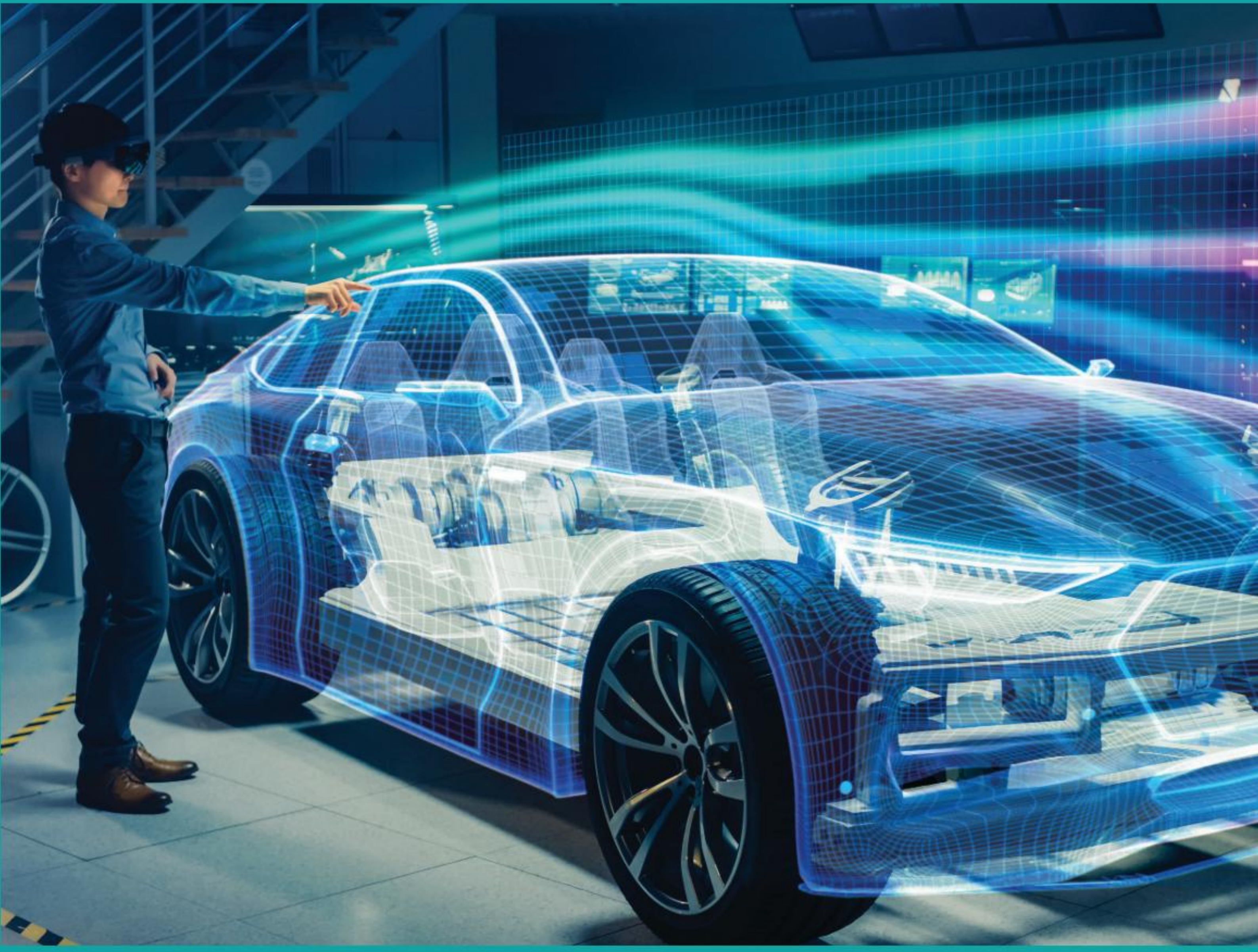
STUDY GUIDE

MATHEMATICS APPLICATIONS

YEAR 11 ATAR COURSE – UNITS 1 & 2

REVISED EDITION



Greg Hill

ACADEMIC ASSOCIATES

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YEAR 11 ATAR COURSE

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ACADEMIC GROUP
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About the Author
Greg Hill has been teaching mathematics for over 30 years at various government and independent schools. Greg has been an ATAR marker and author of study guides and texts, with a keen understanding of the different mathematics levels.

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PREFACE

The purpose of this guide is to assist students in their preparation for tests and examinations in the new Mathematics course—'Mathematics Applications'. The structure is such that students will be able to make use of the book throughout the year.

The **Syllabus Checklist** indicates to the students the skills they need to have and the objectives they need to satisfy under each of the major headings of the course.

The **Introductory Notes** given at the start of each chapter detail all the key ideas associated with each aspect of the course. These notes should be read in conjunction with those given by the classroom teacher.

The **Worked Examples** are presented in a very detailed manner and are often accompanied by brief notes and explanations to enhance students understanding of the particular question types.

The **Problems to Solve** section gives students a wide range of problems and the opportunity to check their understanding of each of the topics. Fully worked solutions are also provided.

The **Trial Tests** are an additional source of questions and can be used when preparing for classroom tests. Suggested times are given for the tests and it is important that students try to work within the stated time constraints. Fully worked solutions are given for these tests and a marking guide is also provided so that students can receive some concrete feedback on their performance. Tests are provided for both the resource free (no calculator) and the resource rich (calculator allowed) components of the assessment.

The **Examination Style Questions** are provided enabling students to practice questions similar to those found in examinations. A wide variety of resource free and resource rich questions on each of the topics in this course are available for students. A fully worked set of solutions are given for each of these questions.

I hope that this study guide will help students to better understand the concepts they will encounter and to achieve greater success in the subject.

Greg Hill
May 2020

PERCENTAGES

1

Syllabus Checklist

By the end of this chapter, you should be able to:

- ☐ apply percentage increase or decrease in contexts, determine the impact of inflation, calculate percentage markups and discounts, calculate GST and calculate profit and loss.

FORMULAE AND DEFINITIONS

Profit and Loss

A profit is the money made on the sale of goods.

Profit = Selling Price − Cost Price.

Percentage profit $= \dfrac{\text{profit}}{\text{cost price}} \times 100\%$

A loss is when the cost price exceeds the selling price.

Loss = Cost price − Selling price.

Percentage loss $= \dfrac{\text{loss}}{\text{cost price}} \times 100\%$

'Break even' is when:

Cost Price = Selling Price

Discount

A discount is a reduction in the price of an item usually given as a percentage.

Commission

A commission is a payment to a salesperson usually as a percentage on the value of goods sold.

Real estate and car salespeople are examples of those who earn commissions. A salesperson may also be paid a flat or base salary called a **retainer**.

Inflation

Inflation is the rate at which the level of prices for goods and services is rising.

Goods and Services Tax (G.S.T.)

The Goods and Services Tax is a value added tax of 10% in Australia on most goods and services transactions.

Worked Examples

1.1 Find the following:

(a) 25% of \$120.

(b) $12\frac{1}{2}$% of 45 grams.

(c) Convert 11 marks out of 25 as a percentage.

(d) If 40% of an amount is \$12, find the amount.

(a) 25% of \$120

$$= \frac{25}{100} \times 120$$

$$= \$30$$

(b) $12\frac{1}{2}$% of 45g

$$= \frac{12.5}{100} \times 45$$

$$= 5.625 \text{ grams}$$

(c) $\frac{11}{25}$ as a percentage (multiply by 100)

$$= \frac{11}{25} \times 100$$

$$= 44\%$$

(d) 40% of an amount is \$12

$$\frac{40}{100} \times x = 12$$

$$x = 12 \times \frac{100}{40}$$

$$x = 30$$

The amount is \$30

1.2 (a) Increase 250 by 30%

(b) Decrease 120 by 25%

(c) A 300g box of Rice Bubbles contains 3% sugar. Determine the amount of sugar contained in the box.

(d) Morgan jumped a height of 185cm at the high jump competition. This was an increase of 13.5%. Determine the original height he jumped.

(a) 250×1.3 *1.3 increases an amount by 30%*
$= 325$ *1.06 increases an amount by 6%*

(b) 120×0.75 *0.75 decreases an amount by 25%*
$= 90$ *0.95 decreases an amount by 5%*

(c) Sugar 3% of 300 g

$$\frac{3}{100} \times 300$$

$$= 9 \text{ g}$$

(d) $$1.135 \times \text{original height} = 185$$

$$\text{original height} = \frac{185}{1.135}$$

$$\text{Morgan's original height} = 163 \text{ cm}$$

1.3 Shoes bought by a store for \$60 were sold at \$100.

Determine:

(a) the profit.

(b) the percentage profit.

(a) Profit = Selling price – cost price

= \$100 – \$60

= \$40

(b) % profit $= \dfrac{\text{profit}}{\text{cost price}} \times 100\%$

$= \dfrac{40}{60} \times 100$

$= 66\dfrac{2}{3}\%$

1.4 A motorbike sold for \$1500, a loss of 25%. Determine the cost price of the motorbike.

Cost price $\times$ 0.75 = 1500 *decrease of 25%*

Cost price $= \dfrac{\$1500}{0.75}$

Cost price = \$2000

1.5 Jeans bought by a local store cost \$40 and are sold with a profit margin of 62%. Determine the selling price of the jeans.

Selling price = \$40 $\times$ 1.62 *increase of 62%*

= \$64.80

1.6 A sofa is marked at $1999 with a discount of 30%.

Determine:

(a) the discount.

(b) the discounted price of the sofa.

(a) Discount $= \frac{30}{100} \times \$1999$

$= \$599.70$

(b) Discounted price $= \$1999 - \$599.70$

$= \$1399.30$

1.7 A light is marked down from $270 to $195. Determine the percentage discount.

Discount $= \$270 - \$195$

$= \$75$

% discount $= \frac{75}{270} \times 100$

$= 27.\dot{7}\%$

1.8 Over the past week a salesman has sold properties worth $625 000. If he is paid a commission of 2% on sales plus a weekly retainer of $250, determine his weekly earnings?

Commission $= 2\% \times \$625\,000$

$= \$12\,500$

Weekly earnings $= \$12\,500 + \mathbf{\$250}$ *retainer*

$= \$12\,750$

1.9 A lounge suite is priced at $4350 today. If inflation is 2.5% p.a. determine the price of the lounge suite next year.

$\$4350 \times 1.025$ *increase of 2.5%*

$= \$4458.75$

1.10 A television costs $2250 plus 10% G.S.T. Calculate the selling price of the television.

$\$2250 \times 1.1$ *increase of 10%*

$= \$2475$

1.11 A car costs $22 500 inclusive of G.S.T. What was the price of the car before the G.S.T. was included.

Original Price $\times\ 1.1 = \$22\,500$

Original price $= \frac{22\,500}{1.1}$

$= \$20\,454.55$

PROBLEMS TO SOLVE

Chapter 1: Percentages

1. Calculate:

   (a) 70% of 42

   (b) 125% of 25 L

   (c) $3\frac{1}{3}$% of \$14

   (d) 17.8% of 15.2 kg

2. Find the quantity if:

   (a) 70% is \$14

   (b) 55.5% is 72.15 g

   (c) $13\frac{1}{3}$% is 11.96 mL

3. A car was purchased for \$29 990 on sale at a 25% reduction. What was the original price of the car?

4. Determine the following:

   (a) Increase \$500 by 5%

   (b) Increase 725 g by 23%

   (c) Increase 16 L by 124.6%

   (d) Decrease \$120 by 6%

   (e) Decrease 40 cm by 34%

   (f) Decrease 7.85 mL by 50.5%

5. Jerry's house is valued at \$625 000. He decides to reduce the price by $12\frac{1}{2}$%. Determine the new price of the house.

6. Fiona earns \$72 000 p.a. She receives a $4\frac{1}{2}$% pay rise. Determine her new yearly salary.

7. Determine the percentage change if there is:

   (a) an increase of 10% followed by a decrease of 10%

   (b) a decrease of 25% followed by an increase of 25%

   (c) an increase of 5% followed by a decrease of 4%.

8. Abi's salary of \$56 000 p.a. is to increase by 3%, 4% and 6% over three years.

   (a) What is her salary at the end of the third year?

   (b) What is the percentage change in Abi's salary for the three years?

9. A sale discount of 20% was offered on the price of all shoes. Find:

   (a) the discount on shoes valued at \$240.

   (b) the sale price of the shoes.

10. A shirt marked at \$80 sells for \$58. Determine the percentage discount.

11. George is a car salesman who is paid 1.5% commission.
Calculate how much George will be paid if he sells a car worth \$230 000.

12. Melissa works for a real estate agency. The agency pays commissions of 4% on the first \$300 000 and $2\frac{1}{2}$% thereafter.
If Melissa sells properties worth \$525 000 calculate the commission she would receive.

13. For each of the following items determine:

- the profit or loss.
- the percentage profit or percentage loss.

	Cost Price	Selling Price
(a)	\$20	\$25
(b)	\$75	\$50
(c)	\$110	\$203.50
(d)	\$150	\$56.25
(e)	\$1.12	\$1.25

14. Find the selling price for each item below:

(a) Chairs costing \$45 each are sold with a profit of 70%.

(b) A fridge costing \$1200 is sold at a loss of 35%.

15. Geoff sold a DVD for \$12.50, at a profit of 25%. At what price should Geoff have sold the DVD in order to make a profit of 35%?

16. John works selling coffee machines and is paid via commission at a rate of $22\frac{1}{2}$%.

If John earns a commission of \$731.43, what was the value of his sales for the week?

17. A used car salesman earns a weekly retainer of \$225 plus 2% commission on the first \$7500 of sales and $1\frac{1}{4}$% on the remainder. If he sells cars in a week valued at \$33 000 calculate his weekly income.

18. By selling computer games for \$235, a profit of 25% was made by John.
How much did the computer games cost John?

19. The selling price of a bike is \$250 and its cost price is \$150. If the salesman sells the bike at a discount of 15%, what profit will be made?

20. A computer store operates on a profit margin of 75%. How much would the computer store have paid for a computer which sells for \$3027.50 ?

21. If the average annual rate of inflation was 2.5%, and the cost of a dozen eggs currently sell for $5.50, calculate the cost

    (a) in one year

    (b) in two years

    (c) in five years

    (d) last year

22. If a dishwasher was marked at $1950 plus 10% G.S.T. and was reduced by 35%, calculate the sale price inclusive of G.S.T.

23. If the G.S.T. on a pair of jeans was $24.95, determine the price of the jeans inclusive of G.S.T.

24. If the inflation rates for 4 years were 5% p.a., 7.2% p.a., 3% p.a. and 4.6% p.a. determine the cost of a loaf of bread in 4 years time, which currently costs $5.25.

2 ALGEBRAIC SKILLS

Syllabus Checklist

By the end of this chapter, you should be able to:

- ☐ substitute numerical values into algebraic expressions and evaluate
- ☐ determine the value of the subject of a formula

FORMULAE AND DEFINITIONS

Simplify	To expand brackets, collect 'like' terms and reduce the expression.
Solve	To find all solutions to an equation, inequality or system of equations.
Factorise	To remove the highest common factor (HCF) outside a set of brackets.
Inequalities	These contain the signs $>$, $<$, $\leq$, or $\geq$.

Worked Examples

2.1 If $x = 2$, $y = -3$, and $z = 4$ calculate:

(a) $2xy$

(b) $3yz - x$

(c) $y^2 + 2x^2z$

(a) $2xy$
$= 2(2)(-3)$
$= -12$

(b) $3yz - x$
$= 3(-3)(4) - (2)$
$= -36 - 2$
$= -38$

(c) $y^2 + 2x^2z$
$= (-3)^2 + 2(2)^2(4)$
$= 9 + 2(4)(4)$
$= 9 + 32$
$= 41$

2.2 A pendulum on a clock swings from side to side. The time (T seconds) to complete one cycle (left to right and back to left again) is

$T = 2\neq\sqrt{\frac{\ell}{g}}$ where

ℓ = length of the swing (cm)
g = gravitational acceleration.

Find T if:

(a) $\ell = 6.2$, $g = 4$

(b) $\ell = 30$, $g = 8.5$

(a) $T = 2\pi\sqrt{\frac{\ell}{g}}$

$= 2\pi\sqrt{\frac{6.2}{4}}$

$=$ 7.82 seconds

(b) $T = 2\pi\sqrt{\frac{30}{8.5}}$

$=$ 11.80 seconds

2.3 Simplify the following:

(a) $4x + 2t + x - t$

(b) $3 \times q \times q^2 \times s \times 2$

(c) $\frac{14r}{21s}$

(a) $4x + 2t + x - t$ *remember to add or subtract 'like' terms*

$= 5x + t$

(b) $3 \times q \times q^2 \times s \times 2$ *remember when multiplying* $p \times p = p^2$

$= 6q^3$s

(c) $\frac{{}^{2}\cancel{14}r}{{}_{3}\cancel{21}s}$

$= \frac{2r}{3s}$

2.4 Expand and simplify the following:

(a) $4(p - 2)$

(b) $-3x(x + 4p)$

(c) $4(2a - 5) - 2(3a + 1)$

Brackets mean multiply

(a) $4(p - 2)$ $= 4 \times p - 4 \times 2$

$= 4p - 8$

(b) $-3x(x + 4p)$ $= (-3x)(x) + (-3x)(4p)$

$= -3x^2 - 12px$

(c) $4(2a - 5) - 2(3a + 1)$ $= 8a - 20 - 6a - 2$ *collect 'like' terms*

$= 2a - 22$

2.5 Expand and simplify:

(a) $(x + 4)(x + 3)$

(b) $(2t - 1)^2$

(c) $-2(a + 6)(a - 3)$

Multiply the terms

(a) $(x + 4)(x + 3)$ $= (x.x) + (x.3) + (4.x) + (4.3)$

$= x^2 + 3x + 4x + 12$

$= x^2 + 7x + 12$

(b) $(2t - 1)^2$ $= (2t - 1)(2t - 1)$

$= 4t^2 - 2t - 2t + 1$

$= 4t^2 - 4t + 1$

(c) $-2(a + 6)(a - 3)$ $= -2(a^2 - 3a + 6a - 18)$

$= -2(a^2 + 3a - 18)$

$= -2a^2 - 6a + 36$

2.6 Factorise the following:

(a) $6 - 18p$

(b) $-18x + 12y$

(c) $-10ab - 20ac$

Remove the highest common factor out the front of a bracket.

(a) $6 - 18p$ $= 6(1 - 3p)$

HCF is 6 *divide* $\frac{6}{6}$ *divide* $\frac{18p}{6}$

(b) $-18x + 12y$ $= -6(3x - 2y)$

HCF is 6 *sign changes as a negative value is removed out the front of the bracket*

(c) $-10ab - 20ac$ $= -10a(b + 2c)$

HCF is 10a *be careful !!*

2.7 Solve each of the following **without** a calculator:

(a) $4p - 7 = 21$

(b) $\frac{2a + 3}{5} = 3$

(c) $\frac{n}{5} + 2 = -1$

Remember
- Variables to the left : numbers to the right.
- Change side : change sign.

(a)
$$\begin{aligned} 4p - 7 &= 21 \\ 4p &= 21 + 7 \quad \textit{add 7} \\ 4p &= 28 \\ p &= \frac{28}{4} \quad \textit{divide by 4} \\ p &= 7 \end{aligned}$$

(b)
$$\begin{aligned} \frac{2a + 3}{5} &= 3 \\ 2a + 3 &= 3 \times 5 \quad \textit{multiply by 5} \\ 2a + 3 &= 15 \\ 2a &= 15 - 3 \quad \textit{subtract 3} \\ 2a &= 12 \\ a &= \frac{12}{2} \quad \textit{divide by 2} \\ a &= 6 \end{aligned}$$

(c)
$$\begin{aligned} \frac{n}{5} + 2 &= -1 \\ \frac{n}{5} &= -1 - 2 \quad \textit{subtract 2} \\ \frac{n}{5} &= -3 \\ n &= -3 \times 5 \quad \textit{multiply by 5} \\ n &= -15 \end{aligned}$$

2.8 Solve the following **without** a calculator:

(a) $6 - 2(4p + 5) = 10$

(b) $\dfrac{5x + 2}{3} = \dfrac{2x - 3}{4}$

(c) $\dfrac{7 - (g + 4)}{3} + 2 = 4(g - 6)$

(a)
$$\begin{aligned} 6 - 2(4p + 5) &= 10 \quad \textit{expand brackets} \\ 6 - 8p - 10 &= 10 \quad \textit{collect like terms} \\ -8p - 4 &= 10 \\ -8p &= 10 + 4 \\ -8p &= 14 \\ p &= -\frac{7}{4} \end{aligned}$$

(b)
$$\begin{aligned} \frac{5x + 2}{3} &= \frac{2x - 3}{4} \quad \textit{cross multiply} \\ 4(5x + 2) &= 3(2x - 3) \\ 20x + 8 &= 6x - 9 \\ 20x - 6x &= -9 - 8 \\ 14x &= -17 \\ x &= -\frac{17}{14} \end{aligned}$$

(c)
$$\begin{aligned} \frac{7 - (g + 4)}{3} + 2 &= 4(g - 6) \quad \textit{expand brackets} \\ \frac{7 - g - 4}{3} + 2 &= 4g - 24 \\ \frac{7 - g - 4}{3} &= 4g - 24 - 2 \quad \textit{collect like terms} \\ \frac{-g + 3}{3} &= 4g - 26 \\ -g + 3 &= 3(4g - 26) \\ -g + 3 &= 12g - 78 \\ g + 12g &= 78 + 3 \\ 13g &= 81 \\ g &= \frac{81}{13} \end{aligned}$$

2.9 Write an equation and solve it.

I think of a number, double it, add 5 and divide the answer by 3.
If the result is 5, what is the number?

Let n be the number

double

$$\frac{2n+5}{3} = 5$$

$$2n + 5 = 15$$

$$2n = 10$$

$$n = 5$$

The number is 5

2.10 Solve the following inequalities:

(a) $4x - 5 \leq 27$

(b) $5(2x + 1) > 4(3x - 2)$

(a)

$$4x - 5 \leq 27$$

$$4x \leq 27 + 5$$

$$4x \leq 32$$

$$x \leq \frac{32}{4}$$

$$x \leq 8$$

(b)

$$5(2x + 1) > 4(3x - 2) \quad \textit{expand brackets}$$

$$10x + 5 > 12x - 8$$

$$10x - 12x > -8 - 5$$

$$-2x > -13 \quad \textit{when dividing by a negative value change the inequality sign}$$

$$x < \frac{-13}{-2}$$

$$x < \frac{13}{2}$$

2.11 Solve the following **without** a calculator.

(a) $2x^2 = 32$

(b) $5x^2 + 4 = 40$

(c) $3^x = 27$

(d) $8^{x+1} = 16$

(a)

$$2x^2 = 32$$

$$x^2 = \frac{32}{2}$$

$$x^2 = 16$$

$$x = \sqrt{16}$$

$$x = \pm 4 \quad \textit{remember - two solutions}$$

(b)
$$\begin{aligned}
5x^2 + 4 &= 40 \\
5x^2 &= 40 - 4 \\
5x^2 &= 36 \\
x^2 &= \frac{36}{5} \\
x &= \sqrt{\tfrac{36}{5}} \\
x &= \pm \tfrac{6}{\sqrt{5}}
\end{aligned}$$

(c)
$$\begin{aligned}
3^x &= 27 \quad \textit{change both values to the same base} \\
3^x &= 3^3 \\
\therefore x &= 3
\end{aligned}$$

(d)
$$\begin{aligned}
8^{x+1} &= 16 \\
(2^3)^{x+1} &= 2^4 \quad \textit{multiply 3 by (x + 1)} \\
2^{3x+3} &= 2^4 \\
\therefore\ 3x + 3 &= 4 \\
3x &= 1 \\
x &= \frac{1}{3}
\end{aligned}$$

PROBLEMS TO SOLVE

Chapter 2: Algebraic Skills

1. If $m = -2$, $n = -6$, $p = 4$

   evaluate

   (a) $2mn - 3p$

   (b) $m^2 + n^2 - p^2$

   (c) $3 - (mnp)$

   (d) $-2n - 3mp$

   (e) $4m^2 - (3n + p - 6)$

2. The compound interest formula is $A = P\left(1+\frac{r}{100}\right)^t$ where

   A = amount in the account

   P = principal

   r% = interest rate p.a.

   t = time in years

   Find the value in the account if:

   (a) $4000 is deposited for 4 years at 6% p.a. compounded annually

   (b) $12 000 is deposited for 6 years at 7.25% p.a. compounded annually

3. The formula for the surface area of a cylinder is $S = 2\pi r^2 + 2\pi rh$ where:

   r = radius

   h = height

   Calculate the surface area of a cylinder if

   (a) radius is 7 cm and height is 10 cm

   (b) radius is 10 cm and height is 0.2 m

4. Converting temperature in degrees Fahrenheit (F) to degrees Celsius (C) is given by:
   $C = \frac{5}{9}(F - 32)$.
   Convert the following to degrees Celsius:

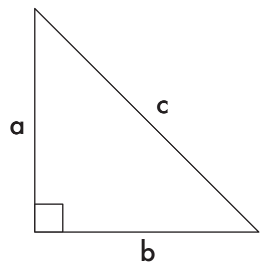
   (a) 42° F

   (b) 0° F

   (c) 100° F

5. The formula to calculate the length of the hypotenuse on a right angled triangle is

$c = \sqrt{a^2 + b^2}$



Calculate the length of the hypotenuse when:

(a) a = 6 m, b = 8 m

(b) a = 2.5 cm, b = 4 cm

6. Write an algebraic expression for:

(a) Four times the number *p* is subtracted from twelve.

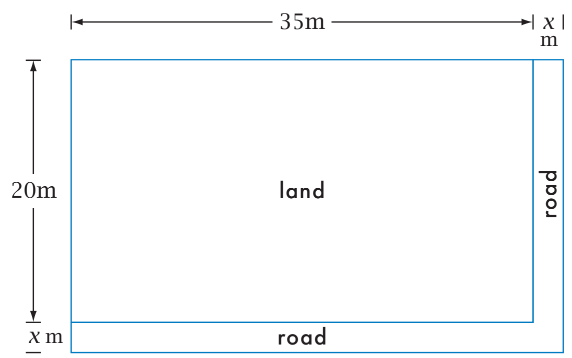
(b) The number *n* is divided by three times the number *t*.

(c) The sum of six and the number *f* is multiplied by negative two.

(d) The difference between the number *p* and the number *q* is divided by the product of *p* and *q*.

7. For each of the following state an algebraic expression:

(a) A rectangular block of land measures 35 metres by 20 metres.
A road is constructed of width *x* metres as shown in the diagram below.
Determine the total area of the land which includes the road.



(b) Sam buys a container with *m* marbles.

i. If she equally divides them amongst *p* people how many marbles will each person receive?

ii. Sam decides to keep $\frac{1}{3}$ of the marbles and divides the remaining ones amongst *t* people. How many marbles will each person receive?

7. (c) For an 8 hour work day Sydney uses the car for 'h' hours. Her brother Tom uses it for $\frac{1}{4}$ of the remaining time. Determine how long Sydney and Tom use the car over an 8 hour day.

8. Simplify the following algebraic expressions:

(a) $m + m + m - n - n$

(b) $t + 6t$

(c) $5m^2 - m^2$

(d) $4x^2 - 2x + 5 - 6x - 8 + 3x^2$

9. Simplify the following:

(a) $-3a \times -5ab \times -10abc$

(b) $(-3xy)^2$

(c) $\frac{6a^2b}{-12a}$

(d) $\frac{-25m}{-35n}$

10. If $a = 6$ and $b = -2$ find the value of:

(a) ab

(b) $3a - 2b$

(c) $\mathrm{a} - b^2$

11. Expand the following:

(a) $5(y - 6)$

(b) $-2x(4 - 3f)$

(c) $3h(2h - 5g)$

(d) $-(3p + 2q)$

12. Expand and simplify each of the following:

(a) $7(2a - 5) - 4a$

(b) $3(2p - 1) + 2(p + 6)$

(c) $5(3e - 2) - 2(4 - e)$

(d) $-3x(2x - 3) - 2x(5x + 1)$

13. Expand and simplify the following:

(a) $(p - 2)(p - 9)$

(b) $-3(2y - 7)^2$

(c) $(8w - 3)(2w - 3)$

(d) $-2(5c - 4)(3c - 2)$

14. Factorise the following:

(a) $7 - 21b$

(b) $-16 - 12y$

(c) $15pq - 25ps$

(d) $x^2 - 10x$

15. Solve each of the following equations **without** a calculator:

(a) $x - 6 = 10$

(b) $2y = 12$

(c) $\frac{p}{7} = -3$

(d) $9 - t = 5$

16. Solve each of the following equations **without** a calculator:

(a) $-9x + 5 = 86$

(b) $\frac{3p - 2}{5} = 8$

(c) $\frac{6}{x} = 5$

(d) $\frac{3}{4}b + 5 = -2$

17. Solve each of the following **without** a calculator:

(a) $x - 9 = 12 - 2x$

(b) $2(z + 4) + 4z = -32$

(c) $-3(4m - 5) + 6m = 9$

(d) $\frac{15 - b}{2} + 3 = 12$

18. Solve the following **without** a calculator:

(a) $7(2x + 3) - 6(3x - 1) = -1$

(b) $\frac{3x - 7}{2} = x + 5$

(c) $\frac{4x + 5}{3} = \frac{2x + 7}{2}$

(d) $5g - 2 = 8 - (10 - g)$

19. Write an equation for each of the following and then solve it:

(a) I think of a number, multiply it by three, add five and the result is negative ten.

(b) Three is subtracted from one quarter of a number and the result is seven.

(c) Twice a number is subtracted from eight and the answer divided by seven giving a result of four.

(d) A man travels '*k*' kilometres on the first day. On each of the next two days he travels twice as far as he did on the first day. On each of the two remaining days he travels 10 more kilometres than what he did on the second day. If the total journey is 524km, how far did he travel on the first day?

20. Solve each of the following inequalities **without** a calculator:

(a) $8 + 3x \geq -13$

(b) $2x - 1 < 5x + 7$

(c) $6(3x - 1) \leq 5(4x - 3)$

(d) $\dfrac{5 - 2x}{6} > -7$

21. Solve the following equations **without** the use of a calculator:

(a) $9x^2 = 25$

(b) $x^2 - \dfrac{1}{25} = 0$

(c) $6(p^2 - 1) = 37$

(d) $-2t^3 + 4 = 132$

22. Solve the following equations **without** a calculator:

(a) $3^{x+1} = 27$

(b) $7^x = \dfrac{1}{49}$

(c) $\dfrac{5^x}{5} = 125$

(d) $10^x = 0.0001$

23. Solve the following equations using a calculator correct to 2 decimal places.

(a) $2^x = 7$

(b) $3^{x+1} = 15$

(c) $7^x = \dfrac{1}{25}$

(d) $\dfrac{4^{x-2}}{3} + 2 = 11$

MATRICES

3

Syllabus Checklist

By the end of this chapter, you should be able to:

- ☐ use matrices for storing and displaying information
- ☐ recognise different types of matrices and determine their size
- ☐ perform matrix addition, subtraction, multiplication by a scalar, multiplication and a power of a matrix
- ☐ use matrices to model and solve problems

FORMULAE AND DEFINITIONS

A rectangular array of numbers is called a MATRIX. The numbers in a matrix are called ELEMENTS. In a matrix there are horizontal *rows* and vertical *columns*. If a matrix has 'm' rows and 'n' columns then the *order* of the matrix is '**m × n**'.

Row matrix

A row matrix has *one* row.

$\begin{bmatrix} -4 & 2 & 6 \end{bmatrix}$ is a 1×3 matrix

Column matrix

A column matrix has *one* column.

$\begin{bmatrix} 3 \\ 1 \\ -5 \end{bmatrix}$ is a 3×1 matrix

Square matrix

A square matrix has the same number of rows as columns.

$\begin{bmatrix} 4 & 1 \\ 3 & 5 \end{bmatrix}$ is a 2×2 matrix

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is a 3×3 matrix. This matrix is also called a diagonal matrix as the elements are all zero except for the leading diagonal (shaded).

Zero matrix

A matrix in which all elements are zero.

$$\begin{bmatrix} 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Identity matrix

A square matrix with leading diagonal all ‘1’s and ‘0’s elsewhere.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Scalar multiplication

$$\text{If} \quad A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$$

$$\text{Then } 3A = 3\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 9 \\ -3 & 12 \end{bmatrix}$$

Matrix multiplication

Two matrices can only be multiplied if the number of columns in the first matrix is equal to the number of rows in the second matrix.

Matrix X: Order 4×2

Matrix Y: Order 2×3

XY: $(4 \times 2) \times (2 \times 3)$

same

order of resulting matrix
$= 4 \times 3$

Matrix Addition/Subtraction

Matrices can be added or subtracted if they are of the same size.

Position of elements in a Matrix

$$\text{Given } A = \begin{bmatrix} 1 & 2 & 5 \\ 0 & -3 & 4 \end{bmatrix}$$

then a_{ij} represents an element of matrix A where i is the row number and j the column number.

Powers of Matrices

A matrix can only be raised to a power if it is a *square matrix.*

Worked Examples

3.1 State the order of the following matrices.

(a) $\begin{bmatrix} 1 & 3 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} -2 & 3 & 6 & 2 \\ 5 & 1 & 0 & -4 \end{bmatrix}$

(a)

$$\begin{array}{cc} \begin{matrix} \downarrow & \downarrow & \downarrow \end{matrix} & \text{3 columns} \\ \begin{bmatrix} 1 & 3 & 2 \end{bmatrix} & \leftarrow 1 \text{ row} \end{array}$$

Order: 'm' rows by 'n' columns

$= 1 \times 3$

(b)

$$\begin{array}{cc} \begin{bmatrix} -2 & 3 & 6 & 2 \\ 5 & 1 & 0 & -4 \end{bmatrix} & \begin{matrix} \leftarrow \\ \leftarrow \end{matrix} \ \text{rows} \\ \begin{matrix} \uparrow & \uparrow & \uparrow & \uparrow \end{matrix} & \text{columns} \end{array}$$

Order: 2×4

3.2 If $x = \begin{bmatrix} 1 & 7 & 3 & 5 \\ -3 & 2 & -4 & 0 \\ -2 & 4 & 6 & -1 \end{bmatrix}$

determine

(a) x_{13}

(b) x_{22}

(c) $x_{34} + x_{12}$

(a)	x_{13}	$=$	3	row 1	column 3
(b)	x_{22}	$=$	2	row 2	column 2
(c)	$x_{34} + x_{12}$	$=$	$-1 + 7$	row 3	column 4
		$=$	6	row 1	column 2

3.3 If $A = \begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix}$

$B = \begin{bmatrix} 1 & 5 & -3 \end{bmatrix}$

$C = \begin{bmatrix} 0 & -2 & 3 \end{bmatrix}$

$D = \begin{bmatrix} -2 & 5 \\ 1 & 4 \end{bmatrix}$

Determine each of the following. If they cannot be determined state this and give the reason why.

(a) A + D

(b) 3B – 2C

(c) D + 3C

(d) 2D

(a) A + D

add

$$= \begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 5 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + (-2) & 3 + 5 \\ 0 + 1 & 4 + 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 8 \\ 1 & 8 \end{bmatrix}$$

(b) 3B – 2C

multiply *multiply*

$$= 3\begin{bmatrix} 1 & 5 & -3 \end{bmatrix} - 2\begin{bmatrix} 0 & -2 & 3 \end{bmatrix}$$

subtract

$$= \begin{bmatrix} 3 & 15 & -9 \end{bmatrix} - \begin{bmatrix} 0 & -4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 3-0 & 15 - (-4) & -9 - 6 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 19 & -15 \end{bmatrix}$$

(c) D + 3C

$$\begin{bmatrix} -2 & 5 \\ 1 & 4 \end{bmatrix} + 3\begin{bmatrix} 0 & -2 & 3 \end{bmatrix}$$

D and C are not the same size. D + 3C cannot be determined.

(d) 2D

$$2\begin{bmatrix} -2 & 5 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 10 \\ 2 & 8 \end{bmatrix}$$

3.4 If $X = \begin{bmatrix} 4 & -2 \\ 0 & 3 \end{bmatrix}$

$Y = \begin{bmatrix} 1 & 5 & -2 \\ 3 & 0 & -1 \end{bmatrix}$

$Z = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$

Determine each of the following. If any cannot be calculated state the reason why.

(a) XY

(b) YX

(c) YZ

(d) X^2

(a) XY

$(2 \times 2) \times (2 \times 3)$

$= (2 \times 3)$ resulting order of matrix.

$$\begin{bmatrix} 4 & -2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 5 & -2 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} (4 \times 1) + (-2 \times 3) & * & * \\ * & * & * \end{bmatrix}$$

$$= \begin{bmatrix} -2 & * & * \\ * & * & * \end{bmatrix}$$

$$\begin{bmatrix} 4 & -2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 5 & -2 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -2 & (4 \times 5) + (-2 \times 0) & * \\ * & * & * \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 20 & * \\ * & * & * \end{bmatrix}$$

$$\begin{bmatrix} 4 & -2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 5 & -2 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -2 & 20 & (4 \times -2) + (-2 \times -1) \\ * & * & * \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 20 & -6 \\ * & * & * \end{bmatrix}$$

$$\begin{bmatrix} 4 & -2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 5 & -2 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -2 & 20 & -6 \\ (0 \times 1) + (3 \times 3) & * & * \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 20 & -6 \\ 9 & * & * \end{bmatrix}$$

$$\begin{bmatrix} 4 & -2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 5 & -2 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -2 & 20 & -6 \\ 9 & (0 \times 5) + (3 \times 0) & * \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 20 & -6 \\ 9 & 0 & * \end{bmatrix}$$

$$\begin{bmatrix} 4 & -2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 5 & -2 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -2 & 20 & -6 \\ 9 & 0 & (0 \times -2) + (3 \times -1) \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 20 & -6 \\ 9 & 0 & -3 \end{bmatrix}$$

(b) YX

$(2 \times 3) \times (2 \times 2)$

not possible since the number of columns in matrix Y does not equal the number of rows in matrix X.

(c) YZ

$(2 \times 3) \times (3 \times 1)$

$= (2 \times 1)$ resulting order of matrix.

$$\begin{bmatrix} 1 & 5 & -2 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} (1 \times 3) + (5 \times 1) + (-2 \times 2) \\ * \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ * \end{bmatrix}$$

$$\begin{bmatrix} 1 & 5 & -2 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ (3 \times 3) + (0 \times 1) + (-1 \times 2) \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

(d) X^2

$= XX$

$(2 \times 2) \times (2 \times 2)$

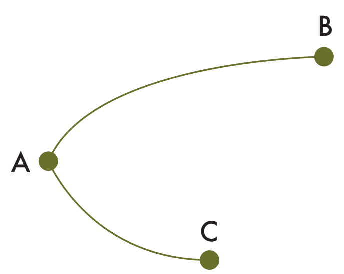
$= (2 \times 2)$ resulting order of matrix.

$$\begin{bmatrix} 4 & -2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 4 & -2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 16 & -14 \\ 0 & 9 \end{bmatrix}$$

or use CAS calculator.

Matrices can be used to model road maps, networks and social interactions.

3.5 The map below shows roads connecting three towns; A, B and C.



(a) Construct a matrix M representing the number of one-step paths.

(b) Determine M^2 and interpret the result.

(c) How many two-step paths exist from town A to town B?

(a) $M = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$

'1' represents the
1 direct path from C to A

'0' represents no direct
path from C to B

(b) $^{o}M^2 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

This matrix represents the number of two-step paths for the road map.
The '2' represents the number of two-step paths from Town A to Town A.

(c) There is 1 two-step path from town B to town B. i.e. Town B $\rightarrow$ Town A $\rightarrow$ Town B.

PROBLEMS TO SOLVE

Chapter 3: Matrices

1. State the order of the following matrices.

(a) $\begin{bmatrix} 2 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 5 \\ -1 & 0 \\ 2 & 1 \\ 4 & 7 \end{bmatrix}$

(c) $\begin{bmatrix} -1 \end{bmatrix}$

2. Determine the value of the variables in each of the following:

(a) $\begin{bmatrix} 2x & -3+y \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ 0 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} 8 \\ a-2b \\ d \end{bmatrix} = \begin{bmatrix} -2a \\ 3 \\ a+b \end{bmatrix}$

(c) $\begin{bmatrix} 10 & 12 \\ 21 & -2 \\ 5 & -p \end{bmatrix} = \begin{bmatrix} 10 & 3t-6 \\ 4t-q & -2 \\ 5 & 2q+5t \end{bmatrix}$

(d) $\begin{bmatrix} 7 & -3x & -4 \\ 4z+y & 2 & 7 \end{bmatrix} = \begin{bmatrix} y-x & 12 & -4 \\ 2z-3 & 2 & 7 \end{bmatrix}$

3. Determine each of the following:

(a) $\begin{bmatrix} 0 & -10 \\ 7 & 3 \end{bmatrix} + \begin{bmatrix} -6 & 5 \\ 2 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} x+1 & 2x-2 & 3 \\ x & 0 & 4x \end{bmatrix} + \begin{bmatrix} 2 & 3-x & 4x \\ -2 & -5x & x+2 \end{bmatrix}$

(c) $\begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix} - \begin{bmatrix} 3 \\ -1 \\ -5 \end{bmatrix}$

(d) $\begin{bmatrix} 2y & -y-5 \\ 3y+1 & 2y-3 \end{bmatrix} - \begin{bmatrix} 3y+1 & -y \\ 2y-5 & -y+4 \end{bmatrix}$

4. Find matrix p in each of the following:

(a) $p + \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$

(b) $\begin{bmatrix} 17 & 5 & 2 \\ -3 & -6 & 5 \end{bmatrix} - p = \begin{bmatrix} 2 & -4 & 5 \\ 5 & 7 & -10 \end{bmatrix}$

5. Determine each of the following:

(a) $3\begin{bmatrix} 2 & -1 \\ 4 & 5 \end{bmatrix}$

(b) $2\begin{bmatrix} 4 & 0 \\ -1 & 3 \end{bmatrix} - 3\begin{bmatrix} 5 & 1 \\ 0 & -2 \end{bmatrix}$

(c) $-2\begin{bmatrix} 0.5 & 0 & 3 \\ 2 & -2.5 & -1 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 3 & -2 & 4 \\ -1 & -4 & 10 \end{bmatrix}$

6. The order of matrices X, Y, and Z are 2×1, 2×2, 1×3 respectively.

State the order of each of the following matrix products, if it exists.

(a) XZ

(b) YZ

(c) Y^2

7. Determine the following matrix products. If the product does not exist state the reason why.

(a) $\begin{bmatrix} 2 & -1 \end{bmatrix}\begin{bmatrix} 5 \\ -2 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 0 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}\begin{bmatrix} 0 & 2 & -1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 5 & -3 \\ 0 & -2 & 2 \end{bmatrix}\begin{bmatrix} 3 & 0 & 2 \end{bmatrix}$

(e) $\begin{bmatrix} 2 & -3 \\ -1 & 4 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} -2 & 7 \\ 5 & -3 \end{bmatrix}$

8. Find the value of the variables in each of the following:

(a) $\begin{bmatrix} x + y & x - y \end{bmatrix} = \begin{bmatrix} -2 & 10 \end{bmatrix}$

(b) $\begin{bmatrix} a \\ b + 2c \\ b - c \end{bmatrix} = \begin{bmatrix} 3b - 2c \\ -b + c \\ 4b + c + 4 \end{bmatrix}$

9. Given $P = \begin{bmatrix} 4 & 2 \\ -1 & 0 \end{bmatrix}$

$Q = \begin{bmatrix} 2 & -3 \\ 2 & -1 \end{bmatrix}$

$R = \begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix}$

determine:

(a) $P + Q + R$

(b) $2P - Q + R$

(c) $2Q - 2(P - 3R)$

(d) $P + 2(R - \frac{1}{2}Q)$

10. If $A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$

$B = \begin{bmatrix} -3 & 6 \\ 1 & -2 \\ -2 & 3 \end{bmatrix}$

determine:

(a) A^2

(b) BA^2

(c) C if $C + B = BA^2$

11. If $T = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

determine:

(a) T^2

(b) T^3

(c) T^4

(d) an expression for the matrix T^n, where n is a positive integer.

(e) T^9 using your expression in part (d)

(f) the value of k if the matrix

$T^k = \begin{bmatrix} 1 & 0 \\ 64 & 1 \end{bmatrix}$

12. Given:

$A = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

$B = \begin{bmatrix} -3 & 2 \\ -5 & 1 \end{bmatrix}$

$C = \begin{bmatrix} 3 & -2 & 1 \\ 0 & -1 & 4 \end{bmatrix}$

$D = \begin{bmatrix} -2 \\ 10 \end{bmatrix}$

Determine:

(a) (i) $(B - A)\,A$

(ii) $(B + A)\,C$

(b) (i) the order of matrix X when $AX = D$

(ii) matrix X

13. If

$$X = \begin{bmatrix} a + b \\ 2a + 2b \\ 2a - 5b \end{bmatrix} \text{ and}$$

$$Y = \begin{bmatrix} c \\ -2a + 2b \\ 4a - 3b + 6 \end{bmatrix} \text{ are equal}$$

determine:

(a) the values of a, b and c

(b) $2X - 4Y$

(c) k, if $kX + (2k - 7)\,Y = 0$
where 0 is a zero matrix of order 3×1

14. Let $F = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

Calculate:

(a) F^2

(b) F^3

(c) F^4

(d) F^5

(e) Is there a relationship between the terms in the Fibonacci sequence and the elements in the above matrices?

15. Given $A = \begin{bmatrix} 4 & 1 \\ 5 & 2 \end{bmatrix}$

$B = \begin{bmatrix} 2 & -1 \\ -5 & 4 \end{bmatrix}$

determine:

(a) AB

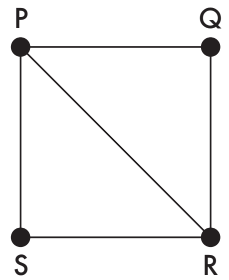
(b) C, such that

$AC = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

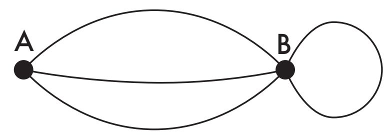
(c) If $A^2B^2 = (AB)^2$ and whether this applies to all 2×2 matrices.

16. Write down a matrix to represent each of the following diagrams:

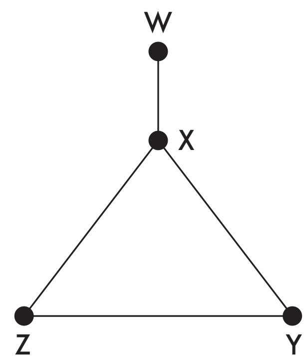
(a)



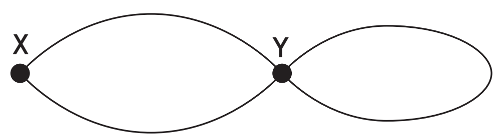
(b)



(c)



17.



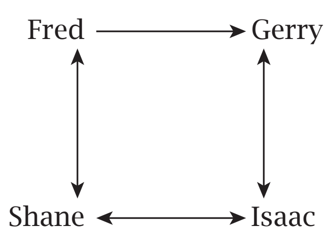
(a) For the diagram above, determine the one-step matrix A.

(b) Determine the two-step matrix.

(c) Calculate A^2 and compare your answer to part (b)

(d) Hence determine the three-step matrix.

18. The number of ways 4 school friends can directly communicate with each other is shown in the diagram below:

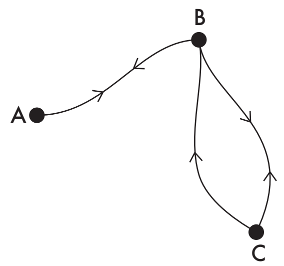


(a) Construct a matrix M to represent the number of ways each person can directly communicate with another person.

(b) Calculate M^2. What does this represent?

(c) Calculate M^3. What does this represent?

19. The diagram below shows a road system between towns A, B and C. The arrows indicate directions of travel.



(a) Complete the one-step matrix below:

$$\text{From}\quad \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \text{To} \\ \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 1 & * & * \\ * & * & * \end{bmatrix} \end{array}$$

(b) Determine the two-step matrix for this road system.

20. The table below shows the weekly sales of flavoured milk in three different sizes, in the school canteen.

	Small	Medium	Large
Chocolate	10	7	14
Vanilla	15	4	8

Small milk sells for \$1.20, medium \$1.50 and large \$2.00.

(a) Construct a 3×2 matrix M, representing weekly sales of flavoured milk.

(b) Construct a 1×3 matrix P, representing the price of the three different sizes of flavoured milk.

(c) Calculate PM and interpret your answer.

(d) Let $X = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Calculate PMX and interpret your answer.

21. The College Production will be held over 3 nights; Wednesday, Thursday and Friday. Ticket prices are Adult Reserve \$35, Adults \$25, Children \$15, Concession \$10.

The number of tickets sold for each night is given in the table below:

	Adult Reserve	Adult	Children	Concession
Wednesday	45	160	100	80
Thursday	60	150	120	90
Friday	90	210	180	140

(a) State two matrices and using matrix multiplication determine the total amount of ticket sales for each of the three nights.

(b) Use matrix multiplication to determine the total ticket sales for the College Production.

22. Boxes of cupcakes and muffins are produced on Thursdays and Fridays by a local baker. The table below shows the ingredients and quantity to make *each box* of muffins and cupcakes. Cost of each ingredient is also given in the table.

	Flour	Eggs	Sugar	Vanilla	Butter
Box of cupcakes	200 g	4	100 g	1 teaspoon	300 g
Box of muffins	400 g	3	200 g	1 teaspoon	400 g
Cost	\$2.50/kg	\$0.50/egg	\$1.25/kg	\$0.10/ teaspoon	\$3.00/kg

The table below indicates the number of boxes sold each day.

	Thursday	Friday
Cupcakes	30	50
Muffins	10	65

Boxes of cupcakes are sold at \$15 each and muffins \$20 each.

(a) Use matrix multiplication to find how much of each ingredient is required on Thursday and Friday.

(b) Find the total cost of the ingredients each day using matrix methods.

(c) If all boxes are sold on each day, use matrix methods to calculate the total revenue for Thursday and Friday.

(d) Hence determine the total profit each day.

FINANCE

4

Syllabus Checklist

By the end of this chapter, you should be able to:

- ☐ calculate weekly or monthly wage from an annual salary, wages from an hourly rate including overtime, allowances, commission and piecework
- ☐ calculate payments based on government allowances and pensions
- ☐ prepare a personal budget for a given income
- ☐ compare prices and values using the unit cost method
- ☐ use currency exchange rates to convert between currencies
- ☐ calculate the dividend paid on a portfolio of shares and compare share values by calculating a price to earnings ratio

FORMULAE AND DEFINITIONS

Best Buy

A best buy is the best **value for money** based on a per unit cost.

Currency Exchange Rates

To exchange Australian currency for currency in another country a *currency exchange rate* is used.

The table below is an example of currency exchange rates for foreign currencies based on $1 Australian.

Currency	$1 Australian buys
United States dollar	0.9306
Japanese yen	95.49
European euro	0.6963
Singapore dollar	1.1619
UK pound sterling	0.5519
NZ dollar	1.1035
Thai baht	29.94
Indonesian rupiah	10 939
Hong Kong dollar	7.2126
Malaysian ringgit	2.9802

Shares

People *invest* money in companies by purchasing *shares*.

A *shareholder* is a person who owns shares in a company.

The process of purchasing or selling shares involves going through a *stockbroker*.

Dividend

A dividend is the profit distributed to shareholders based on the number of shares held. The cash dividend paid is expressed as *cents per share*.

Brokerage

Brokerage is a fee (commission) imposed by stockbrokers when trading shares. This can be a flat fee or a percentage of the value of the shares traded.

Portfolio

A portfolio is all the shares an investor owns in various companies.

Price to Earning Ratio (P/E)

$$P/E = \frac{\text{Price per share}}{\text{Yearly Earnings per share}}$$

P/E is a ratio of a companies share value to its earnings per share. A P/E of 8 means that – every $8 of share value will earn a dividend of $1.

It is useful to compare P/E ratios of companies in similar industries. A company with a lower P/E is usually the better investment.

GOVERNMENT ALLOWANCES AND PENSIONS

There are many government allowances and pensions that are administered by Centrelink. Examples are included below.

Newstart Allowance

Financial help for individuals looking for work. Payments are made to those 22 years or more. *Payment* rates are:

Status	Fortnightly payment
Single (no children)	$510.50
Single (with dependent child or children)	$552.40
Single aged 60 or over	$552.40
Partnered (each)	$460.90

Newstart Allowance customers can each earn an *income* but this reduces the total fortnightly payment according to the following table.

Income per fortnight	Reduction in fortnightly payment
$100–$250	50 cents in the dollar
More than $250	60 cents in the dollar

Age Pension

The age pension is used to support seniors aged 65 or over in their retirement. *Payment* rates are:

Status	Fortnightly payment
Single	$766.00
Couple (each)	$577.40

Age pensioners can earn an *income* but this reduces the fortnightly payment according to the following table:

Status	Income per fortnight	Reduction in fortnightly payment
Single	over $160	50 cents in the dollar
Couple	over $284	50 cents in the dollar

Youth Allowance

Youth Allowance gives financial aid to people aged 16 to 24 studying full time, training, looking for work or sick. Payment rates are:

Status	Fortnightly payment
Single less than 18 years living at home	$226.80
Single less than 18 years not living at home	$414.40
Single older than 18 years living at home	$272.80
Single older than 18 years not living at home	$414.40
Single with children	$542.90

Youth Allowance students can earn an *income* but this reduces the fortnightly payment according to the following table:

Income per fortnight	Reduction in fortnightly payment
over $415 to $498	50 cents in the dollar
over $498	60 cents in the dollar

Assistance for Isolated Children Scheme

Assistance for students who cannot attend school due to isolation, disability or special health needs. Payment rates are:

Allowance	Maximum payment	
Boarding allowance	$9133.00 per year	—Basic $7667 —Additional $1466
Second home allowance	$223.31 per fortnight per student (max 3 students)	
Distance Education Allowance	$3833 per year	

The assistance for isolated children *Additional Boarding Allowance* is income tested. This allowance decreases by $1 for each $5 of parental income over $48 837.

WAGES, SALARIES AND PIECEWORK

Employees are paid a *wage* by an employer. This might be fortnightly or weekly. Employees might also earn *bonuses* through *overtime* and receive *bonuses* for clothing, food etc.

An employee might also be paid a fixed amount per year. This is a *salary* and can be paid at regular intervals such as fortnightly or monthly.

Piecework is a type of employment where employees are paid a fixed piece rate for each unit produced regardless of time.

Worked Examples

4.1 Which jar of vegemite is the best buy?

(a)	80g	at	\$1.75
(b)	150g	at	\$3.25
(c)	225g	at	\$4.10

Convert each to the price per weight, i.e. cents/gram

(a) $\dfrac{175 \text{ cents}}{80\text{g}} = 2.1875$ cents/gram

(b) $\dfrac{325}{150} = 2.1\dot{6}$ cents/gram

(c) $\dfrac{410}{225} = 1.8\dot{2}$ cents/gram

Best value for money is the 225g container of vegemite.

4.2 According to the currency exchange table \$1 Australian is equal to 0.6963 Euro.

Determine:

(a) How many Euros can be bought for \$500 Australian dollars.

(b) How many Australian dollars (AUD) can be purchased for 1200 Euros.

(a)
$$\begin{aligned} \$1 \text{ Australian dollar} &= 0.6963 \text{ Euro} \\ \$500 \text{ AUD} &= 0.6963 \times 500 \\ &= 348.15 \text{ Euro} \end{aligned}$$

(b)
$$\begin{aligned} \$1 \text{ AUD} &= 0.6963 \text{ Euro} \\ \$ \text{ AUD} &= \frac{1200}{0.6963} \text{ Euro} \\ \$ \text{ AUD} &= \$1723.40 \end{aligned}$$

4.3 David purchases 6000 shares in Westcom @ \$3.75 per share. A dividend of \$0.30 per share was paid by Westcom. Brokerage fees of 2.5% were charged by the stockbroker.

Calculate

(a) the total cost of purchasing the shares (include brokerage fee).

(b) the total dividend paid.

(c) the total earnings if David sells all 6000 shares @ \$4.00 per share, at the end of one year.

(a)
$$\begin{aligned} \text{Cost of shares} &= 6000 \times \$3.75 \\ &= \$22\,500 \\ \text{Brokerage fee} &= 2.5\% \times \$22\,500 \\ &= \$562.50 \\ \text{Total cost} &= \$22\,500 + \$562.50 \\ &= \$23\,062.50 \end{aligned}$$

(b) Dividend $= 6000 \times 0.30$
$= \$1800$

(c) Selling price of shares $= 6000 \times \$4$
$= \$24\,000$
Brokerage fee $= \$600$

Earnings $= \$24\,000 - \$600 + \$1800 - \$23\,062.50$
$= \$2137.50$

4.4 Determine the P/E of a company with a share price of \$15.50 and annual earnings per share of \$2.50.

$$P/E = \frac{\text{Price per share}}{\text{Yearly earnings per share}}$$
$$= \frac{\$15.50}{\$2.50}$$
$$= 6.2$$

Price to earnings ratio is 6.2.

Use the tables on pages 34 and 35 for Examples 4.5 and 4.6.

4.5 Calculate the Assistance for Isolated Children payment for each of the following:

(a) John boards at a city school; parents combined income is \$70 000 p.a.
(b) Jenny boards at St Xavier's School; single parent earns \$35 000 p.a.
(c) Jack completes school via Distance Education.

(a) Boarding Allowance: Basic \$7667
Additional: Nil (Parents income too high)

(b) Boarding Allowance: Basic \$7667
Additional: \$1466
Total: \$9133.00

(c) Distance Education Allowance \$3833.00

4.6 Calculate the Newstart Allowance for each situation below.

(a) Scotty, aged 23, no children, earning \$3120 p.a. on various part-time jobs.
(b) Davina, aged 25, dependant child, has no income.
(c) Craig and partner Chloe, both aged 24, combined income of \$16 900 p.a. on odd jobs, no children.

(a) Scotty: Income $\frac{3120}{26}$ fortnight
$= \$120$

Payment $= \$510.50 - 0.5 \times (\$120 - \$100)$
$= \$500$ per fortnight.

As Scotty earns \$20 over \$100 his payment is reduced by 50 cents in the dollar.

(b) Davina:

Payment: \$552.40 per fortnight

(c) Craig and Chloe:

Income: $\frac{16900}{2}$ = \$8450 each

Income per fortnight: $= \frac{8450}{26}$

= \$325

Fortnightly Payment will reduce by 60 cents in the dollar above \$250.

Payment: = \$460.90 – 0.6 (\$325 – \$250)

= \$415.90 per fortnight.

Chloe and Craig will each receive \$415.90 per fortnight.

4.7 Catherine earns \$25.00 per hour. She is paid time and a half for Saturday and double time for Sundays. A meal allowance of \$12.50 per day is also paid. Calculate Catherine's total earnings for the week for the hours shown in the table below.

Day	Hours worked
Monday	8
Tuesday	9
Wednesday	12
Thursday	8
Friday	10
Saturday	4
Sunday	5

Normal time: \$25/hour

Time and a half: \$25 × 1.5 = \$37.50/hour

Double time: \$25 × 2 = \$50/hour

Earnings

Monday–Friday: 47 × \$25 = \$1175

Saturday: 4 × \$37.50 = \$150

Sunday: 5 × \$50 = \$250

Meal Allowance: 7 × \$12.50 = \$87.50

Total: \$1662.50

4.8 For delivering the local paper, Matt is paid 10 cents per paper.

(a) How much will Matt earn for delivering 2250 papers.

(b) If he wishes to earn \$475, how many papers must he deliver.

(a) Total earnings = 2250 × \$0.10
= \$225.00

(b) Number of papers = $\frac{\$475}{\$0.10}$
= 4750

PROBLEMS TO SOLVE

Chapter 4: Finance

1. Which size Weetbix is the best value for money?

1 kg	cost	\$6.95
500 g	cost	\$3.99
375 g	cost	\$2.65

2. Determine which are the 'best buys' for each of the following:

(a) Yoghurt
- i. 6 × 200 g tubs: \$6.34
- ii. 1 kg tub: \$4.49

(b) Steak
- i. 1.068 kg: \$14.94
- ii. 224 g: \$3.15

(c) Batteries
- i. 10 Pack: \$15.03
- ii. 4 Pack: \$7.06

(d) Toilet Paper
- i. 2 Rolls: \$2.36
- ii. 6 Rolls: \$4.59
- iii. 9 Rolls: \$5.61
- iv. 12 Rolls: \$8.49

3. Cola soft drink is sold in various size containers. Determine which size is the best value for money.

600 mL	cost	\$2.87
6 × 375 mL cans	cost	\$6.87
15 × 375 mL cans	cost	\$12.99
24 × 375 mL cans	cost	\$14.66
1.5L	cost	\$2.55
2L	cost	\$3.26

4. Milk is sold in four different size containers:

500 mL	cost	\$1.07
1L	cost	\$1.96
2L	cost	\$3.17
3L	cost	\$4.77

(a) Determine which size container is the best buy.

(b) Milk at a local deli has a 'best before' date printed on the container of 2 days from the date of purchase. If Morgan consumes on average 500mL per day of milk should he purchase a '2L' container or '2 × 500mL' containers as the 1 litre container has sold out?

5. Convert the following using the currency exchange rate table included in the formula and definitions section (page 33)

(a) \$2000 Australian dollars to Singapore dollars

(b) \$550 Australian dollars to UK pound sterling

(c) 4500 Thai baht to Australian dollars

(d) 7000 Hong Kong dollars to Australian dollars.

6. (a) Sarah is travelling to Japan and decides to convert her Australian currency to Japanese Yen. How much Japanese Yen will she receive for \$2500.

(b) On her return journey to Australia, Sarah decides to convert her remaining 19 098 Japanese Yen back to Australian currency. How much Australian currency will she receive?

7. Alex decided to purchase a new computer from Malaysia. The cost of the computer was 3400 Malaysian ringgit. How much did Alex pay for the computer in Australian dollars? (Answer to the nearest dollar).

8. On a round the world trip Jenny accumulates various amounts of different currencies. These amounts are:

40	British pounds sterling
50	Singapore dollars
75	United States dollars
125	New Zealand dollars
45 000	Indonesian rupiah

Jenny decides to convert these to Australian dollars. How much in total will she receive? (Answer to the nearest dollar.)

9. Dorothy has the following portfolio of shares.

Company	Number of shares	Share value per share	Dividend per share
Ajax	8500	\$0.72	\$0.10
Glass	7000	\$5.90	\$0.35
Pac	500	\$24.00	\$1.50

Determine:

(a) the total value of the portfolio.

(b) the total dividend.

10. A company has 3.2 million shares and makes an after tax profit of \$7.5 million. The company agrees to pay a dividend of 12% on the profit.

What is the dividend paid per share?

11. The brokerage fees charged by a stockbroker are:

Share value	Up to $15 000	Over $15 000
Brokerage	$69.95	0.40% of the trade value

Determine the brokerage fee for the following:

(a) 2000 shares @ $7.50 per share

(b) 7500 shares @ $5.25 per share.

12. Victoria purchases 8000 shares in Zedform.

- Shares are $7.25 each
- The company pays a dividend of 4.5% of the share price.
- A brokerage fee of 3% is paid to the stockbroker.

Determine:

(a) the total cost of purchasing the shares (including the brokerage fee).

(b) the total dividend paid.

(c) the total gain if Victoria sells all 8000 shares at the end of one year @ $9.05 per share.

13. Otus Mining shares are valued at $16.50. A dividend of $0.80 per share is paid and 20% of Otus' yearly earnings were paid as a dividend.

Determine:

(a) the total yearly earnings per share.

(b) the price to earning ratio (P/E).

Use the tables on pages 34 and 35 to complete the following questions.

14. Calculate the Youth Allowance for each of the following:

(a) Gerry, single, 17 years old, living at home, earning $10 000 p.a. on part time jobs and studying full time.

(b) Deanna, 19 years old, living at home, has a daughter and studying full time.

(c) Richard, 21 years old, not living at home, studying full time and working part time. His yearly income was $13 520.

15. Calculate the Age Pension for each of the following:

(a) David, aged 75, has no partner and no income.

(b) Lois, aged 81, has no partner and receives a yearly income of $4000 from investments.

(c) Mark and Leonie both aged 76, earn a combined income of $100 per week cleaning a friend's house.

16. Calculate the Newstart Allowance for each situation below:

   (a) Kai, unemployed, single with two children.
   (b) Sally, unemployed, married with one child.
   (c) Tom, unemployed, single, no children and earns $60 per week mowing the neighbour's lawn.

17. Simon is paid a salary of $92 000. How much does he earn:

   (a) per week?
   (b) per month?
   (c) per fortnight?

18. Fran is paid $14.50 per hour for a 36 hour week, time and a half for the first 3 hours overtime and double time thereafter. How much does she earn in a week if she worked from 8 a.m. to 5 p.m., Monday to Friday?

19. Which of the following earn the most?

   - Coleen: Salary $47 515
   - Sam: $856 per week
   - Pam: $23 per hour, 40 hour week

20. Gordon is paid $5.70 per kilogram picking apples. For any additional kilograms of apples picked over 100 kg in a week, the rate increases by 40%. If Gordon picks 150 kg of apples in a week, what is his total earnings?

21. Floyd is paid $7.50 per pizza delivery.

   (a) How much does he earn delivering 36 pizzas?
   (b) How many pizzas must be delivered for Floyd to earn $2000?

22. Debbie worked the following rate and hours.

   Normal rate: 35 hours

   Time and a half: 4 hours

   Double time: 6 hours

   Her total earnings was $1934.50.

   What was Debbie's normal rate of pay?

23. Determine the total weekly earnings for each person below.

(a) Zac – $17.40 per hour. Details of work completed are shown in the table below.

Days	Rate	Hours
Monday–Friday	Normal	36
Saturday	Time and a half	6
Sunday	Double time	5
Overtime	Double time	7

A meal allowance of $10.75 per day and a remote living allowance of an extra 1.5%.

(b) Penny – $15.80 per hour

Normal Rate:	Monday to Friday
Time and a half:	Saturday
Double time	Sunday
Overtime:	Any additional hours above an 8 hour day paid at time and a half.
Clothing allowance:	$120 per week

Listed below are Penny's weekly details.

Day	Hours	Breaks
Monday	8.30 a.m.–5.30 p.m.	1 hour
Tuesday	8.00 a.m.–6.00 p.m.	1 hour
Wednesday	7.00 a.m.–6.00 p.m.	30 minutes
Thursday	8.30 a.m.–9.00 p.m.	30 minutes
Friday	8.30 a.m.–5.00 p.m.	30 minutes
Saturday	10.00 a.m.–5.30 p.m.	1 hour
Sunday	10.30 a.m.–6.00 p.m.	45 minutes

5 PYTHAGORAS

Syllabus Checklist

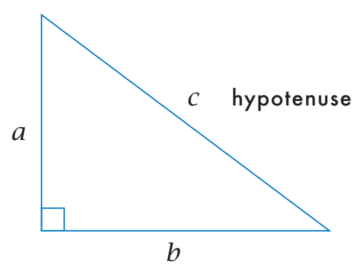
By the end of this chapter, you should be able to:

- ☐ Use Pythagoras' theorem to solve practical problems in two dimensions and for simple applications in three dimensions.

FORMULAE AND DEFINITIONS

Pythagoras

Pythagoras' theorem states that the length of the hypotenuse squared is equal to the sum of the squares of the lengths of the other two sides.

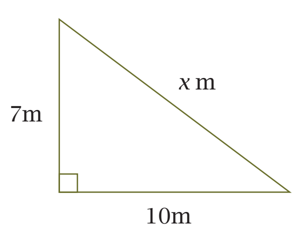


$$c^2 = a^2 + b^2$$

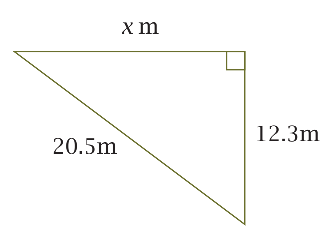
Worked Examples

5.1 Find the value of x correct to 2 decimal places.

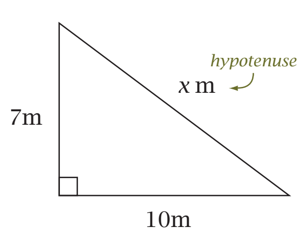
(a)



(b)

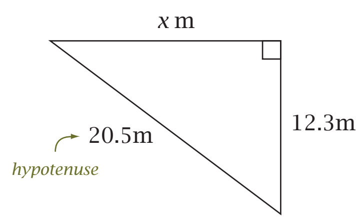


(a)



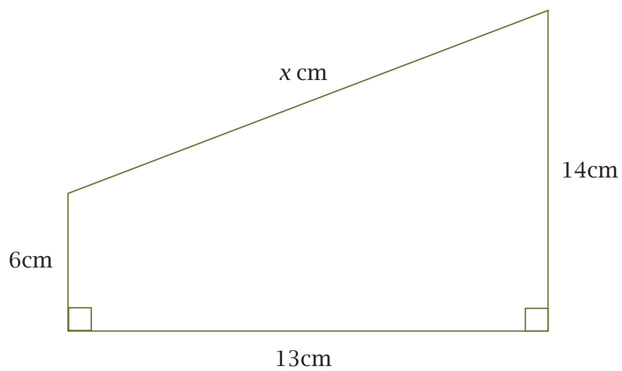
$$x^2 = 7^2 + 10^2$$
$$x^2 = 49 + 100$$
$$x^2 = 149$$
$$x = \sqrt{149}$$
$$x = 12.21\text{m}$$

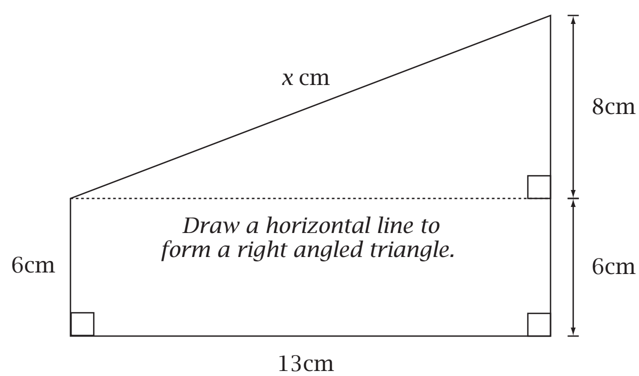
(b)



$$\begin{aligned} 20.5^2 &= x^2 + 12.3^2 \\ x^2 &= 20.5^2 - 12.3^2 \\ x^2 &= 268.96 \\ x &= \sqrt{268.96} \\ x &= 16.40\text{m} \end{aligned}$$

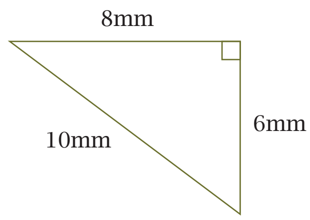
5.2 Find the value of x correct to 2 decimal places in the following diagram.





$$\begin{aligned} \therefore x^2 &= 13^2 + 8^2 \\ x^2 &= 233 \\ x &= \sqrt{233} \\ x &= 15.26\text{cm} \end{aligned}$$

5.3 Is the following triangle right angled?



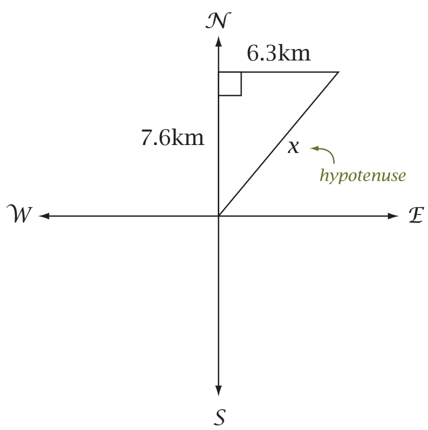
If the longest side squared is equal to the sum of the squares of the other two sides, then the triangle is right angled.

$$10^2 = 6^2 + 8^2$$
$$100 = 36 + 64$$
$$100 = 100$$

The triangle is right angled.

5.4 A hiker leaves camp and travels north for 7.6km and then travels east 6.3km. How far from camp is the hiker?

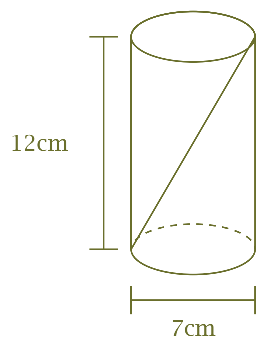
Hint: Draw a diagram

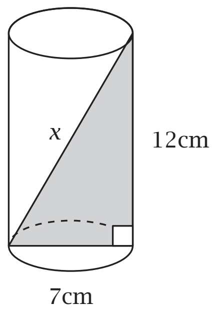


$$x^2 = 7.6^2 + 6.3^2$$
$$x^2 = 97.45$$
$$x = \sqrt{97.45}$$
$$x = 9.87$$

The hiker is 9.87km from camp.

5.5 Calculate the length of the longest rod that will fit inside the cylinder below:





$$x^2 = 12^2 + 7^2$$
$$x^2 = 144 + 49$$
$$x^2 = 193$$
$$x = \sqrt{193}$$
$$x = 13.89\text{cm}$$

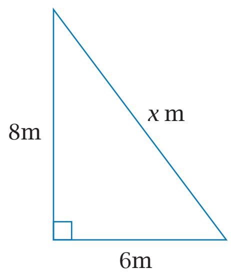
The length of the longest rod in the cylinder is 13.89cm

PROBLEMS TO SOLVE

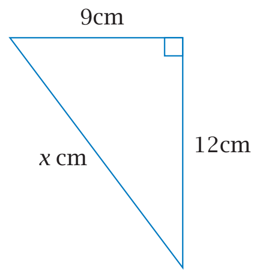
Chapter 5: Pythagoras

1. Find the value of x in each of the following, correct to 2 decimal places where necessary.

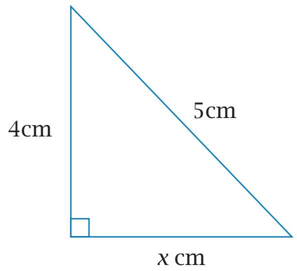
(a)



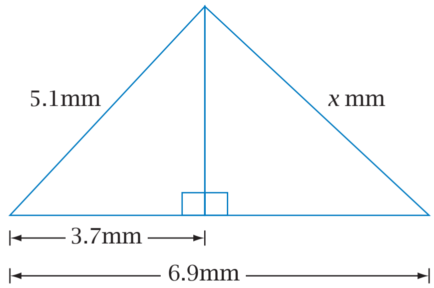
(b)



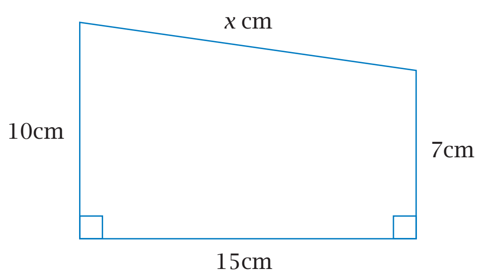
(c)



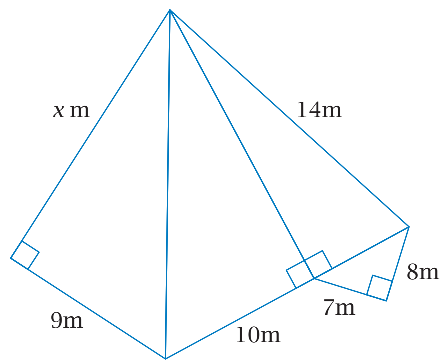
(d)



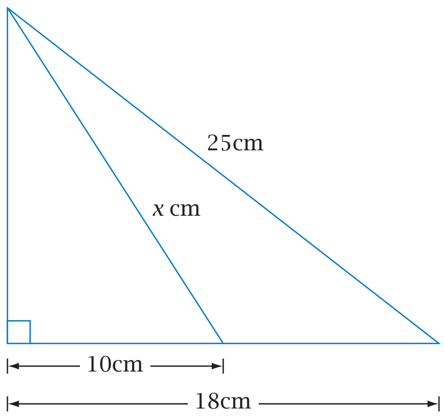
(e)



(f)

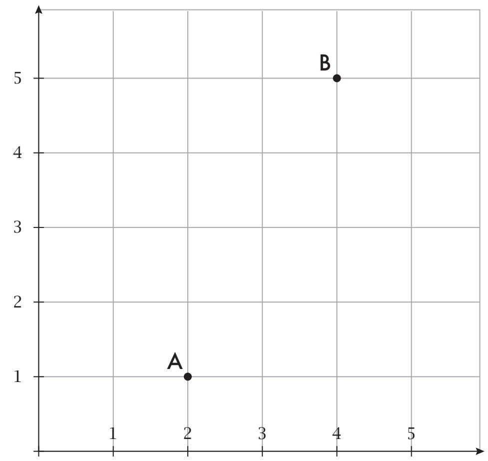


(g)

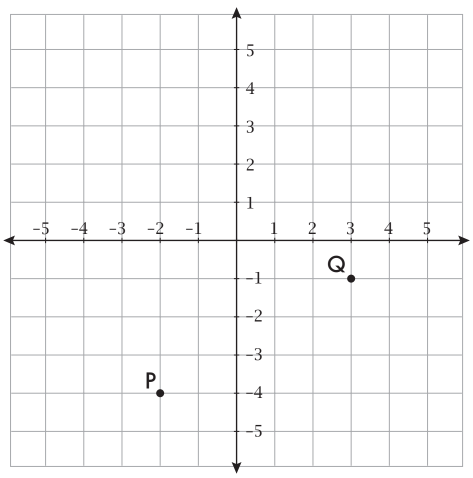


2. Using Pythagoras find the distance between the coordinates marked on the grid below, correct to 2 decimal places.

(a)



(b)



3. For each of the following triangles, find the length of the missing side.

(a) AB = 4.7cm

AC = 5.6cm

$\angle$BAC = 90°

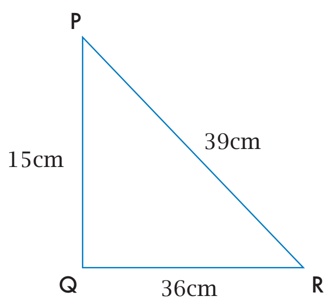
(b) XY = 15m

YZ = 11.4m

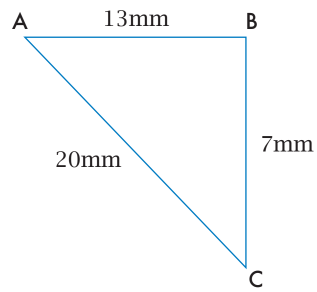
$\angle$XZY = 90°

4. Determine whether or not the following triangles are right angled.

(a)



(b)



5. An explorer walks 7.6km due East from base camp and then 8.3km due South. How far is the explorer from base camp?

6. A gate 1.2m wide by 2.7m high needs a diagonal brace for support. Calculate to 2 decimal places the length of wood required for the brace.

7. A shadow of length 26m is cast by a tree of height 13.5m. Determine the distance from the end of the shadow to the top of the tree.

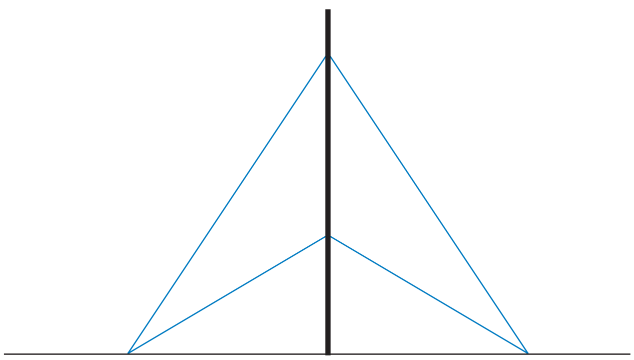
8. A rhombus has diagonals of 14mm and 5mm respectively.
Determine:

   (a) The length of the rhombus.

   (b) The perimeter of the rhombus.

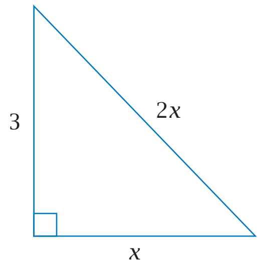
9. A square has an area of 144m^2. Determine the length of the diagonal.

10. For safety reasons a 120m long television antenna requires 4 support wires. Two of the wires are placed one third of the way up the antenna and the remaining two one sixth of the distance below the top of the antenna. All four wires are positioned on the ground floor 30m from the foot of the antenna. Determine the total length of the wire needed to support the television antenna.

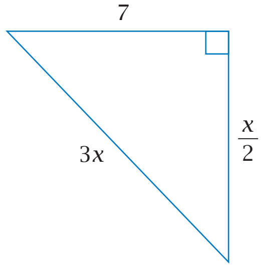


11. Find the value of x, correct to 2 decimal places, in the following, by using Pythagoras' theorem.

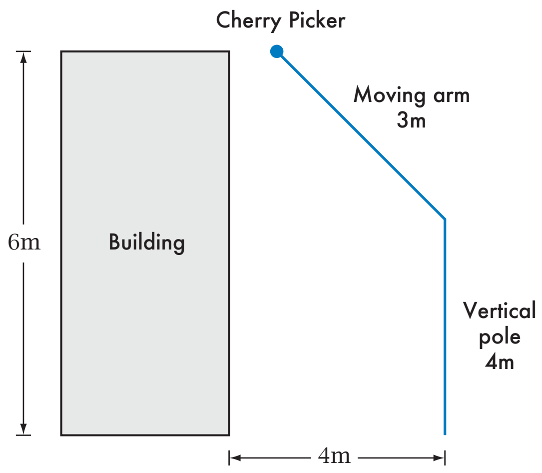
(a)



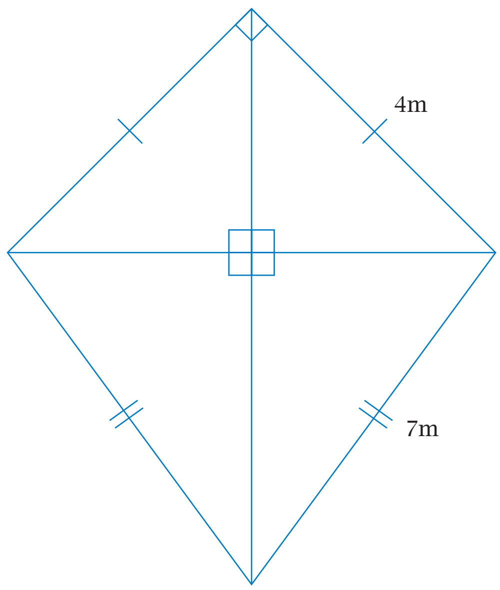
(b)



12. A cherry picker is used to clean gutters on buildings and is elevated on a 4m vertical pole on a 3m moving arm. If the vertical pole is 4m from the base of the 6m high building, how far is the cherry picker from the building?



13. A paddock is divided into 4 main areas according to the following diagram. Each area must be fenced and this will cost $18.50 per metre length. Determine the cost of fencing the paddock



14. A pendulum of a clock swings 3cm horizontally either side of vertical. If the pendulum is 14cm long, determine how far vertically the tip of the pendulum will rise.

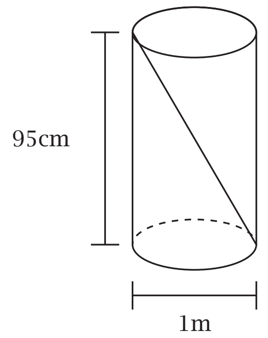
15. A pole 15m long is placed against a brick wall with the following safety condition:

The distance the pole reaches up the brick wall must be twice the distance the foot of the pole is from the brick wall.

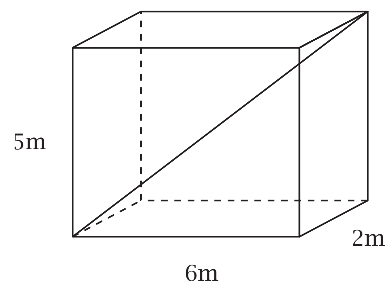
Determine how far up the brick wall the pole must be placed.

16. Find the length of the longest rod that will fit inside the shapes below:

(a)



(b)



TRIGONOMETRY

6

Syllabus Checklist

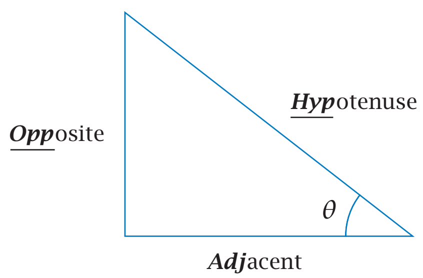
By the end of this chapter, you should be able to:

- ☐ use trigonometric ratios to determine the length of an unknown side, or the size of an unknown angle in a right angled triangle
- ☐ determine the area of a triangle using Area = $\frac{1}{2}$ ab sin c, by using Heron's rule and solve practical problems
- ☐ solve problems involving non-right angled triangles using the sine rule and the cosine rule

FORMULAE AND DEFINITIONS

Trigonometric ratios

To calculate the lengths or angles in a right angled triangle use trigonometric ratios.



$$\sin\theta = \frac{opp}{hyp}$$

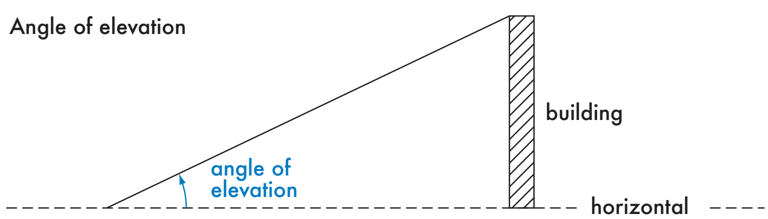
$$\cos\theta = \frac{adj}{hyp}$$

$$\tan\theta = \frac{opp}{adj}$$

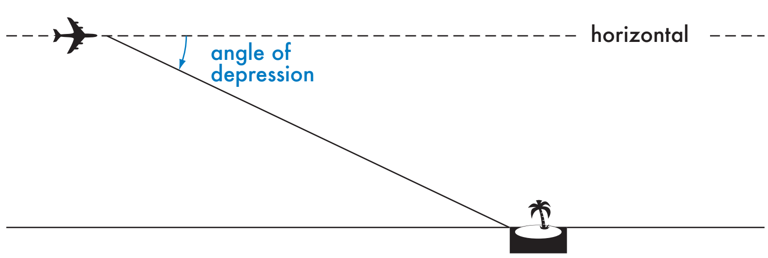
Angles of elevation and depression

Angles of elevation and depression are always measured from the horizontal.

Angle of elevation



Angle of depression

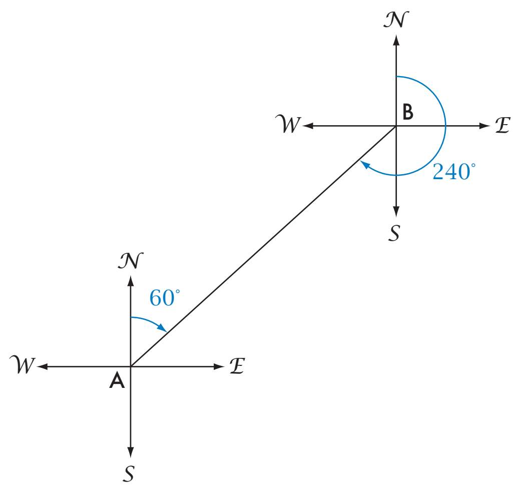


Bearings

Bearings can be expressed as either:

- An angle using three digits clockwise from North
  e.g. 042°, 157°, 248°

- The number of degrees East or West from North or South
  e.g. N32°W, S15°E

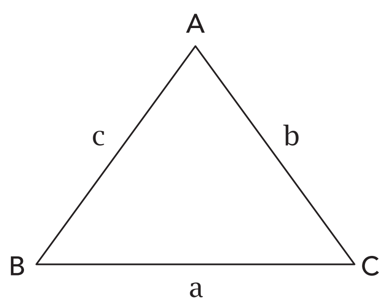


The bearings (left) could be described as:

- the bearing of B **from** A is 060°

- the bearing of A **from** B is 240°

Non right angled triangles



Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Using the sine rule requires

* two side lengths and a non-included angle *or*

* one side length and two angles

Cosine rule (for finding a length)

$a^2 = b^2 + c^2 - 2bc \cos A$

Cosine rule (for finding an angle)

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Using the cosine rule requires

* two side lengths and an included angle *or*

* three side lengths

Area of a triangle

$$\begin{aligned}\text{Area} &= \frac{1}{2}\, ab \sin C \\ &= \frac{1}{2}\, bc \sin A \\ &= \frac{1}{2}\, ac \sin B\end{aligned}$$

Using Heron's Rule

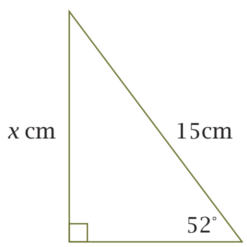
$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{a+b+c}{2}$$

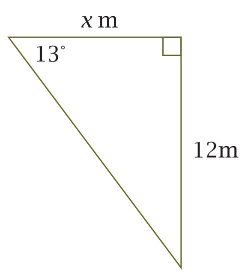
Worked Examples

6.1 For each of the following find the value of x correct to 2 decimal places.

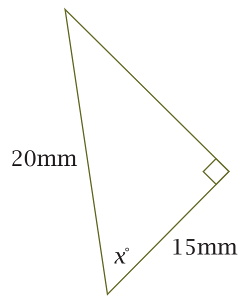
(a)



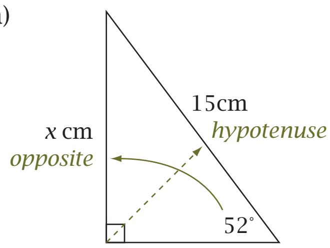
(b)



(c)



(a)



Opposite and hypotenuse – use **sin**

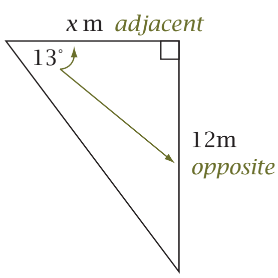
$$\sin\theta = \frac{opp}{hyp}$$

$$\sin 52^\circ = \frac{x}{15}$$

$$x = 15 \times \sin 52^\circ$$

$$x = 11.82\text{cm}$$

(b)



Opposite and adjacent – use **tan**

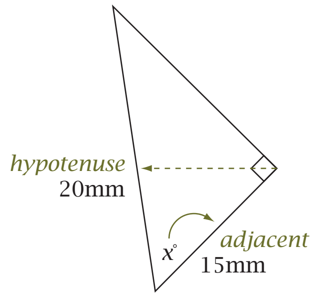
$$\tan\theta = \frac{opp}{adj}$$

$$\tan 13^\circ = \frac{12}{x}$$

$$x = \frac{12}{\tan 13^\circ}$$

$$x = 51.98\text{m}$$

(c)



Adjacent and hypotenuse – use **cos**

$$\cos\theta = \frac{adj}{hyp}$$

$$\cos x = \frac{15}{20}$$

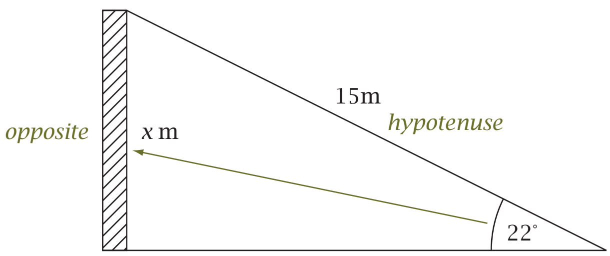
use **Inv cos** *or* **cos^{-1}** *or* **A cos** *to find the angle*

$$x = \text{Inv}\cos\left(\frac{15}{20}\right)$$

$$x = 41.41^\circ$$

6.2 A ladder 15m long rests against a wall and makes an angle of 22° to the horizontal. How far up the wall does the ladder reach?

Hint: Always draw a diagram



$$\sin 22^\circ = \frac{x}{15}$$

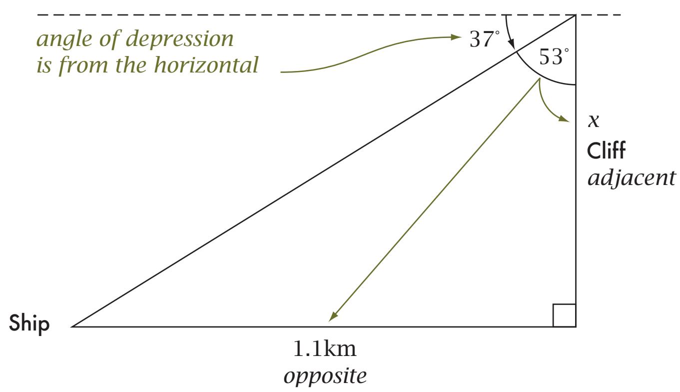
$$x = 15 \times \sin 22^\circ$$

$$x = 5.62$$

The ladder reaches up the wall 5.62m.

6.3 From a cliff the angle of depression to a ship out to sea is 37°. If the ship is 1.1km from the base of the cliff, determine the height of the cliff.

Hint: Always draw a diagram



$$\tan\theta = \frac{opp}{adj}$$

$$\tan 53^\circ = \frac{1.1}{x}$$

$$x = \frac{1.1}{\tan 53^\circ}$$

$$x = 0.83$$

Cliff is 0.83km high.

6.4 Two ships leave port at the same time. Ship A travels for 30km on a bearing of 41° while ship B travels for 45km on a bearing of 131°.

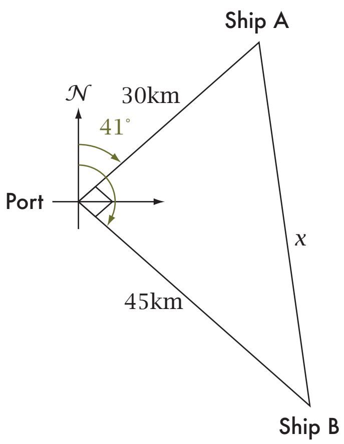
Determine:

(a) The distance from ship A to ship B.

(b) The bearing of ship A from ship B.

Hint:

- Always draw a diagram.
- Use compass directions to assist.



Bearings are from North in a clockwise direction.

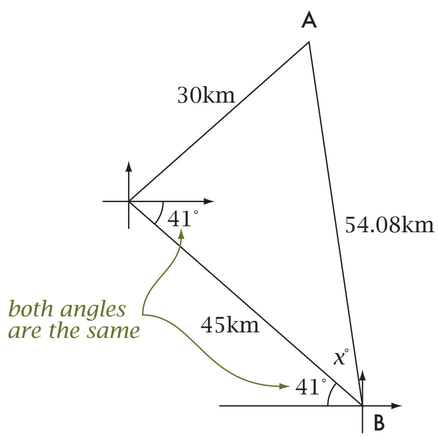
The angle between ship A and ship B is (131°– 41°) = 90°

use pythagoras

(a)

$$x^2 = 30^2 + 45^2$$
$$x^2 = 2925$$
$$x = \sqrt{2925}$$
$$x = 54.08$$

Distance between ship A and ship B is 54.08km.

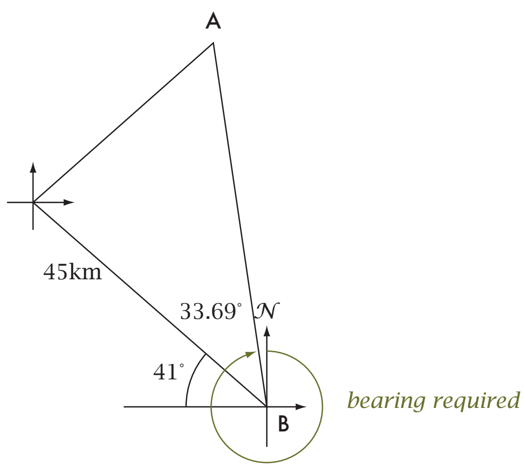


(b) Bearing of ship A **from** ship B.

Place compass directions on B

Find x°

$$\tan x^\circ = \frac{30}{45}$$
$$x^\circ = 33.69^\circ$$

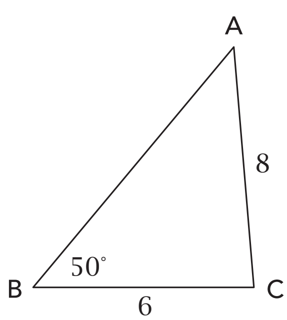


Bearing is calculated from North in a clockwise direction

i.e. 270° + 41° + 33.69°

= 344.69°

6.5 Find the size of angle A in the triangle shown below



Using the sine rule

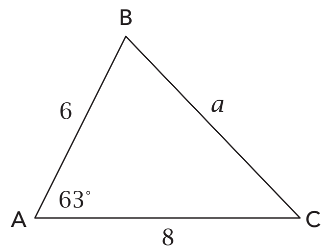
$$\frac{6}{\sin A} = \frac{8}{\sin 50^\circ}$$

$$\sin A = \frac{6 \sin 50^\circ}{8}$$

$$A = 35.07^\circ$$

In this course we *only* consider *acute* angles.

6.6 Find **a** in the triangle shown below



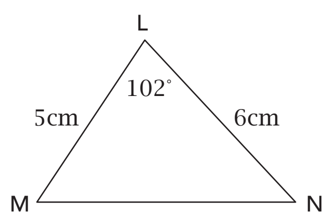
Using the cosine rule

$$a^2 = 8^2 + 6^2 - 2(8)(6) \cos 63^\circ$$

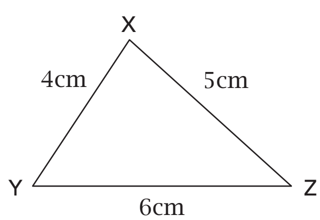
$$a = 7.51$$

6.7 Find the area of each triangle below

(a)



(b)



(a) Area $= \frac{1}{2}$ MN sin L

$$= \frac{1}{2}(5)(6) \sin 102^\circ$$

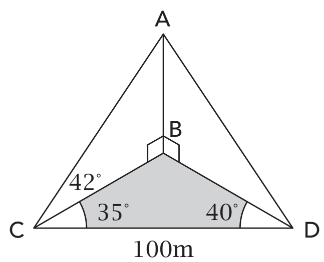
$$= 14.67 \text{ cm}^2$$

(b) Using Heron's rule

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$s = \frac{a+b+c}{2}$$
$$s = \frac{4+5+6}{2}$$
$$s = 7.5$$

$$\text{Area} = \sqrt{7.5(7.5-4)(7.5-5)(7.5-6)}$$
$$= 9.92\,\text{cm}^2$$

6.8 In the diagram below



$\angle BCD = 35^\circ$, $\angle BDC = 40^\circ$, CD = 100m. The angle of elevation of A from C is 42°.

Determine:

(a) BD

(b) BC

(c) AB

(d) the angle of elevation of A from D.

(a) $\frac{BD}{\sin 35^\circ} = \frac{100}{\sin 105^\circ}$

$BD = \frac{100 \sin 35^\circ}{\sin 105^\circ}$

$BD = 59.38\text{m}$

(b) $\frac{BC}{\sin 40^\circ} = \frac{100}{\sin 105^\circ}$

$BC = \frac{100 \sin 40^\circ}{\sin 105^\circ}$

$BC = 66.55\text{m}$

(c) $\tan 42^\circ = \frac{AB}{66.55}$

$AB = 66.55 \times \tan 42^\circ$

$AB = 59.92\text{m}$

(d) $\tan D = \dfrac{AB}{BD}$

$\tan D = \dfrac{59.92}{59.38}$

$D = \tan^{-1}\left(\dfrac{59.92}{59.38}\right)$

$D = 45.26^\circ$

Angle of elevation of A from D is 45.26°.

6.9 Two hikers set off from base camp X. Hiker A travels 42km on a bearing of 310° from X. Hiker B travels 37km on a bearing of 65° from X.

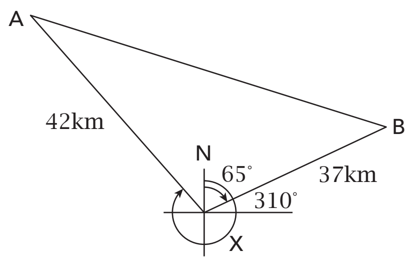
Determine

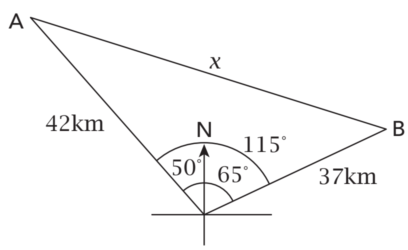
(a) the distance between the two hikers A and B

(b) the bearing of hiker A from hiker B.

Draw a diagram

(a)





Use cosine rule

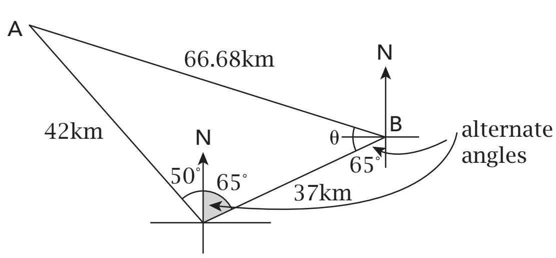
$x^2 = 42^2 + 37^2 - 2(42)(37)\cos 115^\circ$

$x^2 = 4446.50$

$x = \sqrt{4446.50}$

$x = 66.68\text{km}$

(b)



Use sine rule to find θ

$$\frac{42}{\sin\theta} = \frac{66.68}{\sin 115^\circ}$$

$$\sin\theta = \frac{42\sin 115^\circ}{66.68}$$

$$\theta = 34.81^\circ$$

Bearing of A *from* B is

$180^\circ + 65^\circ + 34.81^\circ$

$= 279.81^\circ$

PROBLEMS TO SOLVE

Chapter 6: Trigonometry

1. Use your calculator to find correct to 2 decimal places the values of:

   (a) $\sin 35^\circ$

   (b) $\cos 72^\circ$

   (c) $\tan 68.5^\circ$.

2. Use your calculator to find the acute angle x in degrees correct to 2 decimal places.

   (a) $\sin x^\circ = 0.2381$

   (b) $\cos x^\circ = \frac{7}{8}$

   (c) $\tan x^\circ = 1.4672$.

3. In each of the following determine the value of x correct to 2 decimal places.

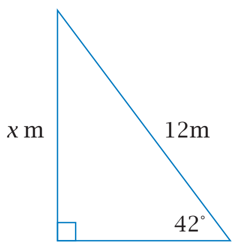
   (a) $\sin 40^\circ = \frac{x}{5}$

   (b) $\frac{21.5}{x} = \cos 12^\circ$

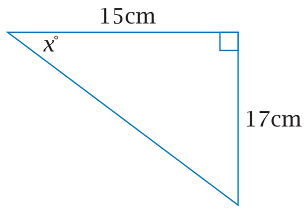
   (c) $\tan 46^\circ = \frac{x}{10}$.

4. For each of the following find the value of x correct to 2 decimal places.

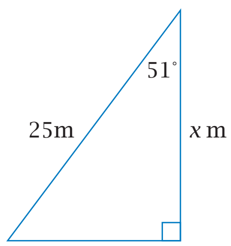
(a)



(b)

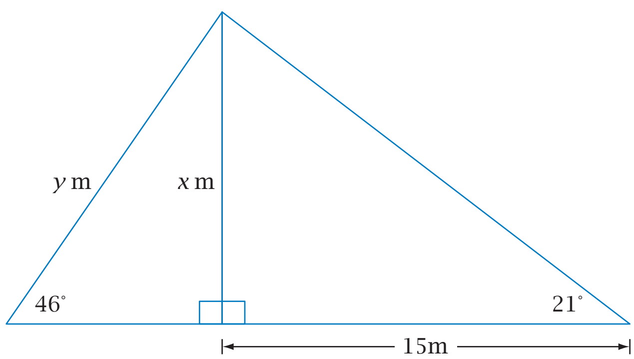


(c)

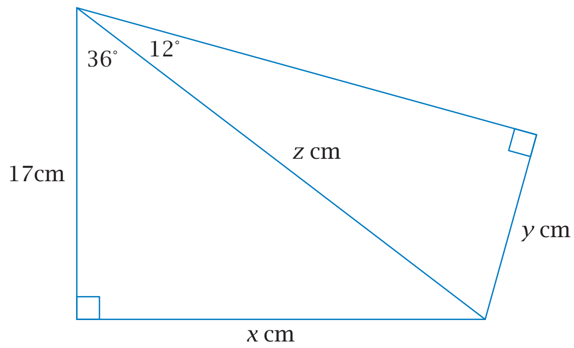


5. Find the values of the pronumerals correct to 2 decimal places for each of the following:

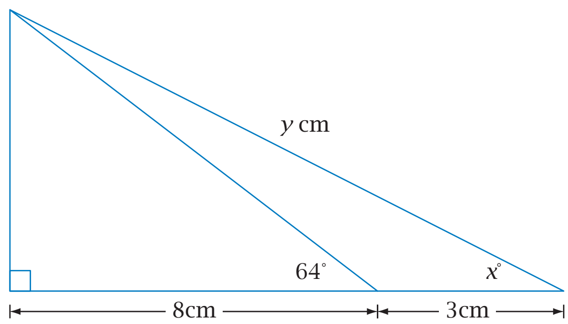
   (a)



(b)



(c)



6. A ladder leans against a wall inclined at an angle of 27° to the horizontal. If the ladder reaches 7.5m up the wall, what is the length of the ladder?

7. A ramp has a steady gradient of 1 in 12.

   (a) What angle does the ramp make with the horizontal ground?

   (b) If a trolley is pushed from the start of the incline for 4 metres, how high vertically will it be?

8. A ship sails from port due South for 17km and then due East for 8km. Find the ships bearing from port?

9. Two fishing boats depart port at the same time. Pegasus travels South at 8km/h while Atlantis travels in a westerly direction at 10km/h. Find:

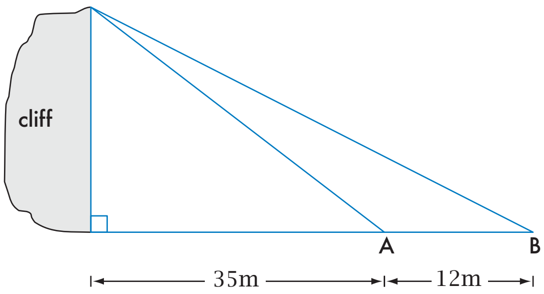
   (a) The distance the ships are apart after 2.5 hours

   (b) The bearing of Pegasus from Atlantis at this time.

10. From the top of a cliff, ship A is observed at an angle of depression of 15°, 35m from the base of the cliff. At the same time ship B is observed 12m from ship A as shown in the diagram below. Determine:

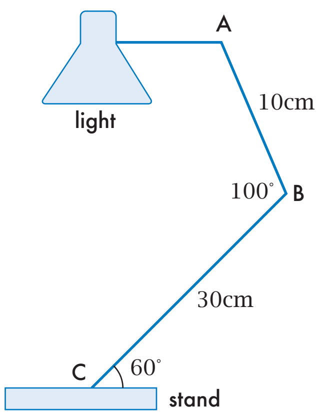
(a) The height of the cliff

(b) The angle of elevation from ship B to the top of the cliff.



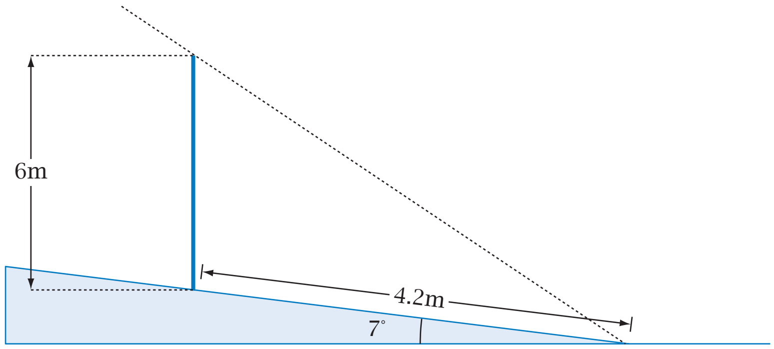
11. An off road rally vehicle leaves a checkpoint on a bearing of 46°, travelling at 120km/h for 1 hour. The vehicle then changes direction and travels on a bearing of 316° for 40 minutes at 150km/h. Find the distance and bearing of the rally vehicle from the checkpoint.

12. A new desktop lamp is shown in the diagram below



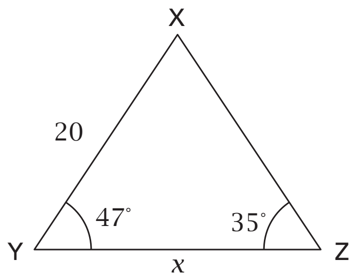
The lamp can swivel at point B and is positioned on a stand at point C. The light can be moved up and down at point A. Calculate the vertical distance from point A to the top of the stand at point C.

13. A flagpole 6m high sits on top of a hill inclined at an angle of 7° to the horizontal. Determine the angle of elevation of the sun if the length of the shadow of the flagpole is 4.2m.

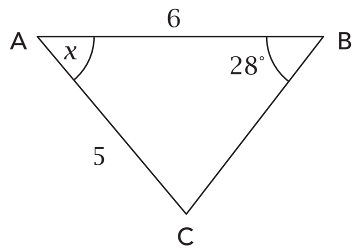


14. Find the value of x in each of the following:

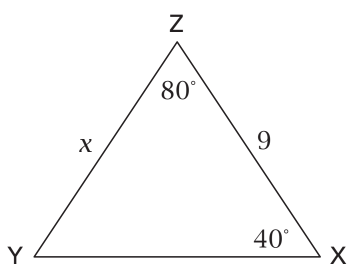
(a)



(b)

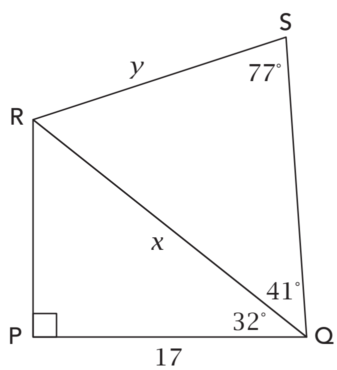


(c)



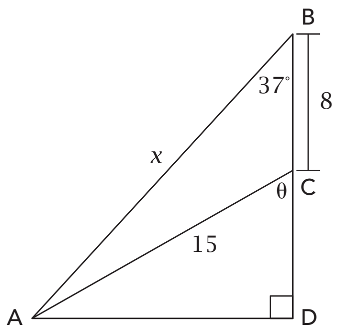
15. In ΔXYZ, $x = 10$cm, $z = 3$cm and $\angle X = 135^\circ$. Find $\angle Y$.

16.



Find the values of x and y.

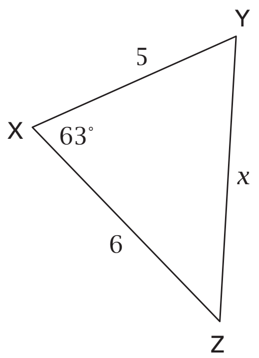
17.



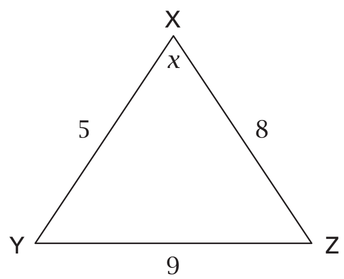
Find the values of x and θ.

18. Find the value of x in each of the following:

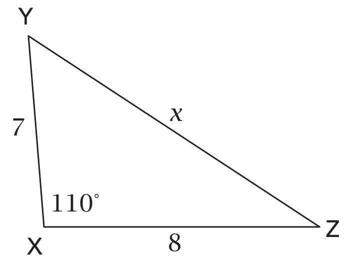
(a)



(b)

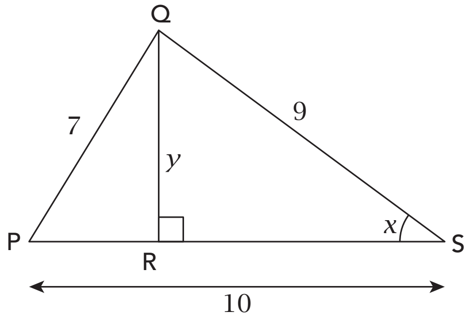


(c)



19. In ΔABC, $a = 7$cm, $b = 6$cm and $c = 4$cm. Find the smallest angle.

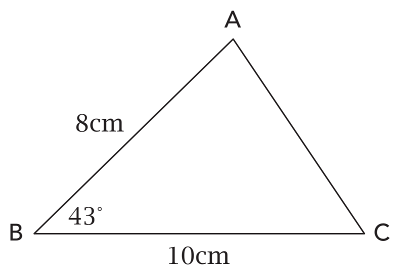
20.



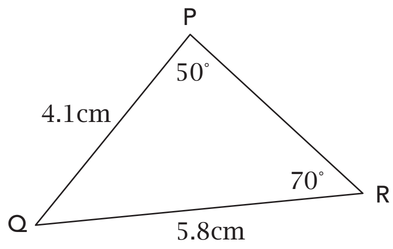
Find the values of x and y.

21. Find the area of the following:

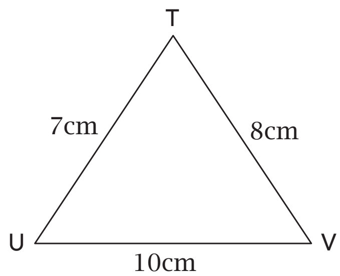
(a)



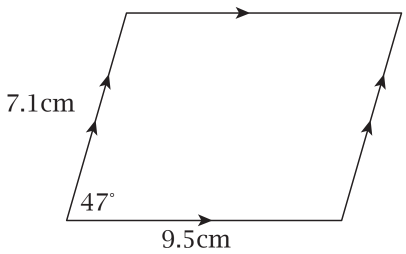
(b)



(c)



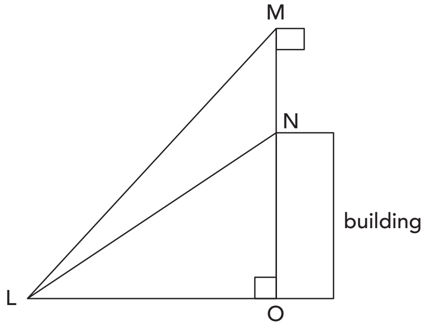
(d)



22. A triangle has an area of 767.69cm^2. The lengths of two of the sides of the triangle are 43.1cm and 38.7cm. Find the size of the angle between these two sides.

23. Find the area of a regular decagon inscribed in a circle of radius 10cm.

24. A flagpole stands vertically on a building. LM = 25m, LN = 20m, and $\angle$MLN = 15°.



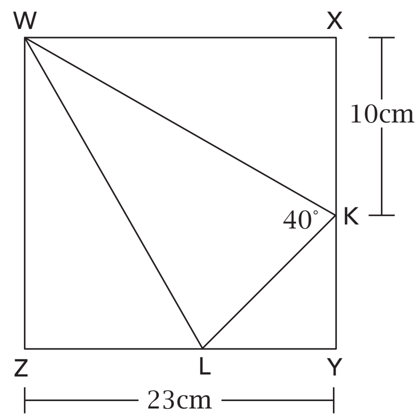
Calculate:

(a) the height of the building

(b) the height of the flagpole

(c) area of ΔLMN.

25.



In rectangle WXYZ, XK = 10cm, KL =12cm, YZ = 23cm and $\angle WKL = 40^\circ$.

Calculate:

(a) WK

(b) WL

(c) WZ

(d) Area ΔWKL.

26. Ship A leaves port P travelling 120 km west. Ship B leaves port on a bearing of 215° travelling 120km.

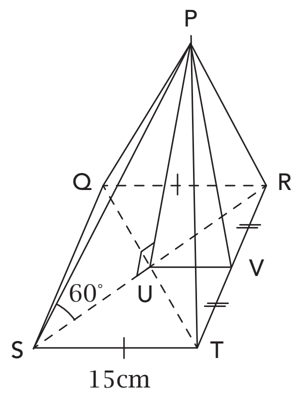
Find:

(a) the distance between the two ships

(b) the bearing of ship A from ship B

(c) the bearing of ship B from ship A

27. A square based pyramid is drawn below.



Determine:

(a) SR

(b) PS

(c) PU

(d) PV

(ε) ∠PVU.

7 PERIMETER, AREA, SURFACE AREA & VOLUME

Syllabus Checklist

By the end of this chapter, you should be able to:

- ☐ solve practical problems requiring the calculation of perimeters and areas of circles, sectors of circles, triangles, rectangles, parallelograms and composites
- ☐ calculate the volumes of standard three dimensional objects
- ☐ calculate the surface areas of standard three dimensional objects

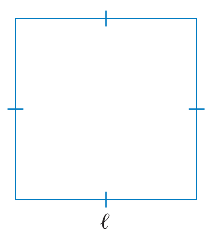
FORMULAE AND DEFINITIONS

Perimeter

The perimeter is the distance around the outside of a two dimensional object.

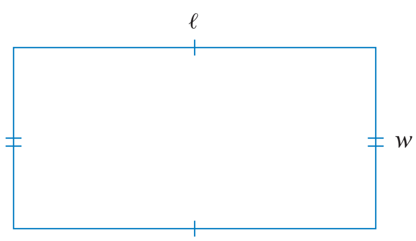
Specific Types

— Square



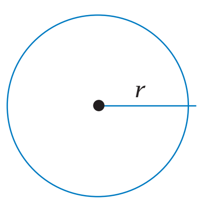
$P = 4\ell$

— Rectangle



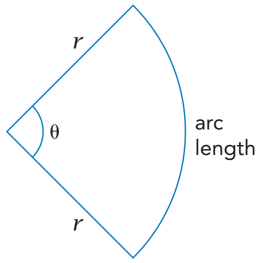
$P = 2\ell + 2w$

— Circle



$C = 2\pi r$ or
$C = \pi d$

— Sector of a circle



$$\text{Arc length} = \frac{\theta}{360} \times 2\pi r$$

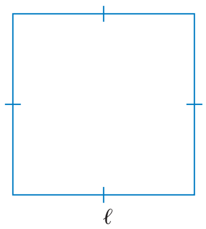
$$\text{Perimeter} = \frac{\theta}{360} \times 2\pi r + 2r$$

Area

The area is the number of square units covering a two dimensional object.

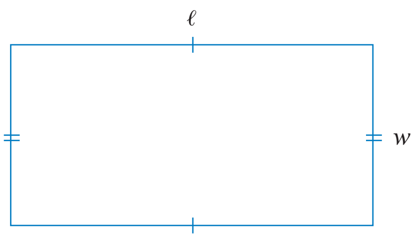
Specific Types

— Square



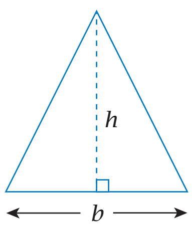
$$A = \ell^2$$

— Rectangle



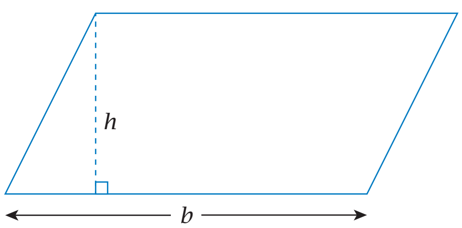
$$A = \ell \times w$$

— Triangle



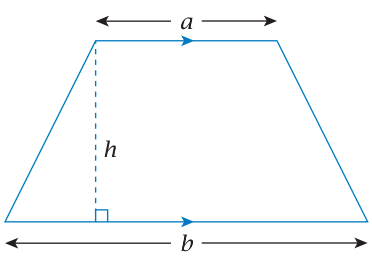
$$A = \frac{1}{2} b \times h$$

— Parallelogram



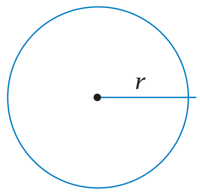
$$A = b \times h$$

— Trapezium



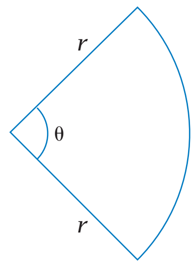
$$A = \frac{1}{2}(a + b).h$$

— Circle



$A = \pi r^2$

— Sector of a circle



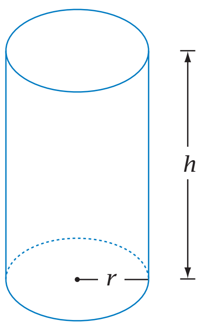
$A = \frac{\theta}{360} \times \pi r^2$

Surface area

The surface area of a three dimensional figure is the sum of the areas of the faces.

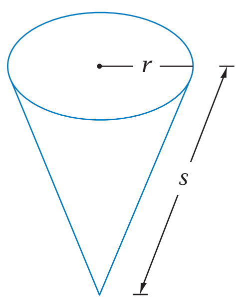
Specific Types

— Cylinder



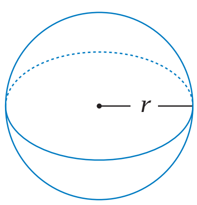
$$SA = \underbrace{2\pi rh}_{\text{curved surface}} + \underbrace{2\pi r^2}_{\text{2 circular ends}}$$

— Cone



$$SA = \underbrace{\pi rs}_{\text{curved surface}} + \underbrace{\pi r^2}_{\text{circular end}}$$

— Sphere

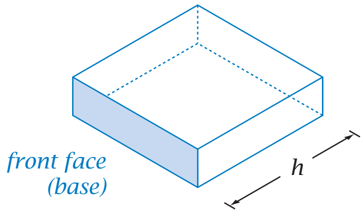


$SA = 4\pi r^2$

Volume

Volume is the amount of space an object occupies.

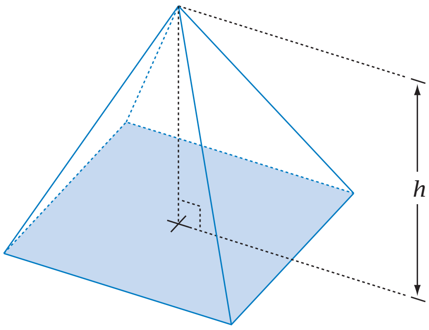
— Prisms



General formula

Volume = area of the front face (base) × height

— Pyramids

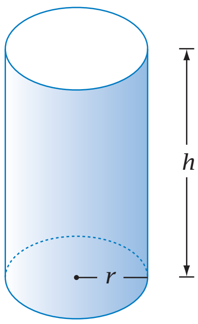


General formula

$$\text{Volume} = \frac{\text{area of the base} \times \text{height}}{3}$$

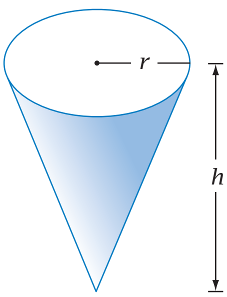
Specific Types

— Cylinder



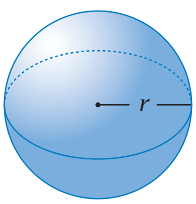
$$\text{Volume} = \pi r^2 h$$

— Cone



$$\text{Volume} = \frac{\pi r^2 h}{3}$$

— Sphere

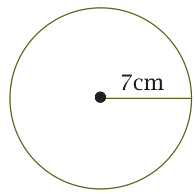


$$\text{Volume} = \frac{4}{3}\pi r^3$$

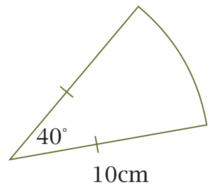
Worked Examples

7.1 Find the perimeter of each of the following giving answers correct to 2 decimal places.

(a)



(b)



(a) Circumference $= 2\pi r$

$= 2 \times \pi \times 7$

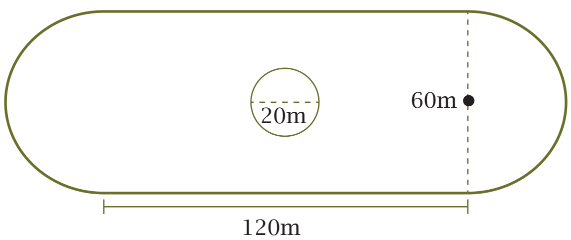
$= 43.98\text{cm}$

(b) $P = \dfrac{\theta}{360} \times 2\pi r + 2r$

$= \dfrac{40}{360} \times 2\pi(10) + 2(10)$

$= 26.98\text{cm}$

7.2 Calculate the cost of fencing the field below, including the central pond, if fencing costs $30 per metre.



- Length of both ends forms a circle

$C = \pi \times d$

$= \pi \times 60$

$= 188.50\text{m}$

- Length around pond

$C = \pi \times d$

$= \pi \times 20$

$= 62.83\text{m}$

- Straight lengths

$P = 2 \times 120$

$= 240\text{m}$

Total length $= 188.50 + 62.83 + 240$

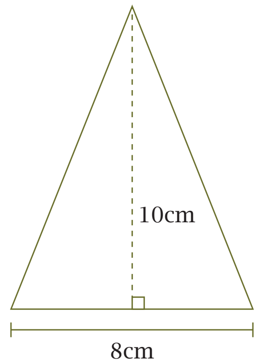
$= 491.33\text{m}$

Total fencing cost $= 491.33 \times \$30$

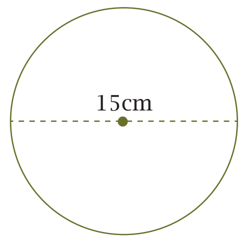
$= \$14\,739.90$

7.3 Calculate the areas of each of the following giving answers correct to 2 decimal places, where appropriate.

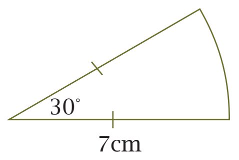
(a)



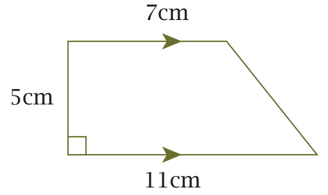
(b)



(c)



(d)



(a) Area triangle $= \frac{1}{2}$ base $\times$ perp. height

$= \frac{1}{2}(8) \times (10)$

$= 40\text{cm}^2$

(b) Area circle $= \pi \times r^2$

$= \pi \times (7.5)^2$

$= 176.71\text{cm}^2$

(c) Area sector $= \frac{\theta}{360} \times \pi r^2$

$= \frac{30}{360} \times \pi(7)^2$

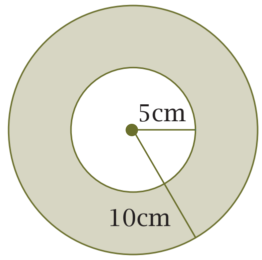
$= 12.83\text{cm}^2$

(d) Area trapezium $= \frac{1}{2}(a + b) \times h$

$= \frac{1}{2}(7 + 11) \times 5$

$= 45\text{cm}^2$

7.4 Find the area of the shaded region between two concentric circles as shown in the diagram below.



Area large circle $= \pi r^2$

$= \pi(10)^2$

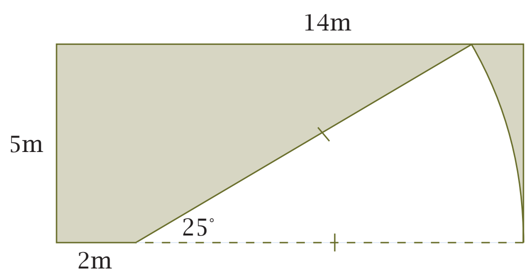
Area small circle $= \pi r^2$

$= \pi(5)^2$

Area shaded region $= \pi(10)^2 - \pi(5)^2$

$= 235.62\text{cm}^2$

7.5 Find the area of the shaded region below.



Area rectangle $= \ell \times w$

$= 14 \times 5$

$= 70\text{m}^2$

Area sector $= \dfrac{\theta}{360} \times \pi r^2$

$= \dfrac{25}{360} \times \pi(12)^2$

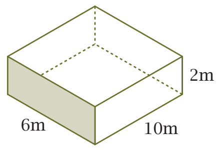
$= 31.42\text{m}^2$

Area shaded region $= 70 - 31.42$

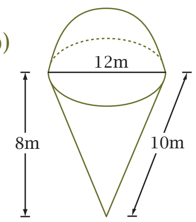
$= 38.58\text{m}^2$

7.6 Calculate correct to 2 decimal places the volume and surface area of the following shapes.

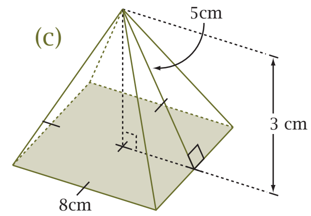
(a)



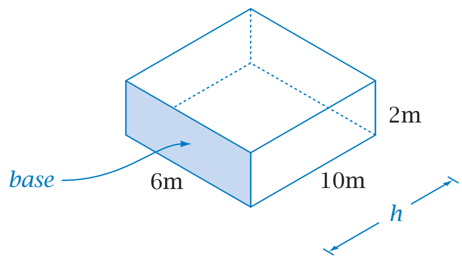
(b)



(c)



(a)



Volume $= \text{area base} \times \text{height}$

$= (6 \times 2) \times 10$

$= 120\text{m}^3$

Surface area

top/bottom $= 2(6 \times 10)$

$= 120\text{m}^2$

front/back $= 2(6 \times 2)$

$= 24\text{m}^2$

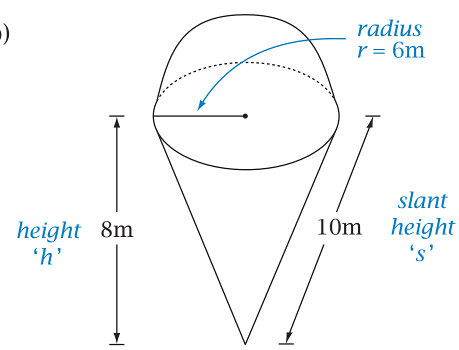
side/side $= 2(10 \times 2)$

$= 40\text{m}^2$

Total surface area $= 120 + 24 + 40$

$= 184\text{m}^2$

(b)



Volume sphere $= \dfrac{4}{3}\pi r^3$

Volume hemisphere $= \dfrac{2}{3}\pi r^3$

Volume $=$ Cone + Hemisphere

$$= \frac{\pi r^2 h}{3} + \frac{2}{3}\pi r^3$$

$$= \frac{\pi(6)^2(8)}{3} + \frac{2}{3}\pi(6)^3$$

$= 753.98\text{m}^3$

Surface Area

Cone $= \underbrace{\pi r^2}_{\text{top}} + \underbrace{\pi rs}_{\text{curved surface}}$

Sphere $= 4\pi r^2$

Hemisphere $= 2\pi r^2$

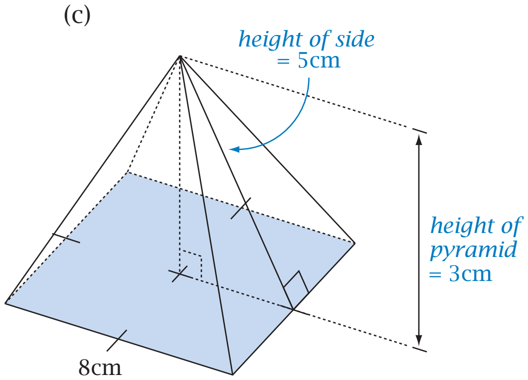
SA $=$ Cone (no top) + hemisphere (no base)

$= \pi rs + 2\pi r^2$

$= \pi(6)(10) + 2\pi(6)^2$

$= 414.69\text{m}^2$

(c)



Volume pyramid $= \dfrac{\text{area base} \times \text{height}}{3}$

$= \dfrac{(8 \times 8) \times 3}{3}$

$= 64\text{cm}^3$

SA pyramid Area Base $= 8 \times 8$

$= 64\text{cm}^2$

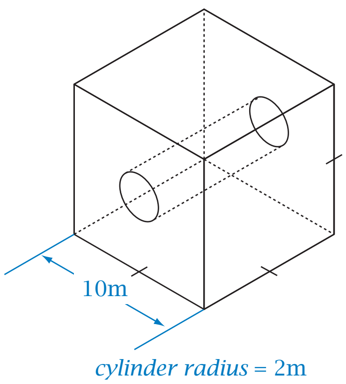
Area 4 sides $= 4 \times (\frac{1}{2} \times 8 \times 5)$ area of triangular side

$= 80\text{cm}^2$

Total surface area $= 64 + 80$

$= 144\text{cm}^2$

7.7 Find the volume and surface area correct to 2 decimal places for the shape below.



Volume = Cube – Cylinder

$= (\text{Area base}) \times \text{height} - \pi r^2 h$

$= (10 \times 10) \times 10 - \pi(2)^2\,(10)$

$= 874.34\text{m}^3$

SA = Cube – 2 circles + curved surface cylinder

$= (\text{Area square} \times 6) - 2\pi r^2 + 2\pi r h$

$= (10 \times 10 \times 6) - 2\pi(2)^2 + 2\pi(2)(10)$

$= 700.53\text{m}^2$

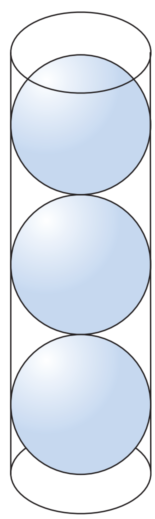
7.8 3 tennis balls each with a radius of 2cm are stored in a cylindrical tin as shown in the diagram below.

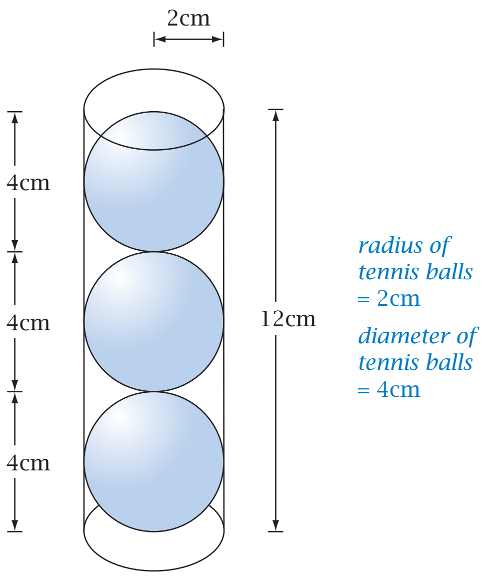
(a) Calculate the volume of all 3 tennis balls.

(b) Calculate the volume of the cylindrical tin.

(c) Calculate the amount of unused space inside the cylindrical tin.

(d) If the material to make the tin costs $\$1.25/\text{cm}^2$, calculate the cost of the cylindrical tin.





(a) Volume (tennis balls)

$$= 3 \times \left(\frac{4}{3}\pi r^3\right)$$

$$= 3 \times \left(\frac{4}{3}\pi (2)^3\right)$$

$$= 100.53\text{cm}^3$$

(b) Volume (tin)

$$= \pi r^2 h$$

$$= \pi(2)^2(12)$$

$$= 150.80\text{cm}^3$$

(c) Volume (unused space)

$$= \text{Volume (tin)} - \text{Volume (balls)}$$

$$= 150.80 - 100.53$$

$$= 50.27\text{cm}^3$$

(d) Surface area (tin)

$$= 2\pi r^2 + 2\pi rh$$

$$= 2\pi(2)^2 + 2\pi(2)(12)$$

$$= 175.93\text{cm}^2$$

Cost: 175.93 × \$1.25 = \$219.91

7.9 Calculate the radius of a sphere if the volume is 219.27cm^3.

Note: This is an inverse problem.

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$219.27 = \frac{4}{3}\pi r^3$$

$$\frac{219.27}{\left(\frac{4}{3}\pi\right)} = r^3$$

Find the cube root of r

$$\sqrt[3]{\frac{219.27}{\left(\frac{4}{3}\pi\right)}} = r$$

$$r = 3.74$$

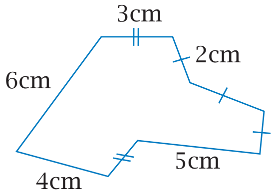
Radius is 3.74cm.

PROBLEMS TO SOLVE

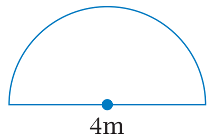
Chapter 7: Perimeter, Area, Surface Area & Volume

1. Find the perimeter of each of the following giving answers correct to 2 decimal places, where necessary.

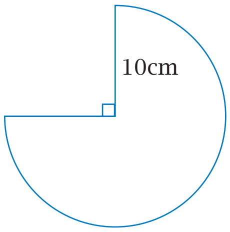
(a)



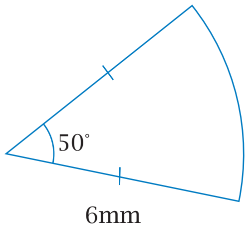
(b)



(c)

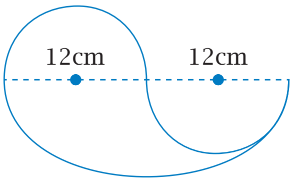


(d)

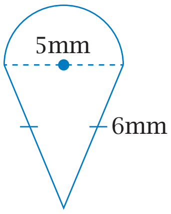


2. Find the perimeter of the following composite shapes expressing answers to 1 decimal place.

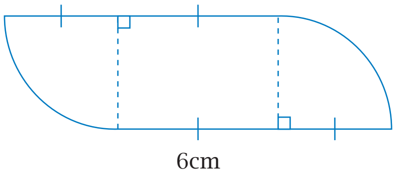
(a)



(b)

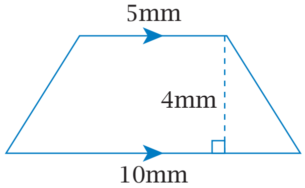


(c)

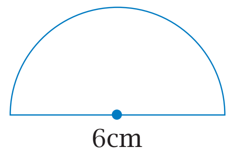


3. Calculate the areas of each of the following, giving answers correct to 2 decimal places.

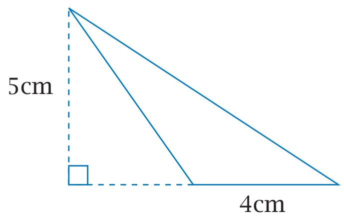
(a)



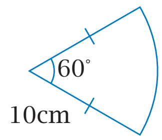
(b)



(c)

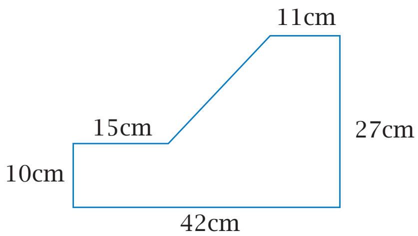


(d)

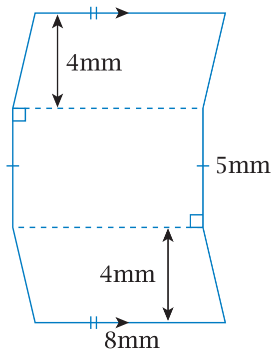


4. Find the area of the following:

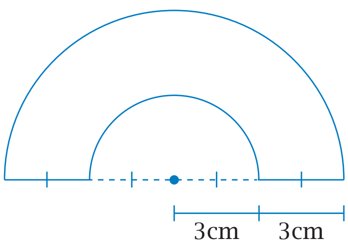
(a)



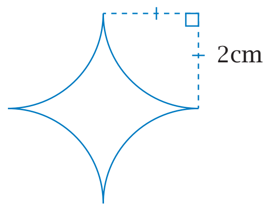
(b)



(c)

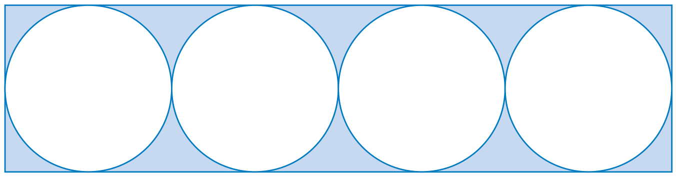


(d)

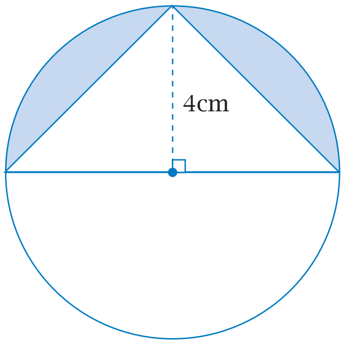


5. Find the shaded area for each of the following:

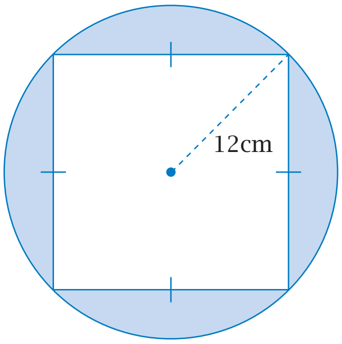
(a)



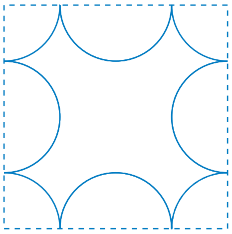
(b)



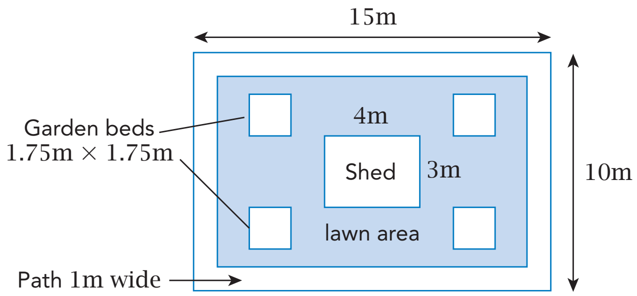
(c)



6. A piece of jewellery (shown below) of side 4cm has four semi-circles and four quarter circles, each of radius 1cm, removed from a square. What is the perimeter of the piece of jewellery?



7. Part of a backyard consisting of a shed, four garden beds, a lawn area and a surrounding path is shown in the diagram below:



Calculate:

(a) the area of the path

(b) the shaded area

(c) what fraction of the backyard is not covered by lawn

(d) the cost of the lawn area if lawn costs \$12.50 per m^2.

8. A floor measuring 27m × 20m is to be tiled. The tiles used are rectangular and measure 1.5m × 1m and cost $5.50 per tile.

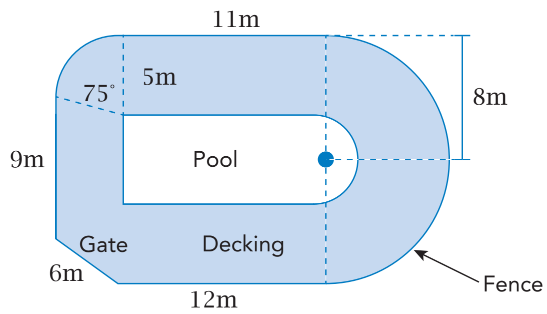
Determine:

(a) the area of the floor

(b) the number of tiles required

(c) the total cost of tiling the floor.

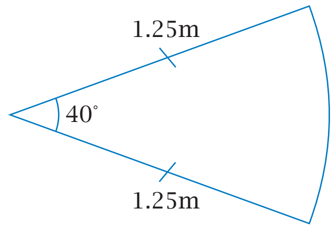
9. John has installed a pool in his backyard surrounded by decking. Fencing and a child proof gate are to be installed around the decking as shown in the diagram below.



(a) Calculate the length of fencing required (exclude gate).

(b) Determine the cost if fencing is $80/m and gates are $12/m.

10. A new discus pitch is to be installed on a school oval as shown below.



The surface is to be covered in non-slip tiles each measuring 4cm × 6cm. An extra 10% more tiles will be needed to cover breakage and for areas requiring cut tiles.

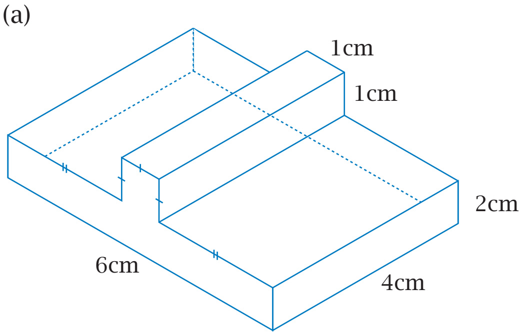
Calculate:

(a) the area of the pitch

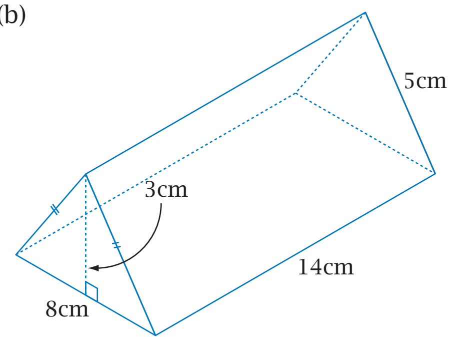
(b) the number of tiles required for the pitch.

11. Calculate the volume and surface area of each of the following solids. Answers correct to 2 decimal places where appropriate.

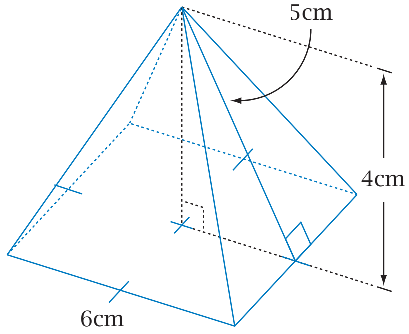
(a)



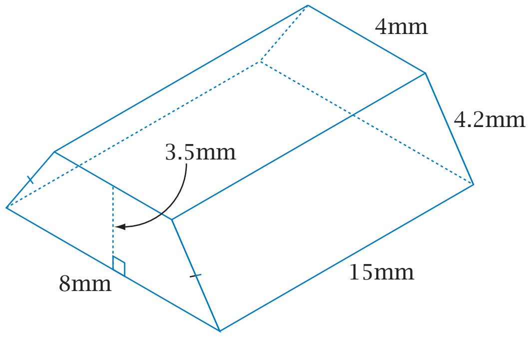
(b)



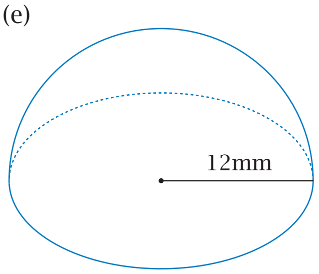
(c)



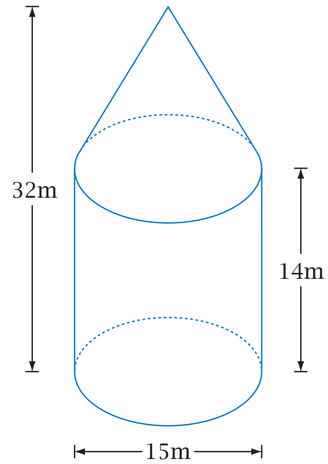
(d)



(e)



(f)

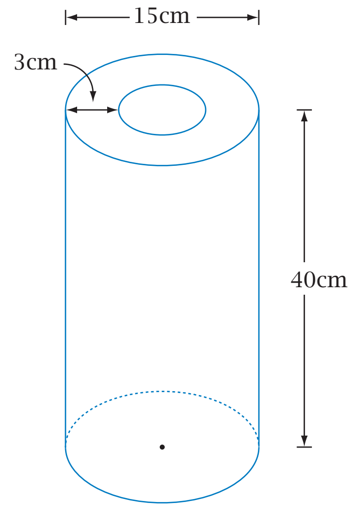


12. A cylindrical pipe of length 40cm has an outer diameter of 15cm and a thickness of 3cm. Find:

(a) The outer curved surface area

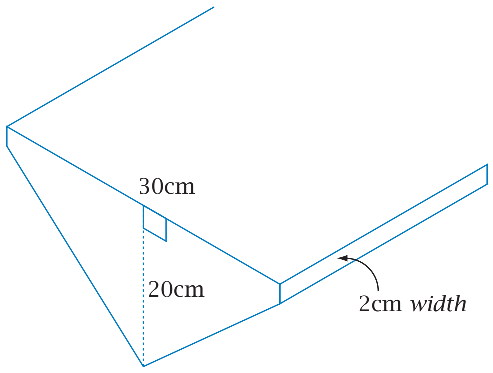
(b) The inner curved surface area

(c) The volume of the pipe.



13. A cylindrical can of baked beans has a base diameter of 14cm and a total surface area of $900cm^2$. Find it's height.

14. A horizontal drinking trough on a cattle station is 3m long. Determine the amount of water in cm^3 it will hold when full.

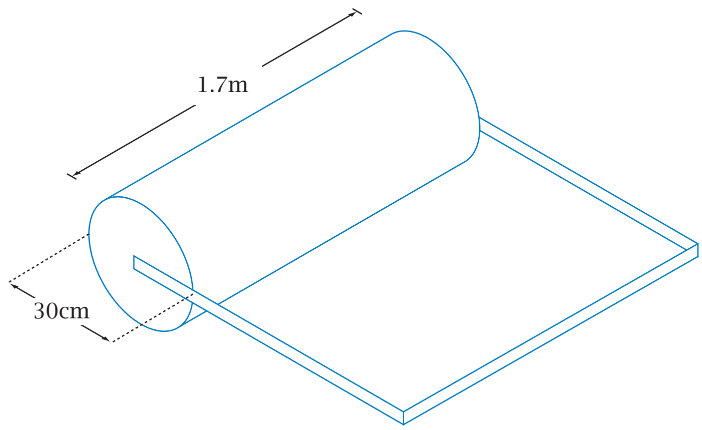


15. A letterbox in the shape of a tetrahedron is 10cm high. If the base triangle measures 6cm with a height of 5cm, determine the volume of the letterbox.

16. Before each lawn bowls regatta, the 'green' must be prepared by rolling the lawn with a cylindrical roller. Determine:

(a) The curved surface of the roller

(b) How many revolutions are required to roll the lawn if the green measures 170m by 50m?



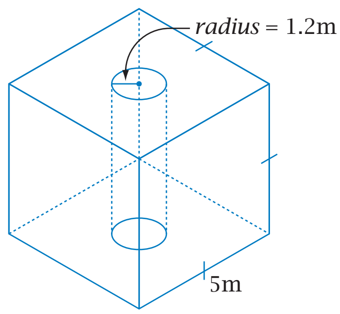
17. The height of a cylinder is 15mm and the curved surface area is 1220mm^2. Determine the base area of the cylinder.

18. A company makes playground equipment out of wood in the following designs. Calculate:

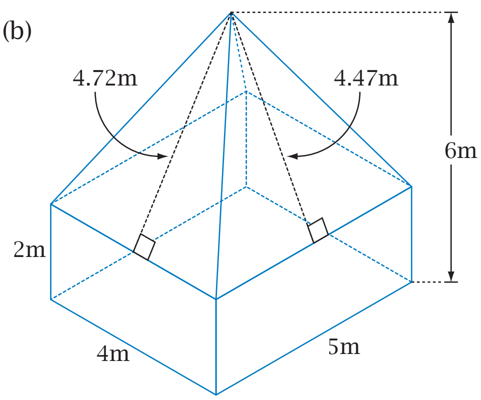
(i) The volume of the shape in m^3

(ii) The cost of painting each piece of equipment if paint costs \$6.50 per square metre.

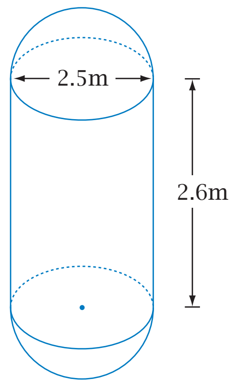
(a)



(b)



(c)

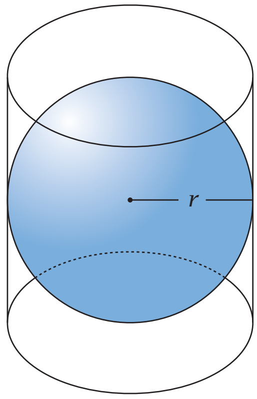


19. (a) Wigwam tents are constructed in the shape of a cone with height 4m and diameter 6 m. If the material to make the tent cost \$9.60 per square metre, determine the cost of making the tent (including the base).

(b) Teepee tents are made of the same material but have a diameter of 8m. If the material costs the same as Wigwam tents and in total the cost of making the Teepee tent is \$1085.73, determine the height of the tent.

20. A rectangular tank 4m long, 2m wide and 1m deep is filled with water to a depth of $\frac{3}{4}$m. How many bricks measuring 20cm × 12.5cm × 10cm can be put into the tank before the water overflows?

21. Single tennis balls are sold in cylindrical tubes. The tennis ball is stored so as to touch all sides of the tube as shown in the diagram below. If the empty space in the cylindrical tube is $18\pi\text{cm}^3$ find the radius 'r' of the tennis ball.

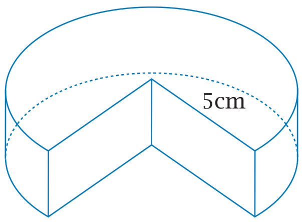


22. A piece of cheese with radius 5cm and thickness 4cm stands on a shop counter. A slice representing $\frac{1}{5}$ of the shape is removed by cutting vertically downwards.

Determine:

(a) The volume and

(b) The total surface area of the remaining piece of cheese.



23. A rectangular fish tank is built in the backyard measuring 3m by 1.5m and 1.2m high.

(a) Black plastic must line the walls and floor of the fish pond. If it costs $4.25 per square metre, how much will it cost for the black plastic?

(b) Water is added at a constant rate of 0.2m^3 per minute. How deep will the water be after 9 minutes?

24. Lead spheres of diameter 10cm are melted down to form spherical fishing weights of radius 0.5cm. How many spherical fishing weights are formed from 1 lead sphere?

8 LINEAR RELATIONSHIPS

Syllabus Checklist

By the end of this chapter, you should be able to:

- ☐ identify and solve linear equations
- ☐ develop a linear formula from a word description and solve the resulting equation
- ☐ construct straight line graphs
- ☐ determine the slope and intercepts of a straight line graph from both its equation and its plot
- ☐ construct and analyse a straight line graph
- ☐ interpret, in context, the slope and intercept of a straight line graph
- ☐ solve a pair of simultaneous linear equations graphically or algebraically
- ☐ solve practical problems that involve determining the point of intersection of two straight line graphs
- ☐ sketch piece-wise linear graphs and step graphs
- ☐ interpret piece-wise linear and step graphs used to model practical situations

FORMULAE AND DEFINITIONS

General equation of a straight line

General form : $y = mx + c$

m = gradient $\frac{rise}{run}$ *vertical Intercept*

x	1	2	3	4	5
y	5	8	11	14	17

3 3 3 3 *1st difference pattern*

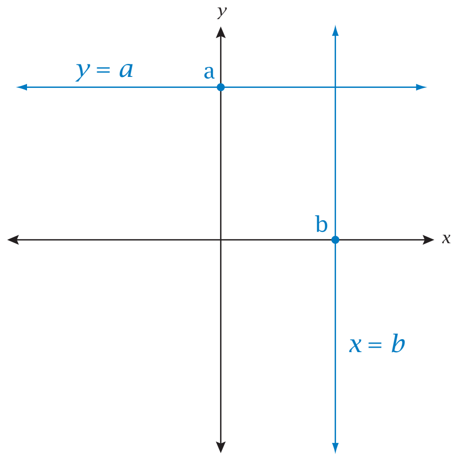
A constant 1st difference pattern indicates the relationship is linear.
The 1st difference pattern is the **gradient** of the linear function.

Line parallel to the x axis

Equation: $y = a$

Line parallel to the y axis

Equation: $x = b$



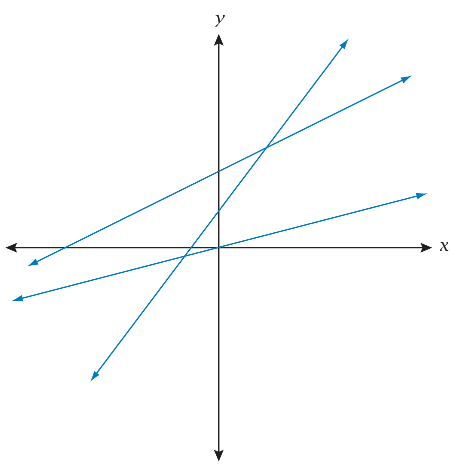
Gradient of a line given two points

$$A\ (x_1, y_1) \quad B\ (x_2, y_2) \qquad \textbf{gradient}:\ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run}$$

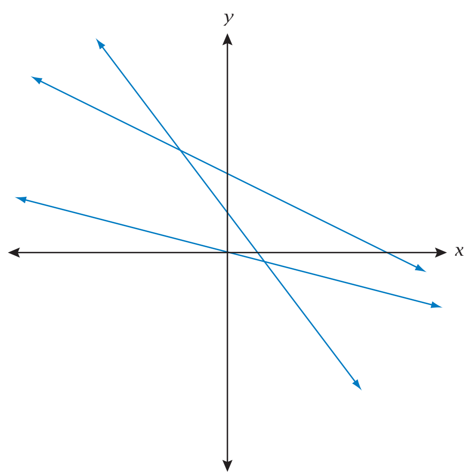
Types of gradients

Gradients can be positive, negative, zero or undefined.
Examples of lines with these gradients are drawn below:

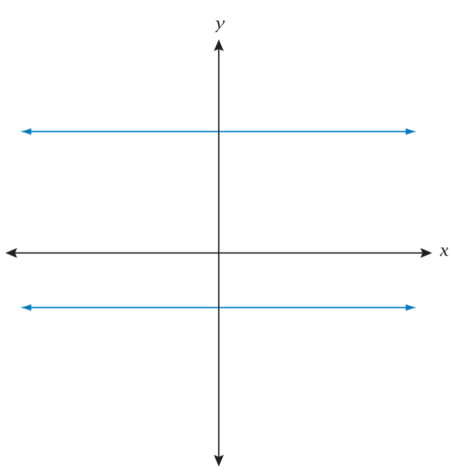
Positive gradients



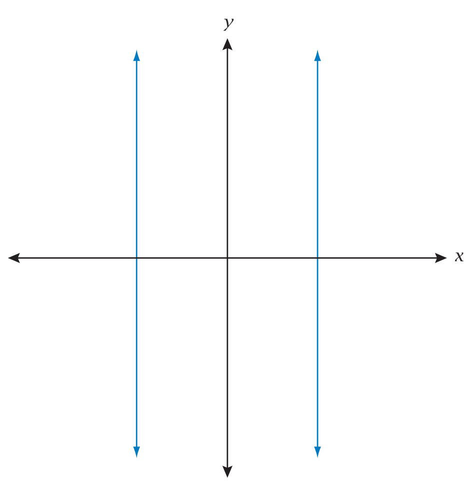
Negative gradients



Zero gradients



Undefined gradients



Intercepts

x intercept is found when $y = 0$

y intercept is found when $x = 0$

Distance or length formula between two points

$A\ (x_1, y_1) \quad B\ (x_2, y_2)$

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Simultaneous equations

To find the intersection of equations either algebraically or graphically.

Direct proportion

If two quantities are directly proportional to one another then the graph of one against the other is a straight line through the origin, so if one doubles, the other doubles.

The quantities are also in the **same ratio**.

The general rule is $y = kx$ where k is a constant.

Piece-wise linear function

A Piece-wise linear graph is comprised of several line segments.

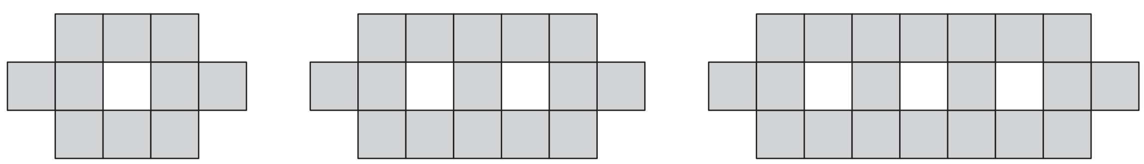
Step graphs

A step graph is comprised of several horizontal line segments.

In both piece-wise and step graphs a closed circle includes the point and an open circle indicates the point is not included.

Worked Examples

8.1 The diagrams below are made up of black and white squares.



(a) Complete all entries in the table below.

Number of white squares w	1	2	3	4	5
Number of black squares b					

(b) Describe the pattern in the number of black squares.

(c) Find a rule that calculates the number of black squares needed from the given number of white squares.

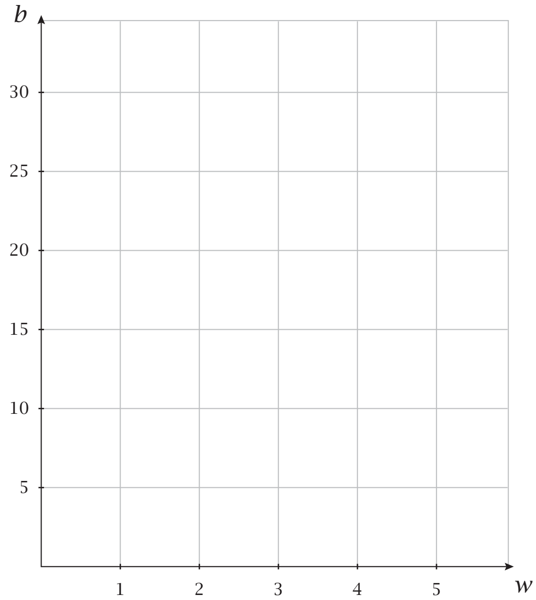
(d) Find the number of black squares if there are 12 white squares.

(e) Determine the number of white squares if there are 95 black squares.

(f) On the axes (right) draw a graph to represent the information in the table.

(g) What do you notice about the points?

(h) Does it make sense to join the points? Why?



(a)

Number of white squares w	1	2	3	4	5
Number of black squares b	10	15	20	25	30

5 5 5 5

(b) The number of black squares is increasing by 5 each time.

(c) 1st difference pattern is 5
$\therefore$ linear

b = diff. pattern x w + constant

$\boldsymbol{b = 5w + \text{constant}}$

If $w = 1$ then $b = 10$ from table

i.e. $b = 5(1) + \text{constant}$

$10 = 5 + \text{constant}$

$\therefore$ constant $= 5$

$\therefore$ $\boldsymbol{b = 5w + 5}$

(d) $b = 5w + 5$

$b = 5(12) + 5$

$b = 65$

$\therefore$ 65 black squares

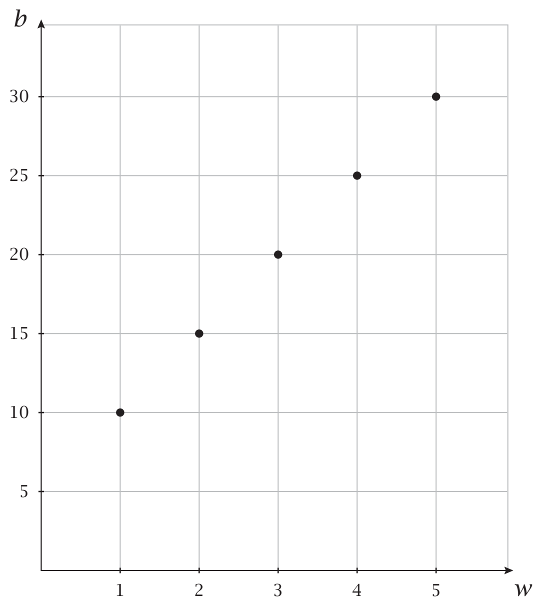
(e) $b = 5w + 5$

$95 = 5w + 5$

$w = 18$

$\therefore$ 18 white squares

(f)



(g) They are in a straight line.

(h) No, since the values deal with whole numbers, i.e. the number of white and black squares. There are **no** part squares.

8.2 A plumber charges a call out fee of \$110 plus an hourly fee of \$95.

(a) Determine a rule which connects the total cost, c, with the number of hours worked, h.

(b) If the plumber works for $3\frac{1}{2}$ hours, determine the total cost of hiring the plumber.

(c) If the plumber charges \$703.75 to fix a blocked drain, how long did it take him to fix it?

(a) $c = \underbrace{95}_{\textit{hourly rate}}h + \underbrace{110}_{\textit{fixed cost}}$

(b) $c = 95(3\frac{1}{2}) + 110$

$c = \$442.50$

Cost is \$442.50

(c) $c = 95h + 110$

$703.75 = 95h + 110$

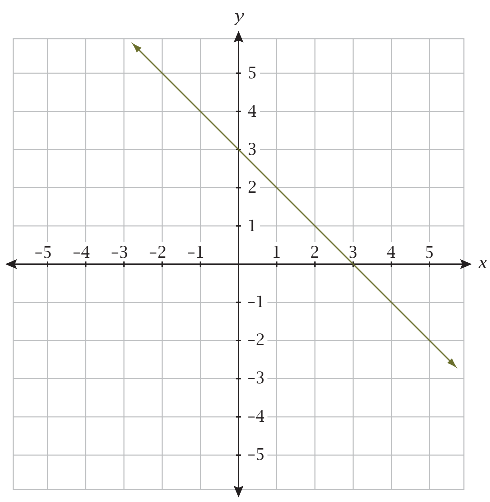
$593.75 = 95h$

$h = \dfrac{593.75}{95}$

$h = 6.25$

Plumber works for $6\frac{1}{4}$ hours.

8.3 The graph of $y = -x + 3$ is drawn below:



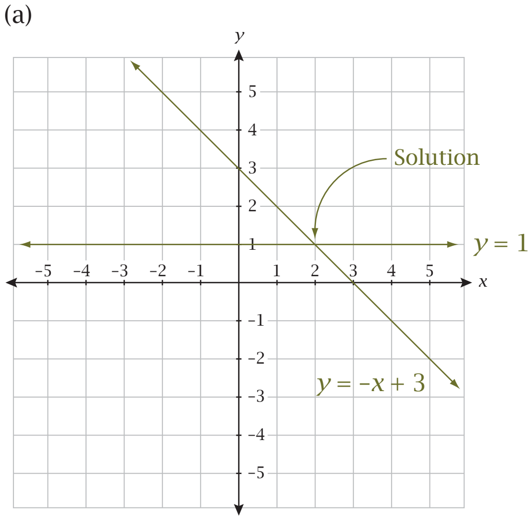
Use the graph to solve correct to 1 decimal place where necessary, the following:

(a) $-x + 3 = 1$

(b) $-x + 3 = x$

(c) $2(-x + 3) = 0$

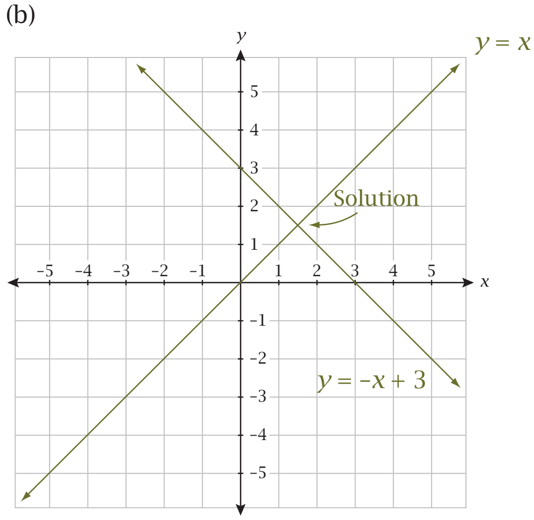
(a)



$\underbrace{-x + 3}_{y = -x + 3} = \underbrace{1}_{y = 1}$

$\therefore x = 2$

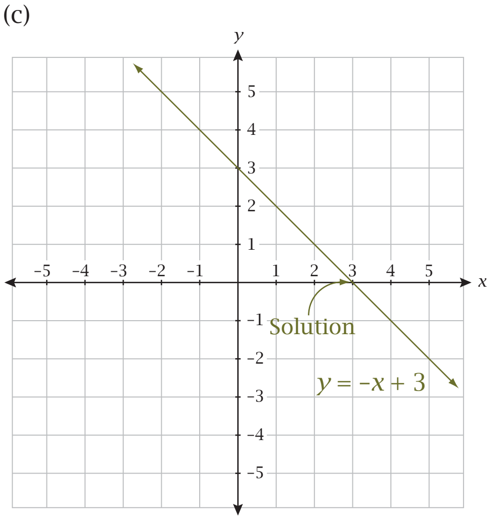
(b)



$\underbrace{-x + 3}_{y = -x + 3} = \underbrace{x}_{y = x}$

$\therefore x = 1.5$

(c)



$$2(-x + 3) = 0$$

$$-x + 3 = \frac{0}{2}$$

$$\underbrace{-x + 3}_{y = -x + 3} = \underbrace{0}_{y = 0}$$

$$\therefore x = 3$$

8.4 Solve the following simultaneous equations:

(a) graphically

$y = 3x - 2$

$2x - y = 1$

(b) algebraically

$y = 3x - 2$

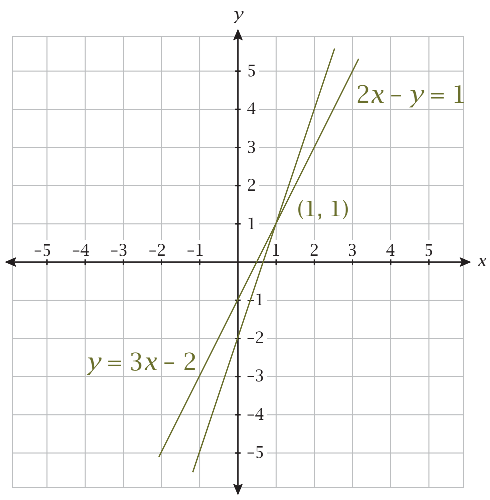
$2x - y = 1$

(c) algebraically

$4x + 5y = 9$

$6x - 2y = 4$

(a) graphically



(b) $y = 3x - 2$ ①

$2x - y = 1$ ②

Sub eqn ① into eqn ②

i.e.

$$\begin{aligned} 2x - y &= 1 \\ 2x - (3x - 2) &= 1 \\ 2x - 3x + 2 &= 1 \\ -x + 2 &= 1 \\ x &= 1 \end{aligned}$$

Sub $x = 1$ into eqn ①

$$\begin{aligned} y &= 3x - 2 \\ y &= 3(1) - 2 \\ y &= 1 \end{aligned}$$

Point of intersection (1, 1)

(c)

$$\begin{aligned} 4x + 5y &= 9 \quad ① \\ 6x - 2y &= 4 \quad ② \end{aligned}$$

$$\begin{aligned} 8x + 10y &= 18 \quad ① \times 2 \\ 30x - 10y &= 20 \quad ② \times 5 \end{aligned}$$

Add both Equations

$38x = 38$

$x = 1$

Sub x into 1

$4(1) + 5y = 9$

$4 + 5y = 9$

$5y = 5$

$y = 1$

Point of intersection (1, 1)

Remember

** Signs are different*
– add both equations

** Signs are the same*
– subtract both equations

8.5 For the following problem:

(i) define the variables
(ii) set up a pair of simultaneous equations
(iii) solve the simultaneous equations
(iv) answer the problem

One glass and two cups cost a total of \$8 whereas two glasses and one cup cost \$7. Find the cost of each item.

Let g be the cost of each glass.

Let c be the cost of each cup.

$g + 2c = 8$ 1

$2g + c = 7$ 2

$2g + 4c = 16$ 1 $\times 2$

$2g + c = 7$ 2

$3c = 9$

$c = 3$

Sub $g + 2(3) = 8$

$g + 6 = 8$

$g = 2$

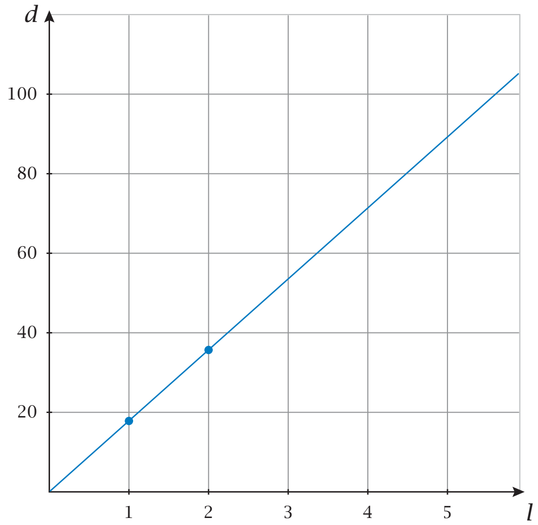
Glasses cost \$2 each and cups cost \$3 each.

8.6 The table below shows the distance travelled (d) km and the number of litres of petrol (l) used by a vehicle.

Number of litres (l)	1	2	3	4	5
Distance travelled (d)	18	36	54	72	90

(a) Graph the information given in the table above. What do you notice?
(b) Is the relationship directly proportional? Why?
(c) Determine the rule connecting the distance travelled and the number of litres of petrol used.
(d) If the vehicle travels 126km, how many litres of petrol will be used?

(a)



The graph is a straight line.

(b) Yes it is directly proportional. If one quantity y is multiplied by a number the other quantity is also multiplied by the same number. Corresponding ratios are also the same:

i.e. 1:18 = 2:36 = 3:54

(c) $d = 18l$

(d) Using ratios:

$$\frac{18}{1} = \frac{126}{x} \qquad \frac{\textit{distance}}{\textit{litres}}$$

$$18x = 126$$

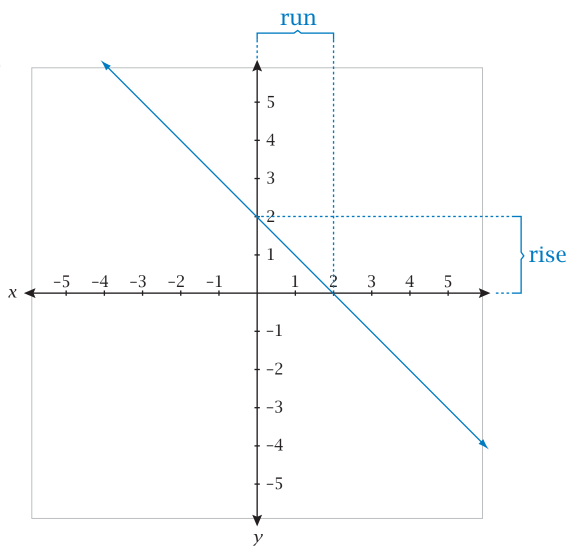
$$x = \frac{126}{18}$$

$$x = 7$$

7 litres of petrol will be used.

8.7 Find the gradient and y intercept for each of the following given:

(a)



$$\text{gradient} = \frac{\textit{rise}}{\textit{run}}$$

$$= -\frac{2}{2}$$

$$= -1$$

y intercept: (0, 2)

(b) The straight line passes through the given points (–1, 4) and (3, –5).

$\underset{}{\overset{x_1, y_1}{(-1, 4)}} \qquad \overset{x_2, y_2}{(3, -5)}$

Use the gradient formula: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{-5 - 4}{3 - (-1)}$$

$$\text{gradient} = -\frac{9}{4}$$

$$y = -\frac{9}{4}x + c$$

Sub in $(-1, 4)$ to find c

$$4 = -\frac{9}{4}(-1) + c$$

$$4 - \frac{9}{4} = c$$

$$\frac{7}{4} = c$$

$\therefore$ y int: $\left(0, \dfrac{7}{4}\right)$

(c) The equation of the line is $y = 4x - 6$
Solution: $y = \underbrace{4}_{\text{gradient: } 4}x \underbrace{- 6}_{y \text{ int } (0,\,-6)}$

gradient: 4 $\quad$ y int: $(0, -6)$

(d) The equation of the line is $2x - 5y = 6$
Solution: Rearrange into the form

$$y = mx + c$$

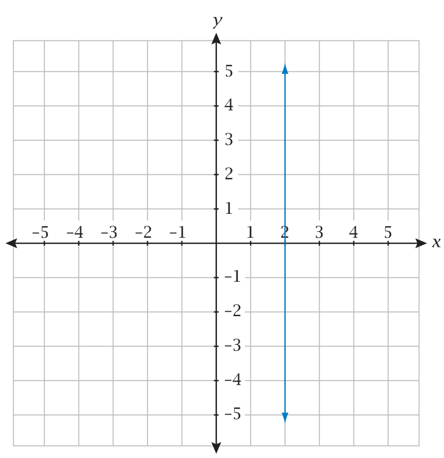
$$2x - 5y = 6$$

$$5y = 2x - 6$$

$$y = \underbrace{\frac{2}{5}}_{\text{gradient}}x - \overbrace{\frac{6}{5}}^{y \text{ intercept}}$$

gradient: $\dfrac{2}{5}$ $\quad$ y int: $\left(0, -\dfrac{6}{5}\right)$

8.8 Find the equation of the line given:

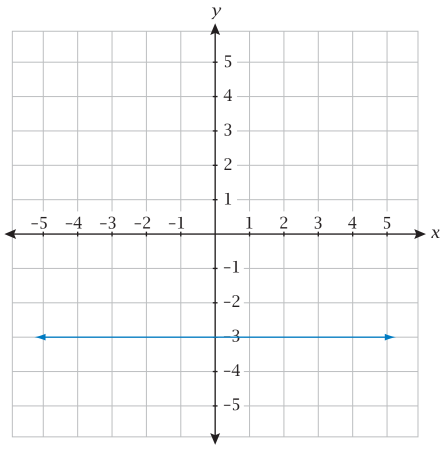


(a) **Solution**:

Lines parallel to the y axis

have equations $x = b$

$x = 2$

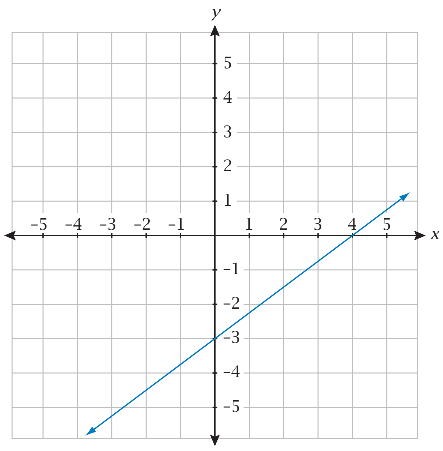


(b) **Solution:**

Lines parallel to the x axis

have equations $y = a$

$y = -3$



(c) **Solution:**

Determine gradient and

y intercept from graph

$$\text{gradient} = \frac{rise}{run}$$
$$= \frac{3}{4}$$

y int: (0, –3)

$$\therefore\ y = \frac{3}{4}x - 3$$

(d) The line has a gradient of $\frac{1}{2}$ and passes through the point (2, –5)

Solution: $y = mx + c$

$$y = \frac{1}{2}x + c \qquad \textit{substitute in gradient}$$

Substitute in (2, –5) to find 'c'

$$-5 = \frac{1}{2}(2) + c$$
$$-5 = 1 + c$$
$$c = -6$$

$\therefore$ Equation of the line is

$$y = \frac{1}{2}x - 6$$

(e) The line passes through the points (1, –5) and (3, –3)

Solution:

Calculate gradient using gradient formula

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \begin{matrix} x_1\ y_1 \\ (1, -5) \end{matrix} \quad \begin{matrix} x_2\ y_2 \\ (3, -3) \end{matrix}$$
$$= \frac{-3 - (-5)}{3 - 1}$$
$$= \frac{2}{2}$$
$$m = 1$$

$$y = mx + c$$
$$y = 1x + c \qquad \textit{substitute in gradient}$$
$$-5 = 1\,(1) + c \qquad \textit{substitute in either of the two given points to find 'c'}$$
$$-6 = c$$

$\therefore$ Equation of the line is

$$y = x - 6$$

8.9 State whether the points A and B below lie on the line $y = -2x + 5$

(a) A (4, –3)

(b) B (–2, 1)

Solution:

(a) If A lies on the line then when the value of x is substituted into the equation the value of y should be obtained.

$$y = -2x + 5$$
$$y = -2(4) + 5$$
$$y = -3$$

Thus the point A (4, –3) **does** lie on the line.

(b) B (–2, 1)

$$y = -2x + 5$$
$$y = -2(-2) + 5$$
$$y = 9$$

Thus the point B (–2, 1) **does not** lie on the line.

8.10 Calculate the distance between the points A (–4, 2) and B (–1, 5)

Solution: $A\ (\overset{x_1}{-4}, \overset{y_1}{2})$ $B\ (\overset{x_2}{-1}, \overset{y_2}{5})$

Distance
$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$= \sqrt{(-4 - (-1))^2 + (2 - 5)^2}$$
$$= \sqrt{(-4 + 1)^2 + (-3)^2}$$
$$= \sqrt{(-3)^2 + (-3)^2}$$
$$= \sqrt{9 + 9}$$
$$= \sqrt{18}$$
$$= 4.24$$

8.11 Determine if ΔABC is isosceles given A (2, –3), B (4, 1) and C (–5, 3)

Solution:
Calculate the distances AB, BC and AC.

$$AB = \sqrt{(2 - 4)^2 + (-3 - 1)^2}$$
$$= \sqrt{(-2)^2 + (-4)^2}$$
$$= \sqrt{4 + 16}$$
$$= \sqrt{20}$$

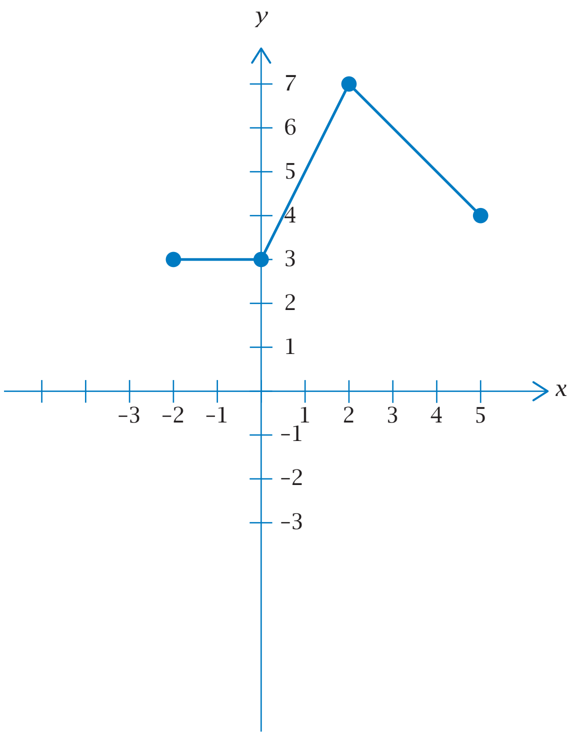
$$AC = \sqrt{(2 + 5)^2 + (-3 - 3)^2}$$
$$= \sqrt{7^2 + (-6)^2}$$
$$= \sqrt{49 + 36}$$
$$= \sqrt{85}$$

$$
\begin{aligned}
BC &= \sqrt{(4+5)^2 + (1-3)^2} \\
&= \sqrt{9^2 + (-2)^2} \\
&= \sqrt{81+4} \\
&= \sqrt{85}
\end{aligned}
$$

As the two sides AC and BC have the same length ΔABC is isosceles.

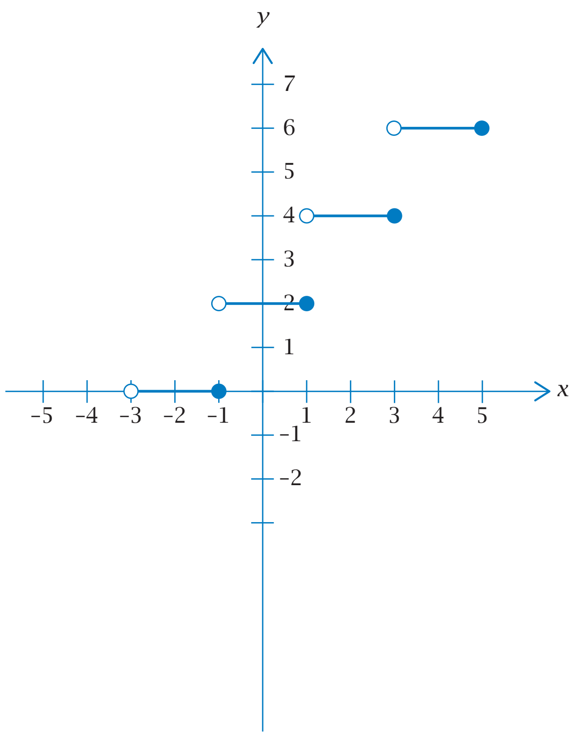
8.12 Draw the following piece-wise function:

$$
y = \begin{cases} 3, & -2 \le x < 0 \\ 2x+3, & 0 \le x < 2 \\ 9-x, & 2 \le x \le 5 \end{cases}
$$



8.13 Graph the following function:

$$
y = \begin{cases} 0, & -3 < x \le -1 \\ 2, & -1 < x \le 1 \\ 4, & 1 < x \le 3 \\ 6, & 3 < x \le 5 \end{cases}
$$



8.14 The following piece-wise function describes the path of an object where d = distance travelled in metres and t = time in seconds.

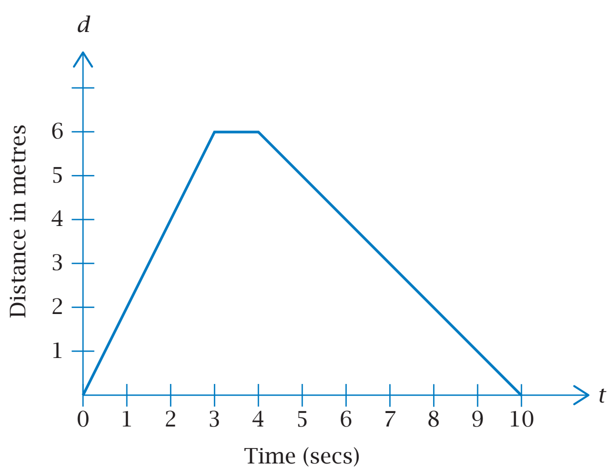
$$d = \begin{cases} 2t, & 0 \le t < 3 \\ 6, & 3 \le t < 4 \\ 10 - t, & 4 \le t \le 10 \end{cases}$$

(a) Graph the piece-wise function

(b) Describe what is happening between t = 3 and t = 4

(c) Calculate the average speed for t = 0 to t = 3

(a)



(b) The object is not moving between t = 3 and t = 4 secs

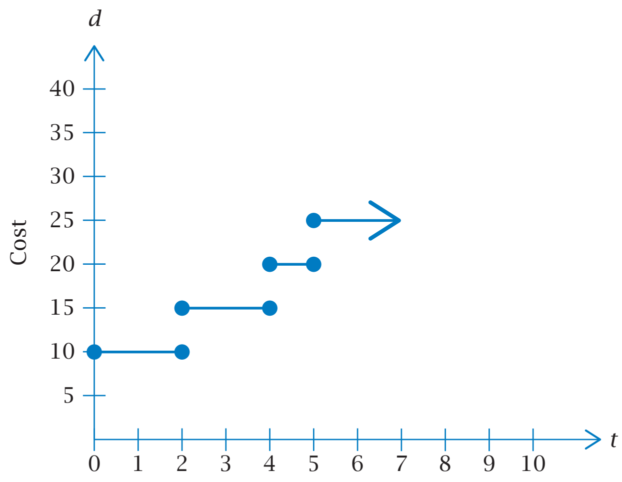
(c) Average speed $= \dfrac{\text{distance}}{\text{time}}$

$= \dfrac{6}{3}$

$= 2$

∴ Average speed is 2m/s

8.15 Parking rates at a local airport can be represented by the following step graph.



(a) Calculate the cost of parking for

(i) 1 hour

(ii) 2 hours

(iii) 4.5 hours

(iv) 7 hours

(b) What does the arrow mean?

(a) (i) \$10

(ii) \$10

(iii) \$20

(iv) \$25

(b) The arrow indicates the cost of parking remains at a constant \$25 when parking for longer than 5 hours.

PROBLEMS TO SOLVE

Chapter 8: Linear Relationships

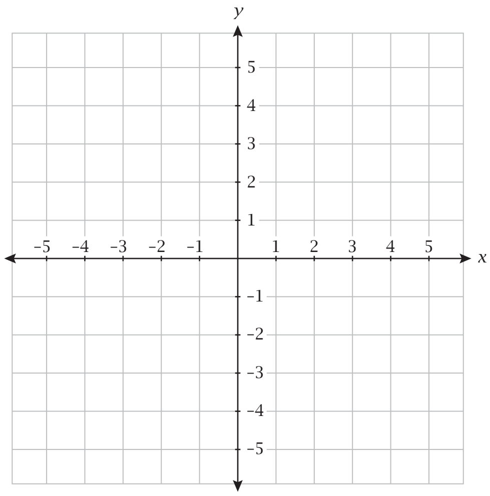
1. On the grid below, plot the following coordinate points and join them together in order

   (0, 0), (–1, 3), (–2, 3), (–1, 0),

   (–1, –1), (–2, –3), (–1, –3),

   (3, 3), (2, 3), (0, 0)

   What did you notice?



2. The following diagrams are made from matchsticks.

(a) How many matchsticks are needed for the 4th diagram?

(b) How many matchsticks will the 5th diagram need?

(c) Describe the pattern in the number of matchsticks needed.

(d) Complete the table of values below.

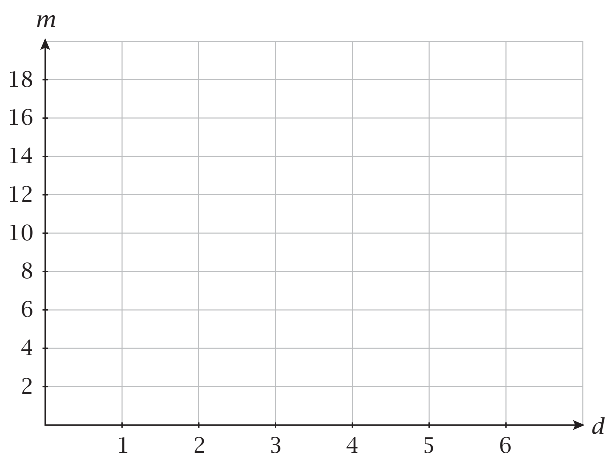
Diagram number (d)	1	2	3	4	5	6
Number of matchsticks (m)	3					

(e) Find a rule that calculates the number of matches needed for any diagram number.

(f) Find the number of matches needed for the 25th diagram.

(g) If 63 matches are used, determine the diagram number.

(h) On the axes below draw a graph to represent the information in the table.



(i) What do you notice about the points?

(j) Does it make sense to join the points? Why?

3. Determine which of the following are linear.

(a) $y = \frac{x}{3} - 4$

(b) $y = 2^x + 5$

(c) $4x + 3y = 9$

(d) $y = \frac{7}{x} - 6$

4. Complete the table of values for each rule below:

(a) $y = 4x - 2$

x	0	1	2	3	6	38	
y							106

(b) $q = \frac{p}{2} + 5$

p	3	4	5	6	20		
q						33	64.5

5. For each of the following:

(i) find the gradient of the line.

(ii) find the vertical intercept.

(iii) graph the line using the gradient and vertical intercept.

(a) $y = -2x + 5$

(b) $3x + 6y = -12$

(c) $2x - 5y - 10 = 0$

(d) $y = -3$

6. For each of the patterns below

(i) use the first difference pattern to show that there is a linear relationship between the variables.

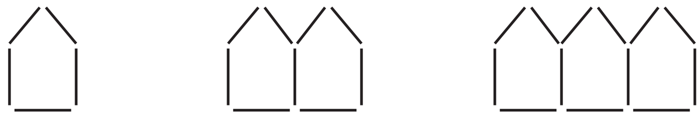
(ii) determine the rule for the linear relationship.

(iii) complete the entries in the table.

(a)

Number of squares (s)	1	2	3	4	12	
Number of matchsticks (m)	4	7	10			109

(b)



Number of houses (h)	1	2	3	4	10	
Number of matchsticks (m)						49

7. For the following tables determine:

(i) if the relationship between x and y is linear.

(ii) for those which are linear the equation connecting x and y.

(a)

x	0	1	2	3	4	5
y	2	5	8	11	14	17

(b)

x	1	2	3	4	5
y	2	3	5	8	12

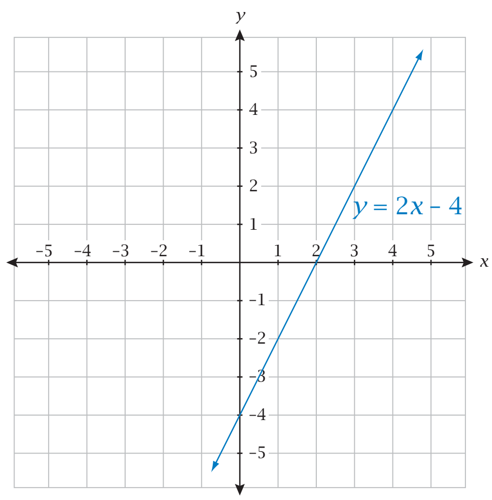
(c)

x	5	6	7	8	9
y	12	10	8	6	4

(d)

p	1	3	5	7	9
q	7	17	27	37	47

8. The graph of $y = 2x - 4$ is drawn below.



Use the graph to solve the following correct to 1 decimal place where necessary.

(a) $2x - 4 = 2$

(b) $2x - 4 = 0$

(c) $\dfrac{2x - 4}{x} = 1$

(d) $x - 2 = 2$

9. Solve the following simultaneous equations algebraically:

(a) $3p + q = 6$

$9p + 2q = 1$

(b) $2m - 5n = 16$

$6m + 2n = -3$

(c) $2a - 3b - 8 = 0$

$5a + 4b - 43 = 0$

(d) $2(5x + 4y) = 4$

$6x + 5y = 3$

10. For the following problems:

(i) define the variables

(ii) set up a pair of simultaneous equations

(iii) solve the simultaneous equations

(iv) answer the question

(a) The length of a rectangle is 6cm longer than the width.
If the perimeter is 100cm, find the dimensions of the rectangle.

(b) Five times the larger number added to twice the smaller is 60 while the larger exceeds the smaller by 12. Find the two numbers.

(c) Three drinks and two chocolates cost \$1.75, while six drinks and three chocolates cost \$2.85. Find the individual cost of a drink and a chocolate.

(d) At a fun fair, each mother had brought 2 children along. At the end of the day, it was found that 38 mothers had lost one or both of their children and 54 children had lost their mothers. How many mothers had lost only one of their children and how many mothers had lost both of their children?

11. A phone company charges a connection fee of \$2 plus 0.20 cents for every minute used.

(a) Determine a rule to calculate the cost (c, \$) of using the phone over a given period of time (m, minutes).

(b) How much would it cost to make a call lasting 7 minutes?

(c) If Mason was charged \$10 for a phone call, how long was his phone call?

12. The owner of a parts manufacturing company for cars believes that a linear relationship exists between the profit, \$$P$, and the number of parts sold 's'.

If 200 parts are sold the profit is \$450, while a \$1700 profit is obtained when 700 parts are sold.

(a) Determine the profit equation in the form of \$$P = ms + c$
where 'm' and 'c' are constants.

(b) Use the equation to determine the profit if 1000 parts are sold.

(c) If only 10 parts are sold, determine the profit and interpret your result.

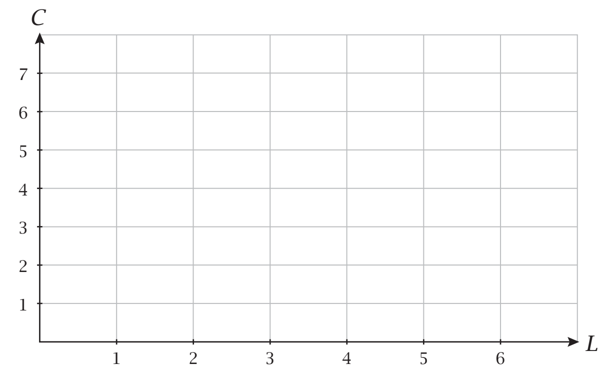
(d) What is the minimum number of parts that need to be sold in order to make a profit?

13. The table below shows the total cost C ($) and the number of litres, L, of petrol.

Number of litres (L)	1	2	3	4	5
Cost C ($)	$1.50	$3.00	$4.50	$6.00	$7.50

(a) Determine the cost of 7 litres of petrol.

(b) Graph the information in the above table on the axes shown below.



(c) What do you notice about the graph of cost versus the number of litres?

(d) Does it appear that the relationship is directly proportional? Why?

(e) Determine the rule connecting the cost with the number of litres.

(f) Given 10 litres of petrol cost $15.00, determine using:

(i) the rule in part (e) the cost of 37.5 litres

(ii) corresponding ratios the cost of 37.5 litres

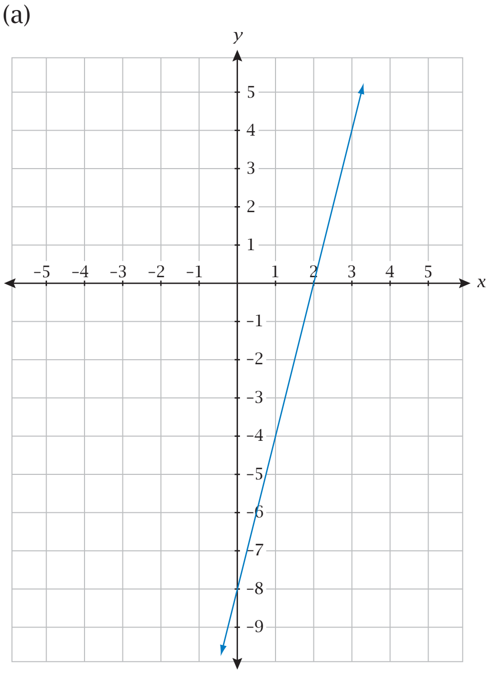
What do you notice about your answers to part (i) and (ii)?

14. Find the:

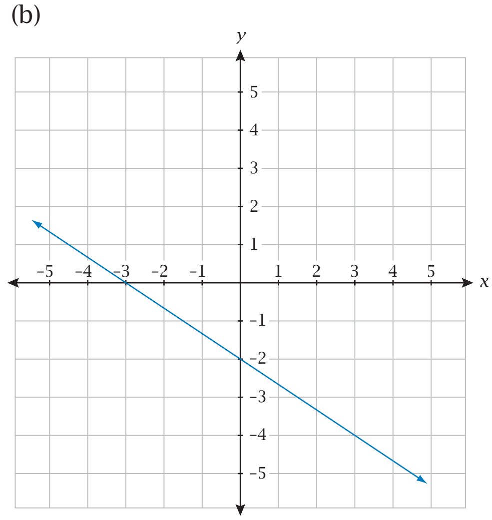
(i) gradient (ii) y intercept

for each of the following lines:

(a)



(b)



15. Find the gradient of the line joining coordinates A and B if:

(a) $A\ (1, 1)$ $B\ (3, 7)$

(b) $A\ (0, 6)$ $B\ (-1, -2)$

(c) $A\ (2, -4)$ $B\ (-3, -1)$

(d) $A\ (4, 6)$ $B\ (0, -4)$

16. Find the gradient and y intercept of the lines with equations:

(a) $y = -2x + 5$

(b) $3y = x - 4$

(c) $2x - y = 9$

(d) $x + 4y - 7 = 0$

17. Find the equation of the line passing through the points:

(a) $(3, 1)$ and $(1, 5)$

(b) $(-4, 1)$ and $(-2, -4)$

(c) $(4, 5)$ and $(4, -5)$

(d) $(p, -4p)$ and $(-2p, 2p)$

18. Find the equation of the line with:

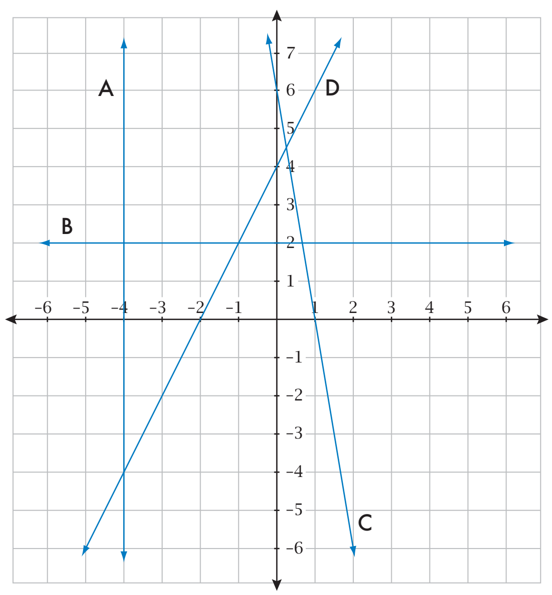
(a) gradient $\frac{1}{3}$ and y intercept -4

(b) gradient of -3 and passing through the point $(1, -1)$

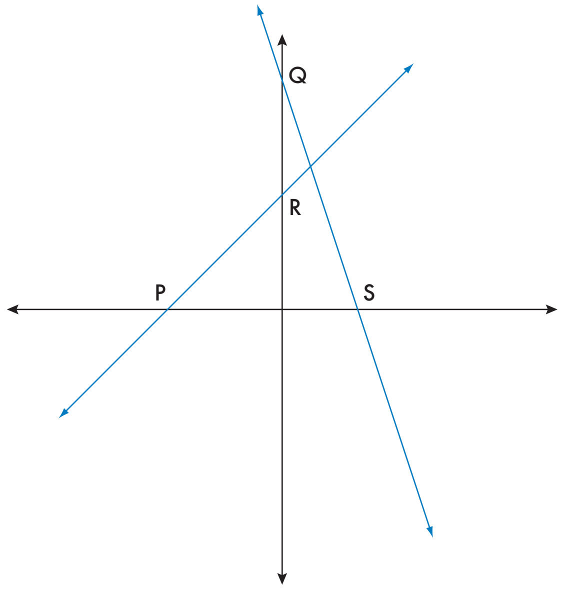
(c) gradient of $\frac{1}{2}$ and passing through the point $(6, 3)$

(d) gradient of $\frac{2}{5}$ and passing through the point $(4, -2)$

19. Find the equations of each of the lines on the graph below:



20. On the graph below is drawn the lines with equations:
$3x + y = 6$ and $x - y = -3$



Determine:

(a) the x intercepts P and S

(b) the y intercepts Q and R

(c) the equation of a line passing through S and the point $(-2, -4)$

21. Use the distance formula to determine the length between the following coordinate points. Answers correct to 2 decimal places.

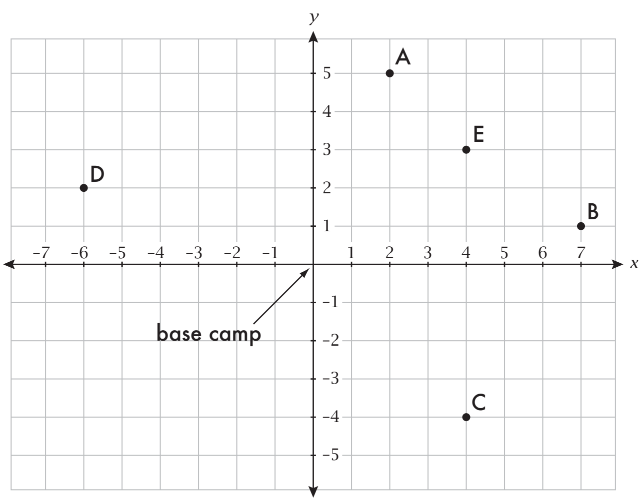
(a) A $(-3, 2)$ B $(4, -7)$

(b) A $(0, 6)$ B $(-1, -2)$

(c) A $(2, -4)$ B $(-3, -1)$

(d) A $(-5, -6)$ B $(-2, 5)$

22. Five orienteering students leave base camp on an expedition. Their positions 2 hours later are shown on the map below.



each unit on the map represents 1km

(a) Determine the distance:

(i) from Base Camp to Student A

(ii) from Student B to Student C

(iii) from Student D to Student E

(b) Determine the equation of the line joining:

(i) Student A to Student B

(ii) Student E to Student C

23. Graph the following functions

(a)

$$y = \begin{cases} 8, & -4 < x \le -1 \\ 5, & -1 < x \le 0 \\ 2, & 0 < x \le 3 \\ 4, & 3 < x \le 4 \end{cases}$$

(b)

$$y = \begin{cases} x + 1, & -7 \le x < -4 \\ -3, & -4 \le x < 0 \\ 2x - 3, & 0 \le x \le 5 \end{cases}$$

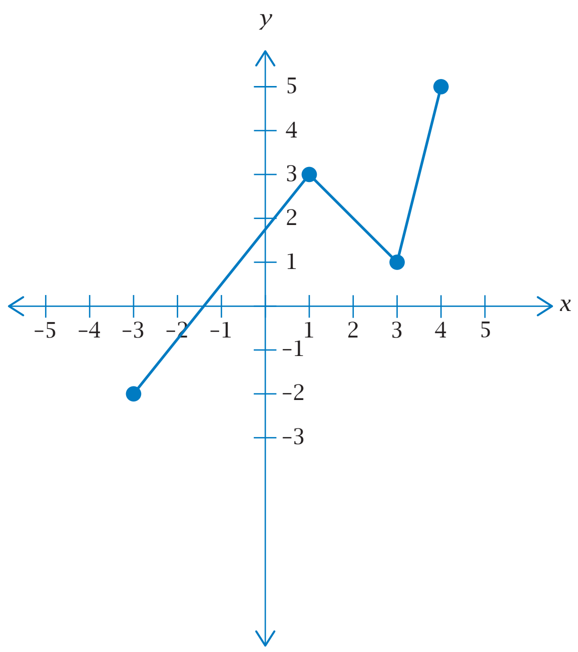
(c)

$$y = \begin{cases} 2 - x, & -3 \le x \le -1 \\ x - 2, & -1 < x \le 2 \\ 2 - x, & 2 < x \le 3 \end{cases}$$

(d)

$$y = \begin{cases} 5, & x > 3 \\ x, & 0 < x \le 3 \\ -x - 1, & -4 < x \le 0 \\ x + 2, & x \le -4 \end{cases}$$

24. Write an equation for the piece-wise function drawn below:

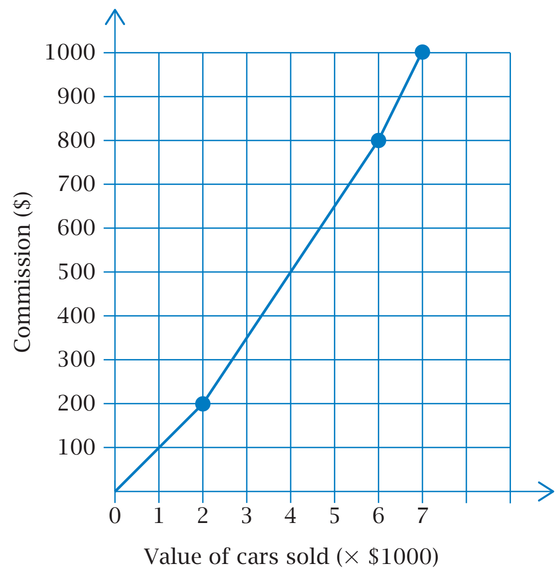


25. John receives a commission for selling second hand cars.

He receives:

- 10% of the first \$2000
- 15% of the next \$4000
- 20% of any remaining amount.

The piece-wise graph below represents John's commission on the value of cars he sells.



(a) How much commission did John earn for selling cars worth

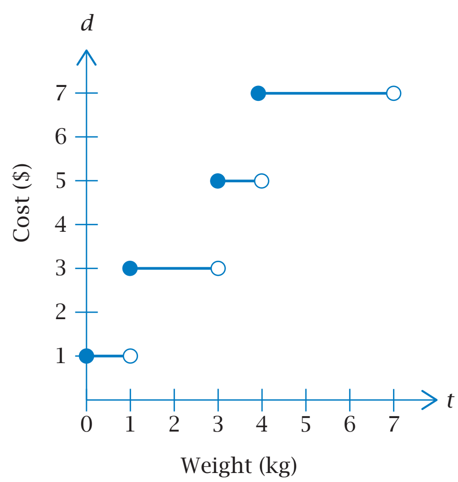
(i) $1500

(ii) $4000

(iii) $6500

(b) John earns a total of $800 commission. What was the value of his car sales?

26. The cost of sending a parcel via post varies according to its weight as shown by the step graph below.



(a) Determine the cost of sending a parcel if it weighs:

(i) 500g

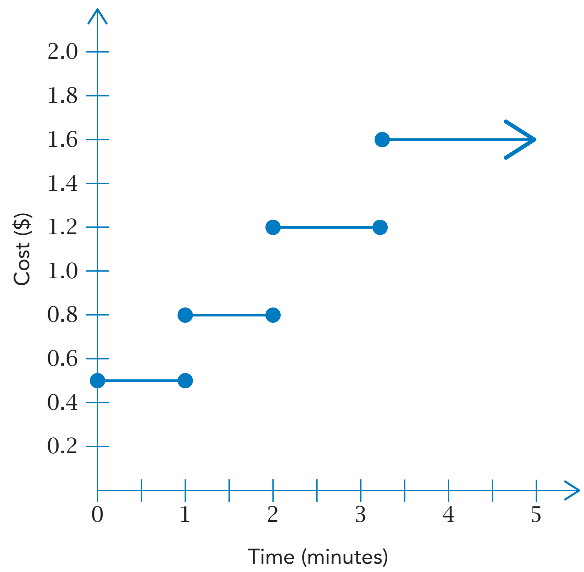
(ii) 1kg

(iii) 3.5kg

(iv) 6.8kg

(b) If a parcel costs $5 to send, determine its possible weight.

27. The cost of mobile phone calls depends on the duration of the call. The step graph below illustrates one mobile provider's cost of mobile calls.



(a) Determine the cost for a mobile phone call which lasts:

(i) 45 seconds

(ii) 2 minutes

(iii) 3 mins 45 seconds

(b) Determine the length of a phone call which costs $1.20.

28. The distance from Derek's house to his friend Peter's house is 80km. Derek leaves one morning for Peter's house at 8.00 am, travelling for 30 minutes at 40 km/h. He stops for ½ hour to buy some supplies.

He leaves the shops and arrives at Peter's house at 10.30 am. The boys study for 1 hour. Derek then travels home at a steady speed arriving at 12.15 pm.

(a) Draw a piece-wise linear graph to represent Derek's journey.

(b) Calculate Derek's average speed from:

(i) 9 am to 10.30 am

(ii) 11.30 am to 12.15 pm

(c) What is the fastest part of the journey? How is this represented on the graph?

9 SIMILARITY AND SCALE

Syllabus Checklist

By the end of this chapter, you should be able to:

- ☐ review conditions for similarity of two dimensional objects, including similar triangles
- ☐ use the scale factor for two similar figures to solve linear scaling problems
- ☐ obtain measurements from scale drawings to solve problems
- ☐ calculate areas of similar figures and surface area and volumes of similar solids

FORMULAE AND DEFINITIONS

Similarity

Similar figures have the same shape but not necessarily the same size.

Similar figures have

- corresponding angles equal
- corresponding sides in the same ratio.

The original object is enlarged or reduced by a scale factor to produce an image.

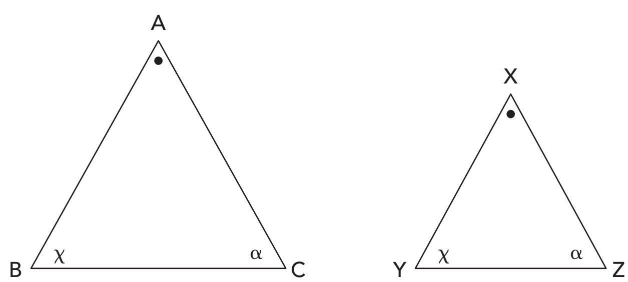
- Scale factor (k) = $\dfrac{\text{length of image}}{\text{corresponding length of the object}}$
- Surface area scale factor is k^2
- Volume scale factor is k^3

The symbol for similarity is '~'.

Similarity tests

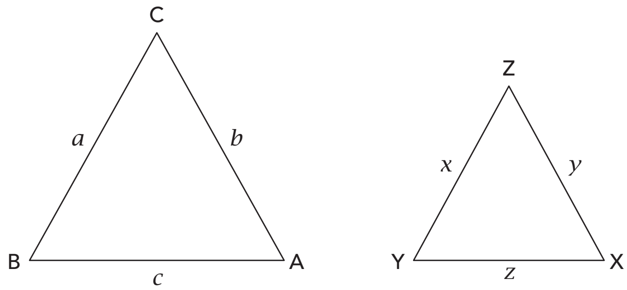
The following are tests for similarity:

1. Three pairs of corresponding angles are equal (A.A.)



$\triangle ABC \sim \triangle XYZ$ (A.A.)

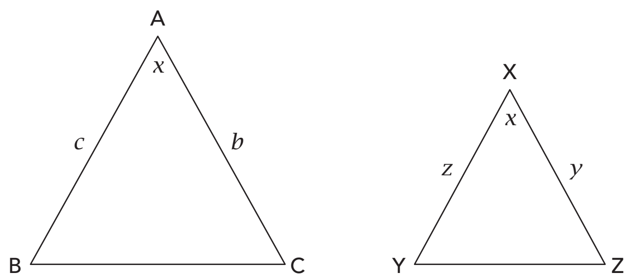
2. All pairs of corresponding sides are in the same proportion (S.S.S.)



$$\frac{a}{x} = \frac{b}{y} = \frac{c}{z}$$

$\therefore$ ABC ~ △ XYZ (S.S.S.)

3. Two pairs of sides are in the same proportion and an included angle (S.A.S.)



$$\frac{c}{z} = \frac{b}{y}; \quad \angle BAC = \angle YXZ$$

$\therefore$ △ABC ~ △XYZ (S.A.S.)

Worked Examples

9.1 A scale drawing has a scale of 1:1000. Find the actual length of an object with a scale length of

(a) 4cm

(b) 0.3m

(a) 1:1000

4cm : 4000cm   ÷100 as

= 40m   1m = 100cm

Actual length = 40m

(b) 0.3m : 300m
Actual length = 300m

9.2 Find the scale distance on a map given a scale and the actual distance.

(a) 1 : 10 000 , 3km

(b) 1 : 15 000 , 600m

(a) $1 : 10\,000$

Convert 3km to cm

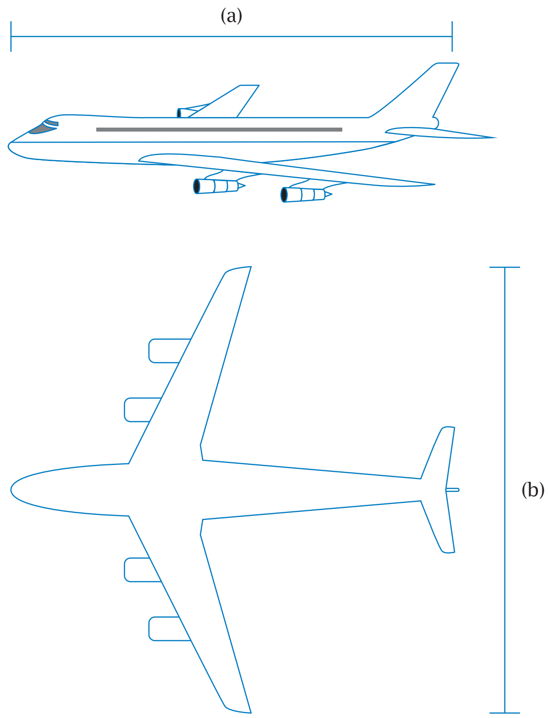
$$\begin{aligned}
3\text{km} &= 300\,000\text{cm} \\
1 : 10\,000 &= x : 300\,000 \\
\frac{1}{10\,000} &= \frac{x}{300\,000} \\
\therefore x &= \frac{300\,000}{10\,000} \\
x &= 30
\end{aligned}$$

Scale distance = 30cm

(b)
$$\begin{aligned}
1 : 15\,000 &= x : 60\,000\text{(cm)} \\
\frac{1}{15\,000} &= \frac{x}{60\,000} \\
x &= \frac{60\,000}{15\,000} \\
x &= 4
\end{aligned}$$

$\therefore$ Scale distance = 4cm

9.3



Scale 1 : 500

Use a ruler and the scale to determine the actual length of

(a) the plane

(b) the wingspan

(a) Scale length : 5.8cm (plane length)

$$\begin{aligned}
1 : 500 &= 5.8 : x \\
\frac{1}{500} &= \frac{5.8}{x} \\
x &= 2900\text{cm}
\end{aligned}$$

$\therefore$ Actual length = 29m

(b) Scale length : 5.85cm (wingspan)

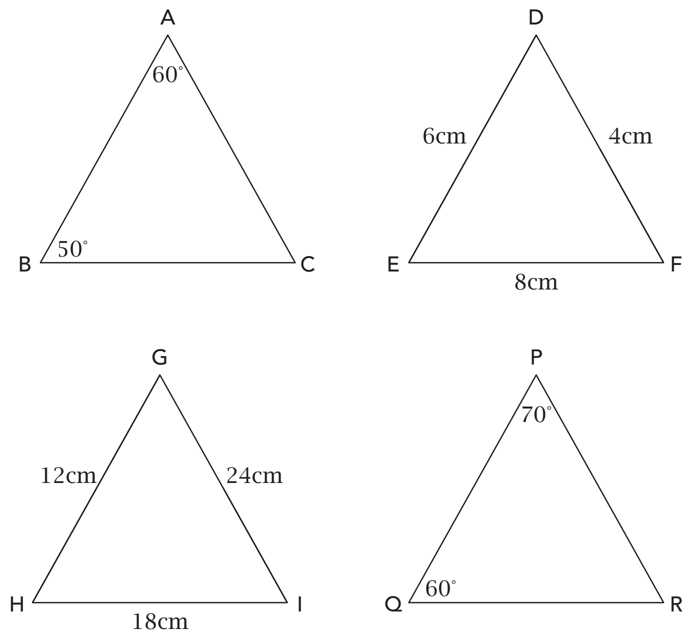
$$1 : 500 = 5.85 : x$$

$$\frac{1}{500} = \frac{5.85}{x}$$

$$x = 2925\text{cm}$$

$\therefore$ Actual length = 29.25m

9.4 For the triangles below state the pair of similar triangles and give the reason for the similarity.

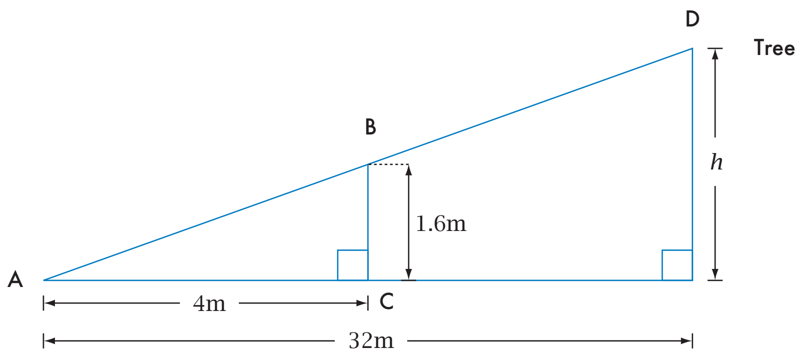


(a) $\triangle ABC \sim \triangle QRP$ (A.A.)

(b) Scale factor $\dfrac{HG}{DF} = \dfrac{HI}{DE} = \dfrac{GI}{EF} = 3$

$\therefore \triangle DEF \sim \triangle HIG$ (S.S.S.)

9.5 A man 1.6m tall casts a shadow 4m long at the same time as a tree of '*h*' height casts a shadow 32m long. Find the height '*h*' of the tree.



$\triangle ABC$ is similar to $\triangle ADE$

Use direct proportion to find '*h*'

$$\frac{\text{Tree height } (DE)}{\text{Tree shadow } (AE)} = \frac{\text{Man's height } (BC)}{\text{Man's shadow } (AC)}$$

$$\frac{h}{32} = \frac{1.6}{4}$$

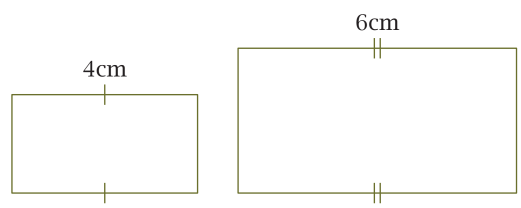
$$h = \frac{1.6 \times 32}{4}$$

$$h = 12.8$$

The height of the tree is 12.8m

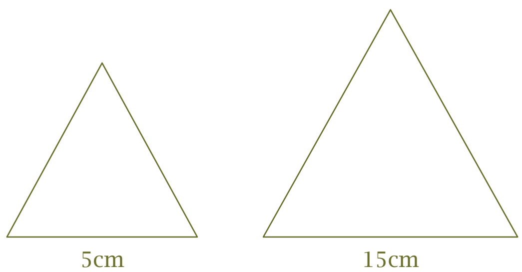
9.6 Determine the value of x in each of the following similar figures

(a)



Area = 50cm^2 Area = $x\text{cm}^2$

(b)



Area = $x\text{cm}^2$ Area = 180cm^2

(a) Scale factor $= \frac{6}{4}$

$= \frac{3}{2}$

(Ratio of corresponding sides)

Area scale factor is length scale factor squared i.e. $\left(\frac{3}{2}\right)^2$

$$= \frac{9}{4}$$

$$\therefore \text{ Area } x = 50 \times \frac{9}{4}$$

$$= 112.5\text{cm}^2$$

(b) Length scale factor $= \frac{15}{5}$

$= 3$

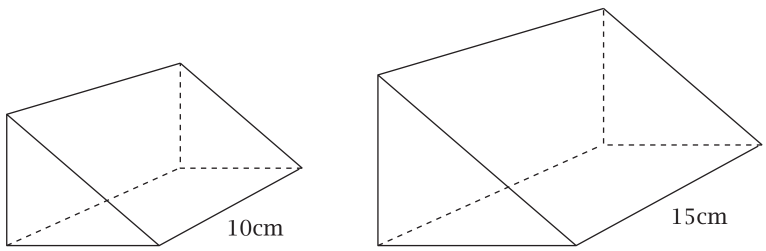
Area scale factor $= 3^2$

$= 9$

$\therefore$ Area x $= 180 \div 9$

$= 20\text{cm}^2$

9.7 Determine the volume of the larger prism.



Volume = 25cm^3

Length scale factor $= \frac{15}{10}$

$= \frac{3}{2}$

Volume scale factor is length scale factor cubed

i.e. $\left(\frac{3}{2}\right)^3 = \frac{27}{8}$

$\therefore$ Volume larger prism = volume small prism × volume scale factor

$= 25 \times \frac{27}{8}$

$= 84.375\text{cm}^3$

PROBLEMS TO SOLVE

Chapter 9: Similarity and Scale

1. A scale drawing has a scale of 1 : 100. Find the actual length of an object with a scale length of

   (a) 1cm

   (b) 10.5cm

   (c) 46mm

2. Write each ratio below in simplest form, without units.

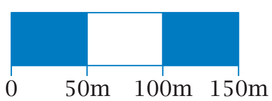
   (a) 1cm : 5m

   (b) 50m : 5km

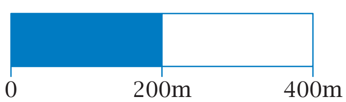
   (c) 12cm : 144km

3. Given the following scales, determine the *actual* distance, if the distance between the two points on the map is 15cm.

   (a)



   (b)



4. Find the scale – distance on a map given a scale and the actual distance.

   (a) 1 : 40 000 , 2km

   (b) 1 : 100 000 , 675m

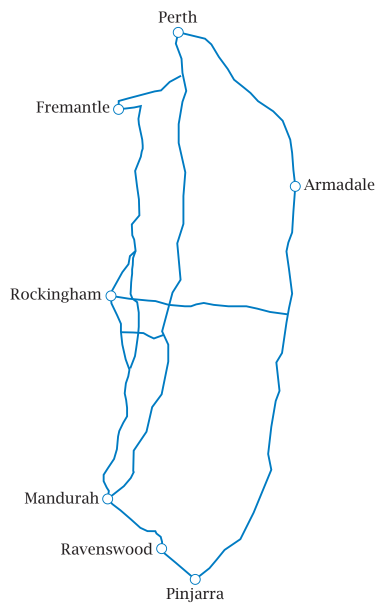
   (c) 1 : 1250 , 250m

5. The scale on a diagram reads 1 : 1 000 000.

   Determine:

   (a) the real distance if the distance on the diagram measured 5cm.

   (b) the scale distance if the real distance was 75km

6. 

The map above shows south-west of Western Australia. The scale of the map is 1 : 1120 000.

Determine:

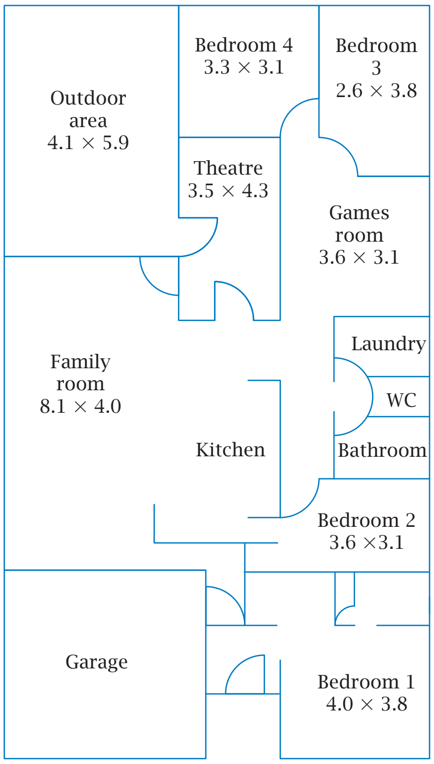
(a) the distance (straight line only) between

(i) Perth and Mandurah

(ii) Mandurah and Rockingham

(b) which two towns are approximately 18km apart.

7. A new house has the following floor plan



All units in metres

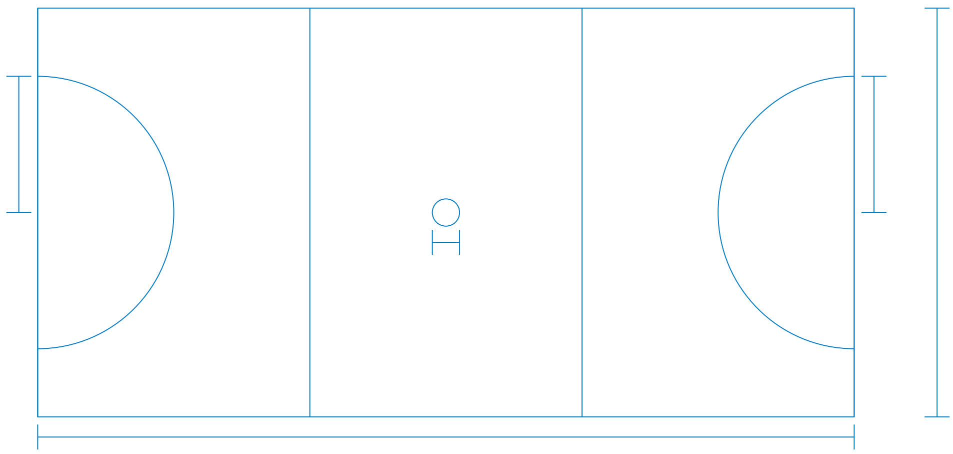
Determine:

(a) the cost of carpeting all bedrooms if carpet costs $73/m^2$

(b) how many tiles are needed to tile the family room if each tile measures 50cm × 50cm?

8. A scale drawing of a netball court is shown below.

1 : 300

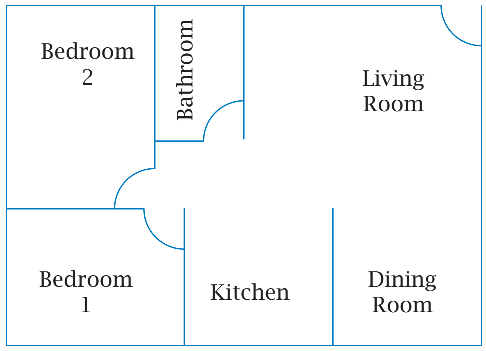


(a) What is the actual length and width of the court?

(b) Calculate the total area of the two half circles.

(c) Determine the cost of covering the court with non-slip material if it costs $45 per square metre.

9.



1 : 300

The diagram is a scale drawing of a 2 bedroom flat.

Determine:

(a) the dimensions of

(i) Bedroom 1

(ii) Kitchen

(b) the cost of carpeting the bedrooms if carpet cost $62/m^2

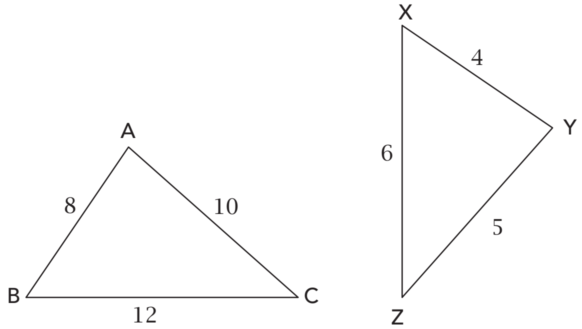
(c) the number of tiles needed to tile the kitchen if each tile measures 20cm $\times$ 20cm.

10. Determine whether the two triangles are similar. If they are:

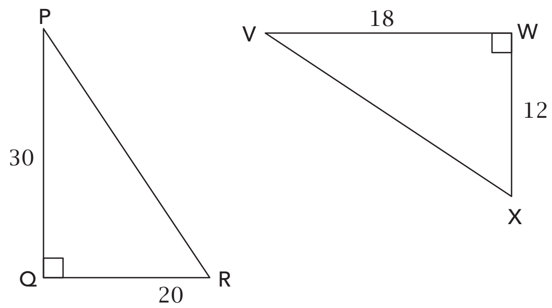
(a) state the scale factor

(b) name the pair of similar triangles and give the reason for the similarity.

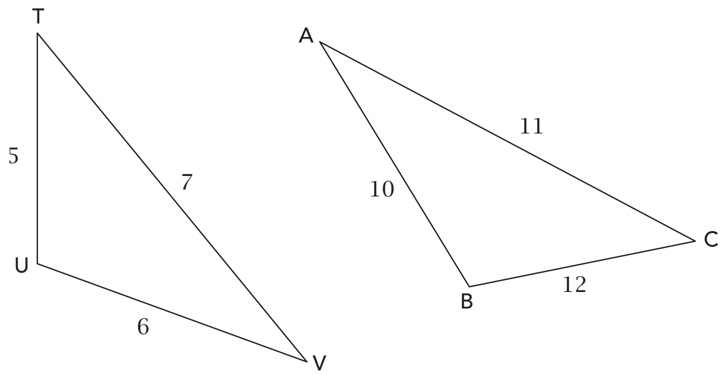
(i)



(ii)

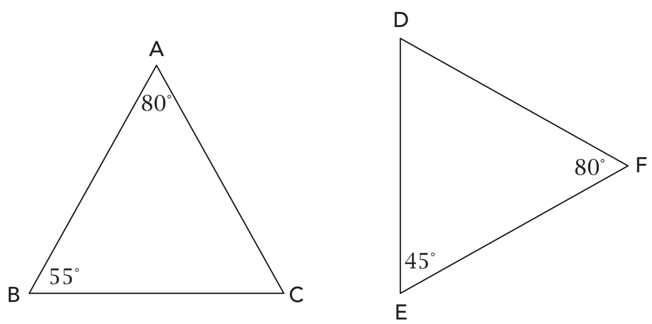


(iii)

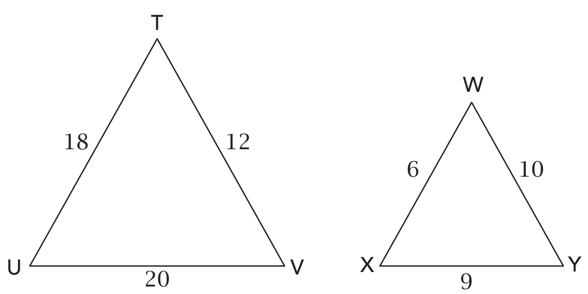


11. Determine whether the two triangles are similar and state the reason for similarity.

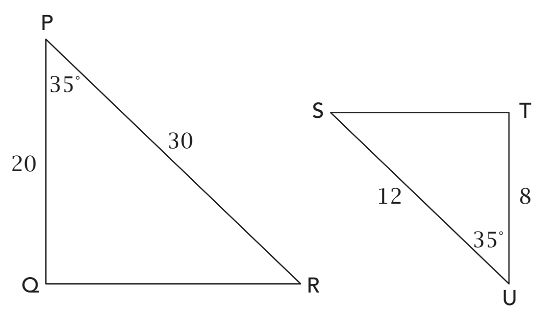
(a)



(b)



(c)

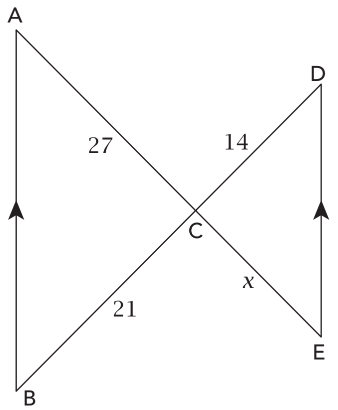


12. For each of the following diagrams

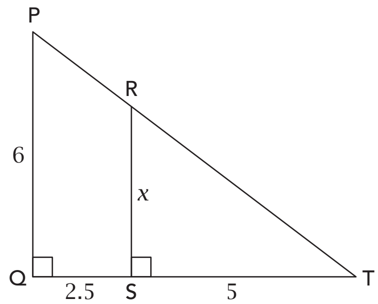
(i) name the pair of similar triangles stating the condition of similarity

(ii) determine the value of x

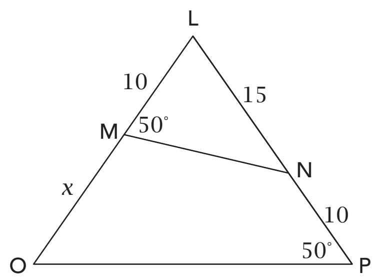
(a)



(b)

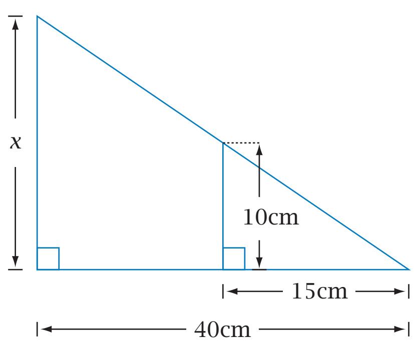


(c)

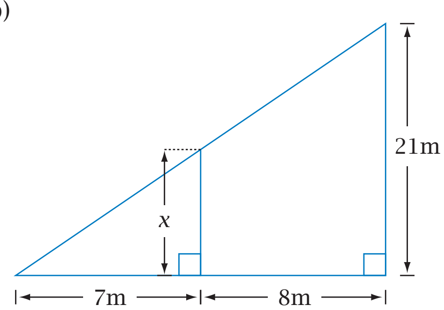


13. Find the value of x

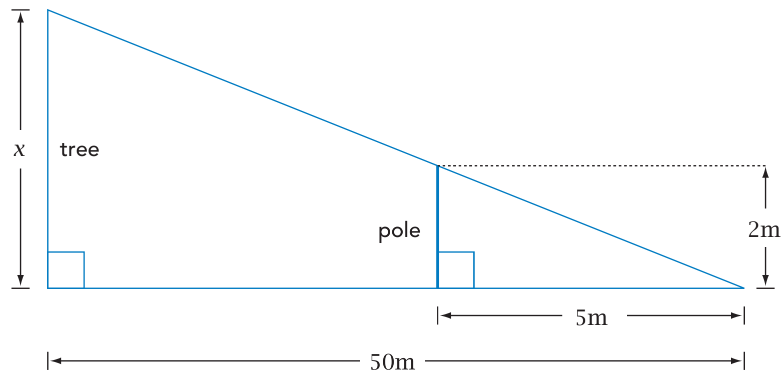
(a)



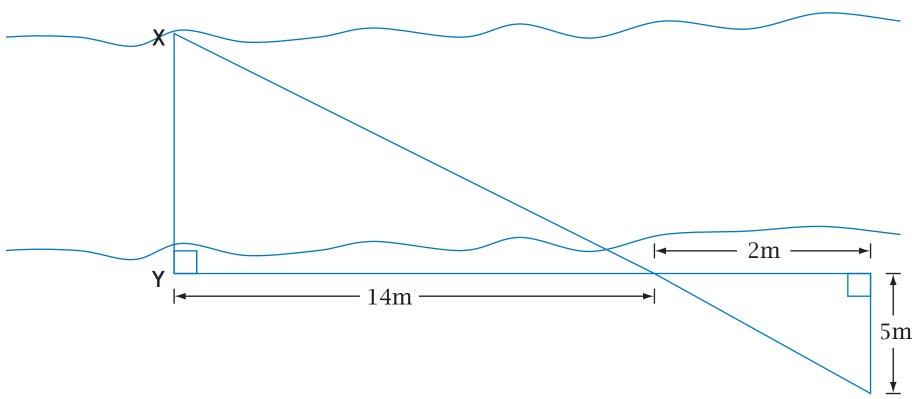
(b)



14. Estimate the height of a tree that casts a shadow 50m long at the same time that a 2m long vertical pole casts a shadow of 5m.



15. Find the distance *XY* across the river

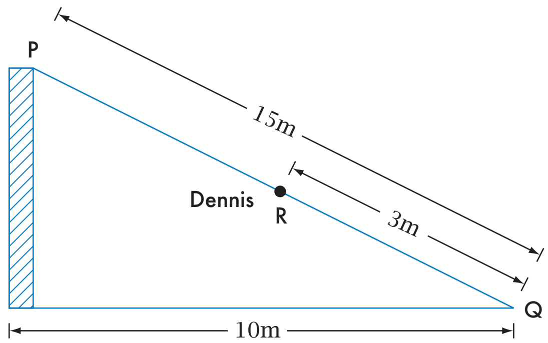


16. John who is 2m high casts a shadow 5m long from a street light 12m above the ground. Determine the horizontal distance John is from the streetlight.

17. A ladder *PQ* 15m long rests against a wall. Dennis walks up the ladder to a point *R* 3m from point *Q*. If the foot of the ladder is 10m from the wall determine:

(a) the horizontal distance of Dennis from the wall

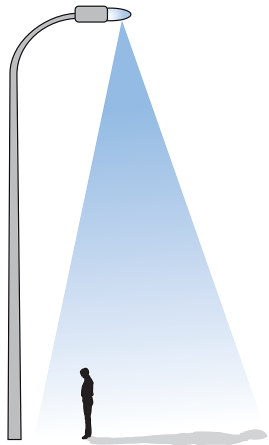
(b) Dennis' height above the ground.



18. Sam who is 2m tall casts a shadow 5m long. He stands 7m from a street pole which casts a shadow finishing at the same place as Sam's shadow.

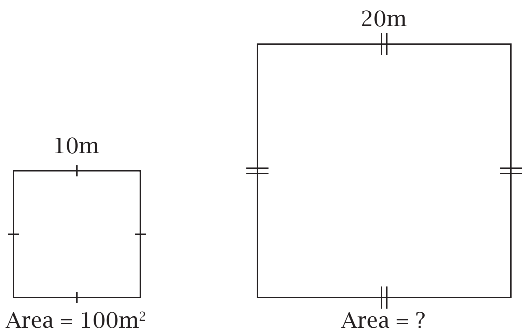
(a) Determine the height of the street pole.

(b) Damien who is 90cm tall stands in a position where his shadow and the pole's shadow meet. How far from the pole is Damien standing?

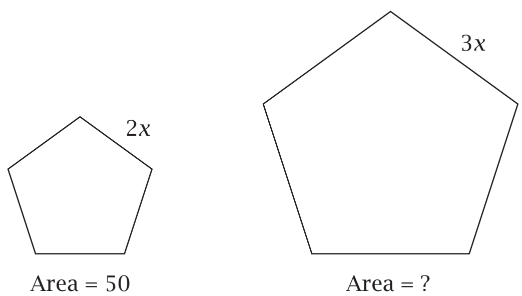


19. Determine the missing area for each of the following pairs of similar figures.

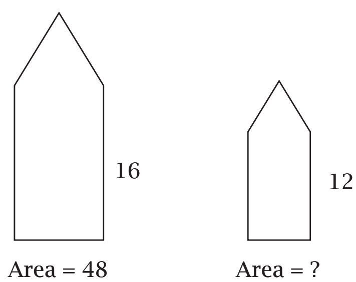
(a)



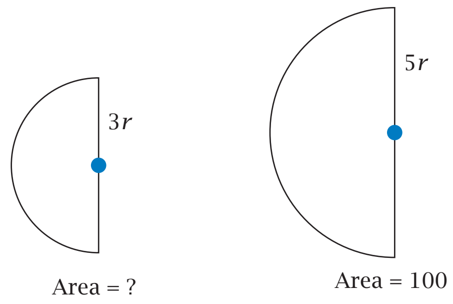
(b)



(c)

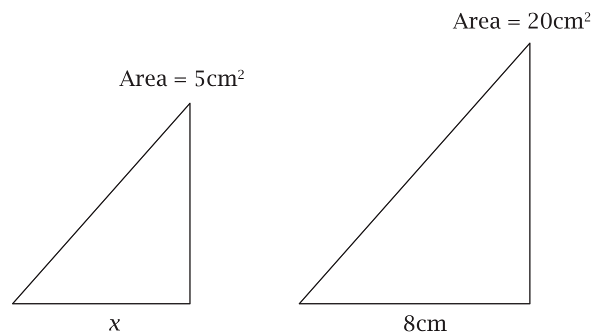


(d)

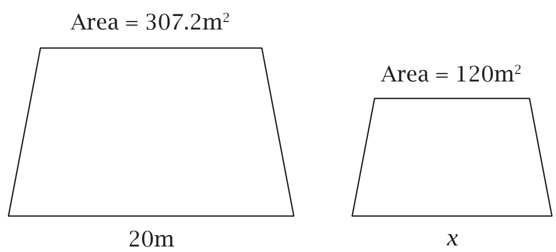


20. Determine the value of x in each pair of similar figures.

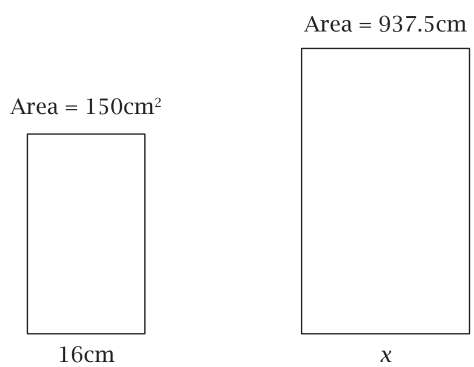
(a)



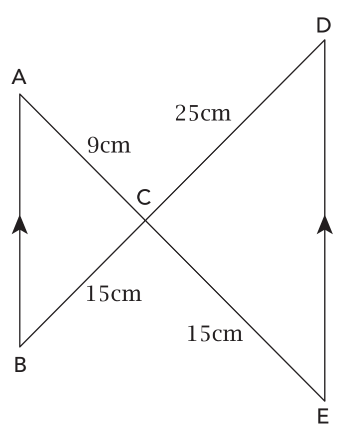
(b)



(c)



21. Given the diagram



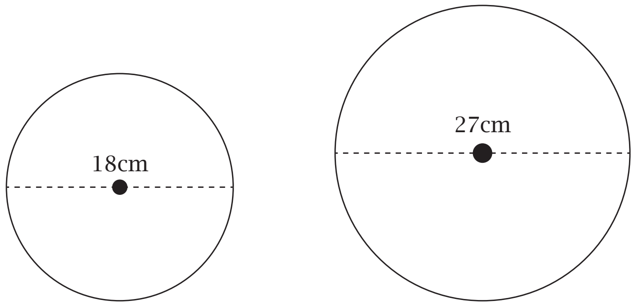
and the area $\triangle CDE = 625cm^2$

Find

(a) the similar triangles and state the condition of similarity

(b) the area of $\triangle ABC$.

22.



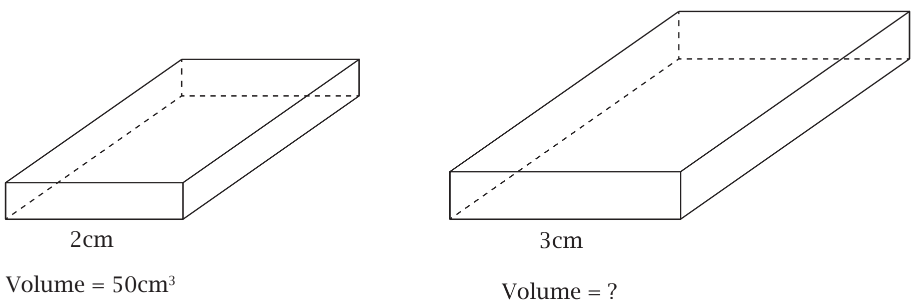
Find the ratio of

(a) the circumference of the two circles

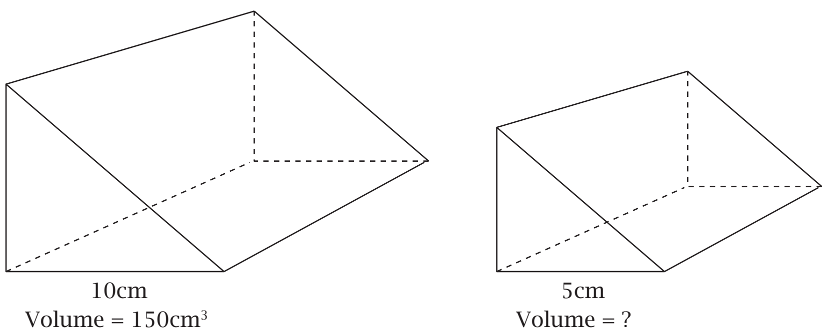
(b) the areas of the two circles.

23. For each of the following similar solids determine the missing volume.

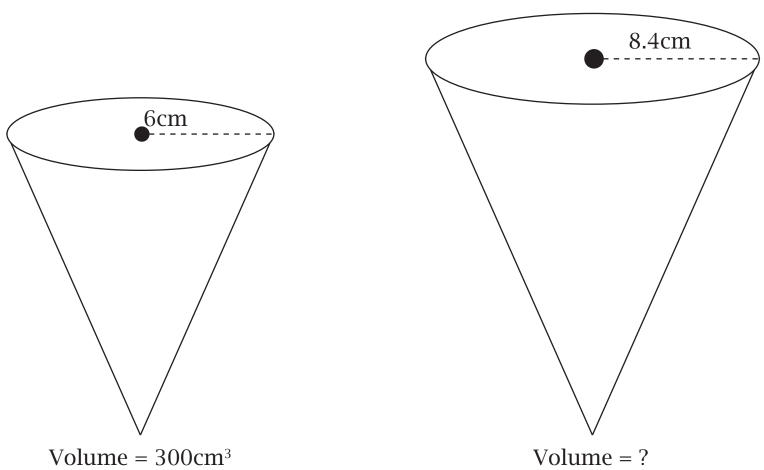
(a)



(b)



(c)



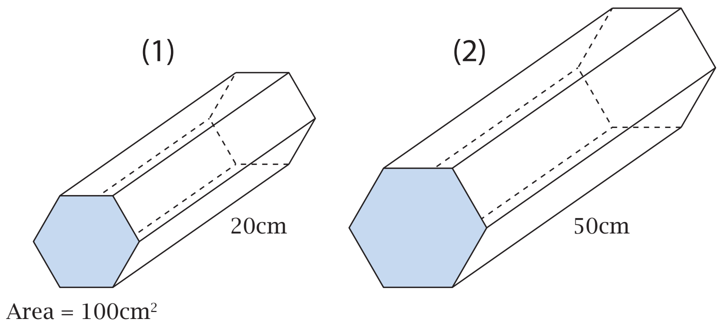
24. The areas of corresponding faces of two cubes are in the ratio of 25 : 9.

Determine:

(a) the ratio of corresponding sides of the two cubes

(b) the ratio of the volumes of the two cubes.

25.



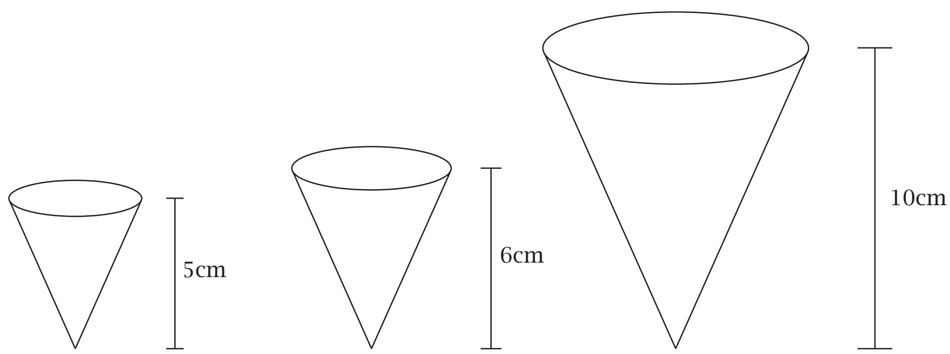
Two hexagonal prisms are drawn above.

Determine:

(a) the volume of hexagonal prism 1

(b) the volume of hexagonal prism 2.

26. Three cones made of ice are shown below.

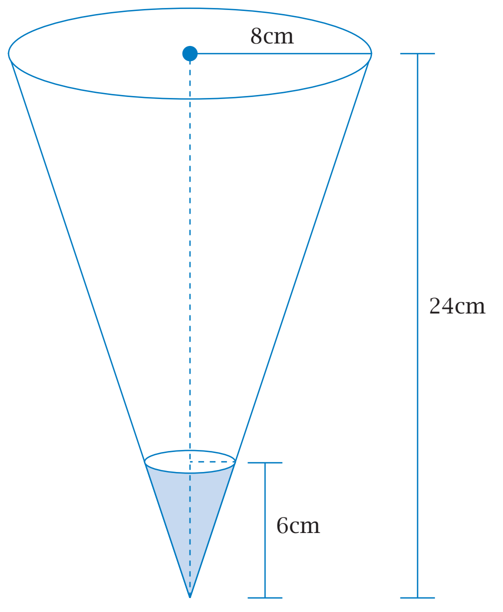


Calculate:

(a) the ratio of their surface areas

(b) the ratio of their volumes

27. A cone is partly filled with water to a depth of 6cm.



Calculate:

(a) the volume of the cone

(b) the volume of water in the cone

(c) the additional amount of water (in mL) needed so that the height of water in the cone is 15cm.

SIMPLE AND COMPOUND INTEREST

10

Syllabus Checklist

By the end of this chapter, you should be able to:

- [ ] calculate simple and compound interest

FORMULAE AND DEFINITIONS

Simple or flat rate interest

Interest is constant and calculated on the *initial* investment.

$$\begin{aligned} SI &= P \times R \times T \\ SI &= \text{simple interest} \\ P &= \text{principal} \\ R &= \text{interest rate (as a decimal)} \\ T &= \text{time (in years)} \end{aligned}$$

Compound Interest

Interest is added to the account and itself earns interest in subsequent time periods. As a result interest grows each year and the account grows exponentially.

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$\begin{aligned} A &= \text{amount in the account} \\ P &= \text{principal} \\ r &= \text{interest rate (as a decimal)} \\ n &= \text{number of compounding periods each year} \\ t &= \text{time (in years)} \end{aligned}$$

$$\text{Compound Interest} = P\left(1+\frac{r}{n}\right)^{nt} - P$$

Worked Examples

10.1 John deposited \$6000 into his savings account. The savings account earns interest at a flat rate of 6% p.a.

Find the amount of interest earned in:

(a) 2 years

(b) 15 months.

(a) Simple Interest $= P \times R \times T$

$= 6000 \times 0.06 \times 2$

$= \$720$

(b) Simple Interest $= P \times R \times T$

$= 6000 \times 0.06 \times 1.25$

$\frac{15}{12}$ months = 1.25 years

$= \$450$

10.2 Jody borrowed \$2800 for a period of 27 months. The amount she repaid was \$3304.

What simple interest rate was Jody charged for the loan?

$$\begin{aligned}
\text{Interest} &: \$3304 - \$2800 \\
&= \$504 \\
\text{Simple Interest} &= P \times R \times T \\
504 &= 2800 \times R \times 2.25 \\
R &= \frac{504}{2800 \times 2.25} \\
&= 0.08 \\
\therefore \text{Interest rate} &= 8\% \text{ p.a.}
\end{aligned}$$

10.3 \$8200 is invested at an interest rate of 9% p.a. for 4 years. Determine the interest if it is calculated as

(a) simple interest

(b) compound interest, compounded yearly

(c) compound interest, compounded monthly.

(a) $SI = P \times R \times T$

$= 8200 \times 0.09 \times 4$

$= \$2952$

(b) $CI = P\left(1+\frac{r}{n}\right)^{nt} - P$

$= 8200\left(1+\frac{0.09}{1}\right)^{(1\times 4)} - 8200$

$= \$3374.97$

(c) $$\text{CI} = \text{P}\left(1+\frac{r}{n}\right)^{nt} - \text{P}$$

$$= 8200\left(1+\frac{0.09}{12}\right)^{(12\times 4)} - 8200$$

compounded 12 times per year

$$= \$3537.52$$

10.4 Sarah invests a sum of money at 5% p.a. compounded yearly for 3 years. Determine the value of the original investment if on maturity it yields \$13 891.50.

$$\text{A} = \text{P}\left(1+\frac{r}{n}\right)^{nt}$$

$$13\,891.50 = \text{P}\left(1+\frac{0.05}{1}\right)^{(1\times 3)}$$

$$\text{P} = \frac{13\,891.50}{(1+0.05)^3}$$

$$\text{P} = \$12\,000$$

PROBLEMS TO SOLVE

Chapter 10: Simple and Compound Interest

1. Fill in the missing entries in the table below:

	Principal P	Interest rate R (%)	Time T	Simple Interest	Amount
(a)	\$2000	5%	3 years		
(b)	\$10 000	9%	18 months		
(c)	\$4200	7%		\$1543.50	
(d)		2.5%	3.5 years	\$306.25	
(e)		12.5%	15 months		\$11 100
(f)	\$8100		3¼ years		\$9679.50

2. Complete the table below on compound interest:

	Principal P	Rate r	Compounded n	Time t	Interest	Amount
(a)	\$3000	6%	yearly	4 years		
(b)	\$8500	5%	daily	3 years		
(c)	\$4600	9%	monthly	5 years		
(d)	\$7000	7%	monthly	3 months		
(e)		7%	yearly	4 years		\$3276.99
(f)		5%	monthly	5 years		\$8341.83

3. Alex borrowed \$5500 to purchase a car. He repaid the full amount including simple interest of \$5747.50 after 9 months.

   What was the simple interest rate?

4. Kelsey deposited \$10 000 into a bank account at a rate of 5% p.a. over 6 years.

   Calculate:

   (a) the simple interest

   (b) the compound interest if compounded monthly

   (c) the difference between the simple interest and compound interest.

5. Simon invests an amount of money received from his grandparents. At the end of the first year, the investment is worth \$6825 and at the end of the second year \$7166.25.

   If interest is compounded annually, determine:

   (a) the interest rate p.a.

   (b) the original investment.

6. A bank offers a choice of two fixed term deposits.

- $\boxed{\text{Option A}}$ earning simple interest at 6.5% p.a. for 4 years.
- $\boxed{\text{Option B}}$ earning simple interest at 8% p.a. for 3.5 years.

David invests the same amount of money in both Option A and Option B. Option B earns \$105 more than Option A.

Determine:

(a) David's original investment

(b) the interest rate if the original investment accumulates to \$8715 after 5 years.

7. A bank offers an account paying 8% p.a. compounded monthly. Georgie opens an account on June 1st 2013 with a deposit of \$3000. Every subsequent year on June 1st Georgie makes a similar deposit.

Determine the amount in the account on:

(a) June 1st 2015

(b) June 1st 2020

(c) May 31st 2021

8. (a) Determine the length of time required for \$6200 to grow to \$7130 if the money is invested at 6% p.a. simple interest.

(b) The interest earned on an amount of money invested for 2 years at 15% p.a. compounded annually is \$2709. What is the simple interest earned on the same amount, time and interest rate.

9. Matt purchases a new TV by paying \$220 per month for 15 months. The cash price of the TV was \$2250.

Determine:

(a) the total amount paid

(b) the interest paid

(c) the simple interest rate.

10. The price of a new washing machine is \$3600. Laura pays a deposit of 25% with the remainder to be paid over 2 years. The monthly repayments are \$220.

Calculate:

(a) the deposit

(b) the total amount paid for the washing machine

(c) the flat rate of interest for the monthly repayments.

11. A company offers a new investment scheme. For the first 4 years investors are paid 6% p.a. simple interest with an additional 3% p.a. simple interest thereafter.

(a) Joanna invests \$10 000 in this new investment scheme for 3 years. How much does she accumulate?

(b) Joseph invests \$$x$ and accumulates after 7 years \$1230 less than Joanna. What was his initial investment?

12. Bob wishes to invest \$20 000. Which plan should he choose and why?

Plan 1: 5.95% p.a. compounded quarterly

Plan 2: 5.9% p.a. compounded monthly.

13. An amount of money is invested at an interest rate of r% compounded annually. This amount of money triples in 6 years time.

Determine the value of r.

11

STATISTICS

Syllabus Checklist

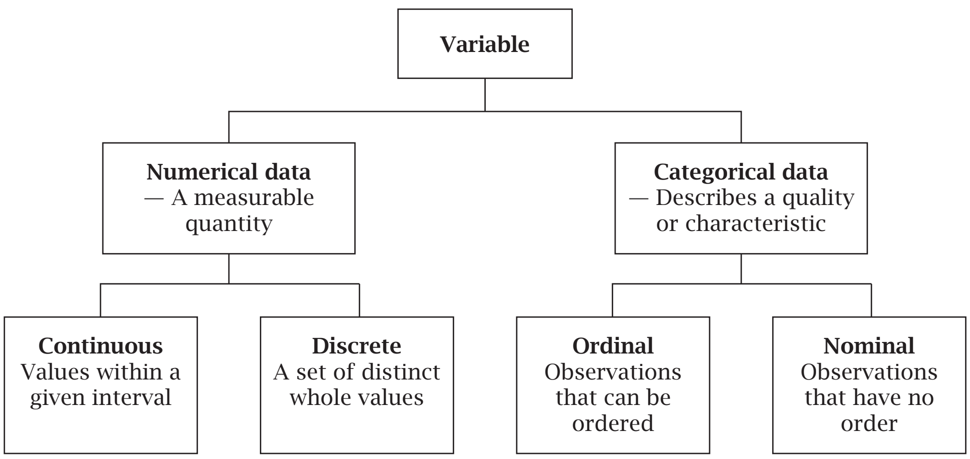
By the end of this chapter, you should be able to:

- ☐ review the statistical investigation process
- ☐ classify a categorical variable as ordinal or nominal and display data
- ☐ classify a numerical variable as discrete or continuous
- ☐ using appropriate graphs describe the distribution of a numerical data set
- ☐ determine the mean and standard deviation of a data set
- ☐ construct, compare and interpret parallel boxplots
- ☐ compare and interpret groups using medians, means, IQR's, ranges or standard deviations

FORMULAE AND DEFINITIONS

Types of data

Collected information is known as *data*. A *variable* is any characteristic, number or quantity that can be measured or counted.



REPRESENTATION OF NUMERICAL DATA

Frequency table

A frequency table is a tabular representation of the data.

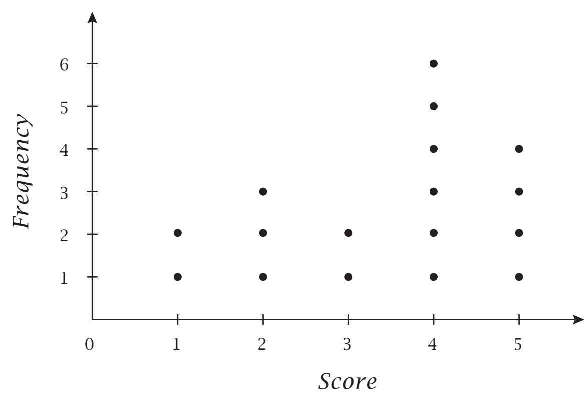
Ungrouped Data		
Score	Tally	Frequency
1	II	2
2	III	3
3	II	2
4	~~IIII~~ I	6
5	IIII	4
		17

Grouped Data	
Score	Frequency
0–4	2
5–9	3
10–14	1
15–19	8
20–24	5
	19

Dot frequency plot

A dot frequency plot gives a clear visual picture of the set of data. From this, data can be analysed to see if it contains:

- *Gaps* spaces between points
- *Outliers* points that are particularly large or small and away from the main data
- *Clusters* isolated groups of points
- *Spread* how far the points are spread or if they are tightly packed

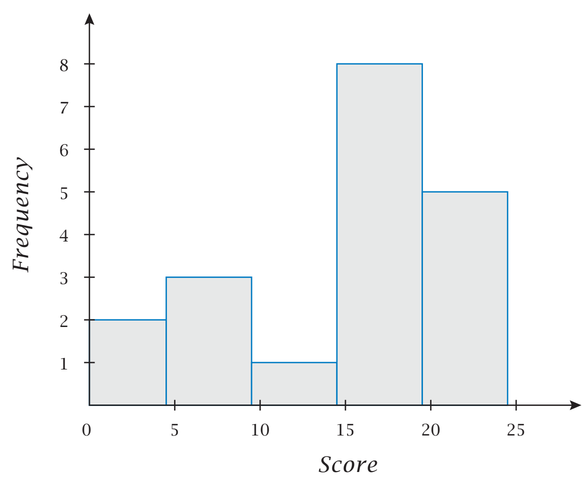


Frequency histograms

A graphical representation of a frequency table. A frequency histogram has:

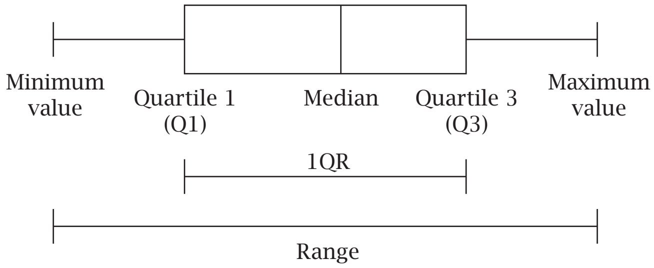
- The frequency on the vertical axis
- No gaps between the columns
- Columns with equal widths

Histograms can be drawn for ungrouped data or grouped data and are usually used to represent continuous distributions.



Boxplots

A boxplot or box and whisker plot is another graphical display of numerical data. Five summary statistics are used to construct a boxplot.



Stem and leaf plot

A visual representation of numerical data. 5|0 represents the number 50.

Example

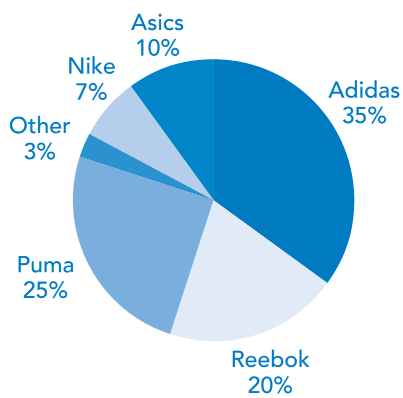
Stem	Leaf
5	0 1 5 9
6	2 4
7	3
8	1 4 7

REPRESENTATION OF CATEGORICAL DATA

Circle graphs

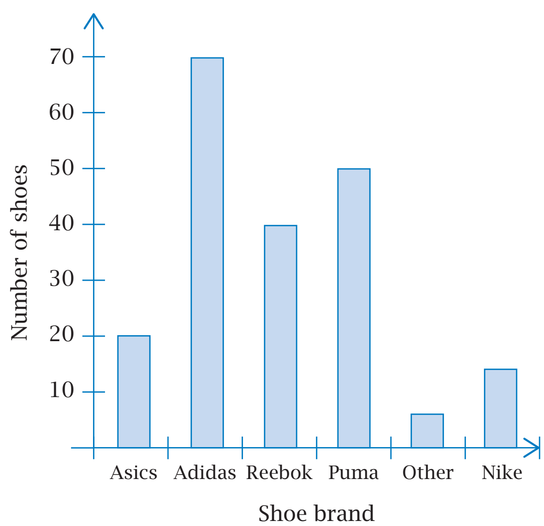
A circle graph is a graph in the form of a circle that is divided into sectors, with each sector representing a part of the data.

Brands of shoes worn by students



Column graph (or bar graph)

Shoe Brand	Number
Asics	20
Adidas	70
Reebok	40
Puma	50
Other	6
Nike	14



Statistics

Measures of central tendency

Mean – The average of a set of data.

Ungrouped data

$$\text{Mean} = \frac{\text{sum of all scores}}{\text{total number of scores}}$$

$$\bar{x} = \frac{\Sigma x_i}{n}$$

If the scores have frequencies other than 1, then use the following formula:

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

Grouped data

The midpoint of the class interval must first be calculated in order to find the mean. The midpoint can be found by averaging the two values of the interval.

i.e. $20 \leq x < 25$

$$\text{midpoint} = \frac{20 + 25}{2}$$

$$= 22.5$$

$$\bar{x} = \frac{\Sigma f_i m}{\Sigma f_i}$$

f = frequency of each grouping
m = midpoint of each grouping
n = number of scores

Advantages/Disadvantages

- Useful for comparing sets of data.
- Influenced by the size of every data value.
- Outliers can have a large effect on the mean.

Mode – The score which occurs most often.

Grouped data
For grouped data, the **modal class** is calculated.

Advantages/Disadvantages

- Gives no information on the other data values.
- Can give a distorted picture of the data.

Median – The middle score when the scores are arranged in ascending or descending order. (For an even number of scores, find the average of the two middle scores). The position of the median is: $\frac{n+1}{2}$th score where n is the number of scores

Grouped data
For grouped data, the **median class** is calculated.

For grouped data, a more accurate value of the median can also be calculated.

Calculation of Median

Score	Frequency
30–39	4
40–49	8
50–59	11
60–69	1
70–79	1
	25

**13th score is located here*

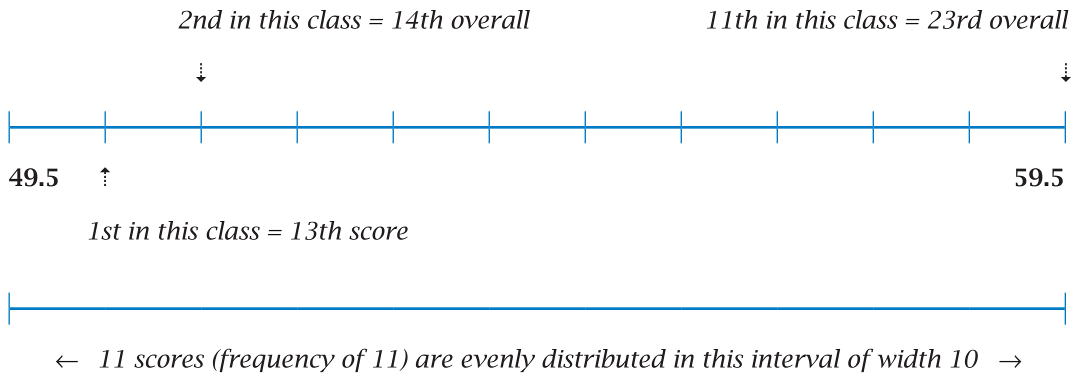
*position of median

$$= \frac{\text{number of scores} + 1}{2}$$

$$= \frac{25 + 1}{2}$$

$$= 13$$

- The median mark is the 13th score when arranged in ascending order.
- The **median class** is 50–59 as there are 12 scores in the previous two class intervals.
- The **continuous** class interval is 49.5–59.5 giving a width of 10.
- The **13th score** is the **1st position** in the interval 49.5–59.5 out of the 11 scores.



1st position in interval *lower class interval*

$$\text{Median score} = \frac{1}{11} \times 10 + 49.5$$

frequency *width*

$$= 50.4$$

Advantages/Disadvantages
- Not affected by outliers
- Can give a more accurate picture of the centre.

Measures of spread

Range – the difference between the highest and lowest score.

Relative frequency

The frequency of a score divided by the number of scores. Relative frequencies are often represented as percentages.

i.e. relative frequency $= \dfrac{f}{n}$

Standard deviation

Standard deviation is a measure of spread. It can be calculated by using the formula:

$$S_x = \sqrt{\frac{\Sigma x^2}{n} - \overline{x}^2}$$

A CAS calculator is also used to calculate the standard deviation.

Interquartile range (IQR)

Interquartile range = Quartile 3 (Q3) – Quartile 1 (Q1). This represents the middle 50% of the scores.

Outlier

An outlier is an extreme score that lies outside the other scores. Outliers can be calculated by:

$$Q_3 + 1.5\ \text{IQR} \leq \text{score} \leq Q_1 - 1.5\ \text{IQR}$$

Worked Examples

11.1 State each of the following variables as:

numerical — discrete / continuous

or

categorical — nominal / ordinal

(a) Arm span

(b) School telephone number

(c) A person's sex

(d) Number of goals scored in an AFL game

(a) Numerical and continuous

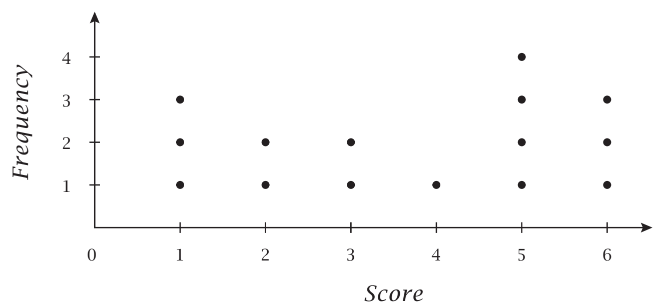
(b) Categorical and ordinal

(c) Categorical and nominal

(d) Numerical and discrete

11.2 For the dot frequency graph, determine the following

(a) the number of scores
(b) the mean
(c) the median
(d) the mode
(e) the range
(f) the standard deviation
(g) the interquartile range



(a) Number of scores = 15. *each dot represents a score*

(b) Mean = $3.\dot{6}$ *add all scores then divide by the number of scores*

(c) Median = 4 *the middle score when scores are arranged in order*

(d) Mode = 5 *the score 5 occurs more frequently*

(e) Range = 5 *highest – lowest score*

(f) St Dev = 1.85 (by calculator)

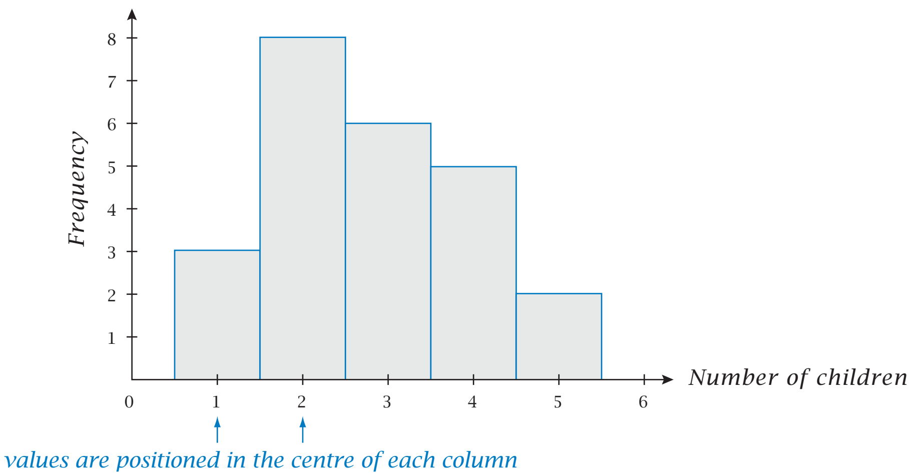
(g) IQR = Q3 – Q1
= 3

11.3 (a) Draw a frequency histogram for the 'number of children in a family' collected in a classroom of 24 students. The information is displayed in the frequency table below.

No of children in family	Frequency
1	3
2	8
3	6
4	5
5	2
	24

(b) Calculate each of the relative frequencies and express them as percentages.

(a)



(b)

Number of children in family	1	2	3	4	5
Frequency	3	8	6	5	2
Relative frequency	$\frac{3}{24}$	$\frac{8}{24}$	$\frac{6}{24}$	$\frac{5}{24}$	$\frac{2}{24}$
Percentage relative frequency	$12\frac{1}{2}\%$	$33\frac{1}{3}\%$	25%	$20\frac{5}{6}\%$	$8\frac{1}{3}\%$

11.4 An analysis of the number of words per sentence in 100 sentences from a novel gives the following frequency table.

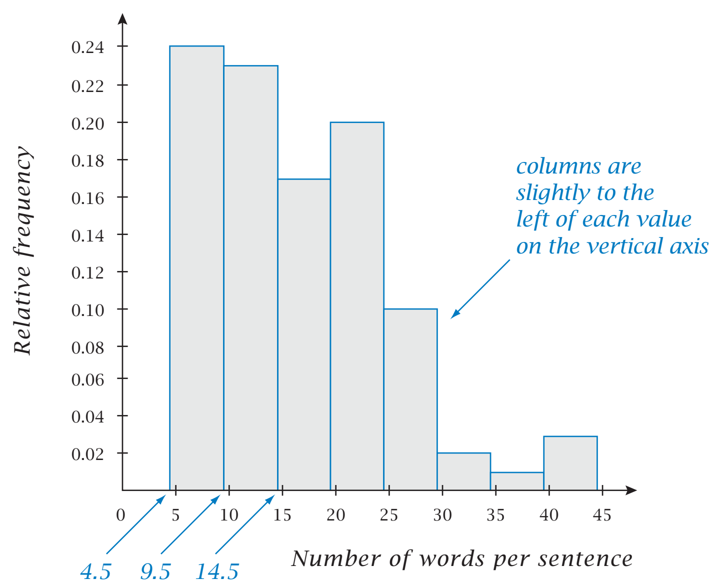
Number of words per sentence	Frequency
5–9	24
10–14	23
15–19	17
20–24	20
25–29	10
30–34	2
35–39	1
40–44	3
	100

Construct a relative frequency histogram.

The first step is to make the intervals continuous. (Subtract 0.5 from the left hand interval and add 0.5 to the right hand interval to make the data continuous)

Continuous intervals	Number of words per sentence	Frequency	Relative frequency
4.5–9.5	5–9	24	$\frac{24}{100} = 0.24$
9.5–14.5	10–14	23	0.23
14.5–19.5	15–19	17	0.17
19.5–24.5	20–24	20	0.2
24.5–29.5	25–29	10	0.1
29.5–34.5	30–34	2	0.02
34.5–39.5	35–39	1	0.01
39.5–44.5	40–44	3	0.03
		100	

divide the frequency by the total number of scores to obtain the relative frequency



11.5 The number of goals scored by the school soccer team in the past 20 matches is displayed in the table below

Number of goals	Frequency
0	2
1	3
2	2
3	6
4	5*
5	1
6	1

(a) Calculate:

(i) The mean (iii) The mode (v) IQR

(ii) The median (iv) The standard deviation

(b) Describe what the '5' represents in the table above (indicated by *)

(a) (i) By calculator: $\bar{x} = 2.8$

By formula: $\bar{x} = \dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$= \dfrac{(0 \times 2) + (1 \times 3) + (2 \times 2) + (3 \times 6) + (4 \times 5) + (5 \times 1) + (6 \times 1)}{20}$

$= \dfrac{56}{20}$

$= 2.8$

$\therefore$ Mean of 2.8 goals per match.

(ii) Median position: $\dfrac{n+1}{2}$

$= \dfrac{20+1}{2}$

$=$ 10.5th score

To help determine the median class calculate the cumulative frequency – the 'running total' of the frequencies

Number of goals	Frequency	Cumulative frequency
0	2	2
1	3	5
2	2	7
3	6	13*
4	5	18
5	1	19
6	1	20

– There are 7 scores less than or equal to 2 goals
– There are 13 scores less than or equal to 3 goals
* Hence the 10.5th score occurs here.

$\therefore$ The median is 3.

(iii) The mode = 3. The **highest frequency** being 6 is when 3 goals are scored.

(iv) By calculator

Standard deviation = 1.568

(v) IQR = 4 – 1.5
= 2.5

(b) The '5' represents 5 times 4 goals were scored.

11.6 From a particular experiment, the length of 43 leaves was measured and the results recorded in the table below.

Length (mm)	20–24	25–29	30–34	35–39	40–44	45–49
Frequency	4	7	8	12	9	3

(a) What does the '8' represent in the table above?
(b) Calculate the mean length of the leaves.
(c) Determine the modal class.
(d) Calculate the median length showing how you obtained your answer.
(e) Calculate the standard deviation.
(f) Calculate the interquartile range.

(a) The 8 represents the fact that there were 8 leaves measured between 30 and 34mm in length.

(b) Find the midpoints of each interval

Midpoint (m)	22	27	32	37	42	47
Length (mm)	20–24	25–29	30–34	35–39	40–44	45–49
Frequency (f)	4	7	8	12	9	3

By calculator: Enter both midpoints and frequency into calculator to find the mean.

$\bar{x} = 34.79$

By formula:

$$\bar{x} = \frac{\Sigma f_i m}{\Sigma f_i}$$

$$= \frac{(22 \times 4) + (27 \times 7) + (32 \times 8) + (37 \times 12) + (42 \times 9) + (47 \times 3)}{43}$$

$$= \frac{1496}{43}$$

$= 34.79$ Average length of leaf is 34.79mm

(c) Most frequent length of leaf has a frequency of 12.

$\therefore$ Modal class is 35–39mm.

(d) * change the intervals to be continuous. See table below

Position of the median is:

$\dfrac{n+1}{2}$ th score

$= \dfrac{43+1}{2}$

= 22nd score

* Continuous interval	19.5–24.5	24.5–29.5	29.5–34.5	34.5①–39.5 ②	39.5–44.5	44.5–49.5
Length (mm)	20–24	25–29	30–34	35–39	40–44	45–49
Frequency	4	7	8	12③	9	3
Cumulative frequency	4	11	19	④		

Median is found in the 35–39 interval.

Calculation of median

$= \dfrac{3^{④}}{12^{③}} \times 5^{②} + 34.5^{①}$

$= 35.75$

① Lower bound of the continuous interval
② The width of the continuous interval
③ The frequency of the interval where the median is located
④ How far into the class interval the median is located

(e) Standard deviation by calculator: 7.013

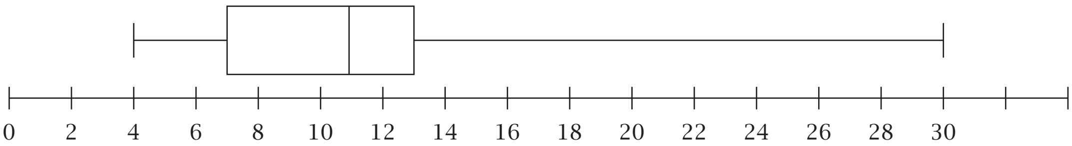
(f) IQR by calculator: 42 – 27 = 15

11.7 Given the following set of scores: 4, 6, 8, 10, 11, 12, 14, 30

(a) construct a boxplot

(b) determine if any outliers exist.

(a) min: 4, max: 30, median: 10.5, Q1: 7, Q3: 13



(b) Outliers:

IQR = 6

$Q_3 + 1.5$ IQR ≤ score ≤ $Q_1 - 1.5$ IQR

13 + 1.5(6) ≤ score ≤ 7 – 1.5(6)

22 ≤ score ≤ –2

Outlier is 30.

PROBLEMS TO SOLVE

Chapter 11: Statistics

1. Classify the following variables as
   (i) categorical or numerical
   (ii) nominal, ordinal, discrete or continuous

   (a) Eye colour
   (b) House numbers
   (c) Weight of a person
   (d) Star movie rating
   (e) The number of people who live in a house
   (f) Country of birth
   (g) The length of time waiting in a supermarket queue.

2. A survey was conducted on preferred car colours. The following categorical data is shown in the table below:

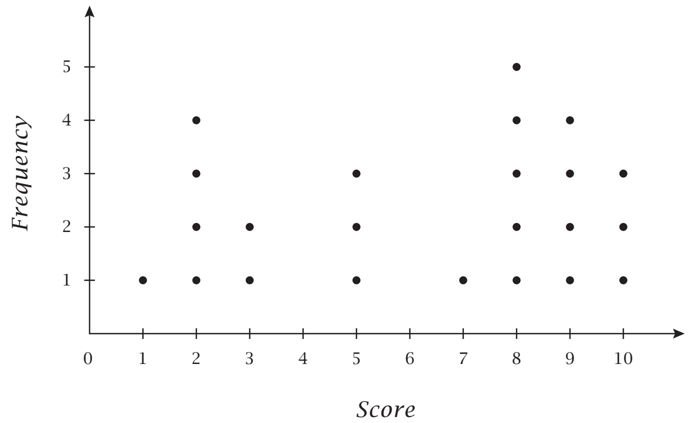
Car Colour	Frequency
Green	5
White	25
Red	20
Black	35
Blue	10
Silver	15

   Construct a column graph to represent this data.

3. From the stem and leaf plot below calculate the:
   (a) mean
   (b) median
   (c) mode
   (d) standard deviation
   (e) IQR.

Stem	Leaf
12	0 1 2 5
13	1 4 6
14	2 5 8 9
15	0 2 2 7
16	1 4 8
17	6

4. For the dot frequency graph drawn below, determine each of the following



(a) the total number of scores

(b) the mean

(c) the mode

(d) the median

(e) the range

(f) the percentage relative frequency of a score equal to 7

(g) identify any outliers, gaps or clusters

(h) IQR

(i) standard deviation.

5. The number of videos watched by a group of 40 adults over a 1 week period is summarised in the table below.

Number of videos	Frequency
0	1
1	4
2	15
3	8
4	6
5	4
6	2
	40

Construct a relative frequency histogram of the information in the table above.

6. The weights in kilograms (kg) of 50 students are shown below:

53	43	38	52	52	54	60	41	37	35
44	50	53	58	48	49	47	38	47	44
40	46	53	61	58	47	47	39	43	54
58	57	47	46	45	55	50	45	45	46
30	56	53	51	40	54	58	58	57	53

(a) Using class intervals 30–34, 35–39 etc., construct a frequency table.

(b) Draw a frequency histogram to represent the above table.

7. The number of typing mistakes per page on a word document was analysed and the results were recorded in the table below.

Number of typing mistakes per page	0	1	2	3	4	5	6	7
Frequency	3	2	5	1	4	6	8	1

(a) How many pages were analysed?

(b) Calculate the

(i) mean

(ii) mode

(iii) median (number of typing mistakes per page)

(iv) range

(v) interquartile range

(vi) standard deviation.

8. The number of runs scored in a cricket season is given in the table below.

Cricket runs scored	Frequency
0–9	9
10–19	13
20–29	25
30–39	46
40–49	18
50–59	13
60–69	2
70–79	1
80–89	3

(a) Calculate the mean number of runs scored.

(b) Determine the modal class.

(c) Calculate the median class.

(d) Estimate the median. Explain how you arrived at your answer.

(e) Calculate the standard deviation.

9. For each of the following calculate

i. the mean.

ii. the median class interval.

iii. the median.

iv. the standard deviation.

(a)

Score	1–12	13–24	25–36	37–48	49–60	61–72	73–84	85–96	97–108	109–120
Frequency	2	7	9	15	26	26	38	19	11	5

(b)

Score	10–19	20–29	30–39	40–49	50–59	60–69
Frequency	1	3	4	9	11	3

10. The test results for John for the past year are:

83, 72, 94, 78, 32, 84, 90, 87

(a) Which measure of central tendency gives an accurate measure of his progress? Explain.

(b) What mark must John attain in his final test so that his yearly average is 78%?

11. The number of points scored by two basketballers on opposing teams over a 6 week period are listed below

Player 1: 10, 12, 15, 11, 14, 14
Player 2: 16, 15, 16, 13, 2, 14

(a) Calculate the mean and range for each player.

(b) Who is the most consistent player? Comment with regard to the statistical information found in part (a).

12. The number of students late to science class was recorded in the frequency table below.

No of students late	0	1	2	3	4	5
Frequency	2	4	3	8	5	5

(a) In how many science classes was student lateness recorded?

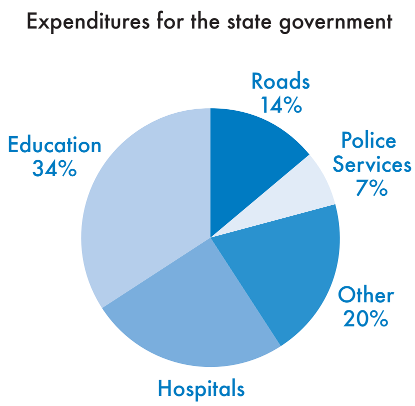
(b) Which measure of central tendency – mean or median best describes the punctuality of students attending science classes? Explain.

13. Based on the circle graph given below, answer the following questions.

(a) Determine what percentage is spent on hospitals.

(b) If the total spending is $150 000, how much was spent on police services?

(c) What fraction of the total expenditure is spent on education and roads combined?



14. The following table shows the marks obtained by the same 150 students sitting the maths A and maths B examination papers.

Mark	1–10	11–20	21–30	31–40	41–50	51–60	61–70	71–80	81–90	91–100
No of students Maths A	2	6	20	28	31	2	17	11	8	7
No of students Maths B	3	12	22	31	33	15	14	9	9	2

(a) Calculate the mean for Maths A and Maths B papers

(b) Estimate the median of both Maths A and Maths B papers

(c) Comment on the difficulty of the two papers.

15. If the average weekly wage for a business is \$520.65, determine the total wage bill if the business has 15 employees.

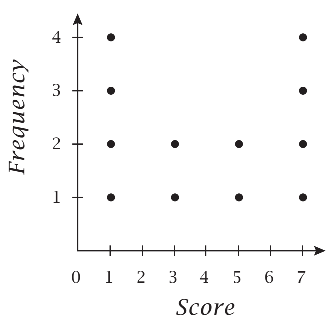
16. If the average for an English test for the 8 girls was 74% and for the 12 boys was 57%, calculate the combined average.

17. The class average for the 15 students who sat the mathematics exam was 56%. Unfortunately Sam was absent on the day but she scored 82%. Calculate the new class average for the maths exam.

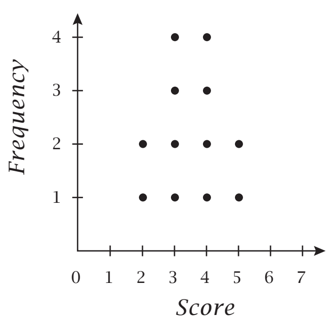
18. For each of the dot frequency plots drawn below, rank each from lowest to highest according to:

(i) the mean (ii) the median (iii) standard deviation

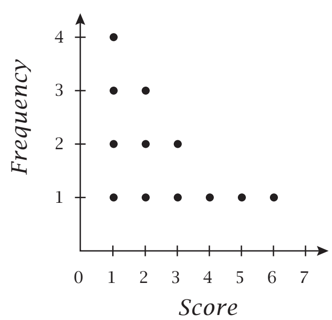
(a)



(b)



(c)



19. Find the mean, mode, median and range for each of the following scores.

(a) 5, 8, 1, 3, 4, 5, 6, 5, 1, 4

(b) 50, 80, 10, 30, 40, 50, 60, 50, 10, 40

(c) 10, 13, 6, 8, 9, 10, 11, 10, 6, 9

(d) Compare your answers in part (b) and (c) to part (a). What do you notice?

20. A set of scores is given by the letters a, b, c, d, e, f. Suppose these scores have a mean of 5 and a range of 12. Determine the new mean and range for each of the following sets:

(a) $a + 4$, $b + 4$, $c + 4$, $d + 4$, $e + 4$, $f + 4$

(b) $2a$, $2b$, $2c$, $2d$, $2e$, $2f$

(c) $3a - 1$, $3b - 1$, $3c - 1$, $3d - 1$, $3e - 1$, $3f - 1$

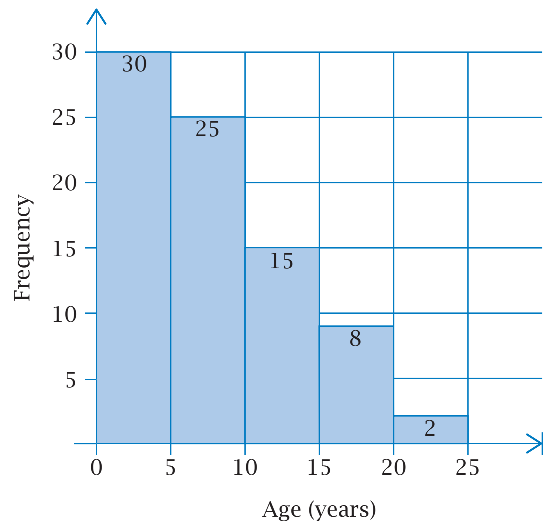
21. For the following data set

a 5 6 6 b c 12 13 14 d 17 24

the mean = 11.25
range = 21
median = 11
interquartile range = 9.5

determine the values of a, b, c, and d.

22. The histogram below shows the age of cars (in years) driven to school by students.



(a) Calculate the

(i) mean age of the cars

(ii) standard deviation of the age

(b) Describe the distribution.

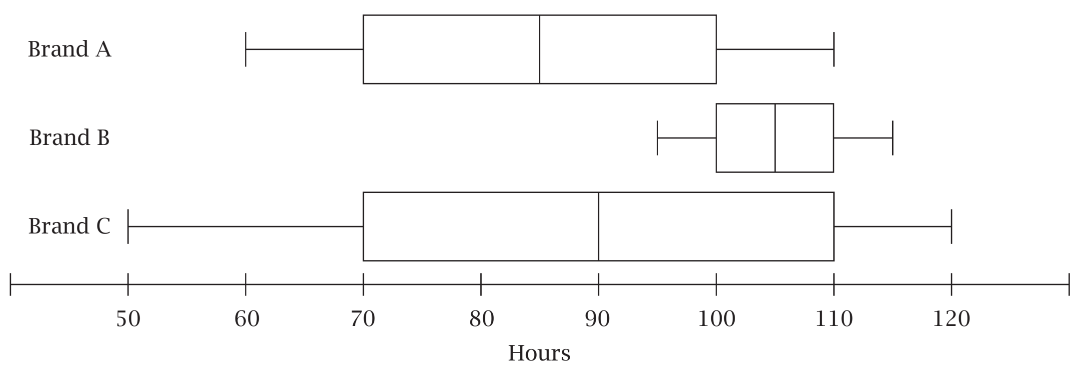
23. Research was conducted on the rental prices in the local suburb. The results for the 20 properties surveyed are listed below.

\$460, \$500, \$650, \$550, \$490, \$700, \$1050, \$600, \$750, \$650, \$500, \$480, \$500, \$550, \$600, \$220, \$660, \$590, \$700, \$650

(a) Explain how to identify any outliers and calculate the outliers, if any.

(b) State the effect on the mean and standard deviation if the outliers are removed.

24. A study was conducted on 3 brands of light bulbs. The length of time (in hours) each bulb lasted was recorded for each brand and the results displayed as parallel boxplots.



(a) What was the median length of time for Brand B?

(b) What is the range of time for Brand A?

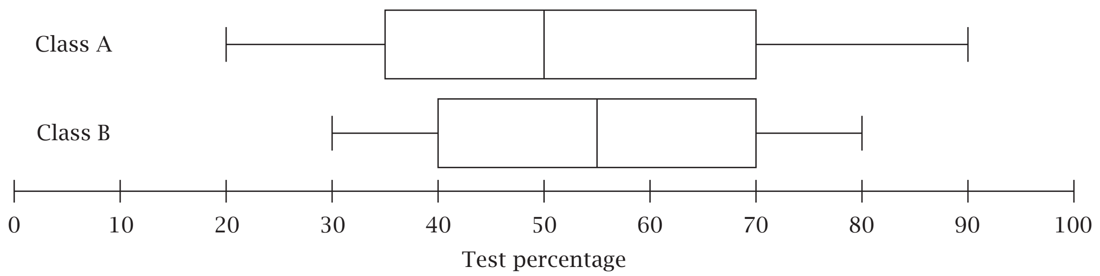
(c) Which light bulb is most effective? Explain.

(d) What percentage of bulbs lasted:

  (i) between 110 and 120 hours for Brand C?

  (ii) less than 100 hours for Brand A?

25. The boxplots below show the test results from Class A and Class B. Each class had the same number of students and an identical test.



Compare the distributions shown in the boxplots above.

NORMAL DISTRIBUTION

12

Syllabus Checklist

By the end of this chapter, you should be able to:

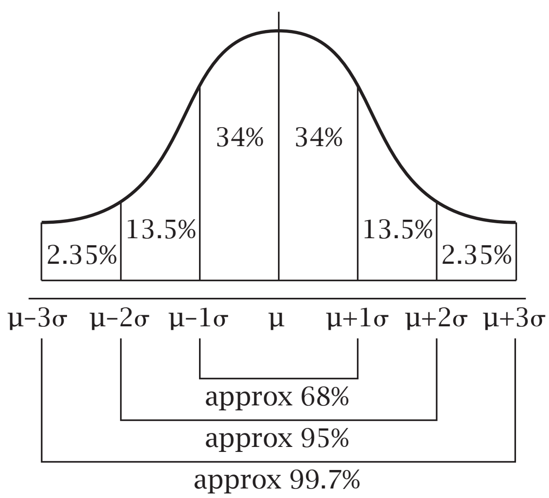
- ☐ use the number of deviations from the mean – standard score
- ☐ calculate quantiles for normally distributed data
- ☐ use the 68%, 95% and 99.7% rule
- ☐ calculate the probabilities for normal distributions

FORMULAE AND DEFINITIONS

The normal distribution

The normal distribution has the following properties:

- half of the population is above the mean and half is below the mean.
- the curve is symmetrical.
- just over $\frac{2}{3}$ (68%) of the population is within one standard deviation of the mean, 95% within two standard deviations and 99.7% within three standard deviations of the mean.
- the area under the curve is 1.
- when standardised, the distribution has a mean of 0 and standard deviation of 1.



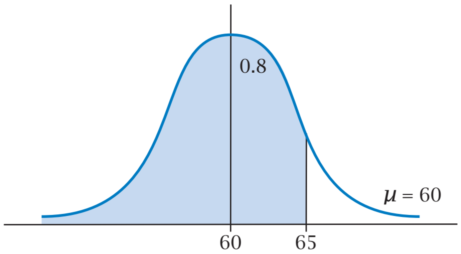
Standard scores (*z* scores)

Scores are standardised when they have a mean of 0 and a standard deviation of 1. The value of z obtained determines the number of standard deviations above or below the mean a particular score is.

If X has a mean of μ and a standard deviation σ then the z score $= \dfrac{x-\mu}{\sigma}$.

Note: all values can be obtained from tables or a calculator.

Quantiles and percentiles



Area under the curve below 65 is 0.8. This is known as the 80th percentile and 65 is the 0.8 quantile.

Well known quantiles are:

- 0.25 quantile known as the lower quartile.
- 0.75 quantile known as the upper quartile.

Worked Examples

12.1 X is a normally distributed variable with a mean of 20 and a standard deviation of 3. Find, *without using a calculator*:

(a) $P(X < 17)$

(b) $P(14 < X < 26)$

(c) $P(17 < X < 26)$

(a) $P(X < 17)$

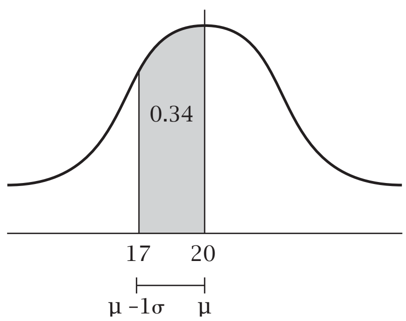
Mean = 20

Standard deviation = 3

One standard below the mean = 17

Using the 68%, 95%, 99.7% rule

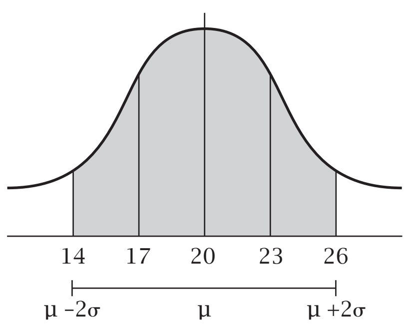
$P(X < 17) = 0.5 - 0.34$
$= 0.16$



(b) $P(14 < X < 26)$

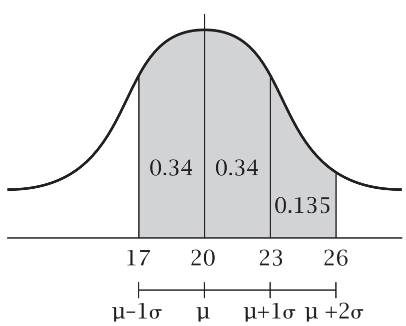
2 standard deviations from the mean

$P(14 < X < 26) = 0.95$



(c) $P(17 < X < 26)$

$P(17 < X < 26) = 0.34 + 0.34 + 0.135$
$= 0.815$



12.2 Let X be a variable which is normally distributed with a mean of 10 and a standard deviation of 2. Calculate the following:

(a) $P(X \leq 11)$

(b) $P(8 \leq X \leq 12)$

(c) $P(X \geq 8.5)$

(d) k, given $P(X \geq k) = 0.9192$

(e) the 0.25 quantile

(f) the 75th percentile

Use calculator to obtain solutions

$X \sim N(10, 2^2)$

(a) $P(X \leq 11) \approx 0.6915$

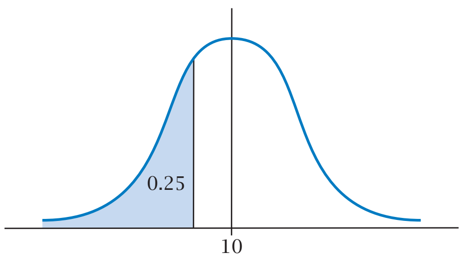
(b) $P(8 \leq X \leq 12) \approx 0.6827$

(c) $P(X \geq 8.5) \approx 0.7734$

(d) k given $P(X \geq k) \approx 0.9192$

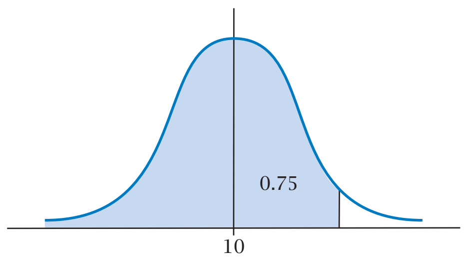
$k \approx 7.2006$

(e)



Using calculator: 0.25 quantile is 8.6510

(f)



Using calculator: 75th percentile is 11.3490

12.3 John has joined the local 'Little Athletics Club' as a specialist discus thrower. His throws are normally distributed with a mean of 26.0 metres and a standard deviation of 2 metres.

(a) What is the probability that John throws between 23 and 30 metres?

(b) Determine the probability he throws no less than 25 metres.

(c) Determine the interquartile range.

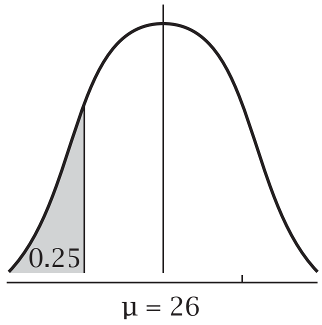
Use calculator to determine the solutions

$X \sim N(26, 2^2)$

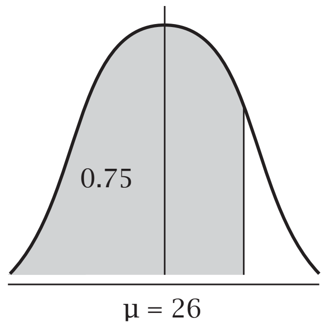
(a) $P(23 \leq X \leq 30) \approx 0.9104$

(b) $P(X \geq 25) \approx 0.6915$

(c)



25th percentile is 24.6510



75th percentile is 27.3490

Interquartile range = 75th percentile – 25th percentile
= 2.698

12.4 Fred achieved the following results in his Semester 1 examinations:

Mathematics	76
English	53
Chemistry	45
Physics	66

All examination results produced the following statistics

Mathematics – Mean: 70
Standard deviation: 12

English – Mean: 55
Standard deviation: 15

Chemistry – Mean: 55
Standard deviation: 22

Physics – Mean: 62
Standard deviation: 10

Standardise each of Fred's scores and rank his subjects from best to worst.

Mathematics: $z = \frac{x - \mu}{\sigma}$

$= \frac{76 - 70}{12}$

$= 0.5$

English: $z = \dfrac{x - \mu}{\sigma}$

$= \dfrac{53 - 55}{15}$

$= -0.1\dot{3}$

Chemistry: $z = \dfrac{x - \mu}{\sigma}$

$= \dfrac{45 - 55}{22}$

$= -0.\overline{45}$

Physics: $z = \dfrac{x - \mu}{\sigma}$

$= \dfrac{66 - 62}{10}$

$= 0.4$

Rank
1. Mathematics
2. Physics
3. English
4. Chemistry.

PROBLEMS TO SOLVE

Chapter 12: Normal Distribution

1. Determine the standard score for each of the following:
   (a) A mark of 52 in a test with a mean of 50 and a standard deviation of 5
   (b) A mark of 46 in a test with a mean of 60 and a standard deviation of 4
   (c) A mark of 88 in a test with a mean of 66 and a standard deviation of 11.

2. John's results for each of the six tests in Mathematics Applications as well as the test means and standard deviations are listed below.

Test	John's mark	Mean	Standard deviation
1	60	65	4
2	68	62	8
3	75	71	6
4	43	50	5
5	51	53	10
6	56	53	9

   (a) Standardise each of John's test marks and rank his tests from best to worst.
   (b) Find a common mark in Test 1 and Test 2 that has the same standardised score.

3. If $X \sim N(30, 5^2)$ determine *without a calculator*:
   (a) $P(X = 30)$
   (b) $P(25 < X < 35)$
   (c) $P(20 < X < 30)$
   (d) $P(20 < X < 35)$
   (e) t, such that $P(X \leq t) = 0.025$.

4. If $X \sim N(70, 4^2)$ determine:
   (a) $P(X < 58)$
   (b) $P(X > 72)$
   (c) $P(68 < X < 75)$
   (d) k, given $P(X \geq k) = 0.9$
   (e) 45th percentile.

5. Scores are normally distributed with a mean of 28 and a standard deviation of 3.75. If the 75th percentile is 35, determine the interquartile range, *without the use of a calculator*.

6. The weight of bricks is normally distributed with a mean of 200g and a standard deviation of 25g.

   (a) Determine *without using a calculator* the probability that a randomly chosen brick:

      (i) weighs exactly 210g

      (ii) weighs between 150g and 250g

      (iii) weighs between 125g and 200g

   (b) Determine the weight, k, such that 4 out of 25 randomly chosen bricks weighs more than k.

7. The number of kilometres a new tyre lasts is normally distributed with a mean of 55 000km and a standard deviation of 10 500km.

   (a) What percentage of tyres will last more than 70 000km?

   (b) What is the probability that a tyre randomly selected will last between 52 000 and 58 250km?

   (c) Determine the distance that will be exceeded by 97% of all tyres.

8. A certain species of spider has a length which is normally distributed with a mean length of 17mm and a standard deviation of 2.25mm. A spider is selected at random.

   (a) What is the probability the spider is more than 20mm?

   (b) What is the probability the spider's length is within 3mm of the mean length?

   (c) What is the maximum length of 95% of the spiders?

9. The lengths (L cm) of fish are normally distributed with a mean of 24.6cm and a standard deviation of 2.7cm.

   (a) If a fish is caught at random, determine:

      (i) $P(L > 25)$

      (ii) $P(22 < L < 27)$

   (b) If 120 fish are caught, determine the expected number that are shorter than 21cm

   (c) One quarter of the fish are longer than t cm. Determine the value of t.

10. A machine fills bags of flour and the weights are normally distributed with a mean of 500g and a standard deviation of 12g.

   (a) Determine the probability that a randomly selected bag weighs:

      (i) more than 520g

      (ii) between 495 and 507g

   (b) 7.5% of the bags weigh less than k grams. Find the value of k

   (c) Determine the median weight of the bags

   (d) Calculate the interquartile weight of the bags of flour.

RESOURCE FREE: TRIAL TEST 1

Calculators NOT allowed
Time Allowed: 20 minutes
Total Marks: 20

1. If $F = \dfrac{2T^2 - P}{\sqrt{L}}$

   Calculate

   (a) F if $T = 3$, $P = 6$ and $L = 4$

   (b) L if $T = 1$, $P = -6$ and $F = -4$

   [3]

2. Tarjet offer a store-wide discount of 15% off everything.

   (a) If a music video was marked at $30.00, calculate the sale price.

   (b) A computer game was marked down from $101.00 to $86.86.
   Was the correct percentage given?

   [3]

3. Simplify:

   (a) $10y - 15y$

   (b) $(4x^2)(3x)(-2xy)$

   [2]

4. Solve the following:

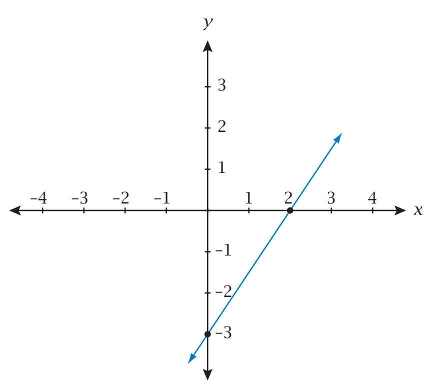
(a) $7 - 2x = -15$

(b) $-5(3 - h) = -10$

(c) $4(m + 3) - (2m + 5) = 10$

[3]

5. The graph of a linear function is drawn on the axes below:



Determine:

(a) The gradient.

(b) The vertical intercept.

(c) The equation of the line.

(d) Using the equation of the line complete the table below.

x	–2	–1	0	1	2	3
y						

(e) Find the 1st difference pattern.

(f) Compare the 1st difference pattern with the gradient in part (a) above. What do you notice?

(g) Draw the graph of the line $x + y = 2$ on the graph above.

(h) Solve simultaneously using algebra the equations in part (c) and part (g). What do you notice? Comment in relation to the graphs of the two lines.

[9]

RESOURCE FREE: TRIAL TEST 2

Calculators NOT allowed
Time Allowed: 25 minutes
Total Marks: 25

1. A recipe for Chilli lamb chops to serve 8 people requires the following ingredients:

- 1 tablespoon of oil
- 2 onions
- ½ cup oyster sauce
- 16 lamb chops
- 3 tablespoons of chilli paste
- ⅔ cup tomato paste

Determine the quantity of each ingredient required for 20 people.

[3]

2. If $A = \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix}$

$B = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

$C = \begin{bmatrix} -2 & 5 \\ 1 & -3 \end{bmatrix}$

$D = \begin{bmatrix} 2 & 5 \end{bmatrix}$

Evaluate, giving a reason if the calculation is not possible

(a) $A - 3C$

(b) DA

(c) BC

[5]

3. For the following data set:

a, 3, 3, b, 6, c, 7, d, 10, 12

The range is 10
median is 6
interquartile range is 5
mean is 6.2

Determine the values of a, b, c and d.

[5]

4. For the following problem:

(i) define the variables

(ii) set up a pair of simultaneous equations

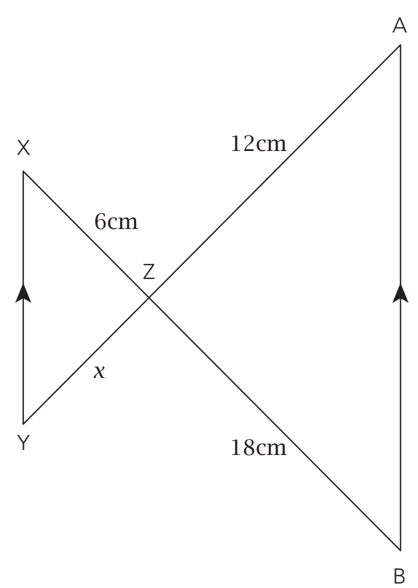
(iii) solve the simultaneous equations

(iv) answer the question

The sum of two numbers is 6 while three times the smaller added to twice the larger is five. Find both numbers.

[4]

5. Given the diagram below and the area $\triangle AZB = 108\text{cm}^2$



find

(a) the similar triangles and state the condition of similarity

(b) the value of x

(c) the area of $\triangle XYZ$

[5]

6. The number of litre bottles of milk sold by a shop per day is represented by a normal distribution with a mean of 140 and standard deviation of 15:

(a) Determine the probability that tomorrow

(i) between 125 and 155 1L bottles of milk will be sold

(ii) less than 125 1L bottles of milk will be sold

(b) Calculate the number sold 's' such that the probability of at least 's' is 0.025.

[3]

RESOURCE FREE: TRIAL TEST 3

Calculators NOT allowed
Time Allowed: 25 minutes
Total Marks: 25

1. (a) In triangle PQR, $\sin Q = 0.6$, $PQ = 5$cm, $PR = 8$cm and $QR = 7$ cm. Determine the area of the triangle.

   (b) In triangle XYZ, $\sin X = 0.4$, $\sin Z = 0.5$ and $XY = 15$cm. Determine the length of YZ.

   [5]

2. Expand and simplify the following:

   (a) $(x+7)(x-5)$

   (b) $-6(7x-2)(2x+3)$

   [3]

3. A piece of rope is 100cm long. State an algebraic expression if:

   (a) John cuts off x cm, how much rope remains?

   (b) John cuts off $\frac{1}{3}$ of the remaining rope, how much rope has been cut off?

   [2]

4. The cost of repairs for a car can be worked out using the formula
$C = 100t + 120$ where t = time in hours and C = cost in dollars

(a) What does the value '120' represent?

(b) What does the '$100t$' represent?

(c) If the repairer worked for 6 hours determine the cost.

(d) If the cost to repair the car was \$1350,
how many hours was required to fix the car?

[4]

5. A carpet 5m by 6m cost \$360.
Find the cost of a 2m × 3m carpet of the same quality?

[2]

6. (a) Find the values of x, y and z such that:

$$\begin{bmatrix} 10 \\ y \\ -3x + 2y \end{bmatrix} = \begin{bmatrix} x - 3y \\ 2x - 5 \\ z \end{bmatrix}$$

(b) Find matrix T such that:

$$\begin{bmatrix} 1 & 2 \\ -6 & 4 \end{bmatrix} - \tfrac{1}{2}\, T = \begin{bmatrix} 7 & 3 \\ -1 & 4 \end{bmatrix}$$

[5]

7. A survey was completed in a classroom of the number of children in a family.
The results are displayed in the table below:

Number of Children in a Family	Frequency
1	5
2	6
3	7
4	1
12	1

(a) Calculate:

(i) the mean number of children in a family.

(ii) the median number of children in a family.

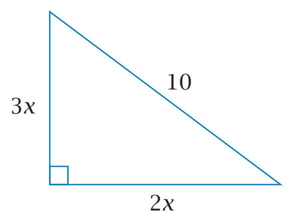
(b) Which measure of central tendency gives a better representation of the average number of children in a family?

[4]

RESOURCE FREE: TRIAL TEST 4

Calculators NOT allowed
Time Allowed: 20 minutes
Total Marks: 20

1. Find the exact value of x by using Pythagoras' Theorem.



[3]

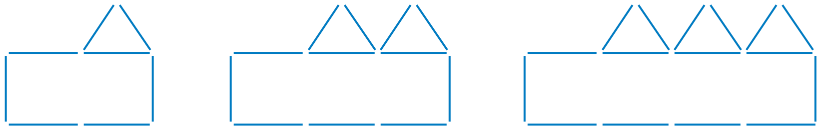
2. Factorise the following:

(a) $-4x + 8$

(b) $15x^2 - 25xyz$

[2]

3. The diagrams below are made from matchsticks.



(a) Use the diagrams above to complete the table below:

Diagram number (d)	1	2	3	4	5
Number of matchsticks (m)	8				

(b) Describe the pattern in the number of matchsticks needed.

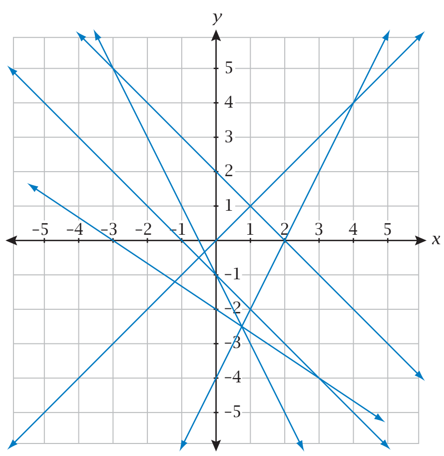
(c) Find a rule that connects the diagram number (d) with the number of matchsticks (m).

(d) Use your rule to find the number of matchsticks needed for the 12th diagram.

(e) Use your rule to find the diagram number if 84 matchsticks are used.

[5]

4. Use the graphs drawn below to solve the following simultaneous equations:



(a) $y = x$ *and* $y = -x - 1$

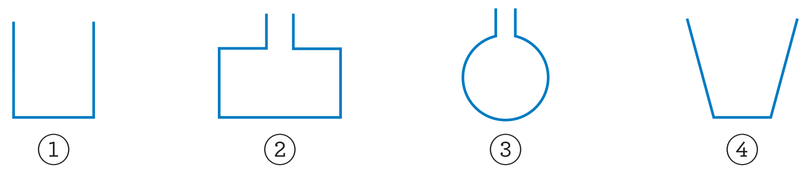
(b) $y = 2x - 4$ *and* $y = -x - 1$

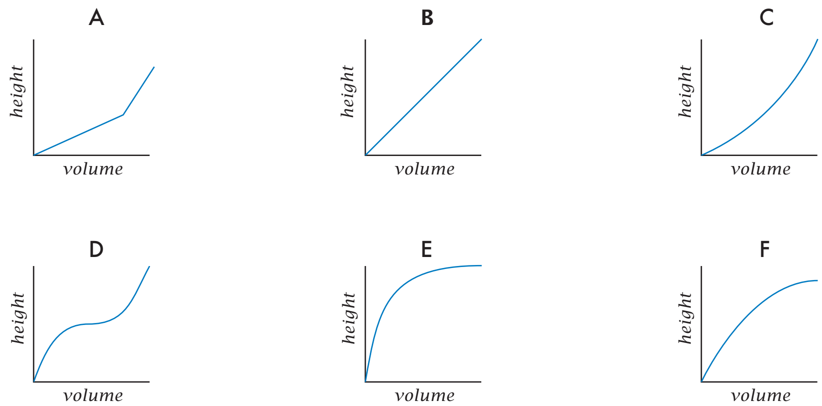
(c) $2x + y = -1$ *and* $x + y = 2$

[3]

5. Water is poured into the following containers at a steady rate.

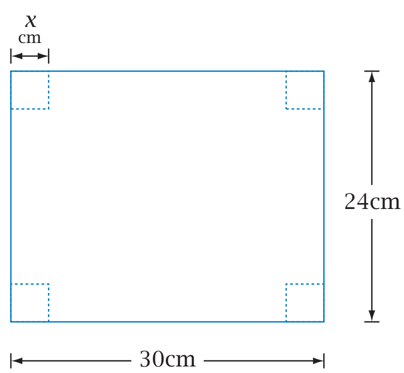
For each container below write down the letter of the graph which best shows the relationship between the height of the liquid and the volume of the container.





[4]

6. A template for a box is to be cut out of a rectangular piece of card 24cm by 30cm. A square of length x cm is to be removed from each of the corners to allow for ease of folding. The diagram below illustrates the template.



Determine *(with the squares removed)*:

(a) an expression for the length

(b) an expression for the width and

(c) an expression for the area of the card.

[3]

RESOURCE RICH: TRIAL TEST 5
FINANCE AND PERCENTAGE

Calculators allowed
Time Allowed: 40 minutes
Total Marks: 40

1. Jo is paid a weekly retainer of $350 plus a 5% commission on sales.
If she sells goods worth $700 in a week, calculate her weekly pay.

[2]

2. Kylie purchased an old table for $50. She spent $37.50 on parts for repair and $12 on paint. She then advertised in the local newspaper at a cost of $21.00 and sold the refurbished table for $125.00

Determine:

(a) the profit or loss Kylie made on the table

(b) the percentage profit or loss.

[4]

3. By selling books for $476 a saleswoman lost 15% on her outlay.
What should she have sold the books for in order to obtain a 10% profit?

[4]

4. Which purchase of lemonade is the best buy?

A	B	C
600mL	375mL	1L
cost	cost	cost
\$2.40	\$1.80	\$3.75

[7]

5. A property tycoon buys land for \$250 000. He subdivides the land into 5 blocks. The cost of subdivision is \$110 000, the council rates \$900p.a. per block, the upkeep on each block \$300p.a. per block and the real estate agents commission, 2% on the sale price of each block.

If the blocks were sold 3 years later for \$210 000, \$185 000, \$150 000, \$142 000 and \$95 000 determine the percentage profit or percentage loss.

[5]

6. (a) Determine the average speed (in km/h) and fuel consumption (in km/L) of a car travelling 300km for 3 hours and using 50 litres of petrol.

(b) If a truck travels a distance of 1260km at an average speed of 84km/h and uses 94.5 litres of diesel, calculate the fuel consumption (in L/100km) and the time taken (in hours) for the journey.

[4]

7. A man bought a surfboard for \$350. By selling the surfboard later at a discount of 30% on the list price, he made a profit of 20% on the cost. Find the list price.

[4]

8. A pizza is purchased at $9.90 including a 10% GST.
Determine the price of the pizza without GST.

[2]

9. A discount of 17% reduced the price of a book by $10.20. Determine:

(a) the original price of the book

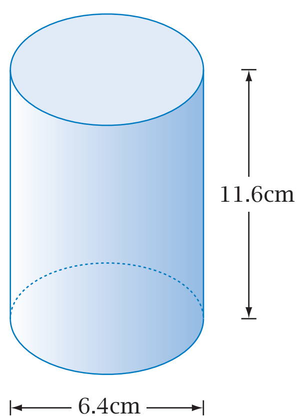
(b) the discounted price of the book.

[4]

10. Cans of soft drink have a height of 11.6cm and a diameter of 6.4cm.
The can is filled with drink at a rate of $30cm^3$/second.

(a) How long will it take to fill a can with soft drink?

(b) If there are 24 cans of soft drink to a carton,
how long will it take to fill all cans in a carton with soft drink.



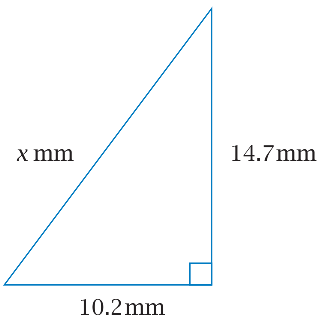
[4]

RESOURCE RICH: TRIAL TEST 6
PYTHAGORAS

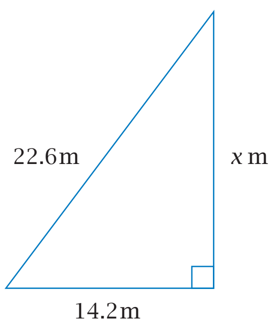
Calculators allowed
Time Allowed: 40 minutes
Total Marks: 40

1. Find the value of x in each of the following, correct to 2 decimal places where necessary.

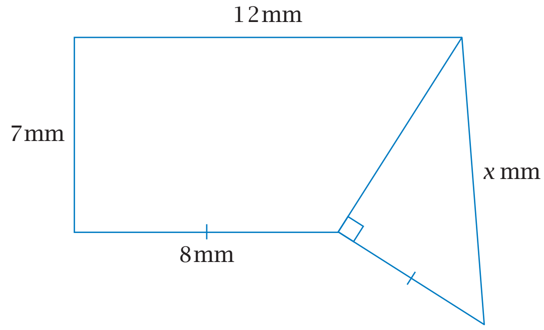
(a)



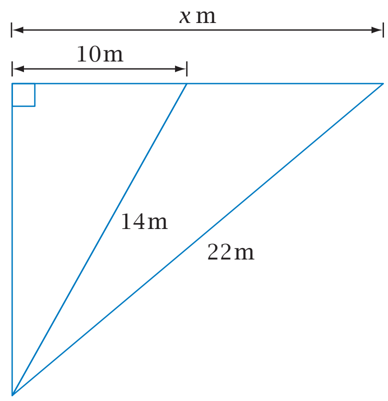
(b)



(c)



(d)



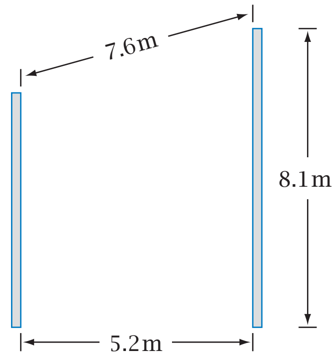
[10]

2. Determine whether or not the triangle ABC with lengths as indicated below, is right angled.

$$AB = 2.5\text{m}$$
$$BC = 6.8\text{m}$$
$$AC = 4.3\text{m}$$

[3]

3. Two flagpoles 5.2m apart are joined at the top by a 7.6m support wire.
If the longest flagpole is 8.1m long, determine the length of the shortest flagpole.



[3]

4. The length of a rectangle is three times its width and its diagonal measures 15m.

Determine:

(a) The width.

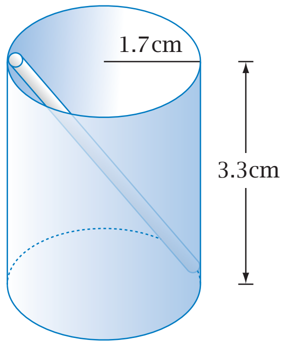
(b) The length.

(c) The area of the rectangle.

[5]

5. Determine the length of a straw that will fit inside a cylinder with height 3.3cm and radius of 1.7cm. Answer to 1 decimal place.

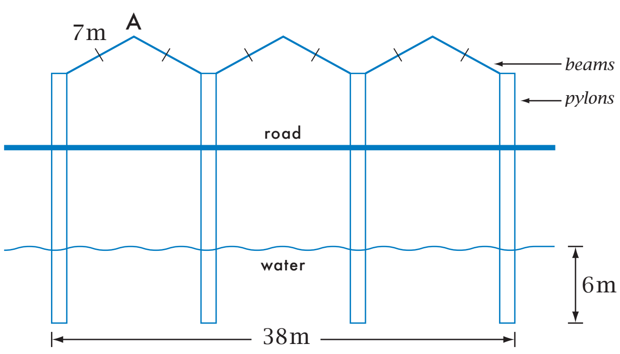
(a)



(b) If the same length straw was placed inside a rectangular box 2cm by 3cm, determine the height of the box so that the straw fits entirely inside the container. Answer correct to 2 decimal places.

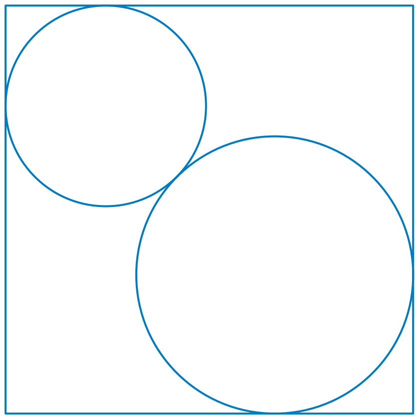
[7]

6. The diagram below shows a bridge supported by 4 pylons. The road is 14m above the water line and the 4 pylons are 2m wide and 25m high. The beams of the bridge are of equal length and measure 7 metres. Determine the distance from the top of the bridge (point *A*) to the road.



[5]

7. Two circles of radii 5cm and 7cm respectively are drawn inside a square.
Find correct to 2 decimal places the area of the square.



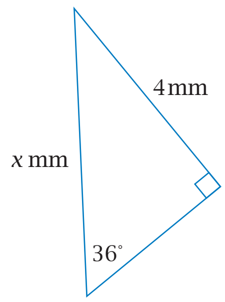
[7]

RESOURCE RICH: TRIAL TEST 7
TRIGONOMETRY

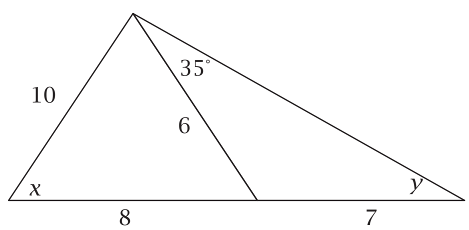
Calculators allowed
Time Allowed: 40 minutes
Total Marks: 40

1. For each of the following find the value of x correct to 2 decimal places.

(a)

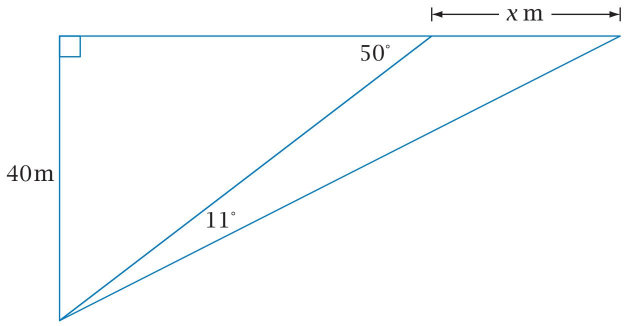


(b) Find the values of x and y in the diagram below.



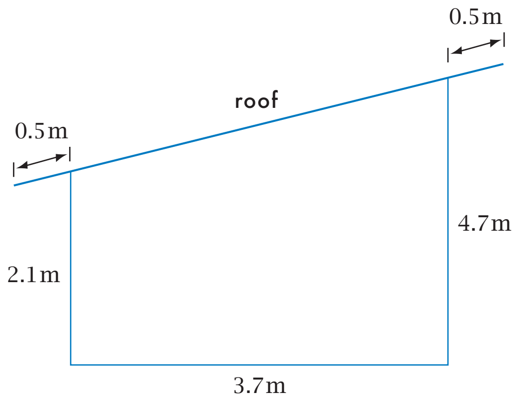
[6]

2. Find the value of the pronumeral correct to 2 decimal places for the following:



[6]

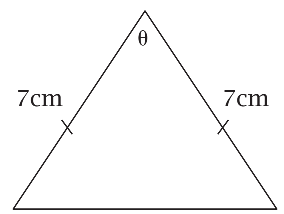
3. (a) A shed is to be built as shown in the diagram below.
Calculate the length of the sloping roof, correct to 2 decimal places.



(b) Calculate the angle of the roof to the horizontal.

[4]

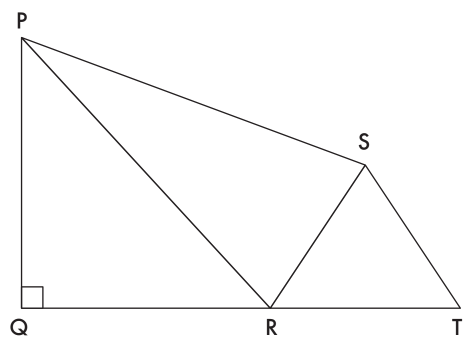
4. The isosceles triangle below has an area of $15cm^2$



Determine the value of the two equal angles.

[4]

5. The sketch shows part of a pulley system designed to lift loads from boats suspended at point *P*.



$SR = ST = 6$m and

$\angle RST = 28^\circ$

$\angle SRP = 45^\circ$

$\angle RPS = 25^\circ$

Calculate

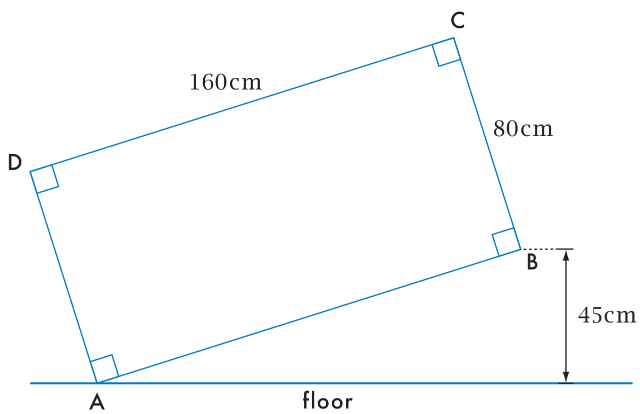
(a) length of *PR*

(b) height of *P* above the ground

(c) the length of RT

[9]

6. The diagram below shows the side view of a refrigerator tilted about A until B is 45cm above the horizontal. The dimensions of the refrigerator are 80cm by 160cm.



Determine

(a) The angle of inclination of AB to the horizontal.

(b) The height of the fridge at point C above the floor.

The refrigerator must pass through a door and for this to happen point C must be vertically above A.

(c) What increase in the angle of the fridge to the horizontal will enable this to happen?

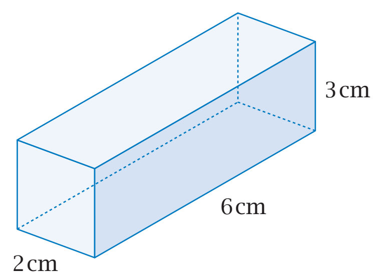
[11]

RESOURCE RICH: TRIAL TEST 8
VOLUME AND SURFACE AREA

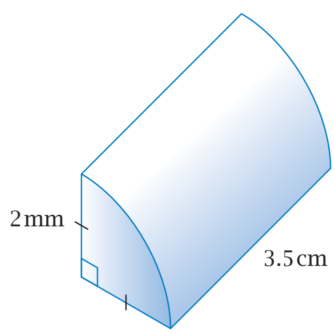
Calculators allowed
Time Allowed: 40 minutes
Total Marks: 40

1. Calculate the volume and surface area of each of the following solids.
Answers correct to 2 decimal places where appropriate.

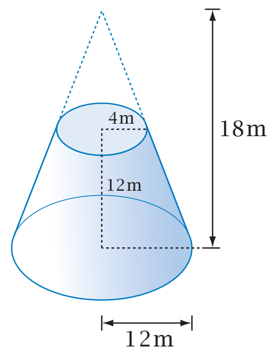
(a)



(b)



(c)

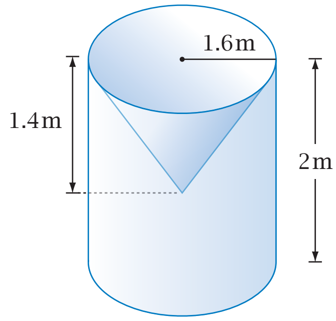


[15]

2. A hemispherical blob of gelati is placed on top of a cone.
If the volume of the gelati is $126cm^3$ determine its diameter.

[3]

3. A rain gauge is constructed out of galvanised iron according to the diagram below:



Calculate:

(a) the volume of the gauge.

(b) The cost of constructing the rain gauge if galvanised iron costs $6.50 per square metre.

[7]

4. One of the seven wonders of the ancient world "Khufu's Pyramid" has a base length and width of 230.36m and a height of 146.59m. The pyramid was constructed with 2.4 million stones. Determine the average volume of each stone.

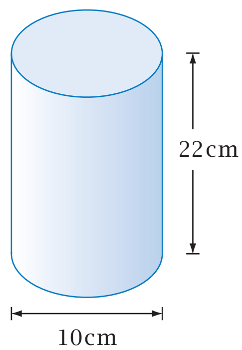
[3]

5. Plain flour is now packaged in a conical shaped container for easy storage. If the container has a diameter twice its height and the volume of flour is 80cm^3, what is the height of the container?

[4]

6. A metal cylinder has a height of 22cm and a base diameter of 10cm.

(a) Determine its volume and surface area.



The metal cylinder is melted down to form a solid hemisphere.

Determine:

(b) The radius of this hemisphere.

(c) The surface area of this hemisphere.

(d) The percentage increase or percentage decrease in the change of the surface area from cylinder to hemisphere.

[8]

RESOURCE RICH: TRIAL TEST 9
LINEAR RELATIONSHIPS

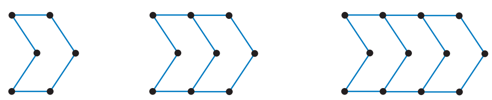
Calculators allowed
Time Allowed: 40 minutes
Total Marks: 40

1. For the pattern below:

   (i) complete the 1st four entries in the table.

   (ii) use the first difference pattern to show that there is a linear relationship between the variables.

   (iii) determine the rule for the linear relationship and complete the table.



Number of diagram (d)	1	2	3	4		20		
Number of matchsticks (m)								46

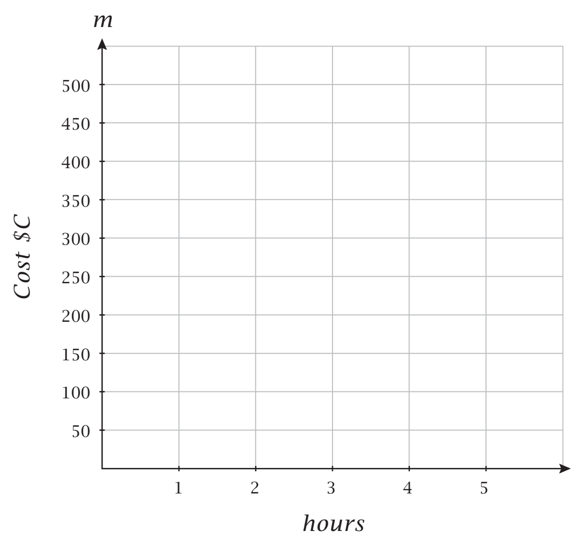
[6]

2. George, the electrician, charges a call out fee of \$90 plus an hourly fee of \$75.

   (a) Determine the rule connecting the total cost, $\$C$, with the number of hours (h) worked.

   (b) If George works for 4 hours repairing a switch, determine the total cost of repair.

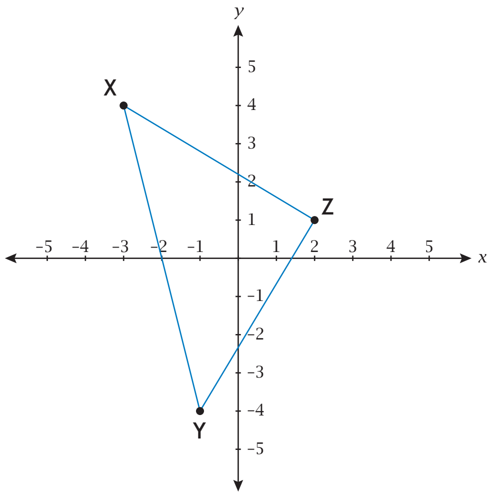
(c) Graph the relationship on the axes below.
Is it suitable to join the points? Why?



(d) Another electrician Harry charges a call out fee of $50 plus an hourly rate of $100. Is this electrician cheaper to use? Explain in detail using the graph above.

[8]

3. On the diagram below are 3 coordinate points $X(-3, 4)$ $Y(-1, -4)$ and $Z(2, 1)$ which when joined form a triangle.



(a) Find the lengths XY, XZ and YZ.

(b) Using Pythagoras's Theorem determine if $\triangle XYZ$ is right angled.

(c) What type of triangle is $\triangle XYZ$?

(d) Determine the area of $\triangle XYZ$.

[7]

4. (a) Show that the points $(3, 6)$, $(5, 14)$ and $(-1, -10)$ are collinear.

(b) Which of the points $A(2, 4)$, $B(3, 7)$ and $C(-2, -16)$ lie on the line $y = 5x - 6$?

(c) Given that the points $X(2, p)$ and $Y(q, -4)$ lie on the line $x - 2y + 4 = 0$, find the values of p and q.

(d) State whether the point $T(-2, 1)$ lies on the line passing through $(4, 3)$ and $(-11, -2)$.

[13]

5. For the following problem:

(i) define the variables

(ii) set up a pair of simultaneous equations

(iii) solve the simultaneous equations

(iv) answer the question

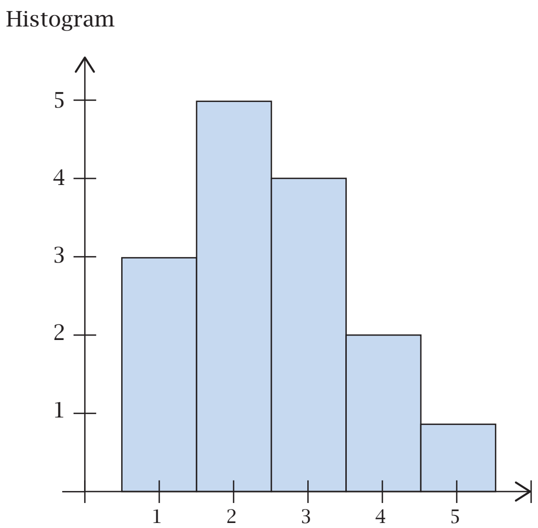
The incomes of A and B are in the ratio $5:3$, while their expenditures are in the ratio $9:5$. Each saves \$300 a month, find their monthly incomes.

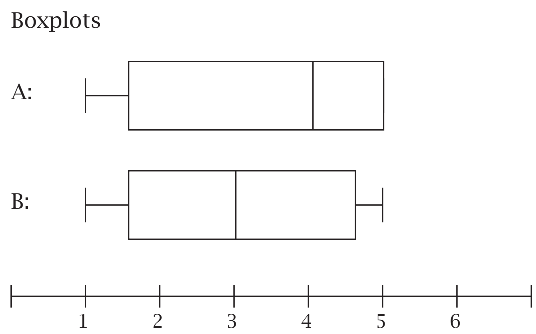
[6]

RESOURCE RICH: TRIAL TEST 10
STATISTICS

Calculators allowed
Time Allowed: 40 minutes
Total Marks: 40

1. Given the following:





(a) Match the correct boxplot to the histogram with reasons.

(b) State one advantage of using

(i) a boxplot rather than a histogram

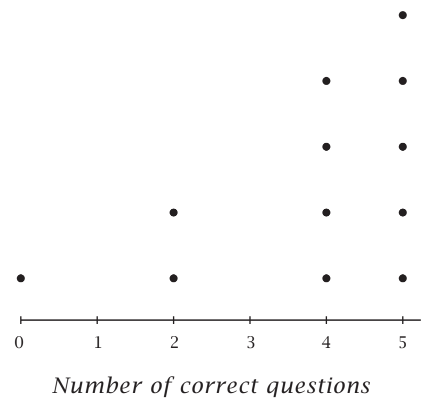
(ii) a histogram rather than a boxplot

(c) Write down a set of 8 scores that could represent Boxplot A.

[6]

2. Students in two year 7 classes completed a 5 question quiz. The results for the number of questions answered correctly are displayed for each class below.

CLASS 1



CLASS 2

Number of correct questions	Frequency
0	3
1	2
2	1
3	0
4	1
5	5

(a) Comment on any similarities or differences between the two classes.

(b) What advantages or disadvantages are there in displaying the data as either a dot frequency plot or a frequency table?

(c) Are there any outliers, gaps or clusters found in any of the two classes?

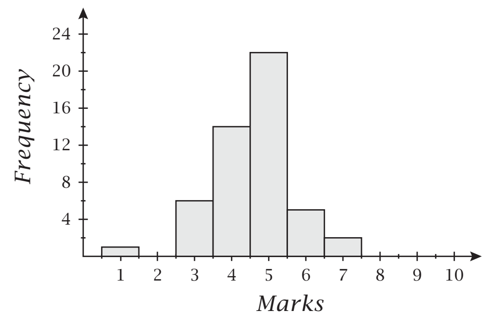
[6]

3. Year 10 classes at a school completed two school maths tests. The marks out of 10 are summarised below for each test.

TEST 1

Marks	Frequency
0	4
1	1
2	3
3	5
4	2
5	8
6	9
7	5
8	2
9	3
10	7

TEST 2



(a) Draw a frequency histogram for Test 1.

(b) Calculate for each test the

(i) mean

(ii) mode

(iii) median

(iv) range

(v) standard deviation

(c) Which test was the most difficult?
Why? Explain using your answers in part (a).

(d) The tests were to be used to select students for the top extension group. Which test was the better indicator? Why?

[16]

4. Individuals attending a rock concert had their ages (in years) recorded. The table below summarises this data.

Age (Years)	18–24	25–31	32–38	39–45	46–52	53–59	60–66
Frequency	35	25	20	18	0	1	1

(a) Calculate the average age of people attending the concert.

(b) Determine the most common age.

(c) Calculate the median age of those attending the concert.

(d) Which indicator of the average age is better? Mean or median. Discuss.

(e) Calculate the standard deviation.

[8]

5. The class average for a maths assignment was 64%. One of Tom's questions was incorrectly marked. His mark of 53% changes to 72%. Determine the new class average if there are 24 students in the class.

[4]

TRIAL TEST 11
MISCELLANEOUS

Calculators allowed
Time Allowed: 36 minutes
Total Marks: 36

1. (a) John is travelling to Hong Kong and needs to convert Australian dollars. The exchange rate is \$1AUD = 6.952 Hong Kong dollars. How many Hong Kong dollars will he receive for \$1725 Australian?

(b) On his return journey, John decides to convert his remaining 560 Hong Kong dollars back to Australian currency. How much will he receive?

[3]

2. Determine the total weekly earnings for Phoebe who has the following working conditions.

- \$16.20 per hour
- A meal allowance of \$9.50 per day
- A clothing allowance of 8.5% on top of her weekly pay (excluding meals)

Days	Rate	Hours
Monday–Friday	Normal	35
Saturday	Time and a half	6
Sunday	Double time	5
Overtime	Triple time	3

Determine Phoebe's weekly take home pay.

[5]

3. Percy decides to invest $25 000 into a bank account and must decide between two options.

Option A: 7.5% simple interest

Option B: 7.05% compounded annually

(a) Which option should Percy choose if he invests for 3 years.

(b) If Option B was changed to compounded monthly or daily which one should he choose for maximum return?

(c) Percy's friend invests, at another bank, $4000 and after 2 years accumulates a total of $4800 from an investment compounded annually. Should Percy invest in this bank?

[9]

4. A canteen sells bottles of apple and orange juice in three sizes – small, medium and large. The amounts sold in one day are shown in the table below.

	Small	Medium	Large
Apple juice	18	32	35
Orange juice	12	15	20

The cost of each small, medium and large bottle is $0.85, $1.50 and $3.00 respectively.

Calculate using *matrix multiplication* the total earnings on sales of juice.

[4]

5. A machine fills bottles of water and the amounts are normally distributed with a mean of 375mL and a standard deviation of 12mL. A bottle is randomly selected.

(a) Determine the probability it contains:

(i) at least 390mL

(ii) between 360mL and 387mL

(b) 2.5% of the bottles contain more than k mL. Find the value of k.

(c) Calculate the interquartile range capacity of the bottles.

[6]

6. A scale drawing has a scale of 1:1 000 000.

(a) Find the actual length of an object with a scale length of 5cm.

(b) The actual length of an object is 650m. What length would this be on the scale drawing?

[4]

7. Joseph purchases 4000 shares in Wireless PT.

- Shares are $10.50 each
- The company pays a dividend of 2.5% on the share price.
- A brokerage fee of 3% is paid to the stockbroker.

Determine:

(a) the total cost of purchasing the shares including the brokerage fee.

(b) the total dividend paid.

[5]

MIXED EXAMINATION STYLE QUESTIONS

RESOURCE FREE

1. Solve the equations below:

(a) $2(3x - 2) = 6 - (x + 7)$

(b) $-3x + 7 \leq -11$

(c) $2^{3x-1} = 32$

RESOURCE RICH

2. (a) Calculate 33.5% of 450kg.

(b) If 6% is 30, what is 100%?

(c) What 'net' percentage change is equivalent to a 14% increase followed by a 16% decrease?

(d) Increase \$175 by 45% and then by another 30%.

RESOURCE FREE

3. The length of a certain eucalyptus leaf is normally distributed with a mean of 11.5cm and a standard deviation of 3cm.

   Determine the probability using the 68%, 95% or 99.7% rule for the length of this type of eucalyptus leaf to be:

   (a) More than 8.5cm.

   (b) Between 2.5 and 17.5cm.

RESOURCE RICH

4. The angle of depression from the top of a 25m tall tree to a spike located on level ground is 18°.

   (a) Calculate the distance from the base of the tree to the spike.

The spike is moved 15m closer to the base of the tree.

(b) Calculate the angle of depression from the spike to a point 8m below the top of the tree.

RESOURCE RICH

5. An electrical retailer purchases a 4K TV for $5000 and sells it for $12 250.

(a) Determine the percentage profit.

(b) Jarod decides to purchase this TV. He pays a deposit of 20% with the remainder financed over 3 years at 17.4% p.a. simple interest.

Calculate:

(i) the deposit.

(ii) the interest paid in the first month.

(iii) the total paid for the TV including the deposit.

(c) The price of $12 250 includes a GST component of 10%. Calculate the price of the TV without GST.

RESOURCE FREE

6. The centripetal force (f) of an object in Newtons N is calculated using the formula: $f = \frac{mv^2}{r}$ where m is the mass in kg, r is the radius in metres (m) and v is the velocity in metres per second (m/s).

(a) Calculate the centripetal force when:

(i) $m = 5$kg, $v = 3$m/s and $r = 9$m.

(ii) $m = 16$kg, $v = \frac{1}{4}$ m/s and $r = 2$m.

(b) Determine v when $m = 12$kg, $f = 8$N and $r = 6$m.

(c) Calculate m when $f = 20\text{N}$, $v = 5\text{m/s}$ and $r = 0.5$ m.

RESOURCE FREE

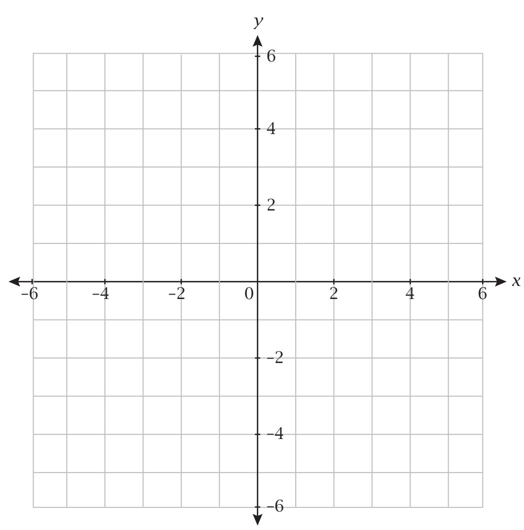
7. Given the equation of a straight line is $2x - 5y = 10$ determine each of the following:

(a) the coordinate of the y intercept.

(b) the coordinate of the x intercept.

(c) the gradient.

(d) Draw the line on the axis below.



8. Given the matrices below:

$P = [1\ \ -2]$ $\quad Q = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$ $\quad R = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ $\quad S = \begin{bmatrix} 5 & -2 \\ 4 & -3 \end{bmatrix}$

(a) Calculate each of the following, giving reasons if not possible.

(i) $2Q + 3S$

(ii) $P - R$

(iii) QS

(iv) SR

(b) Determine the value of p if $[4\ \ 2\ \ 5p+3]\begin{bmatrix} 2p \\ 3 \\ -1 \end{bmatrix} = [4p]$.

9. The cost of sending a parcel via post varies according to its weight as shown in the table below:

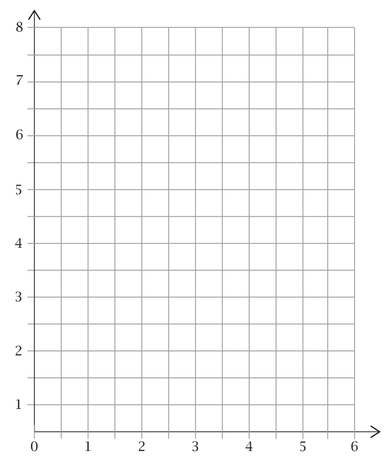
Weight (x kg)	$0 \le x < 1$	$1 \le x < 3$	$3 \le x < 4$	$x \ge 4$
Cost ($)	3	3.5	5	7.25

(a) Determine the cost of sending a parcel with the following weights:

(i) 1kg

(ii) 5.5kg

(b) Sketch a step graph for the cost of sending parcels according to the weight on the axes below:



(c) Two parcels weigh 900 grams and 3kg respectively.
Which option below is cheaper? Show working.

- Sending the parcels individually <u>or</u>
- Boxing the two parcels together and sending as one.

10. (a) In triangle XYZ, $\angle XYZ = 68^\circ$, $\angle XZY = 55^\circ$ *and* $YZ = 15$cm.

Calculate XZ using trigonometry.

(b) In triangle PQR, $PQ = 12$cm, $PR = 18$cm and $QR = 22$cm. Determine the value of the smallest angle.

(c) Use Heron's rule to find the area of ΔPQR in part (b) above.

11. A cylindrical water tank has a radius of 3.2 metres and a height of 5.6 metres.

(a) Calculate the surface area of the water tank.

(b) Determine the capacity of the tank in litres.

(c) Given initially the water tank is full, calculate the height of the water in the tank if 20% of water is removed.

12. Variables can be classified as either:

A: categorical
B: numerical

and one of

I: continuous
II: discrete
III: nominal
IV: ordinal

(a) Determine the classification using **A or B** and **I, II, III or IV** for each of the following variables:

(i) TV show star rating

(ii) Weight of a person

(iii) Country of birth

(b) Give an example of a variable that is:

(i) numerical and discrete

(ii) categorical and ordinal

RESOURCE FREE

13. The staff at a local drive thru coffee shop record the number of people they serve during their shift. The data is listed below:

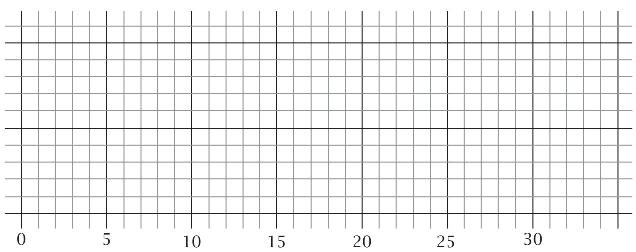
12	15	22	26	5	16	18	21	19	13	14
15	12	10	11	17	19	21	20	15	17	13

(a) Determine the range of this data.

(b) Determine the five number summary for this data.

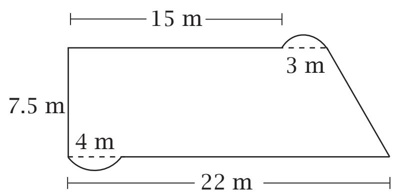
(c) Is the value 5 an outlier? Show working.

(d) Use the five number summary to construct a box plot on the scale below.



RESOURCE RICH

14. A swimming pool is designed in the following shape which includes two semi-circles of diameters 3m and 4m:



(a) Determine the perimeter of the swimming pool.

(b) The main part of the swimming pool has a constant depth of 1.75 metres. The two semi-circles have a depth of 35cm. Determine the amount of water in the pool to the nearest kilolitre.

RESOURCE RICH

15. From the top (T) of a vertical 9.4m cliff, a boat (B) is 35 metres from the base of the cliff. The boat then moves in a straight line away from the cliff before dropping anchor at (A). The angle of depression from the top of the cliff (T) to the anchored boat (A) is 11.3°,

(a) Draw a diagram indicating all the above information.

(b) Determine the distance from the anchored boat (A) to the top of the cliff (T).

(c) Calculate the angle of depression from the top of the cliff (*T*) to the initial position of the boat (*B*).

(d) Determine the distance the boat travels from (*B*) to (*A*).

RESOURCE RICH

16. Victoria decides to purchase some shares. She has shortlisted the following.

Share Name	Share value per share	Annual Dividend per share	Price to earning ratio (P/E)
PPR	$0.65	$0.56	24
ATA	$5.50	1.5%	12
QAM	$12.00	$1.30	18

(a) Calculate the cost of purchasing 5000 QAM shares.

(b) Determine the annual dividend payable on:

(i) 1250 PPR shares.

(ii) 3000 ATA shares.

(c) As an investor which share above has the better price to earning ratio? Explain.

(d) Brokerage fees of 2.5% were charged by the stockbroker on the purchase of shares. Victoria purchases 3000 PPR and 2000 QAM shares. Determine the brokerage fee.

RESOURCE RICH

17. 'Cakes R Us' and 'Cakes Delight' sells muffins and cakes. The table below shows their daily sales.

	Cakes R Us	Cakes Delight
Muffins	300	280
Cupcakes	260	315

It costs \$0.50 and \$0.65 to produce the muffin and cupcake respectively. Muffins sell for \$2.50 and cupcakes \$3.00.

(a) Write down a matrix S which represents the number of muffins and cakes sold by both shops.

(b) Matrix $C = \begin{bmatrix} 0.5 \\ 0.65 \end{bmatrix}$ and Matrix $R = \begin{bmatrix} 2.5 \\ 3 \end{bmatrix}$

Explain the meaning of both matrices C and R.

(c) Calculate matrix Q where $Q = S(R - C)$.

(d) Explain what information matrix Q displays.

RESOURCE RICH

18. (a) John's current weekly wage is \$1250. What is his new weekly wage if he receives a 4.2% increase?

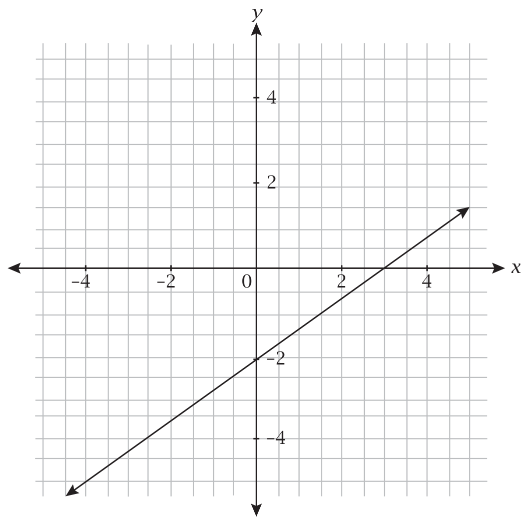
(b) Phoebe purchases a car for \$21 000 and sells it 8 years later for \$12 500. Calculate her percentage loss.

(c) A car salesman earns a weekly retainer of \$450 plus 3% commission on sales. If he sells cars valued at \$125 000 in a week, what is his weekly income?

(d) The selling price of a bike is \$1200 inclusive of 10% GST and its cost price is \$450. If the bike is discounted by 35%, what profit will be made?

RESOURCE FREE

19. (a) The graph of $ax + by = c$ is shown below. Determine the values of a, b, and c, where $a > 0$.



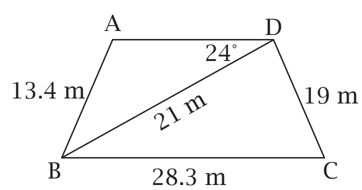
(b) On the axes above draw the graph of $y = -2x + 6$.

(c) State the coordinate point of intersection between the two graphs above.

(d) State the gradient of $3x - 2y = 7$.

RESOURCE RICH

20. *A*, *B*, *C* and *D* are four points on a playing field as shown on the diagram below.



Calculate:

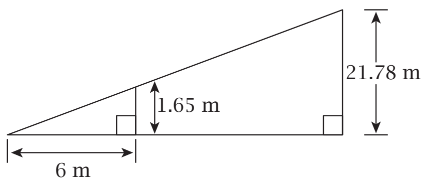
(a) $\angle BAD$ given that the angle is obtuse.

(b) $\angle DBA$.

(c) the area of the field $ABCD$.

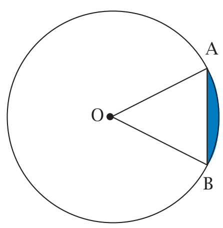
RESOURCE RICH

21. (a) At a particular time during the day, a TV tower 21.78m tall casts a shadow. At the same time, a person 1.65m tall casts a shadow 6m long.



Determine the length of the shadow cast by the tower.

(b) Given the following diagram:



The radius of the circle is 7cm and the chord AB is 4cm. Determine the area of the shaded region.

22. The daily wages of 75 workers at a certain factory are given in the table below:

Daily Wages ($)	Midpoint of interval	Number of workers
$10 < x \leq 15$	12.5	5
$15 < x \leq 20$		6
$20 < x \leq 25$		28
$25 < x \leq 30$	27.5	27
$30 < x \leq 35$		5
$35 < x \leq 40$		

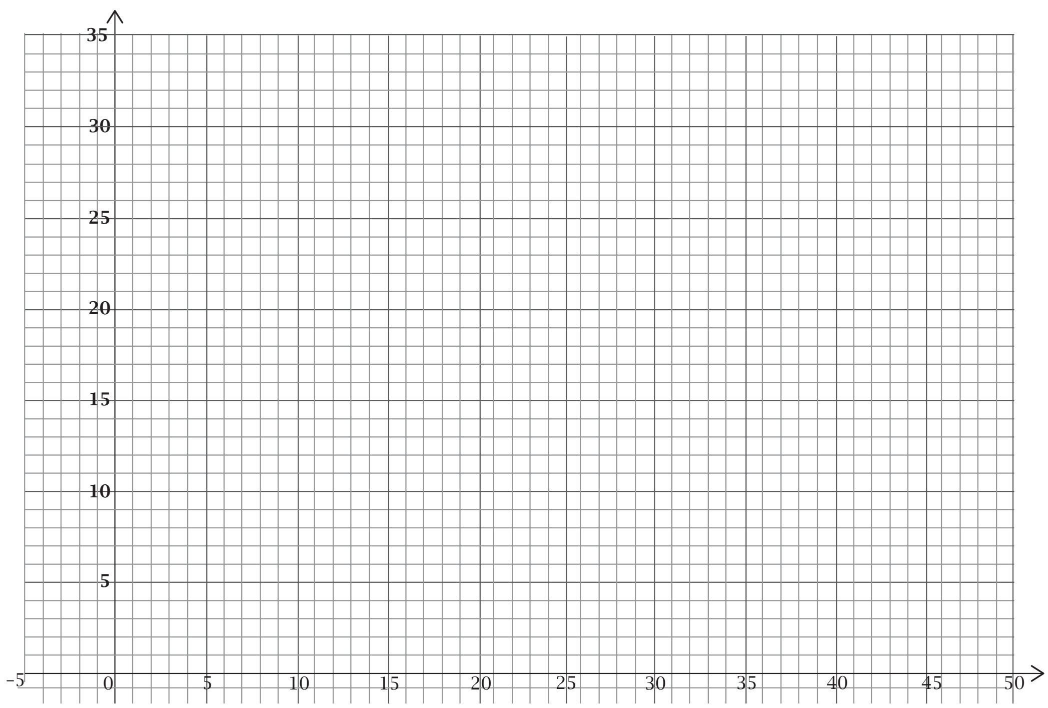
(a) Complete the midpoint and number of workers column in the table above.

(b) Use the midpoint to determine:

(i) the mean daily wage.

(ii) the standard deviation of daily wages.

(c) Draw a histogram on the axes below to display the daily wages of the 75 workers.



(d) Comment on the distribution of daily wages. Include comments on shape and modality.

RESOURCE RICH

23. The weight of a particular species of bird is normally distributed with a mean of 1.2kg with a standard deviation of 130 grams.

(a) If a bird is selected at random from this species, determine the probability that:

(i) Its weight is exactly 1.2kg.

(ii) Its weight is more than 1.5kg.

(iii) Its weight is within 0.2kg of the mean.

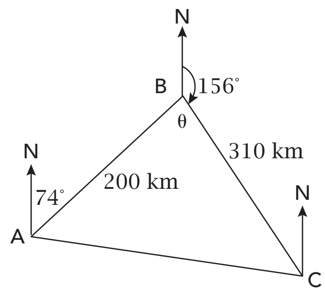
(b) 95% of all weights were between 1.2 – W kg and 1.2 + W kg.

Determine W.

(c) A sample of 500 birds were collected. How many would you expect to weigh less than 1000 grams?

RESOURCE RICH

24. A ship sails from point A on a bearing of 074° for 200km until it reaches point B. The ship alters course and sails on a bearing of 156° for 310km until it reaches point C.



(a) Find angle θ.

(b) Calculate the distance from point C to point A.

(c) The ship travels from point C to point A. Calculate its bearing.

RESOURCE FREE

25. Use the matrices below to calculate, if possible the following. State a reason why for those that are not possible.

$A = \begin{bmatrix} -2 & 1 \\ 5 & -3 \end{bmatrix}$ $\quad B = \begin{bmatrix} 0 & 1 \end{bmatrix}$ $\quad C = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ $\quad D = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$ $\quad E = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

(a) AD

(b) EB

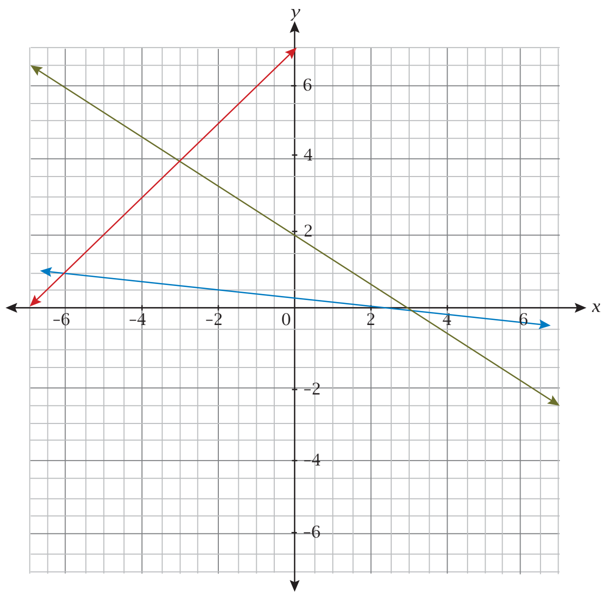
(c) *BE*

(d) *DC*

(e) the value of f_{21} given $F = A^2$.

RESOURCE FREE

26. Three linear graphs are shown below.



(a) Use the graph above to solve the simultaneous equations below:

(i) $x + 9y = 3$ *and* $y = x + 7$

(ii) $3y = 6 - 2x$ *and* $x + 9y = 3$

(b) Solve the simultaneous equations $2x - 4y = 19$ *and* $3x + 5y = 1$.

RESOURCE RICH

27. The company 'Olivio' specialises in production of olive oil. The cost to produce each bottle is \$7.50 and each bottle sells for \$17.50. The weekly fixed cost for the company is \$3500.

'Olivio' sells 750 bottles in a week. Determine:

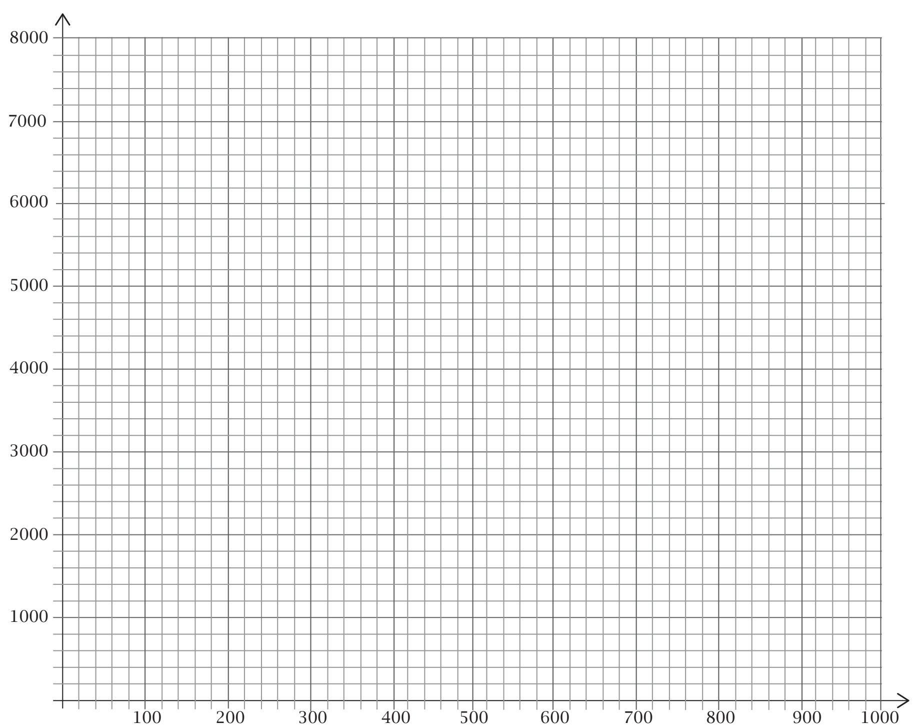
(a) The total cost of producing the 750 bottles in the week.

(b) The revenue obtained by the company in the sale of the 750 bottles of olive oil.

(c) Sketch on the axes below the following linear relationships:

(i) the total cost $\$C$ of producing 'b' bottles of olive oil.

(ii) the total revenue $\$R$ received for selling 'b' bottles of olive oil.



(d) If the company sold 300 bottles of oil in a week comment on its profitability.

(e) How many bottles of olive oil must be sold in a week for the company to break even?

RESOURCE RICH

28. (a) Bill decides to deposit \$150 000 into a bank for 5 years. 'Abank' offers a rate of 2.3% p.a. simple interest while 'BBank' offers a rate of 2.29% p.a. compounded monthly. Determine which bank offers the best investment.

(b) A table is sold at a 40% discount during a sale. A further reduction of 20% is given if cash is paid. The original price of the table is \$3750. Calculate the price of the table if a customer pays cash.

(c) The original price of a car is \$82 500. Two schemes are available for purchase.

Scheme 1: Cash payment with a discount of 10%
Scheme 2: Hire purchase with a cash deposit of 25% followed by monthly installments of \$750 for 6 years and 3 months.

Which is the better scheme. Explain with appropriate working.

29. Karen walks 5km on a bearing of 110° from point P to Q. She then walks 13.8km from Q to R on a bearing of 200°. Point R is due South of point P.

(a) Draw a diagram indicating clearly all distances and bearings.

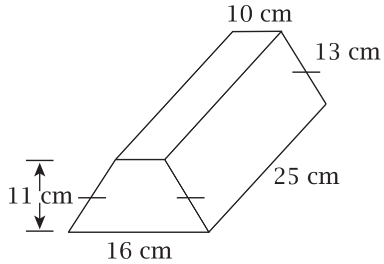
(b) Determine the size of $\angle QPR$.

(c) The bearing of Q from R is 20°. Show how this value is calculated.

(d) Calculate the bearing of P from Q.

(e) Determine the distance from P to R.

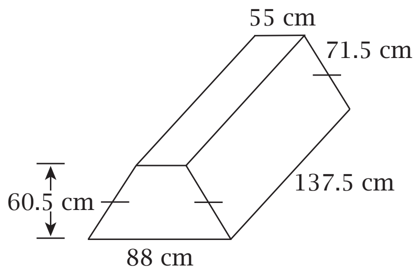
30. Given the following trapezoidal prism:



(a) Determine the total surface area of the above prism.

(b) Calculate the volume of the prism.

The above prism is a scale model drawing of a larger model with a scale factor of 1: k. The actual measurements are represented by the following diagram.



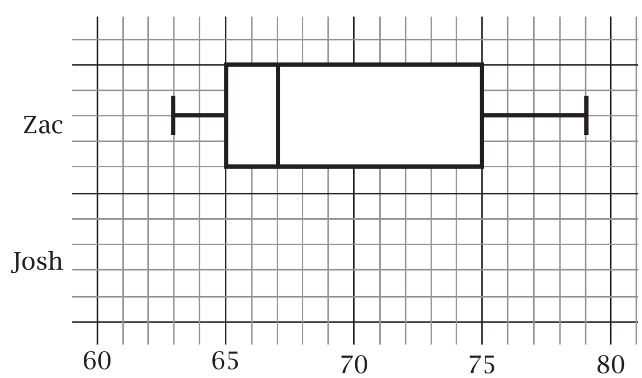
(c) Determine the scale factor k.

(d) The surface area of the larger model can be calculated by multiplying the scale model surface area by a factor. Determine this factor.

(e) The volume will also change. Calculate the new volume using the value obtained in part (c) above.

RESOURCE FREE

31. Twin brothers Zac and Josh each sat fifteen tests throughout the semester. Zac's results are displayed as a box and whisker plot below.



(a) Josh's 5 number summary for the tests are 63, 67, 70, 74, 80. Add a box and whisker plot on the above diagram using Josh's summary statistics.

(b) Determine the percentage of Josh's test results above 74.

(c) Zac is confident that his results are better than Josh's. Compare the test performance between the two brothers commenting on the distribution referring to the statistics and skewness of the distributions.

RESOURCE RICH

32. John and Alicia are planning an overseas trip from Australia to Japan, Indonesia, Europe and USA. The following table highlights the current exchange rates for 1 Australian dollar.

Country	Japan	Indonesia	Europe	USA
$1 AUD	68.78 JPY	10 250.67 IDR	0.62 Euro	$0.76 USD

(a) How many Japanese Yen (JPY) can be exchanged for $750 Australian dollars (AUD)?

(b) How many Australian dollars (AUD) can be exchanged for 85 000 Indonesian Rupiah (IDR)?

(c) On leaving Indonesia, John and Alicia want to convert $250 AUD and 126 560 IDR to United States Dollars. How many USD will John and Alicia have?

(d) In Europe, accommodation for 5 nights costs 3 500 Euro. Calculate the cost per night in Australian dollars.

(e) Alicia wants to buy some perfume on her overseas trip. She researches the cost in both USA and Japan. The cost in Japan is 7800 Yen and in USA is $98 USD. Where should she purchase the perfume?

RESOURCE FREE

33. X and Y are straight lines.

- The equation for line X is $3x - y = -2$
- Line Y passes through coordinates (2, 8) *and* (–3, –2)

Determine if lines X and Y intersect and if so, find the point of intersection. Show full working.

RESOURCE FREE

34. The weight of boxes of 'Go Munch' cereals are normally distributed. The mean is 700 grams with a standard deviation of 4 grams.

Determine:

(a) the z-score of a box of 'Go Munch' cereal with a weight of 708 grams.

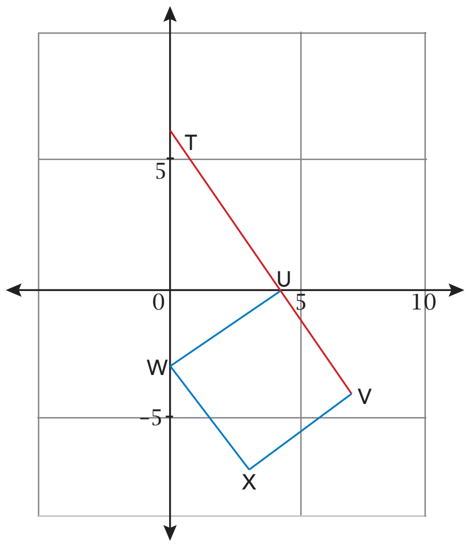
(b) the weight of a box of 'Go Munch' cereal with a z-score of –1.

(c) the percentage of 'Go Munch' boxes with a weight more than 712 grams.

RESOURCE FREE

35. The graph shows a square UVXW where T and W lie on the y axis and U on the x axis.

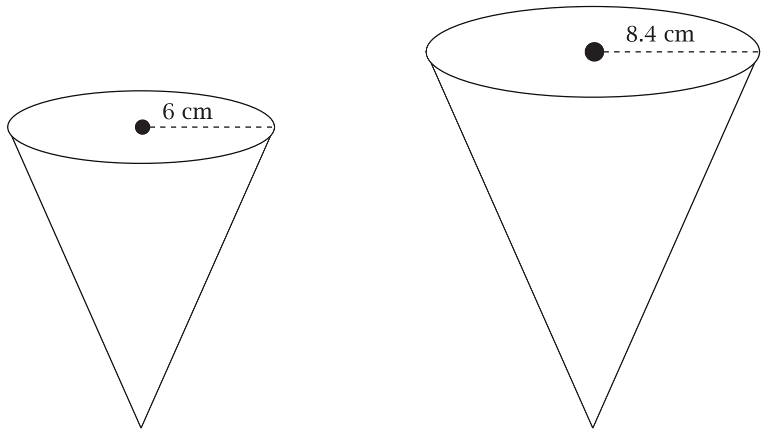
The equation of the line passing through points UW is $y = \frac{3}{4}x - 3$



(a) Determine the length of TW.

(b) Calculate the area of square UVXW.

36. Two **similar** cones have the following dimensions:



The volume of the smaller cone is 300cm^3.

(a) As the cones are similar, determine the heights of both the smaller and larger cone.

(b) Determine the surface area of the smaller cone.

(c) Using a scale factor determine the surface area of the large cone.

(d) Using a scale factor determine the volume of the larger cone.

37. (a) Sixty students in a school had an overall average in a recent Maths test of 72%. Class 1 with 29 students had an average of 65%. Determine the average of Class 2.

(b) A traffic survey recorded the number of people travelling in each car. Details are shown in the table below.

Number of people in each car	1	2	3	4	5	6	7	8
Frequency	55	20	12	3	1	0	0	1

Determine:

(i) the mean number of people in a car.

(ii) the standard deviation of the number of people in a car.

(iii) the total number of cars recorded by the survey.

(iv) Is '8' an outlier? Justify your answer.

(v) Comment on the distribution of the data.

38. (a) The time T taken for a chemical reaction was calculated using the formula $T = \sqrt{2d} - p^2$.

Calculate T when:

(i) $d = 18$ and $p = 2$.

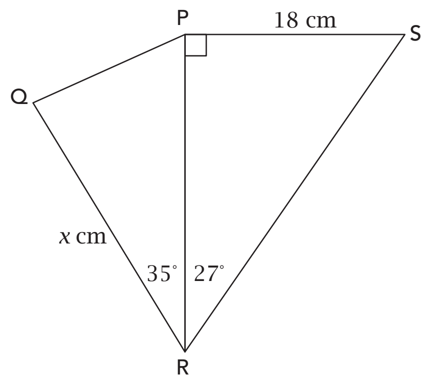
(ii) $d = 0.125$ and $p = 0.1$.

(b) The formula for the chemical reaction above was then refined to $T = \dfrac{\sqrt{2d}}{x} - \dfrac{p^2}{y}$

Determine T when $d = 50$, $x = 0.5$, $p = -4$ *and* $y = -16$.

RESOURCE RICH

39. The diagram below shows two triangles. The area of triangle *PQR* is 52cm^2.



Calculate the value of *x*.

RESOURCE RICH

40. (a) Joel takes the bus each day to school and the journey takes 35 minutes. He turns 18 and now drives to school. The journey takes 20 minutes and 45 seconds. Determine the percentage reduction in Joel's journey to school.

(b) A computer game costs $145 including 10% GST. Calculate the amount of GST included in the price of the game.

RESOURCE RICH

41. The data below is the number of hours students spent using their mobile phones.

Number of hours	0	1	2	3	4	5	6
Number of students	3	x	8	5	6	3	2

(a) If the median is 3, determine the largest possible of x.

(b) Determine x, if the mean number of hours spent on the mobile phone is 2.8965.

(c) Write down the largest value of x if the mode is 2.

RESOURCE RICH

42. The daily temperatures in Atown have found to be approximately normally distributed with a mean of 28.3˚C and a standard deviation of 3.7˚C.

(a) Find the probability that a randomly chosen day from Atown will have a daily temperature:

(i) less than 26.1˚C.

(ii) between 22.7˚C and 31.4˚C.

(b) Days in Atown are often referred to as 'scorchers'. These scorchers are the hottest 2% of days.

Calculate the lowest possible daily temperature of a 'scorcher'.

(c) Determine the interquartile range of daily temperatures in Atown.

RESOURCE RICH

43. Honey is sold to shops in different size jars according to the following table:

Honey Jar size	Price $ per carton	Jars per carton
100 grams	$70.00	20
250 grams	$65.00	10
500 grams	$90.00	8
1 kg	$79.80	4

(a) Determine which jar size is the best buy. Show working.

(b) A 15% discount is offered for sales above $500. A business purchases the following:

Honey Jar size	Number of cartons
100 grams	6
250 grams	3
500 grams	2
1 kg	1

Calculate the purchase cost for this order.

44. 'Model A Toys' produces model cars and trains. Each toy is made of wood (m^2) and then painted. The cost of each toy is illustrated in the table below.

	Wood (m^2)	Paint (number of cans)	Labour (hours)
Model Car	3	4	4
Model Train	2	3	7

Labour costs \$10 per hour, Wood costs \$5 per square metre and paint costs \$8 per can.

The percentage profit for a model train is 15% of its cost price and for a model car is 12% of its cost price.

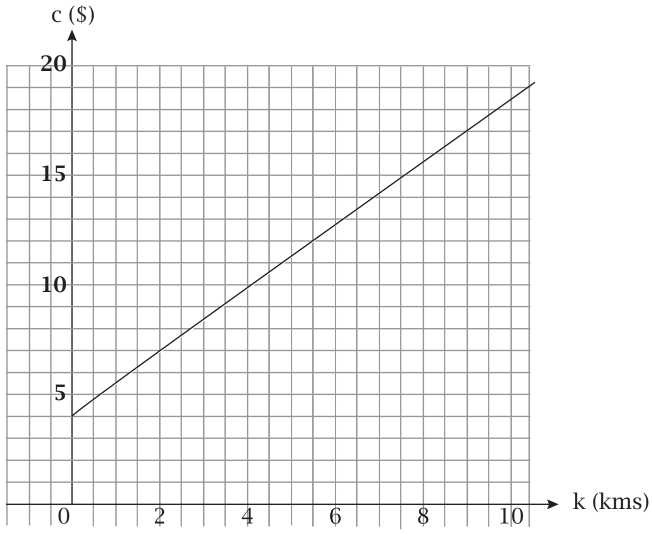
Given $T = \begin{bmatrix} 3 & 4 & 4 \\ 2 & 3 & 7 \end{bmatrix}$ and $C = \begin{bmatrix} 5 \\ 8 \\ 10 \end{bmatrix}$

(a) Calculate TC.

(b) Determine a matrix S where the elements of matrix STC will give the selling price of the model train and car. Calculate STC.

(c) Joanna wishes to purchase both a model train and car. A discount of 15% on the total selling price is offered. Determine matrix Q where the elements of the matrix $QSTC$ give the discounted total selling price. Hence evaluate matrix $QSTC$.

45. The cost of a journey via 'John's taxi' is shown in the graph below. Cost is determined by adding a fixed 'flag fall' fee with a rate based on the number of kilometres travelled.



(a) Calculate:

(i) the 'flagfall' – the fixed fee.

(ii) the cost per kilometre 'John's taxi' charges.

(b) Determine:

(i) the cost for travelling 6km.

(ii) the number of kilometres travelled if 'John's taxi' charges $7.

(c) Determine the cost equation (C) for 'John's taxi' based on the number of kilometres travelled k.

Another company 'Oober Ex' charges \$11 for travelling 4 kilometres and \$17 for 10 kilometres.

(d) State the cost equation (E) for 'Oober Ex' based on the number of kilometres k travelled. Add this line to the graph above.

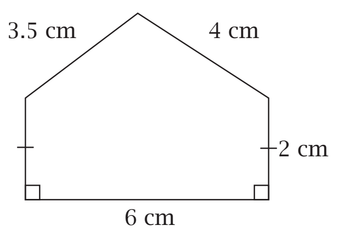
(e) Determine:

(i) the 'flag fall' fixed fee for 'Oober Ex'.

(ii) when 'Oober Ex' is cheaper than 'John's taxi' based on the number of kilometres travelled.

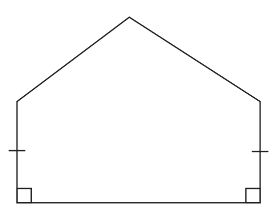
RESOURCE FREE

46. A builder designs a window and draws the following **scale diagram**.



Scale factor 1:300

(a) Determine the real (actual) dimensions of the diagram above in metres and add to the diagram below.



(b) Calculate the total area of the shape in part (a).

(c) Glass costs \$26 per square metre. Determine the cost of the glass for the window.

RESOURCE RICH

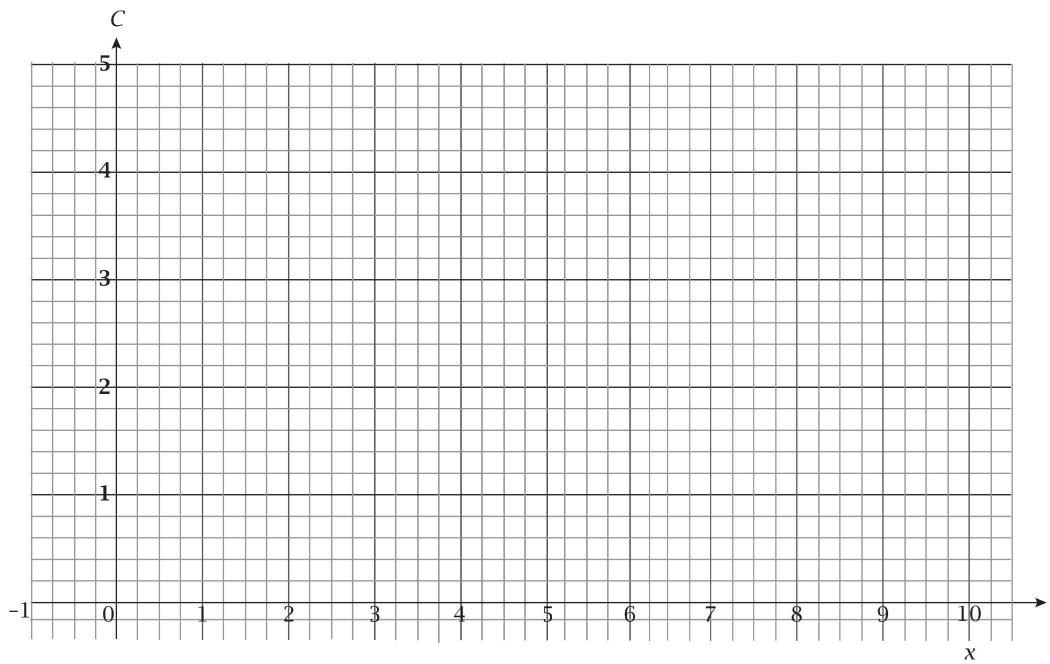
47. The cost of mulch supplied to households is given in the table below:

Mulch (x per m^3)	$0 \le x < 1$	$1 \le x < 5$	$x \ge 5$
Cost (C\$ per m^3)	\$4.50	\$3.80	\$2.80

Trailer hire: \$35.00

(a) Determine the cost of $3.6m^3$ mulch.

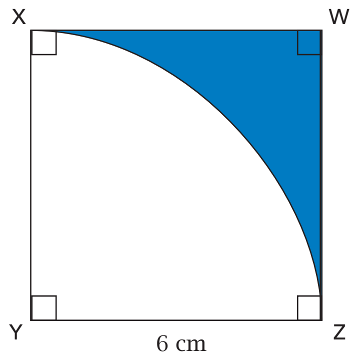
(b) Draw a piece-wise graph below showing the cost per m^3 against the quantity of mulch (per m^3).



(c) Determine the total cost of $5m^3$ of mulch including trailer hire which is currently discounted by 15%.

RESOURCE RICH

48. A sector XYZ of a circle with radius of 6 cm is cut from a square WXYZ as shown in the diagram below.



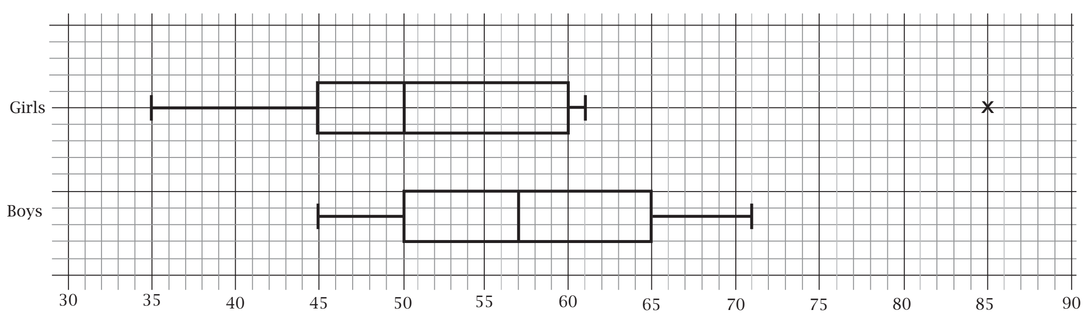
(a) Determine the area of the sector XYZ.

(b) Determine the area of the shaded region XWZ.

(c) Calculate the area of the shaded region if the radius was increased by 50%.

RESOURCE RICH

49. At a cross country carnival, the time taken for the participants are collected. Each male and female runner had times (in minutes) recorded on the completion of the run. The results are summarized in the box and whisker plots below.



(a) Explain what is meant by the cross 'X'. Justify the existence of this cross.

(b) Ignoring the cross, calculate the range of times for both the boys and girls. Comment by comparing the two values.

(c) Determine the interquartile range of the boys and girls.

(d) Compare the performance of both the boys and girls at the cross country carnival using any of the information above.

RESOURCE FREE

50. Abbie's marks for her exams in Maths and Physics were 70 and 62 respectively. The mean and standard deviation for Maths were $\mu = 65$ and $\sigma = 2.5$ while Physics were $\mu = 72$ and $\sigma = 5$.

(a) Use z scores to compare Abbie's exam marks to determine which was her better result.

(b) What percentage of the class scored more than Abbie in the Maths exam?

RESOURCE FREE

51. Xander, Yolanda and Zebedee had \$4.50 between them. Xander had x dollars, Zebedee had three times as much as Yolanda. Yolanda had \$2.25 less than Xander. How much did each person have?

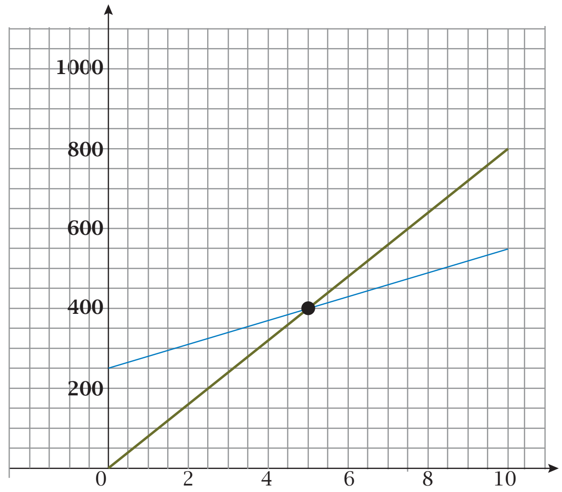
RESOURCE RICH

52. (a) Paul wants to buy a drone that normally costs \$4750. So far, he has saved 55%. An electronics retailer advertises the drone on sale for 20% off. How much more does he need to save, to the nearest cent, to buy the drone at the discounted price?

(b) Maggie sells a dining table at a loss of 65% from the initial cost. She sells the table for \$2750. What was the original price of the dining table?

53. 'ABC Lights' replaces halogen light bulbs with LED light systems for \$80 each. The cost per LED light is \$30 and the labour, machinery and tools required cost \$250.

The revenue, R and cost C, functions for x number of replacement LED light systems are shown on the axes below.



(a) Determine the Revenue R and Cost C equations.

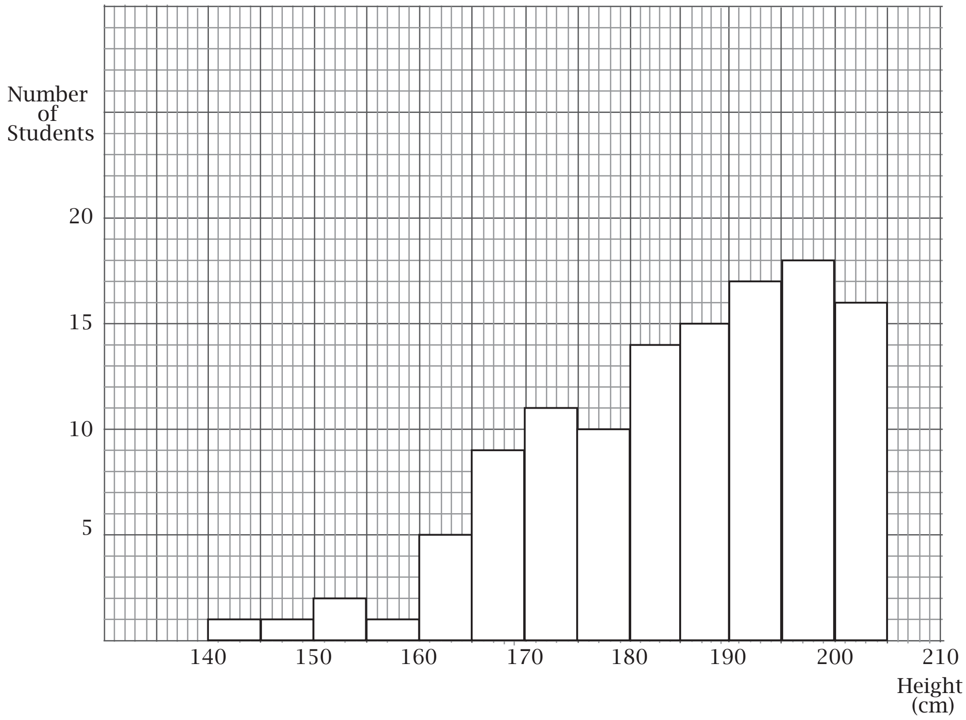
(b) Determine the profit of replacing halogen light bulbs with:

(i) 4 LED light systems. Comment on your answer.

(ii) 8 LED light systems. Comment on your answer.

(c) Calculate the 'break-even' point.

54. A histogram is drawn below showing the heights of 120 Year 12 students in a school.



(a) Complete the table below which is a summary of the heights of Year 12 students

Height of students (cm)	Midpoint of interval	Frequency
$140 < x \le 145$	142.5	1
$145 < x \le 150$	147.5	1
		2
$155 < x \le 160$	157.5	1
$160 < x \le 165$	162.5	
$165 < x \le 170$	167.5	9
		11
$175 < x \le 180$	177.5	
$180 < x \le 185$	182.5	14
$185 < x \le 190$	187.5	15
$190 < x \le 195$	192.5	
$195 < x \le 200$	197.5	18
		16

(b) Describe the distribution of the data.

(c) Use the midpoint to determine the:

(i) mean height of the students.

(ii) standard deviation of the height of students.

RESOURCE RICH

55. The number of direct roads between towns W, X, Y and Z is given in the table below.

from \ to	W	X	Y	Z
W	0	0	1	1
X	0	0	1	1
Y	1	1	0	0
Z	1	0	0	0

(a) Construct a route map to illustrate the above information using arrows to show directions of travel.

(b) Construct matrix M representing the number of one-step paths.

(c) Determine the number of two-step paths between the towns W, X, Y and Z.

(d) Calculate M^3 and explain the meaning of $M^3{}_{23}$.

RESOURCE RICH

56. Employees working at a local supermarket are paid normal time from Monday to Friday, time and a half on Saturdays and double time and a quarter on Sundays.

(a) Desmond works the following hours and is paid $22.50 per hour.

Days	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Number of hours	5	0	4	6	8	4	3

Calculate Desmond's weekly pay.

(b) Evelyn works the following hours. She is paid $22.50 per hour and a supervisor allowance of 18%.

Days	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Number of hours	3	7	8	8	8	A	4

Evelyn's weekly wage was $1380.60. Determine the value of A.

(c) Joey earns $717.60 and works the following hours.

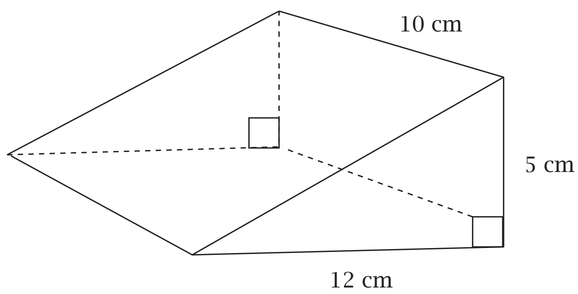
Days	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Number of hours	3	8	0	0	4	4	8

Calculate Joey's hourly rate of pay.

(d) Management must pay 9.5% superannuation to its employees. Calculate the total superannuation payment for Desmond, Evelyn and Joey for the week.

RESOURCE FREE

57. A wooden wedge has dimensions as shown in the diagram below:



(a) Determine the surface area of the wooden wedge.

(b) Calculate the volume of the wedge.

(c) Wooden lacquer required to paint the wedge costs \$2.50 per cm^2.
Determine the cost of painting the wooden wedge.

58. Billy decides to invest $10 000 into shares and two bank accounts.

Investment A: $2500 into a bank account paying 7% p.a. simple interest.
Investment B: $3750 into another bank account earning 6.5% p.a. compounded monthly.
Investment C: The remainder in AAQ share company valued at $3.75 per share and paying an annual dividend of 5.6% of the share value.

(a) Determine the interest earned over 3 years for Investment A.

(b) Calculate the interest earned over 4 years for Investment B.

(c) Calculate the first year annual dividend payable for Investment C.

(d) Billy's shares including the dividends received at the end of three years is valued at $17 250.25. Determine the percentage increase.

59. The number of bags of potatoes sold at 'Mashed Spud' per day is recorded over a two week period and the results are shown below.

21 15 12 28 34 41 6 12 20 14 26 52 30 14

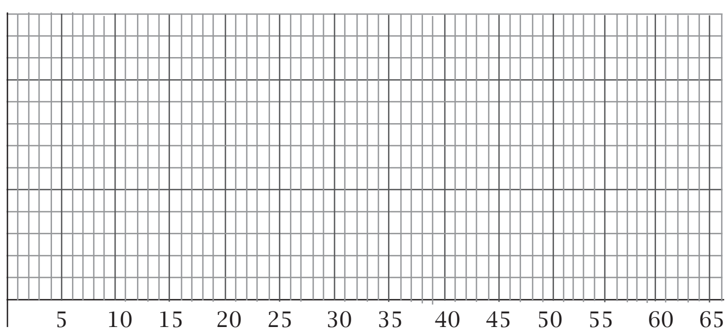
(a) Calculate the mean number of bags sold per day.

(b) Draw a stem and leaf plot to represent this data.

(c) Calculate the 5-number summary.

(d) Determine whether any outliers exist. Justify your answer.

(e) Draw a box plot on the axis below to represent this data.



(f) Comment on the distribution of this data.

60. A retail store purchases 'Wiff' perfume at \$120. This price is marked up by 125% and then 10% GST is added.

(a) What price does the store sell 'Wiff'?

(b) Determine the percentage markup including GST the retail store makes on the sale of the perfume.

(c) The retail store has a '***30% off everything sale***'. Jolie purchases 'Wiff'. How much does she pay?

ANSWERS

CHAPTER 1: *Percentages*

1. (a) 29.4
   (b) 31.25L
   (c) $0.47
   (d) 2.7056kg

2. (a) 70% of $x = \$14$
$$\frac{70}{100} \times x = \$14$$
$$x = \$14 \times \frac{100}{70}$$
$$x = 20$$
   (b) 55.5% of $x = 72.15\text{g}$
$$x = 130\text{g}$$
   (c) $13\frac{1}{3}\%$ of $x = 11.96\text{mL}$
$$x = 89.7\text{mL}$$

3. $$\text{Original Price} \times 0.75 = \$29\,990$$
$$\text{Original Price} = \frac{\$29\,990}{0.75}$$
$$= \$39\,986.67$$

4. (a) $\$500 \times 1.05 = \$525$
   (b) $725\text{g} \times 1.23 = 891.75\text{g}$
   (c) $16\text{L} \times 2.246 = 35.936$
   (d) $\$120 \times 0.94 = \$112.80$
   (e) $40\text{cm} \times 0.66 = 26.4\text{cm}$
   (f) $7.85\text{mL} \times 0.495 = 3.88575\text{mL}$

5. $$\text{New price} = \$625\,000 \times 0.875$$
$$= \$546\,875$$

6. $$\text{Salary} = \$72\,000 \times 1.045$$
$$= \$75\,240$$

7. (a) $$\%\text{ change} = 1.1 \times 0.9$$
$$= 0.99$$
   Decrease of 1%
   (b) $$\%\text{ change} = 0.75 \times 1.25$$
$$= 0.9375$$
   Decrease of 6.25%
   (c) $$\%\text{ change} = 1.05 \times 0.96$$
$$= 1.008$$
   Increase of 0.8%

8. (a) $$\text{Salary} = \$56\,000 \times 1.03 \times 1.04 \times 1.06$$
$$= \$63\,586.43$$
   (b) $$\%\text{ change} = 1.03 \times 1.04 \times 1.06$$
$$= 1.135\,472$$
   % increase of 13.5472%

9. (a) $$\text{Discount} = \$240 \times 20\%$$
$$= \$48$$
   (b) $$\text{Sale Price} = \$240 - \$48$$
$$= \$192$$

10. $$\text{Discount} = \$80 - \$58$$
$$= \$22$$
$$\%\text{ discount} = \frac{22}{80} \times 100$$
$$= 27.5\%$$

11. $$\text{Commission} = \frac{1.5}{100} \times \$230\,000$$
$$= \$3450$$

12. Commission (first $300 000)
$$= \frac{4}{100} \times \$300\,000$$
$$= \$12\,000$$
   Remaining Commission ($225 000)
$$= \frac{2.5}{100} \times \$225\,000$$
$$= \$5625$$
$$\text{Total Commission} = \$17\,625$$

13. (a) $$\text{Profit} = \$5$$
$$\%\text{ profit} = \frac{5}{20} \times 100$$
$$= 25\,\%$$

(b) Loss = $25

$$\% \text{ loss} = \frac{25}{75} \times 100$$
$$= 33\frac{1}{3}\%$$

(c) Profit = $93.50

$$\% \text{ profit} = \frac{93.5}{110} \times 100$$
$$= 85\%$$

(d) Loss = $93.75

$$\% \text{ loss} = \frac{93.75}{150} \times 100$$
$$= 62\frac{1}{2}\%$$

(e) Profit = 0.13

$$\% \text{ profit} = \frac{0.13}{1.12} \times 100$$
$$= 11.6\%$$

14. (a) Selling price = $45 × 1.7
= $76.50

(b) Selling price = $1200 × 0.65
= $780

15. Original cost $= \frac{\$12.50}{1.25}$
= $10

Selling price (with profit 35%):
= $10 × 1.35
= $13.50

Should sell DVD at $13.50.

16. Sales $= \frac{\$731.43}{0.225}$
= $3250.80

17. Commission (first $7500)
$$= \frac{2}{100} \times 7500$$
= $150

Remaining commission ($25 500)
$$= \frac{1.25}{100} \times 25\,500$$
= $318.75

Weekly Income = $150 + $318.75 + $225
= $693.75

18. Cost price $= \frac{\$235}{1.25}$
= $188

Computer games cost $188.

19. Discounted price = $250 × 0.85
= $212.50

Profit = $212.50 – $150
= $62.50

20. Cost price $= \frac{\$3027.50}{1.75}$
= $1730

Computer originally costs $1730.

21. (a) $5.50 × 1.025
= $5.64

(b) $\$5.50 \times (1.025)^2$
= $5.78

(c) $\$5.50 \times (1.025)^5$
= $6.22

(d) $\$5.50 \times (1.025)^{-1}$
= $5.37

22. Sale price = $1950 × 1.1 × 0.65
= $1394.25

23. Cost of jeans = $249.50 + $24.95 (GST)
= $274.45

24. Cost = $5.25 × 1.05 × 1.072 × 1.03 × 1.046
= $6.37

CHAPTER 2: *Algebraic Skills*

1. (a) $2(-2)(-6) - 3(4)$
$= 12$

(b) $(-2)^2 + (-6)^2 - (4)^2$
$= 24$

(c) $3 - (-2)(-6)(4)$
$= -45$

(d) $-2(-6) - 3(-2)(4)$
$= 36$

(e) $4(-2)^2 - (3(-6) + (4) - 6)$
$= 36$

2. (a) $A = 4000\left(1 + \frac{6}{100}\right)^4$

$= \$5049.91$

(b) $A = 12\,000\left(1 + \frac{7.25}{100}\right)^6$

$= \$18\,262.70$

3. (a) $S = 2\pi(7)^2 + 2\pi(7)(10)$

$= 747.70\text{cm}^2$

(b) $S = 2\pi(10)^2 + 2\pi(10)(20)$

$= 1884.96\text{cm}^2$

4. (a) $C = \frac{5}{9}(42 - 32)$

$= 5.56^\circ\text{C}$

(b) $C = \frac{5}{9}(0 - 32)$

$= -17.78^\circ\text{C}$

(c) $C = \frac{5}{9}(100 - 32)$

$= 37.78^\circ\text{C}$

5. (a) $C = \sqrt{6^2 + 8^2}$

$C = \sqrt{100}$

$C = 10\text{m}$

(b) $C = \sqrt{2.5^2 + 4^2}$

$C = 4.717\text{cm}$

6. (a) $12 - 4p$

(b) $\frac{n}{3t}$

(c) $-2(f + 6)$

(d) $\frac{p - q}{pq}$

7. (a) Total Area $= (x + 35)(x + 20)\text{m}^2$

(b) (i) $\frac{m}{p}$

(ii) $\frac{2m}{3t}$

(c) Sydney : 'h' hours

Tom : $\frac{1}{4}(8 - h)$ hours

Total : $h + \frac{1}{4}(8 - h)$

$= h + 2 - \frac{1}{4}h$

$= \left(\frac{3}{4}h + 2\right)$ hours

8. (a) $3m - 2n$

(b) $7t$

(c) 4m^2

(d) $7x^2 - 8x - 3$

9. (a) $-150a^3b^2c$

(b) $9x^2y^2$

(c) $-\frac{ab}{2}$

(d) $\frac{5m}{7n}$

10. (a) -12

(b) $3(6) - 2(-2)$

$= 22$

(c) $6 - (-2)^2$

$= 2$

11. (a) $5y - 30$

(b) $-8x + 6fx$

(c) $6h^2 - 15gh$

(d) $-3p - 2q$

12. (a) $7(2a - 5) - 4a$

$= 14a - 35 - 4a$

$= 10a - 35$

(b) $3(2p - 1) + 2(p + 6)$

$= 6p - 3 + 2p + 12$

$= 8p + 9$

(c) $5(3e - 2) - 2(4 - e)$

$= 15e - 10 - 8 + 2e$

$= 17e - 18$

(d) $-3x(2x - 3) - 2x(5x + 1)$

$= -6x^2 + 9x - 10x^2 - 2x$

$= -16x^2 + 7x$

13. (a) $p^2 - 9p - 2p + 18$

$= p^2 - 11p + 18$

(b) $-3(2y - 7)^2$

$= -3(2y - 7)(2y - 7)$

$= -3(4y^2 - 14y - 14y + 49)$

$= -3(4y^2 - 28y + 49)$

$= -12y^2 + 84y - 147$

(c) $(8w - 3)(2w - 3)$

$= 16w^2 - 24w - 6w + 9$

$= 16w^2 - 30w + 9$

(d) $-2(5c - 4)(3c - 2)$

$= -2(15c^2 - 10c - 12c + 8)$

$= -2(15c^2 - 22c + 8)$

$= -30c^2 + 44c - 16$

14. (a) $7(1 - 3b)$

(b) $-4(4 + 3y)$

(c) $5p(3q - 5s)$

(d) $x(x - 10)$

15. (a) $x - 6 = 10$

$x = 10 + 6$

$x = 16$

(b) $2y = 12$

$y = \frac{12}{2}$

$y = 6$

(c) $\frac{p}{7} = -3$

$p = -3 \times 7$

$p = -21$

(d) $9 - t = 5$

$-t = 5 - 9$

$-t = -4$

$t = 4$

16. (a) $-9x + 5 = 86$

$-9x = 86 - 5$

$-9x = 81$

$x = -\frac{81}{9}$

$x = -9$

(b) $\frac{3p - 2}{5} = 8$

$3p - 2 = 8 \times 5$

$3p - 2 = 40$

$3p = 42$

$p = 14$

(c) $\frac{6}{x} = 5$

$6 = 5x$

$\frac{6}{5} = x$

(d) $\frac{3}{4}b + 5 = -2$

$\frac{3}{4}b = -7$

$b = -\frac{28}{3}$

17. (a) $x - 9 = 12 - 2x$

$x + 2x = 12 + 9$

$3x = 21$

$x = 7$

(b) $2(z + 4) + 4z = -32$

$2z + 8 + 4z = -32$

$6z = -40$

$z = -\frac{20}{3}$

(c) $-3(4m - 5) + 6m = 9$

$-12m + 15 + 6m = 9$

$-6m = 9 - 15$

$-6m = -6$

$m = 1$

(d) $\frac{15 - b}{2} + 3 = 12$

$\frac{15 - b}{2} = 9$

$15 - b = 18$

$b = -3$

18. (a) $7(2x + 3) - 6(3x - 1) = -1$

$14x + 21 - 18x + 6 = -1$

$-4x = -1 - 21 - 6$

$-4x = -28$

$x = 7$

(b) $\frac{3x - 7}{2} = x + 5$

$3x - 7 = 2(x + 5)$

$3x - 7 = 2x + 10$

$3x - 2x = 10 + 7$

$x = 17$

(c) $\dfrac{4x+5}{3} = \dfrac{2x+7}{2}$

$2(4x + 5) = 3(2x + 7)$

$8x + 10 = 6x + 21$

$8x - 6x = 21 - 10$

$2x = 11$

$x = \dfrac{11}{2}$

(d) $5g - 2 = 8 - (10 - g)$

$5g - 2 = 8 - 10 + g$

$5g - g = 8 - 10 + 2$

$4g = 0$

$g = 0$

19. (a) Let n be the number

$3n + 5 = -10$

$3n = -15$

$n = -5$

$\therefore$ the number is -5

(b) Let n be the number

$\dfrac{1}{4}n - 3 = 7$

$\dfrac{1}{4}n = 10$

$n = 40$

$\therefore$ the number is 40

(c) Let n be the number

$\dfrac{8-2n}{7} = 4$

$8 - 2n = 28$

$-2n = 20$

$n = -10$

$\therefore$ the number is -10

(d) Let k be the number of kilometres travelled

$k + 2k + 2k + 2k + 10 + 2k + 10 = 524$

$9k + 20 = 524$

$9k = 504$

$k = 56$

$\therefore$ the number of kilometres travelled on the first day is 56.

20. (a) $8 + 3x \geq -13$

$3x \geq -21$

$x \geq -7$

(b) $2x - 1 < 5x + 7$

$2x - 5x < 7 + 1$

$-3x < 8$

$x > -\dfrac{8}{3}$

(c) $6(3x - 1) \leq 5(4x - 3)$

$18x - 6 \leq 20x - 15$

$18x - 20x \leq -15 + 6$

$-2x \leq -9$

$x \geq \dfrac{9}{2}$

(d) $\dfrac{5-2x}{6} > -7$

$5 - 2x > -42$

$-2x > -47$

$x < \dfrac{47}{2}$

21. (a) $9x^2 = 25$

$x^2 = \dfrac{25}{9}$

$x = \pm\dfrac{5}{3}$

(b) $x^2 - \dfrac{1}{25} = 0$

$x^2 = \dfrac{1}{25}$

$x = \pm\dfrac{1}{5}$

(c) $6(p^2 - 1) = 37$

$6p^2 - 6 = 37$

$6p^2 = 43$

$p^2 = \dfrac{43}{6}$

$p = \pm\sqrt{\dfrac{43}{6}}$

(d) $-2t^3 + 4 = 132$

$-2t^3 = 128$

$t^3 = -64$

$t = -4$

22. (a) $3^{x+1} = 27$

$3^{x+1} = 3^3$

$\therefore x + 1 = 3$

$\therefore x = 2$

(b) $7^x = \dfrac{1}{49}$

$\therefore x = -2$

(c) $\dfrac{5^x}{5} = 125$

$5^x = 625$

$5^x = 5^4$

$\therefore\ x = 4$

(d) $10^x = 0.0001$

$10^x = 10^{-4}$

$\therefore\ x = -4$

23. (a) $2^x = 7$

$x = 2.81$

(b) $3^{x+1} = 15$

$x = 1.46$

(c) $7^x = \dfrac{1}{25}$

$x = -1.65$

(d) $\dfrac{4^{x-2}}{3} + 2 = 11$

$x = 4.38$

CHAPTER 3: *Matrices*

1. (a) Order: 1×2

(b) Order: 4×2

(c) Order: 1×1

2. (a) x = -2

y = 5

(b) a = -4

b = -3.5

d = -7.5

(c) t = 6

q = 3

p = -36

(d) x = -4

y = 3

z = -3

3. (a) $\begin{bmatrix} -6 & -5 \\ 9 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} x+3 & x+1 & 4x+3 \\ x-2 & -5x & 5x+2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ -1 \\ 8 \end{bmatrix}$

(d) $\begin{bmatrix} -y-1 & -5 \\ y+6 & 3y-7 \end{bmatrix}$

4. (a) $P = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$

(b) $P = \begin{bmatrix} 15 & 9 & -3 \\ -8 & -13 & 15 \end{bmatrix}$

5. (a) $\begin{bmatrix} 6 & -3 \\ 12 & 15 \end{bmatrix}$

(b) $\begin{bmatrix} -7 & -3 \\ -2 & 12 \end{bmatrix}$

(c) $\begin{bmatrix} -2.5 & 1 & -8 \\ -3.5 & 7 & -3 \end{bmatrix}$

6. (a) $\underset{(2 \times 1)}{X} \times \underset{(1 \times 3)}{Z} = \underset{(2 \times 3)}{XZ}$

(b) $\underset{(2 \times 2)}{Y} \times \underset{(1 \times 3)}{Z}$ does not exist

(c) $\underset{(2 \times 2)}{Y} \times \underset{(2 \times 2)}{Y} = \underset{(2 \times 2)}{Y^2}$

7. (a) $\begin{bmatrix} 12 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 6 \\ -5 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 4 & -2 \\ 0 & 2 & -1 \\ 0 & -6 & 3 \end{bmatrix}$

(d) Does not exist. The number of columns in the first matrix does not equal the number of rows in the second matrix.

(e) $\begin{bmatrix} -19 & 23 \\ 22 & -19 \\ 15 & -9 \end{bmatrix}$

8. (a) x = 4 y = -6

(b) a = 28 b = 4 c = -8

9. (a) $\begin{bmatrix} 7 & -4 \\ 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 7 & 4 \\ -4 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & -28 \\ 6 & 10 \end{bmatrix}$

(d) $\begin{bmatrix} 4 & -1 \\ -3 & 5 \end{bmatrix}$

10. (a) $\begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix}$

(b) $\begin{bmatrix} -15 & -36 \\ 5 & 12 \\ -8 & -21 \end{bmatrix}$

(c) $\begin{bmatrix} -12 & -42 \\ 4 & 14 \\ -6 & -24 \end{bmatrix}$

11. (a) $\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 \\ 6 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 8 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 \\ 2n & 1 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 0 \\ 18 & 1 \end{bmatrix}$

(f) $k = 32$

12. (a) (i) $\begin{bmatrix} -6 & -11 \\ -3 & -23 \end{bmatrix}$

(ii) $\begin{bmatrix} -6 & -1 & 18 \\ -9 & 6 & -3 \end{bmatrix}$

(b) (i) $AX = D$

$(2 \times 2) \times (?) = (2 \times 1)$

Order $X = 2 \times 1$

(ii) $X = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

13. (a) $a = 0$

$b = -3$

$c = -3$

(b) $\begin{bmatrix} 6 \\ 12 \\ -30 \end{bmatrix}$

(c) $k\begin{bmatrix} -3 \\ -6 \\ 15 \end{bmatrix} + (2k - 7)\begin{bmatrix} -3 \\ -6 \\ 15 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$-3k - 3(2k - 7) = 0$$
$$-3k - 6k + 21 = 0$$
$$-9k + 21 = 0$$
$$k = \frac{21}{9}$$

14. (a) $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$

(d) $\begin{bmatrix} 8 & 5 \\ 5 & 3 \end{bmatrix}$

(e) For F^n

$\begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix}$

$F_1 = 1$

$F_2 = 1$

$F_3 = 2$

$F_4 = 3$

15. (a) $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{5}{3} & \frac{4}{3} \end{bmatrix}$

(c) $A^2B^2 = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$

$(AB)^2 = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$

Does not apply to all 2×2 matrices.

16. (a)
$$\begin{array}{c} \\ P \\ Q \\ R \\ S \end{array}\begin{array}{c} \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

(b)
$$\begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{bmatrix} 0 & 3 \\ 3 & 1 \end{bmatrix} \end{array}$$

(c)
$$\begin{array}{c} \\ W \\ X \\ Y \\ Z \end{array}\begin{array}{c} \begin{array}{cccc} W & X & Y & Z \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

17. (a)
$$A = \begin{array}{c} \\ X \\ Y \end{array}\begin{array}{c} \begin{array}{cc} X & Y \end{array} \\ \begin{bmatrix} 0 & 2 \\ 2 & 1 \end{bmatrix} \end{array}$$

(b) $\begin{bmatrix} 4 & 2 \\ 2 & 5 \end{bmatrix}$

(c) $A^2 = \begin{bmatrix} 4 & 2 \\ 2 & 5 \end{bmatrix}$ same answer as part *(b)*

(d) $A^3 = \begin{bmatrix} 4 & 10 \\ 10 & 9 \end{bmatrix}$

18. (a)
$$M = \begin{array}{c|cccc} & F & G & S & I \\ \hline F & 0 & 1 & 1 & 0 \\ G & 0 & 0 & 0 & 1 \\ S & 1 & 0 & 0 & 1 \\ I & 0 & 1 & 1 & 0 \end{array}$$

(b)
$$M^2 = \begin{array}{c|cccc} & F & G & S & I \\ \hline F & 1 & 0 & 0 & 2 \\ G & 0 & 1 & 1 & 0 \\ S & 0 & 2 & 2 & 0 \\ I & 1 & 0 & 0 & 2 \end{array}$$

The number of two step communications eg there is 1 two step path that allows Fred to communicate with himself via a third person.
$F \rightarrow S \rightarrow F$

(c)
$$M^3 = \begin{array}{c|cccc} & F & G & S & I \\ \hline F & 0 & 3 & 3 & 0 \\ G & 1 & 0 & 0 & 2 \\ S & 2 & 0 & 0 & 4 \\ I & 0 & 3 & 3 & 0 \end{array}$$

The number of three step communications.

19. (a)
$$\text{From} \begin{array}{c|ccc} & \multicolumn{3}{c}{\text{To}} \\ & A & B & C \\ \hline A & 0 & 1 & 0 \\ B & 1 & 0 & 1 \\ C & 0 & 2 & 0 \end{array}$$

(b)
$$\text{From} \begin{array}{c|ccc} & \multicolumn{3}{c}{\text{To}} \\ & A & B & C \\ \hline A & 1 & 0 & 1 \\ B & 0 & 3 & 0 \\ C & 2 & 0 & 2 \end{array}$$

20. (a)
$$M = \begin{array}{c|cc} & C & V \\ \hline S & 10 & 15 \\ M & 7 & 4 \\ L & 14 & 8 \end{array}$$

(b)
$$P = \begin{array}{ccc} S & M & L \\ \hline 1.2 & 1.5 & 2 \end{array}$$

(c) $PM = \begin{bmatrix} 50.5 & 40 \end{bmatrix}$

Revenue from weekly sales of chocolate milk is \$50.50 and vanilla milk \$40

(d) $PMX = \begin{bmatrix} 90.5 \end{bmatrix}$

Total revenue from weekly sales of milk is \$90.50

21. (a) Cost Matrix
$$C = \begin{array}{cccc} AR & A & Ch & Co \\ \hline 35 & 25 & 15 & 10 \end{array}$$

Sales Matrix
$$S = \begin{array}{c|ccc} & W & T & F \\ \hline AR & 45 & 60 & 90 \\ A & 160 & 150 & 210 \\ Ch & 100 & 120 & 180 \\ Co & 80 & 90 & 140 \end{array}$$

$$CS = \begin{array}{ccc} W & T & F \\ \hline 7875 & 8550 & 12\,500 \end{array}$$

Wednesday \$7875

Thursday \$8550

Friday \$12 500

(b) Let $X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$CSX = \begin{bmatrix} 28\,925 \end{bmatrix}$

Total sales \$28 925

22. (a)
$$\begin{array}{c|cc} & C & M \\ \hline Th & 30 & 10 \\ F & 50 & 65 \end{array} \quad \begin{array}{c|ccccc} & F & E & S & V & B \\ \hline C & 0.2 & 4 & 0.1 & 1 & 0.3 \\ M & 0.4 & 3 & 0.2 & 1 & 0.4 \end{array}$$

$$= \begin{array}{c|ccccc} & F & E & S & V & B \\ \hline Th & 10 & 150 & 5 & 40 & 13 \\ F & 36 & 395 & 18 & 115 & 41 \end{array}$$

(b)
$$\begin{array}{c|ccccc} & F & E & S & V & B \\ \hline Th & 10 & 150 & 5 & 40 & 13 \\ F & 36 & 395 & 18 & 115 & 41 \end{array} \quad \begin{array}{c|c} \text{Cost} & \\ \hline 2.5 & F \\ 0.5 & E \\ 1.25 & S \\ 0.1 & V \\ 3 & B \end{array}$$

$$Cost = \begin{array}{c} \text{Th} \\ \text{F} \end{array} \begin{bmatrix} 149.25 \\ 444.5 \end{bmatrix}$$

(c)

$$\begin{array}{cc} & \begin{array}{cc} \text{C} & \text{M} \end{array} \\ \begin{array}{c} \text{Th} \\ \text{F} \end{array} & \begin{bmatrix} 30 & 10 \\ 50 & 65 \end{bmatrix} \end{array} \begin{bmatrix} 15 \\ 20 \end{bmatrix} \begin{array}{c} \text{C} \\ \text{M} \end{array}$$

$$Revenue = \begin{array}{c} \text{Th} \\ \text{F} \end{array} \begin{bmatrix} 650 \\ 2050 \end{bmatrix}$$

(d) Profit: Thursday $650 – $149.25

= $500.75

Friday = $2050 – $444.50

= $1605.50

CHAPTER 4: *Finance*

1. Weetbix

1kg: $\frac{695}{1000}$ = 0.695 cents/g

500g: $\frac{399}{500}$ = 0.798 cents/g

375g: $\frac{265}{375}$ = $0.70\dot{6}$ cents/g

Best value for money is the 1kg size of Weetbix.

2. *(a)* Yoghurt

(i) $\frac{634}{1200}$ = $0.528\dot{3}$ cents/g

(ii) $\frac{449}{1000}$ = 0.449 cents/g

1kg tub is best buy.

(b) Steak

(i) $\frac{1494}{1068}$ = 1.3989 cents/g

(ii) $\frac{315}{224}$ = 1.40625 cents/g

The best buy is the 1.068kg of steak.

(c) Batteries

(i) $\frac{15.03}{10}$ = $1.503/battery

(ii) $\frac{7.06}{4}$ = $1.765/battery

The 10 pack is the best buy.

(d) Toilet paper

(i) $\frac{2.36}{2}$ = $1.18/roll

(ii) $\frac{4.59}{6}$ = $0.765/roll

(iii) $\frac{5.61}{9}$ = $\$0.62\dot{3}$/roll

(iv) $\frac{8.49}{12}$ = $0.7075/roll

The best buy is 9 rolls of toilet paper.

3. **Cola**:

600mL: $\frac{287}{600}$ = $0.478\dot{3}$ cents/mL

6 × 375mL: $\frac{687}{2250}$ = $0.305\dot{3}$ cents/mL

15 × 375mL: $\frac{1299}{5625}$ = $0.2309\dot{3}$ cents/mL

24 × 375mL: $\frac{1466}{9000}$ = $0.162\dot{8}$ cents/mL

1.5L: $\frac{255}{1500}$ = 0.17 cents/mL

2L: $\frac{326}{2000}$ = 0.163 cents/mL

Best buy is the 24 × 375mL cans

4. *(a)* Milk

500mL: $\frac{107}{500}$ = 0.214 cents/mL

1L: $\frac{196}{1000}$ = 0.196 cents/mL

2L: $\frac{317}{2000}$ = 0.1585 cents/mL

3L: $\frac{477}{3000}$ = 0.159 cents/mL

Best buy is the 2L container of milk.

(b) A 2L container of milk @ 500mL per day will last for 4 days.

Cost $3.17

The 500mL container of milk for the same period of time, i.e., 4 days

Cost $4.28

Best buy – the 2L container, saving $1.11

However, with only 2 days to consume the milk the 2L container has a wastage of 2 days, i.e., $1.59.

∴ It is advisable to buy 2 × 500mL containers.

5. (a) $\$2000 \times 1.1619$

$= \$2323.80$ Singapore dollars

(b) $\$550 \times 0.5519$

303.55 UK Sterling

(c) $\dfrac{4500}{29.94} = \$150.30$ Australian dollars

(d) $\dfrac{7000}{7.2126} = \$970.52$ Australian dollars

6. (a) $\$2500 \times 95.49 = 238\,725$ Japanese Yen

(b) $\dfrac{19\,098}{95.49} = \$200$ Australian dollars

7. $\dfrac{3400}{2.9802} = \$1141$ Australian dollars (nearest dollar)

8. $\dfrac{40}{0.5519} = \$72.48$ AUD

$\dfrac{50}{1.1619} = \$43.03$ AUD

$\dfrac{75}{0.9306} = \$80.59$ AUD

$\dfrac{125}{1.1035} = \$113.28$ AUD

$\dfrac{45\,000}{10\,939} = \$4.11$ AUD

Total: \$313 (nearest dollar)

9. (a) Value $= (8500 \times 0.72) + (7000 \times 5.90) + (500 \times 24)$

$= \$59\,420$

(b) Dividend $= (8500 \times 0.1) + (7000 \times 0.35) + (500 \times 1.5)$

$= \$4050$

10. Dividend per share $= \dfrac{7\,500\,000}{3\,200\,000} \times 12\%$

$= \$0.28$

11. (a) $2000 \times \$7.50 = \$15\,000$

Brokerage fee $= \$69.95$

(b) $7500 \times \$5.25 = \$39\,375$

Brokerage fee $= 0.4\% \times 39\,375$

$= \$157.50$

12. (a) Cost $= 8000 \times \$7.25$

$= \$58\,000$

Brokerage fee $= \$58\,000 \times 3\%$

$= \$1740$

Total cost $= \$59\,740$

(b) Dividend $= \$7.25 \times 4.5\%$

$= \$0.32625$ per share

Total dividend $= \$0.32625 \times 8000$

$= \$2610$

(c) Selling $= 8000 \times \$9.05$

$= \$72\,400$

Brokerage fee $= \$72\,400 \times 3\%$

$= \$2172$

Gain: $\$72\,400 + \$2610 - \$59\,740 - \$1740 - \$2172$

$= \$11\,358$

13. (a) Total earnings per share $= \dfrac{\$0.80}{0.2}$

$= \$4$

(b) $P/E = \dfrac{\$16.50}{\$4}$

$= 4.125$

14. (a) Gerry:

Earns $\dfrac{\$10\,000}{26} = \$384.62$ per fortnight

This is less than \$415 – no reduction.

Allowance \$226.80/fortnight

(b) Deanna: no income

Allowance \$542.90/fortnight

(c) Richard:

Earns $\dfrac{\$13\,520}{26} = \$520$ per fortnight

This involves a reduction of 60c/dollar above \$498

Allowance: $\$414.40 - 0.6(\$520 - \$498)$

$= \$401.20$/fortnight

15. *(a)* David: no income

Allowance: \$766.00/fortnight

(b) Lois:

Earns $\frac{\$4000}{26}$ = \$153.85 per fortnight

This is below \$160 – no reduction

Allowance: = \$766.00/fortnight

(c) Mark/Leonie:

Earns: \$200 per fortnight

This is below \$284 – no reduction

Allowance: \$1154.80/fortnight

16. *(a)* Kai:

Allowance = \$522.40/fortnight

(b) Sally:

Allowance = \$460.90/fortnight

(c) Tom:

Earnings \$120 per fortnight

This involves a reduction of 50c in the dollar above \$100

Allowance = \$510.50 – 0.5(120 – 100)

= \$500.50

17. *(a)* ~ \$1769.23

(b) ~ \$7666.67

(c) ~ \$3538.46

18. Fran works 9 hours a day for 5 days ie 45 hours

Normal: 36hrs @ \$14.50 = \$522

Time and a half: 3hrs @ \$21.75 = \$65.25

Double time: 6hrs @ \$29.00 = \$174.00

Total: \$761.25

19. Coleen: \$913.75/week

Sam: \$856.00/week

Pam: \$920.00/week

Pam earns the most.

20. Earnings:

(\$100 × \$5.70) + (\$50 × \$5.70 × 1.4)

= \$969

21. *(a)* \$270

(b) $\frac{2000}{7.50} \sim 267$

267 pizzas

22. *Normal rate*: x

(35 × x) + (4 × 1.5x) + (6 × 2x) = 1934.50

x = \$36.50/hour

23. *(a)* *Zac*

Normal: 36hrs @ \$17.40	=	\$626.40
Time and a half: 6hrs @ \$26.10	=	\$156.60
Double: 12hrs @ \$34.80	=	\$417.60
Meal allowance \$10.75 × 7	=	\$75.25
		\$1275.85
Living allowance 1.5%		\$19.14
Total		\$1294.99

(b) Penny:

Normal: 40hrs @ \$15.80	=	\$632.00
Time and a half: 6.5hrs @ \$23.70	=	\$154.05
Double: 6.75hrs @ \$31.60	=	\$213.30
Overtime: 8.5hrs @ \$23.70	=	\$201.45
Clothing allowance	=	\$120
Total		\$1320.80

CHAPTER 5: *Pythagoras*

1. *(a)* $x^2 = 8^2 + 6^2$

$x^2 = 100$

$x = 10\text{m}$

(b) $x^2 = 9^2 + 12^2$

$x^2 = 225$

$x = 15\text{cm}$

(c) $x^2 = 5^2 - 4^2$

$x^2 = 9$

$x = 3\text{cm}$

(d) $y^2 = 5.1^2 - 3.7^2$

$y^2 = 12.32$

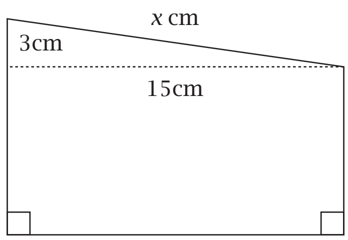
$y = \sqrt{12.32}$

$x^2 = \left(\sqrt{12.32}\right)^2 + (3.2)^2$

$x^2 = 22.56$

$x = 4.75\text{mm}$

(e)



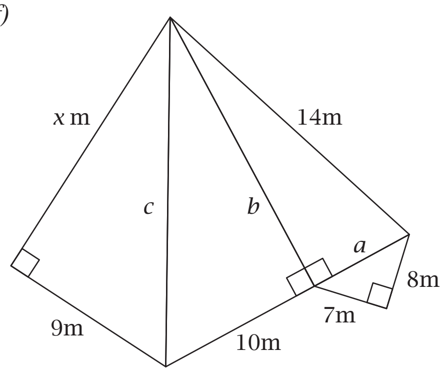
$x^2 = 3^2 + 15^2$

$x^2 = 9 + 225$

$x^2 = 234$

$x = 15.30\text{cm}$

(f)



$a^2 = 8^2 + 7^2$

$a^2 = 64 + 49$

$a^2 = 113$

$a = \sqrt{113}$

$b^2 = 14^2 - \left(\sqrt{113}\right)^2$

$b^2 = 83$

$b = \sqrt{83}$

$c^2 = \left(\sqrt{83}\right)^2 + 10^2$

$c^2 = 183$

$c = \sqrt{183}$

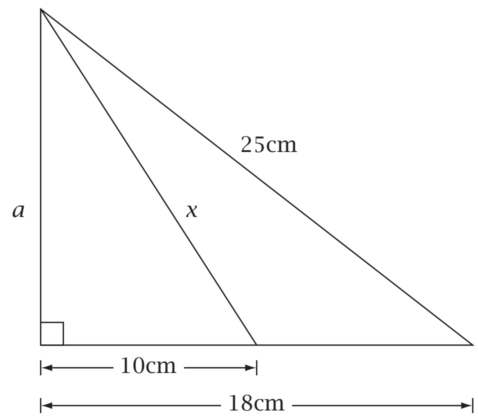
$x^2 = \left(\sqrt{183}\right)^2 - 9^2$

$x^2 = 102$

$x = \sqrt{102}$

$x = 10.1\text{m}$

(g)



$a^2 = 25^2 - 18^2$

$a^2 = 301$

$a = \sqrt{301}$

$x^2 = \left(\sqrt{301}\right)^2 + 10^2$

$x^2 = 401$

$x = 20.02\text{cm}$

2. (a) $AB^2 = 4^2 + 2^2$

$AB^2 = 16 + 4$

$AB = \sqrt{20}$

$AB = 4.47$

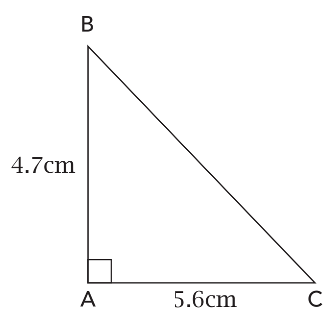
(b) $PQ^2 = 3^2 + 5^2$

$PQ^2 = 9 + 25$

$PQ = \sqrt{34}$

$PQ = 5.83$

3. (a)



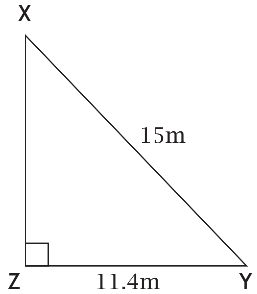
$BC^2 = AC^2 + AB^2$

$BC^2 = 5.6^2 + 4.7^2$

$BC^2 = 53.45$

$BC = 7.31\text{cm}$

(b)



$XZ^2 = XY^2 - ZY^2$

$XZ^2 = 15^2 - 11.4^2$

$XZ^2 = 95.04$

$XZ = 9.75\text{m}$

4. *(a)* $39^2 = 36^2 + 15^2$

$1521 = 1521$

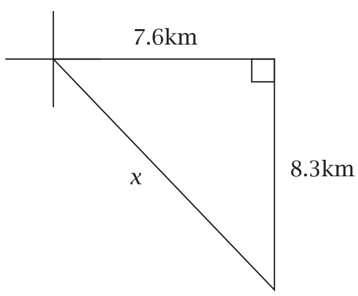
ΔPQR is right angled

(b) $20^2 = 13^2 + 7^2$

$400 \neq 218$

ΔABC is not right angled

5.



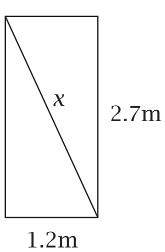
$x^2 = 7.6^2 + 8.3^2$

$x^2 = 126.65$

$x = 11.25$

Explorer is 11.25km from base camp.

6.



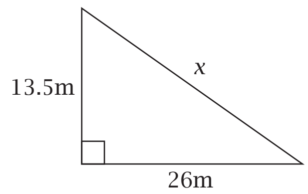
$x^2 = 2.7^2 + 1.2^2$

$x^2 = 8.73$

$x = 2.95$

Length of brace required is 2.95m.

7.



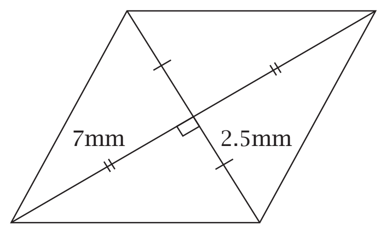
$x^2 = 13.5^2 + 26^2$

$x^2 = 858.25$

$x = 29.3$

Distance is 29.3m.

8.



(a) $x^2 = 7^2 + 2.5^2$

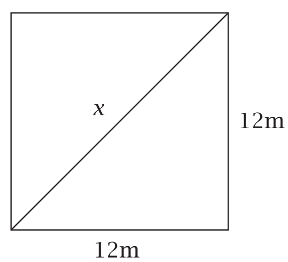
$x^2 = 55.25$

$x = 7.43\text{mm}$

Length of rhombus is 7.43mm.

(b) Perimeter is 29.72mm.

9.



$x^2 = 12^2 + 12^2$

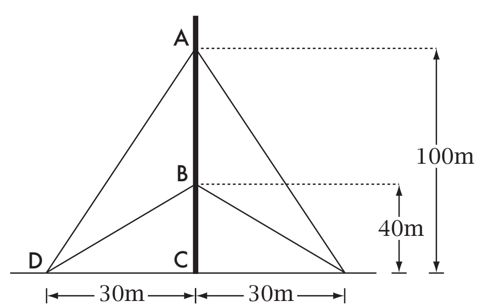
$x^2 = 144 + 144$

$x = \sqrt{288}$

$x = 16.97$

Diagonal is 16.97m long.

10.



$BD^2 = 30^2 + 40^2$

$BD^2 = 2500$

$BD = 50$

$$AD^2 = 100^2 + 30^2$$
$$AD^2 = 10\,900$$
$$AD = 104.4\text{m}$$

Total length of wire required is 308.8m.

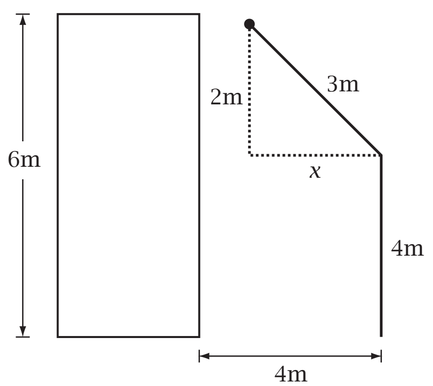
11. *(a)*

$$3^2 + x^2 = (2x)^2$$
$$9 + x^2 = 4x^2$$
$$3x^2 = 9$$
$$x^2 = 3$$
$$x = \sqrt{3}$$
$$x = 1.73$$

(b)

$$7^2 + \left(\frac{x}{2}\right)^2 = (3x)^2$$
$$49 + \frac{x^2}{4} = 9x^2$$
$$9x^2 - \frac{x^2}{4} = 49$$
$$\frac{35}{4}x^2 = 49$$
$$x^2 = \frac{28}{5}$$
$$x = \sqrt{\frac{28}{5}}$$
$$x = 2.37$$

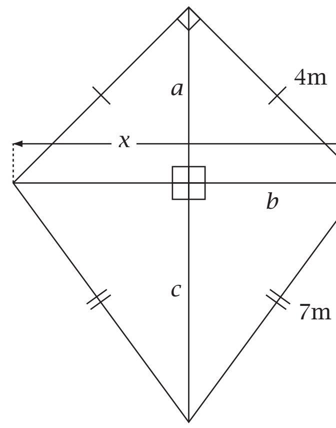
12.



$$x^2 = 3^2 - 2^2$$
$$x^2 = 5$$
$$x = \sqrt{5}$$
$$x = 2.24\text{m}$$

The cherry picker is 1.76m from the building.

13.



$$x^2 = 4^2 + 4^2$$
$$x^2 = 32$$
$$x = \sqrt{32}$$
$$x = 5.66\text{m}$$

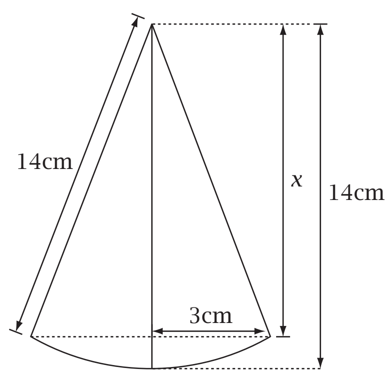
$$b = \sqrt{8}\text{m}$$
$$b = 2.83\text{m}$$

$$a^2 = 4^2 - \left(\sqrt{8}\right)^2$$
$$a^2 = 8$$
$$a = \sqrt{8}$$
$$a = 2.83$$

$$c^2 = 7^2 - \left(\sqrt{8}\right)^2$$
$$c^2 = 41$$
$$c = \sqrt{41}$$
$$c = 6.40\text{m}$$

Total length of fencing is 36.89m.
Cost = 36.89 × \$18.50 = \$682.47

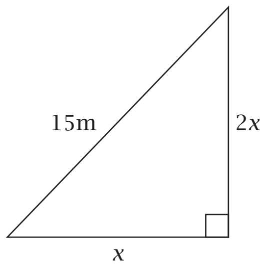
14.



$$x^2 = 14^2 - 3^2$$
$$x^2 = 187$$
$$x = \sqrt{187}$$
$$x = 13.67\text{cm}$$

Pendulum will rise 14 – 13.67cm. i.e. 0.33cm.

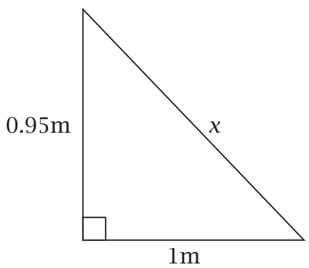
15.



$$(2x)^2 + (x)^2 = 15^2$$
$$4x^2 + x^2 = 225$$
$$5x^2 = 225$$
$$x^2 = 45$$
$$x = \sqrt{45}$$
$$x = 6.71\text{m}$$

Pole must be placed 13.42m up the wall.

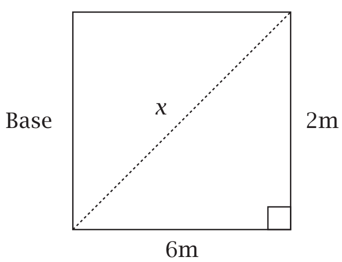
16. (a)



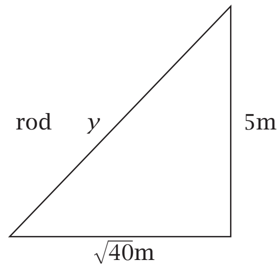
$$x^2 = 0.95^2 + 1^2$$
$$x^2 = 1.9025$$
$$x = \sqrt{1.9025}$$
$$x = 1.379\text{m}$$

Longest rod is 1.379m.

(b)



$$x^2 = 6^2 + 2^2$$
$$x^2 = 36 + 4$$
$$x = \sqrt{40}$$



$$y^2 = 5^2 + 40$$
$$y^2 = 25 + 40$$
$$y = \sqrt{65}$$
$$y = 8.062\text{m}$$

Longest rod is 8.062m.

CHAPTER 6: *Trigonometry*

1. (a) 0.57
   (b) 0.31
   (c) 2.54

2. (a) 13.77
   (b) 28.96
   (c) 55.72

3. (a) $\sin 40^\circ = \dfrac{x}{5}$

$$x = 5\sin 40^\circ$$
$$x = 3.21$$

(b) $\dfrac{21.5}{x} = \cos 12^\circ$

$$x = \frac{21.5}{\cos 12^\circ}$$
$$x = 21.98$$

(c) $\tan 46^\circ = \dfrac{x}{10}$

$$x = 10\tan 46^\circ$$
$$x = 10.36$$

4. (a) $\sin 42^\circ = \dfrac{x}{12}$

$$x = 12\sin 42^\circ$$
$$x = 8.03\text{m}$$

(b) $\tan x^\circ = \dfrac{17}{15}$

$$x = 48.58^\circ$$

(c) $\cos 51^\circ = \dfrac{x}{25}$

$$x = 25\cos 51^\circ$$
$$x = 15.73\text{m}$$

5. (a) $\tan 21^\circ = \dfrac{x}{15}$

$x = 15 \tan 21^\circ$

$x = 5.758\text{m}$

$\sin 46^\circ = \dfrac{5.758}{y}$

$y = \dfrac{5.758}{\sin 46^\circ}$

$y = 8\text{m}$

(b) $\tan 36^\circ = \dfrac{x}{17}$

$x = 17 . \tan 36^\circ$

$x = 12.35\text{cm}$

$\cos 36^\circ = \dfrac{17}{z}$

$z = \dfrac{17}{\cos 36^\circ}$

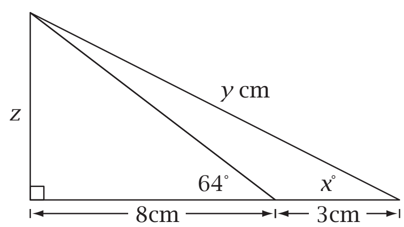
$z = 21.01\text{cm}$

$\sin 12^\circ = \dfrac{y}{21.01}$

$y = 21.01 \sin 12^\circ$

$y = 4.37\text{cm}$

(c)



$\tan 64^\circ = \dfrac{z}{8}$

$z = 8 \tan 64^\circ$

$z = 16.40$

$y^2 = 11^2 + (16.40)^2$

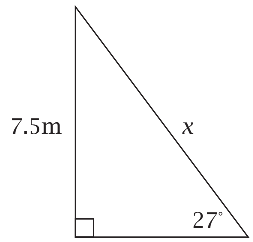
$y^2 = 389.96$

$y = 19.75\text{cm}$

$\tan x^\circ = \dfrac{16.40}{11}$

$x = 56.15^\circ$

6.



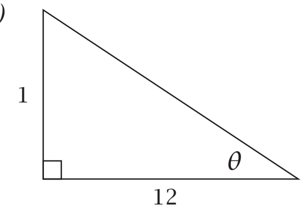
$\sin 27^\circ = \dfrac{7.5}{x}$

$x = \dfrac{7.5}{\sin 27^\circ}$

$x = 16.52$

The ladder is 16.52m long.

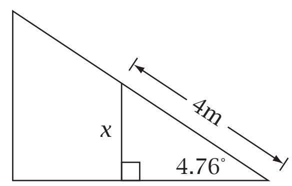
7. (a)



$\tan\theta = \dfrac{1}{12}$

$\theta = 4.76^\circ$

(b)

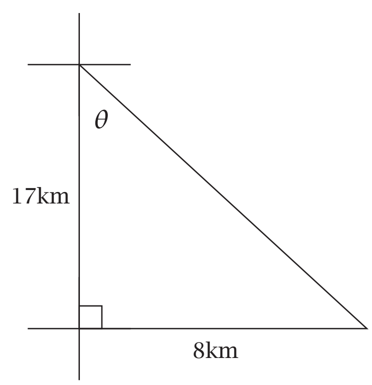


$\sin 4.76^\circ = \dfrac{x}{4}$

$x = 4 \sin 4.76^\circ$

$x = 0.33\text{m}$

8.



$\tan\theta^\circ = \dfrac{8}{17}$

$\theta = 25.2^\circ$

$\text{Bearing} = 180^\circ - 25.2^\circ$

$= 154.8^\circ$

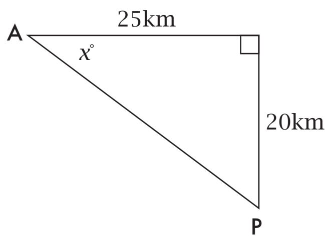
9. *(a)* Distance Pegasus: 20km
Distance Atlantis: 25km

$$\text{Distance}^2 = 20^2 + 25^2$$

$$\text{Distance}^2 = 1025$$

$$\text{Distance} = \sqrt{1025}$$

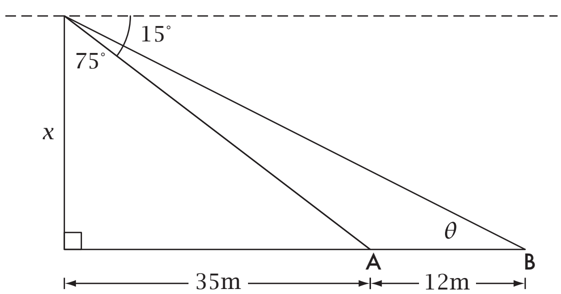
Distance ships are apart is 32.02km



(b) $\tan x = \dfrac{20}{25}$

$x^\circ = 38.66^\circ$

Bearing of Pegasus from Atlantis is 128.66°.

10. 

(a) $\tan 75^\circ = \dfrac{35}{x}$

$x = \dfrac{35}{\tan 75^\circ}$

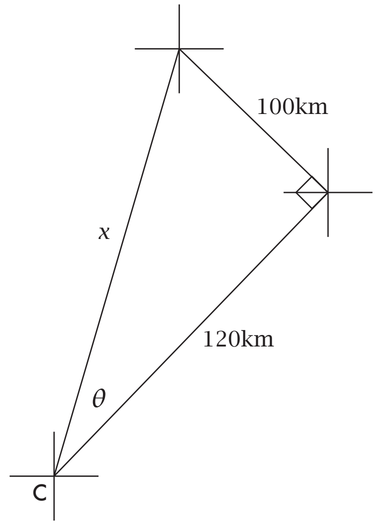
$x = 9.38\text{m}$

Cliff is 9.38m high.

(b) $\tan\theta = \dfrac{9.38}{47}$

$\theta = 11.3^\circ$

Angle of elevation is 11.3°

11. 

$x^2 = 100^2 + 120^2$

$x^2 = 24\,400$

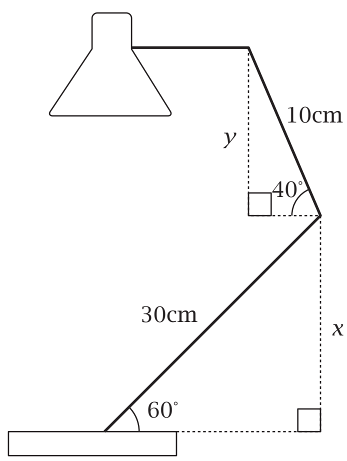
$x = 156.20\text{km}$

Distance is 156.20km from checkpoint.

$\tan\theta^\circ = \dfrac{100}{120}$

$\theta = 39.81^\circ$

Bearing is $46^\circ - 39.81^\circ$
$= 6.19^\circ$

12. 

$\sin 60^\circ = \dfrac{x}{30}$

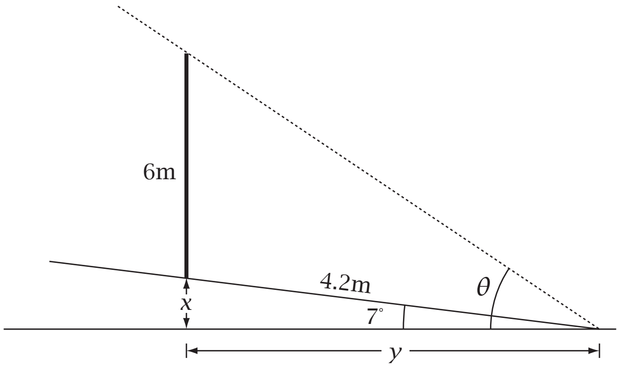
$x = 25.98\text{cm}$

$\sin 40^\circ = \dfrac{y}{10}$

$y = 6.43\text{cm}$

Total distance from base to point A is 32.41cm.

13.



$$\sin 7^\circ = \frac{x}{4.2}$$

$$x = 4.2\sin 7^\circ = 0.51$$

$$\cos 7^\circ = \frac{y}{4.2}$$

$$y = 4.2\cos 7^\circ = 4.17$$

$$\tan\theta = \frac{6 + 4.2\sin 7^\circ}{4.2\cos 7^\circ}$$

$$\theta = 57.4^\circ$$

Angle of elevation is 57.4°

14. *(a)*

$$\frac{x}{\sin 98^\circ} = \frac{20}{\sin 35^\circ}$$

$$x = \frac{20\sin 98^\circ}{\sin 35^\circ}$$

$$x = 34.53$$

(b)

$$\frac{6}{\sin C} = \frac{5}{\sin 28^\circ}$$

$$\sin C = \frac{6\sin 28}{5}$$

$$C = 34.29^\circ$$

$$\therefore\ x = 117.71^\circ$$

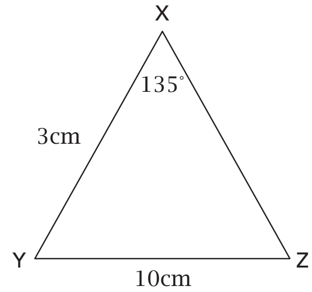
(c)

$$\frac{x}{\sin 40^\circ} = \frac{9}{\sin 60^\circ}$$

$$x = \frac{9\sin 40^\circ}{\sin 60^\circ}$$

$$x = 6.68$$

15.



$$\frac{3}{\sin Z} = \frac{10}{\sin 135^\circ}$$

$$\sin Z = \frac{3\sin 135^\circ}{10}$$

$$Z = 12.25^\circ$$

$$\therefore\ \angle Y = 32.75^\circ$$

16.

$$\cos 32^\circ = \frac{17}{x}$$

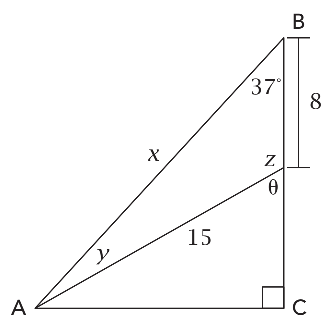
$$x = \frac{17}{\cos 32^\circ}$$

$$x = 20.046$$

$$\frac{y}{\sin 41^\circ} = \frac{20.046}{\sin 77^\circ}$$

$$y = 13.50$$

17.



$$\frac{8}{\sin y} = \frac{15}{\sin 37^\circ}$$

$$\sin y = \frac{8\sin 37^\circ}{15}$$

$$y = 18.72^\circ$$

$$z = 124.28^\circ$$

$$\therefore\ \theta = 55.72^\circ$$

$$\frac{x}{\sin 124.28^\circ} = \frac{8}{\sin 18.72^\circ}$$

$$x = 20.6$$

18. *(a)* $x^2 = 5^2 + 6^2 - 2(5)(6)\cos 63^\circ$

$x = 5.81$

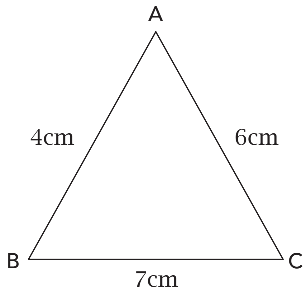
(b) $\cos x = \dfrac{5^2 + 8^2 - 9^2}{2(5)(8)}$

$x = 84.26^\circ$

(c) $x^2 = 7^2 + 8^2 - 2(7)(8)\cos 110^\circ$

$x = 12.3$

19.



$\cos C = \dfrac{7^2 + 6^2 - 4^2}{2(7)(6)}$

$C = 34.77^\circ$

20. $\cos x = \dfrac{10^2 + 9^2 - 7^2}{2(10)(9)}$

$x = 42.83^\circ$

$\sin 42.83^\circ = \dfrac{y}{9}$

$y = 6.12$

21. *(a)* Area $= \dfrac{1}{2}(8)(10)\sin 43^\circ$

$= 27.28\text{cm}^2$

(b) Area $= \dfrac{1}{2}(4.1)(5.8)\sin 60^\circ$

$= 10.297\text{cm}^2$

(c) $\cos T = \dfrac{7^2 + 8^2 - 10^2}{2(7)(8)}$

$T = 83.33^\circ$

Area $= \dfrac{1}{2}(7)(8)\sin 83.33^\circ$

$= 27.81\text{cm}^2$

(d) Area $= \dfrac{1}{2}(7.1)(9.5)\sin 47^\circ$

$= 24.665\text{cm}^2$

2 Triangles

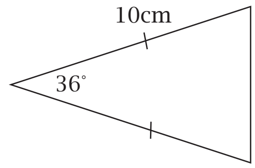
Total Area $= 49.33\text{cm}^2$

22. Area $= \dfrac{1}{2}ab\sin C$

$767.69 = \dfrac{1}{2}(43.1)(38.7)\sin C$

$C = 67^\circ$

23.



Central angle $= \dfrac{360^\circ}{10}$

$= 36^\circ$

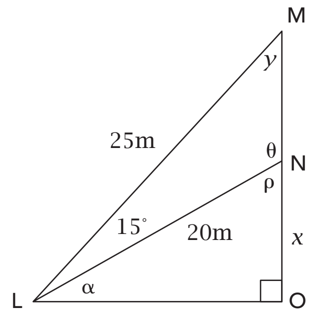
Area $= \dfrac{1}{2}(10)(10)\sin 36^\circ$

(1 triangle) $= 29.389\text{cm}^2$

Area $= 29.389 \times 10$

(decagon) $= 293.89\text{cm}^2$

24.



$MN^2 = 25^2 + 20^2 - 2(25)(20)\cos 15^\circ$

$MN = 7.69\text{m}$

$\dfrac{20}{\sin y} = \dfrac{7.69}{\sin 15}$

$y = 42.31^\circ$

$\theta = 122.69^\circ$

$p = 57.31^\circ$

$\alpha = 32.69^\circ$

$\sin 32.69^\circ = \dfrac{x}{20}$

$x = 10.802\text{m}$

(a) Height of building 10.802m

(b) Height of flagpole 7.69m

(c) Area $\triangle LMN = \dfrac{1}{2}(20)(25)\sin 15^\circ$

$= 64.705\text{m}^2$

25. *(a)* $WK^2 = WX^2 + XK^2$

$WK^2 = 23^2 + 10^2$

$WK = \sqrt{629}$

$= 25.08\text{cm}$

$\tan\angle WKX = \dfrac{23}{10}$

$\angle WKX = 66.5^\circ$

$\angle YKL = 73.5^\circ$

$\cos 73.5^\circ = \dfrac{KY}{12}$

$KY = 3.408\text{cm}$

$\sin 73.5^\circ = \dfrac{LY}{12}$

$LY = 11.506\text{cm}$

(b) $ZL = 11.494\text{cm}$

$\therefore WL^2 = WZ^2 + ZL^2$

$WL^2 = 13.408^2 + 11.494^2$

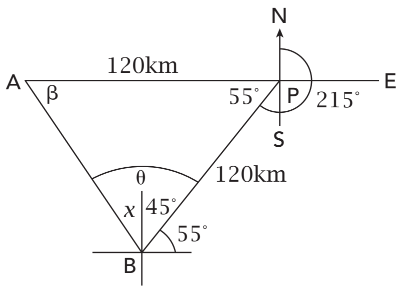
$WL = 17.660\text{cm}$

(c) $WZ = 13.408\text{cm}$

(d) Area $\triangle WKL = \frac{1}{2}(25.08)(17.66)\sin 40^\circ$

$= 142.35\text{cm}^2$

26.



(a) Distance $AB^2 = 120^2 + 120^2 - 2(120)(120)\cos 55^\circ$

$AB = 110.82\text{km}$

(b) $\frac{120}{\sin\theta} = \frac{110.82}{\sin 55^\circ}$

$\theta = 62.5^\circ$

$x = 62.5^\circ - 45^\circ$

$x = 17.5^\circ$

Bearing of ship A from ship B is $360 - 17.5^\circ$

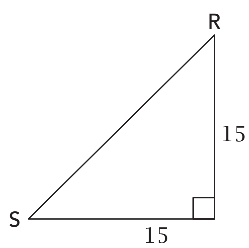
$= 342.5^\circ$

(c) $\beta = 62.5^\circ$

Bearing of ship B from ship A is $90^\circ + 62.5^\circ$

$= 152.5^\circ$

27. (a)

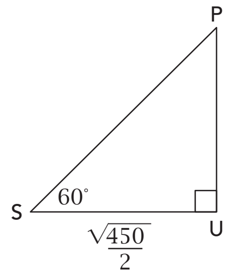


$SR^2 = 15^2 + 15^2$

$SR^2 = 450$

$SR = 21.21\text{cm}$

(b)



$\cos 60^\circ = \dfrac{\frac{\sqrt{450}}{2}}{PS}$

$PS = 21.21\text{cm}$

(c) $PU^2 = PS^2 - SU^2$

$PU^2 = 450 - 112.50$

$PU^2 = 337.5$

$PU = 18.37\text{cm}$

(d) $PV^2 = PU^2 + UV^2$

$= 337.5 + (7.5)^2$

$PV^2 = 393.75$

$PV = 19.84\text{cm}$

(e) $\sin\angle PVU = \frac{18.37}{19.84}$

$\angle PVU = 67.8^\circ$

CHAPTER 7: *Perimeter, Area, Surface Area & Volume*

1. (a) Perimeter $= 27\text{cm}$

(b) Perimeter $= \frac{\pi \times 4}{2} + 4$

$= 10.28\text{m}$

(c) Perimeter $= \frac{270}{360} \times 2\pi(10) + 20$

$= 67.12\text{cm}$

(d) Perimeter $= \frac{50}{360} \times 2\pi(6) + 12$

$= 17.24\text{mm}$

2. (a) $P = \pi \times 12 + \dfrac{\pi \times 24}{2}$

$= 75.4\text{cm}$

(b) $P = \dfrac{\pi \times 5}{2} + 12$

$= 19.85\text{mm}$

(c) $P = \dfrac{\pi \times 12}{2} + (4 \times 6)$

$= 42.85\text{cm}$

3. (a) $\text{Area} = \dfrac{(5+10)}{2} \times 4$

$= 30\text{mm}^2$

(b) $\text{Area} = \dfrac{\pi(3)^2}{2}$

$= 14.14\text{cm}^2$

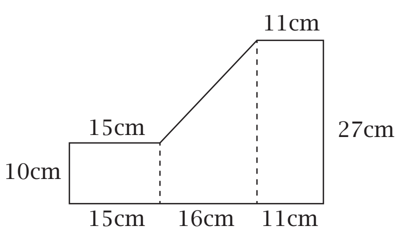
(c) $\text{Area} = \dfrac{1}{2}(4)(5)$

$= 10\text{cm}^2$

(d) $\text{Area} = \dfrac{60}{360} \times \pi(10)^2$

$= 52.36\text{cm}^2$

4. (a)



$\text{Area} = (15 \times 10) + (11 \times 27) + \left(\dfrac{10+27}{2} \times 16\right)$

$= 150 + 297 + 296$

$= 743\text{cm}^2$

(b) $\text{Area} = (4 \times 8) \times 2 + (5 \times 8)$

$= 104\text{mm}^2$

(c) $\text{Area} = \dfrac{\pi(6)^2}{2} - \dfrac{\pi(3)^2}{2}$

$= 42.41\text{cm}^2$

(d) $\text{Area} = 16 - \pi(2)^2$

$= 3.43\text{cm}^2$

5. (a) $A = (28 \times 7)$ rectangle

$A = (\pi \times 3.5^2 \times 4)$ circles

Shaded area $= 196 - 153.94$

$= 42.06\text{cm}^2$

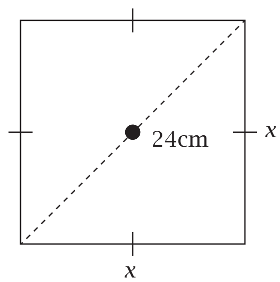
(b) $A = \dfrac{\pi(4)^2}{2}$ $\left(\dfrac{1}{2}\text{ circle}\right)$

$A = \dfrac{1}{2}(8)(4)$ (triangle)

Shaded area $= 25.13 - 16$

$= 9.13\text{cm}^2$

(c)



$x^2 + x^2 = 24^2$

$2x^2 = 576$

$x^2 = 288$

Area square $= 288\text{cm}^2$

Area circle $= \pi(12)^2$

$= 452.39$

Shaded area $= 452.39 - 288$

$= 164.39\text{cm}^2$

6. Perimeter : $\dfrac{1}{2}$ circles

$= \dfrac{4 \times \pi \times 2}{2}$

$= 4\pi$

Perimeter : $\dfrac{1}{4}$ circles

$= \dfrac{4 \times \pi \times 2}{4}$

$= 2\pi$

Total $= 18.8\text{cm}$

7. (a) Area path $=$ Area backyard – Area lawn

$= (15 \times 10) - (13 \times 8)$

$= 46\text{m}^2$

(b) Shaded area

$= (13 \times 8) - (4 \times 1.75^2) - (4 \times 3)$

$= 79.75\text{m}^2$

(c) Area (not lawn)

$= 46 + (4 \times 1.75^2) + (4 \times 3)$

$= 70.25\text{m}^2$

Fraction $\dfrac{70.25}{150} = \dfrac{281}{600}$

(d) Cost : $\$12.50 \times 79.75$

$= \$996.88$

8. (a) Area $= 27 \times 20$

$= 540\text{m}^2$

(b) Area (each tile) $= 1.5\text{m}^2$

Number of tiles $= \dfrac{540}{1.5}$

$= 360$

(c) Total cost $= 360 \times \$5.50$

$= \$1980$

9. (a) Length fencing

$= 12 + 11 + 9 + \dfrac{\pi \times 16}{2} + \dfrac{75}{360} \times 2\pi(5)$

$= 63.68\text{m}$

(b) Total cost

$= (6 \times \$12) + (63.68 \times \$80)$

$= \$5166.40$

10. (a) Area $= \dfrac{40}{360} \times \pi(1.25)^2$

$= 0.545\text{m}^2$

(b) Area each tile $= 24\text{cm}^2$

$= 0.0024\text{m}^2$

Number of tiles required (extra 10%)

$= \dfrac{0.545}{0.0024} \times 1.1$

$\approx$ 250 tiles

11. (a) Volume $= (6\times4\times2) + (1\times1\times4)$

$= 52\text{cm}^3$

SA $= (6\times2\times2) + (4\times2\times2) + (6\times4)$
$+ (2.5\times4\times2) + (1\times1\times2)$
$+ (1\times4\times3)$

$= 98\text{cm}^2$

(b) Volume $= \left(\dfrac{1}{2}\times8\times3\right) \times 14$

$= 168\text{cm}^3$

SA $= (8\times14) + \left(\dfrac{1}{2}\times8\times3\times2\right)$

$+ (14\times5\times2)$

$= 276\text{cm}^2$

(c) Volume $= \dfrac{(6\times6\times4)}{3}$

$= 48\text{cm}^3$

SA $= (6\times6) + \left(\dfrac{1}{2} \times 6 \times 5\right) \times 4$

$= 96\text{cm}^2$

(d) Volume $=$ Area $\times$ Base $\times$ Height

$= \left(\dfrac{4+8}{2} \times 3.5\right) \times 15$

$= 315\text{mm}^3$

SA $= (8 \times 15) + \left(\dfrac{4+8}{2} \times 3.5 \times 2\right)$

$+ (15 \times 4.2 \times 2) + (4 \times 15)$

$= 348\text{mm}^2$

(e) Volume $= \dfrac{2}{3}\pi r^3$

$= \dfrac{2}{3}\pi(12)^3$

$= 3619.11\text{mm}^3$

SA $= 2\pi r^2 + \pi r^2$

$= 3\pi r^2$

$= 3\pi(12)^2$

$= 1357.17\text{mm}^2$

(f) Volume $=$ Cone $+$ Cylinder

$= \dfrac{\pi r^2 h}{3} + \pi r^2 h$

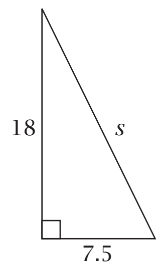
$= \dfrac{\pi(7.5)^2(18)}{3} + \pi(7.5)^2(14)$

$= 3534.29\text{m}^3$

SA $=$ Cone $+$ Cylinder

$= \pi rs + \pi r^2 + 2\pi rh$

Calculate slant height of cone using Pythagoras



$$s^2 = 7.5^2 + 18^2$$
$$s = \sqrt{380.25}$$
$$= 19.5\text{m}$$
$$\text{SA} = \pi(7.5)(19.5) + \pi(7.5)^2 + 2\pi(7.5)(14)$$
$$= 1295.91\text{m}^2$$

12. *(a)* Curved outer surface area

$$= 2\pi rh$$
$$= 2\pi(7.5)(40)$$
$$= 1884.96\text{cm}^2$$

(b) Inner curved surface area

$$= 2\pi rh$$
$$= 2\pi(4.5)(40)$$
$$= 1130.97\text{cm}^2$$

(c) $\text{Volume} = \pi(7.5)^2(40) - \pi(4.5)^2(40)$

$$= 4523.89\text{cm}^3$$

13.
$$\text{SA} = 2\pi r^2 + 2\pi rh$$
$$900 = 2\pi(7)^2 + 2\pi(7).h$$
$$h = \frac{900 - 2\pi(7)^2}{2\pi.7}$$
$$h = 13.46\text{cm}$$

14.
$$\text{Volume} = \left(\frac{1}{2}\times30\times18\right)\times300+(2\times30\times300)$$
$$= 99\,000\text{cm}^3$$

15.
$$\text{Volume} = \frac{\text{area of the base} \times \text{height}}{3}$$
$$= \frac{(\frac{1}{2} \times 6 \times 5) \times 10}{3}$$
$$= 50\text{cm}^3$$

16. *(a)* $\text{Curved surface} = 2\pi rh$

$$= 2\pi(0.15)(1.7)$$
$$= 1.602\text{m}^2$$

(b) Area of 'green'

$$= 170 \times 50$$
$$= 8500\text{m}^2$$

Number of revolutions

$$= \frac{8500}{1.602}$$
$$= 5305.87$$

Minimum number required

$$= 5306$$

17. $\text{Curved surface area} = 2\pi rh$

$$1220 = 2.\pi(15).r$$
$$r = \frac{1220}{2\pi(15)}$$
$$r = 12.944\text{mm}$$
$$\text{Area Base} = \pi r^2$$
$$= \pi(12.944)^2$$
$$= 526.4\text{mm}^2$$

18. *(a)* *(i)* $\text{Volume} = (5\times5\times5) - \pi(1.2)^2(5)$

$$= 102.38\text{m}^3$$

(ii) $\text{SA} = (5\times5\times6)-2\pi(1.2)^2+2\pi(1.2)(5)$

$$= 178.65\text{m}^2$$
$$\text{Cost} = 178.65 \times \$6.50$$
$$= \$1161.23$$

(b) *(i)* $\text{Volume} = (4\times5\times2) + \frac{(4\times5\times4)}{3}$

$$= 66.67\text{m}^3$$

(ii) $\text{SA} = (4\times5) + (5\times2\times2) + (4\times2\times2) + (2\times4.72\times2) + (2.5\times4.47\times2)$

$$= 97.23\text{m}^2$$
$$\text{Cost} = 97.23 \times \$6.50$$
$$= \$632.00$$

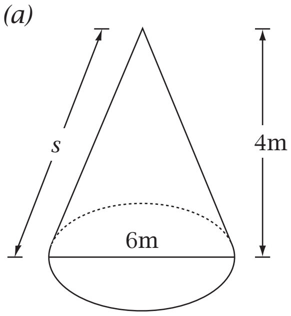
(c) *(i)* $\text{Volume} = \frac{4}{3}\pi(1.25)^3 + \pi(1.25)^2(2.6)$

$$= 20.94\text{m}^3$$

(ii) $\text{SA} = 4\pi(1.25)^2 + 2\pi(1.25)(2.6)$

$$= 40.055\text{m}^2$$
$$\text{Cost} = 40.055 \times \$6.50$$
$$= \$260.36$$

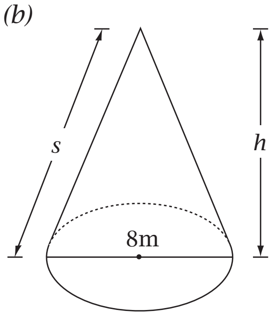
19. *(a)*



Slant height

$$s^2 = 4^2 + 3^2$$
$$s^2 = 16 + 9$$
$$s^2 = 25$$
$$s = 5m$$
$$SA = \pi r^2 + \pi rs$$
$$= \pi(3)^2 + \pi(3)(5)$$
$$= 75.398\text{m}^2$$
$$\text{Cost} = 75.398 \times \$9.60$$
$$= \$723.82$$

(b)



$$\text{Surface area} = \frac{1085.73}{9.60}$$
$$= 113.097\text{m}^2$$
$$\text{Surface area} = \pi r^2 + \pi rs$$
$$113.097 = \pi(4)^2 + \pi(4).s$$
$$s = 5\text{m}$$
$$\therefore h^2 = 5^2 - 4^2$$
$$h^2 = 25 - 16$$
$$h^2 = 9$$
$$h = 3$$

height of the tent is 3m.

20. Volume tank $= 4\times2\times1$

$$= 8\text{m}^3$$

Volume of 1 brick $= 0.0025\text{m}^3$

Amount of missing water

$$= \frac{1}{4} \times 8$$
$$= 2\text{m}^3$$

To overflow:

number of bricks

$$= \frac{2}{0.0025}$$
$$= 800 \text{ bricks}$$

21. Volume Cylinder $= \pi r^2 h$

$$= \pi r^2 (2r)$$
$$= 2\pi r^3$$
$$\text{Volume Sphere} = \frac{4}{3}\pi r^3$$
$$\text{Empty Space} = 2\pi r^3 - \frac{4}{3}\pi r^3$$
$$18\pi = \frac{2}{3}\pi r^3$$
$$r^3 = 27$$
$$r = 3$$

Radius of tennis ball is 3cm.

22. *(a)* Volume of whole cheese

$$= \pi r^2 h$$
$$= \pi(5)^2(4)$$
$$= 314.16\text{cm}^3$$

Volume remaining piece

$$= 314.16 - \frac{314.16}{5}$$
$$= 251.33\text{cm}^3$$

(b)

$$SA = \left(\frac{4}{5} \times 2\pi(5)(4)\right)$$
$$+ (2\times5\times4) + \left(2 \times \frac{4}{5} \times \pi(5)^2\right)$$
$$= 266.19\text{cm}^2$$

23. *(a)*

$$SA = (3\times1.5) + (1.5\times1.2\times2)$$
$$+ (3\times1.2\times2)$$
$$= 15.3\text{m}^2$$
$$\text{Cost} = 15.3 \times \$4.25$$
$$= \$65.03$$

(b)

$$\text{Volume} = 0.2 \times 9$$
$$= 1.8\text{m}^3$$
$$\text{Volume} = l \times w \times h$$
$$1.8 = 3 \times 1.5 \times h$$
$$h = \frac{1.8}{4.5}$$
$$h = 0.4$$

Height is 0.4m.

24. Volume lead sphere

$= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(5)^3$

Volume fishing weights

$= \frac{4}{3}\pi(0.5)^3$

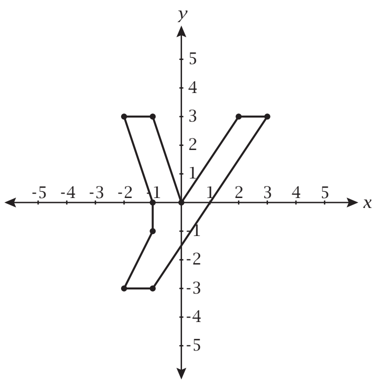
Number of weights

$= \dfrac{\frac{4}{3}\pi(5)^3}{\frac{4}{3}\pi(0.5)^3}$

$= 1000$

CHAPTER 8: *Linear Relationships*

1.



The letter Y is formed

2. (a) 9 matchsticks.

(b) 11 matchsticks.

(c) Increasing by 2 matchsticks each time.

(d)

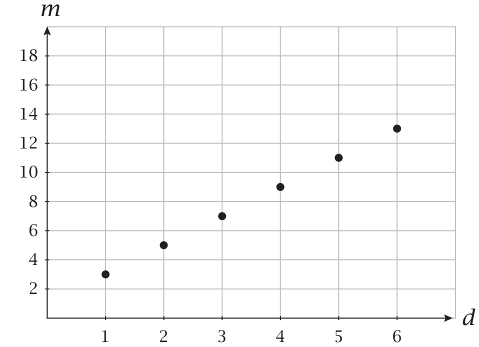
Diagram number (d)	1	2	3	4	5	6
Number of matchsticks (m)	3	5	7	9	11	13

(e) $m = 2d + 1$

(f) $m = 2(25) + 1$
$m = 51$
$\therefore$ 51 matchsticks

(g) $63 = 2d + 1$
$62 = 2d$
$31 = d$
$\therefore$ the 31st diagram

(h)



(i) They form a straight line.

(j) No, since 'd' represents the diagram number and must be a whole number. No part diagrams.

3. The equations which are linear are:

(a) and (c)

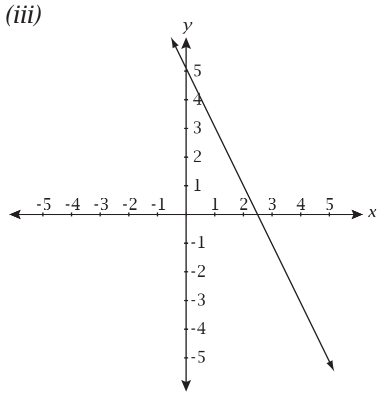
4. (a)

x	0	1	2	3	6	38	27
y	-2	2	6	10	22	150	106

(b)

p	3	4	5	6	20	56	119
q	6.5	7	7.5	8	15	33	64.5

5. (a) (i) $m = -2$
(ii) vertical intercept: $y = 5$
(iii)



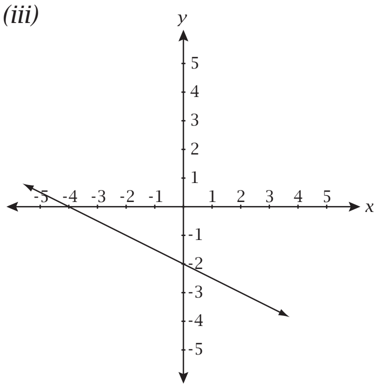
(b) $3x + 6y = -12$

$6y = -3x - 12$

$y = -\frac{1}{2}x - 2$

(i) $m = -0.5$
(ii) vertical intercept: $y = -2$

(iii)



(c) $2x - 5y - 10 = 0$

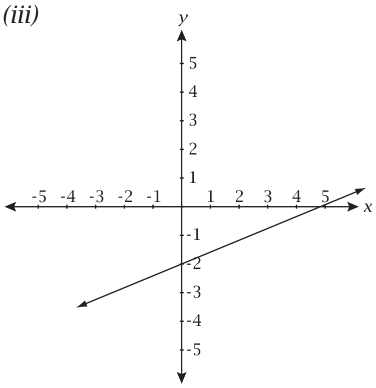
$$5y = 2x - 10$$

$$y = \frac{2}{5}x - 2$$

(i) $m = \dfrac{2}{5}$

(ii) vertical intercept: $y = -2$

(iii)

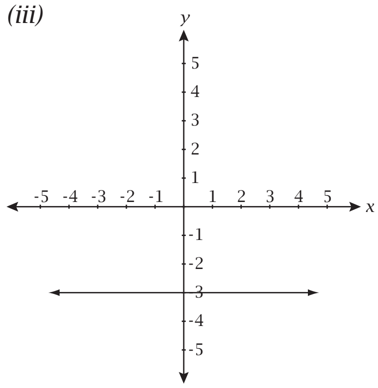


(d) $y = -3$

(i) $m = 0$

(ii) vertical intercept: $y = -3$

(iii)



6. *(a)* *(i)* 1st difference pattern is 3

$\therefore$ linear

(ii) $m = 3s + 1$

(iii)

Number of squares (s)	1	2	3	4	12	36
Number of matchsticks (m)	4	7	10	13	37	109

(b) *(i)* 1st difference pattern is 4

$\therefore$ linear

(ii) $m = 4h + 1$

(iii)

Number of houses (h)	1	2	3	4	10	12
Number of matchsticks (m)	5	9	13	17	41	49

7. *(a)* *(i)* linear as 1st difference pattern is constant

(ii) $y = 3x + 2$

(b) Not linear.

(c) *(i)* linear as 1st difference pattern is constant

(ii) $y = -2x + 22$

(d) *(i)* linear as 1st difference pattern is constant

(ii) $q = 5p + 2$

8. *(a)* $x = 3$

(b) $x = 2$

(c) $\dfrac{2x - 4}{x} = 1$

$2x - 4 = x$

Draw the graph of $y = x$

Find intersection between

$y = x$ and $y = 2x - 4$

$\therefore x = 4$

(d) $x - 2 = 2$

$2(x - 2) = 2(2)$

$2x - 4 = 4$

$\therefore x = 4$

9. *(a)* $3p + q = 6$ ①

$9p + 2q = 1$ ②

$6p + 2q = 12$ ① × 2

$9p + 2q = 1$

$-3p = 11$ ① − ②

$p = -\dfrac{11}{3}$

Sub into ①

$3\left(-\dfrac{11}{3}\right) + q = 6$

$-11 + q = 6$

$q = 17$

$\therefore \left(-\dfrac{11}{3}, 17\right)$

(b) $2m - 5n = 16$ ①

$6m + 2n = -3$ ②

$6m - 15n = 48$ ① × 3

$6m + 2n = -3$

$$-17n = 51 \quad ① - ②$$
$$n = -3$$
$$\text{Sub } 2m - 5(-3) = 16$$
$$2m = 1$$
$$m = \frac{1}{2}$$
$$\therefore \left(\frac{1}{2}, -3\right)$$

(c) $2a - 3b = 8$ ①

$5a + 4b = 43$ ②

$10a - 15b = 40$ ① × 5

$10a + 8b = 86$ ② × 2

$$-23b = -46 \quad ① - ②$$
$$b = 2$$
$$\text{Sub } 2a - 3(2) = 8$$
$$2a - 6 = 8$$
$$2a = 14$$
$$a = 7$$
$$\therefore (7, 2)$$

(d) $5x + 4y = 2$ ①

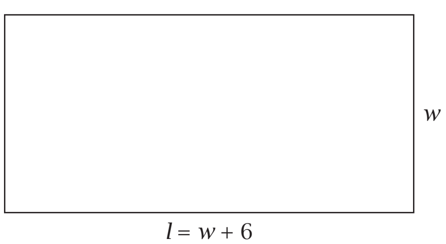
$6x + 5y = 3$ ②

$30x + 24y = 12$ ① × 6

$30x + 25y = 15$ ② × 5

$$-y = -3 \quad ① - ②$$
$$y = 3$$
$$\text{Sub } 5x + 4(3) = 2$$
$$5x + 12 = 2$$
$$5x = -10$$
$$x = -2$$
$$\therefore (-2, 3)$$

10\. *(a)*



Let:
w be the width
l be the length

$$\therefore l = w + 6$$

$$2l + 2w = 100$$
$$\therefore 2(w + 6) + 2w = 100$$
$$2w + 12 + 2w = 100$$
$$4w = 88$$
$$w = 22$$
$$\therefore l = w + 6$$
$$l = 22 + 6$$
$$l = 28$$

$\therefore$ width = 22cm
length = 28cm

(b) Let
x be the larger number
y be the smaller number

$5x + 2y = 60$ ①

$x - y = 12$ ②

$5x + 2y = 60$

$2x - 2y = 24$ ② × 2

$$7x = 84 \quad ① + ②$$
$$x = 12$$
$$\therefore x - y = 12$$
$$12 - y = 12$$
$$y = 0$$

Larger is 12
Smaller is 0

(c) Let:

d be cost of one drink
c be cost of one chocolate

$3d + 2c = 1.75$ ①

$6d + 3c = 2.85$ ②

$6d + 4c = 3.5$ ① × 2

$6d + 3c = 2.85$

$$c = 0.65 \quad ① - ②$$
$$\therefore 3d + 2(0.65) = 1.75$$
$$3d + 1.30 = 1.75$$
$$3d = 0.45$$
$$d = 0.15$$

Drinks cost \$0.15 each
Chocolates cost \$0.65 each

(d) Let:

x be the number of mothers losing one child

y be the number of mothers losing both children

$$x + y = 38$$
$$x + 2y = 54$$

$$-y = -16 \quad ① - ②$$
$$y = 16$$

$$\therefore x = 22$$

22 mothers lost one child
16 mothers lost both children

11. (a) $c = 0.2m + 2$

(b) $c = 0.2(7) + 2$

Cost: \$3.40

(c) $10 = 0.2(m) + 2$

$$8 = 0.2m$$
$$m = \frac{8}{0.2}$$
$$m = 40$$

40 minutes

12. (a) (200, 450) (700, 1700)
$\$P = 2.5s - 50$

(b) $\$P = 2.5(1000) - 50$
Profit = \$2450

(c) $\$P = 2.5(10) - 50$
$\$P = -25$

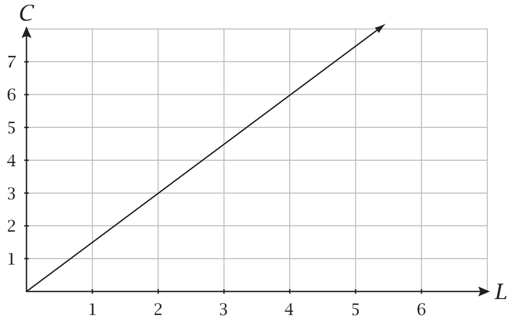
$\therefore$ Loss of \$25

(d) $2.5s - 50 = 0$
$s = 20$

Minimum number needed for a profit is 21 parts.

13. (a) \$10.50

(b)



(c) It is linear.

(d) Yes it is directly proportional.

It has a constant difference pattern increasing by 1.5 and corresponding ratios are constant.

$$\frac{3.00}{2} = \frac{4.50}{3} = \frac{6}{4}$$

(e) $C = 1.5L$

(f) (i) $C = 1.5(37.5)$
$C = \$56.25$

(ii) $\frac{15}{10} = \frac{C}{37.5}$

$$C = \frac{37.5 \times 15}{10}$$
$$C = \$56.25$$

Identical costs.

14. (a) (i) $m = \frac{rise}{run}$

$$= \frac{8}{2}$$
$$= 4$$

(ii) y int: (0, –8)

(b) (i) $m = \frac{rise}{run}$

$$= -\frac{2}{3}$$

(ii) y int: (0, –2)

15. (a) $m = \frac{7 - 1}{3 - 1}$

$$= \frac{6}{2}$$
$$= 3$$

(b) $m = \frac{-2 - 6}{-1 - 0}$

$$= \frac{-8}{-1}$$
$$= 8$$

(c) $m = \frac{-1 + 4}{-3 - 2}$

$$= -\frac{3}{5}$$

(d) $m = \frac{-4 - 6}{0 - 4}$

$$= \frac{-10}{-4}$$
$$= \frac{5}{2}$$

16. (a) gradient: -2 y int: $(0, 5)$

(b) $y = \frac{x}{3} - \frac{4}{3}$

gradient: $\frac{1}{3}$ y int: $\left(0, -\frac{4}{3}\right)$

(c) $y = 2x - 9$

gradient: 2 y int: $(0, -9)$

(d) $4y = -x + 7$

$y = -\frac{x}{4} + \frac{7}{4}$

gradient: $-\frac{1}{4}$ y int: $\left(0, \frac{7}{4}\right)$

17. (a) $m = \frac{5-1}{1-3}$

$m = \frac{4}{-2}$

$m = -2$

$y = -2x + c$

$1 = -2(3) + c$

$c = 7$

$\therefore y = -2x + 7$

(b) $m = \frac{-4-1}{-2+4}$

$m = -\frac{5}{2}$

$y = -\frac{5}{2}x + c$

$1 = -\frac{5}{2}(-4) + c$

$c = -9$

$\therefore y = -\frac{5}{2}x - 9$

(c) $m = \frac{-5-5}{4-4}$

m is undefined

$\therefore x = 4$

(d) $m = \frac{2p - (-4p)}{-2p - p}$

$m = \frac{6p}{-3p}$

$m = -2$

$y = -2x + c$

$-4p = -2(p) + c$

$c = -2p$

$\therefore y = -2x - 2p$

18. (a) $y = \frac{1}{3}x - 4$

(b) $y = -3x + c$

$-1 = -3(1) + c$

$c = 2$

$\therefore y = -3x + 2$

(c) $y = \frac{1}{2}x + c$

$3 = \frac{1}{2}(6) + c$

$c = 0$

$\therefore y = \frac{1}{2}x$

(d) $y = \frac{2}{5}x + c$

$-2 = \frac{2}{5}(4) + c$

$c = -\frac{18}{5}$

$\therefore 5y = 2x - 18$

19. A: $x = -4$

B: $y = 2$

C: $y = -6x + 6$

D: $y = 2x + 4$

20. (a) P: $(-3, 0)$ S: $(2, 0)$

(b) Q: $(0, 6)$ R: $(0, 3)$

(c) $m = 1$
$y = x - 2$

21. (a) $d = \sqrt{(-3-4)^2 + (2+7)^2}$

$d = \sqrt{130}$

$d = 11.40$

(b) $d = \sqrt{(0+1)^2 + (6+2)^2}$

$d = \sqrt{65}$

$d = 8.06$

(c) $d = \sqrt{(2+3)^2 + (-4+1)^2}$

$d = \sqrt{34}$

$d = 5.83$

(d) $d = \sqrt{(-5+2)^2 + (-6-5)^2}$

$d = \sqrt{130}$

$d = 11.40$

22. *(a)* *(i)* $\sqrt{29} = 5.39\text{km}$

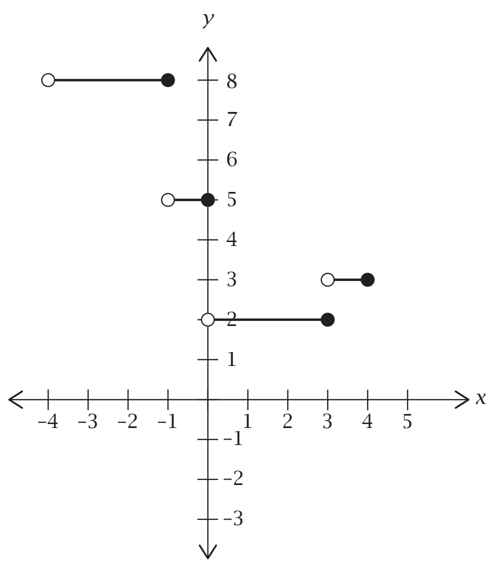
(ii) $\sqrt{34} = 5.83\text{km}$

(iii) $\sqrt{101} = 10.05\text{km}$

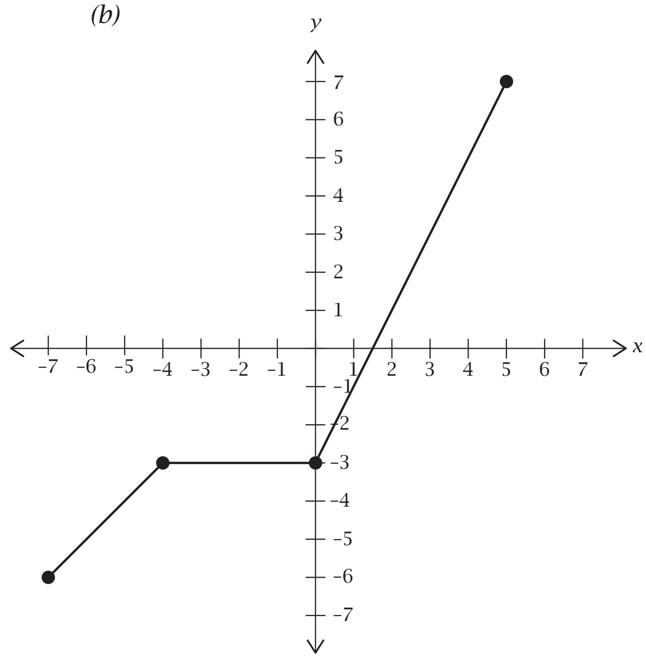
(b) *(i)* $5y = -4x + 33$

(ii) $x = 4$

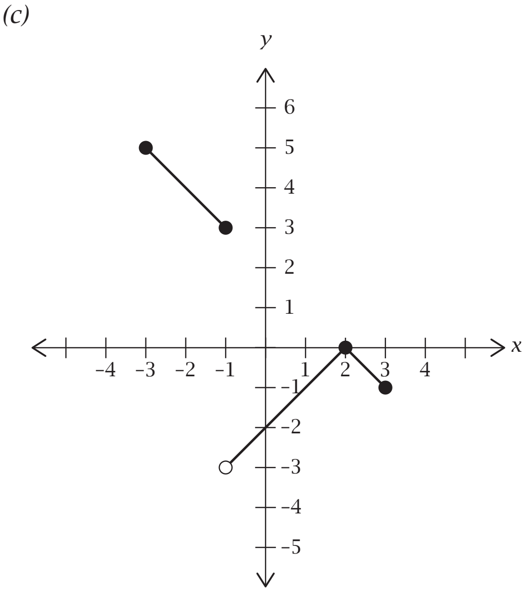
23. *(a)*



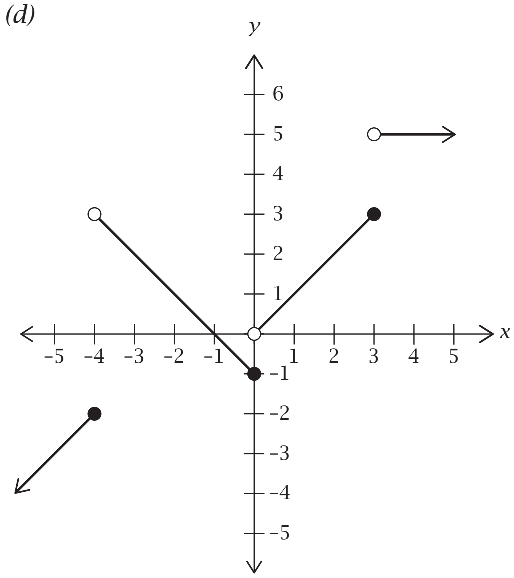
(b)



(c)



(d)



24.

$$y = \begin{cases} 1.25x + 1.75 & -3 \le x < 1 \\ -x + 4 & 1 \le x < 3 \\ 4x - 11 & 3 \le x \le 4 \end{cases}$$

25. *(a)* *(i)* \$150

(ii) \$500

(iii) \$900

(b) \$6000

26. *(a)* *(i)* \$1
(ii) \$3
(iii) \$5
(iv) \$7

(b) From 3kg but less than 4kg

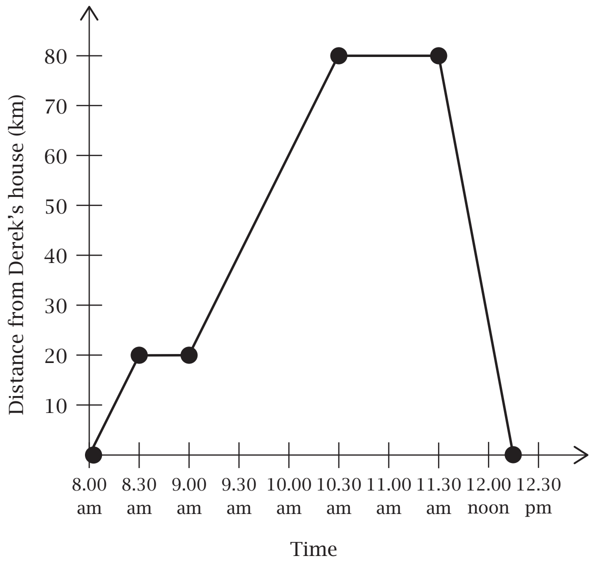
27. *(a)* *(i)* \$0.50

(ii) \$1.20

(iii) \$1.60

(b) Length of call from 2 mins and less 3 mins.

28. *(a)*



(b) *(i)* $\text{speed} = \dfrac{\text{distance}}{\text{time}}$

$= \dfrac{60}{1.5}$

$= 40\text{km/h}$

(ii) $\text{speed} = \dfrac{\text{distance}}{\text{time}}$

$= \dfrac{80}{0.75}$

$= 106\frac{2}{3}$ km/h

(c) Fastest part of the journey is the return journey $\left(106\frac{2}{3}\text{ km/h}\right)$. This is the line with the steepest gradient.

CHAPTER 9: *Similarity and Scale*

1. *(a)* 1m

(b) 10.5m

(c) 4.6m

2. *(a)* 1cm : 5m
1cm : 500cm
1 : 500

(b) 50m : 5km
50m : 5000m
1 : 100

(c) 12cm : 144km
12cm : 14 400 000cm
1 : 1 200 000

3. *(a)* 1cm = 50m
15cm = 750 metres

(b) 2cm = 200m
1cm = 100m
15cm = 1500m
(1.5km)

4. *(a)* 1 : 40 000
2km = 200 000cm
1 : 40 000 = x : 200 000

$$\frac{1}{40\,000} = \frac{x}{200\,000}$$

x = 5cm
Scale distance is 5cm

(b) 1 : 100 000
675m = 67 500cm
∴ 1 : 100 000 = x : 67 500

$$\frac{1}{100\,000} = \frac{x}{67\,500}$$

x = 0.675
Scale distance 0.675cm
6.75mm

(c) 1 : 1250
250m = 25 000cm
∴ 1 : 1250 = x : 25 000

$$\frac{1}{1250} = \frac{x}{25\,000}$$

x = 20
Scale distance 20cm

5. *(a)* 1 : 1 000 000
5cm = 5 000 000cm
= 50 000m
= 50km
Distance: 50km

(b) 75km = 7 500 000
∴ 1 : 1 000 000 = x : 7 500 000

$$\frac{1}{1\,000\,000} = \frac{x}{7\,500\,000}$$

x = 7.5cm
Scale distance 7.5cm

6. *(a)* *(i)* Scale distance : 6.2cm
Actual distance : 69.44km

(ii) Scale distance : 2.7cm
Actual distance: 30.24km

(b) Mandurah and Pinjarra

7. *(a)* Area of bedrooms

Bed 1 : 15.2m^2
Bed 2 : 11.16m^2
Bed 3 : 9.88m^2
Bed 4 : 10.23m^2
Total : 46.47m^2

Carpet cost $= 46.47 \times \$73$
$= \$3392.31$

(b) Area of family room $= 32.4\text{m}^2$
Area of each tile $= 0.25\text{m}^2$

Number of tiles $= \dfrac{32.4}{0.25}$
≈ 130 tiles

8. *(a)* 1 : 300
Scale length : 10.8cm
Actual length : 32.40m

Scale width : 5.4cm
Actual width : 16.20m

(b) Scale radius : 1.8cm
Actual radius : 5.4m

Total Area $= \pi r^2$
$= \pi(5.4)^2$
$= 91.61\text{m}^2$

(c) Area Court $= 524.88\text{m}^2$
Cost $= 524.88 \times \$45$
$= \$23\,619.60$

9. *(a)* *(i)* Bedroom 1 : 7.05×5.4m
(ii) Kitchen : 5.85×5.4m

(b) Area Bedroom 1 $= 38.07\text{m}^2$
Bedroom 2 $= 46.51\text{m}^2$
84.58m^2

Cost $= 84.58 \times \$62$
$= \$5243.96$

(c) Area kitchen $= 31.59\text{m}^2$
Area each tile $= 0.04\text{m}^2$

Number of tiles $= \dfrac{31.59}{0.04}$
≈ 790

10. *(i)* *(a)* Scale factor

$$\frac{BC}{XZ} = \frac{AC}{ZY} = \frac{AB}{XY} = 2$$

(b) $\triangle ABC \sim \triangle YXZ$ (S.S.S)

(ii) *(a)* Scale factor

$$\frac{\text{PQ}}{\text{VW}} = \frac{\text{QR}}{\text{WX}} = \frac{5}{3}$$

(b) $\triangle PQR \sim \triangle VWX$ (S.A.S)

(iii) Scale factor not consistent
$\therefore$ triangles are not similar

11. *(a)* $\triangle ABC \sim \triangle FDE$ (A.A.)

(b) $\triangle TUV \sim \triangle XYW$ (S.S.S)

(c) $\triangle QPR \sim \triangle TUS$ (S.A.S)

12. *(a)* *(i)* $\triangle CBA \sim \triangle CDE$ (S.A.S)

(ii) $\dfrac{27}{x} = \dfrac{21}{14}$

$x = 18$

(b) *(i)* $\triangle PQT \sim \triangle RST$ (A.A)

(ii) $\dfrac{7.5}{5} = \dfrac{6}{x}$

$x = 4$

(c) *(i)* $\triangle LMN \sim \triangle LPO$ (A.A)

(ii) $\dfrac{25}{10} = \dfrac{LO}{15}$

$LO = 37.5$

$\therefore\ x = 27.5$

13. *(a)* $\dfrac{x}{40} = \dfrac{10}{15}$

$x = \dfrac{400}{15}$

$x = 26\dfrac{2}{3}$cm

(b) $\dfrac{x}{7} = \dfrac{21}{15}$

$x = \dfrac{147}{15}$

$x = 9.8$m

14. Let x be the height of the tree

$\dfrac{x}{50} = \dfrac{2}{5}$

$x = 20$

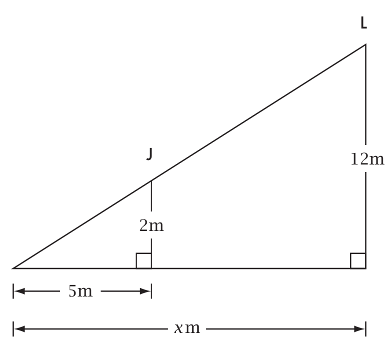
The height of the tree is 20m.

15. $\dfrac{XY}{14} = \dfrac{5}{2}$

$XY = 35$

The width of the river is 35m.

16.

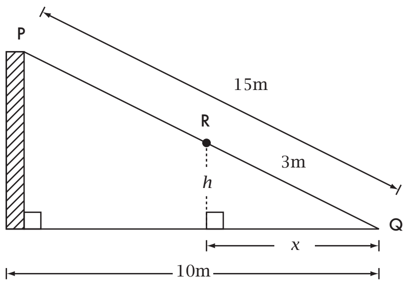


$\dfrac{12}{2} = \dfrac{x}{5}$

$x = 30$

John is 25m from the streetlight.

17.



(a) $\dfrac{x}{3} = \dfrac{10}{15}$

$x = 2$

Horizontal distance from wall is 8m.

(b) $h^2 = 3^2 - 2^2$

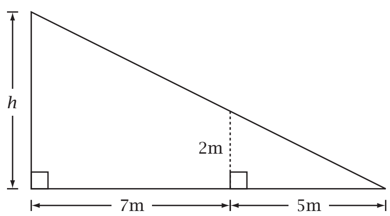
$h^2 = 9 - 4$

$h = \sqrt{5}$

$h = 2.24$

Dennis is 2.24m above the ground.

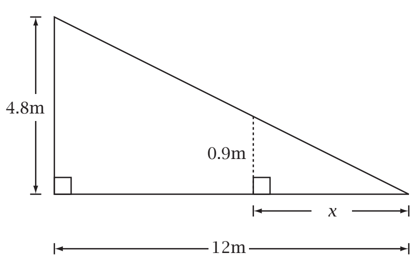
18.



(a) $\dfrac{h}{12} = \dfrac{2}{5}$

$h = 4.8\text{m}$

Street pole is 4.8m high



(b) $\dfrac{x}{0.9} = \dfrac{12}{4.8}$

$x = 2.25\text{m}$

Damien is standing 9.75m from the street pole.

19. (a) Area $= 100 \times 2^2$
$= 400\text{m}^2$

(b) Area $= 50 \times (1.5)^2$
$= 112.5$

(c) Area $= 48 \times \left(\dfrac{3}{4}\right)^2$
$= 27$

(d) Area $= 100 \times \left(\dfrac{3}{5}\right)^2$
$= 36$

20. (a) $x = 4\text{cm}$

(b) $x = 12.5\text{m}$

Scale factor is $\sqrt{\dfrac{120}{307.2}}$

$= \dfrac{5}{8}$

(c) $x = 40\text{cm}$

Scale factor is $\sqrt{\dfrac{937.5}{150}}$

$= \dfrac{5}{2}$

21. (a) $\triangle ACB \sim \triangle ECD$ (S.A.S)

Scale factor

$\dfrac{ED}{AC} = \dfrac{CD}{CB} = \dfrac{5}{3}$

(b) Area $\triangle ABC = \dfrac{625}{\left(\frac{5}{3}\right)^2}$

$= 225\text{cm}^2$

22. (a) 27 : 18
= 3 : 2

(b) $A = \pi(13.5)^2$ (large)
$A = \pi(9)^2$ (small)
$\pi(13.5)^2 : \pi(9)^2$
$= 9 : 4$

23. (a) Volume $= 50 \times \left(\dfrac{3}{2}\right)^3$
$= 168.75\text{cm}^3$

(b) Volume $= 150 \div \left(\dfrac{10}{5}\right)^3$
$= 18.75\text{cm}^3$

(c) Volume $= 300 \times \left(\dfrac{8.4}{6}\right)^3$
$= 823.2\text{cm}^3$

24. (a) 5 : 3

(b) 125 : 27

25. (a) Volume 1 = Area base × height
$= 100 \times 20$
$= 2000\text{cm}^3$

(b) Volume 2 $= 2000 \times \left(\dfrac{50}{20}\right)^3$
$= 31\,250\text{cm}^3$

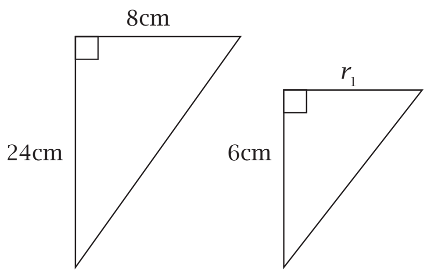
26. Height ratio 5 : 6 : 10

(a) Area ratio = 25 : 36 : 100

(b) Volume ratio = 125 : 216 : 1 000 000

27. (a)
$$\text{Volume cone} = \frac{\pi r^2 h}{3} = \frac{\pi(8)^2(24)}{3} = 1608.50\text{cm}^3$$

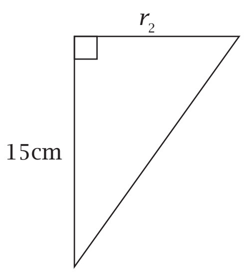
(b)



Using similar triangles

$$\frac{r_1}{8} = \frac{6}{24}$$
$$r_1 = 2\text{cm}$$
$$\text{Volume water} = \frac{\pi(2)^2(6)}{3} = 25.13\text{cm}^3$$

(c)



$$\frac{r_2}{8} = \frac{15}{24}$$
$$r_2 = 5\text{cm}$$
$$\text{Volume water} = \frac{\pi(5)^2(15)}{3} = 392.7\text{cm}^3 = 392.7\text{mL of water}$$

Additional amount of water is
392.7 – 25.13
= 367.6mL

CHAPTER 10: *Simple and Compound Interest*

1. (a)
$$\text{Simple Interest} = \$2000 \times 0.05 \times 3 = \$300$$
$$\text{Total} = \$2300$$

(b)
$$\text{SI} = \$10\,000 \times 0.09 \times 1.5 = \$1350$$
$$\text{Total} = \$11\,350$$

(c)
$$\$1543.50 = \$4200 \times 0.07 \times \text{T}$$
$$\text{T} = 5.25\text{yrs}$$
$$\text{Total} = \$5743.50$$

(d)
$$306.25 = \text{P} \times 0.025 \times 3.5$$
$$\text{P} = \$3500$$
$$\text{Total} = \$3806.25$$

(e)
$$(\text{P} \times 0.125 \times 1.25) + \text{P} = \$11\,100$$
$$\text{P} = \$9600$$
$$\text{SI} = \$1500$$

(f)
$$\text{SI} = 9679.50 - \$8100 = \$1579.50$$
$$1579.50 = 8100 \times \text{R} \times 3.25$$
$$\text{R} = 0.06\ (6\%)$$

2. (a)
$$\text{A} = \$3000\left(1 + \frac{0.06}{1}\right)^4 = \$3787.43$$
$$\text{I} = \$787.43$$

(b)
$$\text{A} = \$8500\left(1 + \frac{0.05}{365}\right)^{(365\times3)} = \$9875.49$$
$$\text{I} = \$1375.49$$

(c)
$$\text{A} = \$4600\left(1 + \frac{0.09}{12}\right)^{(12\times5)} = \$7202.13$$
$$\text{I} = \$2602.13$$

(d)
$$\text{A} = \$7000\left(1 + \frac{0.07}{12}\right)^3 = \$7123.22$$
$$\text{I} = \$123.22$$

(e)
$$3276.99 = \text{P}\left(1 + \frac{0.07}{1}\right)^4$$
$$\text{P} = \$2500$$
$$\text{I} = \$776.99$$

(f)
$$8341.83 = \text{P}\left(1 + \frac{0.05}{12}\right)^{(5\times12)}$$
$$\text{P} = \$6500$$
$$\text{I} = \$1841.83$$

3. Interest : \$5747.50 – \$5500
= \$247.50

$$\$247.50 = \$5500 \times \text{R} \times \frac{9}{12}$$
$$\text{R} = 0.06$$

Simple Interest rate is 6%

4. (a) $SI = \$10\,000 \times 0.05 \times 6$
$= \$3000$

(b) $CI = \$10\,000\left(1 + \frac{0.05}{12}\right)^{72} - \$10\,000$

$= \$3490.18$

(c) Difference $= \$490.18$

5. (a) Interest Rate $= \frac{7166.25}{6825}$
$= 1.05$

Rate $= 5\%$ p.a.

(b) $6825 = P\left(1 + \frac{0.05}{1}\right)^{1}$

$P = \frac{6825}{1.05}$

$P = 6500$

Original investment \$6500

6. (a) $(P \times 0.08 \times 3.5) - (P \times 0.065 \times 4) = 105$
$P = \$5250$

(b) $3465 = 5250 \times R \times 5$

$R = 0.132$

Interest Rate $= 13.2\%$

7. Using ‘financial’ tab on classpad

(a) \$9767.66

(b) \$32 257.68

(c) \$34 935.05

8. (a) $930 = 6200 \times 0.06 \times T$
$T = 2.5$yrs

(b) $2709 = P\left(1 + \frac{0.15}{1}\right)^{2} - P$
$P = \$8400$

$SI = \$8400 \times 0.15 \times 2$
$= \$2520$

9. (a) Total paid $= \$220 \times 15$
$= \$3300$

(b) Interest paid $= \$3300 - \$2250$
$= \$1050$

(c) $1050 = 2250 \times R \times \frac{15}{12}$
$R = 37\frac{1}{3}\%$

10. (a) Deposit $= \$3600 \times 25\%$
$= \$900$

(b) Total paid $= \$220 \times 24 + \$900$
$= \$6180$

(c) $\$2580 = \$2700 \times R \times 2$

$R = 47\frac{7}{9}\%$

11. (a) Joanna:
$SI = \$10\,000 \times 0.06 \times 3$
$= \$1800$
Accumulates \$11 800

(b) Joseph accumulates \$10 570
$(\$10\,570 - P) = (P \times 0.06 \times 4) + (P \times 0.09 \times 3)$
$P = \$7000$

Initial investment \$7000

12. *Plan 1* : $\$20\,000\left(1 + \frac{0.0595}{4}\right)^{4}$
$= \$21\,212.44$

Plan 2 : $\$20\,000\left(1 + \frac{0.059}{12}\right)^{12}$
$= \$21\,212.44$

Plan 1 should be used to invest the \$20 000 as it provides a greater return.

13. $A = P\left(1 + \frac{r}{100}\right)^{t}$

$3000 = 1000\left(1 + \frac{r}{100}\right)^{6}$

$\therefore r = 20.09$

CHAPTER 11: *Statistics*

1. (a) Categorical and nominal

(b) Categorical and ordinal

(c) Numerical and continuous

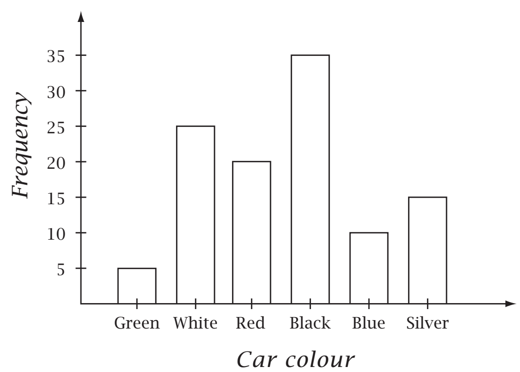
(d) Categorical and ordinal

(e) Numerical and discrete

(f) Categorical and nominal

(g) Numerical and continuous

2.



3. (a) 144.89

(b) 148

(c) 152

(d) 16.12

(e) 26

4\. (a) 23

(b) 6.22

(c) 8

(d) 8

(e) 9

(f) $\frac{1}{23} \times 100$

$= 4.35\%$

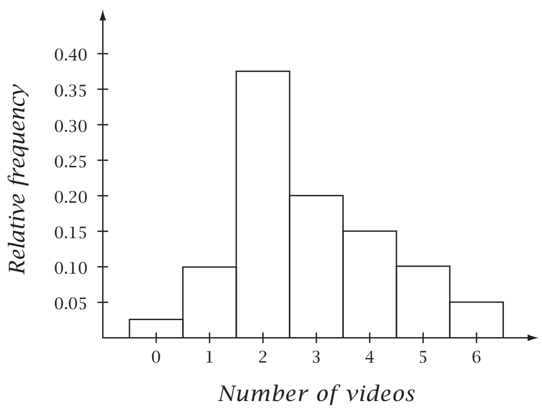
(g) Clusters at scores of 1—3 and 7—10. Gaps at scores of 4 and 6.

(h) 6

(i) 3.05

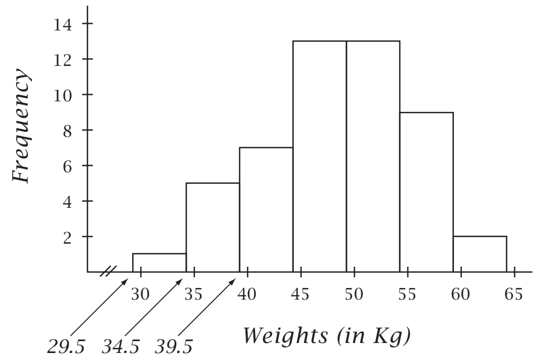
5\.

Number of videos	Frequency	Relative frequency
0	1	0.025
1	4	0.1
2	15	0.375
3	8	0.2
4	6	0.15
5	4	0.1
6	2	0.05



6\.

Continuous intervals	Class intervals	Frequency
29.5–34.5	30–34	1
34.5–39.5	35–39	5
39.5–44.5	40–44	7
44.5–49.5	45–49	13
49.5–54.5	50–54	13
54.5–59.5	55–59	9
59.5–64.5	60–64	2
		50



7\. (a) 30 pages

(b) (i) Mean: $3.8\dot{6}$
(ii) Mode: 6
(iii) Median: 4.5
(iv) 7
(v) 4
(vi) 2.11

8\. (a) Mean: 33.81 runs

(b) Modal class: 30–39 runs

(c) Median position: $\frac{130 + 1}{2}$

$= 65.5$th score

Located in the 30–39th class interval

(d) Median $= \frac{18.5}{46} \times 10 + 29.5$

$= 33.52$

(e) St Deviation: 16.09

9\. (a) (i) Mean $= 67.11$
(ii) Median class: 61–72
(iii) Median $= \frac{20.5}{26} \times 12 + 60.5$
$= 69.96$
(iv) St Dev: 23.80

(b) (i) Mean $= 45.79$
(ii) Median class: 40–49
(iii) Median $= \frac{8}{9} \times 10 + 39.5$
$= 48.39$
(iv) St Dev: 12.38

10\. (a) The median gives a more accurate measure of his central progress. The mean is affected by the outlier.

(b) $\frac{620 + x}{9} = 78$

$x = 82$

Must score a mark of 82%

11. *(a)* Player 1: Mean–$12.\dot{6}$
Range–5

Player 2: Mean–$12.\dot{6}$
Range–14

(b) The most consistent player is the player with the lowest range. i.e. player 1.

12. *(a)* 27 classes

(b) The mean as it takes into account all students that are late, especially the number of times there are 4 or 5 late students.

13. *(a)* 25%

(b) $7\% \times \$150\,000$
$= \$10\,500$

(c) $\frac{48}{100}$
$= \frac{12}{25}$

14. *(a)* Maths A paper mean: 48.7
Maths B paper mean: $45.0\dot{3}$

(b) Median Maths A
$\frac{19.5}{31} \times 10 + 40.5 = 46.79$
Median maths B
$\frac{7.5}{33} \times 10 + 40.5 = 42.77$

(c) Both the mean and the median as measures of central tendency indicate that – Maths B paper is more difficult.

15. Wages bill $= \$520.65 \times 15$
$= \$7809.75$

16. Class average $= \frac{(8 \times 74) + (12 \times 57)}{20}$
$= 63.8\%$

17. Sum of exam marks for the 15 students
$= 15 \times 56$
$= 840$

Add in Sam $= 840 + 82$
$= 922$

New class mean $= \frac{922}{16}$

$= 57.625\%$

18. *(i)* C, B, A

(ii) C, B, A

(iii) B, C, A

19. *(a)* Mean = 4.2
Mode = 5
Median = 4.5
Range = 7

(b) Mean = 42
Mode = 50
Median = 45
Range = 70

(c) Mean = 9.2
Mode = 10
Median = 9.5
Range = 7

(d) *(i)* Each score in part (b) was obtained by **multiplying** each score in part (a) by 10. The mean, mode, median and range values in part (a) were also multiplied by 10 to obtain the answers in part (b).

Summary: Mean, mode, median and range are affected by the **multiplication** of all scores by a constant value.

(ii) Each score in part (c) was obtained by **adding** each score in part (a) by 5. The mean, mode, median values were also affected by the addition of 5. The range was not changed.

Summary: Mean, mode and median are affected by the **addition** of all scores by a constant value. The range is **not** affected.

20. *(a)* Mean: 9
Range: 12

(b) Mean: 10
Range: 24

(c) Mean: 14
Range: 36

21. $a = 3$
$b = 8$
$c = 10$
$d = 17$

22. *(a)* *(i)* 7.9375 years
(ii) 5.431 years

(b) Skewed to the right

23. (a) Outliers can be determined by

$Q_3 + 1.5 \text{ IQR} \le \text{score} \le Q_1 - 1.5 \text{ IQR}$

$Q_1 = 500 \qquad \text{IQR} = 155$
$Q_3 = 655$

$655 + 1.5(155) \le \text{score} \le 500 - 1.5(155)$

$887.5 \le \text{score} \le 267.5$

Outliers are \$220 and \$1050

(b) Removing outliers will decrease both the mean and standard deviation

24. (a) 105 hours

(b) 50 hours

(c) Brand B – higher median
– smaller IQR

(d) (i) 25%
(ii) 75%

25. General comments
- Class B has higher median
- Class A has lowest and highest scores
- Class A and B have both the same Q_3
- Class B is more symmetrical than Class A
- Class B has smaller range and IQR
- Class B is more consistent – lower IQR, higher median

CHAPTER 12: *Normal Distribution*

1. (a) $z = \dfrac{52 - 50}{5}$
$= 0.4$

(b) $z = \dfrac{46 - 60}{4}$
$= -3.5$

(c) $z = \dfrac{88 - 66}{11}$
$= 2$

2. (a) Test 1: $z = \dfrac{60 - 65}{4}$
$= -1.25$

Test 2: $z = \dfrac{68 - 62}{8}$
$= 0.75$

Test 3: $z = \dfrac{75 - 71}{6}$
$= 0.67$

Test 4: $z = \dfrac{43 - 50}{5}$
$= -1.4$

Test 5: $Z = \dfrac{51 - 53}{10}$
$= -0.2$

Test 6: $Z = \dfrac{56 - 53}{9}$
$= 0.33$

Rank: Test 2
Test 3
Test 6
Test 5
Test 1
Test 4

(b) Let x be the common mark

$\dfrac{x - 65}{4} = \dfrac{x - 62}{8}$

$x = 68$

3. (a) $P(X = 30) = 0$

(b) $P(25 < X < 35) = 0.68$
$(\pm 1\ \sigma)$

(c) $P(20 < X < 30) = 0.5 - 0.34 - 0.135$
$(-2\ \sigma) = 0.025$

(d) $P(20 < X < 35) = 0.34 + 0.34 + 0.135$
$(-2\ \sigma) \quad (+1\ \sigma) = 0.815$

(e) $P(X \le t) = 0.025$
0.025 represents 2 st deviations below the mean
$t = 30 - 2(5)$
$t = 20$

4. (a) $P(X < 58) \approx 0.00135$

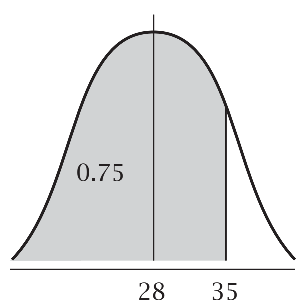
(b) $P(X > 72) \approx 0.3085$

(c) $P(68 < X < 75) \approx 0.5858$

(d) $k \approx 64.8738$

(e) $P(X \le t) = 0.45$
$t \approx 69.4974$

5.



25th percentile must be 21
∴ Interquartile range is 35 – 21 = 14

6. *(a)* $X \sim N(200, 25^2)$

(i) $P(X = 210) = 0$

(ii) $P(150 < X < 250) = 0.95$ $(\pm 2\ \sigma)$

(iii) $P(125 < X < 200) = 0.4985$ $(-3\ \sigma)$

(b) $P(X \geq k) = \dfrac{4}{25}$

$P(X \geq k) = 0.16$

score greater than 1 standard deviation

$k = 200 + 25$
$k = 225$

By calculator
$k = 224.86$g

7. $X \sim N(55\,000, 10\,500^2)$

(a) $P(X > 70\,000) \approx 0.0766$
$\therefore$ 7.66%

(b) $P(52\,000 < X < 58\,250) \approx 0.2340$

(c) $P(X \geq k) = 0.97$
$k \approx 35\,251.67$km

8. $X \sim N(17, 2.25^2)$

(a) $P(X > 20) \approx 0.0912$

(b) $P(14 < X < 20) \approx 0.8176$

(c) $P(X \leq k) = 0.95$
$k \approx 20.701$

Maximum length is 20.701mm.

9. $L \sim N(24.6, 2.7^2)$

(a) *(i)* $P(L > 25) \approx 0.4411$

(ii) $P(22 < L < 27) \approx 0.6452$

(b) $P(L < 21) = 0.09121$

Expected number of fish

$\approx 0.09121 \times 120$
≈ 10.95

Approx. 11 fish.

(c) $P(L \geq t) = 0.25$
$t \approx 26.421$cm

10. *(a)* $X \sim N(500, 12^2)$

(i) $P(X > 520) \approx 0.0478$

(ii) $P(495 < X < 507) \approx 0.3817$

(b) $P(X \leq k) = 0.075$
$k \approx 482.73$g

(c) Median weight $= 500$g

(d) IQR $=$ 75th – 25th percentile
$\approx 508.094 - 491.906$
≈ 16.188

SOLUTIONS TO TRIAL TESTS

RESOURCE FREE: *Trial Test 1*

1. (a) 6 ✓

   (b) 4 ✓✓

2. (a) Sale Price $= \$30 \times 0.85$
   $= \$25.50$ ✓

   (b) Sale Price $= \$101 \times 0.85$
   $= \$85.85$ ✓

   The percentage discount was **NOT** correct. ✓

3. (a) $-5y$ ✓

   (b) $-24x^4y$ ✓

4. (a)
$$\begin{aligned} 7 - 2x &= -15 \\ 2x &= 15 + 7 \\ 2x &= 22 \\ x &= 11 \end{aligned}$$ ✓

   (b)
$$\begin{aligned} -5(3-h) &= -10 \\ 3 - h &= \frac{-10}{-5} \\ 3 - h &= 2 \\ h &= 1 \end{aligned}$$ ✓

   (c)
$$\begin{aligned} 4(m+3) - (2m+5) &= 10 \\ 4m + 12 - 2m - 5 &= 10 \\ 2m &= 10 - 12 + 5 \\ 2m &= 3 \\ m &= \frac{3}{2} \end{aligned}$$ ✓

5. (a) $m = \dfrac{rise}{run}$

   $m = \dfrac{3}{2}$ ✓

   (b) vertical intercept: $y = -3$ ✓

   (c) equation of the line: $y = \dfrac{3}{2}x - 3$ ✓

(d)

x	–2	–1	0	1	2	3
y	–6	–4.5	–3	–1.5	0	1.5

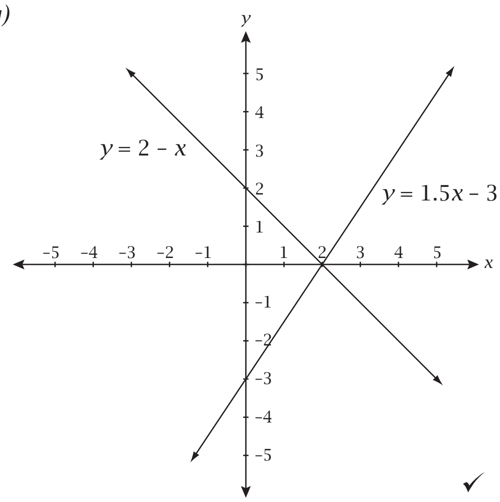
1.5 1.5 1.5 1.5

✓

(e) 1st difference pattern is 1.5 ✓

(f) The 1st difference pattern and gradient are the same. ✓

(g)



✓

(h)
$$\begin{aligned} y &= 2 - x \\ y &= 1.5x - 3 \\ \therefore\ 2 - x &= 1.5x - 3 \\ 2.5x &= 5 \\ x &= 2 \\ \therefore\ y &= 0 \quad (2, 0) \end{aligned}$$ ✓

This is the point of intersection of the two lines on the graph. ✓

RESOURCE FREE: *Trial Test 2*

1. Oil: 2.5 tablespoons
   Lamb Chops: 40
   Onions: 5
   Chilli paste: $7\frac{1}{2}$ tablespoons
   Oyster sauce: $1\frac{1}{4}$ cups
   Tomato paste: $1\frac{2}{3}$ cups ✓✓✓

2. (a) $\begin{bmatrix} 7 & -11 \\ -3 & 11 \end{bmatrix}$ ✓✓

   (b) $\begin{bmatrix} 2 & 18 \end{bmatrix}$ ✓✓

   (c) Not possible since the number of columns in B does not equal the number of rows in C ✓

3. $a = 2$ ✓
$b = 5$ ✓
$c = 6$ ✓
$d = 8$ ✓✓

4. Let x be the smaller number
Let y be the larger number ✓

$$\begin{aligned} x + y &= 6 \quad ① \\ 3x + 2y &= 5 \quad ② \end{aligned}$$

$$\begin{aligned} 3x + 3y &= 18 \quad ① \times 3 \\ 3x + 2y &= 5 \quad ✓ \end{aligned}$$

$$\begin{aligned} y &= 13 \; ✓ \\ \therefore\; x + y &= 6 \\ x + 13 &= 6 \\ x &= -7 \end{aligned}$$

$\therefore$ the smaller number is -7
larger number is 13 ✓

5. *(a)* $\Delta AZB \sim \Delta YZX$ ✓ (A.A.) ✓

(b) $x = 4\text{cm}$ ✓

(c) Area $\Delta XYZ = \frac{108}{(3)^2}$ ✓

$= 12\text{cm}^2$ ✓

6. *(a)* (i) 68% ✓
(ii) 16% ✓

(b) 170 bottles ✓

RESOURCE FREE: *Trial Test 3*

1. *(a)* $A = ½\,(5)(7)(0.6)$ ✓
$= 10.5\text{cm}^2$ ✓

(b) $\frac{YZ}{\sin X} = \frac{XY}{\sin Z}$

$\frac{x}{0.4} = \frac{15}{0.5}$ ✓✓

$x = 12\text{cm}$ ✓

2. *(a)* $(x + 7)(x - 5)$
$= x^2 - 5x + 7x - 35$
$= x^2 + 2x - 35$ ✓

(b) $-6(7x-2)(2x+3)$
$= -6(14x^2+21x-4x-6)$ ✓
$= -6(14x^2+17x-6)$
$= -84x^2 - 102x + 36$ ✓

3. *(a)* $100 - x$ cm ✓

(b) $\frac{1}{3}(100 - x)$ cm

Total cutoff is:

$x + \frac{1}{3}(100 - x)$

$= (\frac{2}{3}x - \frac{100}{3})$ cm ✓

4. *(a)* The fixed cost. ✓

(b) \$100 per hour. ✓

(c) $C = 100(6) + 120$
$= \$720$ ✓

(d) $1350 = 100t + 120$
$1230 = 100t$
$t = 12.3$
12.3 hours ✓
or 12hr 18min

5. $5\text{m} \times 6\text{m} = 30\text{m}^2$ Cost \$360
$\therefore\; 1\text{m}^2$ Cost \$12 ✓

$2\text{m} \times 3\text{m} = 6\text{m}^2$ Cost \$72 ✓

6. *(a)* $x = 1$ ✓, $y = -3$ ✓, $z = -9$ ✓

(b) $-½T = \begin{bmatrix} 7 & 3 \\ -1 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ -6 & 4 \end{bmatrix}$

$-½T = \begin{bmatrix} 6 & 1 \\ 5 & 0 \end{bmatrix}$ ✓

$T = \begin{bmatrix} -12 & -2 \\ -10 & 0 \end{bmatrix}$ ✓

(c) (i) Mean is 2.7 (3) children ✓
(ii) Median is 2 children per family ✓

(d) The best measure of average is the median ✓ as there is an outlier ✓ which has affected the mean value.

RESOURCE FREE: *Trial Test 4*

1. $(2x)^2 + (3x)^2 = 10^2$ ✓
$4x^2 + 9x^2 = 100$ ✓
$13x^2 = 100$
$x^2 = \frac{100}{13}$
$x = \sqrt{\frac{100}{13}}$ ✓

2. *(a)* $-4(x-2)$ ✓

(b) $5x(3x-5yz)$ ✓

3. (a)

Diagram number (d)	1	2	3	4	5
Number of matchsticks (m)	8	12	16	20	24

✓

(b) Increase of 4 matchsticks each time. ✓

(c) $m = 4d + 4$ ✓

(d) $m = 4(12) + 4$
$m = 52$

$\therefore$ 52 matchsticks required. ✓

(e) $84 = 4(d) + 4$
$80 = 4d$
$d = 20$

$\therefore$ 20th diagram ✓

4. (a) (−0.5, −0.5) ✓

(b) (1, −2) ✓

(c) (−3, 5) ✓

5. ① B ✓
② A ✓
③ D ✓
④ C ✓

6. (a) Length $= (30 - 2x)$ cm ✓

(b) Width $= (24 - 2x)$ cm ✓

(c) Area $= (30 - 2x)(24 - 2x)\,\text{cm}^2$ ✓

RESOURCE RICH: *Trial Test 5 Finance and Percentage*

1. Commission $= \frac{5}{100} \times \$700$

$= \$35$ ✓

Weekly Pay $= \$350 + \$35$

$= \$385$ ✓

2. Table cost $= \$50 + \$37.50 + \$12 + \$21$

$= \$120.50$ ✓

(a) Profit $= \$4.50$ ✓

(b) % profit $= \frac{4.5}{120.5} \times 100$ ✓

$= 3.73\%$ ✓

3. Cost books $= \frac{476}{0.85}$ ✓

$= \$560$ ✓

With a 10% profit:

Selling price $= \$560 \times 1.1$ ✓

$= \$616$ ✓

Books should have sold for $616

4. Lemonade

600mL : $\frac{240}{600} = 0.4$ cents/mL ✓✓

375mL : $\frac{180}{375} = 0.48$ cents/mL ✓✓

1L : $\frac{375}{1000} = 0.375$ cents/mL ✓✓

The best buy is the 1L container of lemonade. ✓

5. Cost:

$250 000	Land
$110 000	Subdivision
$13 500	Rates
$4 500	Upkeep
$15 640	Commission
$393 640	✓✓

Total selling price: **$782 000** ✓

Profit $= \$388\,360$

% profit $= \frac{388\,360}{393\,640} \times 100$ ✓

$= 98.66\%$ ✓

6. (a) 100km/h average speed ✓
6 km/L fuel consumption ✓

(b) 15 hours time ✓
7.5L/100km fuel consumption ✓

7. Profit $= \$350 \times 1.2$ ✓

Sold at $= \$420$ ✓

List price $= \frac{420}{0.7}$ ✓

$= \$600$ ✓

List price for surfboard is $600

8. Pizza cost $= \frac{\$9.90}{1.1}$ ✓

$= \$9.00$ ✓

9. (a) Original cost $= \frac{\$10.20}{0.17}$ ✓

$= \$60$ ✓

(b) Discounted price $= \$60 - \$10.20$

$= \$49.80$ ✓✓

10. $V = \pi r^2 h$

$V = \pi (3.2)^2 (11.6)$

$V = 373.17\text{cm}^3$ ✓✓

(a) $\dfrac{373.17}{30} = 12.44$ seconds ✓

(b) $12.44 \times 24 = 298.56$ seconds ✓

$= 4.976$mins

or 4min 59secs

RESOURCE RICH: *Trial Test 6*
Pythagoras

1. (a) $x^2 = 14.7^2 + 10.2^2$ ✓

$x^2 = 320.13$

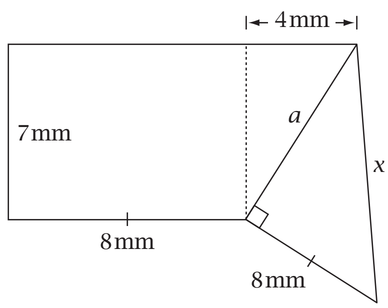
$x = 17.89$mm ✓

(b) $x^2 = 22.6^2 - 14.2^2$ ✓

$x^2 = 309.12$

$x = 17.58$m ✓

(c)



$a^2 = 4^2 + 7^2$

$a^2 = 16 + 49$

$a^2 = 65$

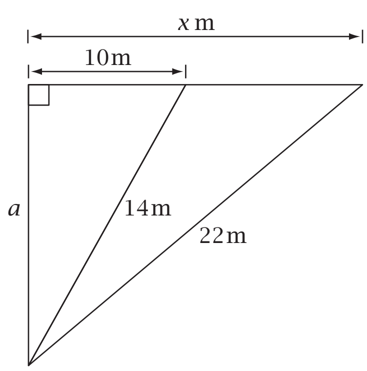
$a = \sqrt{65}$ ✓

$x^2 = (\sqrt{65})^2 + 8^2$ ✓

$x^2 = 129$

$x = 11.36$mm ✓

(d)



$a^2 = 14^2 - 10^2$

$a^2 = 96$

$a = \sqrt{96}$ ✓

$x^2 = 22^2 - (\sqrt{96})^2$ ✓

$x^2 = 388$

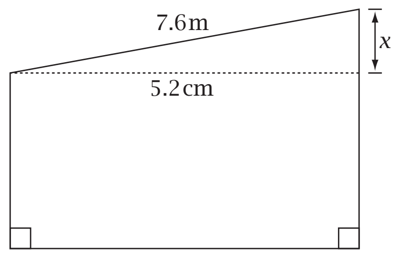
$x = 19.70$m ✓

2. $6.8^2 = 2.5^2 + 4.3^2$ ✓

$46.24 \neq 24.74$ ✓

ΔABC is **not** right angled ✓

3.



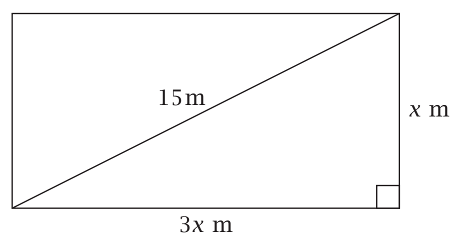
$x^2 = 7.6^2 - 5.2^2$ ✓

$x^2 = 30.72$

$x = 5.5$ ✓

Length of smaller flagpole is 2.6m ✓

4. (a)



Let x be the width

$x^2 + (3x)^2 = 15^2$ ✓

$x^2 + 9x^2 = 225$

$10x^2 = 225$

$x = 4.743$m ✓

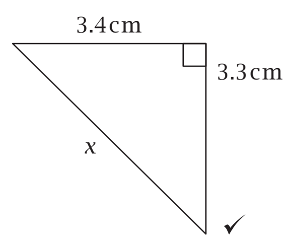
Width is 4.743m

(b) Length is 14.229m ✓

(c) Area $= 4.743 \times 14.229$ ✓

$= 67.488\text{m}^2$ ✓

5. (a)



✓

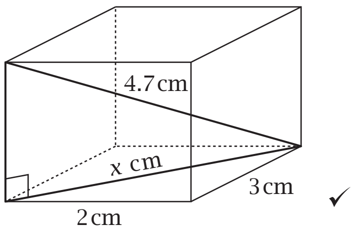
$x^2 = 3.4^2 + 3.3^2$ ✓

$x^2 = 22.45$

$x = 4.7$

Straw is 4.7cm long ✓

(b)



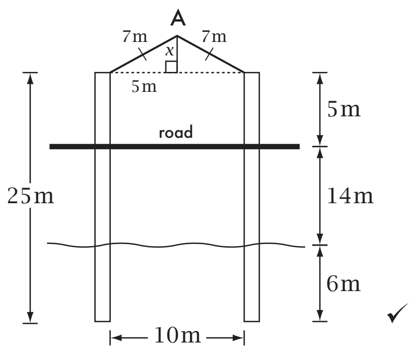
✓

$$x^2 = 3^2 + 2^2$$
$$x^2 = 9 + 4$$
$$x^2 = 13$$
$$x = \sqrt{13} \checkmark$$

$$h^2 = 4.7^2 - (\sqrt{13})^2 \checkmark$$
$$h^2 = 9.09$$
$$h = 3.01$$

Height of the box is 3.01cm ✓

6.

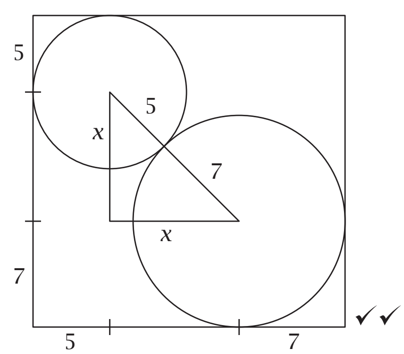


✓

$$x^2 = 7^2 - 5^2 \checkmark$$
$$x^2 = 49 - 25$$
$$x^2 = 24$$
$$x = \sqrt{24} \checkmark$$
$$x = 4.9\text{m}$$

Point A is 9.9m above the road ✓✓

7.



✓✓

$$x^2 + x^2 = 12^2 \checkmark$$
$$2x^2 = 12^2$$
$$x^2 = 72$$
$$x = \sqrt{72}$$
$$x = 8.49 \checkmark$$

Length of square is

$$5 + 7 + \sqrt{72} \approx 20.485\text{cm} \checkmark\checkmark$$

Area of square is 419.6cm^2 ✓

RESOURCE RICH: *Trial Test 7 Trigonometry*

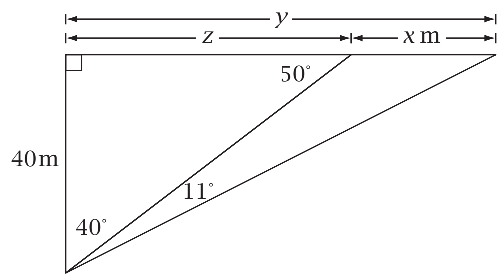
1. *(a)* $\sin 36^\circ = \dfrac{4}{x}$ ✓

$$x = \frac{4}{\sin 36^\circ}$$
$$x = 6.81\text{mm} \checkmark$$

(b) $\cos x = \dfrac{8^2 + 10^2 - 6^2}{2(8)(10)}$ ✓

$$x = 36.87^\circ \checkmark$$
$$\frac{\sin y}{6} = \frac{\sin 35}{7} \checkmark$$
$$y = 29.45^\circ \checkmark$$

2.

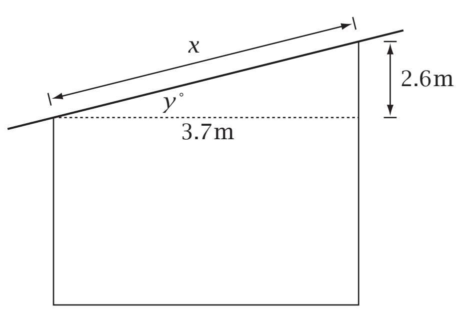


$$\tan 50^\circ = \frac{40}{z} \checkmark$$
$$z = \frac{40}{\tan 50^\circ}$$
$$z = 33.56\text{m} \checkmark$$

$$\tan 51^\circ = \frac{y}{40} \checkmark$$
$$y = 40\tan 51^\circ$$
$$y = 49.40\text{m} \checkmark$$

$$x = y - z$$
$$= 49.40 - 33.56 \checkmark$$
$$= 15.84\text{m} \checkmark$$

3.



(a) $x^2 = 2.6^2 + 3.7^2$ ✓

$x^2 = 20.45$

$x = 4.52$ ✓

The length of the roof is 5.52m.

(b) $\tan y^\circ = \dfrac{2.6}{3.7}$ ✓

$y = 35.1^\circ$

The roof is at an angle of 35.1° to the horizontal. ✓

4. $15 = \frac{1}{2}(7)(7) \sin \theta$ ✓
$\theta = 37.75^\circ$ ✓
[Equal angles are 71.125° each ✓✓

5. (a) $\dfrac{PR}{\sin 110^\circ} = \dfrac{6}{\sin 25}$ ✓✓

$PR = 13.34$m ✓

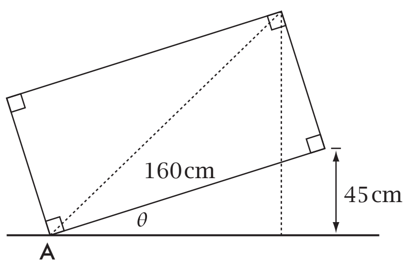
(b) $\angle PRQ = 59^\circ$ ✓

$\sin 59^\circ = \dfrac{PQ}{13.34}$ ✓

$PQ = 11.43$m ✓

(c) $RT^2 = 6^2 + 6^2 - 2(6)(6) \cos 28^\circ$ ✓✓
$RT = 2.90$m ✓

6. (a)

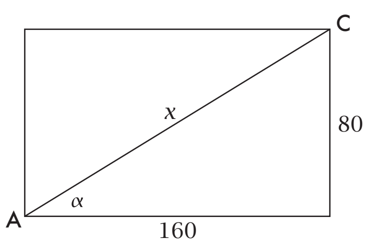


$\sin\theta = \dfrac{45}{160}$ ✓

$\theta = 16.3^\circ$ ✓

Angle of inclination is 16.3°

(b)



$x^2 = 80^2 + 160^2$ ✓

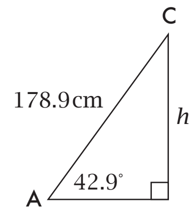
$x = \sqrt{32000}$

$x = 178.9$cm

$AC = 178.9$cm ✓

$\tan \alpha = \dfrac{80}{160}$ ✓

$\alpha = 26.6^\circ$ ✓



$\sin 42.9^\circ = \dfrac{h}{178.9}$ ✓

$h = 121.8$

Height of C above floor is 121.8cm ✓✓

(c) Angle for CA to be vertical is 47.1° ✓✓

RESOURCE RICH: *Trial Test 8*
Volume and Surface Area

1. (a) Volume $= 2 \times 6 \times 3$
$= 36\text{cm}^3$ ✓

SA $= 2(2\times6) + 2(6\times3) + 2(2\times3)$
$= 72\text{cm}^2$ ✓

(b) Volume $= \dfrac{\pi r^2 h}{4}$

$= \dfrac{\pi(2)^2(35)}{4}$ ✓

$= 109.96\text{mm}^3$ ✓

SA $= \dfrac{2\pi r^2 + 2\pi r h}{4} + 2(l \times w)$ ✓✓

$= \dfrac{2\pi(2)^2 + 2\pi(2)(35)}{4} + 2(2\times35)$

$= 256.24\text{mm}^2$ ✓

(c) Volume = Volume large cone – Volume small cone

$= \dfrac{\pi(12)^2(18)}{3} - \dfrac{\pi(4)^2(6)}{3}$ ✓

$= 2613.81\text{m}^3$

Surface Area: Calculate slant height of both large and small cones using Pythagoras

Large cone $S_1 = 12^2 + 18^2$

$S_1 = \sqrt{468}$ ✓

Small cone $S_2 = 4^2 + 6^2$

$S_2 = \sqrt{52}$ ✓

SA $= \pi r S_1 - \pi r S_2 + \pi r^2$ (large) + πr^2 (small)

$= \pi(12)(\sqrt{468}) - \pi(4)(\sqrt{52}) + \pi(12)^2 + \pi(4)^2$ ✓✓

$= 1227.59\text{m}^2$ ✓✓

2. Volume $= \frac{2}{3}\pi r^3$

$126 = \frac{2}{3}\pi r^3$ ✓

$189 = \pi r^3$

$r^3 = 60.16$

$r = 3.92$ ✓

Diameter is 7.84cm ✓

3. (a) Volume $= \pi(1.6)^2(2) - \frac{\pi(1.6)^2(1.4)}{3}$ ✓✓

$= 12.33\text{m}^3$

(b) $\text{SA} = \pi r^2 + 2\pi rh + \pi rs$

$s^2 = 1.4^2 + 1.6^2$ ✓

$s^2 = 4.52$

$s = \sqrt{4.52}$ ✓

$\text{SA} = \pi(1.6)^2 + 2\pi(1.6)(2)$

$+ \pi(1.6)\sqrt{4.52}$ ✓

$= 38.835\text{m}^2$ ✓

Cost $= 38.835 \times \$6.50$

$= \$252.43$ ✓

4. Volume Pyramid

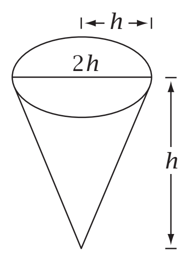
$= \frac{\text{Area Base} \times \text{height}}{3}$

$= \frac{(230.36 \times 230.36 \times 146.59)}{3}$ ✓

$= 2\,592\,968.434\,\text{m}^3$ ✓

$= 1.08\,\text{m}^3$ per stone ✓

5.



Volume $= \frac{\pi r^2 h}{3}$

$80 = \frac{\pi(h)^2.h}{3}$ ✓

$240 = \pi h^3$ ✓

$h^3 = \frac{240}{\pi}$

$h = \sqrt[3]{\frac{240}{\pi}}$ ✓

$h = 4.24$

Height of container is 4.24cm ✓

6. (a) Volume $= \pi(5)^2 \times 22$

$= 1727.88\text{cm}^3$ ✓

$\text{SA} = 2\pi r^2 + 2\pi rh$

$= 2\pi(5)^2 + 2\pi(5)(22)$

$= 848.23\text{cm}^2$ ✓

(b) V (hemisphere) $= \frac{2}{3}\pi r^3$

$1727.88 = \frac{2}{3}\pi r^3$ ✓

$r^3 = 825$

$r = 9.38\text{cm}^3$ ✓

(c) SA (hemisphere) $= 3\pi r^2$

$= 3\pi(9.38)^2$ ✓

$= 829.23\text{cm}^2$ ✓

(d) SA (cylinder – hemisphere)

$= 848.23 - 829.23$

$= 19\text{cm}^2$

% age decrease $= \frac{19}{848.23} \times 100$ ✓

$= 2.24\%$ ✓

RESOURCE RICH: *Trial Test 9*
Linear Relationships

1. (i)

Diagram number (d)	1	2	3	4	20	11
Number of matchsticks (m)	6	10	14	18	82	46

✓✓

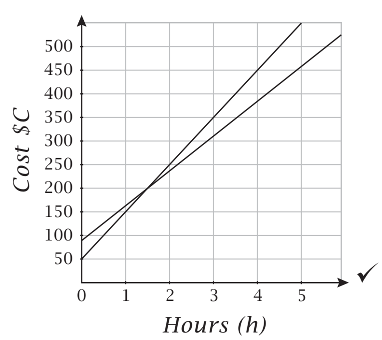
(ii) 1st difference pattern is 4 ✓
∴ linear ✓

(iii) $m = 4d + 2$ ✓✓

2. (a) $c = 75h + 90$ ✓

(b) $c = 75(4) + 90$
$c = \$390$
Cost \$390 ✓

(c)



✓

It is sensible to join the points as the electrician can work part of an hour. ✓

(d) $c = 100h + 50$

Graph rule on the axes in part *(c)* to compare electricians. ✓

- Harry is cheaper from 0 to 1.6 hours. ✓
- At exactly 1.6 hours they both charge the same: \$210 ✓
- After 1.6 hours George is cheaper. ✓

3. (a) $XY = \sqrt{(-3+1)^2 + (4+4)^2}$

$= \sqrt{68}$ ✓

$XZ = \sqrt{(-3-2)^2 + (4-1)^2}$

$= \sqrt{34}$ ✓

$YZ = \sqrt{(-1-2)^2 + (-4-1)^2}$

$= \sqrt{34}$ ✓

(b) $XY^2 = XZ^2 + YZ^2$

$(\sqrt{68})^2 = (\sqrt{34})^2 + (\sqrt{34})^2$ ✓

$68 = 34 + 34$

$\therefore$ ΔXYZ is right angled. ✓

(c) Right Isosceles Triangle. ✓

(d) Area $\Delta XYZ = \frac{1}{2} b \times p.h$

$= \frac{1}{2}\sqrt{34}.\sqrt{34}$

$= 17\text{units}^2$ ✓

4. (a) Equation of line through (3,6) and (5,14)

$y = 4x - 6$ ✓

Sub (–1, –10) into $y = 4x - 6$ ✓

$y = 4(-1) - 6$
$y = -10$

$\therefore$ points are collinear ✓

(b) *A* (2, 4) ✓ and *C* (-2, -16) ✓

(c) $2 - 2(p) + 4 = 0$
$2 - 2p + 4 = 0$
$2p = 6$
$p = 3$ ✓✓

$q - 2(-4) + 4 = 0$
$q + 8 + 4 = 0$
$q = -12$ ✓✓

(d) Equation of line passing through (4, 3) and (–11, –2)

$3y = x + 5$ ✓✓

Sub *T* (–2, 1) into equation
$3y = -2 + 5$
$y = 1$ ✓

$\therefore$ point *T* lies on the line ✓

5. Let x be income
y be expenditure

A: $\frac{5}{8}x - \frac{9}{14}y = 300$ ① ✓

B: $\frac{3}{8}x - \frac{5}{14}y = 300$ ② ✓

$\frac{15}{8}x - \frac{27}{14}y = 900$ ① × 3

$\frac{15}{8}x - \frac{25}{14}y = 1500$ ② × 5

$-\frac{2}{14}y = -600$

$y = 4200$ ✓

$\therefore x = 4800$ ✓

Income *A*: $\frac{5}{8} \times 4800 = \$3000$ ✓

B: $\frac{3}{8} \times 4800 = \$1800$ ✓

RESOURCE RICH: *Trial Test 10 Statistics*

1. (a) Boxplot B ✓ – lower median ✓

(b) (i) A boxplot shows range, median and interquartile range easily. ✓

(ii) A histogram can be used to find many statistics while a boxplot shows 5 summary values. ✓

(c) 1, 1, 2, 4, 4, 5, 5, 5
(other answers may be possible) ✓✓

2. (a) **Similarities**

- 5 students achieved five correct answers in both classes.
- No student scored three correct answers ✓

(Other answers possible)

Differences

- In class 1 most students answered 4 or 5 questions correctly. Class 2 was more spread. ✓

(Other answers possible)

(b) **Advantages for dot frequency plots**

- Easier to visualise
- Easier to see outliers, gaps, and clusters. ✓

(Other answers possible)

Disadvantages for dot frequency plots

- A frequency table can take up less space.
- A frequency table can be drawn up faster ✓

(Other answers possible)

(c) Class 1

Outlier at 0

Gaps at 0–2, 2–4

Cluster at 4–5 ✓

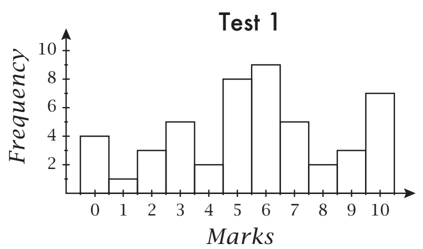
Class 2

No outliers

Gap at 2–4

Clusters at 0–2, 4–5 ✓

3. *(a)*



✓

(b)

	Test 1	Test 2
Mean:	5.55 ✓	4.58 ✓
Mode:	6 ✓	5 ✓
Median:	6 ✓	5 ✓
Range:	10 ✓	6 ✓
Standard deviation:	2.93 ✓	1.08 ✓

(c) Test 2 was the most difficult ✓
Lower mean and median ✓✓

(d) The better indicator was Test 1 ✓ with the higher range. ✓ Students were far more spread out allowing for easier selection.

4. *(a)* Mean: 30.1 ✓

(b) Modal class: 18–24 ✓

(c) Median

position: $\frac{101}{2}$

$= 50.5$ ✓

Median position falls in class interval 25–35

$\therefore \frac{15.5}{25} \times 7 + 24.5$ ✓

$= 28.84$ ✓

(d) Median is the better indicator as the mean is being affected by the two outliers. ✓✓

(e) Standard deviation: 8.83 ✓

5. Total class scores $= 24 \times 64$
$= 1536$ ✓

Remove Tom's old mark and add in his new mark. ✓

$1536 - 53 + 72 = 1555$

New class mean $= \frac{1555}{24}$ ✓

$= 64.79\%$ ✓

RESOURCE RICH: *Trial Test 11 Miscellaneous*

1. *(a)* $\$1725 \times 6.952 = 11\,992.20$ HKD ✓

(b) $\frac{560}{6.952}$ ✓ $= \$80.55$ AUD ✓

2. Phoebe

M–F	$16.20 × 35 =	$567
Sat	$24.30 × 6 =	$145.80
Sun	$32.40 × 5 =	$162
Overtime	$48.60 × 3 =	$145.80 ✓✓✓
		$1020.60

Allowances:

Meal	$66.50
Clothing	$86.75 ✓
	$1173.85 ✓

3. *(a)* Option A:

$SI = \$25\,000 \times 0.075 \times 3$
$= \$5625$ ✓

$CI = 25\,000\,(1 + 0.0705)^3 - 25\,000$
$= \$5669.03$ ✓

Compound interest is best. ✓

(b) $CI = 25\,000(1 + \frac{0.0705}{12})^{36} - 25\,000$

Monthly $= \$5869.14$ ✓

$CI = 25\,000(1 + \frac{0.0705}{365})^{1095} - 25\,000$

Daily $= \$5887.62$ ✓

Daily is best. ✓

(c) $\$4800 = \$4000(1 + r)^2$

$r = 0.0954$ ✓

$\therefore$ 9.54% interest rate ✓

Percy should invest in this bank. ✓

4.

$$\begin{array}{c} \begin{array}{ccc} S & M & L \end{array} \\ \begin{bmatrix} 0.85 & 1.5 & 3.00 \end{bmatrix} \\ \text{Cost } \checkmark \end{array} \begin{array}{c} \\ \begin{array}{c|cc} & A & O \\ \hline S & 18 & 12 \\ M & 32 & 45 \\ L & 35 & 20 \end{array} \\ \text{Quantity } \checkmark \end{array} \begin{array}{c} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ \checkmark \end{array}$$

$= \begin{bmatrix} 261 \end{bmatrix}$ ✓

Total Earnings: $261

5. (a) (i) 0.1056 ✓
(ii) 0.7357 ✓

(b) k = 398.52mL ✓✓

(c) IQR = 383.09 – 366.91
= 16.18 ✓✓

6. (a) 1:1 000 000
5cm:5 000 000cm
Actual length = $5^{o}0$km ✓✓

(b) 650m = 65 000cm
1:1 000 000 = x: 65 000

$$\frac{1}{1\,000\,000} = \frac{x}{65\,000}$$

x = 0.065cm

Scale length = 0.65mm ✓✓

7. (a) Shares: 4000 × \$10.50 = \$42 000 ✓
Brokerage fee \$42 000 × 3% = \$1260 ✓
Total cost = \$43 260 ✓

(b) Dividend paid = 2.5% × \$10.50
= 0.2625 cents ✓
Total paid = 4000 × 0.2625
= \$1050 ✓

MIXED EXAMINATION STYLE QUESTIONS

1. (a) $6x - 4 = 6 - x - 7$

$6x + x = 6 - 7 + 4$

$7x = 3$

$x = \frac{3}{7}$

(b) $-3x \leq -11 - 7$

$-3x \leq -18$

$x \geq 6$

(c) $2^{3x-1} = 2^5$

$3x - 1 = 5$

$x = 2$

2. (a) 150.75 kg

(b) 500

(c) 1.14 × 0.84 = 0.9576

Decrease of 4.24%

(d) \$175 × 1.45 × 1.3

= \$329.88

3. $N(11.5, 3^2)$

(a) $P(X > 8.5) = P(z > -1)$

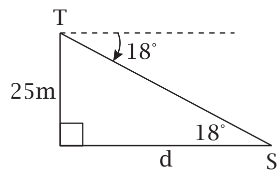
$= 0.84$

(b) $P(2.5 < X < 17.5) = P(-3 < z < 2)$

$= 0.4985 + 0.475$

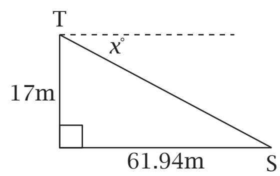
$= 0.9735$

4.



(a) $tan\,(18^\circ) = \frac{25}{d}$

$d = 76.94$m



(b) $tan\,(x) = \frac{17}{61.94}$

$x = 15.35^\circ$

5. (a) Profit = \$7250

% profit $= \frac{7250}{5000}$

= 145%

(b) (i) 0.2 × 12 250 = \$2450

\$9800 × 0.174 × $\frac{1}{12}$

= \$142.10

(ii) Interest = \$9800 × 0.174 × 3

= \$5115.60

Total: \$12 250 + \$5115.60

= \$17 365.60

(c) $\frac{12250}{1.1}$ = \$11 136.36

6. (a) (i) $f = 5\ N$

(ii) $f = 0.5\ N$

(b) $v = 2$ m/s

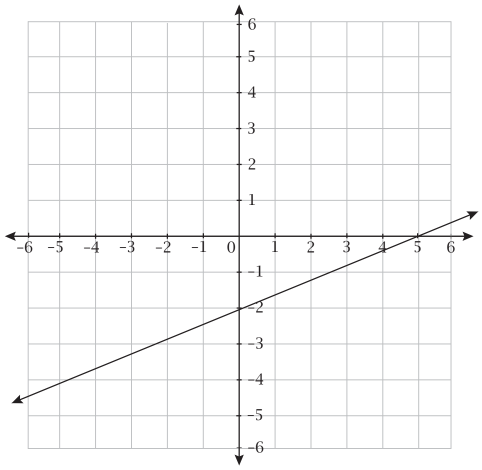
(c) $m = \frac{2}{5}$ kg

7. *(a)* (0, –2)

(b) (5, 0)

(c) $m = \dfrac{2}{5}$

(d)



8. *(a)* (i) $2\begin{bmatrix}3 & 0\\-1 & 2\end{bmatrix} + 3\begin{bmatrix}5 & -2\\4 & -3\end{bmatrix}$

$= \begin{bmatrix}6 & 0\\-2 & 4\end{bmatrix} + \begin{bmatrix}15 & -6\\12 & -9\end{bmatrix}$

$= \begin{bmatrix}21 & -6\\10 & -5\end{bmatrix}$

(ii) Not possible – dimensions must be the same for subtraction

(iii) $\begin{bmatrix}3 & 0\\-1 & 2\end{bmatrix}\begin{bmatrix}5 & -2\\4 & -3\end{bmatrix}$

$= \begin{bmatrix}15 & -6\\3 & -4\end{bmatrix}$

(iv) $\begin{bmatrix}5 & -2\\4 & -3\end{bmatrix}\begin{bmatrix}4\\2\end{bmatrix}$

$= \begin{bmatrix}16\\10\end{bmatrix}$

(b) $8p + 6 - 5p - 3 = 4p$

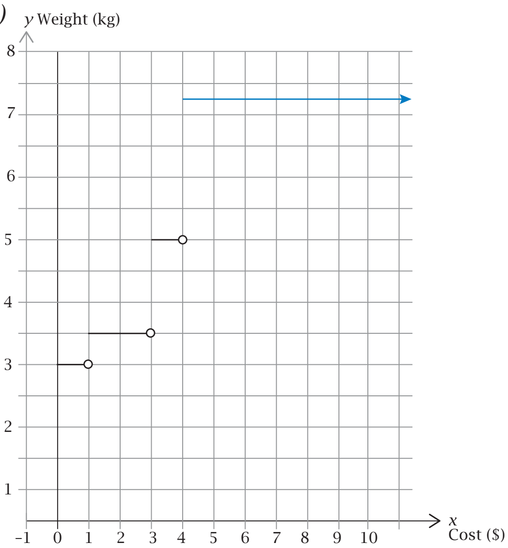
$8p - 5p - 4p = -6 + 3$

$p = 3$

9. *(a)* (i) \$3.50

(ii) \$7.25

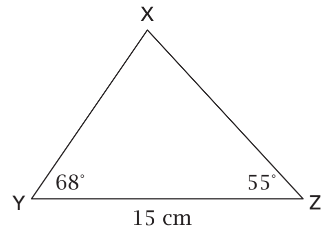
(b)



(c) Individually: \$3 + \$5 = \$8

As one parcel: \$5 ** Cheaper

10. *(a)*

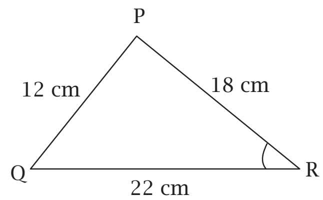


$\angle YXZ = 57^\circ$

$\dfrac{XZ}{\sin(68^\circ)} = \dfrac{15}{\sin(57^\circ)}$

$XZ = 16.58$ cm

(b)



$\cos R = \dfrac{18^2 + 22^2 - 12^2}{2 \times 18 \times 22}$

$R = 33.03^\circ$

(c) $s = \dfrac{12 + 18 + 22}{2}$

s = 26

Area $= \sqrt{26(26 - 12)(26 - 22)(26 - 18)}$

$= 107.93\text{cm}^2$

11. *(a)* $r = 3.2$m and $h = 5.6$m

$SA = 2\pi(3.2)^2 + 2\pi(3.2)(5.6)$

$= 176.93\text{m}^2$

(b) $V = \pi(3.2)^2(5.6)$

$= 180.15\text{m}^3$

= 180.15kL

= 180 151.49 L

(c) $V = \pi r^2 h$

$180.15 \times 0.8 = \pi(3.2)^2 h$

$h = 4.48$m

12. *(a)* (i) *A and IV*

(ii) *B and I*

(iii) *A and III*

(b) (i) Number of people living in a house.

(Many possible solutions)

(ii) House numbers.

(Many possible solutions)

13\. *(a)* 21

(b) Min: 5

Q_1: 13

Median: 15.5

Q_3: 19

Max: 26

(c) IQR = 6

Outlier calculation: $Q_1 - 1.5 \times \text{IQR}$

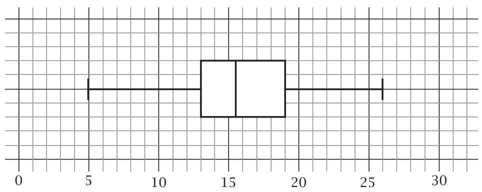
$= 13 - 1.5 \times 6$

$= 4$

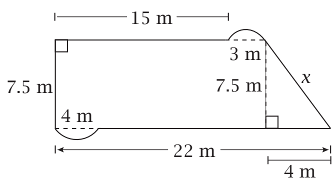
No scores are less than 4.

Hence 5 is not an outlier.

(d)



14\.



(a) $x^2 = 4^2 + 7.5^2$

$x = 8.5\text{m}$

$$\text{Perimeter} = 7.5 + 15 + \frac{\pi(3)}{2} + 8.5 + 18 + \frac{\pi(4)}{2}$$

$= 59.996\text{m}$

(b) $\text{Area Main} = 7.5 \times 18 + \dfrac{7.5 \times 4}{2}$

$= 150\text{m}^2$

Volume $= 150 \times 1.75$

$= 262.5\text{m}^3$

$$\text{Area Halves} = \frac{\pi \times 1.5^2}{2} + \frac{\pi \times 2^2}{2}$$

$= 9.82\text{m}^2$

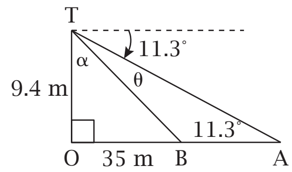
Volume $= 9.82 \times 0.35$

$= 3.44\text{m}^3$

Total volume $= 265.94\text{m}^3$

$= 266\text{kL}$

15\. *(a)*



(b) $\sin(11.3^\circ) = \dfrac{9.4}{AT}$

$AT = 47.97\text{m}$

(c) $\tan \alpha = \dfrac{35}{9.4}$

$\alpha = 74.97^\circ$

$\therefore \quad \theta = 3.73^\circ$

Angle of depression is 15.03°

(d) $\tan(78.7^\circ) = \dfrac{OA}{9.4}$

$OA = 47.04\text{m}$

$AB = 12.04\text{m}$

16\. *(a)* $5000 \times \$12 = \$60\ 000$

(b) (i) $1250 \times 0.56 = \$700$

(ii) $3000 \times 1.5\% = \$45$

(c) A lower P/E ratio is better. Hence ATA shares.

(d) PPR: $3000 \times \$0.65 = \$1950$

QAM: $2000 \times \$12 = \$24\ 000$

Total: \$25 950

Brokerage fee: $\$25\ 950 \times 0.025 = \$648.75$

17\. *(a)* $S = \begin{bmatrix} 300 & 280 \\ 260 & 315 \end{bmatrix}$

(b) C = cost matrix
R = revenue matrix

(c) $Q = \begin{bmatrix} 300 & 260 \\ 280 & 315 \end{bmatrix} [\begin{bmatrix} 2.5 \\ 3 \end{bmatrix} - \begin{bmatrix} 0.5 \\ 0.65 \end{bmatrix}]$

$Q = \begin{bmatrix} 1211.00 \\ 1300.25 \end{bmatrix}$

(d) Profit earned by each shop selling muffins and cupcakes.

18\. *(a)* $\$1250 \times 1.042 = \$1302.50$

(b) Loss: $\$21\ 000 - \$12\ 500 = \$8500$

$\% \text{ loss} = \dfrac{8500}{21000} \times 100$

$= 40.48\%$

(c) Income $= 3\% \times \$125\ 000 + \$450$

$= \$4200$

(d) Price ex GST $= \dfrac{1200}{1.1}$

$= \$1090.91$

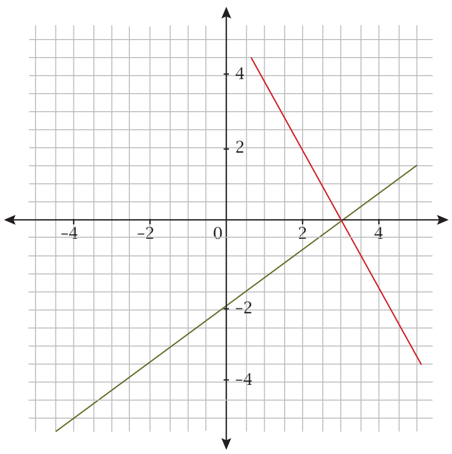
Discounted price: $\$1090.91 \times 0.65$

$= \$709.09$

Profit = \$259.09

19. *(a)* $a = 2$, $b = -3$ *and* $c = 6$

(b)



(c) (3, 0)

(d) $2y = 3x - 7$

$y = \frac{3}{2}x - \frac{7}{2}$

gradient is $\frac{3}{2}$

20. *(a)* $\frac{\sin A}{21} = \frac{\sin(24^\circ)}{13.4}$

$A = 39.6^\circ$

Obtuse angle $\angle BAD = 140.4^\circ$

(b) $\angle DBA = 15.6^\circ$

(c) Area $\Delta ABD = 0.5 \times 13.4 \times 21 \times \sin(15.6^\circ)$

$= 37.84\text{m}^2$

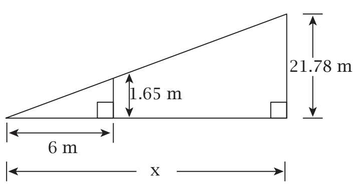
Area $\Delta BDC = \sqrt{34.15(34.15 - 28.3) \times (34.15 - 19)(34.15 - 21)}$

Heron's

$= 199.50\text{m}^2$

Total Area $= 237.34\text{m}^2$

21. *(a)*



$\frac{x}{6} = \frac{21.78}{1.65}$

$x = 79.2\text{m}$

(b) $\cos(\angle AOB) = \frac{7^2 + 7^2 - 4^2}{2 \times 7 \times 7}$

$\angle AOB = 33.2^\circ$

Area sector $= \frac{33.2}{360} \times \pi(7)^2$

$= 14.20\text{cm}^2$

Area $\Delta AOB = 0.5 \times 7 \times 7 \times \sin(33.2^\circ)$

$= 13.42\text{cm}^2$

Area segment $= 0.78\text{cm}^2$

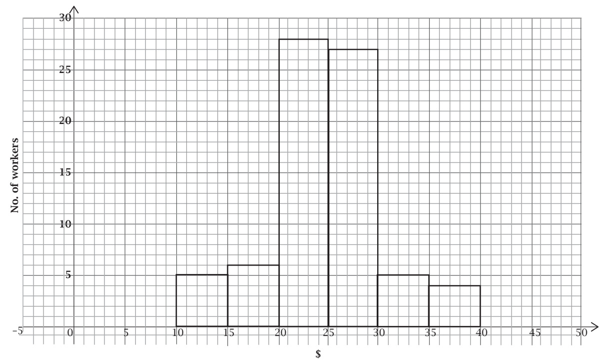
22. *(a)*

Daily Wages ($)	Midpoint of interval	Number of workers
$10 < x \le 15$	12.5	5
$15 < x \le 20$	17.5	6
$20 < x \le 25$	22.5	28
$25 < x \le 30$	27.5	27
$30 < x \le 35$	32.5	5
$35 < x \le 40$	37.5	4

(b) (i) $24.70

(ii) $5.61

(c)



(d) Spread almost symmetrical.

Unimodal.

Most scores between 20 and 30.

23. $X \sim N(1.2, 0.13^2)$

(a) (i) $P(X = 1.2) = 0$

(ii) $P(X > 1.5) = 0.011$

(iii) $P(1 < X < 1.4) = 0.8761$

(b) $W = 0.2548$

(c) $P(X < 1) = 0.062$

Number $= 500 \times 0.062$

$= 31$

24. *(a)* $\theta = 98^\circ$

(b) $AC^2 = 200^2 + 310^2 - 2 \times 200 \times 310 \times \cos(98^\circ)$

$AC = 391.61\text{km}$

(c) $\frac{\sin C}{200} = \frac{\sin(98^\circ)}{391.6}$

$C = 30.38^\circ$

Bearing: $360^\circ - 24^\circ - 30.38^\circ = 305.62^\circ$

25. *(a)* $\begin{bmatrix} 0 & -7 \\ -1 & 18 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

(c) [0]

(d) $\begin{bmatrix} -1 \\ 5 \end{bmatrix}$

(e) $F = \begin{bmatrix} 9 & -5 \\ -25 & 14 \end{bmatrix}$

$f_{21} = -25$

26. (a) (i) (−6, 1)

(ii) (3, 0)

(b) $2x - 4y = 19 \ldots.$ (1)

$3x + 5y = 1 \ldots\ldots$ (2)

$6x - 12y = 57 \ldots.$ (1) × 3

$6x + 10y = 2 \ldots.$ (2) × 2

$-22y = 55 \ldots.$ (1) − (2)

$y = -2.5$

$\therefore\ x = 4.5$

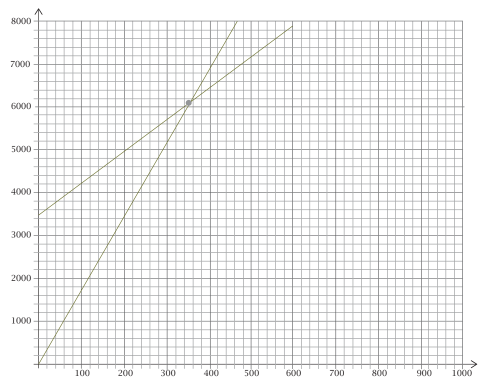
(4.5, −2.5)

27. (a) Cost: \$7.50 × 750 + \$3500 = \$9125

(b) Revenue: \$17.50 × 750 = \$13 125

(c) (i) $C = 7.5b + 3500$

(ii) $R = 17.5b$



(d) Loss of \$500

(e) 350 bottles required to break even.

28. (a) Abank interest: \$150 000 × 2.3% × 5 = \$17 250

BBank interest: \$150 000 × $\left(1 + \dfrac{0.0229}{12}\right)^{60}$ −\$150 000 = \$18 178.54

BBank is best investment – greater interest.

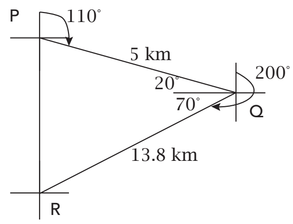
(b) Cash price: \$3750 × 0.6 × 0.8 = \$1800

(c) Scheme 1: \$82 500 × 0.9 = \$74 250

Scheme 2: \$82 500 × 0.25 + \$750 × (6 × 12 + 3) = \$76 875

Scheme 1 is cheaper.

29. (a)



(b) 70°

(c) 180° − 90° − 70° = 20°

(d) 290°

(e) $PR^2 = 5^2 + 13.8^2$

$PR = 14.68$km

30. (a) SA = (25 × 16) + 2(25 × 13) + (25 × 10) + 2 $\left(\dfrac{10 + 16}{2} \times 11\right)$

= 1586cm^2

(b) Vol = $\left(\dfrac{10 + 16}{2} \times 11\right) \times 25$

= 3575cm^3

(c) $k = 5.5$

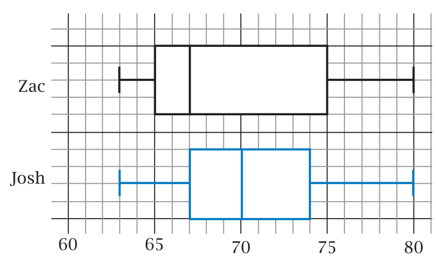
(d) Scale factor = 5.5^2

= 30.25

(e) Vol = 3575×5.5^3

= 594 790.625cm^3

31. (a)



(b) 25%

(c) Zac's results are positively skewed while Josh's are more symmetrical.

Zac's median is 67 while Josh's is 70.

75% of Josh's scores are above 67 while Zac has only 50% over 67.

Zac's claim is incorrect.

32. (a) \$750 × 68.78 = 51 585 JPY

(b) $\dfrac{85\ 000}{10\ 250.67}$ = \$8.29 AUD

(c) $\dfrac{126\ 560}{10\ 250.67}$ = \$12.35 AUD

Total AUD: \$262.35

Total USD = \$262.35 × 0.76
= \$199.39

(d) $\dfrac{3500}{0.62} = \$5645.16$ AUD

Cost per night \$1129.03

(e) Japan: $\dfrac{7800}{68.78} = \$113.41$ AUD

USA: $\dfrac{98}{0.76} = \$128.95$ AUD

Cheaper in Japan.

33. Gradient of line Y is 2.

Equation of line Y: $y = 2x + c \quad (2, 8)$

$8 = 2(2) + c$

$c = 4$

$\therefore y = 2x + 4$

Point of intersection: $3x - y = -2$

Sub in line Y $\quad 3x - (2x + 4) = -2$

$x = 2$

$y = 8$

Pt of intersection: (2, 8)

34. $X \sim N(700, 4^2)$

(a) $z = \dfrac{708 - 700}{4}$

$z = 2$

(b) z score of –1 means 1 standard deviation below the mean.

Hence weight is 696 g.

(c) 712 g is 3 standard deviations above the mean.

$P(X > 712) = 0.15\%$

35. *(a)* $U = (4, 0)$ Perpendicular gradient to UW is $-\dfrac{4}{3}$

Equation of TV is: $y = -\dfrac{4}{3}x + c$

$0 = -\dfrac{4}{3}(4) + c$

$c = \dfrac{16}{3}$

Length of TW $= \dfrac{16}{3} + 3$

$= \dfrac{25}{3}$ units

(b) Using Pythagoras

$UW = \sqrt{3^2 + 4^2}$

$UW = 5$

Hence area square UVXW 25 units^2

36. *(a)* $V = \dfrac{\pi r^2 h}{3}$

$300 = \dfrac{\pi(6)^2 h}{3}$

$h = \dfrac{25}{\pi}$ cm

Scale factor: $\dfrac{8.4}{6} = \dfrac{7}{5}$

Height of large cone $h = \dfrac{25}{\pi} \times \dfrac{7}{5}$

$h = \dfrac{35}{\pi}$ cm

(b) Slant height (small cone)

$s = \sqrt{\left(\dfrac{25}{\pi}\right)^2 + 6^2}$

$s = 9.97$ cm

SA (small cone) $= \pi(6)^2 + \pi(6)(9.97)$

$= 301.03$ cm^2

(c) SA (large cone) $= 301.03 \times \left(\dfrac{7}{5}\right)^2$

$= 590.02$ cm^2

(d) Vol (large cone) $= 300 \times \left(\dfrac{7}{5}\right)^3$

$= 823.2$ cm^3

37. *(a)* Overall total: $60 \times 72 = 4320$

Class 1 total: $29 \times 65 = 1885$

Class 2 total: $4320 - 1885 = 2435$

Class 2 average: $\dfrac{2435}{31} = 78.55\%$

(b) (i) 1.7

(ii) 1.12

(iii) 92

(iv) $IQR = 1$

Outlier if score $> Q_3 + 1.5\ IQR$

Is $8 > 2 + 1.5 \times 1$?

Yes ‘8’ is an outlier.

(v) Skewed to the right (positively skewed).

38. *(a)* (i) $T = \sqrt{2 \times 18} - 2^2$

$T = 2$

(ii) $T = \sqrt{2 \times 0.125} - 0.1^2$

$T = 0.49$

(b) $T = \dfrac{\sqrt{2 \times 50}}{0.5} - \dfrac{(-4)^2}{-16}$

$T = 21$

39. $\tan(27^\circ) = \dfrac{18}{PR}$

$PR = 35.33$ cm

Area $\Delta PQR = 0.5pq \sin(R)$

$52 = 0.5 \times x \times 35.33 \times \sin(35^\circ)$

$x = 5.13$cm

40. *(a)* Reduction: $35 - 20.75 = 14.25$ mins

% loss $= \dfrac{14.25}{35} \times 100$

40.71%

(b) Price ex GST: $\dfrac{145}{1.1} = \$131.82$

GST amount: \$13.18

41. *(a)* $x = 4$

(b) $x = 2$

(c) $x = 7$

42. $X \sim N(28.3, 3.7^2)$

(a) (i) $P(X < 26.1) = 0.2761$

(ii) $P(22.7 < X < 31.4) = 0.7339$

(b) $P(X > k) = 0.02$

$k = 35.9^{\circ}C$

Lowest possible scorcher is $35.9^{\circ}C$.

(c) *IQR* is middle 50%.

Lowest: 25.8044˚C

Highest: 30.7956˚C

IQR = 4.9912˚C

43. *(a)* 100 g: \$3.50 / 100 g

250 g: \$2.60 / 100 g

500 g: \$2.25 / 100 g

1 kg: \$1.995 / 100 g

Cheapest is 1 kg honey.

(b) Purchase cost = (6 × \$70) + (3 × \$65) + (2 × \$90) + (1 × \$79.80)

= \$874.80

Discounted cost = \$874.80 × 0.85

= \$743.58

44. *(a)* $\begin{bmatrix} 87 \\ 104 \end{bmatrix}$

(b) $S = \begin{bmatrix} 1.12 & 0 \\ 0 & 1.15 \end{bmatrix}$

$STC = \begin{bmatrix} 97.44 \\ 119.60 \end{bmatrix}$

(c) $Q = [0.85 \quad 0.85]$

$QSTC = [184.48]$

45. *(a)* (i) \$4

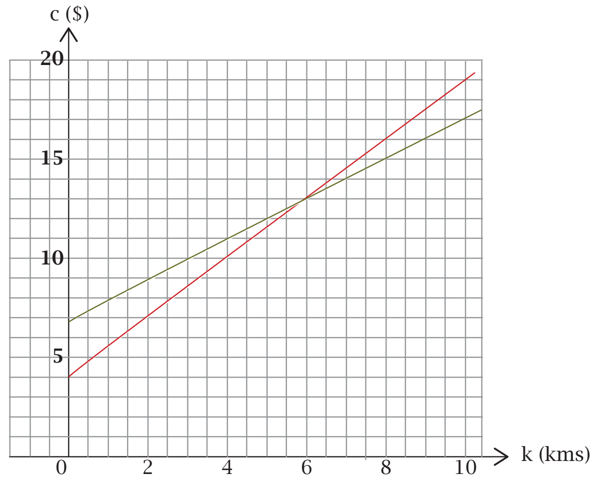
(ii) \$1.50 / km

(b) (i) \$13

(ii) 2km

(c) $C = 1.5k + 4$

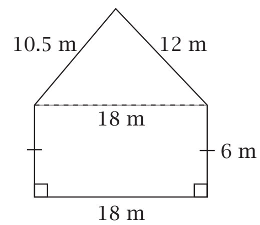
(d) $E = k + 7$



(e) (i) \$7

(ii) Travelling more than 6km.

46. *(a)*



(b) Area rectangle = 18 × 6 = 108m^2

Area triangle using Herons'

$s = 20.25$

Area triangle = $\sqrt{20.25(20.25 - 18) \times (20.25 - 12)(20.25 - 10.5)}$

= 60.54m^2

Total area = 168.54m^2

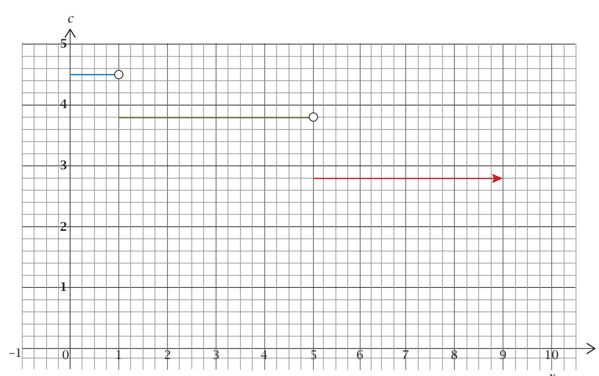
(c) Cost = 168.54 × \$26

= \$4382.04

47. *(a)* Cost = 3.6 × \$3.80

= \$13.68

(b)



(c) Cost = 5 × \$2.80 + \$35 × 0.85

= \$43.75

48. *(a)* Area sector: $\frac{90}{360} \times \pi(6)^2 = 28.27\text{cm}^2$

(b) Area WXYZ = 36cm^2

Area shaded = 7.726cm^2

(c) New area: $7.726 \times 1.5^2 = 17.38\text{cm}^2$

49. *(a)* X is an outlier.

$IQR = 15$. $Q_3 = 60$

Outlier if a score > $Q_3 + 1.5\ IQR$

i.e. $60 + 1.5 \times 15$

$= 82.5$

85 is greater than 82.5 hence an outlier.

(b) Range: Boys – 26 Girls – 26
Both have the same range.

(c) *IQR*: Boys – 15 Girls – 15.
Identical IQR values.

(d) Range and *IQR* values are the same.

75% boys > 50 while 50% girls > 50.

Boys median > girls median.

Girls had a better performance – faster times.

50. *(a)* Maths: $z = \frac{70 - 65}{2.5}$

$z = 2$

Physics: $z = \frac{62 - 72}{5}$

$z = -2$

Abbie performed better in Maths.

(b) 2 standard deviations above the mean.

Percentage scored more than Abbie = 2.5%

51. Xander: x

Yolanda: y

Zebedee: z

Equations are:

$x + y + z = 4.5$ …(1)

$y = x - 2.25$ …(2)

$z = 3y$ …(3)

Sub (2) and (3) into (1)

$y + 2.25 + y + 3y = 4.5$

$y = 0.45$

Xander = \$2.70

Yolanda = \$0.45

Zebedee = \$1.35

52. *(a)* Saved: \$4750 × 55% = \$2612.50

Discounted price: \$4750 × 0.8 = \$3800

Must save a further \$1187.50

(b) Original price: $\frac{2750}{0.35}$ = \$7857.14

53. *(a)* $C = 30x + 250$

$R = 80x$

(b) (i) –\$50 *a loss of* \$50.

(ii) \$150 *a profit of* \$150.

(c) Breakeven point is 5 LED light systems.

54. *(a)*

Height of students (cm)	Midpoint of interval	Frequency
$140 < x \le 145$	142.5	1
$145 < x \le 150$	147.5	1
$150 < x \le 155$	**152.5**	2
$155 < x \le 160$	157.5	1
$160 < x \le 165$	162.5	**5**
$165 < x \le 170$	167.5	9
$170 < x \le 175$	**172.5**	11
$175 < x \le 180$	177.5	**10**
$180 < x \le 185$	182.5	14
$185 < x \le 190$	187.5	15
$190 < x \le 195$	192.5	**17**
$195 < x \le 200$	197.5	18
$200 < x \le 205$	**202.5**	16

(b) Negative skew.

Unimodal.

Median > Mean.

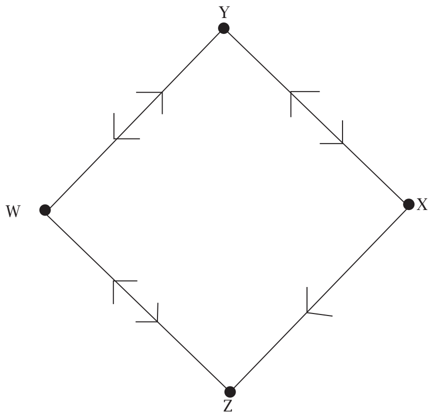
Not symmetrical.

Other possible answers.

(c) (i) 184.83 cm

(ii) 13.692 cm

55. *(a)*



(b) $M = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

(c) $M^2 = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

(d) $M^3 = \begin{bmatrix} 0 & 0 & 3 & 3 \\ 0 & 0 & 3 & 3 \\ 4 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \end{bmatrix}$

$M^3_{23} = 3$ *which indicates there are 3 three-step paths from town X to town Y.*

56. *(a)* Desmond's pay: $22.50 × (23 + 4 × 1.5 + 3 × 2.25) = $804.38

(b) Evelyn's pay: 1380.60 = ($22.50 × 1.18) × (34 + 1.5*A* + 4 × 2.25)

A = 6 hours

(c) Joey's pay: $\frac{717.6}{39}$ = $18.40 per hour

(d) Total pay: $2902.58

Superannuation payment:
$2902.58 × 0.095 = $275.75

57. *(a)* Oblique length using Pythagoras is 13cm.

SA = (13 × 10) + (12 × 10) + (10 × 5) + 2($\frac{1}{2}$ × 5 × 12)
= 360cm^2

(b) V = $\frac{1}{2}$ × 12 × 5 × 10
= 300cm^3

(c) Cost = 360 × $2.50
= $900

58. *(a)* Interest: $2500 × 0.07 × 3 = $525

(b) Interest: $\$3750 \times \left(1 + \frac{0.065}{12}\right)^{48} - \$3750 = \$1110.08$

(c) Dividend: $3750 × 5.6% = $210

(d) Gain: $17 250.25 – $3750 = $13 500.25

% increase = $\frac{13\,500.25}{3750} \times 100$
= 360%

59. *(a)* 23.21

(b)

0	6				
1	2	2	4	4	5
2	0	1	6	8	
3	0	4			
4	1				
5	2				

(c) Min = 6

Q_1 = 14

Median = 20.5

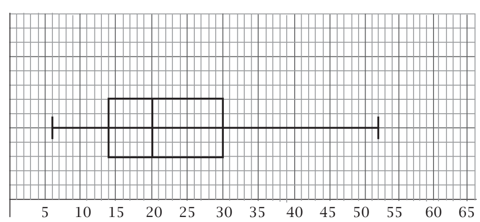
Q_3 = 30

Max = 52

(d) IQR = 16

Outliers when
Q_1 – 1.5 IQR < *score* < Q_3 + 1.5 IQR
14 – 1.5 × 16 < *score* < 30 + 1.5 × 16
–10 < *score* < 54 *No scores are outliers*

(e)



(f) Skewed to the right (positively skewed).

60. *(a)* Price: $120 × 2.25 × 1.1 = $297

(b) % markup: $\frac{177}{120}$ × 100 = 147.5%

(c) Sale price: $297 × 0.7 = $207.90

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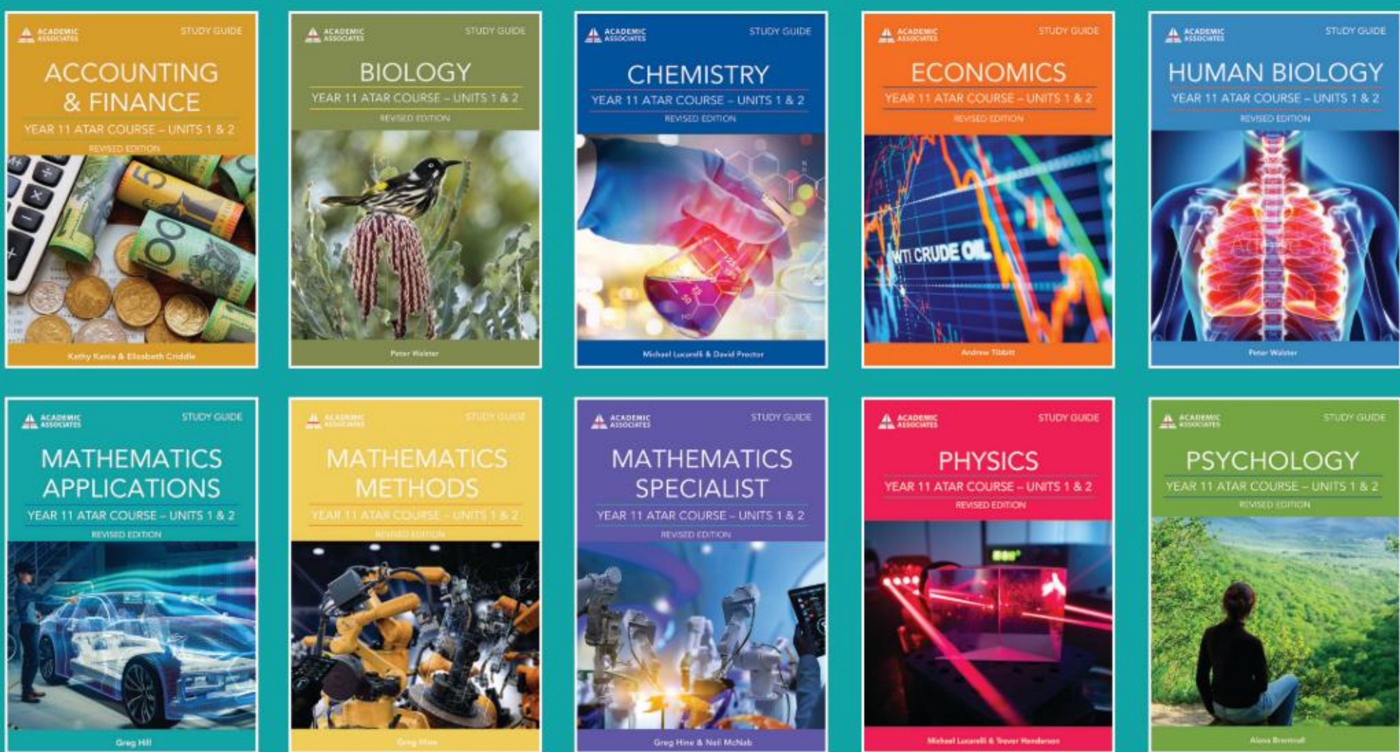

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