triangle
e Acute-angled triangle **f** Scalene triangle
g Right-angled triangle
h Right-angled isosceles triangle
3 **a** 50° **b** 90° **c** 101° **d** 202° **e** 180° **f** 360°
4 **a** **i** 125° **ii** 35°
b **i** 149° **ii** 59°
c **i** 106° **ii** 16°
d **i** 170° **ii** 80°
e **i** 91° **ii** 1°
f **i** 158° **ii** 68°
g **i** 142° **ii** 52°
h **i** 115° **ii** 25°
i **i** 133° **ii** 43°
j **i** 103° **ii** 13°
5 **a** $a = 63$ **b** $a = 71$ **c** $a = 38$ **d** $a = 147$
e $a = 233$ **f** $a = 33$
6 **a** Obtuse isosceles, $a = 40$ **b** Acute scalene, $b = 30$
c Right-angled scalene, $c = 90$ **d** Equilateral, $d = 60$
e Obtuse isosceles, $e = 100$
f Right-angled isosceles, $f = 45$
g Obtuse scalene, $g = 100$ **h** Equilateral, $h = 60$
i Obtuse isosceles, $i = 120$ **j** Obtuse isosceles, $j = 35$
k Right-angled scalene, $k = 90$ **l** Acute scalene, $l = 70$
7 **a** **i** $\angle BAC$ **ii** Obtuse **iii** 120°
b **i** $\angle PRQ$ **ii** Acute **iii** 30°
c **i** $\angle XYZ$ **ii** Reflex **iii** 315°
d **i** $\angle SRT$ **ii** Straight angle **iii** 180°
e **i** $\angle ROB$ **ii** Obtuse **iii** 103°
f **i** $\angle AOB$ **ii** Right **iii** 90°
8 **a** $s = 120$ **b** $t = 20$ **c** $r = 70$ **d** $a = 60, x = 120$
e $a = 100, b = 140$ **f** $c = 115, d = 65$ **g** $x = 56$
h $x = 155$
9 $AO = BO$ (radii)
ΔAOB is isoceles, 2 sides equal, $\angle AOB = 116°$
$\angle OAB = 32°$, base angles of isosceles triangle
10 **a** 360° **b** 90°
c 60° **d** 90°
e 432° **f** 6°
g 720° **h** 8640°
11 **a** 90° **b** 150°
c 15° **d** 165°
e 157.5° **f** 80°
g 177.5° **h** 171°

8B

Now you try

Example 4
a No. The two cointerior angles are not supplementary.
b Yes. The alternate angles are equal.
c Yes. The corresponding angles are equal.

Example 5
a $a = 59$ (straight line) $b = 59$ (alternate)
b $a = 128$ (cointerior) $b = 128$ (vertically opposite)

Exercise 8B

1 **a** Corresponding **b** Alternate **c** Cointerior
d Alternate **e** Cointerior **f** Corresponding
2 **a** equal **b** equal **c** supplementary
3 **a** No, alternate angles are not equal.
b Yes, corresponding angles are equal.
c Yes, alternate angles are equal.
d No, cointerior angles don't add to 180°.
e Yes, cointerior angles add to 180°.
f Yes, corresponding angles are equal.
g No, corresponding angles are not equal.
h No, alternate angles are not equal.
i No, cointerior angles do not add to 180°.
4 **a** $x = 125$, alternate angles in || lines
b $y = 110$, cointerior angles in || lines
c $r = 80$, corresponding angles in || lines
d $s = 66$, alternate angles in || lines
e $t = 96$, vertically opposite
f $v = 126$, corresponding angles in || lines
g $w = 62$, angles on a straight line
h $p = 115$, corresponding angles in || lines
i $q = 116$, cointerior angles in || lines
5 **a** $a = 60, b = 120$ **b** $c = 95, d = 95$
c $e = 100, f = 100, g = 100$ **d** $a = 110, b = 70$
e $a = 100, b = 80, c = 80$
f $e = 140, f = 140, d = 140$
6 $a = 40, b = 140, c = 40, d = 40, e = 140$
7 **a** $x = 70, y = 40$ **b** $t = 58, z = 122$
c $u = 110, v = 50, w = 50$ **d** $x = 118$
e $x = 295$ **f** $x = 79$
8 **a** 56 **b** 120 **c** 265
9 **a** 105° **b** 105° **c** 56° **d** 105° **e** 90° **f** 85°

8C

Now you try

Example 6
a Parallelograms, rhombus, rectangle and square **b** Kite

Example 7
$x = 80$

Example 8
a $m = 40$ **b** $n = 104$

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Exercise 8C

1 Parallelogram, rhombus, rectangle, square, kite, trapezium
2 b, c, e, g
3 a, b, c, e, g, h
4 **a** Parallelogram, rectangle, rhombus, square
b Rectangle, square **c** Square, rectangle
d Square, rhombus, kite **e** Trapezium
f Parallelogram, rhombus, kite
5 **a** 115 **b** 159 **c** 30 **d** 121
e 140 **f** 220
6 **a** $a = 110$ **b** $x = 70$ **c** $b = 54$ **d** $a = 33$
e $y = 63$ **f** $a = 109$
7 **a** Yes **b** No **c** Yes **d** No
8 60°
9 A parallelogram has opposite sides parallel and equal and rectangles, squares and rhombi have these properties (and more) and are therefore all parallelograms.
10 **a** 110 **b** 55 **c** 120
11 **a** 255 **b** 80 **c** 115 **d** 37 **e** 28 **f** 111

8D

Now you try

Example 9
$S = 900°$
$a = 250$

Example 10
$a = 144$

Exercise 8D

1 **a** hexagon **b** regular **c** non-convex
d $S = (n - 2) \times 180°$
2 **a** 5 **b** 7 **c** 4 **d** 11 **e** 9 **f** 12
3 **a** 720° **b** 1080° **c** 1620° **d** 900°
4 **a** Convex quadrilateral **b** Non-convex hexagon
c Non-convex heptagon **d** Non-convex decagon
5 **a** 140 **b** 100 **c** 100 **d** 120
6 **a** $S = 540°, a = 110$ **b** $S = 900°, a = 150$
c $S = 720°, a = 210$ **d** $S = 720°, a = 130$
e $S = 360°, a = 25$ **f** $S = 540°, a = 285$
7 **a** 90° **b** 108° **c** 140° **d** 120°
e 128.57° **f** 135°
8 **a** 16 **b** 25 **c** 102
9 **a** Yes **b** Yes
10 **a** 135 **b** 240 **c** 85 **d** 150
11 **a** **i** No **ii** Yes **iii** No
b **i** One **ii** Two **iii** Five
c $(n - 3)$

Progress quiz

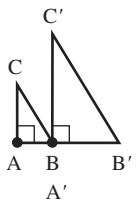
1 **a** 27° **b** 117°
2 **a** Right-angled scalene, $a = 55$
b Acute-angled isosceles, $b = 40$
c Obtuse-angled scalene, $c = 154$
3 **a** Alternate angles are not equal.
b Cointerior angles are not supplementary.
4 **a** $a = 55$ (supplementary angles), $b = 55$ (corresponding angles in parallel lines)
b $a = 65$ (cointerior angles in parallel lines), $b = 65$ (vertically opposite)
c $a = 40$ (alternate angles in parallel lines and angle sum of a triangle)
5 **a** $a = 100$ **b** Parallelogram, $b = 58$
c Kite, $c = 80$
6 **a** $S = 720, a = 109$ **b** $S = 540, a = 108$
7 **a** 180° **b** 360° **c** 540° **d** 720°

8E

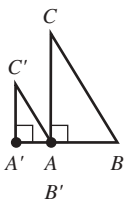
Now you try

Example 11
a 2.5 **b** 40 **c** 13

Example 12
a

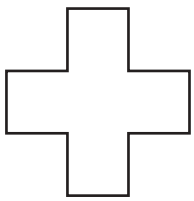


b



Exercise 8E

1 **a** $HG = GF = 15$ cm **b** Scale factor = 2 cm
c $FE = 10$ cm, $FD = ED = 15$ cm
d $EH = 2.5$ cm, $HG = 4$ cm **e**



(all sides are 1.5 cm, all angles are 90°)

2 **a** $\angle F$ **b** $\angle D$ **c** GH
d AE **e** 2
3 **a** 18 cm by 12 cm
b 12 cm by 8 cm
c 9 cm by 6 cm
d 7.5 cm by 5 cm
e 3 cm by 2 cm
f 4.5 cm by 3 cm
4 **a** T **b** F **c** T **d** F **e** T
5 **a** **i** 2 **ii** 14 **iii** 10
b **i** 1.5 **ii** 10.5 **iii** 8
c **i** 1.5 **ii** 45 **iii** 24
d **i** 2.5 **ii** 1 **iii** 1.4
e **i** 2.5 **ii** 0.6 **iii** 2
f **i** 1.75 **ii** 3.5 **iii** 3
6 All sides 1.5 times longer.
7 All sides half as long as original.
8 **a** Yes **b** 8 cm **c** 25 cm
9 3.7 cm
10 1.41; Note: On a copier, to enlarge A3 to A4 the setting is 141%

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11 a All angles of any square equal 90°, with only 1 side length
b All angles in any equilateral triangle equal 60°, with only 1 side length
c The ratio of the length to the width may vary between different rectangles.
d 2 isosceles triangles do not have to have the same size equal angles.

12 a 12 500 mm **b** 100 mm **c** 125 **d** 24 mm
e 3000 mm

13 a 100 000 cm = 1 km **b** 24 cm

8F

Now you try

Example 13
10 m

Exercise 8F

1 a 2 **b** 12

2 a E **b** C **c** DF **d** BC **e** $\angle A$ **f** $\angle E$

3 a $\Delta ABC \text{ ||| } \Delta GHI$ **b** $\Delta ABC \text{ ||| } \Delta MNO$
c $\Delta ABC \text{ ||| } \Delta ADE$ **d** $\Delta HFG \text{ ||| } \Delta HJI$
e $\Delta ADC \text{ ||| } \Delta AEB$ **f** $\Delta ABD \text{ ||| } \Delta ECD$

4 a 3 **b** 3.6 m

5 a All internal angles are the same **b** 40 m

6 6 m

7 a All internal angles are the same **b** 7.5 m

8 24 m

9 7.2 m

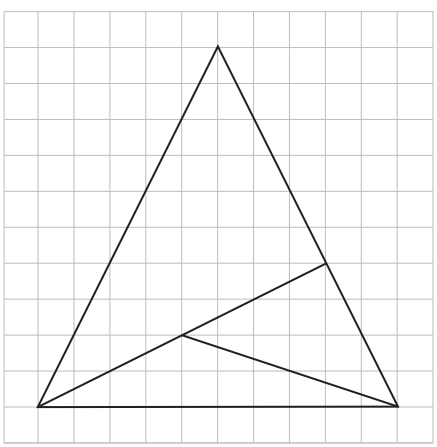
10 20 m

11 a i 3.6 m **ii** 9 m **iii** 2.7 m
b i 5.4 m **ii** 6.3 m
c i 4 m **ii** 6 m

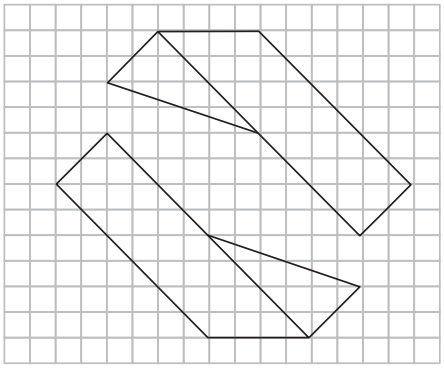
12 a 5 : 7, 3 : 4, 4 : 5, 2 : 3, 11 : 14, 3 : 4, 4 : 5, 5 : 6, 2 : 3
b 20 by 30; scale factor is 5
c 12 by 16; scale factor is 2
d 16 by 20; scale factor is 2

Maths@Work: Animator

2 a



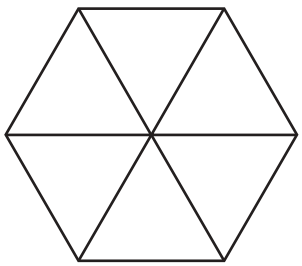
b



3 12 pairs of congruent triangles; 2 pairs of similar triangles
4 30° clockwise; 60° clockwise; 20° clockwise
5 Answers will vary.

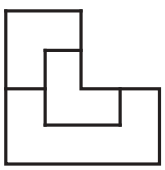
Puzzles and challenges

1



2 110
3 10
4 11
5 Base of altitudes (an altitude is a line through a vertex that meets the opposite side at right angles)

6



7

	5	3	
2	8	1	7
	6	4	

Chapter checklist with success criteria

1 155° and 65°
2 Right isosceles, $a = 45°$, $b = 135°$
3 $a = 113°$ complimentary
$b = 67°$ alternate angles
$c = 113°$ complimentary
$d = 67°$ corresponding angles
4 No, co-interior angles sum to 180°
5 Square, rhombus and kite
6 50°

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7 30°
8 285°
9 135°
10 Scale factor $= 1.5$, $x = 6$, $y = 3$
11 8.75
12 **a** Matching angles are equal **b** 16 m

Chapter 8 review

Short-answer questions

1 **a** $a = 142$ **b** $b = 40$ **c** $c = 120$
2 **a** Acute-angled isosceles, $x = 50$, $y = 80$
b Right-angled scalene, $x = 25$
c Obtuse-angled scalene, $x = 30$, $y = 110$
3 **a** No – corresponding angles are not equal.
b Yes – cointerior angles are supplementary.
c Yes – alternate angles are equal.
4 **a** $a = 60, b = 120, c = 120$ **b** $a = 40, b = 80$
5 **a** Square, rectangle **b** Square, rhombus, kite
c Parallelogram, rectangle, rhombus, square
6 **a** $a = 70, b = 110$ **b** $x = 115$ **c** $x = 15$
7 **a** $x = 95$ **b** $a = 120$ **c** $x = 135$
8 For image $\Delta A'B'C'$, $OA' = 3OA, OB' = 3OB, OC' = 3OC$
9 **a** 3.5 **b** 4 **c** 18
10 **a** 3 **b** 4.8 m

Multiple-choice questions

1 D **2** A **3** B **4** D **5** C **6** A
7 E **8** C **9** D **10** A

Extended-response questions

1 **a** Isosceles, right-angled
b $a = 40$, angle sum in isosceles triangle
$b = 70$, angles in parallel lines
$c = 120$, angle sum of a quadrilateral
$d = 25$, angles on a straight line
$e = 65$, angle sum of a triangle
2 **a** $\angle ECD$
b 2.44
c 19.8 m

Chapter 9

Warm-up quiz

1 **a** 4 **b** 6 **c** −2 **d** −7
2 **a** 5 **b** 2 **c** 1 **d** −6
3 **a** −10 **b** 16 **c** 11 **d** −100
e 2 **f** 48 **g** −20 **h** −70
4 **a** $2x + 6$ **b** $3a - 15$ **c** $12x - 8xy$ **d** $3 - 6b$
5 **a** 2 **b** 6 **c** $2x$ **d** $3y$
e $5x$ **f** ab
6 **a** $2(a + 3)$ **b** $3(x + 4y)$ **c** $5x(x - 3)$
d $2m(2 - 3n)$
7 **a** $\frac{5}{7}$ **b** $-\frac{1}{9}$ **c** $\frac{7}{6}$ **d** $-\frac{1}{8}$
8 **a** $\frac{8}{15}$ **b** $\frac{1}{2}$ **c** $\frac{1}{3}$ **d** 1
e 2 **f** $\frac{3}{2}$ **g** $\frac{9}{8}$ **h** $\frac{7}{12}$
9 **a** $3x + 2$ **b** $x + 4$ **c** $-10 - 3x$
10 $x(x + 2), x^2 + 2x$

9A

Now you try

Example 1
a $2m - n$ **b** $-2x^2y - xy$

Example 2
a $-15xy^2$ **b** $\frac{a}{2}$

Example 3
a $-3x - 6$ **b** $6x^2 - 4x$

Exercise 9A

1 **a** **i** $\$11x$ **ii** $50n$ cents
b **i** $x + y$ **ii** $b + g$
c $\$(50 + 60n)$
2 **a** 5 **b** −2 **c** −7 **d** No
e −10 **f** 6
3 **a** $5 + 2x$ **b** $4a - 7$ **c** $y^2 - 1$ **d** xz
e $a^2 + b^2$ **f** $\sqrt{x + 8}$
4 **a** $-a$ **b** $11x$ **c** $8ab$ **d** $-4xy$
e $a - 1$ **f** $2x - 4y$ **g** $6x - 4y$ **h** $2ab$
i $-4a^2b - 3$ **j** $-3xy$ **k** $12ar^2$ **l** $st^2 - st$
5 **a** $15x$ **b** $14a$ **c** $-8xy$ **d** $-42a^2$
e $-3ab^2$ **f** $-12a^2b^2$ **g** $2x$ **h** 7
i 3 **j** $2a$ **k** $\frac{10}{a}$ **l** $4x$
m ab **n** $\frac{3a}{c}$ **o** $\frac{1}{3y}$
6 **a** $3x + 6$ **b** $8 + 2x$ **c** $-3x - 12$ **d** $-6x - 6$
e $-2x + 6$ **f** $-x^2 - x$ **g** $-6y + 9y^2$
h $-15x^2 - 6y - 12$ **i** $2x + 1$ **j** $4 - x$
k $3 - 4x$ **l** $2x + 6$ **m** $4x - 9$
n $-17x - 3$ **o** $13x - 40$ **p** $3x - 11$
q $2y^2 - 6y - 20$ **r** $2y^2 - 14y + 20$
s $14x + 8y + 10$ **t** $10x + 8y + 6$
u $10 + 4x$ **v** $26 - 4x$
7 **a** **i** $2a + 2b$ or $2(a + b)$ **ii** ab
b **i** $4x$ **ii** x^2
c **i** $a + b + c$ **ii** $\frac{1}{2}ab$
8 **a** 4 **b** 24 **c** −1 **d** 10
e 13 **f** −8 **g** $\frac{25}{2}$ **h** 1
i −2 **j** 35 **k** −12 **l** $\sqrt{13}$
9 **a** **i** \$260 **ii** \$820
b $100 + 80n$

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10 a $2x^2 - x$ **b** $x^2 + 6x$ **c** $x^2 + x + 2$

11 a $-3x + 3$ **b** $\frac{a}{2}$ **c** $\frac{2}{a}$ **d** 9

12 a **i** \$120 **ii** \$600

b **i** \$140 **ii** \$540

c $60n

d $(40 + 50n)

e 4 hours

9B

Now you try

Example 4

$x^2 + 11x + 18$

Example 5

a $x^2 - 3x - 18$ **b** $3x^2 - 14x + 8$

c $18x^2 - 9x - 20$

Exercise 9B

1 a $x^2, 2x, 3x, 6$ **b** $x^2 + 3x + 2x + 6 = x^2 + 5x + 6$

2 a $2x^2, 2x, 3x, 3$

b $(2x + 3)(x + 1) = 2x^2 + 2x + 3x + 3$

$= 2x^2 + 5x + 3$

3 a $x^2 + 5x + x + 5 = x^2 + 6x + 5$

b $x^2 + 2x - 3x - 6 = x^2 - x - 6$

c $21x^2 + 6x - 14x - 4 = 21x^2 - 8x - 4$

d $12x^2 - 16x - 3x + 4 = 12x^2 - 19x + 4$

4 a $x^2 + 7x + 10$ **b** $b^2 + 7b + 12$

c $t^2 + 15t + 56$ **d** $p^2 + 12p + 36$

e $x^2 + 15x + 54$ **f** $d^2 + 19d + 60$

g $a^2 + 8a + 7$ **h** $y^2 + 12y + 20$

i $m^2 + 16m + 48$ **j** $a^2 + 10a + 9$

k $6 + 5a + a^2$ **l** $n^2 + 12n + 36$

5 a $x^2 - x - 12$ **b** $x^2 + 3x - 10$

c $x^2 - 4x - 32$ **d** $x^2 - 4x - 12$

e $x^2 + 9x - 10$ **f** $x^2 + 2x - 63$

g $x^2 + 5x - 14$ **h** $x^2 - 3x + 2$

i $x^2 - 9x + 20$ **j** $8x^2 + 26x + 15$

k $6x^2 + 7x + 2$ **l** $15x^2 + 17x + 4$

m $6x^2 + x - 15$ **n** $24x^2 + 23x - 12$

o $6x^2 - x - 2$ **p** $10x^2 - 31x - 14$

q $6x^2 + 5x - 6$ **r** $16x^2 - 16x - 5$

s $18x^2 - 27x + 10$ **t** $15x^2 - 11x + 2$

u $21x^2 - 37x + 12$

6 a **i** $x + 10$ **ii** $x + 7$

b $x^2 + 17x + 70$

c 108 m^2

7 a $x^2 + 9x + 20$

b **i** 56 m^2 **ii** 36 m^2

8 a **i** $2x + 3$ **ii** $2x + 2$

b $4x^2 + 10x + 6$

c 20 m^2

9 a 3 **b** 2 **c** 6, 6 **d** 2, 18

e 2, 6 **f** 3, 15

10 a $a^2 + ac + ab + bc$ **b** $a^2 + ac - ab - bc$

c $ab + bc - a^2 - ac$ **d** $2x^2 - 3xy - 2y^2$

e $2a^2 - ab - b^2$ **f** $6x^2 + xy - y^2$

11 a $2x^2$ **b** $2x^2 - 30x + 100$

12 a **i** $15 - 2x$ **ii** $10 - 2x$

b $150 - 50x + 4x^2$

c 66 m^2

d $x = 2.5$

9C

Now you try

Example 6

a $x^2 + 16x + 64$ **b** $x^2 - 10x + 25$

Example 7

$25x^2 - 30x + 9$

Exercise 9C

1 a $x^2, 3x, 3x, 9$ **b** $x^2 + 6x + 9$

c $(x + 3)(x + 3) = x^2 + 3x + 3x + 9$

$= x^2 + 6x + 9$

2 a $+4x + 16 = x^2 + 8x + 16$

b $+5x + 25 = x^2 + 10x + 25$ **c** $-2x + 4 = x^2 - 4x + 4$

d $-7x + 49 = x^2 - 14x + 49$

3 a **i** $x^2 + 6x + 9$ **ii** $x^2 + 22x + 121$

b **i** $x^2 - 4x + 4$ **ii** $x^2 - 18x + 81$

4 a $x^2 + 2x + 1$ **b** $x^2 + 6x + 9$

c $x^2 + 4x + 4$ **d** $x^2 + 10x + 25$

e $x^2 + 8x + 16$ **f** $x^2 + 18x + 81$

g $x^2 + 14x + 49$ **h** $x^2 + 20x + 100$

i $x^2 - 4x + 4$ **j** $x^2 - 12x + 36$

k $x^2 - 2x + 1$ **l** $x^2 - 6x + 9$

m $x^2 - 18x + 81$ **n** $x^2 - 14x + 49$

o $x^2 - 8x + 16$ **p** $x^2 - 24x + 144$

5 a $4x^2 + 4x + 1$ **b** $4x^2 + 20x + 25$

c $9x^2 + 12x + 4$ **d** $9x^2 + 6x + 1$

e $25x^2 + 20x + 4$ **f** $16x^2 + 24x + 9$

g $49 + 28x + 4x^2$ **h** $25 + 30x + 9x^2$

i $4x^2 - 12x + 9$ **j** $9x^2 - 6x + 1$

k $16x^2 - 40x + 25$ **l** $4x^2 - 36x + 81$

6 a $9 - 6x + x^2$ **b** $25 - 10x + x^2$

c $1 - 2x + x^2$ **d** $36 - 12x + x^2$

e $121 - 22x + x^2$ **f** $16 - 8x + x^2$

g $49 - 14x + x^2$ **h** $144 - 24x + x^2$

i $64 - 32x + 4x^2$ **j** $4 - 12x + 9x^2$

k $81 - 36x + 4x^2$ **l** $100 - 80x + 16x^2$

7 a $x + 12$

b $x^2 + 24x + 144$

c **i** 196 m^2 **ii** 289 m^2

8 a $10 - x$

b $x^2 - 20x + 100$

c **i** 64 cm^2 **ii** 16 cm^2

d $x = 5$

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9 **a** $20 - 2x$

b $(20 - 2x)(20 - 2x) = 400 - 80x + 4x^2$

c 196 cm^2

d 588 cm^3

10 **a** $a^2 - b^2$

b **i** $(a - b)^2 = a^2 - 2ab + b^2$ **ii** $b(a - b) = ab - b^2$

iii $b(a - b) = ab - b^2$

c Yes, $a^2 - 2ab + b^2 + ab - b^2 + ab - b^2 = a^2 - b^2$

11 **a** $a + b$ **b** $(a + b)(a + b) = a^2 + 2ab + b^2$

c $a - b$ **d** $(a - b)(a - b) = a^2 - 2ab + b^2$

e $4ab$ **f** ab so, yes, four courts' area is $4ab$

9D

Now you try

Example 8

a $x^2 - 25$ **b** $x^2 - 225$

Example 9

$144x^2 - 25y^2$

Exercise 9D

1 Because $-3x + 3x = 0$

2 **a** Single **b** Difference **c** Difference **d** Single

3 **a** $+4x - 16, x^2 - 16$ **b** $+2x - 2x - 1, 4x^2 - 1$

4 **a** $x^2 - 1$ **b** $x^2 - 9$ **c** $x^2 - 64$ **d** $x^2 - 16$

e $x^2 - 144$ **f** $x^2 - 121$ **g** $x^2 - 81$ **h** $x^2 - 25$

i $x^2 - 36$ **j** $25 - x^2$ **k** $4 - x^2$ **l** $49 - x^2$

5 **a** $9x^2 - 4$ **b** $25x^2 - 16$ **c** $16x^2 - 9$

d $49x^2 - 9y^2$ **e** $81x^2 - 25y^2$ **f** $121x^2 - y^2$

g $64x^2 - 4y^2$ **h** $100x^2 - 81y^2$

i $49x^2 - 25y^2$ **j** $36x^2 - 121y^2$

k $64x^2 - 9y^2$ **l** $81x^2 - 16y^2$

6 **a** 899 **b** 1599 **c** 624 **d** 2499

e 396 **f** 391 **g** 875 **h** 2484

7 **a** **i** x^2 **ii** $x^2 - 4$

b No, they differ by 4.

8 **a** $x^2 - 1$ **b** No, area of rectangle is 1 square unit less.

9 **a** **i** x^2 **ii** $x + 3$ **iii** $x - 3$

b $x^2 - 9$

c No, 9 m^2 less

9E

Now you try

Example 10

a $5b$ **b** $7xy$

Example 11

a $5(3a + 4)$ **b** $2x(x - 3)$

Example 12

$-11a(1 + 2a)$

Exercise 9E

1 **a** 4 **b** 10 **c** 5 **d** 6

2 **a** x **b** x **c** a **d** $2a$ **e** $-2y$ **f** $-3x$

3 **a** **i** 6 **ii** $3x$ **iii** $6x$

b iii

4 **a** $2x$ **b** $6a$ **c** 2 **d** 4 **e** 3

f 1 **g** $3x$ **h** $3n$ **i** $2y$ **j** $2x$

k $2xy$ **l** $5ab$

5 **a** $7(x + 1)$ **b** $3(x + 1)$ **c** $4(x - 1)$

d $5(x - 1)$ **e** $4(1 + 2y)$ **f** $5(2 + a)$

g $3(1 - 3b)$ **h** $2(3 - x)$ **i** $3(4a + b)$

j $6(m + n)$ **k** $2(5x - 4y)$ **l** $4(a - 5b)$

m $x(x + 2)$ **n** $a(a - 4)$ **o** $y(y - 7)$

p $x(1 - x)$ **q** $3p(p + 1)$ **r** $8x(1 - x)$

s $4b(b + 3)$ **t** $2y(3 - 5y)$

6 **a** $-4(2x + 1)$ **b** $-2(2x + 1)$ **c** $-5(2x + y)$

d $-7(a + 2b)$ **e** $-3(3x + 4)$ **f** $-2(3y + 4)$

g $-5(2x + 3y)$ **h** $-4(m + 5n)$ **i** $-3x(x + 6)$

j $-4x(2x + 3)$ **k** $-2y(8y + 3)$ **l** $-5a(a + 2)$

7 **a** 3 **b** y **c** a **d** $5x$

e -7 **f** $-12a$

8 **a** $4(x + 2)$ **b** $2(x + 3)$ **c** $10(x + 2)$ **d** $2(x + 7)$

e $2(2x + 3)$ **f** $2(x + 7)$

9 $4x$

10 **a** $t(5 - t)$

b **i** 0 m **ii** 6 m **iii** 4 m

c 5 s

11 **a** 63 **b** 72 **c** -20 **d** -70

e 69 **f** 189

12 **a** $3(a^2 + 3a + 4)$ **b** $z(5z - 10 + y)$

c $x(x - 2y + xy)$ **d** $2b(2y - 1 + 3b)$

e $-4y(3x + 2z + 5xz)$ **f** $ab(3 + 4b + 6a)$

13 **a** $(x + 3)(4 + x)$ **b** $(x + 1)(3 + x)$

c $(m - 3)(7 + m)$ **d** $(x - 7)(x + 2)$

e $(a + 4)(8 - a)$ **f** $(x + 1)(5 - x)$

g $(y + 3)(y - 2)$ **h** $(x + 2)(a - x)$

i $(2t + 5)(t + 3)$ **j** $(5m - 2)(m + 4)$

k $(4y - 1)(y - 1)$ **l** $(7 - 3x)(1 + x)$

Progress quiz

1 **a** $8x - 6y$ **b** $7ab^2 - ab$ **c** $20a^2b$ **d** $\frac{y}{3}$

2 **a** $3x - 12$ **b** $6y^2 + 10y$ **c** $3 - 6x$

3 **a** $x^2 + 10x + 24$ **b** $x^2 + x - 6$

c $2x^2 + 5x - 12$ **d** $6x^2 - 19x + 10$

4 **a** $x^2 + 12x + 36$ **b** $x^2 - 16x + 64$

c $9x^2 + 42x + 49$ **d** $25 - 10x + x^2$

5 **a** $(2x^2 - 5x - 3)$ m^2 **b** $(9x^2 - 6x + 1)$m^2

6 **a** $x^2 - 100$ **b** $x^2 - 1$ **c** $9x^2 - 4$

d $16x^2 - 25y^2$

7 $2x + 2$

8 **a** 4 **b** $3a$ **c** $5ab$ **d** $3x$

9 **a** $6(x + 2)$ **b** $5(3y - 4)$ **c** $4a(2b + 5)$

d $2x(2x - 1)$ **e** $-5(2x + 1)$ **f** $-2x(2x + 5)$

10 **a** $2(4x + 3)$ **b** $6(x + 2)$

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9F

Now you try

Example 13

a $\frac{-ab}{2}$ **b** $-(x+7)$ or $-x-7$

Example 14

a $a-6$ **b** -7

Example 15

a $6a$ **b** 2

Example 16

a $\frac{-3x}{2}$ **b** $\frac{2}{15}$

Exercise 9F

1 **a** 2 **b** $2x$ **c** $2(x+2)$ **d** $2(x-1)$

2 **a** $\frac{3}{2}$ **b** $\frac{3}{4}$ **c** $\frac{2}{7x}$ **d** $\frac{4a^2}{5}$

e $\frac{1}{7}$ **f** $\frac{1}{6x}$

3 **a** $\frac{8}{9}$ **b** $\frac{7}{6}$ **c** $\frac{3}{2}$ **d** $\frac{1}{4}$

e 6 **f** 1

4 **a** $\frac{x}{2}$ **b** $\frac{x}{2}$ **c** $\frac{-ab}{3}$ **d** $\frac{a}{2b}$

e $x-1$ **f** $-2(2x+1)$ **g** $\frac{-1}{3}$ **h** $\frac{a}{10}$

i 3 **j** -2 **k** $\frac{1}{2}$ **l** $\frac{-2}{3}$

m $x-4$ **n** $2(x-1)$ **o** $-2(x+2)$ **p** -1

5 **a** $x+2$ **b** $x-3$ **c** $x-4$ **d** $\frac{1}{x+1}$

e $\frac{-1}{2x+3}$ **f** $\frac{a}{3a-2}$ **g** 2 **h** 7

i -5 **j** -2 **k** 6 **l** -5

6 **a** 1 **b** $\frac{1}{2}$ **c** $\frac{6a}{5}$ **d** $\frac{3}{2a}$

e $\frac{7xy}{3}$ **f** $\frac{5x}{6}$ **g** 4 **h** 5

i $\frac{9}{2}$ **j** $\frac{x+1}{4}$ **k** 1 **l** $\frac{x+6}{2(x+7)}$

7 **a** $\frac{7x}{6}$ **b** $\frac{2x}{3}$ **c** $\frac{-21}{8}$ **d** $\frac{-a^2}{2}$

e 6 **f** $\frac{4}{3}$ **g** 11 **h** $\frac{-3}{2}$

i $\frac{-(2x-4)}{2(2x-7)} = \frac{-x+2}{2x-7}$

8 **a** $x+1$ **b** $-(x+2)$ **c** $\frac{3}{2}$ **d** $x(x+2)$

9 **a** 4 **b** 5 **c** -18 **d** 15

e -34 **f** 14

10 **a** $x+3$ **b** $2x-1$ **c** $-5(x-3)$ **d** $\frac{-1}{3(x-4)}$

11 **a** -1 **b** -1 **c** $\frac{-4}{3}$ **d** 2

e $\frac{-1}{x-2}$ **f** $-4(x-6)$

9G

Now you try

Example 17

a $\frac{7a}{10}$ **b** $\frac{5x}{24}$

Example 18

$\frac{7x-17}{12}$

Exercise 9G

1 **a** 24 **b** 15 **c** 143 **d** 36

2 **a** $\frac{5}{6}$ **b** $\frac{17}{20}$ **c** $\frac{13}{8}$ **d** $\frac{7}{10}$

e $-\frac{2}{15}$ **f** $\frac{1}{12}$

3 **a** $4x$ **b** $21x$ **c** 3 **d** 2

e 8 **f** 90

4 **a** $\frac{3x}{12} + \frac{8x}{12} = \frac{11x}{12}$

b $\frac{25x}{35} - \frac{14x}{35} = \frac{11x}{35}$

5 **a** 15 **b** 14 **c** 8 **d** 6

e 10

6 **a** $\frac{9x}{14}$ **b** $\frac{2x}{5}$ **c** $\frac{x}{8}$ **d** $\frac{14x}{45}$

e $\frac{y}{56}$ **f** $\frac{13a}{22}$ **g** $\frac{2b}{9}$ **h** $\frac{m}{6}$

i $\frac{11m}{12}$ **j** $\frac{15a}{28}$ **k** $\frac{x}{2}$ **l** $-\frac{20p}{63}$

m $-\frac{5b}{18}$ **n** $\frac{61y}{40}$ **o** $\frac{13x}{35}$ **p** $\frac{5x}{12}$

7 **a** $\frac{7x+11}{10}$ **b** $\frac{7x}{12}$ **c** $\frac{15a-51}{56}$ **d** $\frac{11y+9}{30}$

e $\frac{13m+28}{40}$ **f** $\frac{5x-13}{24}$ **g** $\frac{11b-6}{24}$ **h** $\frac{7x}{6}$

i $\frac{7y-8}{14}$ **j** $\frac{5t-4}{16}$ **k** $\frac{34-10x}{21}$ **l** $\frac{8m-9}{12}$

8 **a** $\frac{4x}{8} + \frac{6x}{8} = \frac{10x}{8} = \frac{5x}{4}$

b $\frac{2x}{4} + \frac{3x}{4} = \frac{5x}{4}$

c Using denominator 8 does not give answer in simplified form and requires extra steps. Preferable to use actual LCD.

9 **a** Didn't make a common denominator and subtracted denominators, $\frac{7x}{15}$

b Didn't use brackets: $2(x+1) = 2x+2$, $\frac{7x+2}{10}$

10 **a** $\frac{3x}{2}$ **b** $\frac{5x}{4}$ **c** $\frac{7x}{4}$ **d** $\frac{9x}{2}$

11 **a** $-\frac{43x}{30}$ **b** $\frac{5x}{12}$ **c** $\frac{13x}{24}$ **d** $\frac{43x-5}{60}$

e $\frac{23x-35}{42}$ **f** $\frac{29x+28}{40}$

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12 a $\frac{x}{3}$ **b** $\frac{x}{8}$

c $\frac{x}{2}$ **d** $\frac{x}{5}$

e $\frac{8x}{9}$ **f** $\frac{x}{4}$

Maths@Work: Automotive technology

1 a $A = 2lb + 2hb + 2hl$

$= 2(lb + hb + hl)$

b $A = ah + bh + ch + ab$ **c** $A = 2\pi r^2 + 2\pi rh$

$A = 2\pi r(r + h)$

d $A = \frac{\pi d^2}{2} + \pi dh$

$A = \pi d\left(\frac{d}{2} + h\right)$

2 i c ii a iii b iv d

3 a Area of cross-section $= \pi R^2 - \pi r^2$

$= \pi(R^2 - r^2)$

$= \pi\left(\frac{D^2}{2^2} - \frac{d^2}{2^2}\right)$

$= \frac{\pi}{4}(D^2 - d^2)$

$= \frac{\pi}{4}(D + d)(D - d)$

b i 76π mm^2

ii 31π mm^2

iii $\frac{\pi}{2}$ cm^2

iv 3π cm^2

4

Pipe material	Area of cross-section in cm^2	Length in cm	Weight in g	Weight in kg
Poly pipe a	3.77	100	360.03	0.36
Poly pipe b	11.50	100	1098.08	1.10
Poly pipe c	141.37	100	13500.99	13.50
Copper pipe a	0.35	100	308.94	0.31
Copper pipe b	0.31	100	273.84	0.27
Copper pipe c	1.54	100	1376.21	1.38
Steel pipe a	2.89	100	2268.86	2.27
Steel pipe b	8.64	100	6781.91	6.78
Steel pipe c	25.13	100	19729.20	19.73

a Copper pipe c = 1.38 kg/m; steel pipe b = 6.78 kg/m
Steel pipe b is 5.4 kg/m heavier than copper pipe c.

b 3.5 m of poly pipe a = 1.26 kg; 3.5 m of steel pipe a = 7.94 kg
3.5 m of steel pipe a is 6.68 kg heavier than 3.5 m of poly pipe a.

c 2.75 m of poly pipe c = 37.13 kg; 5 m of steel pipe c = 98.65 kg.
5 m of steel pipe c is 61.52 kg heavier than 2.75 m of poly pipe c.

Puzzles and challenges

1 RECIPROCAL

2 a $4x^2 - 80x + 400 = 100$ **b** $x = 5$

3 a n^2 **b** $n(n + 1)$ **c** $\frac{1}{2}n(n + 1)$

4 a $\frac{x + 10}{12}$ **b** $\frac{-11x - 2}{15}$

5 a 48, 49 **b** 33, 35 **c** 12, 15

6 Ryan

Chapter checklist with success criteria

1 a $2a^2 - ab + 1$ **b** $-x^2y - xy$

2 35

3 a $14a^2b$ **b** $\frac{1}{3}y$

4 a $-3x + 6$ **b** $-8a^2 + 10a$

5 a $2x^2 - x - 15$ **b** $6x^2 - 29x + 35$

6 a $x^2 + 14x + 49$ **b** $16x^2 - 24x + 9$

7 a $x^2 - 16$ **b** $9x^2 - 25$

8 $6xy$

9 a $5(a + 3)$ **b** $11x(x - 3y)$ **c** $-7m(2n + 1)$

10 2

11 $\frac{x^2}{6}$

12 $\frac{1}{6}$

13 a $\frac{5a}{6}$ **b** $\frac{x}{15}$

Chapter 9 review

Short-answer questions

1 a $6x^2$ **b** $-18ab^2$ **c** $\frac{b}{2}$ **d** $-4x$

2 a $3x - 12$ **b** $-2x - 12$ **c** $-x^2 + x$

d $-8x^2 + 12x$

3 a $7x - 1$ **b** $5ab$ **c** $-3a^2b$ **d** $8x^2 - xy$

4 a $x^2 + x - 12$ **b** $x^2 - 9x + 14$

c $6x^2 - 5x - 6$ **d** $3x^2 + x - 4$

5 a $x^2 + 6x + 9$ **b** $x^2 - 8x + 16$

c $9x^2 - 12x + 4$ **d** $x^2 - 25$

e $49 - x^2$ **f** $121x^2 - 16$

6 a $4(a + 3b)$ **b** $-3(1 + 3x)$ **c** $x(x + 1)$

d $3x(2 - 3x)$ **e** $-5xy(x + 2)$

f $11ab(a - 3)$

7 a $\frac{x - 1}{2}$ **b** $x + 1$ **c** $x + 4$

d $\frac{-(x + 3)}{2}$ **e** $\frac{2}{3}$ **f** $2(x - 1)$

8 a $\frac{1}{4}$ **b** $\frac{a}{2}$ **c** $\frac{x - 4}{2}$

d $\frac{2a}{3}$ **e** 4 **f** $\frac{2}{5}$

9 a $\frac{11x}{12}$ **b** $-\frac{x}{24}$ **c** $\frac{a}{4}$

d $\frac{5x - 2}{6}$ **e** $\frac{10x - 5}{12}$ **f** $\frac{7x + 5}{24}$

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Multiple-choice questions

1 E 2 C 3 B 4 B
5 C 6 A 7 E 8 A
9 E 10 E

Extended-response questions

1 a $(x + 3)$ m
b i No change ii $(x^2 - 1)$ m^2, 1 square metre less in area
c i 3 m^2 ii $x = 4$
2 a 400 m^2
b i $L = W = (20 + 2x)$ m ii $(4x^2 + 80x + 400)$ m^2
c $4x(x + 20)$ m^2
d $x = 5$

Chapter 10

Warm-up quiz

1 a 0.1 b 0.25 c 0.3 d 0.85 e 0.237
2 a $\frac{1}{2}$ b $\frac{1}{3}$ c $\frac{2}{3}$ d 1 e 0 f $\frac{1}{12}$
g $\frac{1}{2}$ h $\frac{18}{29}$ i $\frac{2}{3}$ j $\frac{2}{7}$
3 a 1, 2, 3, 4, 5 or 6
b i 3 ii 2 iii 3 iv 5 v 5 vi 3
4 a 5 b 7 c 6 d 7 e 4 f 6
5 a 8 b 20 c $\frac{3}{20}$
6 a 9 b 6 c 5 d 5 e 12
f i $\frac{2}{9}$ ii $\frac{7}{9}$ iii $\frac{5}{9}$

10A

Now you try

Example 1

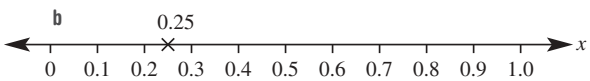
a {1, 2, 3} b $\frac{1}{3}$ c $\frac{1}{2}$ d $\frac{1}{2}$ e $\frac{5}{6}$

Example 2

a $\frac{1}{10}$ b $\frac{7}{10}$ c $\frac{2}{5}$ d $\frac{3}{5}$
e $\frac{9}{10}$ f $\frac{1}{2}$

Exercise 10A

1 a sample space b event
c probability d impossible, certain e even
f complement
2 a i $\frac{1}{4}$ ii 0.25 iii 25%
b



3

	Percentage	Decimal	Fraction	Number line
a	50%	0.5	$\frac{1}{2}$	0 0.5 1
b	25%	0.25	$\frac{1}{4}$	0 0.25 0.5 0.75 1
c	75%	0.75	$\frac{3}{4}$	0 0.25 0.5 0.75 1
d	20%	0.2	$\frac{1}{5}$	0 0.2 0.4 0.6 0.8 1

4 0.15, $\frac{2}{9}$, 1 in 4, 0.28, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{7}$, 2 in 3, 0.7, 0.9

5 a i {1, 2, 3, 4, 5, 6, 7} ii $\frac{1}{7}$ iii $\frac{6}{7}$ iv $\frac{2}{7}$ v $\frac{6}{7}$
b i {2, 6, 7} ii $\frac{1}{2}$ iii $\frac{1}{2}$ iv $\frac{1}{2}$ v 1
c i {1, 2, 3} ii $\frac{2}{3}$ iii $\frac{1}{3}$ iv $\frac{5}{6}$ v $\frac{5}{6}$
d i {1, 2, 3} ii $\frac{1}{3}$ iii $\frac{2}{3}$ iv $\frac{2}{3}$ v $\frac{2}{3}$
e i {1, 2, 3, 4} ii $\frac{1}{7}$ iii $\frac{6}{7}$ iv $\frac{3}{7}$ v $\frac{5}{7}$
f i {2} ii 1 iii 0 iv 1 v 1
6 a $\frac{1}{2}$ b $\frac{3}{8}$ c $\frac{1}{6}$ d $\frac{1}{4}$ e 1 f 0
7 a $\frac{1}{2}$ b $\frac{5}{8}$ c $\frac{5}{6}$ d $\frac{3}{4}$ e 0 f 1
8 a $\frac{1}{8}$ b $\frac{7}{8}$
9 a $\frac{1}{8}$ b $\frac{1}{4}$ c $\frac{3}{8}$ d $\frac{3}{8}$ e $\frac{5}{8}$ f 1 g 0
h $\frac{1}{4}$ i $\frac{3}{4}$ j $\frac{3}{4}$
10 a {Hayley, Alisa, Rocco, Stuart}
b i $\frac{1}{4}$ ii $\frac{1}{2}$ iii $\frac{1}{2}$
11 a $\frac{1}{52}$ b $\frac{1}{13}$ c $\frac{1}{26}$ d $\frac{1}{2}$
e $\frac{2}{13}$ f $\frac{12}{13}$ g $\frac{23}{26}$ h $\frac{25}{26}$
12 a $\frac{1}{6}$ b $\frac{1}{6}$ c $\frac{5}{6}$ d $\frac{1}{3}$
e $\frac{2}{3}$ f 1 g $\frac{1}{3}$ h $\frac{1}{2}$
13 a $\frac{2}{11}$ b $\frac{9}{11}$ c $\frac{4}{11}$ d $\frac{7}{11}$
e $\frac{7}{11}$ f $\frac{3}{11}$ g $\frac{7}{11}$ h $\frac{4}{11}$
14 a 31 min
b $\frac{3}{31}$
c i $\frac{4}{31}$ ii $\frac{4}{31}$ iii $\frac{20}{31}$ iv $\frac{5}{31}$ v $\frac{8}{31}$ vi $\frac{11}{31}$

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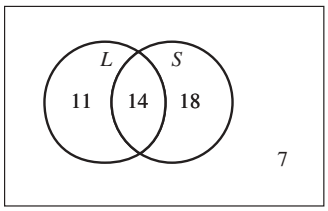
10B

Now you try

Example 3

a



b i 43 ii 25 iii 18

c i $\frac{7}{25}$ ii $\frac{7}{50}$ iii $\frac{11}{50}$

Example 4

a

	Natural	Not Natural	Total
Artificial	20	25	45
Not artificial	45	10	55
Total	65	35	100

b i 55 ii 10

c i $\frac{13}{20}$ ii $\frac{9}{20}$ iii $\frac{9}{10}$

Exercise 10B

1 a 26

b i 10 ii 14 iii 5 iv 9 v 4 vi 7 vii 19

c i 12 ii 17

2 a 22 b 12 c 10 d 39

e 13 f 1

3 a B b D c A d C

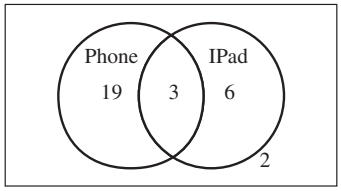
4 a

	A	Not A	Total
B	7	8	15
Not B	3	1	4
Total	10	9	19

b

	A	Not A	Total
B	2	5	7
Not B	9	4	13
Total	11	9	20

5 a



b i 28 ii 21 iii 6

c i $\frac{1}{10}$ ii $\frac{1}{15}$ iii $\frac{19}{30}$

6 i Not A: diagram T ii A or B: diagram R

iii A and B: diagram U iv B: diagram P

v B only: diagram Q vi Neither A nor B: diagram S

7 a i $\frac{2}{5}$ ii $\frac{1}{3}$ iii $\frac{7}{15}$ iv $\frac{1}{15}$ v $\frac{13}{15}$ vi $\frac{2}{15}$

b i $\frac{3}{7}$ ii $\frac{12}{35}$ iii $\frac{13}{35}$ iv $\frac{3}{35}$ v $\frac{34}{35}$ vi $\frac{1}{35}$

8 a

	Cream	Not cream	Total
Ice cream	5	20	25
Not ice cream	16	9	25
Total	21	29	50

b i 29 ii 9

c i $\frac{21}{50}$ ii $\frac{8}{25}$ iii $\frac{41}{50}$

9 a

	A	Not A	Total
B	3	1	4
Not B	2	2	4
Total	5	3	8

i $\frac{5}{8}$ ii $\frac{3}{8}$ iii $\frac{3}{8}$ iv $\frac{3}{4}$ v $\frac{1}{8}$ vi $\frac{1}{4}$

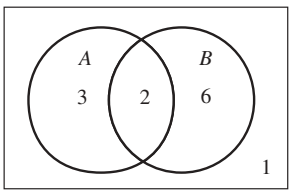
b

	A	Not A	Total
B	11	4	15
Not B	6	5	11
Total	17	9	26

i $\frac{17}{26}$ ii $\frac{9}{26}$ iii $\frac{11}{26}$ iv $\frac{21}{26}$ v $\frac{2}{13}$ vi $\frac{5}{26}$

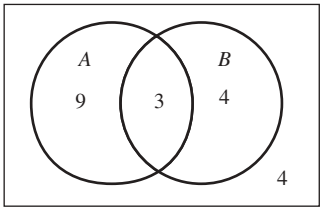
10 a

	A	Not A	Total
B	2	6	8
Not B	3	1	4
Total	5	7	12



b

	A	Not A	Total
B	3	4	7
Not B	9	4	13
Total	12	8	20



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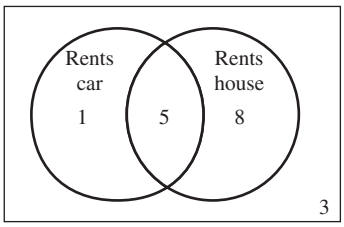
11 a



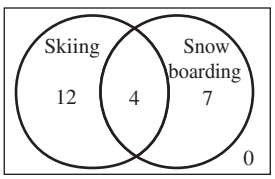
b 5 **c** $\frac{1}{5}$

12 a

	Tank water	No tank water	Total
Tap water	12	36	48
No tap water	11	41	52
Total	23	77	100

b 12 **c** $\frac{9}{25}$ **d** $\frac{59}{100}$

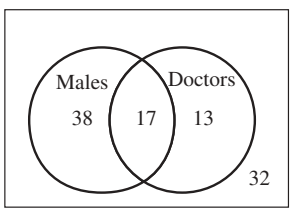
13 a



b 23 **c** $\frac{11}{23}$ **d** $\frac{4}{16} = \frac{1}{4}$ **e** $\frac{4}{11}$

14 a $\frac{3}{10}$ **b** $\frac{1}{3}$

15



a 32 **b** 45 **c** 70 **d** 38
e 13 **f** 13 **g** 62

10C

Now you try

Example 5

a

	1	2	3	4	5	6	7	8
H	H1	H2	H3	H4	H5	H6	H7	H8
T	T1	T2	T3	T4	T5	T6	T7	T8

b 16 **c** $\frac{1}{16}$ **d** $\frac{3}{16}$

Example 6

a

	3	5	6
3	×	(3, 5)	(3, 6)
5	(5, 3)	×	(5, 6)
6	(6, 3)	(6, 5)	×

b $\frac{1}{6}$ **c** $\frac{1}{3}$

Exercise 10C

1

	B	Y	E
H	HB	HY	HE
I	IB	IY	IE

2 a

	1	2	3	4
H	H1	H2	H3	H4
T	T1	T2	T3	T4

b 8 outcomes in the sample space
c $\frac{1}{8}$ **d** $\frac{1}{8}$

3 a 10 **b** H2, H4, T2, T4 **c** $\frac{2}{5}$
d T1, T3, T5 **e** $\frac{3}{10}$

4 a $\frac{1}{4}$ **b** HH, TT **c** $\frac{1}{2}$

5 a

	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

b 12 **c** $\frac{1}{12}$ **d** $\frac{1}{4}$

6 a

	R	I	D	E
L	LR	LI	LD	LE
I	IR	II	ID	IE
N	NR	NI	ND	NE
E	ER	EI	ED	EE

b 16 **c** $\frac{1}{16}$ **d** $\frac{1}{16}$ **e** $\frac{1}{4}$
f $\frac{1}{4}$ **g** $\frac{1}{8}$

7 a

	R	P	B
R	RR	RP	RB
P	PR	PP	PB
G	GR	GP	GB
B	BR	BP	BB

b $\frac{1}{12}$ **c** $\frac{1}{12}$ **d** $\frac{1}{6}$ **e** $\frac{1}{4}$

8 a

Sum	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

36 outcomes **b** $\frac{1}{9}$ **c** $\frac{1}{2}$
d 7 **e** 2 and 12

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9 a

	Y	W	B	B	B
W	WY	WW	WB	WB	WB
O	OY	OW	OB	OB	OB
O	OY	OW	OB	OB	OB

b $\frac{2}{15}$ **c** $\frac{1}{15}$ **d** $\frac{2}{15}$ **e** $\frac{1}{5}$

f $\frac{1}{15}$

10 a

	D	O	G
D	X	(O, D)	(G, D)
O	(D, O)	X	(G, O)
G	(D, G)	(O, G)	X

b $\frac{1}{6}$ **c** $\frac{2}{3}$

11 a

		1st			
		1	**2**	**3**	**4**
2nd	**1**	X	(2, 1)	(3, 1)	(4, 1)
	2	(1, 2)	X	(3, 2)	(4, 2)
	3	(1, 3)	(2, 3)	X	(4, 3)
	4	(1, 4)	(2, 4)	(3, 4)	X

b i $\frac{1}{12}$ **ii** $\frac{1}{12}$

c i $\frac{1}{6}$ **ii** $\frac{5}{6}$ **iii** $\frac{1}{6}$ **iv** $\frac{1}{2}$

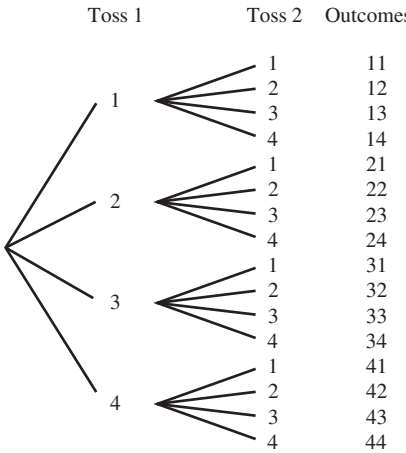
12 a $\frac{1}{8}$ **b** $\frac{1}{3}$

10D

Now you try

Example 7

a

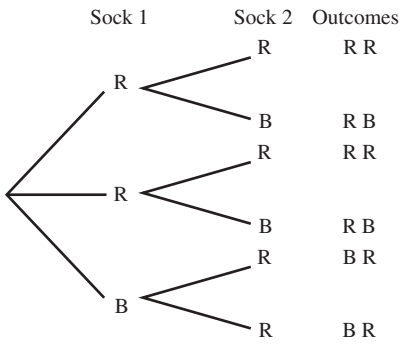


b 16

c i $\frac{1}{16}$ **ii** $\frac{3}{8}$ **iii** $\frac{15}{16}$

Example 8

a



b i $\frac{1}{3}$

ii $\frac{2}{3}$

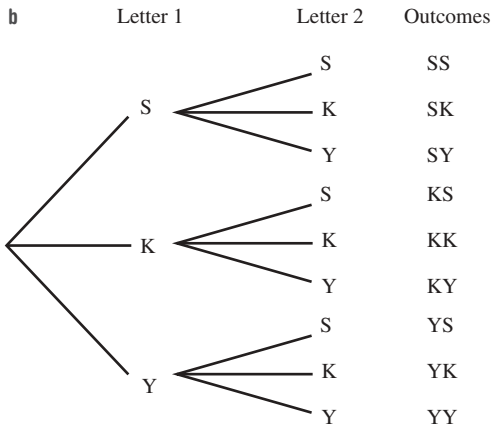
iii 1

Exercise 10D

1 a A **b** B

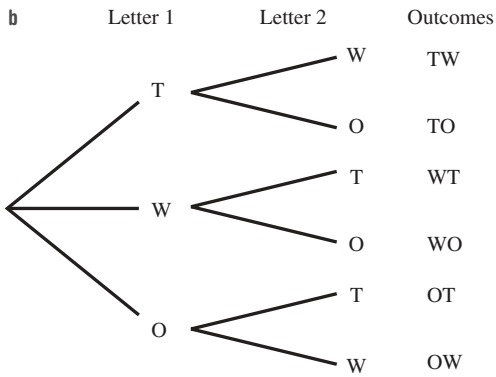
2 a i replacement **ii** 3

b



3 a i Without replacement **ii** 2

b



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4 a

Toss 1	Toss 2	Outcomes
H	H	HH
	T	HT
T	H	TH
	T	TT

b 4

c **i** $\frac{1}{4}$

ii $\frac{1}{2}$

iii $\frac{3}{4}$

iv $\frac{3}{4}$

5 a

Spin 1	Spin 2	Outcomes
1	1	1, 1
	2	1, 2
	3	1, 3
2	1	2, 1
	2	2, 2
	3	2, 3
3	1	3, 1
	2	3, 2
	3	3, 3

b $\frac{1}{9}$ **c** $\frac{1}{9}$ **d** $\frac{2}{9}$ **e** $\frac{1}{3}$

f $\frac{1}{3}$

6 a

Person 1	Person 2	Outcomes
D	E	DE
	F	DF
E	D	ED
	F	EF
F	D	FD
	E	FE

b **i** $\frac{1}{3}$ **ii** $\frac{2}{3}$ **iii** 1

7 a

Sock 1	Sock 2	Outcomes
R	R	RR
	B	RB
	Y	RY
R	R	RR
	B	RB
	Y	RY
B	R	BR
	R	BR
	Y	BY
Y	R	YR
	R	YR
	B	YB

b **i** $\frac{1}{3}$ **ii** $\frac{1}{6}$ **iii** $\frac{1}{6}$ **iv** $\frac{5}{6}$

8 a

Coin 1	Coin 2	Coin 3	Outcomes
H	H	H	HHH
		T	HHT
	T	H	HTH
		T	HTT
T	H	H	THH
		T	THT
	T	H	TTH
		T	TTT

b $\frac{1}{8}$ **c** $\frac{1}{8}$ **d** $\frac{3}{8}$

e Getting exactly 2 tails

9 a $\frac{3}{5}$ **b** $\frac{2}{5}$ **c** $\frac{7}{10}$

10 a $\frac{1}{9}$ **b** $\frac{1}{9}$ **c** $\frac{7}{18}$ **d** $\frac{7}{18}$

e $\frac{1}{2}$ **f** 0 **g** $\frac{1}{6}$ **h** $\frac{2}{3}$

i $\frac{1}{9}$ **j** $\frac{17}{18}$

10E

Now you try

Example 9

a $\frac{4}{5}$ or 0.8 **b** 40

Example 10

a $\frac{5}{24}$ **b** 200

Exercise 10E

1 a Observed **b** Theoretical **c** Theoretical
d Observed **e** Expected

2 a $\frac{9}{10}$ **b** $\frac{4}{5}$ **c** $\frac{5}{9}$

3 a $\frac{2}{5}$ **b** 40

4 a 5 **b** 10 **c** 50 **d** 4

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5 **a** $\frac{35}{90} = \frac{7}{18}$ **b** 1050

6 **a** 0.6

b **i** 60 **ii** 120 **iii** 360

7 **a** $\frac{7}{8}$

b **i** 350 **ii** 4375 **iii** 35

8 **a** $\frac{1}{15}$ **b** $\frac{2}{5}$ **c** $\frac{11}{60}$

9 **a** B: $\frac{5}{20} = 0.25$ C: $\frac{30}{100} = 0.3$

b C as it is a larger sample size

10 0.41, from the 100 throws as the more times an experiment is carried out the closer the observed probability becomes to the actual/theoretical probability

11 **a** 20 **b** 40 **c** 60 **d** 40

12 **a** 20

b **i** $\frac{1}{4}$ **ii** $\frac{7}{20}$

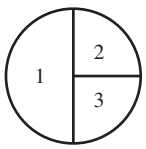
c **i** 25 **ii** 20 **iii** 45

13 **a** **i** $\frac{13}{25}$ **ii** $\frac{12}{25}$ **iii** $\frac{39}{50}$

b 78

c $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$

d



14 **a** $\frac{\text{Shaded area}}{\text{Total area}} = 0.225$, $\therefore$ 100 shots ≈ 23

b $\frac{1}{10} \times 100 = 10$

c $\frac{150 - 32}{150} \times 100 \approx 79$

d $\frac{225\pi - 25\pi}{225\pi} \times 100 \approx 89$

10F

Now you try

Example 11

a **i** 5 **ii** 4 **iii** 7 **iv** 6

b **i** 0.75 **ii** 0.5 **iii** 0 **iv** 9

Example 12

a 87 **b** 97

Exercise 10F

1 **a** mean

b **i** 7.8 **ii** 9.9

2 **a** common (or frequent)

b **i** 2 **ii** 10

3 **a** middle

b **i** 5 **ii** 8

4 **a** range

b **i** 21 **ii** 22

5 **a** outlier

b **i** 36 **ii** 3

6

	Mean	Median	Mode	Range
a	6	7	8	7
b	8	6	5, 10	13
c	6	6	2	10
d	11	12	none	13
e	4	3.5	2.1	6.2
f	5	4.5	none	9.7
g	3	3.5	−3	12
h	0	2	3	11

7 **a** Outlier = 33, mean = 12, median = 7.5

b Outlier = 2, mean = 35.2, median = 42

c Outlier = −1.1 , mean = 1.075, median = 1.4

d Outlier = −4, mean = −49, median = −59

8 **a** Yes **b** No **c** No **d** Yes

9 **a** 12 **b** 5.5 **c** 6

d Bimodal; 4, 7

10 **a** 24.67 s **b** 24.8 s

11 **a** $1 700 000

b ($354 500 and $324 000) drops $30 500

c ($570 667 and $344 800) drops $225 867

12 **a** 15 **b** 26

13 **a** 90 **b** 60

14 **a** 76.75

b **i** 71.4, B^+ **ii** 75, B^+ **iii** 80.2, A

c 81.4, he cannot get an A^+

d **i** 43 **ii** 93

Progress quiz

1 **a** {1, 2, 4, 5, 7}

b **i** $\frac{1}{3}$ **ii** $\frac{2}{3}$ **iii** $\frac{1}{2}$ **iv** $\frac{5}{6}$

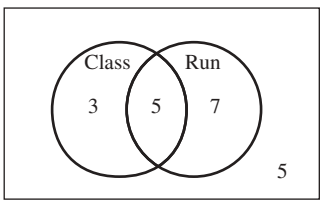
2 **a** $\frac{1}{8}$ **b** $\frac{1}{4}$ **c** $\frac{3}{4}$ **d** $\frac{1}{4}$ **e** $\frac{1}{2}$

3 **a**

	A	Not A	Total
B	4	5	9
Not B	4	7	11
Total	8	12	20

b **i** $\frac{2}{5}$ **ii** $\frac{1}{5}$ **iii** $\frac{1}{4}$ **iv** $\frac{13}{20}$

4 **a**



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b i 8 ii 15 iii 3

c i $\frac{1}{4}$ ii $\frac{1}{4}$ iii $\frac{7}{20}$

d

	Run	Not run	Total
Class	5	3	8
Not class	7	5	12
Total	12	8	20

5 a

	1	2	3	4
H	H1	H2	H3	H4
T	T1	T2	T3	T4

b 8

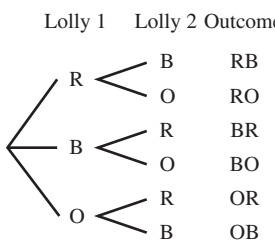
c i $\frac{1}{8}$ ii $\frac{3}{8}$

6 a

		1st choice		
		B	**E**	**E**
2nd choice	**B**	×	(E, B)	(E, B)
	E	(B, E)	×	(E, E)
	E	(B, E)	(E, E)	×

b $\frac{1}{3}$ **c** $\frac{2}{3}$

7 a



b i $\frac{1}{3}$ ii $\frac{2}{3}$

8 a $\frac{3}{5}$ **b** 240

9 a i 12 ii 10 iii 8 iv 15

b i 16 ii 15.5 iii No mode iv 17

10 a 25° **b** 32°

10G

Now you try

Example 13

a i 125 ii 200 iii 175

b 75

c 144°

Exercise 10G

1 Mean, range

2 A skewed, B symmetrical

3 A, the gap looks larger because the vertical scale does not start at zero.

4 a 17

b 3, 5, 6, 6, 7, 8, 8, 8, 9, 9, 9, 9, 9, 10, 10, 10, 10 **c** 8

d 9 **e** 7 **f** Skewed **g** $\frac{12}{17}$

5 a 25 **b** 30 cm **c** 173 cm **d** 170 cm

e 168.88 cm **f** 186 cm

6 a 44% **b** 10 560/year **c** 29/day **d** 15.1/day

e 108°

7 a 1.42 m **b** 0847, 8:47 a.m.

c 1 hour 21 minutes **d** 31 cm or 0.31 m

e 11 hours 55 minutes

8 a 41 years **b** 24 years **c** 16 years

d 10 years, 14.5%

9 a 18°C **b** 43 mm **c** 5°C **d** 47 mm

e January, February, March, December

f February, March, May **g** August, September

h 9°C

10 a 45 m

b 25 m

c 20 m

d 69 m

e 35 m − 14 m = 21 m more

11 a Channel 15, 27; channel 16, 24; channel 17, 26; channel 18, 24

b Graph A

c Graph B

d The scale on graph A starts at 23 but the scale on graph starts at 0.

e Graph A is misleading as the scale expands the difference in column heights.

12 a Energy and Yum-crunch equal, Chunky, Sunshine

b No, it was equal to Energy.

c The Yum-crunch column has wider dimensions, a different colour and the cereal name on the column.

d All columns should be the same width, same colour and either all or none have the name.

13 a Rugby

b Rugby, soccer, basketball, cricket, tennis

c Rugby: 700 000; Soccer: 600 000

d Cricket, 800 000

e Cricket, rugby, soccer, basketball and tennis equal

f The length of each row is misleading because the ball sizes are not equal.

g Each picture should be an equal size.

14 a 10

b $560 000, $560 000, $750 000, $750 000, $750 000, $750 000, $870 000, $870 000, $870 000, $2 300 000

c $903 000

d $750 000

e Mean

f 9 out of 10 house prices are less than the mean of $903 000.

g The median is a better measure of 'average' as it is the middle value. The mean is increased by one very large value.

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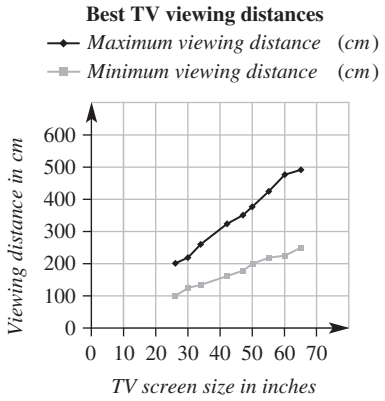
15 a



b 26 inch, 30 inch, 34 inch

c 42 inch, 47 inch, 50 inch, 55 inch, 60 inch, 65 inch

10H

Now you try

Example 14

a

Stem	Leaf
0	9
1	1 3 6
2	1 4 6 9 9
3	4 5 7
4	2 6
5	0 2

3 | 5 means 35

b 29

c 29

d Almost symmetrical

Example 15

a

Class A		Class B
	0	3
9 9 7 5	1	1 2 3 4 6 8
9 4 1	2	0
6 1	3	

2 | 4 means 24 pages

b The number of pages read by class A is between 15 and 36 while for class B it is between 11 and 20 except for a lower value of 3. In general class A is reading more.

Exercise 10H

1 a 3.5, 3.7, 4.1, 4.3, 4.8

b 52, 60, 61, 67, 73, 75

2 a 9

b i 8 min **ii** 35 min

c 21 min

d 21 min

3 a 26

b i 0 mm **ii** 6 mm

c i 21 mm **ii** 24 mm

d i 8 mm **ii** 15 mm

e Skewed

f Symmetrical

4 a i

Stem	Leaf
1	9 9
2	3 4 6 6 8 8 8
3	2 2 3 5
4	1
5	4

2 | 3 means 23

ii 28

iii 28

iv Skewed

b i

Stem	Leaf
1	5 5
2	1 3 3 3 6
3	0 1 1 3 4 5 5 7 9
4	2 2 5 5 5 8
5	0 0 1

4 | 2 means 42

ii 35 **iii** 23, 45 **iv** Almost symmetrical

c i

Stem	Leaf
32	0
33	3 7 8
34	3 4 5 5 7 8 9
35	2 2 4 5 8
36	1 3 5
37	0

34 | 3 means 34.3

ii 34.85 **iii** 34.5, 35.2 **iv** Almost symmetrical

d i

Stem	Leaf
15	7 8 9 9 9 9
16	1 4 7 7 7
17	3 5 7 7
18	5 5 7 9
19	3 8
20	0 2

17 | 7 means 177

ii 173 **iii** 159 **iv** Skewed

5 a

Stem	Leaf
0	5 6 6 8 8 9
1	0 1 1 2 2 2 4 4 5 6 7
2	0 1 2

1 | 2 means 12

b 9 **c** 12

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6 a i

Set A		Set B
9 3 2	3	1
8 7 6 5 3 3 0 0	4	0 1 3 4 4 6 7 8 8 9 9
4 4 3	5	1 3

3 | 2 means 32

ii Set A has values spread between 32 and 54 while set B has most of its values between 40 and 53 with an outlier at 31.

b i

Set A		Set B
8 7 4 3	0	1 1 1 2 3 6 6 9 9
9 9 7 6 4	1	2 3 5 8 9
6 5 3 1	2	1 5 6
9 9 6 4 3 2	3	3 4 9
7 3 1	4	3 7 8
7 3 2	5	2 3 7
2 1	6	1 2
8 3	8	3 8
1	9	

4 | 1 means 4.1

ii Set A and B are similar. Set A has values between 0.3 and 9.1 and set B has values between 0.1 and 8.8. Both sets have many small numbers and fewer large numbers.

7 a

Collingwood		St Kilda
8 3	6	6 8 8
8 7 2 1 0	7	8
9 9 8 2 0	8	0 0 2 2 3 4 7 8
8 7 5 1	9	0 4
4 3 3 3	10	6 9
9 8 5	11	1 3 7 8
7	12	2 5 6
	13	8

10 | 6 means 106

b Collingwood $33\frac{1}{3}\%$ St Kilda $41\frac{2}{3}\%$

c Collingwood is almost symmetrical data. St Kilda is not symmetrical.

d Collingwood results seem to be consistent. St Kilda has groups of similar scores and, while less consistent, that team has higher scores.

8 a 16.1 s **b** 2.3 s **c** Yes, 0.05 s lower

9 a

Battery lifetime

Brand A		Brand B
3	7	2 3 4
	7	5 6 8 9 9
4 2	8	0 1 3
9 8 7 5	8	
4 4 3 2 1 0	9	0 1 2 3 4
9 9 8 7 6 5 5	9	5 6 8 8

8 | 3 represents 8.3 hours

b Brand A, 12; brand B, 8

c Brand A consistently performs better than brand B.

10 a 52 **b** 17.8

11 a 48% **b** 15%

c In general, birth weights of babies are lower for mothers who smoke.

10I

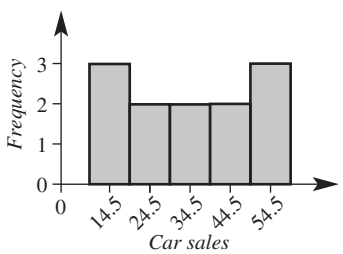
Now you try

Example 16

a

Class interval	Frequency	Percentage frequency
10–19	3	25
20–29	2	$16\frac{2}{3}$
30–39	2	$16\frac{2}{3}$
40–49	2	$16\frac{2}{3}$
50–59	3	25
Total	12	100

b



c 3

d 25%

Exercise 10I

1 a class interval **b** frequency table
c histogram **d** no spaces **e** percentage frequency

2 a 20 **b** 20

3 a 3 **b** 360 000–370 000

4 a 2

b 20

c i 30% **ii** 65%

5 a

Class	Frequency	Percentage frequency
0–9	23	46
10–19	10	20
20–29	6	12
30–39	7	14
40–49	4	8
	50	100

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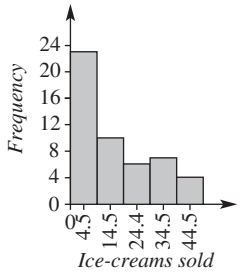
b



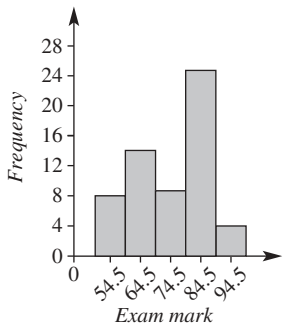
c i 33 ii 11

d 34%

6 a

Class	Frequency	Percentage frequency
50–59	8	$13\frac{1}{3}$
60–69	14	$23\frac{1}{3}$
70–79	9	15
80–89	25	$41\frac{2}{3}$
90–99	4	$6\frac{1}{3}$
	60	100

b

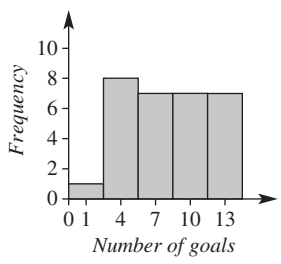


c i 22 ii $48\frac{1}{3}\%$

7 a

Number of goals	Frequency
0–2	1
3–5	8
6–8	7
9–11	7
12–14	7
	30

b



c 9 **d** 7

8 a Symmetrical data **b** Skewed data

9 a $a = 15$ $b = 2$ $c = 30$ $d = 40$ $e = 20$ $f = 100$

b $a = 50$ $b = 28$ $c = 12$ $d = 40$ $e = 100$

10 85.5%

11 a i 20% ii 55% iii 80% iv 75%
v 50%

b i 30 ii 45

c i 2 ii 22

12 a Student A

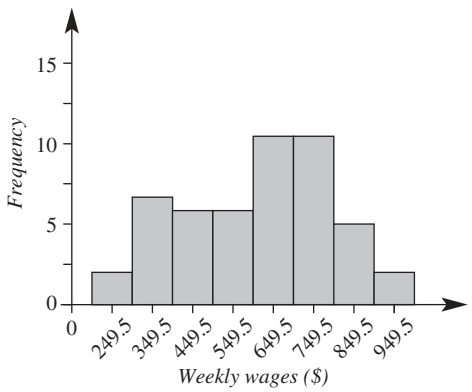
b Make the intervals for their groups of data smaller so that the graph conveys more information.

13 a Minimum wage: \$204; Maximum wage: \$940

b i

Weekly wages (\$)	Frequency
200–299	2
300–399	7
400–499	6
500–599	6
600–699	11
700–799	11
800–899	5
900–999	2
Total	50

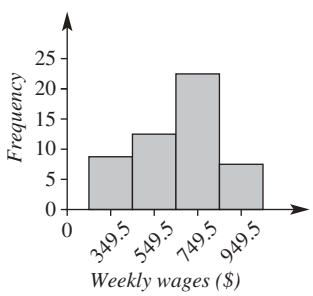
ii



c i

Weekly wages	Frequency
200–399	9
400–599	12
600–799	22
800–999	7
Total	50

ii



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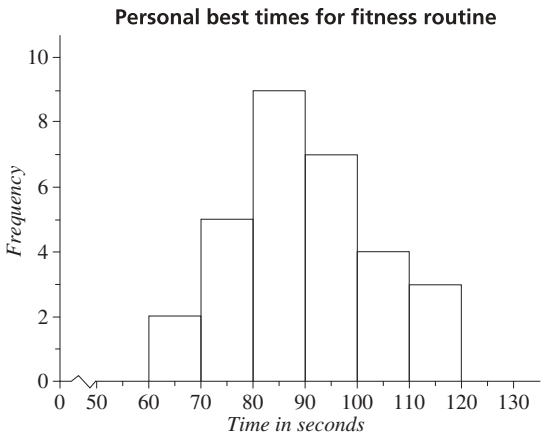
d More intervals shows greater detail. Since the first graph has each pair of intervals quite similar, these two graphs are quite similar.

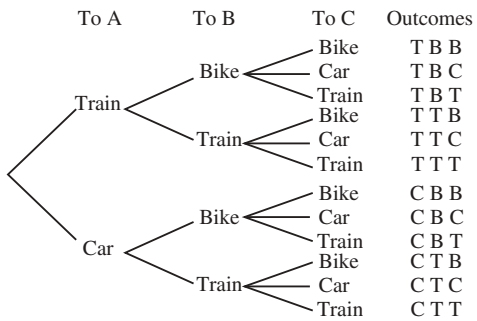
Maths@Work: Personal trainer

1 **a** 88 bpm **b** 84 bpm **c** 98 bpm **d** 119 bpm
2 **a** Yes **b** No **c** Yes **d** No
e Yes
3 Mean 94.6 bpm, mode 90 bpm, median 93.5 bpm, range 35 bpm
4 Mean 105.8 bpm, mode 98 bpm, median 104.5 bpm, range 59 bpm
The data for heart rate after exercise has a higher mean, median and mode, and is more
spread out (range is much higher).
5 Graph should look like the following.



Puzzles and challenges

1 IT'S RANDOM
2 1 – C, 2 – B, 3 – E, 4 – D, 5 – A
3 6 own both cat and a dog
4 2, 3, 4, 8, 8
5 7, 8, 9, 10 and 11
6 **a**



b 12 **c** $\frac{1}{4}$

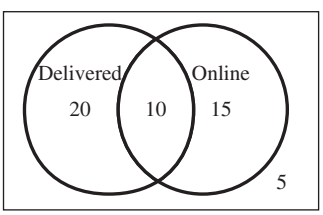
Chapter checklist with success criteria

1 **a** $\frac{1}{5}$ **b** $\frac{7}{10}$ **c** $\frac{9}{10}$
2 **a** 10 **b** $\frac{28}{30}$
3 **a** 2 **b** $\frac{11}{20}$
4 **a** $\frac{1}{4}$ **b** $\frac{5}{8}$ **c** $\frac{7}{8}$
5 **a** $\frac{1}{6}$ **b** $\frac{1}{3}$
6 **a** $\frac{1}{2}$ **b** $\frac{2}{4}$
7 **a** $\frac{3}{5}$ **b** $\frac{7}{10}$
8 **a** $\frac{1}{4}$ **b** 625
9 **a** 3 and 18.5 **b** 3 and 19.5 **c** 3 and 21 **d** 4 and 13
10 68 and 78
11 **a** 4 **b** 2.5 and 2.5 **c** 70%
12 a is misleading because it uses only 2 tick marks to make the gap look bigger
13 mode = 26, median = 27
14 Team A has a wider spread of scores that are on average higher and on average score more than team B
15 75%

Chapter 10 review

Short-answer questions

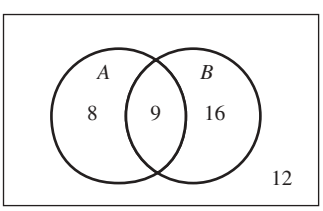
1 **a** $\frac{2}{3}$ **b** $\frac{5}{9}$ **c** $\frac{3}{5}$
2 **a**



b 15
c **i** $\frac{1}{5}$ **ii** $\frac{2}{5}$ **iii** $\frac{3}{10}$
3 **a**

	A	Not A	Total
B	9	16	25
Not B	8	12	20
Total	17	28	45

b



c **i** $\frac{4}{9}$ **ii** $\frac{1}{5}$ **iii** 8 **iv** 33

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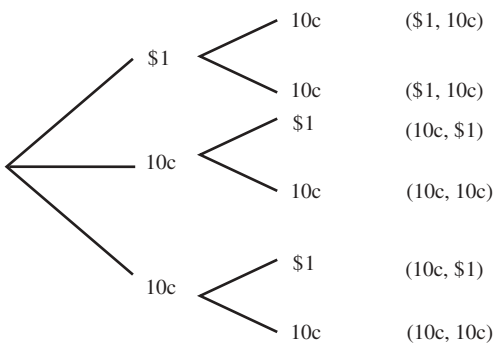
4 a 12 outcomes

	1	2	3	4
red	(red, 1)	(red, 2)	(red, 3)	(red, 4)
green	(green, 1)	(green, 2)	(green, 3)	(green, 4)
blue	(blue, 1)	(blue, 2)	(blue, 3)	(blue, 4)

b i $\frac{1}{6}$ **ii** $\frac{1}{6}$ **iii** $\frac{2}{3}$

5 a



b $\frac{2}{3}$

6 a $\frac{48}{120} = \frac{2}{5}$ **b** 14

7 a 26 **b** 26.5 **c** 23, 31 **d** 18

8 a

Employee 1		Employee 2
9	1	7 7
5 3 1	2	0 4 8 9
9 8 6 5 5 4	3	2 7 7 8
6 5 0	4	0 1 2 5 8
3 1	5	

2 | 4 means 24

b i Employee 1: 36, employee 2: 37
ii Employee 1: 36, employee 2: 33

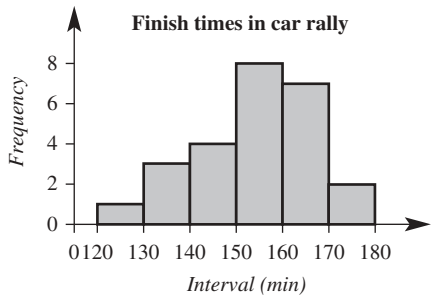
c Employee 1; they have a higher mean and more sales at the high end.

d Employee 1 symmetrical, employee 2 skewed

9 a

Class interval	Frequency	Percentage frequency
120–	1	4
130–	3	12
140–	4	16
150–	8	32
160–	7	28
170–	2	8
Total	25	100

b



c i 4 **ii** 60%

10 a Mean = $760 000

Median = $590 000

b The median is better because the mean has been inflated by the outlier of $2 900 000.

Multiple-choice questions

1 B
2 A
3 i A **ii** B **iii** D **iv** C
4 B
5 D
6 i B **ii** C
7 E

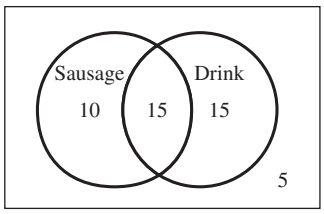
Extended-response questions

1 a i

		1st spin				
		1	**2**	**3**	**4**	**5**
2nd spin	**1**	(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)
	2	(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)
	3	(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)
	4	(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)
	5	(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)

ii $\frac{12}{25}$ **iii** $\frac{4}{25}$ **iv** 24 **v** $68

b i



ii 5 **iii** 10 **iv** $\frac{1}{3}$ **v** $\frac{4}{9}$

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2 a

Airline A		Airline B
4 4 3 2 2 1 1 0	0	1 2 4 4
9 8 7 7 6	0*	5 5 6 7 7 8 9
4 4 4 3 3 3 2 2 1	1	0 0 0 0 1 2 2
9 9 8 8 7	1*	5 5 8 8 8 9 9
4 3	2	1 2 2
	2*	5 6
	3	
	3*	
	4	
	4*	
2	5	

1 | 2 means 12 1* | 5 means 15

b Yes, 52 min

c i Airline A: 12, airline B: 10.5

ii Airline A: 10.6, airline B: 12.4

d No, the median time is 12 min so half the flights have less than 12 min delay.

Semester review 2

Chapter 6: Length, area, surface area and volume

Short-answer questions

1 a 22 cm^2 **b** 12 cm^2 **c** 25.92 m^2

2 a 25.13 m, 50.27 m^2 **b** 62.83 cm, 314.16 cm^2

c 13.19 cm, 13.85 cm^2

3 a 54.82 cm^2 **b** 26 m^2 **c** 24.57 m^2 **d** 36 cm^2

4 a 220 m^2 **b** 216 m^2 **c** 25.45 m^2

5 a 12 m^3 **b** 42 m^3 **c** 100.53 cm^3

Multiple-choice questions

1 C **2** C **3** D **4** C **5** A

Extended-response question

a 12.57 m^2 **b** 77.43 m^2 **c** $628

Chapter 7: Indices

Short-answer questions

1 a $15p^3q^2$ **b** $\frac{a^2b}{2}$ **c** $54x^9y^6$ **d** -2

2 a $3 \times 4^2 = 48$ **b** $\frac{4}{10^3} = 0.004$

3 a 2400 **b** 1 080 000 **c** 0.0071

d 0.0000206

4 a 6.03×10^4 **b** 2.7×10^6 **c** 4×10^{-3}

d 7.03×10^{-4}

5 a 3.07×10^{-2} kg **b** 4.24×10^6 kg

c 1.22×10^4 s **d** 7.45×10^{-6} years

Multiple-choice questions

1 C **2** B **3** D **4** E **5** C

Extended-response question

a i 74 000 000 000 **ii** 7.4×10^{10}

b 1.87×10^{17}

c 8.72×10^{-7}

Chapter 8: Properties of geometrical figures

Short-answer questions

1 a 180° **b** 360° **c** 540°

d 900° **e** 1080° **f** 1440°

2 a $x = 70$ **b** $x = 70$ **c** $a = 100, b = 140$

d $a = 70, b = 110$ **e** $x = 100, y = 110$

f $x = 67, y = 98$ **g** $x = 120$

h $a = 135$

3 $x = 4$

4 2

5 2.4

Multiple-choice questions

1 C **2** B **3** A **4** A **5** B

Extended-response question

a 6 m **b** $\frac{5}{3} = 1.\dot{6}$ **c** 7.5 m **d** 3.75 m

Chapter 9: Quadratic expression and algebraic figures

Short-answer questions

1 a $-3x - 6$ **b** $x - x^2$ **c** $x^2 - 1$

d $x^2 - 8x + 16$ **e** $25x^2 - 4$

f $9x^2 - 24x + 16$ **g** $8x^2 + 10x - 3$

h $2x + 3$ **i** $4 - 4x + x^2$

2 a $-6x^2y$ **b** $42a^2b^2$ **c** $\frac{a}{2}$ **d** $\frac{-6x}{y}$

e $\frac{a}{b}$ **f** $\frac{1}{2y}$

3 a $x^2 - 9$ **b** $x^2 + 4x + 4$ **c** $6x^2 - 17x + 12$

4 a $3(x - 4)$ **b** $-7(x + 2)$ **c** $x(5x + 2)$

d $7x(2x - 3)$

5 a $\frac{3x}{2}$ **b** $\frac{3}{4}$ **c** 4 **d** $-x$

e $\frac{3}{2}$ **f** $\frac{9a}{x}$

6 a $\frac{x}{12}$ **b** $\frac{53x}{20}$ **c** $\frac{7x + 5}{10}$ **d** $\frac{11x + 7}{15}$

Multiple-choice questions

1 E **2** E **3** A **4** B **5** D

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Extended-response question

a $10 - 2x$ and $8 - 2x$

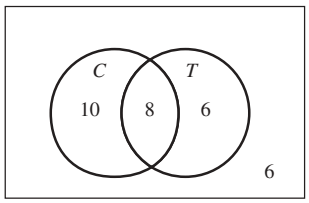
b $(10 - 2x)(8 - 2x) = 80 - 36x + 4x^2$

c 48 m^2

Chapter 10: Probability and data analysis

Short-answer questions

1 a



b i 24 **ii** 16

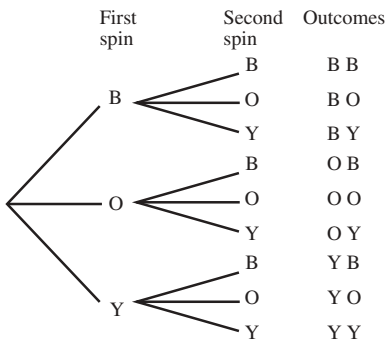
c i $\frac{1}{5}$ **ii** $\frac{1}{3}$

2 a

	1	2	3	4
H	H1	H2	H3	H4
T	T1	T2	T3	T4

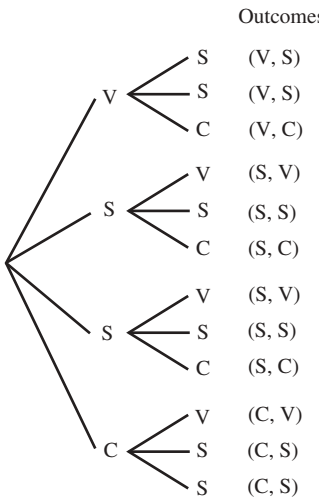
b 8 **c** $\frac{1}{8}$ **d** $\frac{1}{4}$

3 a



b $\frac{1}{9}$ **c** $\frac{1}{3}$ **d** $\frac{2}{9}$

4 a



b i $\frac{1}{3}$ **ii** $\frac{1}{6}$ **iii** $\frac{1}{2}$

5 a

Stem	Leaf
1	0 1 1 2 3 5 7 8
2	2 5 5 5 6
3	1 2 2

1 | 3 means 13 aces

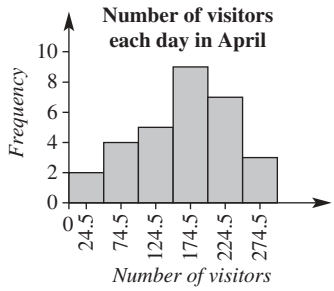
b Mode = 25, median = 20

c Skewed

6 a

Class interval	Class centre	Frequency	Relative frequency (%)
0–49	24.5	2	6.7
50–99	74.5	4	13.3
100–149	124.5	5	16.7
150–199	174.5	9	30
200–249	224.5	7	23.3
250–299	274.5	3	10
Total	30	100	100

b



c i 6 **ii** 60%

Multiple-choice questions

1 D

2 A

3 a C **b** D

4 B

5 B

Extended-response question

a

		Red		
		1	2	3
Green	1	(1, 1)	(1, 2)	(1, 3)
	2	(2, 1)	(2, 2)	(2, 3)
	3	(3, 1)	(3, 2)	(3, 3)

b $\frac{4}{9}$

c i $\frac{1}{3}$ **ii** 18

d i 17 years **ii** 92

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